Title: Small thermal machines

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Abstract: The second law of thermodynamics tells that physics imposes a fundamental constraint on the efficiency of all thermal machines. Here I will address the question of whether size imposes further constraints upon thermal machines, namely whether there is a minimum size below which no machine can run, and whether when they are small if they can still be efficient? I will present a simple model which shows that there is no size limitation and no limit on the efficiency of thermal machine and that this leads to a unified view of small refrigerators, pumps and engines.

Small thermal machines

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Joint work with Sandu Popescu, Noah Linden, Nicolas Brunner

University of BRISTOL

outline

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- introduction & motivation
- basic functioning principle
- refrigerator - model and efficiency

introduction

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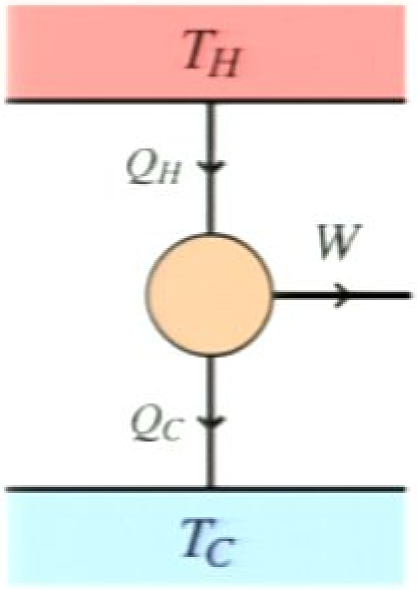
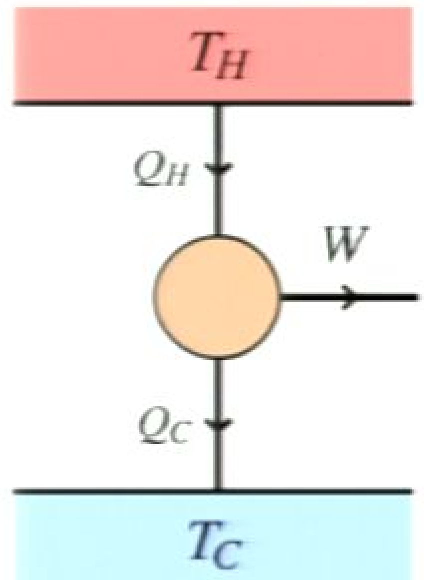
- Sadi Carnot: abstract, model independent machines



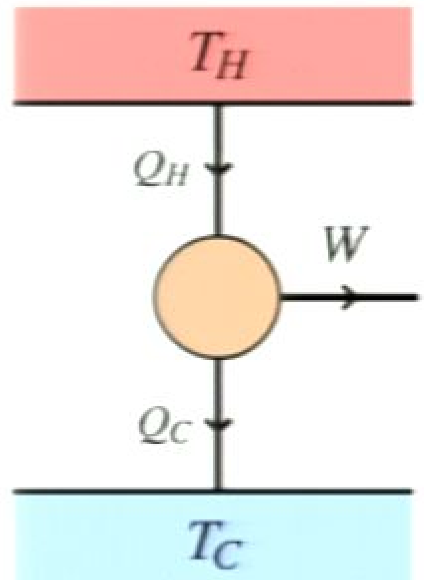
introduction

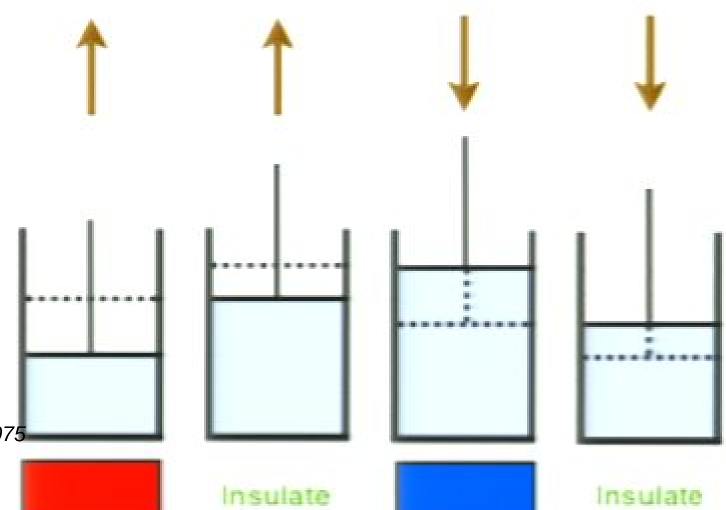
- Sadi Carnot: abstract, model independent machines
    - nature imposes a fundamental limit on efficiency of all thermal machines

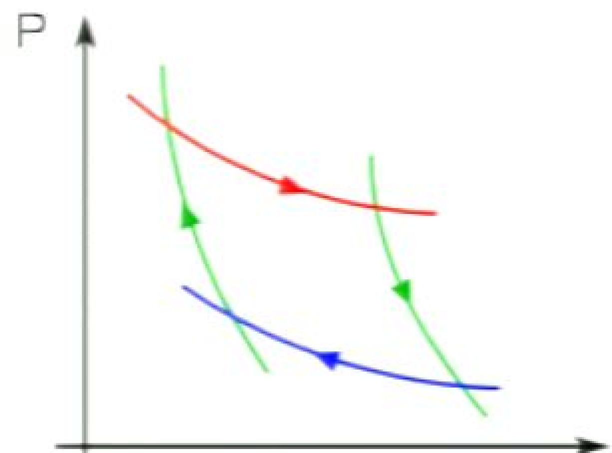


introduction

- Sadi Carnot: abstract, model independent machines
  - nature imposes a fundamental limit on efficiency of all thermal machines
    - machines become maximally efficient when they become reversible







additional fundamental constraints

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- question: what happens if physical properties are specified?

  specifically 'size'

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  → is there a minimum size below which no thermal machine can exist?

additional fundamental constraints

- question: what happens if physical properties are specified?

  specifically 'size'

  ➔ is there a minimum size below which no thermal machine can exist?

  ➔ if a thermal machine is small does this constrain its performance? in particular its efficiency?

small and self-contained

- size: dimension of Hilbert space of machine

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important: '*self-contained*' - i.e. must ensure that the counting is fair

→ no time dependent interactions: t external parameter

small and self-contained

- size: dimension of Hilbert space of machine

important: '*self-contained*' - i.e. must ensure that the counting is fair

→ no time dependent interactions: t external parameter

↳ macroscopic control

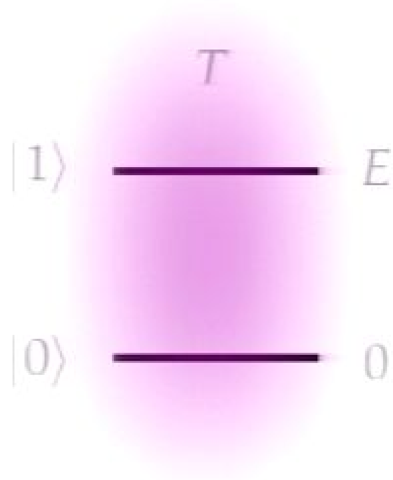
→ only external interaction: incoherent interaction with thermal baths

basic functioning principle I

how can you cool a qubit?

basic functioning principle I

how can you cool a qubit?



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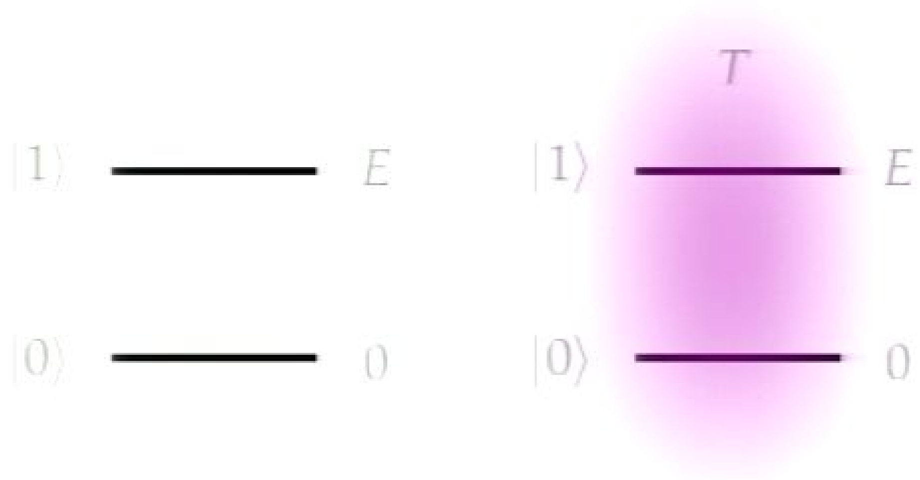

$$\rho = \frac{1}{Z} e^{-H_0/kT}$$

$$= \frac{1}{1+e^{-E/kT}} |0\rangle\langle 0| + \frac{e^{-E/kT}}{1+e^{-E/kT}} |1\rangle\langle 1|$$

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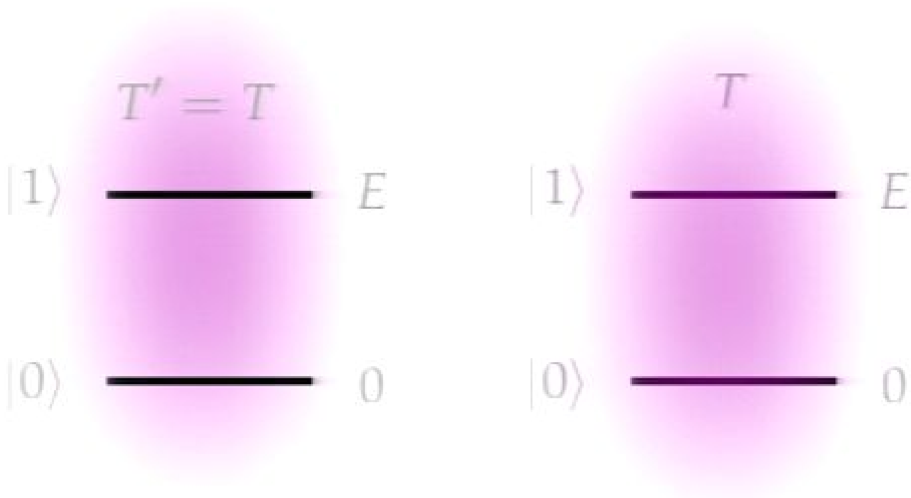

$T' = T$

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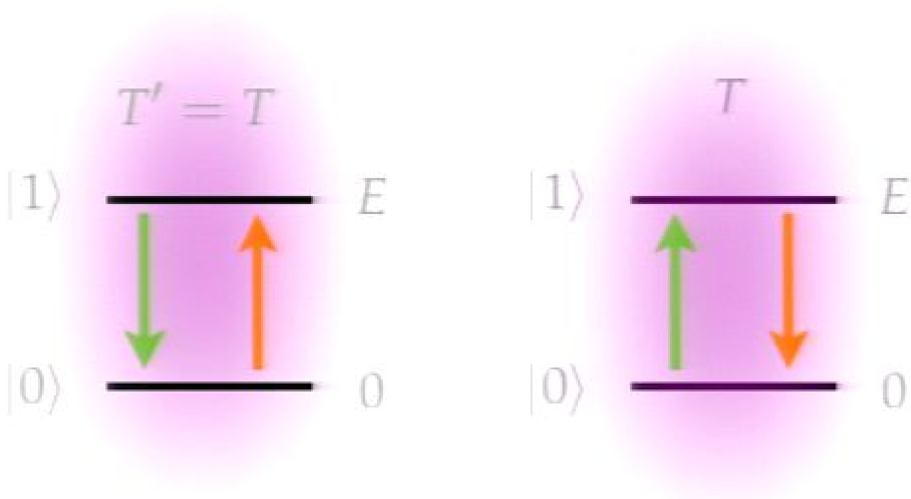

$T' = T$ $\quad p(10) = p(01)$

nothing happens

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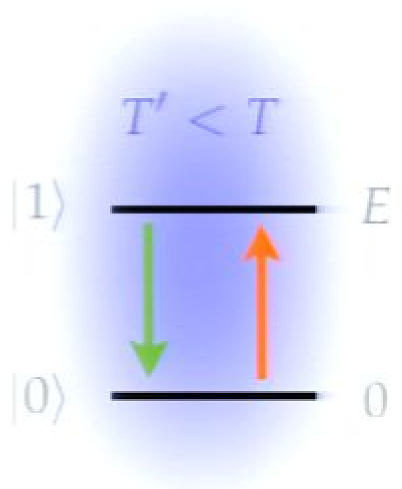

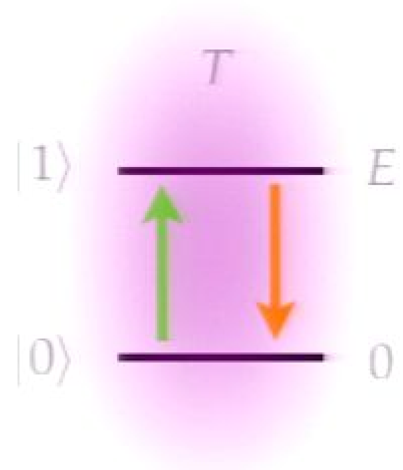

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$T' < T$ $\quad p(10) < p(01)$

cools down qubit

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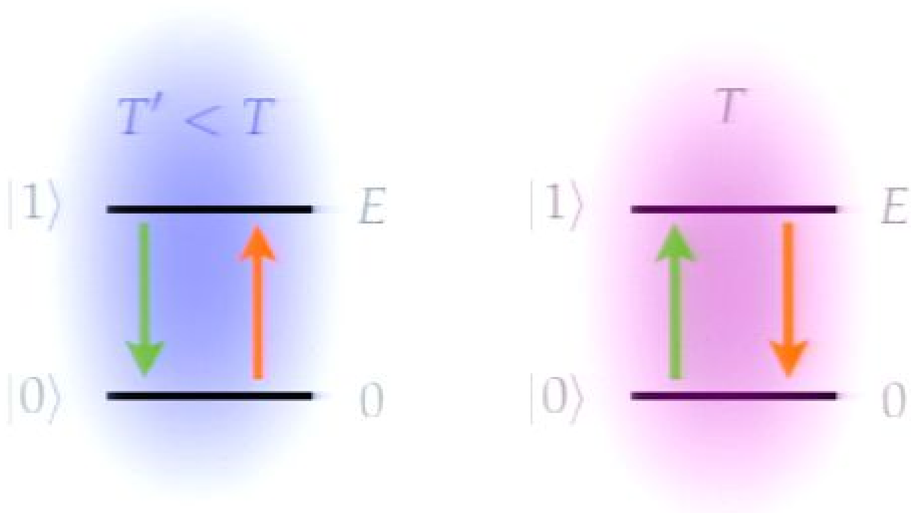

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cools down qubit

qubit 1 is like an ice-cube

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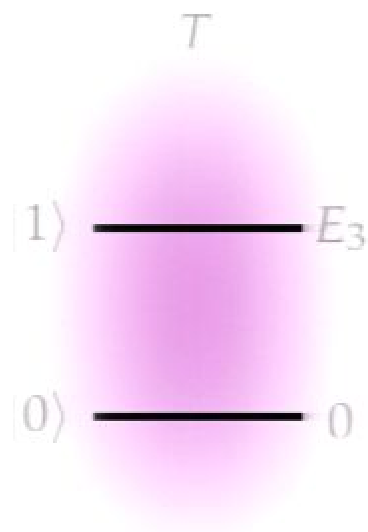
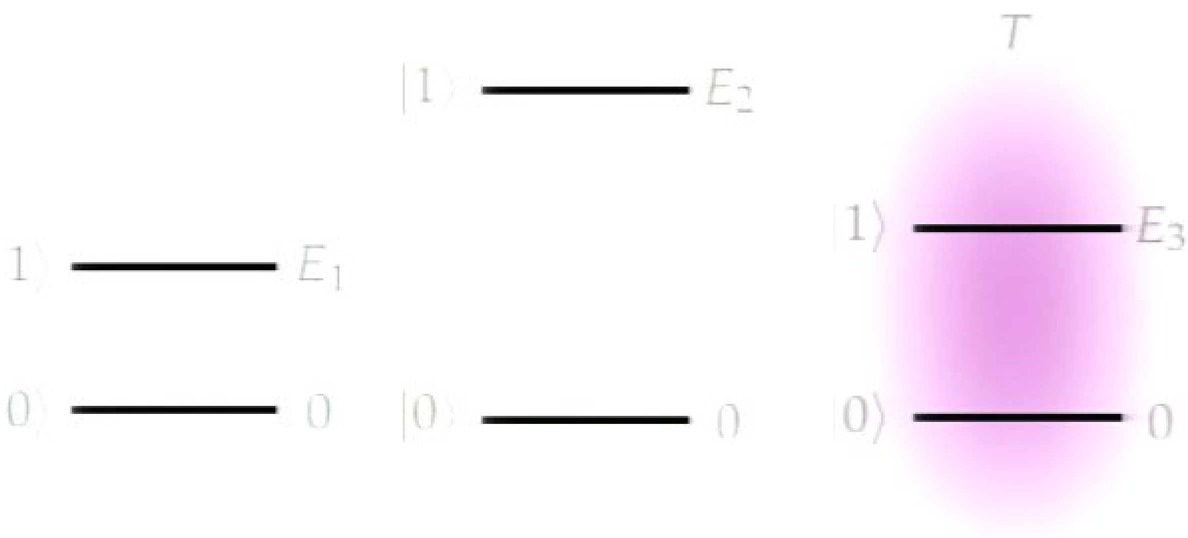
basic functioning principle II



basic functioning principle II



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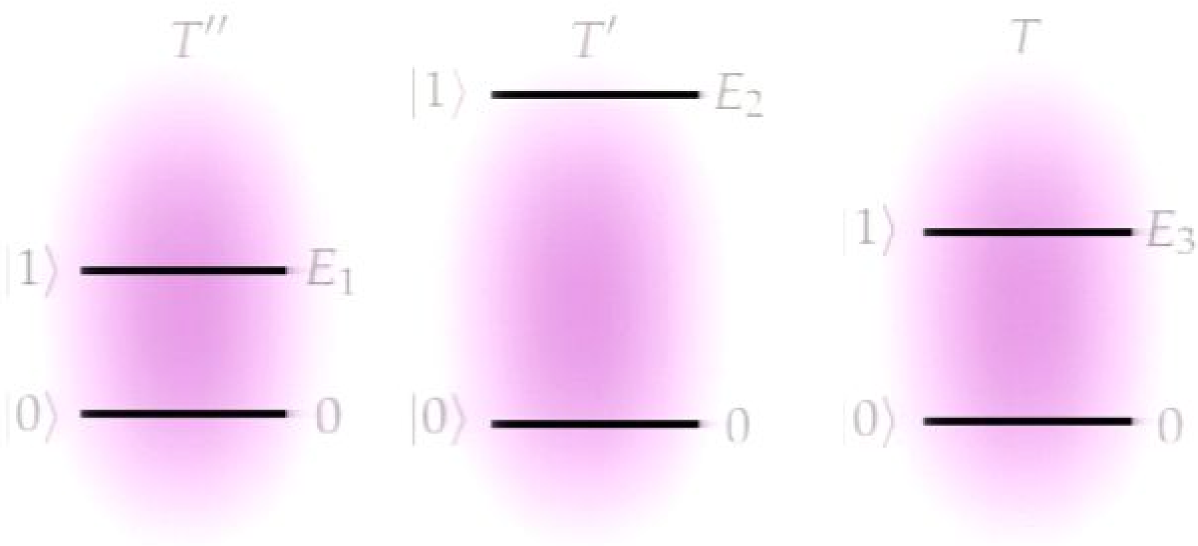
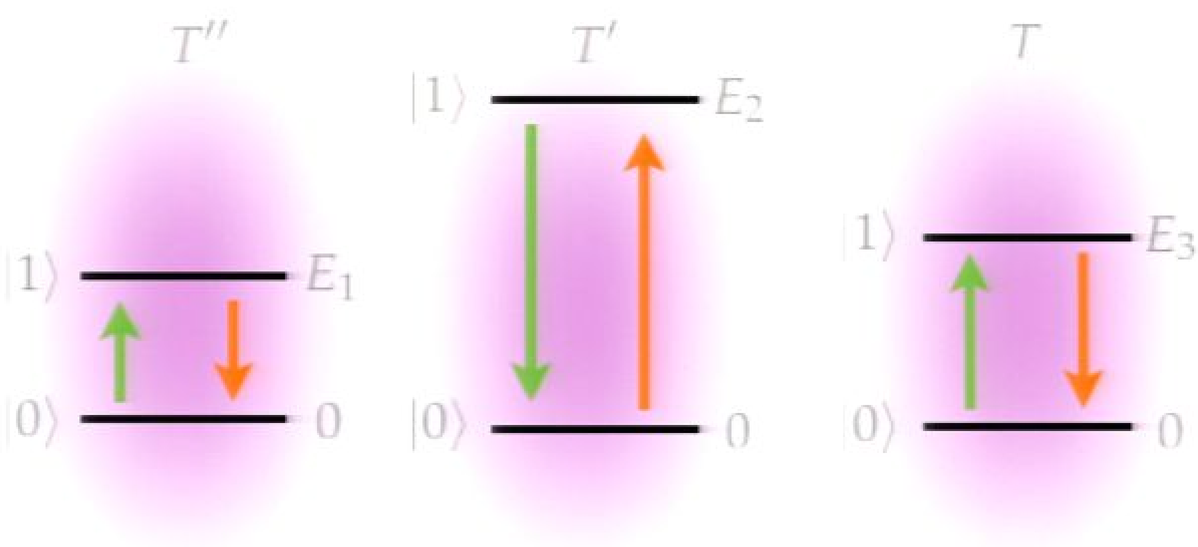


$$T'' = T' = T$$

$$E_2 = E_1 + E_3$$

basic functioning principle II



$E_2 = E_1 + E_3$

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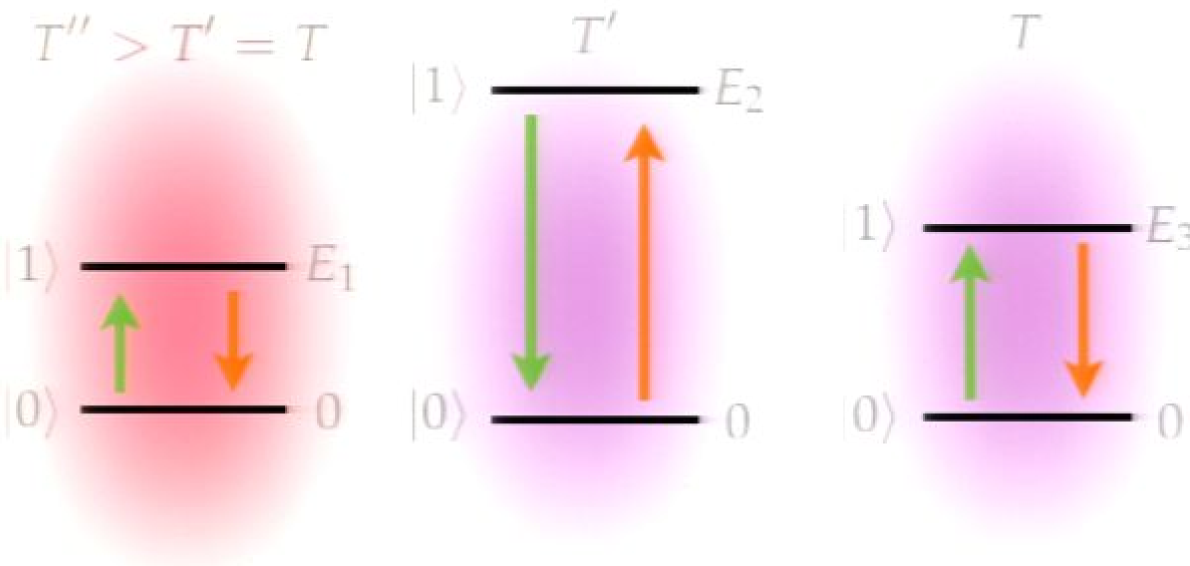
$p(101) = p(010)$

nothing happens

basic functioning principle II

$T'' > T' = T$



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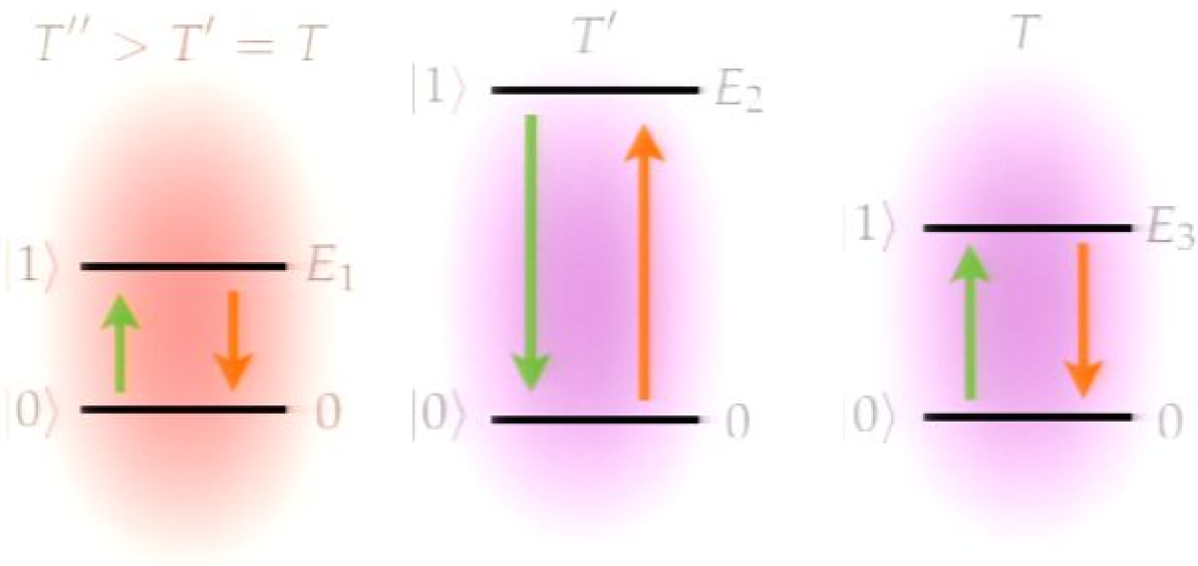

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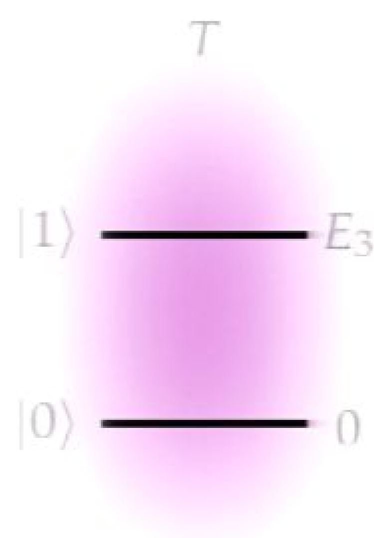
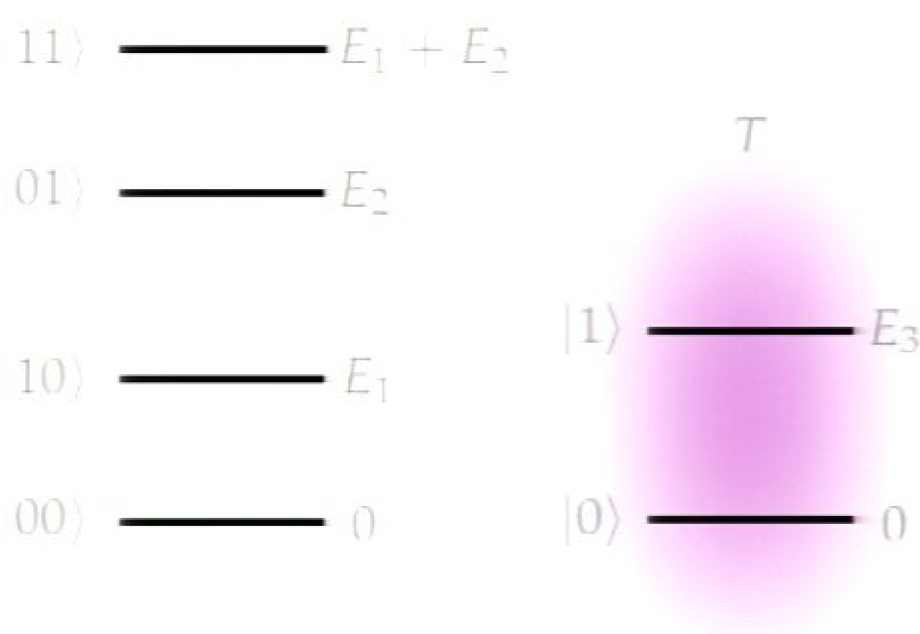


basic functioning principle III



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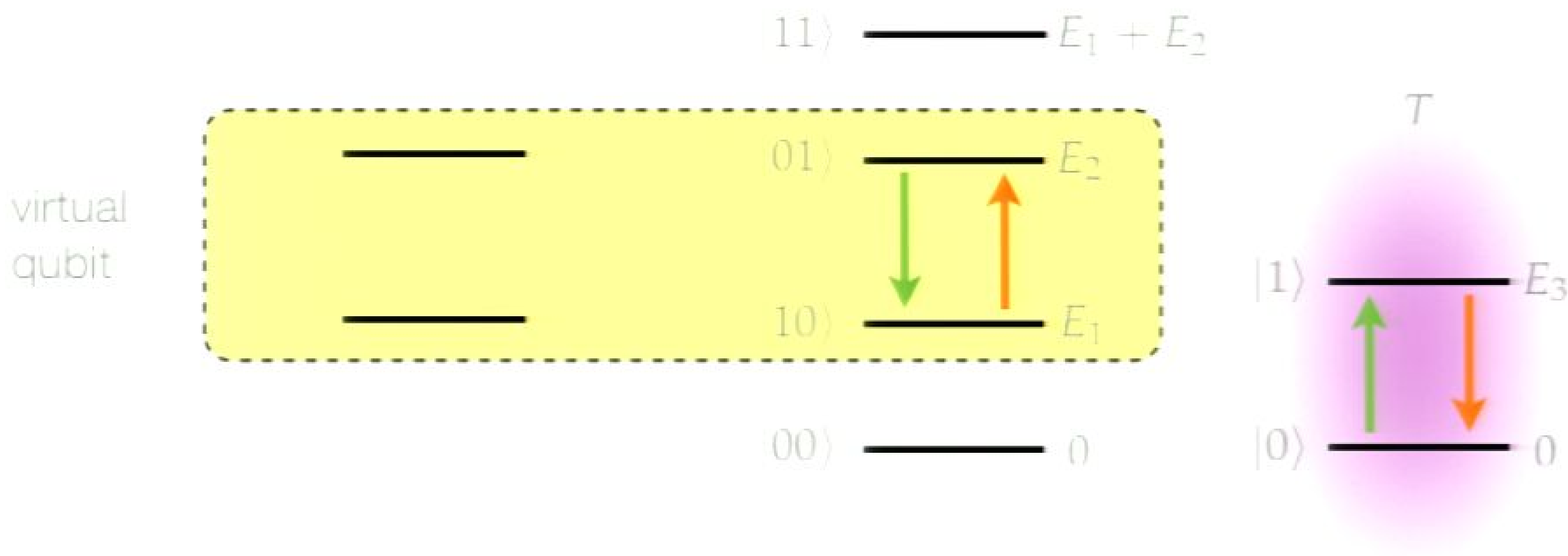

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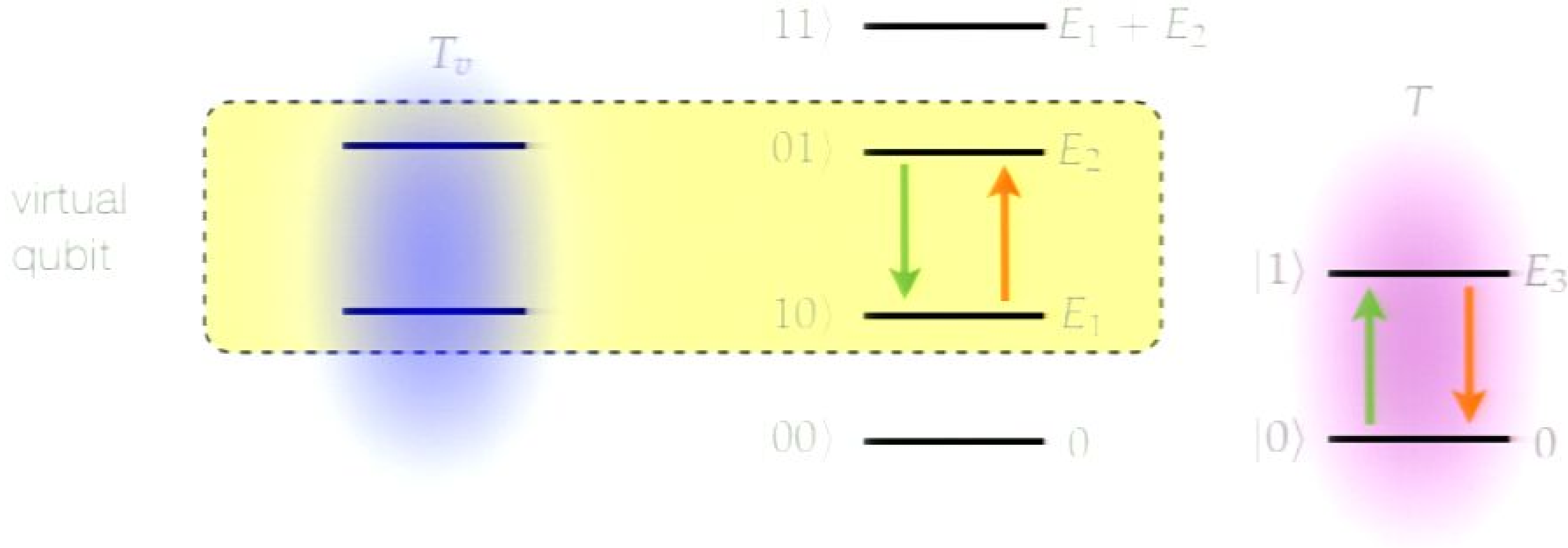

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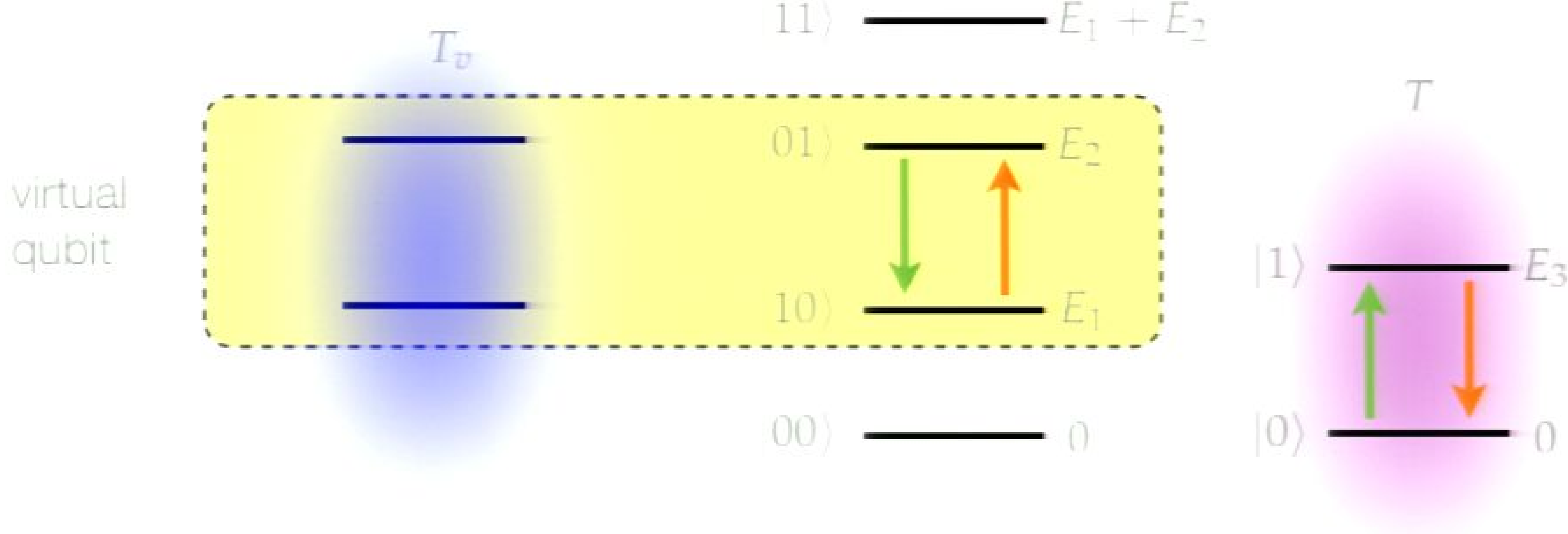
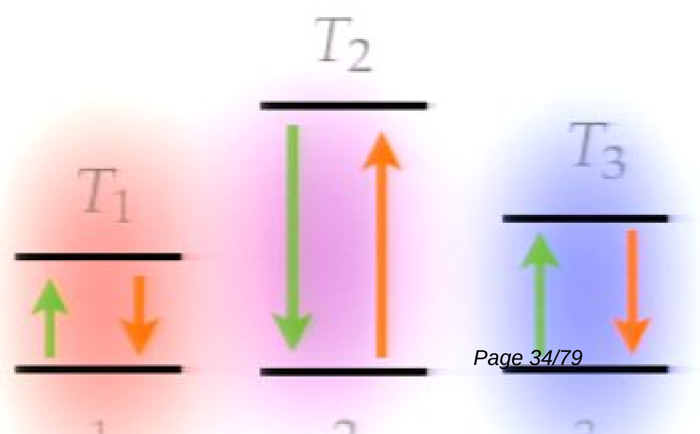


$$\frac{p(01)}{p(10)} = \frac{e^{-E_2/T'}}{e^{-E_1/T''}} = e^{-E_3/T_v}$$

$$T_v = \frac{E_2 - E_1}{\frac{E_2}{T'} - \frac{E_1}{T''}}$$

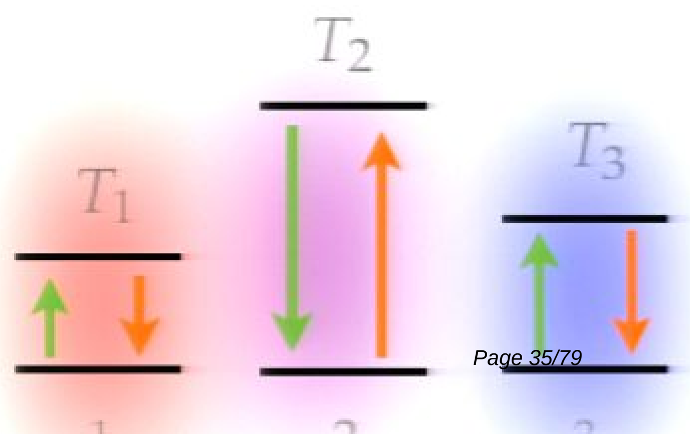
model refrigerator



model refrigerator

- dissipation: prob. $p_i \delta t$ $\quad \rho \mapsto \tau_i \otimes \mathrm{tr}_i \rho$ $\qquad \tau_i = \frac{1}{Z_i} e^{-H_i/kT_i}$
  $1 - p_i \delta t$ $\quad \rho \mapsto \rho$

- interaction Hamiltonian: $H_{int} = g\left(|010\rangle\langle 101| + |101\rangle\langle 010|\right)$



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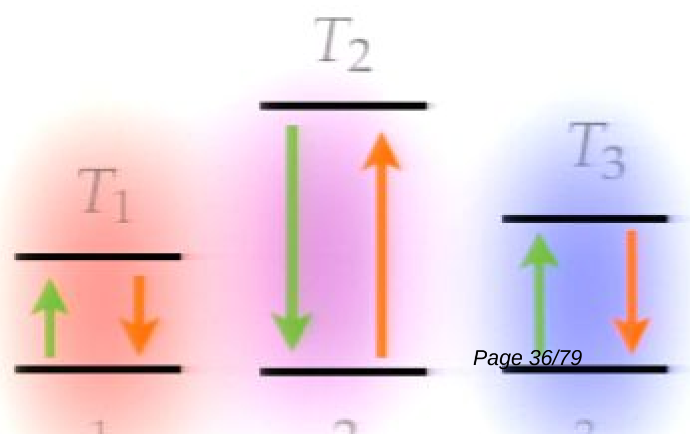

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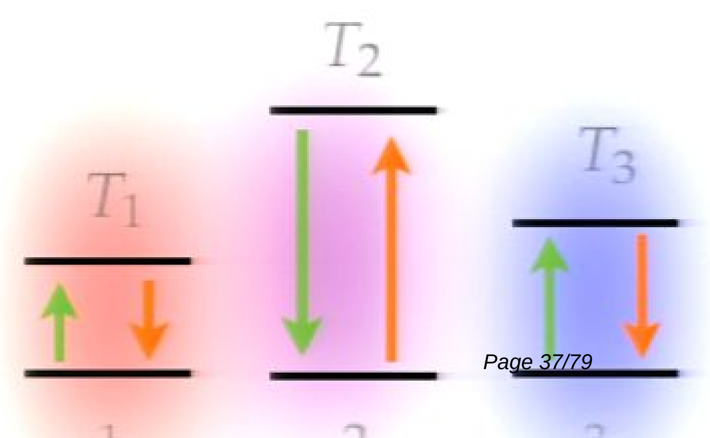
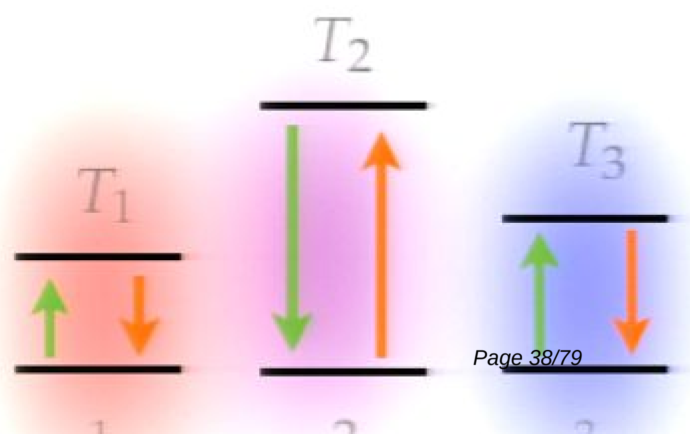
subtleties:

- T defined relative to H_0 not $H_0 + H_{int}$
  ↳ $g \ll E_i$

- no coupling between interaction and dissipation



refrigerator solution



model refrigerator

- dissipation: prob. $p_i \delta t$ $\quad \rho \mapsto \tau_i \otimes \mathrm{tr}_i \rho$
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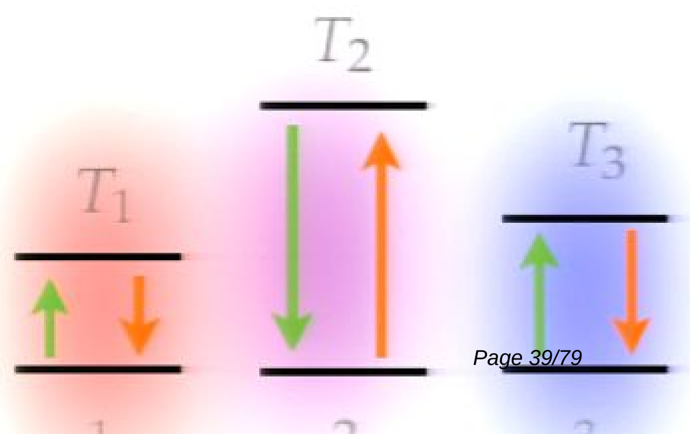
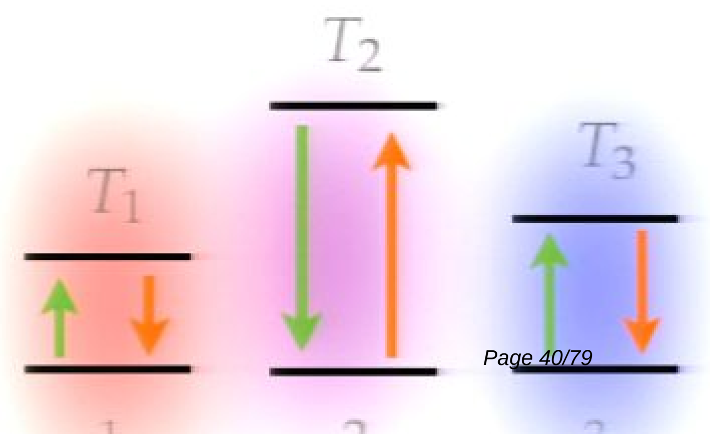
subtleties:

- T defined relative to H_0 not $H_0 + H_{int}$
  ↳ $g \ll E_i$

- no coupling between interaction and dissipation
  ↳ corrections of order $g p_i$ negligible



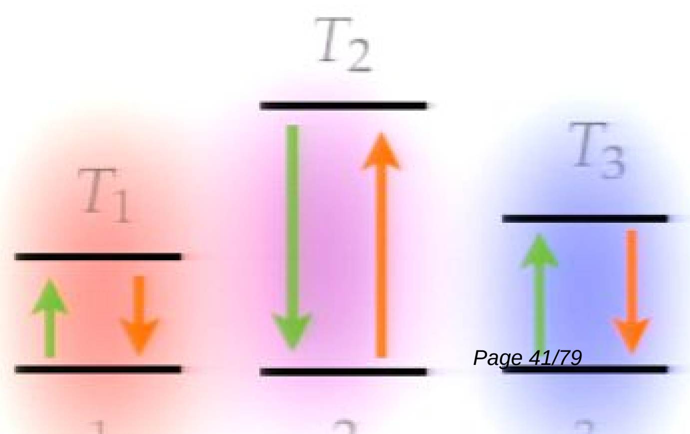
refrigerator solution



refrigerator solution

- stationary solution to Master equation: $\frac{\partial \rho^S}{\partial t} = 0$

  ↳ $0 = -i[H_0 + H_{int}, \rho^S] + \sum_i p_i(\tau_i \otimes \mathrm{tr}_i \rho^S - \rho^S)$



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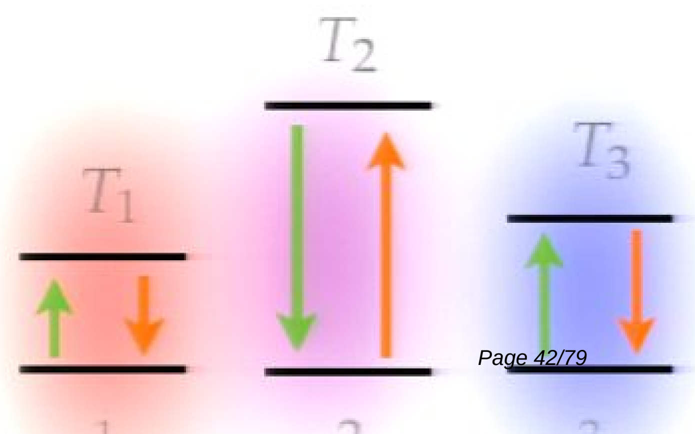
such that $T(\rho_3^S) < T_3$

- analytic solution $\rho_3^S = \tau_3 + p_3 \gamma \sigma_z$ $\gamma(g, p_i, E_i, T_i)$



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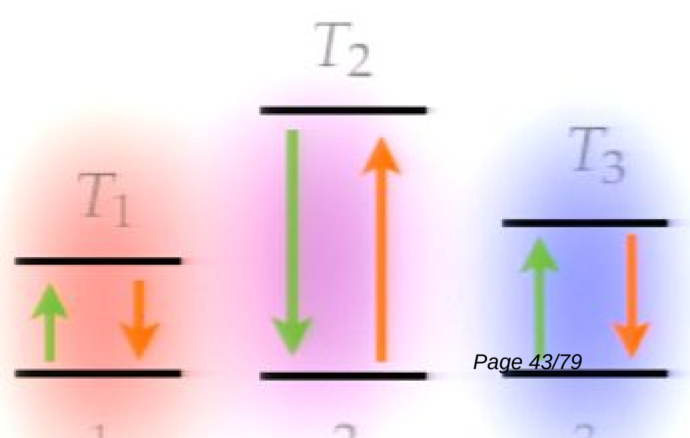
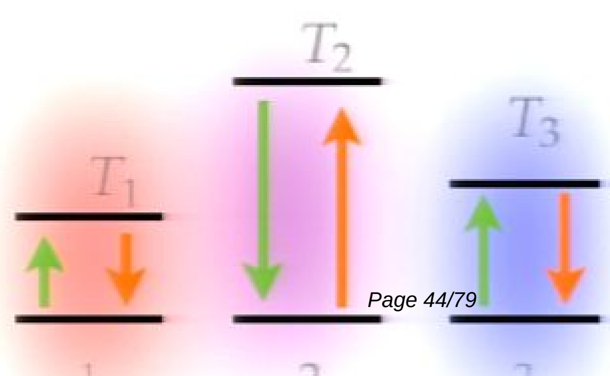
$$\gamma \propto p(101) - p(010) \propto e^{-E_3/T_v} - e^{-E_3/T_3}$$

$$\boxed{T(\rho_3^S) < T_3 \quad \text{iff} \quad T_v < T_3}$$



efficiency



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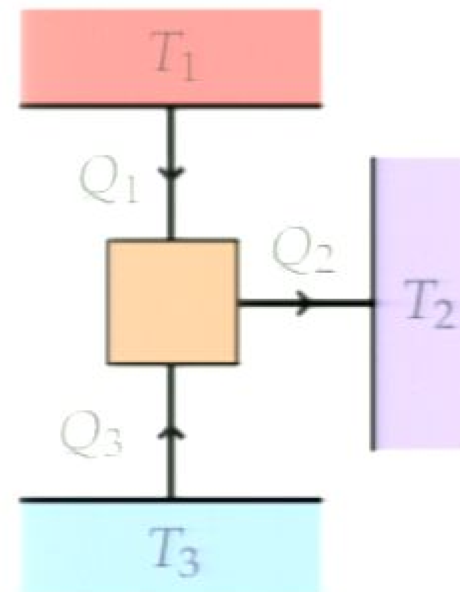
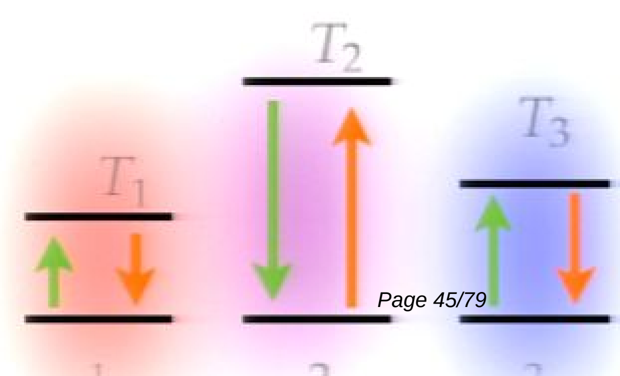


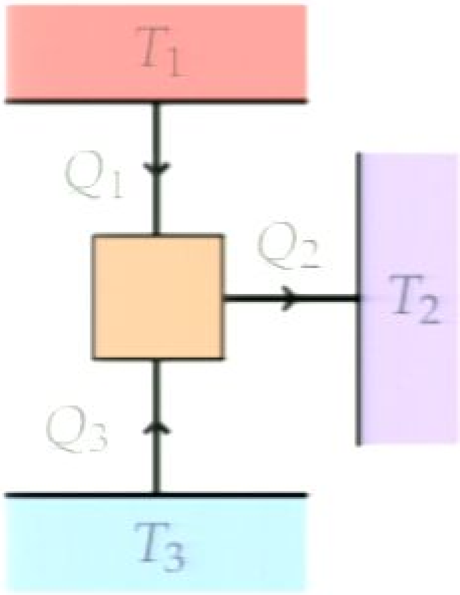


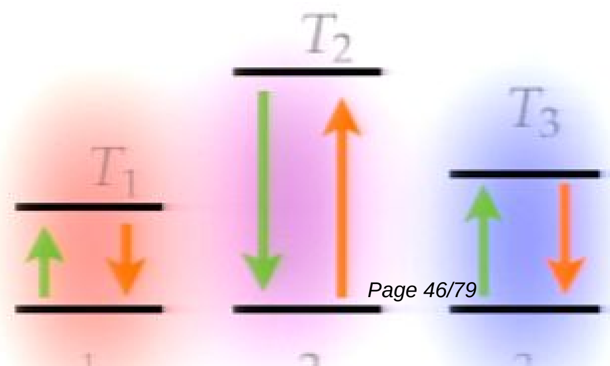
efficiency

$$\eta = \frac{Q_3}{Q_1} \quad \rightarrow \quad \eta^c = \frac{1 - \frac{T_2}{T_1}}{\frac{T_2}{T_3} - 1}$$

$$\frac{dQ_i}{dt} = \mathrm{tr}(H_i \mathcal{D}_i(\rho)) \qquad \mathcal{D}_i(\rho) = p_i(\tau_i \otimes \mathrm{tr}_i \rho - \rho)$$

$$\frac{dQ_i}{dt} = \gamma E_i \quad \rightarrow \quad \eta^q = \frac{E_3}{E_1}$$





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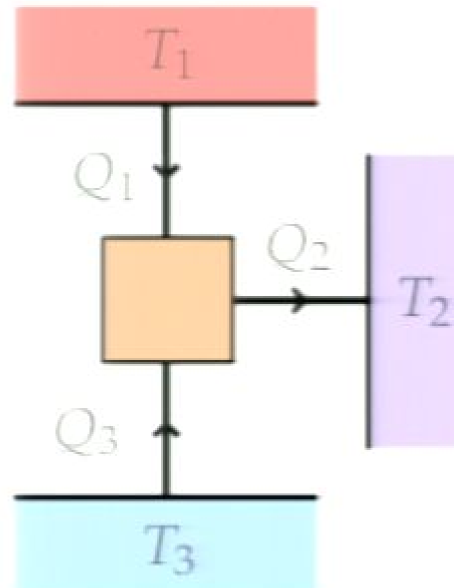

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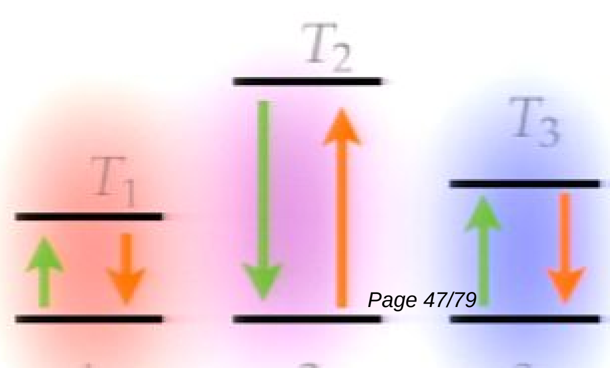
- $T_v < T_3$ ↘
- $T_v = \dfrac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}}$ → $\dfrac{E_3}{E_1} < \dfrac{1 - \frac{T_2}{T_1}}{\frac{T_2}{T_3} - 1}$ ↗



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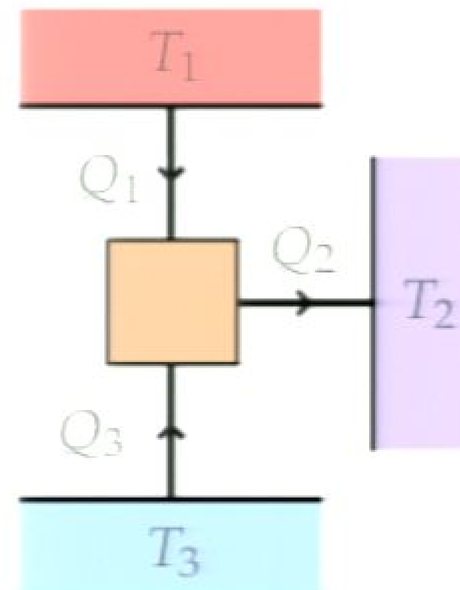

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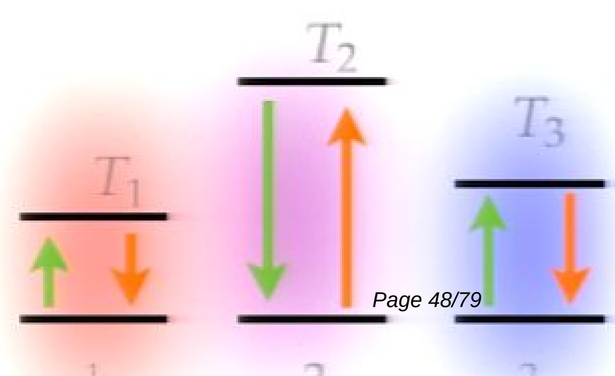
$$\boxed{\eta^q < \eta^c}$$

- $T_v < T_3$
- $T_v = \dfrac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}} \quad \rightarrow \quad \dfrac{E_3}{E_1} < \dfrac{1 - \frac{T_2}{T_1}}{\frac{T_2}{T_3} - 1}$



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Sadi Carnot, reflexions sur la puissance motrice du feu

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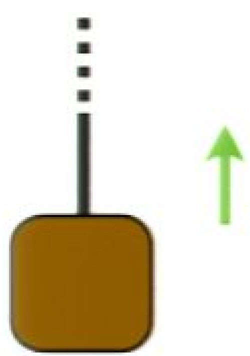
>
> *Sadi Carnot, reflexions sur la puissance motrice du feu*



but how to model an engine?

- question: what is *work* in this context?

"motive power (work) is the useful effect that a motor is capable of producing. This effect can always be likened to the elevation of a weight to a certain height."

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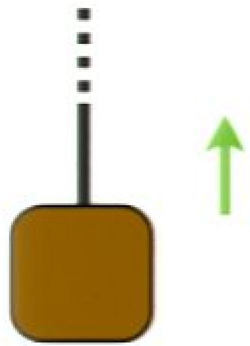

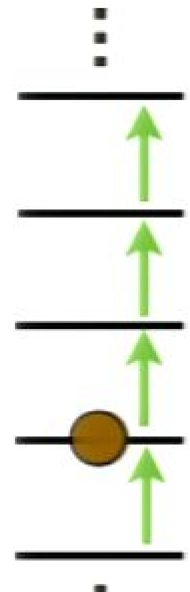

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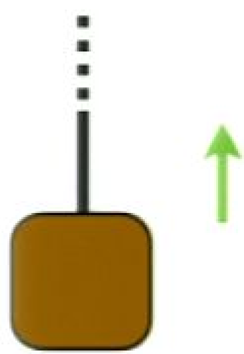

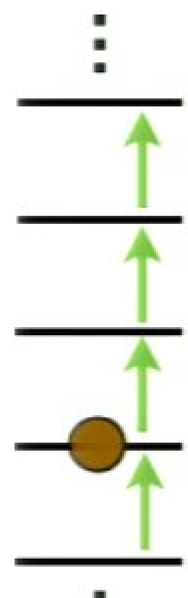

model engine

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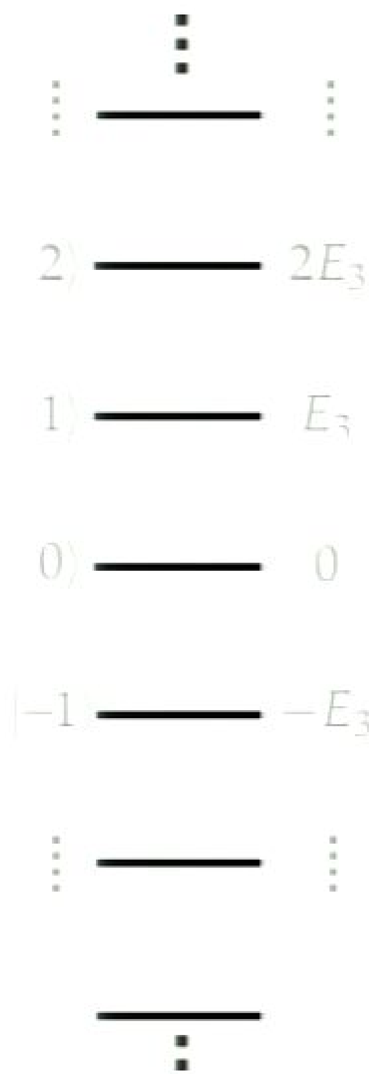

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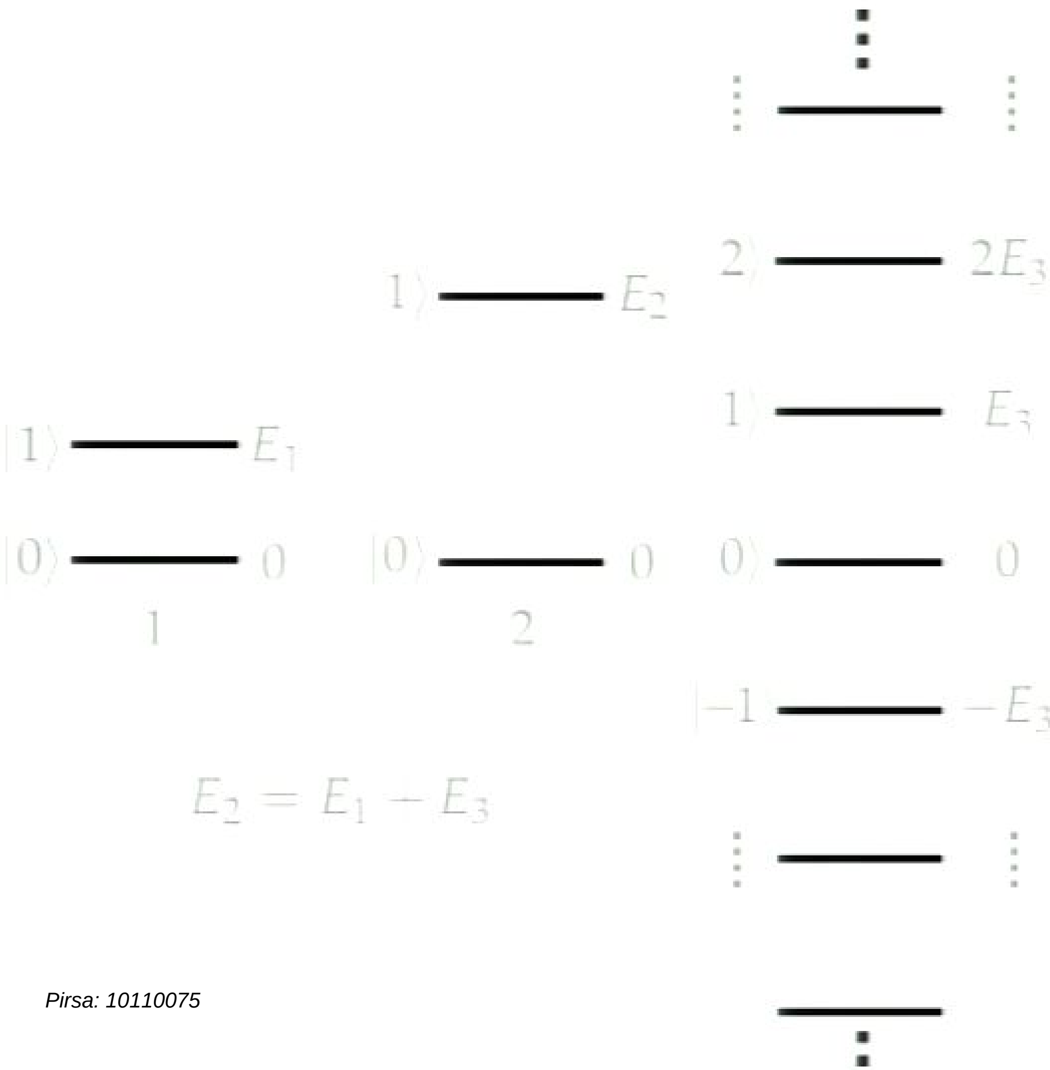
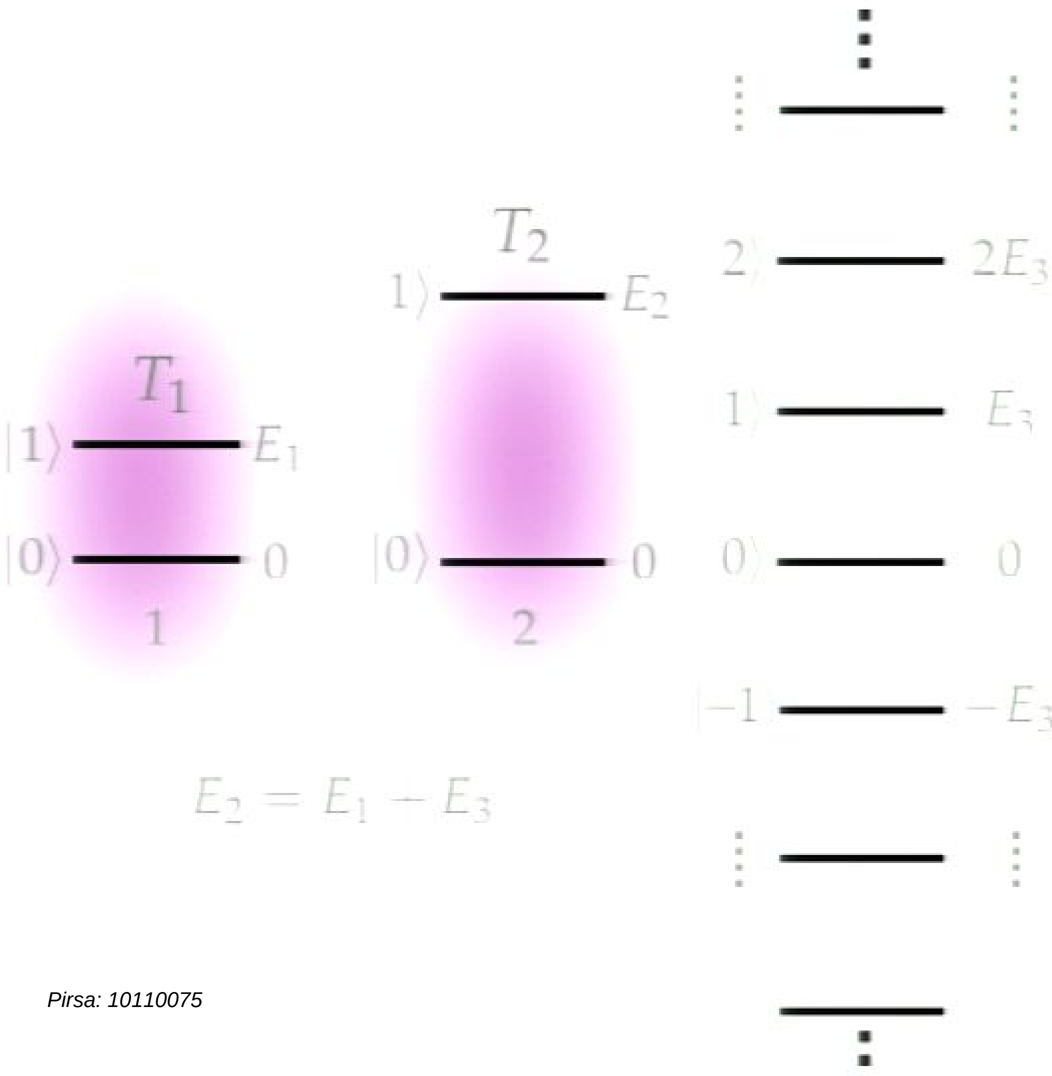


$$E_2 = E_1 - E_3$$

model engine



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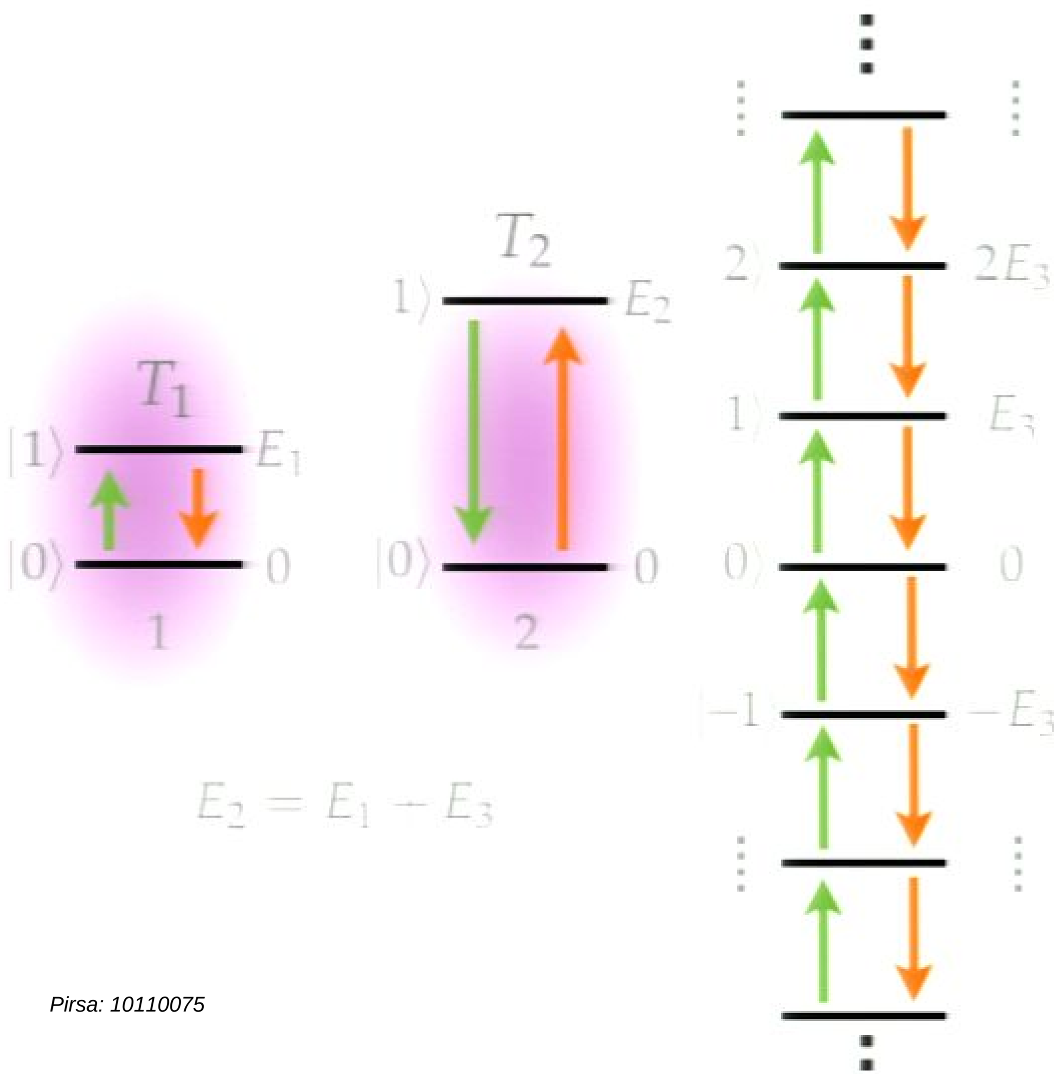
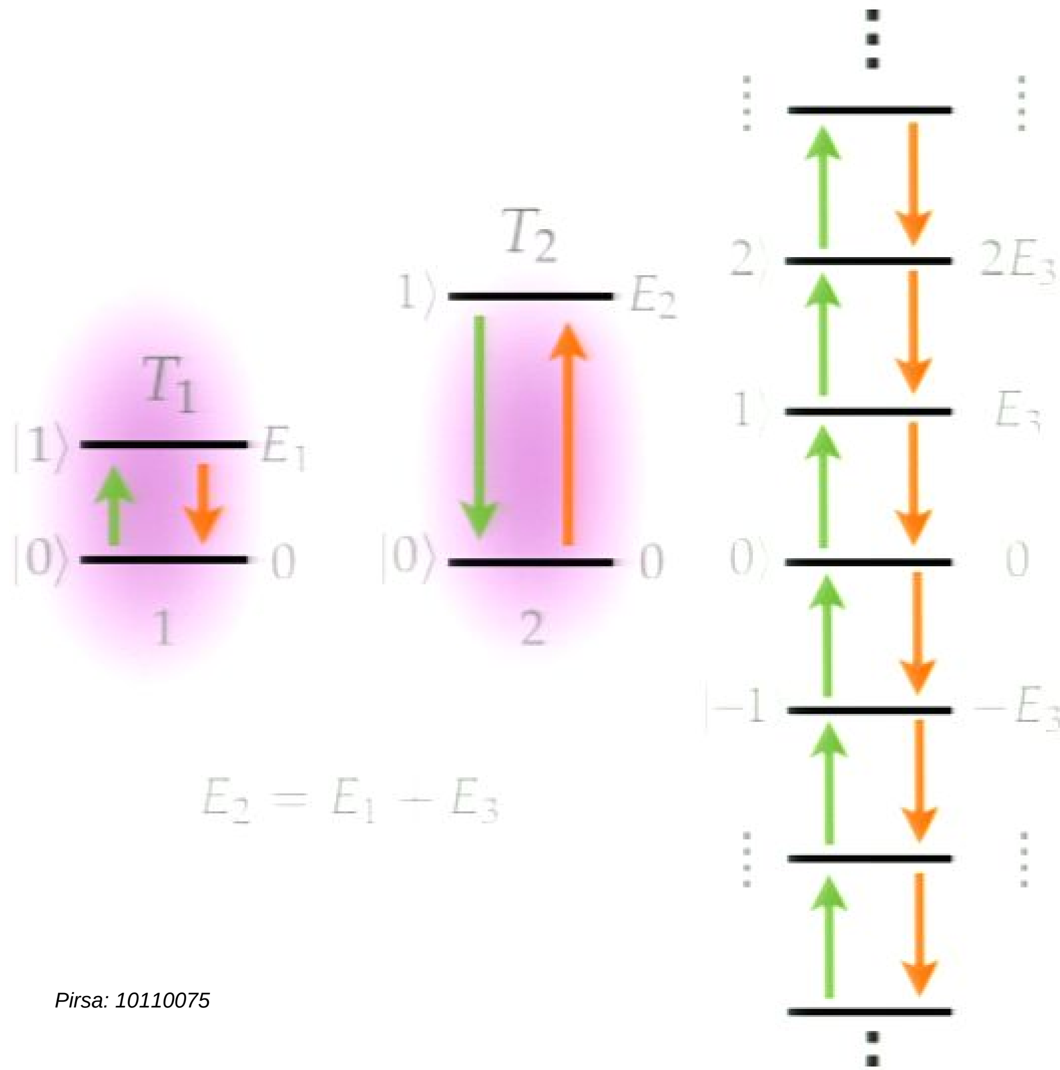


model engine



$E_2 = E_1 - E_3$

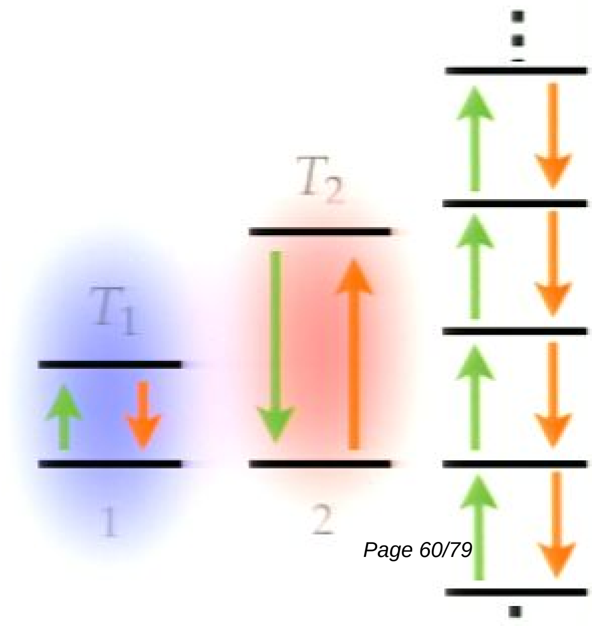
want $p(\text{raise}) > p(\text{lower})$

i.e. $p(01) > p(10)$

$$e^{-E_2/T_2} > e^{-E_1/T_1}$$

$$\boxed{\frac{E_1}{T_1} > \frac{E_2}{T_2}}$$

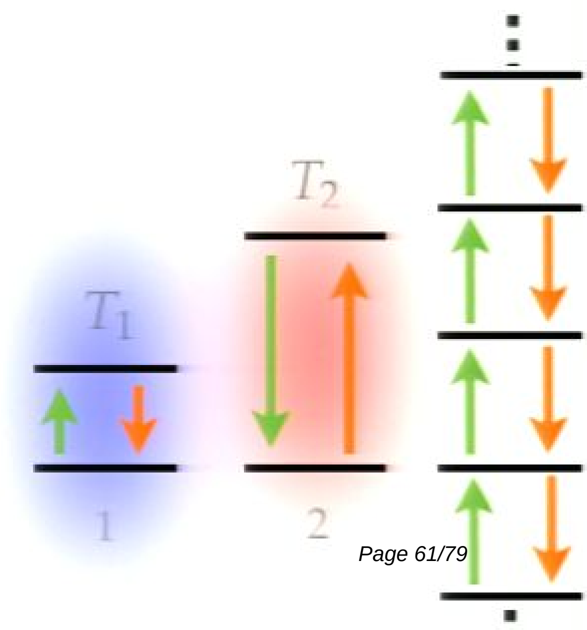
engine solution



engine solution

$$\frac{\partial\rho}{\partial t} = -i[H_0 + H_{int}, \rho] + \sum_i p_i(\tau_i \otimes \mathrm{tr}_i\rho - \rho)$$

$$H_{int} = g \sum_{n=-\infty}^{\infty} \left(|01, n\rangle\langle 10, n+1| + |10, n+1\rangle\langle 01, 1| \right)$$



engine solution

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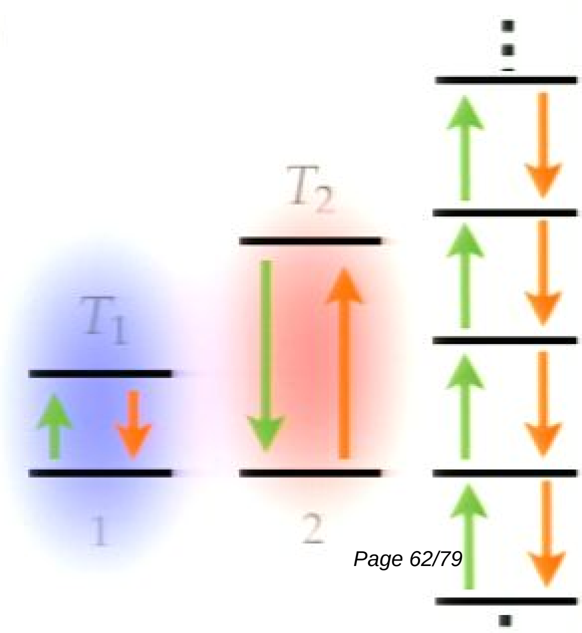
$$H_{int} = g \sum_{n=-\infty}^{\infty} (|01, n\rangle\langle 10, n+1| + |10, n+1\rangle\langle 01, 1|)$$

$$\lim_{t\to\infty} \frac{d}{dt}\langle E_w\rangle = \frac{2E_3 g^2 p_1 p_2}{(p_1+p_2)(2g^2+p_1 p_2)}(p(01) - p(10))$$



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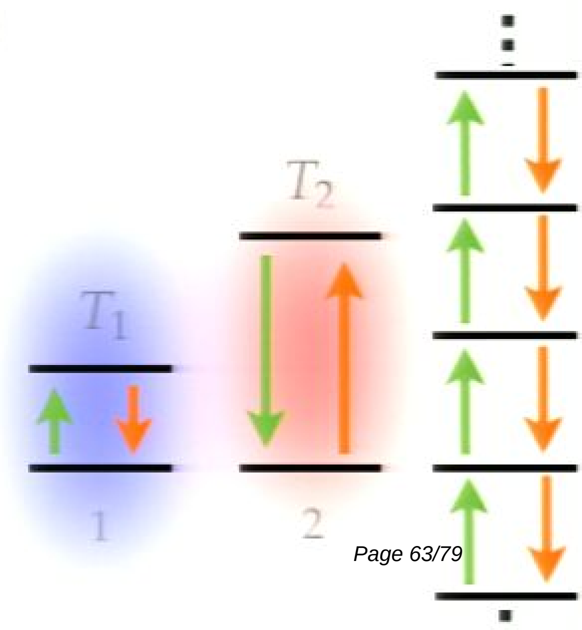
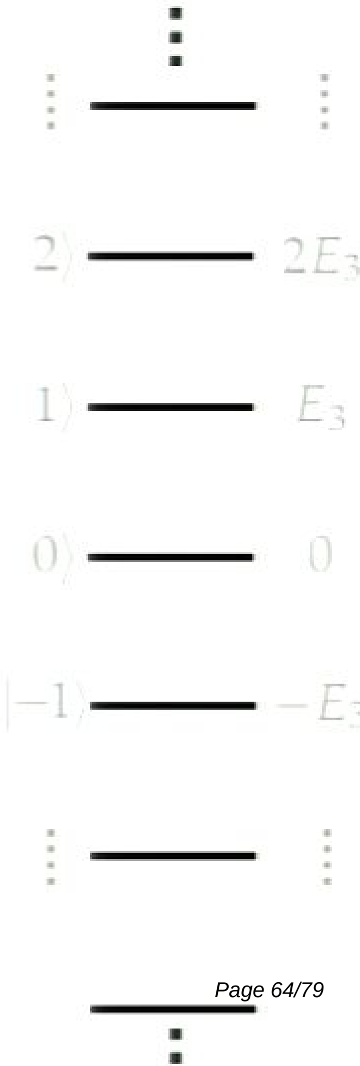
$$\boxed{\frac{d}{dt}\langle E_w \rangle > 0 \quad \text{iff} \quad \frac{E_1}{T_1} > \frac{E_2}{T_2}}$$



virtual transition



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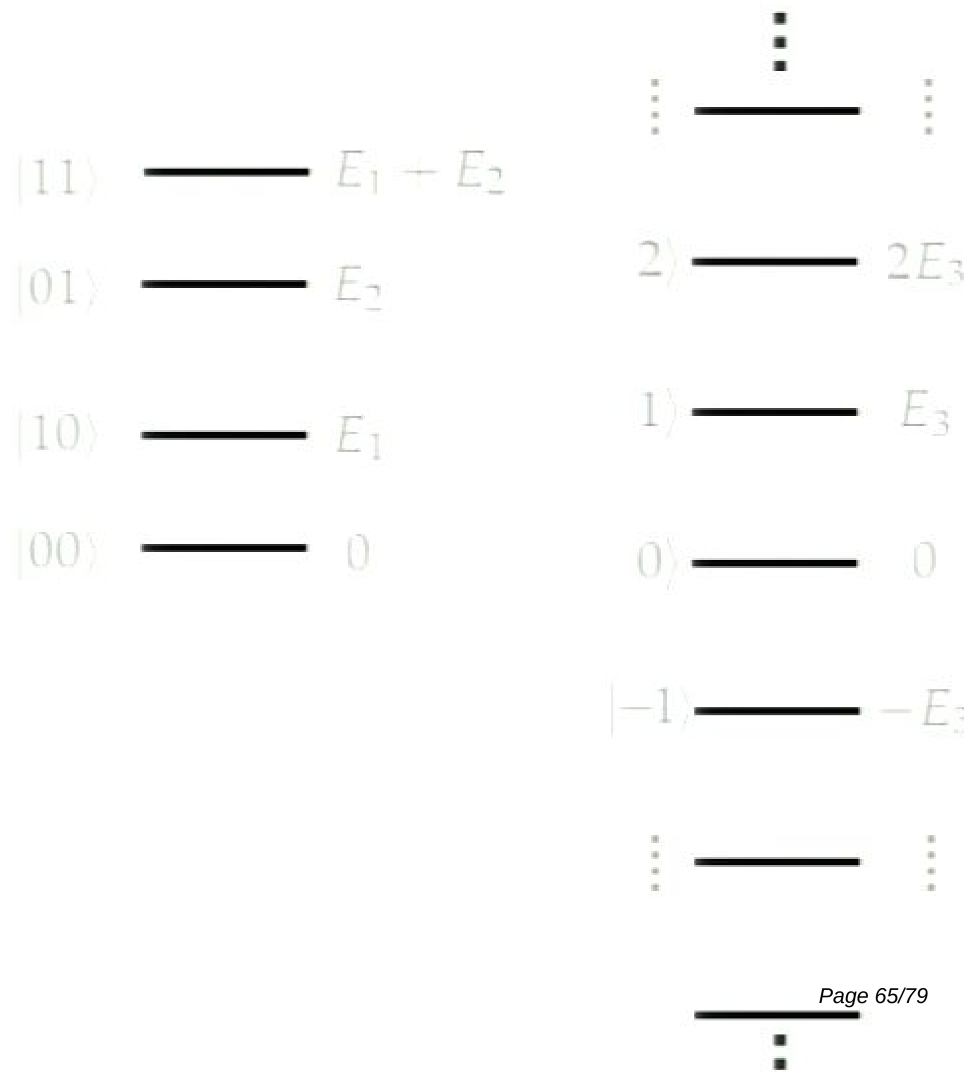

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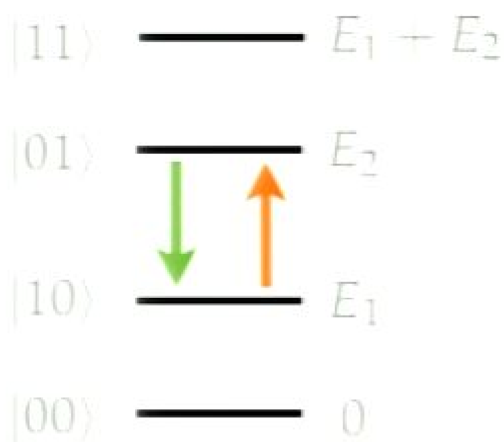
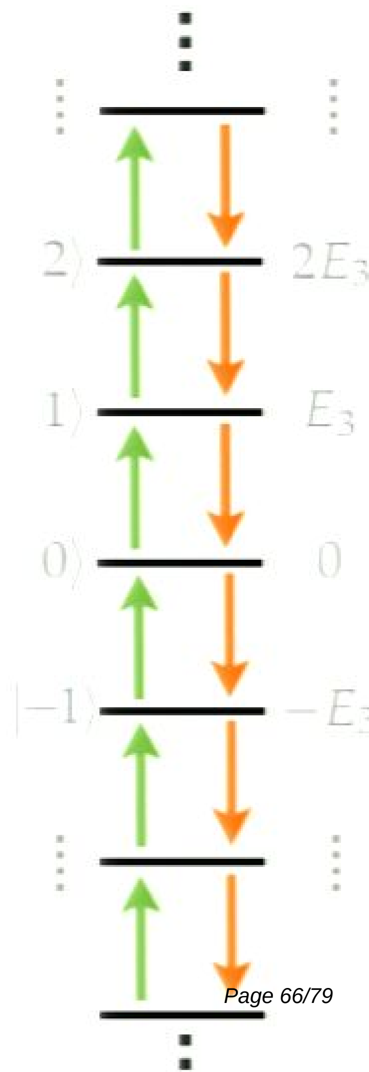
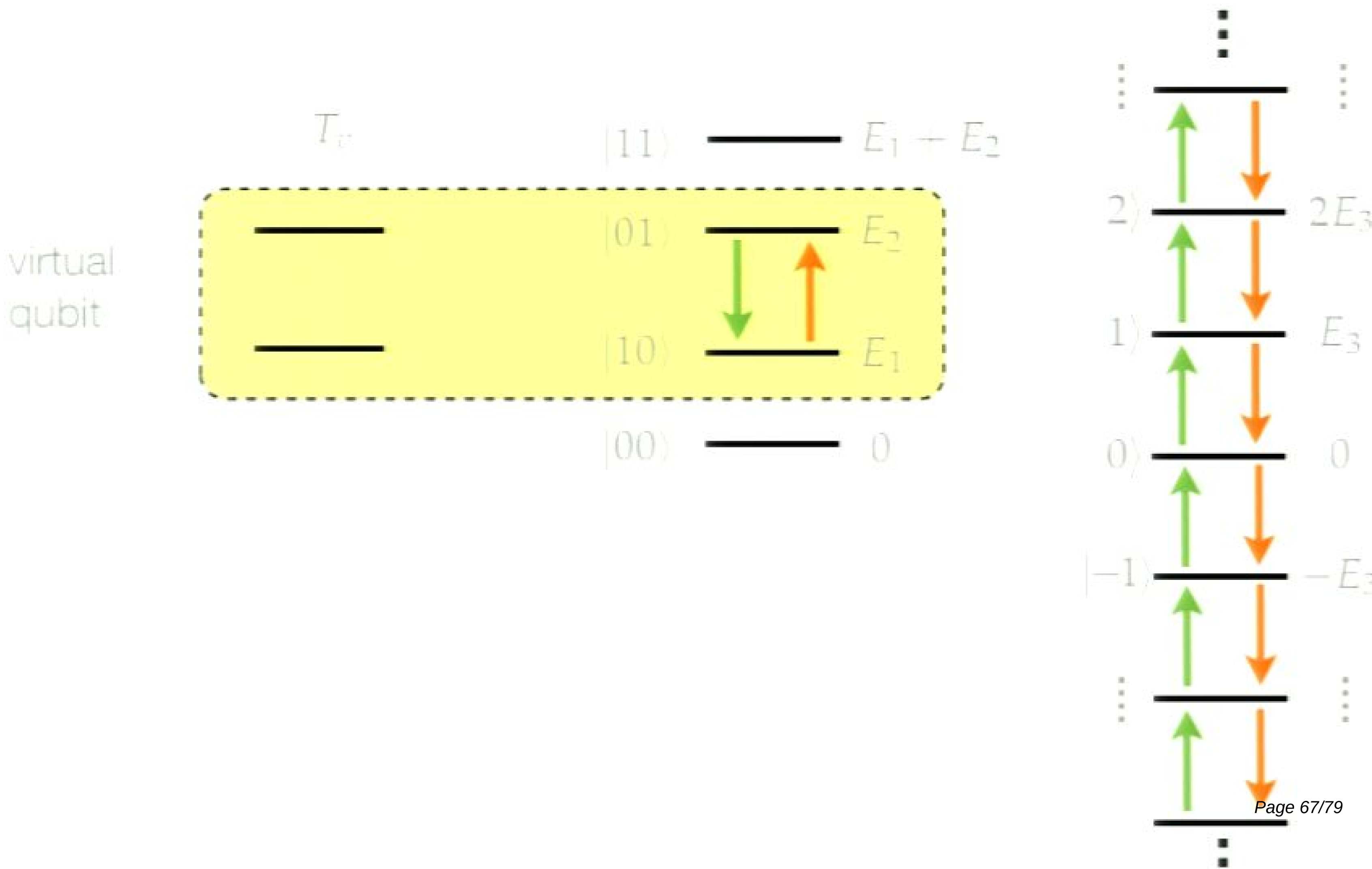




virtual transition



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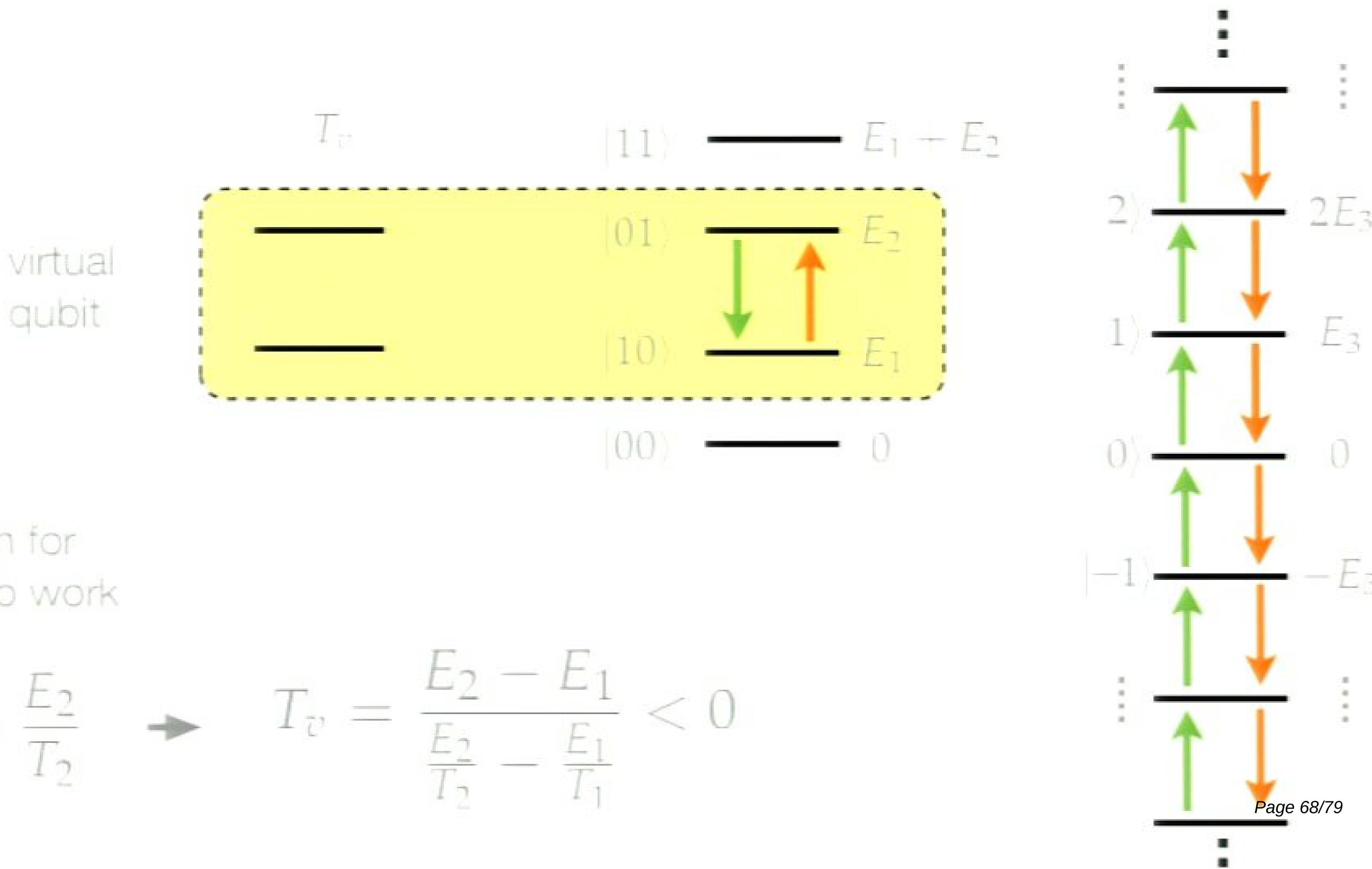
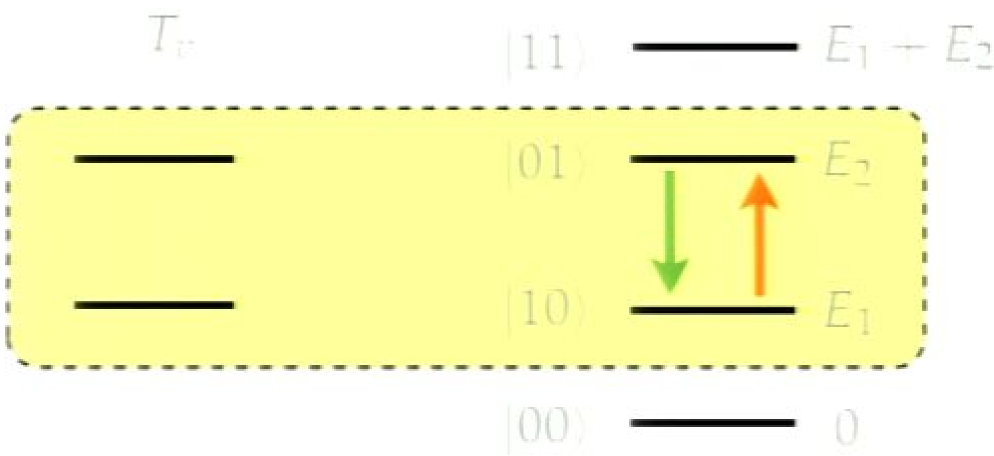


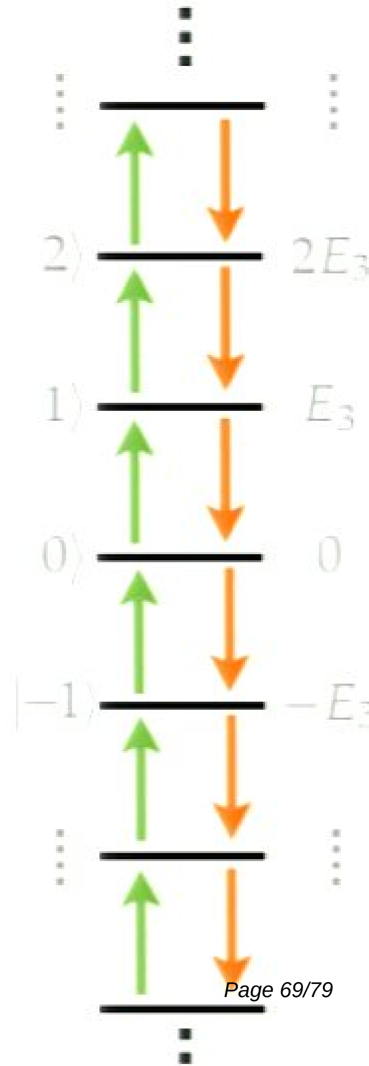
condition for
engine to work

$$\frac{E_1}{T_1} > \frac{E_2}{T_2} \rightarrow T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}} < 0$$

virtual transition

virtual qubit





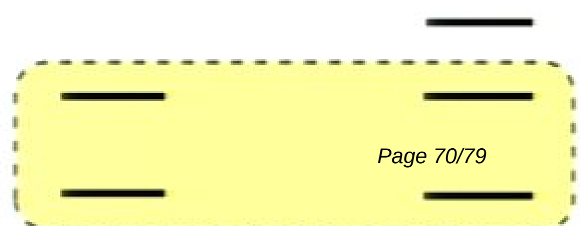
condition for engine to work

$$\frac{E_1}{T_1} > \frac{E_2}{T_2} \rightarrow T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}} < 0$$

in other words: **population inversion**

unified viewpoint

$$T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}}$$

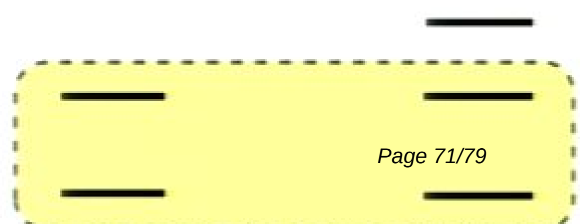


unified viewpoint

$$0 \le T_2 < T_1 \quad \rightarrow \quad 0 \le T_v < T_2$$ **refrigerator**

$$0 \le T_1 < T_2 < \frac{E_2}{E_1} T_1$$

$$T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}}$$



unified viewpoint

$0 \leq T_2 < T_1$ → $0 \leq T_v < T_2$ **refrigerator**

$0 \leq T_1 < T_2 < \frac{E_2}{E_1} T_1$ → $T_v \geq T_2$ **heat pump**

$0 \leq \frac{E_2}{E_1} T_1 < T_2$ → $T_v < 0$ **heat engine**

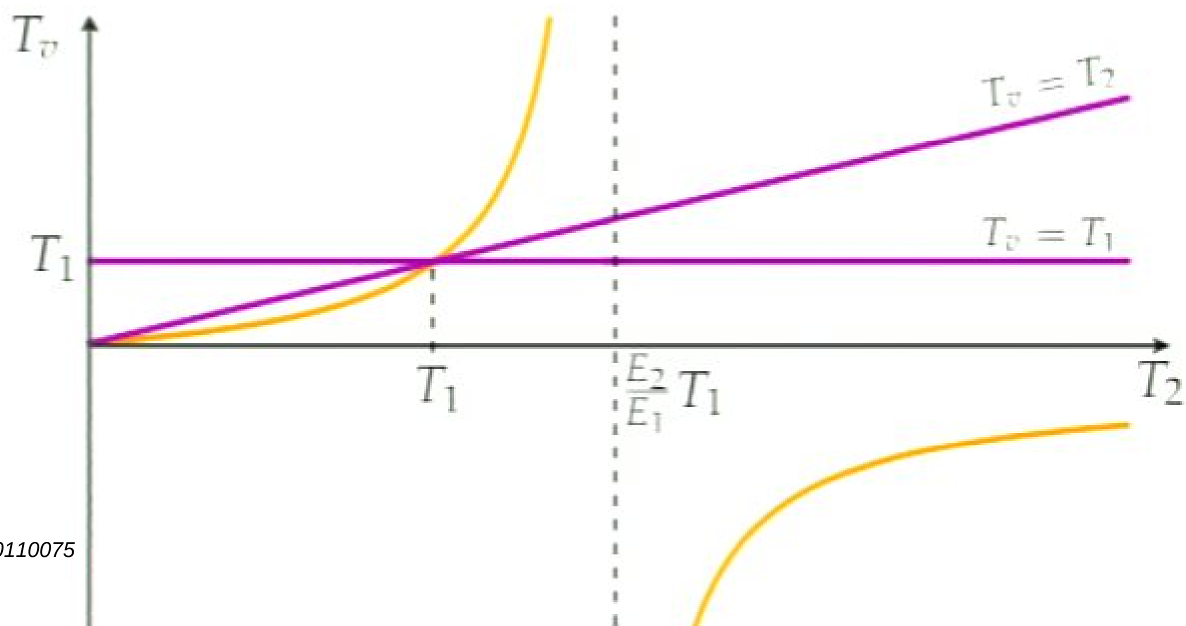
$$T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}}$$

unified viewpoint

$0 \leq T_2 < T_1 \rightarrow 0 \leq T_v < T_2$ **refrigerator**

$0 \leq T_1 < T_2 < \frac{E_2}{E_1} T_1 \rightarrow T_v \geq T_2$ **heat pump**

$0 \leq \frac{E_2}{E_1} T_1 < T_2 \rightarrow T_v < 0$ **heat engine**



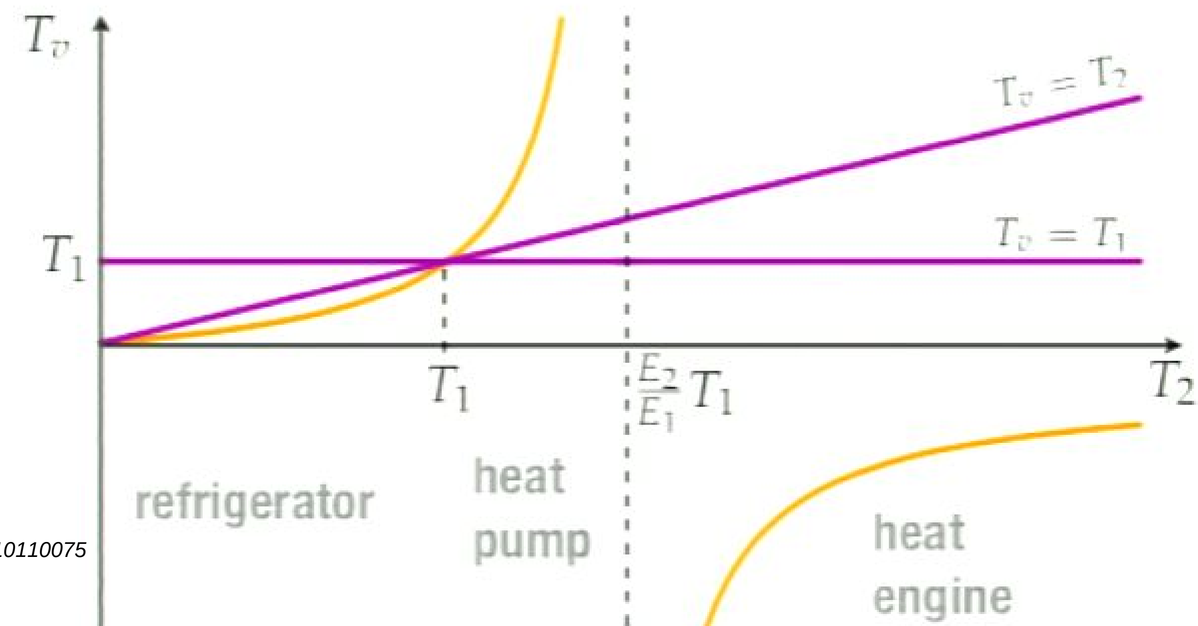
$$T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}}$$

unified viewpoint

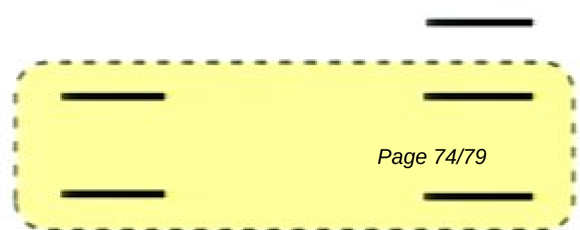
$0 \leq T_2 < T_1$ ➔ $0 \leq T_v < T_2$ **refrigerator**

$0 \leq T_1 < T_2 < \frac{E_2}{E_1} T_1$ ➔ $T_v \geq T_2$ **heat pump**

$0 \leq \frac{E_2}{E_1} T_1 < T_2$ ➔ $T_v < 0$ **heat engine**



$$T_v = \frac{E_2 - E_1}{\frac{E_2}{T_2} - \frac{E_1}{T_1}}$$



open questions

- how far does the idea that work is inversion go?

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- quantify the 'quality' of work
- third law of thermodynamics

conclusions

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- no limitation on the size of thermal machines
- no complementarity between size and efficiency

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- no limitation on the size of thermal machines
- no complementarity between size and efficiency
- work intimately related to population inversion
- unified view of thermal machines at quantum level