Title: Is temperature the speed of time? Thermal time and the Tolman effect

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Abstract: Why is a vertical column of gas at thermal equilibrium slighly hotter at the bottom than a the top? My answer in this talk will be that time runs slower in a deeper gravitational potential, and temperature is nothing but the (inverse) speed of time. Specifically, I will (i) introduce Rovelli's notion of thermal time, (ii) use it to provide a "principle" characterization of thermal equilibrium in stationary spacetimes, and (iii) effortlessly derive the Tolman-Ehrenfest relation. This approach contrasts with the "constructive" accounts of thermal equilibrium in curved spacetimes given in the literature, and vindicates the time-temperature relationship cropping up in the Hawking-Unruh and Kubo-Martin-Schwinger relations.

Is temperature the speed of time?

Thermal time and the Tolman effect

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November, 2010

Joint work with Carlo Rovelli
[arXiv:1005.2985]

Prologue: time and temperature

In differents parts of physics, a connection between time and temperatures crops up:

- an accelerated observer in the vacuum measures a temperature

$$T_{\mathrm{HU}} = \frac{\hbar a}{2\pi c k_B}$$

- QFT correlation functions at thermal equilibrium are periodic in imaginary time, with period

$$\tau = \frac{2\pi k_B T}{\hbar}$$

- Chern-Simons time in Euclidean quantum gravity is periodic with the same period

[Smolin, Soo (94)]

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Introduction

I will argue, in the context of thermal equilibrium in stationary spacetimes, that is useful to think of

"temperature as the speed of time".

Specifically, I will

- introduce the Connes-Rovelli notion of thermal time, and
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Outline

Tolman's law

In 1930, Tolman realized that, within general relativity, in a stationary gravitational field, temperature is not constant at equilibrium:

$$T(\vec{x}) \propto \frac{1}{\sqrt{g_{00}(\vec{x})}}$$

[Tolman, Tolman-Ehrenfest, PR (30)]

in stationary coordinates. In the Newtonian limit, this means that

$$\frac{\nabla T}{T} = \frac{\vec{g}}{c^2}$$

with $\vec{g}$ the acceleration of gravity.

The lower, the hotter.

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Thermal equilibrium in stationary spacetimes I

To make sense of Tolman's law, one needs a characterization of thermal equilibrium. In non-relativistic statistical mechanics, we know many:

- thermodynamically, by Kelvin's second law: no work from a single heat source
- information-theoretically, by the maximization of entropy
- dynamically, by a stability condition w.r.t. perturbations
- stochastically, by the condition of detailed balance of microscopic fluxes
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- ...

All of them neglect gravity.

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Thermal equilibrium in stationary spacetimes II

Extensions of the notion of thermal equilibrium to stationary spacetimes have been proposed. All use the Einstein mass-energy relation $E = mc^2$ ('heat has weight'), and

- a dynamical input (Einstein field equations) [Tolman, Tolman-Ehrenfest (30)]
- a thermodynamical input ($\partial S/\partial E = 1/T$) [Balazs (58), Balazs-Dawson (65), Landau-Lifschitz (59)]
- a study of relativistic Carnot cycles [Ebert, Gobel (73)]
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There is one, using as only input that temperature is the speed of time.

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Time in non-relativistic physics

In non-relativistic physics, time plays both a dynamical and thermodynamical rôle

- dynamics: time-reversible differential equations in t (Newton, Lagrange or Hamilton)
- thermodynamics: time-irreversible PDE's in τ (heat equation, entropy balance equation).

We might called them mechanical time t and thermal time τ respectively.

They coincide in non-relativistic physics, but their confusion generates longstanding paradoxes (Loschmidt, Zermelo).

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Time in relativistic physics

The coincidence of mechanical and thermal time breaks down in relativistic physics.

- special relativity: mechanical time is Lorentz covariant, thermal time is not
- curved spacetime: mechanical time (proper time) is local, and metric-dependent; what is thermal time?
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Thermal time: heuristics

Connes and Rovelli propose a fully relativistic notion of thermal time.

[Rovelli (93), Connes-Rovelli (94)]

Heuristically, the passing of thermal time is associated to the ignorance of the microscopic dynamics. This is represented by statistical states in statistical mechanics. Hence

the thermal time flow is induced by a statistical state.

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Thermal time: definition

A relativistic system can be described by a Poisson manifold $\mathcal{A}$ and a set of constraints $C \in \mathcal{C}^\infty(\mathcal{A})$. Let ρ be a statistical state on $\mathcal{A}$ such that

$$\{\rho, C\} \approx 0.$$

The thermal time flow on $\mathcal{A}$ induced by ρ is defined as the

Hamiltonian vector field of $(-\ln \rho)$.

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Thermal time: example

As an limit example, consider a non-relativistic equilibrium state:

$$\rho = Z^{-1} e^{-\beta H}.$$

Then the thermal time flow matches the mechanical time flow, up to a constant:

$$X_{-\ln\rho} = \beta X_H.$$

- This identity characterizes thermal equilibrium in this setting.
- The (inverse) temperature β sets the relative scale of thermal time w.r.t. mechanical time. Pictorially, the "speed of time".

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Outline

Thermal equilibrium in stationary spacetimes: heuristics

The identity

$$X_{-\ln\rho} = \beta X_H$$

can be used to characterize thermal equilibrium in stationary spacetimes. Roughly speaking, it says that

$$\text{thermal time} = \beta\ (\text{mechanical time})$$

- Thermal time is defined in general, given a statistical state,
- mechanical time is proper time along stationary worldlines.

What is needed is a way to replace X_H by $\frac{d}{ds}$ along stationary worldlines.

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Thermal equilibrium in stationary spacetimes: criteria

Thermal time is defined as a vector field on phase space, while proper time is defined on spacetime. The bridge between the two is through local observables A_x. Example:

$$n_x = \text{density of a gas about the spacetime time point x.}$$

Then the two criteria expressing the relationship between thermal and proper time at equilibrium are

1. $X_{-\ln\rho} A_x = \mathcal{L}_{\xi^\rho} A_x$ for some timelike Killing ξ^ρ
2. $\xi_\rho = \beta \frac{d}{ds}$ along stationary worldlines.

This is the precise meaning of the statement that, at equilibrium, temperature is the speed of time.

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The Tolman law

The Tolman law follows immediately. From

$$\xi_\rho = \beta \frac{d}{ds},$$

we have that

$$T \propto \|\xi_\rho\|^{-1},$$

and in stationary coordinates w.r.t. ξ_ρ

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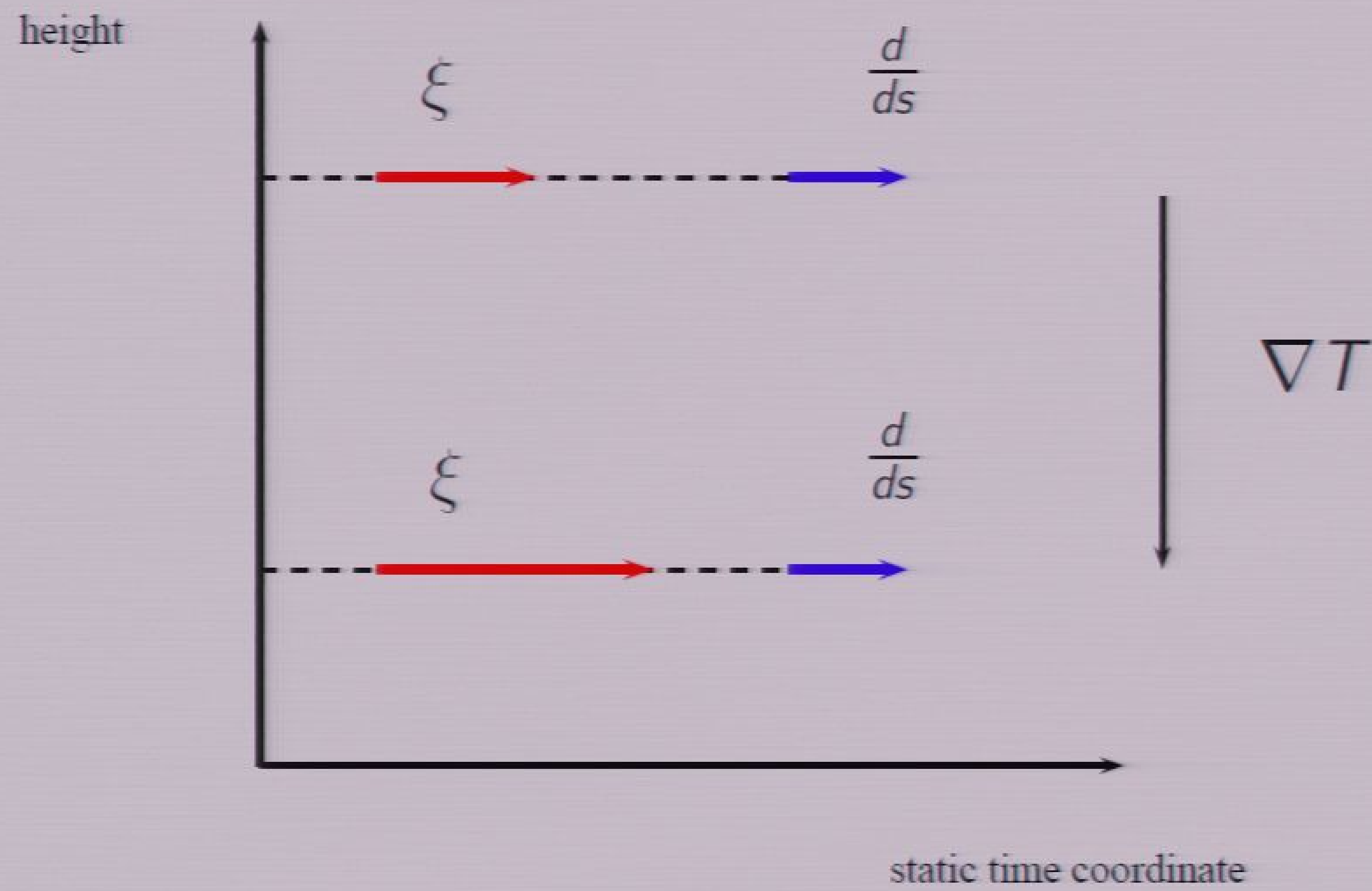

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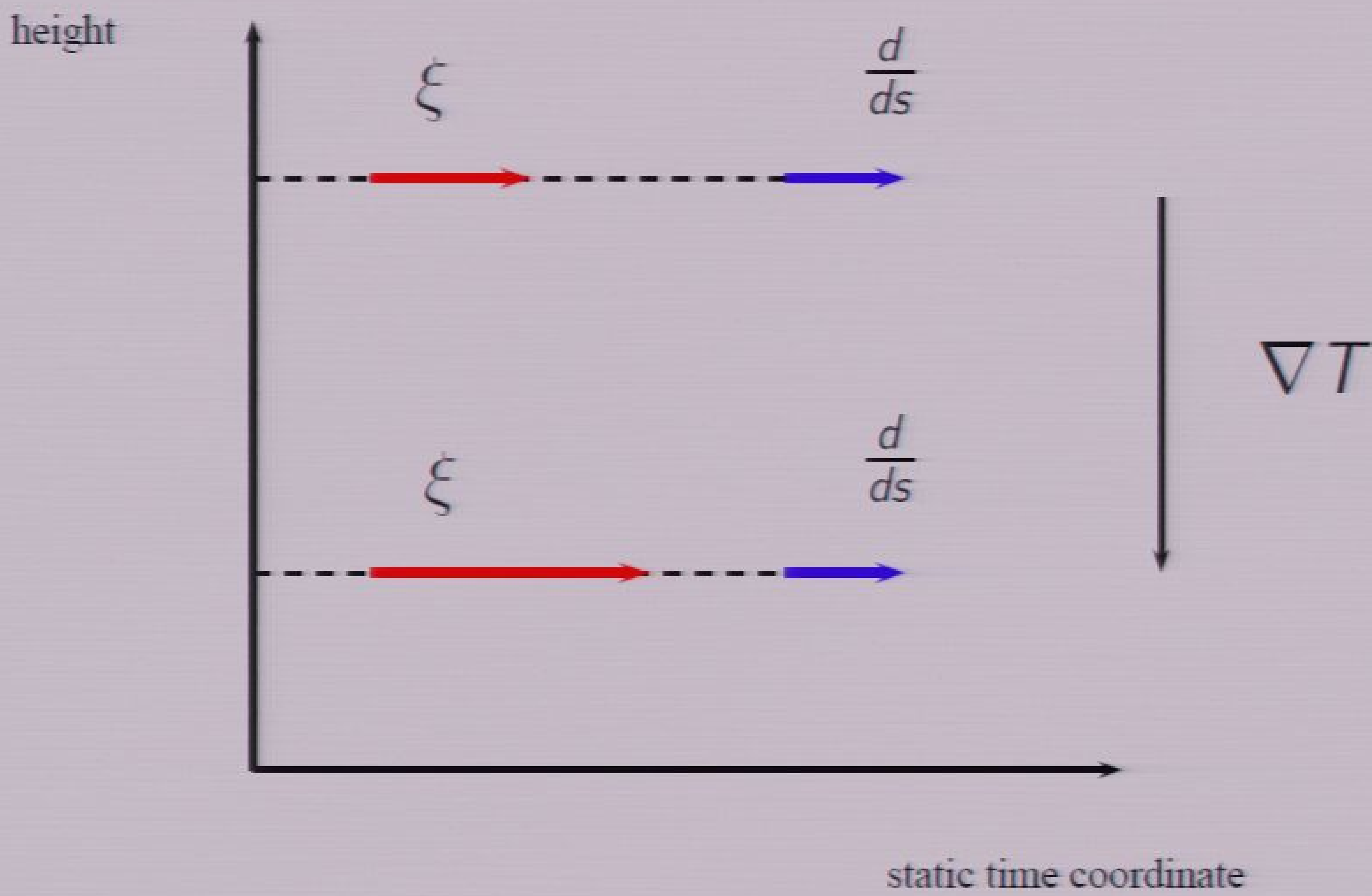

Conclusion

In our opinion, this argument shows that

- There is well-defined notion of thermal time in general relativity.
- On a fixed stationary spacetime, it yields a natural and minimalist characterization of thermal equilibrium.
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... and a wink.

... and not just physics! In biology, the “thermal time hypothesis” refers to the widely observed linear relationship between development rate and temperature.

[Réaumur (1735)]

Figure: Crop and insect growth rate versus temperature.

For crops and insects too, temperature is the speed of time!

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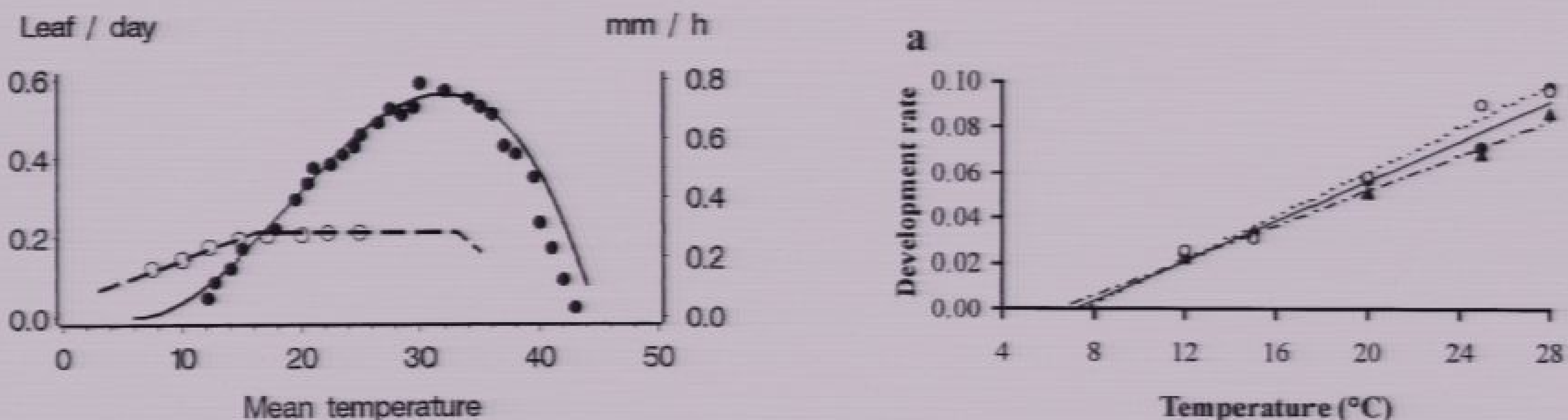

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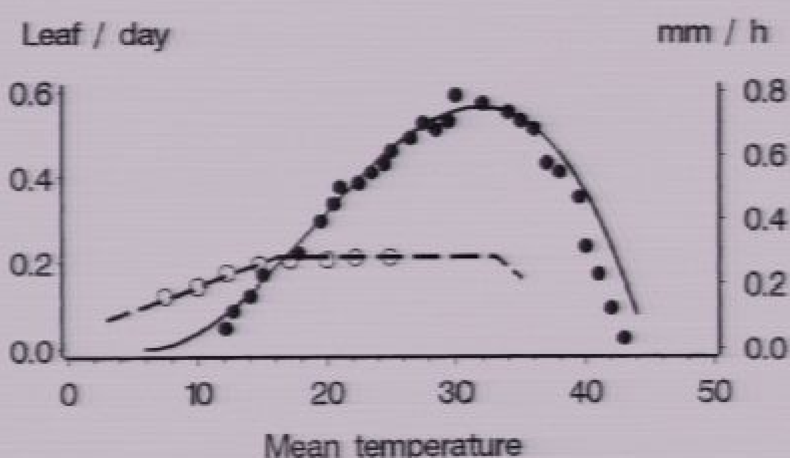

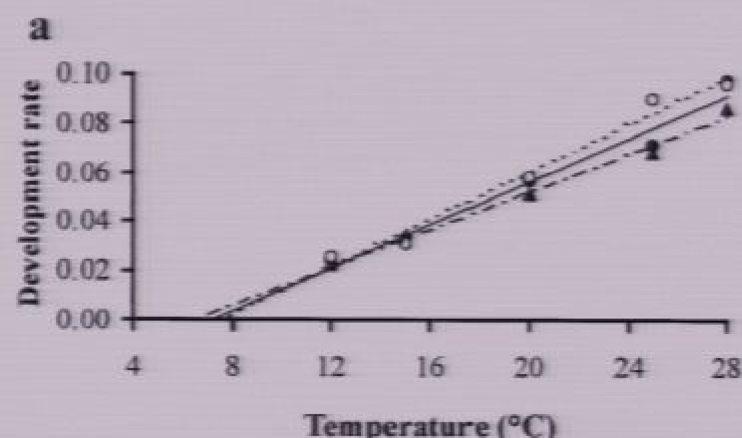

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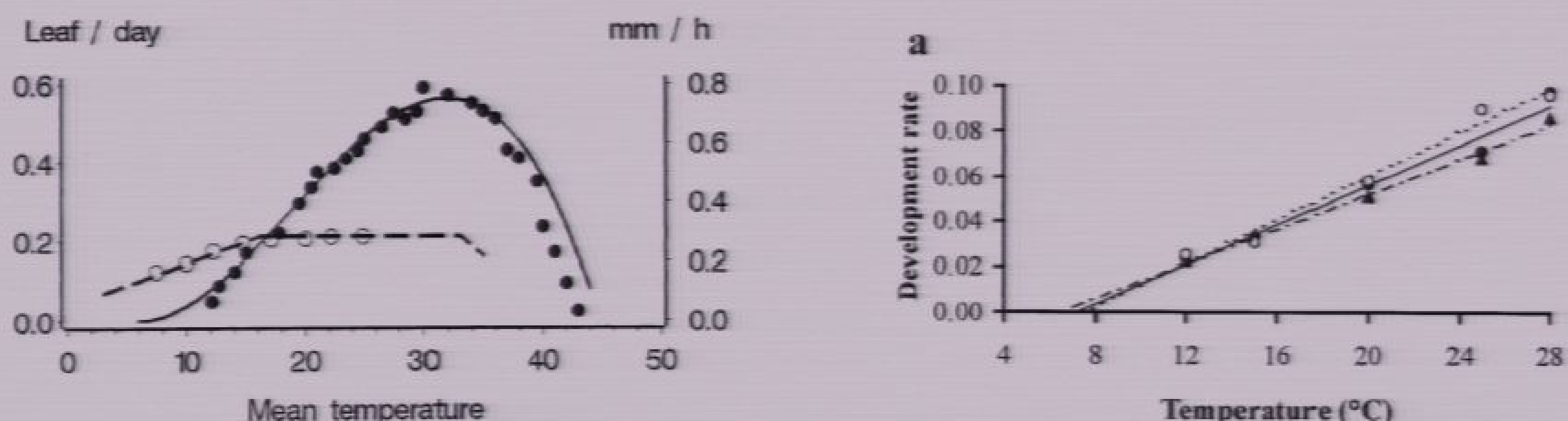

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Thermal equilibrium in stationary spacetimes: criteria

Thermal time is defined as a vector field on phase space, while proper time is defined on spacetime. The bridge between the two is through local observables A_x. Example:

$$n_x = \text{density of a gas about the spacetime time point x.}$$

Then the two criteria expressing the relationship between thermal and proper time at equilibrium are

1. $X_{-\ln\rho} A_x = \mathcal{L}_{\xi^\rho} A_x$ for some timelike Killing ξ^ρ
2. $\xi_\rho = \beta \frac{d}{ds}$ along stationary worldlines.

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The Tolman law

The Tolman law follows immediately. From

$$\xi_\rho = \beta \frac{d}{ds},$$

we have that

$$T \propto \|\xi_\rho\|^{-1},$$

and in stationary coordinates w.r.t. ξ_ρ

$$\|\xi_\rho\| = \|\partial_t\| = \sqrt{g_{00}}.$$

This derivation makes use of no dynamical or kinematical input (except that spacetime is stationary).

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Thermal time: example

As an limit example, consider a non-relativistic equilibrium state:

$$\rho = Z^{-1} e^{-\beta H}.$$

Then the thermal time flow matches the mechanical time flow, up to a constant:

$$X_{-\ln \rho} = \beta X_H.$$

- This identity characterizes thermal equilibrium in this setting.
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