

Title: Gravity and a Geometrization of Turbulence: An Intriguing Correspondence: Part 3

Date: Nov 12, 2010 10:00 AM

URL: <http://pirsa.org/10110070>

Abstract: The dynamics of fluids is a long standing challenge that remained as an unsolved problem for centuries. Understanding its main features, chaos and turbulence, is likely to provide an understanding of the principles and non-linear dynamics of a large class of systems far from equilibrium. We consider a conceptually new viewpoint to study these features using black hole dynamics. Since the gravitational field is characterized by a curved geometry, the gravity variables provide a geometrical framework for studying the dynamics of fluids: A geometrization of turbulence. We present new experimental predictions for relativistic and non-relativistic turbulent flows and for heavy ion collisions.

lecture 1 - $\langle (\delta_{ij}(F))^\alpha \rangle \sim t^{-3n}$

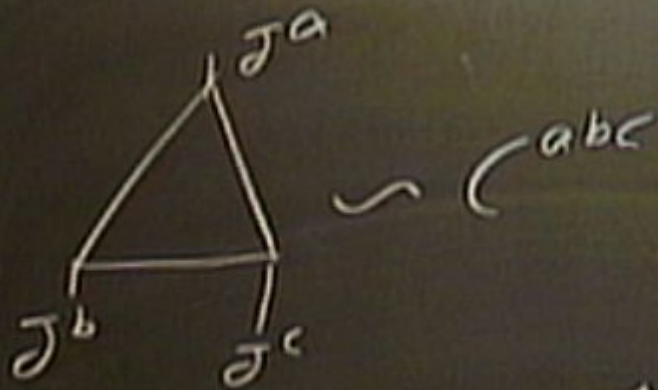
lecture 1 - $\langle (\delta\phi(F))^n \rangle \sim T^{\frac{3n}{2}}$

lecture 2 - CFT Hydro.

lecture 1 - $\langle (\delta\mathcal{O}(F))^n \rangle \sim \mathcal{Z}_n$

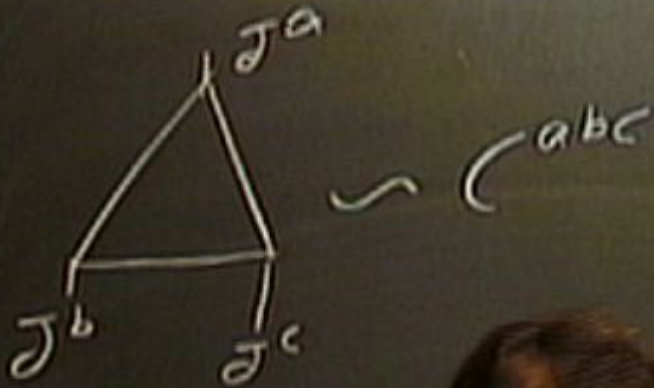
lecture 2 - CFT Hydro

$$\partial_\mu T^{\mu\nu} = 0 \quad \partial_\mu J^{\mu a} = 0$$



$$J_\mu J^{\mu a} = \epsilon^{abc} F_{\mu\nu}^b \hat{F}^{\mu\nu c}$$

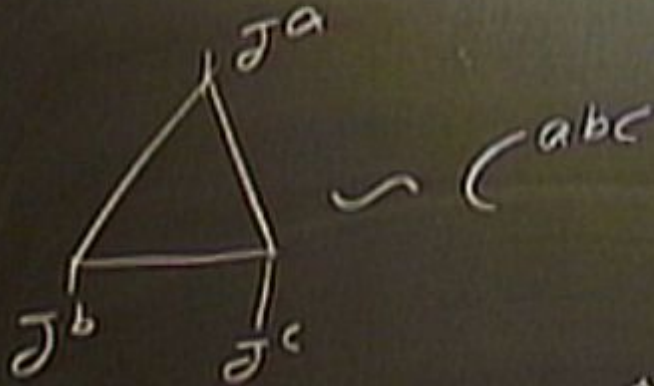
(a-11)



$$J_\mu J^{\mu a} = \epsilon^{abc} J^{\mu b c}$$

$$J^{\mu a} = \gamma^{\mu \nu} \dots + \int^a \omega^{\mu}$$

CAUTION
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 www.pearson.com

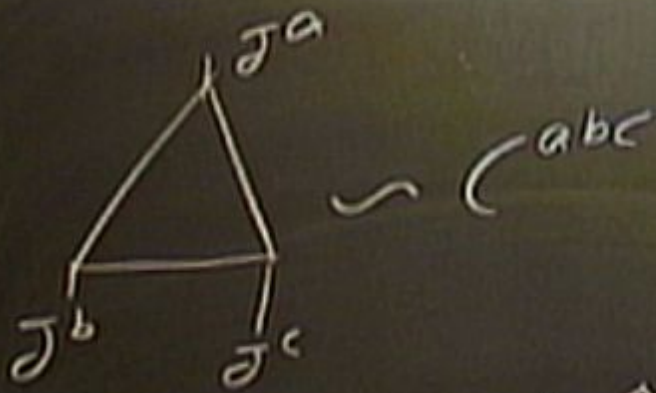


$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta}$$

$$J_\mu J^{\mu a} = \epsilon^{abc} F_{\mu\nu} \hat{F}^{\mu\nu c}$$

$$J^{\mu a} = \eta^a U^\mu + \dots + \mathcal{J}^a \omega^\mu$$

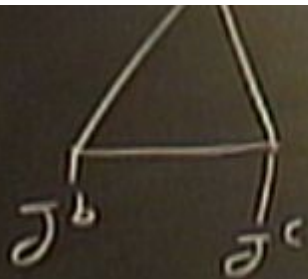
(a)



$$\omega^\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} U_\nu \omega_\alpha U_\beta$$

$$J_\mu J^\mu = C^{abc} F_{\mu\nu} \hat{F}^{\mu\nu}$$

$$J^{\mu a} = \eta^a U^\mu + \dots$$



$\sim C^{abc}$

$\omega = \partial C$

$$Z^a = C^{abc} \mu^b \mu^c$$

$$J_\mu J^\mu = C^{abc} F_{\mu\nu} \hat{F}^{\mu\nu c}$$

$$- \frac{2}{3} n^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + \rho}$$

$$J^{\mu a} = n^a U^\mu + \dots + Z^a \omega^\mu$$

(11)

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$\sim C^{abc}$

$$\mathcal{Z}^a = C^{abc} \mu^b \mu^c$$

$$\partial_\mu \mathcal{Z}^a = C^{abc} F_{\mu\nu}^b \hat{F}^{\mu\nu c}$$

$$- \frac{2}{3} n^a C^{bcd} \frac{\mu^b \mu^c \mu^d}{\epsilon + \rho}$$

$$\mathcal{T}^{\mu a} = n^a u^\mu + \dots + \underline{\mathcal{Z}^a} \omega^\mu$$

μ^a - chemical potentials

potentials

lecture 3 - Singularities

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potentials

Lecture 3 - Singularities

NS eq. : $\partial_t \tau^i + \tau^j \partial_j \tau^i = \partial^i p +$

τ^j

potentials

Lecture 3 - Singularities

$$\text{NS } \epsilon \quad \partial_t \tau^i + \tau^j \partial_j \tau^i = -\partial^i \rho + \dots + \dots$$

Lecture 3 - Singularities

$$NS \text{ eq. : } \begin{cases} \partial_k \tau^i + \tau^j \partial_j \tau^i = -\partial^i \rho + \\ \partial_i \tau^i = 0 \end{cases} \quad + \nu \partial_j \partial_j \tau^i$$

Lecture 3 - Singularities

$$NS \text{ eq. : } \begin{cases} \partial_t \tau^i + \tau^j \partial_j \tau^i = -\partial^i \rho + \\ \partial_j \tau^i = 0 \end{cases} \quad + \nu \partial_j \partial_j \tau^i$$

Lecture 3 - Singularities

$$\text{NS eq. : } \begin{cases} \partial_t \psi^i + \psi^j \partial_j \psi^i = -\partial^i \rho + \\ \partial_j \psi^i = 0 \end{cases} + \psi^j \partial_j \psi^i$$

$$\psi^i(\pm 0)$$

Lecture 3 - Singularities

$$\text{NS eq. : } \begin{cases} \partial_t \psi^i + \psi^j \partial_j \psi^i = -\partial^i \rho + \\ \partial_j \psi^i = 0 \end{cases} + \psi^j \partial_j \psi^i$$

$\psi^i(\pm 0)$ "bounded"

$$NS \text{ eq. : } \begin{cases} \Delta \psi + \psi \Delta \psi = -\rho + \\ \Delta \psi = 0 \end{cases} + \psi \Delta \psi$$

$\psi(\pm\infty)$ "bounded"

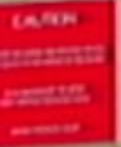
Line: $|\psi| \rightarrow \infty$

Lecture 3 - Singularities

$$\text{NS eq. : } \begin{cases} \partial_k \psi^i + \psi^j \partial_j \psi^i = -\partial^i \rho + \\ \partial_i \psi^i = 0 \end{cases} + \psi^j \partial_j \psi^i$$

$\psi^i(\pm 0)$ "bounded"

Finite time: $|\partial \psi| \rightarrow \infty$



Cosmic Censorship

Cosmic Censatship
Cauchy data (M_3, \mathcal{G}, K)
4d

Cosmic Censorship
Cauchy data (M_3, ∂, K) \rightarrow Naked Sing
4d



potentials

Lecture 3 - Singularities

$$\begin{aligned}
 \text{N.S. eq. : } & \left\{ \begin{aligned} \Delta \psi + \psi \Delta \psi &= -\psi \rho + \\ & + \psi \Delta \psi \end{aligned} \right. \\
 & \Delta \psi = 0
 \end{aligned}$$

"boundary"

$$\text{Line: } |\psi| \rightarrow \infty$$

Lecture 3 - Singularities

NS eq. :
$$\begin{cases} \Delta \psi + \psi \Delta \psi = -\psi \rho + \\ \Delta \psi = 0 \end{cases} + \psi \Delta \psi$$

$\psi(\pm 0)$ "bounded" $\psi \ll \psi_s \ll \psi$

Finite time: $|\psi| \rightarrow \infty$

Pentose inequality (1973)

Pentose inequality (1973)

- Initial Condition M_0, A_0

Asy
Fl
Pentose inequality (1973)
Initial Condition (M_0, A_0)
late times

3+1 Pentose inequality (1973)
Asym Flat - Initial Condition (M_0, A_0)
late - Kerr Solution

3+1 Pentose inequality (1973)
Asym Flat - Initial Condition (M_0, A_0)

late times - Kerr Solution

$$M \geq \left(\frac{A}{16\pi}\right)^{1/2} \quad G_N = 1 = c$$

3+1 Pentose inequality (1973)
 Asym - Im Condition (M_0, A_0)
 Flat - late - Kott Section

$$M \geq \left(\frac{A}{4\pi}\right)^{1/2} \geq$$

3+1 Pentose inequality (1973)

Asymptotic Condition (M_0, A_0)

Lines - Kottler Section

$$M \geq \left(\frac{A}{16\pi}\right)^{1/2} \geq \left(\frac{A_0}{16\pi}\right)^{1/2}$$

371 Pentose inequality (1973)

Asymptotic

Initial condition (M_0, A_0)

late times - Kottler Solution

$$M_0 \geq M \geq \left(\frac{A}{16\pi}\right)^{1/2} \geq \left(\frac{A_0}{16\pi}\right)^{1/2}$$

371
Asym
Pentose inequality (1973)
Initial condition (M_0, A_0)

Keenes - Kott Section

$$M_0 \geq M \geq \left(\frac{A}{4\pi}\right)^{1/2} \geq \left(\frac{A_0}{4\pi}\right)^{1/2}$$

3+1 Pentose inequality (1973)
Asym Flat - Initial Condition (M_0, A_0)

late times - Kerr Solution

$$M_0 \geq M \geq \left(\frac{A}{16\pi}\right)^{1/2} \geq \left(\frac{A_0}{16\pi}\right)^{1/2}$$

potentials

Fluid / Gravity Correspondence

[The rest of the chalkboard is heavily obscured by large, dense, and mostly illegible white chalk scribbles.]

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IF YOU HAVE A PROBLEM
CALL THE HELP DESK

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potentials

Fluid / Gravity Correspondence

$d+1$ -dim
 (x^μ, t)

$$R_{mn} + d g_{mn} = 0$$

potentials

$$(d+1)\text{-dim } R_{mn} + d^2 g_{mn} = 0$$

(x^μ, t)

$$dS^2 = -2u_\mu dx^\mu dt + \frac{1}{b} \frac{d}{dt} u_\mu u_\nu dx^\mu dx^\nu + t^2 \sum_{\mu, \nu} dx^\mu dx^\nu$$

$$U^{\mu} = (\gamma, \gamma \beta^i)$$

$$b^{-1} = \frac{4\pi T}{\alpha}$$

$$(d+1)\text{-dim } R_{mn} + d g_{mn} = 0$$

(x^μ, t)

$$dS^2 = -2U_\mu dx^\mu dt + \frac{1}{b} \frac{d}{dt} U_\mu U_\nu dx^\mu dx^\nu + t^2 \sum_{\mu, \nu} dx^\mu dx^\nu$$

Solution: U^μ const
 T const

$$U^{\mu} = (\gamma, \gamma \beta^i)$$

$$b^{-1} = \frac{4\pi T}{\alpha}$$

$$U^{\mu}(x, \epsilon)$$

$$T(x, \epsilon)$$

$$U^\mu = (\gamma, \gamma \beta^i)$$

$$b^{-1} = \frac{4\pi T}{d}$$

$$U^\mu(t, \epsilon)$$

$$T(t, \epsilon)$$

$$ds^2 = ds_0^2 + ds_1^2 + \dots$$

provided:

$$U^\mu = (\gamma, \gamma \beta^i)$$

$$b^{-1} = \frac{4\pi T}{d}$$

$$\begin{pmatrix} t, \epsilon \\ t, \epsilon \end{pmatrix}$$

$$ds^2 = ds_0^2 + ds_1^2 + \dots$$

provided:

$$G_m^{\mu=0} \rightarrow \partial_\mu T^{\mu\nu} = 0$$

$$b^{-1} = \frac{4\pi T}{d}$$

$U^\mu(x)$

provided:

$$G_M^{t=0} \rightarrow \partial_\mu T^{\mu\nu} = 0$$

$$T_{\mu\nu} = \frac{1}{16\pi} d \left(\partial_\mu \partial_\nu U_\alpha U_\alpha - \frac{1}{2} \partial_\mu U_\alpha \partial_\nu U_\alpha \right) - \frac{1}{8\pi} b^{d-1} \sigma_{\mu\nu} +$$

late time solution

$$M \geq \left(\frac{A}{16\pi} \right)^{1/2} \geq \left(\frac{A_0}{16\pi} \right)^{1/2}$$

$$b^{-1} = \frac{4\pi}{d}$$

$$U^\mu(x, \epsilon)$$

provided:

$$G_m^{t=0} \rightarrow d_\mu T^{\mu\nu} = 0$$

$$T_{\mu\nu} = \frac{1}{16\pi b^d} (\dot{\Sigma}_{\mu\nu} + d U_\mu U_\nu) - \frac{1}{8\pi b^{d-1}} \sigma_{\mu\nu} +$$

- 25

later - Kerr Solution

$$\geq M \geq \left(\frac{A}{16\pi}\right)^{1/2} \geq \left(\frac{A_0}{16\pi}\right)^{1/2}$$

$$T_{\mu\nu} = \frac{1}{16\pi b^d} (2\mu\nu^T d\mu_{\mu\nu}) \quad \left(\frac{1}{8\pi b^d} \right)$$

-25

Horizon at $r = b^{-1}$



$$g_{\mu\nu} = \frac{1}{16\pi b^d} (\xi_{\mu\nu} + u_\mu u_\nu) \quad \left(\frac{1}{8\pi b^d} \right) - 2\xi$$

Horizon at $r = b^{-1}$

$$ds^2 = b^{-2} (\xi_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

$\underbrace{\hspace{10em}}_{\rho_{\mu\nu}} \quad \rho_{\mu\nu} u^\nu = 0$

$$T_{\mu\nu} = \frac{1}{16\pi b^d} (\xi_{\mu\nu} + u_\mu u_\nu) \quad \left(\frac{1}{8\pi b^d} \right) - 2\xi$$

Horizon at $r = b^{-1}$ $F = t - b^{-1}$

$$ds^2|_H = b^{-2} (\xi_{\mu\nu} + u_\mu u_\nu) dx^\mu dx^\nu$$

$$g^m = g^{mn} \partial_n F \quad \rho_{\mu\nu} \quad \rho_{\mu\nu} u^\nu = 0$$

$$g^\mu = u^\mu$$

potentials



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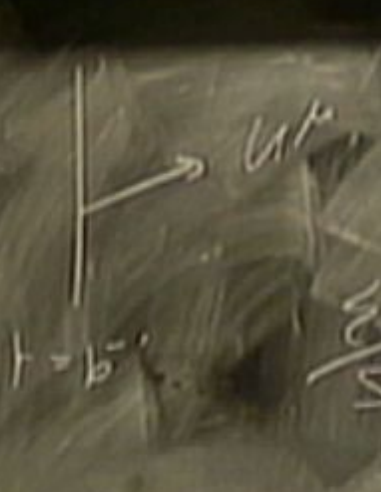
potentials

$\int_{\Sigma} \omega = \int_{\Sigma} \omega^H$

$\int_{\Sigma} \omega = \int_{\Sigma} \frac{1}{4} b^{d-1}$

$\int_{\Sigma} \omega = \frac{1}{16\pi} b^{d-1}$

$\frac{1}{4} b^{d-1} = \frac{1}{4\pi}$

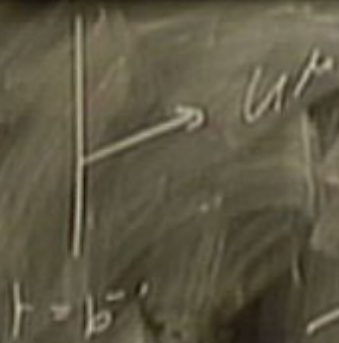


CAUTION

CAUTION

potentials

$$S = \int \mathcal{L} dt$$
$$S = \int \frac{1}{4} b^{d-1}$$
$$\frac{2}{S} = \frac{1}{16\pi b^{d-1}}$$
$$\frac{1}{4} b^{d-1} = \frac{1}{4\pi}$$



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OR THE CHALK

CAUTION
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OR THE CHALK

Handwritten notes on the top chalkboard panel, including the number 25 with a minus sign and some illegible scribbles.

A large, dense area of the chalkboard is completely obscured by thick, overlapping layers of white chalk, rendering any original content illegible.



$$E = \int T_{00}$$

$$S = \int S^0$$

$\frac{1}{2} \frac{d}{dt} \left(\frac{1}{2} \frac{d}{dt} \right)$

E
 T_{00}
 S^0

$$U_{\mu} = (1, 0, \dots, 0)$$

$$E = \int T_{00}$$

$$S = \int S^0$$

$$U_\mu = (1, 0, \dots, 0)$$

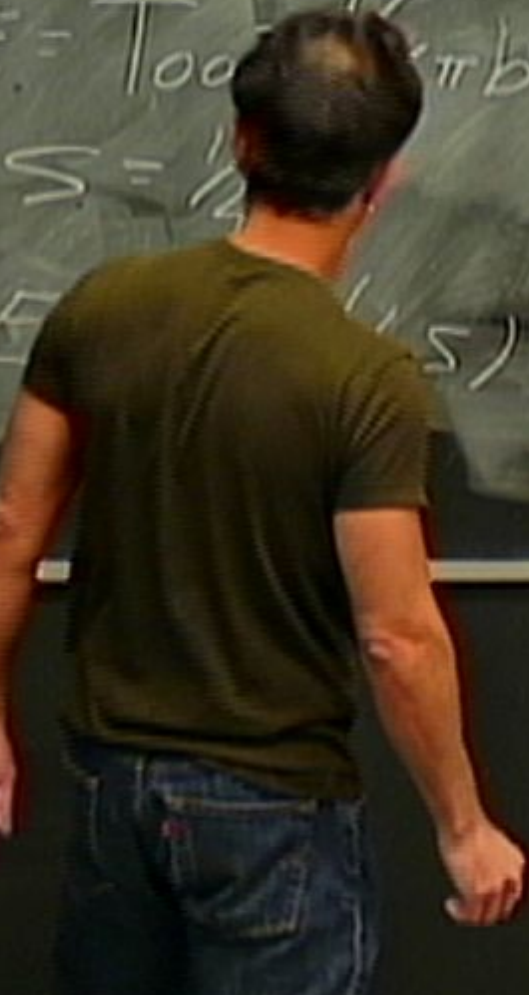
$$T_{00} = \frac{1}{16\pi b^d} (-1 + d)$$

$1/\mu$ $10/\mu$ $(1/\mu)$ $(1/\mu)$ $(1/\mu)$ $(1/\mu)$
 -25

$$E = \int T_{00}$$

$$U_\mu = (1, 0, \dots, 0)$$

$$S = \int S_0 \cdot E = T_{00} \cdot \pi b^d (-1+d)$$



$$E = \int T_{00}$$

$$U_\mu = (1, 0, \dots, 0)$$

$$E = T_{00} = \frac{1}{16\pi} b^d (-1 + d)$$

$$S = \int$$

$$S = \frac{1}{4} b^{d-1}$$

$$E = \frac{d-1}{16\pi} (4S)^{d/d-1}$$

$$E + p = TS$$

$$E = (d-1)p$$

$$E = \int T_{00}$$

$$U_\mu = (1, 0, \dots, 0)$$

$$E = T_{00} = \frac{1}{16\pi} b^d (-1 + d)$$

$$S = \int$$

$$S = \frac{1}{4} b^{d-1}$$

$$E = \frac{d-1}{16\pi} (4\pi)^{d/2-1}$$

$$\begin{aligned} E + p &= TS \\ E &= (d-1)p \end{aligned}$$

$$E = \int T_{00}$$

$$U_\mu = (1, 0, \dots, 0)$$

$$E = T_{00} = \frac{1}{16\pi} b^d (-1 + d)$$

$$S = \int S_0 \quad S = \frac{1}{4} b^{d-1}$$

$$E \Rightarrow \frac{d-1}{16\pi} (4\pi)^{d/2-1}$$

$$\begin{aligned} E + p &= TS \\ E &= (d-1)p \end{aligned}$$

$$E = \int T_{00}$$

$$u_\mu = (1, 0, \dots, 0)$$

$$S = \int S_0 \quad E = T_{00} = \frac{1}{16\pi} b^d (-1 + d)$$

$$S = \frac{1}{4} b^{d-1}$$

$$E \Rightarrow \frac{d-1}{16\pi} (4\pi)^{d/2-1}$$

$$\begin{aligned} E + p &= TS \\ E &= (d-1)p \end{aligned}$$

$$E = \int T_{00} \quad U_\mu = (1, 0, \dots, 0)$$

$$S = \int S^0 \quad E = T_{00} = \frac{1}{16\pi} b^d (-1 + d)$$

$$S = \frac{1}{4} b^{d-1}$$

$E \Rightarrow \frac{d-1}{16\pi} (4S)^{d/d-1}$	$\begin{aligned} \epsilon + p &= TS \\ \epsilon &= (d-1)p \end{aligned}$
--	--

potentials

Boosted - $E = \sqrt{6\pi b^d (dt^2 - 1)}$



potentials

Boosted - $E = \frac{1}{6\pi b^2} (\dot{t}^2 - 1)$
 $S = \frac{1}{4\pi b^2} \dot{t}$



potentials

Boosted -

$$E = \frac{1}{6\pi b^2} (\dot{t}^2 - 1)$$
$$S = \frac{1}{4\pi b^2} \dot{t}$$

potentials

Boosted -

$$\left\{ \begin{aligned} E &= \frac{1}{6\pi} b d (\beta^2 - 1) \\ S &= \frac{1}{40} d \beta \end{aligned} \right.$$

Equality for $\beta = 1$

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potentials

Viscous drag:

$$\Delta F = \frac{1}{2} \pi b d \left(\frac{d^2}{t^2} + 1 \right) \frac{1}{16 \pi \eta d^2}$$

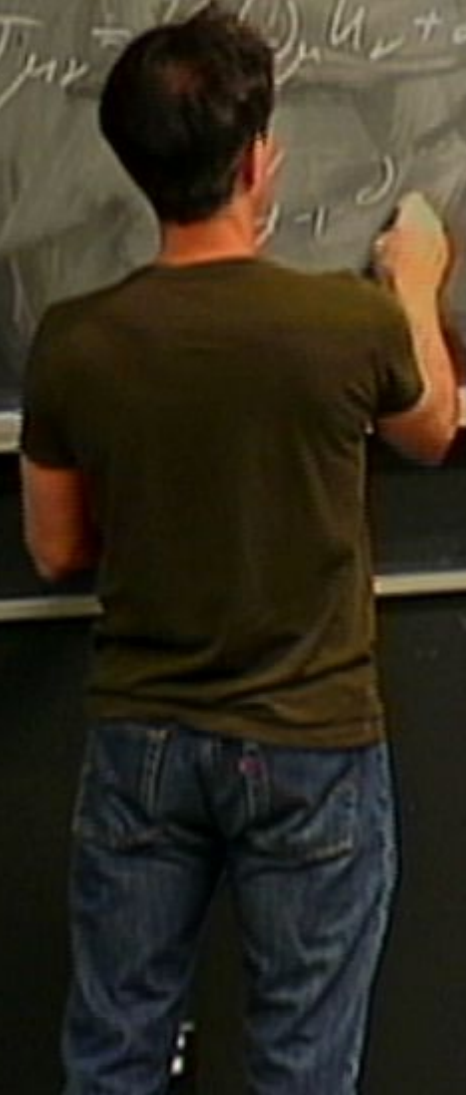
potentials

Viscous order:

$$\Delta \Phi = \frac{1}{16\pi} b^d \left(d^2 \Phi - \Phi \right) = \frac{1}{16\pi} b^{d-1} \sigma_{00}$$

potentials

$$\Delta E = \frac{1}{16\pi b^d} (\partial_0 - 1) \dots \frac{1}{16\pi b^d}$$
$$T_{\mu\nu} = \dots + \frac{1}{2} (\dots)$$



potentials

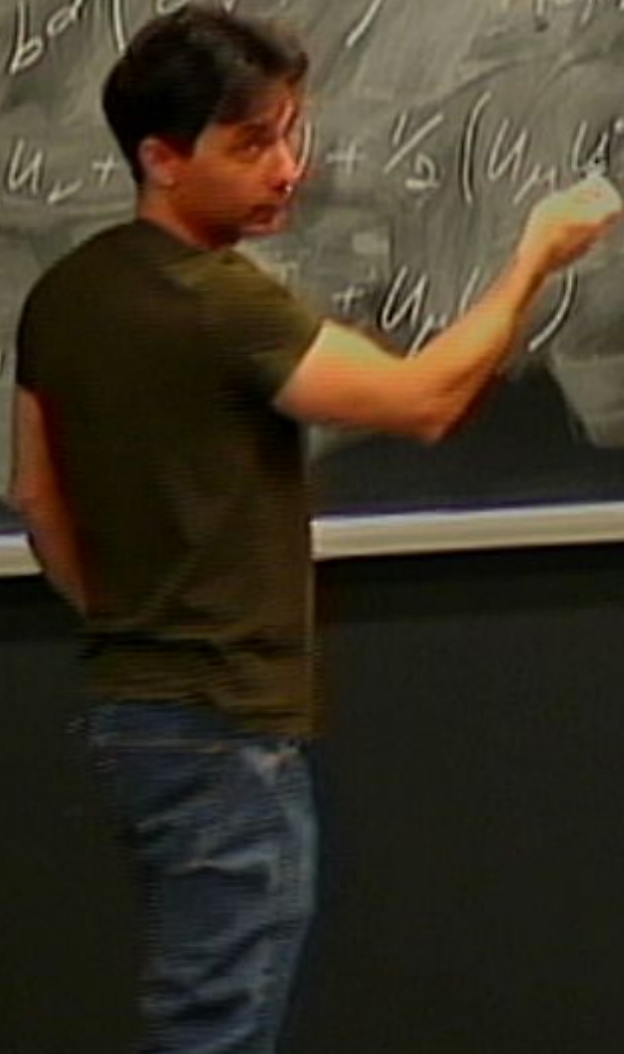
$$\mathcal{L} = \frac{1}{16\pi b^d} (\partial\phi)^2 + \frac{1}{16\pi b^d} \sigma_{\mu\nu} + \frac{1}{2} (\partial_\mu \psi_\nu + \partial_\nu \psi_\mu) + \frac{1}{2} (\psi_\mu \psi^\mu \psi_\nu + \psi_\nu \psi^\mu \psi_\mu) - \frac{1}{2} (\partial_\mu \psi^\mu) (\psi_\mu + \psi_\mu)$$

potentials

Viscous order:

$$\Delta \Phi = \frac{1}{16\pi b^d} (d^2 - 1) \frac{1}{16\pi b^{d-1}} \sigma_{00}^{-2\epsilon}$$

$$\sigma_{\mu\nu} = \frac{1}{2} (\partial_\mu u_\nu + \partial_\nu u_\mu) + \frac{1}{2} (u_{\mu\lambda} \partial_\lambda u_\nu + u_{\nu\lambda} \partial_\lambda u_\mu) - \frac{1}{d-1} (\partial_\mu u^\mu \partial_\nu u_\lambda + \partial_\nu u^\mu \partial_\mu u_\lambda)$$



potentials

Viscous order:

$$\Delta \Phi = \frac{1}{16\pi b^d} (d^2 - 1) \frac{1}{16\pi b^{d-1}} \sigma_{00}^{-2\epsilon}$$

$$\sigma_{\mu\nu} = \frac{1}{2} (\partial_\mu u_\nu + \partial_\nu u_\mu) + \frac{1}{2} (u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu) - \frac{1}{d-1} (\partial_\alpha u^\alpha) (\xi_{\mu\nu} + u_\mu u_\nu)$$

$$\int \frac{1}{16\pi b^{d-1}} (d^2 - 1) - 2\zeta_0$$

15

$$\left(\frac{1}{16\pi b^{d-1}} (d^2 - 1) - 2\zeta_{00} - \frac{d-1}{16\pi b^{d-1}} \right) \geq 0$$



potentials

$$T_{\mu\nu} = T_{\mu\nu}^{ideal} - 2\zeta\sigma_{\mu\nu}$$

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OR MATERIALS
IN THIS AREA

potentials

$$E = \frac{1}{16\pi b d} (d^2 - 1) - 22000$$



potentials

$$F = \frac{1}{16\pi b d} (d\delta^2 - 1) - 2\sigma_{00}$$

$$= \frac{1}{4b d} \delta^2 - \frac{1}{2} \sigma_{00}$$

$$+ \frac{1}{2} \sigma_{00}^2$$

$$\frac{\partial \mathcal{L}}{\partial \delta} = \frac{1}{2} \sigma_{00}^2 - \frac{1}{2} \sigma_{00}$$
$$= \sigma_{00} \left(\frac{\sigma_{00}}{2} - \frac{1}{2} \right)$$
$$= \rho_{00} \epsilon^2$$

potentials

$$E = \frac{1}{16\pi b^2 a} (d\tau^2 - 1) - 2\tau\sigma_{00}$$

$$S = \frac{1}{4b^2 a} \int d\tau \int d\sigma \left(\frac{1}{2} \dot{\tau}^2 - \frac{1}{2} \dot{\sigma}^2 \right)$$

$$L_{eff} = \left(1 + \frac{1}{2} \sigma'^2 \right) \sigma'$$

$\partial_\tau \tau \sim \frac{1}{c^2}$
 $\partial_\tau \sigma \sim \frac{1}{c}$
 $\partial_\sigma \tau \sim \frac{1}{c}$
 $\partial_\sigma \sigma \sim \frac{1}{c^2}$

potentials

$$E = \frac{1}{16\pi b d} (d\delta^2 - 1) - 2\gamma\sigma_{00}$$

$$S = \frac{1}{4b d} \delta - \frac{d\gamma}{2} \epsilon^2 - \frac{1}{c^2}$$

$$U^{\mu\nu} = (1 + \frac{1}{2}\sigma^2, \sigma^i)$$

$$T = T_0(1 + \rho \dots)$$

$\frac{d\gamma}{2} \epsilon^2 - \frac{1}{c^2}$
 $\sigma \sim \epsilon$
 $\rho \sim \epsilon^2$

CAUTION

CAUTION

potentials

$$E = \frac{1}{16\pi b d} (d\delta^2 - 4) - 2\gamma\sigma_{00}$$

$$S = \frac{1}{4b d} \delta \quad \frac{\partial \gamma}{\partial x} \epsilon^2 \sim \frac{1}{c^2}$$

$$\sigma \sim \epsilon$$

$$T = T_0 (1 + \rho \dots) \quad \rho \sim \epsilon^2$$



potentials

$$\left(\frac{1}{16\pi b^{d-1}} (d^2 - 1) - 2\zeta\sigma_0 \right. \\ \left. - \frac{d-1}{16\pi b} \right) \geq 0$$

$$\left(\frac{2\zeta\sigma_0 + \dots}{\epsilon^3} \right)$$

$$I_{\mu\nu} = \frac{1}{16\pi b^d} \left(\sum_{\mu\nu} + d \delta_{\mu\nu} \right) \left(\frac{1}{8\pi b^{d-1}} \right) \mu\nu +$$

- 25

$$b = b_0(1 - \rho)$$

$$T_{\mu\nu} = \frac{1}{16\pi b^d} (\sum_{\mu\nu} + d U_{\mu\nu}) - \frac{1}{8\pi b^{d-1}} U_{\mu\nu} +$$

- 25

$$b = b_0(1 - \rho)$$

$$E = \frac{1}{16\pi b^d} (\sum_{\mu\nu} + d U_{\mu\nu})$$

$$T_{\mu\nu} = \frac{1}{16\pi b^d} (\Sigma_{\mu\nu} + d U_\mu U_\nu) - \frac{1}{8\pi b^{d-1}} \Theta_{\mu\nu} +$$

- 25

$$b = b_0(1 - \rho)$$

$$= \frac{1}{16\pi b_0^d} (1 + d\rho) (d + d\frac{v^2}{2} - 1)$$

$$\frac{1}{16\pi b_0^d} ($$

$$h_{\mu\nu} = \frac{1}{16\pi b^d} (\xi_{\mu\nu} + d h_{\mu\nu}) - \frac{1}{8\pi b^{d-1}} \xi_{\mu\nu} +$$

- 2\xi

$$b = b_0(1 - \rho)$$

$$E = \frac{1}{16\pi b_0^d} (1 + d\rho) \left(d + d \frac{v^2}{2} - 1 \right)$$

$$+ \frac{1}{16\pi b_0^d} \left(1 + \frac{d^2}{d-1} \rho + \frac{d}{2(d-1)} v^2 \right)$$

$$T_{\mu\nu} = \frac{1}{16\pi b^d} \left(\sum_{\mu\nu} + d \eta_{\mu\nu} \right) - \frac{1}{8\pi b^{d-1}} \Theta_{\mu\nu} +$$

- 2\epsilon

$$b = b_0(1 - \rho)$$

$$E = \frac{1}{16\pi b_0^d} (1 + d\rho) \left(d + d\frac{v^2}{2} - 1 \right)$$

$$\approx d + \frac{d^2}{d-1} \rho + \frac{d}{2(d-1)} v^2$$

$$\mathcal{D}' = \sum_{i=1}^n \mu_i \mathcal{U}_i + \dots + \sum_{j=1}^m \mu_j \mathcal{U}_j \quad \mu^a - \text{chemical potentials}$$

$$E = \frac{1}{16\pi b^2 d} (\dot{\delta}^2 - 1) - 2\sigma_0 \sigma_0$$

$$\mathcal{D} = \frac{1}{4b^2 d} \delta$$

$$\frac{1}{4b^2 d} (1 + (\partial_t \delta)^2) \left(1 + \frac{1}{2} \sigma_0^2\right) \sigma_0 \sim \epsilon$$

$\frac{\partial \mathcal{D}}{\partial \delta} \epsilon^2 \sim \frac{1}{c^2}$
 $\frac{\partial \mathcal{D}}{\partial \sigma_0} \epsilon$
 $\rho \sim \epsilon^2$

$$J' = \sum \mu_i n_i + \dots + \sum \mu_j n_j \quad \mu^a - \text{chemical potentials}$$

$$E = \frac{1}{16\pi b^2 d} (d\delta^2 - 1) - 2\sigma_0 \sigma_{00}$$

$$S = \frac{1}{4b^2 d} \delta \quad \text{with } \delta \sim \epsilon^2 \sim \frac{1}{c^2}$$

$$S = \frac{1}{4b^2 d} (1 + (d-1)\rho) \left(1 + \frac{1}{2}\epsilon^2\right) \sigma \sim \epsilon$$

$$S' = N^a U^a + \dots + \sum \mu^a \psi^a$$

μ^a - chemical potentials

$$E = \frac{1}{16\pi b^d} (d\dot{\sigma}^2 - 1) - 2\dot{\sigma}^2 \sigma_{00}$$

$$S = \frac{1}{4b^d} \int dt \left(\dot{\sigma}^2 \epsilon^2 - \frac{1}{c^2} \right)$$

$$S = \frac{1}{4b_0^d} \left(1 + (d-1)\rho \right) \left(1 + \frac{1}{2} \sigma^2 \right) \sigma' \sim \epsilon$$

$\rho \sim \epsilon^2$

$$= \int \sigma^2 \geq 0$$