Title: Gravity and a Geometrization of Turbulence: An Intriguing Correspondence: Part 3

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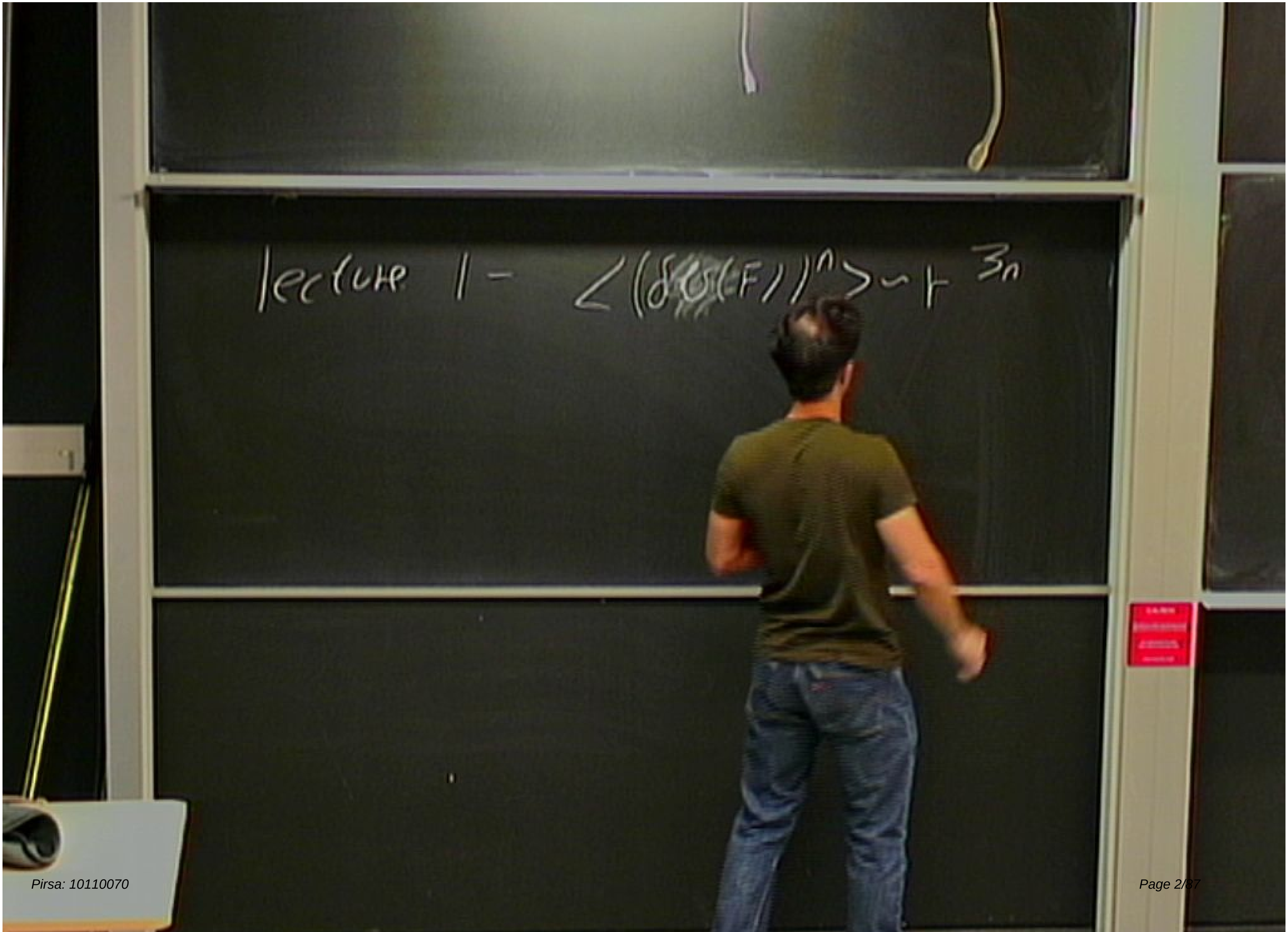
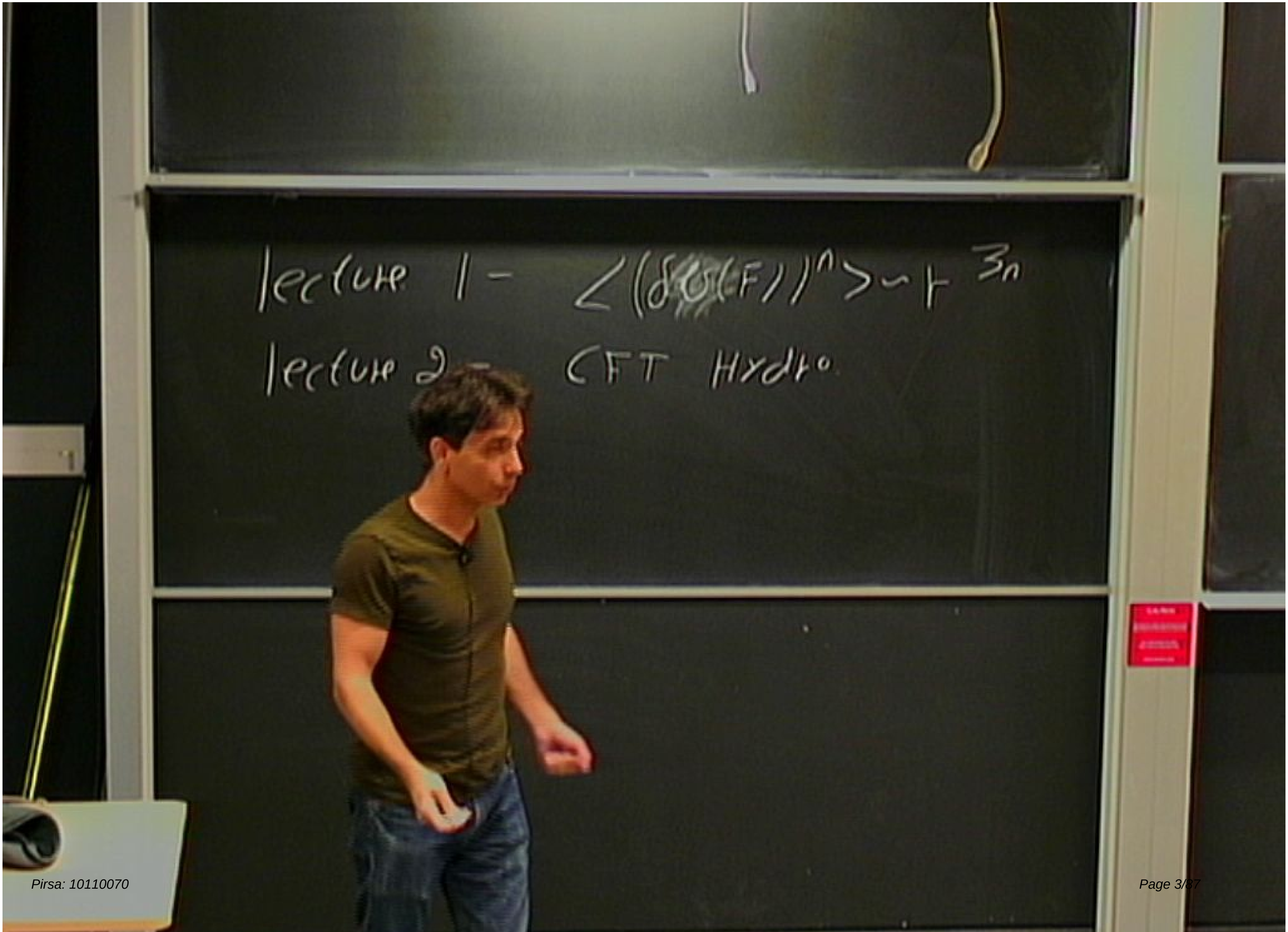
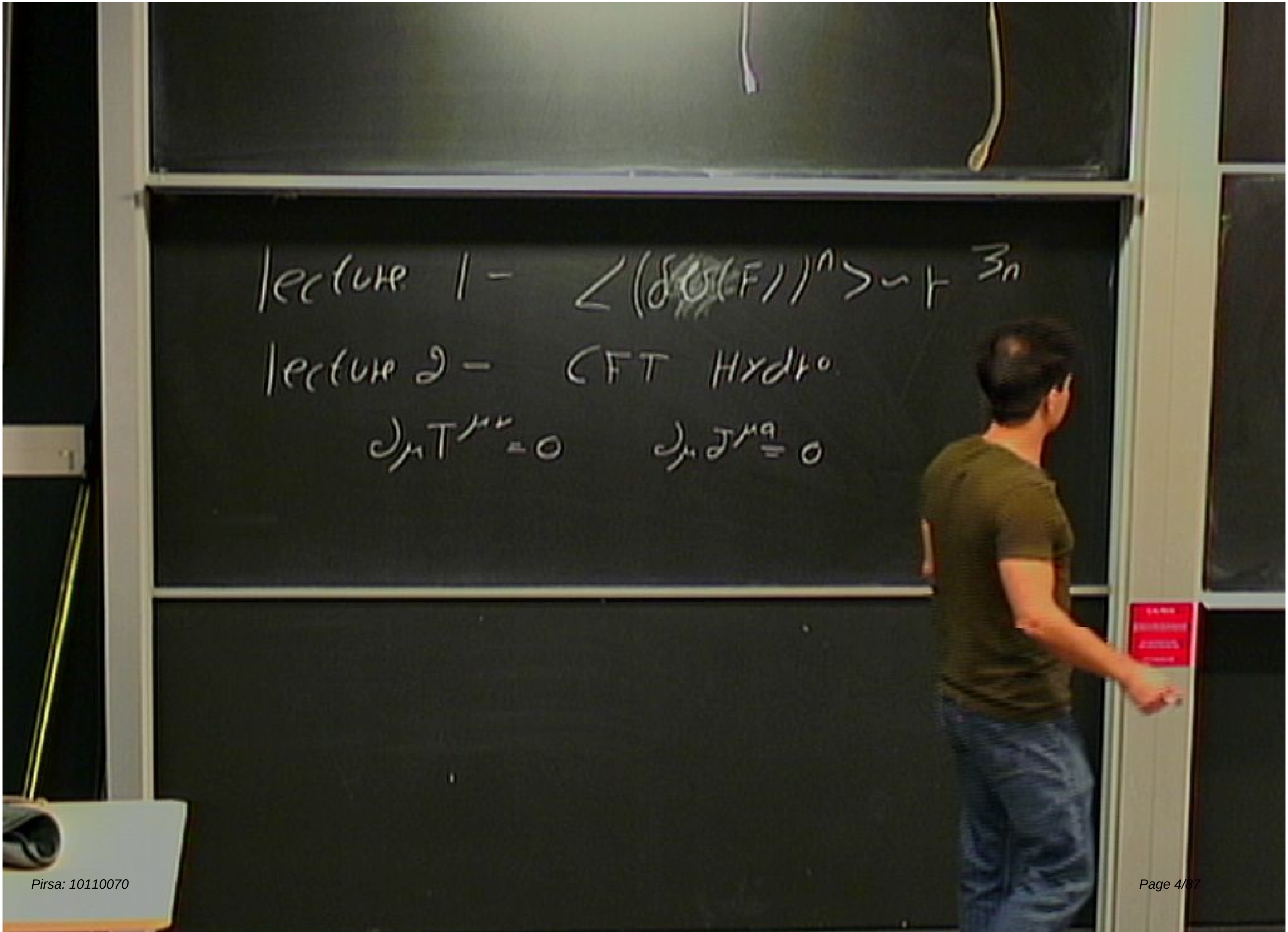
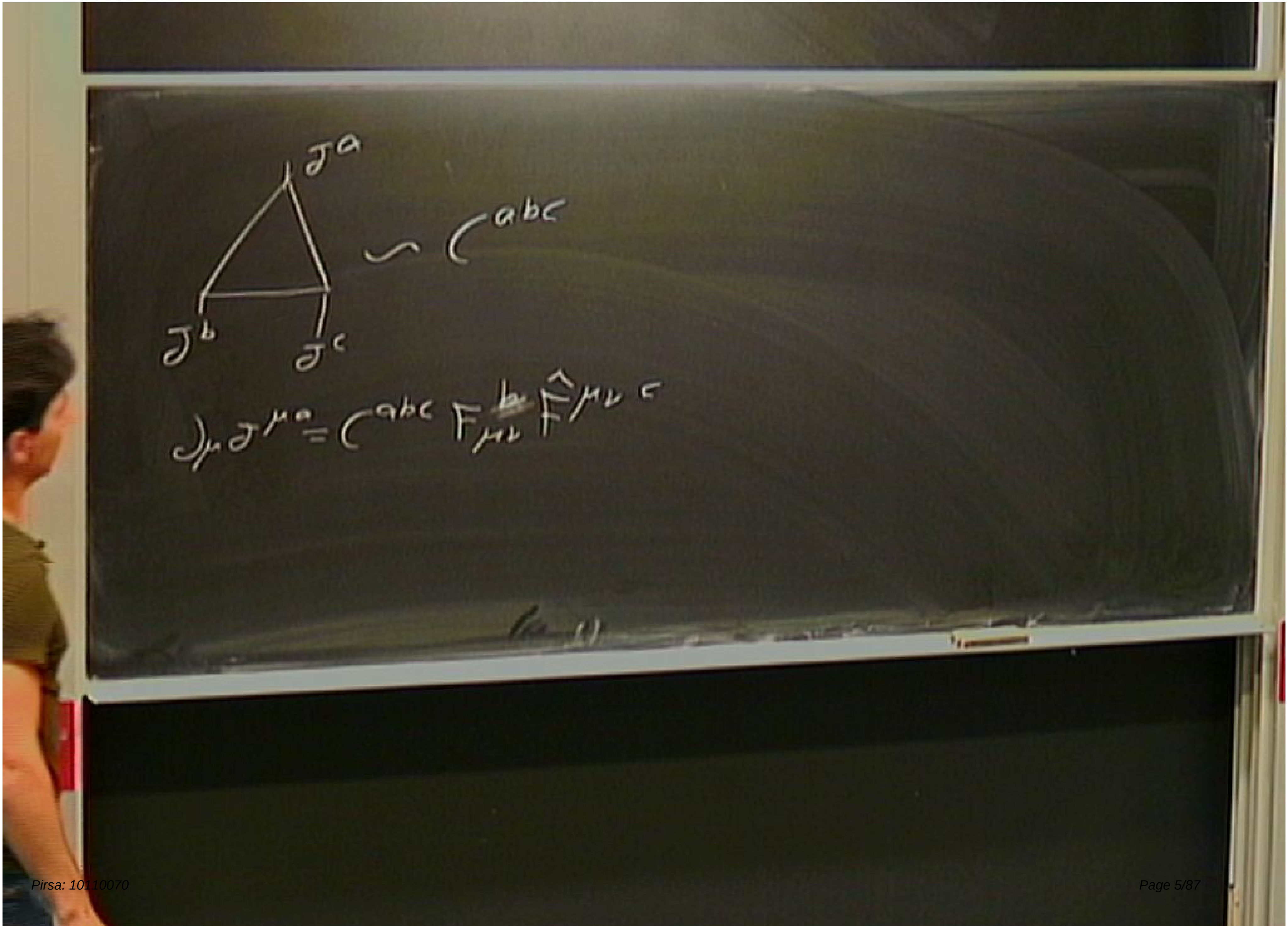
URL: http://pirsa.org/10110070

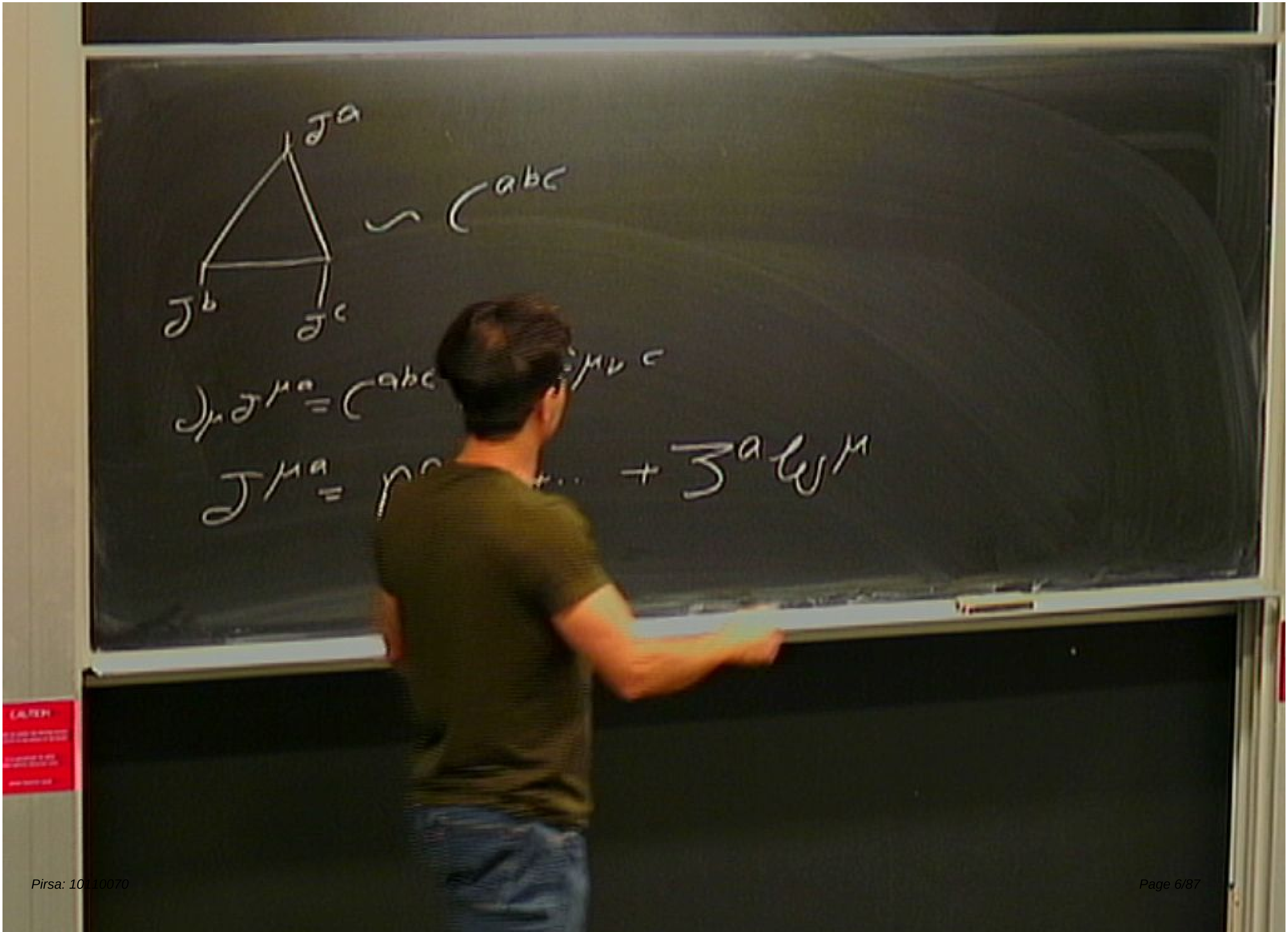
Abstract: The dynamics of fluids is a long standing challenge that remained as an unsolved problem for centuries. Understanding its main features, chaos and turbulence, is likely to provide an understanding of the principles and non-linear dynamics of a large class of systems far from equilibrium. We consider a conceptually new viewpoint to study these features using black hole dynamics. Since the gravitational field is characterized by a curved geometry, the gravity variables provide a geometrical framework for studying the dynamics of fluids: A geometrization of turbulence. We present new experimental predictions for relativsitic and non-relativistic turbulent flows and for heavy ion collisions.


lecture 1 -

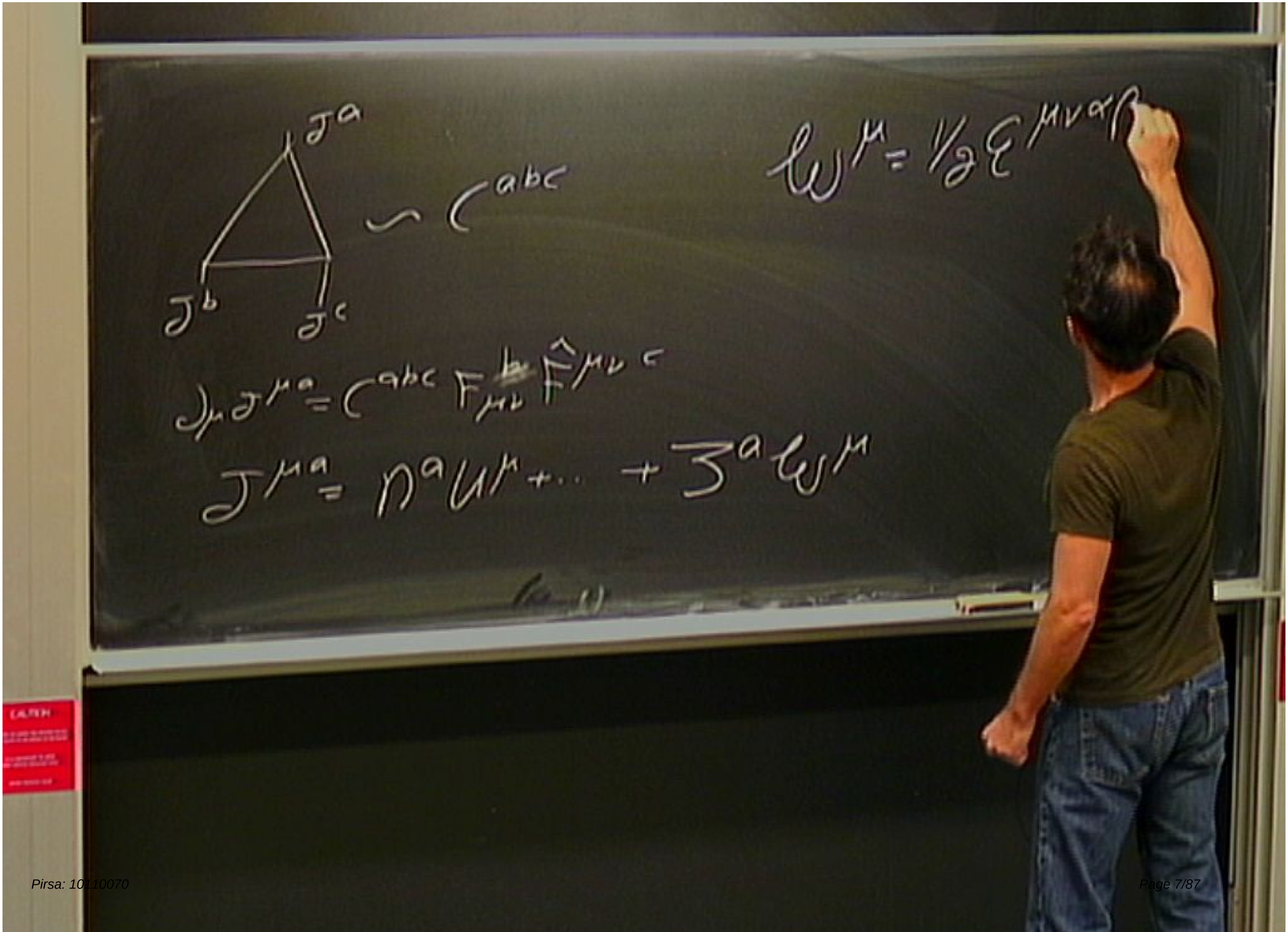

lecture 1 –
lecture 2 – CFT Hydro

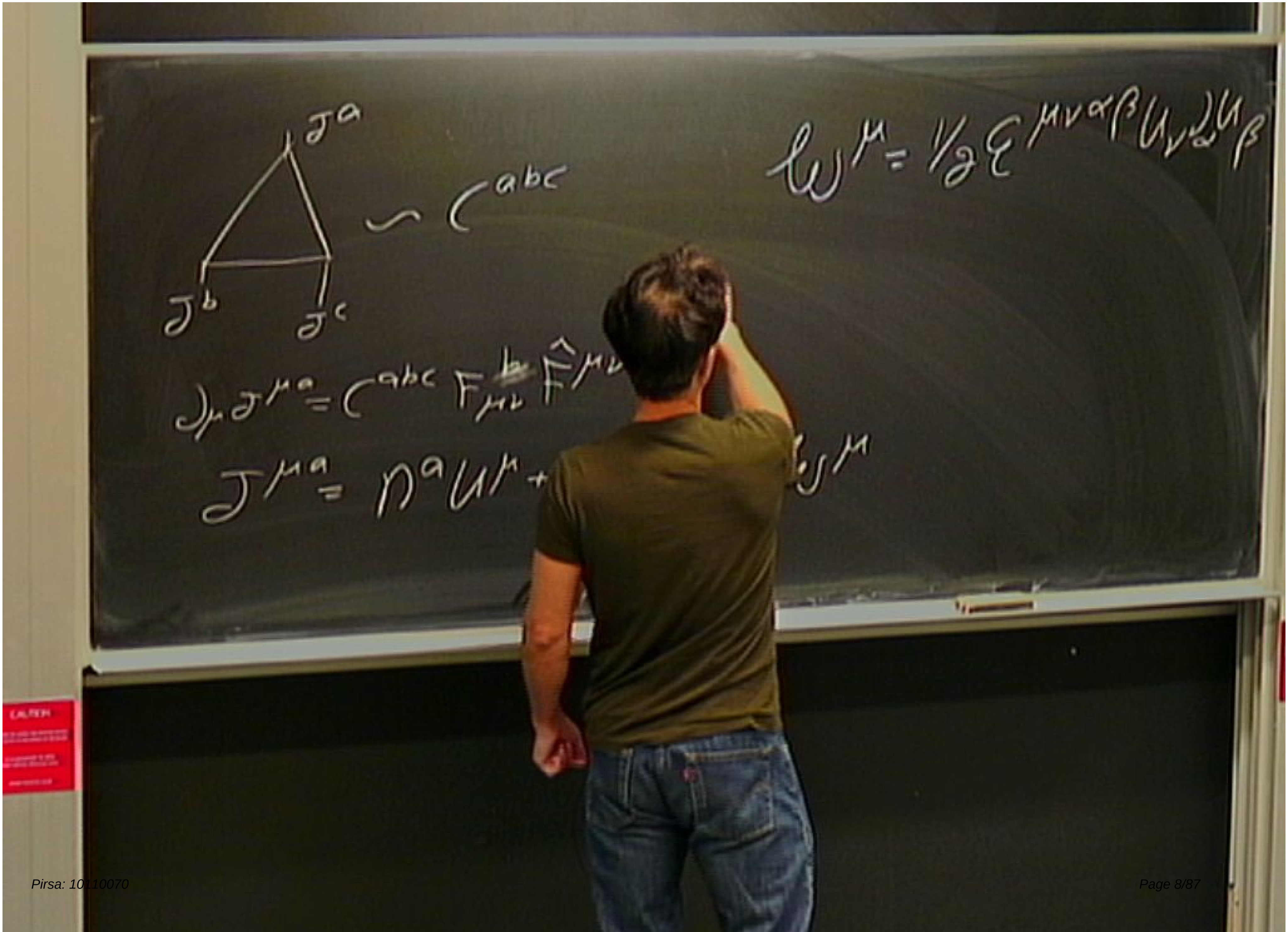

lecture 1 –
lecture 2 – CFT Hydro
$\partial_\mu T^{\mu\nu} = 0$ $\partial_\mu J^{\mu a} = 0$

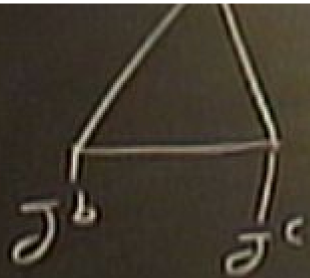

J^a
J^b
J^c
$\sim C^{abc}$
$\partial_\mu J^{\mu a} = C^{abc} F_{\mu\nu} \hat{F}^{\mu\nu c}$



J^a
~ C^abc
J^b
J^c


J^a
~ C^abc
J^b
J^c
ω^μ = 1/2 ε^μναβ
∂_μ J^μa = C^abc F_μν^b F̃^μν c
J^μa = n^a u^μ + ... + ξ^a ω^μ


J^a
J^b
J^c
~ C^abc
ω^μ = ½ ε^μναβ u_ν ∂_α u_β
∂_μ J^μa = C^abc F^b_μν F̃^μν
J^μa = n^a u^μ +

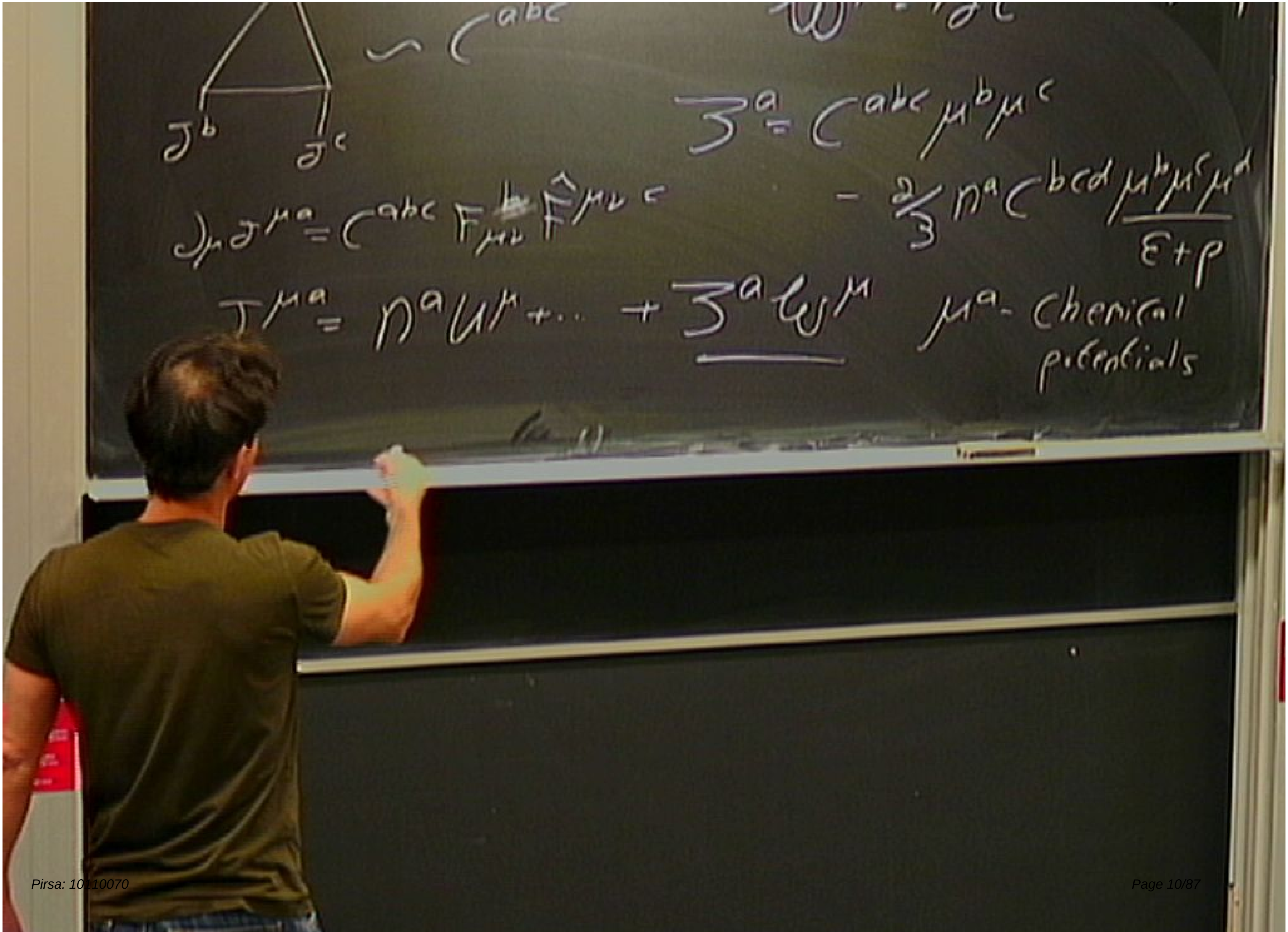


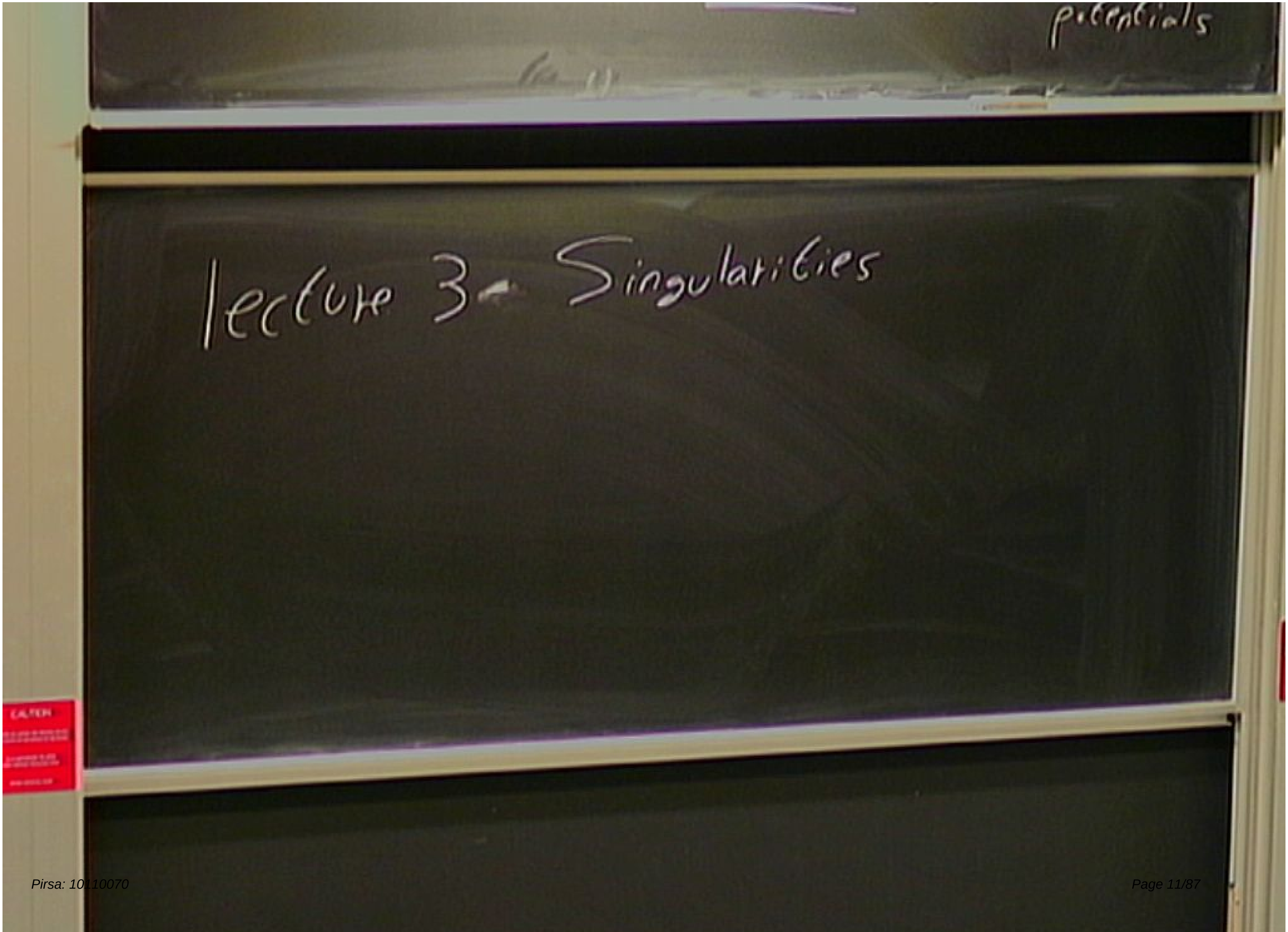
$\sim C^{abc}$

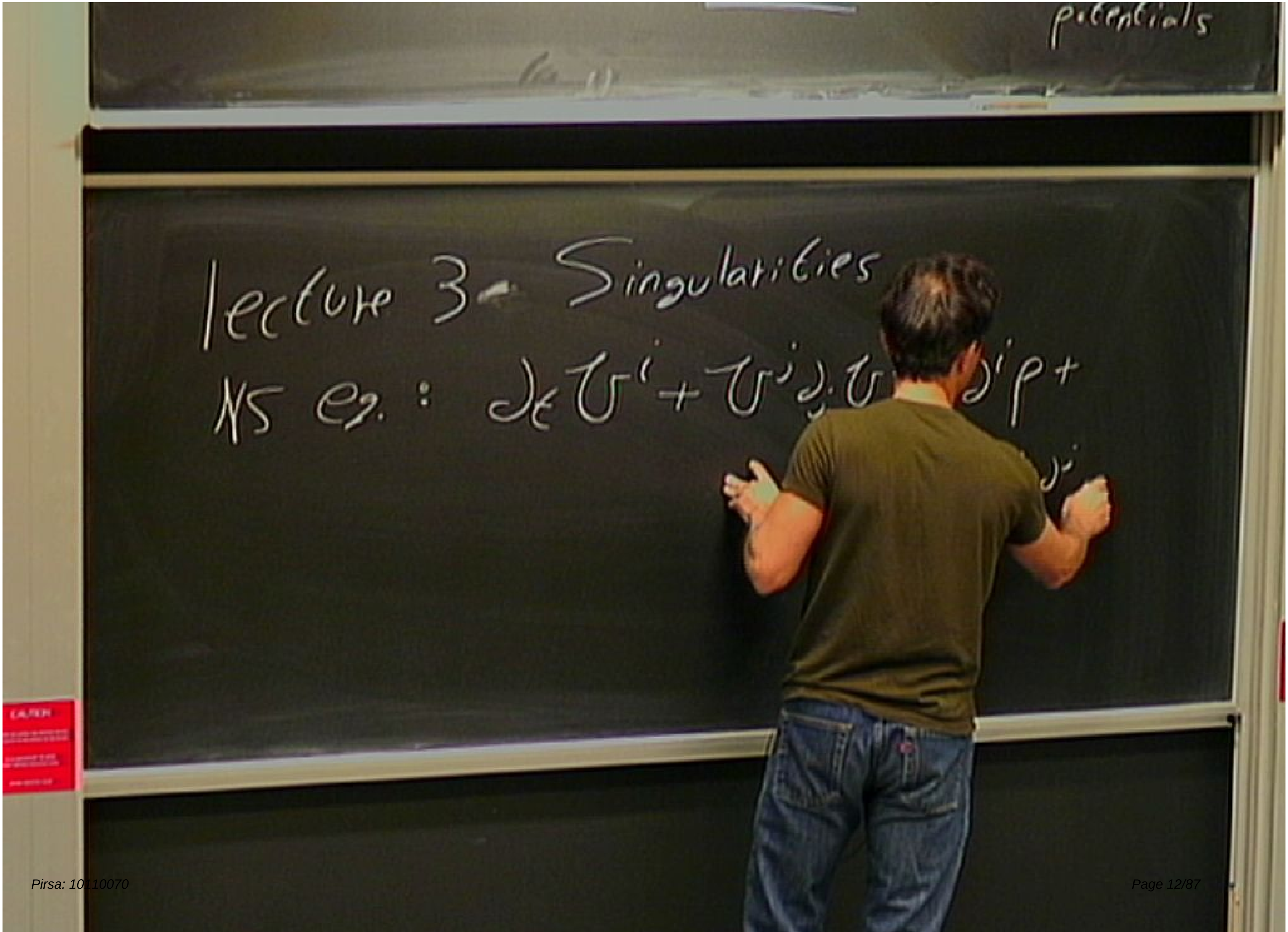
$$\xi^a = C^{abc}\mu^b\mu^c$$

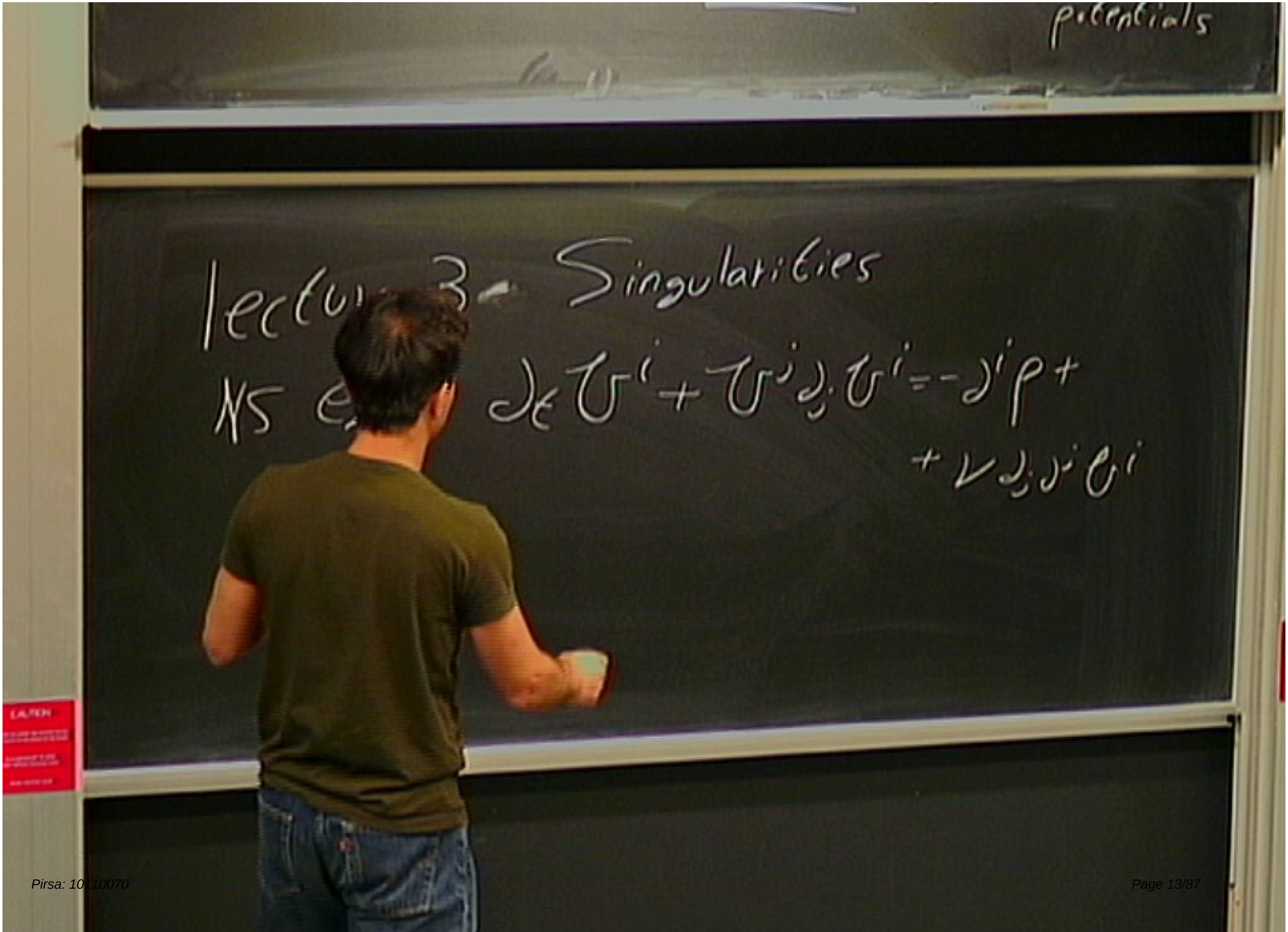
$$\partial_\mu J^{\mu a} = C^{abc} F^b_{\mu\nu}\tilde{F}^{\mu\nu c} - \frac{2}{3} n^a C^{bcd}\frac{\mu^b\mu^c\mu^d}{\varepsilon+p}$$

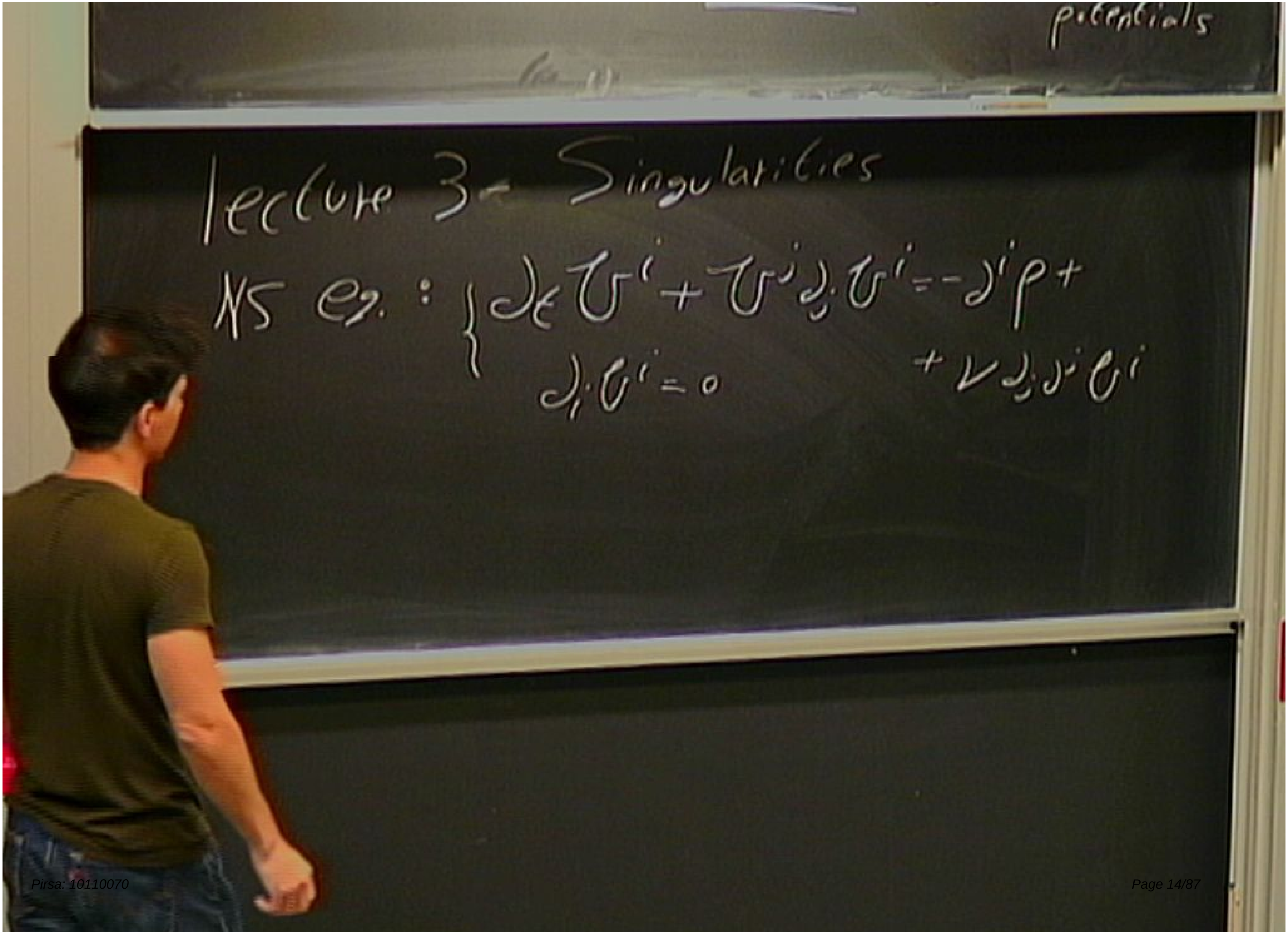
$$J^{\mu a} = n^a u^\mu + \dots + \xi^a \omega^\mu$$


J^b J^c
$\sim C^{abc}$
$\xi^a = C^{abc}\mu^b\mu^c$
$\partial_\mu J^{\mu a} = C^{abc} F^b_{\mu\nu}\tilde{F}^{\mu\nu c} - \frac{2}{3} n^a C^{bcd}\frac{\mu^b\mu^c\mu^d}{\varepsilon+p}$
$J^{\mu a} = n^a u^\mu + \dots + \xi^a \omega^\mu$
μ^a - chemical potentials


potentials
lecture 3 - Singularities


potentials
lecture 3 - Singularities
NS eq. :


potentials
Singularities
NS
∂_t v^i + v^j ∂_j v^i = -∂^i p +
+ ν ∂_j ∂^j v^i


potentials
lecture 3 - Singularities
NS eq. : { ∂t v^i + v^j ∂j v^i = - ∂^i p +
+ ν ∂j ∂^j v^i
∂i v^i = 0

potentials

lecture 3 - Singularities

NS eq.:

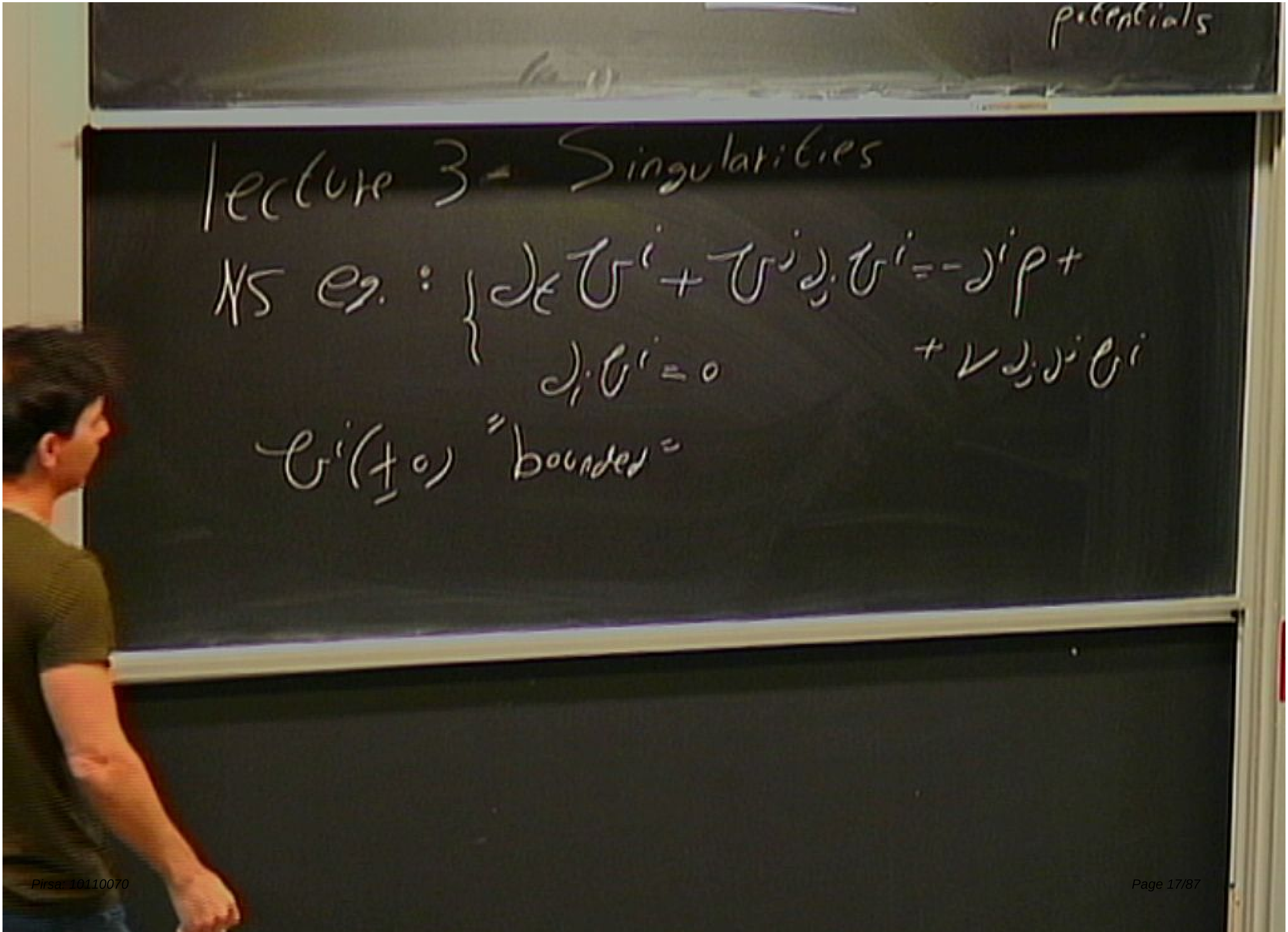
$$\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i \\ \partial_i v^i = 0 \end{cases}$$

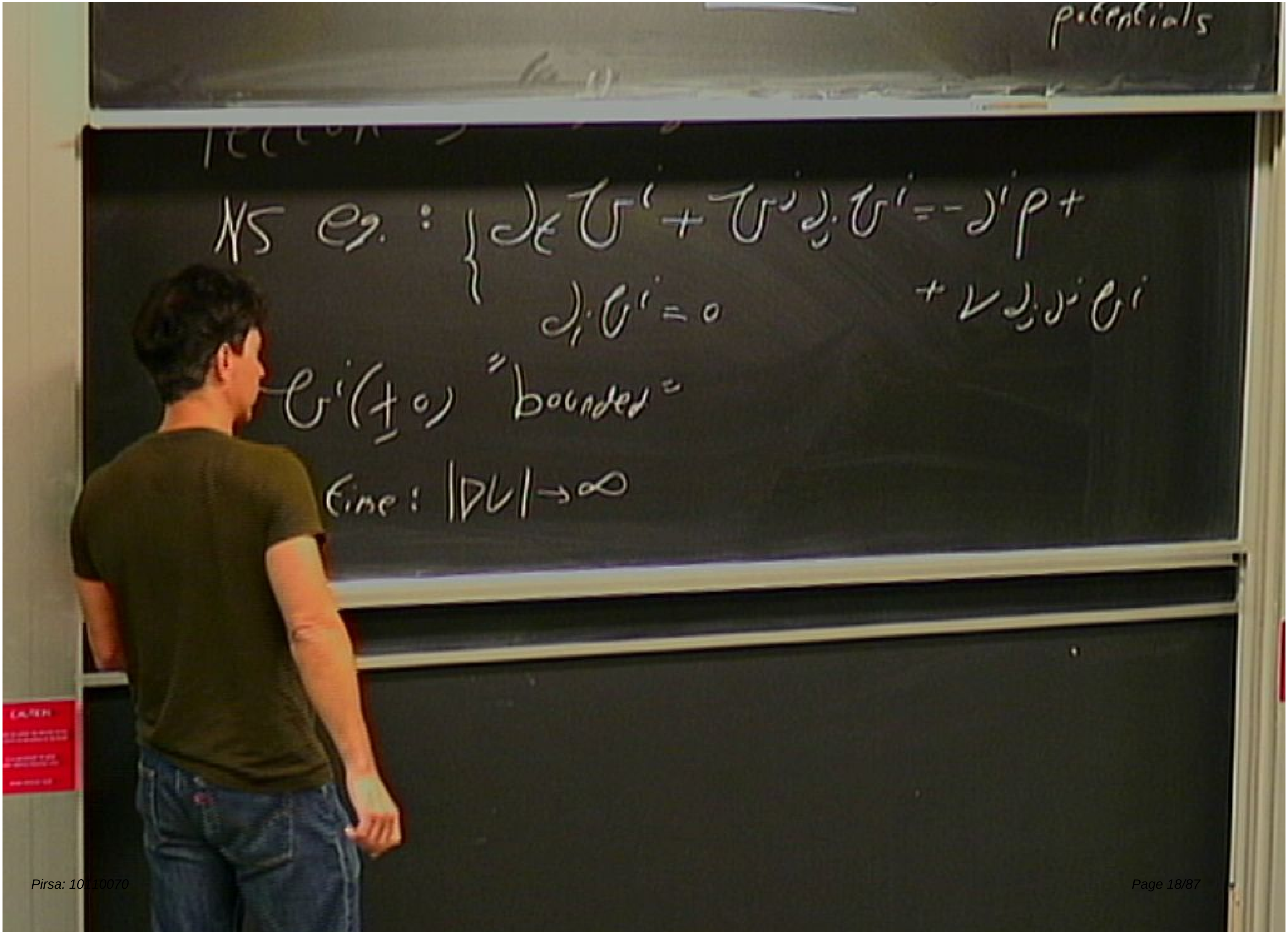
potentials

Lecture 3 - Singularities

NS eq.: $\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i \\ \partial_i v^i = 0 \end{cases}$

$v^i(t=0)$


potentials
lecture 3 - Singularities
NS eq. : $\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i$
$\partial_i v^i = 0$
$v^i(t=0)$ "bounded"


potentials
NS eq. :
$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i$
$\partial_i v^i = 0$
$v^i(t=0)$ "bounded"
time: $|\nabla v| \to \infty$

potentials

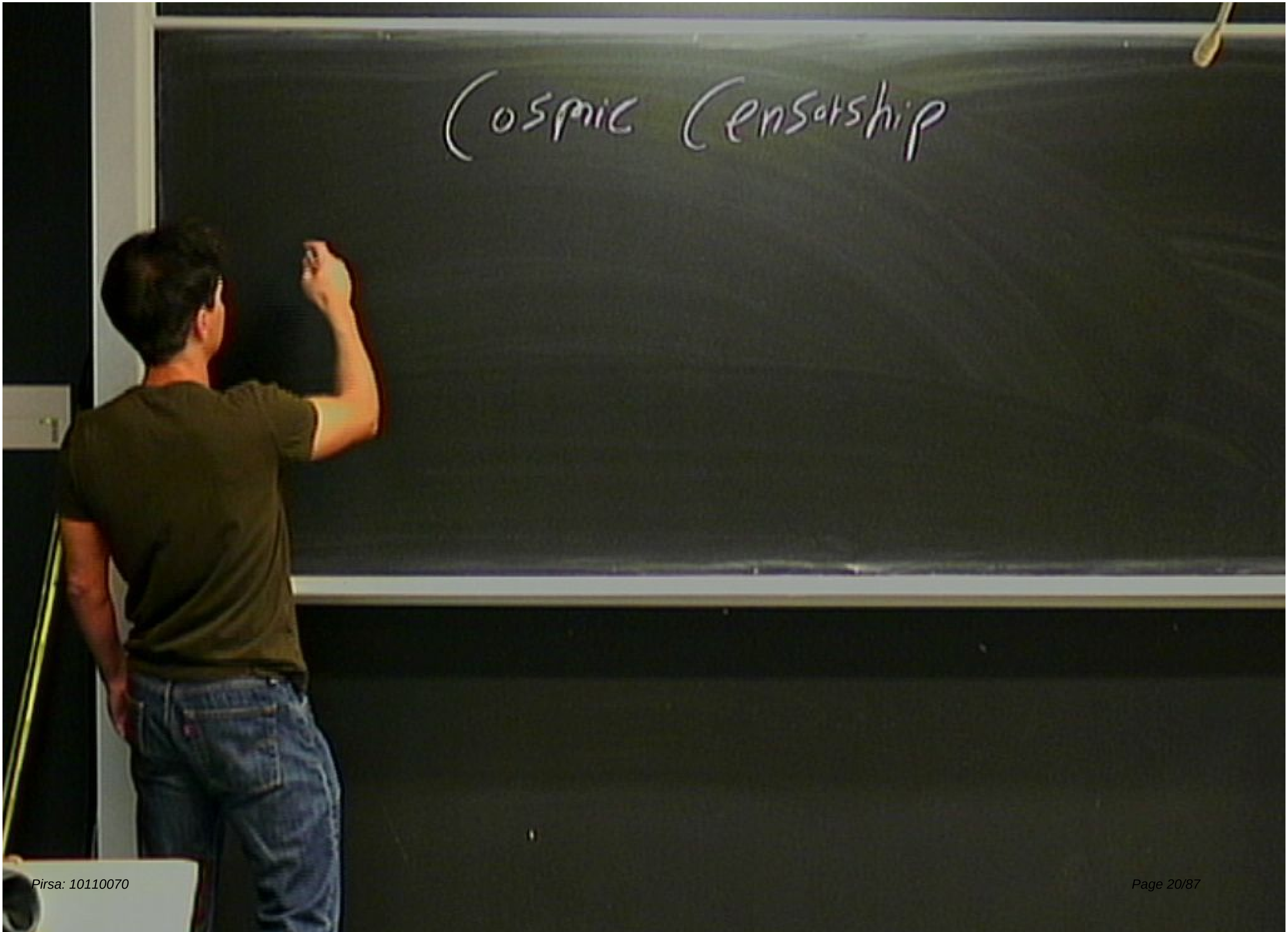
Lecture 3 - Singularities

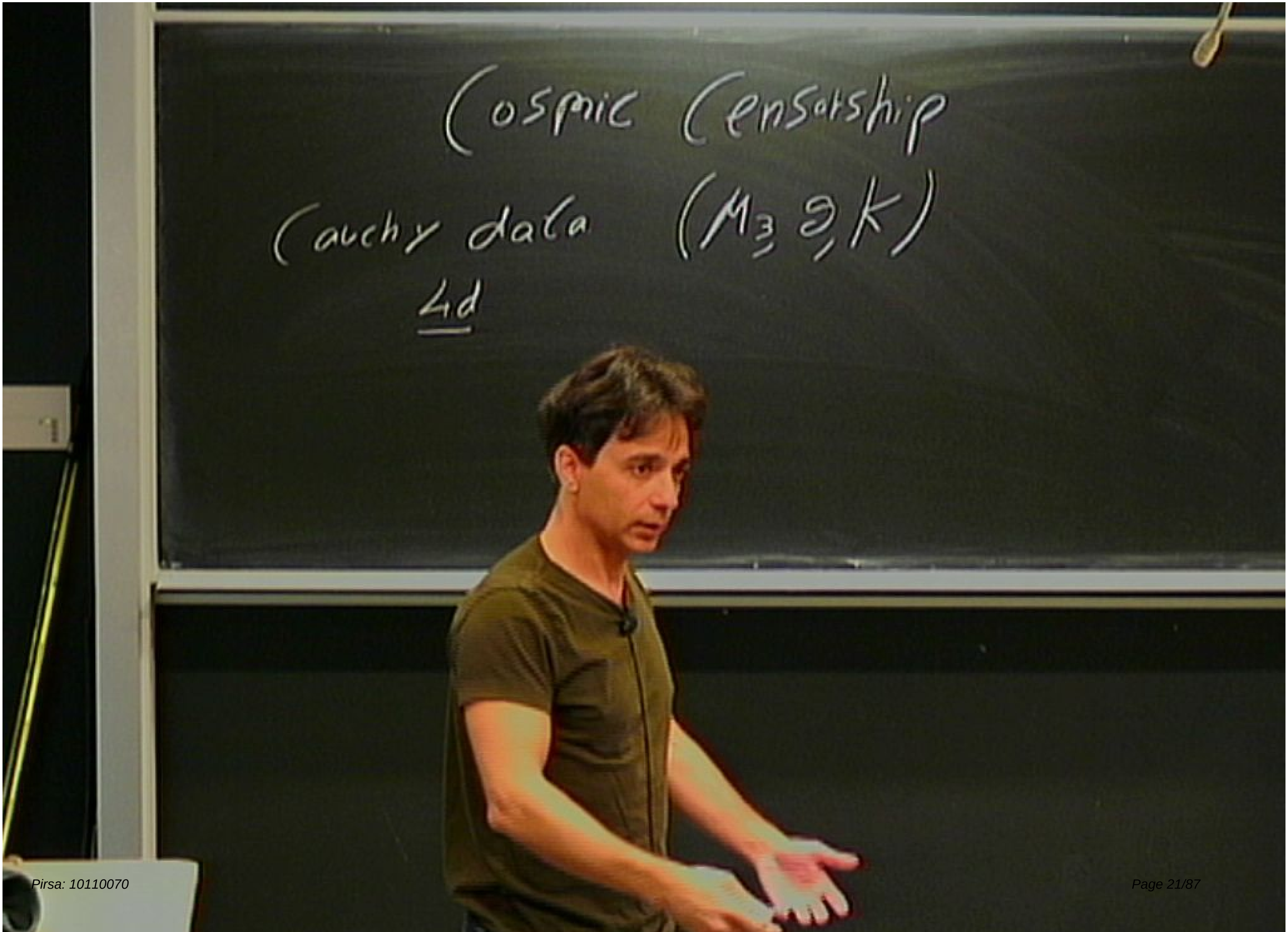
NS eq.:

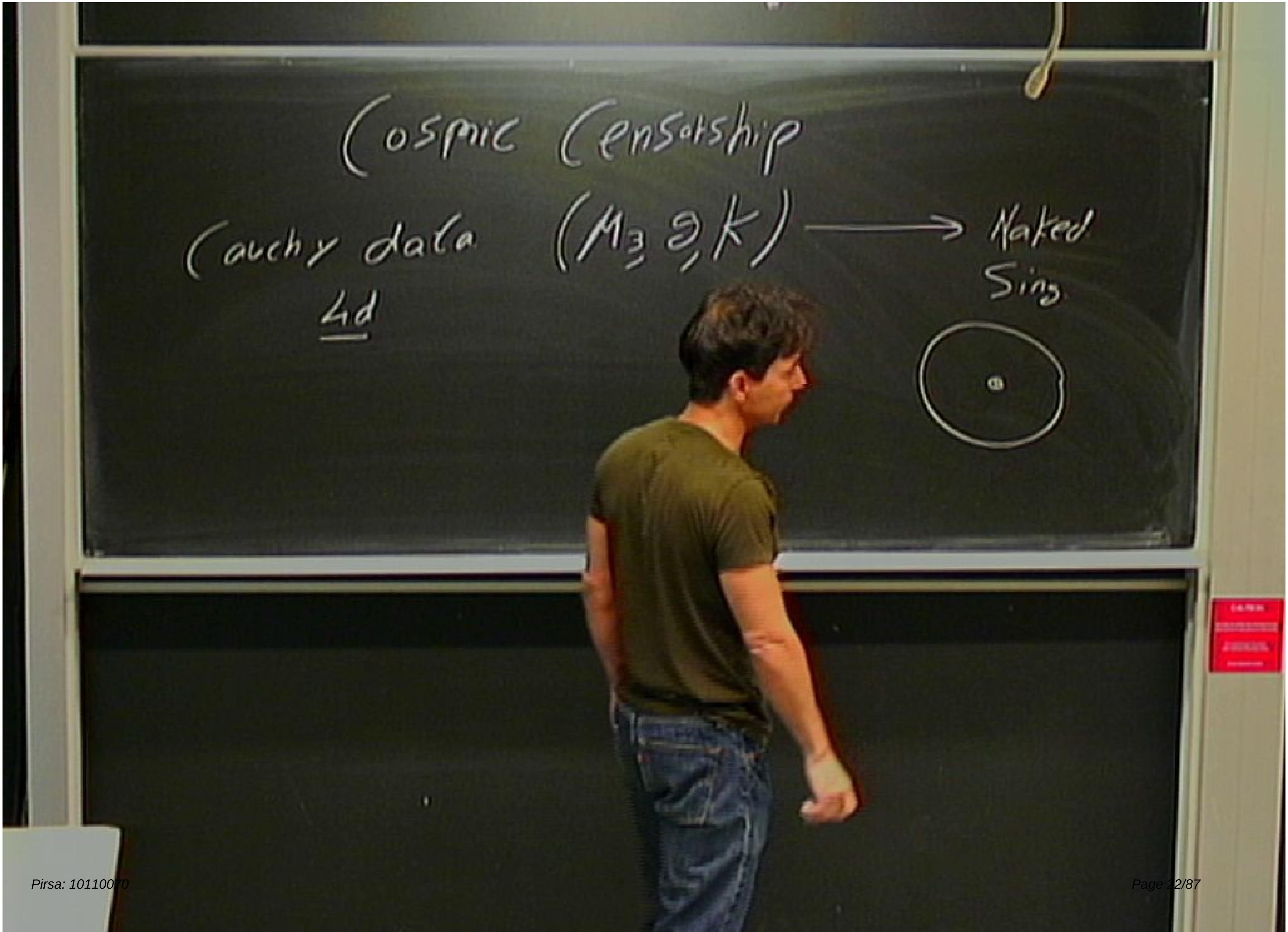
$$\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i \\ \partial_i v^i = 0 \end{cases}$$

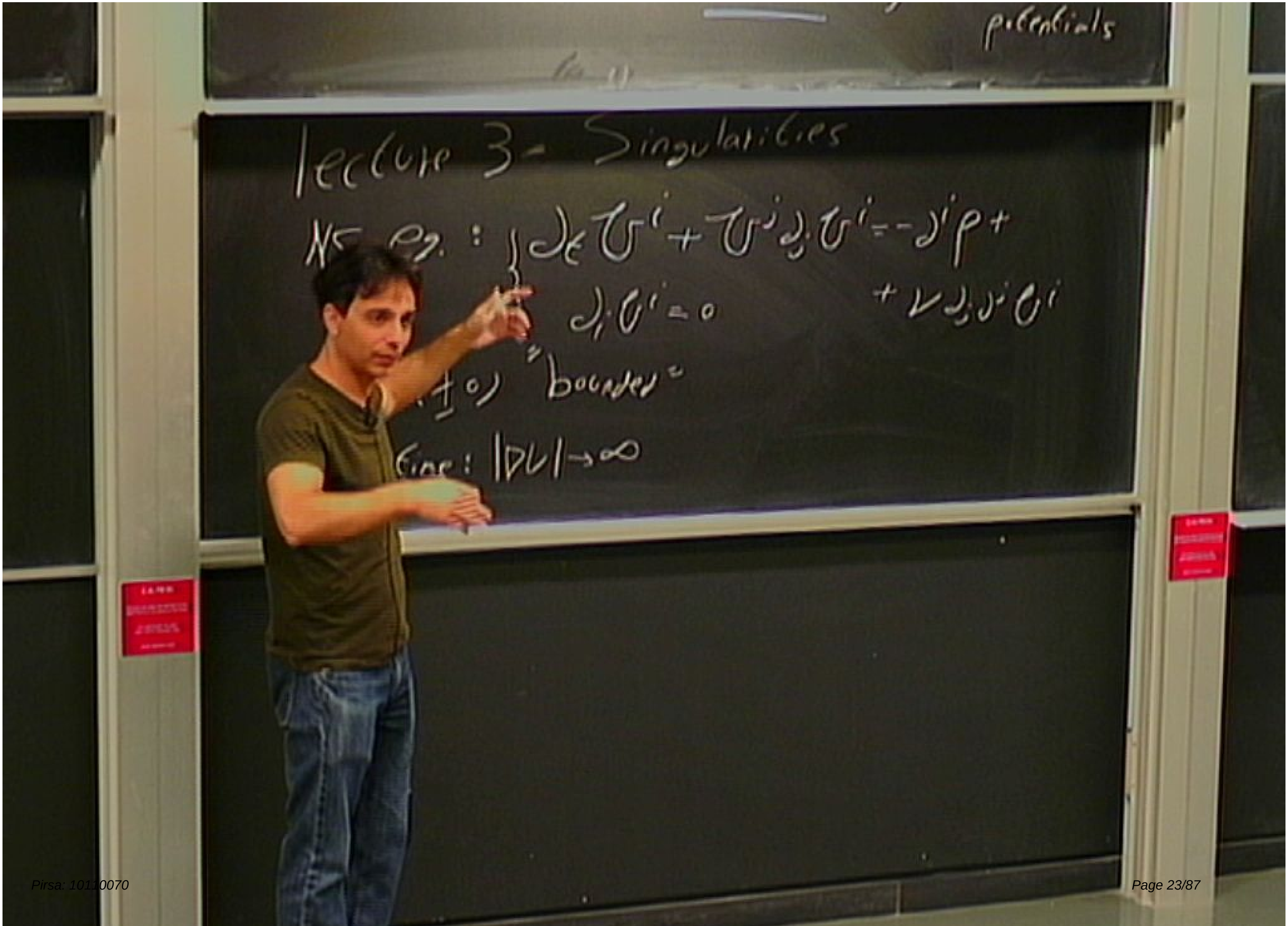
$v^i(t=0)$ "bounded"

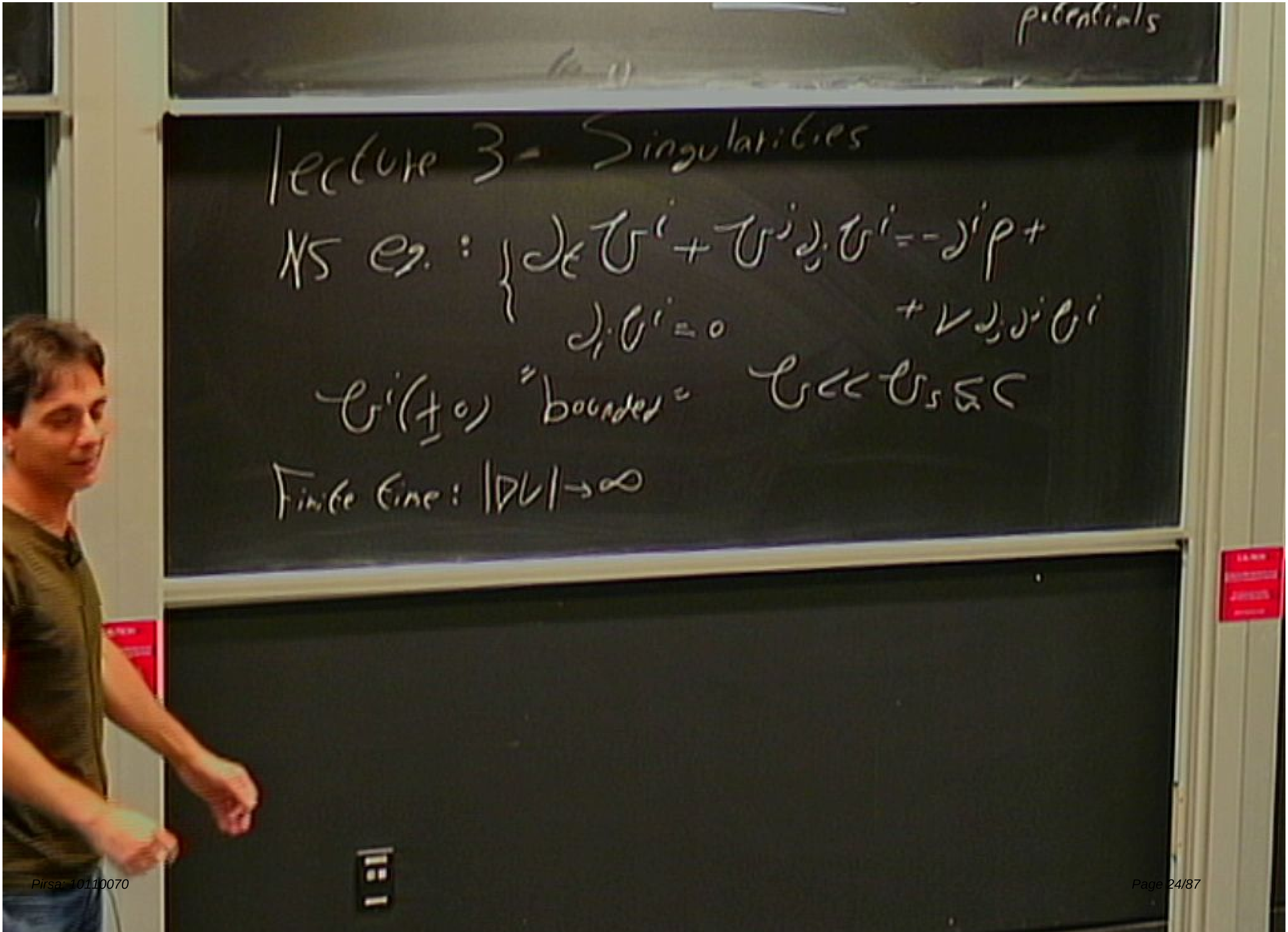
Finite time: $|\nabla v| \to \infty$

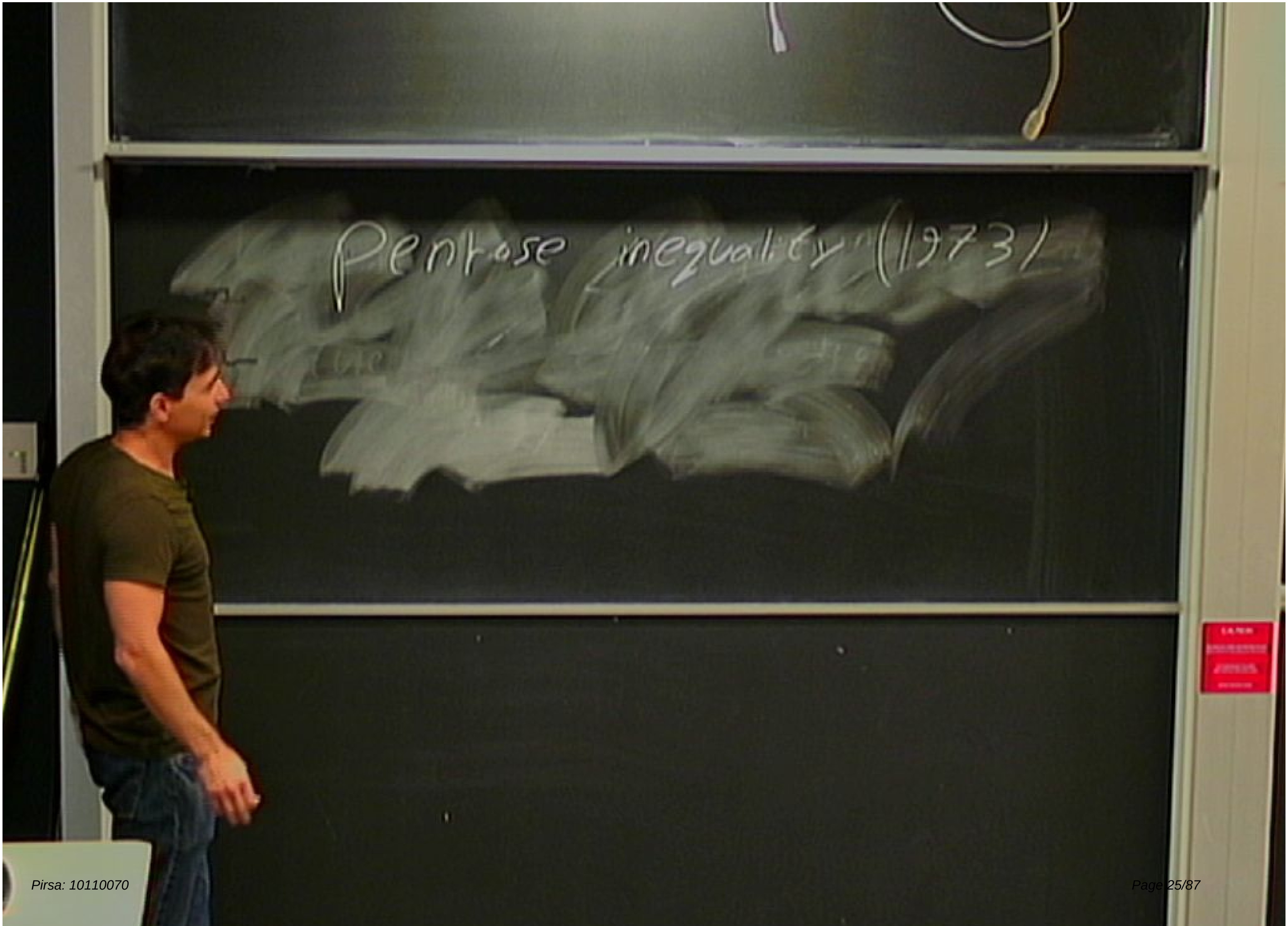

Cosmic Censorship

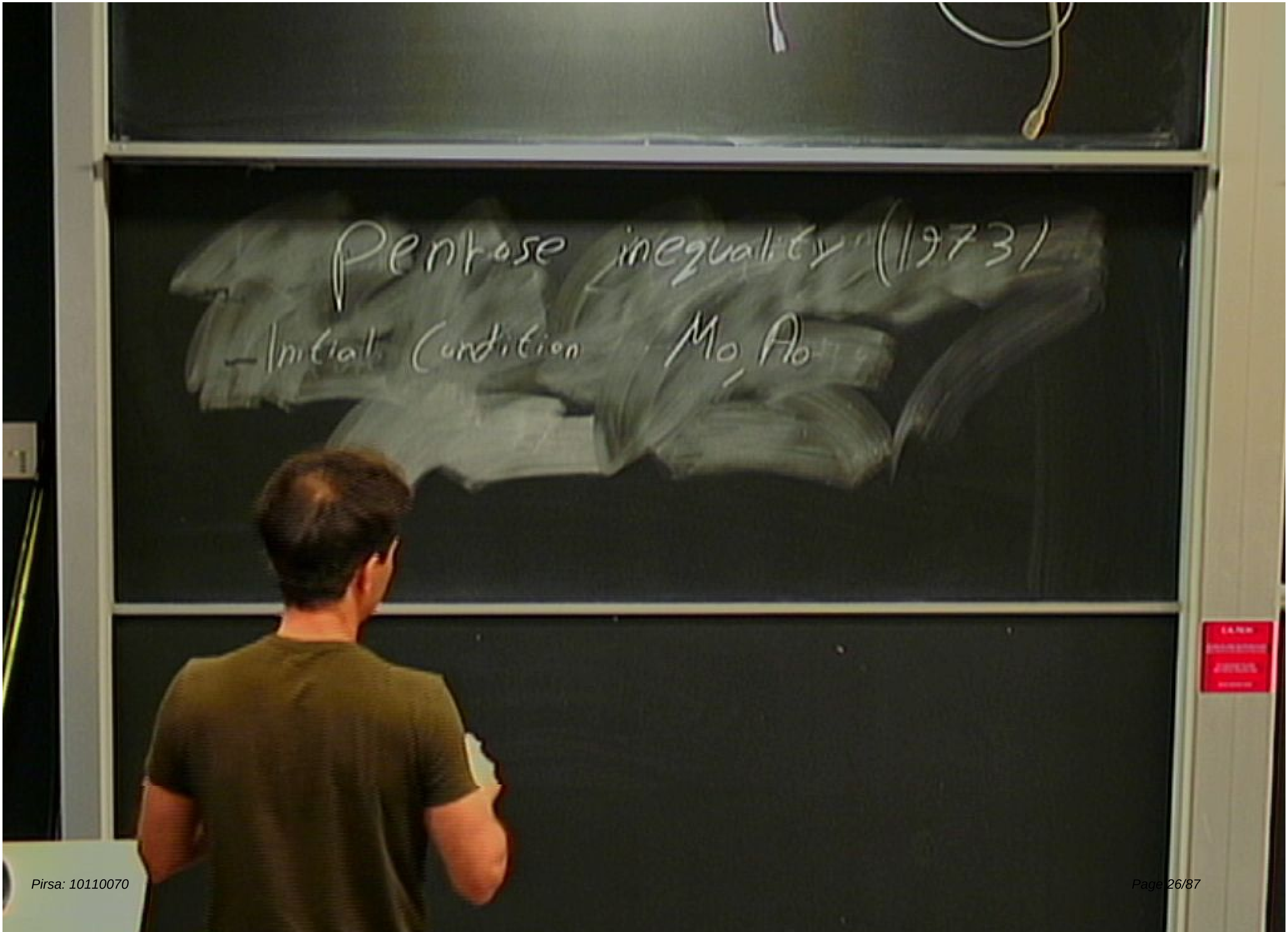

Cosmic Censorship
Cauchy data (M3, g, K)
4d

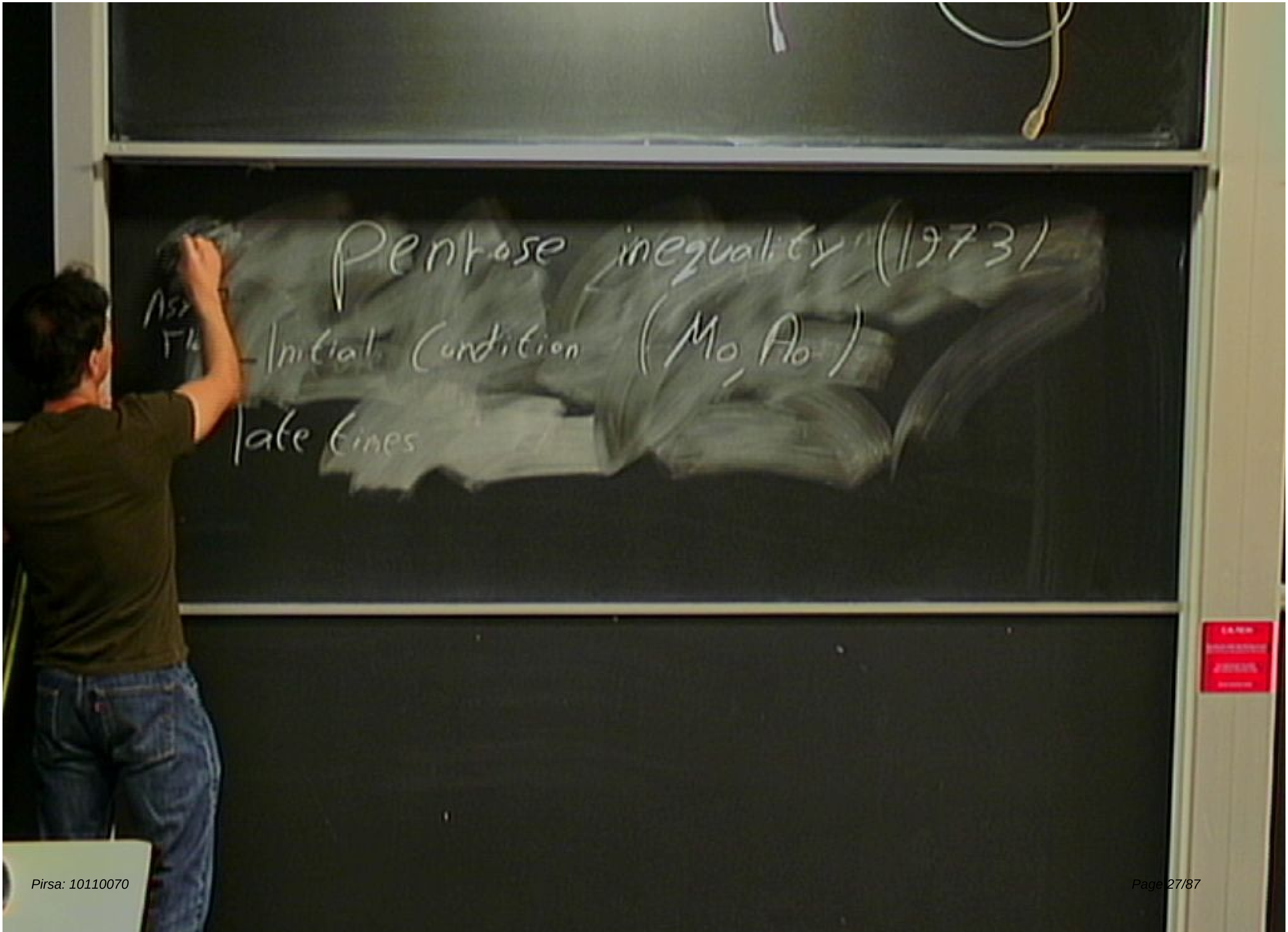

Cosmic Censorship
Cauchy data
4d
(M3 g, k)
Naked
Sing

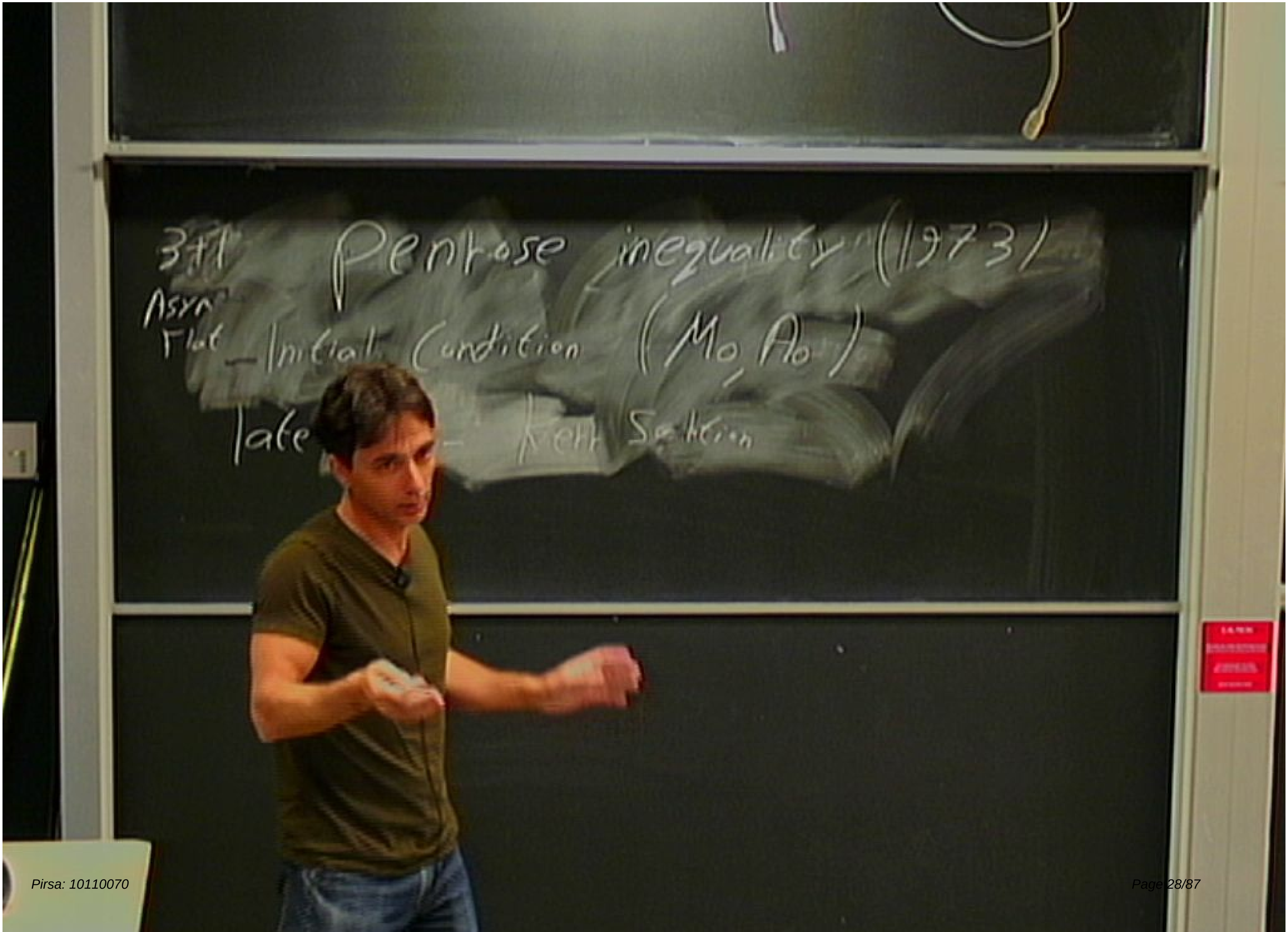

potentials
lecture 3 - Singularities
NS eq. : $\partial_t v^i + v^j \partial_j v^i = -\partial^i p +$
$+ \nu \partial_j \partial^j v^i$
$\partial_i v^i = 0$
"bounded"
$|\nabla v| \to \infty$

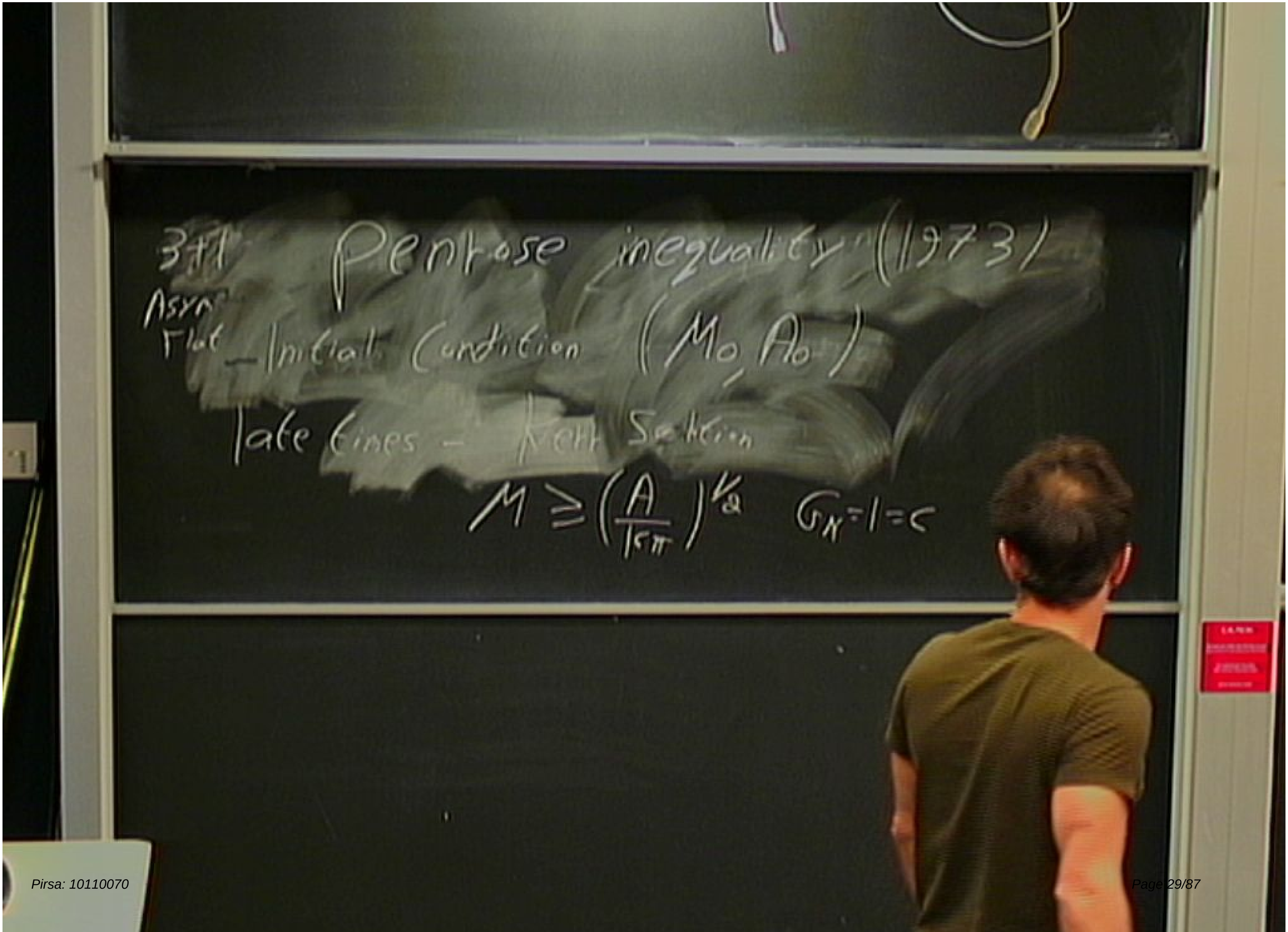

potentials
lecture 3 - Singularities
NS eq.: { ∂t v^i + v^j ∂j v^i = -∂^i p + ν ∂j ∂^j v^i
∂i v^i = 0
v^i(t=0) "bounded" v << vs ≈ c
Finite time: |∇v| → ∞

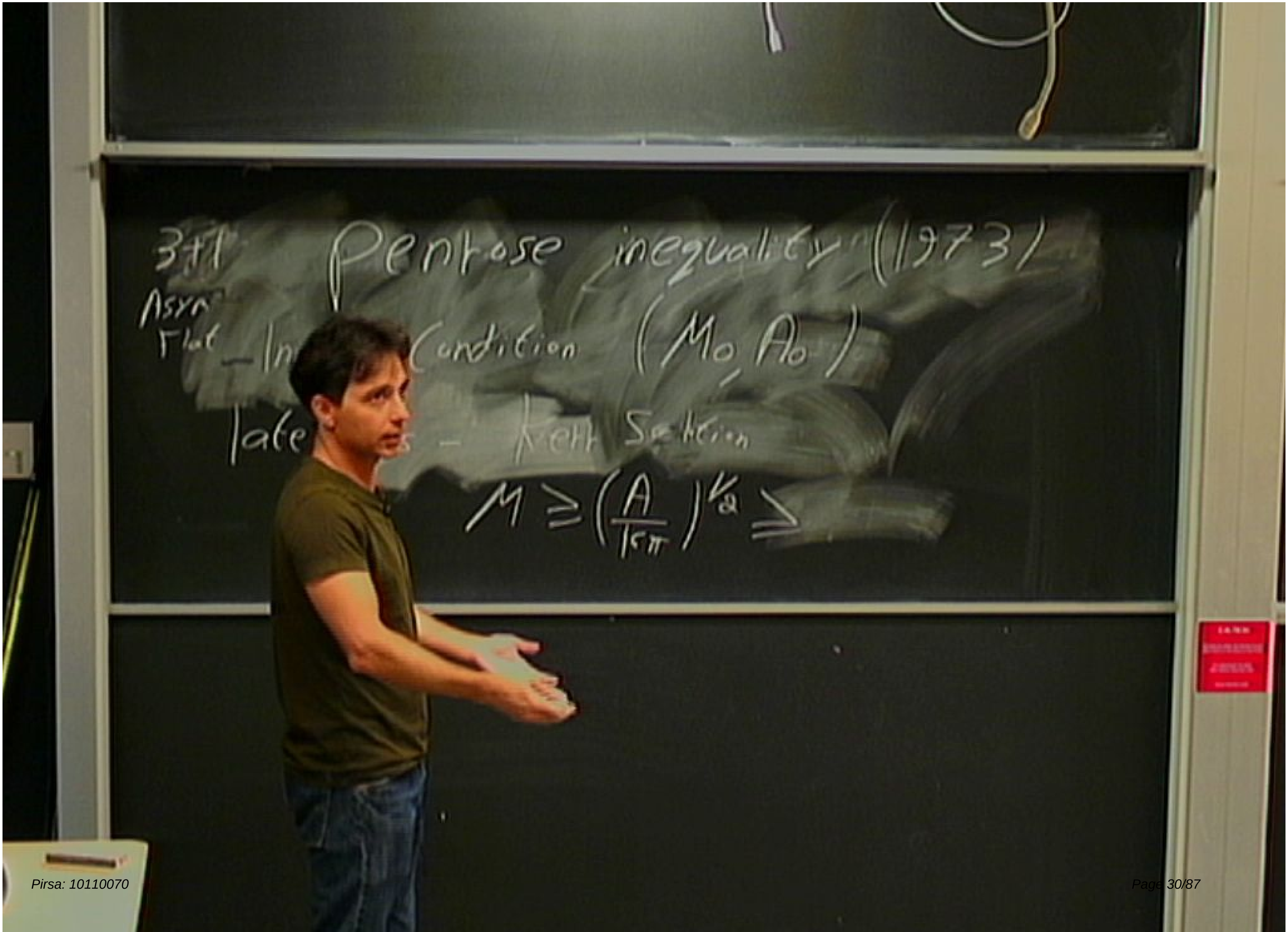

Penrose inequality (1973)


Penrose inequality (1973)
- Initial Condition M0, A0

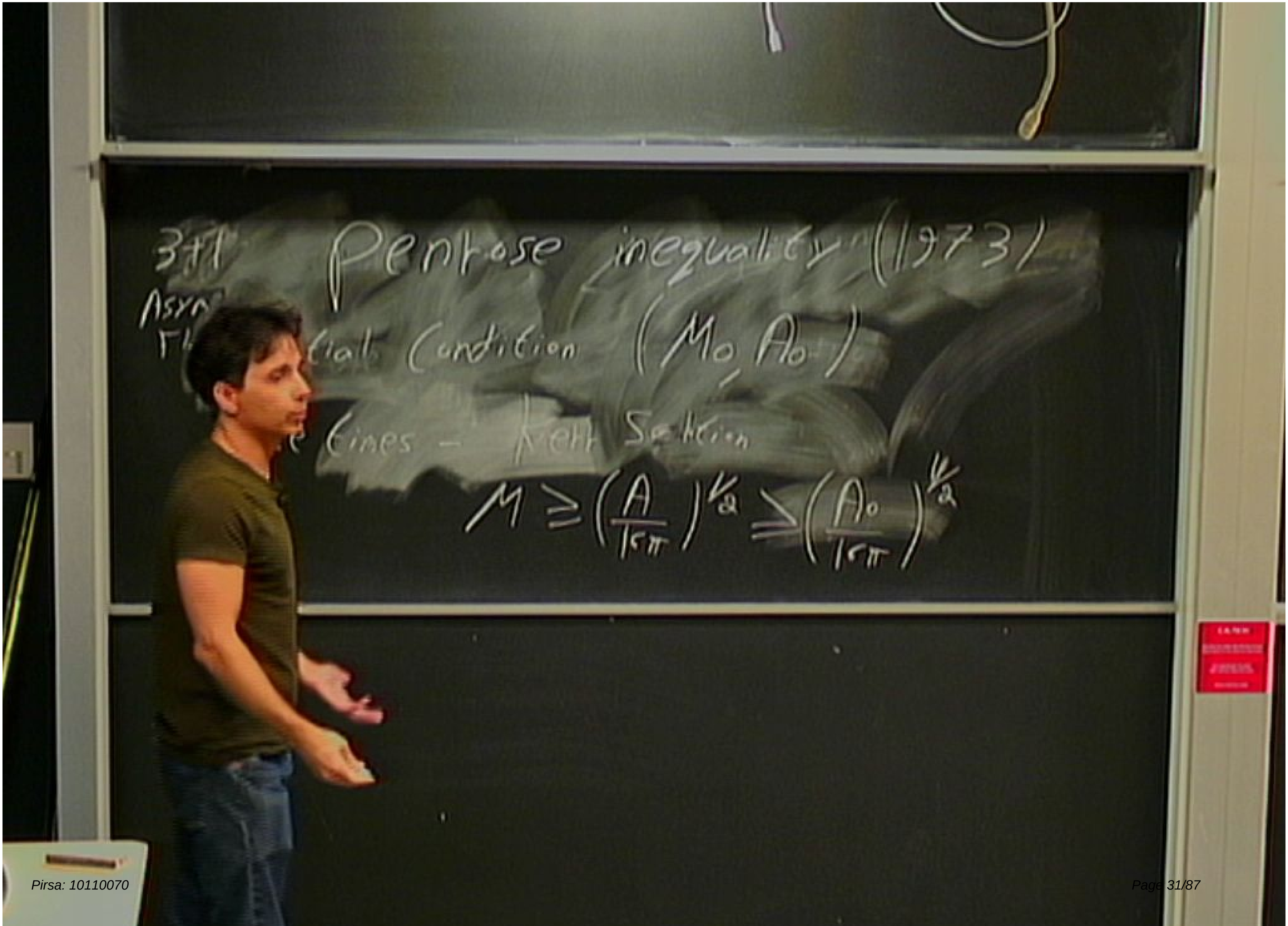

Penrose inequality (1973)
Initial Condition (M0, A0)
late times

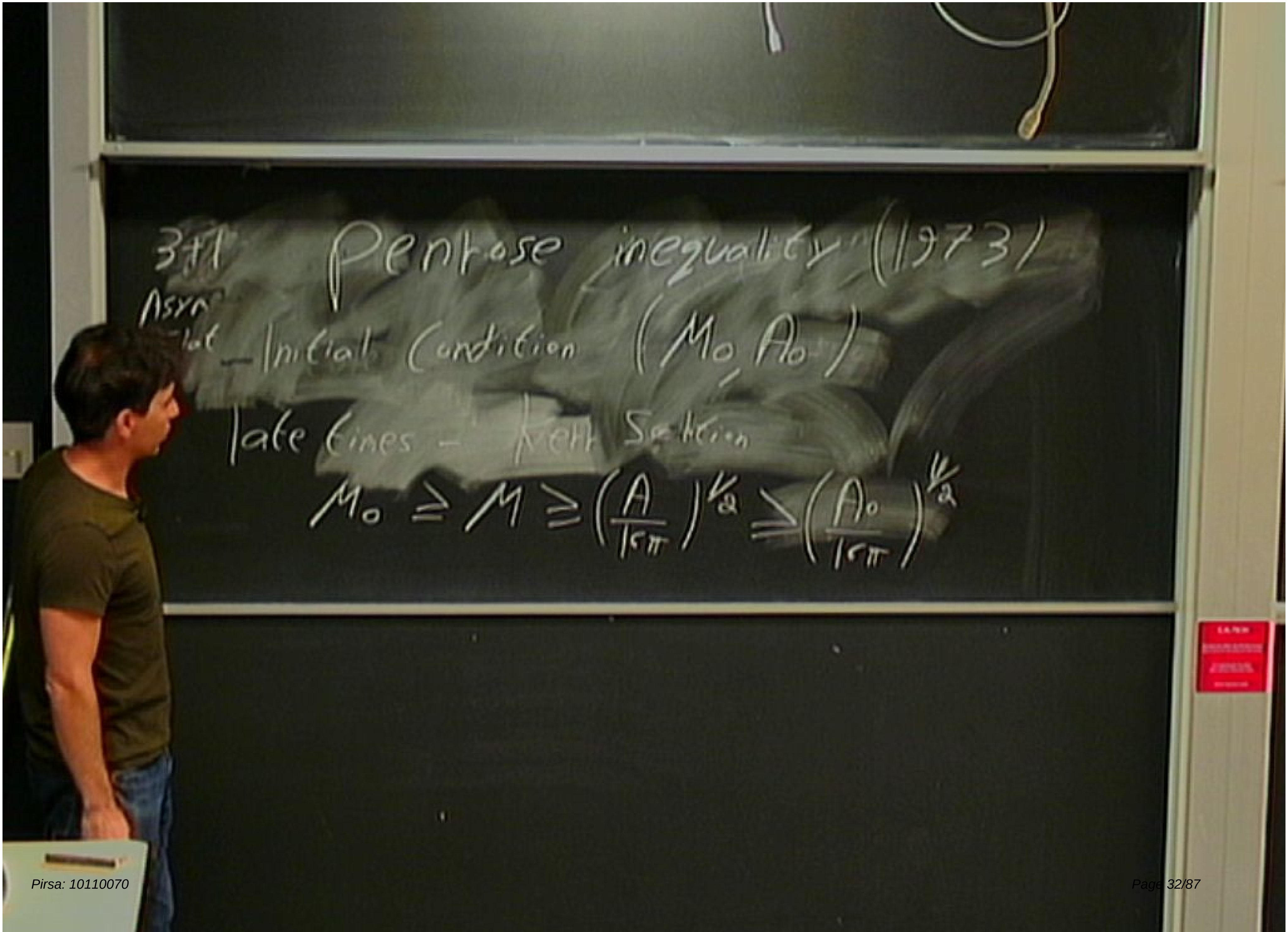

3+1 Penrose inequality (1973)
Initial Condition (M_0, A_0)
Kerr Solution

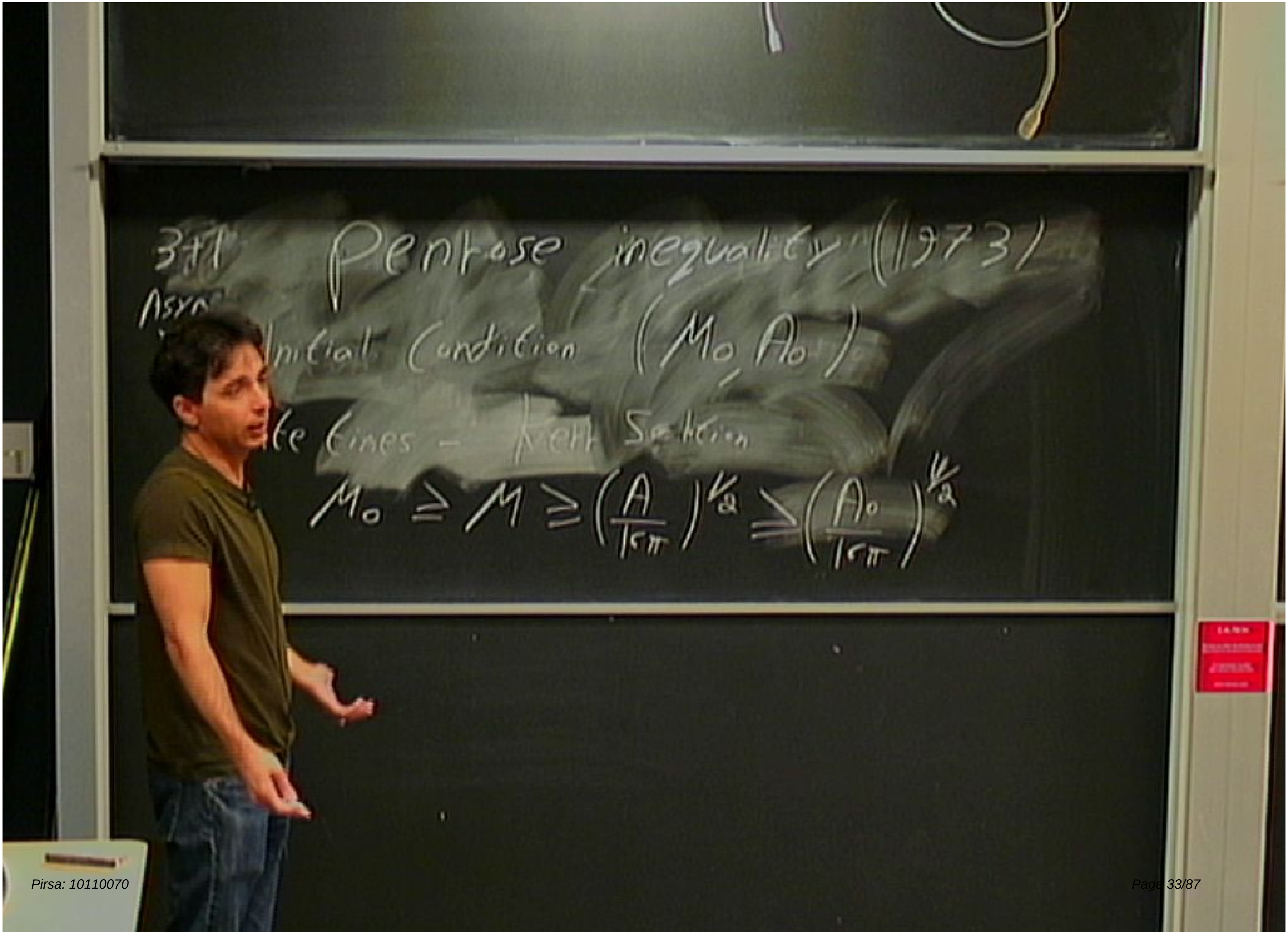

3+1 Penrose inequality (1973)
Asym Flat — Initial Condition (M0, A0)
late times — Kerr Solution
M ≥ (A/16π)^1/2   G_N = 1 = c

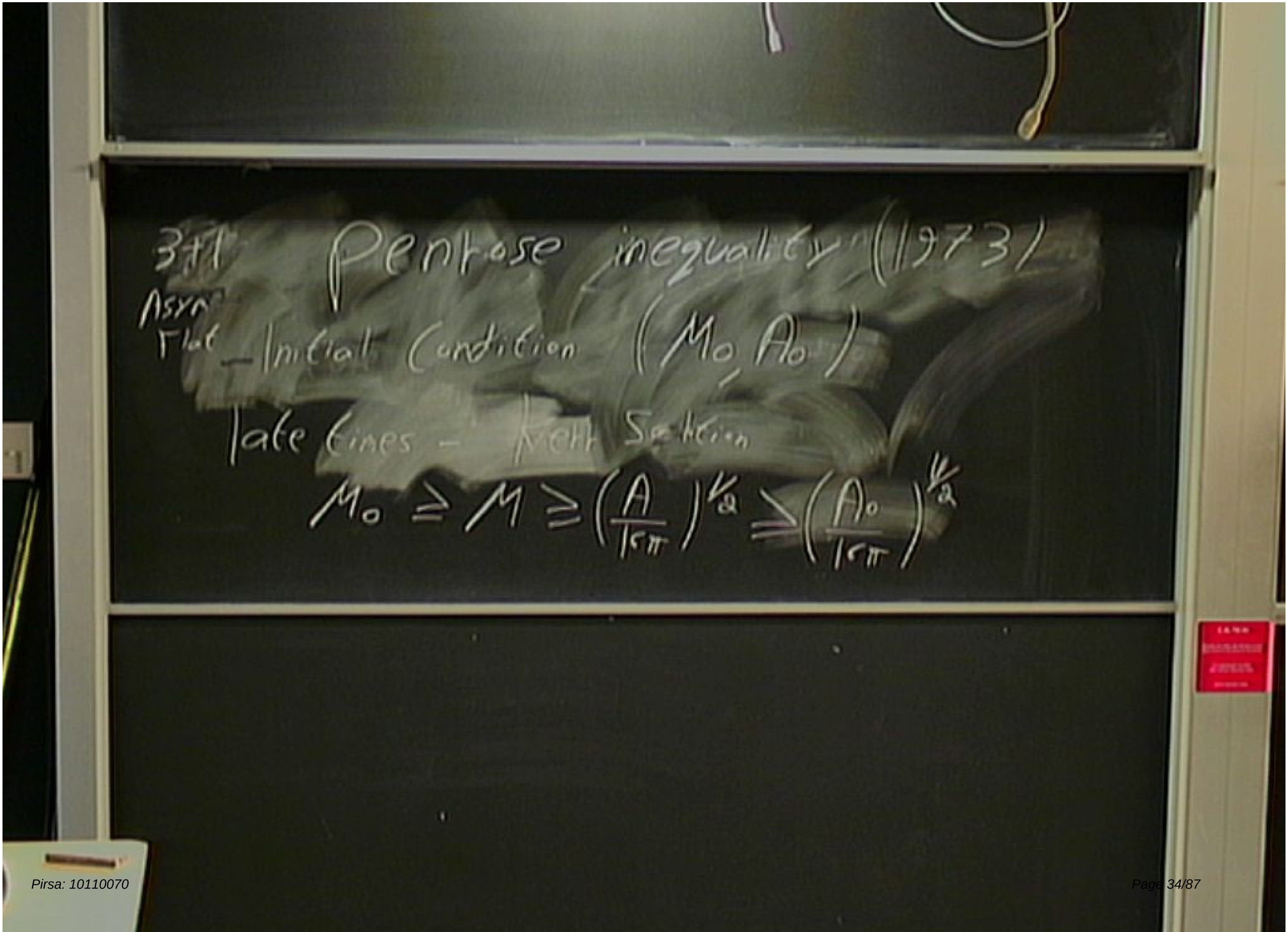


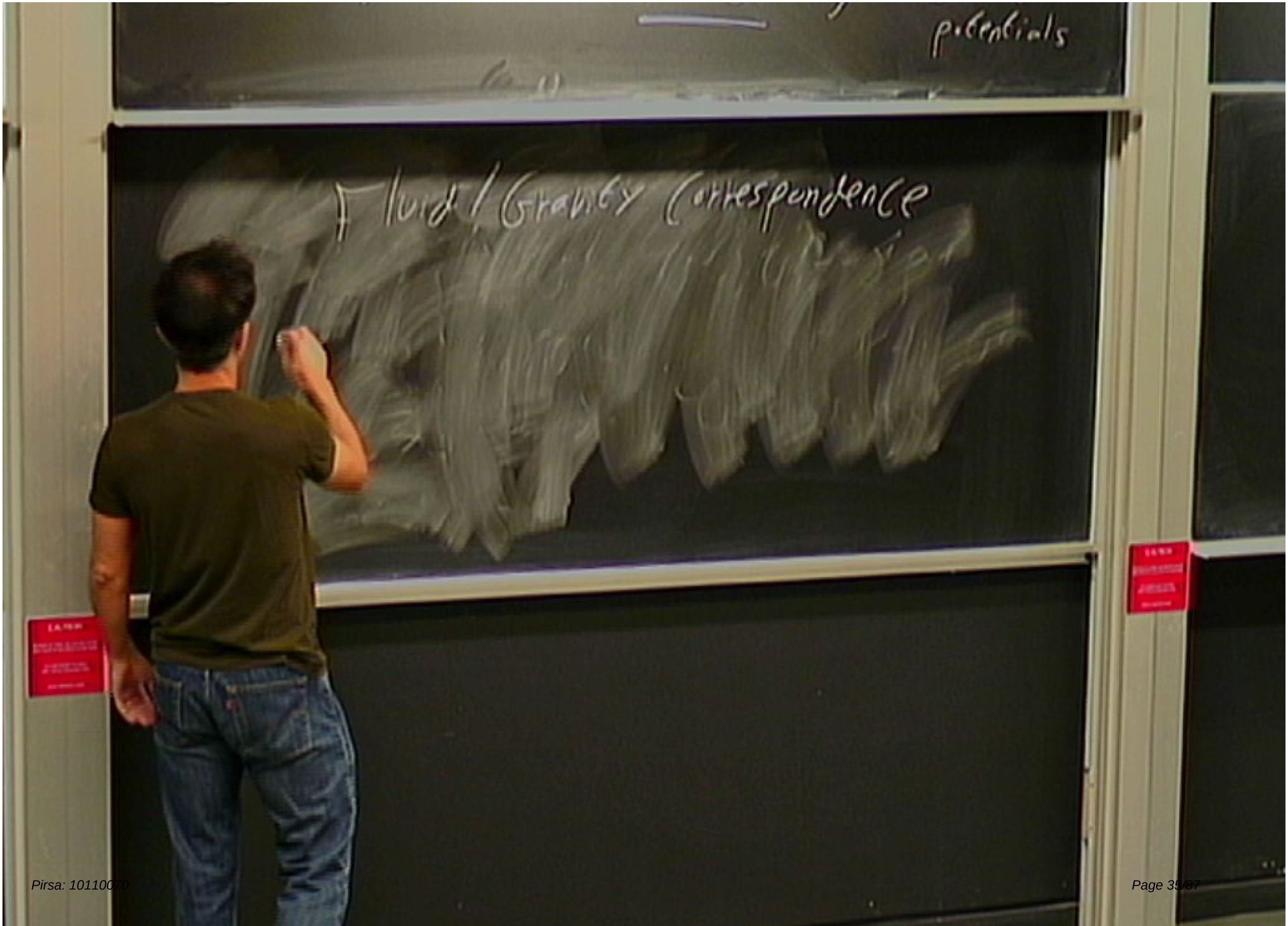
3+1 Penrose inequality (1973)
Asym Flat
Condition (M_0, A_0)
Kerr Solution
$M \geq \left(\frac{A}{16\pi}\right)^{1/2} \geq$

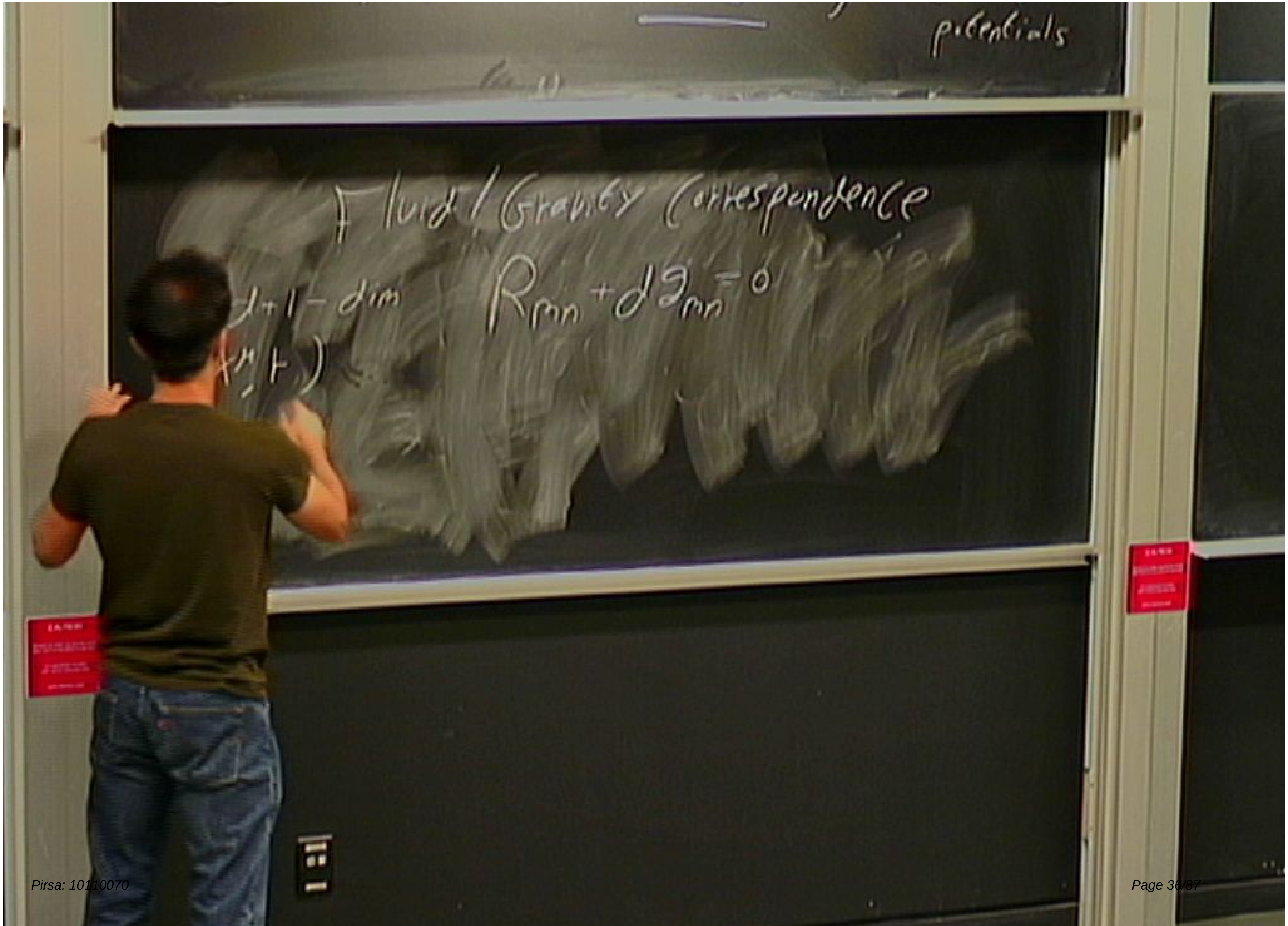

3+1 Penrose inequality (1973)
Condition (M0, A0)
Kerr Solution
M ≥ (A/16π)^1/2 ≥ (A0/16π)^1/2

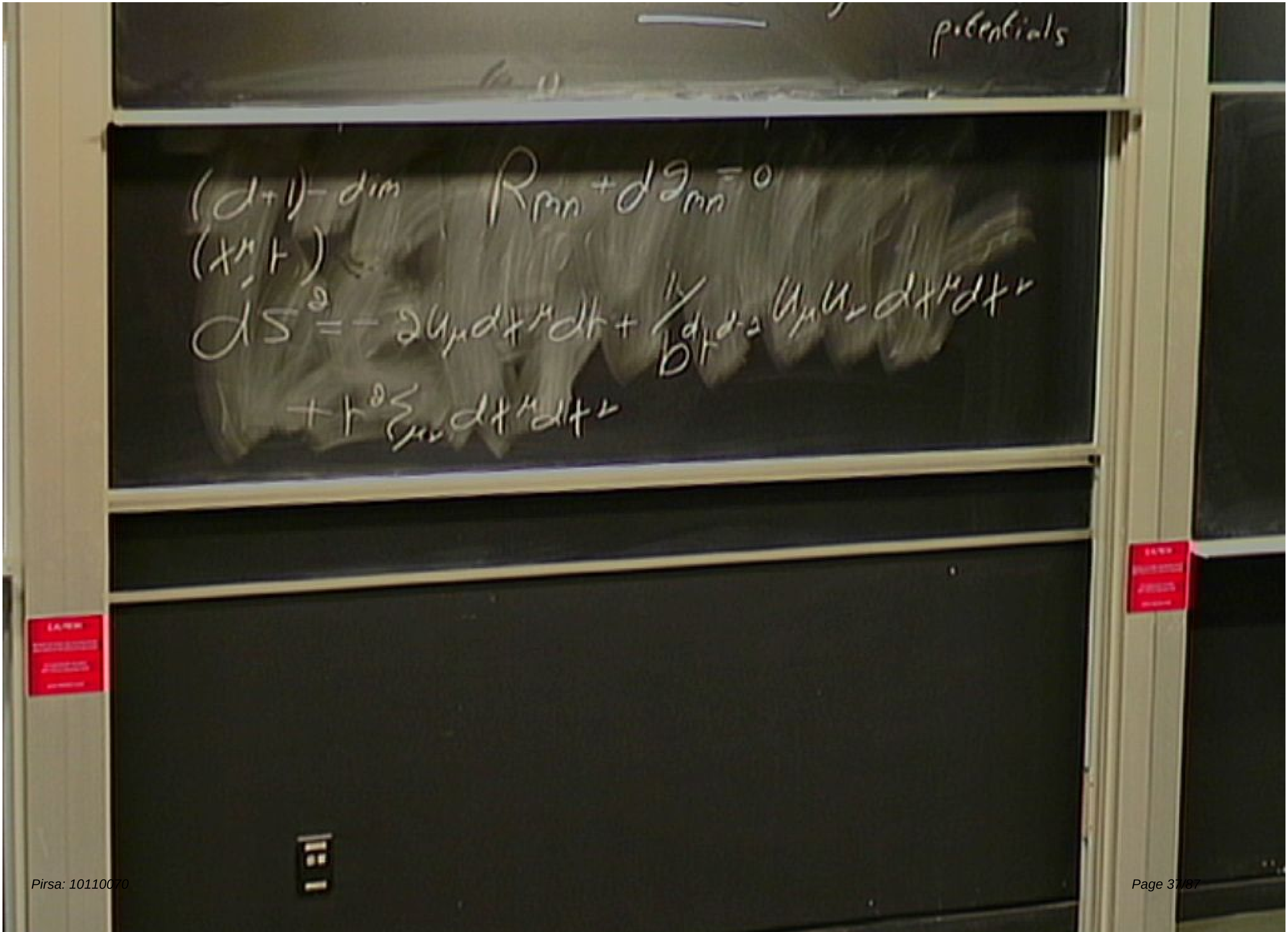

3+1 Penrose inequality (1973)
Initial Condition (M0, A0)
late times – Kerr Solution
M0 ≥ M ≥ (A/16π)^1/2 ≥ (A0/16π)^1/2

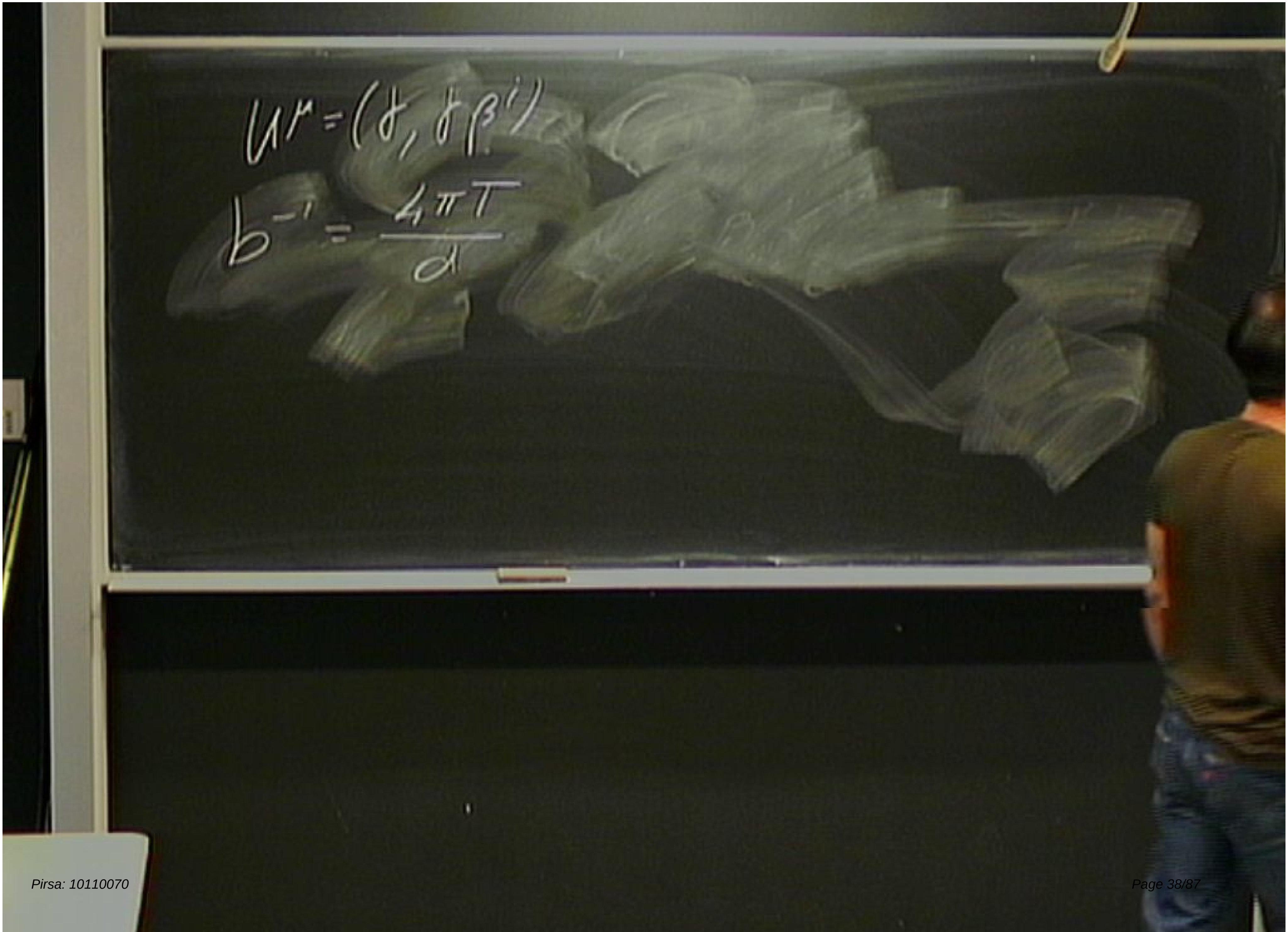

3+1 Penrose inequality (1973)
Initial Condition (M_0, A_0)
Kerr Solution
M_0 ≥ M ≥ (A/16π)^1/2 ≥ (A_0/16π)^1/2

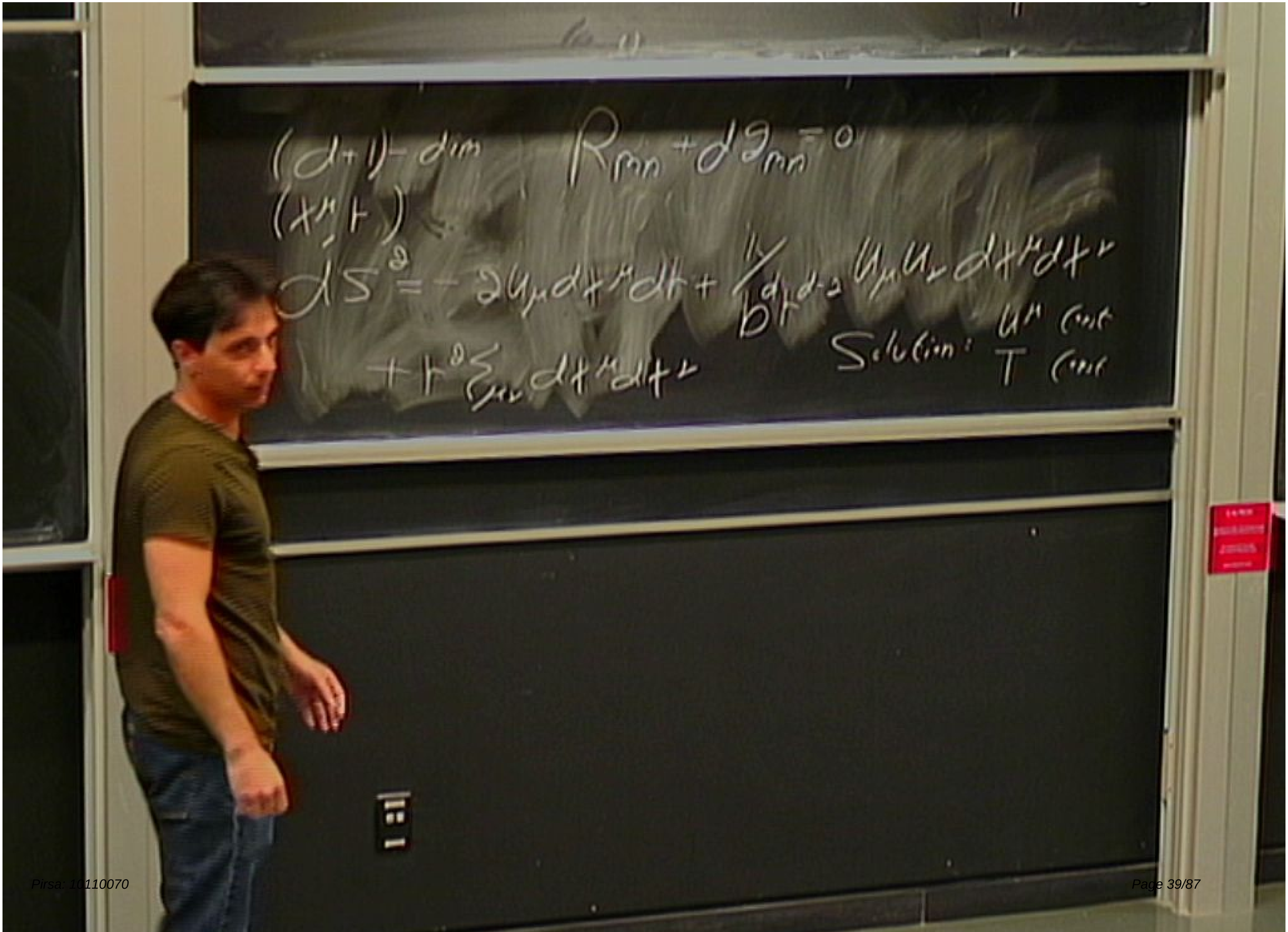

3+1 Penrose inequality (1973)
Asym Flat – Initial Condition (M0, A0)
late times – Kerr Solution
M0 ≥ M ≥ (A/16π)^1/2 ≥ (A0/16π)^1/2

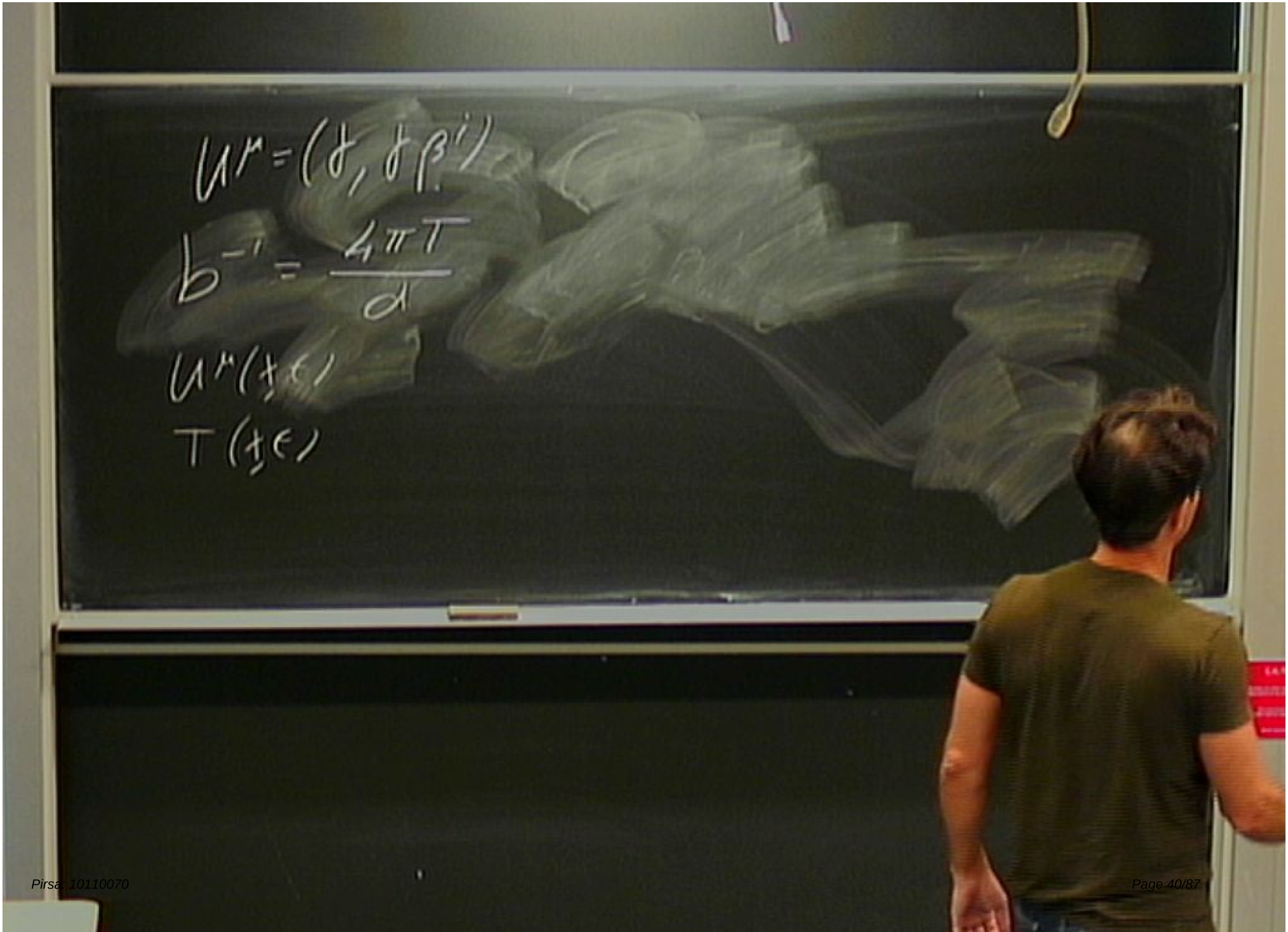

potentials
Fluid / Gravity Correspondence

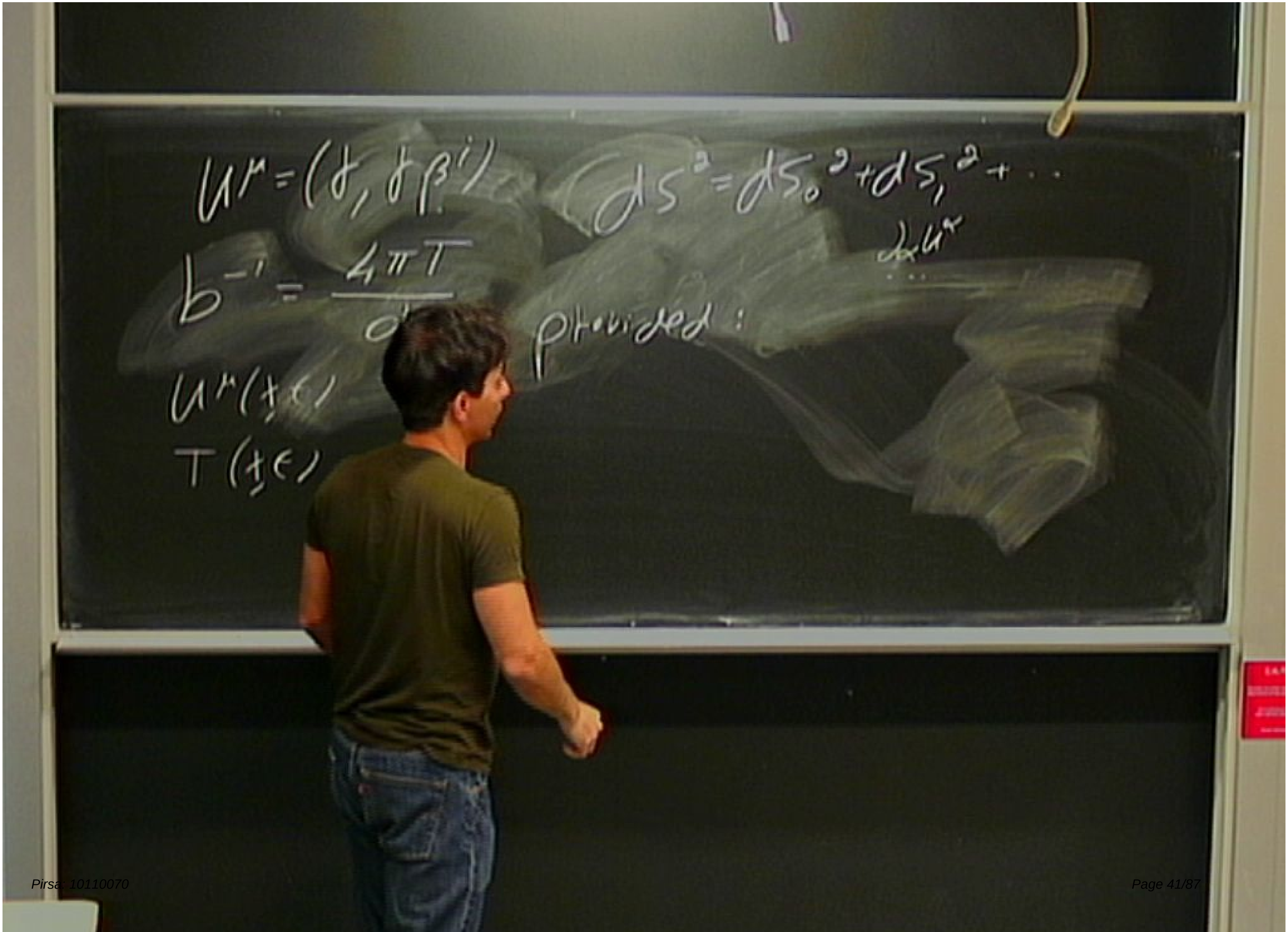

potentials
Fluid / Gravity Correspondence
$R_{mn} + d g_{mn} = 0$

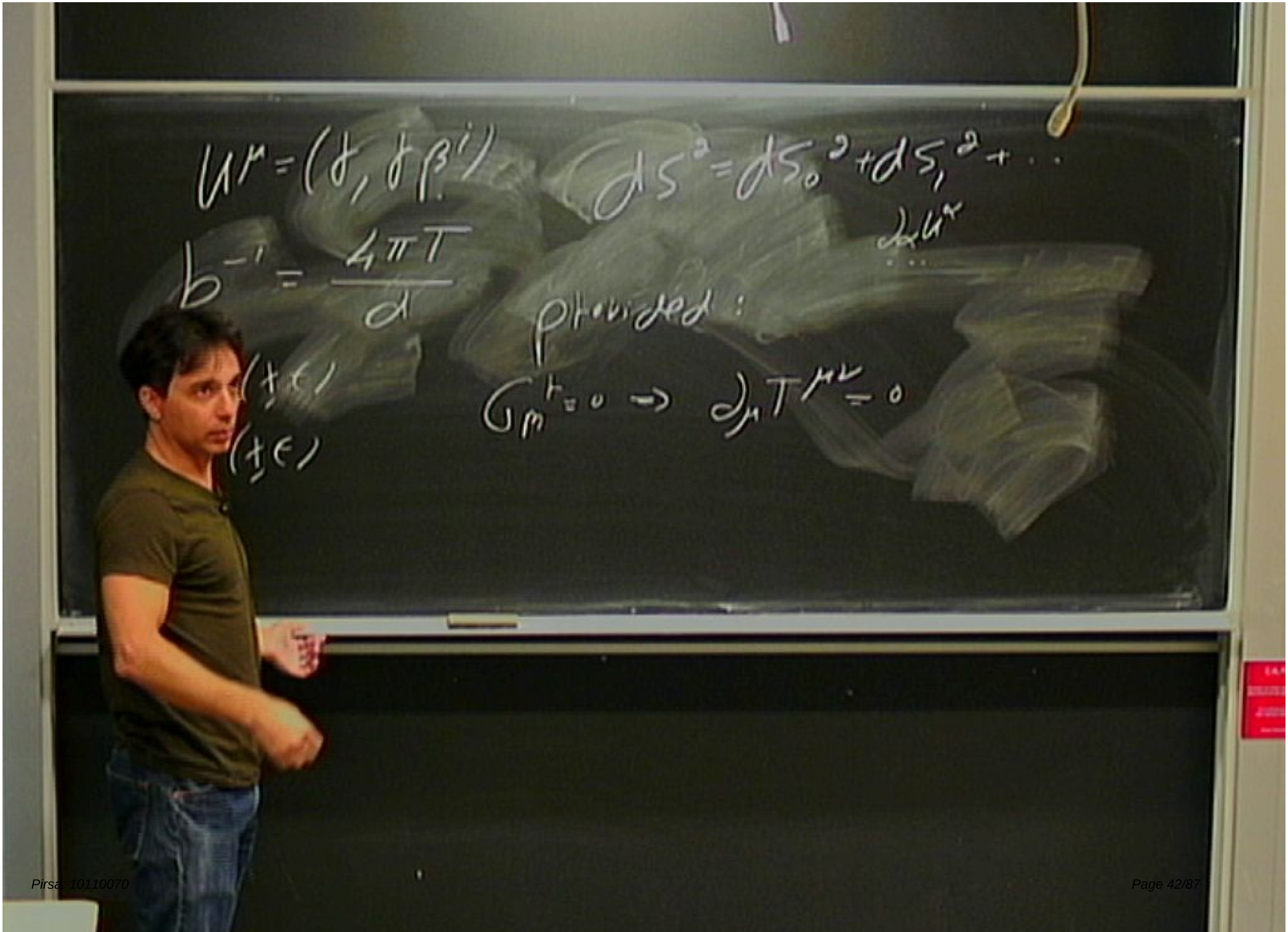

potentials
(d+1)-dim R_mn + d g_mn = 0
(x^μ, r)
ds² = -2u_μ dx^μ dr + r/(b^d r^{d-2}) u_μ u_ν dx^μ dx^ν
+ r² Σ_μν dx^μ dx^ν

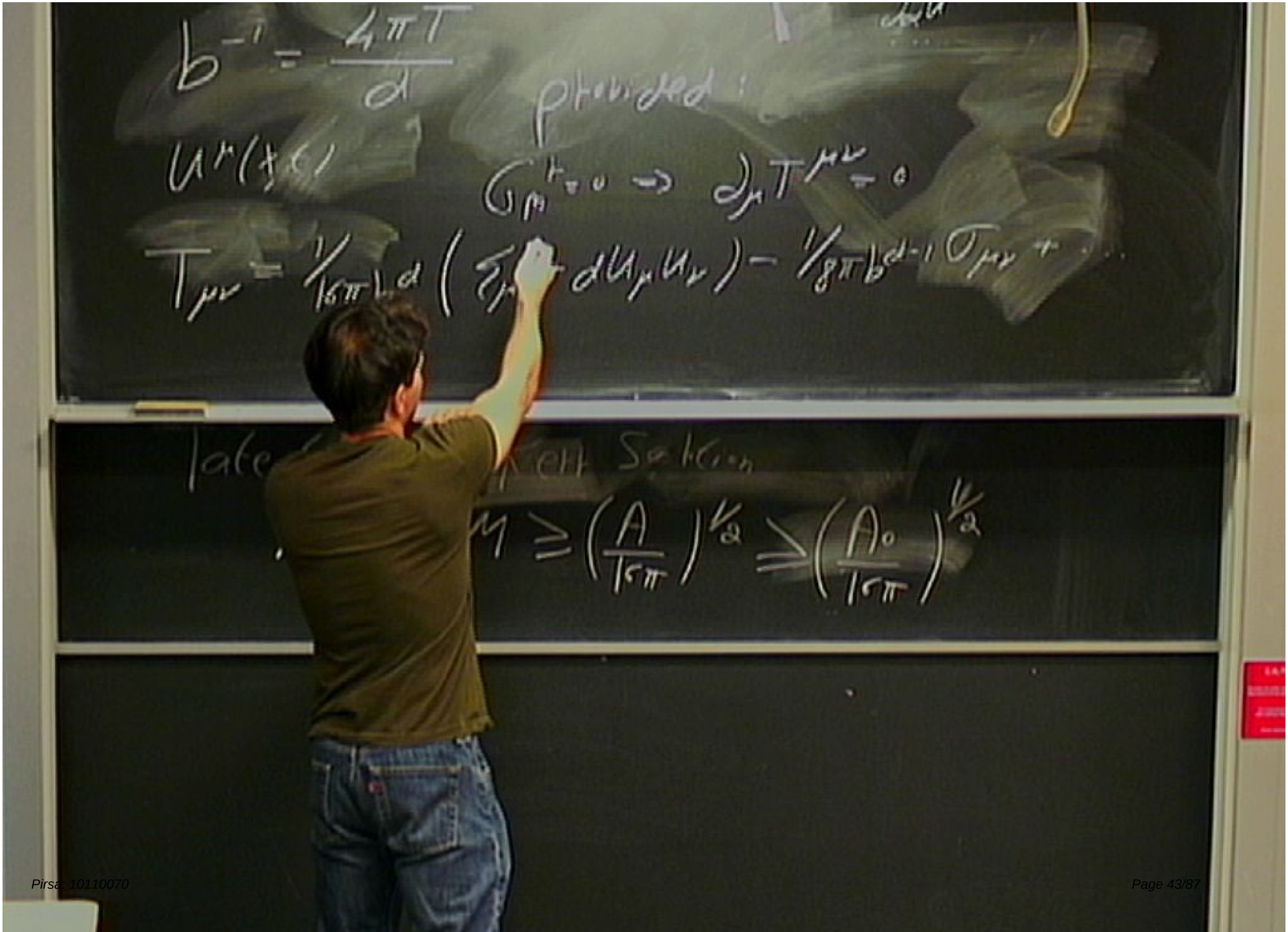

u^μ = (γ, γβ^i)
b^-1 = 4πT / d

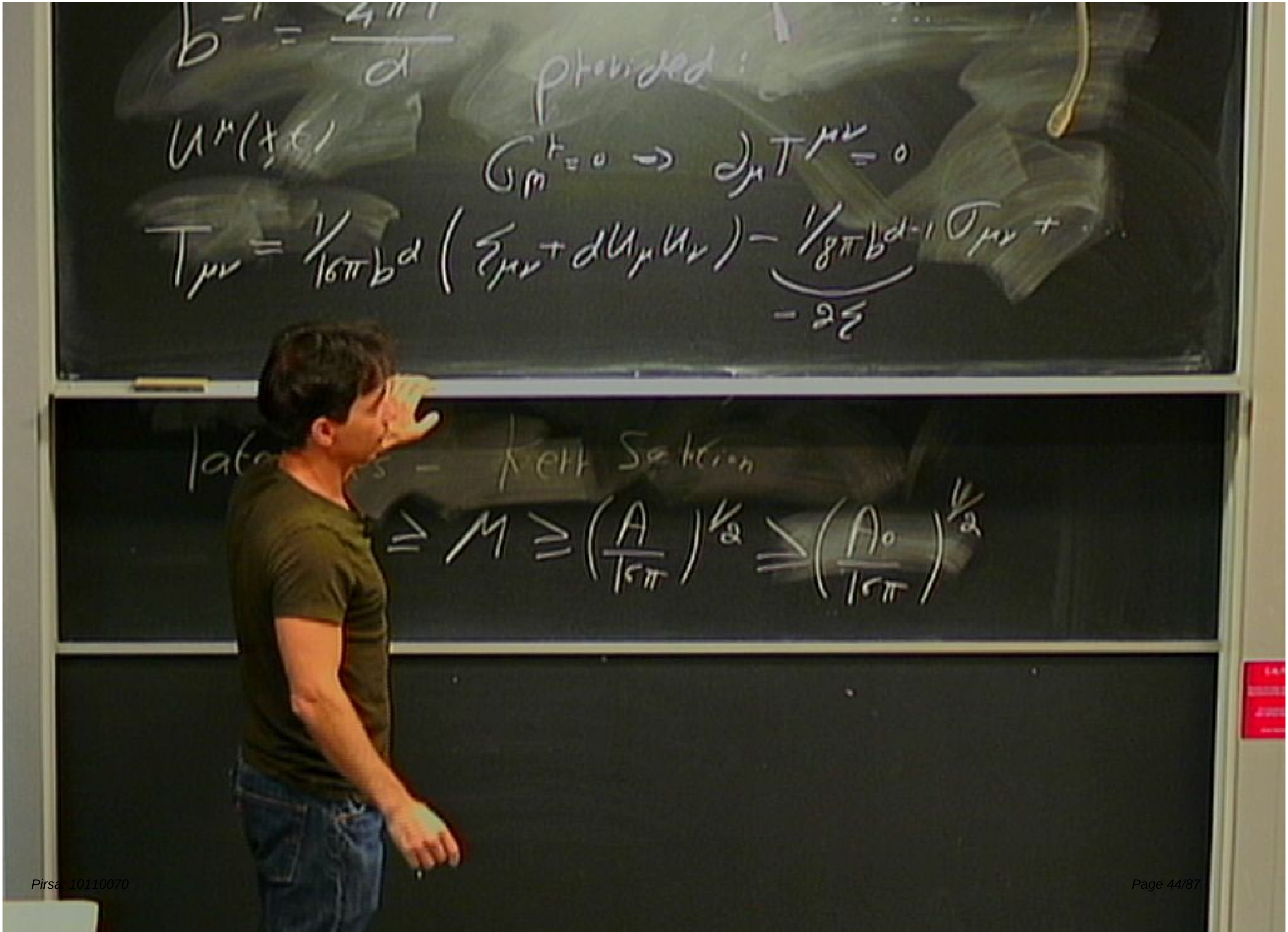

(d+1)-dim
R_mn + d g_mn = 0
(x^μ, r)
ds² = -2u_μ dx^μ dr + r²/(b^d r^{d-2}) u_μ u_ν dx^μ dx^ν
+ r² δ_μν dx^μ dx^ν
Solution: u^μ const
T const

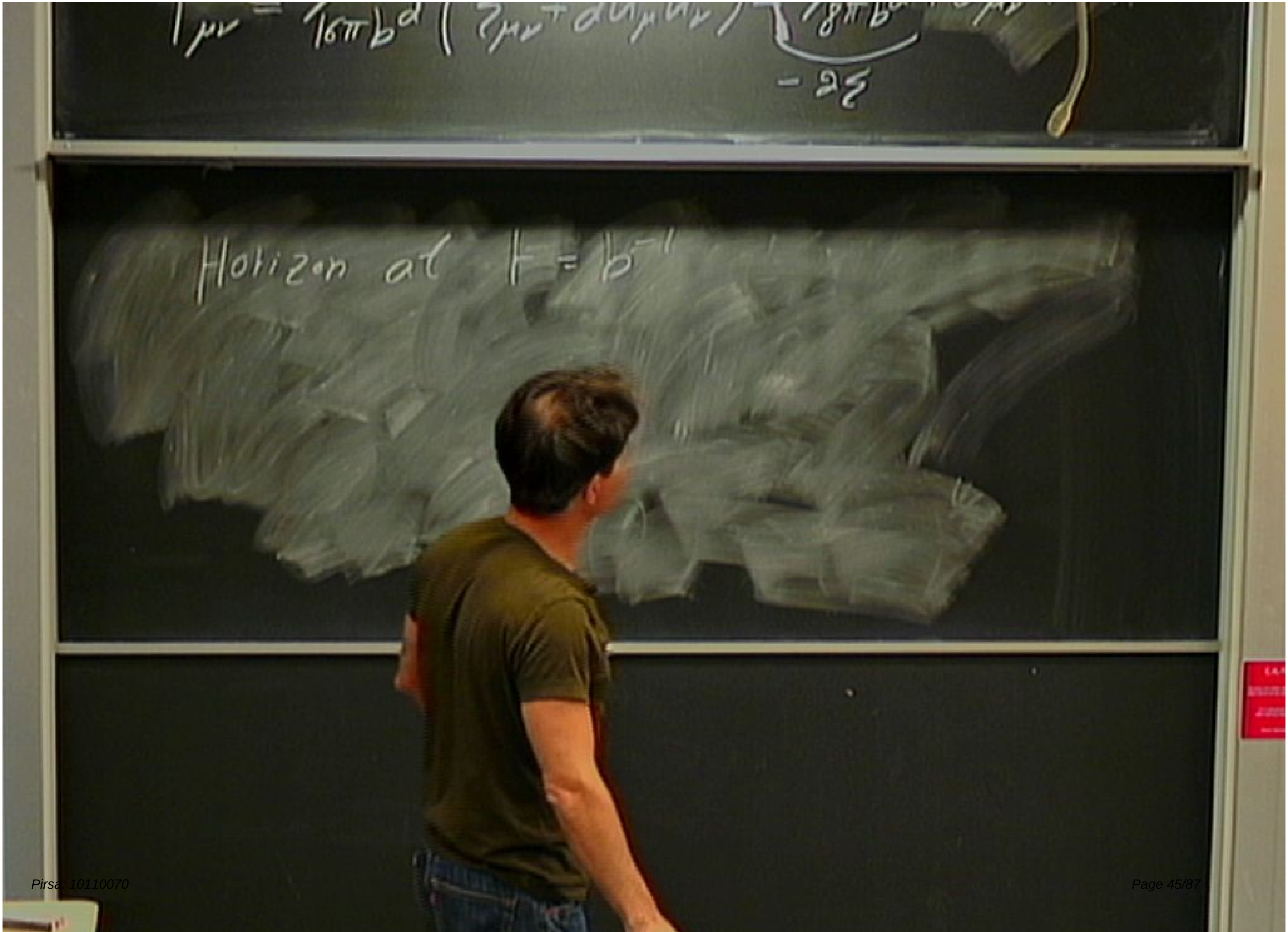

u^μ = (γ, γβ^i)
b^{-1} = 4πT / d
u^μ(t, ε)
T(t, ε)

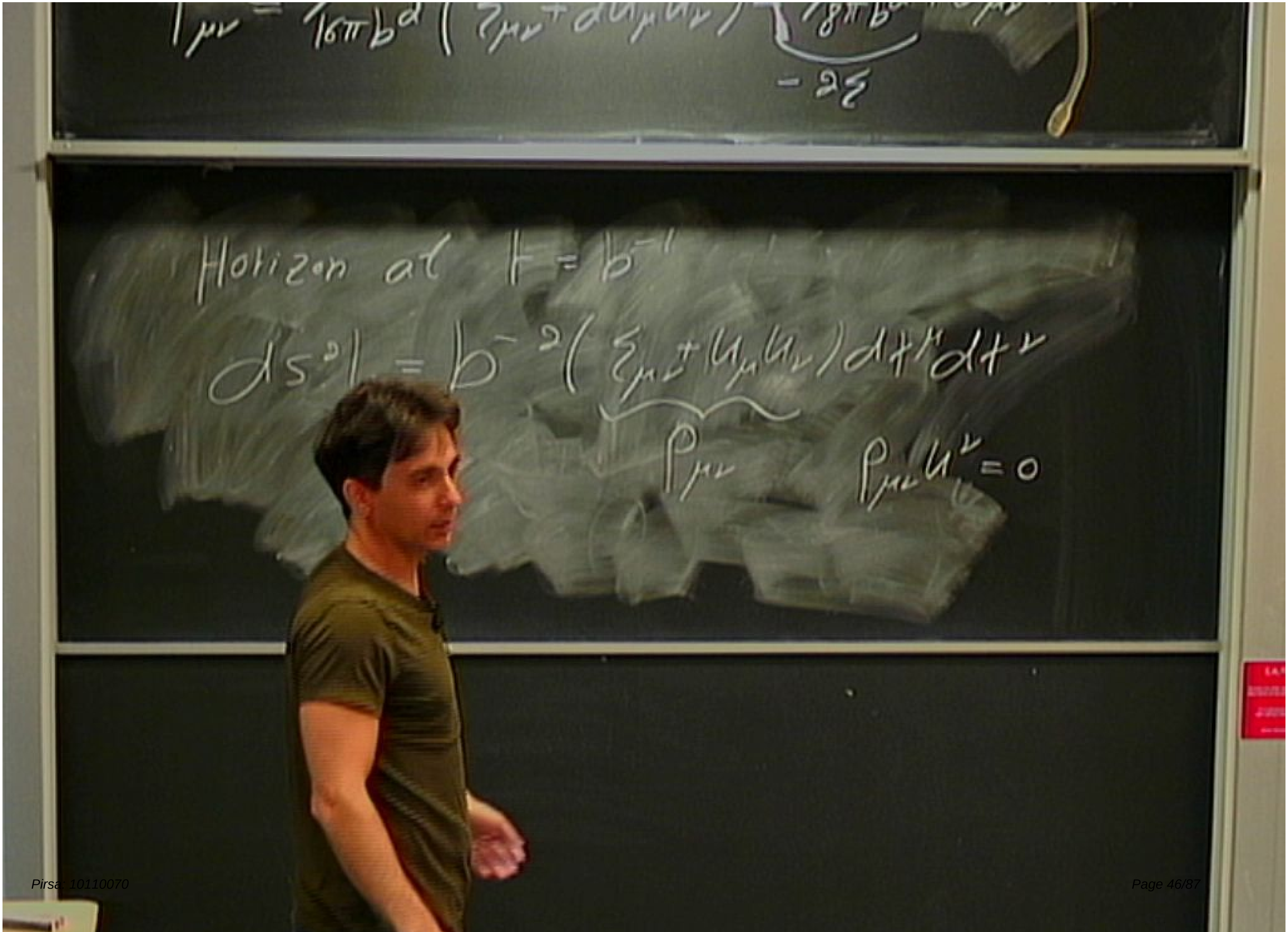

uμ = (γ, γβi)
ds² = ds0² + ds1² + ...
b⁻¹ = 4πT / d
provided:
uμ(t,ε)
T(t,ε)


u^μ = (γ, γβ^i)
ds^2 = ds_0^2 + ds_1^2 + . . .
b^-1 = 4πT / d
provided :
G_μ^ν = 0 ⇒ ∂_μ T^μν = 0


provided :


provided :
Kerr Solution


Horizon at r = b^-1



-2σ
Horizon at r = b^-1
ds^2| = b^-2(η_μν + u_μ u_ν) dx^μ dx^ν
P_μν
P_μν u^ν = 0

$$T_{\mu\nu} = \frac{1}{16\pi b^{d}}\left(\eta_{\mu\nu} + d\,u_\mu u_\nu\right) - \frac{1}{8\pi b^{\dots}} \dots$$

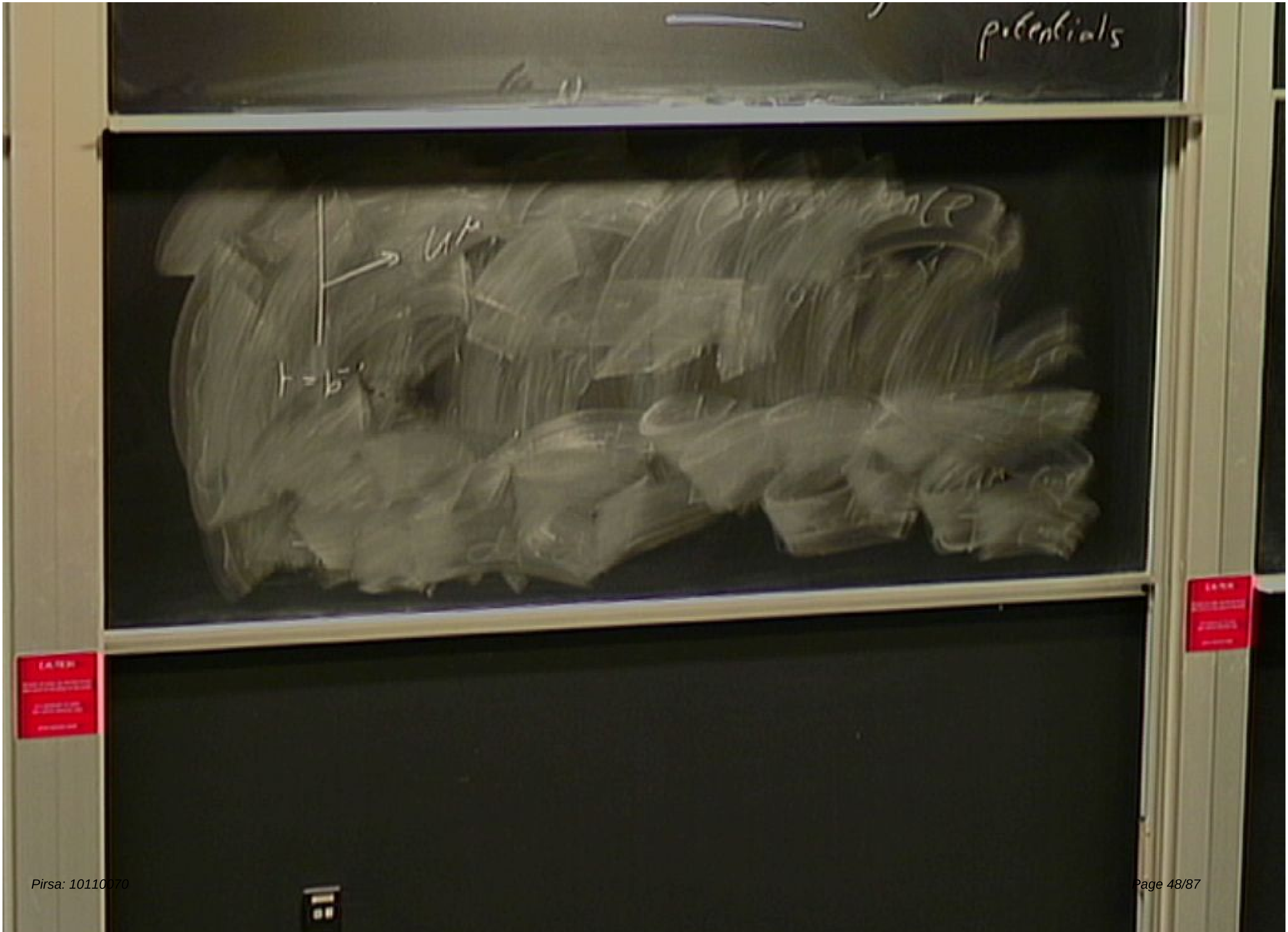
$$-2\xi$$

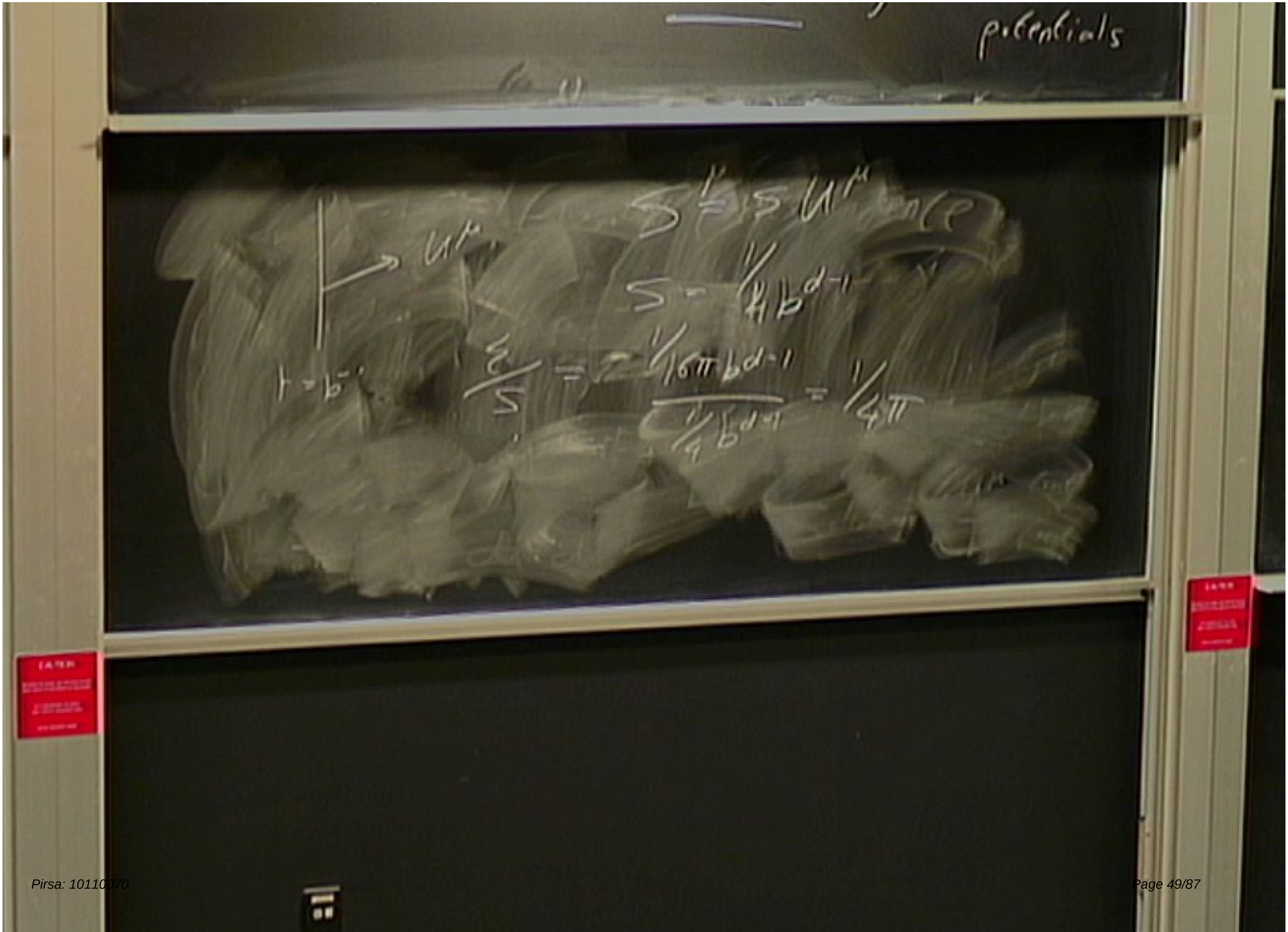
Horizon at $r = b^{-1}$ $\quad F = r - b^{-1}$

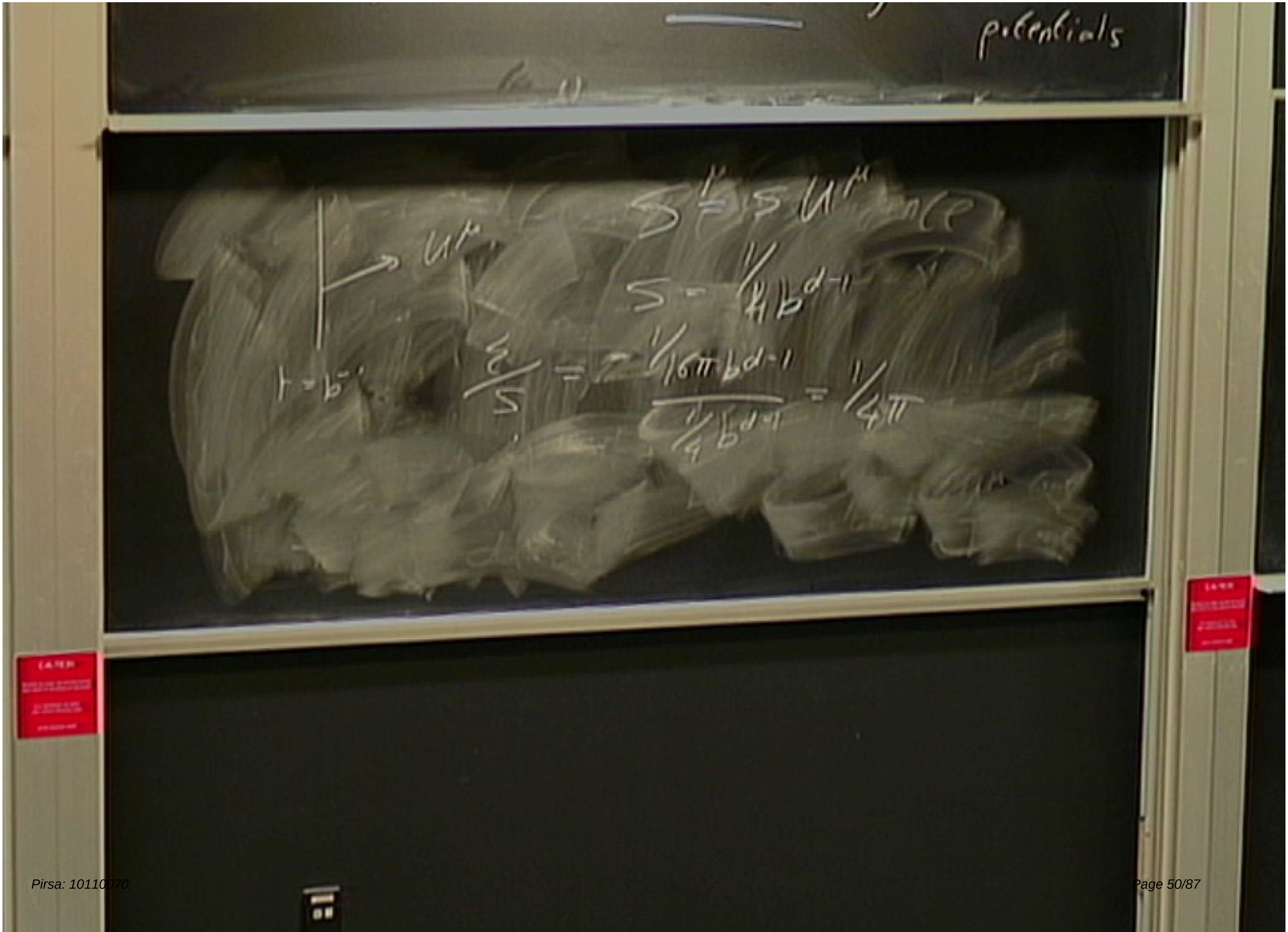
$$ds^2\big|_H = b^{-2}\underbrace{\left(\eta_{\mu\nu} + u_\mu u_\nu\right)}_{P_{\mu\nu}} dx^\mu dx^\nu$$

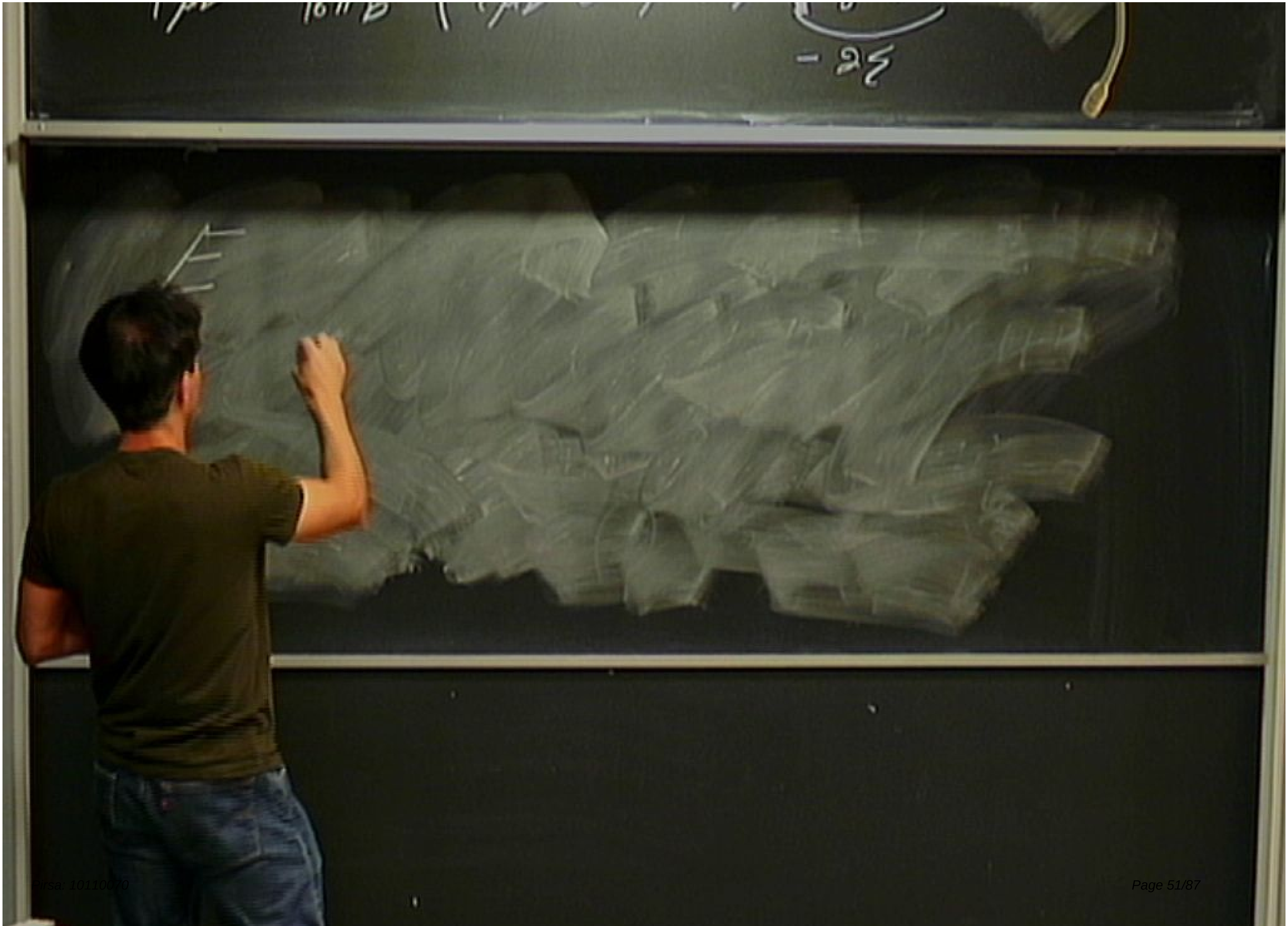
$$\ell^m = g^{mn}\partial_n F \qquad P_{\mu\nu} \qquad P_{\mu\nu}u^\nu = 0$$

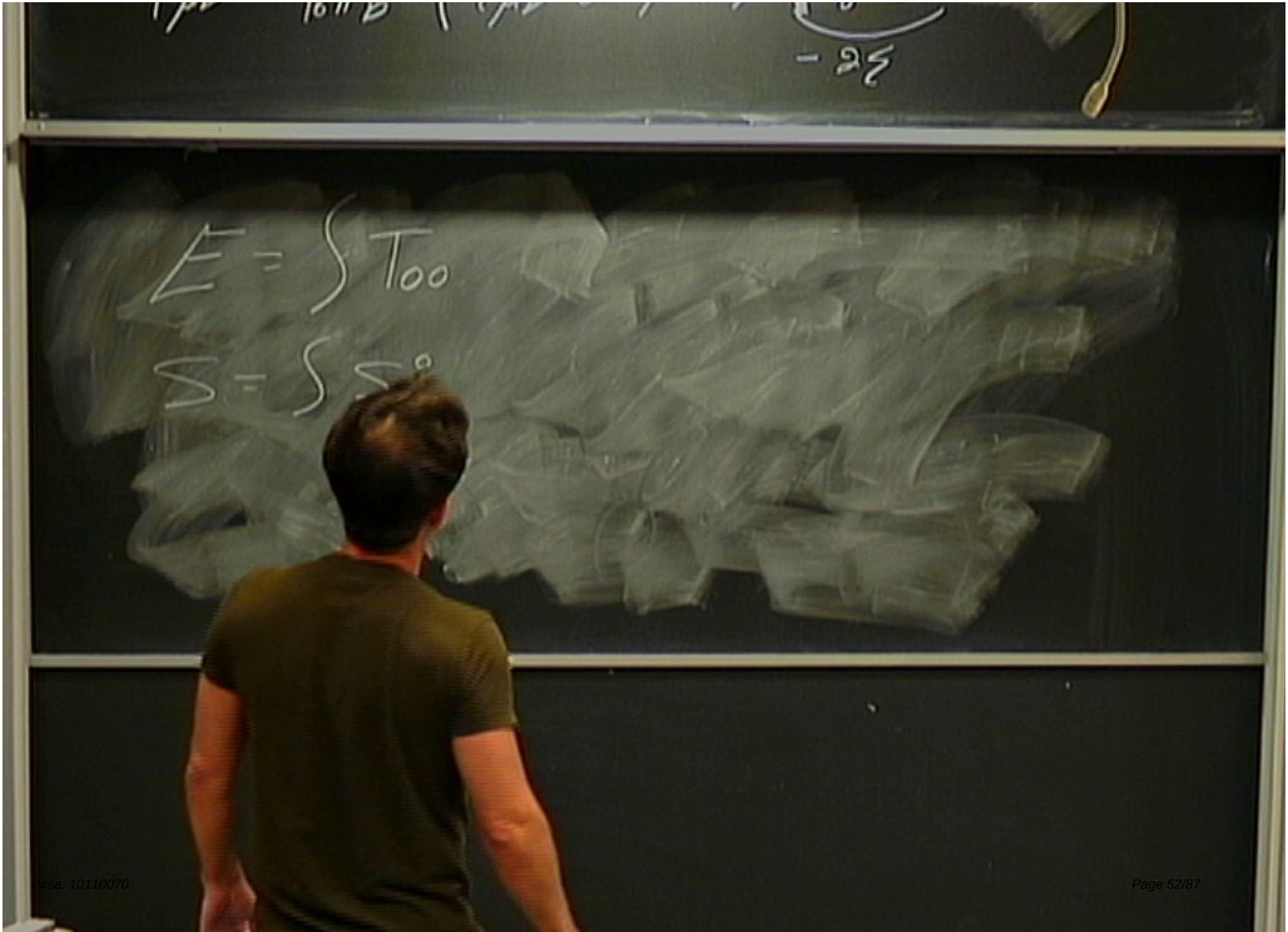
$$\ell^\mu = u^\mu$$

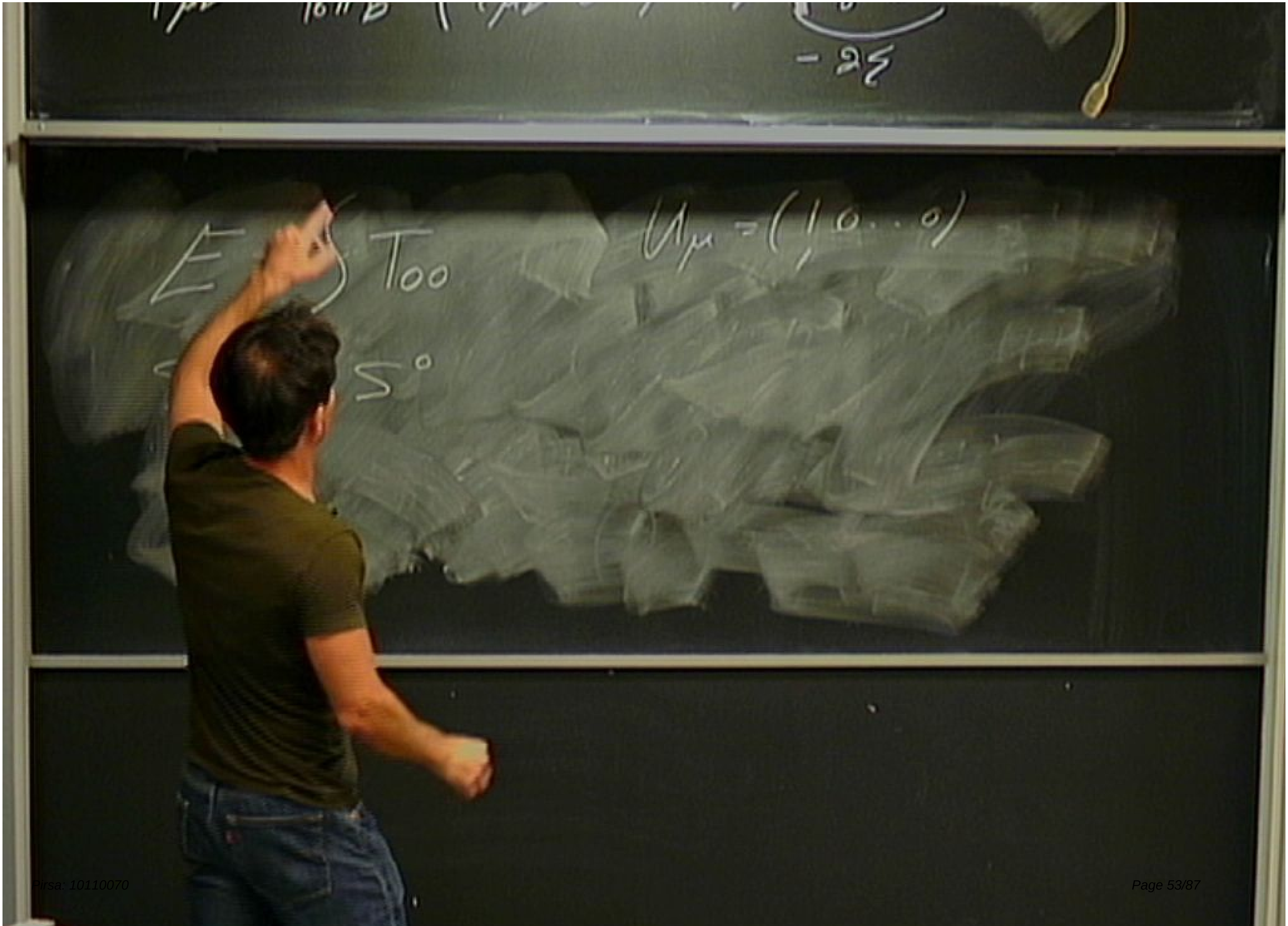

potentials

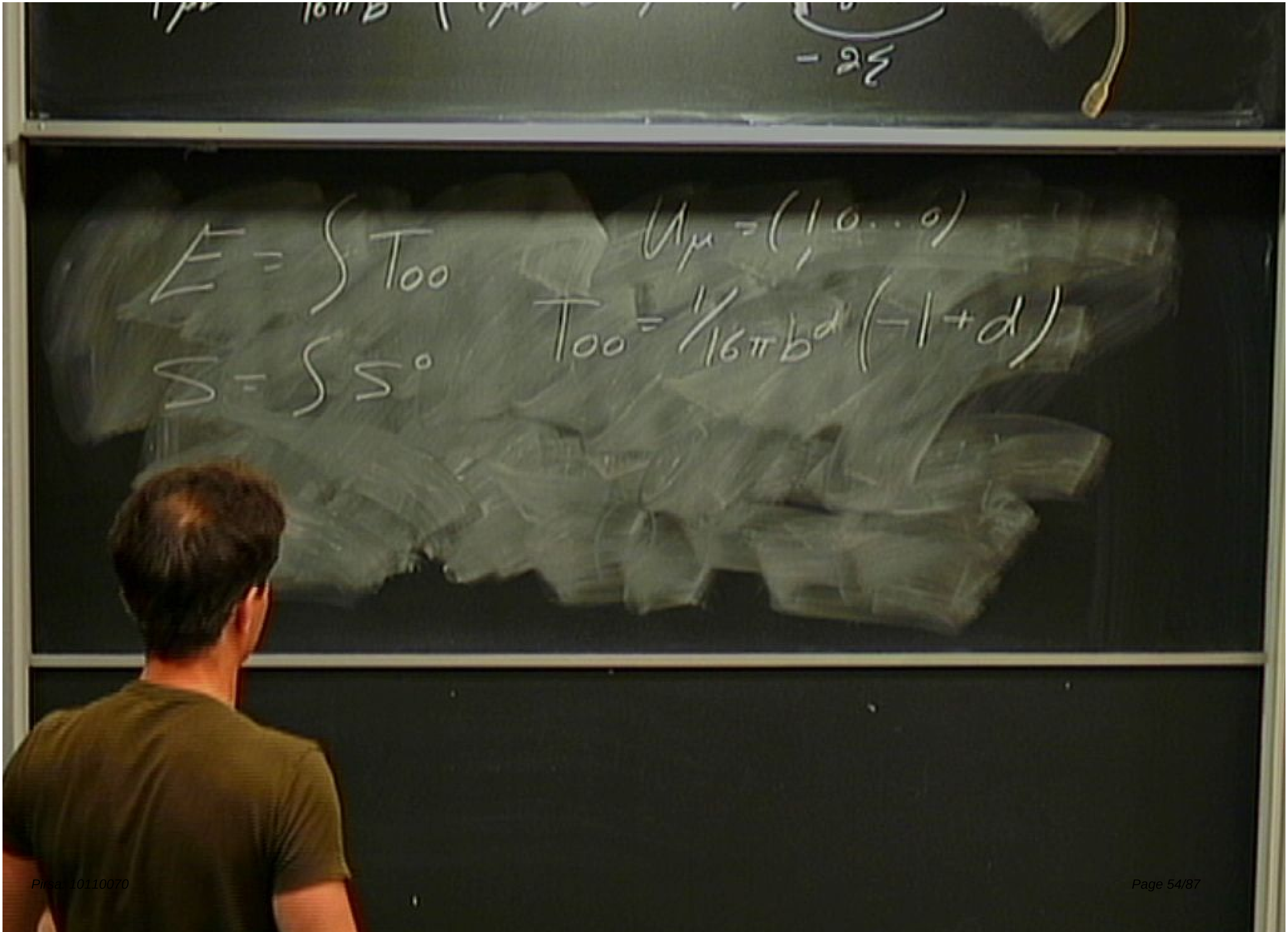

potentials

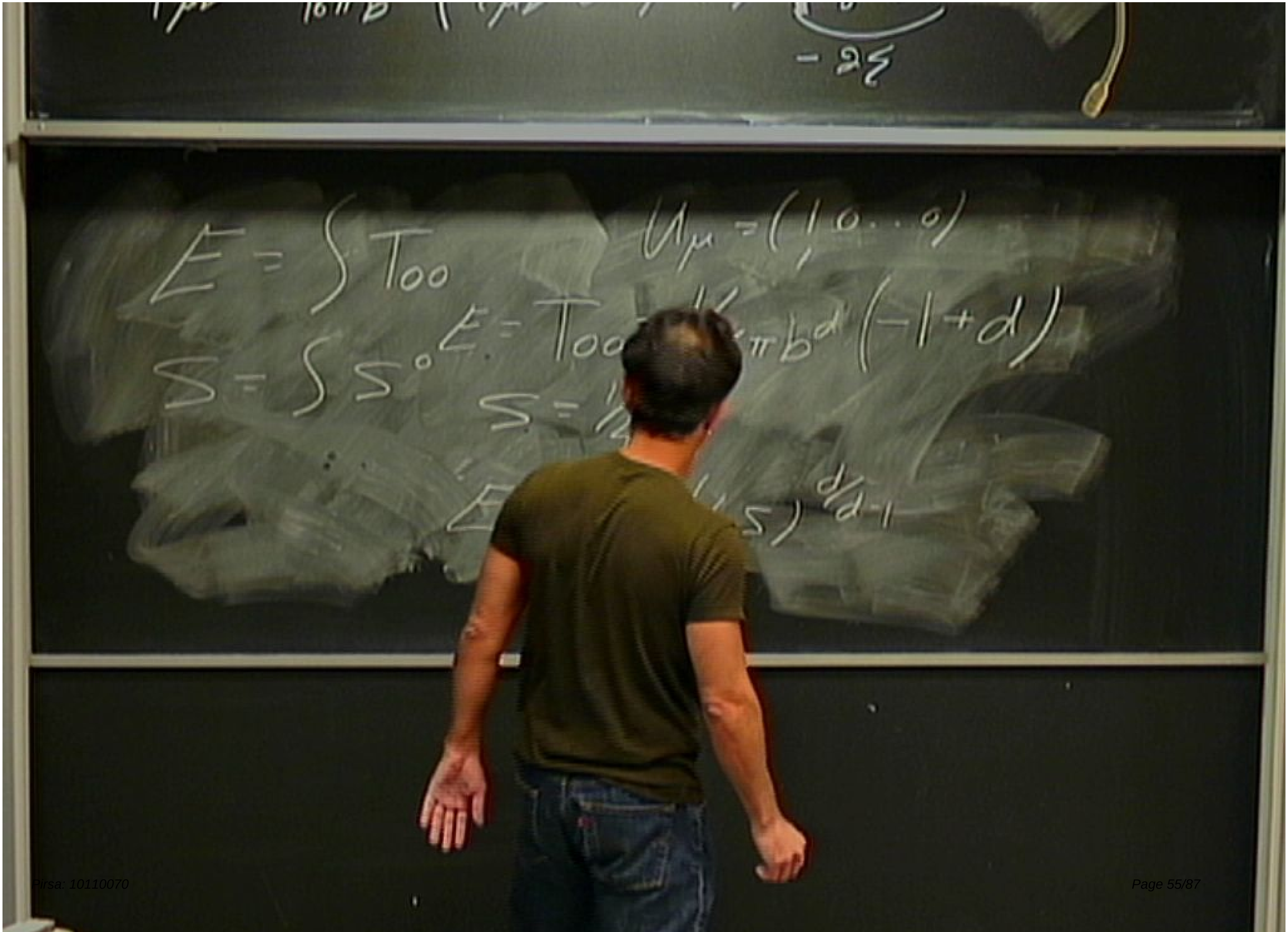

potentials

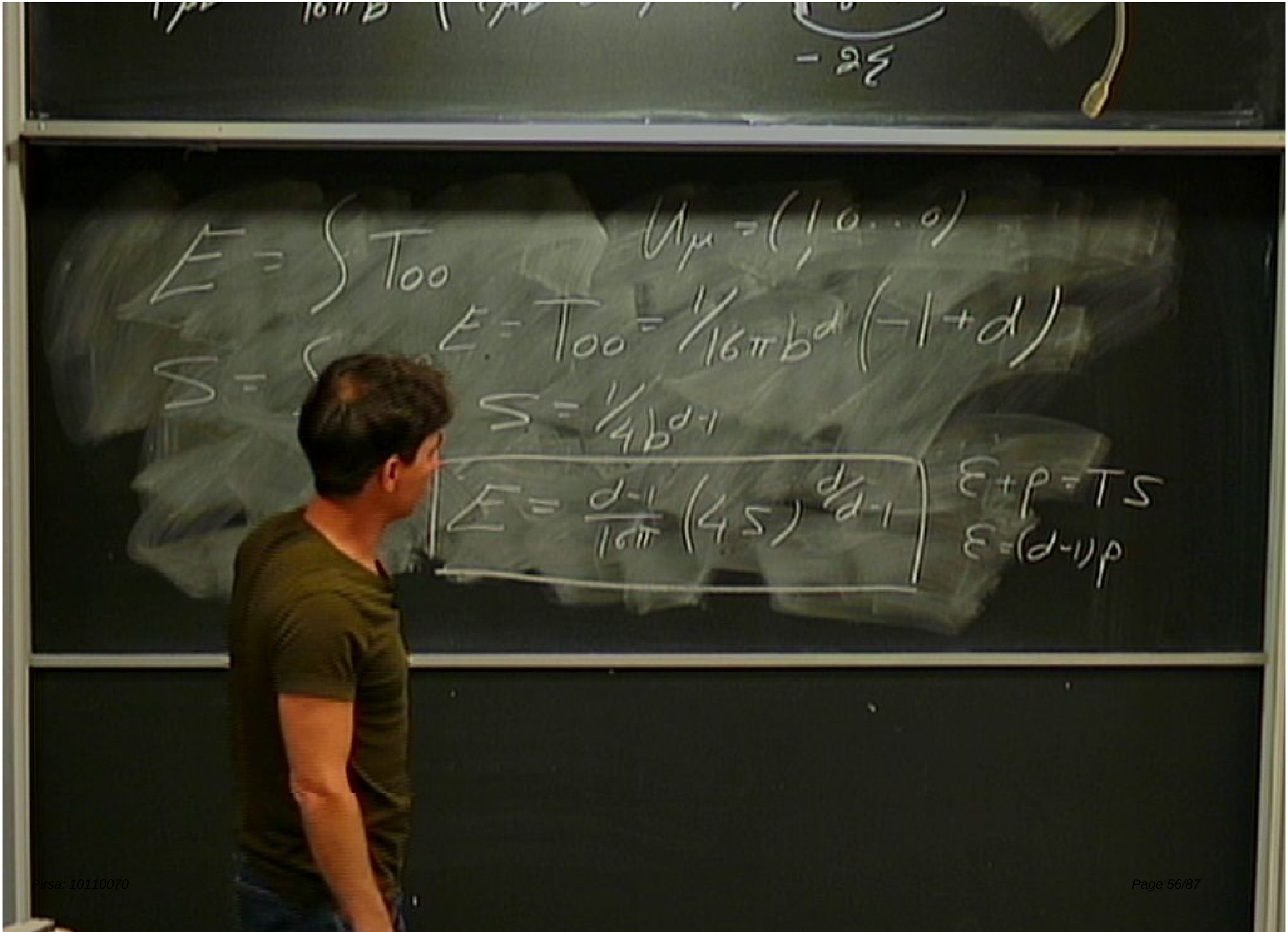



- 2ε
E = ∫ T00
Σ = ∫ S

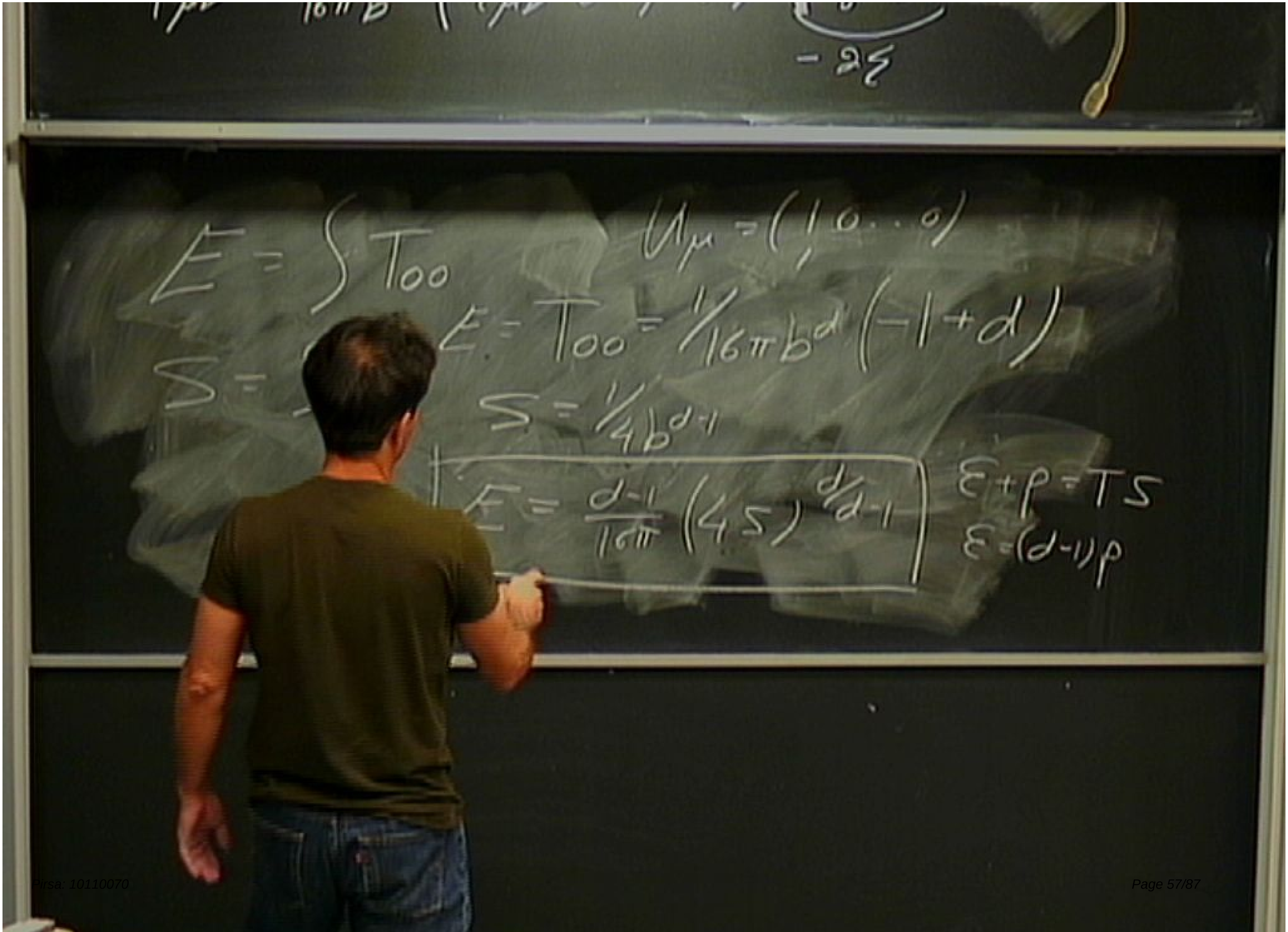

T00
u_μ = (1 0 . . 0)
- 2ξ


- 2ξ
E = ∫ T00
u_μ = (1, 0 . . 0)
T00 = 1/16πb^d (-1 + d)
S = ∫ s^0


- 2ξ
E = ∫ T00
u_μ = (1, 0 . . 0)
S = ∫ s^0



- 2ξ
E = ∫ T00
u_μ = (1, 0 . . . 0)
E = T00 = 1/16πb^d (-1 + d)
S = 1/4b^{d-1}
E = (d-1)/16π (4S)^{d/d-1}
ε + p = TS
ε = (d-1)p


- 2ξ
E = ∫ T00
uμ = (1, 0 . . 0)
E = T00 = 1/16πb^d (-1+d)
S =
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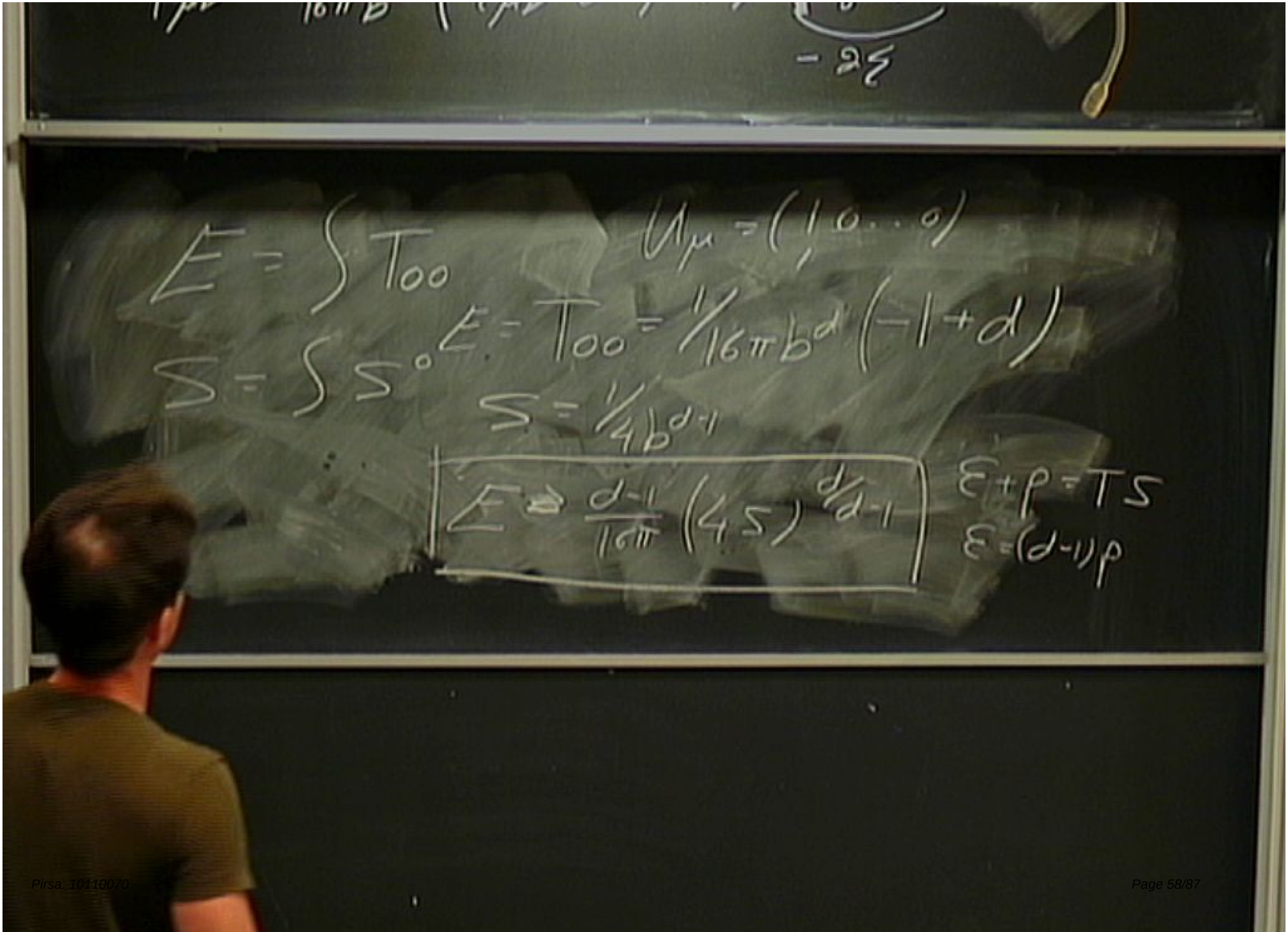

E = ∫ T00
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− 2ξ

-2ξ

$$E = \int T_{00} \qquad u_\mu = (1, 0 \ldots 0)$$

$$S = \int s_0 \qquad \varepsilon = T_{00} = \frac{1}{16\pi b^d}(-1+d)$$

$$s = \frac{1}{4 b^{d-1}}$$

$$\boxed{E = \frac{d-1}{16\pi}(4S)^{d/d-1}} \qquad \begin{aligned} \varepsilon + p &= TS \\ \varepsilon &= (d-1)p \end{aligned}$$

$$E = \int T_{00} \qquad u_\mu = (1, 0 \ldots 0)$$

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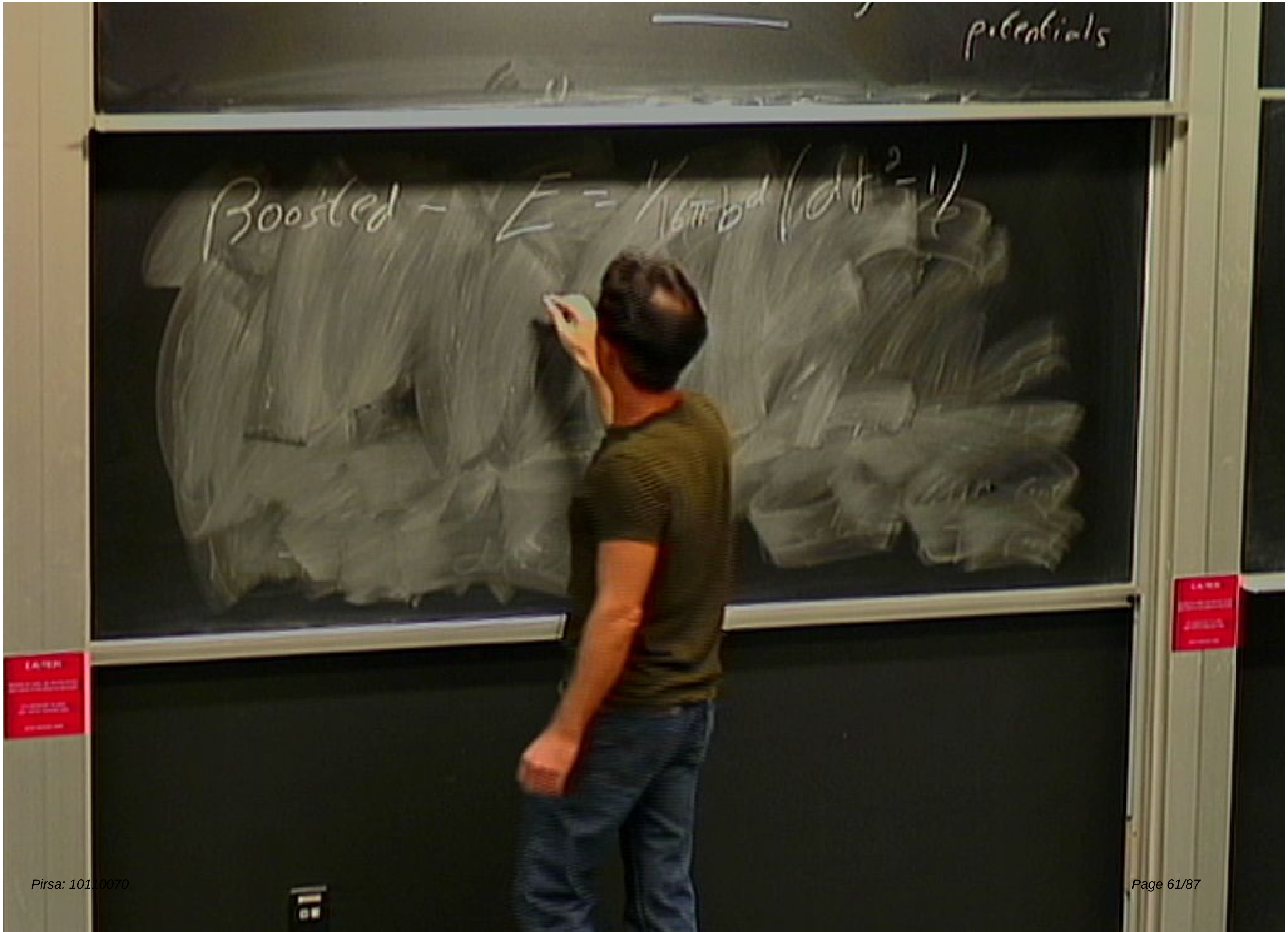
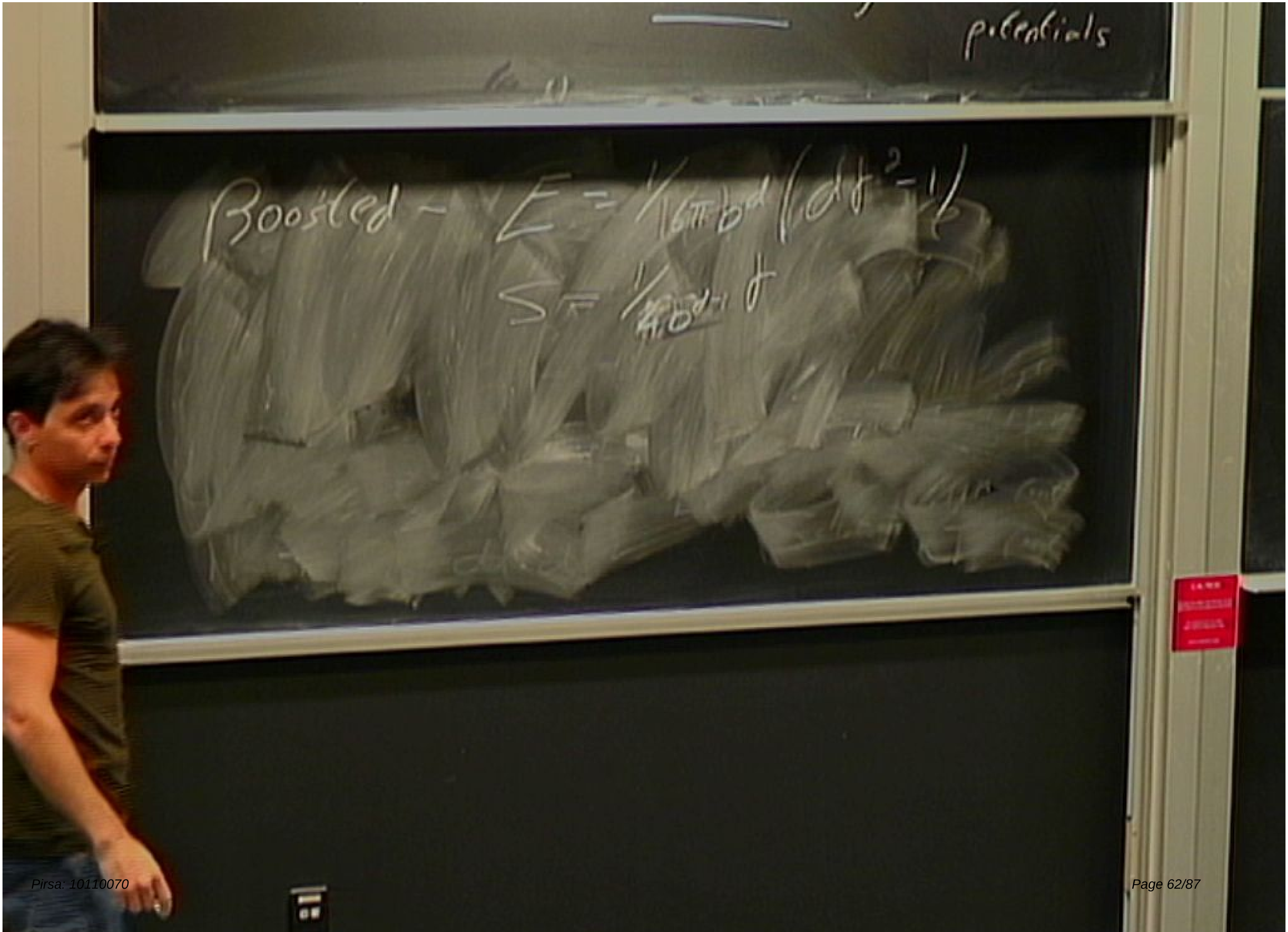
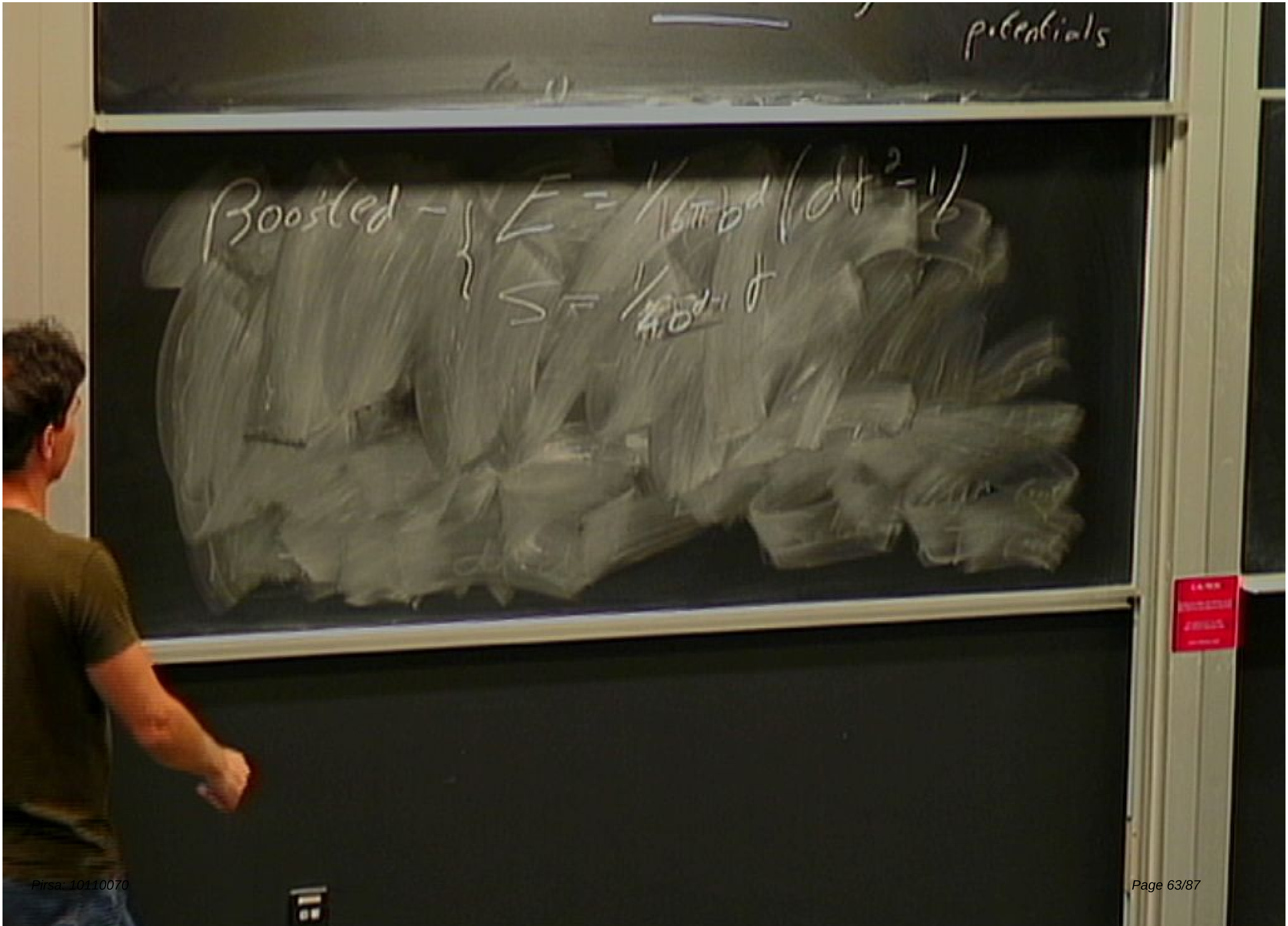
$$s = \frac{1}{4 b^{d-1}}$$

$$\boxed{E = \frac{d-1}{16\pi}(4S)^{d/d-1}} \qquad \begin{array}{l} \varepsilon + p = Ts \\ \varepsilon = (d-1)p \end{array}$$


potentials
Boosted


potentials
Boosted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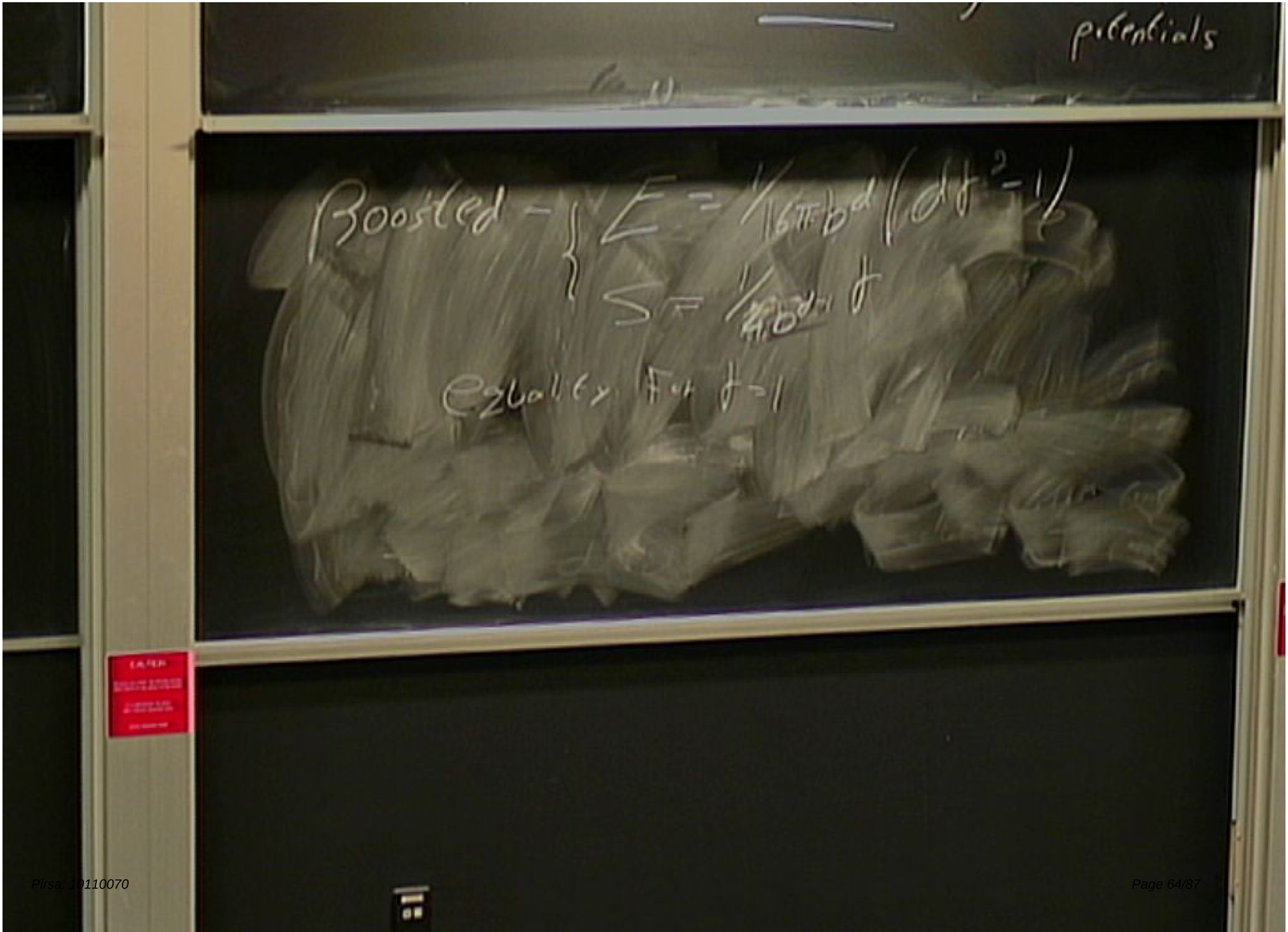
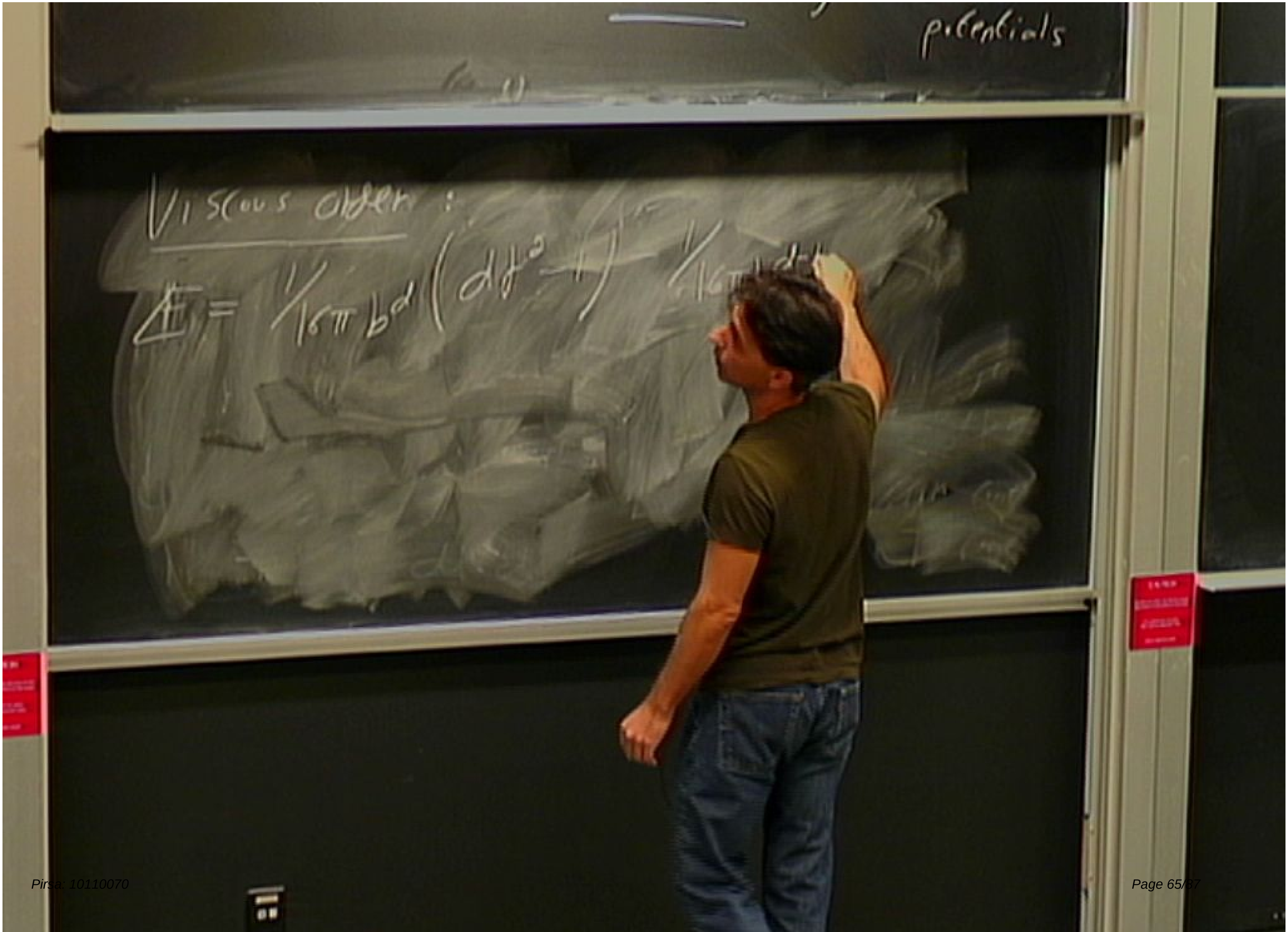
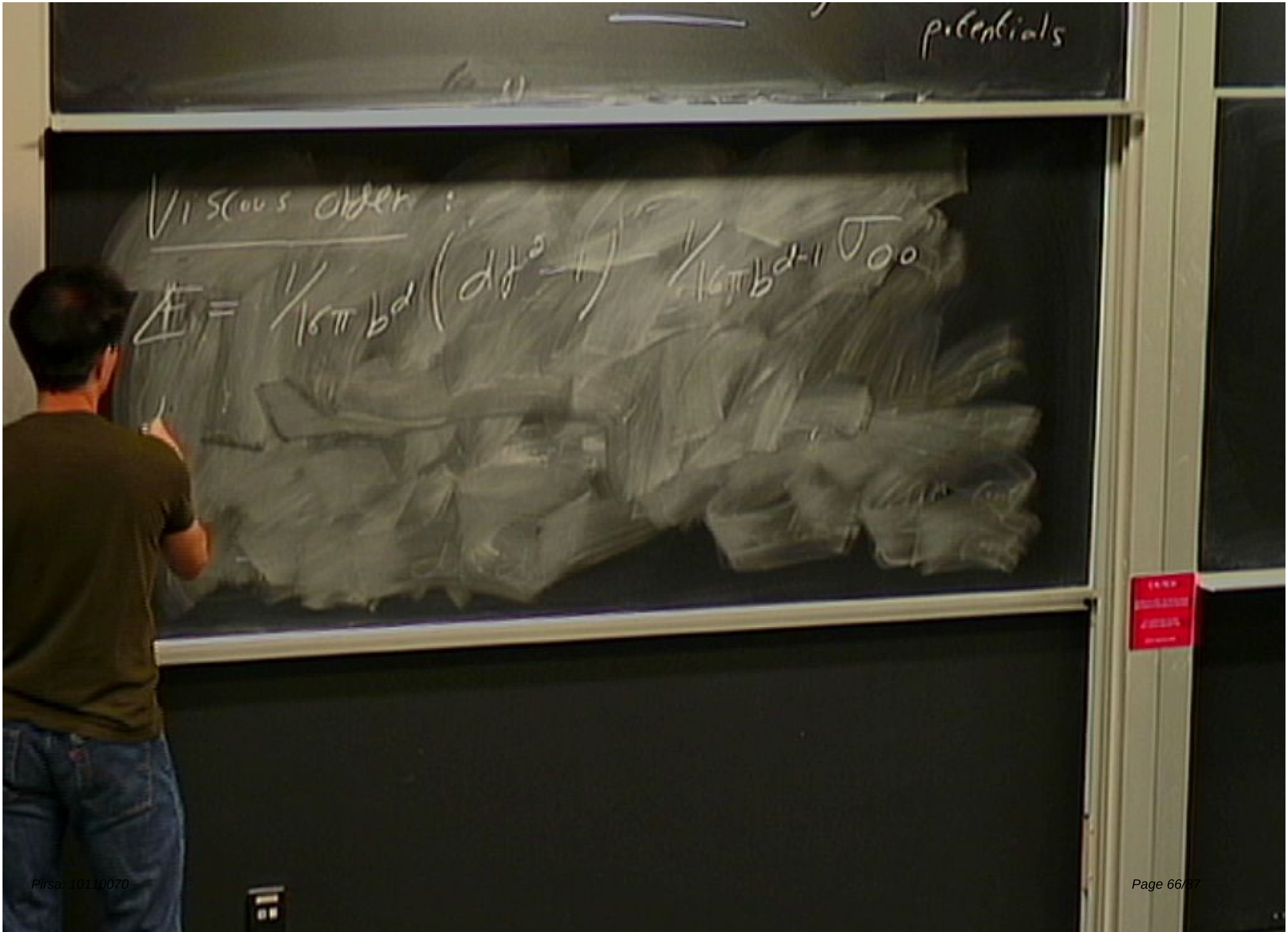
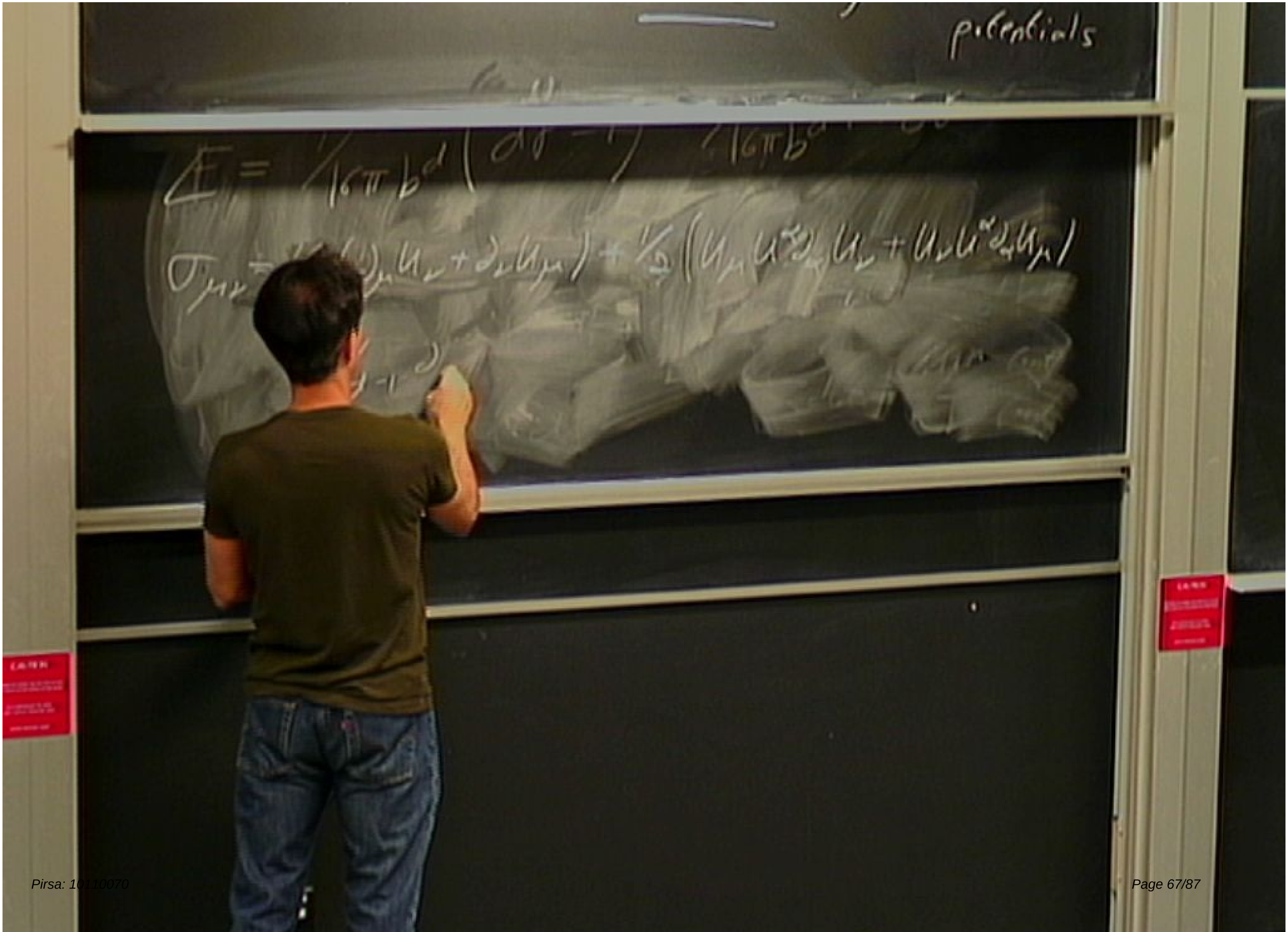
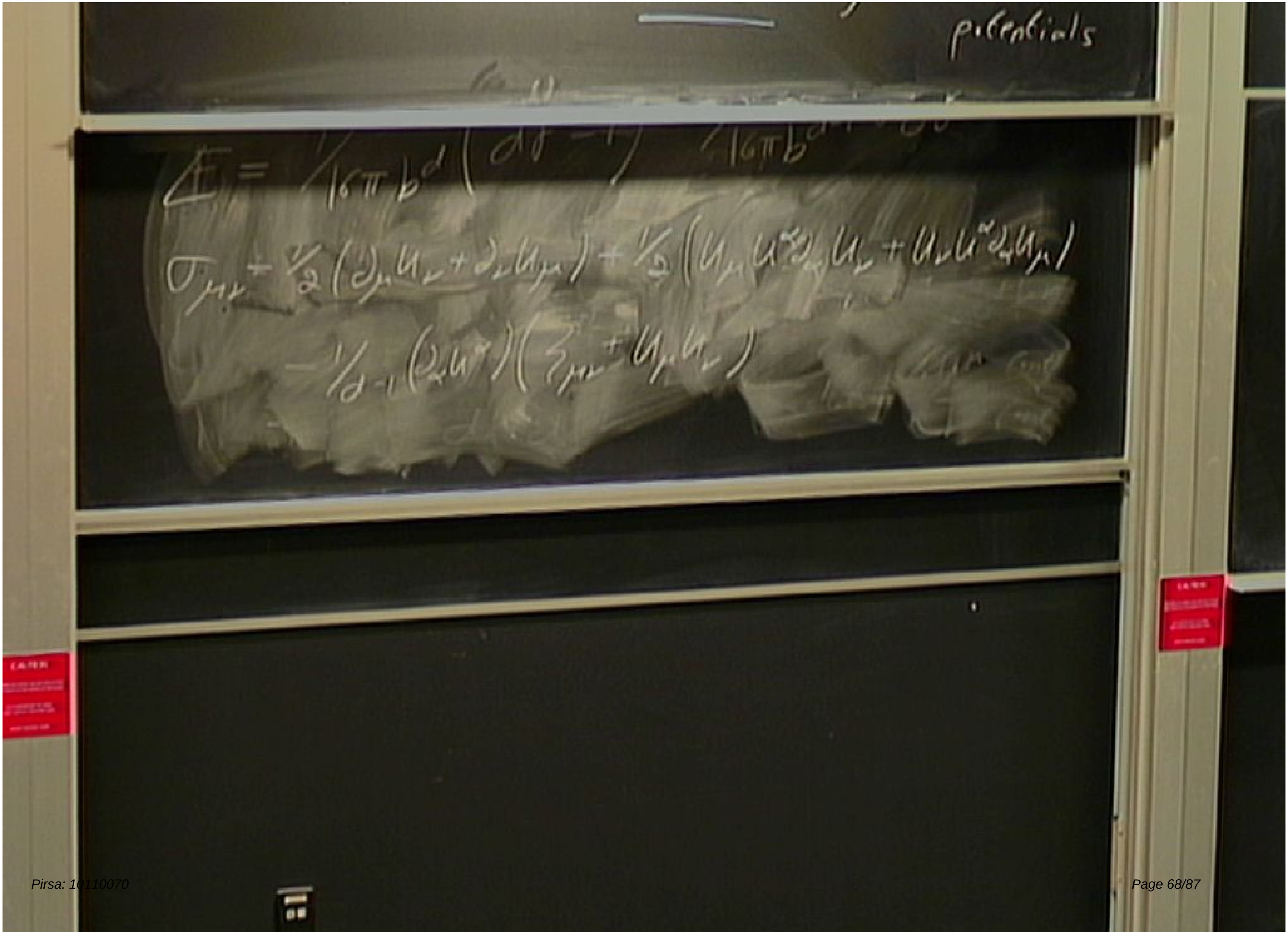
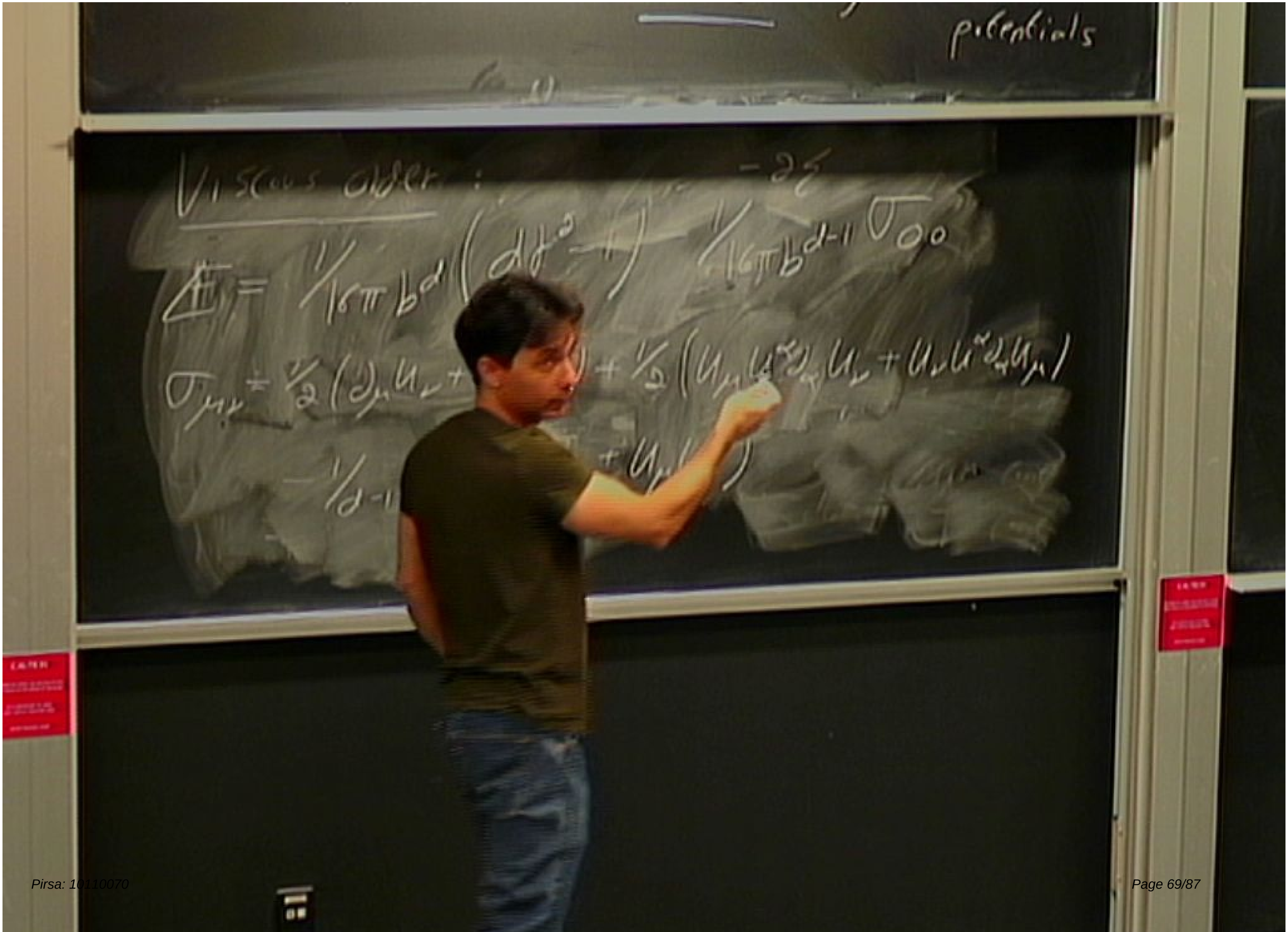

potentials
Boosted
equality for


potentials
Viscous order :


potentials
Viscous order :


potentials


potentials
$\sigma_{\mu\nu} = \frac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu) + \frac{1}{2}(u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu)$
$- \frac{1}{d-1}(\partial_\alpha u^\alpha)(\eta_{\mu\nu} + u_\mu u_\nu)$


potentials
Viscous order :

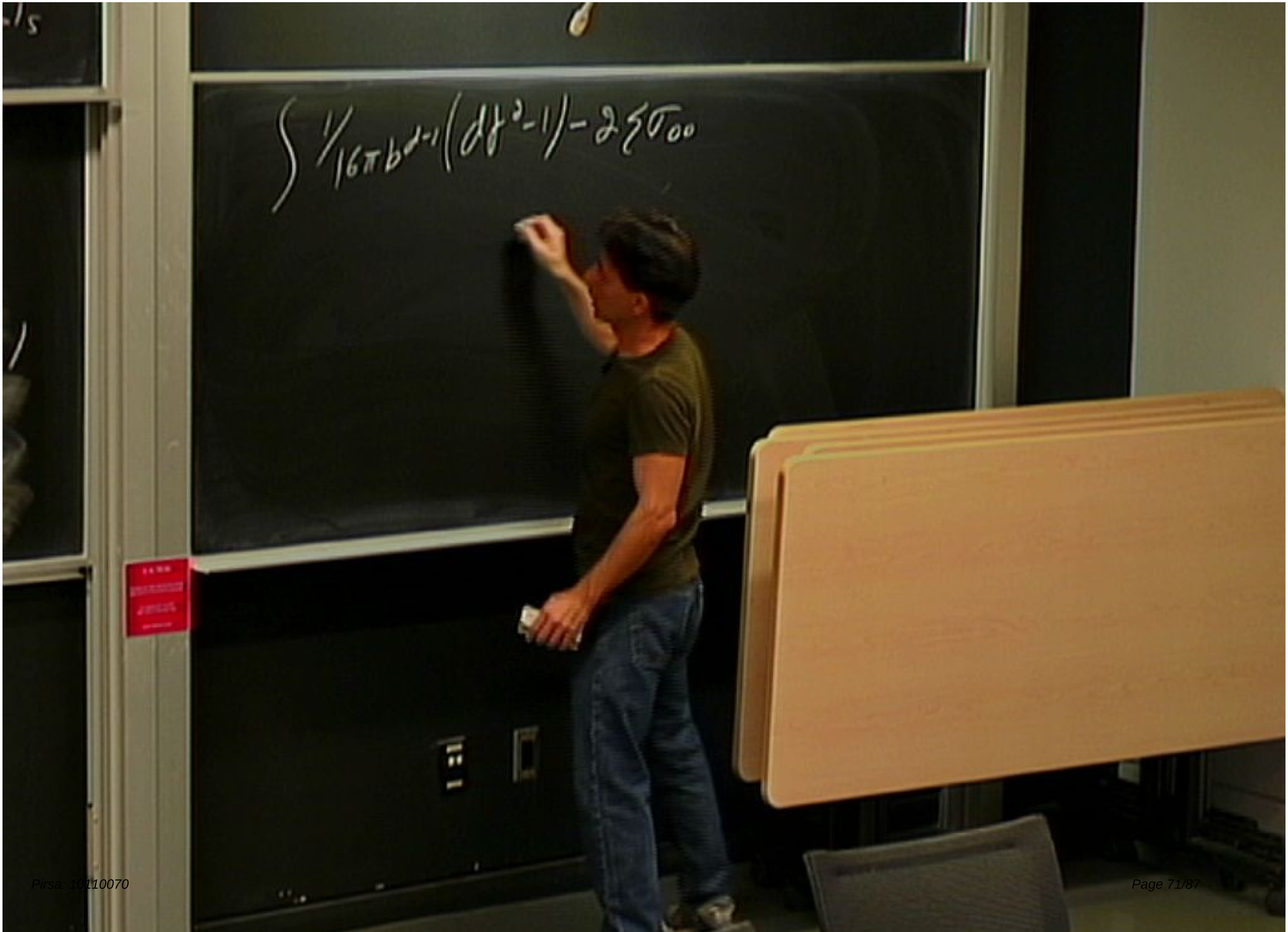
potentials

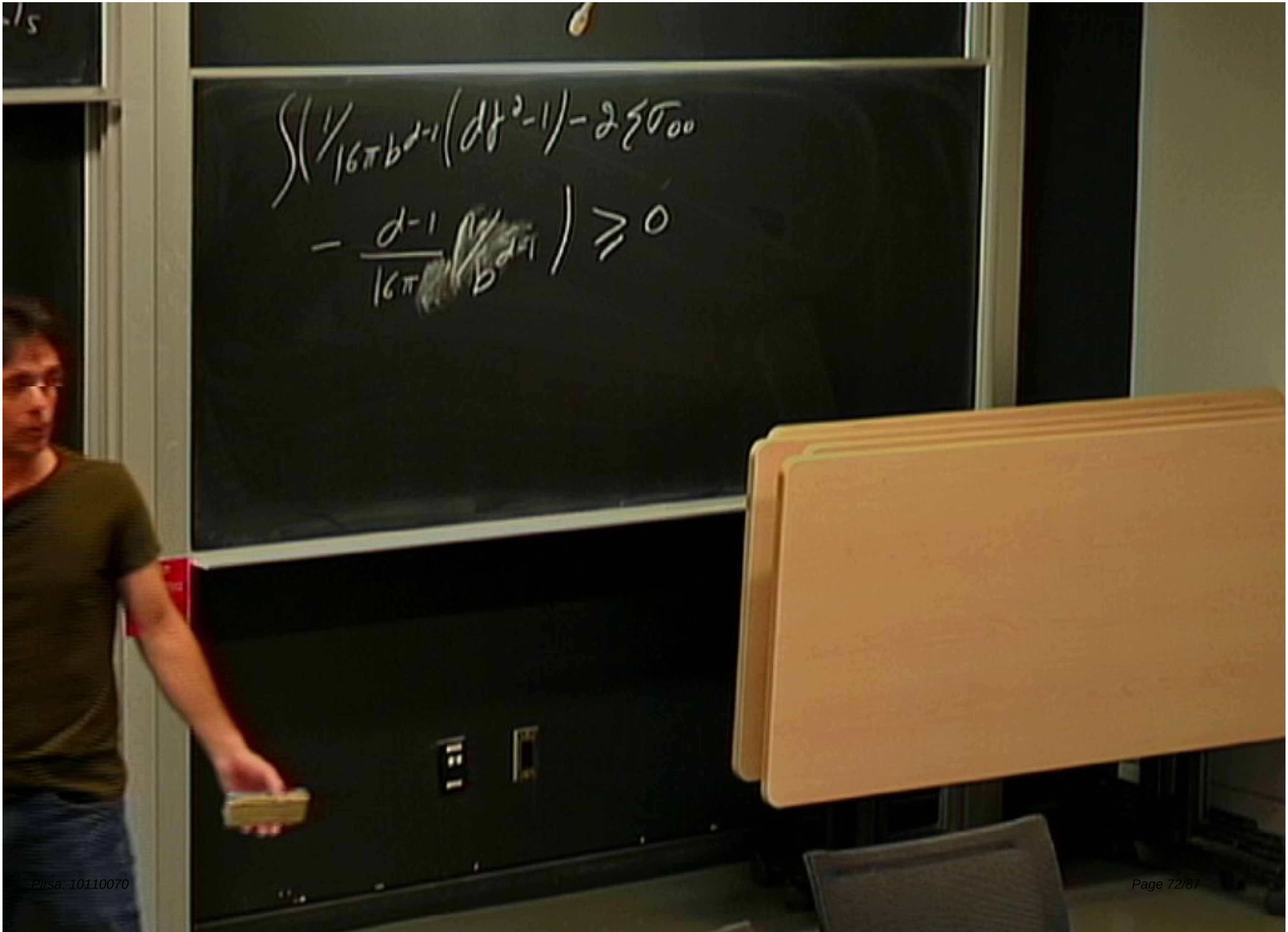
Viscous order:

$$-\partial \xi$$

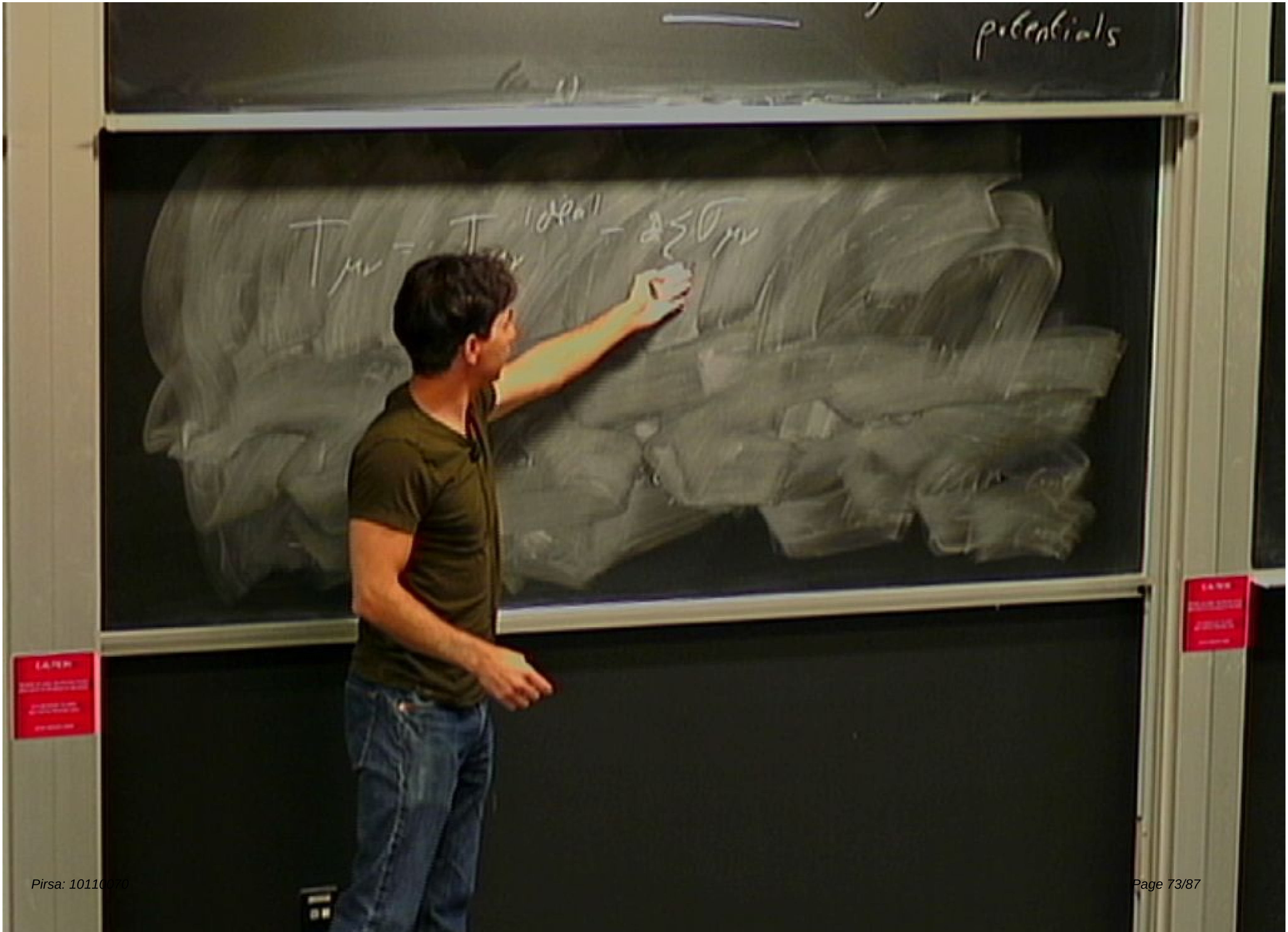
$$\Delta \Xi = \frac{1}{16\pi b^{d}}\left(d \gamma^{0} - 1\right) \qquad \frac{1}{16\pi b^{d-1}}\,\sigma_{00}$$

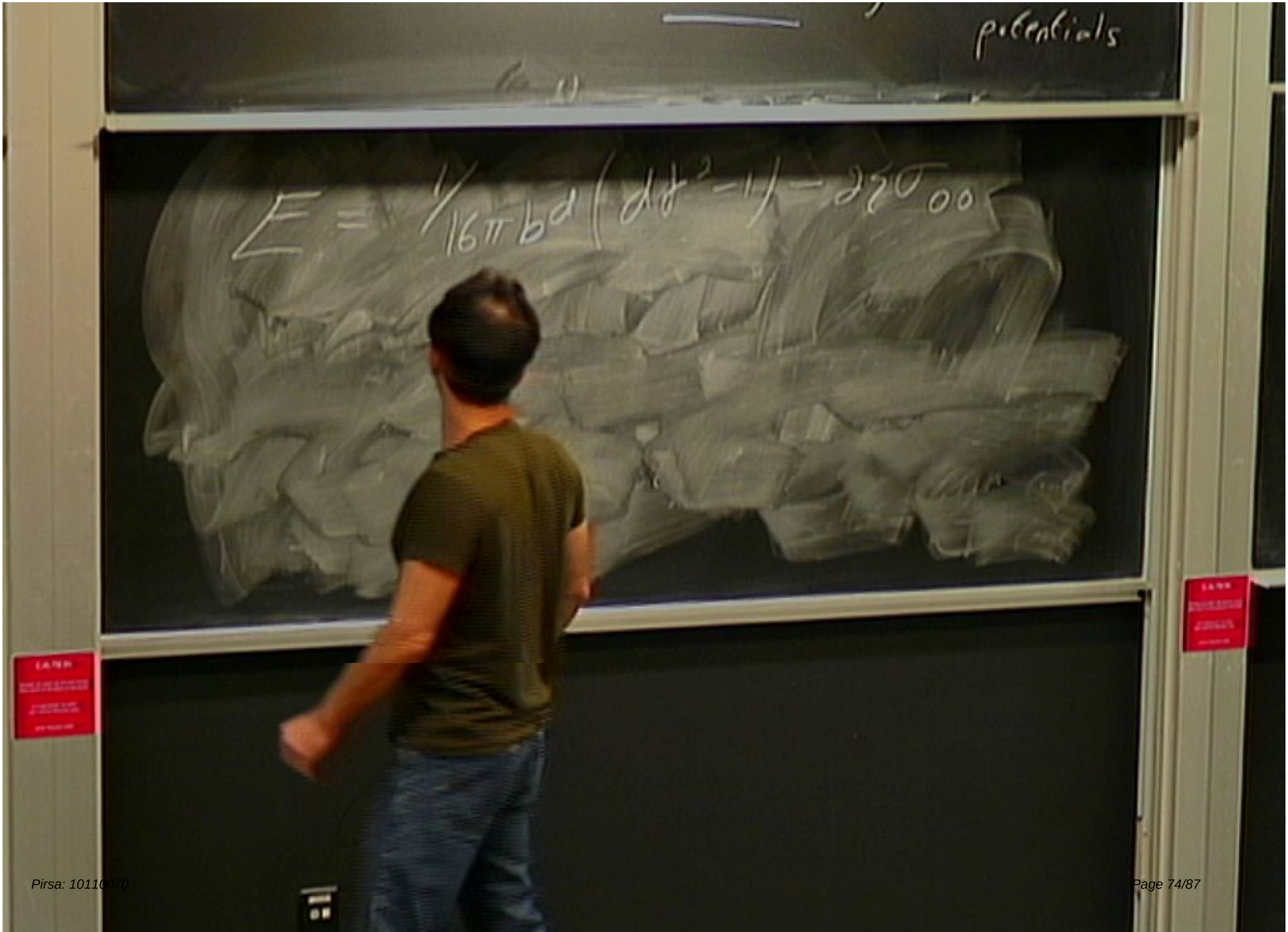
$$\sigma_{\mu\nu} = \frac{1}{2}\left(\partial_\mu u_\nu + \partial_\nu u_\mu\right) + \frac{1}{2}\left(u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu\right) - \frac{1}{d-1}\left(\partial_\alpha u^\alpha\right)\left(\eta_{\mu\nu} + u_\mu u_\nu\right)$$





$\int\left(\frac{1}{16\pi b^{d-1}}\left(d\gamma^2-1\right)-2\zeta\sigma_{00}\right.$
$\left.-\frac{d-1}{16\pi}\right)\geq 0$


potentials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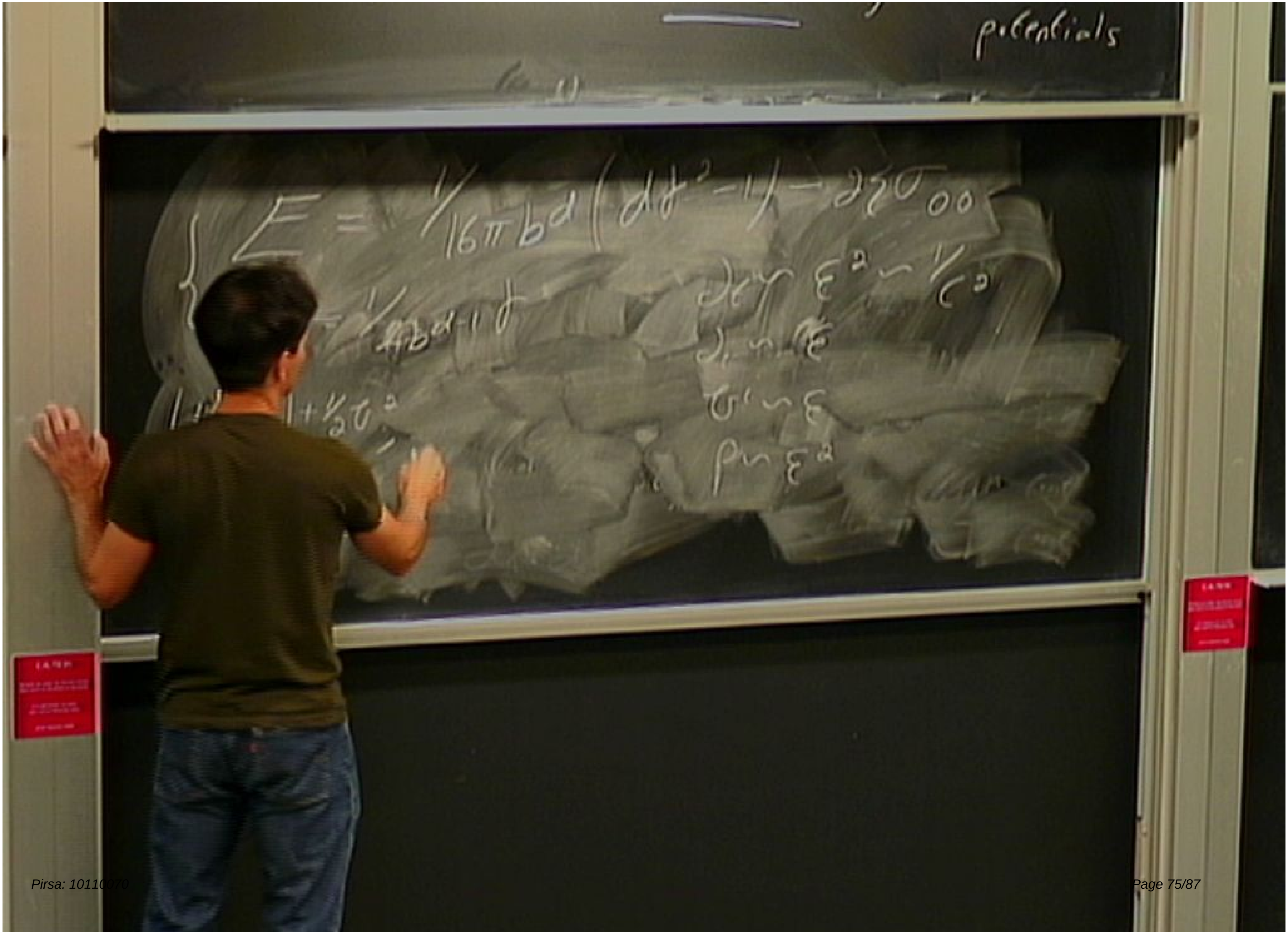
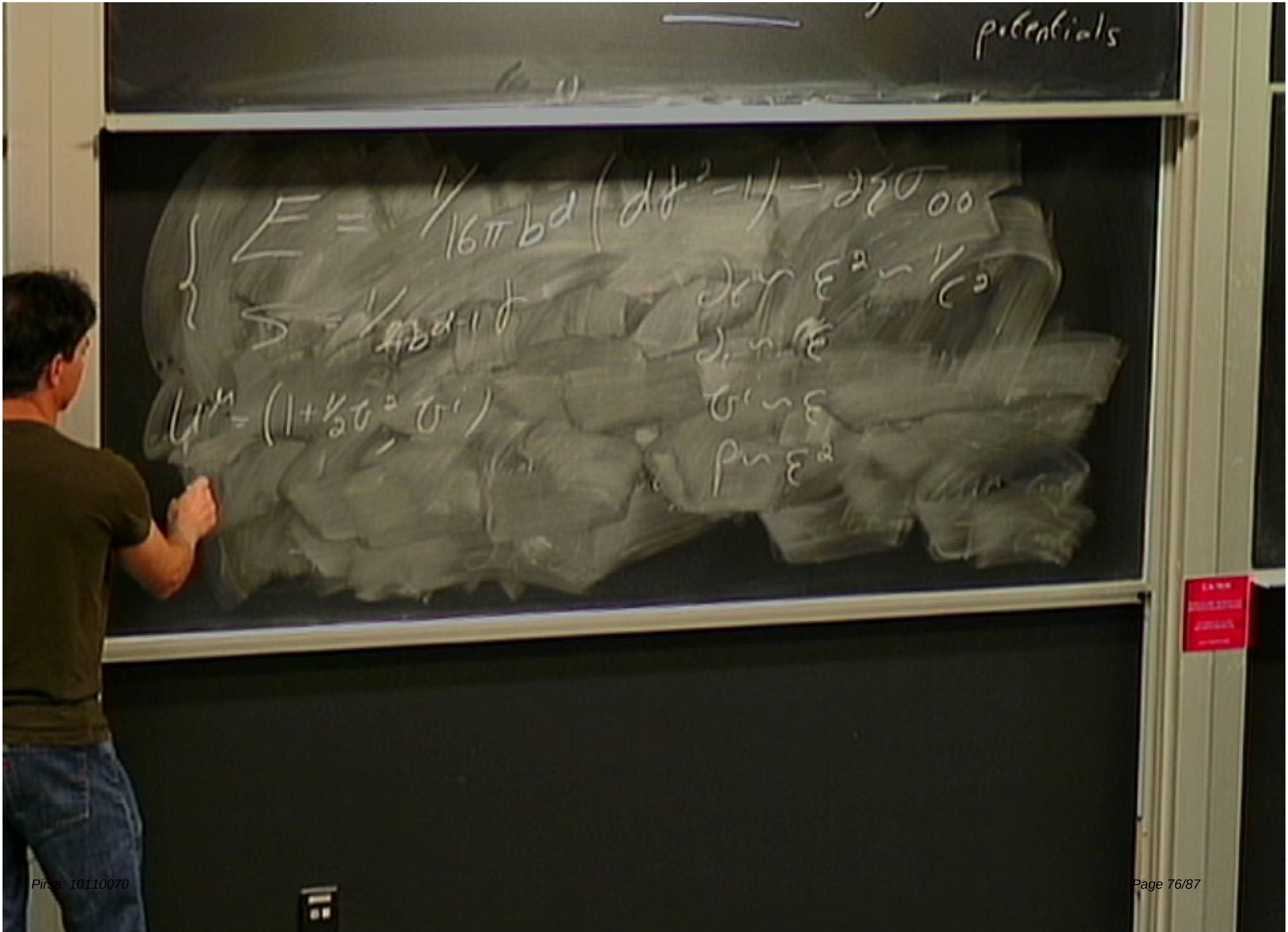
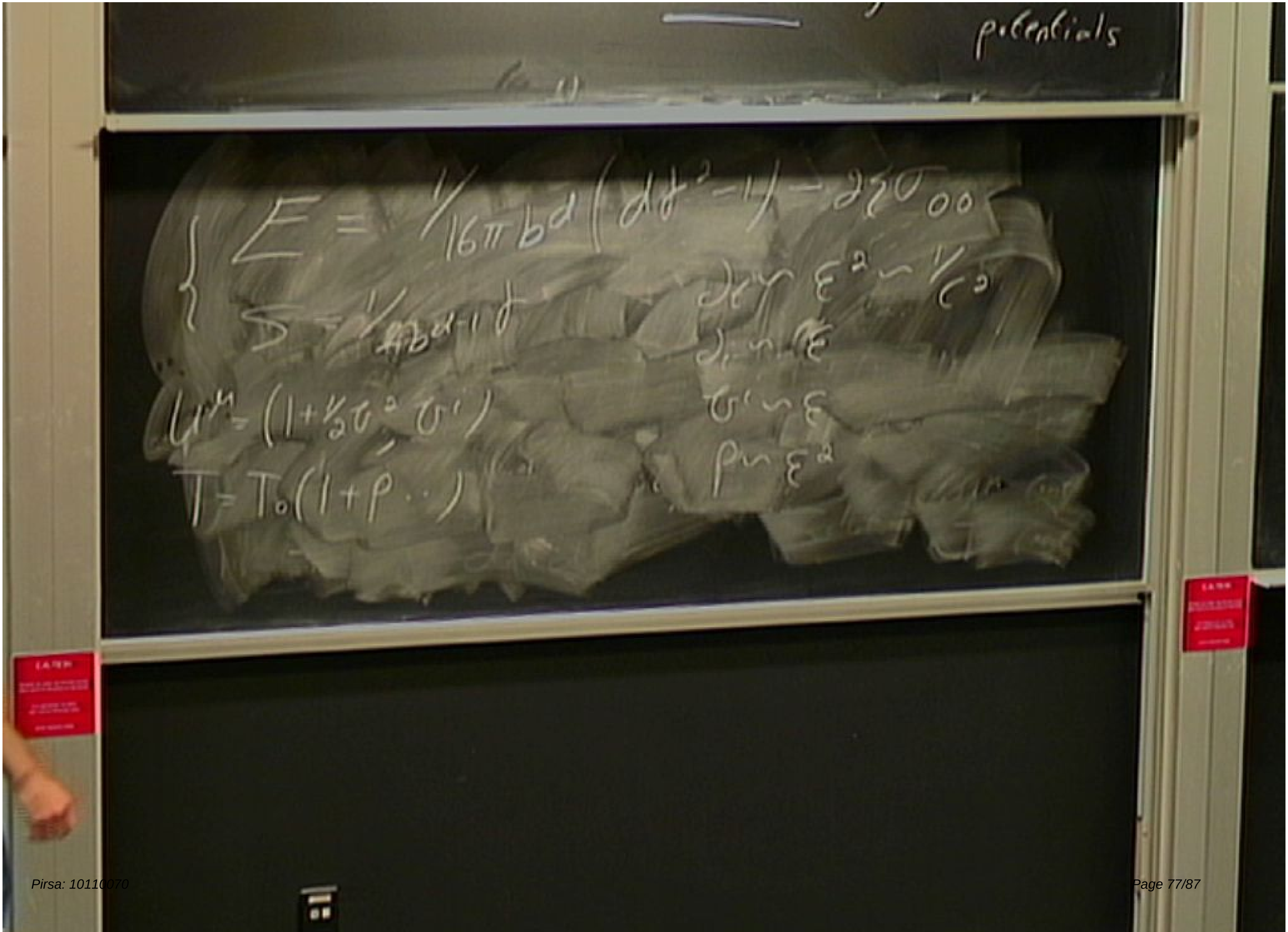

potentials


potentials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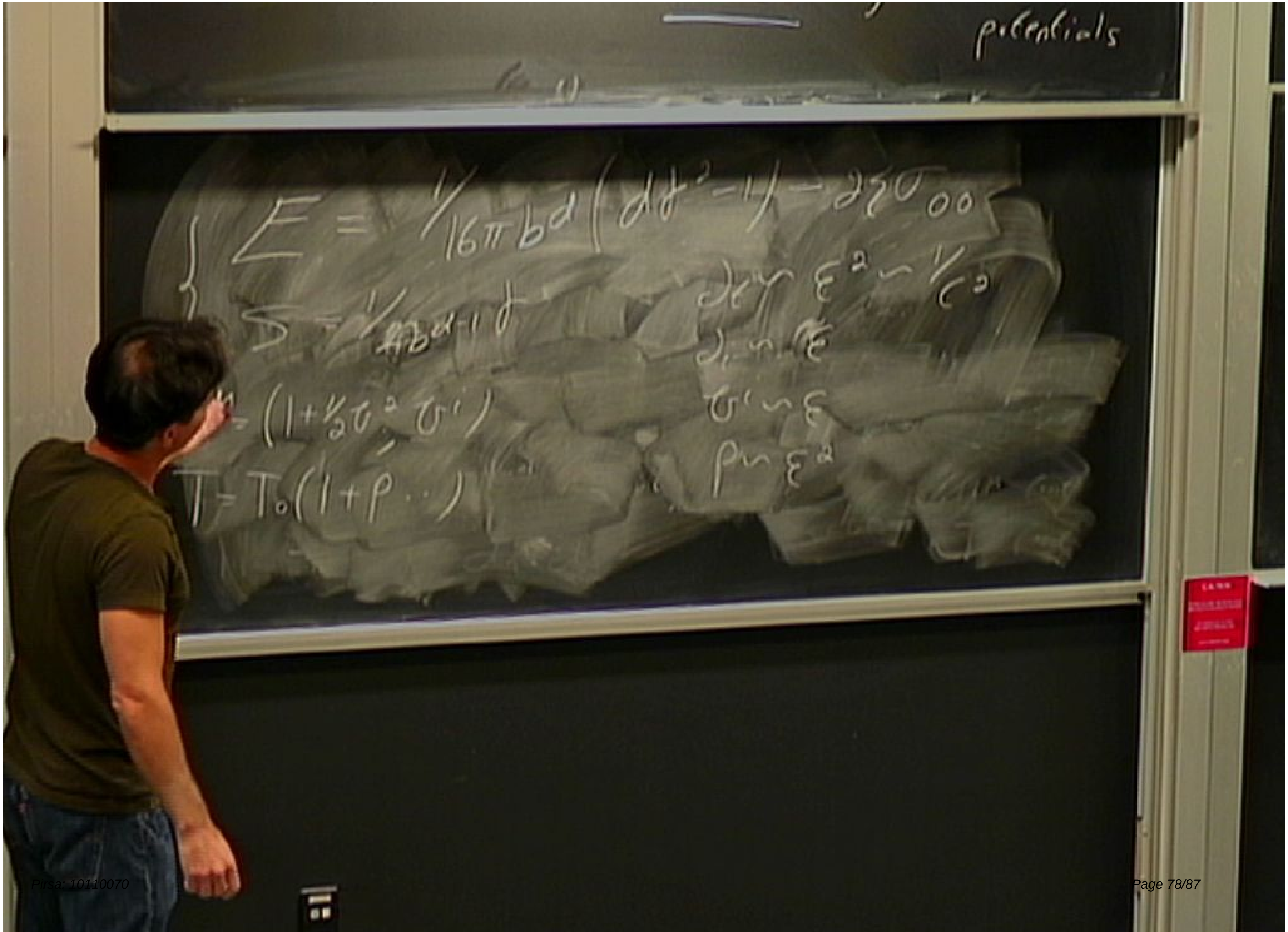
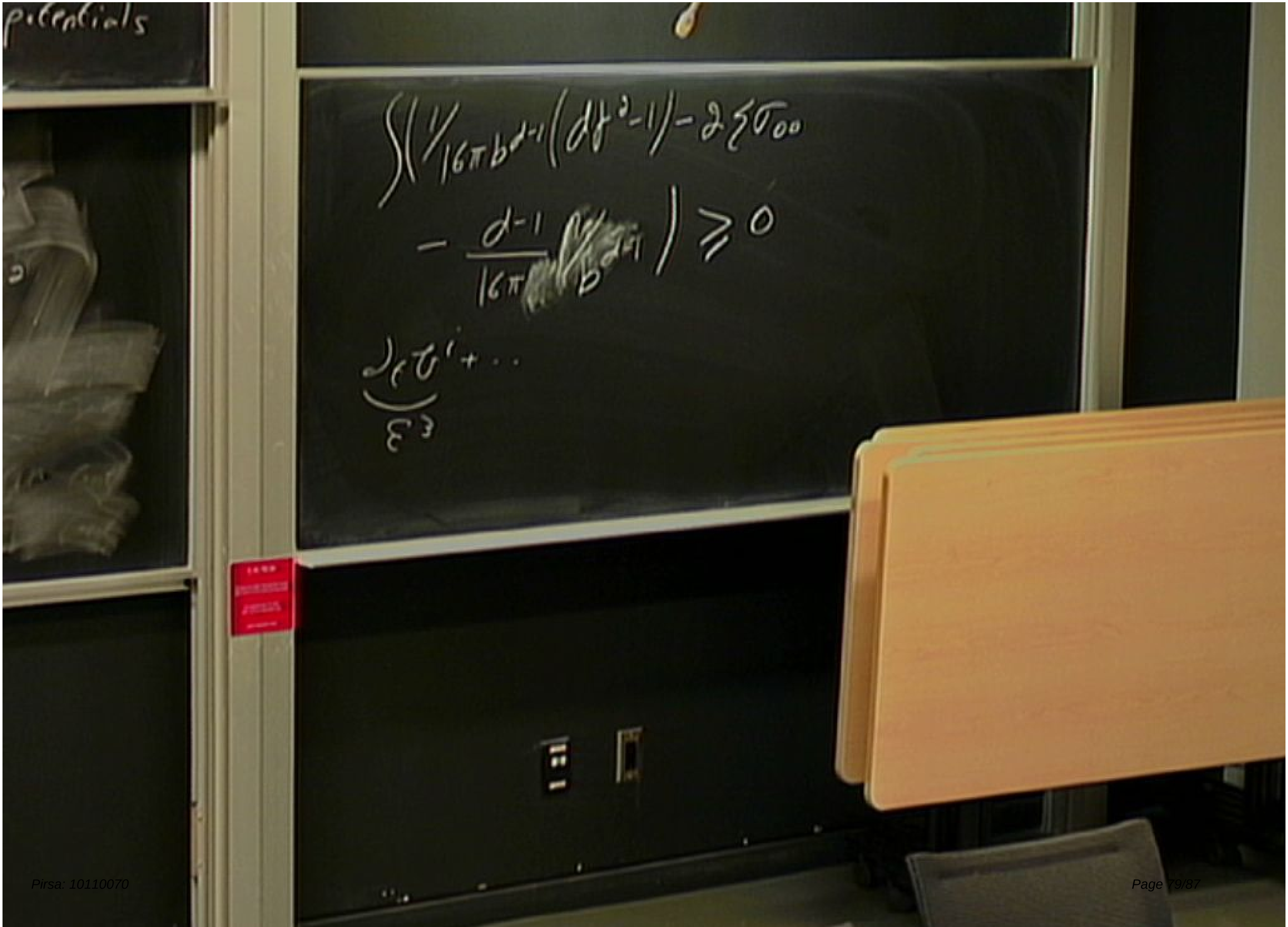
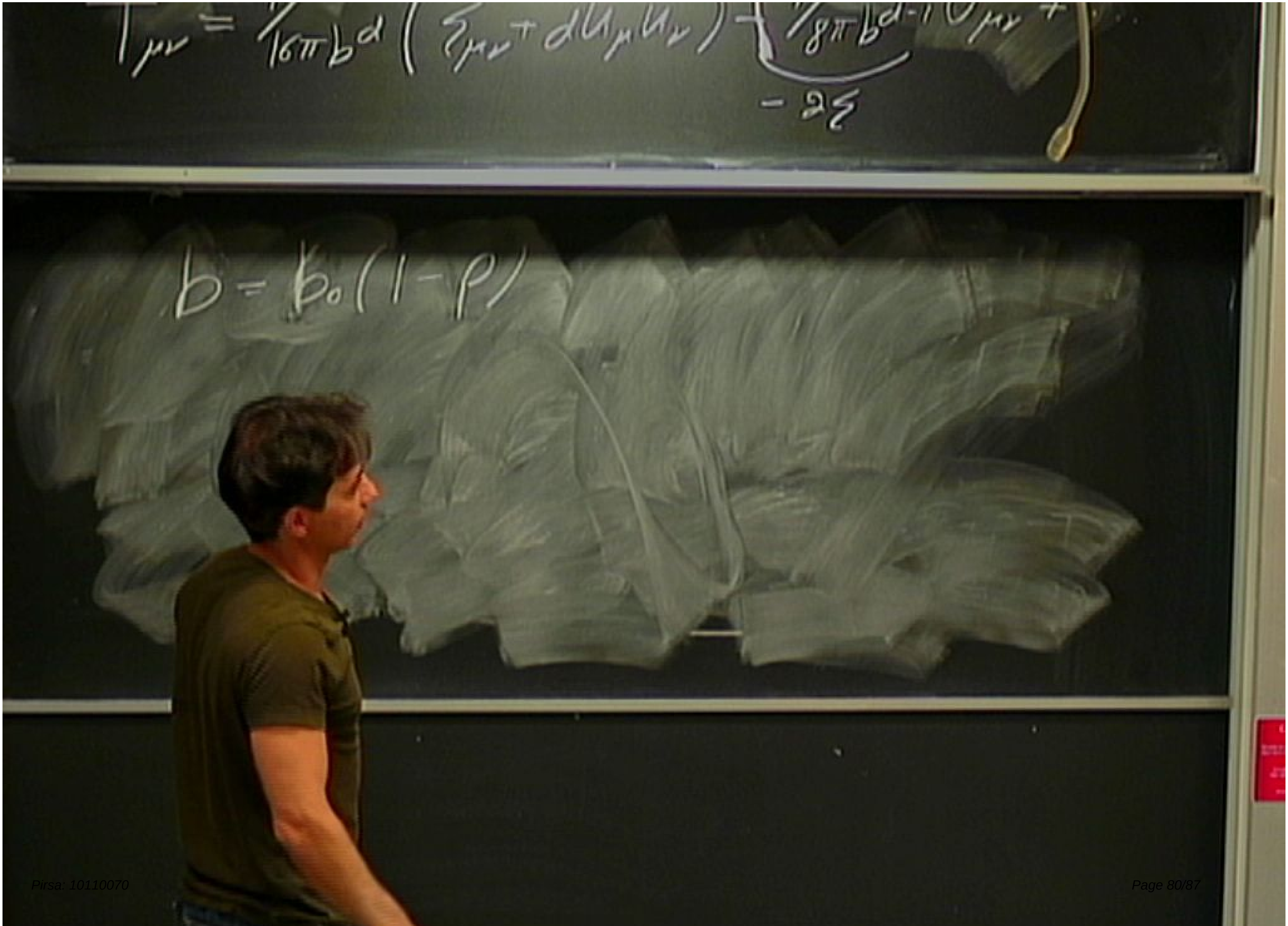
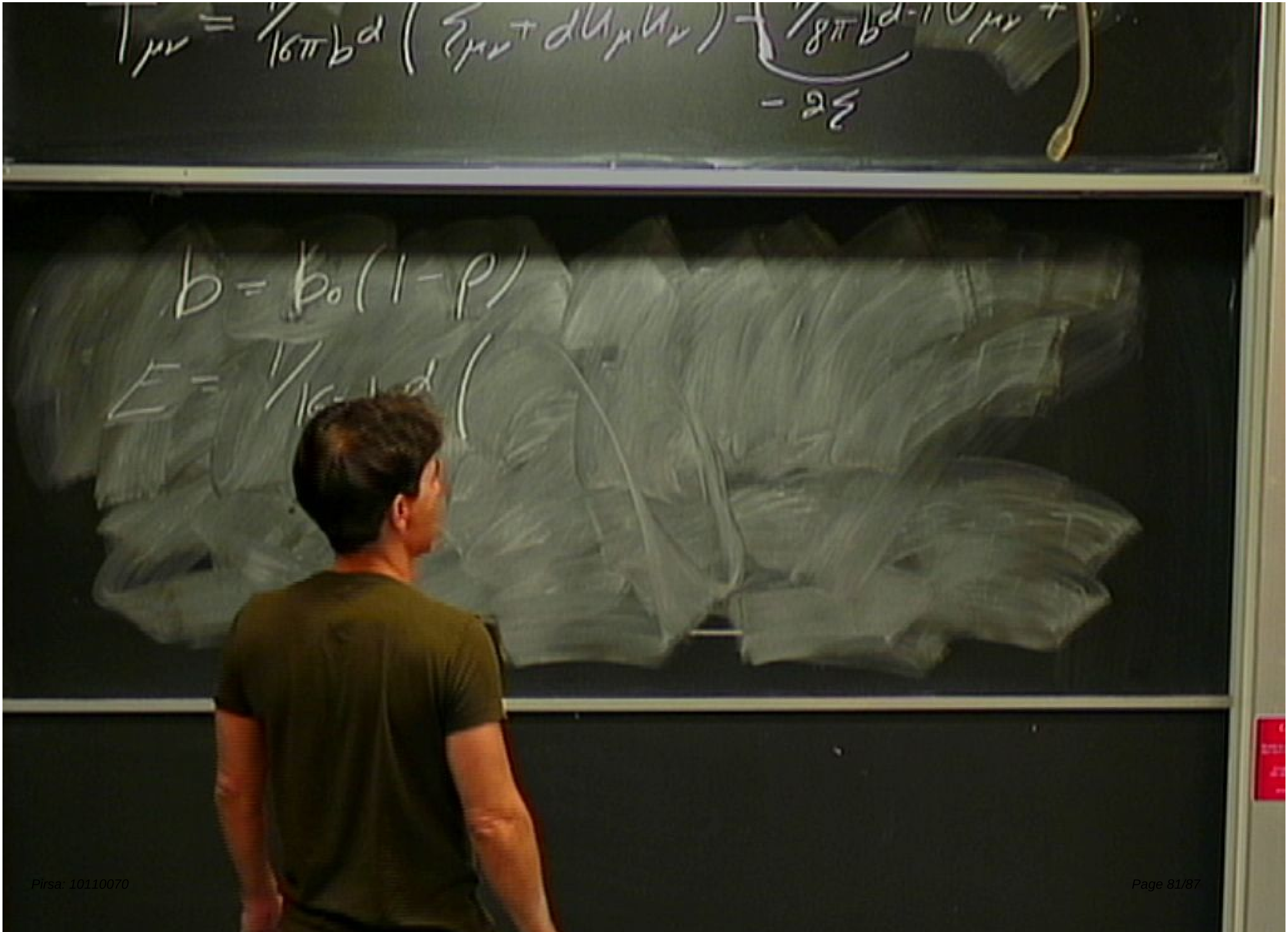
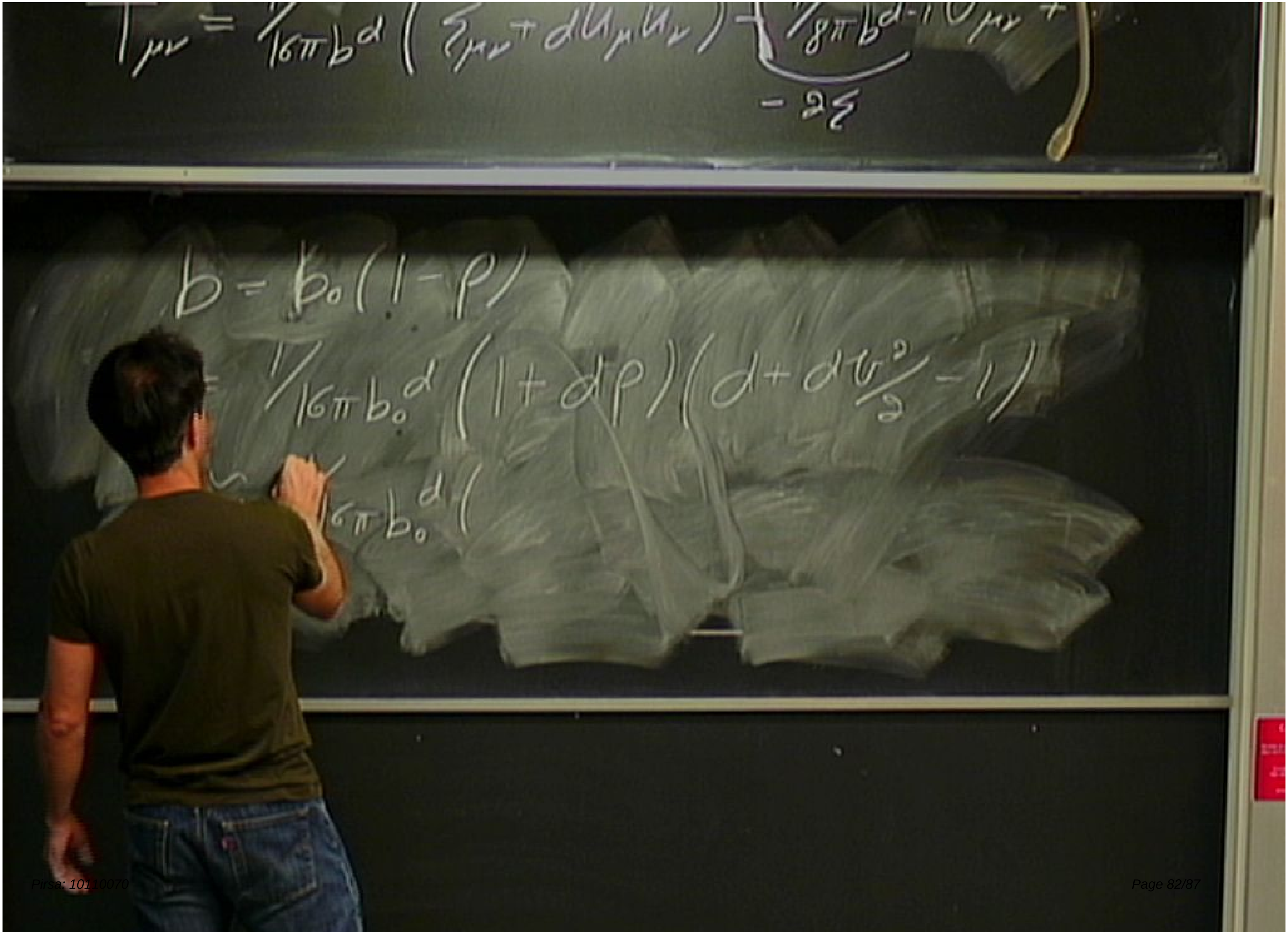

potentials



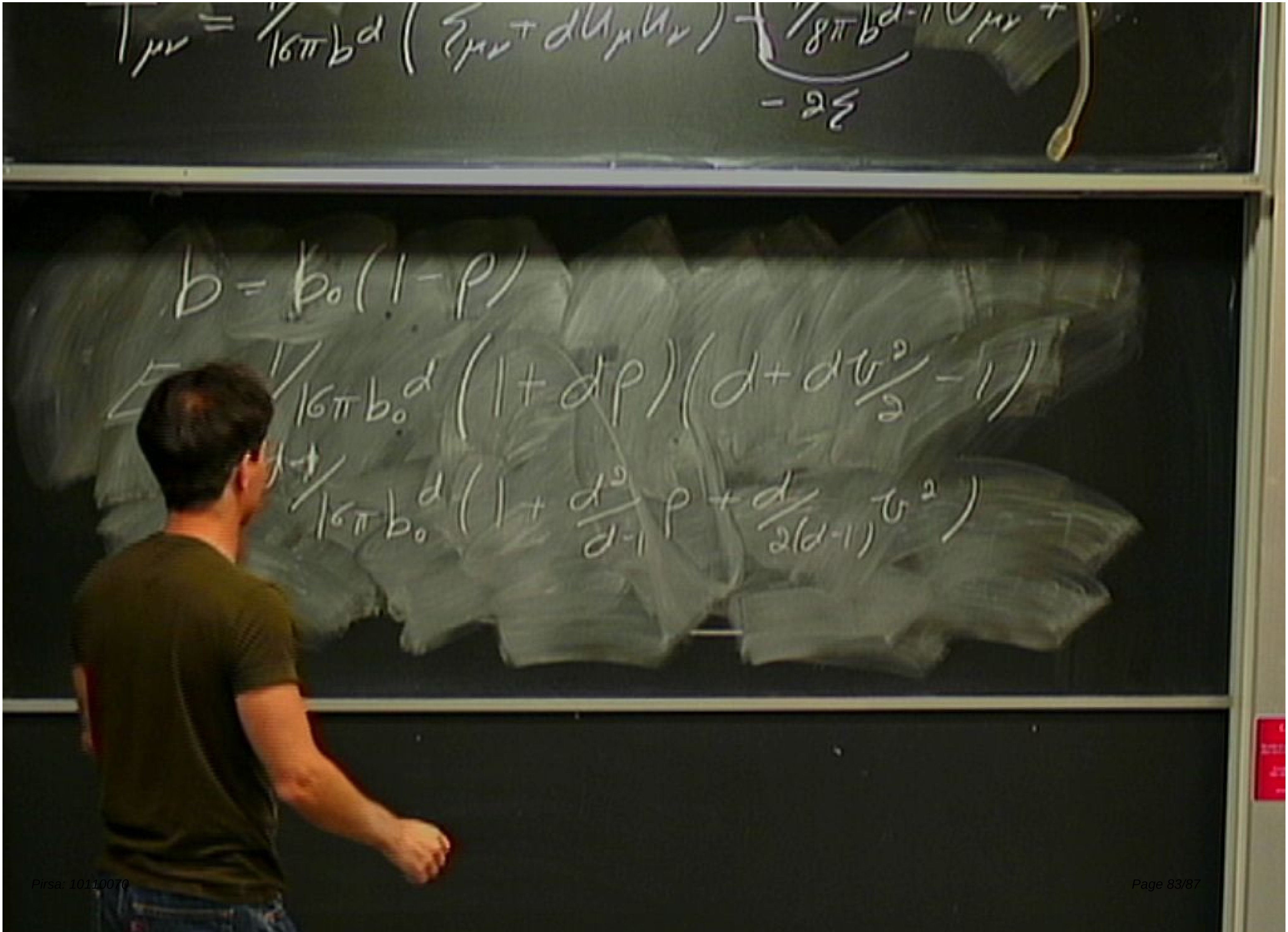

$b = b_0(1-\rho)$
-2ξ



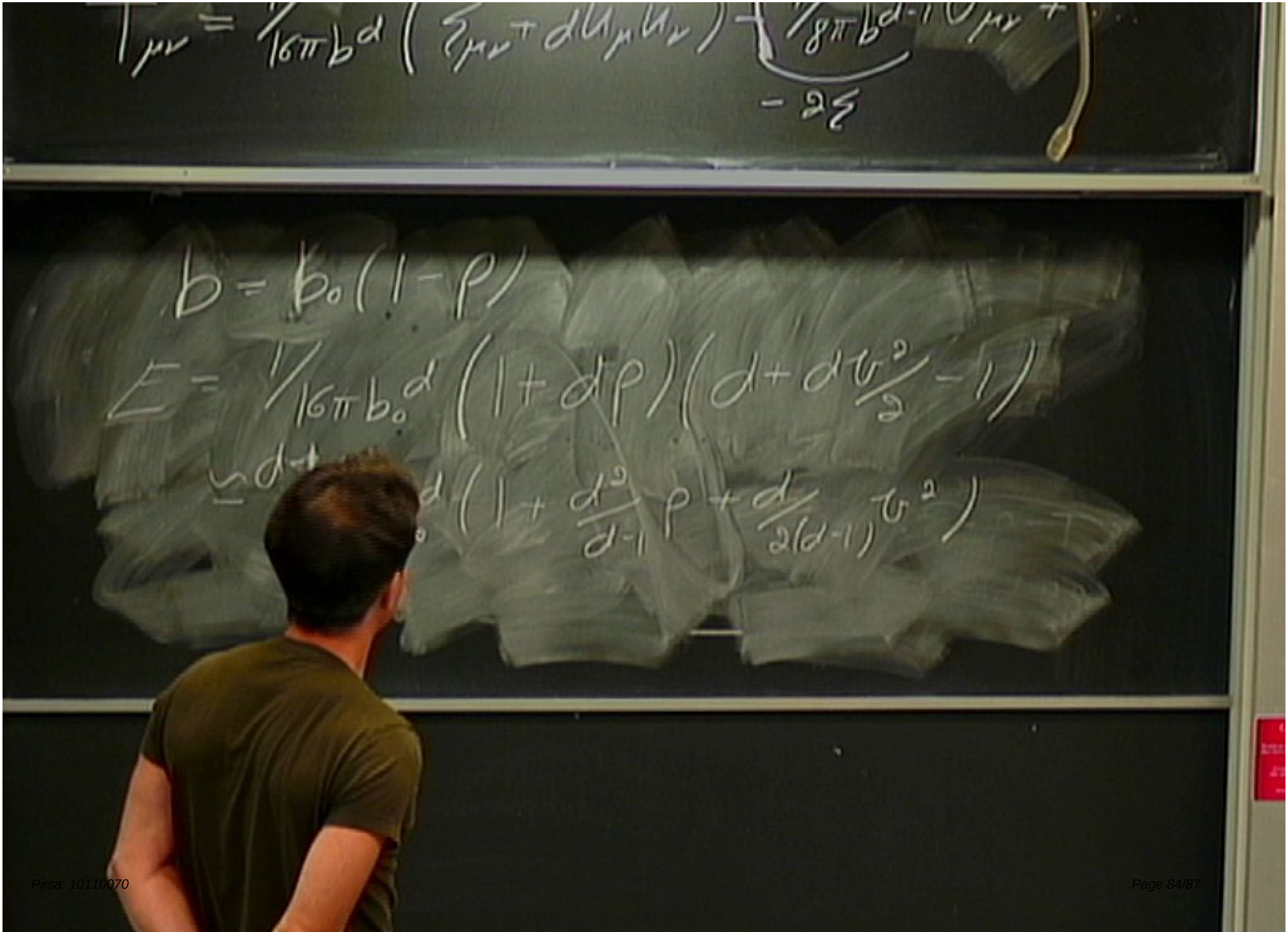
$T_{\mu\nu} = \frac{1}{16\pi b^d}(\eta_{\mu\nu} + d u_\mu u_\nu) - \frac{1}{8\pi b^{d-1}}\sigma_{\mu\nu} +$
-2ζ
$b = b_0(1-\rho)$
$E = \frac{1}{16\pi b^d}($

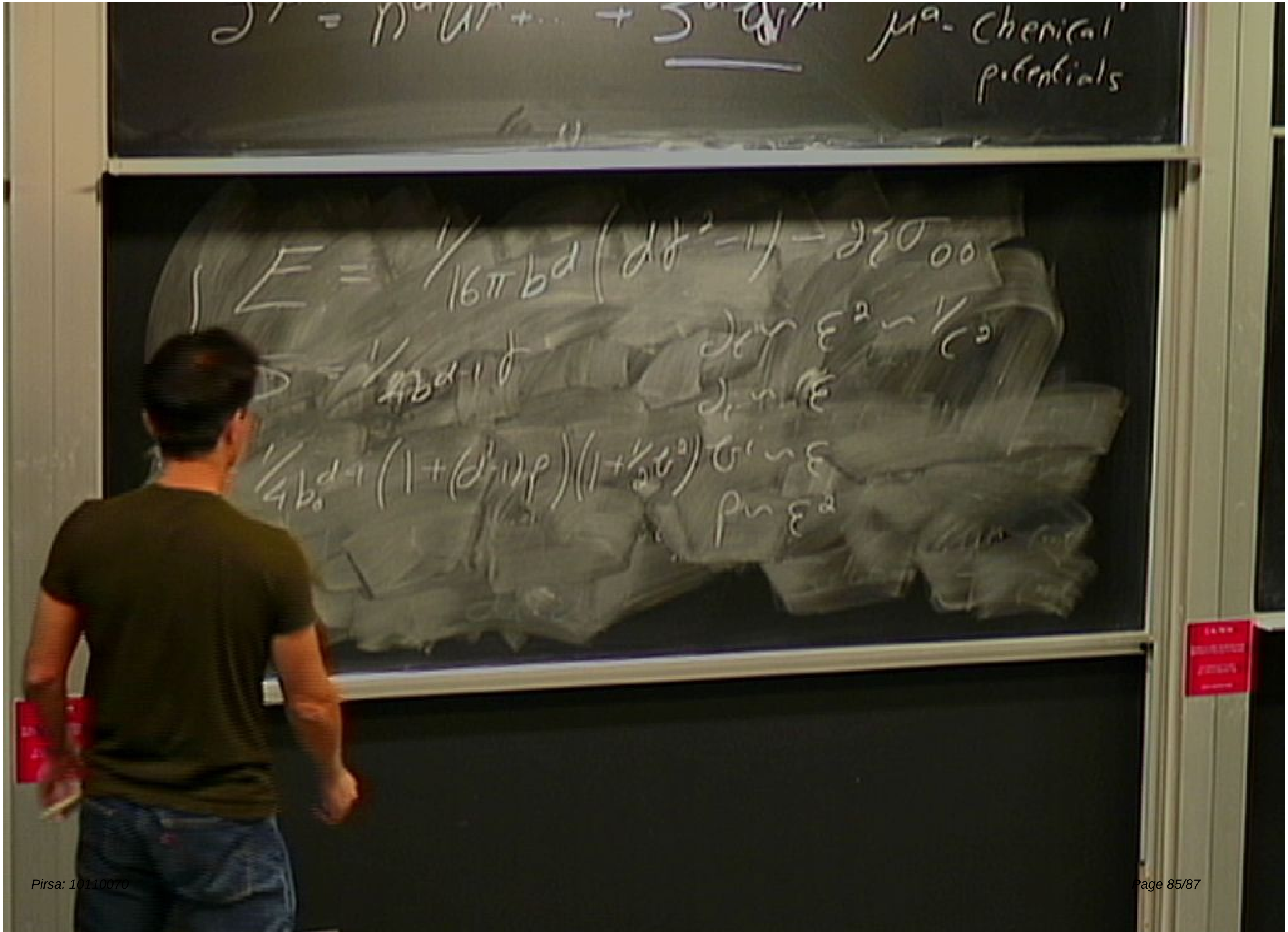


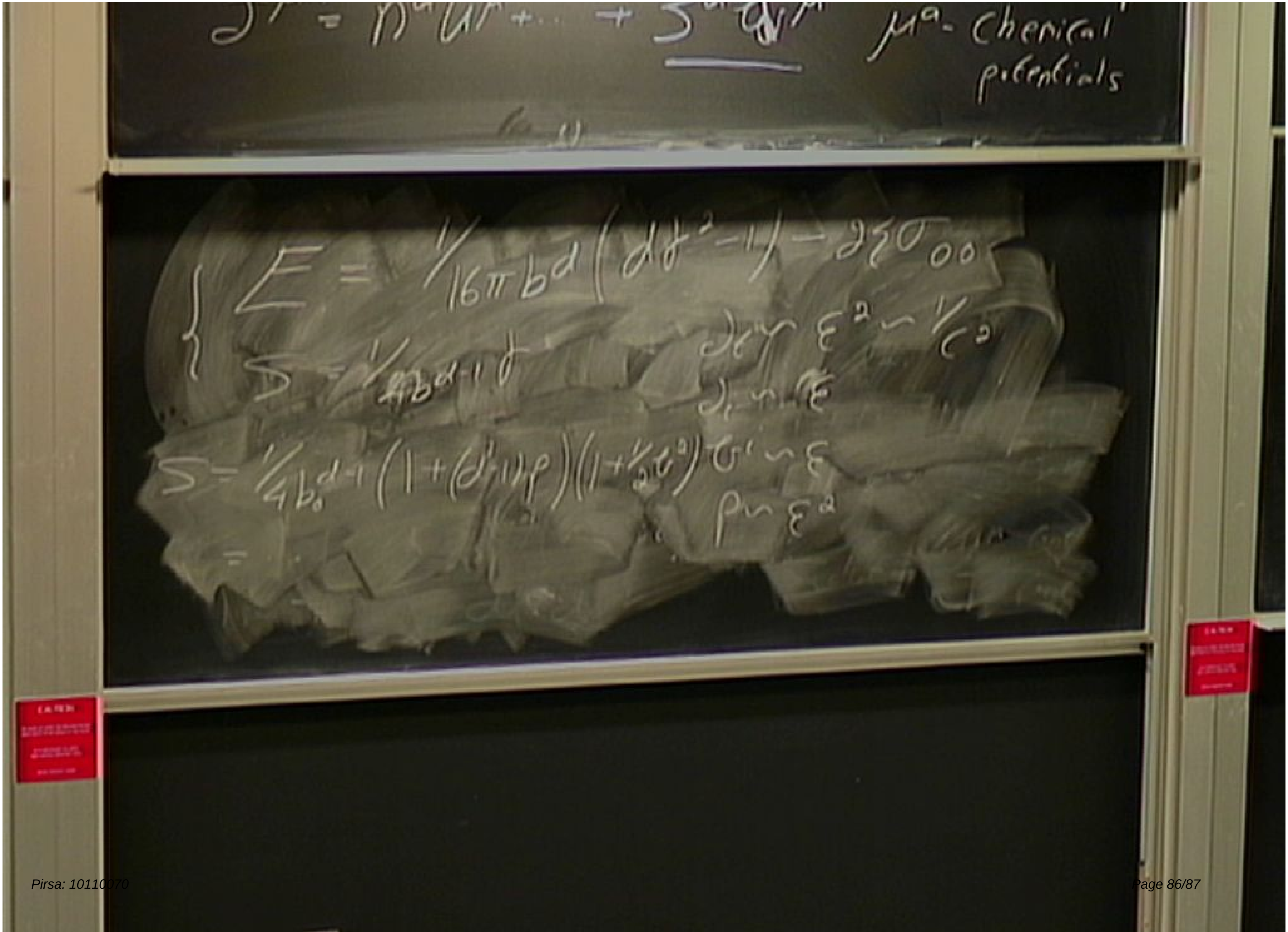
b = b0(1 - ρ)
= 1/16πb0^d (1 + dρ)(d + dv²/2 - 1)
1/16πb0^d (

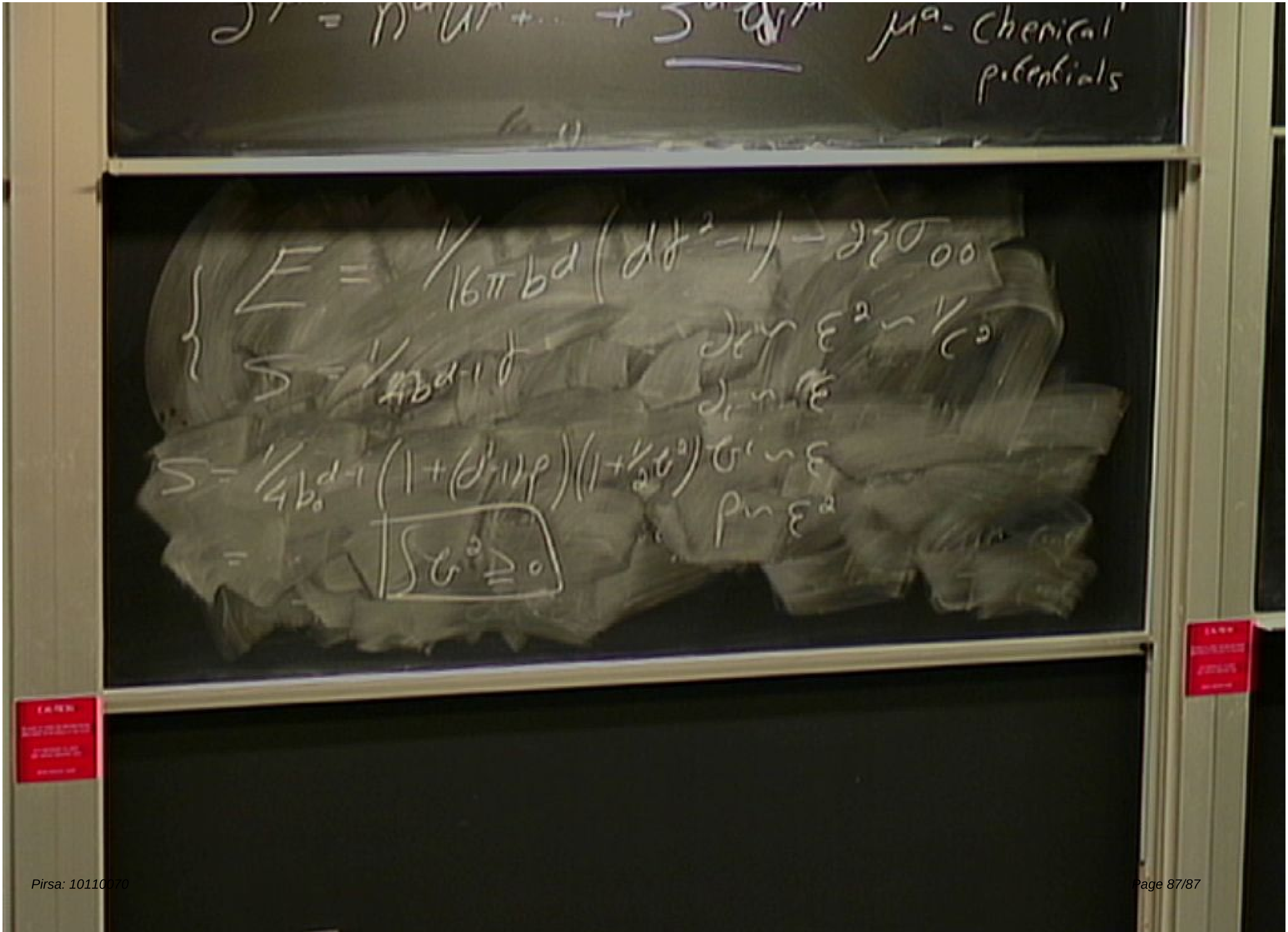


$T_{\mu\nu} = \frac{1}{16\pi b^d}(\eta_{\mu\nu} + d u_\mu u_\nu) - \frac{1}{8\pi b^{d-1}}\sigma_{\mu\nu} +$
-2ξ
$b = b_0(1-\rho)$
$E = \frac{1}{16\pi b_0^d}(1+d\rho)(d + d\frac{v^2}{2} - 1)$
$\frac{d-1}{16\pi b_0^d}(1 + \frac{d^2}{d-1}\rho + \frac{d}{2(d-1)}v^2)$


T_μν = 1/(16π b^d) (η_μν + d u_μ u_ν) − 1/(8π b^(d−1)) σ_μν +
−2η
b = b_0 (1 − ρ)
E = 1/(16π b_0^d) (1 + dρ)(d + d v^2/2 − 1)
≃ d−1 … (1 + d^2/(d−1) ρ + d/(2(d−1)) v^2)


μa- chemical
potentials


μa - chemical potentials


μ^a - chemical potentials