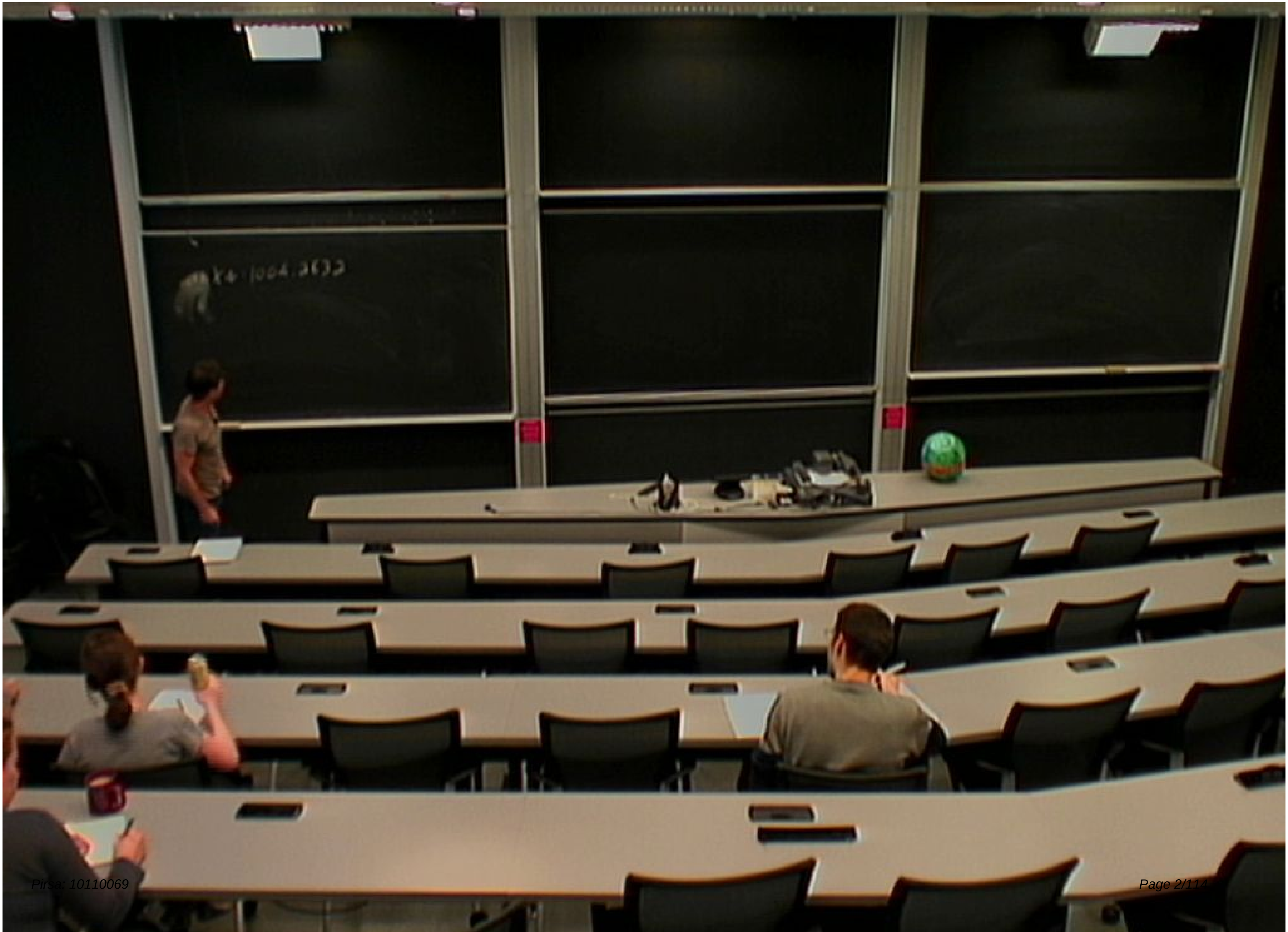
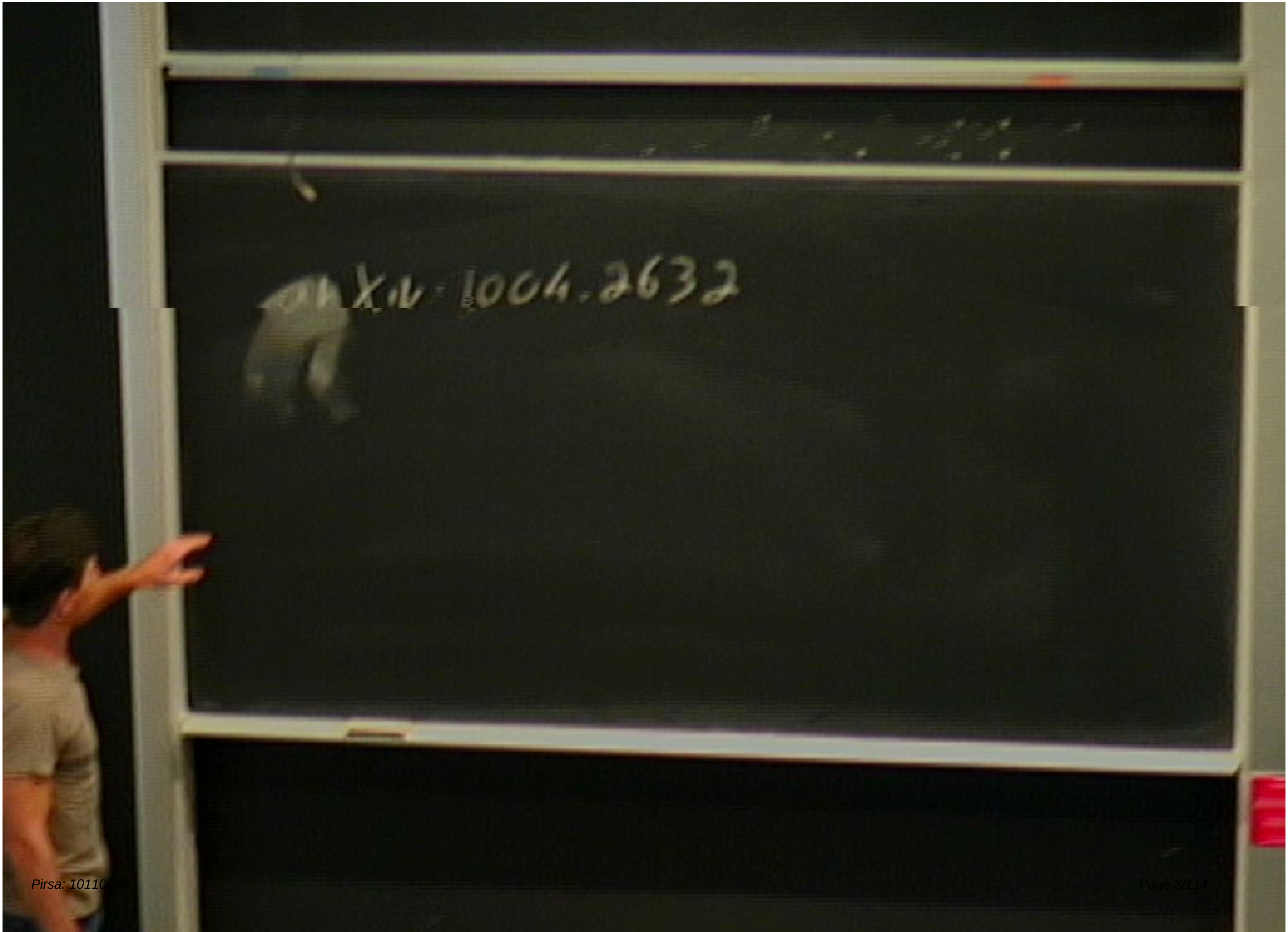
Title: Gravity and a Geometrization of Turbulence: An Intriguing Correspondence: Part 2

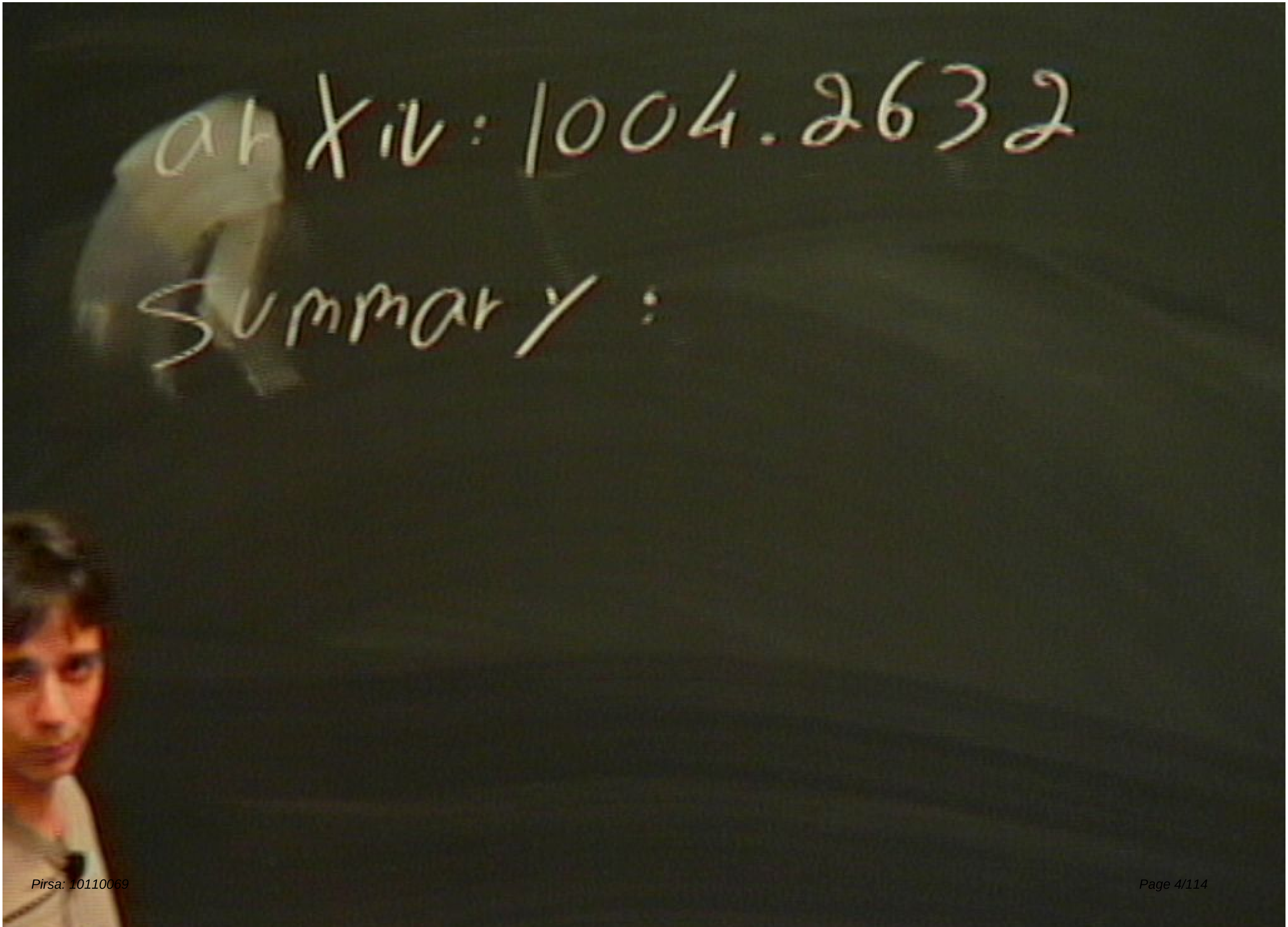
Date: Nov 11, 2010 11:00 AM

URL: http://pirsa.org/10110069

Abstract: The dynamics of fluids is a long standing challenge that remained as an unsolved problem for centuries. Understanding its main features, chaos and turbulence, is likely to provide an understanding of the principles and non-linear dynamics of a large class of systems far from equilibrium. We consider a conceptually new viewpoint to study these features using black hole dynamics. Since the gravitational field is characterized by a curved geometry, the gravity variables provide a geometrical framework for studying the dynamics of fluids: A geometrization of turbulence. We present new experimental predictions for relativsitic and non-relativistic turbulent flows and for heavy ion collisions.




1004.2632


arXiv: 1004.2632
Summary :

arXiv:1004.2632

Summary:

$$\langle (\delta v(r))^n \rangle \sim r^{3n}$$

$$\ll r \ll L$$

arXiv:1004.2632

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$$\langle (\delta v(r))^n \rangle \sim r^{\zeta_n}$$

$$\ell \ll r \ll L \qquad \zeta_3 = 1$$

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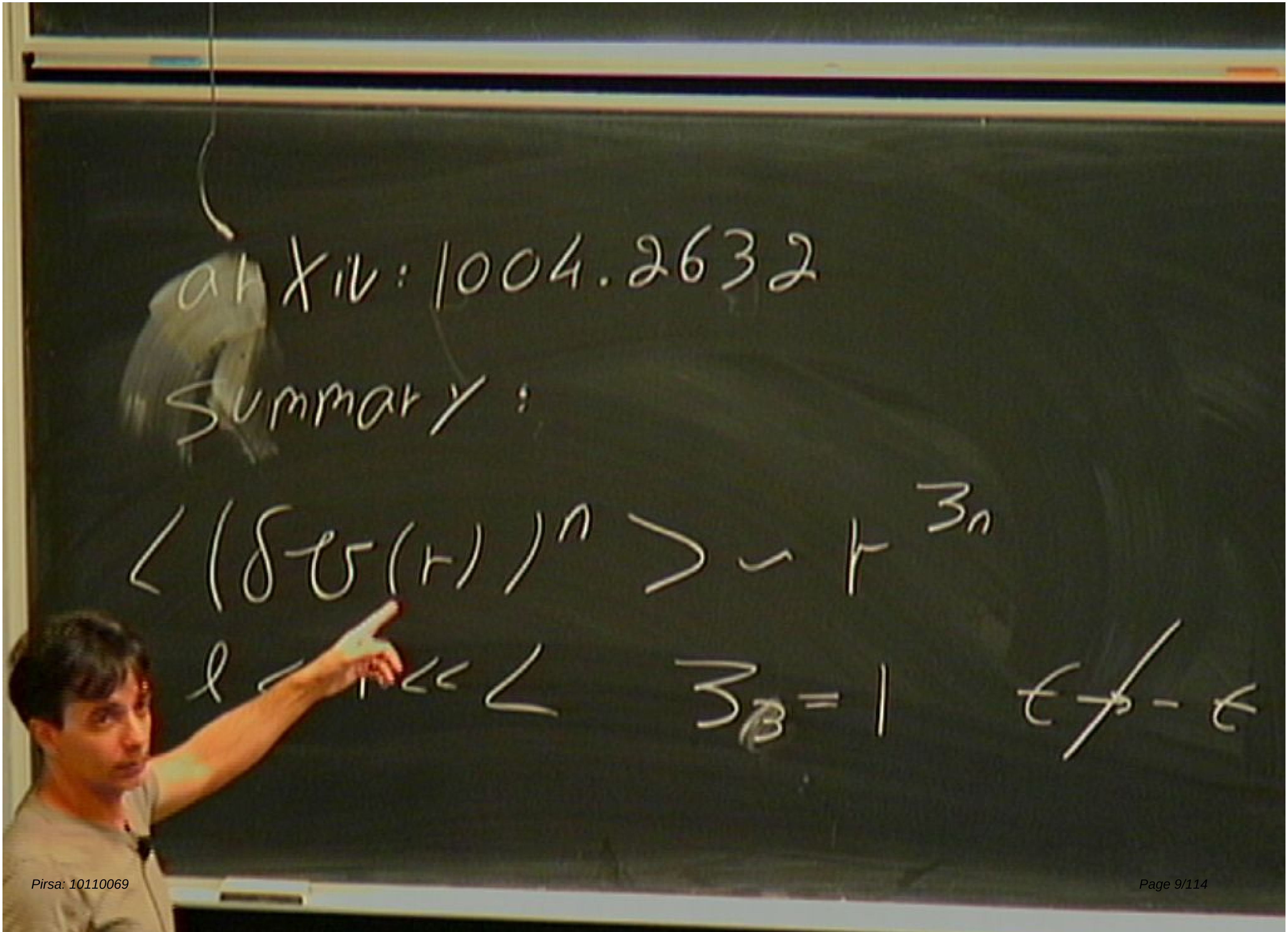
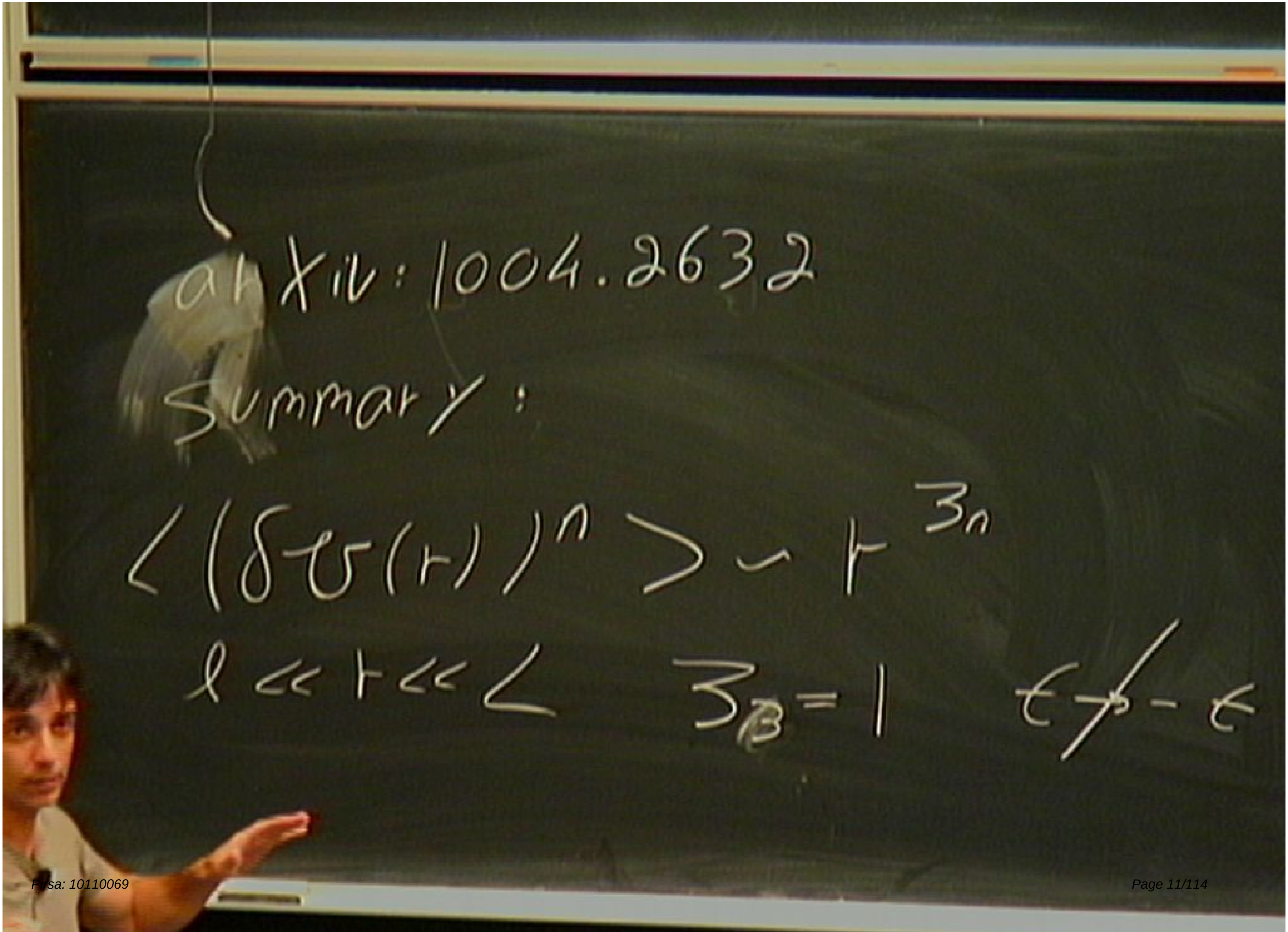

arXiv:1004.2632
Summary:
$\langle (\delta v(r))^n \rangle \sim r^{3n}$
$\zeta_B = 1$
$t \not\to -t$

arXiv:1004.2632

Summary:

$$\langle (\delta v(r))^n \rangle \sim r^{\zeta_n}$$

$\ell \ll r \ll L$ $\quad \zeta_3 = 1 \quad t \not\to -t$

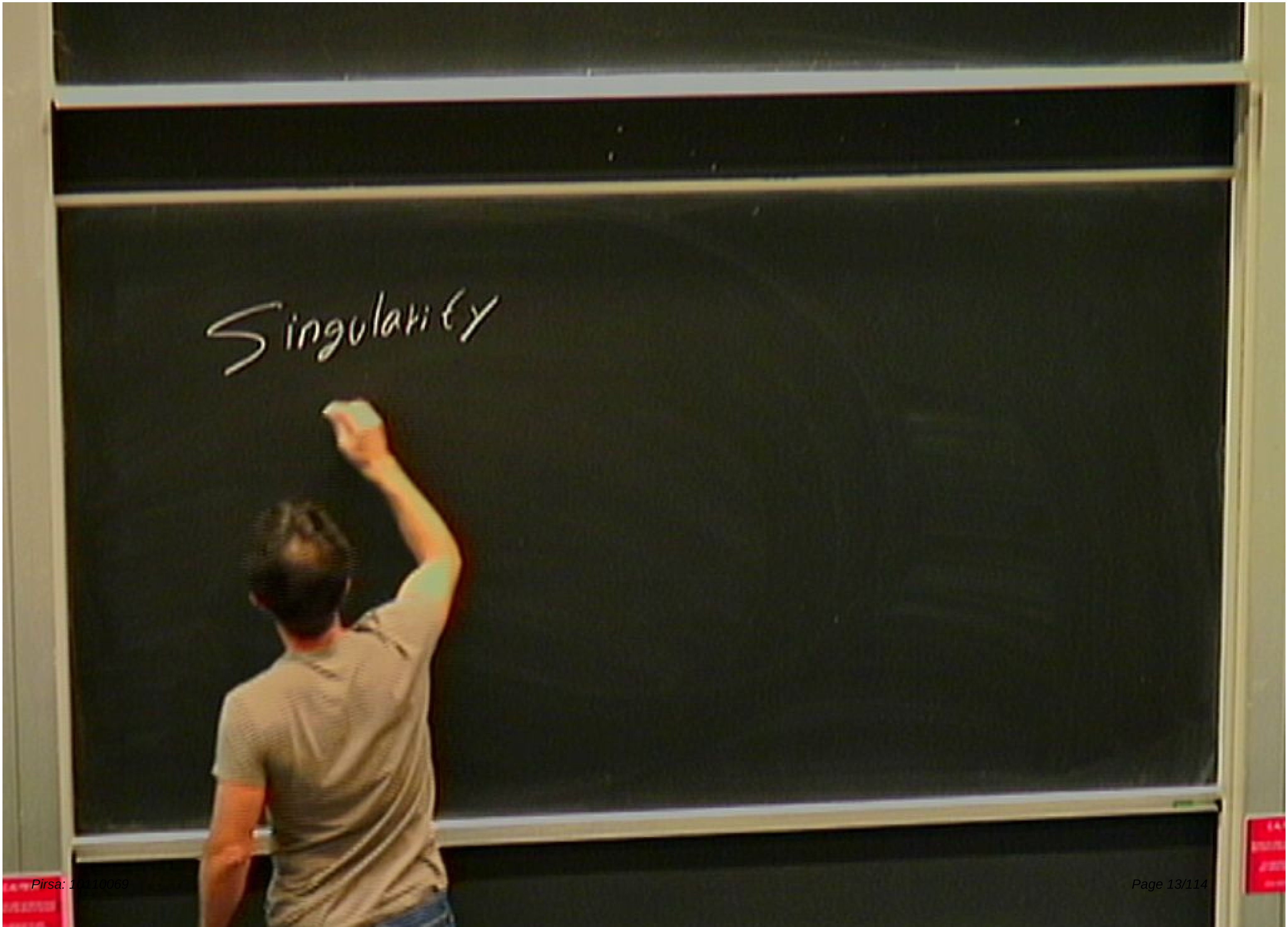

arXiv:1004.2632
Summary:
<(δv(r))^n> ~ r^3n
l << r << L
3_B = 1
ε ↛ -ε

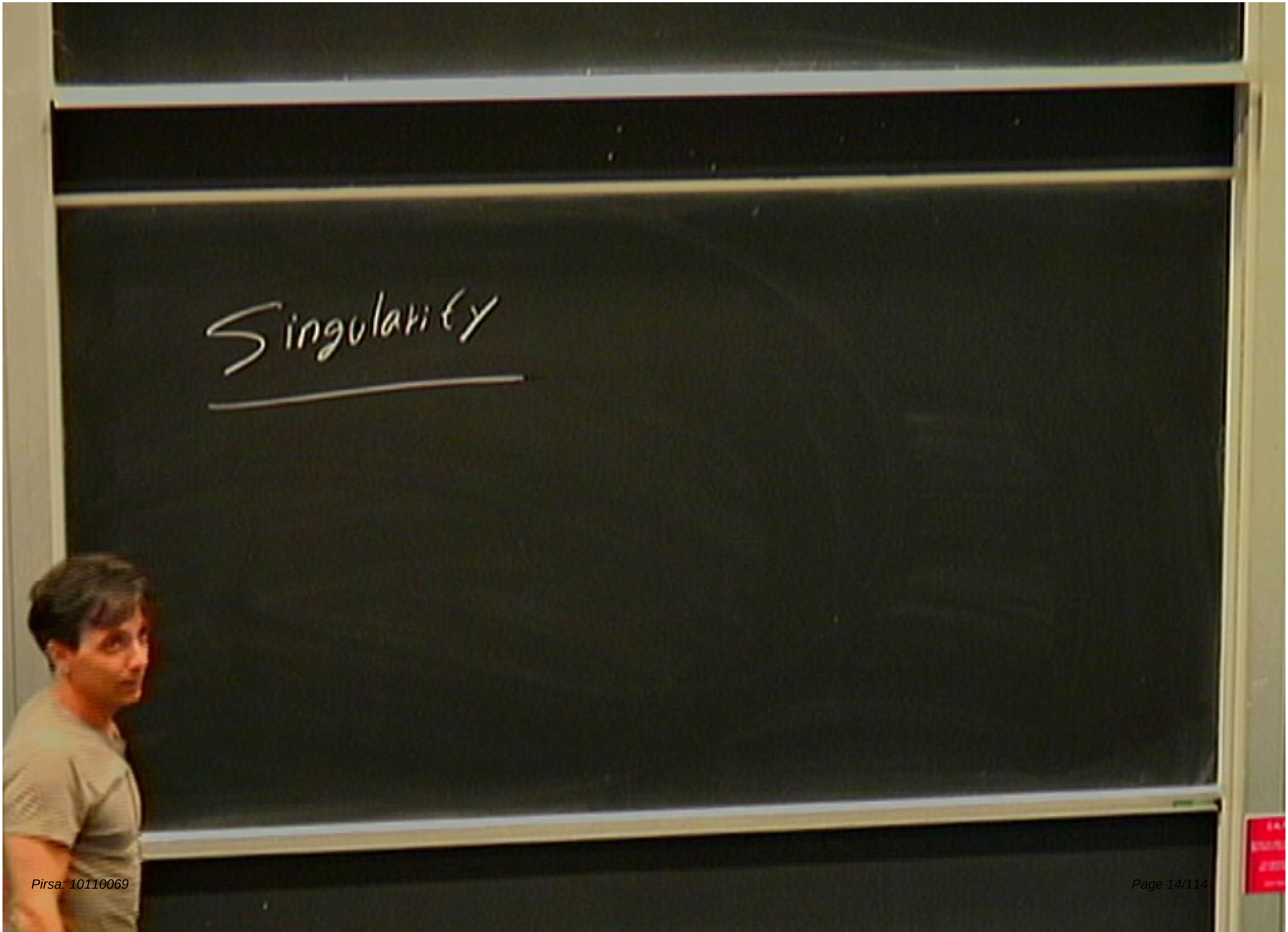
arXiv: 1004.2632

Summary:

$$\langle (\delta v(r))^n \rangle \sim r^{\zeta_n}$$

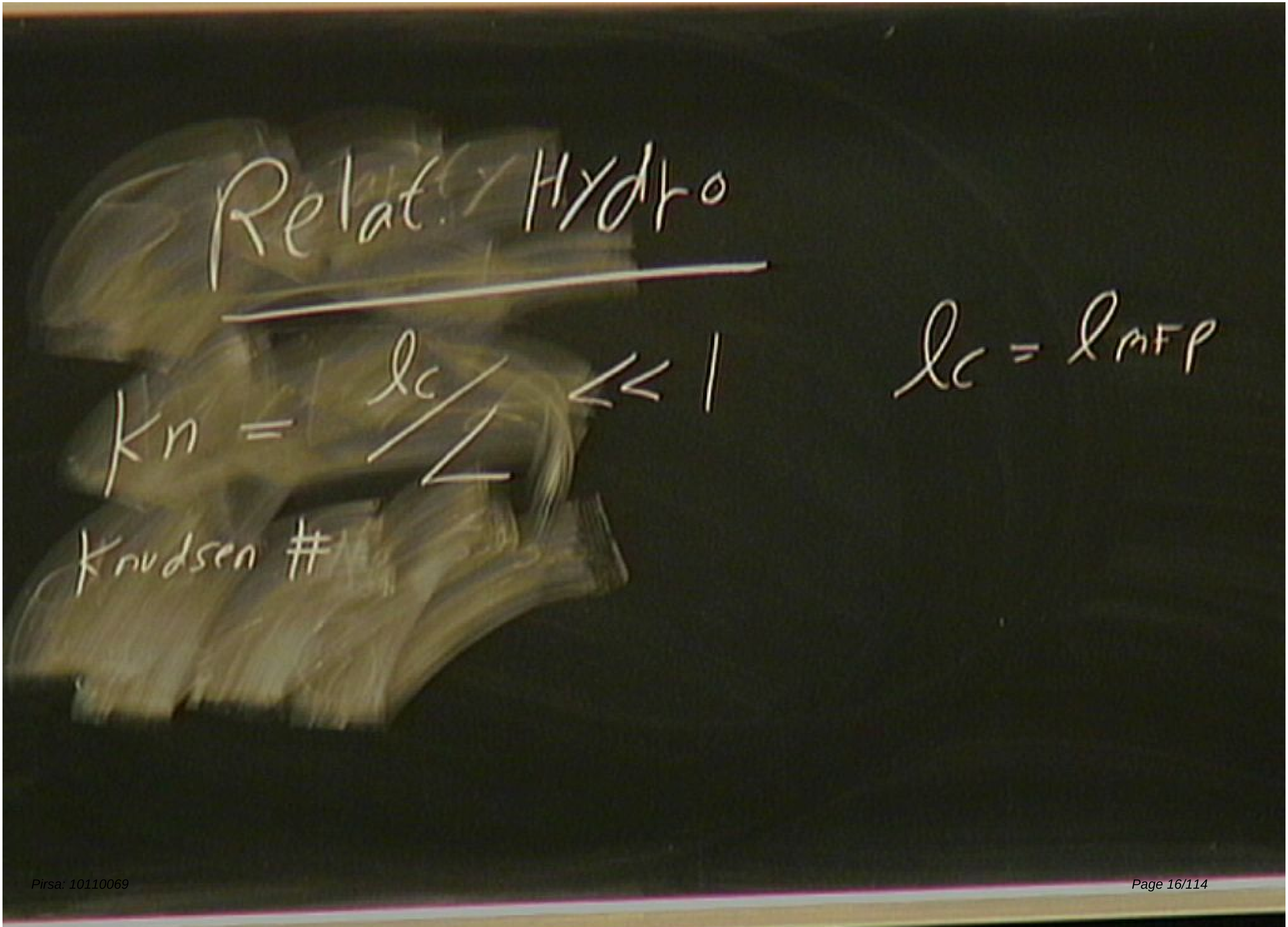
$\ell \ll r \ll L$ $\qquad \zeta_B = 1$ $\qquad t \not\to -t$


Singularity


Singularity

Singularity

$|\nabla V| \to \infty$


Relat. Hydro
Kn = lc/L << 1
Knudsen #
lc = lMFP

Relat. Hydro

$$Kn = l_c / L \ll 1 \qquad l_c = l_{mfp}$$

Knudsen #

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\mu J^{\mu a} = 0 \end{cases}$$

Relat. Hydro

$Kn = l_c / L \ll 1 \qquad l_c = l_{mfp}$

Knudsen #

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\mu J^{\mu a} = 0 \end{cases}$$

ε – energy density $\quad P$ – pressure

Relat. Hydro

$Kn = \frac{l_c}{L} \ll 1$ $\quad l_c = l_{mfp}$

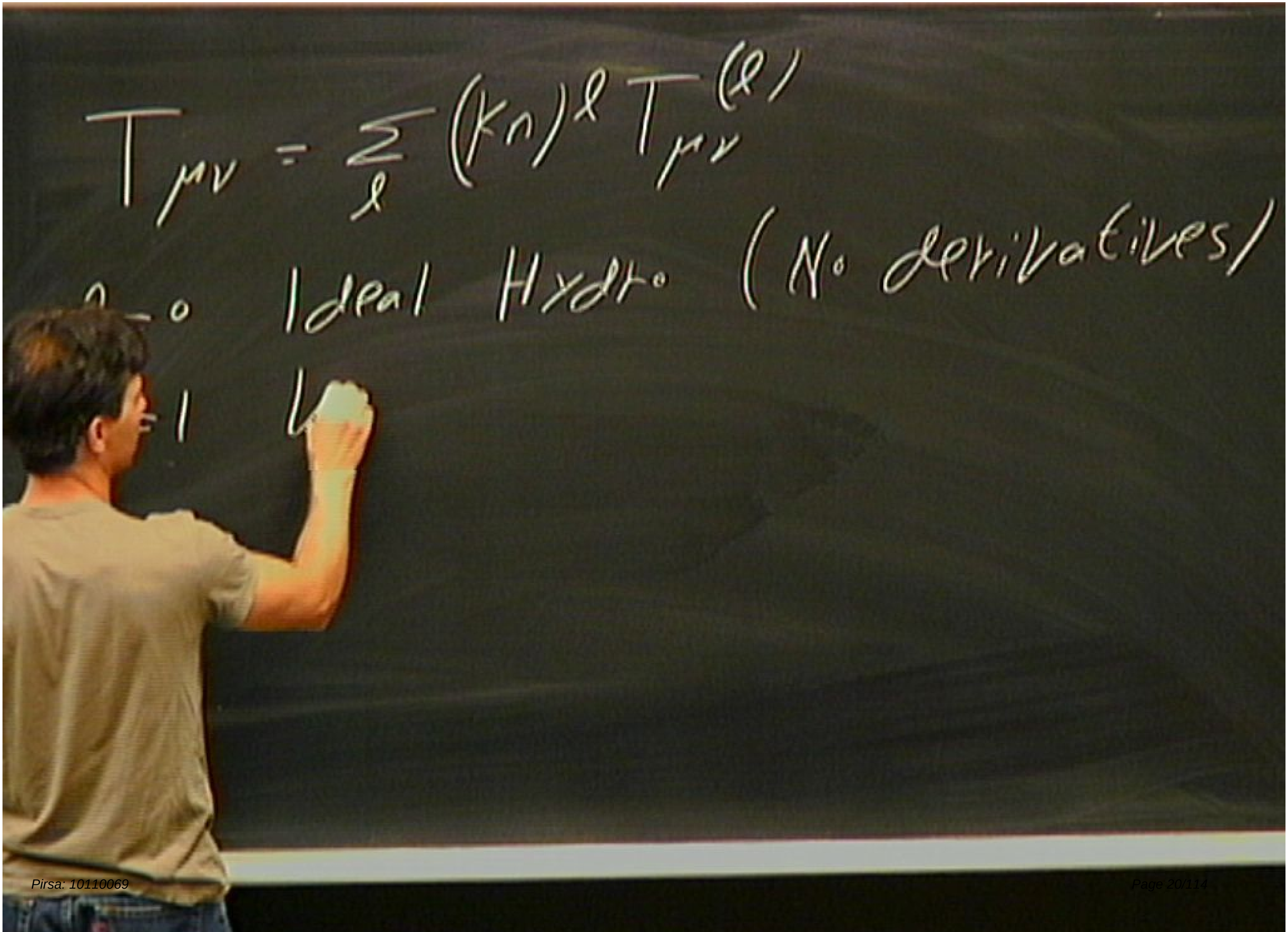
Knudsen #

$$\begin{cases} \partial_\mu T^{\mu\nu} = 0 \\ \partial_\mu J^{\mu a} = 0 \end{cases}$$

$u^\mu = (\gamma, \gamma \beta^i)$

ε – energy density $\quad P$ – pressure

$u_\mu u^\mu = -1$


$T_{\mu\nu} = \sum_l (Kn)^l \, T_{\mu\nu}^{(l)}$
$l=0$ Ideal Hydro (No derivatives)

$$T_{\mu\nu} = \sum_{\ell} (Kn)^{\ell} \, T_{\mu\nu}^{(\ell)}$$

$\ell = 0$ Ideal Hydro (No derivatives)

$\ell = 1$ Viscous Hydro (First ")

$\ell = 2$

$$T_{\mu\nu} = \sum_{\ell} (Kn)^{\ell}\, T_{\mu\nu}^{(\ell)}$$

$\ell = 0$ Ideal Hydro (No derivatives)

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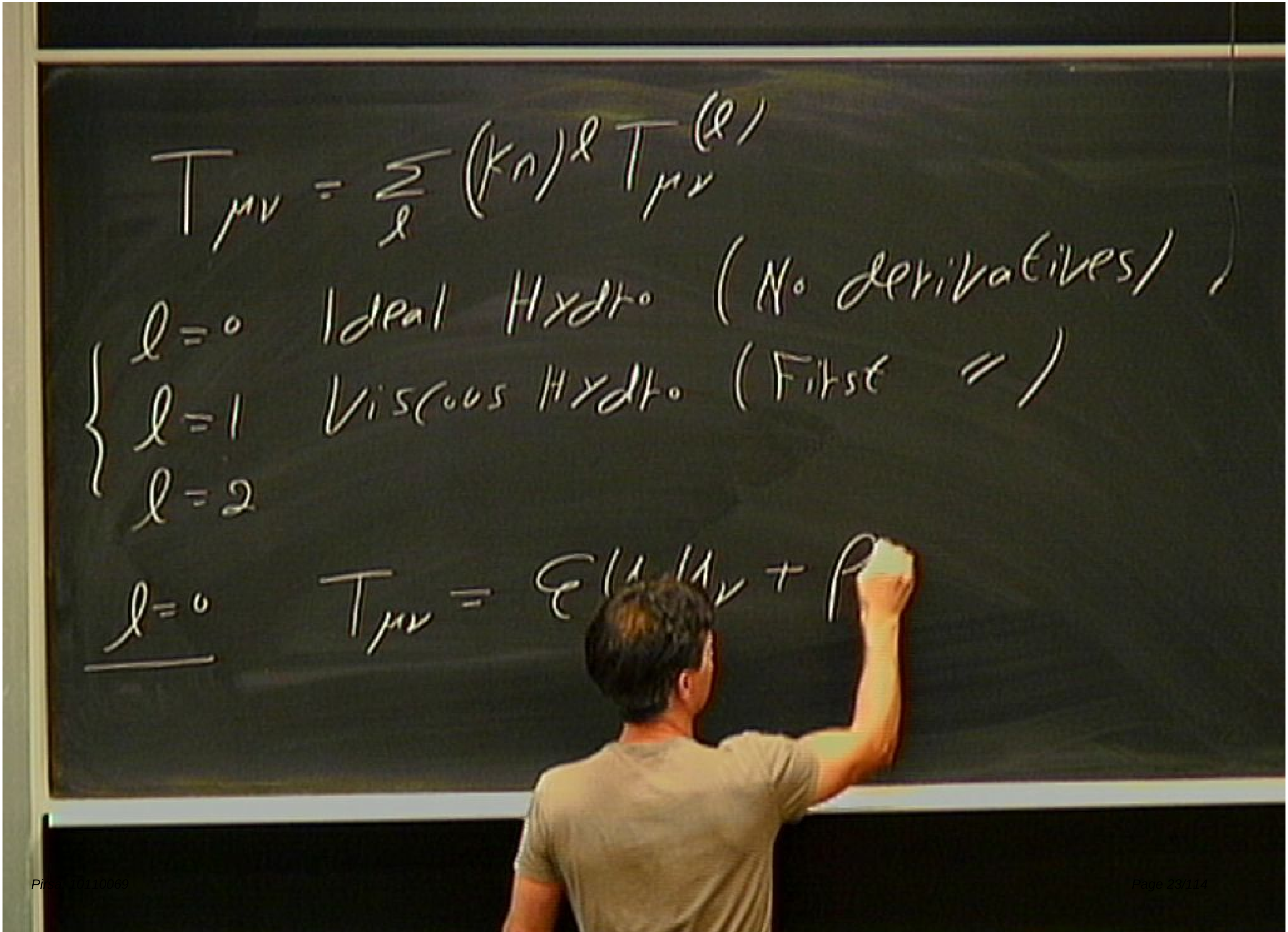
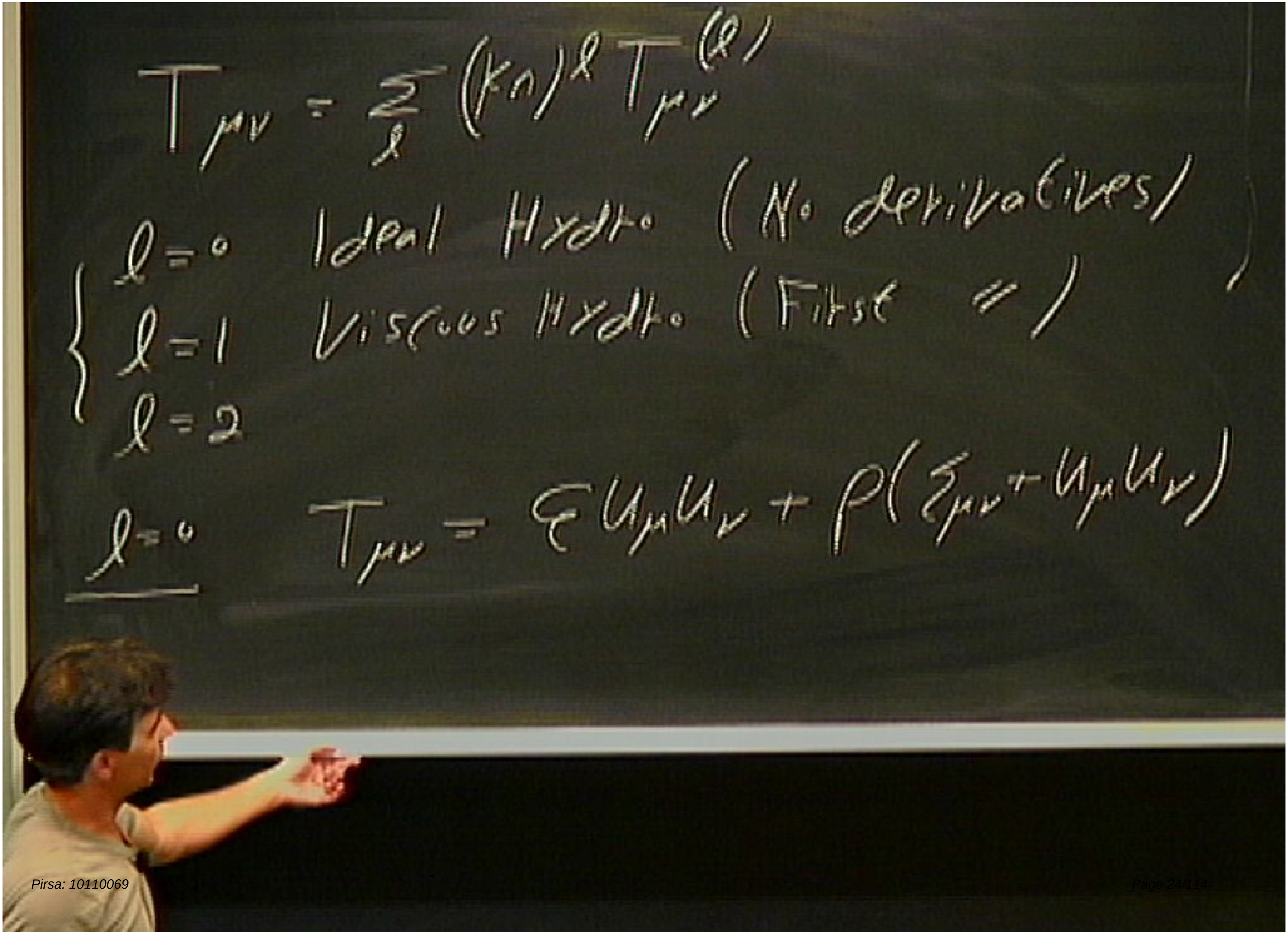

$T_{\mu\nu} = \sum_{\ell} (Kn)^{\ell} T_{\mu\nu}^{(\ell)}$
$\ell = 0$ Ideal Hydro (No derivatives)
$\ell = 1$ Viscous Hydro (First ")
$\ell = 2$
$\ell = 0$ $T_{\mu\nu} = \varepsilon u_\mu u_\nu + P$



$T_{\mu\nu} = \sum_{l} (Kn)^{l}\, T_{\mu\nu}^{(l)}$
$l=0$ Ideal Hydro (No derivatives)
$l=1$ Viscous Hydro (First ")
$l=2$
$l=0$ $T_{\mu\nu} = \epsilon u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu)$

$$T_{\mu\nu} = \sum_{\ell} (Kn)^{\ell}\, T^{(\ell)}_{\mu\nu}$$

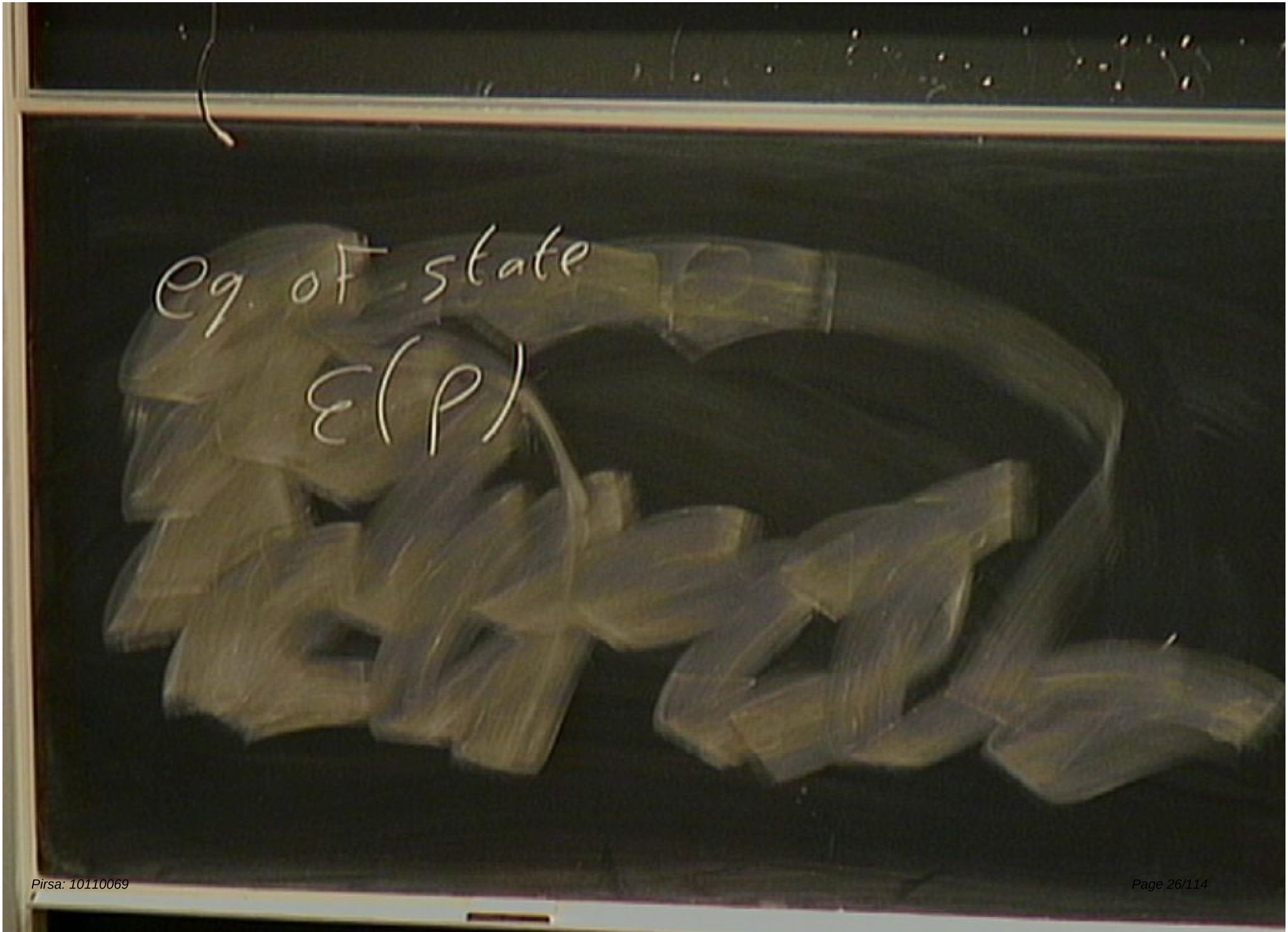
$\ell = 0$ Ideal Hydro (No derivatives)

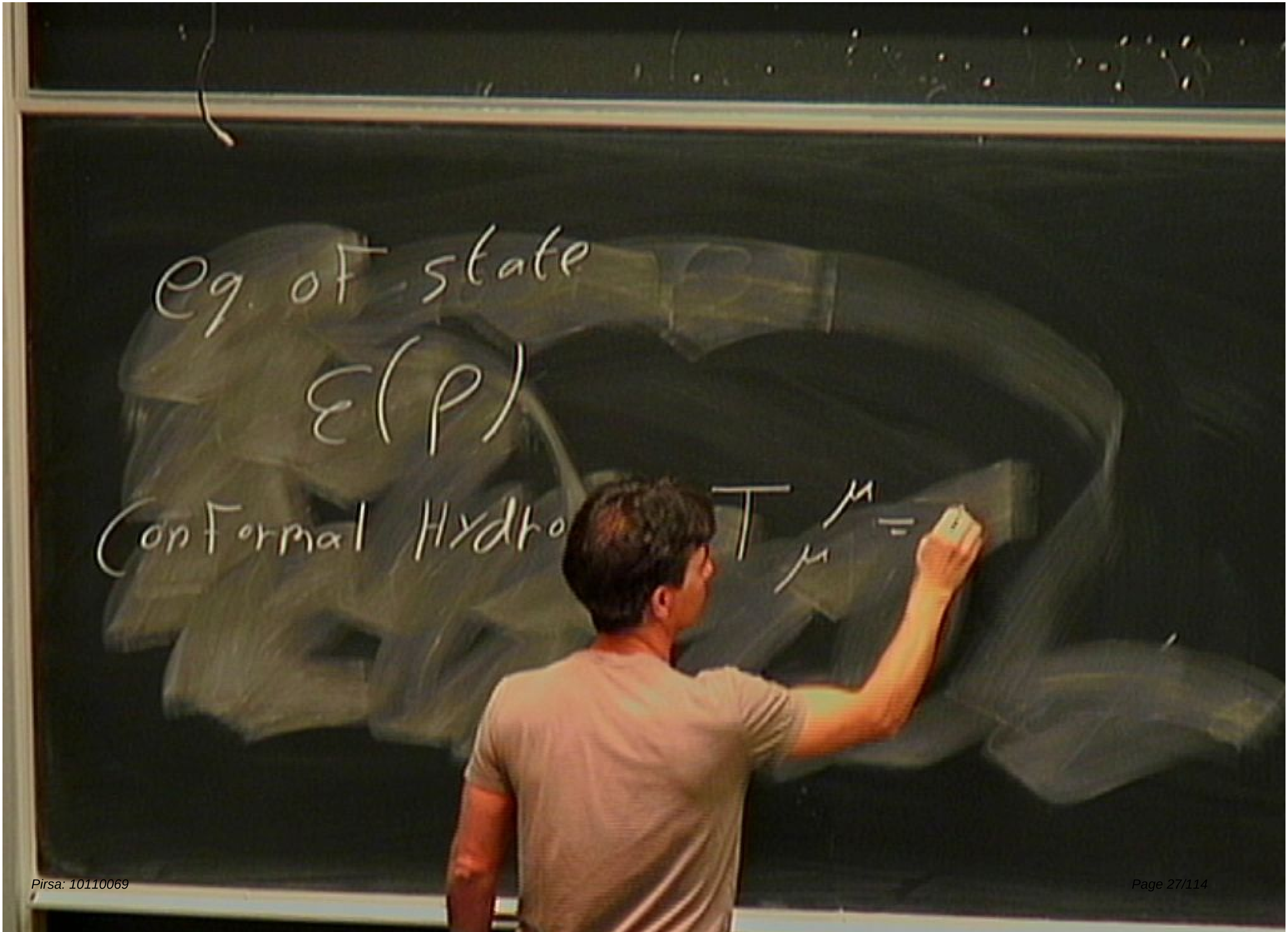
$\ell = 1$ Viscous Hydro (First ")

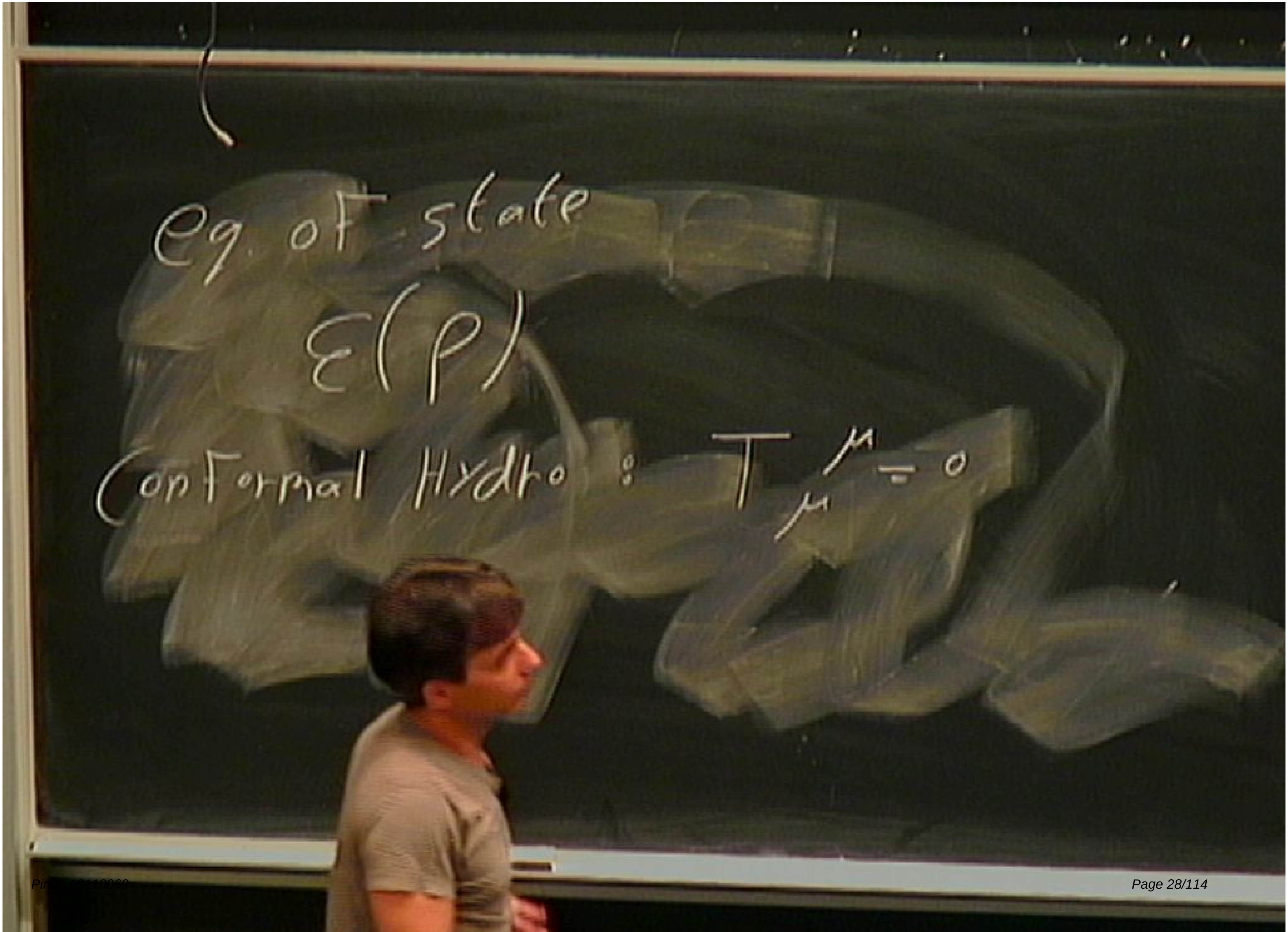
$\ell = 2$

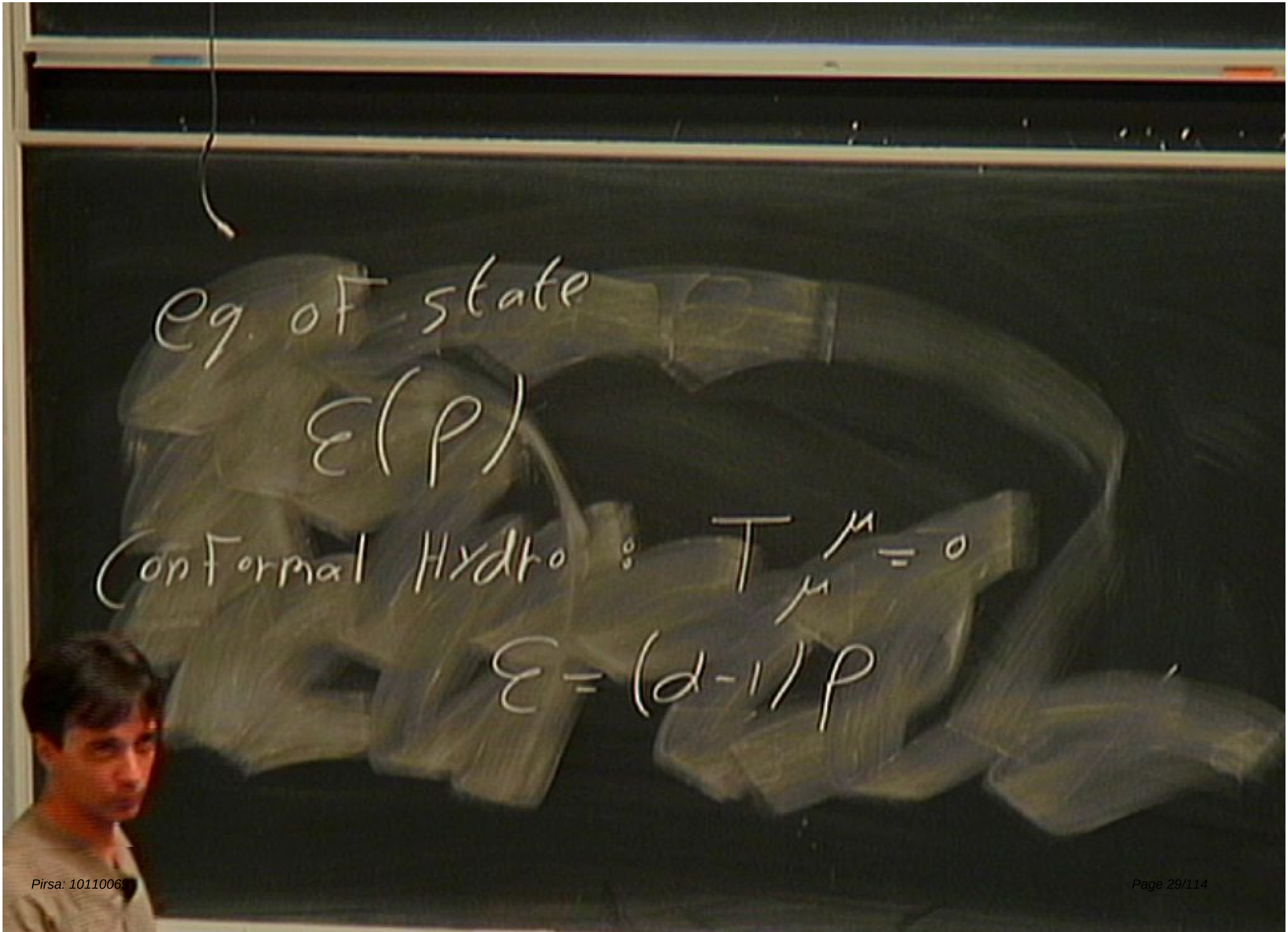
<u>$\ell = 0$</u> $\quad T_{\mu\nu} = \epsilon\, u_\mu u_\nu + p\,(g_{\mu\nu} + u_\mu u_\nu)$

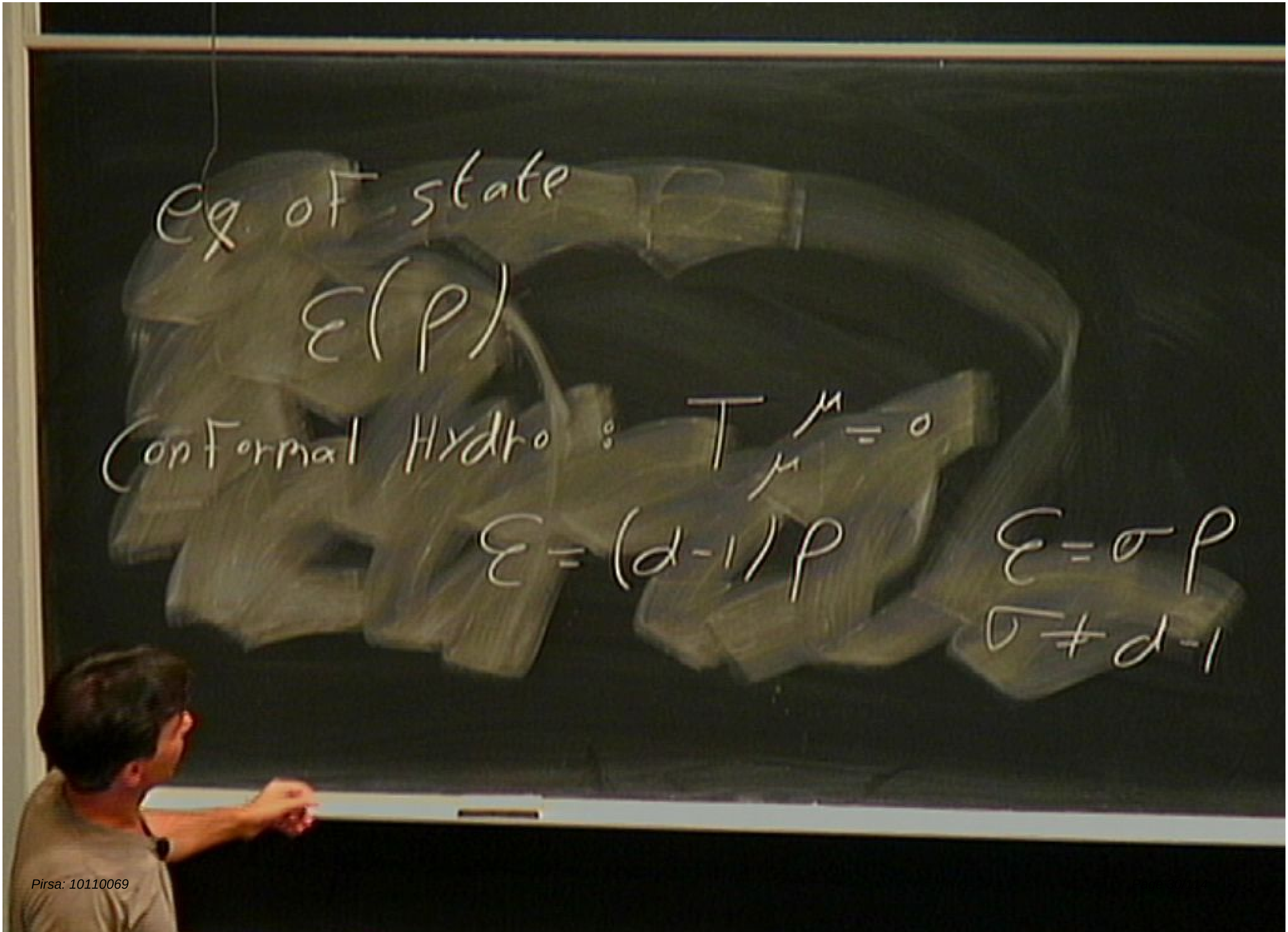
Rest Frame $\quad u_\mu = (1\ 0 \ldots 0) \quad T_{\mu\nu} = \begin{pmatrix} \epsilon & & \\ & p & \\ & & \ddots \end{pmatrix}$


eq. of state
ε(ρ)


eq. of state
ε(p)
Conformal Hydro
T μ μ =


eq. of state
ε(p)
Conformal Hydro : $T_\mu^{\ \mu} = 0$


eq. of state
$\varepsilon(p)$
Conformal Hydro : $T_\mu^{\ \mu} = 0$
$\varepsilon = (d-1)p$


Eq. of state
ε(p)
Conformal Hydro : T_μ^μ = 0
ε = (d-1) p
ε = σ p
σ ≠ d-1

eq. of state

$\varepsilon(p)$

Conformal Hydro : $T_\mu^{\ \mu} = 0$

$\varepsilon = (d-1)p \qquad \varepsilon = \sigma p$

Speed of Sound $\quad v_s^2 = \partial p / \partial \varepsilon = 1/(d-1) \qquad \sigma \neq d-1$

eq. of state

($\varepsilon(P)$

Conformal Hydro: $T_\mu^{\ \mu} = 0$

$$\varepsilon = (d-1)P \qquad \varepsilon = \sigma P$$

$$\sigma \neq d-1$$

Speed of sound $v_s^2 = \frac{\partial P}{\partial \varepsilon} = 1/(d-1)$

$\boxed{c = 1}$

eq. of state

($\varepsilon(P)$

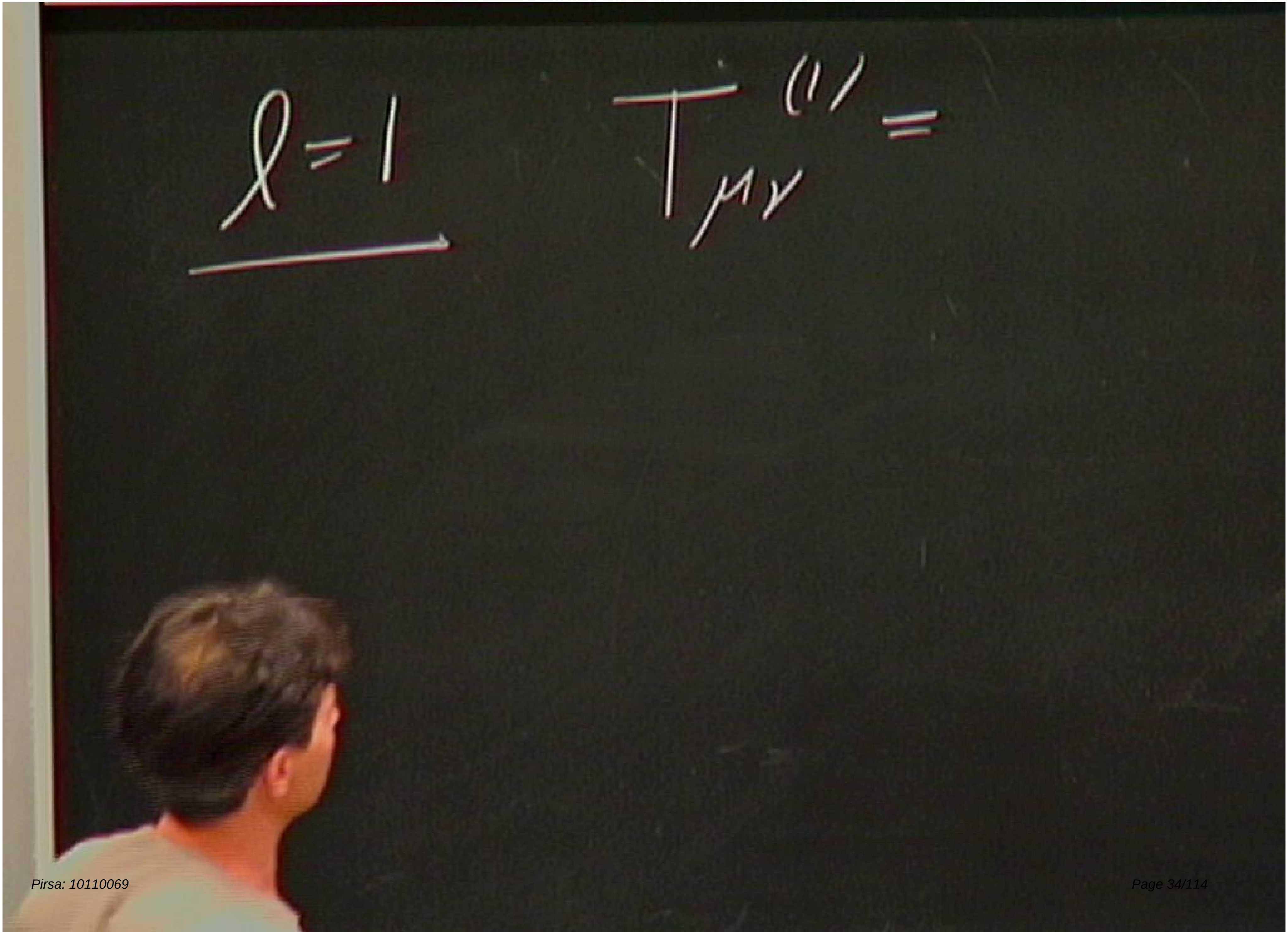
Conformal Hydro: $T_\mu^{\ \mu} = 0$

$$\varepsilon = (d-1)P \qquad \varepsilon = \sigma P$$

$$\sigma \neq d-1$$

Speed of sound $\quad v_s^2 = \frac{\partial P}{\partial \varepsilon} = \frac{1}{d-1}$

$\boxed{c = 1}$


$\ell = 1$
$T_{\mu\nu}^{(1)} =$

$\underline{\ell = 1}$

$$T_{\mu\nu}^{(1)} = -2\xi\,\sigma_{\mu\nu} - 2\zeta\,(\partial_\alpha u^\alpha)\cdot(\xi_{\mu\nu} + u_\mu u_\nu)$$

$$P_{\mu\nu} = \xi_{\mu\nu} + u_\mu u_\nu$$

$\underline{l=1}$ $\quad T^{(1)}_{\mu\nu} = -2\xi\,\sigma_{\mu\nu} - 2\zeta(\partial_\alpha u^\alpha)\cdot(\Sigma_{\mu\nu} + u_\mu u_\nu)$

$P_{\mu\nu} = \Sigma_{\mu\nu} + u_\mu u_\nu$

$P_{\mu\nu} u^\nu = 0$

$\underline{\ell = 1}$

$$T^{(1)}_{\mu\nu} = -2\xi\,\sigma_{\mu\nu} - 2\zeta\,(\partial_\alpha u^\alpha)\cdot(g_{\mu\nu} + u_\mu u_\nu)$$

$$P_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$$P_{\mu\nu} u^\nu = 0$$

ξ shear viscosity

ζ bulk viscosity

$\underline{l=1}$

$$T^{(1)}_{\mu\nu} = -2\xi\sigma_{\mu\nu} - 2\zeta(\partial_\alpha u^\alpha)\cdot(g_{\mu\nu}+u_\mu u_\nu)$$

$$P_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

$$P_{\mu\nu}u^\nu = 0$$

ξ shear viscosity

ζ bulk viscosity

$\underline{\ell = 1}$ $$T^{(1)}_{\mu\nu} = -2\xi\,\sigma_{\mu\nu} - 2\zeta\,(\partial_\alpha u^\alpha)\cdot(\xi_{\mu\nu} + u_\mu u_\nu)$$

$$P_{\mu\nu} = \xi_{\mu\nu} + u_\mu u_\nu$$

$$P_{\mu\nu} u^\nu = 0$$

ξ shear viscosity

ζ bulk viscosity

$\underline{l=1}$

$$T^{(1)}_{\mu\nu} = -2\xi\sigma_{\mu\nu} - 2\zeta(\partial_\alpha u^\alpha)\cdot(g_{\mu\nu}+u_\mu u_\nu)$$

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Landau Frame

ξ shear viscosity

ζ bulk viscosity

$\underline{l=1}$ $T^{(1)}_{\mu\nu} = -2\xi\sigma_{\mu\nu} - 2\zeta(\partial_\alpha u^\alpha) \cdot (\Sigma_{\mu\nu} + u_\mu u_\nu)$

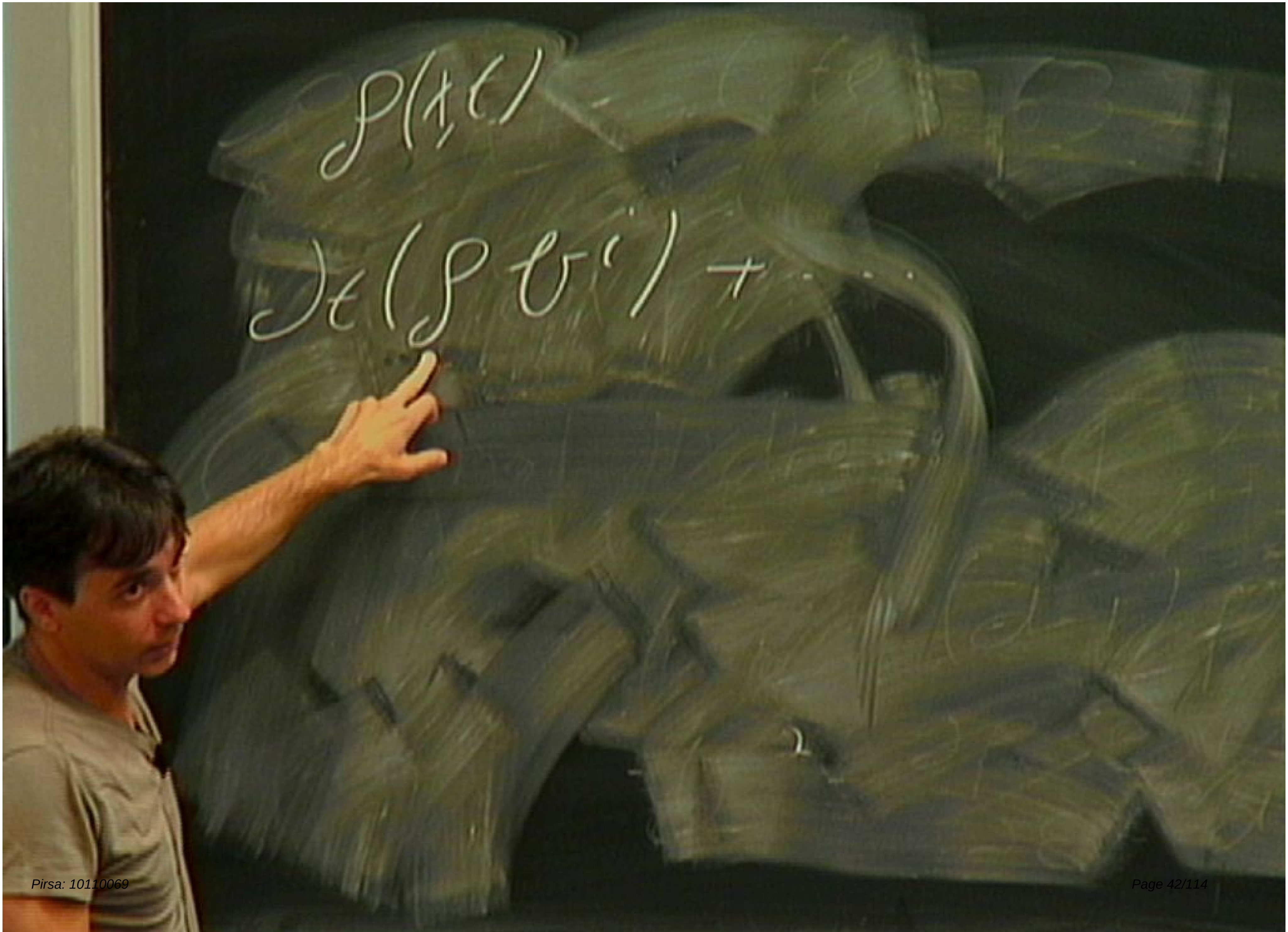
$P_{\mu\nu} = \Sigma_{\mu\nu} + u_\mu u_\nu$

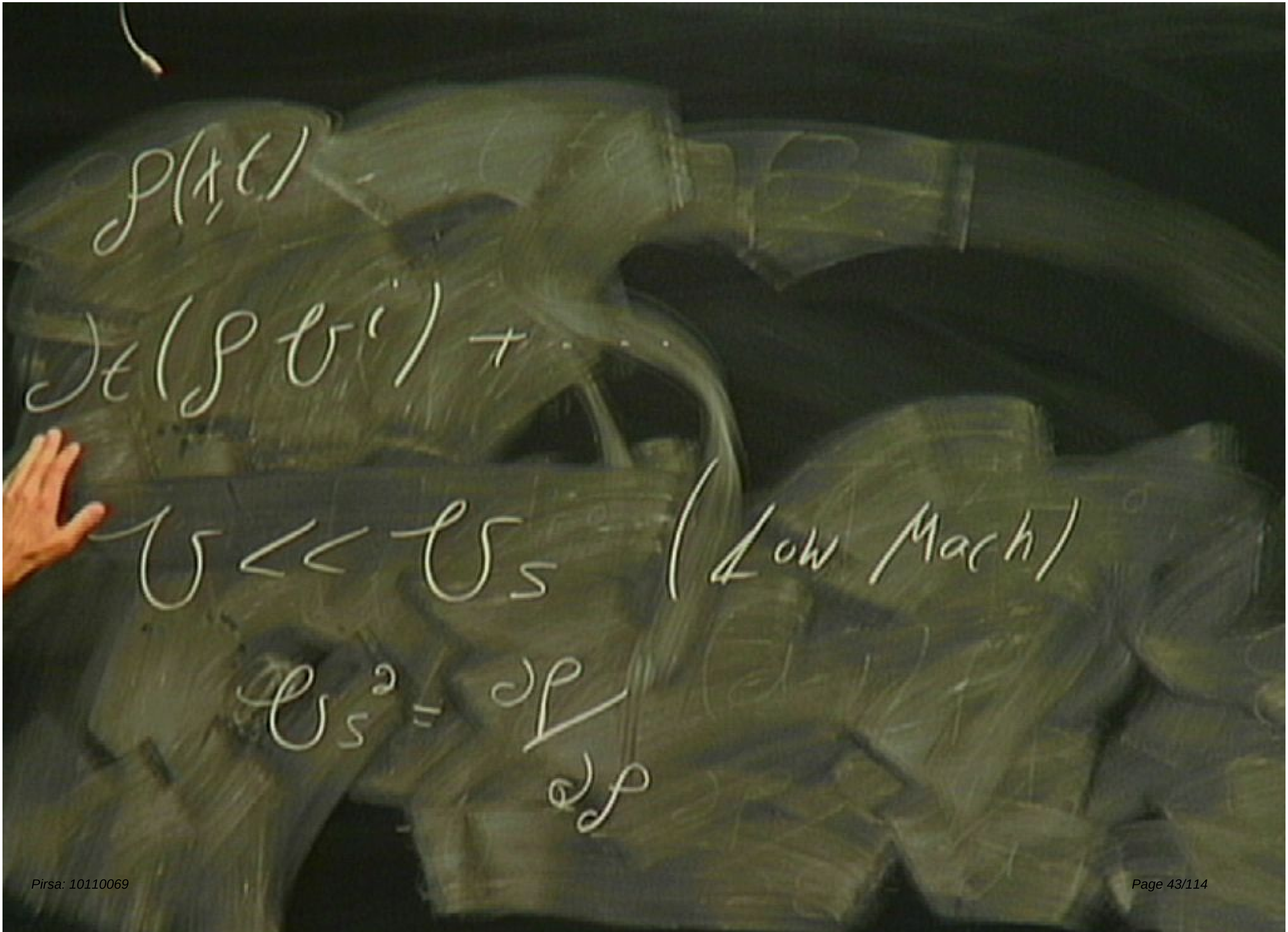
$P_{\mu\nu} u^\nu = 0$

Landau Frame

ξ shear viscosity

ζ bulk viscosity


ρ(t,ℓ)
∂t(ρ v^i) + . . .



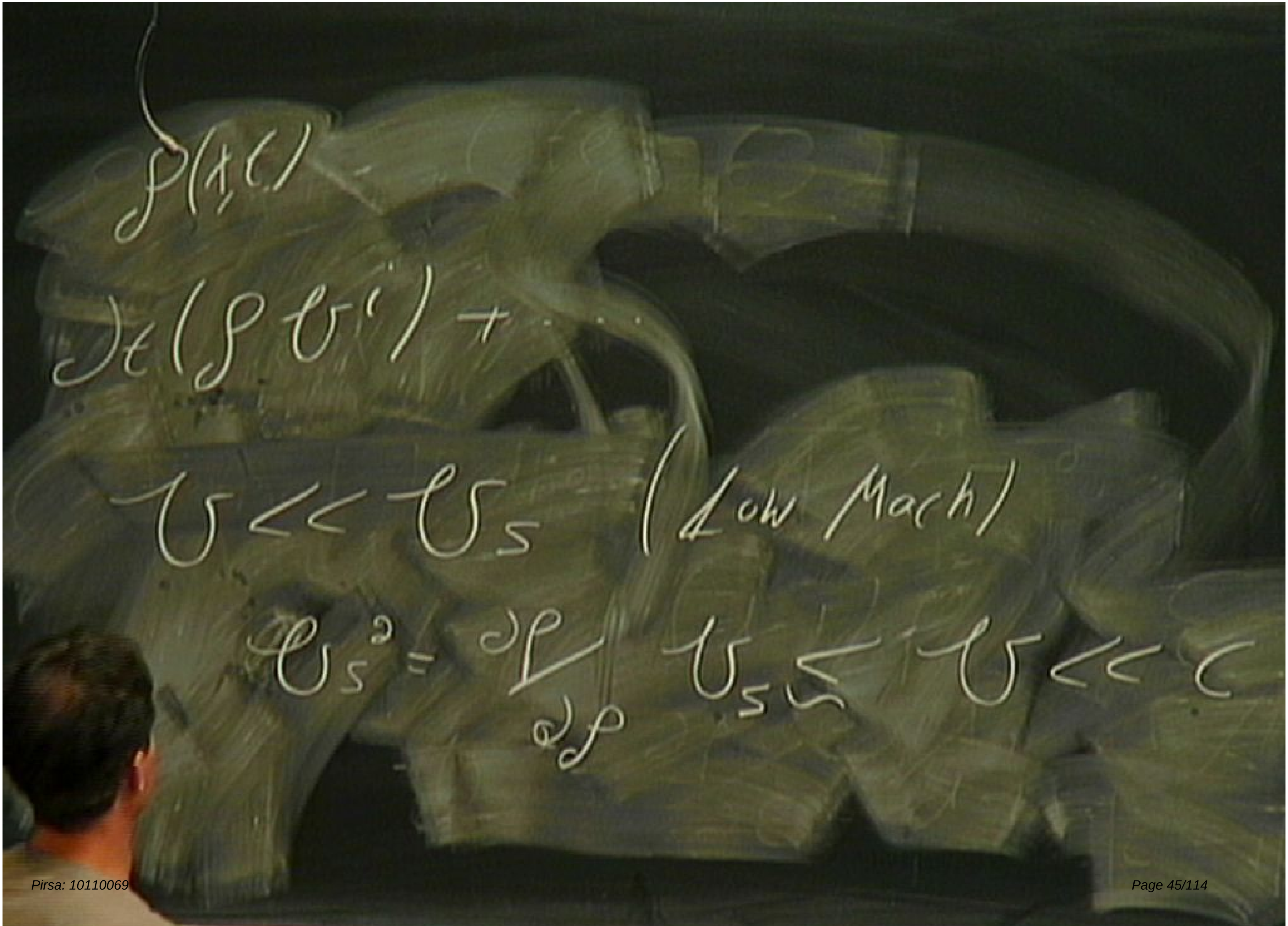
$\rho(x,t)$
$\partial_t(\rho v^i) + \dots$
$v << v_s$ (Low Mach)
$v_s^2 = \frac{\partial p}{\partial \rho}$

$\rho(x,t)$

$\partial_t(\rho v^i) + \ldots$

$v \ll v_s$ (Low Mach)

$$v_s^2 = \frac{\partial P}{\partial \rho}$$


ρ(x,t)
∂t(ρ v^i) + . . .
v << v_s (Low Mach)
v_s^2 = ∂P/∂ρ v_s ≲ v << c

$\underline{l=1}$ $T^{(1)}_{\mu\nu} = -2\xi\sigma_{\mu\nu} - 2\zeta(\partial_\alpha u^\alpha)\cdot(\eta_{\mu\nu} + u_\mu u_\nu)$

$P_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$ $\quad\xi$ shear viscosity

$P_{\mu\nu} u^\nu = 0$ $\quad\zeta$ bulk viscosity

Landau Frame $u_\mu T^{\mu\nu(n)} = 0 \quad n \geq 1$

$\underline{l=1}$ $\quad T_{\mu\nu}^{(1)} = -2\xi\,\sigma_{\mu\nu} - 2\zeta\,(\partial_\alpha u^\alpha)\cdot(g_{\mu\nu}+u_\mu u_\nu)$

$P_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ $\quad \xi$ shear viscosity

$P_{\mu\nu} u^\nu = 0$ $\quad \zeta$ bulk viscosity

Landau Frame $\quad u_\mu T^{\mu\nu(n)} = 0 \quad n \geq 1$

$l = 1$

$$T^{(1)}_{\mu\nu} = -2\xi\sigma_{\mu\nu} - 2\zeta(\partial_\alpha u^\alpha)\cdot(\xi_{\mu\nu} + u_\mu u_\nu)$$

$$P_{\mu\nu} = \xi_{\mu\nu} + u_\mu u_\nu$$

$$P_{\mu\nu} u^\nu = 0$$

ξ shear viscosity

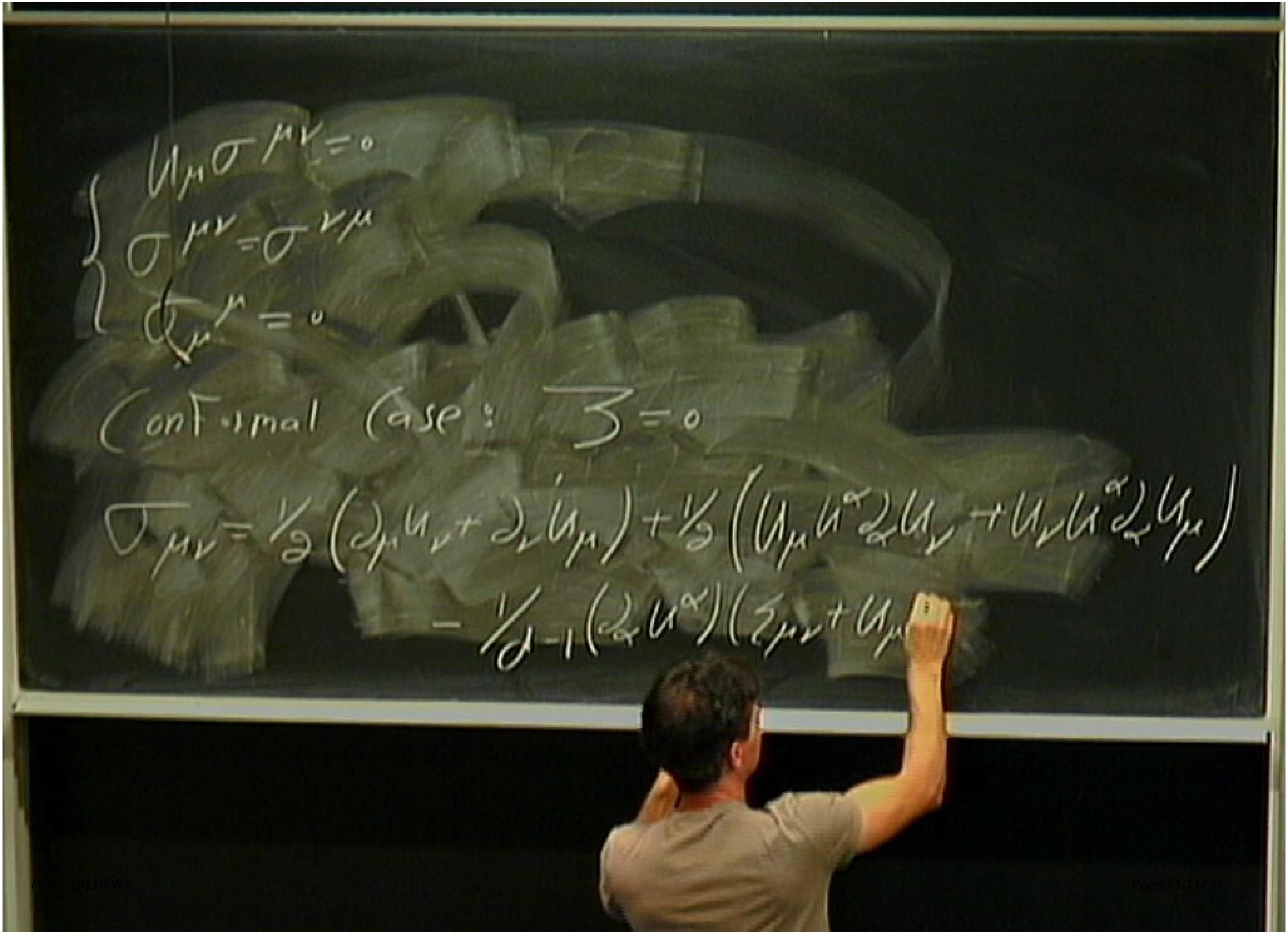
ζ bulk viscosity

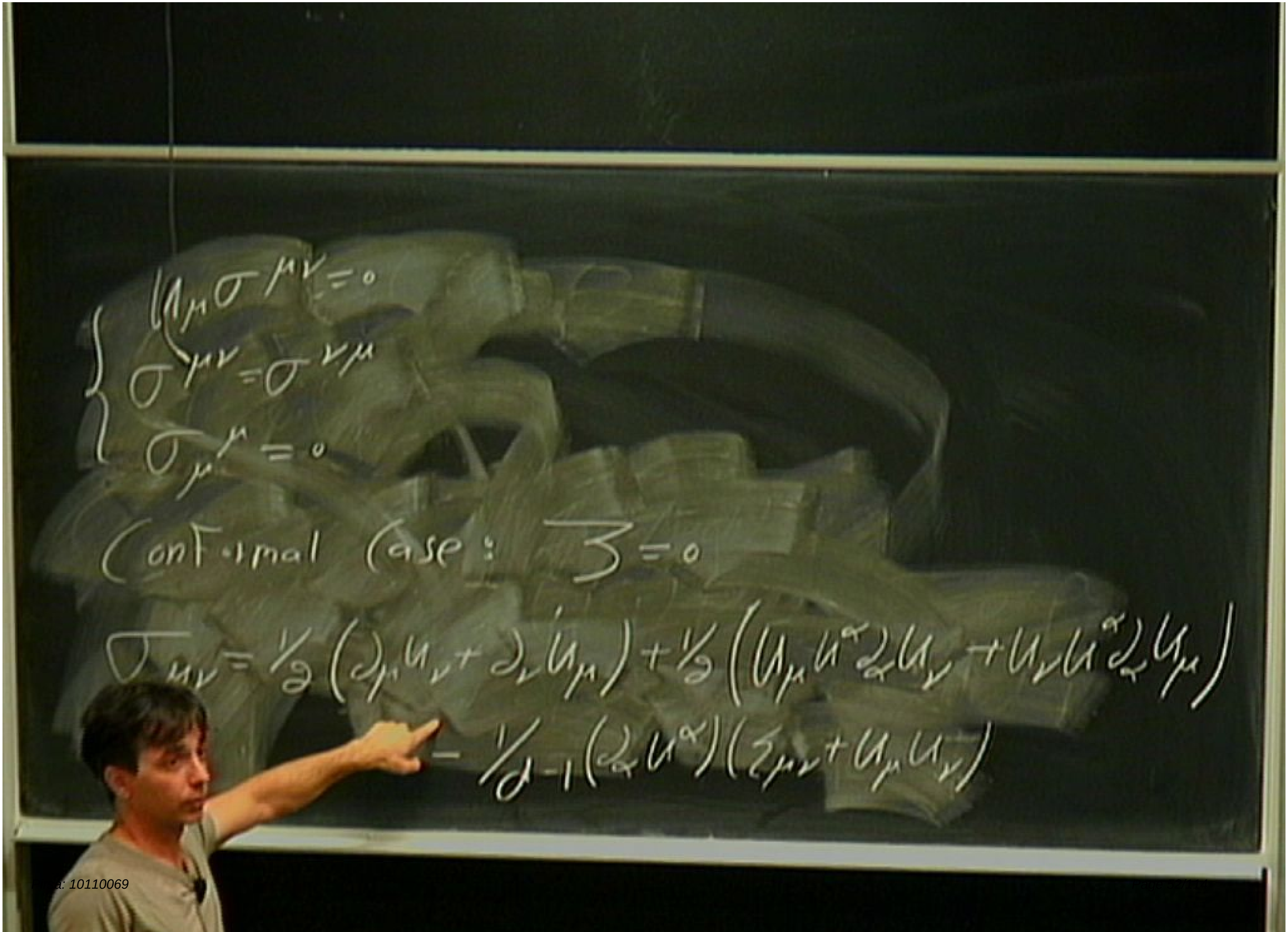
Landau Frame $u_\mu T^{\mu\nu(n)} = 0 \quad n \geq 1$

$$\begin{cases} u_\mu \sigma^{\mu\nu} = 0 \\ \sigma^{\mu\nu} = \sigma^{\nu\mu} \\ \sigma_\mu^{\ \mu} = 0 \end{cases}$$

$$\begin{cases} u_\mu \sigma^{\mu\nu} = 0 \\ \sigma^{\mu\nu} = \sigma^{\nu\mu} \\ \sigma_\mu^{\ \mu} = 0 \end{cases}$$

Conformal Case: $\zeta = 0$


$u_\mu \sigma^{\mu\nu} = 0$
$\sigma^{\mu\nu} = \sigma^{\nu\mu}$
$\sigma_\mu{}^\mu = 0$
Conformal Case: $\zeta = 0$
$\sigma_{\mu\nu} = \frac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu) + \frac{1}{2}(u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu)$
$- \frac{1}{d-1}(\partial_\alpha u^\alpha)(g_{\mu\nu} + u_\mu$


$u_\mu \sigma^{\mu\nu} = 0$
$\sigma^{\mu\nu} = \sigma^{\nu\mu}$
$\sigma_\mu{}^\mu = 0$
Conformal case: $\zeta = 0$
$\sigma_{\mu\nu} = \frac{1}{2}(\partial_\mu u_\nu + \partial_\nu u_\mu) + \frac{1}{2}(u_\mu u^\alpha \partial_\alpha u_\nu + u_\nu u^\alpha \partial_\alpha u_\mu)$
$- \frac{1}{d-1}(\partial_\alpha u^\alpha)(g_{\mu\nu} + u_\mu u_\nu)$
a: 10110069

Conformal Case :

$$\mathcal{E} = (d-1)\, p$$

$$\mathcal{E} + p = T S$$

$$\mathcal{E} \sim T^{d}$$

Conformal Case:

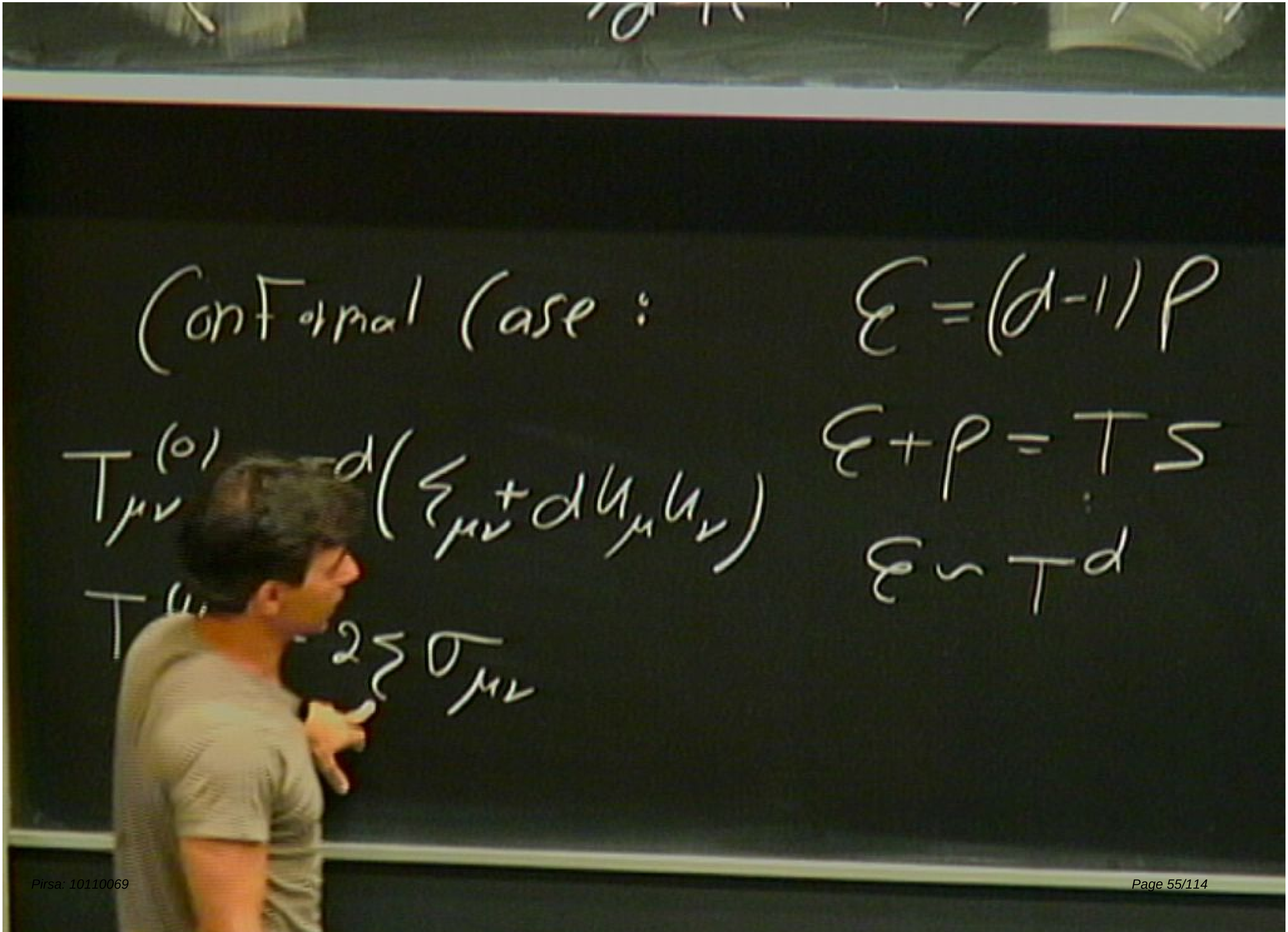
$$\mathcal{E} = (d-1)P$$

$$T^{(0)}_{\mu\nu} = T^d(\eta_{\mu\nu} + d\, u_\mu u_\nu)$$

$$\mathcal{E} + P = TS$$

$$\mathcal{E} \sim T^d$$

T


Conformal Case :
$\epsilon = (d-1) P$
$\epsilon + P = T s$
$\epsilon \sim T^d$
$T^{(0)}_{\mu\nu}$
$d(\eta_{\mu\nu} + d u_\mu u_\nu)$
$2\eta \sigma_{\mu\nu}$

Conformal Case:

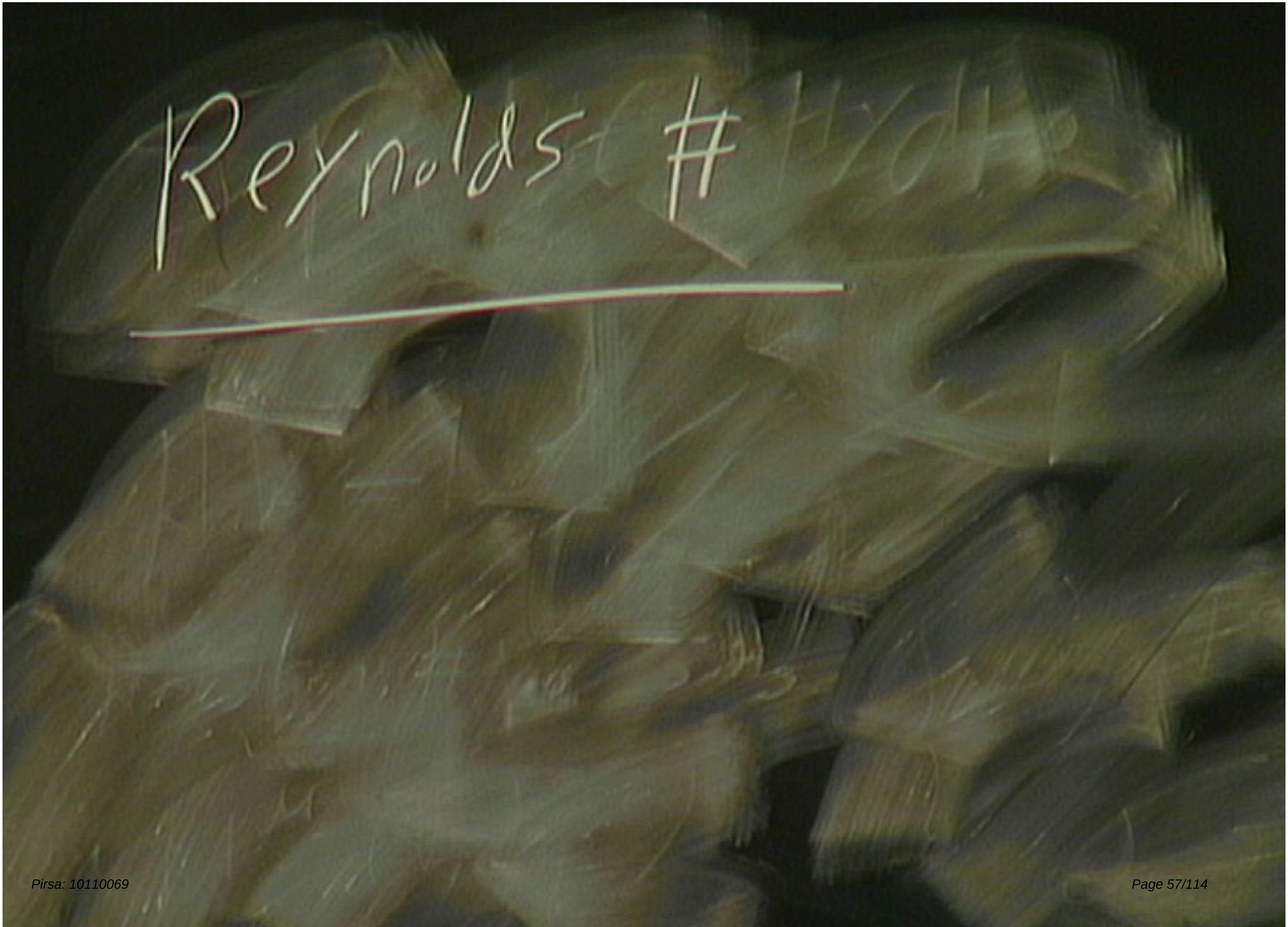
$$\varepsilon = (d-1)\,P$$

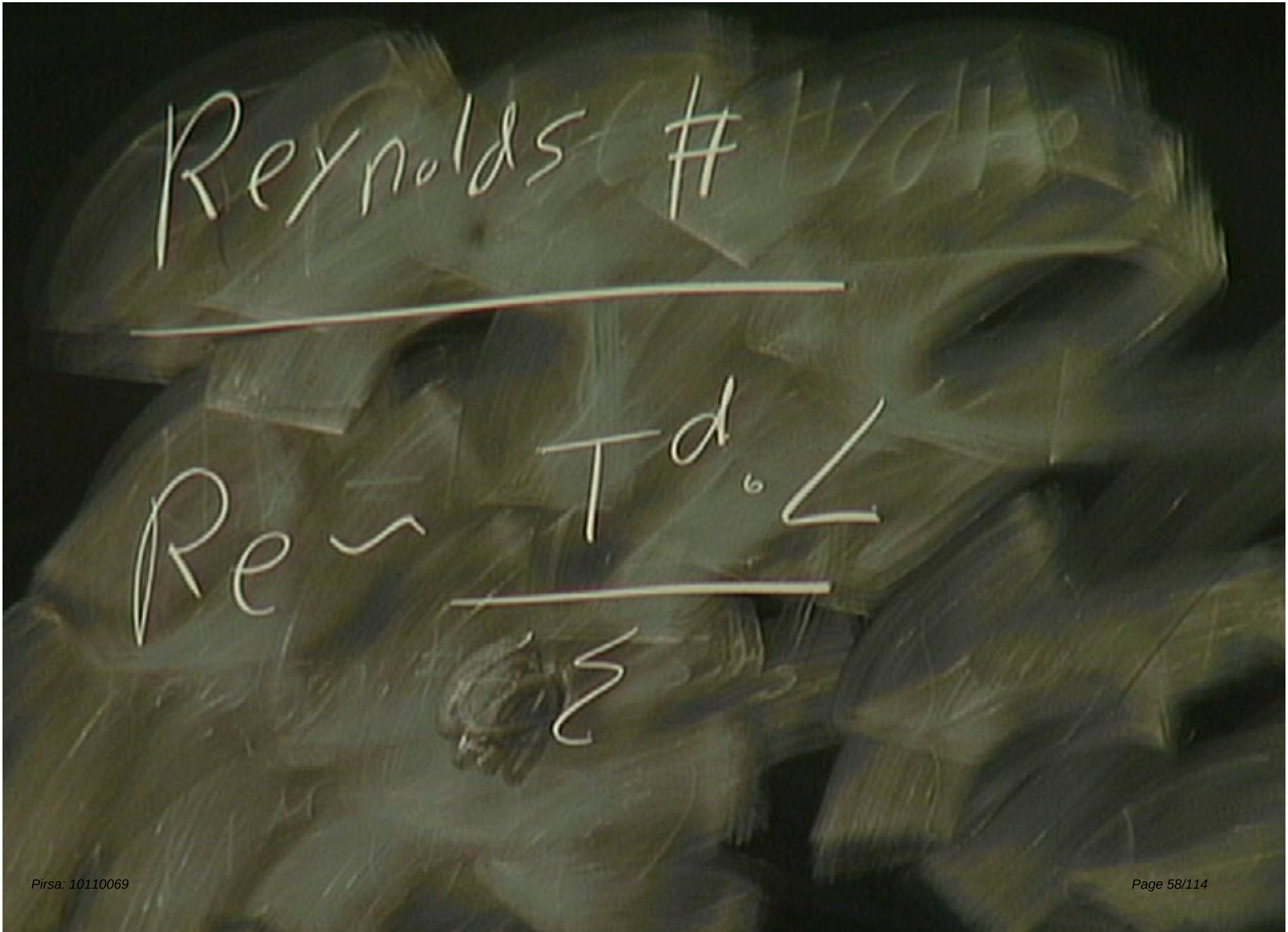
$$T^{(0)}_{\mu\nu} = T^{d}\left(\eta_{\mu\nu} + d\, u_\mu u_\nu\right)$$

$$\varepsilon + P = T\,s$$

$$T^{(1)}_{\mu\nu} = -2\,\eta\,\sigma_{\mu\nu}$$

$$\varepsilon \sim T^{d}$$

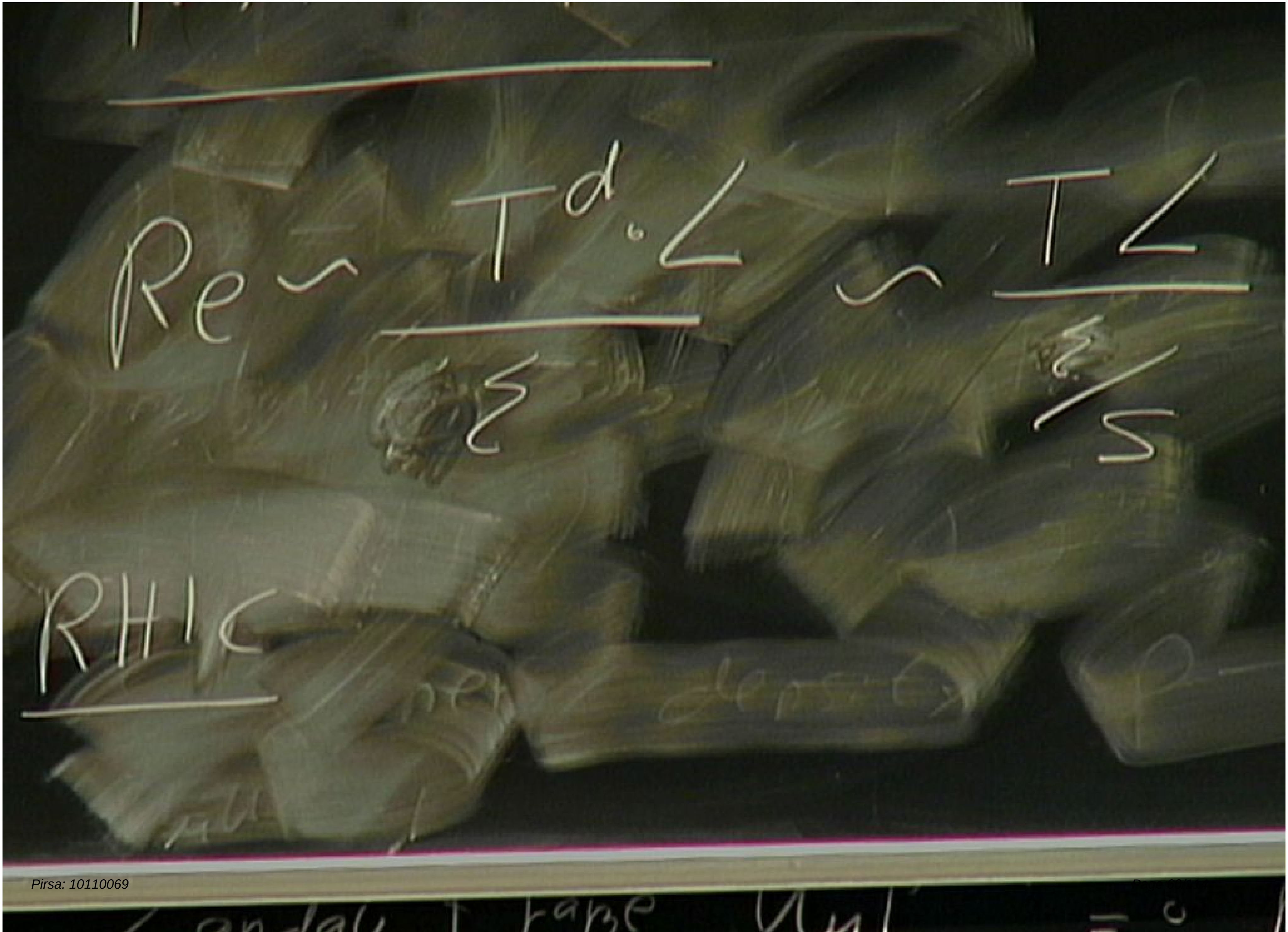

Reynolds #

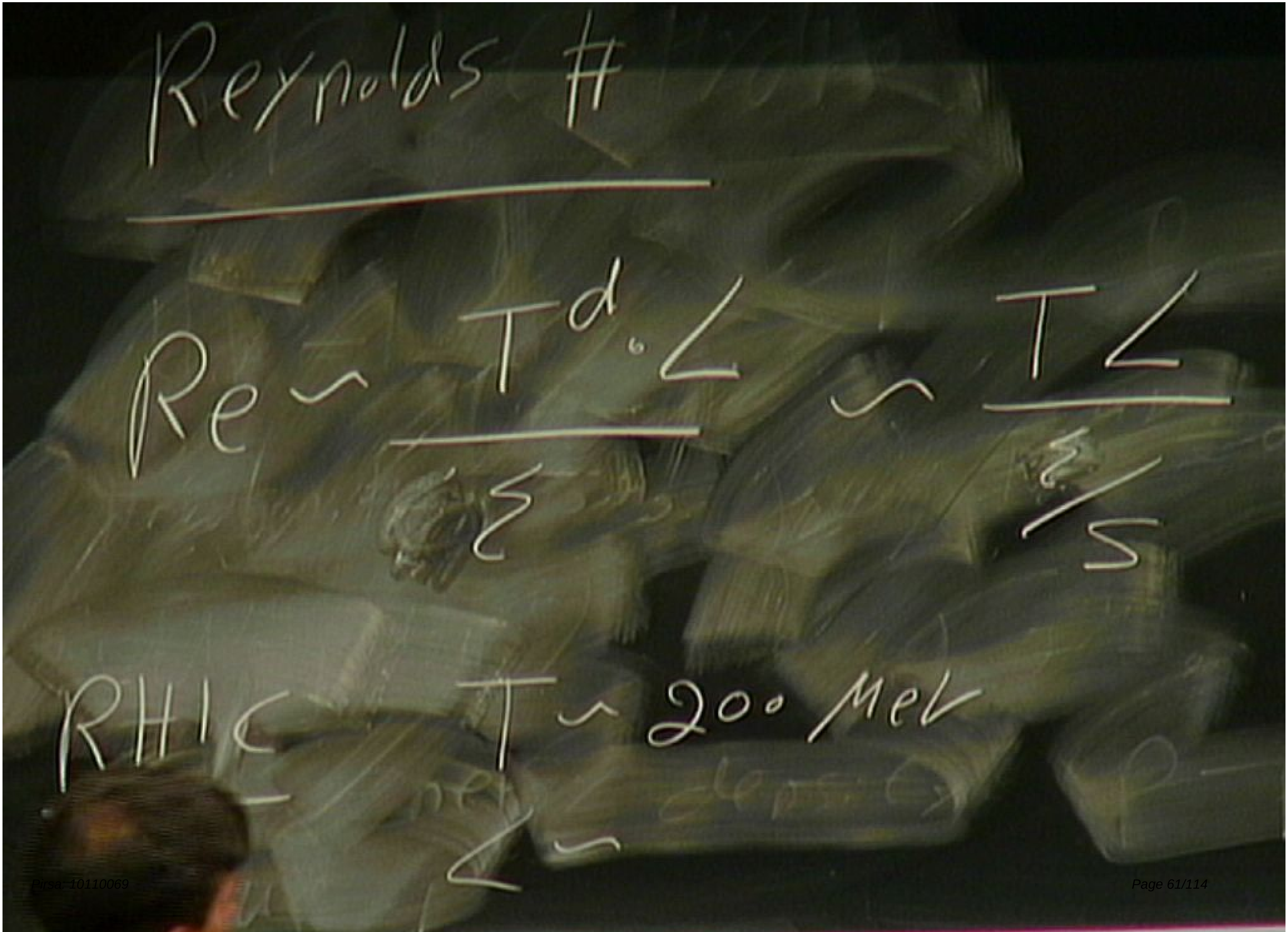

Reynolds #
Re ~ Td·L / ε

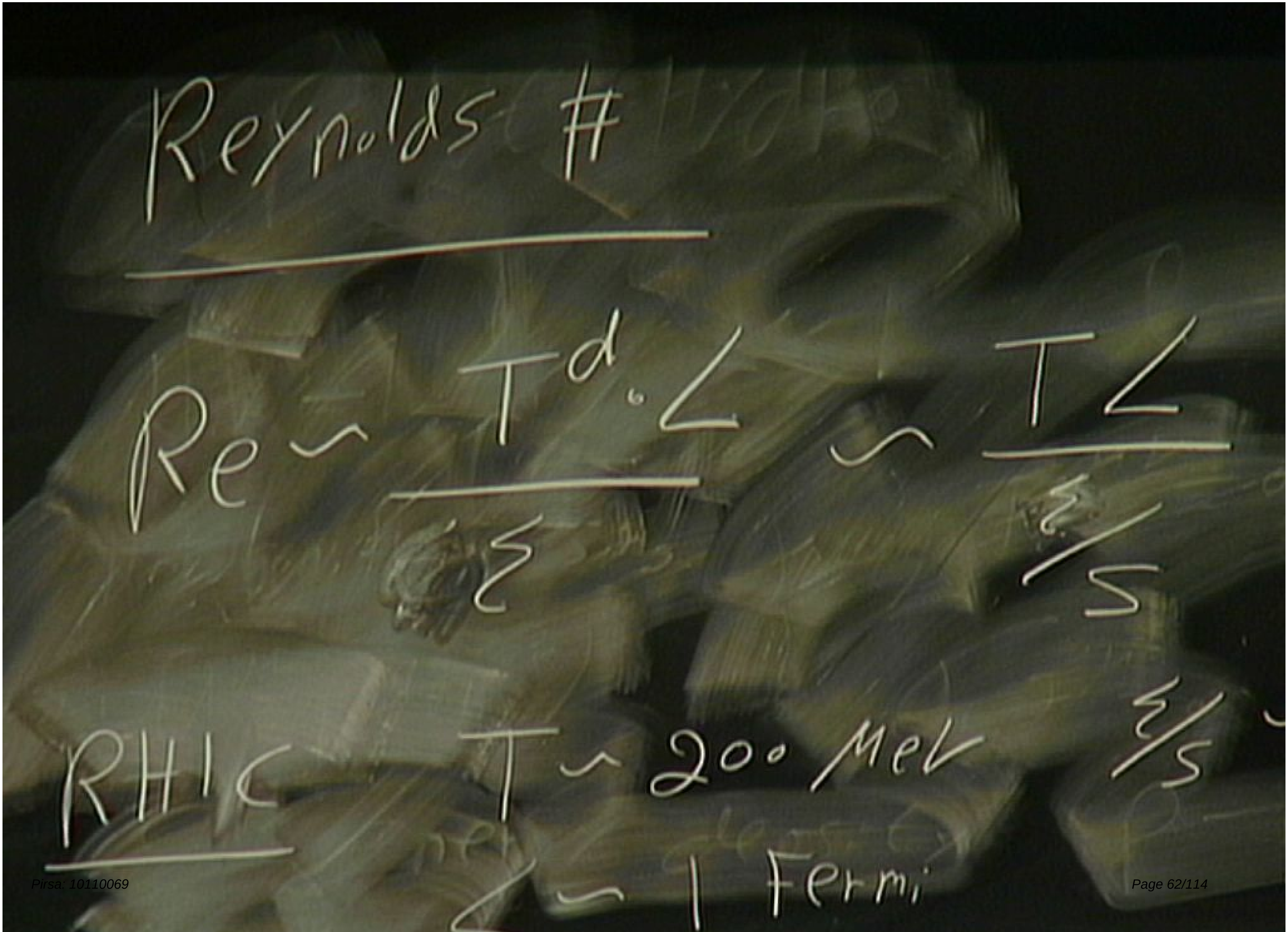
Reynolds #

$$Re \sim \frac{T^d \cdot L}{\varepsilon} \sim \frac{T L}{s}$$

$$s \sim T^{d-1}$$


$Re \sim \frac{T^d L}{\varepsilon} \sim \frac{TL}{s}$
RHIC


Reynolds #
Re ~ $\frac{T^d \cdot L}{\varepsilon}$ ~ $\frac{TL}{\varepsilon/s}$
RHIC T ~ 200 MeV


Reynolds #
Re ~ T d · L / ε ~ T L / (ε/s)
RHIC T ~ 200 MeV ε/s
L ~ 1 Fermi

Reynolds

$$Re \sim \frac{T^{d} \cdot L}{\varepsilon} \sim \frac{T L}{\varepsilon / s}$$

RHIC $T \sim 200\ \text{MeV}$ $\quad \varepsilon / s$

$L \sim 1$ Fermi

Reynolds

$$Re \sim \frac{T^d L}{\varepsilon} \sim \frac{T L}{\varepsilon / s} \sim O(10)$$

RHIC $\quad T \sim 200 \text{ MeV} \quad \varepsilon/s \sim 1/4\pi$

$L \sim 1$ Fermi

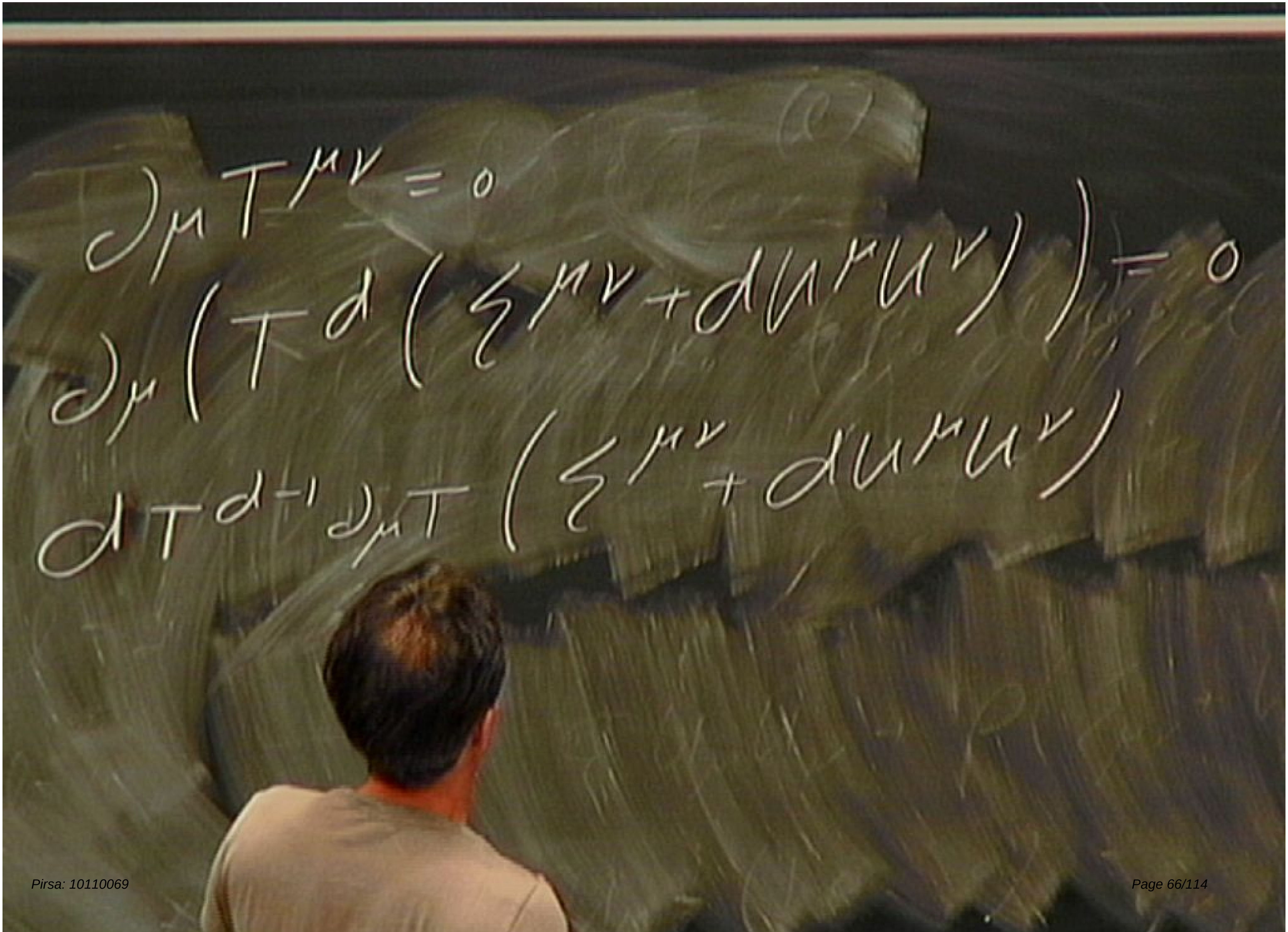
Landau Frame $\quad u_\mu T \quad = 0 \quad n \geq 1$

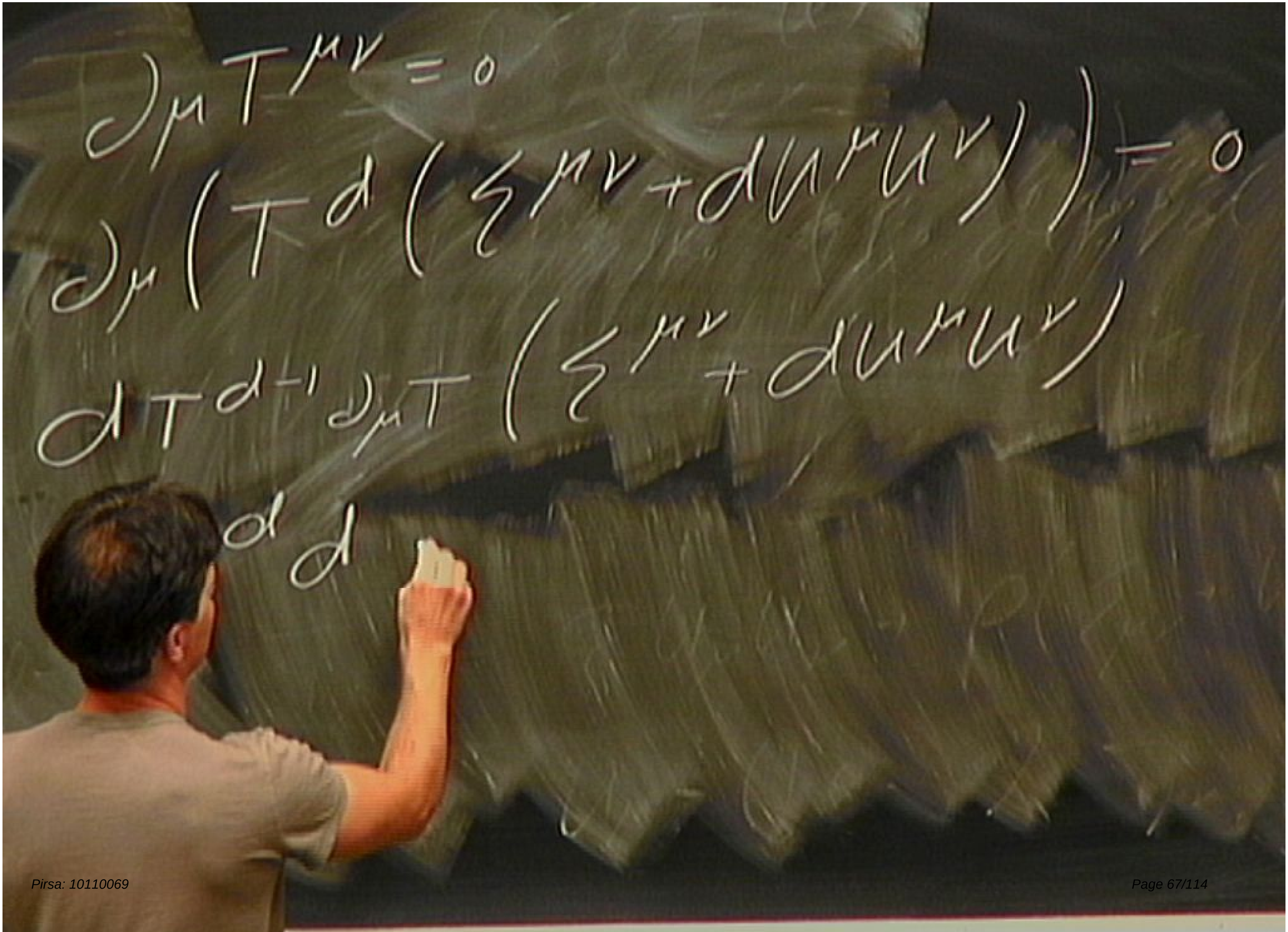
Reynolds #

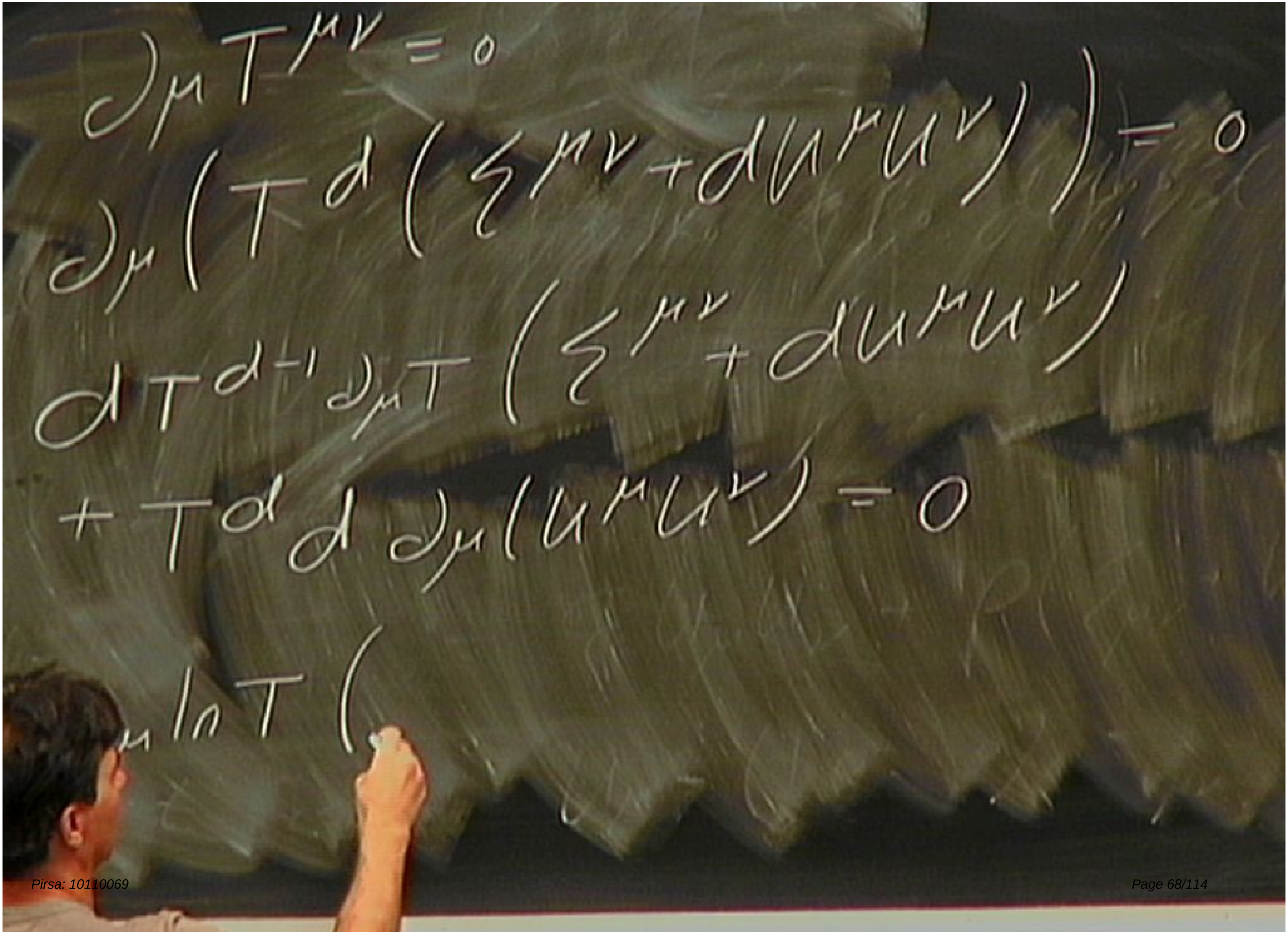
$$Re \sim \frac{T^d \cdot L}{\varepsilon} \sim \frac{T L}{\varepsilon / s} \sim O(10)$$

RHIC $T \sim 200$ MeV $\quad \varepsilon/s \sim 1/4\pi$

$L \sim 1$ Fermi


$\partial_\mu T^{\mu\nu} = 0$
$\partial_\mu \left(T^d \left(\eta^{\mu\nu} + d u^\mu u^\nu \right) \right) = 0$
$d T^{d-1} \partial_\mu T \left(\eta^{\mu\nu} + d u^\mu u^\nu \right)$


$\partial_\mu T^{\mu\nu} = 0$
$\partial_\mu \left(T^d \left(g^{\mu\nu} + d u^\mu u^\nu \right) \right) = 0$
$d T^{d-1} \partial_\mu T \left(g^{\mu\nu} + d u^\mu u^\nu \right)$
$d \; d$


$\partial_\mu T^{\mu\nu} = 0$
$\partial_\mu \left(T^d \left(\xi^{\mu\nu} + d u^\mu u^\nu\right)\right) = 0$
$d T^{d-1} \partial_\mu T \left(\xi^{\mu\nu} + d u^\mu u^\nu\right)$
$+ T^d d \, \partial_\mu (u^\mu u^\nu) = 0$
$\partial_\mu \ln T ($

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu \left(T^d \left(\xi^{\mu\nu} + d u^\mu u^\nu \right) \right) = 0$$

$$d T^{d-1} \partial_\mu T \left(\xi^{\mu\nu} + d u^\mu u^\nu \right)$$

$$+ T^d d \, \partial_\mu \left(u^\mu u^\nu \right) = 0$$

$$\partial_\mu \ln T \left(\xi^{\mu\nu} + d u^\mu u^\nu \right) + \partial_\mu (u$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu \left(T^d \left(\eta^{\mu\nu} + d\, u^\mu u^\nu \right) \right) = 0$$

$$d\, T^{d-1} \partial_\mu T \left(\eta^{\mu\nu} + d\, u^\mu u^\nu \right)$$

$$+ T^d d\, \partial_\mu (u^\mu u^\nu) = 0$$

$$\partial_\mu \ln T \left(\eta^{\mu\nu} + d\, u^\mu u^\nu \right) + \partial_\mu (u^\mu u^\nu) = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\mu \left(T^d \left(\eta^{\mu\nu} + d u^\mu u^\nu \right) \right) = 0$$

$$d T^{d-1} \partial_\mu T \left(\eta^{\mu\nu} + d u^\mu u^\nu \right)$$

$$+ T^d d\, \partial_\mu \left(u^\mu u^\nu \right) = 0$$

$$\partial_\mu \ln T \left(\eta^{\mu\nu} + d u^\mu u^\nu \right) + \partial_\mu \left(u^\mu u^\nu \right) = 0$$

$$u_\nu , \; P^\sigma_\nu$$

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

$$u^\mu \partial_\mu \ln T \, (1-d) + \partial_\mu u^\mu = 0$$

$$\partial_\mu u^\mu + (d-1) u^\mu \partial_\mu \ln T = 0$$

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

$$u^\mu \partial_\mu \ln T \,(1-d) - \partial_\mu u^\mu = 0$$

$$\partial_\mu u^\mu + (d-1)\, u^\mu \partial_\mu \ln T = 0$$

Entr

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

$$u^\mu \partial_\mu \ln T \,(1-d) + \partial_\mu u^\mu = 0$$

$$\partial_\mu u^\mu + (d-1)\, u^\mu \partial_\mu \ln T = 0$$

Entropy $\quad S^\mu = T^{d-1} u^\mu$

Current $\quad \partial_\mu S^\mu = 0$

$$+ T \ldots \alpha\, \partial_\mu (u^\mu u^\nu) \ldots = 0$$

$$\partial_\mu \ln T \left(\xi^{\mu\nu} + \alpha u^\mu u^\nu \right) + \partial_\mu (u^\mu u^\nu) = 0$$

$$u_\nu, \; P^\sigma_\nu$$

$$P^\sigma_\nu \, \partial_\mu T^{\mu\nu} = 0$$

$$P^\sigma_\nu \, \partial^\nu \ln T$$

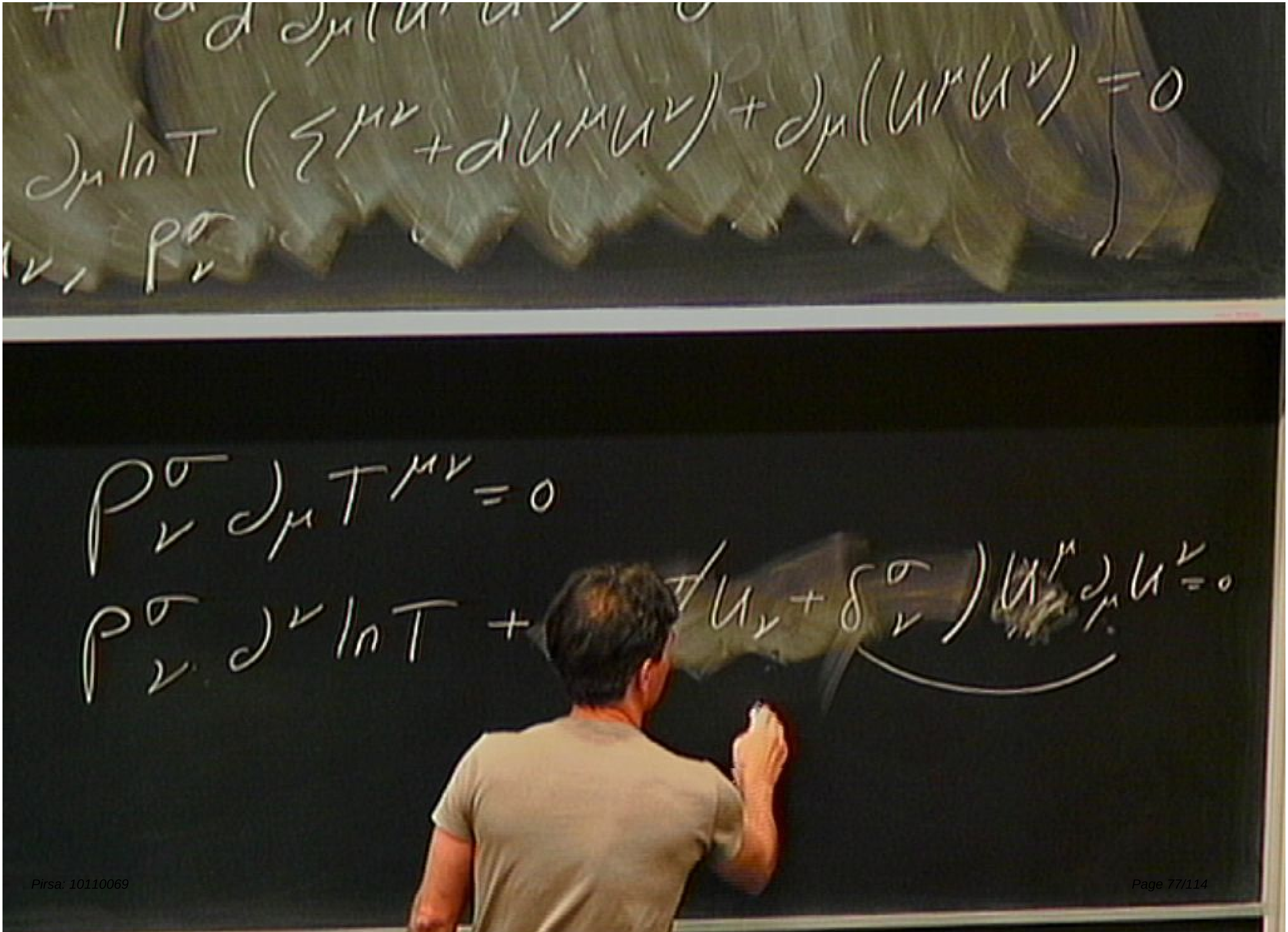
$$+\ T\,\alpha\,\partial_\mu(u^\mu u^\nu) = 0$$

$$\partial_\mu \ln T\,(\xi^{\mu\nu} + \alpha u^\mu u^\nu) + \partial_\mu(u^\mu u^\nu) = 0$$

$u_\nu,\ P^\sigma_\nu$

$$P^\sigma_\nu\,\partial_\mu T^{\mu\nu} = 0$$

$$P^\sigma_\nu\,\partial^\nu \ln T + u^\mu \partial_\mu P^\sigma_\nu u^\nu = 0$$

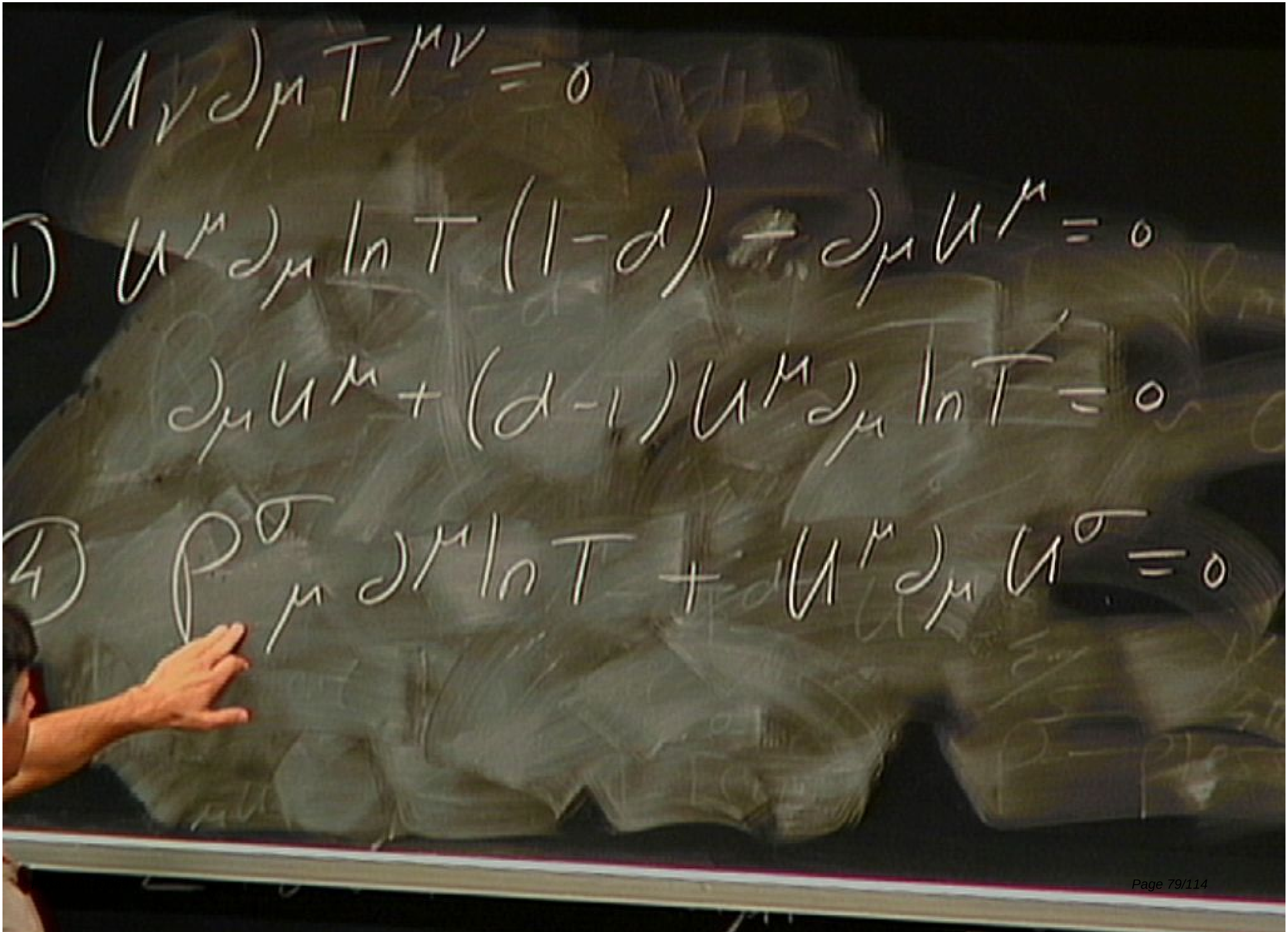

$\partial_\mu \ln T \,(\xi^{\mu\nu} + a u^\mu u^\nu) + \partial_\mu (u^\mu u^\nu) = 0$
$\rho^\sigma_\nu \partial_\mu T^{\mu\nu} = 0$

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

$$u^\mu \partial_\mu \ln T (1-\alpha) - \partial_\mu u^\mu = 0$$

$$\partial_\mu u^\mu + (\alpha-1) u^\mu \partial_\mu \ln T = 0$$

$$P^\sigma{}_\mu \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$$

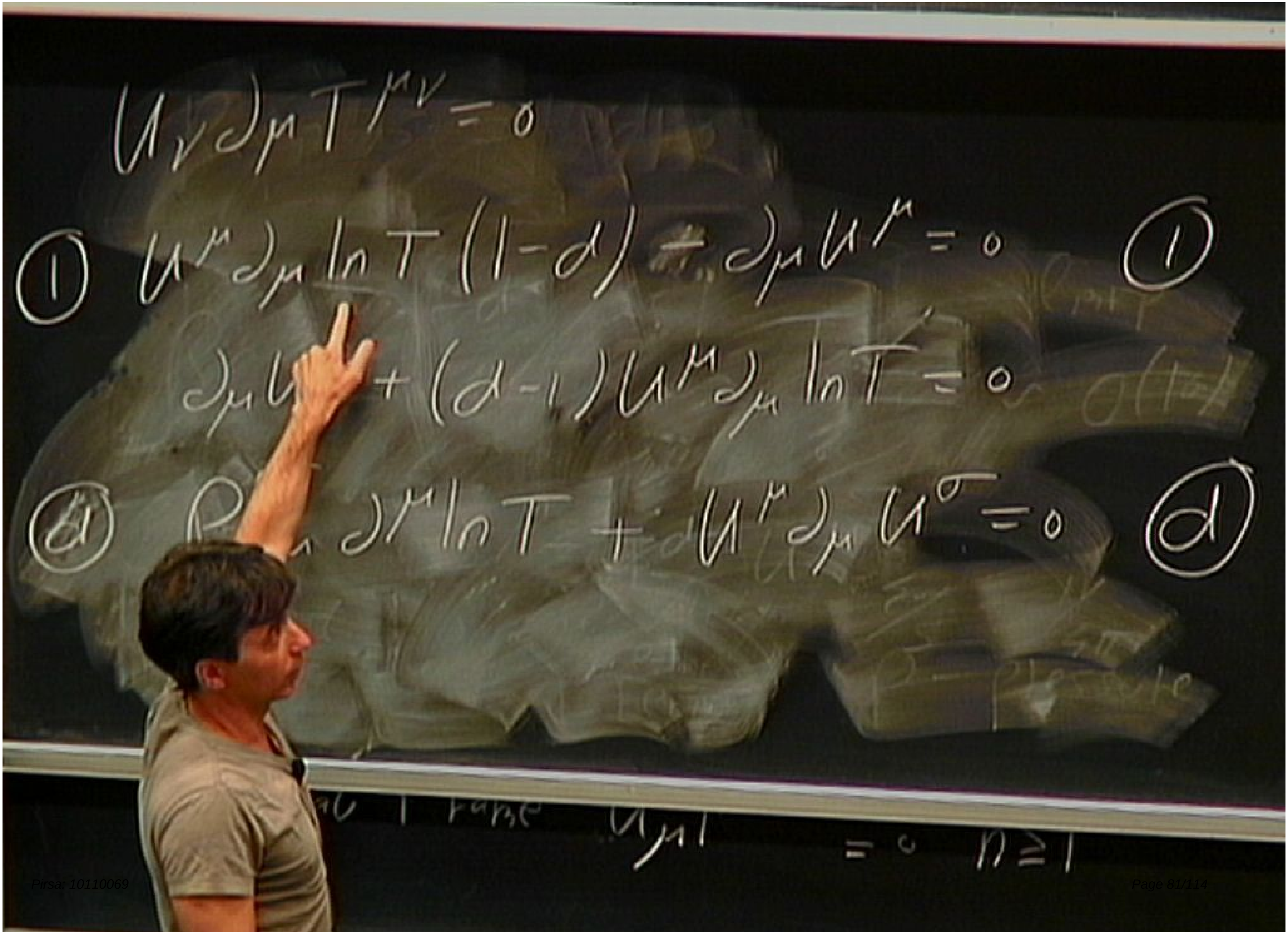

$u_\nu \partial_\mu T^{\mu\nu} = 0$
(1) $u^\mu \partial_\mu \ln T \, (1-\alpha) + \partial_\mu u^\mu = 0$
$\partial_\mu u^\mu + (\alpha-1) u^\mu \partial_\mu \ln T = 0$
(4) $P^\sigma{}_\mu \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$

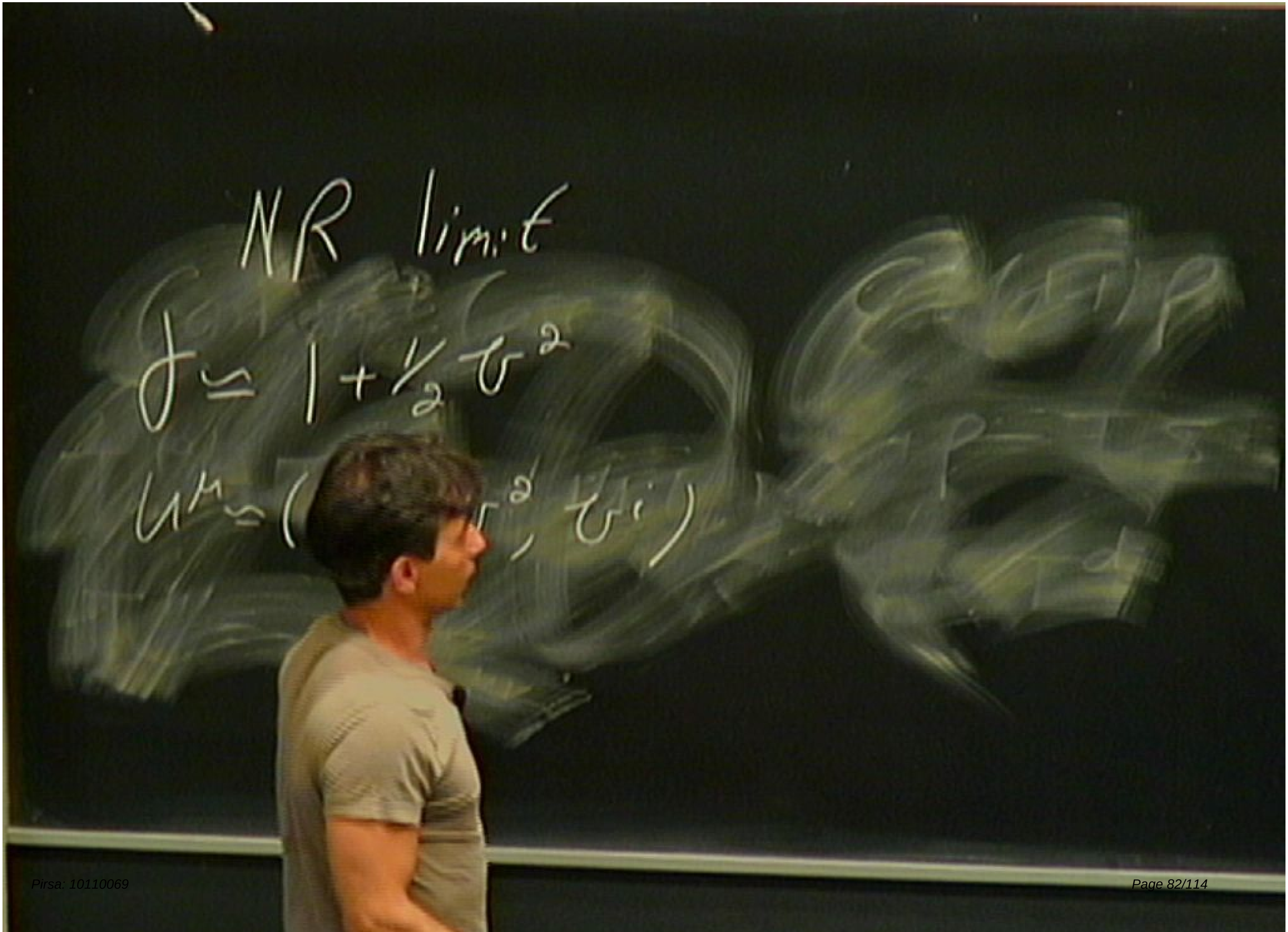
$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

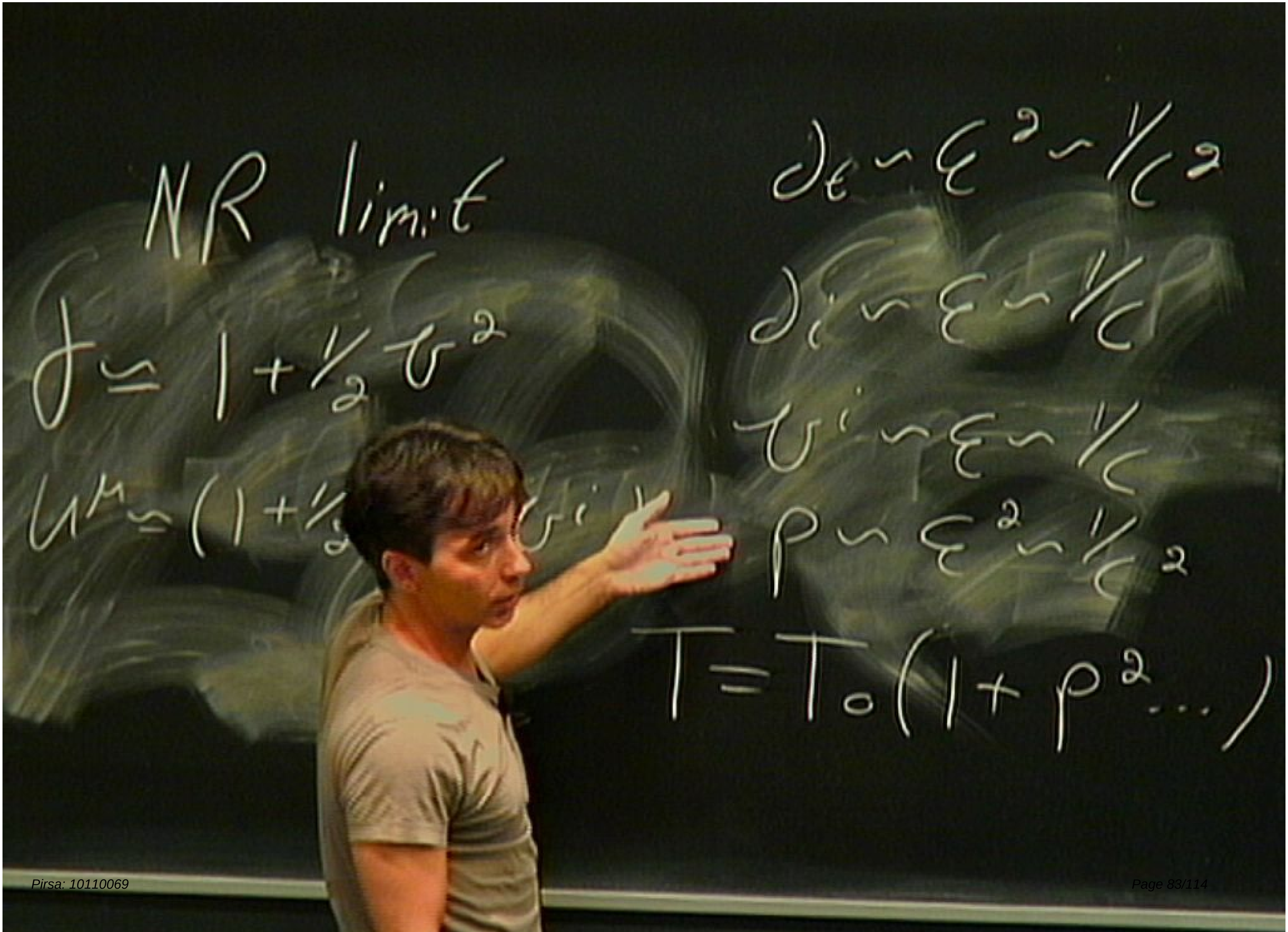
(1) $$u^\mu \partial_\mu \ln T \,(1-d) - \partial_\mu u^\mu = 0$$ (1)

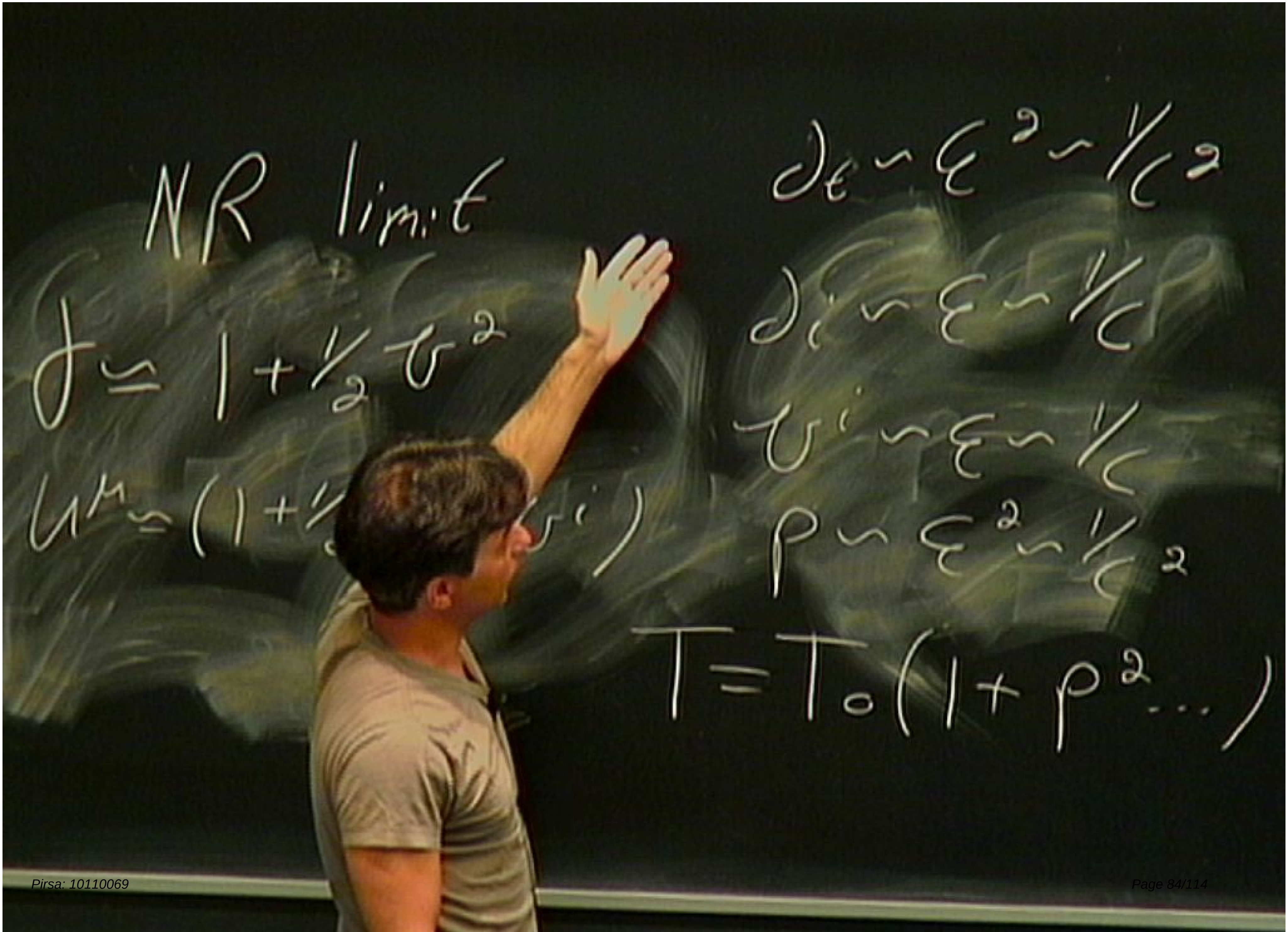
$$\partial_\mu u^\mu + (d-1)\, u^\mu \partial_\mu \ln T = 0$$

(d) $$P^\sigma{}_\mu \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$$ (d)


$u_\nu \partial_\mu T^{\mu\nu} = 0$
① $u^\mu \partial_\mu \ln T (1-d) + \partial_\mu u^\mu = 0$ ①
$\partial_\mu u^\mu + (d-1) u^\mu \partial_\mu \ln T = 0$
ⓓ $P^\sigma_\mu \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$ ⓓ
$= 0 \quad n \geq 1$


NR limit
γ ≃ 1 + ½ v²


NR limit
γ ≃ 1 + ½ v²
u^μ ≃ (1 + ½
∂t ~ ε² ~ 1/c²
∂i ~ ε ~ 1/c
v^i ~ ε ~ 1/c
ρ ~ ε² ~ 1/c²
T = T0 (1 + ρ² ...)


NR limit
$\gamma \simeq 1 + \frac{1}{2} v^2$
$\partial_t \sim \epsilon^2 \sim 1/c^2$
$\partial_i \sim \epsilon \sim 1/c$
$v^i \sim \epsilon \sim 1/c$
$\rho \sim \epsilon^2 \sim 1/c^2$
$T = T_0 (1 + \rho^2 \ldots)$

NR limit

$$\gamma \simeq 1 + \tfrac{1}{2} v^2$$

$$u^\mu \simeq \left(1 + \tfrac{1}{2} v^2, v^i\right)$$

$$\partial_t \sim \epsilon^2 \sim 1/c^2$$

$$\partial_i \sim \epsilon \sim 1/c$$

$$v^i \sim \epsilon \sim 1/c$$

$$\rho \sim \epsilon^2 \sim 1/c^2$$

$$T = T_0\left(1 + \rho^2 \ldots\right)$$

NR limit

$\gamma \simeq 1 + \frac{1}{2} v^2$

$u^\mu \simeq (1 + \frac{1}{2} v^2, v^i)$

$\partial_t \sim \epsilon^2 \sim 1/c^2$

$\partial_i \sim \epsilon \sim 1/c$

$v^i \sim \epsilon \sim 1/c$

$\rho \sim \epsilon^2 \sim 1/c^2$

$T = T_0 (1 + \rho^2 \ldots)$

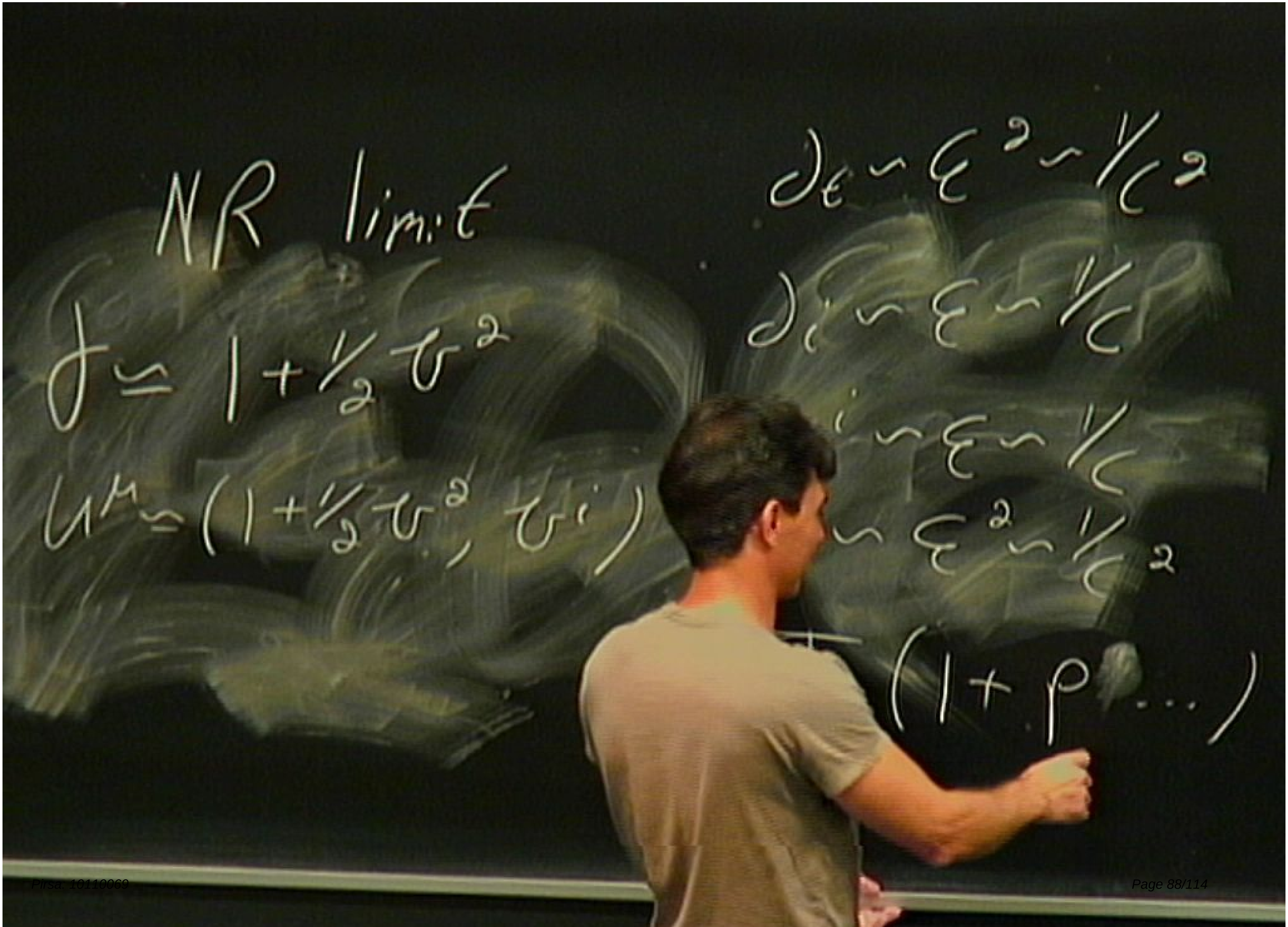
① $u^\mu \partial_\mu \ln T (1-d) + \partial_\mu u^\mu = 0$ ①

$$\partial_\mu u^\mu + (d-1) u^\mu \partial_\mu \ln T = 0$$

ⓓ $P^\sigma_{\ \mu} \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$ ⓓ

$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p$$

Landau Frame $u_\mu T^{\mu\nu}_{(n)} = 0 \quad n \geq 1$


NR limit
γ ≃ 1 + ½ v²
u^μ ≃ (1 + ½ v², v^i)
∂_t ~ ε² ~ 1/c²
∂_i ~ ε ~ 1/c
~ ε ~ 1/c
~ ε² ~ 1/c²
(1 + ρ ...)

NR limit

$$\gamma \simeq 1 + \tfrac{1}{2} v^2$$

$$u^\mu \simeq \left(1 + \tfrac{1}{2} v^2, v^i\right)$$

$$\partial_t \sim \epsilon^2 \sim 1/c^2$$

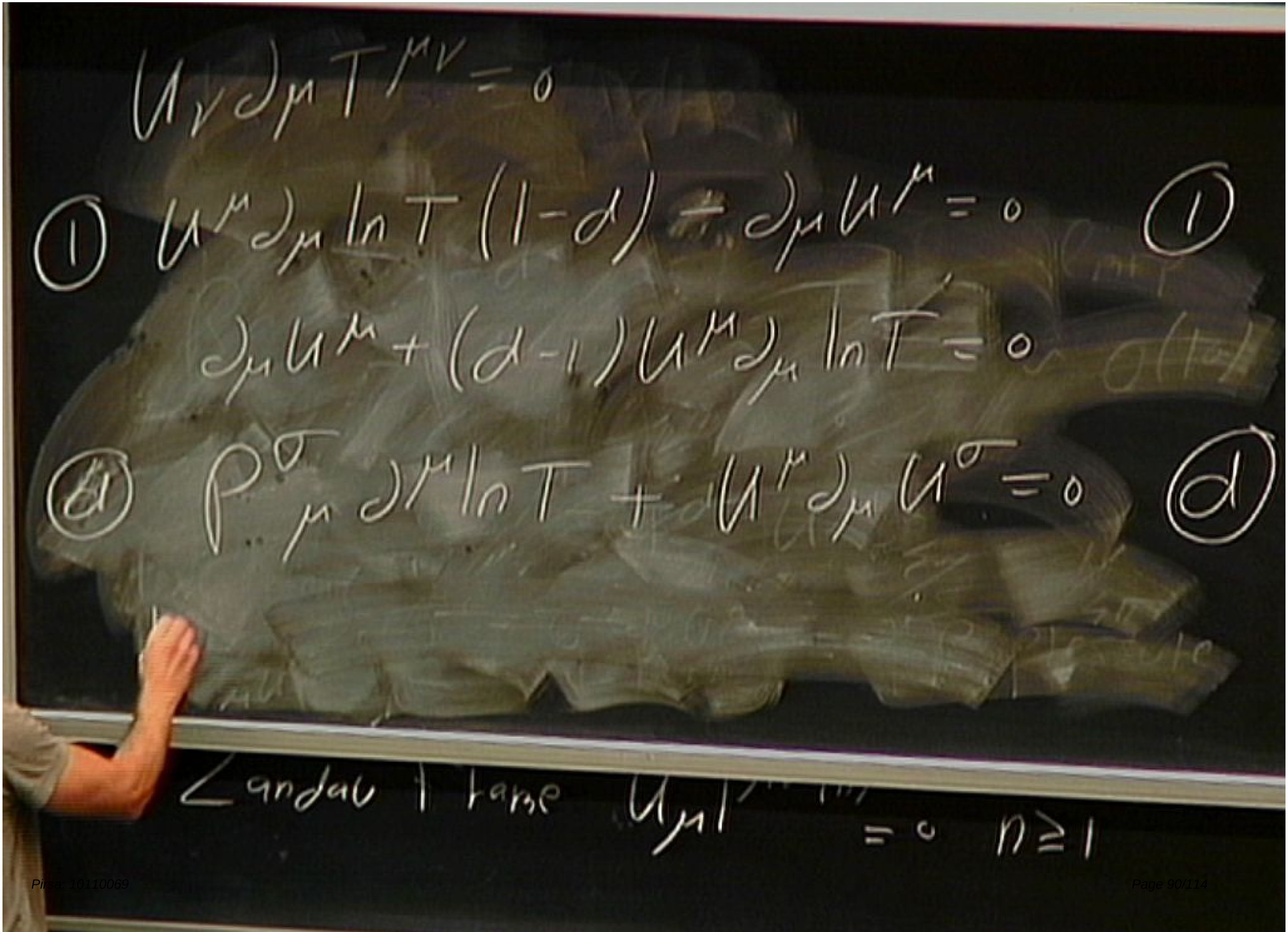
$$\partial_i \sim \epsilon \sim 1/c$$

$$v^i \sim \epsilon \sim 1/c$$

$$\rho \sim \epsilon^2 \sim 1/c^2$$

$$T = T_0 \left(1 + \rho \ldots\right)$$

#2



u_ν ∂_μ T^{μν} = 0
① u^μ ∂_μ ln T (1−d) + ∂_μ u^μ = 0 ①
∂_μ u^μ + (d−1) u^μ ∂_μ ln T = 0
ⓓ P^σ_μ ∂^μ ln T + u^μ ∂_μ u^σ = 0 ⓓ
Landau Frame u_μ T^{μν (n)} = 0 n ≥ 1

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

(1) $u^\mu \partial_\mu \ln T \,(1-d) + \partial_\mu u^\mu = 0$ (1)

$$\partial_\mu u^\mu + (d-1)\, u^\mu \partial_\mu \ln T = 0$$

(d) $P^\sigma{}_\mu \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$ (d)

(1) $\partial_i v^i = 0$

Landau Frame $u_\mu T^{\mu\nu}_{(n)} = 0 \quad n \geq 1$

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

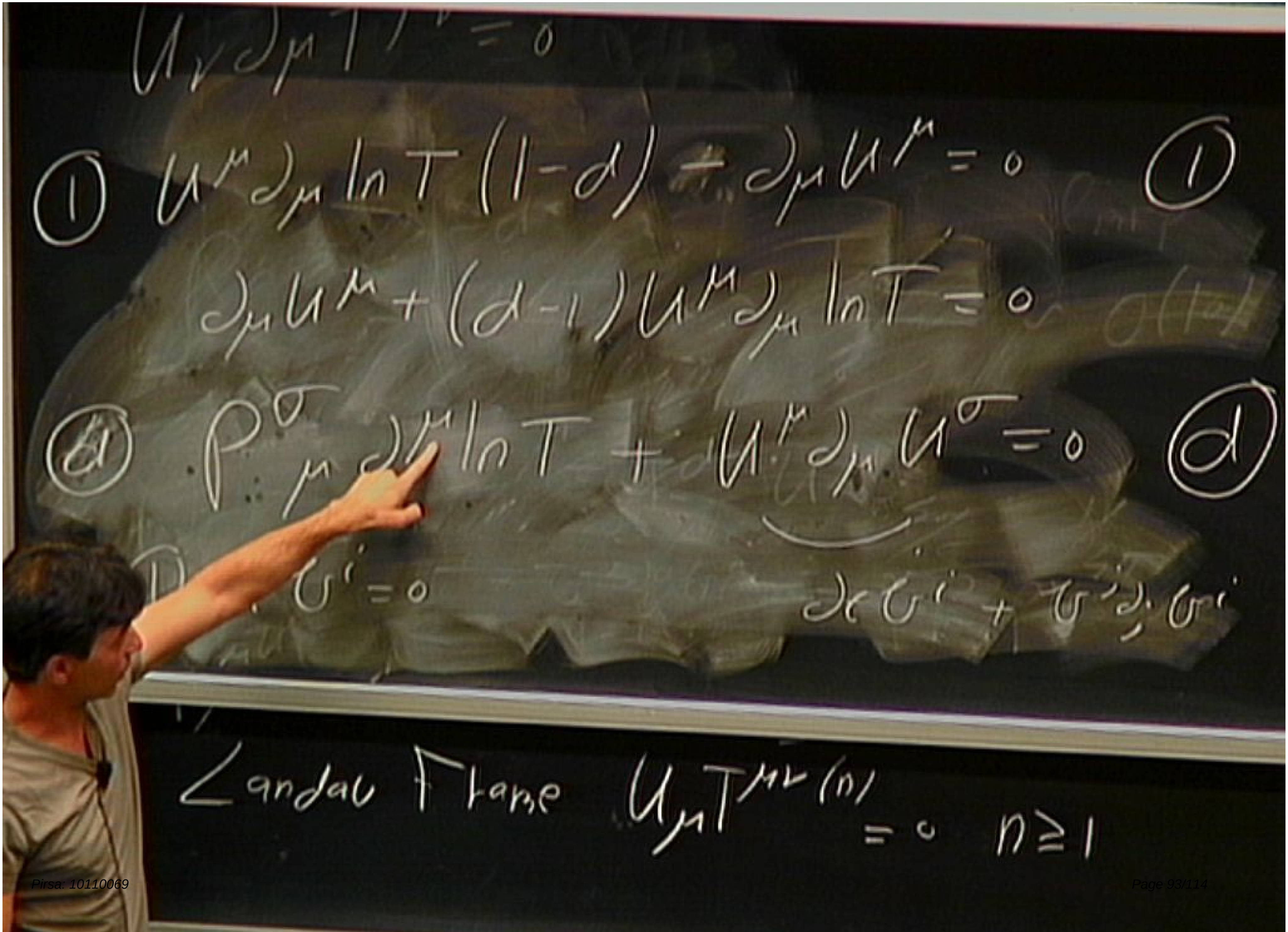
(1) $u^\mu \partial_\mu \ln T (1-d) + \partial_\mu u^\mu = 0$ (1)

$$\partial_\mu u^\mu + (d-1) u^\mu \partial_\mu \ln T = 0$$

(d) $P^{\sigma}_{\ \mu} \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$ (d)

(1) $\partial_i v^i = 0$ $\quad \partial_t v^i + v^j \partial_j v^i$

Landau Frame $u_\mu T^{\mu\nu(n)} = 0 \quad n \geq 1$



$u_\nu \partial_\mu T^{\mu\nu} = 0$
① $u^\mu \partial_\mu \ln T (1-d) + \partial_\mu u^\mu = 0$ ①
$\partial_\mu u^\mu + (d-1) u^\mu \partial_\mu \ln T = 0$
② $P^{\sigma}{}_{\mu} \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$ ②
$\partial_i v^i = 0$
$\partial_t v^i + v^j \partial_j v^i$
Landau Frame $u_\mu T^{\mu\nu (n)} = 0 \quad n \geq 1$

$$u_\nu \partial_\mu T^{\mu\nu} = 0$$

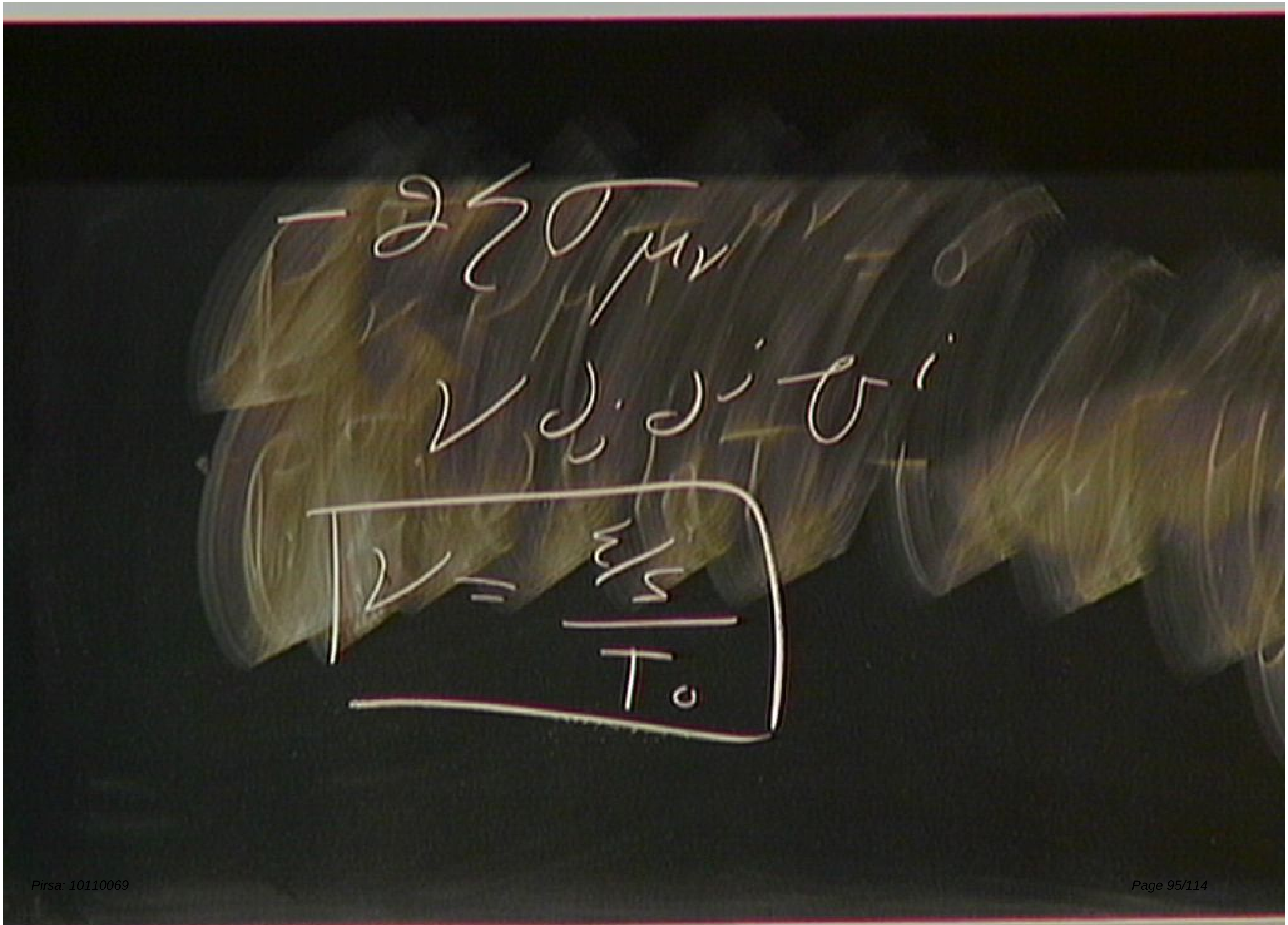
(1) $u^\mu \partial_\mu \ln T (1-d) + \partial_\mu u^\mu = 0$ (1)

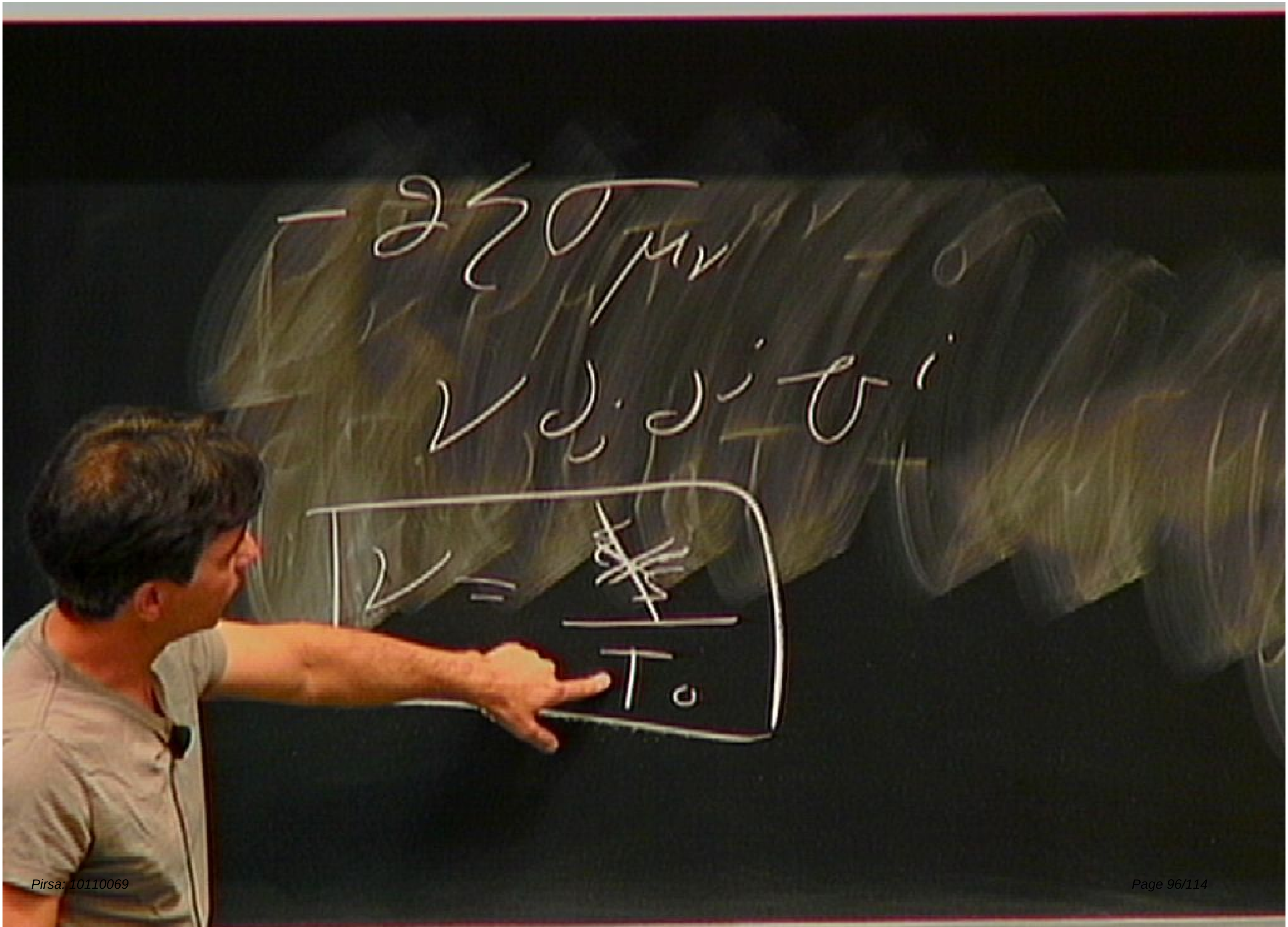
$\partial_\mu u^\mu + (d-1) u^\mu \partial_\mu \ln T = 0$

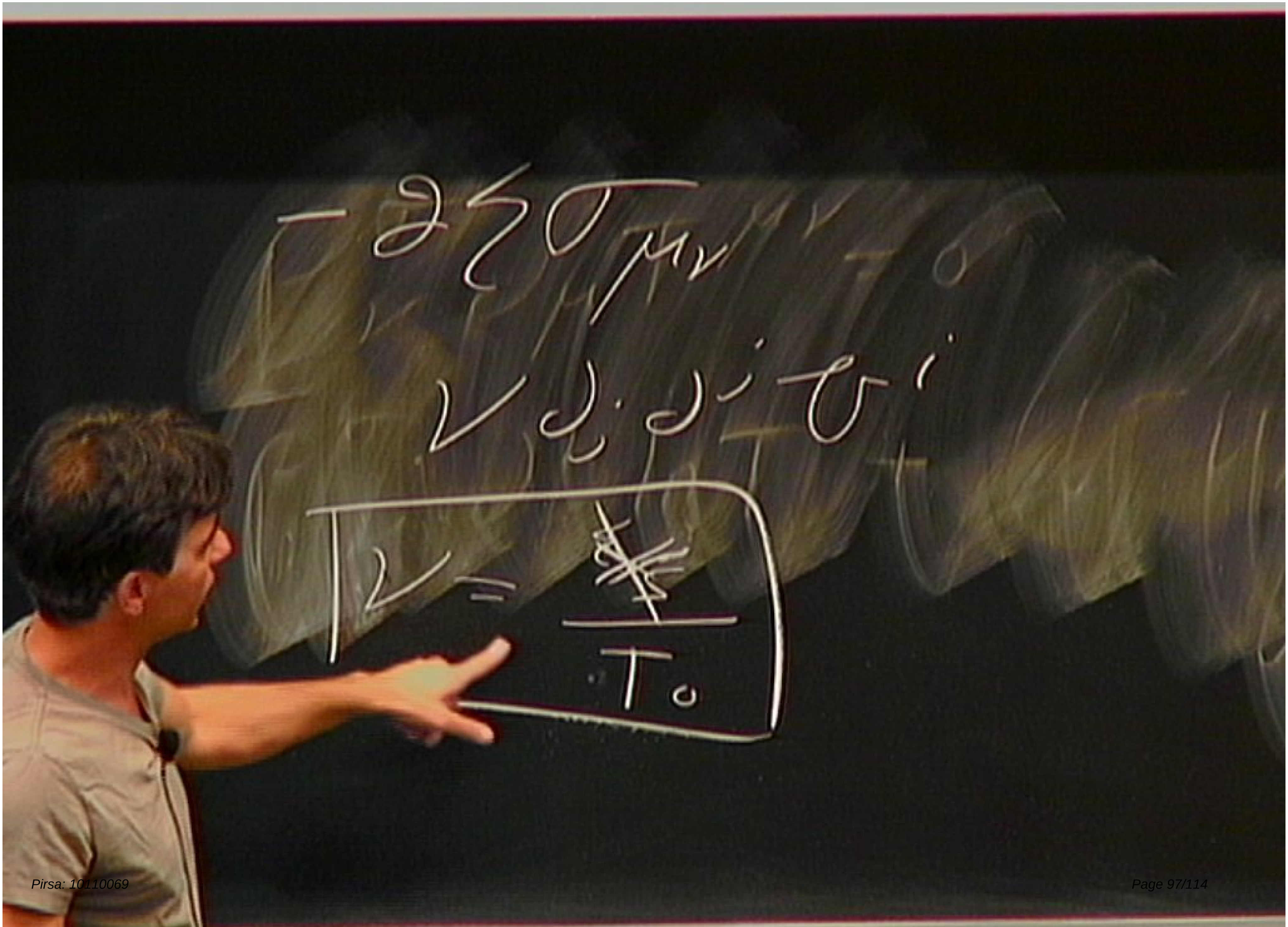
(d) $P^\sigma{}_\mu \partial^\mu \ln T + u^\mu \partial_\mu u^\sigma = 0$ (d)

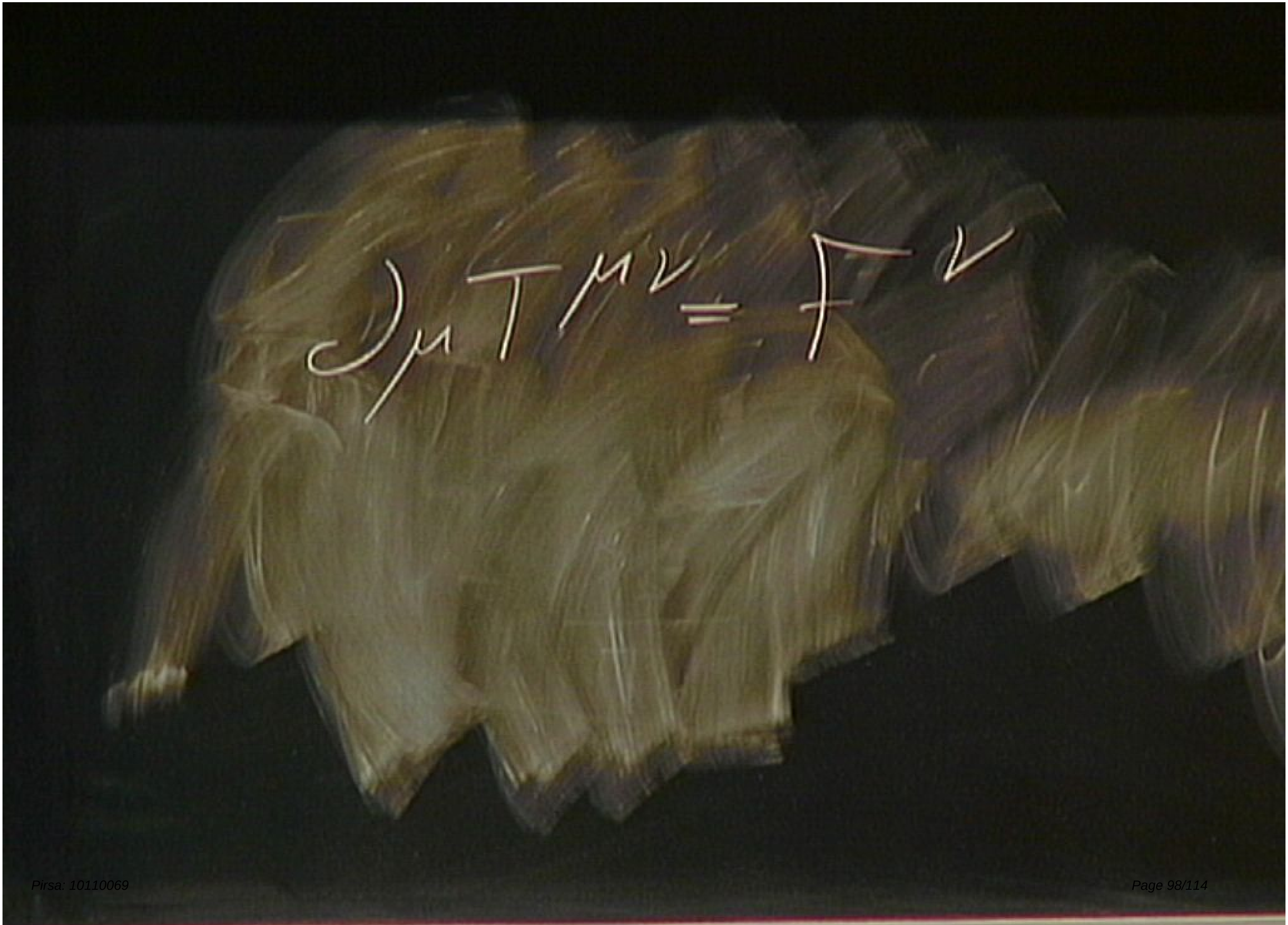
(1) $\partial_i v^i = 0$ $\quad -\partial^i p + \partial_t v^i + v^j \partial_j v^i = 0$

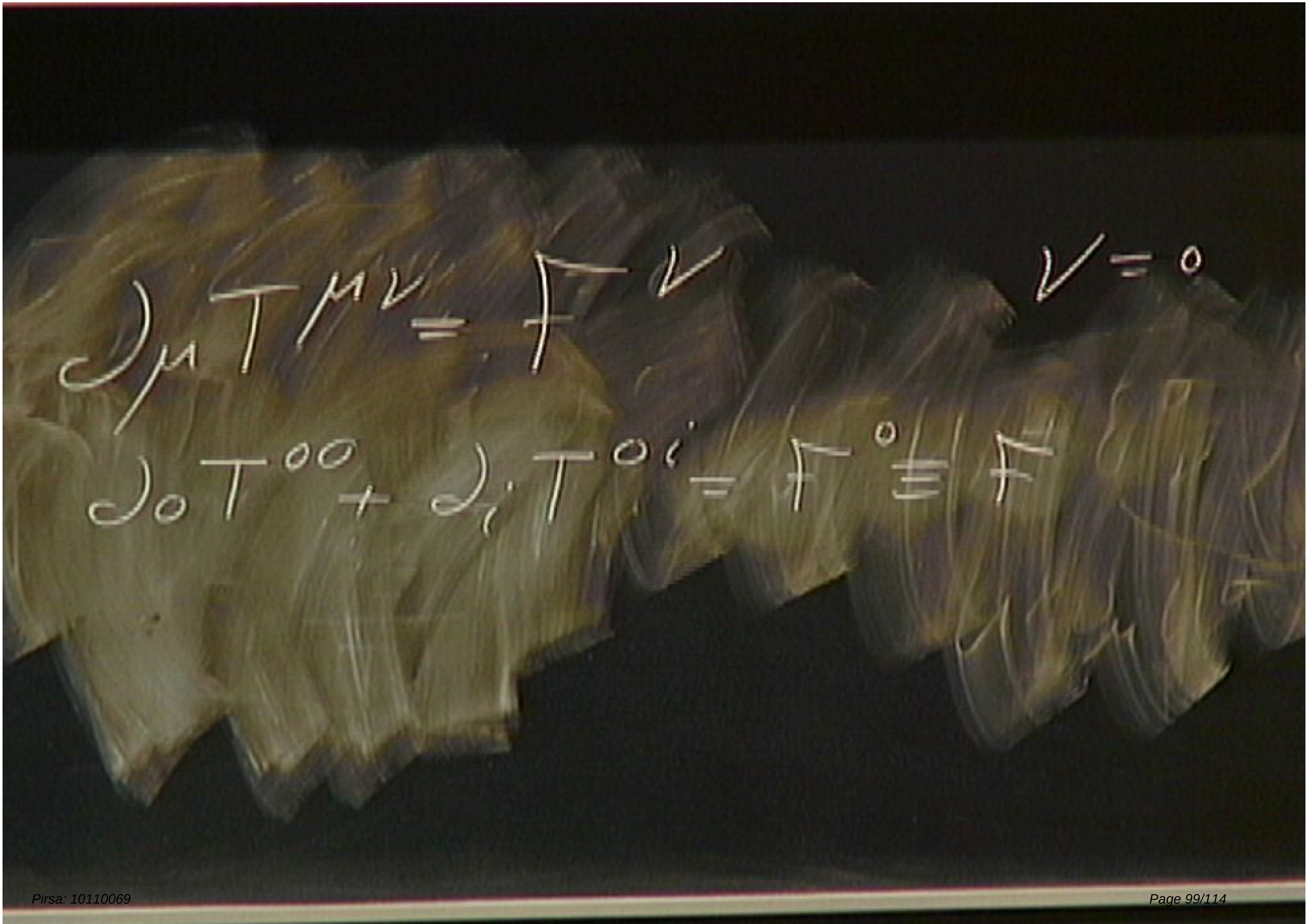
Landau Frame $u_\mu T^{\mu\nu(n)} = 0 \quad n \geq 1$








$\partial_\mu T^{\mu\nu} = F^\nu$


$\partial_\mu T^{\mu\nu} = F^\nu$
$\nu = 0$
$\partial_0 T^{00} + \partial_i T^{0i} = F^0 \equiv F$

$$\partial_\mu T^{\mu\nu} = F^\nu \qquad \nu = 0$$

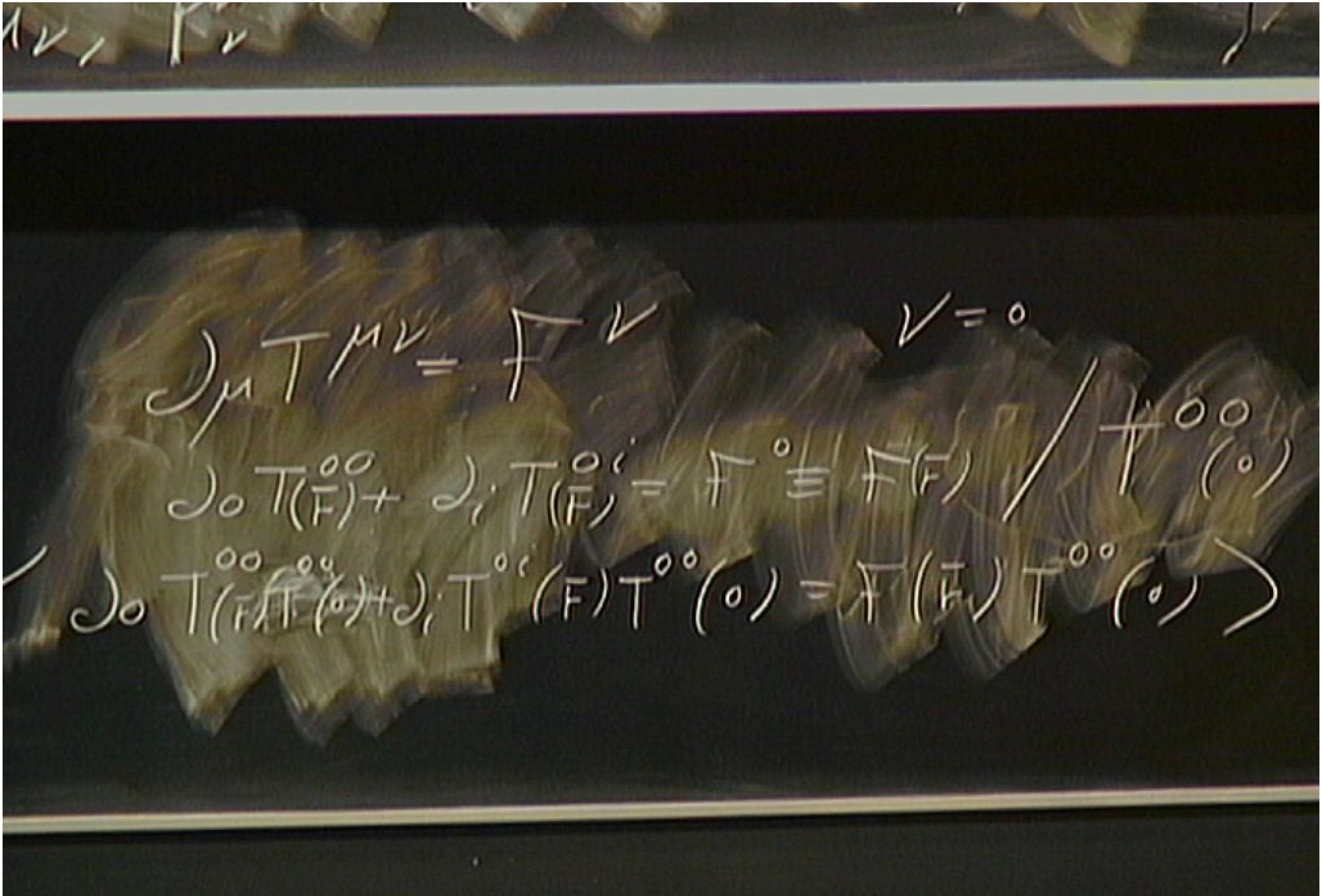
$$\partial_0 T^{00}_{(F)} + \partial_i T^{0i}_{(F)} = F^0 \equiv F_{(F)} \; / \; t^{00}_{(0)}$$

$\mu\nu,\ F_\nu$

$$\partial_\mu T^{\mu\nu} = F^\nu \qquad \nu = 0$$

$$\partial_0 T^{00}_{(F)} + \partial_i T^{0i}_{(F)} = F^0 \equiv F(F) \,/\, T^{00}(0)$$

$$\partial_0 T^{00}_{(F)} T^{00}(0) + \partial_i T^{0i}(F)\, T^{00}(0) = F(F)\, T^{00}(0)$$


$\partial_\mu T^{\mu\nu} = F^\nu$
$\nu = 0$
$\partial_0 T^{00}(F) + \partial_i T^{0i}(F) = F^0 \equiv F(F)$ / $T^{00}(0)$
$\partial_0 T^{00}(F) T^{00}(0) + \partial_i T^{0i}(F) T^{00}(0) = F(F) T^{00}(0)$

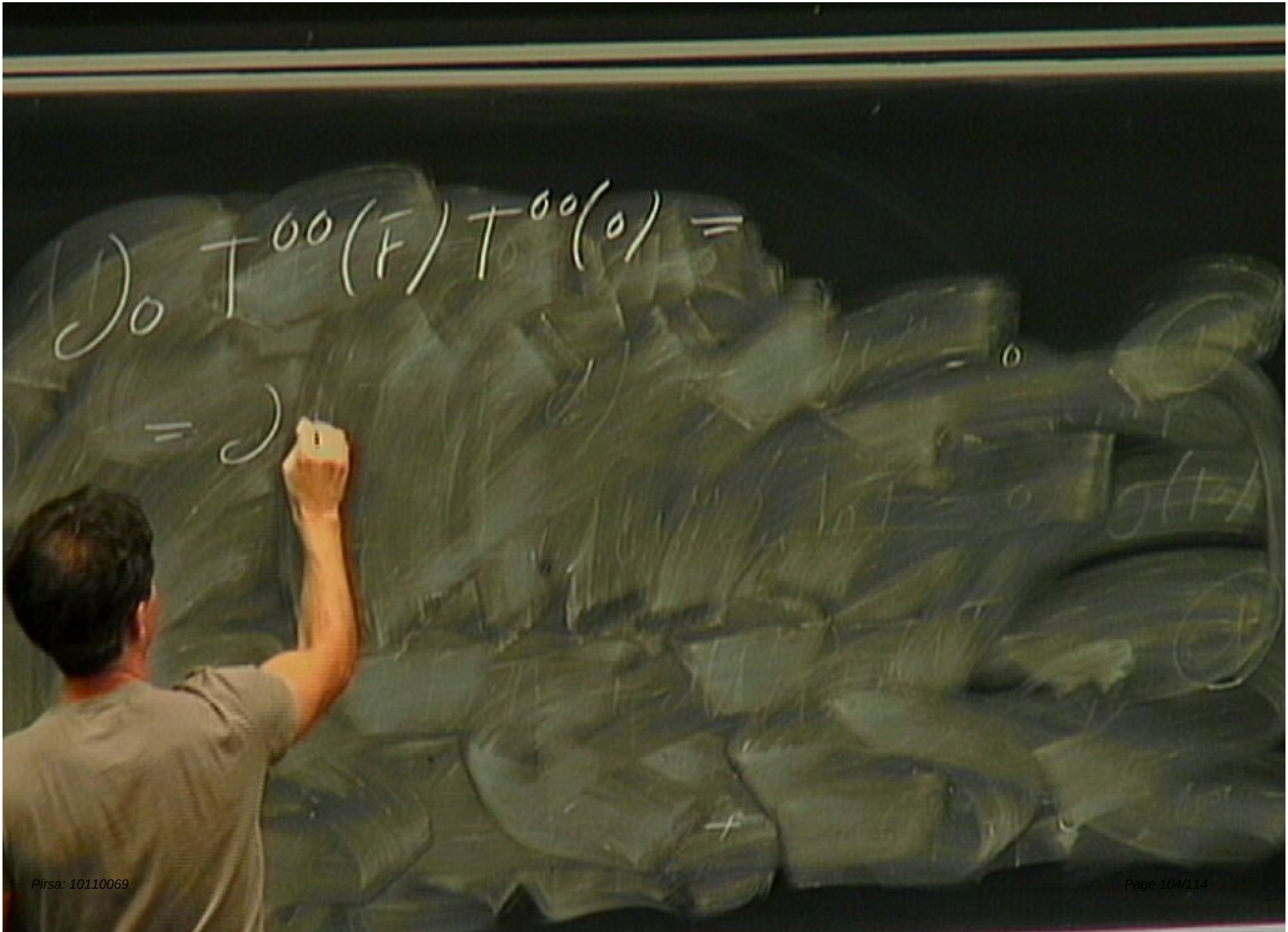
$u_\nu, \; \rho^\sigma_\nu$

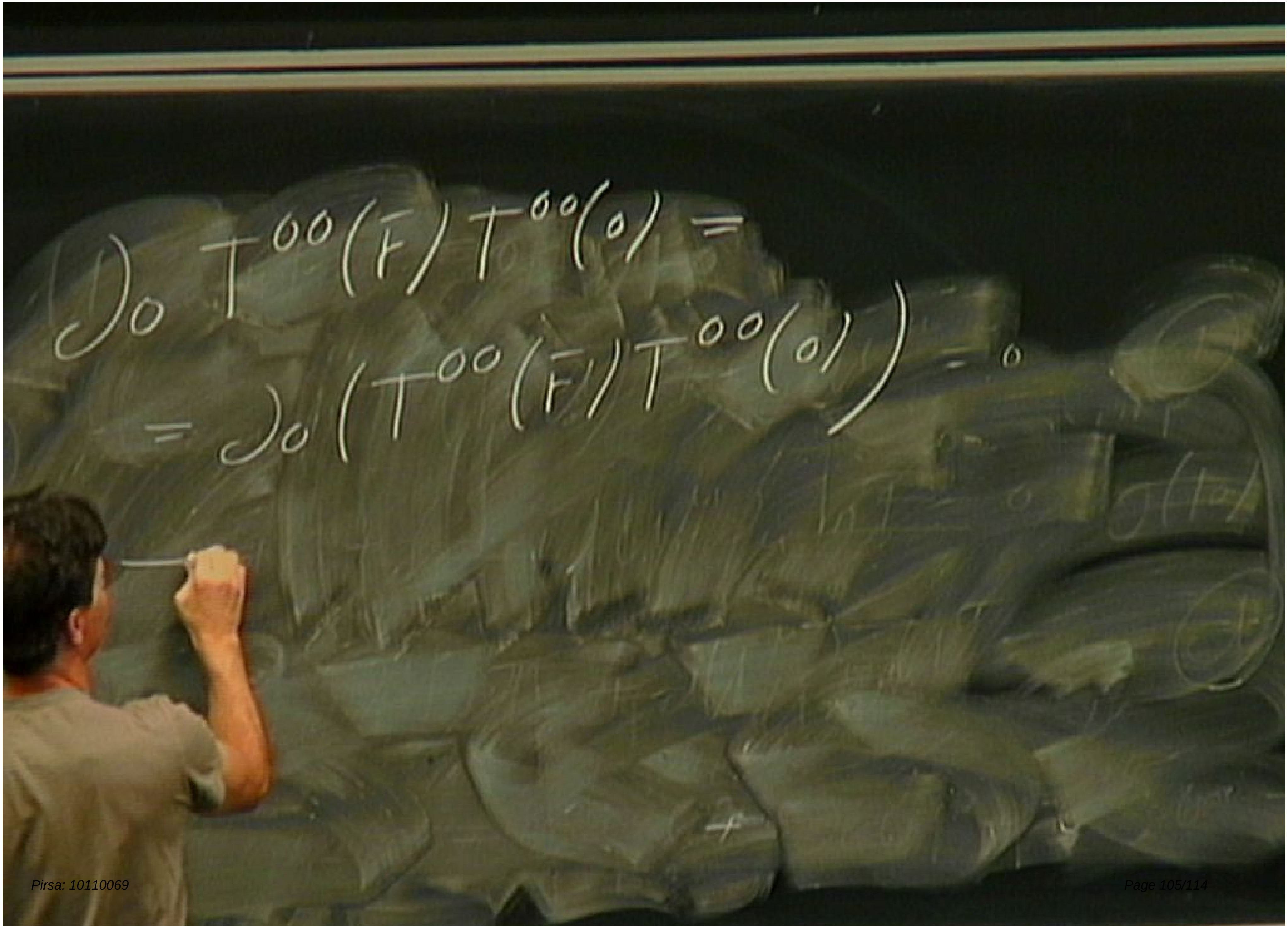
$$\partial_\mu T^{\mu\nu} = F^\nu \qquad \nu = 0$$

$$\partial_0 T^{00}(F) + \partial_i T^{0i}(F) = F^0 \equiv F(F) \; / \; T^{00}(0)$$

$$\langle \partial_0 T^{00}(F) T^{00}(0) + \partial_i T^{0i}(F) T^{00}(0) = F(F) T^{00}(0) \rangle$$

$$\ell \ll |F| \ll L \qquad \langle F(0) T^{00}(0) \rangle = \epsilon$$

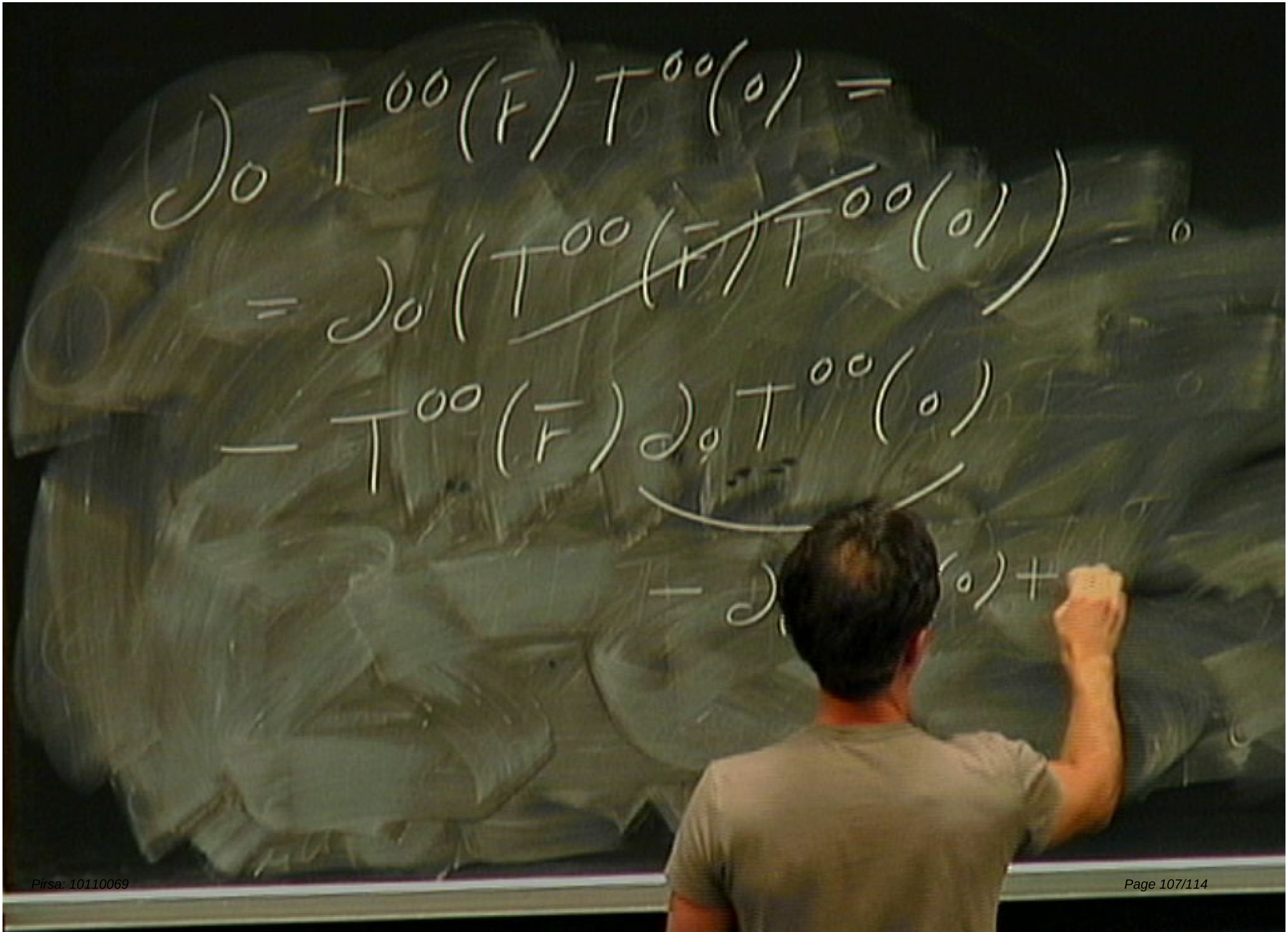

∂0 T00(t) T00(0) =


$\partial_0 T^{00}(F) T^{00}(0) =$
$= \partial_0 (T^{00}(F) T^{00}(0)) = 0$

$$\partial_0 T^{00}(F)\, T^{00}(0) =$$

$$= \partial_0 \left(T^{00}(F)\, T^{00}(0) \right) = 0$$

$$- T^{00}(F)\, \partial_0 T^{00}(0)$$


$\partial_0 T^{00}(F) T^{00}(0) =$
$= \partial_0 \left(T^{00}(F) T^{00}(0) \right) = 0$
$- T^{00}(F) \partial_0 T^{00}(0)$

$$\partial_0 T^{00}(F)\, T^{00}(0) =$$

$$= \partial_0 \left(T^{00}(F)\, T^{00}(0) \right) = 0$$

$$- T^{00}(F)\, \underbrace{\partial_0 T^{00}(0)}$$

$$- \partial_i T^{0i}(0) + F(0)$$

$$\partial_0 T^{00}(F)/T^{00}(0) =$$

$$= \partial_0 \left(T^{00}(F)/T^{00}(0) \right) = 0$$

$$+ T^{00}(F)\, \partial_0 T^{00}(0)$$

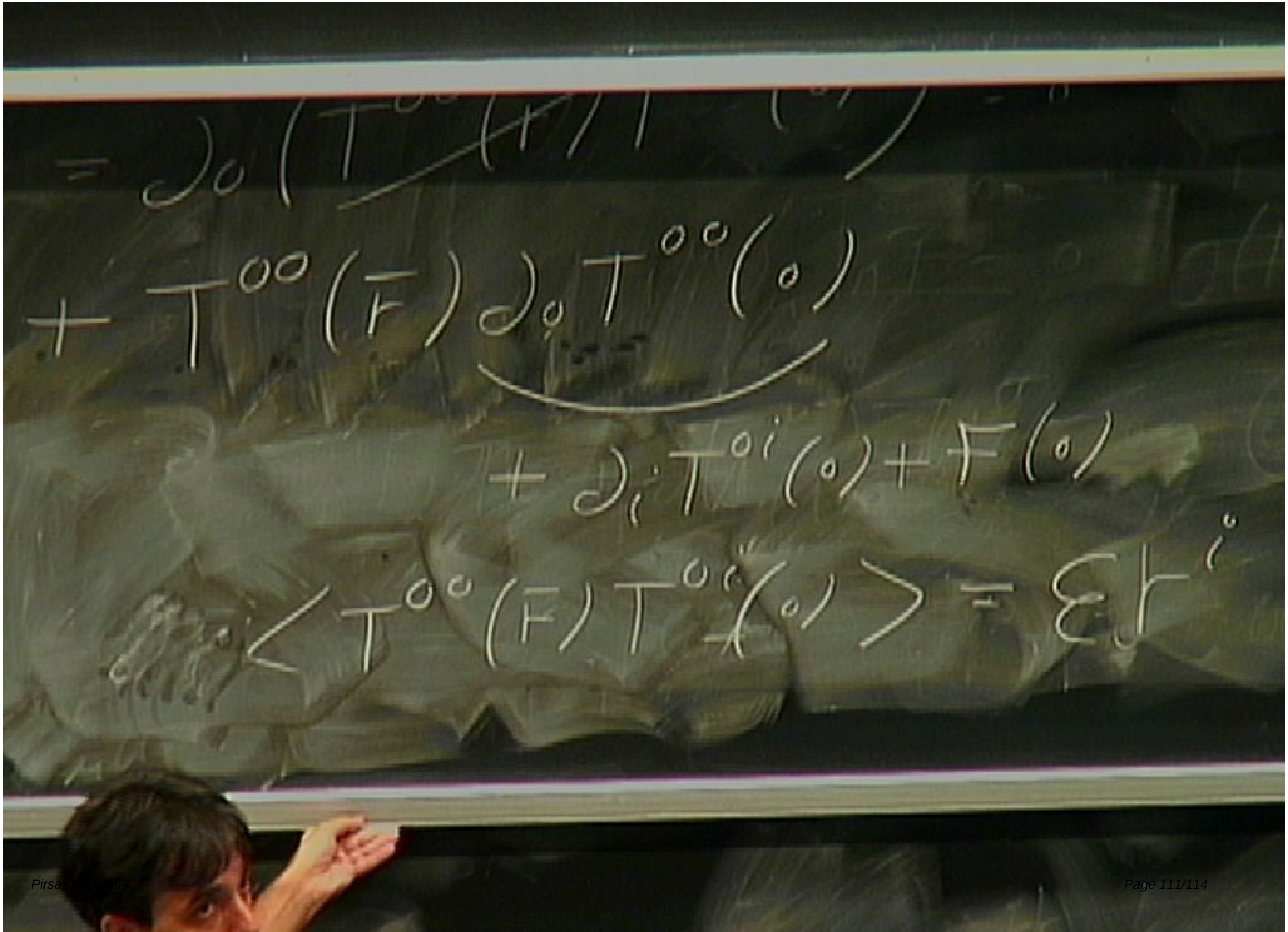
$$+ \partial_i T^{0i}(0) + F(0)$$

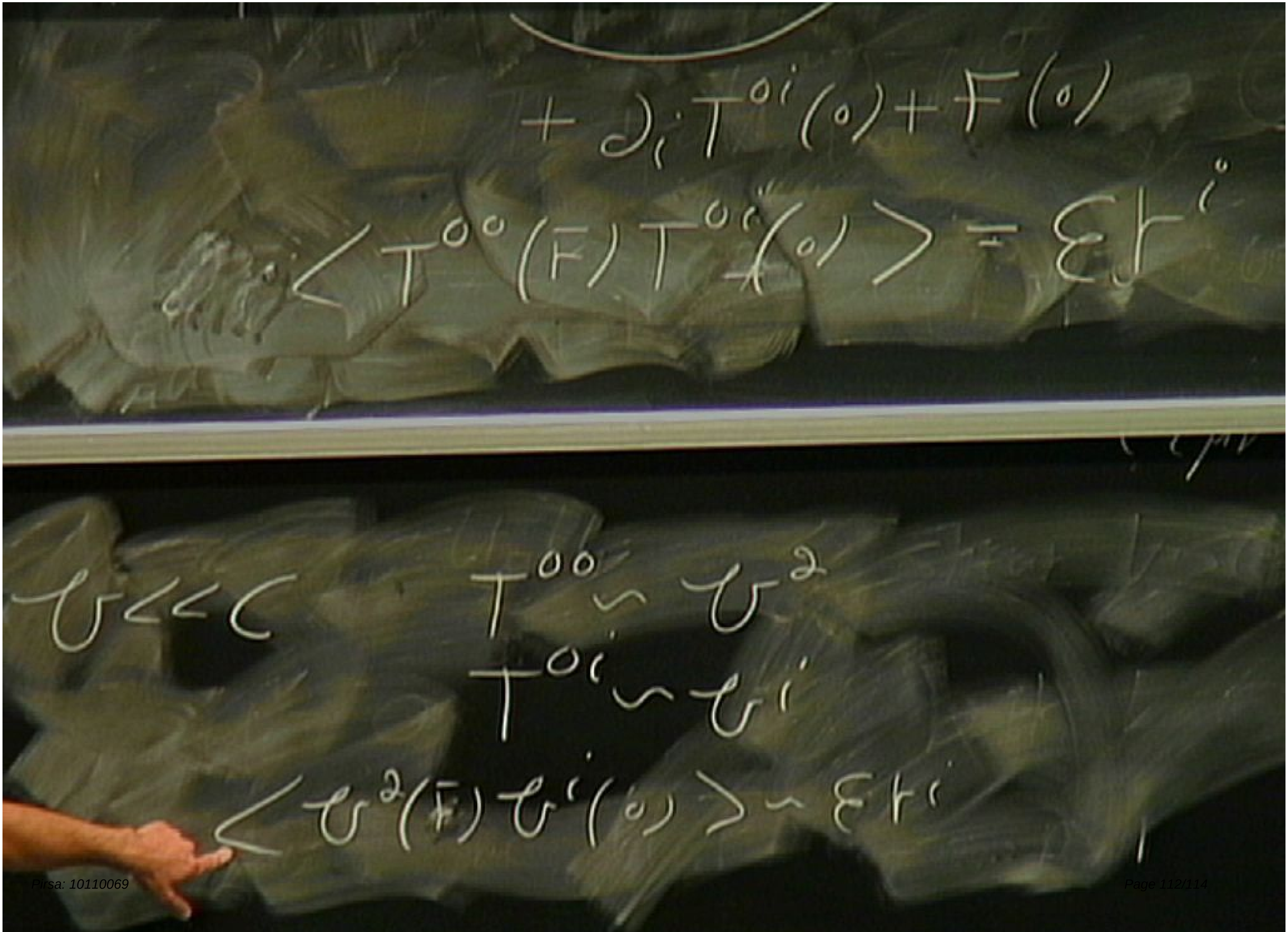
$$= \partial_0 \left(T^{00}(F)\, T^{00}(0) \right) = 0$$

$$+\, T^{00}(F)\, \partial_0 T^{00}(0)$$

$$+\, \partial_i T^{0i}(0) + F(0)$$

$$\langle T^{00}(F)\, T^{0i}(0) \rangle = E r^i$$


$+ T^{00}(F)\, \partial_0 T^{00}(0)$
$+ \partial_i T^{0i}(0) + F(0)$
$\langle T^{00}(F)\, T^{0i}(0) \rangle = E t^i$


$+ \partial_i T^{0i}(0) + F(0)$
$\langle T^{00}(F)\, T^{0i}(0) \rangle = \varepsilon t^i$
$v \ll c$
$T^{00} \sim v^2$
$T^{0i} \sim v^i$
$\langle v^2(F)\, v^i(0) \rangle \sim \varepsilon t^i$

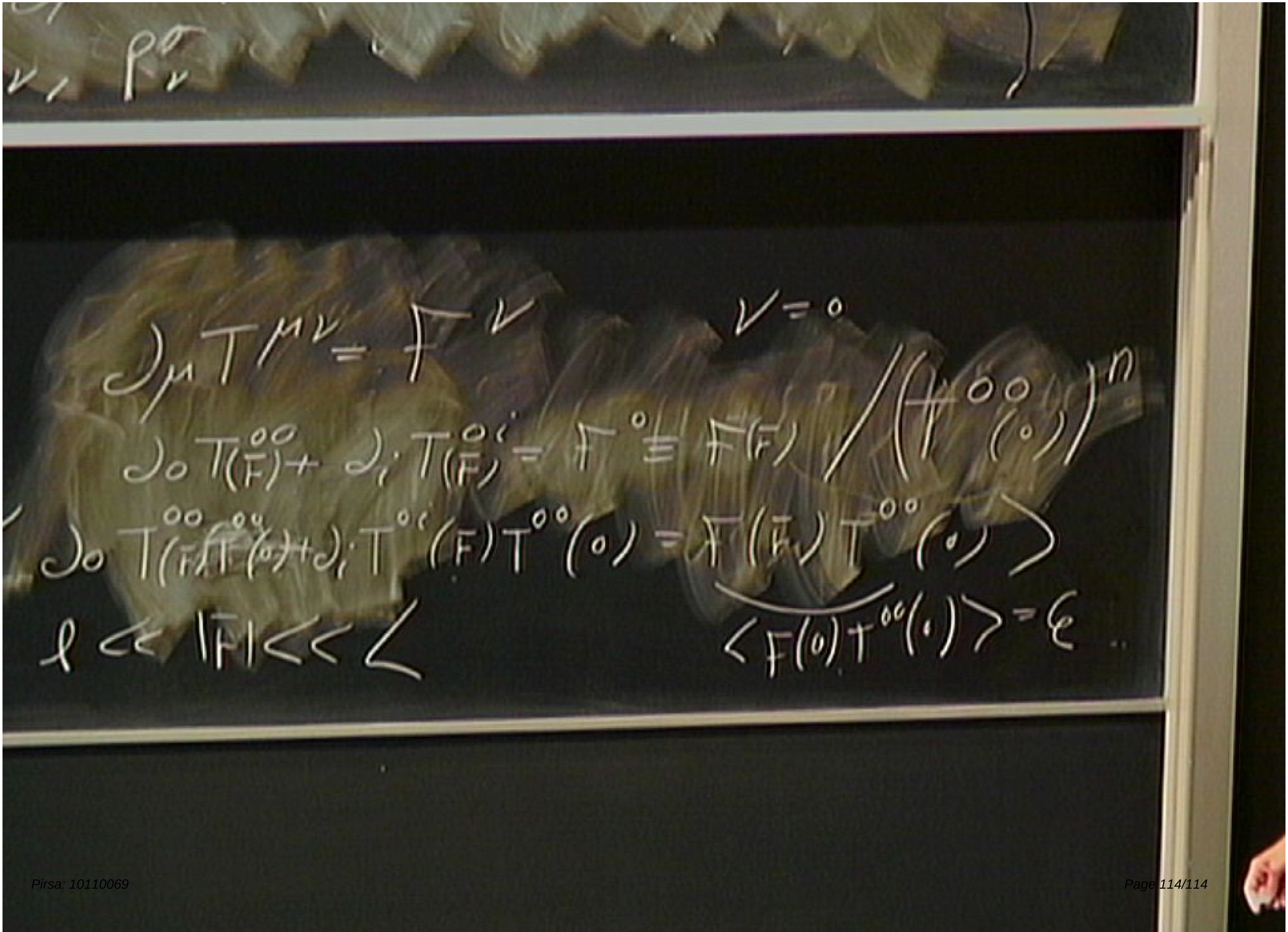
$u_\nu, \ \rho^\sigma_\nu$

$$\partial_\mu T^{\mu\nu} = F^\nu \qquad \nu = 0$$

$$\partial_0 T^{00}(F) + \partial_i T^{0i}(F) = F^0 \equiv F(F) \Big/ \left(T^{00}(0) \right)^n$$

$$\langle \partial_0 T^{00}(F) T^{00}(0) + \partial_i T^{0i}(F) T^{00}(0) = F(F) T^{00}(0) \rangle$$

$$\ell \ll |F| \ll L \qquad \langle F(0) T^{00}(0) \rangle = \epsilon \ \ldots$$



∂μ T^μν = F^ν
ν = 0
ℓ << |F| << L
< F(0) T^00(0) > = ε