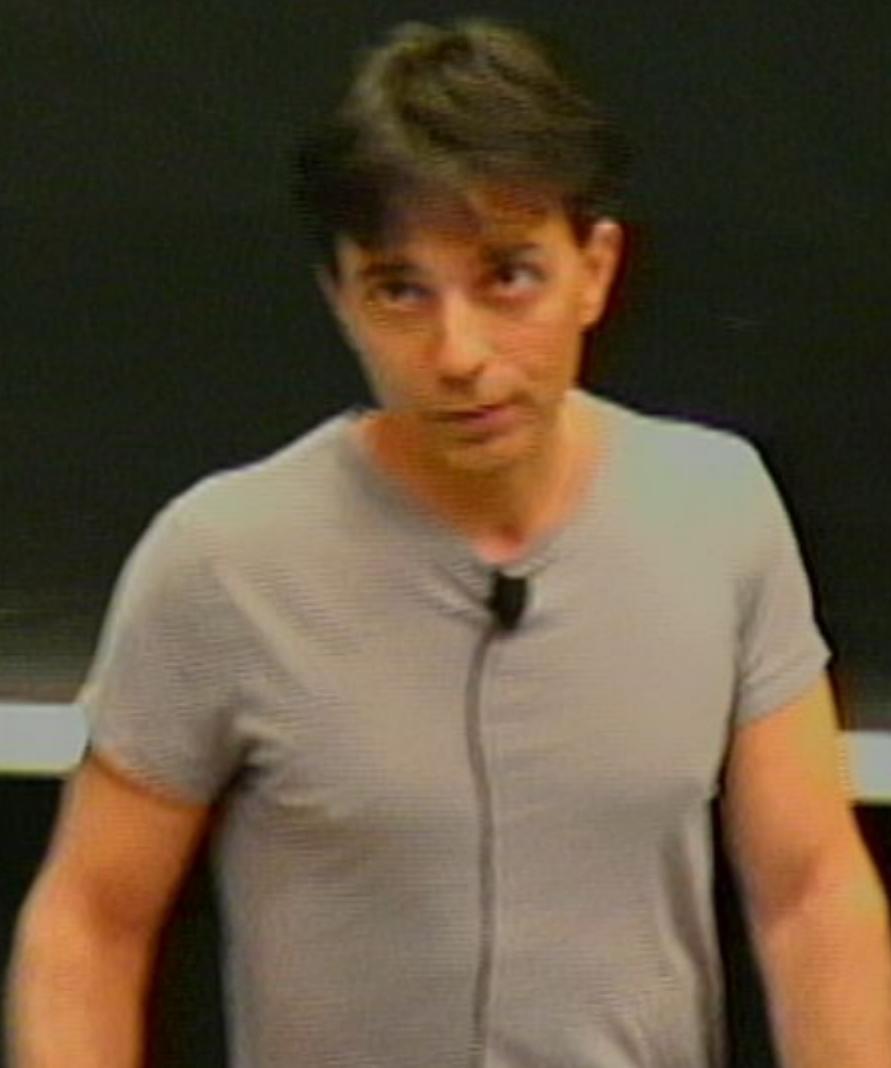


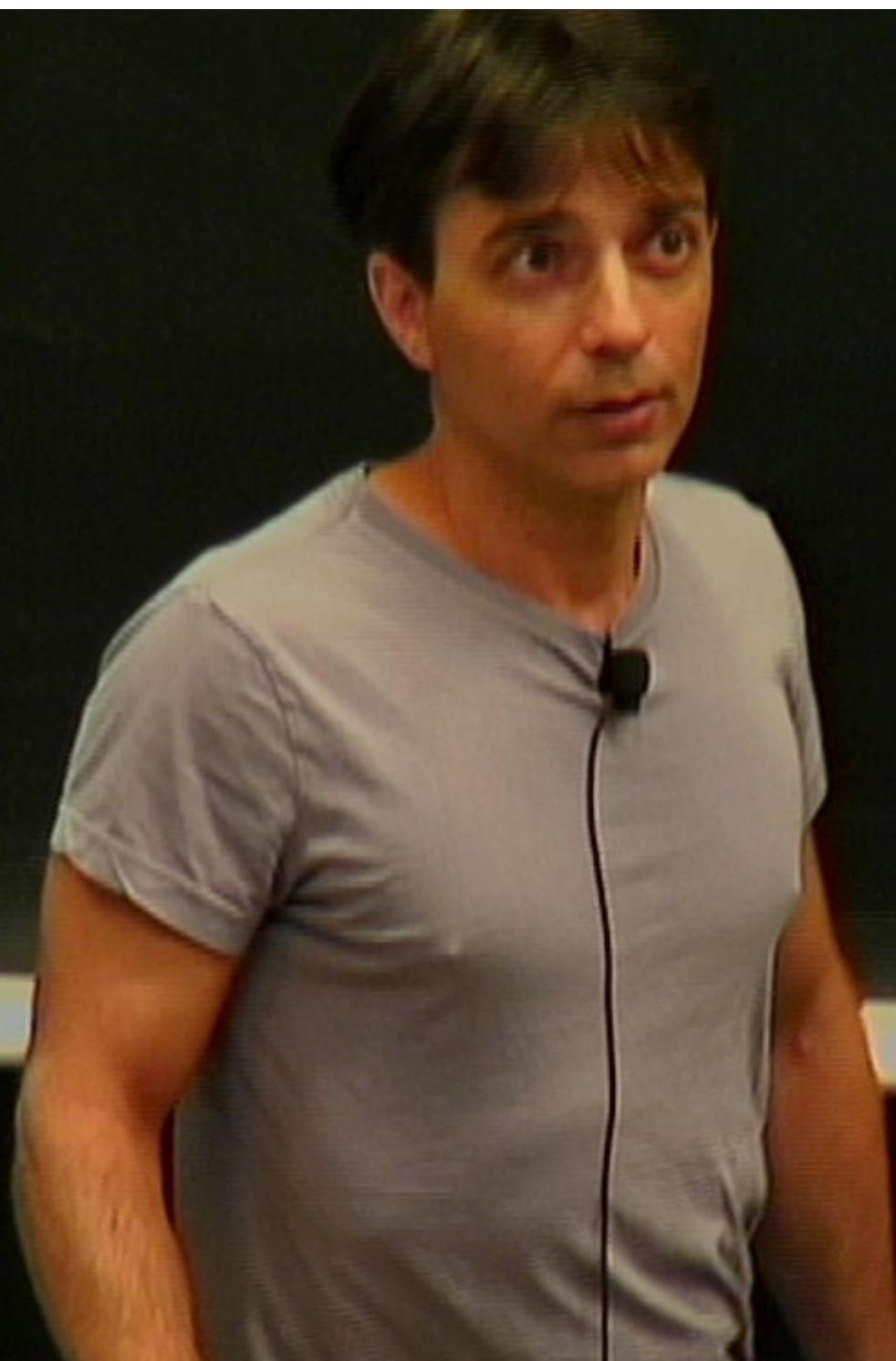
Title: Gravity and a Geometrization of Turbulence: An Intriguing Correspondence: Part 1

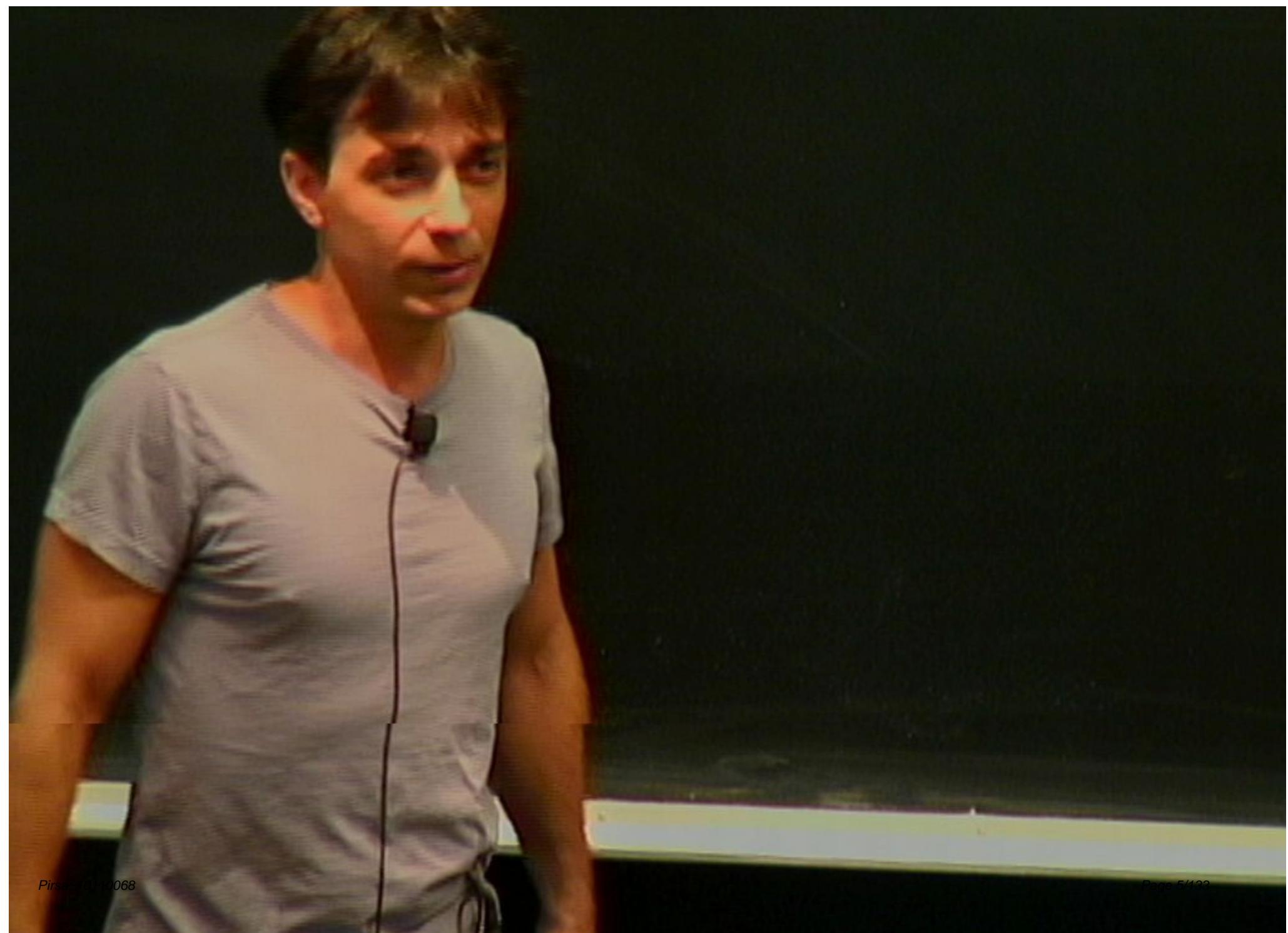
Date: Nov 10, 2010 11:00 AM

URL: <http://pirsa.org/10110068>

Abstract: The dynamics of fluids is a long standing challenge that remained as an unsolved problem for centuries. Understanding its main features, chaos and turbulence, is likely to provide an understanding of the principles and non-linear dynamics of a large class of systems far from equilibrium. We consider a conceptually new viewpoint to study these features using black hole dynamics. Since the gravitational field is characterized by a curved geometry, the gravity variables provide a geometrical framework for studying the dynamics of fluids: A geometrization of turbulence. We present new experimental predictions for relativistic and non-relativistic turbulent flows and for heavy ion collisions.







$$AdS = CFT$$

$$\text{AdS} = \text{CFT } (N=4)$$

$AdS = CFT \quad (N=4)$

$T \neq 0$ Thermal
 CFT

$$AdS = CFT \quad (N=4)$$

$$\text{Black brane} = T + o \quad \begin{matrix} \text{Thermal} \\ \text{CFT} \end{matrix}$$

$$AdS = CFT \quad (N=4)$$

$$\text{Black brane} = T + o \quad \begin{matrix} \text{Thermal} \\ \text{FT} \end{matrix}$$
$$T(\pm \epsilon)$$
$$U^\mu(\pm \epsilon) \quad U^\mu = (\phi, \rho^i)$$

$$AdS = CFT \quad (N=4)$$

Black brane = $T \neq 0$ Thermal
FT

Deformation = $T(\pm\epsilon)$
of BB $U^\mu(\pm\epsilon), U^\mu(\phi, \phi^i)$

$T(t)$ = Constant

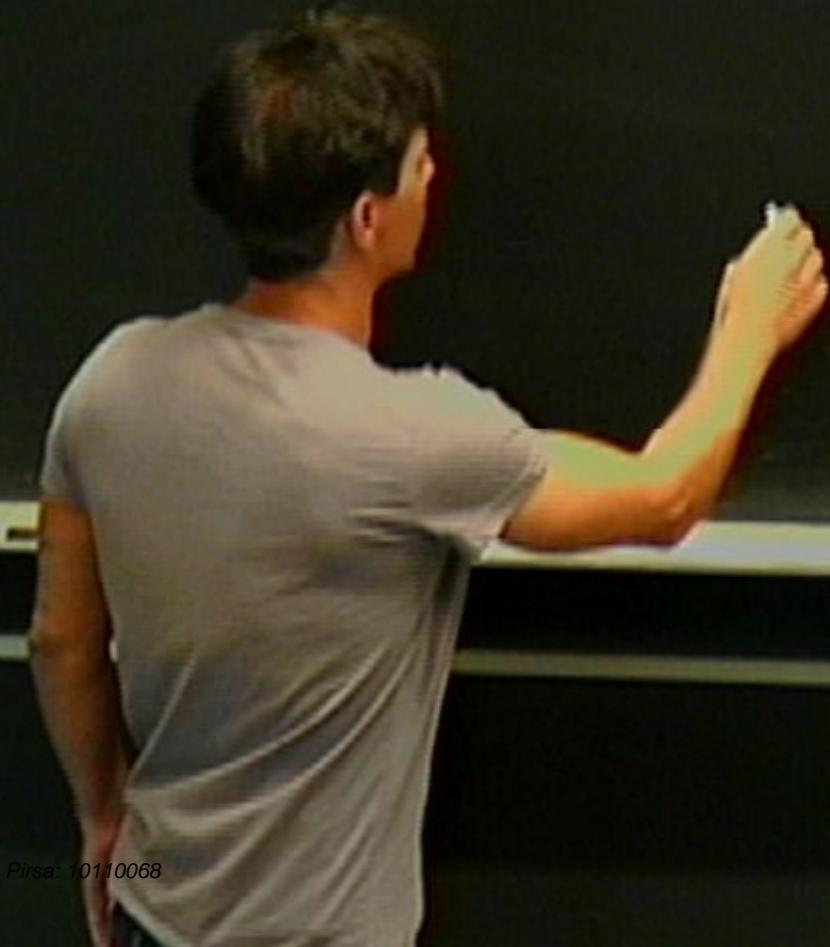
Black brane = $T + \sigma$ Thermal
FT

Deformation = $T(\pm \epsilon)$

of BB $U^\mu(\pm \epsilon)$ $U^\mu(\phi, \delta \rho^i)$

NR limit $T \ll C$

Navier-Stokes ϕ



Black brane = $T + \phi$ Thermal
FT

Deformation = $T(\pm \epsilon)$
of BB $U^\mu(\pm \epsilon), U^\mu(\phi, \delta \rho^i)$

Geometry = NR limit $T \ll C$
Navier-Stokes eq.

$$AdS = CFT \quad (N=4)$$

Black brane = $T \neq 0$ Thermal
FT

Deformation = $T(\pm \epsilon)$
of BB $U^M(\pm \epsilon), U^M(\pm, \beta \rho^i)$

Geometry = NR limit $T \ll C$
Hawking-Entropy \rightarrow

$$AdS = CFT \quad (N=4)$$

Black brane = $T \neq 0$ Thermal
FT

Deformation = $T(\pm\epsilon)$
of BB $U^a(\pm\epsilon), U^m(\pm\epsilon, p^i)$

Geometry = NR limit $T \ll C$
Majer-Sethres

Membrane Paradigm

Pamour

Gauss-Codazzi

Membrane Paradigm
Pamour
Gauss-Codazzi \leftrightarrow Fluid dynamics eq.

Membrane Paradigm
Pamour
Gauss-Codazzi \leftrightarrow Fluid dynamics eq.

$$AdS = CFT \quad (N=4)$$

Black brane = $T + o$ Thermal
CFT

Deformation = $T(\pm\epsilon)$
of BB $U^a(\pm\epsilon), U^m(\phi, \delta p^i)$

Geometry = NR limit $T \ll C$
Kaluza-Klein \rightarrow

$$AdS = CFT \quad (N=4)$$

Black brane = $T + o$ Thermal
CFT

Deformation = $T(\pm\epsilon)$
of BB $U^r(\pm\epsilon), U^M(\phi, \delta\rho^i)$

Geometry = NR limit $T \ll C$
Kaluza-Klein ϕ

Membrane Paradigm

Pamour

Gauss-Codazzi \leftrightarrow Fluid dynamics eq.

$$\left\{ \begin{array}{l} \partial_i U^i + U^j \partial_j U^i = - \partial^i \rho + \gamma \partial_j \partial^j U^i \\ \partial_i U^i = 0 \end{array} \right.$$

Membrane Paradigm

Pamour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_i U^i + U^j \partial_j U^i = - \partial^i \rho + \gamma \partial_j \partial^j U^i \\ \partial_i U^i = 0 \quad U^i(x^i) \quad i=1\dots d \end{array} \right.$$

Membrane Paradigm

Painour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t \mathbf{U}^i + \mathbf{U}^j \partial_j \mathbf{U}^i = - \partial^i \rho + \gamma \partial_j \partial^j \mathbf{U}^i \\ \partial_i \mathbf{U}^i = 0 \end{array} \right. \quad \begin{array}{l} \mathbf{U}^i(t) \quad i=1\dots d \\ \text{Velocity} \end{array}$$
$$+ \mathbf{F}^i$$

$d=2, 3$ experimental results

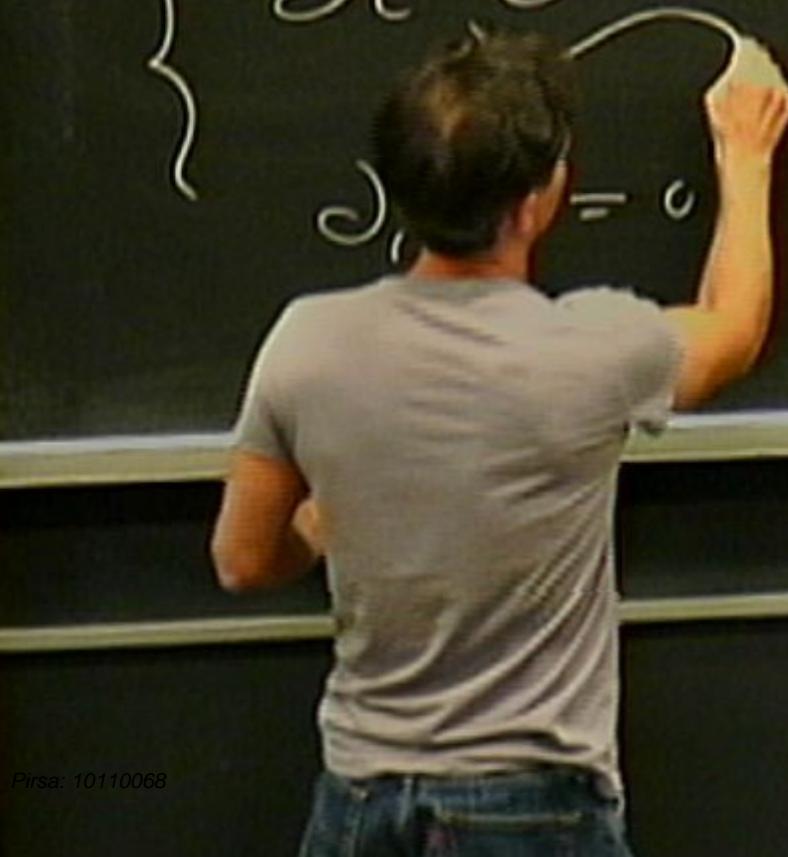
Membrane Paradigm

Pamour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t \bar{U}^i + \bar{U}^j \partial_j \bar{U}^i = - \partial^i \rho + \gamma \partial_j \bar{U}^j \\ \partial_i \bar{U}^i = 0 \end{array} \right. + F^i$$

$\bar{U}^i(t)$ $i=1\dots d$
Velocity



Membrane Paradigm

Papour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t \tilde{U}^i + \tilde{U}^j \partial_j U^i = - \partial^i \rho + \gamma \partial_j \tilde{U}^j \\ \partial_i U^i = 0 \end{array} \right. \quad \begin{array}{l} \tilde{U}^i(t) \quad i=1..d \\ \text{Velocity} \end{array}$$

$d=2, 3$ experimental results

$\delta_{de} \sigma^i$

Membrane Paradigm

Papour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t \tilde{U}^i + \tilde{U}^j \partial_j U^i = - \partial^j \rho + \gamma \partial_j \tilde{\sigma}^i \\ \partial_i U^i = 0 \end{array} \right. \quad \begin{array}{l} U^i(t) \quad i=1\dots d \\ \text{Velocity} \end{array}$$

$d=2, 3$ experimental results

$\partial d/\partial r^i$

$(\times \epsilon)$ pressure



Membrane Paradigm

Papour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t \mathbf{U}^i + \mathbf{U}^j \partial_j \mathbf{U}^i = - \partial^i \rho + \gamma \partial_j \partial^j \mathbf{U}^i \\ \partial_i \mathbf{U}^i = 0 \end{array} \right. \quad \begin{array}{l} \mathbf{U}^i(t) \quad i=1\dots d \\ \text{Velocity} \end{array}$$

Membrane Paradigm

Papour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t \tilde{U}^i + \tilde{U}^j \partial_j \tilde{U}^i = - \partial^i \rho + \gamma \partial_j \partial^j + F^i \\ \partial_i \tilde{U}^i = 0 \end{array} \right. \quad \begin{array}{l} \tilde{U}^i(t) \quad i=1\dots d \\ \text{Velocity} \end{array}$$



$\sigma = \sigma_0 + C \tau$

$d\sigma/d\epsilon$

$P(t; \epsilon)$ Pressure

ν Kinematic Viscosity

$\nabla F = \rho a =$ Incompressible case

$\rho =$

$d=2, 3$ experimental results

$d/d\epsilon \sigma'$

$\rho(\epsilon)$ pressure

ν kinematic viscosity

$\rho = \text{Incompressible case}$ $P = \text{const}$

$d=2, 3$ experimental results

$$\frac{d\dot{\epsilon}}{dt}(\rho\sigma') = \dots$$

$\rho(\dot{\epsilon}, \epsilon)$ pressure

ν kinematic viscosity

" $F = m\alpha'$ " Incompressible case $P = \text{const}$

Speed of Sound

$$U_s^2 = \frac{\rho}{\rho}$$

U_s

Wavelength
Velocity

Speed of Sound

$$U_s^2 = \frac{\rho}{\rho}$$

$$U < U_s \leq C$$

In comples Fluids are NR

Membrane param.

Damour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \underbrace{\partial_t U^i + U^j \partial_j U^i}_{\text{Divergence}} = - \partial^i \rho + \gamma \partial_j \partial^j U^i \\ \partial_i U^i = 0 \quad U^i(t^*) \quad i=1\dots d \\ \text{Velocity} \end{array} \right.$$



Membrane param.

Ramour

Gauss-Codazzi \leftrightarrow Fluid dynamics eq.

$$\left\{ \begin{array}{l} \partial_t U^i + \underbrace{U^j \partial_j U^i}_{\text{Dissipation}} = - \partial^j \rho + \nu \partial_j \partial_i U^j \\ \quad + F^i \end{array} \right.$$

$$\begin{array}{ll} \partial_i U^i = 0 & U^i(t=0) \quad i=1..d \\ (\partial_t \rho + \partial_i U^i) = 0 & \text{Velocity} \end{array}$$



Membrane param. eqn

Damour

Gauss-Codazzi \leftrightarrow Fluid dynamics eq.

$$\left\{ \begin{array}{l} \partial_t U^i + U^j \partial_j U^i = - \partial^i \rho + \nu \partial_j \partial^j U^i \\ \quad + F^i \\ \partial_i U^i = 0 \quad U^i(t^*) \quad i=1\dots d \\ (\partial_t \rho + \partial_i U^i = 0) \quad \text{Velocity} \end{array} \right.$$

Membrane paradigm

Damour

Gauss-Codazzi \leftrightarrow Fluid dynamics of

$$\left\{ \begin{array}{l} \partial_t U^i + U^j \partial_j U^i = - \partial^i \rho + \nu \partial_j \nu U^j \\ \quad + F^i \\ \partial_i U^i = 0 \end{array} \right.$$

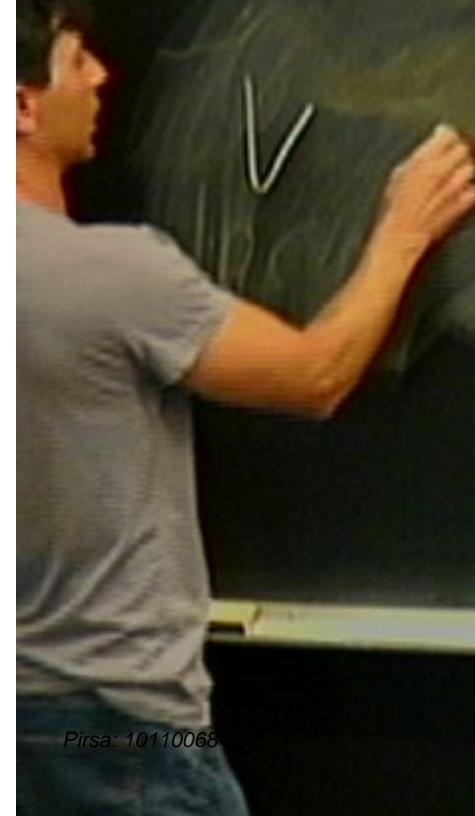
$$\left(\begin{array}{l} \partial_t \rho + \partial_i U^i = 0 \\ \quad \end{array} \right) \quad \begin{array}{l} U^i(t) \quad i=1\dots d \\ \text{Velocity} \end{array}$$

Reynolds #

$$Re = \frac{V \cdot L}{\nu} = \frac{V \cdot L}{\nu}$$

Reynolds #

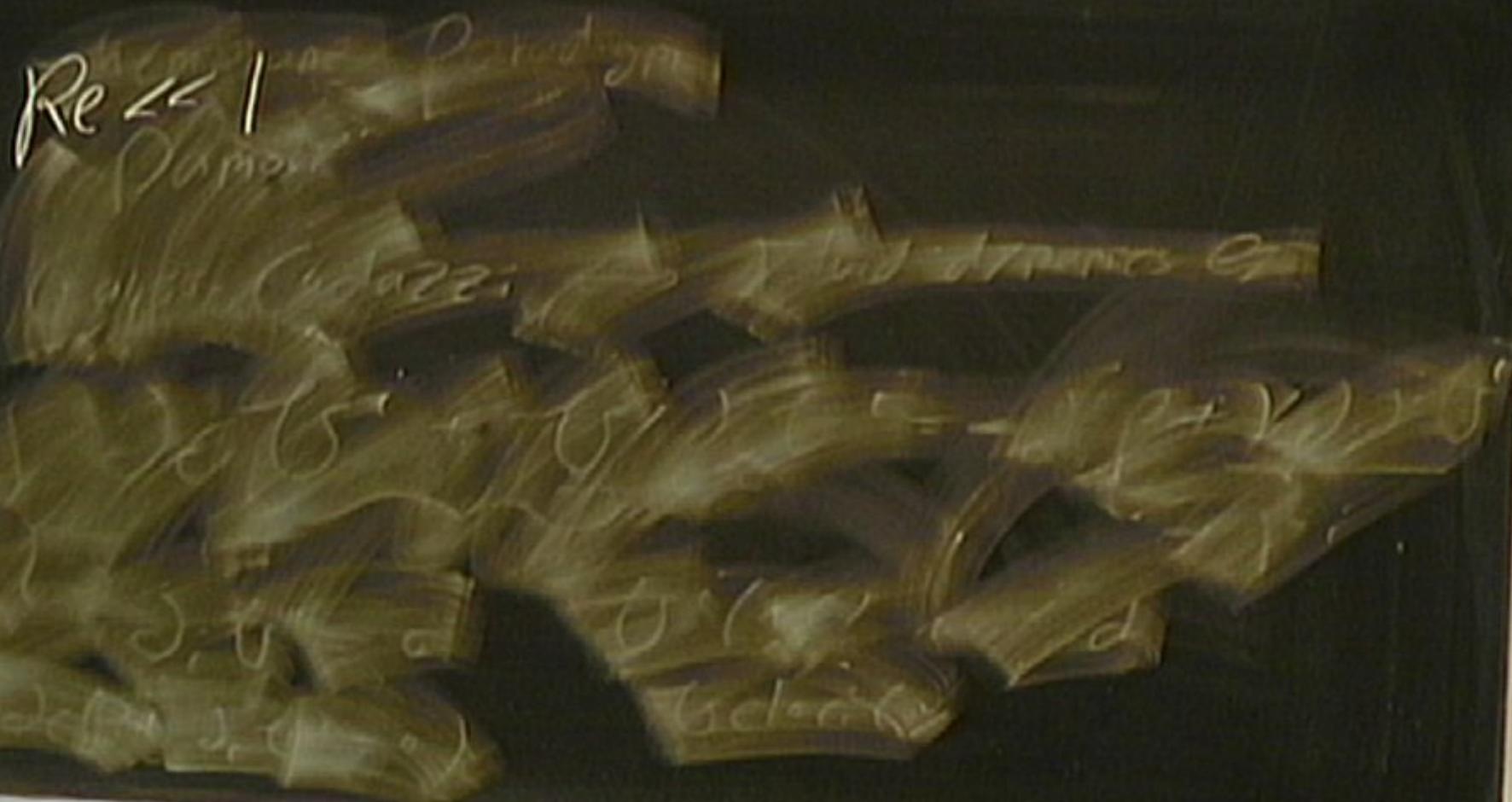
$$Re = \frac{V \cdot d}{\eta} = \frac{V \cdot d}{\nu}$$



Reynolds #

$$Re = \frac{V \times L}{\eta} = \frac{VL}{\eta}$$

V Characteristic Velocity
 L length



$Re \ll 1$ Laminar

$1 < Re \leq 100$



$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

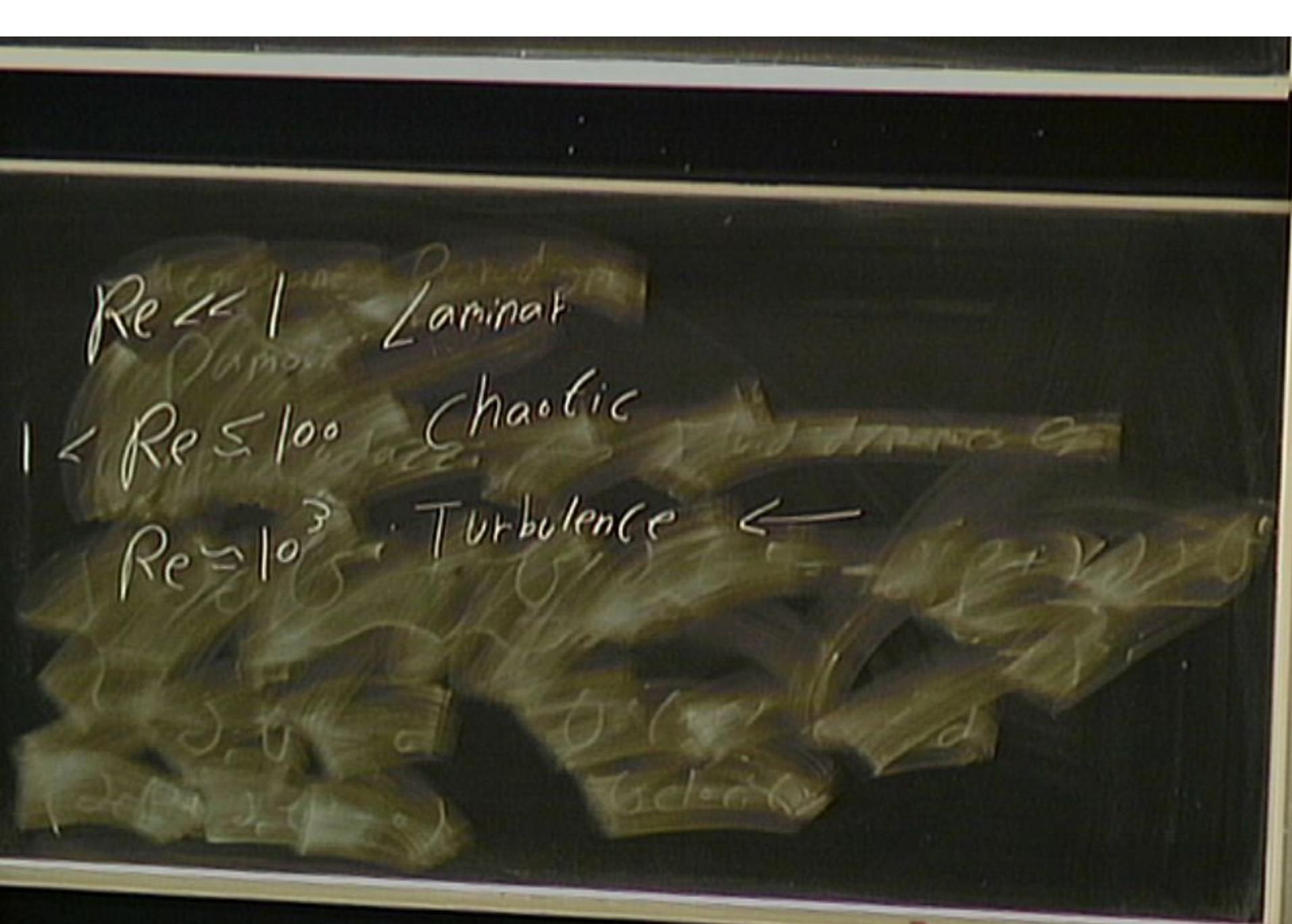
$Re \approx 10^3$, Turbulence



$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

$Re \approx 10^3$, Turbulence



$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

$Re \approx 10^3$ Turbulence

$$\nu_{\text{Water}} = 10^{-6} \frac{\text{m}^2}{\text{sec}}$$

$$\nu_{\text{Air}} \approx 1.5 \cdot 10^{-5} =$$

Reynolds #

$$Re = \frac{V \cdot L}{\nu}$$

Characteristic length

$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

$Re \approx 10^3$ Turbulence

$V_{water} = 10^{-6} \frac{m^3}{sec}$

$V_{air} \approx 1.5 \cdot 10^{-5}$

River $\approx \frac{10^4}{10^{-6}} \sim 10^7$

$Re \ll 1$ Laminar

$1 < Re \leq 10^0$ Chaotic

$Re = 10^3$ Turbulence

$$V_{\text{water}} = 10^{-6} \frac{\text{m}^3}{\text{sec}}$$

$$V_{\text{air}} \leq 1.5 \cdot 10^{-5} =$$

$$\text{River} \approx \frac{10^4}{10^{-6}} \sim 10^7$$

$Re \ll 1$ Laminar

$1 < Re \leq 10^0$ Chaotic

$Re = 10^3$ Turbulence \leftarrow Generic

$$V_{water} = 10^{-6} \frac{m^3}{sec}$$

$$V_{air} \approx 1.5 \cdot 10^{-5} =$$

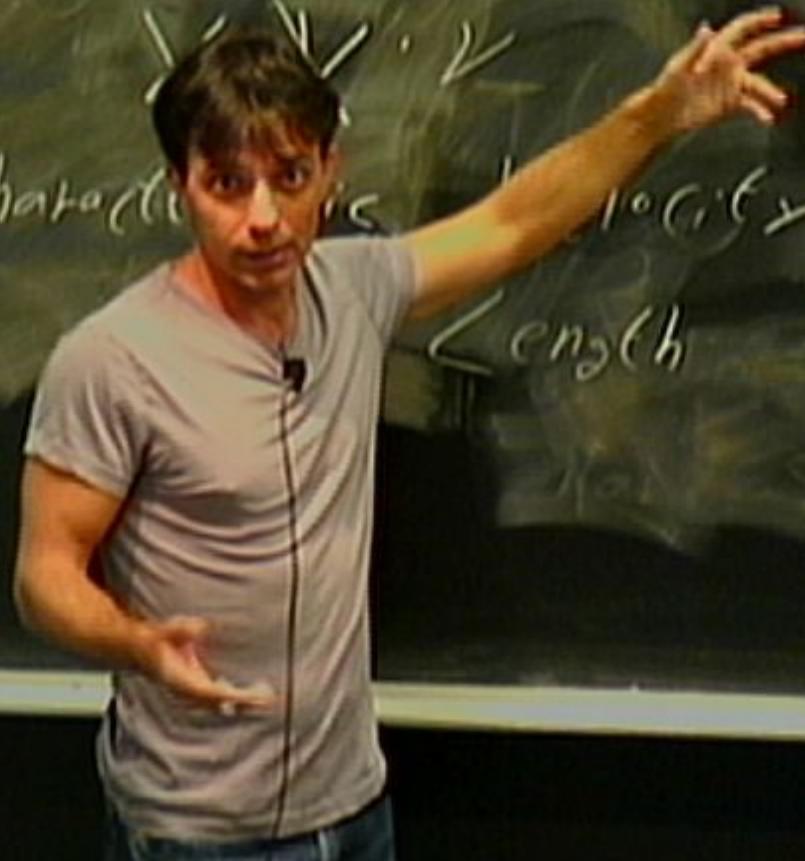
$$\text{River} \approx \frac{10^{-1}}{10^{-6}} \sim 10^7$$

Reynolds #

$$Re = \frac{V \cdot L}{\nu} = \frac{V \cdot L}{\eta / \rho}$$

Characteristics velocity

length



Reynolds #

$$Re = \frac{V \times L}{\eta} = \frac{V L}{\eta}$$

Characteristic Velocity

Length

$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

$Re = 10^3$: Turbulence ← Generic

$$V_{water} = 10^{-6} \frac{m^3}{sec}$$

$$V_{air} \approx 1.5 \cdot 10^{-5}$$

$$\text{River} \approx \frac{10^6}{10^{-6}} \sim 10^7$$

$Re \ll 1$ Laminar

$1 < Re \leq 10^0$ Chaotic

$Re = 10^3$ Turbulence \leftarrow Generic

$$V_{\text{water}} = 10^{-6} \frac{\text{m}^3}{\text{sec}}$$

$$V_{\text{air}} \approx 1.5 \cdot 10^{-5}$$

$$\text{River} \approx \frac{10^1}{10^{-6}} \sim 10^7$$

Statics

$$V^i(F) \quad V^i(\bar{F})$$

$$\bar{F} - Y - \bar{F}$$
$$+ |F|$$



Statistics

$$T - \bar{Y} = \bar{F}$$

$$T - |\bar{F}|$$

$$\delta U^i = -U^i(\bar{F}) - U^i(\bar{\bar{F}})$$

$$\delta U^i = \bar{U}^i(\bar{x}) - U^i(\bar{x}) \\ (U^i(\bar{x}) - U^i(\bar{v})) \cdot \frac{\bar{v}^i}{\bar{x}^i}$$

$$\bar{x} - \bar{y} - \bar{F} \\ \bar{x} - |F|$$

$$U^i(\bar{x}) \quad U^i(\bar{v})$$

$$\begin{aligned} \bar{x} - \bar{y} &= \bar{F} \\ |x - y| &= |\bar{F}| \\ \delta \sigma^i &= \bar{\sigma}^i(\bar{x}) - \bar{\sigma}^i(\bar{y}) \\ &< (\bar{\sigma}^i(\bar{x}) - \bar{\sigma}^i(\bar{y}')) \cdot \frac{\bar{x} - \bar{y}'}{|\bar{x}|} \end{aligned}$$



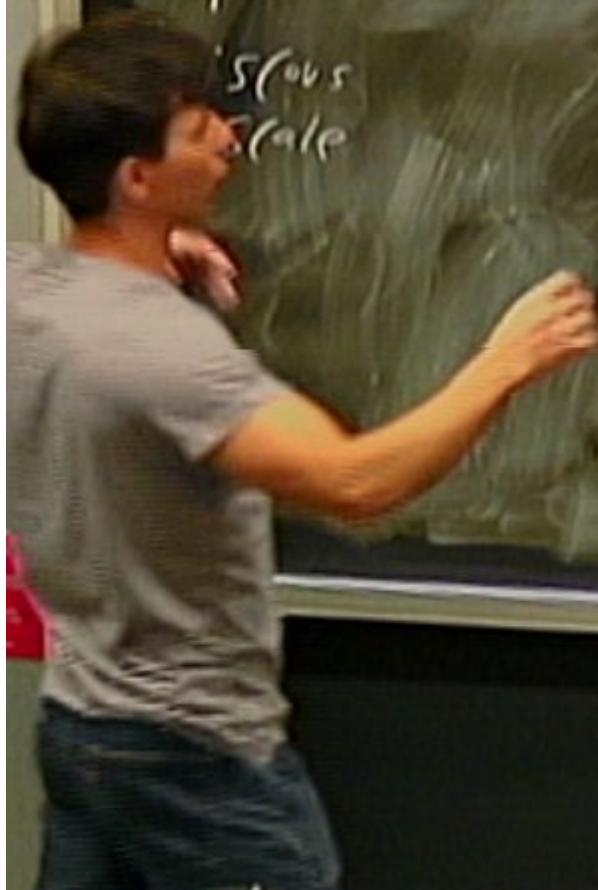
$\delta \ll 1 \ll$

Inertial range



Scales
scale

Galaxy



$\delta \ll T \ll L$

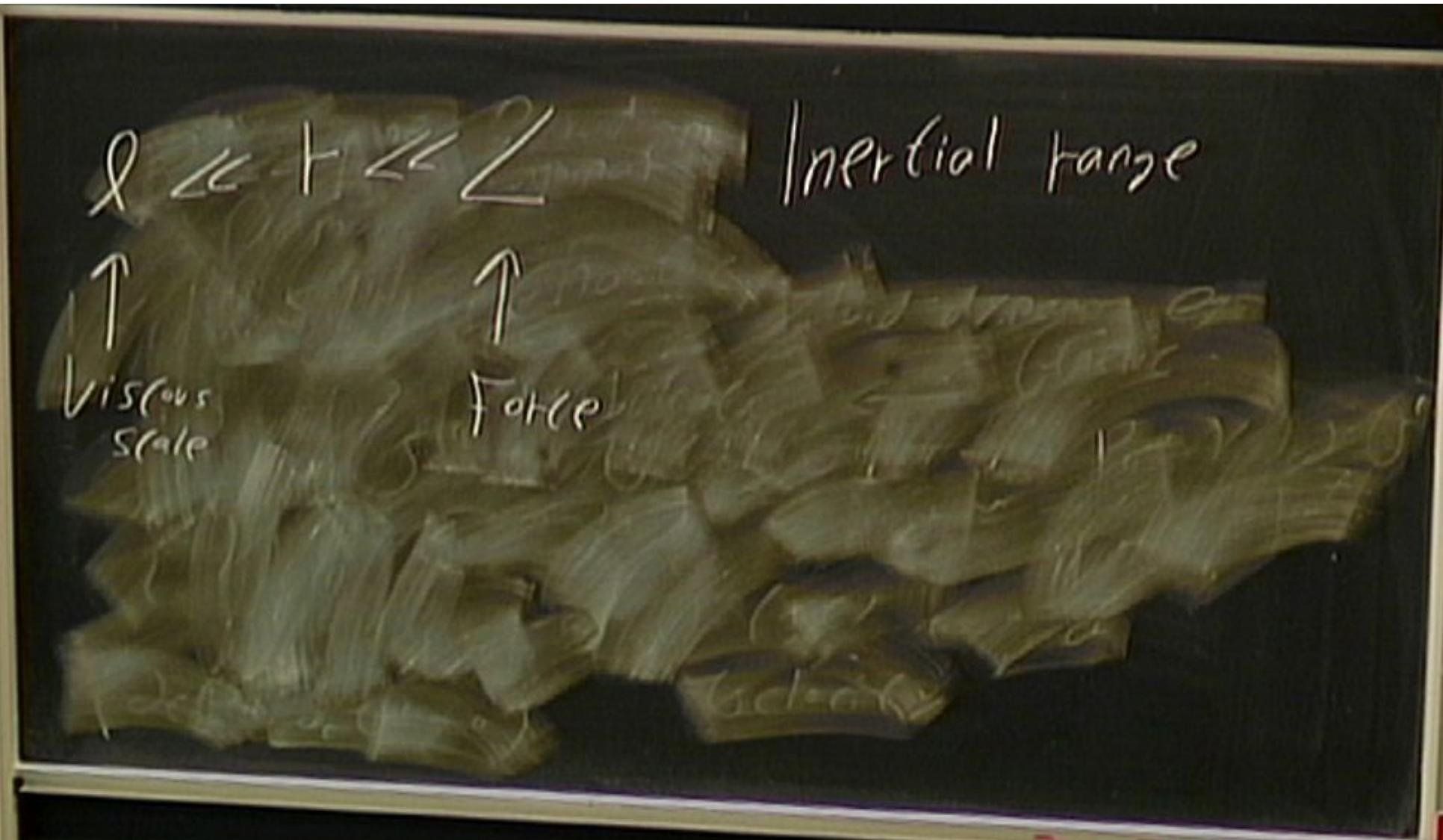
Inertial range



Viscous
scale



Force



$$V^*(F) \quad V^*(\bar{F})$$

$$\bar{X} - \bar{Y} = \bar{F}$$

$$T = |F|$$

$$\delta U^* = U^*(F) - U^*(\bar{F})$$

$$\langle ((U^*(F) - U^*(\bar{F})) \cdot \frac{T}{F}) \rangle \sim T^{3_n}$$

$\delta \ll T \ll \zeta$

Inertial range



Viscous
scale



Force

β_n real numbers

Galaxy

$$\begin{aligned} \bar{x} - \bar{y} &= \bar{F} \\ T &= |F| \\ \delta U' &= U'(F) - U'(\bar{F}) \\ <((U'(F) - U'(\bar{F})) \cdot \frac{T}{F})> &\sim T^{\beta_n} \end{aligned}$$

Statistics

$$\bar{F} - \bar{Y} = \bar{F}$$

$$T = |\bar{F}|$$

$$V^*(\bar{F})$$

$$V^*(\bar{Y})$$

$$\delta V^* = V^*(\bar{F}) - V^*(\bar{Y})$$
$$\left\langle \left((V^*(\bar{F}) - V^*(\bar{Y})) \cdot \frac{\bar{Y}}{\bar{F}} \right) \right\rangle \sim T^{3n}$$

Anomalous Exponents of the Range

Statistics

$$V'(F) \quad V'(\bar{F})$$

$$\bar{F} - \bar{\bar{Y}} = \bar{F}$$

$$r = |\bar{F}|$$

$$\delta v' = V'(F) - V'(\bar{F})$$

$$\left\langle \left((V'(F) - V'(\varphi)) \cdot \frac{t}{F} \right) \right\rangle \sim r^3 n$$

Anomalous Exponents

Franz

Kolmogorov 1941 (K41)

Anomalous Exponents

Kolmogorov 1941 (K41)

$$\log \frac{E(k)}{k^3} \sim -\beta \log k$$

Anomalous Exponents (1/tau)

Kolmogorov α_{41} (κ_4)

$\log \frac{\alpha_{41}}{1 - \alpha_{41}}$

Anomalous exponents

Kolmogorov 1941 (K41)

$$\log \phi_0 = \dots + \frac{1}{r}$$

Anomalous exponents

Kolmogorov 1941 (K41)

$$\log \zeta \sim \frac{1}{r} \quad \text{for } r \gg 1$$

Cascade

Anomalous exponents

Kolmogorov 1941 (K_41)



$$\log \langle (\delta v)^3 \rangle \sim \rho$$

decay

Anomalous exponents

Kolmogorov 1941 (k_4)

$$\log \langle (\delta r)^3 \rangle \propto \log T$$

$$\langle (\delta r)^3 \rangle \propto T^{\beta}$$

$$[\varepsilon] = \frac{C}{T^\beta}$$

$$\frac{C}{\sigma}$$



Kolmogorov

1941

(K41)

$\log \sigma_0$

cascade

$$\overline{\sigma}_n = \frac{n}{3}$$

$$\langle (\delta v)^3 \rangle \propto \epsilon t$$

$$[\epsilon] = \frac{C^2}{T^3}$$

$$\frac{C^3}{C}$$

18T-13 experimental results

$\langle \delta v \rangle \sim k^{\frac{2}{3}}$

experimental results

$$\langle v^2 \rangle \sim \langle (\delta v)^2 \rangle \sim k^{\frac{2}{3}}$$

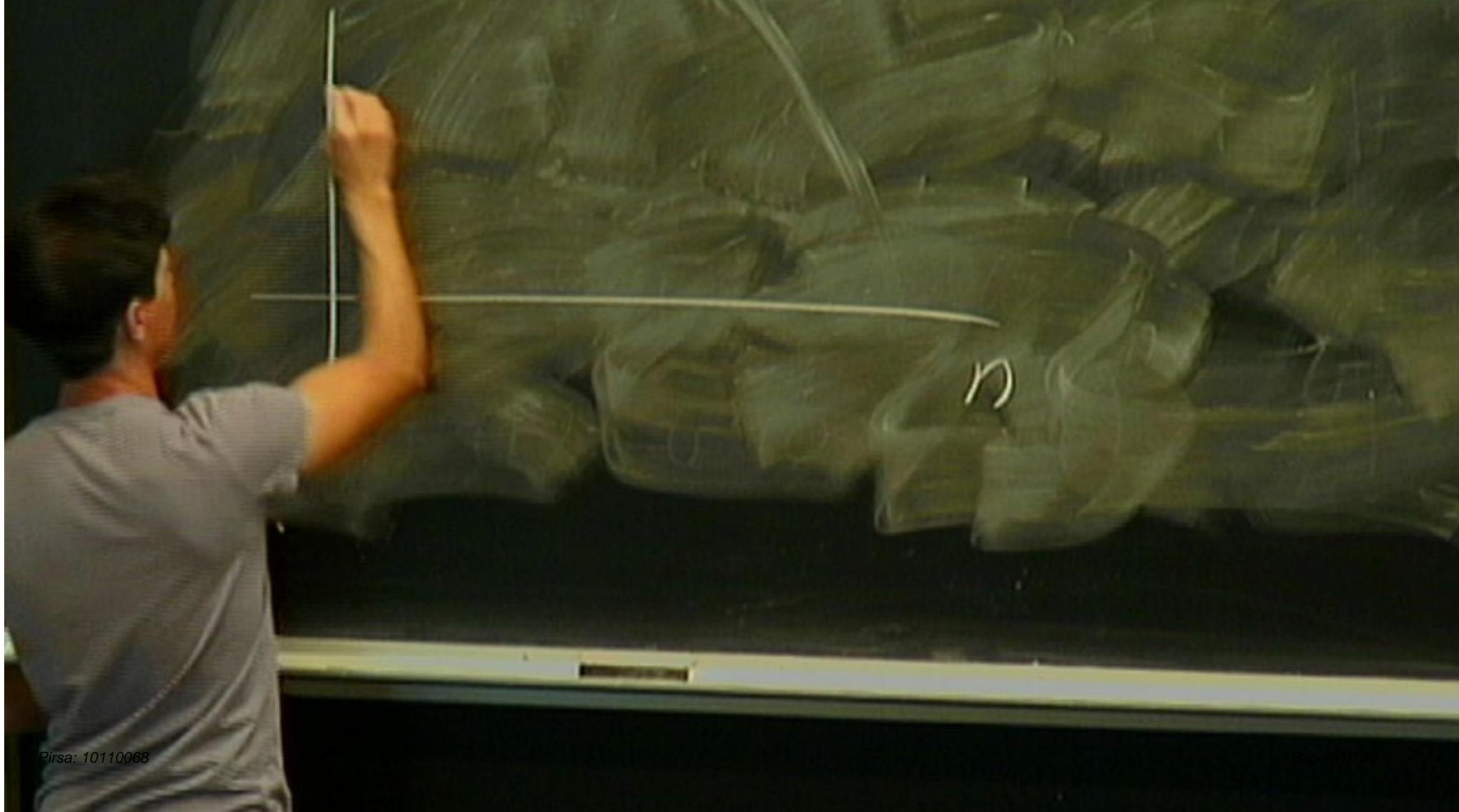
$$E(k) \sim \int e^{ikr} r^{\frac{2}{3}} dr \sim k^{-\frac{5}{3}}$$

$\delta U \sim k^{\frac{2}{3}}$ experimental results

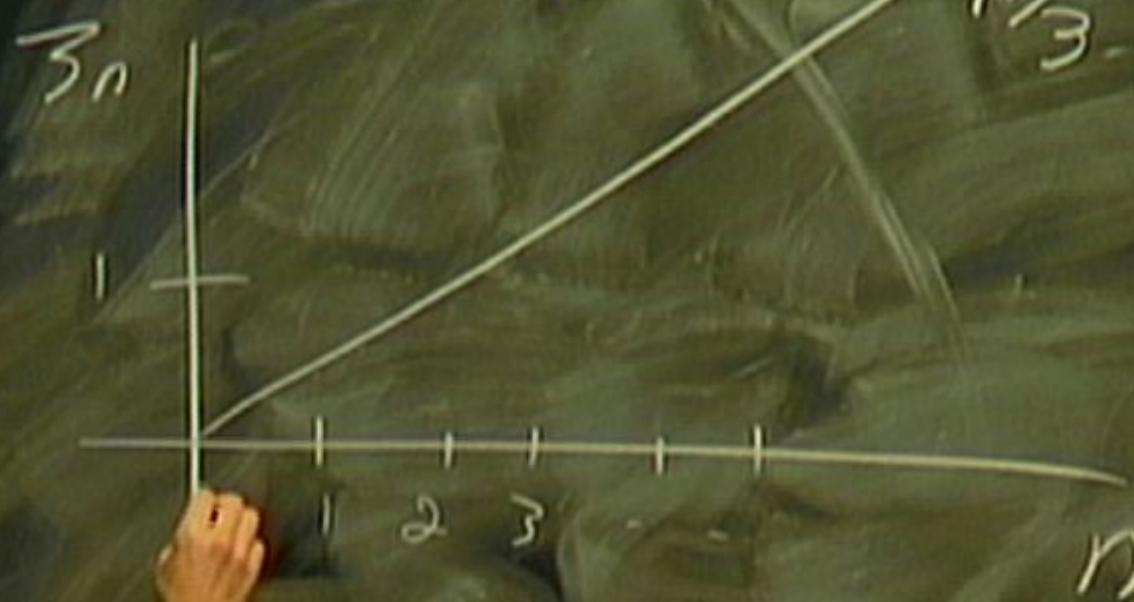
$$\langle E_n < (\delta U)^2 \rangle \sim k^{\frac{2}{3}}$$

$$E(k) \sim S e^{ikt} k^{\frac{2}{3}} dt \sim k^{-\frac{5}{3}}$$

$$\beta_3 = 1 \quad \checkmark$$



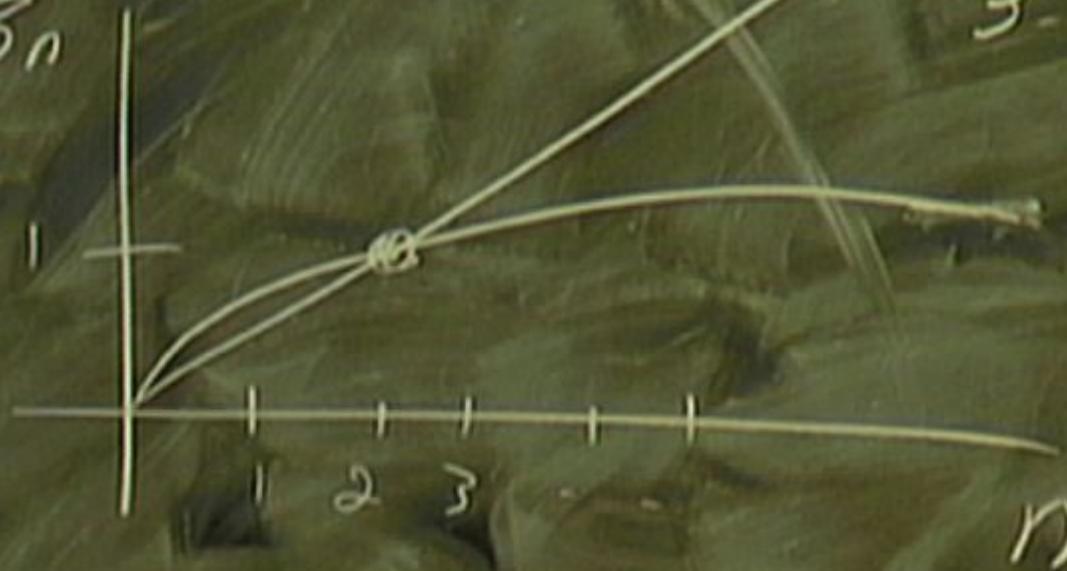
$$\beta_3 = 1 \quad \checkmark$$

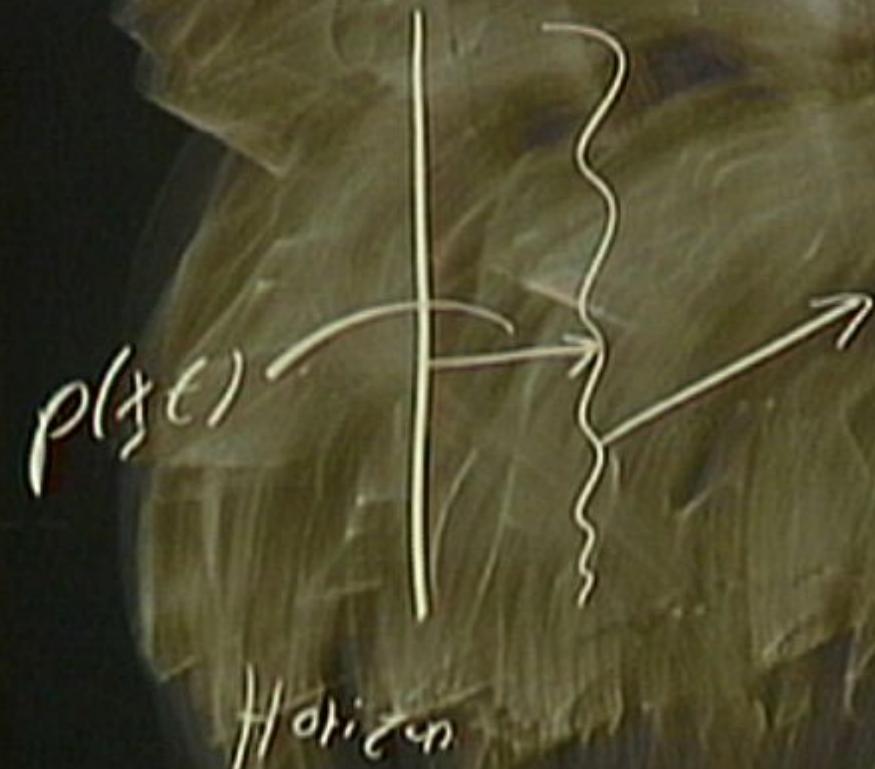


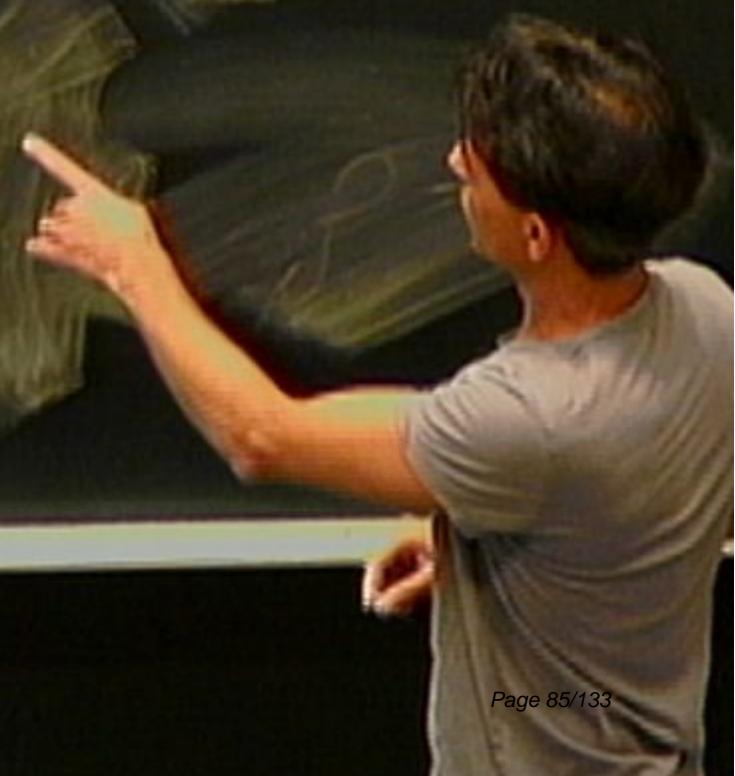
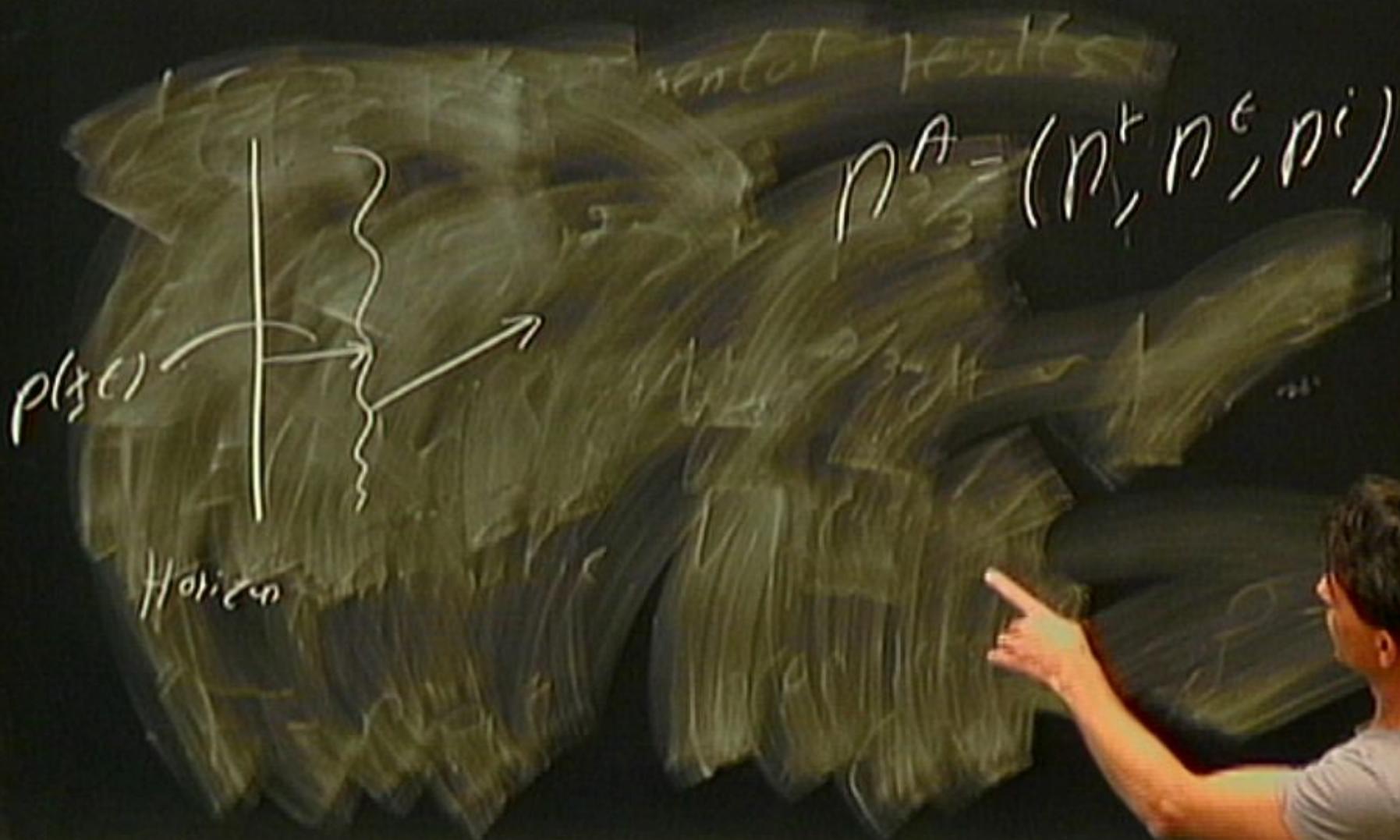
$$\beta_3 = 1$$

✓

$$\beta_n$$







$$\rho(x^c) \quad T_c \quad T(x^c)$$

$$\rho^A - (\rho^r, \rho^e, \rho^i)$$

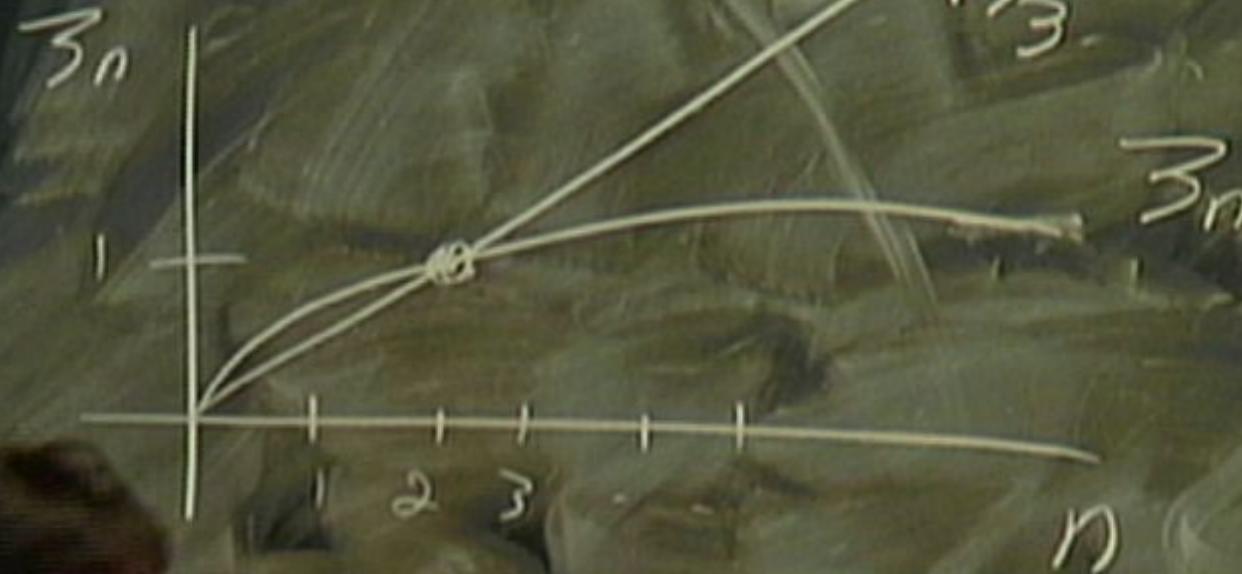
$$\rho^{(x^c)} \xrightarrow{T_c} \left\{ \begin{array}{l} T \\ T^{(x^c)} \end{array} \right\} \xrightarrow{\rho^A} \rho^A$$

$$\rho^A = (\rho^t, \rho^e, \rho^i) \\ - (o, l, v')$$

$$\rho(x^c) \xrightarrow{T_c} \left\{ \begin{array}{l} T(x^c) \\ \vdots \\ T^k(x^c) \end{array} \right\} \xrightarrow{\rho^A} \rho^A$$

$$\rho^A = (\rho^t, \rho^e, \rho^i) \\ - (o, l, v')$$

$$\beta_3 = 1 \quad \checkmark$$

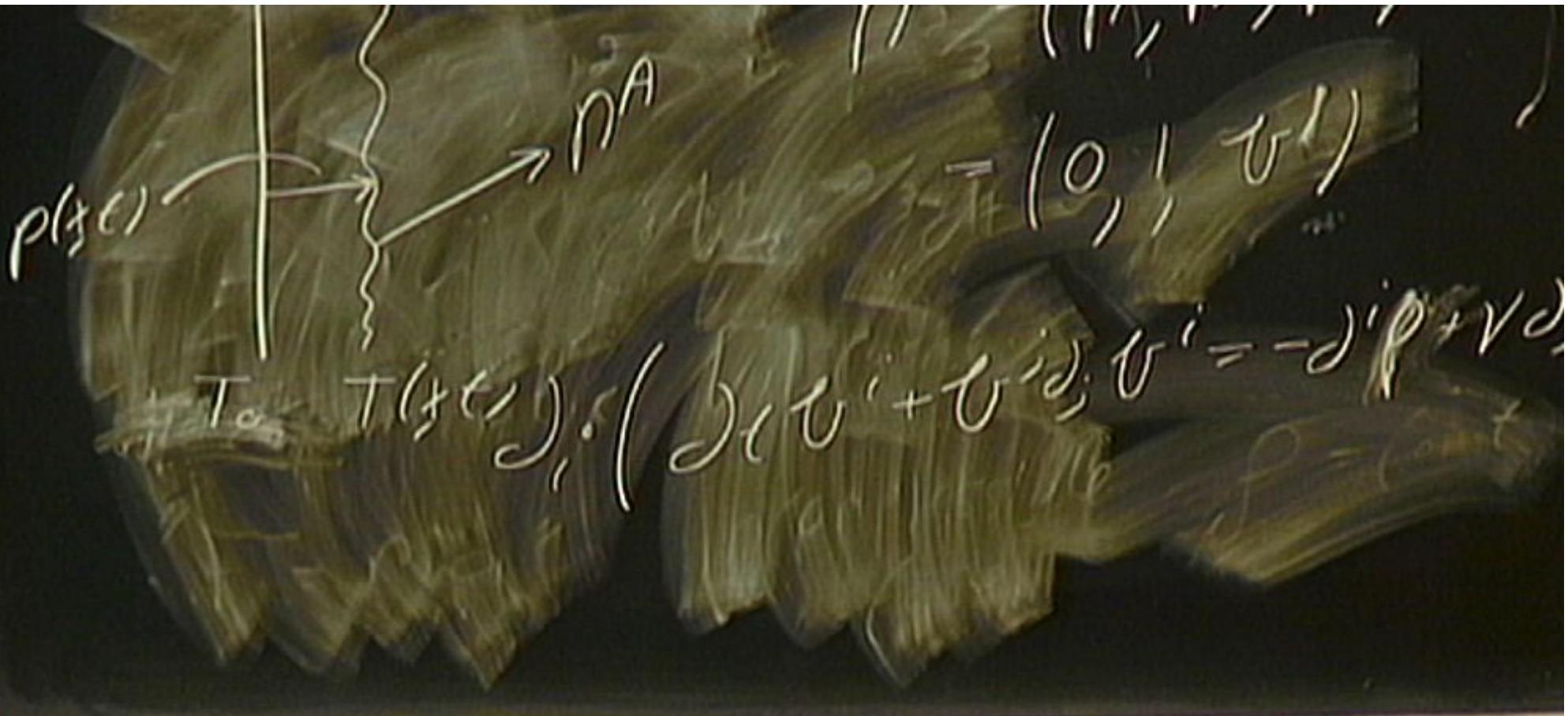


center results

$$\rho^A = (\rho^t, \rho^\epsilon, \rho^i)$$
$$- (o, l, v')$$

The diagram illustrates the decomposition of a vector field $\rho(x)$ into its tangential (ρ^t), normal (ρ^ϵ), and binormal (ρ^i) components along a curve T . A vertical line segment at a point on the curve is shown with three arrows pointing outwards, labeled ρ^t , ρ^ϵ , and ρ^i respectively. The curve is labeled T , and the overall vector field is labeled $\rho(x)$.





$$\rho^{(j,c)} \xrightarrow{T_c} \left\{ \begin{array}{l} \rho^A \\ \rho^A - (\rho^t, \rho^e, \rho^i) \\ - (o, l, v') \end{array} \right\}$$
$$T_{c'} \xrightarrow{T^{(j,c')}} \left(\frac{\partial f}{\partial v'} + v' \frac{\partial}{\partial v'} v' = - \frac{\partial}{\partial v'} \rho + v \frac{\partial}{\partial v} v' \right)$$

$$\begin{aligned} & \left. \rho^{(j,c)} \right|_{T_c} \xrightarrow{\quad} \rho^A \\ & \left. T^{(j,c)} \right|_{T_c} \cdot \left(\frac{\partial}{\partial} \rho^A - \left(\rho^A, \nu^A \right) \right) = - \frac{\partial}{\partial} \rho^A + \nu^A \cdot \frac{\partial}{\partial} \nu^A \end{aligned}$$



$$\rho^{(j,c)} \xrightarrow{T_c} \left\{ \begin{array}{l} \rho^A = (\rho^t, \rho^e, \rho^i) \\ - (o, l, v) \end{array} \right.$$
$$T_c(\rho^{(j,c)}) \cdot \left(\frac{\partial f}{\partial U^i} + U^i \frac{\partial}{\partial U^i} - \frac{\partial f}{\partial P} \cdot V \frac{\partial}{\partial U^i} \right)$$
$$\Delta P = - \frac{\partial f}{\partial U^i} \cdot U^i$$

Feynman diagrammatic results

$$\rho^{(3c)} \rightarrow \rho^A \rightarrow \rho^A - (\rho^r, \rho^e, \rho^i)$$
$$= (0, 1, v^I)$$
$$T_c = T(x^c), \quad (v^i + v^j) \partial_i v^j = - \partial^i \rho^r \cdot v^j \partial_j v^i$$
$$\Delta \rho^r = \partial^i v^j \partial_j v^i$$

sentral results

$$\rho^{(3c)} \xrightarrow{T} \left\{ \begin{array}{l} \rho^A \\ \rho^B \\ \rho^C \end{array} \right\} \xrightarrow{\rho^A = (\rho^t, \rho^e, \rho^i)}$$
$$= (0, 1, \rho^A)$$
$$T^{(3c)} \cdot \left(\frac{\partial}{\partial U^i} + U^j \frac{\partial}{\partial U^j} \right) U^i = - \frac{\partial}{\partial U^i} \rho^A \cdot \nu^i$$
$$\Delta \rho = - \frac{\partial}{\partial U^i} U^i \cdot \nu^i$$

$\rho^{(j,c)} \rightarrow \left\{ \begin{array}{l} T_c \\ T^{(j,c)} \end{array} \right\} \rightarrow \rho^A$

$\rho^A = (\rho^t, \rho^e, \rho^i)$
 $= (0, 1, \rho^i(j,c))$
 $\partial/\partial U^i + U^i \partial/\partial U^i = -\partial/\partial \rho^i \cdot \nabla \rho^i(j,c)$
 $\boxed{\Delta \rho^i = \partial_i U^i \cdot \nabla \rho^i}$

$$\frac{d=1}{1+1dm}$$

$$\frac{d=1}{}$$

$$1+1 \text{ dim}$$

$$\frac{d}{dt}$$

Butgers

$$J_C$$

$$d=1$$

Burgers

1+1 d.m

$$\partial_t U + U \partial_x U = \nu \partial_x^2 U$$

$$\partial_x U^x = 0$$

1) open problem - dual stability



$$\partial = 1$$

Butter's

1 + 1 d.m.

$$\partial_t U + U \partial_x U = \sqrt{d_x}^2 U$$

$$\partial_x U^F = 0$$

1) open problem - dual stability



$$\partial_t U + \partial_x U = 0$$

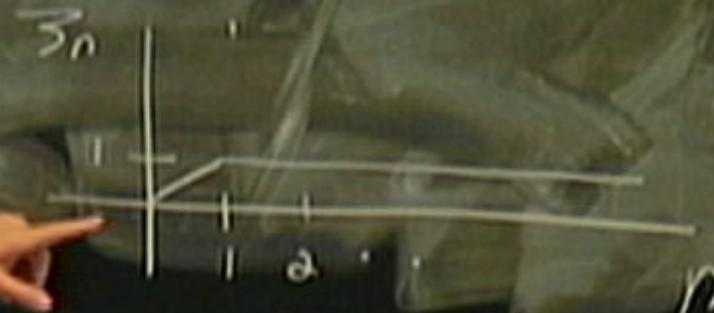
Burgers

1+1 d.m

$$\partial_t U + U \partial_x U = \nu \partial_x^2 U$$

$$\partial_t U + U \partial_x U = 0$$

1) open problem - dual stability



$$\partial = 1$$

Burgers

$$1 + 1 \text{ d.o.f.}$$

$$\partial_t U + U \partial_x U = \nu \partial_x^2 U$$

$$\partial_x U^2 = 0$$

1) open problem - dual stability



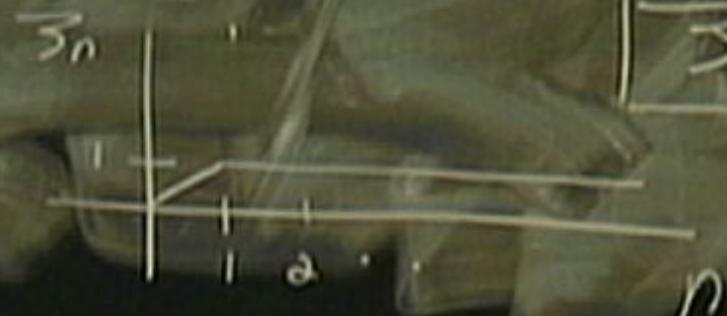
$$\delta = 1$$

Burgers

$$1+1 \text{ d.o.m} \quad \partial_t U + U \partial_x U = \nu \partial_x^2 U$$

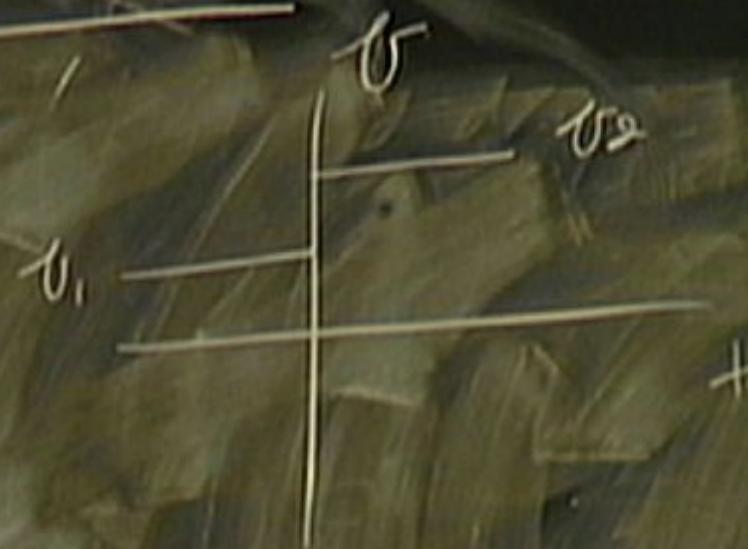
$$\partial_x U^T = 0$$

1) open problem - dual stability



Anomalous exponents

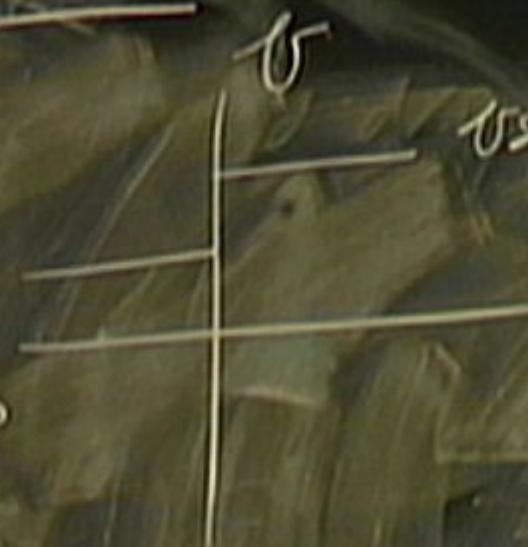
Shock Waves



Anomalous exponents

Shock Waves

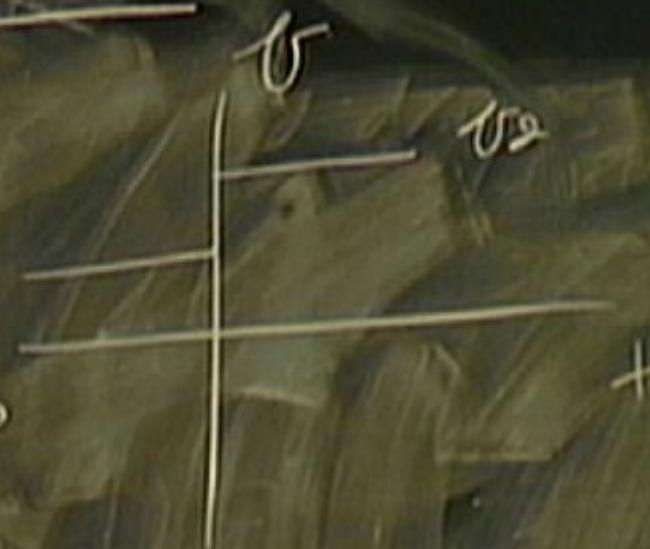
$$\langle (U(+\Delta t) - U(+))^n \rangle$$



Anomalous exponents

Shock Waves

$$\frac{\langle (U(+\Delta t) - U(+)) \rangle^{n_{U_1}}}{\Delta t}$$



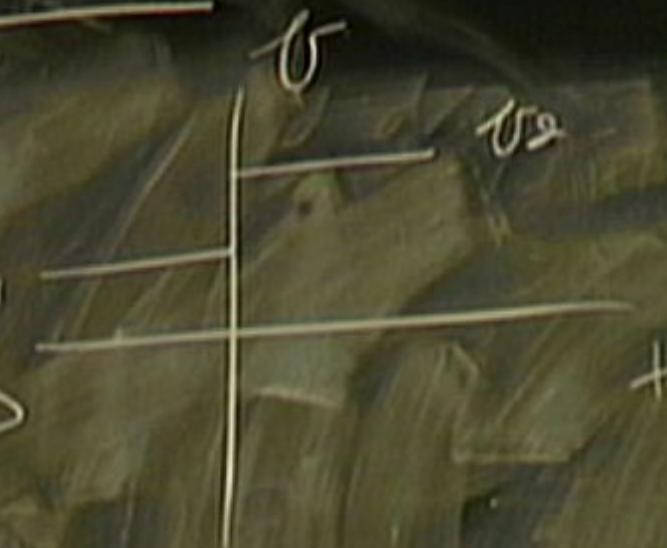
Anomalous exponents

Shock Waves

$$\langle (U(+ + \Delta t) - U(+))^n \rangle$$

$\sim \Delta t$

Conjecture - NS e.g. when $d \rightarrow \infty$ $\beta_n = 1$



Anomalous exponents

Shock Waves

$$\langle (U(t+\Delta t) - U(t))^n \rangle$$

$\sim \Delta t$

Conjecture - NS \Rightarrow when $d \rightarrow \infty$ $\beta_n = 1$
open problem

Anomalous exponents

Shock Waves

$$\langle (U(t + \Delta t) - U(t))^n \rangle$$

Δt

Conjecture - NS \Rightarrow when $d \rightarrow \infty$ $\beta_0 = 1$
open problem.

Anomalous exponents

Shock Waves

$$\langle (U(t + \Delta t) - U(t))^n \rangle$$

$\Delta t \rightarrow 0$

Conjecture - NS es when $d \rightarrow \infty$ $\beta_n = 1$
open problem



$$\langle (\delta\sigma)^3 \rangle \sim \sigma$$

$$\partial_t \sigma' + \text{[higher order terms]} =$$

$$\langle (\delta v)^3 \rangle \sim 0$$

$$\partial_i v^i + v^i \partial_i v^i = -\partial^i \rho$$

Euler

$$e \rightarrow -e$$

$$v \rightarrow -v$$

$$\langle (\delta v)^3 \rangle \sim \sigma$$
$$\partial_t v' + v' \partial_x v' = - \partial_x p + \underbrace{K \partial_x v'}$$

Euler

$$C \rightarrow -C$$

$$v' \rightarrow -v'$$

Parity

$$\langle (\delta v)^3 \rangle \sim 0$$

$$\partial_t v^i + v^j \partial_j v^i = - \partial^i p + \underbrace{v^j \partial_j v^i}_{\gamma \rightarrow 0}$$

Euler

$$t \rightarrow -t$$

$$v^i \rightarrow -v^i$$

Adiabatic

$$\langle (\delta r)^3 \rangle \sim r^0$$
$$\partial_r v' + v' \partial_r v = -\partial_r p + \underbrace{v \partial_r v'}_{r \rightarrow 0}$$
$$v \rightarrow -v$$
$$v' \rightarrow -v'$$

Apology

$$\langle (\delta r)^3 \rangle \sim r^0$$

$$\partial_t U' + U' \partial_x U' = - \partial_x P + \underbrace{V \partial_x U'}_{=0}$$

Euler

$$C \rightarrow -C$$

$$Y \rightarrow 0$$

$$U' \rightarrow -U'$$

Adiabatic

$$\frac{d^3}{\Delta(8r)^3} > \sim r^0$$
$$U' + U' U' = - \partial^i P + \underbrace{V \partial_i \partial^i}_{\sim} U'$$

Euler

$$U \rightarrow -U$$
$$U' \rightarrow -U'$$
$$V \rightarrow 0$$

$$J_\mu J^{\mu=0}$$



$$J_\mu J^\mu = 0$$

$$J_\mu T^{\mu\nu} = 0$$

$$\epsilon U' - \dots$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\nu (U^\mu \partial_\mu - U^\nu \partial_\nu) V^\rho = -\partial^\rho \rho + \nu \partial_\nu \partial^\rho V^\rho + F^\rho / U^\nu \partial_\nu$$

$$\partial_\mu J^\mu = 0$$

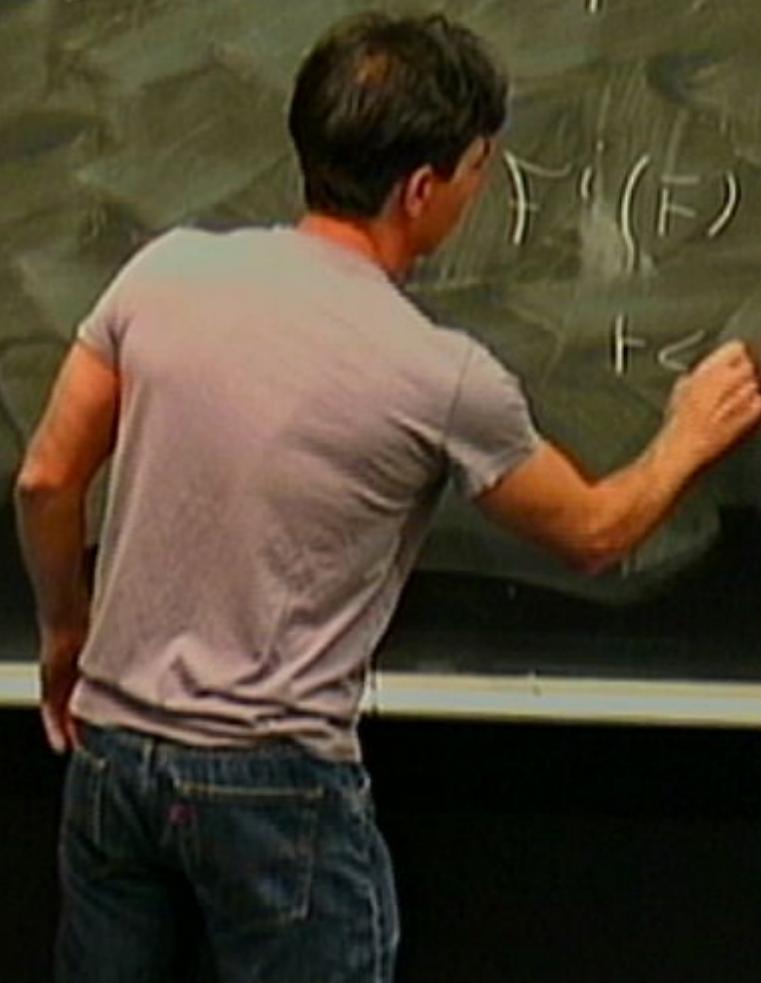
$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_\nu (U^\mu \partial_\mu U^\nu) - \partial_\nu U^\mu \partial_\mu U^\nu = -\partial^\mu \rho + \nu \partial_\mu \partial^\mu U^\nu + F^\nu / U^\nu$$

$$J_\mu J^\mu = 0$$

$$J_\mu T^{\mu\nu} = 0$$

$$\partial_\nu (U^i \partial_\mu - U^j \partial_\nu) U^i = - \partial^i \rho + \nu \partial_\nu U^i + F^i > U^i$$
$$F^i(F) U^i(0) >$$
$$+ <$$



$$J_\mu J^\mu = 0$$

$$J_\mu T^{\mu\nu} = 0$$

$$\partial_i U'(m - \sigma') \partial_i U' = -\partial_i \rho + \nu \partial_i \sigma' U' + F' > / U'(\circ)$$

$$\{ F'(F) U'(\circ) \}$$

+ <<

$$J_\mu J^\mu = 0$$

$$J_\mu T^{\mu\nu} = 0$$

$$\partial_t (U'(F) - U'(0)) \cdot U' = -\partial/\rho + \nu \partial_x U' + F' > |U'(0)|$$
$$\langle F'(F) U'(0) \rangle$$
$$\leq \epsilon \langle F'(0) U'(0) \rangle = \epsilon$$

$$J_\mu J^\mu = 0$$

$$J_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} J_\nu (T^{\mu\nu} - \sigma^{\mu\nu})_v v^\nu &= -\cancel{\rho} + \cancel{v \cdot F} v^\nu \\ &\quad + F^\nu \cancel{v^\mu} / v^\nu \\ &\quad - v^\nu (F^\mu) v^\nu (v^\mu) \\ &\quad - \langle F^\nu (F^\mu) v^\mu (v^\nu) \rangle \\ &\leq \langle F^\nu (v^\mu) v^\nu (v^\mu) \rangle - \epsilon \end{aligned}$$



$$\begin{aligned} & \langle U'(f)U'(e) \rangle \leq \langle F'(f)U'(e) \rangle \\ & \langle U'(U^*)U'(U') \rangle = \varepsilon \leq \langle F'(e)U'(e) \rangle = \varepsilon \end{aligned}$$



$$\begin{aligned} & \langle U'(F)U'(0) \rangle \quad \langle F'(F)U'(0) \rangle \\ - & \langle U'(U)U'(0') \rangle = \epsilon = \langle F'(0)U'(0) \rangle = \epsilon \\ & \langle U'U'U' \rangle = \epsilon + \end{aligned}$$

$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\partial_t (U'(F) - U'(0)) + U' = -\partial/\rho + \nu \partial_x U' + F' > |U'|$$

$$\partial_t \langle U'(F) U'(0) \rangle$$

$$\langle F'(F) U'(0) \rangle$$

$$\partial_t \langle U'(F) U'(0) \rangle = \epsilon \stackrel{?}{=} \langle F'(F) U'(0) \rangle = \epsilon$$

$$\langle U' U' U' \rangle - \epsilon r t$$



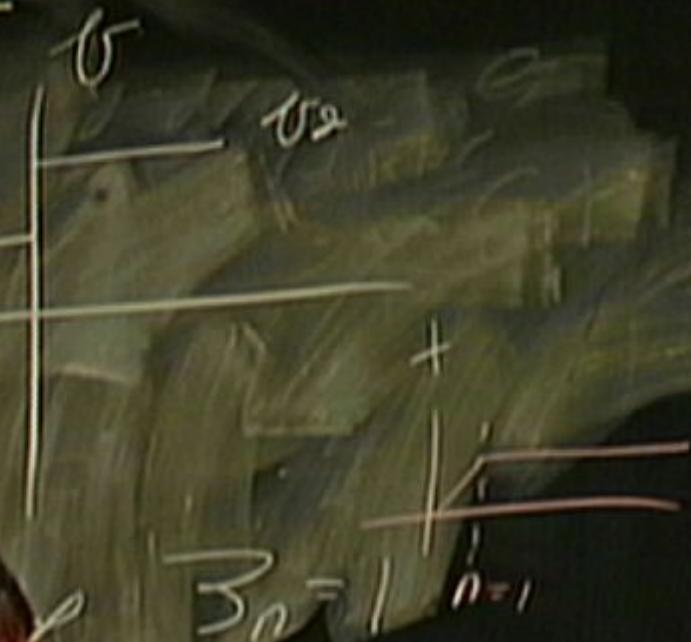
Anomalous exponents

Shock Waves

$$\langle (U(t+\Delta t) - U(t))^n \rangle$$

$$\sim \Delta t^{\alpha}$$

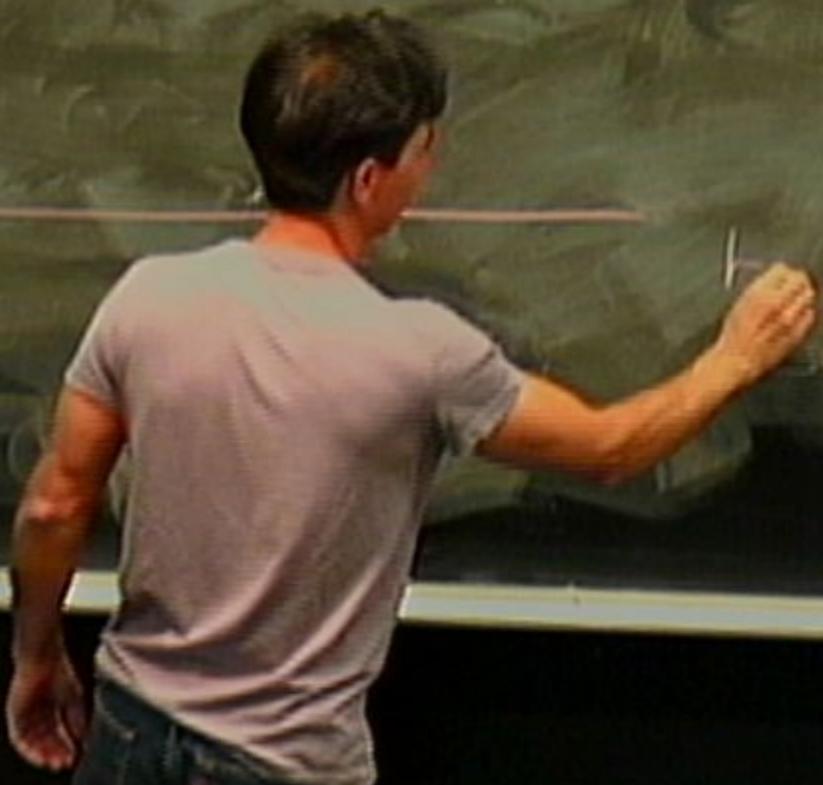
Conjecture - NS \Rightarrow when
open problem.



$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\left\langle \partial_t U'(\tau), -U' \right\rangle_U U' = -\partial_t P + \text{[other terms]} + F'$$



$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\begin{aligned} \partial_\mu (T^{\mu\nu} - \partial^\nu J^\mu) \cdot \partial_\nu U' = & -\partial^\nu P + \nabla_{\mu\nu} \partial^\mu U' \\ & + F^\nu \cdot \nabla_{\mu\nu} U' \end{aligned}$$



$$\partial_\mu J^\mu = 0$$

$$\partial_\mu T^{\mu\nu} = 0$$

$$\langle \partial_\nu (U^\mu_{\alpha\beta} - U^\mu_{\beta\alpha}) \cdot U^\nu - - \partial_\nu P + V^\mu_{\alpha\beta} U^\nu + F^\nu \rangle / U^\mu_{\alpha\beta}$$

