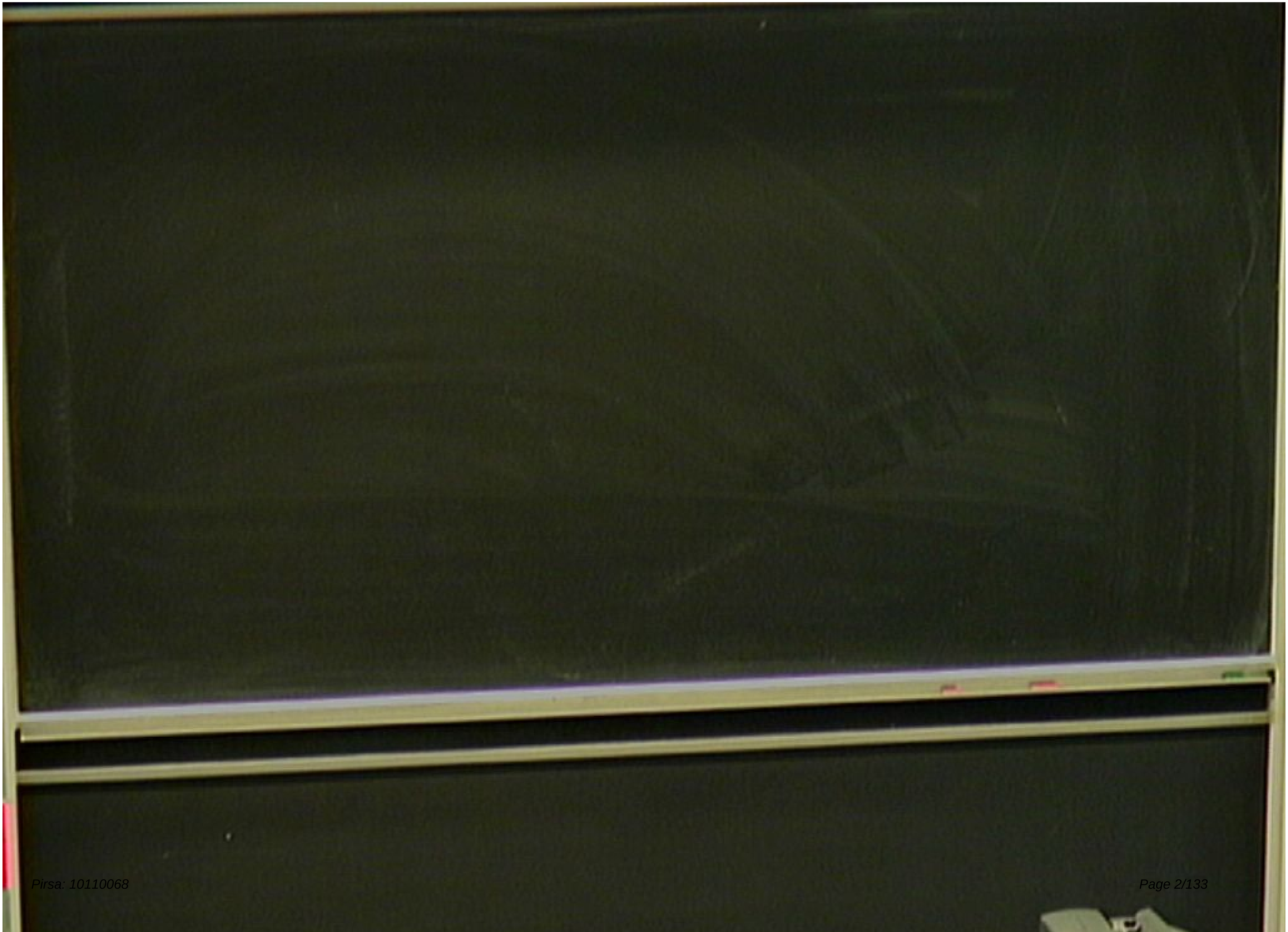
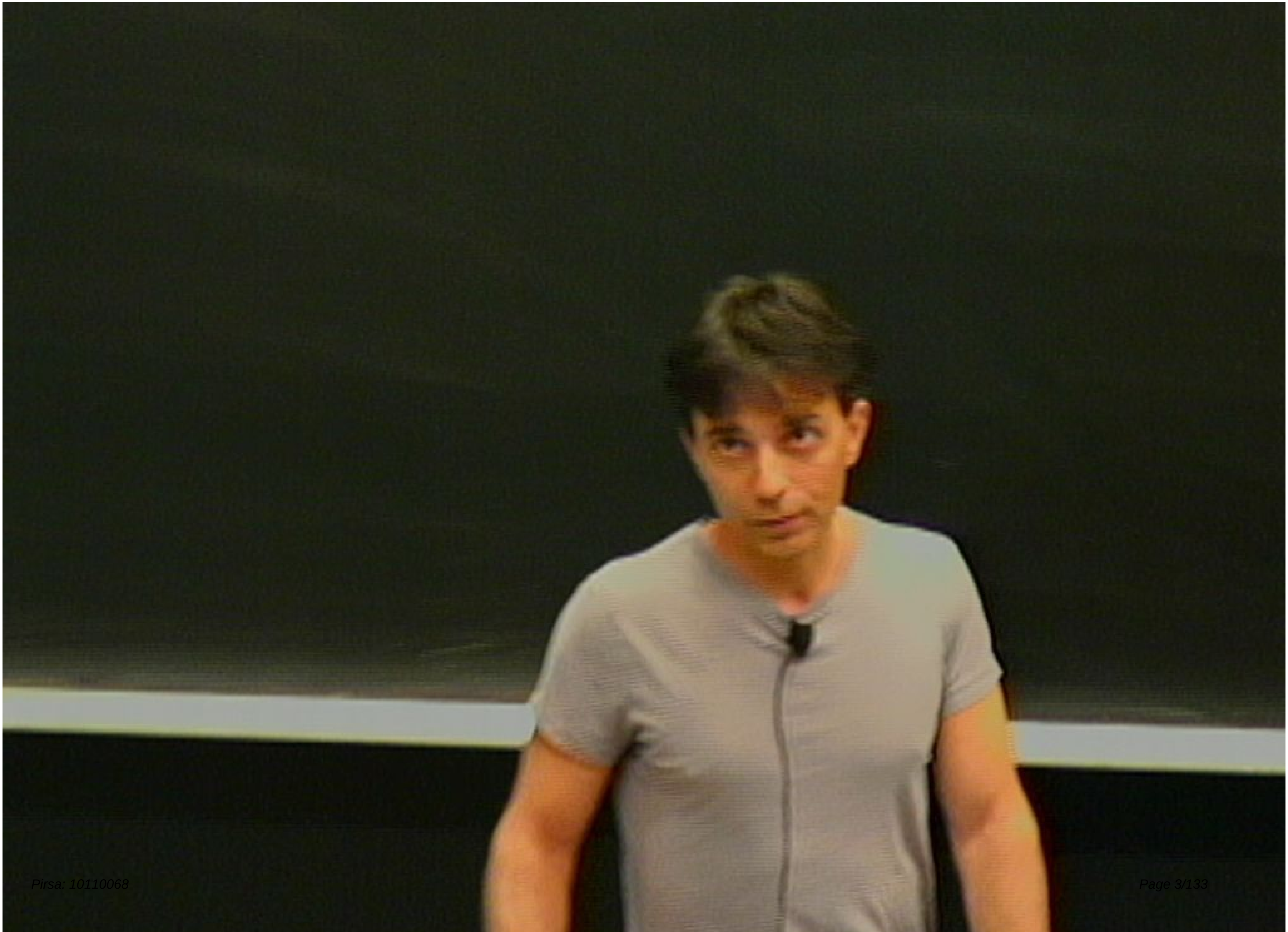
Title: Gravity and a Geometrization of Turbulence: An Intriguing Correspondence: Part 1

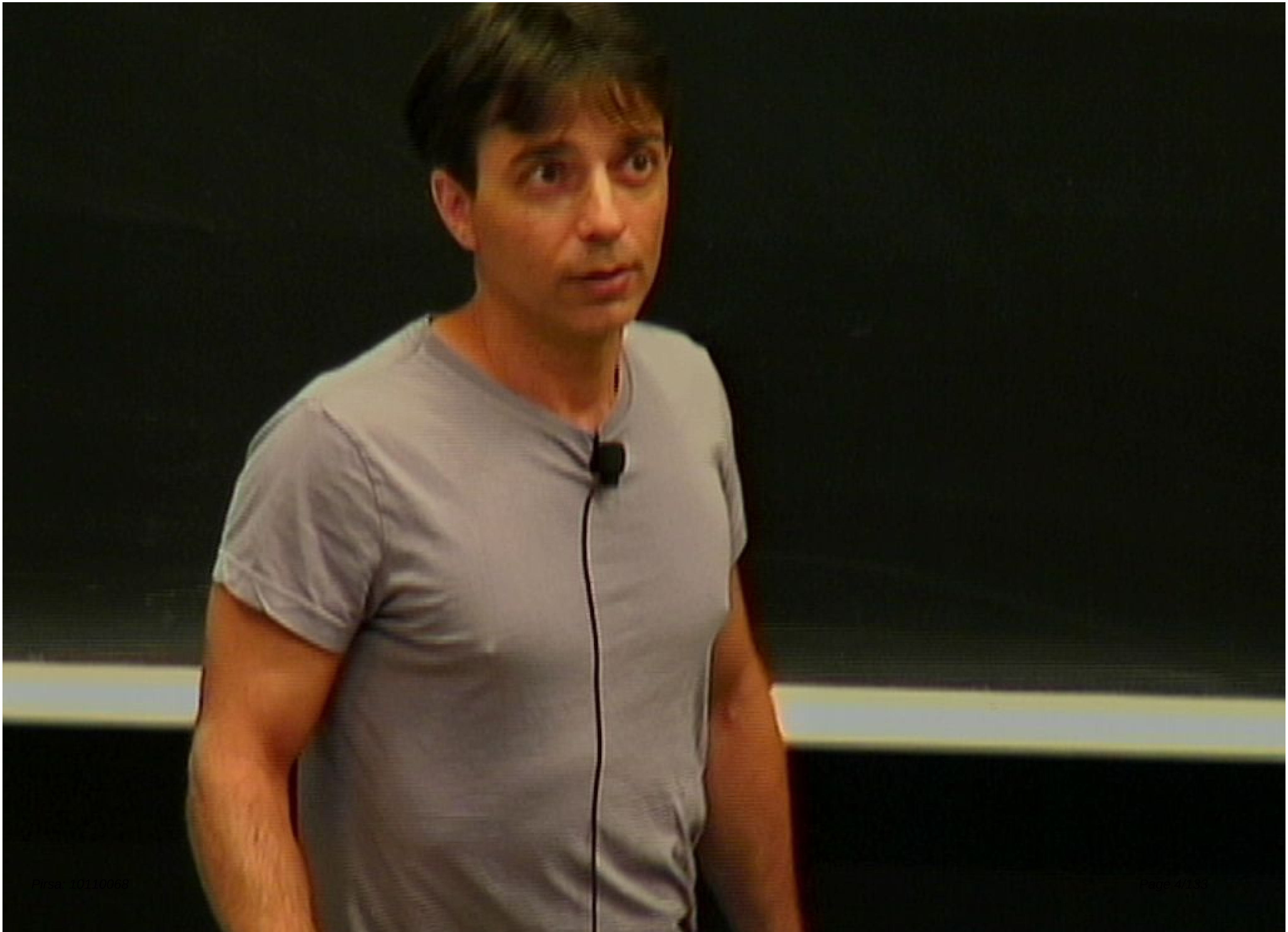
Date: Nov 10, 2010 11:00 AM

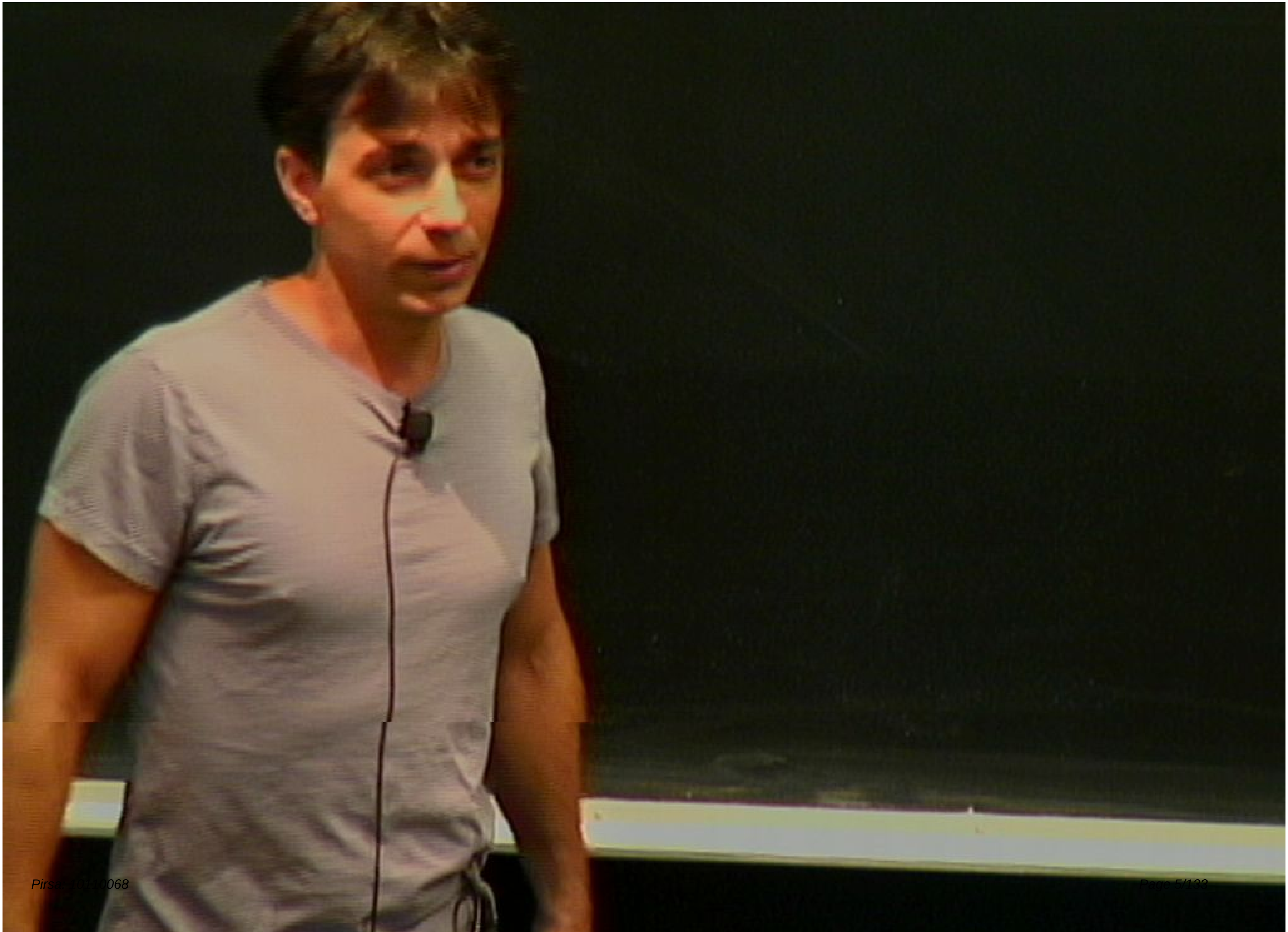
URL: http://pirsa.org/10110068

Abstract: The dynamics of fluids is a long standing challenge that remained as an unsolved problem for centuries. Understanding its main features, chaos and turbulence, is likely to provide an understanding of the principles and non-linear dynamics of a large class of systems far from equilibrium. We consider a conceptually new viewpoint to study these features using black hole dynamics. Since the gravitational field is characterized by a curved geometry, the gravity variables provide a geometrical framework for studying the dynamics of fluids: A geometrization of turbulence. We present new experimental predictions for relativsitic and non-relativistic turbulent flows and for heavy ion collisions.









AdS = CFT

AdS = CFT ($N=4$)

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$T \neq 0$ Thermal FT

$$\text{AdS} = \text{CFT} \quad (\mathcal{N}=4)$$

$$\text{Black brane} = T \neq 0 \quad \text{Thermal FT}$$

$AdS = CFT \quad (N=4)$

Black brane = $T \neq 0$ Thermal FT

$T(\pm\epsilon)$

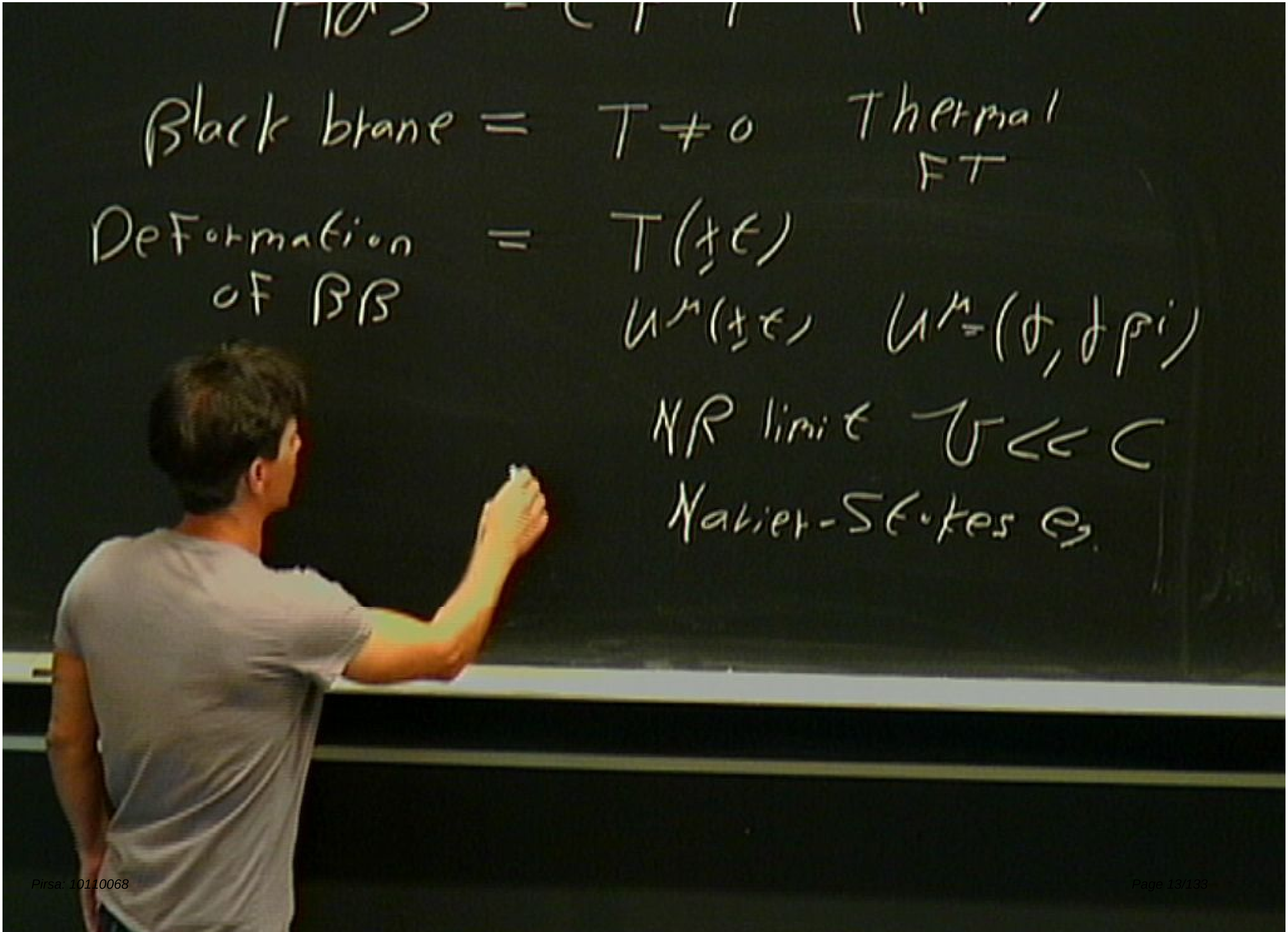
$u^\mu(\pm\epsilon) \quad u^\mu = (1, \partial\beta^i)$

$AdS = CFT \quad (N=4)$

Black brane $=$ $T \neq 0$ Thermal FT

Deformation of BB $=$ $T(x,t)$

$u^\mu(x,t) \quad u^\mu = (\gamma, \gamma \beta^i)$


Black brane = T ≠ 0 Thermal FT
Deformation of BB = T(x, t)
u^μ(x, t) u^μ = (v, v^i)
NR limit v << c
Navier-Stokes eq.

Black brane = $T \neq 0$ Thermal FT

Deformation of BB = $T(\underline{x}, t)$

$u^\mu(\underline{x}, t)$ $\quad u^\mu = (\gamma, \gamma \beta^i)$

Geometry = NR limit $v \ll c$

Navier-Stokes eq.

$$\text{AdS} = \text{CFT} \quad (\mathcal{N}=4)$$

Black brane $=$ $T \neq 0$ Thermal FT

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Navier-Stokes eq.

$$AdS = CFT \quad (N=4)$$

Black brane = $T \neq 0$ Thermal FT

Deformation of BB = $T(x,t)$

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Geometry = NR limit $v \ll c$

Navier-Stokes eq.

Membrane Paradigm

Damour

Gauss-Codazzi

Membrane Paradigm

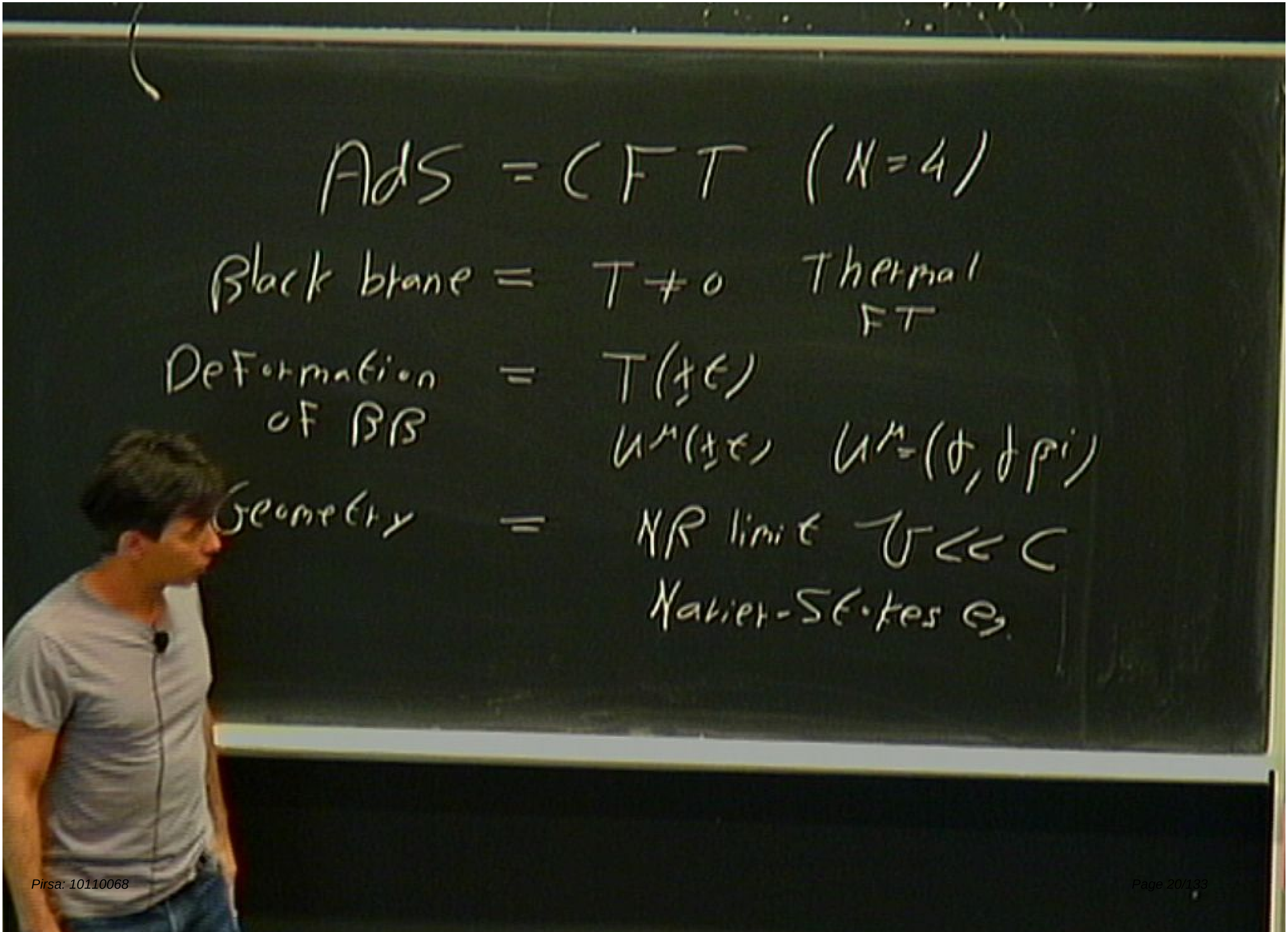
Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq.

Membrane Paradigm

Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq.


AdS = CFT (N=4)
Black brane = T ≠ 0 Thermal FT
Deformation of BB = T(x,t)
u^μ(x,t) u^μ=(γ, γβ^i)
Geometry = NR limit v << c
Navier-Stokes eq

$$\text{AdS} = \text{CFT} \quad (N=4)$$

Black brane = $T \neq 0$ Thermal FT

Deformation of BB = $T(x,t)$

$u^\mu(x,t) \quad u^\mu = (v, v\beta^i)$

Geometry = NR limit $v \ll c$

Navier-Stokes eq

Membrane Paradigm

Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq.

$$\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{cases}$$

Membrane Paradigm

Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq

$$\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{cases}$$

$v^i(\underline{x}, t) \quad i = 1 \ldots d$

Membrane Paradigm

Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq.

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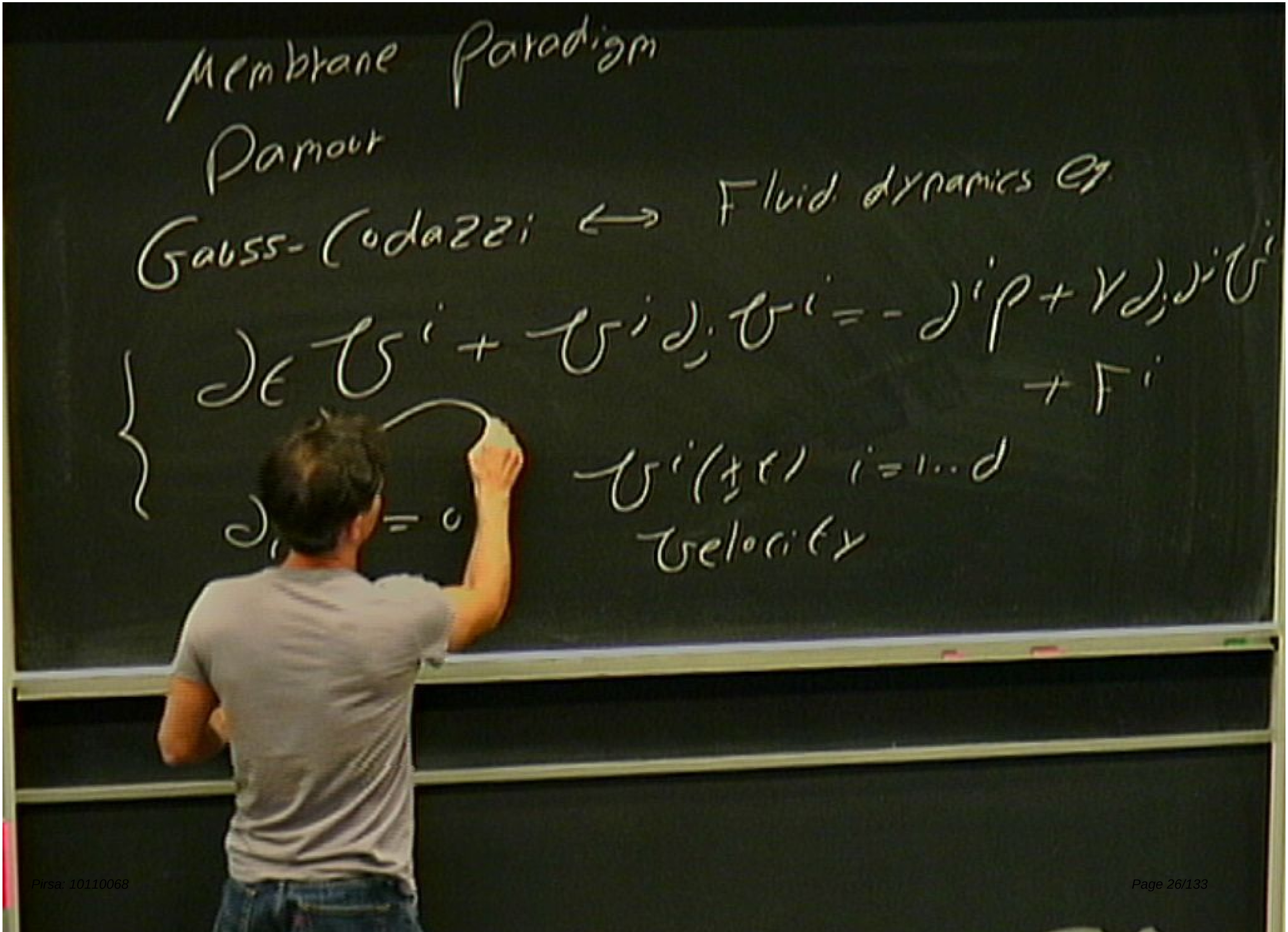
$v^i(t,x)$ $i = 1 \ldots d$

velocity

$d = 2, 3$ experimental results


Membrane Paradigm
Damour
Gauss-Codazzi ⟷ Fluid dynamics eq.
$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i$
$v^i(t,x)$ $i = 1 \ldots d$
velocity

Membrane Paradigm

Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq

$$
\begin{cases}
\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\
\partial_i v^i = 0
\end{cases}
$$

$v^i(t, x)$ $i = 1 \ldots d$

velocity

$d = 2, 3$ experimental results

$d/dt \; \theta^{i}$

Membrane Paradigm

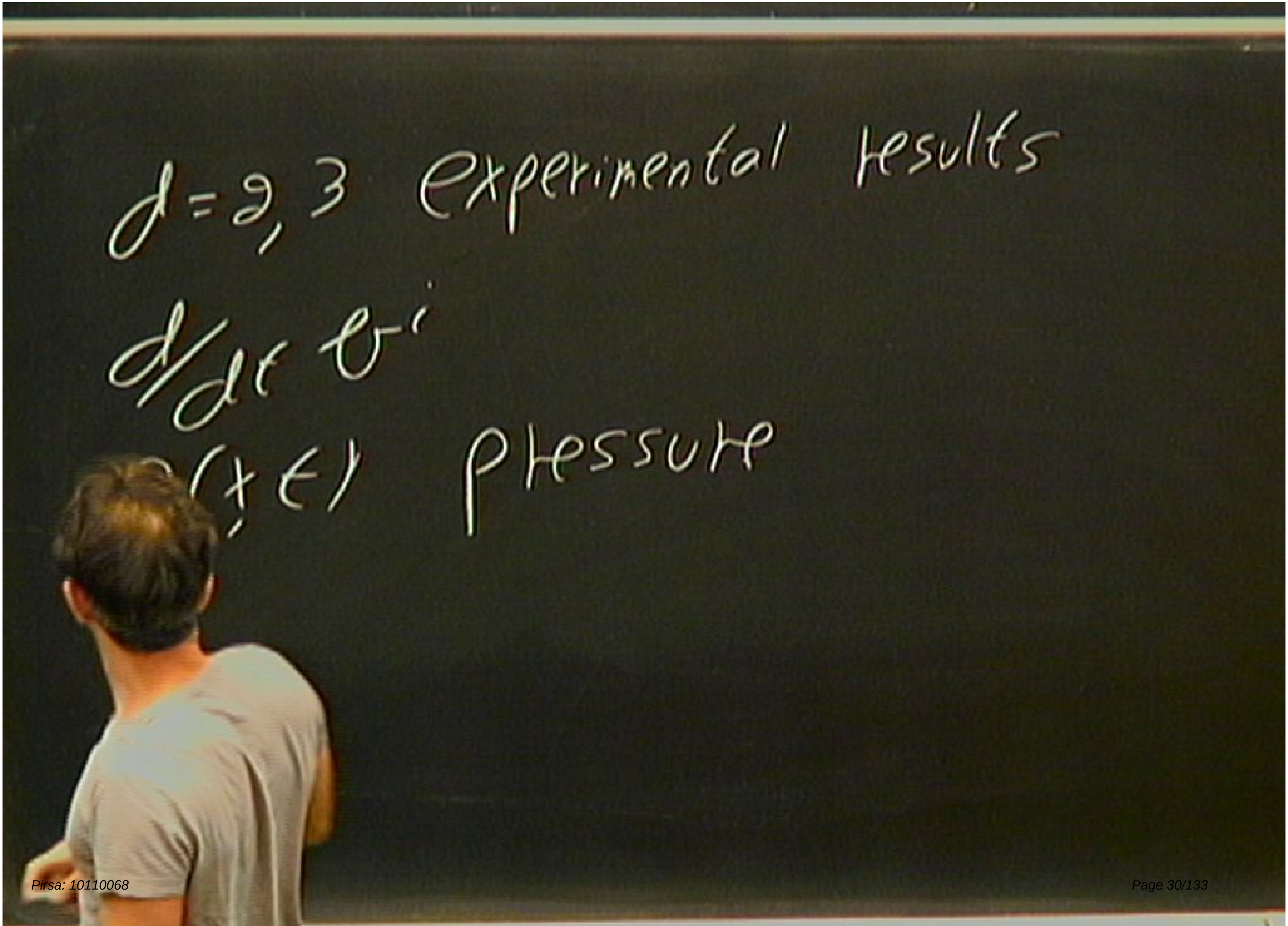
Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq

$$\begin{cases} \partial_t \underbrace{v^i + v^j \partial_j v^i}_{} = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{cases}$$

$v^i(t,x)$ $i = 1 \ldots d$

velocity


d=2, 3 experimental results
pressure

Membrane Paradigm

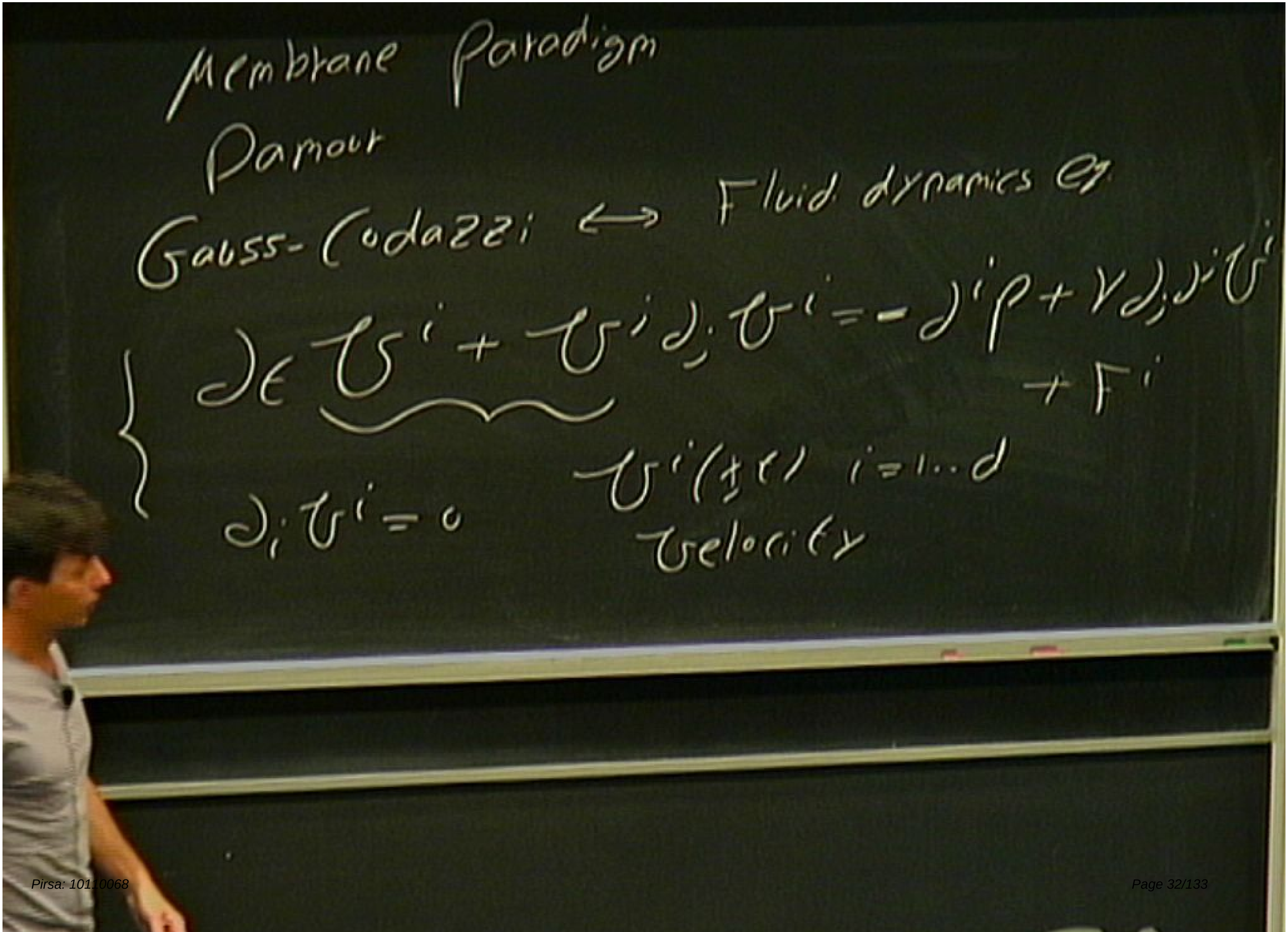
Damour

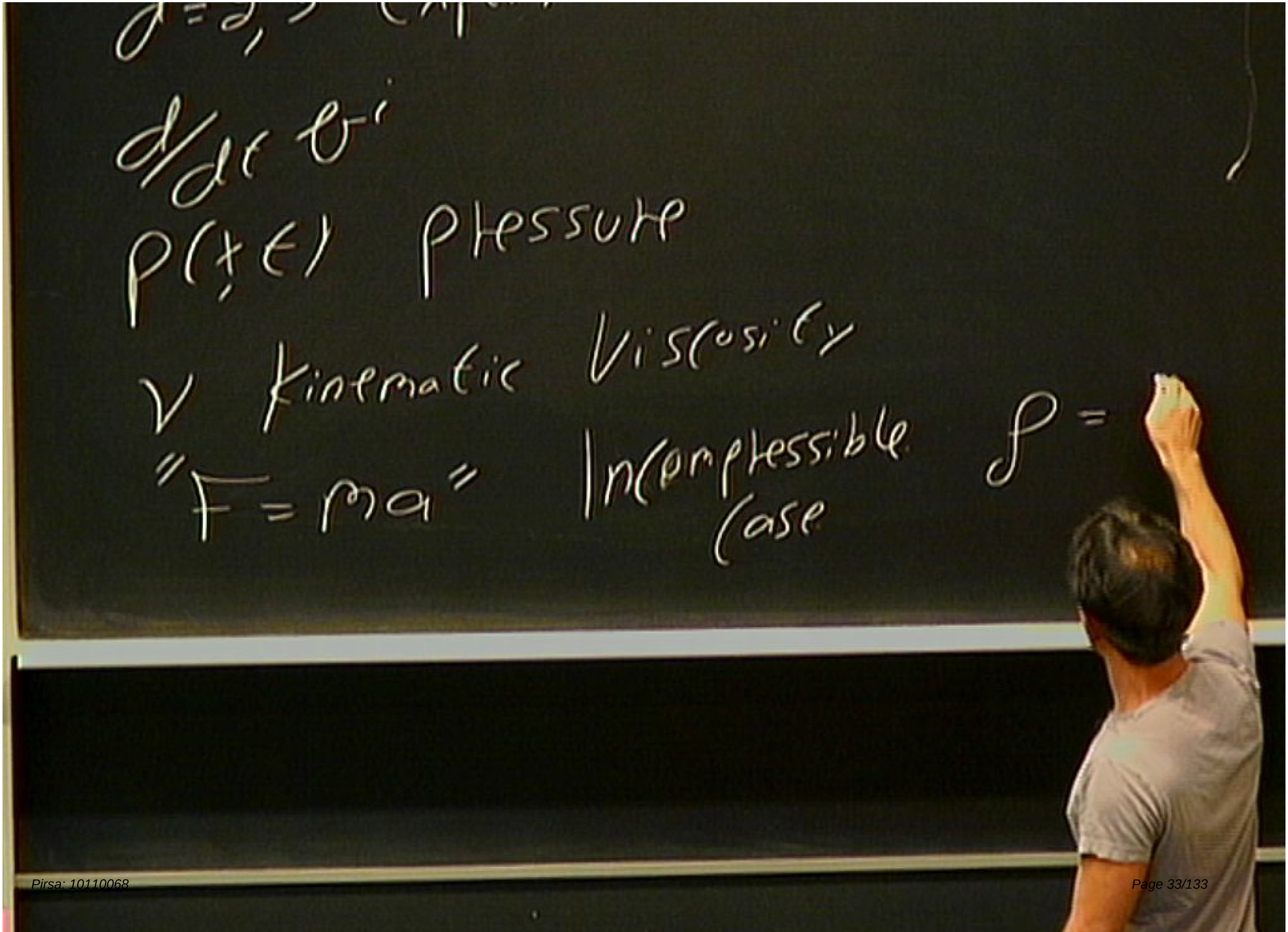
Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq

$$\left\{\begin{array}{l} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{array}\right.$$

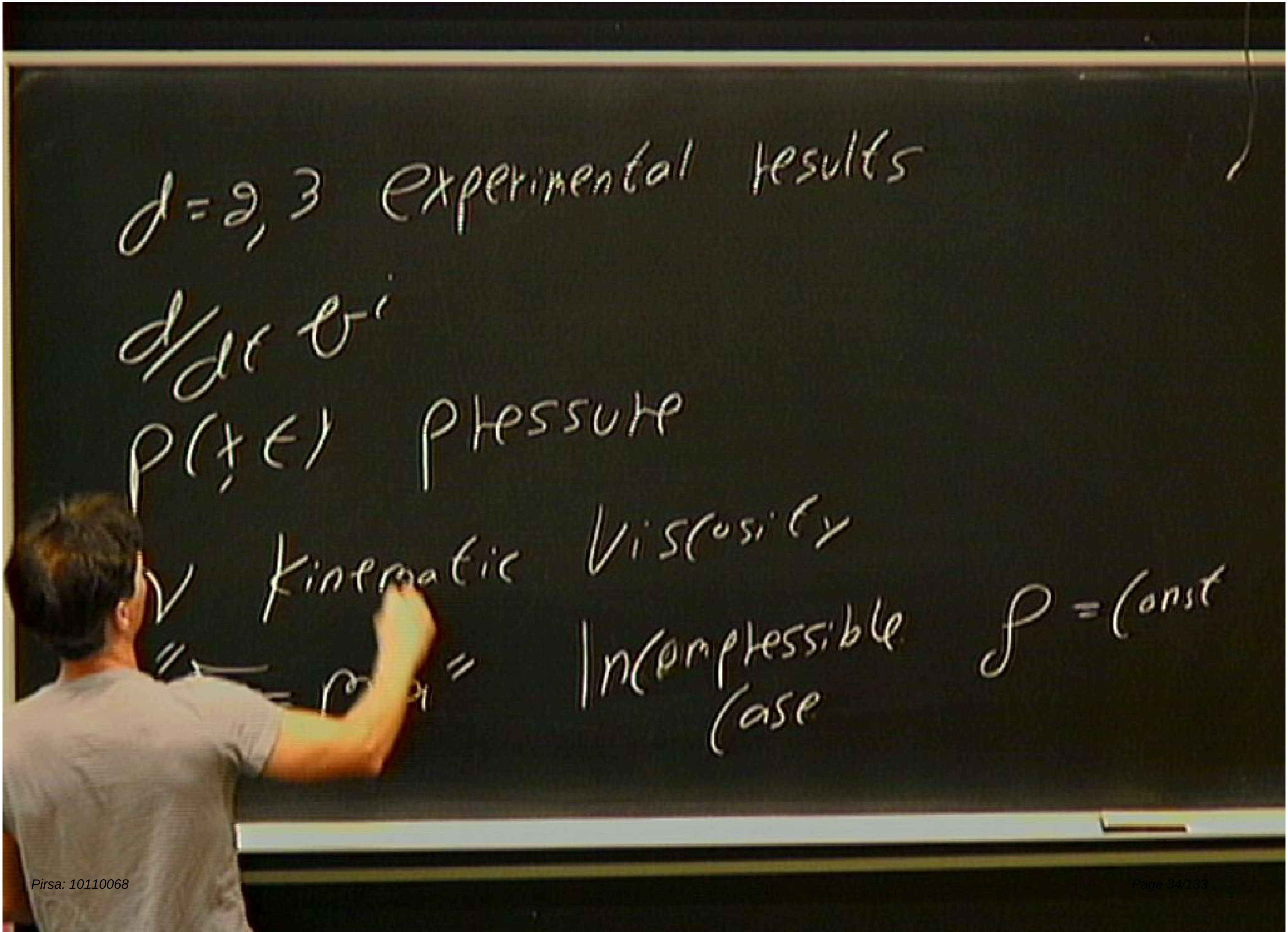
$v^i(\underline{x}, t) \quad i = 1 \ldots d$

velocity


Membrane Paradigm
Damour
Gauss-Codazzi ⟷ Fluid dynamics eq
$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i$
$\partial_i v^i = 0$
$v^i(t,x)$ $i = 1 \ldots d$
velocity



$d/dt\, v^i$
$p(t,x)$ pressure
$\nu$ Kinematic Viscosity
"F = ma"
Incompressible case
$\rho =$


d=2,3 experimental results
d/dt v^i
p(x,t) pressure
ν kinematic viscosity
Incompressible case
ρ = const

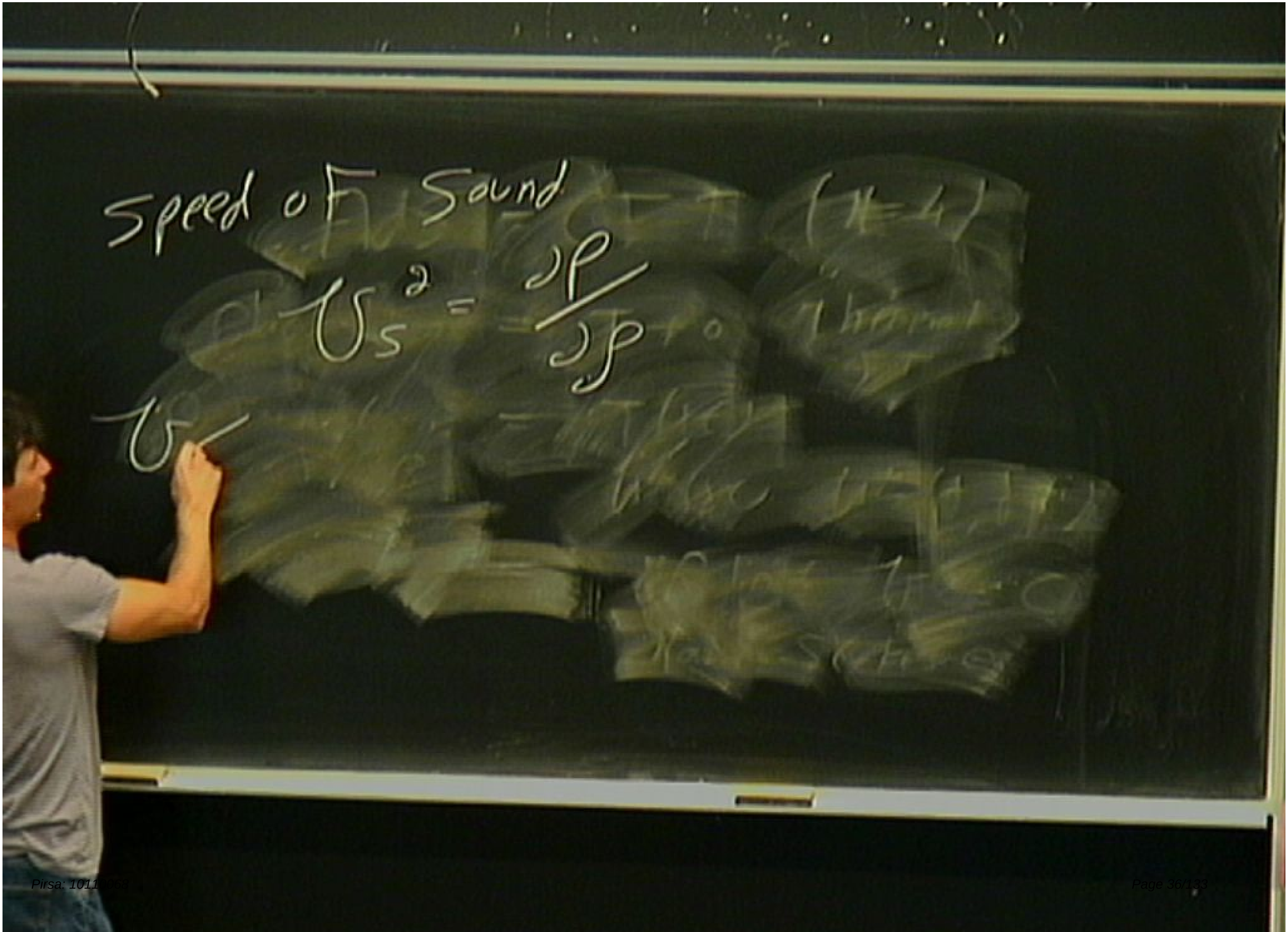
$d = 2, 3$ experimental results

$d/dr (\beta e^{-i}) = \ldots$

$p(\vec{x}, t)$ pressure

$\nu$ kinematic viscosity

"$F = ma$" Incompressible case $\rho = $ const


Speed of Sound
$v_s^2 = \partial p / \partial \rho$
$v_s$

Speed of Sound

$$v_s^2 = \frac{\partial P}{\partial \rho}$$

$$v \ll v_s \leq c$$

Incomples. Fluids are NR

Membrane Paradigm

Damour

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$$\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{cases}$$

$v^i(t,x)$ $i = 1 \ldots d$

velocity

Membrane Paradigm

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Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq.

$$\begin{cases} \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{cases}$$

$v^i(t,x)$ $i = 1 \ldots d$

velocity

$(\partial_t \rho + \partial_i v^i = 0$

Membrane Paradigm

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$$\begin{cases} \partial_t \underbrace{v^i} + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \\ \partial_i v^i = 0 \end{cases}$$

$v^i(t, x)$ $i = 1 \dots d$

velocity

$(\partial_t \rho + \partial_i v^i = 0)$

Membrane Paradigm

Damour

Gauss-Codazzi $\longleftrightarrow$ Fluid dynamics eq.

$$\begin{cases} \partial_t \underbrace{v^i + v^j \partial_j v^i} = -\partial^i p + \nu \partial_j \partial^j v^i \\ \qquad\qquad\qquad\qquad + F^i \\ \partial_i v^i = 0 \end{cases}$$

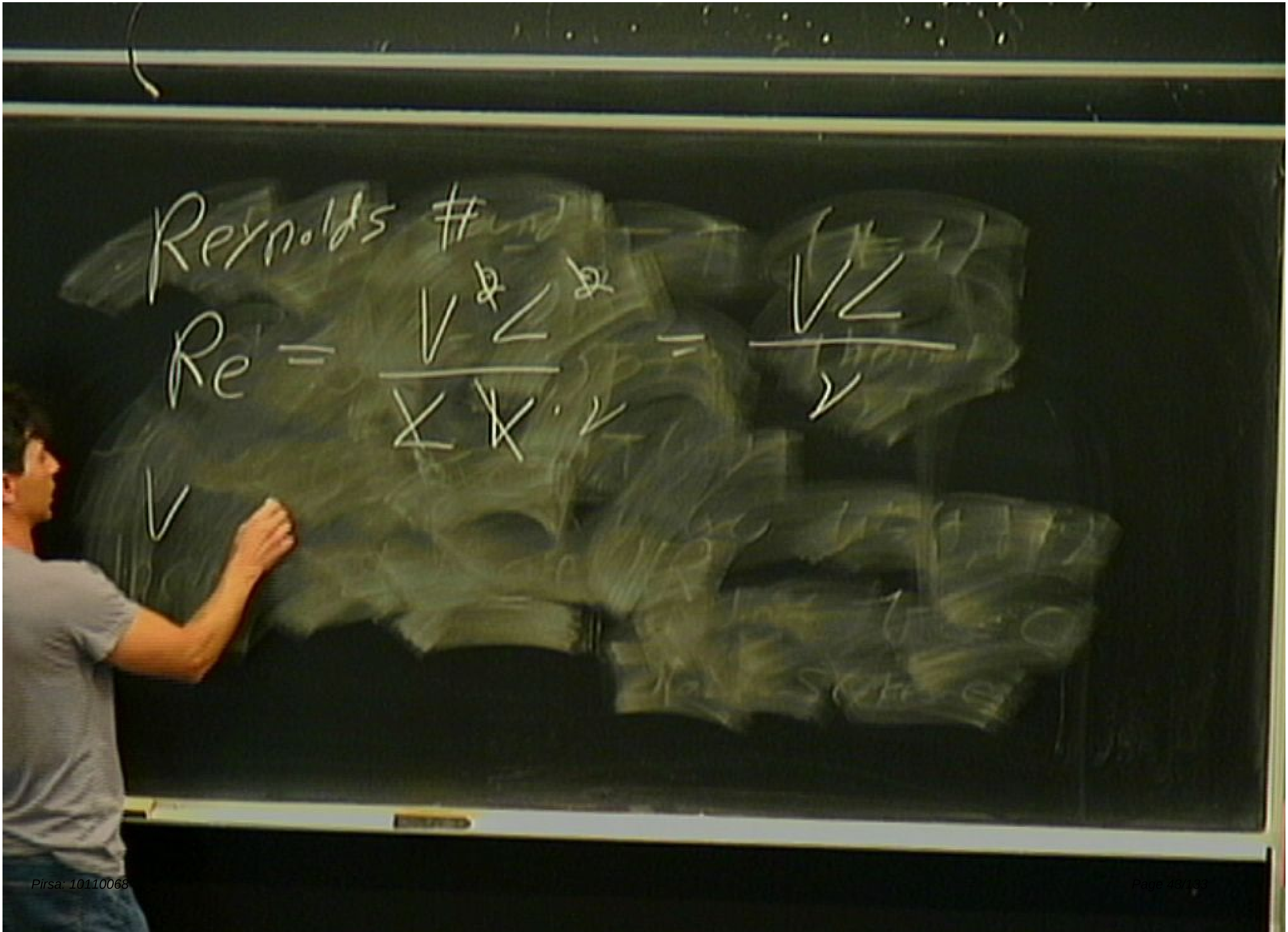
$v^i(t,x) \quad i = 1 \ldots d$

velocity

$(\partial_t \rho + \partial_i v^i = 0)$

Reynolds #

$$Re = \frac{V^2 L^2}{\cancel{L}\,\cancel{V}\cdot \nu} = \frac{VL}{\nu}$$

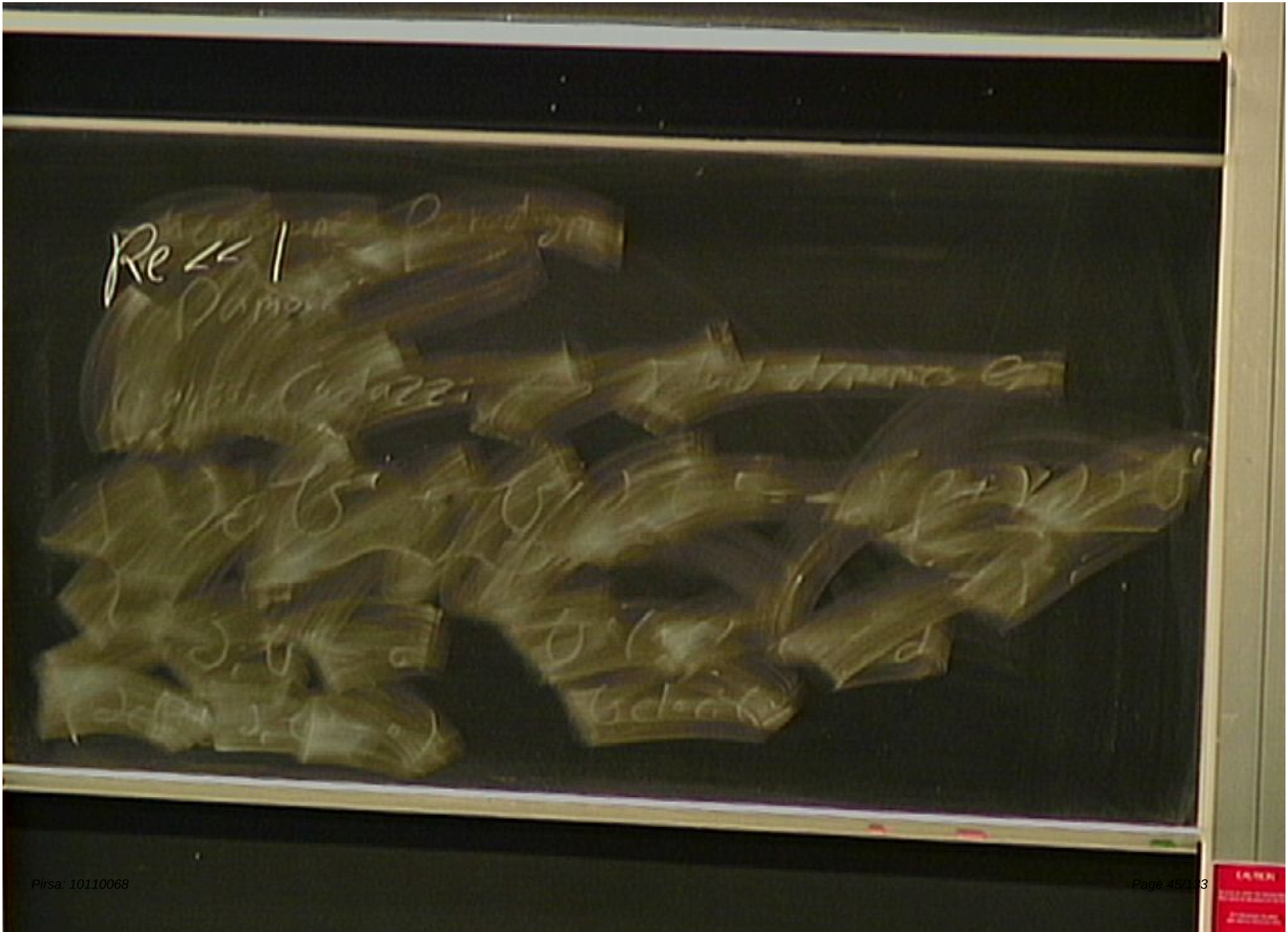

Reynolds #
Re = V² L² / L ν L = VL / ν
V

Reynolds #

$$Re = \frac{V^{2} L^{2}}{L \nu \cdot L} = \frac{VL}{\nu}$$

$V$ characteristic Velocity

$L$ = Length


$Re \ll 1$

$Re \ll 1$ Laminar

$1 < Re \lesssim 100$

$Re \ll 1$ Laminar

$1 < Re \lesssim 100$ Chaotic

$Re \simeq 10^3$ Turbulence

$Re \ll 1$ Laminar

$1 < Re \lesssim 100$ Chaotic

$Re \gtrsim 10^3$ Turbulence ←

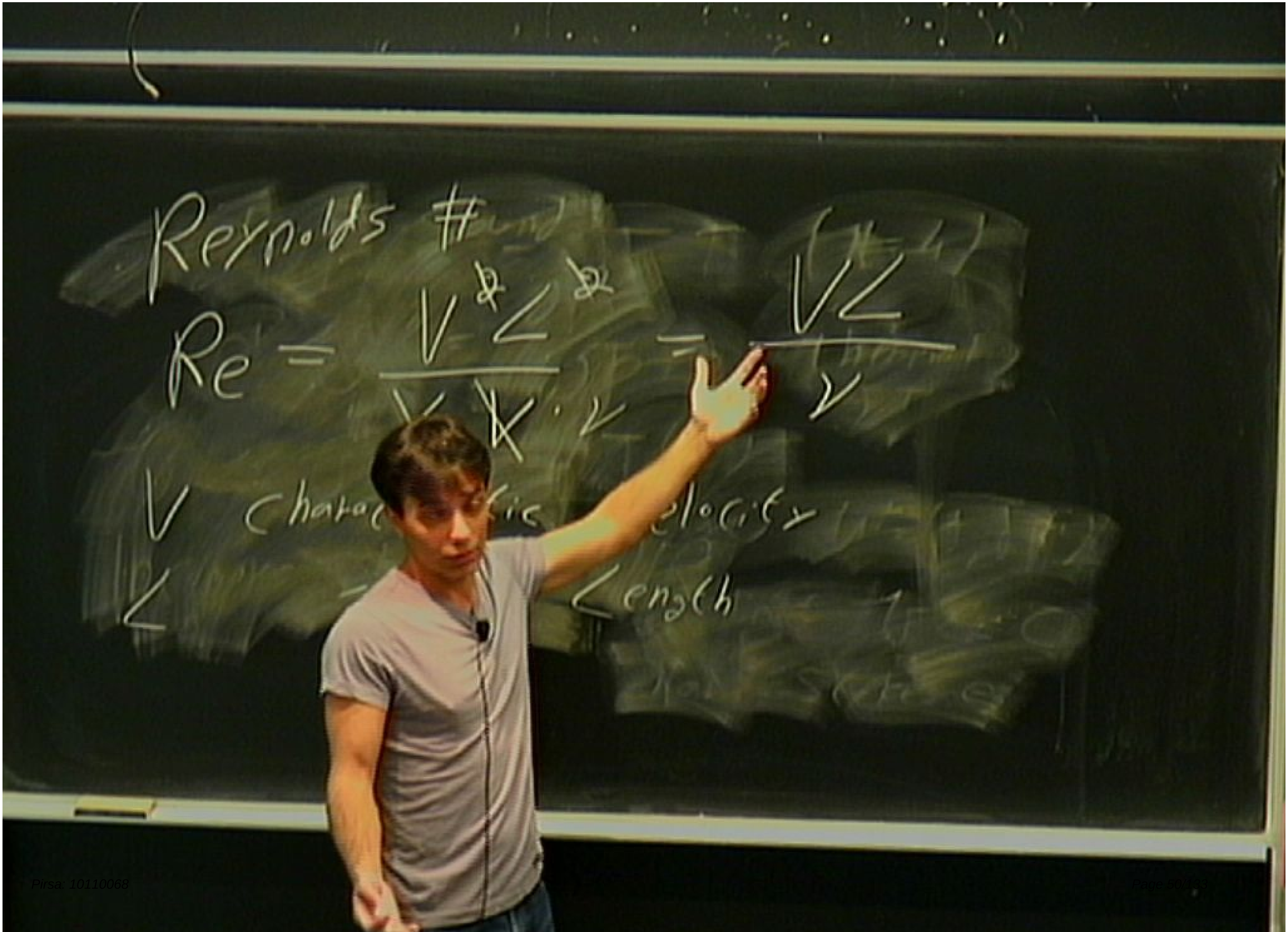
$Re \ll 1$ Laminar

$1 < Re \lesssim 100$ Chaotic

$Re \simeq 10^3$ Turbulence ←

$\nu_{water} = 10^{-6}\ m^2/sec$

$\nu_{air} \simeq 1.5 \times 10^{-5} =$


Reynolds #
Re = V^2 L^2 / V ν L = VL / ν
Length

$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

$Re \simeq 10^3$ Turbulence $\leftarrow$

$\nu_{water} = 10^{-6}\ m^2/sec$ River $\simeq \frac{10^{1}}{10^{-6}} \sim 10^7$

$\nu_{air} \simeq 1.5 \times 10^{-5} =$

$Re \ll 1$ Laminar

$1 < Re \leq 100$ Chaotic

$Re \simeq 10^3$ Turbulence $\longleftarrow$

$\nu_{water} = 10^{-6}\ m^2/sec$ River $\simeq \frac{10}{10^{-6}} \sim 10^7$

$\nu_{air} \simeq 1.5 \times 10^{-5} =$

$Re \ll 1$ Laminar

$1 < Re \lesssim 100$ Chaotic

$Re \simeq 10^3$ Turbulence $\longleftarrow$ Generic

$\nu_{water} = 10^{-6}\ m^2/sec$ River $\simeq \frac{10}{10^{-6}} \sim 10^7$

$\nu_{air} \simeq 1.5 \times 10^{-5} =$

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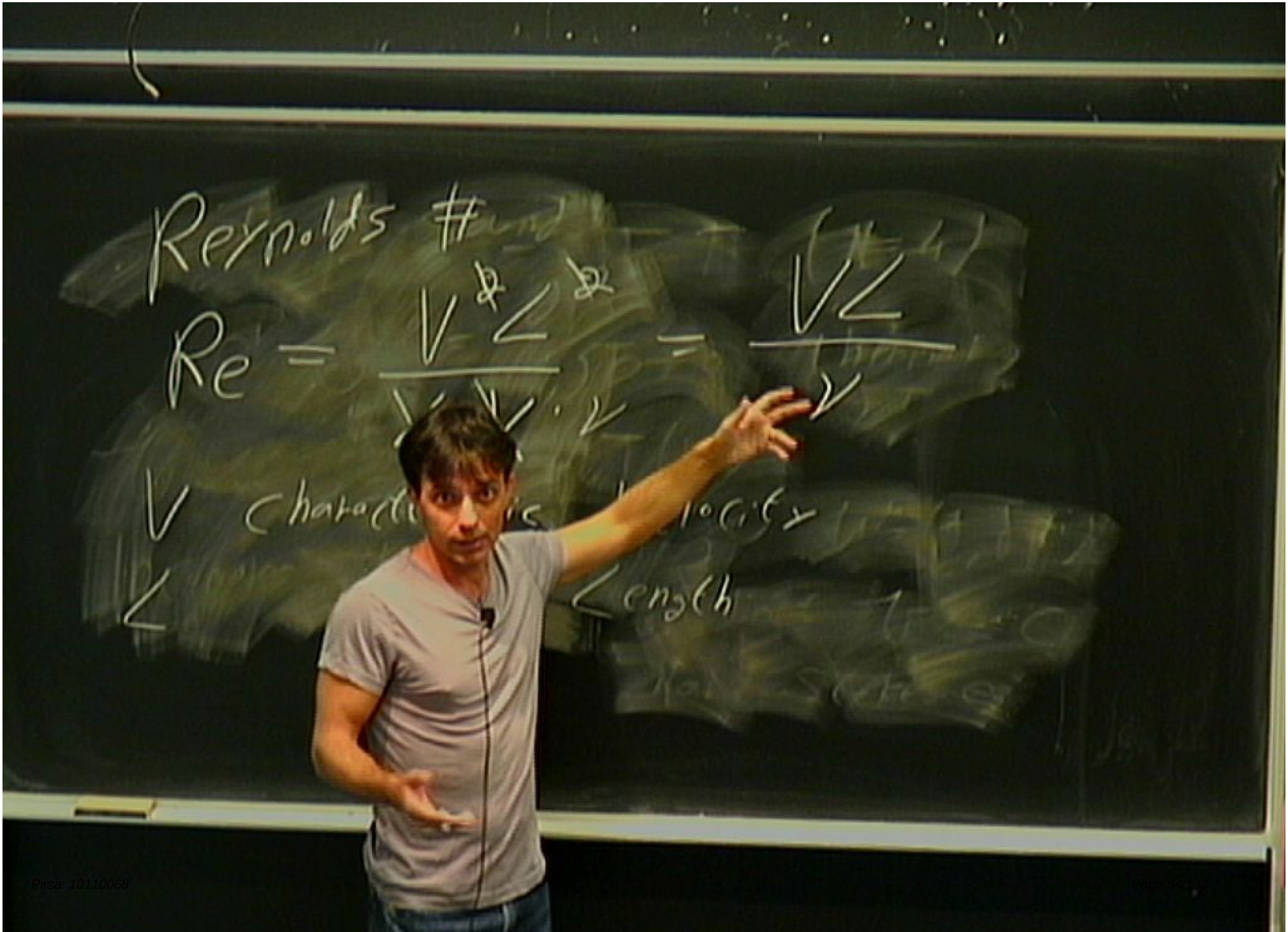
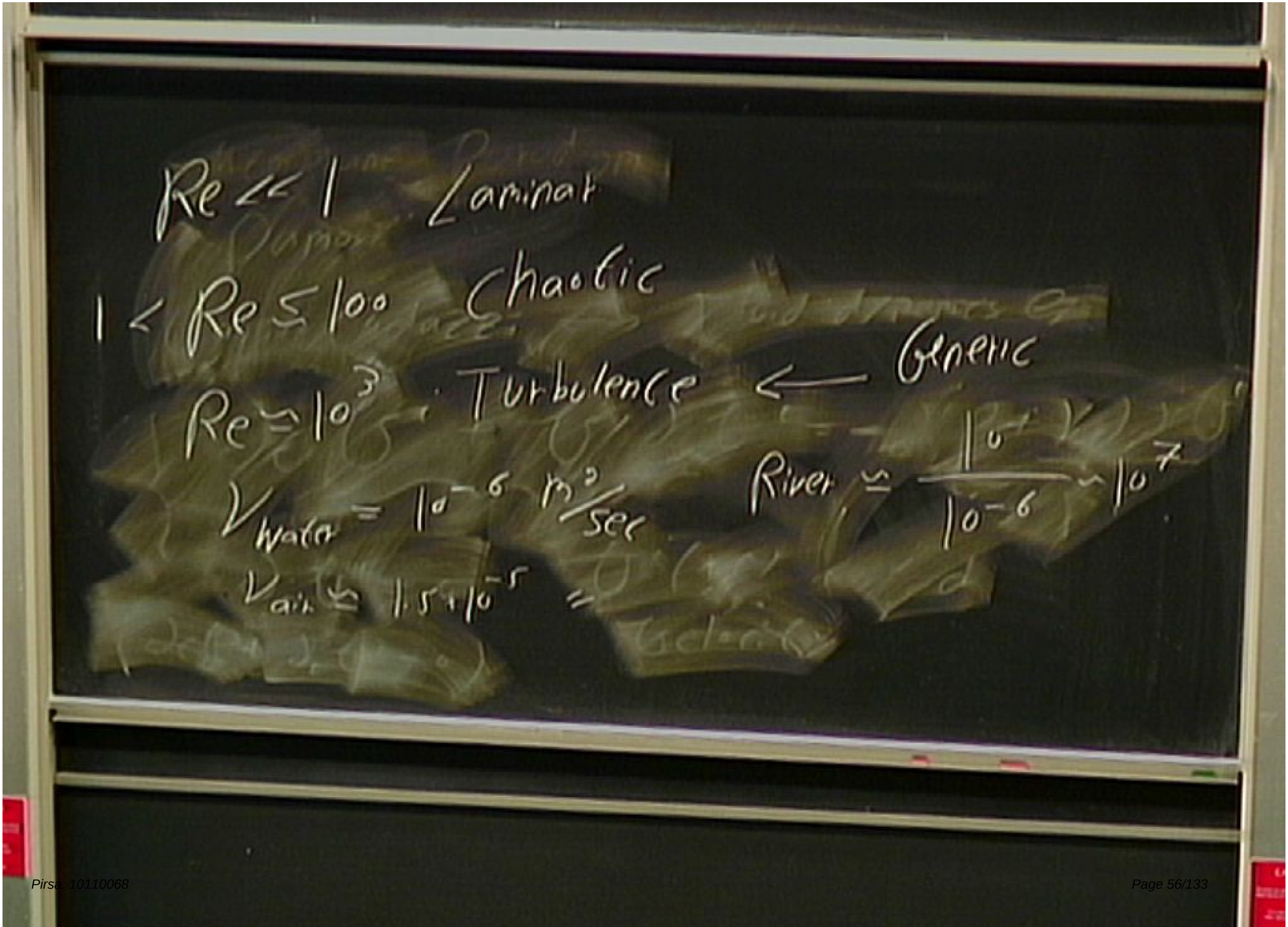

Reynolds #
Length

Reynolds #

$$Re = \frac{V^2 L^2}{L V \cdot \nu L} = \frac{VL}{\nu}$$

$V$ characteristic Velocity

$L$ = Length


$Re \ll 1$ Laminar
$1 < Re \lesssim 100$ Chaotic
$Re \simeq 10^3$ Turbulence ← Generic
$\nu_{water} = 10^{-6}\ m^2/sec$
River $\simeq \frac{10^{1}}{10^{-6}} \sim 10^7$
$\nu_{air} \simeq 1.5 \times 10^{-5}$

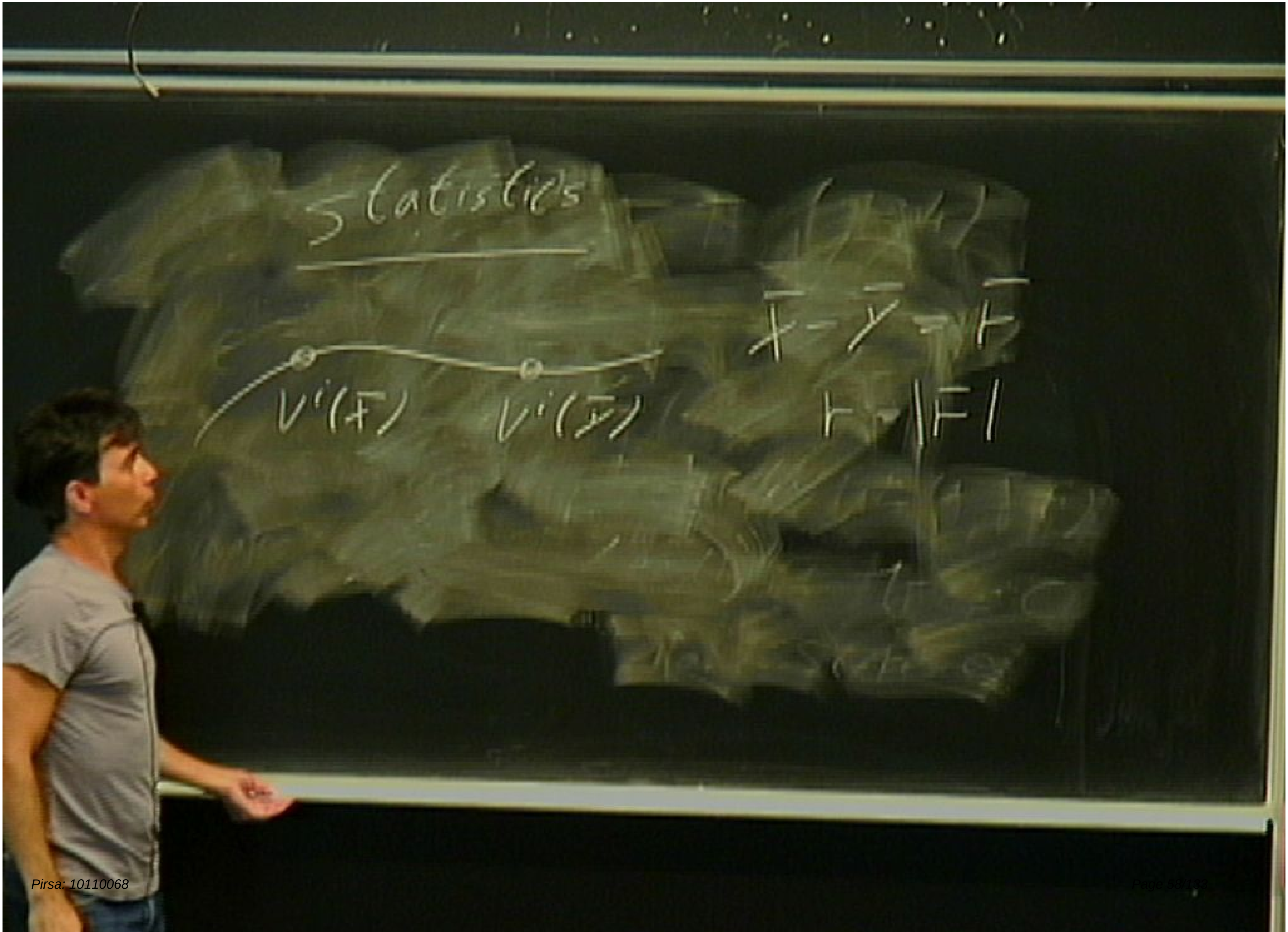
$Re \ll 1$ Laminar

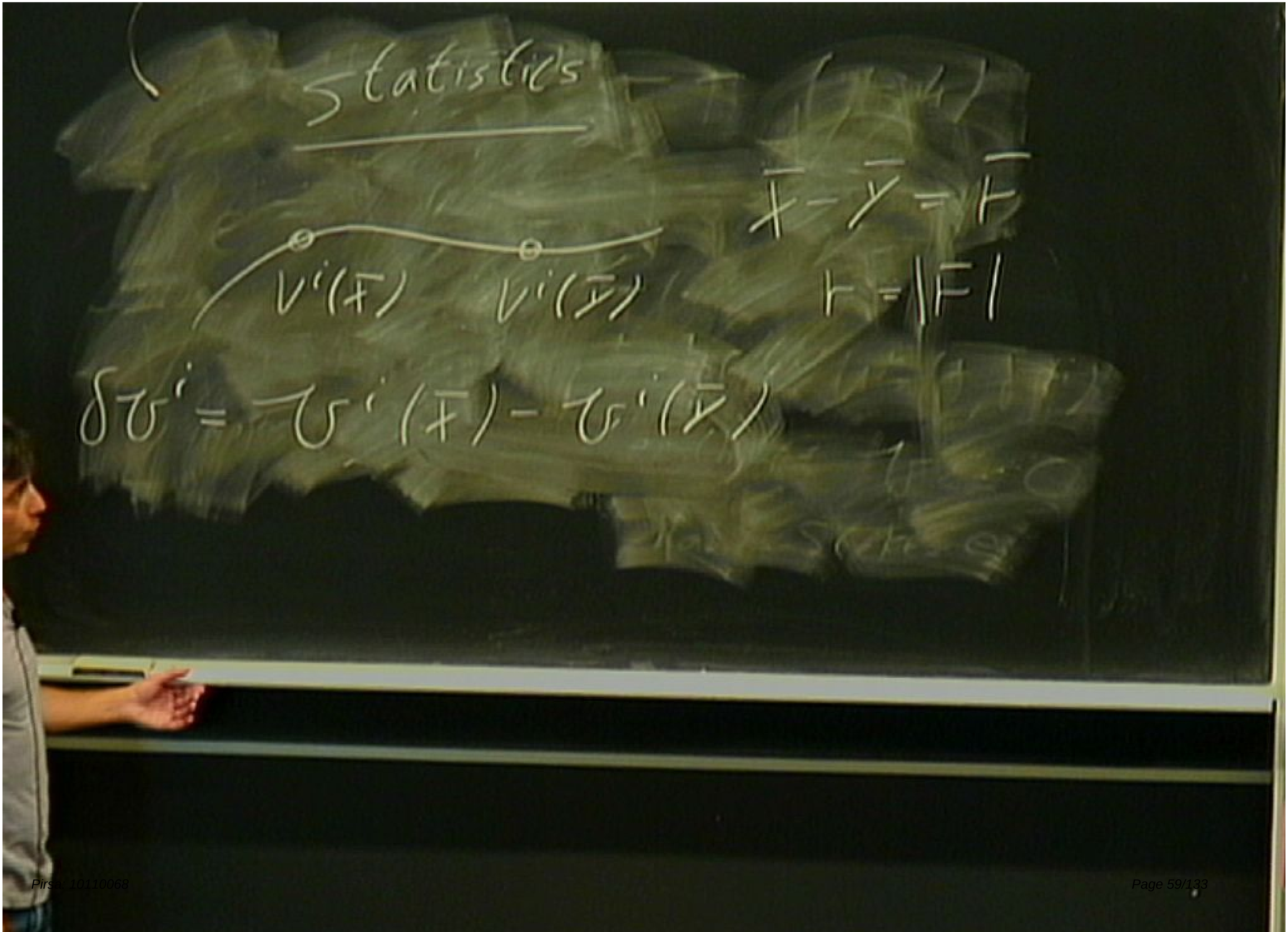
$1 < Re \lesssim 100$ Chaotic

$Re \simeq 10^3$ Turbulence $\longleftarrow$ Generic

$\nu_{water} = 10^{-6}\ m^2/sec$ River $\simeq \frac{10^{1}}{10^{-6}} \sim 10^7$

$\nu_{air} \simeq 1.5 \times 10^{-5} =$


statistics


Statistics
$\bar{X} - \bar{Y} = \bar{F}$
$v^i(X)$ $v^i(Y)$
$\delta v^i = v^i(X) - v^i(Y)$

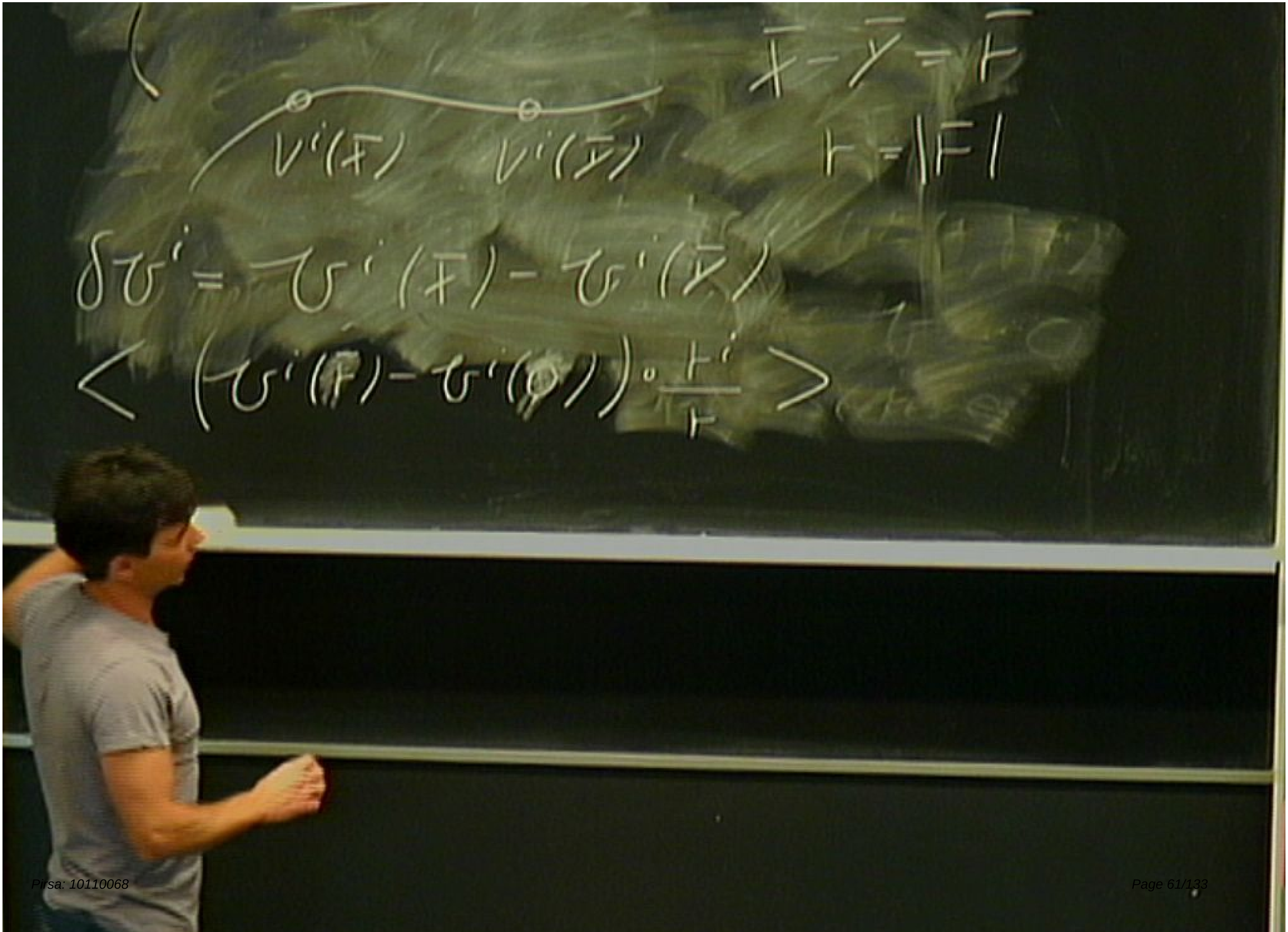
Statist...

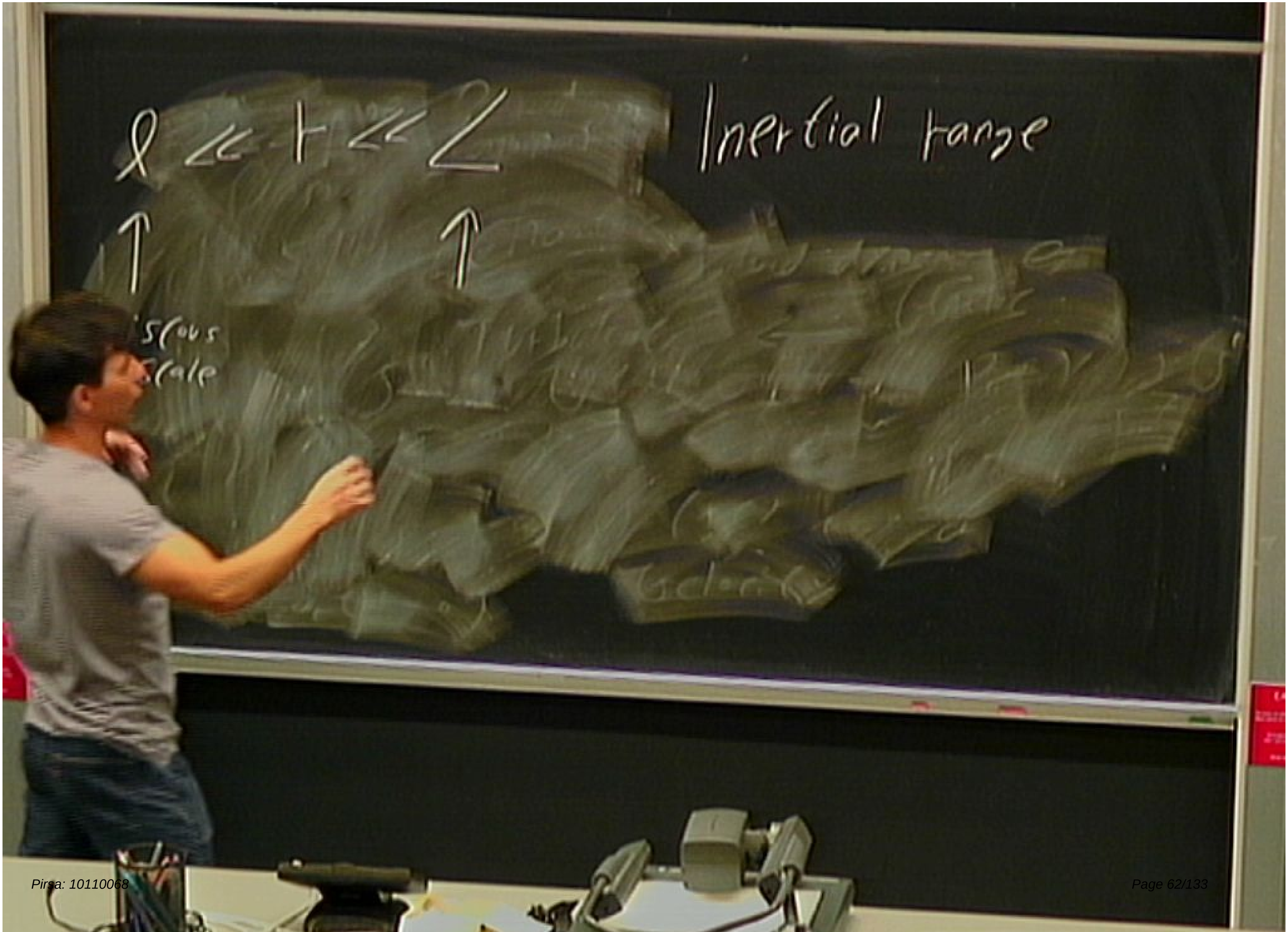
$$\bar{x} - \bar{y} = \bar{r}$$

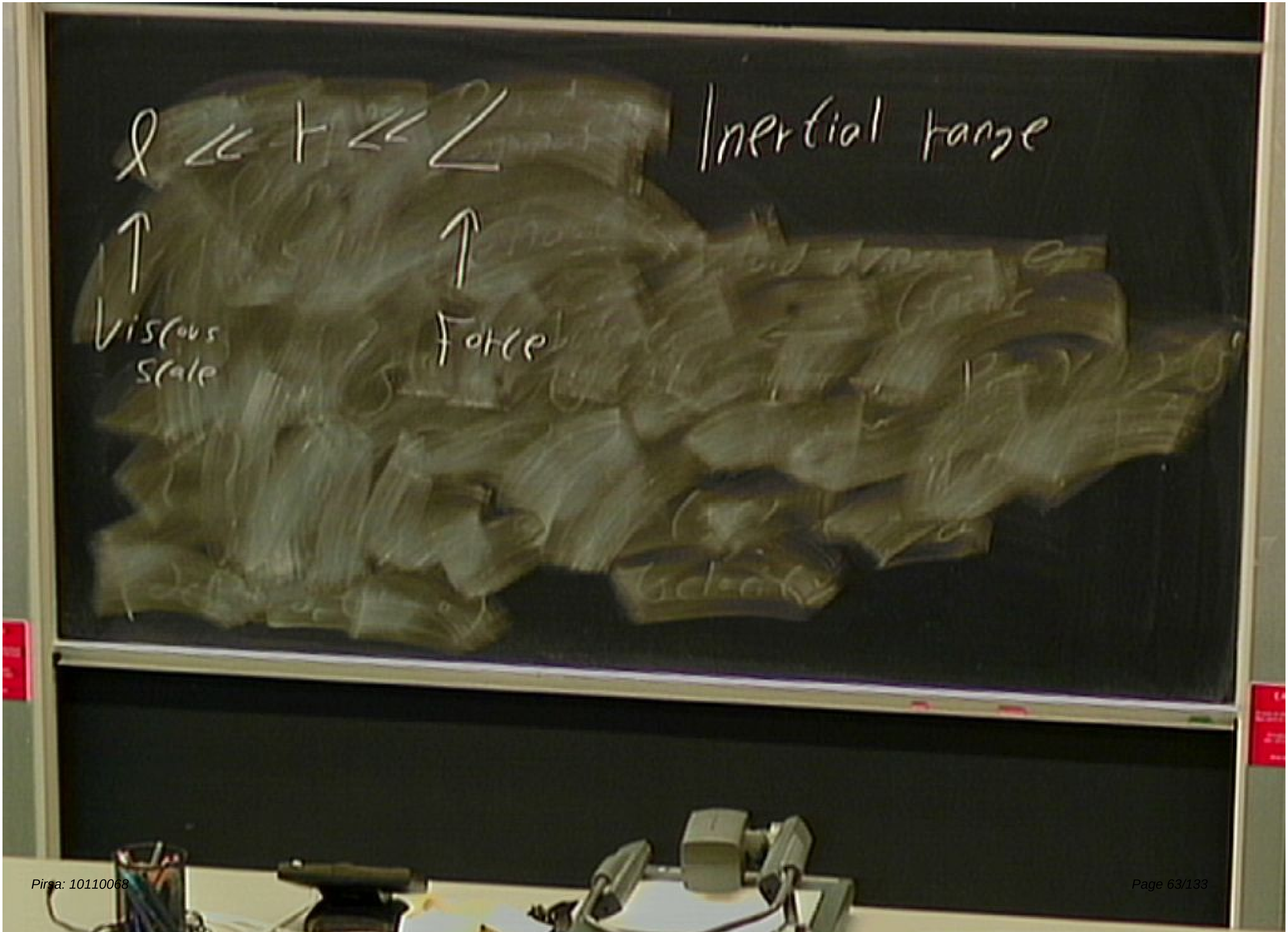
$v^i(\bar{x})$ $\quad v^i(\bar{y})$ $\quad r = |\bar{r}|$

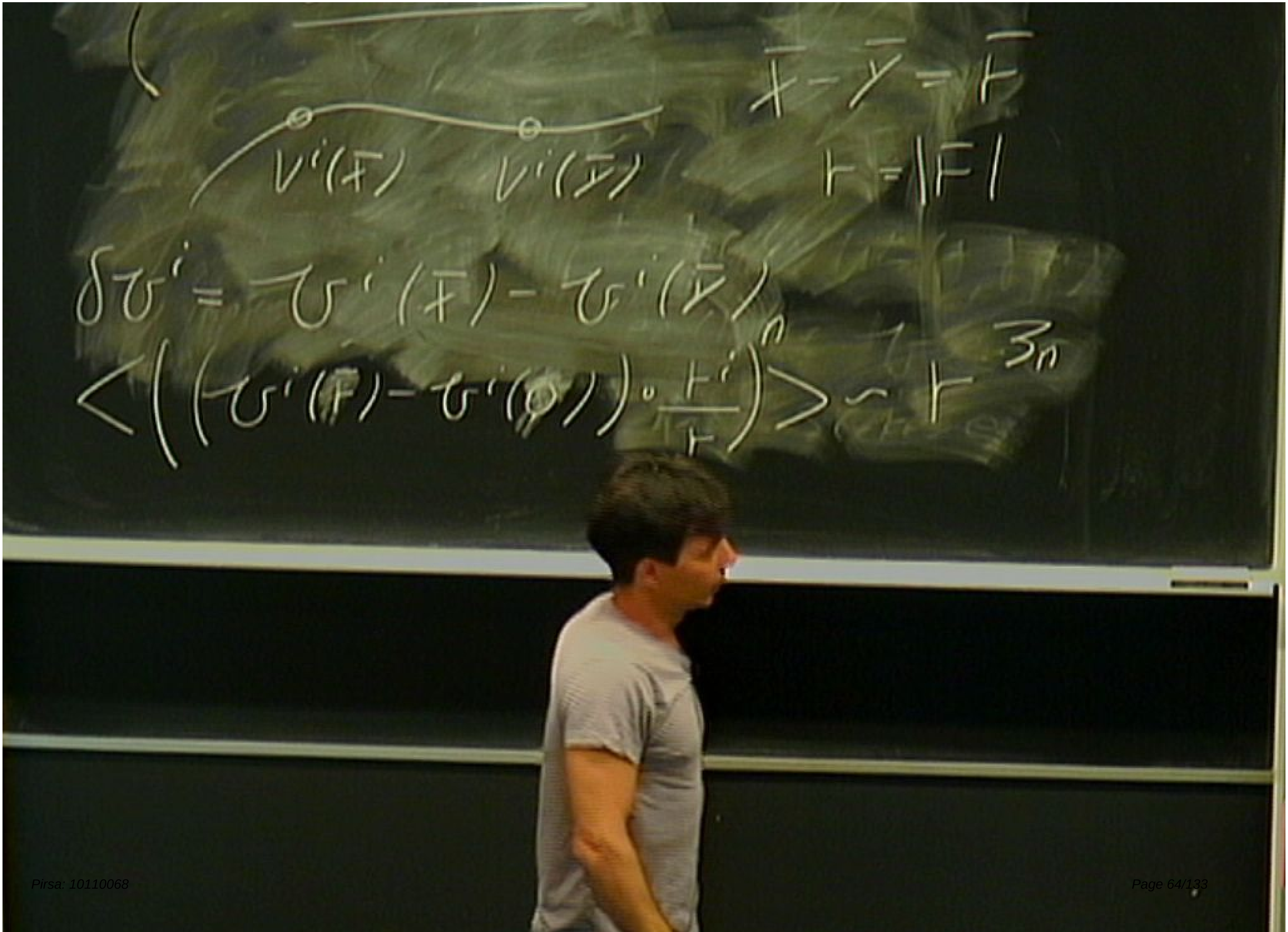
$$\delta v^i = v^i(\bar{x}) - v^i(\bar{y})$$

$$\left(v^i(\bar{x}) - v^i(\bar{y})\right) \cdot \frac{r^i}{r}$$


$\bar{x} - \bar{y} = \bar{r}$
$v^i(\bar{x})$ $v^i(\bar{y})$
$r = |\bar{r}|$
$\delta v^i = v^i(\bar{x}) - v^i(\bar{y})$
$\langle (v^i(\bar{r}) - v^i(0)) \cdot \frac{r^i}{r} \rangle$


$\ell \ll r \ll L$
Inertial range


ℓ << r << L
Inertial range
Viscous scale
Force


x̄ − ȳ = r̄
v^i(x̄)
v^i(ȳ)
r = |r̄|
δv^i = v^i(x̄) − v^i(ȳ)
<((v^i(r̄) − v^i(0)) · r^i/r)^n> ~ r^{3n}

$\ell \ll r \ll L$ Inertial range

Viscous scale

Force

$3n$ real numbers

$$\bar{x} - \bar{y} = \bar{r}$$


$$r = |\bar{r}|$$

$$\delta v^i = v^i(\bar{x}) - v^i(\bar{y})$$

$$\left\langle \left( \left( v^i(\bar{x}) - v^i(\bar{y}) \right) \cdot \frac{r^i}{r} \right)^n \right\rangle \sim r^{\zeta_n}$$

Statistics

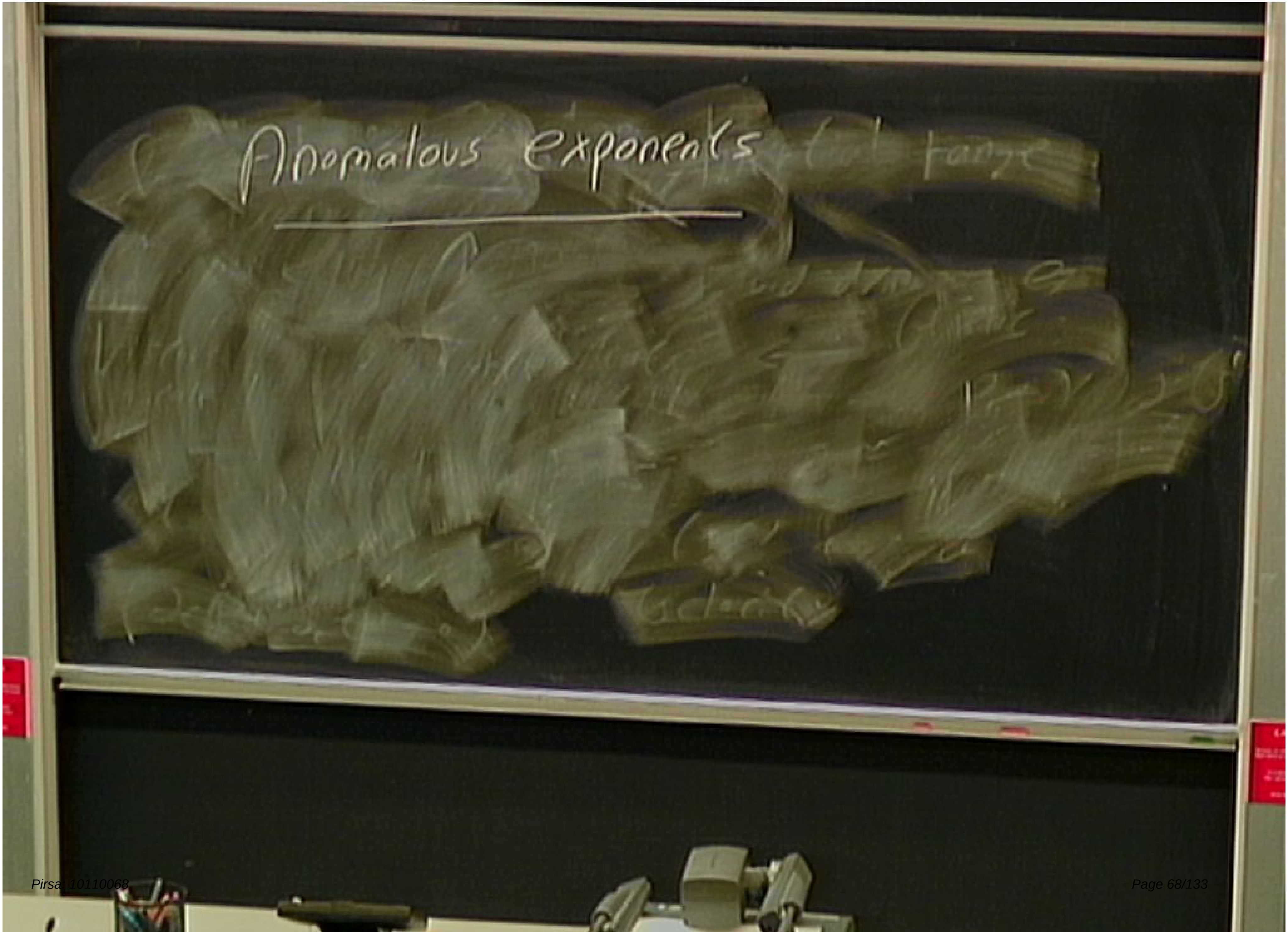
$$\vec{x} - \vec{y} = \vec{r}$$

$$r = |\vec{r}|$$

$v^i(\vec{x})$ $\quad$ $v^i(\vec{y})$

$$\delta v^i = v^i(\vec{x}) - v^i(\vec{y})$$

$$\left\langle \left( \left( v^i(\vec{r}) - v^i(0) \right) \cdot \frac{r^i}{r} \right)^n \right\rangle \sim r^{3n}$$


Anomalous exponents

# Statistics

$$\bar{x} - \bar{y} = \bar{r}$$

$$r = |\bar{r}|$$

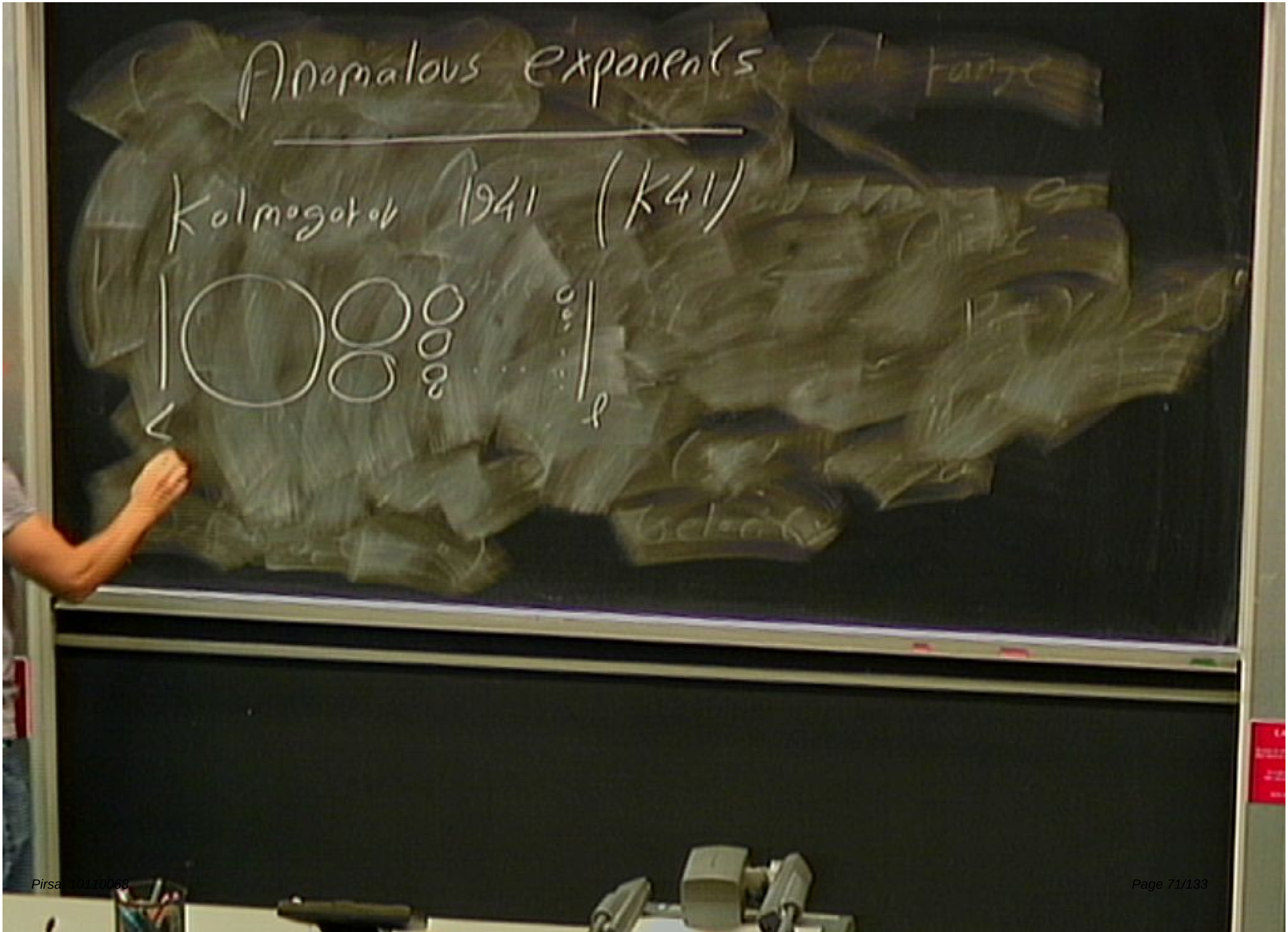
$$v^i(\bar{x}) \qquad v^i(\bar{y})$$

$$\delta v^i = v^i(\bar{x}) - v^i(\bar{y})$$

$$\left\langle \left( \left( v^i(\bar{r}) - v^i(0) \right) \cdot \frac{r^i}{r} \right)^n \right\rangle \sim r^{3n}$$

Anomalous exponents

Kolmogorov 1941 (K41)


Anomalous exponents
Kolmogorov 1941 (K41)
L
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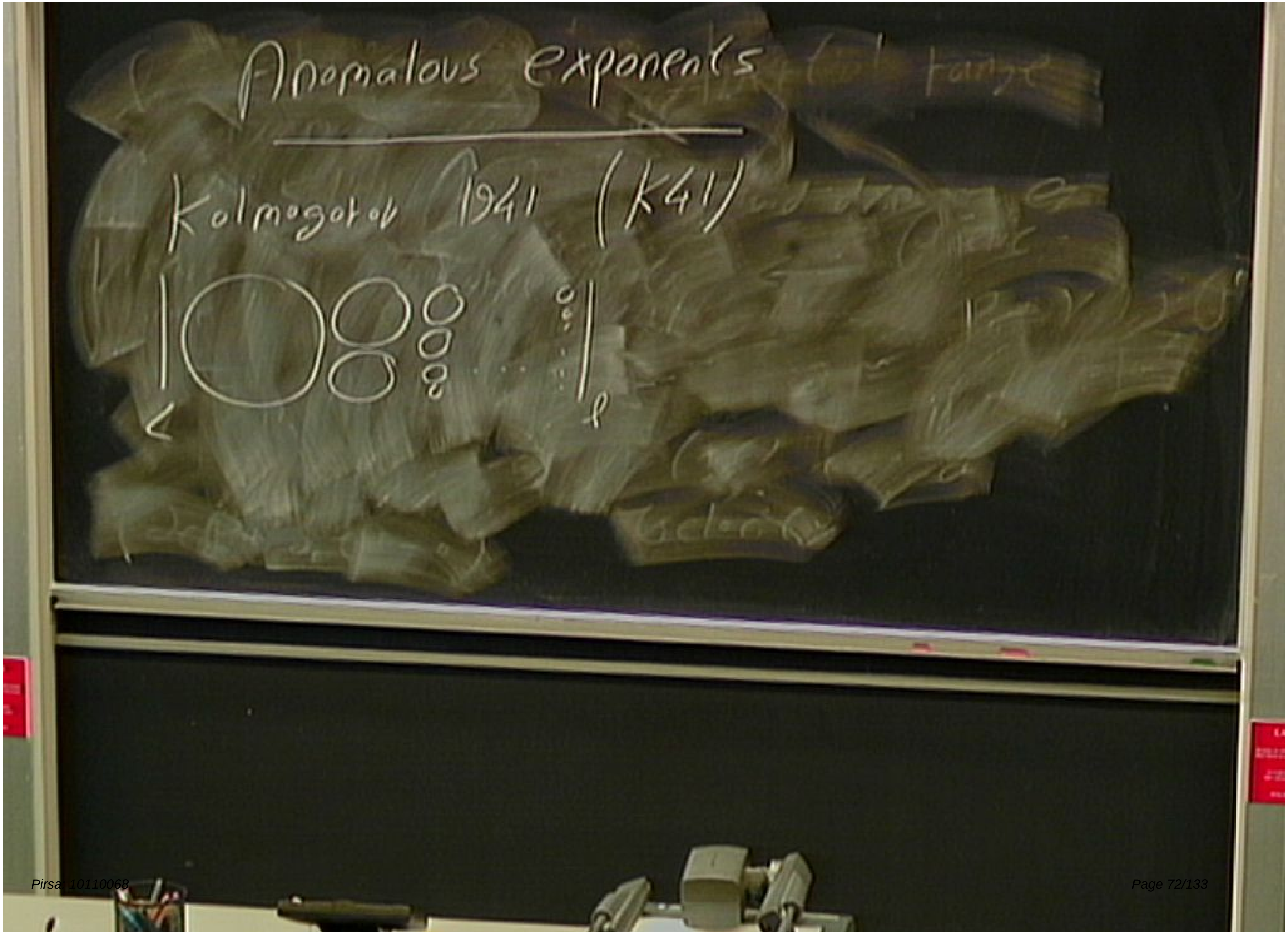
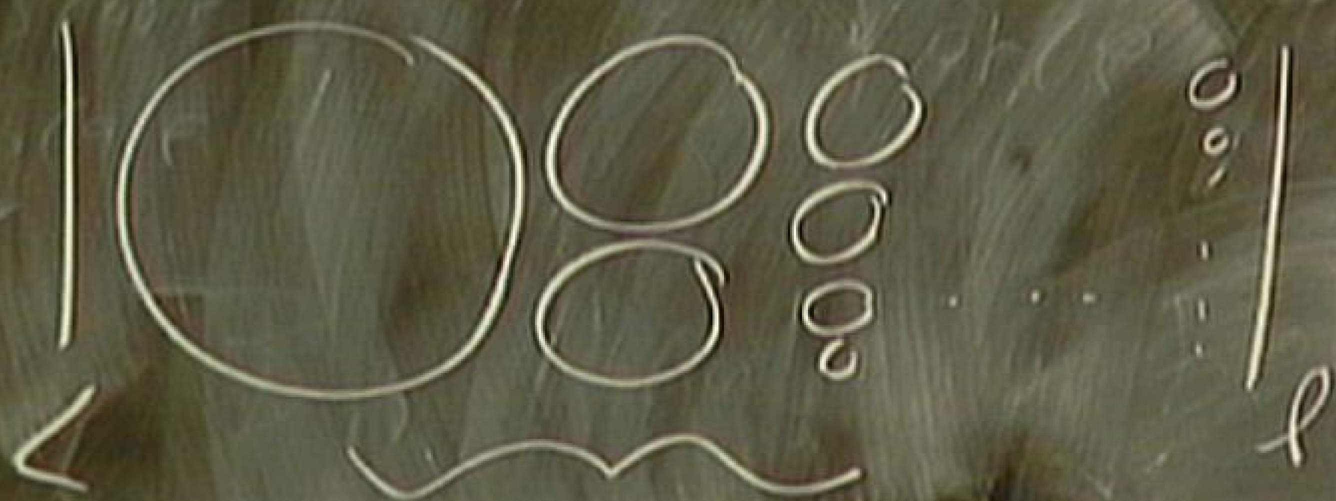

Anomalous exponents
Kolmogorov 1941 (K41)

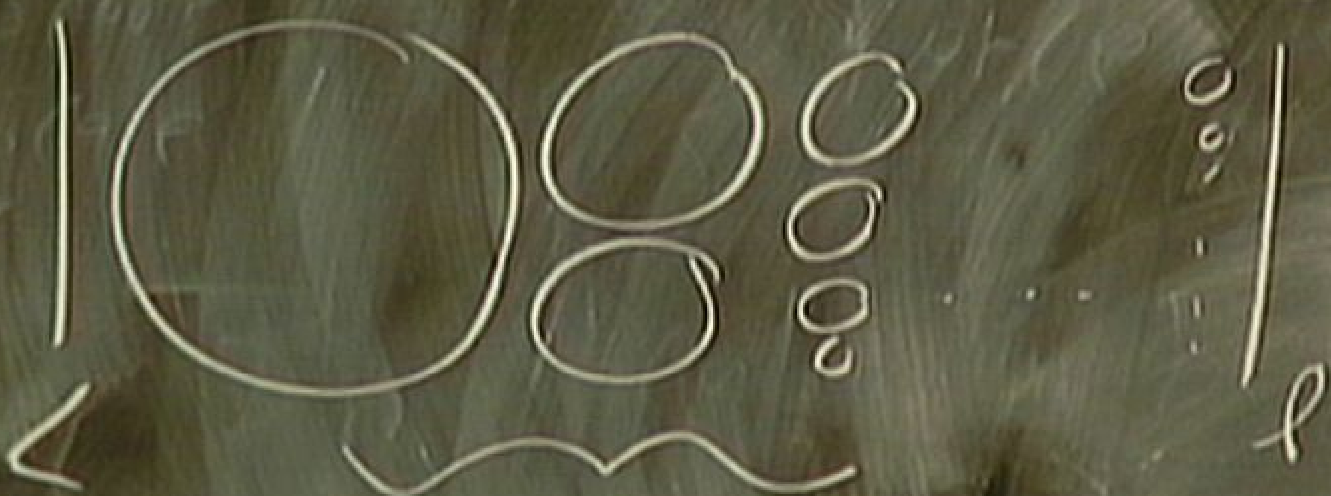
# Anomalous exponents

Kolmogorov 1941 (K41)



# Anomalous exponents

Kolmogorov 1941 (K41)



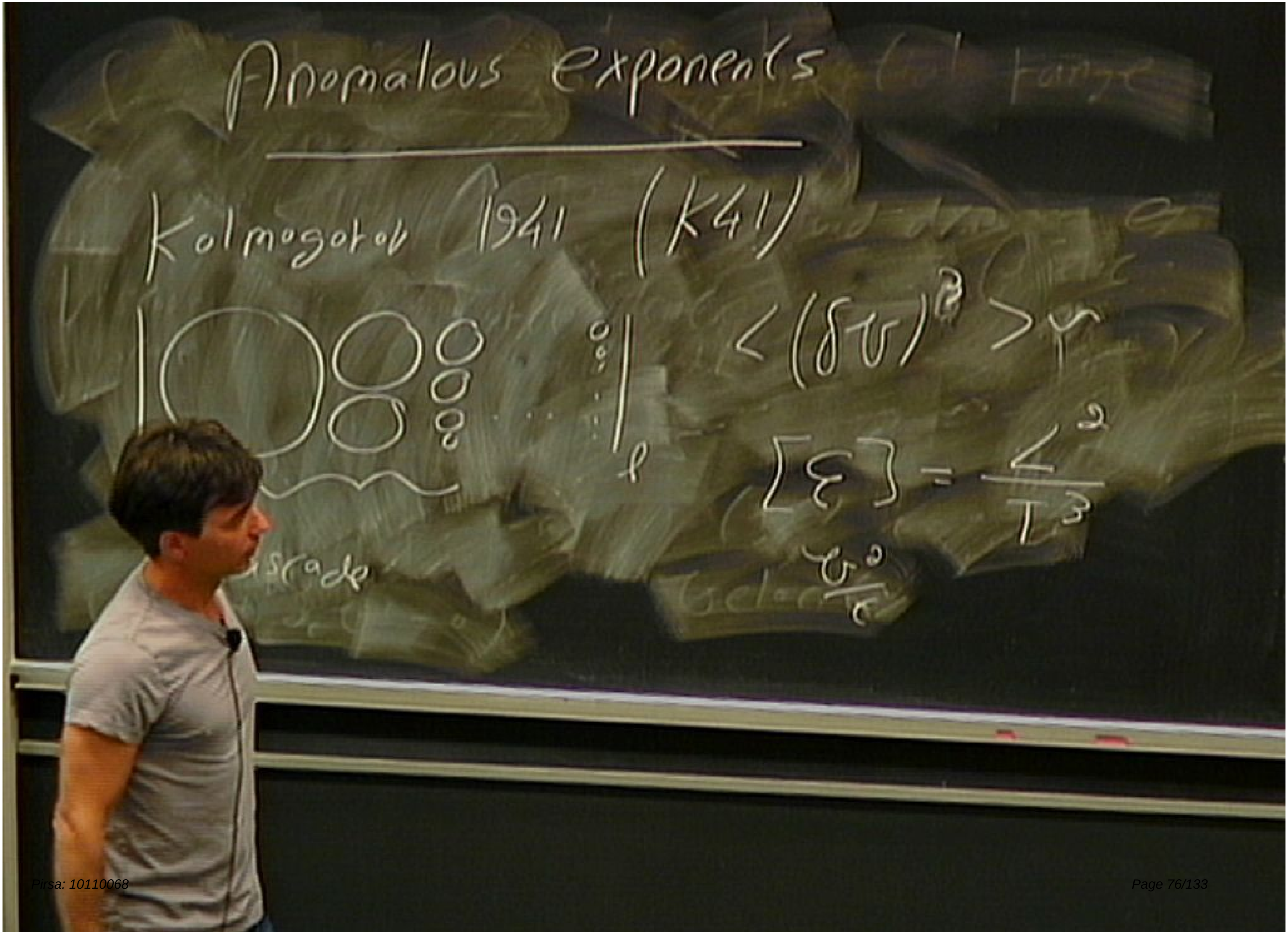

Cascade

# Anomalous exponents

Kolmogorov 1941 (K41)

$\langle (\delta v)^3 \rangle \sim r$

Cascade


Anomalous exponents
Kolmogorov 1941 (K41)
$\langle (\delta v)^n \rangle \sim$
$[\varepsilon] = \frac{L^2}{T^3}$

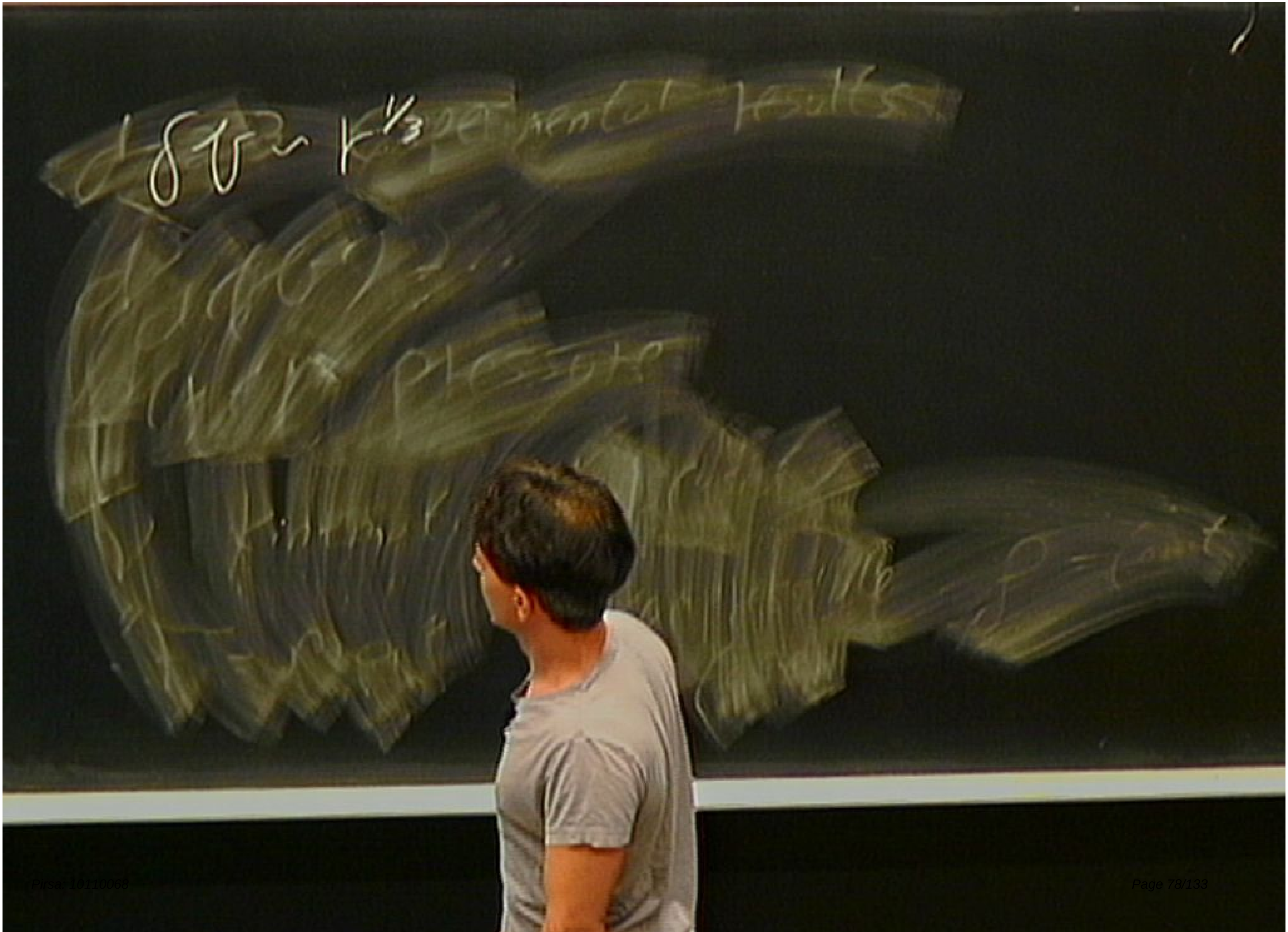
Kolmogorov 1941 (K41)


$$\langle (\delta v)^2 \rangle \sim \varepsilon \cdot t$$

$$[\varepsilon] = \frac{L^2}{T^3}$$

$$\sim \frac{v^2}{\varepsilon}$$

Cascade $\boxed{\zeta_n = \frac{n}{3}}$


$\delta v \sim k^{1/3}$

$$\delta v \sim r^{1/3}$$

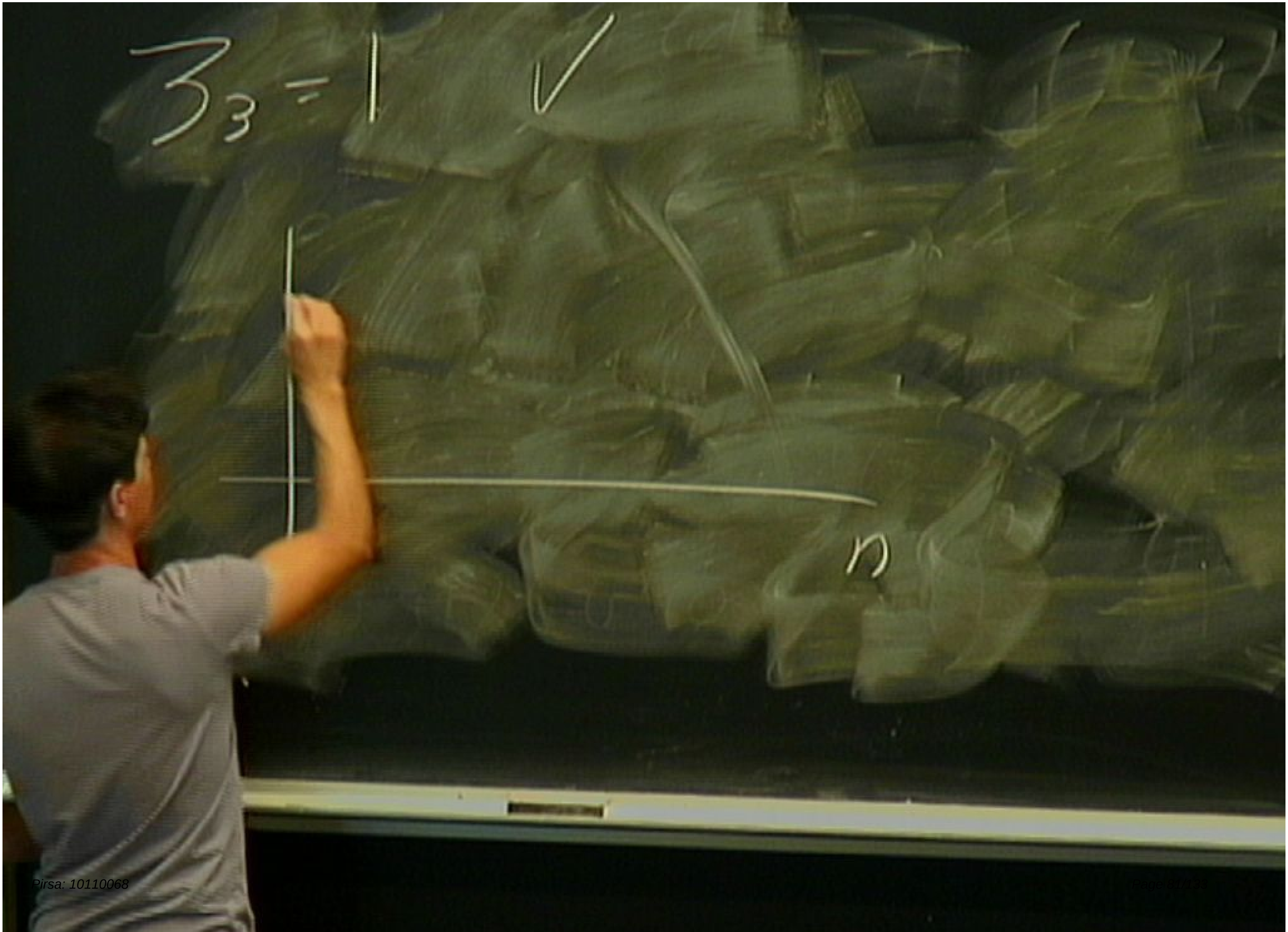
$$E \sim \langle (\delta v)^2 \rangle \sim r^{2/3}$$

$$E(k) \sim \int e^{ikr}\, r^{2/3}\, dr \sim k^{-5/3}$$

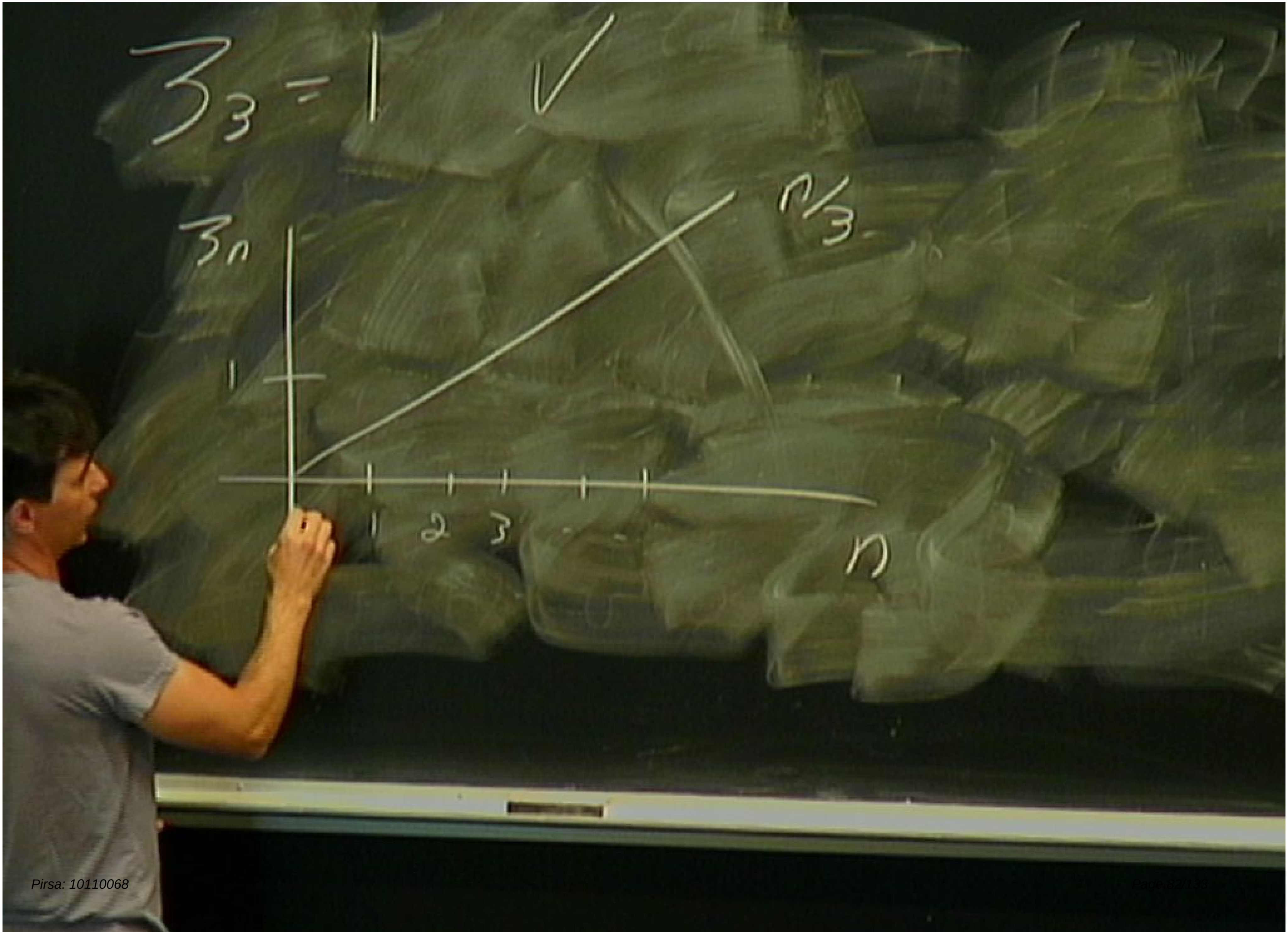
$$\delta v \sim r^{1/3}$$

$$E \sim \langle (\delta v)^2 \rangle \sim r^{2/3}$$

$$E(k) \sim \int e^{ikr} \, r^{2/3} \, dr \sim k^{-5/3}$$

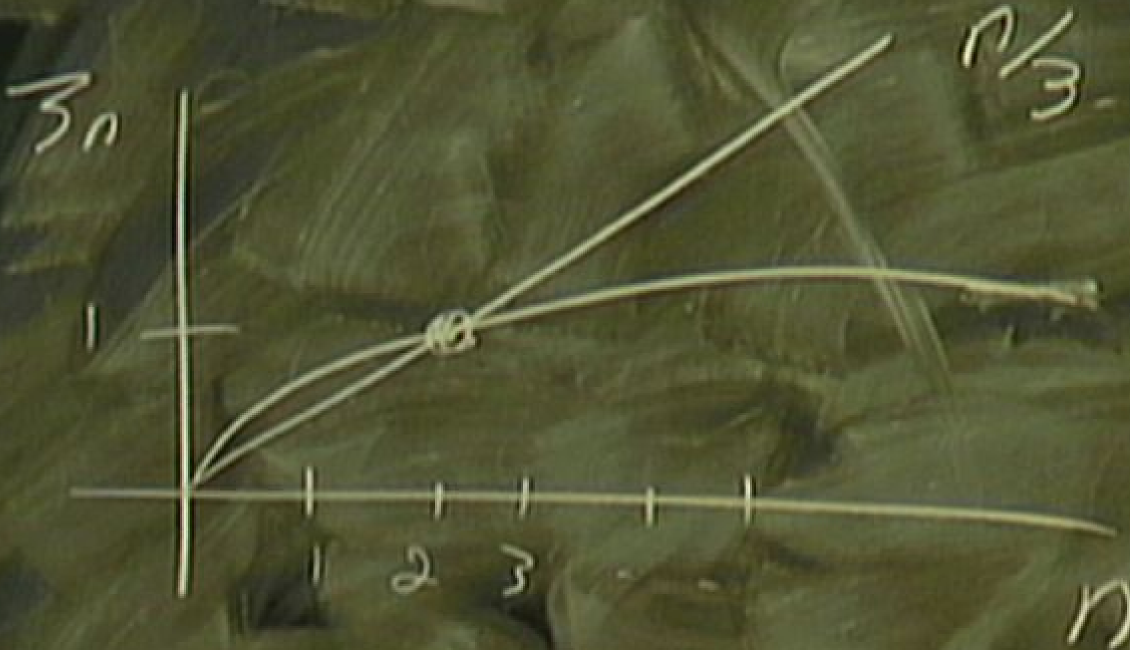


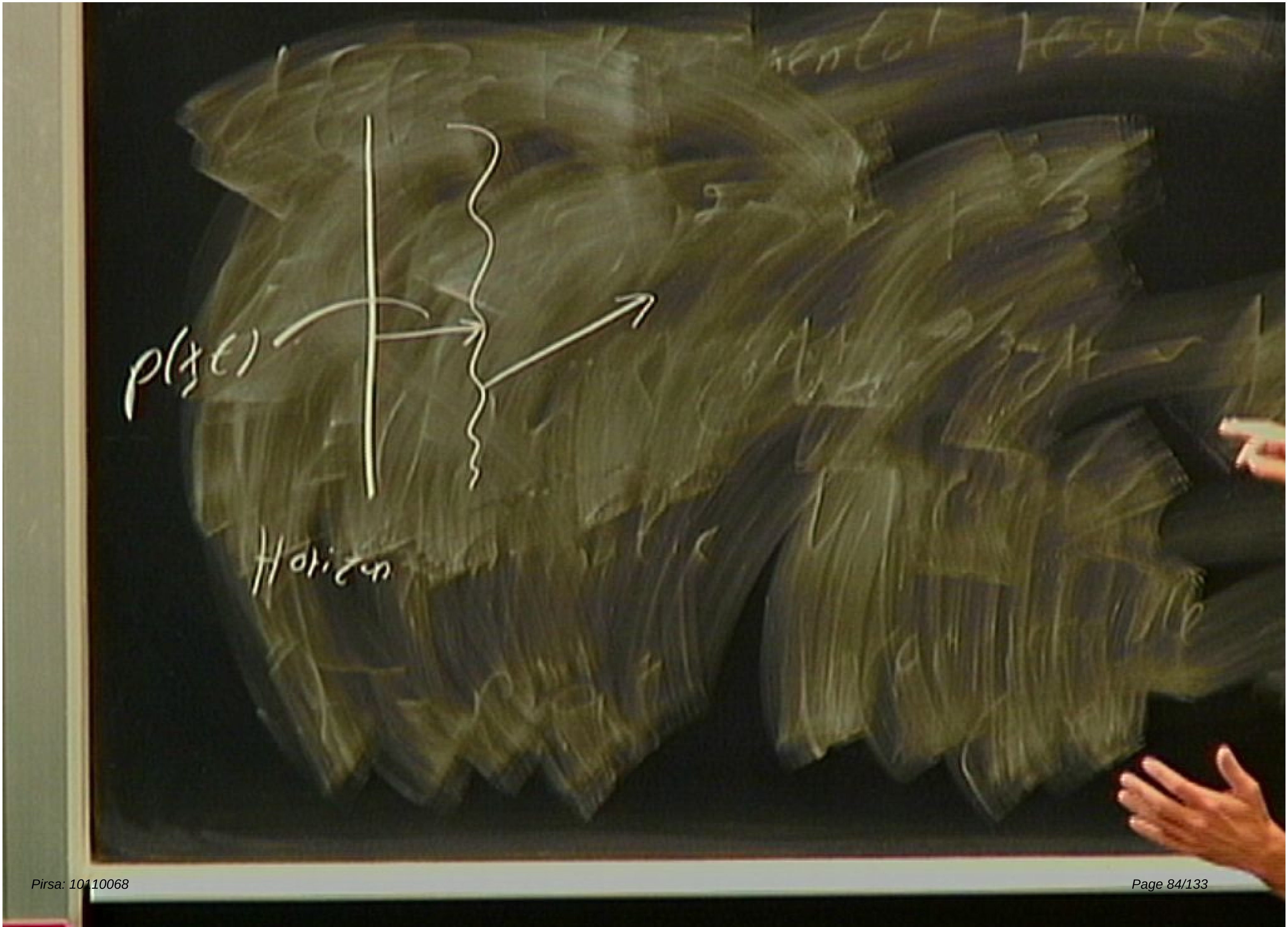
$Z_3 = 1$
n

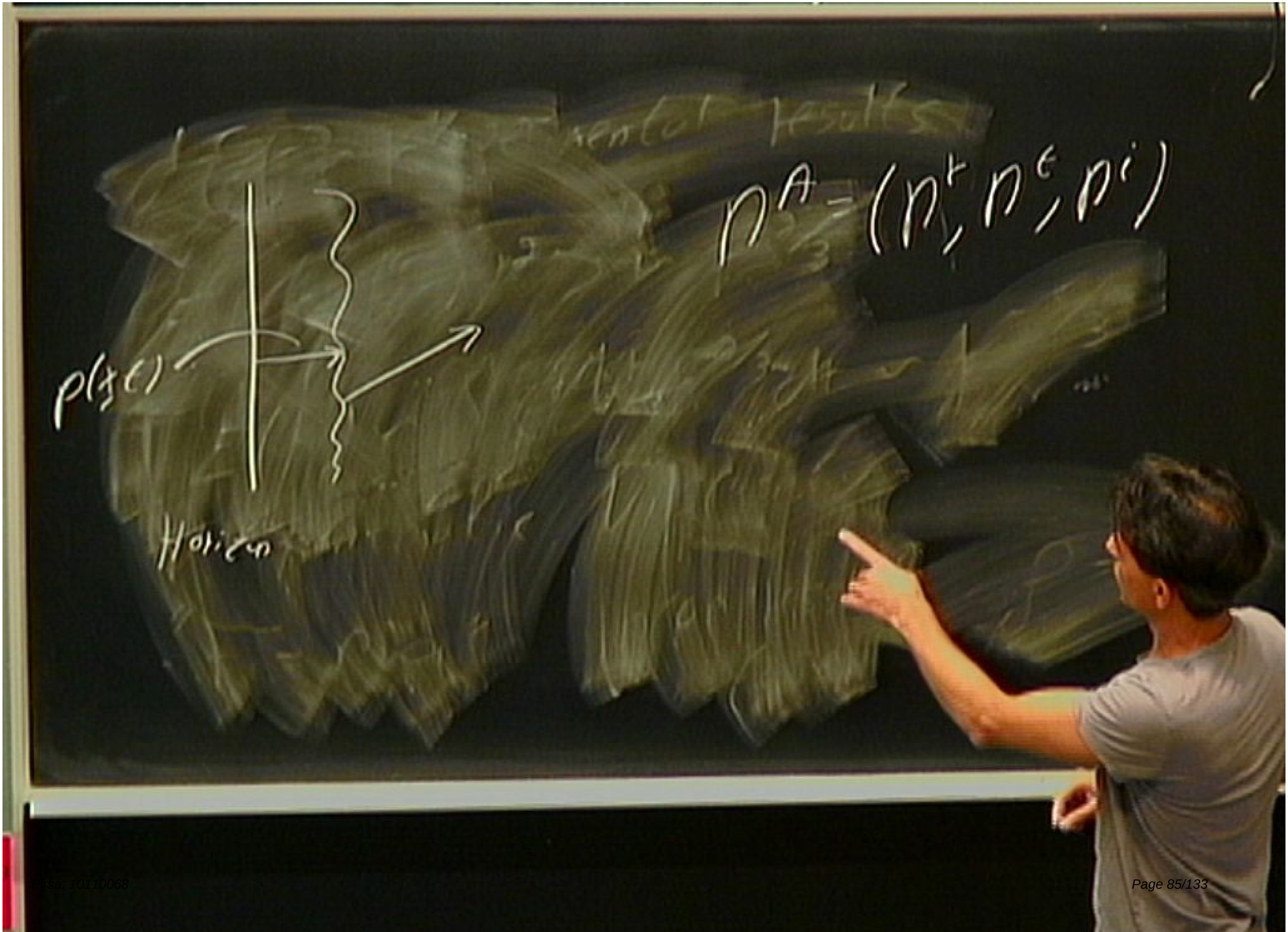


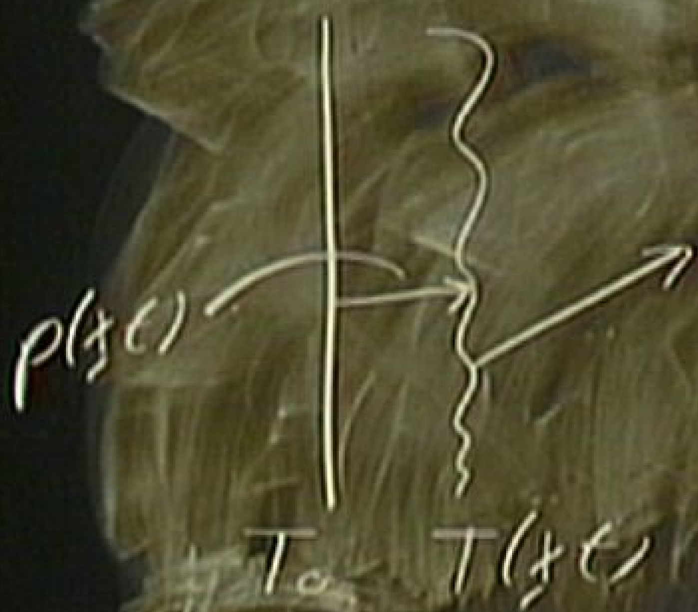
$z_3 = 1$
$z_n$
$n/3$
1
1 2 3 . .
n

$\zeta_3 = 1$ ✓

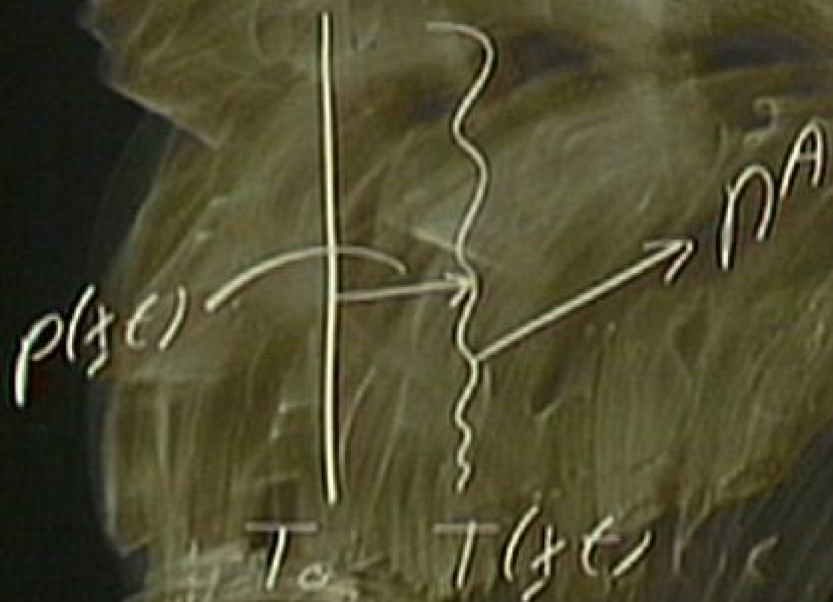



Horizon

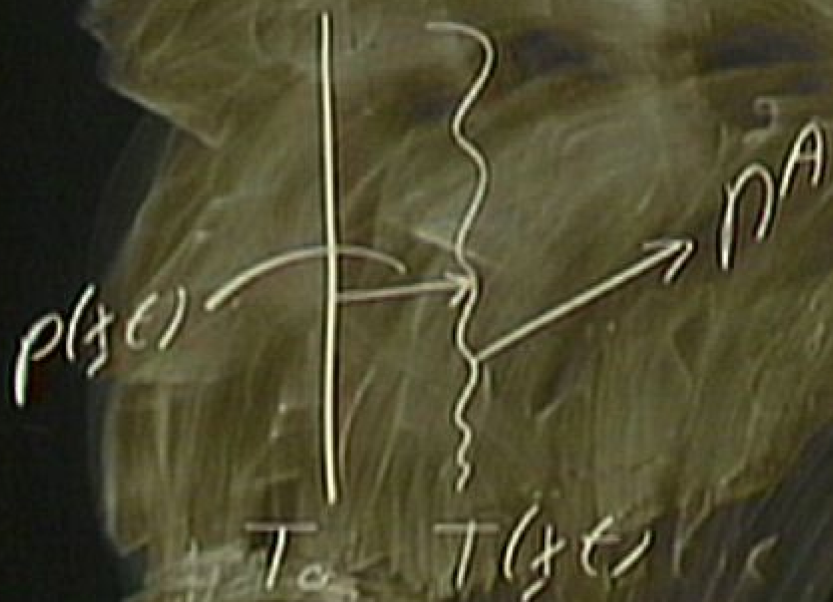

$p^A = (p^+, p^-, p^i)$
Horizon




$$p^A = (p^+, p^-, p^i)$$

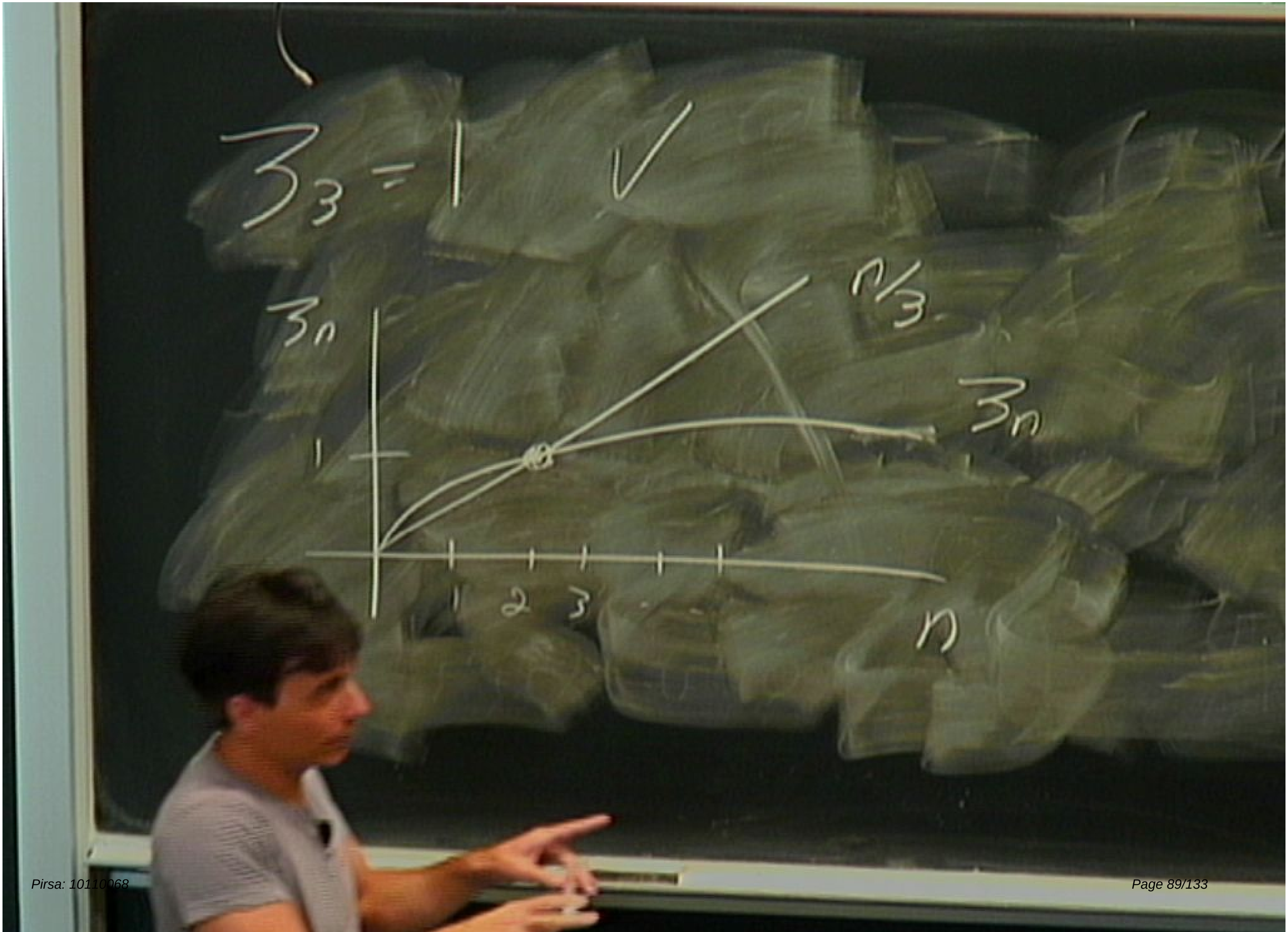


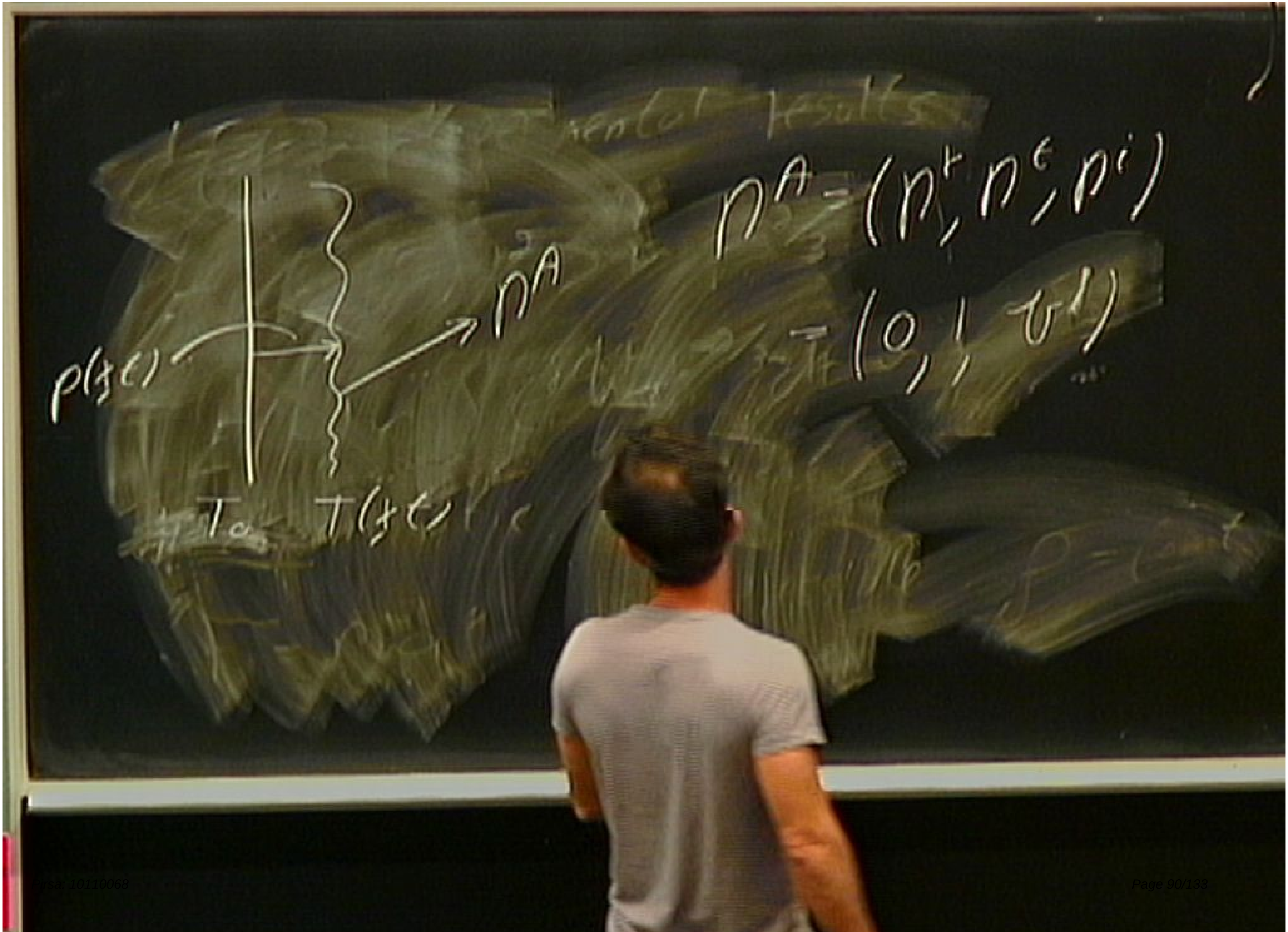

$$n^A = (n^t, n^z, n^i)$$
$$= (0, 1, v^i)$$




$$n^A = (n^t, n^{\epsilon}, n^i)$$

$$= (0, 1, v^i)$$


ζ3 = 1 ✓
ζn
n/3
ζn
1
1 2 3
n

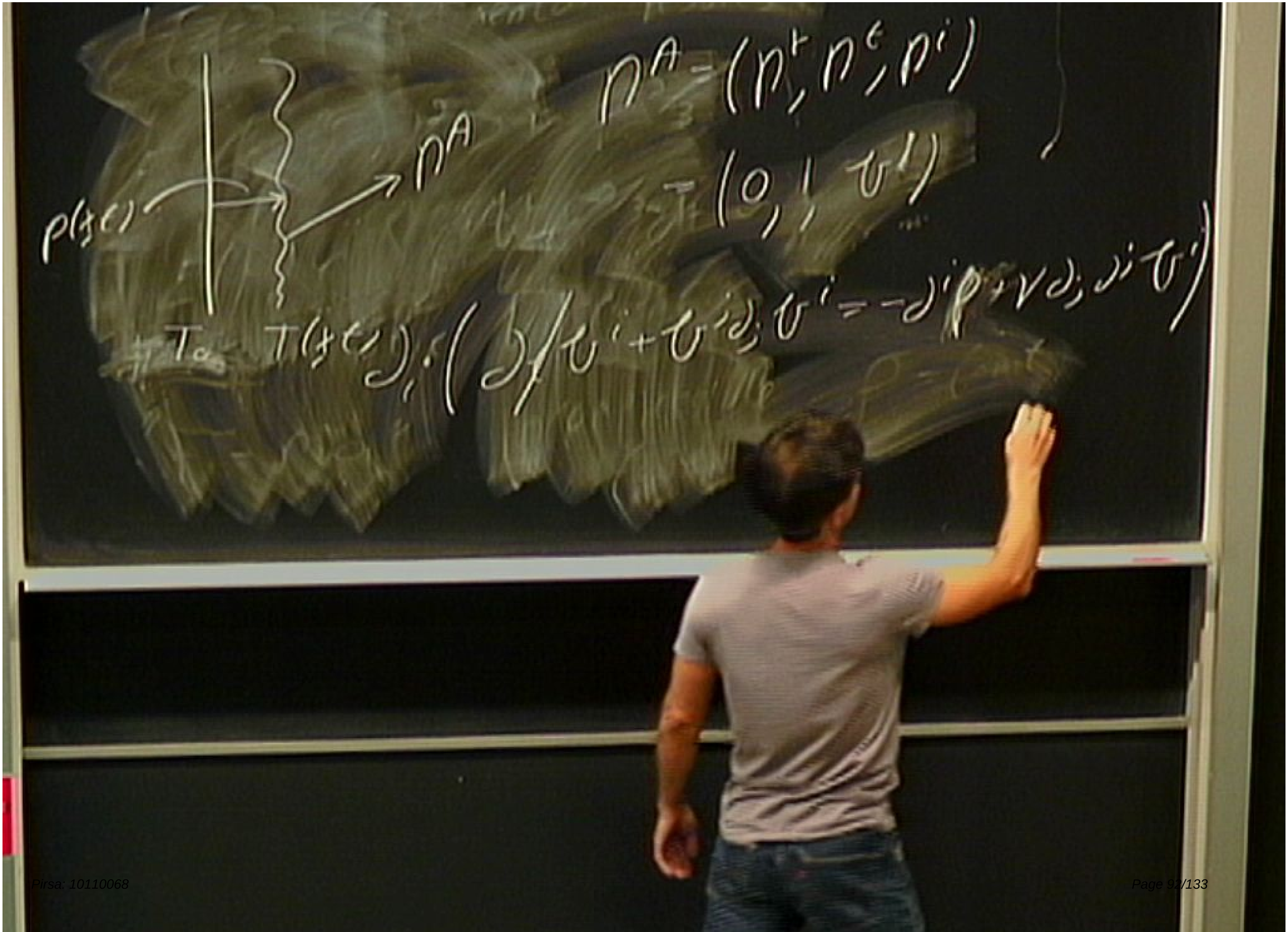


p(ξ,t)
p^A
p^A = (p^t, p^ξ, p^i)
= (0, 1, v^i)
T_0
T(ξ,t)

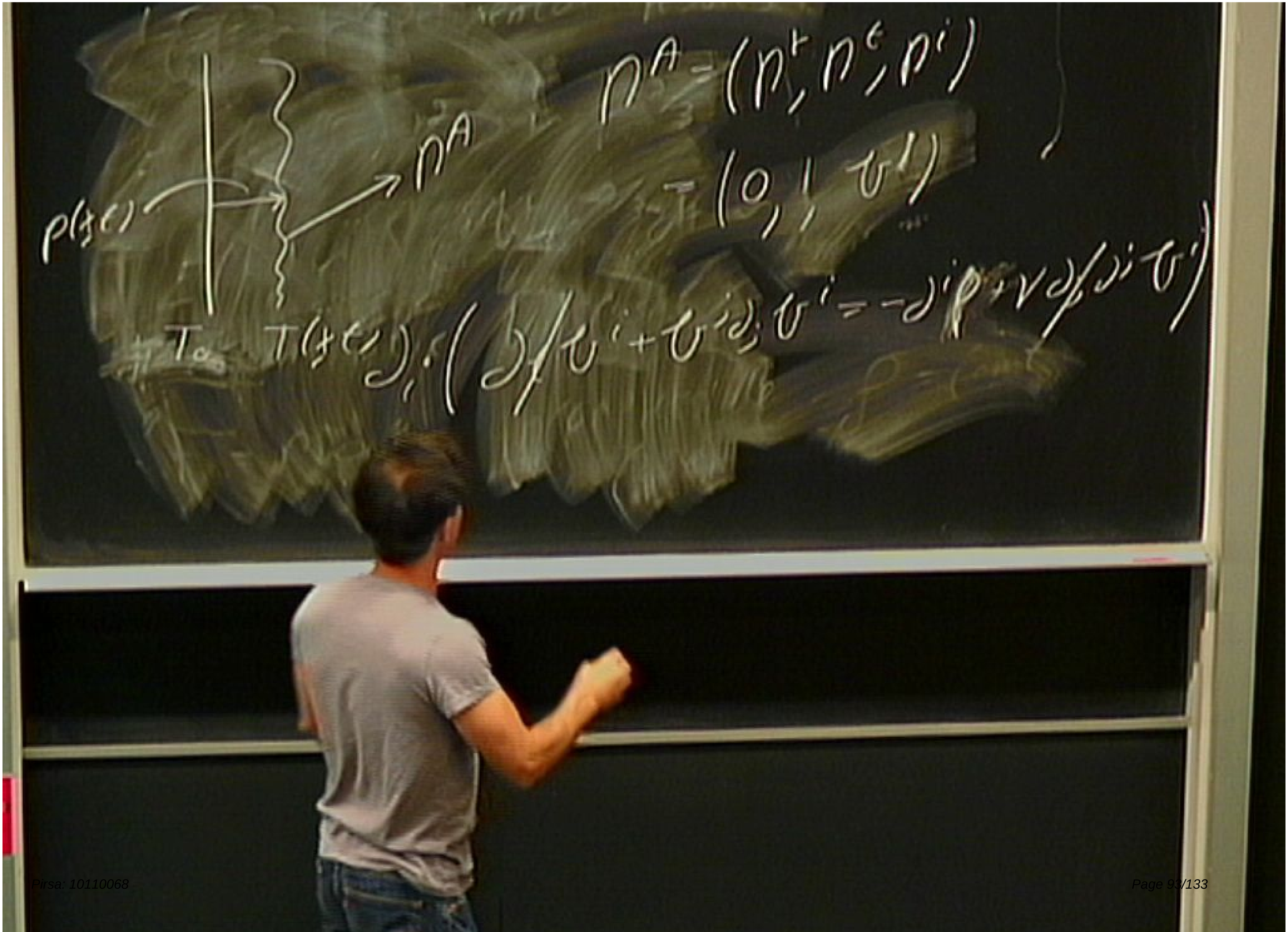
$n^A$

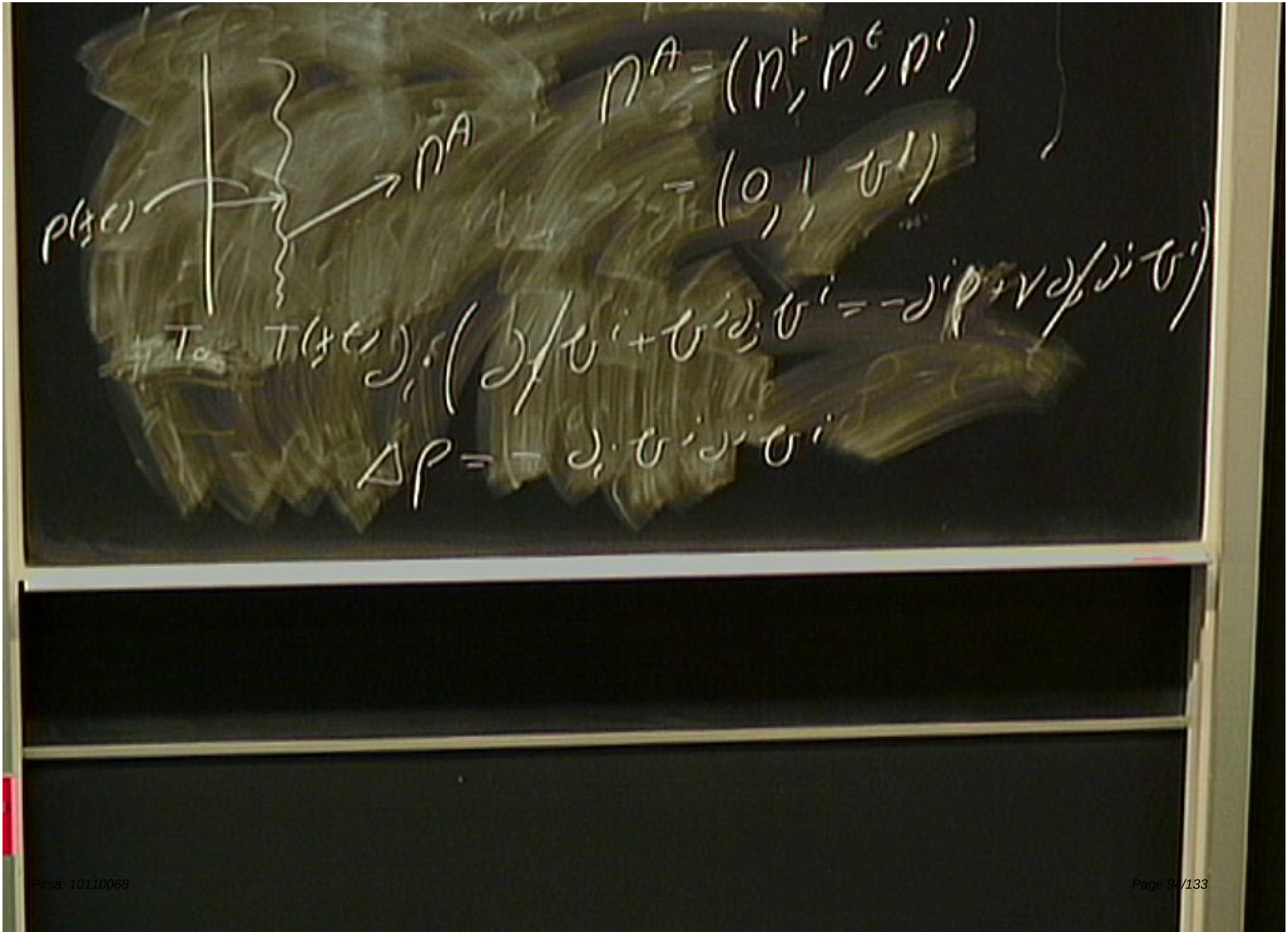
$\rho(t,\epsilon)$ $= (0, 1, v^i)$

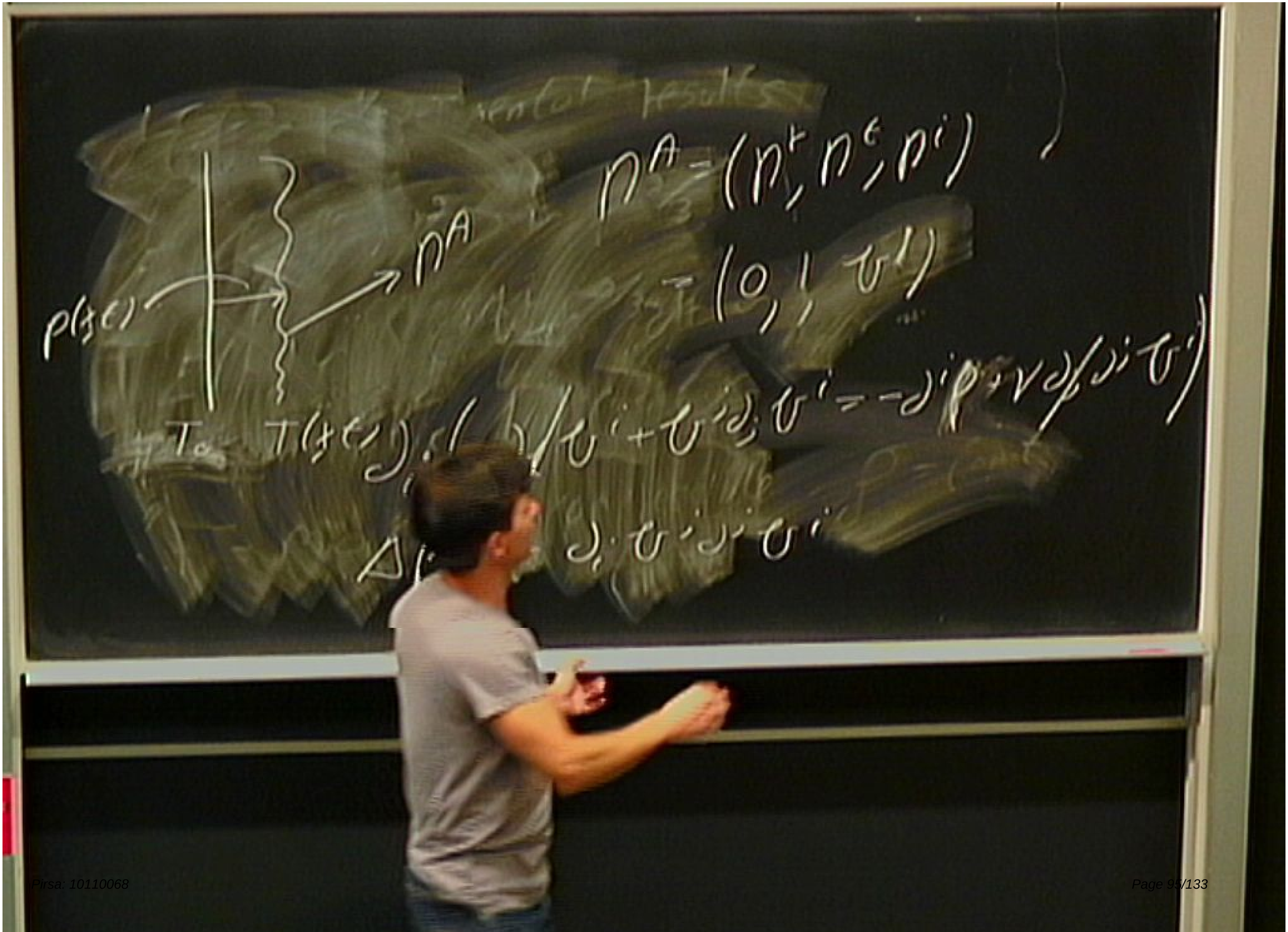
$T_0$ $T(t,\epsilon)\, \partial_i \cdot \left( \partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial\right.$

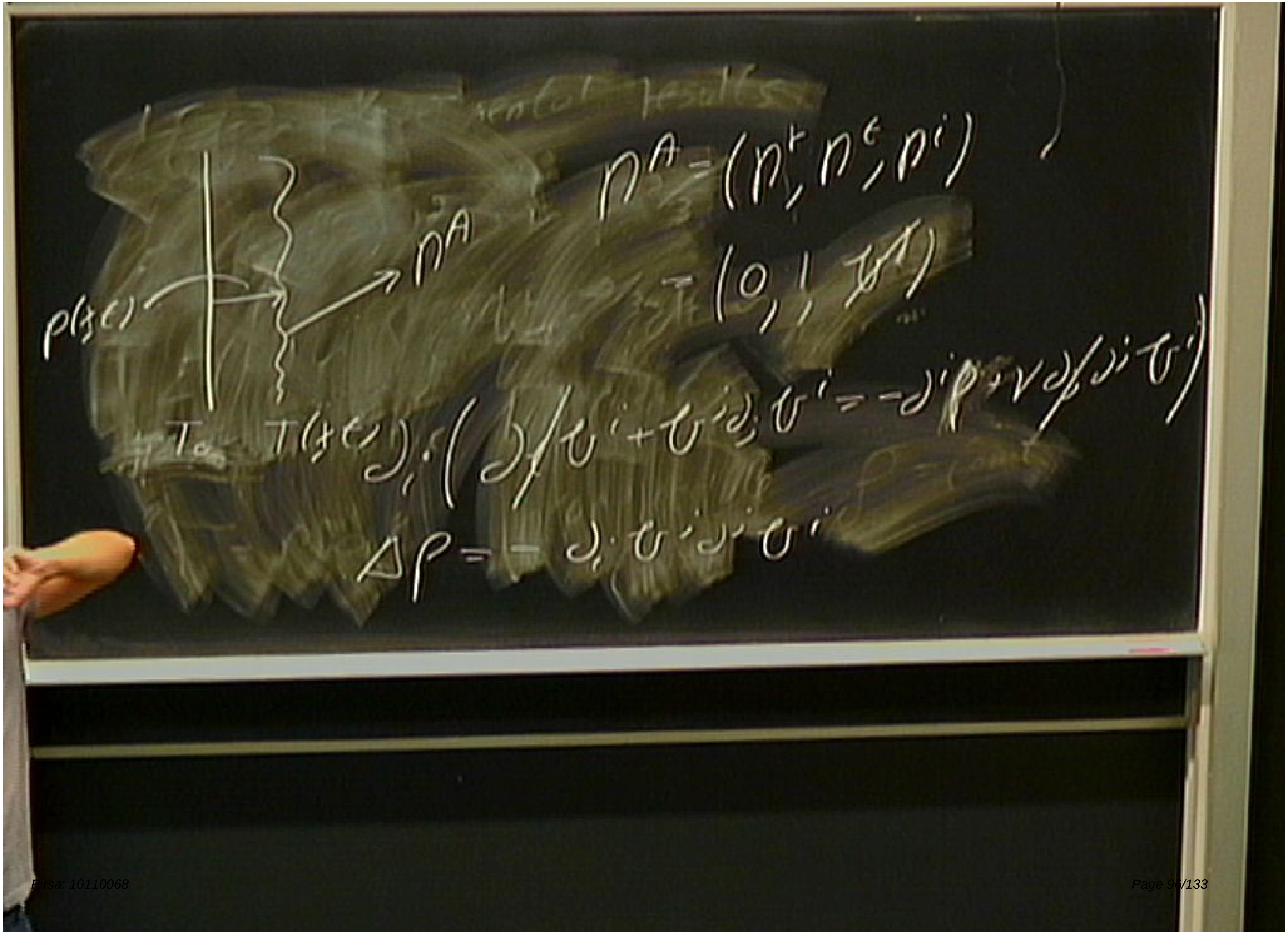


n^A = (n^t, n^t, n^i)
= (0, 1, v^i)
n^A
∂_t v^i + v^j ∂_j v^i = −∂^i p + ν ∂_j ∂^j v^i


n^A = (n^t, n^c, n^i)
= (0, 1, v^i)


n^A = (n^t, n^c, n^i)
= (0, 1, v^i)
∂_t v^i + v^j ∂_j v^i = −∂^i p + ν ∂_j ∂^j v^i
Δp = −∂_i v^j ∂_j v^i
p(t,c)
T(t,c)




$n^A = (n^t, n^\xi, n^i)$
$= (0, 1, v^i)$
$n^A$
$p(\xi(t))$
$T(\xi(t))$
$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i$
$\Delta p = -\partial_i v^j \partial_j v^i$


$$n^A = (n^t, n^c, n^i)$$

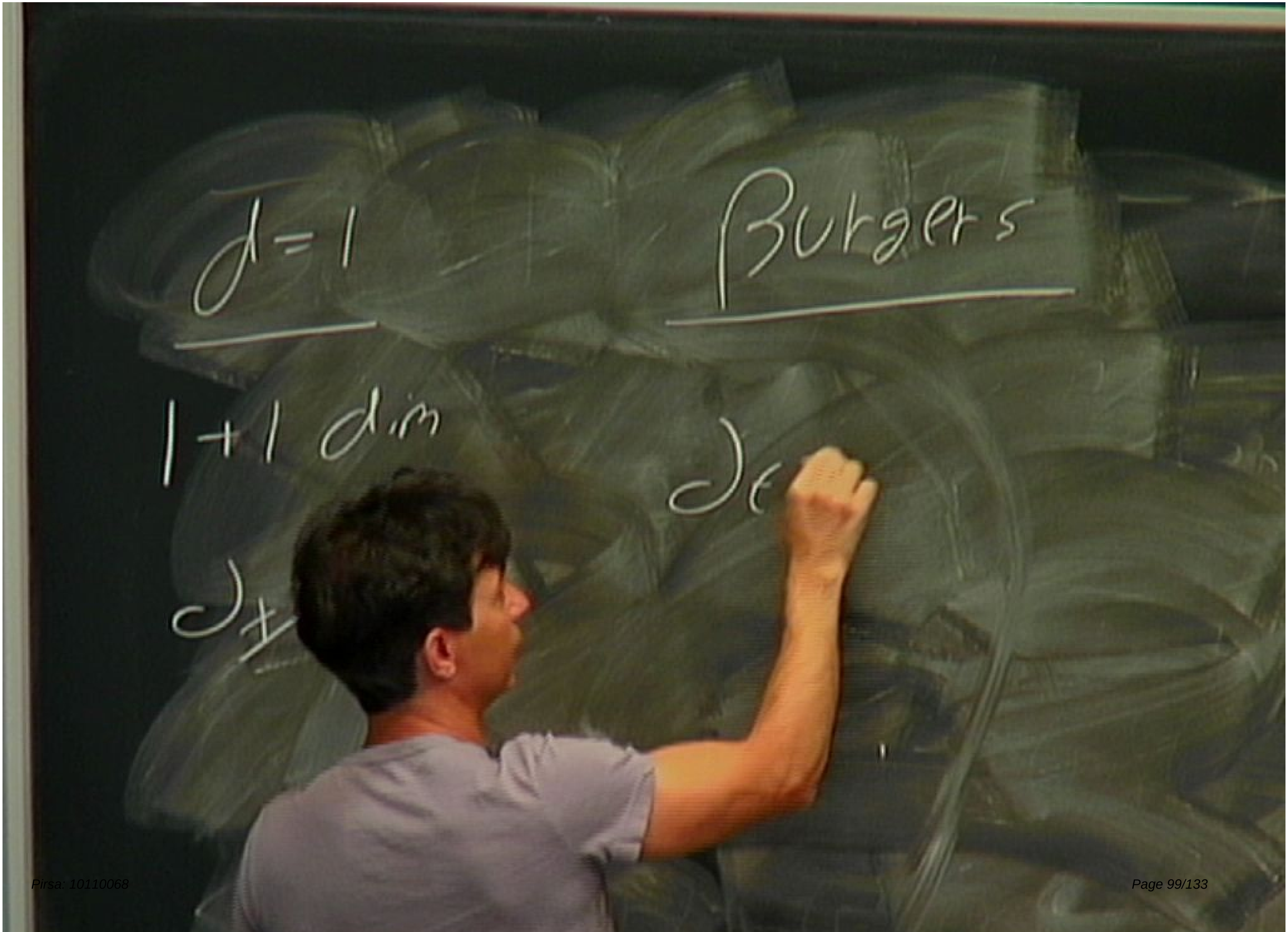
$$= (0, 1, v^i(\xi(c)))$$

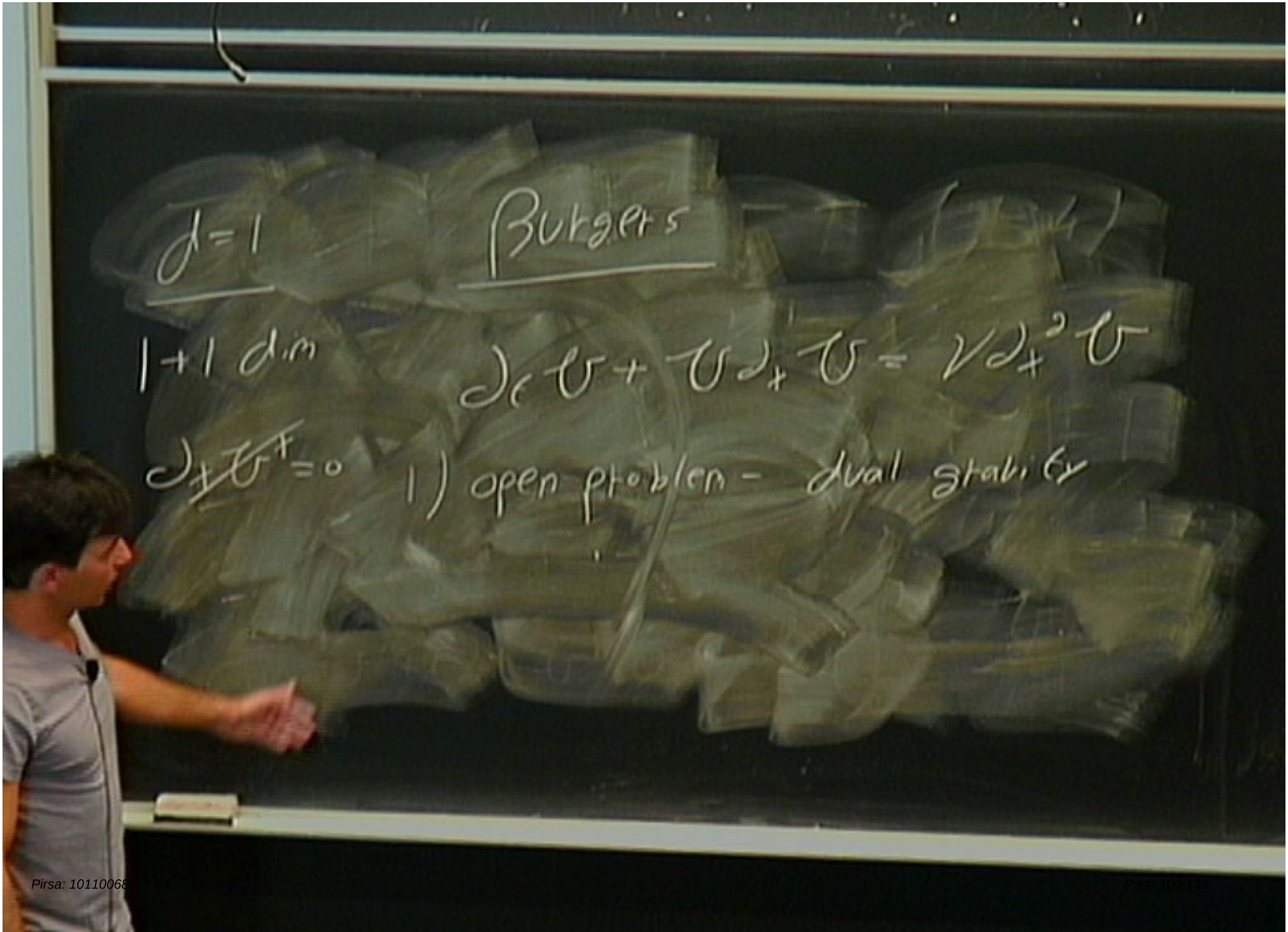
$$T(\xi(c)),\ \partial_i\left(\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i\right)$$

$$\boxed{\Delta p = -\partial_i v^j \partial^j v^i}$$

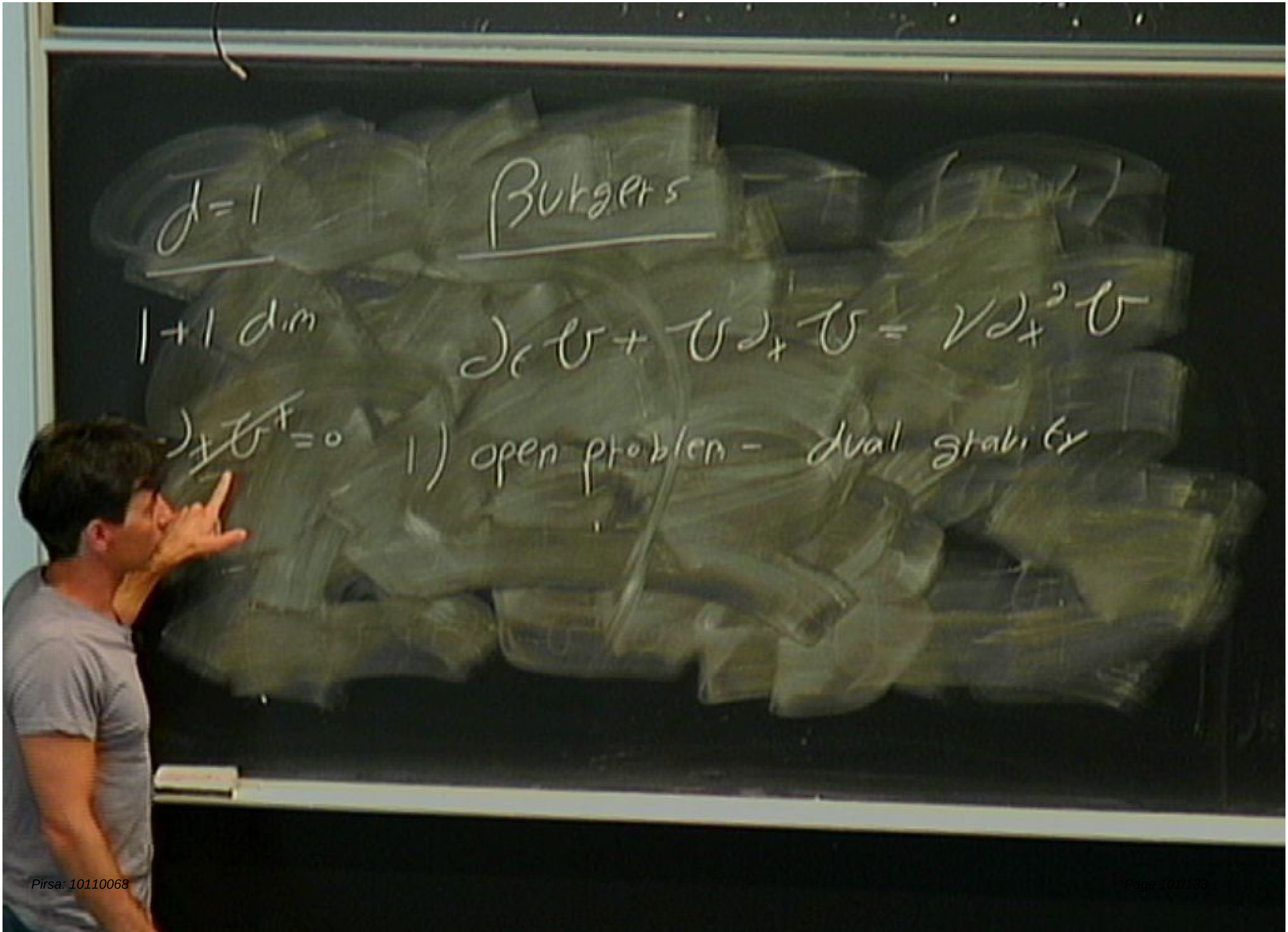
$d = 1$

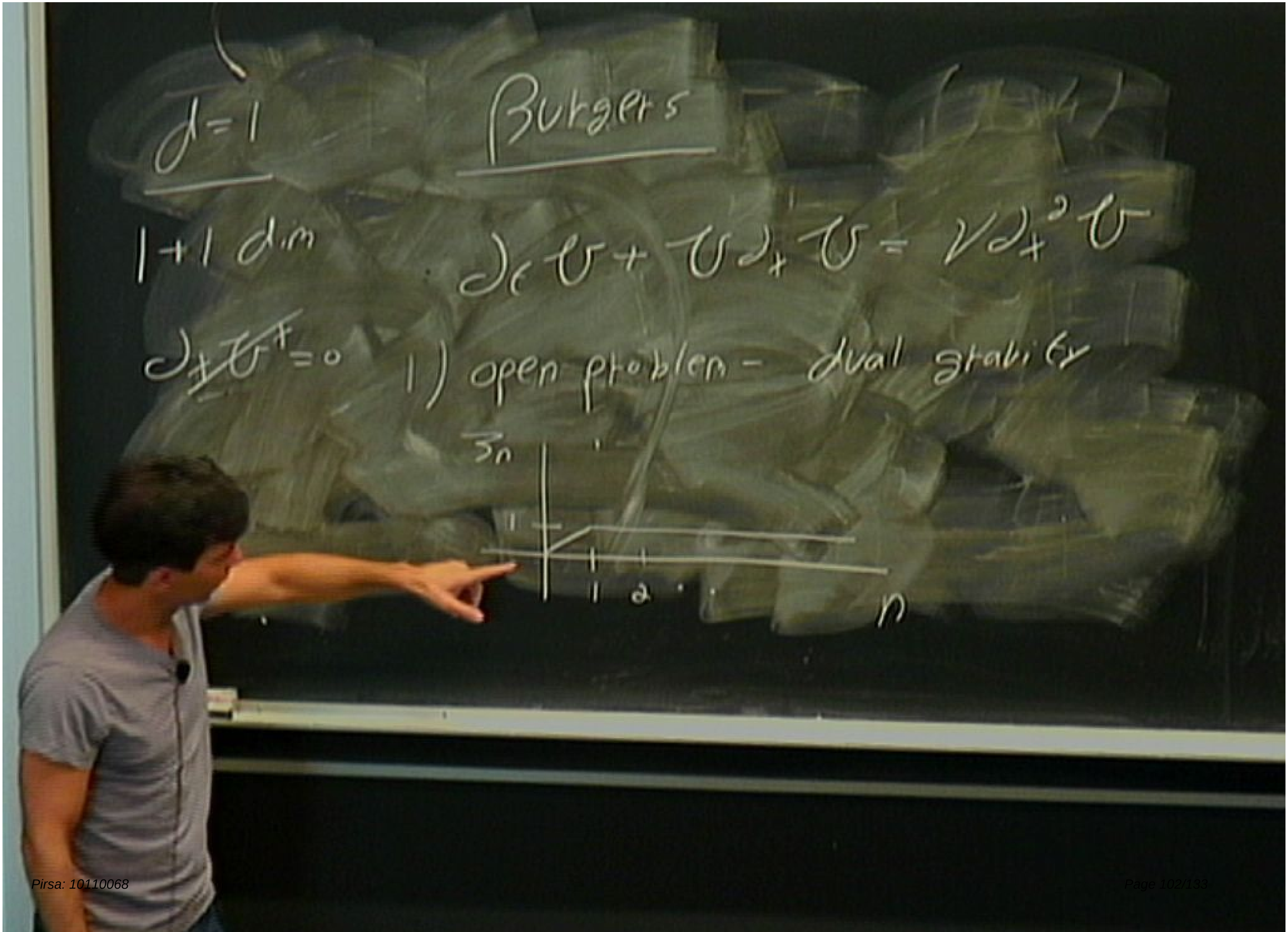
$1+1$ dim


d=1
Burgers
1+1 dim

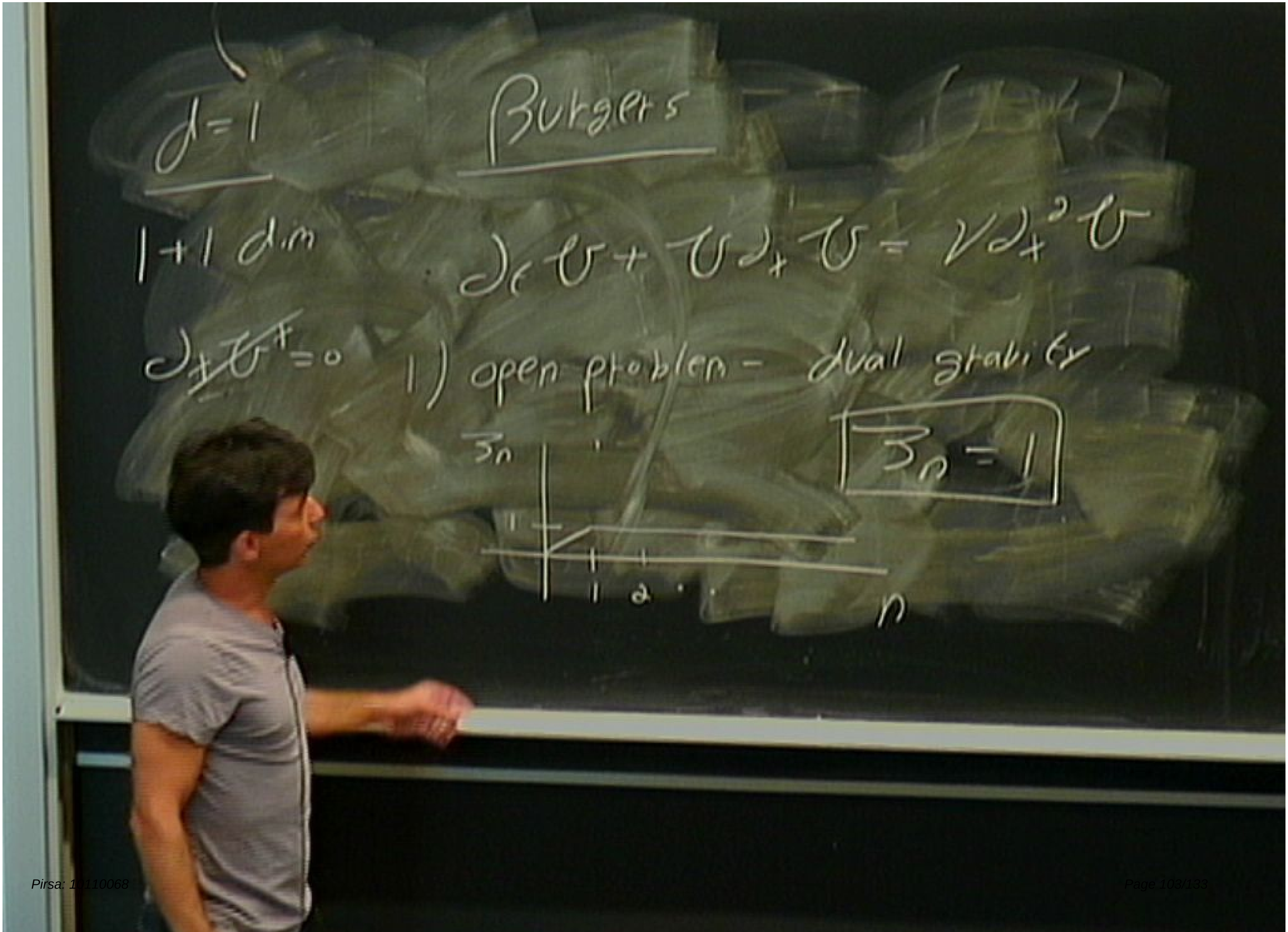


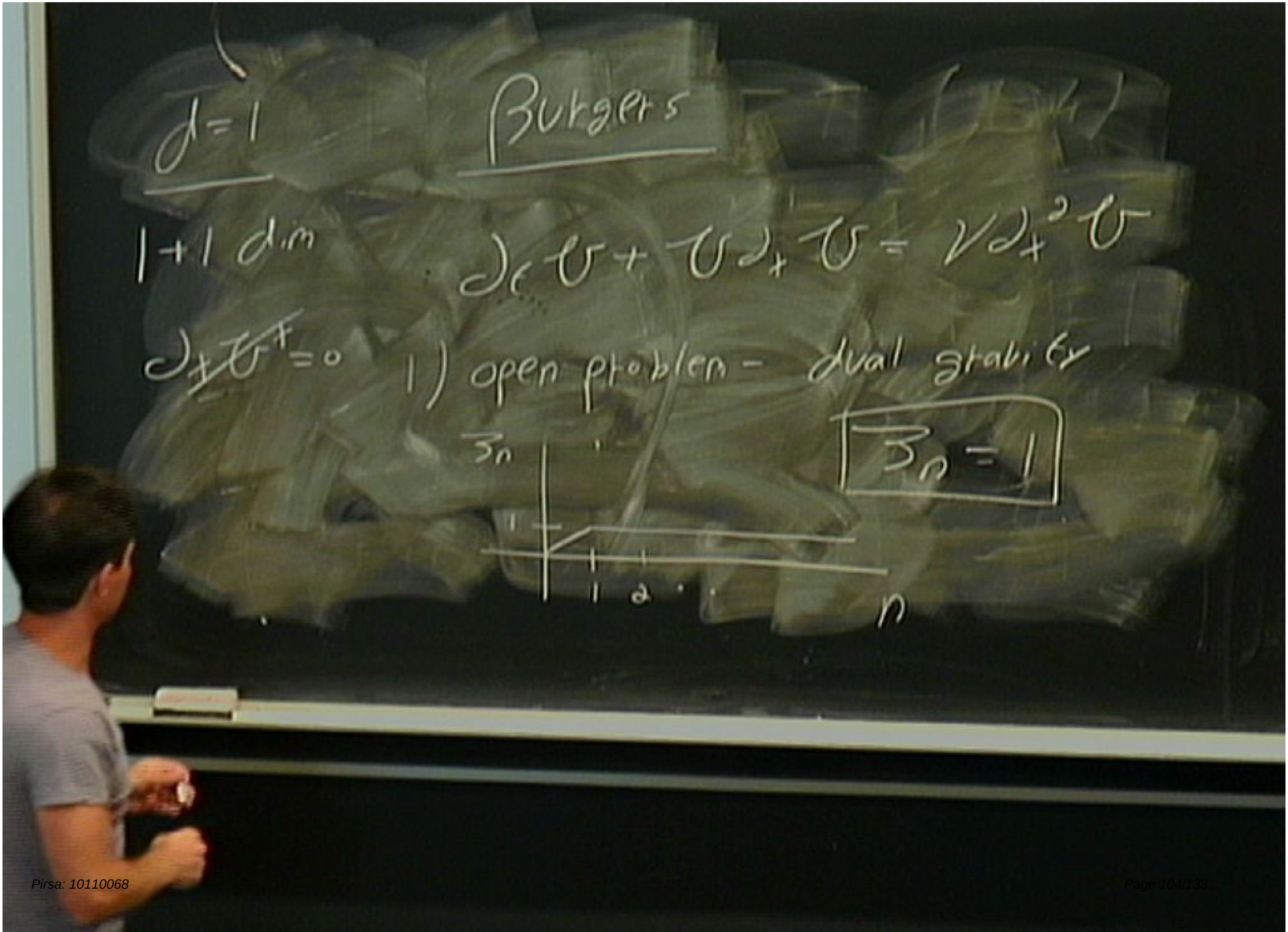
$d=1$
Burgers
1+1 dim
$\partial_t v + v \partial_x v = \nu \partial_x^2 v$
$\partial_\pm v^\mp = 0$
1) open problem - dual stability


d=1
Burgers
1+1 dim
∂t v + v∂x v = ν∂x² v
1) open problem - dual stability

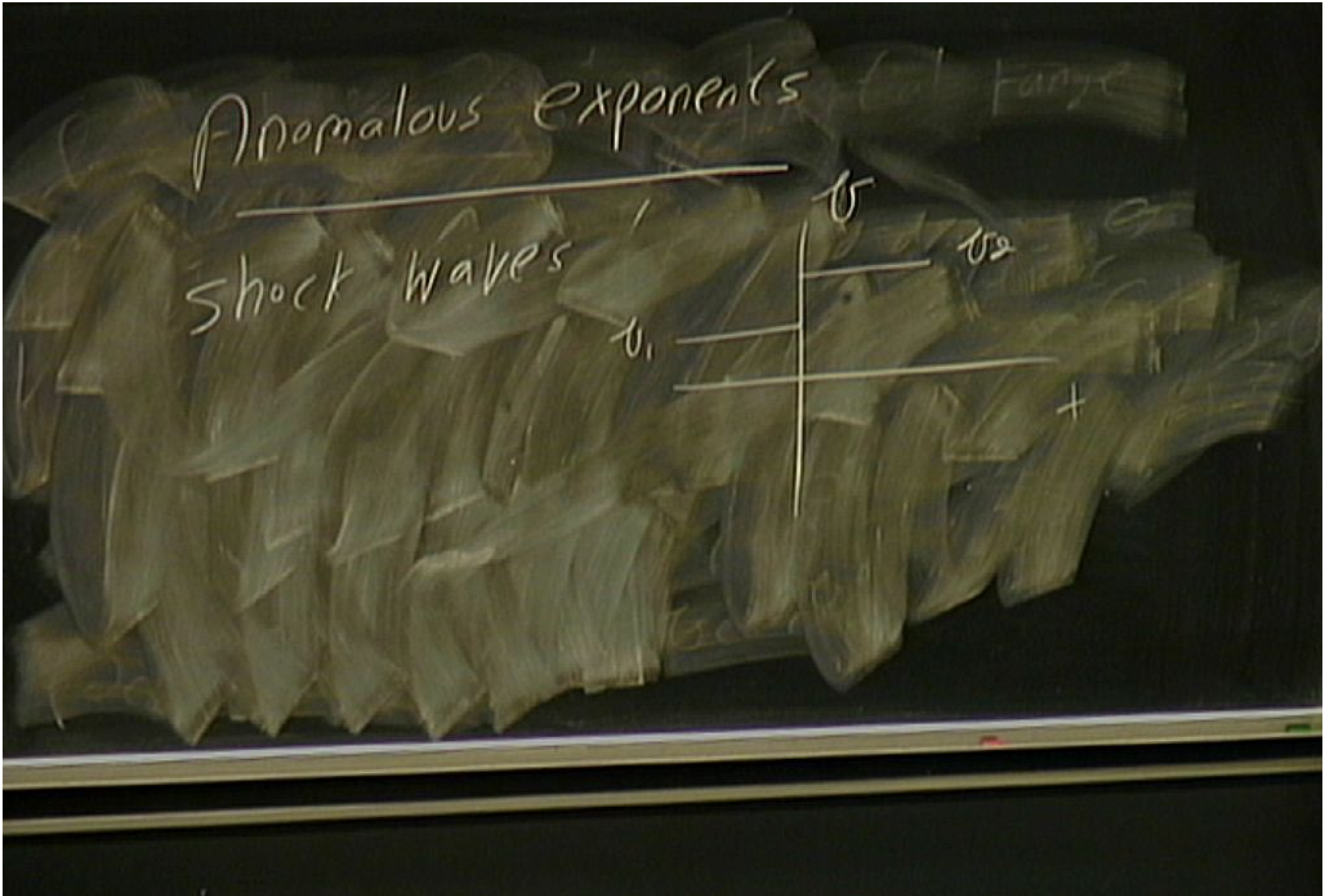


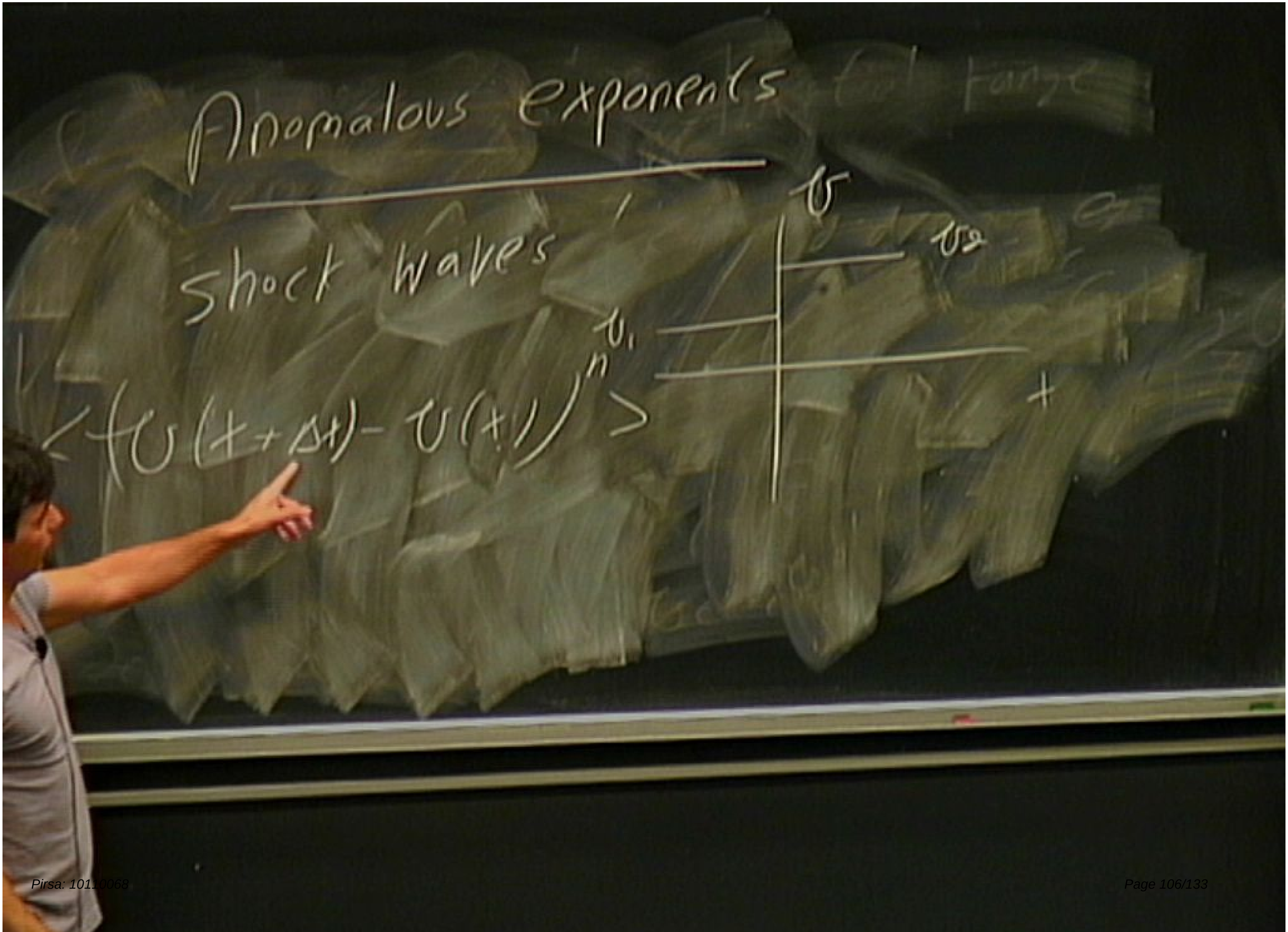
d=1
Burgers
1+1 dim
$\partial_t v + v \partial_x v = \nu \partial_x^2 v$
1) open problem - dual stability


d=1
Burgers
1+1 dim
$\partial_t v + v\partial_x v = \nu \partial_x^2 v$
$\partial_\pm v^\mp = 0$
1) open problem - dual stability
$B_n$
$\boxed{B_n = 1}$
1 2
n



d=1
Burgers
1+1 dim
$\partial_t v + v \partial_x v = \nu \partial_x^2 v$
$\partial_x v^x = 0$
1) open problem - dual stability
$\boxed{3_n = 1}$
1 2 n


Anomalous exponents
shock waves
v
v2
v1


Anomalous exponents
shock waves
<(v(t+Δt) − v(t))ⁿ>
v
v2
v1
t

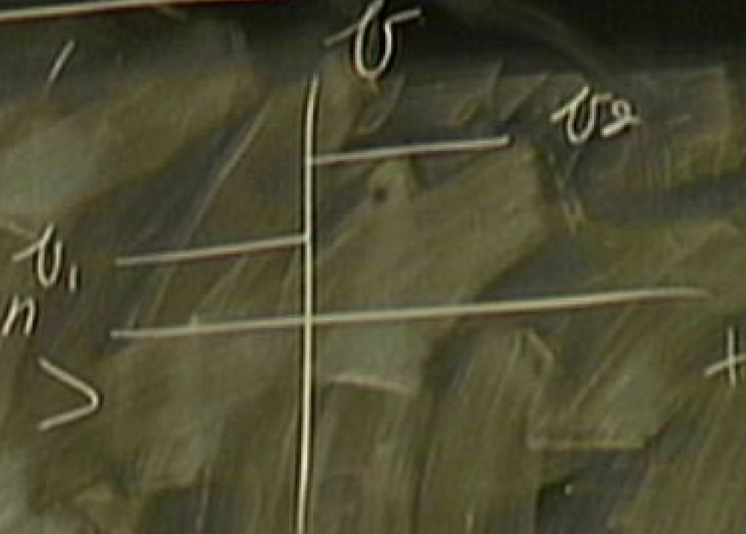
## Anomalous exponents

shock waves


$$\langle (v(t+\Delta t) - v(t))^n \rangle \sim \Delta t$$

# Anomalous exponents

Shock waves

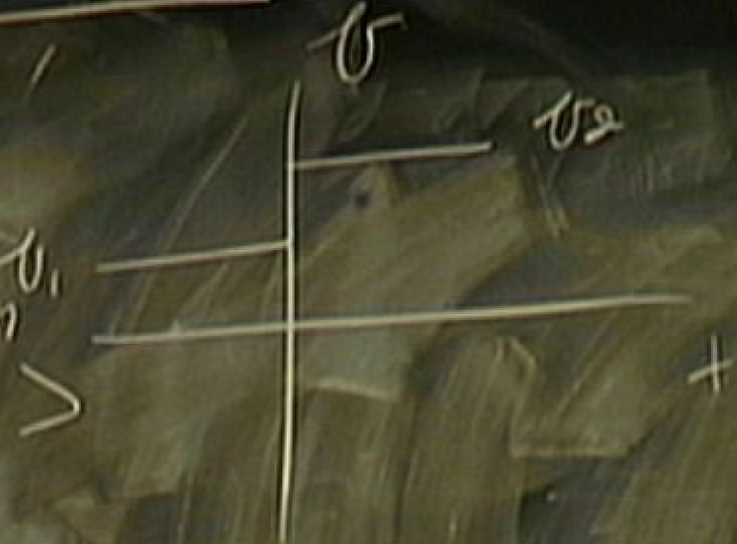



$$\langle (v(t+\Delta t) - v(t))^n \rangle \sim \Delta t$$

Conjecture - NS eq when $d \to \infty$ $\zeta_n = 1$

# Anomalous exponents

shock waves




$$\langle (v(t+\Delta t) - v(t))^n \rangle \sim \Delta t \qquad \zeta_n = 1$$

Conjecture - NS eq when $d \to \infty$

open problem

# Anomalous exponents

Shock waves

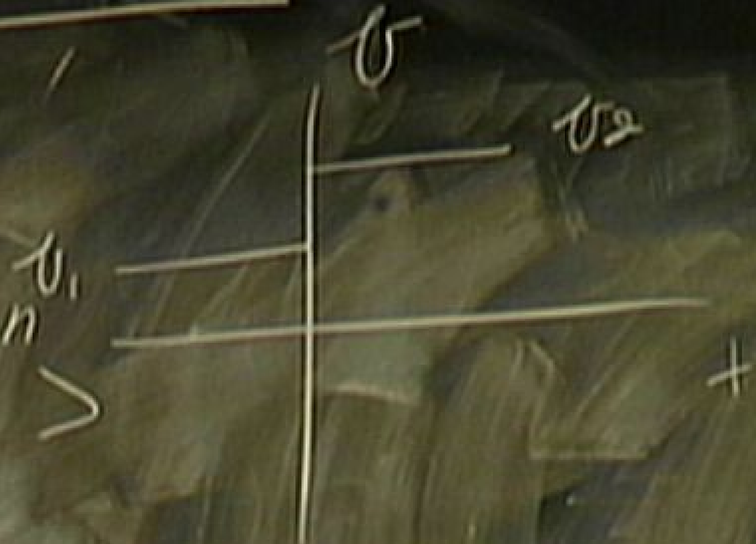

$$\langle (v(t+\Delta t) - v(t))^n \rangle \sim \Delta t$$

Conjecture – NS eq when $d \to \infty$ $\zeta_n = 1$

open problem

# Anomalous exponents

shock waves

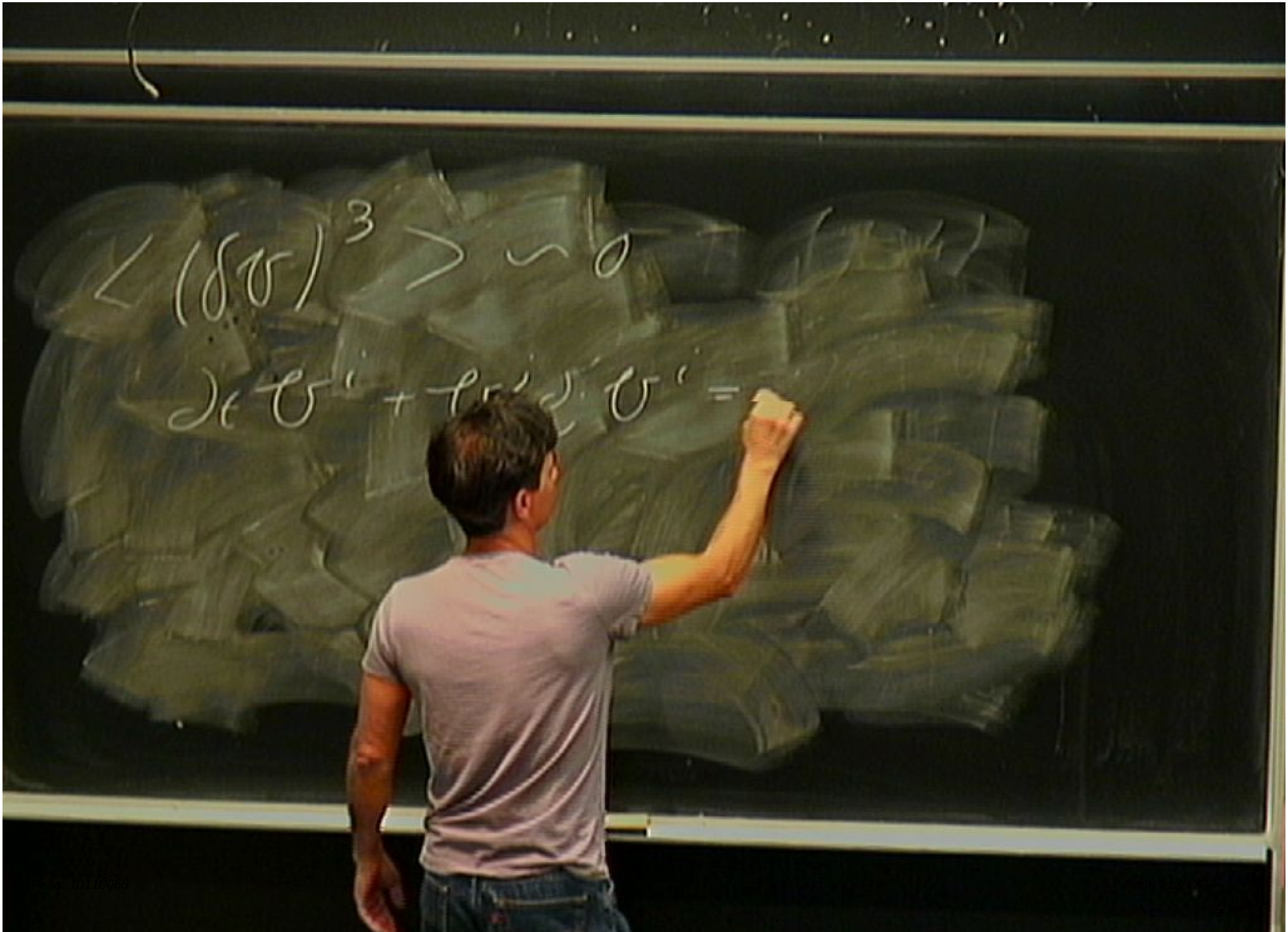



$$\langle (v(t+\Delta t) - v(t))^n \rangle$$

$$\sim \Delta t^{\zeta_n} \qquad \zeta_n = 1$$

Conjecture - NS eq when $d \to \infty$

open problem


$\langle (\delta v)^3 \rangle \sim$
$\partial_t v^i + v^j \partial_j v^i =$

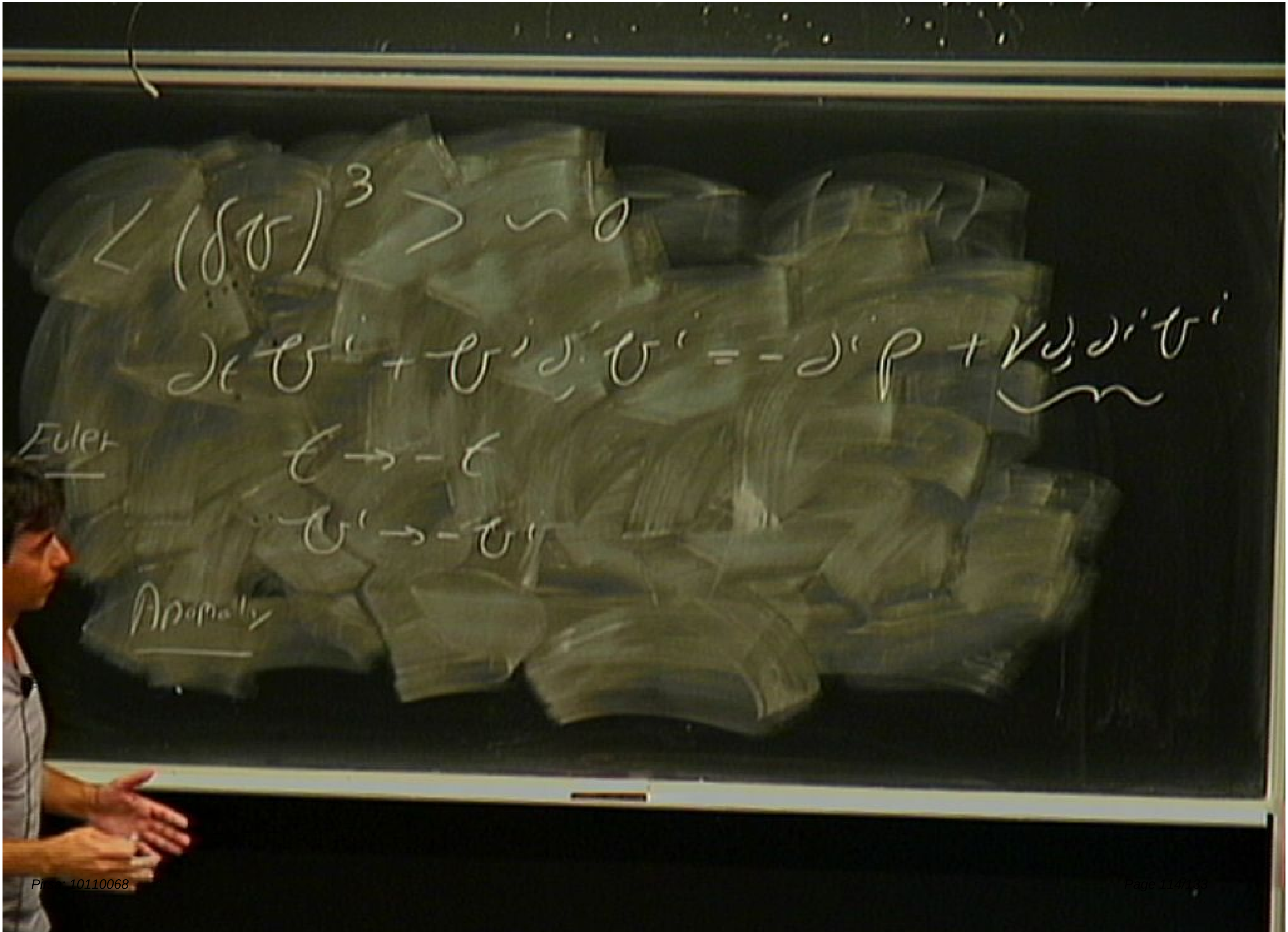
$$\langle (\delta v)^3 \rangle \sim l$$

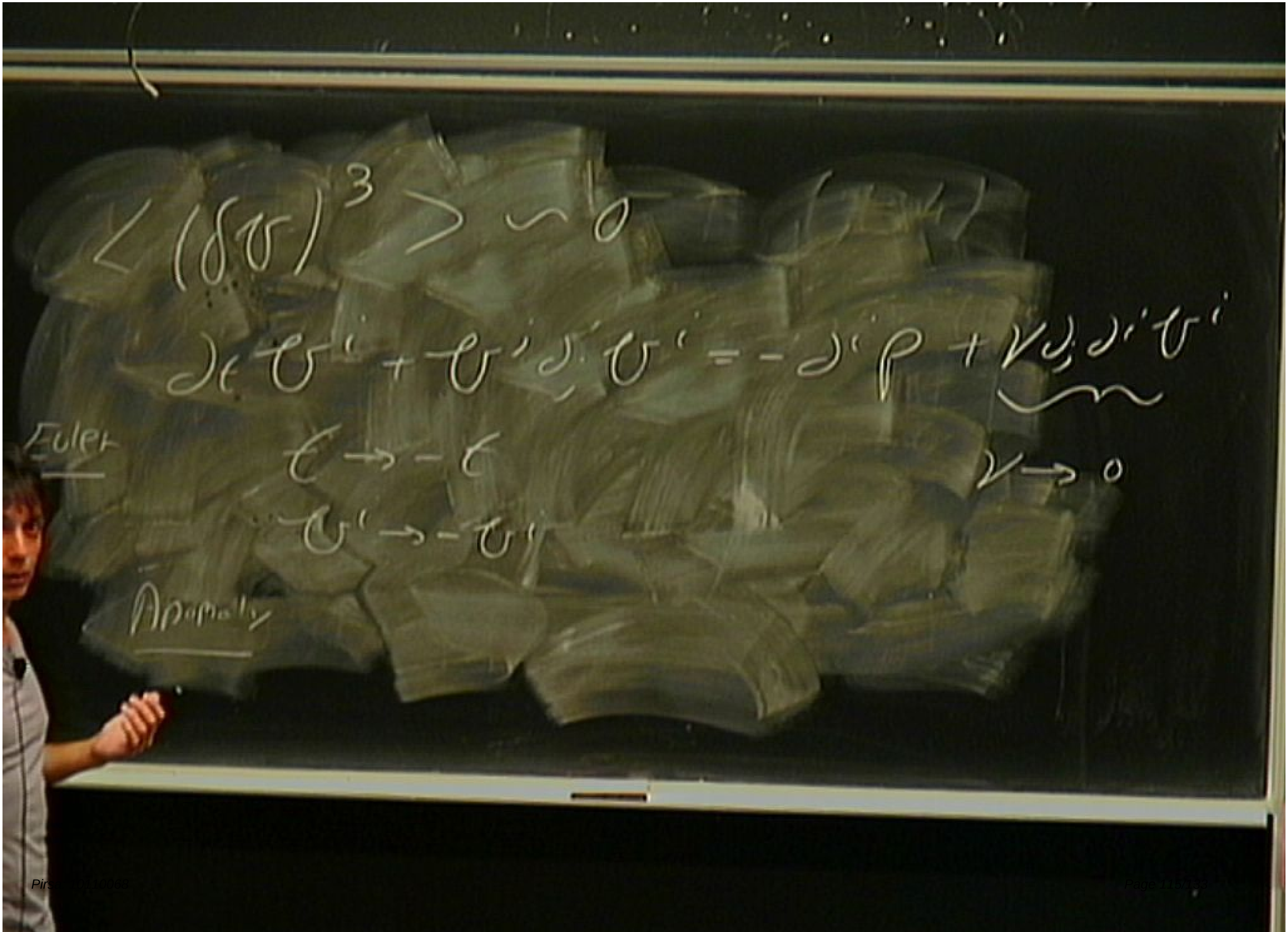
$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p$$

Euler

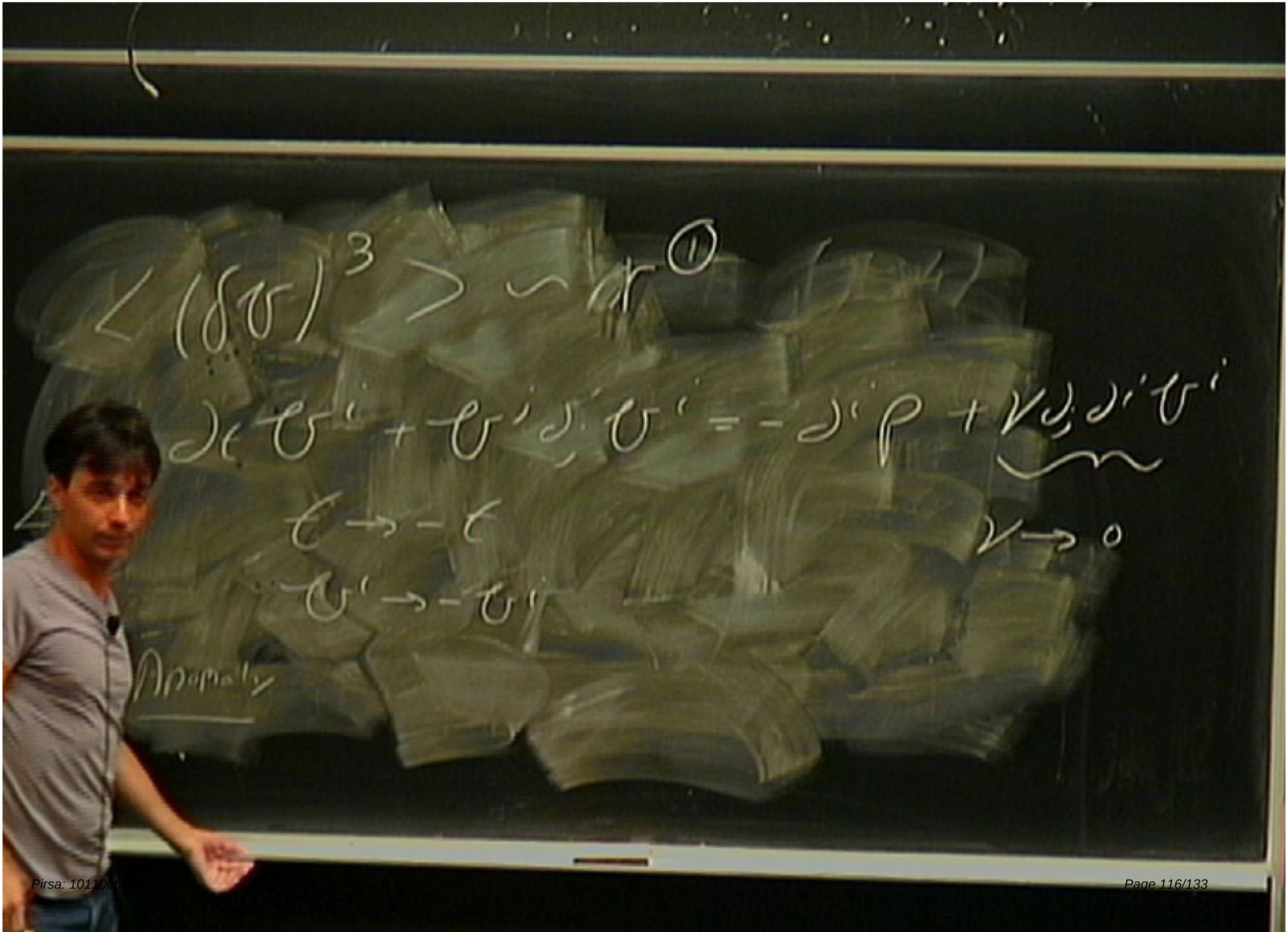
$$t \to -t$$

$$v^i \to -v^i$$


$\langle (\delta v)^3 \rangle \sim \epsilon$
$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i$
Euler
$t \to -t$
$v^i \to -v^i$
Anomaly



⟨(δv)³⟩ ∼ ε
∂ₜvⁱ + vʲ∂ⱼvⁱ = −∂ⁱp + ν∂ⱼ∂ʲvⁱ
Euler
t → −t
vⁱ → −vⁱ
ν → 0
Anomaly



$\langle (\delta v)^3 \rangle \sim r^{1}$
$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i$
$t \to -t$
$\nu \to 0$
$v^i \to -v^i$
Anomaly

$$\langle (\delta v)^3 \rangle \sim \# r^{①}$$

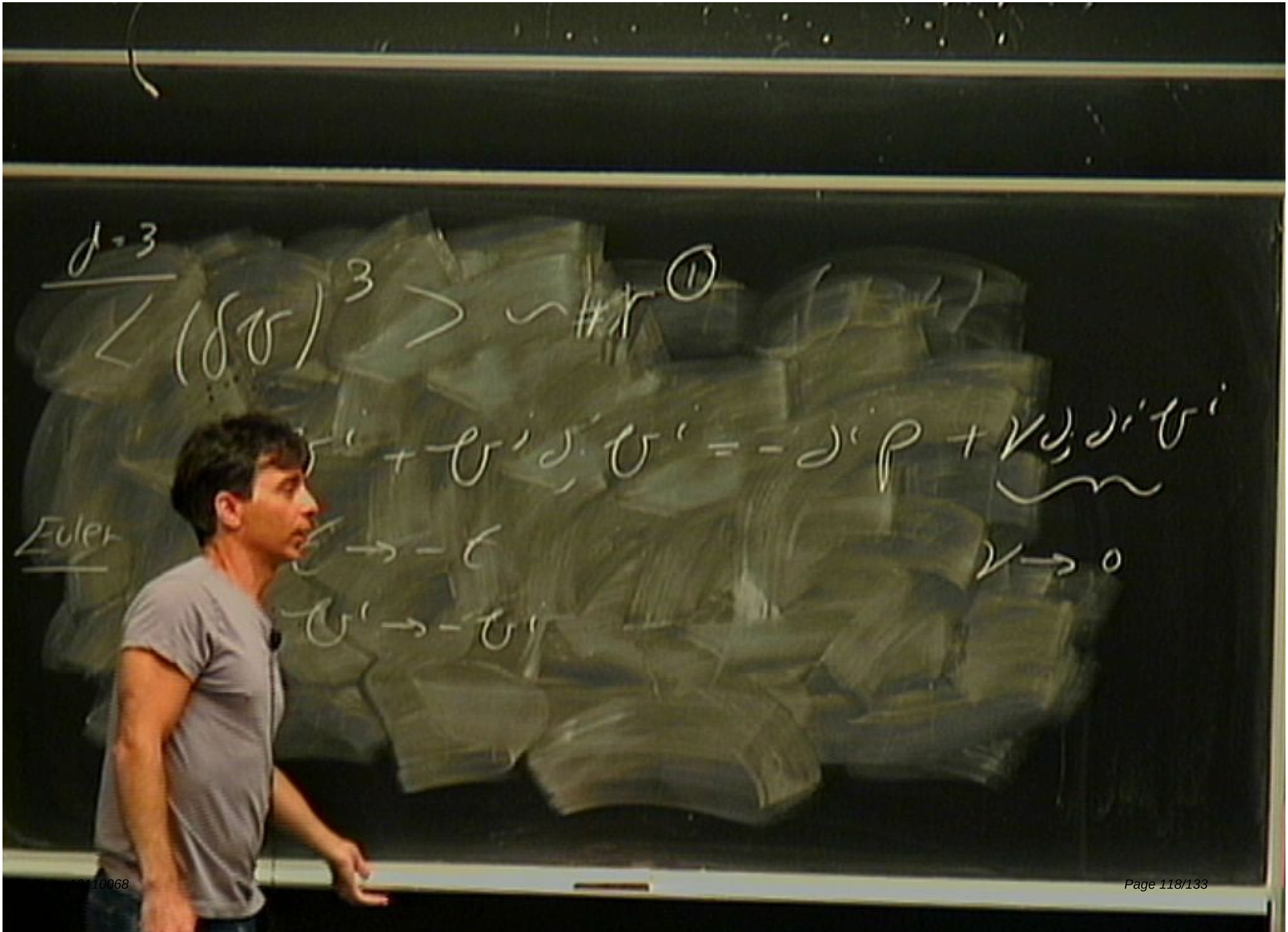
$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \underbrace{\nu \partial_j \partial^j v^i}$$

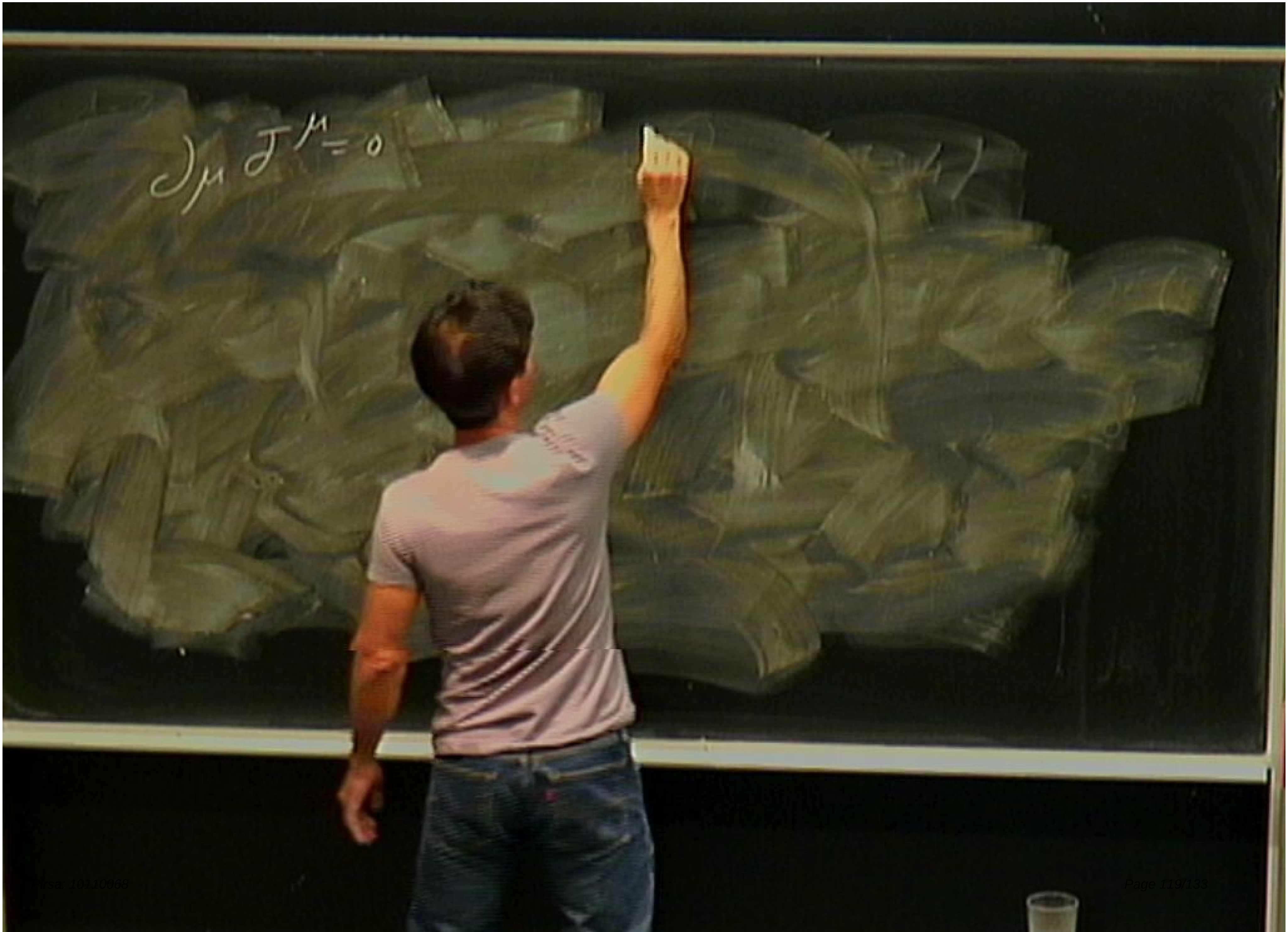
Euler

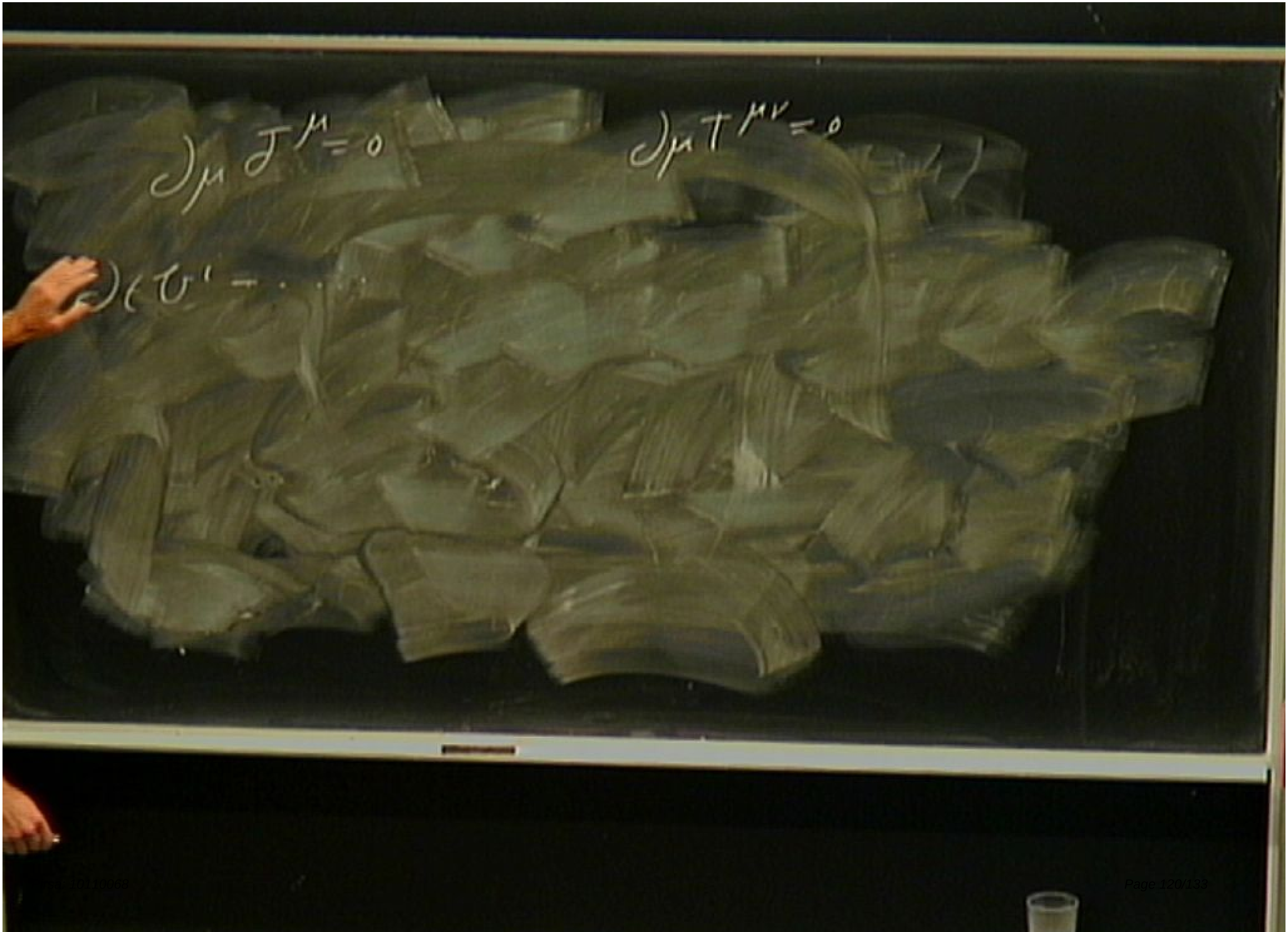
$t \to -t$ $\qquad$ $\nu \to 0$

$v^i \to -v^i$

Anomaly

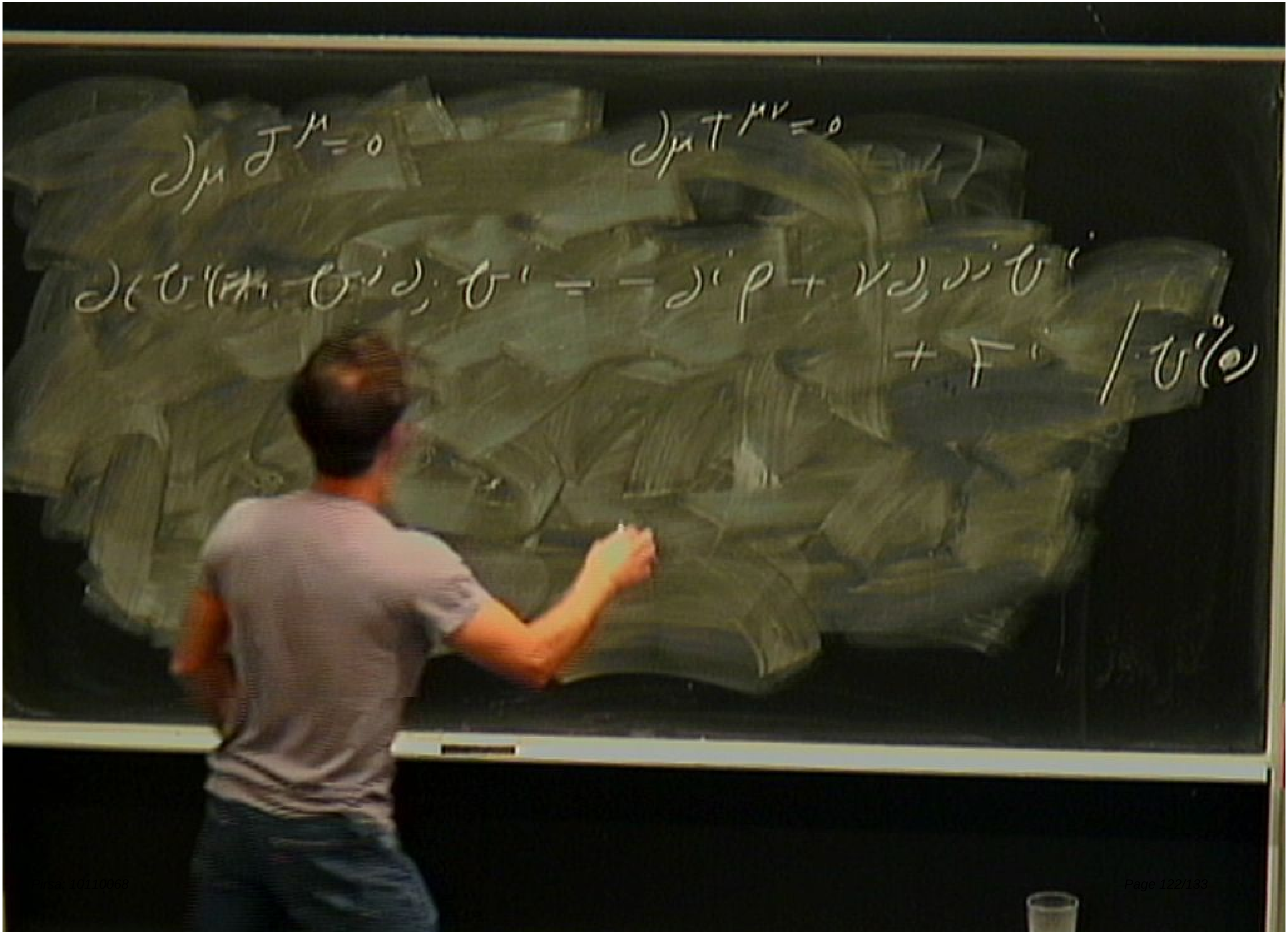

d = 3
Euler
ν → 0


$\partial_\mu J^\mu = 0$

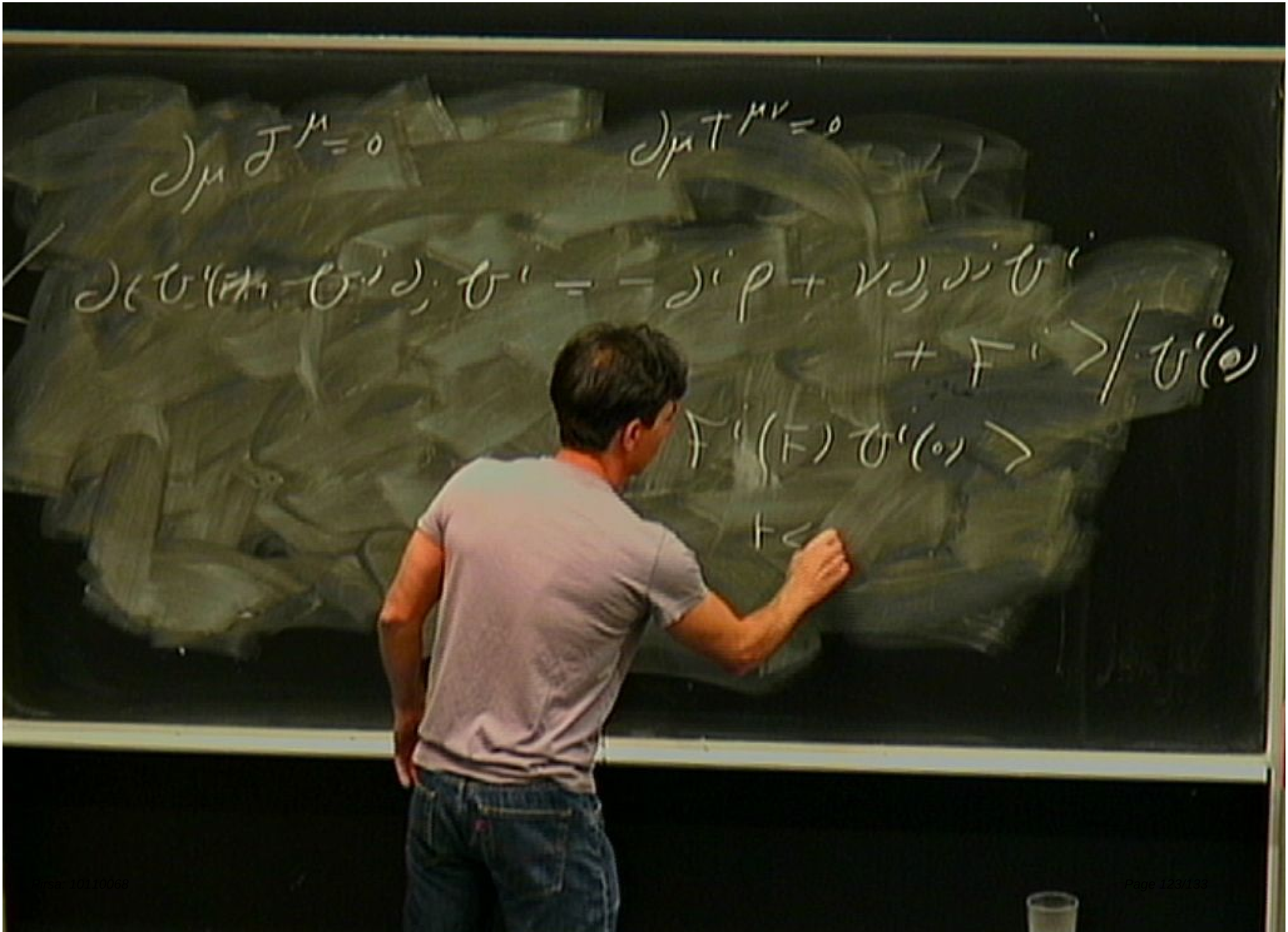

$\partial_\mu J^\mu = 0$
$\partial_\mu T^{\mu\nu} = 0$

$$\partial_\mu J^\mu = 0 \qquad \partial_\mu T^{\mu\nu} = 0$$

$$\partial_t v^i + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i + F^i \quad / \quad v^i(0)$$



$\partial_\mu J^\mu = 0$
$\partial_\mu T^{\mu\nu} = 0$


$\partial_\mu J^\mu = 0$
$\partial_\mu T^{\mu\nu} = 0$

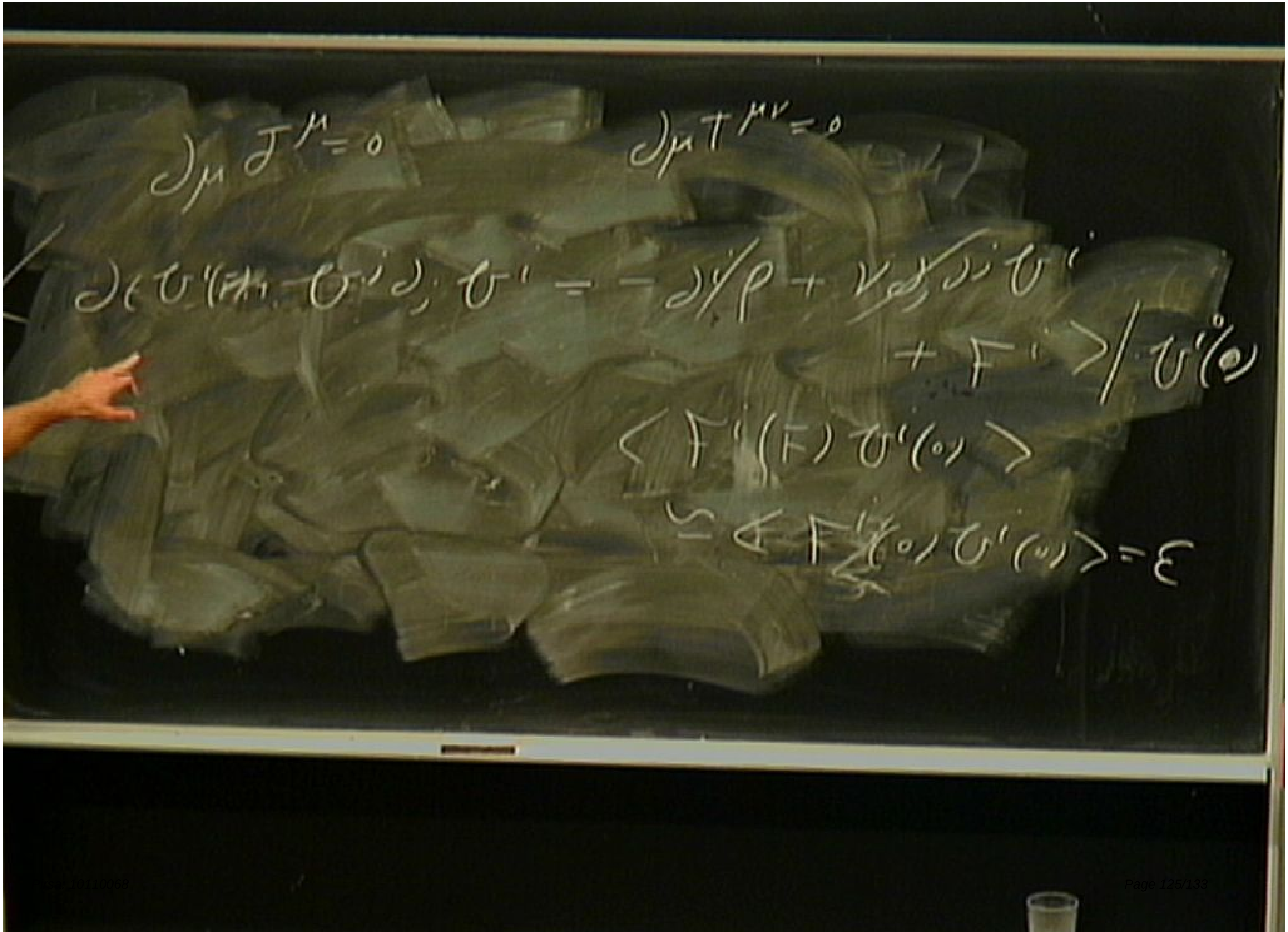
$$\partial_\mu J^\mu = 0 \qquad \partial_\mu T^{\mu\nu} = 0$$

$$\partial_t v^i(t,x) + v^j \partial_j v^i = -\partial^i p + \nu \partial_j \partial^j v^i$$

$$+ F^i \rangle / v^i(0)$$

$$\langle F^i(t)\, v^i(0) \rangle$$

$$+ cc$$


$\partial_\mu J^\mu = 0$
$\partial_\mu T^{\mu\nu} = 0$
$\partial_t v^i + v^j \partial_j v^i = -\partial^i P/\rho + \nu \partial_j \partial^j v^i$
$+ F^i \rangle / v^i(0)$
$\langle F^i(t)\, v^i(0) \rangle$
$\simeq \langle F^i(0)\, v^i(0) \rangle = \varepsilon$

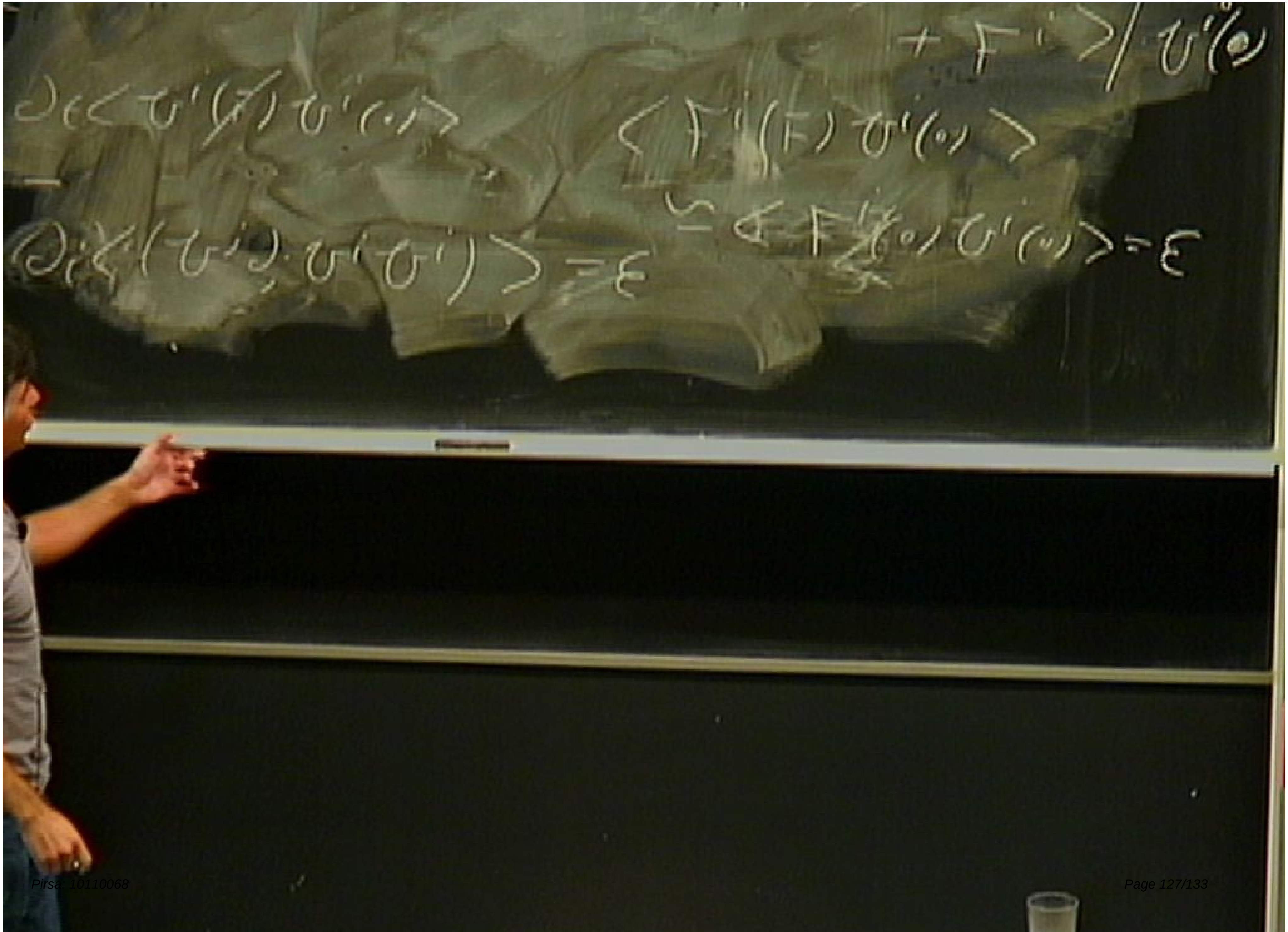
$$\partial_\mu J^\mu = 0 \qquad \partial_\mu T^{\mu\nu} = 0$$

$$\partial_t v^i(x) + v^j \partial_j v^i = -\partial^i p/\rho + \nu \partial_j \partial_j v^i$$

$$+ F^i \rangle / v^i(0)$$

$$\langle v^i(x)\, v^i(0) \rangle \qquad \langle F^i(x)\, v^i(0) \rangle$$

$$\simeq \langle F^i(0)\, v^i(0) \rangle = \varepsilon$$



$\partial_t \langle \sigma'(t)\, \sigma'(0) \rangle$ $\quad \langle F'(t)\, \sigma'(0) \rangle$

$\partial_t \langle (-\sigma' \partial \cdot \sigma'(\sigma')) \rangle = \epsilon$ $\quad \simeq \langle F'(0)\, \sigma'(0) \rangle = \epsilon$

$\langle \sigma' \sigma' \sigma' \rangle \sim \epsilon r l$

$\partial_\mu J^\mu = 0 \qquad \partial_\mu T^{\mu\nu} = 0$

$\partial_t \left( v^i(\vec{r}) \right) + v^j \partial_j v^i = -\partial^i p/\rho + \nu \partial_j \partial_j v^i$

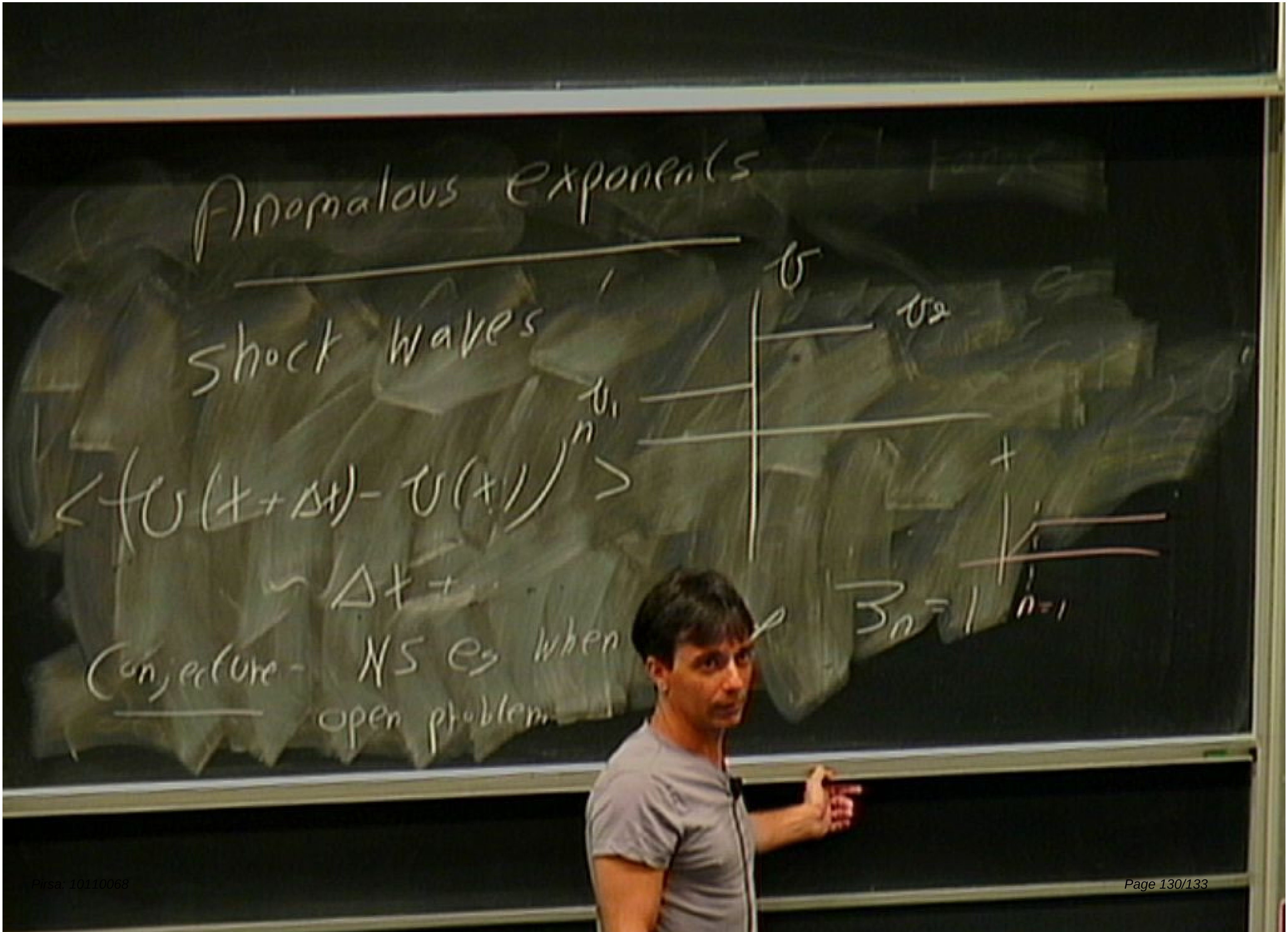
$+ F^i \rangle / v^i(0)$

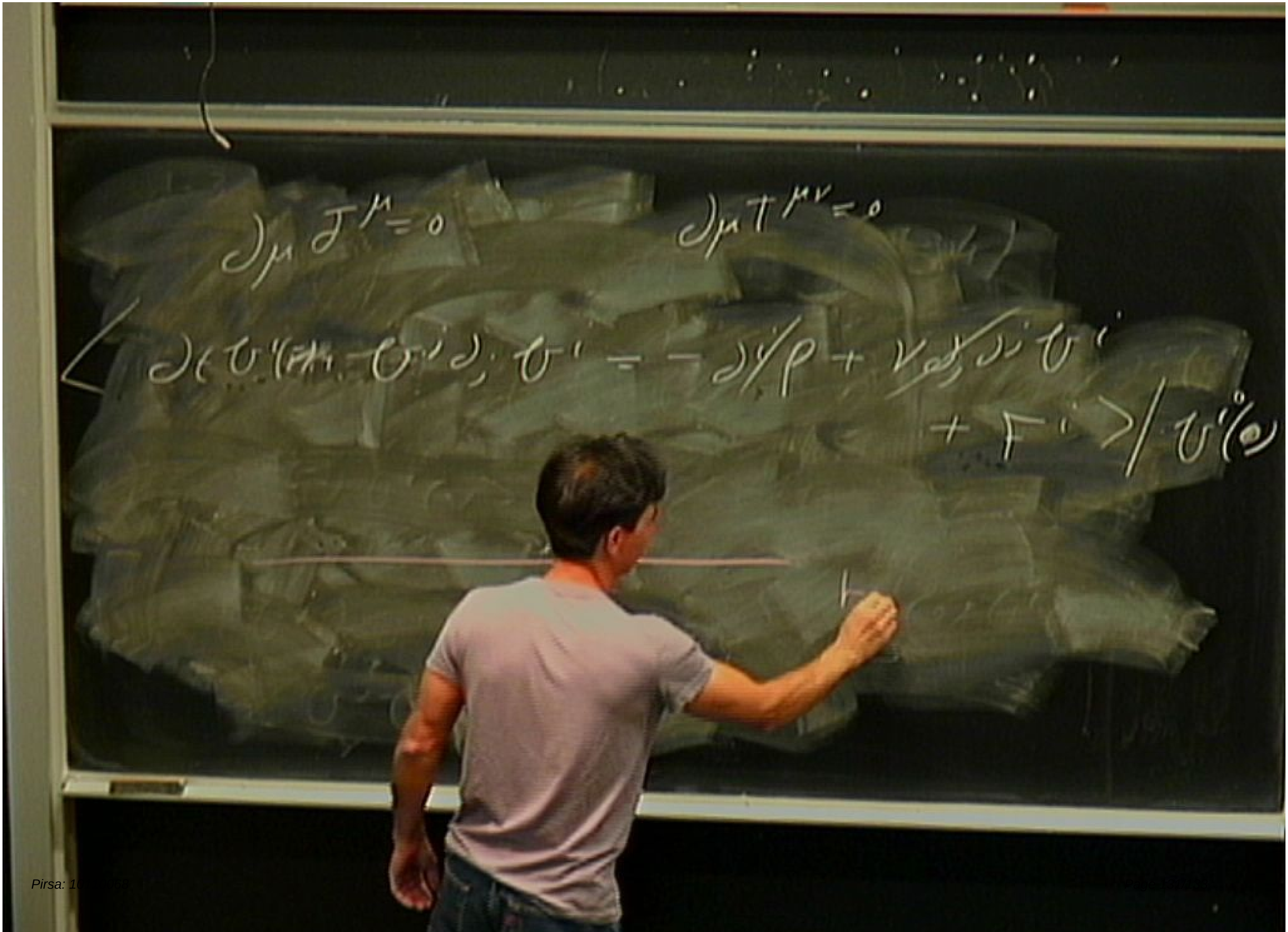
$\partial_t \langle v^i(\vec{r})\, v^i(0) \rangle \qquad \langle F^i(\vec{r})\, v^i(0) \rangle$

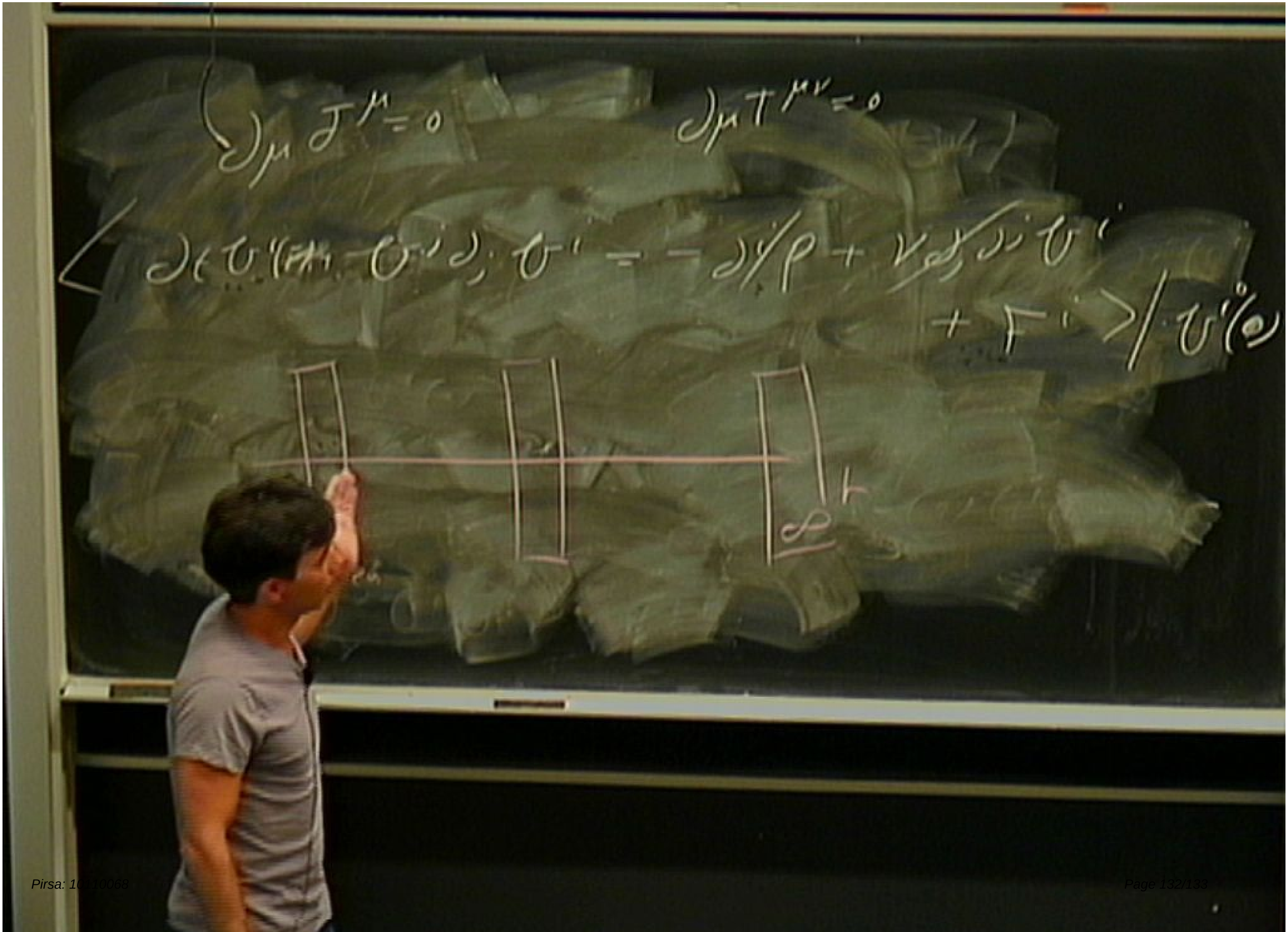
$\simeq \langle F^i(0)\, v^i(0) \rangle = \varepsilon$

$\partial_j \langle (v^i \partial_j v^i (v^i) \rangle = \varepsilon$

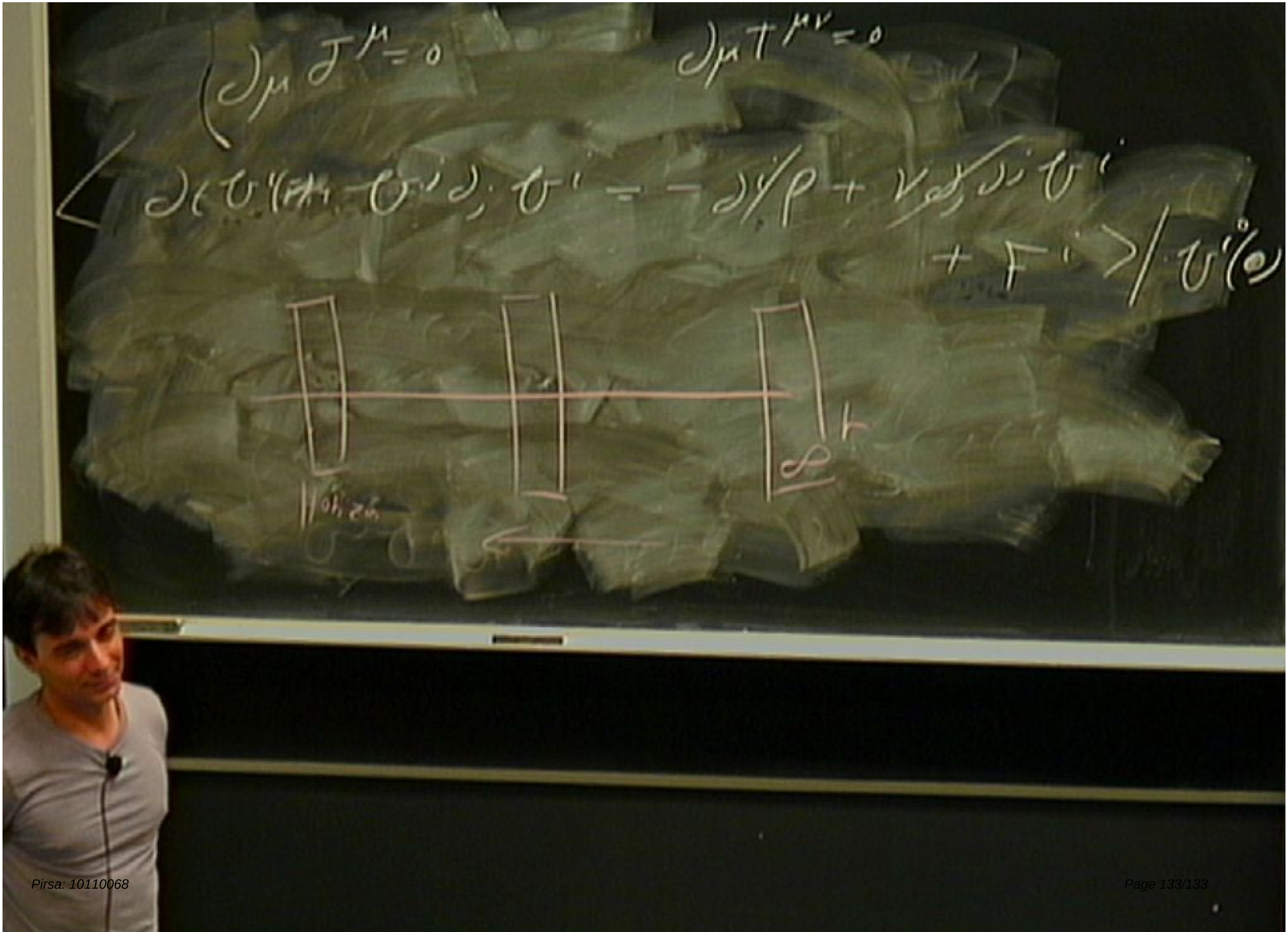
$\langle v^i v^i v^i \rangle \sim \varepsilon r^{l}$


Anomalous exponents
shock waves
$v_1$
$v_2$
$\langle (v(t+\Delta t) - v(t))^n \rangle$
$\sim \Delta t$
Conjecture - NS eq when
open problem
$\beta_n = 1$
$n=1$


$\partial_\mu J^\mu = 0$
$\partial_\mu T^{\mu\nu} = 0$



∂μ J^μ = 0
∂μ T^μν = 0


$\partial_\mu J^\mu = 0$
$\partial_\mu T^{\mu\nu} = 0$
Horizon