

Title: A Tomimatsu-Sato/CFT Correspondence

Date: Nov 16, 2010 11:00 AM

URL: <http://pirsa.org/10110066>

Abstract: We analyze the  $\delta = 2$  Tomimatsu-Sato spacetime in the context of the proposed Kerr/CFT correspondence. This 4-dimensional vacuum spacetime is asymptotically flat and has a well-defined ADM mass and angular momentum, but also involves several exotic features including a naked ring singularity, and two disjoint Killing horizons separated by a region with closed timelike curves and a rod-like conical singularity. We demonstrate that the near horizon geometry belongs to a general class of Ricci-flat metrics with  $SL(2, \mathbb{R}) \times U(1)$  symmetry that includes both the extremal Kerr and extremal Kerr-bolt geometries. We calculate the central charge and temperature for the CFT dual to this spacetime and confirm the Cardy formula reproduces the Bekenstein-Hawking entropy. We find that all of the basic parameters of the dual CFT are most naturally expressed in terms of charges defined intrinsically on the horizon, which are distinct from the ADM charges in this geometry.

Tomimatsu - Sato / CFT Correspondence?

with H. Lin, S. Saha, B. Tippett

hep-th/10102803

Tomimatsu - Sato / CFT Correspondence?

with H. Lin, S. Saha, B. Tippett

hep-th/10102803

Tomimatsu - Sato / CFT Correspondence?

with H. Lin, S. Seahra, B. Tippett

hep-th/10102803

Kerr / CFT : CFT dual to physics  
in Extremal Kerr

Tomimatsu - Sato / CFT Correspondence?

with H. Liu, S. Seahra, B. Tippett

hep-th/10102803

Kerr / CFT : CFT dual to physics  
' ' ' in Extremal Kerr  
CFT lives on boundary of NHEK.

$ADS_5 / CFT_4$

CFT on boundary of  $ADS_5$  - well understood.

AdS<sub>5</sub>/CFT<sub>4</sub>

CFT on boundary of AdS<sub>5</sub> - well understood.

Kerr = 1 soln among many stationary axisymm. asympt. flat vacuum

AdS<sub>5</sub>/CFT<sub>4</sub>

CFT on boundary of AdS<sub>5</sub> - well understood.

Kerr = 1 soln among many of stationary axisymmetric asymptotically flat Einstein eqns.



AdS<sub>5</sub>/CFT<sub>4</sub>

CFT on boundary of AdS<sub>5</sub> - well understood.

Kerr = 1 soln among many of Stationary axisymm. asympt. flat of vacuum Einstein eqns.

AdS<sub>5</sub>/CFT<sub>4</sub>

CFT on boundary of AdS<sub>5</sub> - well understood.

$\chi_{\text{evr}} = 1$  soln among many of Stationary axisymm. asympt. flat of vacuum Einstein eqns.

AdS<sub>5</sub>/CFT<sub>4</sub>

CFT on boundary of AdS<sub>5</sub> - well understood.

Keer = 1 soln among many of Stationary  
axisymm. asympt. flat of vacuum  
Einstein eqns.  
Come from Ernst eqn. integrable

nonlinear superposition of solutions

2. Classical TS

3. Quick review of Kerr/CFT

4. NHTS (NHES)

nonlinear superposition of solutions

2. Classical TS

3. Quick review of Kerr/CFT

4. NHTS (NHES)

nonlinear superposition of solutions

2. Classical TS

3. Quick review of Kerr/CFT

4. NHTS (NHES)

nonlinear superposition of solutions

2. Classical TS

3. Quick review of Kerr/CFT

4. NHTS (NHES)

5. CFT dual desc

6. Conclusions

nonlinear superposition of solutions

2. Classical TS

3. Quick review of Kerr/CFT

4. NHTS (NHES)

5. CFT dual description

6. Conclusions



# Classical TS

# Classical TS

1970's: Kramers et al.

Maisson

Belinskii & Zakharov,

...

# Classical TS

1970's: Kramers et al.

Maison

Belinskii & Zakharov,

...

WLP metric

$$ds^2 = f^{-1} \left[ e^{2b} (dt^2 + d\rho^2) + \rho^2 d\phi^2 \right] + f (dt - \omega d\phi)^2$$

## classical TS

1970's: Kramer's et al.

Maison

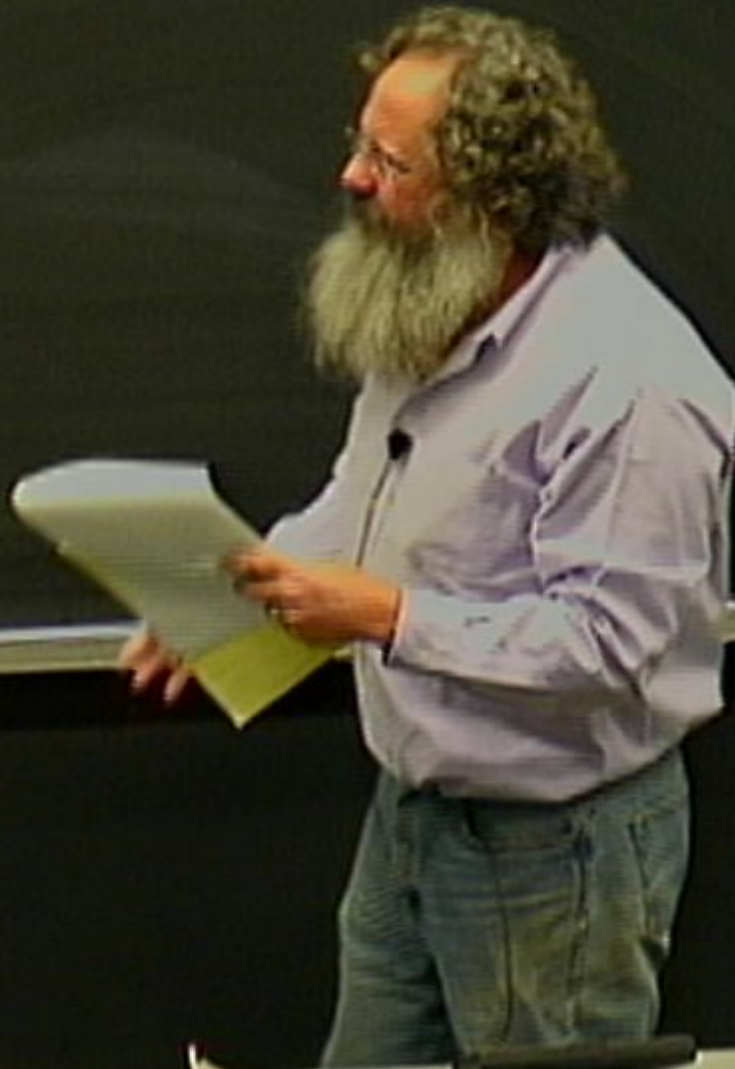
Belinskii & Zakharov,

...

WLP metric

$$ds^2 = f^{-1} \left[ e^{2b} (dt^2 + dr^2) + r^2 d\phi^2 \right] + f (dt - \omega d\phi)^2$$

$g$  = complex  $2 \times 2$  matrix  
= from  $f, \omega$



LECTURE  
SERIES  
PERSA  
10/10/06

$g$  = complex  $2 \times 2$  matrix  
= from  $f, \omega$

$g$  = complex  $2 \times 2$  matrix  
= from  $f, \omega$

$g$  obeys E-nsf:

$$\Re(g_{13} g^{-1})_{\bar{3}} + \Re(g_{\bar{3}3} g^{-1})_{3} = 0$$

$g$  = complex  $2 \times 2$  matrix  
= from  $f, \omega$

$g$  obeys E-nsf:

$$\left( (P g, \xi g^{-1}) \right)_{\bar{\xi}} + \left( (P g, \bar{\xi} g^{-1}) \right)_{\xi} = 0$$

$$\xi = 2 + iP$$



$g$  = complex  $2 \times 2$  matrix  
= from  $f, \omega$

$g$  obeys Ernst:

$$\Re(g, \xi g^{-1})_{,\bar{\xi}} + \Re(g, \bar{\xi} g^{-1})_{,\xi} = 0$$

$$\xi = z + i\rho$$

Ernst: integrable, soliton solutions

Kerr = 1-soliton

TS = superposition of two 1-solitons

$g$  = complex  $2 \times 2$  matrix  
= from  $f, \omega$

$g$  obeys Ernst:

$$\left( \rho g, \xi g^{-1} \right)_{,\bar{\xi}} + \left( \rho g, \bar{\xi} g^{-1} \right)_{,\xi} = 0$$

$$\xi = z + i\rho$$

Ernst: integrable, soliton solutions

Kerr = 1-soliton

TS = superposition of two 1-solitons

3 real parameters:  $\sigma, p, q, p^2 + q^2 = 1$

$\delta$  - soliton number

3 real parameters:  $\sigma, p, q, p^2 + q^2 = 1$

$\delta$  = soliton number

$\delta = 1$  Kerr

$\delta = 2$  TS

3 real parameters:  $\sigma, p, q, p^2 + q^2 = 1$

$\delta = 1$  soliton number

$\delta = 1$  Kerr

$\delta = 2$  TS (2)

Far field region

3 real parameters:  $\sigma, p, q, p^2 + q^2 = 1$

$\delta = 1$  soliton number

$\delta = 1$  Kerr

$\delta = 2$  TS (2)

Far field region

$$M_{\text{ADM}} = \frac{2\sigma}{Gp}$$

$$I_{\text{ADM}} = \frac{4\sigma^2 q}{Gp^2} = q G M_{\text{ADM}}^2$$

3 real parameters:  $\sigma, p, q, p^2 + q^2 = 1$

$\delta = 1$  soliton number

$\delta = 1$  Kerr

$\delta = 2$  TS (2)

Far field region

$$M_{\text{ADM}} = \frac{2\sigma}{Gp}$$

$$|J_{\text{ADM}}| = \frac{4\sigma^2 q}{Gp^2} = q G M_{\text{ADM}}^2$$

3 real parameters:  $\sigma, p, q, p^2 + q^2 = 1$

$\delta = 1$  soliton number

$\delta = 1$  Kerr

$\delta = 2$  TS (2)

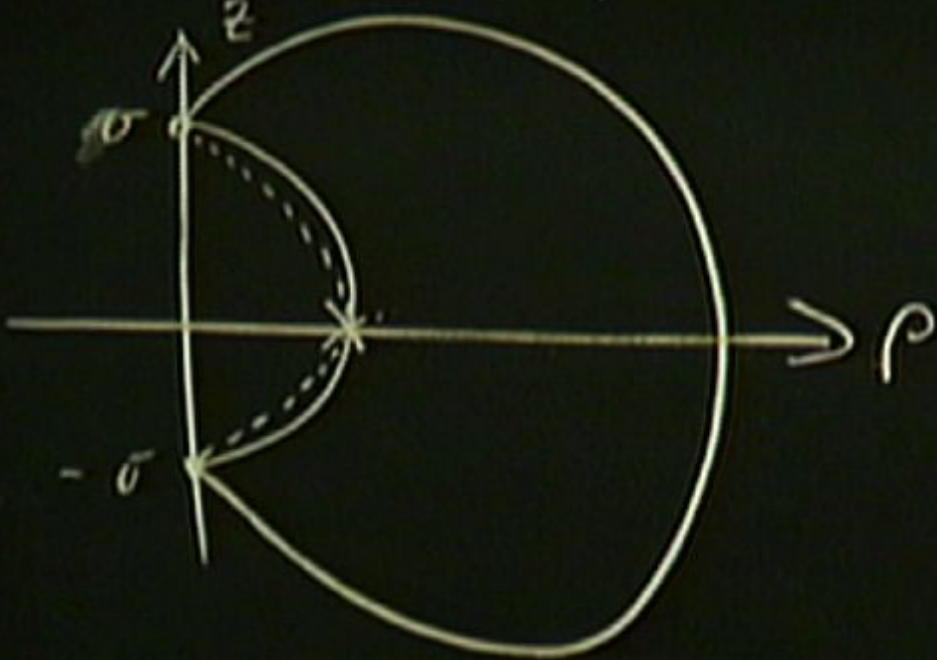
Far field region

$$M_{\text{ADM}} = \frac{2\sigma}{Gp}$$

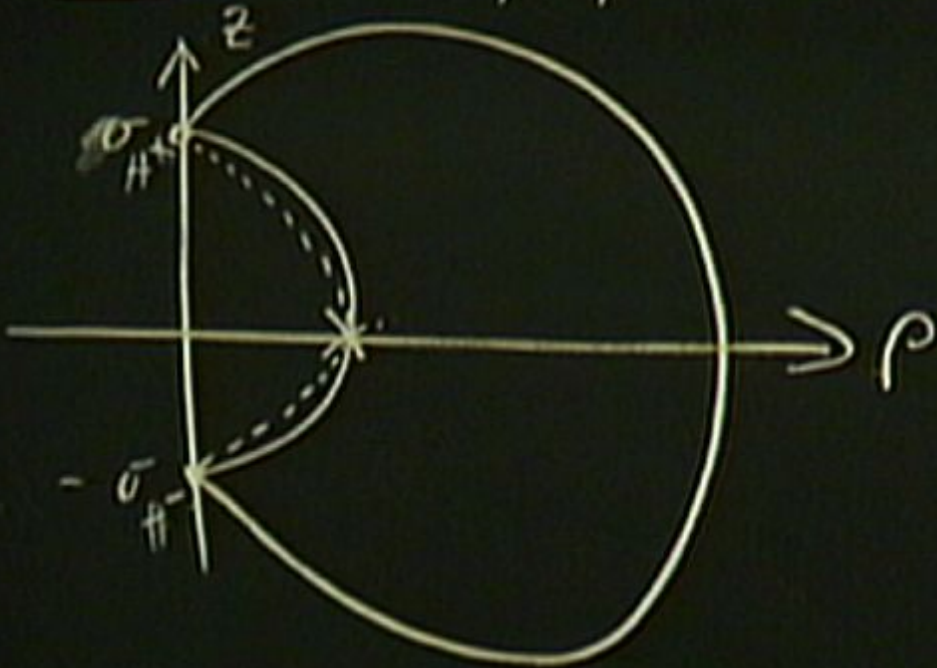
$$|J_{\text{ADM}}| = \frac{4\sigma^2 q}{Gp^2} = q G M_{\text{ADM}}^2$$



$KV's$   $\partial_L, \partial_H$

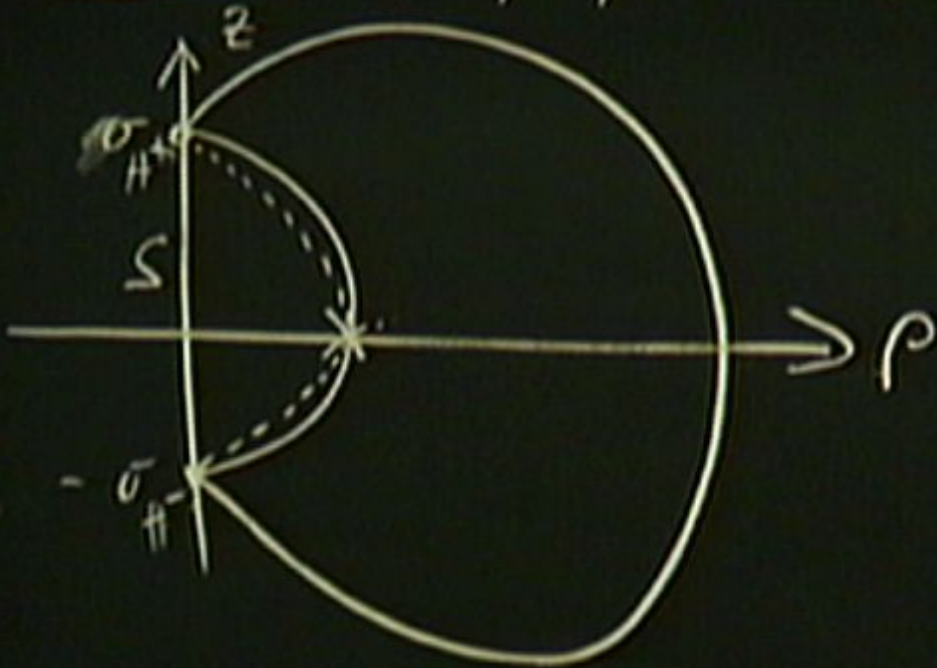


$KV's$   $\partial_L, \partial_P$



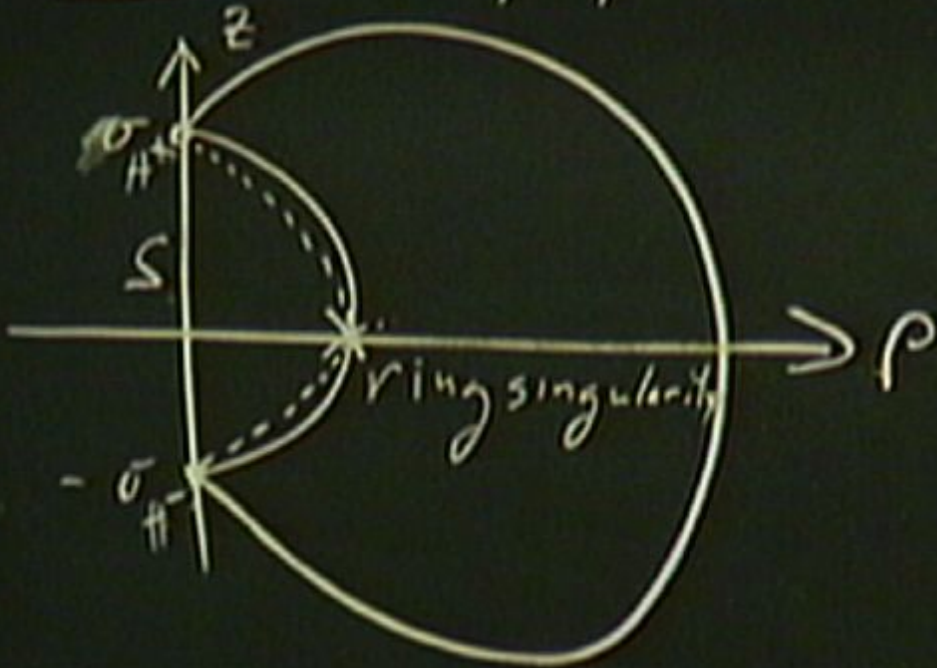
$KV's$   $\partial_t, \partial_\phi$

$H^\pm$  Killing horizon  
 $S$  strut



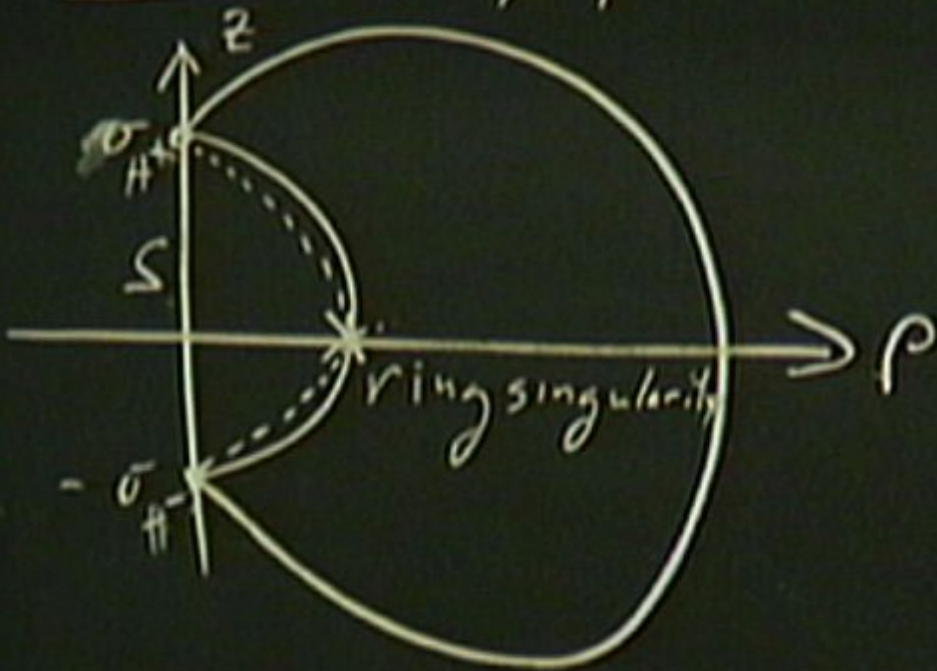
KV's  $\partial_t, \partial_\phi$

$H^\pm$  Killing horizon  
 $S$  strat



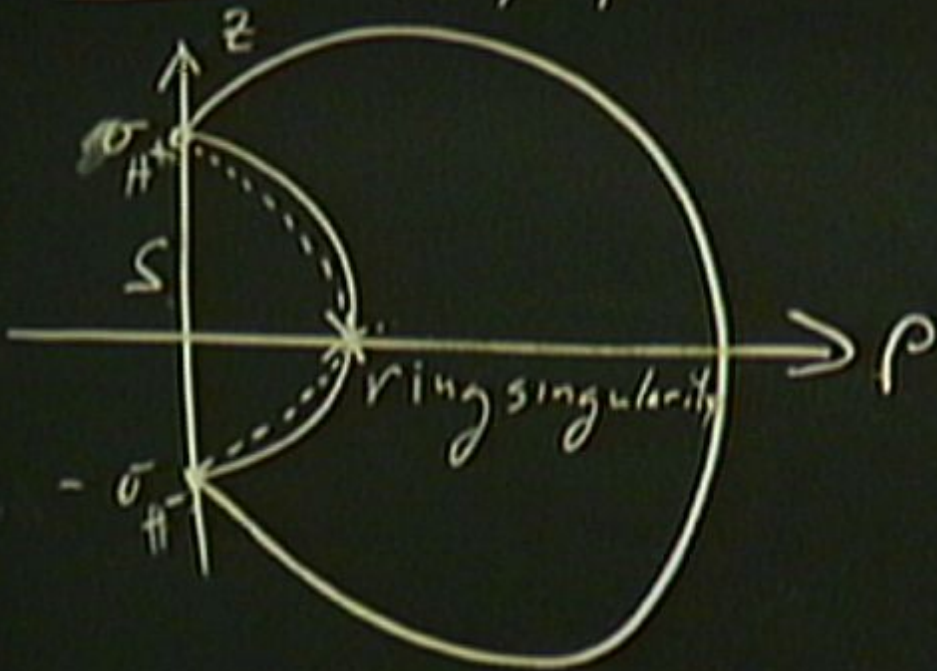
$KV's$   $\partial_t, \partial_\phi$

$H^\pm$  Killing horizon  
 $S$  strut



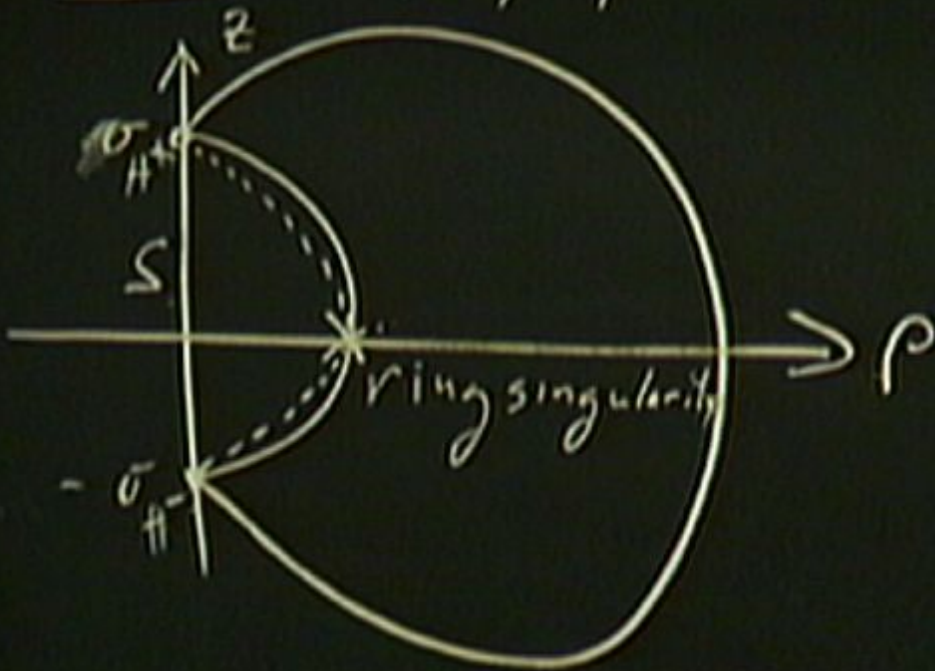
KV's  $\partial_t, \partial_\phi$

$H^\pm$  Killing horizon  
S strut



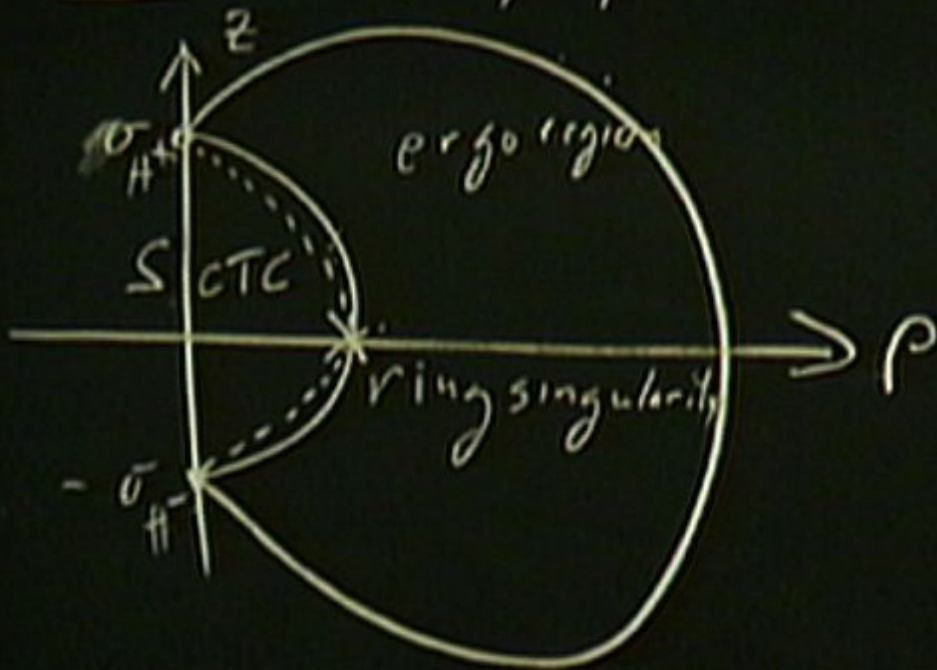
KV's  $\partial_t, \partial_\phi$

$H^\pm$  Killing horizon  
S strut

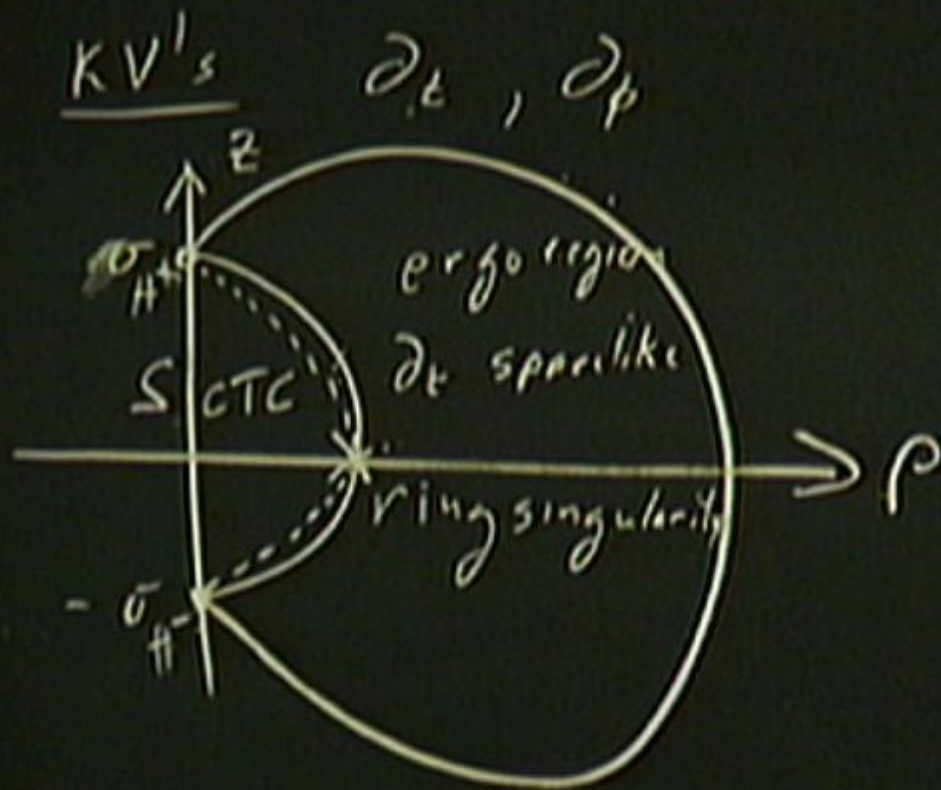


KV's  $\partial_t, \partial_\phi$

$H^\pm$  Killing horizon  
S strut







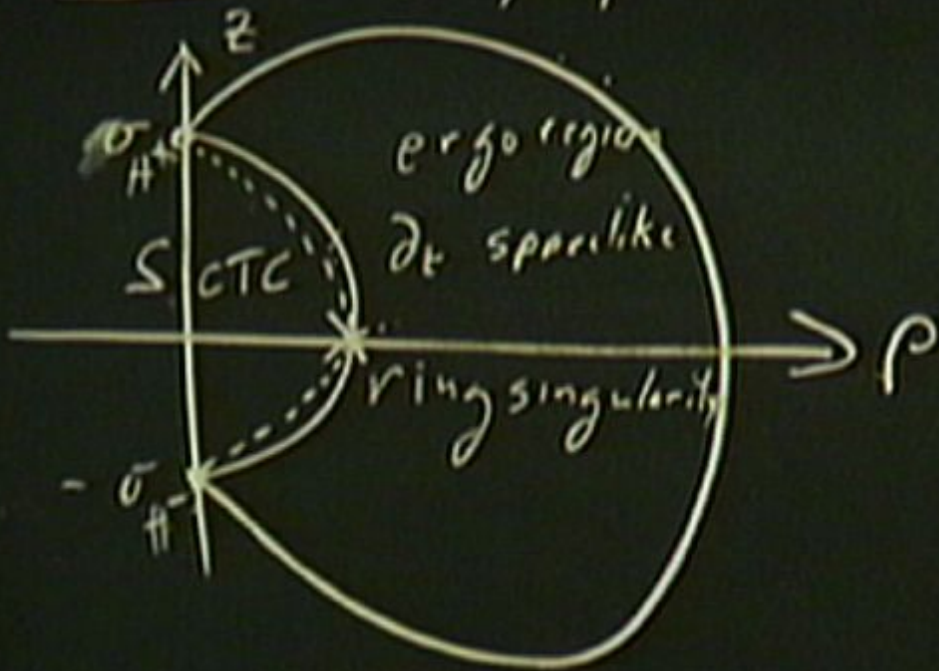
$H^\pm$  Killing horizon  
S start  
Folium

KV's

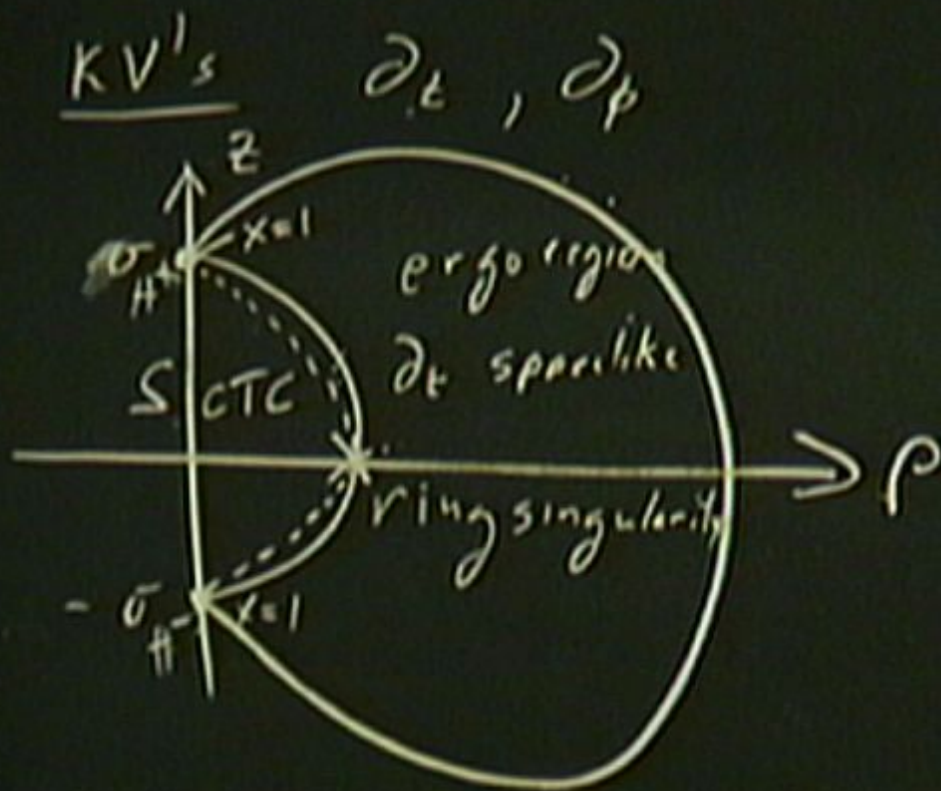
$\partial_t, \partial_\phi$

$H^\pm$

Killing horizon  
start



Fu - field  
S



$H^\pm$  Killing horizon  
 S strat  
 Pen. field

---

Compute Komar Mass/Ang.  
 $\partial \Sigma \rightarrow \rho, t$   
 probe  
 $\chi = \chi_0 > 1$   
 $\partial \Sigma = \{ \chi = \chi_0 > 1 \}$

$M_{x_0} = M_{ADM}$  | All mass & ang. mom. contained  
 $J_{x_0} = J_{ADM}$  | inside  $H^{\pm}$ ,  $S$ .

$M_{x_0} = M_{ADM}$  | All mass & ang. mom. contained  
 $J_{x_0} = J_{ADM}$  | inside  $H_{\pm}^{\pm}$ ,  $S$ .

$M_{\pm} = \text{mass inside } H_{\pm}^{\pm} = 0$

$$M_{x_0} = M_{ADM} \quad \left| \quad \text{All mass \& ang. mom. contained} \right.$$

$$J_{x_0} = J_{ADM} \quad \left| \quad \text{inside } H_{\pm}^{\pm}, S. \right.$$

$$M_{\pm} = \text{mass inside } H_{\pm}^{\pm} = 0$$

$$M_S = M_{ADM} \quad \left| \quad J_S = \left(1 - \frac{p}{2}\right) J_{ADM} \right.$$

$$J_I = \frac{\sigma^2}{6p} \sqrt{\frac{1+p}{1-p}}$$

$M_{x_0} = M_{ADM}$  | All mass & ang. mom. contained  
 $J_{x_0} = J_{ADM}$  | inside  $H^{\pm}$ ,  $S$ .

$M_{\pm} = \text{mass inside } H_{\mp}^{\pm} = 0$

$M_S = M_{ADM}$  |  $J_S = \left(1 - \frac{p}{2}\right) J_{ADM}$   
 $J_I = \frac{\sigma^2}{6p} \sqrt{\frac{1+p}{1-p}}$

3. Kerr/CFT

Kerr characterized by M&T



### 3. Kerr/CFT

Kerr characterized by  $M$  &  $J$ .

Extreme Kerr:  $J = M^2/G$ .

NHEK

### 3. Kerr/CFT

Kerr characterized by  $M$  &  $J$ .

Extreme Kerr:  $J = M^2/G$ .

NHEK

$r=0$  Horizon



### 3. Kerr/CFT

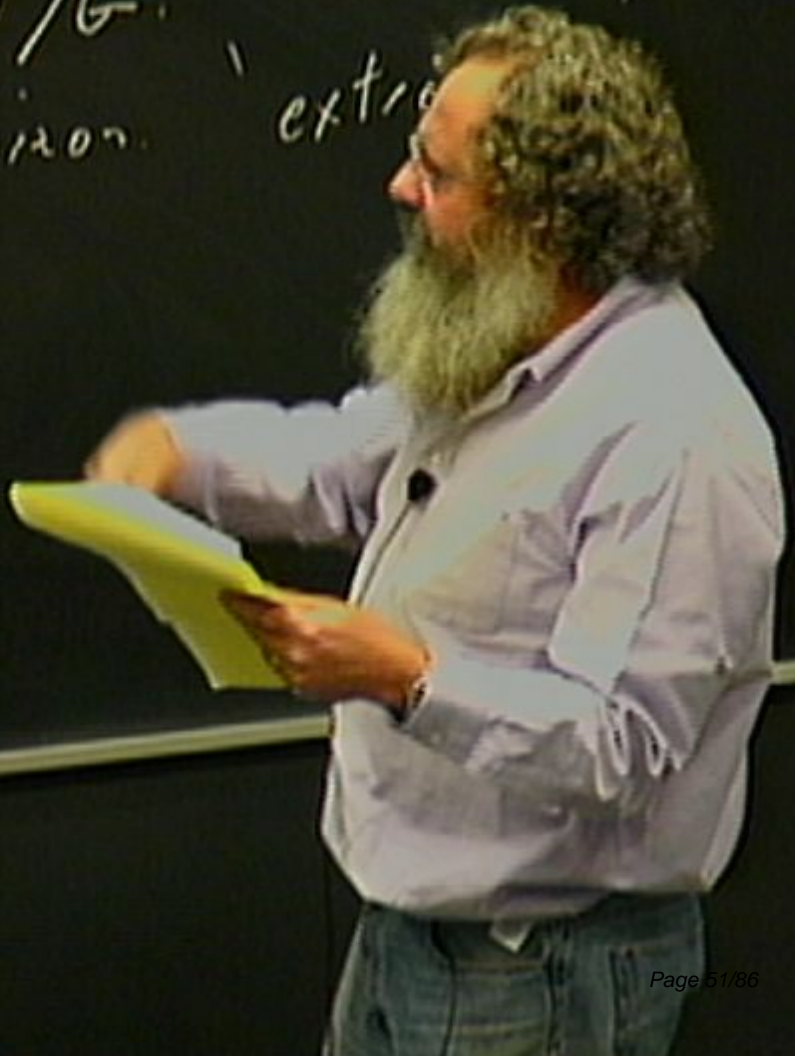
Kerr characterized by  $M$  &  $J$ .

Extreme Kerr:  $J = M^2/G$ .

NHEK

$r=0$  Horizon

'extremal'



### 3. Kerr/CFT

Kerr characterized by  $M$  &  $J$ .

Extreme Kerr:  $J = M^2/G$ .

NHEK  $r=0$  Horizon 'extremal'

$$ds^2 = \left( \frac{1 + \cos^2 \theta}{2} \right) \left[ -\frac{r^2}{r_0^2} dt^2 + \frac{r_0^2}{r^2} dr^2 + r_0^2 d\theta^2 \right] + \frac{2r_0 \sin^2 \theta}{(1 + \cos^2 \theta)} \left[ d\phi + \frac{r}{r_0} dt \right]^2$$

Symm. group  $\cong (SU(2,1) \times U(1))$

Symm. group  $\cong (SU(2,1) \times U(1))$

AdS<sub>2</sub> × S<sup>2</sup>

deform. :  $g_{\mu\nu} = g_{\mu\nu}^{EK} + h_{\mu\nu}$

Symm. group  $\cong (SU(2,1) \times U(1))$

AdS<sub>2</sub> × S<sup>2</sup>

deform:  $g_{\mu\nu} = g_{\mu\nu}^{EK} + h_{\mu\nu}$   
fall-off for  $h_{\mu\nu}$  prescribed

Asympt. Symm group of  $g_{\mu\nu}$

Symm. group  $\cong (SU(2,1) \times U(1))$

$AdS_3 \times S^2$

deform. :  $g_{\mu\nu} = g_{\mu\nu}^{EK} + h_{\mu\nu}$   
fall-off for  $h_{\mu\nu}$  prescribed

Asympt. Symm group of  $g_{\mu\nu}$  = conformal group.



Symm. group  $\cong (SL(2, \mathbb{R}) \times U(1))$

AdS<sub>2</sub>  $\times$  S<sup>2</sup>

deform:  $g_{\mu\nu} = g_{\mu\nu}^{EK} + h_{\mu\nu}$

fall-off for  $h_{\mu\nu}$  prescribed

Asympt. Symm group of  $g_{\mu\nu}$  = conformal group.  
central charge

$$C_R = 0, \quad C_L = \frac{12J}{\hbar}$$

CFT in asympt. region of NHEK.

4. NAT'S (NHES)

---

#### 4. NATS (NHES)

'bipolar coords'  $(\bar{r}, \bar{\theta}) \longleftrightarrow (x, y)$  prolate

$$x^2 = \frac{0}{\sigma - \sqrt{2} \bar{r} \cos^2 \theta/2}, \quad y^2 = \frac{\sigma - \sqrt{2} \bar{r}}{\sigma - \sqrt{2} \bar{r} \cos^2 \theta/2}$$

#### 4. NATS (NHES)

'bipolar coords'  $(\bar{r}, \bar{\theta}) \longleftrightarrow (x, y)$  prolate

$$x^2 = \frac{0}{\sigma - \sqrt{2} \bar{r} \cos^2 \bar{\theta}/2}, \quad y^2 = \frac{\sigma - \sqrt{2} \bar{r}}{\sigma - \sqrt{2} \bar{r} \cos^2 \bar{\theta}/2}$$

Expand around  $\bar{r} = 0$

#### 4. NATS (NHES)

'bipolar coords'  $(\bar{r}, \bar{\theta}) \longleftrightarrow (x, y)$  prolate

$$x^2 = \frac{0}{\sigma - \sqrt{2} \bar{r} \cos^2 \theta/2}, \quad y^2 = \frac{\sigma - \sqrt{2} \bar{r}}{\sigma - \sqrt{2} \bar{r} \cos^2 \theta/2}$$

Expand around  $\bar{r} = 0$  |  $H^\pm$

$$ds^2 = \Gamma(\theta) \left[ \frac{-\bar{r}^2}{r_0} dt^2 + \frac{r_0}{\bar{r}^2} d\bar{r}^2 + r_0 d\theta^2 \right] + \frac{\frac{1}{2} \sin^2 \theta}{\Gamma(\theta)} \left( \frac{\bar{r}}{r_0} dt - r_0 d\phi \right)^2$$

$$\Gamma(\theta) = \frac{\alpha}{2} (1 + \cos^2 \theta) + \beta \cos \theta$$

$$\gamma = \sqrt{\alpha^2 - \beta^2}$$

For TS:  $\alpha = \frac{1}{2\rho^2} \rightarrow \beta = \frac{1}{\rho^2}$

$$\Gamma(\theta) = \frac{\alpha}{2} (1 + \cos^2 \theta) + \beta \cos \theta$$

$$\gamma = \sqrt{\alpha^2 - \beta^2}$$

For TS:  $\alpha = \frac{1}{2\rho^2} \Rightarrow \beta = 1 - \frac{1}{2\rho^2}$

$$r_0^2 = \frac{\sqrt{2} \sigma \sqrt{\rho+1}}{\rho} = \sqrt{2\gamma J_\Delta}$$

$J_\Delta = \text{ang. mom.}$   
horizon

$$\Gamma(\theta) = \frac{\alpha}{2} (1 + \cos^2 \theta) + \beta \cos \theta$$

$$\gamma = \sqrt{\alpha^2 - \beta^2}$$

For TS:  $\alpha = \frac{1}{2\rho^2}$ ,  $\beta = 1 - \frac{1}{2\rho^2}$

$$r_0^2 = \frac{\sqrt{2} \sigma \sqrt{\rho+1}}{\rho} = \frac{\sqrt{2\gamma J_\Delta}}{\rho}$$

$$\text{conical deficit} = 2\pi [1 + (\alpha \pm \beta)]$$

$J_\Delta = \text{ang. mom. on horizon}$



NHTS

NHEK

NHEKB

$$\alpha = \frac{1}{2\rho^2}$$

$$\beta = 1 - \frac{1}{2\rho}$$

$$\gamma = \frac{1}{\rho}$$

$$\gamma_0 = \frac{1}{2}$$

a



	$\alpha$	$\beta$	$\gamma$	$\gamma_0$
NHTS	$\frac{1}{2\rho^2}$	$1 - \frac{1}{2\rho^2}$	$\frac{1}{\rho}$	$2\delta J_0$
NHEK	1	0	1	2J
NHEKB				



	$\alpha$	$\beta$	$\gamma$	$\gamma_0$
NHTS	$\frac{1}{2\rho^2}$	$1 - \frac{1}{2\rho^2}$	$\frac{2}{\rho}$	$2\delta J\Delta$
NHEK	1	0	1	$2J$
NHEKB	1	$\frac{N}{a}$	$\left(1 - \frac{N^2}{a^2}\right)^{1/2}$	$2a^2$



<u>NHES</u>	$\alpha$	$\beta$	$\gamma$	$\gamma_0$
NHTS	$\frac{1}{2\rho^2}$	$1 - \frac{1}{2\rho^2}$	$\frac{2}{\rho}$	$2\delta J_0$
NHEK	1	0	1	$2J$
NHEKB	1	$\frac{N}{a}$	$\left(1 - \frac{N^2}{a^2}\right)^{1/2}$	$2a^2$

a. NHES:  $R_{\mu\nu} = 0$

<u>NHES</u>	$\alpha$	$\beta$	$\gamma$	$\gamma_0$
NHTS	$\frac{1}{2\rho^2}$	$1 - \frac{1}{2\rho^2}$	$\frac{2}{\rho}$	$2\delta J_0$
NHEK	1	0	1	$2J$
NHEKB	1	$\frac{N}{a}$	$(1 - \frac{N^2}{a^2})^{1/2}$	$2a^2$

a. NHES:  $R_{\mu\nu} = 0$

NHTS:  $A_{\Delta} = 4\pi r_0^2$   
 $A_{\Delta} = 4\pi \left[ G^2 M^2 + \sqrt{G^4 M^4 - G^2 J^2} \right] = \frac{1}{2} A^{Kerr}$

<u>NHES</u>	$\alpha$	$\beta$	$\gamma$	$\gamma_0$
NHTS	$\frac{1}{2\rho^2}$	$1 - \frac{1}{2\rho^2}$	$\frac{2}{\rho}$	$2\delta J_\Delta$
NHEK	1	0	1	$2J$
NHEKB	1	$\frac{N}{a}$	$(1 - \frac{N^2}{a^2})^{1/2}$	$2a^2$

a. NHES:  $R_{\mu\nu} = 0$

NHTS:  $A_\Delta = 4\pi r_0^2$   
 $A_\Delta = 4\pi \left[ G^2 M^2 + \sqrt{G^4 M^4 - G^2 J^2} \right] = \frac{1}{2} A^{Kerr}$

<u>NHES</u>	$\alpha$	$\beta$	$\gamma$	$\gamma_0$
NHTS	$\frac{1}{2\rho^2}$	$1 - \frac{1}{2\rho^2}$	$\frac{2}{\rho}$	$2\delta J_0$
NHEK	1	0	1	$2J$
NHEKB	1	$\frac{N}{a}$	$(1 - \frac{N^2}{a^2})^{1/2}$	$2a^2$

a. NHES:  $R_{\mu\nu} = 0$

NHTS:  $A_{\Delta} = 4\pi r_0^2$   
 $A_{\Delta}^{(total)} = 4\pi \left[ \sqrt{G^2 M^2 + \sqrt{G^4 M^4 - G^2 J^2}} \right] = \frac{1}{2} A^{Kerr}$

15. CFT Description

(1)  $\mu$

H+



VS. CFT description

Most general diffeos preserving 'fall-off'  
as  $r \rightarrow \infty$



# 45. CFT description

Most general diffeos preserving 'fall-off'

as  $r \rightarrow \infty$

$$\xi^\alpha = \rho^\alpha + \chi^\alpha$$

$$\rho^\alpha \partial_\alpha = [\varepsilon(\phi) + o(r^{-1})] \partial_\phi + [r \varepsilon'(\phi) + o(r^0)] \partial_r$$

$$\chi^\alpha \partial_\alpha = [c + o(r^{-1})] \partial_\tau$$

# 45. CFT description

Most general diffeos preserving 'fall-off'

$$\xi^\alpha = \rho^\alpha + \chi^\alpha \quad \text{periodic } \sim 2\pi/\delta$$

$$\rho^\alpha \partial_\alpha = [\varepsilon(\phi) + o(r^{-1})] \partial_\phi + [r \varepsilon'(\phi) + o(r^0)] \partial_r$$

$$\chi^\alpha \partial_\alpha = [c + o(r^{-1})] \partial_\tau$$

$$\tau = \frac{\Omega_S t - \phi}{\Omega_S}$$

$$\Omega_S := \frac{\rho}{4G} \sqrt{\frac{1-f}{11f}}$$

# 45. CFT description

Most general diffeos preserving 'fall-off'

$$\xi^\alpha = \rho^\alpha + \chi^\alpha \quad \text{periodic } \sim 2\pi/\delta$$

$$\rho^\alpha \partial_\alpha = [\epsilon(\phi) + o(r^{-2})] \partial_\phi + [r \epsilon'(\phi) + o(r^0)] \partial_r$$

$$\chi^\alpha \partial_\alpha = [c + o(r^{-2})] \partial_\tau \quad \left| \tau = \frac{\Omega_S t - \phi}{\Omega_S} \right.$$

$$\Omega_S := \frac{p}{4\delta} \sqrt{\frac{1-f}{11f}}$$

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\delta m \phi}$$

$$m = 0, \pm 1, \dots$$

$$y_m = r^{-1} e^{i\delta m \phi} (\partial_\phi - i\delta m r \partial_r)$$

$$\Sigma(\phi) = \frac{1}{2} e^{i\delta m \phi}$$

$$m = 0, \pm 1, \dots$$

$$y_m = r^{-1} e^{i\delta m \phi} (\partial_\phi - i\delta m r \partial_r)$$

Satisfies vir

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\gamma m \phi}$$

$$m = 0, \pm 1, \dots$$

$$y_m = \gamma^{-1} e^{i\gamma m \phi} \left( \frac{\partial \phi}{\partial r} - i\gamma m r \frac{\partial}{\partial r} \right)$$

Satisfy virasoro

$$\psi(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\delta m \phi}$$

$m = 0, \pm 1, \dots$

$$y_m = r^{-1} e^{i\delta m \phi} (\partial_\phi - i\delta m r \partial_r)$$

Sat

$$-\frac{1}{4} \epsilon_{\alpha\beta\gamma\delta} [ \dots ] dx^\alpha \wedge dx^\beta \wedge dx^\gamma \wedge dx^\delta$$

key L

Q



$$\zeta(\phi) = \frac{1}{\sqrt{2\pi}} e^{i\delta m \phi}$$

$m = 0, \pm 1, \dots$

$$y_m = \gamma^{-1} e^{i\delta m \phi} (\partial \phi - i\delta m r \partial r)$$

Satisfy Virasoro

$$L_y[h, g] = -\frac{1}{4} \int \Sigma_{\alpha\beta\gamma\nu} \left[ \gamma^{\nu\lambda} \nabla^{\alpha} h + \dots \right] dx^{\alpha} dx^{\beta}$$

$$Q_{y_m} \text{ satisfy } \{ Q_{y_m}, Q_{y_n} \} = i(m-n) Q_{y_m} y_n$$

$$+ \frac{1}{8\pi G} \int ds L_{y_m}[f, g, g]$$

$$\epsilon(\phi) = \frac{1}{\hbar} \mathcal{L}(\phi) + i\delta m b$$

$$m = 0, \pm 1, \dots$$

$$y_m = \gamma^{-1} e^{i\delta m \phi} (\partial \phi - i\delta m r \partial r)$$

Satisfy Virasoro

$$L_y[h, g] = -\frac{1}{4} \epsilon_{\alpha\beta\gamma\nu} \left[ \dots \right] dx^\alpha dx^\beta$$

$$Q_{y_m} \text{ satisfy } \{ Q_{y_m}, Q_{y_n} \} = -i(m-n) Q_{y_m} y_n$$

$$+ \frac{1}{8\pi G} \int ds \text{ (const } t, r)$$

$$\Sigma(\phi) = \frac{1}{4} \sum_{m=0, \pm 1, \dots} e^{i\sigma m \phi}$$

$$y_m = \gamma^{-1} e^{i\sigma m \phi} (\partial \phi - i\sigma m r \partial r)$$

Satisfy Virasoro

$$L_y[h, g] = -\frac{1}{4} \sum_{\alpha\beta\gamma\nu} [y^\nu \Delta^\alpha h + \dots] dx^\alpha dx^\beta$$

$$Q_{y_m} \text{ satisfy } \{Q_{y_m}, Q_{y_n}\} = i(m-n)Q_{y_m} y_n$$

$$+ \frac{1}{8\pi G} \int_{\partial\Sigma} \text{const } t, r, r =$$

$$C = \frac{6\gamma r_0^2}{G} \approx 12\gamma^2 J_{\Delta}$$

$$\epsilon(\phi) = \frac{1}{2} \epsilon^{im} \partial_\mu \phi \partial^\mu \phi$$

$m = 0, \pm 1, \dots$

$$\psi_m = \gamma^{-1} e^{i\alpha m \phi} (\partial_\mu \phi - i\alpha m r \partial_r)$$

Satisfy Virasoro

$$L_\mu [h, g] = -\frac{1}{4} \epsilon_{\alpha\beta\gamma\nu} \left[ \gamma^\nu \nabla^\alpha h + \dots \right] dx^\alpha dx^\beta$$

$Q_{\psi_m}$  satisfy  $\{ Q_{\psi_m}, Q_{\psi_n} \} = i(m-n) Q_{\psi_m} Q_{\psi_n}$

PB  $\left[ L_\mu [f, g, g] \right]$   
 $+\frac{1}{8\pi G} \partial_\mu \text{const } t, r, r =$

$$C = \frac{6\gamma r_0^2}{G} \approx 12\gamma^2 J_{\Delta}$$