Title: Torsion as a Probe in Condensed Matter Systems

Date: Nov 12, 2010 11:00 AM

URL: http://pirsa.org/10110065

Abstract: : In this talk I will review the common appearance of torsion in solids as well as some new developments.
Torsion typically appears in condensed matter physics associated to topological defects known as dislocations. Now we are beginning to uncover new aspects of the coupling of torsion to materials. Recently, a dissipationless viscosity has been studied in the quantum Hall effect. I will connect this viscosity to a 2+1-d torsion Chern-Simons term and discuss possible thought experiments in which this could be measured. Additionally I will discuss a new topological defect in 3+1-d, the torsional monopole, which does not require a lattice deformation to exist. If present, torsional monopoles are likely to impact the behavior of materials with strong spin-orbit coupling such as topological insulators.

Applications of Torsion in Condensed Matter Physics

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UIUC

Perimeter Institute

November 12, 2010

In collaboration with Rob Leigh and Eduardo Fradkin, and Andy Randono

Hughes, Leigh, Fradkin (in preparation)

Randono, Hughes arxiv: 1010.1031

Overview

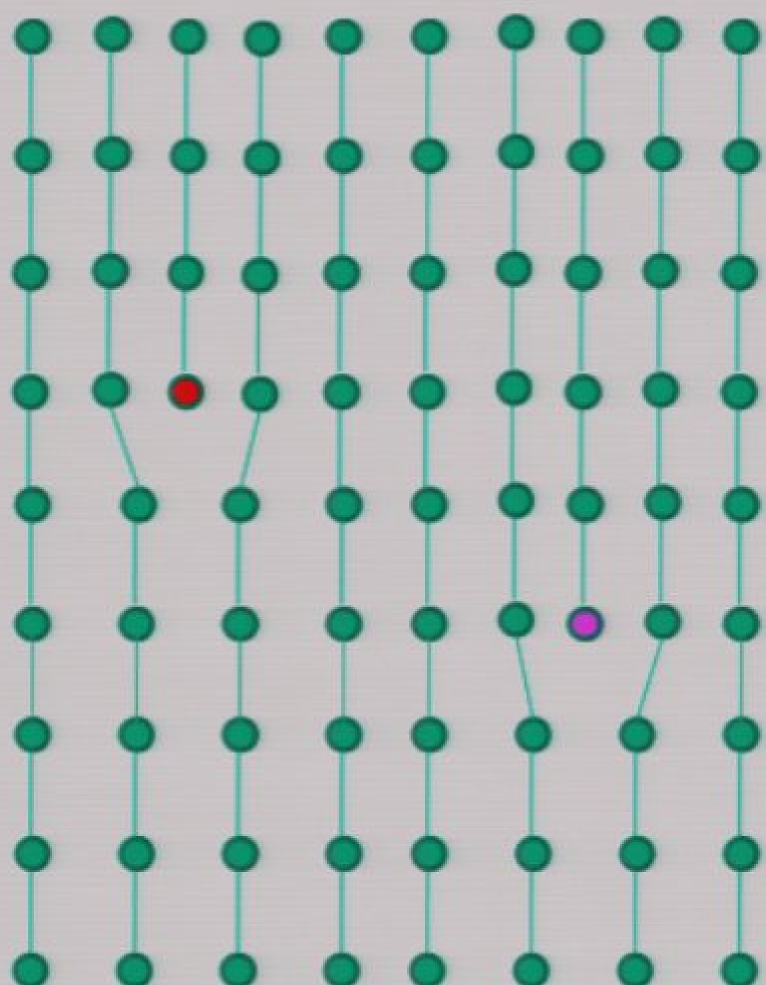
Part 1: Dislocations as sources of torsion

Part 2: Torsional Response of Topological Insulators

Part 3: Torsional Monopoles in 3+1-d

Part 1: Dislocations

Crystal Dislocations



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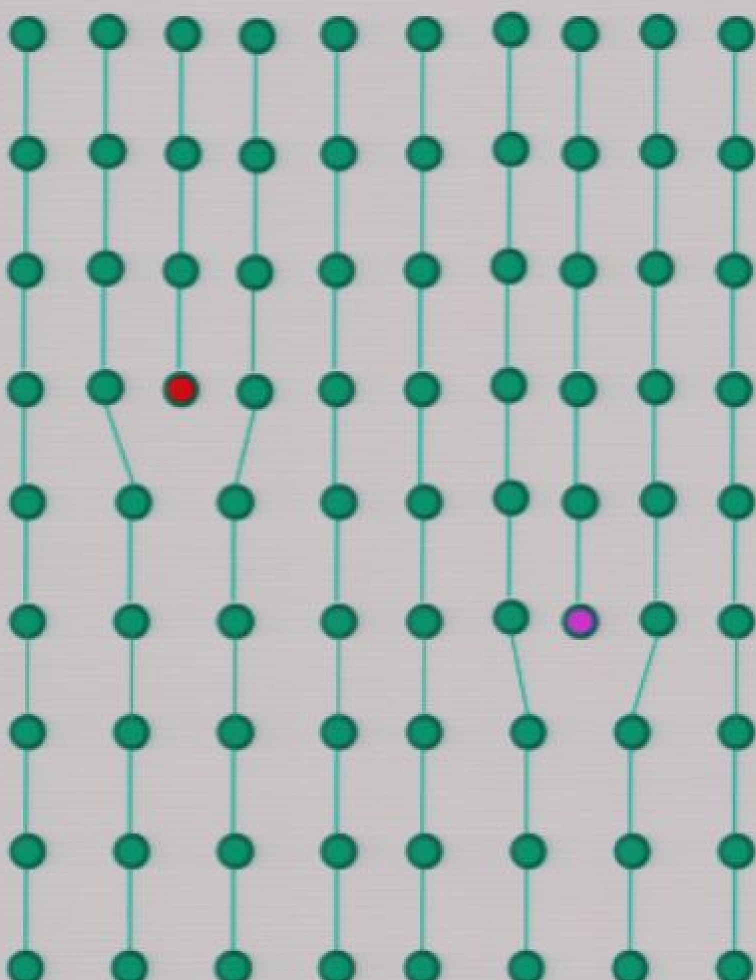

Let's take a path in the lattice
3 steps right
3 steps up
3 steps left
3 steps down
This path is closed in the reference state.

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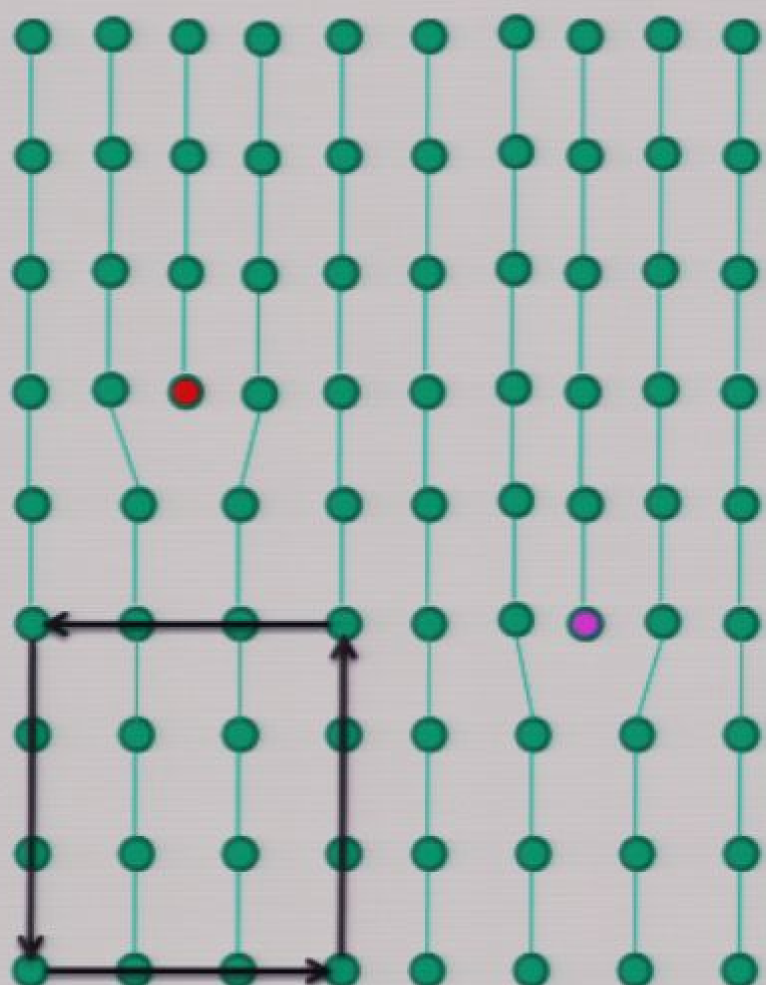

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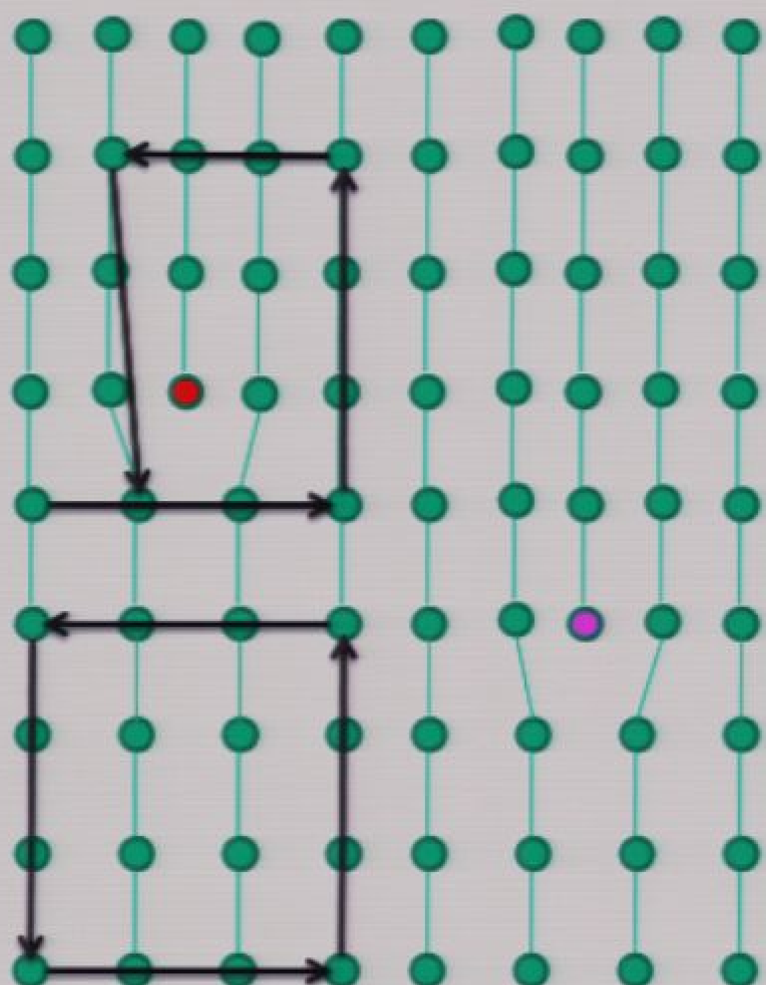

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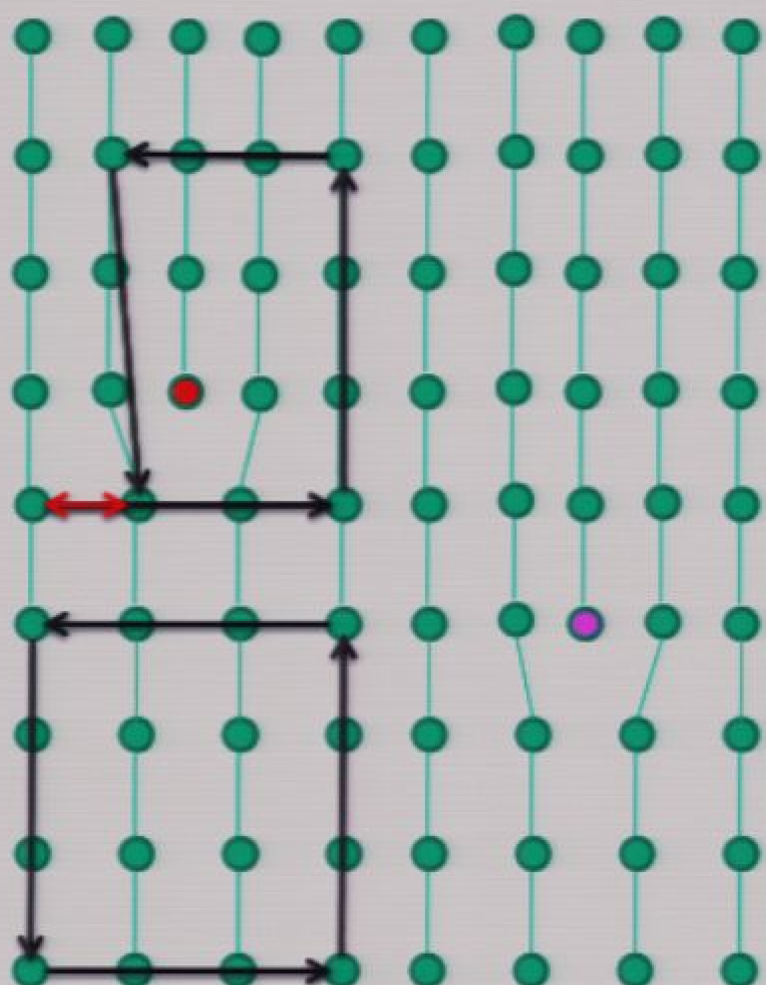

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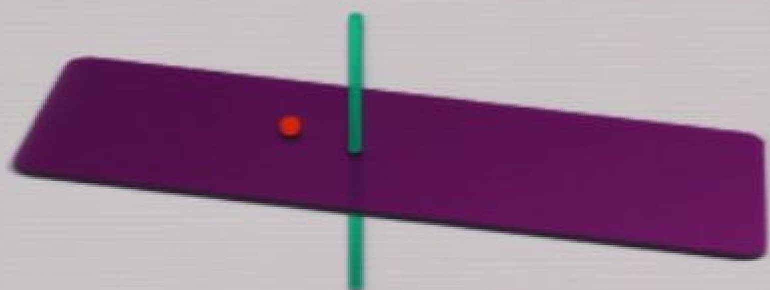
The amount of translation is the Burgers vector and it is a vector of topological charges. It doesn't change if you continuously deform the dislocation.

Crystal Dislocations and Aharonov-Bohm Phases



Magnetic flux gives a U(1) phase

$$U = \exp\left[i\phi\right]$$

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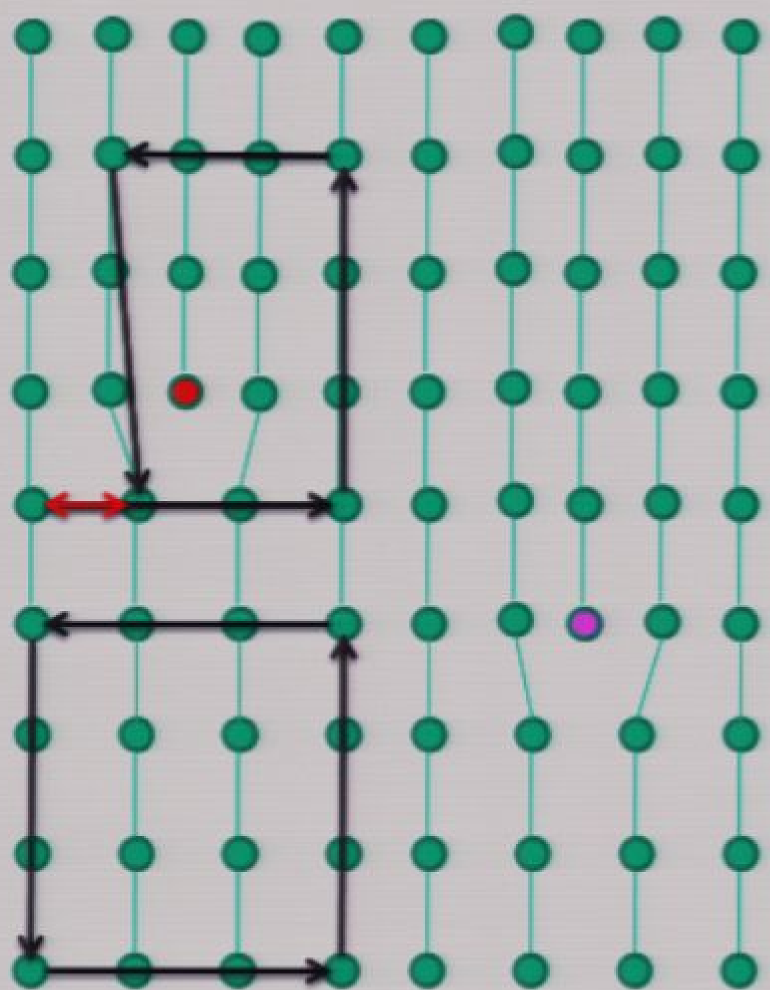

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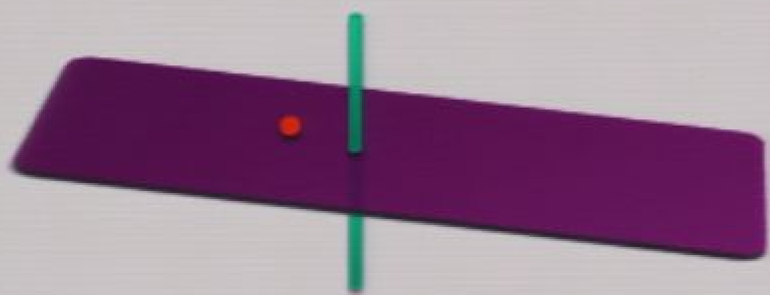

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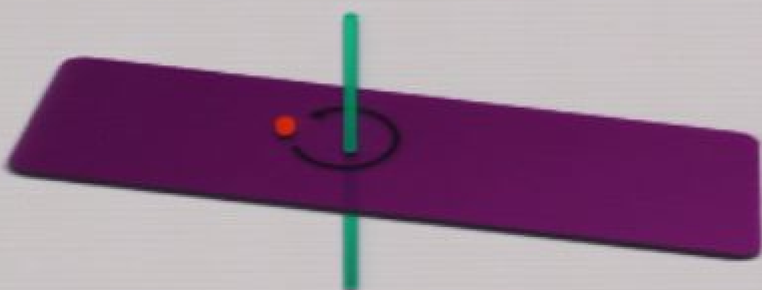

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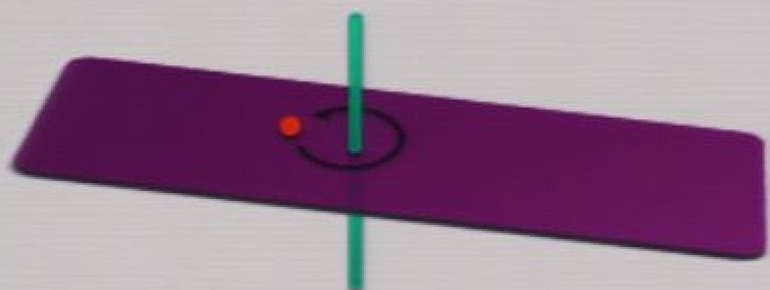

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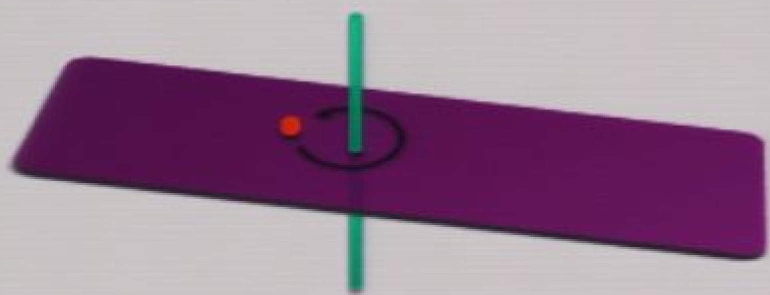
$$\exp\left[\frac{i}{\hbar} p_a b^a\right] \psi(x^a) = \sum_p \phi(p) \exp\left[\frac{i}{\hbar} p_a (x^a + b^a)\right]$$

Crystal Dislocations and Aharonov-Bohm Phases



Gauge potential and Wilson loop for electro-magnetic field:

$$\mathbf{B} = \nabla \times \mathbf{A}$$

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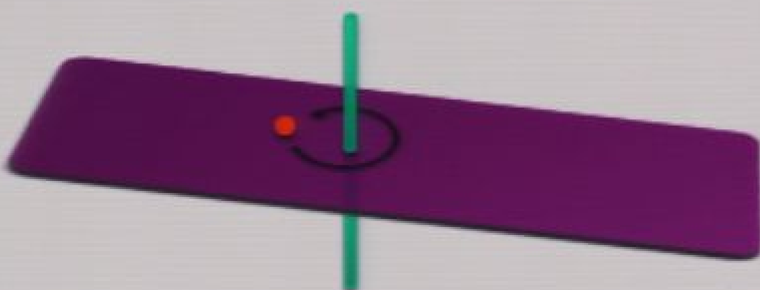

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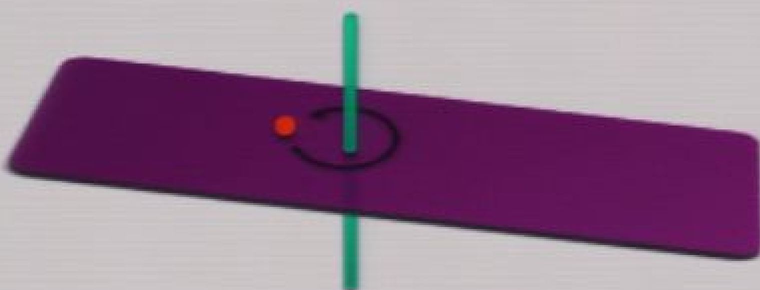

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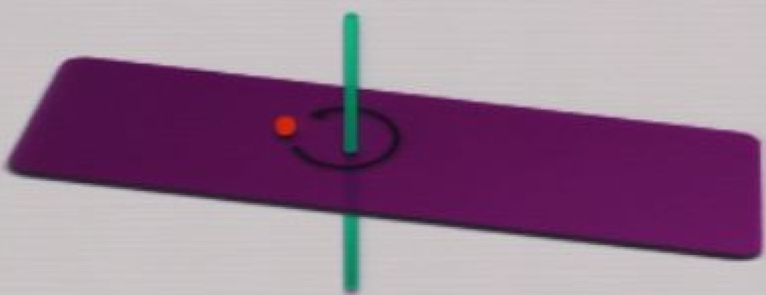

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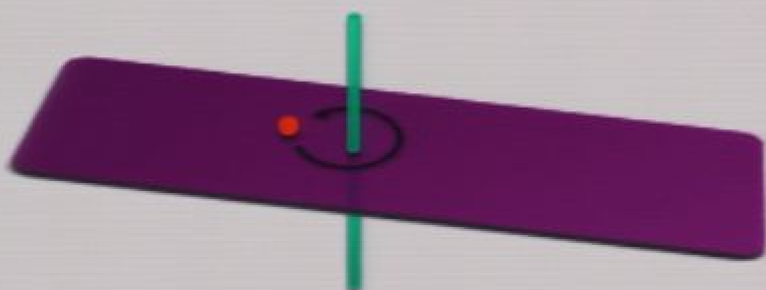

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$$[A_\mu] = \frac{1}{\text{Length}}$$

$$[e^a_\mu] = 1$$

Example: 2+1-d Topological Insulator (QHE)

Take Dirac Hamiltonian in 2+1-d

$$H_{2d} = \sum_k c_k^\dagger (k_x \sigma^x + k_y \sigma^y + m\sigma^z) c_k = \sum_k c_k^\dagger \begin{pmatrix} m & k_x - ik_y \\ k_x + ik_y & -m \end{pmatrix} c_k$$

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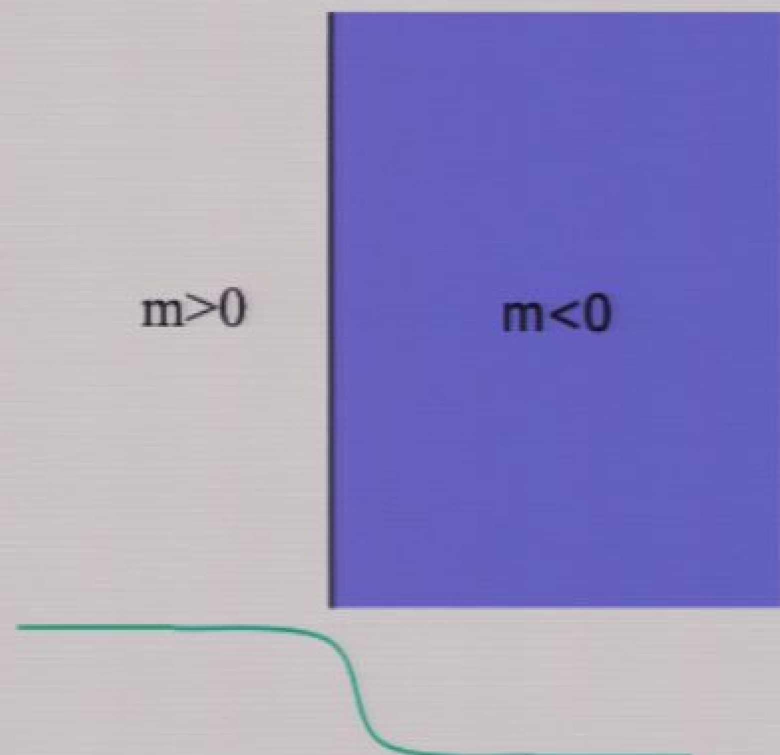

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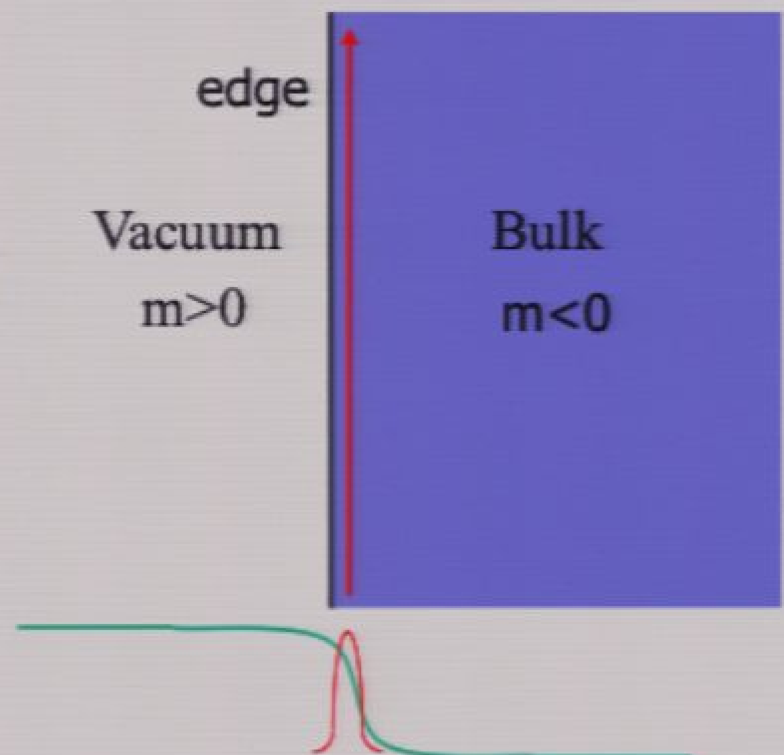

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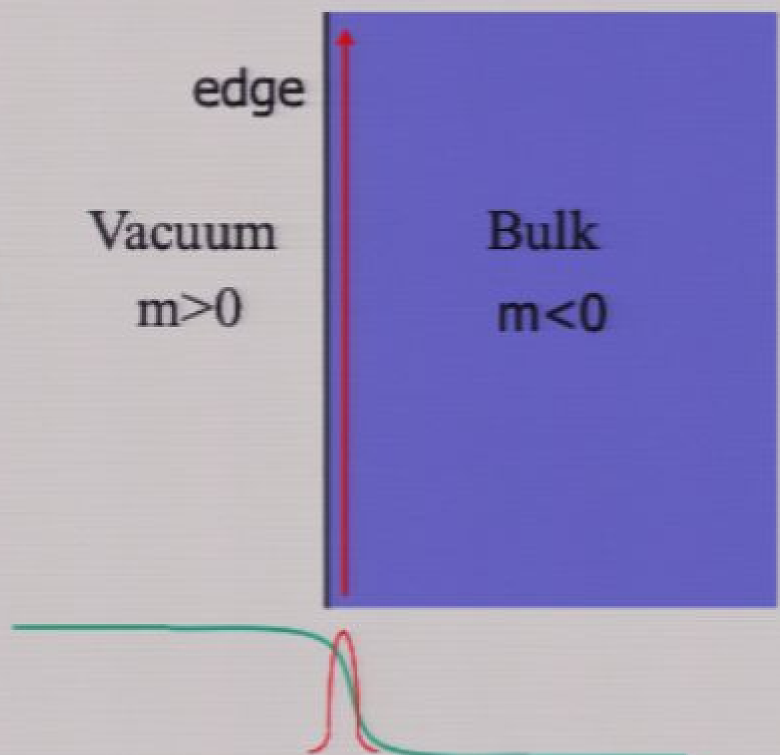
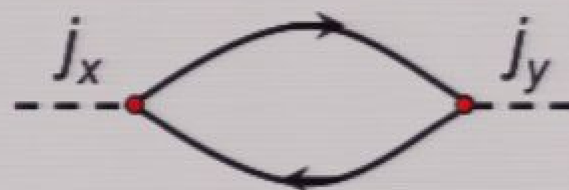


Bulk described by massive Dirac fermions, boundary described by massless chiral fermions in one lower dimension, Clifford algebra dimension cut in half

Electro-magnetic Response

Electromagnetic linear response:



$$S_{eff}[A_\mu] = \frac{n}{4\pi}\int d^3x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho$$

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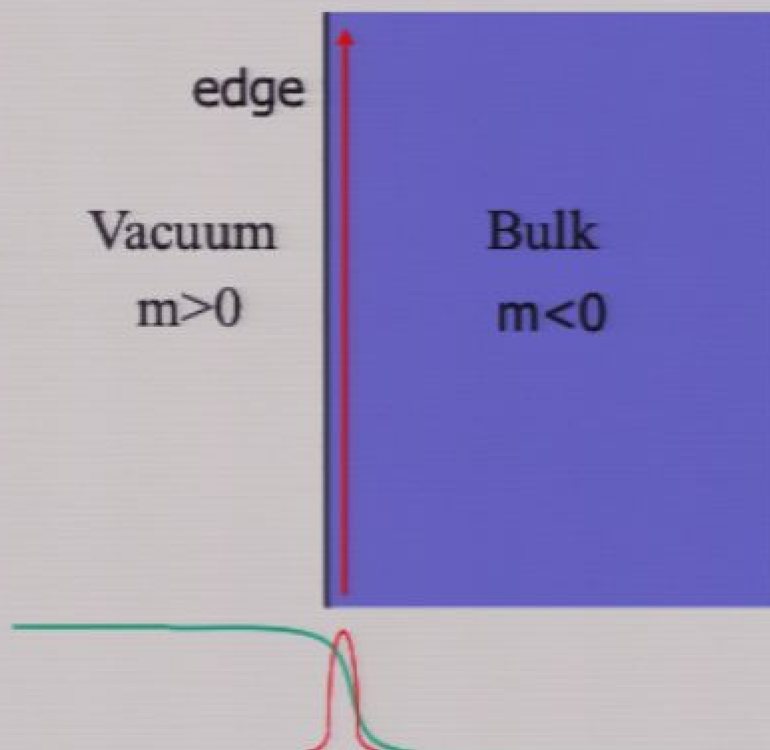

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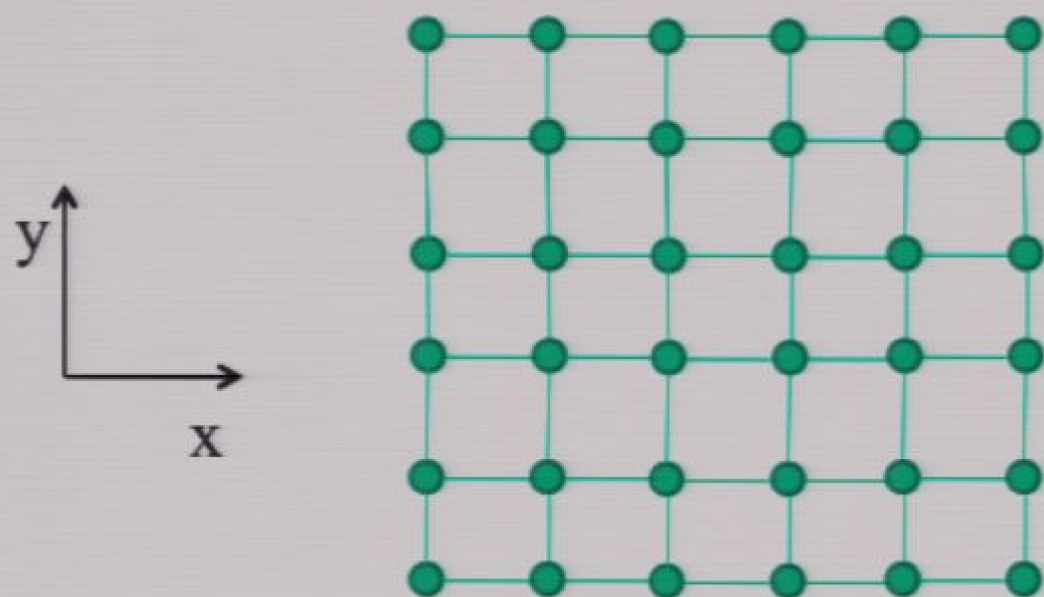

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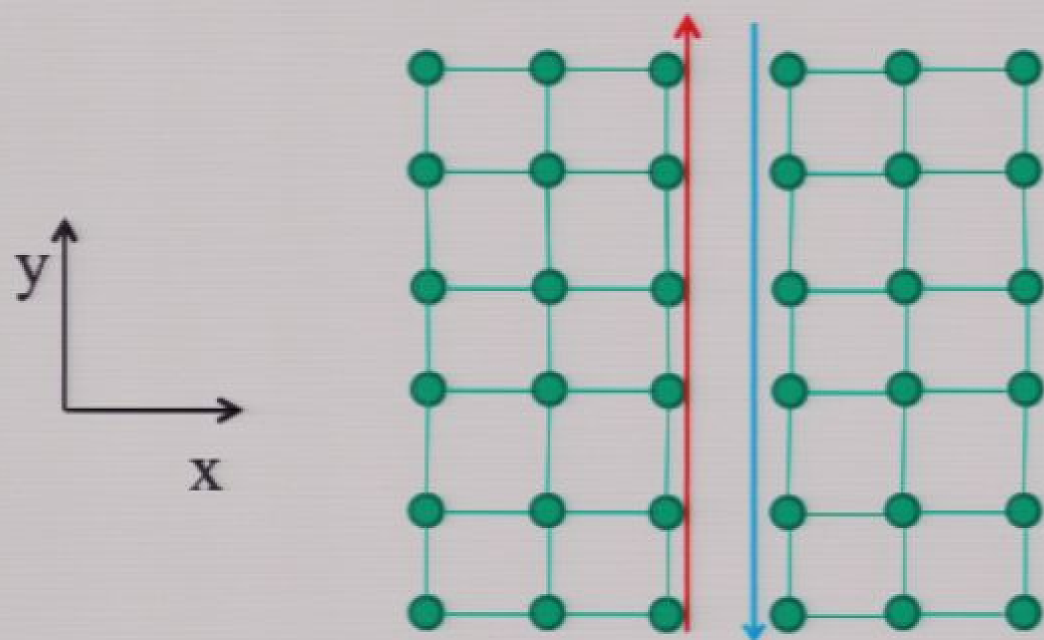
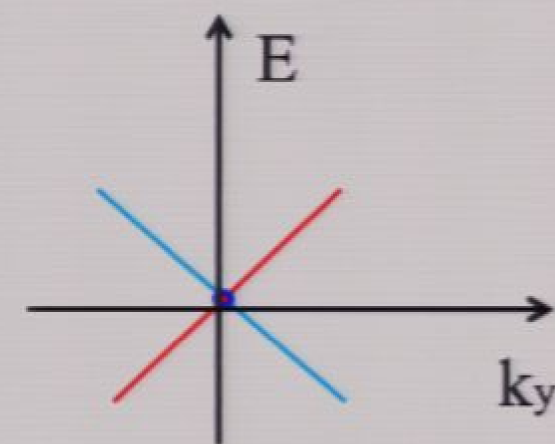


Gapless fermion spectrum on cut



$$H = \begin{pmatrix} p_y & 0 \\ 0 & -p_y \end{pmatrix}$$

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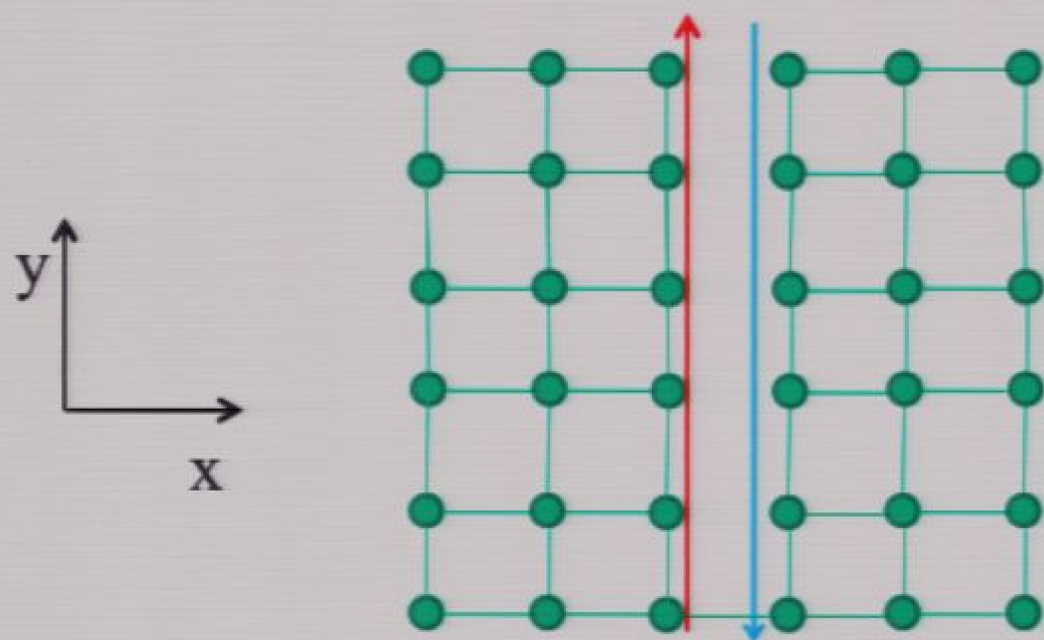

$$m(y) = m$$

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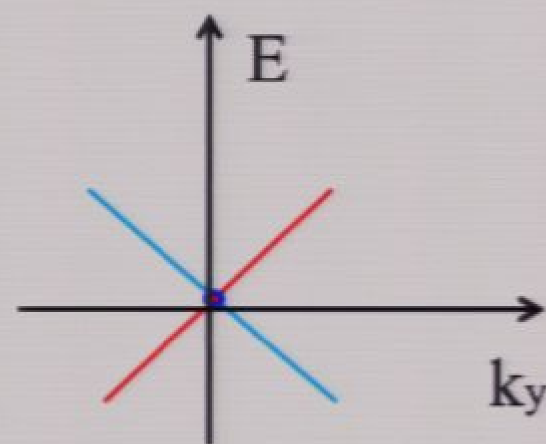

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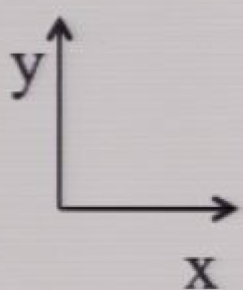

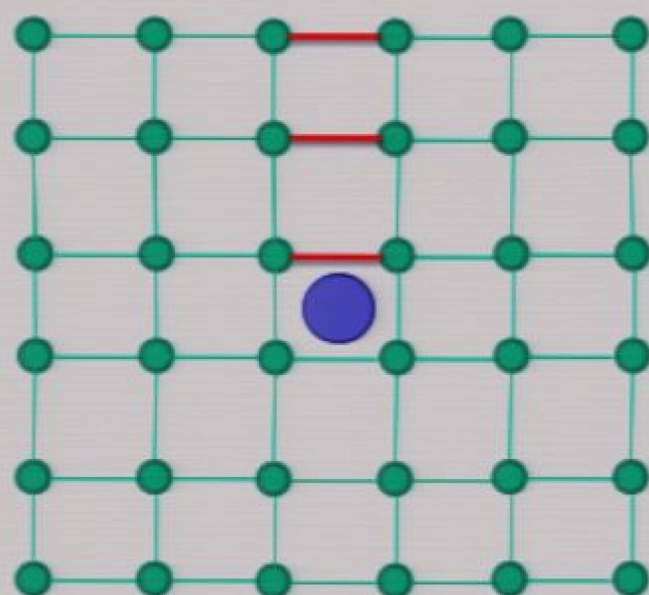

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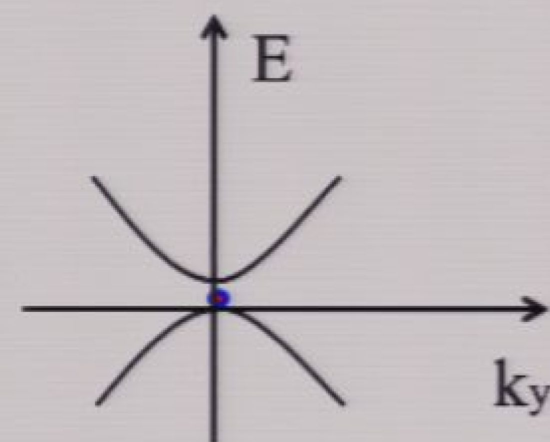

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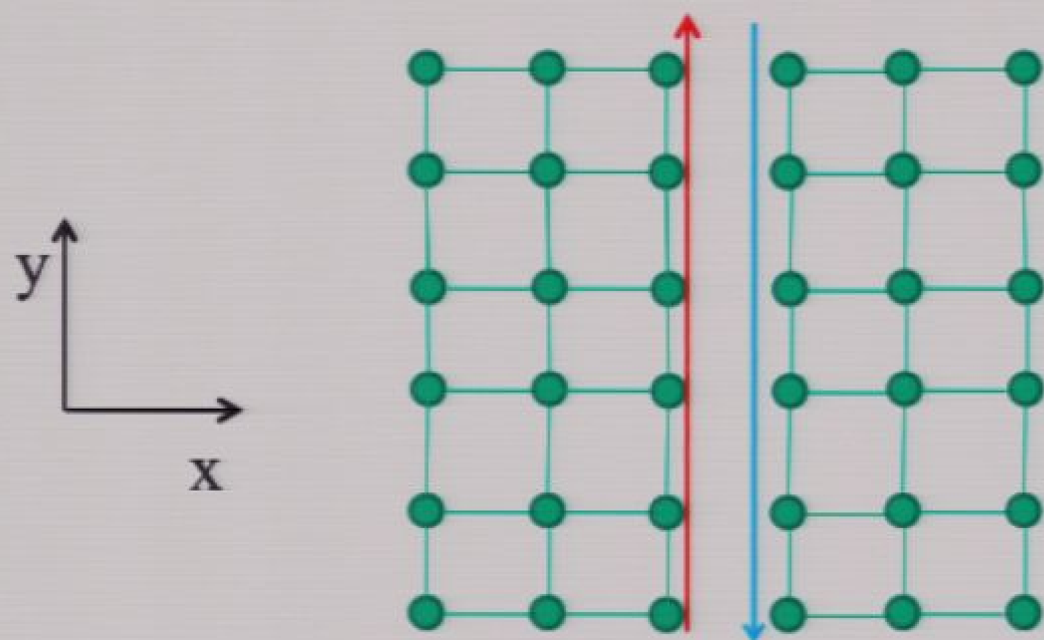

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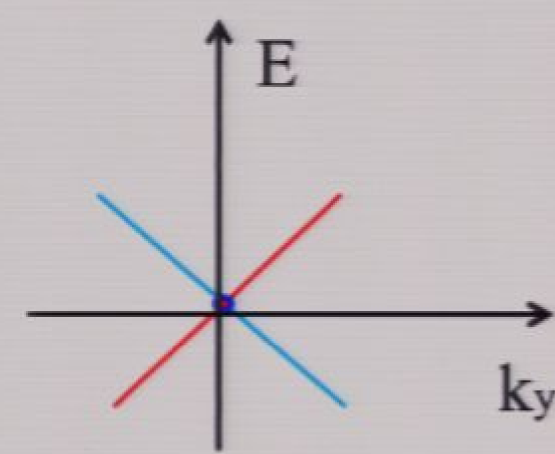

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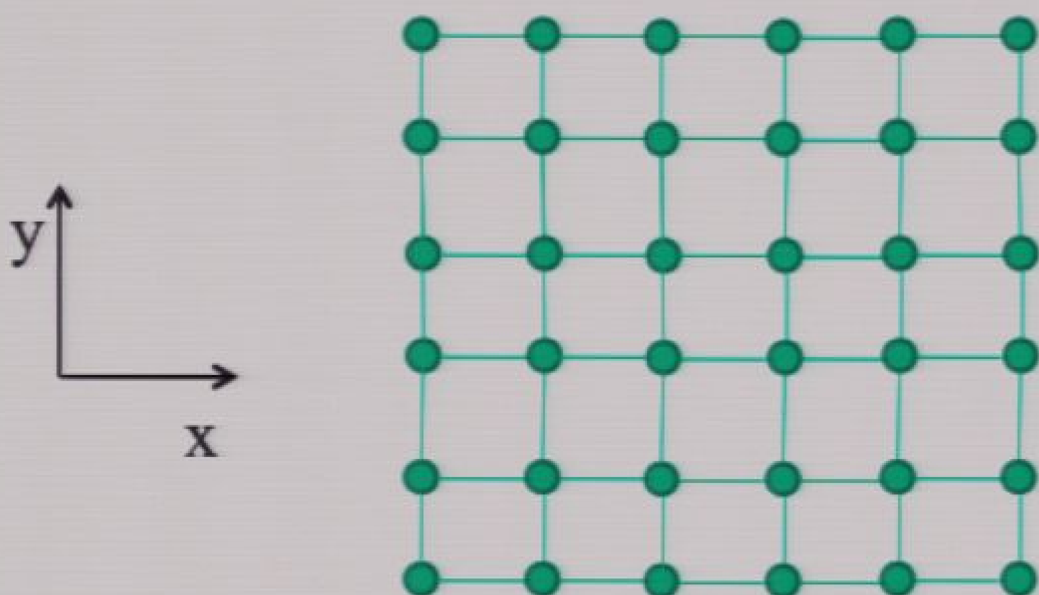

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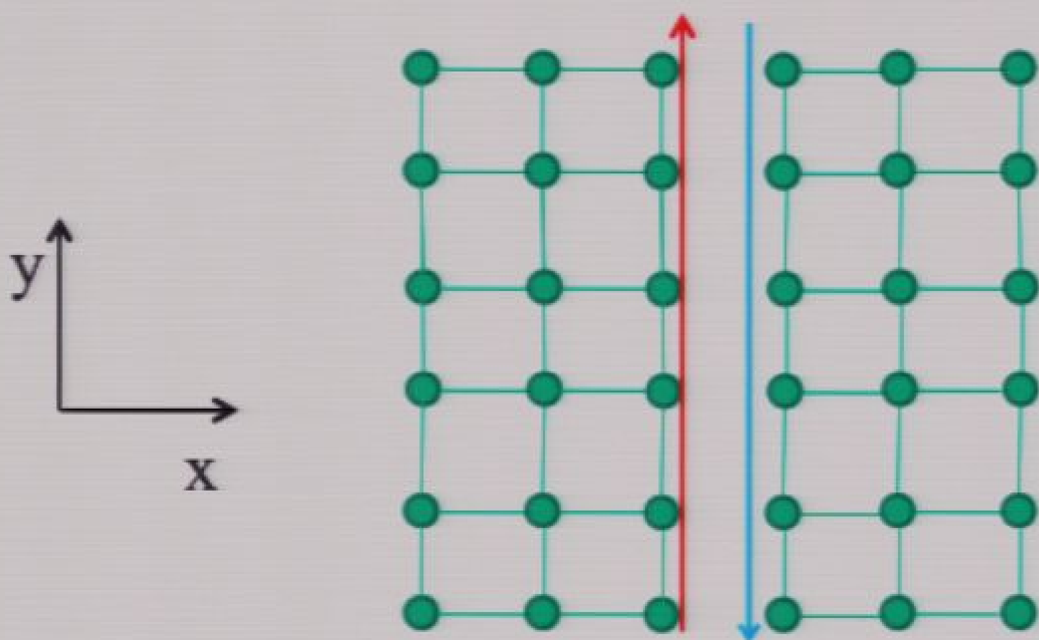

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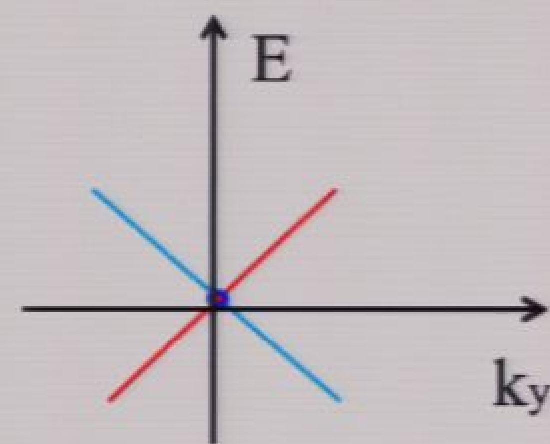

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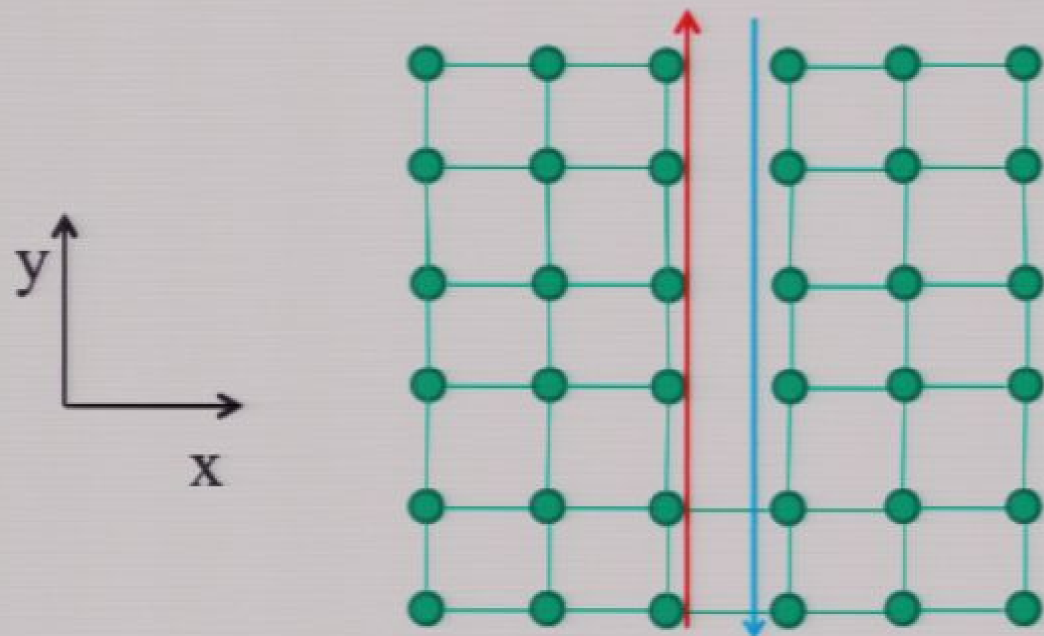

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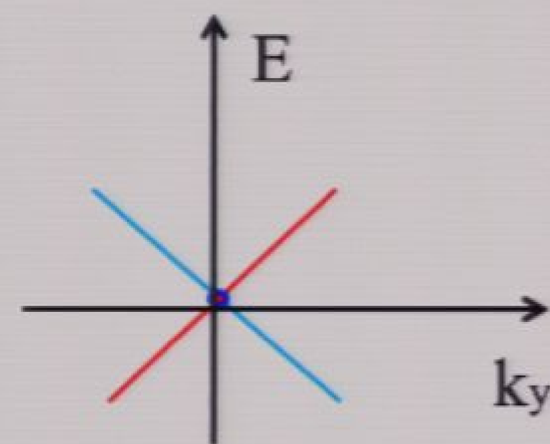

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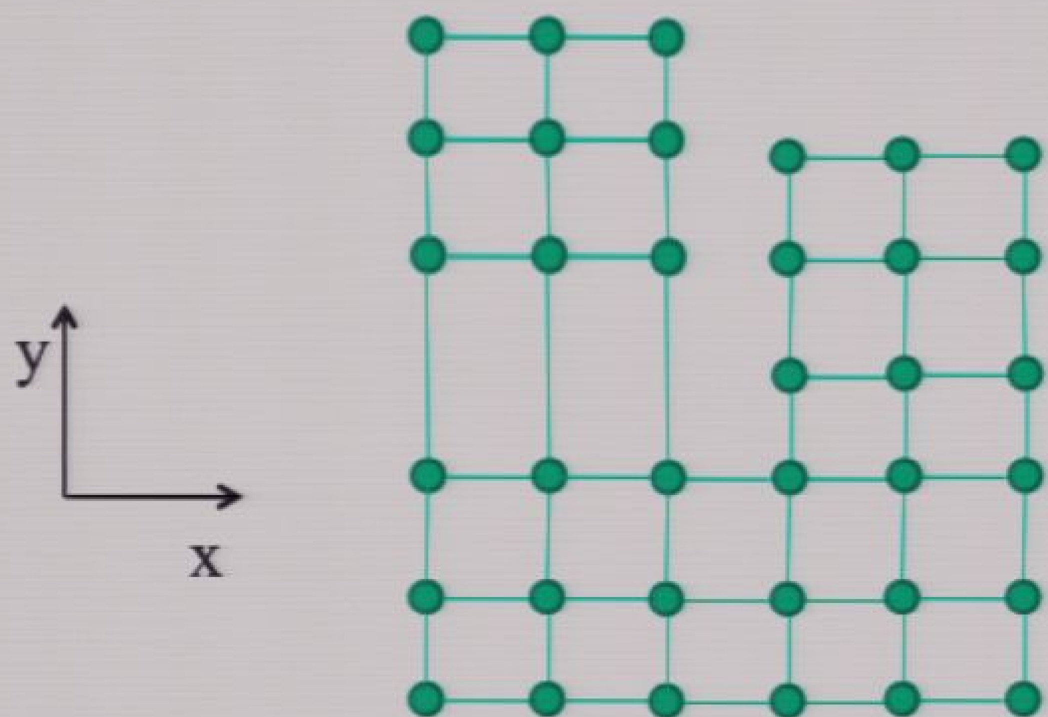

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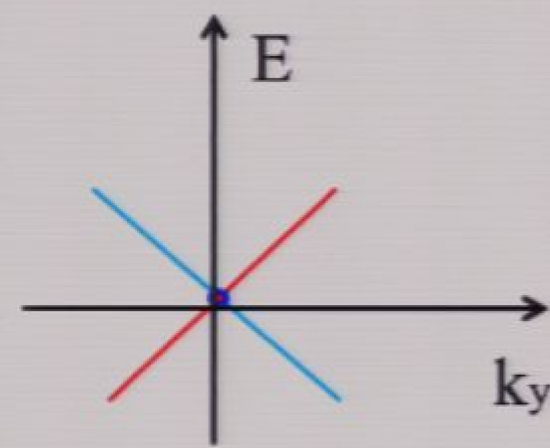

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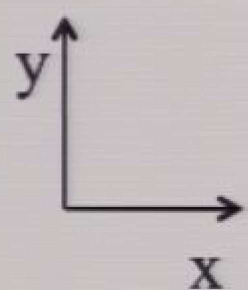
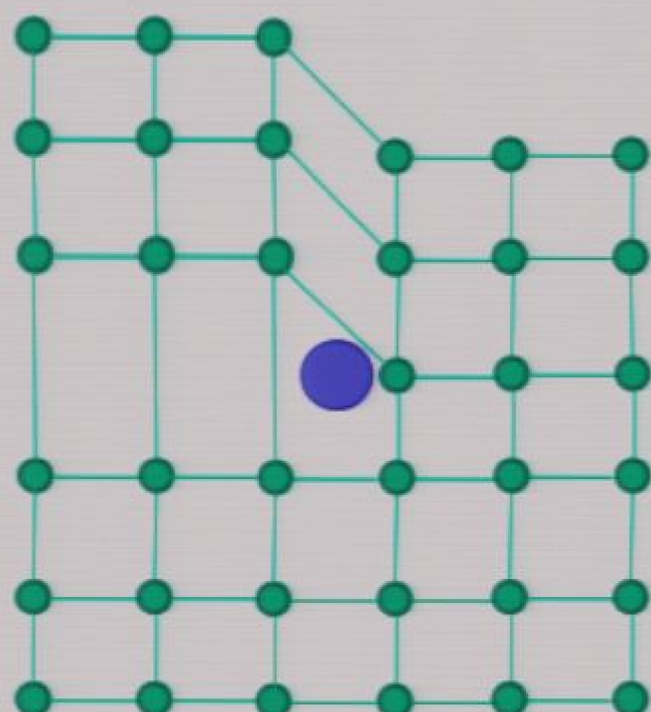
Bound States on Dislocations

$$m(y) = me^{i\mathbf{b}\cdot\mathbf{K}}$$





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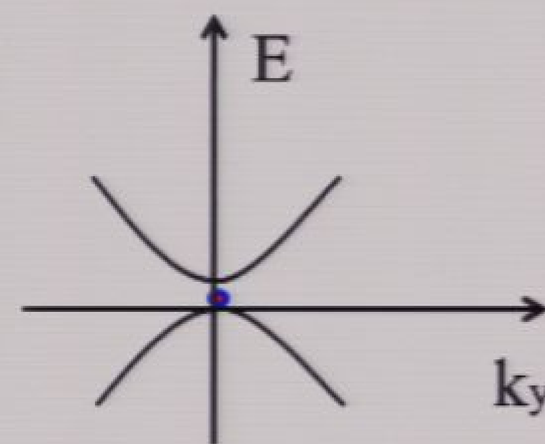

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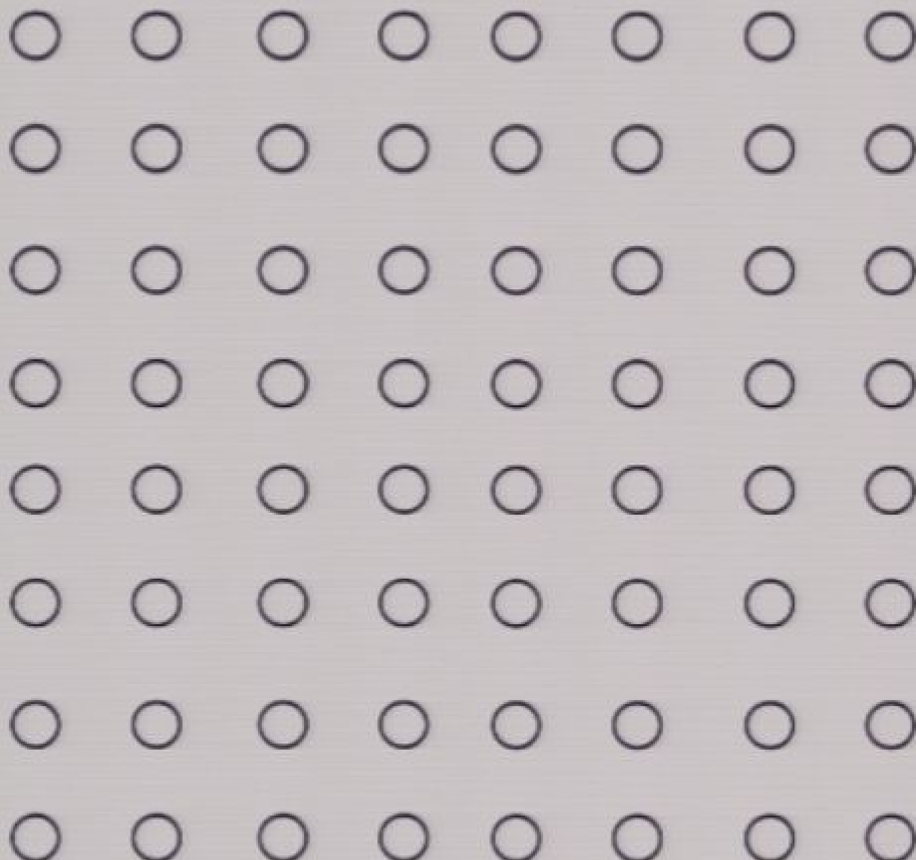
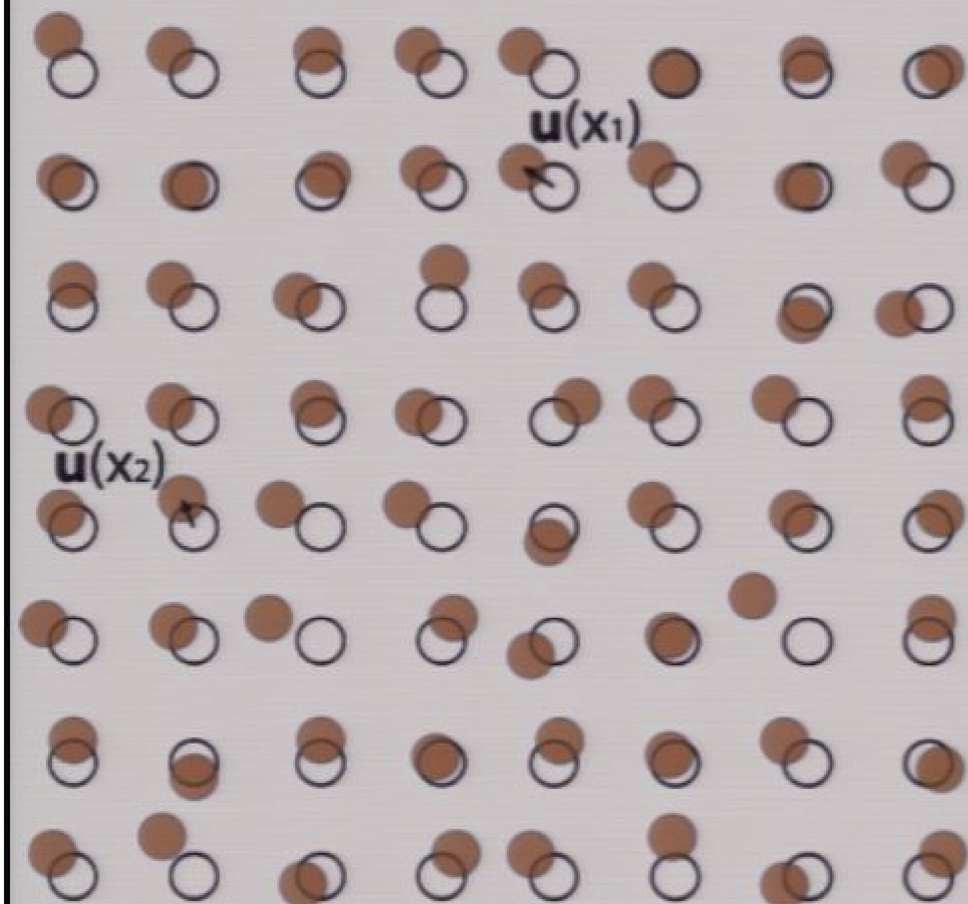
Part 2: Topological Viscosity

Elasticity and Viscosity



We begin with an elastic medium in a reference state, and look at perturbations around the reference state.

Elasticity and Viscosity



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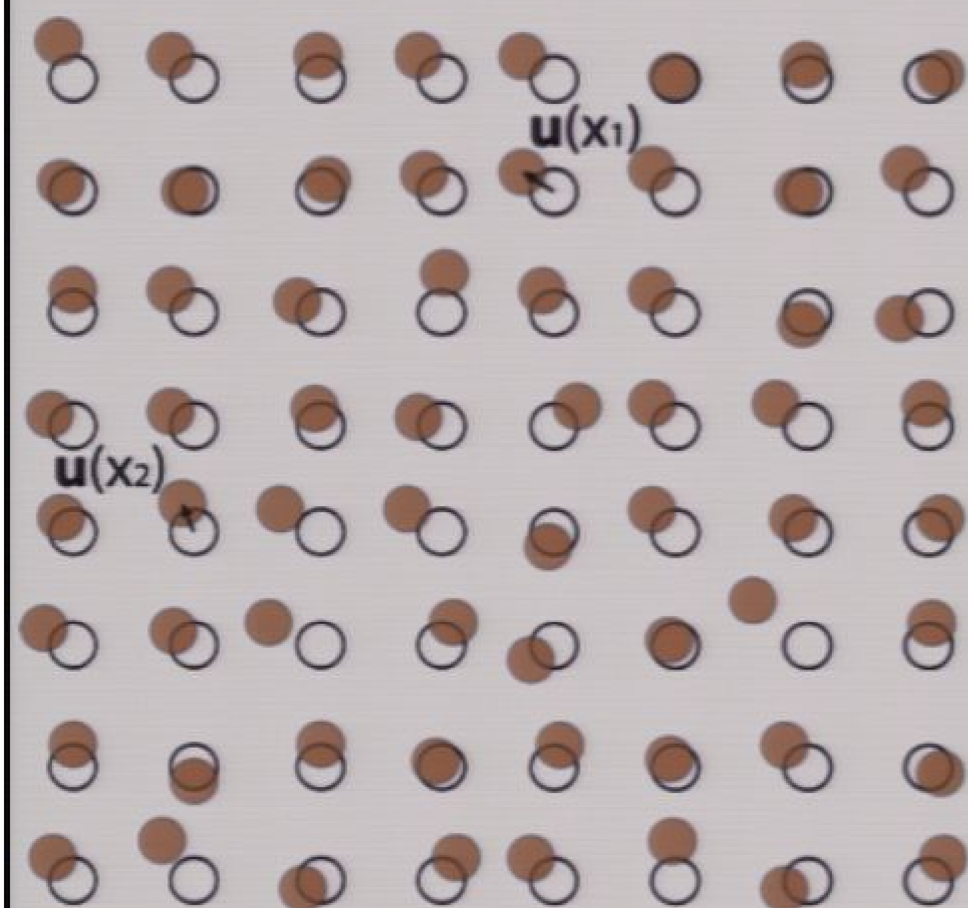
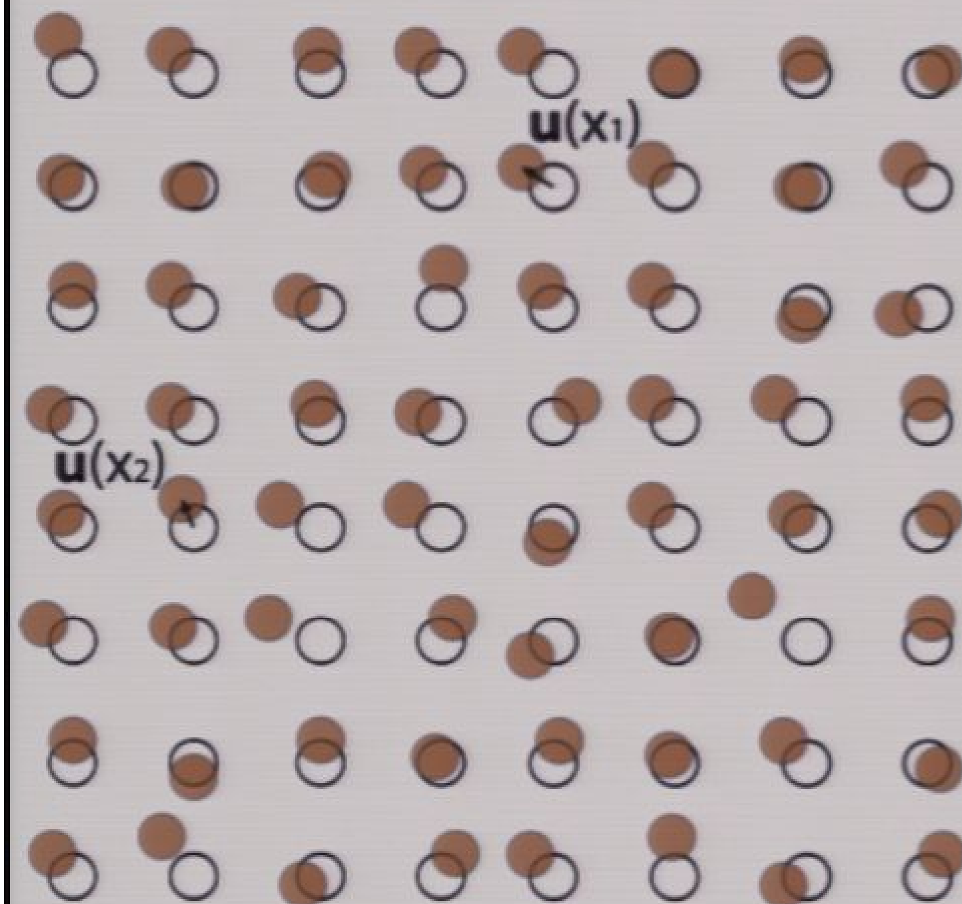


We begin with an elastic medium in a reference state, and look at perturbations around the reference state.

$$T^{ij} = \Lambda^{ijk\ell} u_{k\ell} + \eta^{ijk\ell} \dot{u}_{k\ell}$$

$$u_{k\ell} = \frac{1}{2}\left(\frac{\partial u^k}{\partial x^\ell} + \frac{\partial u^\ell}{\partial x^k}\right)$$

Elasticity and Viscosity



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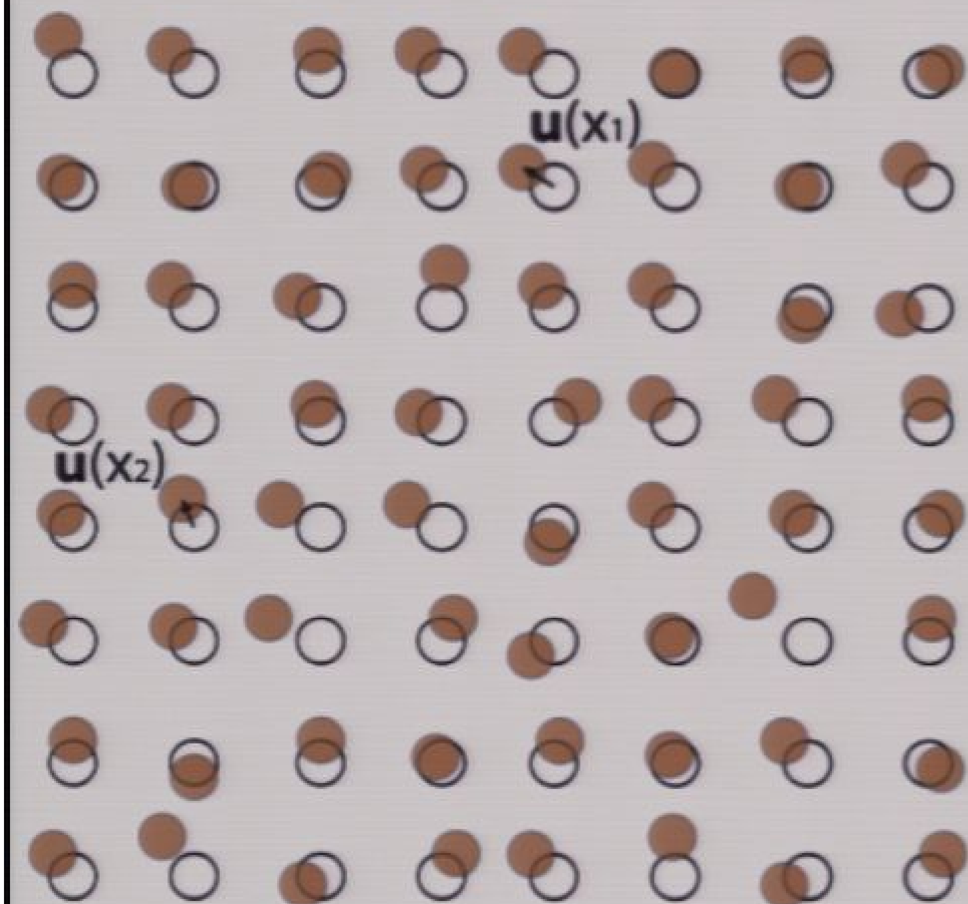

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$$e^a_\mu = \delta^a_\mu + w^a_\mu$$

$$w^a_\mu = \frac{\partial u^a}{\partial x^\mu}$$

Elasticity and Viscosity

Formulate in terms of an effective action:

$$S_{eff}[u_{ij}] = \frac{1}{2} \int d^3x \left(u_{ij} \Lambda^{ijk\ell} u_{k\ell} + u_{ij} \eta^{ijk\ell} \dot{u}_{k\ell} \right)$$

Elasticity and Viscosity

Formulate in terms of an effective action:

$$S_{eff}[u_{ij}] = \frac{1}{2}\int d^3x\left(u_{ij}\Lambda^{ijk\ell}u_{k\ell} + u_{ij}\eta^{ijk\ell}\dot{u}_{k\ell}\right)$$

Isotropic Viscosities:

Shear: $\delta_{ik}\delta_{j\ell} + \delta_{i\ell}\delta_{jk}$

Bulk: $\delta_{ij}\delta_{k\ell}$

These terms are symmetric under (ij) -> (kl)

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This symmetry implies that the viscosity term cannot be derived From an action (*i.e.* the term vanishes identically)

This makes sense since the viscosity is dissipative.

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In 2d there is one more isotropic term we can write:

$$\eta^{ijk\ell} = \eta_3\left(\epsilon^{ik}\delta^{j\ell} + \epsilon^{i\ell}\delta^{jk} + \epsilon^{jk}\delta^{i\ell} + \epsilon^{j\ell}\delta^{ik}\right)$$

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This is anti-symmetric under (ij) -> (kl)

This is a non-dissipative viscosity called the QH viscosity or odd viscosity. To be non-zero, time-reversal symmetry must be broken.

Topological Viscosity

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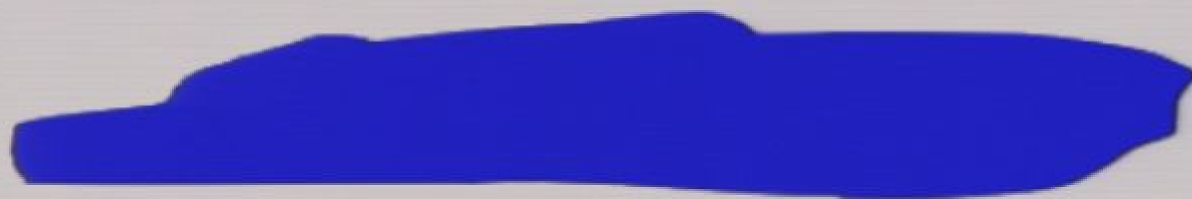
The viscosity is known to be non-zero in quantum Hall fluids. We want to also consider massive Dirac fermions in 2+1-d which exhibit a QHE.

Non-Dissipative Viscosity



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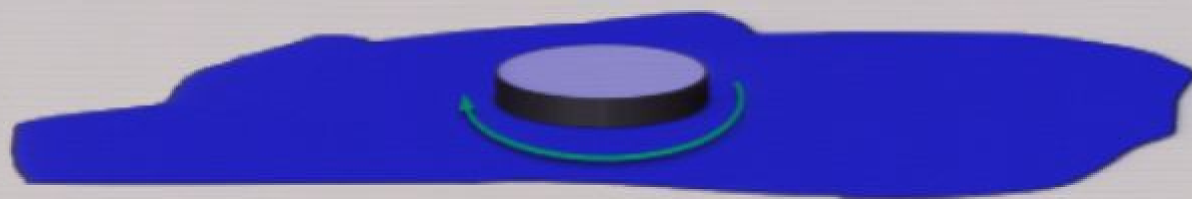

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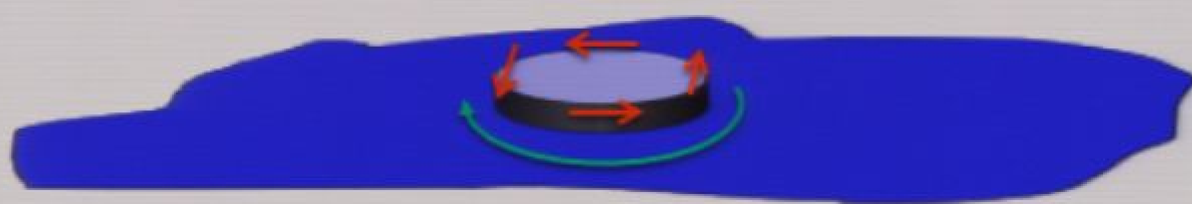

Shear viscosity: Force tangent to motion

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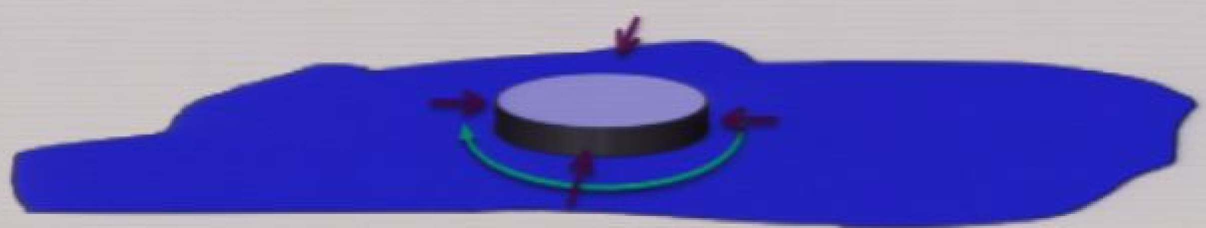


Shear viscosity: Force tangent to motion

Non-dissipative viscosity: Force perpendicular to motion

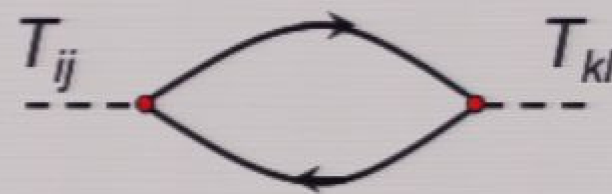
Calculating Viscosity

We will be considering massive fermions coupled to external gravitational fields so we can just integrate out the fermions:

$$S = \int d^3x \, \det(e)\bar{\psi} \left(iD_\mu e_a^\mu \gamma^a - m\right) \psi$$

$$D_\mu = \partial_\mu - i\omega_{\mu ab}\Sigma^{ab}$$

Stress Tensor response:



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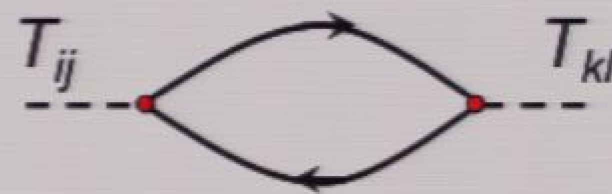

We find:

$$S_{eff}^{(3d)}[e^a, \omega_b^a] = \frac{\sigma_3}{2}\int \mathrm{Tr}\left[\omega \wedge d\omega + \frac{2}{3}\omega \wedge \omega \wedge \omega\right]$$

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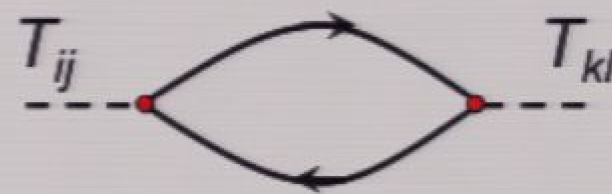

We find:

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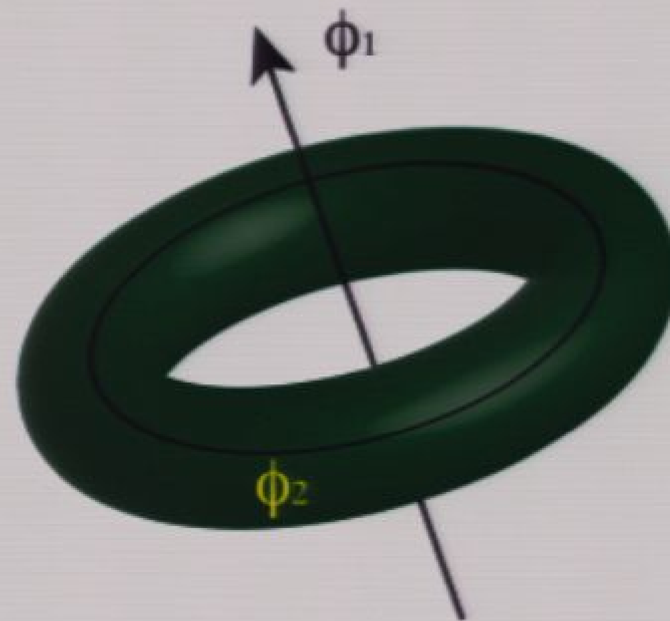
$$T^a = de^a + \omega^a_b \wedge e^b$$

Calculating Viscosity via Adiabatic Transport

For gapped systems you can calculate conductivities and other transport coefficients by looking at the behavior of the ground state under adiabatic deformations.

(Hall) Conductivity is the response to E-M flux insertion Or twisting of boundary conditions (due to Faraday effect)



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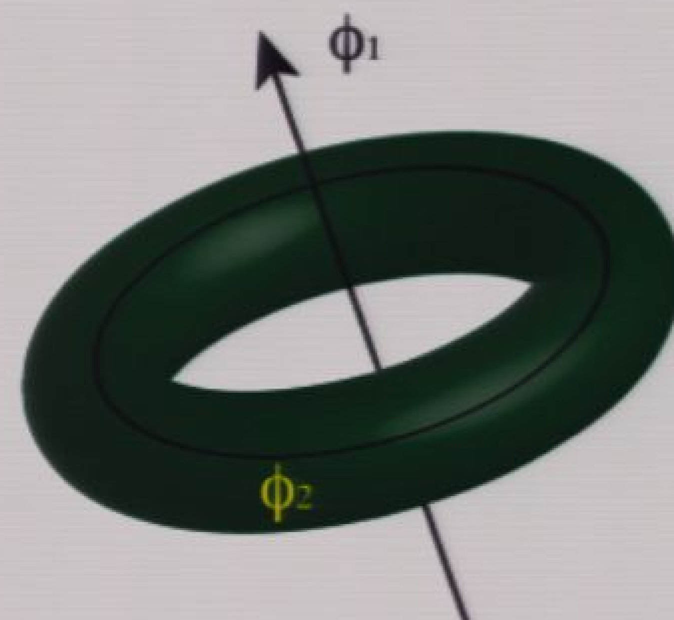

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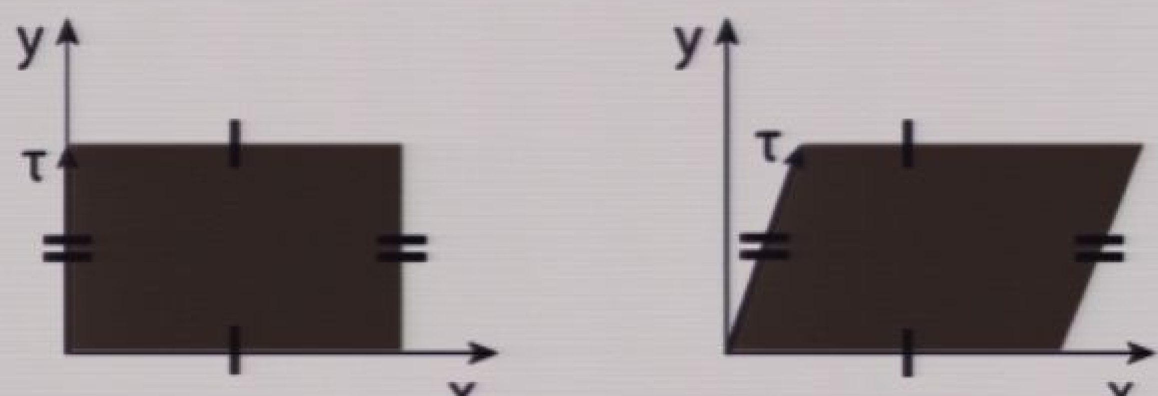
For the viscosity we calculate the response to deformations of the modular parameter of the torus



Topological Viscosity

- We will only look at the torsion term and to simplify the description we focus on a flat background where we pick a gauge where the spin-connection vanishes:

$$S_{eff} = \frac{1}{2}\eta_3 \int d^3x \epsilon^{\mu\nu\rho} e^a_\mu \partial_\nu e^b_\rho \eta_{ab}$$

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$$\eta_3 = \frac{\hbar}{4\pi}\left(\frac{|m|c}{\hbar}\right)^2 \mathrm{sgn}(m) \equiv \frac{\hbar}{4\pi\ell^2}$$

Topological Viscosity

We can calculate the stress-energy tensor and find:

$$T^i_a = \eta_3 \epsilon^{ij} (\partial_j e^b_0 - \partial_0 e^b_j) \eta_{ab} \equiv \eta_3 \epsilon^{ij} \mathcal{E}^b_j \eta_{ab}$$

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Torsion Magnetic Field:

$$\mathcal{B}^a = -\sum_i b^a_{(i)} \delta(\mathbf{x} - \mathbf{x}_{(i)})$$

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The torsion magnetic field is simply tied to the dislocation density

Magnetic Torsion Response

$$T_a^0 = \eta_3 \epsilon^{ij} \partial_i e_j^b \eta_{ab} \equiv \eta_3 \mathcal{B}^b \eta_{ab}$$

This torsion response implies that momentum *density* in the *a-th* direction is bound to a frame field flux *i.e.* a dislocation

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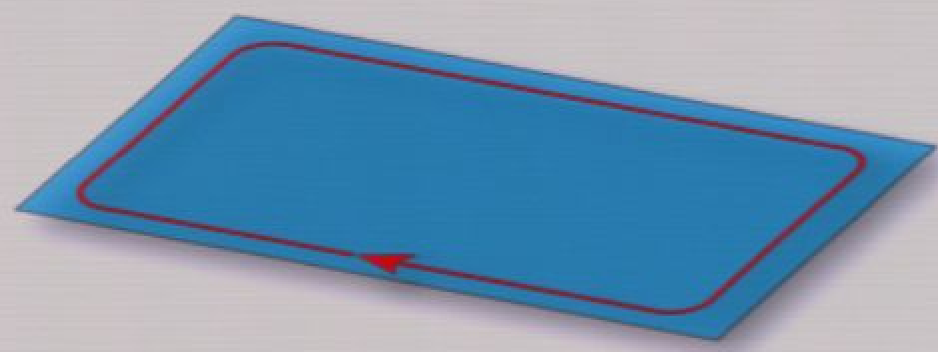

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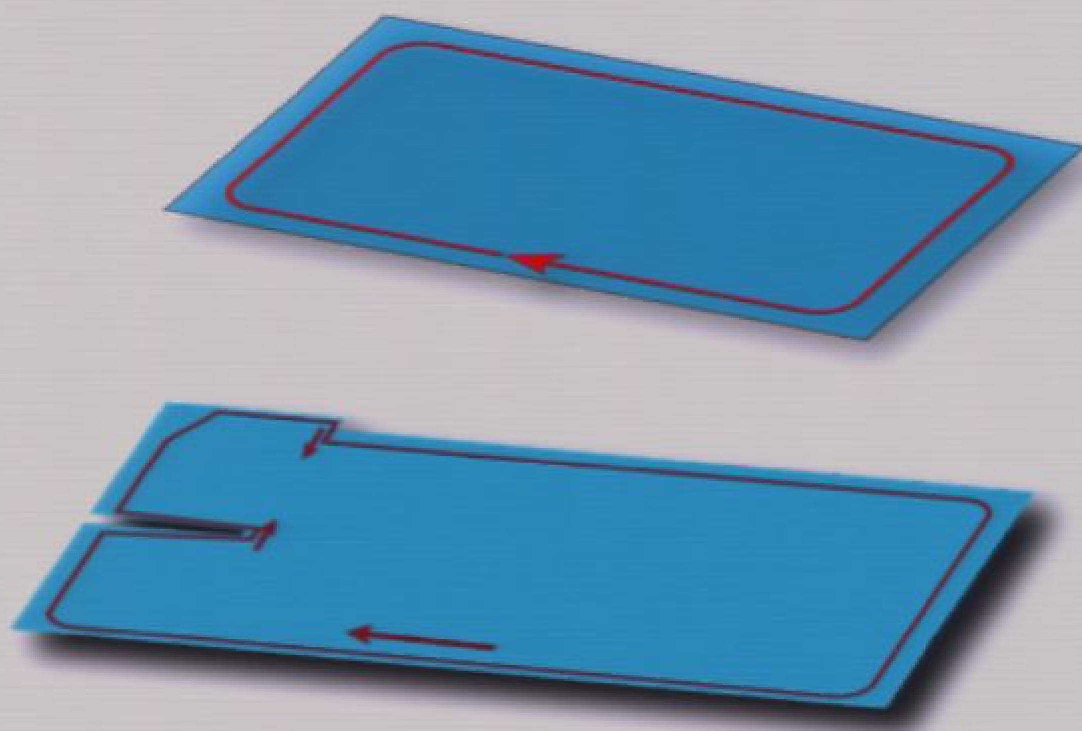

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How can we create a torsion electric field? Through the Faraday effect. That is a time-dependent torsion magnetic field will produce a torsion electric field.

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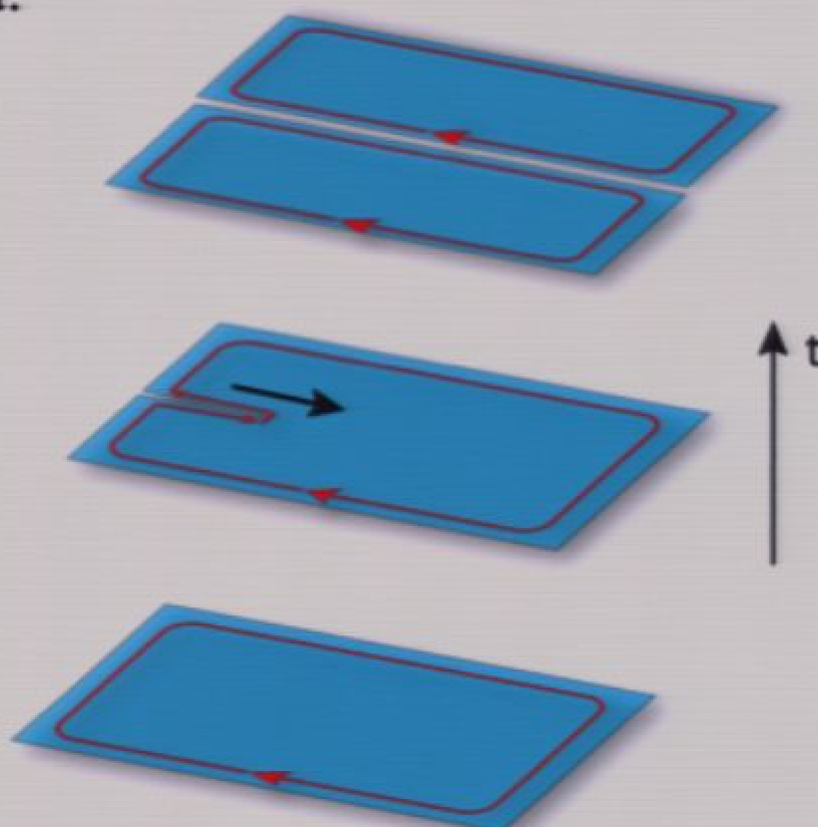
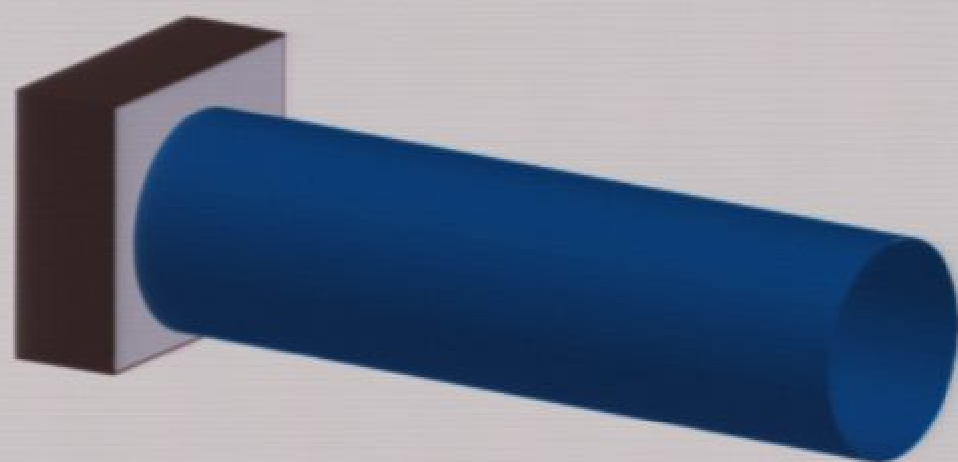


Electric Torsion Response

Alternative way: Thread a torsion flux through the cylinder.



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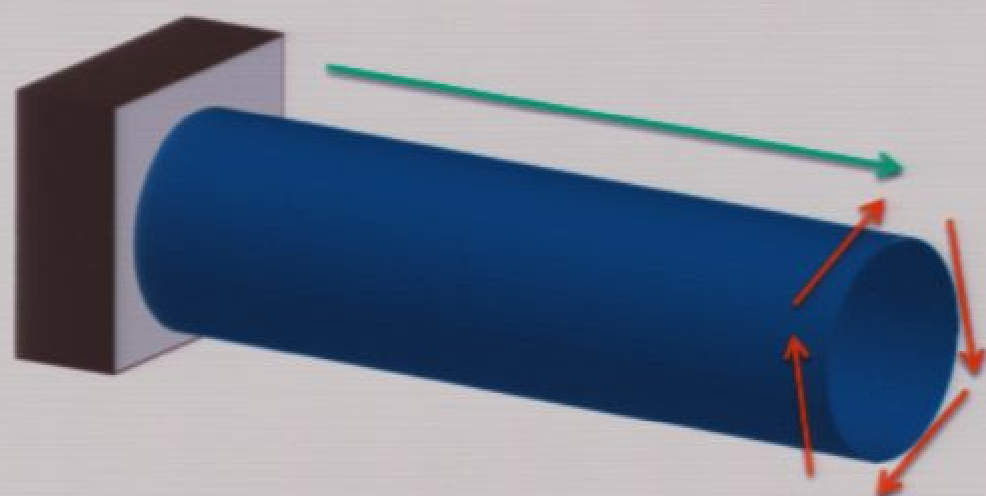

Torsion in 3+1-d

- In 3+1-d we have the Nieh-Yan term which simply implies there is a quantum Hall viscosity on an axion domain wall.

$$S^{(4d)}_{eff}[e^a, \omega^a_b] = \sigma_4 \int R^{ab} \wedge R_{ab} + \eta_4 \int \theta \; [T^a \wedge T_a - R_{ab} \wedge e^a \wedge e^b]$$

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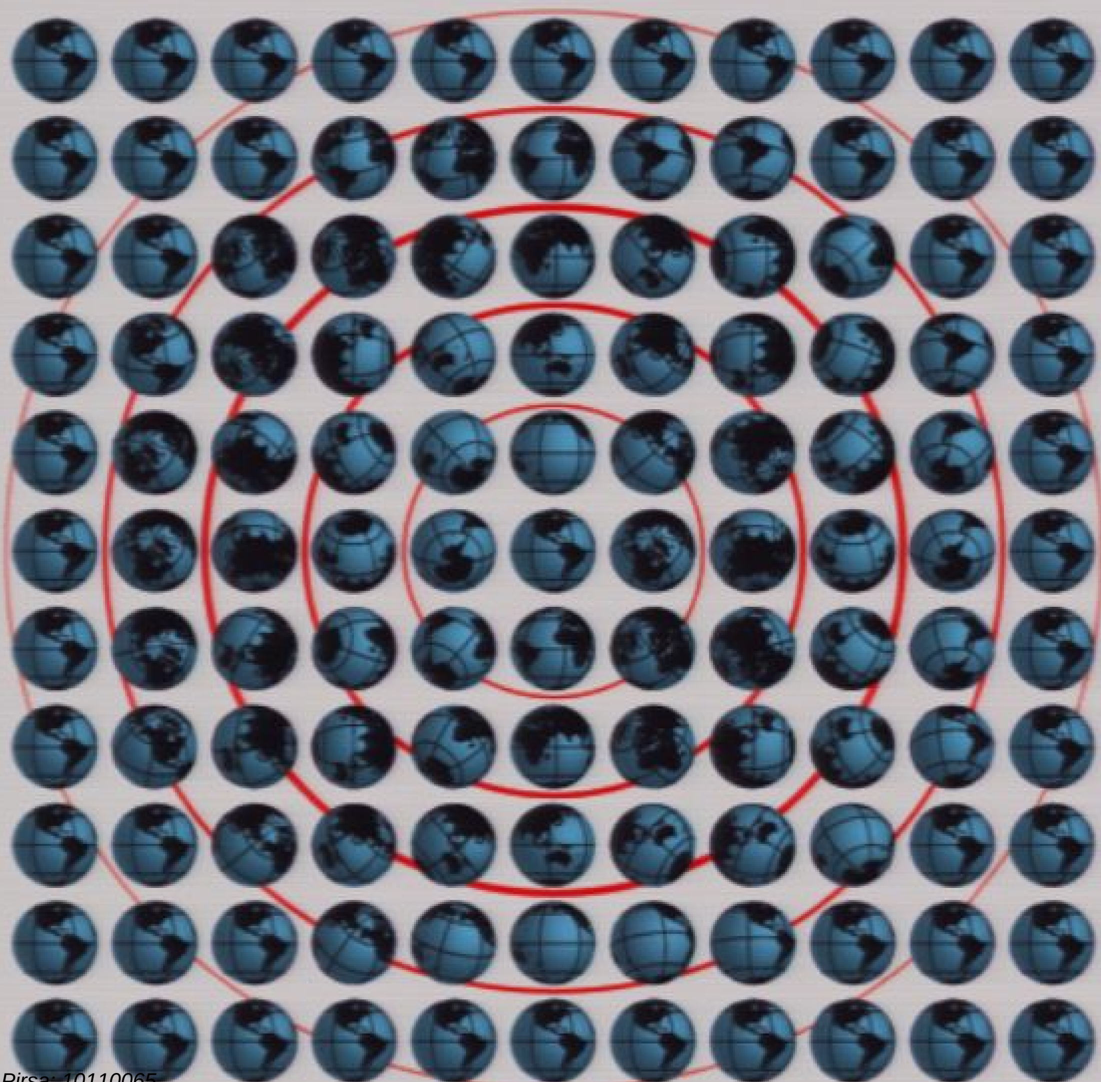
Again the coefficient of the torsion piece has units of 1/[Length]^2.

So topological insulators in 3+1-d should exhibit a quantum Hall viscosity on the surface. It goes hand in hand with the QHE on the surface. 'Axion visco-elasticity.'

Part 3: Torsional Monopole

Heuristic Picture



We begin in 3+1-d flat space in the first order formalism:

$$e^a,\ \omega^a_b$$

To simplify the description we pick a gauge where

$$\omega^a_b \equiv 0$$

Mathematical Description

What we want is a completely torsional defect which can exist with or without space-time curvature. Only consider flat space for now.

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$$\omega^{a}_{\mu\ b}$$
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$$\omega = g\Gamma g^{-1} - dg\, g^{-1}$$

Mathematical Description

With this setup we want to define a current:

$$*J \equiv \frac{1}{4\pi^2}\mathrm{Tr}_D\left(\omega\wedge d\omega+\frac{2}{3}\omega\wedge\omega\wedge\omega-\Gamma\wedge d\Gamma-\frac{2}{3}\Gamma\wedge\Gamma\wedge\Gamma\right)$$

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$$Q = \frac{1}{12\pi^2}\int_\Sigma \mathrm{Tr}_D\left(dgg^{-1} \wedge dgg^{-1} \wedge dgg^{-1}\right)$$

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Torsion Monopoles in Solids

We can gauge transform our monopole solution so that the spin connection becomes the Levi-Civita connection and all of the torsion is contained in the tetrad. When this is done we do not have to deform the underlying lattice:

However, simple Schrodinger electrons won't even feel the defect:

$$H = \frac{e^i_a e^j_a p_i p_j}{2m} = \frac{g^{ij} p_i p_j}{2m}$$

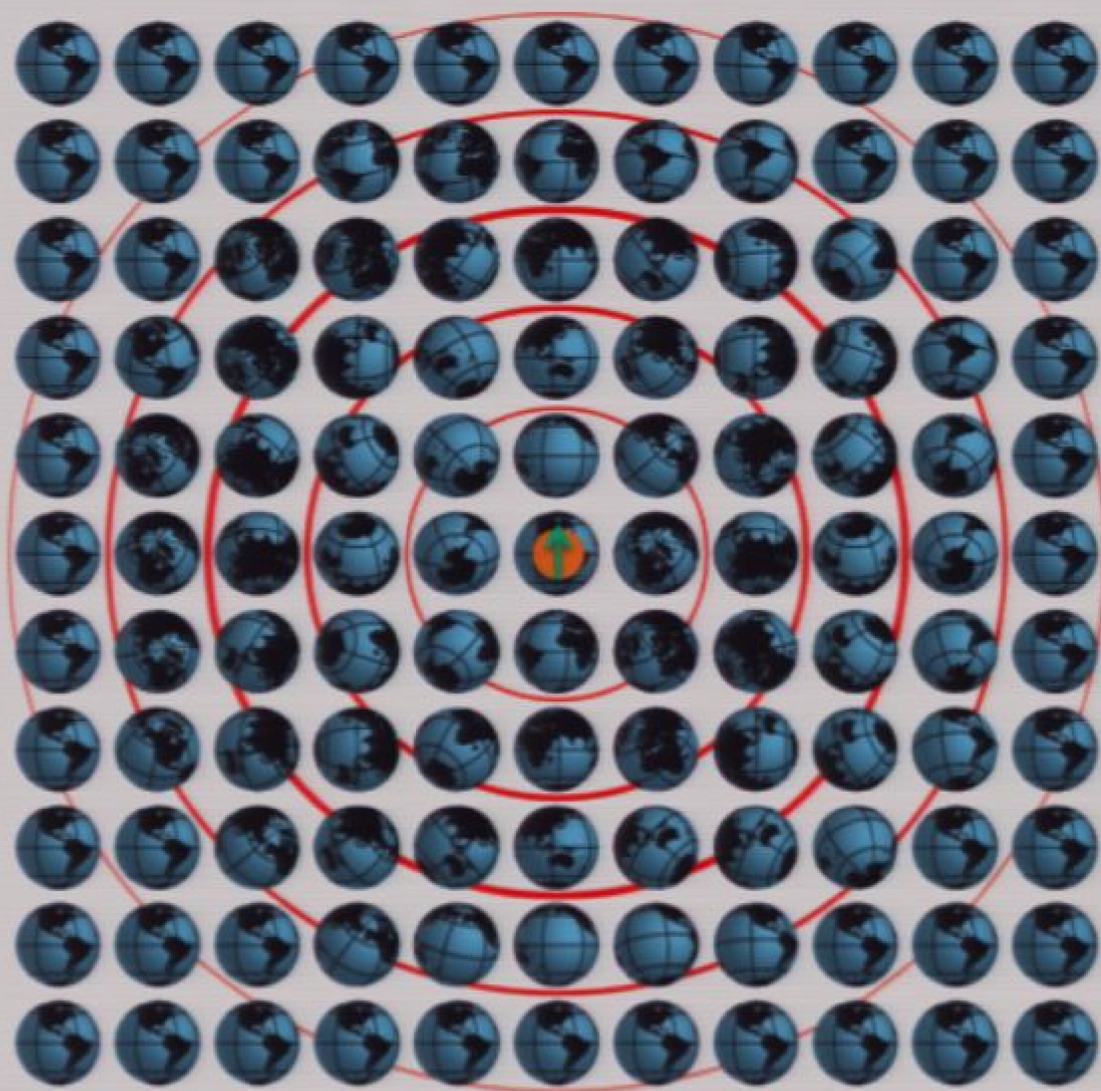
Torsion Monopoles in Solids

The place to look for the effects of such defects is in materials which have strong spin-orbit coupling. This means that you want the motion/momentum coupled to spin degrees of freedom:

$$H = p_i e^i_a \Gamma^a + m\Gamma^0$$ Dirac model/Topological Insulator

$$H = p_i p_j e^i_a e^j_b S^a S^b$$ Luttinger model for common semi-conductors (spin 3/2)

Possible Interference Effects



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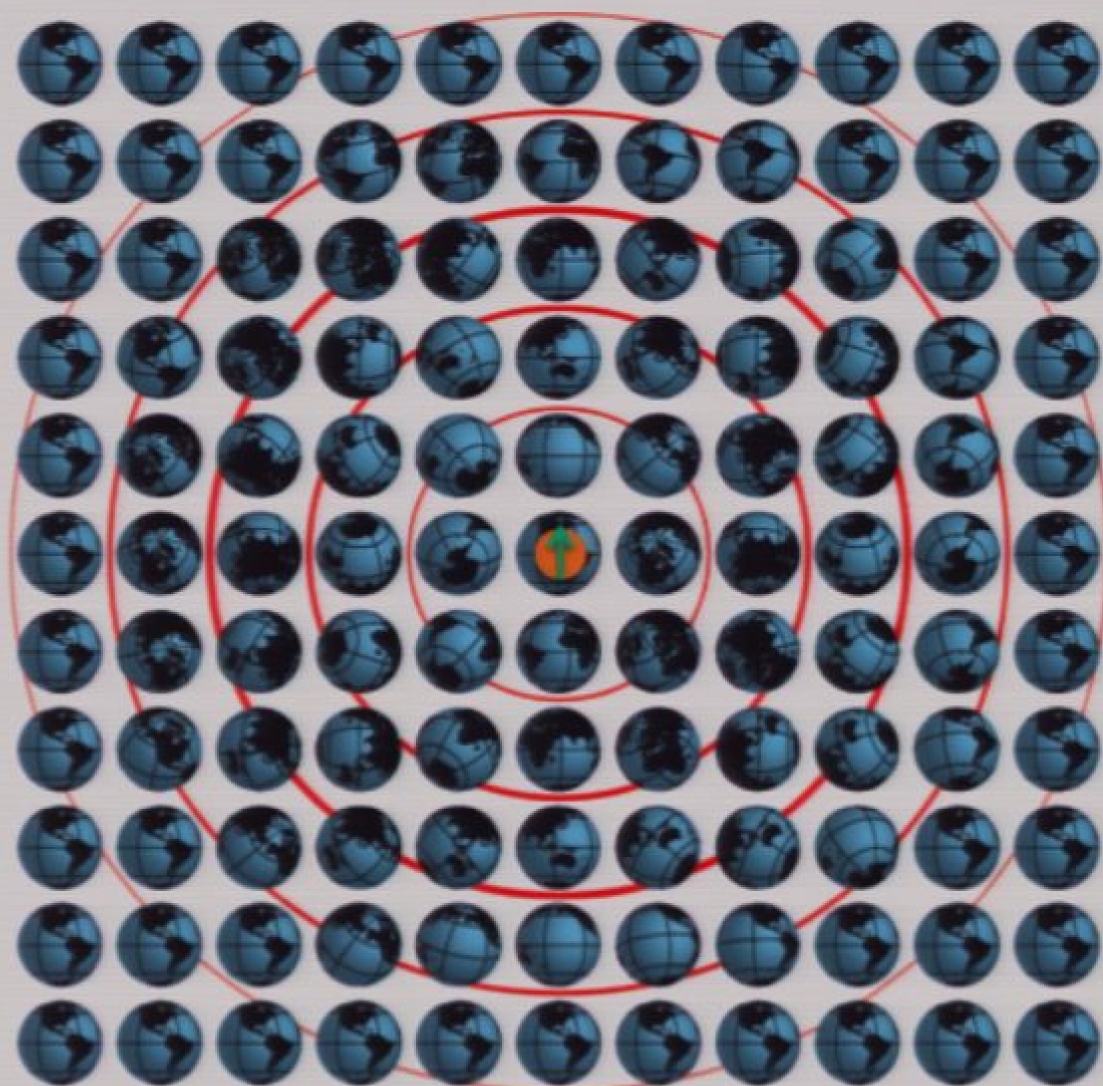

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