Title: From the Abstract to the Concrete: Extracting Physics from Mathematics

Date: Nov 19, 2010 11:00 AM

URL: http://pirsa.org/10110063

Abstract: TBA

From the Abstract to the Concrete: Extracting Physics from Mathematics.

Margaret Morrison
Philosophy Dept.
University of Toronto

Outline:

0. Stage Setting and Background

1. The Abstract
    1.1 What is the Renormalization Group?
    1.2 RG – QFT
    1.3 RG Extended

2. Extracting Physics from the RG Framework.

3. What is Explained?

0. Stage Setting and Background

<u>WIGNER'S PROBLEM</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>**WIGNER'S PROBLEM**</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>WIGNER'S PROBLEM</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>WIGNER'S PROBLEM</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>WIGNER'S PROBLEM</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>WIGNER'S PROBLEM</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>**WIGNER'S PROBLEM**</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

0. Stage Setting and Background

<u>WIGNER'S PROBLEM</u>: THE UNREASONABLE EFFECTIVENESS OF MATHEMATICS IN PHYSICS.

What does 'effectiveness' mean here?

1. Appropriateness of the language of mathematics for the formulation of laws. (symmetries, idealisations etc. French, Batterman and others)
2. Explanation of 'physical' phenomena/effects via mathematics (Baker, Bueno, Colyvan, Pincock).
3. Prediction of phenomena/effects (Bangu and others).

- All of these focus on the applicability of mathematics, whether it is eliminable, and if not what we are committed to in accepting its role.

- Status of mathematics and mathematical **entities** is the focus in these debates.

- Importance of strategies like IBE in establishing the status of these entities.

QUESTION: Given that mathematics figures prominently in scientific explanations/predictions what constitutes a “mathematical” as opposed to a “physical explanation”?

Baker 2005 – life cycle of cicadas

- Cicada case involves evolutionary accounts that require appeal to mathematical facts about prime numbers and the notion of a lowest common multiple.

- This isn't simply a "mathematical" explanation but the mathematics seems crucial.
- So how does that bear on the status of the math entities? (Phil of Math issues, realism)

What about cases where the mathematics alone seems to ‘generate’ new physics or physical explanation ? (Not simply the prediction of new phenomena based on mathematical structures – eightfold way)

E.G. Gauge fields; $T \rightarrow \infty$/RG (critical behaviour)

Focus on:

1. How the physics and mathematics become intertwined.
2. How we should understand the status of the **physical** entities and explanatory features that emerge from the mathematics.

- This represents an *inversion* of the typical problem because the focus is on the status of the *physical* features rather than mathematical ones.

- **Obvious response**: External physical reality *is* a mathematical structure; the physics is an epiphenomenon.

An elementary particle is simply a unitary irreducible representation of the inhomogenous Lorentz group.

- **Obvious problem**: This doesn't fit well with physics/life as we know or practice it.

What work does this kind of metaphysical reduction do if it doesn't help you understand the physics? (e.g. Versions of SR; Platonism)

There are more interesting questions to answer.

2 QUESTIONS:

1. **How** does Mathematics generate Physical explanations? *

2. **What** are the differences between physical explanations that require mathematics and mathematical techniques/frameworks that produce physical answers?

1. The Abstract

Two different contexts where mathematics "produces" physics/physical explanations:

1. Group theory/gauge theory (mathematical entities, symmetries)

2. Renormalization Group (mathematical method)

"There are symmetry principles that dictate the very existence of all the known forces of nature" (Steven Weinberg)

Problematic Issues:

1. Physical status of "gauge invariance". If it's not interpreted realistically then the accompanying forces (interactions) are rendered fictitious.
2. Is the gauge field a physical field?
3. Isn't it largely put in by hand? Are there other considerations required to get out the physics?

Renormalization Group Methods have produced an understanding of critical phenomena and aspects of quantum field theory that was impossible without them.

Problematic Issues:

1. The role of the thermodynamic limit (Batterman, Earman, Callender, Belot)
2. Isn't RG simply a calculational method?

- **Batterman (2010):** $T \rightarrow \infty$ is an explanatory essential mathematical **operation.**
- There is a genuine mathematical explanation of physical phenomenon.
- No appeal to existence of mathematical entities or properties.
- Appeal to a mathematical **idealization** resulting from a limit operation.
- These explain without representing the system in question.

Points of difference:

1. The mathematical idealization is a mathematical description of a physical system and as such involves a mathematical properties.

2. These kinds of idealisations play and explanatory role by representing the system in question.

- **Batterman (2010):** $T \to \infty$ is an explanatory essential mathematical **operation.**
- There is a genuine mathematical explanation of physical phenomenon.
- No appeal to existence of mathematical entities or properties.
- Appeal to a mathematical **idealization** resulting from a limit operation.
- These explain without representing the system in question.

Points of difference:

1. The mathematical idealization is a mathematical description of a physical system and as such involves a mathematical properties.

2. These kinds of idealisations play and explanatory role by representing the system in question.

- Clarify how $T \rightarrow \infty$ figures in the physical explanation (i.e. why it's necessary for a phase transition).
- As such, the mathematics in its application becomes intimately connected with the physics.

1.1 What is the RG?

- **Renormalization group (RG)**: a mathematical framework used to investigate the changes of a physical system viewed at different *distance scales*.

- In particle physics it reflects changes in the underlying force laws as one varies the *energy scale* at which physical processes occur (scale transformation).

- Scale invariance: a symmetry by which the system appears the same at all scales (self-similarity).

- The system will generally make a self-similar copy of itself, with slightly different parameters describing the components (particles, spins etc.) of the system and the interactions between them.

- E.G. coupling constants that measure the strength of various forces, or mass parameters.
- Electron at very short distances has a slightly different electric charge than does the "dressed electron" seen at large distances.

- This change, or "running," in the value of the electric charge is determined by the renormalization group equation(s).

Distinguish between **old fashioned renormalization** vs. RG:

- Renormalization involves a theory of mass and charge where the infinity is cut-off by an implicit ultra-large mass scale, Λ.
- The dependence of physical quantities, such as the electric charge or electron mass, on Λ is hidden, effectively swapped for the scales at which the physical quantities are measured.

1.2 Emergence of RG: QFT

- Gell-Mann and Low (1954): the effective scale can be arbitrarily defined (μ) and can vary to define the theory at any other scale.
- Proposed the existence of a mathematical function of the coupling parameter g of a theory $\psi(g)$, where the function determines the differential change of the coupling constant with a small change in energy scale μ by the RG equation (several different forms)
- $\psi(g) = \beta(g)/g$
- Specifically a **beta function** β(g) encodes the dependence of a coupling parameter, g, on the energy scale μ of a given physical process. It is defined by the relation:
- $\beta(g) = \mu\, \partial g/\partial \mu$

- The main point of the RG: as the scale μ varies the theory makes a self-similar replica of itself, with a small change in *g* given by the RG equation and $\psi(g)$.

- The self-similarity stems from the fact that $\psi(g)$ depends only upon the parameter(s) of the theory, not upon the scale μ.

- Earlier versions of renormalization – parameters like mass, charge etc. were specified at the beginning and were not the outcome of renormalization calculation.
- The change in length scale simply changed the values from the bare values appearing in the basic Hamiltonian to the renormalized values.
- In RG the number and type of relevant parameters is determined by the outcome of the renormalization calculation.

1.3 RG Extended

- RG is also used to derive properties of the behaviour of **critical phenomena** that exhibit a divergence in correlation length and a general slowing down of the dynamics.

- The phenomena of 2^{nd} order phase transitions – RG shows how to *understand and explain* experimental data in terms of scaling and universality. (E.G. Why do fluids and magnets behave the same way at critical point?)

- **Phase transition** – system exhibits the effects of a singularity over the entire spatial extent of the system.
- Occurs only in infinite systems (infinite particles, volume or strong interactions).
- Variation over a vast range of length scales.
- RG methods allow one to investigate the changes of a physical system as one views it at different distance scales.

- Phase changes involve and **Order parameter** that distinguishes between the two phases (in ferromagnets, homogenous magnetization) which describes the degree of order below Tc.

1.3 RG Extended

- RG is also used to derive properties of the behaviour of **critical phenomena** that exhibit a divergence in correlation length and a general slowing down of the dynamics.

- The phenomena of 2^{nd} order phase transitions – RG shows how to *understand and explain* experimental data in terms of scaling and universality. (E.G. Why do fluids and magnets behave the same way at critical point?)

- Phase changes involve and **Order parameter** that distinguishes between the two phases (in ferromagnets, homogenous magnetization) which describes the degree of order below Tc.

- Properties near critical point are determined primarily by the **correlation length** for fluctuations in the order parameter.
- The **correlation function** $\Gamma(r)$ measures how the value of the order parameter at one point is correlated to its value at some other point.

- Two points separated by a distance larger than the correlation length will each have fluctuations which are relatively independent.
- Experimentally, the correlation length is found to diverge at the critical point.
- The divergence means that distant points become correlated and long-wavelength fluctuations dominate.
- The system 'loses memory' of its microscopic structure and begins to display new long-range macroscopic correlations and new behaviour.

- RG provides scaling equations for the correlation functions used in the statistical description.
- RG links physical behaviour across different scales and where fluctuations on many different scales interact.
- *Why is this important and what is its impact on mathematical / physical explanation?*

2. Extracting physics from the RG equation(s).

Momentum/Fourier space – QFT

- Change in μ (scale) changes the scope of the degrees of freedom in the calculations.
- To avoid large logarithms take μ to be the order of the energy E that is relevant to the process under investigation.
- Integrate out high wave number modes from a continuous spin field.

- The problem is broken down into a sequence of sub-problems with each one involving only a few length scales.
- Each one has a characteristic length and you get rid of the degrees of freedom you don't need.
- Reducing the degrees of freedom gives you a sequence of corresponding Hamiltonians pictured as a trajectory in a space spanned by the system parameters (temperature, external fields and coupling constants).

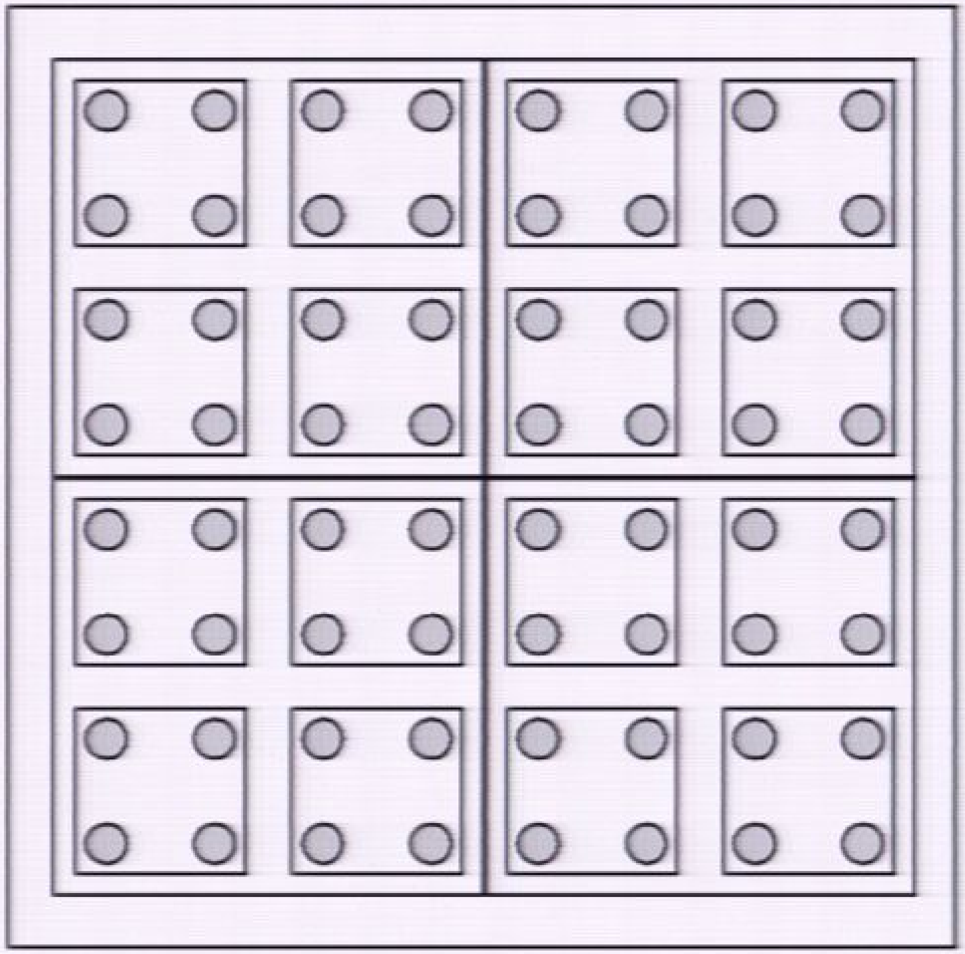
RG gives us a transformation that looks like this: $\mathcal{H}' = R[\mathcal{H}]$

$\mathcal{H}$ is the original Hamiltonian with N degrees of freedom.

Rescale the fields to maintain invariance of $\mathcal{H}$.

- The real space approach (definite lattice) : Wilson–Kadanoff
- Scaling relations involves a lattice of interacting spins (ferromagnetic transition) and transformations from a site lattice with the Hamiltonian $H_a(S)$ to a block lattice with Hamiltonian $H_{2a}(S)$.
- Each block is considered as a new basic entity.
- Calculate the effective interactions between them and construct a family of corresponding Hamiltonians.
- Starting from a lattice model of lattice size *a* sum over degrees of freedom at size *a* while maintaining their average on the sub–lattice of size 2*a* fixed.

- $H_a(S)$ on the initial lattice one would generate an effective Hamiltonian $H_{2a}(S)$ on the lattice of double spacing.
- This transformation is repeated as long as the lattice spacing remains small compared to the correlation length.
- The transition from $H_a(S)$ to $H_{2a}(S)$ can be regarded as a rule for obtaining the parameters of $H_{2a}(S)$ from those of $H_a(S)$.
- The process can be repeated with the lattice of small blocks being treated as a site lattice for a lattice of larger blocks.



–O––O––O––O–

–O––O––O––O–

–O––O––O––O–

–O––O––O––O–

- Technically, the procedure involves carrying out statistical averages over thermal fluctuations on all size scales.
- Average out the fluctuations in sequence starting with fluctuations on an atomic scale and then moving to successively larger scales until fluctuations on all scales have been averaged out.

- Close to critical point the correlation length (the distance over which the fluctuations of one microscopic variable are associated with another) far exceeds the lattice constant *a* which is the difference between neighbouring spins.

- For each new block lattice one constructs effective interactions and finds their connection with the interactions of the previous lattice. (Kadanoff)

Wilson:

1. Showed how the coupling constants at different length scales could be computed.
2. How critical components could be estimated and hence
3. How to understand universality, which follows from the fact that the process can be iterated (i.e. universal properties follow from the limiting behaviour of such iterative processes).

How does this work?

- The behaviour of thermodynamic parameters near critical point are also characterized by critical indices.
- Phase transitions with the same set of critical indices are said to belong to the same universality class. (magnetism and gas-liquid)
- These systems behave in exactly the same way at critical point regardless of their micro-structure.

Wilson:

1. Showed how the coupling constants at different length scales could be computed.
2. How critical components could be estimated and hence
3. How to understand universality, which follows from the fact that the process can be iterated (i.e. universal properties follow from the limiting behaviour of such iterative processes).

How does this work?

- The behaviour of thermodynamic parameters near critical point are also characterized by critical indices.
- Phase transitions with the same set of critical indices are said to belong to the same universality class. (magnetism and gas-liquid)
- These systems behave in exactly the same way at critical point regardless of their micro-structure.

- The correlation length ξ increases as $T \to T_c$ (provided all other parameters are fixed).
- Diverges at Tc – the fluctuations become completely dominant and we are left without a characteristic length scale because all lengths are equally important.
- Reducing the number of degrees of freedom with RG amounts to establishing a correspondence between one problem having a given correlation length and another whose length is smaller by a certain factor.

- As the process is iterated the Hamiltonian becomes more and more insensitive to what happens on smaller length scales.

- In the long wave-length/large space-scale limit the scaling process leads to a fixed point when the system is at a critical point.

- The properties of this fixed point determine the critical exponents that characterize the fluctuations at the critical point.
- The same fixed point interactions can describe a number of different types of systems. (Universality)

- RG shows that different kinds of transitions have the same critical exponents and can be understood in terms of the same fixed-point interaction that describes all these systems.
- Liquid-gas transitions, magnetic transitions, alloy transitions, etc. all show the same critical exponents experimentally.

- The basis of universality is that the fixed points are a *property of transformations* that are not particular sensitive to the original Hamiltonian.

- What the fixed points do is determine the kinds of cooperative behaviour that are possible.

- The important point isn't just the elimination of irrelevant degrees of freedom but also *the existence of cooperative behaviour and its relation to the order parameter (symmetry breaking) that characterizes the different kinds of systems.*

3. What is Explained?

- RG enables us to explain:
1. How/Why the phenomena at critical point behave the way they do
2. The relation between symmetry breaking (the order parameter) and phase transitions
3. Why that behaviour is insensitive to the underlying microphysics.

- But, and perhaps most importantly, the RG framework gives us a physical understanding of the relation between the dynamics of the system and assumptions about $T \rightarrow \infty$.

- It shows us how phase transitions are connected not only to symmetry (fluctuations in the order parameter) that are part of the **microscopic** features of the system (as in the case of spin directions in a ferromagnet) but also to the topology of an infinite spatial region, and why the latter is crucial for determining, within the RG calculations, that a phase transition has taken place.

- This is evident when comparisons with Mean Field theory are made.

- One of the advantages of RG methods over mean field theory was to extend the list of possible couplings to include all the different terms that might be found in the Hamiltonian of the system.

- Each transformation increases the size of the length scale so that the transformation would eventually extend to information about the parts of the system that are infinitely far away.

- Hence the infinite spatial extent of the system became part of the calculation and it was this behaviour at the far reaches of the system would determine the thermodynamic singularities, thereby including them in the calculation.
- The phase transition was then identified as the place where the transformations brought the coupling to a fixed point with further iterations producing no changes in either the couplings or the correlation length.
- The key here is that the transformation multiplies the length scale by a factor that *depends on the details of the transformation*, not on the microphysical dynamics.

- So, from a mathematical method we get genuine physical understanding.
- The issue here is not simply taking the thermodynamic limit, although that is part of the explanation.
- It involves going beyond that to explain how the dynamical features of universal behaviour arise and why we need TL, not just as a calculational device.

- RG encodes the relation between the microscopic features of the system and the macroscopic in the sense that the iteration of the scaling operation represents how one moves from the very small to TL.

- Also shows that this isn't simply an operation of taking limits – it is a special kind of limit that contains important physical behaviour.

- Physics and mathematics are intertwined but the physical explanation clearly arises out of the mathematical framework.
- The idealisation $T \rightarrow \infty$ constitutes a representation that embodies a dynamical explanation.
- Not simply a calculational device that we need to reify in order to get the right answers.

The How (technical) question and the why (conceptual) question merge into one explanation.

APPENDIX

QFT ANALOGUE

In SM we have a microscopic length scale like crystal lattice spacing.

Cooperative phenomena near critical point create a correlation length

In the limit of critical point the ratio of these two lengths tends toward infinity.

The How (technical) question and the why (conceptual) question merge into one explanation.