Title: Viscosity bound and causality in a superfluid plasma

Date: Nov 30, 2010 11:00 AM

URL: http://pirsa.org/10110054

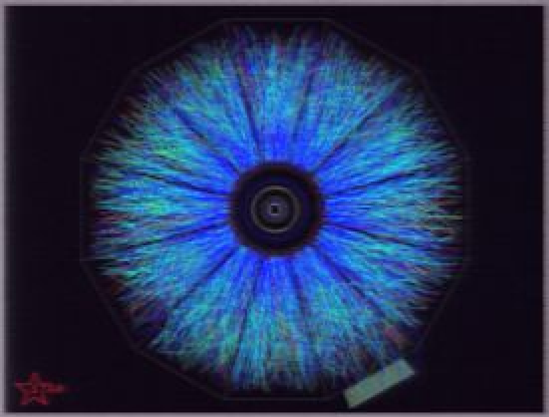
Abstract: In this talk I will discuss the applications of the gauge/gravity duality to the strongly coupled quark gluon plasma, focusing in particular on the role of the shear viscosity to entropy ratio.
It has been argued that the lower bound on the shear viscosity to entropy density in strongly coupled plasmas can be understood in terms of microcausality violation in the dual gravitational description.
However, since the transport properties of the system characterize its infrared dynamics, while the causality of the theory is determined by its ultraviolet behavior, the link between the viscosity bound and microcausality should not be applicable in theories that undergo low temperature phase transitions.
I will discuss an explicit holographic model confirming this fact, in which there is a ``decoupling" of UV from IR physics.

Today's talk based on:

- arXiv:0812.3572
- arXiv:0903.3244
- arXiv:0910.5159
- arXiv:1007.2963

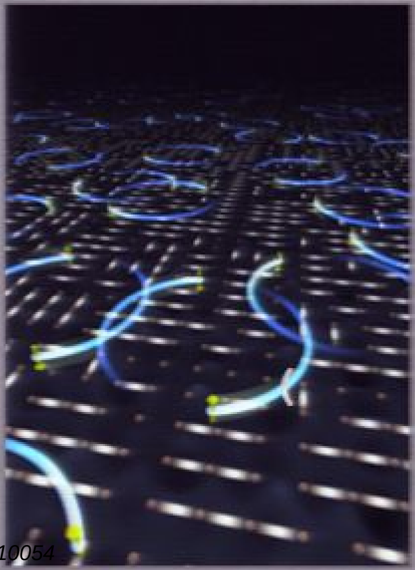
In collaboration with:
A. Buchel
J. Liu, K. Hanaki, P. Szepietowski (Michigan)

and some work in progress…



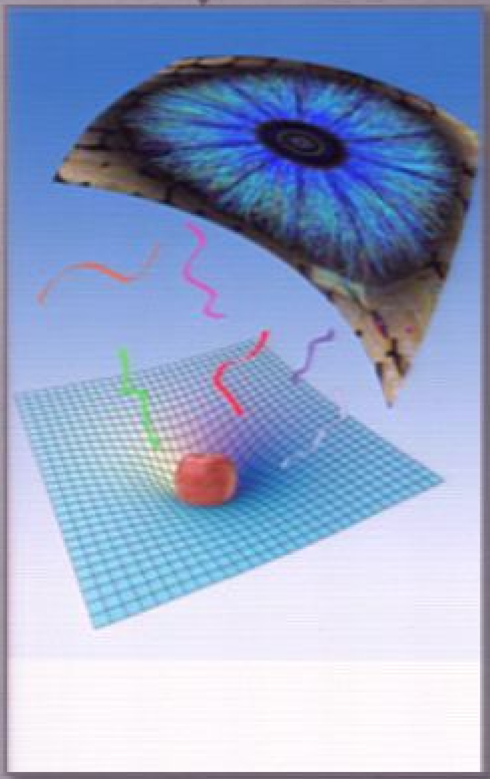
The behavior of many important physical phenomena is governed by the physics of interacting many-body systems, whose dynamics involves a very large number of constituents

Typically, one is interested in the macroscopic behavior (at large distances and long time scales)



In this regime, a system generically exhibits features which are universal (independent of the fine details of the underlying microscopic description)

Thanks to holographic methods emerging from AdS/CFT, connection between gravitational systems and strongly coupled gauge theories



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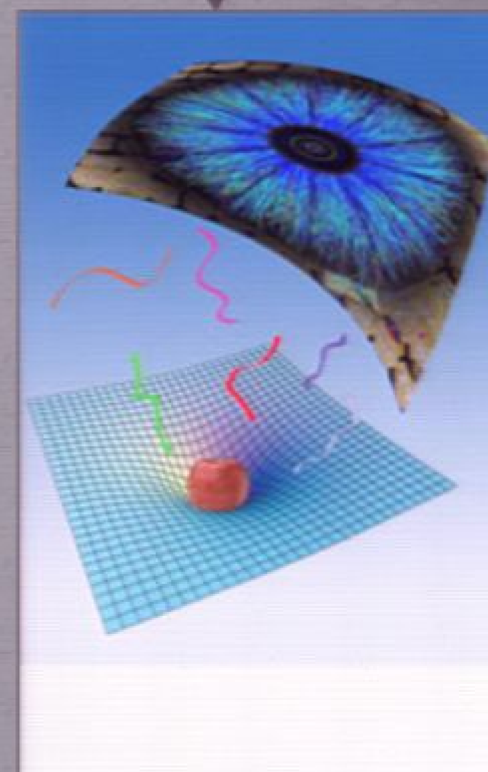

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thermal and hydrodynamical properties
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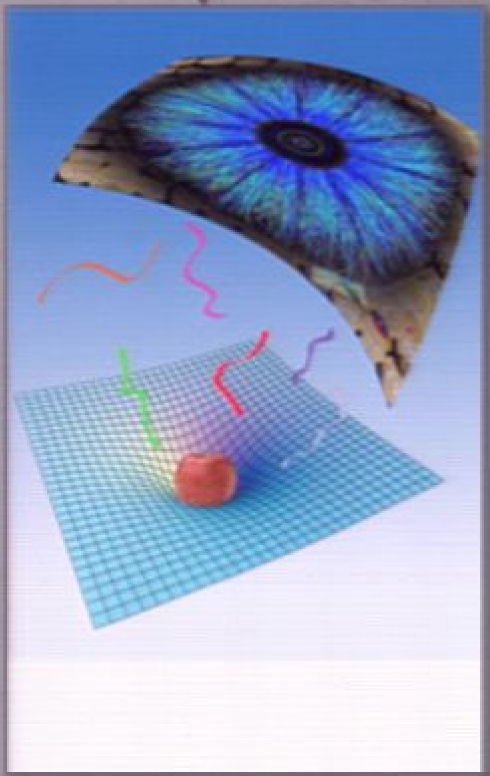

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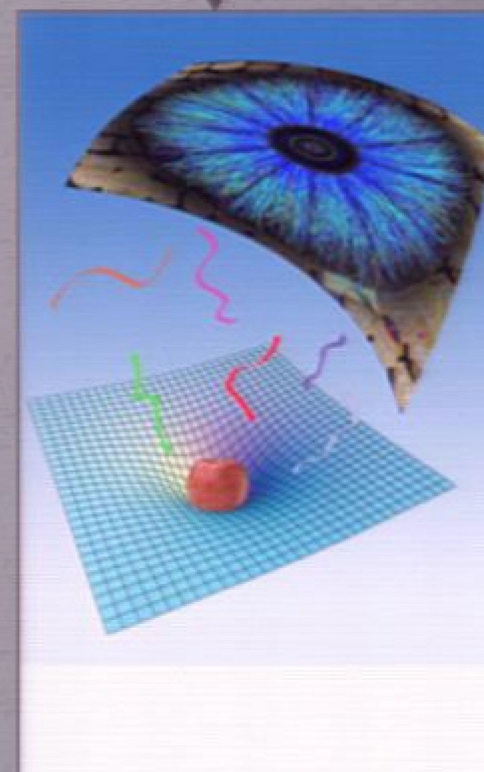

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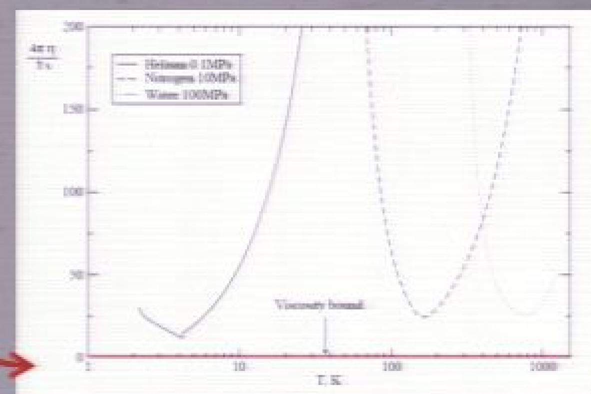
In this program, it is particularly important to find universal features that might hold in realistic systems

Plan

Focus on a "universal" quantity that has played key role in studies of the QCD quark gluon plasma:

shear viscosity/entropy ratio η/s



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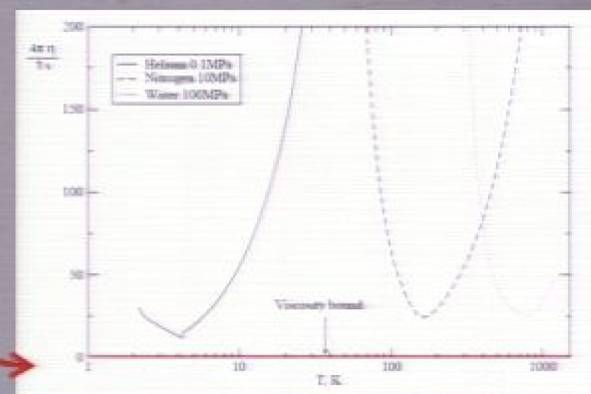

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$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

QGP value measured at RHIC close to that predicted by gauge/gravity duality

Outline

- Background on η/s :
  the QGP plasma and why so much attention from AdS/CFT
- What AdS/CFT has taught us about η/s
  → focus on **higher derivative corrections**
- It's now well understood that the **bound is violated**
  - features in string theory-based models
- Apparent **link between causality violation and violation of the viscosity bound is <u>not</u> of fundamental nature**
  - Toy model with a decoupling of UV from IR physics

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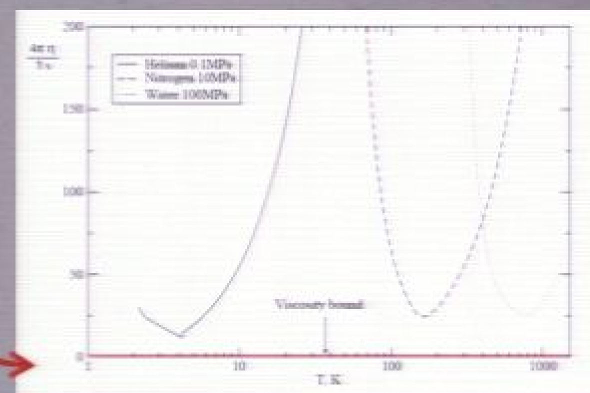

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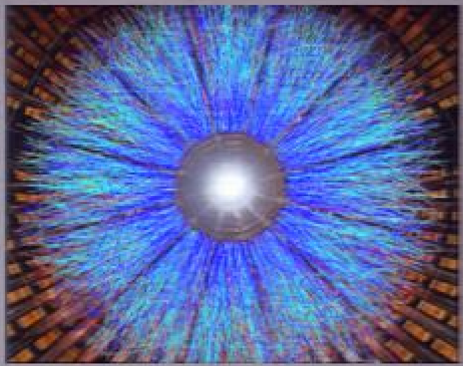
Part I

The quark gluon plasma and the shear viscosity to entropy ratio

Insight into the quark gluon plasma



RHIC (Au+Au, ~200 GeV per nucleon):

creates hot and dense nuclear matter

→ probe QGP behavior (transport properties)

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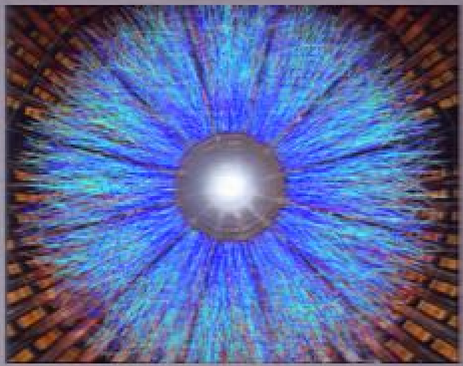

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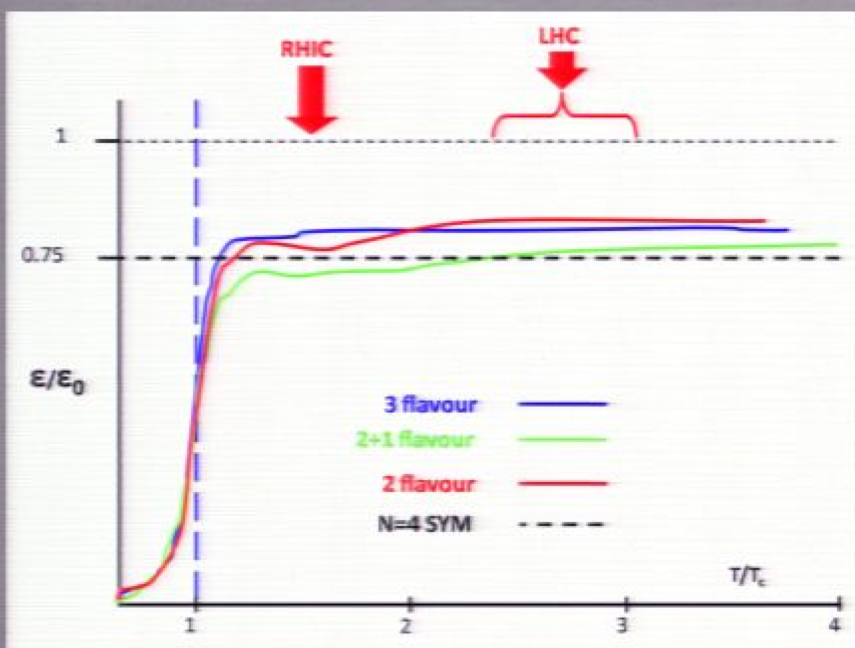

Karsch, hep-lat/0106019

N = 4 SYM at finite T is not QCD but:

- Some features *qualitatively* similar to QCD (for T ~ T_c - $3T_c$)
  - strongly coupled
  - nearly conformal (small bulk viscosity away from T_c)
- Some properties may be *universal*

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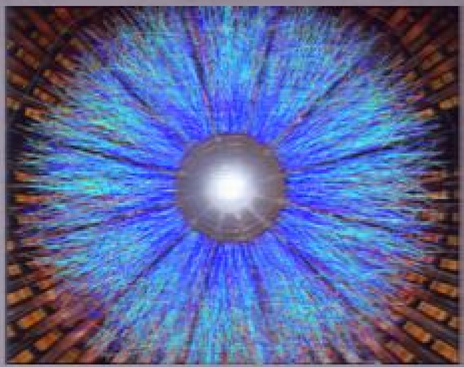

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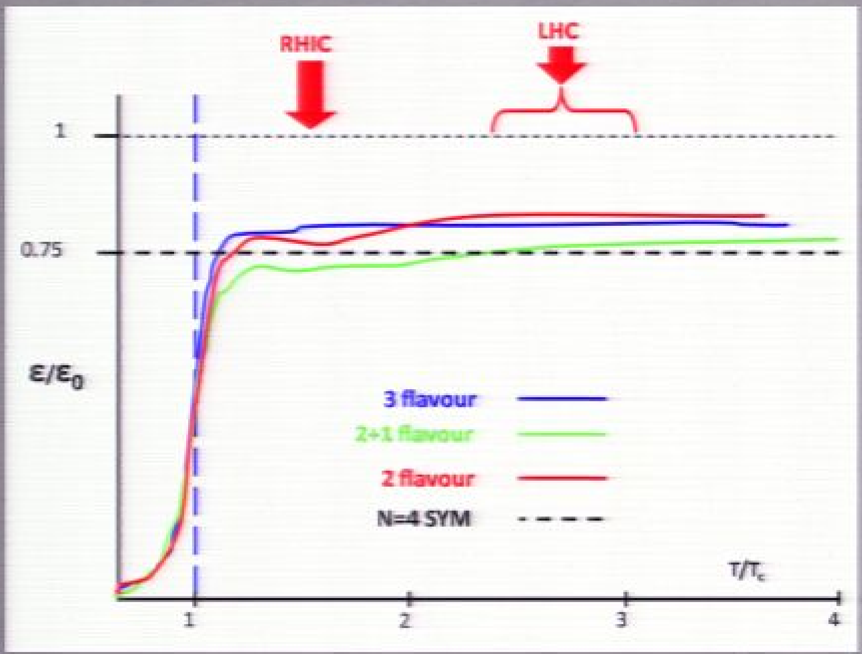

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such generic relations might provide INPUT into realistic simulations of sQGP

Elliptic Flow at RHIC

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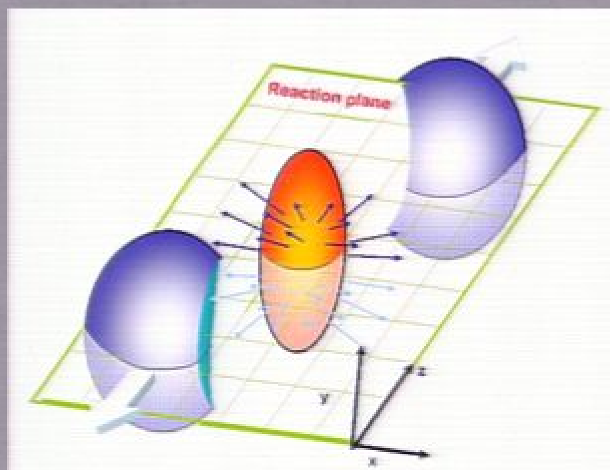
Off-central heavy-ion collisions at RHIC:



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(large pressure gradient
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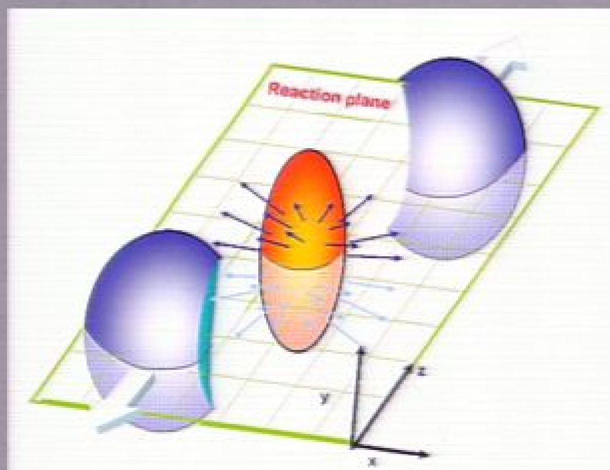

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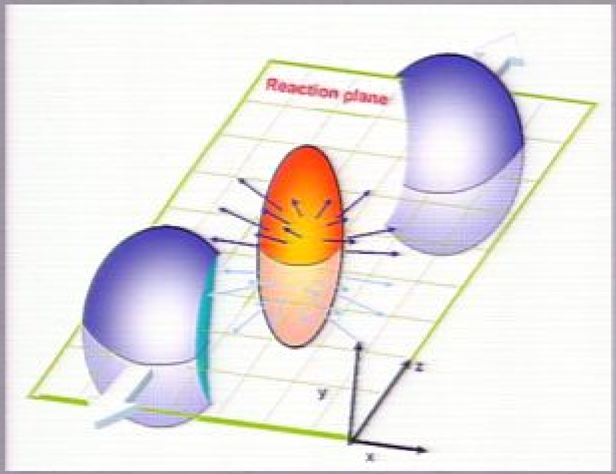

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RHIC data favors $\eta/s < 5/4\pi$ (Heinz/Song 0909.1549)

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$$\frac{\eta}{s} \sim \frac{1}{\lambda^4 \log 1/\lambda^2} \gg 1$$

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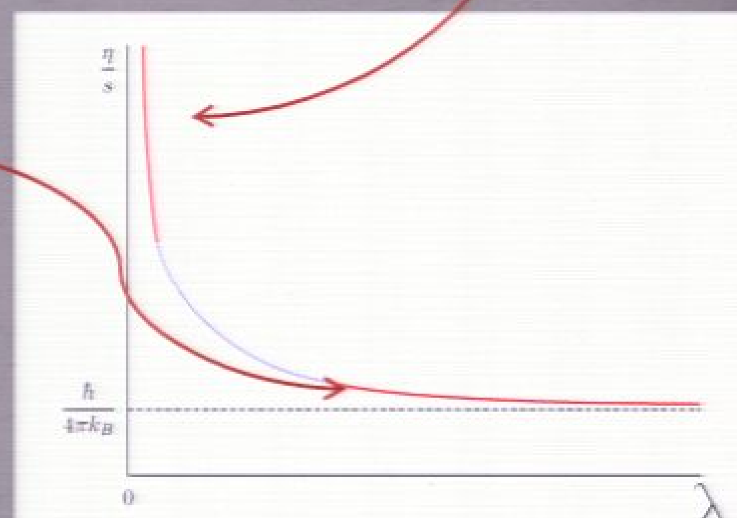

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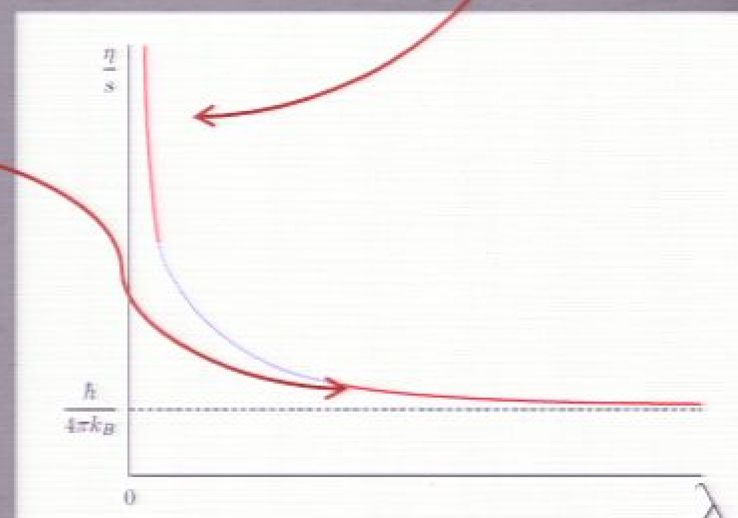

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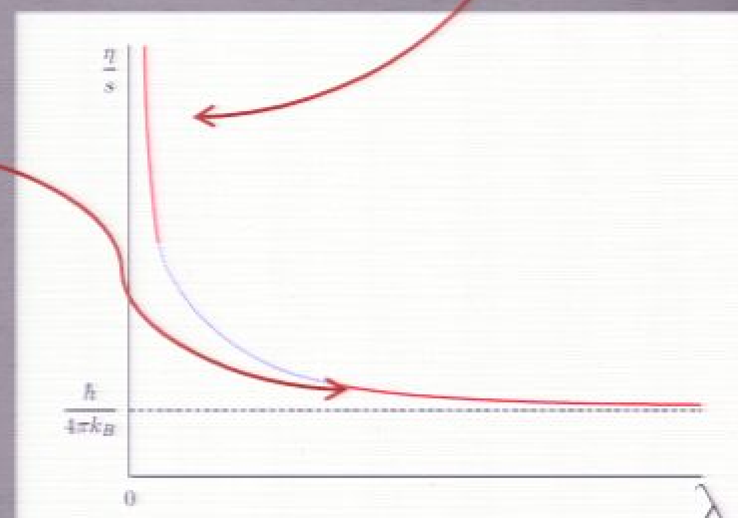

Strong coupling → natural setting for AdS/CFT applications

Universal Viscosity/Entropy Ratio

For $\mathcal{N}=4$ SU(N) SYM plasma:

planar limit, infinite 't Hooft coupling [Policastro,Son,Starinets hep-th/0104066]

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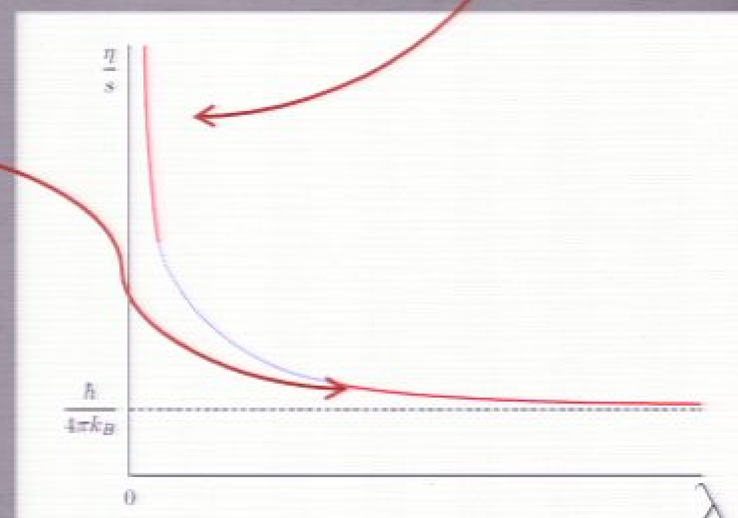

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Result is **universal** in all gauge theories whose gravity duals are dictated by **Einstein gravity**

[Buchel & Liu th/0311175]

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A simple dilute gas estimate seems to suggest a QM bound:

$$\frac{\eta}{s} \sim p\, l_{mfp} \Rightarrow \frac{\eta}{s} \gtrsim O(\hbar)$$

Shear Viscosity/Entropy Bound

Conjectured lower bound for field theories at finite T

[Kovtun,Son,Starinets th-0309213]

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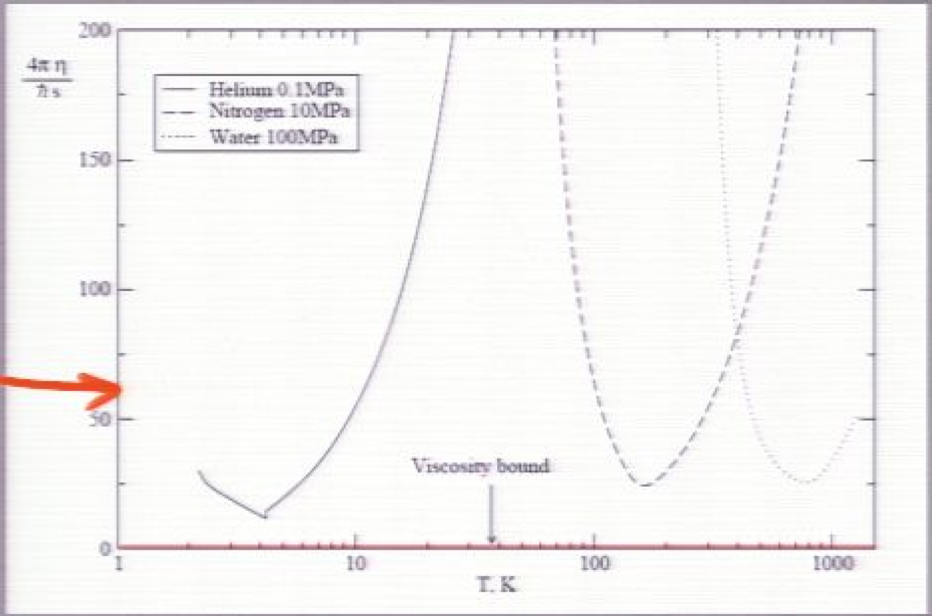

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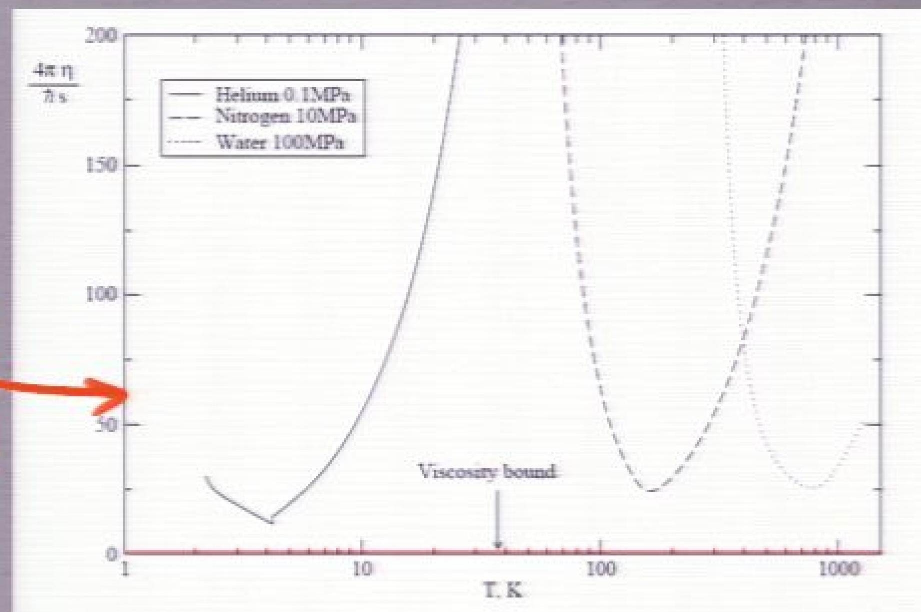

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$$\frac{\eta}{s} = \frac{1}{4\pi} \sim .08$$

RHIC value is at most a few times

Ratio $\frac{\eta}{s} = \frac{1}{4\pi}$ is universal in Einstein GR $\mathcal{L} = R - \frac{1}{2n!} F_n^2 + \ldots$

How does it change with higher derivative corrections?

$$\mathcal{L} = R - \frac{1}{2n!} F_n^2 + \ldots + \alpha' R^2 + \alpha'^2 R^3 + \alpha'^3 R^4 + \ldots$$

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Why include higher derivatives?

Gravity side:

curvature corrections

CFT side:

finite λ, N corrections

- Natural from EFT point of view:

  Einstein GR is only low-energy description of string theory

- More "phenomenological" point of view:

  corrections might bring observable quantities closer to measured values

Pathologies of higher derivative gravity

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R_{\mu\nu}^2 + \alpha_3 R_{\mu\nu\rho\sigma}^2 + \ldots$$

Higher derivatives can lead to undesirable features:

- Modify graviton propagator
- ill-poised Cauchy problem (no generalization of Gibbons-Hawking term)

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arXiv:0910.5159
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Part II

Black holes with higher derivatives
and
the violation of the viscosity bound

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Testing The Bound

- Leading α' correction on $AdS_5 \times S^5$ ($\mathcal{N} = 4$ SYM) increased the ratio [Buchel,Liu,Starinets th/0406264]

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + 15\zeta(3)\lambda^{-3/2} + \ldots\right]$$

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come back to this later

String Construction Violating Bound

Kats & Petrov (arXiv:0712.0743)

- Type IIB on $\mathrm{AdS}_5 \times \mathrm{S}^5/\mathbb{Z}_2$
- Decoupling limit of N D3′s sitting inside 8 D7′s coincident on O7 plane

$$S = \int d^D x \sqrt{-g} \left(\frac{R}{2\kappa} - \Lambda + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + c_3 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \right)$$

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Couplings → determined by (fundamental) matter content of theory and sensitive to 1/N corrections

[Buchel et al. 0812.2521 for large class of CFTs violating bound]

Our interest in this story

SC, K. Hanaki, J. Liu, P. Szepietowski, 0812.3572, 0903.3244, 0910.5159

- Higher derivative corrections at finite electric chemical potential (R-charged)
- Role of chemical potential on the bound?
  - at two-derivative level, it has no effect (universality)
  - with higher derivatives, is bound restored with sufficiently large chemical potential?
- Corrections constrained by supersymmetry

$$R^2 + ARR + F^4 + \ldots$$

- Role of SUSY (string/susy constraints)?
- Phenomenological applications:
  - one more parameter (chemical potential) to better fit data

Corrections to η/s at finite chemical potential

[arXiv:0903.3244, SC,K.Hanaki,J.Liu,P.Szepietowski]

The setup: D=5 $\mathcal{N}=2$ gauged SUGRA (electrically charged black holes)

To leading order:

$$\mathcal{L}_0 = -R - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{12\sqrt{3}}\epsilon^{\mu\nu\rho\lambda\sigma}F_{\mu\nu}F_{\rho\lambda}A_\sigma + 12g^2$$

$$ds^2 = H^{-2}f dt^2 - H\left(f^{-1}dr^2 + r^2 d\Omega_{3,k}^2\right) \quad H(r) = 1 + \frac{Q}{r^2},$$

$$A = \sqrt{\frac{3(kQ+\mu)}{Q}}\left(1 - \frac{1}{H}\right)dt, \quad f(r) = k - \frac{\mu}{r^2} + g^2 r^2 H^3$$

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- In this theory higher derivative corrections start at R^2 (sensitive to amount of SUSY)
- They include the mixed gauge-gravitational CS term:

$$\mathbf{A} \wedge \mathrm{Tr}(\mathbf{R} \wedge \mathbf{R})$$

SUSY R^2 terms in 5D

- We are interested in consistent string theory reductions
  → want R^2 terms constrained by *SUSY*
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- Off-shell formulation of $\mathcal{N}$=2, D=5 gauged SUGRA (superconformal formalism). End Result

off shell action, lots of auxiliary fields,
supersymmetric curvature-squared term in 5D

On-shell Lagrangian (minimal SUGRA)

[arXiv:0812.3572, SC,K.Hanaki,J.Liu,P.Szepietowski]

$$\mathcal{L} = -R - \frac{1}{4}F^2 + \frac{1}{12\sqrt{3}}\Big(1 - \frac{1}{6}c_2 g^2\Big)\epsilon^{\mu\nu\rho\lambda\sigma} A_\mu F_{\nu\rho} F_{\lambda\sigma} + 12g^2 + \frac{c_2}{24}\Big[\frac{1}{48}RF^2 + \frac{1}{576}(F^2)^2\Big] + \mathcal{L}_1^{\text{ungauged}}.$$

$$\mathcal{L}_1^{\text{ungauged}} = \frac{c_2}{24}\Big[\frac{1}{16\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}A^\mu R^{\nu\rho\delta\gamma}R^{\lambda\sigma}{}_{\delta\gamma} + \frac{1}{8}C^2_{\mu\nu\rho\sigma} + \frac{1}{16}C_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda} - \frac{1}{3}F^{\mu\rho}F_{\rho\nu}R^\nu_\mu - \frac{1}{24}RF^2 + \frac{1}{2}F_{\mu\nu}\nabla^\nu\nabla_\rho F^{\mu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\mu F_{\nu\rho} + \frac{1}{4}\nabla^\mu F^{\nu\rho}\nabla_\nu F_{\rho\mu} + \frac{1}{32\sqrt{3}}\epsilon_{\mu\nu\rho\lambda\sigma}F^{\mu\nu}(3F^{\rho\lambda}\nabla_\delta F^{\sigma\delta} + 4F^{\rho\delta}\nabla_\delta F^{\lambda\sigma} + 6F^\rho{}_\delta\nabla^\lambda F^{\sigma\delta}) + \frac{5}{64}F_{\mu\nu}F^{\nu\rho}F_{\rho\lambda}F^{\lambda\mu} - \frac{5}{256}(F^2)^2\Big].$$

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controls strength of higher derivative terms

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Interpretation on dual gauge theory side?

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Wireless Radio Status

All wireless radios are now off.

OK

$$\mathcal{L}_1^{\text{ungauged}} = \frac{c_2}{24}\Big[\frac{1}{16\sqrt{3}} \cdots F^{\mu\nu}F^{\rho\lambda} - \frac{1}{3}F^{\mu\rho}F_{\rho\nu}R^{\nu}_{\mu}$$

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Use AdS/CFT to relate c_2 to central charges of dual CFT via:

- Holographic trace anomaly
- R-current anomaly ← See paper

Using the dual CFT

For us: 4D CFT with N=1 SUSY

- 4D CFT central charges a, c defined in terms of trace anomaly: (CFT coupled to external metric)

$$\langle T^{\mu}_{\mu} \rangle = \frac{c}{16\pi^2} C - \frac{a}{16\pi^2} E$$

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Prescription for extracting trace anomaly for higher derivative GR:
Blau, Narain, Gava th/9904179, Nojiri, Odintsov th/9903033

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$\Updownarrow$

$$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R^2_{\mu\nu} + \alpha_3 R^2_{\mu\nu\rho\sigma} + \ldots$$

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⇕

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$$\Rightarrow \quad c_2 = \frac{24}{g^2} \frac{c - a}{a}$$

Thermodynamics and Hydrodynamics of R-charged black-holes

Now we have all the ingredients we need to compute η/s

We expect:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + f\left(\frac{c-a}{a}, Q\right)\right]$$

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- **entropy**
  Einstein GR: area of event horizon
  Higher derivative GR: Wald's entropy formula

- **shear viscosity**
  can be extracted from boundary stress tensor

Shear Viscosity

Relativistic Hydrodynamics:

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \sigma^{\mu\nu}$$

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effective description of dynamics of system at large wavelengths and long time scales

Bound Violation

electrically charged
black holes

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reminiscent of Wald's entropy formula

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Part III

Microcausality violation and the link to η/s

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Click to add notes

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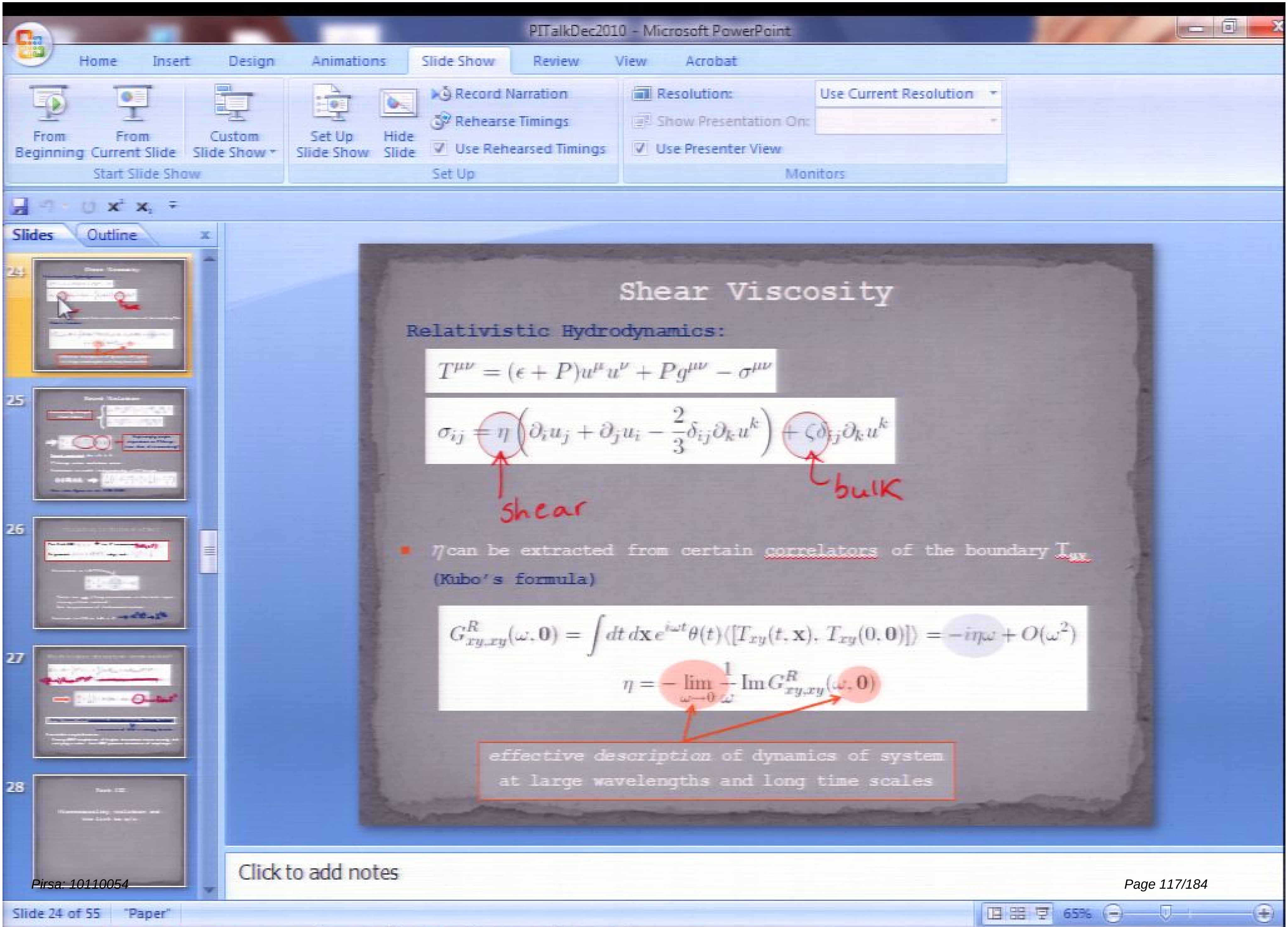
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From Beginning
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Start Slide Show
Set Up Slide Show
Hide Slide
Record Narration
Rehearse Timings
Use Rehearsed Timings
Set Up
Resolution:
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Show Presentation On:
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effective description of dynamics of system at large wavelengths and long time scales
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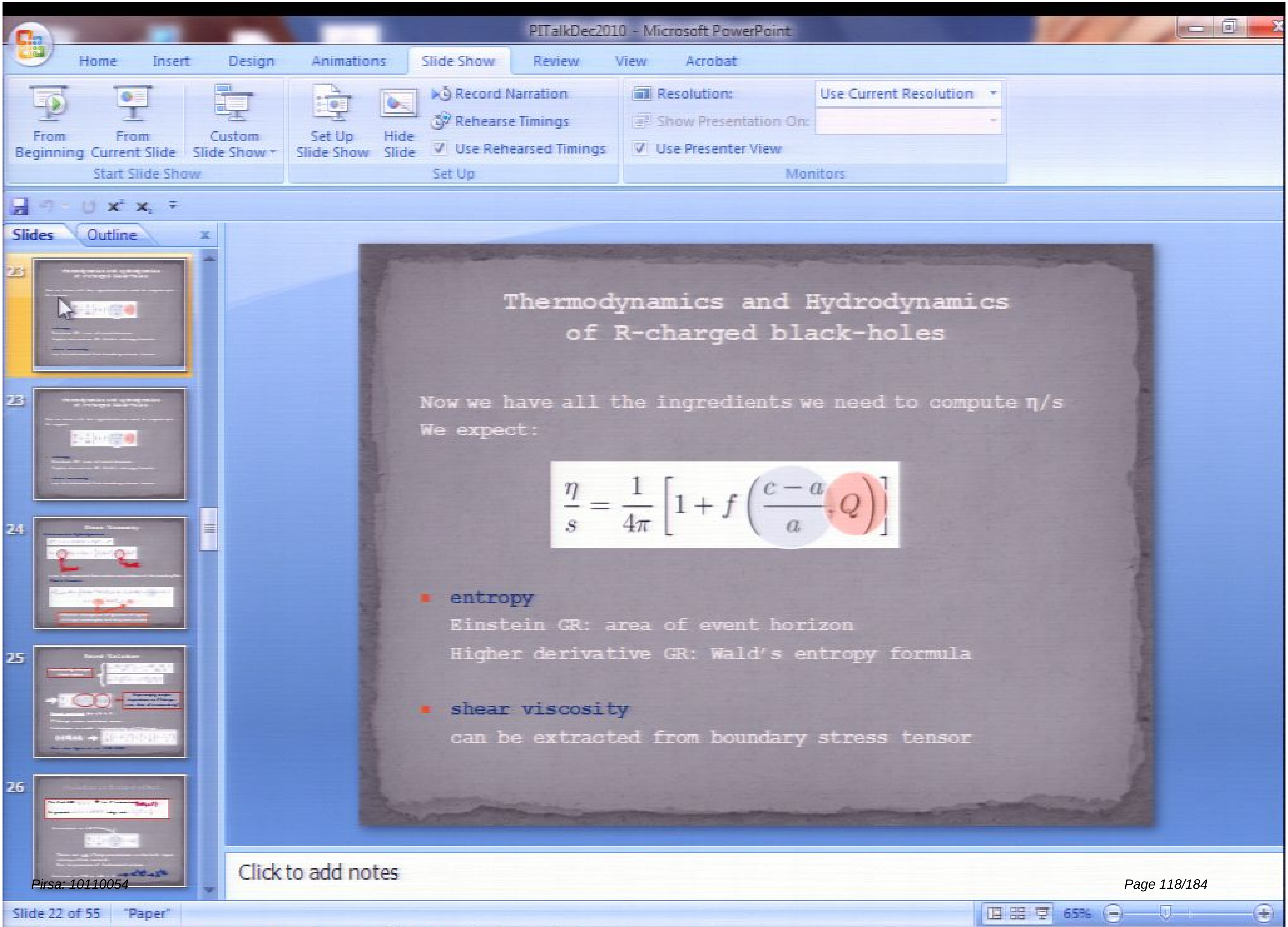
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Outline
Thermodynamics and Hydrodynamics of R-charged black-holes
Now we have all the ingredients we need to compute η/s
We expect:
$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 + f\left(\frac{c-a}{a}, Q\right)\right]$$
entropy
Einstein GR: area of event horizon
Higher derivative GR: Wald's entropy formula
shear viscosity
can be extracted from boundary stress tensor
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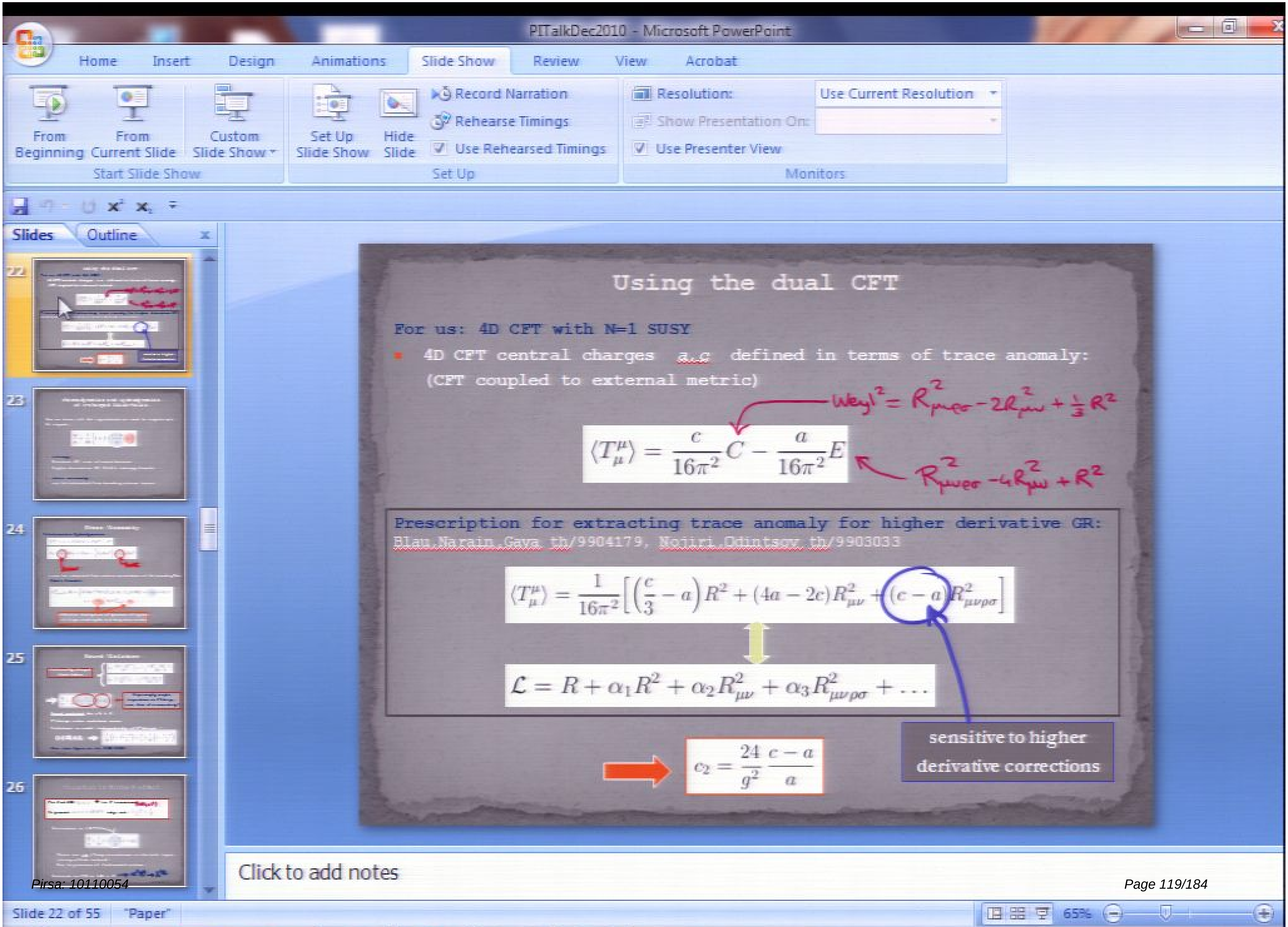
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Using the dual CFT
For us: 4D CFT with N=1 SUSY
4D CFT central charges a,c defined in terms of trace anomaly:
(CFT coupled to external metric)
$\langle T^\mu_\mu \rangle = \frac{c}{16\pi^2} C - \frac{a}{16\pi^2} E$
$\mathrm{Weyl}^2 = R^2_{\mu\nu\rho\sigma} - 2R^2_{\mu\nu} + \frac{1}{3}R^2$
$R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$
Prescription for extracting trace anomaly for higher derivative GR:
Blau,Narain,Gava th/9904179, Nojiri,Odintsov th/9903033
$\langle T^\mu_\mu \rangle = \frac{1}{16\pi^2}\left[\left(\frac{c}{3} - a\right)R^2 + (4a - 2c)R^2_{\mu\nu} + (c - a)R^2_{\mu\nu\rho\sigma}\right]$
$\mathcal{L} = R + \alpha_1 R^2 + \alpha_2 R^2_{\mu\nu} + \alpha_3 R^2_{\mu\nu\rho\sigma} + \ldots$
$c_2 = \frac{24}{g^2}\frac{c - a}{a}$
sensitive to higher derivative corrections
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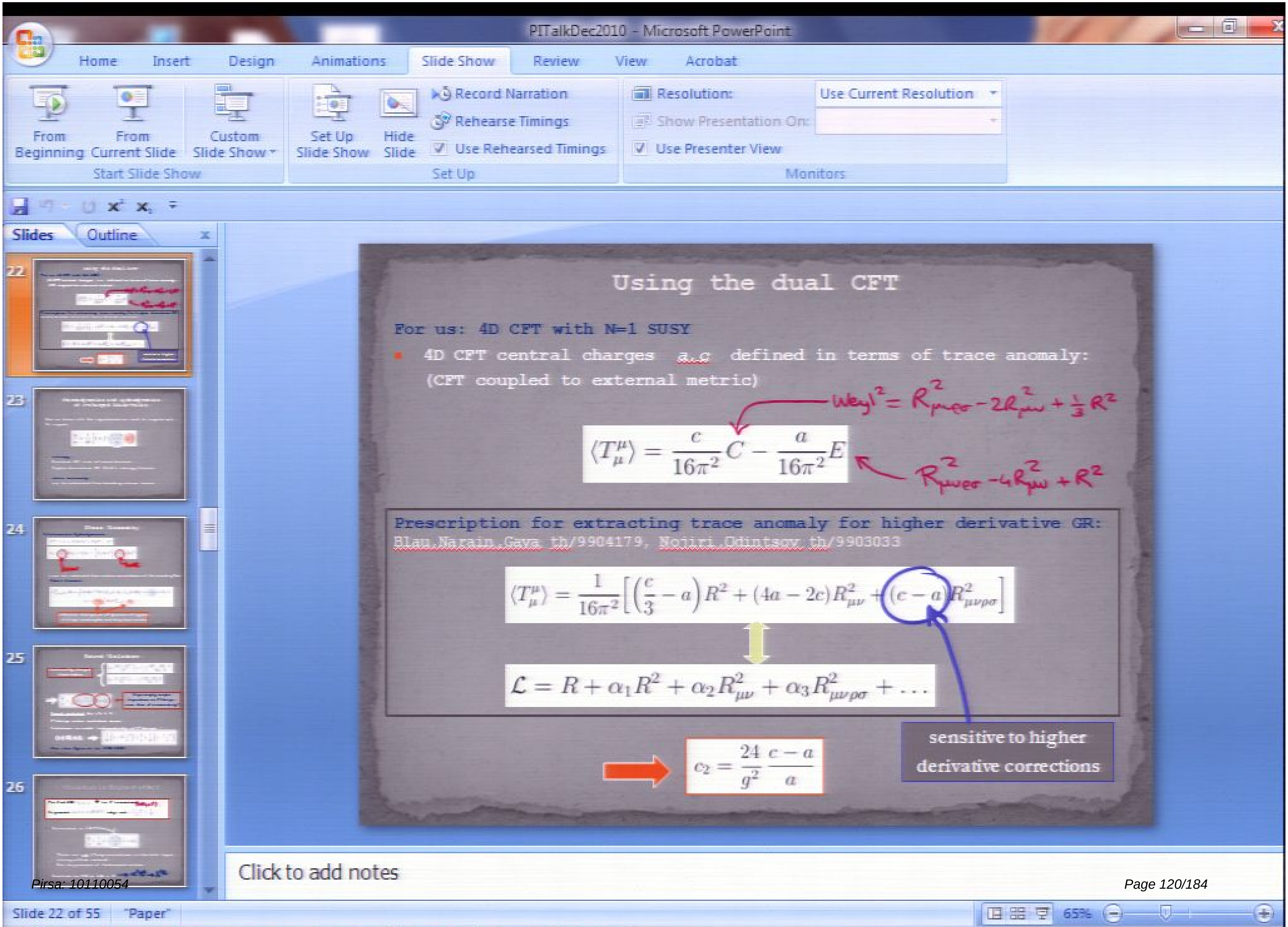
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$\Rightarrow$ $$c_2 = \frac{24}{g^2}\frac{c - a}{a}$$

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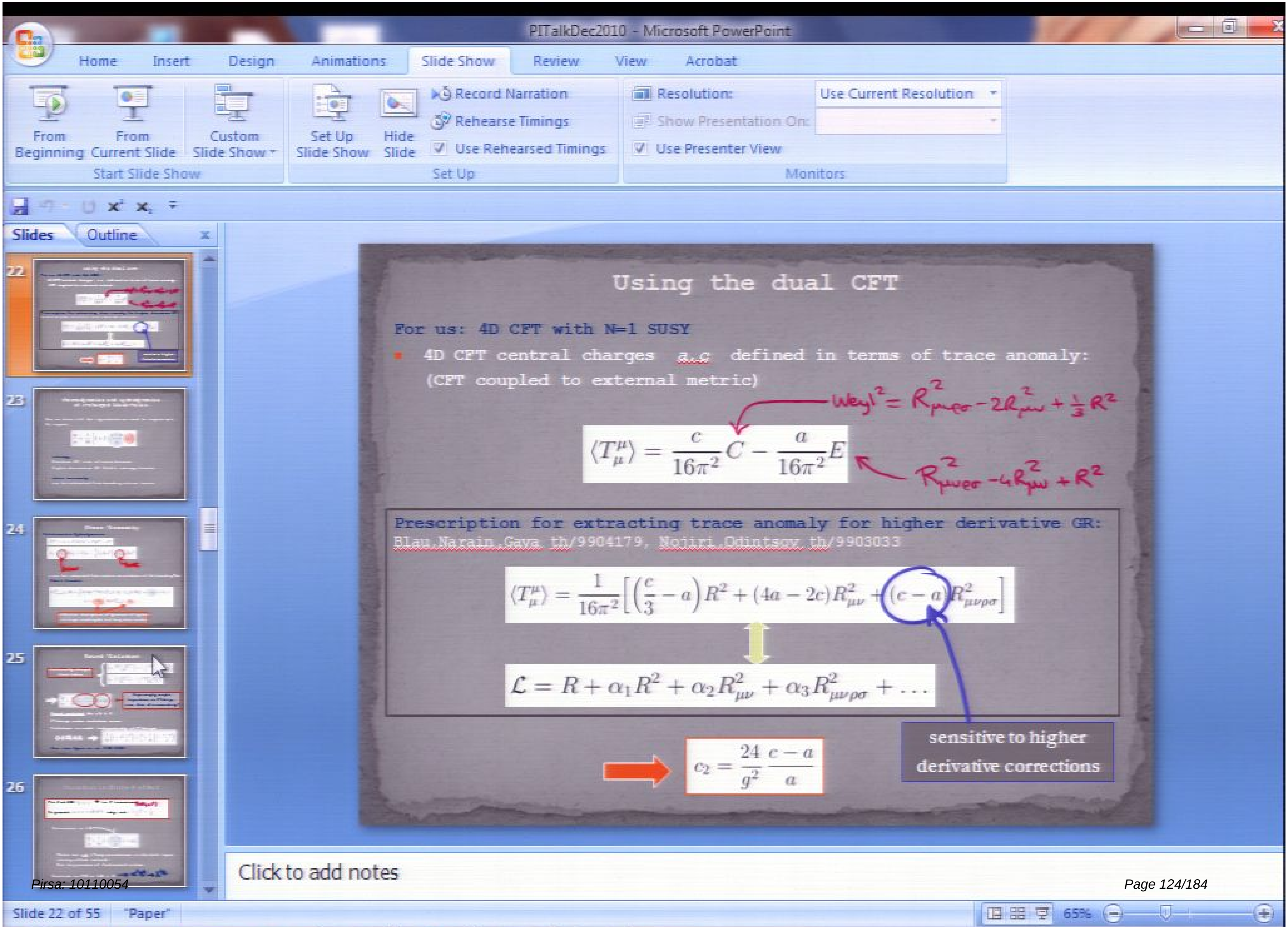

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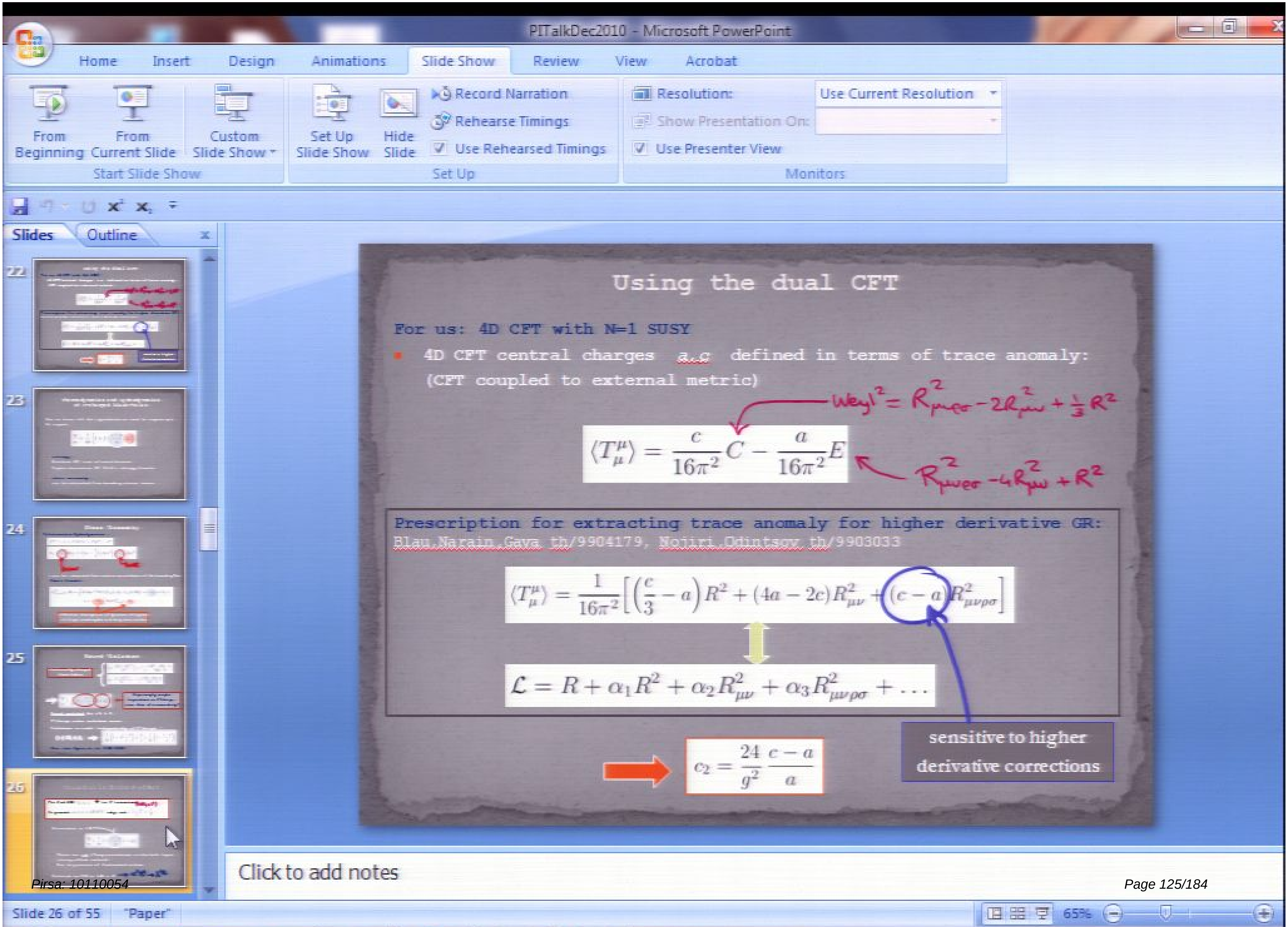
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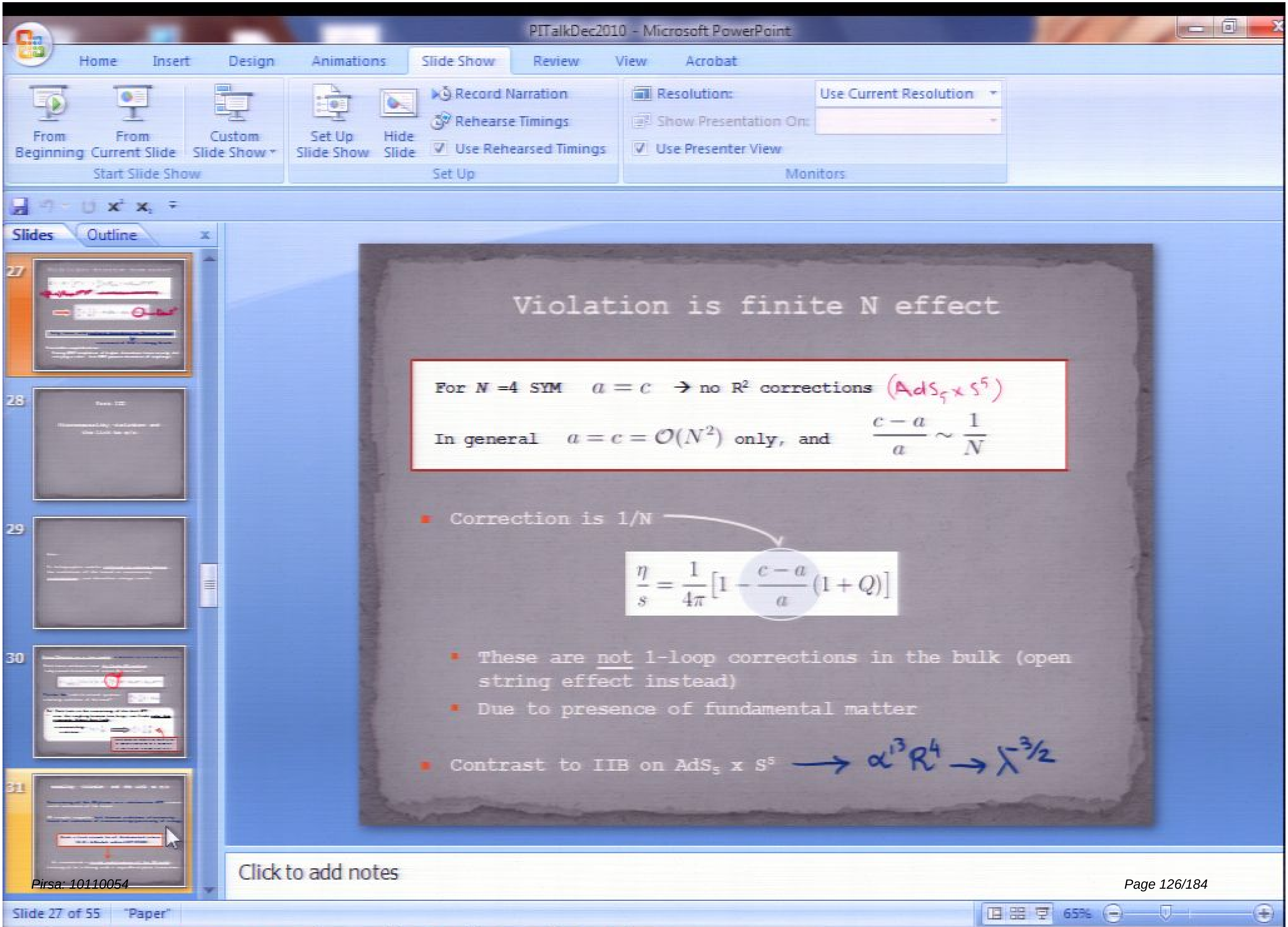
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Violation is finite N effect
For N =4 SYM a = c → no R² corrections (AdS5 x S5)
In general a = c = O(N²) only, and (c − a)/a ~ 1/N
Correction is 1/N
η/s = 1/4π [1 − (c − a)/a (1 + Q)]
These are not 1-loop corrections in the bulk (open string effect instead)
Due to presence of fundamental matter
Contrast to IIB on AdS5 x S5 → α'³R⁴ → λ^(−3/2)
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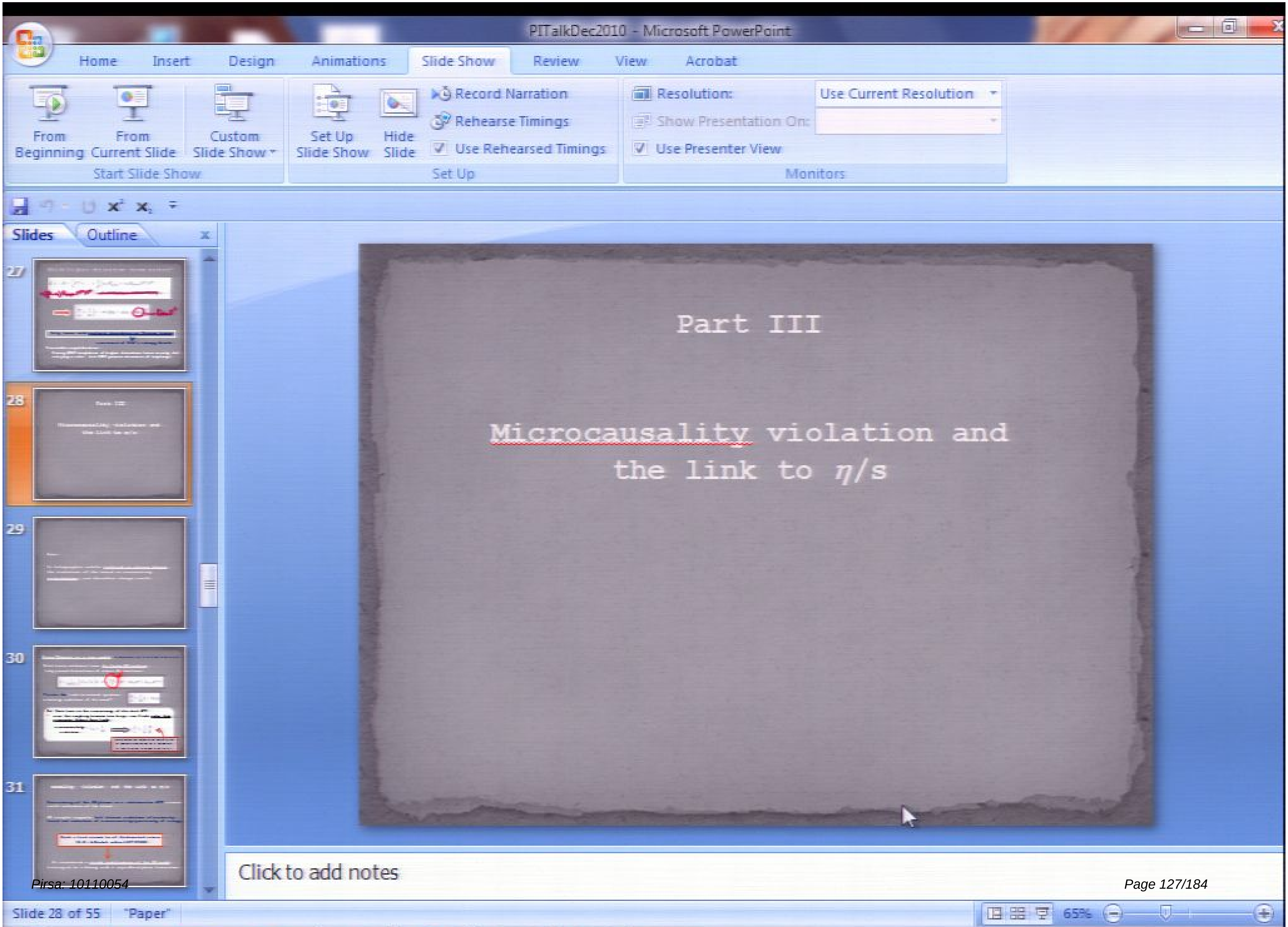
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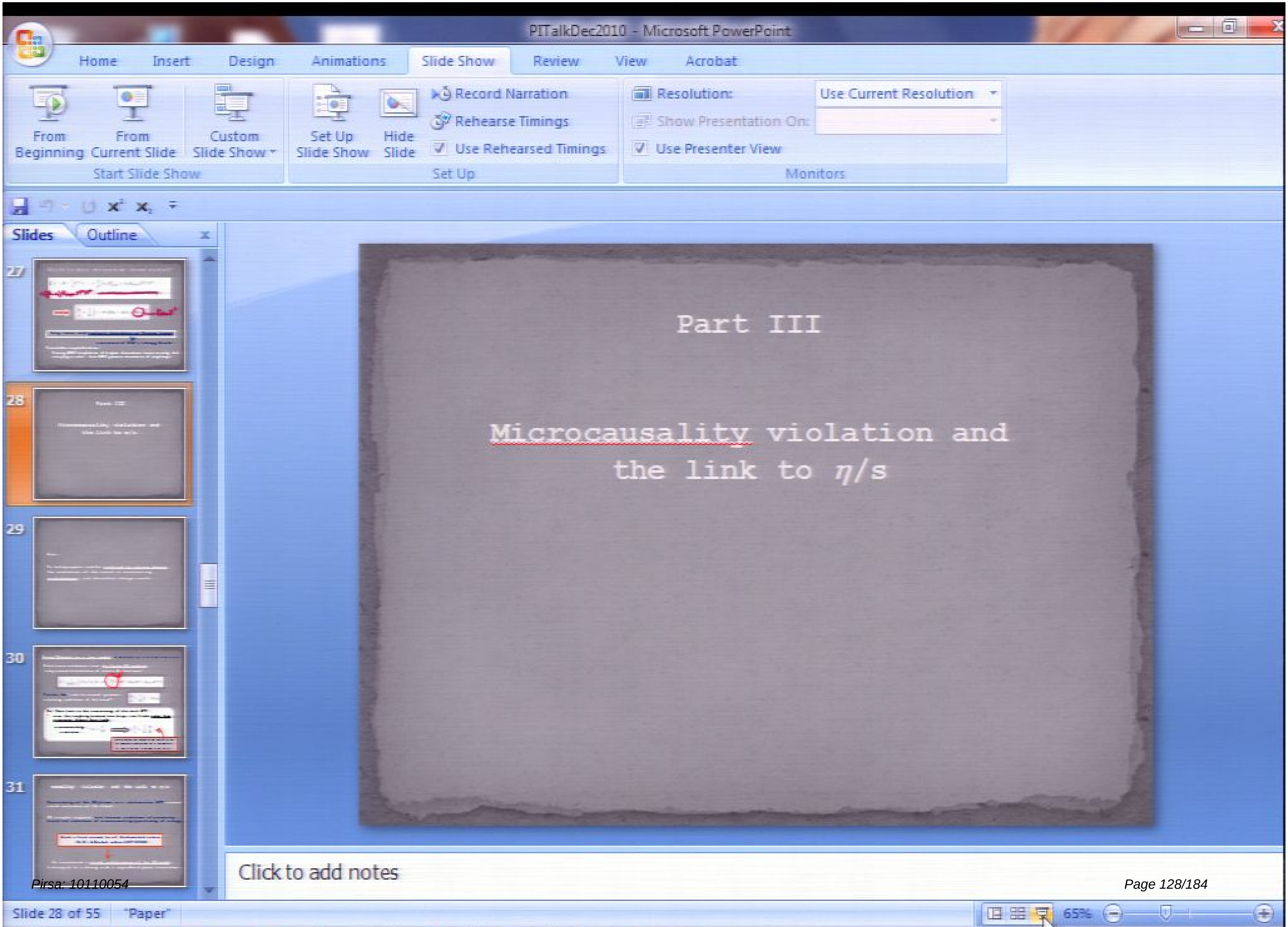
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Part III
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Part III

Microcausality violation and the link to η/s

Note:

In holographic models realized in string theory the violation of the bound is necessarily perturbative, and therefore always small.

Gauss-Bonnet as a toy model [Brigante et al, 0712.0805, 0802.3318]

Black brane solutions known for finite GB coupling
(only second derivatives of metric fluctuations)

$$I = \frac{1}{16\pi G_N}\int d^5x\,\sqrt{-g}\,\left[R - 2\Lambda + \frac{\lambda_{GB}}{2}L^2(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma})\right]$$

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Finite λ_{GB} leads to natural question:
arbitrary violation of the bound?

$$\frac{\eta}{s} = \frac{1}{4\pi}[1 - 4\lambda_{GB}]$$

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No! Must look at the consistency of the dual QFT:

- once the coupling becomes too large, one finds modes that propagate faster than light

microcausality violation $\lambda_{GB} > \frac{9}{100}$ $\Rightarrow$ $\frac{\eta}{s} > \frac{1}{4\pi}\frac{16}{25}$

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same bound by requiring positivity of energy measured by a detector in the plasma (Hofman 0907.1625)

Causality Violation and the Link to η/s

- Consistency of the GB plasma as a relativistic QFT ensures small violation of the bound
- GB example suggests link between violation of viscosity bound and violation of microcausality/positivity of energy

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[S.C.,A.Buchel arXiv:1007.2963]

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We considered a slight modification of the GB model, linking it to a theory with a superfluid phase transition

Idea is generic:

While transport properties are determined by the IR features of the theory, causality is determined by the propagation of UV modes (whose dynamics is not that of hydro)

IR vs. UV Physics

- shear viscosity: coupling of effective hydro description at low momentum and frequency

$$\omega \ll \min(T, \mu, \cdots)\,, \qquad |\vec{k}| \ll \min(T, \mu, \cdots)$$

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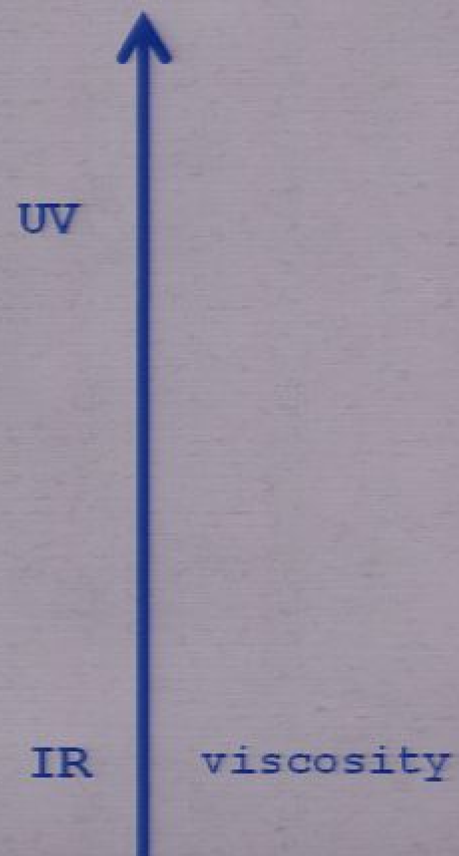

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- microcausality: determined by propagation of modes in UV

$$\omega \gg \max(T, \mu, \cdots)\,, \qquad |\vec{k}| \gg \max(T, \mu, \cdots)$$

CFT

UV microcausality

IR viscosity

IR vs. UV Physics

- shear viscosity: coupling of effective hydro description at low momentum and frequency

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CFT

UV — microcausality

IR — viscosity

Link only if same phase of the theory extends over all energy scales
(e.g. no phase transitions decoupling UV from IR)

Features of our model [S.C.,A.Buchel arXiv:1007.2963]

- Plasma with 2nd order phase transition below some T_c associated with:
  - spontaneous breaking of global U(1)
  - generation of condensate of an operator:

$$\langle \mathcal{O}_c \rangle \begin{cases} = 0\,, & T > T_c \\ \neq 0\,, & T < T_c \end{cases}$$

- Dual GR theory coupled to GB term, engineered so that:

$$\lambda_{GB}\Big|^{effective} \propto \mathcal{O}_c \quad \Longrightarrow \quad \lambda_{GB}\Big|^{effective} \begin{cases} = 0\,, & \text{UV} \\ \neq 0\,, & \text{IR}\,. \end{cases}$$

Expectation:

Because of phase transition, proposed connection between microcausality violation and viscosity bound will be lost

Motivation

Holographic model of superfluidity proposed by GHPT 0907.3510
(consistent truncation of Type IIB)

$$\mathcal{L}_{superfluid} = R - \frac{L^2}{3} F_{\mu\nu} F^{\mu\nu} + \left(\frac{2L}{3}\right)^3 \frac{1}{4} \epsilon^{\lambda\mu\nu\sigma\rho} F_{\lambda\mu} F_{\nu\sigma} A_\rho + \mathcal{L}^{SUGRA}_{scalar}$$

$$\mathcal{L}^{SUGRA}_{scalar} = -\frac{1}{2}\left[(\partial_\mu \phi)^2 + \sinh^2\phi\,(\partial_\mu\theta - 2A_\mu)^2 - \frac{6}{L^2}\,\cosh^2\frac{\phi}{2}\,(5 - \cosh\phi)\right]$$

ψ

dual to $\mathcal{O}_\Delta$

$$V(\phi) = -\frac{12}{L^2} - \frac{3}{2L^2}\phi^2 + \ldots$$

$m^2L^2 = \Delta(\Delta - 4)$

Below some critical temperature the operator develops a VEV:

$$\langle \mathcal{O}_c \rangle \begin{cases} = 0\,, & T > T_c \\ \neq 0\,, & T < T_c \end{cases}$$

Our Toy Model

In analogy with GHPT (slight simplification but same physics)

$$\mathcal{L} = R - \frac{L^2}{3} F_{\mu\nu} F^{\mu\nu} + \left(\frac{2L}{3}\right)^3 \frac{1}{4} \epsilon^{\lambda\mu\nu\sigma\rho} F_{\lambda\mu} F_{\nu\sigma} A_\rho + \mathcal{L}_{scalar} + \mathcal{L}_{GB}$$

$$\mathcal{L}_{scalar} = -\frac{1}{2}\left[(\partial_\mu \phi)^2 + 4\phi^2 A_\mu A^\mu\right] + \frac{12}{L^2} + \frac{3}{2L^2}\phi^2$$

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Additional GB higher derivative corrections:

$$\mathcal{L}_{GB} = \beta \phi^4 L^2 \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\lambda}R^{\mu\nu\rho\lambda} \right)$$

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λ^{eff}

λ_{GB}

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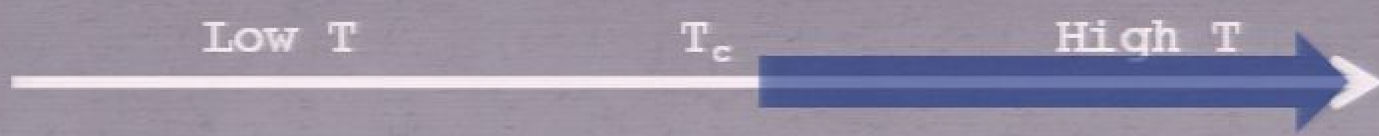
λ^{eff}_{GB}

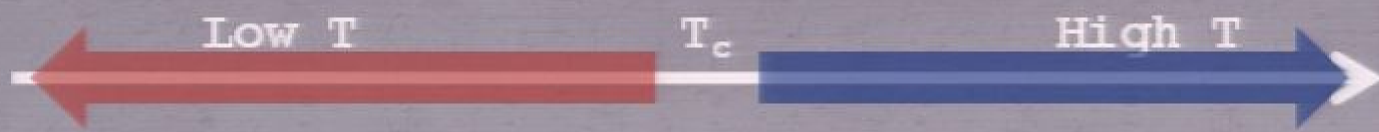
turned on only below T_c

Low T T_c High T



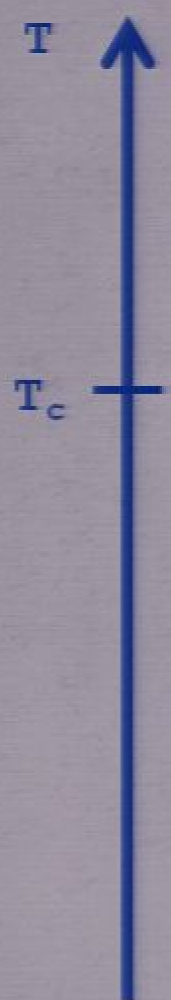
$T > T_c$

- unbroken phase
- $\lambda_{GB} = 0$
- no higher derivatives (Einstein GR with U(1) gauge field)
- electrically charged AdS black hole



$T < T_c$	$T > T_c$
▪ broken symmetry phase	▪ unbroken phase
▪ $\lambda_{GB} \neq 0$	▪ $\lambda_{GB} = 0$
▪ Gauss-Bonnet higher-derivative corrections	▪ no higher derivatives (Einstein GR with U(1) gauge field)
▪ Black hole develops scalar hair	▪ electrically charged AdS black hole

The shear viscosity bound [arXiv:1007.2963]



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expected from universality

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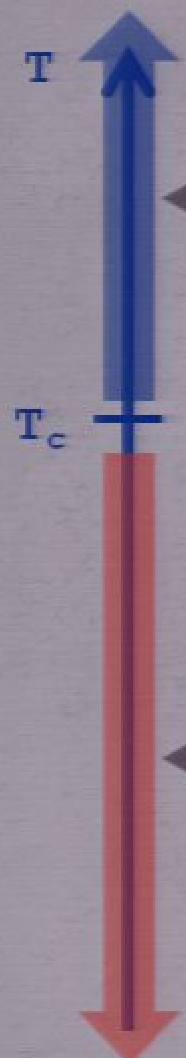

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when $\lambda_{GB} = 0$ we have standard superconductor (no higher derivatives)

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

still from universality

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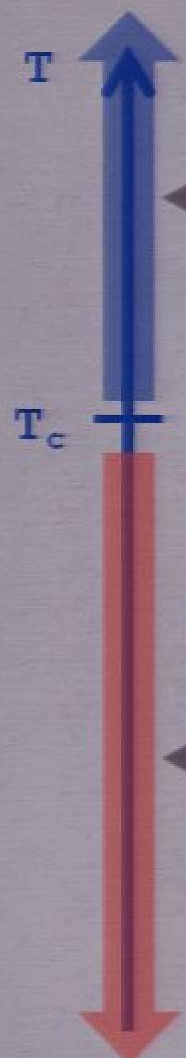
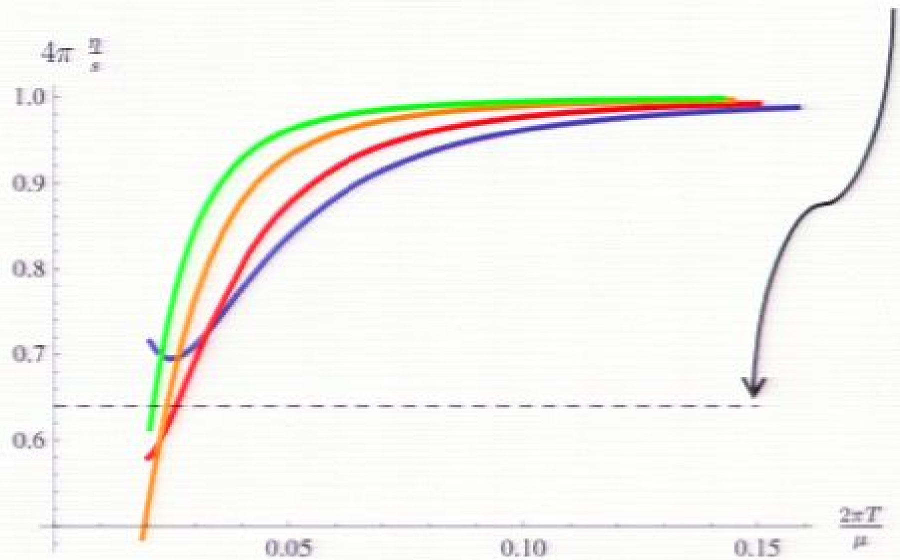


$$\frac{\eta}{s} = \frac{1}{4\pi}$$

expected from universality

when λ_{GB} is non-zero:

- no micro-causality violation (scalar channel)
- eta/s goes well below pure GB bound (finite λ_{GB})



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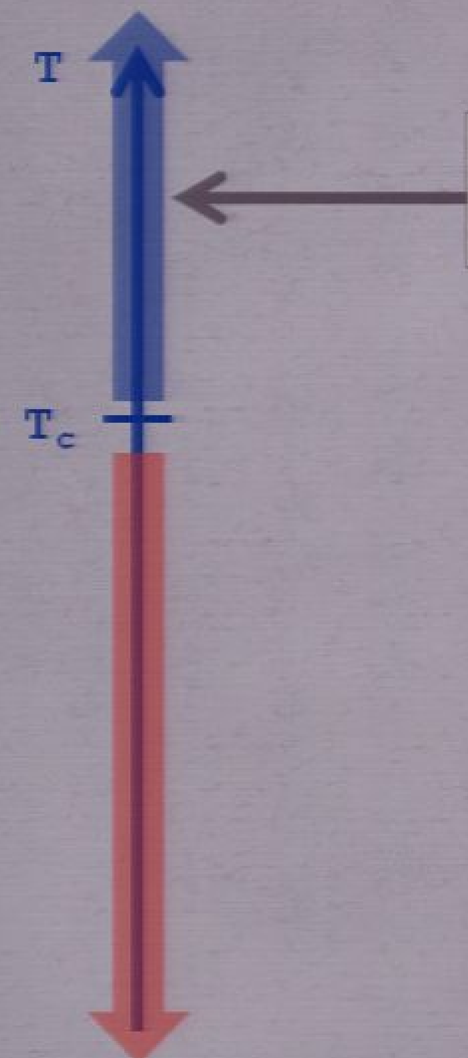

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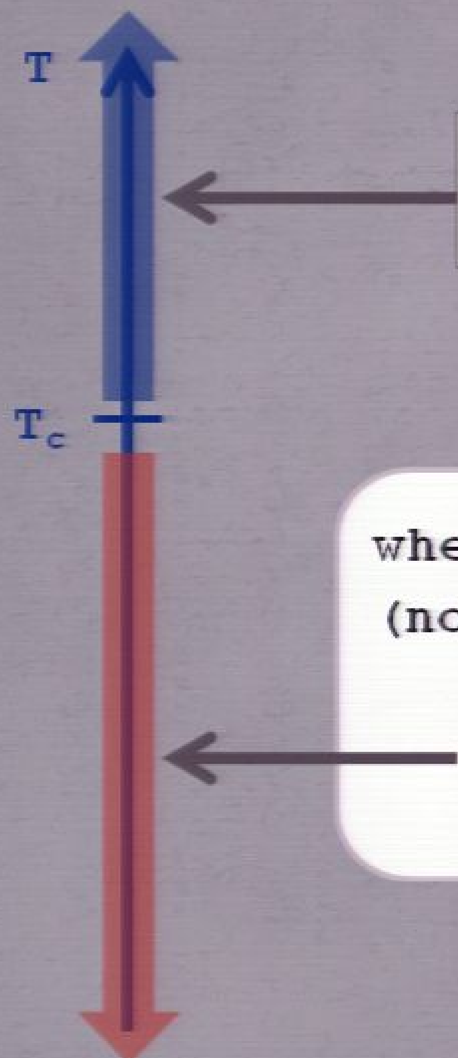
The shear viscosity bound [arXiv:1007.2963]



$$\frac{\eta}{s} = \frac{1}{4\pi}$$

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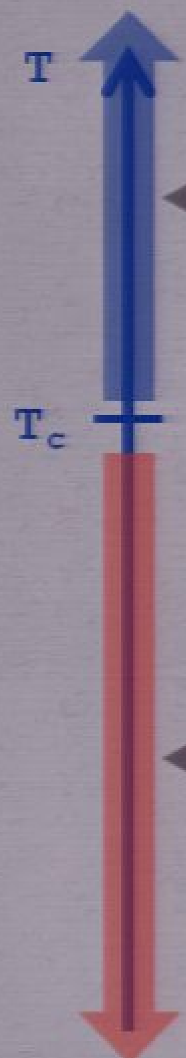
still from universality

The shear viscosity bound [arXiv:1007.2963]



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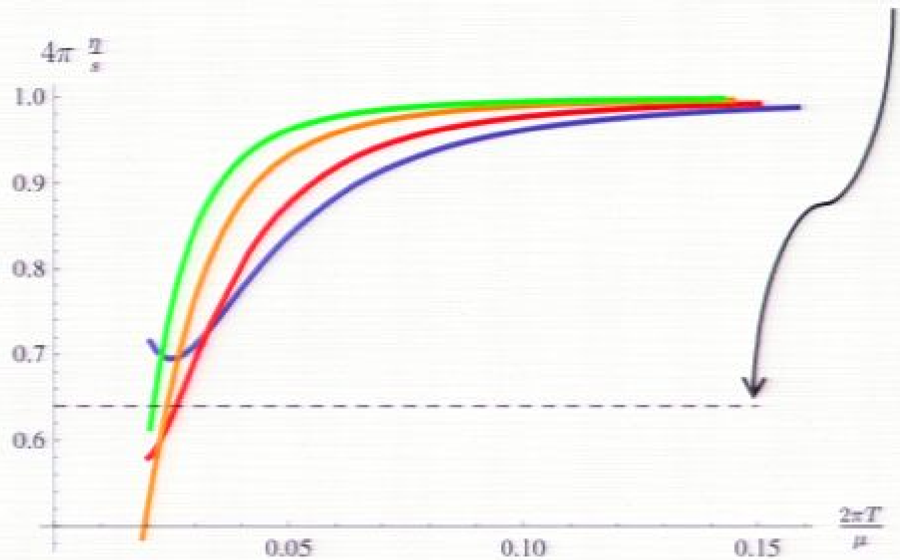

In summary…

The "link" between eta/s and causality violation is not fundamental.

Here there is no link between eta/s and the central charges of the dual theory:

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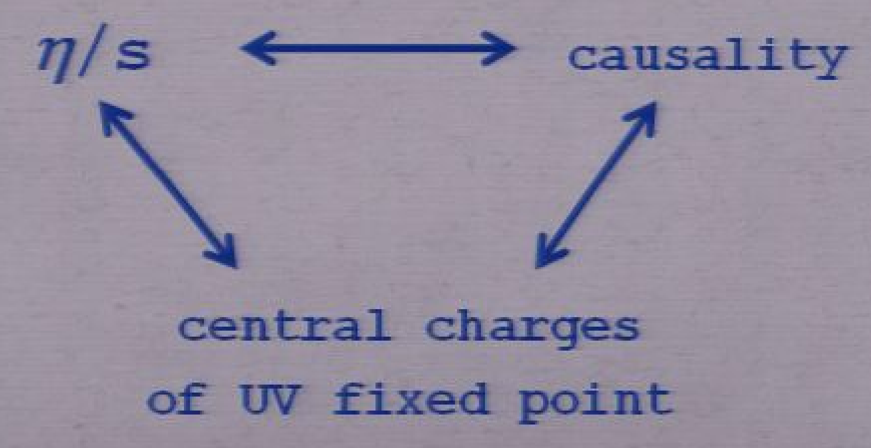

Recently: several attempts at refining/developing Wilsonian approach to gauge/gravity duality

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In conclusion...

- **AdS/CFT : rich framework to explore strongly coupled FTs:** important to understand how much mileage we can get from gravity setups to model interesting field theory systems (and any potential constraints arising from consistency of the theory)

- **development of universal relations particularly important:** useful for providing inputs into realistic simulations of strongly coupled systems

- **Higher derivatives: finite N and λ corrections**
  For eta/s:

$$\frac{\eta}{s} = \frac{1}{4\pi}\left[1 - \frac{\#}{N}(1+Q) + \frac{\#}{\lambda^{3/2}} + \cdots\right]$$

As for the "bound"...

- The original KSS bound is clearly violated. With higher derivative corrections the **universality of eta/s is seemingly lost.**

- Idea behind GB superfluid is **generic**: transport coefficients are IR features of the theory, while causality/central charges are a property of the UV.

- **Microscopic constraints** (causality, positivity of energy), while important for the general consistency of the plasma as a relativistic field theory, **are NOT responsible for setting the lower bound on η/s.**

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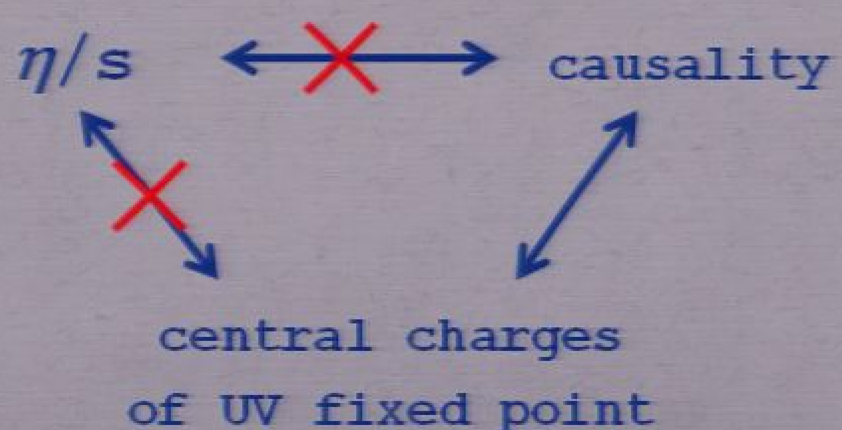

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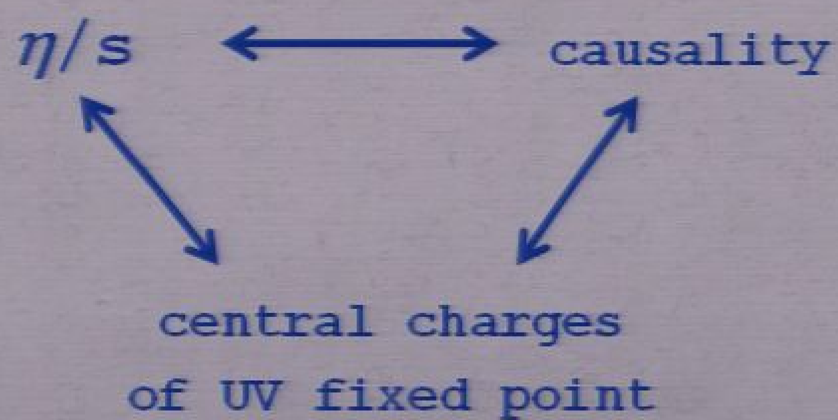

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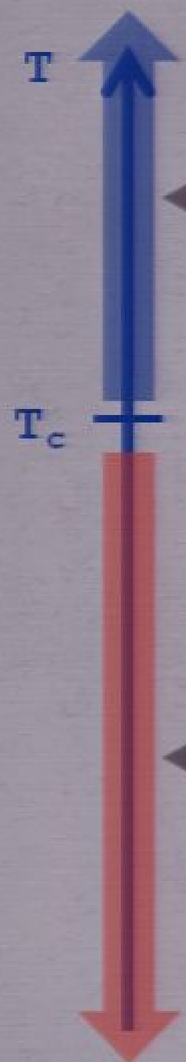

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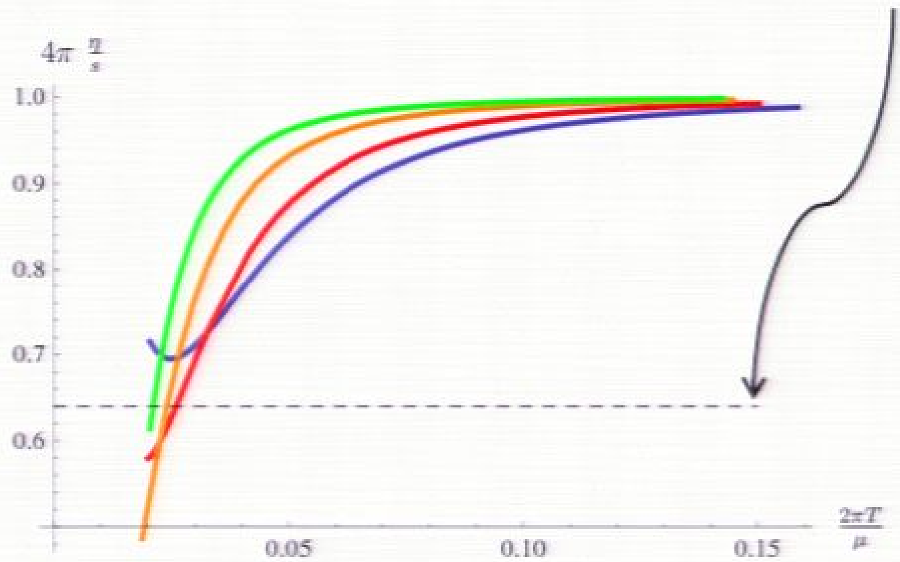

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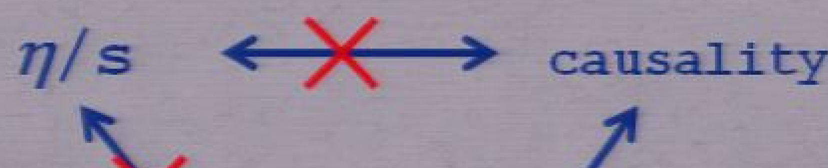

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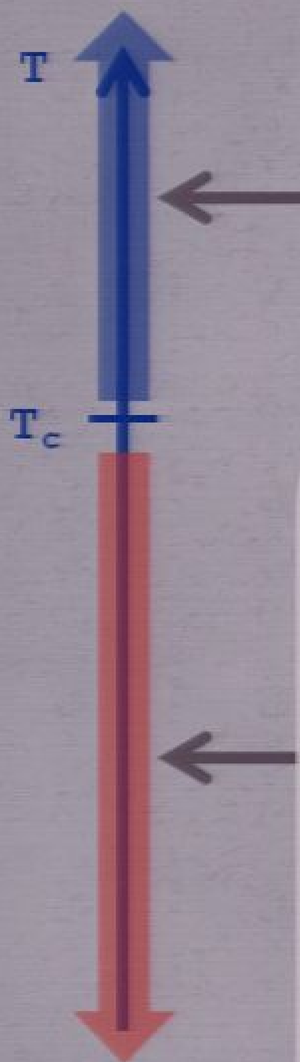

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