

Title: Lumps and bumps in the early universe

Date: Nov 09, 2010 02:00 PM

URL: <http://pirsa.org/10110050>

Abstract: I will discuss the emergence of large, localized, pseudo-stable configurations (oscillons) from inflaton fragmentation at the end of inflation. Remarkably, the emergent oscillons take up  $\gtrsim 50$  per cent of the energy density of the inflaton. First, I will give an overview of oscillons, provide some analytic solutions and discuss their stability. Then, I will discuss the conditions necessary for their emergence and provide estimates for their cosmological number density. I will show results from detailed 3+1-dimensional numerical simulations and compare them to the analytic estimates. Finally, I discuss possible observational consequences of oscillons in the early universe.

# LUMPS AND BUMPS IN THE EARLY UNIVERSE

- 1) 1002.3380 PHYS. REV. D, (WITH D. SHIROKOFF)
- 2) 1006.3075 ACCEPTED PHYS REV D. (MA)
- 3) 1009.2505 ACCEPTED JCAP (MA, EASTHER AND FINKEL)

MUSTAFA AMIN (MIT)

SUPPORTED BY A PAPPALARDO FELLOWSHIP  
9.11.2010

# LUMPS AND BUMPS IN THE EARLY UNIVERSE

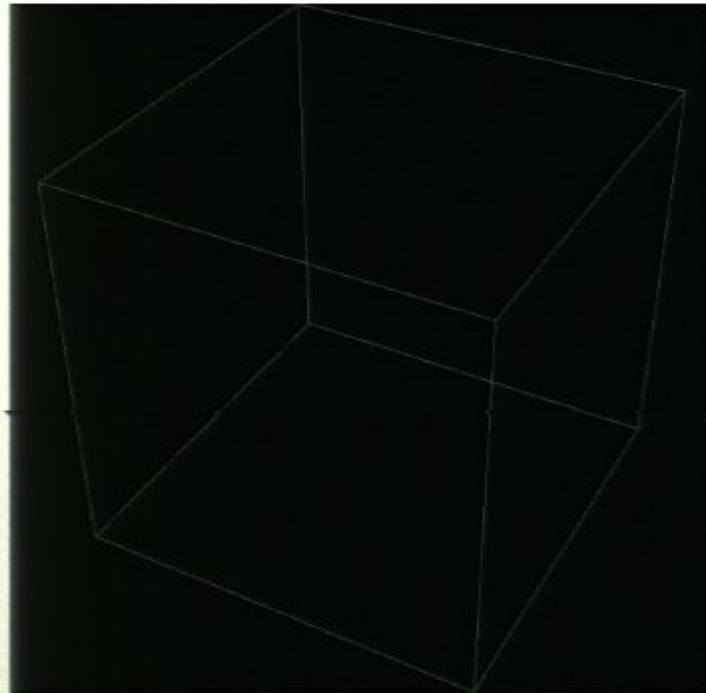
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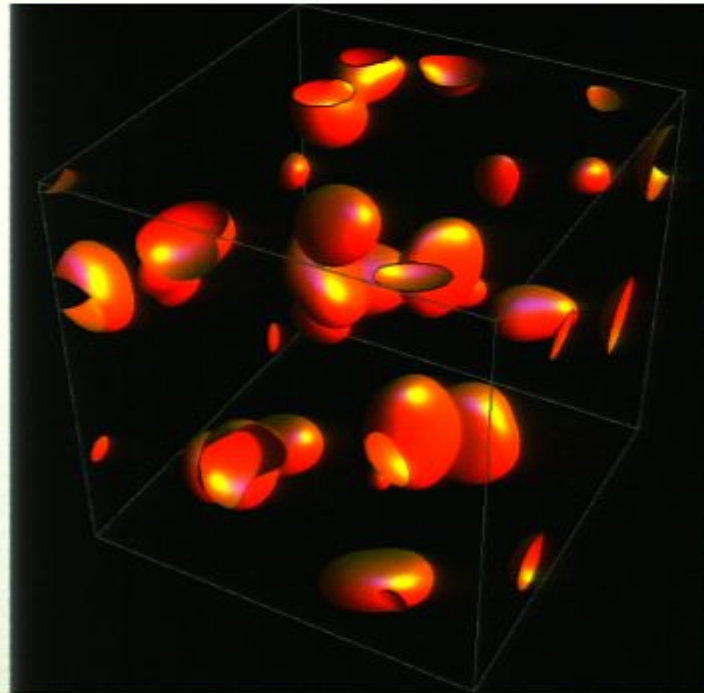
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- inflaton fragmentation into pseudo-solitons: **oscillons**
- populating the universe after inflation :  
**reheating**

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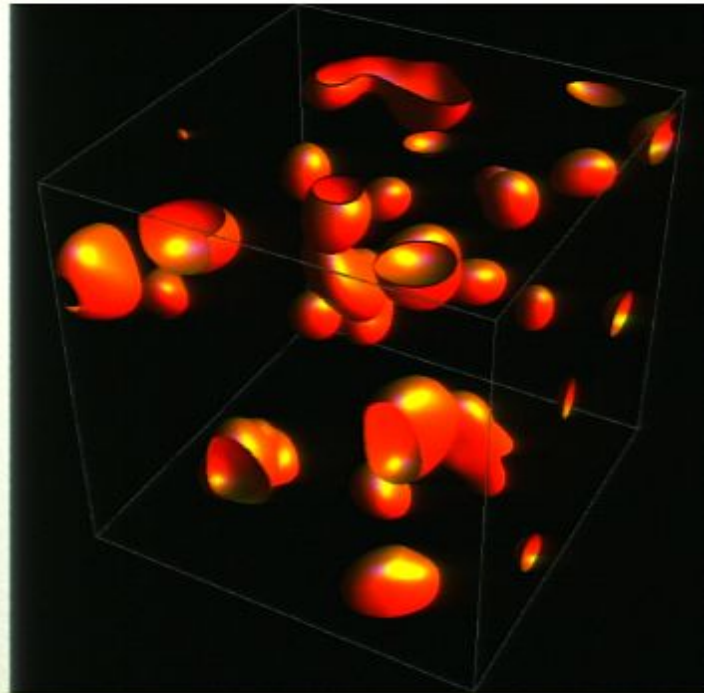
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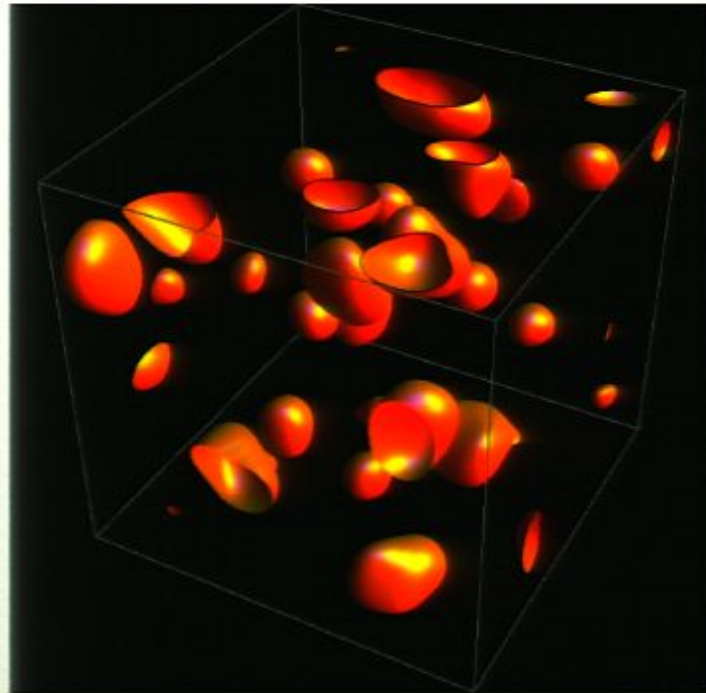
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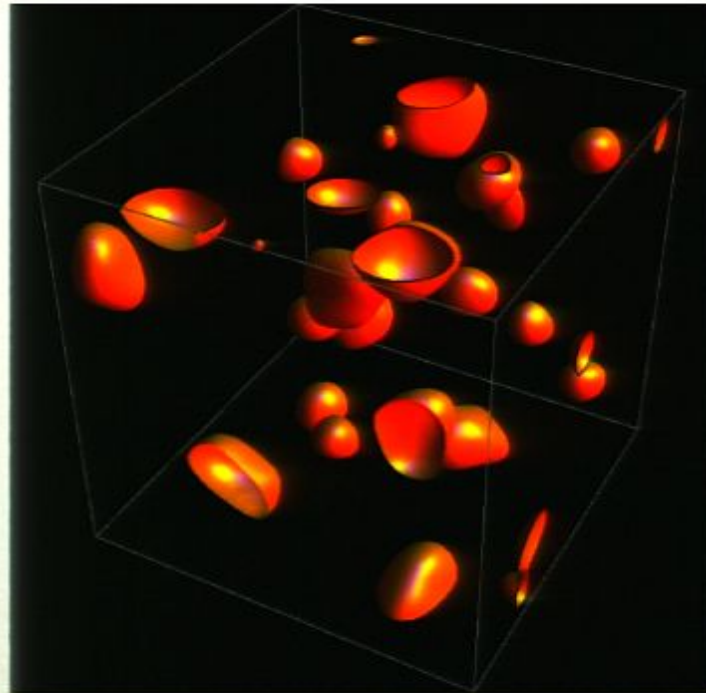
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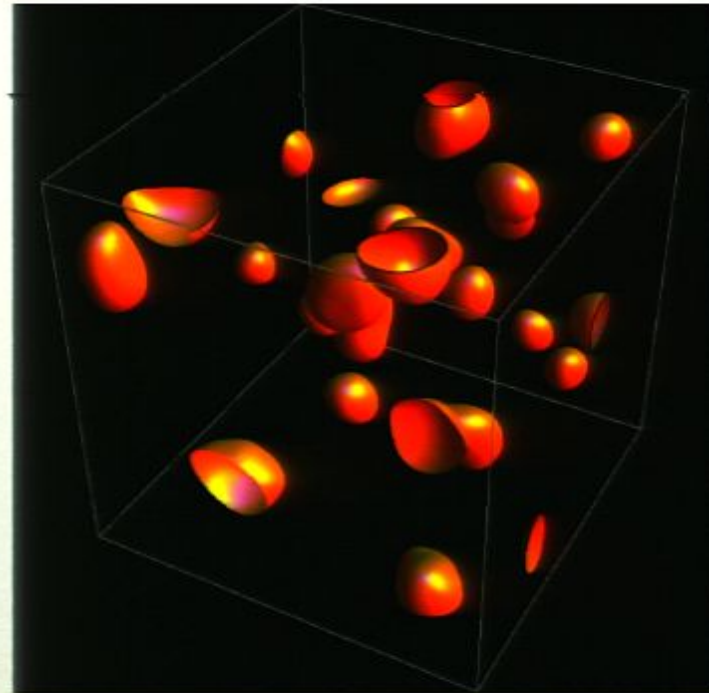


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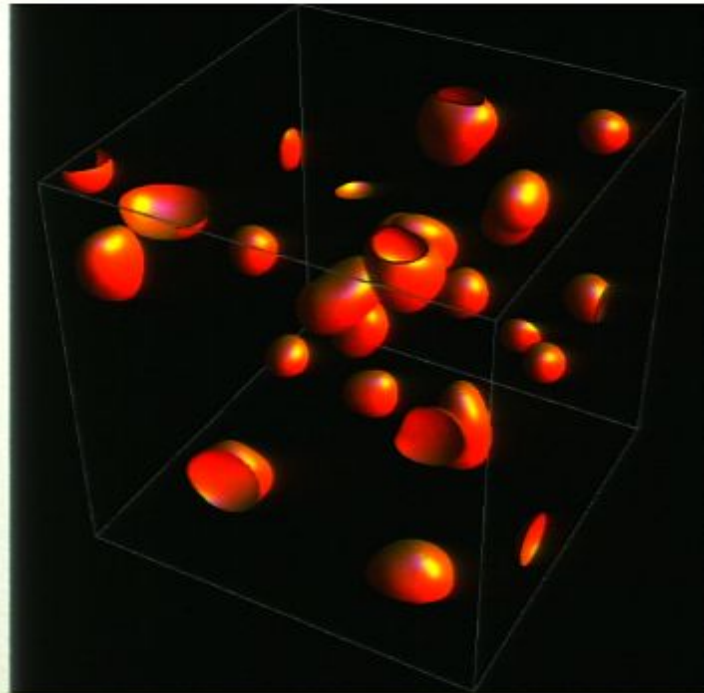
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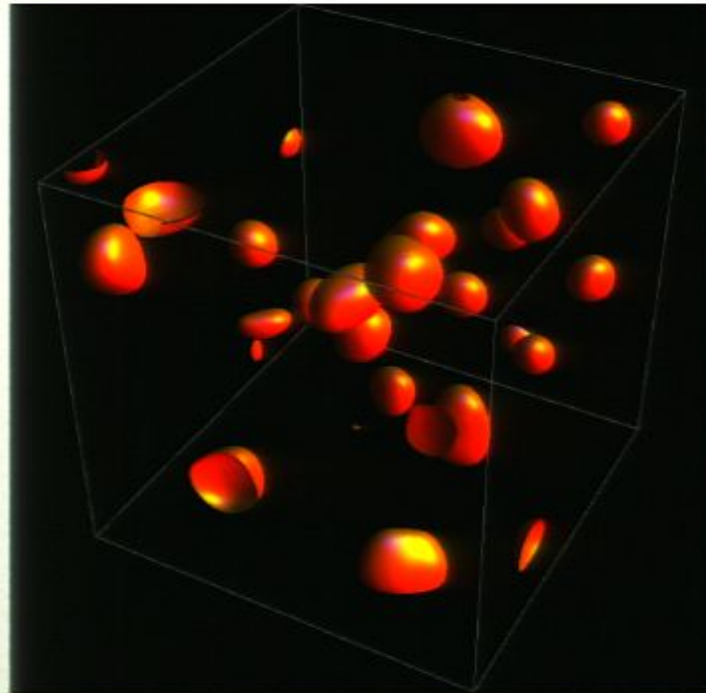
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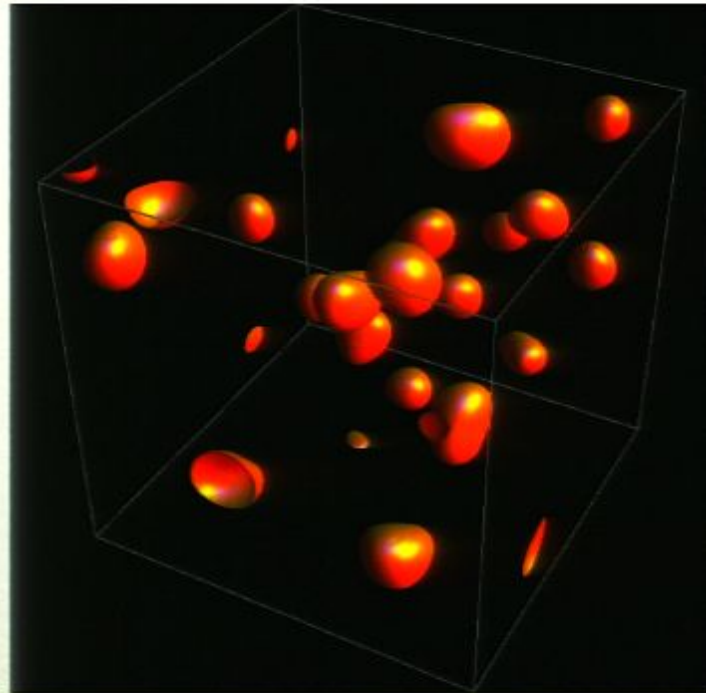
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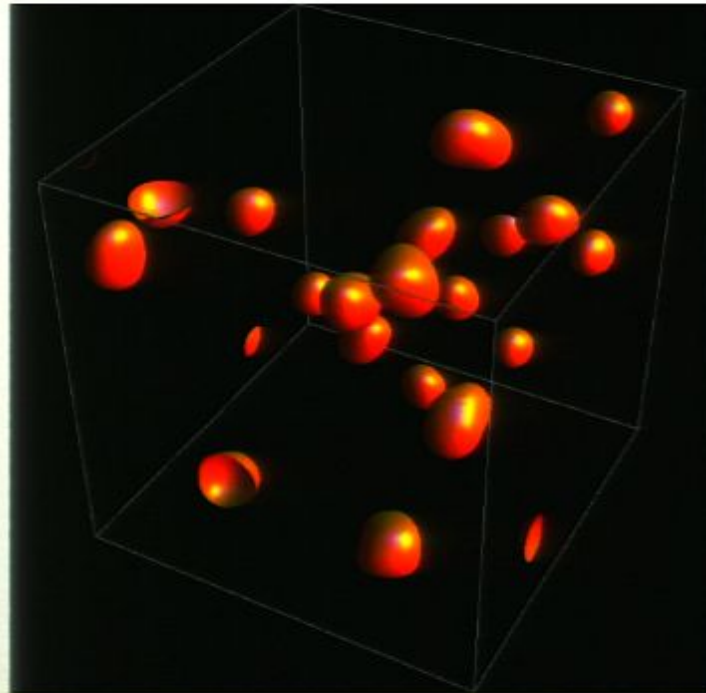
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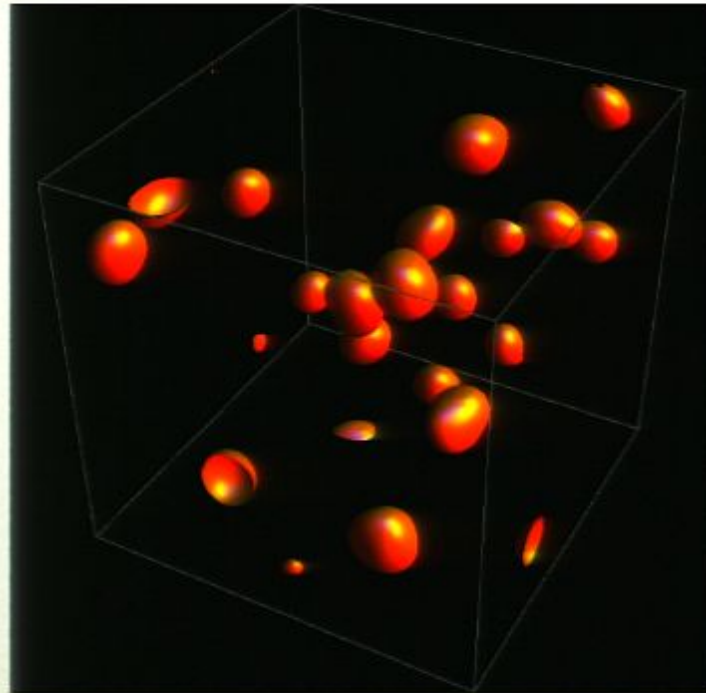
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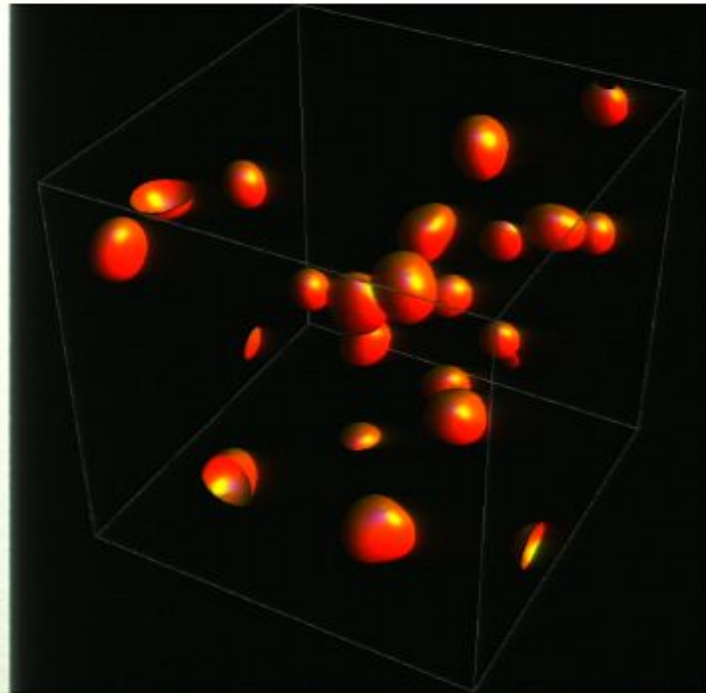
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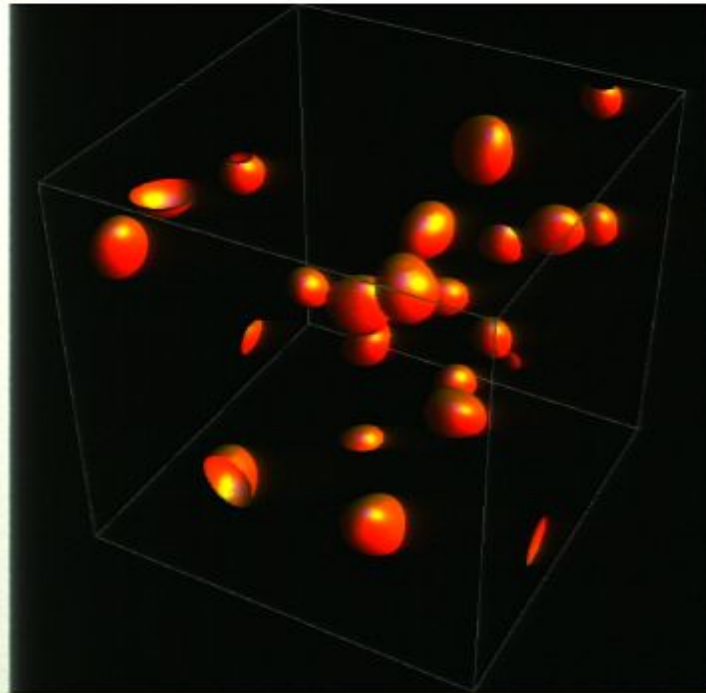
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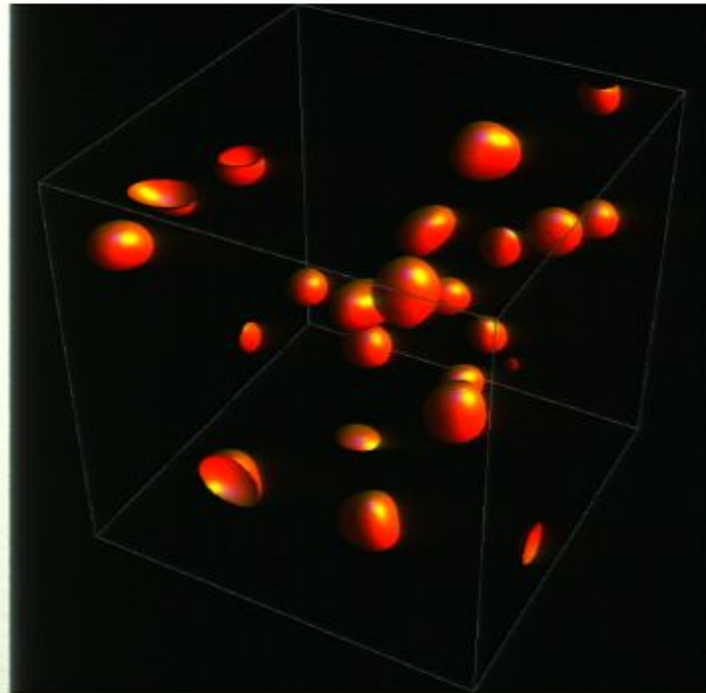


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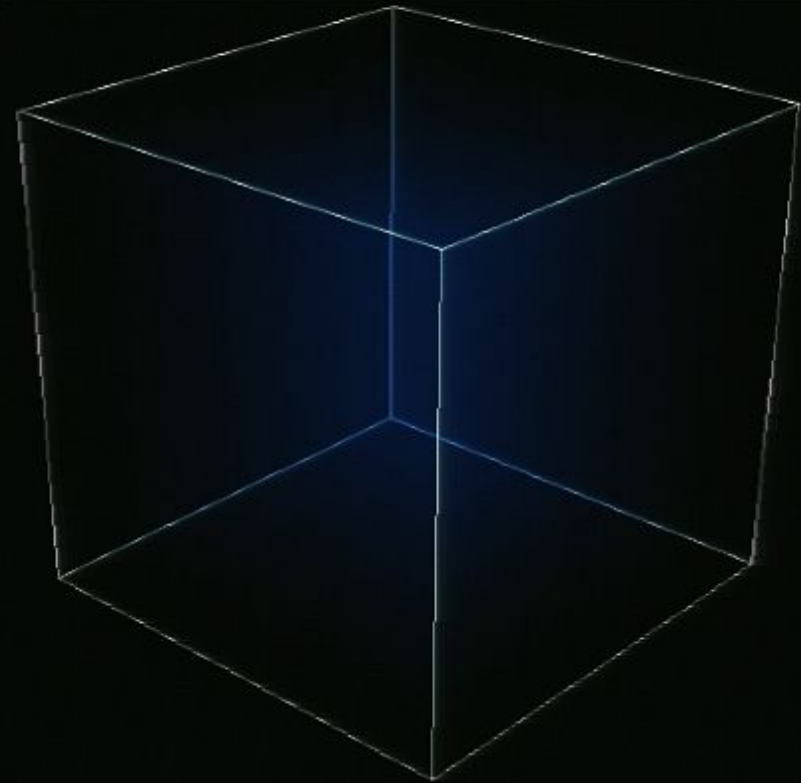
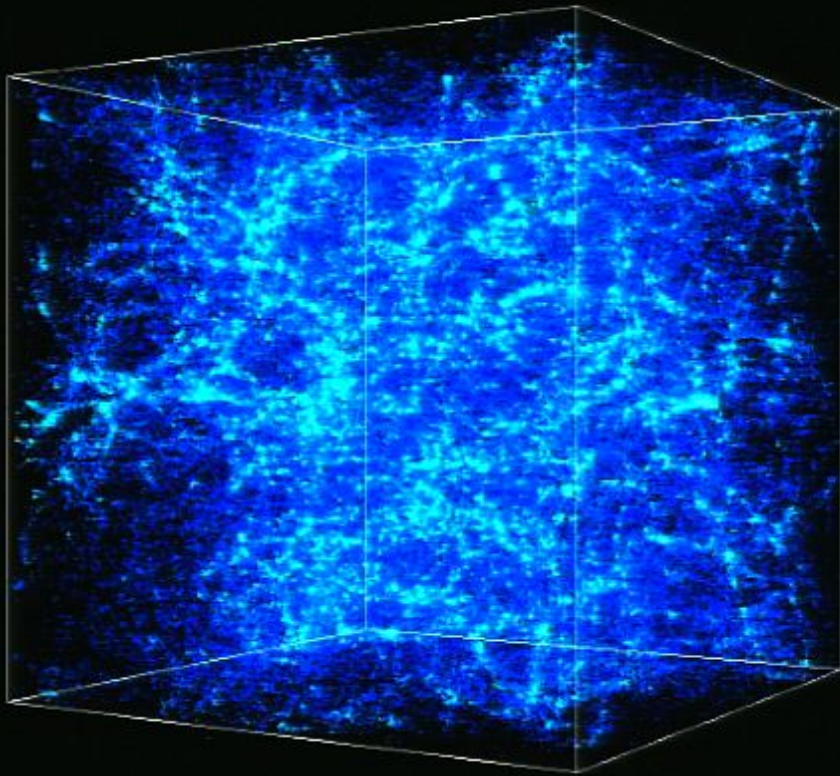
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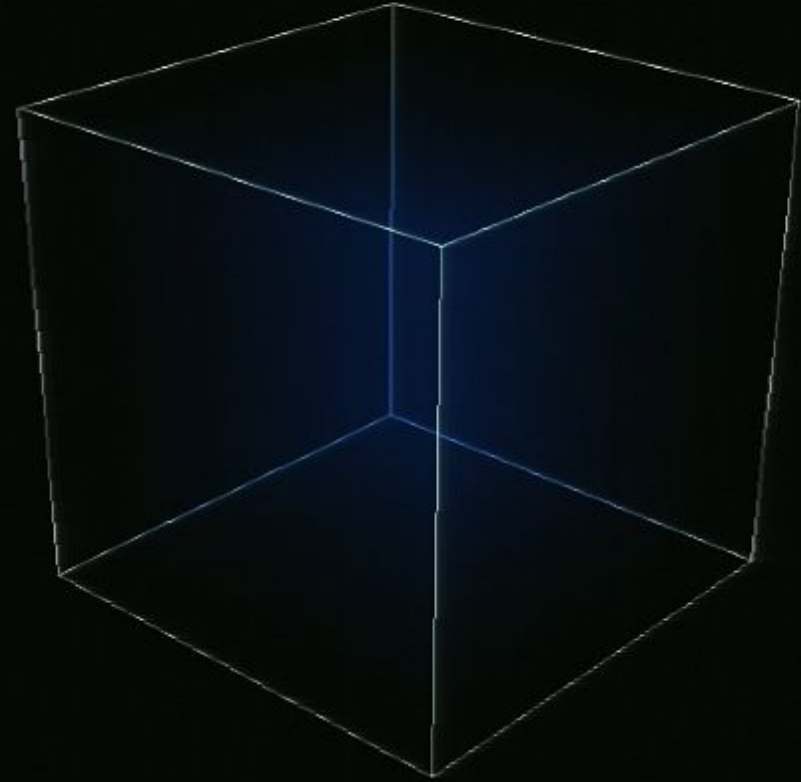
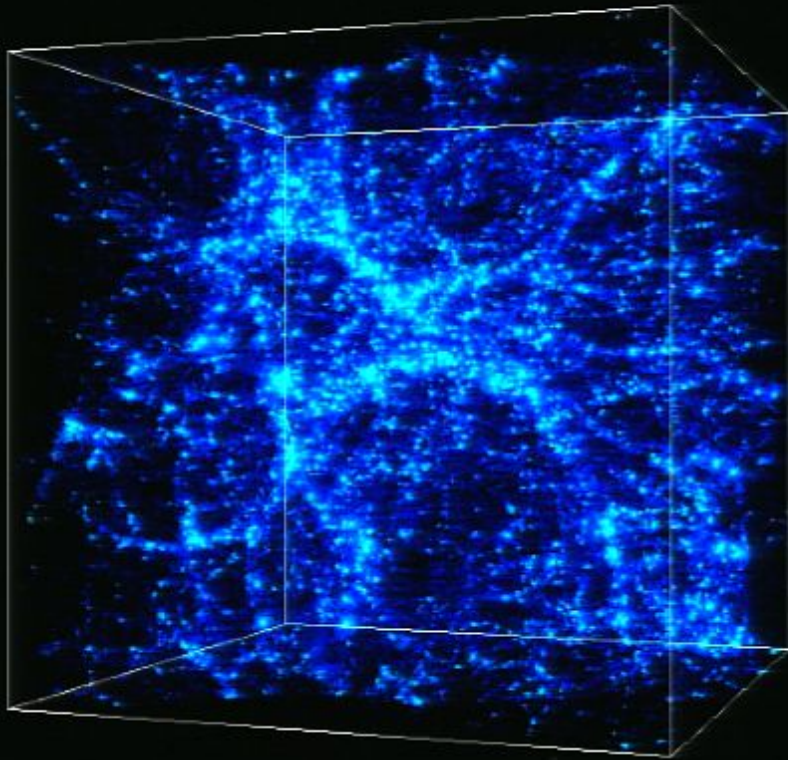
# Structure formation

$z < 100$



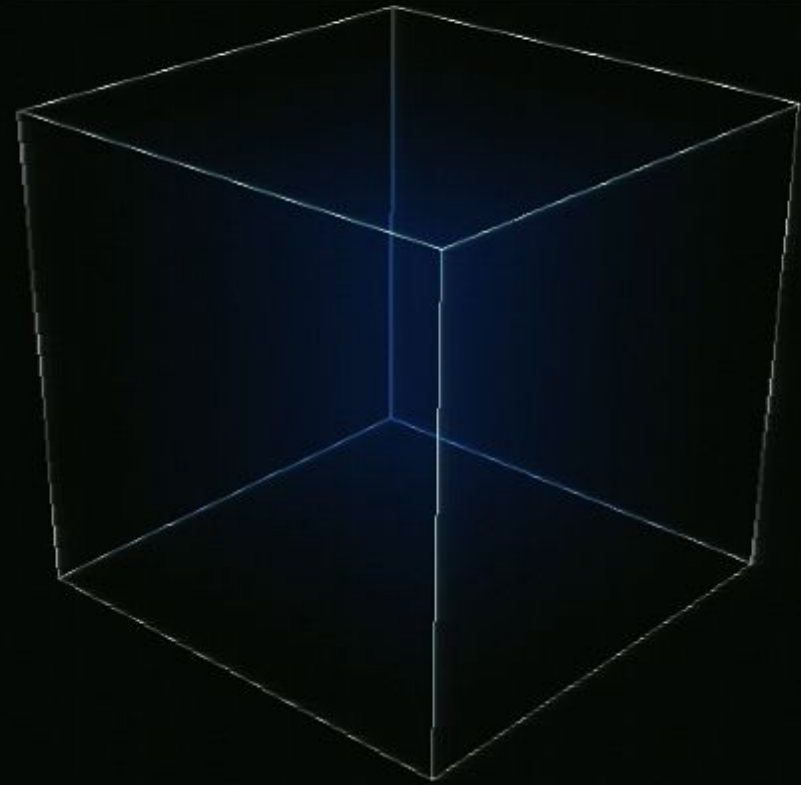
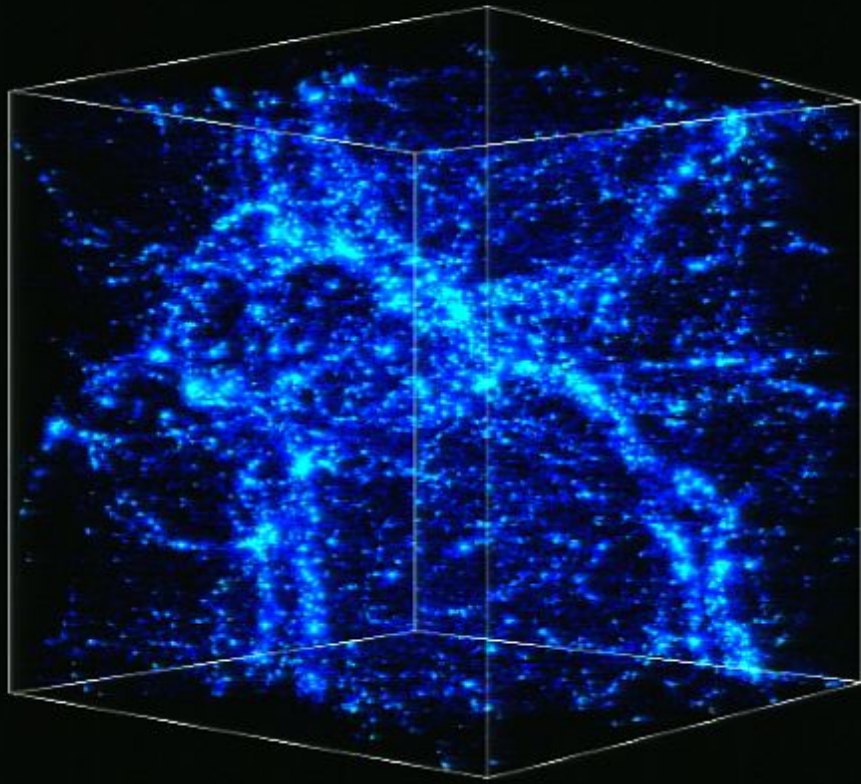
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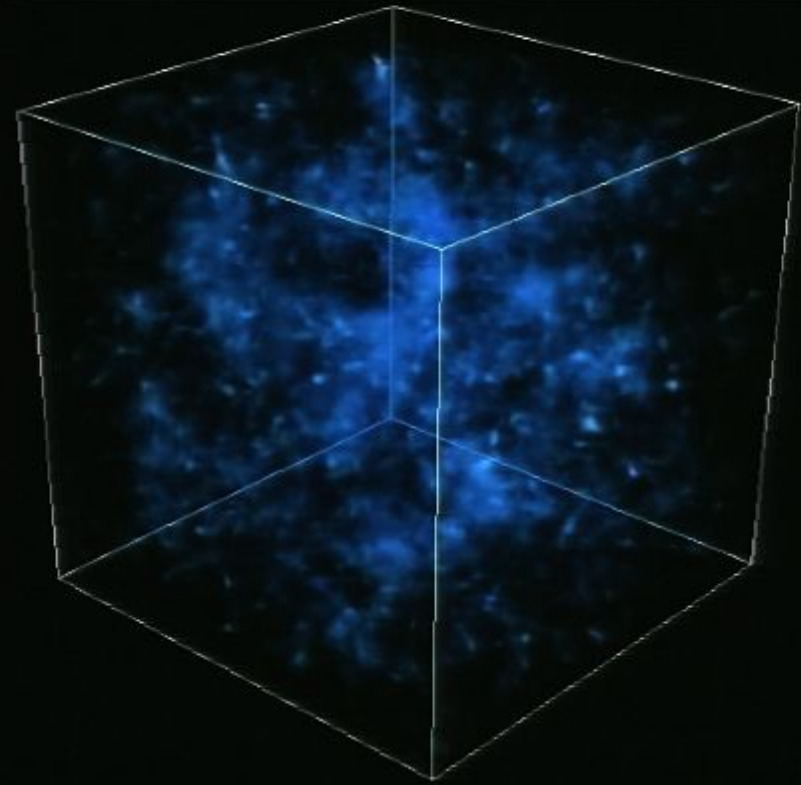
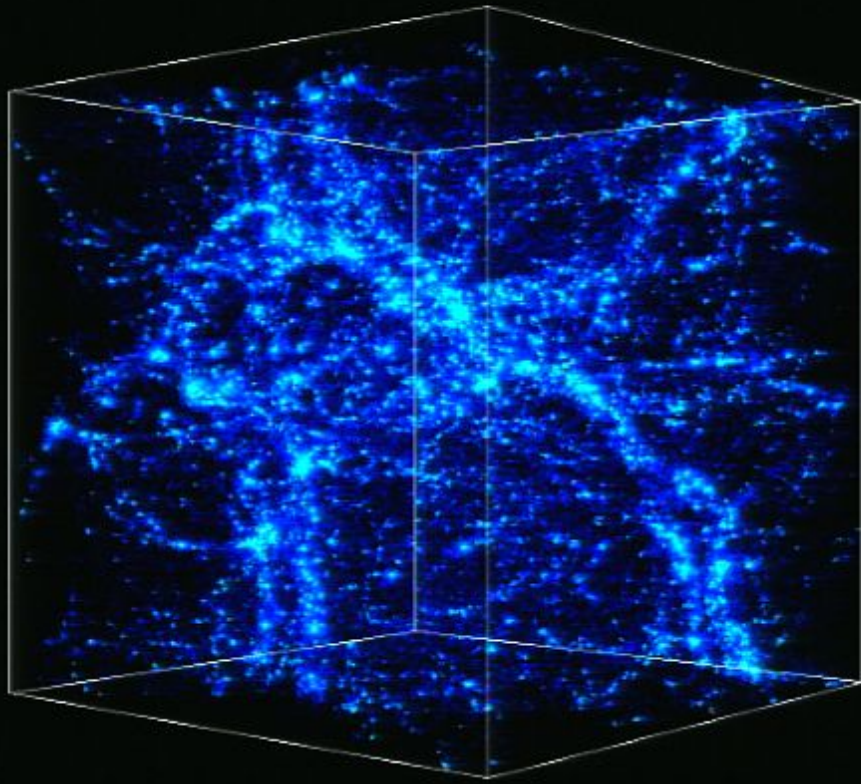
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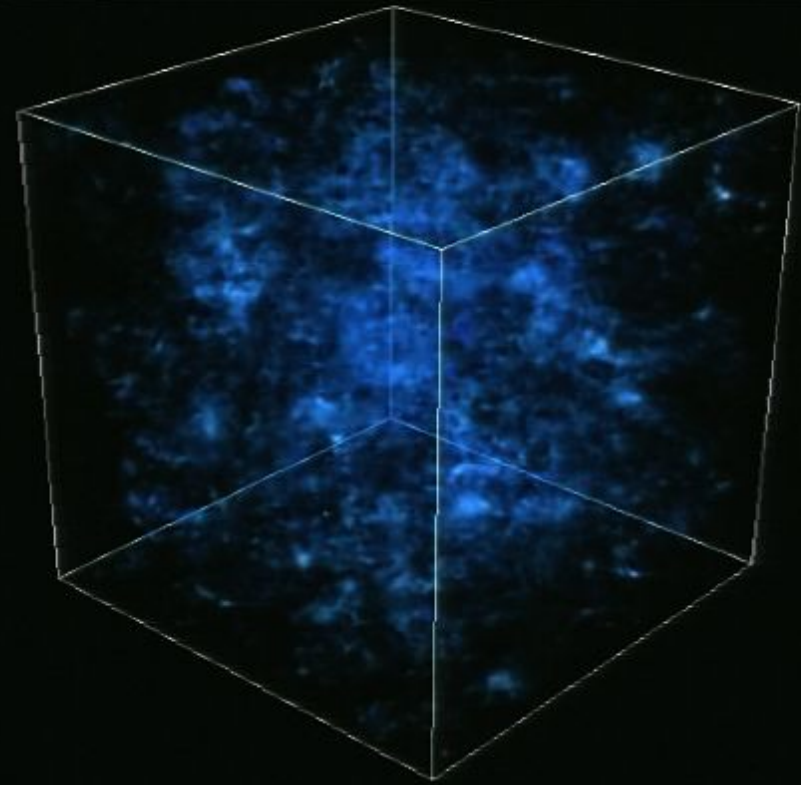
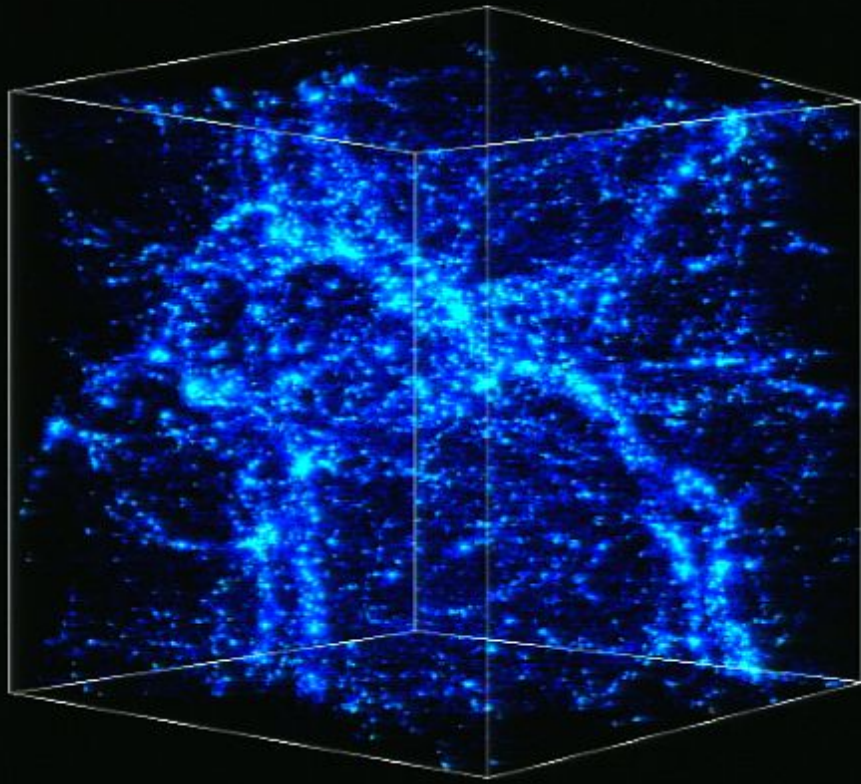
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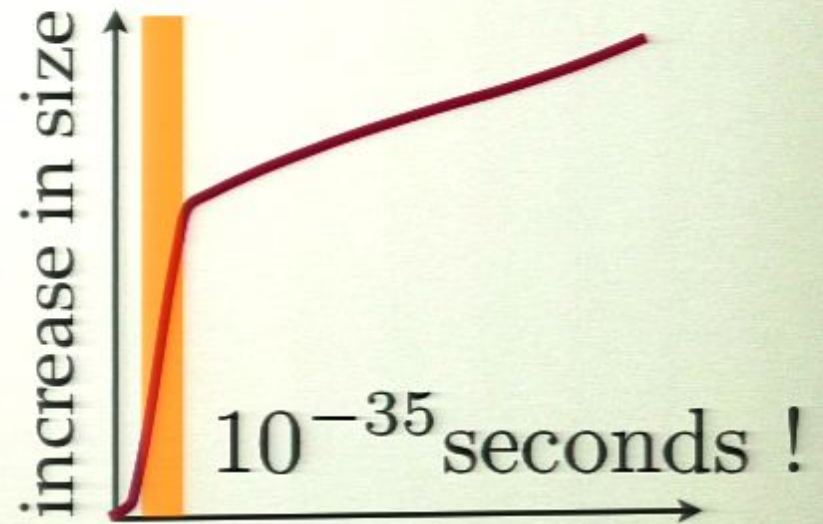


# SYNOPSIS

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- inflation and its end
- fragmentation  $\longrightarrow$  localized clumps !
- clumps ?
- generic emergence ?
- number densities and energy fraction ?
- consequences

# INFLATION

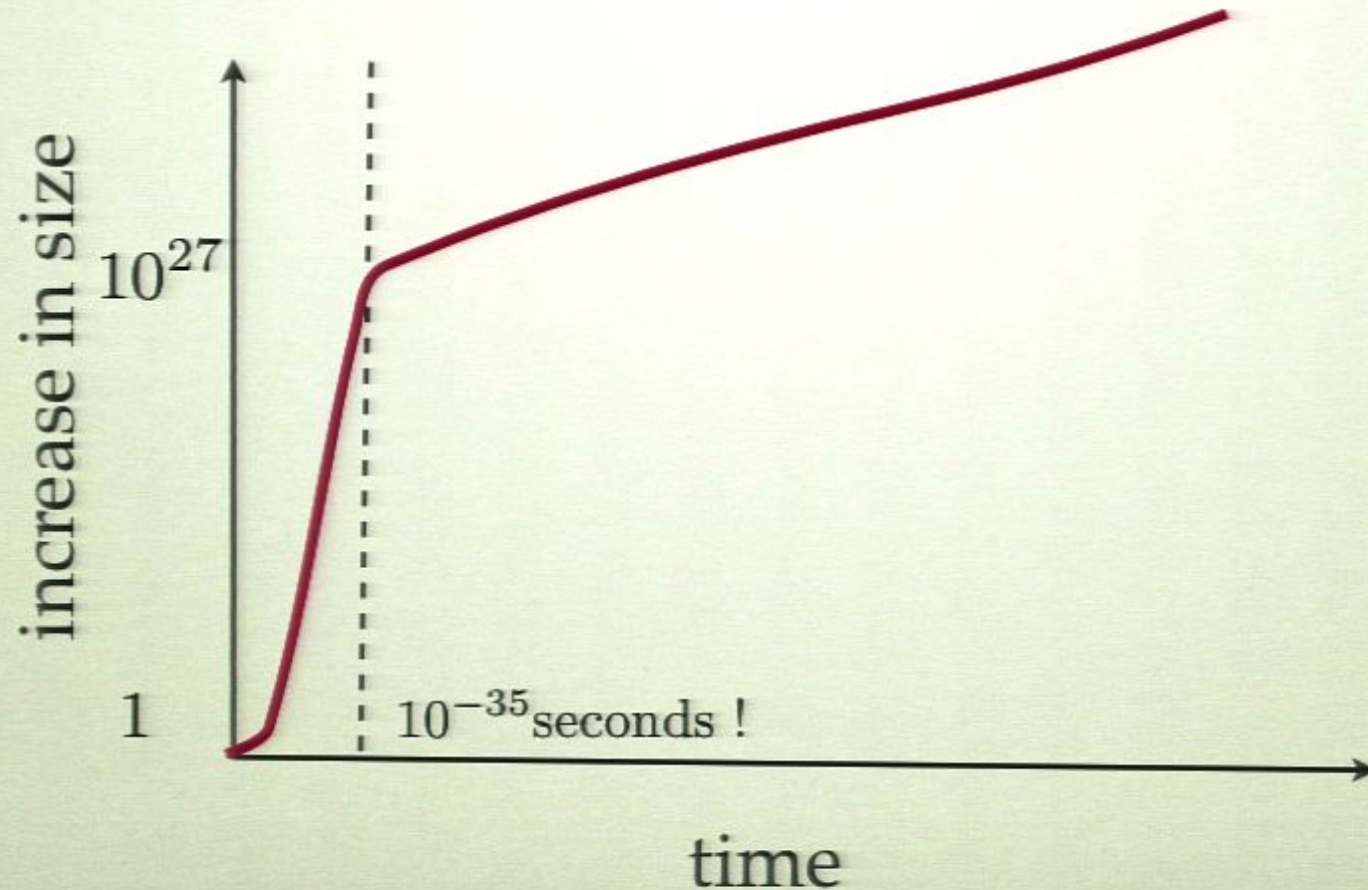


H1N1 flu virus

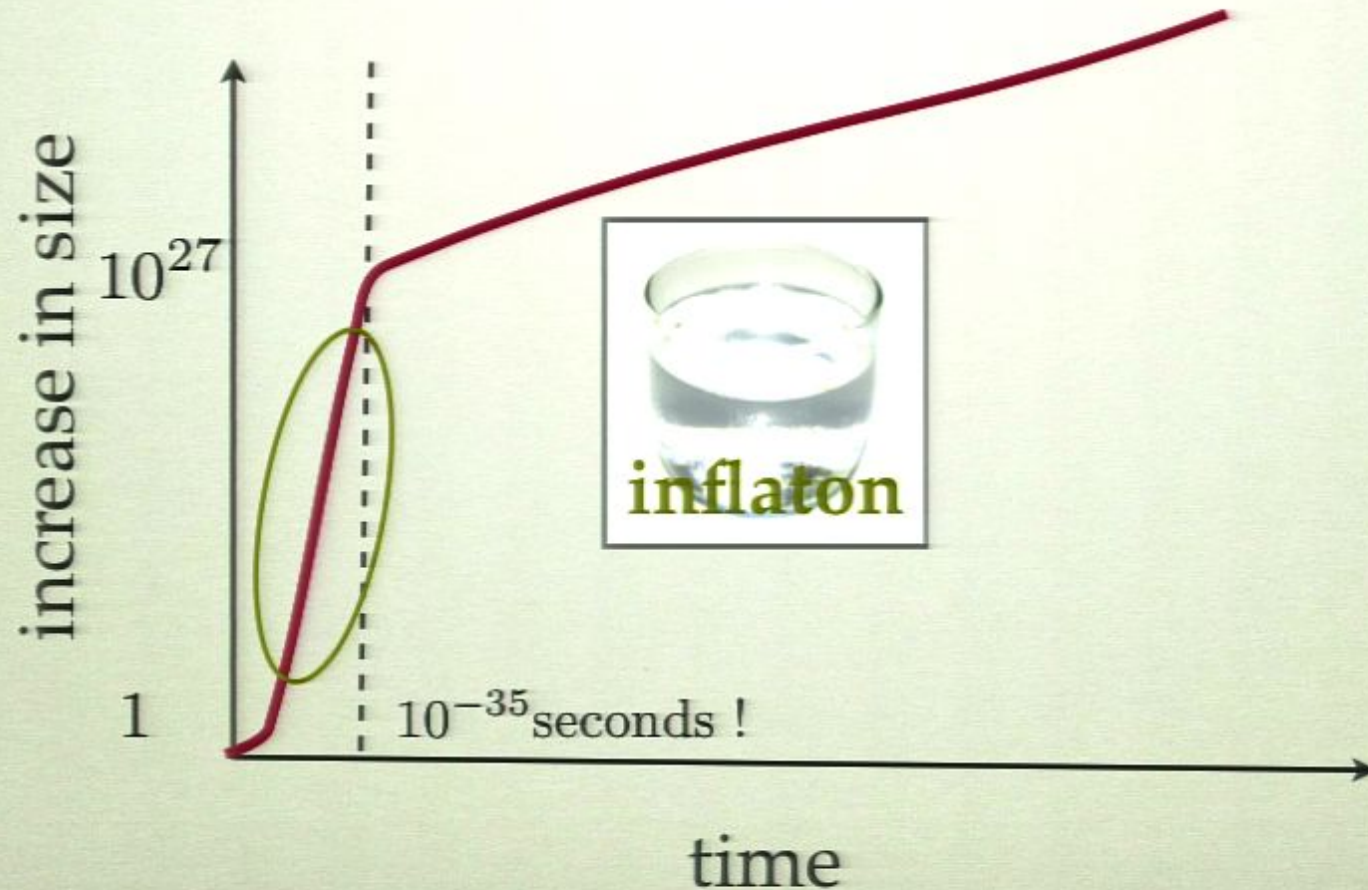
galaxy!



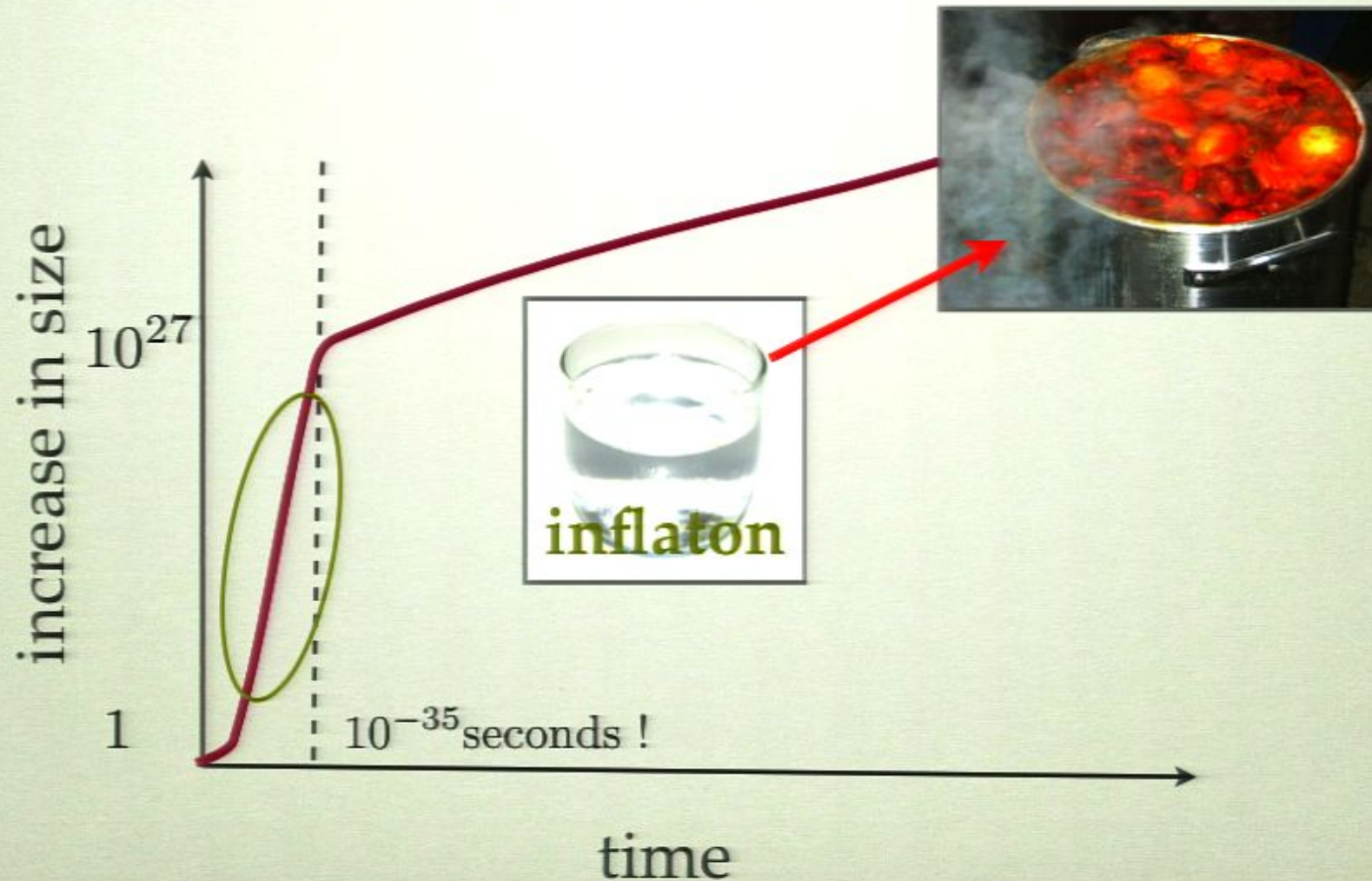
# POPULATING THE UNIVERSE



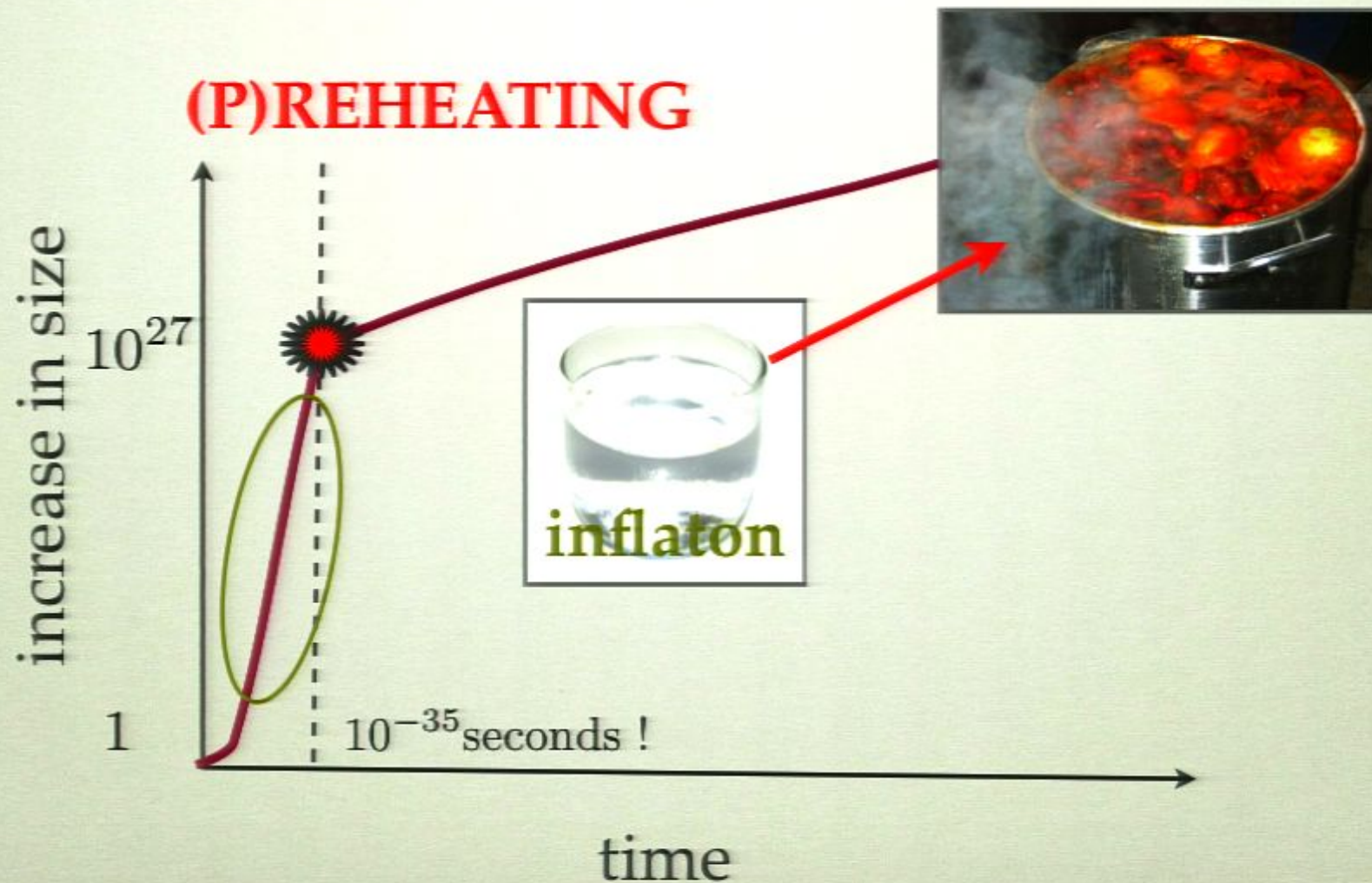
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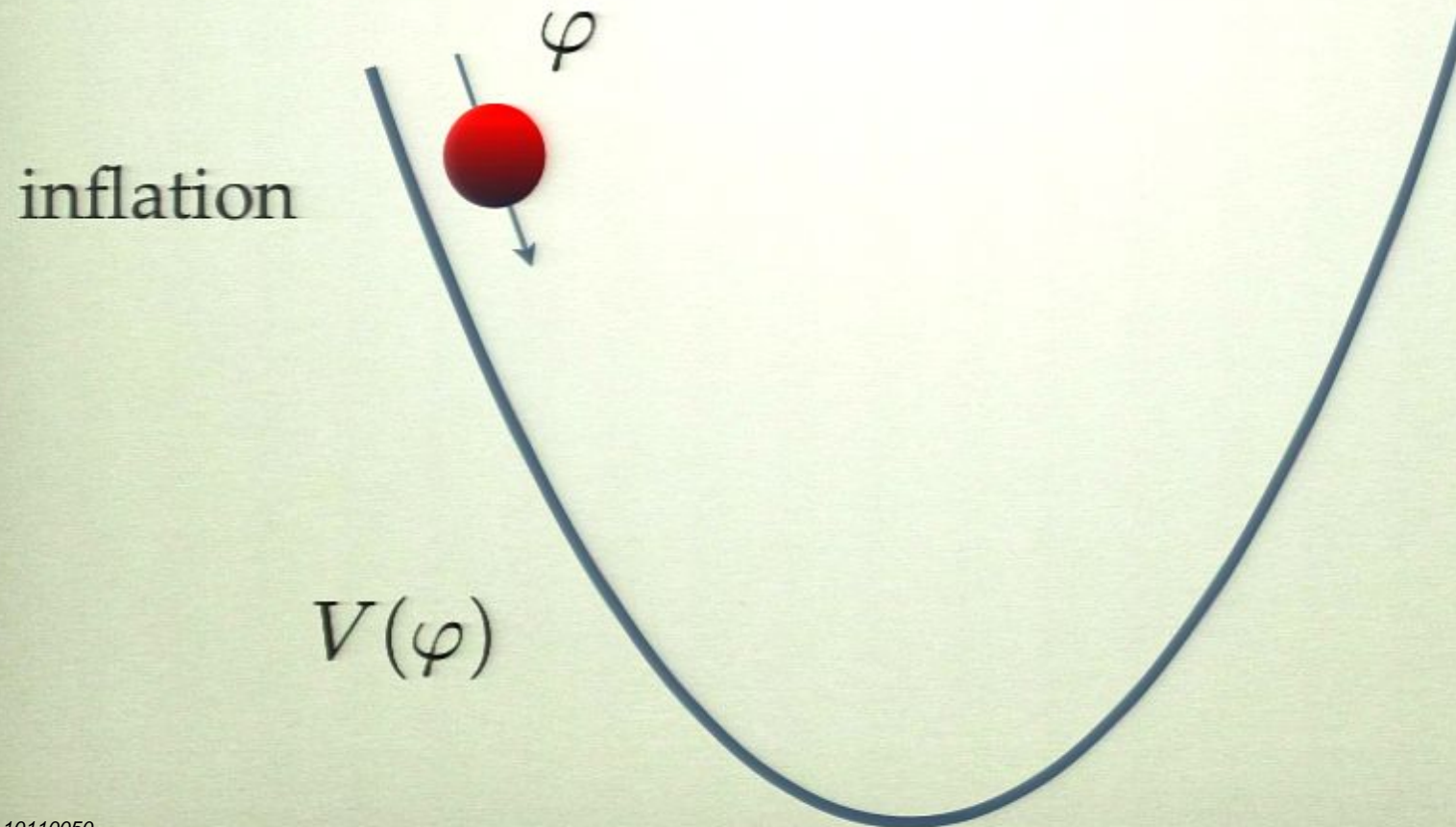


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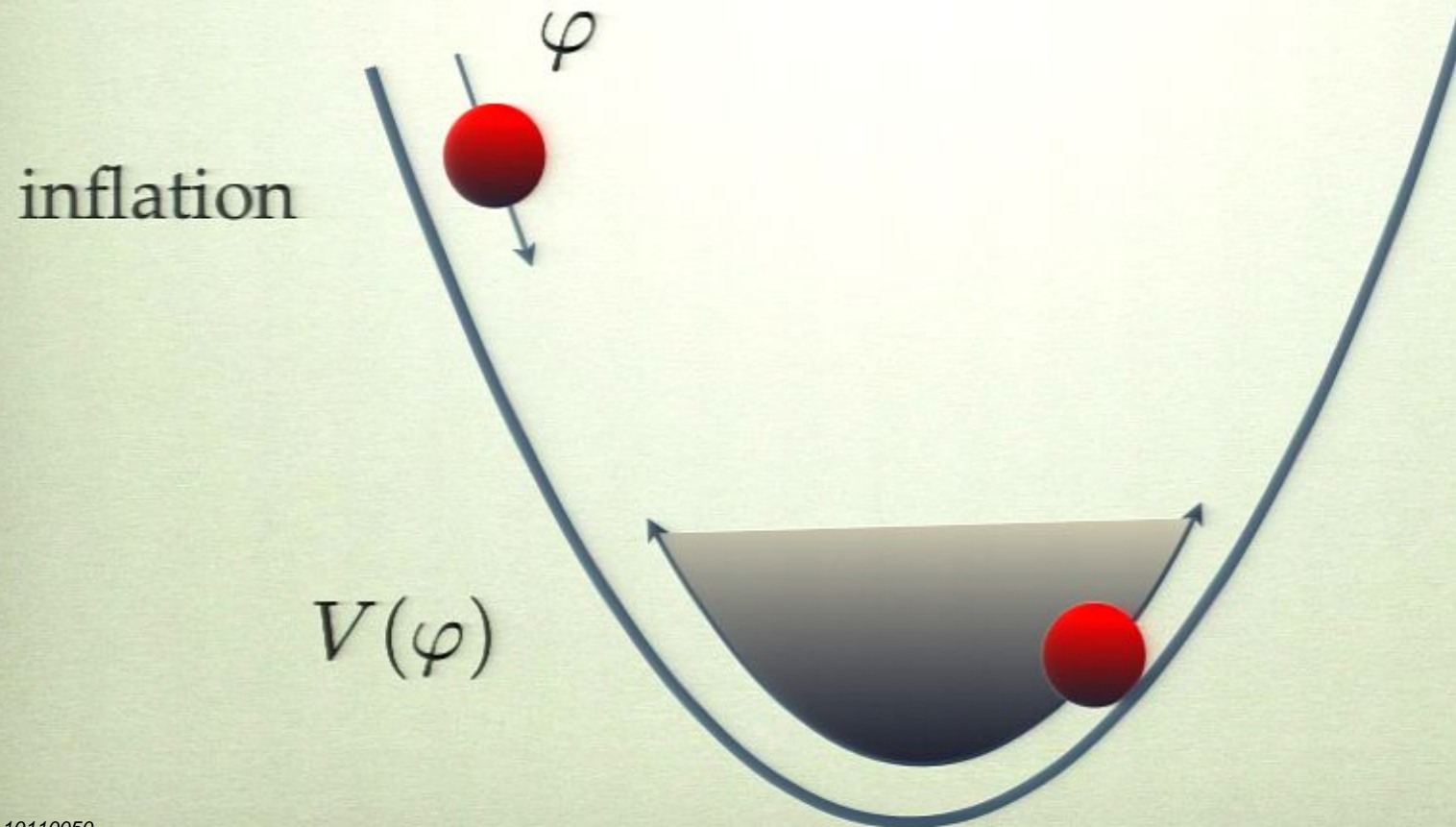
# INFLATION AND ITS END

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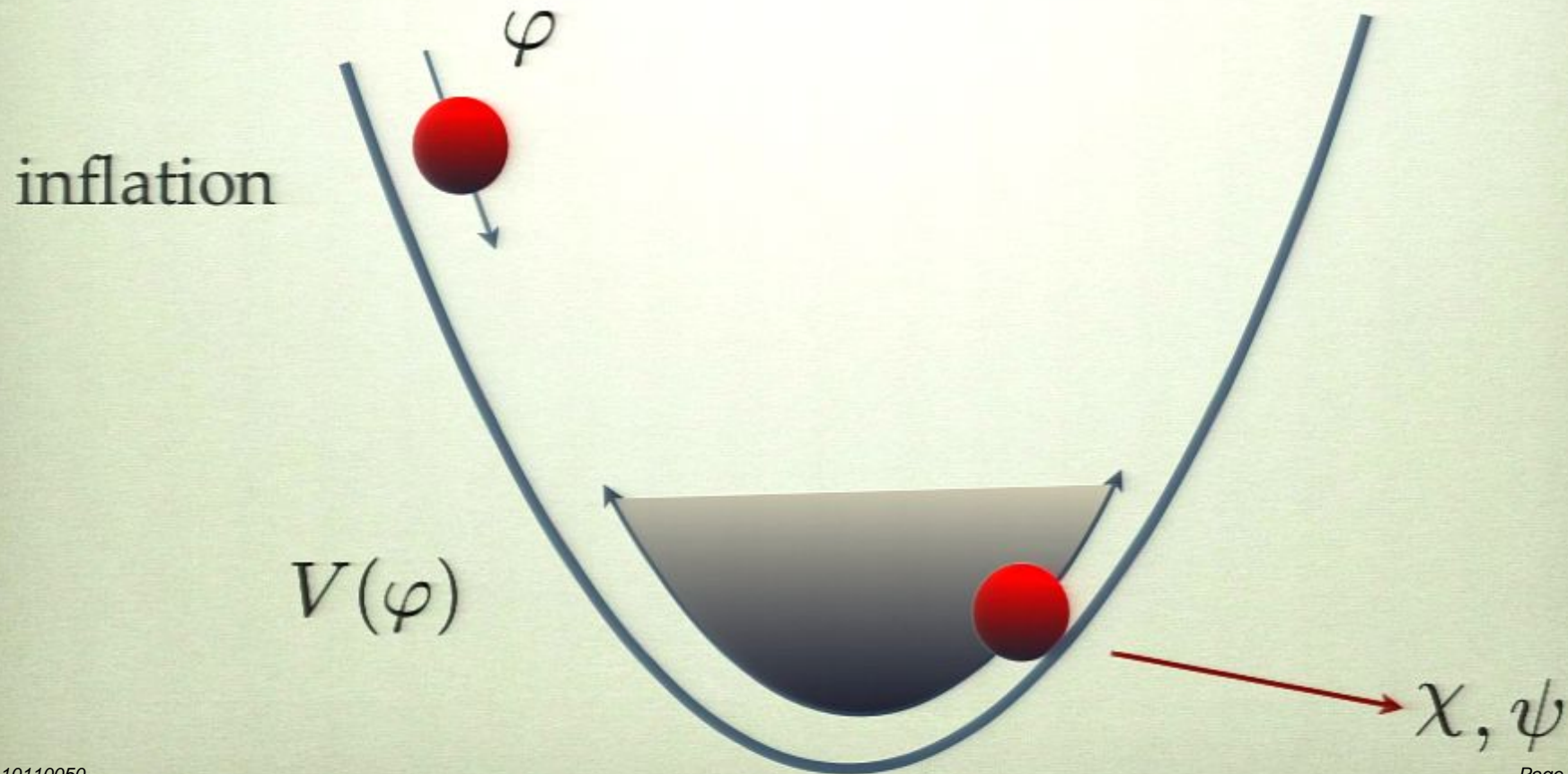


# INFLATION AND ITS END

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# INFLATION AND ITS END



# REHEATING

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$$V \sim \frac{1}{2} m_\varphi^2 \varphi^2 + g^2 \varphi^2 \chi^2 + h \varphi \bar{\psi} \psi + \dots$$

*slow* perturbative decay

$$\Gamma(\varphi \rightarrow \psi\psi) \sim \frac{h^2 m_\varphi}{8\pi} \ll H$$

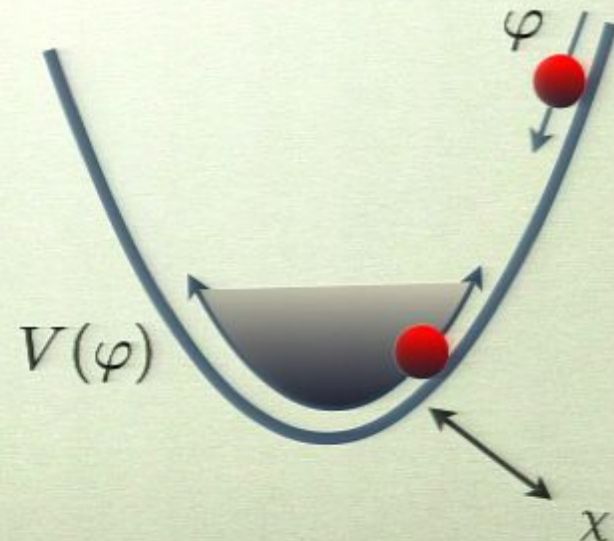


# PREHEATING

$$V \sim \frac{1}{2}m_\varphi^2\varphi^2 + g^2\varphi^2\chi^2 + h\varphi\bar{\psi}\psi + \dots$$

*explosive* particle production

$$\square\chi = V_{,\chi} = g^2\varphi^2\chi$$

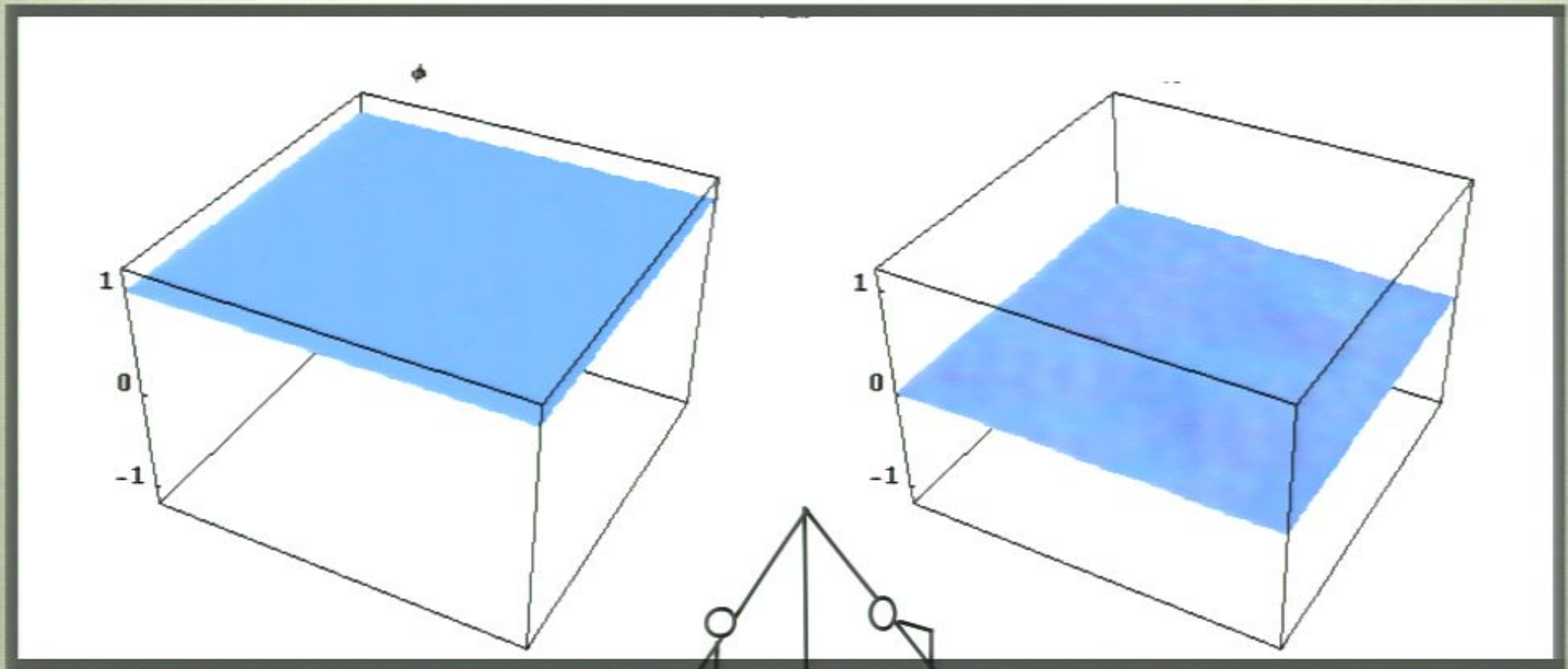


similarly for  $\chi = \delta\varphi$

# PREHEATING

inflaton

daughter fields

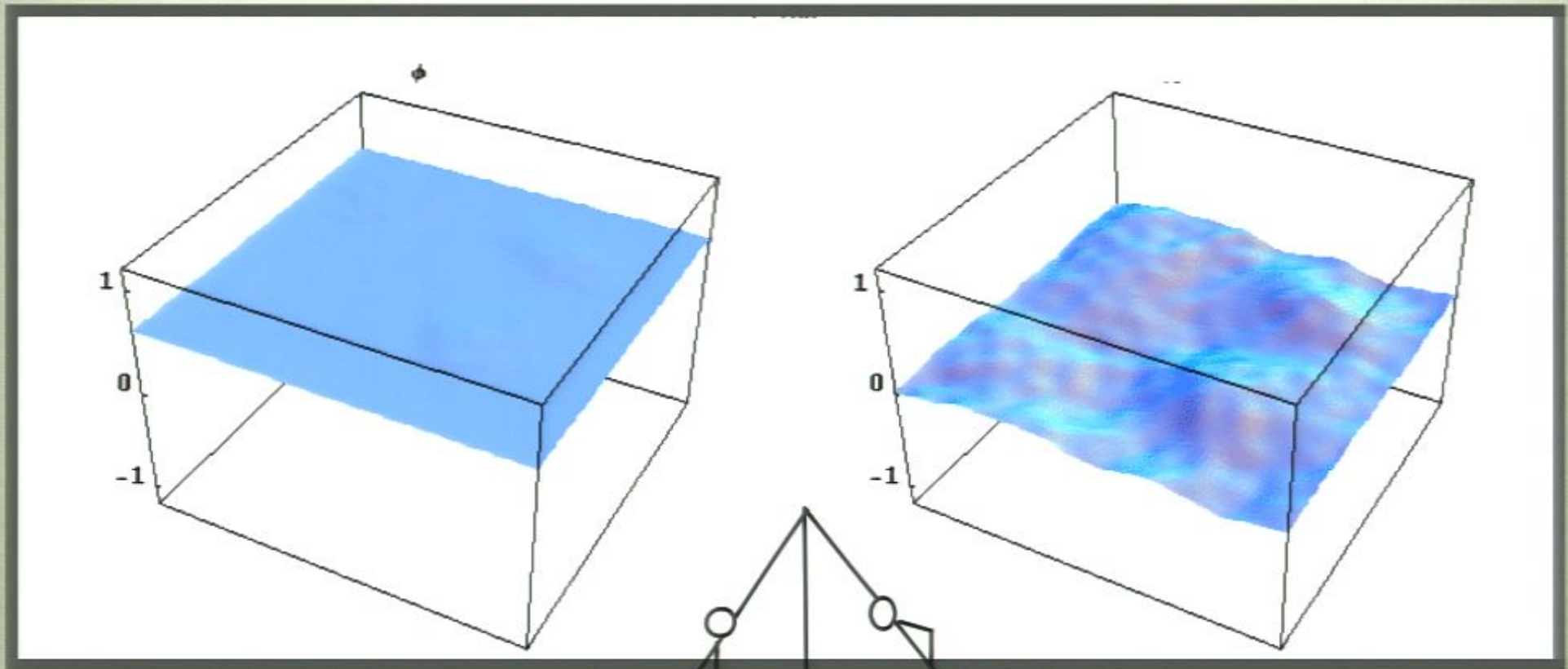


Felder and Kofman 2006

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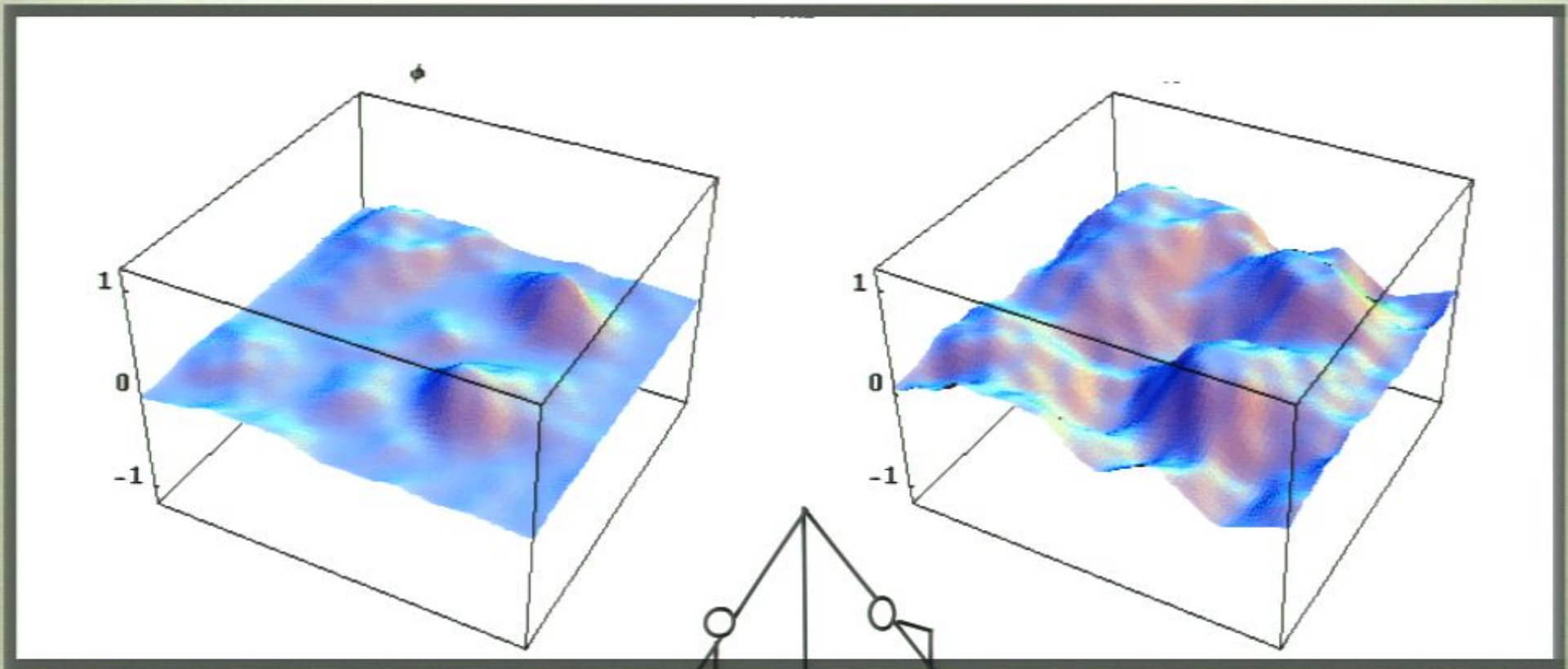


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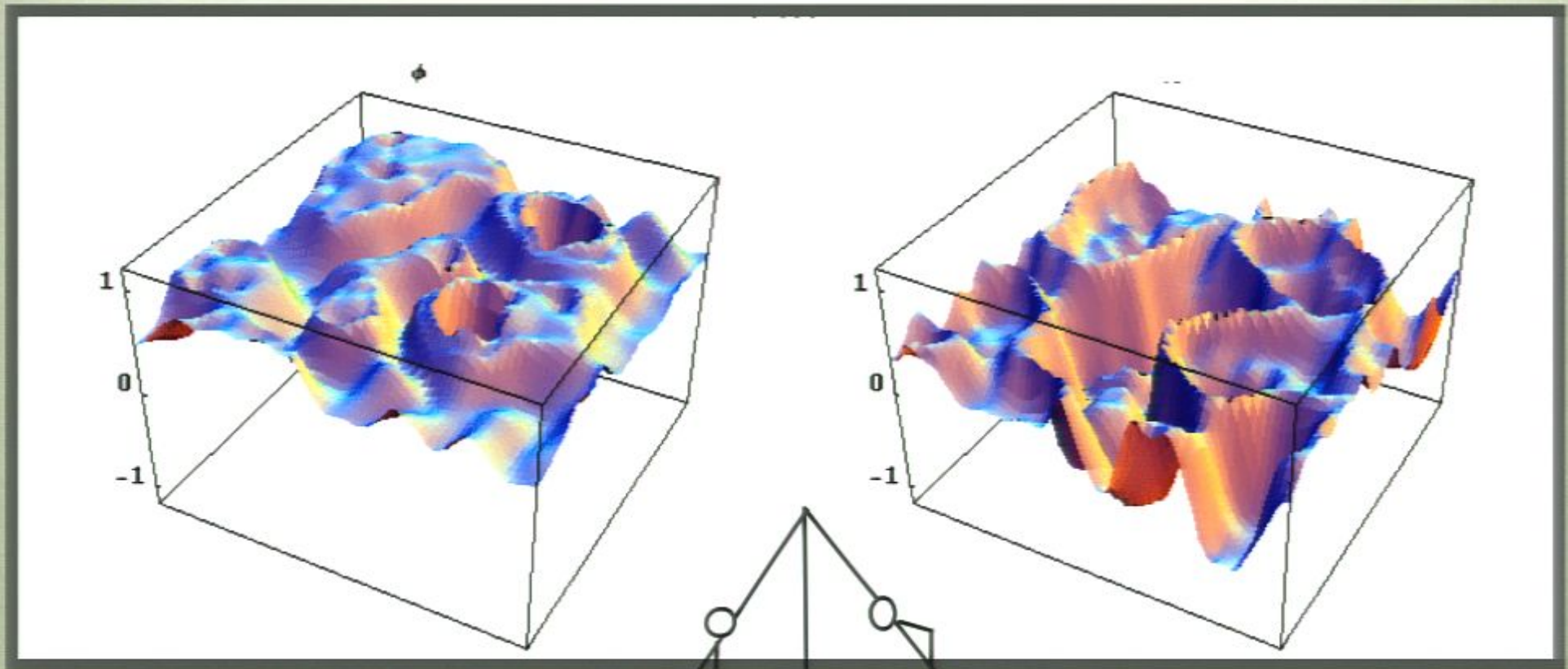


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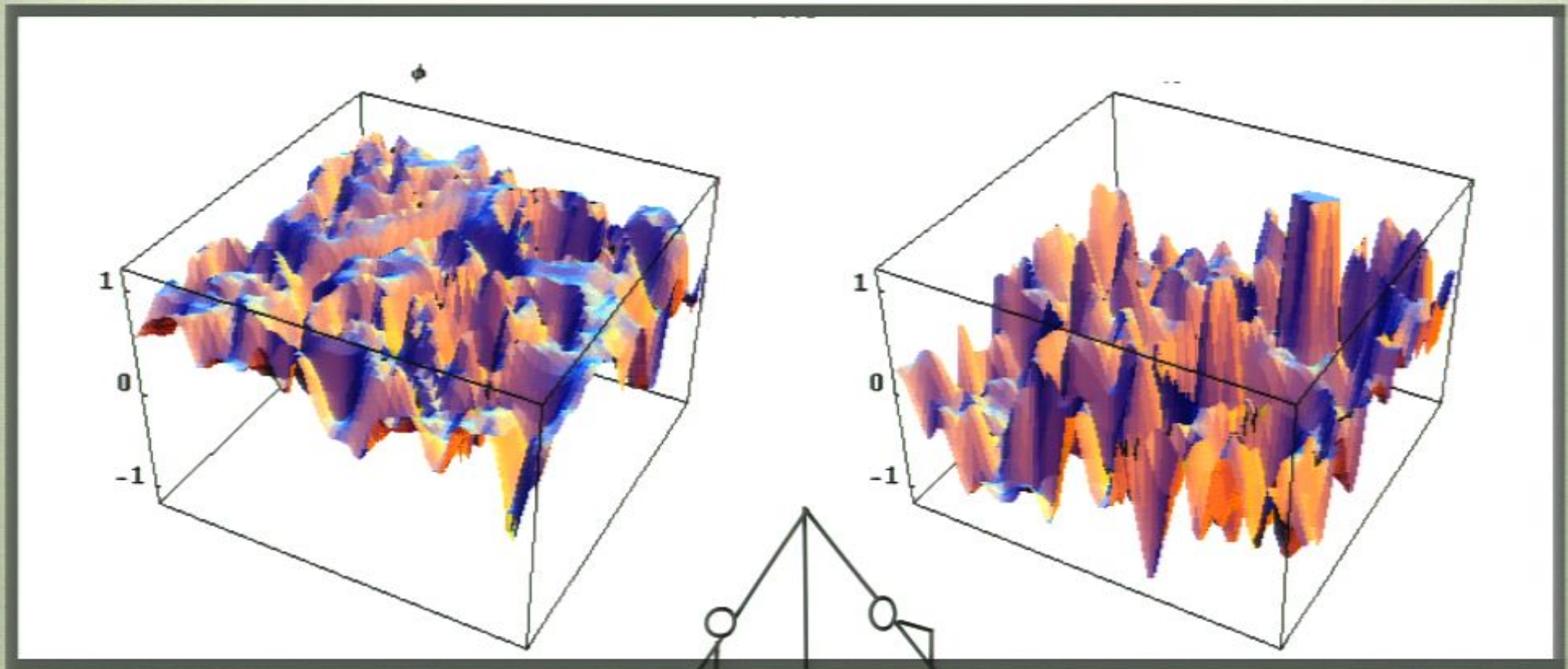


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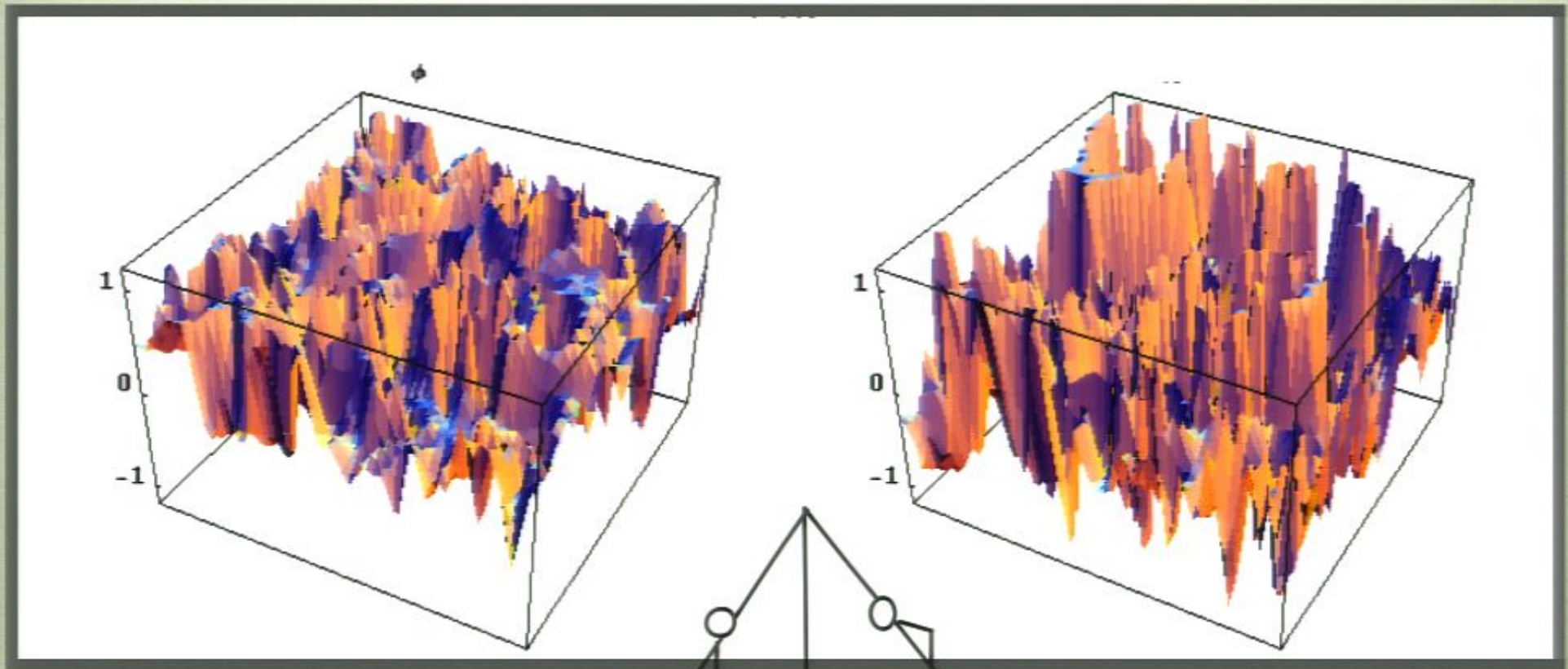


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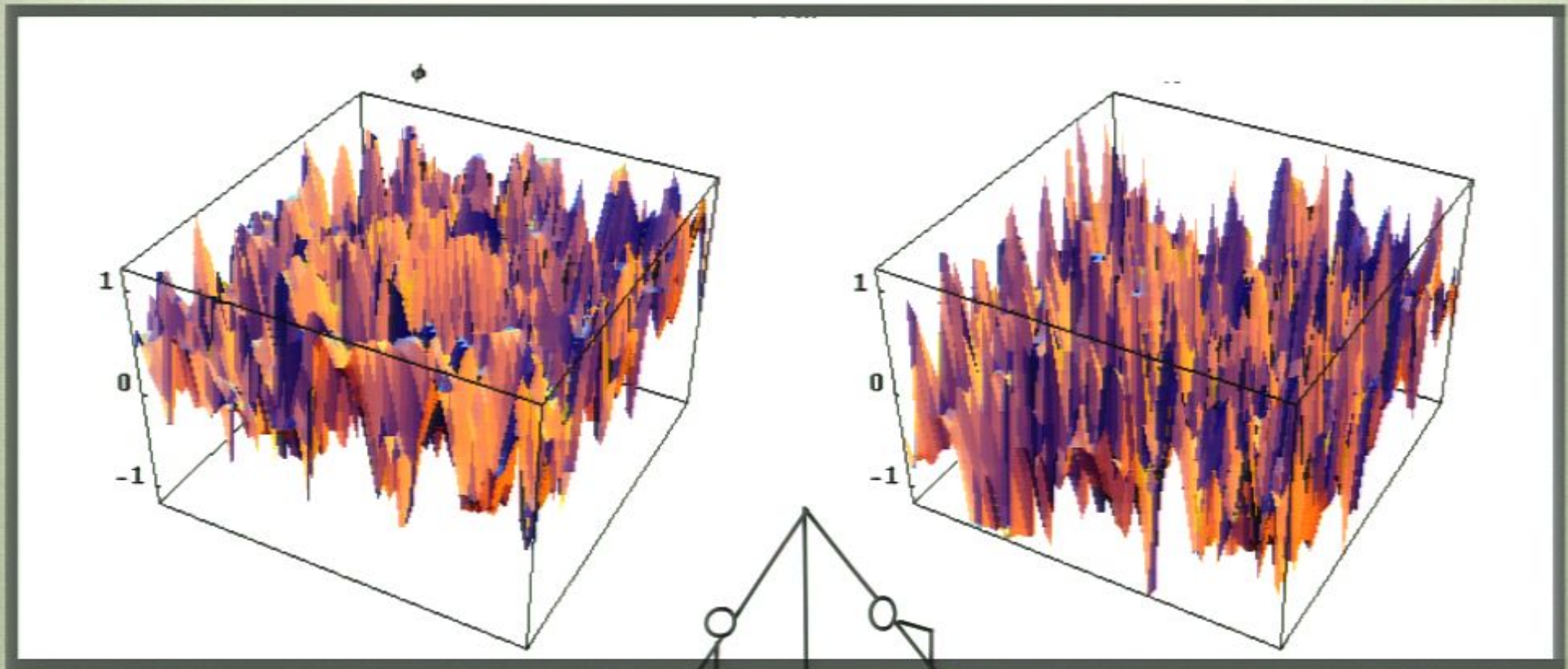


Felder and Kofman 2006

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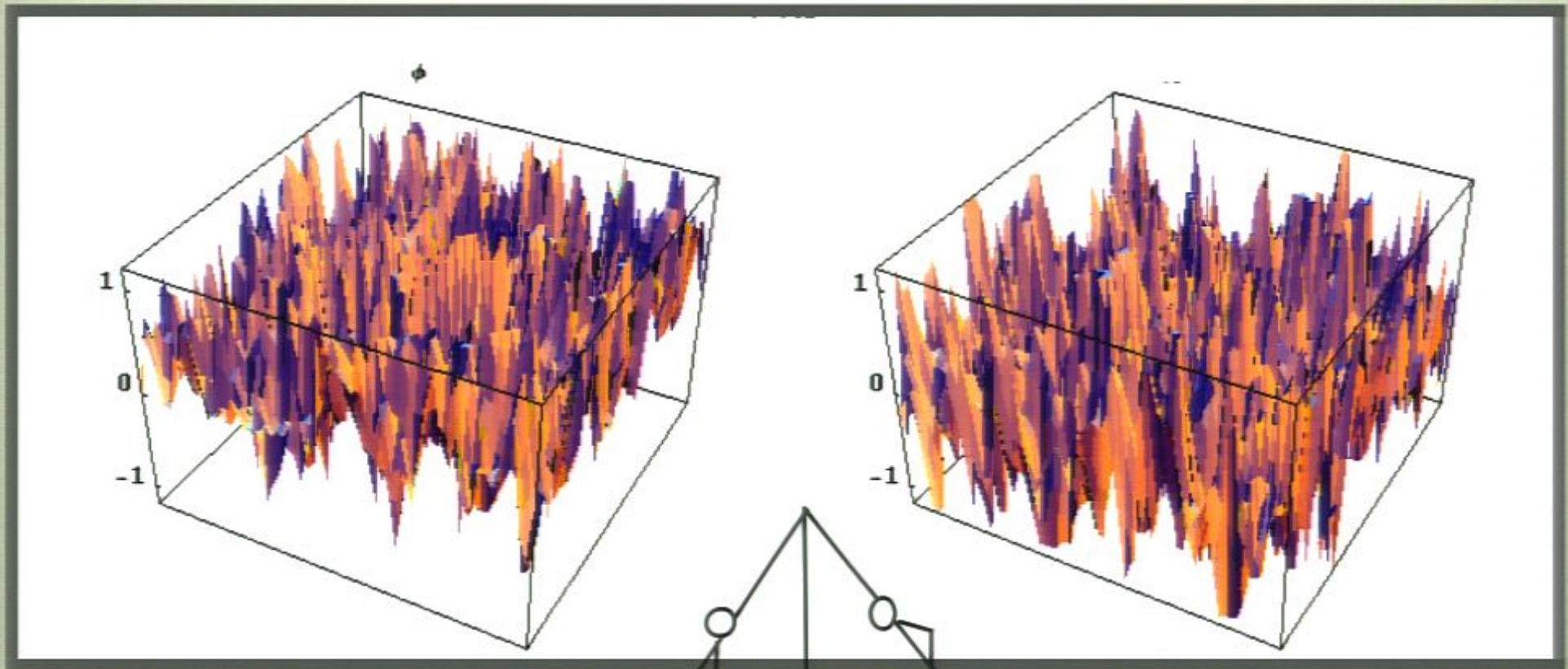
Felder and Kofman 2006



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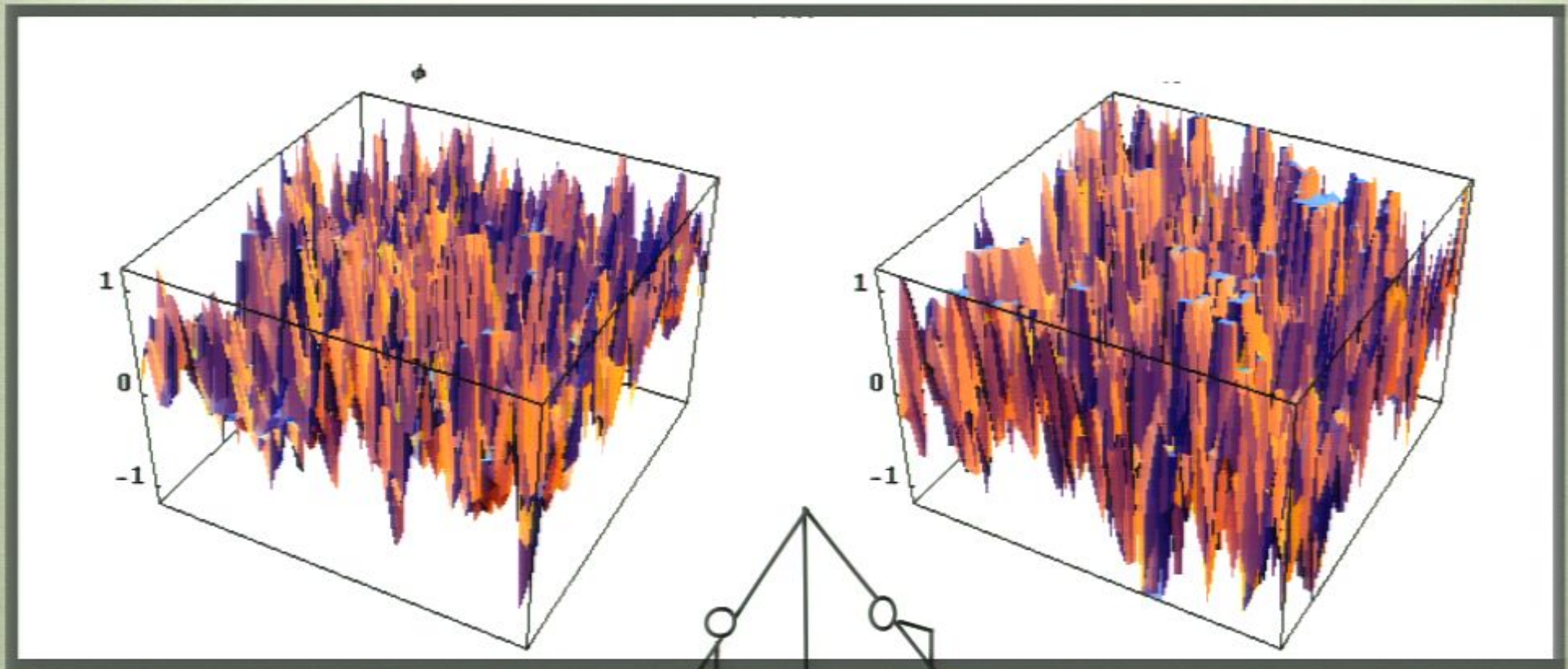


Felder and Kofman 2006

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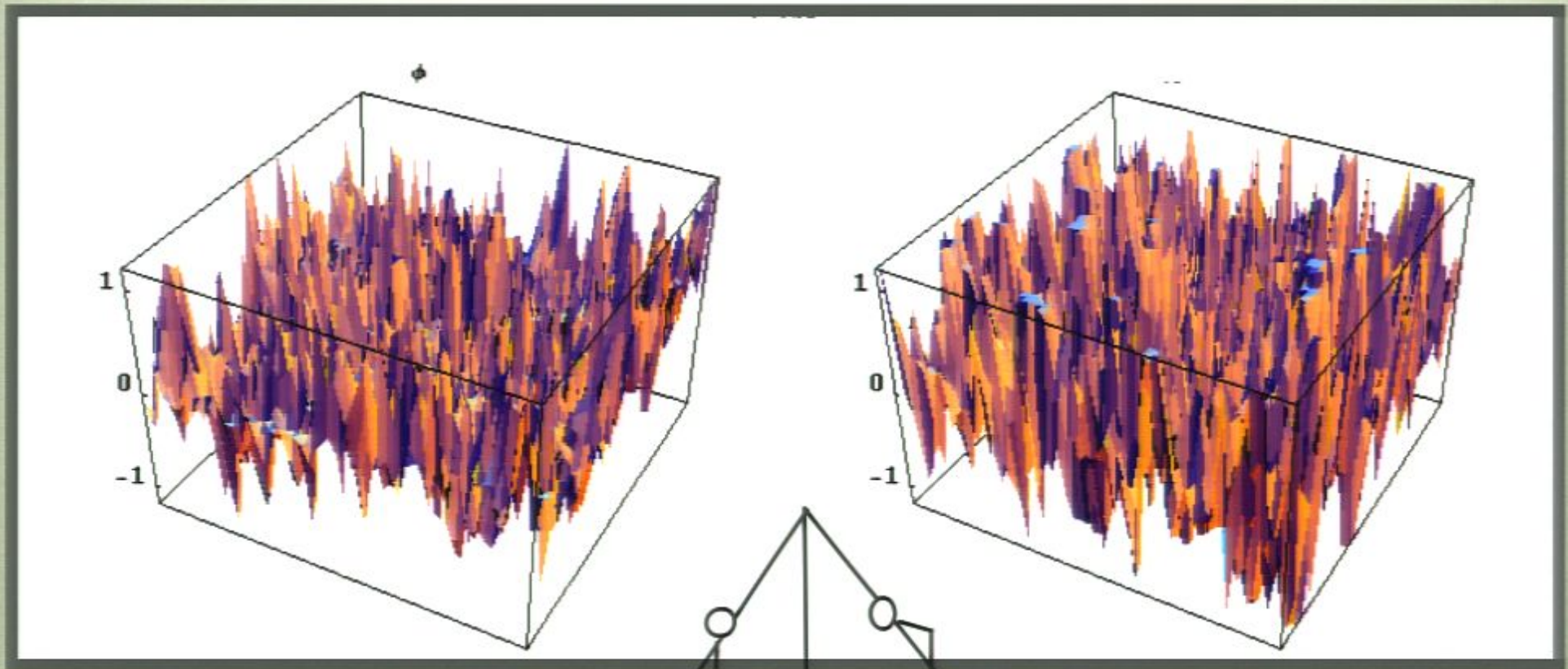


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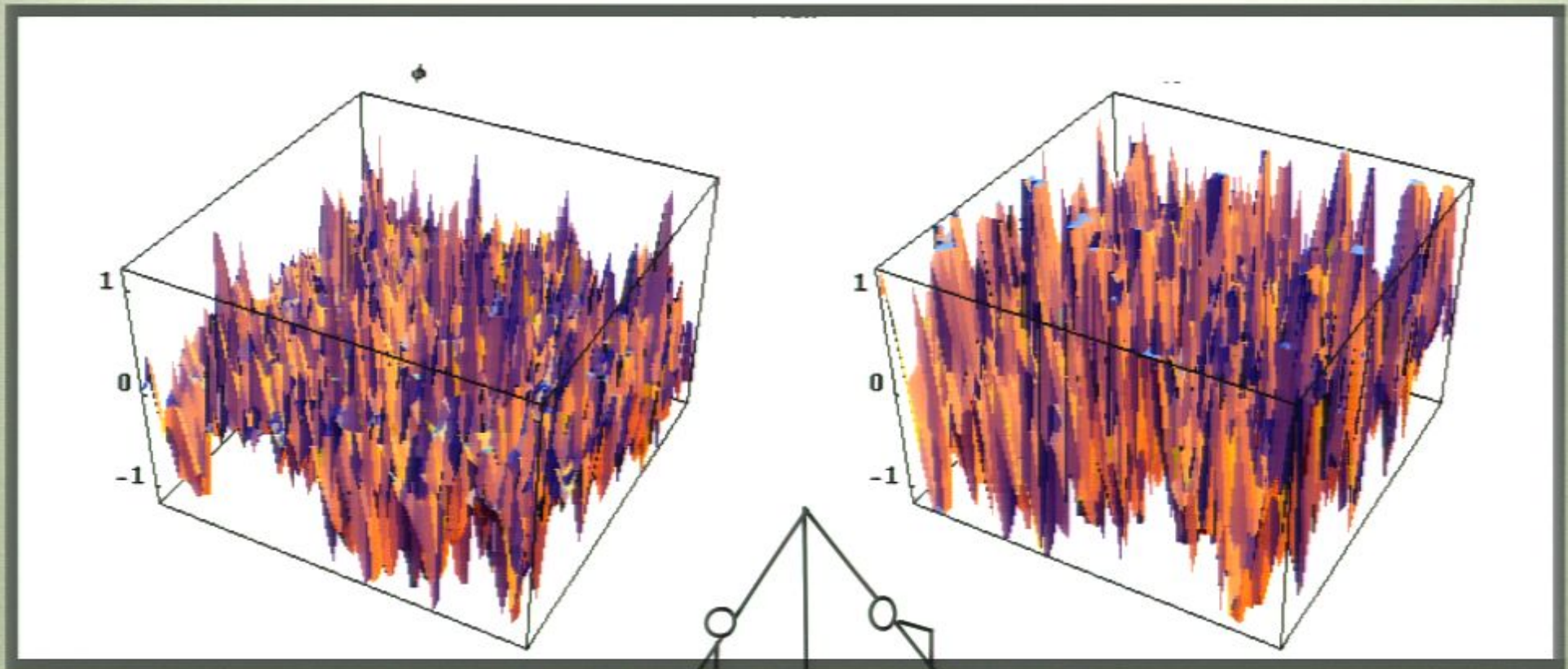


Felder and Kofman 2006

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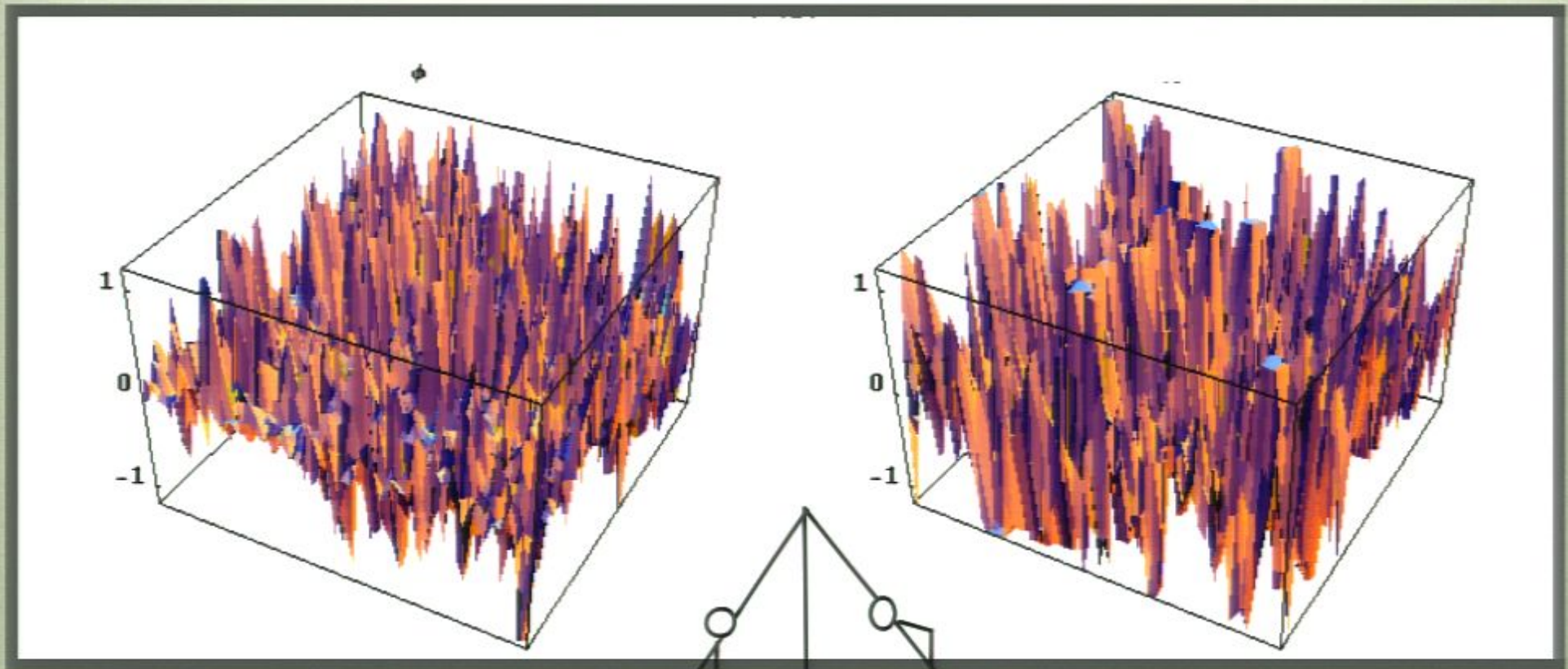


Felder and Kofman 2006

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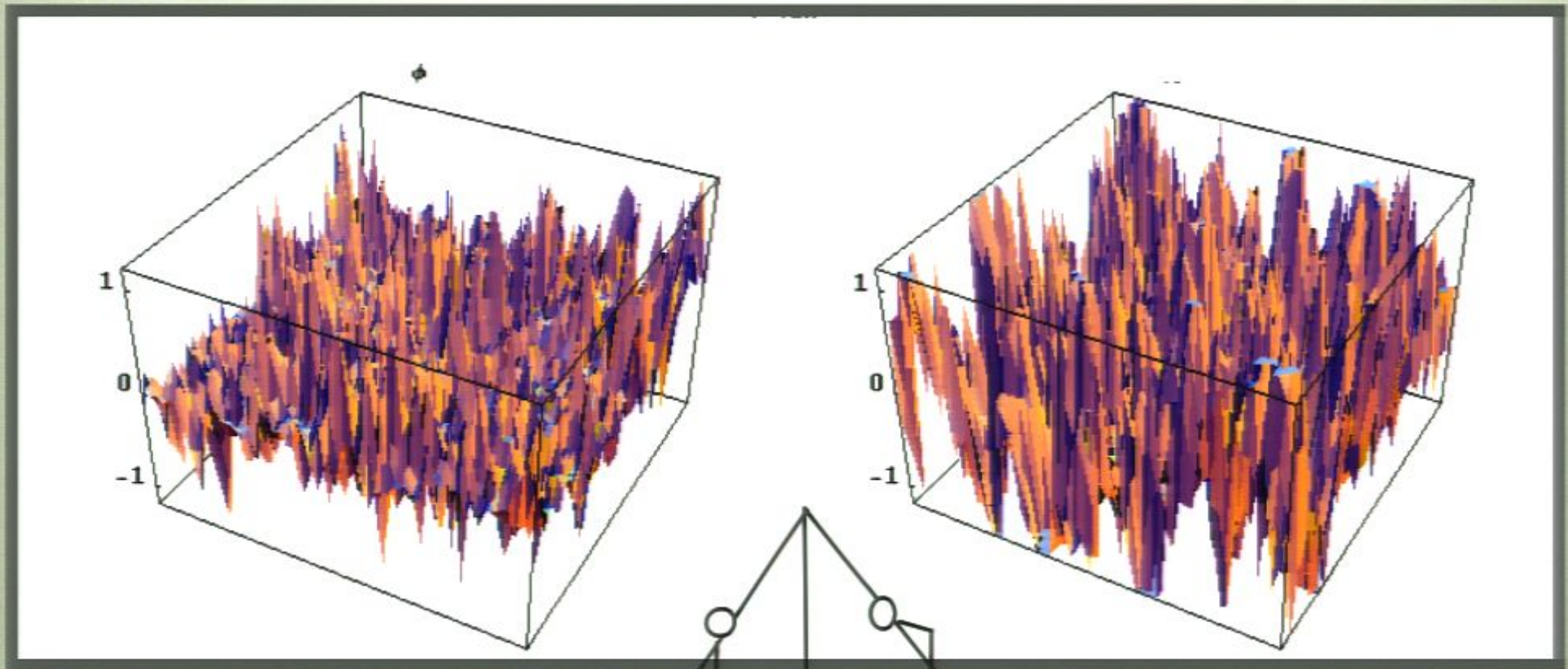


Felder and Kofman 2006

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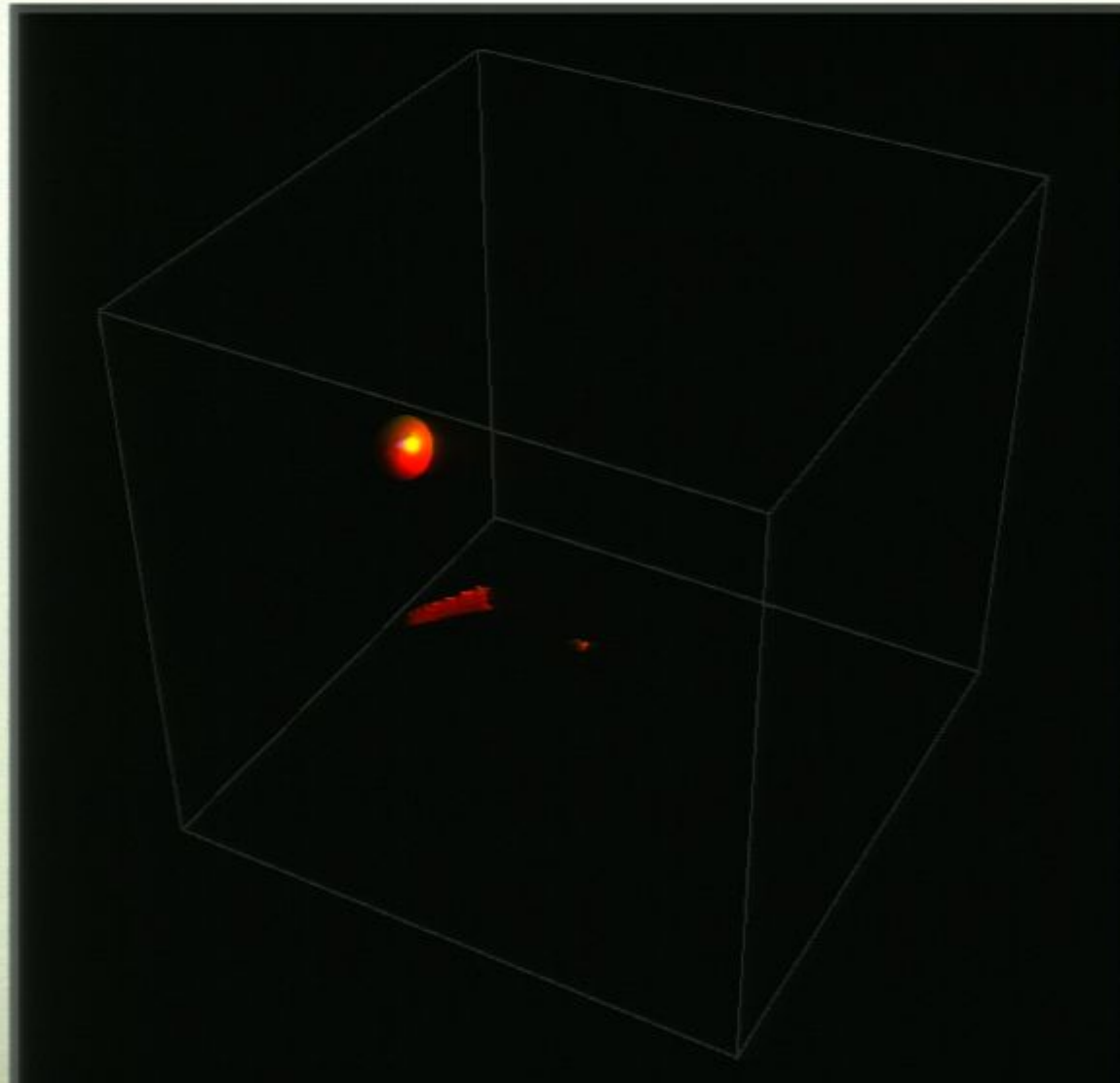
inflaton

daughter fields



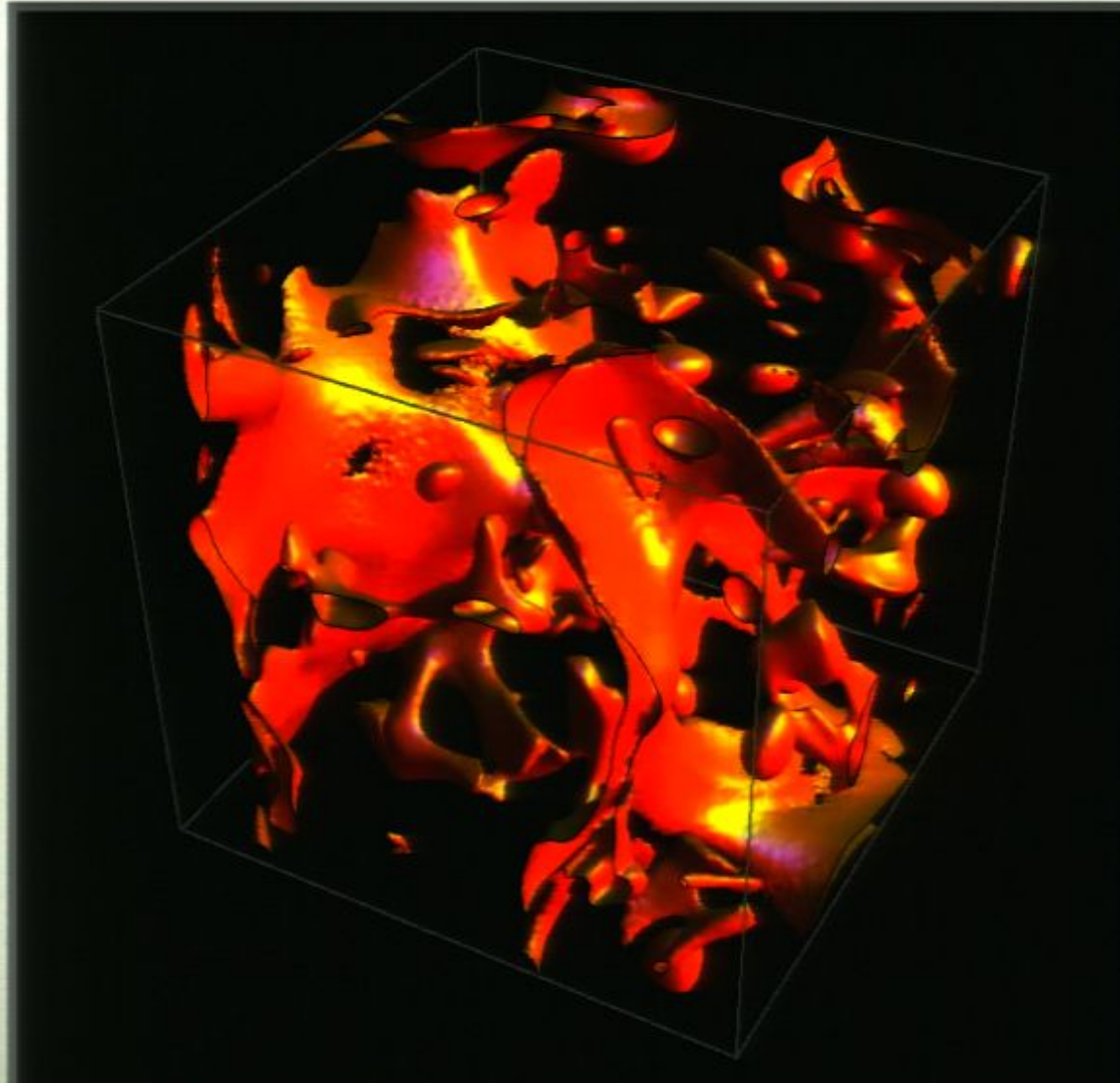
Felder and Kofman 2006

# FRAGMENTATION: TURBULENT MESS ...



4 Hubble radii at the end of inflation  
( a few meters today)

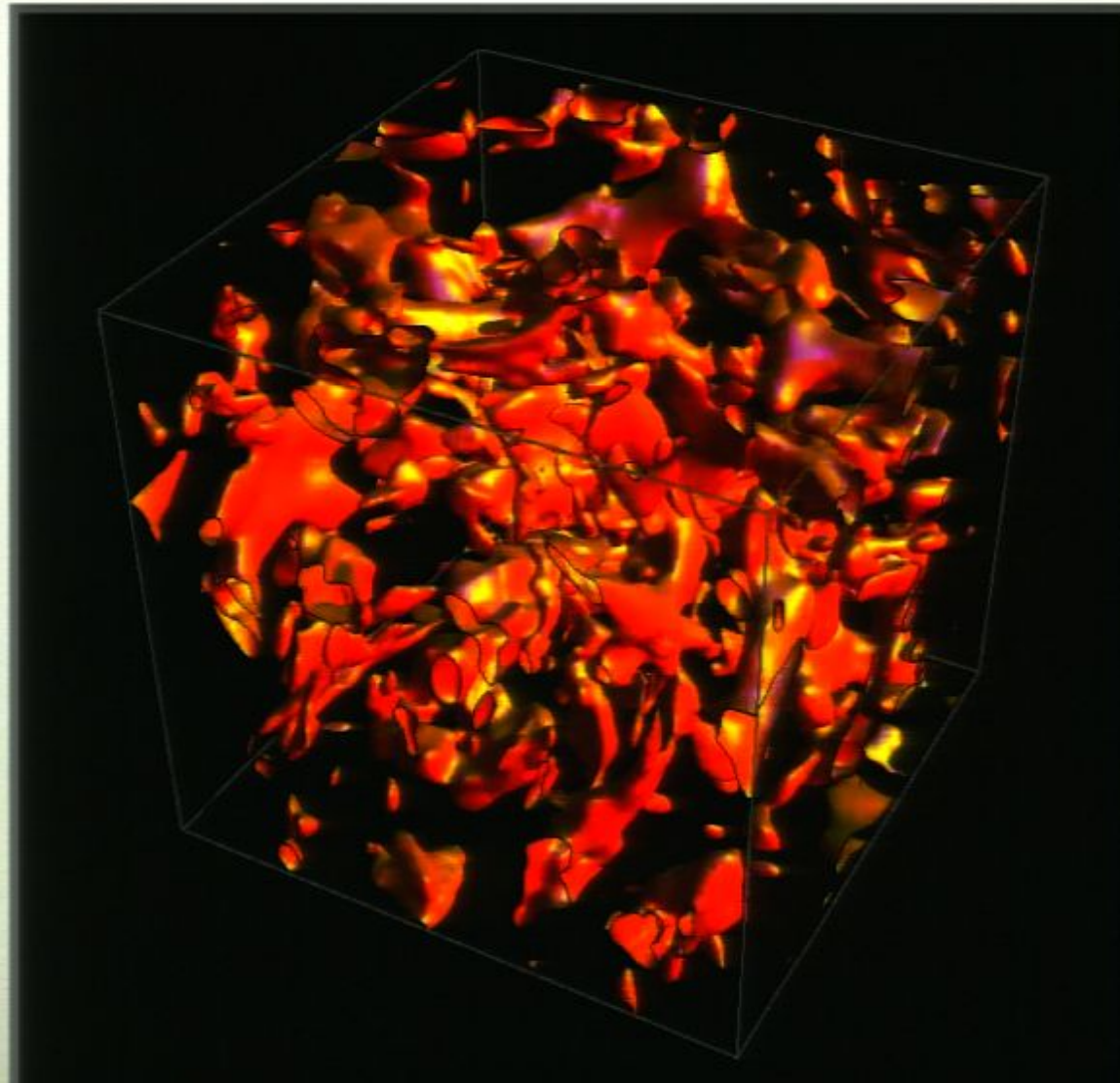
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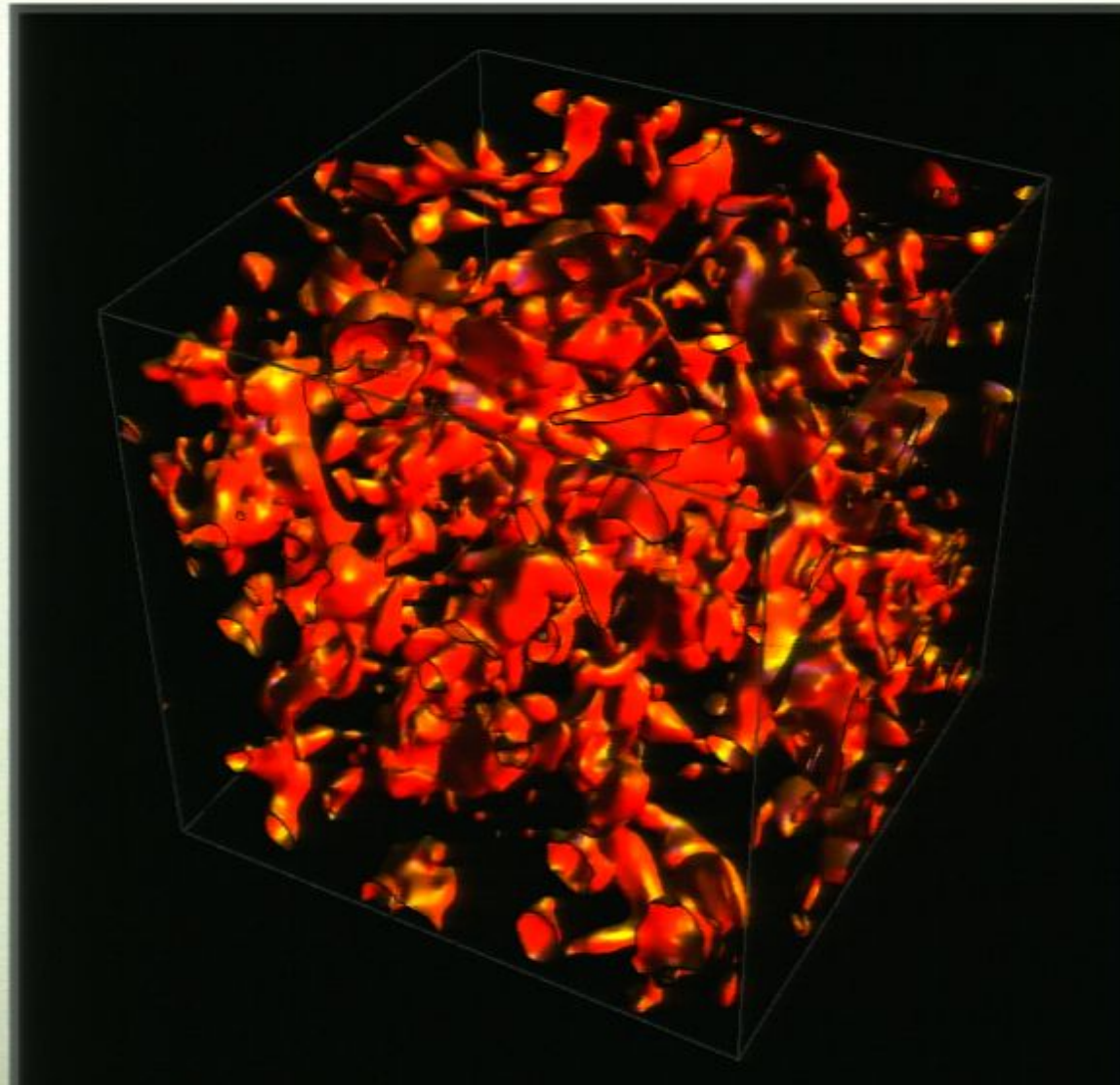


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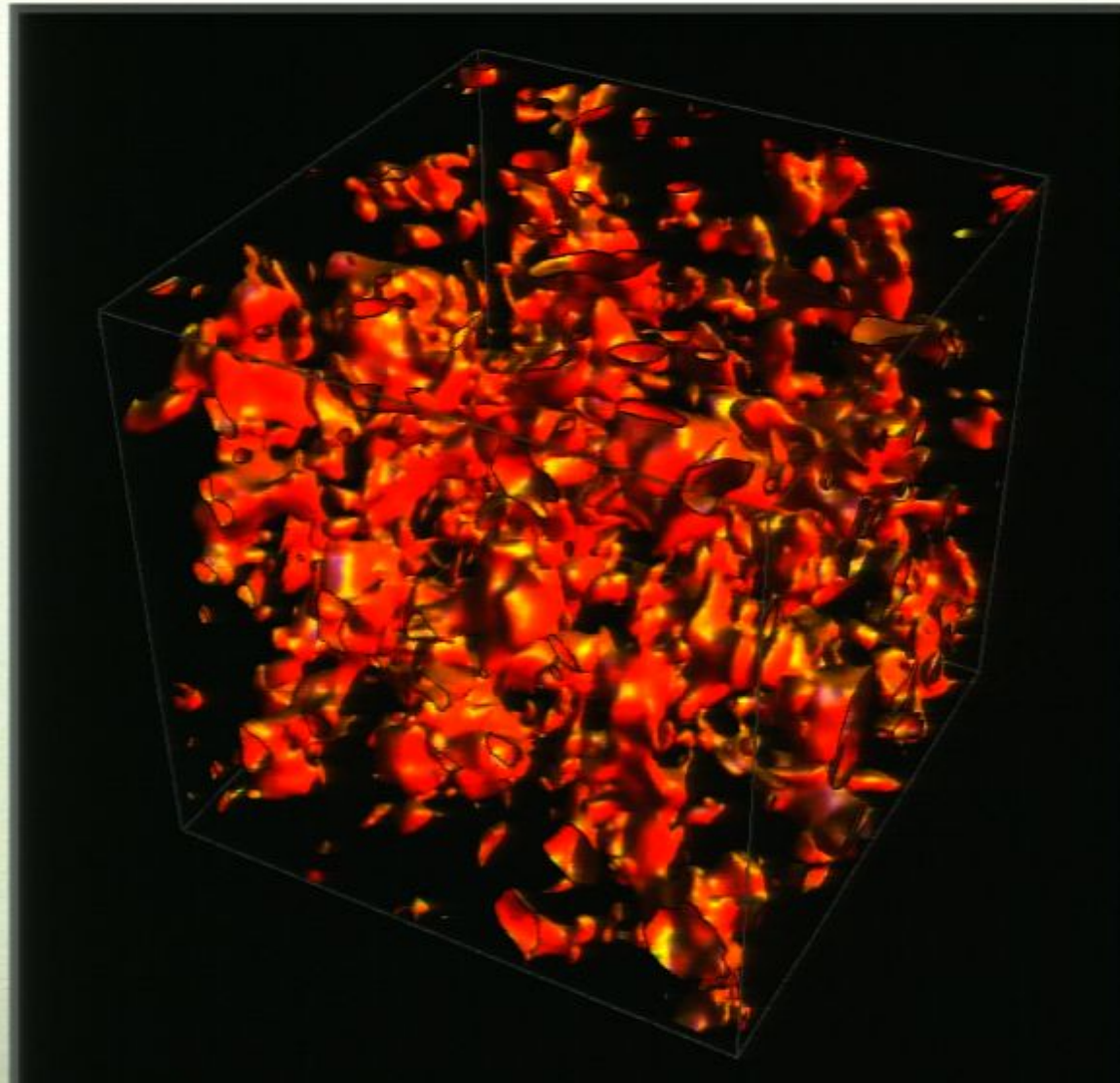
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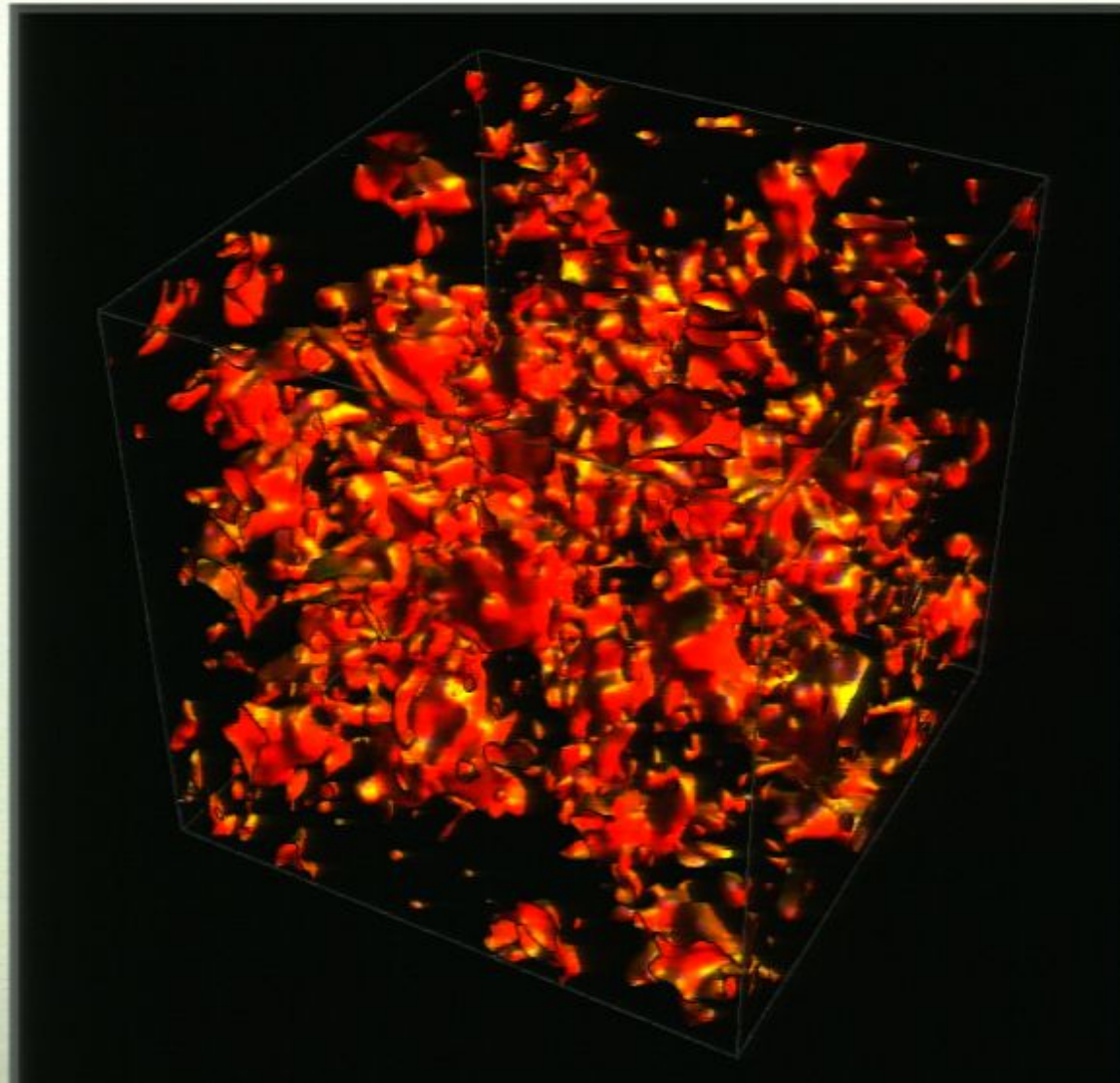
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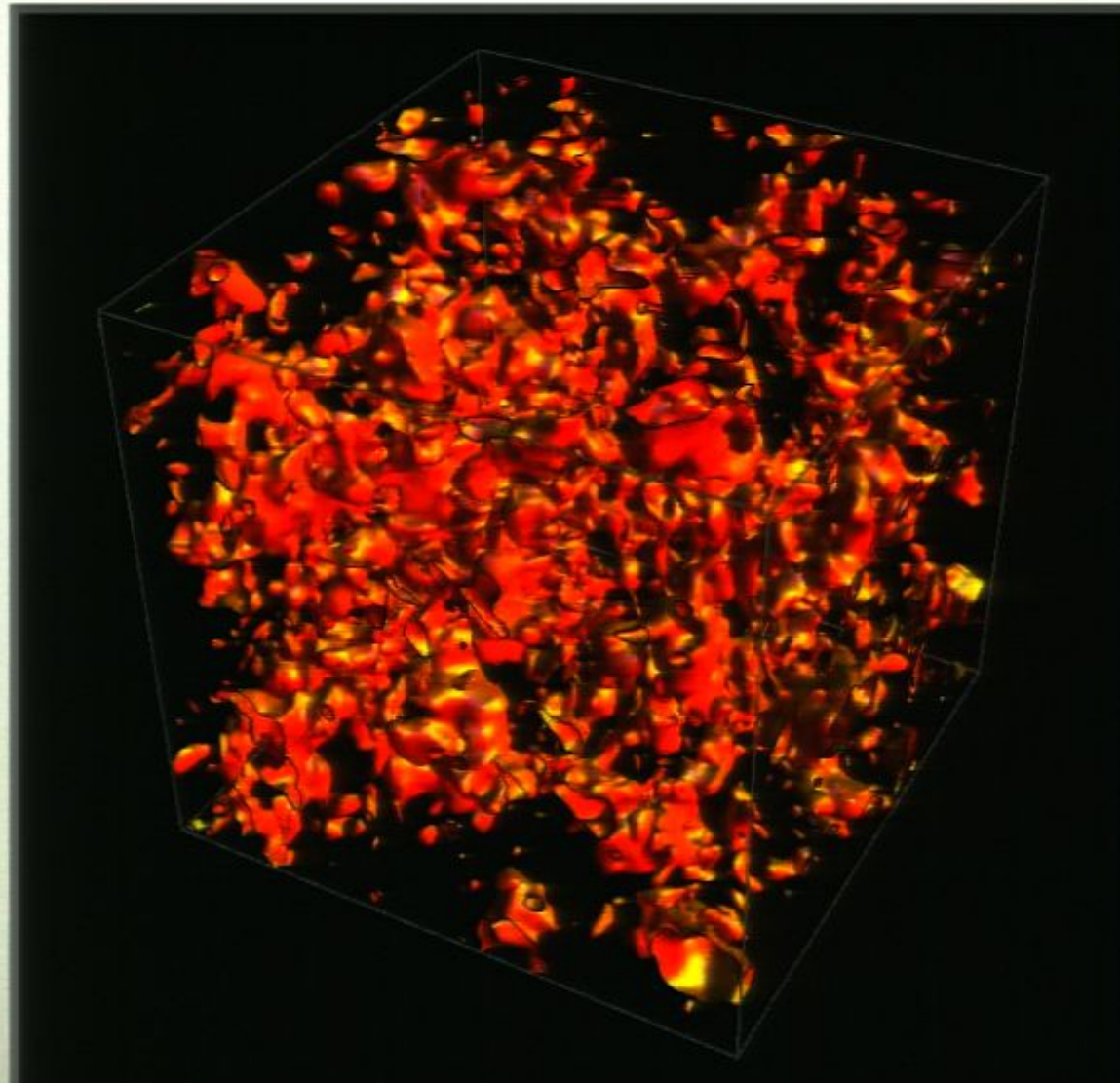
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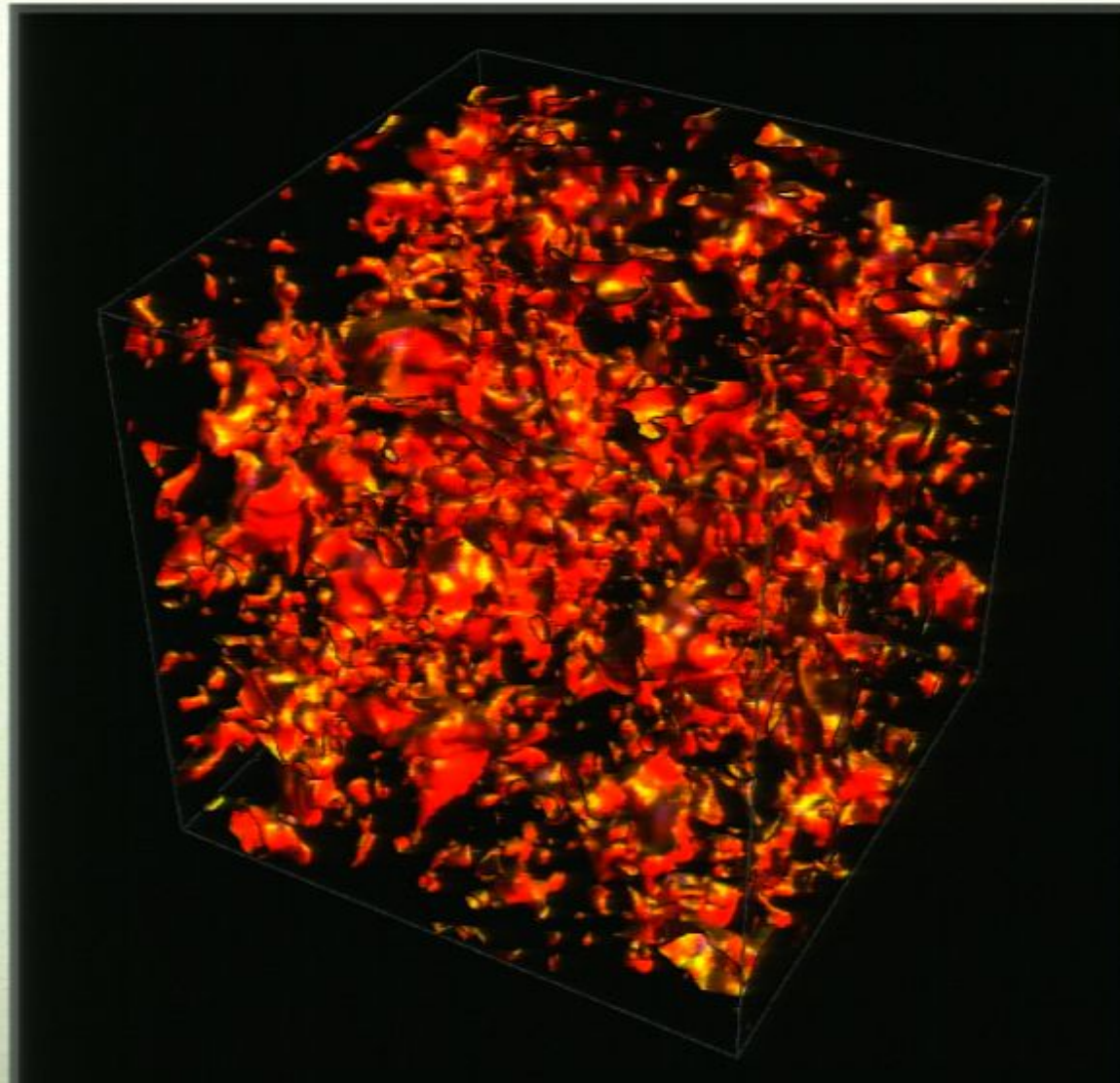
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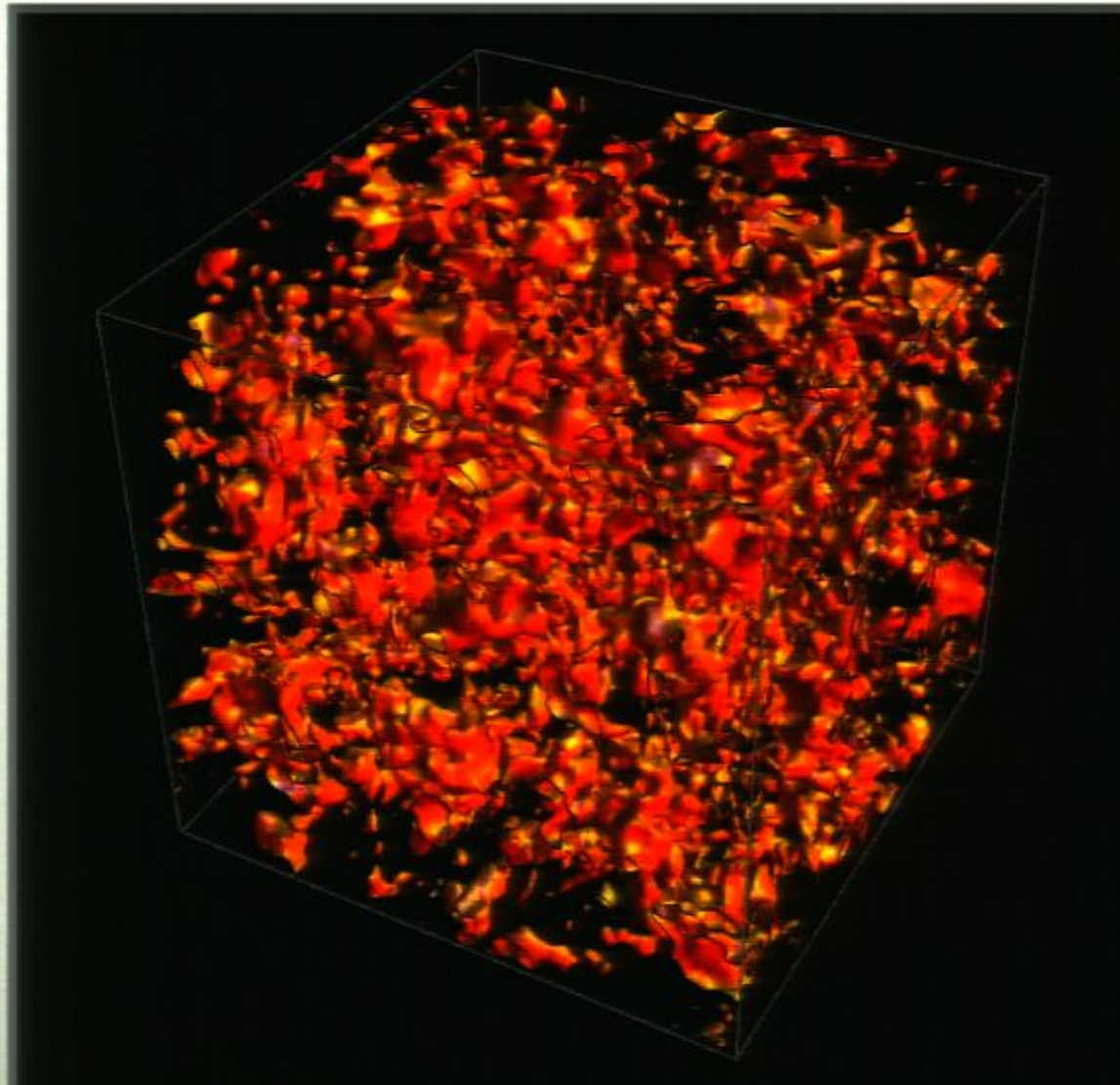
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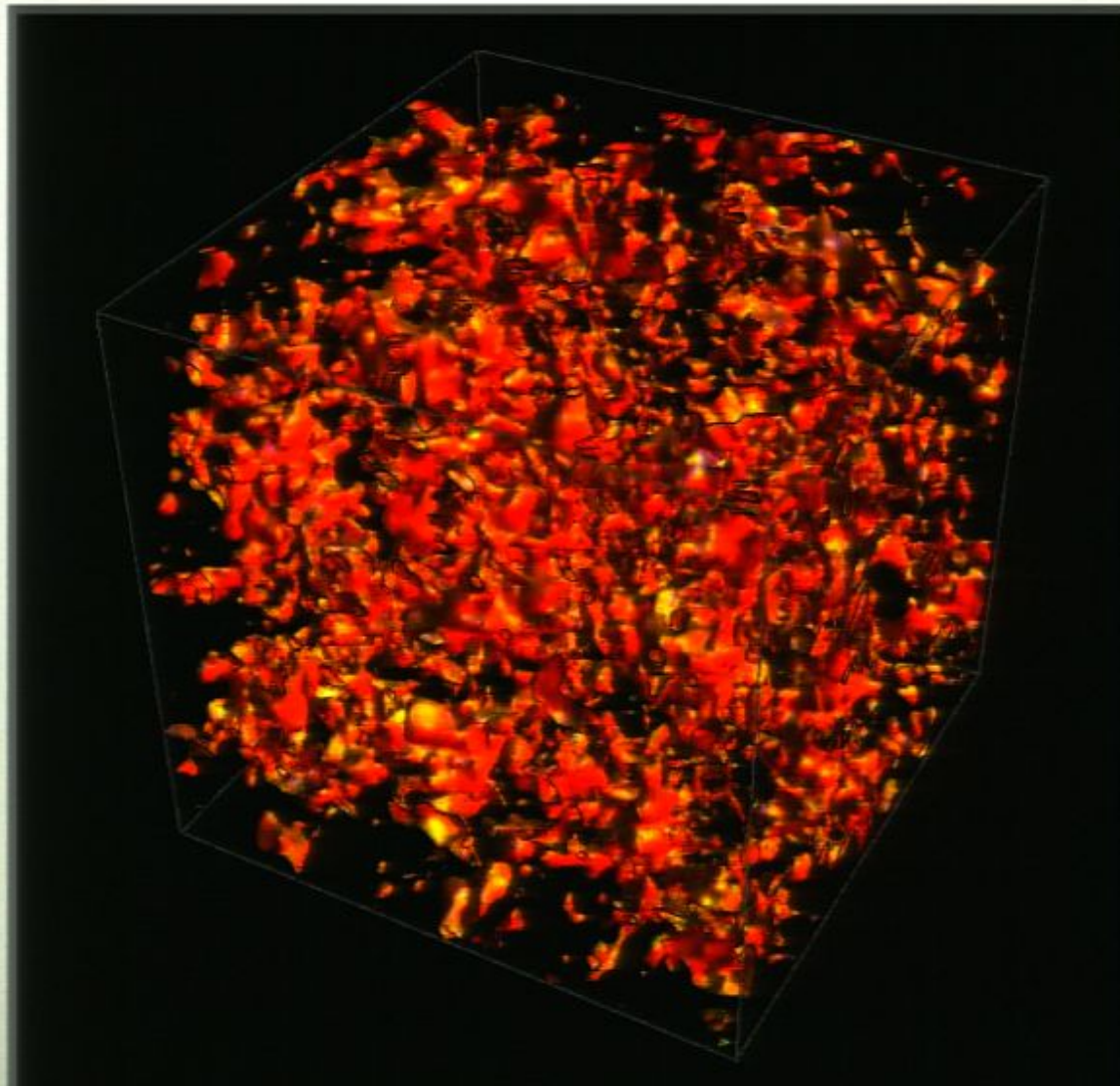
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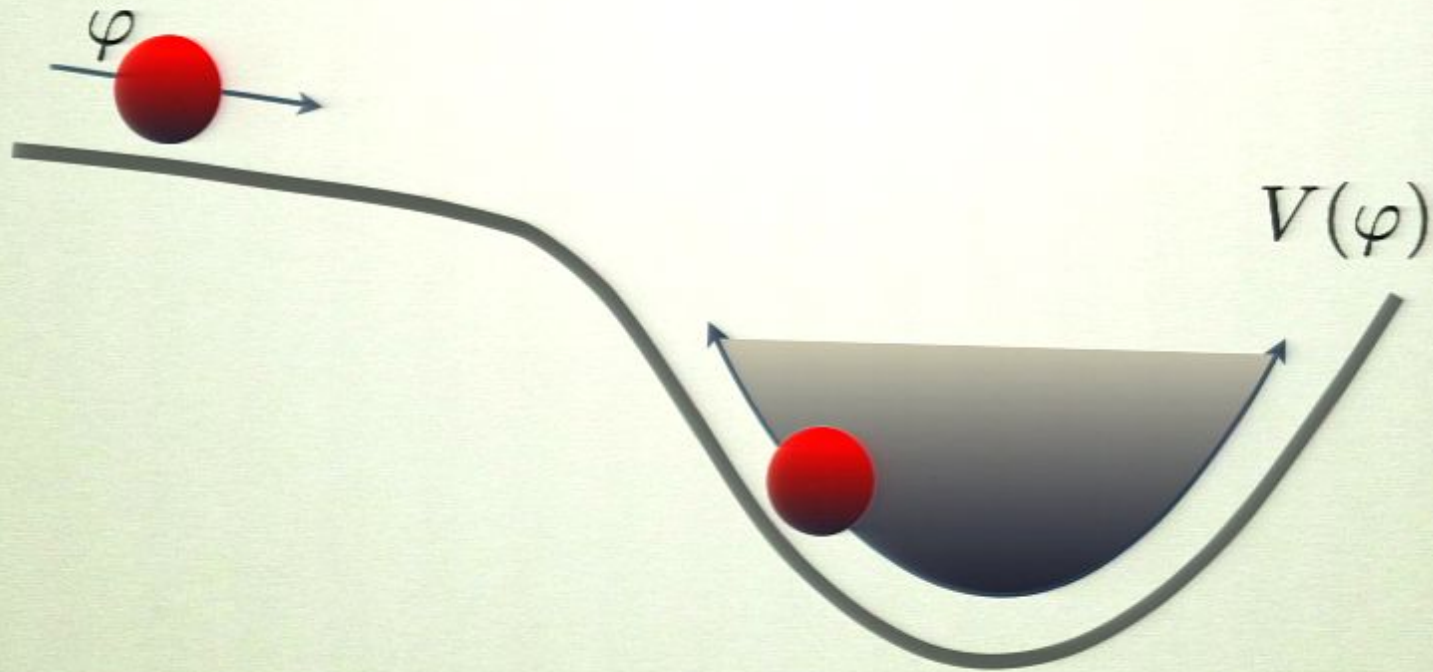
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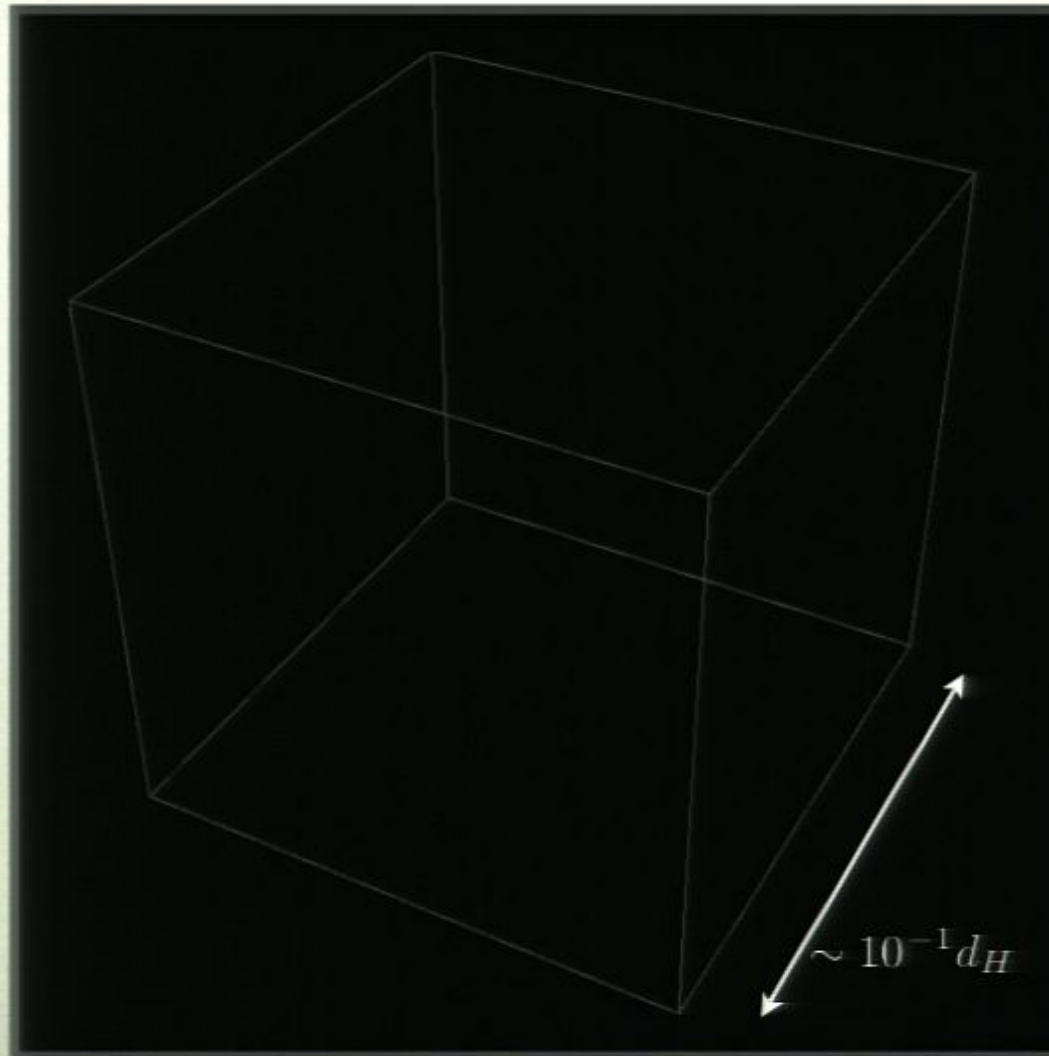
**BUT ...**



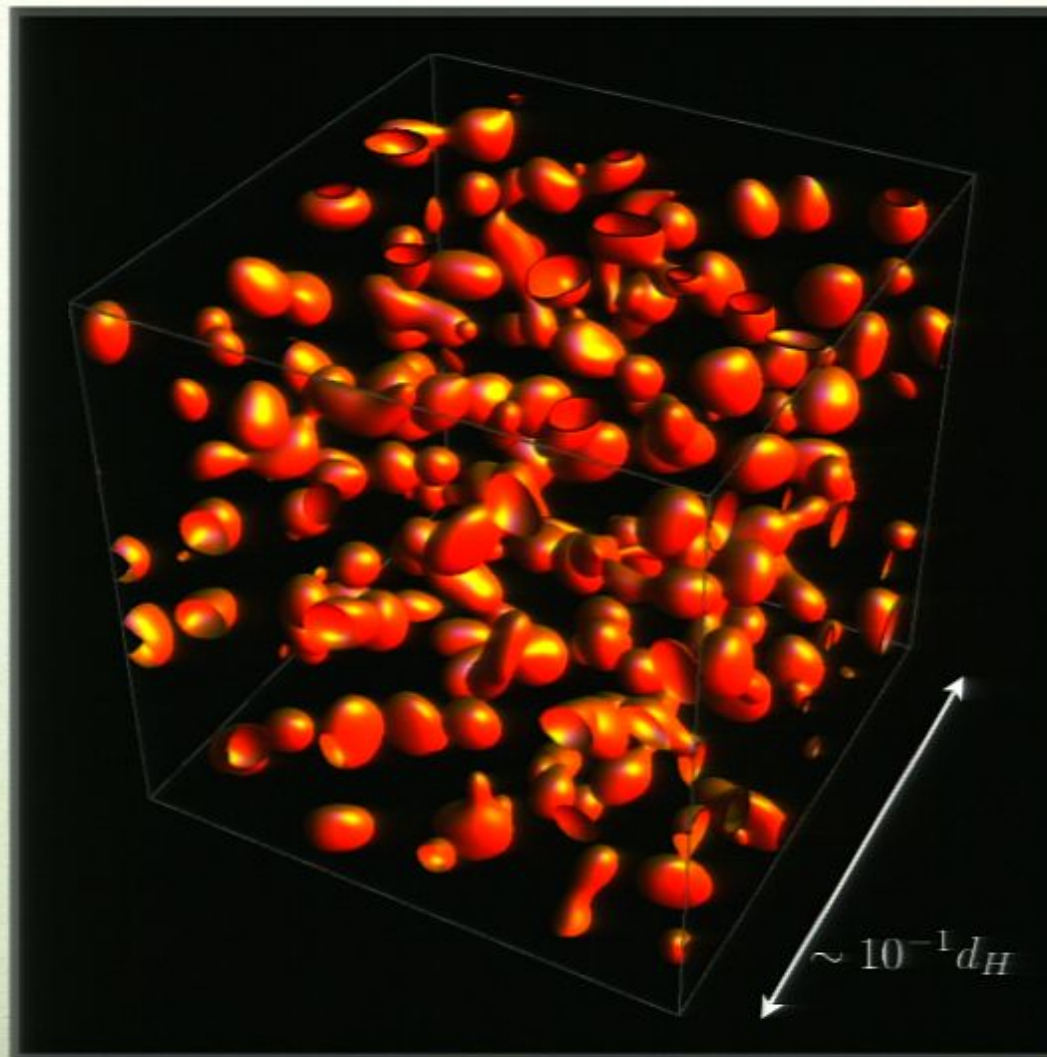
$$V \sim \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \frac{g^2}{6m^2}\varphi^6 + \dots + h\varphi\bar{\psi}\psi$$

# PSEUDO STABLE LUMPS

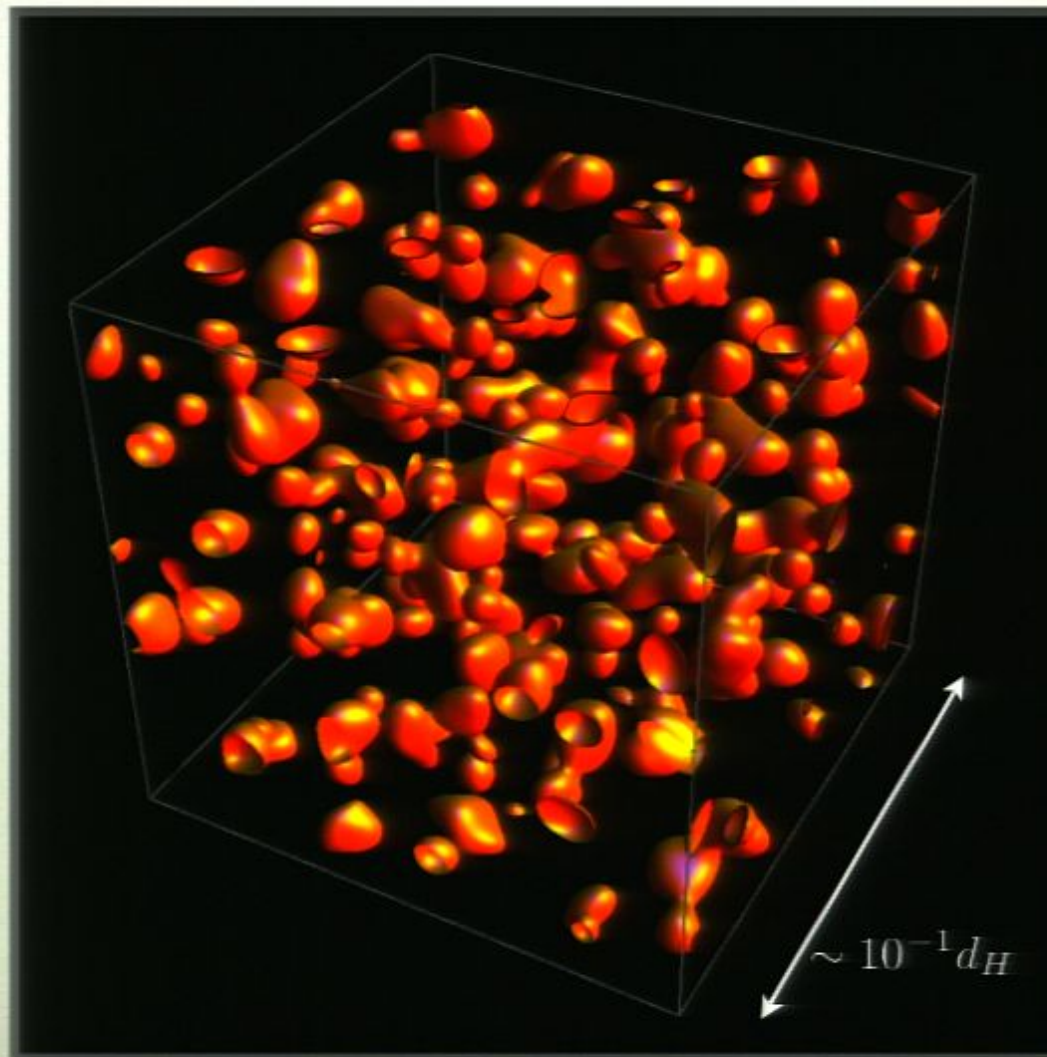
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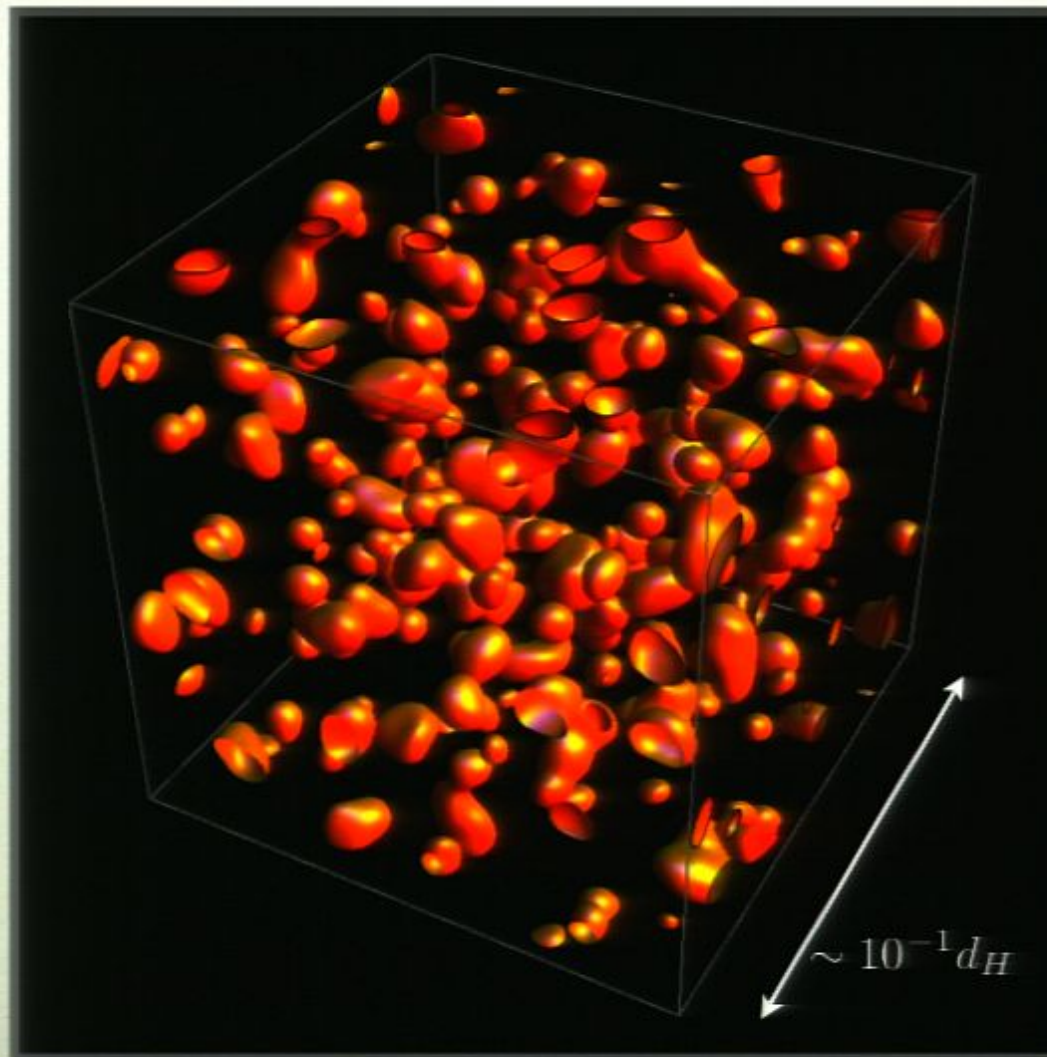
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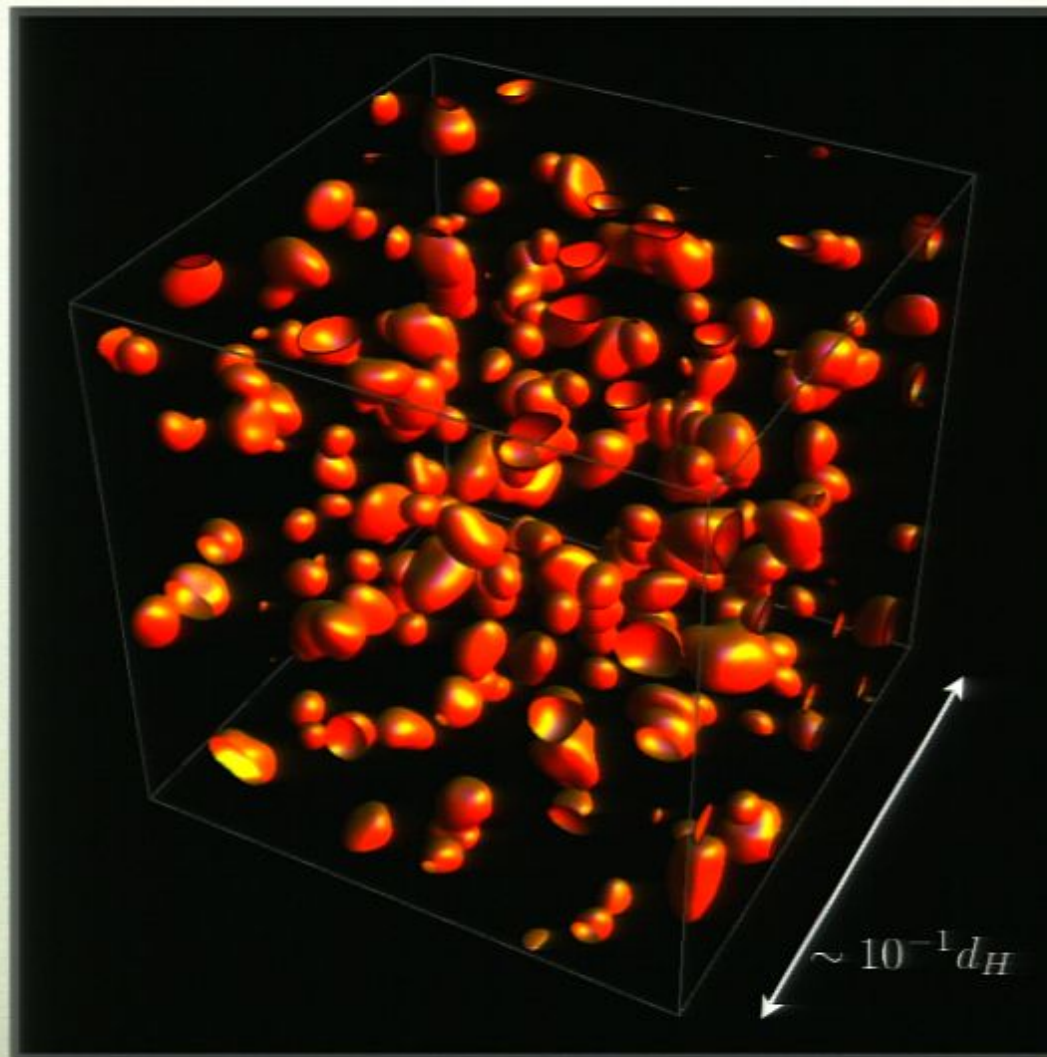
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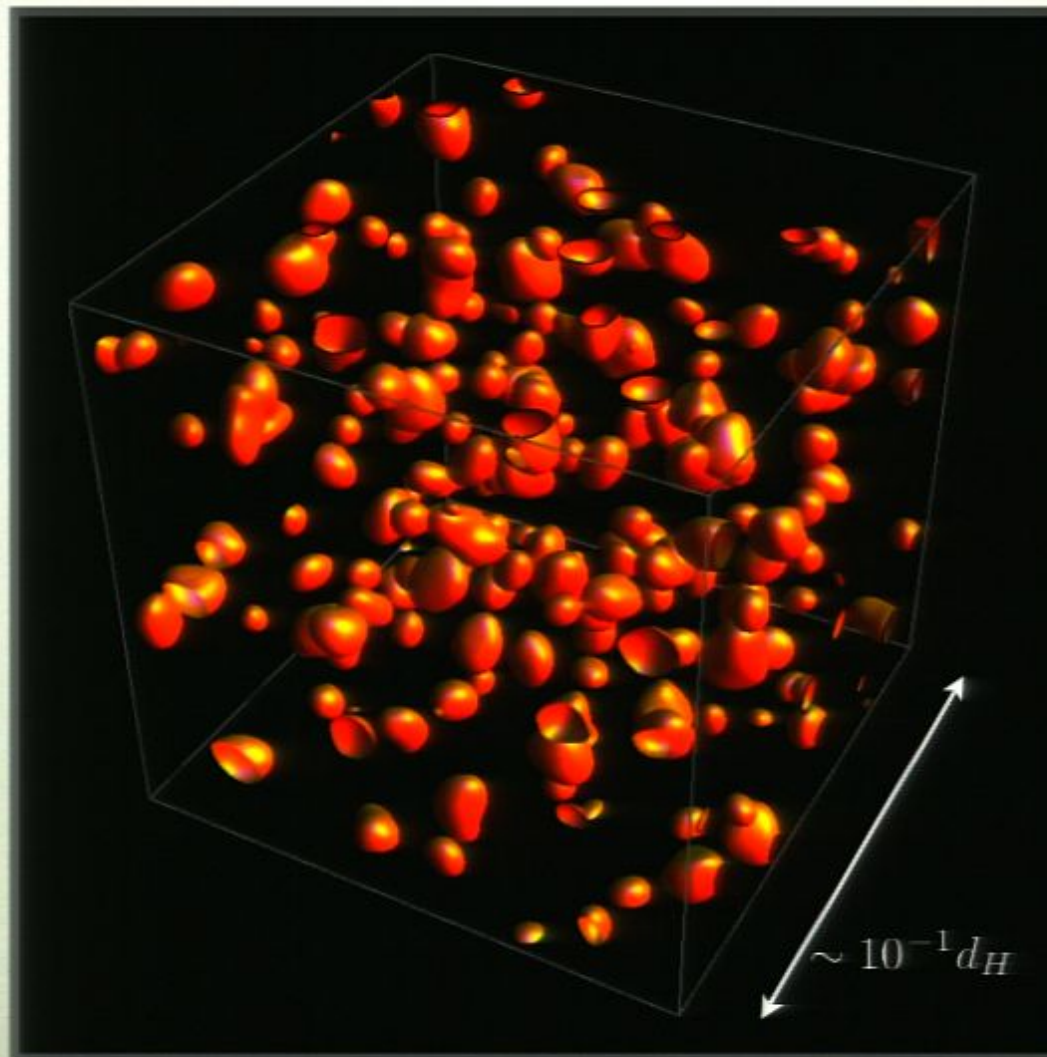
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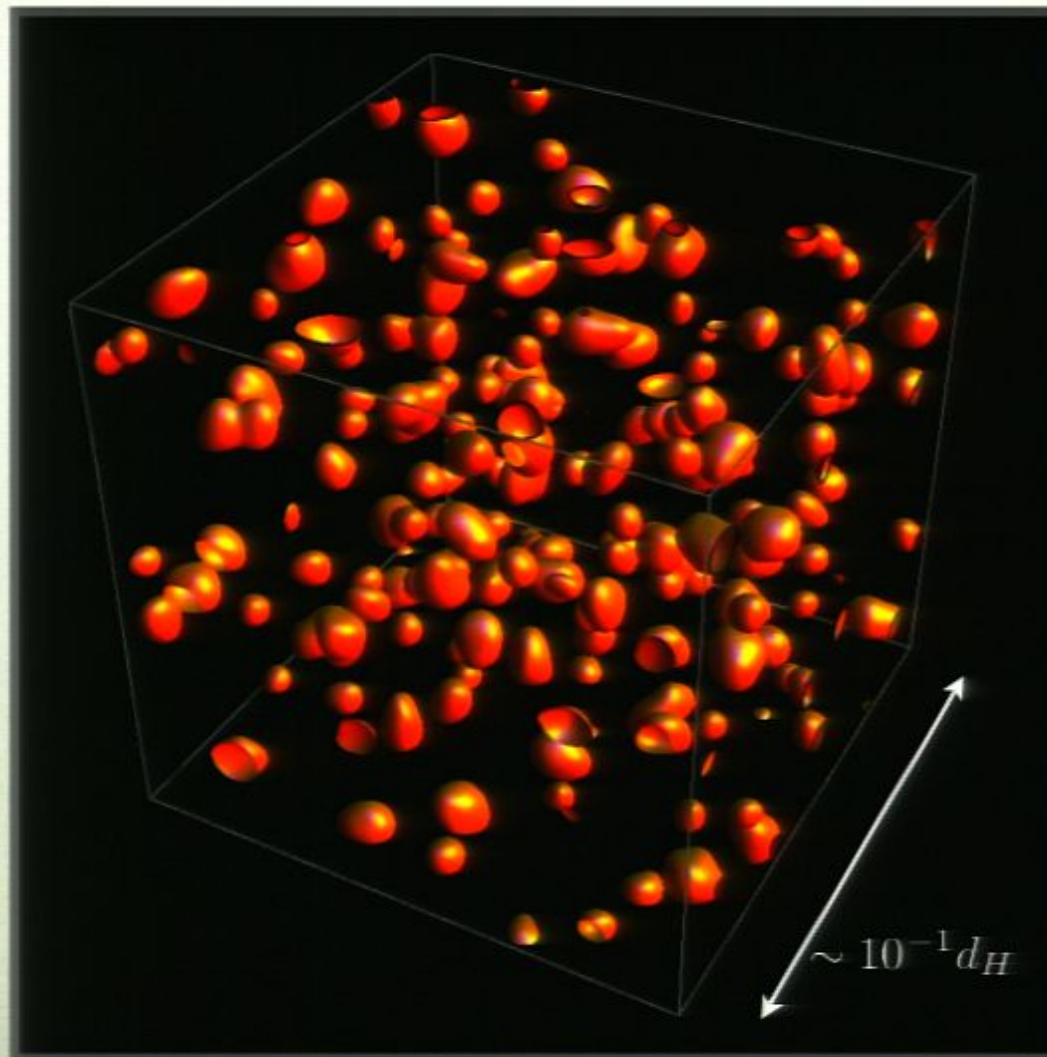
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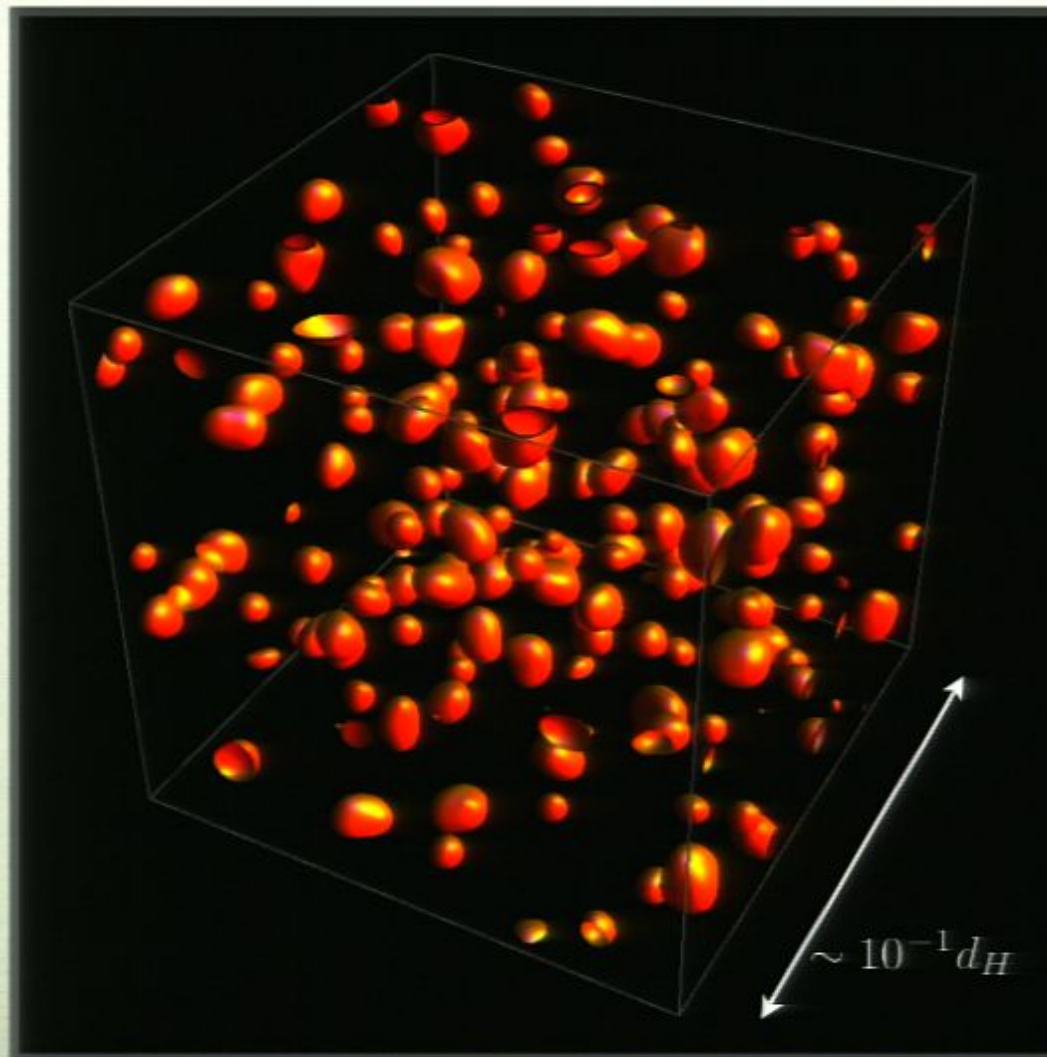


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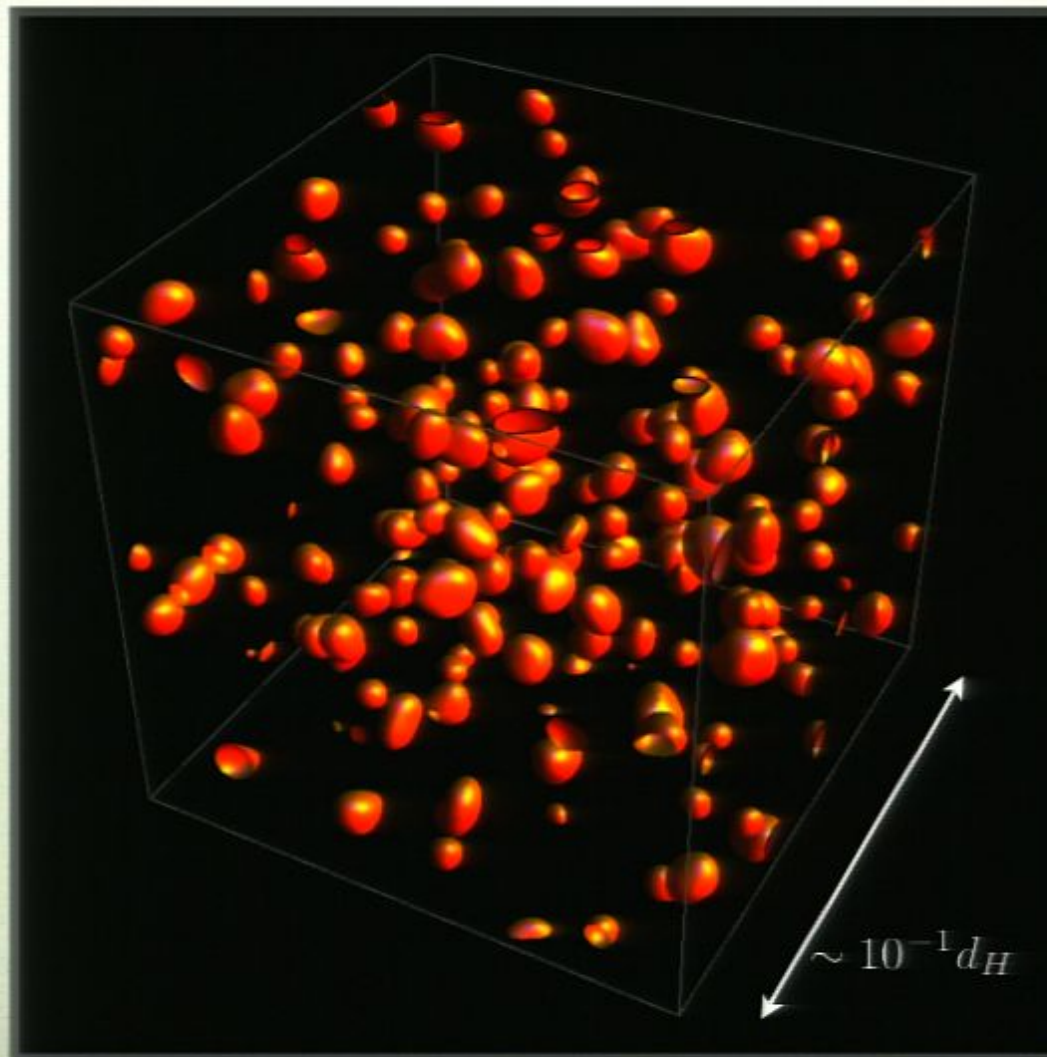




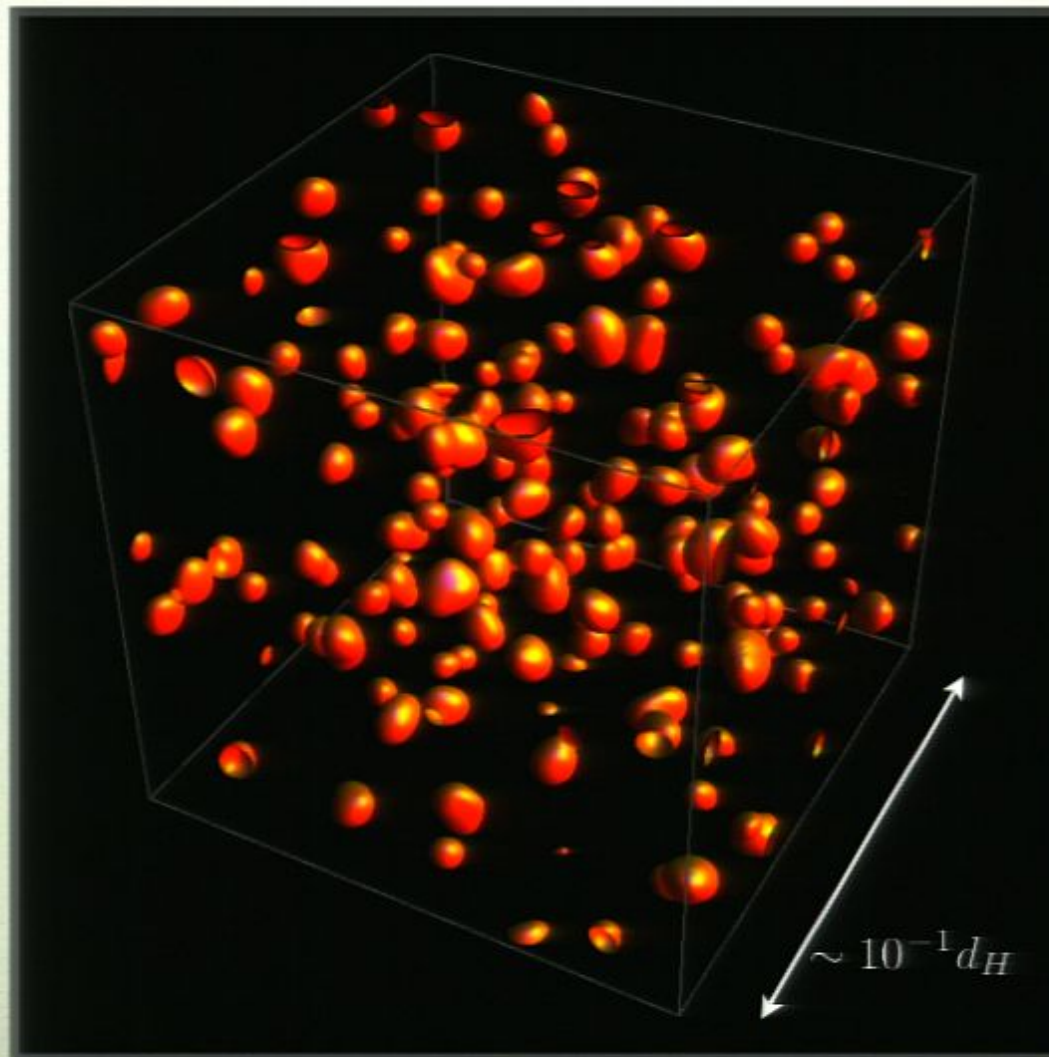
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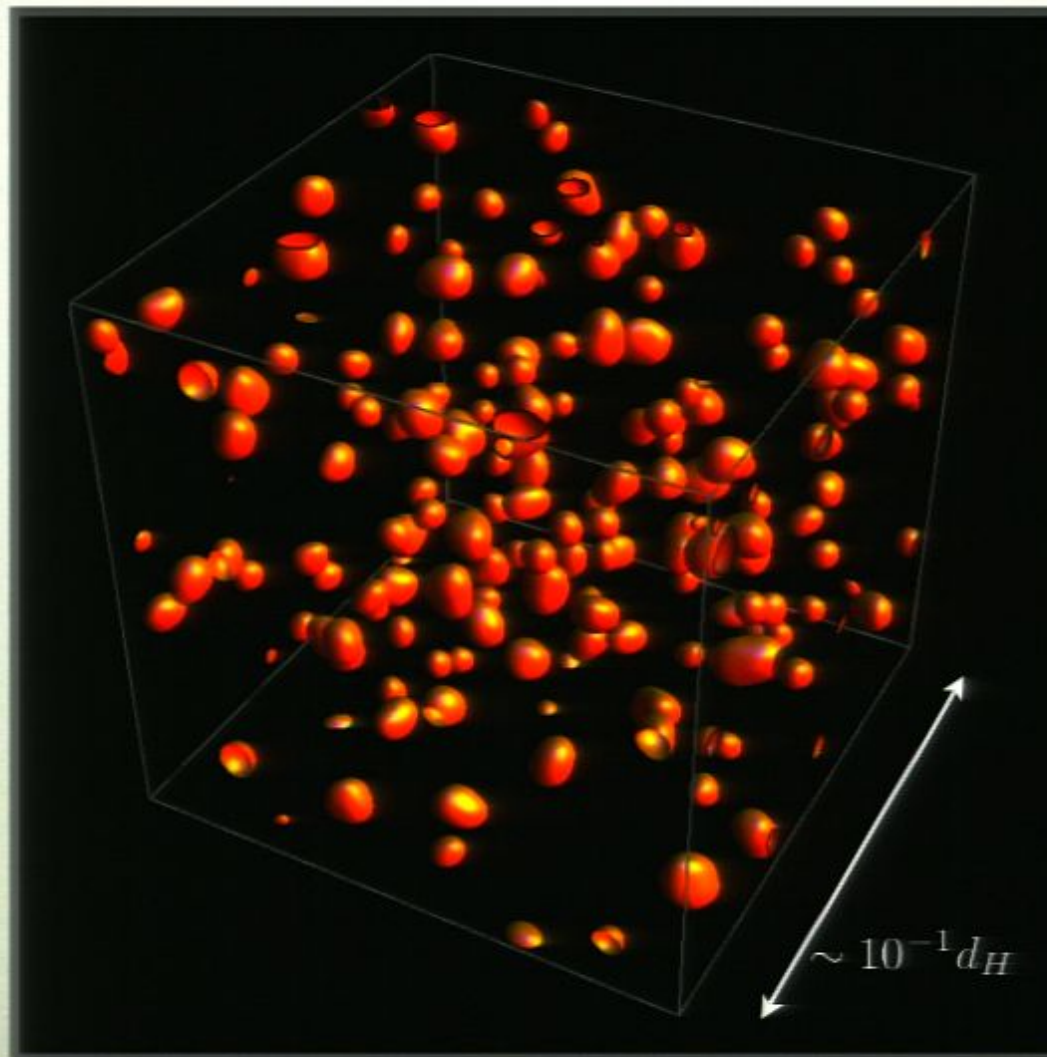
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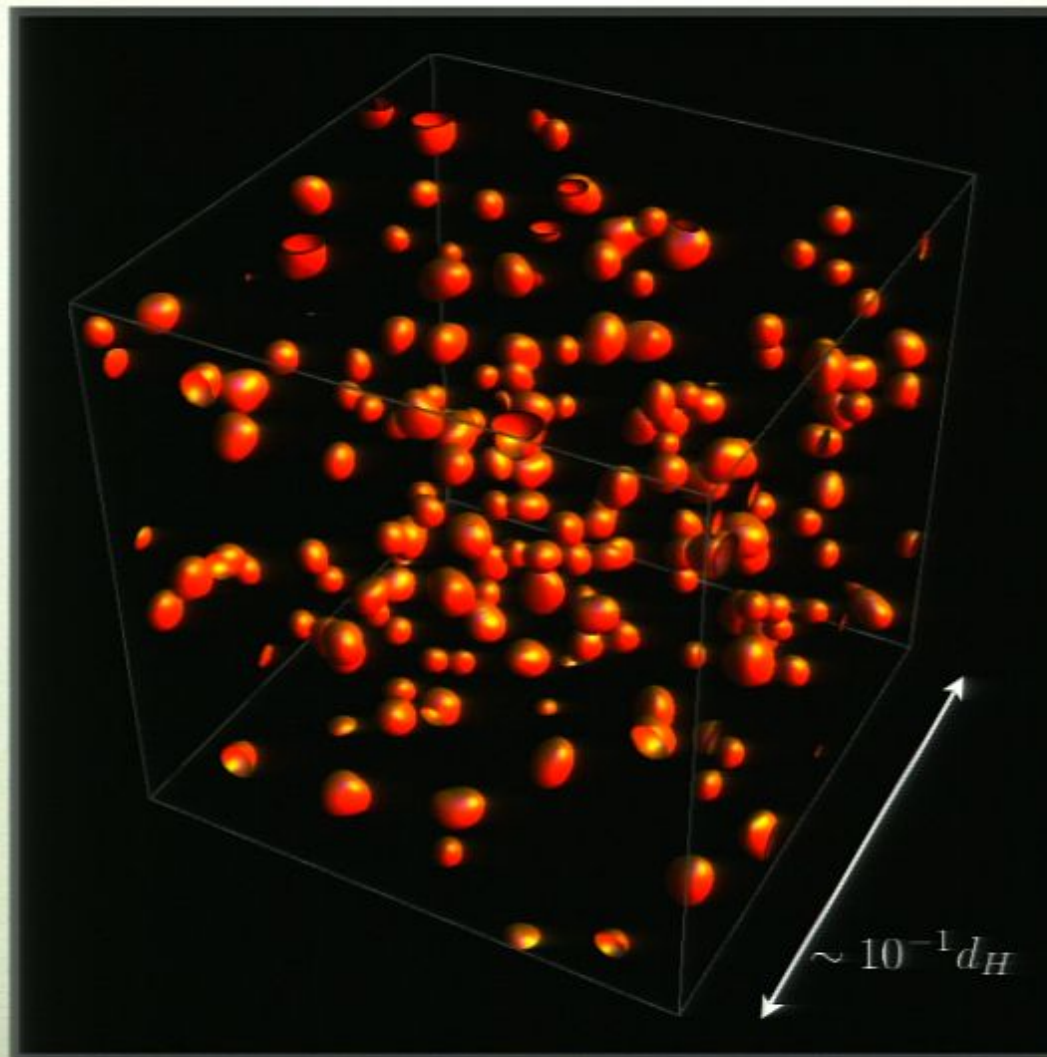
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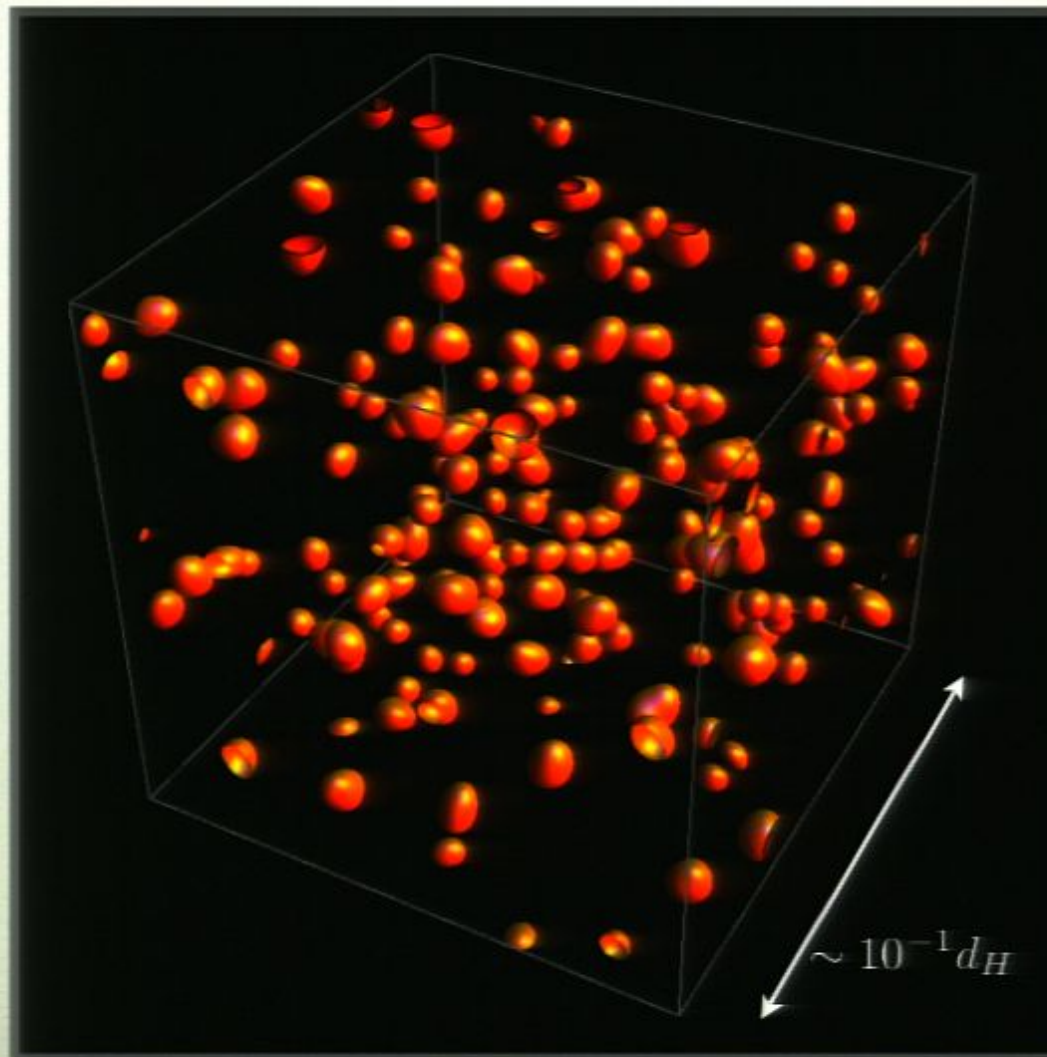
# PSEUDO STABLE LUMPS



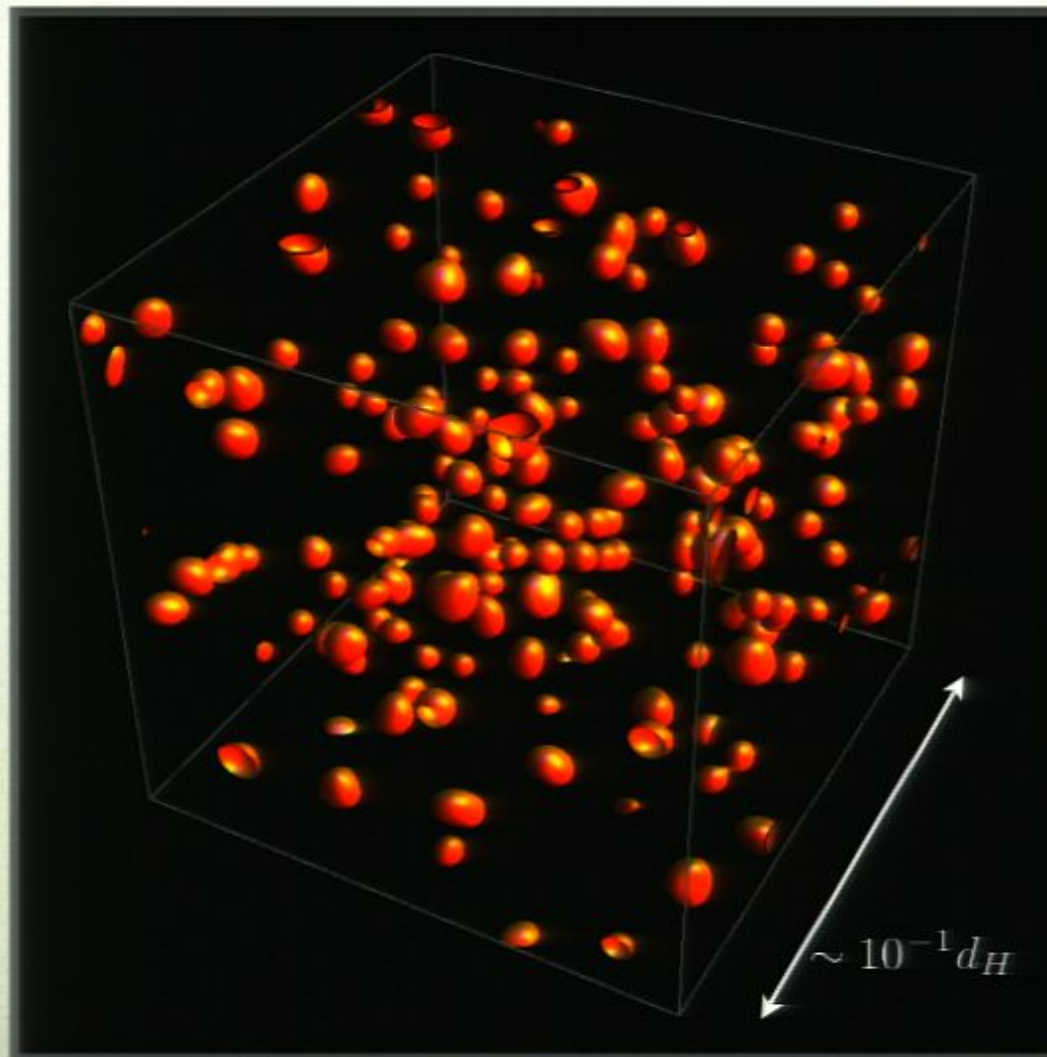
# PSEUDO STABLE LUMPS



# PSEUDO STABLE LUMPS

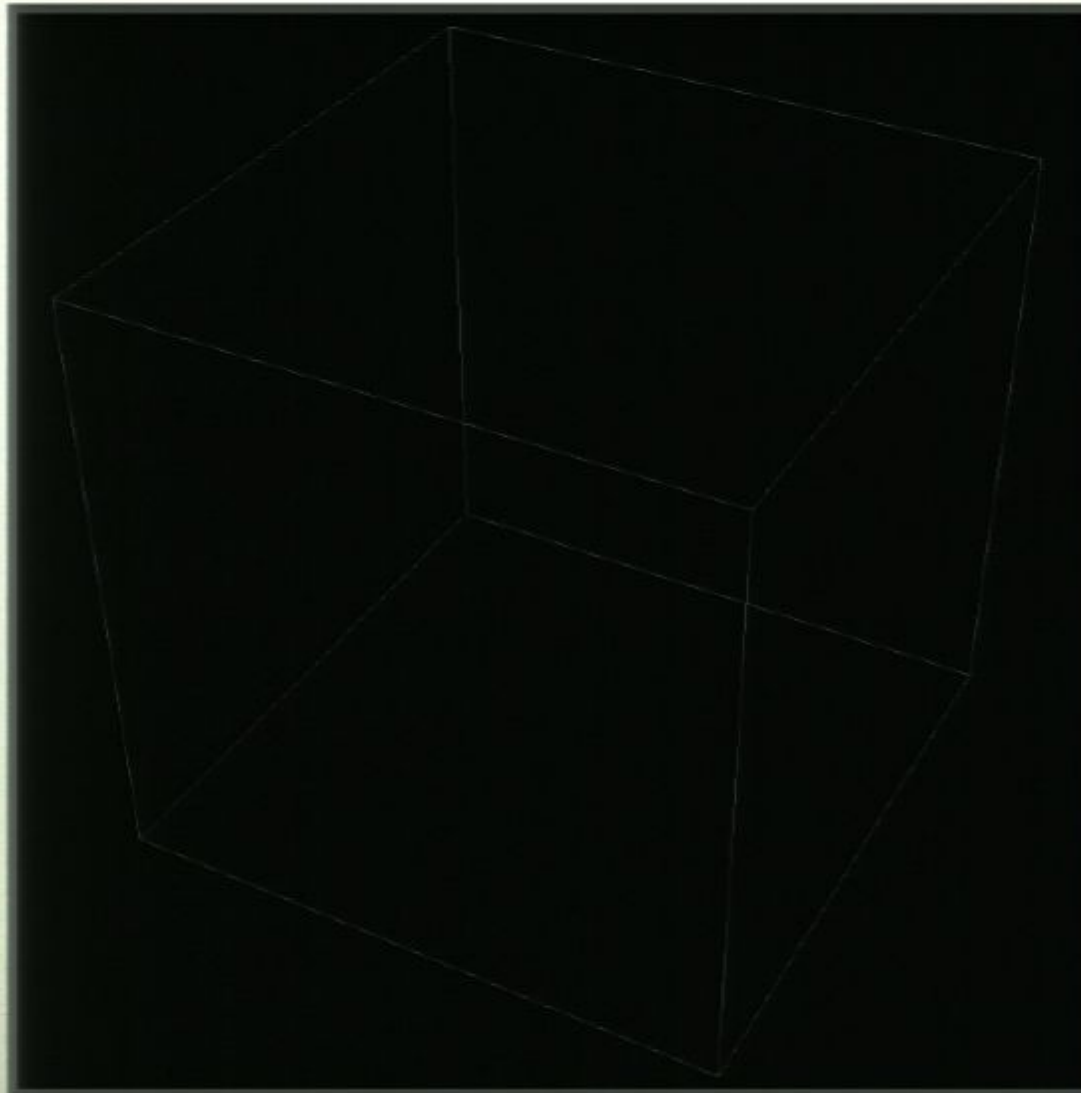


# PSEUDO STABLE LUMPS



# CLOSER LOOK

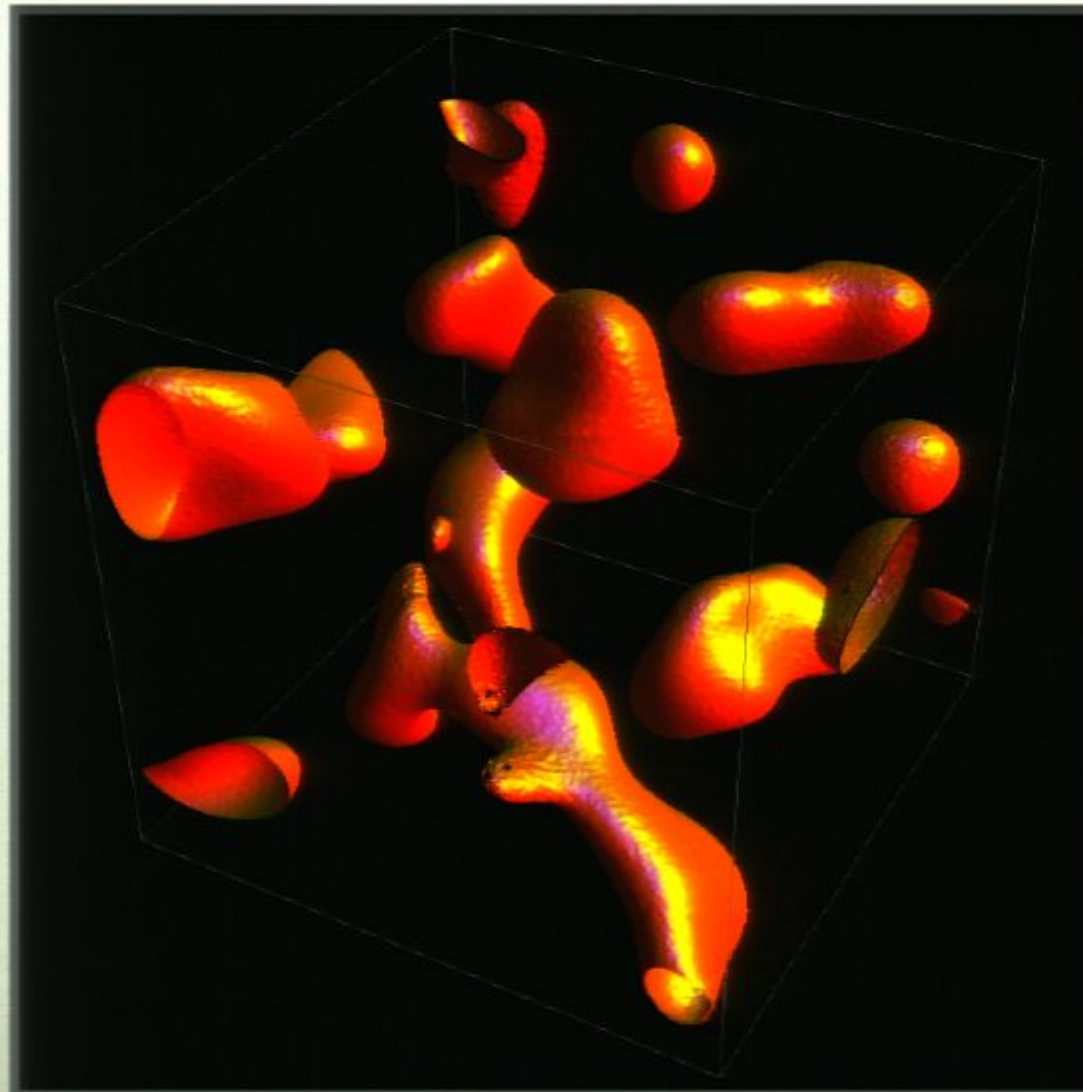
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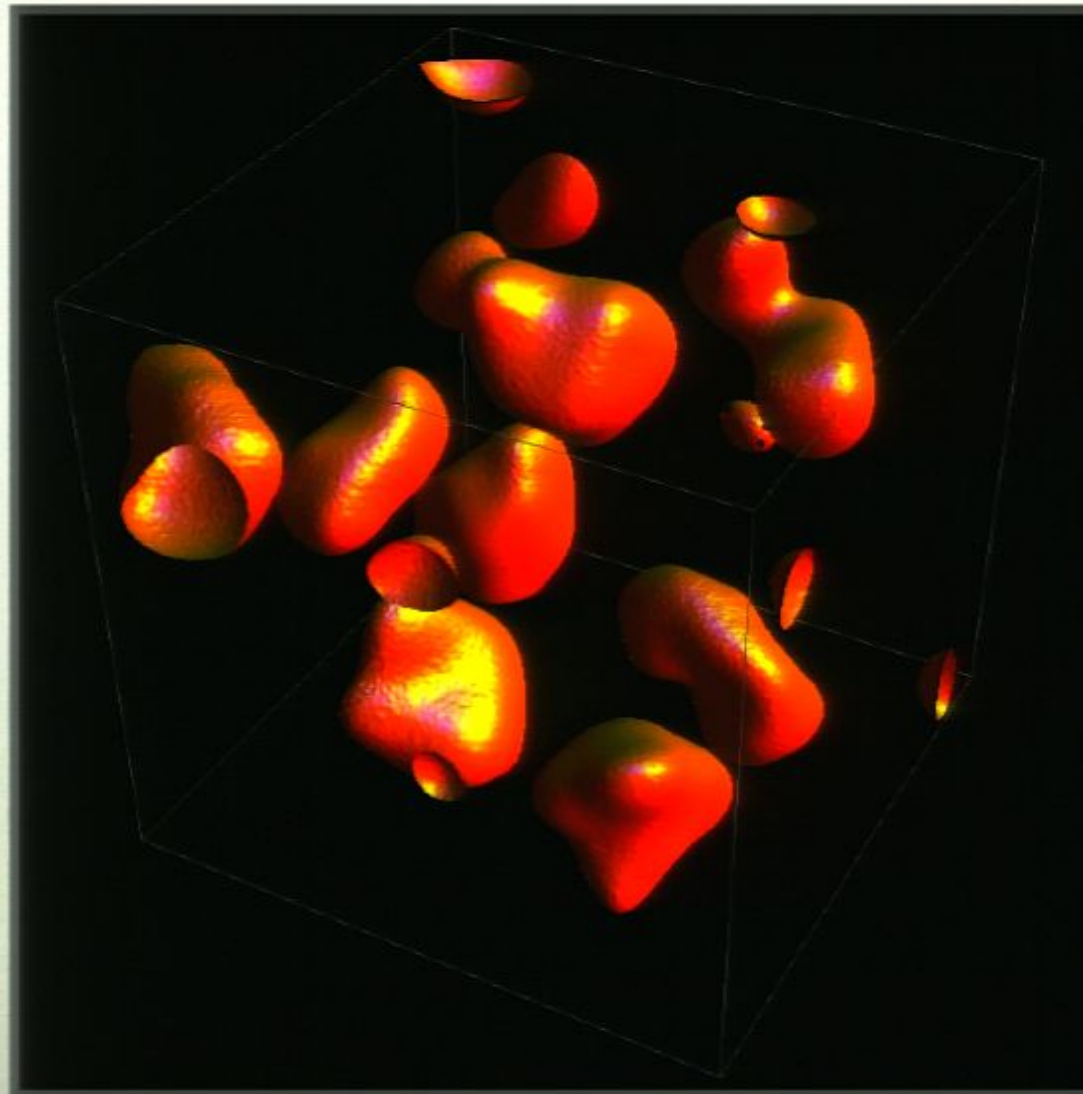
# CLOSER LOOK

---



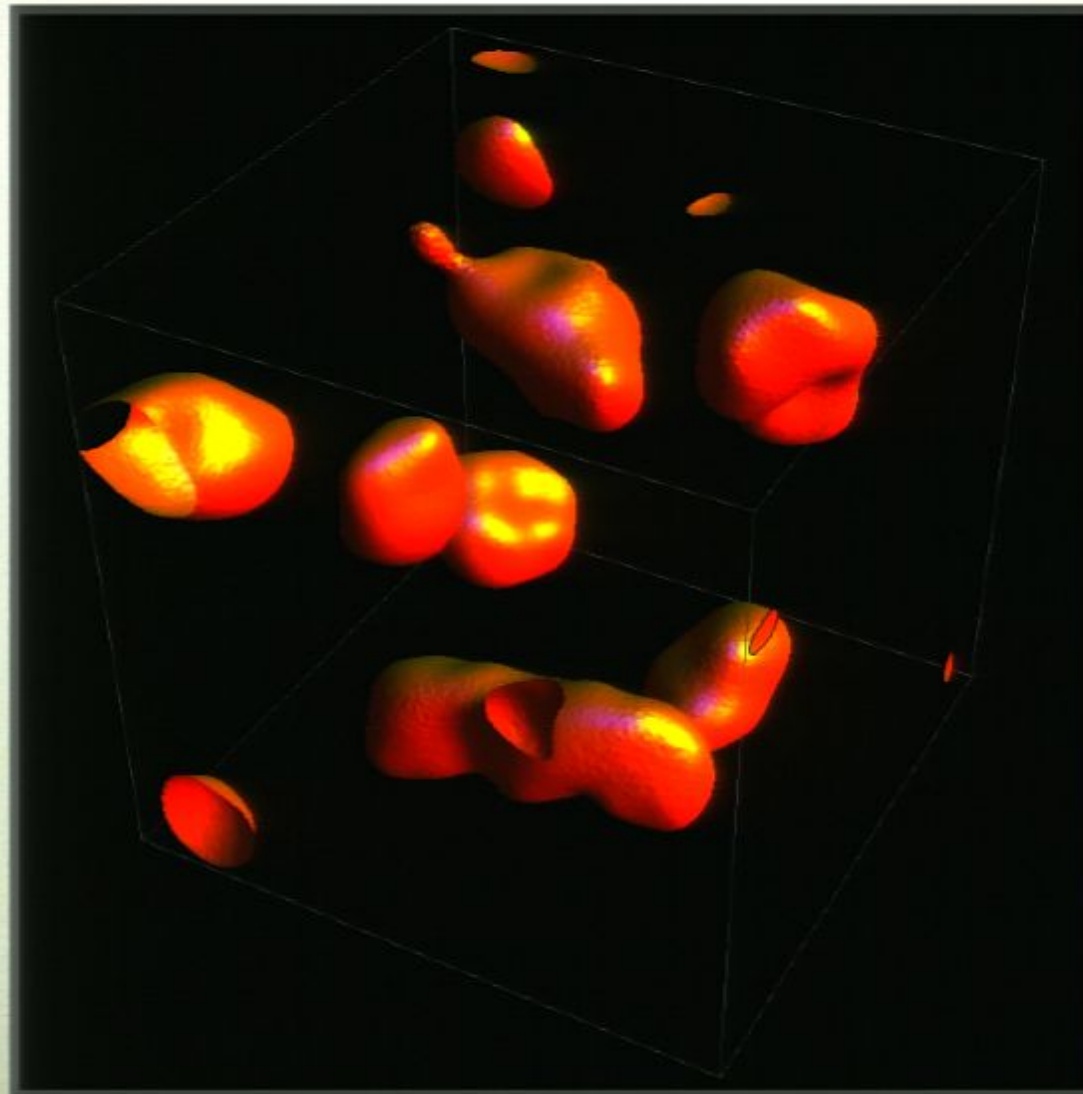
# CLOSER LOOK

---



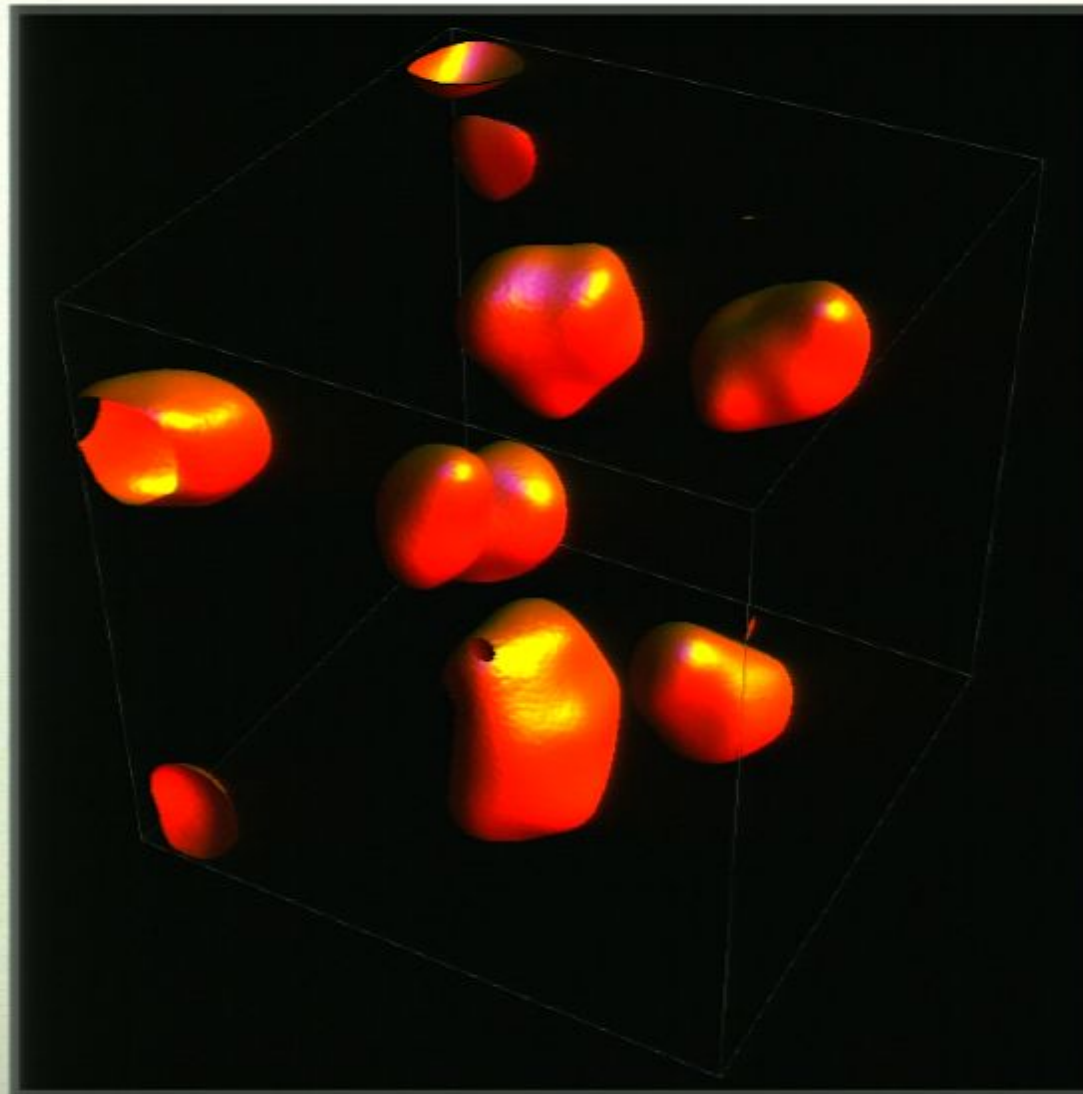
# CLOSER LOOK

---



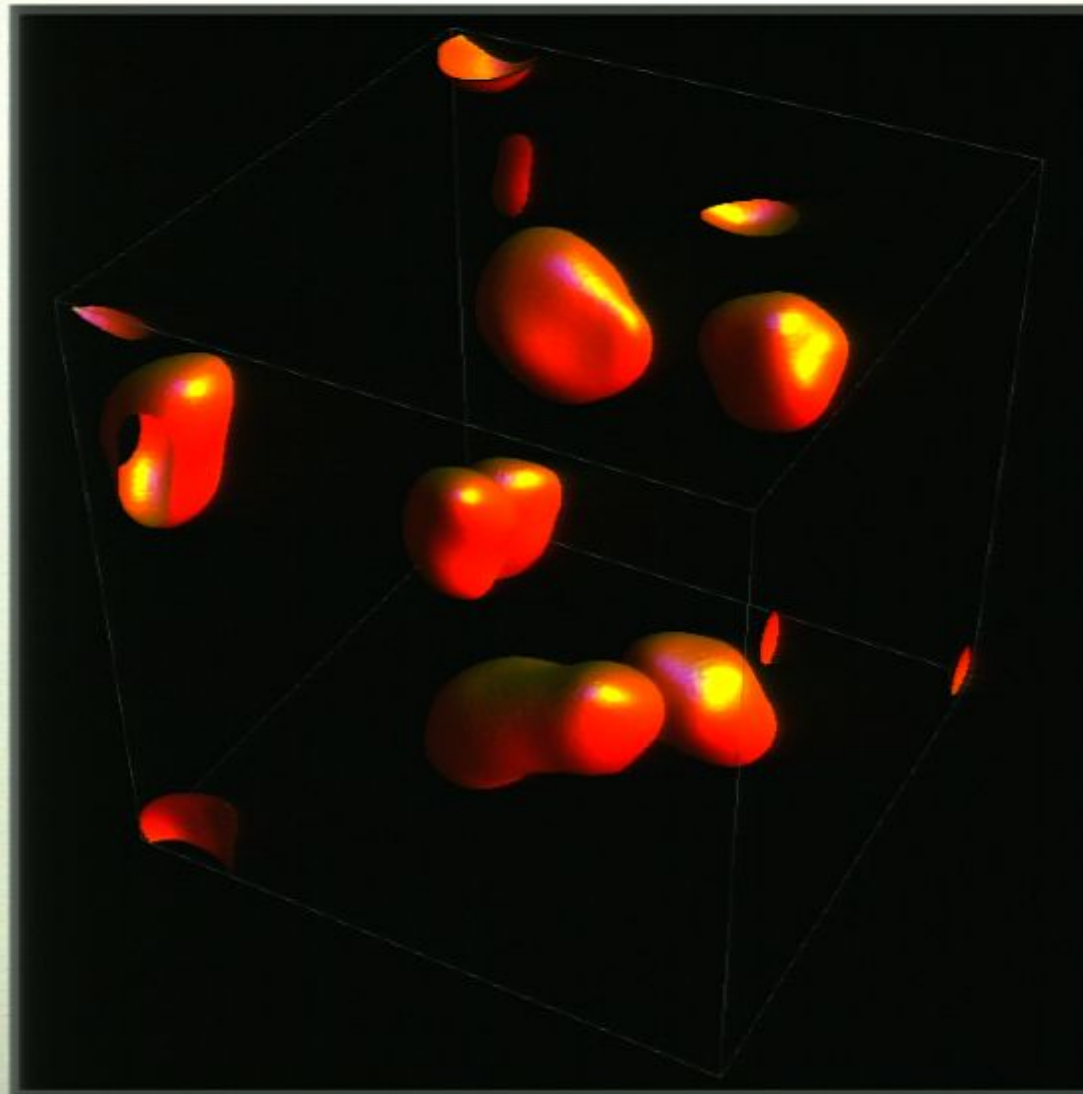
# CLOSER LOOK

---



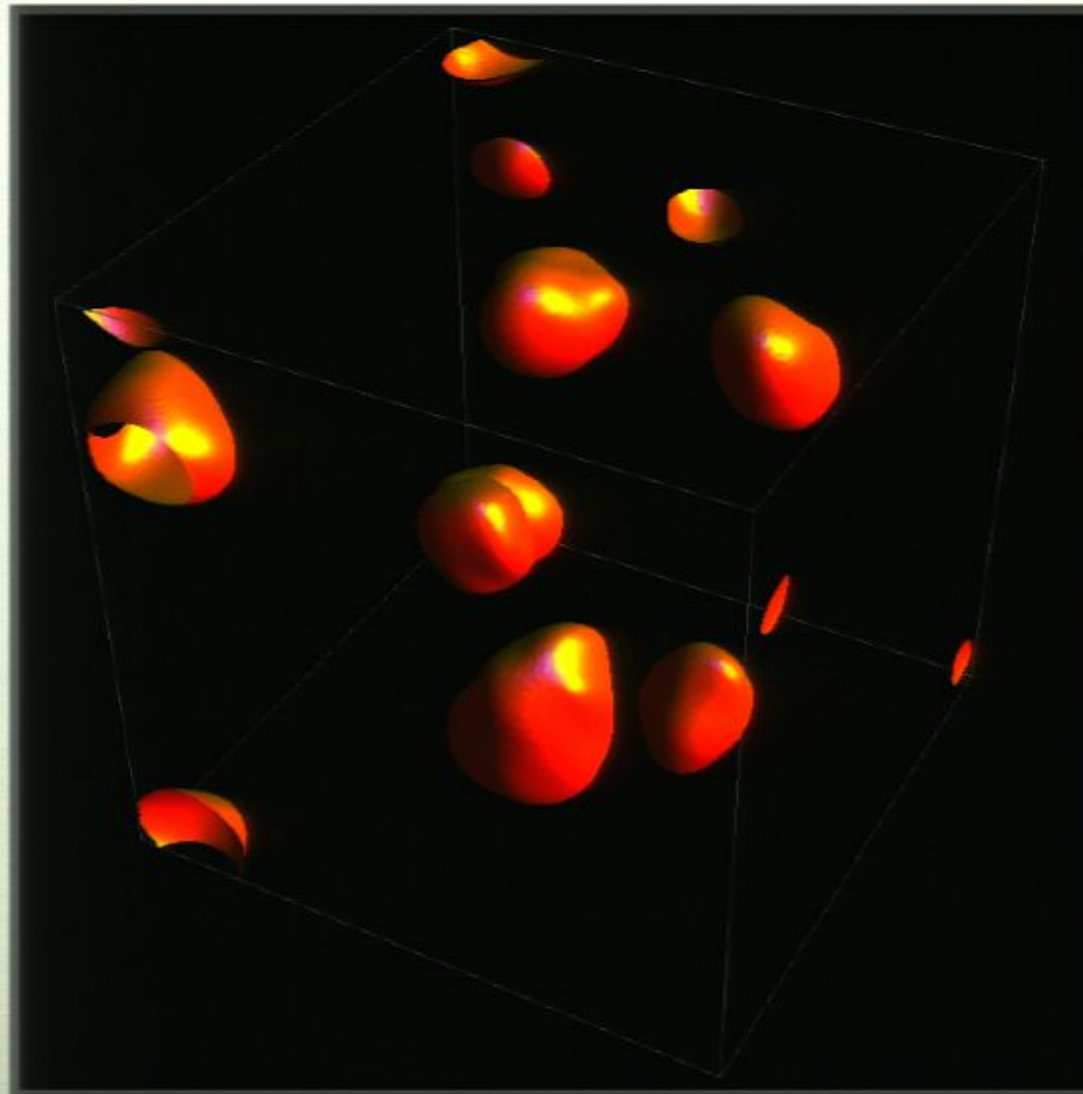
# CLOSER LOOK

---



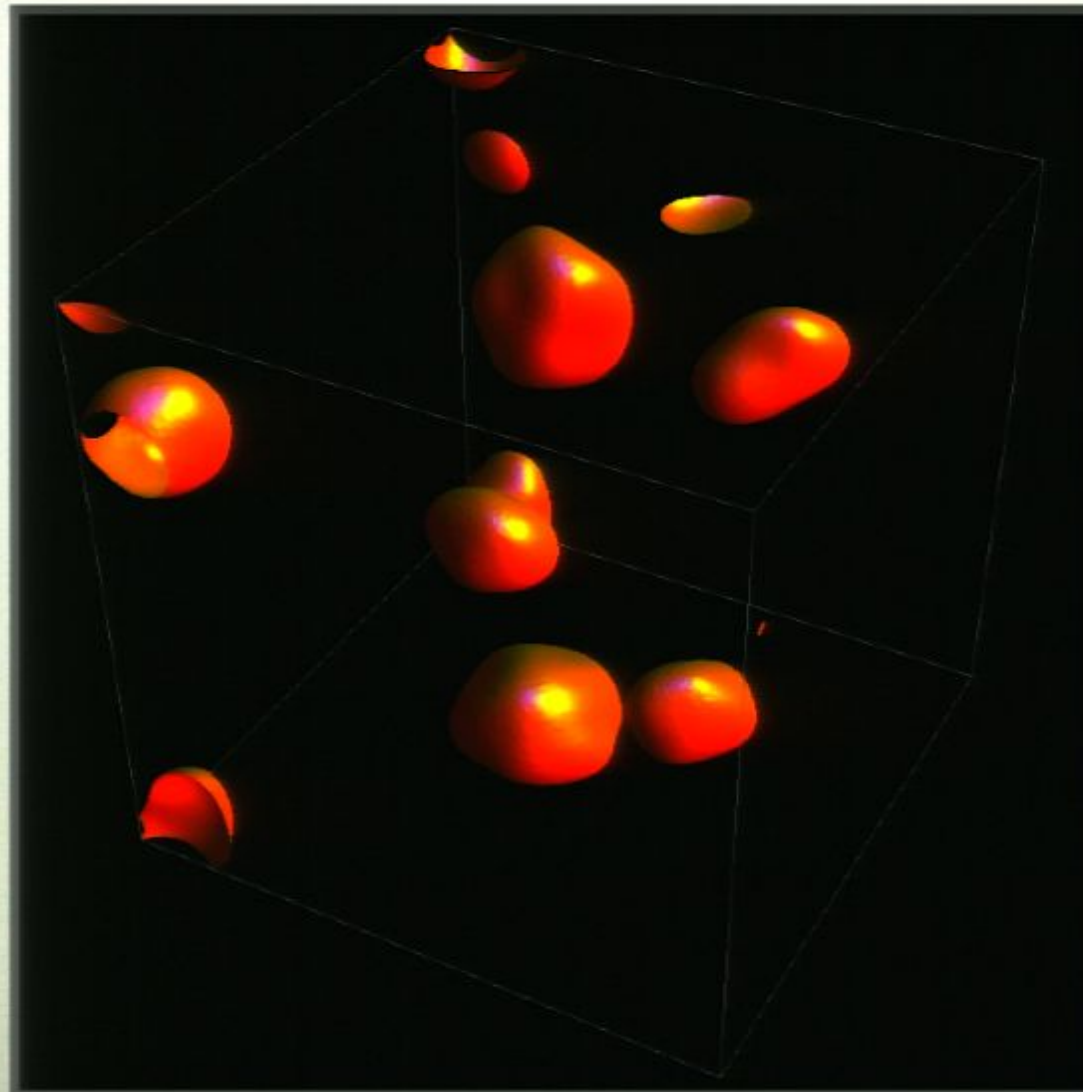
# CLOSER LOOK

---



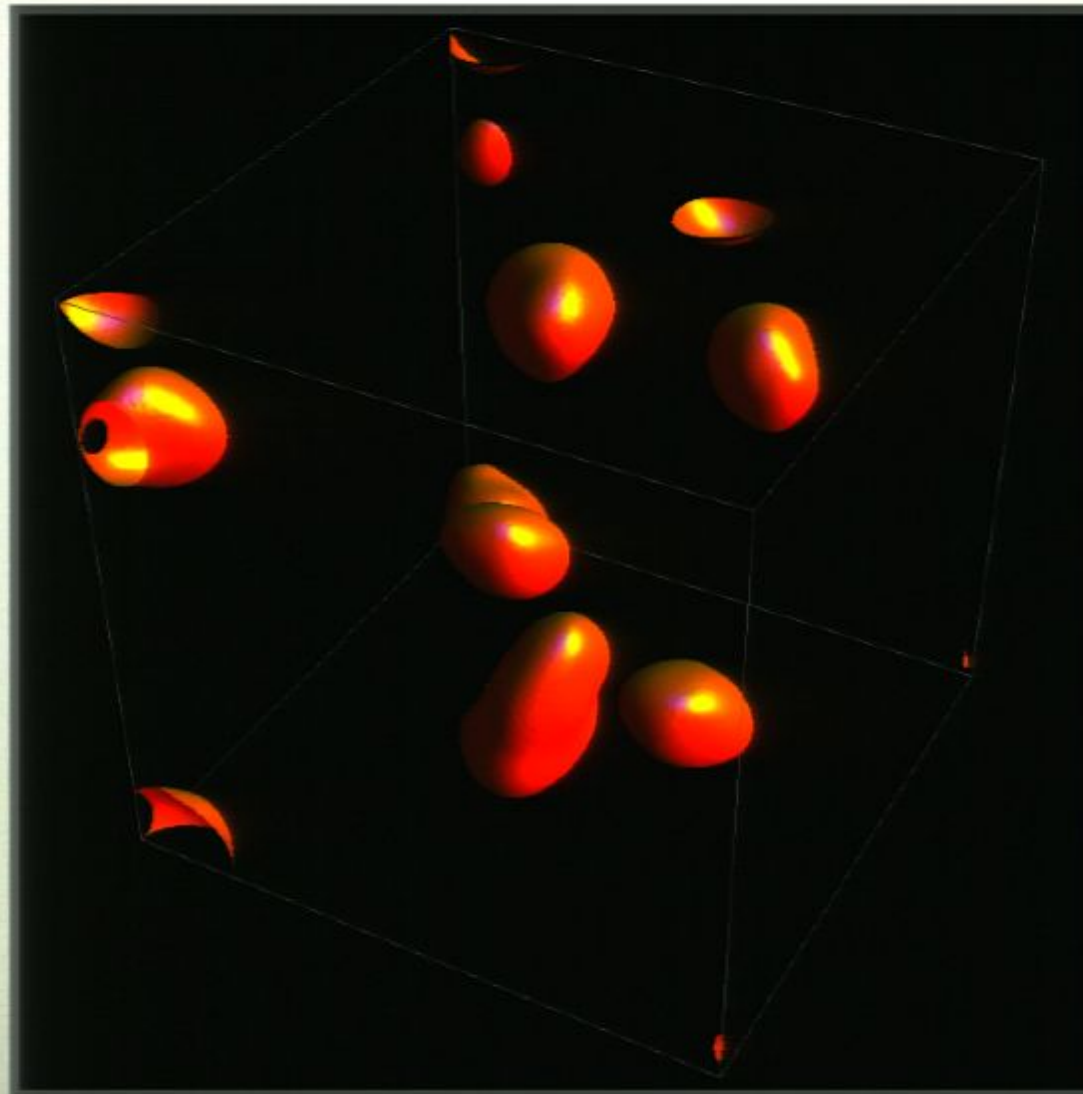
# CLOSER LOOK

---



# CLOSER LOOK

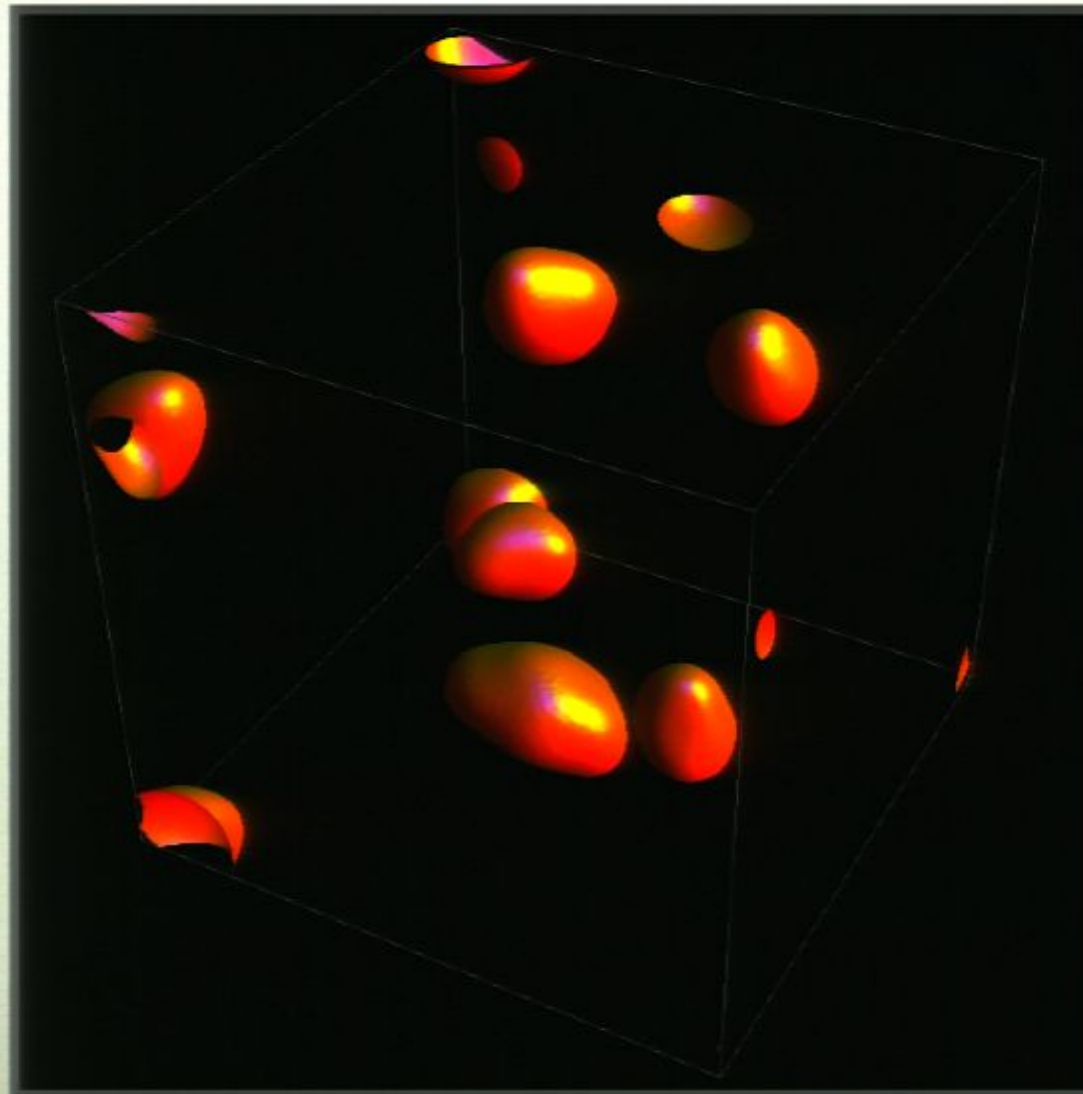
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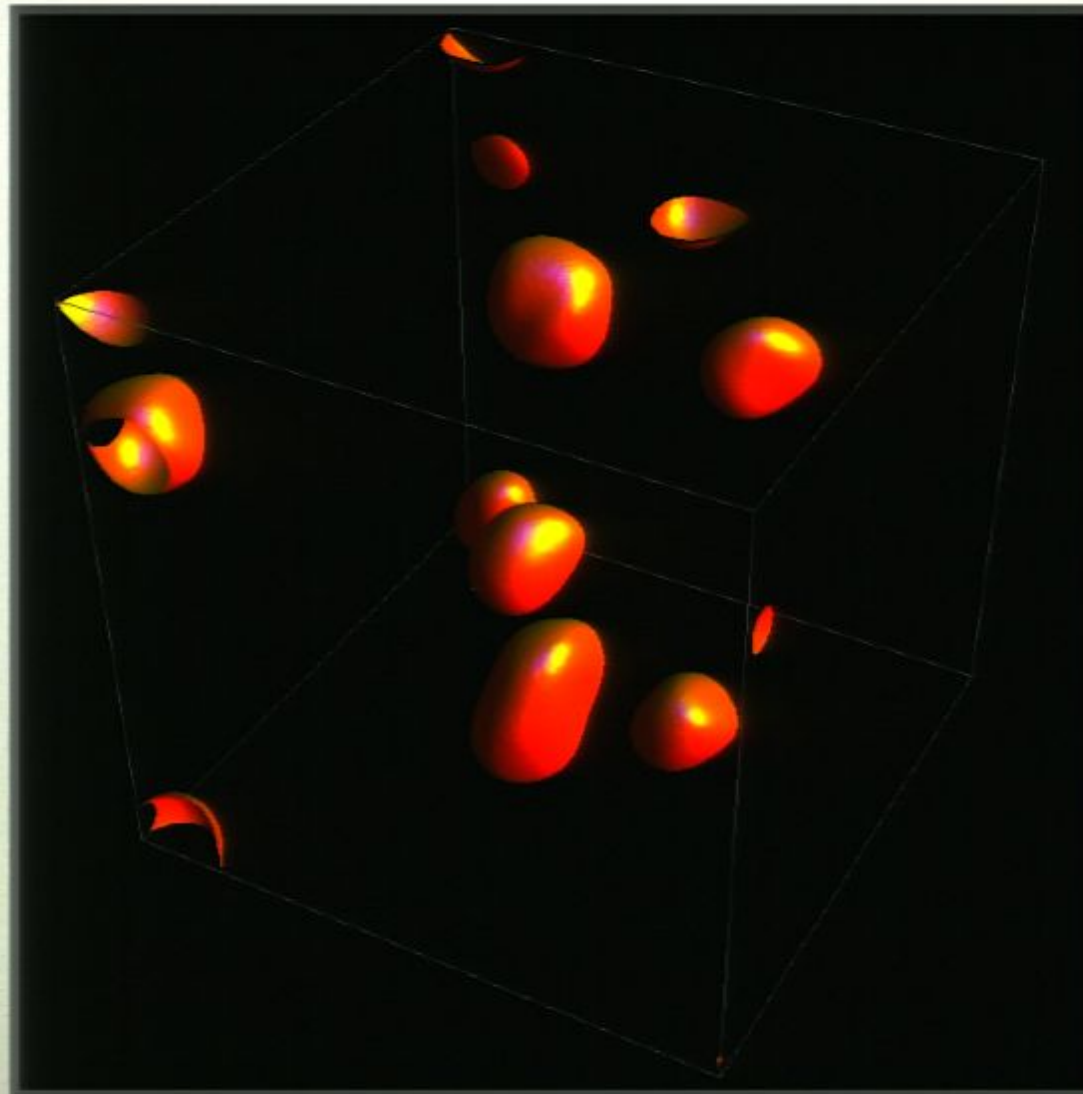
# CLOSER LOOK

---



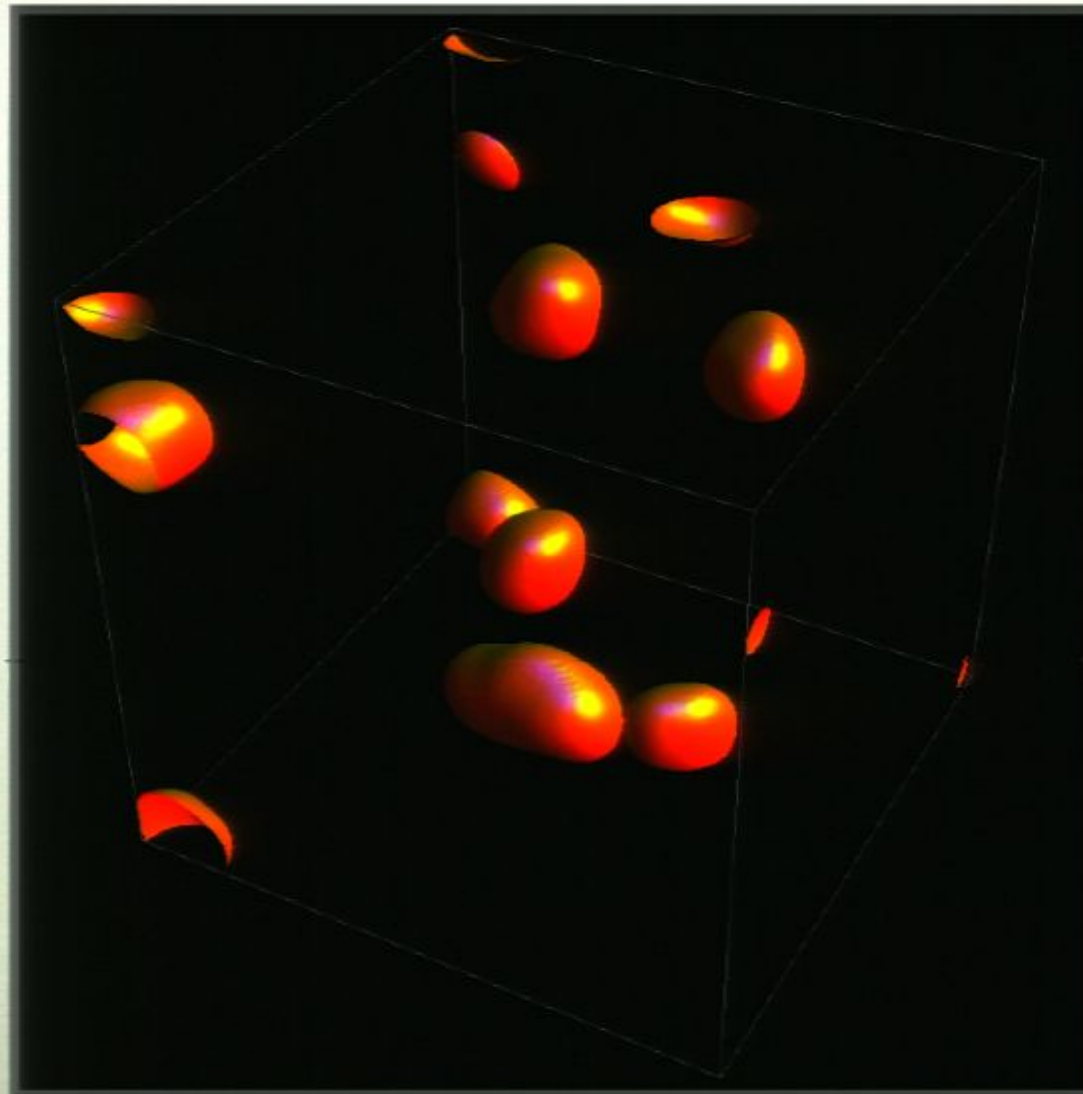
# CLOSER LOOK

---



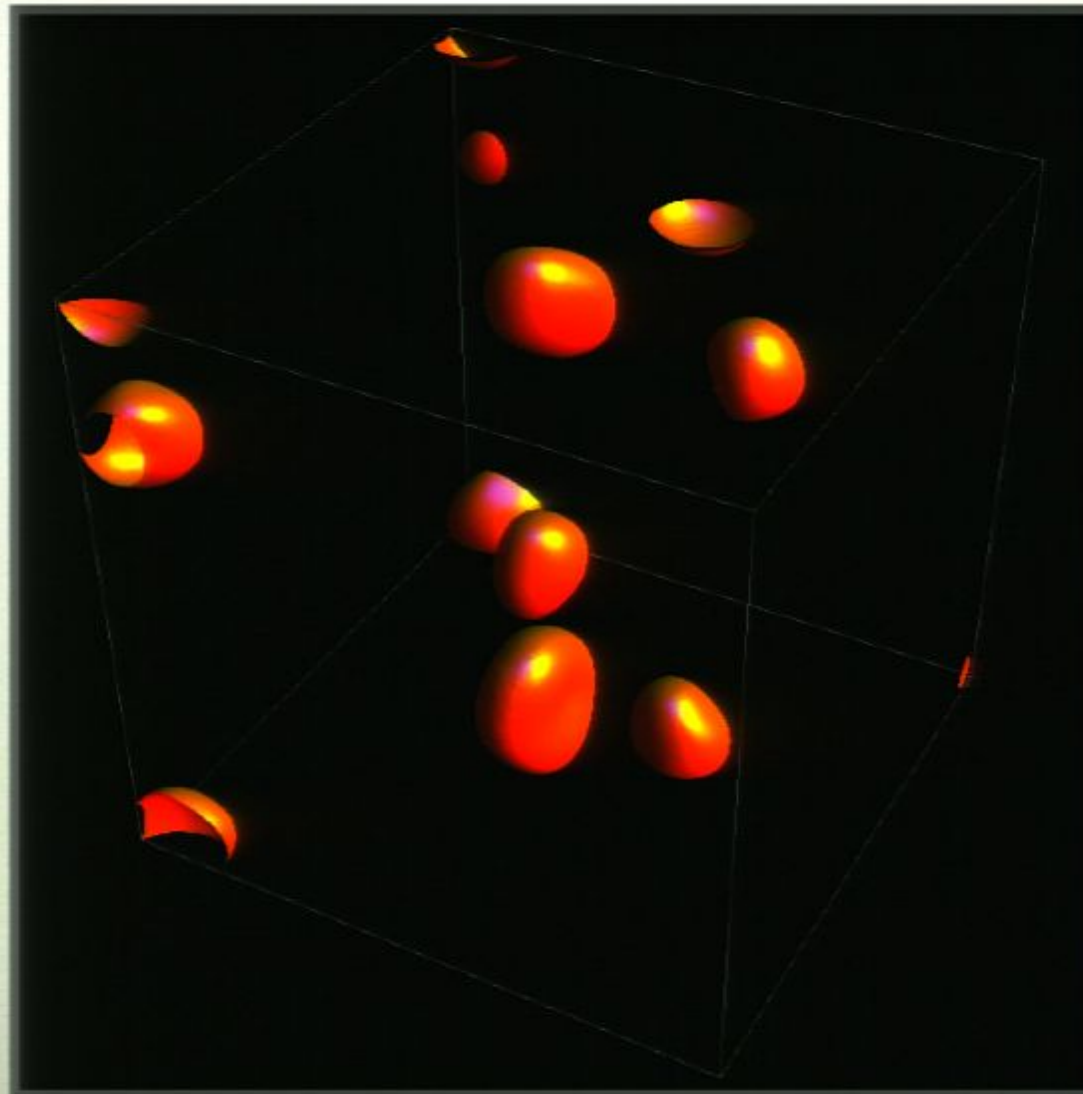
# CLOSER LOOK

---



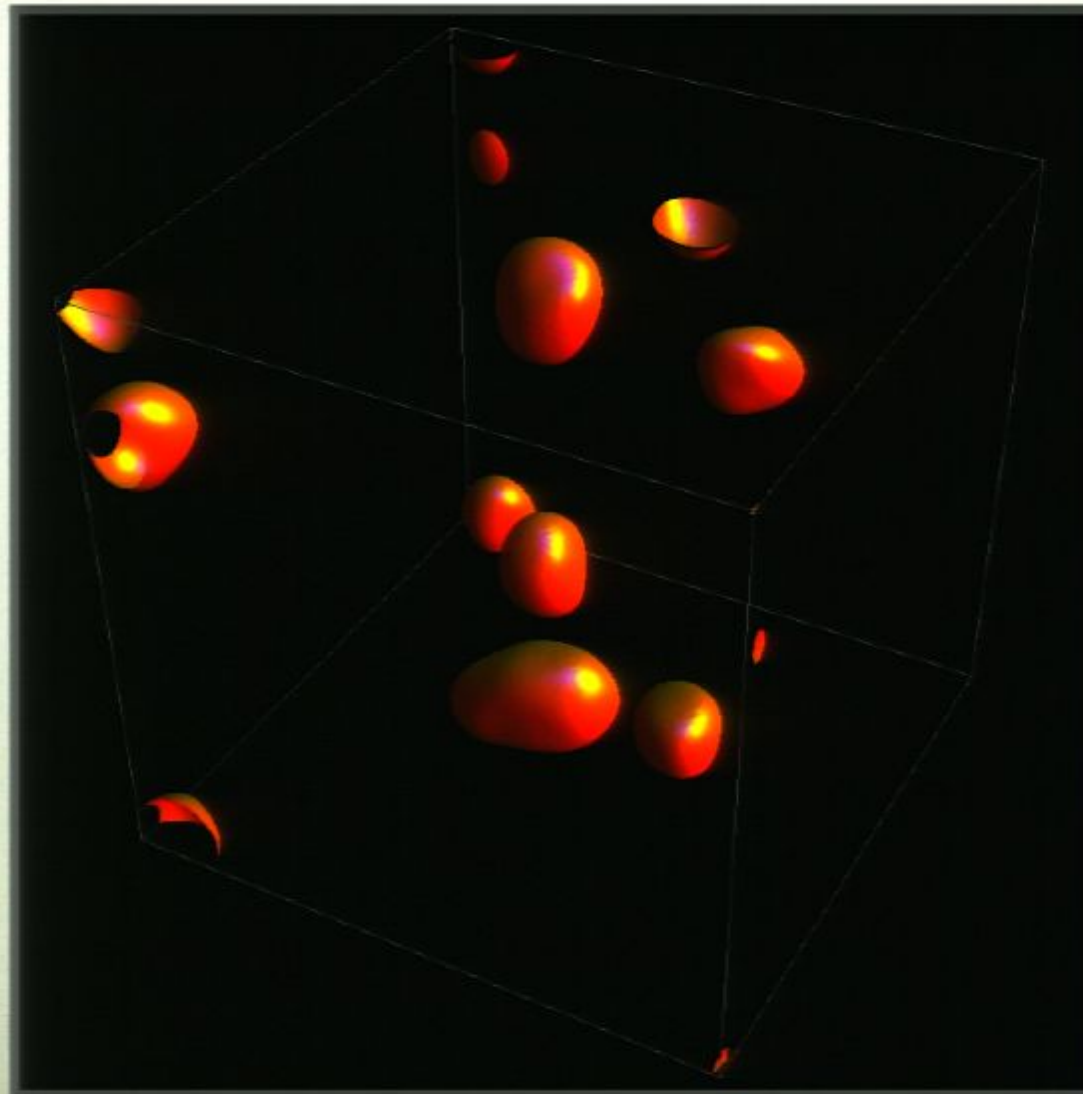
# CLOSER LOOK

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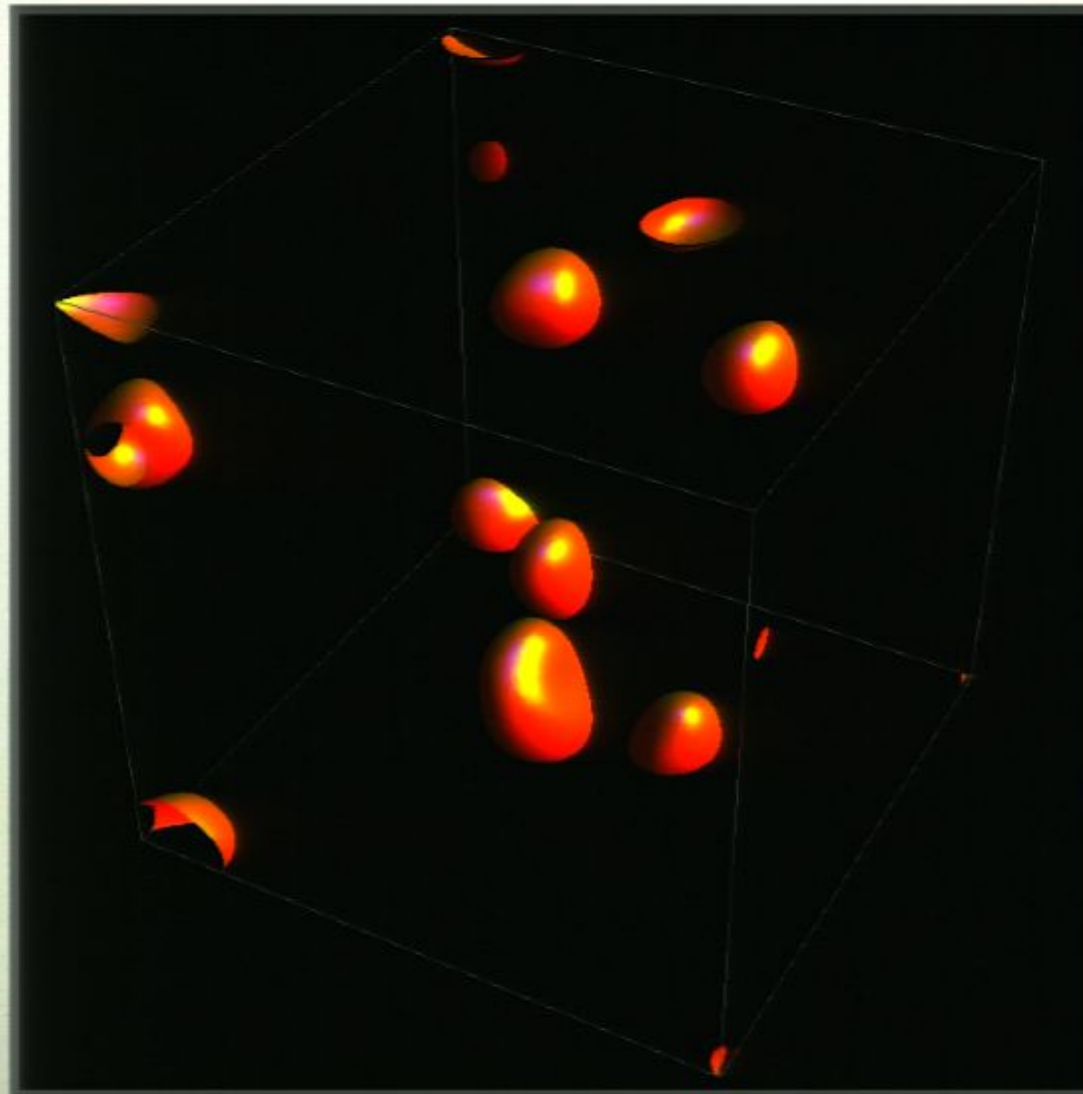
# CLOSER LOOK

---



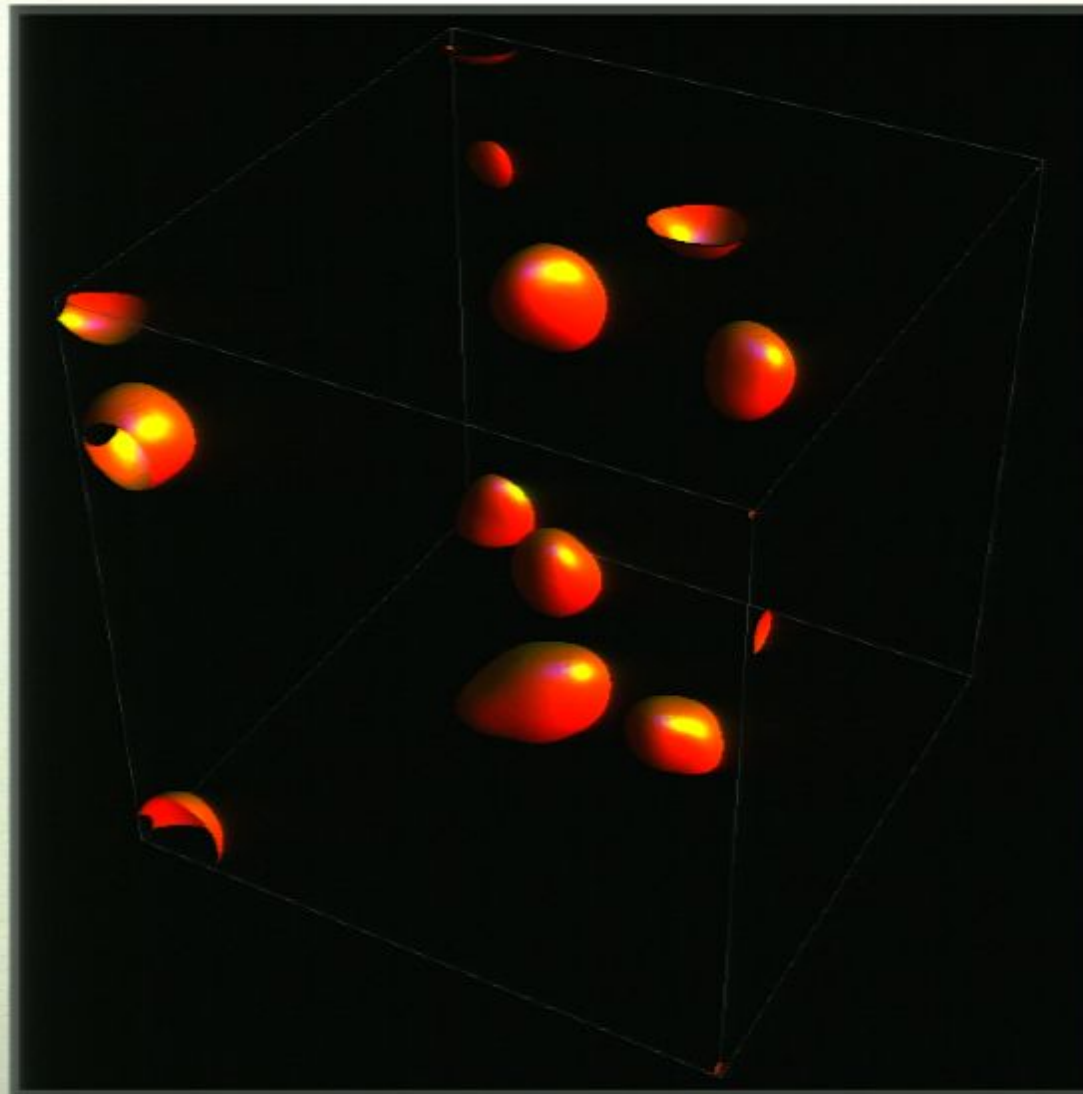
# CLOSER LOOK

---



# CLOSER LOOK

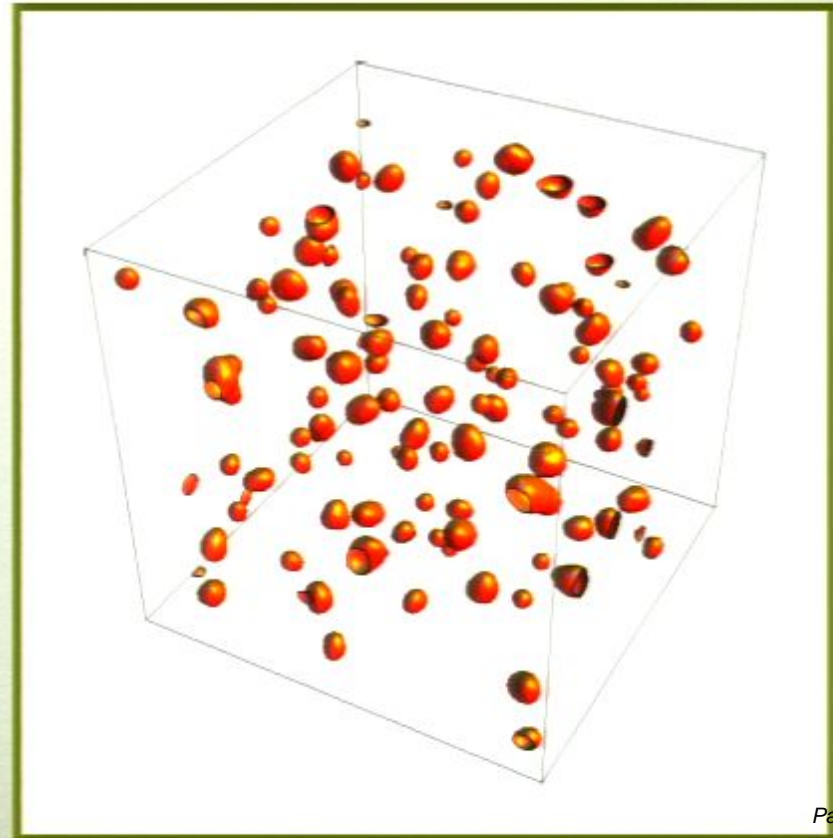
---



# LUMPS?

---

- consequences ?
- what are they ?





3+1

# CONSEQUENCES IF ...

---

3+1

## CONSEQUENCES IF ...

---

- last for many Hubble times (at that time)

3+1

## CONSEQUENCES IF ...

---

- last for many Hubble times (at that time)
- dominate the energy density

# PLAN

---

- **individual blobs:** *MA & Shirokoff 2010*
  - solution & stability
- **post inflationary cosmological emergence:**
  - analytics and 1+1 d simulations MA (2010)
- **lattice simulations**
  - 3+1 d simulations MA, Easter and Finkel (2010)



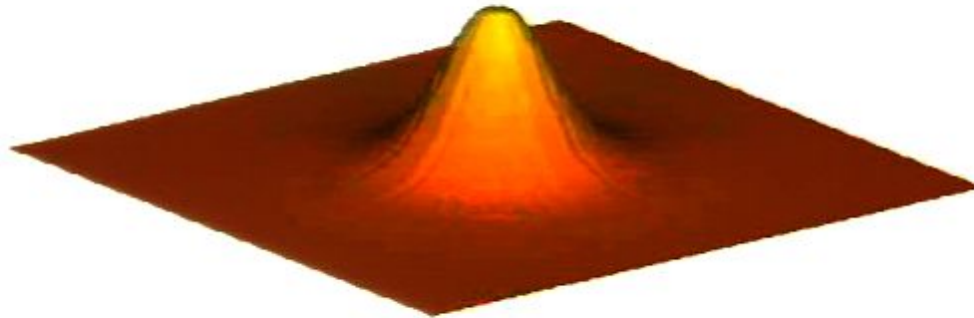
# INDIVIDUAL OSCILLONS



# MASSIVE FREE FIELD

---

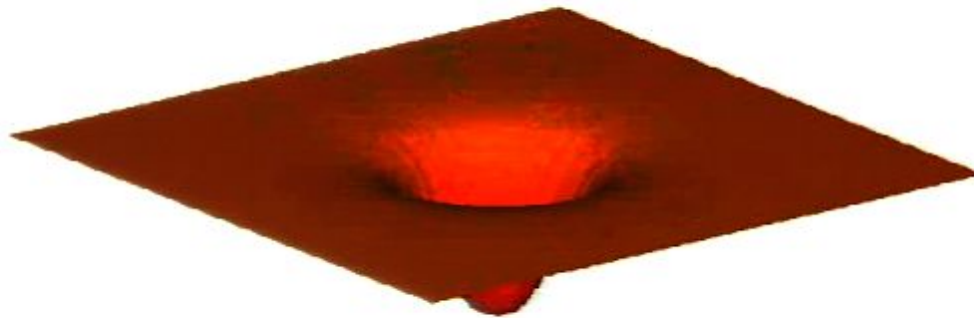
$$t = 20m^{-1}$$



# MASSIVE FREE FIELD

---

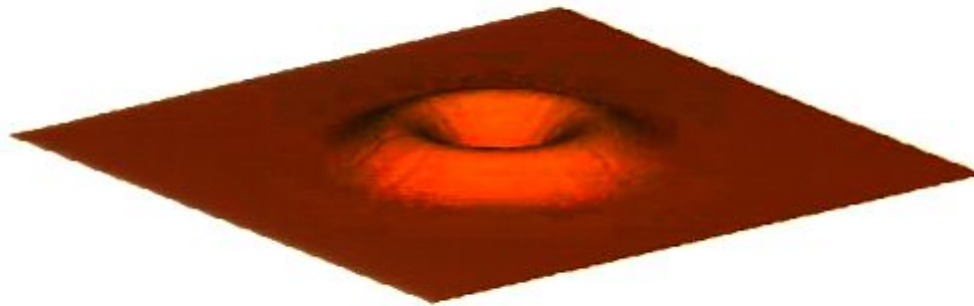
$$t = 34m^{-1}$$



# MASSIVE FREE FIELD

---

$$t = 77m^{-1}$$

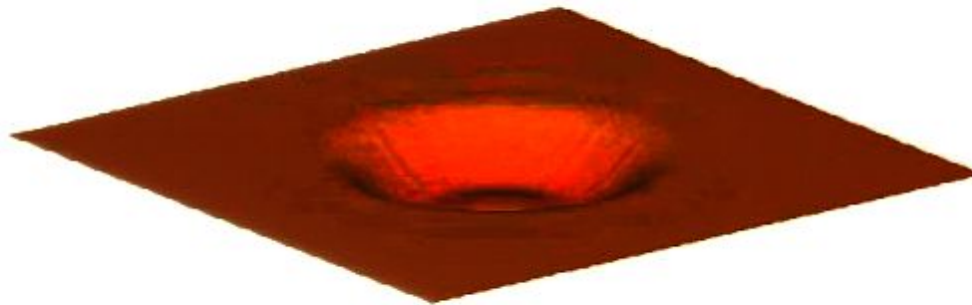




# MASSIVE FREE FIELD

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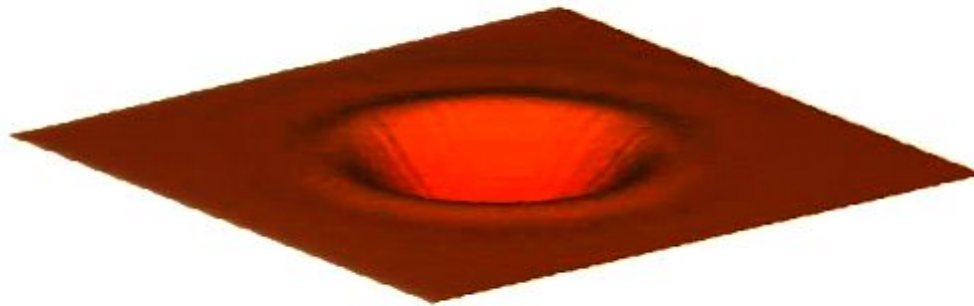
$$t = 92m^{-1}$$



# MASSIVE FREE FIELD

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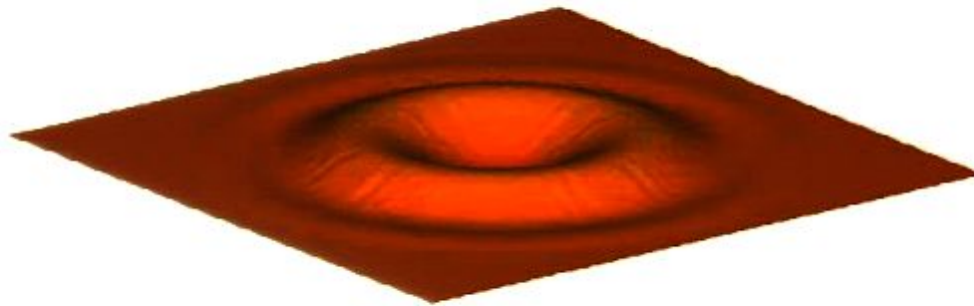
$$t = 110m^{-1}$$



# MASSIVE FREE FIELD

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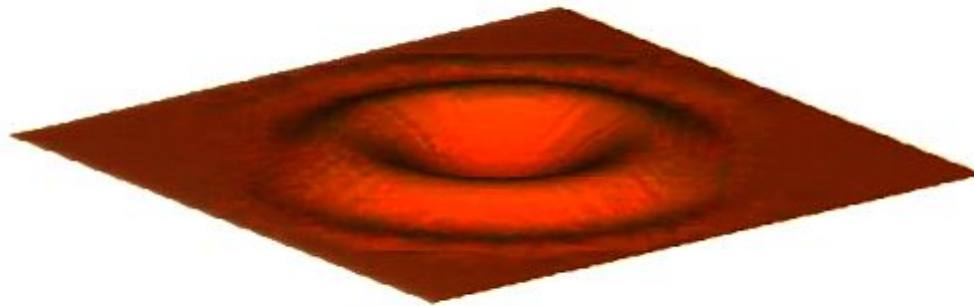
$$t = 146m^{-1}$$



# MASSIVE FREE FIELD

---

$$t = 165m^{-1}$$



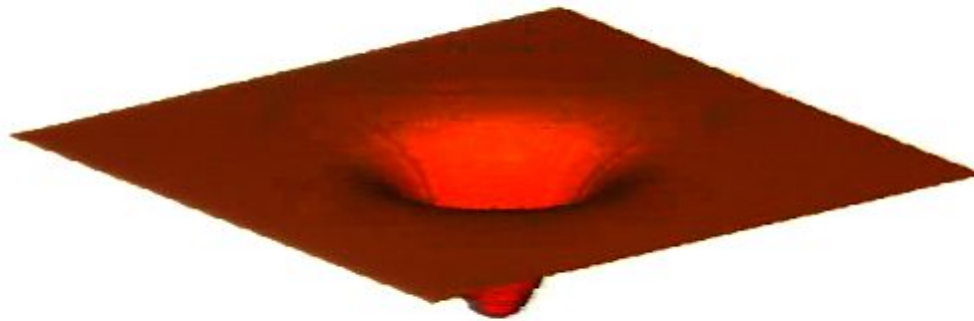
# INDIVIDUAL OSCILLONS



# MASSIVE FREE FIELD

---

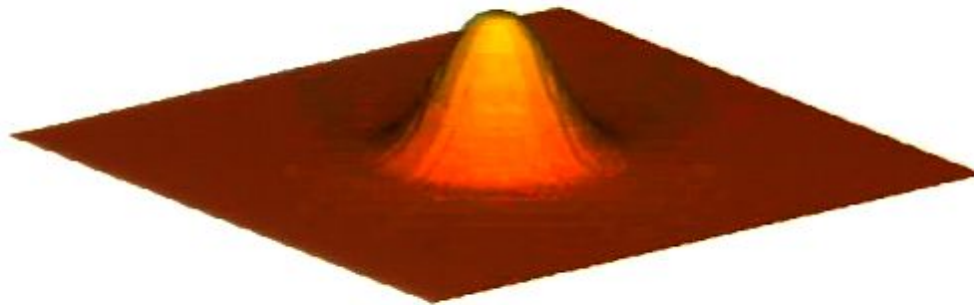
$$t = 20m^{-1}$$



# MASSIVE FREE FIELD

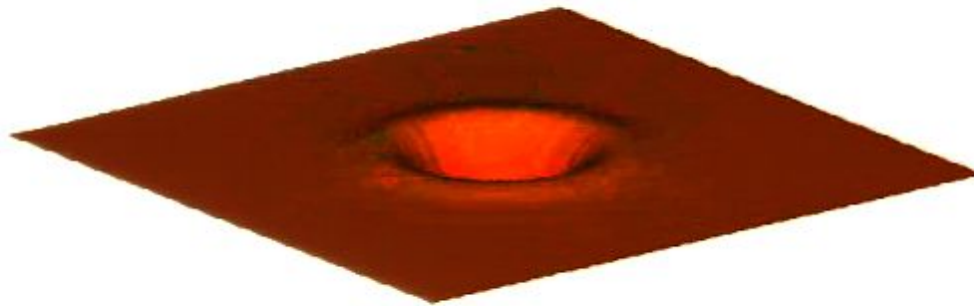
---

$$t = 44m^{-1}$$



# MASSIVE FREE FIELD

$$t = 59m^{-1}$$

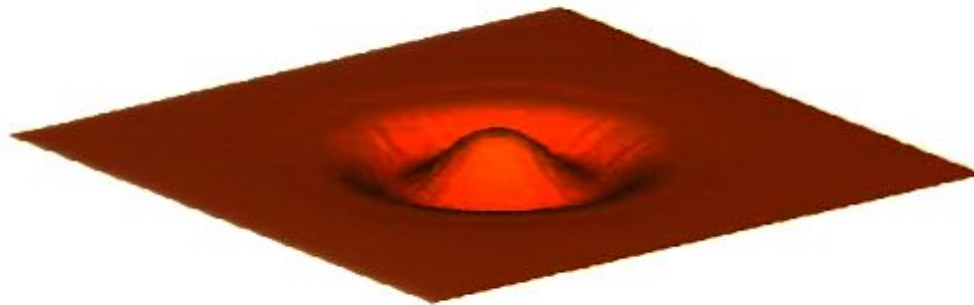




# MASSIVE FREE FIELD

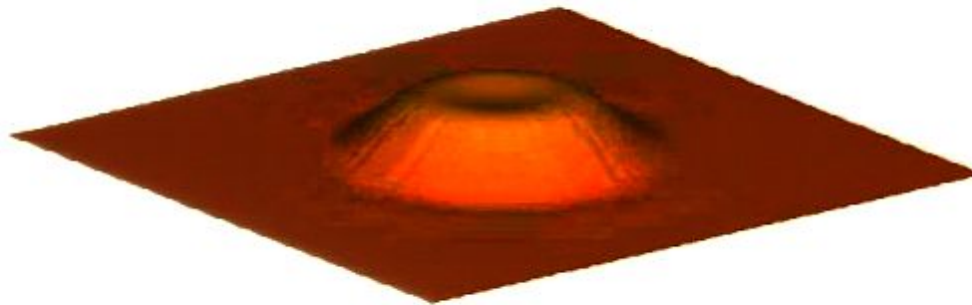
---

$$t = 74m^{-1}$$



# MASSIVE FREE FIELD

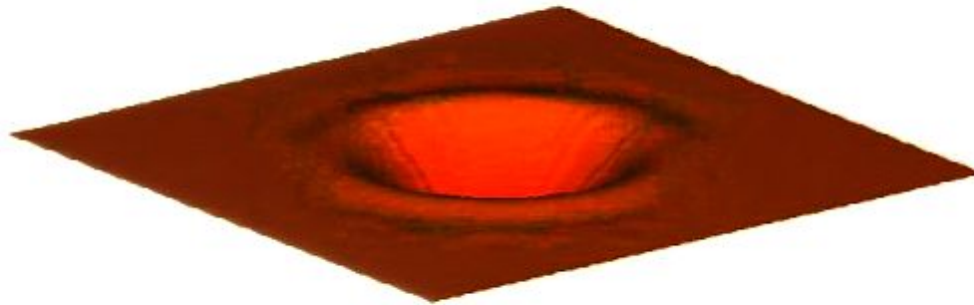
$$t = 95m^{-1}$$



# MASSIVE FREE FIELD

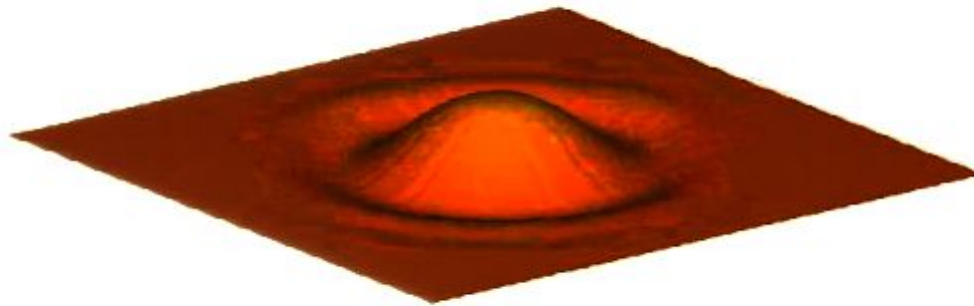
---

$$t = 116m^{-1}$$



# MASSIVE FREE FIELD

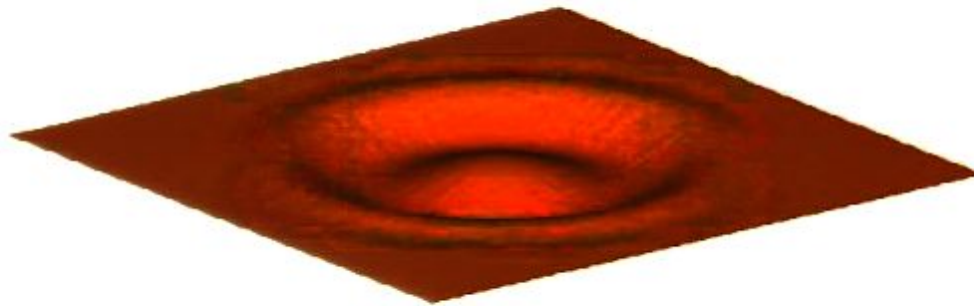
$$t = 131m^{-1}$$



# MASSIVE FREE FIELD

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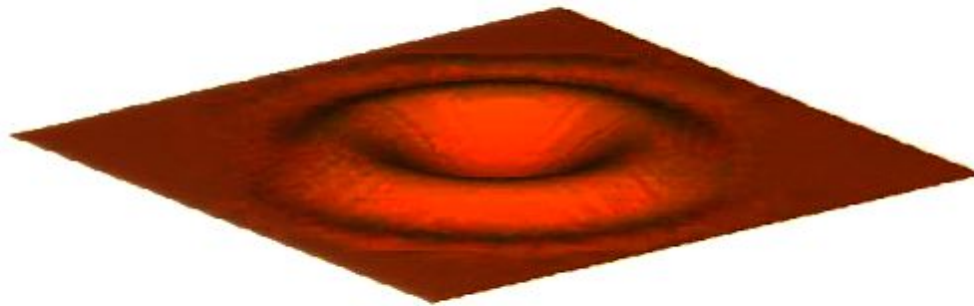
$$t = 155m^{-1}$$



# MASSIVE FREE FIELD

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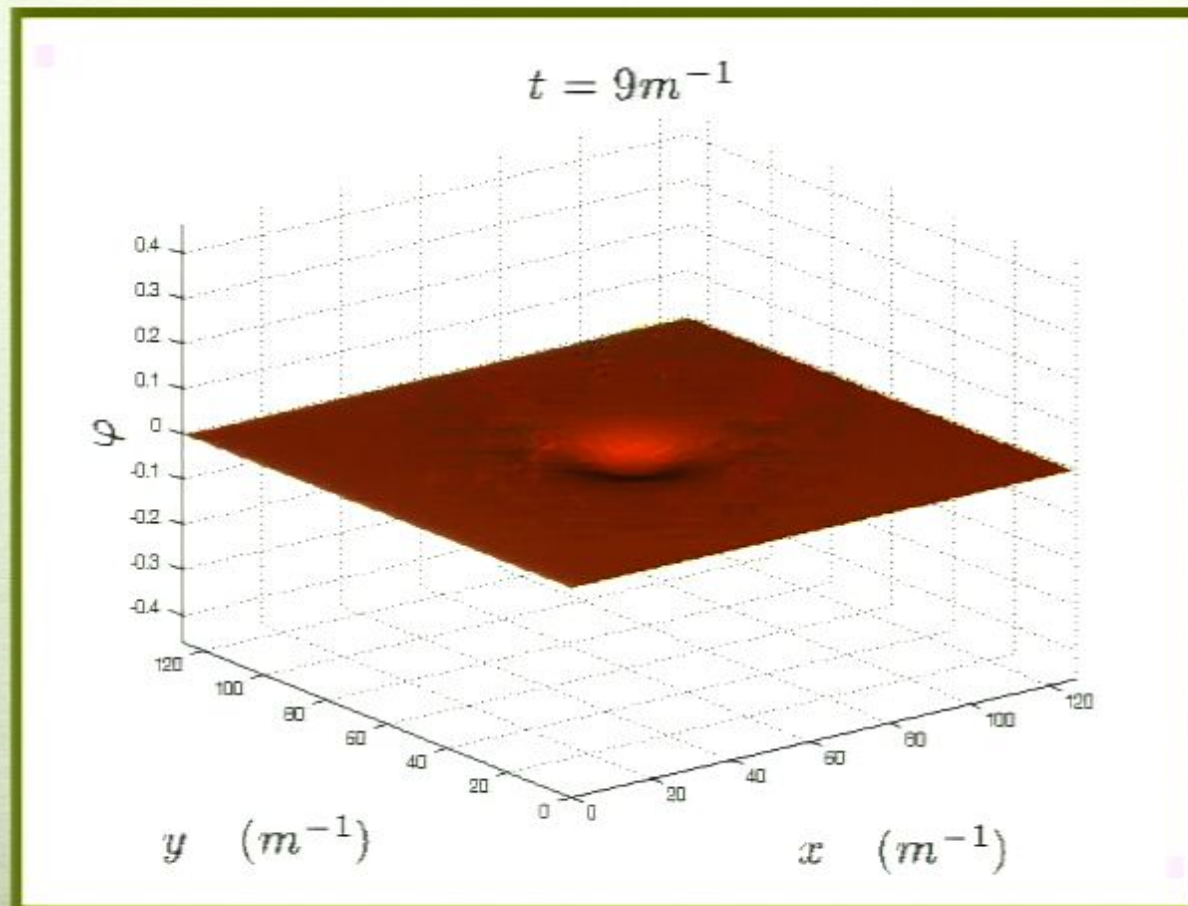
$$t = 165m^{-1}$$



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

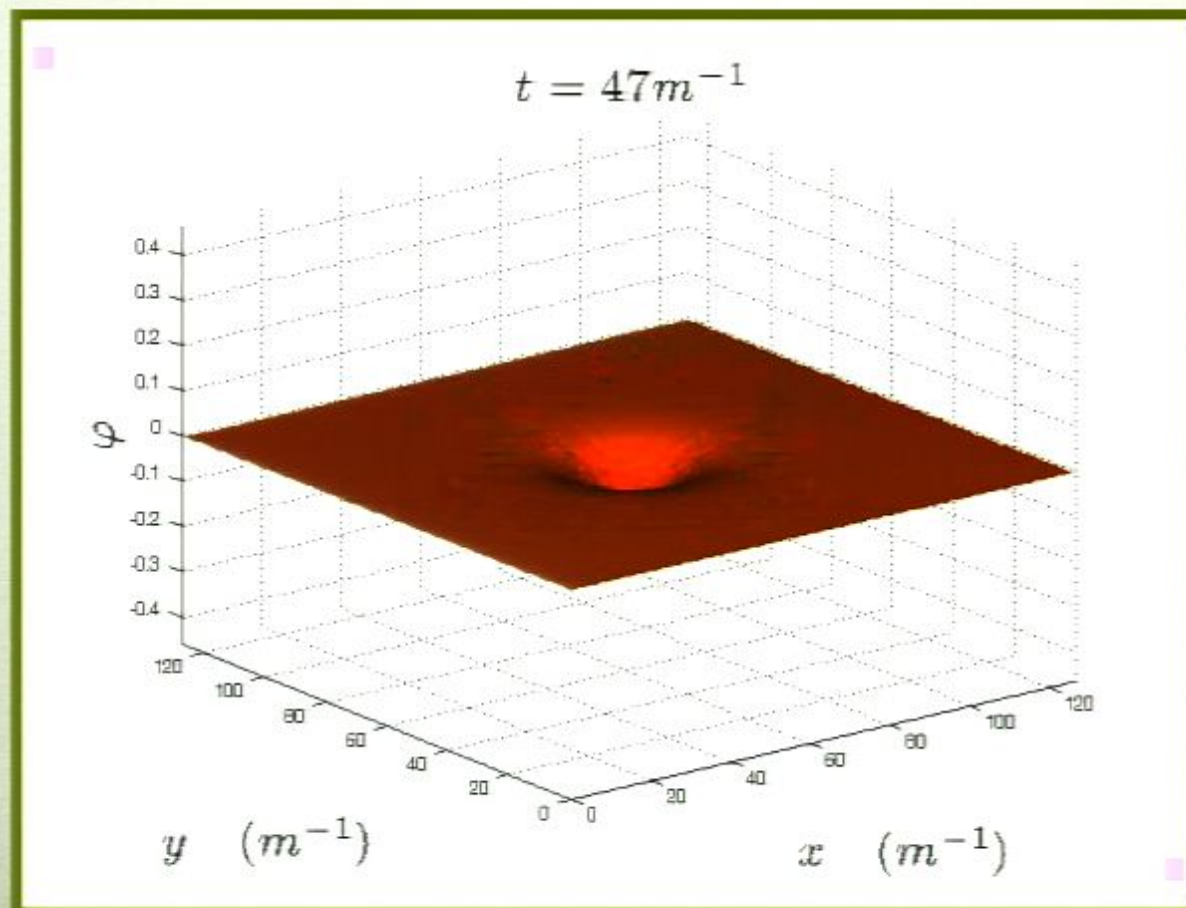
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

(1) oscillatory (2) spatially localized (3) **very long lived**

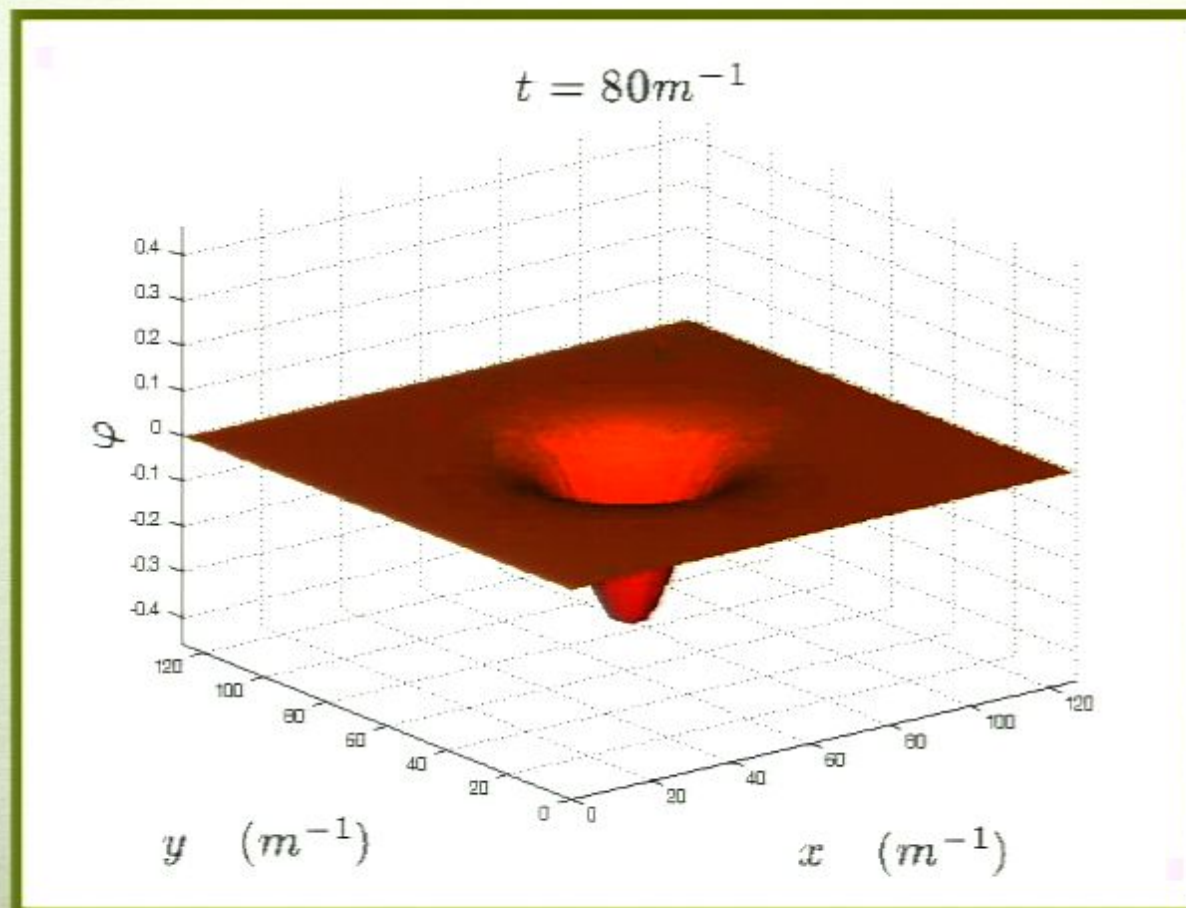




# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

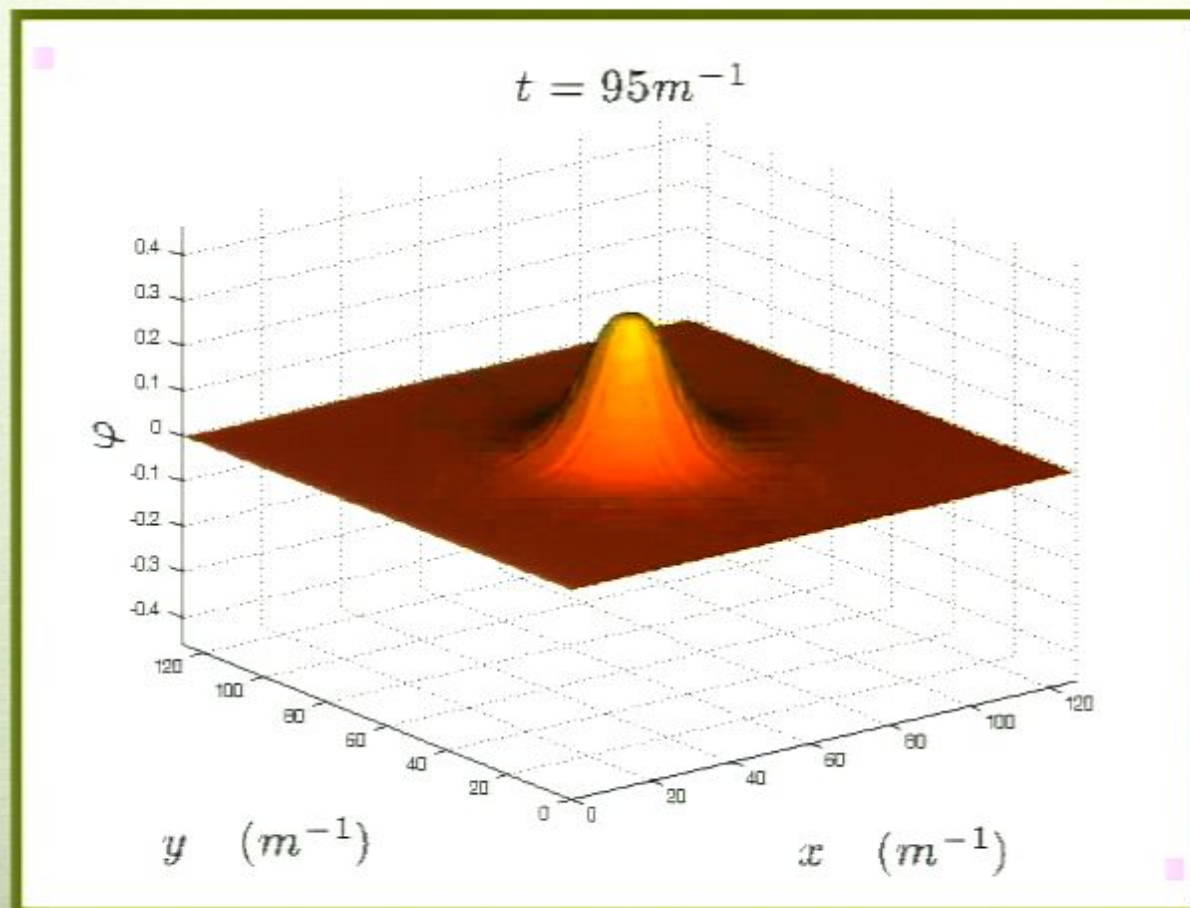
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

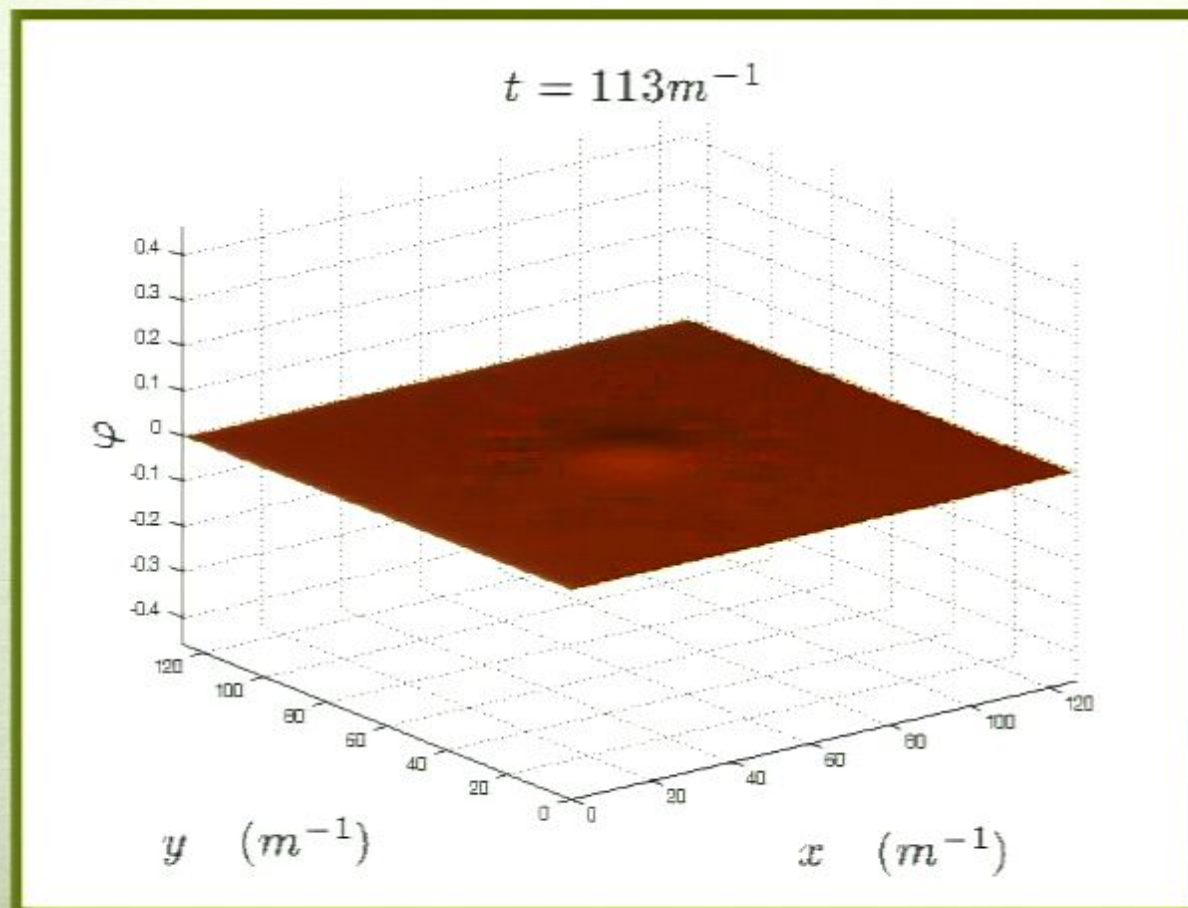
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

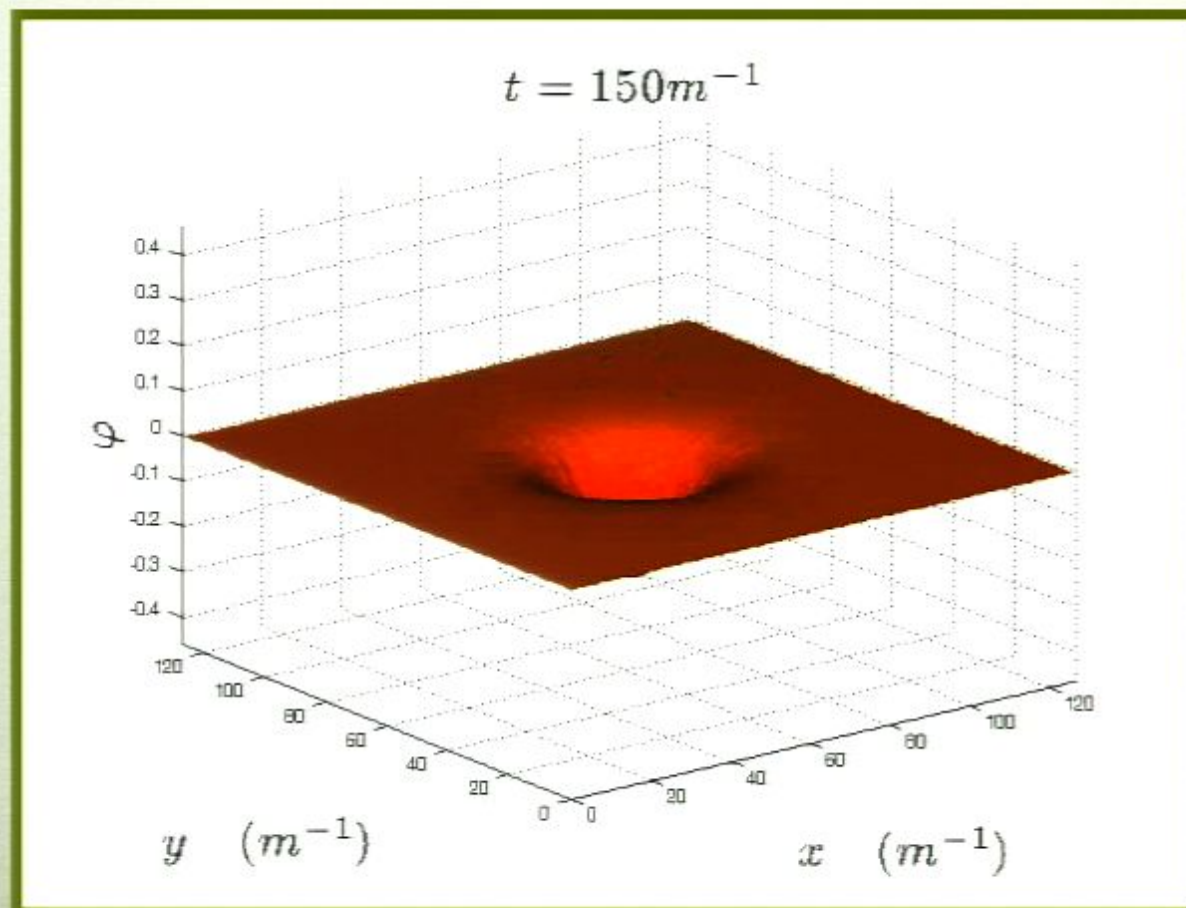
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

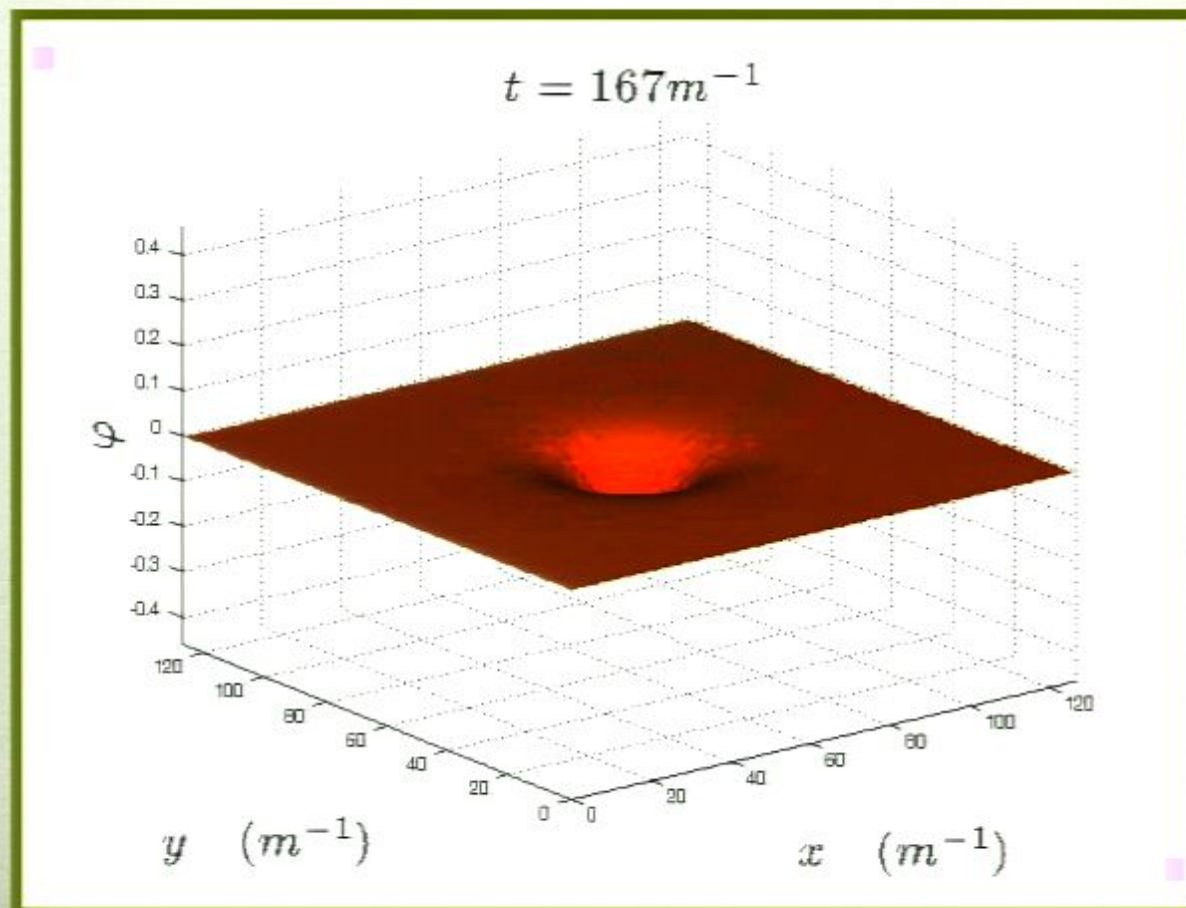
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

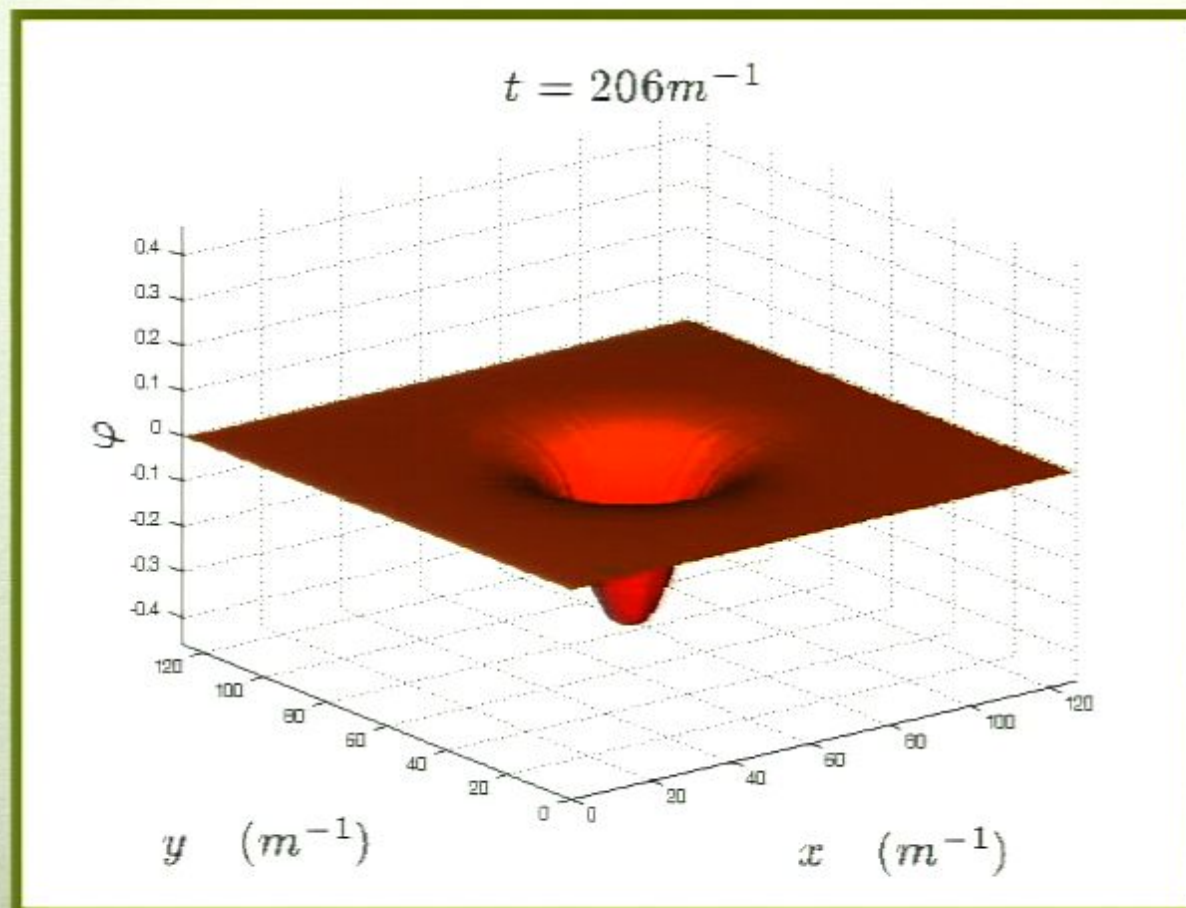
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

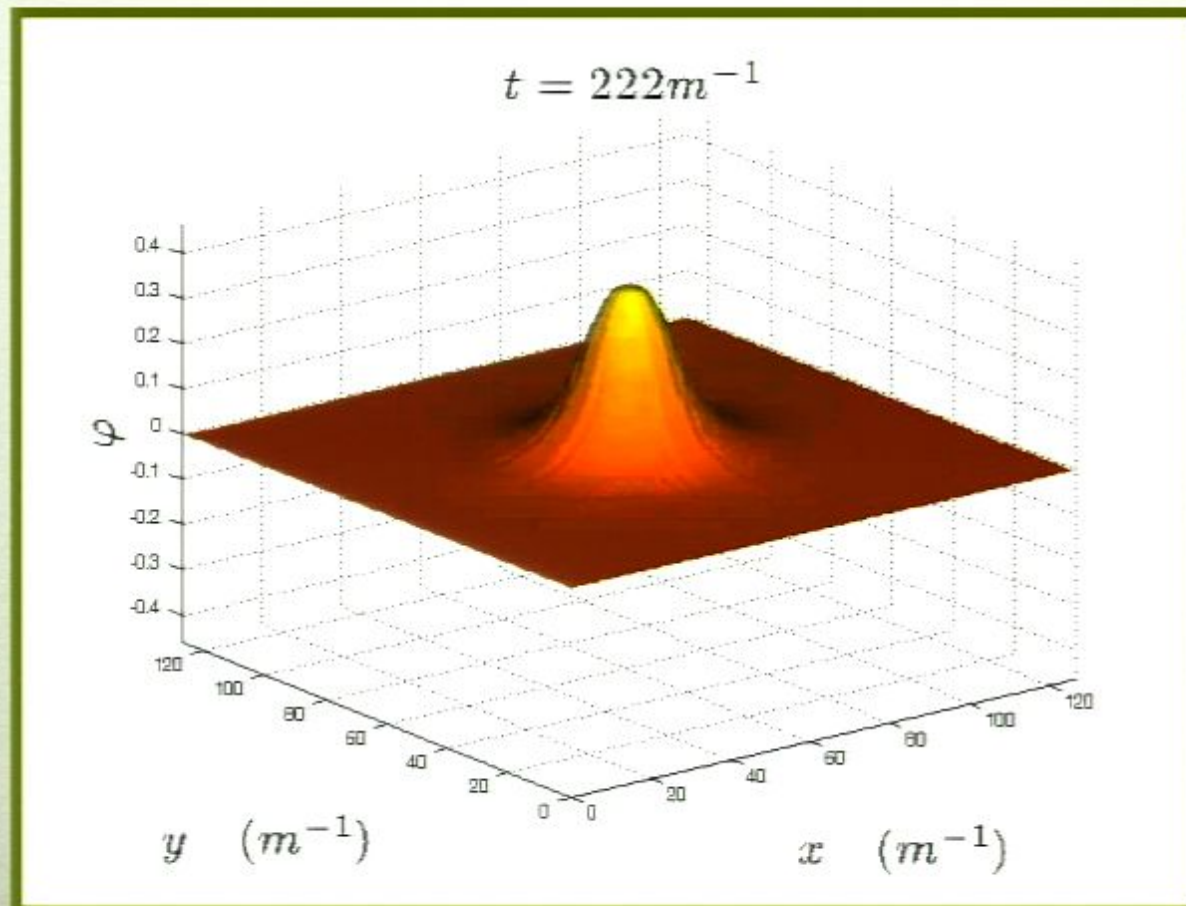
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

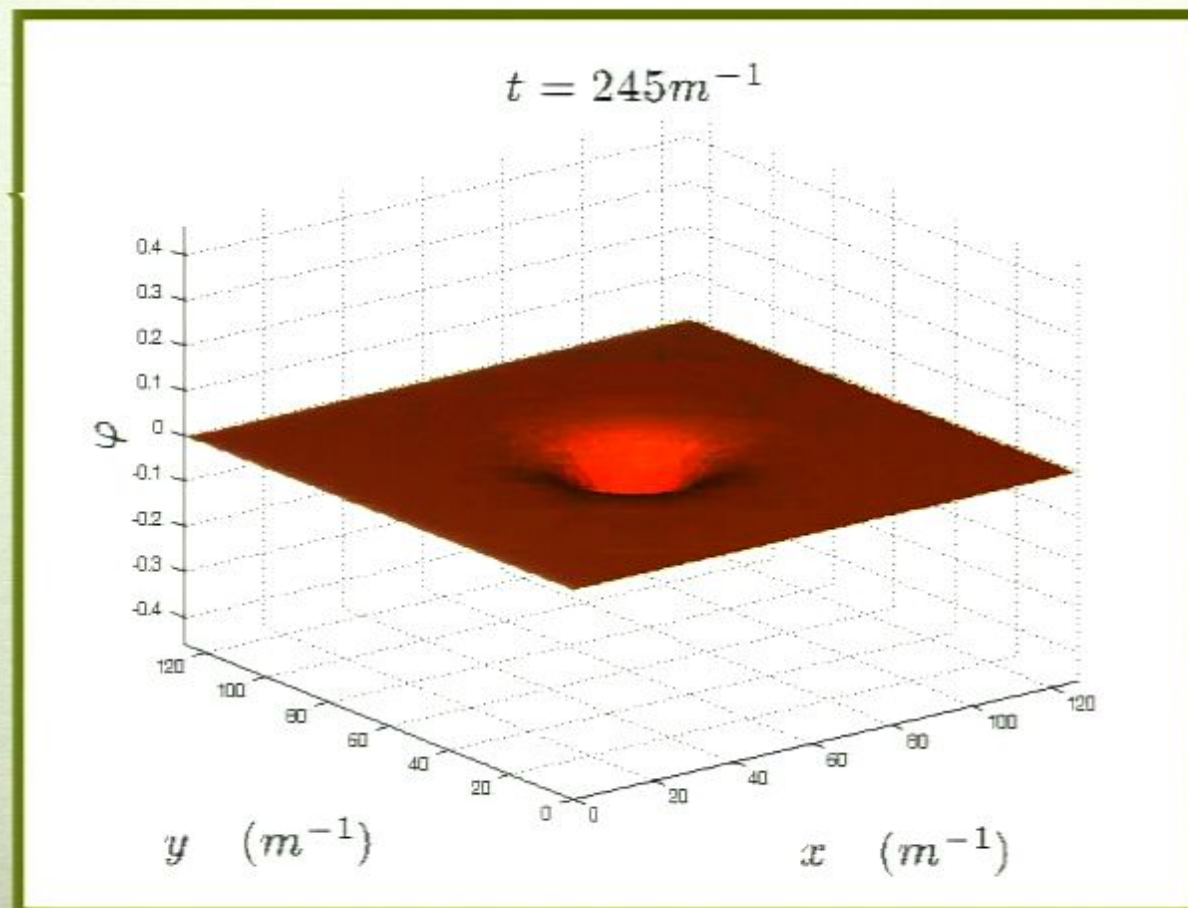
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

(1) oscillatory (2) spatially localized (3) **very long lived**

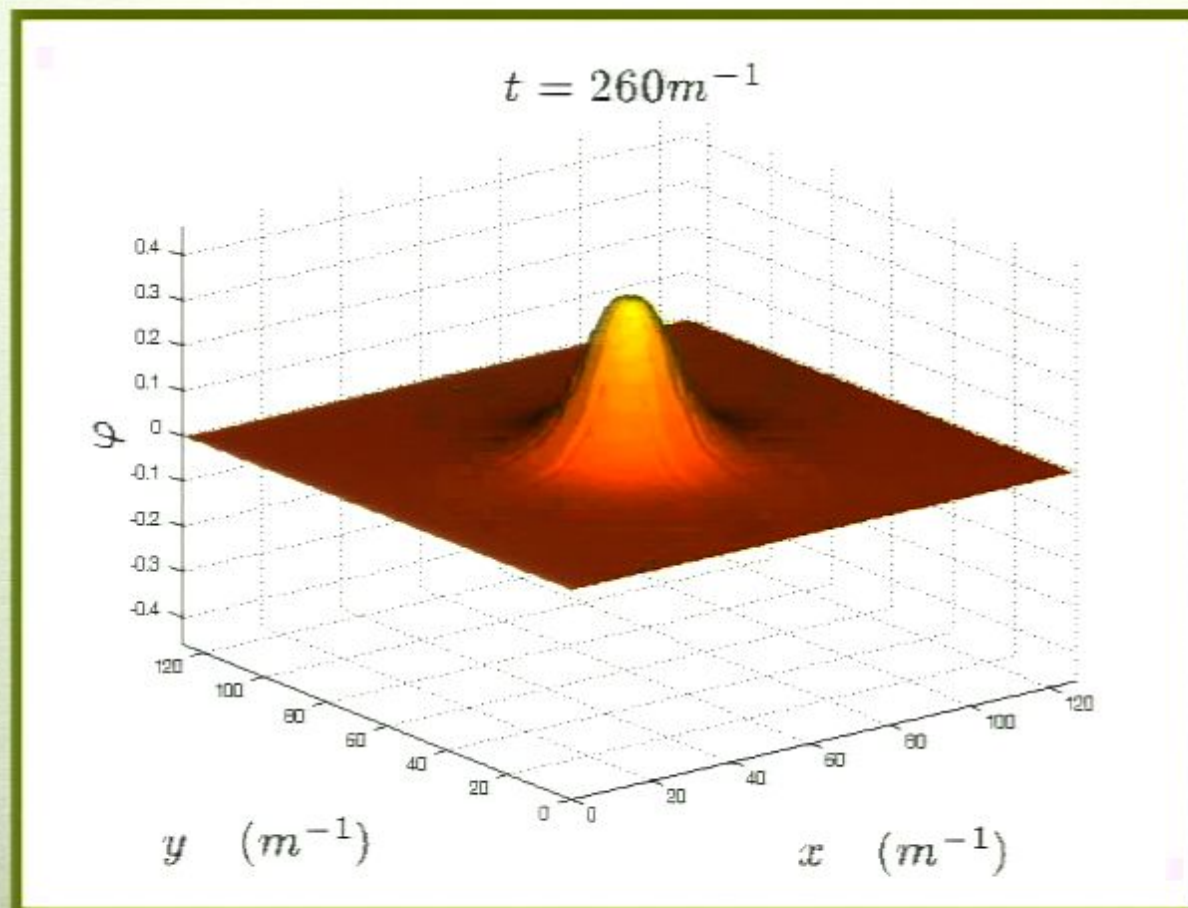




# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

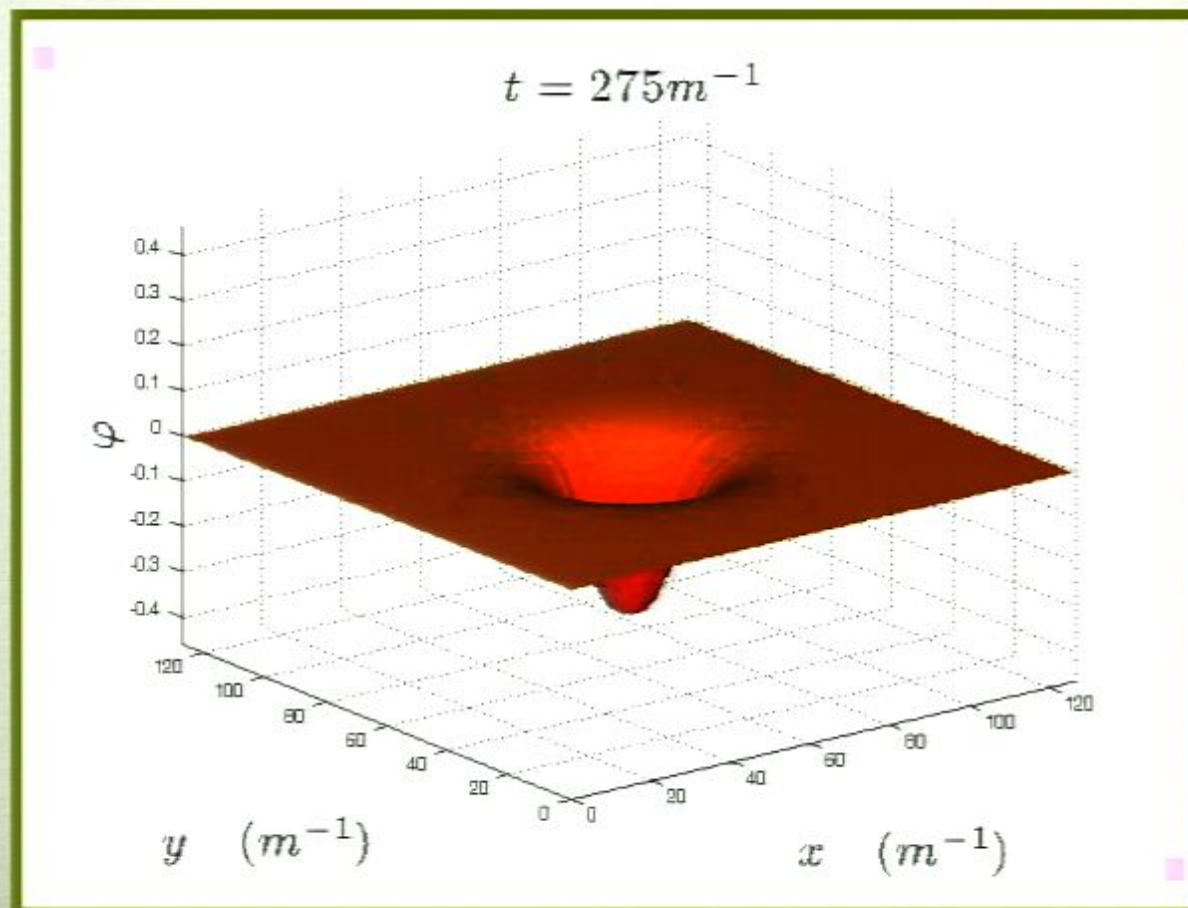
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

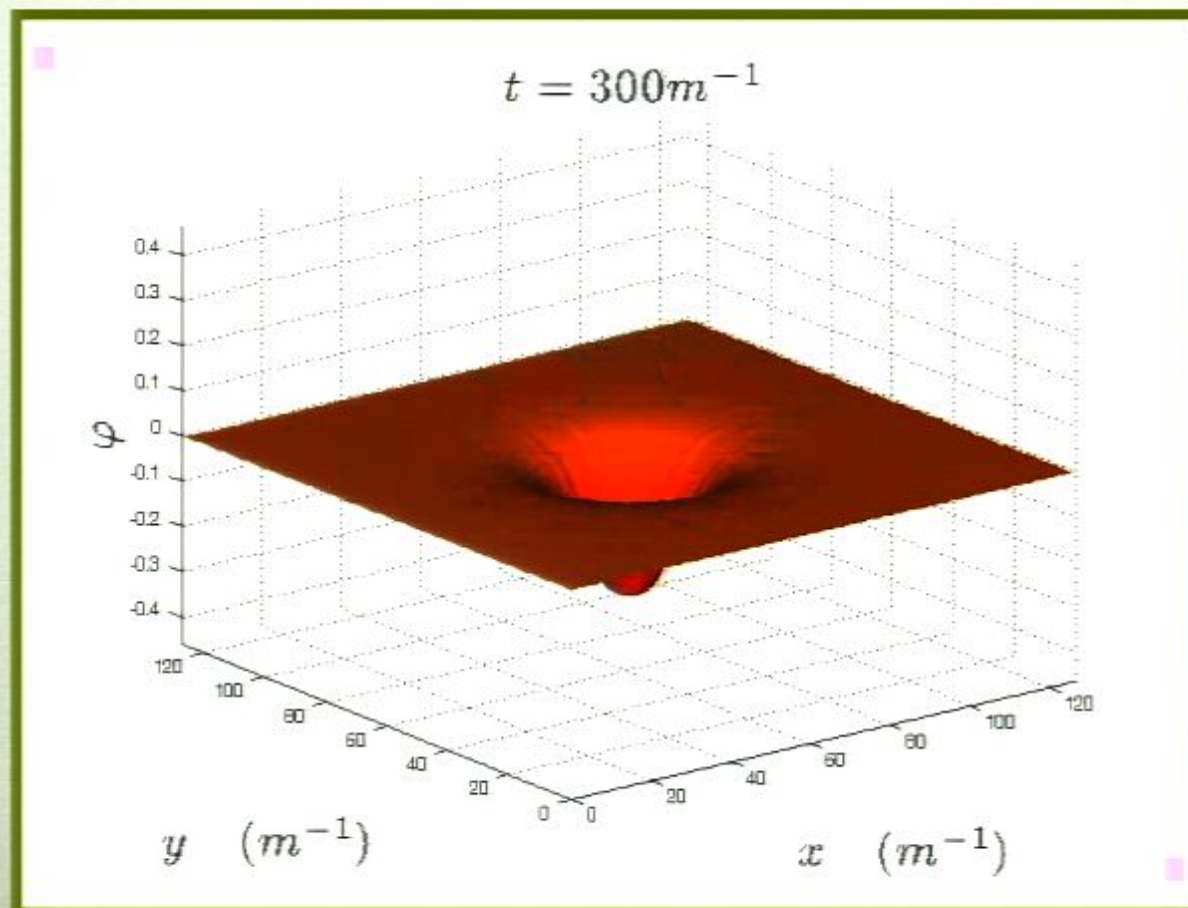
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

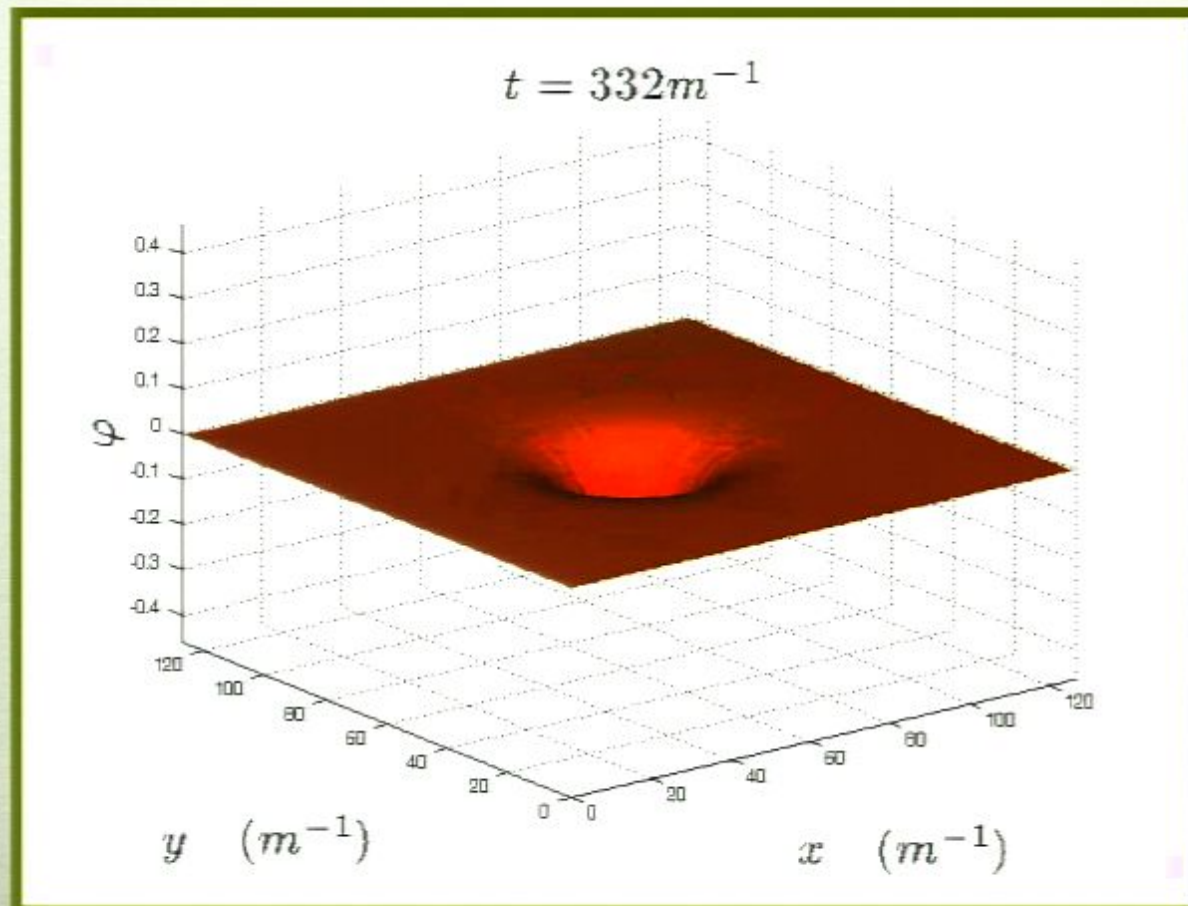
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

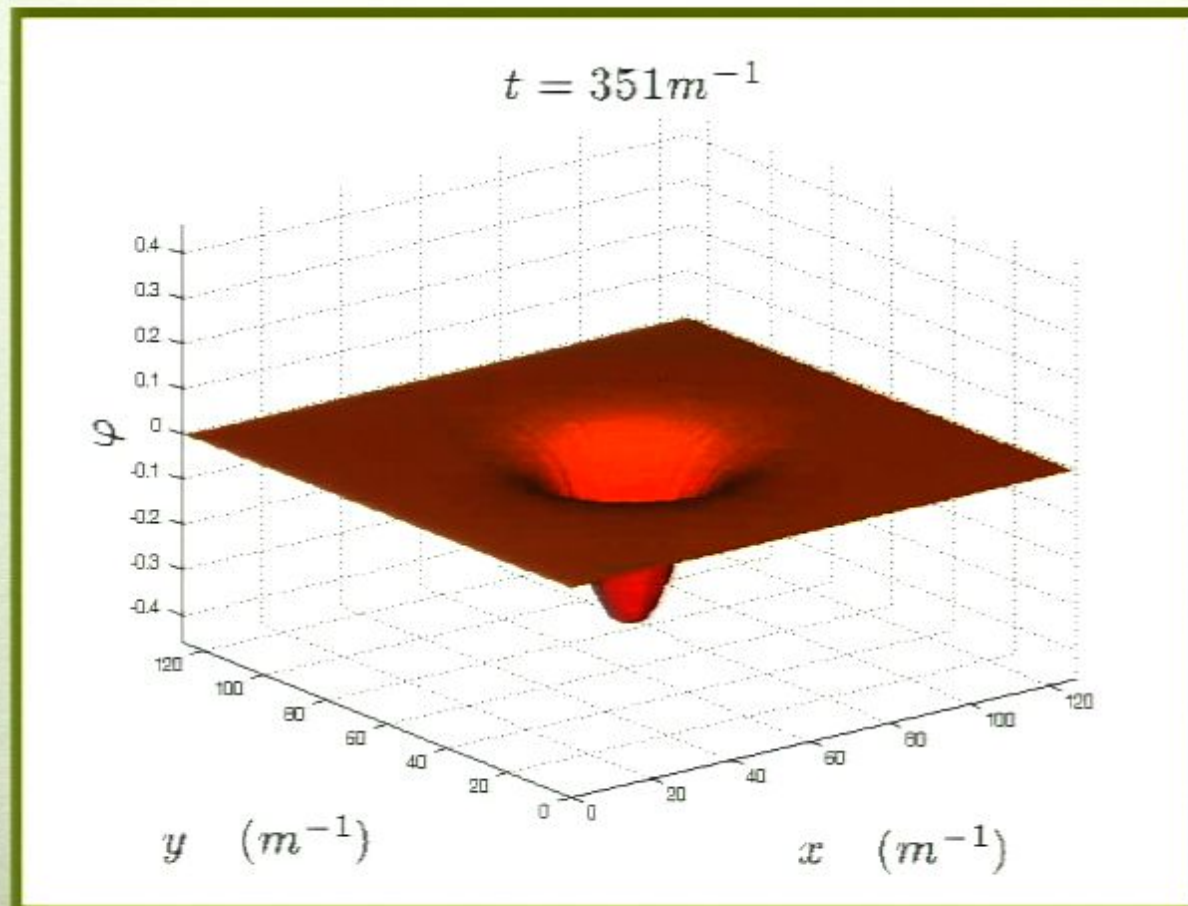
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(BREATHERS IN CONDENSED MATTER)

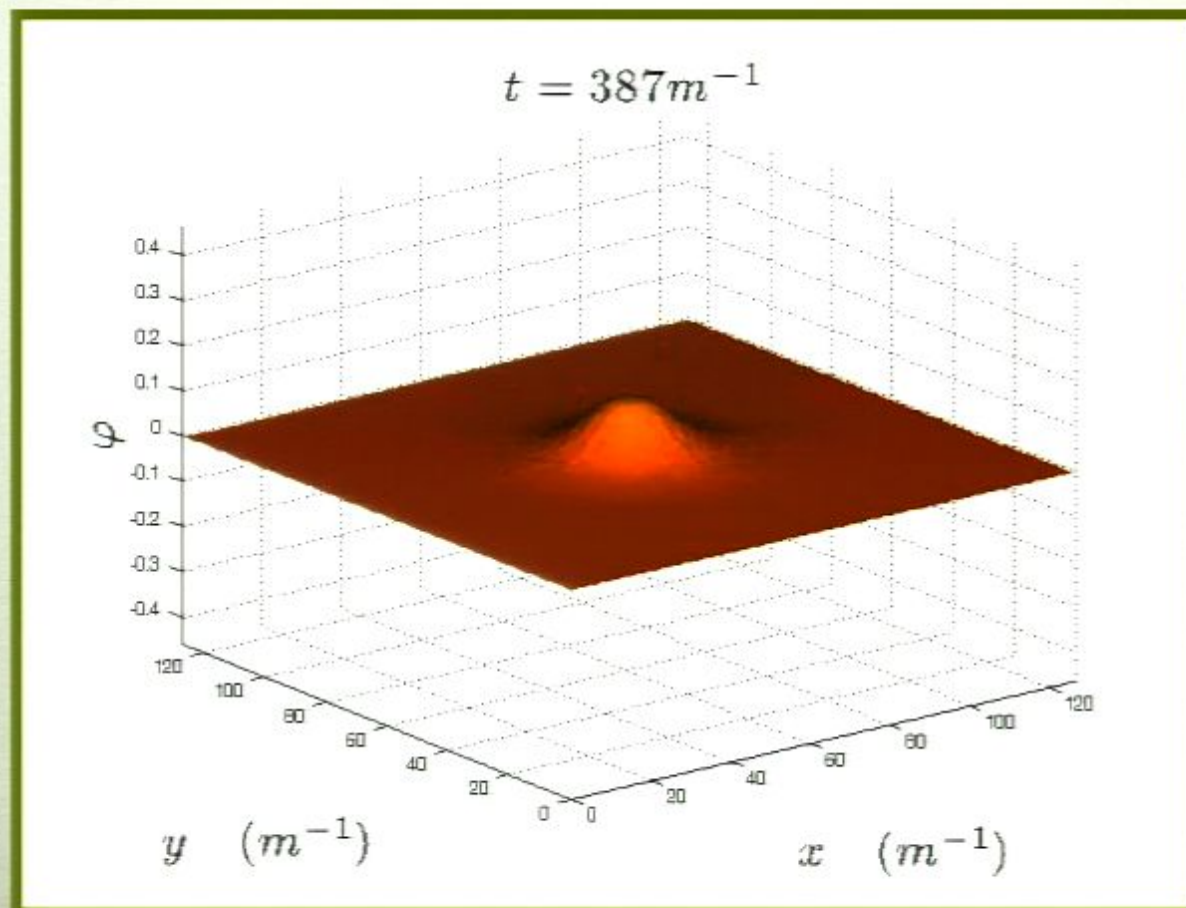
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

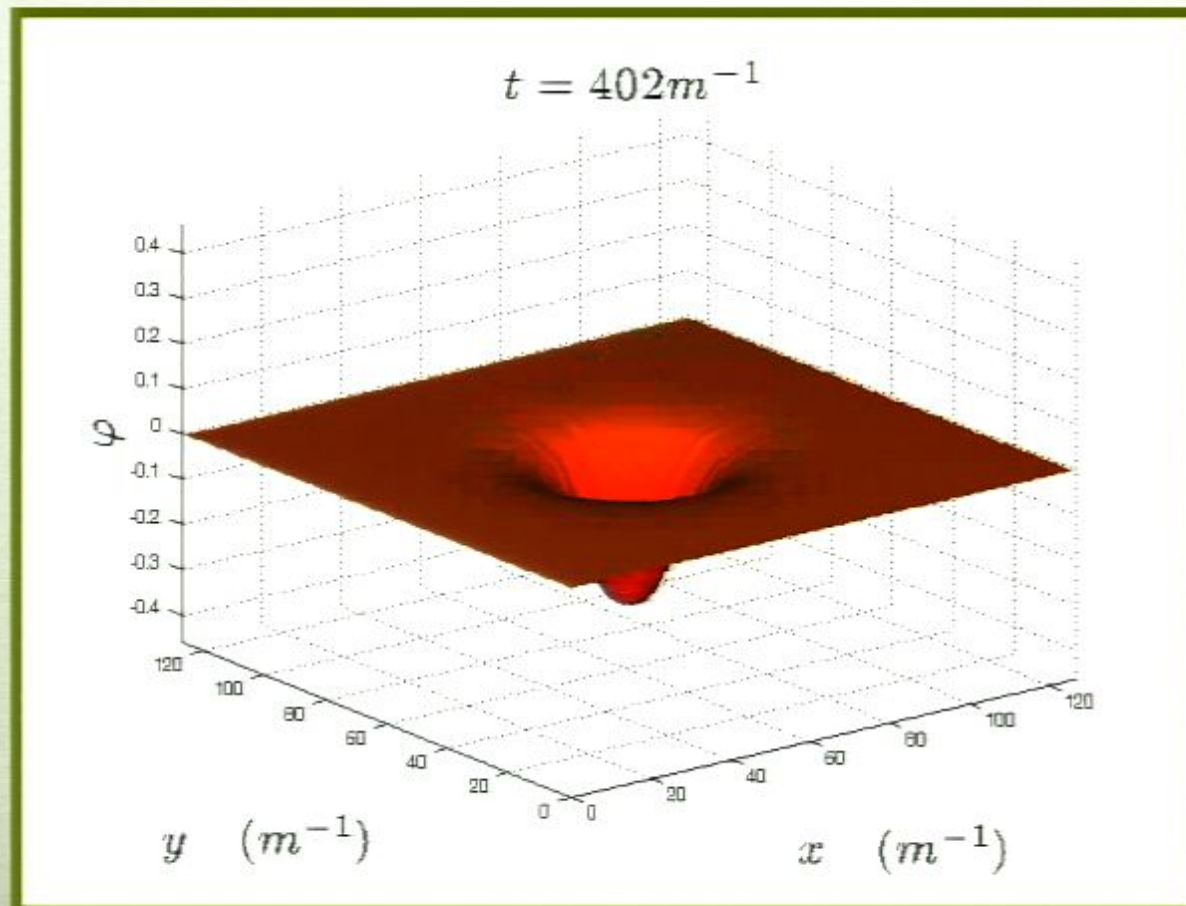
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# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

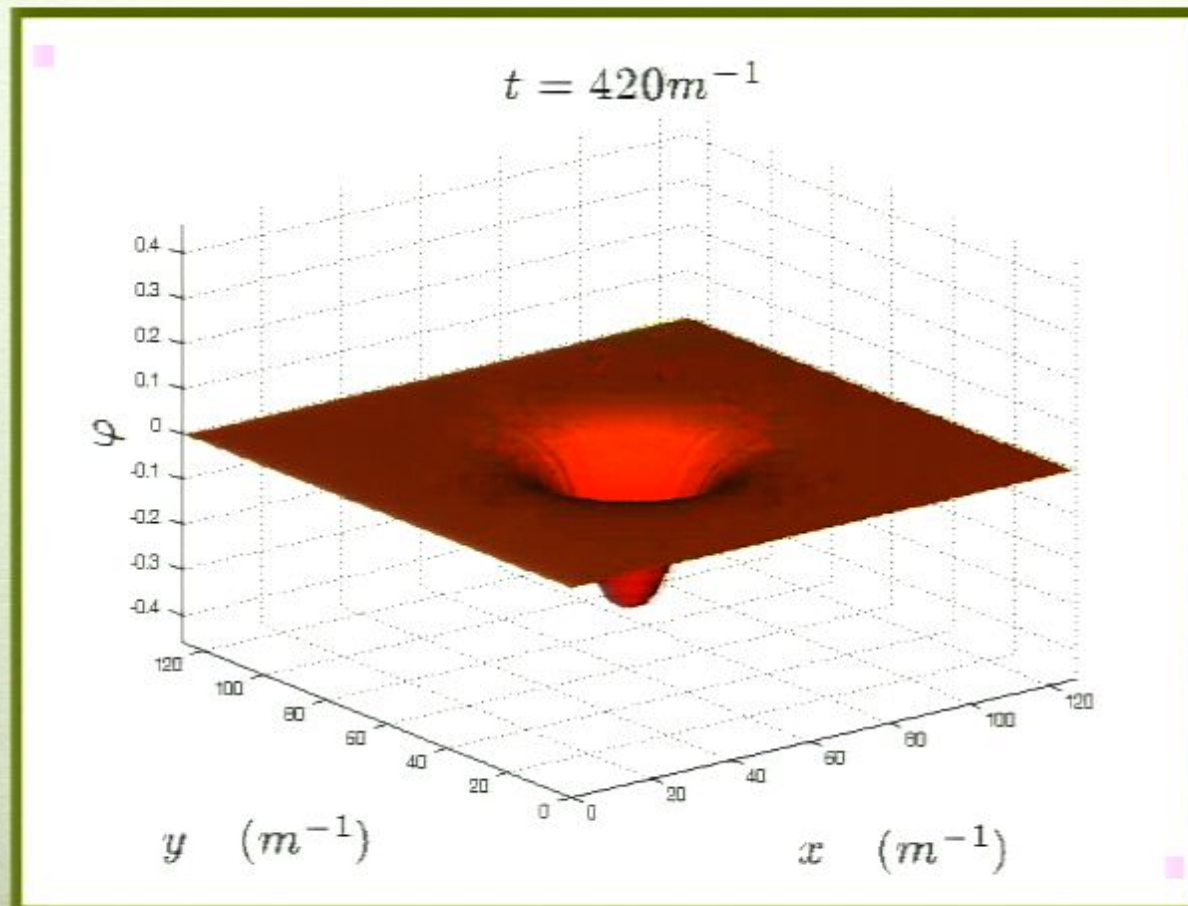
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(BREATHERS IN CONDENSED MATTER)

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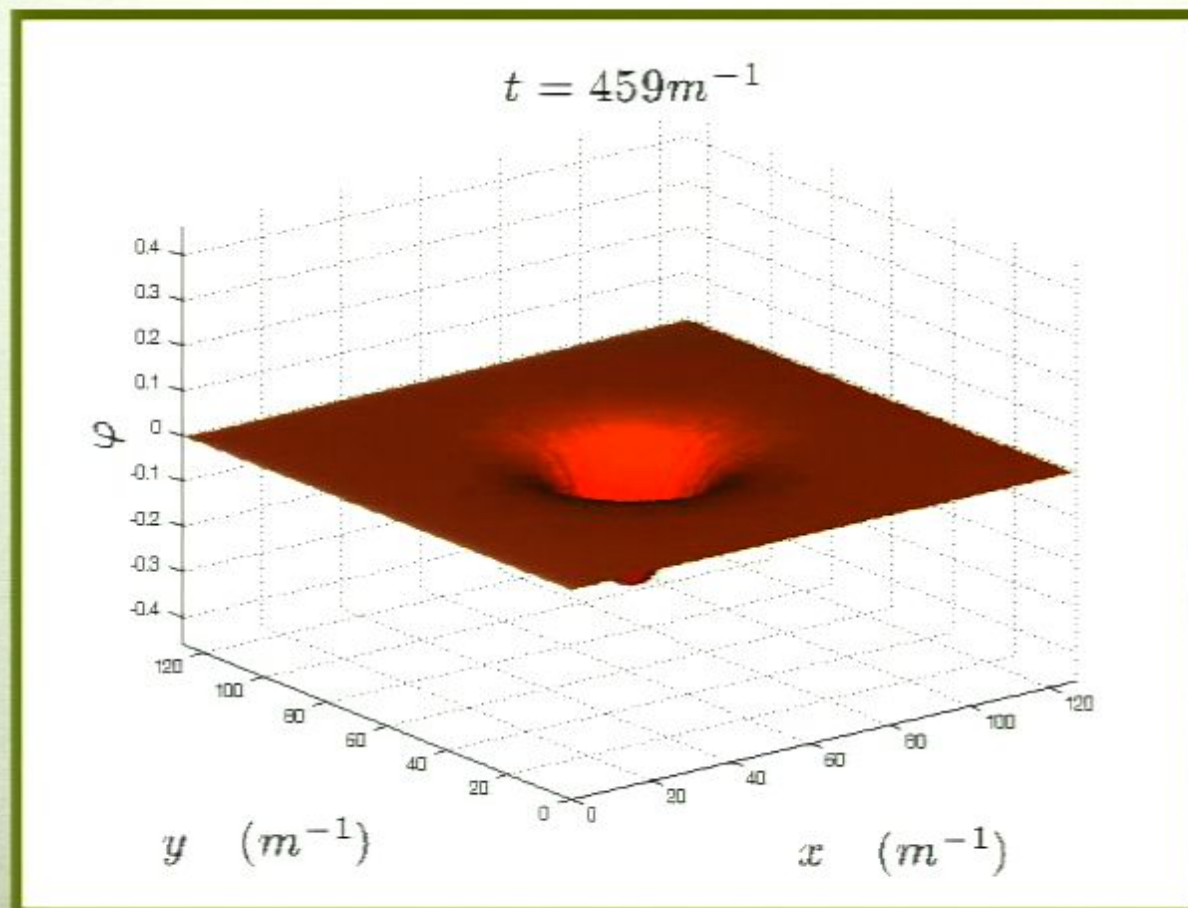




# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

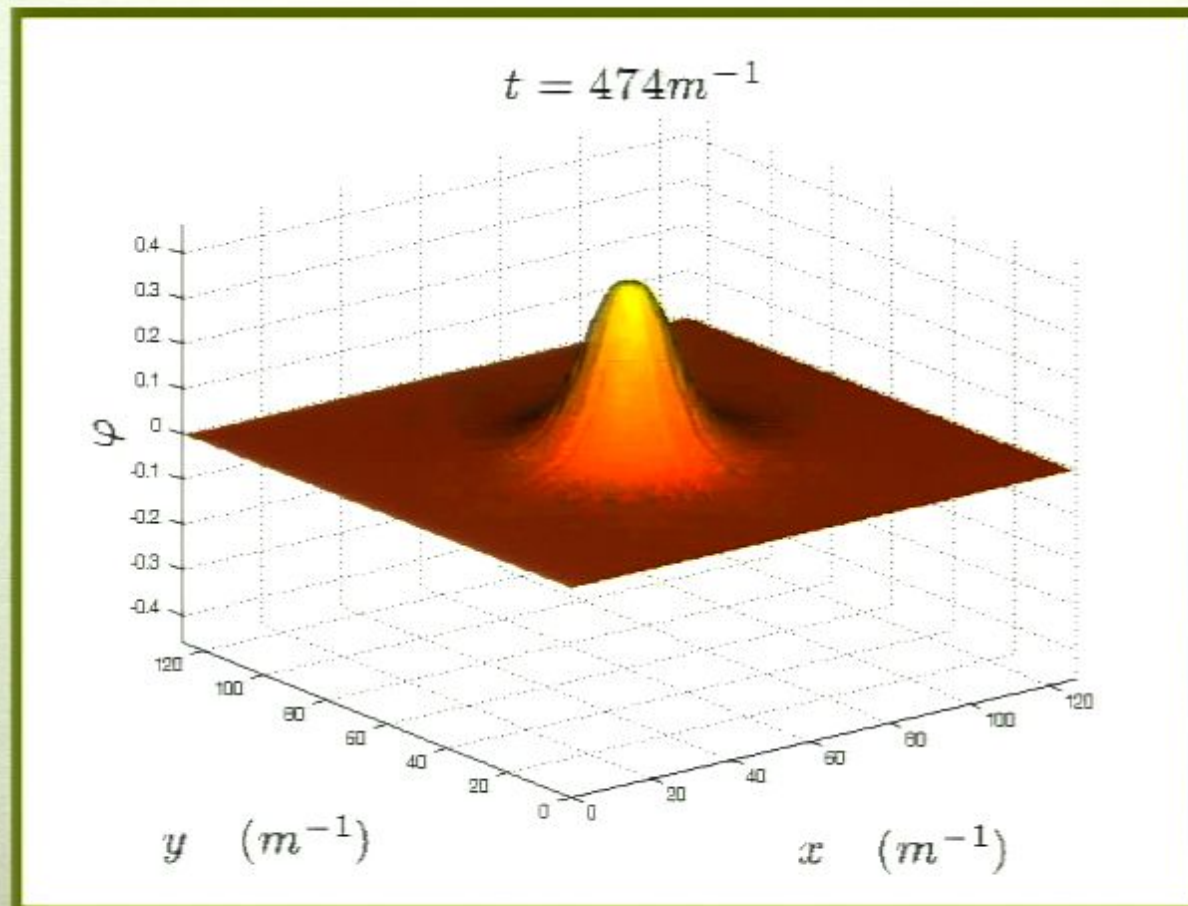
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# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

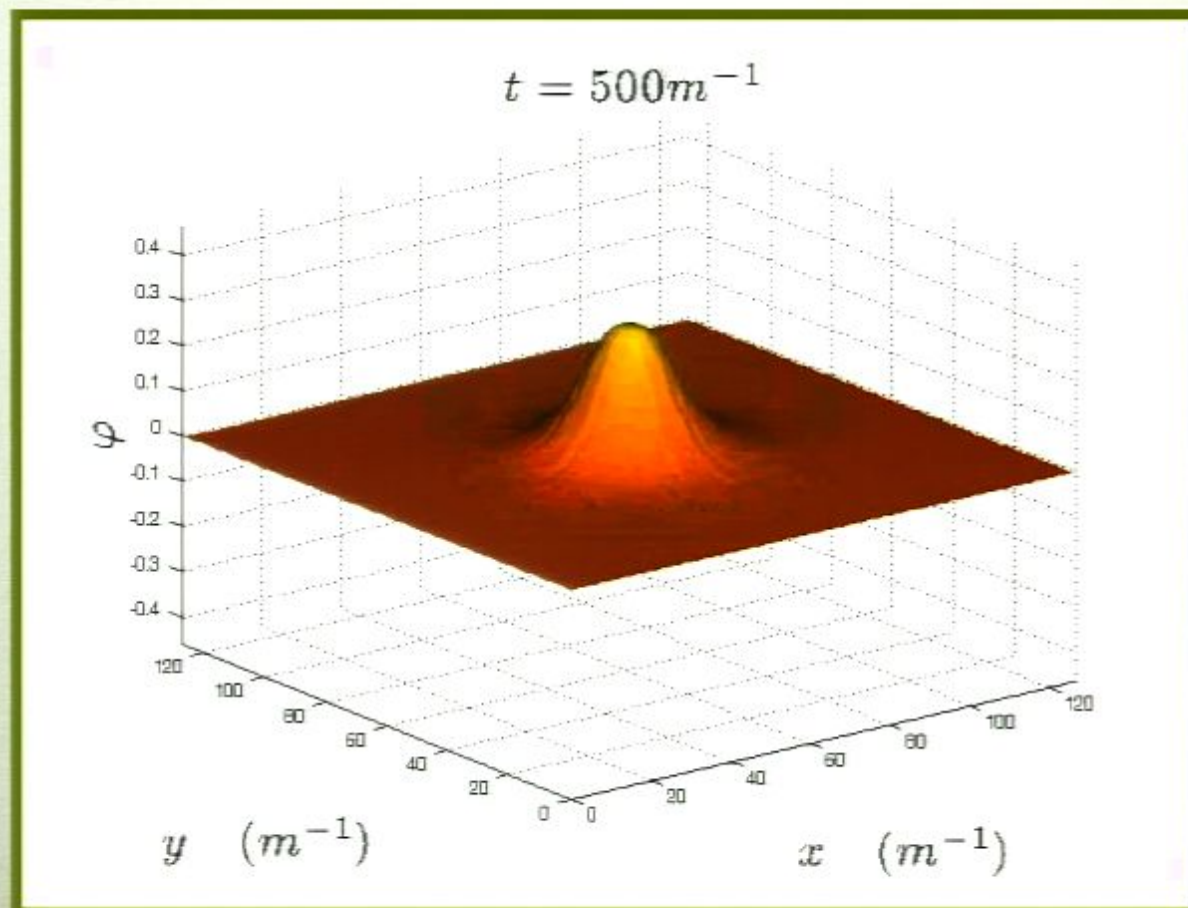
(1) oscillatory (2) spatially localized (3) **very long lived**



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(BREATHERS IN CONDENSED MATTER)

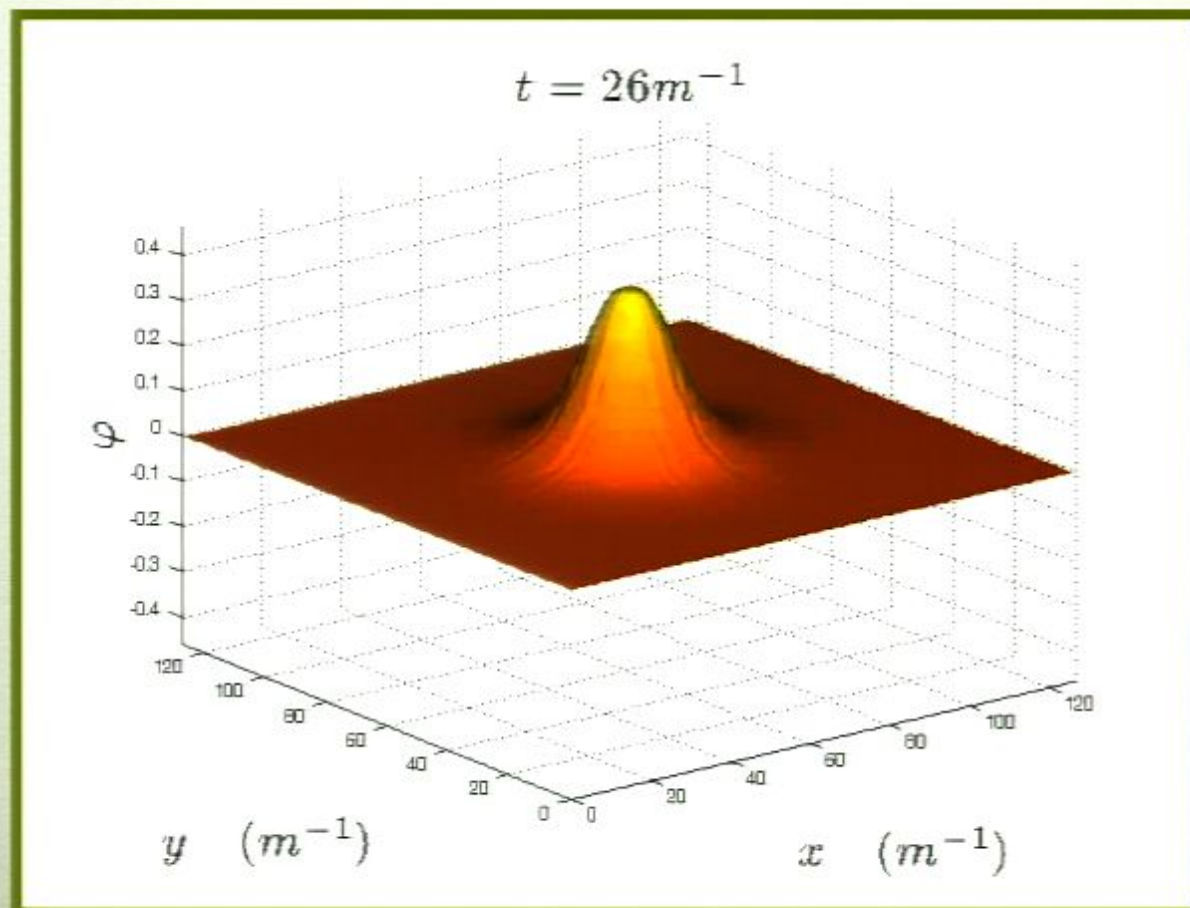
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# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

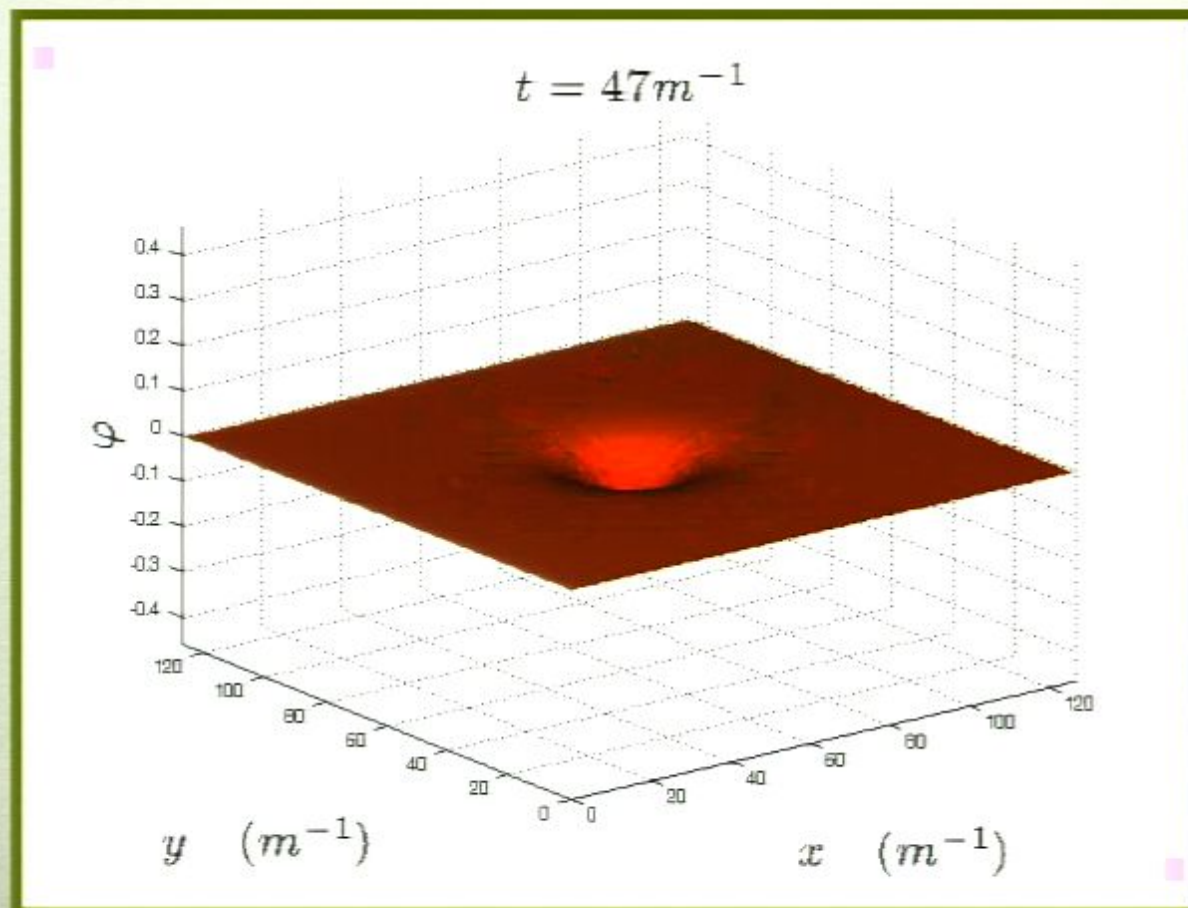
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(BREATHERS IN CONDENSED MATTER)

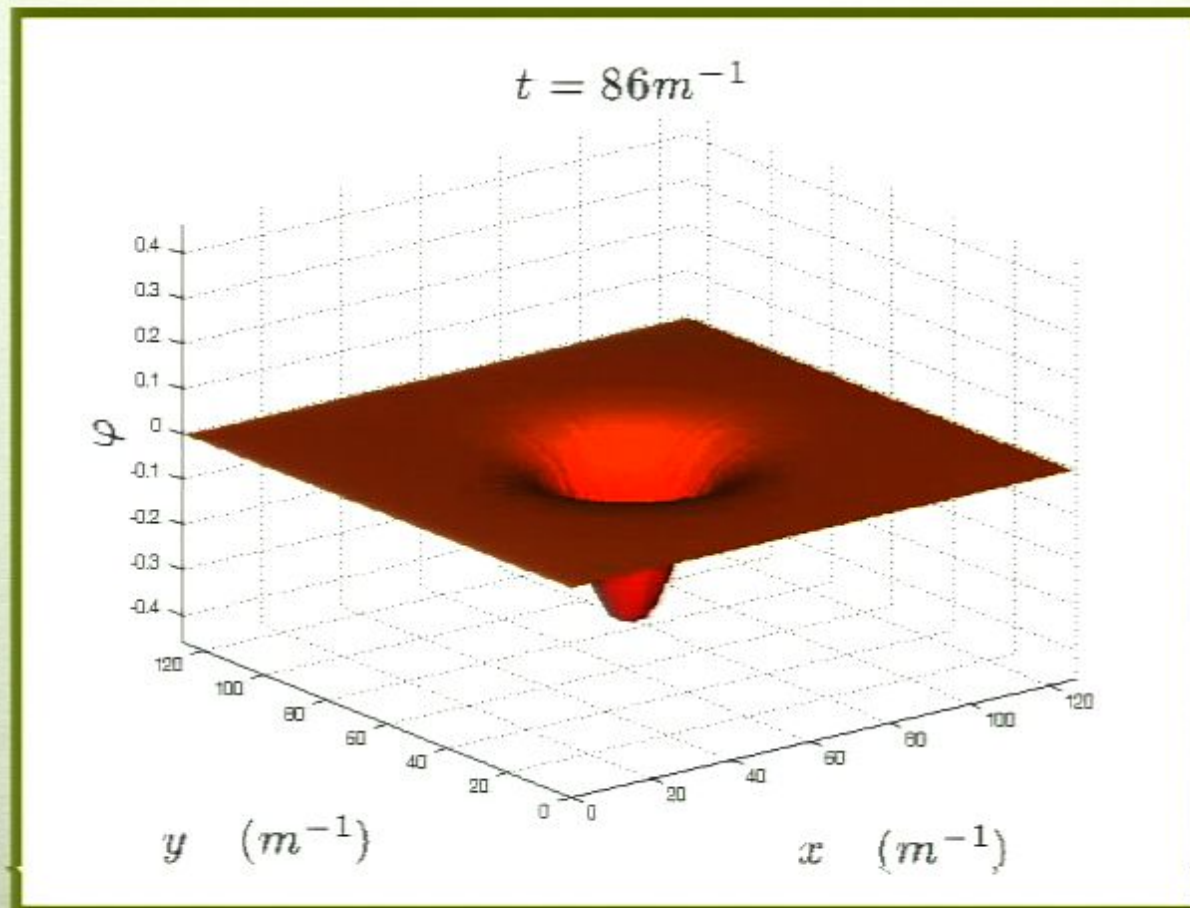
(1) oscillatory (2) spatially localized (3) **very long lived**



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(BREATHERS IN CONDENSED MATTER)

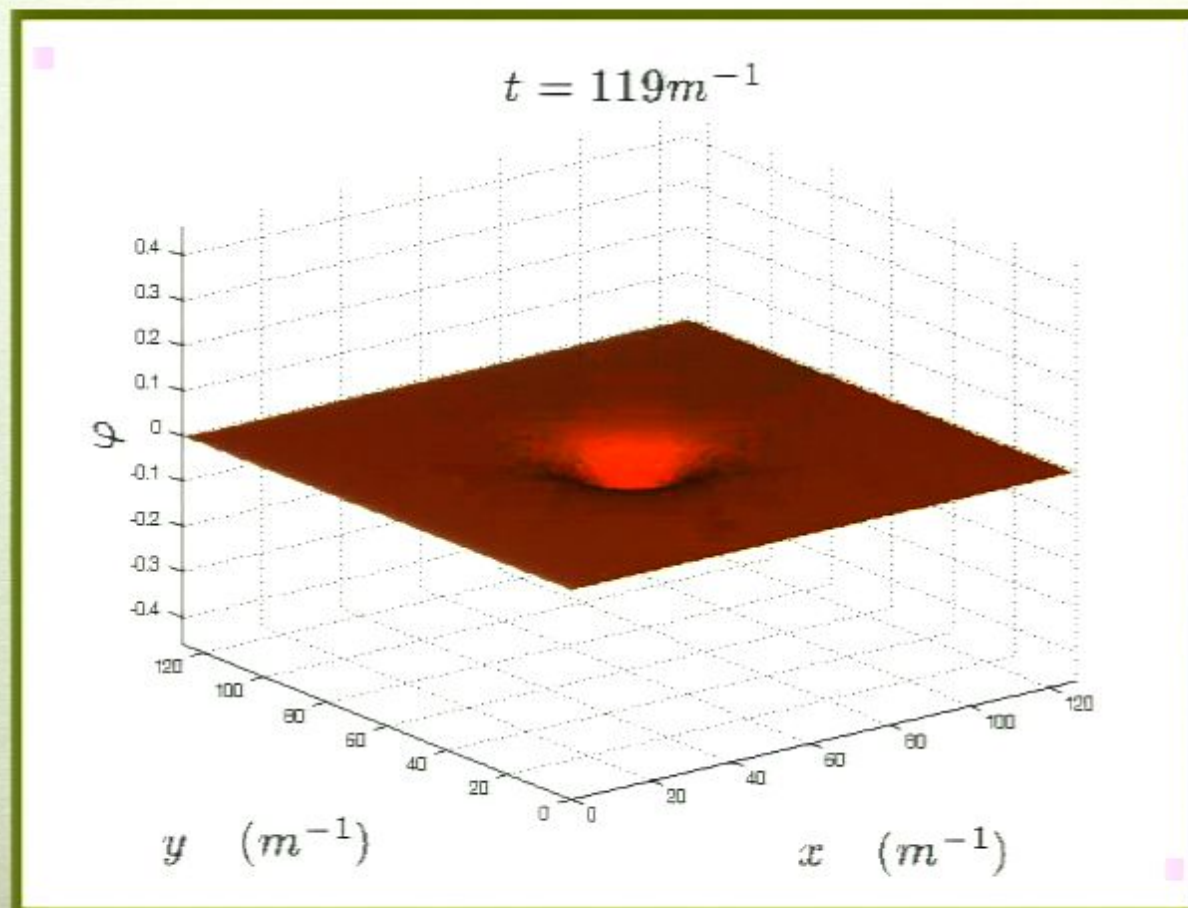
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

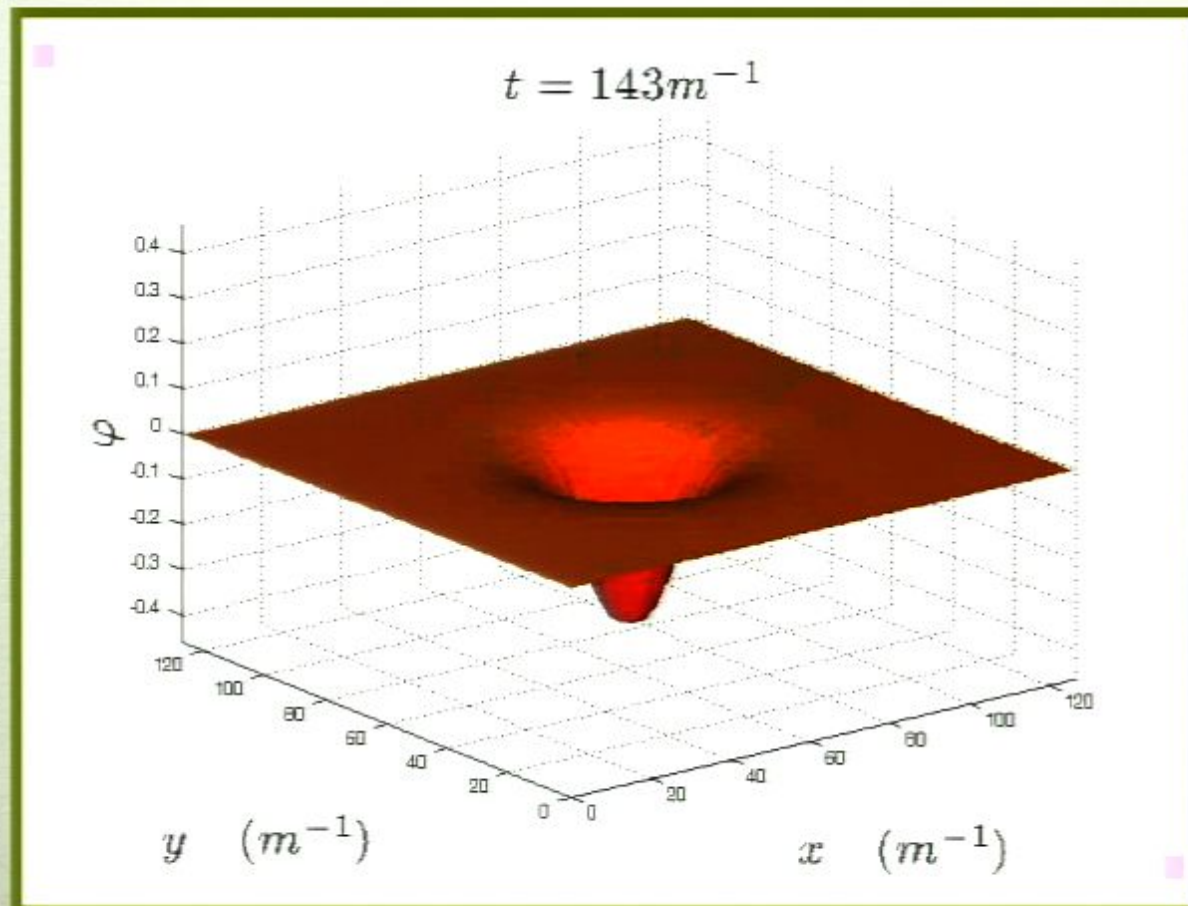
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

(1) oscillatory (2) spatially localized (3) **very long lived**

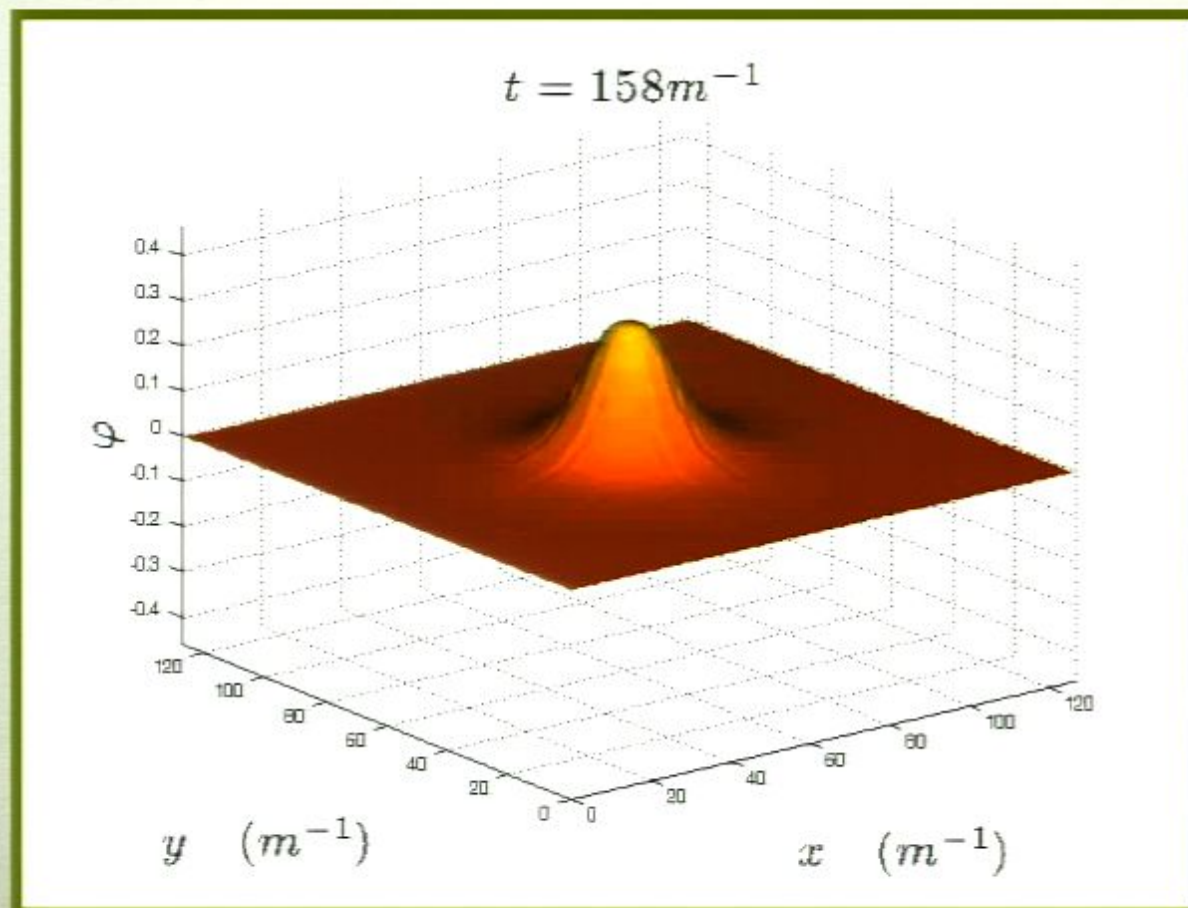




# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

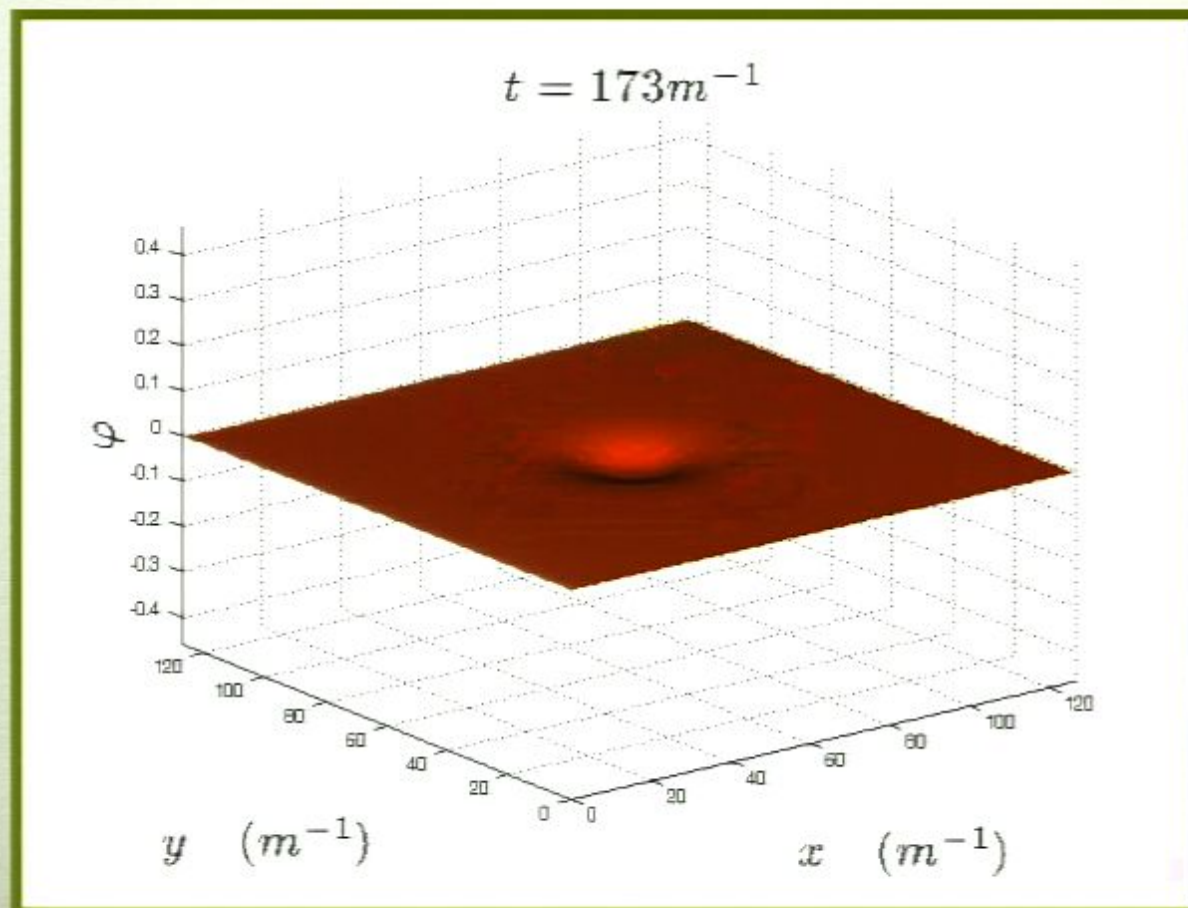
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

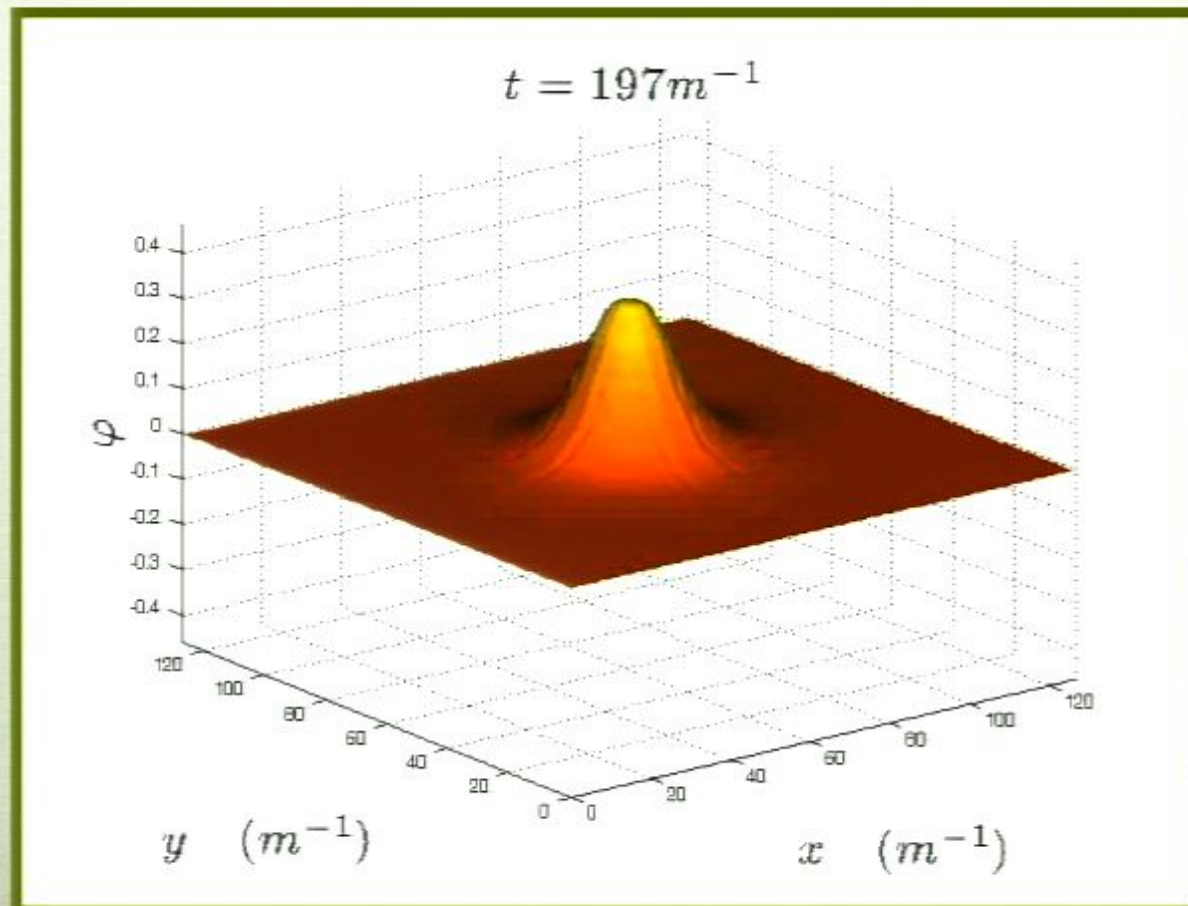
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(BREATHERS IN CONDENSED MATTER)

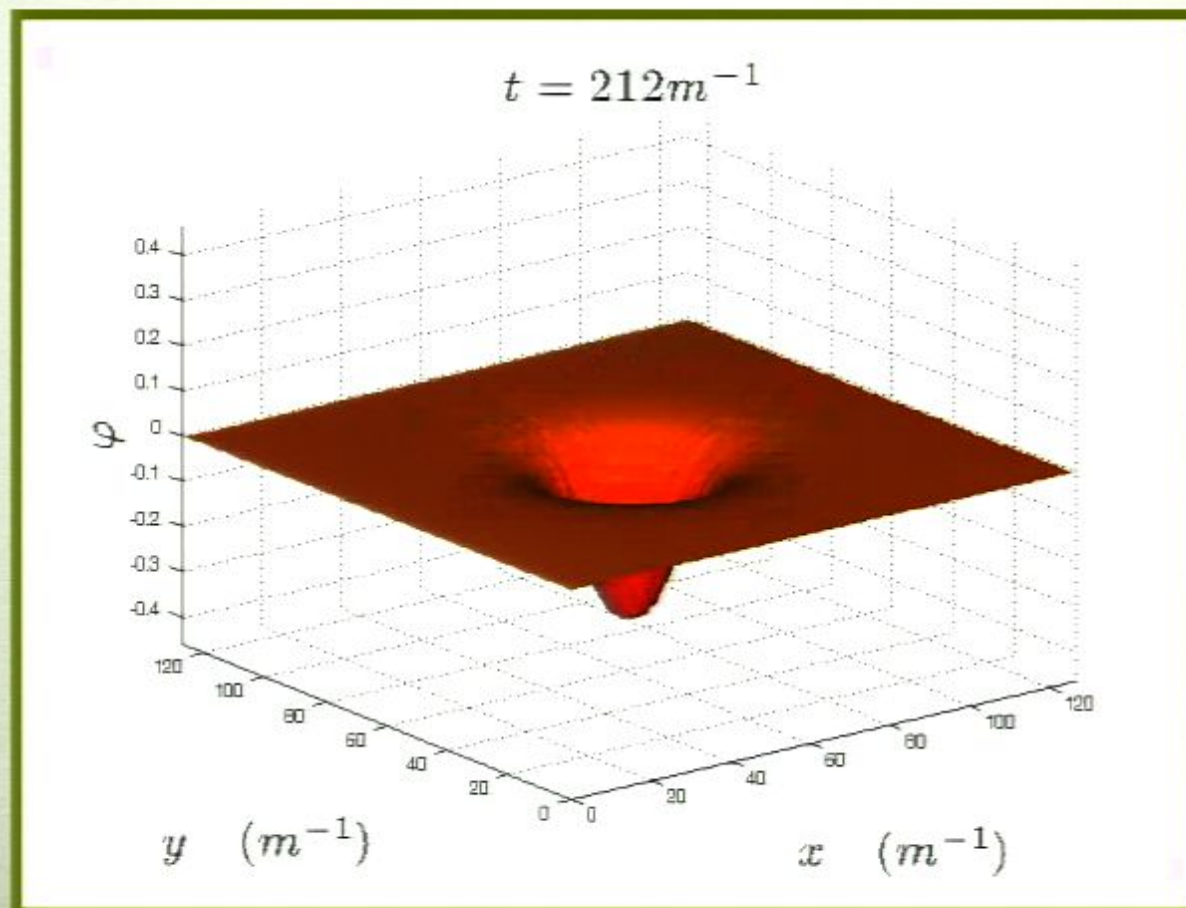
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# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

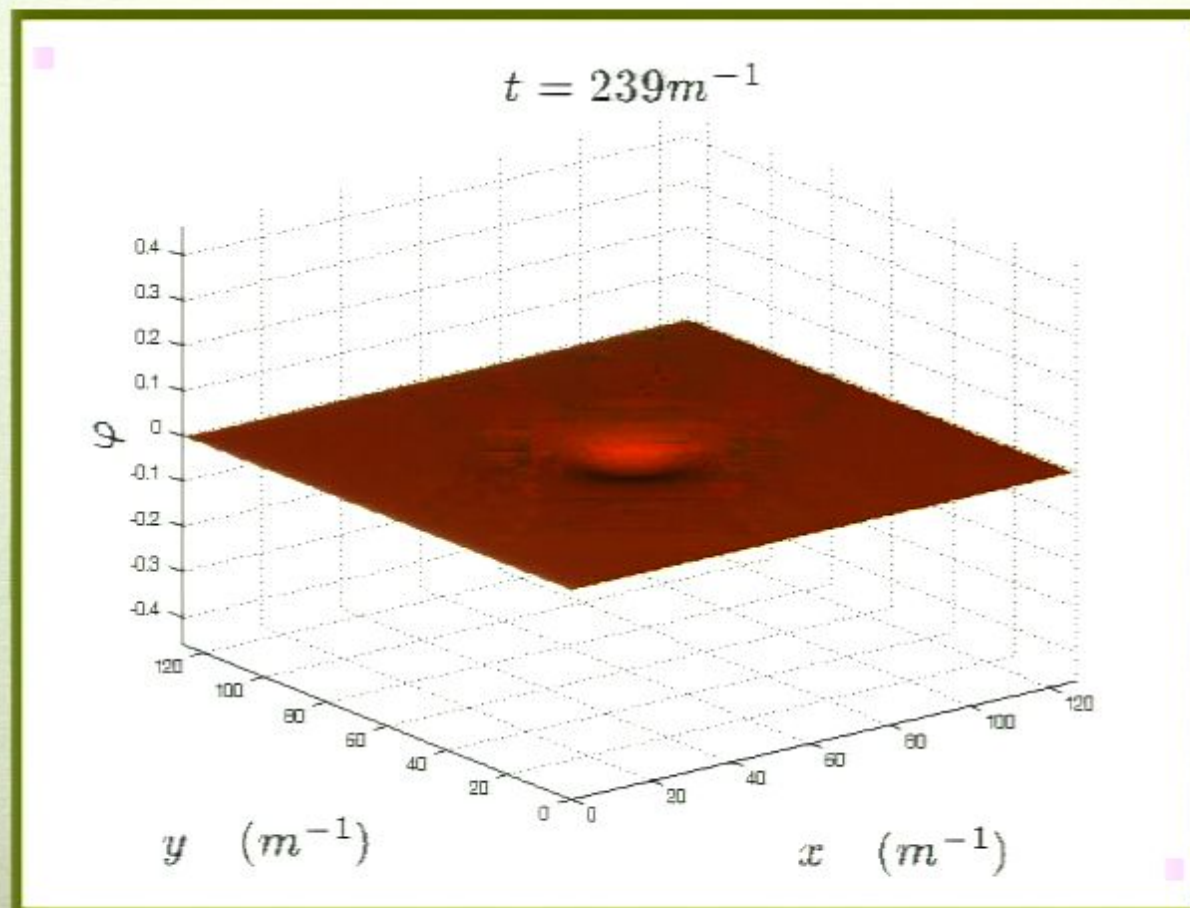
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# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

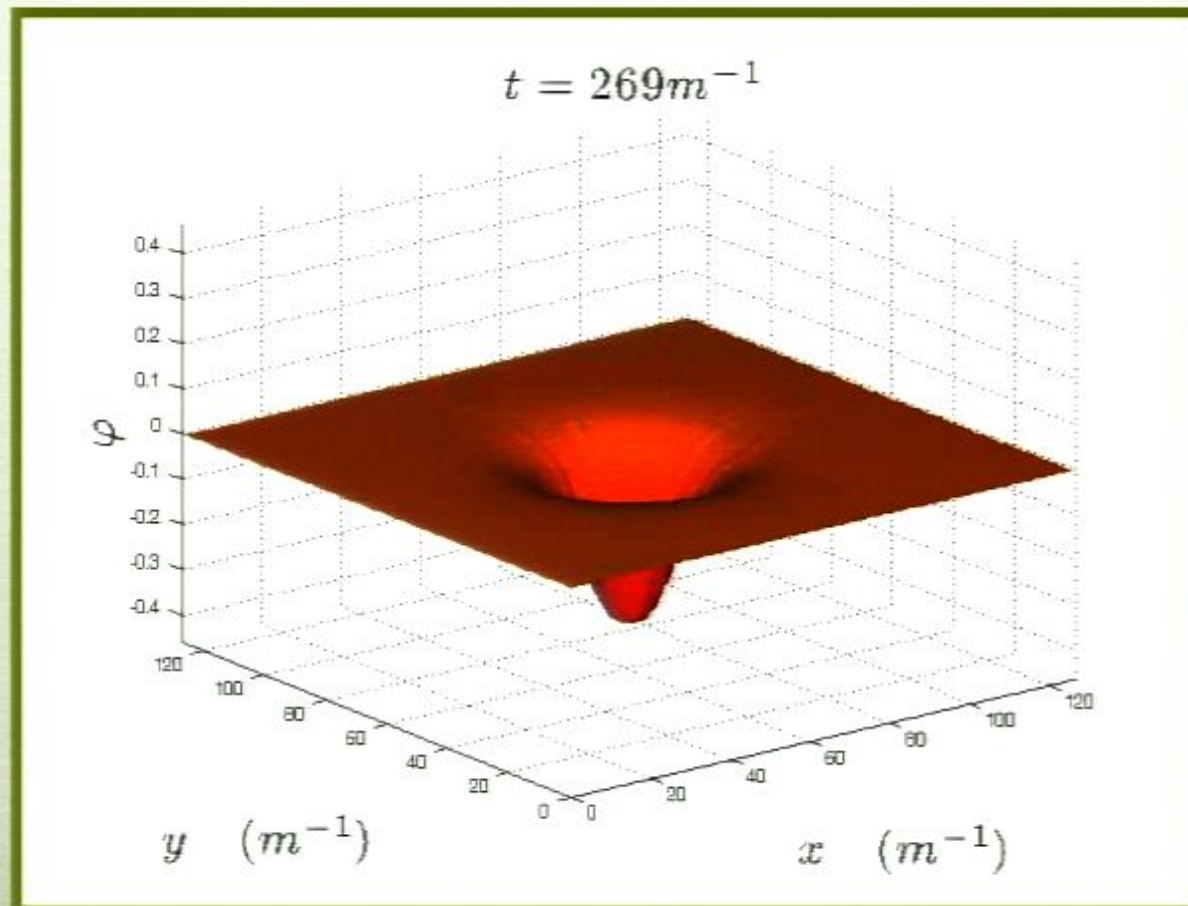
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(BREATHERS IN CONDENSED MATTER)

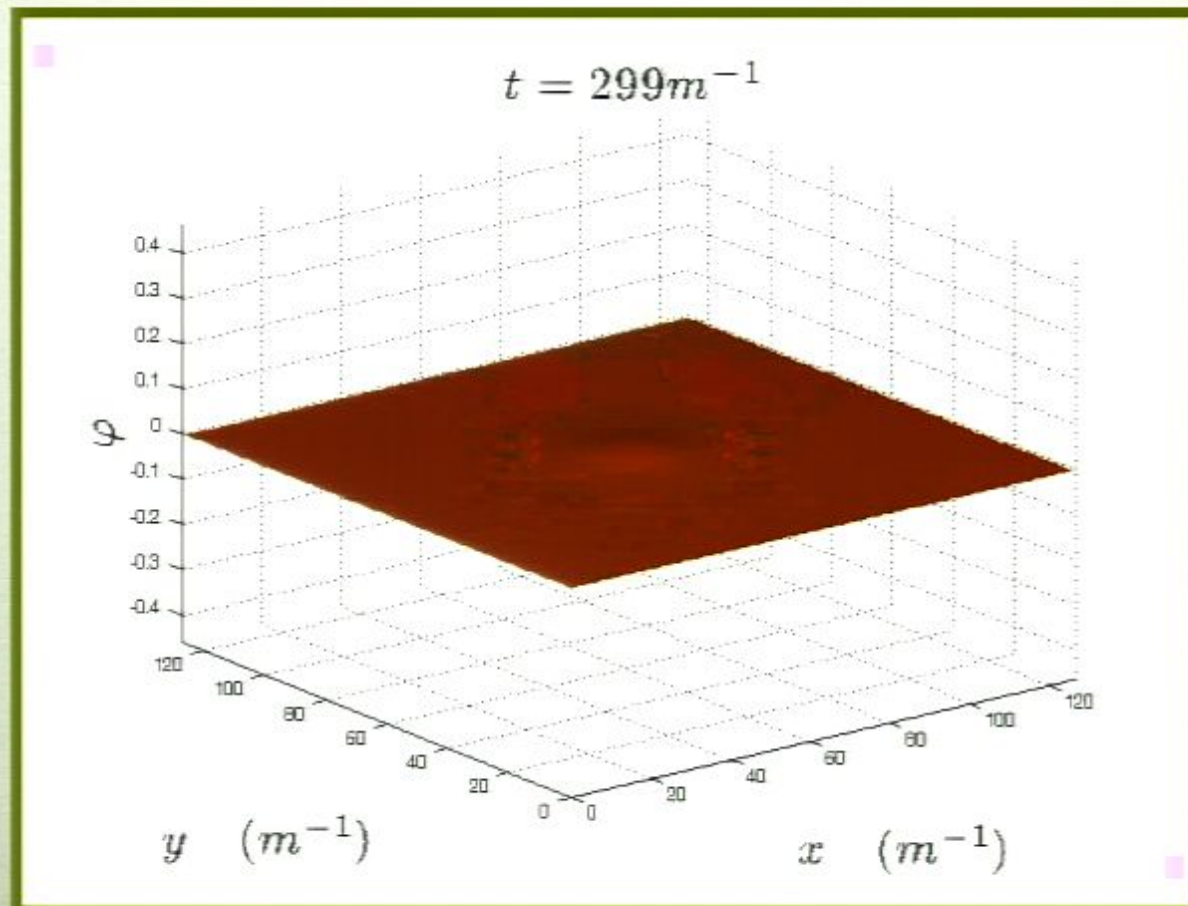
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(BREATHERS IN CONDENSED MATTER)

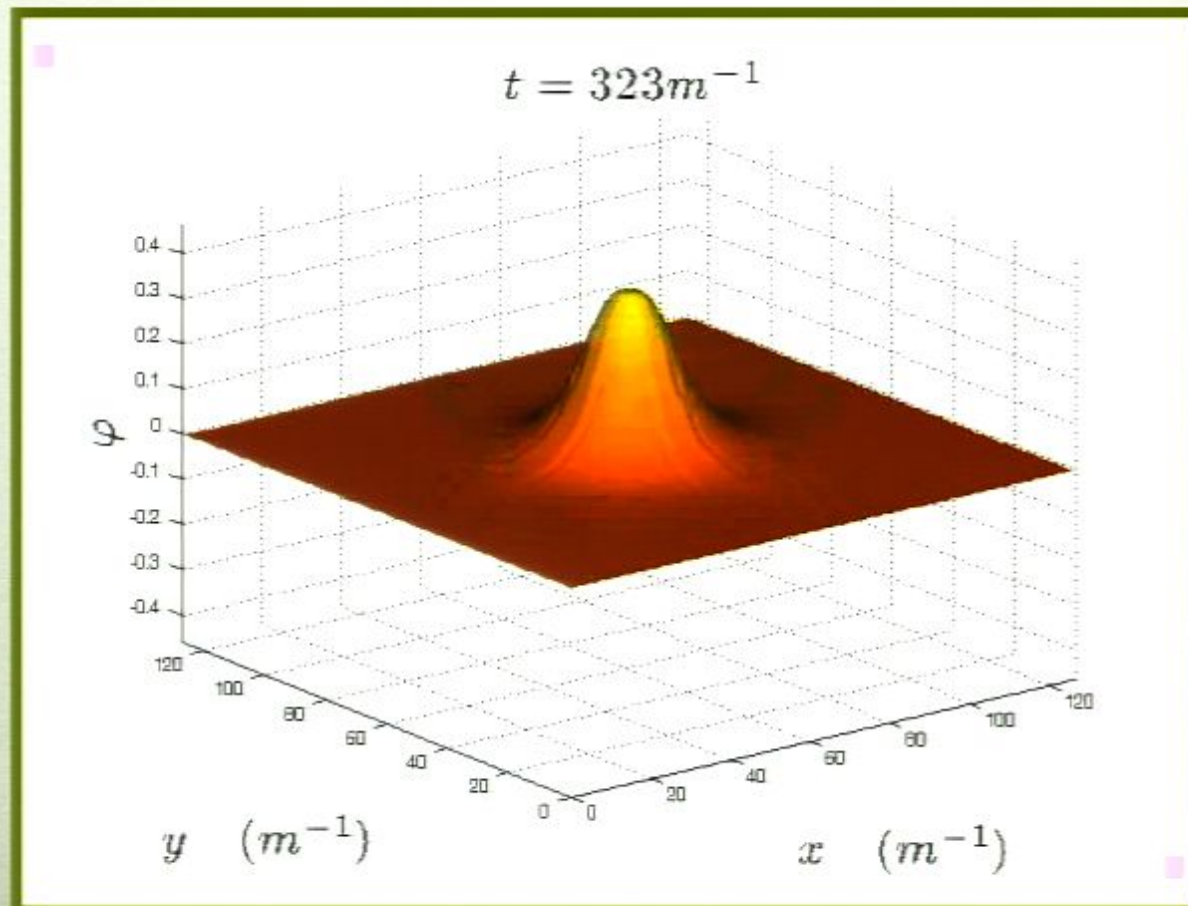
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(BREATHERS IN CONDENSED MATTER)

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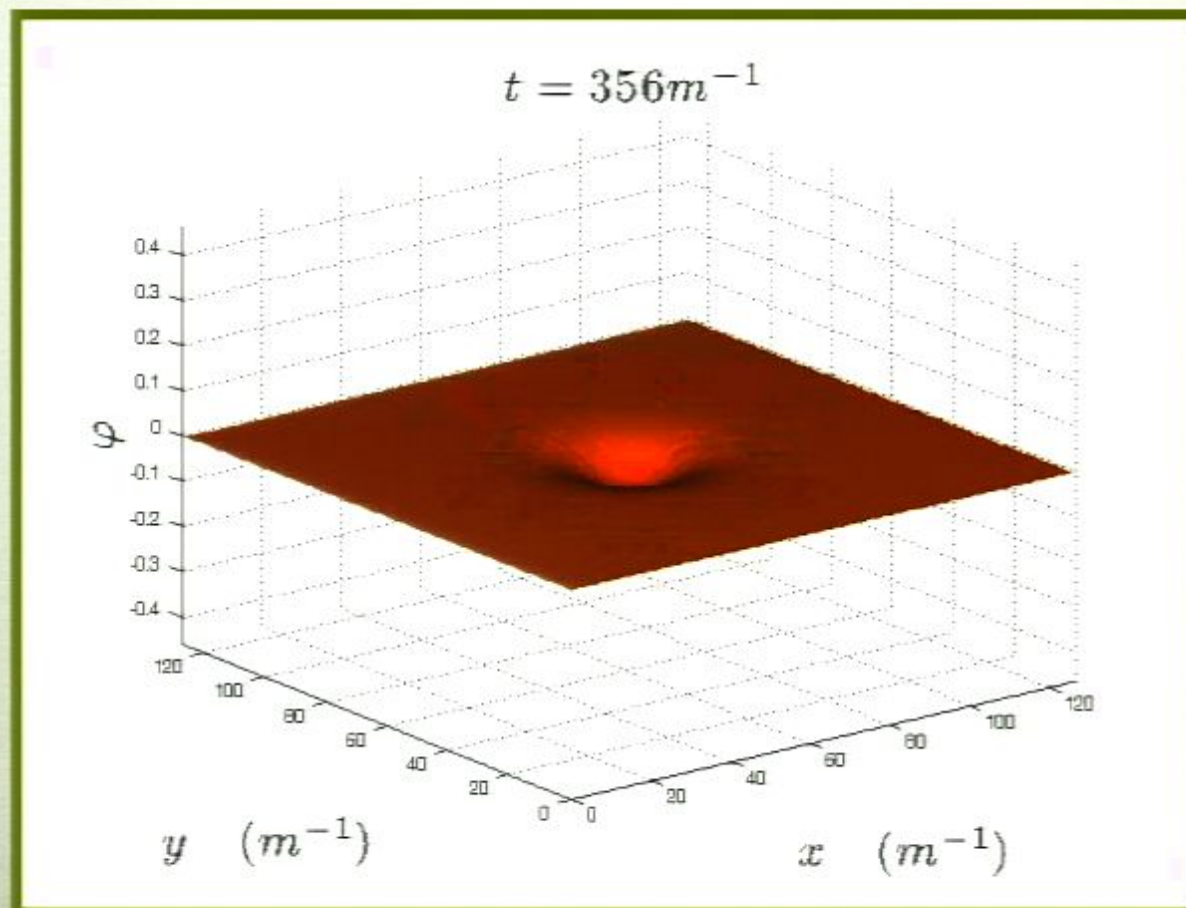




# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

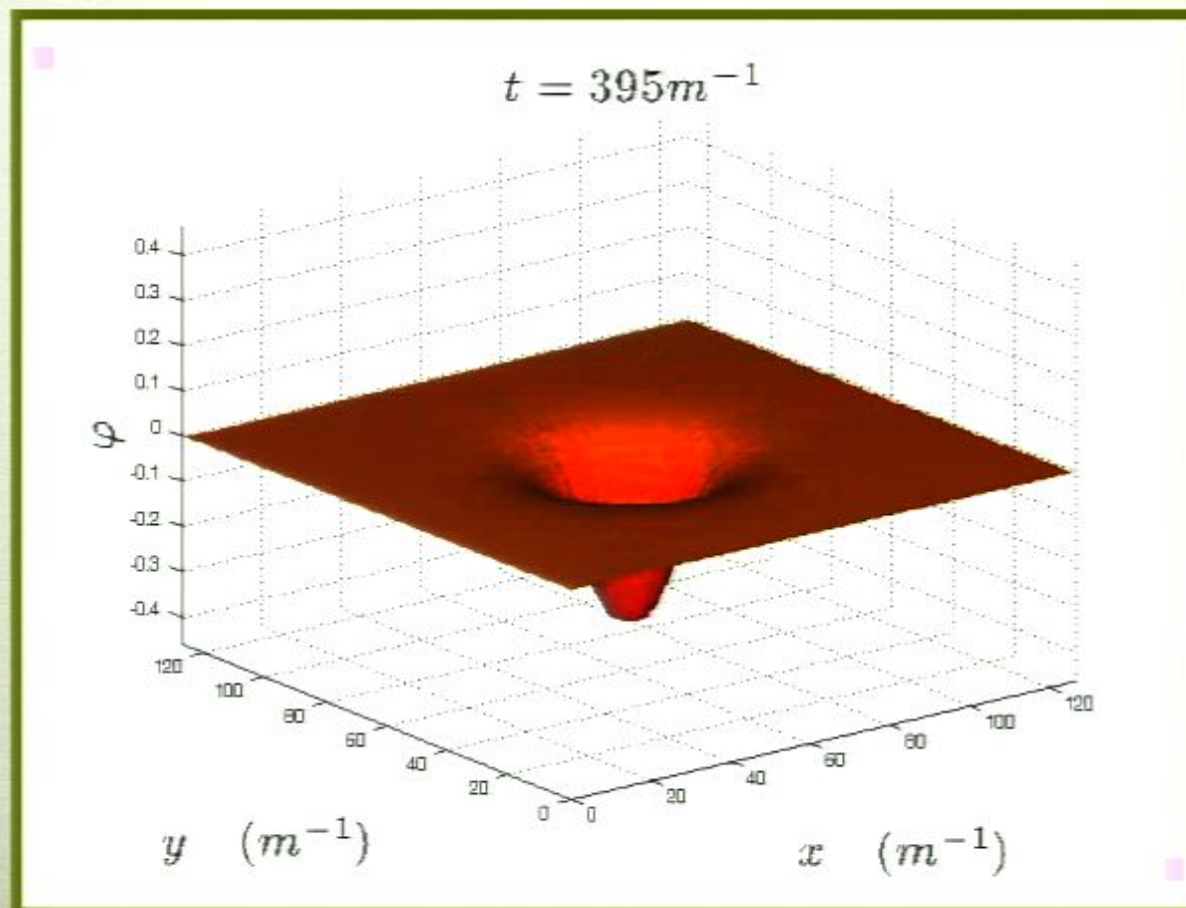
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

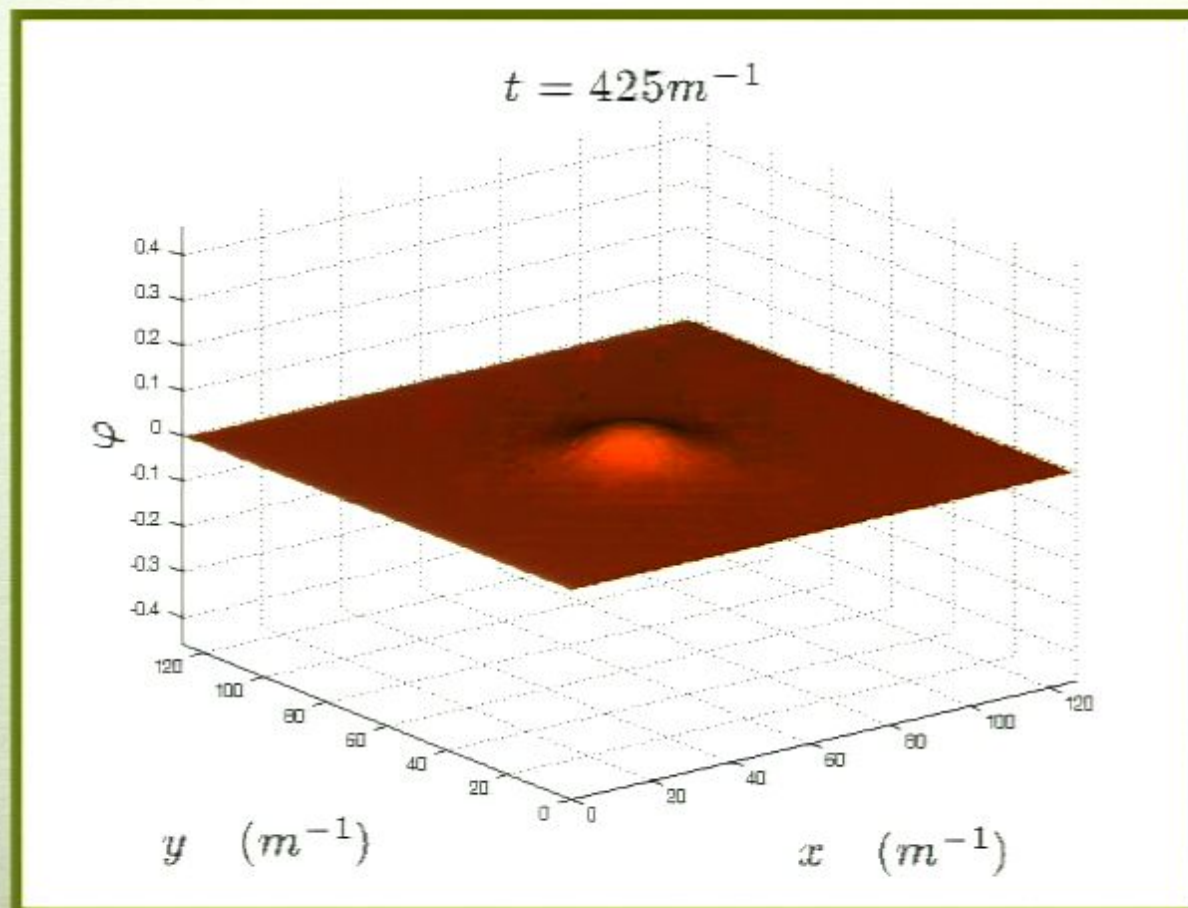
(1) oscillatory (2) spatially localized (3) **very long lived**



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(BREATHERS IN CONDENSED MATTER)

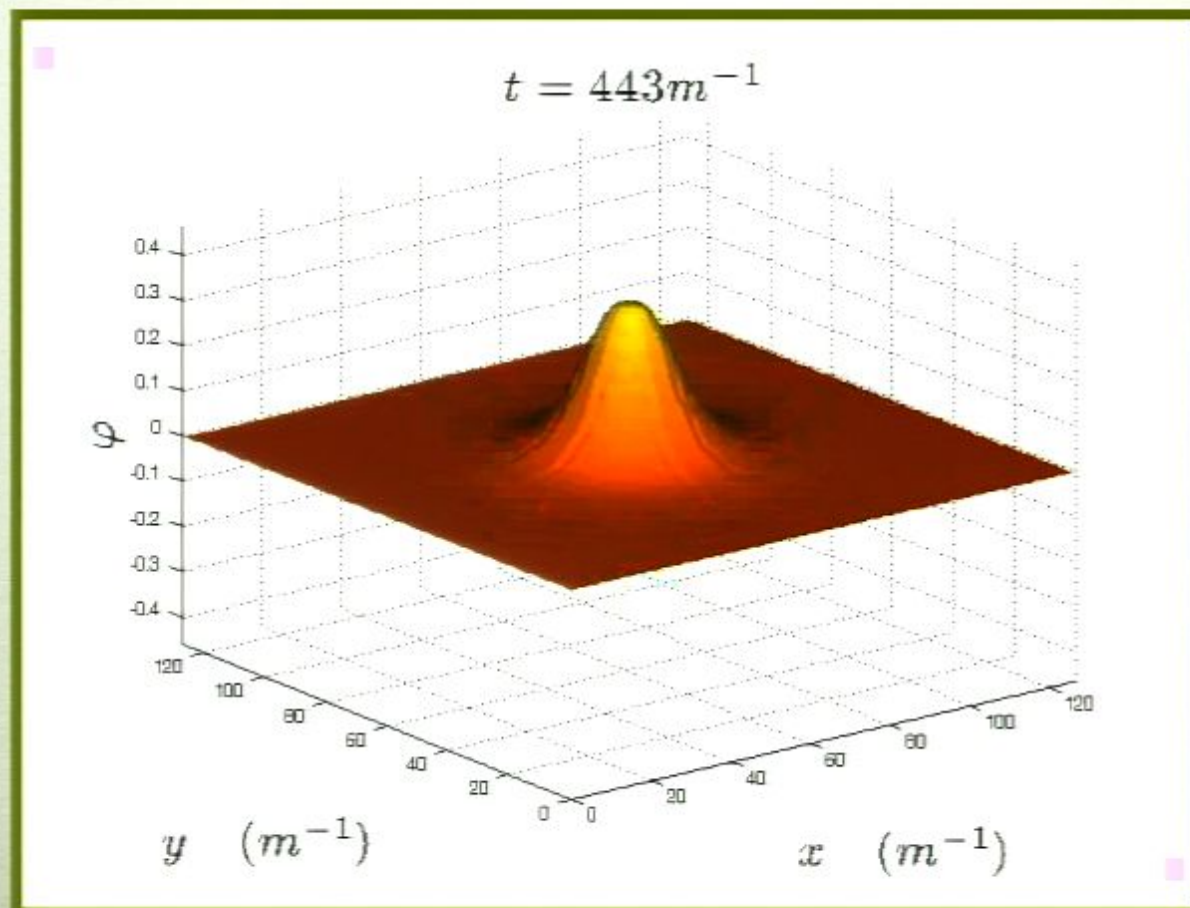
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(BREATHERS IN CONDENSED MATTER)

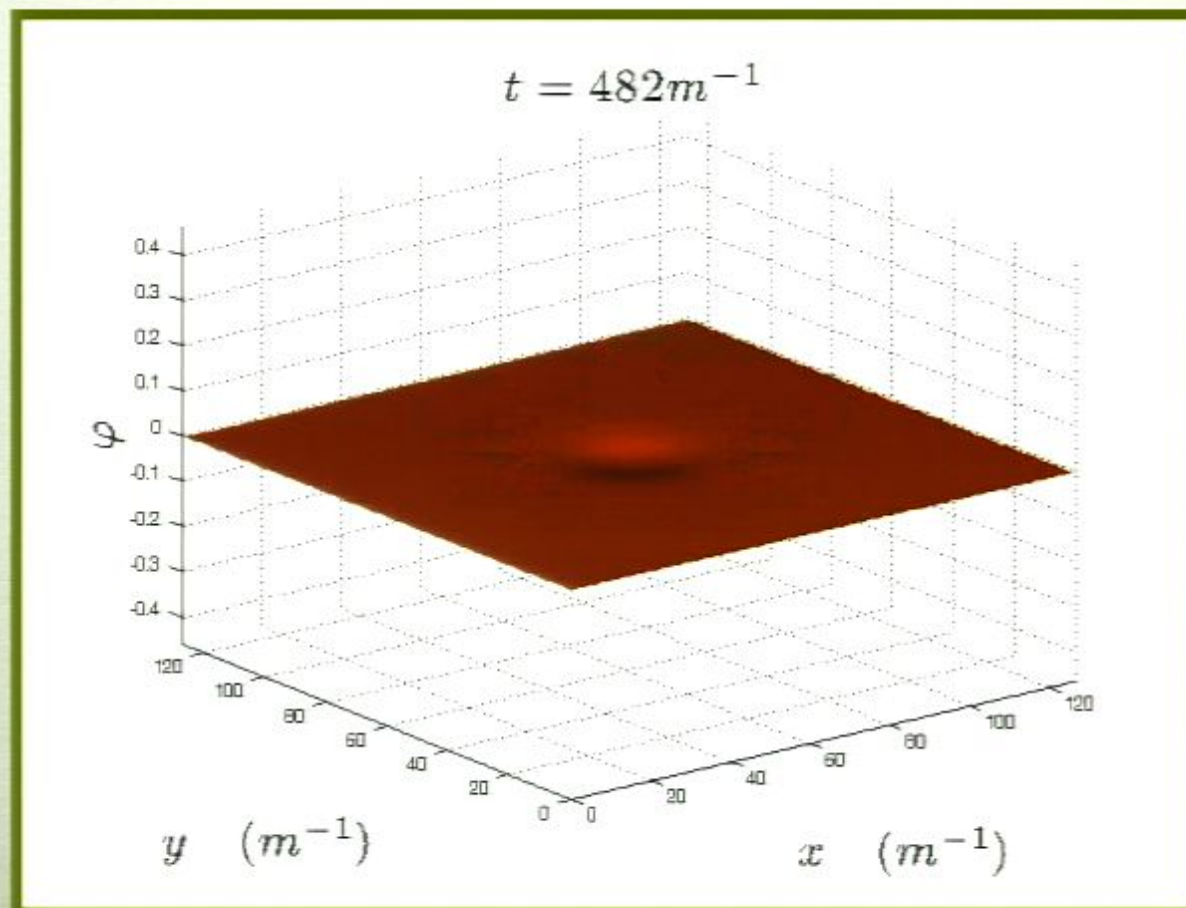
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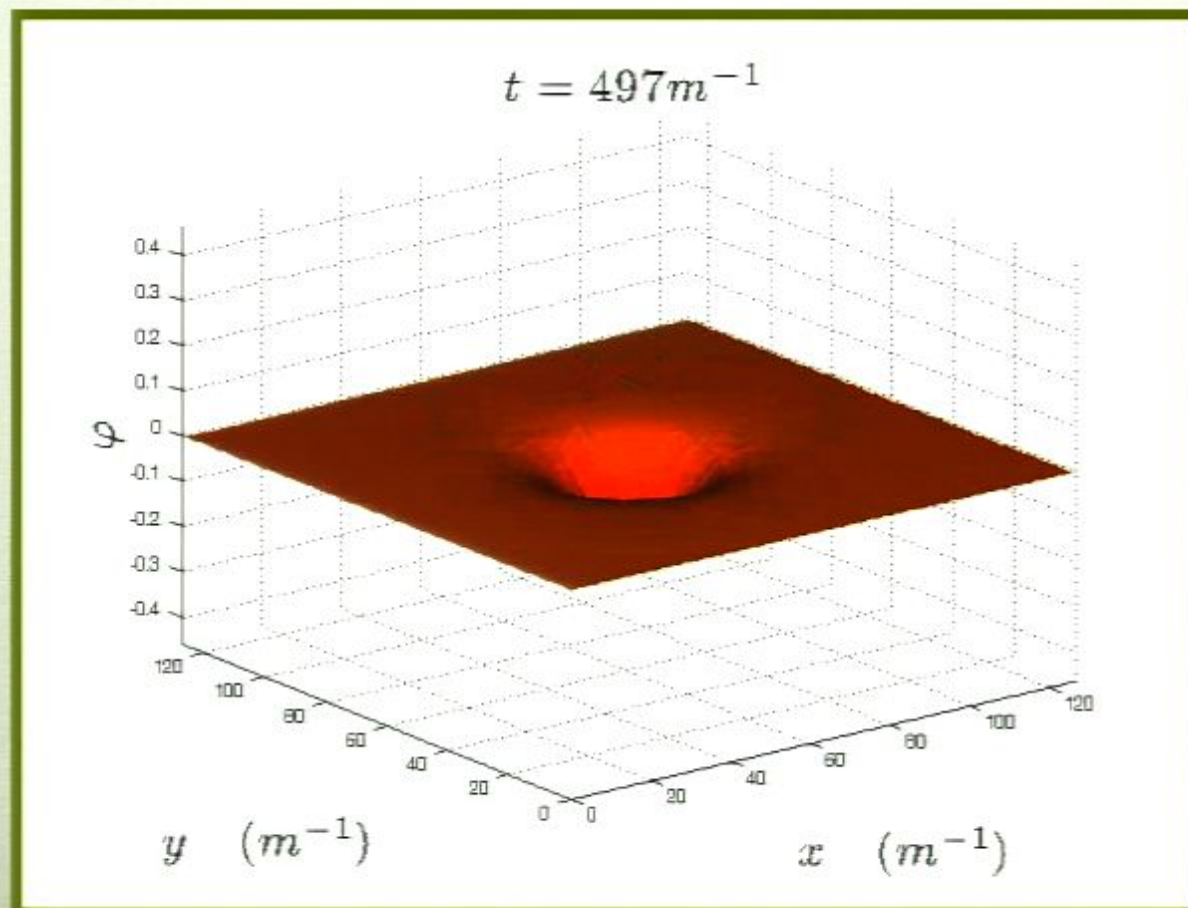
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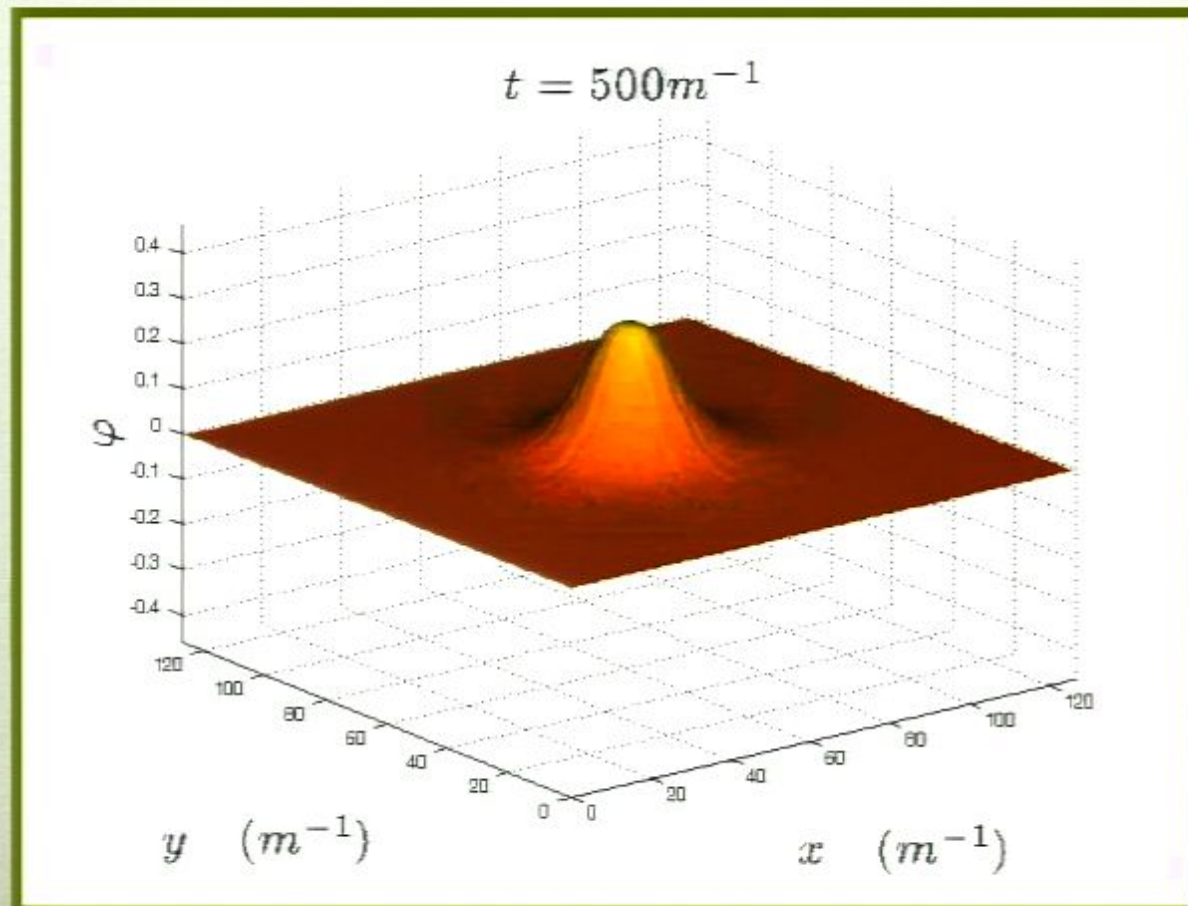
(1) oscillatory (2) spatially localized (3) **very long lived**



# OSCILLONS

(BREATHERS IN CONDENSED MATTER)

(1) oscillatory (2) spatially localized (3) **very long lived**



# NECESSARY CONDITION

---

$$V(\varphi) - \frac{1}{2}m^2\varphi^2 < 0$$

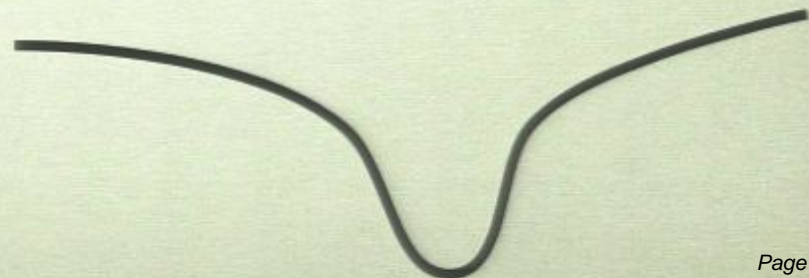
for some range of  $\varphi$



Examples:



symmetry breaking



axion monodromy





## WHY ?

$$\partial_t^2 \varphi - \partial_x^2 \varphi + V'(\varphi) = 0$$

localized periodic solution:  $\varphi(t, x) \sim \Phi(x) \cos \omega t$



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$$[(m^2 - \omega^2)\varphi] + [-\partial_x^2 \varphi] + [V'(\varphi) - m^2 \varphi] \sim 0$$

frequency    curvature    non-linearity



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frequency      curvature      non-linearity

+ve

+ve



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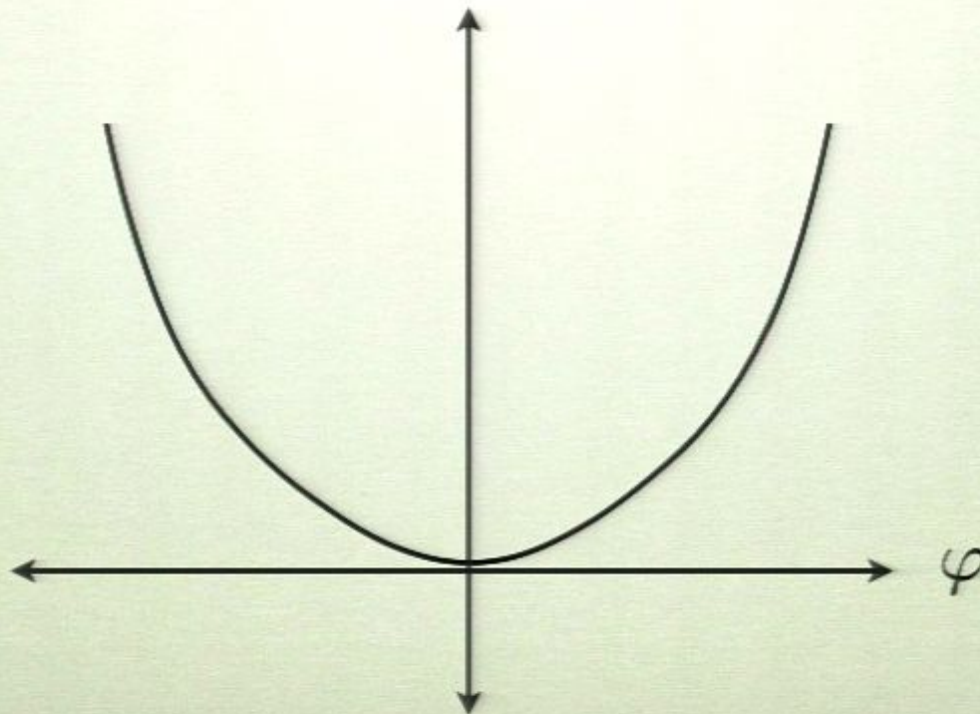
frequency      curvature      non-linearity

+ve

+ve

## EXAMPLE

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \frac{g^2}{6m^2}\varphi^6 \quad \lambda^2 \ll g^2$$



## HOW TO SOLVE -2

$$\phi = \left(\frac{\lambda}{g}\right) \phi_1 + \left(\frac{\lambda}{g}\right)^2 \phi_2 + \dots \quad \text{asymptotic expansion}$$

$$\partial_\tau^2 \phi_1 + \phi_1 = 0 \quad \Rightarrow \quad \phi_1 = \Phi_\alpha(\rho) \cos \tau \quad \text{oscillatory soln}$$

$$\partial_\tau^2 \phi_3 + \phi_3 = \alpha^2 \partial_\tau^2 \phi_1 + \partial_\rho^2 \phi_1 + \frac{2}{\rho} \partial_\rho \phi_1 + \phi_1^3 - \phi_1^5$$

$$\Rightarrow \quad \partial_\tau^2 \phi_3 + \phi_3 = \left[ -\alpha^2 \Phi + \partial_\rho^2 \Phi + \frac{2}{\rho} \partial_\rho \Phi + \frac{3}{4} \Phi^3 - \frac{5}{8} \Phi^5 \right] \cos \tau, \quad \text{kill resonance}$$

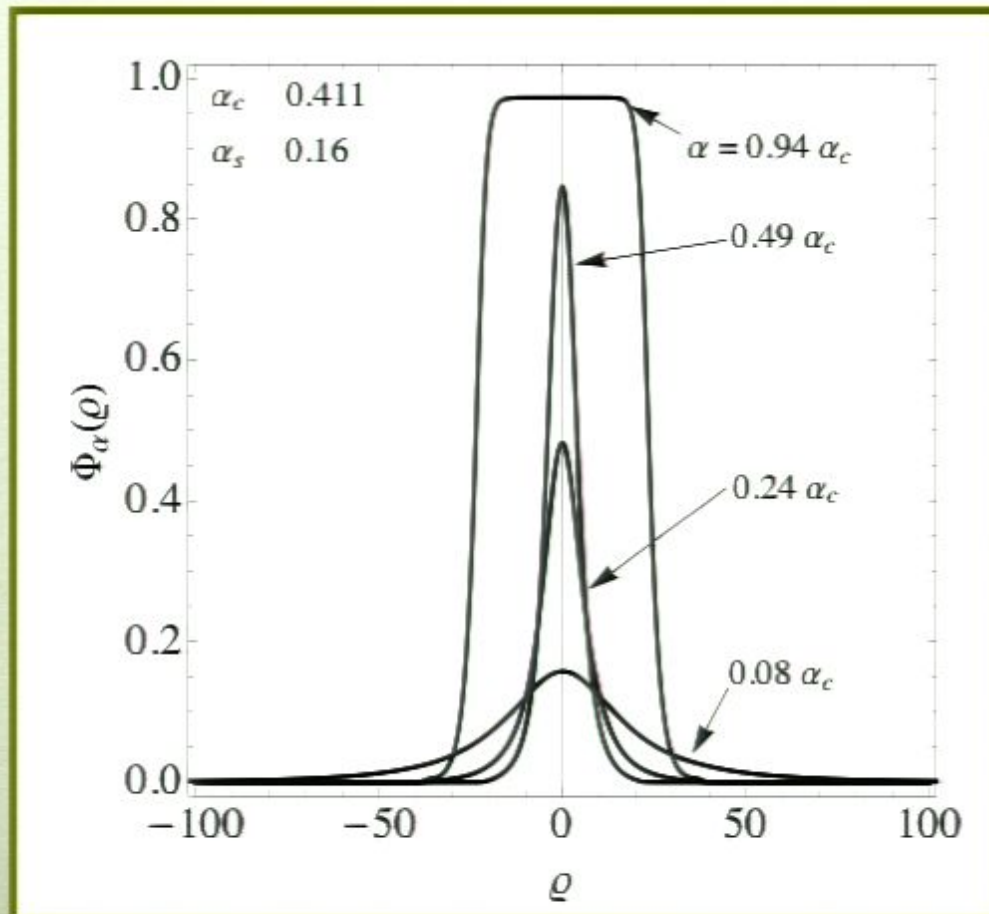
$$+ \left[ \frac{1}{4} \Phi^3 - \frac{5}{16} \Phi^5 \right] \cos 3\tau - \frac{1}{16} \Phi^5 \cos 5\tau.$$

$$\Rightarrow \quad \partial_\rho^2 \Phi_\alpha + \frac{2}{\rho} \partial_\rho \Phi_\alpha - \alpha^2 \Phi_\alpha + \frac{3}{4} \Phi_\alpha^3 - \frac{5}{8} \Phi_\alpha^5 = 0 \quad \text{profile equation}$$

# PROFILES

$$\partial_{\rho}^2 \Phi_{\alpha} + \frac{2}{\rho} \partial_{\rho} \Phi_{\alpha} - \alpha^2 \Phi_{\alpha} + \frac{3}{4} \Phi_{\alpha}^3 - \frac{5}{8} \Phi_{\alpha}^5 = 0$$

field amplitude





# HOW TO SOLVE -1

---

$$\square\varphi = V'(\varphi)$$

$$\partial_t^2\varphi - \partial_r^2\varphi - \frac{2}{r}\partial_r\varphi + m^2\varphi - \lambda\varphi^3 + \frac{g^2}{m^2}\varphi^5 = 0$$

$$\phi = \frac{\sqrt{\lambda}}{m}\varphi$$

$$\varrho = (\lambda/g)mr$$

$$\tau = \omega t$$

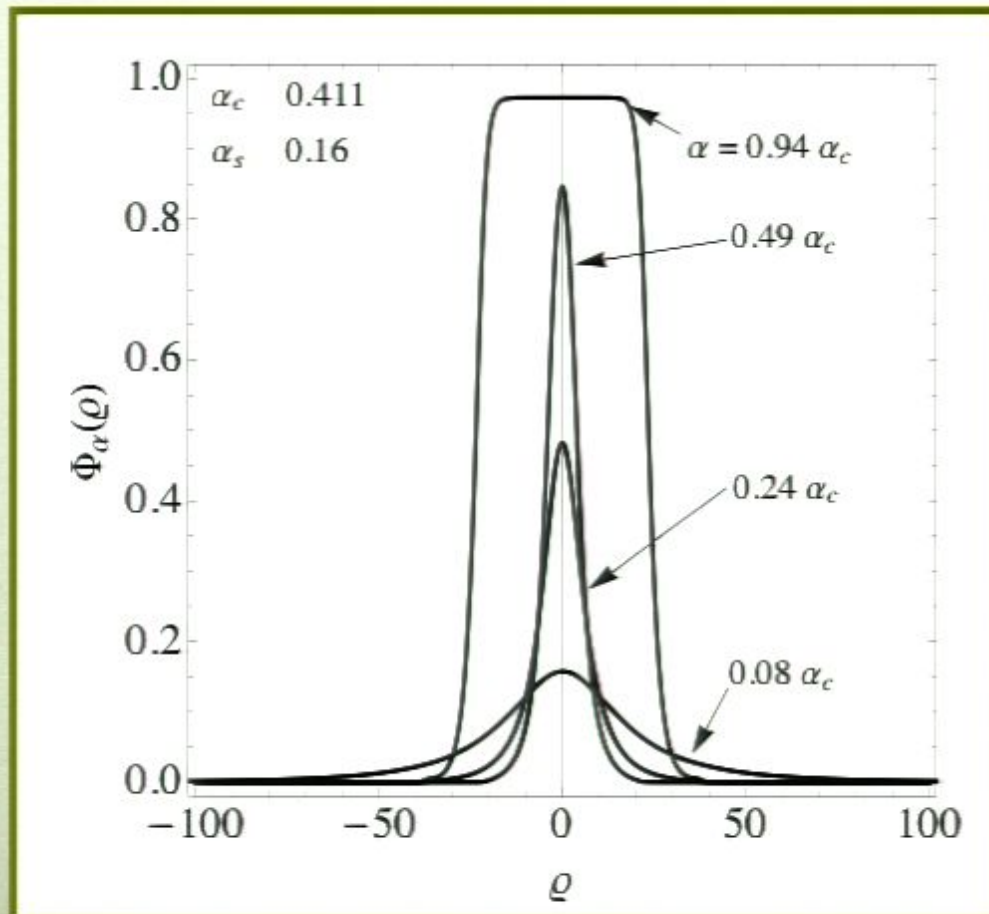
$$\omega^2 = m^2 [1 - (\lambda/g)^2\alpha^2]$$

$$\partial_\tau^2\phi + \phi + \frac{\lambda}{g} \left[ -\alpha^2\partial_\tau^2\phi - \partial_\varrho^2\phi - \frac{2}{\varrho}\partial_\varrho\phi - \phi^3 + \phi^5 \right] = \mathcal{O}[(\lambda/g)^3]$$

# PROFILES

$$\partial_{\rho}^2 \Phi_{\alpha} + \frac{2}{\rho} \partial_{\rho} \Phi_{\alpha} - \alpha^2 \Phi_{\alpha} + \frac{3}{4} \Phi_{\alpha}^3 - \frac{5}{8} \Phi_{\alpha}^5 = 0$$

field amplitude



# HOW TO SOLVE

---

$$\square\varphi = V'(\varphi)$$

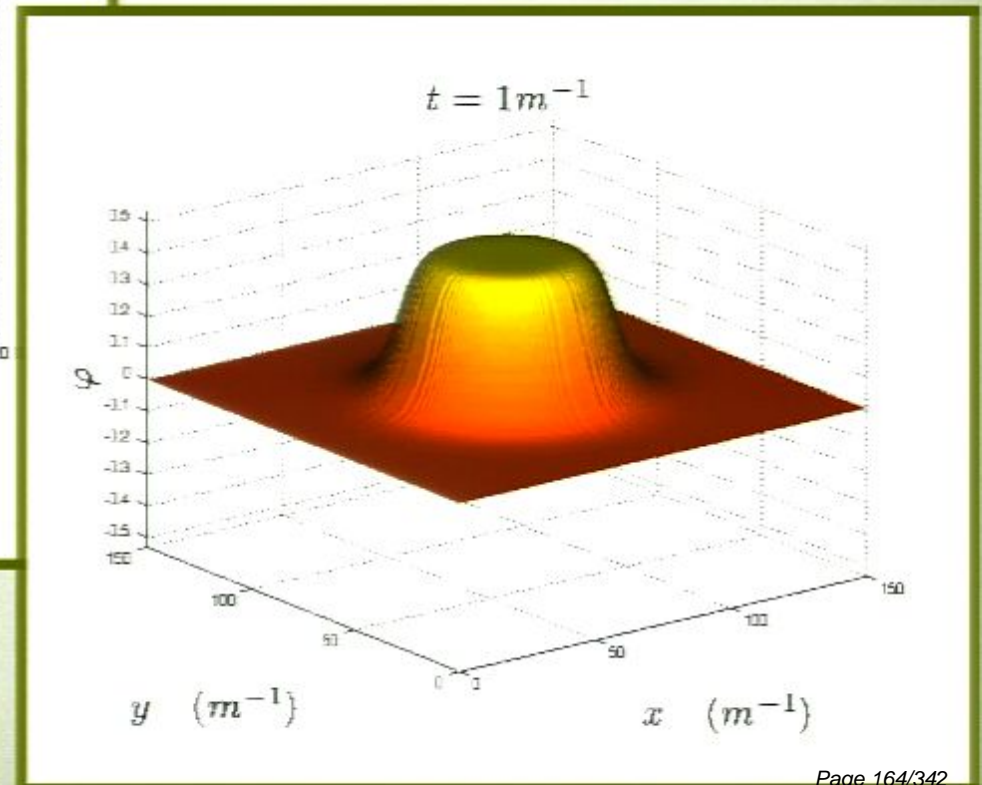
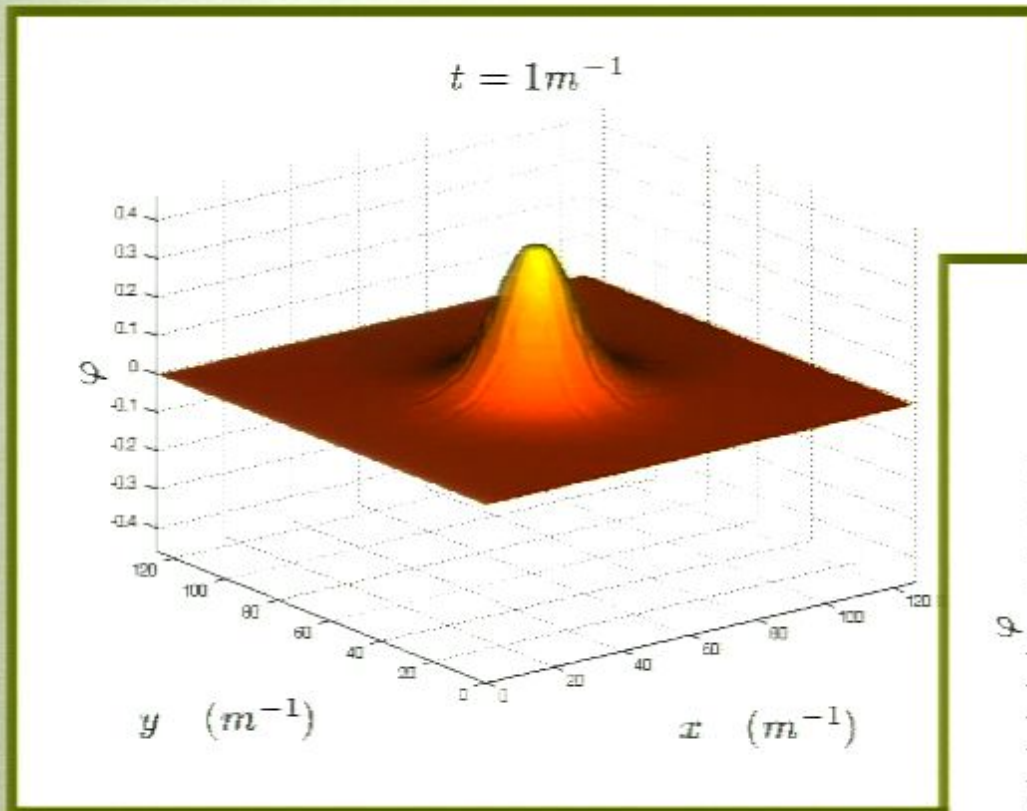
$$\varrho = (\lambda/g)mr$$

$$\omega^2 = m^2 [1 - (\lambda/g)^2\alpha^2]$$

$$\varphi(t, r) \approx \frac{m}{\sqrt{\lambda}} \left[ \left( \frac{\lambda}{g} \right) \Phi_\alpha(\varrho) \cos \omega t + \mathcal{O}[(\lambda/g)^3] \right]$$

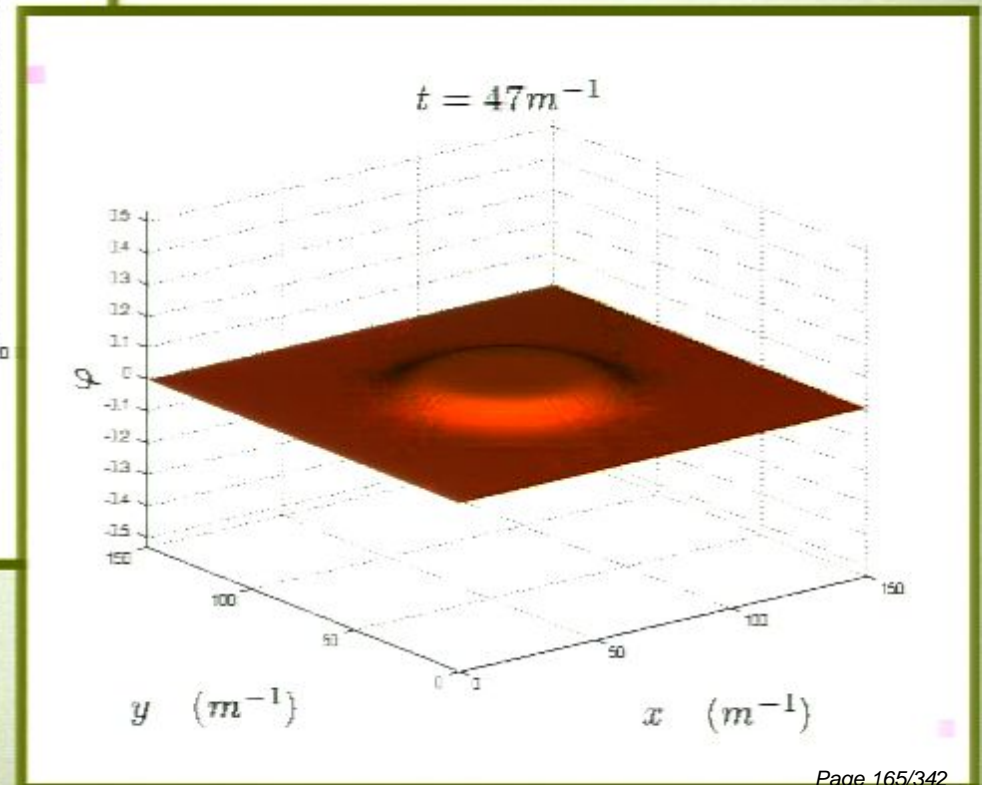
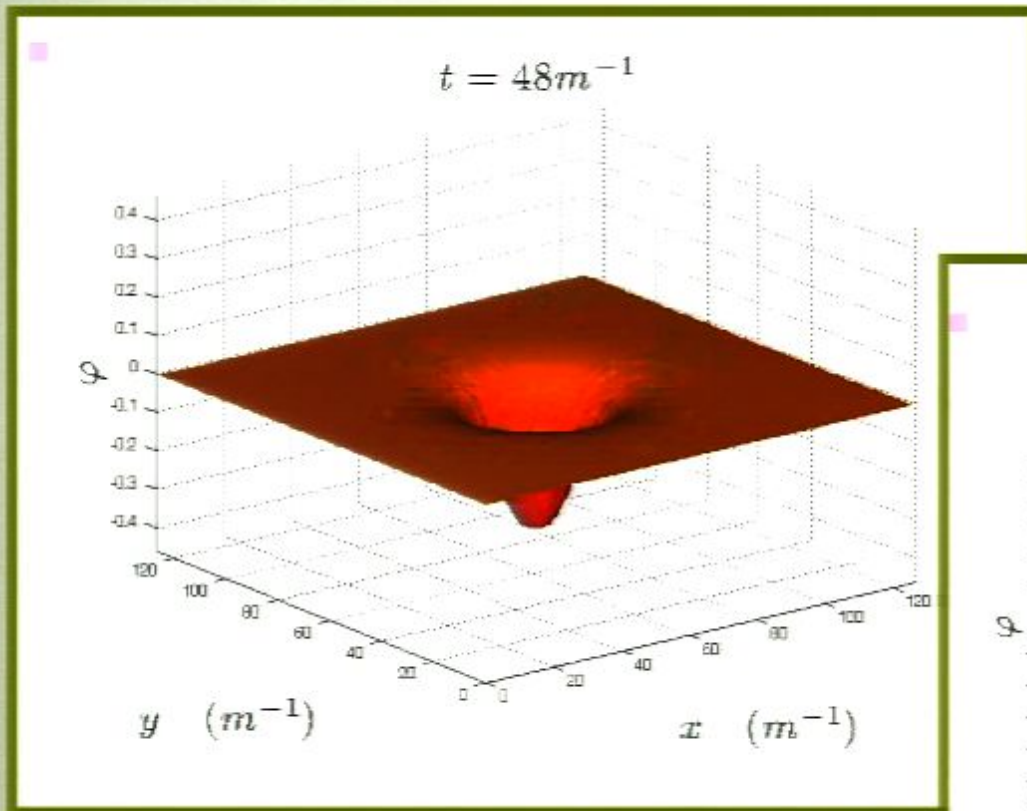
3+1

# SOLUTIONS



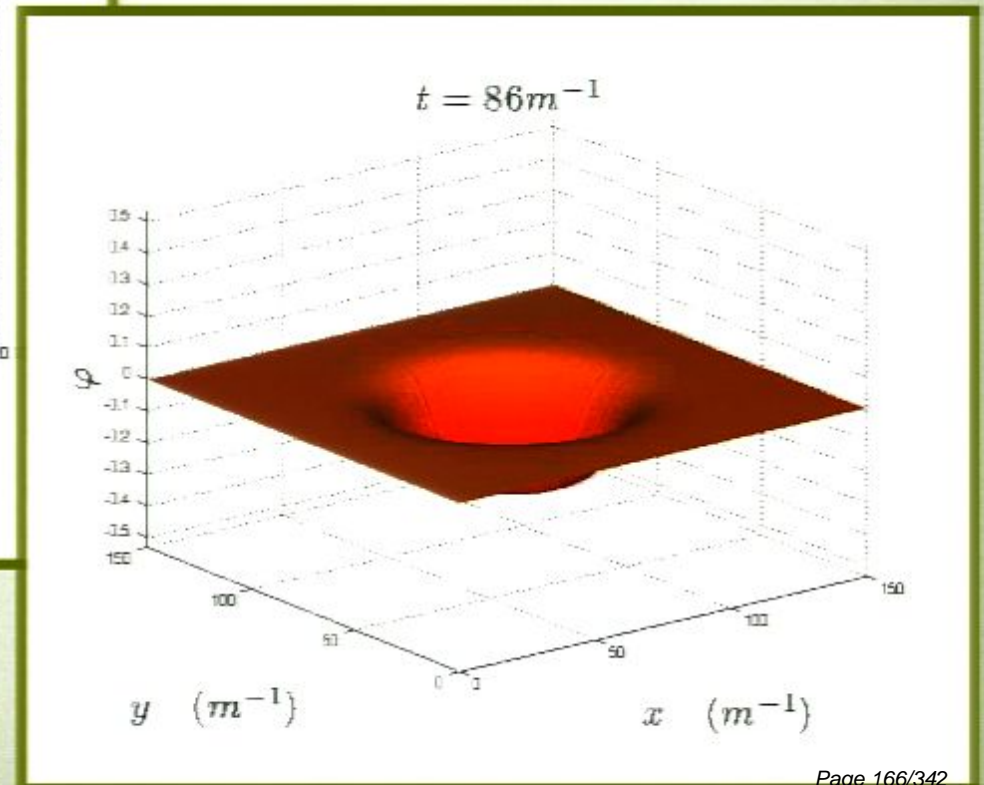
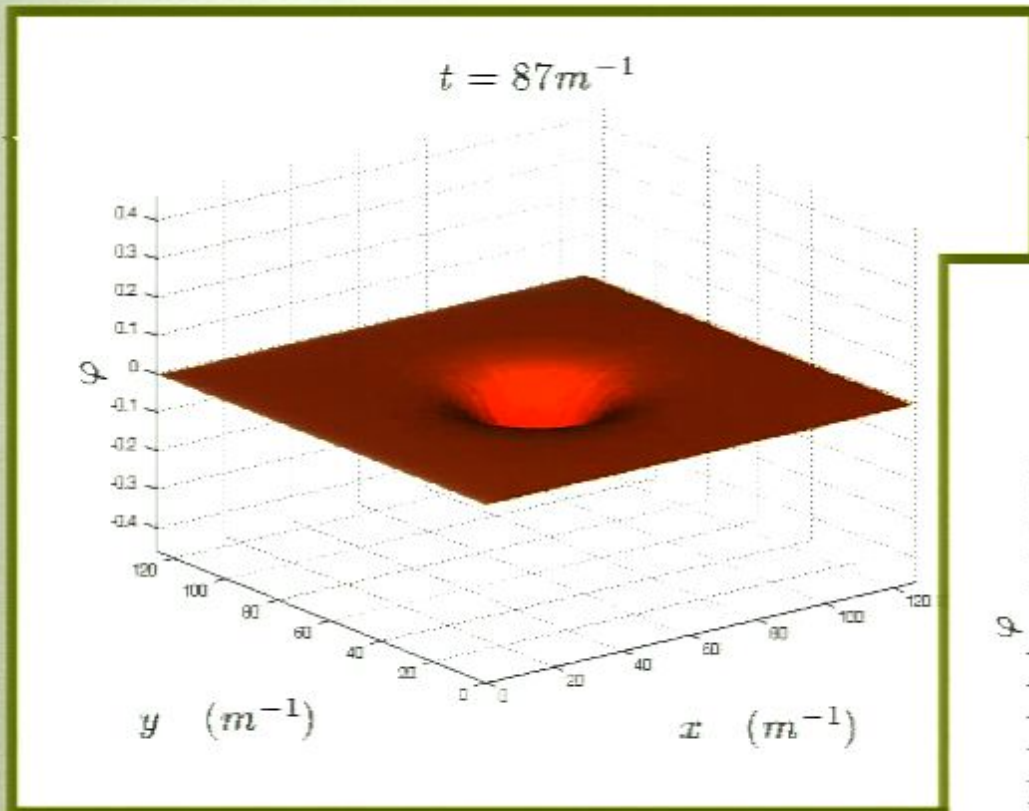
3+1

# SOLUTIONS



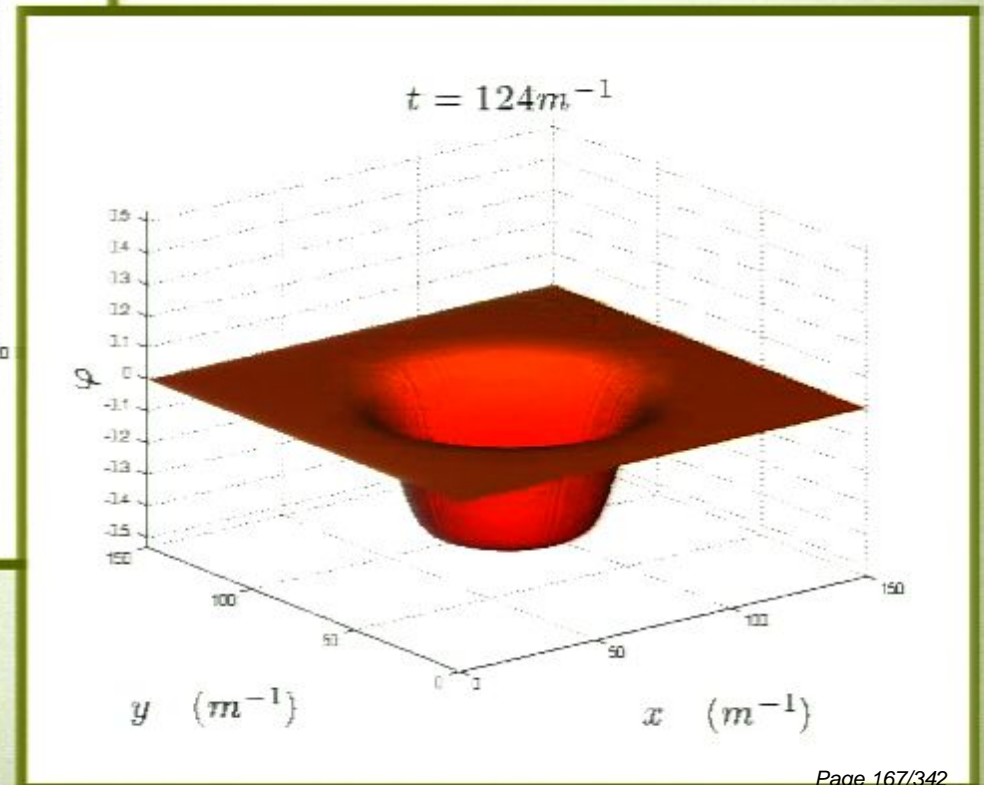
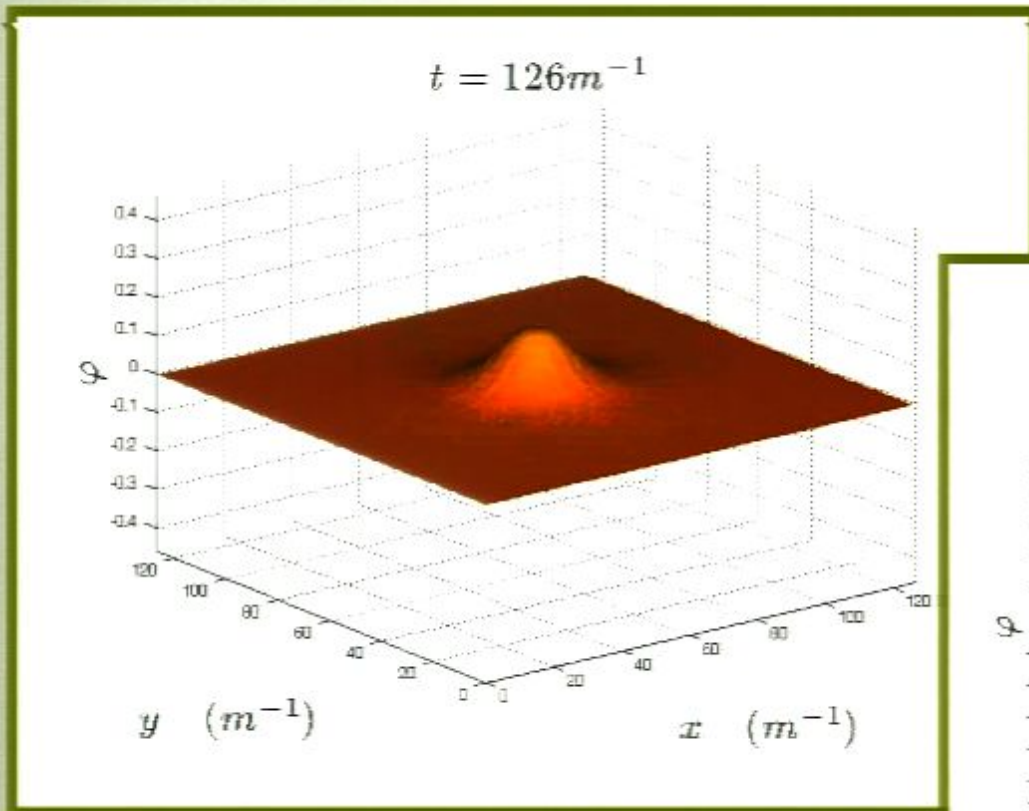
3+1

# SOLUTIONS



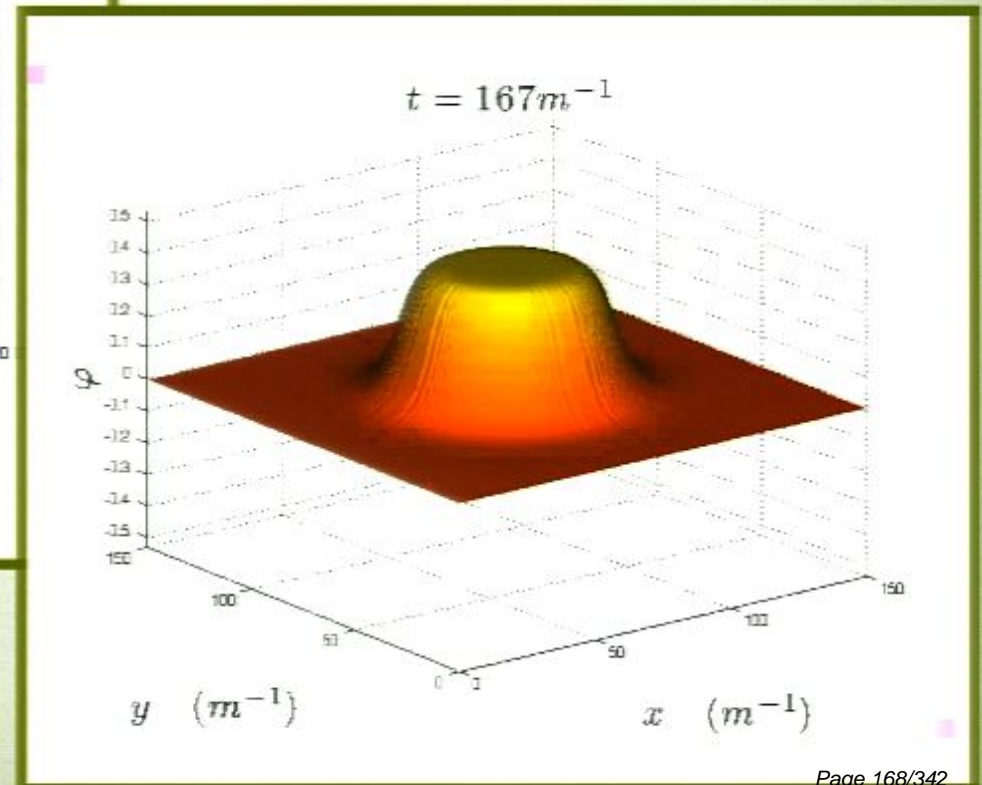
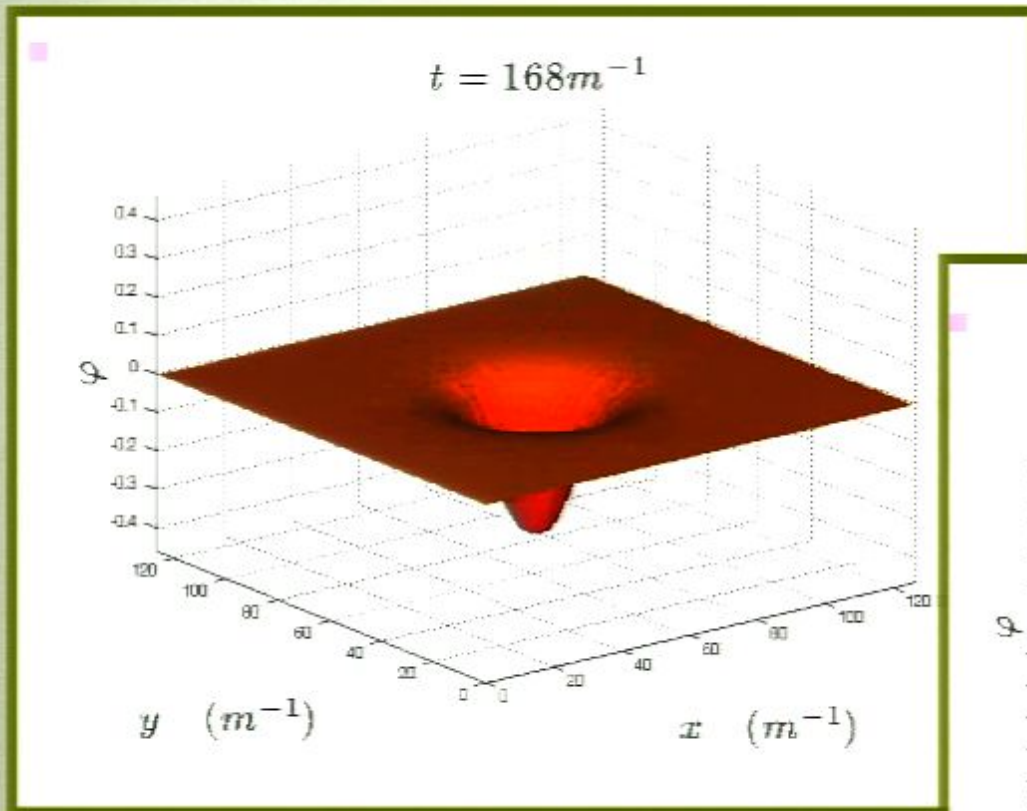
3+1

# SOLUTIONS



3+1

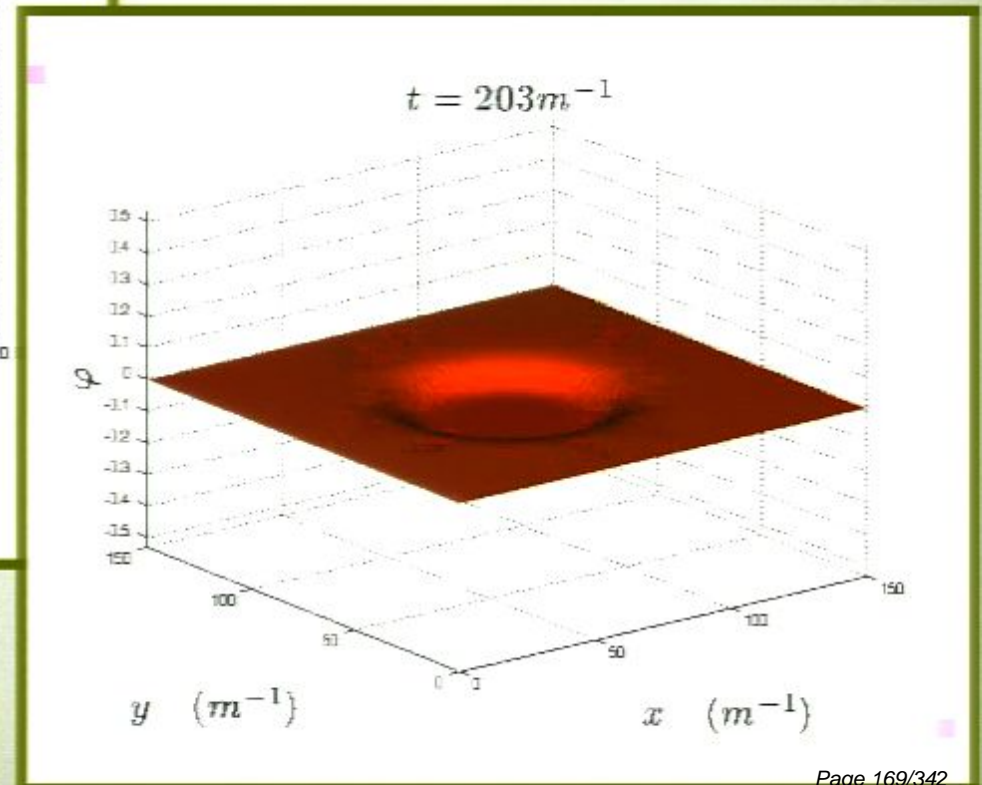
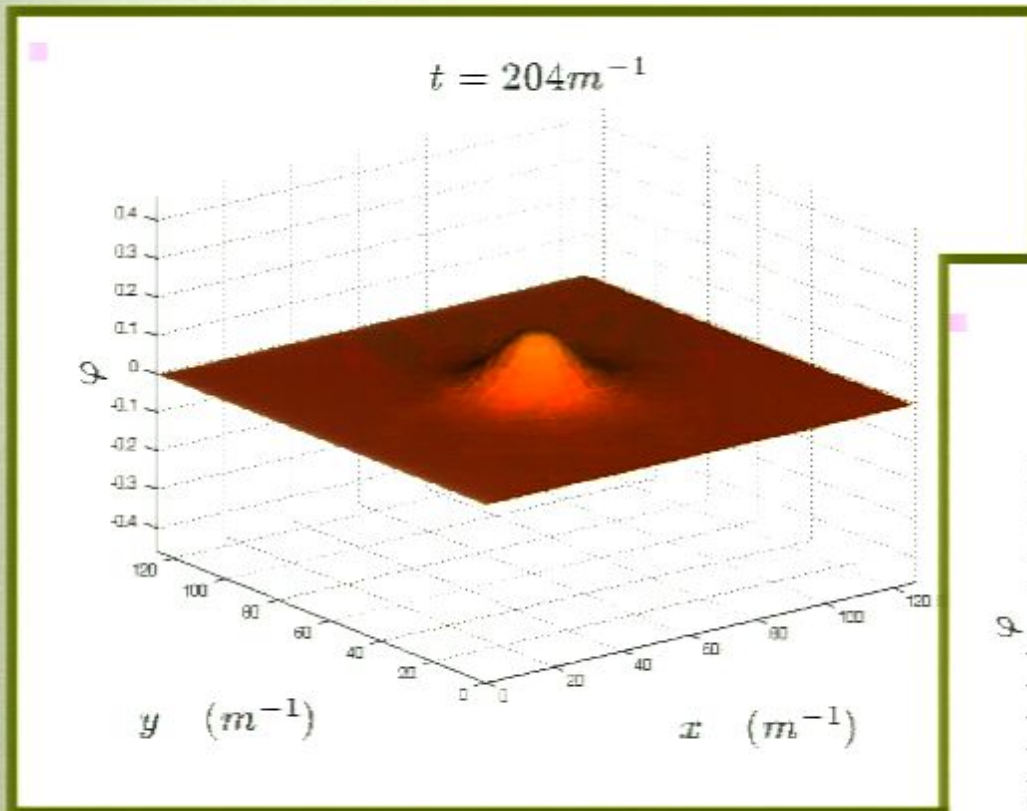
# SOLUTIONS





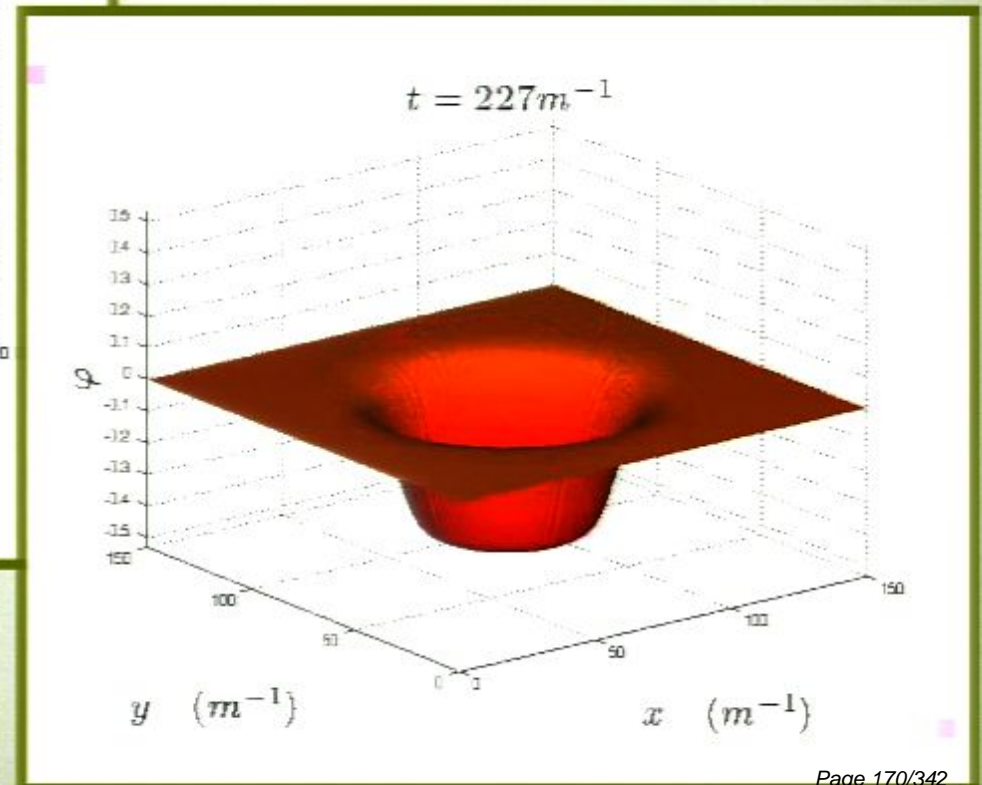
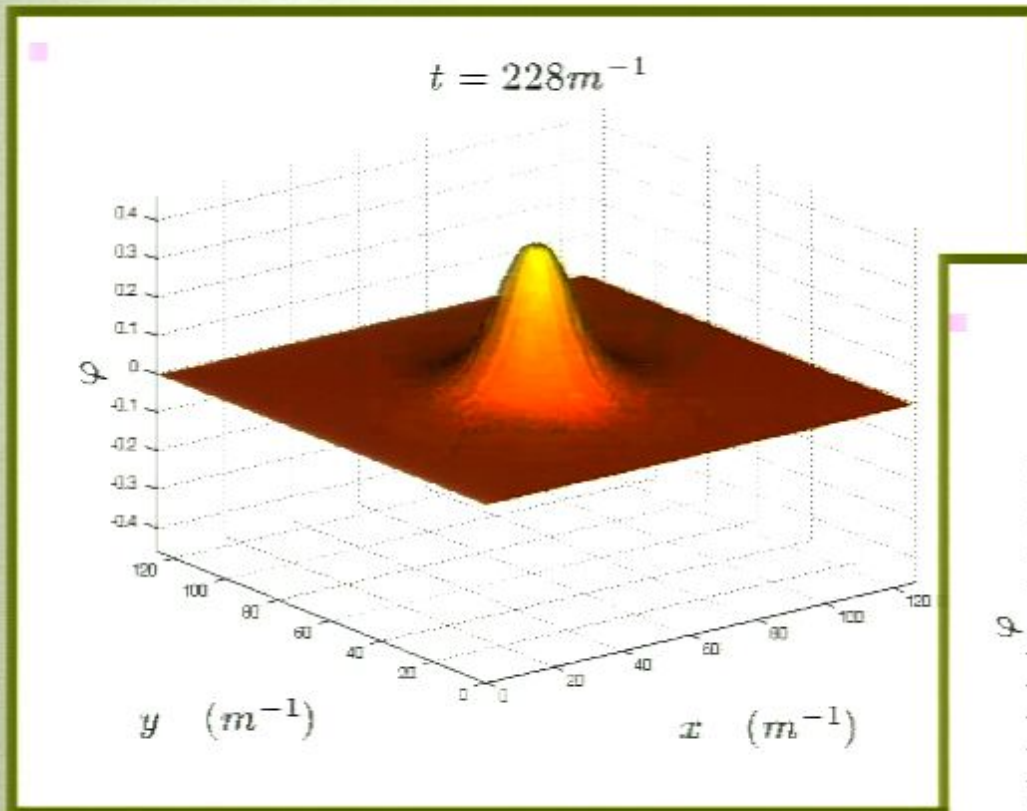
3+1

# SOLUTIONS



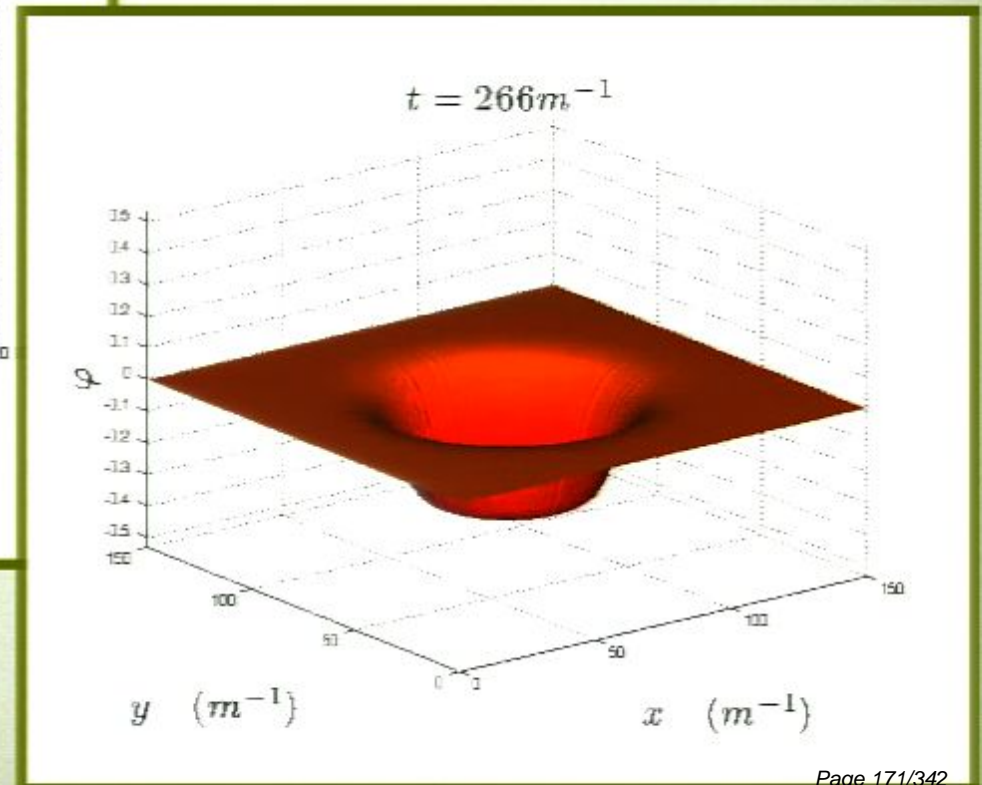
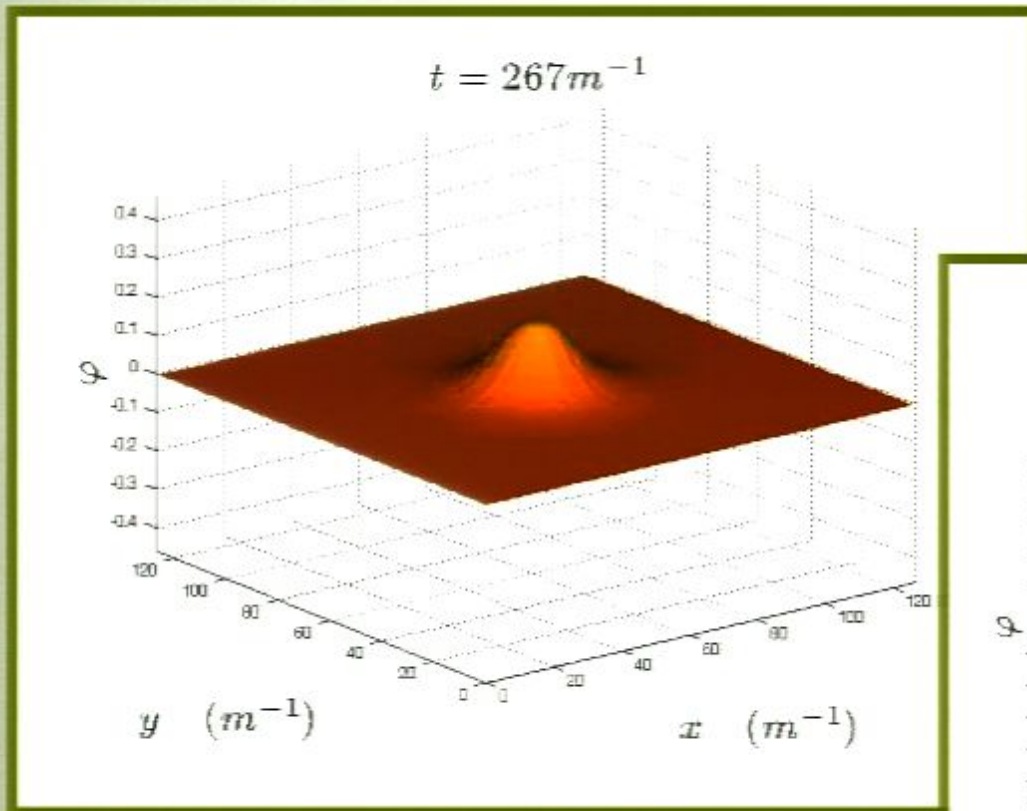
3+1

# SOLUTIONS



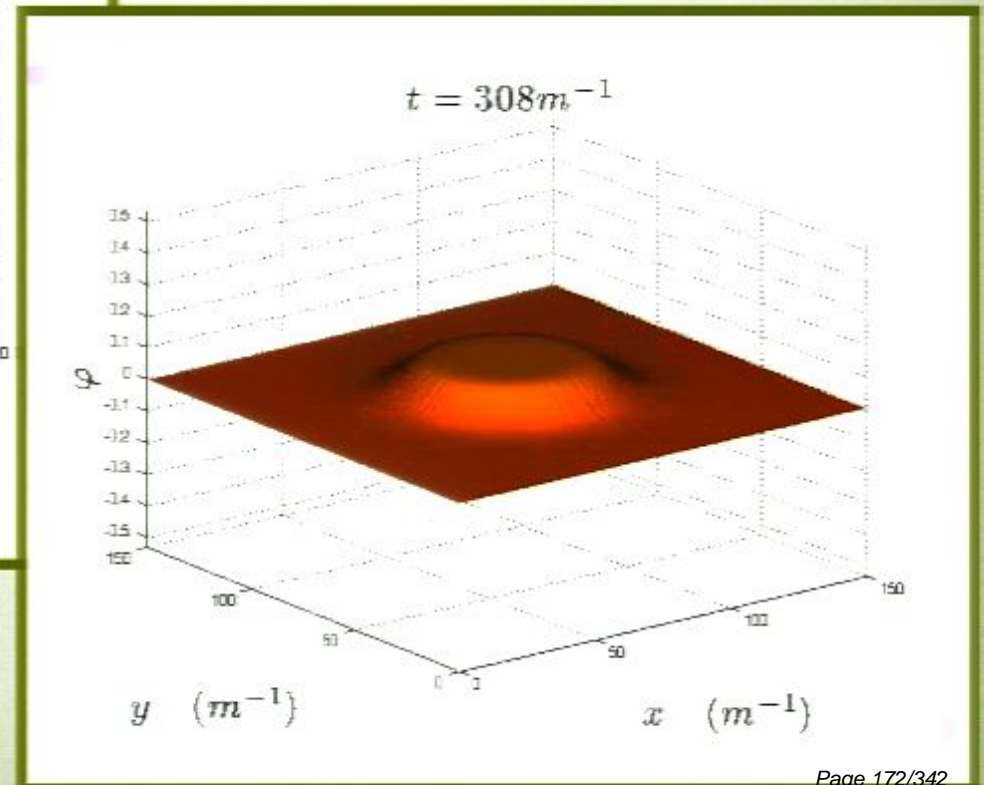
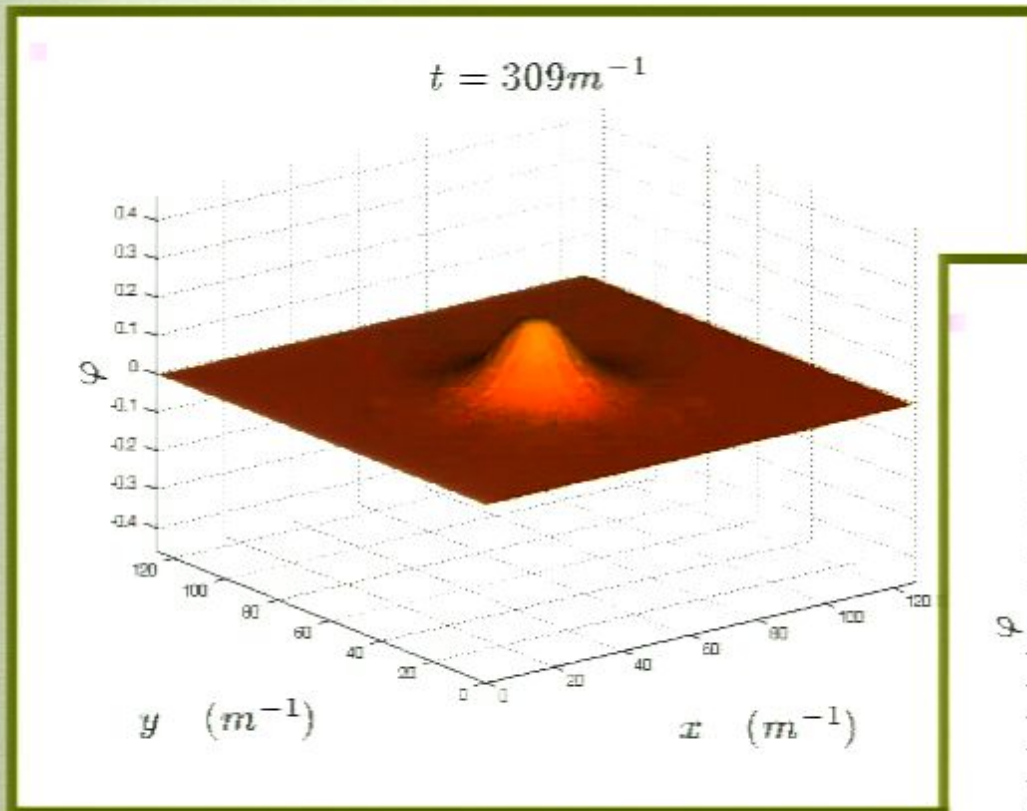
3+1

# SOLUTIONS



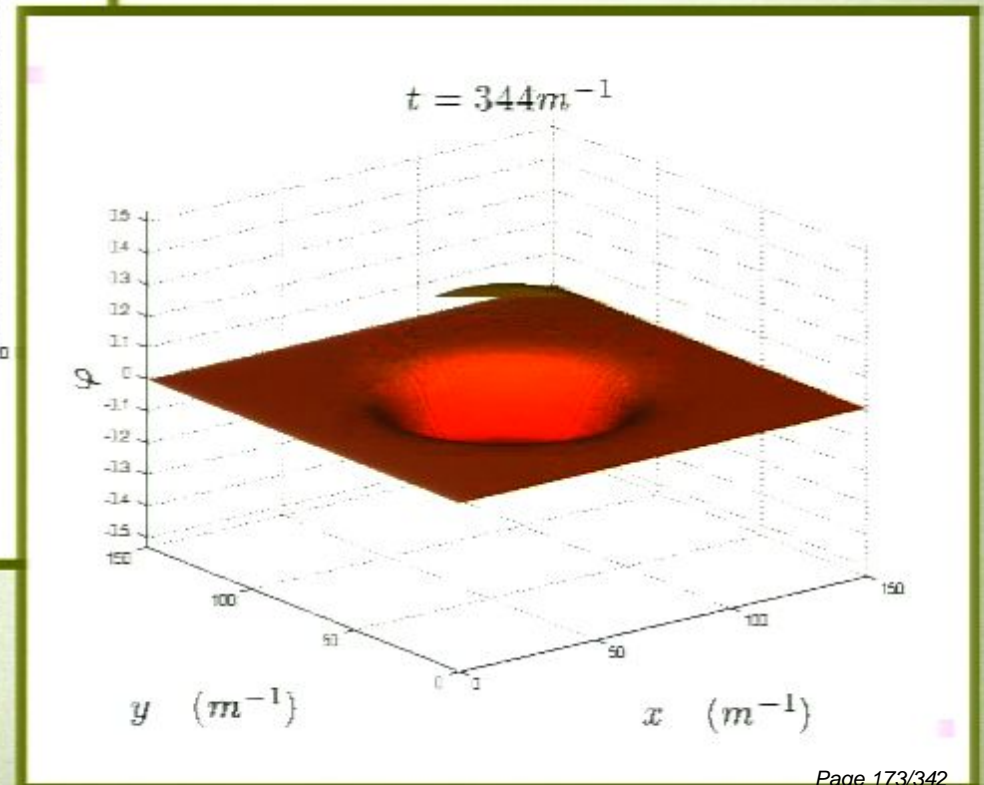
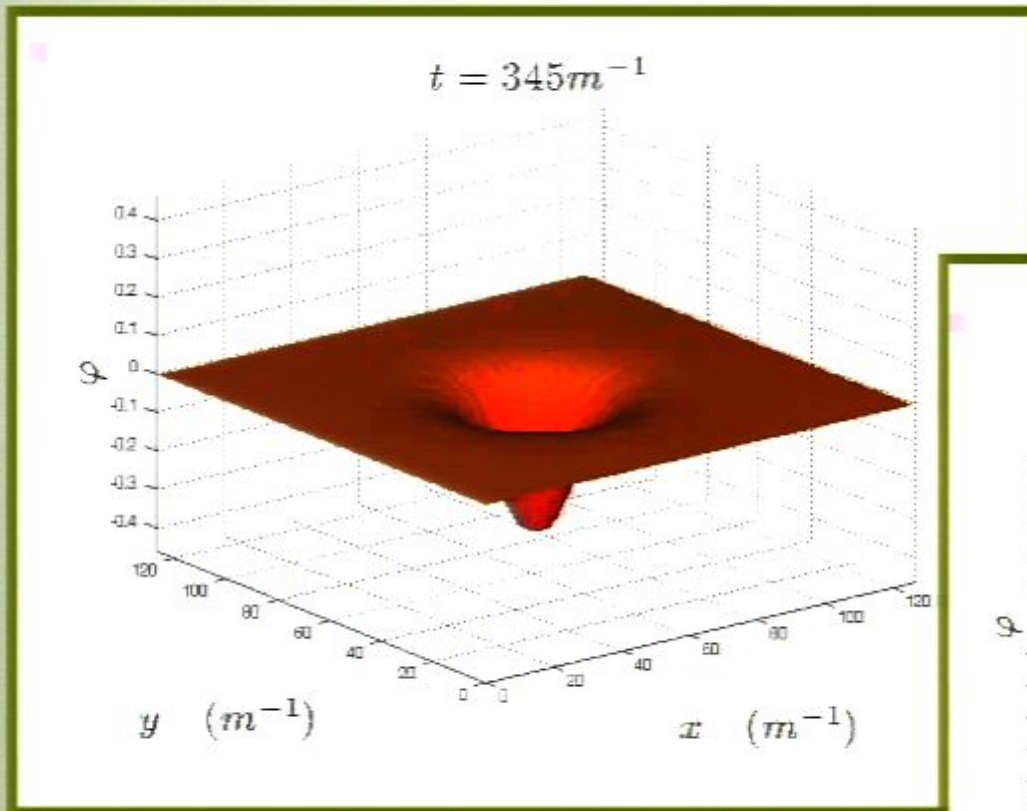
3+1

# SOLUTIONS



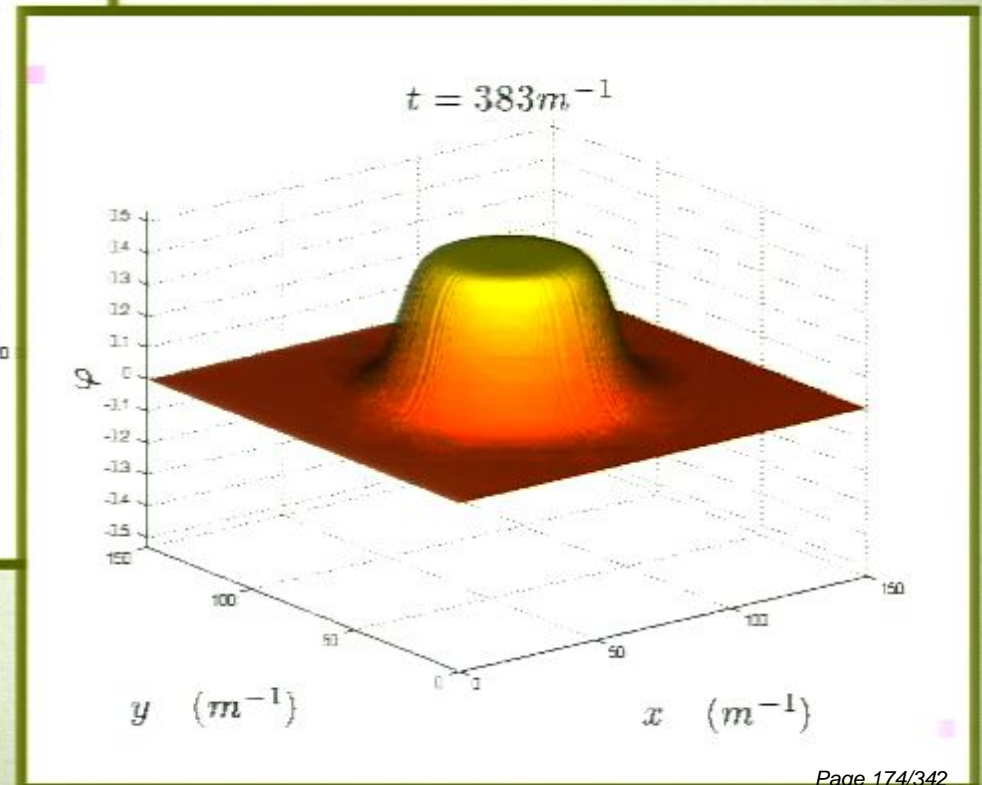
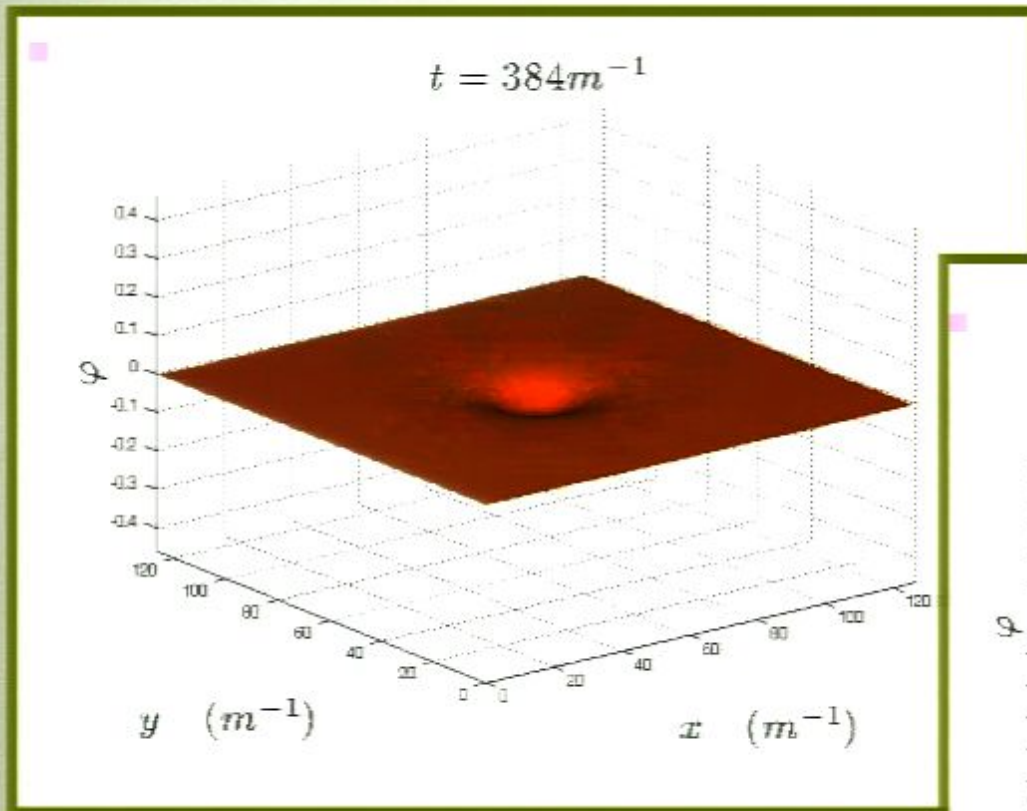
3+1

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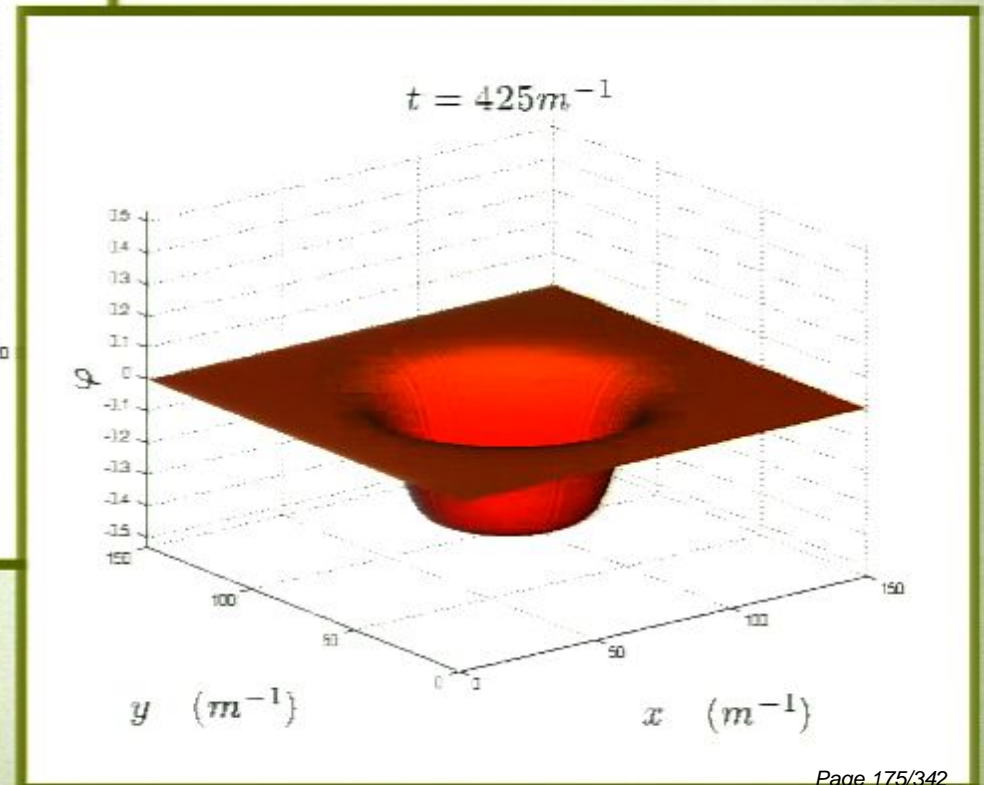
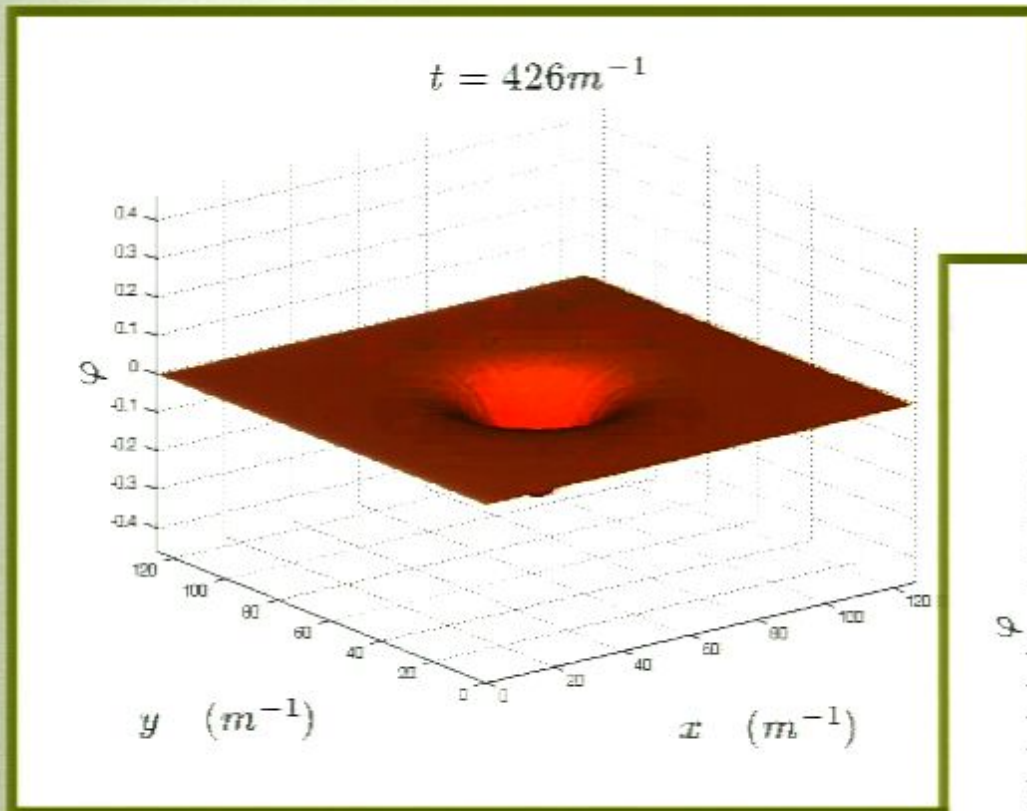
3+1

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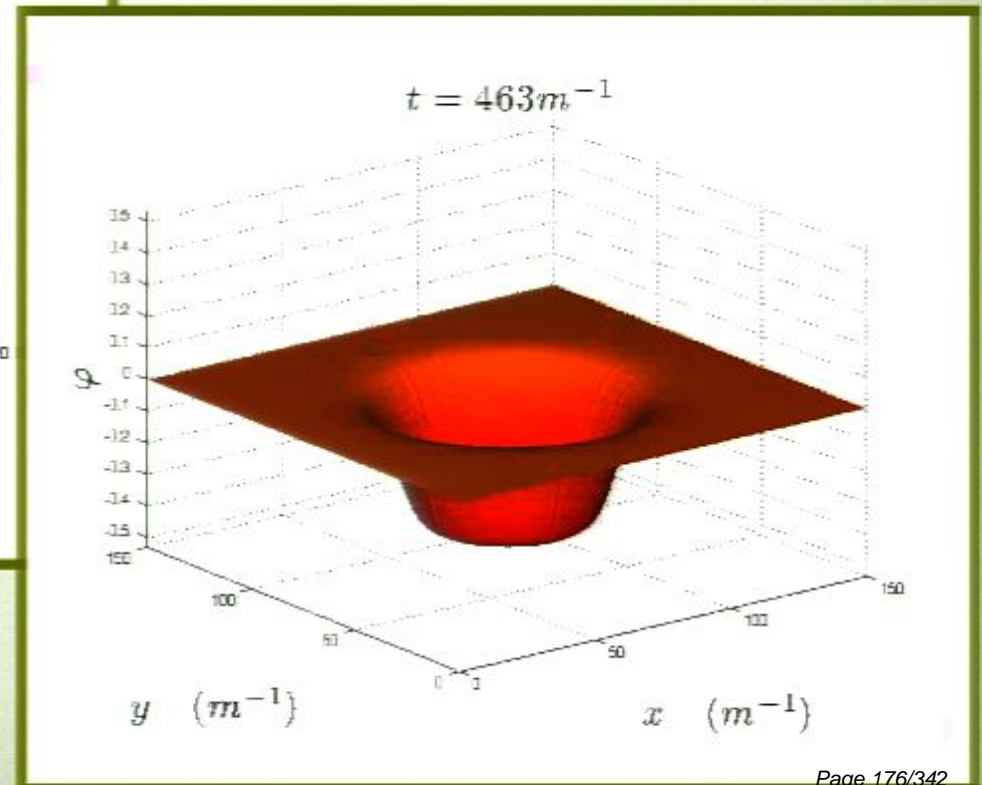
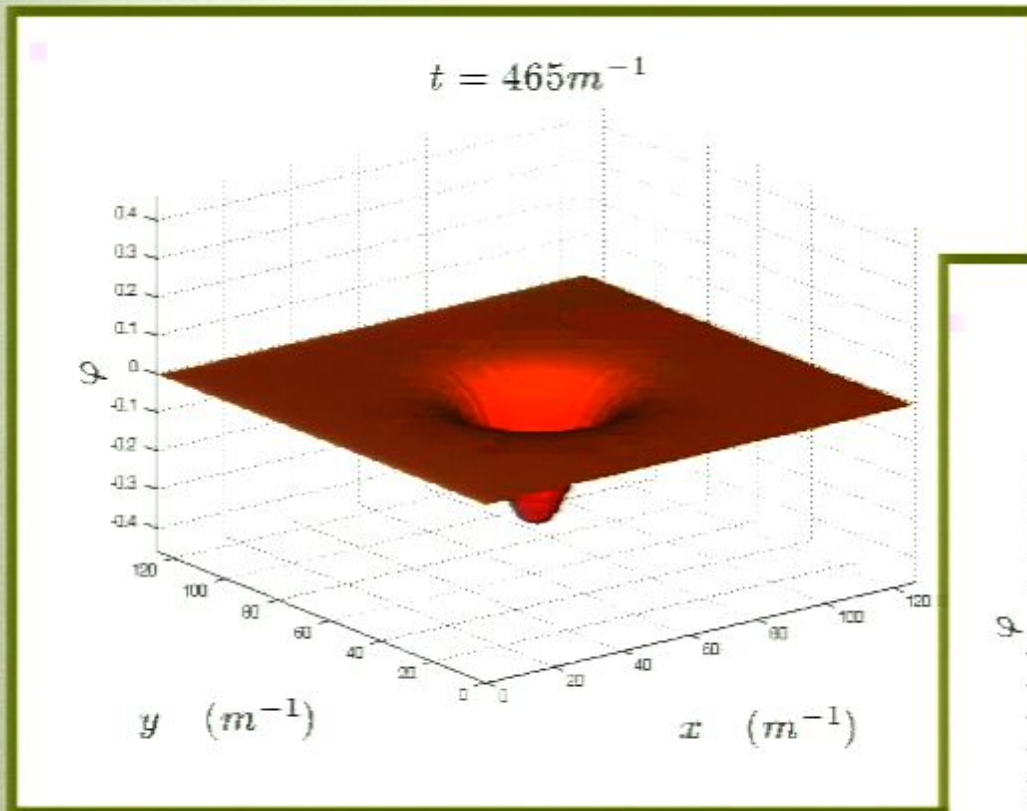
3+1

# SOLUTIONS



3+1

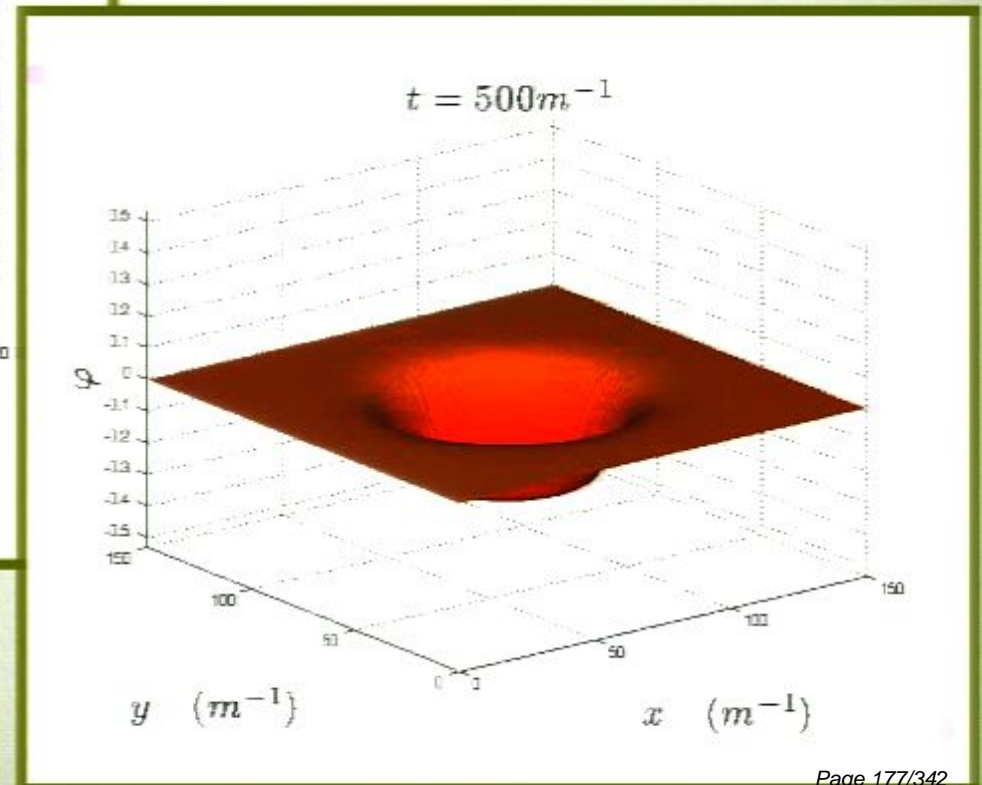
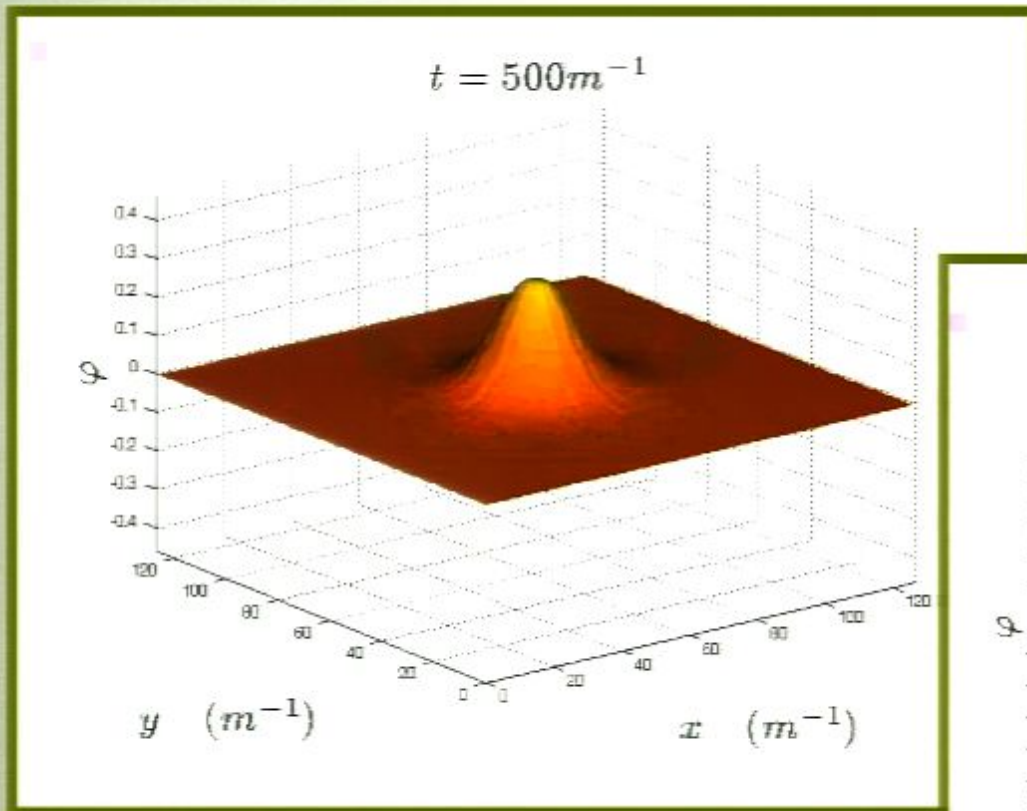
# SOLUTIONS





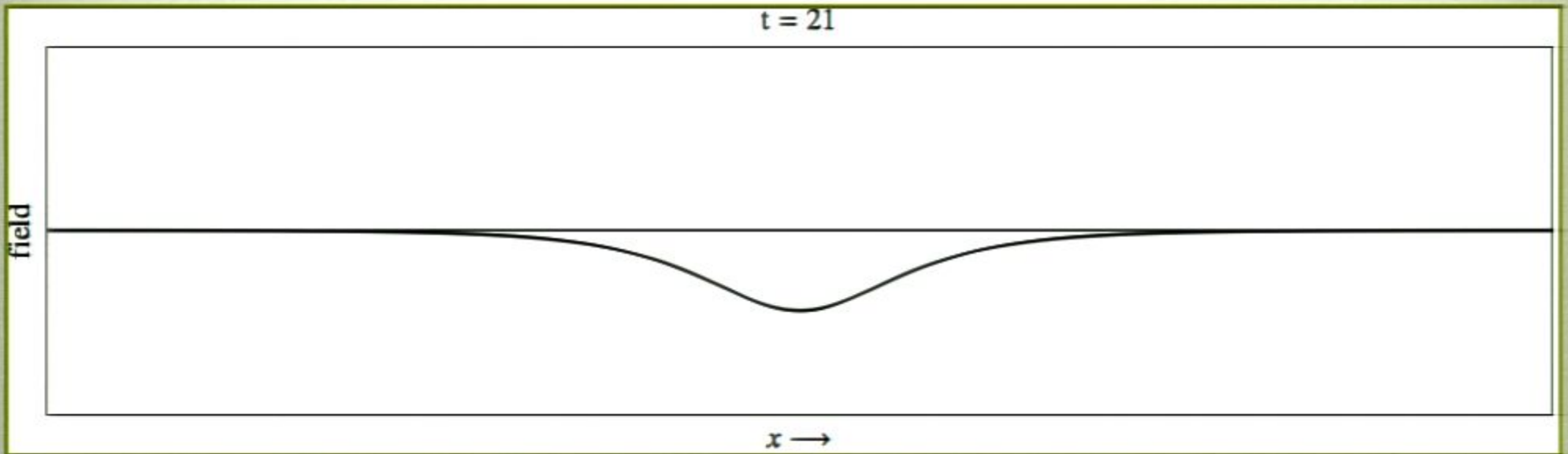
3+1

# SOLUTIONS



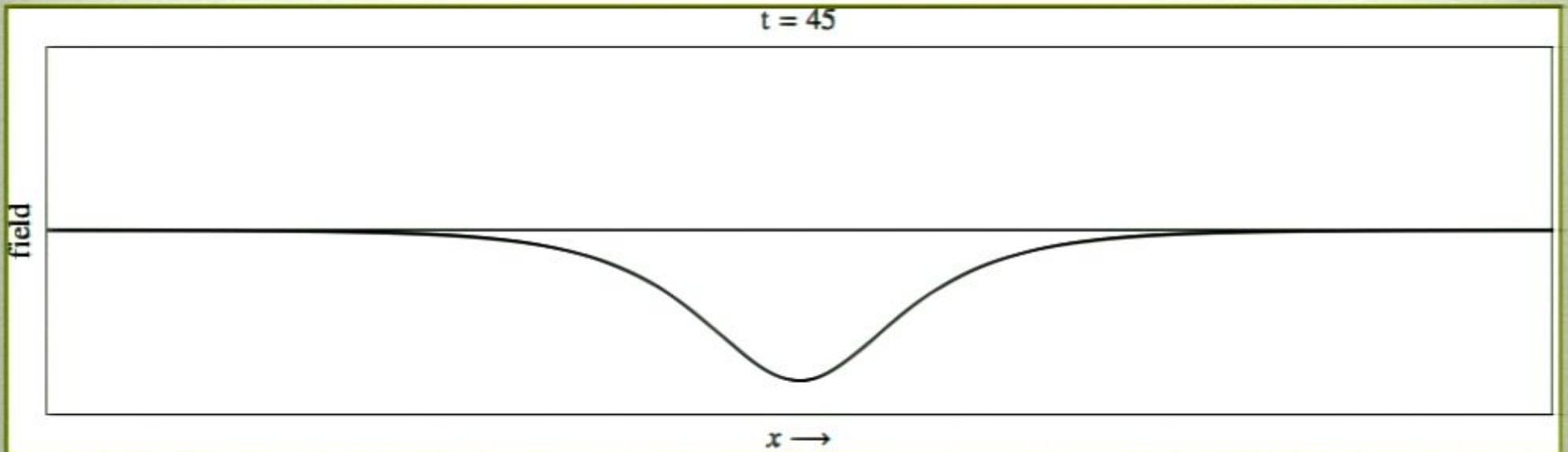
# WHAT ABOUT STABILITY ?

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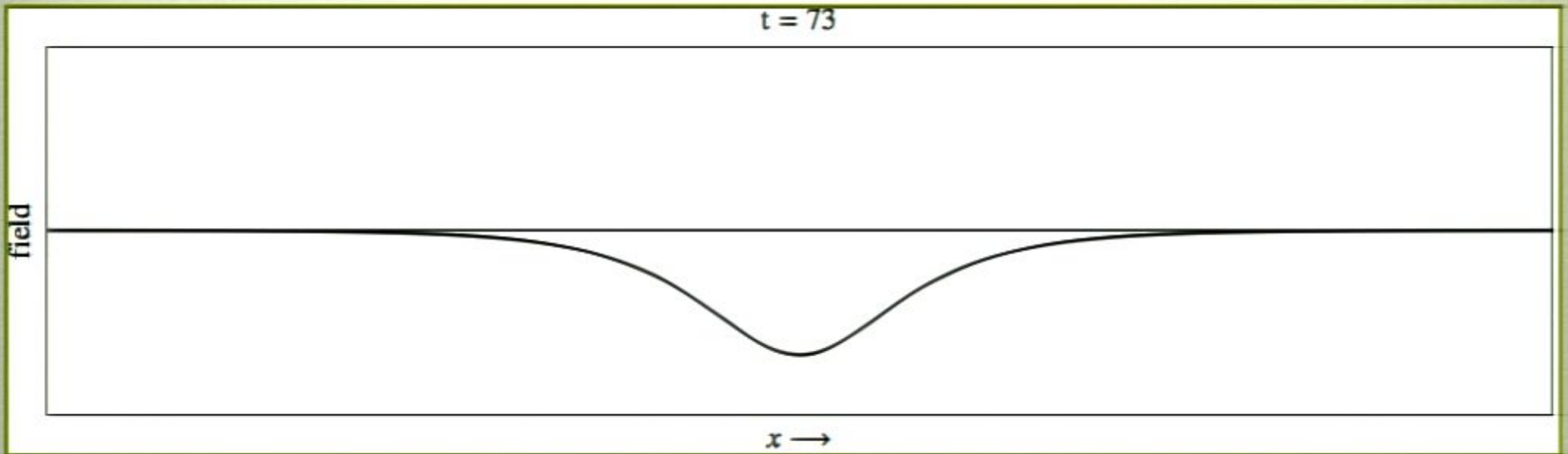
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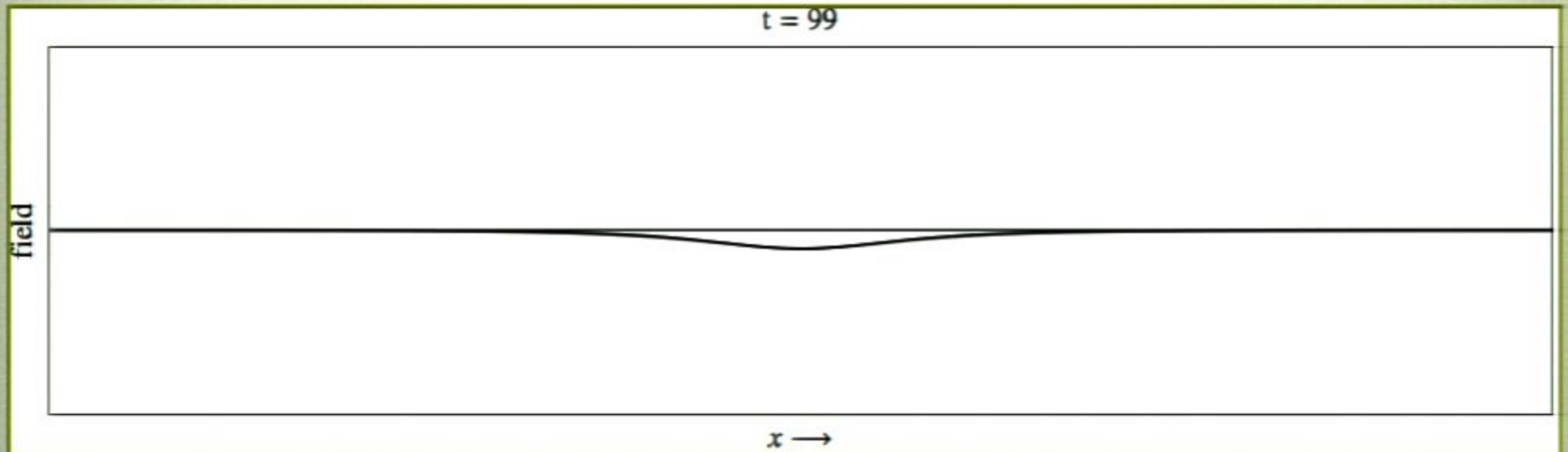
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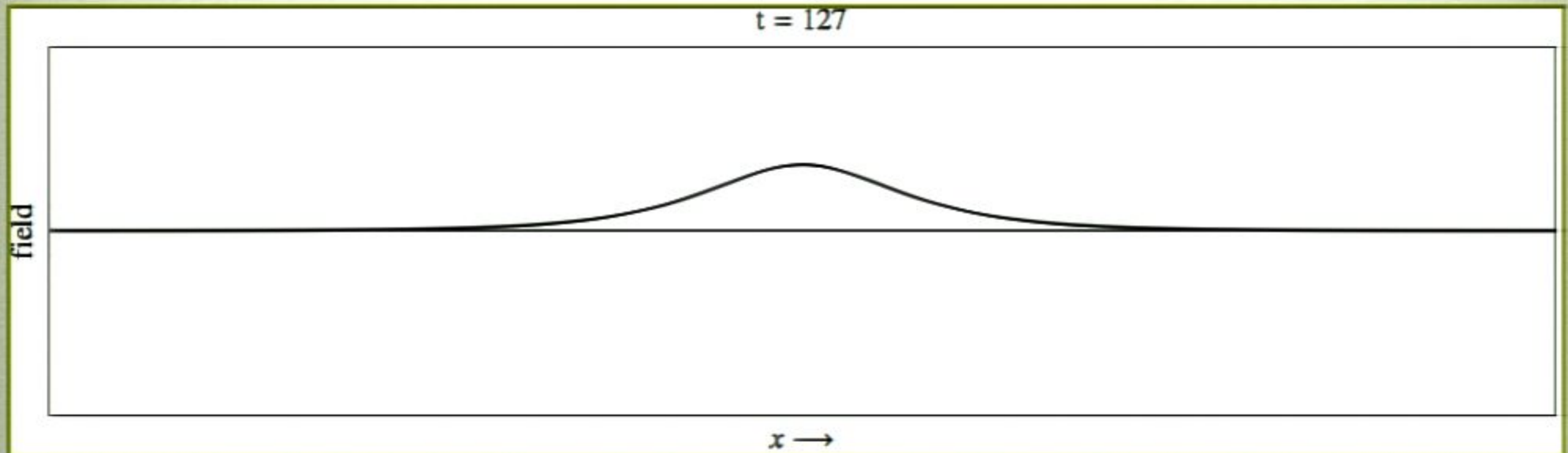
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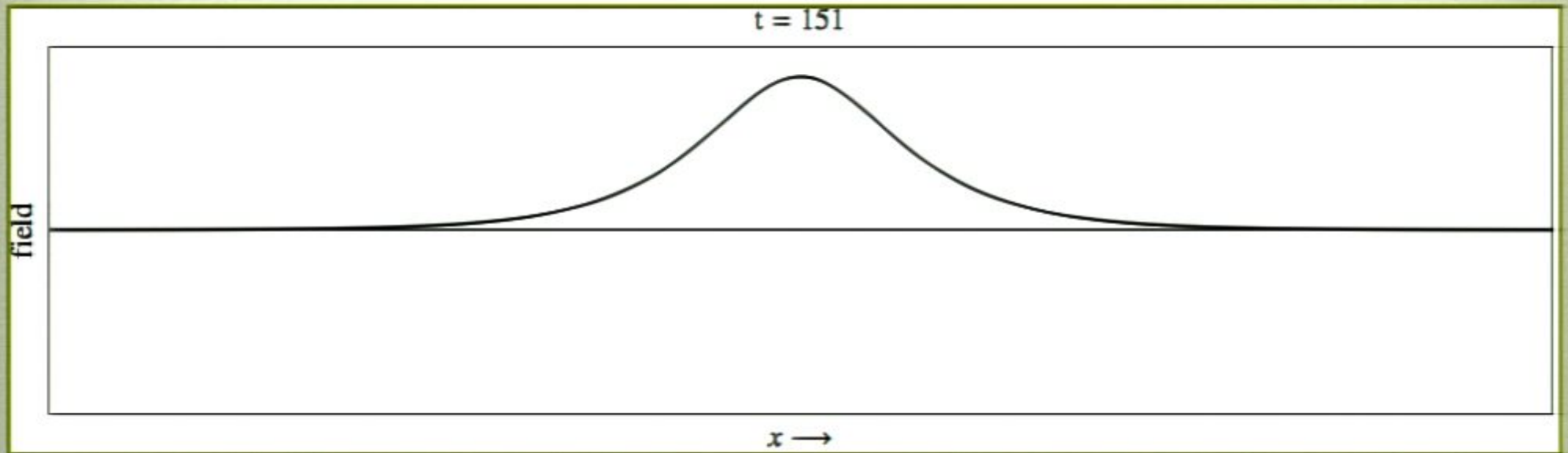
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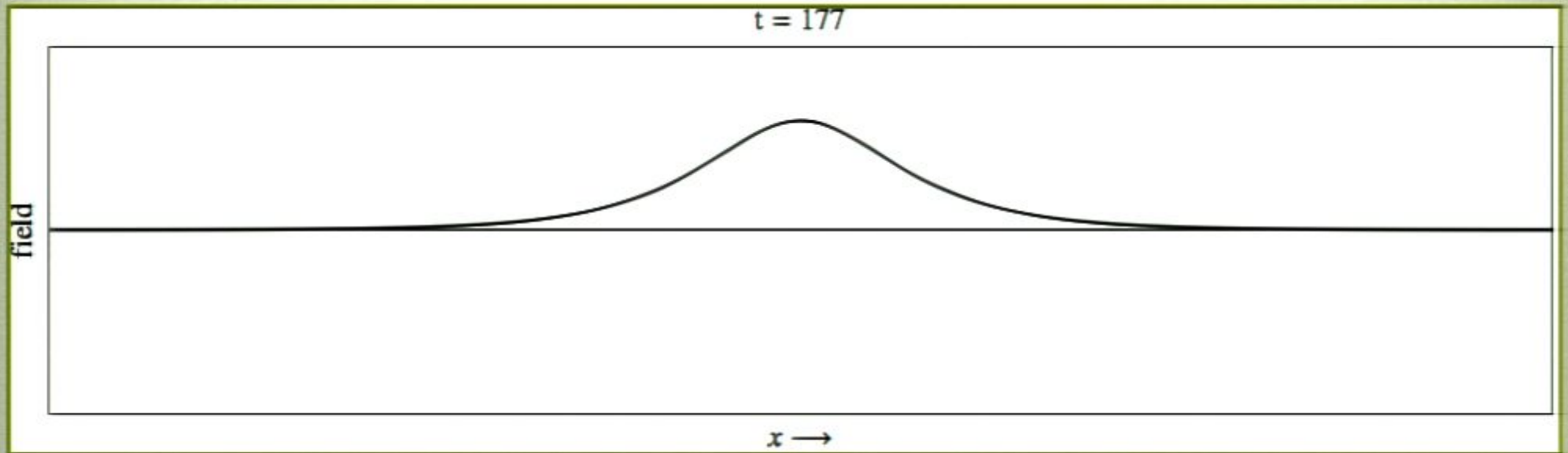
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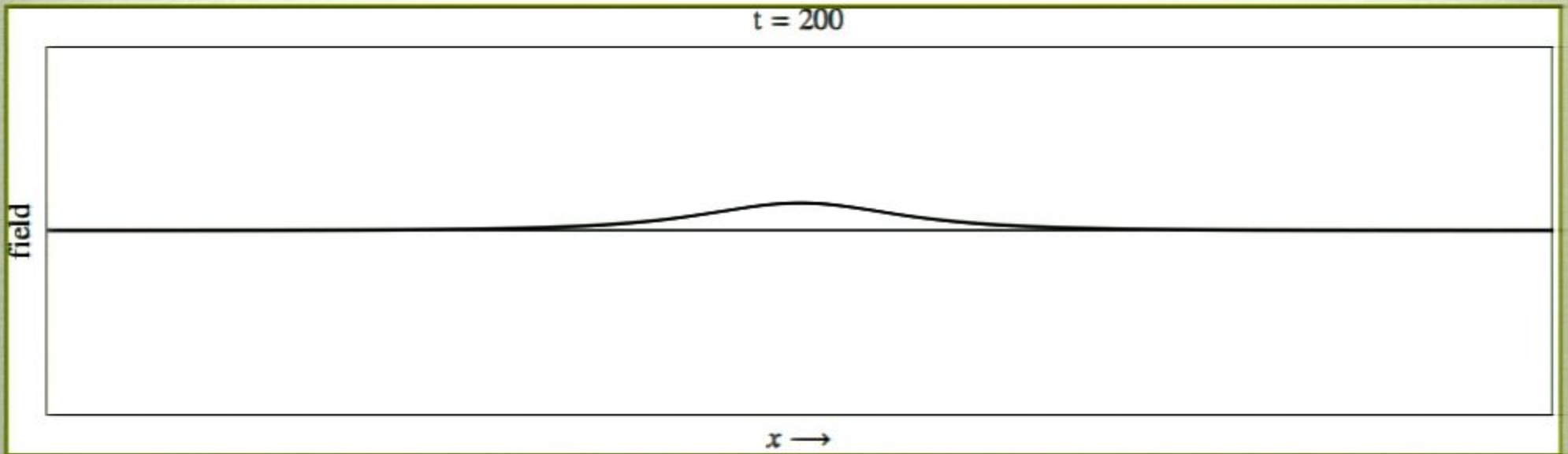
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# WHAT ABOUT STABILITY ?

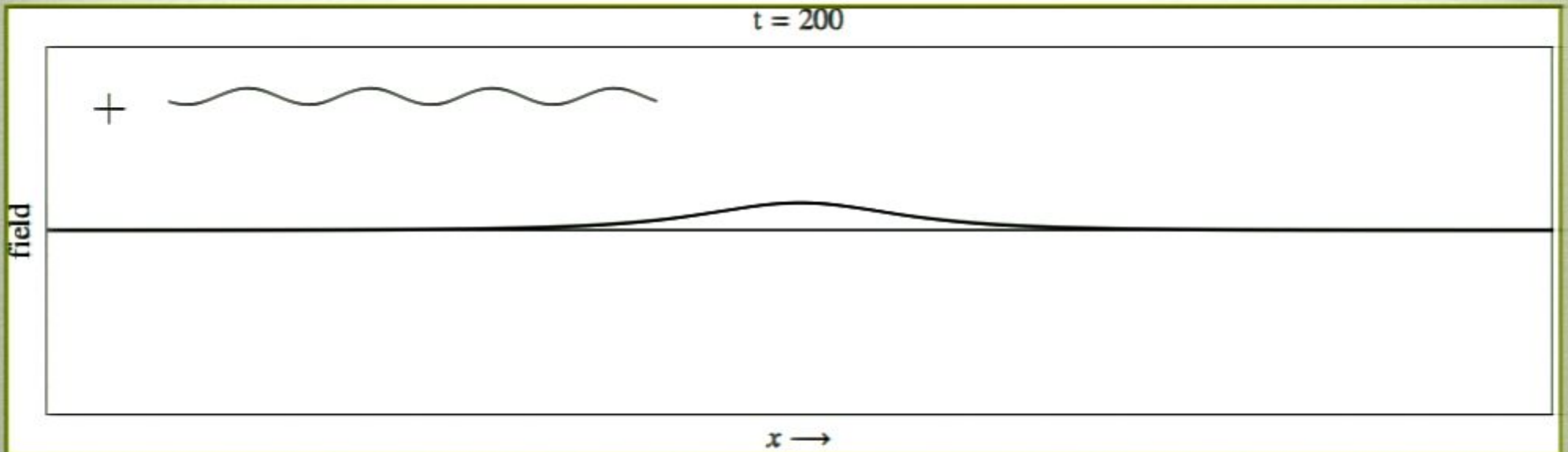
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# WHAT ABOUT STABILITY ?

---

perturbations

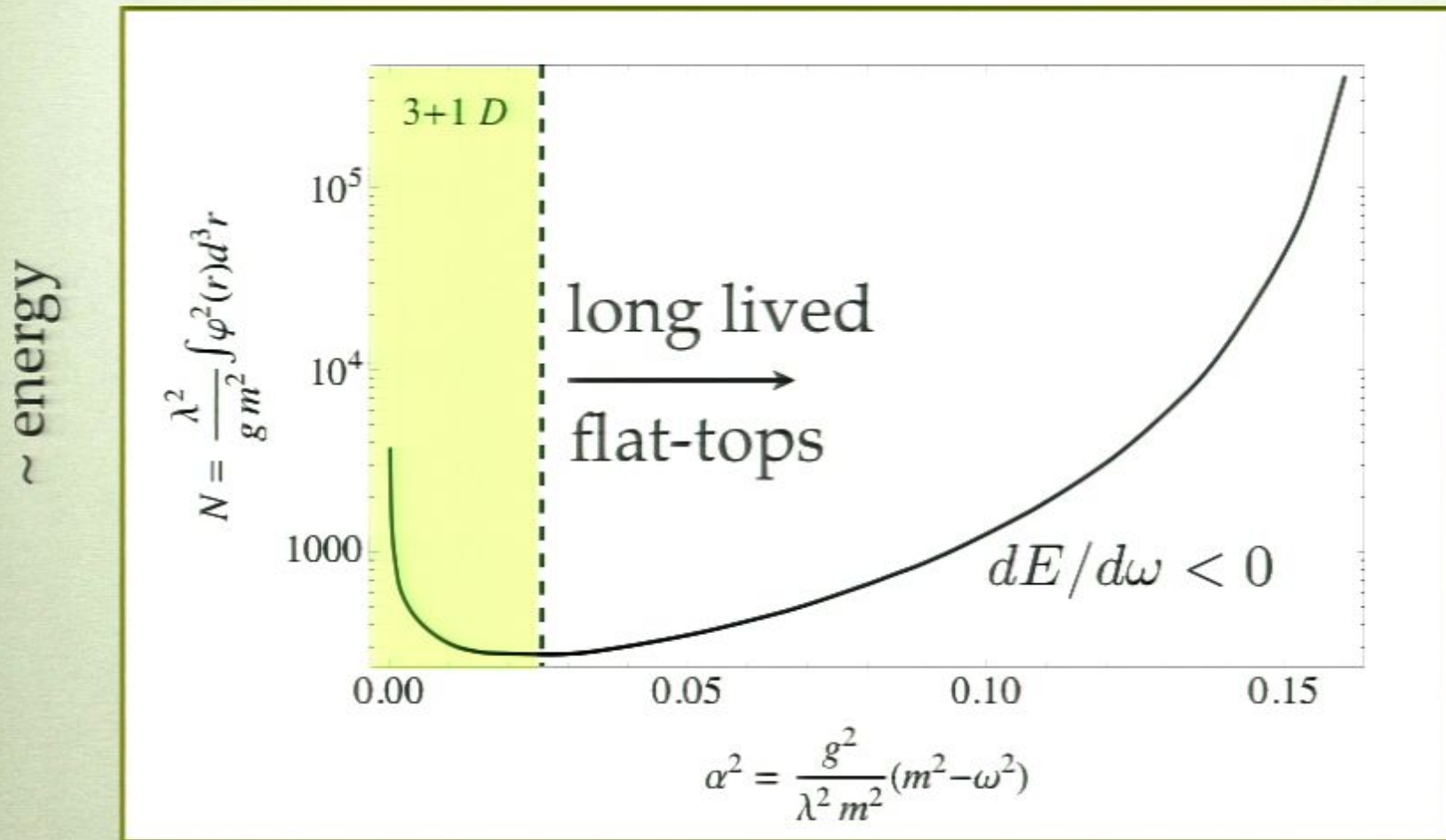


# LINEAR STABILITY ANALYSIS

---

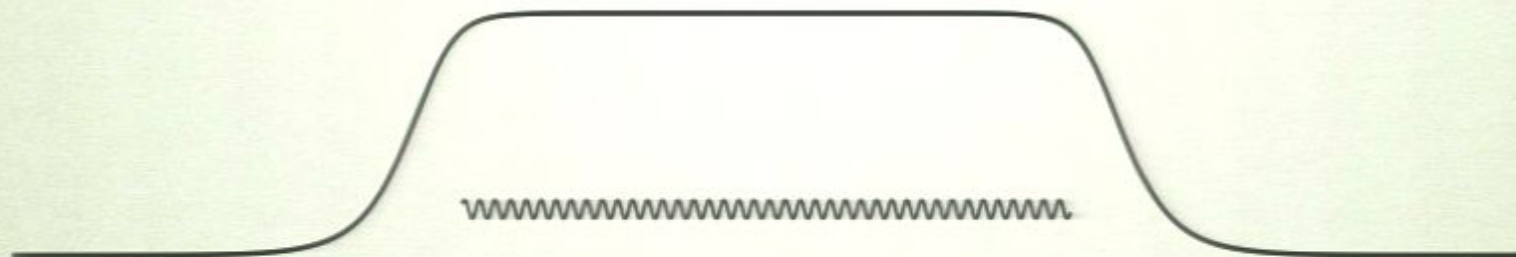
- *collapse* instability ?      wavelength  $\sim$  width
- *Floquet* instability ?      wavelength  $\ll$  width

# COLLAPSE INSTABILITY ?

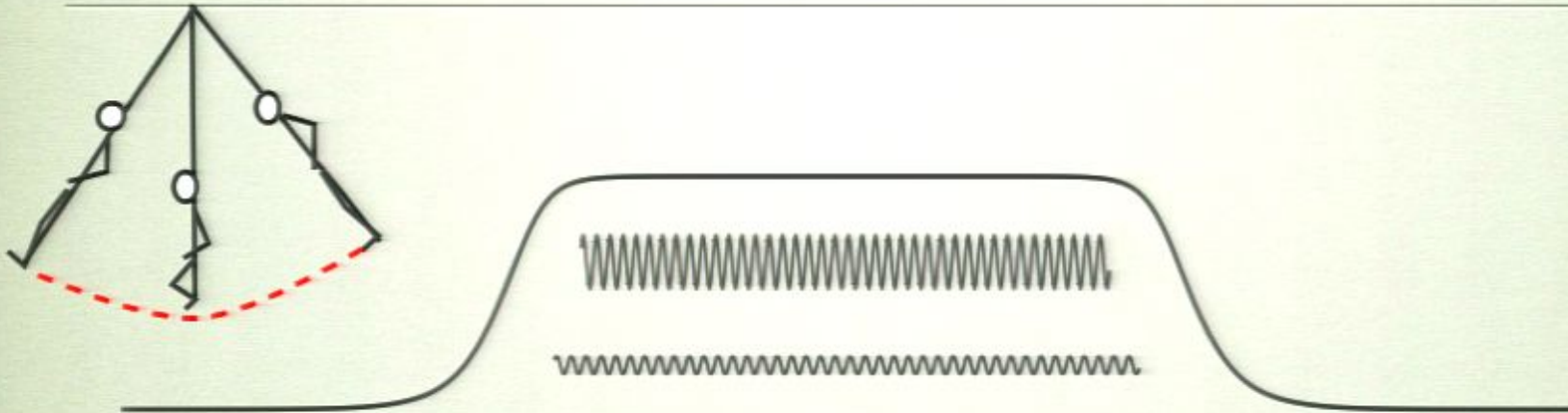


# FLOQUET INSTABILITY ?

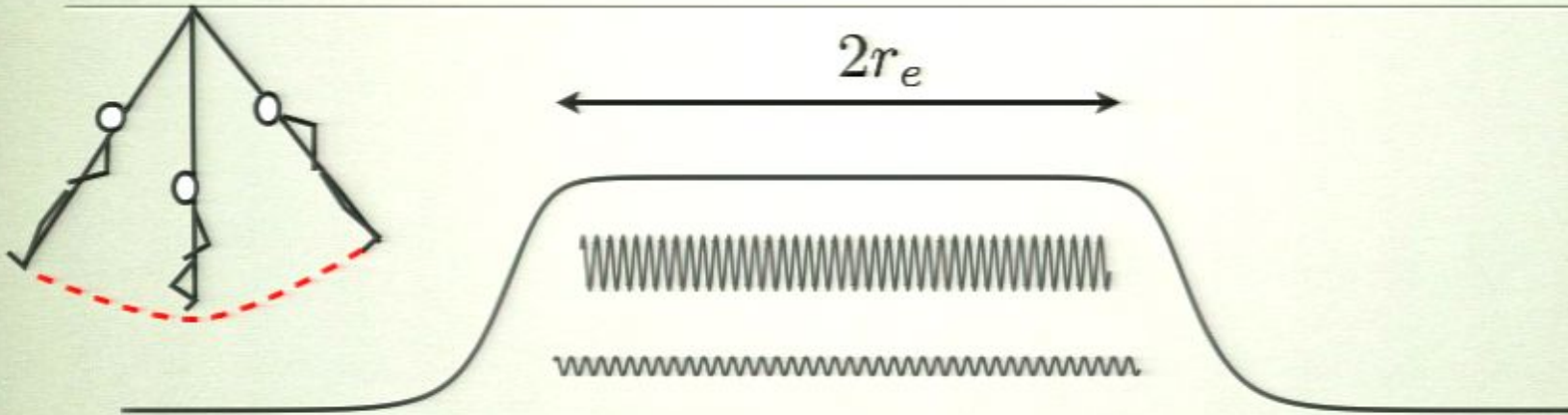
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# FLOQUET INSTABILITY ?

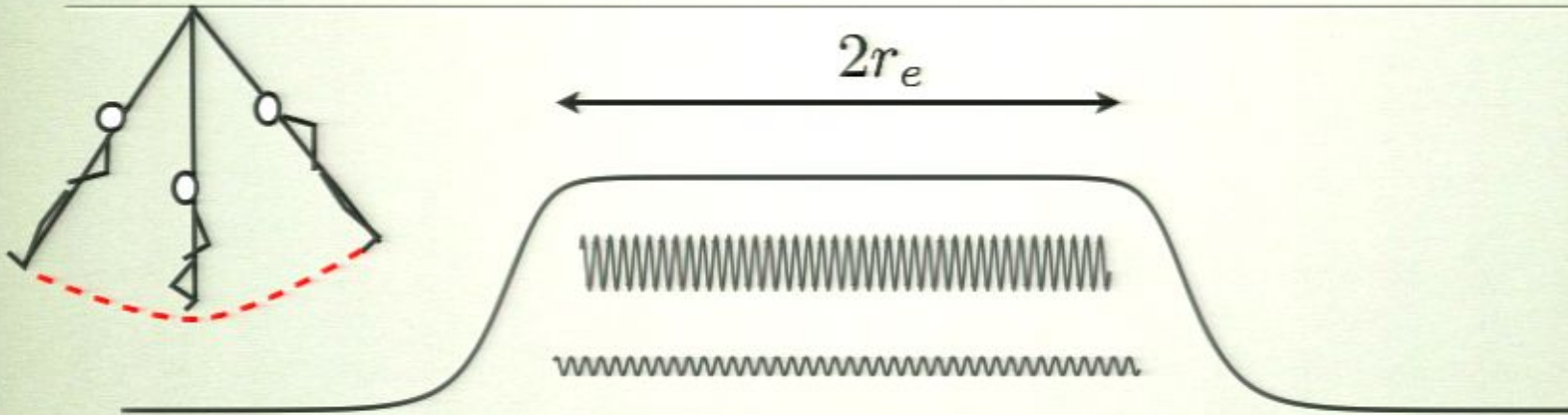


# FLOQUET INSTABILITY ?



$$\mu_{max} \sim \begin{cases} (\lambda/g)^2 m, & \text{if } (\lambda/g)^2 m > 1/2r_e \\ 0 & \end{cases}$$

# FLOQUET INSTABILITY ?



$$\mu_{max} \sim \begin{cases} (\lambda/g)^2 m, & \text{if } (\lambda/g)^2 m > 1/2r_e \\ 0 & \end{cases}$$

narrow band instability

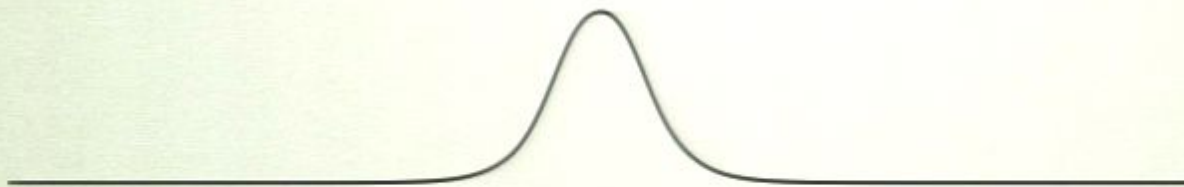


# LINEAR STABILITY

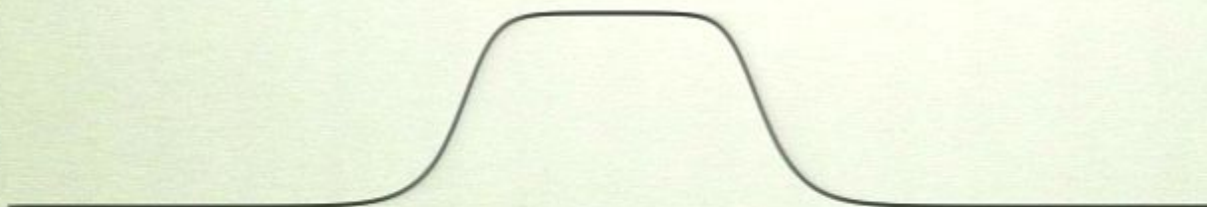
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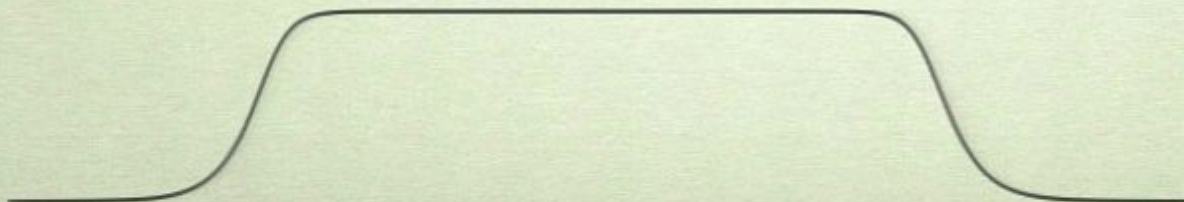
**X**  
collapse



✓



✓

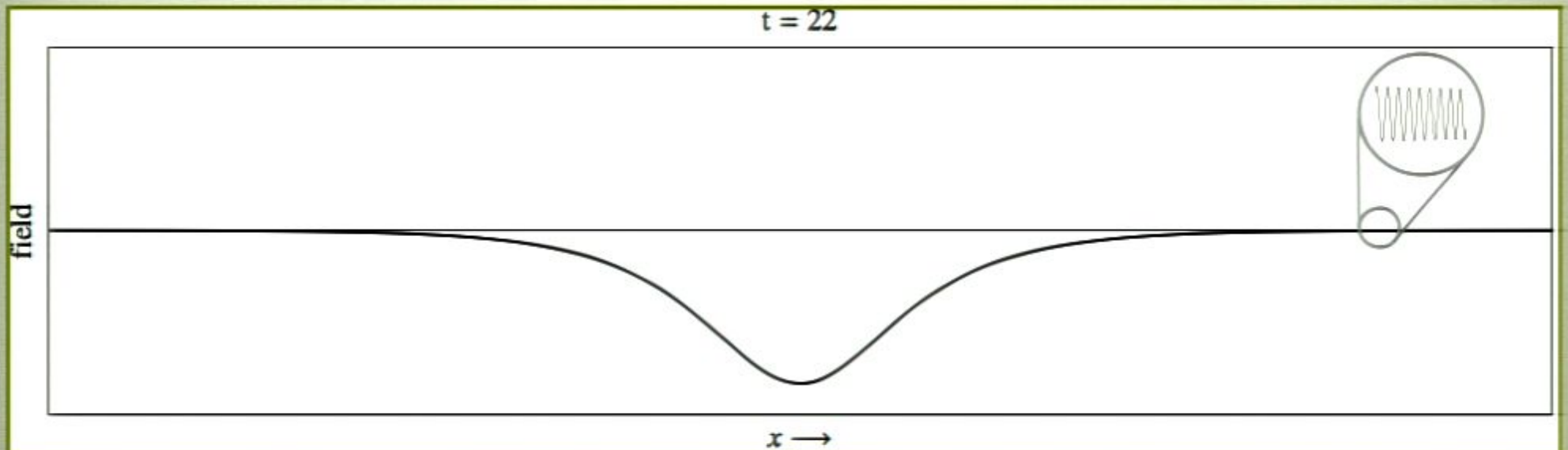


**X**  
floquet

# OUTGOING RADIATION

- classical : *Kruskal and Segur (1987)*

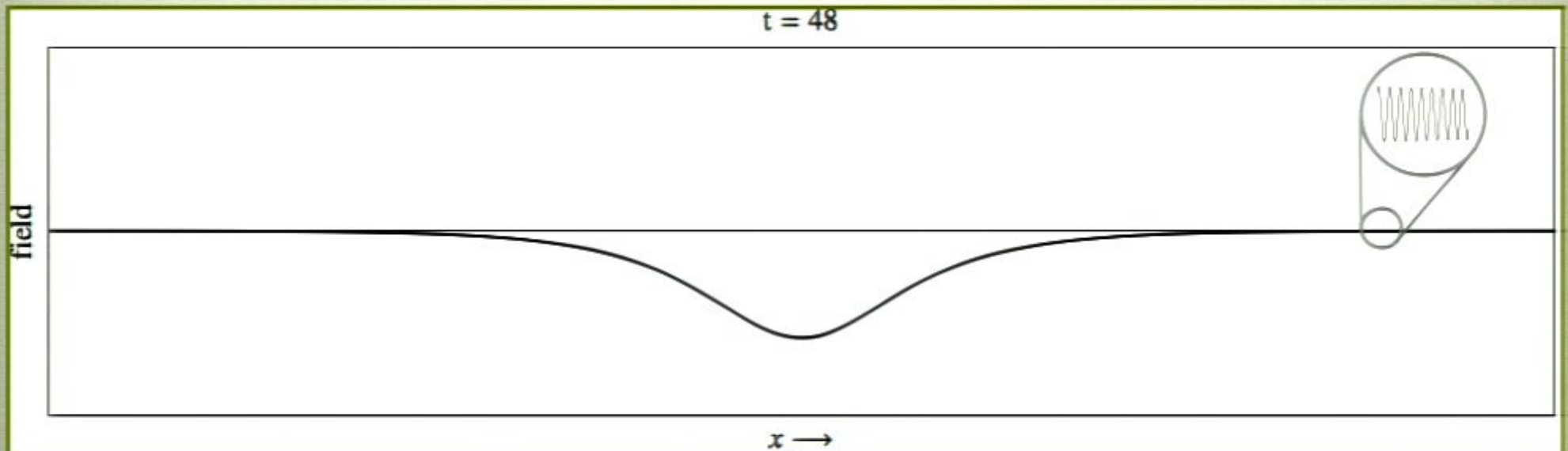
radiative tail



# OUTGOING RADIATION

- classical : *Kruskal and Segur (1987)*

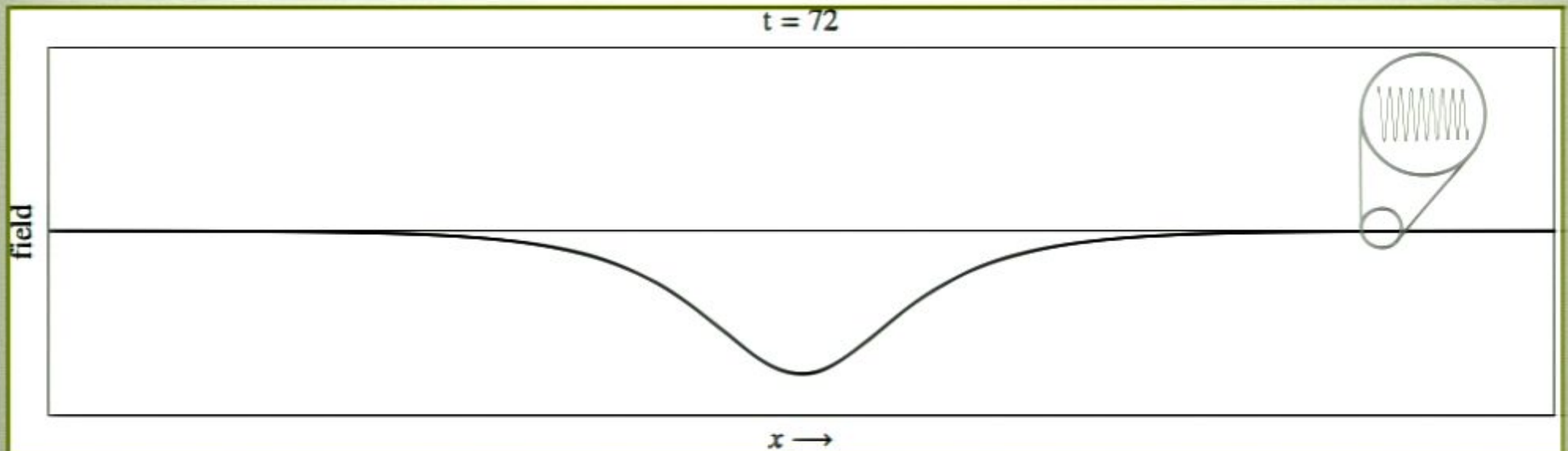
radiative tail



# OUTGOING RADIATION

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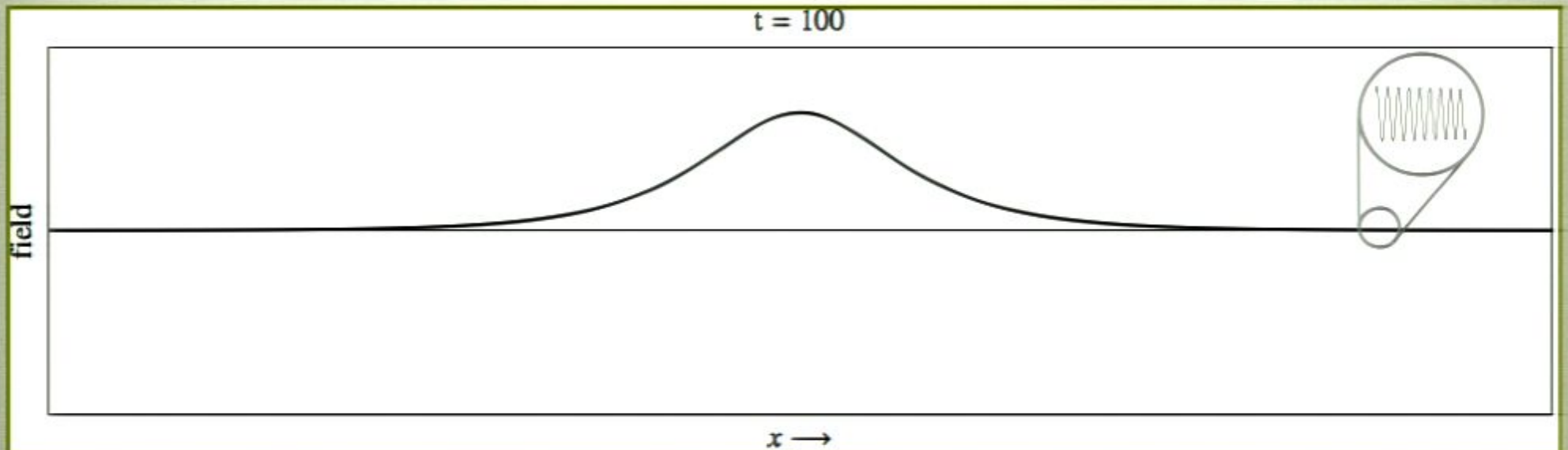
radiative tail



# OUTGOING RADIATION

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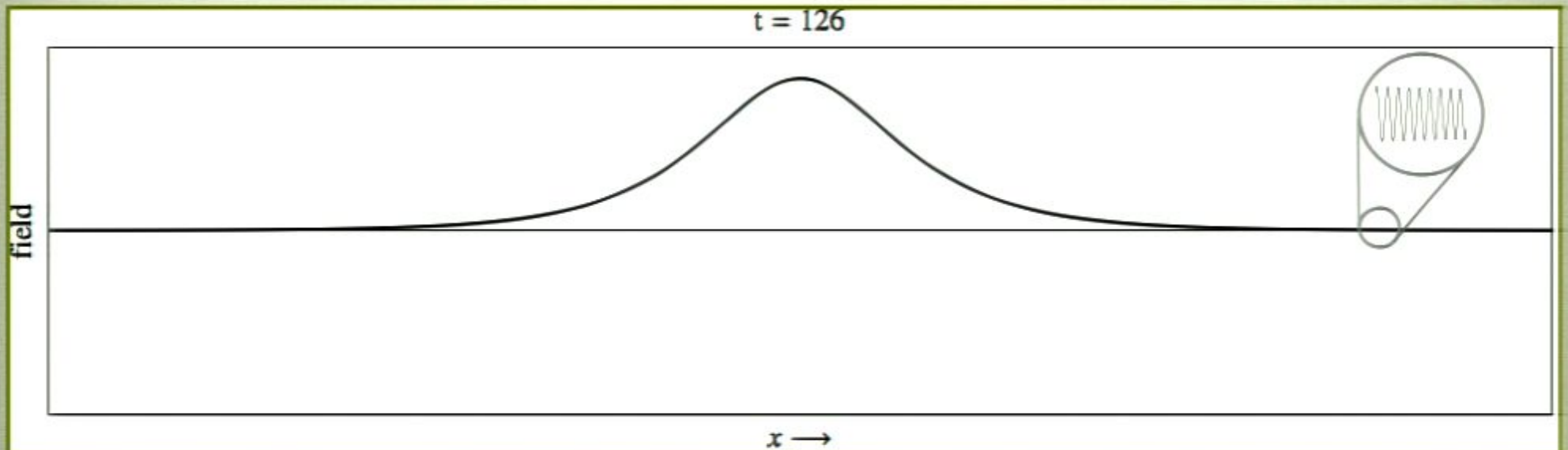
radiative tail



# OUTGOING RADIATION

- classical : *Kruskal and Segur (1987)*

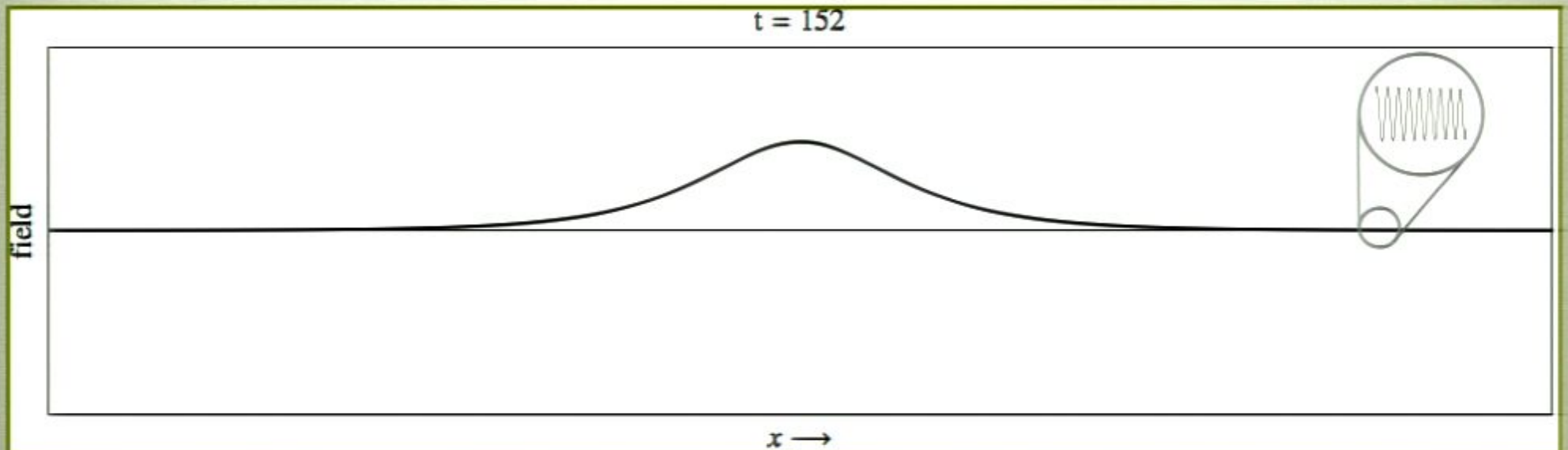
radiative tail



# OUTGOING RADIATION

- classical : *Kruskal and Segur (1987)*

radiative tail



# DECAY RATES

---

$$\Gamma_{\text{classical}} \sim m \frac{\lambda}{g} e^{-b \frac{g}{\lambda}}$$

$$\Gamma_{\text{quantum}} \sim m \frac{\lambda^3}{g^2}$$



# EXPANDING BACKGROUND

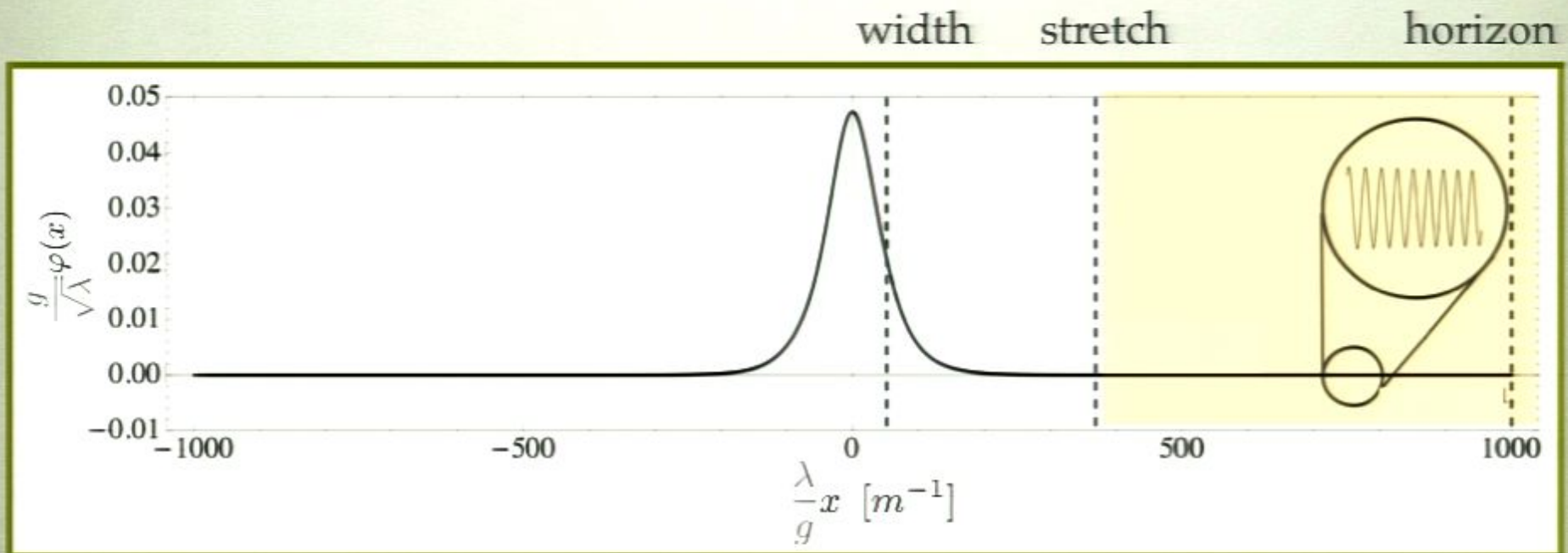
---

$$\partial_t^2 \varphi - \nabla^2 \varphi + 3H \partial_t \varphi + m^2 \varphi - \lambda \varphi^3 + \frac{g^2}{m^2} \varphi^5 = 0$$

- *stretching* instability ?
- *radiative* tail

# INCLUDING EXPANSION

- loss of energy  $\Gamma_H \sim m e^{-mH^{-1}} (\alpha\lambda/g)^2$



## LONG LIVED IF ...

---

$$\Gamma \sim \Gamma(\textit{expansion}) + \Gamma(\textit{radiation})$$

$$\Gamma \ll H \ll m$$

**possible for inflaton** (and axions)

# SO FAR ...

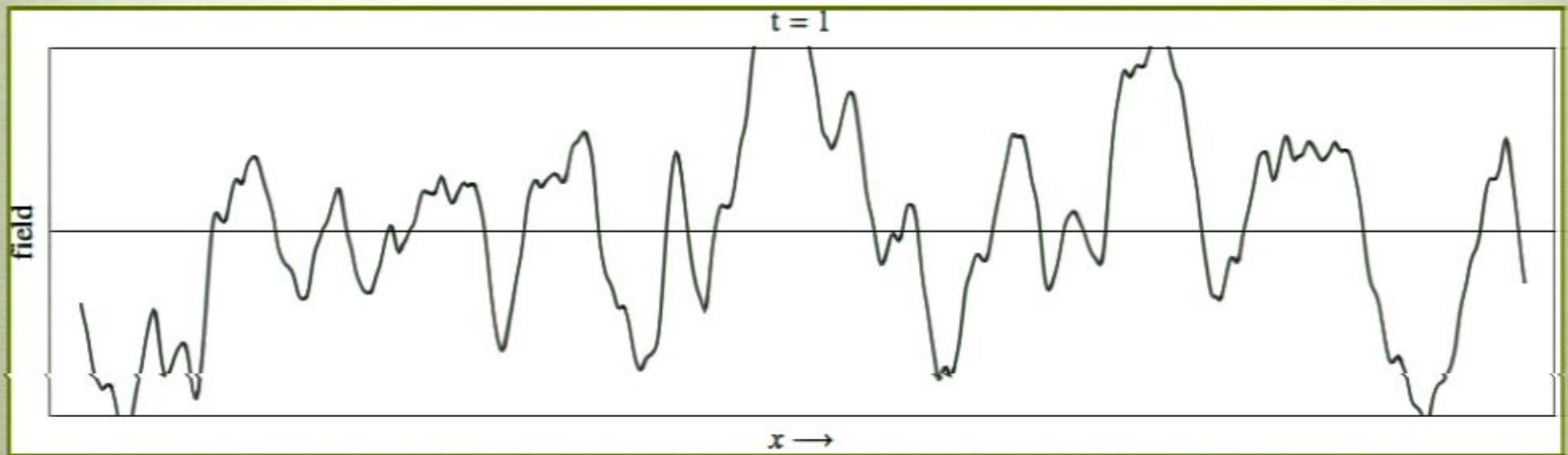
---

- **individual blobs:** *MA & Shirokoff 2010*
  - 3+1 expanding universe
- **cosmological emergence:**
  - conditions for emergence ?
  - number density ? *MA (2010)*

# COSMOLOGICAL EMERGENCE



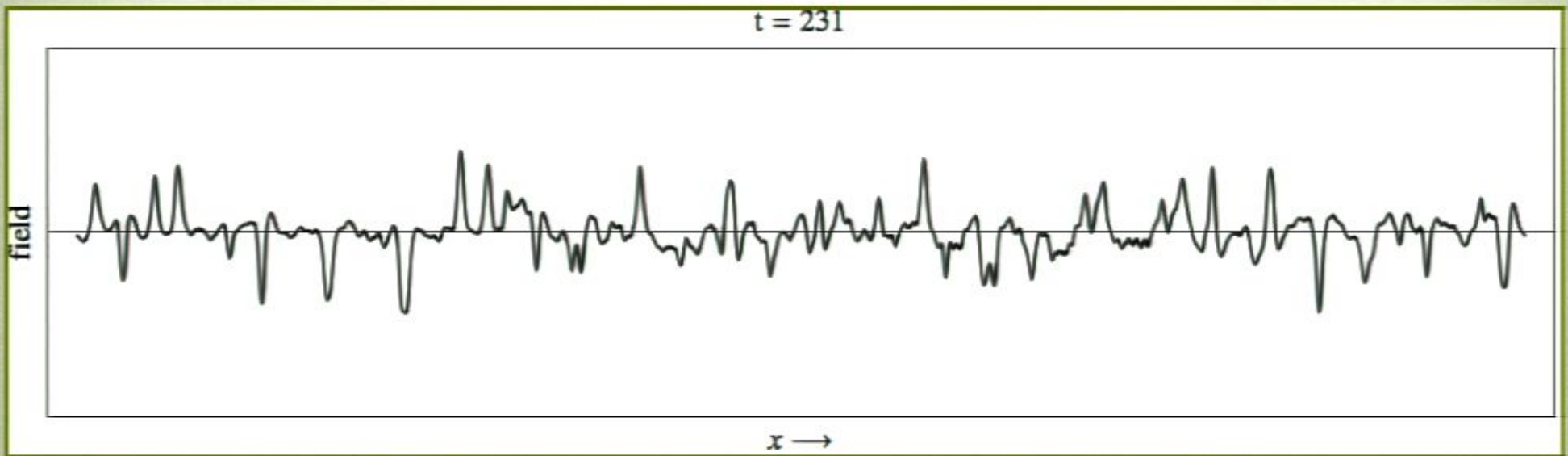
# GENERIC EMERGENCE



> 80% energy density in oscillons !

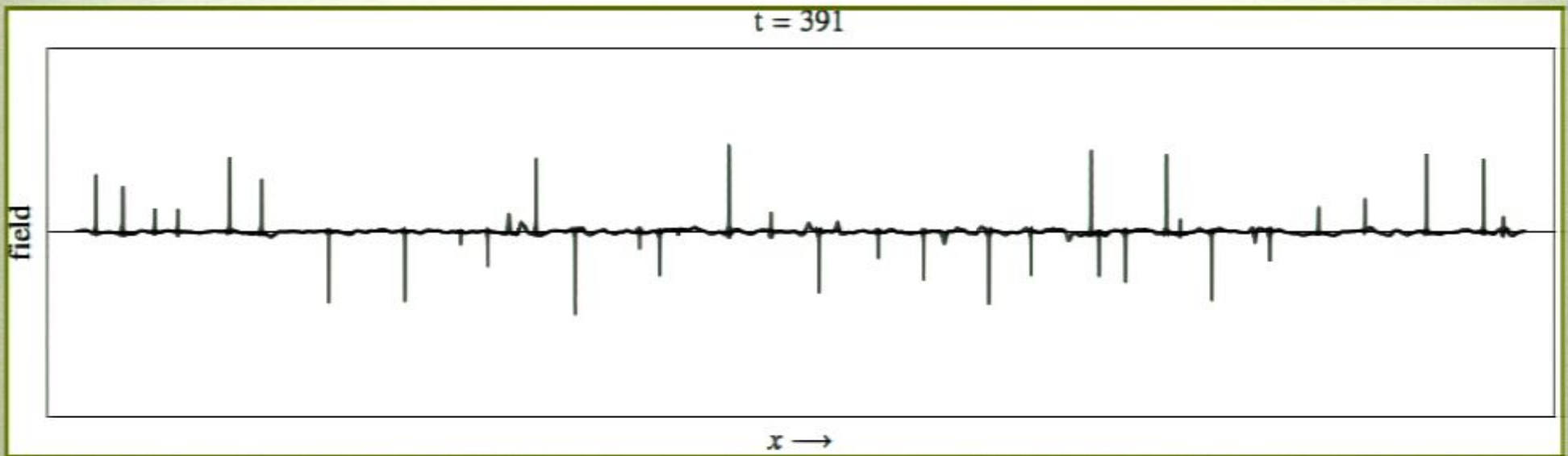
# GENERIC EMERGENCE

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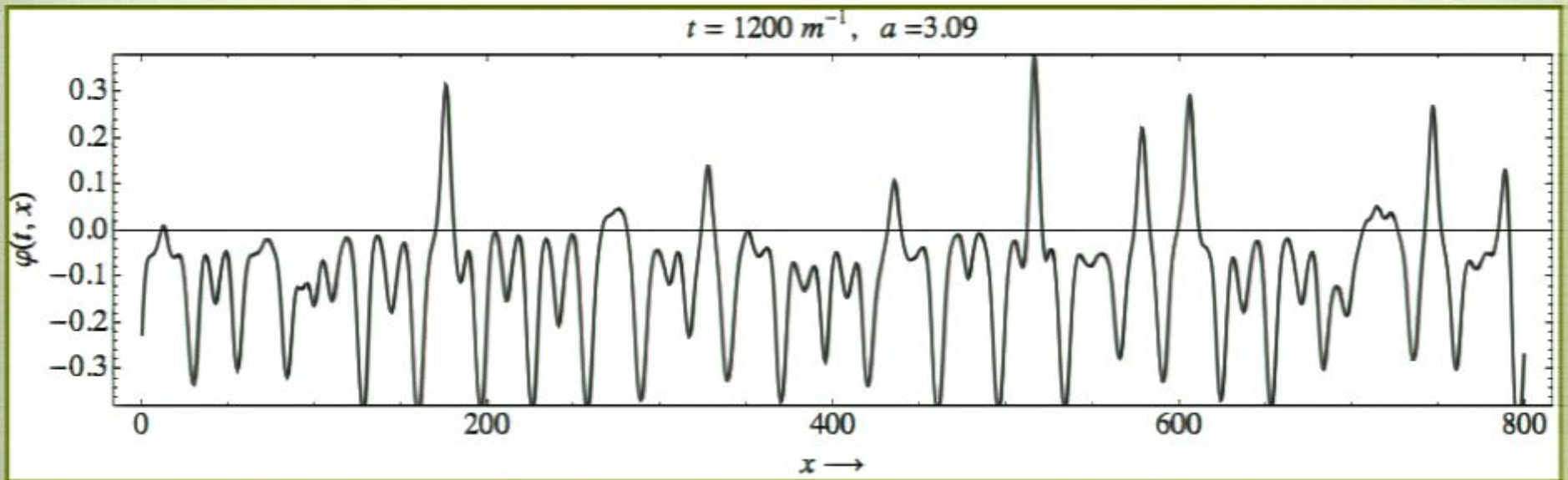
# NEED LARGE FLUCTUATIONS

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- start with tiny “quantum” fluctuations
- amplify them via resonance

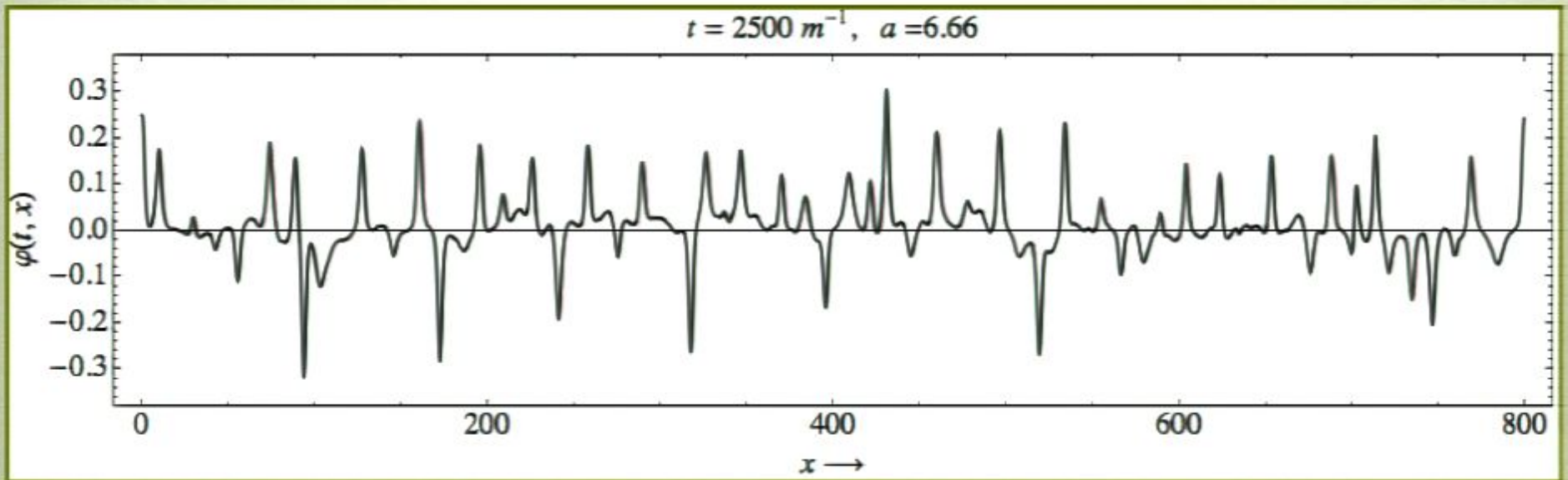
1+1

# POST-INFLATIONARY EMERGENCE



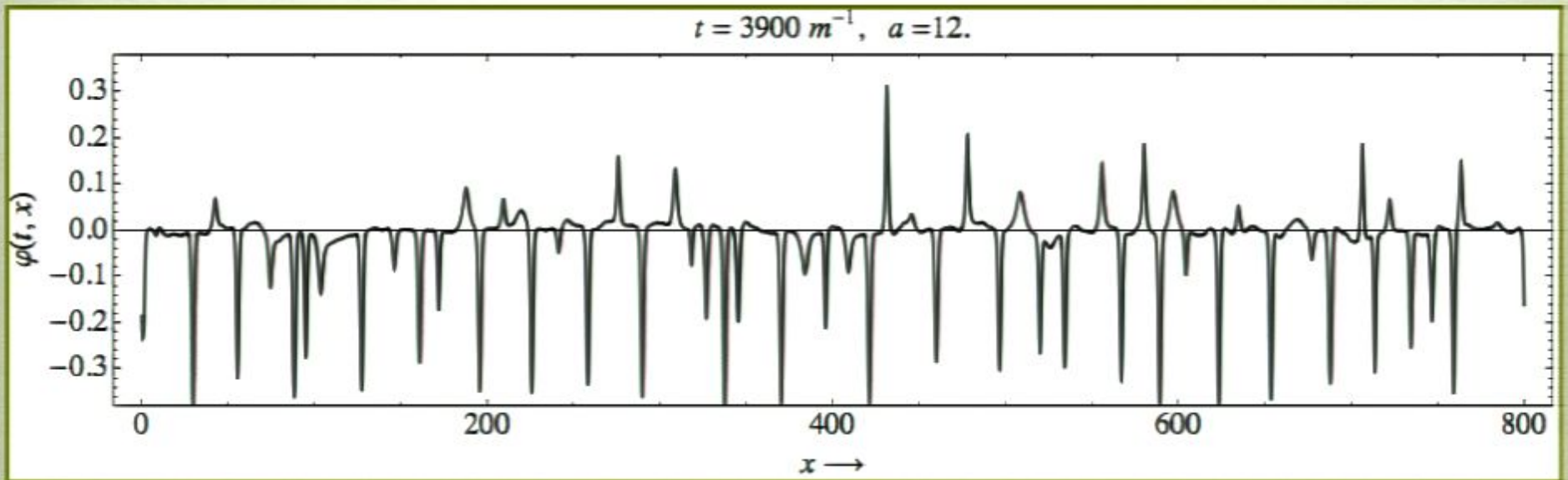
1+1

# POST-INFLATIONARY EMERGENCE



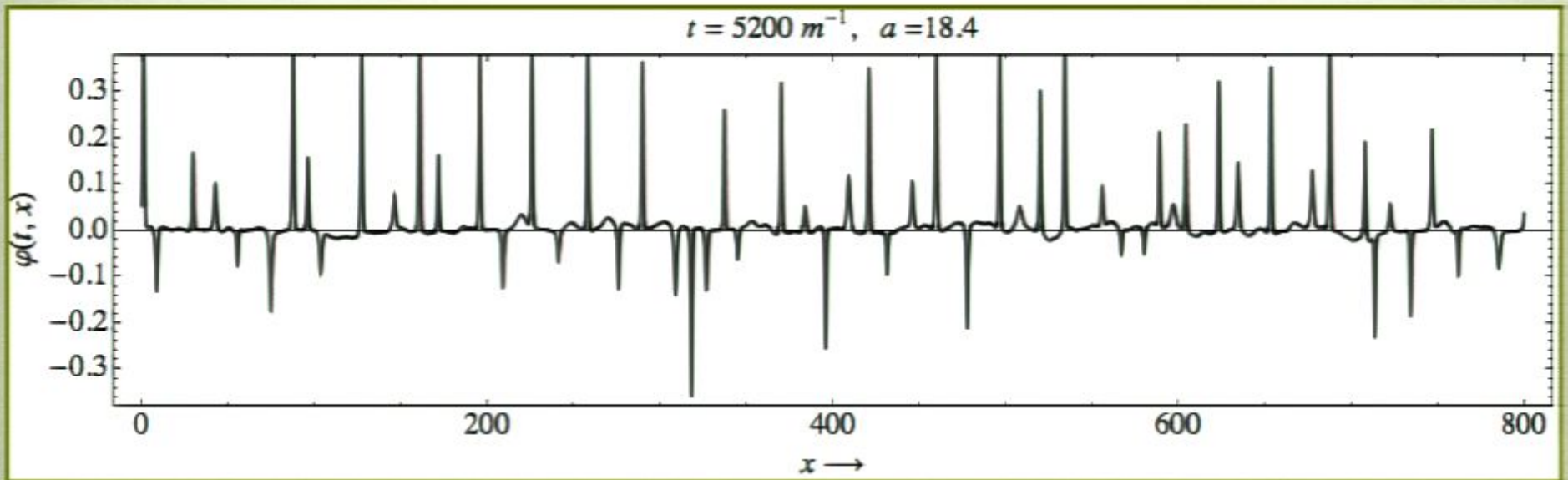
1+1

# POST-INFLATIONARY EMERGENCE



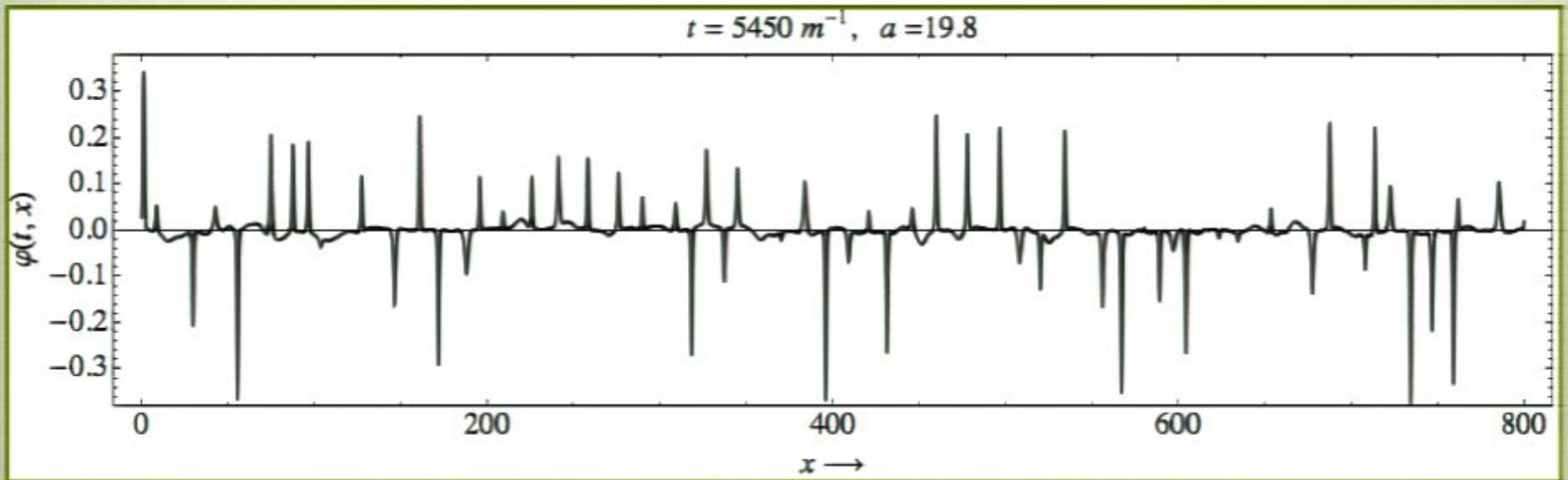
1+1

# POST-INFLATIONARY EMERGENCE



1+1

# POST-INFLATIONARY EMERGENCE



## AMPLIFICATION IF ...

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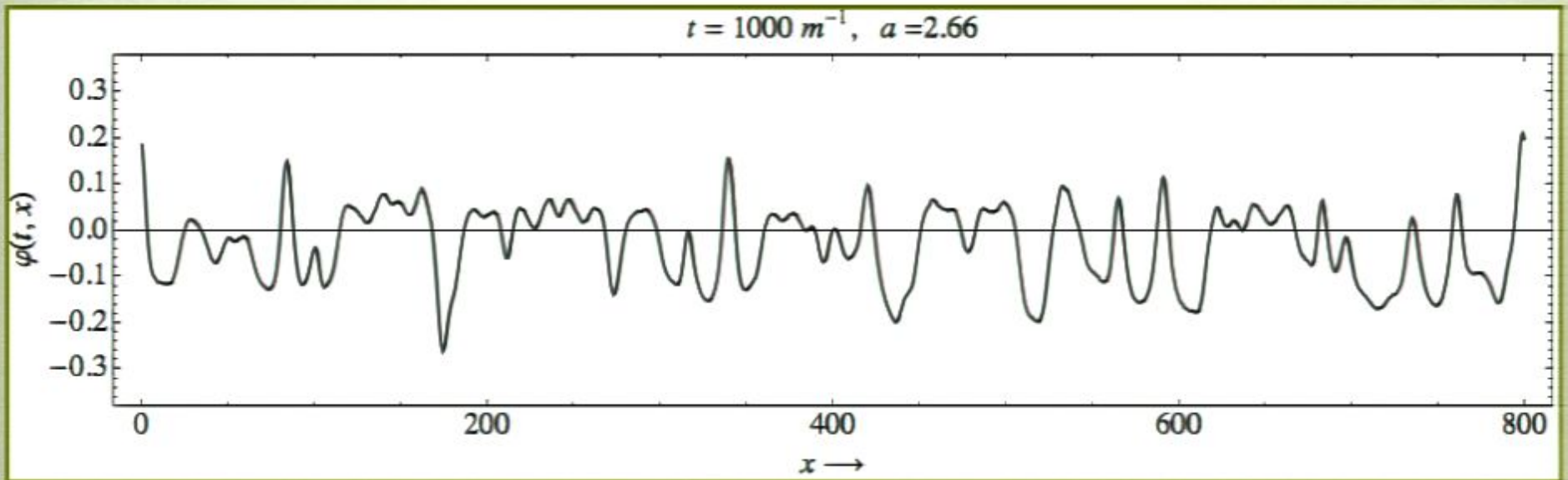
- growth rate  $\mu_k$
- expansion rate  $H$

$$\mu_k \gg H$$

$$\frac{\mu_k(a)}{H} \approx \underbrace{\left[ \left( \frac{m_{pl}}{m} \right) \frac{\lambda^{3/2}}{g} \right]}_{\beta} f(k, a) \gg 1$$

1+1

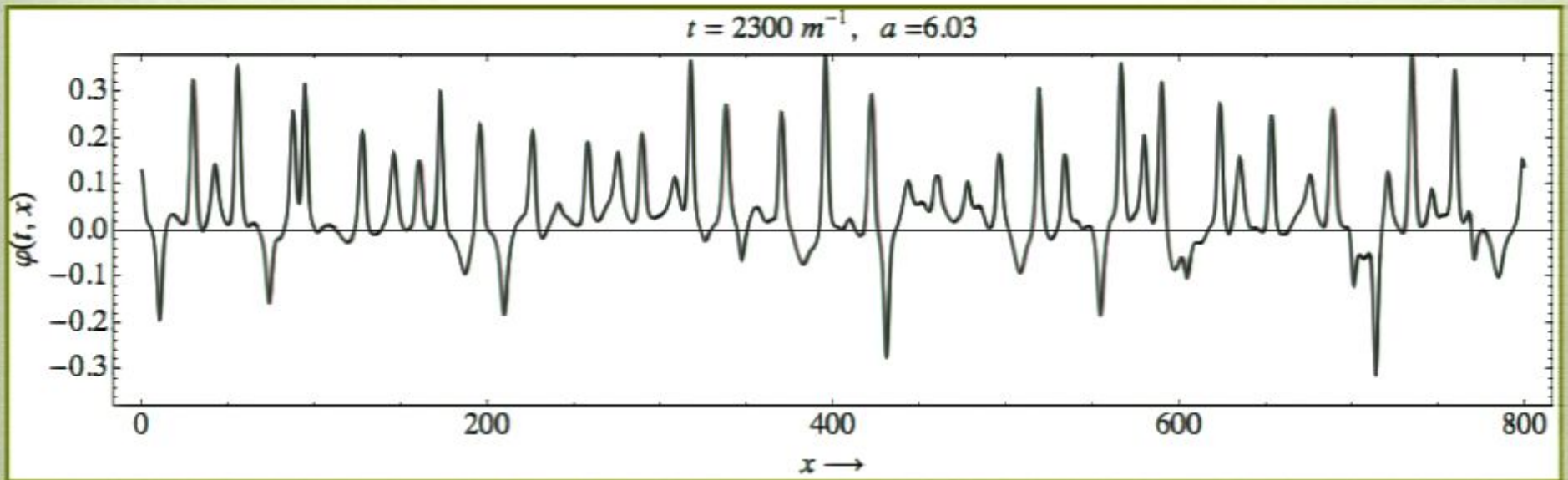
# POST-INFLATIONARY EMERGENCE





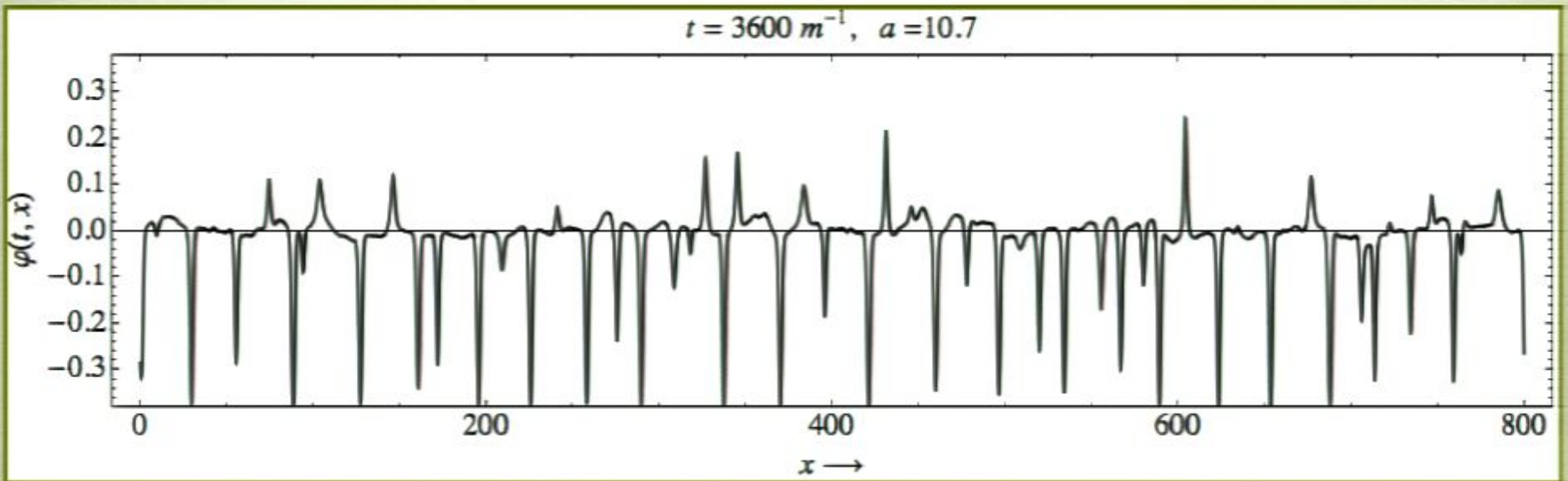
1+1

# POST-INFLATIONARY EMERGENCE



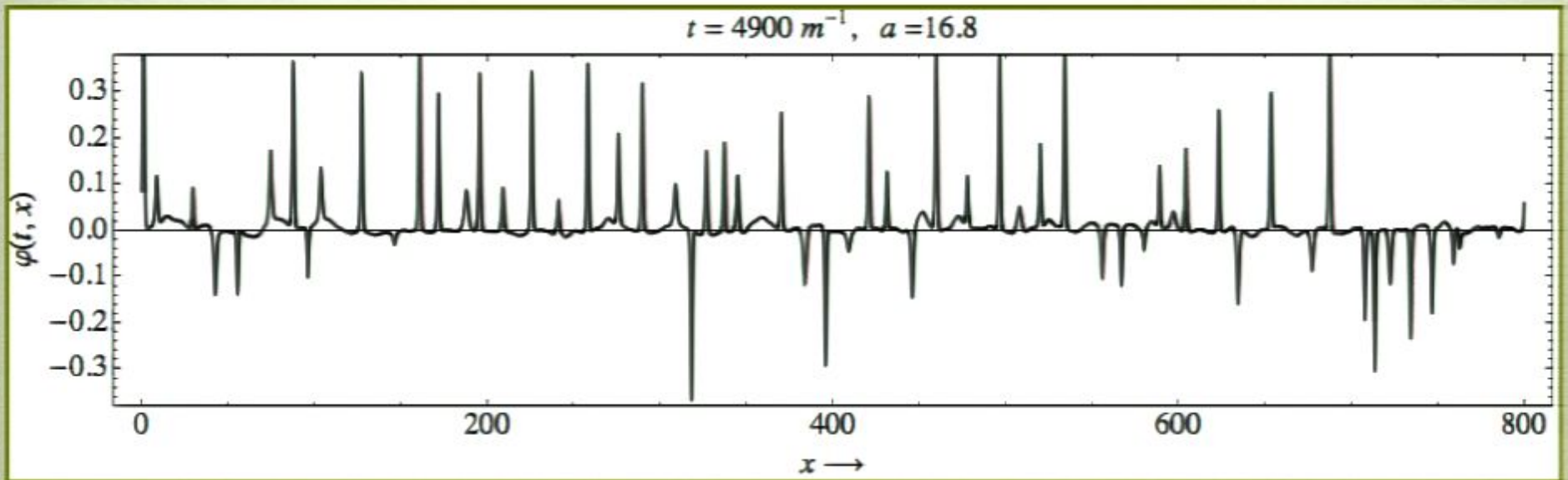
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# POST-INFLATIONARY EMERGENCE



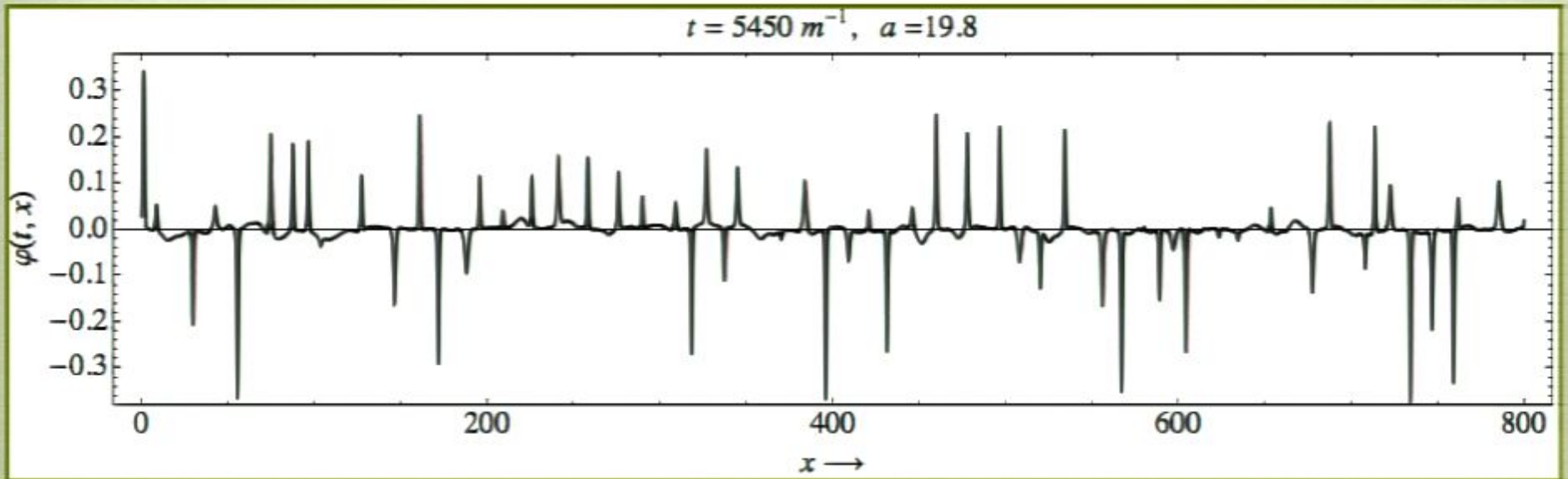
1+1

# POST-INFLATIONARY EMERGENCE



1+1

# POST-INFLATIONARY EMERGENCE




## AMPLIFICATION IF ...

- growth rate  $\mu_k$
- expansion rate  $H$

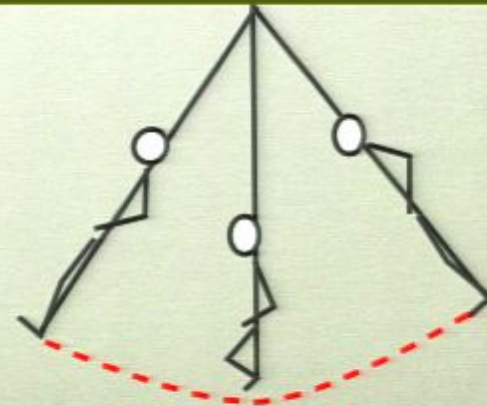
$$\mu_k \gg H$$

$$\frac{\mu_k(a)}{H} \approx \underbrace{\left[ \left( \frac{m_{pl}}{m} \right) \frac{\lambda^{3/2}}{g} \right]}_{\beta} f(k, a) \gg 1$$

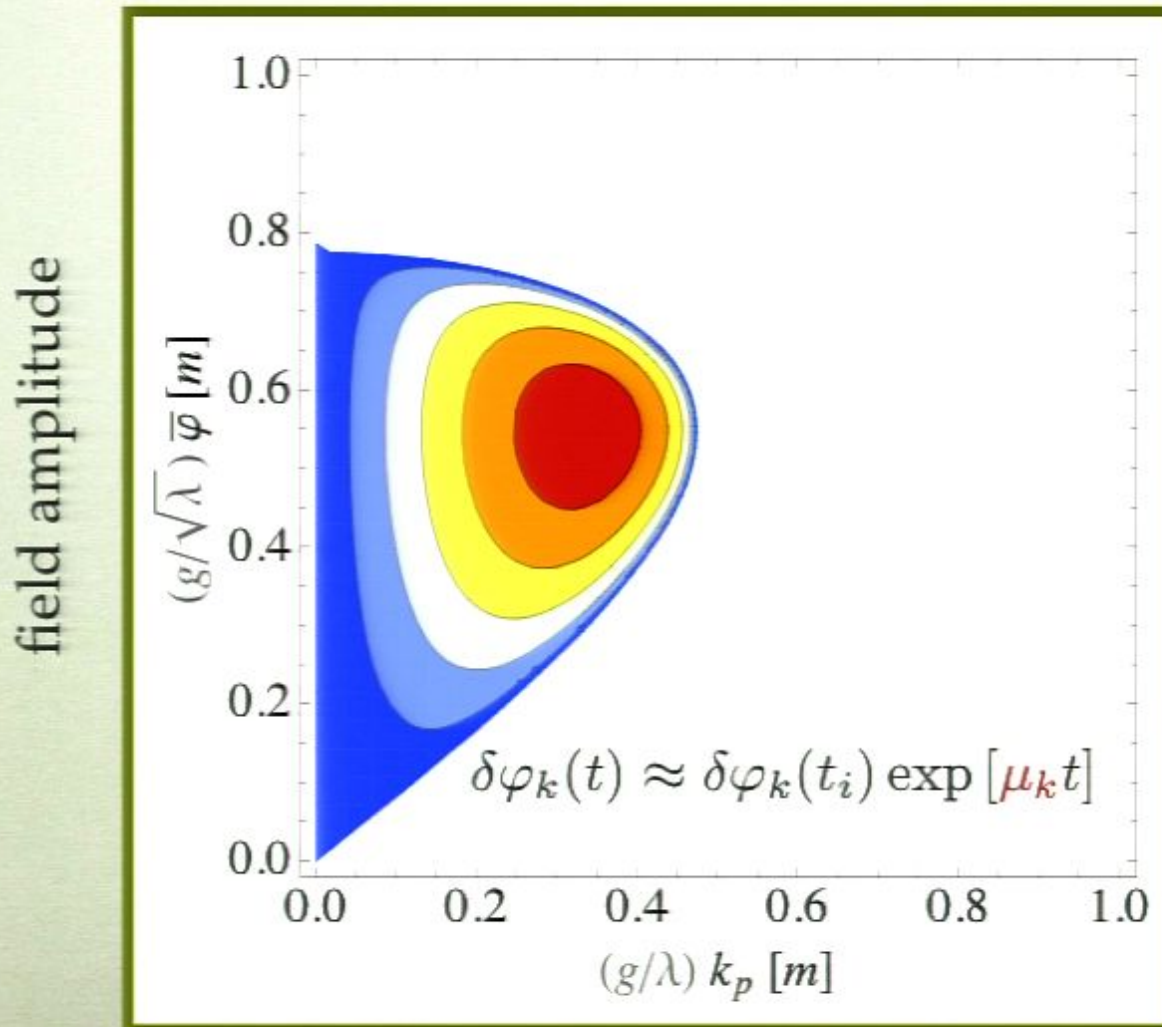
# LINEARIZED FLUCTUATIONS

$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{\lambda}{4}\varphi^4 + \frac{g^2}{6m^2}\varphi^6$$


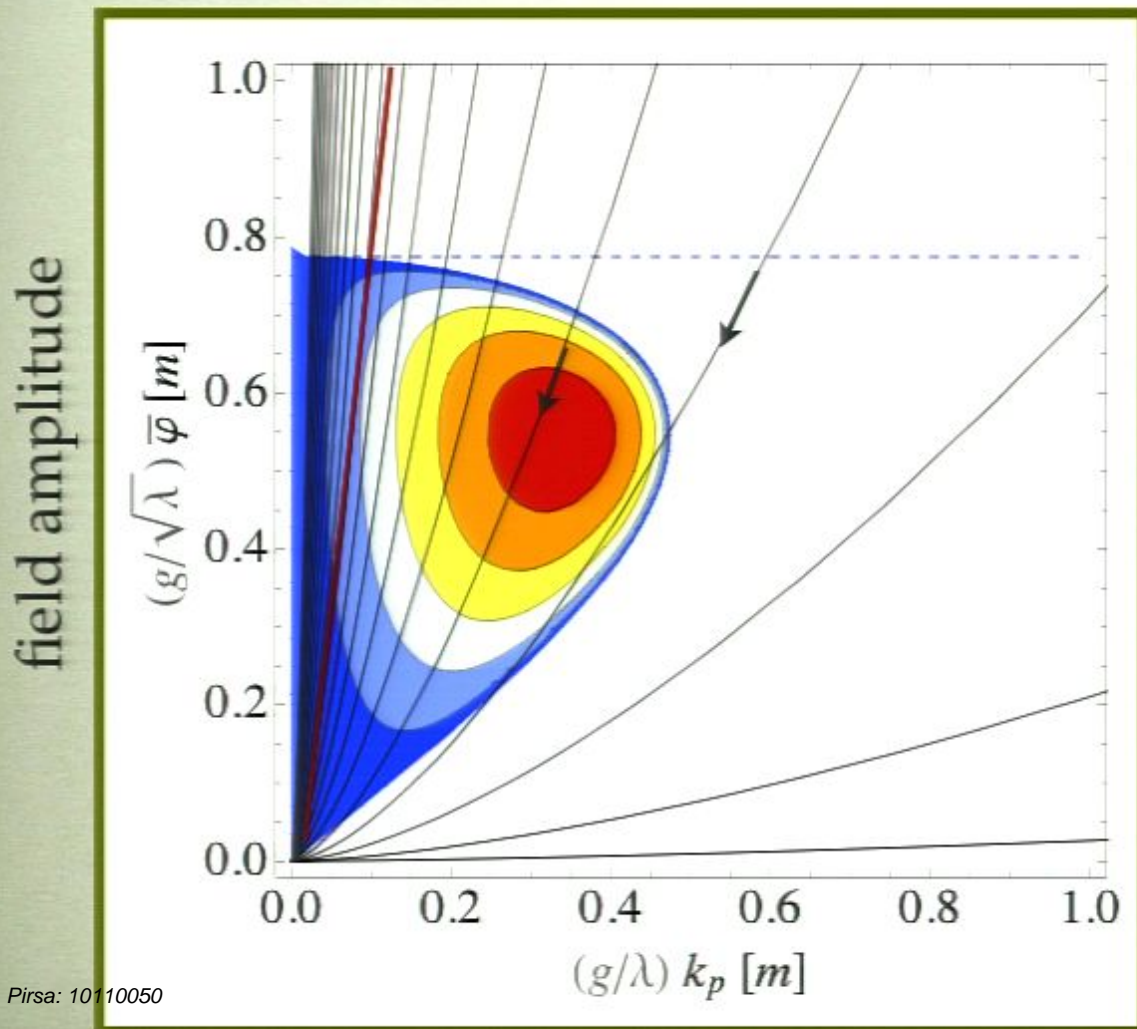
$$\partial_t^2 \delta\varphi_k + 3H\partial_t \delta\varphi_k + \left( \frac{k^2}{a^2} + V'''(\bar{\varphi}) \right) \delta\varphi_k = 0$$



# FLOQUET GROWTH: FLAT SPACE



# FLOQUET ANALYSIS: EXPANDING UNIVERSE



$$\delta\varphi_k \approx \frac{\delta\varphi_k(t_i)}{a^{3/2}(t)} \exp \left[ \int dt \mu_k(t) \right]$$

$$= \frac{\delta\varphi_k(a_i)}{a^{3/2}} \exp \left[ \int d \ln a \frac{\mu_k(a)}{H(a)} \right]$$



# LINEAR THEORY BREAKS DOWN

---

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---

- end of *quasi-exponential* growth

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---

- end of *quasi-exponential* growth
- fragmentation

# LINEAR THEORY BREAKS DOWN

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- end of *quasi-exponential* growth
- fragmentation
- rapid mode-mode coupling

# LINEAR THEORY BREAKS DOWN

---

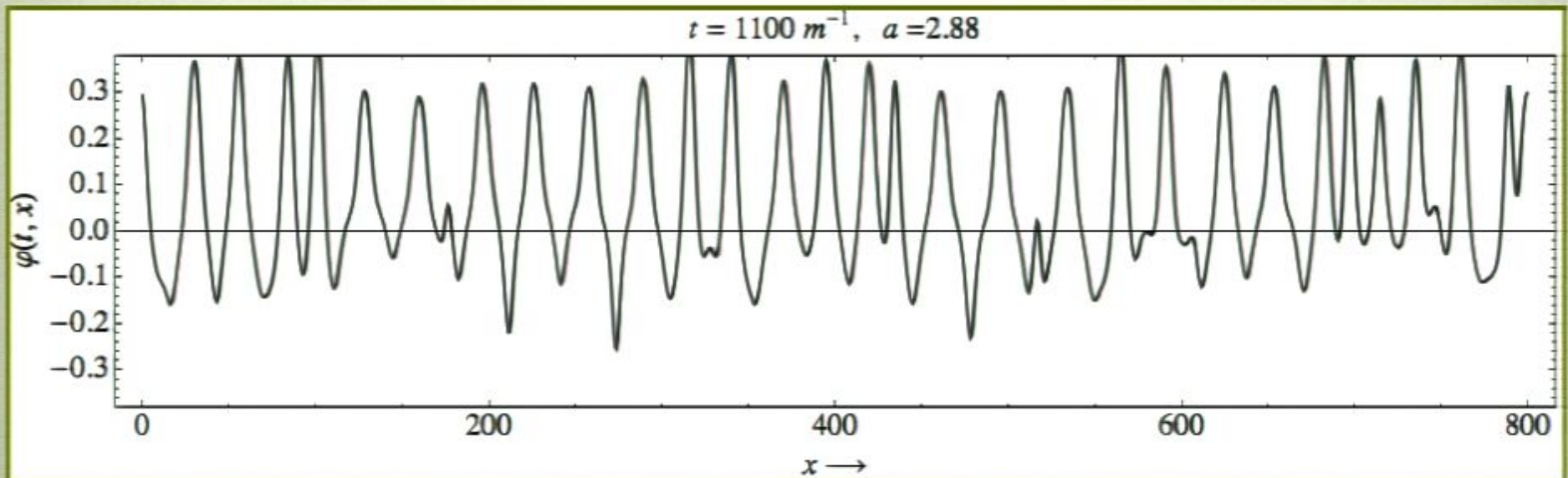
- end of *quasi-exponential* growth
- fragmentation
- rapid mode-mode coupling
- oscillon formation

---

# PREDICTING THE NUMBER DENSITY

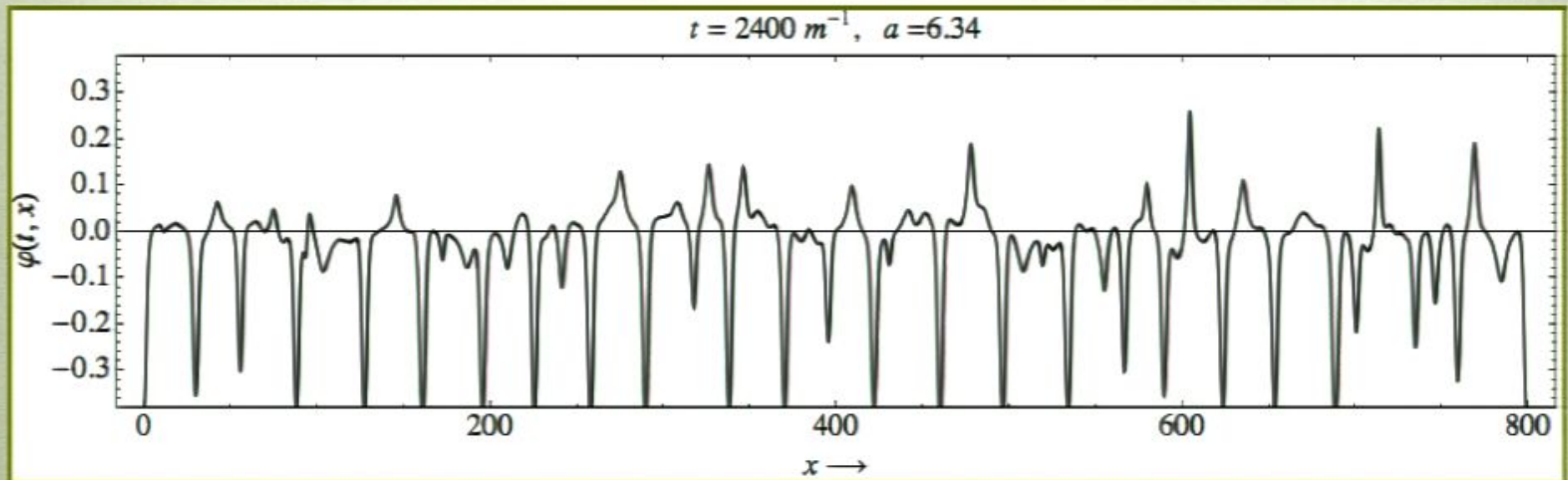
# ONE IMPORTANT SCALE

$k_{nl}$



# ONE IMPORTANT SCALE

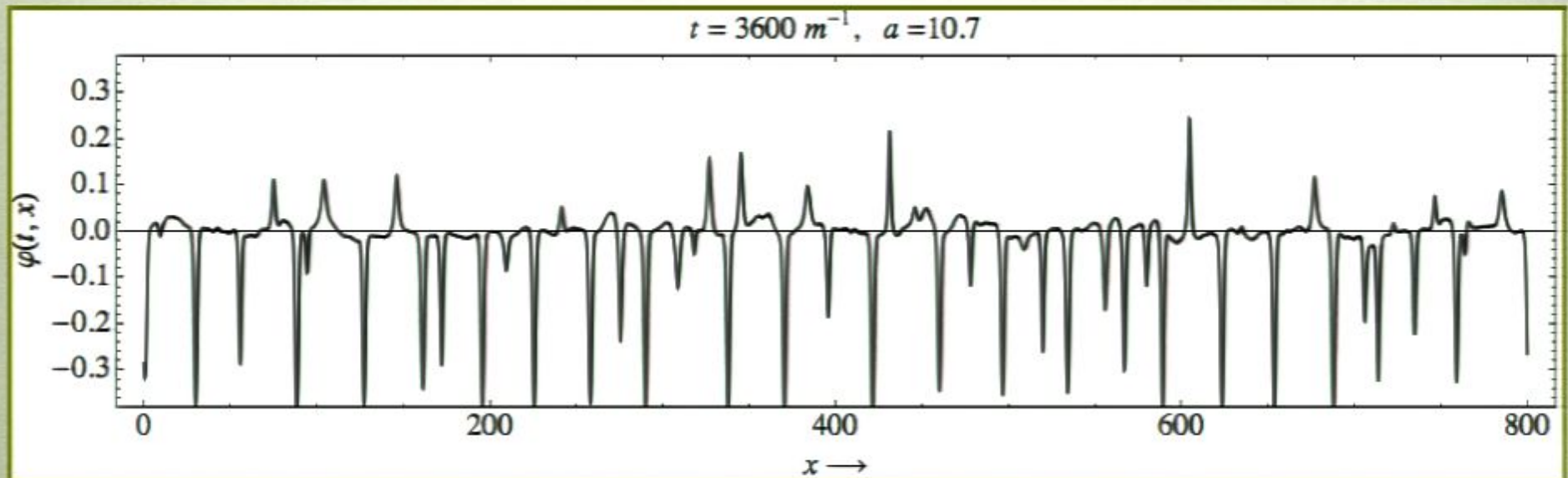
$k_{nl}$





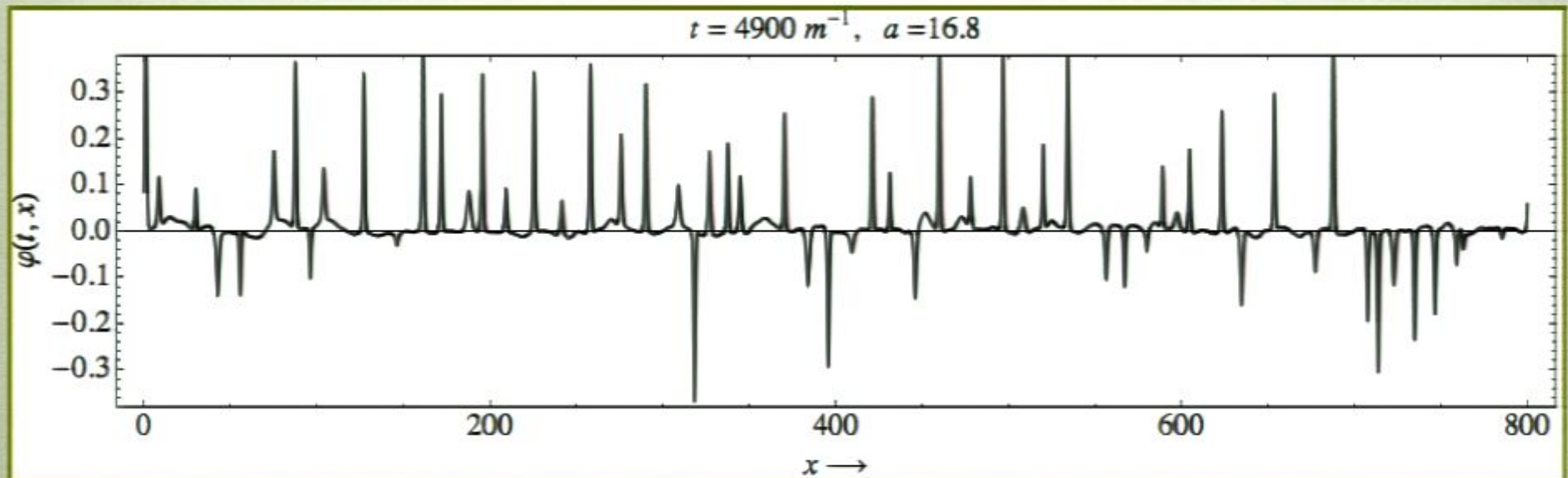
# ONE IMPORTANT SCALE

$k_{nl}$



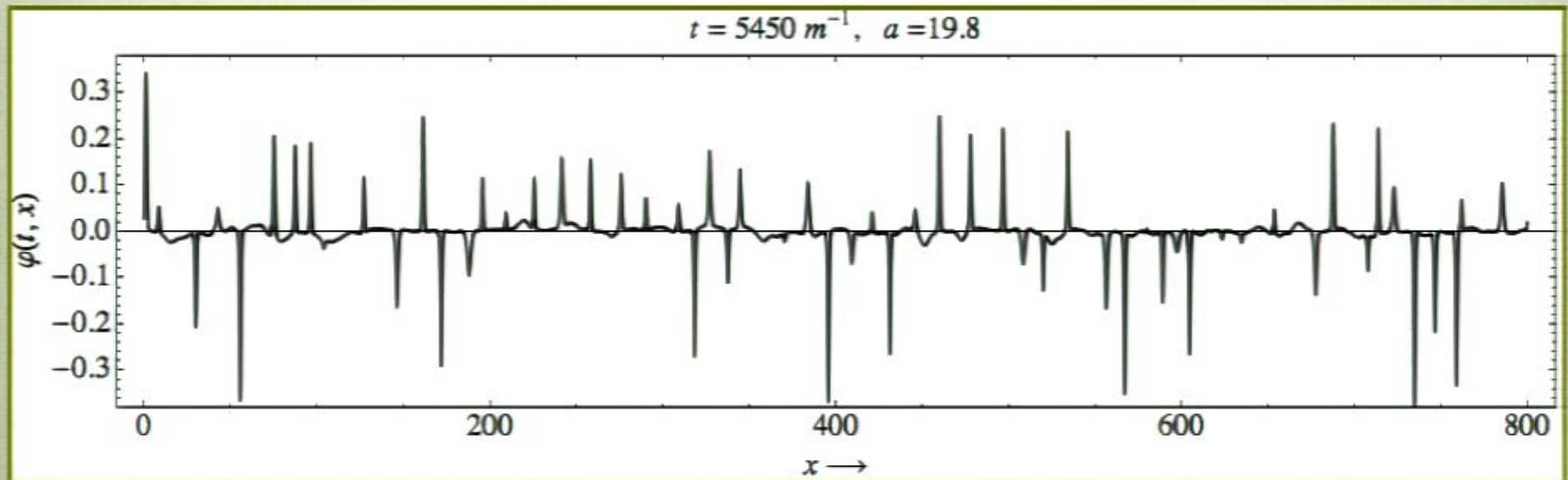
# ONE IMPORTANT SCALE

$k_{nl}$



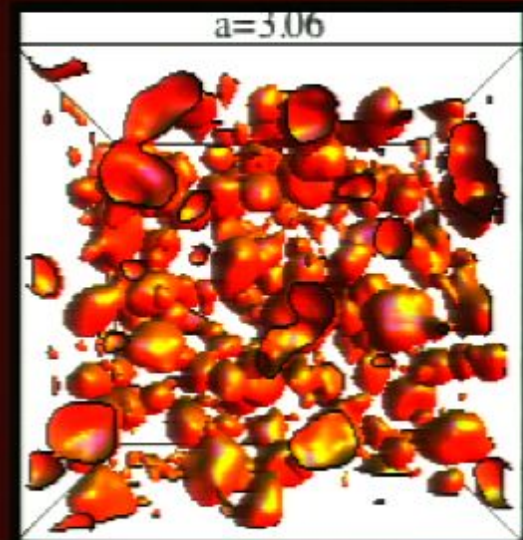
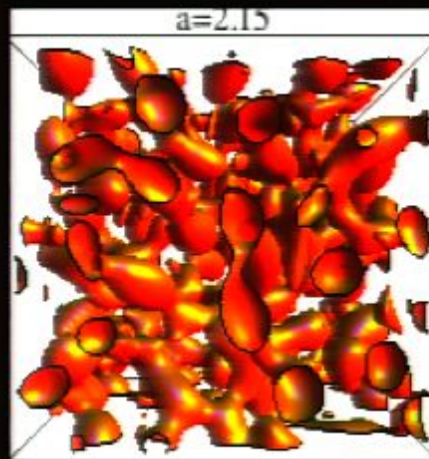
# ONE IMPORTANT SCALE

$k_{nl}$

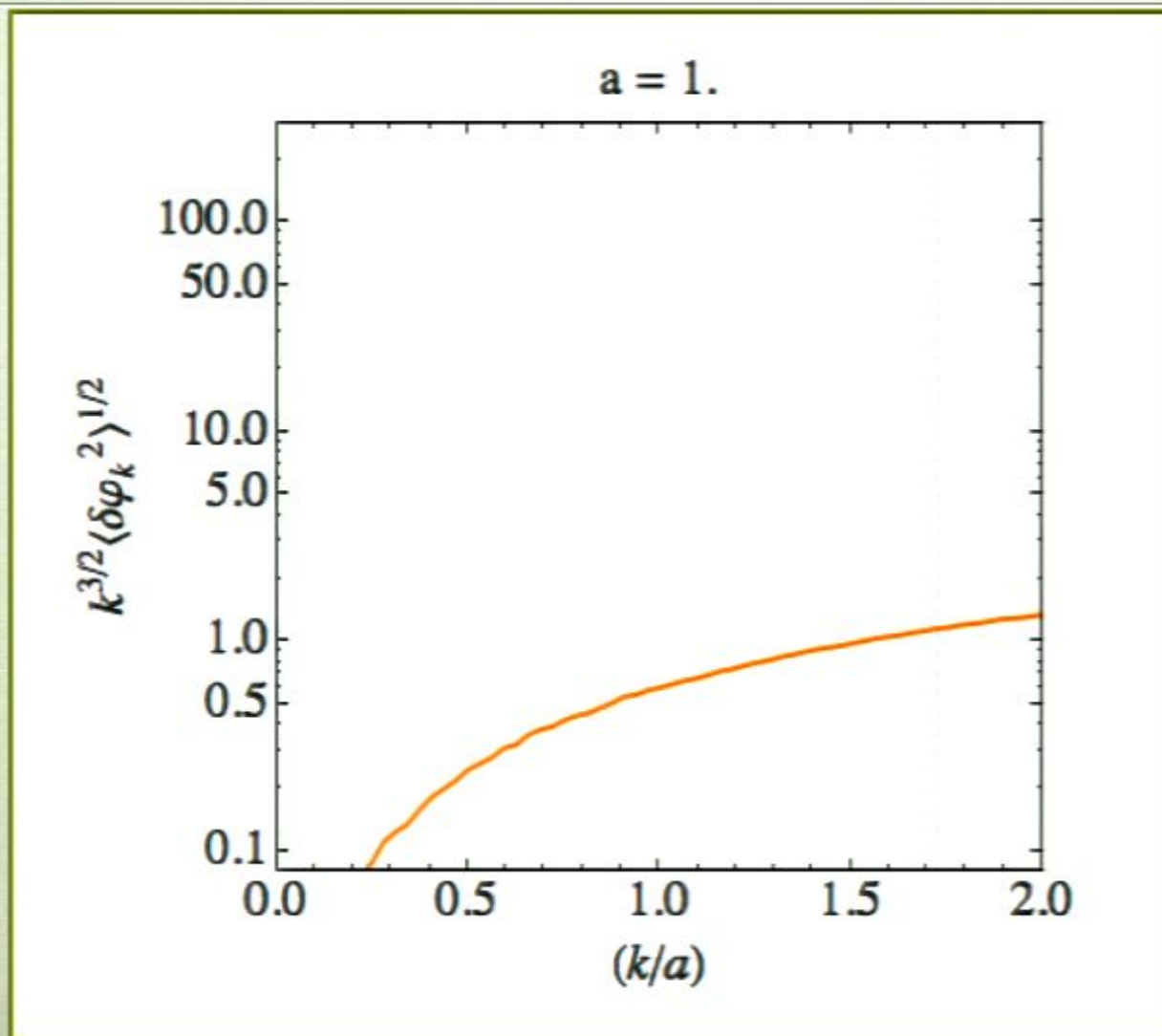


# HOW MANY PEAKS ?

$k_{nl}$



# POWER SPECTRUM

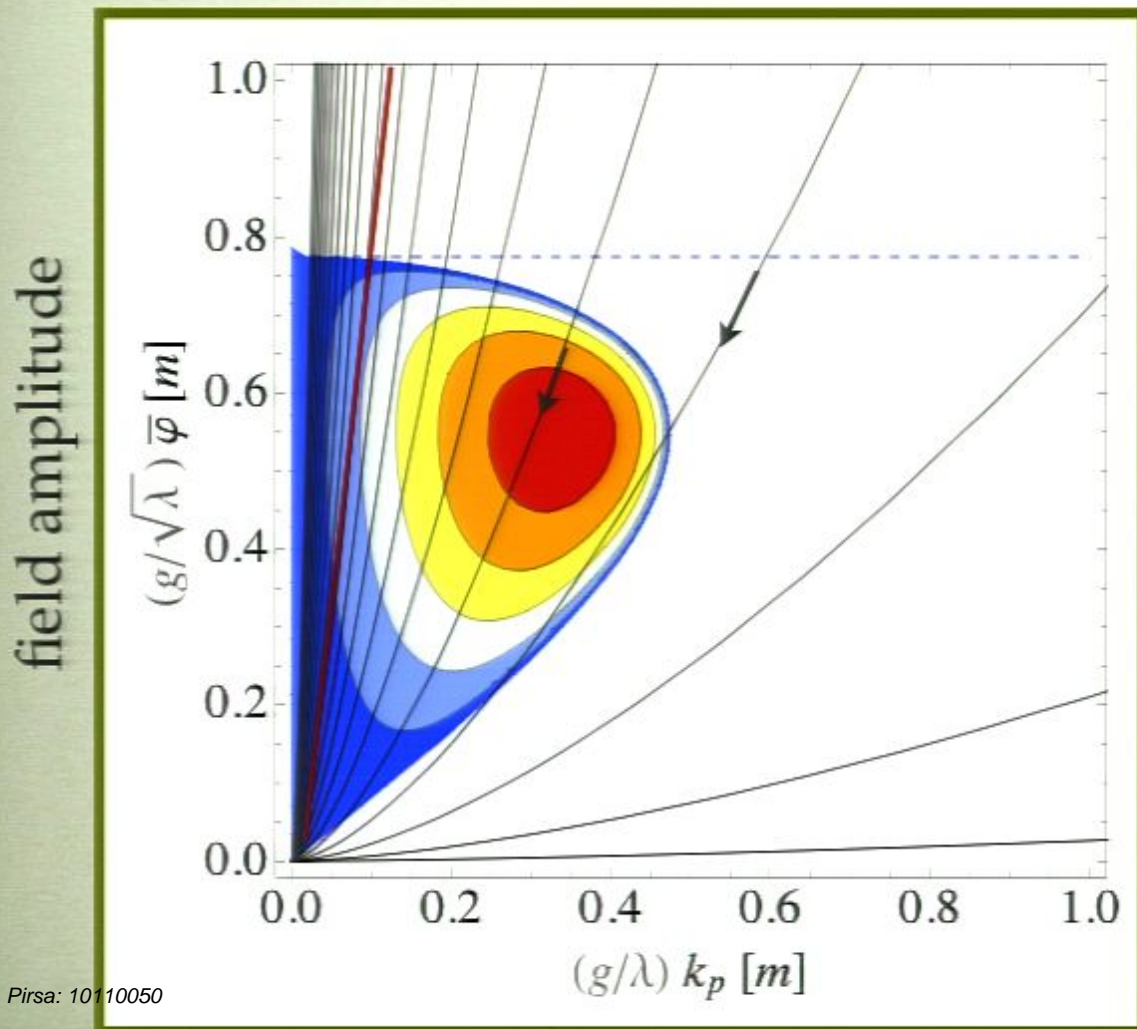


# NEED LARGE FLUCTUATIONS

---

- start with tiny “quantum” fluctuations
- amplify them via resonance

# FLOQUET ANALYSIS: EXPANDING UNIVERSE



$$\begin{aligned} \delta\varphi_k &\approx \frac{\delta\varphi_k(t_i)}{a^{3/2}(t)} \exp \left[ \int dt \mu_k(t) \right] \\ &= \frac{\delta\varphi_k(a_i)}{a^{3/2}} \exp \left[ \int d \ln a \frac{\mu_k(a)}{H(a)} \right] \end{aligned}$$

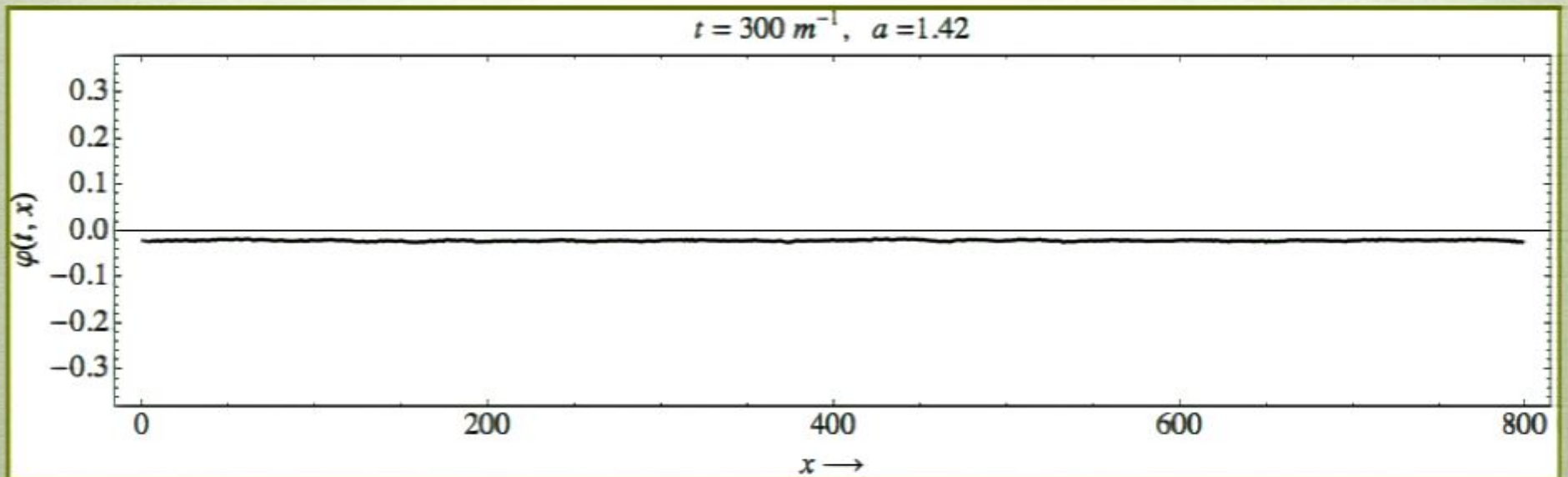
# LINEAR THEORY BREAKS DOWN

---

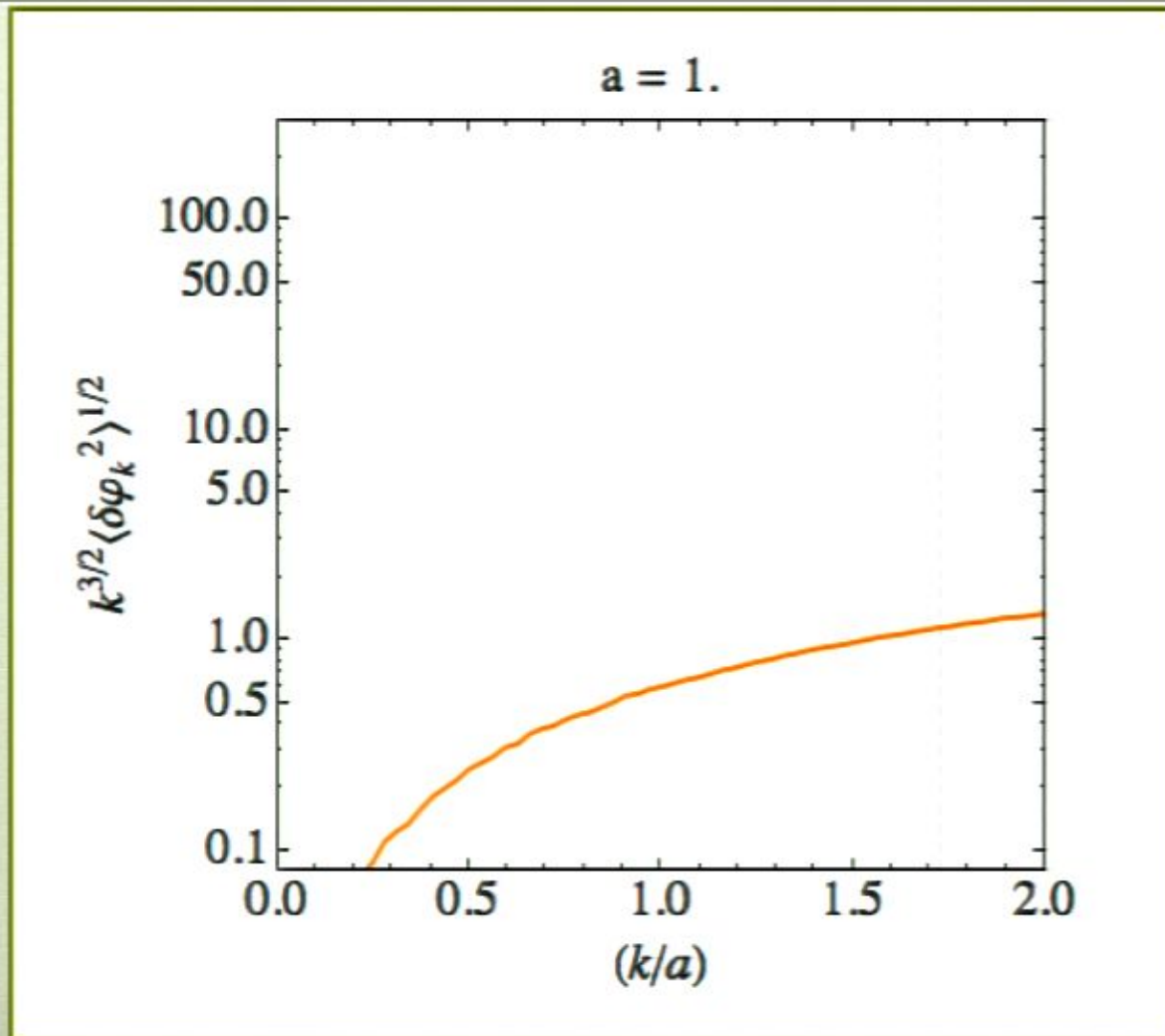


# ONE IMPORTANT SCALE

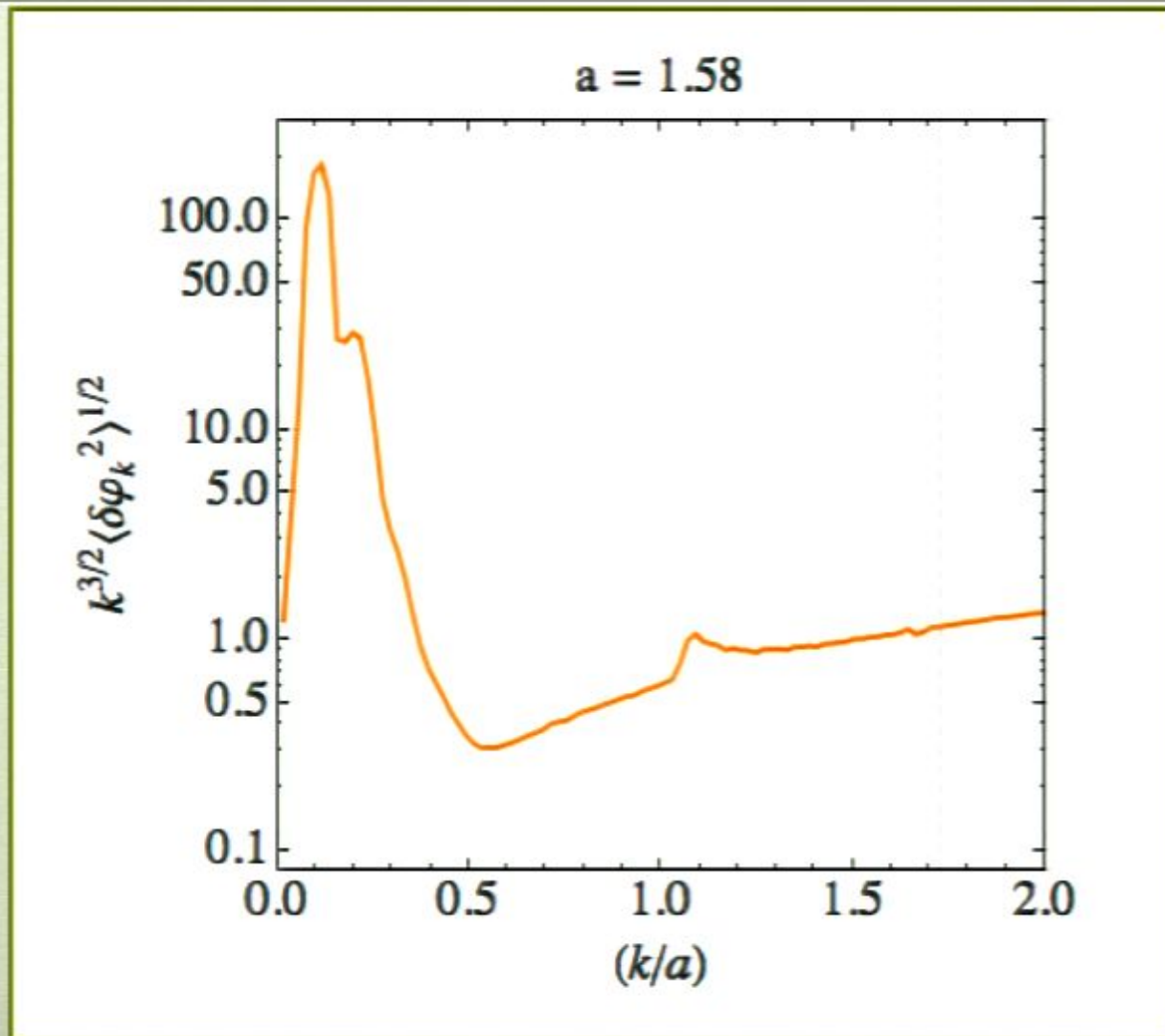
$k_{nl}$



# POWER SPECTRUM



# POWER SPECTRUM



# NUMBER DENSITY

---

$$n_{osc} a^3 \sim \left( \frac{k_{nl}}{2\pi} \right)^3$$
$$\approx \beta^{-3/5} \left( \frac{\lambda}{g} \frac{m}{2\pi} \right)^3$$

MA (2010)

$$\frac{\mu_k}{H} \beta \equiv \left[ \left( \frac{m_{pl}}{m} \right) \frac{\lambda^{3/2}}{g} \right]$$

# NUMERICAL SIMULATIONS



# NUMERICAL SIMULATIONS

---

- verify individual characteristics
- confirm predictions for number densities and determine energy fraction
- more realistic inflationary potentials where analytics are difficult (eg. monodromy models)

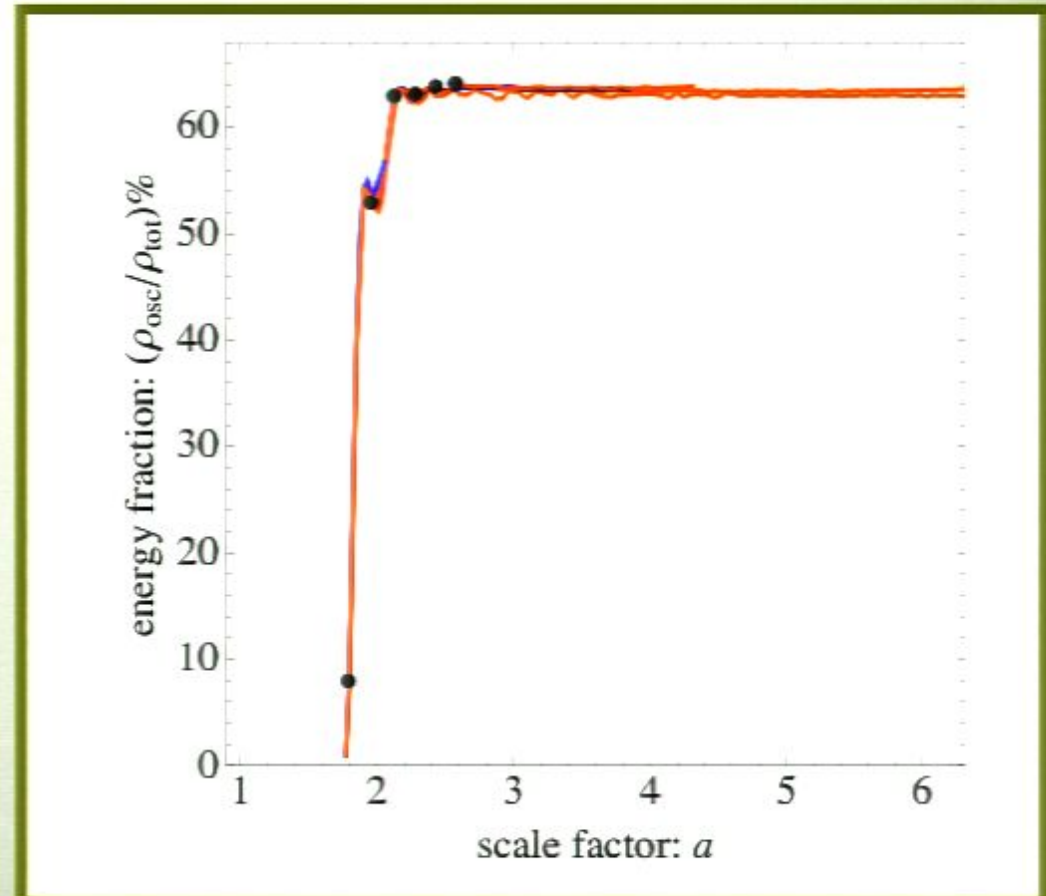
# SIMULATION CODES

---

- used modified versions of
  - **PSpectRe** (*Pseudo-Spectral* by Easther, Finkel & Roth 2008)
  - **Defrost** (*Finite Difference* by Frolov 2007)

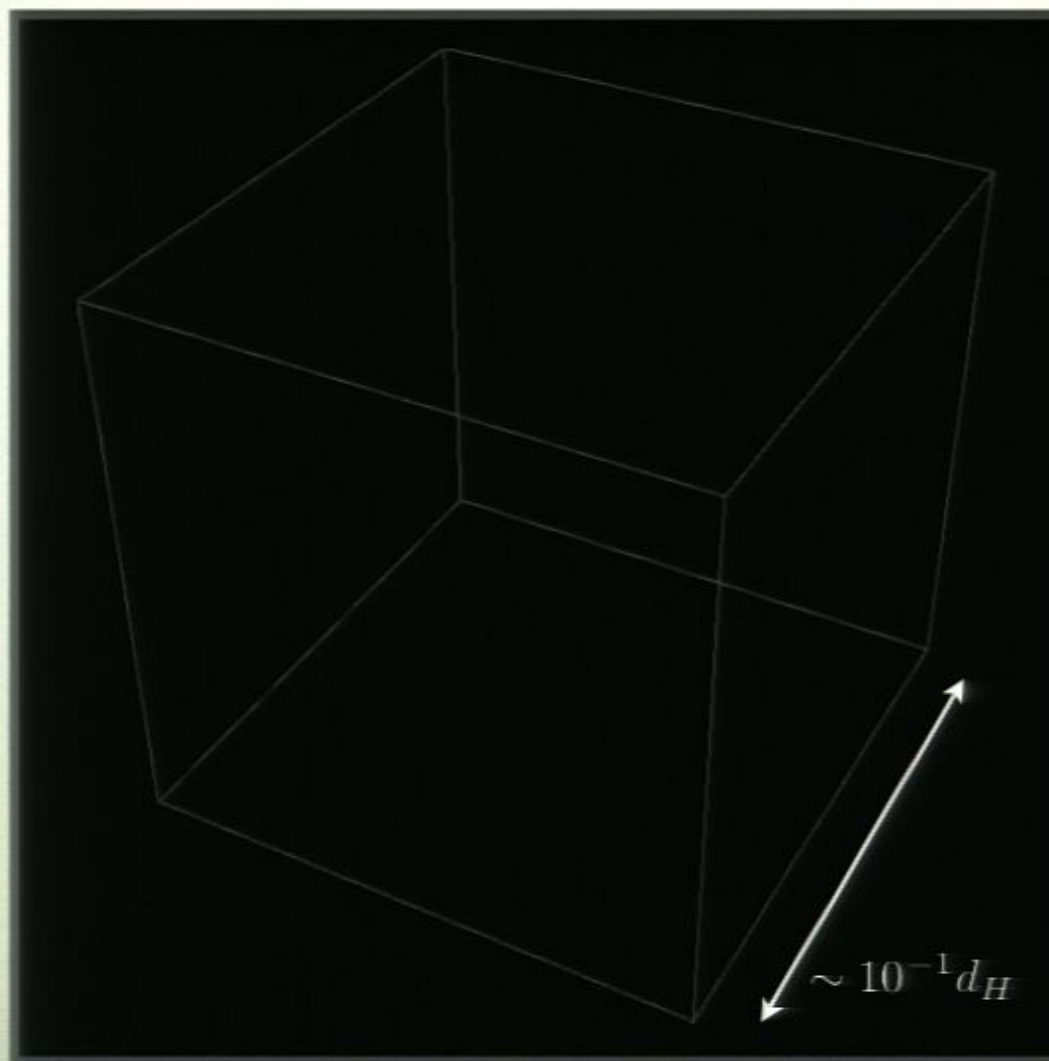
# ROBUSTNESS OF NUMERICS

- black points: defrost  
( $N = 1024$ ,  $L = 400$ )
- orange curves  
PSpectRe  
( $N = 256$ ,  
 $L = 200, 400, 600$ )
- blue curve PSpectRe  
( $N = 384$ ,  $L = 400$ )

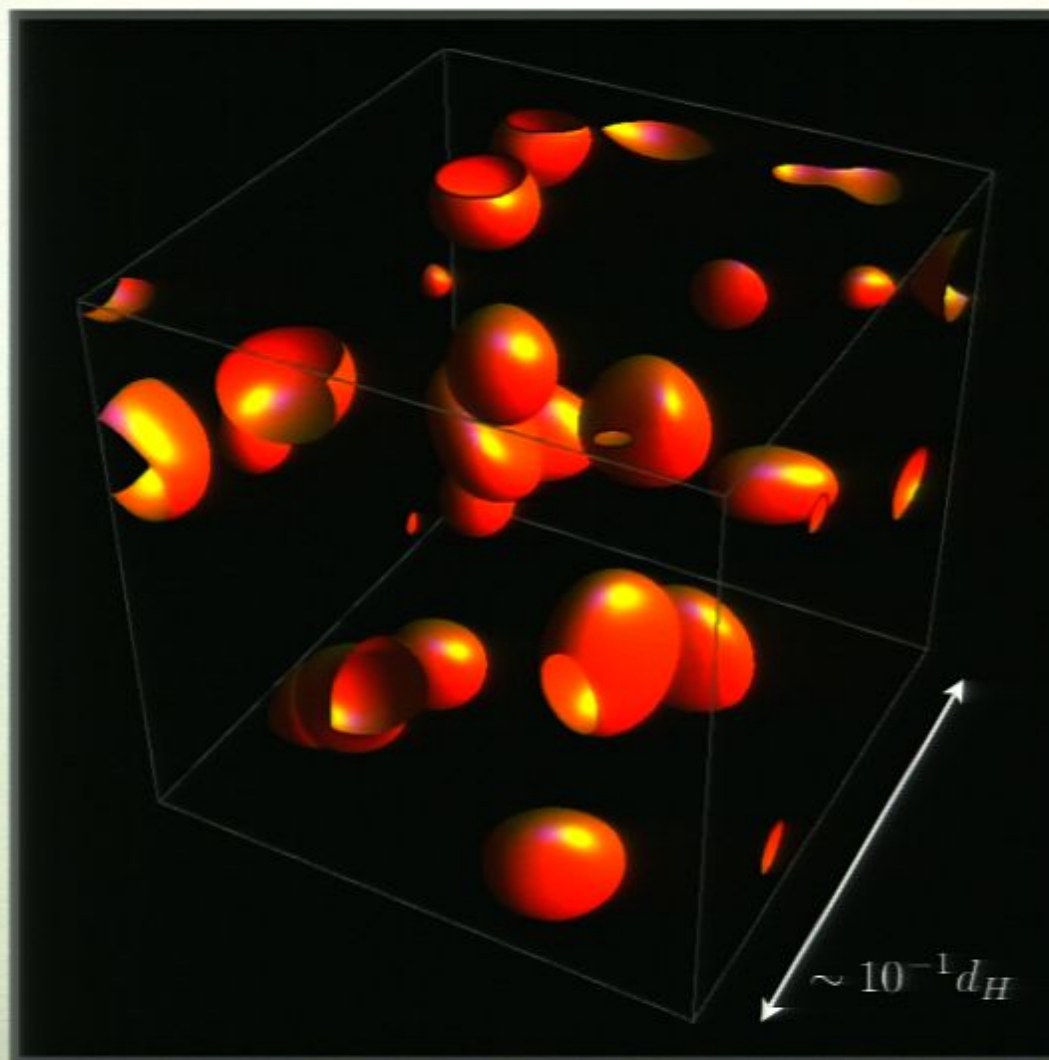




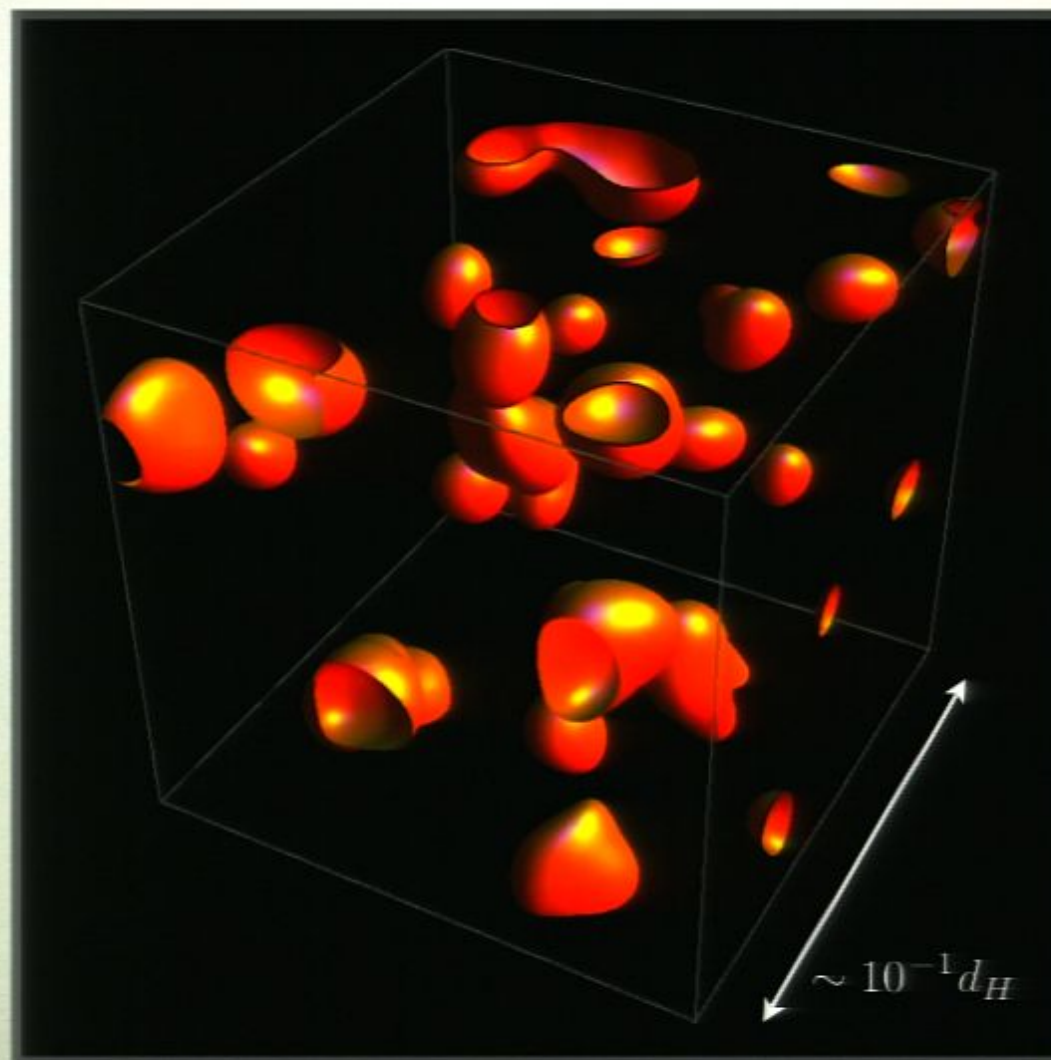
# OSCILLONS?



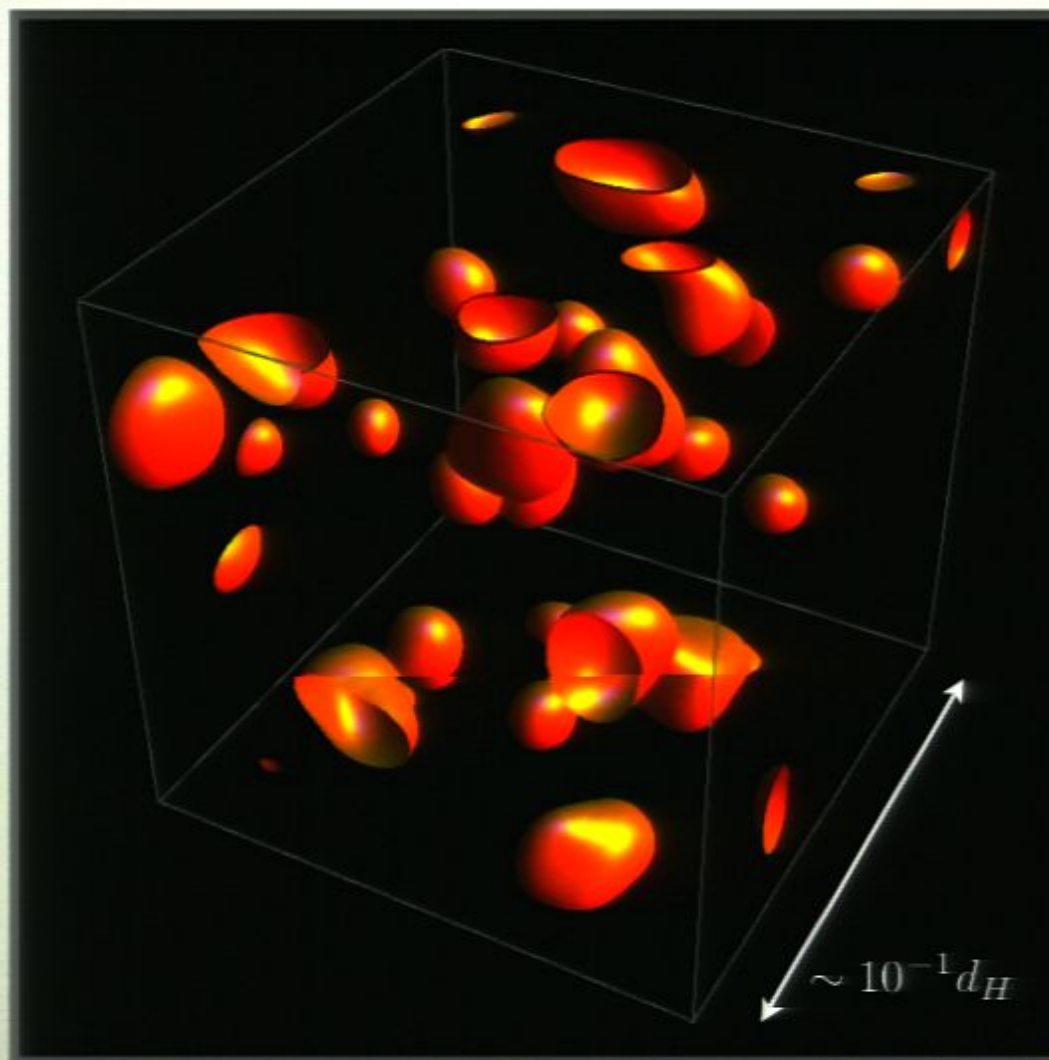
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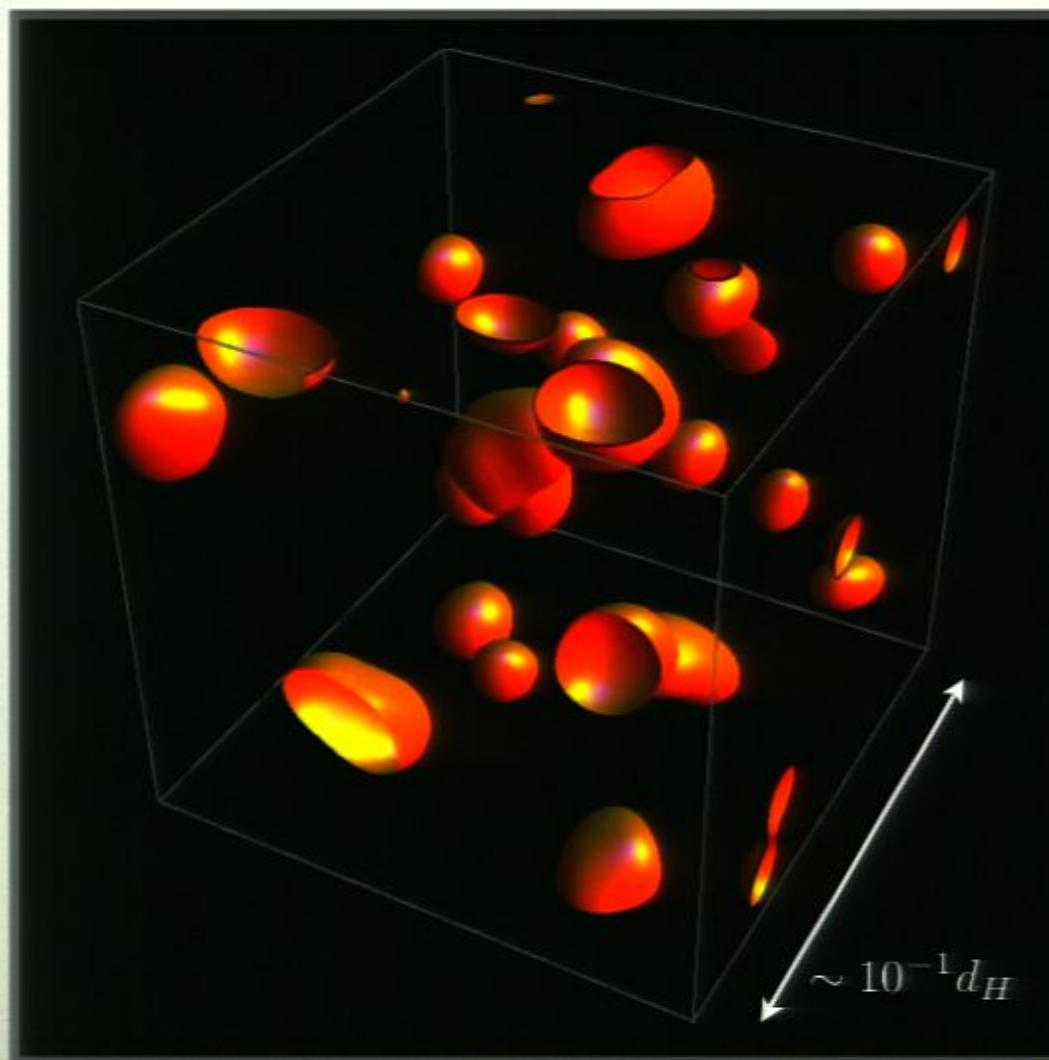
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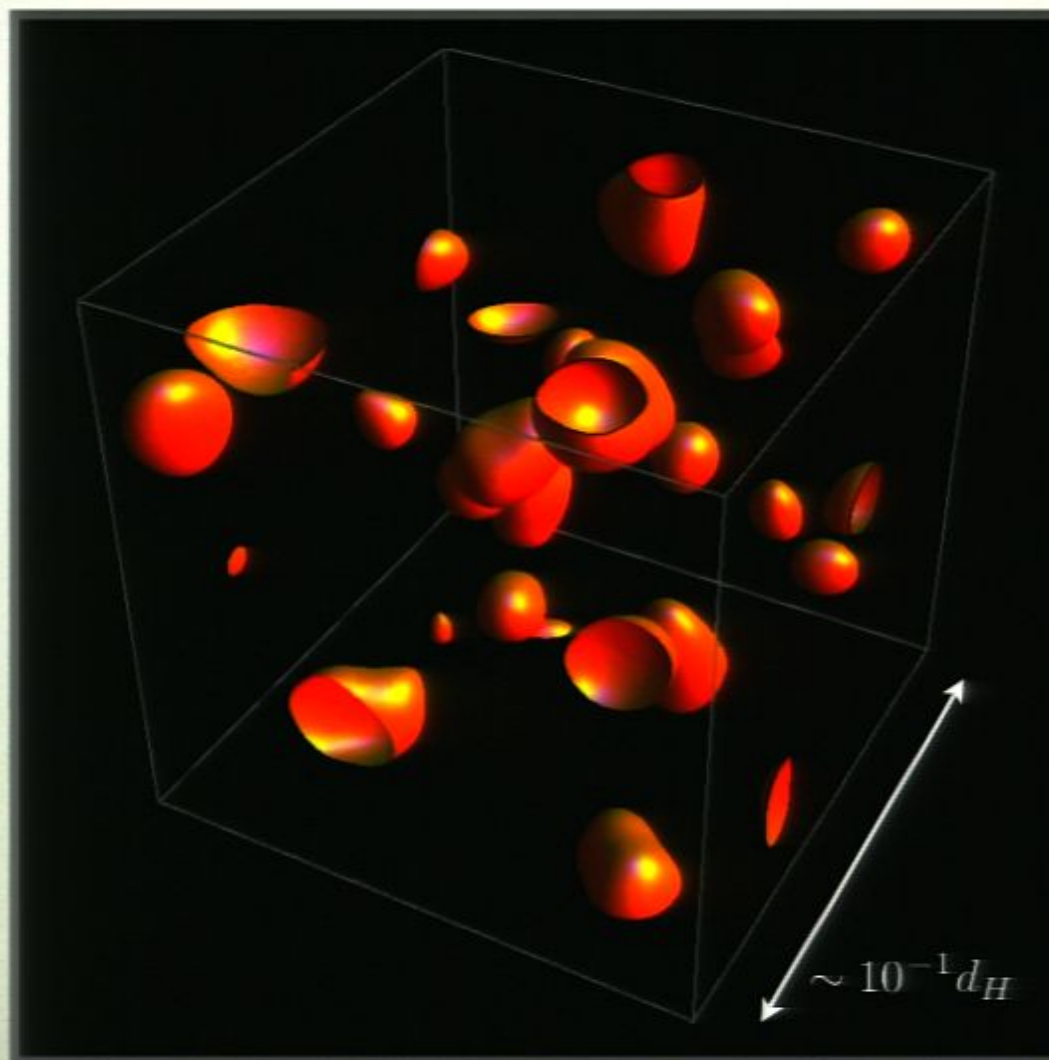
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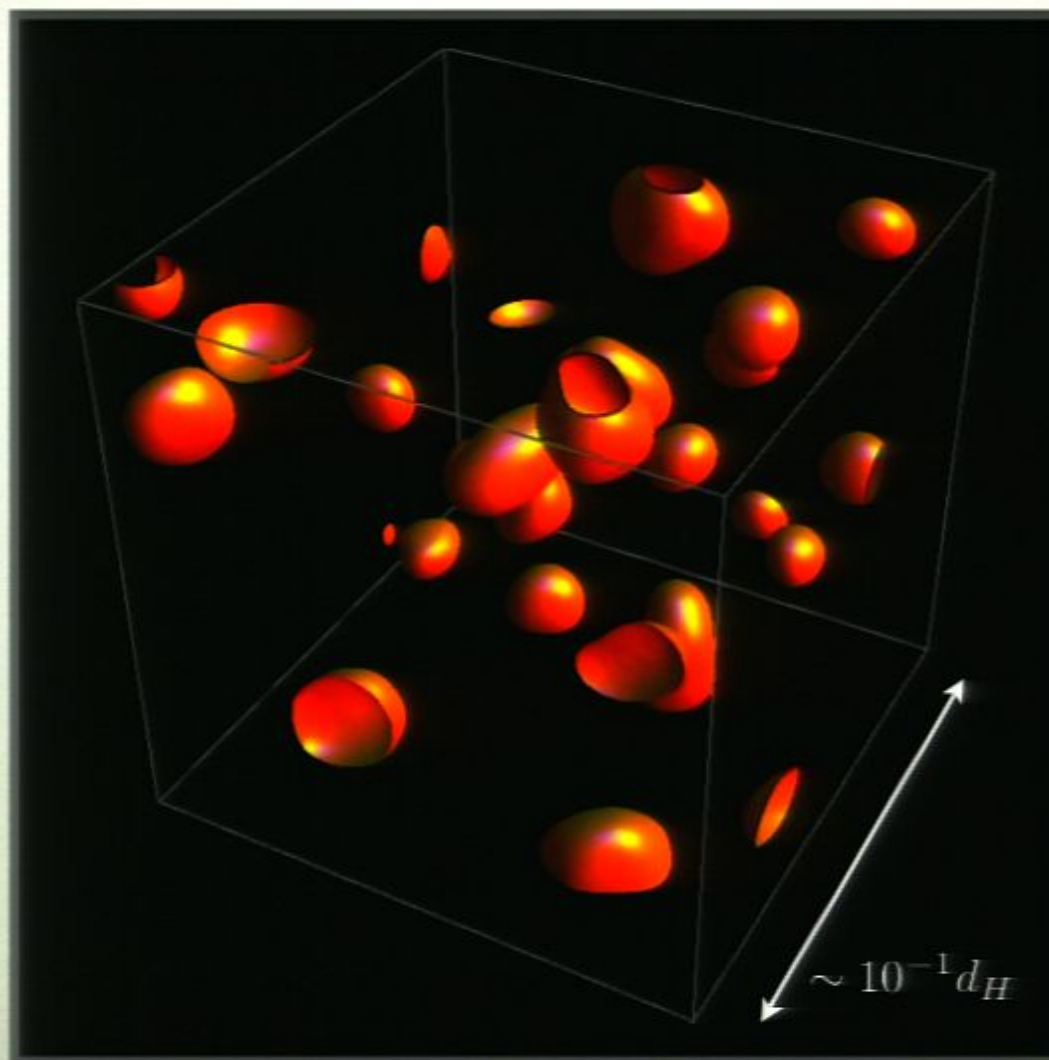
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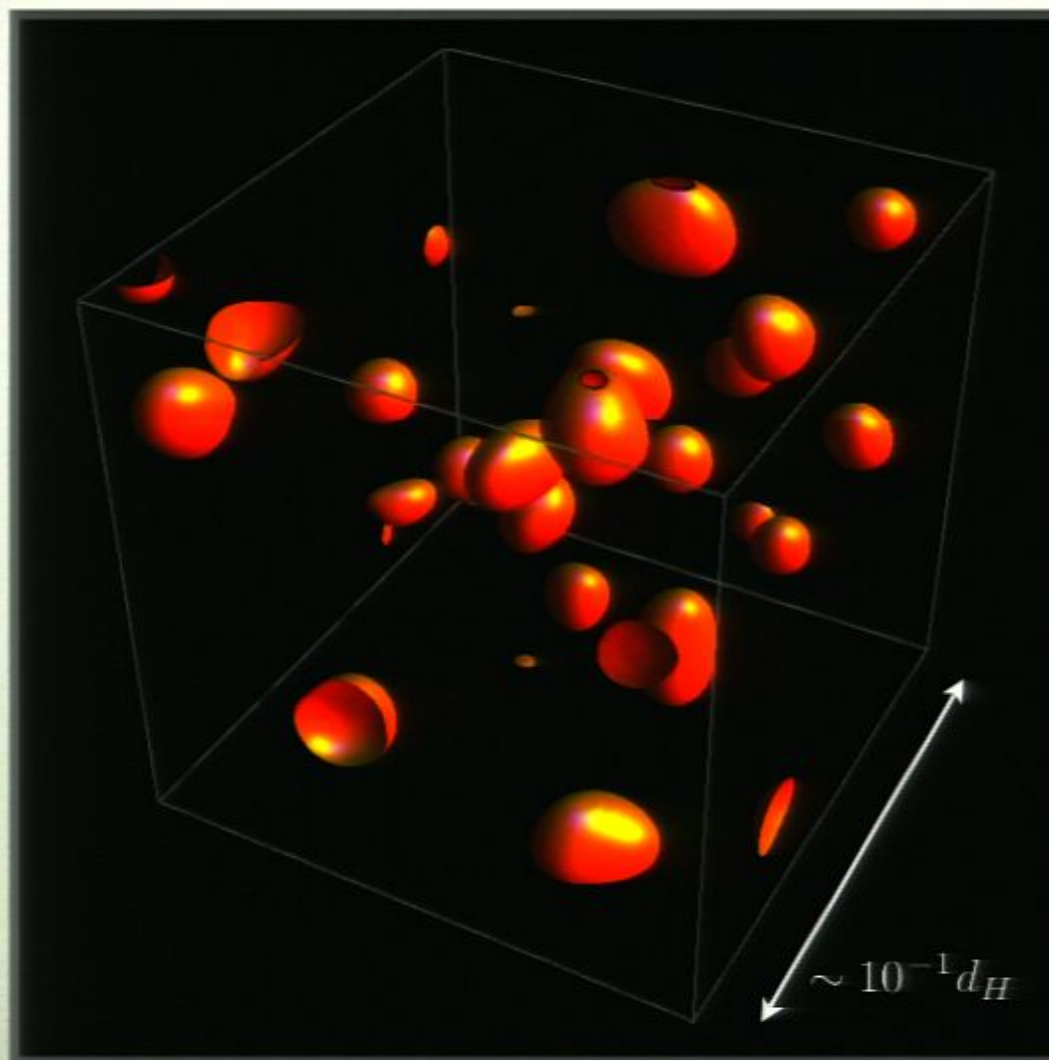
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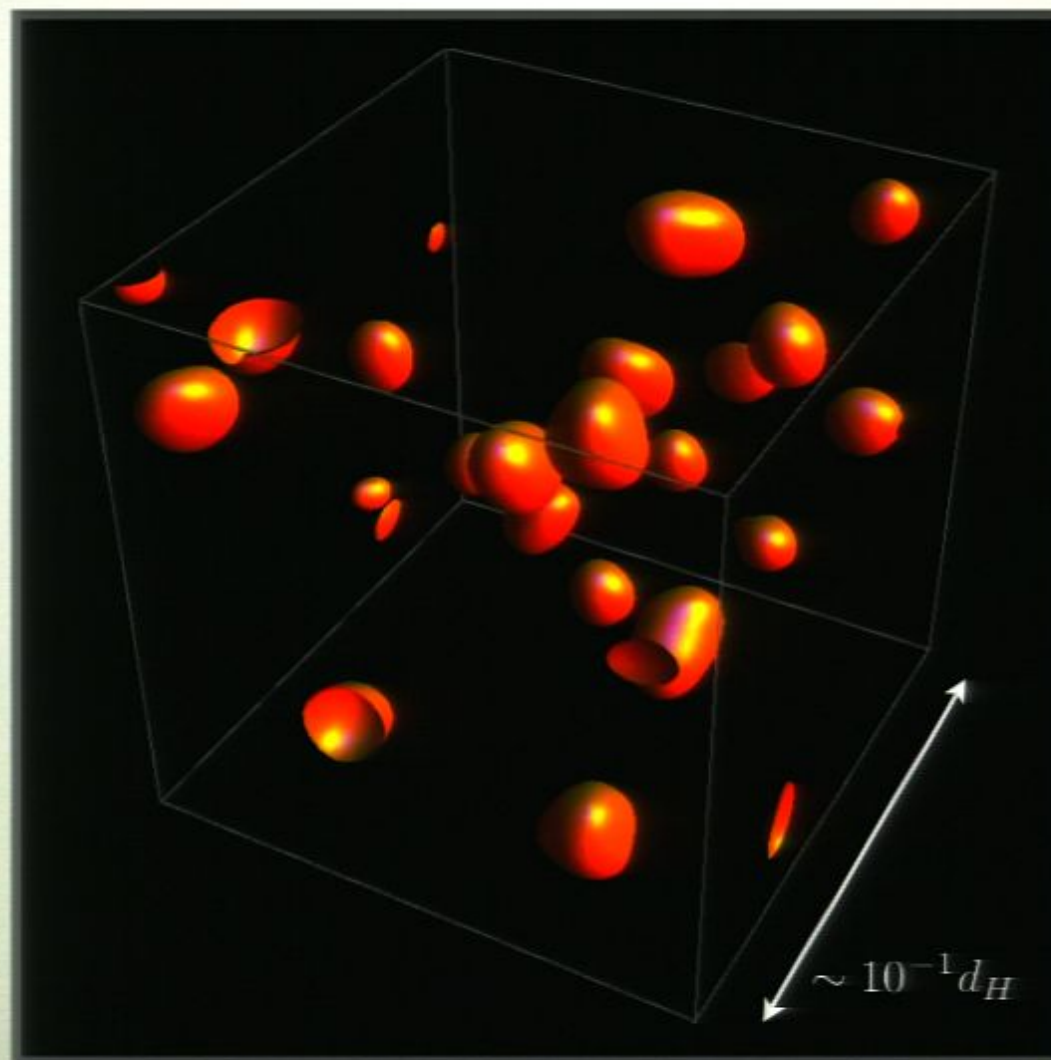


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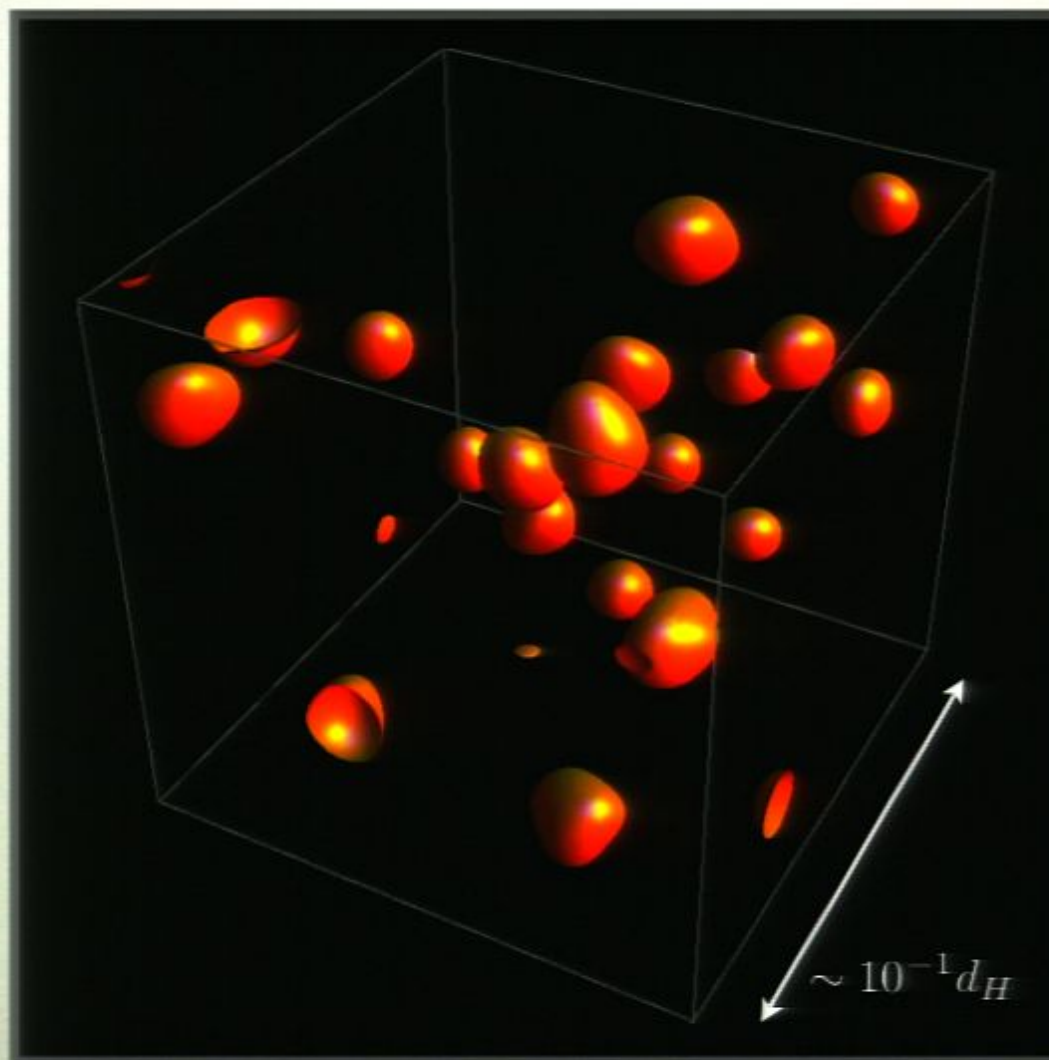




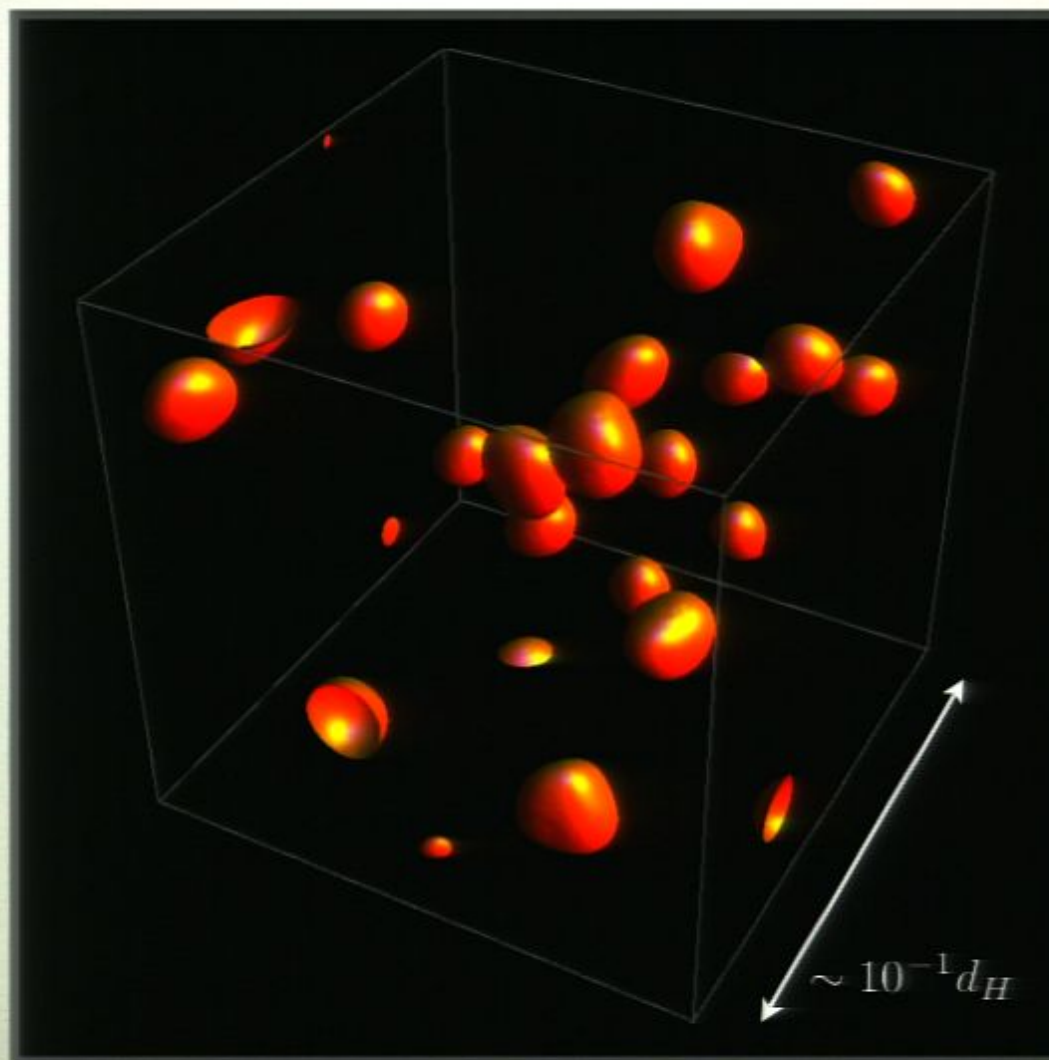
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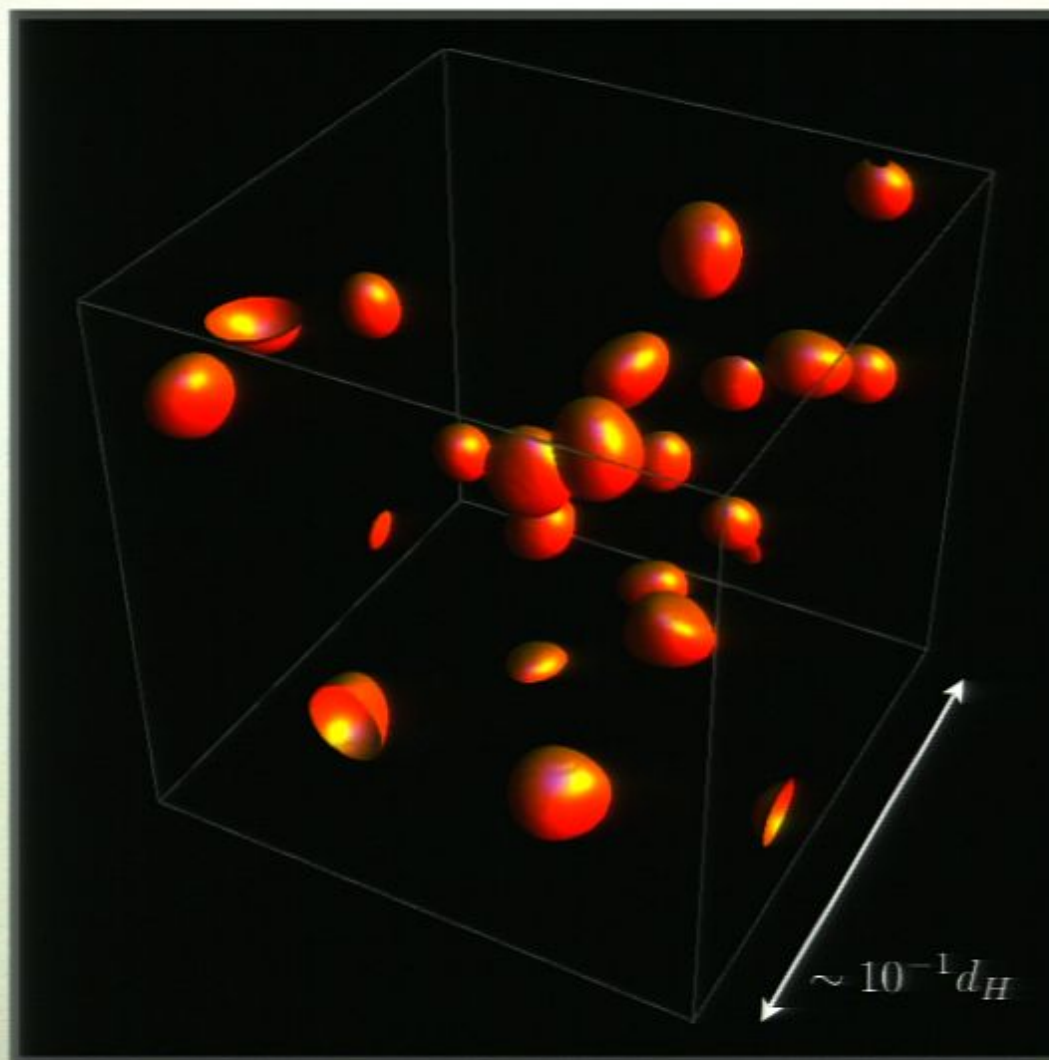
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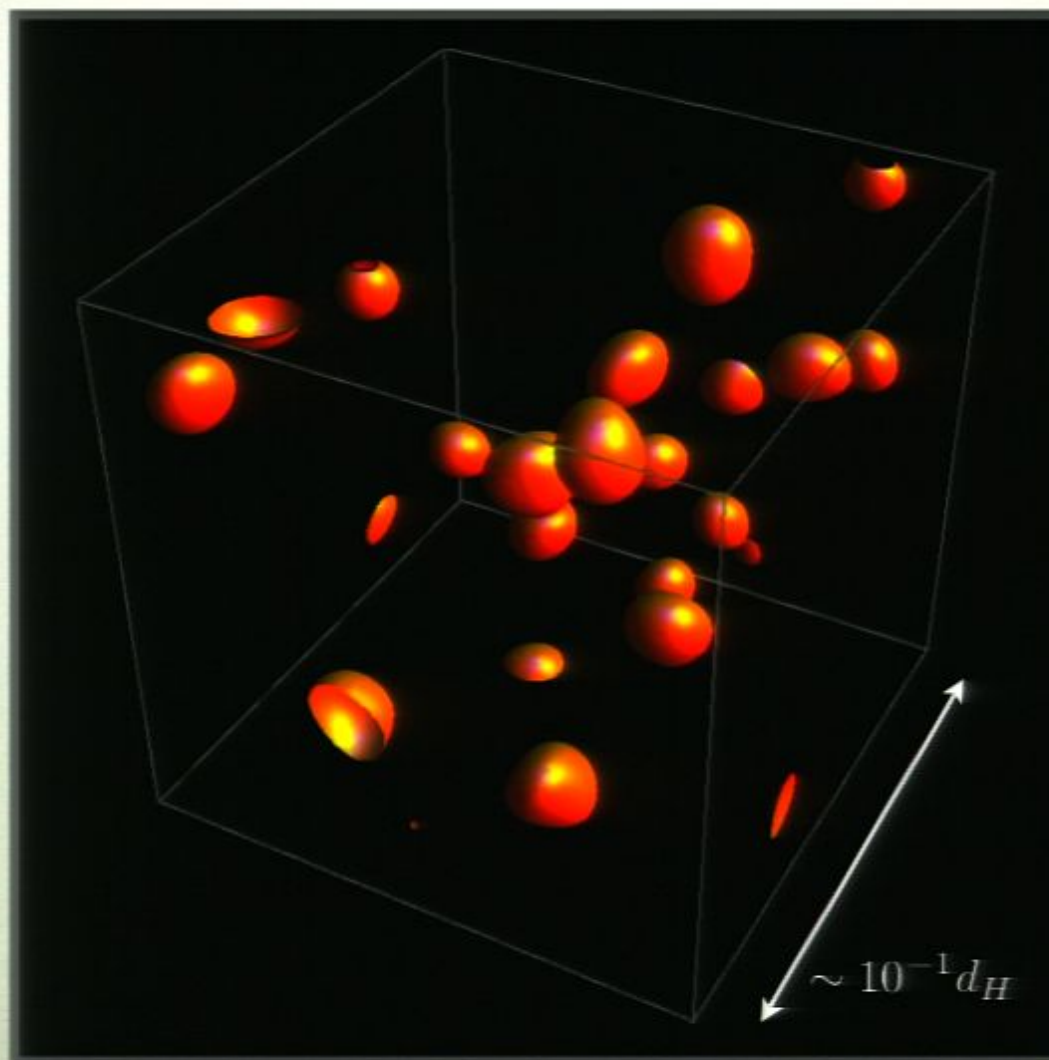
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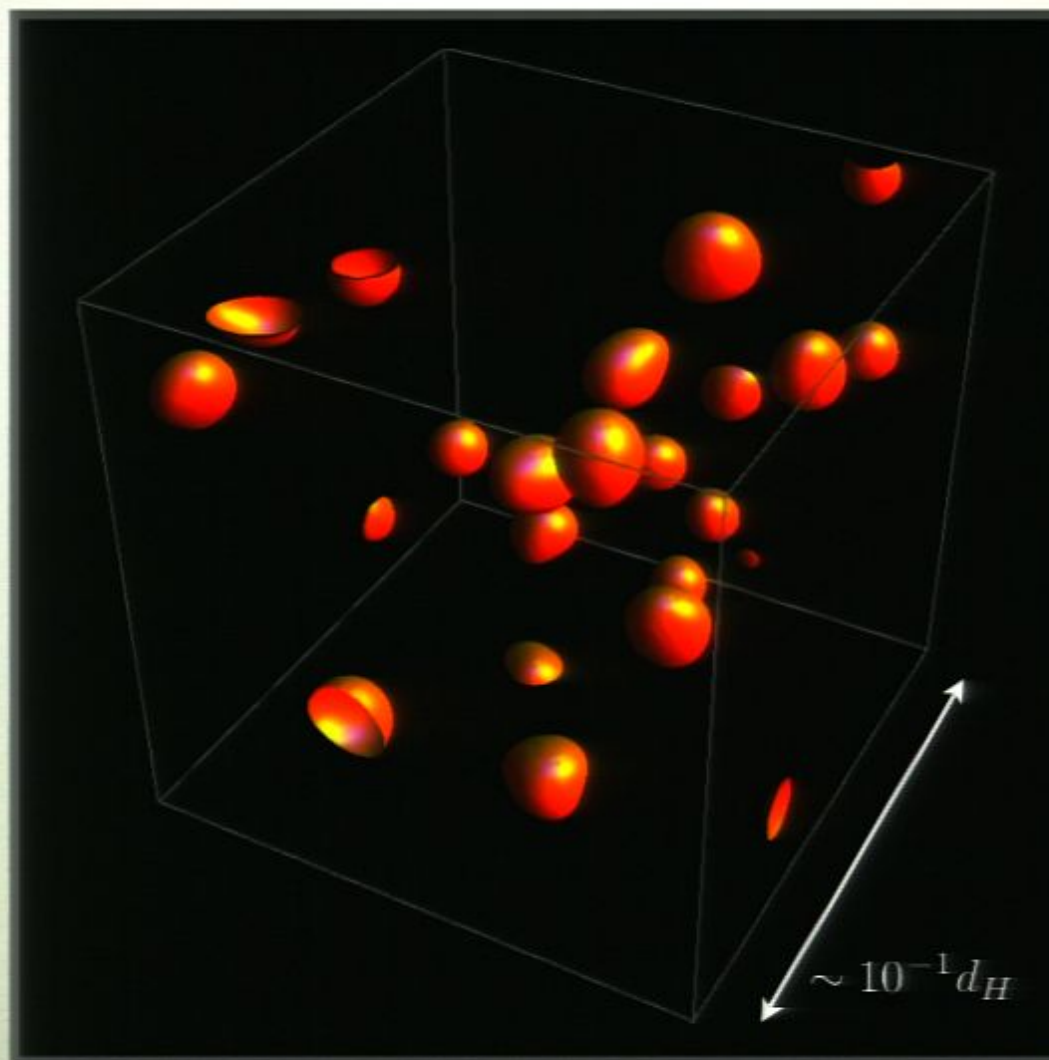
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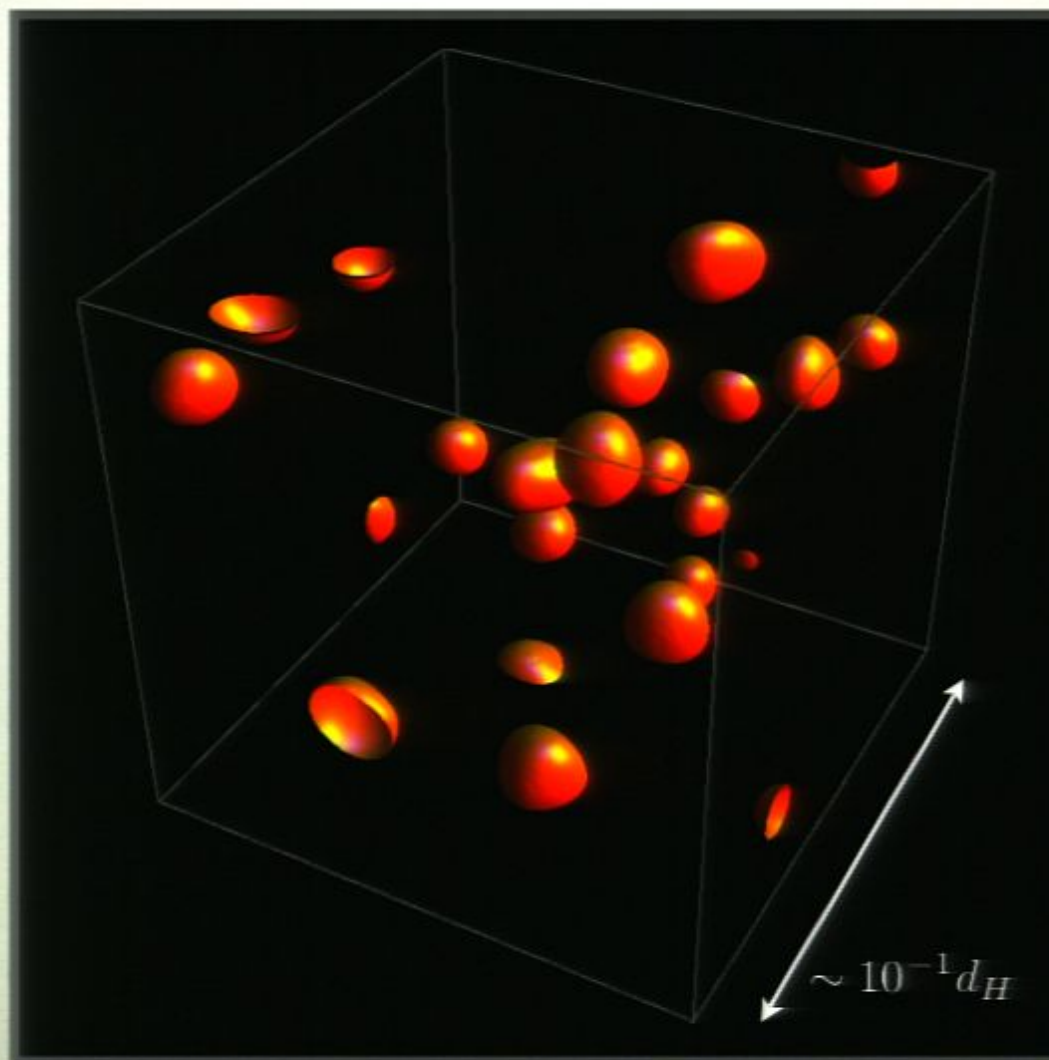
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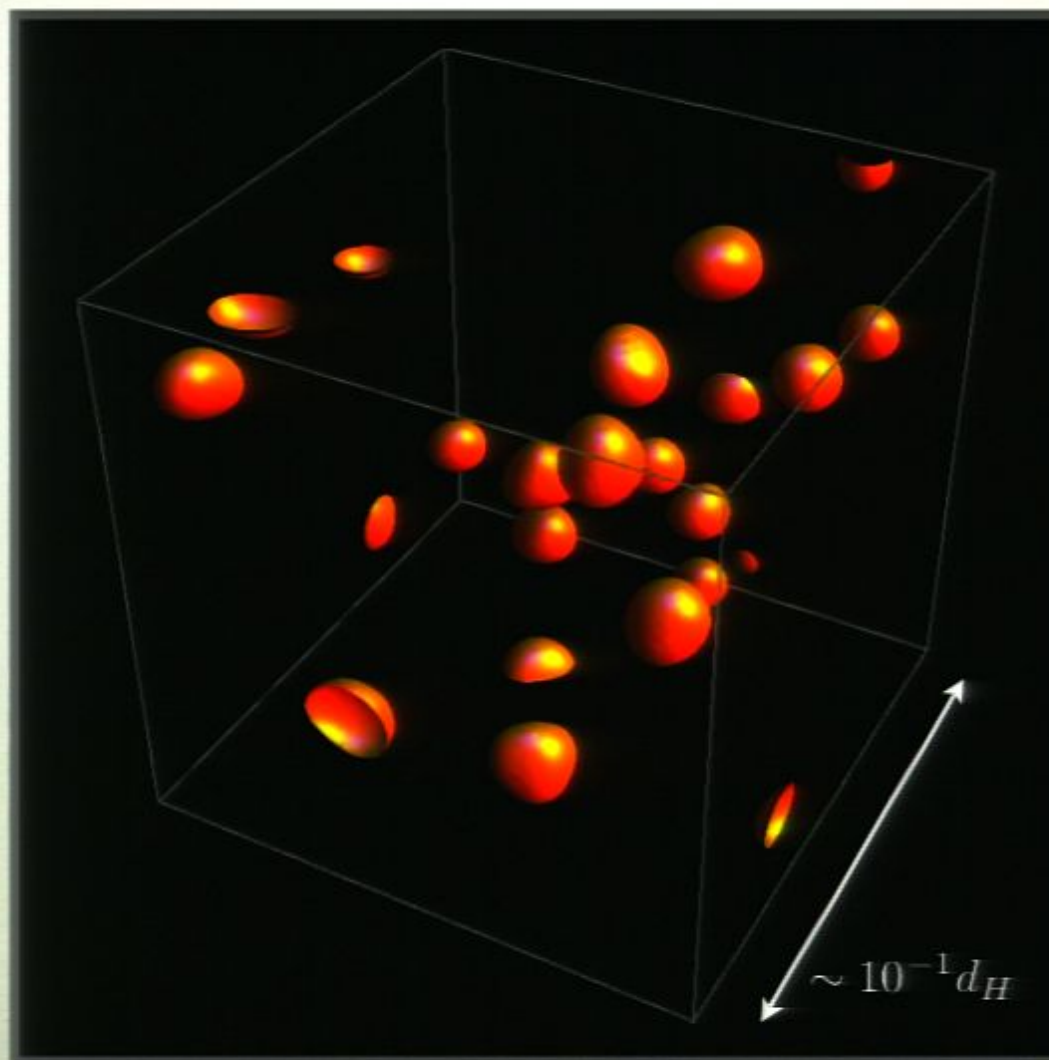
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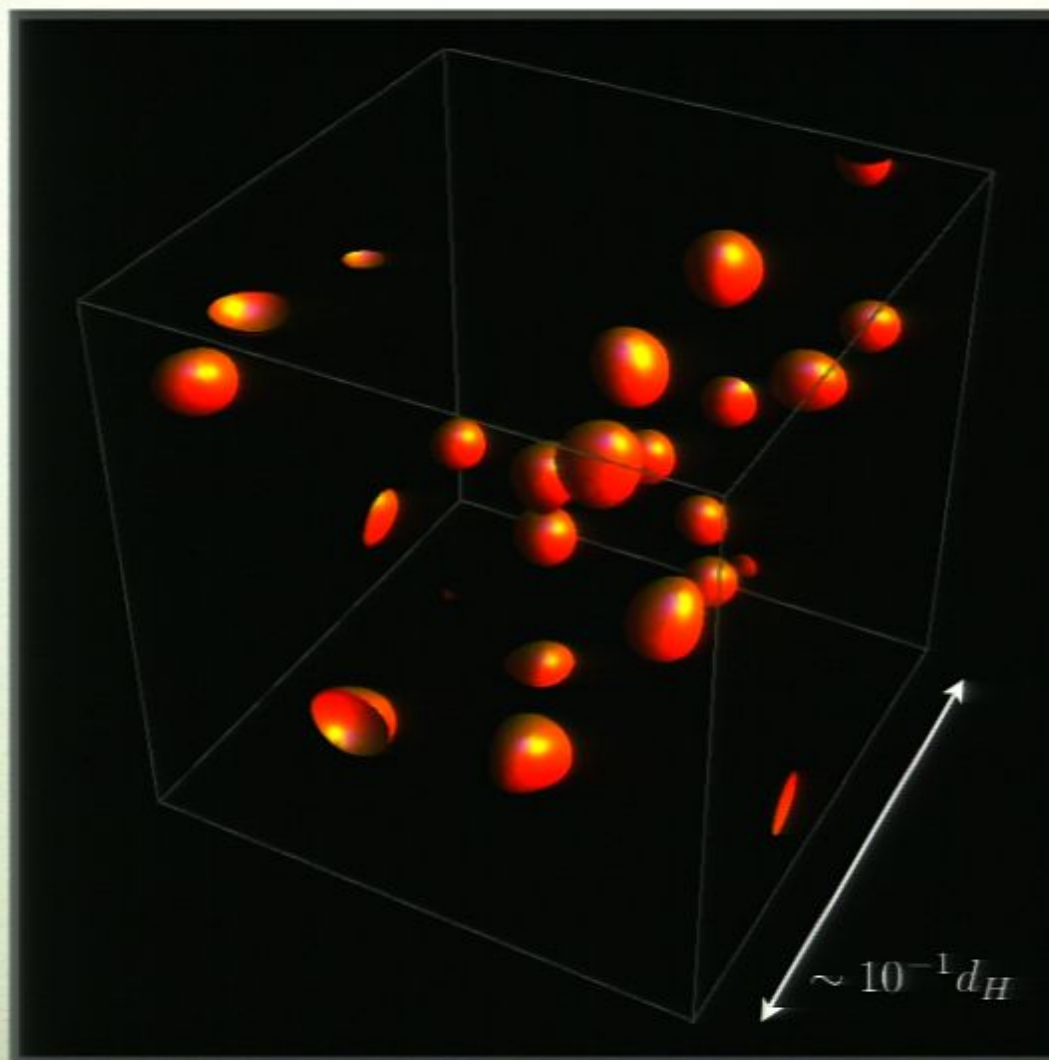


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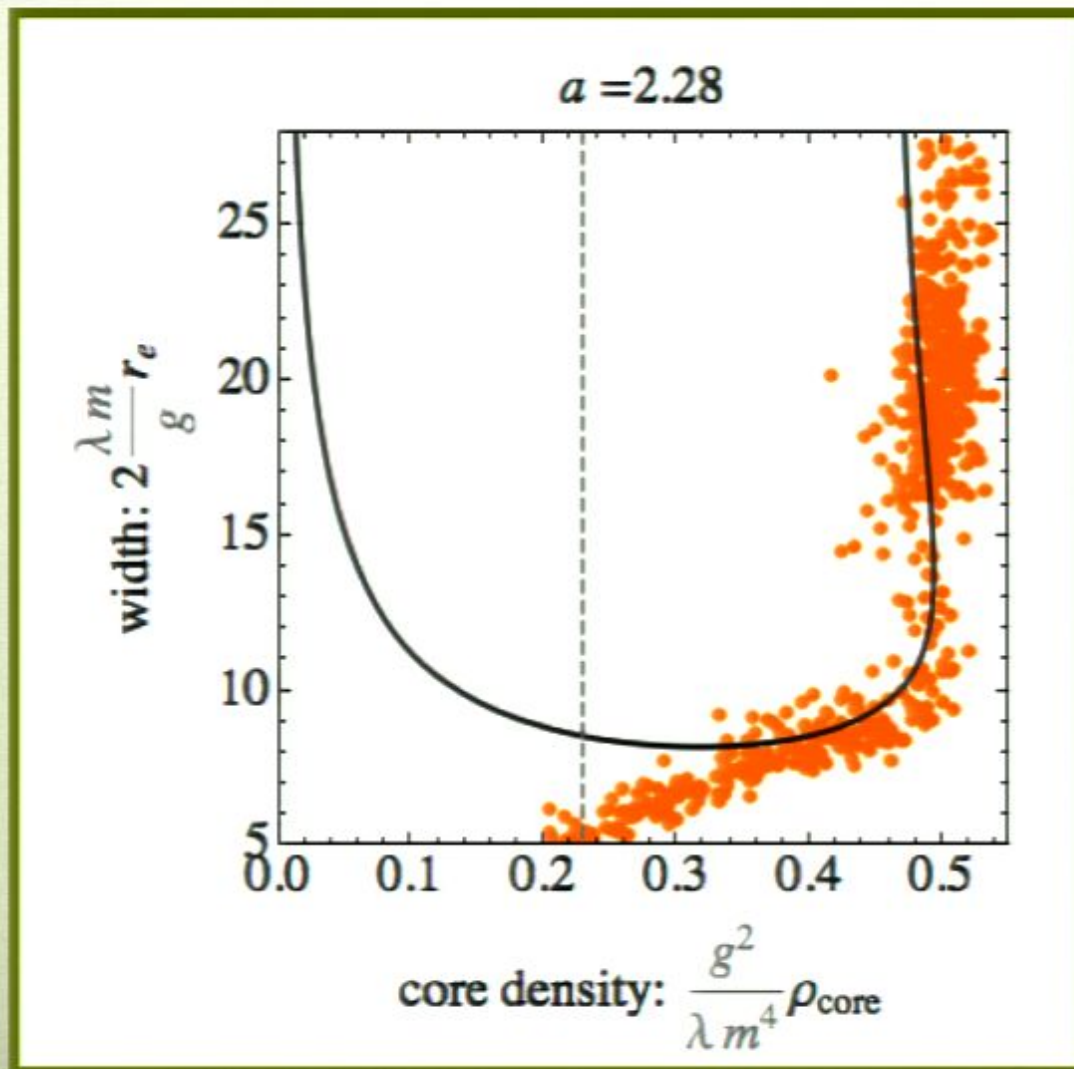




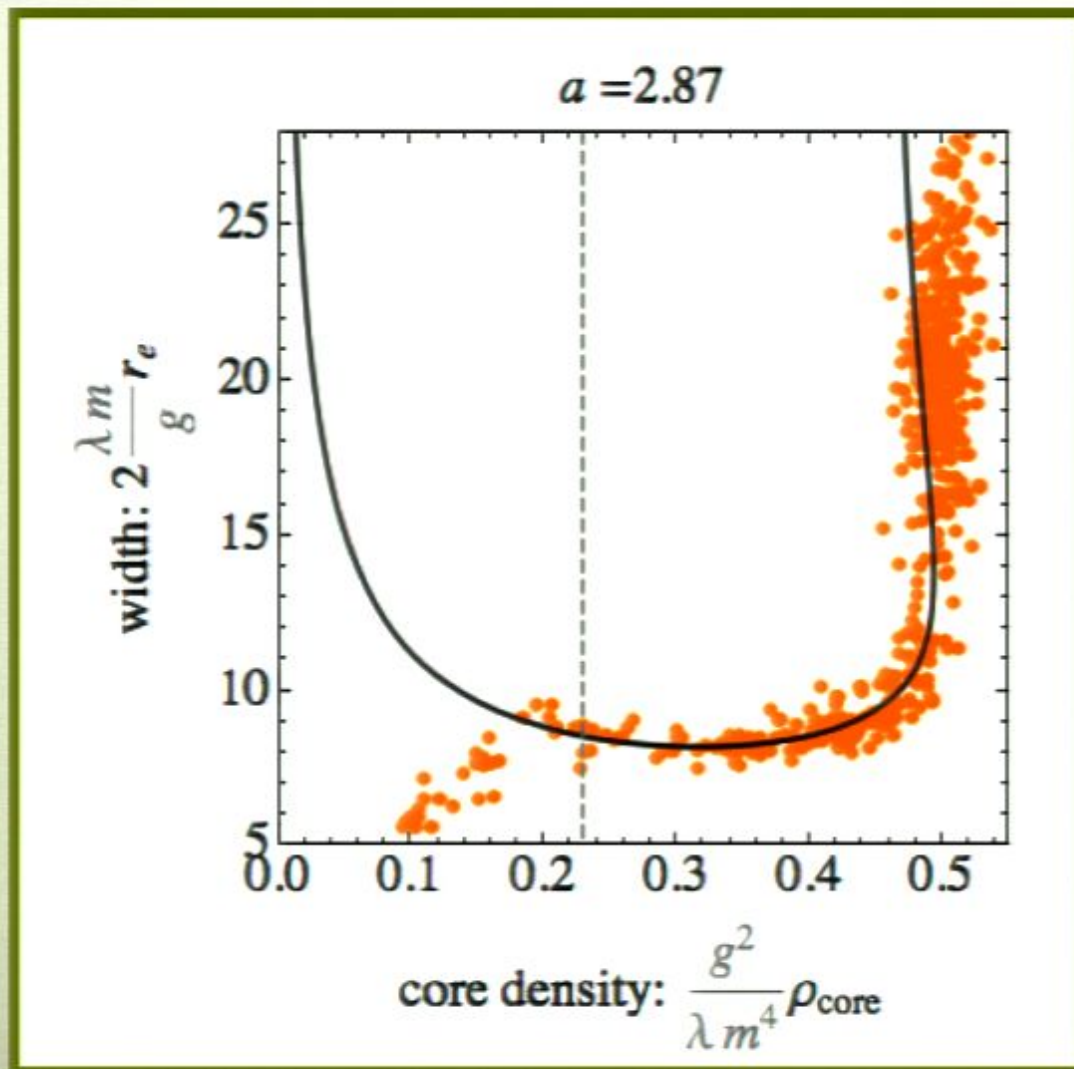
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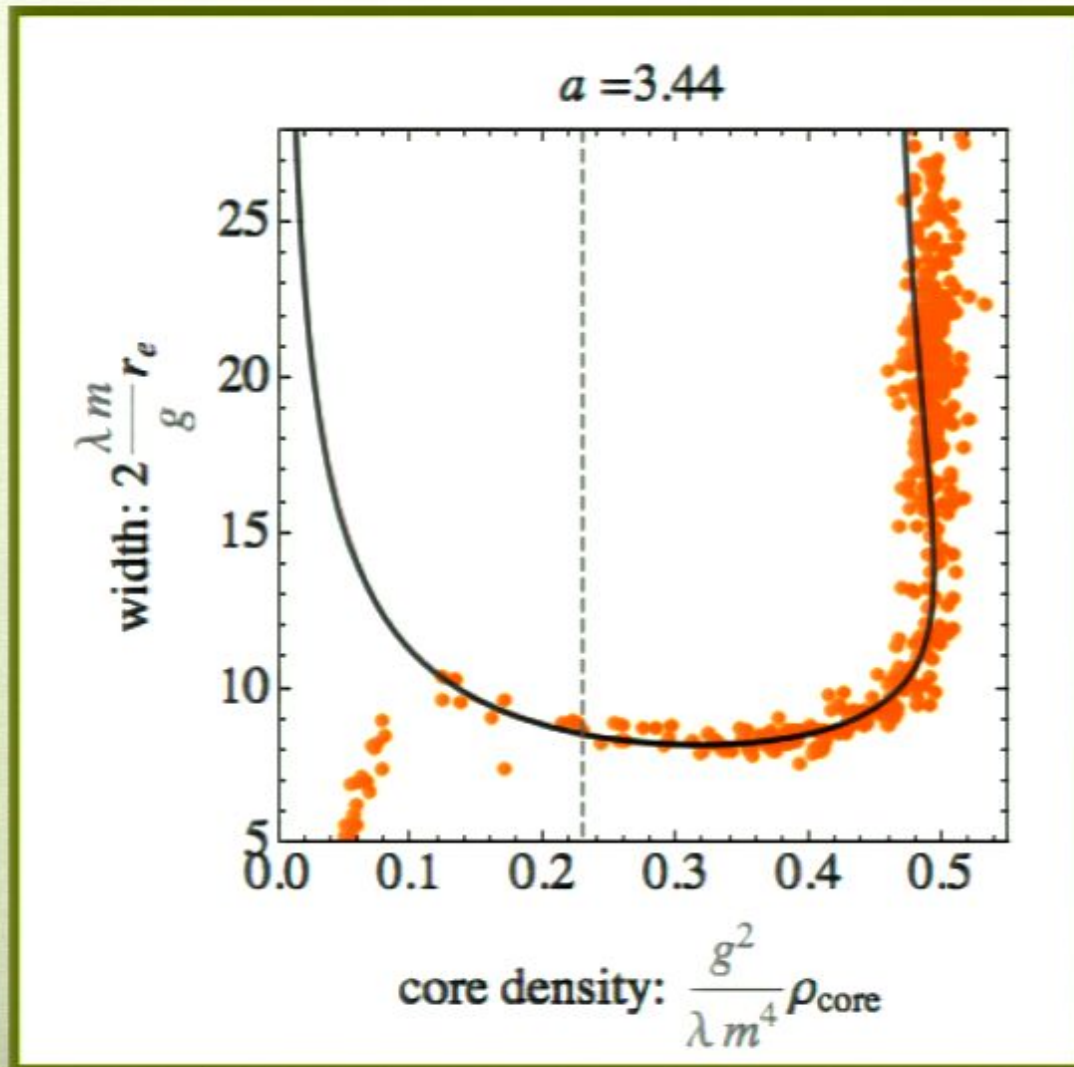
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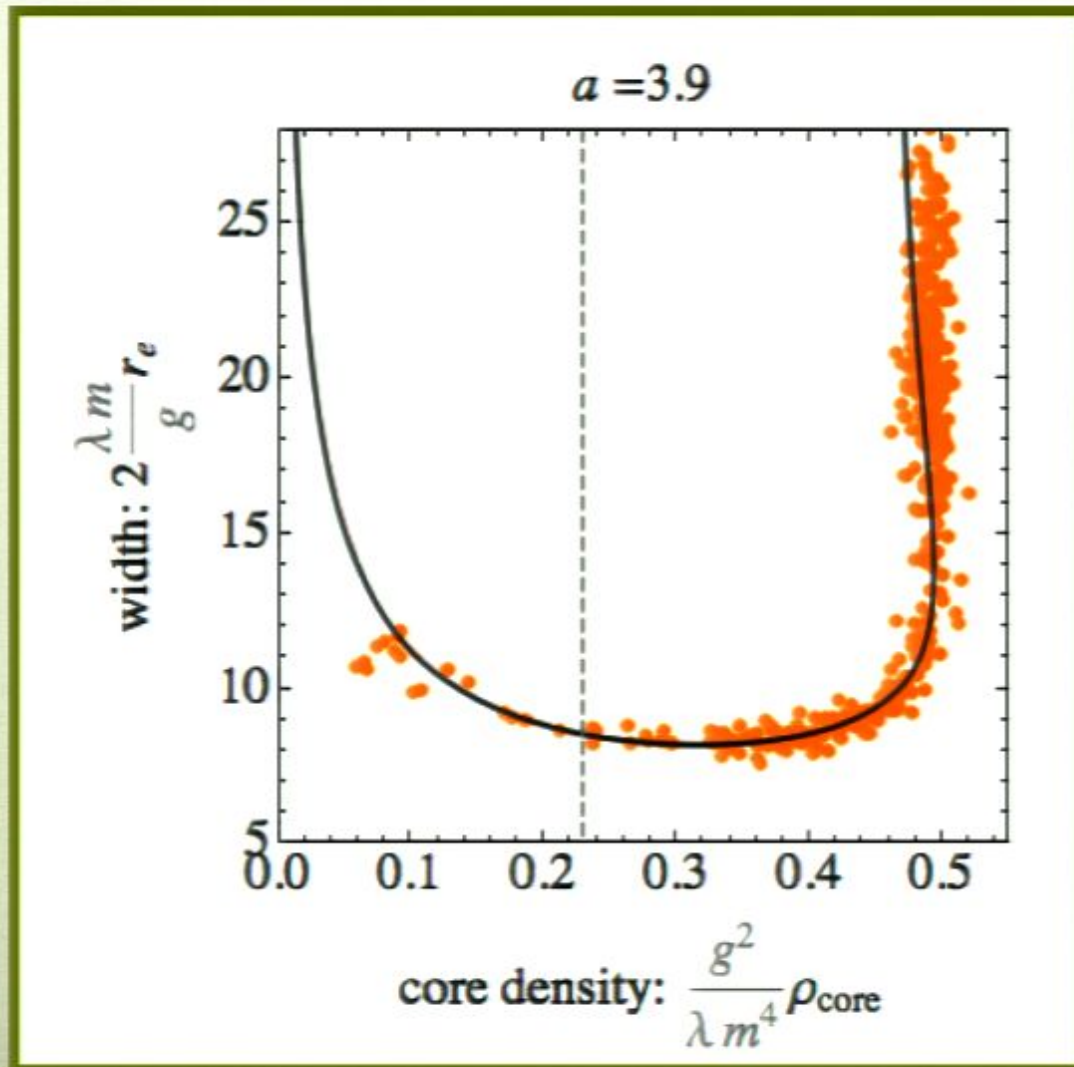
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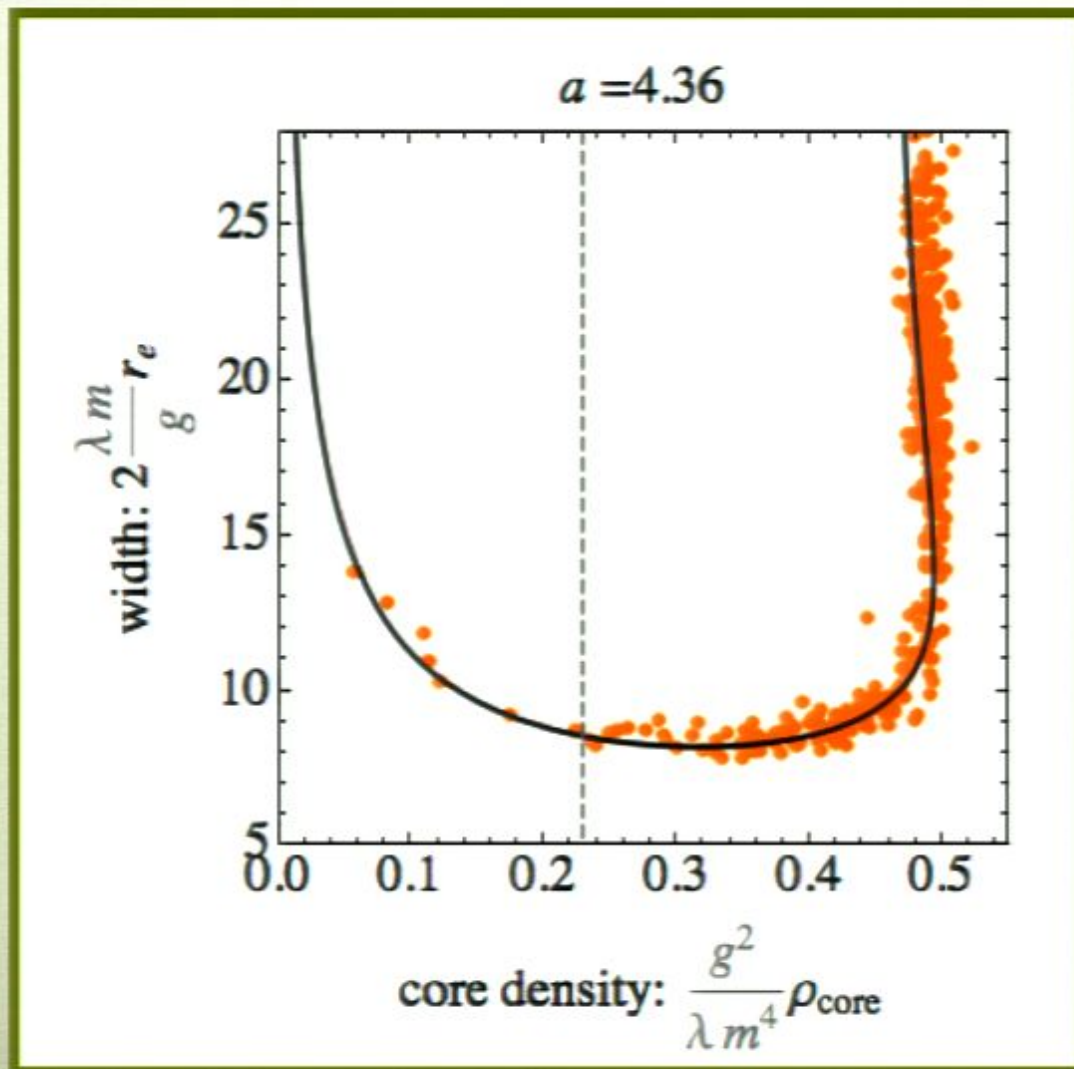
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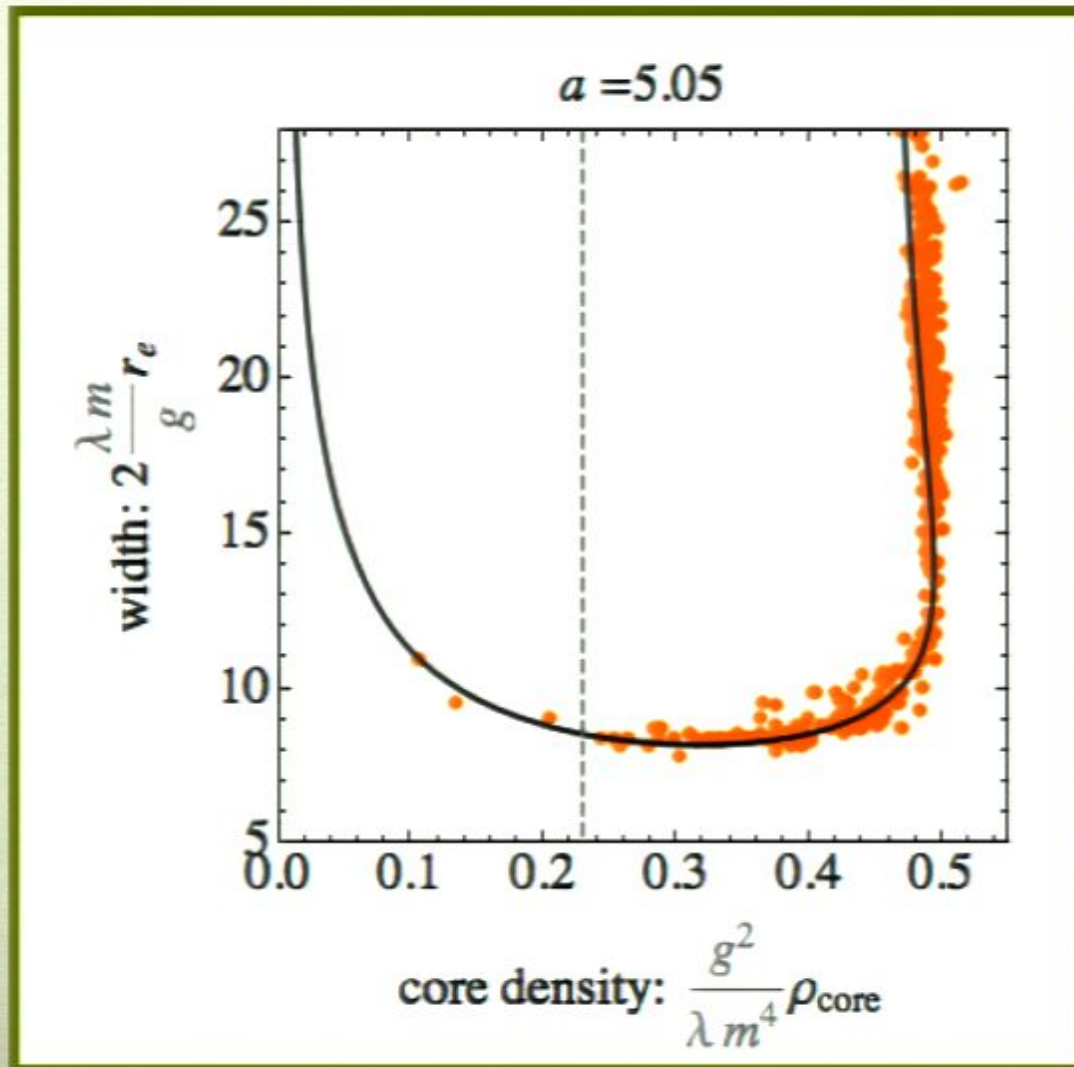
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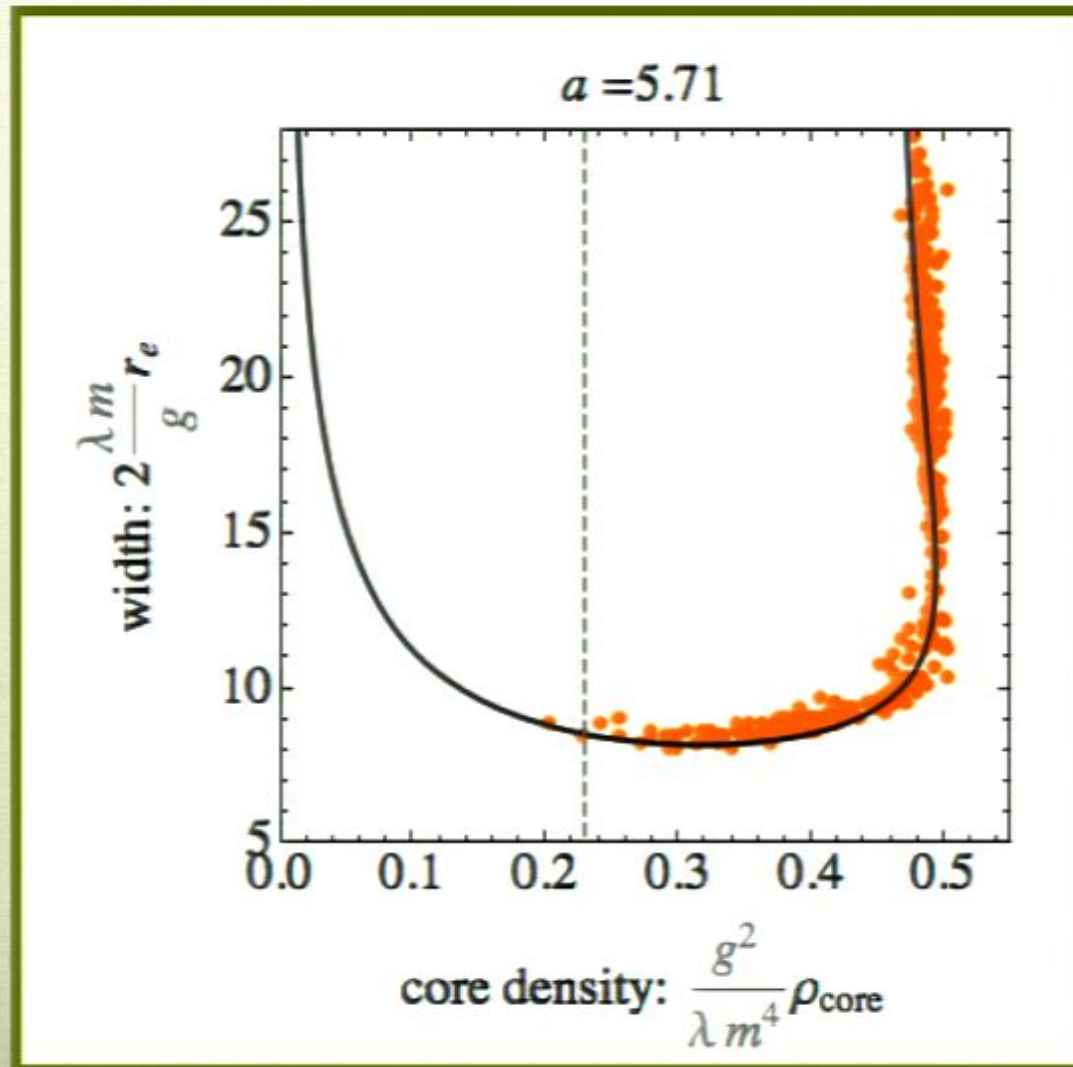
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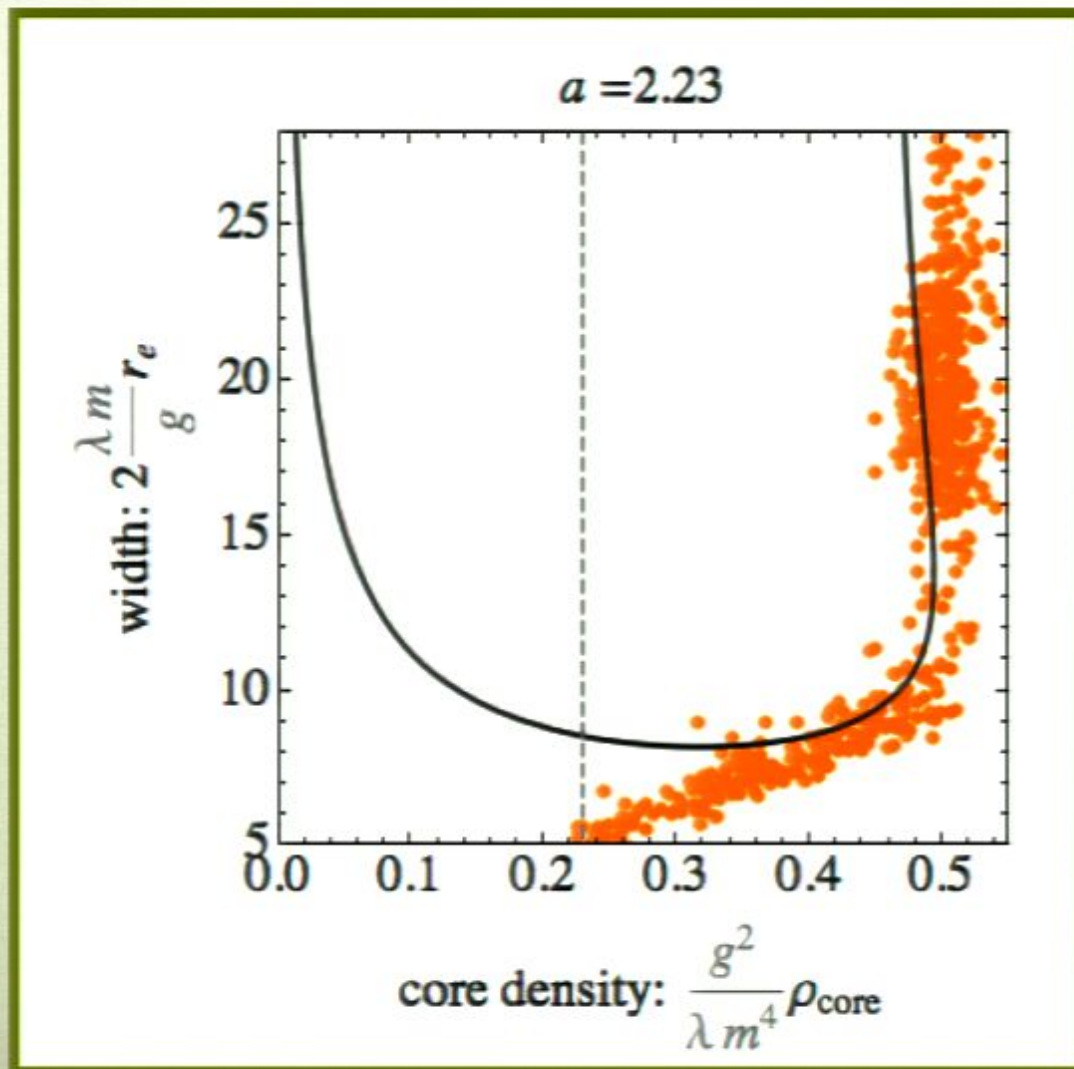


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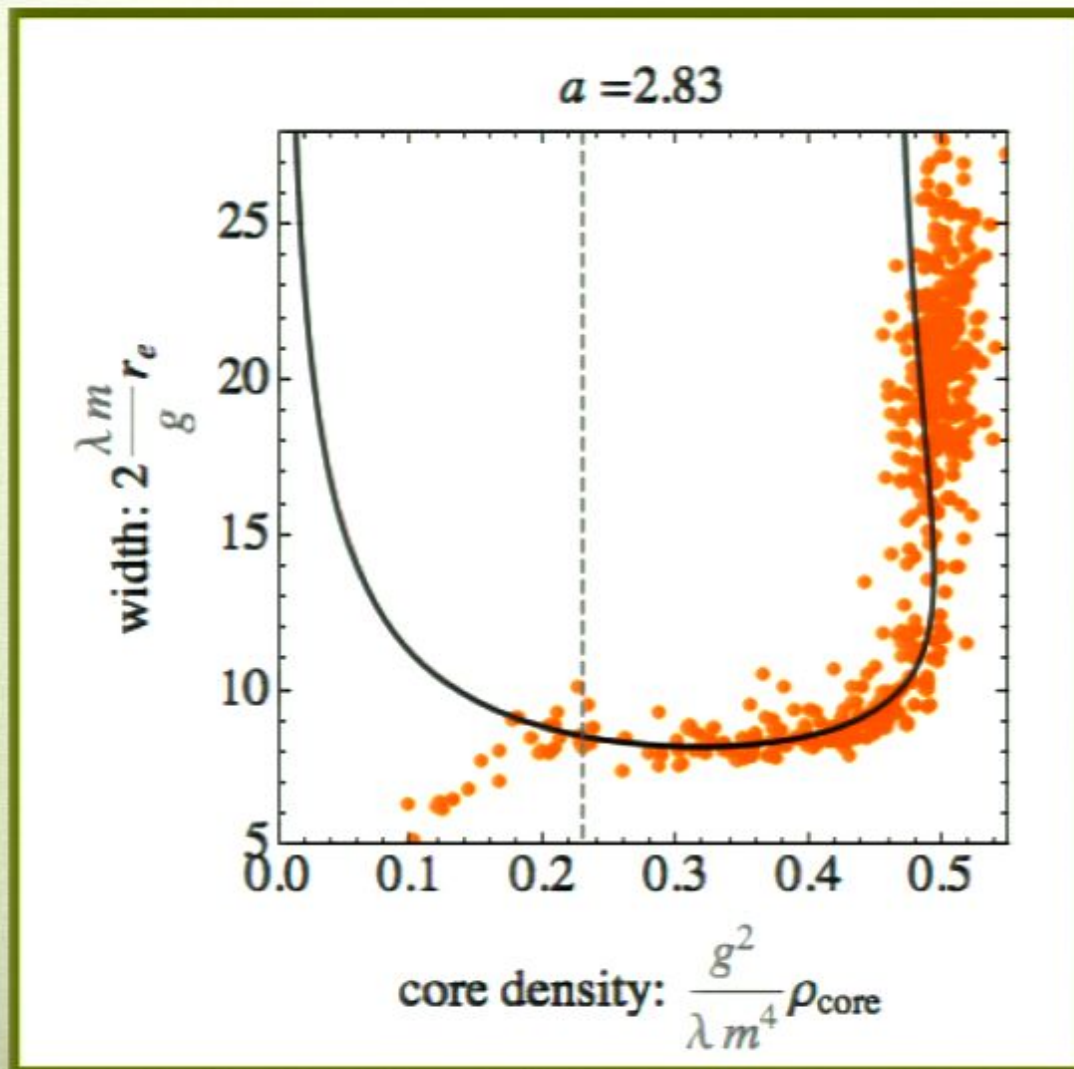




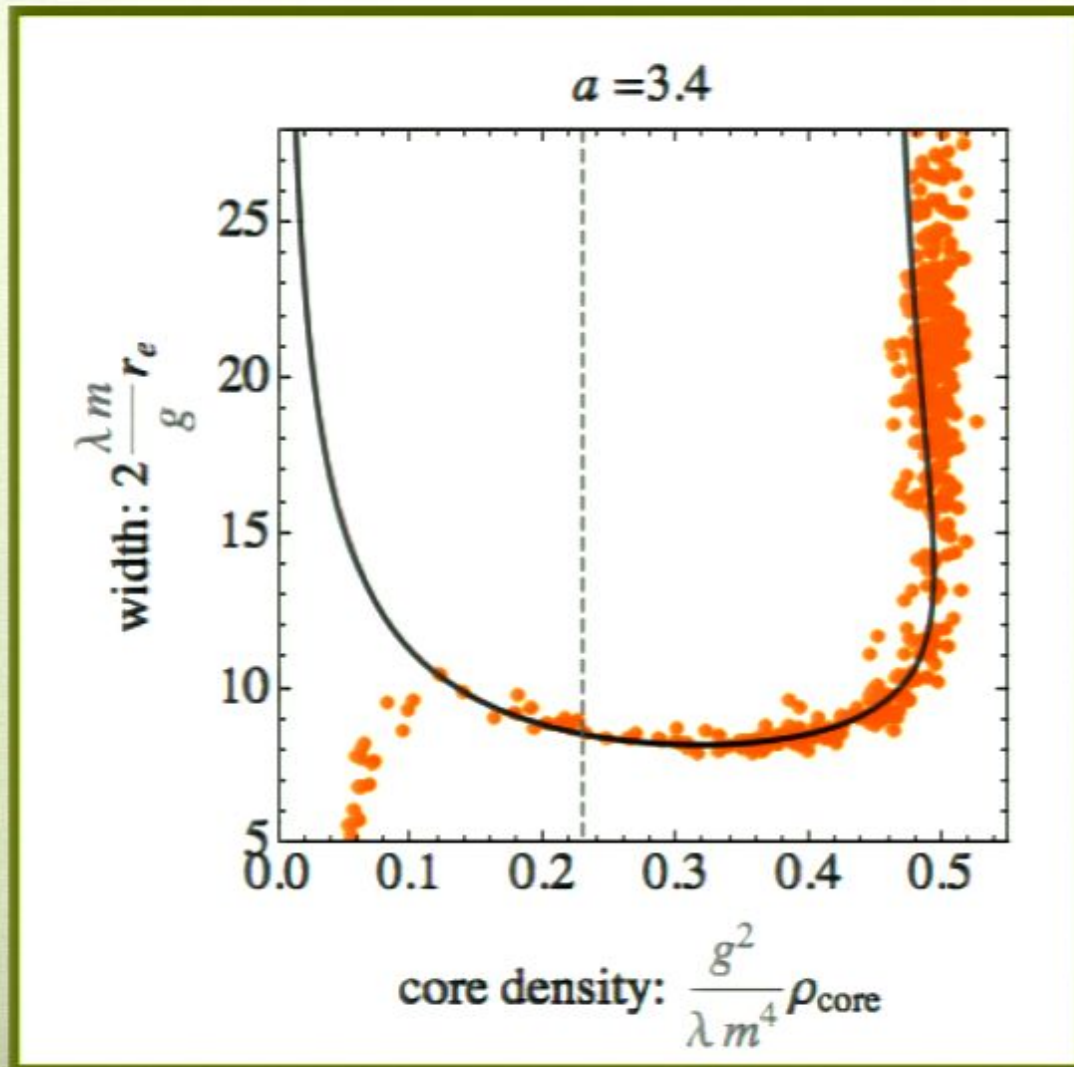
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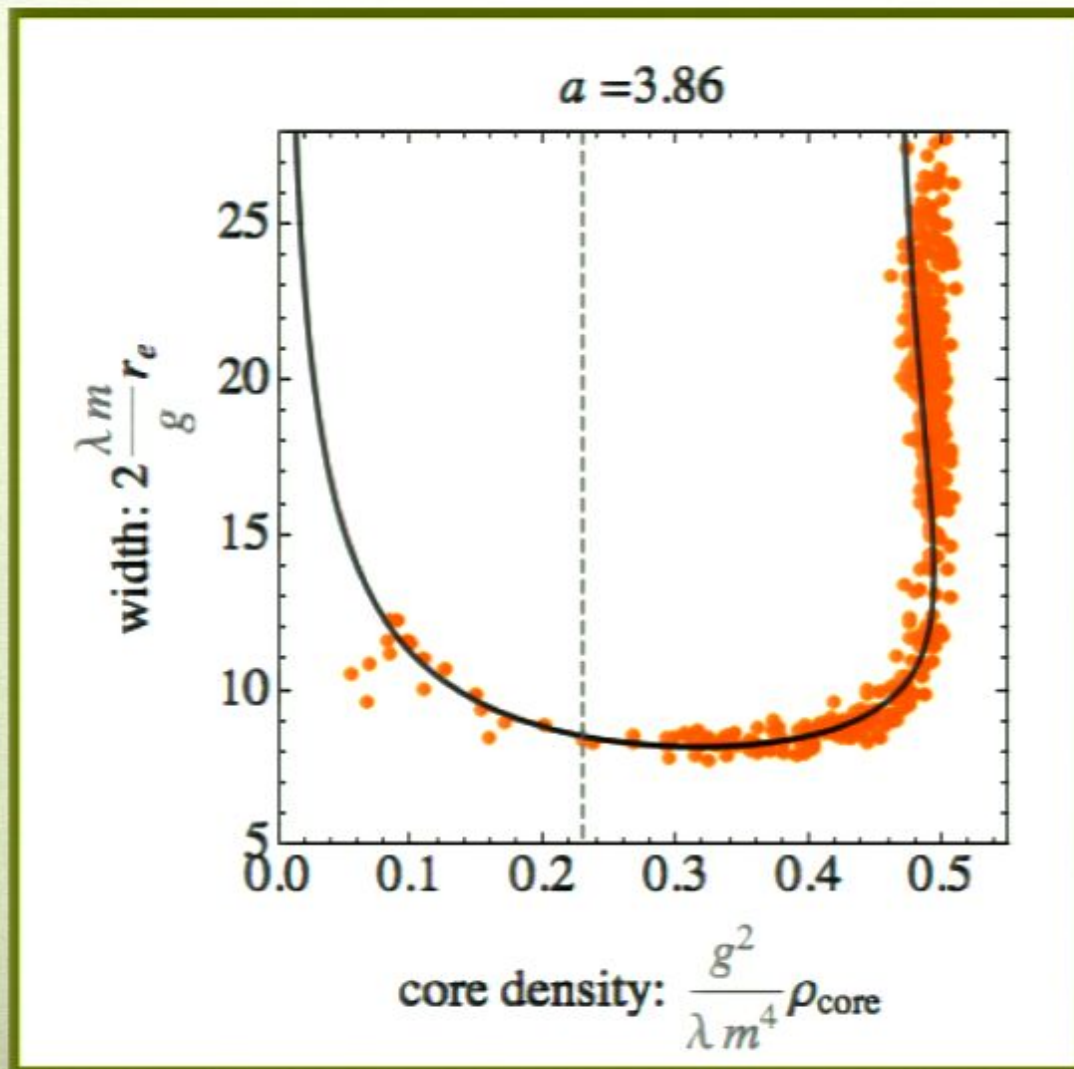
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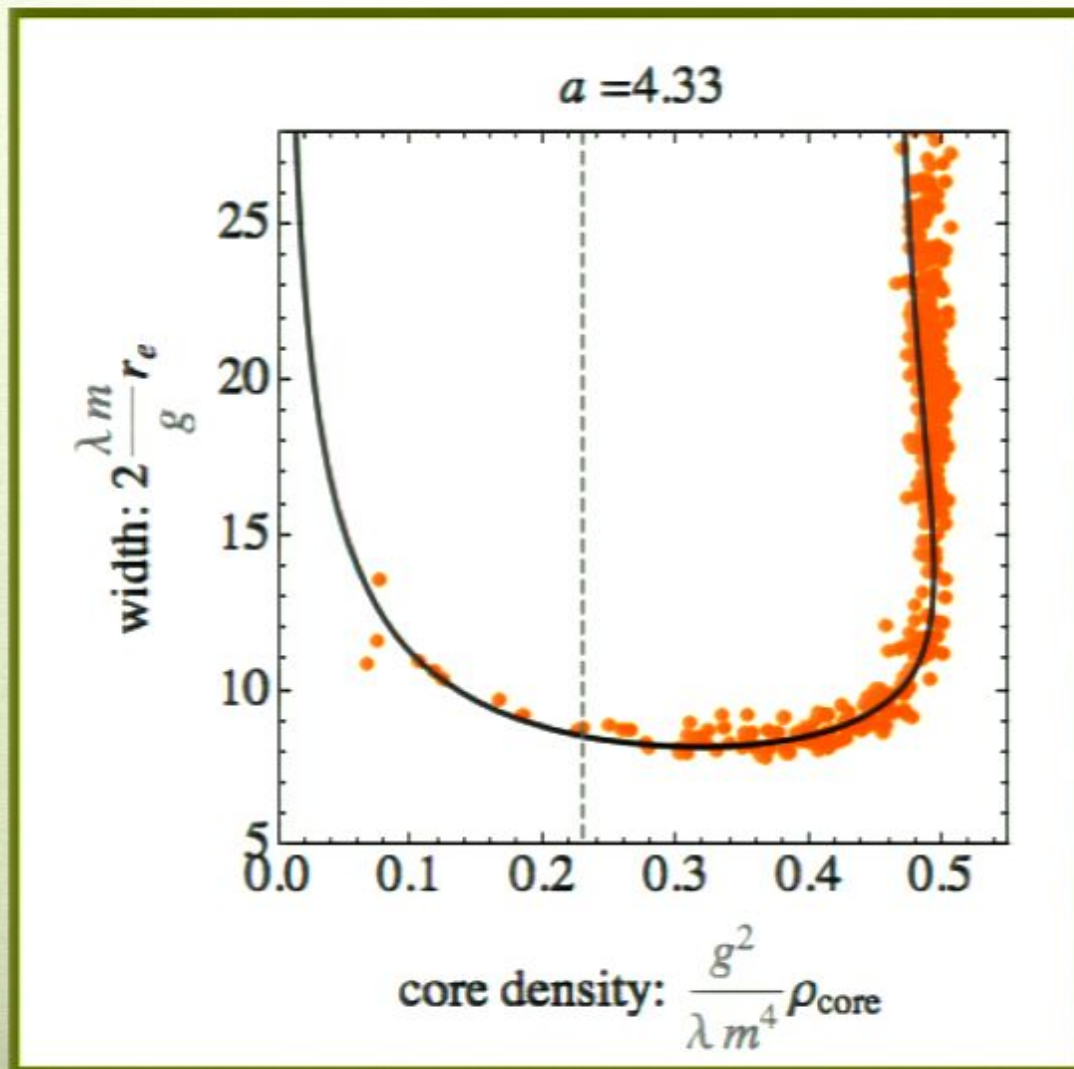
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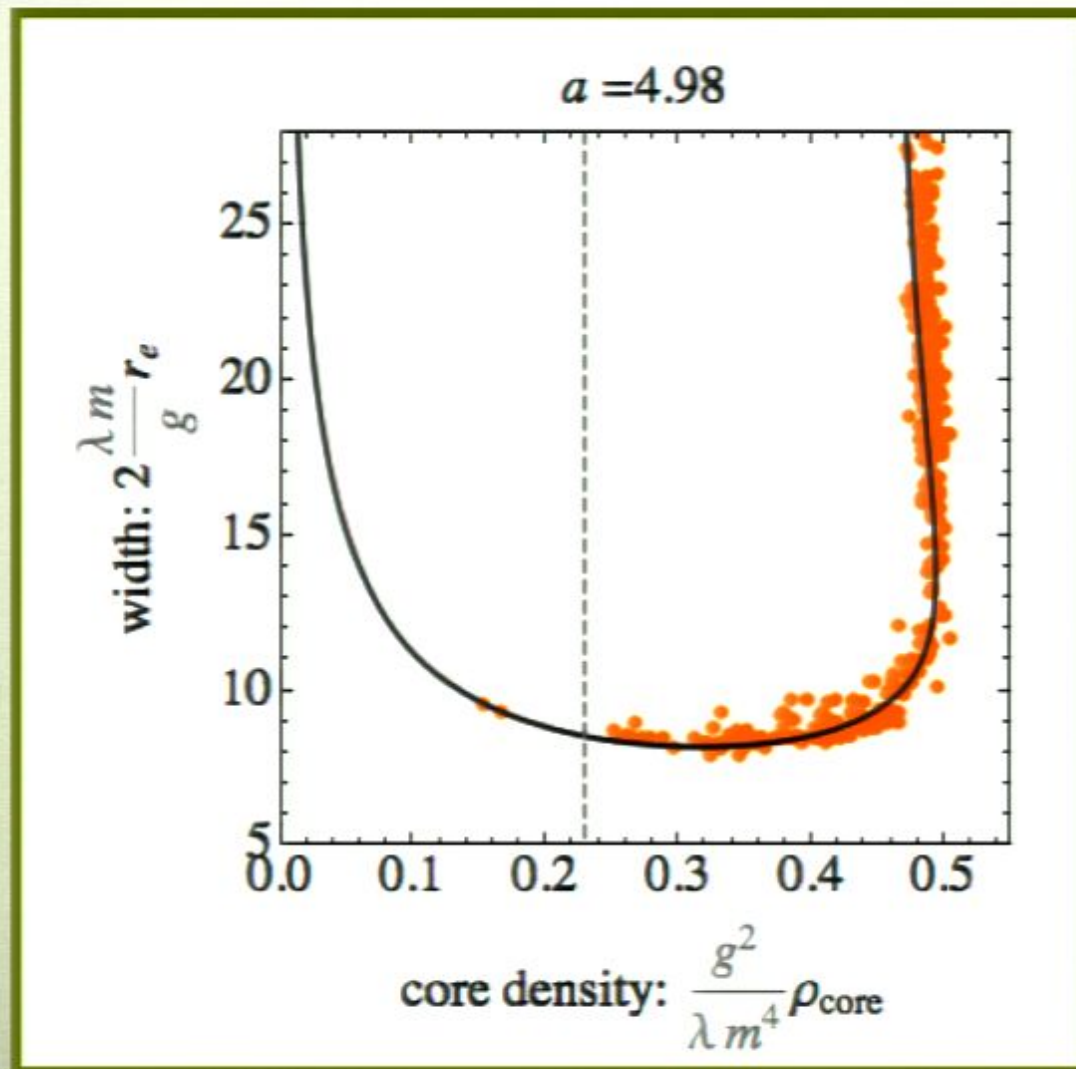
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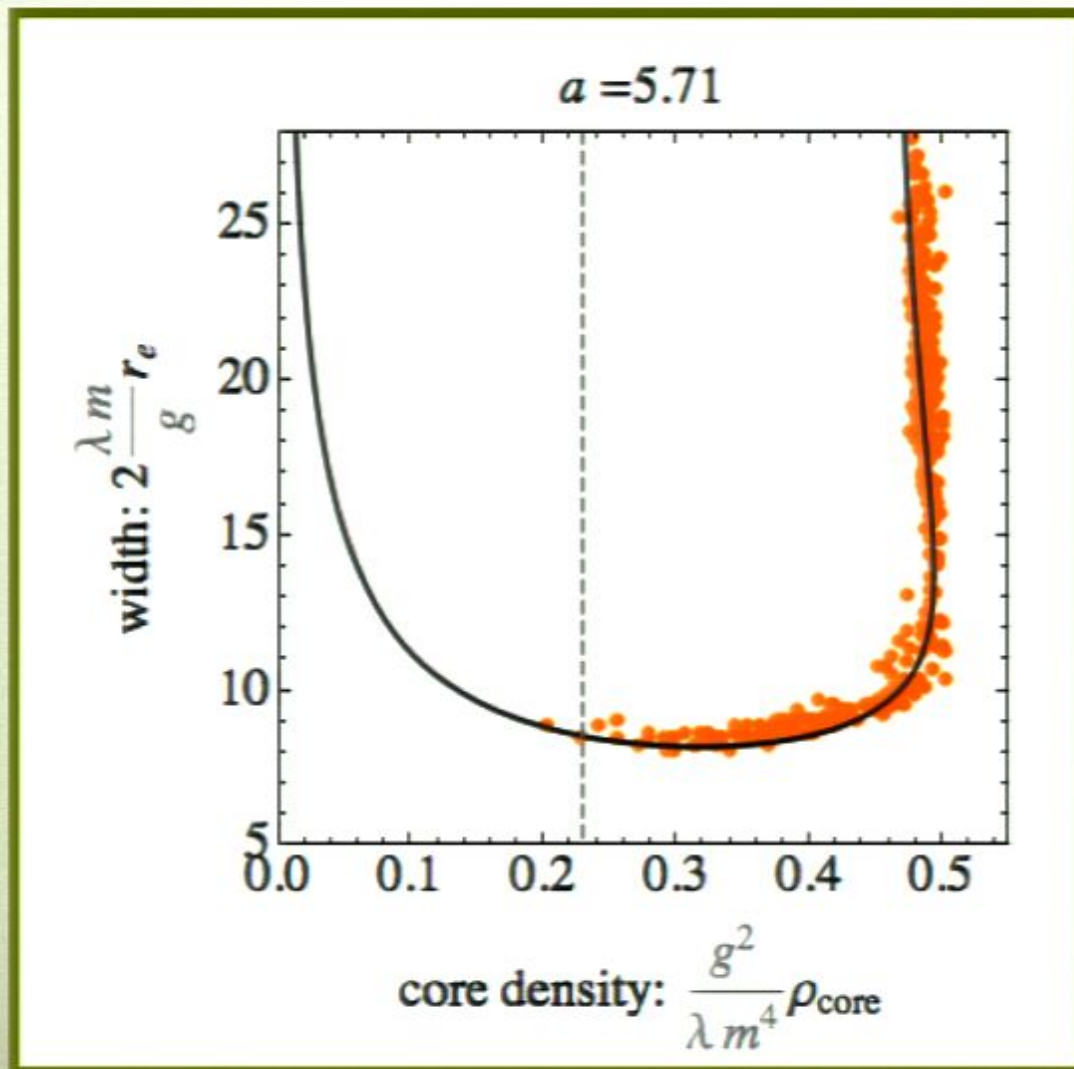
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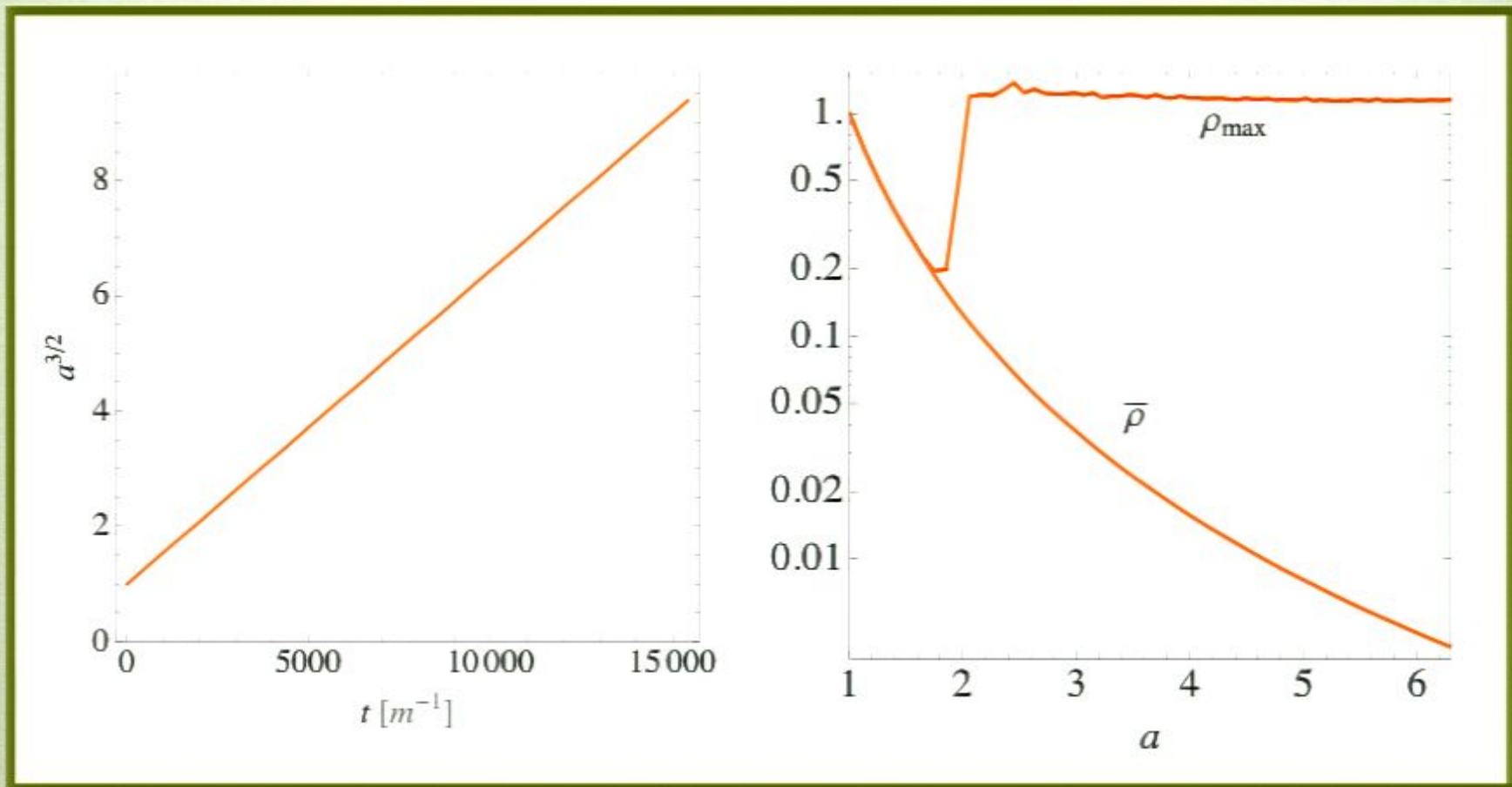
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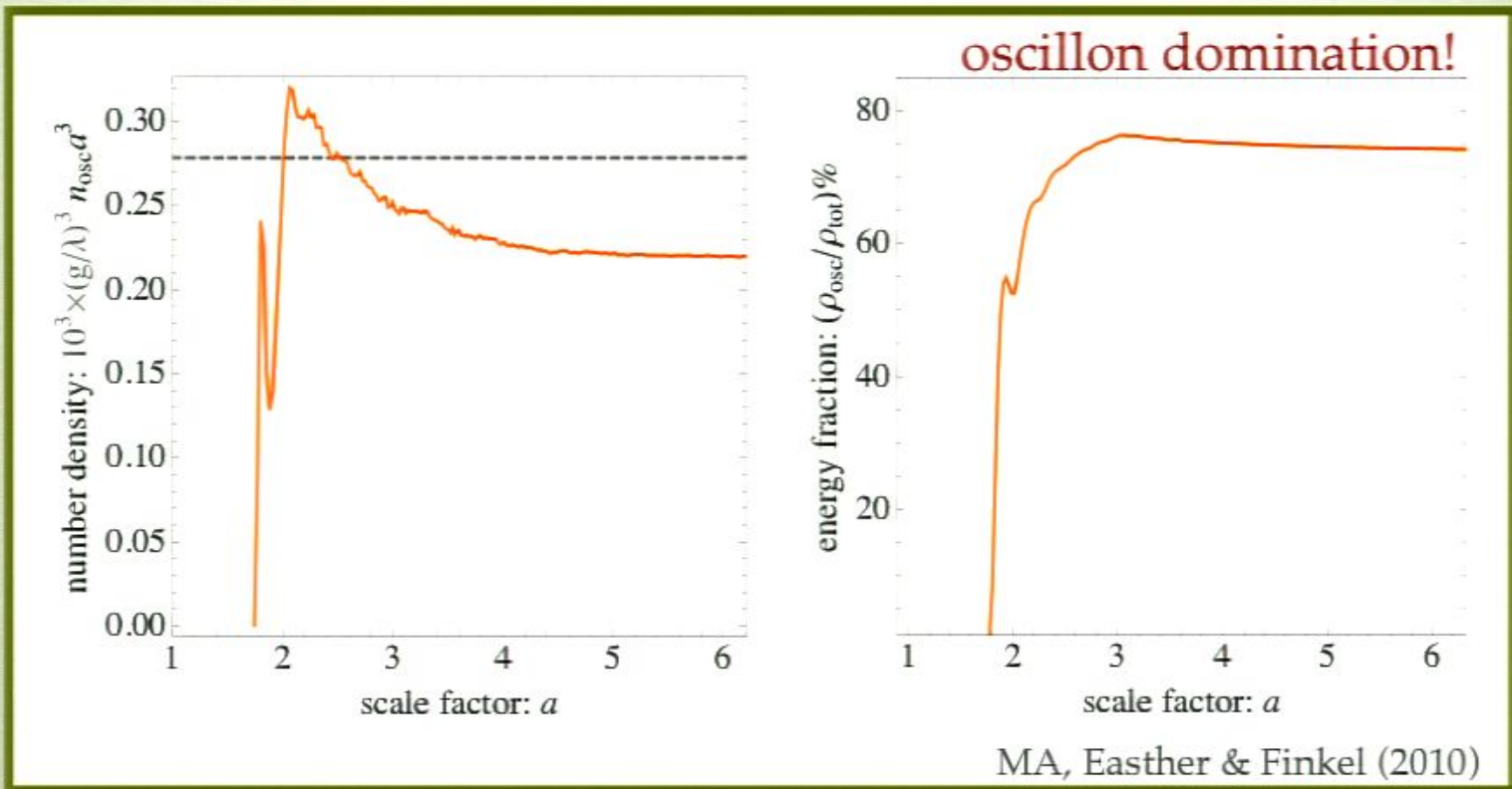


# EXPANSION HISTORY



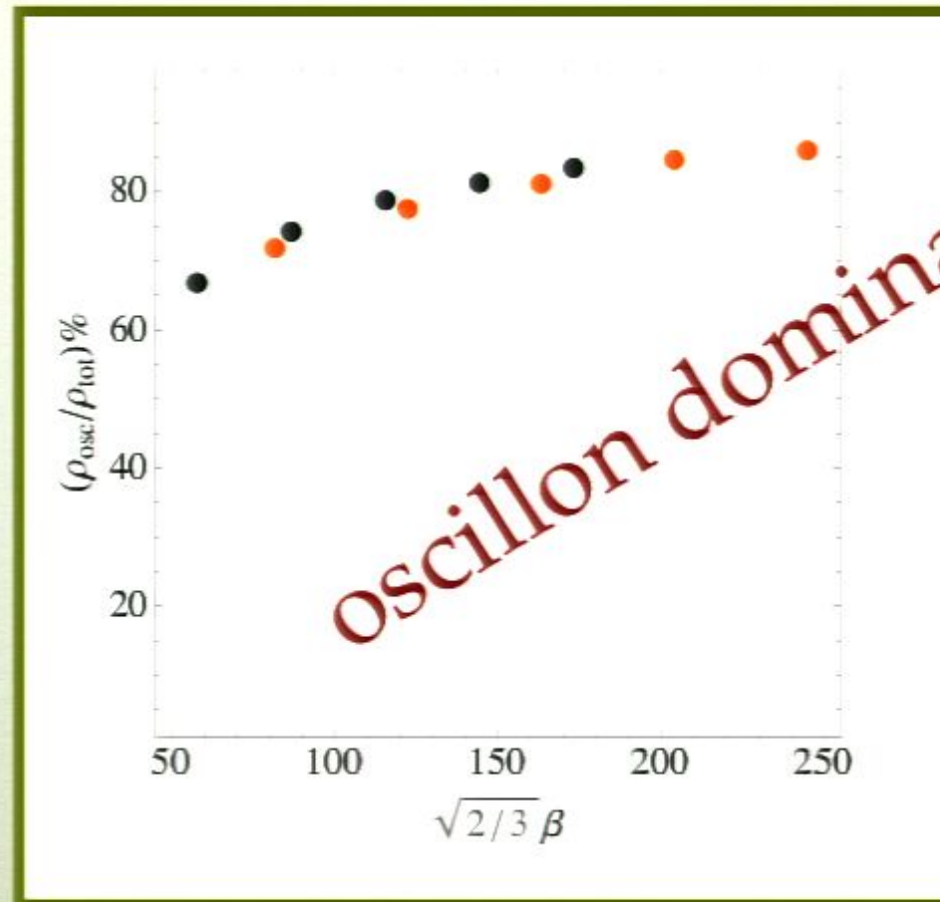


# NUMBER DENSITY AND ENERGY FRACTION EVOLUTION



# ENERGY FRACTION

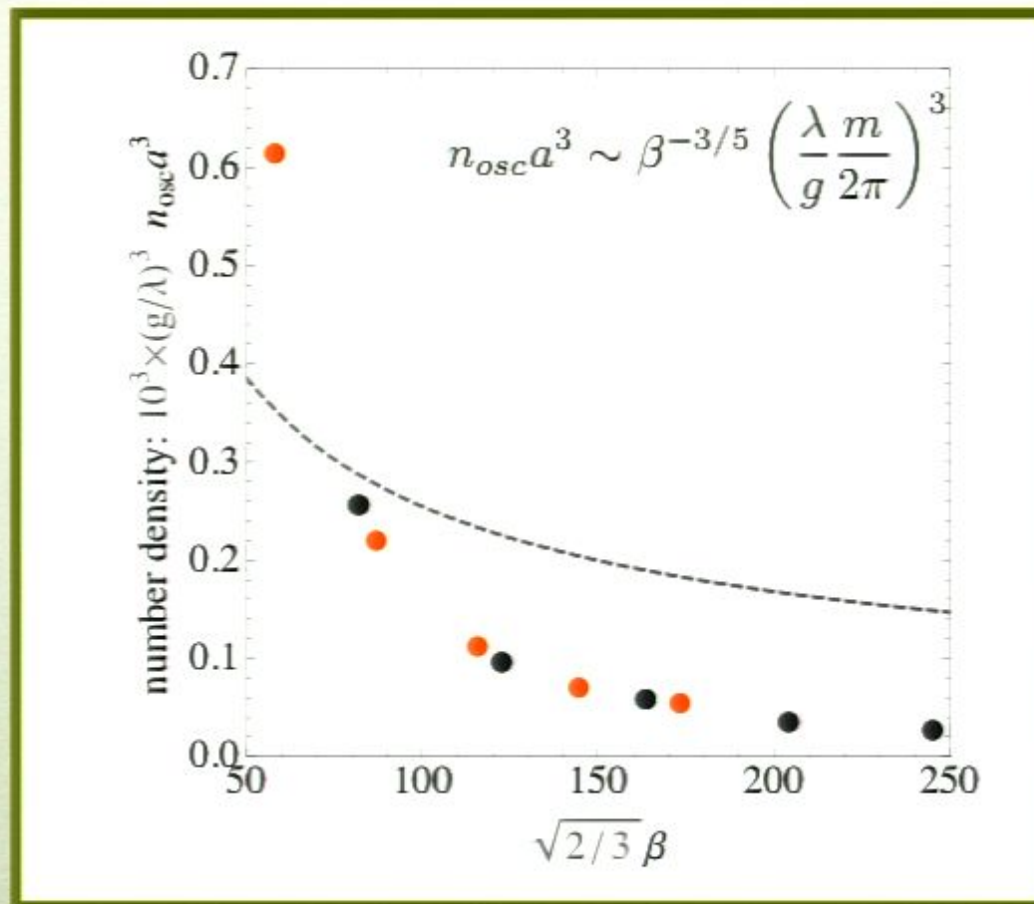
energy fraction in oscillons



Pirsa: 10110050

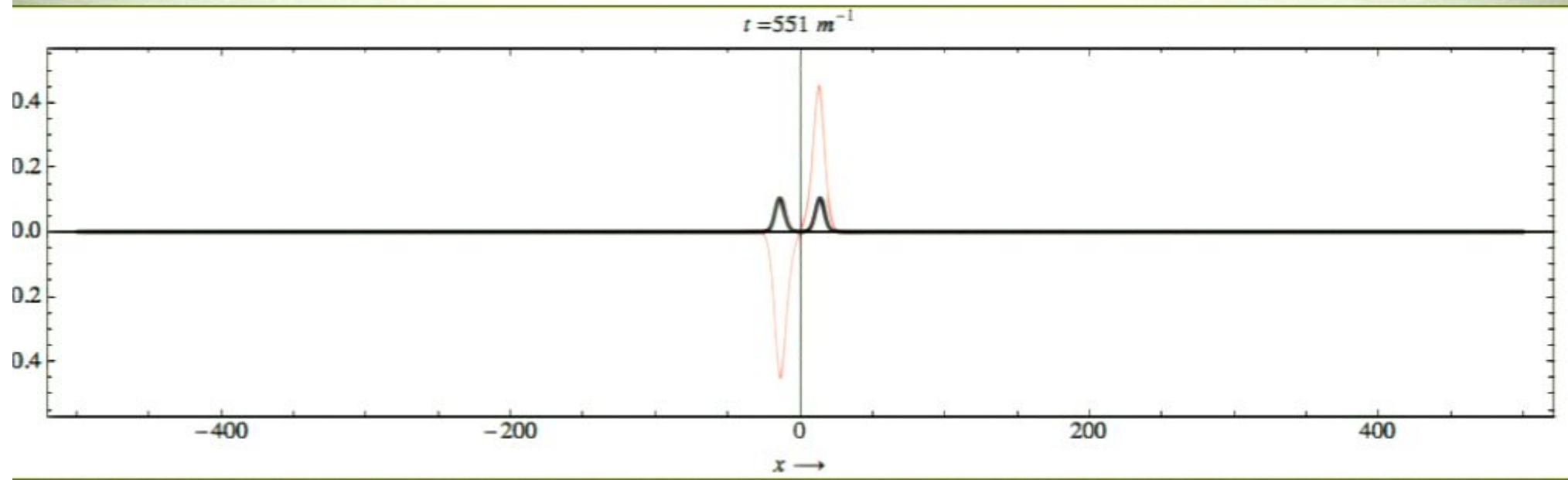
$$\frac{\mu_k}{H} \sim \beta \equiv \left[ \left( \frac{m_{pl}}{m} \right) \frac{\lambda^{3/2}}{c} \right]$$

# NUMBER DENSITY

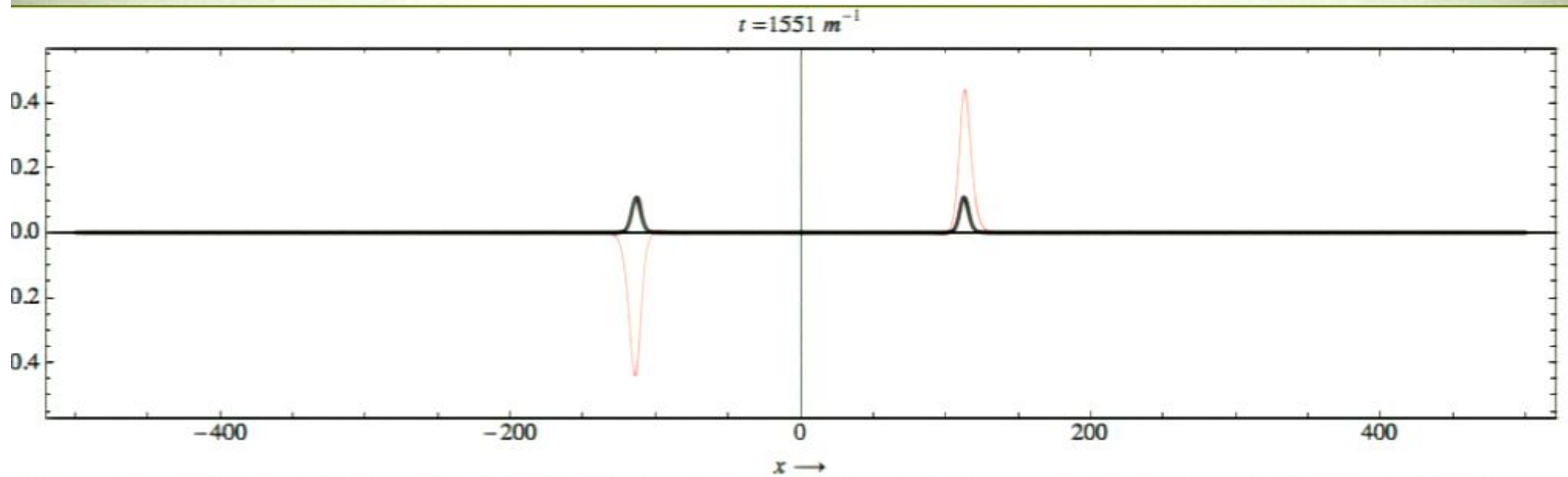


$$\frac{\mu_k}{\lambda} \sim \beta \equiv \left[ \left( \frac{m_{pl}}{\lambda} \right) \frac{\lambda^{3/2}}{g} \right]$$

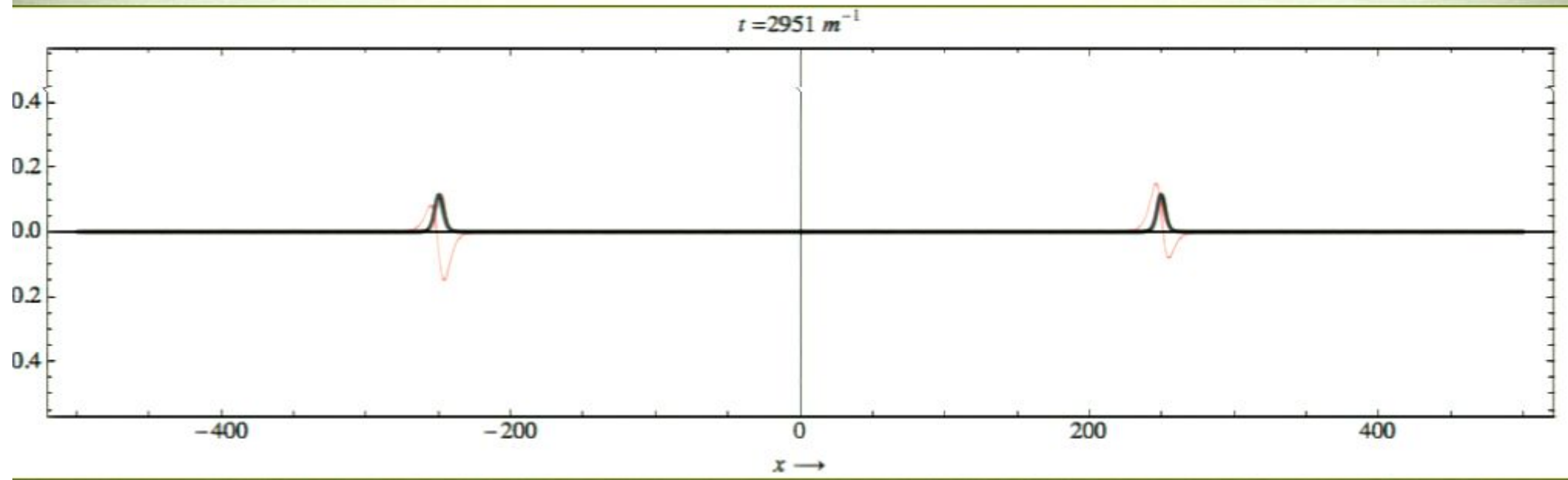
# COLLISIONS



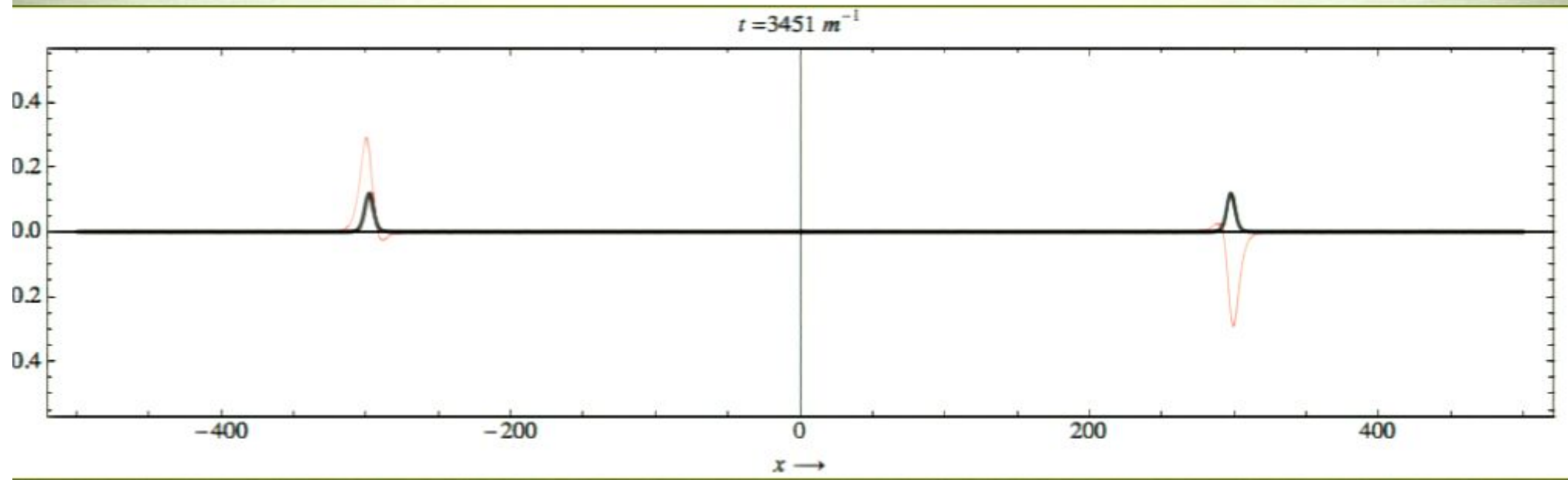
# COLLISIONS



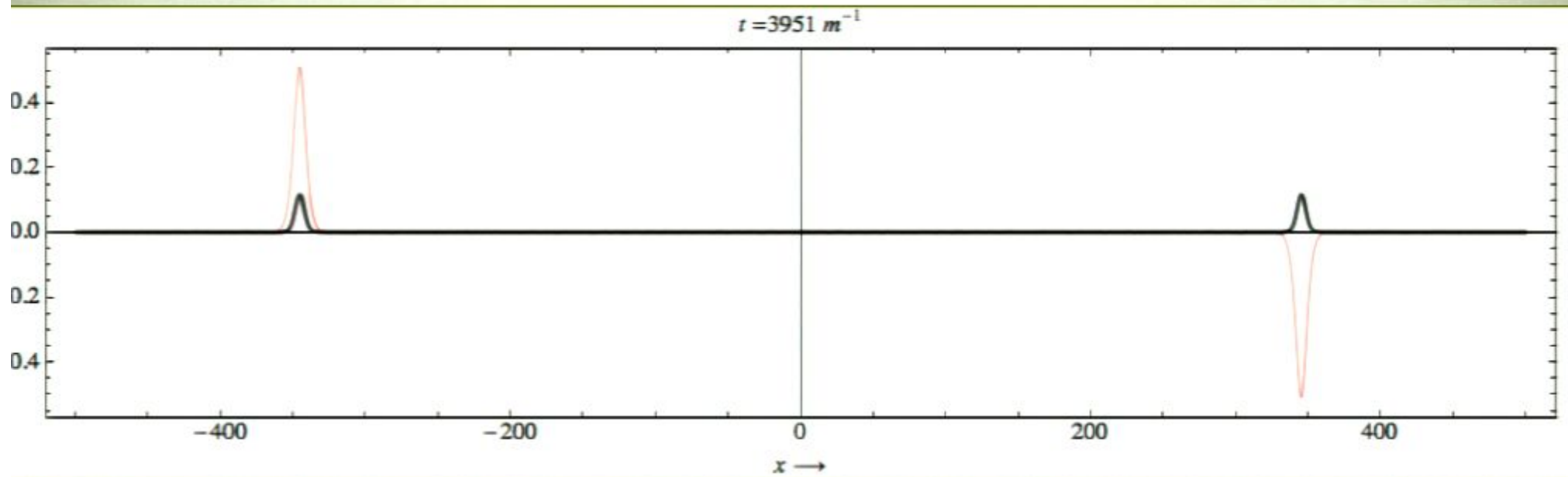
# COLLISIONS



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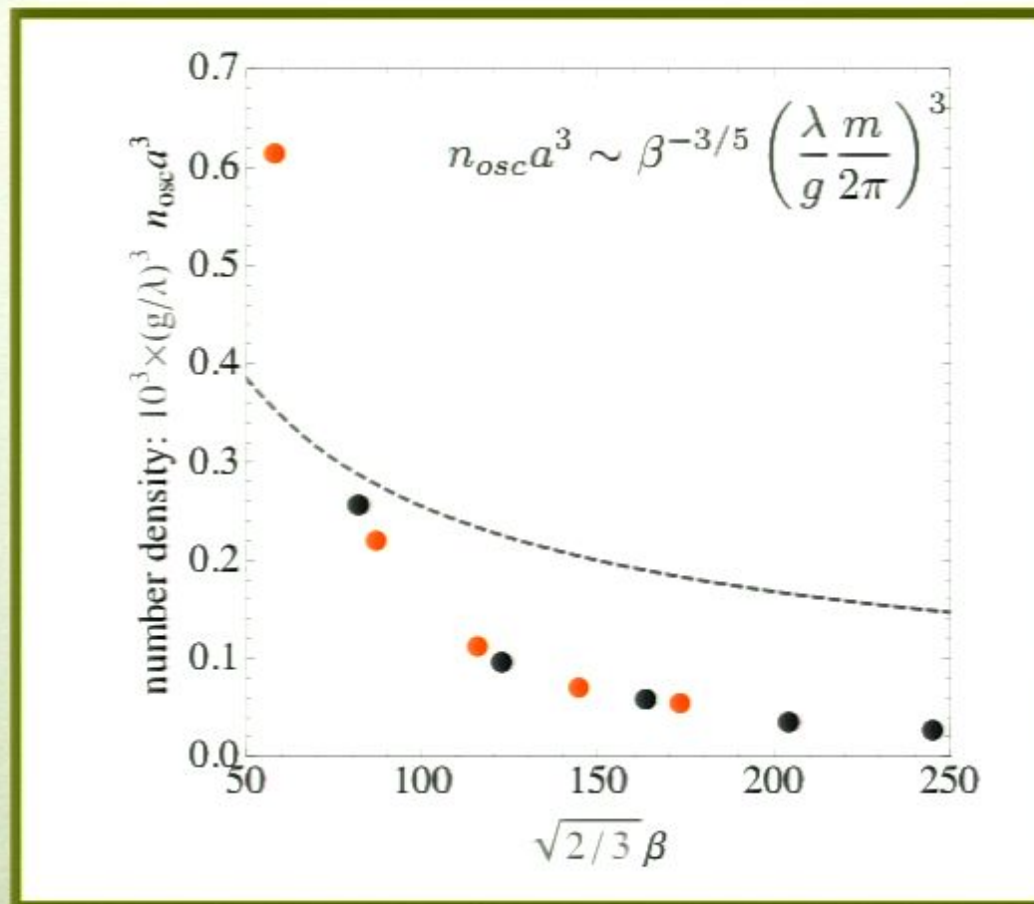


# COLLISIONS



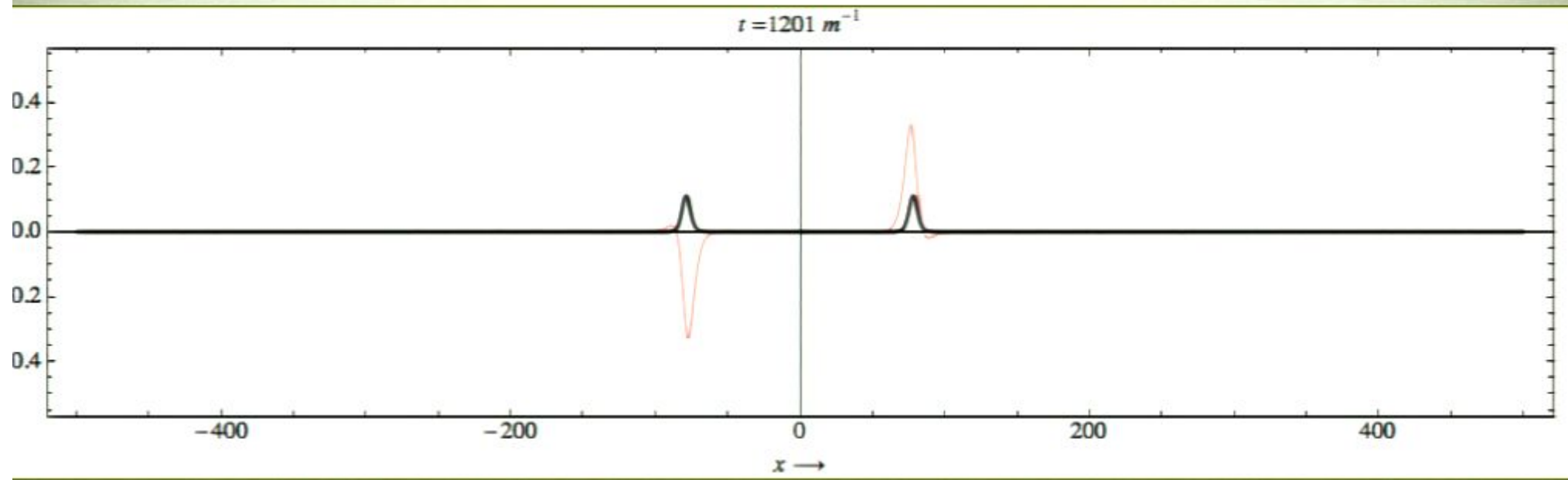


# NUMBER DENSITY

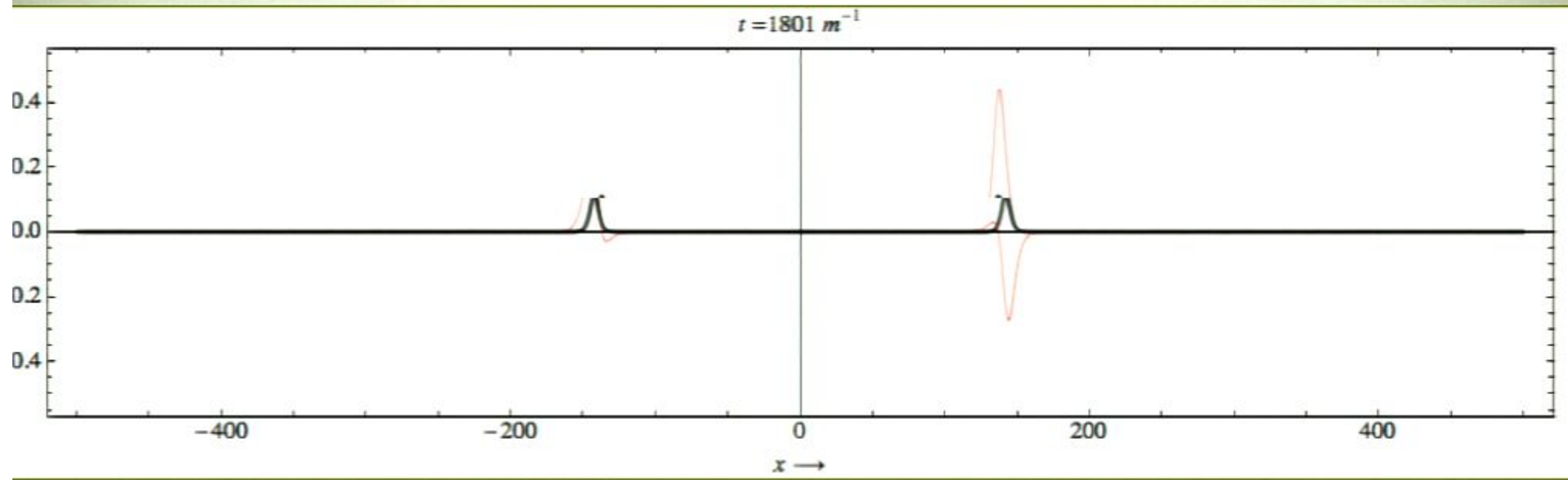


$$\frac{\mu_k}{\lambda} \sim \beta \equiv \left[ \left( \frac{m_{pl}}{\lambda} \right) \frac{\lambda^{3/2}}{g} \right]$$

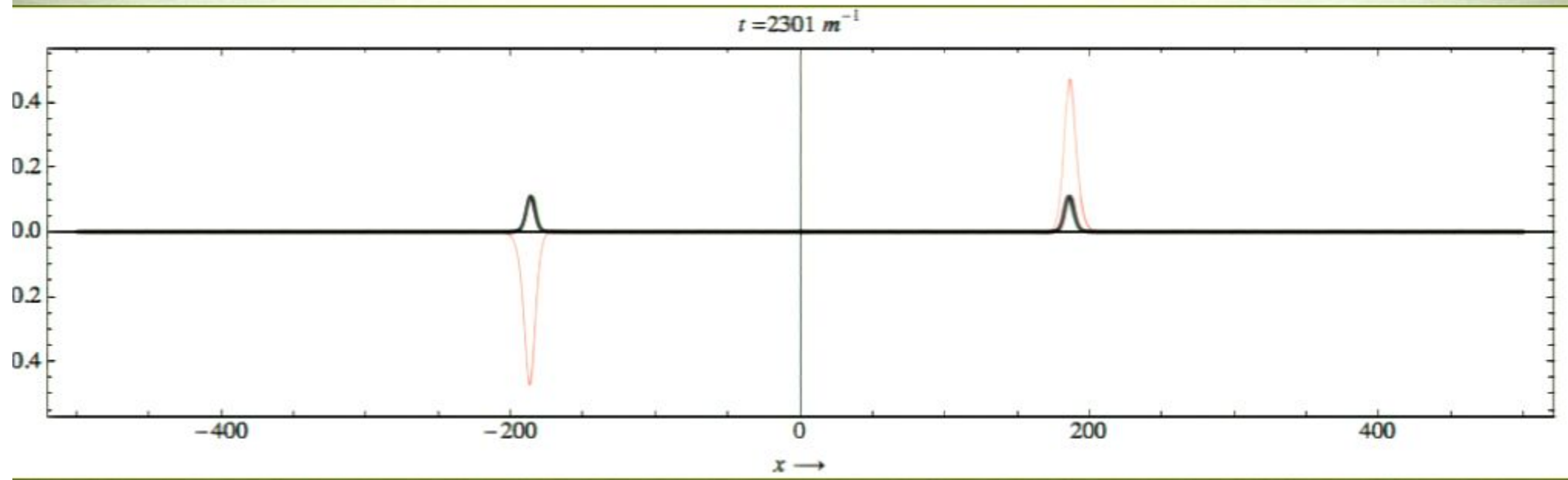
# COLLISIONS



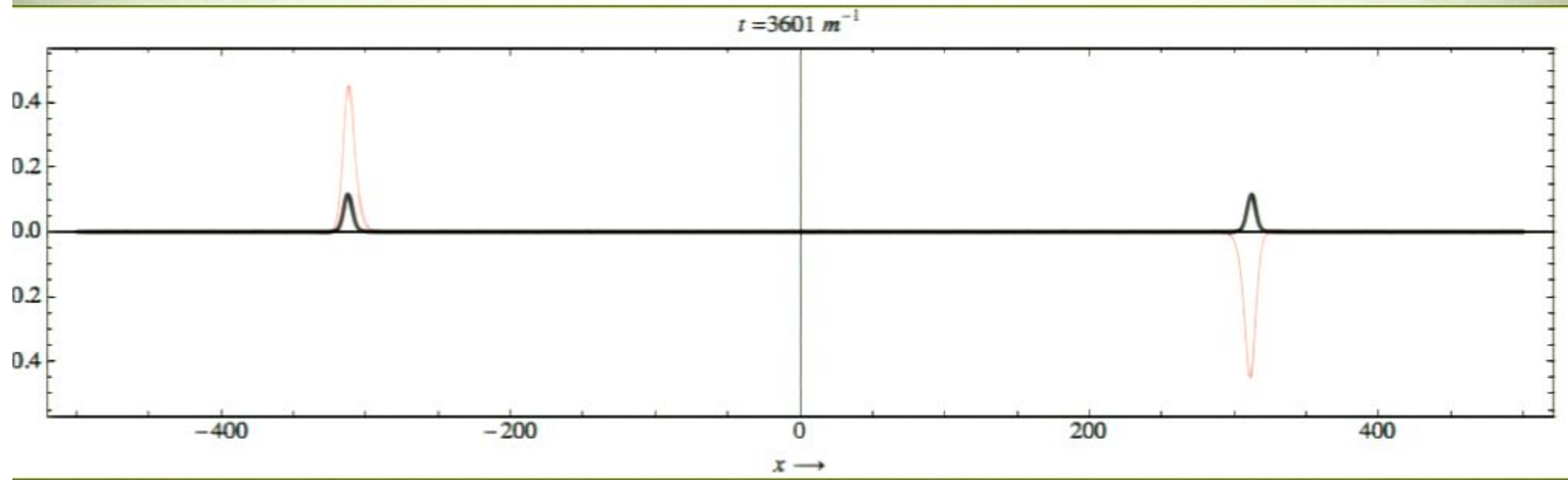
# COLLISIONS



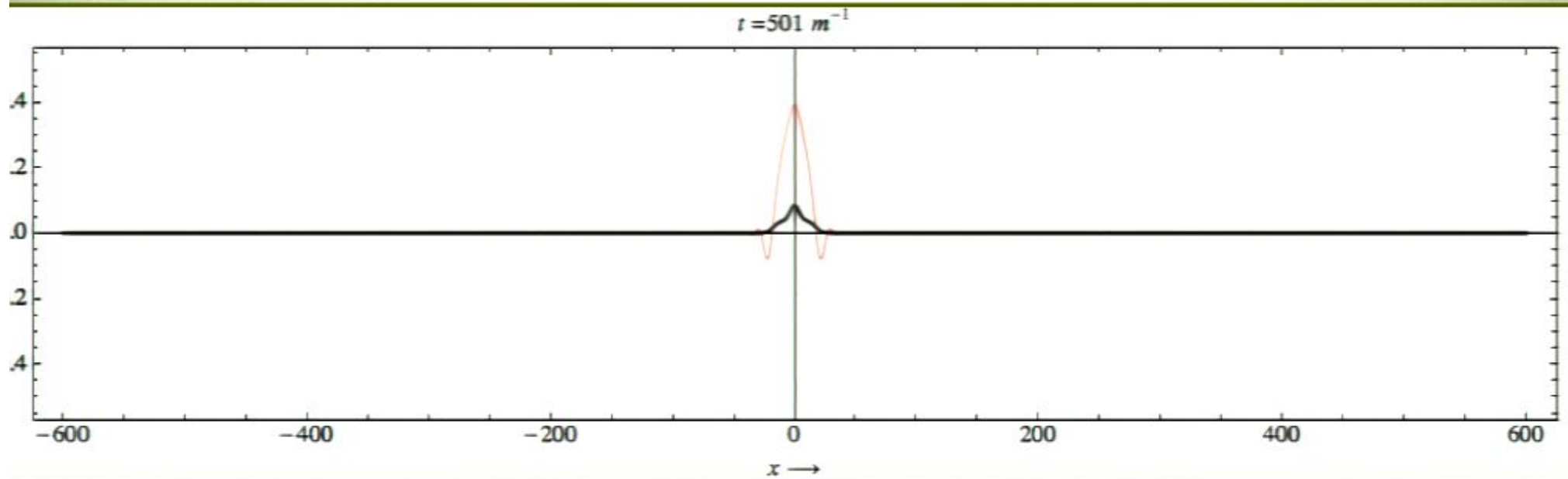
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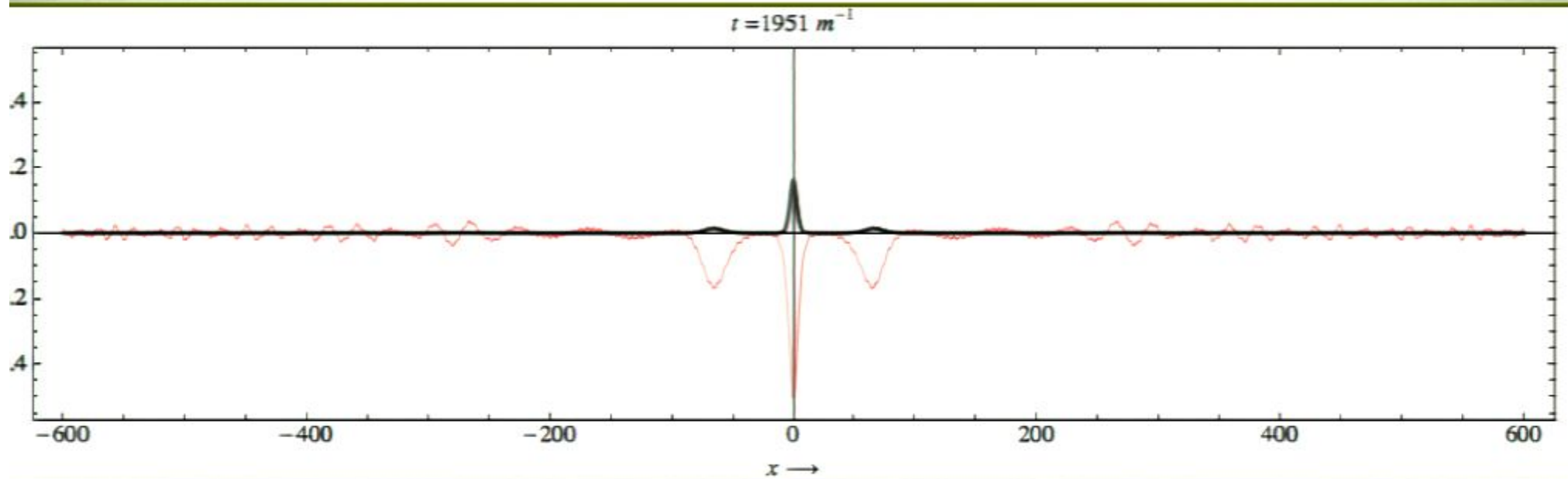
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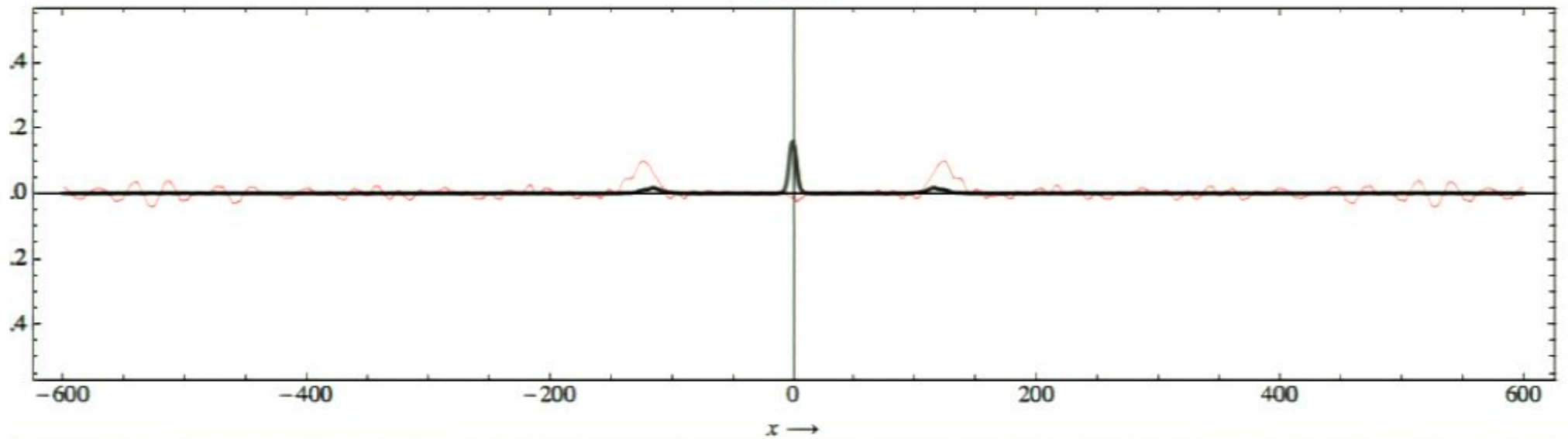


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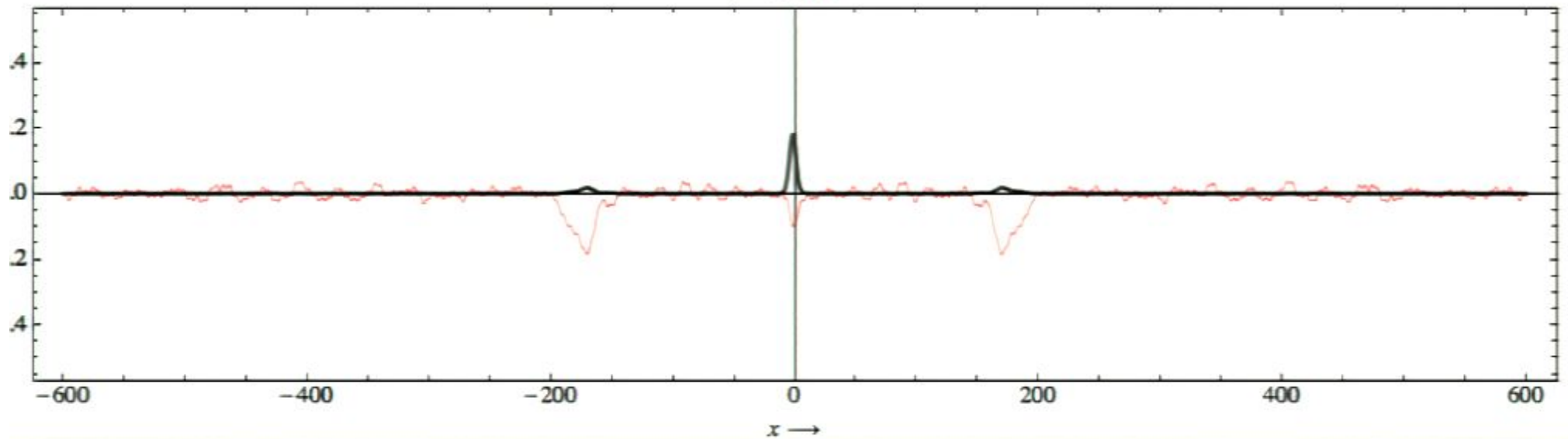
$t = 3201 \text{ m}^{-1}$



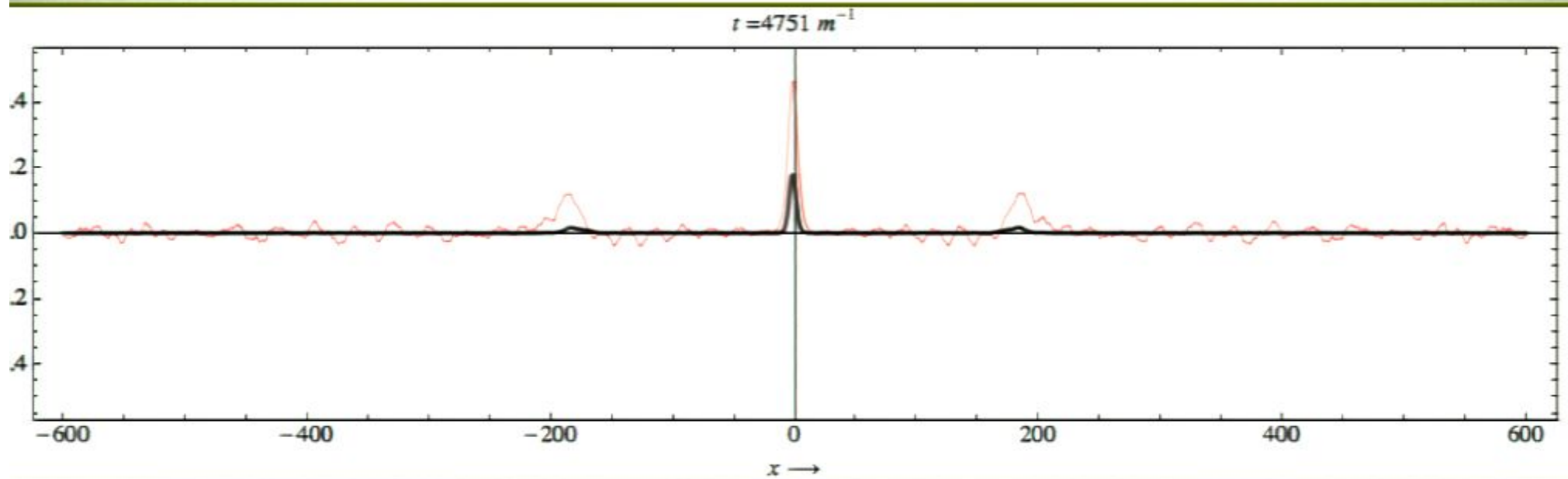


# COLLISIONS

$t = 4551 \text{ m}^{-1}$



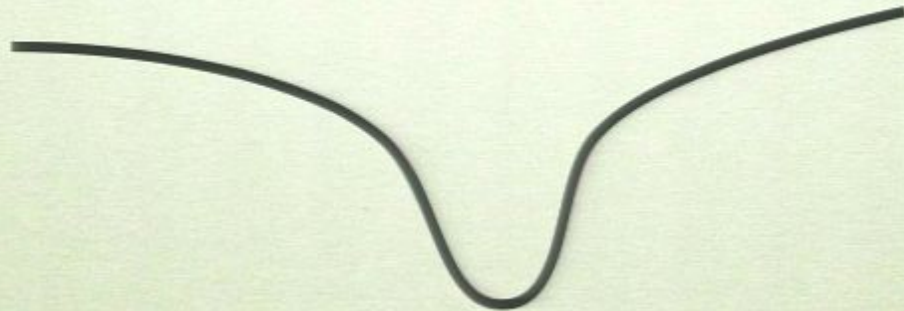
# COLLISIONS



# REALISTIC EXAMPLES

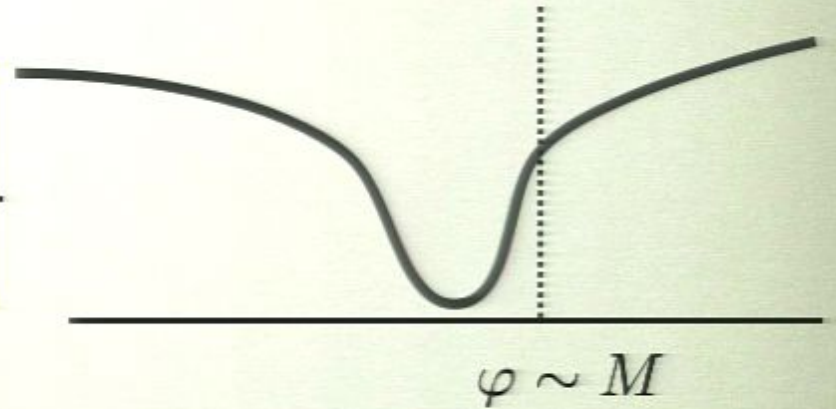
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- satisfy relevant WMAP constraints
- examples: axion monodromy (Silverstein & Westphal), supergravity (Kallosh & Linde)



# WMAP CONSTRAINTS

$$V(\varphi) = m^2 M^2 \left( \sqrt{1 + \left( \frac{\varphi}{M} \right)^2} - 1 \right).$$



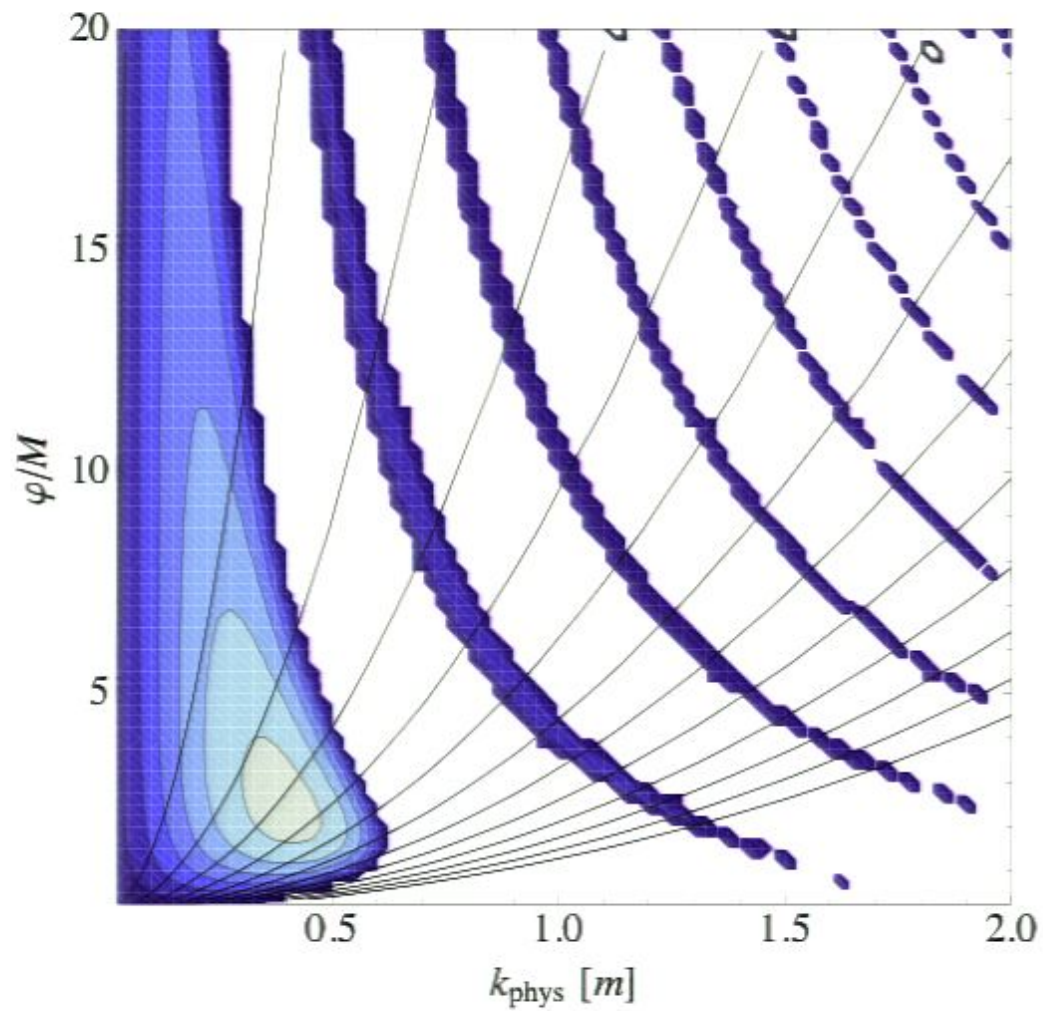
$$\Delta_{\mathcal{R}}^2 = (2.445 \pm 0.096) \times 10^{-9}$$

$$k = 0.002 \text{ Mpc}^{-1}$$

$$m^2 M \approx 2.2 \times 10^{-10} m_{pl}^3.$$

$$\varphi \gg M$$

# MONODROMY



# EMERGENCE CONSTRAINT

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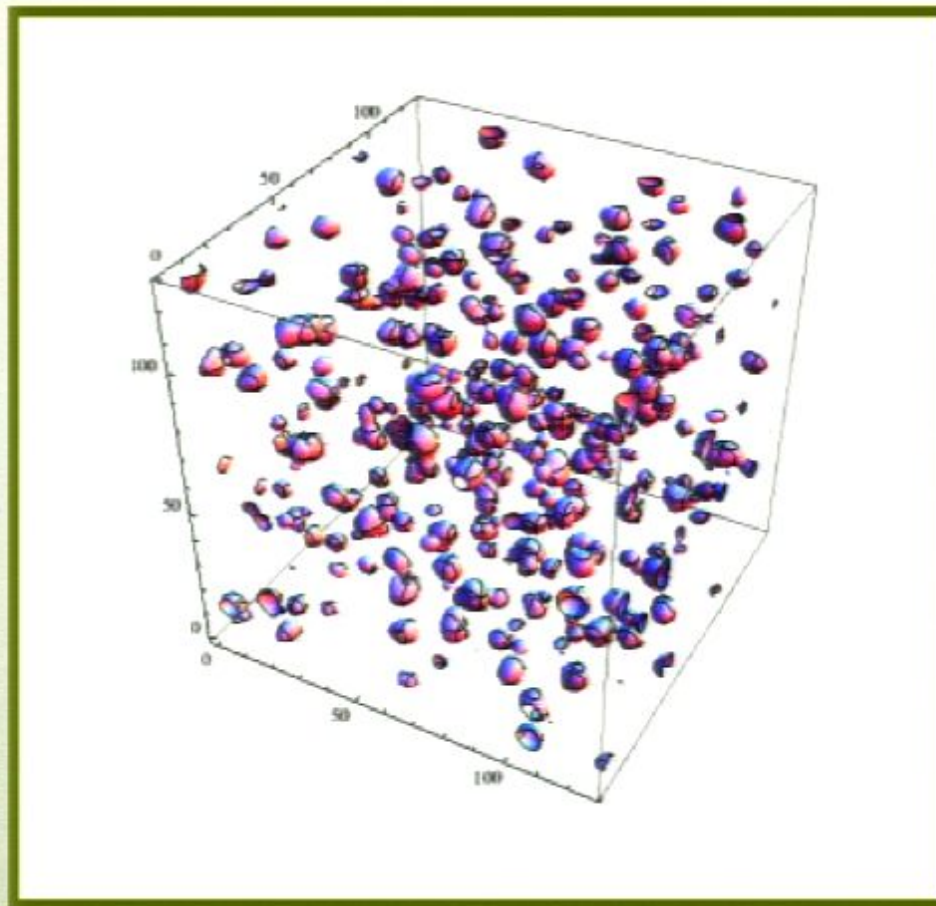
$$V(\varphi) = \frac{1}{2}m^2\varphi^2 - \frac{1}{4}\lambda\varphi^4 + \frac{1}{6}\frac{g^2}{m^2}\varphi^6 - \dots$$

$$\lambda = \frac{1}{2}\frac{m^2}{M^2}, \quad g = \sqrt{\frac{3}{2}}\lambda \dots$$

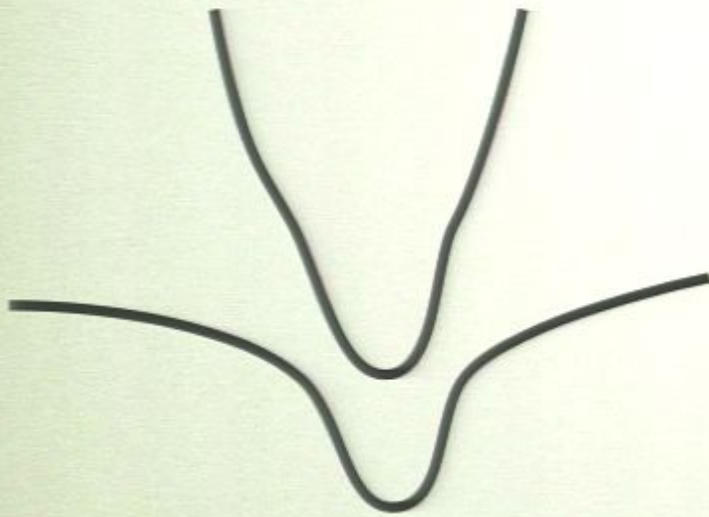
$$\mu_k \gg H$$

$$M \approx 10^{-2}m_{pl}$$

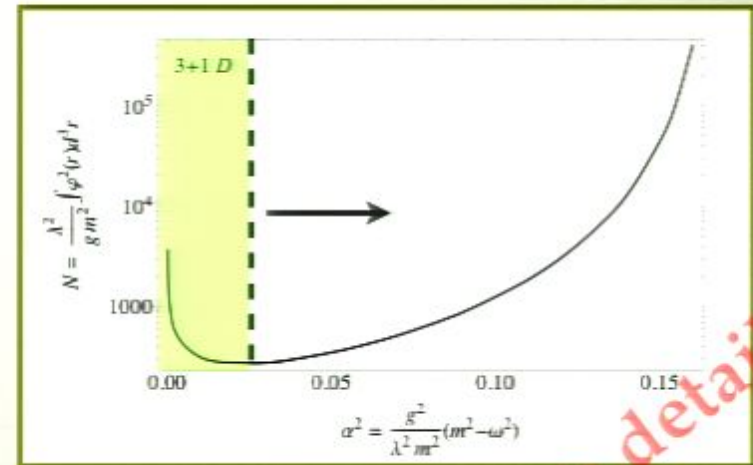
# MONODROMY OSCILLONS!



# A HEURISTIC GUIDE ...



necessary



stability

$$\mu_k \gg H$$

emergence

$$\Gamma \ll H \ll m$$

cosmologically relevant



3+1

# WHAT TO DO WITH THEM?

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3+1

## WHAT TO DO WITH THEM?

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- gravitational effects:

3+1

## WHAT TO DO WITH THEM?

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  - early universe structure formation

3+1

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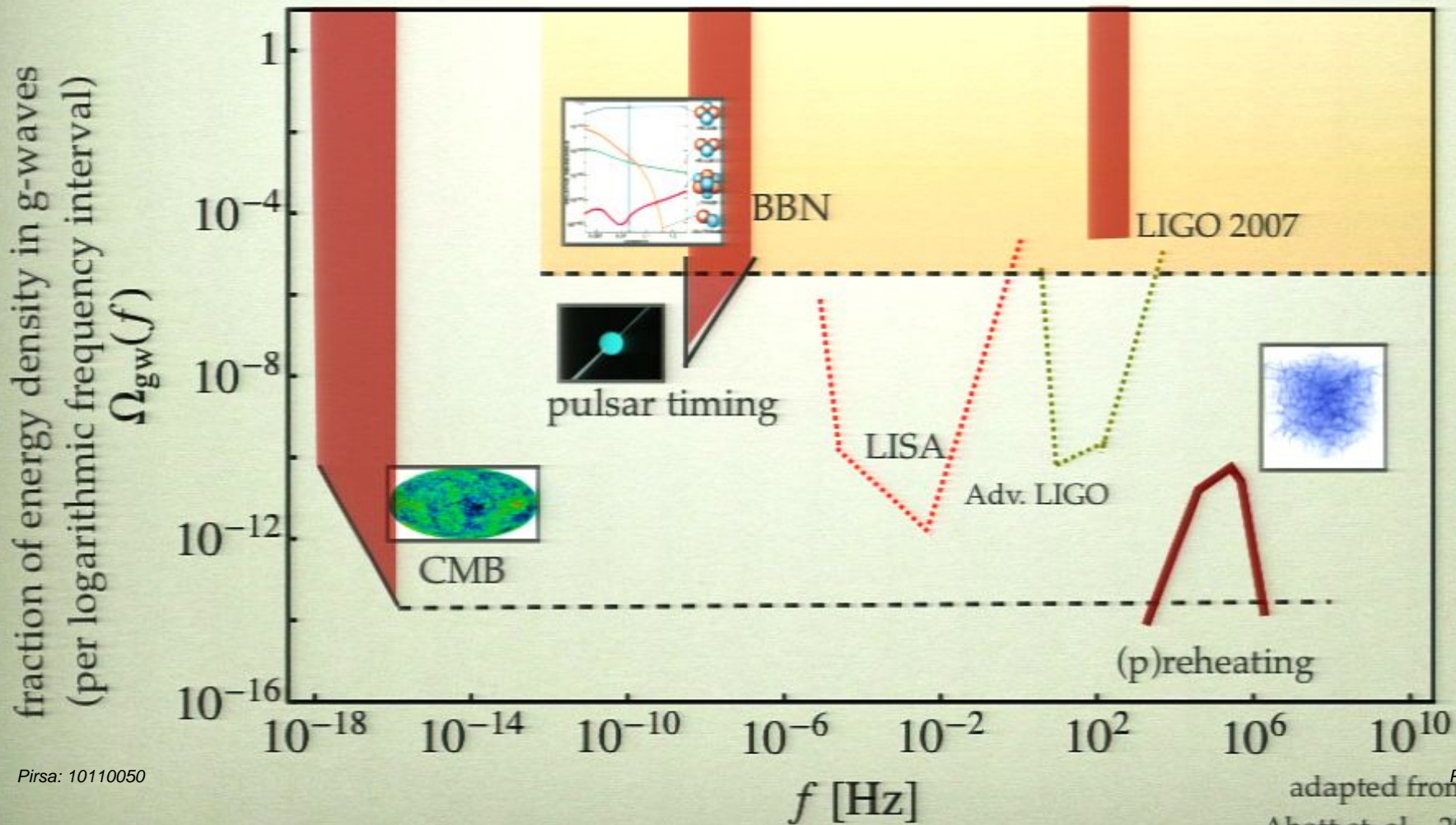


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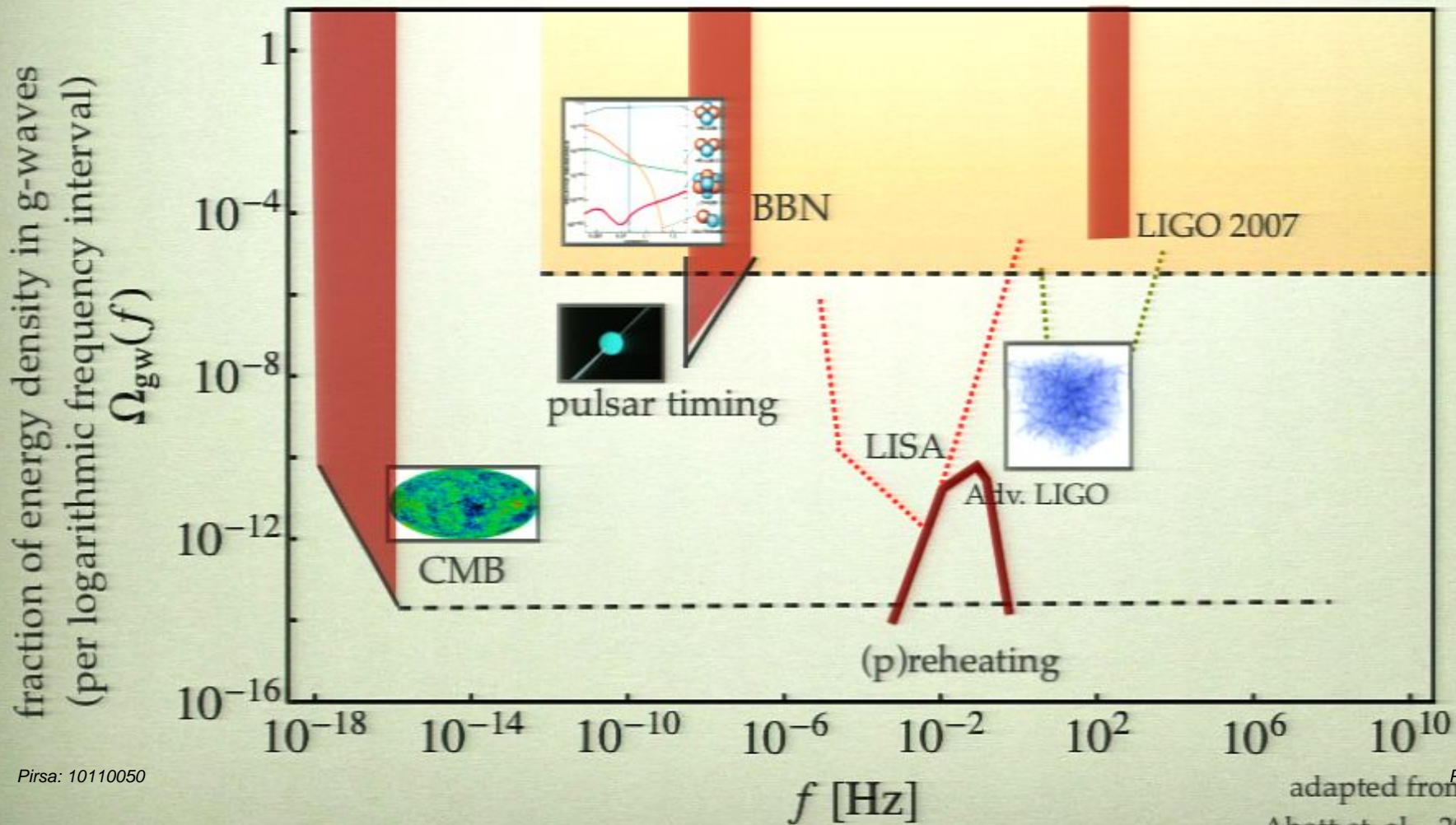
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  - black holes ?
  - expansion history and influence on inflationary observables ?
  - g-waves ?
- axions *(Kolb & Tkachev 1992)*
- thermalization, enhanced decay rates
- SU(2) oscillons *(Graham & Farhi 2007)*

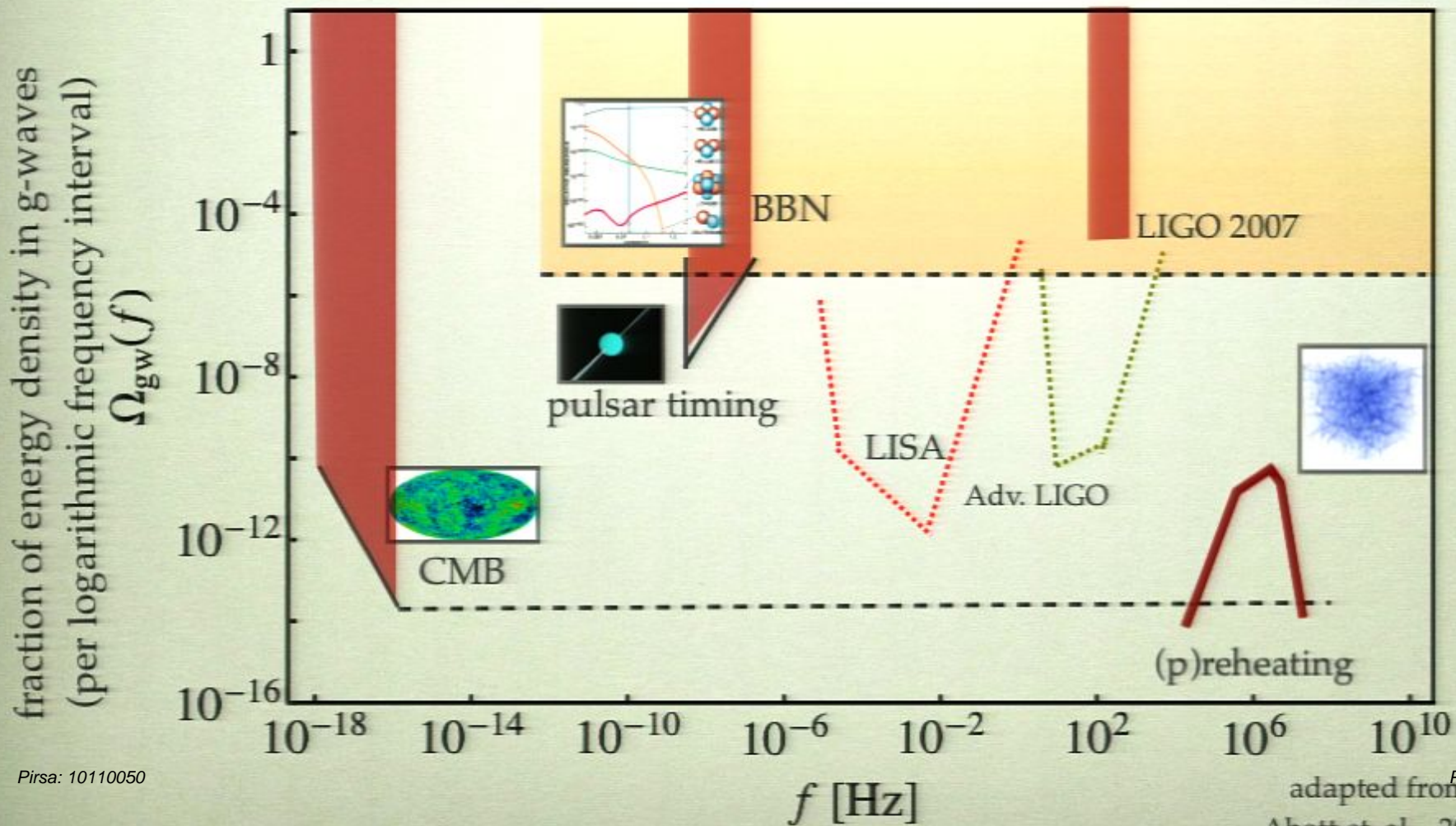
# GRAVITATIONAL WAVES



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# SUMMARY

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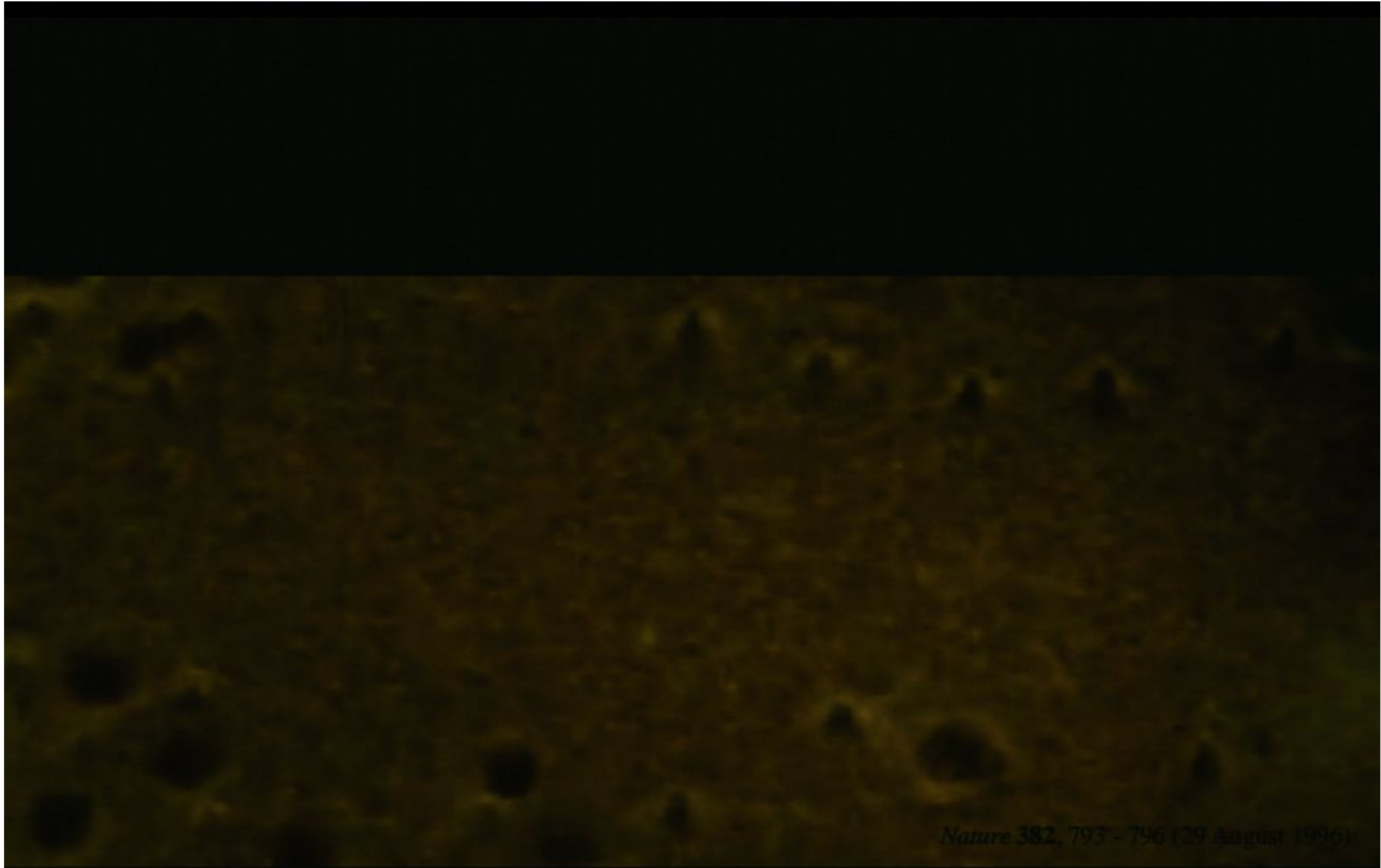
- can *emerge naturally* from inflaton fragmentation in many models
- *existence, emergence and stability conditions*
- possible implications: enhanced g-waves, structure formation, blackholes, CMB observables ?



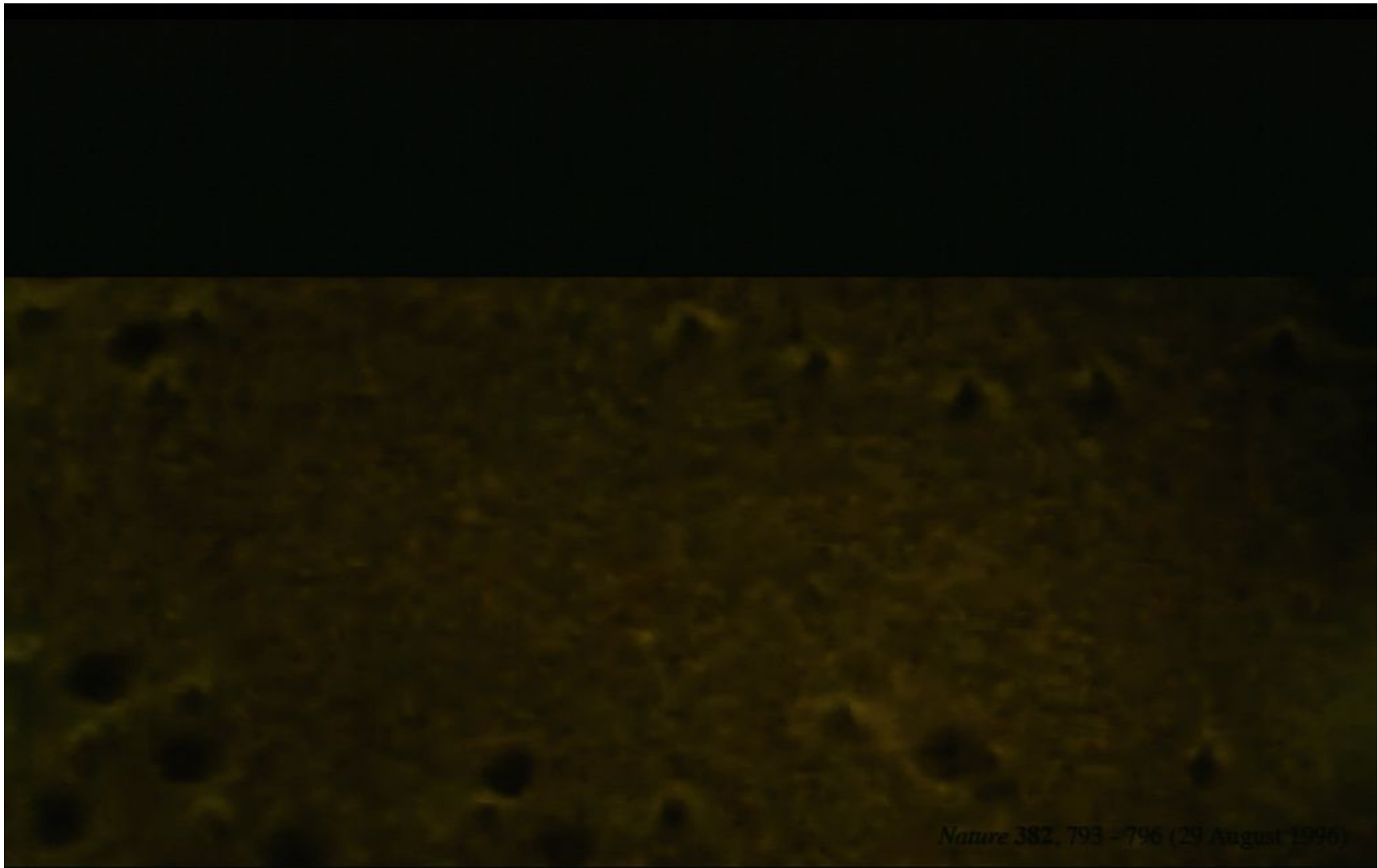
# ACKNOWLEDGEMENTS

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- **Alan Guth**, Eddie Farhi, Ruben Rosales, Evangelos Sfakianakis, Mark Mezei, David Shirokoff & Antony Speranza (MIT)
- Richard Easter & Hal Finkel (Yale)
- Mark Hertzberg (Stanford)

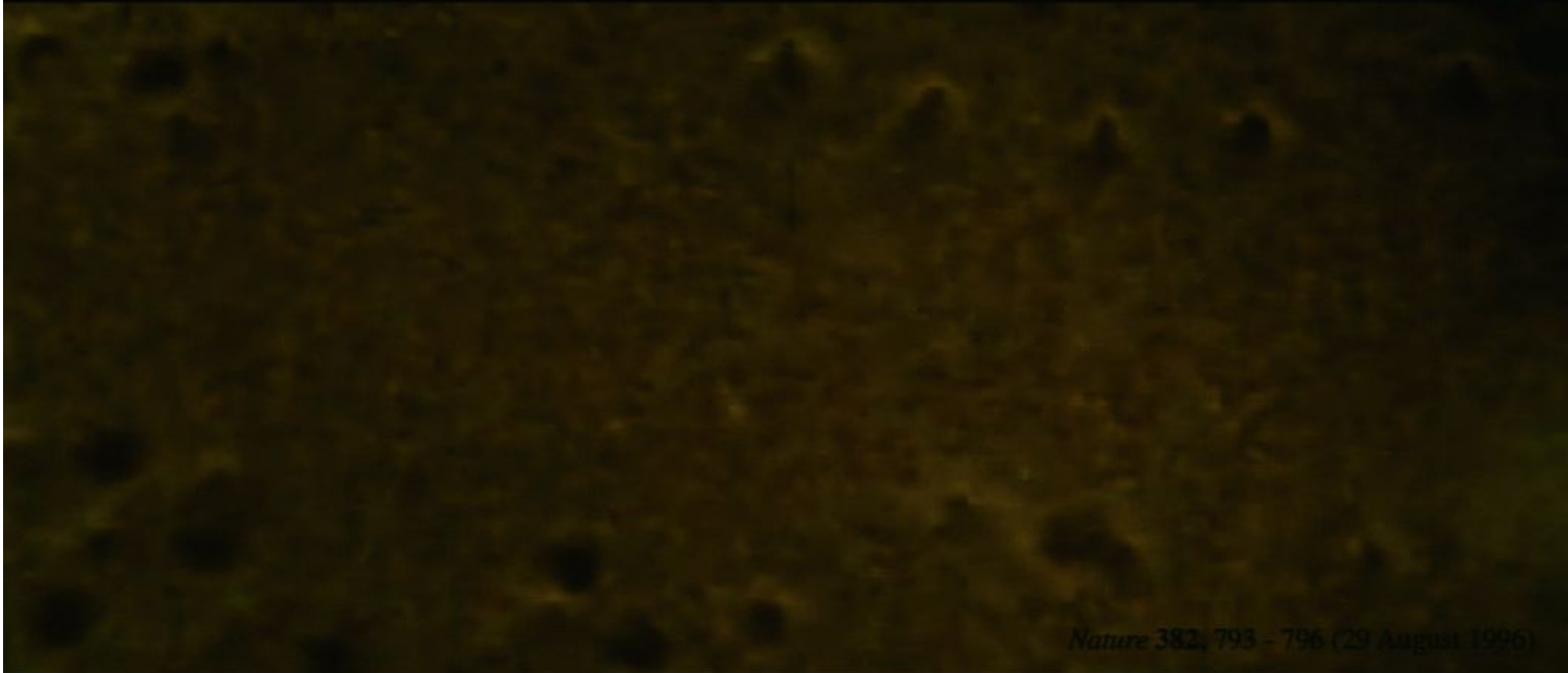
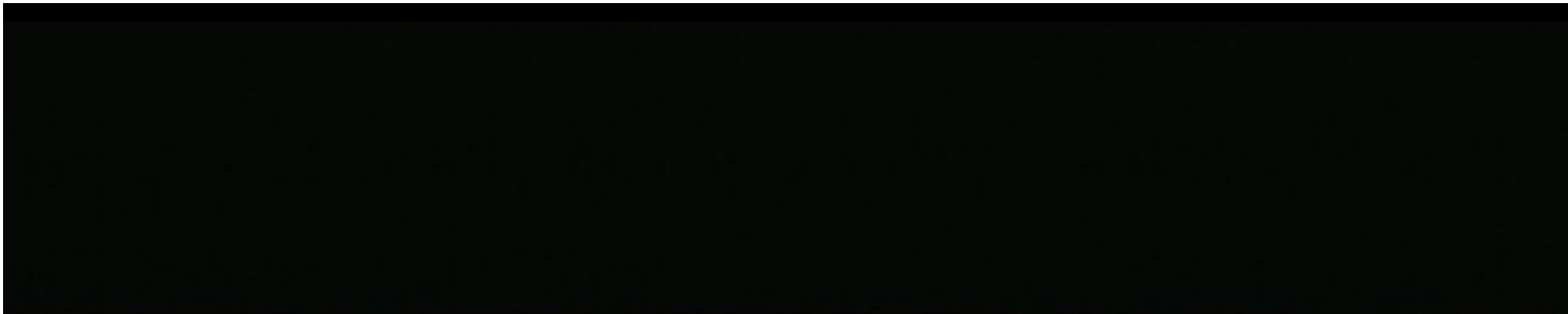


*Nature* 382, 793 - 796 (29 August 1996)



*Nature* 382, 793–796 (29 August 1996)

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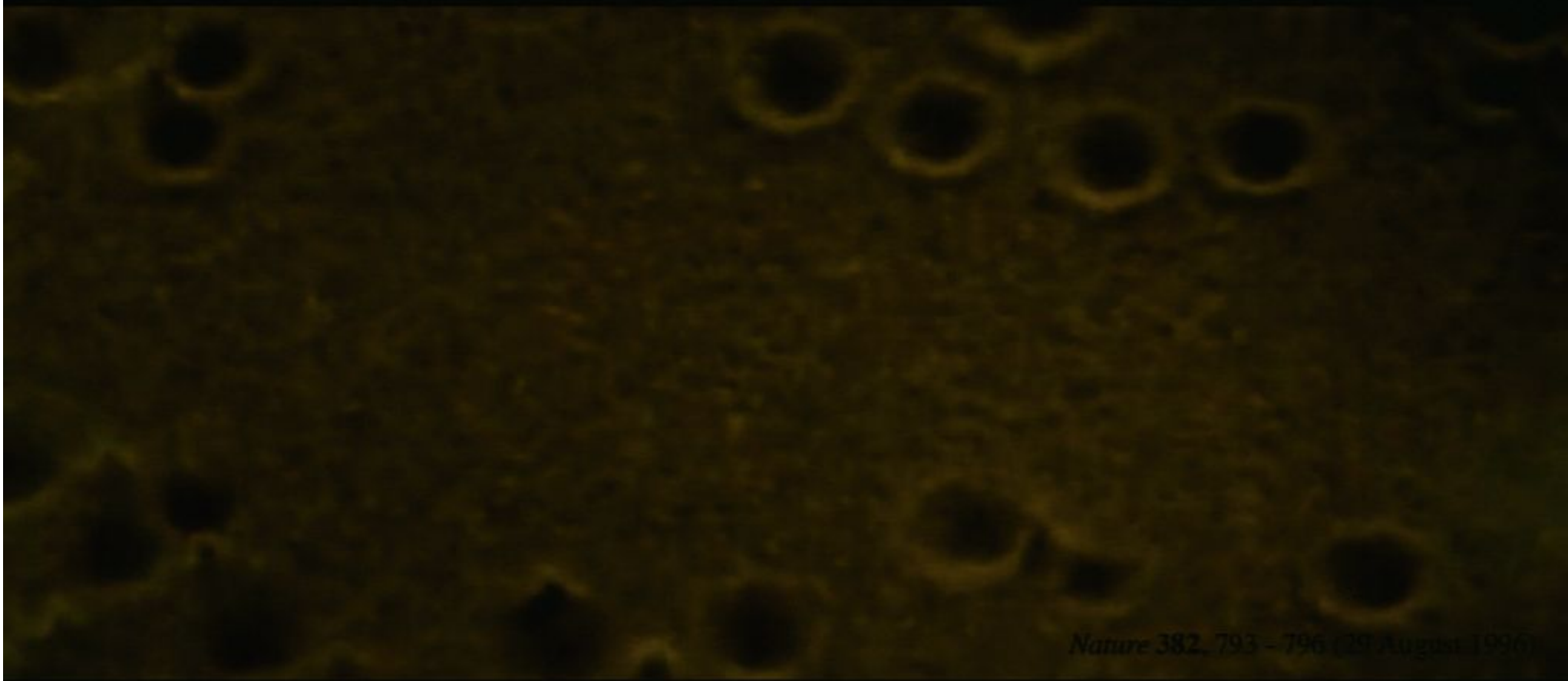


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