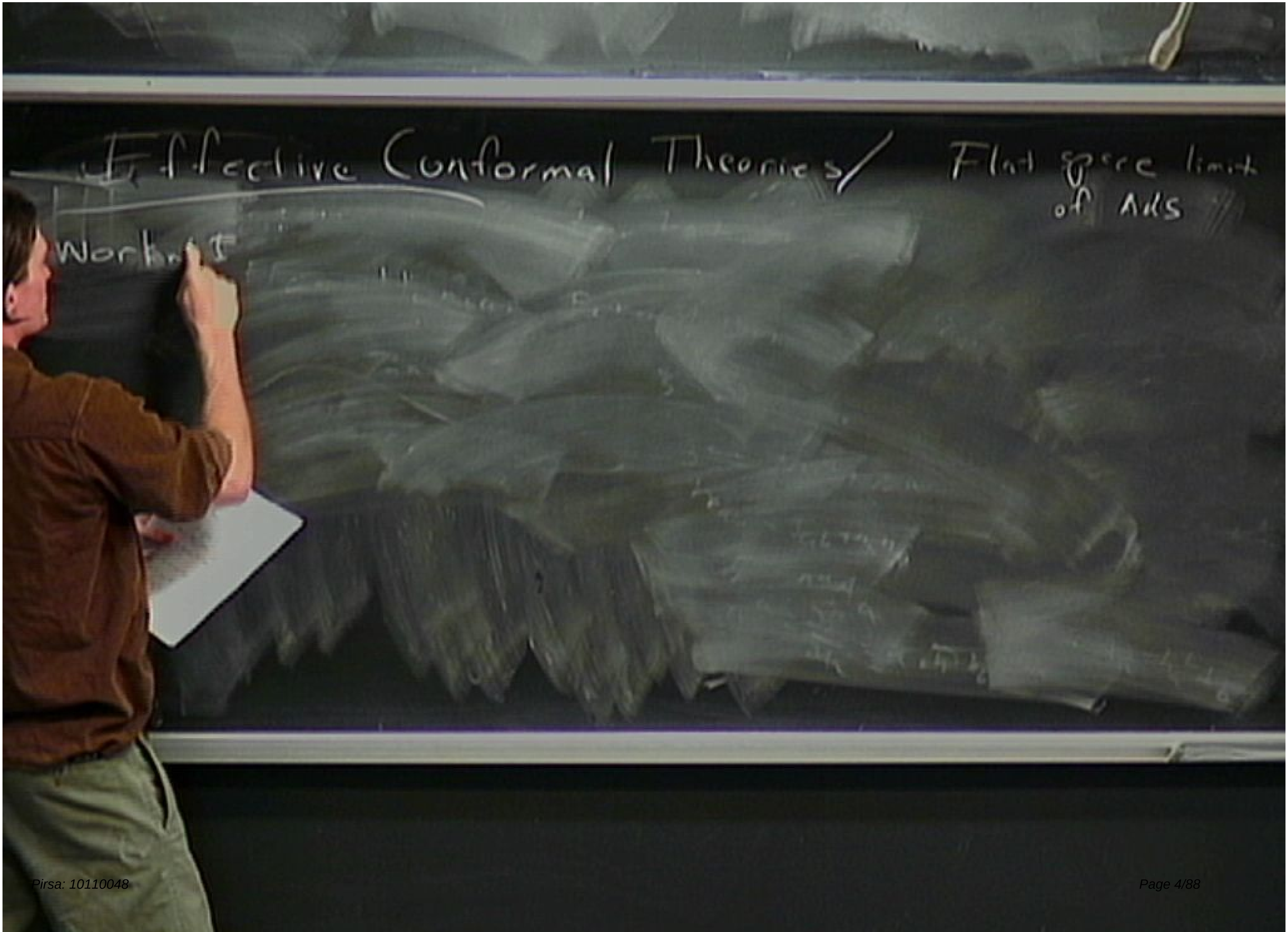
Title: Effective Conformal Theory and the Flat-space Limit of AdS

Date: Nov 19, 2010 02:30 PM

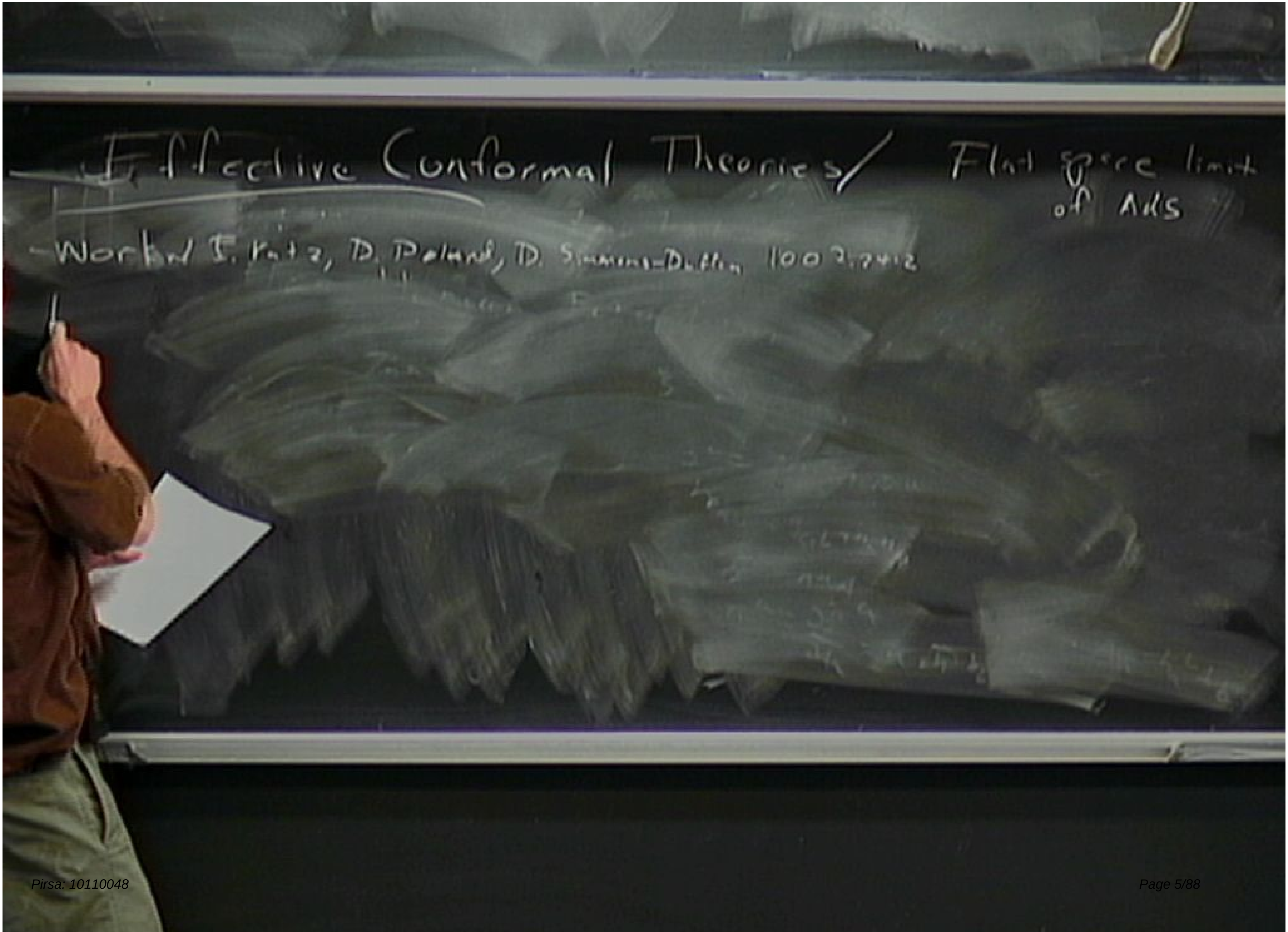
URL: http://pirsa.org/10110048

Abstract: The idea of an effective conformal theory describing the low-lying spectrum of the dilatation operator in a CFT is developed. Such an effective theory is useful when the spectrum contains a hierarchy in the dimension of operators, and a small parameter whose role is similar to that of 1/N in a large N gauge theory. These criteria insure that there is a regime where the dilatation operator is modified perturbatively. Global AdS is the natural framework for perturbations of the dilatation operator respecting conformal invariance, much as Minkowski space naturally describes Lorentz invariant perturbations of the Hamiltonian. Assuming that the lowest-dimension single-trace operator is a scalar, O, I consider the anomalous dimensions, gamma(n,l), of the double-trace operators of the form O (del^2)^n (del)^l O. Purely from the CFT, perturbative unitarity places a bound on these dimensions; non-renormalizable AdS interactions lead to violations of the bound at large values of n. I also consider the case that these interactions are generated by integrating out a heavy scalar field in AdS. The presence of the heavy field "unitarizes" the growth in the anomalous dimensions, and leads to a resonance-like behavior in gamma(n,l) when n is close to the dimension of the CFT operator dual to the heavy field. Finally, I demonstrate that bulk flat-space S-matrix elements can be extracted from the large n behavior of the anomalous dimensions. This leads to a direct connection between the spectrum of anomalous dimensions in d-dimensional CFTs and flat-space S-matrix elements in d+1 dimensions



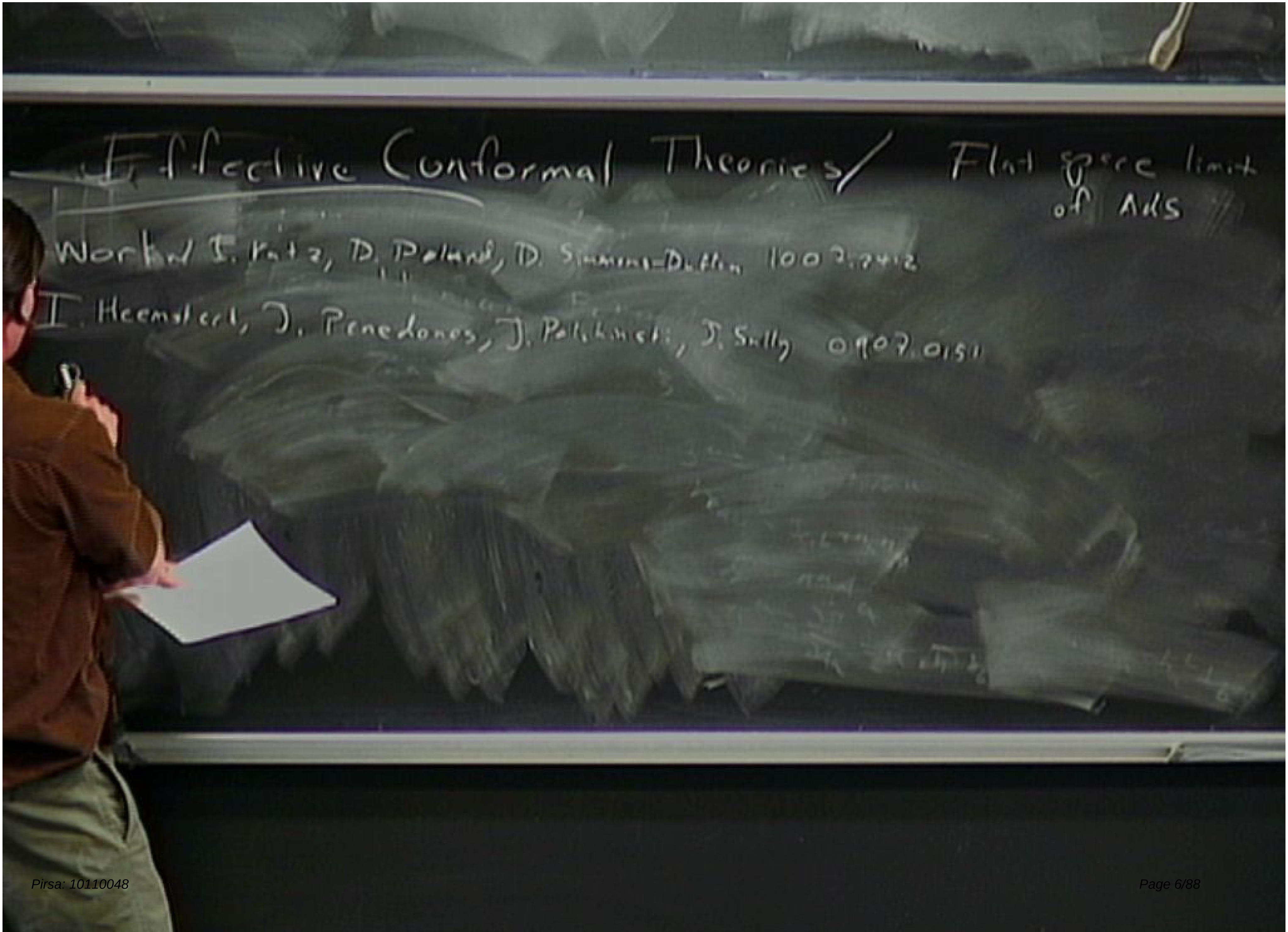
Effective Conformal Theories / Flat space limit of AdS

Work I



Effective Conformal Theories / Flat space limit of AdS

- Work w/ I. Katz, D. Poland, D. Simmons-Duffin 1007.2412



Effective Conformal Theories / Flat space limit of AdS

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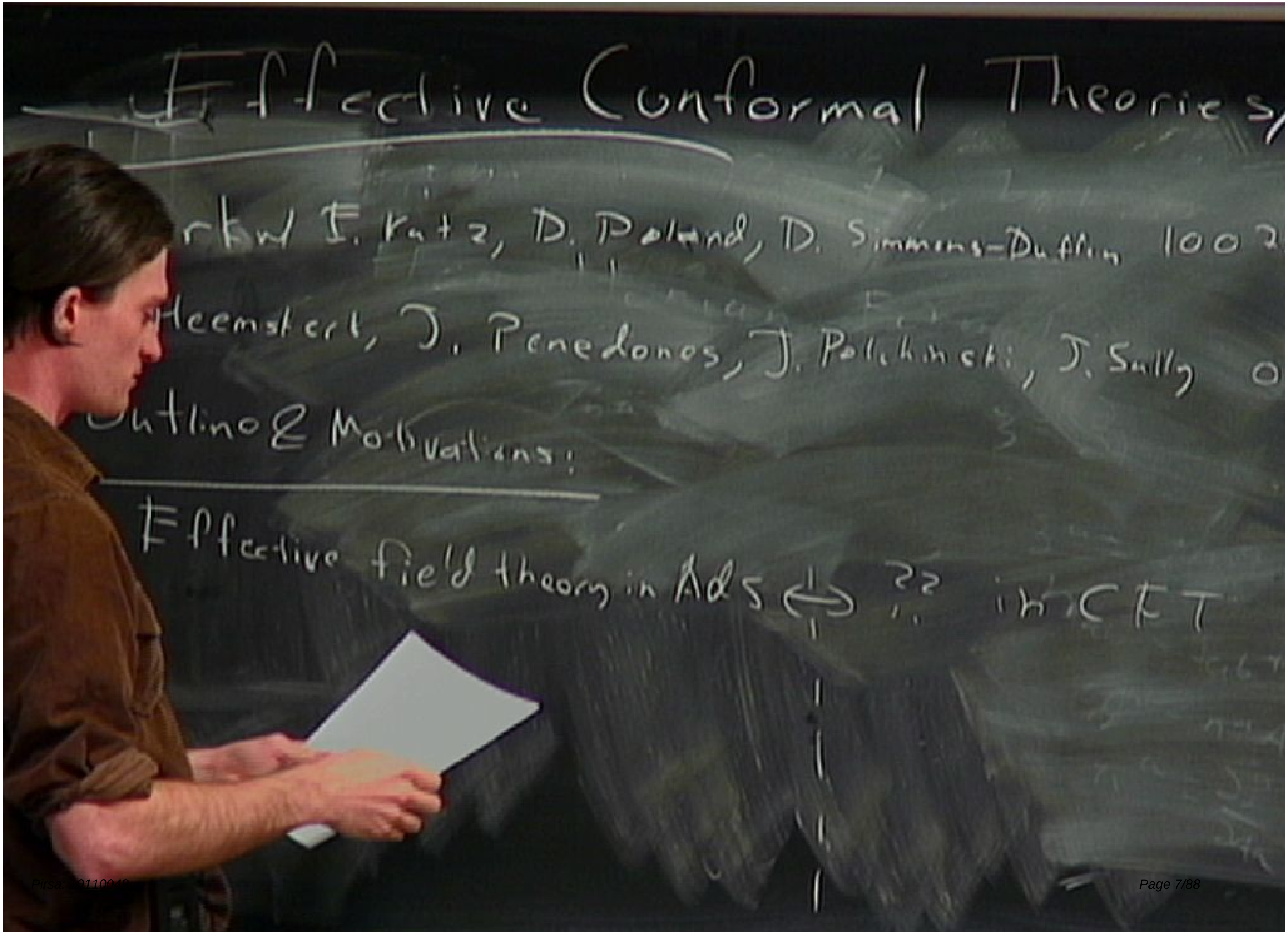
I. Heemskerk, J. Penedones, J. Polchinski, J. Sully 0907.0151



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Heemskerk, J. Penedones, J. Polchinski, J. Sully 0

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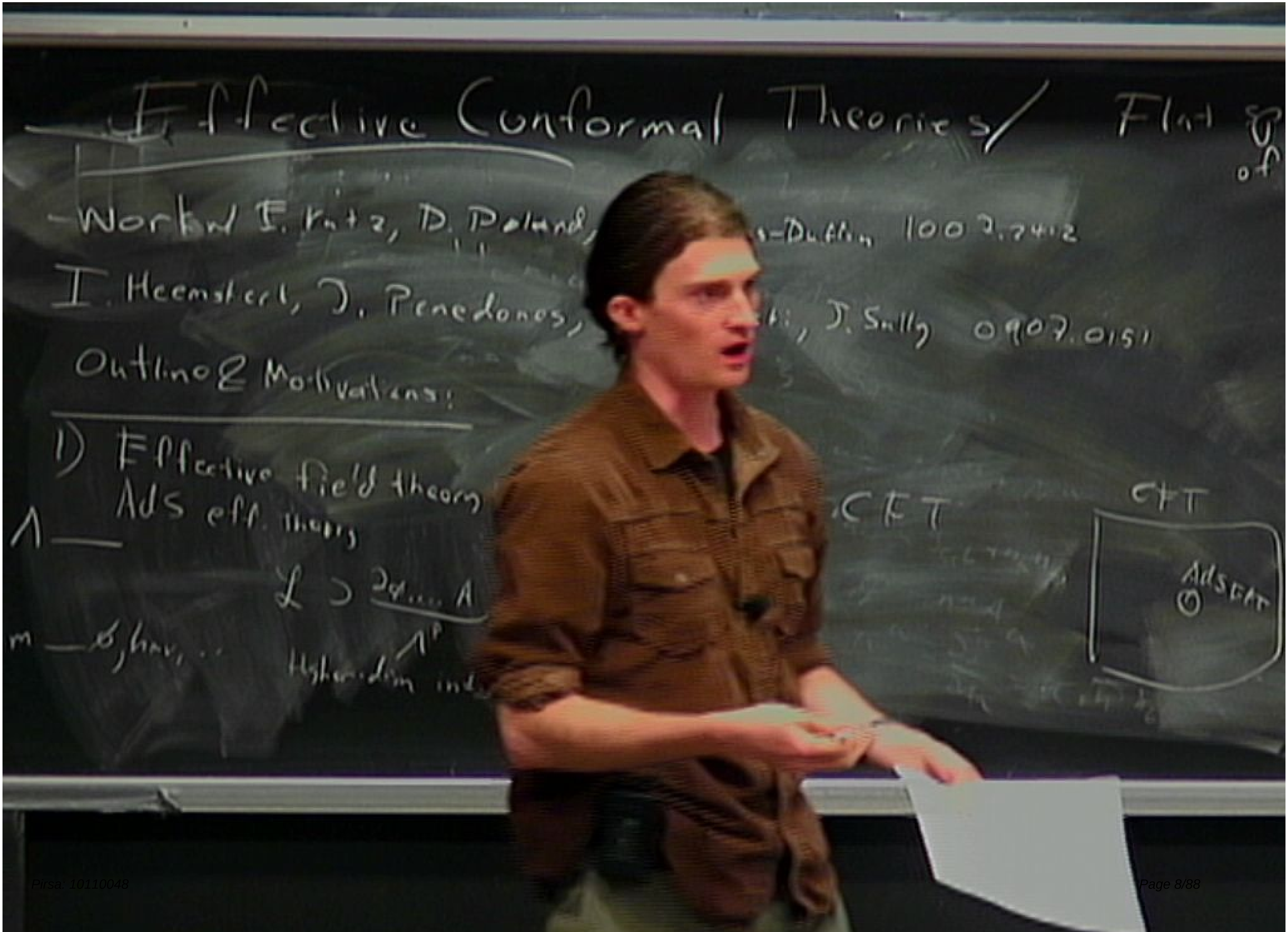
Effective field theory in AdS $\longleftrightarrow$?? in CFT



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I. Heemskerk, J. Penedones, , J. Sully 0907.0151

Outline & Motivations:

1) Effective field theory CFT

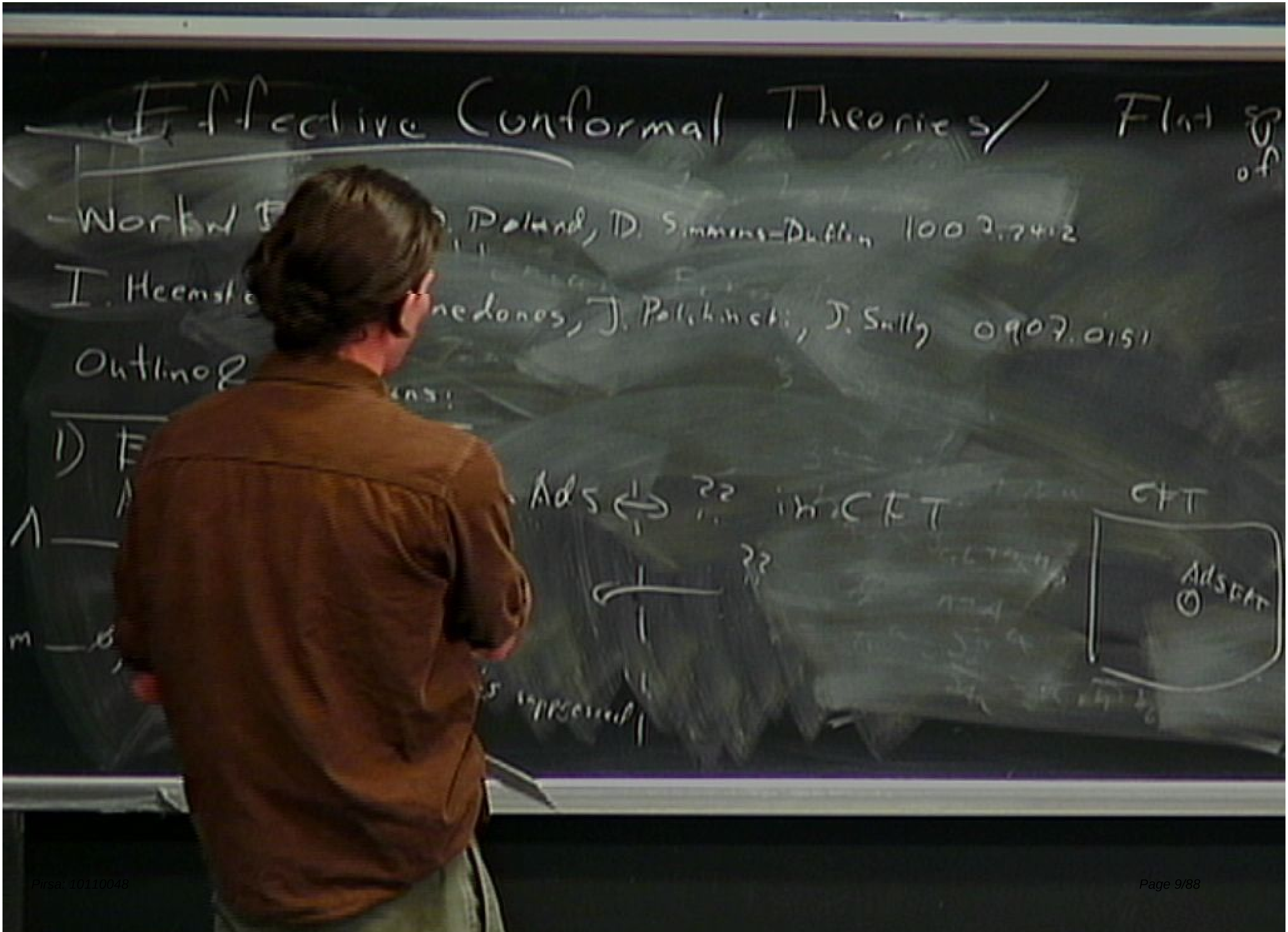
AdS eff. theory

Λ ___

$\mathcal{L} \supset \frac{\partial\phi\ldots}{\Lambda^p}$

m ___ 0, hier, ..

Higher-dim ind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CFT
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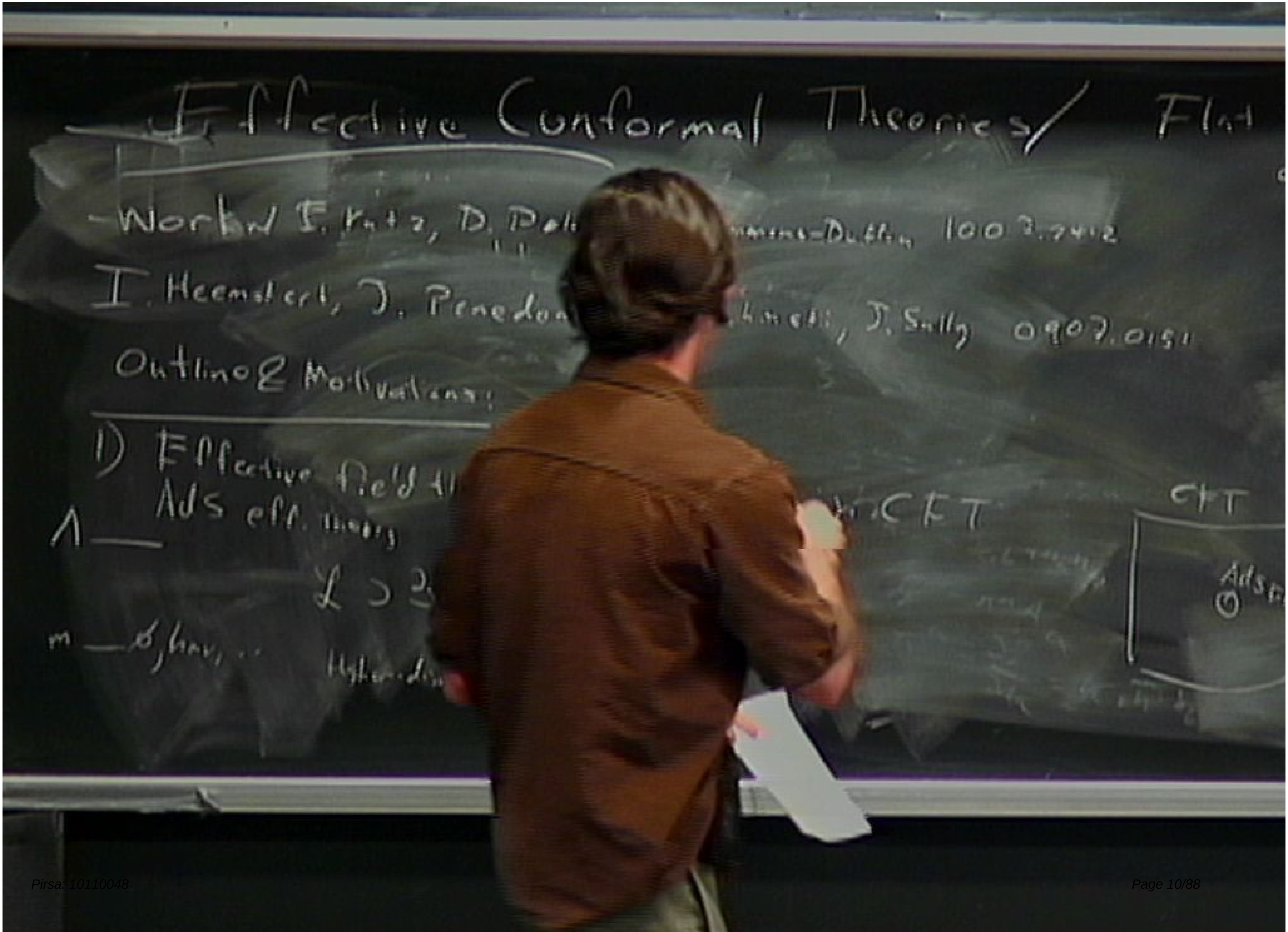

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I. Heemskerk, J. Penedon… …chinski, J. Sully 0907.0151

Outline & Motivations:

1) Effective field th… …CFT

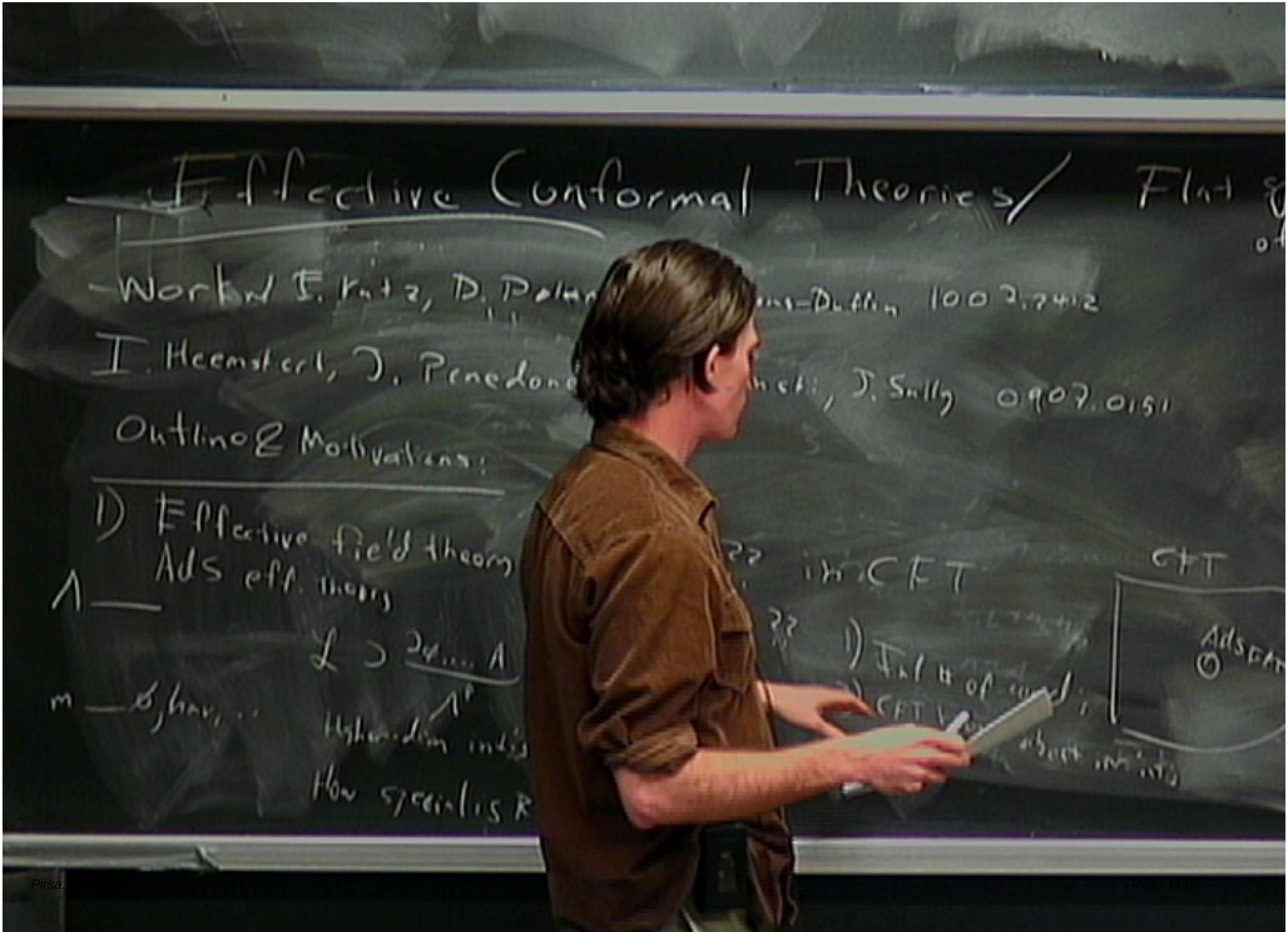
AdS eff. theory

Λ ___

$\mathcal{L} \supset$ …

m ___ ϕ, hav, ..

Higher-dim…

Effective Conformal Theories/ Flat
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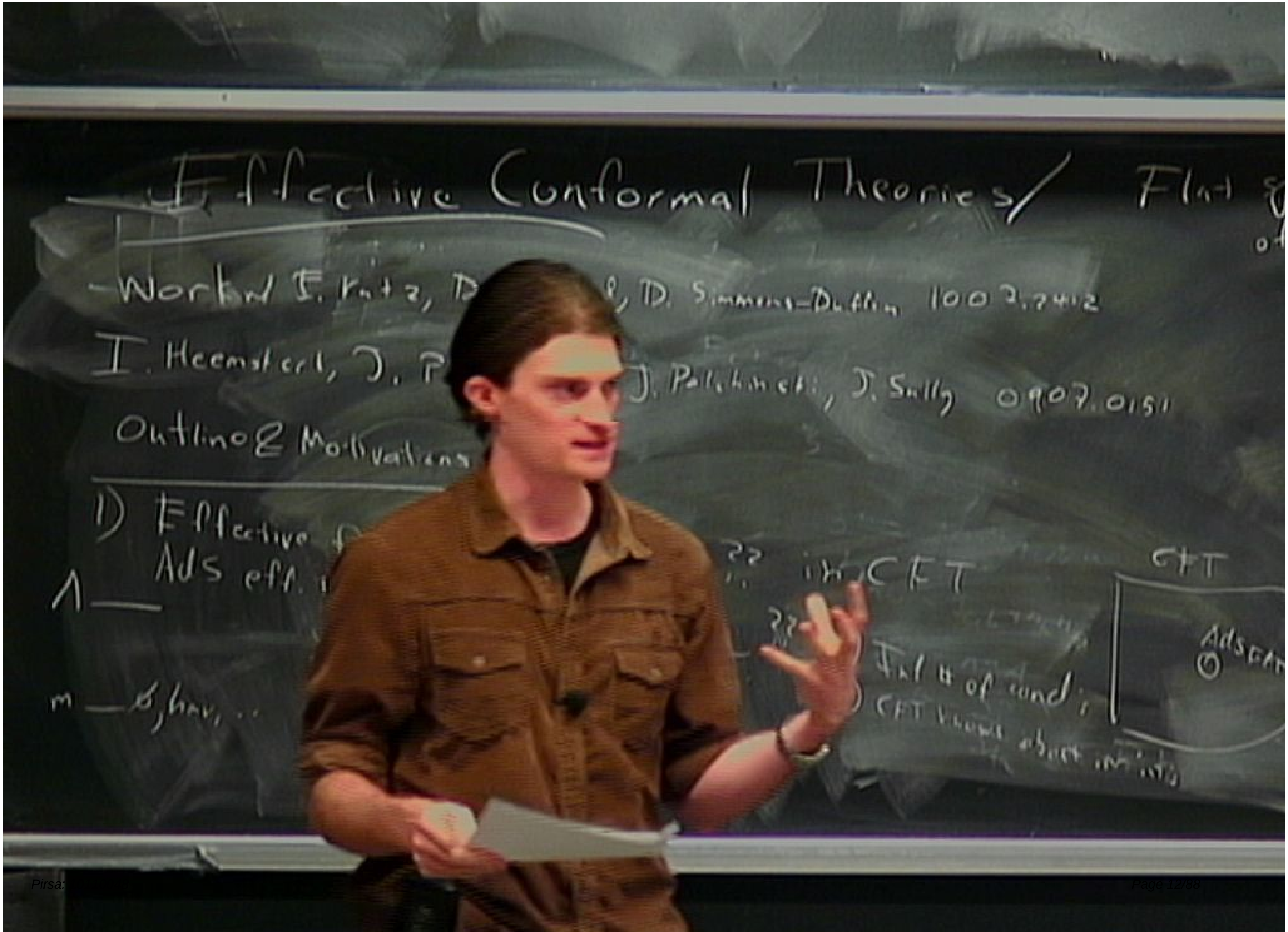
-Duffin 1007.2412
I. Heemskerk, J. Penedones
J. Sully 0907.0151
Outline & Motivations:
1) Effective field theory
AdS eff. theory
?? in CFT
Λ
??
1)
2) CFT
CFT
AdS
m
Higher-dim
How special is R
infinity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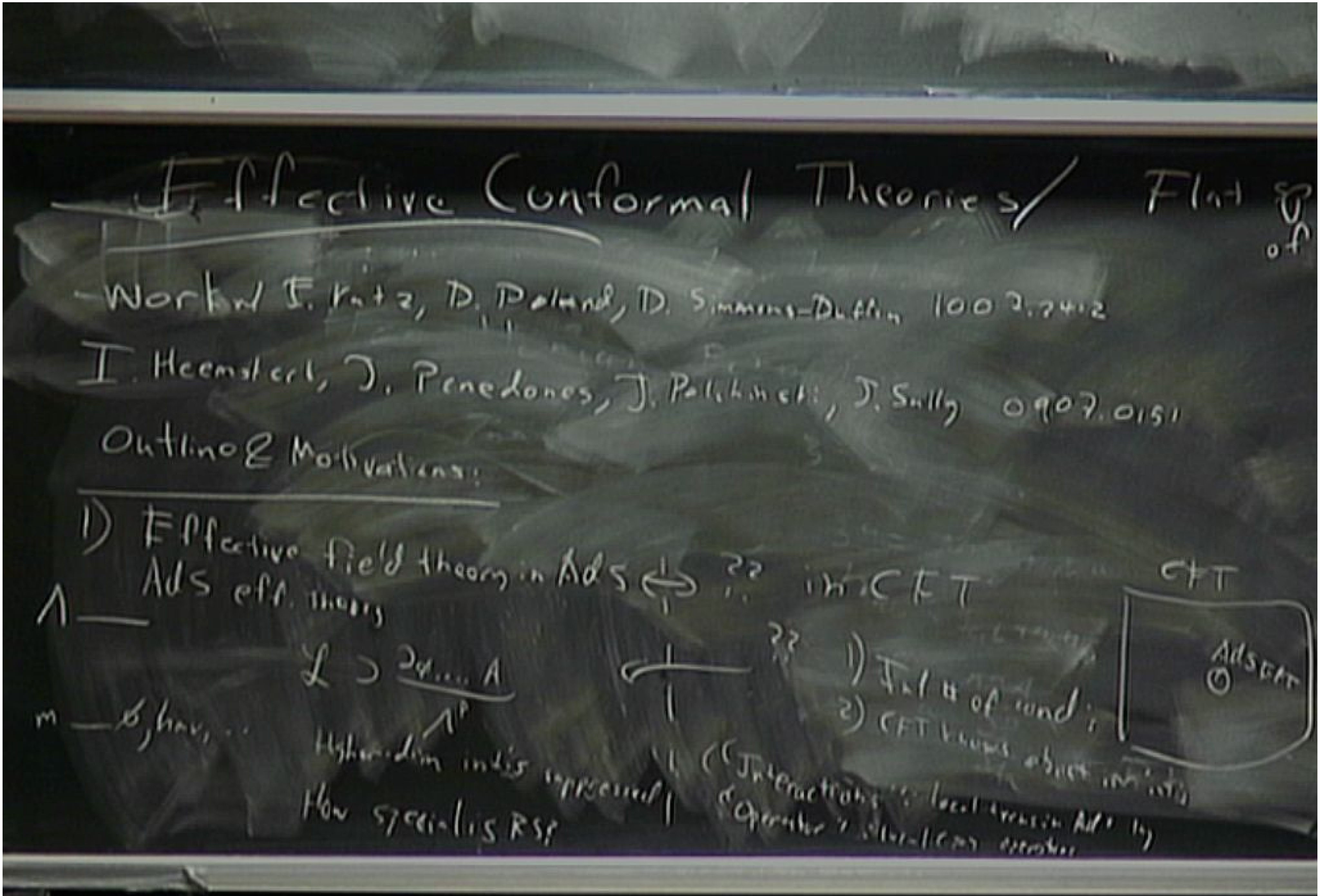
Outline & Motivations
1) Effective f
AdS eff.
?? in CFT
??
Λ
m g, hav, ..
fnd # of cond;
CFT knows about infinity
CFT
AdS



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Outline & Motivations:

1) Effective field theory in AdS ⟺ ?? in CFT
AdS eff. theory

Λ —

m — 0, heavy, ...

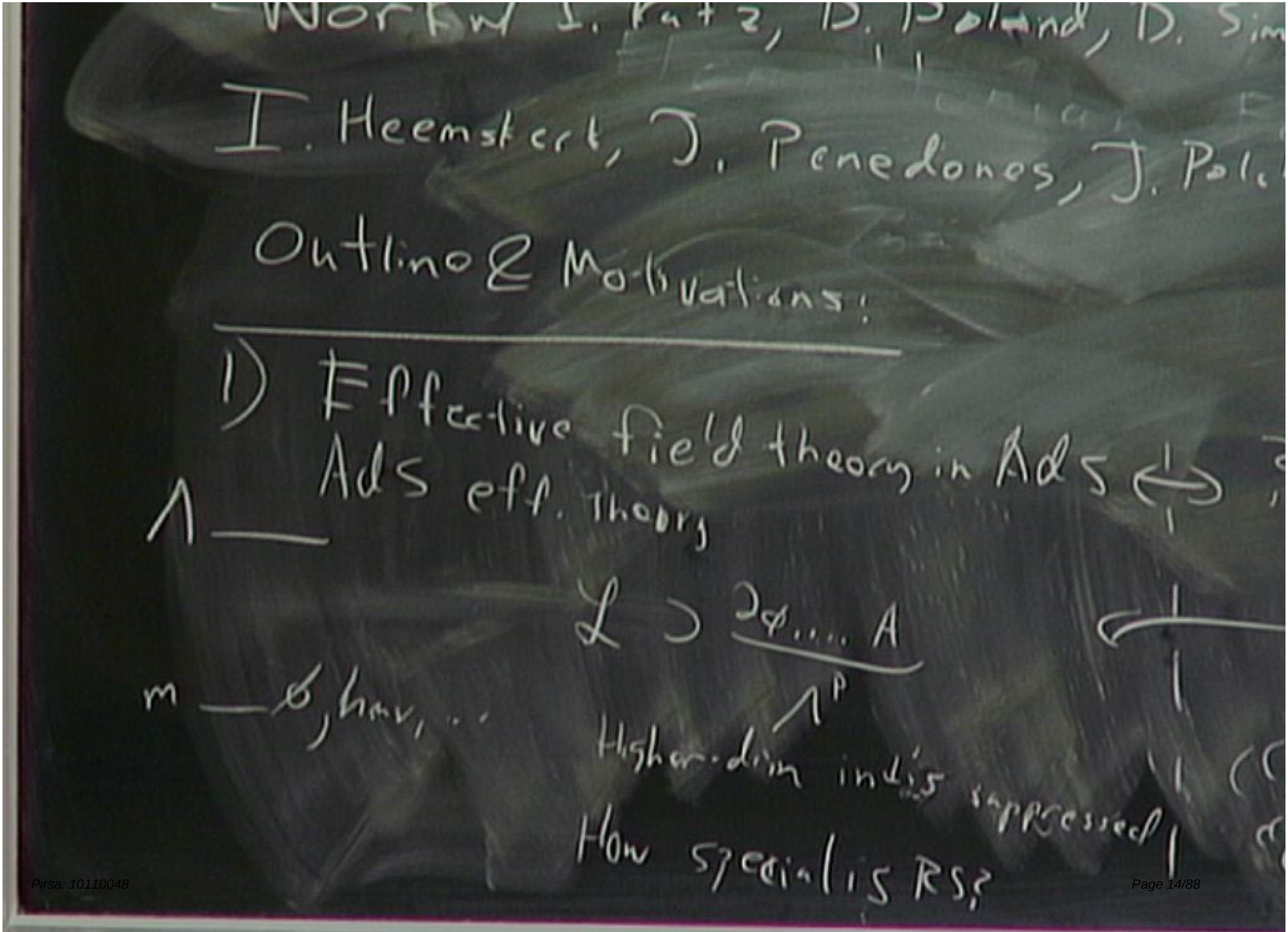
$\mathcal{L} \supset \frac{\partial\phi \ldots A}{\Lambda^p}$

Higher-dim ints suppressed

How special is RS?

??
1) Small # of cond.;
2) CFT knows about [illegible]

"Interactions" ~ local [illegible] in AdS [illegible]
& Operator [illegible] operator



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I. Heemskerk, J. Penedones, J. Pol

Outline & Motivations:

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1) Effective field theory in AdS ⟺

AdS eff. theory

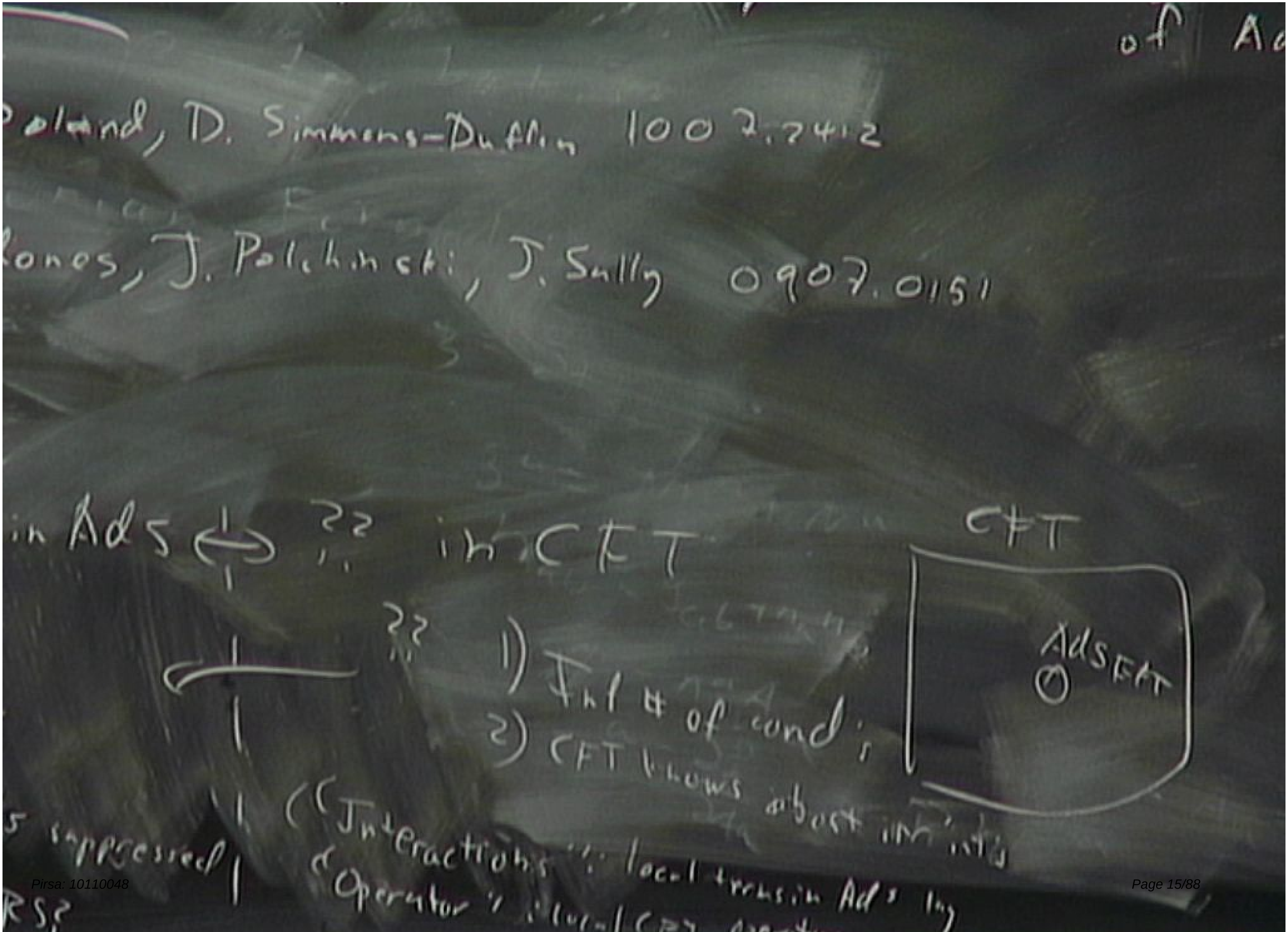
Λ ___

$$\mathcal{L} \supset \frac{\partial\phi \dots A}{\Lambda^p}$$

m ___ $\phi, h_{\mu\nu}, \dots$

Higher-dim int's suppressed

How special is RS?



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in AdS $\not\Leftrightarrow$?? in CFT

$\not\Leftarrow$??

1) Inf # of cond;

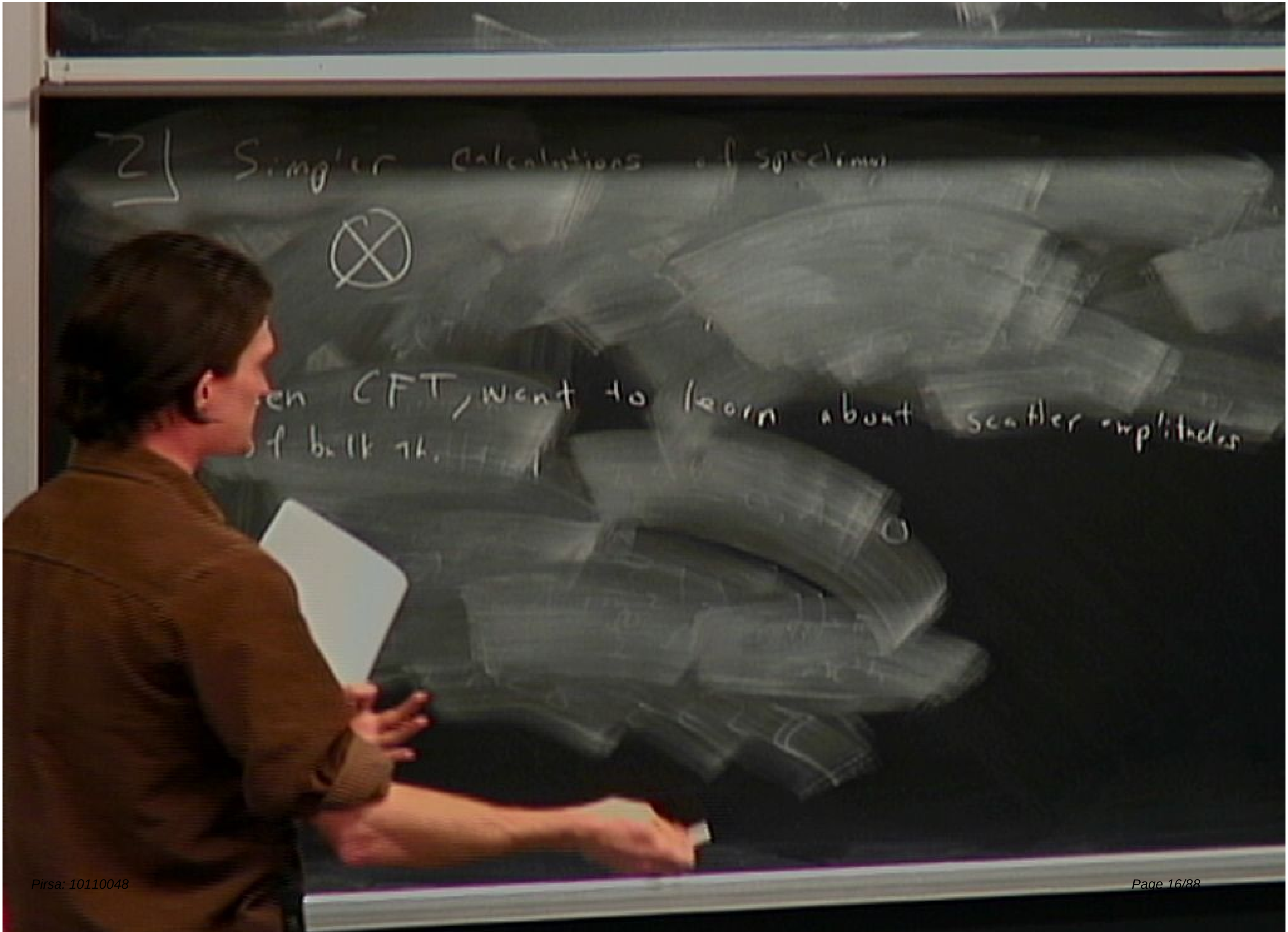
2) CFT knows about infinity

5 suppressed

RS?

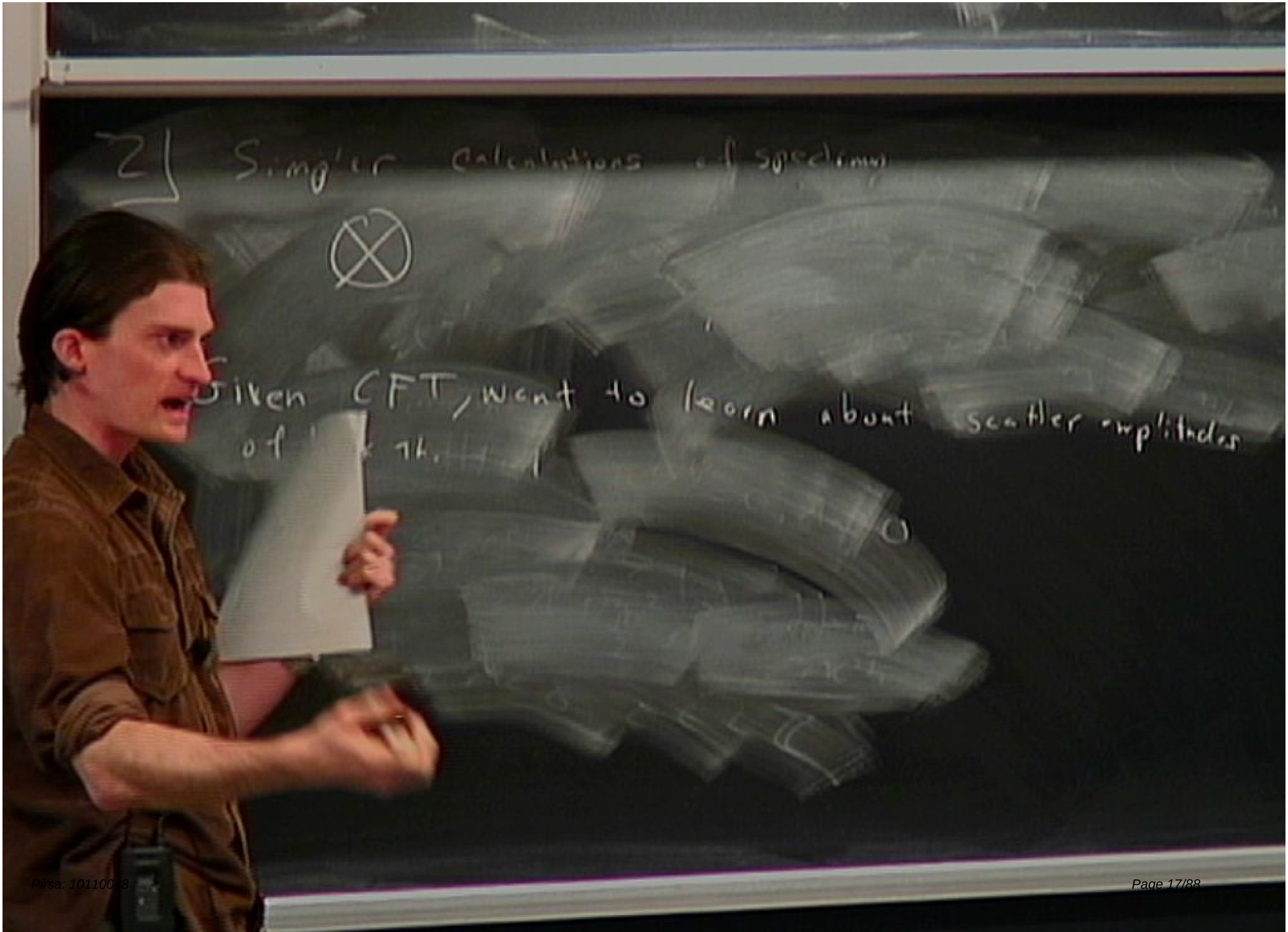
((Interactions'' : local trans in AdS by

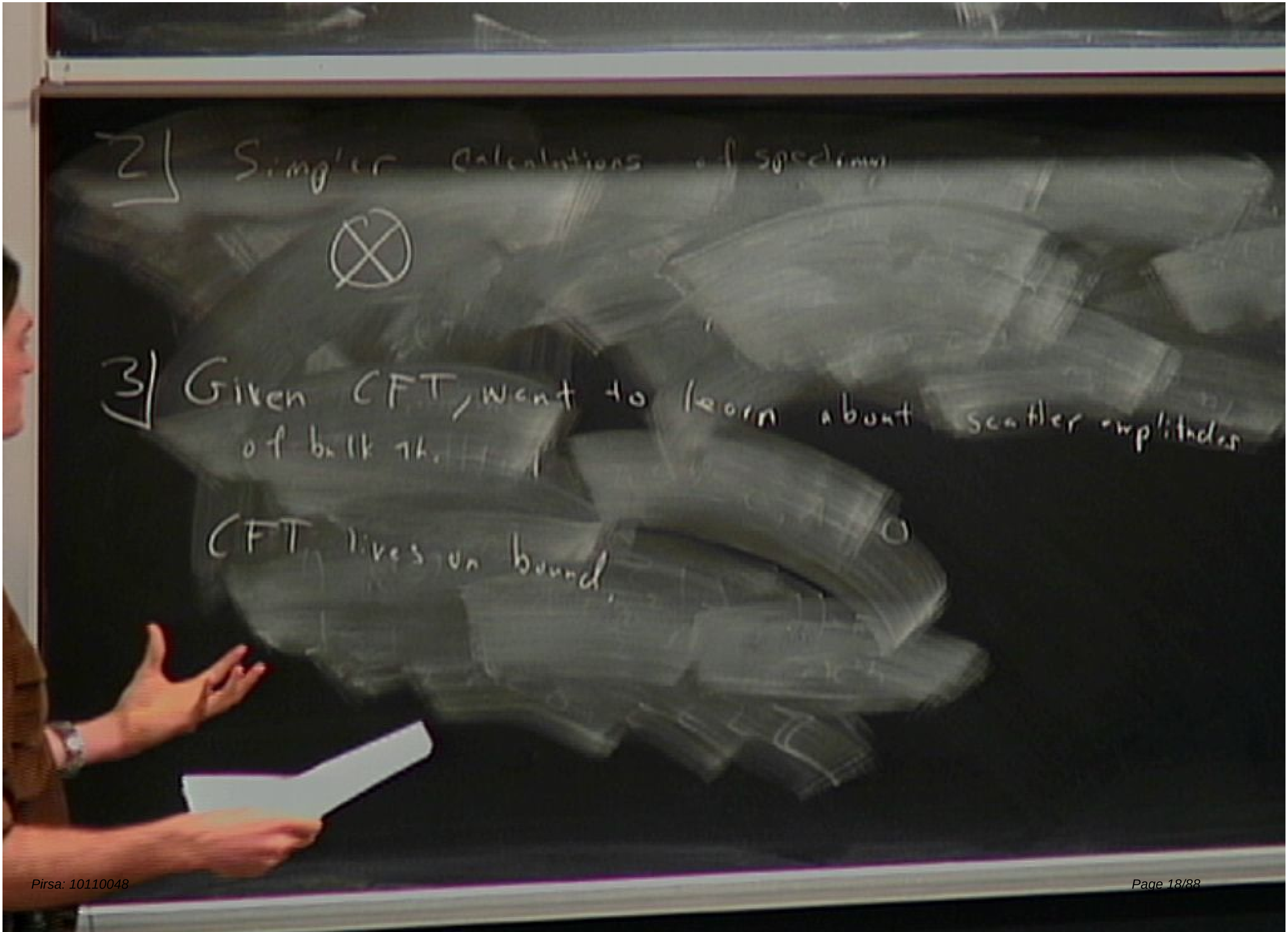
((Operator '' : local CFT operator



2] Simpler calculations of spectrum

en CFT, want to learn about scatter amplitudes
f bulk th.


2] Simpler calculations of spectrum
Given CFT, want to learn about scatter amplitudes
of [illegible] th.



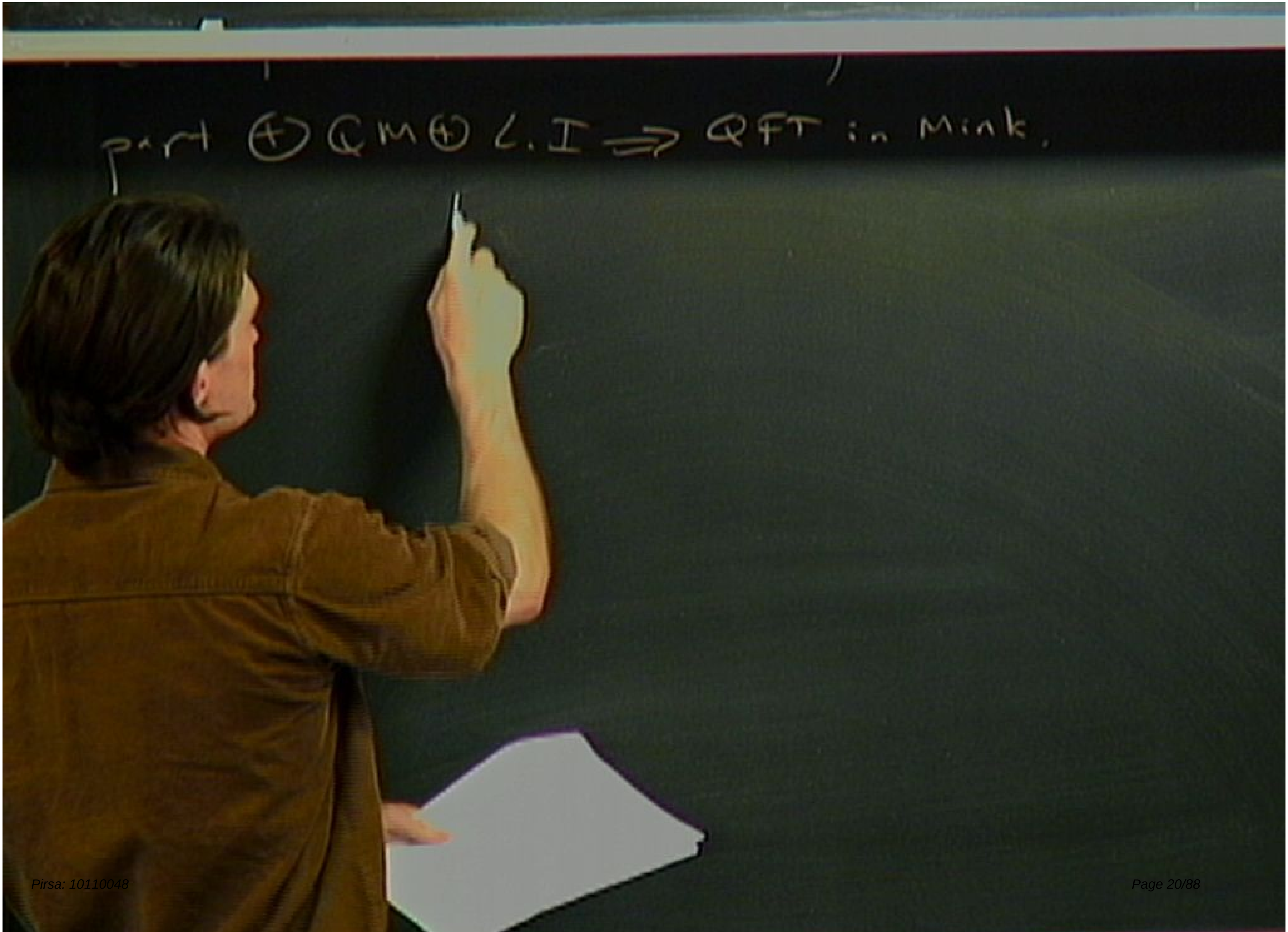
2] Simpler calculations of spectrum

$\otimes$

3] Given CFT, want to learn about scatter amplitudes of bulk th.

CFT lives on bound.

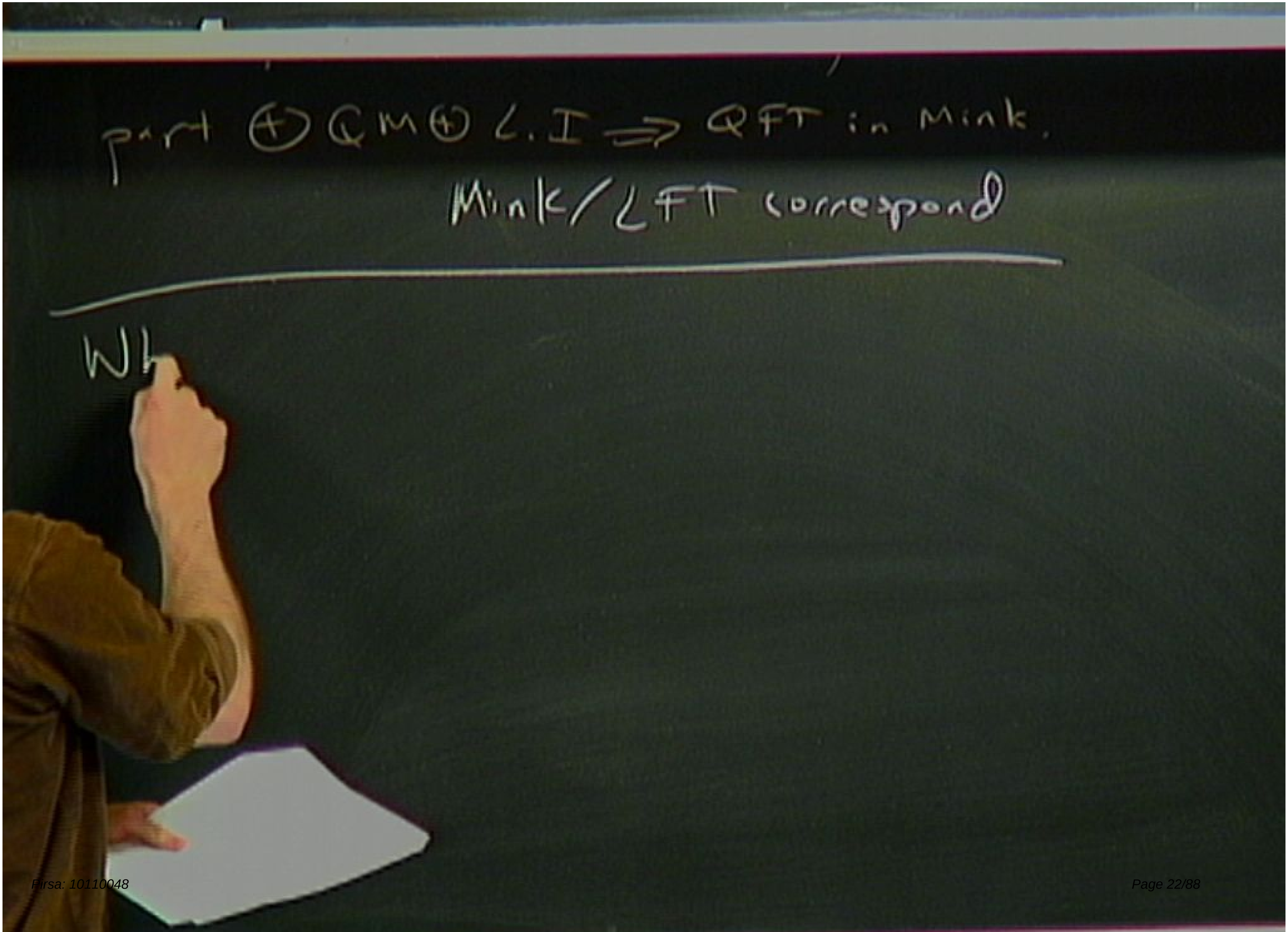
Develop [illegible]



part ⊕ QM ⊕ L.I $\Longrightarrow$ QFT in Mink.

part $\oplus$ QM $\oplus$ L.I $\Longrightarrow$ QFT in Mink.

Mink/LFT correspond



part ⊕ QM ⊕ L.I $\Rightarrow$ QFT in Mink.

Mink/LFT correspond

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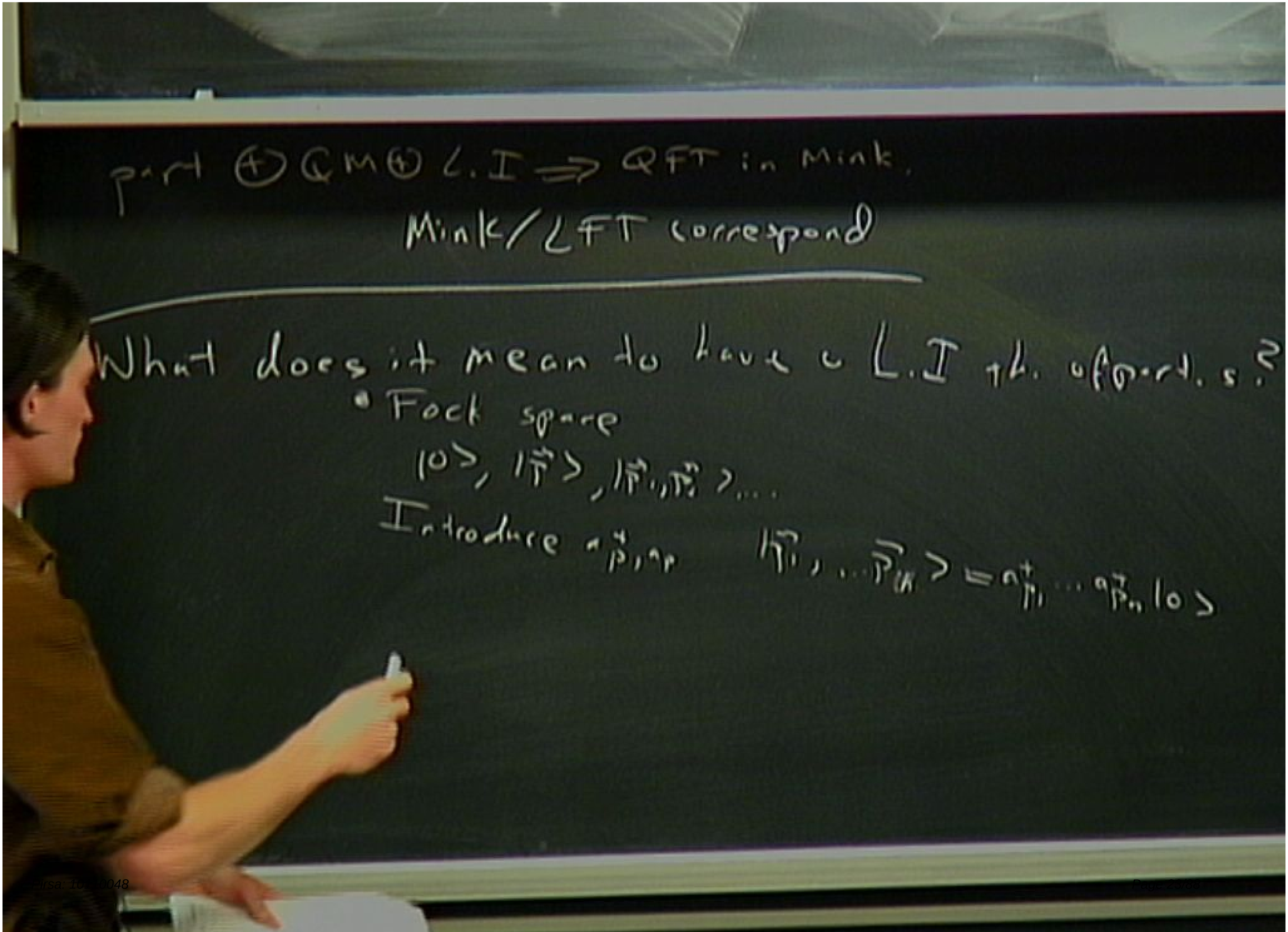


part ⊕ QM ⊕ L.I ⟹ QFT in Mink.

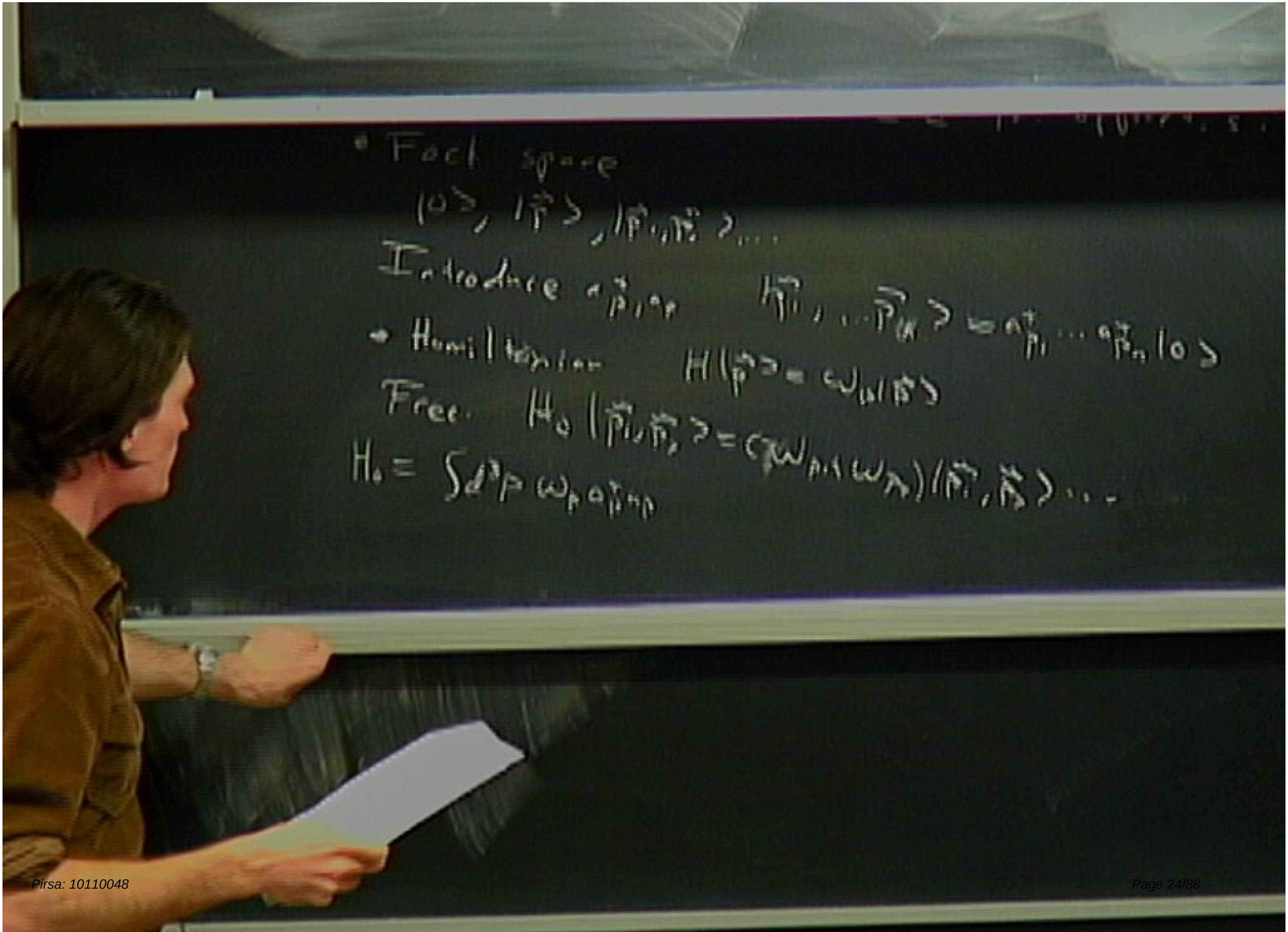
Mink/LFT correspond

What does it mean to have a L.I th. of part.s?

- Fock space

$|0\rangle, |\vec{p}\rangle, |\vec{p}_1, \vec{p}_2\rangle, \ldots$

Introduce $a^\dagger_{\vec{p}}, a_{\vec{p}}$ $\quad |\vec{p}_1, \ldots \vec{p}_n\rangle = a^\dagger_{\vec{p}_1} \cdots a^\dagger_{\vec{p}_n} |0\rangle$



- Fock space

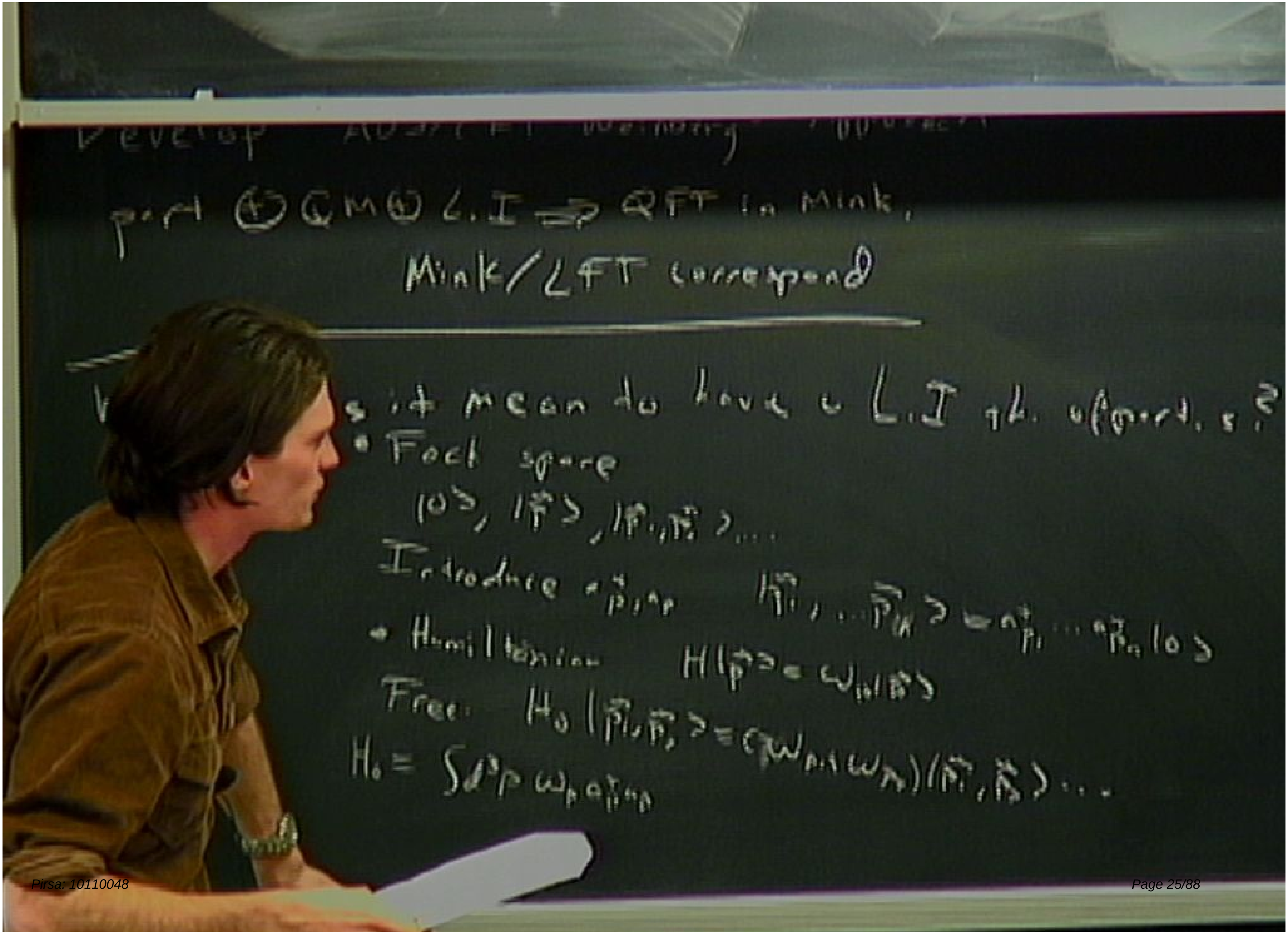
$|0\rangle, |\vec{p}\rangle, |\vec{p}_1, \vec{p}_2\rangle, \dots$

Introduce $a_{\vec{p}}, a^\dagger_{\vec{p}}$ $\quad |\vec{p}_1, \dots \vec{p}_n\rangle = a^\dagger_{\vec{p}_1} \cdots a^\dagger_{\vec{p}_n} |0\rangle$

- Hamiltonian $\quad H|\vec{p}\rangle = \omega_p |\vec{p}\rangle$

Free. $\quad H_0 |\vec{p}_1, \vec{p}_2\rangle = (\omega_{p_1} + \omega_{p_2}) |\vec{p}_1, \vec{p}_2\rangle \dots$

$H_0 = \int d^3p \, \omega_p a^\dagger_{\vec{p}} a_{\vec{p}}$



Develop [illegible]

pert ⊕ QM ⊕ L.I ⇒ QFT in Mink.

Mink/LFT correspond

[illegible]s it mean to have a L.I th. of particles?

- Fock space

  $|0\rangle, |\vec{p}\rangle, |\vec{p}_1, \vec{p}_2\rangle, \ldots$

  Introduce $a^\dagger_{\vec{p}}, a_{\vec{p}}$ $\quad |\vec{p}_1, \ldots \vec{p}_n\rangle = a^\dagger_{\vec{p}_1} \cdots a^\dagger_{\vec{p}_n}|0\rangle$

- Hamiltonian $\quad H|\vec{p}\rangle = \omega_{|\vec{p}|}|\vec{p}\rangle$

  Free: $\quad H_0|\vec{p}_1, \vec{p}_2\rangle = (\omega_{p_1} + \omega_{p_2})|\vec{p}_1, \vec{p}_2\rangle \ldots$

$H_0 = \int d^3p \, \omega_p a^\dagger_{\vec{p}} a_{\vec{p}}$

part ⊕ QM ⊕ L.I $\Rightarrow$ QFT in Mink.

Mink/LFT correspond

What does it mean to have a L.I th. of part.s?

- Fock space

  $|0\rangle, |\vec{p}\rangle, |\vec{p}_1, \vec{p}_2\rangle, \ldots$

Introduce $a^\dagger_{\vec{p}}, a_{\vec{p}}$ $\quad |\vec{p}_1, \ldots \vec{p}_n\rangle = a^\dagger_{\vec{p}_1} \cdots a^\dagger_{\vec{p}_n} |0\rangle$

- Hamiltonian $\quad H|\vec{p}\rangle = \omega_{|p|} |\vec{p}\rangle$

Free. $\quad H_0 |\vec{p}_1, \vec{p}_2\rangle = (\omega_{p_1} + \omega_{p_2}) |\vec{p}_1, \vec{p}_2\rangle \ldots$

$$H_0 = \int d^3p \, \omega_p a^\dagger_{\vec{p}} a_{\vec{p}}$$

part $\oplus$ QM $\oplus$ L.I $\Longrightarrow$ QFT in Mink.

Mink/LFT correspond

What does it mean to have a L.I th. of part.s?

- Fock space

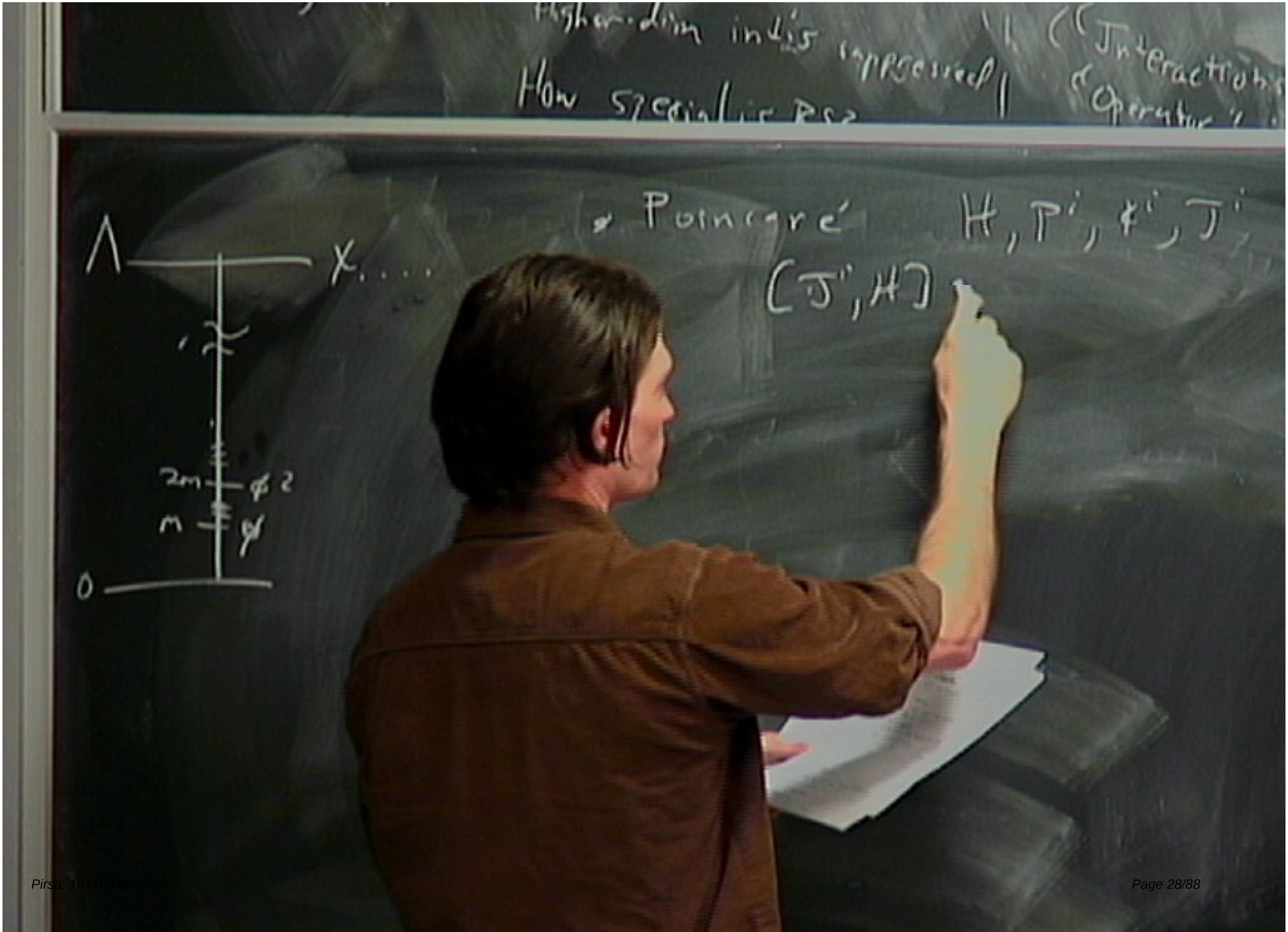
$|0\rangle, |\vec p\rangle, |\vec p_1, \vec p_2\rangle, \dots$

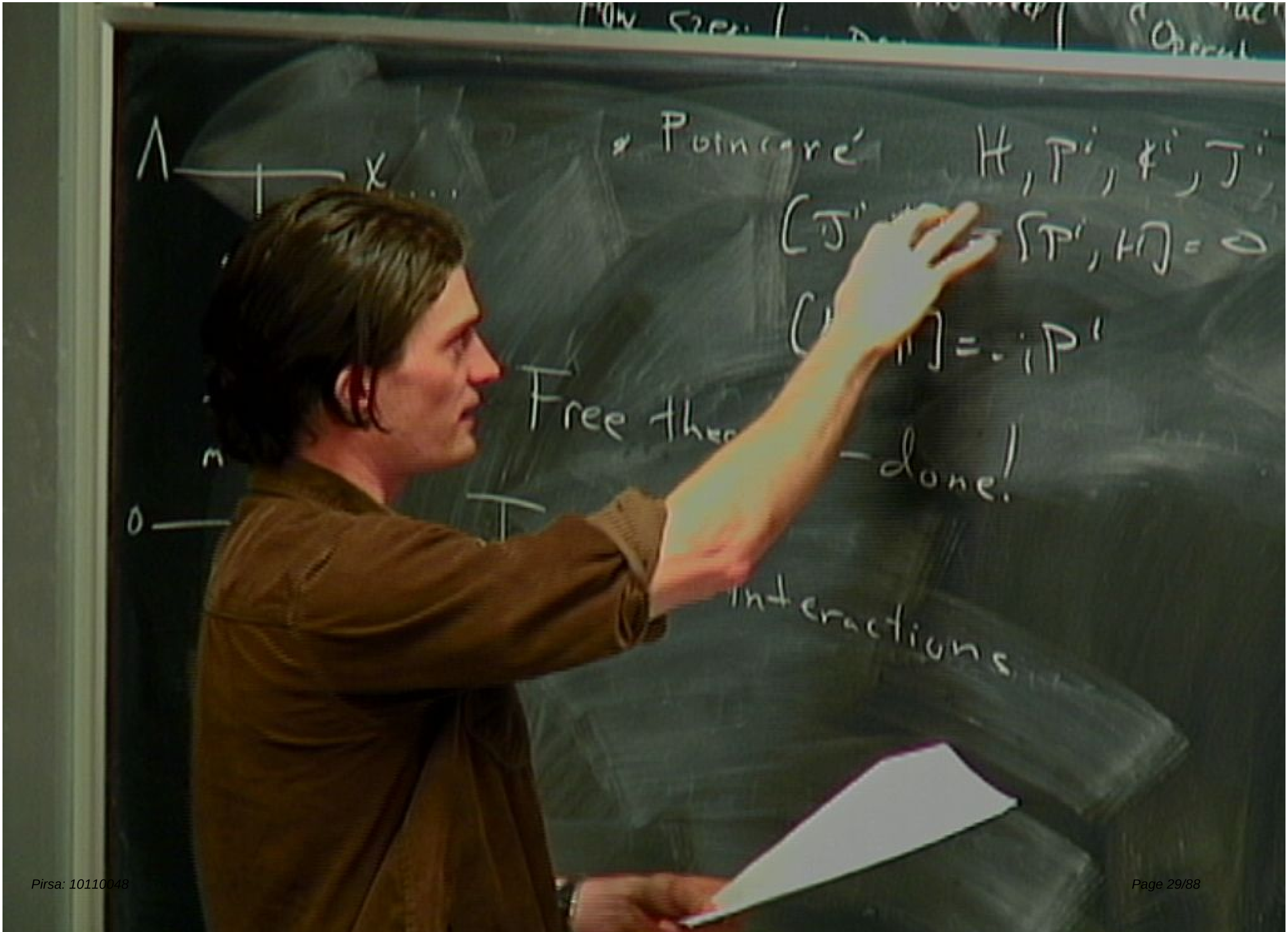
Introduce $a^\dagger_{\vec p}, a_{\vec p}$ $\quad |\vec p_1, \dots \vec p_n\rangle = a^\dagger_{\vec p_1} \cdots a^\dagger_{\vec p_n}|0\rangle$

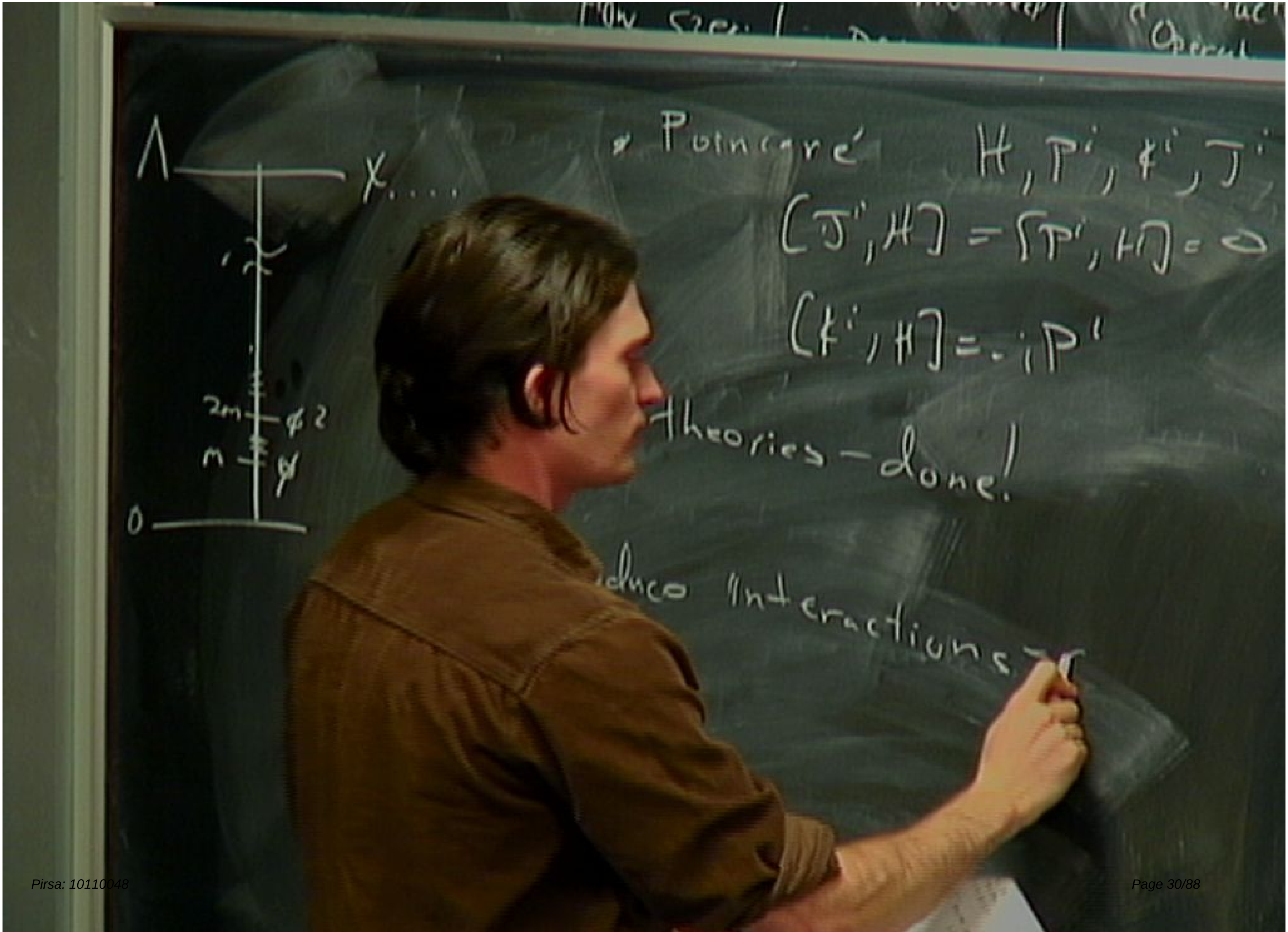
- Hamiltonian $\quad H|\vec p\rangle = \omega_p|\vec p\rangle$

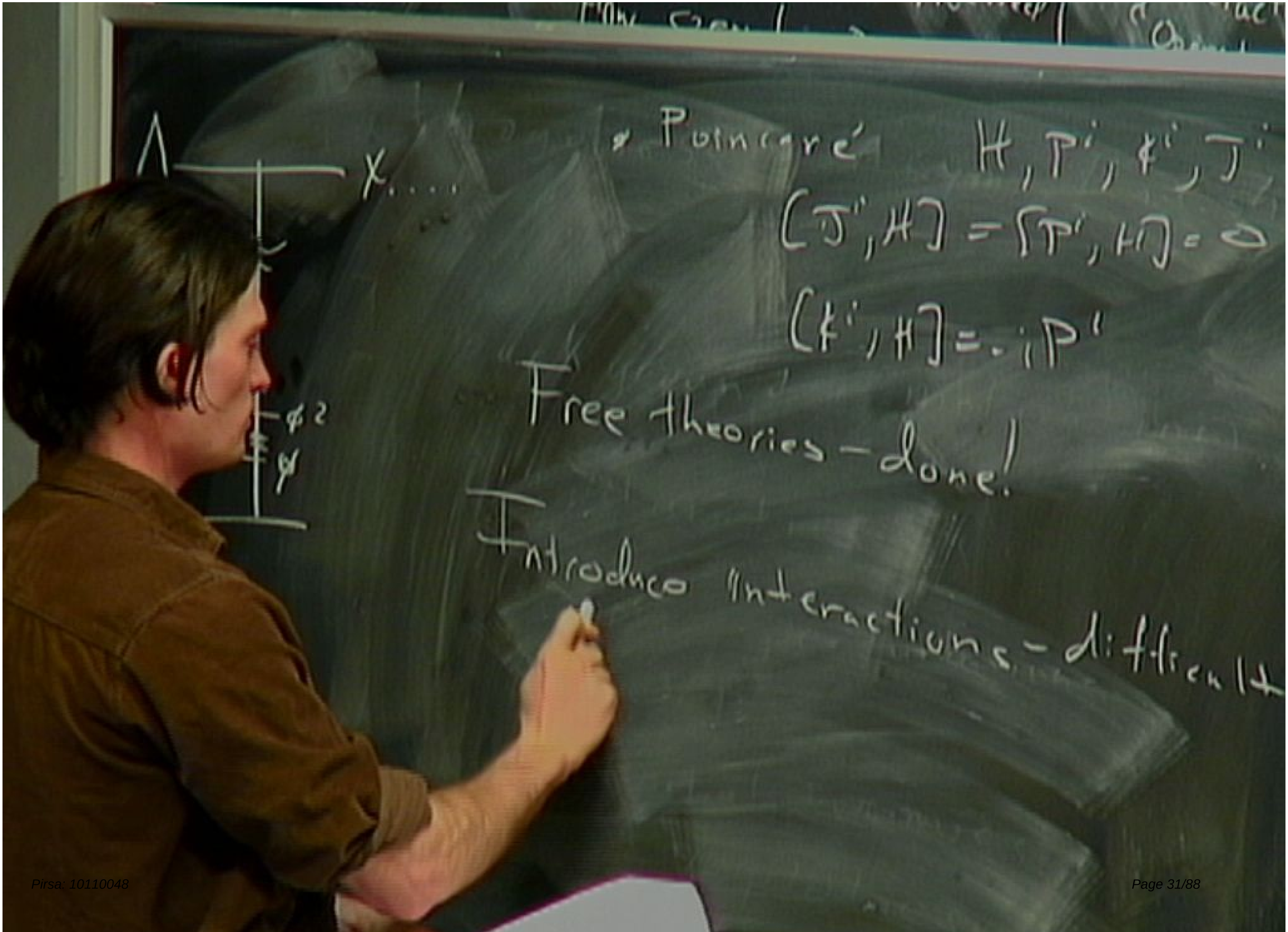
Free. $\quad H_0|\vec p_1, \vec p_2\rangle = (\omega_{p_1} + \omega_{p_2})|\vec p_1, \vec p_2\rangle \dots$

$H_0 = \int d^3p\, \omega_p a^\dagger_p a_p$


higher-dim ind's
unpreserved
How special is RS?
Interactions
Operators
* Poincaré H, P^i, K^i, J^i
$[J^i, H]$
Λ
X.....
2m — ϕ 2
m — ϕ
0


Λ
x
m
0
Poincaré
H, P^i, K^i, J^i
[J^i, ... = [P^i, H] = 0
[... H] = iP^i
Free theo... — done!
Interactions


Λ
X....
2m φ2
m φ
0
* Poincaré H, P^i, K^i, J^i
[J^i, H] = [P^i, H] = 0
[K^i, H] = -iP^i
theories - done!
duce interactions



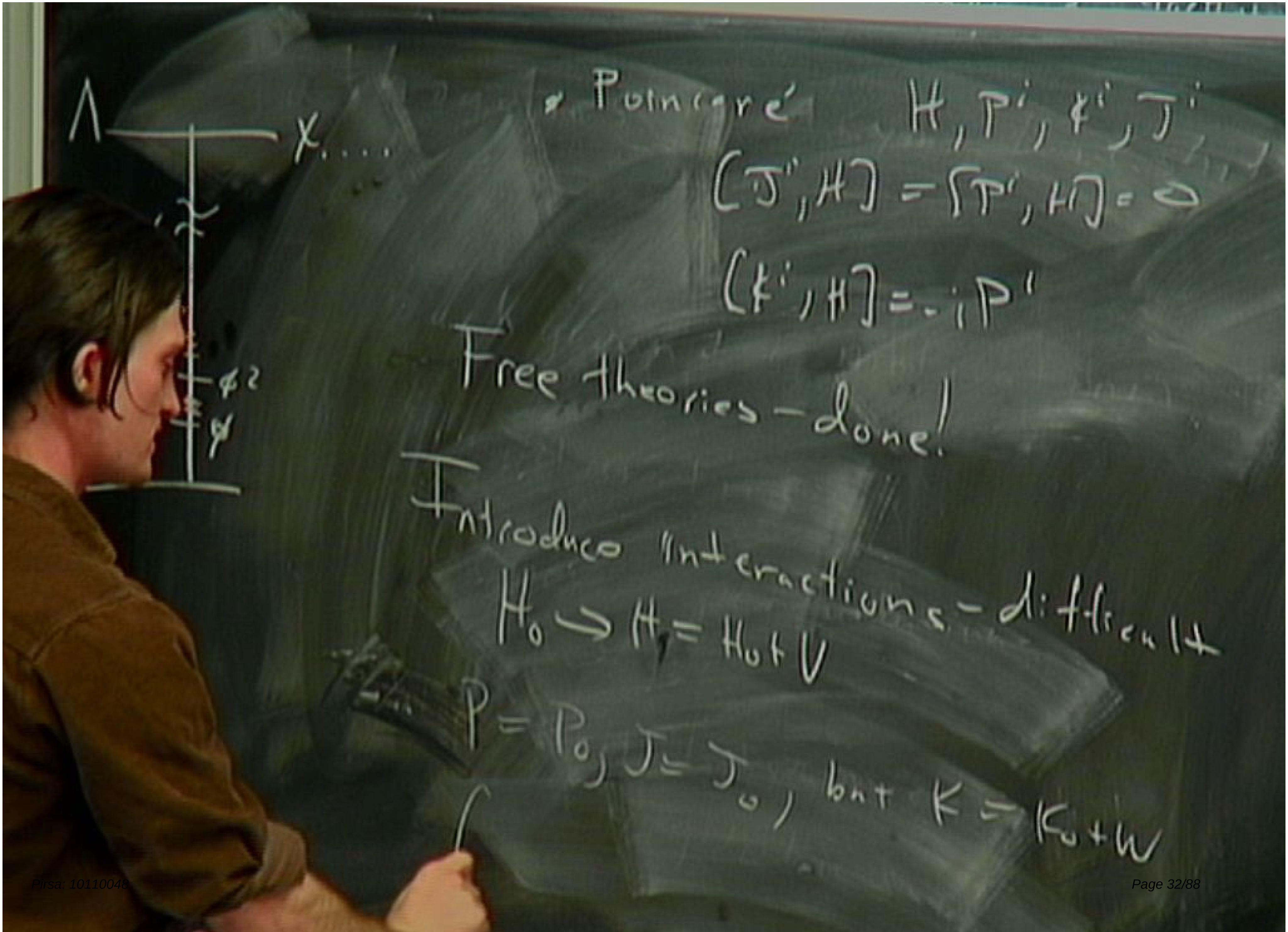
• Poincaré H, P^i, K^i, J^i

$[J^i, H] = [P^i, H] = 0$

$[K^i, H] = . \, i P^i$

Free theories - done!

Introduce interactions - difficult



* Poincaré H, P^i, K^i, J^i

$[J^i, H] = [P^i, H] = 0$

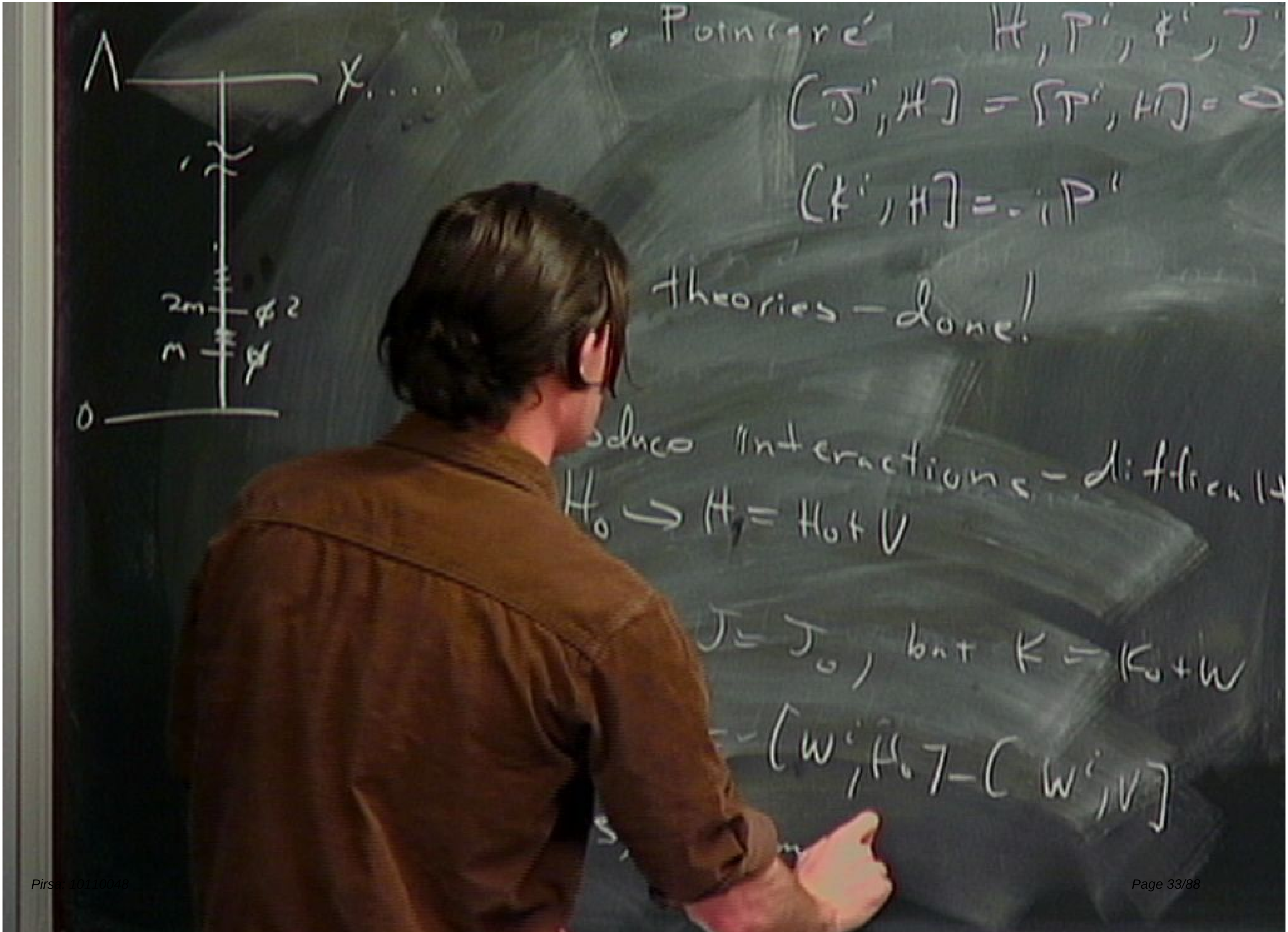
$[K^i, H] = -iP^i$

Free theories – done!

Introduce interactions – difficult

$H_0 \to H = H_0 + V$

$P = P_0$, $J = J_0$, but $K = K_0 + W$



Poincaré H, P^i, K^i, J^i

$$[J^i, H] = [P^i, H] = 0$$

$$[K^i, H] = -iP^i$$

theories - done!

oduce interactions - difficult

$$H_0 \to H = H_0 + V$$

$$J = J_0, \text{ but } K = K_0 + W$$

$$= -[W^i, H_0] - [W^i, V]$$

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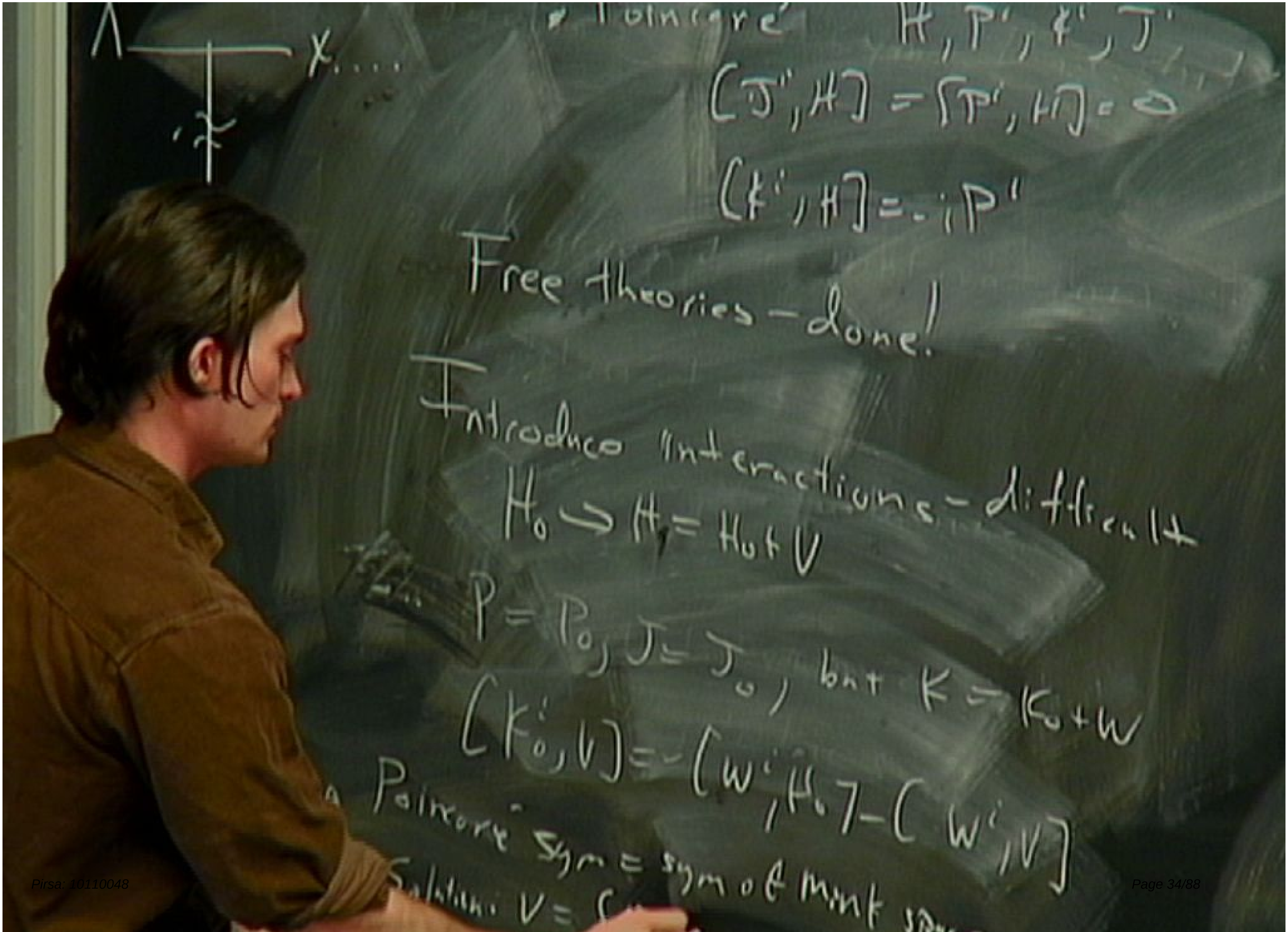


Poincaré H, P^i, K^i, J^i

$$[J^i, H] = [P^i, H] = 0$$

$$[K^i, H] = -iP^i$$

Free theories - done!

Introduce interactions - difficult

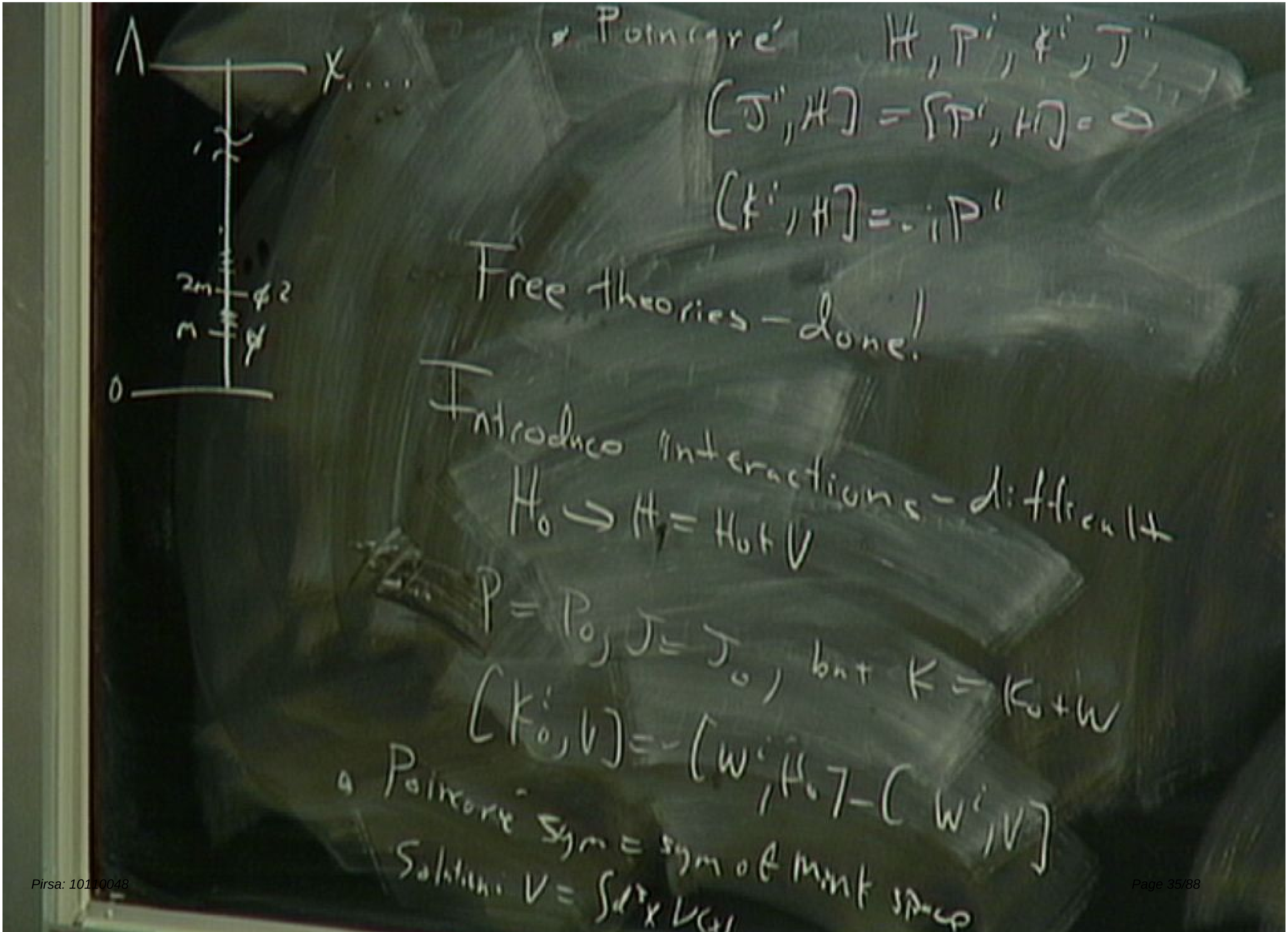
$$H_0 \to H = H_0 + V$$

$$P = P_0,\ J = J_0,\ \text{but } K = K_0 + W$$

$$[K_0^i, V] = -[W^i, H_0] - [W^i, V]$$

Poincaré sym = sym of Mink space

Solution: $V = \ldots$



• Poincaré H, P^i, K^i, J^i

$$[J^i, H] = [P^i, H] = 0$$

$$[K^i, H] = -iP^i$$

Free theories – done!

Introduce interactions – difficult

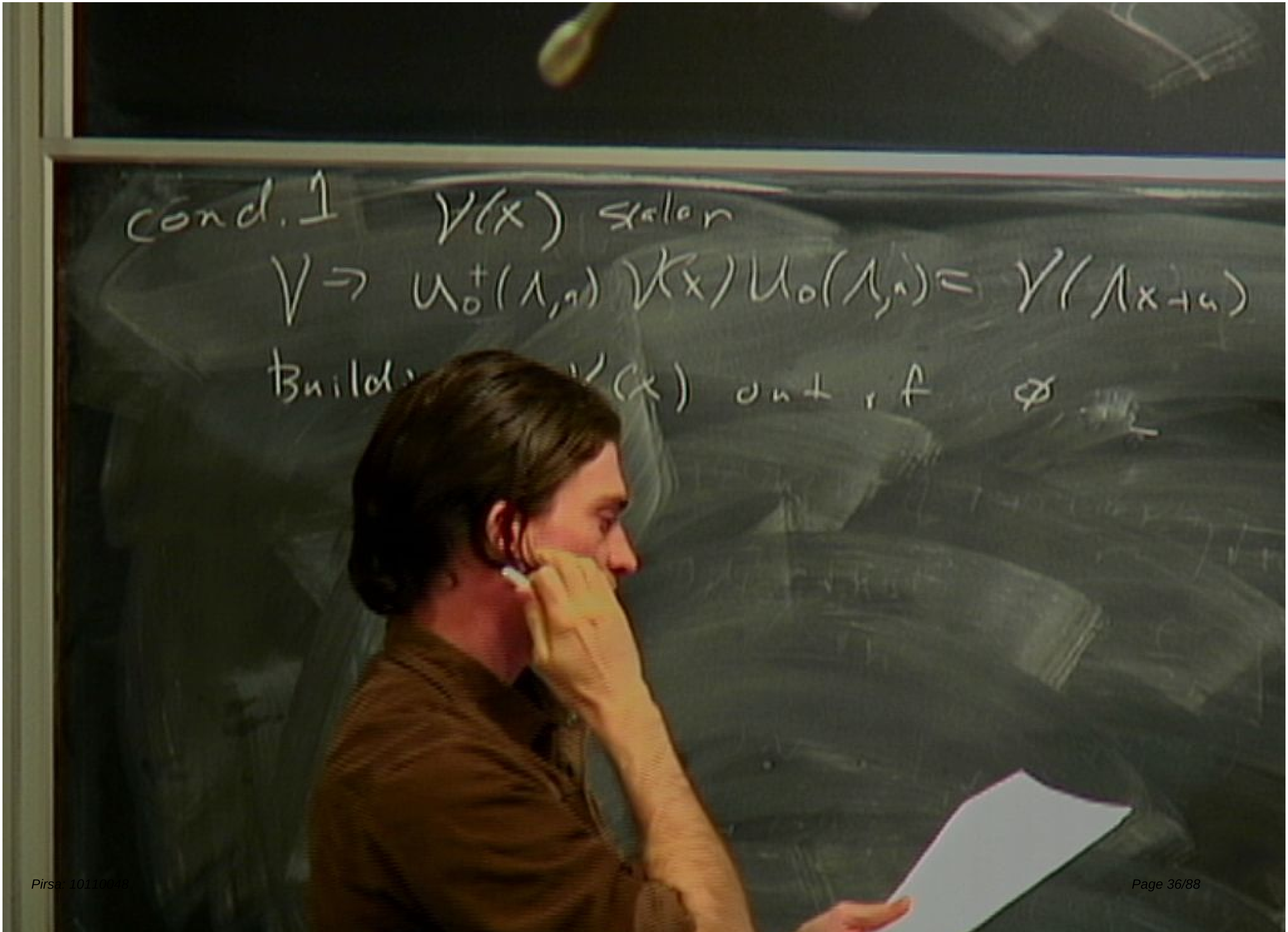
$$H_0 \to H = H_0 + V$$

$P = P_0, J = J_0$, but $K = K_0 + W$

$$[K_0^i, V] = -[W^i, H_0] - [W^i, V]$$

• Poincaré sym = sym of Mink space

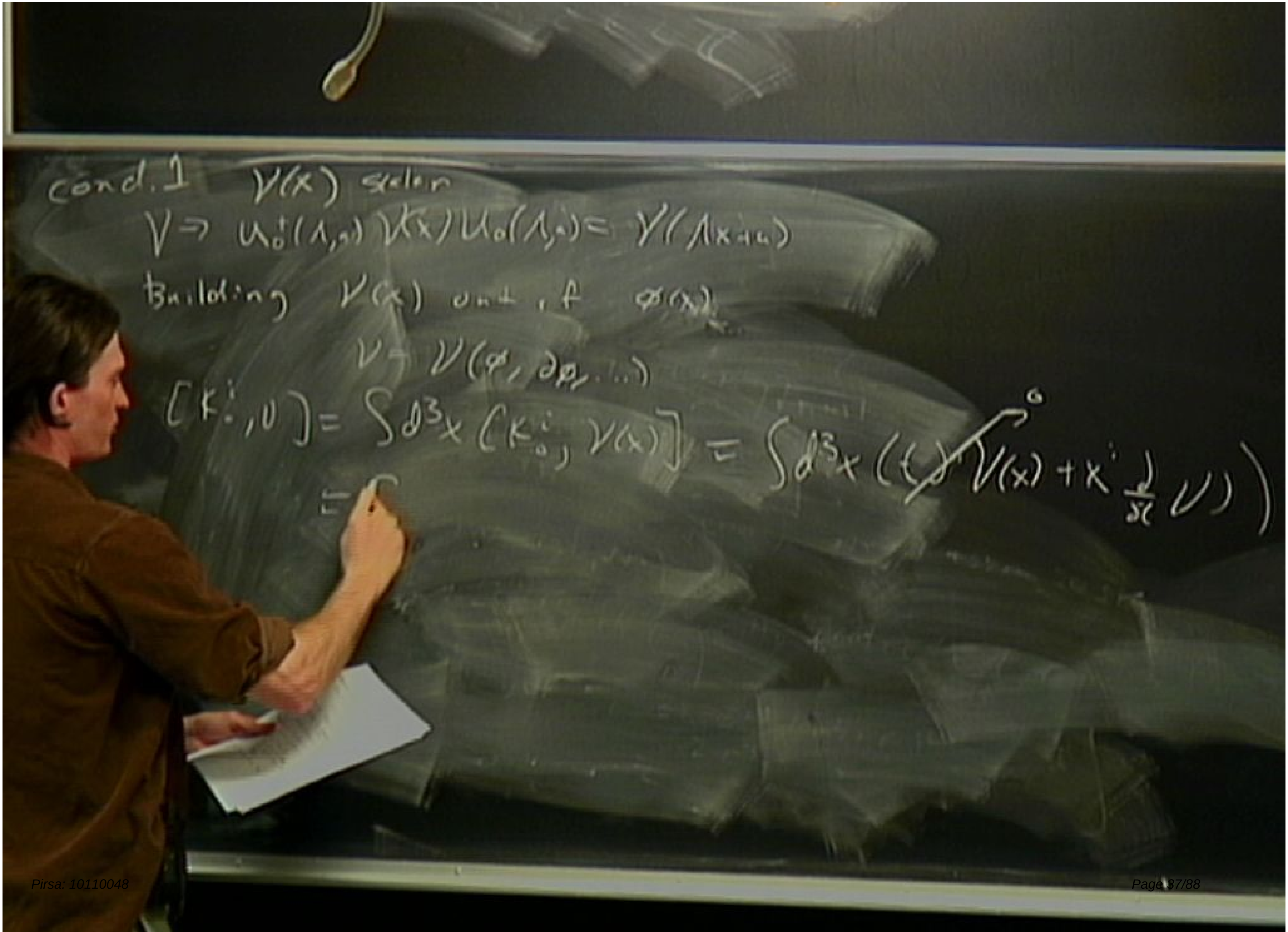
Solution: $V = \int d^3x\, V(x)$



cond. 1 $V(x)$ scalar

$$V \to U_0^{\dagger}(\Lambda,a)\, V(x)\, U_0(\Lambda,a) = V(\Lambda x + a)$$

Buildi[illegible] $V(x)$ out of ϕ.



cond. 1 $V(x)$ scalar

$V \Rightarrow U_0^\dagger(\Lambda,a)\, V(x)\, U_0(\Lambda,a) = V(\Lambda x + a)$

Building $V(x)$ out of $\phi(x)$

$V = V(\phi, \partial\phi, \ldots)$

$[K^i_0, V] = \int d^3x\, [K^i_0, V(x)] = \int d^3x \left(\cancelto{0}{(t\partial)} V(x) + x^i \frac{\partial}{\partial x} V) \right)$

$= \int$

cond. 1 $V(x)$ scalar

$$V \rightarrow U_0^\dagger(\Lambda, a)\, V(x)\, U_0(\Lambda, a) = V(\Lambda x + a)$$

Building $V(x)$ out of $\phi(x)$

$$V = V(\phi, \partial\phi, \ldots)$$

Then

$t=0$

$$[K_0^i, V] = \int d^3x\, [K_0^i, V(x)] = \int d^3x\, \Big(\cancelto{0}{t\,\partial} V(x) + x^i \frac{\partial}{\partial x} V \Big) \Big)$$

$$= [H^0, W^i] \qquad W^i = -\int d^3x\, x^i\, V(x)$$

cond. 1 $\mathcal{V}(x)$ scalar

$$\mathcal{V} \to U_0^\dagger(\Lambda,a)\,\mathcal{V}(x)\,U_0(\Lambda,a) = \mathcal{V}(\Lambda x + a)$$

Building $\mathcal{V}(x)$ out of $\phi(x)$

Then $\mathcal{V} = \mathcal{V}(\phi, \partial\phi, \dots)$

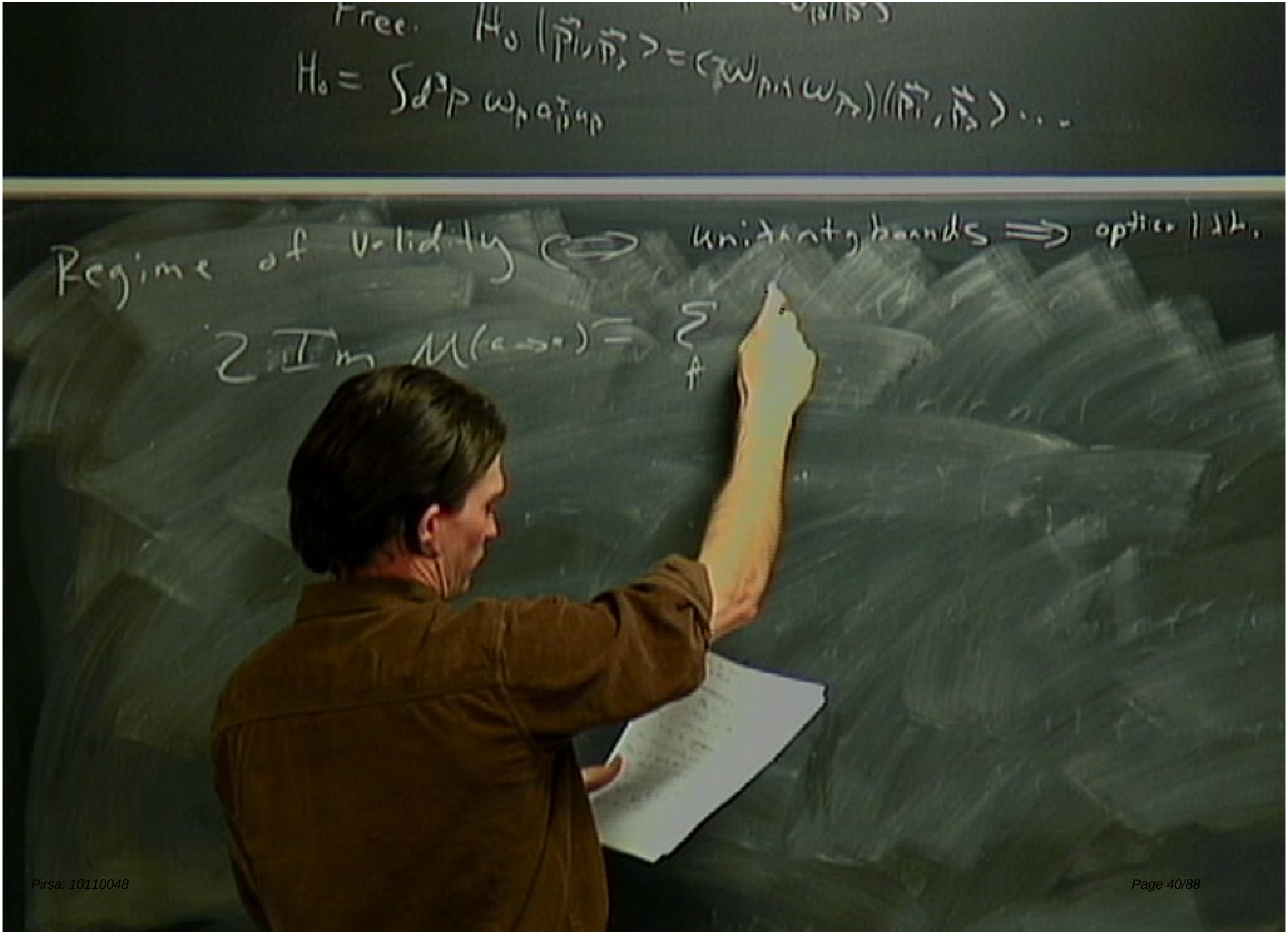
$t=0$

$$[K_0^i, V] = \int d^3x\,[K_0^i, \mathcal{V}(x)] = \int d^3x\,(\overset{0}{\cancel{t\partial}}\,\mathcal{V}(x) + x^i \tfrac{\partial}{\partial x}\mathcal{V})$$

$$= [H^0, W^i] \qquad W^i = -\int d^3x\, x^i\,\mathcal{V}(x)$$

cond 2: $[\mathcal{V}(x), \mathcal{V}(x')] = 0 \quad (x-x')^2 > 0$

$$[W^i, V] = \int d^3x\, d^3y\, x^i\,[\mathcal{V}(x), \mathcal{V}(y)] = 0$$



Free. $H_0 |\vec{p}_1, \vec{p}_2\rangle = (\omega_{p_1} + \omega_{p_2}) |\vec{p}_1, \vec{p}_2\rangle \ldots$

$H_0 = \int d^3p \, \omega_p a_p^\dagger a_p$

Regime of validity $\Longleftrightarrow$ unitarity bounds $\Longrightarrow$ optical th.

$2 \, \mathrm{Im} \, \mathcal{M}(a \to a) = \sum_A$

Free. $H_0 |\vec{p}_1, \vec{p}_2\rangle = (\omega_{p_1} + \omega_{p_2}) |\vec{p}_1, \vec{p}_2\rangle \ldots$

$$H_0 = \int d^3p \, \omega_p a_p^\dagger a_p$$

Regime of validity $\Longleftrightarrow$ unitarity bounds $\Longrightarrow$ optical th.

$$2 \, \mathrm{Im} \, \mathcal{M}(a \to a) = \sum_f \int d\Pi_f \, |\mathcal{M}(a \to f)|^2$$

Partial waves: $|a_{\ell m} - \frac{i}{2}| \leq \frac{1}{2}$

If

Regime of validity $\Longleftrightarrow$ unitarity bounds $\Longrightarrow$ optical thm.

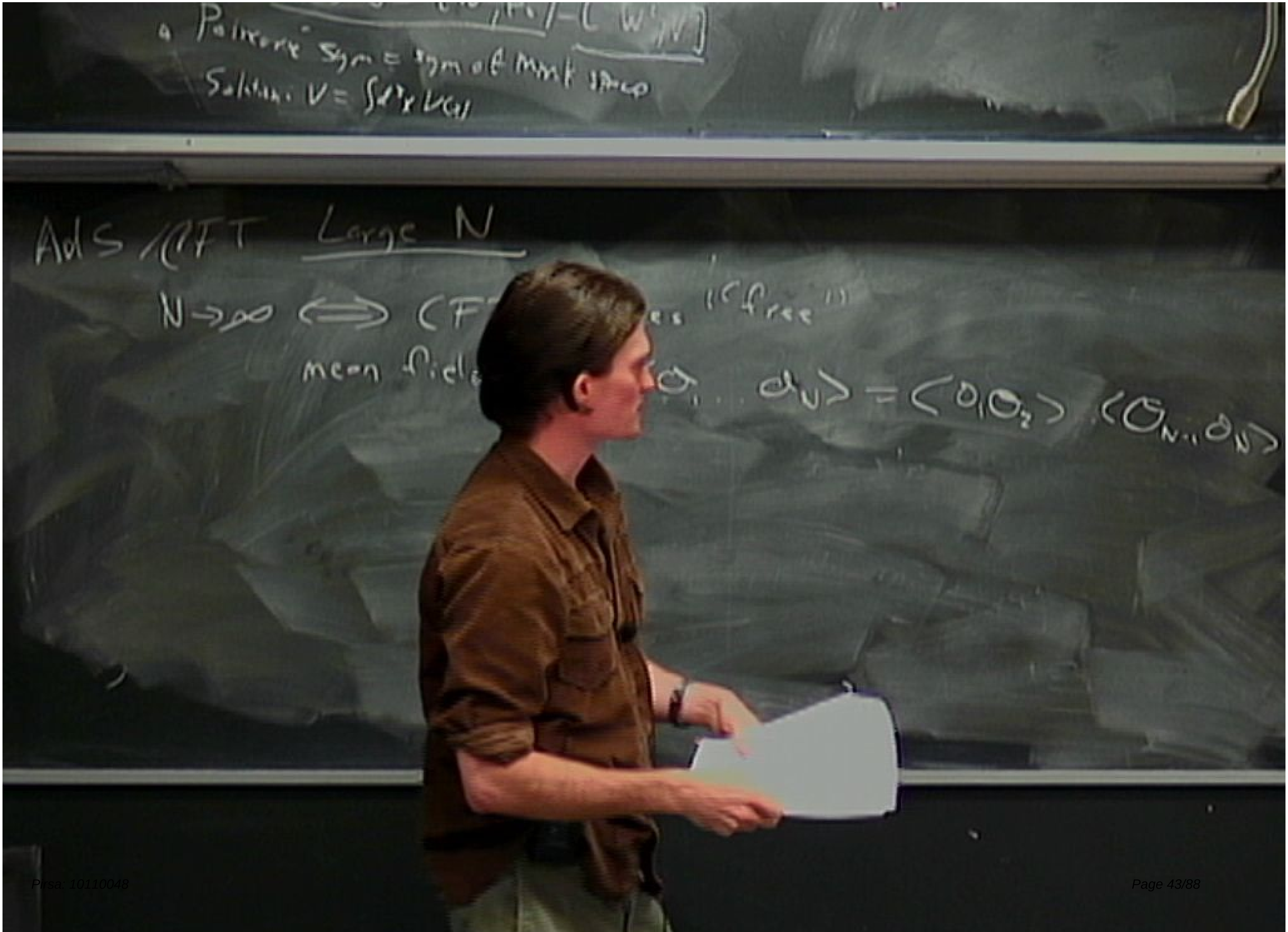
$$2\,\mathrm{Im}\,\mathcal{M}(a\to a) = \sum_f \int d\Pi_f\, |\mathcal{M}(a\to f)|^2$$

partial waves: $|a_{\ell m} - \frac{i}{2}| \le \frac{1}{2}$

If we know part cont up to Λ

$$\Longrightarrow \frac{\partial\phi \ldots \partial x}{\Lambda^p}$$

$\Longrightarrow$ new part's at $m_{\text{Heavy}} \Rightarrow M_{\text{Heavy}} \le \Lambda$



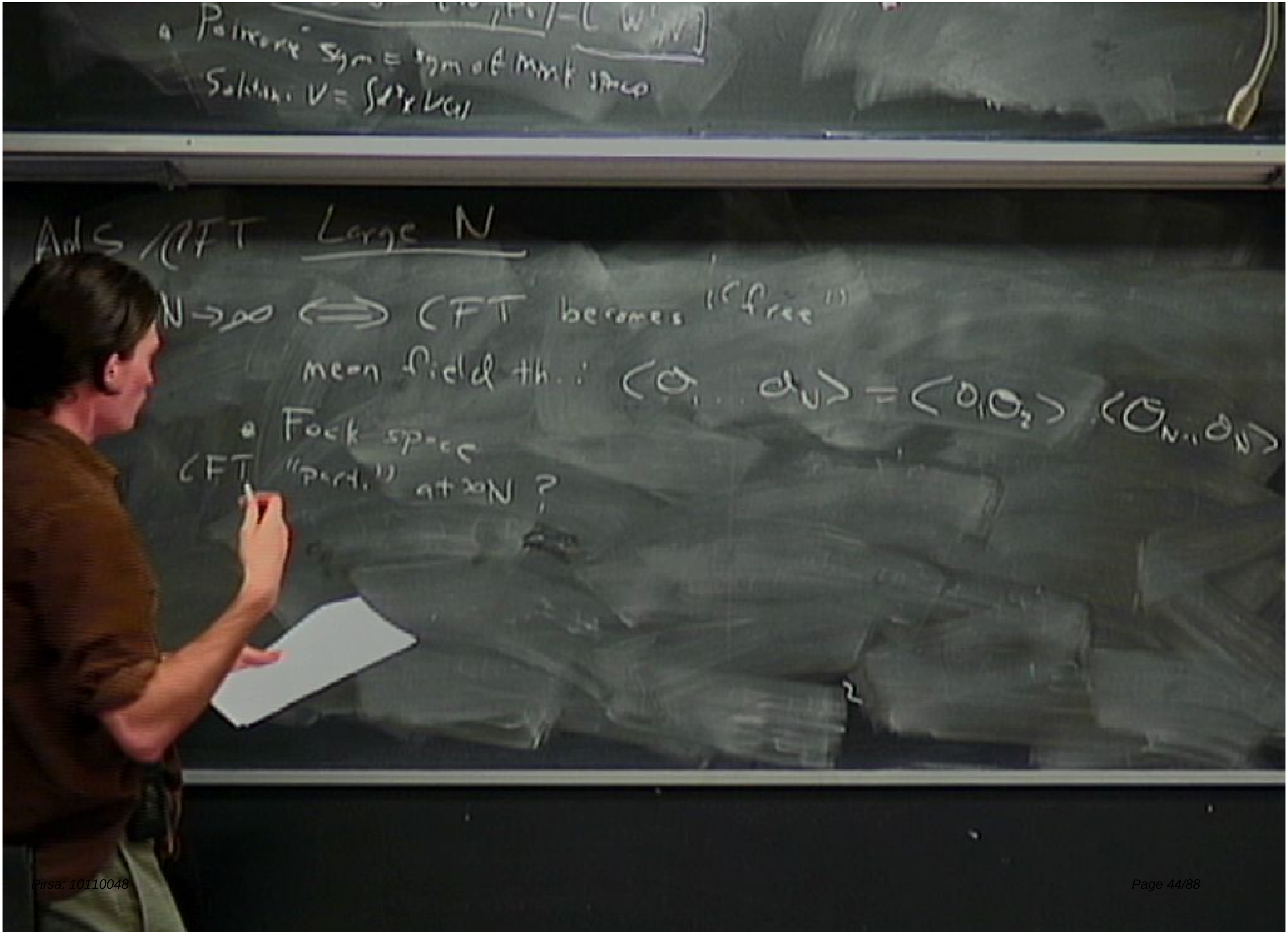
• Poincaré sym = sym of Mink space

Solution: $V = \int d^dx\, V(x)$

AdS/CFT Large N

$N \to \infty \Longleftrightarrow$ (CF… …es "free"

mean field… $\langle O_1 \ldots O_N \rangle = \langle O_1 O_2 \rangle \ldots \langle O_{N-1} O_N \rangle$



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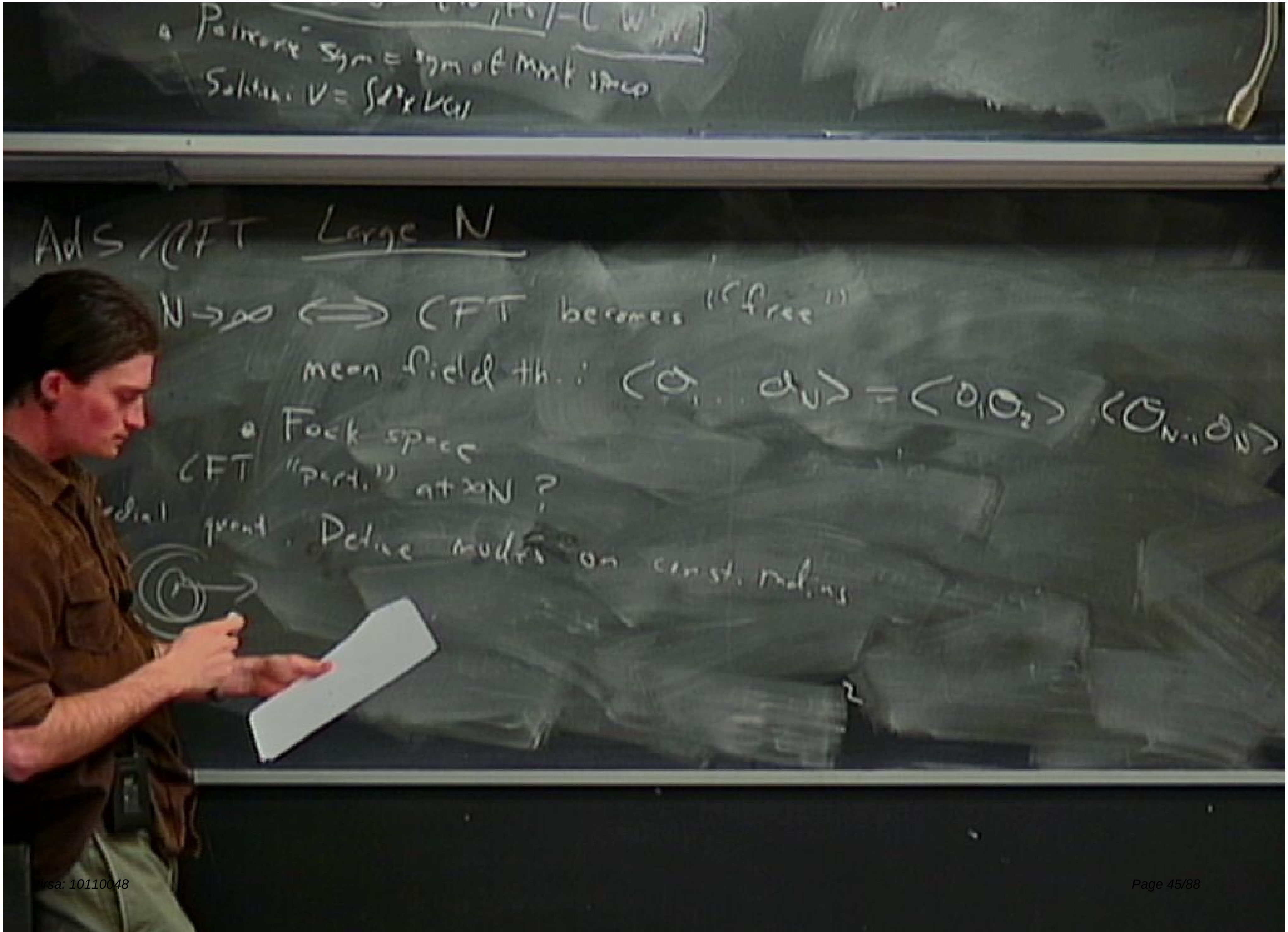
- Fock space

CFT "pert." at ∞N ?



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AdS/CFT **Large N**

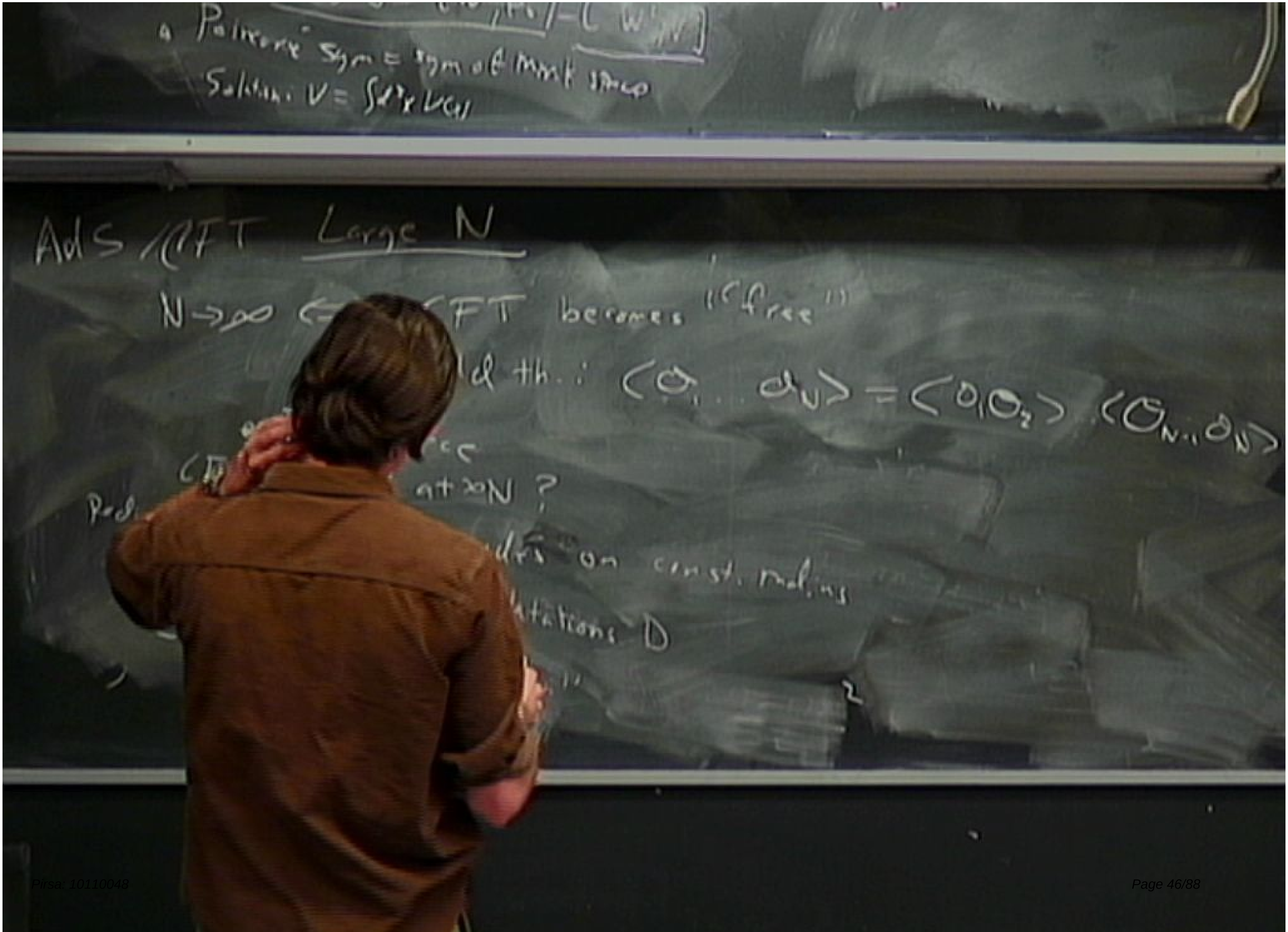
$N \to \infty \iff$ CFT becomes "free"

mean field th.: $\langle O_1 \ldots O_N \rangle = \langle O_1 O_2 \rangle \langle O_{N-1} O_N \rangle$

- Fock space

CFT "pert." at ∞N ?

dial quant. Define modes on const. radius



Poincaré sym = sym of Mink space

Solution: $V = \int d^3x\, V(x)$

AdS/CFT Large N

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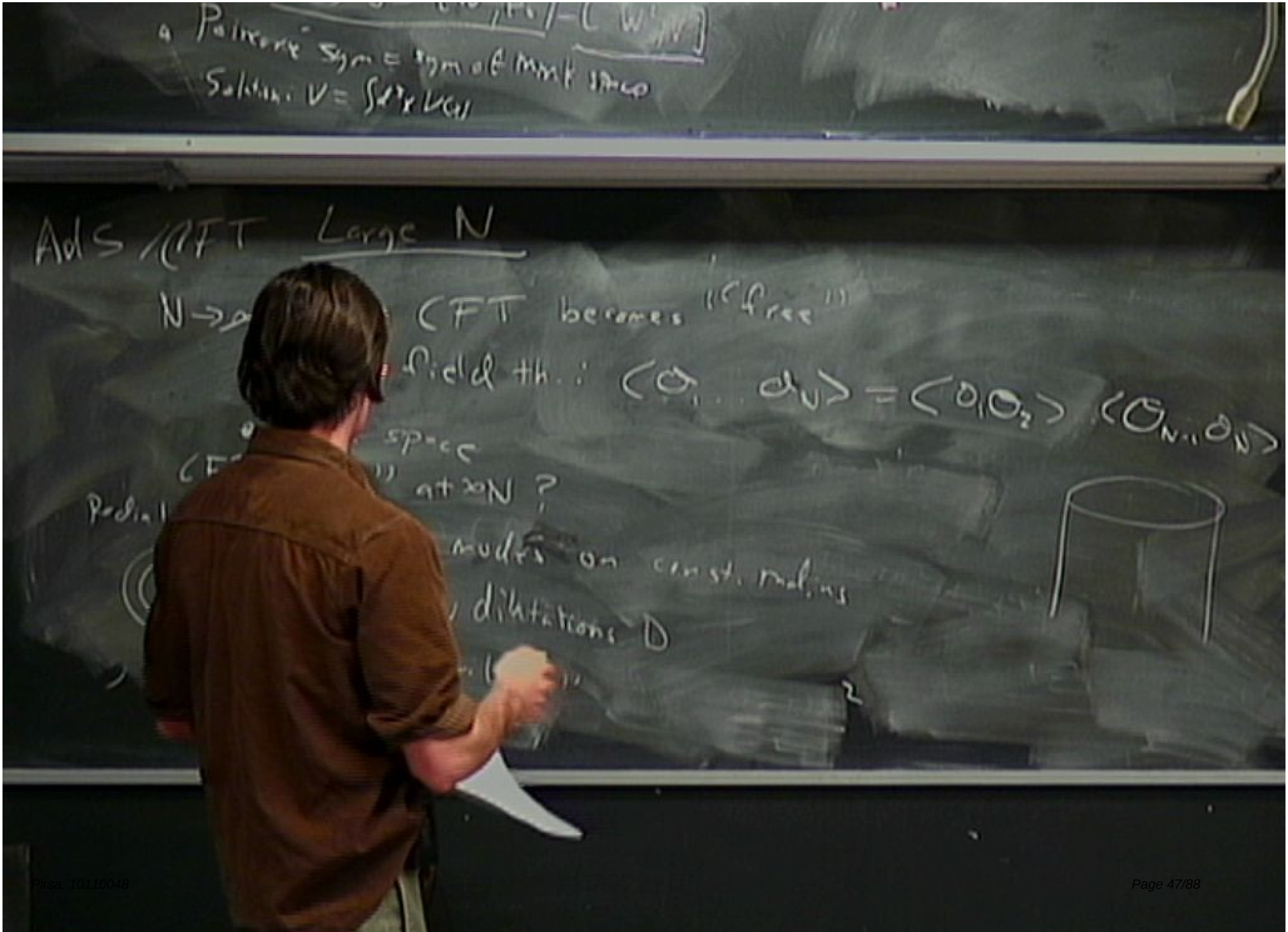
free

at ∞N ?

on const. modes

tations D



Poincaré sym = sym of mink space

Solution: $V = \int d^dx\, V(x)$

AdS/CFT Large N

$N \to \infty$ CFT becomes "free"

field th.: $\langle O_1 \ldots O_N \rangle = \langle O_1 O_2 \rangle \langle O_{N-1} O_N \rangle$

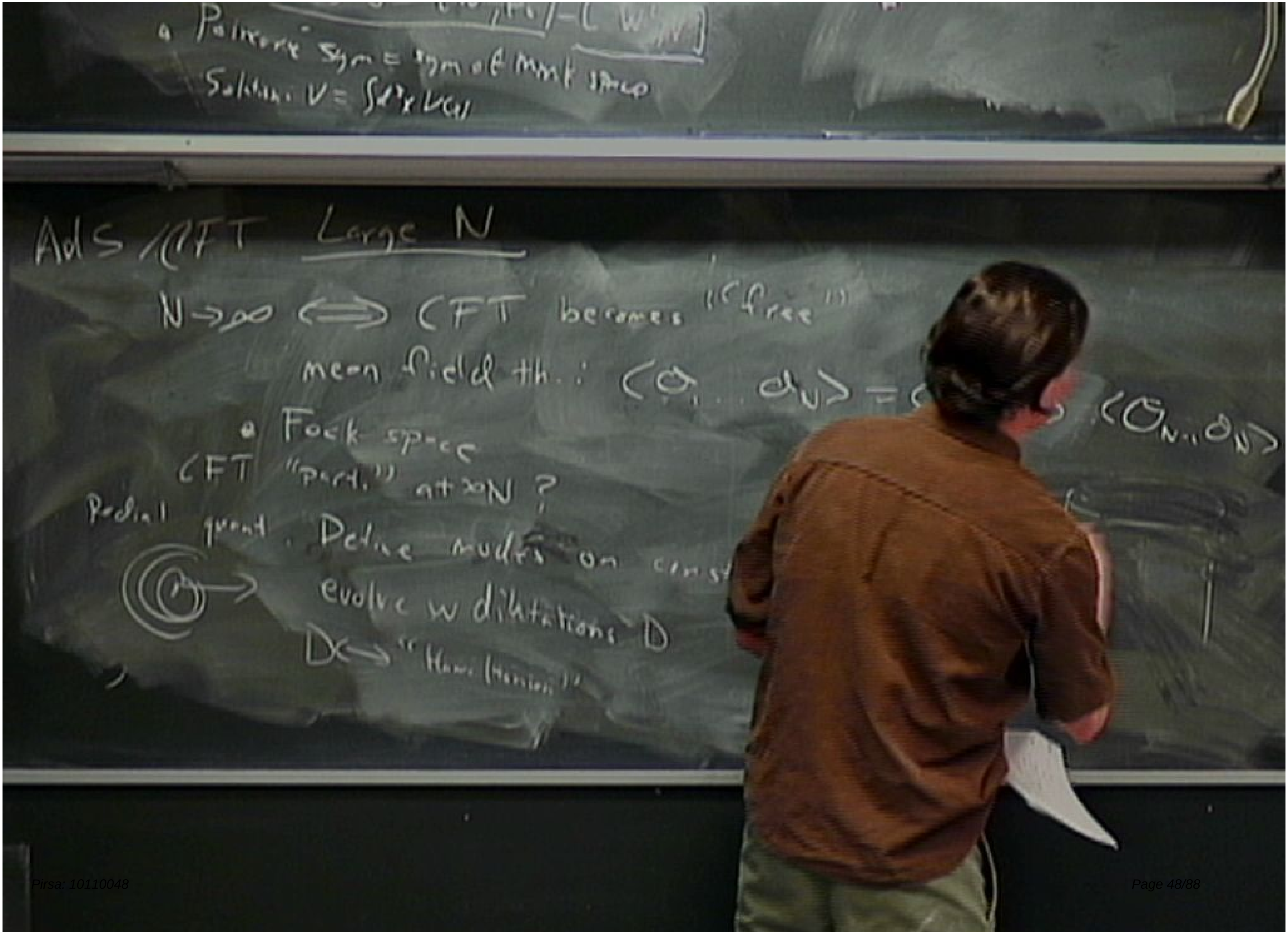
space

CF " at ∞N ?

Radial

modes on const. radius

dilatations D



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AdS / CFT Large N

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mean field th.: $\langle O_1 \ldots O_N \rangle = \langle$ $\rangle \langle O_N \ldots O_N \rangle$

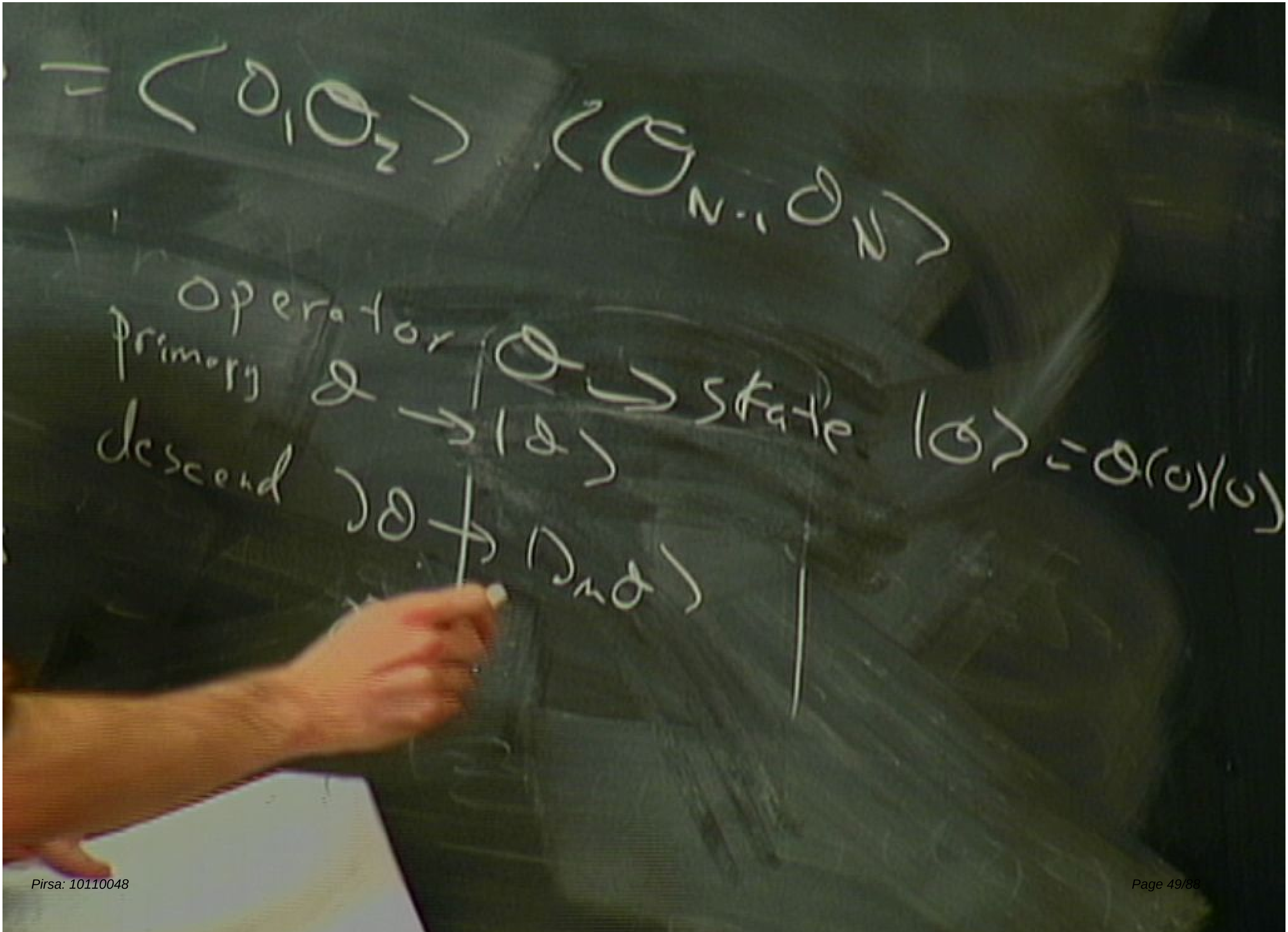
- Fock space

CFT "pert." at ∞N ?

Radial quant.: Define modes on const

evolve w dilatations D

$D \longleftrightarrow$ "Hamiltonian"



$= \langle O_1 O_2 \rangle \langle O_{N-1} O_N \rangle$

operator $\mathcal{O} \to$ state $|\mathcal{O}\rangle = \mathcal{O}(0)|0\rangle$

primary $\mathcal{O} \to |\mathcal{O}\rangle$

descend $\partial\mathcal{O} \to |\partial_\mu \mathcal{O}\rangle$

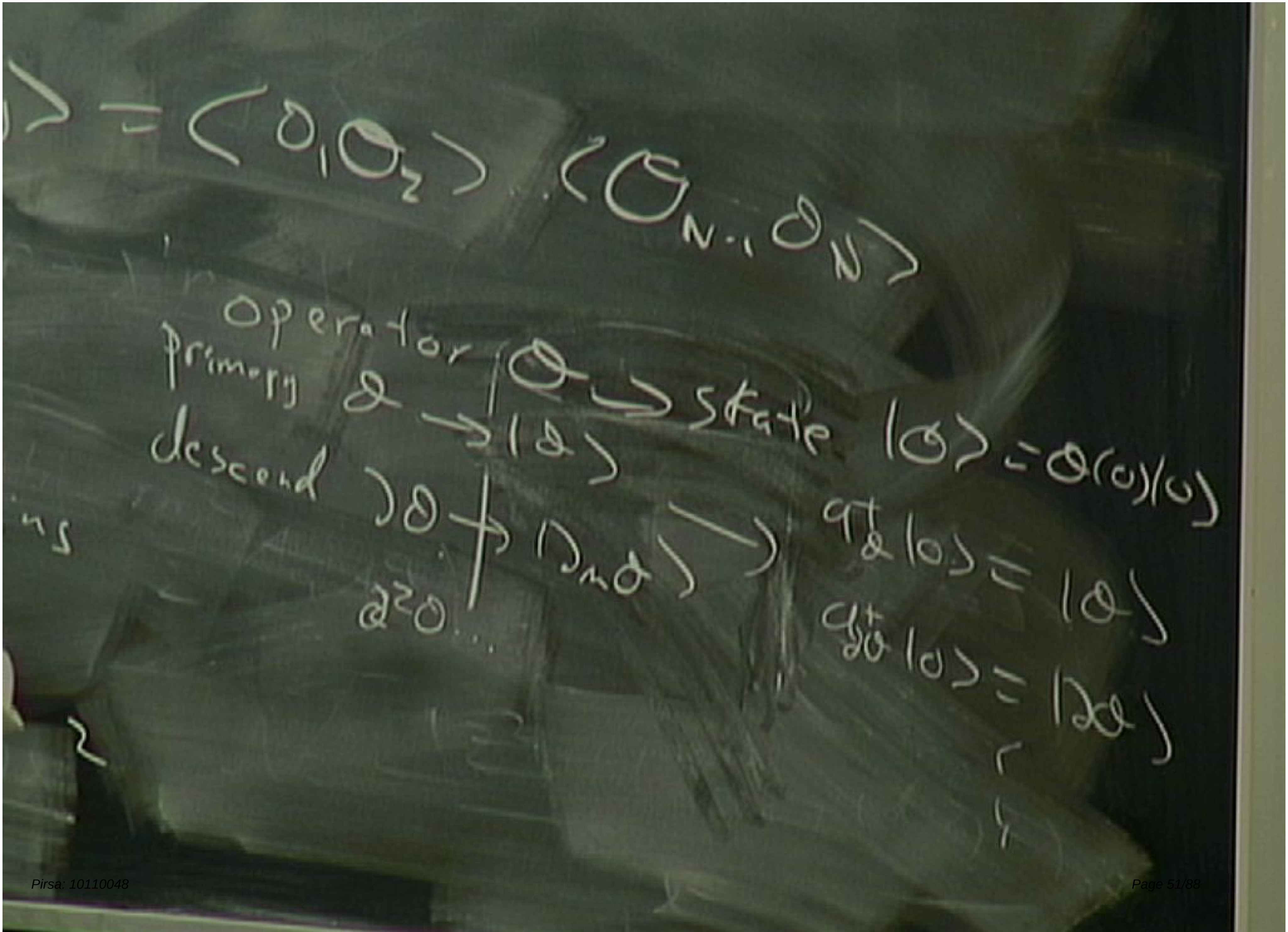
$= \langle O_1 O_2 \rangle \ldots \langle O_{N-1} O_N \rangle$

operator $O \Rightarrow$ state $|O\rangle = O(0)|0\rangle$

primary $O \rightarrow |O\rangle$

descend $\partial O \rightarrow |\partial_\mu O\rangle$

$\partial^2 O \ldots$



$$)\rangle = \langle O_1 O_2 \rangle \langle O_{N-1} O_N \rangle$$

operator $\mathcal{O}$ → state $|\mathcal{O}\rangle = \mathcal{O}(0)|0\rangle$

primary $\mathcal{O} \rightarrow |\mathcal{O}\rangle$

descend $\partial\mathcal{O} \rightarrow |\partial_\mu \mathcal{O}\rangle$

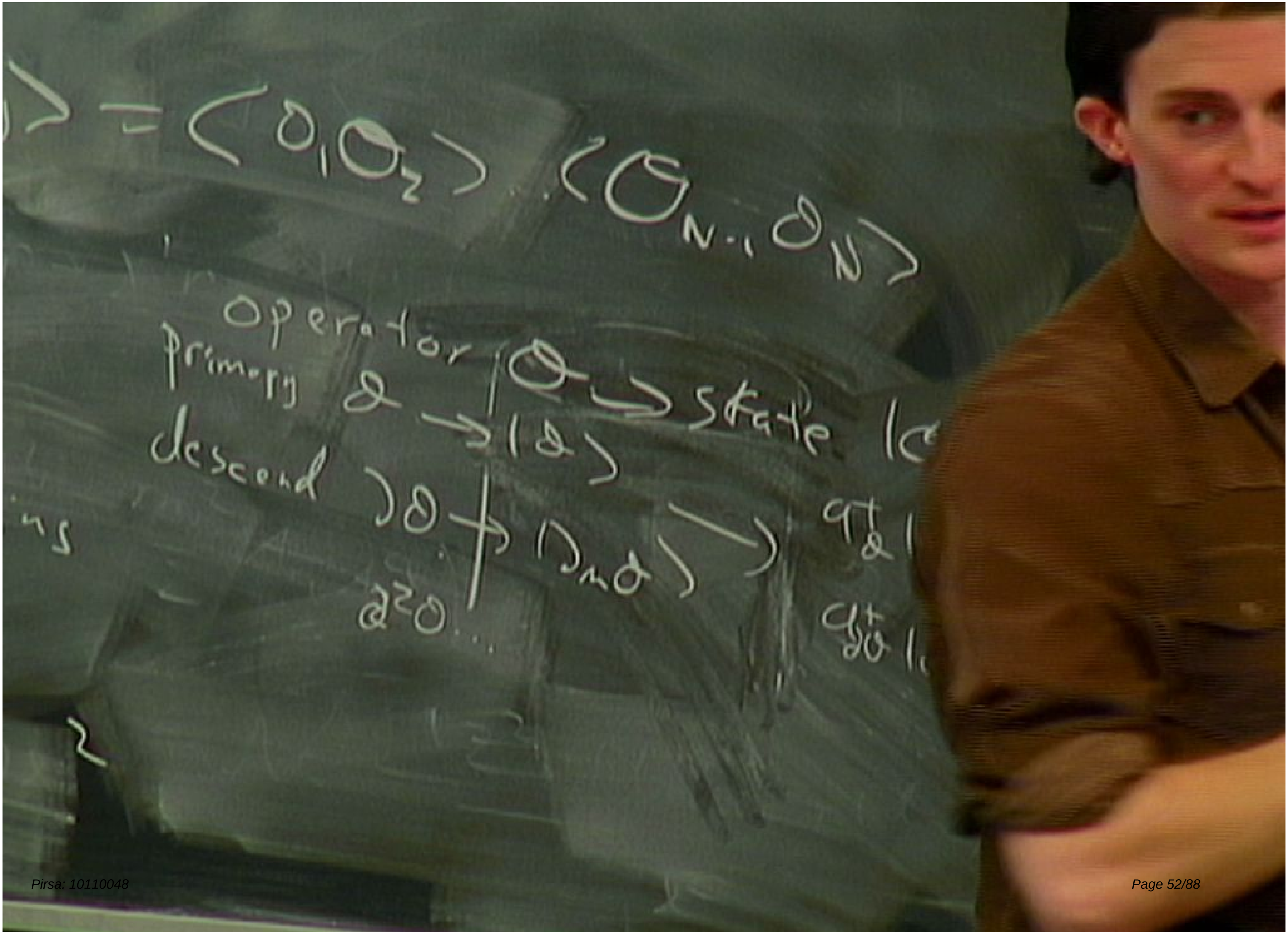
$\partial^2 \mathcal{O} \ldots$

$$a^\dagger_{\mathcal{O}}|0\rangle = |\mathcal{O}\rangle$$

$$a^\dagger_{\partial\mathcal{O}}|0\rangle = |\partial\mathcal{O}\rangle$$

ns

2



$$\rangle = \langle O_1 O_2 \rangle \cdots \langle O_{N-1} O_N \rangle$$

operator $O \Rightarrow$ state $|O$

primary $O \to |O\rangle$

descend $\partial O \to |\partial_\mu O\rangle$

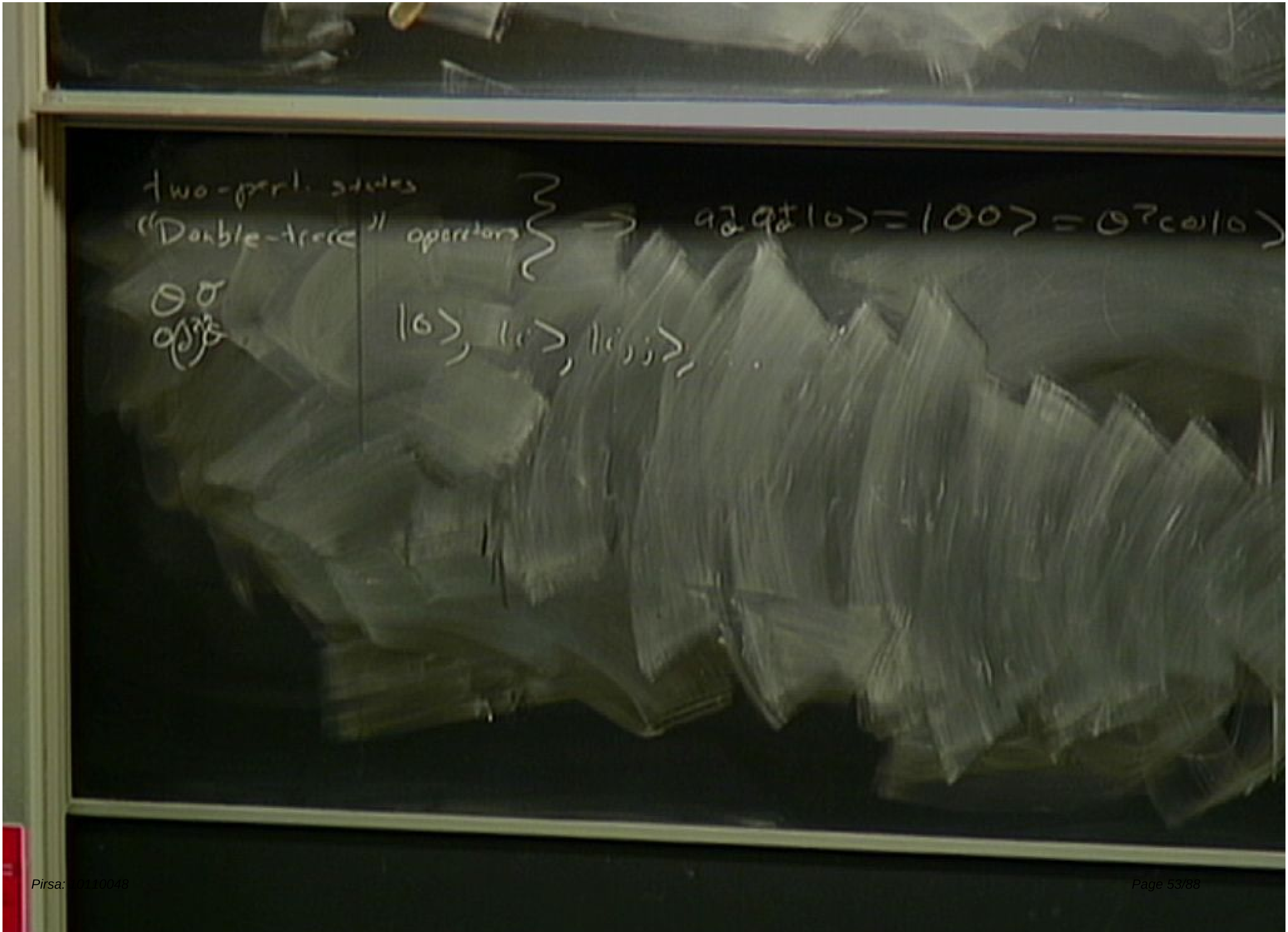
$\partial^2 O \ldots$

$a^\dagger_O |$

$a^\dagger_{\partial O} |$

ns

2



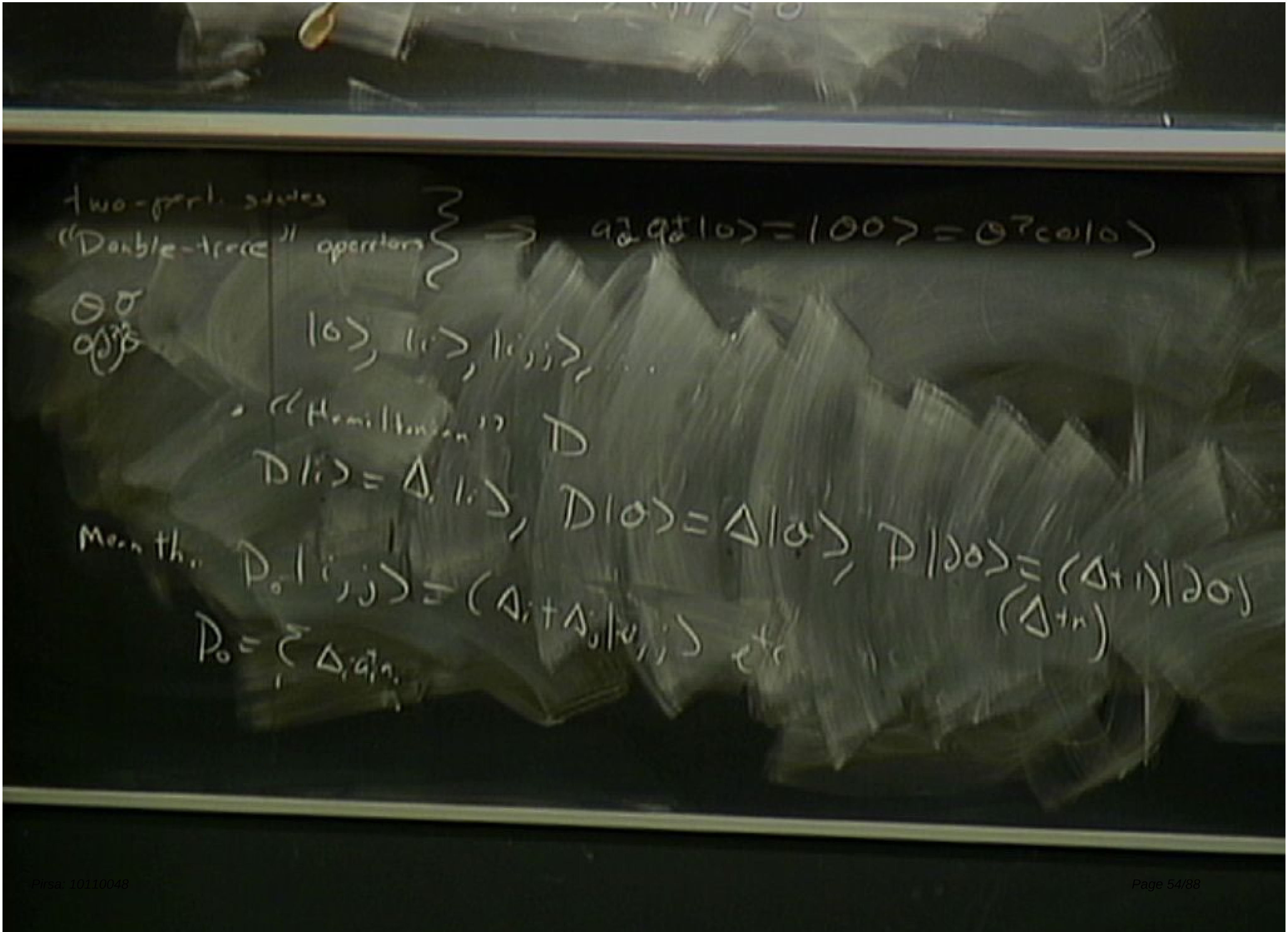
two-part. states

"Double-trace" operators $\Big\} \rightarrow \quad a_0^\dagger a_0^\dagger|0\rangle = |00\rangle = \mathcal{O}^2(0)|0\rangle$

$\mathcal{O}\,\mathcal{O}$

$\mathcal{O}\,\partial^n\mathcal{O}$

$|0\rangle,\ |i\rangle,\ |ij\rangle,\ \ldots$



two-part. states

"Double-trace" operators $\Big\} \Rightarrow a_0^\dagger a_0^\dagger|0\rangle = |00\rangle = O^2(0)|0\rangle$

$O\,O$

$O\partial^n O$

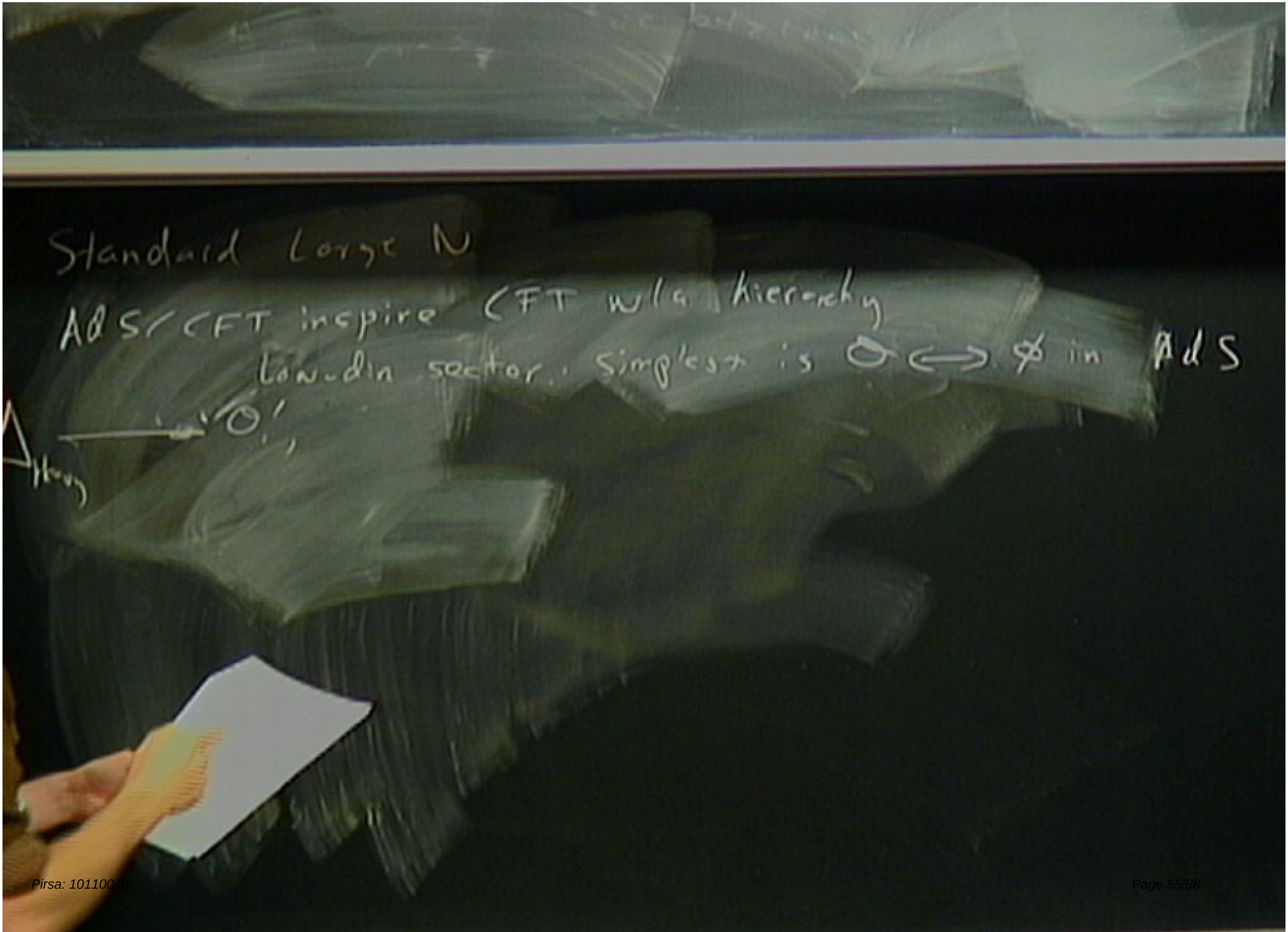
$|0\rangle, |i\rangle, |i,j\rangle, \ldots$

• "Hamiltonian" D

$D|i\rangle = \Delta_i|i\rangle, \quad D|O\rangle = \Delta|O\rangle, \quad D|\partial O\rangle = \underset{(\Delta+n)}{(\Delta+1)}|\partial O\rangle$

Mean th. $D_0|i,j\rangle = (\Delta_i+\Delta_j)|i,j\rangle$ etc.

$D_0 = \sum_i \Delta_i a_i^\dagger a_i$

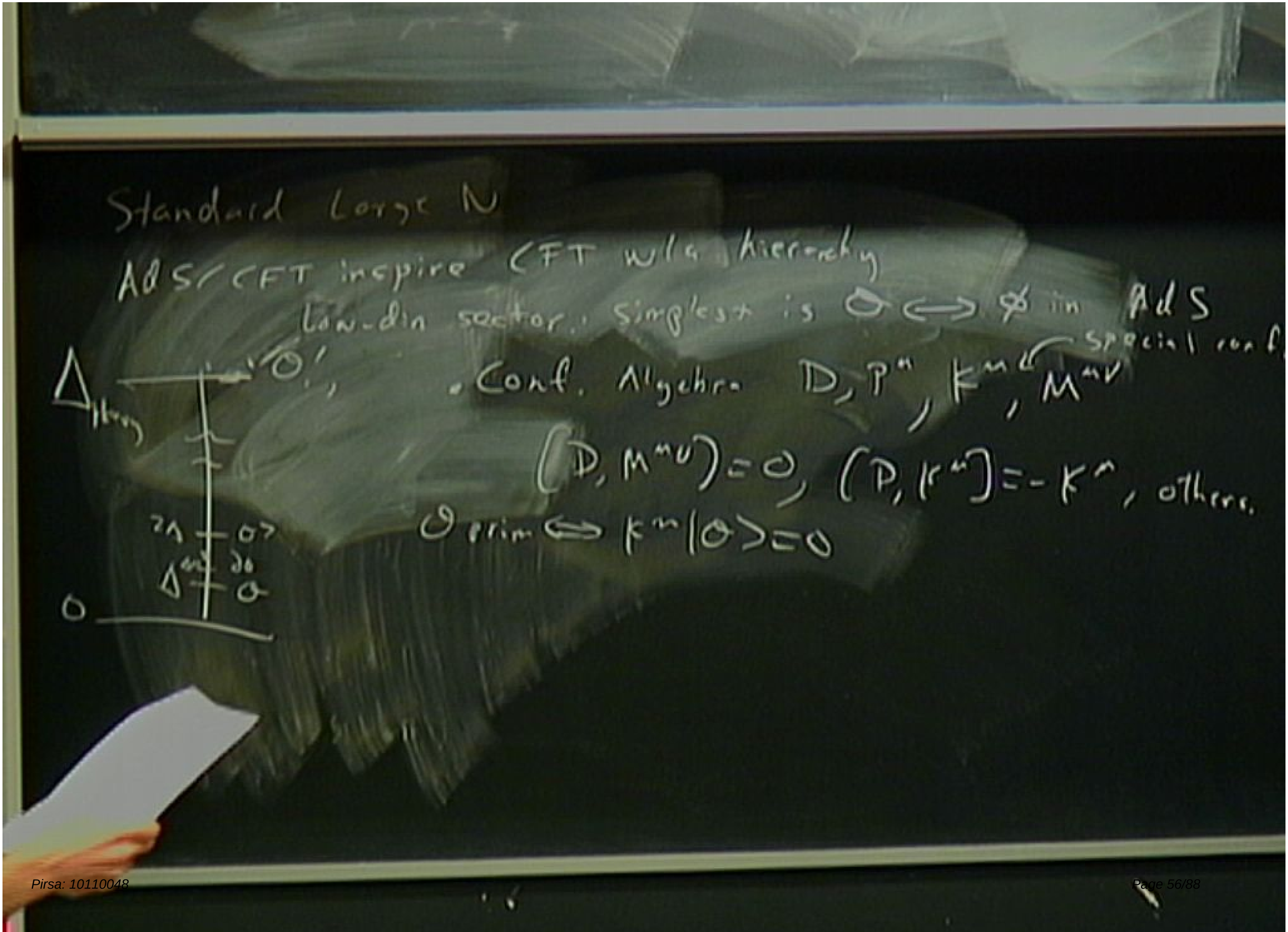


Standard Large N

AdS/CFT inspire CFT w/a hierarchy

low-dim sector. Simplest is $\mathcal{O} \longleftrightarrow \phi$ in AdS

Δ_{heavy} $\mathcal{O}'$



Standard Large N

AdS/CFT inspire CFT w/ a hierarchy

low-dim sector; simplest is $\mathcal{O} \Longleftrightarrow \phi$ in AdS

• Conf. Algebra $D, P^\mu, K^\mu, M^{\mu\nu}$ special conf.

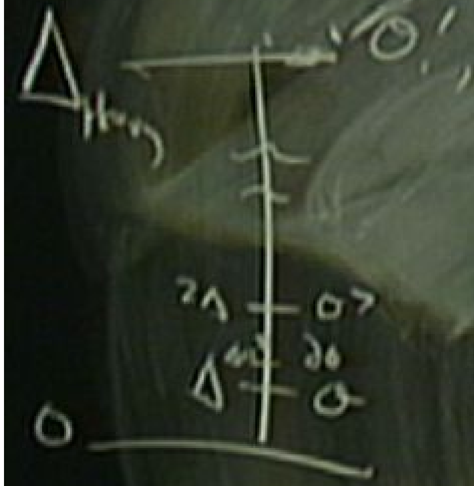
$[D, M^{\mu\nu}] = 0$, $[D, K^\mu] = -K^\mu$, others.

$\mathcal{O}$ prim $\Longleftrightarrow K^\mu|\mathcal{O}\rangle = 0$

Standard Large N

AdS/CFT inspire CFT w/ a hierarchy

low-dim sector. Simplest is $\mathcal{O} \Longleftrightarrow \phi$ in AdS



special conf.

• Conf. Algebra $D, P^\mu, K^\mu, M^{\mu\nu}$

$[D, M^{\mu\nu}] = 0, \; [P, K^\mu] = -K^\mu$, others.

$\mathcal{O}$ prim $\Longleftrightarrow K^\mu|\mathcal{O}\rangle = 0$

Introduce finite N: $D_0 \to D = D_0 + V$

$K_0 \to K_0 + K^{(1)}$

$[V, K_0^\mu] + [D_0, K^{(1)\mu}] = -K^{(1)\mu} - [V, K^{(1)\mu}]$

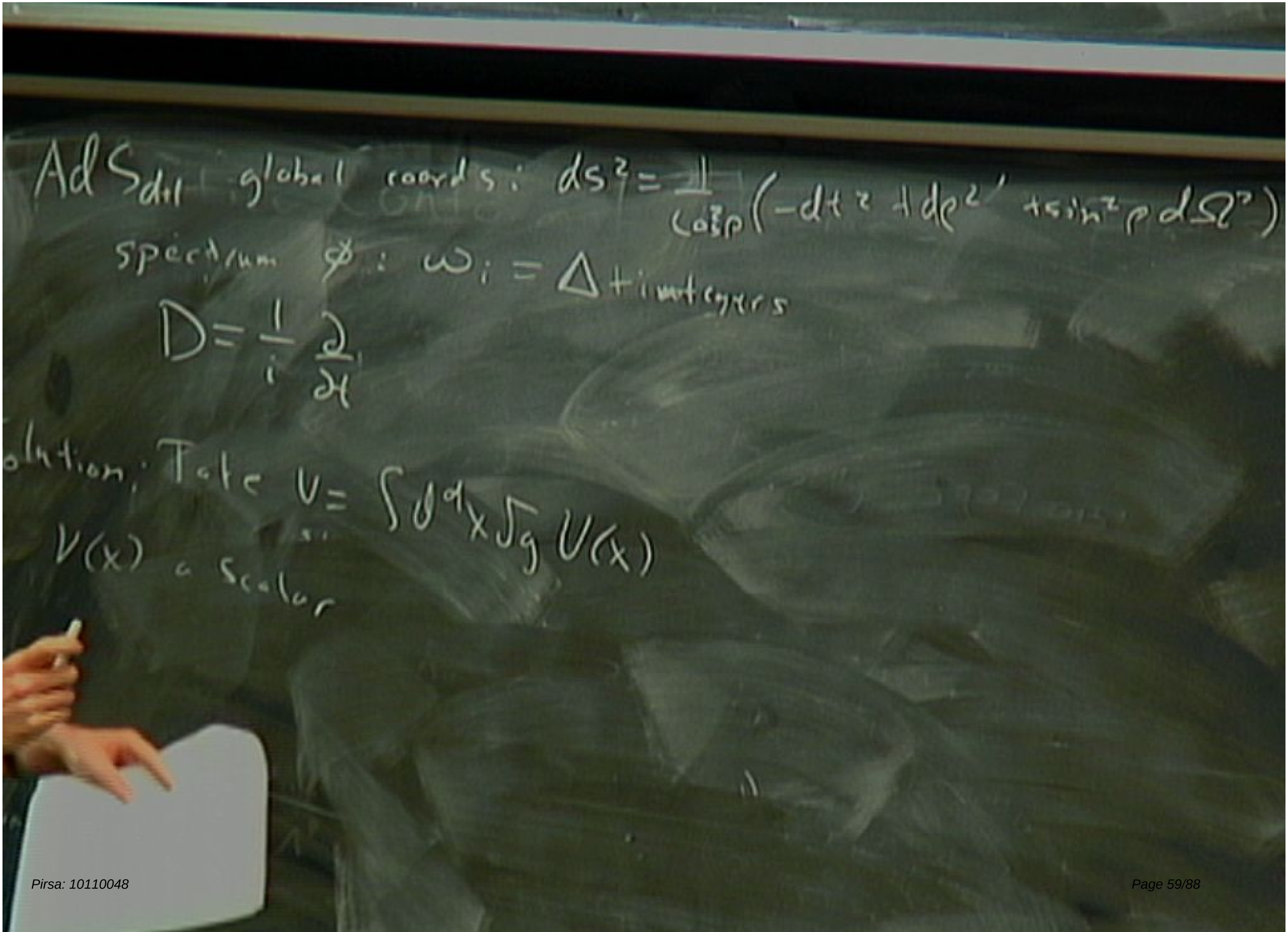
AdS_{d+1} global coords: $ds^2 = \frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho \, d\Omega^2\right)$

spectrum ϕ : $\omega_i = \Delta +$ integers

$$D = \frac{1}{i}\frac{\partial}{\partial t}$$

olution: Take $V = \int d^dx \sqrt{g}\, V(x)$

$V(x)$ scalar



AdS_{d+1} global coords: $ds^2 = \frac{1}{\cos^2\rho}\left(-dt^2 + d\rho^2 + \sin^2\rho \, d\Omega^2\right)$

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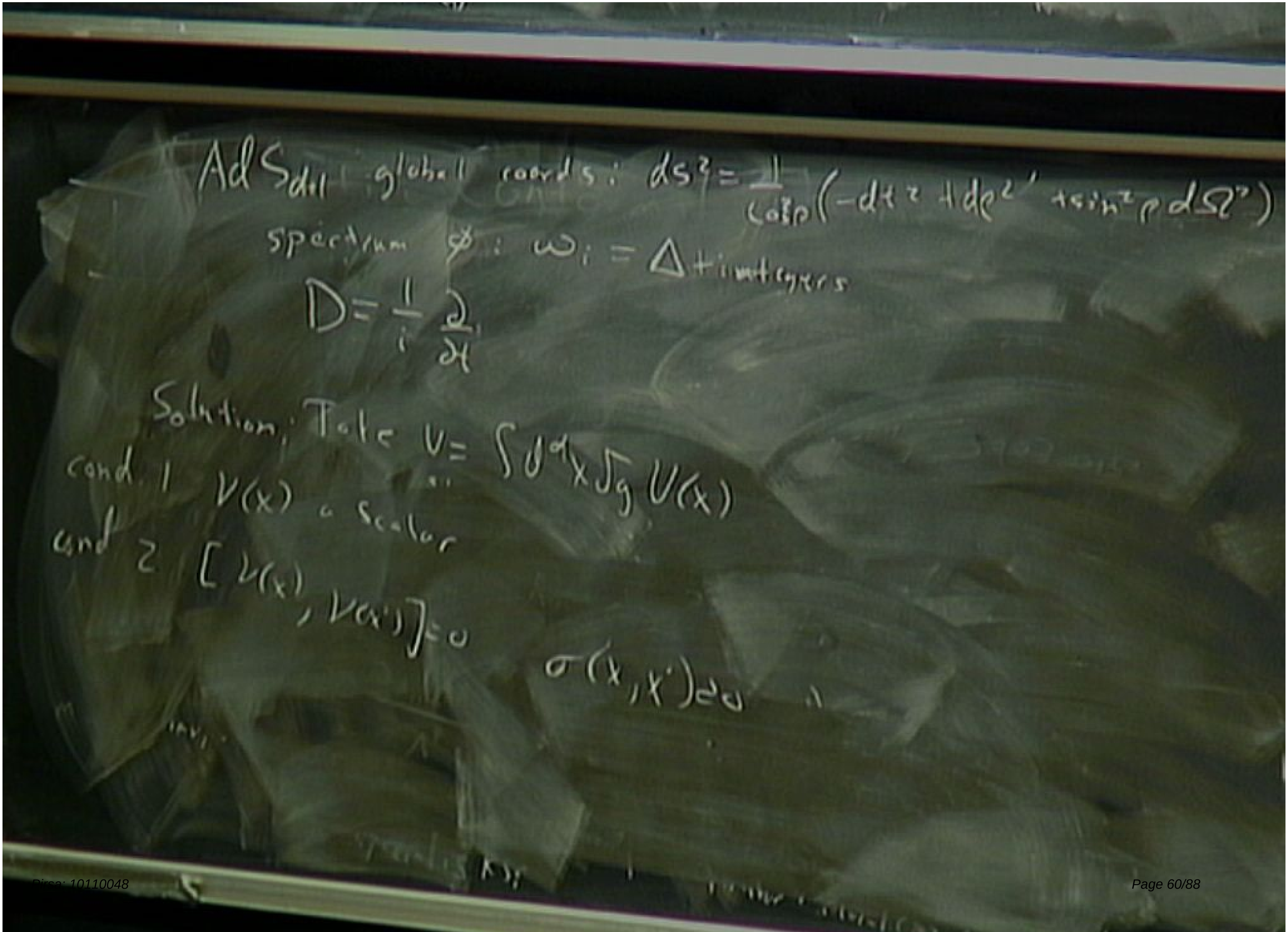


AdS_{d+1} global coords: $ds^2 = \frac{1}{\cos^2\rho}(-dt^2 + d\rho^2 + \sin^2\rho \, d\Omega^2)$

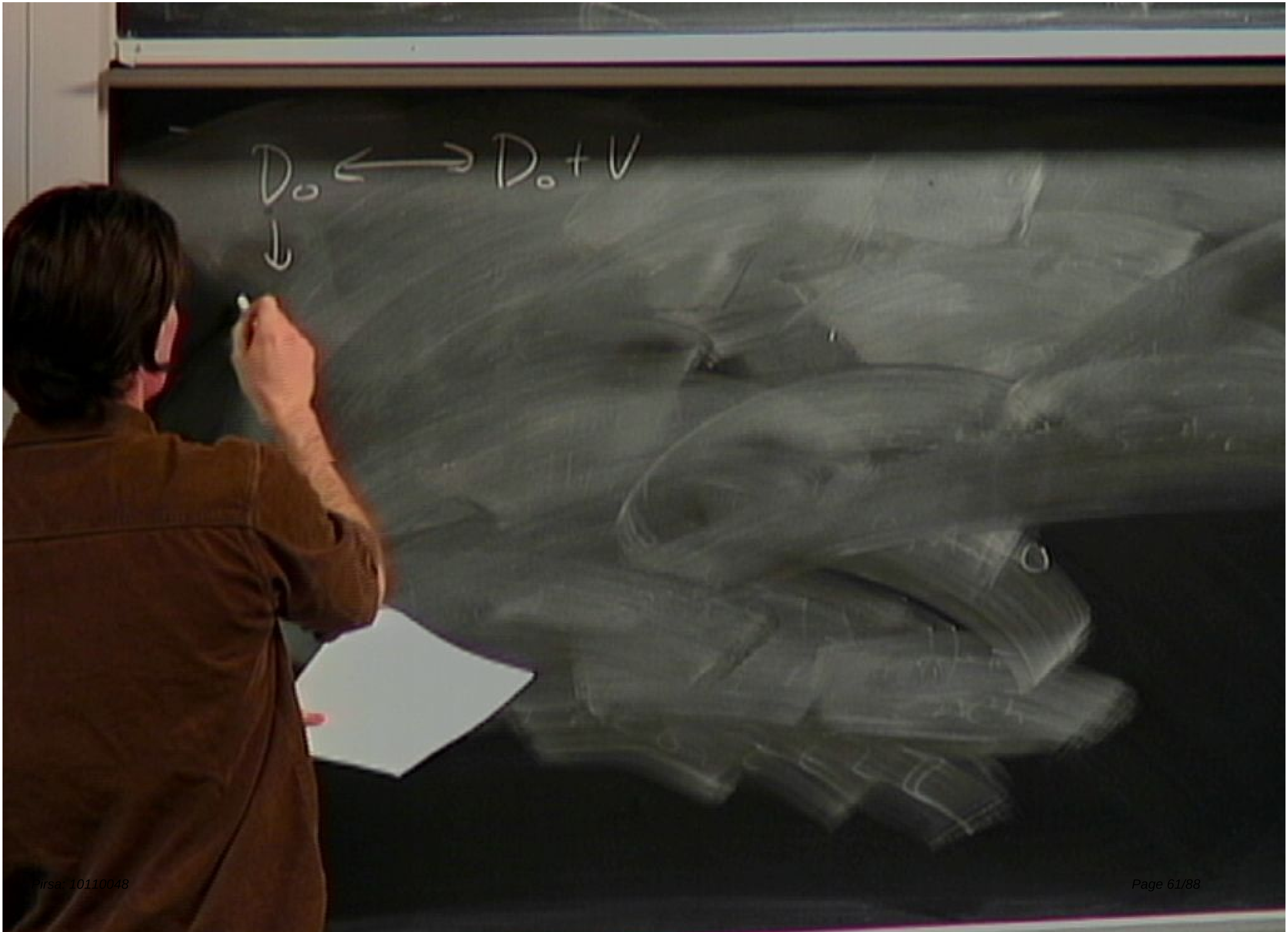
spectrum ϕ: $\omega_i = \Delta + $ integers

$D = \frac{1}{i}\frac{\partial}{\partial t}$

Solution: Take $V = \int d^d x \sqrt{g}\, V(x)$

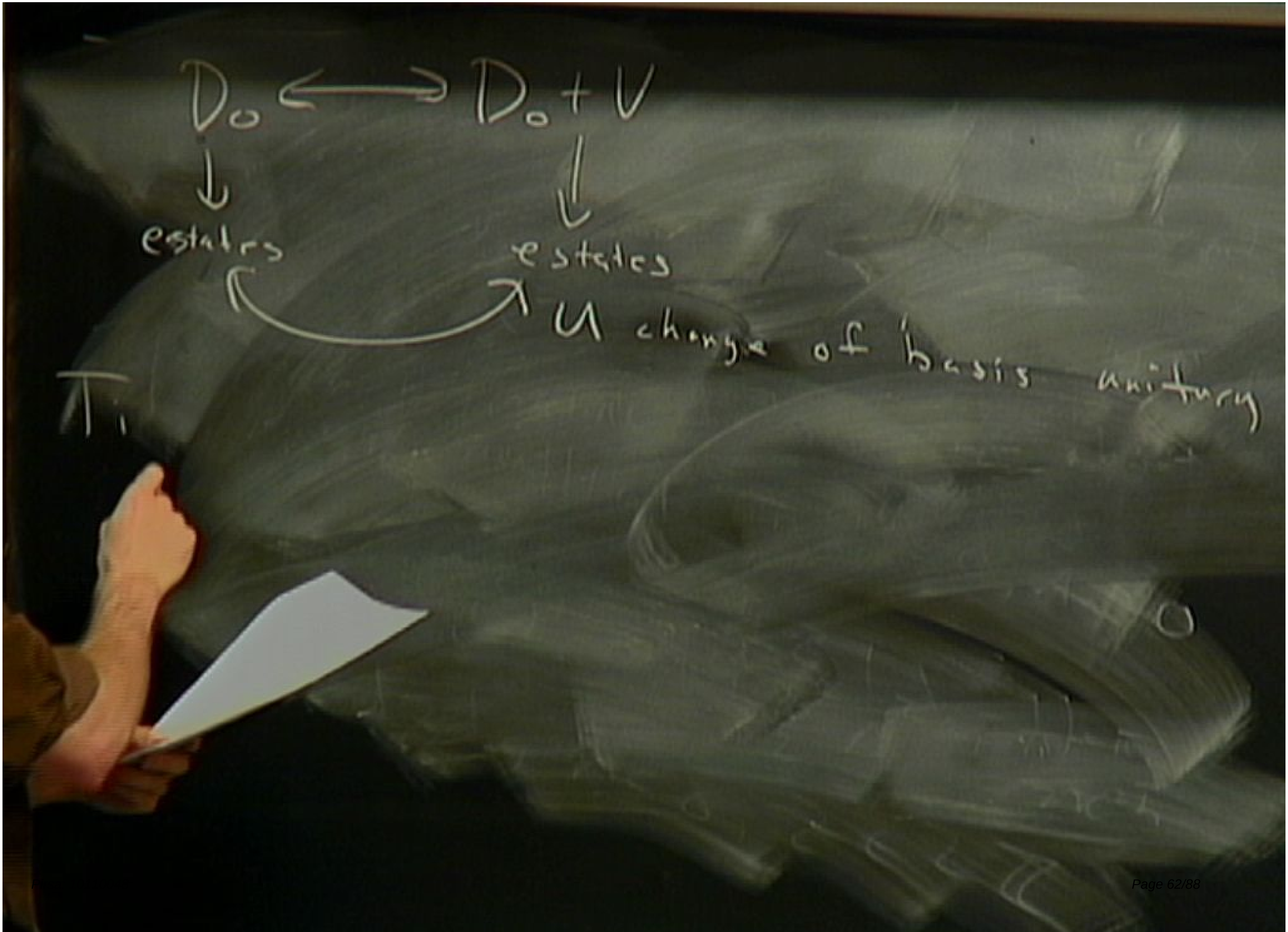
cond. 1 $V(x)$ a scalar

and 2 $[V(x), V(x')] = 0$ $\sigma(x, x') < 0$



$$D_0 \longleftrightarrow D_0 + V$$

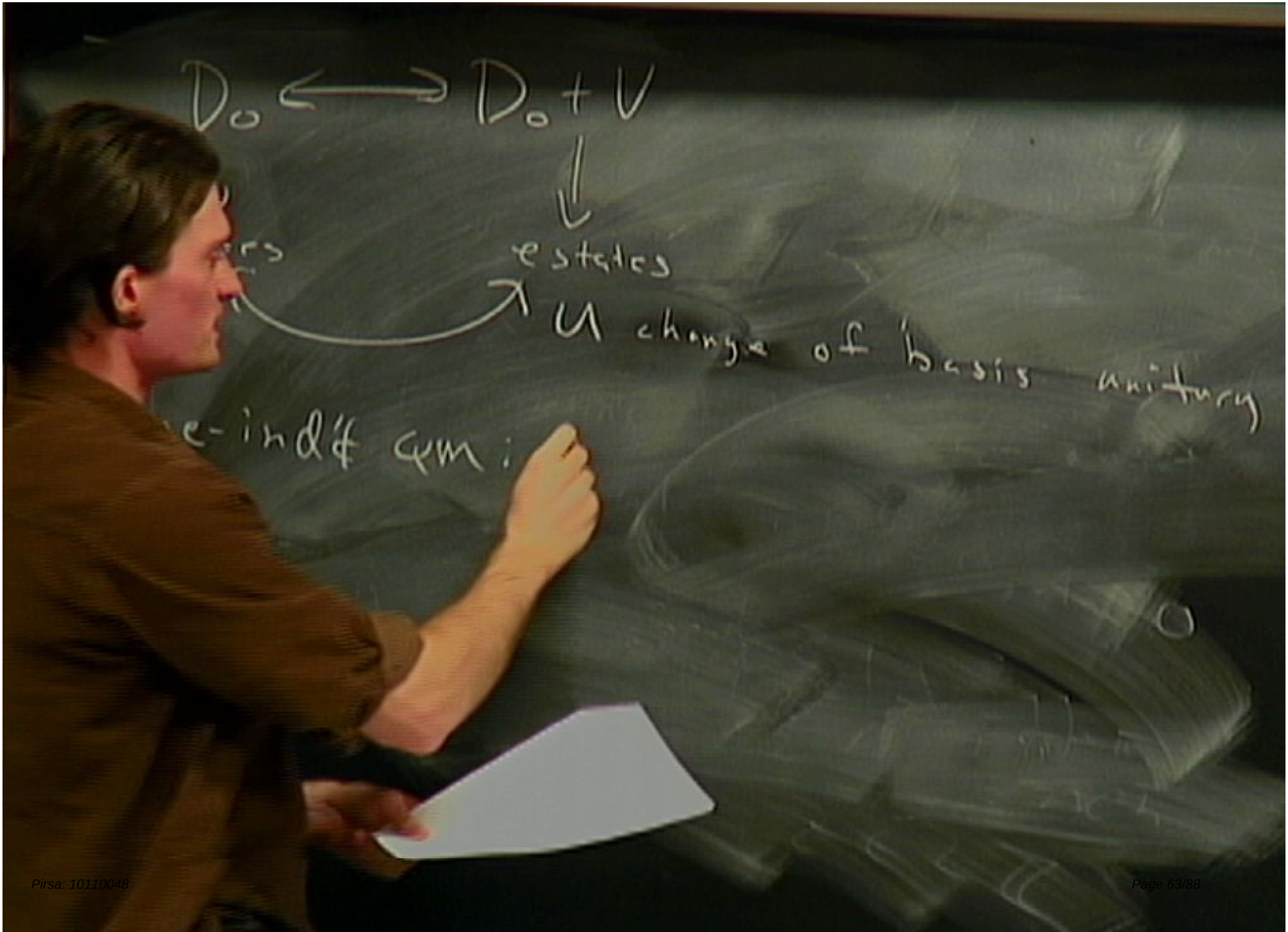
$$\Downarrow$$



$$\begin{tikzcd}
D_0 \arrow[r, leftrightarrow] \arrow[d] & D_0 + V \arrow[d] \\
\text{estates} \arrow[r, leftrightarrow, bend right, "U"'] & \text{estates}
\end{tikzcd}$$

U change of basis unitary

Ti



$D_0 \longleftrightarrow D_0 + V$

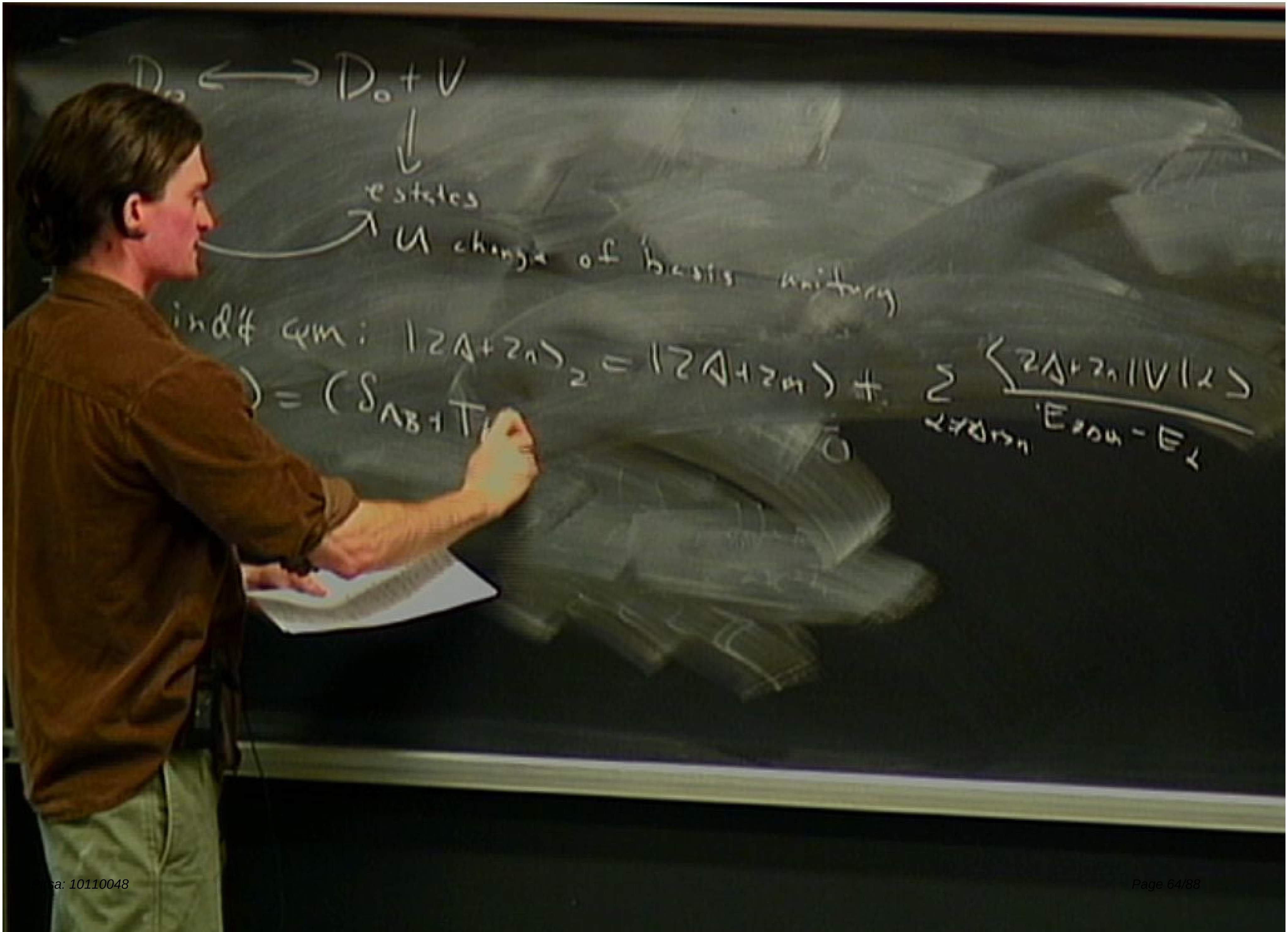
$\downarrow$

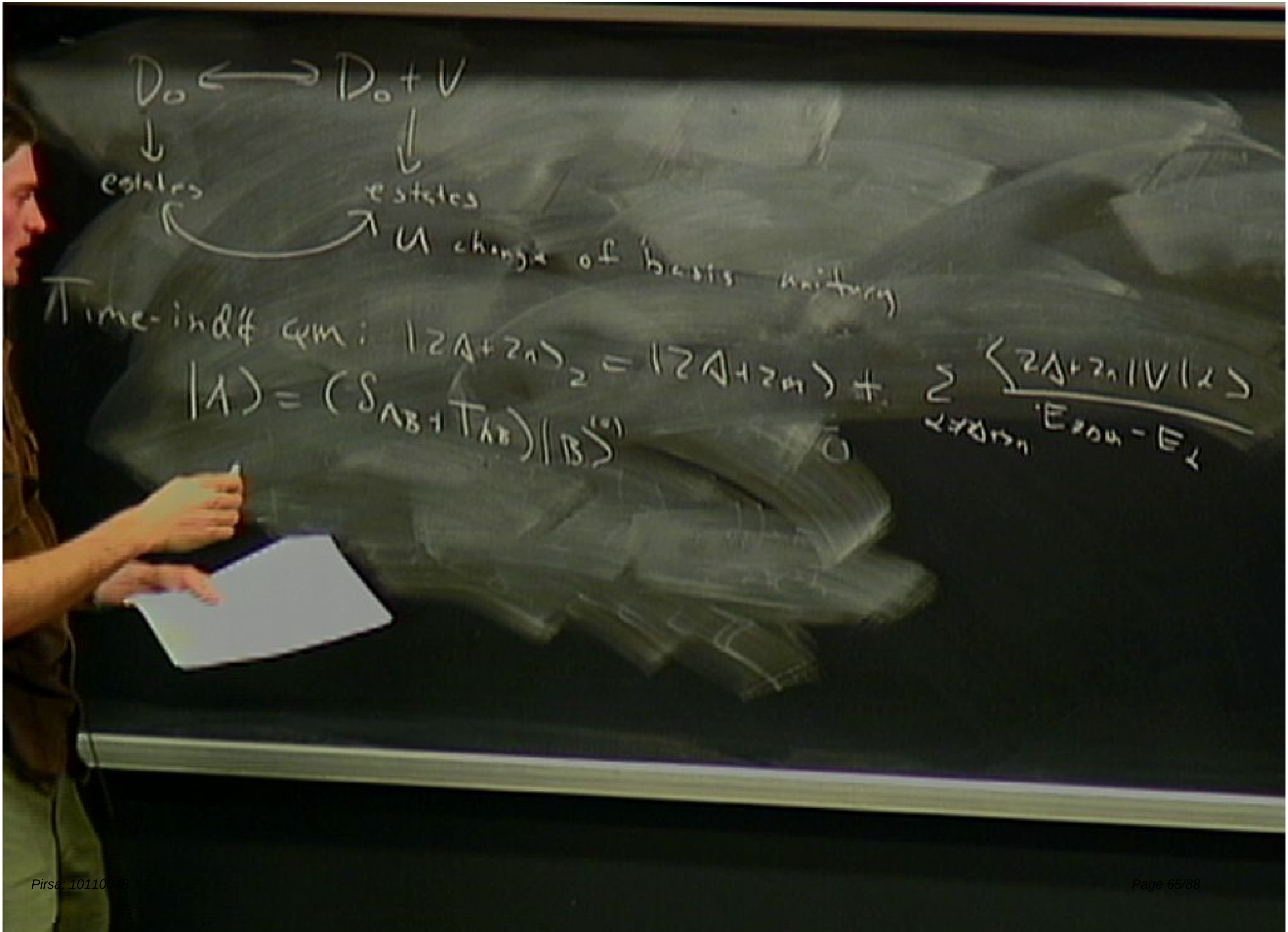
es

estates

$\curvearrowright$ U change of basis unitary

e-ind't qm:


D₀ ⟷ D₀ + V
e states
U change of basis unitary
ind't qm: |2Δ+2n⟩₂ = |2Δ+2n⟩ + Σ ⟨2Δ+2n|V|λ⟩ / E₂Δn − Eλ
λ≠2Δ+2n
) = (δ_AB + T

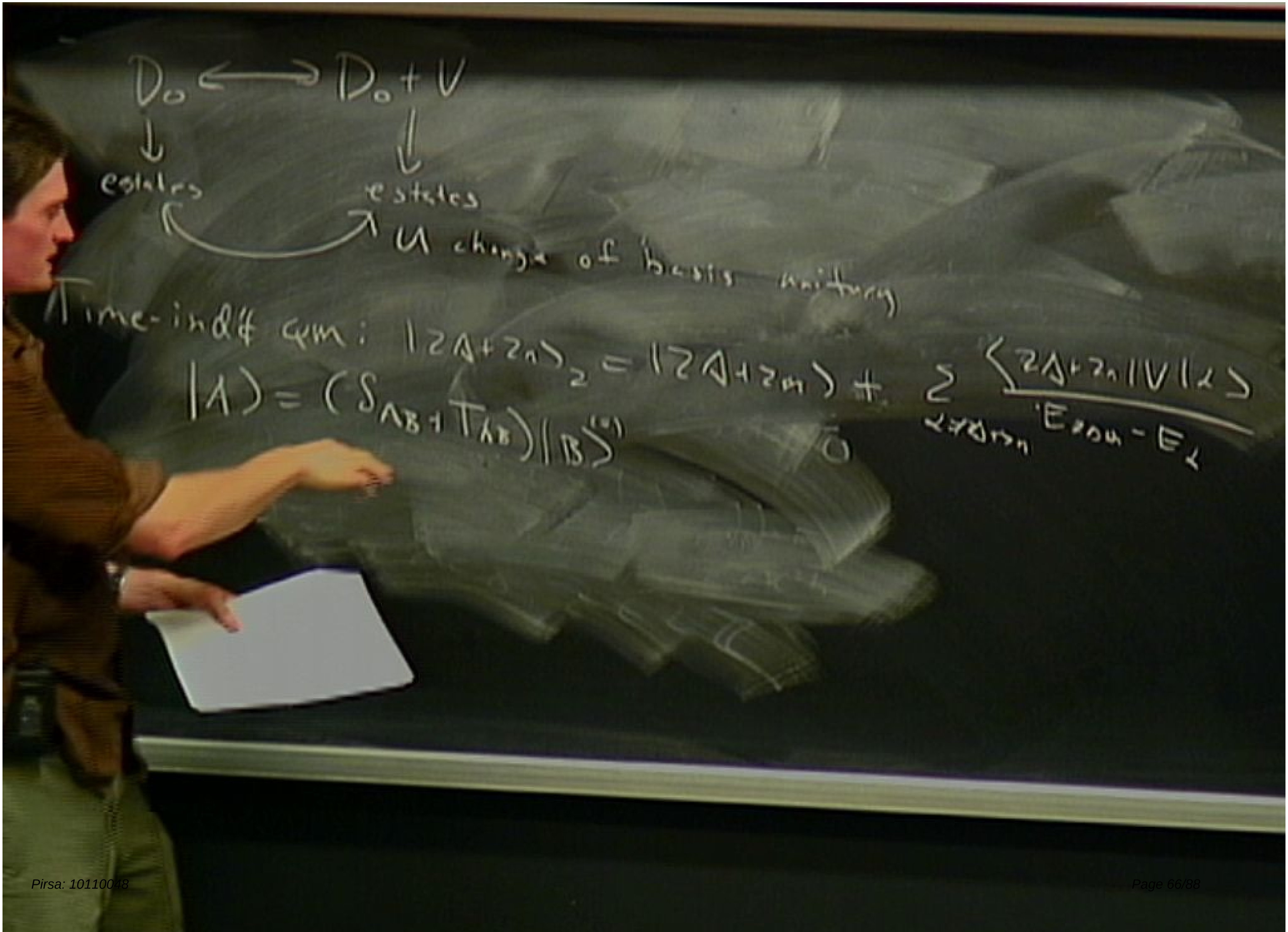


$$\begin{tikzcd}
D_0 \arrow[r, leftrightarrow] \arrow[d] & D_0+V \arrow[d] \\
\text{estates} \arrow[r, bend right, leftrightarrow] & \text{e states}
\end{tikzcd}$$

U change of basis unitary

Time-indep qm: $|2\Delta+2n\rangle_2 = |2\Delta+2m\rangle \underset{0}{+} \sum_{\ell\neq\Delta+n} \frac{\langle 2\Delta+2n|V|\ell\rangle}{E_{\Delta n}-E_\ell}$

$|A\rangle = (S_{AB}+T_{AB})|B\rangle^{(0)}$



$D_0 \longleftrightarrow D_0 + V$

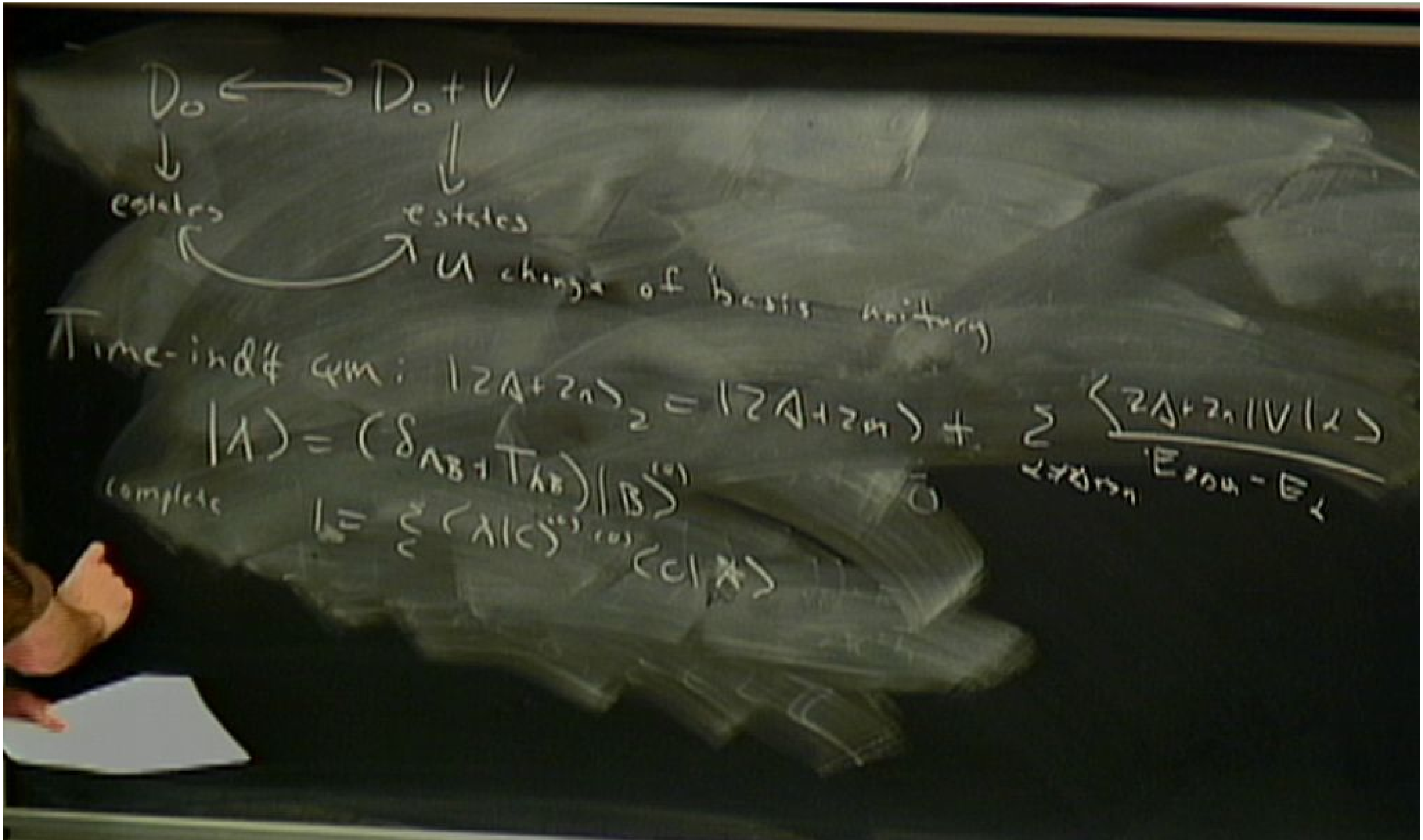
$\downarrow \qquad \downarrow$

estates estates

U change of basis unitary

Time-indep qm: $|2\Delta+2n\rangle_2 = |2\Delta+2n\rangle_0 + \sum_{\alpha\neq\Delta+n} \frac{\langle 2\Delta+2n|V|\alpha\rangle}{E_{2\Delta+2n}-E_\alpha}$

$|A\rangle = (\delta_{AB} + T_{AB})|B\rangle^{(0)}$



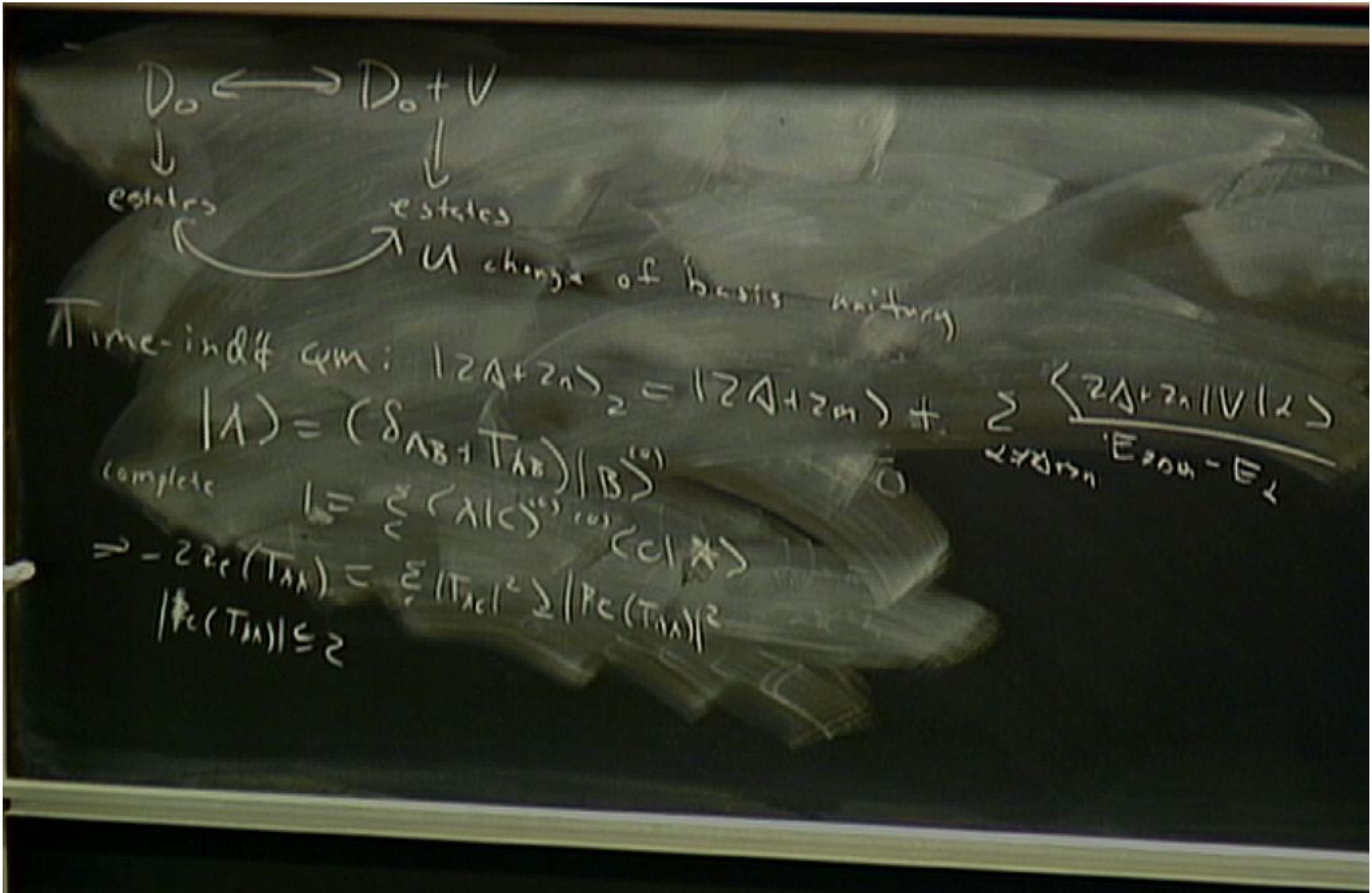
$$\begin{tikzcd} D_0 \arrow[r, leftrightarrow] \arrow[d] & D_0+V \arrow[d] \\ \text{estates} \arrow[r, bend right, "\mathcal{U}"'] & \text{estates} \end{tikzcd}$$

$\mathcal{U}$ change of basis unitary

Time-indep qm: $|2\Delta+2n\rangle_2 = |2\Delta+2n\rangle + \sum_{\alpha\neq\Delta+n} \frac{\langle 2\Delta+2n|V|\alpha\rangle}{E_{2\Delta n}-E_\alpha}$

$|A\rangle = (\delta_{AB}+T_{AB})|B\rangle^{(0)}$

complete $1 = \sum_C \langle A|C\rangle^{(0)}\,{}^{(0)}\langle C|\ast\rangle$



$$\begin{tikzcd} D_0 \arrow[r, leftrightarrow] \arrow[d] & D_0+V \arrow[d] \\ \text{estates} \arrow[r, bend right] & \text{estates} \end{tikzcd}$$

$\mathcal{U}$ change of basis unitary

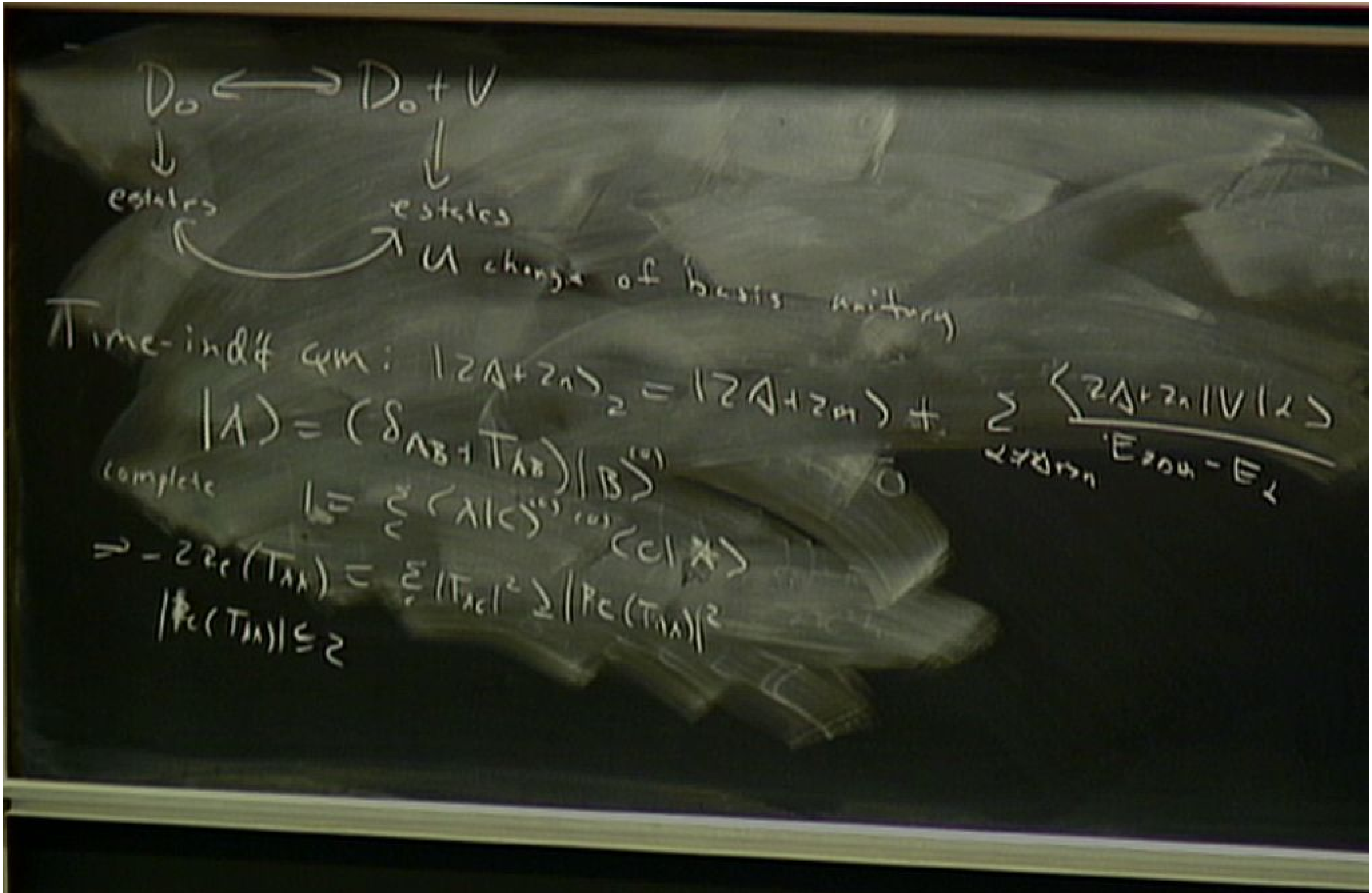
Time-ind't qm: $|2\Delta+2n\rangle_2 = |2\Delta+2n\rangle + \sum_{\alpha\neq\Delta+n} \frac{\langle 2\Delta+2n|V|\alpha\rangle}{E_{\Delta n}-E_\alpha}$

$|A\rangle = (\delta_{AB} + T_{AB})|B\rangle^{(0)}$

complete $1 = \sum_C |A|C\rangle^{(0)} \, {}^{(0)}\langle C|A\rangle$

$\Rightarrow -2\mathrm{Re}(T_{AA}) = \sum_C |T_{AC}|^2 \geq |\mathrm{Re}(T_{AA})|^2$

$|\mathrm{Re}(T_{AA})| \leq 2$



$$\begin{tikzcd} D_0 \arrow[r, leftrightarrow] \arrow[d] & D_0+V \arrow[d] \\ \text{estates} \arrow[r, bend right, leftrightarrow, "\mathcal{U}"'] & \text{e states} \end{tikzcd}$$

$\mathcal{U}$ change of basis unitary

Time-ind't qm: $|2\Delta+2n\rangle_2 = |2\Delta+2n\rangle + \sum_{\alpha\neq\Delta+2n} \frac{\langle 2\Delta+2n|V|\alpha\rangle}{E_{\Delta+2n}-E_\alpha}$

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complete $\quad 1 = \sum_C \langle A|C\rangle^{(0)\,(0)}\langle C|A\rangle$

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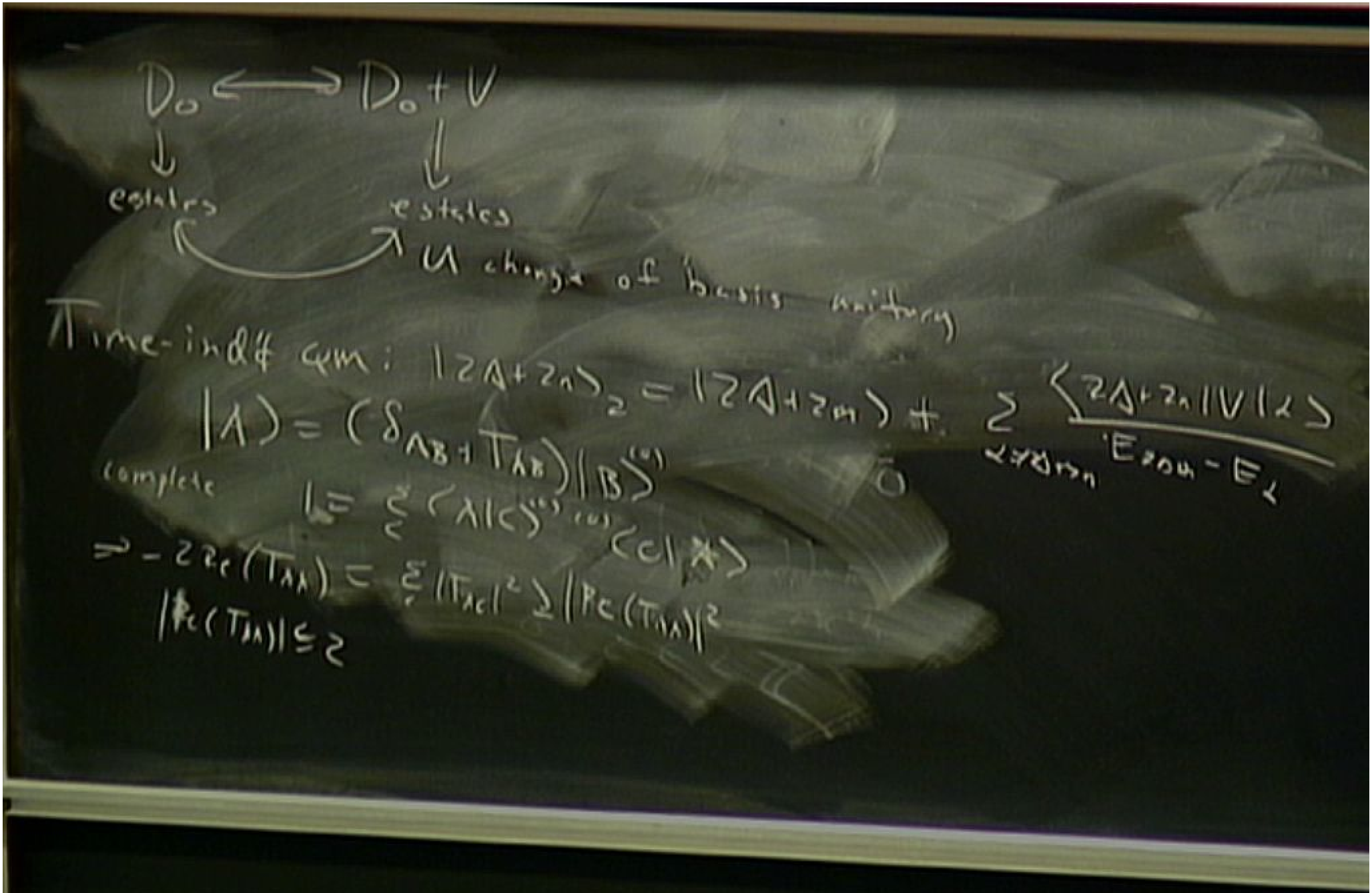

$$\begin{tikzcd} D_0 \arrow[r, leftrightarrow] \arrow[d] & D_0+V \arrow[d] \\ \text{estates} \arrow[r, bend right, "U"'] & \text{estates} \end{tikzcd}$$

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Time-ind't qm: $|2\Delta+2n\rangle_2 = |2\Delta+2n\rangle + \sum_{\alpha\neq\Delta+n} \frac{\langle 2\Delta+2n|V|\alpha\rangle}{E_{\Delta n}-E_\alpha}$

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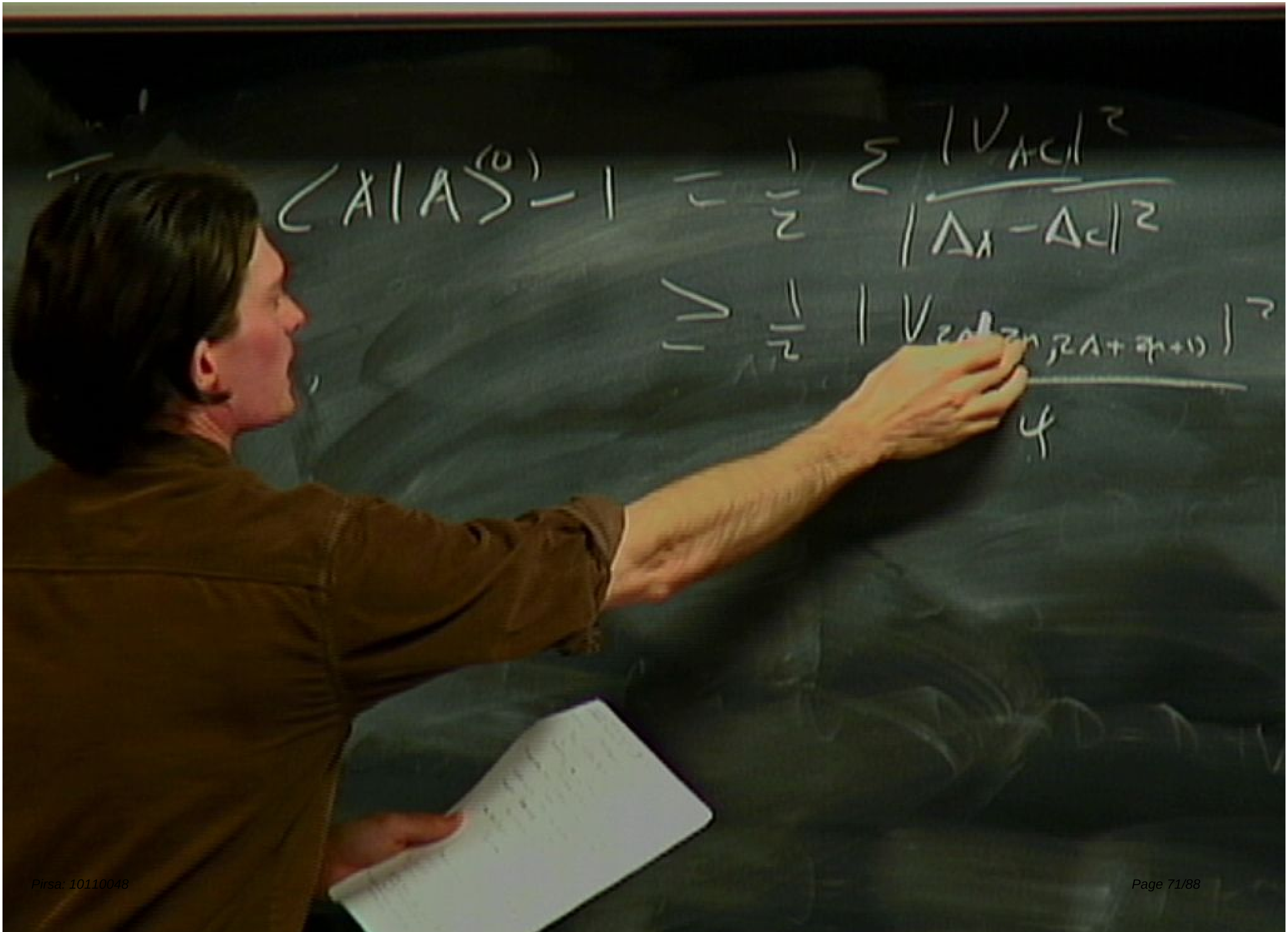
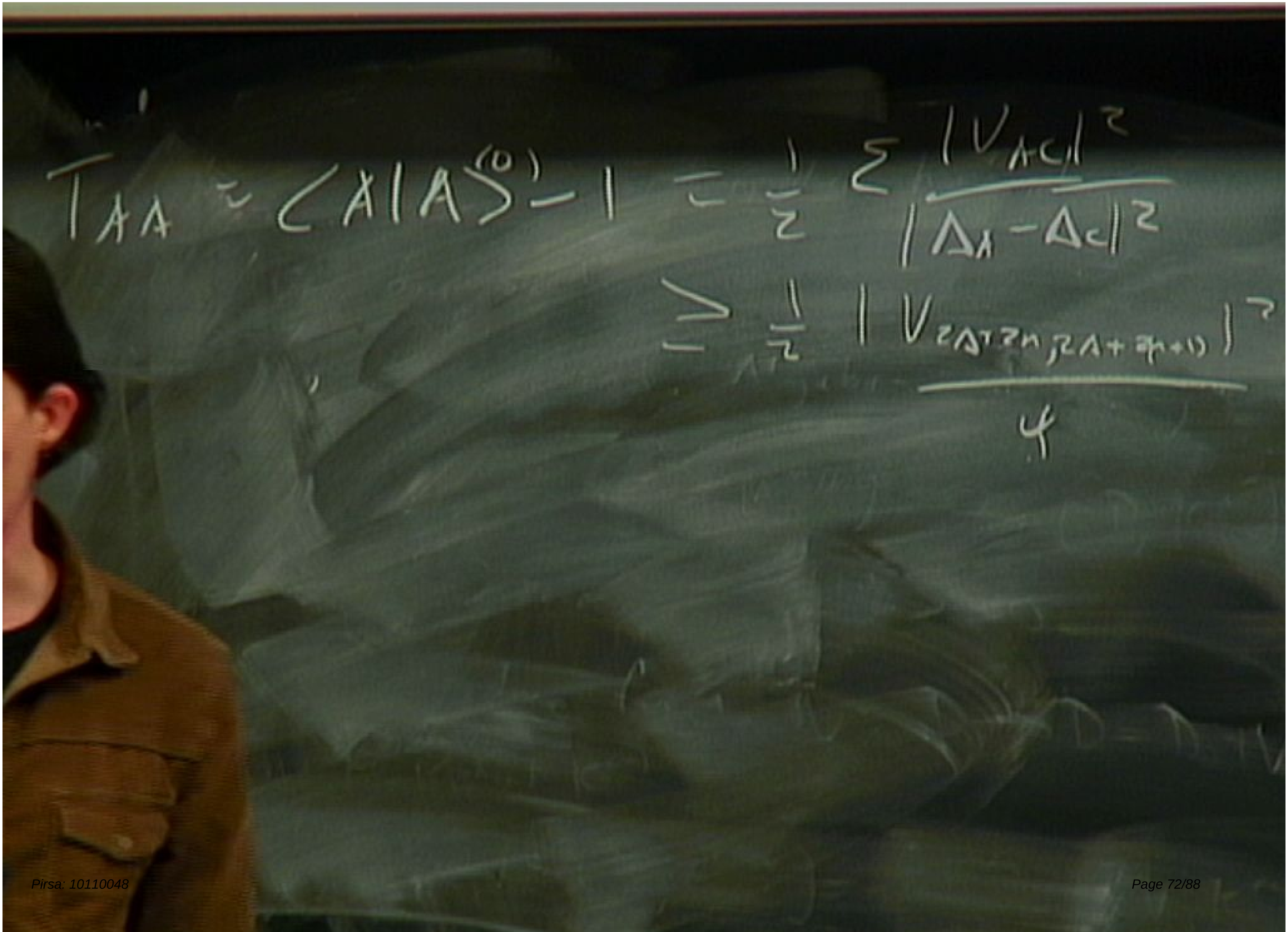


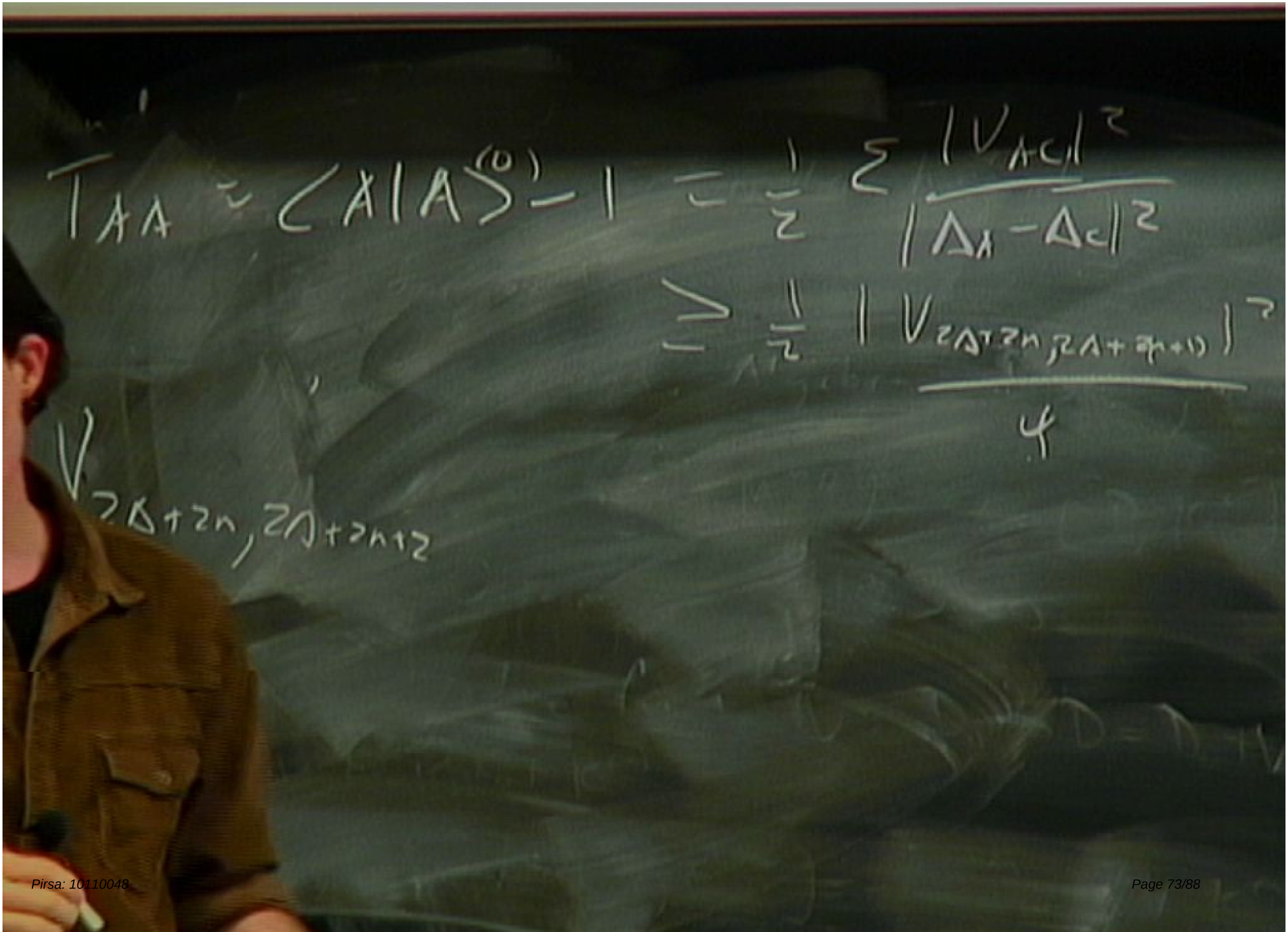
$$\langle A|A\rangle^{(0)} - 1 = \frac{1}{2}\sum \frac{|V_{AC}|^2}{|\Delta_A - \Delta_C|^2}$$

$$\geq \frac{1}{2}\,\frac{|V_{2\Lambda\,2n,\,2\Lambda+2(n+1)}|^2}{4}$$



$$T_{AA} \simeq \langle A|A\rangle^{(0)} - 1 = \frac{1}{2}\sum \frac{|V_{AC}|^2}{|\Delta_A - \Delta_C|^2}$$

$$\geq \frac{1}{2} \frac{|V_{2\Delta+2n,\, 2\Delta+2(n+1)}|^2}{4}$$



$$T_{AA} \simeq \langle A | A \rangle^{(0)} - 1 = \frac{1}{2} \sum \frac{|V_{AC}|^2}{|\Delta_A - \Delta_C|^2}$$

$$\geq \frac{1}{2} \frac{|V_{2\Delta+2n,2\Delta+2(n+1)}|^2}{4}$$

$$V_{2\Delta+2n,2\Delta+2n+2}$$

$$T_{AA} = \langle A | A \rangle^{(0)} - 1 = \frac{1}{2} \sum \frac{|V_{Ac}|^2}{|\Delta_A - \Delta_c|^2} \qquad 2\Delta + 2n + \gamma(n)$$

$$\geq \frac{1}{2} \frac{|V_{2\Delta+2n, 2\Delta+2n+2}|^2}{4} \qquad \mathcal{O}(\partial^2)^n \mathcal{O}$$

$$V_{2\Delta+2n, 2\Delta+2n+2} = V_{2\Delta+2n, 2\Delta+2n} = \gamma(n) \quad \text{anomalous dim of } |2\Delta+2n\rangle$$

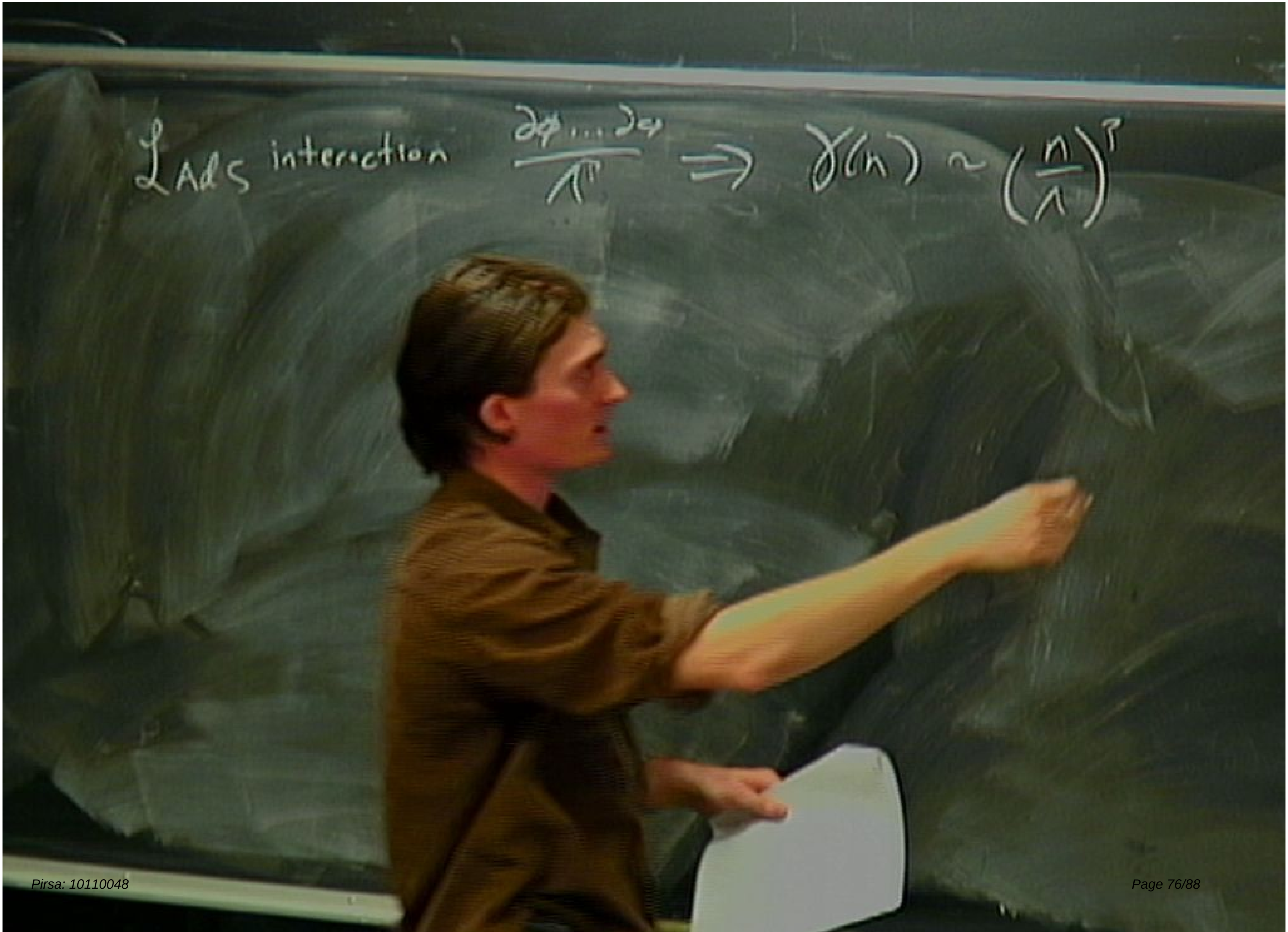
$$\Rightarrow |\gamma(n)| < 4$$

$$T_{AA} = \langle A|AS^{(1)} - 1 = \frac{1}{2}\sum \frac{|V_{AC}|^2}{|\Delta_A - \Delta_C|^2} \qquad 2\Delta + 2n + \gamma(n)$$

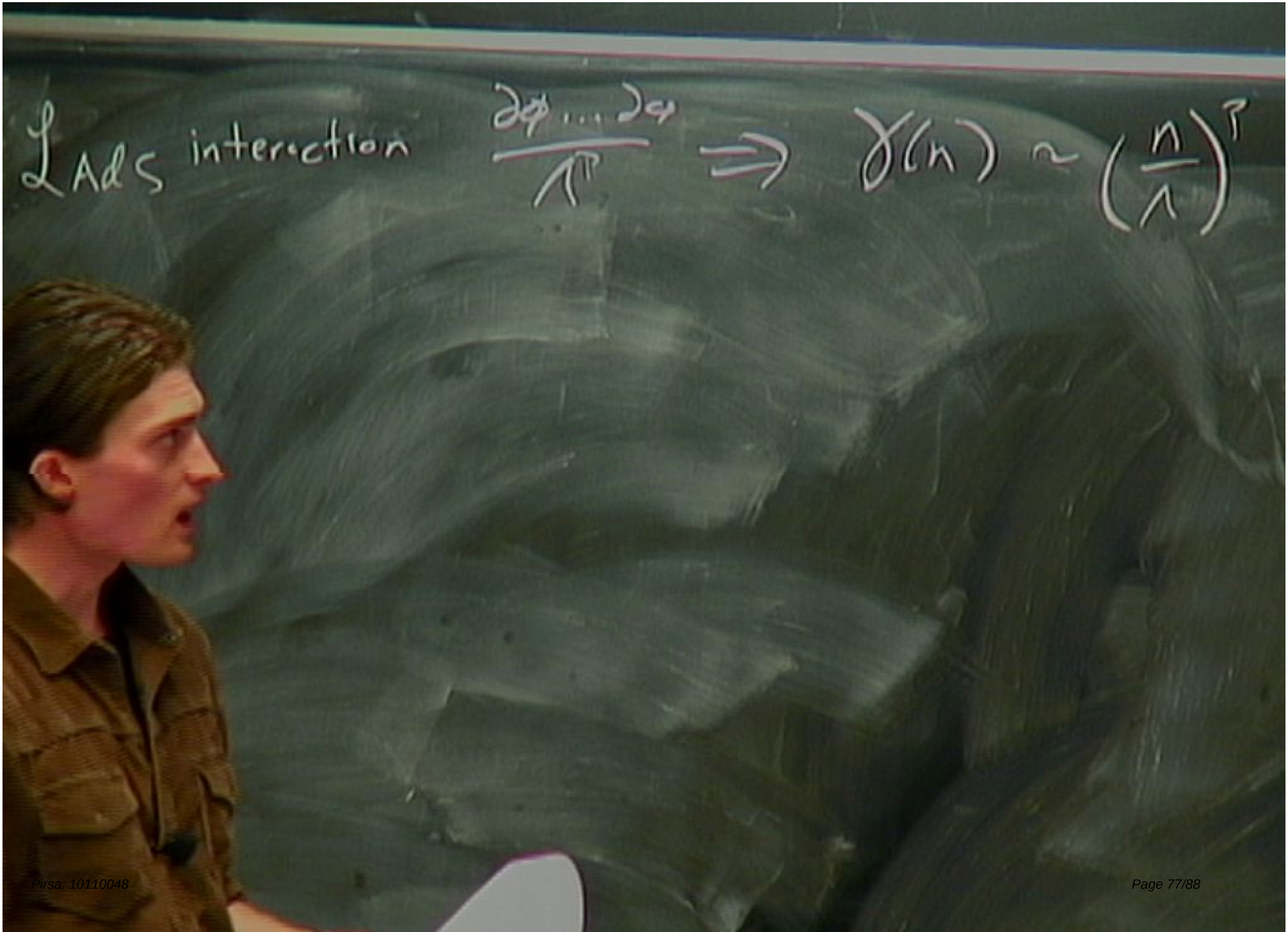
$$\geq \frac{1}{2}\frac{|V_{2\Delta+2n,2\Delta+\gamma(n)}|^2}{4} \qquad \mathcal{O}(\partial^2)^n\mathcal{O}$$

$$V_{2\Delta+2n,2\Delta+2n+2} = V_{2\Delta+2n,2\Delta+2n} = \gamma(n)$$ anomalous dim of $|2\Delta+2n\rangle$

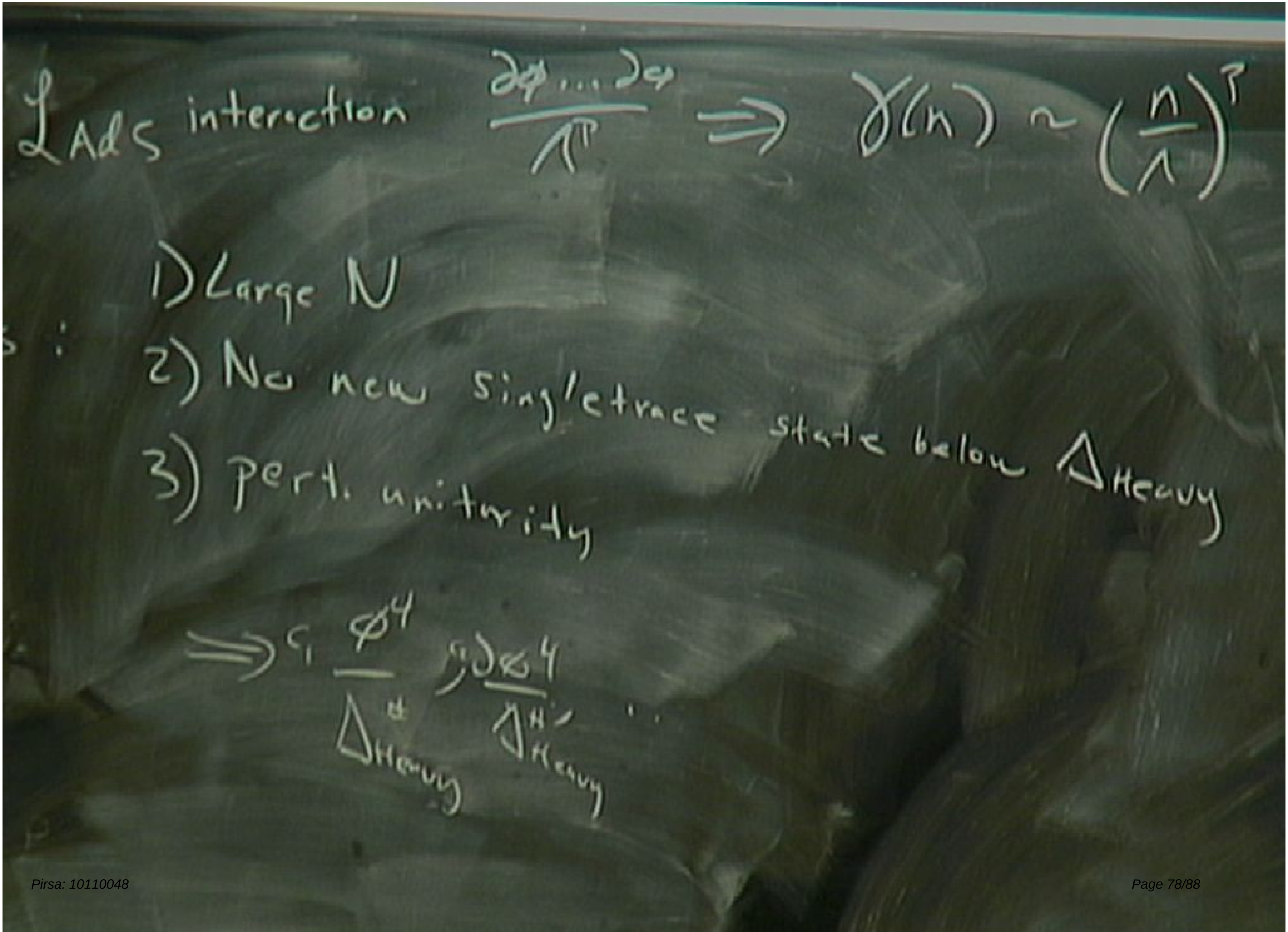
$$\Rightarrow |\gamma(n)| < 4$$



$\mathcal{L}_{AdS}$ interaction $\frac{\partial\phi \ldots \partial\phi}{\Lambda^{n}} \Rightarrow \gamma(n) \sim \left(\frac{n}{\Lambda}\right)^{?}$



$$\mathcal{L}_{AdS}\ \text{interaction}\quad \frac{\partial\phi\ldots\partial\phi}{\Lambda^{p}} \Rightarrow \gamma(n) \sim \left(\frac{n}{\Lambda}\right)^{p}$$

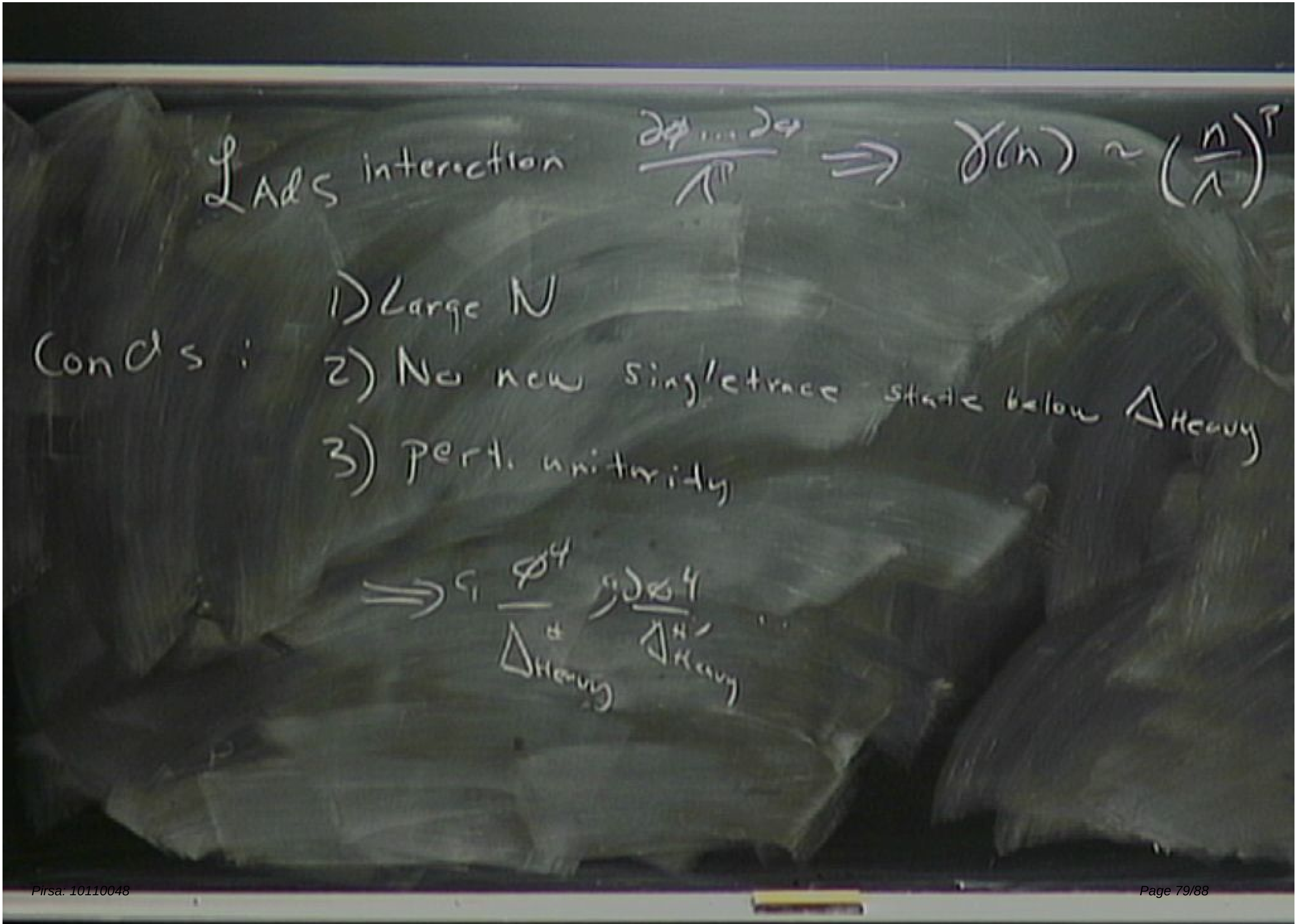


$\mathcal{L}_{AdS}$ interaction $\frac{\partial\phi \ldots \partial\phi}{\Lambda^P} \Rightarrow \gamma(n) \sim \left(\frac{n}{\Lambda}\right)^P$

s :

1) Large N
2) No new single trace state below Δ_{Heavy}
3) pert. unitarity

$$\Rightarrow c_1 \frac{\phi^4}{\Delta_{Heavy}^{\#}} , c_2 \frac{\partial\phi^4}{\Delta_{Heavy}^{\#'}} \ldots$$



$\mathcal{L}_{AdS}$ interaction $\frac{\partial\phi \ldots \partial\phi}{\Lambda^p} \Rightarrow \gamma(n) \sim \left(\frac{n}{\Lambda}\right)^p$

Cond's :

1) Large N
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$\mathcal{L}_{AdS}$ interaction $\frac{\partial\phi \ldots \partial\phi}{\Lambda^{\#}} \Rightarrow \gamma(n) \sim \left(\frac{n}{\Lambda}\right)^{\#}$

Cond's :

1) Large N
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$$\Rightarrow c_1 \frac{\phi^4}{\Delta_{\text{Heavy}}^{\#}} , c_2 \frac{\partial\phi^4}{\Delta_{\text{Heavy}}^{\#'}} \ldots$$

$\mathcal{L}_{AdS}$ interaction $\frac{\partial\phi \ldots \partial\phi}{\Lambda^{r}} \Rightarrow \gamma(n) \sim \left(\frac{n}{\Lambda}\right)^{r}$ N

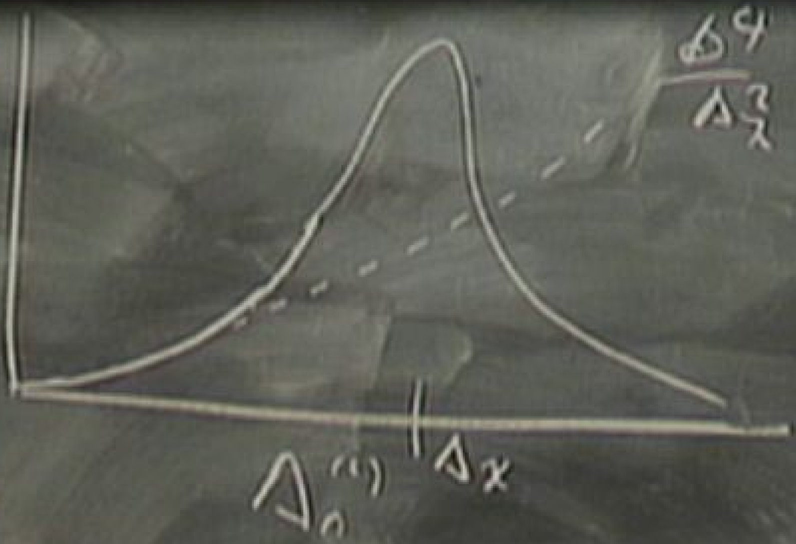
s :

1) Large N
2) No new single trace state below Δ_{Heavy}
3) pert. unitarity

$\Rightarrow c_1 \frac{\phi^4}{\Delta_{Heavy}^{\#}},\ c_2 \frac{(\partial\phi)^4}{\Delta_{Heavy}^{\#'}} \ldots$

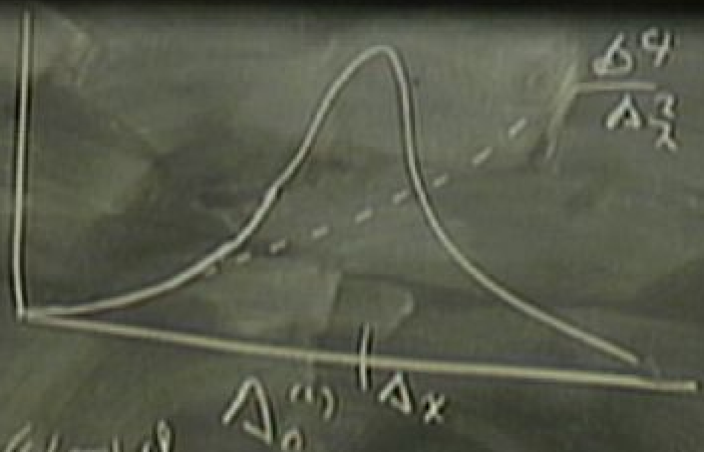
$\langle 0|\psi(x)|2\Delta+2n\rangle$
$\sim \cos^{2\Delta+2n}$

$\mathcal{L}_{AdS_5} \supset \phi^2 \chi \Rightarrow \gamma(n)$



(AdS_5)

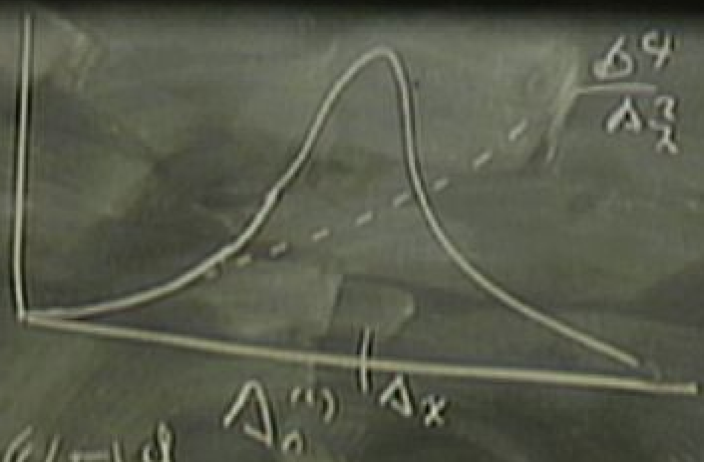
$\mathcal{L}_{AdS_5} \supset \phi^2 \chi \Rightarrow \gamma(n)$



(AdS_5)

$$\mathcal{M}^{(d+1)}_{\substack{\text{flat-space}\\ 2\to 2}} = \frac{(4\pi)^d}{\mathrm{Vol}(S^{d+1})} \frac{E_r}{(E_r^2-4(m^2))^{\frac{d-2}{2}}} \sum \gamma(n,\ell)\Big|^{\frac{1}{2}}_{n\gg\ell} P_\ell^{(d+1)}(\cos\theta)$$

$\mathcal{L}_{AdS_5} \supset \phi^2 \chi \Rightarrow \gamma(n)$

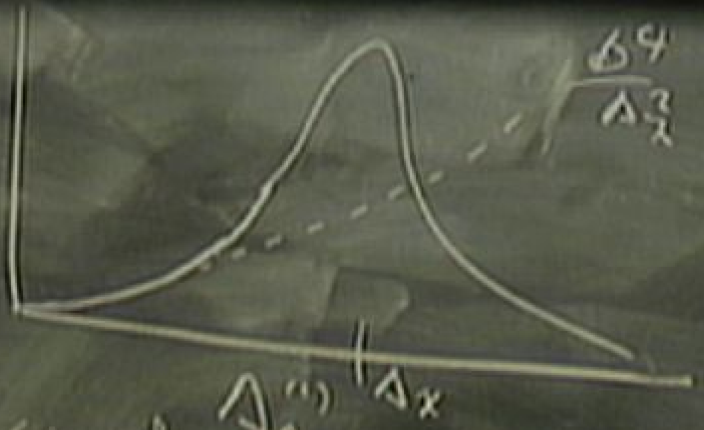


$$\mathcal{M}^{(d+1)}_{\substack{\text{flat-space}\\ 2\to 2}}(s,\gamma_n) = \frac{(4\pi)^d}{\text{Vol}(S^{d-1})} \frac{E_r}{(E_r^2-4m^2)^{\frac{d-2}{2}}} \sum \gamma(n,\ell)\big|_{n\gg\ell} P^{(d+1)}_{\ell}(\cos\theta)$$

$\mathcal{M} \sim T$-matrix [illegible]

$\gamma(n)$ [illegible]

$\mathcal{L}_{AdS_5} \supset \phi^2 \chi \Rightarrow \gamma(n)$

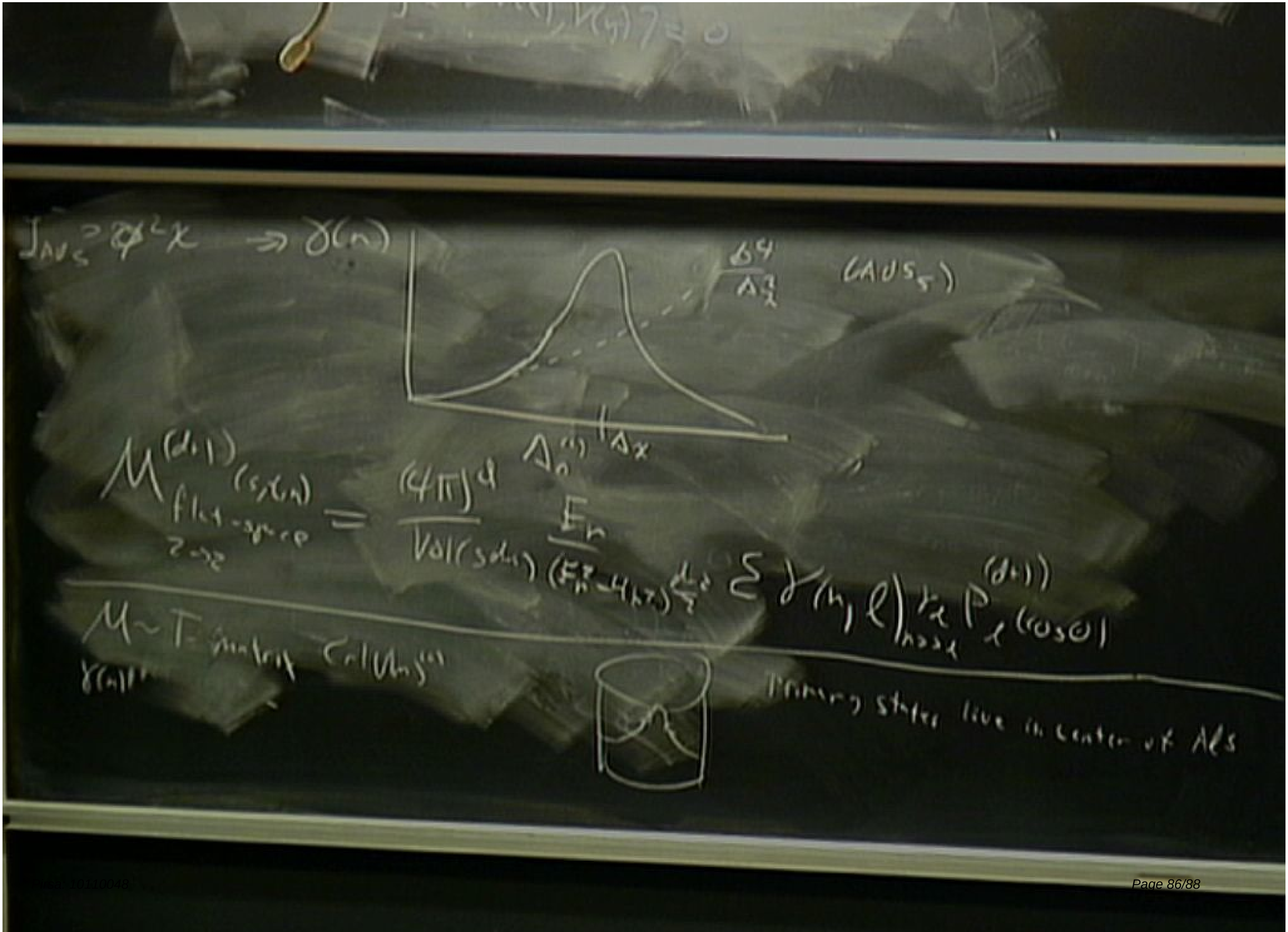


(AdS_5)

$$M^{(d+1)}_{\substack{\text{flat-space} \\ 2\to 2}}(s,\gamma_n) = \frac{(4\pi)^d}{\text{Vol}(S^{d+1})} \frac{E_n}{(E_n^2-4m^2)^{\frac{d-3}{2}}} \sum \gamma(n,\ell)\Big|_{n\gg x} P_\ell^{(d+1)}(\cos\theta)$$

$M \sim$ T-matrix [illegible]

$\gamma(n)|$



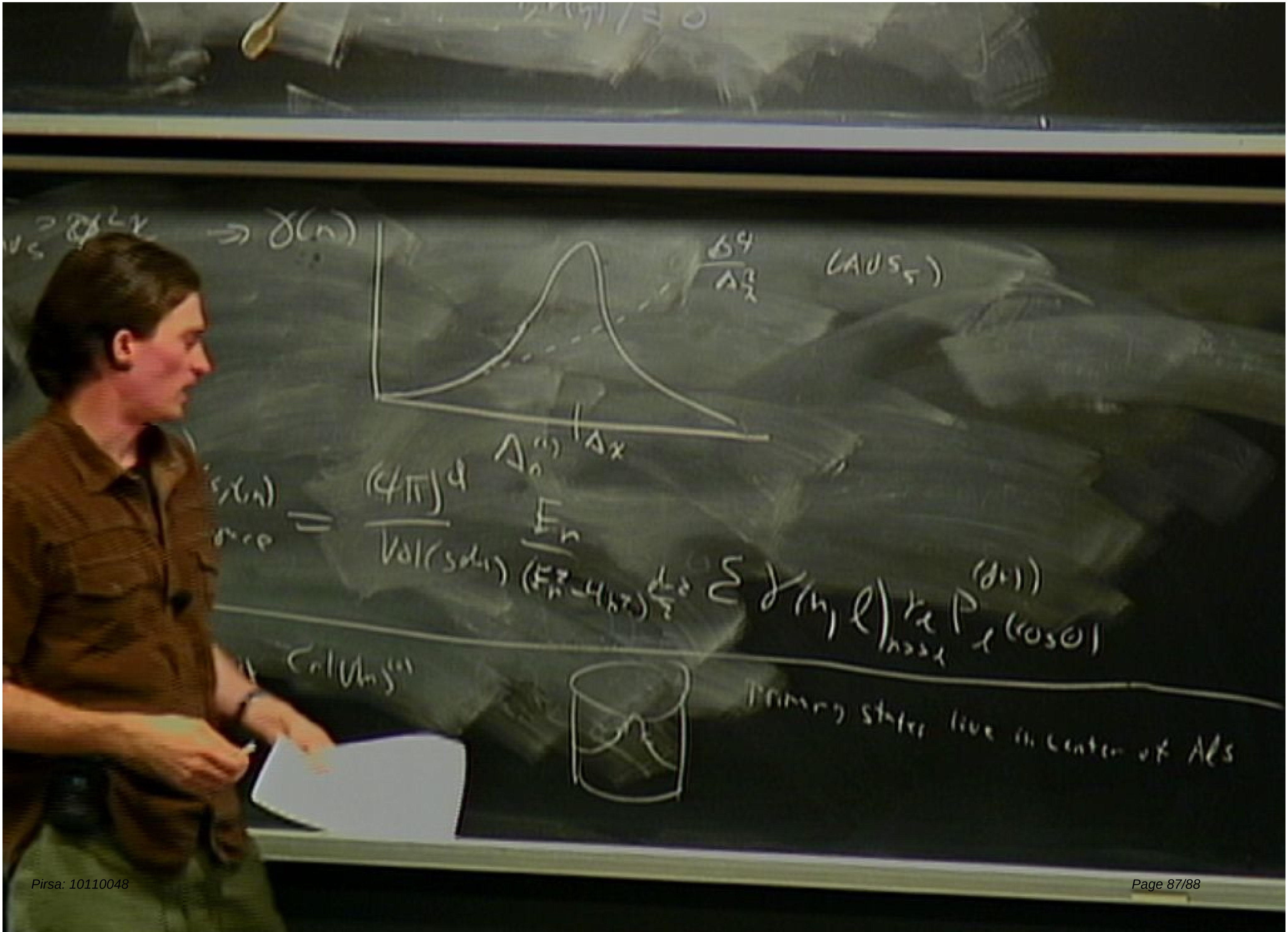
$J_{AdS_5} \supset \phi^2 \chi \Rightarrow \gamma(n)$

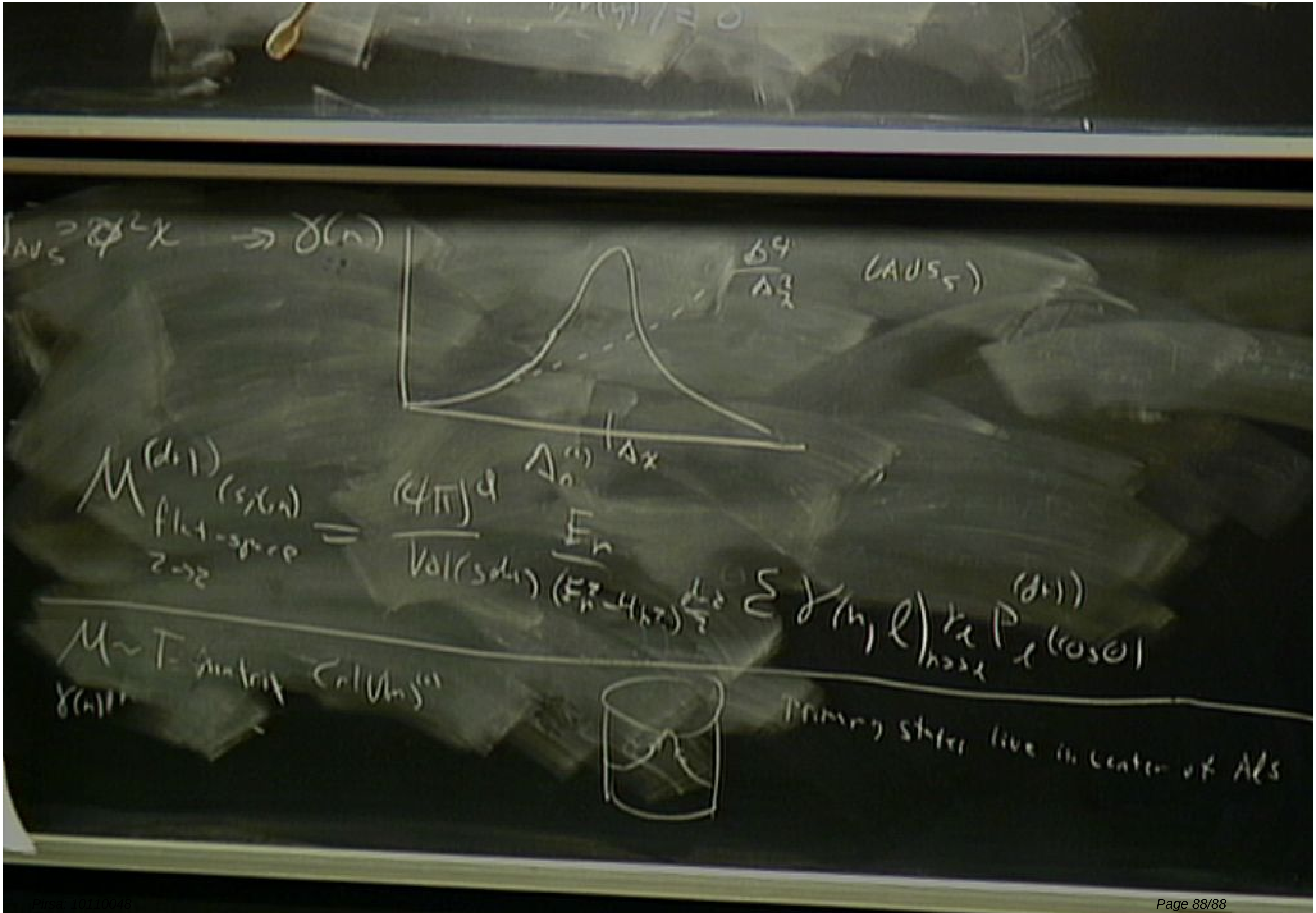
$$M^{(d+1)}_{\substack{\text{flat-space} \\ R\to\infty}}(s,t_n) = \frac{(4\pi)^d}{\text{Vol}(S^{d-1})} \frac{E_r}{(E_r^2-4m^2)^{\frac{d-2}{2}}} \sum \gamma(n,\ell)\Big|_{n\gg\ell}^{E_r} P_\ell^{(d+1)}(\cos\theta)$$

$M \sim T$-matrix Coll(Am)$^{(1)}$

$\gamma(n)|$

Primary states live in center of AdS


$\Rightarrow \gamma(n)$
Δ^4 / Δ_χ^2
(AdS$_5$)
$\Delta_n^{(n)}$
Δ_χ
$(4\pi)^d$
Vol(S^{d-1})
E_n
$\sum \gamma(n,\ell)\big|_{n\gg\ell}$
$P_\ell^{(d-1)}(\cos\theta)$
Primary states live in center of AdS



$\int_{AdS_5} \phi^2 \chi \Rightarrow \gamma(n)$

$$\mathcal{M}^{(d+1)}_{\substack{\text{flat-space}\\ 2\to 2}}(s,\gamma(n)) = \frac{(4\pi)^d}{\mathrm{Vol}(S^{d+1})} \frac{E_n}{(E_n^2-4(m^2))^{\frac{d-2}{2}}} \sum \gamma(n,\ell)\Big|^{r_\ell}_{n\gg\ell} P_\ell^{(d+1)}(\cos\theta)$$

$\mathcal{M} \sim T$-matrix Coll(Am)

$\gamma(n)$

Primary states live in center of AdS