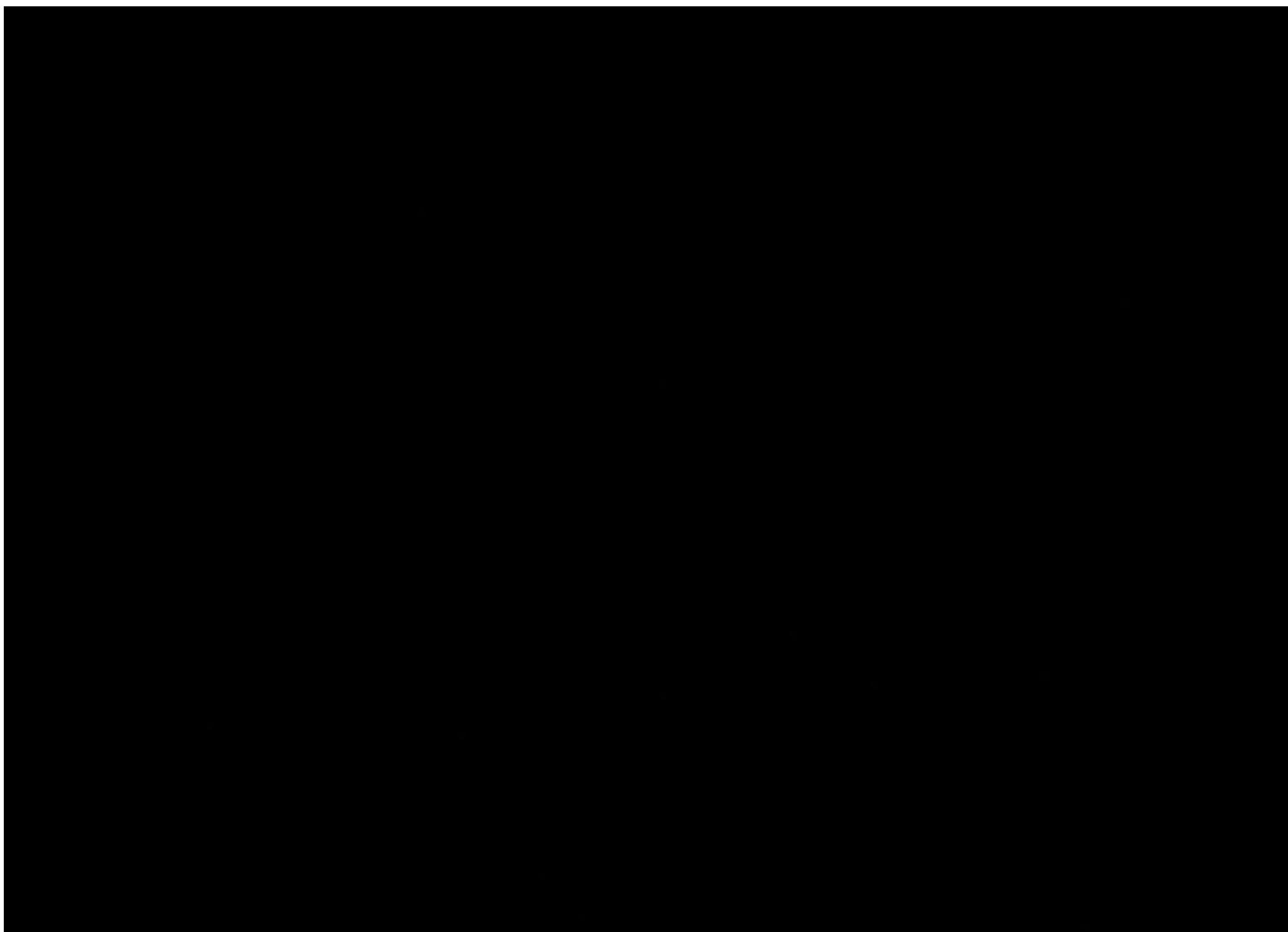


Title: Effective Conformal Theory and the Flat-space Limit of AdS

Date: Nov 19, 2010 02:30 PM

URL: <http://pirsa.org/10110048>

Abstract: The idea of an effective conformal theory describing the low-lying spectrum of the dilatation operator in a CFT is developed. Such an effective theory is useful when the spectrum contains a hierarchy in the dimension of operators, and a small parameter whose role is similar to that of $1/N$ in a large N gauge theory. These criteria insure that there is a regime where the dilatation operator is modified perturbatively. Global AdS is the natural framework for perturbations of the dilatation operator respecting conformal invariance, much as Minkowski space naturally describes Lorentz invariant perturbations of the Hamiltonian. Assuming that the lowest-dimension single-trace operator is a scalar, O , I consider the anomalous dimensions, $\gamma(n,l)$, of the double-trace operators of the form $O (\partial^2)^n (\partial)^l O$. Purely from the CFT, perturbative unitarity places a bound on these dimensions; non-renormalizable AdS interactions lead to violations of the bound at large values of n . I also consider the case that these interactions are generated by integrating out a heavy scalar field in AdS. The presence of the heavy field "unitarizes" the growth in the anomalous dimensions, and leads to a resonance-like behavior in $\gamma(n,l)$ when n is close to the dimension of the CFT operator dual to the heavy field. Finally, I demonstrate that bulk flat-space S-matrix elements can be extracted from the large n behavior of the anomalous dimensions. This leads to a direct connection between the spectrum of anomalous dimensions in d -dimensional CFTs and flat-space S-matrix elements in $d+1$ dimensions



Effective Conformal Theories / Flat space limit
of AdS

Workshop

Effective Conformal Theories

Flat space limit
of AdS

Work by E. Katz, D. Poland, D. Simmons-Duffin 1002.2412



Effective Conformal Theories / Flat space limit of AdS

Work by S. Katz, D. Poland, D. Simmons-Duffin 1002.2412

I. Heemskerk, J. Penedones, J. Polchinski, J. Sully 0907.0151

Effective Conformal Theories

Mark I. Katz, D. Poland, D. Simmons-Duffin, 1002

Hemelrijk, J., Penedones, J., Polchinski, J., Sully, O
Outline & Motivations:

Effective field theory in AdS \hookrightarrow ?? in CFT

Effective Conformal Theories

Flor S

- Work by E. Kiritsis, D. Poland,

- Dutta, 1002.24.12

I. Heemskerk, J. Penedones,

, J. Smit, 0907.01.1

Outline & Motivations:

1) Effective field theory

AdS eff. theory

$L \supset \partial_\mu \dots A^\mu$

Higher-dim int.

$\sigma, h_{\mu\nu}, \dots$

CFT

CFT



Effective Conformal Theories / First of

- Work by E. Witten, D. Poland, D. Simmons-Duffin, 1003.2412

I. Heemskerk, J. Maldacena, J. Neddermann, J. Polchinski, J. Sully, 0907.0151

Outline 2

DF

A

m - s,

AdS \leftrightarrow CFT



CFT



Effective Conformal Theories / Flat

- Work by F. Ruzz, D. Park, J. Zaanen-Dutten, 1003.2412

I. Heemskerk, J. Penedones, J. van Loon, J. Sully 0907.0191

Outline & Motivation

D Effective Field Theory
AdS eff. theory

A $L \gg 2$
 $m = \delta, h_{\mu\nu}, \dots$
Higher-order

CFT

CFT

AdS₅

Effective Conformal Theories / Flows

- Work w/ E. Kiritsis, D. Polarski
- M. Gabella, J. Zaanen-Duvenhage, 1002.7412

I. Heemskerk, J. Penedones, J. van Rees, J. Smillie, 0907.051

Outline & Motivation:

D Effective field theory
AdS eff. theory

A

m

$$L \supset \frac{d_4}{\lambda} \dots \Lambda$$

Higher-dim terms

How specialize?

?? in CFT

??

) Trial it out;

) CFT

CFT

AdS_{D+1}

Effective Conformal Theories / Flat S

- Work by E. Kiritsis, D. Kleban, D. Simmons-Duffin, 1002.27412

I. Heemskerk, J. P. M. G. Veltman, J. Polchinski, D. Smit, 0907.0151

Outline & Motivation

D Effective CFT
AdS eff.

A

$m = \delta, h, \dots$

? in CFT

? in CFT

Fault of wind;

CFT views about in

CFT

AdS₅

Effective Conformal Theories

Fluct.

- Work w/ E. Kiritsis, D. Poland, D. Simmons-Duffin 1003.2412

I. Heemskerk, J. Penedones, J. Polchinski, D. Sully 0907.0151

Outline & Motivations:

1) Effective field theory in AdS₅ \Leftrightarrow 2) in-CFT

AdS eff. theory

$$\mathcal{L} \supset \partial_\mu \dots A^\mu$$



- ??
- 1) Twists of fields,
- 2) CFT vacua above in-CFT

$\delta, h_{\mu\nu}, \dots$

Higher-dim inst. appears

How special is RSP?

((Interactions & local operators in AdS))
Operators & structures operator



Worlfram, S. Carlitz, D. Poland, D. Sim

I. Heemskerk, J. Penedones, J. Pol.

Outline & Motivations:

D) Effective field theory in AdS

↑ —

AdS eff. Theory

$$\mathcal{L} \supset \frac{\partial_\mu \dots A}{\lambda^P}$$

m — $\delta_{\text{grav}}, \dots$

Higher-dim inst.

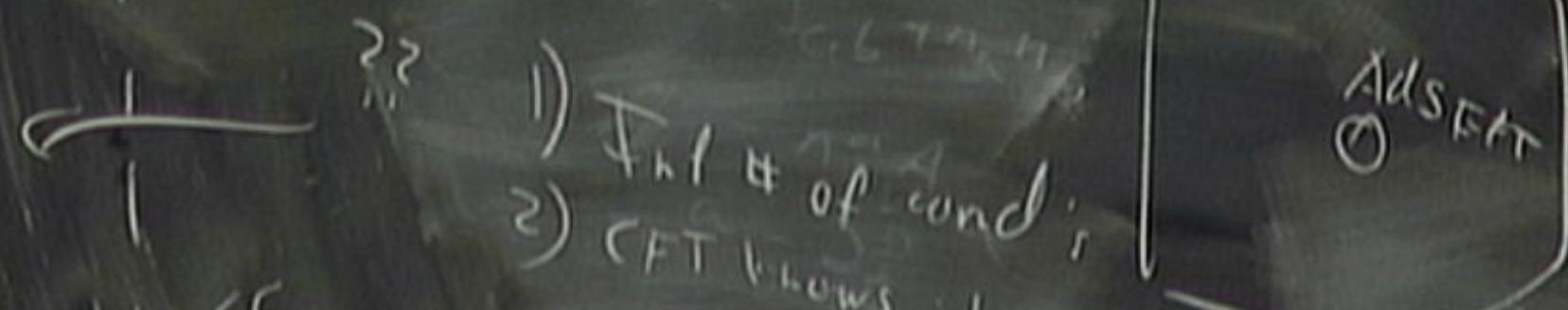
How special is RS?

of Ad

Poland, D. Simmons-Duffin 1002.24.2

Konos, J. Polchinski, J. Sally 0907.01.51

in AdS \leftrightarrow ?? in CFT

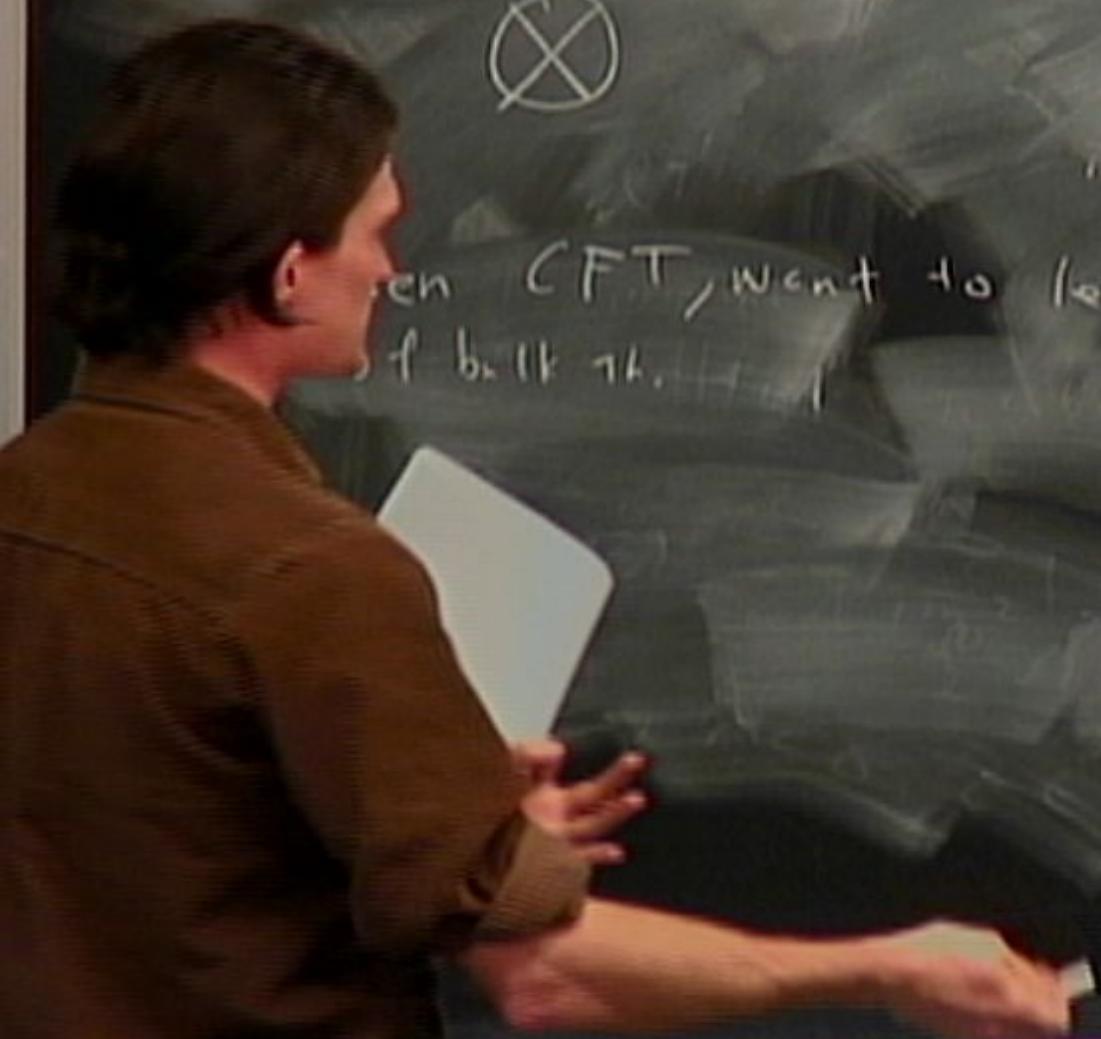


5 suppressed | ((Interactions are local terms in AdS),
RSS | Operator is local in CFT sense)

2) Singlet calculations of spinons



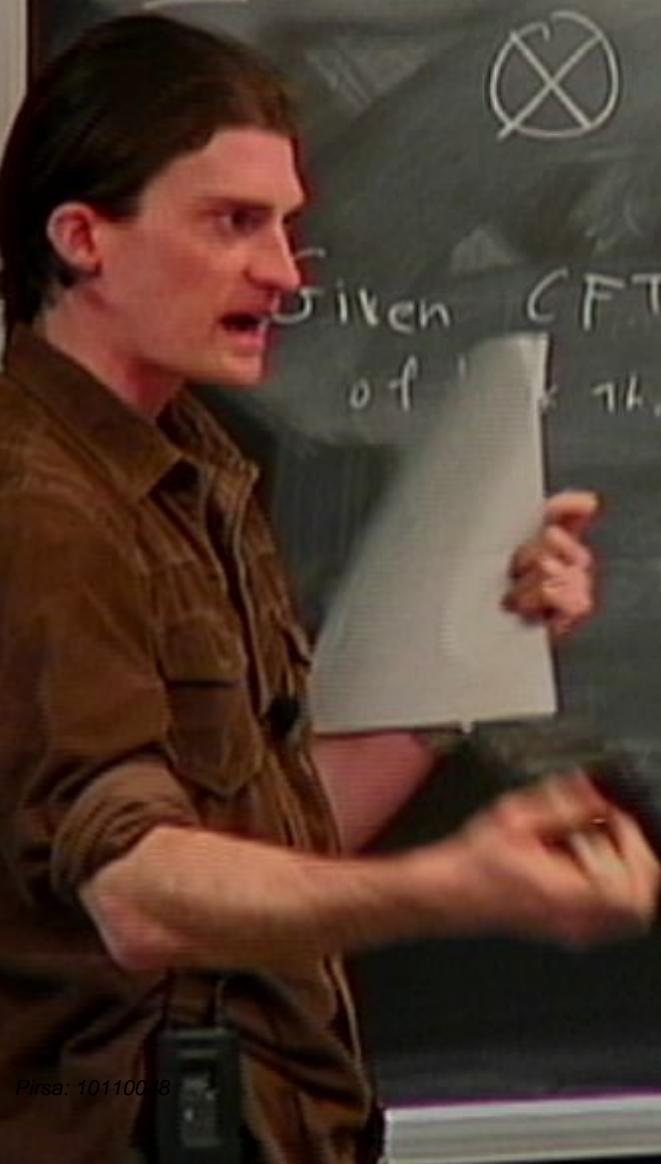
then CFT, want to learn about scatter amplitudes
of bulk th.



2) Singular calculations of spectrum



Given CFT, want to learn about scatter amplitudes
of $\langle \dots \rangle_{\text{th}}$.



2) Singular calculations of spectrum



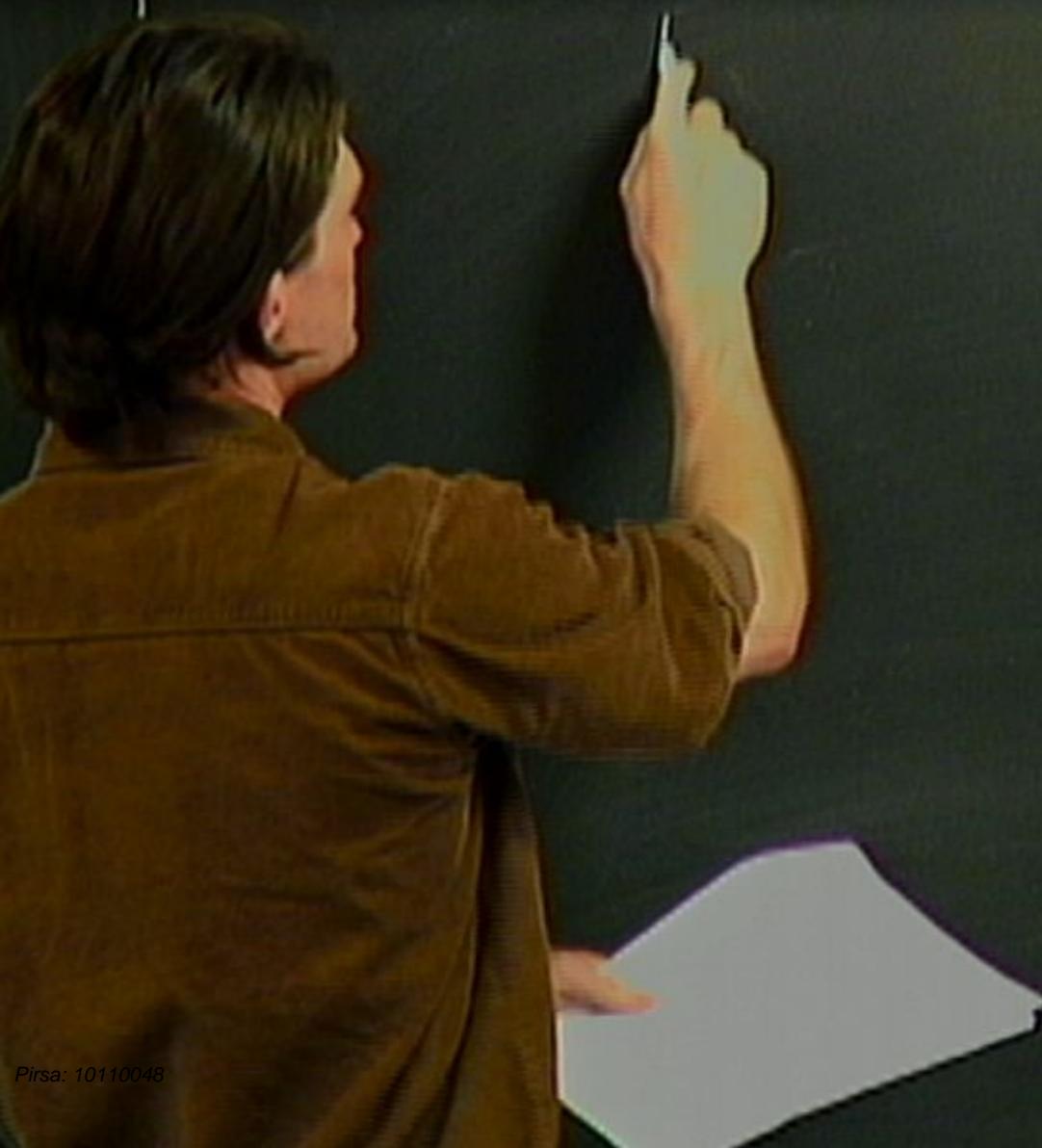
3) Given CFT, want to learn about scattering amplitudes
of bulk th.

CFT lives on bound-



Develop Architectural Drawing's "00000000"

part \oplus QM \oplus L.I \Rightarrow QFT in Mink.



part \oplus QM \oplus L.I \Rightarrow QFT in Mink.

Mink/LFT correspond

part 4) QM + L.I. \Rightarrow QFT in Mink.

Mink/LFT correspond

W.H.

Part ④ QM + L.I. \Rightarrow QFT in Mink.
Mink/LFT correspond

What does it mean to have c L.I. th. operat. s?

• Fock space

$|0\rangle, |1\rangle, |2\rangle, \dots$

Introduce $a_{\vec{p}_1}^*, a_{\vec{p}_1}, a_{\vec{p}_2}^*, a_{\vec{p}_2}, \dots, a_{\vec{p}_K}^*, a_{\vec{p}_K}$

- Fact: $\omega_p \propto \vec{p}$
 - $(\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \dots)$
 - Introduce $\alpha_{\vec{p}_1, \vec{p}_2} \vec{p}_1, \dots \vec{p}_N \propto \alpha_{\vec{p}_1, \dots \vec{p}_N} \vec{p}_1, \dots \vec{p}_N$
 - Homogeneous $H(\vec{p}) = \omega_0(\vec{p})$
 - Free $\mu_0(\vec{p}_1, \vec{p}_2) = C \omega_{p_1, p_2} / (\vec{p}_1, \vec{p}_2) \dots$
- $$H_0 = \int d^3p \omega_p \alpha_{\vec{p}, \vec{p}}$$

Development of quantum field theory in Minkowski space

Part 1: QM & L.I. \Rightarrow QFT in Mink.

Mink/LFT correspond

• i + mean do have a L.I. op. or. &?

• Fact: $\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \dots$

$\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4, \dots$

Introduce $\vec{p}_1, \vec{p}_2, \vec{p}_3, \dots, \vec{p}_k, \omega_1, \omega_2, \dots$

• Hamiltonian $H(\vec{p}_1, \omega_1, \vec{p}_2, \omega_2, \dots)$

Free: $H_0(\vec{p}_1, \vec{p}_2) = \sum \omega_p (\omega_p)(\vec{p}_1, \vec{p}_2)$

$$H_0 = \sum p^3 \omega_p (\omega_p)(\vec{p}_1, \vec{p}_2, \dots)$$

$\text{P.A.} \oplus \text{Q.M.} \oplus \text{L.I.} \Rightarrow \text{QFT}$ in Mink.

Mink/LFT correspond

What does it mean to have a L.I. op. r.l.s.?

- Fock space

$$|0\rangle, |\vec{p}\rangle, |\vec{p}_1, \vec{p}_2\rangle, \dots$$

Introduce $|\vec{p}_1, \vec{p}_2, \dots, \vec{p}_K\rangle = a_{\vec{p}_1}^{\dagger} \dots a_{\vec{p}_K}^{\dagger} |0\rangle$

- Hamiltonian $H(\vec{p}) = \omega_{\vec{p}} |\vec{p}\rangle$

$$\text{Free } H_0(|\vec{p}_1, \vec{p}_2\rangle = \omega_{\vec{p}_1} \omega_{\vec{p}_2} |\vec{p}_1, \vec{p}_2\rangle)$$

$$H_0 = \sum d^3 p \omega_{\vec{p}} a_{\vec{p}}^{\dagger} a_{\vec{p}}$$

Part 4) QM + L.I. \Rightarrow QFT in Mink.

Mink/LFT correspond

What does it mean to have a L.I. op. r.s.?

- Fock space

- $|0\rangle, |\vec{r}\rangle, |\vec{r}_1, \vec{r}_2\rangle, \dots$

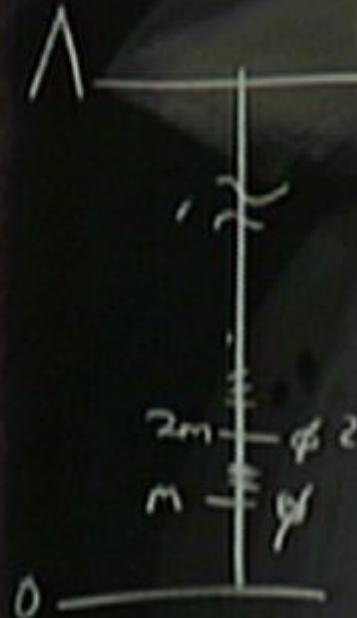
Introduce $|\vec{p}_1, \vec{p}_2, \vec{p}_3, \dots, \vec{p}_n\rangle = a_{\vec{p}_1}^{\dagger} \dots a_{\vec{p}_n}^{\dagger} |0\rangle$

- Hamiltonian $H(\vec{p}) = \omega_1 |\vec{p}_1\rangle \langle \vec{p}_1| + \dots$

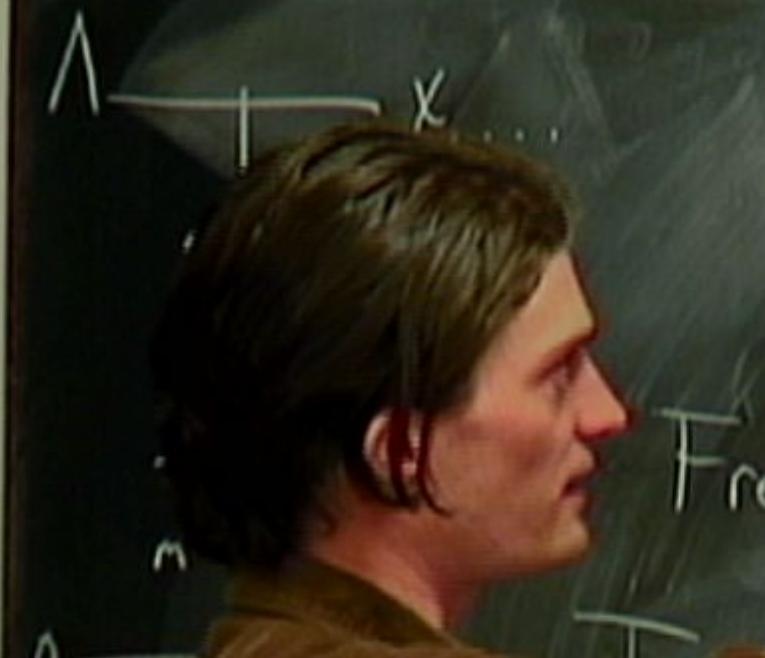
Free. $H_0 (\vec{p}_1, \vec{p}_2) = \frac{1}{2} \omega_1 |\vec{p}_1\rangle \langle \vec{p}_1| + \frac{1}{2} \omega_2 |\vec{p}_2\rangle \langle \vec{p}_2|$

$$H_0 = \int d^3 p \omega_p a_{\vec{p}}^{\dagger} a_{\vec{p}}$$

Higher-dim in 1D is “represented”
How special is \mathcal{P} ? \mathcal{H} , \mathcal{T}^i , \mathcal{J}^i , \mathcal{J}^i
“Interaction”
“Operator”



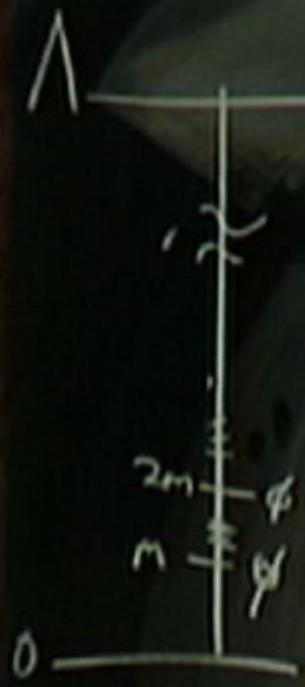
Poincaré $\mathcal{H}, \mathcal{T}^i, \mathcal{J}^i, \mathcal{J}^i$
 $(\mathcal{J}^i, \mathcal{H})$



$$\begin{aligned} & \xrightarrow{\text{Poincaré}} H, T^i, \epsilon^{ijk} J^j \\ & [J^i] = -\nabla T^i, HJ = 0 \\ & [T^i] = -\epsilon^{ijk} P^j \end{aligned}$$

Free theory - done!

Interactions



• Poincaré H, T^i, ψ, J^i
 $[J^i, H] = i[T^i, H] = 0$
 $[t^i, H] = -iP^i$

Theories - done!

Induce interactions -



• Poincaré H, T^i, ψ, J^i
 $[J^i, H] = i[T^i, H] = 0$
 $[J^i, J^j] = iP^i_{ij}$

Free theories - done!

Introduce interactions - difficult



Poincaré H, P^i, ψ, J^i

$$[J^i, H] = [P^i, H] = 0$$

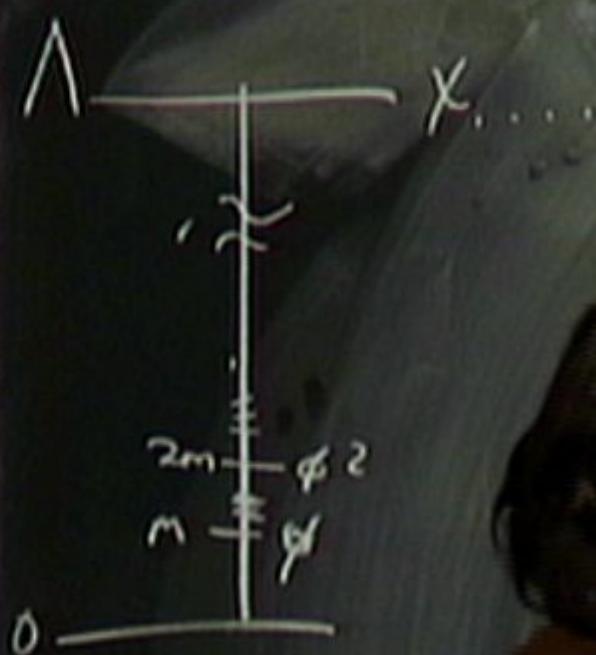
$$[\psi, H] = -i P^i$$

Free theories - done!

Introduce interactions - difficult

$$H_0 \cup H = H_0 + V$$

$$P = P_0, J = J_0, b_n + K = K_0 + W$$



$$\rightarrow \text{Poincaré } H, T, \text{ & } J$$

$$[J, H] = [T, H] = 0$$

$$[L, H] = -iP$$

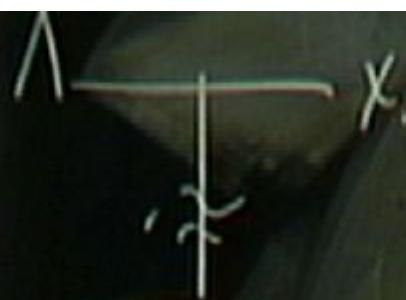
Theories - done!

Sources interactions - diffraction

$$H_0 \cup H = H_0 + V$$

$$J = J_0, \text{ but } K = K_0 + w$$

$$= -(w, H, T) - (w, V)$$



• Ioincare $\mathbb{R}, \mathbb{P}', \mathbb{K}' \mathcal{T}'$

$$[\mathcal{T}', \mathbb{K}] = [\mathbb{P}', \mathbb{K}] = 0$$

$$[\mathbb{K}', \mathbb{K}] = -[\mathbb{P}']$$

Free theories - done!

Introduce interactions - difficult

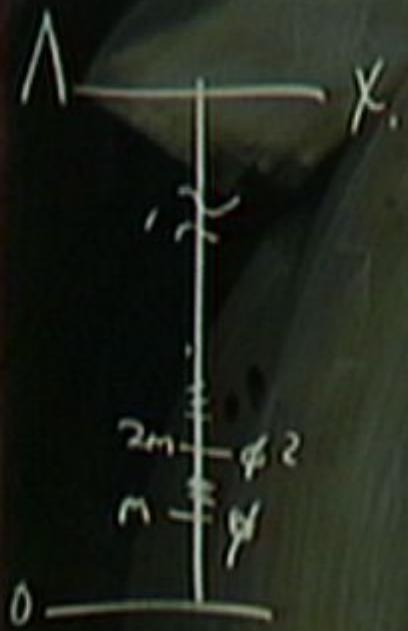
$$H_0 \cup H_1 = H_0 + V$$

$$\mathbb{P} = \mathbb{P}_0, \mathcal{T} = \mathcal{T}_0$$

$$[\mathbb{K}_0, V] = -[\mathbb{W}, \mathbb{H}, \mathcal{T}] - [\mathbb{W}', V]$$

• Pollock $S_{\text{sym}} = \text{sym of } M \otimes I$

$$\text{Solutions: } V = \dots$$



• Poincaré H, T, f, \bar{T}
 $[T, H] = [P, H] = 0$
 $[f, H] = -iP'$

Free theories - done!

Introduce interactions - difficult
 $H_0 \cup H = H_0 + V$

$P = P_0, T = T_0, b_n + k = k_0 + w$
 $[k_0, v] = -[w, H, T] - [w, V]$

• Poincaré $S_{sym} = sym \circ M^{-1} \circ S_{sym}$

Solutions: $V = S_{sym} L_{(1)} L_{(2)}$

cond. 1 $V(x)$ scalar

$$V \rightarrow U_0^+(1, \gamma) V(x) U_0(1, \gamma) = V(\gamma x \gamma^{-1})$$

Build $V(x)$ out of f & φ

cond. 1 $V(x)$ scalar

$$V \rightarrow U_0^\dagger(\lambda, \mu) V(x) U_0(\lambda, \mu) = V(\lambda x + \mu)$$

Building $V(x)$ out of $\varphi(x)$

$$V \rightarrow V(\varphi, \partial\varphi, \dots)$$

$$[F_i, \psi] = \int d^3x [F_i, V(x)] = \int d^3x (\cancel{\partial} V(x) + x \frac{d}{dx} V)$$

cond. 1 $V(x)$ scalar

$$\nabla \rightarrow \mathcal{U}_0^*(\lambda, \cdot) V(x) \mathcal{U}_0(\lambda, \cdot) \in Y(\lambda_{X+u})$$

Building $V(x)$ such that $\varphi(x)$

Then

$$\begin{aligned} c=0 \quad [k^i, v] &= \int d^3x [k^i, V(x)] = \int d^3x (\cancel{\partial} V(x) + x^i \cancel{\frac{\partial}{\partial x}} V) \\ &= [A^i, w] \quad w^i = - \int d^3x x^i V(x) \end{aligned}$$

cond. 1 $V(x)$ scalar

$$V \rightarrow u_0(\lambda_x) V(x) u_0(\lambda_y) = V(\lambda_{x+y})$$

Building $V(x)$ out of $\varphi(x)$

Then

$$V \rightarrow V(\varphi, \partial\varphi, \dots)$$

$$\begin{aligned} c=0 \quad [k^i, v] &= \int d^3x [k^i, V(x)] = \int d^3x (\cancel{\partial} V(x) + k^i \cancel{\frac{\partial}{\partial x}} V) \\ &= [A^i, w^i] \quad w^i = - \int d^3x x^i V(x) \end{aligned}$$

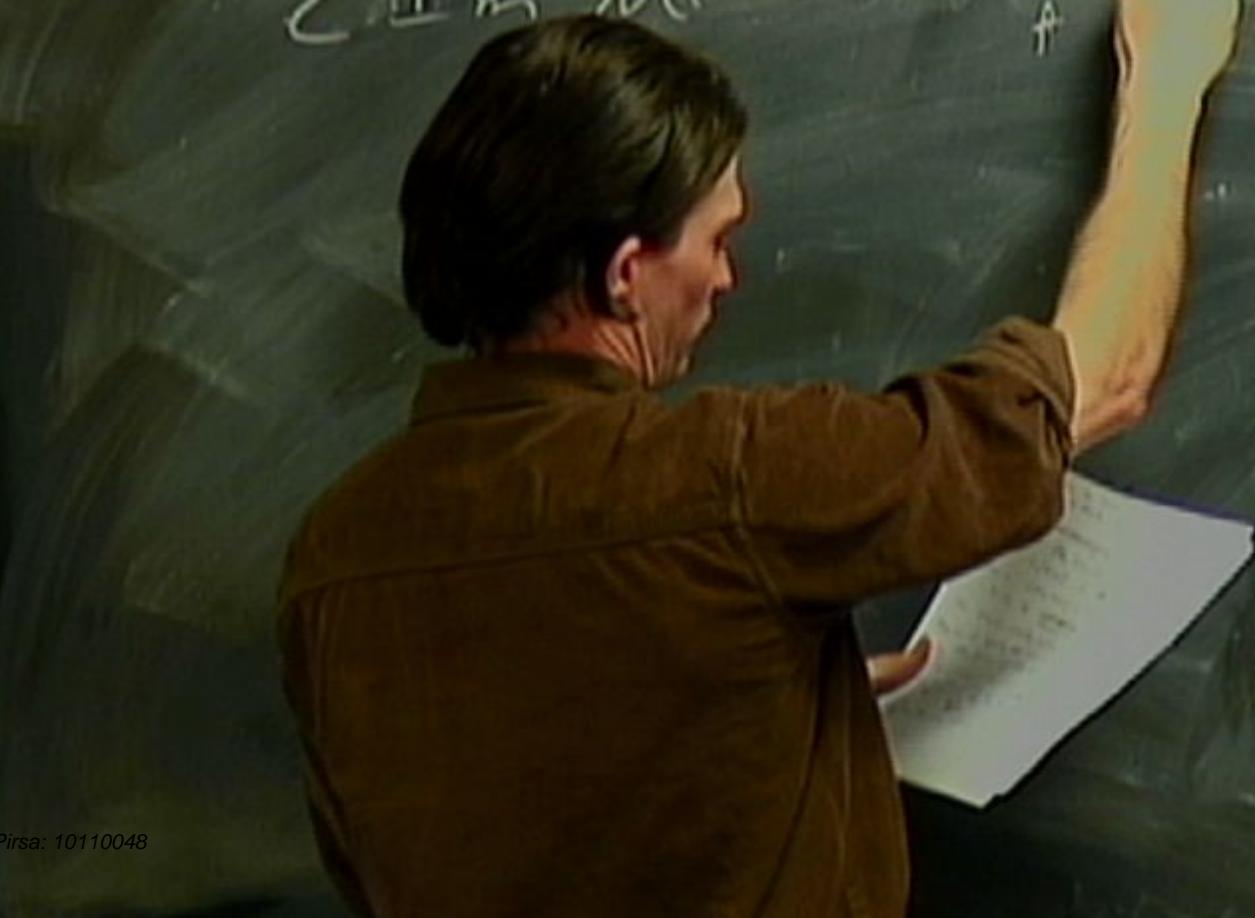
cond 2: $(V(x), V(x')) = 0 \quad (x-x')^2 \underset{x \rightarrow x'}{\rightarrow} 0$

$$(w^i, V) = \int d^3x d^3y k^i [V(x), V(y)] = 0$$

$$\text{Free } H_0 \langle \vec{p}_1, \vec{p}_2 \rangle = \zeta \omega_{\text{min}} \omega_{\text{max}} \langle \vec{p}_1, \vec{p}_2 \rangle \dots$$
$$H_0 = \int d^3 p \omega_p a_{\vec{p}} a_{\vec{p}}$$

Regime of validity \Leftrightarrow unitarity bounds \Rightarrow optical theorem.

$$2 \operatorname{Im} M(s) = \sum_A$$



$$\text{Free } H_0 \langle \vec{p}_1, \vec{p}_2 \rangle = \int d^3 p \omega_{p1} \omega_{p2} \langle \vec{p}_1, \vec{p}_2 \rangle \dots$$

Regime of validity \Leftrightarrow unitarity bounds \Rightarrow optical lab.

$$2 \operatorname{Im} M(a \rightarrow r) = \sum_p \int d\Gamma_p |M(a \rightarrow r)|^2$$

$$\text{Partial waves: } |a_{JM} - \frac{1}{2}| \leq \frac{1}{2}$$

T_P

Regime of validity (\hookrightarrow unitarity bounds \Rightarrow optical th.).

$$2 \operatorname{Im} M(c \rightarrow s) = \sum_f \int d\Gamma_f |M(f \rightarrow s)|^2$$

Partial waves: $|a_{km} - \frac{1}{2}| \leq \frac{1}{2}$

T^P we know part cont up to Λ

$$\Rightarrow \frac{\partial \phi}{\partial P}, \frac{\partial \phi}{\partial \bar{P}}$$

\Rightarrow New part's at $M_{\text{Heavy}} \Rightarrow M_{\text{Heavy}} \lesssim 1$

• Patient Sym = Sym of Minf Space
Solutn. $V = \sum p_i V_{G_i}$

Ans CFT Large N

$N \rightarrow \infty \Leftrightarrow$ CFT becomes "free"

mean field

$$\langle \sigma_1, \sigma_N \rangle = \langle \sigma_1 \rangle \langle \sigma_N \rangle$$

• Palindromic Sym = sym of Mink space
Solitons, $V = \int d^3x L_{GJ}$

AdS/CFT Large N

$N \rightarrow \infty \iff$ CFT becomes "free"

mean field th.: $\langle \phi_i, \phi_j \rangle = \langle \phi_i \phi_j \rangle \langle \phi_N, \phi_N \rangle$

• Fock space

CFT "part." at ∞N ?

• Palermo Sym = sym of minf space
Solutn. $V = \int d^3x L_{\text{eff}}$

AdS/CFT Large N

$N \rightarrow \infty \Leftrightarrow$ CFT becomes "free"

mean field th.: $\langle \phi_1 \dots \phi_N \rangle = \langle \phi_1 \rangle \dots \langle \phi_N \rangle$

• Fock sp.

CFT "pert." at ∞N ?

radial quant. Define multi. on const. mul. ns

• Palindrome Sym = sym of min. space
Solutions: $V = \sum_{i,j} L_{ij}$

AdS/CFT Large N

$N \rightarrow \infty$ \leftarrow CFT becomes "free"

Id th.: $\langle \phi_1 \dots \phi_N \rangle = \langle \phi_1 \phi_2 \rangle \langle \phi_{N-1} \phi_N \rangle$

Prob

CFT

at ∞N ?

adS on const. radius

rotations: D

4 Palermo Sym = sum of min since
Sakhar. $V = \sum_{\mu} V_{\mu}$

AdS/CFT Large N

$N \rightarrow \infty$ CFT becomes "free"

Field th.: $\langle \phi_i, \phi_j \rangle = \langle \phi_i \rangle \langle \phi_j \rangle$

spacetime
CFT at ∞ ?
at ∞N ?

moduli on const. modulus
dilatations D

• Partition Sym = sum of many terms
Sakhar. $V = \int d^3x L_{QJ}$

AdS/CFT Large N

$N \rightarrow \infty \Leftrightarrow$ CFT becomes "free"

mean field th.: $\langle \phi_1, \phi_N \rangle = c \rightarrow \langle \phi_N, \phi_N \rangle$

• Fock space

CFT "part." at ∞N ?

Pediat. quant.

$\circlearrowleft \rightarrow$ Define moduli on const.

evolve w/ dilatations: D

$D \rightarrow$ "Haw. (Renormal.)"

$$= \langle \phi_1, \phi_2 \rangle, \langle \phi_N, \phi_N \rangle$$

operator
primary ϕ \rightarrow state $|\phi\rangle = \phi(0)/\sqrt{c}$
descendant $\phi + D_N \phi$

$$= \langle \delta_1 \phi_1 \rangle, \langle \phi_N, \delta_N \rangle$$

operator
primary δ \rightarrow state $|\delta\rangle = \phi(0)/\sqrt{c}$
descend $\delta + D_N \delta$
also

$$|> = \langle \phi_1 \phi_2 \rangle \langle \phi_{N-1} \phi_N \rangle$$

operator
primary & $\rightarrow |\phi\rangle$ state $|\phi\rangle = \phi(0)|0\rangle$
secondary $\rightarrow |\phi + \phi_{N\phi}\rangle$

$$\phi(0)|0\rangle = |\phi\rangle$$

$$\phi(0)|0\rangle = |\phi\rangle$$

$$|> = \langle \phi_1 \phi_2 \rangle \cdot \langle \phi_{N-1} \phi_N \rangle$$

operator
primary & secondary state (c)
descendant operator $\langle \phi_1 \phi_2 \rangle$
 $\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle$

Two-part. states

"Double-track" operators

$$\begin{matrix} \sigma \\ \sigma \\ \sigma \end{matrix}$$

{



$$q_1 q_2 |0\rangle = |00\rangle = \sigma^z_{\text{coll}} |0\rangle$$

$$|0\rangle, |+\rangle, |0,j\rangle, \dots$$

two-part states

"Double-trace" operators $\left\{ \hat{q}_a^{\dagger} \hat{q}_a^{\dagger} |0\rangle = |00\rangle = |\psi_{\text{cold}}\rangle \right.$

$$\begin{matrix} \sigma \\ \sigma \\ \hat{q}_a^{\dagger} \hat{q}_a^{\dagger} \end{matrix}$$

$$|0\rangle, |+\rangle, |0,j\rangle, \dots$$

- "Hamiltonian" D

$$D|i\rangle = \Delta_i |i\rangle, D|0\rangle = \Delta_0 |0\rangle, D|00\rangle = (\Delta_1 \Delta_0) |00\rangle$$

More th..

$$D_0 |0,j\rangle = (\Delta_j + \Delta_0) |0,j\rangle \quad \text{etc.}$$

$$D_0 = \sum_i \Delta_i q_i \rho_i$$

Standard Large N

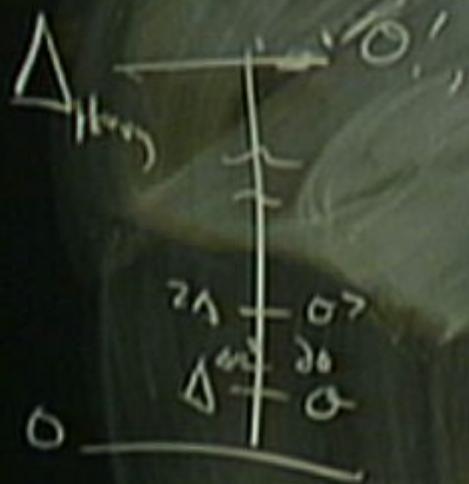
AdS/CFT inspire CFT w/ hierarchy
Lambdabin sector. Simplest is $\sigma \leftrightarrow \phi$ in AdS

$$\Delta_{\text{theory}} = \frac{\partial \mathcal{L}}{\partial \dot{\phi}}$$

Standard Large N

AdS/CFT inspire CFT with hierarchy

low-dim sector. Simplest is $\phi \leftrightarrow \phi$ in AdS



Conf. Algebra: $D, P^\mu, K^\mu{}_\nu, M^{\mu\nu}$ special rel.

$$[D, M^{\mu\nu}] = 0, [D, K^\mu] = -K^\mu, \text{ others.}$$

$$\phi_{\text{prim}} \Leftrightarrow K^\mu |\phi\rangle = 0$$

Standard Large N

AdS/CFT inspire CFT w/ a hierarchy
Low-dim sector. Simplest is $\Delta \hookrightarrow \phi$ in AdS

$$\Delta_{\text{phys}} \leftarrow \phi'$$

$$0 \xrightarrow{\Delta} \phi \xrightarrow{\Delta} \phi'$$

Conf. Algebra D, P^a, K^m, M^{uv} special const.

$$(D, M^{uv}) = 0, (P, K^m) = -K^a, \text{ others.}$$

$$\phi_{\text{prim}} \hookrightarrow K^m |\phi\rangle = 0$$

Introduce finite N: $D_0 \rightarrow D = D_0 + V$

$$K_0 \rightarrow K_0 + K^{(1)}$$

$$(V, K_0^m) + (D_0, K^{(1)m}) = -K^{(1)m} - (V, K^{(1)m})$$

AdS_{dil} global coords: $ds^2 = \frac{1}{\cosh^2 r} (-dt^2 + dr^2 + \sin^2 r d\Omega^2)$

spectrum ω : $\omega_i = \Delta + i \text{ integers}$

$$D = \frac{1}{i} \frac{2}{\lambda}$$

solution: $T \in U = \bigcap J^0 \times \bigcap g U(x)$

$V(x)$

scalar

$$\text{AdS}_d \text{ in global coords: } ds^2 = \frac{1}{\cosh r} (-dt^2 + dr^2 + \sin^2 r d\Omega^2)$$

$$\text{spectrum } \omega : \omega_i = \Delta + i \text{ integers}$$

$$D = \frac{1}{2} \frac{2}{\lambda}$$

$$\text{solution: Total } V = \int J^a \times \nabla_g V(x)$$

$$V(x) \text{ a scalar}$$

AdS_{d+1} global coords: $ds^2 = \frac{1}{\cosh^2 r} (-dt^2 + d\Omega^2 + \sin^2 r d\Omega^2)$

spectrum ϕ : $\omega_i = \Delta + i$ integers

$$D = \frac{1}{i} \frac{\partial}{\partial t}$$

Solution: Take $V = \int d^d x \sqrt{g} V(x)$

cond 1 $V(x)$ scalar

and 2 $[V(x), V(x')] = 0$

$$\sigma(x, x') = 0$$

$$D_0 \longleftrightarrow D_0 + V$$



$$D_0 \longleftrightarrow D_0 + V$$

\downarrow
estates

\downarrow

estates

$\rightarrow U$ change of basis unitary

T

$$D_0 \longleftrightarrow D_0 + V$$



e states

$\rightarrow U$ change of basis unitary

$e^{-i\pi d' q_m}$:

$$D_0 \longleftrightarrow D_0 + V$$



e states

\rightarrow U change of basis unitary

indif qm: $|2A+2n\rangle_2 = |2A+2n\rangle +$

$$= (S_{AB+T} \sum_{\lambda, m} \frac{\langle 2A+2n | V | \lambda \rangle}{E_{204} - E_\lambda}$$

$$D_0 \longleftrightarrow D_0 + V$$

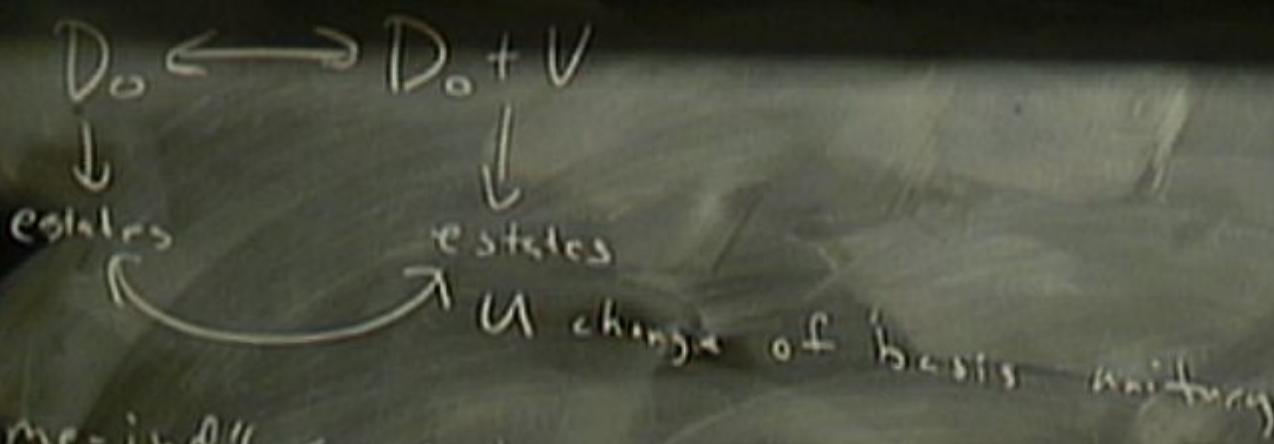
↓
estates

↓
estates

↗ U change of basis unitary

Time-indep qm: $|2A+2n\rangle_2 = |2A+2n\rangle + \sum_{\lambda > 0, n} \frac{\langle 2A+2n|V|\lambda\rangle}{E_{204}-E_\lambda}$

$$|\lambda\rangle = (S_{AB} + T_{AB}) |\beta\rangle$$



Time-ind&f qm:

$$|\psi\rangle = (S_{AB} + T_{AB}) |\psi_B\rangle + \sum_{k \neq 0, n} \frac{\langle 2A+2n|V|k\rangle}{E_{2n} - E_k}$$

$$D_0 \longleftrightarrow D_0 + V$$

\downarrow
estates
 \curvearrowright

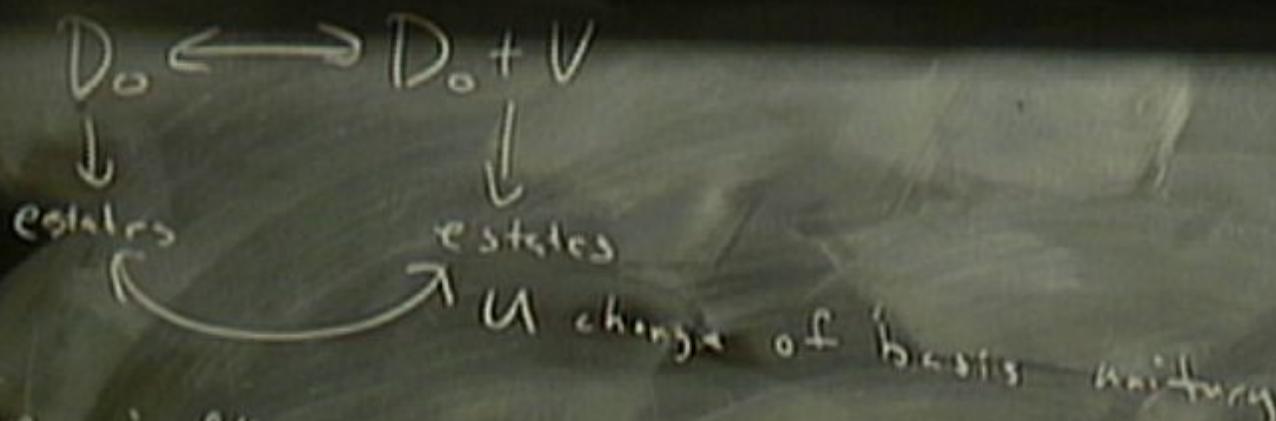
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estates

\curvearrowright U change of basis unitary

Time-ind't qm: $|2A+2n\rangle_2 = |2A+2n\rangle + \sum_{\text{levels}} \frac{\langle 2A+2n | V | k \rangle}{E_{2A+2n} - E_k}$

$$|A\rangle = (\delta_{AB} + T_{AB}) |B\rangle$$

complete
 $| = \sum \langle A | C \rangle^{(n)} \langle C | *$



Time-indep qm: $|2A+2n\rangle_2 = |2A+2n\rangle + \sum_{\text{levels}} \langle 2A+2n|V|2\rangle$

$$|\Lambda\rangle = (\delta_{AB} + T_{AB}) |\beta\rangle$$

complete $\lambda = \sum \langle \lambda | C^* \rangle \langle C | \lambda \rangle$
 $\Rightarrow -2\mu_c(T_{AB}) = \sum |\Gamma_{AC}|^2 \sum |\mu_c(T_{AB})|^2$
 $|\mu_c(T_{AB})| \leq \sqrt{\sum |\Gamma_{AC}|^2}$

$$D_0 \longleftrightarrow D_0 + V$$

\downarrow
estates

\downarrow
estates

$\rightarrow U$ change of basis unitary

Time-indep qm: $|2A+2n\rangle_2 = |2A+2n\rangle + \sum_{\text{new}} \langle 2A+2n|V|2\rangle$

$$|A\rangle = (\delta_{AB} + T_{AB})|B\rangle$$

complete

$$I = \sum \langle A | \langle \dots, \dots | C | \dots \rangle$$

$$\Rightarrow -2\epsilon_c(T_{AA}) = \sum |\Gamma_{AC}|^2 \sum |\Gamma_C(T_{AA})|^2$$

$$|\Gamma_C(T_{AA})| \leq \sqrt{\dots}$$

$$D_0 \longleftrightarrow D_0 + V$$

\downarrow
estates

\downarrow
estates

$\rightarrow U$ change of basis unitary

Time-indep qm: $|2A+2_n\rangle_2 = |2A+2_m\rangle + \sum_{n' \neq m} \frac{\langle 2A+2_n|V|2_m\rangle}{E_{2m} - E_{2n}}$

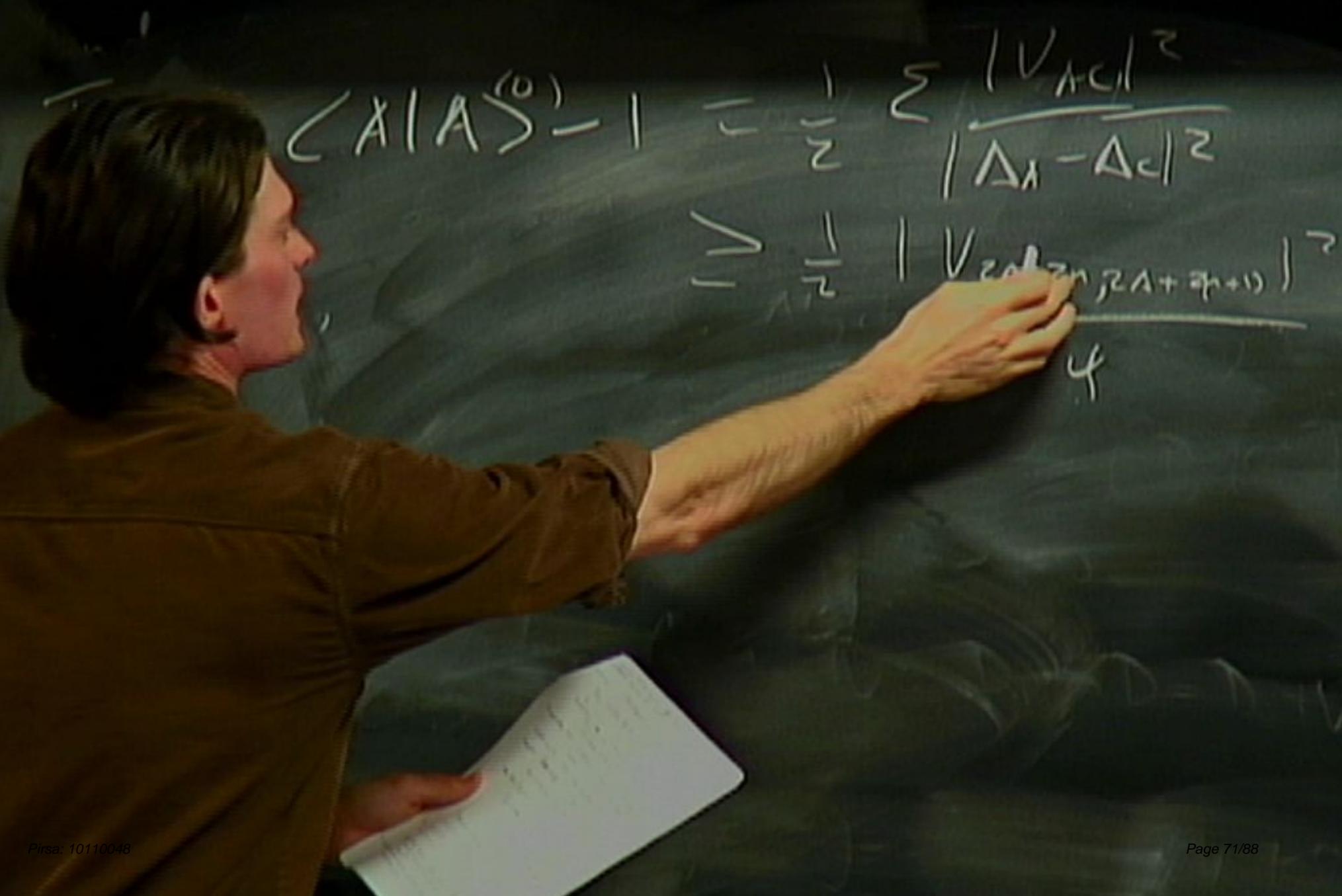
$$|A\rangle = (\delta_{AB} + T_{AB})|B\rangle$$

complete

$$I = \sum C_i |C_i\rangle$$

$$\Rightarrow -2\omega_c(T_{AA}) = \sum |\Gamma_{AC}|^2 \sum |P_C(T_{AA})|^2$$

$$|\Gamma_{AC}(T_{AA})| \leq \sqrt{2}$$



$$\begin{aligned} T_{AA} = & \langle A | A^{\dagger} \rangle - 1 = \frac{1}{2} \sum \frac{|V_{AC}|^2}{|\Delta_A - \Delta_C|^2} \\ & \geq \frac{1}{2} \left| \frac{V_{e_{A1} e_{n,2A+2n+1}}}{4} \right|^2 \end{aligned}$$

$$\begin{aligned} T_{AA} = \langle A | A^2 \rangle - 1 &= \frac{1}{2} \sum \frac{|V_{AC}|^2}{|\Delta_A - \Delta_C|^2} \\ &\geq \frac{1}{2} \left| \frac{V_{2\Delta+2n, 2\Delta+2n+1}}{4} \right|^2 \end{aligned}$$

$$T_{AA} = \langle A | A \rangle - 1 = \frac{1}{2} \sum \left| \frac{|V_{A\bar{A}}|^2}{\Delta_A - \Delta_{\bar{A}}/2} \right| \leq \frac{1}{2} \left| V_{2\Delta_{12n}, 2\Delta_{12n+12}} \right|^2 \mathcal{O}(n)^4$$

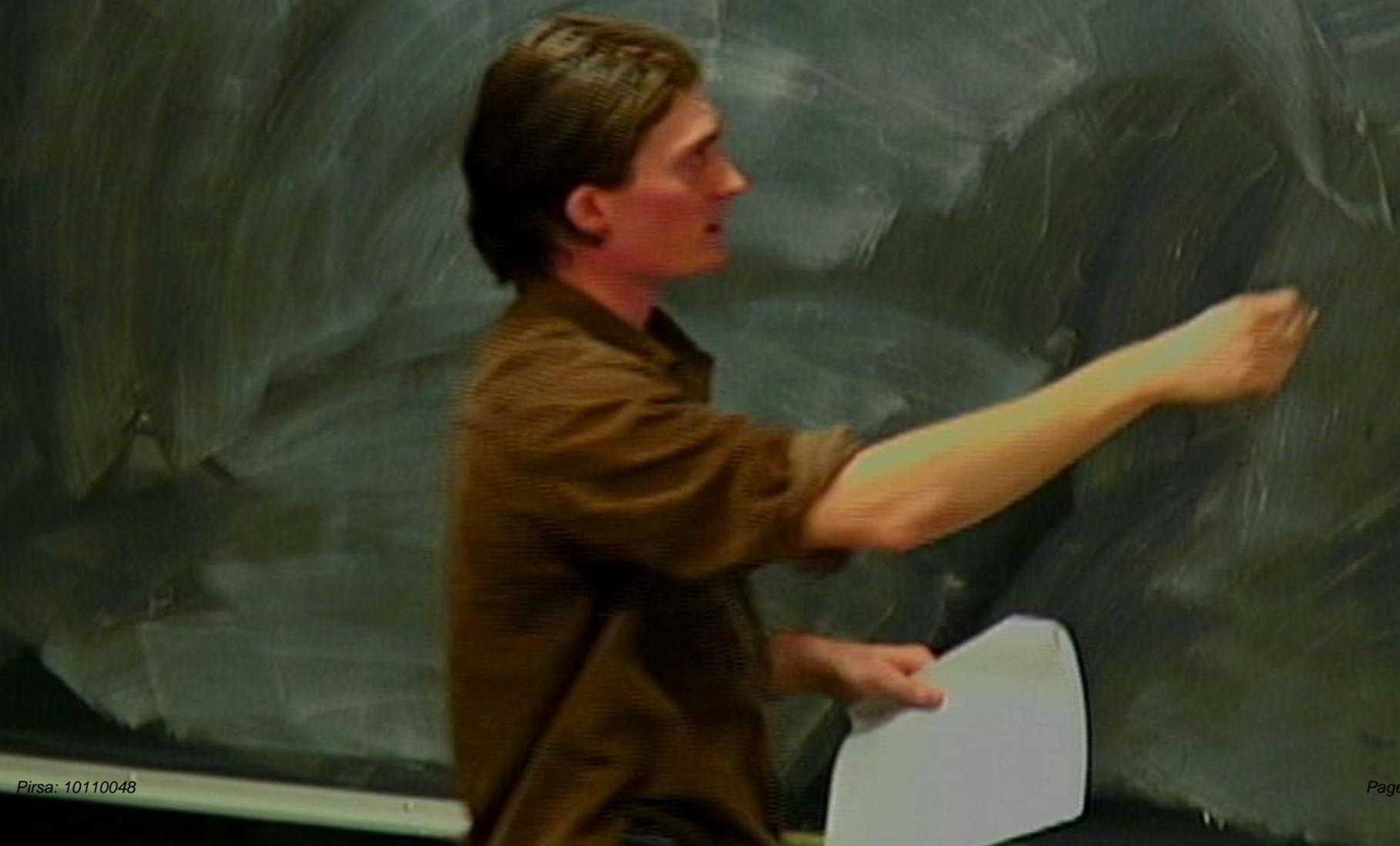
$$V_{2\Delta_{12n}, 2\Delta_{12n+12}} = V_{2\Delta_{12n}, 2\Delta_{12n}} = \mathcal{O}(n) \quad \text{armonious dim of } 1/2\Delta_{12n}$$

$$\Rightarrow |\mathcal{O}(n)| < 4$$

$$\begin{aligned}
 T_{AA} = \langle A | A^S \rangle - 1 &= \frac{1}{2} \sum \frac{|V_{\text{inel}}|^2}{|\Delta_A - \Delta_d|^2} \\
 &\geq \frac{1}{\pi} \left| V_{2\Delta+2n, 2\Delta+2n+2} \right|^2 \quad \mathcal{O}(1) \text{ of } \\
 V_{2\Delta+2n, 2\Delta+2n+2} &= V_{2\Delta+2n, 2\Delta+2n} = \mathcal{O}(n) \quad \text{aromatic dim} \\
 \Rightarrow |\mathcal{O}(n)| &< 4 \quad \text{of } |2\Delta+2n>
 \end{aligned}$$

$\mathcal{L}_{\text{AdS interaction}}$

$$\frac{\partial^p \dots \partial^q}{\pi} \Rightarrow \gamma(n) \sim (\lambda)^p$$



L_{AdS} interaction

$$\frac{d\phi_1 \dots d\phi_n}{R} \Rightarrow \gamma(n) \sim \left(\frac{A}{R}\right)^n$$



$$\mathcal{L}_{AdS \text{ interaction}} \xrightarrow{\frac{\partial \phi_1 \dots \partial \phi_r}{\pi}} \mathcal{Y}(n) \sim \left(\frac{n}{\lambda}\right)^r$$

1) Large N

2) No new singlet trace states below Δ_{Heavy}

3) Pert. unitarity

$$\Rightarrow \frac{\phi^4}{\Delta_{Heavy}} \propto \frac{1}{\Delta_{Heavy}^2}$$

\mathcal{L}_{AdS} interaction

$$\frac{\partial \phi_1 \dots \partial \phi_n}{R} \Rightarrow \mathcal{V}(n) \sim (\frac{R}{\lambda})^n$$

$D_{Large N}$

- conds:
- 2) No new singlet trace state below Δ_{Heavy}
 - 3) Pert. unitarity

$$\Rightarrow \frac{\phi^n}{\Delta_{Heavy}} \text{ is } \frac{\phi^n}{\Delta_{Heavy}}$$

\mathcal{L}_{AdS} interaction

$$\frac{\partial \phi \dots \partial \phi}{R} \Rightarrow \mathcal{V}(n) \sim (\hat{A})^n$$

Large N

conds:

- 2) No new sing'ltre states below Δ_{Heavy}
- 3) Pert. unitarity

$$\Rightarrow \frac{\phi^4}{\Delta_{Heavy}} \text{ vs } \frac{\phi^4}{\Delta_{Heavy}}$$

$$\mathcal{L}_{AdS \text{ interaction}} \frac{\partial \phi_1 \dots \partial \phi_n}{\pi} \Rightarrow \mathcal{V}(n) \sim (\frac{n}{\lambda})^r$$

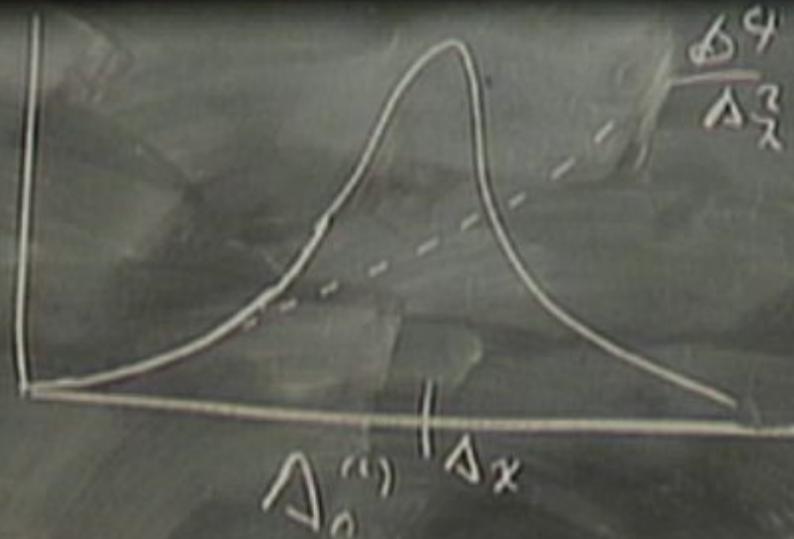
1) Large N

- s :
 2) No new singlet trace state below Δ_{Heavy}
 3) Pert. unitarity

$$\Rightarrow \frac{\phi^4}{\Delta_{\text{Heavy}}} \frac{\partial \phi^4}{\Delta_{\text{Heavy}}}$$

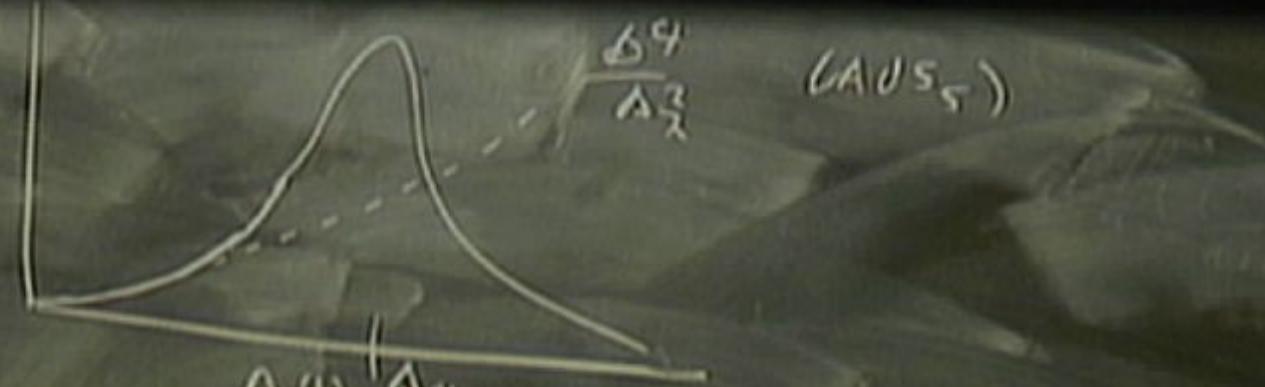
$$\langle \partial \phi(x) / \partial A_{1234} \rangle \\ \sim \cos^{2\alpha m}$$

$J_{\Delta \nu_s} \circ \varphi^2 \chi \rightarrow \chi(n)$



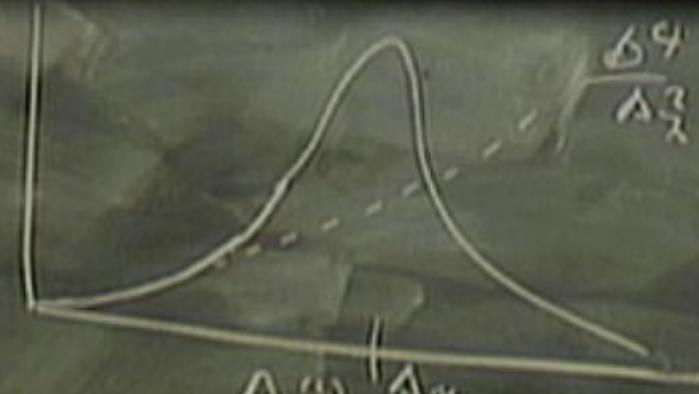
(GAUS5)

$J_{AVS} \circ \varphi^* \chi \rightarrow \mathcal{J}(n)$



$$\mathcal{M}_{\text{flux-sol}}^{(d+1)} = \frac{(4\pi)^d}{\text{Vol}(\text{Sol}_n)} \frac{\mathcal{E}_n}{(\mathcal{E}_n - 4\pi)^{d-1}} \sum_{l=1}^{d-1} \mathcal{J}(n, l) \in \mathbb{P}_n^{(d+1)}$$

$$J_{AVS} = \varphi^2 \chi \rightarrow \mathcal{D}(n)$$

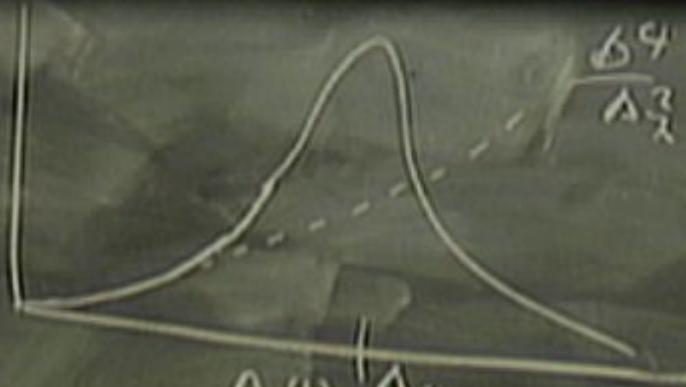


CAUSAL

$$\mathcal{M}_{\substack{\text{flat-space} \\ z \rightarrow \infty}}^{(d+1)}(\epsilon, \chi_n) = \frac{(4\pi)^4}{\text{Vol}(\text{Sol}_n)} \frac{\Delta_n^{(d+1)}}{E_n} \frac{1}{(E_n^2 - 4\mu^2)^{\frac{d+1}{2}}} \sum \mathcal{D}(n, \ell) r_\ell P_\ell^{(d+1)}(\cos \theta)$$

$\mathcal{M} \sim T^{-\frac{1}{2}}$ matrix elements

$$J_{AVS} = \nabla^2 X \rightarrow \mathcal{D}(n)$$



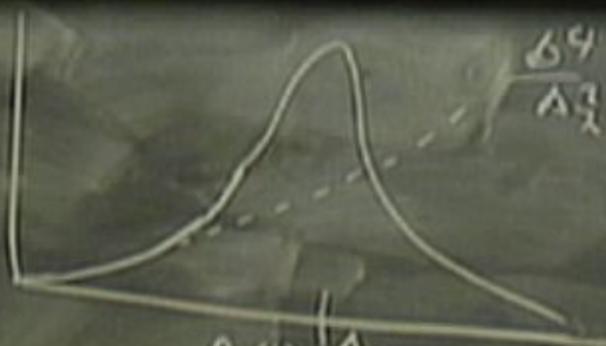
$$\frac{\delta^4}{\Delta_x^3} \text{ (AUS)}$$

$$M_{\substack{\text{flat-space} \\ \text{2-d}}}^{(d+1)}(x_{n+1}) = \frac{(4\pi)^4}{Vol(S^{d+1})} \frac{\Delta_n^{(d)}}{F_n} \frac{F_n}{(F_n^2 - 4\pi^2)^{\frac{d-1}{2}}} \sum \mathcal{D}_{(n, l)} P_l^{(d+1)}$$

$M \sim T$ matrix $\mathcal{D}_{(n, l)}$



$$\int_{A\in S} \phi^2 \chi \rightarrow \delta(n)$$



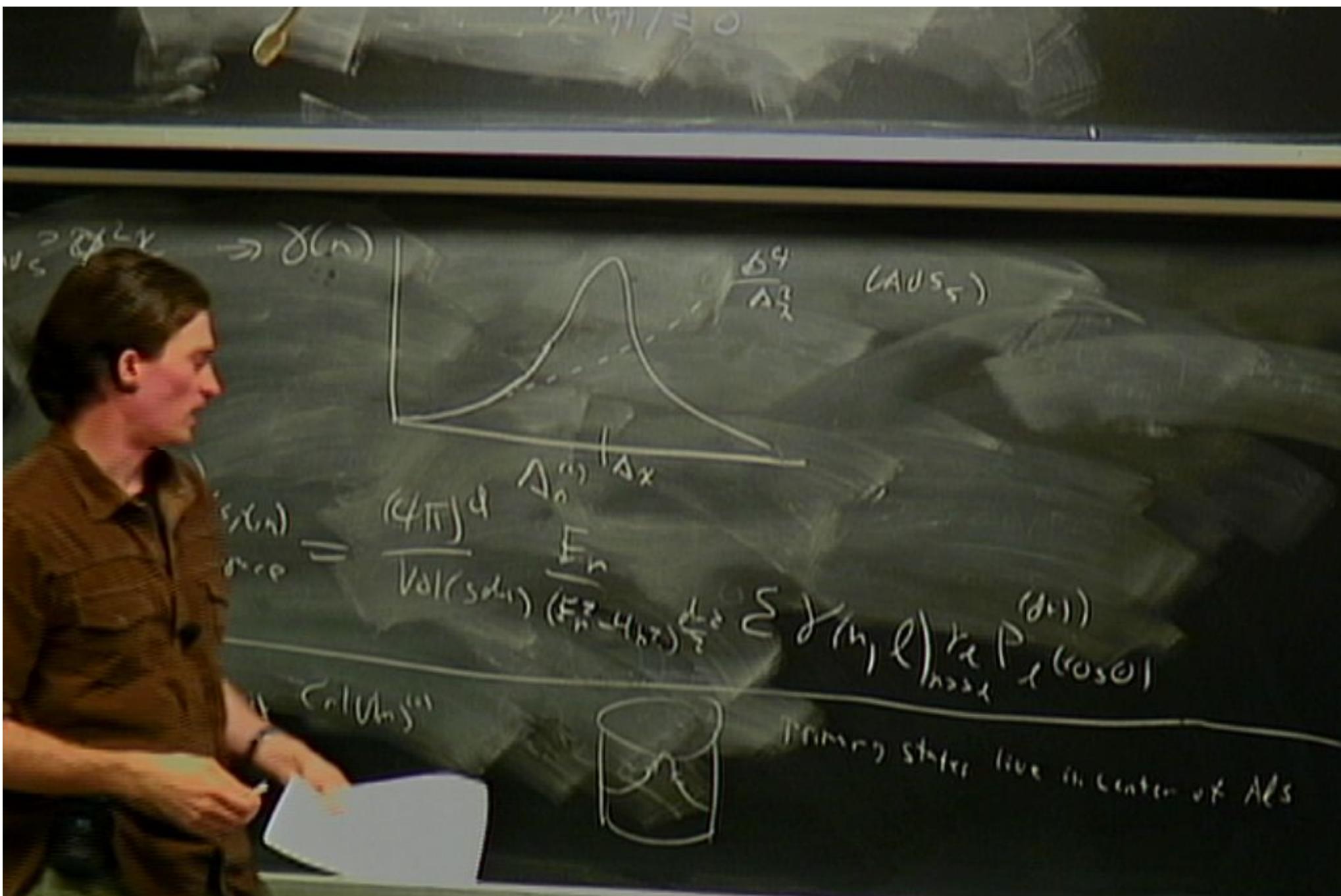
$\frac{\delta^4}{\Delta_\lambda^2}$ GAUS₅)

$$M_{\text{flat-space}}^{(d+1)}(s, t_n) = \frac{(4\pi)^4}{\text{Vol}(S^{d+1})} \frac{A_0^{(d+1)}}{E_n} \sum_{k=1}^{\infty} J_{n+k}(\ell) r_k P_{\ell}^{(d+1)}(\cos\theta)$$

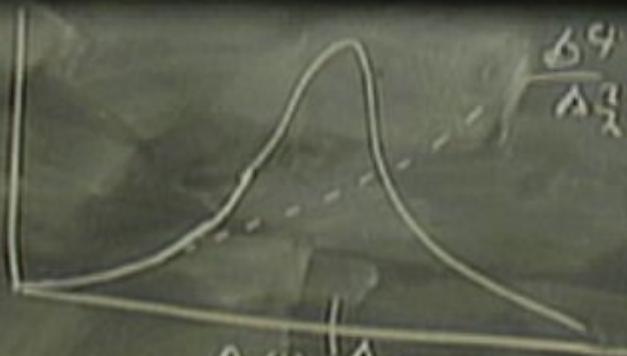
$M \sim T$ -matrix Calculus



Primary states live in center of AdS



$$S_{\text{AUS}} = \Phi^L X \rightarrow \mathcal{D}(n)$$



$$\frac{69}{\Delta_x^2}$$

(AUS₅)

$$M_{\text{flat-sphere}}^{(d+1)}(s_{(l,n)}) = \frac{(4\pi)^d}{\text{Vol}(\text{solid}_l)} \frac{\Delta_n^{(d)}}{E_n} \sum_{k=0}^{d-1} \sum_{m=-k}^{k} J_{(n,l,m)} P_l(\cos\theta)$$

$M \sim T^{-\frac{1}{2}}$ Matrix Calculus



Primary states live in center of AUS