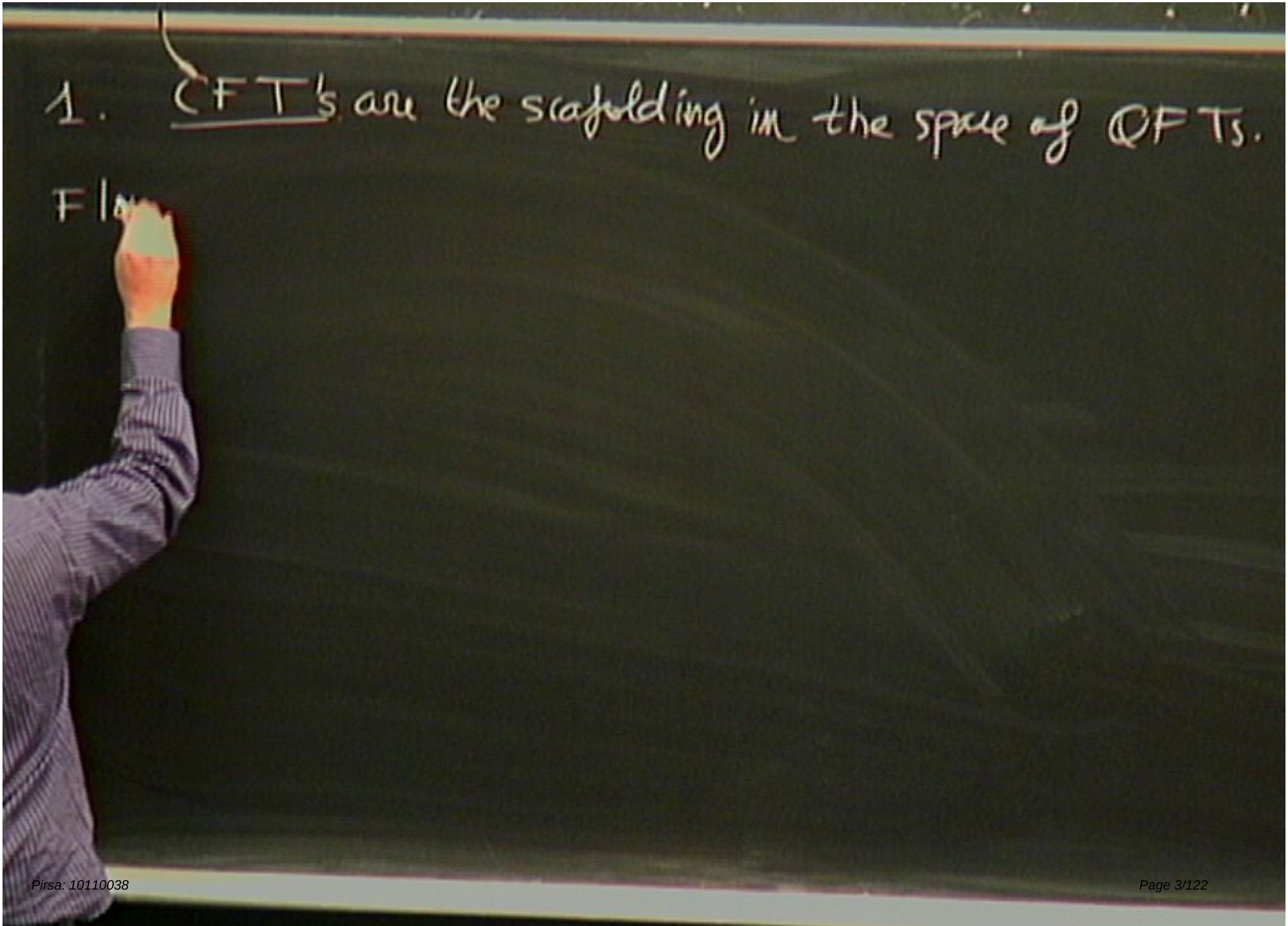
Title: Conformal Field Theory (PHYS 609) - Lecture 2

Date: Nov 23, 2010 10:30 AM

URL: http://pirsa.org/10110038

Abstract:

1. CFT's are the scaffolding in the space of QFTs.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RG flows in the space of couplings.

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$$E \frac{d g_i}{dE}$$

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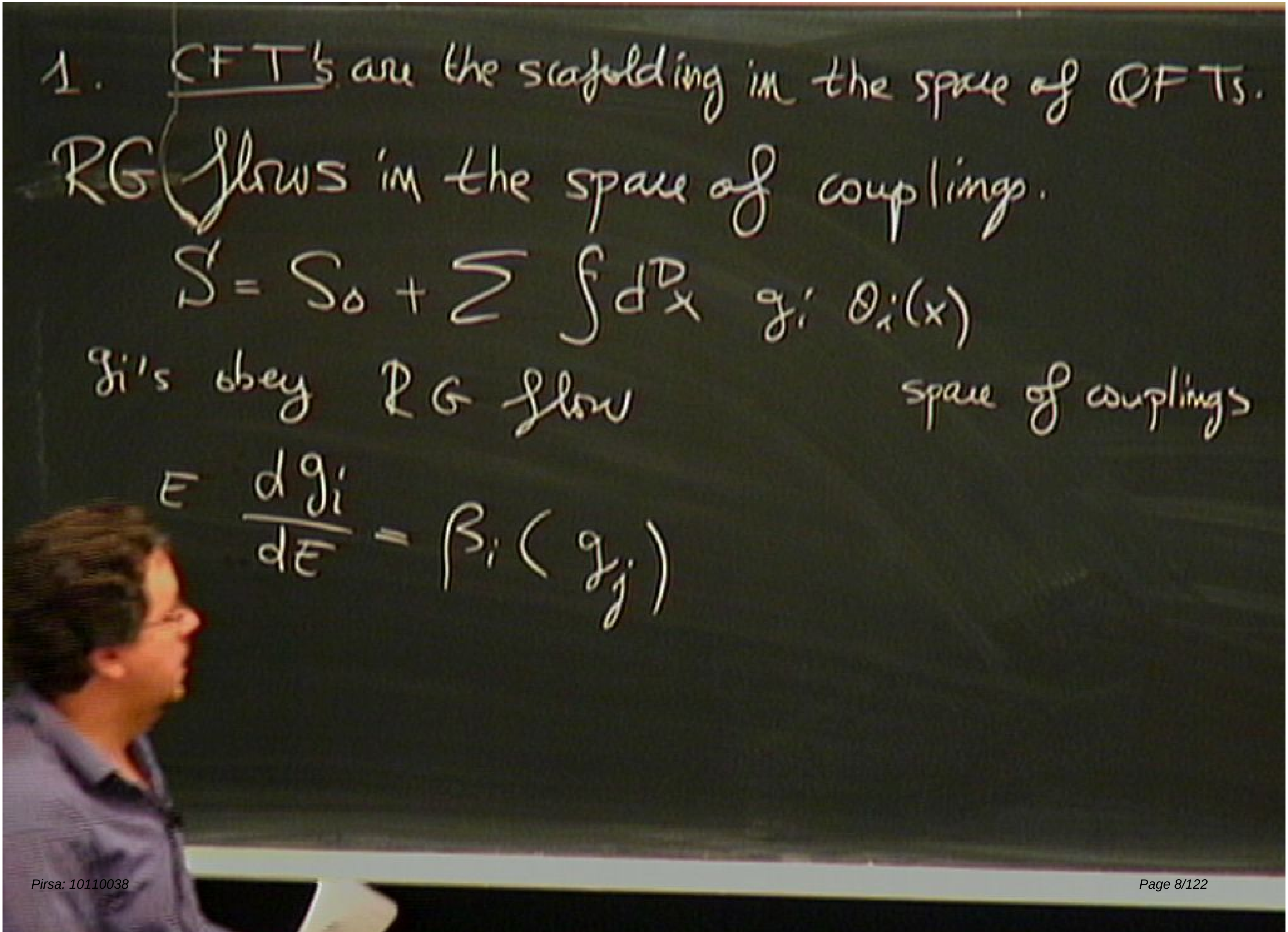
space of couplings

$$\frac{d g_i}{dE} = \beta_i ( g_j )$$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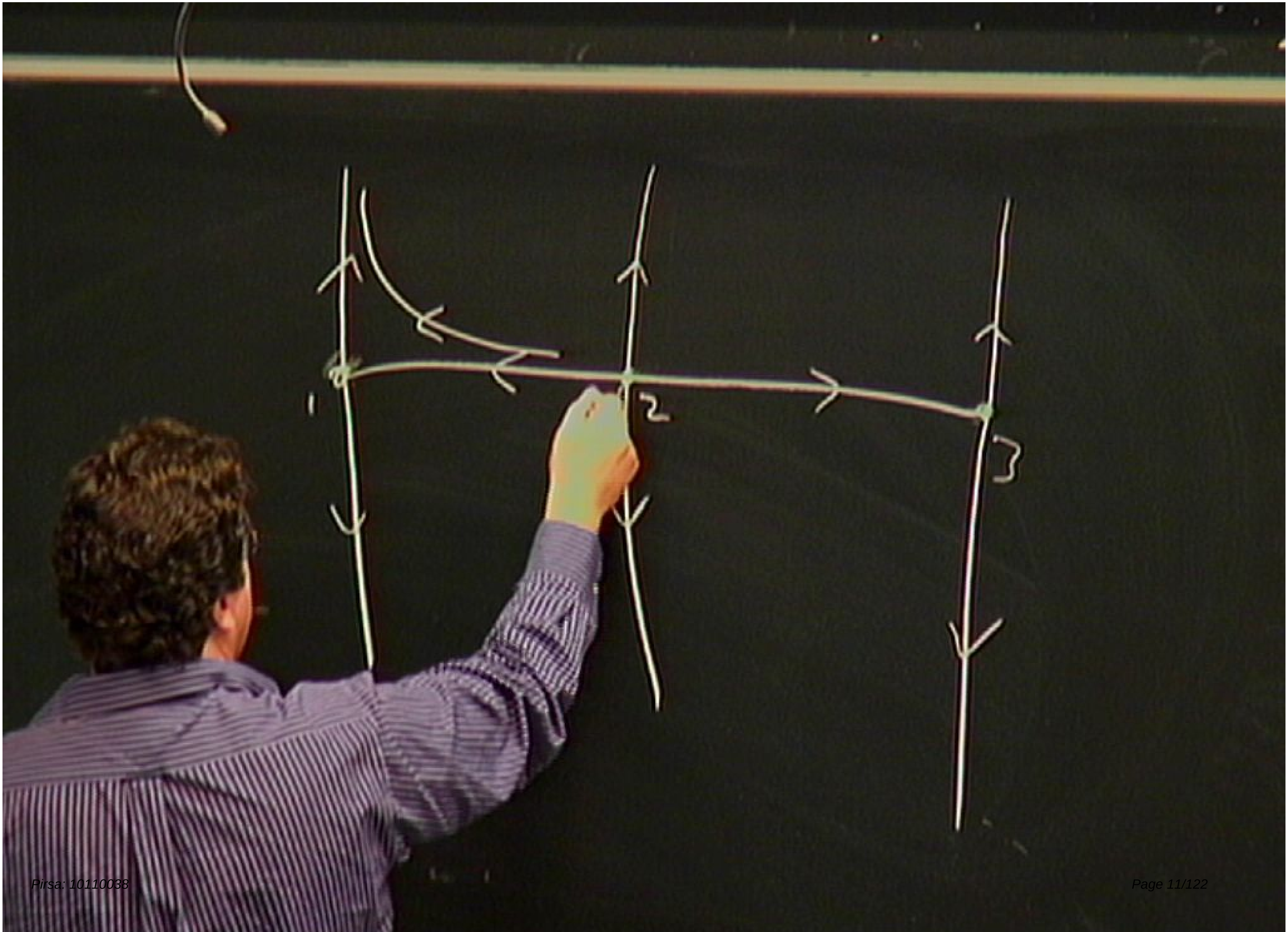
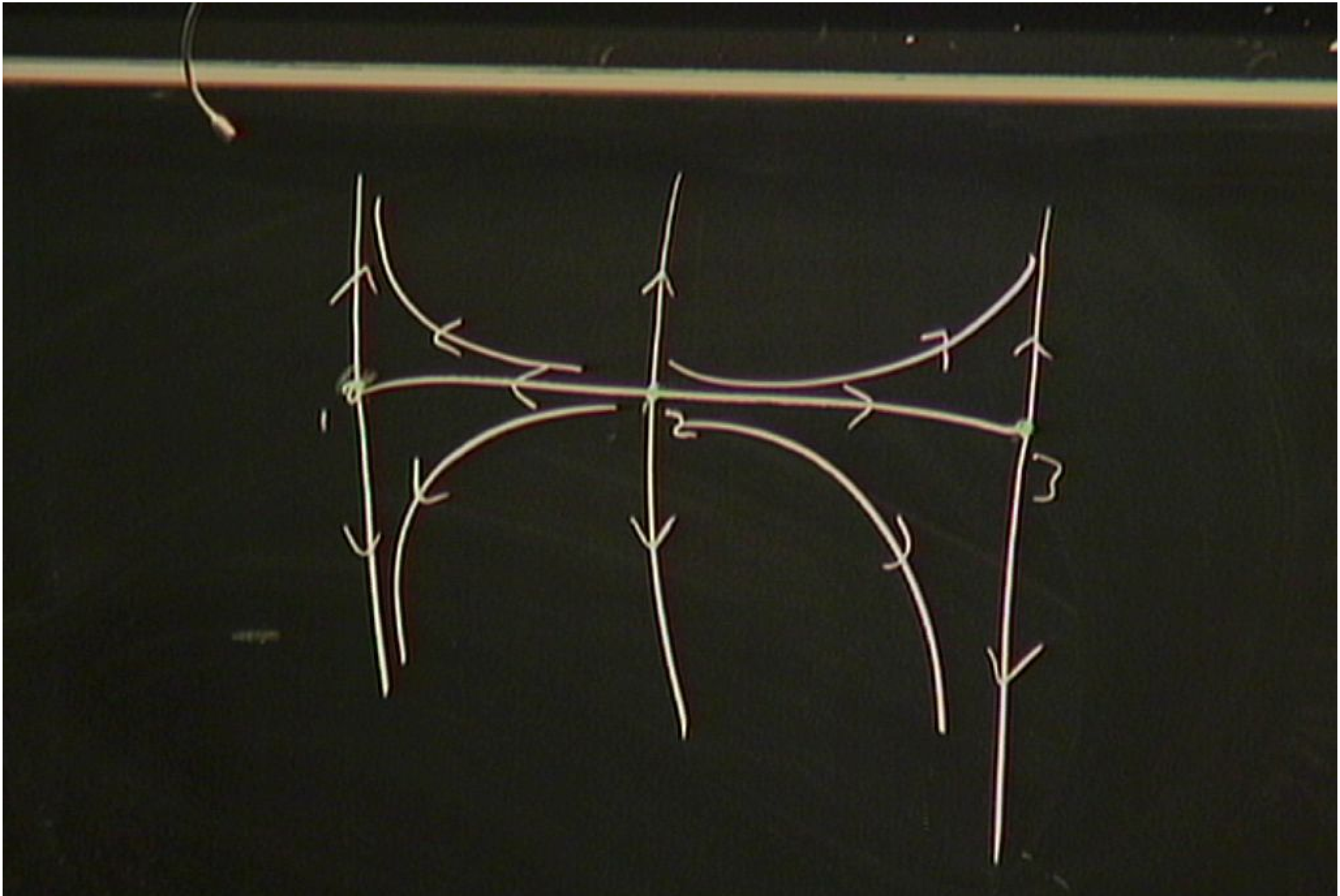
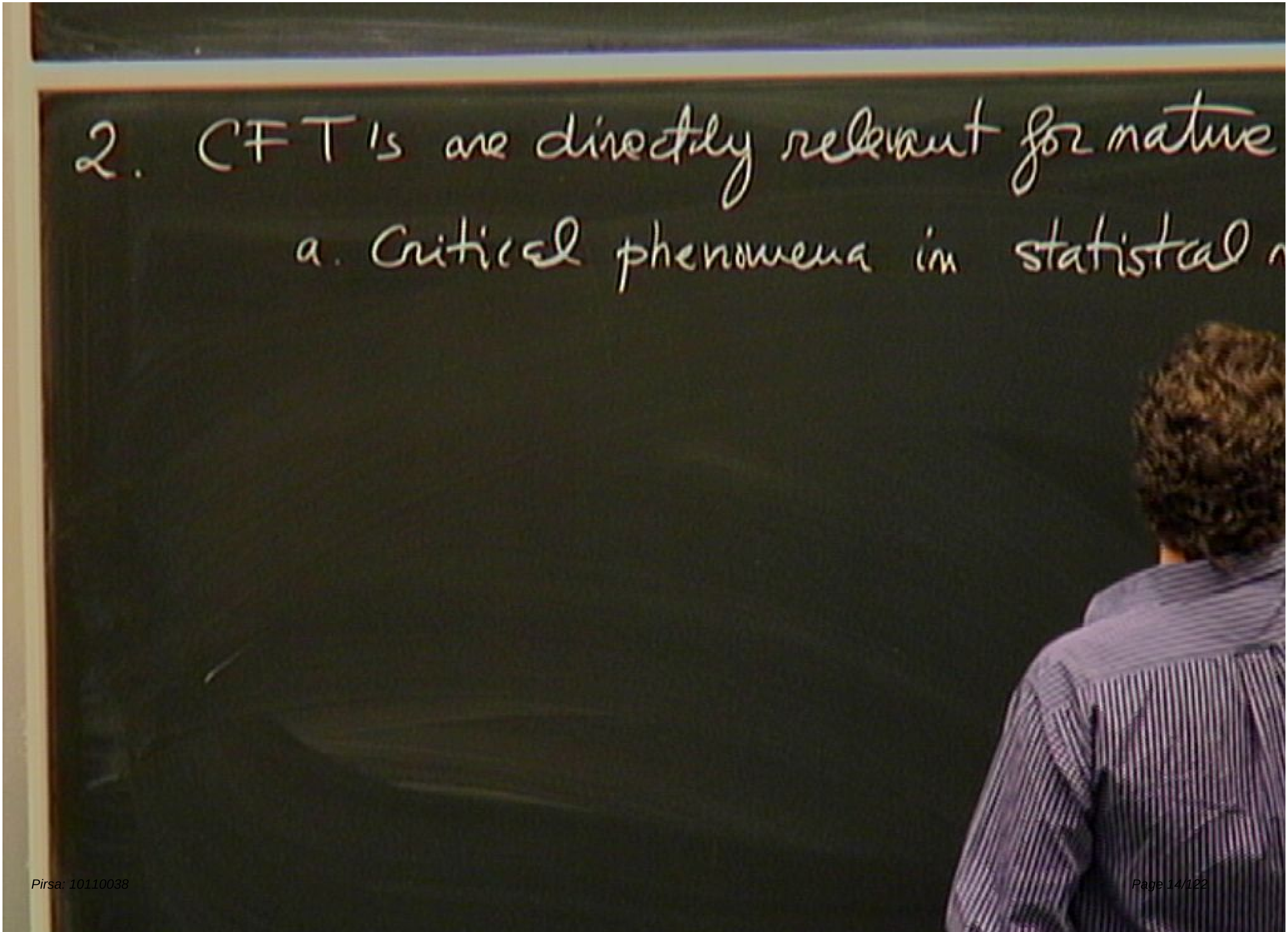
$$E \frac{d g_i}{dE} = \beta_i(g_j)$$

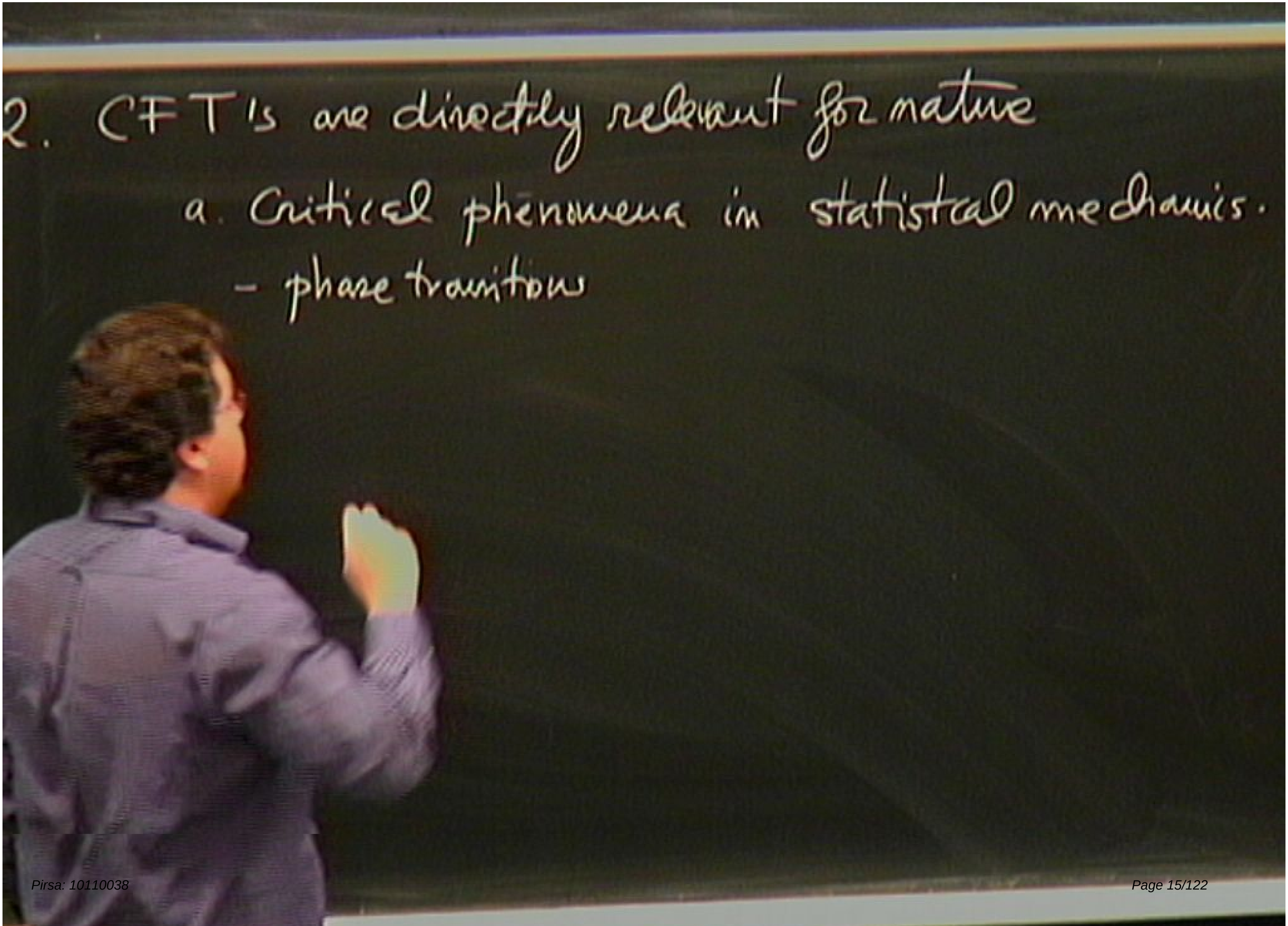
Fixed pts: $\beta_i(g_j^*) = 0 \Longrightarrow$ CFT


1
2
3


1
2
3

2. CFT's are directly relevant for nature


2. CFT's are directly relevant for nature
a. Critical phenomena in statistical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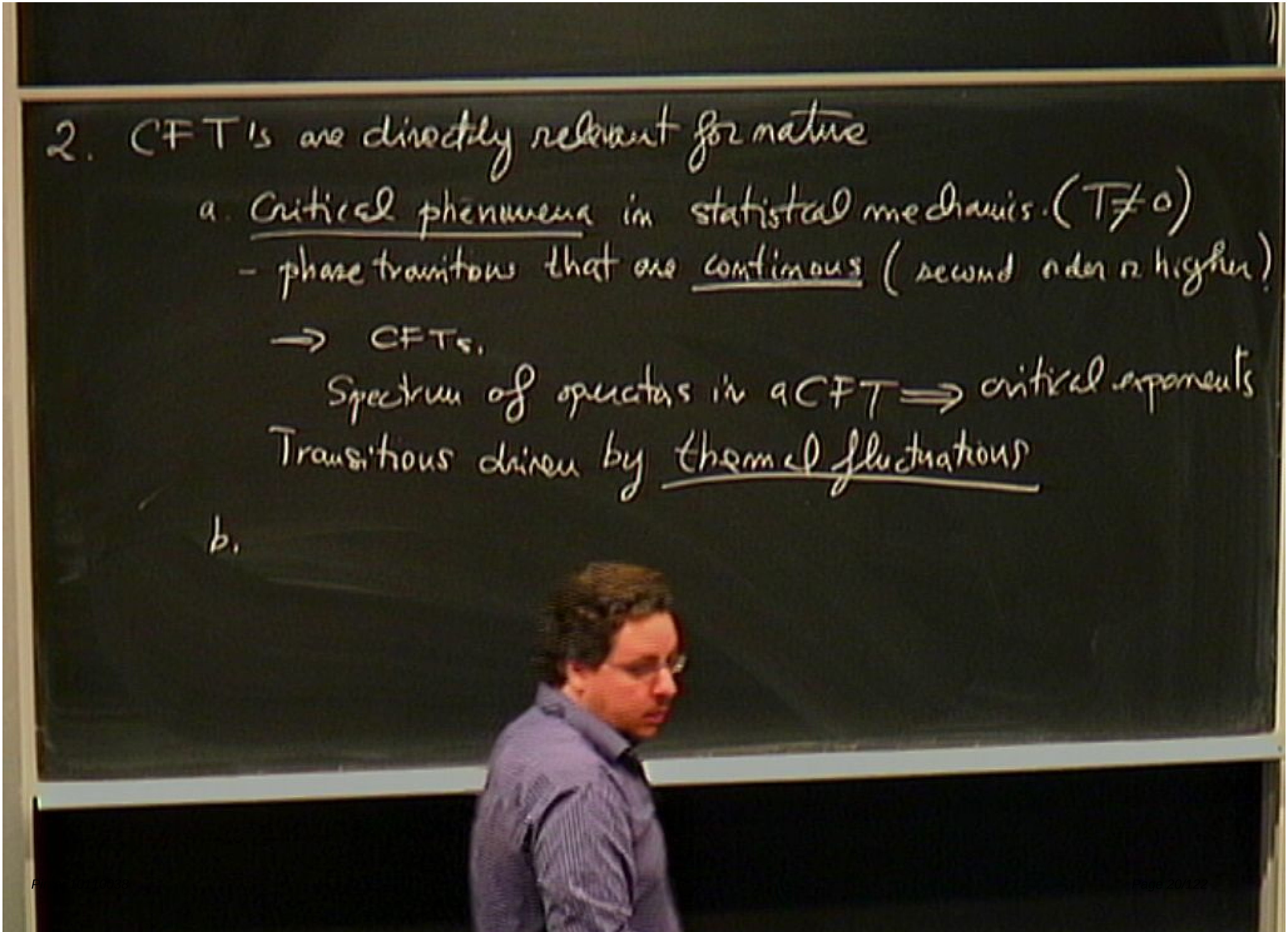
- phase transitions that are <u>continuous</u> (second order or higher)

$\longrightarrow$ CFTs.

Spectrum of operators in a CFT $\Longrightarrow$ critical exponents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a. Critical phenomena in statistical mechanics. (T≠0)
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⟹ CFTs.
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Transitions driven by thermal fluctuations
b.

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b. Quantum Critical Points. ($T = 0$)

Transition driven by quantum fluctuations

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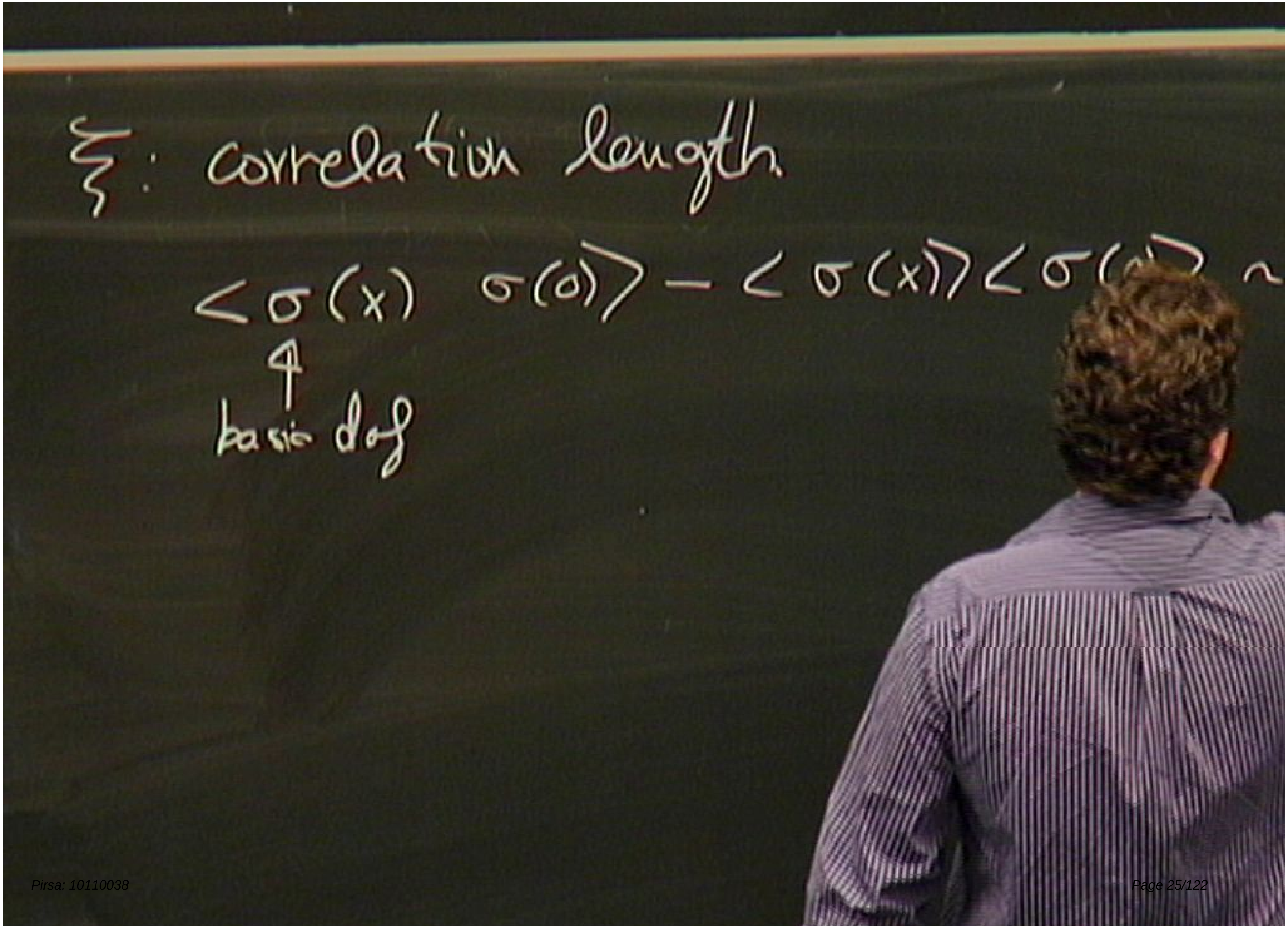
$\xi$: correlation length


ξ : correlation length
<σ(x) σ(0)> − <σ(x)><σ(0)> ~
basic dof

$\xi$ : correlation length

$$\langle \sigma(x)\ \sigma(0)\rangle - \langle \sigma(x)\rangle\langle \sigma(0)\rangle \sim \exp\left(-|x|/\xi\right).$$

↑ basic dof

$\xi$: correlation length

$$\langle \sigma(x)\ \sigma(0)\rangle - \langle \sigma(x)\rangle\langle \sigma(0)\rangle \sim \exp\left(-|x|/\xi\right).$$

$\uparrow$ basic dof

massive scalar field of mass $m$ $\quad \xi = \frac{1}{m}$

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($\sigma$ ↑ basic dof)

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- Tune external parameter

$\xi$ : correlation length

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massive scalar field of mass $m$ $\quad \xi = \frac{1}{m}$

- Tune external parameter (Pressure, Magnetic Field, Temperature)

$T \to T_c$, $\xi \to \infty$ } $\Rightarrow$ critical behaviour

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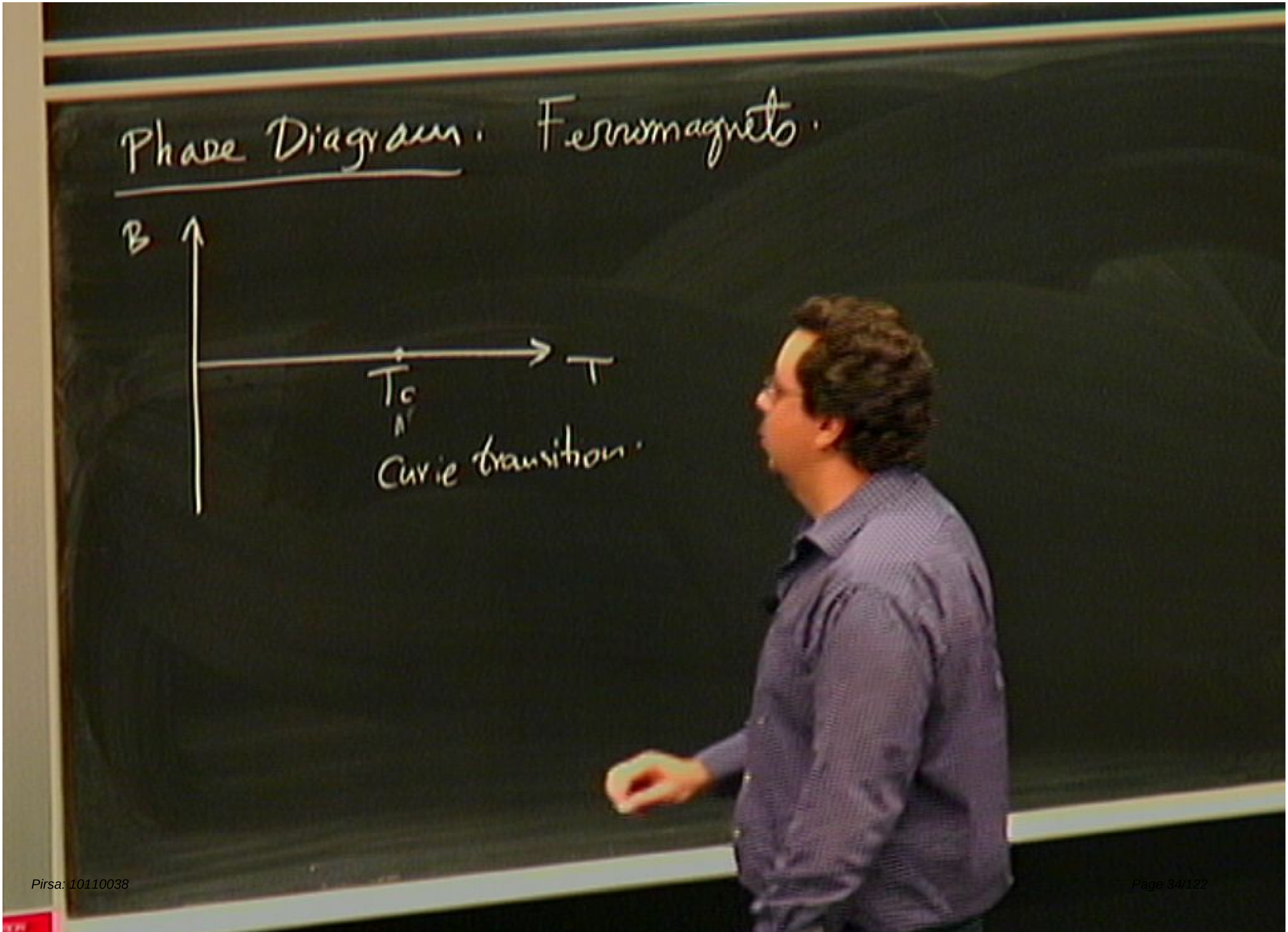
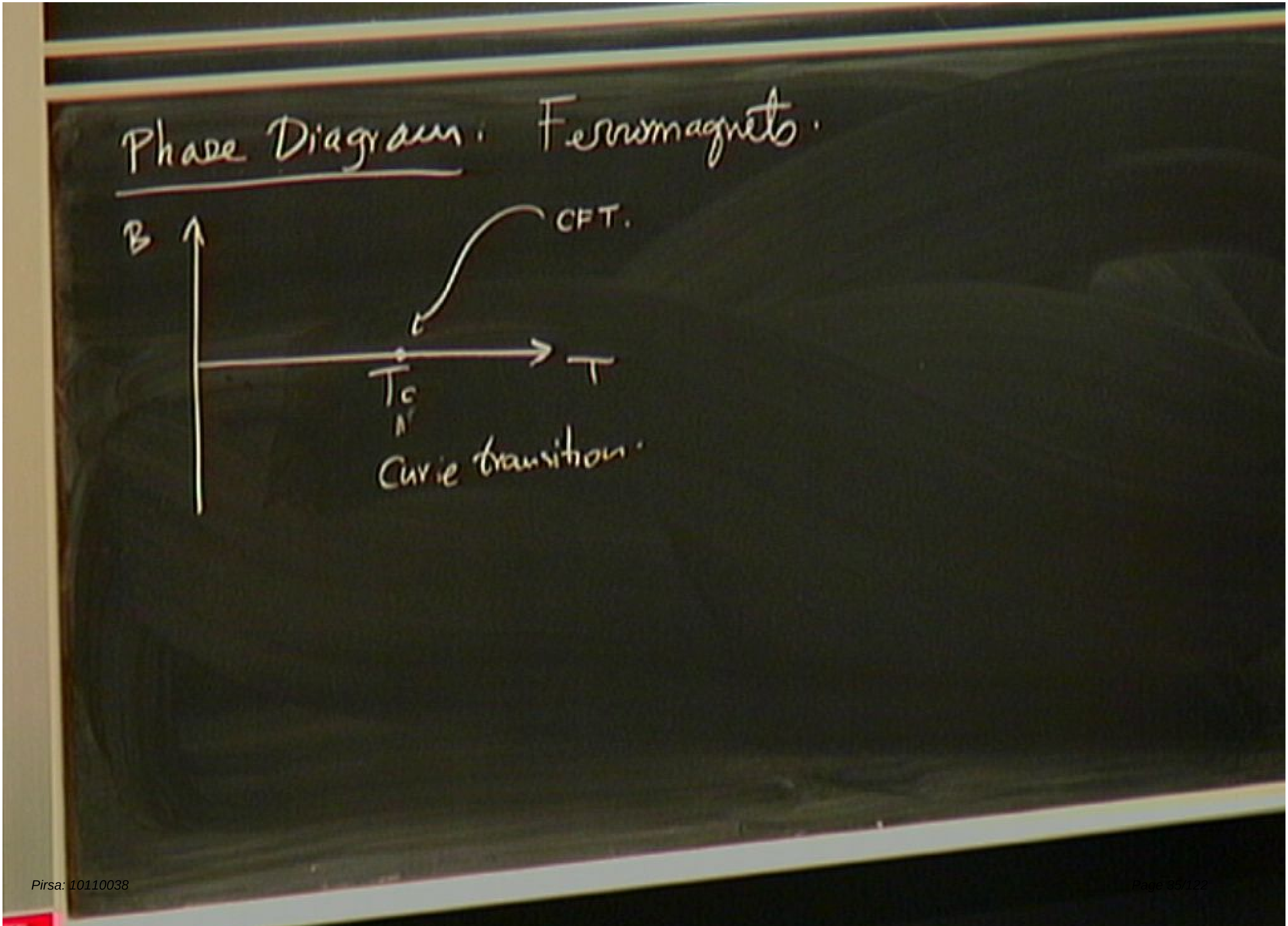
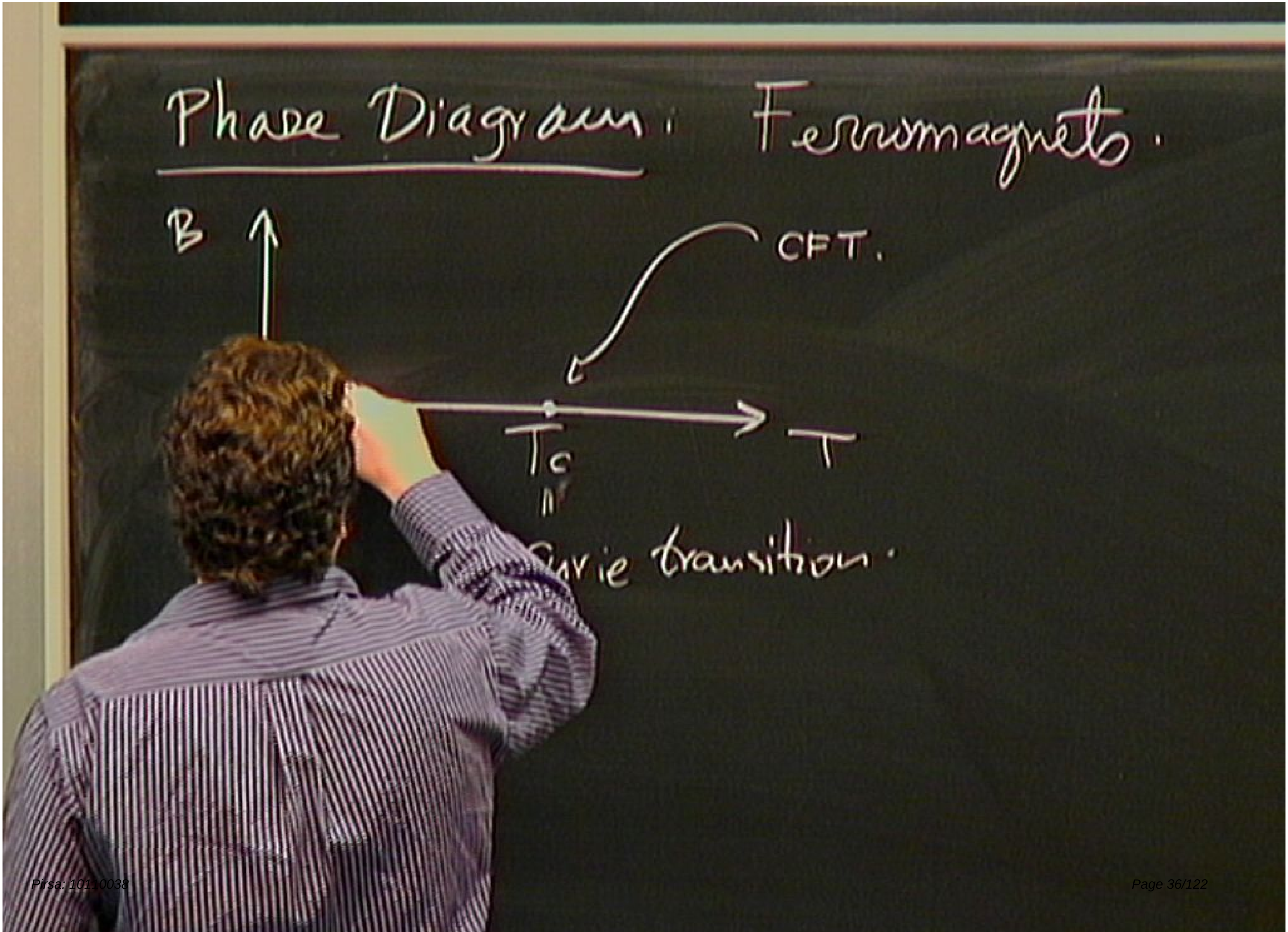

Phase Diagram: Ferromagnets.
B
T
$T_c$
Curie transition.


Phase Diagram: Ferromagnets.
B
CFT.
T
Tc
Curie transition.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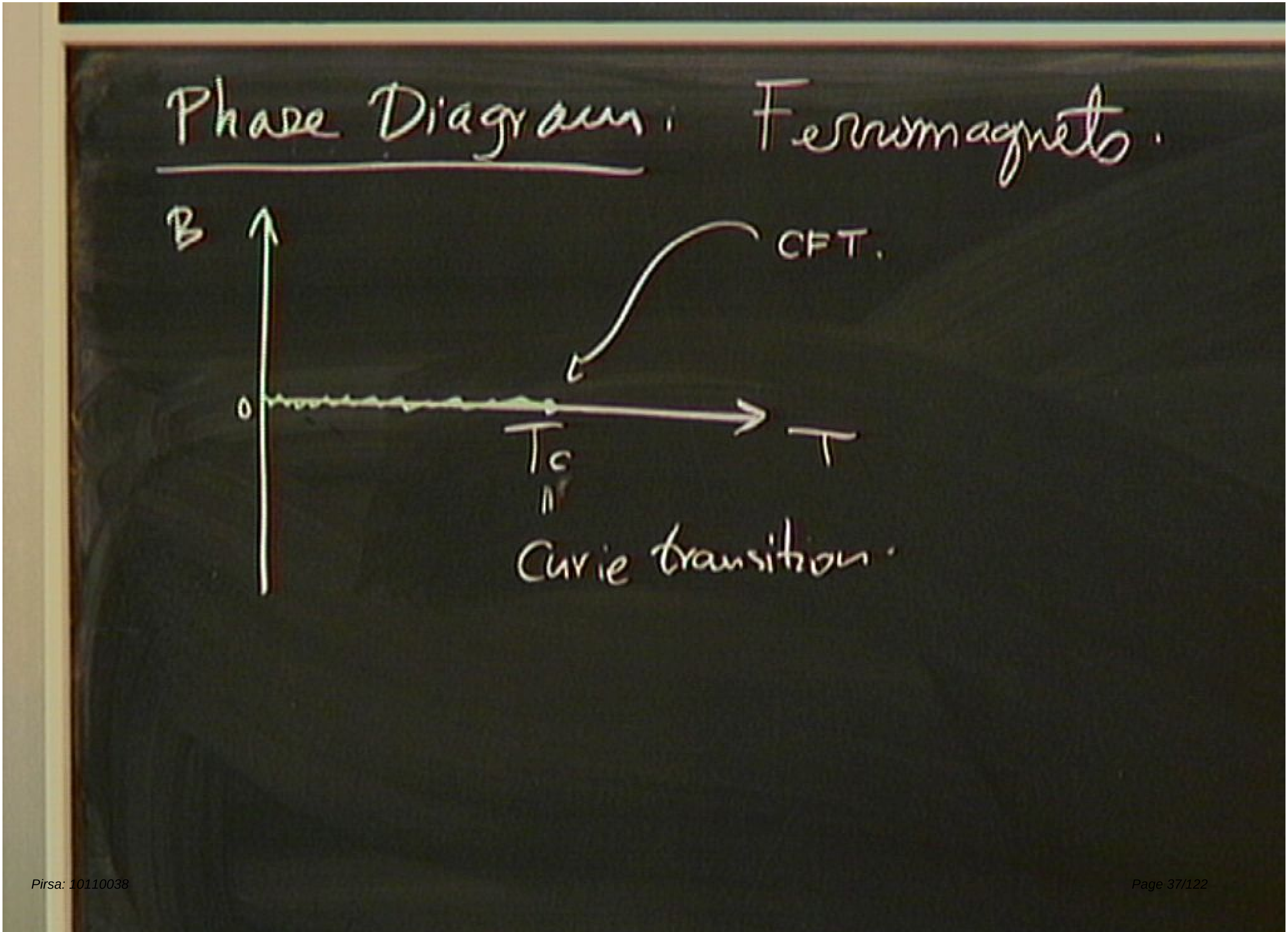
Phase Diagram: Ferromagnets.
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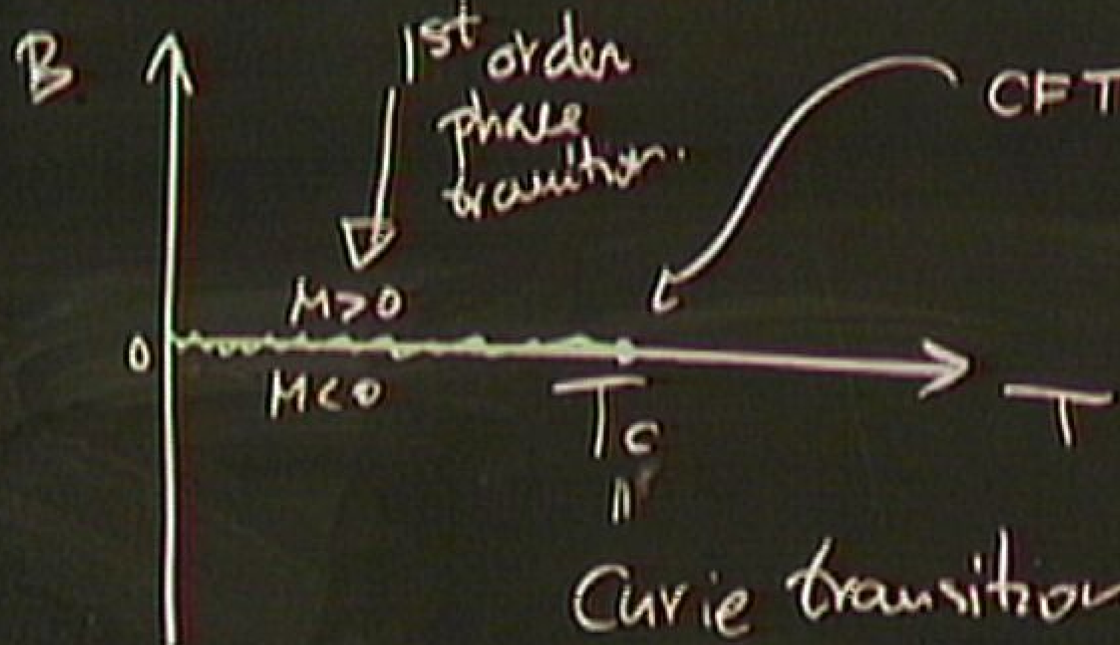
CFT.
0
$T_c$
T
Curie transition.

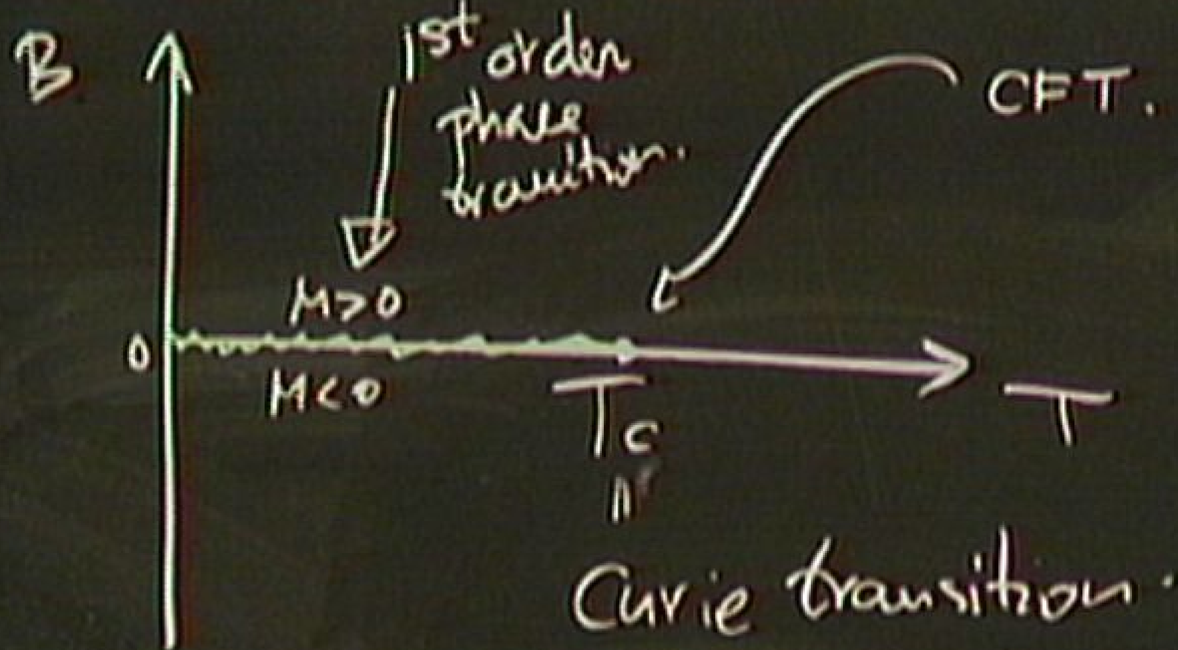
# Phase Diagram: Ferromagnets.




Curie transition.

ordered.

## Phase Diagram: Ferromagnets.

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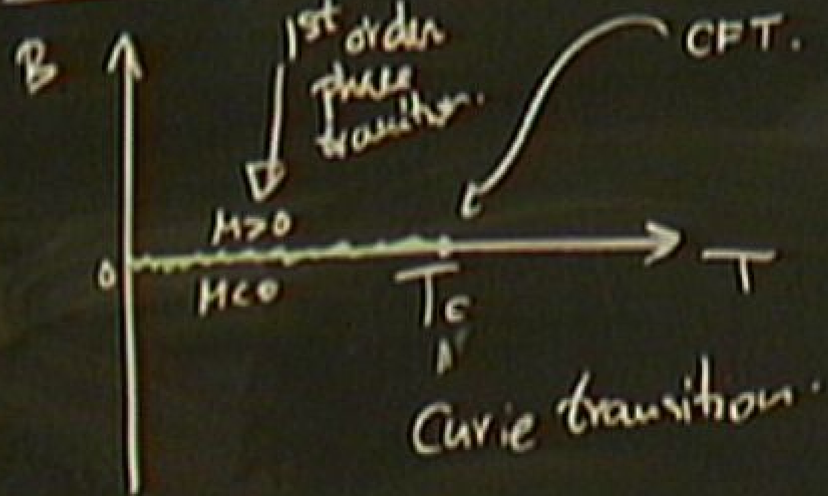
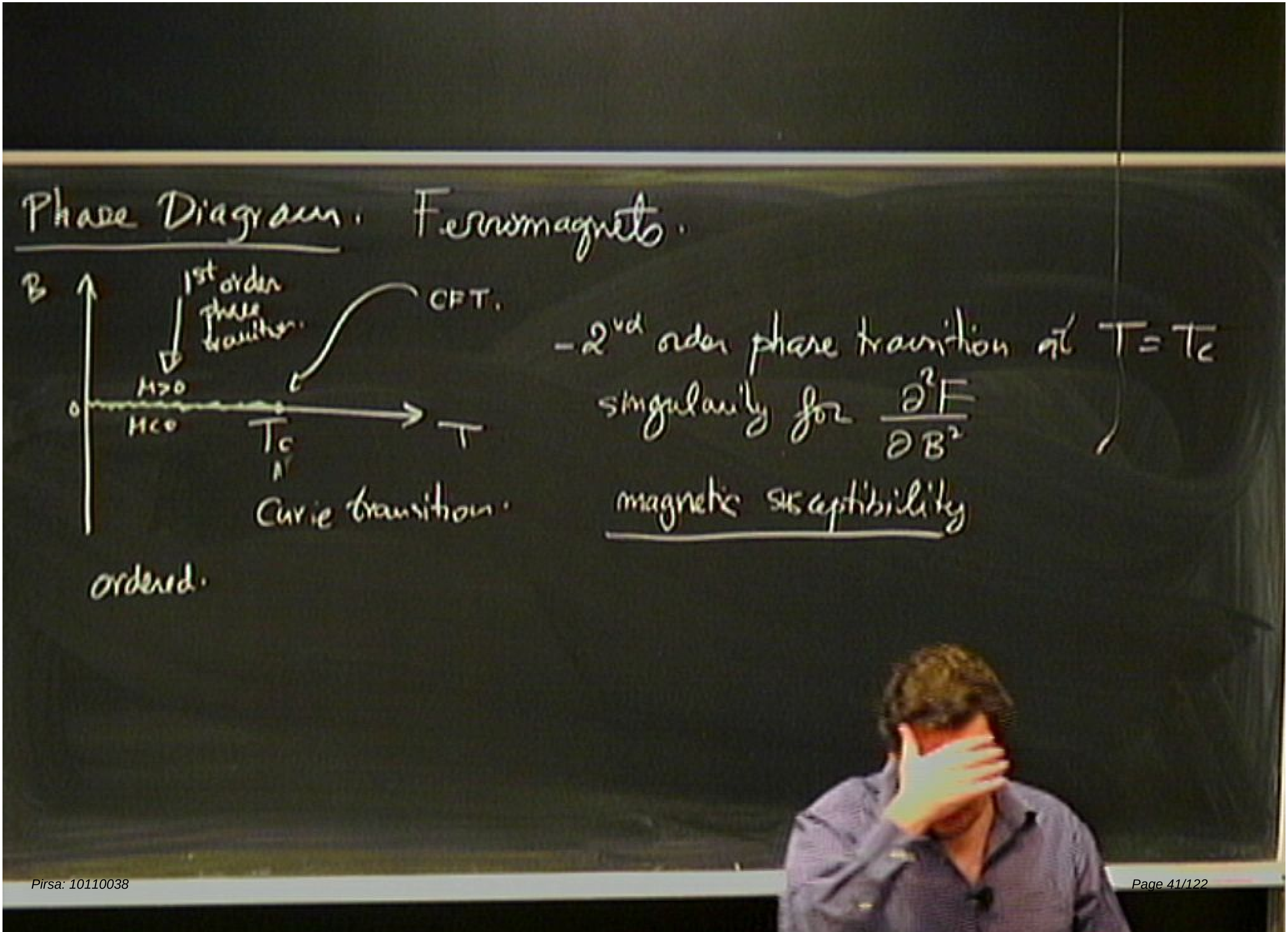
## Phase Diagram: Ferromagnets.

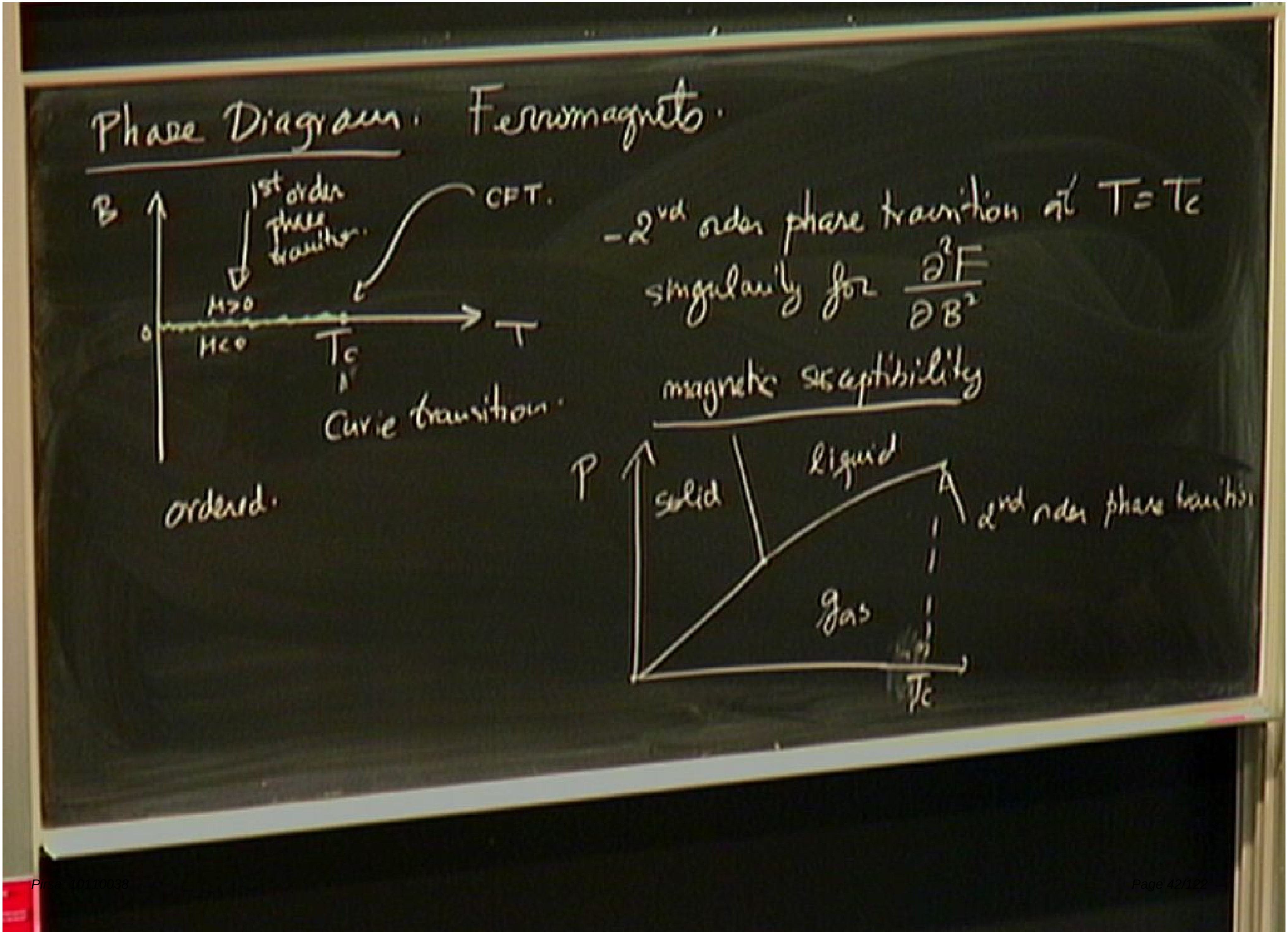



ordered.

- 2nd order phase transition at $T = T_c$

singularity for


Phase Diagram. Ferromagnets.
B
1st order phase transition.
CFT.
M>0
M<0
0
Tc
T
Curie transition.
ordered.
- 2nd order phase transition at T = Tc
singularity for ∂²F/∂B²
magnetic susceptibility


Phase Diagram: Ferromagnets.
B
1st order phase transition
CPT.
M>0
M<0
0
Tc
T
Curie transition.
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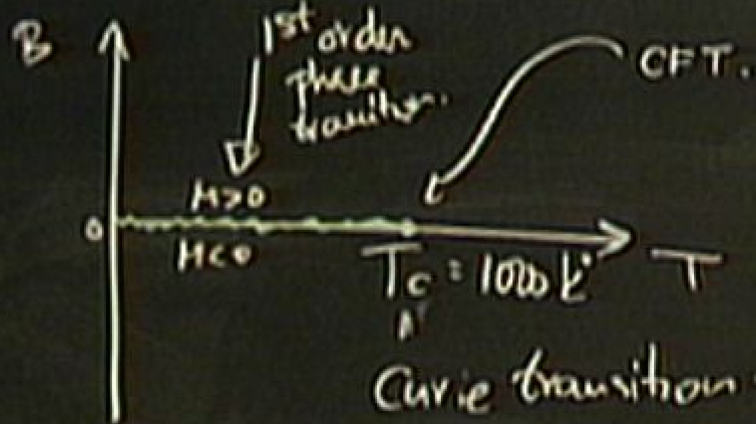
- 2nd order phase transition at T = Tc
singularity for ∂²F/∂B²
magnetic susceptibility
P
solid
liquid
gas
2nd order phase transition
Tc

# Phase Diagram. Ferromagnets.

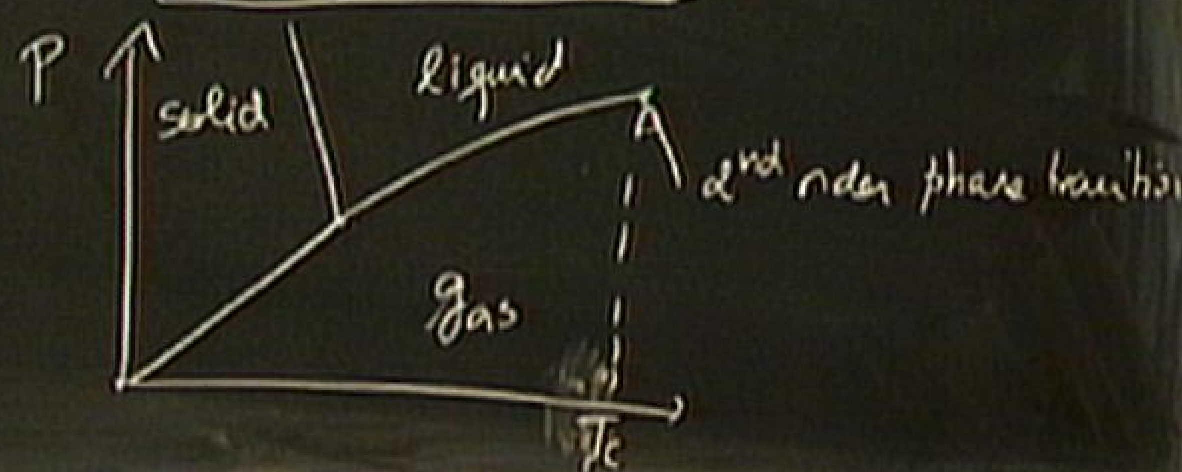


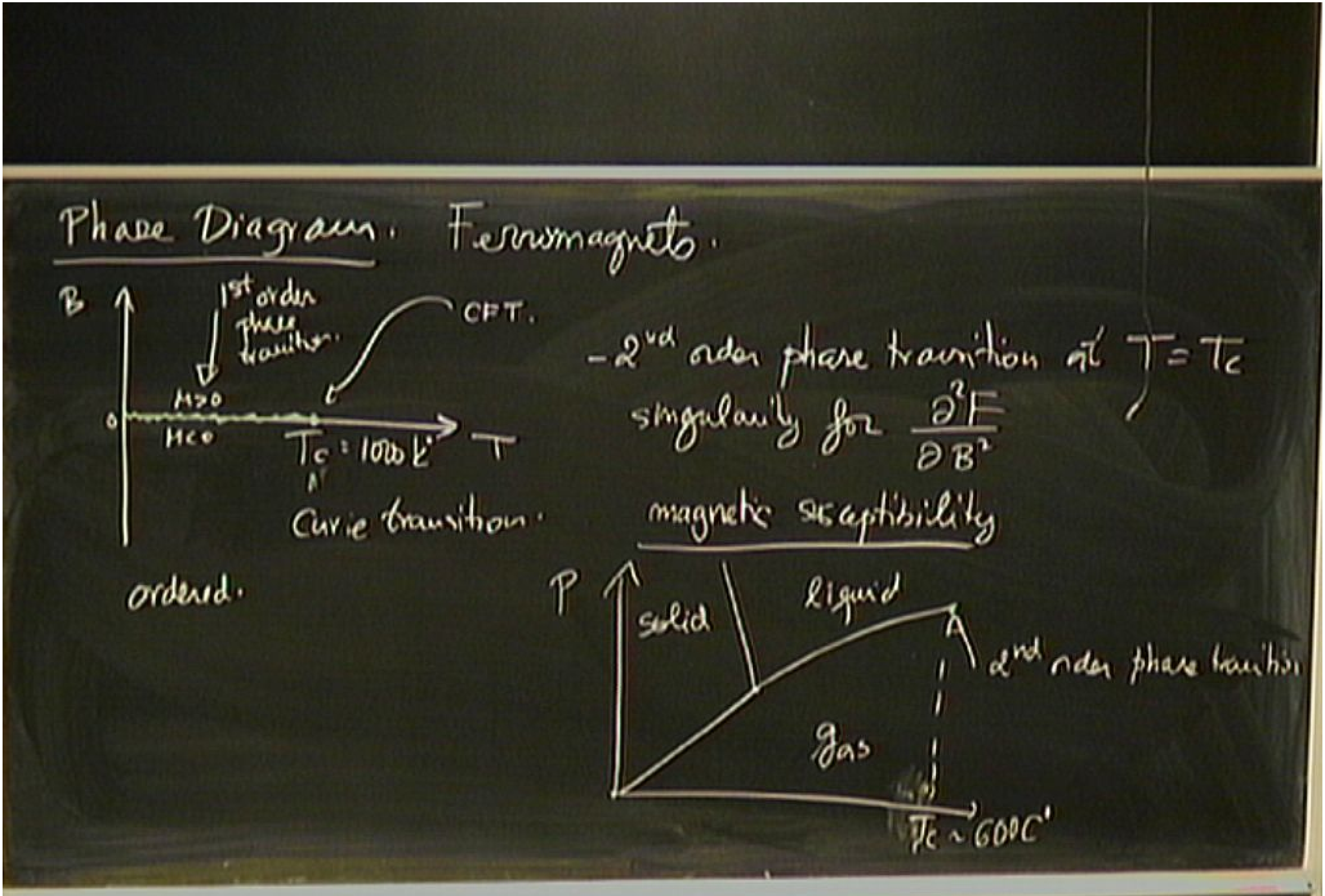

– 2nd order phase transition at $T = T_c$

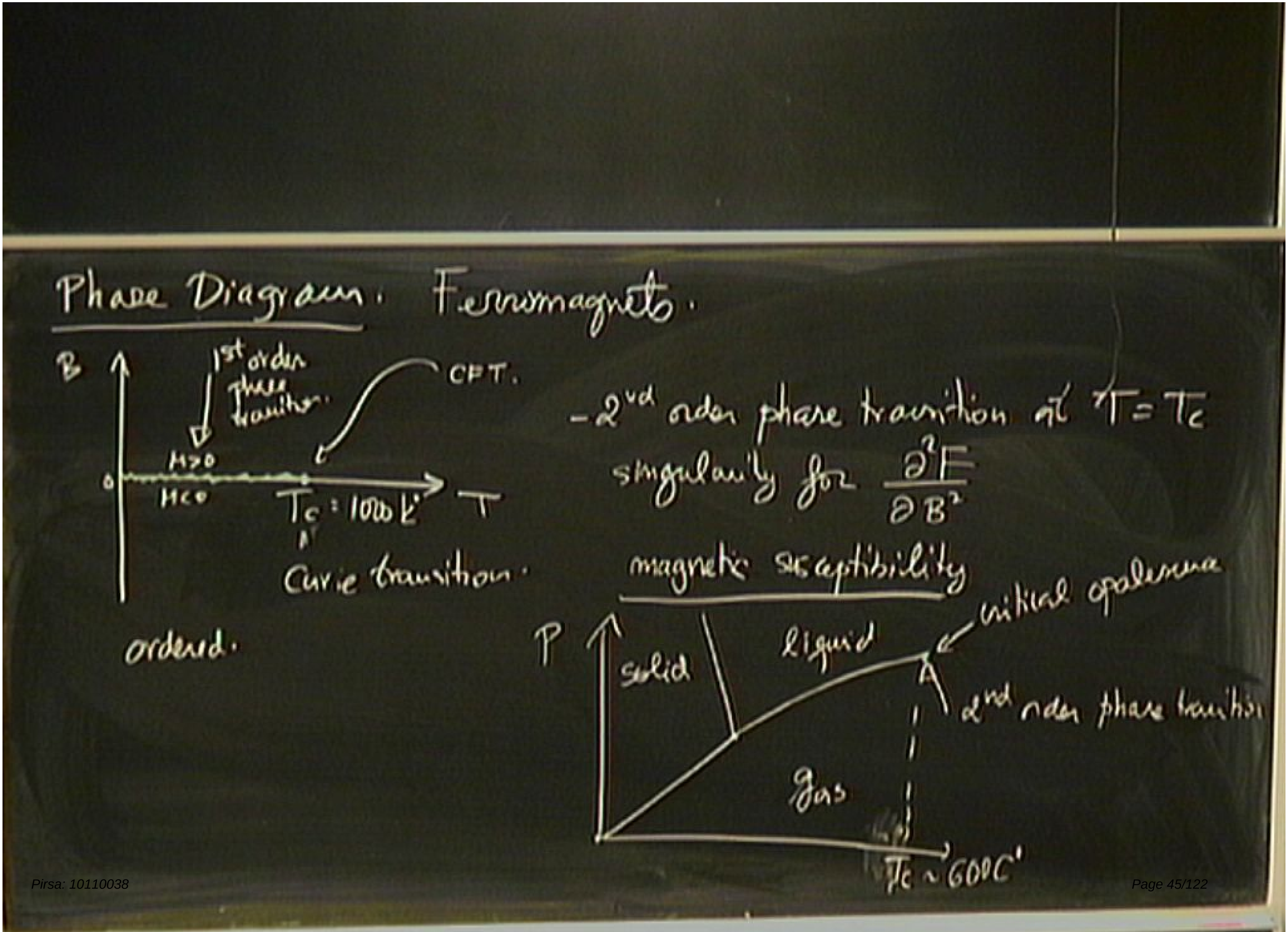
singularity for $\frac{\partial^2 F}{\partial B^2}$

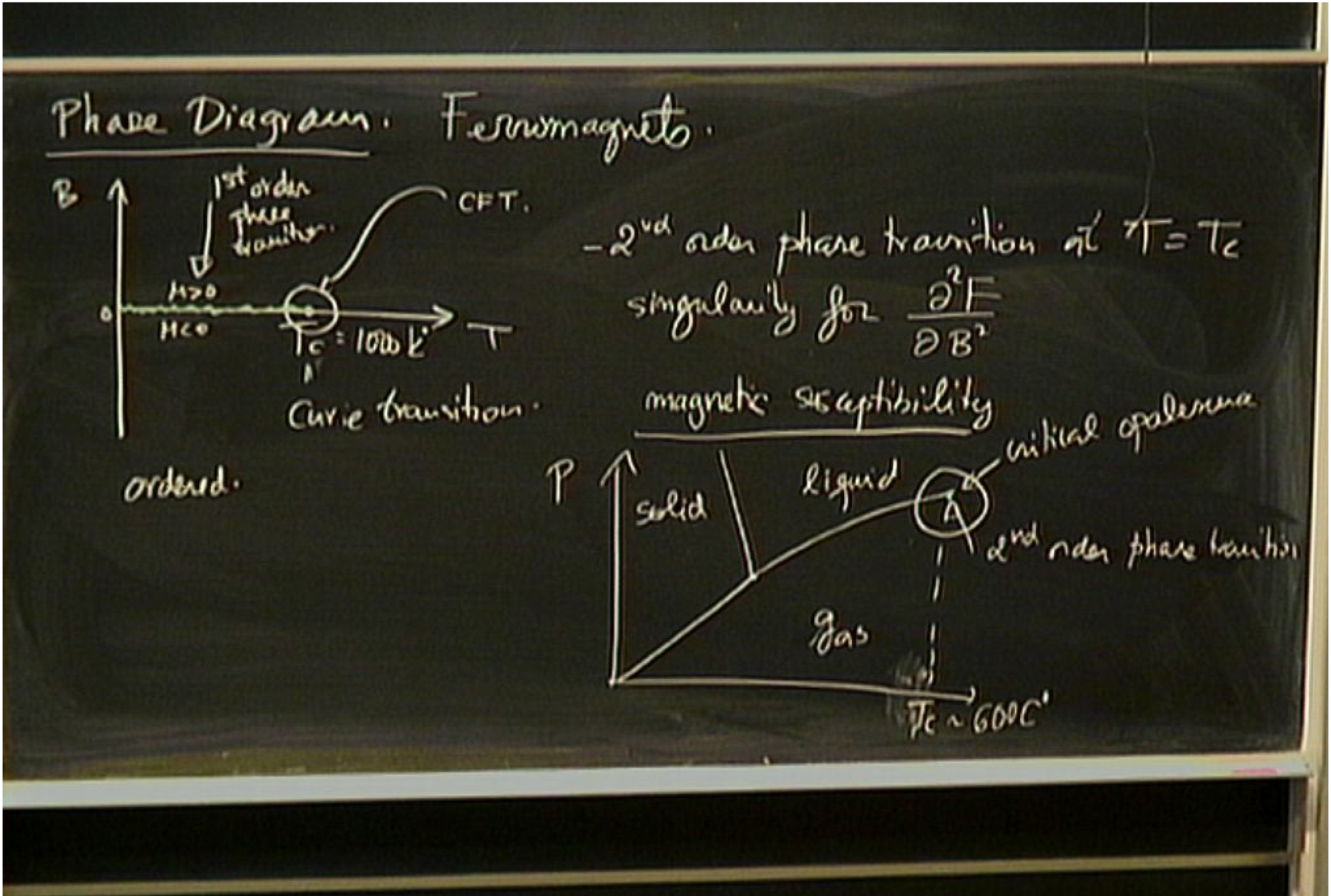
magnetic susceptibility

ordered.




Phase Diagram. Ferromagnets.
B
1st order phase transition.
CFT.
M>0
M<0
0
T
Tc : 1000 K
Curie transition.
ordered.
- 2nd order phase transition at T = Tc
singularity for ∂²F/∂B²
magnetic susceptibility
P
solid
liquid
gas
2nd order phase transition
Tc ~ 600 C


Phase Diagram. Ferromagnets.
B
1st order phase transition
CFT.
H>0
H<0
0
$T_c = 1000K$
T
Curie transition.
ordered.
– 2nd order phase transition at $T = T_c$
singularity for $\frac{\partial^2 F}{\partial B^2}$
magnetic susceptibility
critical opalescence
P
solid
liquid
gas
2nd order phase transition
$T_c \sim 600C$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1st order phase transition.
CFT.
H>0
H<0
0
Tc = 1000 K
T
Curie transition.
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singularity for ∂²F/∂B²
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solid
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Tc ~ 600C

$T_c \sim 600C$'

- Phase transitions are characterized by non-analyticities of macroscopic thermodynamic quantities

$T_c \sim 600C$

- Phase transitions are characterized by non-analyticities of macroscopic thermodynamic quantities.

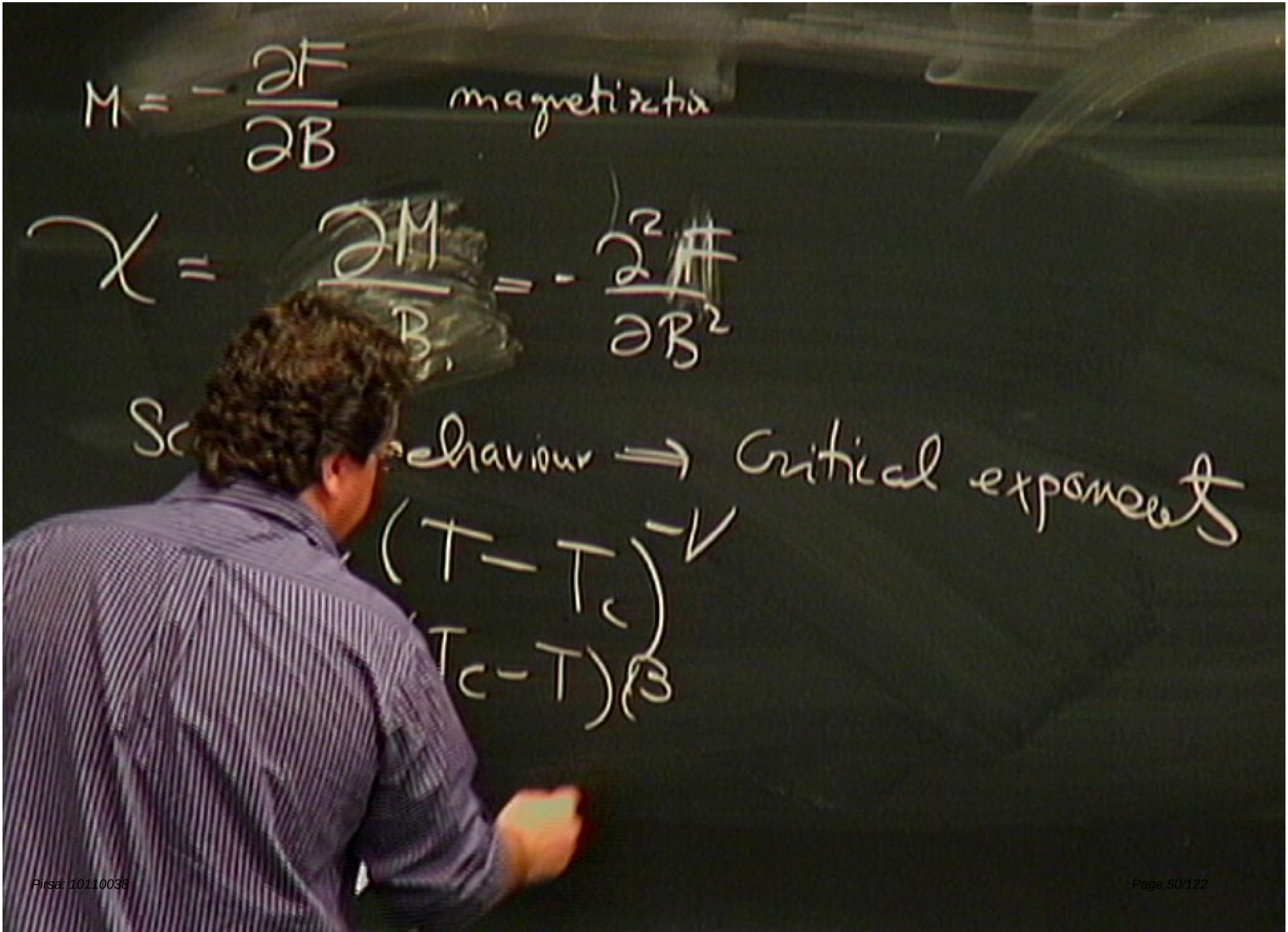
$$Z = \sum_{s} e^{-\frac{H(s, B, P, \ldots)}{T}} \equiv e^{-F/T}$$

$Z$ ↑ partition function

$\sum_s$ ↑ states.

$F$ ↑ Free energy.

$$M = -\frac{\partial F}{\partial B}$$ magnetization



$M = -\frac{\partial F}{\partial B}$ magnetizatia
$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2}$
Sc
ehaviour ⇒ Critical exponents
$(T - T_c)^{-\nu}$
$T_c - T)^\beta$

$$M = -\frac{\partial F}{\partial B} \quad \text{magnetization}$$

$$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2}$$

Scaling Behaviour $\Rightarrow$ Critical exponents

$$\xi \sim (T - T_c)^{-\nu}$$

$$M \sim (T_c - T)^{\beta}$$

$$M = -\frac{\partial F}{\partial B} \quad \text{magnetization}$$

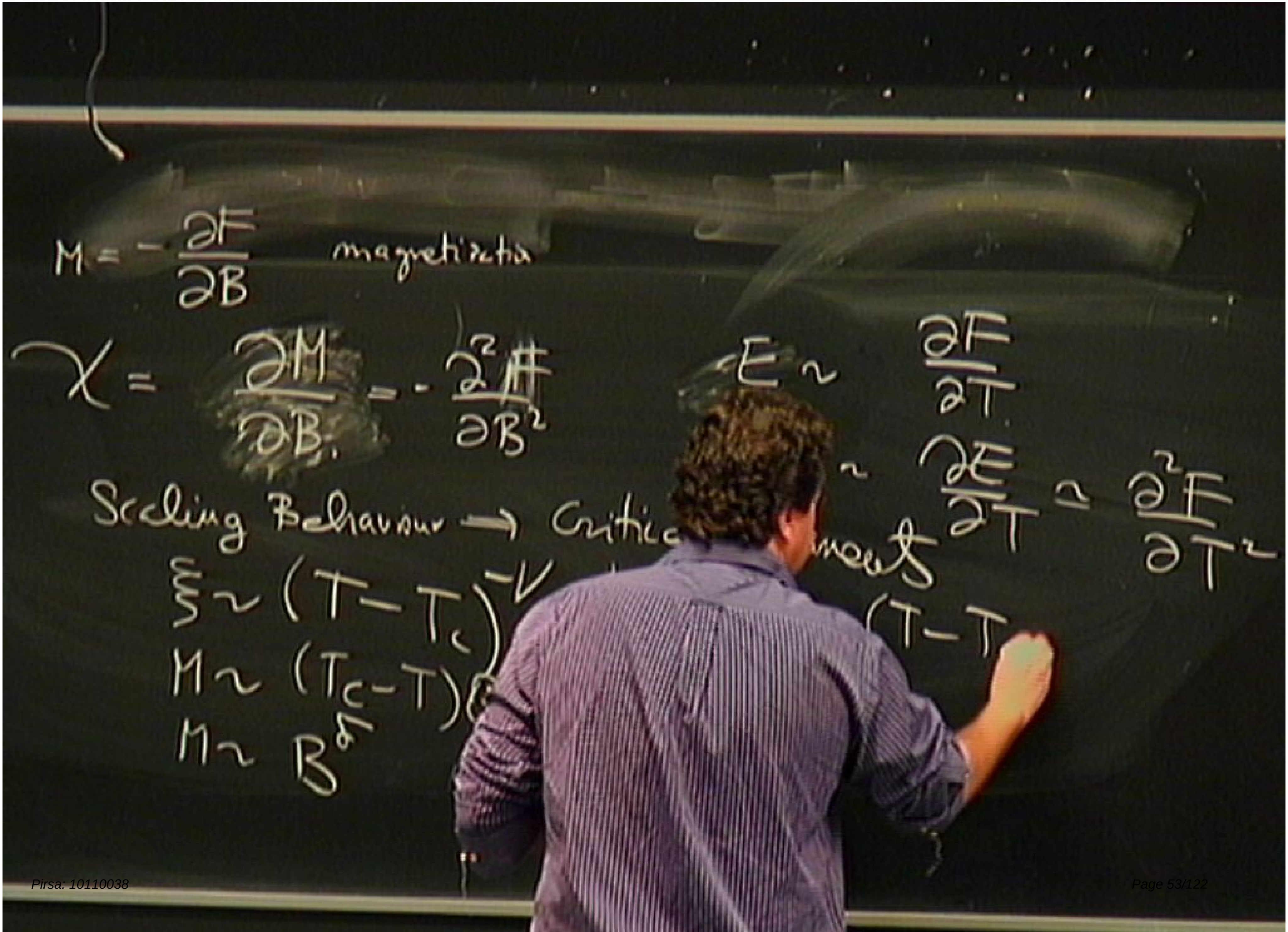
$$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2} \qquad C_V$$

Scaling Behaviour $\Rightarrow$ Critical exponent

$$\xi \sim (T - T_c)^{-\nu} \qquad C$$

$$M \sim (T_c - T)^{\beta}$$

$$M \sim B^{\delta}$$


M = - ∂F/∂B magnetization
χ = ∂M/∂B = - ∂²F/∂B²
E ~ ∂F/∂T
~ ∂E/∂T ≃ ∂²F/∂T²
Scaling Behaviour →
ξ ~ (T−T_c)^-ν
M ~ (T_c−T)
M ~ B

$M = -\frac{\partial F}{\partial B}$ magnetization

$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2}$

$E \sim \frac{\partial F}{\partial T}$

$C_V \sim \frac{\partial E}{\partial T} \simeq \frac{\partial^2 F}{\partial T^2}$

Scaling Behaviour $\Rightarrow$ Critical exponents

$\xi \sim (T - T_c)^{-\nu}$

$M \sim (T_c - T)^{\beta}$

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Scaling Behaviour $\Rightarrow$ Critical exponents

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$M \sim (T_c - T)^{\beta}$

$M \sim B^{\delta}$

$C_V \sim (T - T_c)^{-\alpha}$

$\langle \sigma(x)\, \sigma(0) \rangle \sim \frac{1}{x}$

$M = -\frac{\partial F}{\partial B}$ magnetization

$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2}$

$E \sim \frac{\partial F}{\partial T}$

$C_V \sim \frac{\partial E}{\partial T} \sim \frac{\partial^2 F}{\partial T^2}$

Scaling Behaviour $\Rightarrow$ Critical exponents

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$M \sim (T_c - T)^{\beta}$

$M \sim B^{\delta}$

$C_V \sim (T - T_c)^{-\alpha}$.

$\langle \sigma(x)\, \sigma(0) \rangle \sim \frac{1}{|x|^{D-2+\eta}}$

$M = -\frac{\partial F}{\partial B}$ magnetization

$\chi = \frac{\partial M}{\partial B} = -\frac{\partial^2 F}{\partial B^2}$

$E \sim \frac{\partial F}{\partial T}$

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Scaling Behaviour $\rightarrow$ Critical exponents

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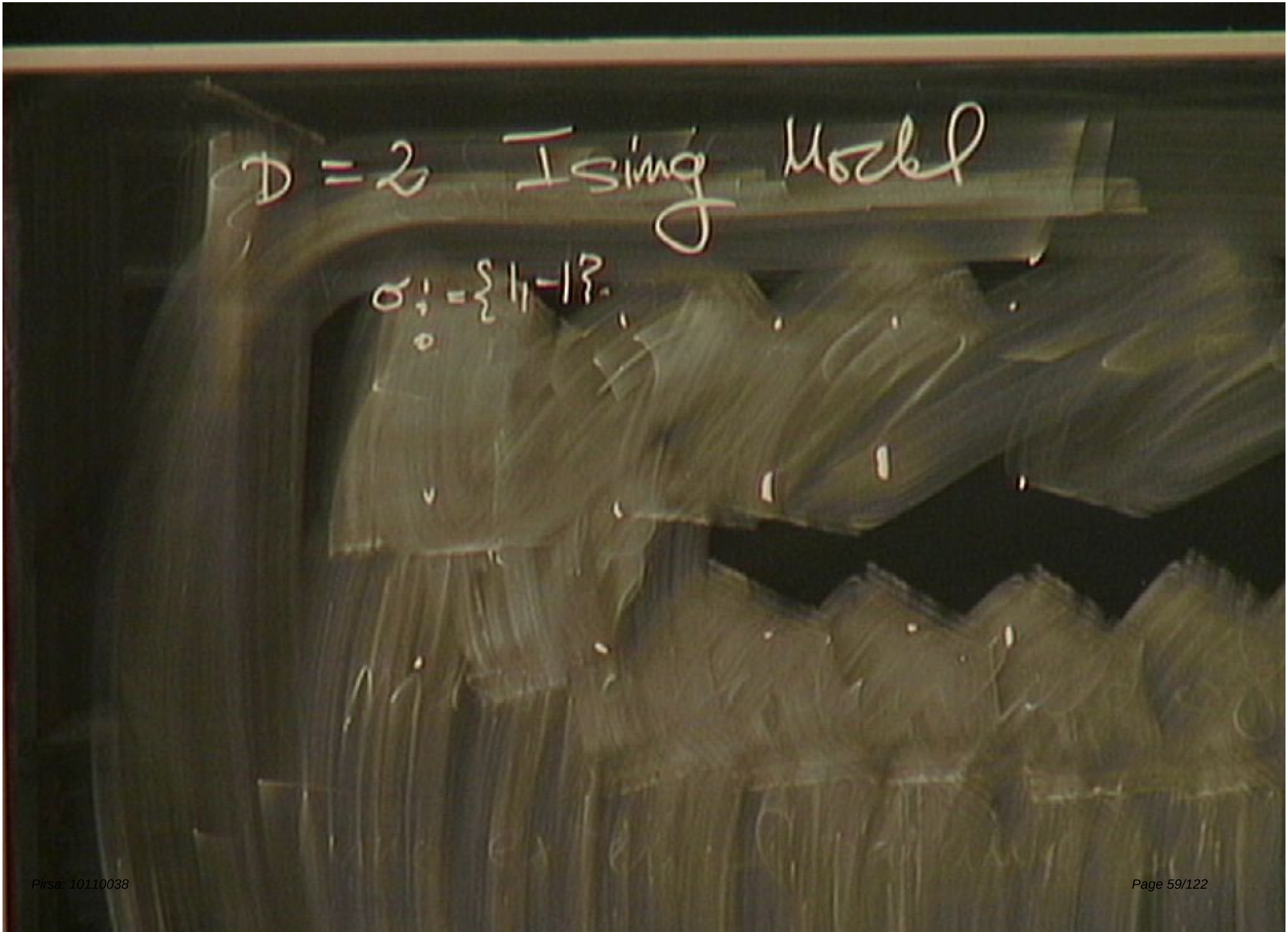
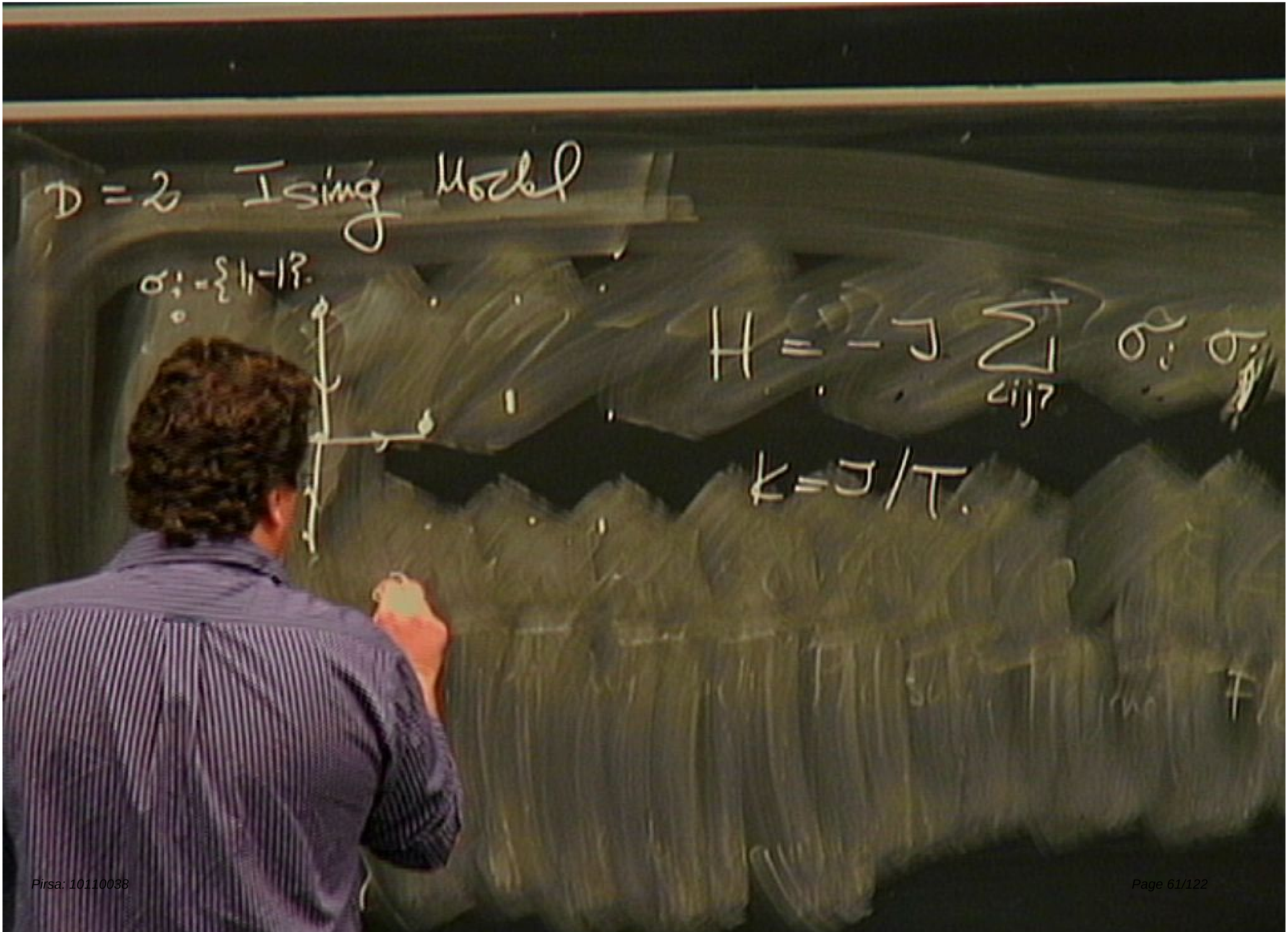

D = 2 Ising Model
σ_i = {1, -1}.

# $D = 2$ Ising Model

$\sigma_i = \{1, -1\}$

$$H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$


D = 2 Ising Model
σi = {1, -1}.
H = -J Σ<ij> σi σj
K = J/T.

## D = 2 Ising Model

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$$K = J/T.$$

2nd order phase transition when $\sinh(2K_c) = 1$

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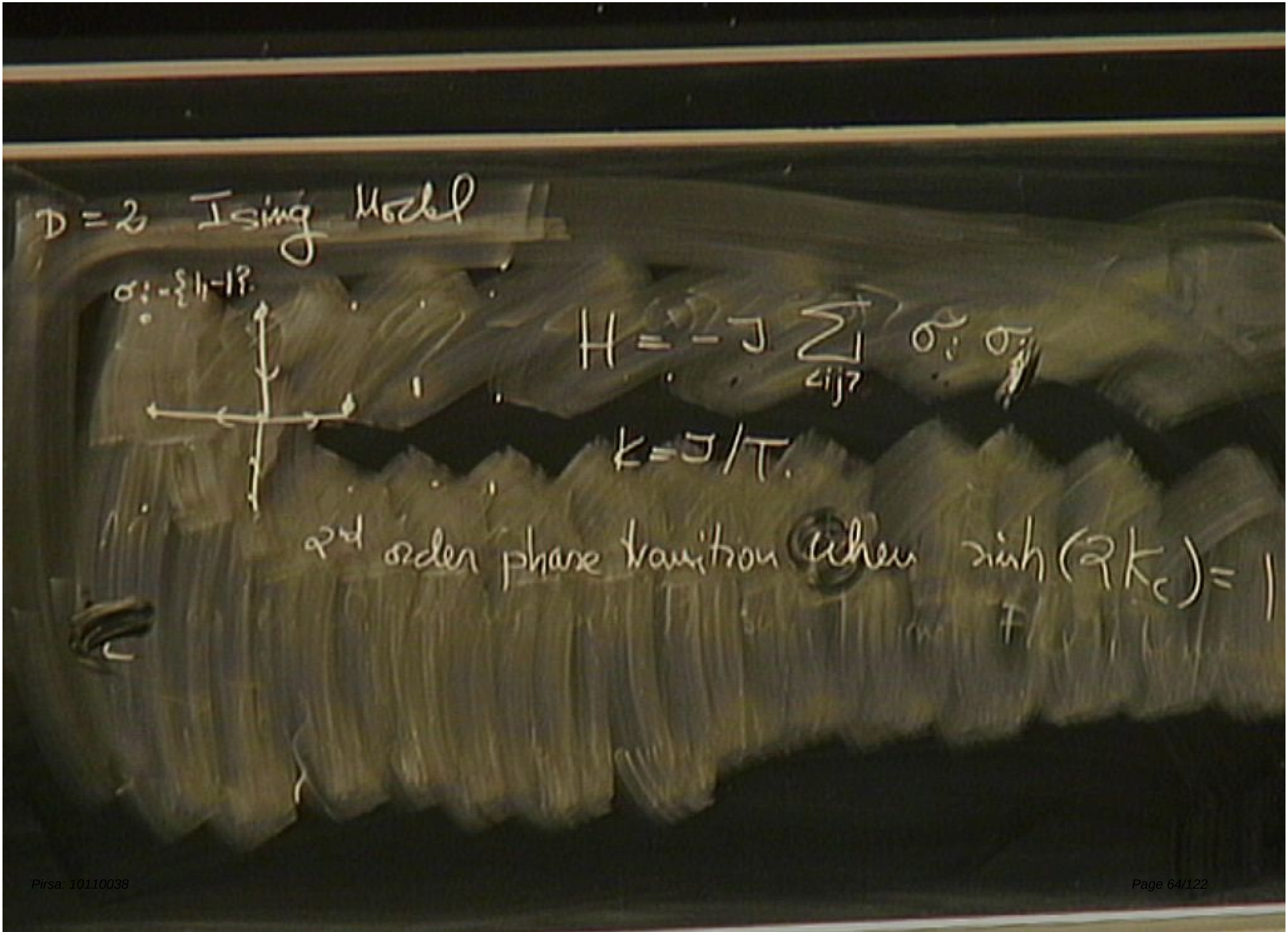
$$K = J/T.$$

2nd order phase transition when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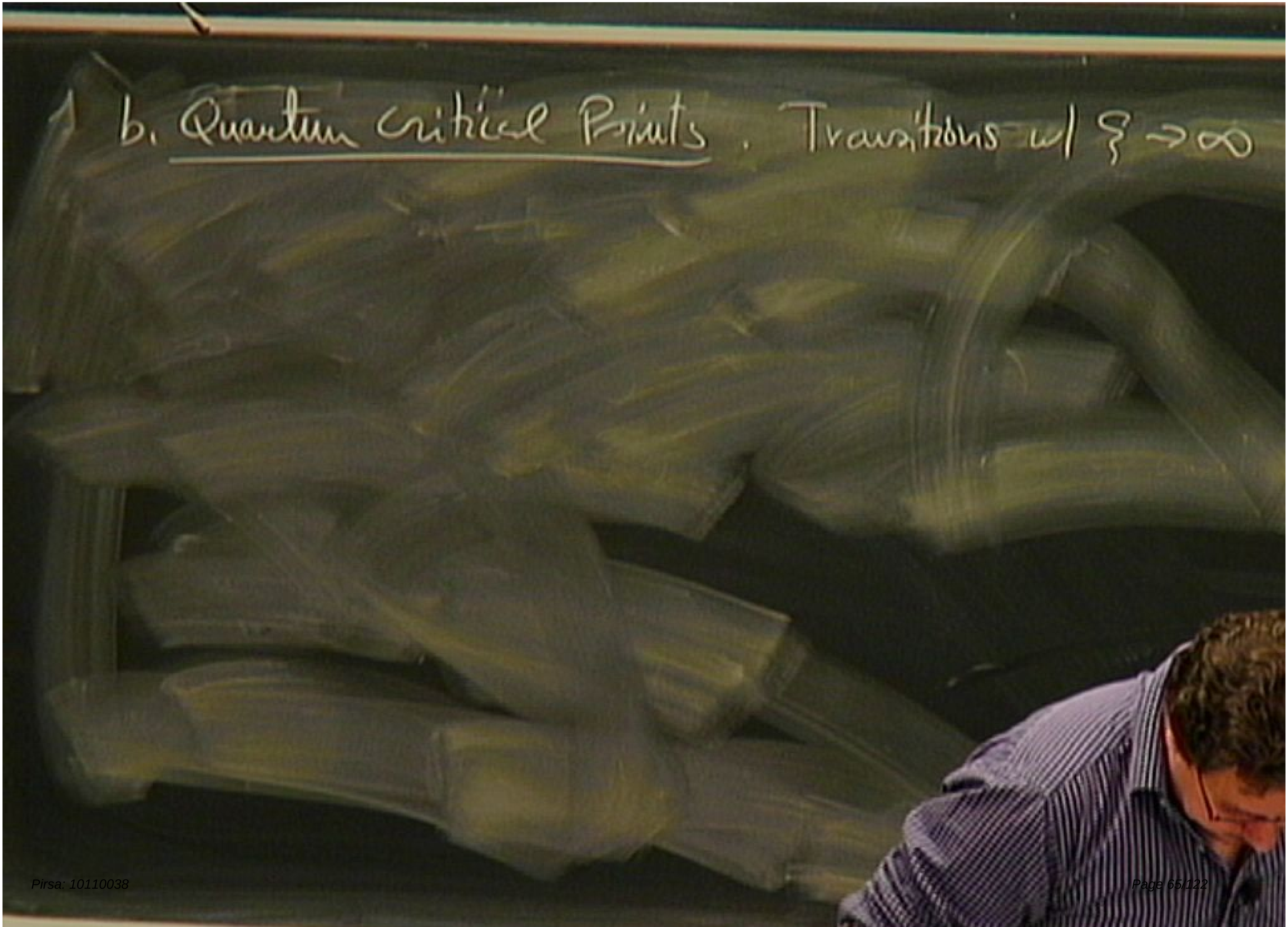
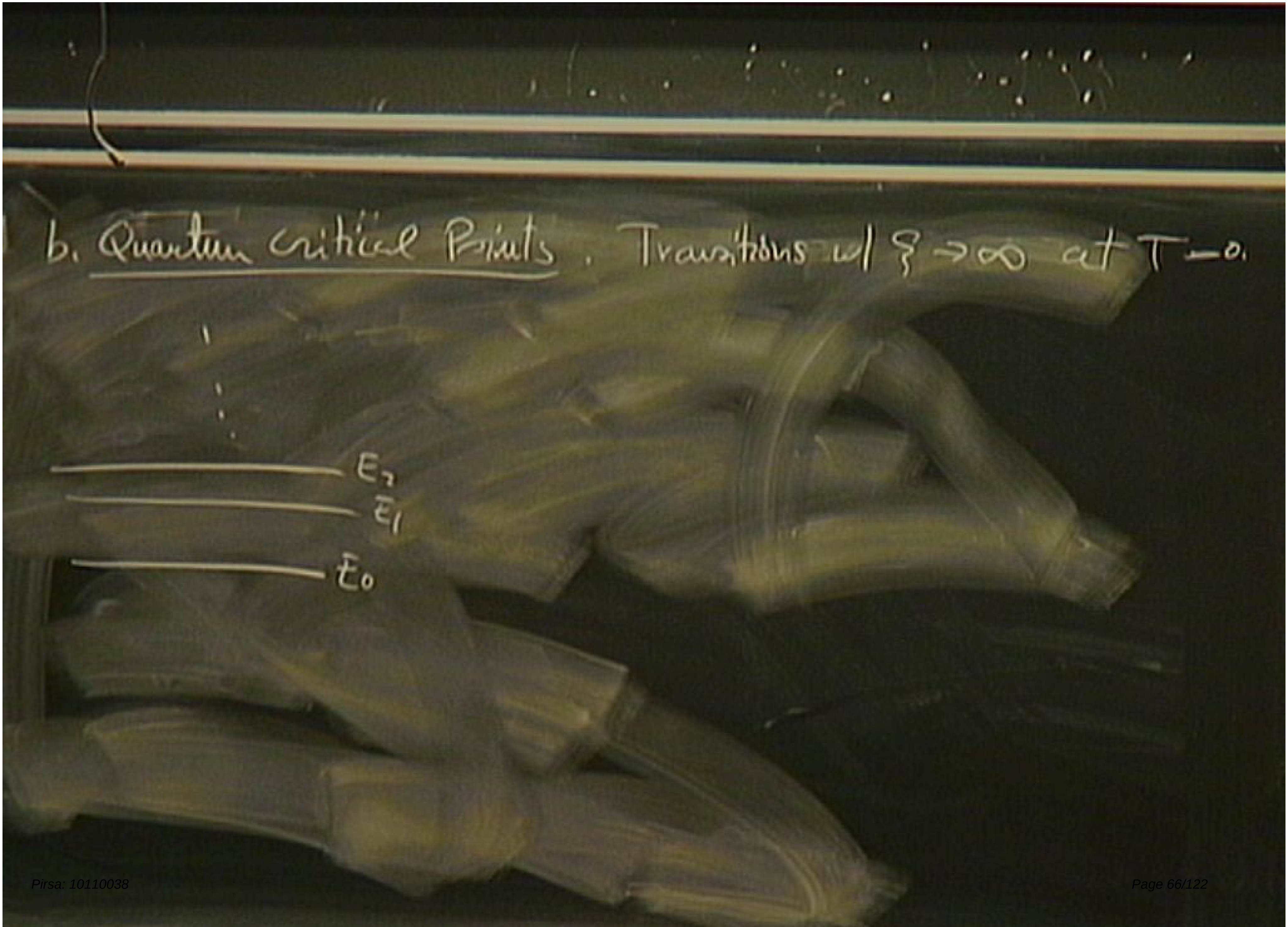

b. Quantum Critical Points. Transitions w/ ξ → ∞


b. Quantum critical Points. Transitions w/ $\xi \to \infty$ at T = 0.
$E_2$
$E_1$
$E_0$

## b. Quantum Critical Points. Transition

$\vdots$

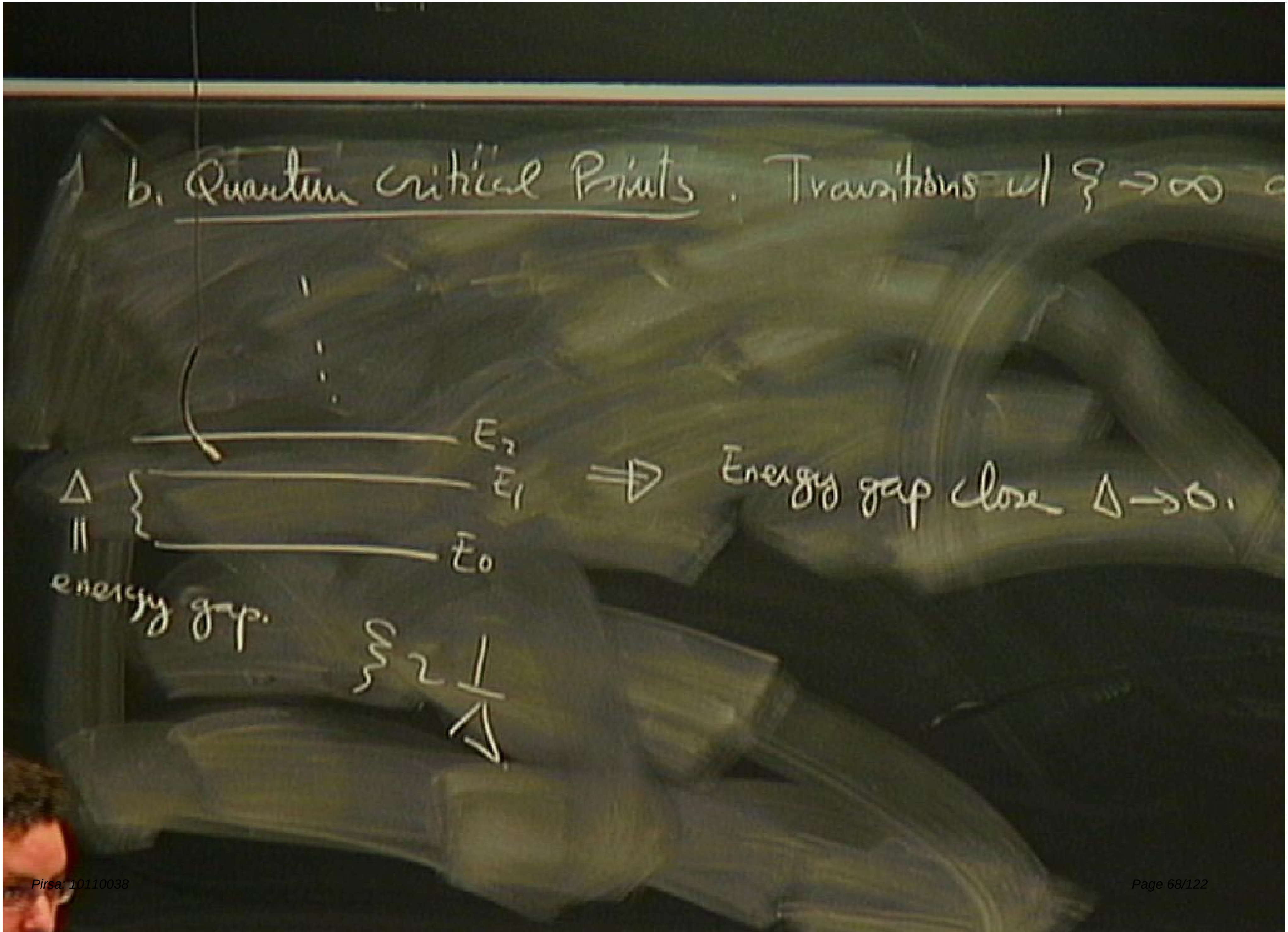
$E_2$

$E_1$

$E_0$

$\Delta$ = energy gap ($\Delta = E_1 - E_0$)

$$\Rightarrow \quad \xi \sim \frac{1}{\Delta}$$



b. Quantum Critical Points. Transitions w/ $\xi \to \infty$
$E_2$
$E_1$
$E_0$
$\Delta$
$\|$
energy gap.
$\Rightarrow$ Energy gap close $\Delta \to 0$.
$\xi \sim \frac{1}{\Delta}$

b. Quantum Critical Points. Transitions w/ $\xi \to \infty$ at $T = 0$.

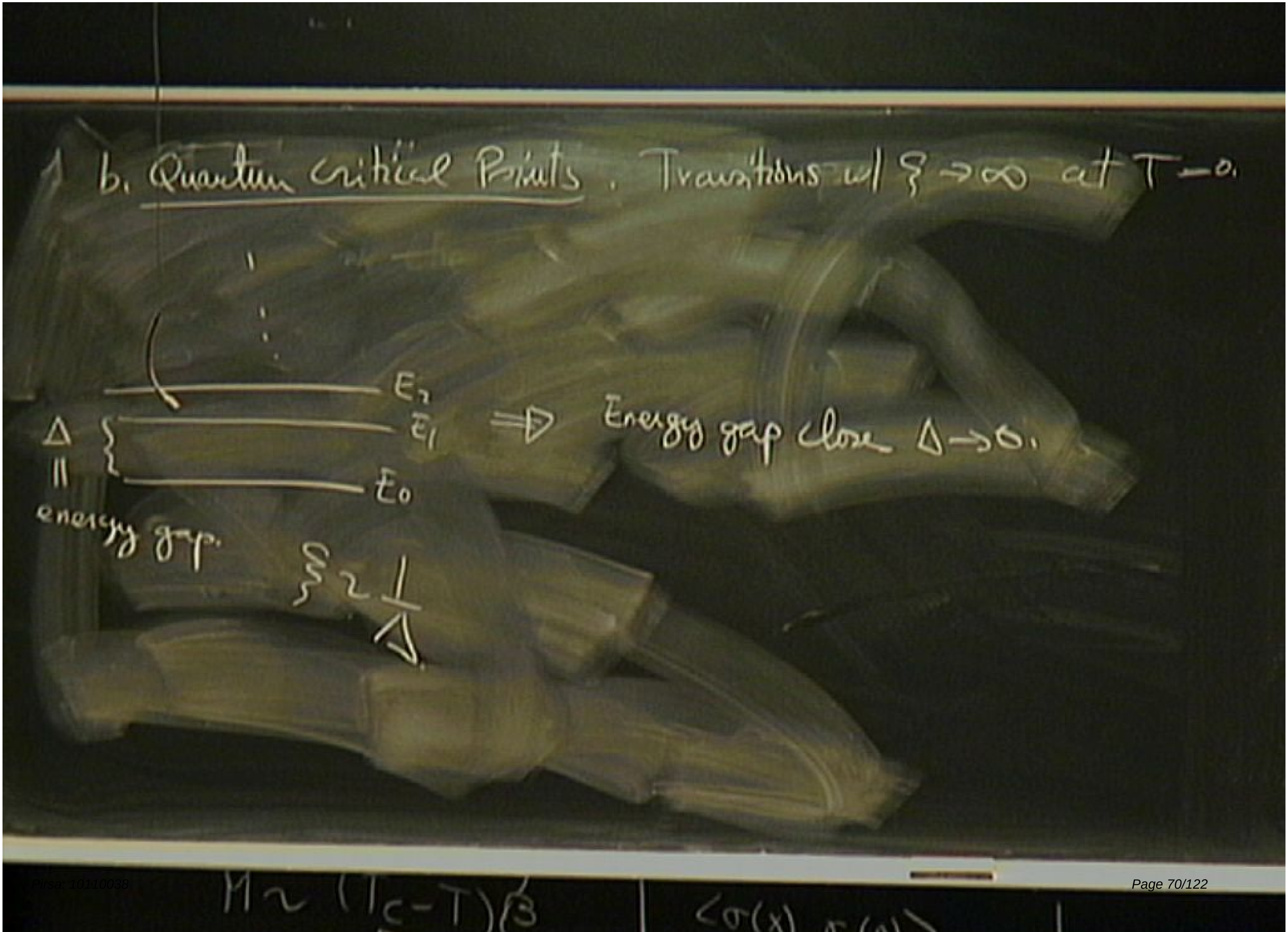
$E_2$

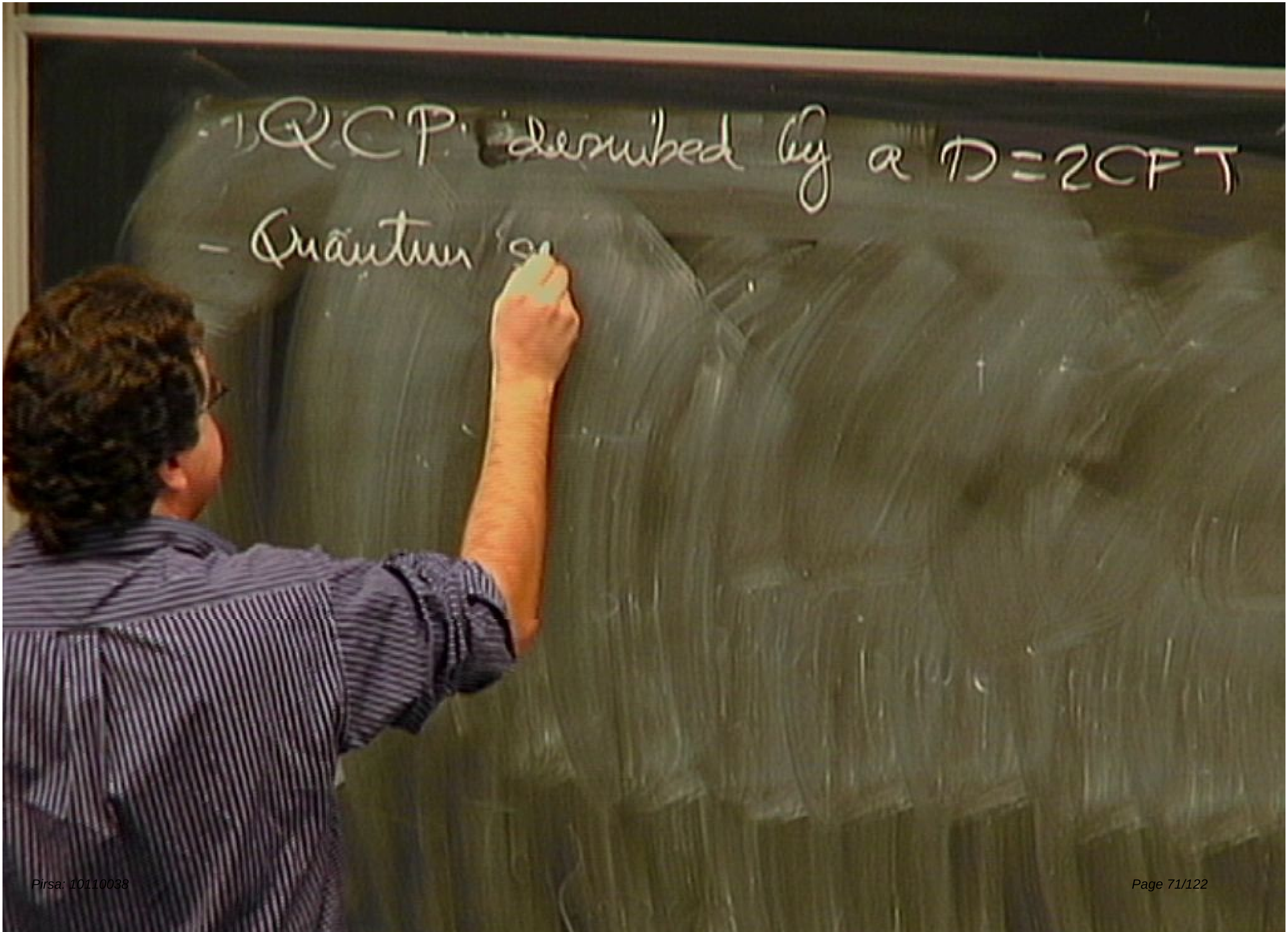
$\Delta \{$ $E_1$ $\Rightarrow$ Energy gap close $\Delta \to 0$.

$\parallel$ $E_0$

energy gap.

$$\xi \sim \frac{1}{\Delta}$$


b. Quantum Critical Points. Transitions w/ ξ → ∞ at T = 0.
E2
E1
E0
Δ
energy gap.
⇒ Energy gap close Δ → 0.
ξ ~ 1/Δ
M ~ (Tc − T)β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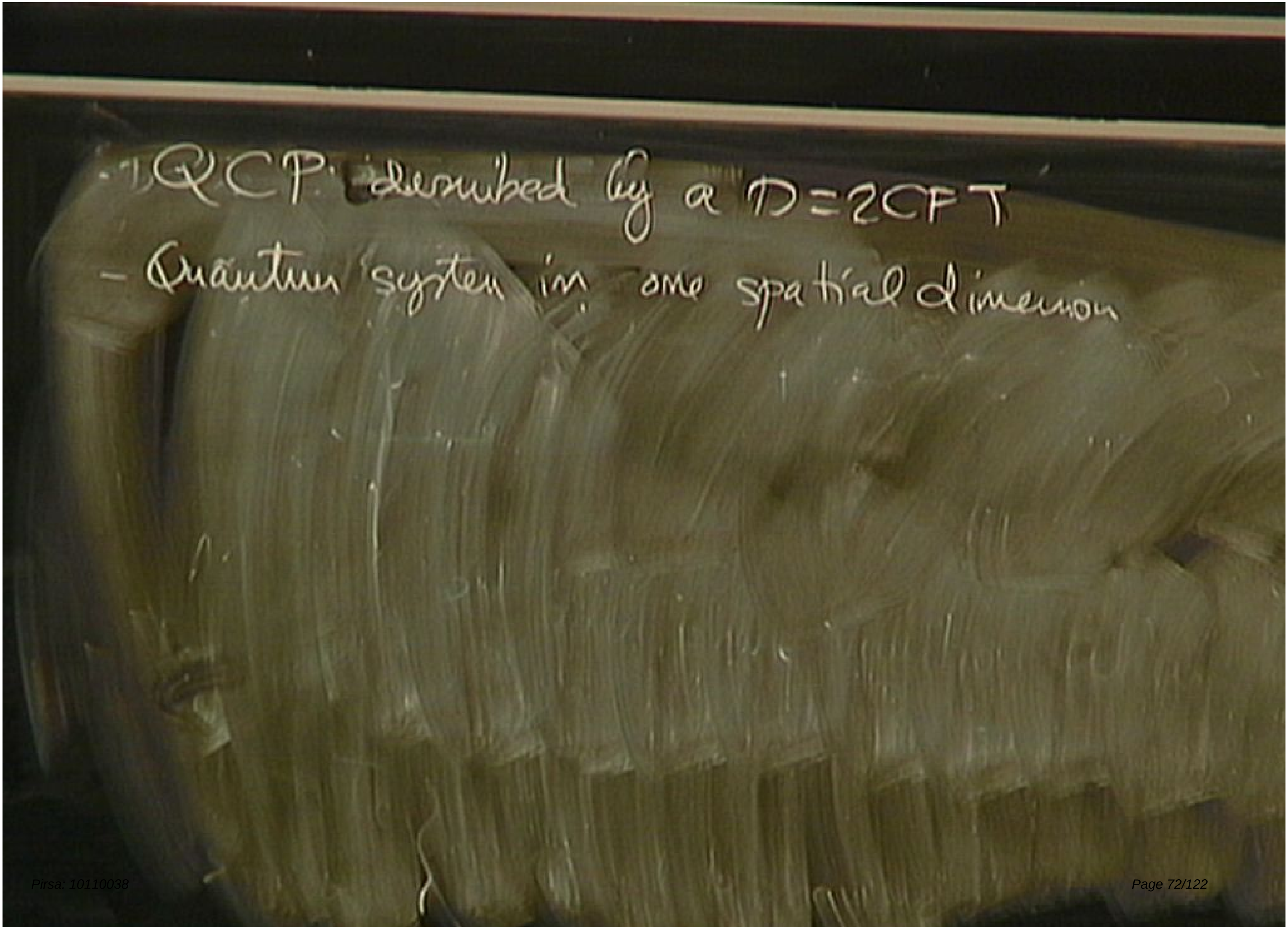
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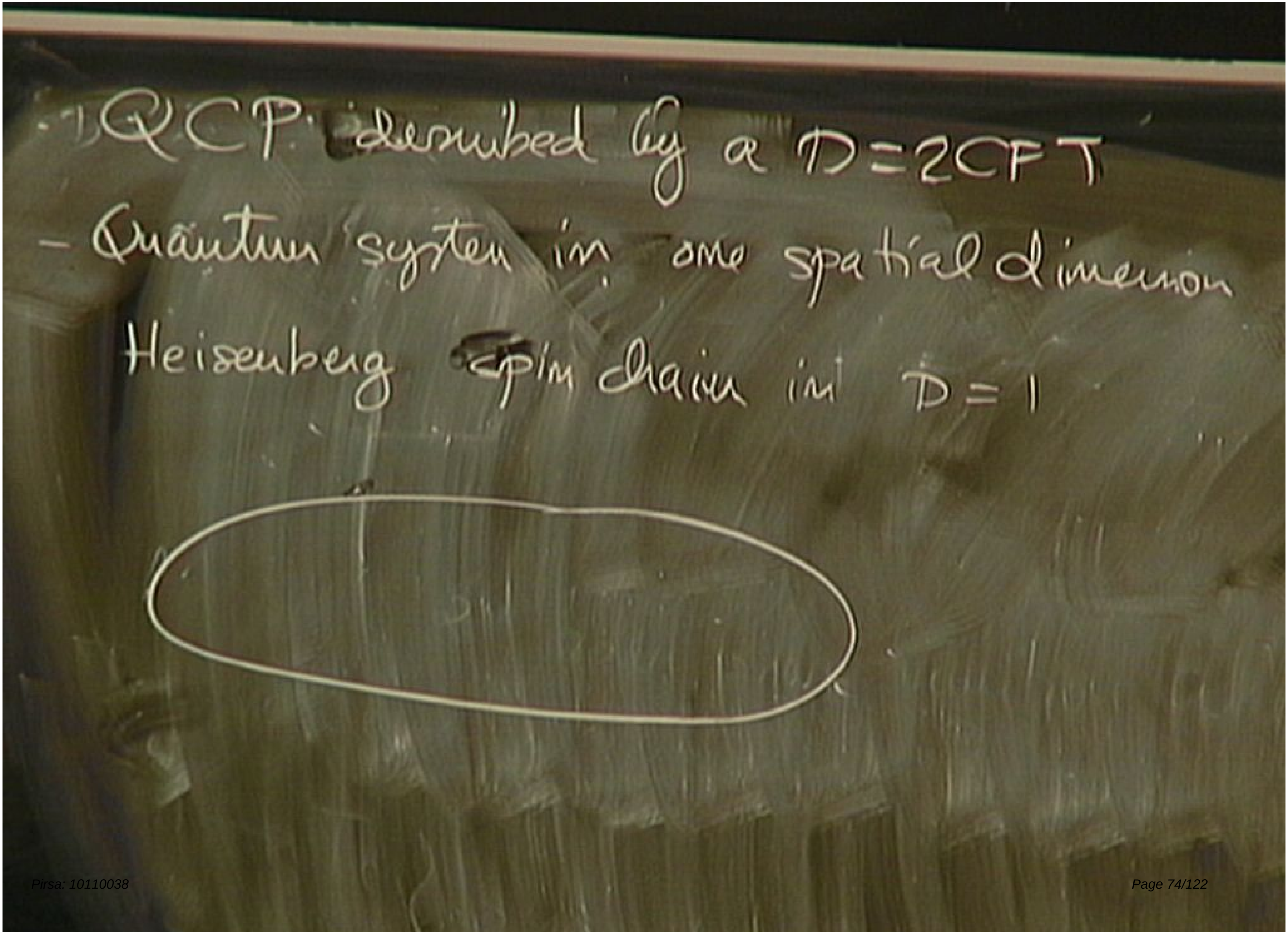
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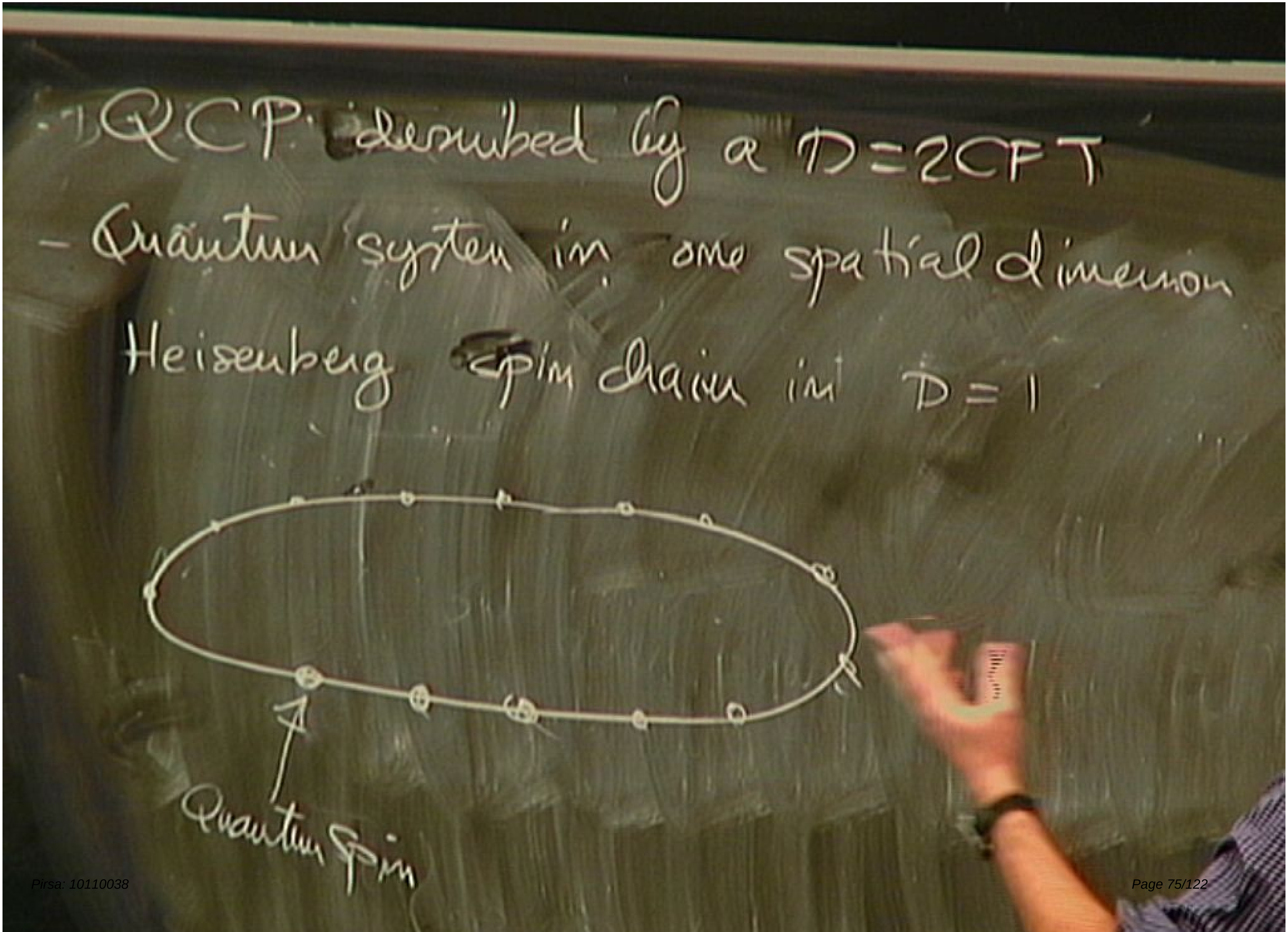
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- QCP described by a $D=2$ CFT
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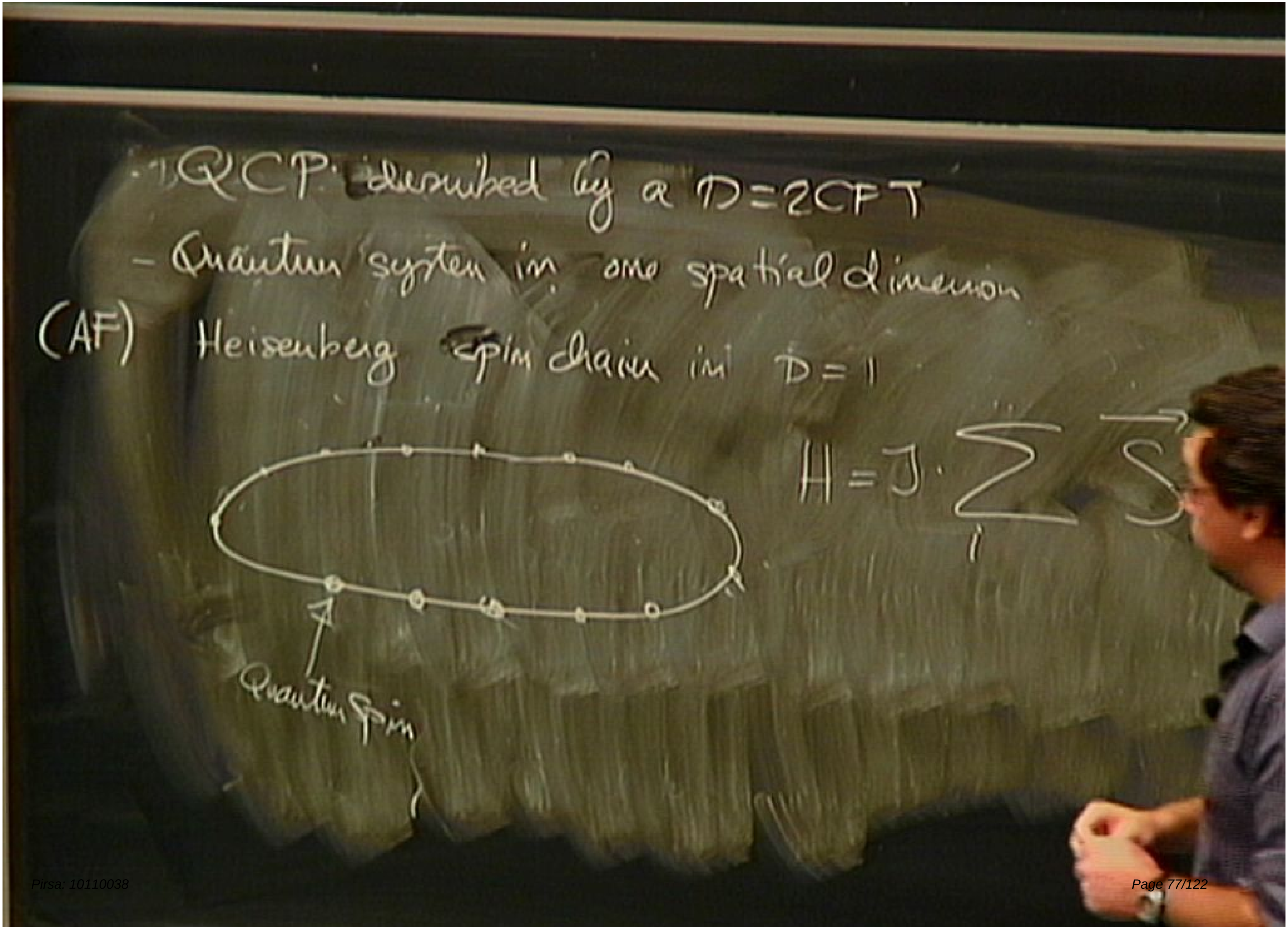
(AF) Heisenberg spin chain in $D=1$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$H = J \cdot \sum_i \vec{S}$
Quantum spin

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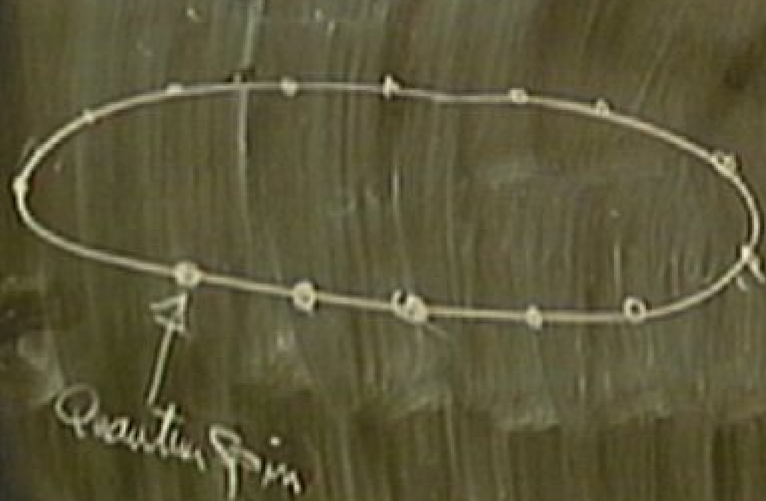
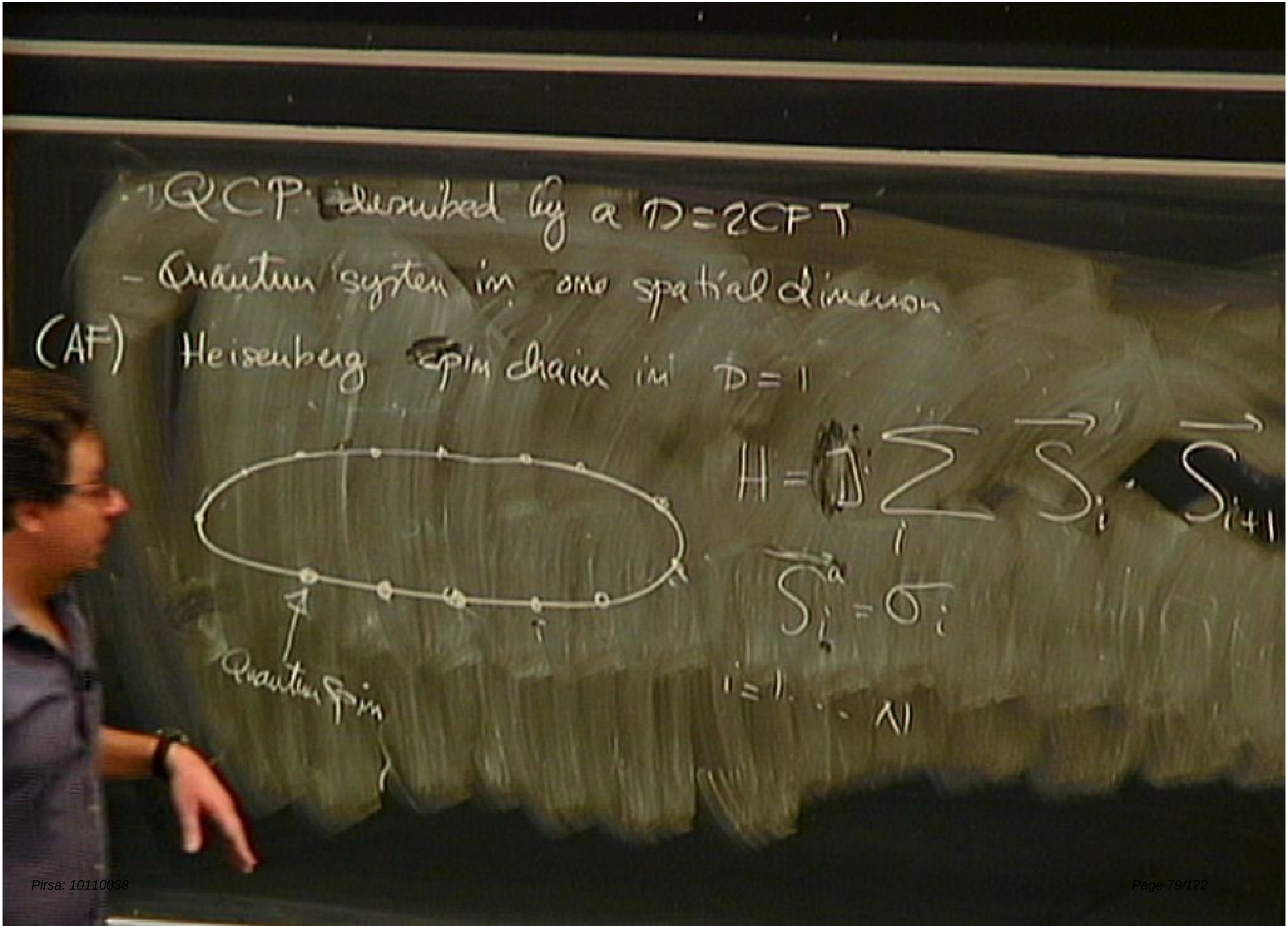
(AF) Heisenberg spin chain in D=1

$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$




QCP described by a D=2CFT
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(AF) Heisenberg spin chain in D=1
H = Σ S_i · S_{i+1}
i
S_i^a = σ_i^a
i = 1 . . . N
Quantum spin
i

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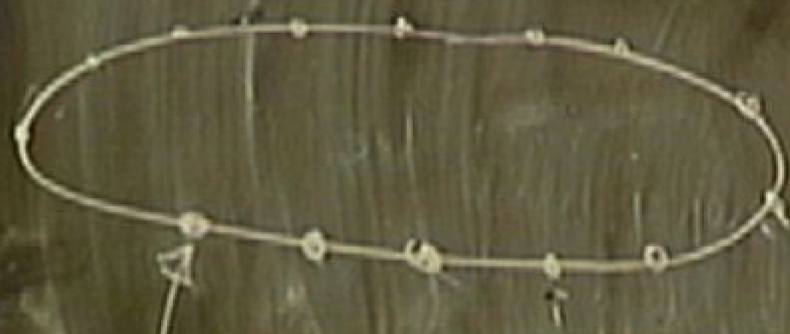


$$H = J \sum_i \vec{S}_i \cdot \vec{S}_{i+1}$$

$$\vec{S}_i^a = \sigma_i^a \qquad \sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$i = 1 \ldots N$

$a = 1,2,3$

$$\sigma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$\sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

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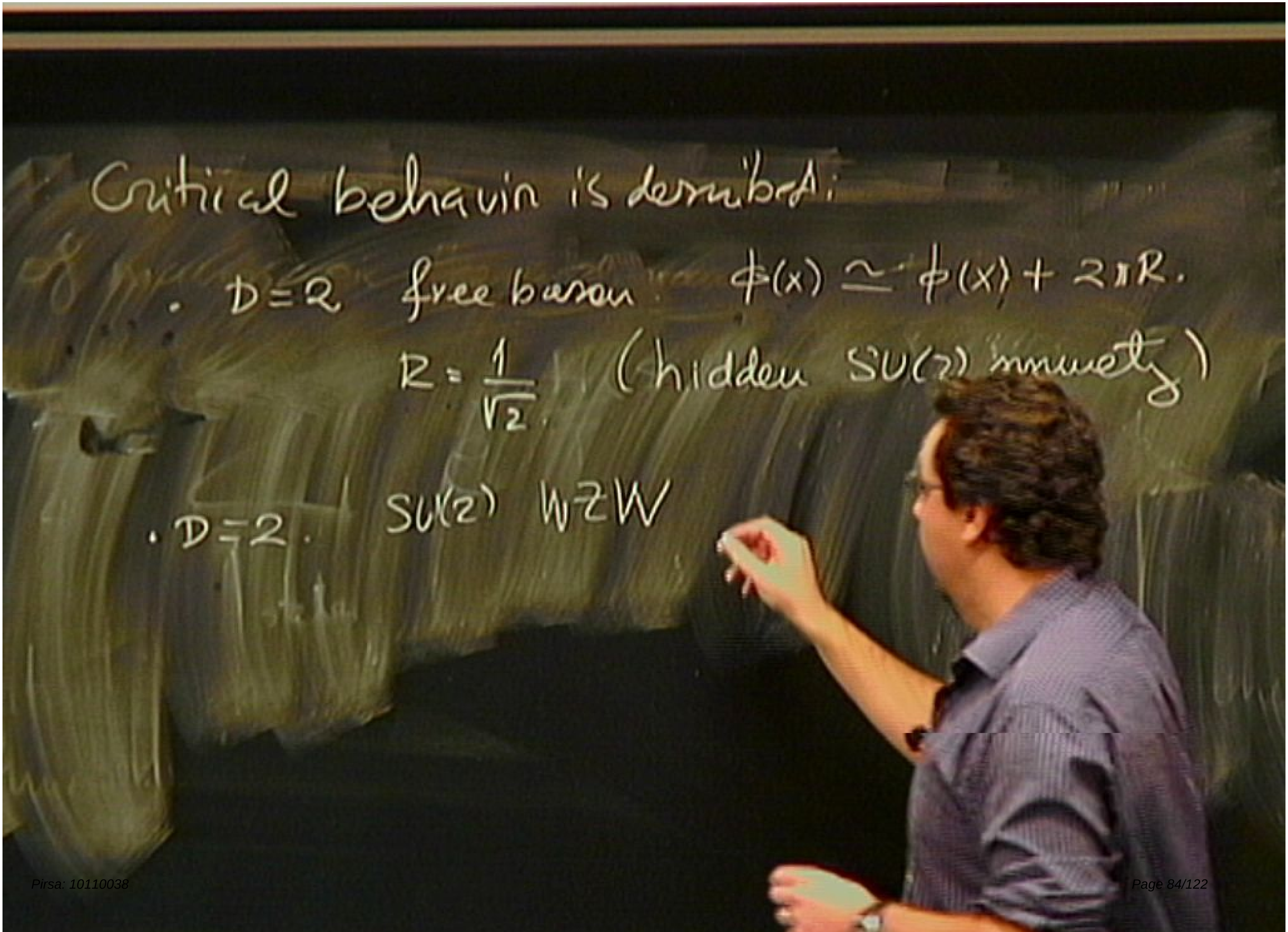
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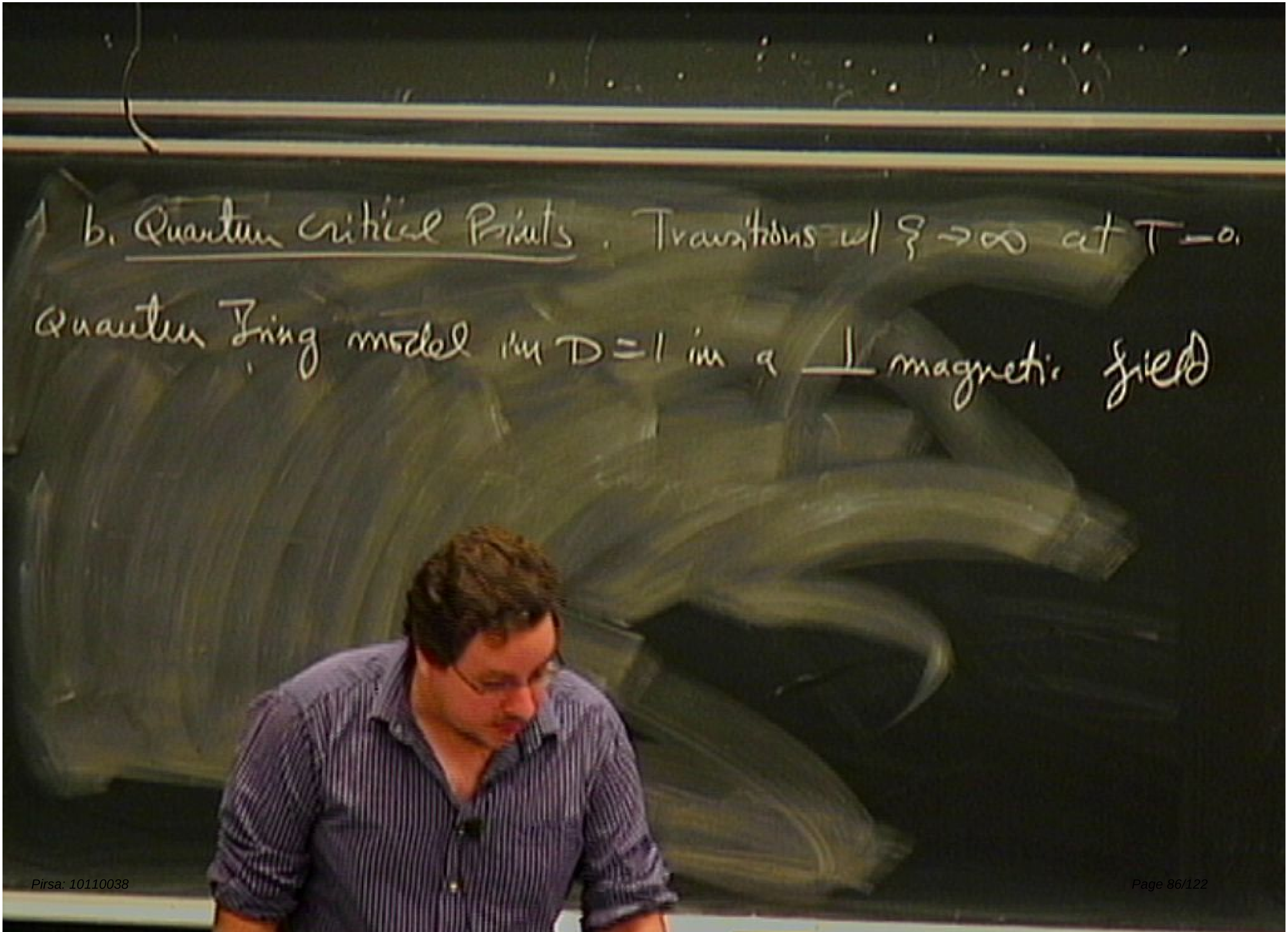
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Quantum Ising model in $D=1$ in a $\perp$ magnetic field

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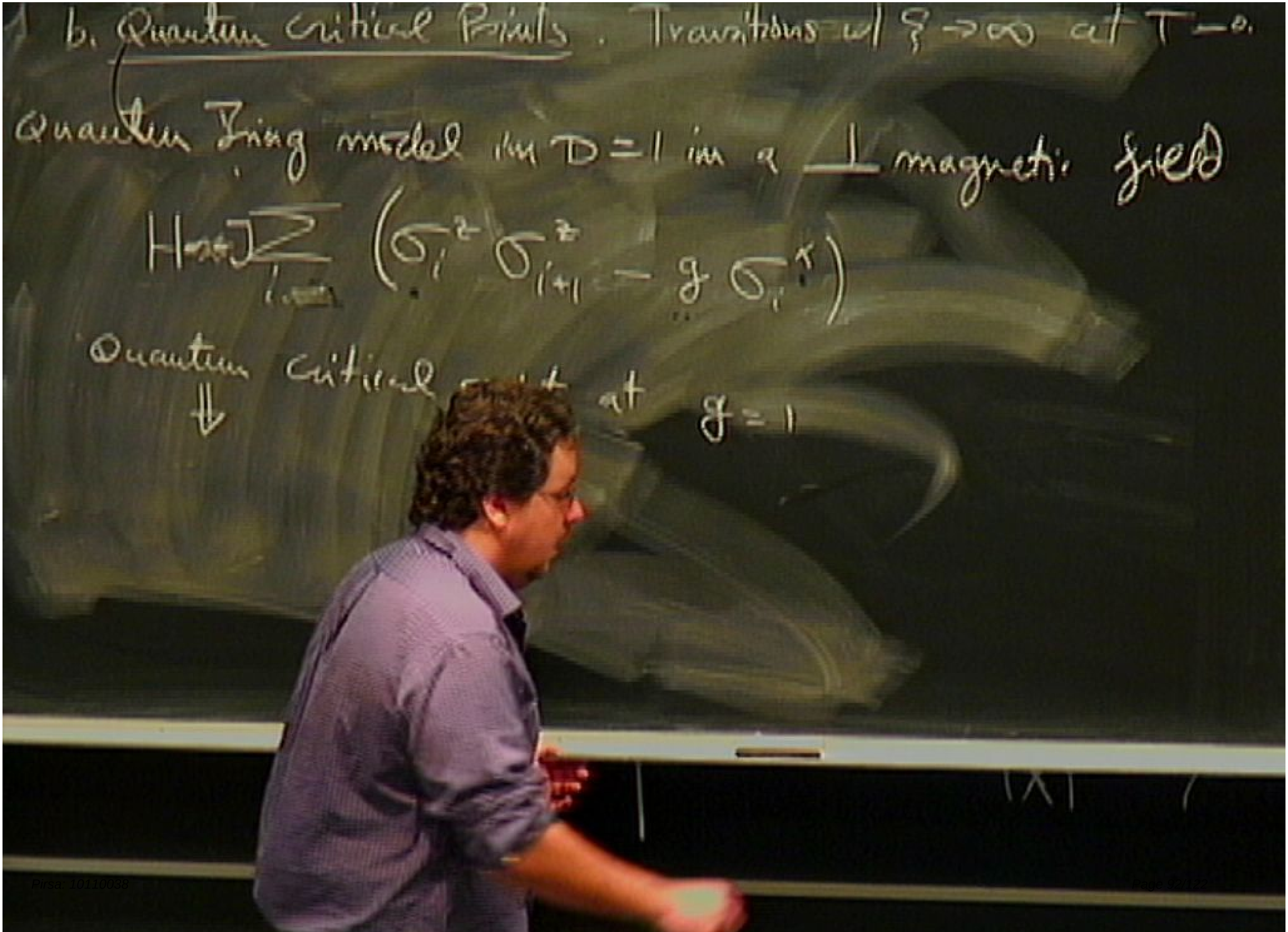
$\Downarrow$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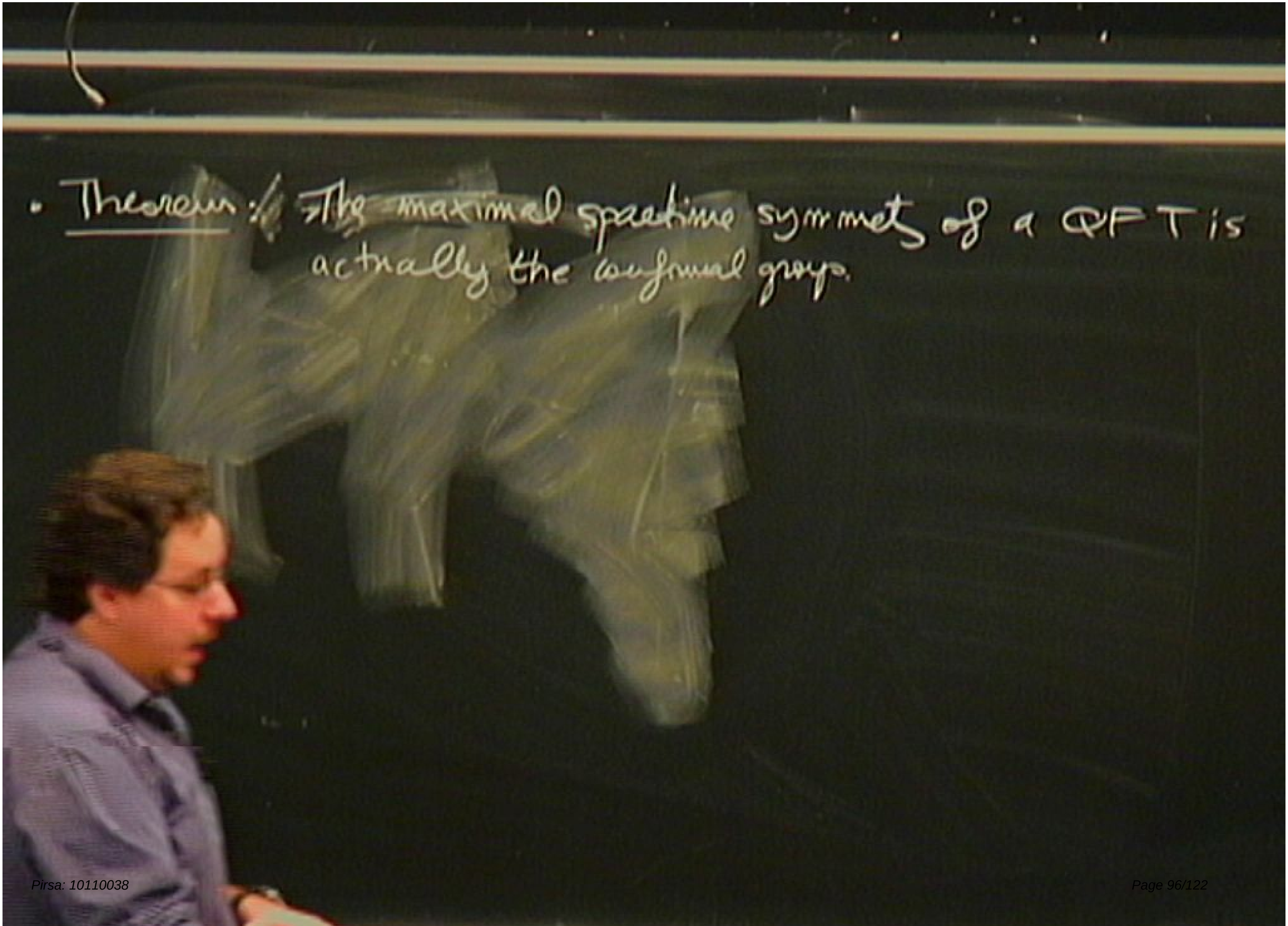
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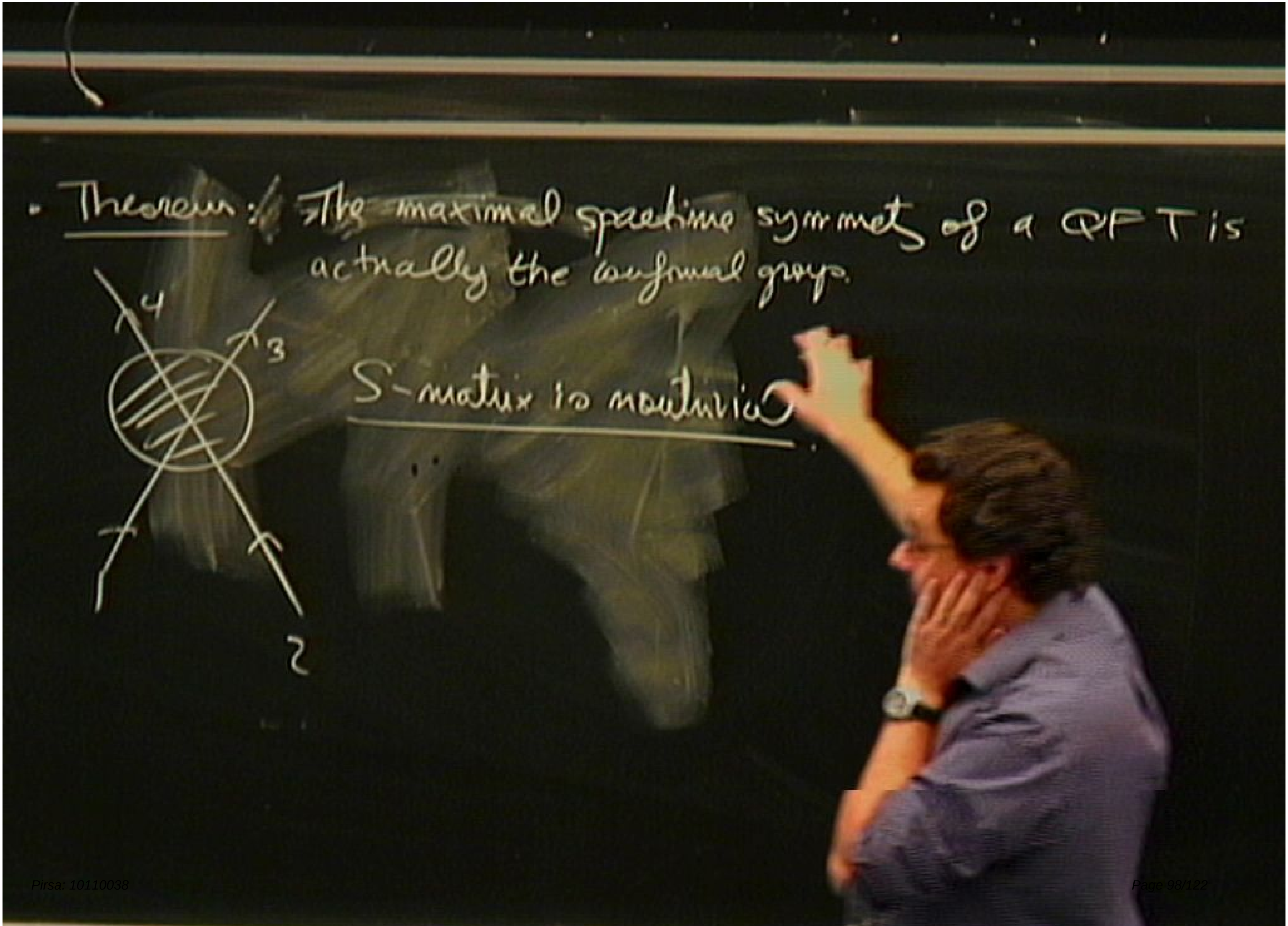
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S-matrix is nontrivial
4
3
2

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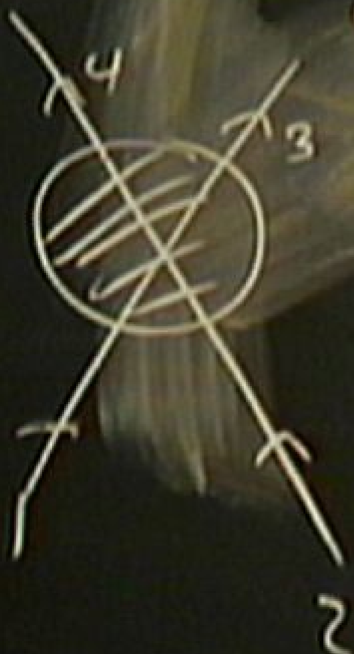


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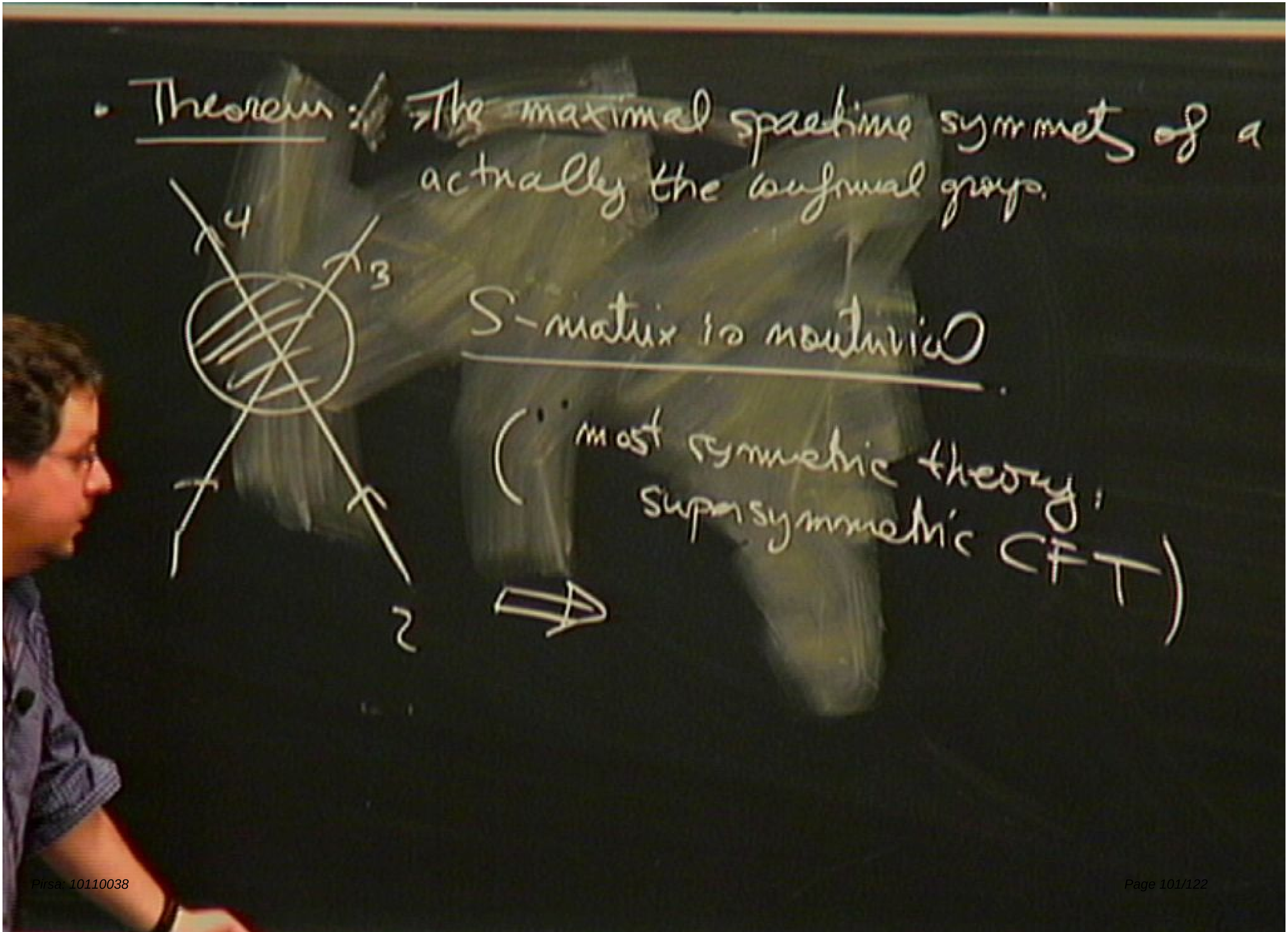
("most symmetric theory", supersymmetric CFT)



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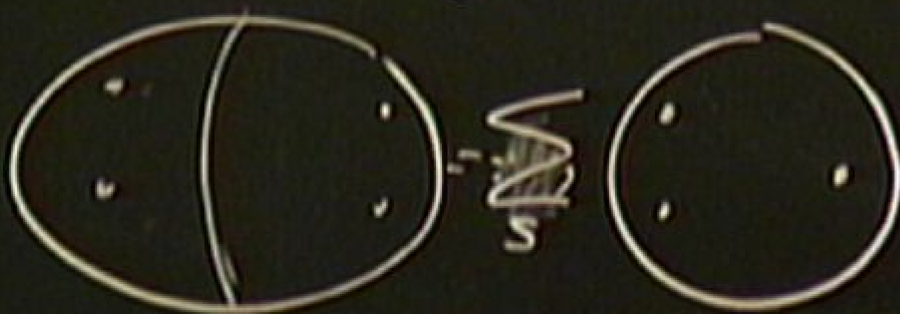

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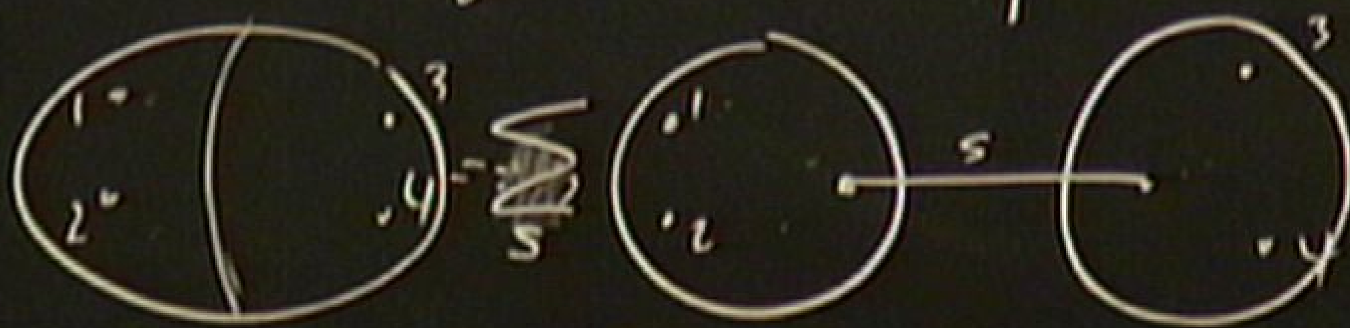

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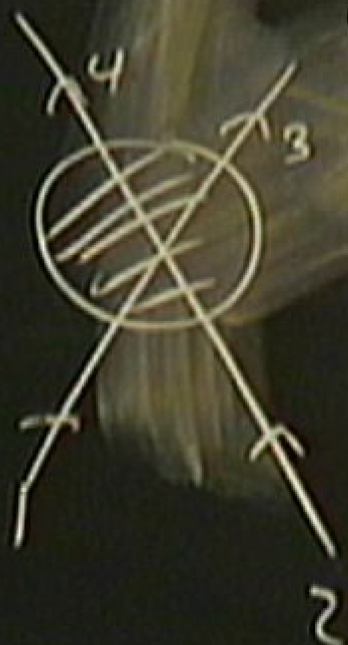


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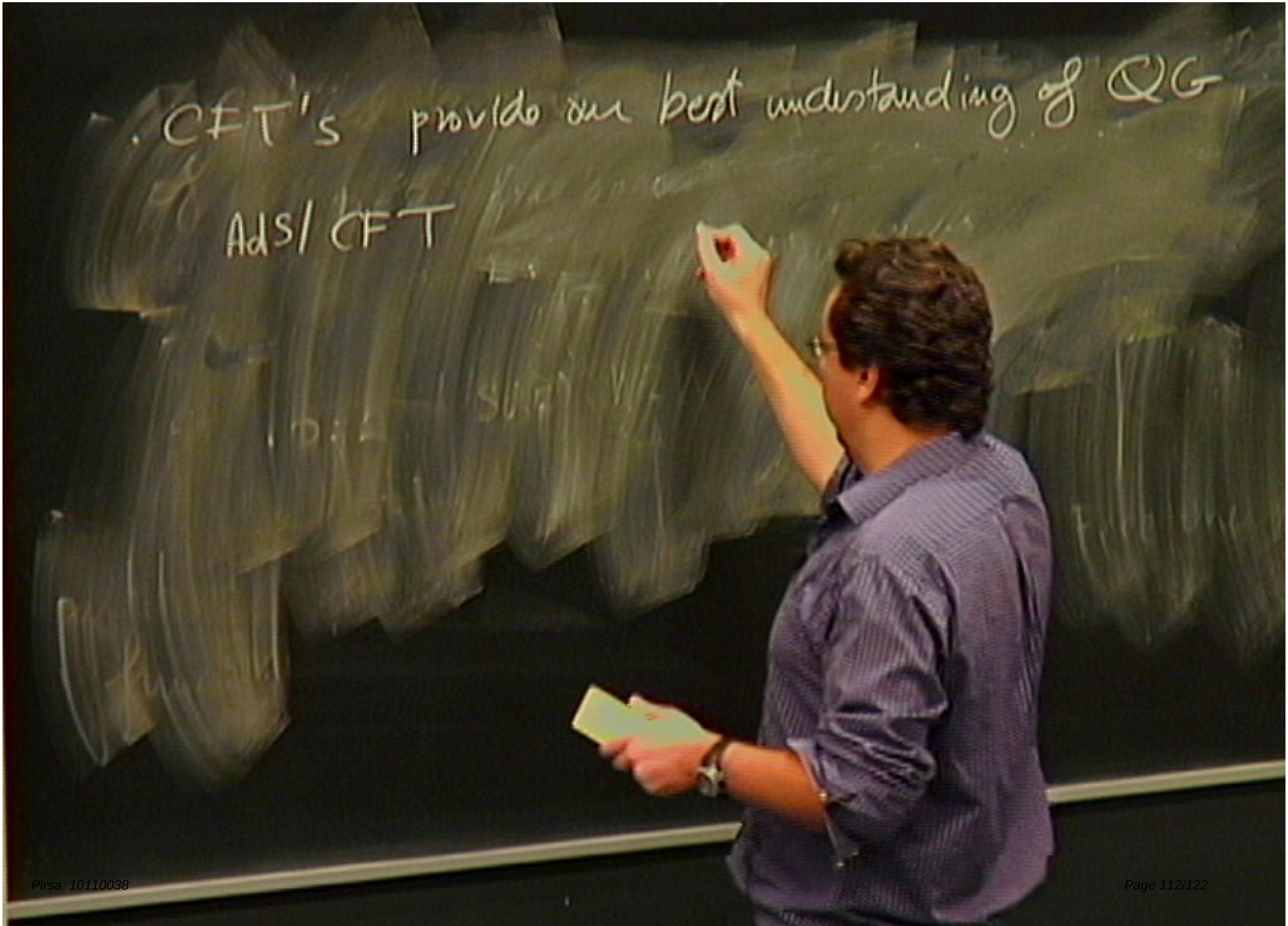
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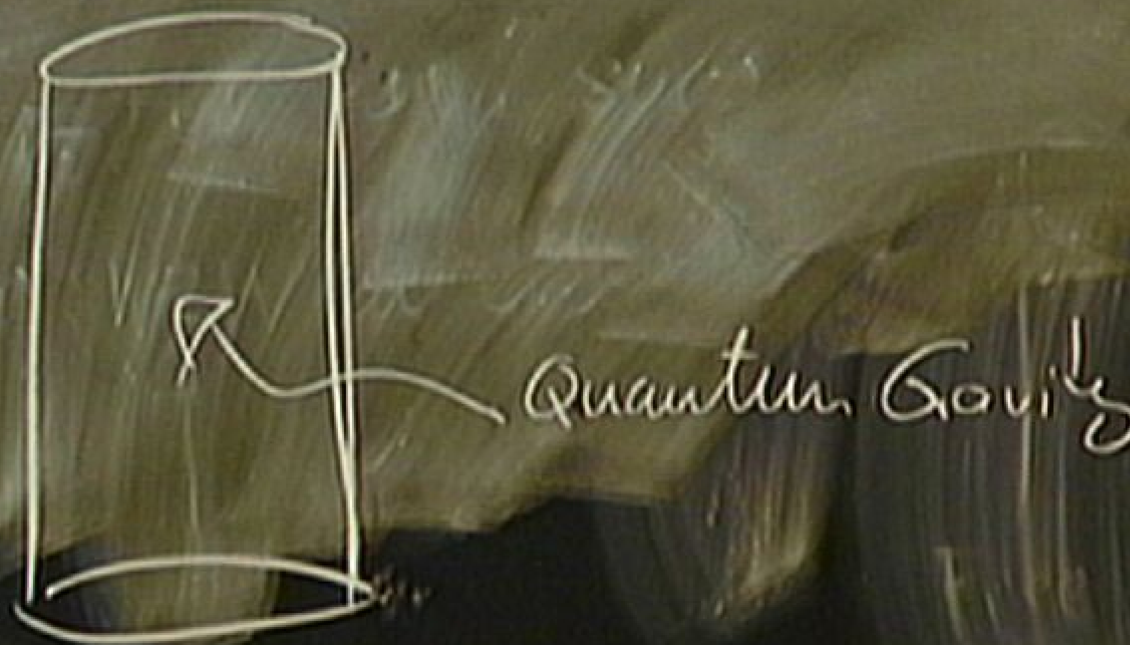

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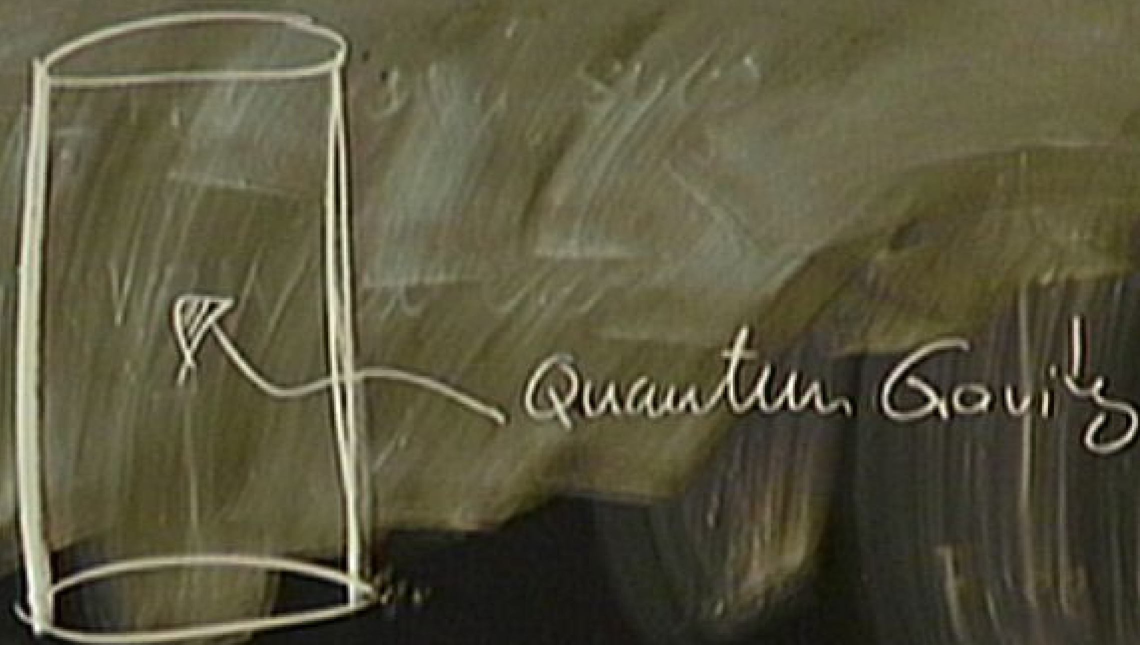

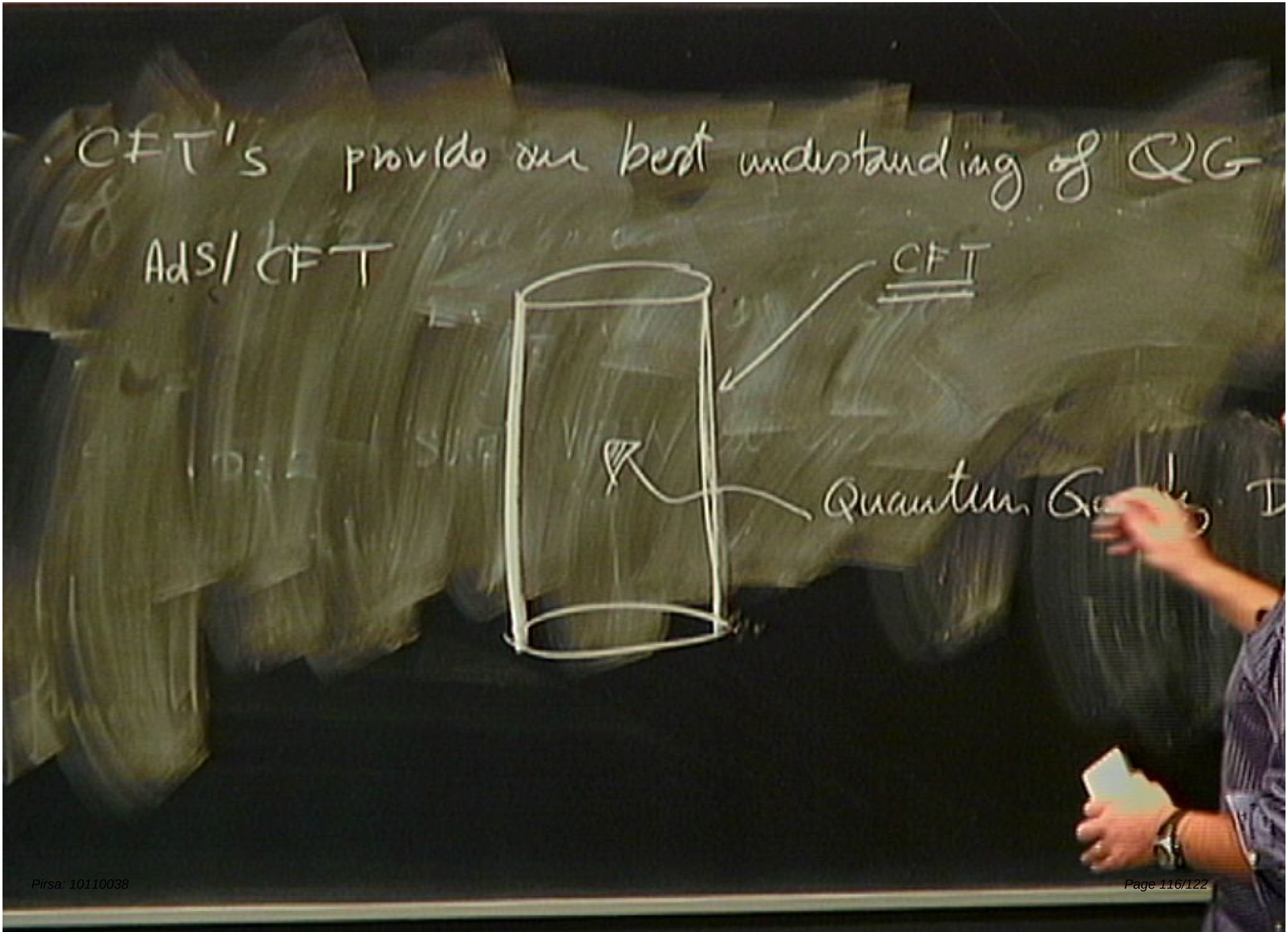
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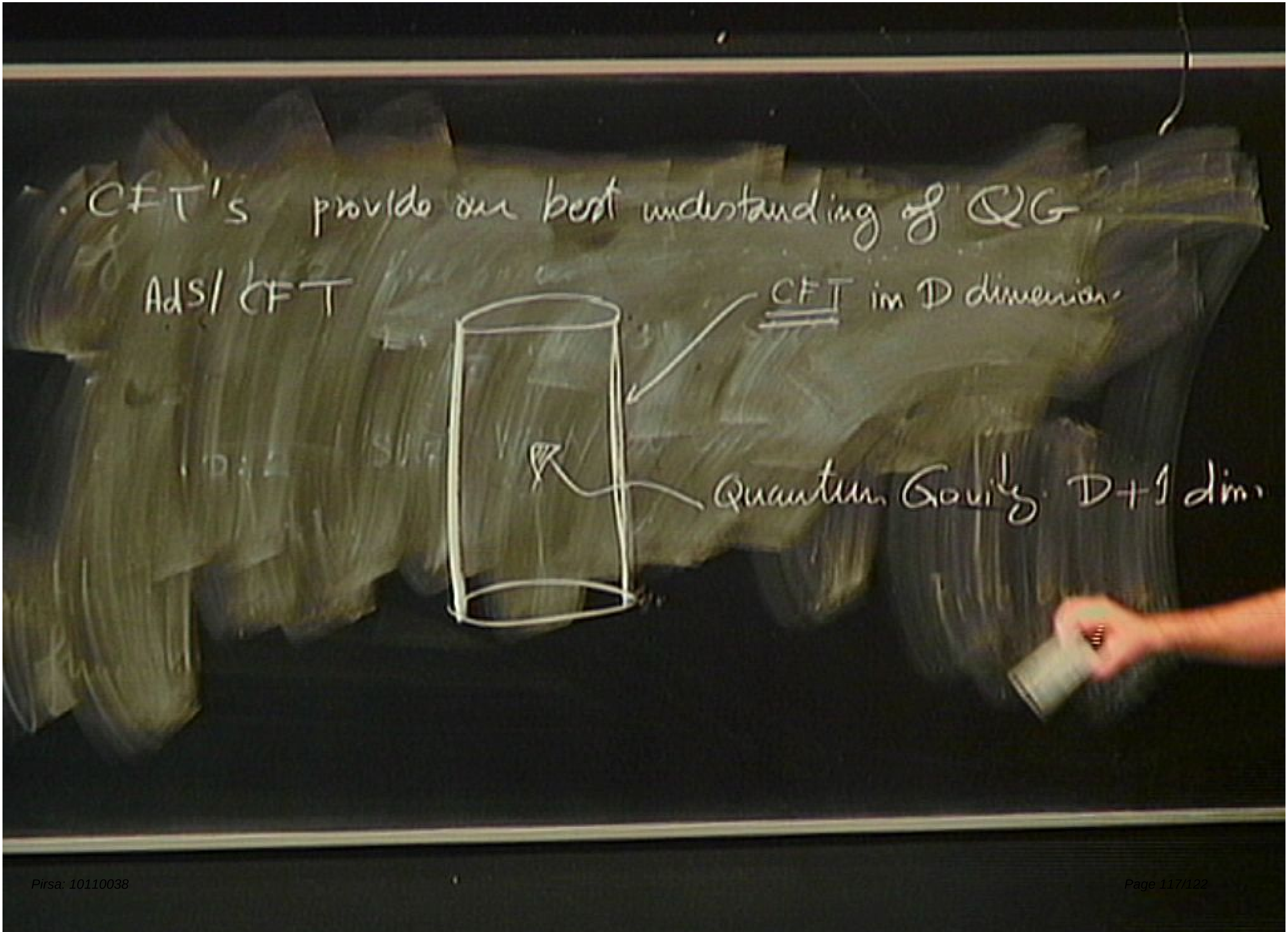
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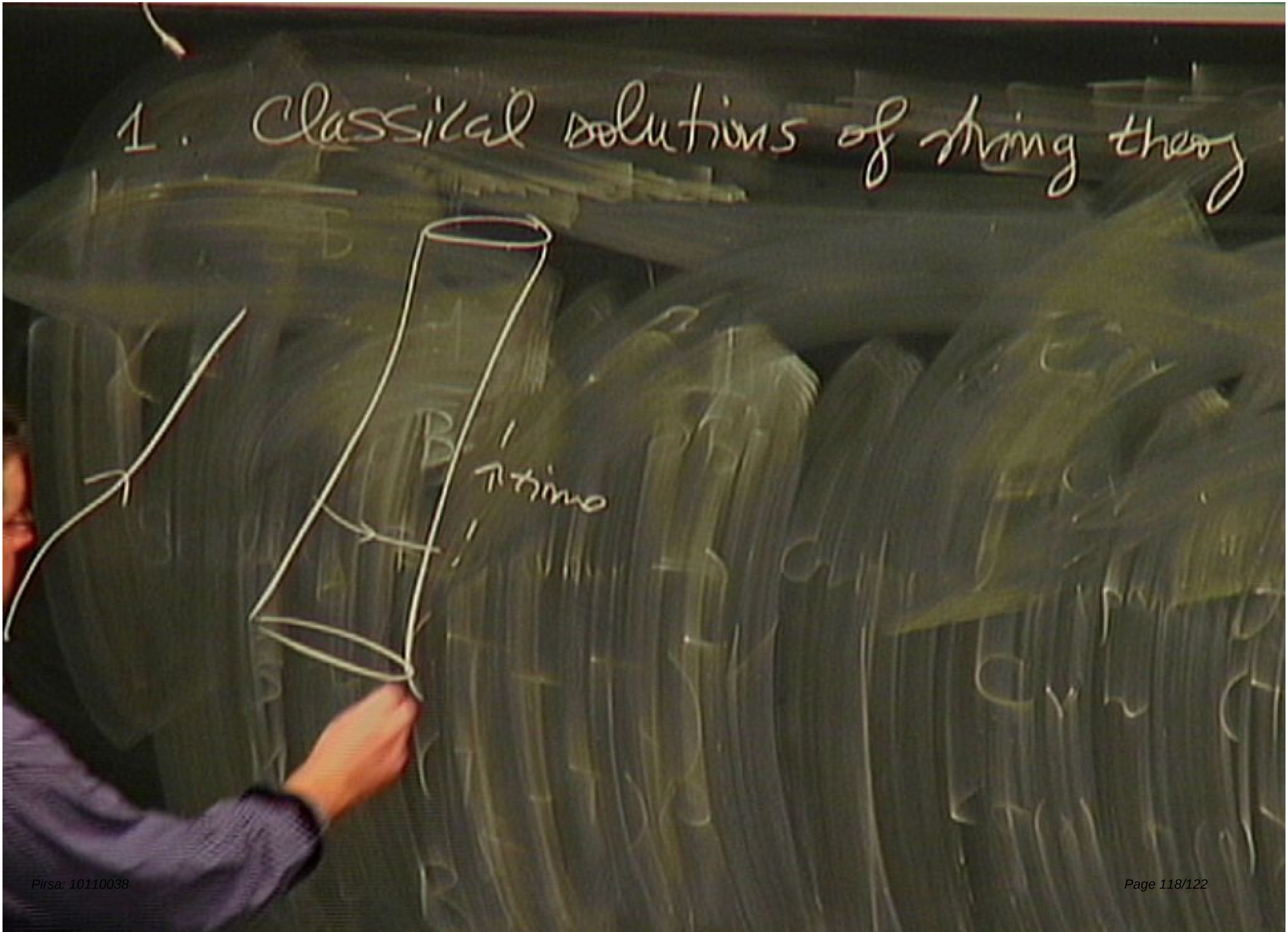
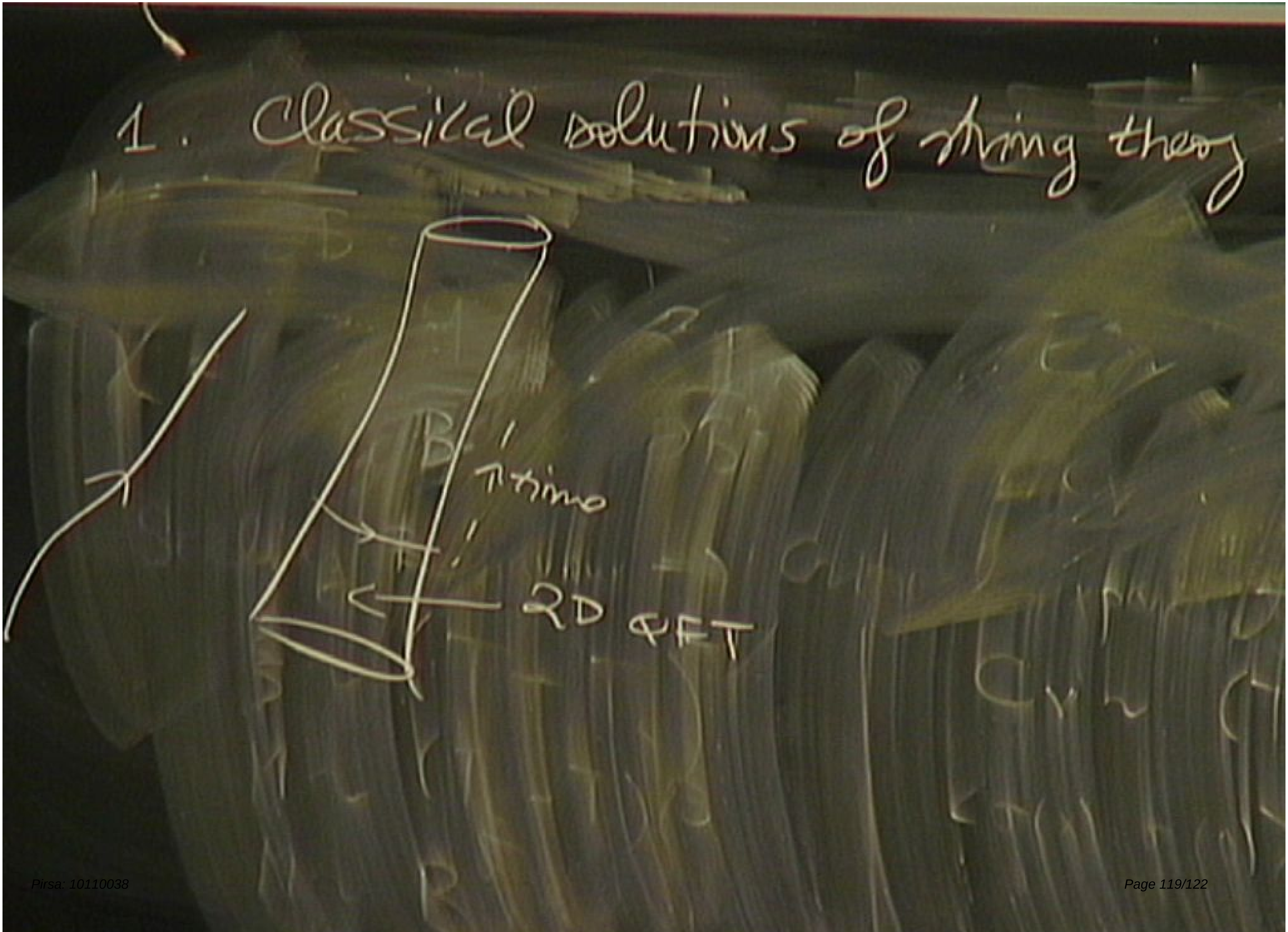
Quantum Gravity D+1 dim


1. Classical solutions of string theory
time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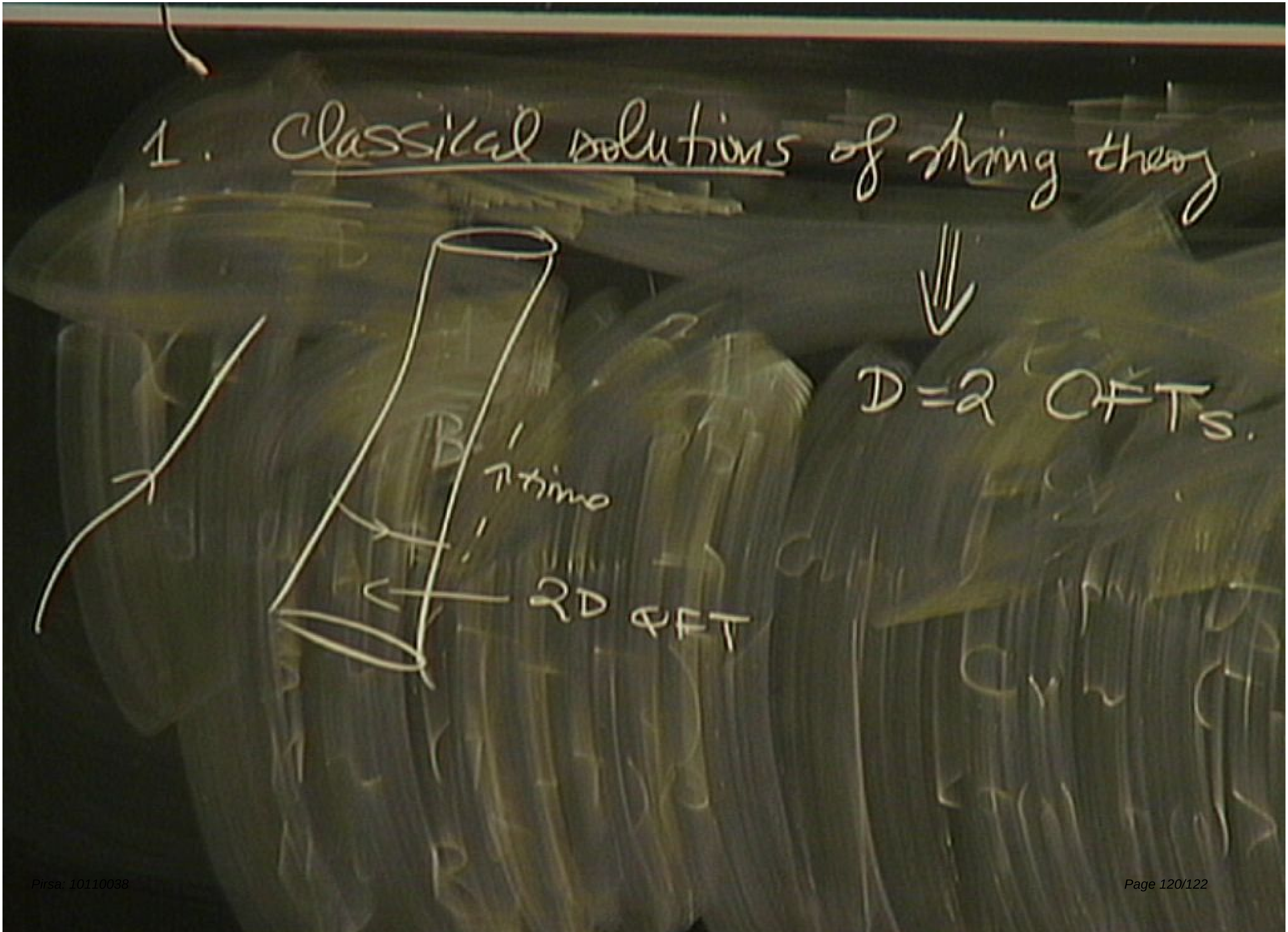
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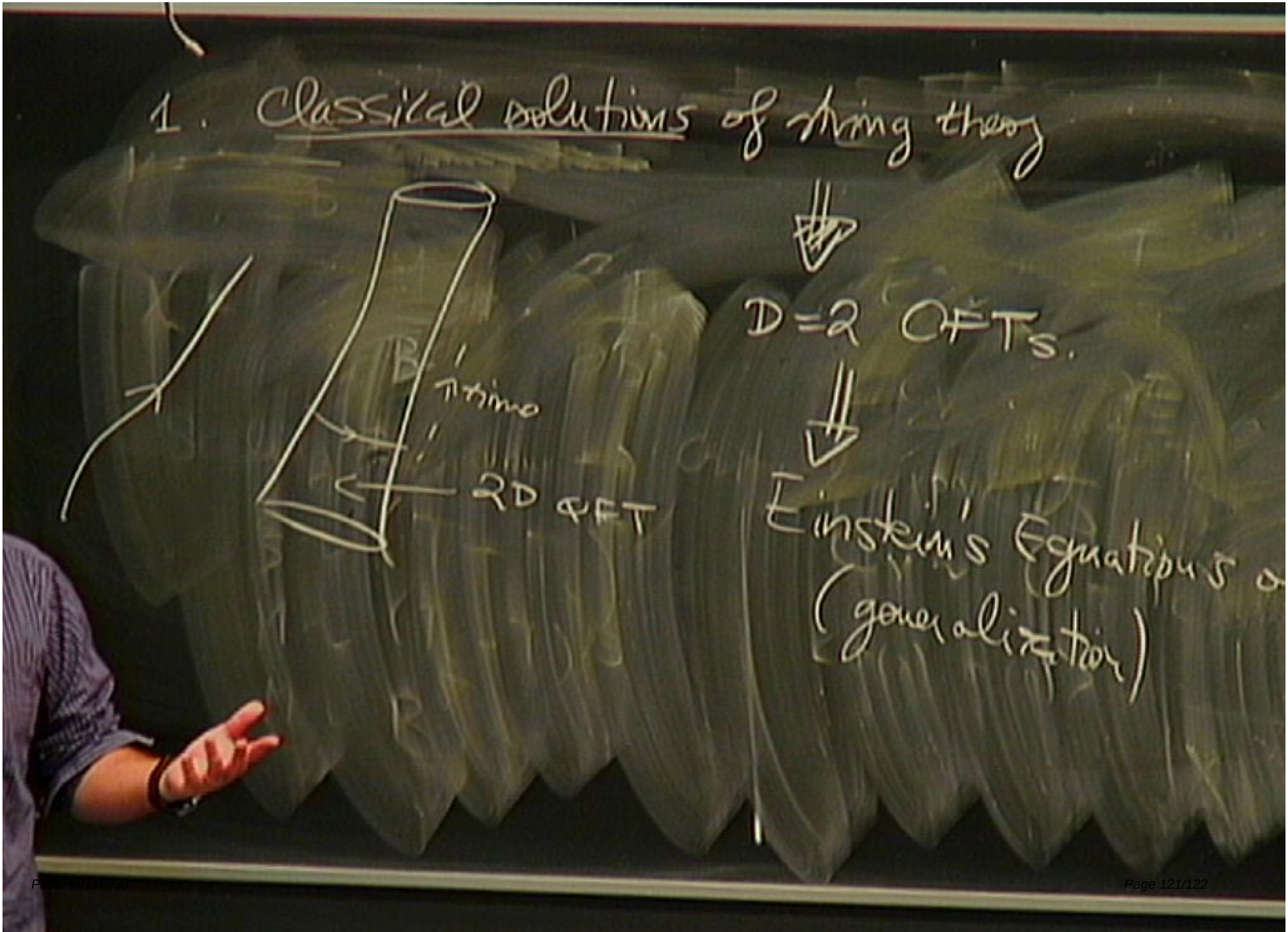
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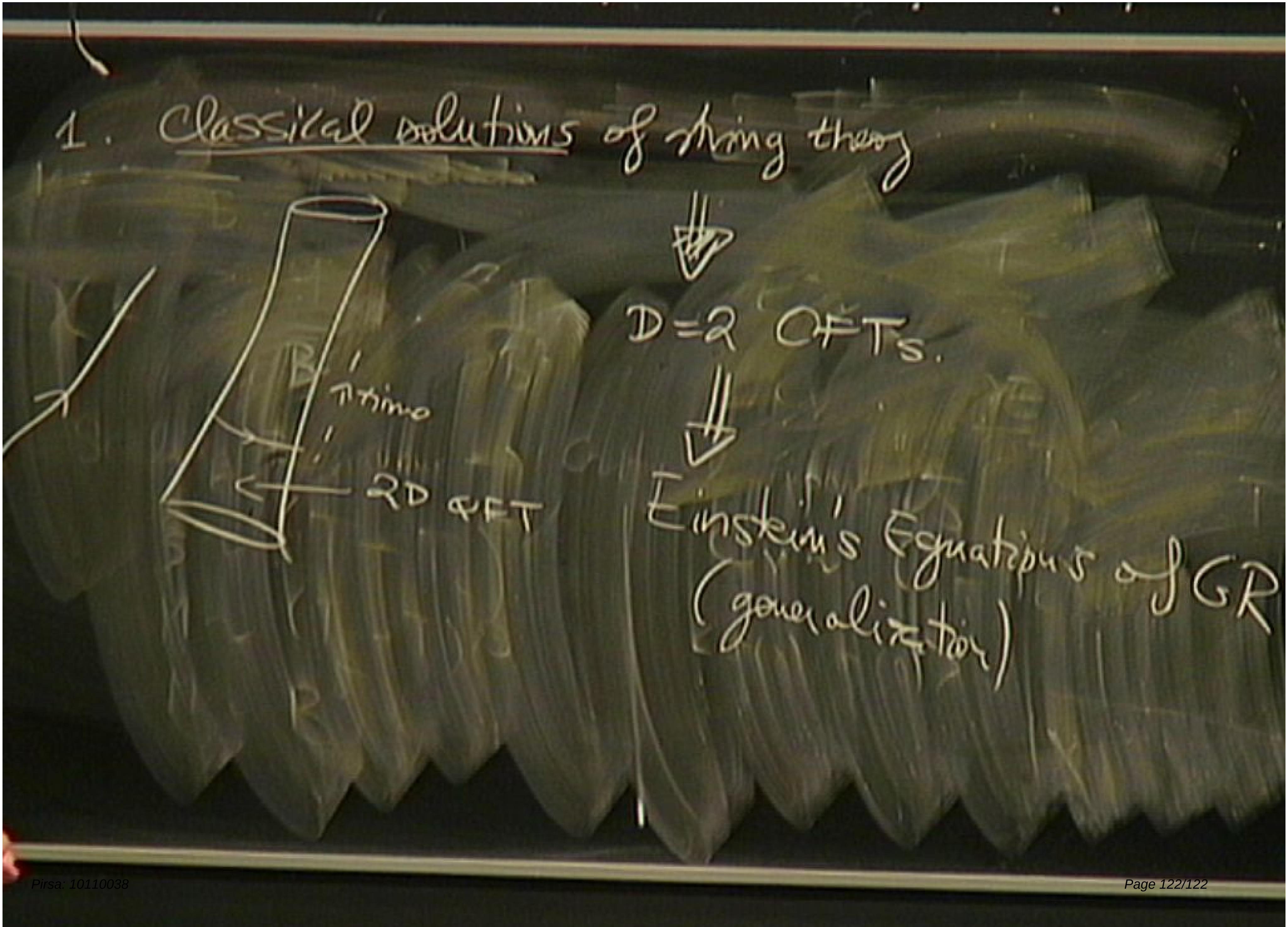
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