Title: Mathematical Physics (PHYS 624) - Lecture 4

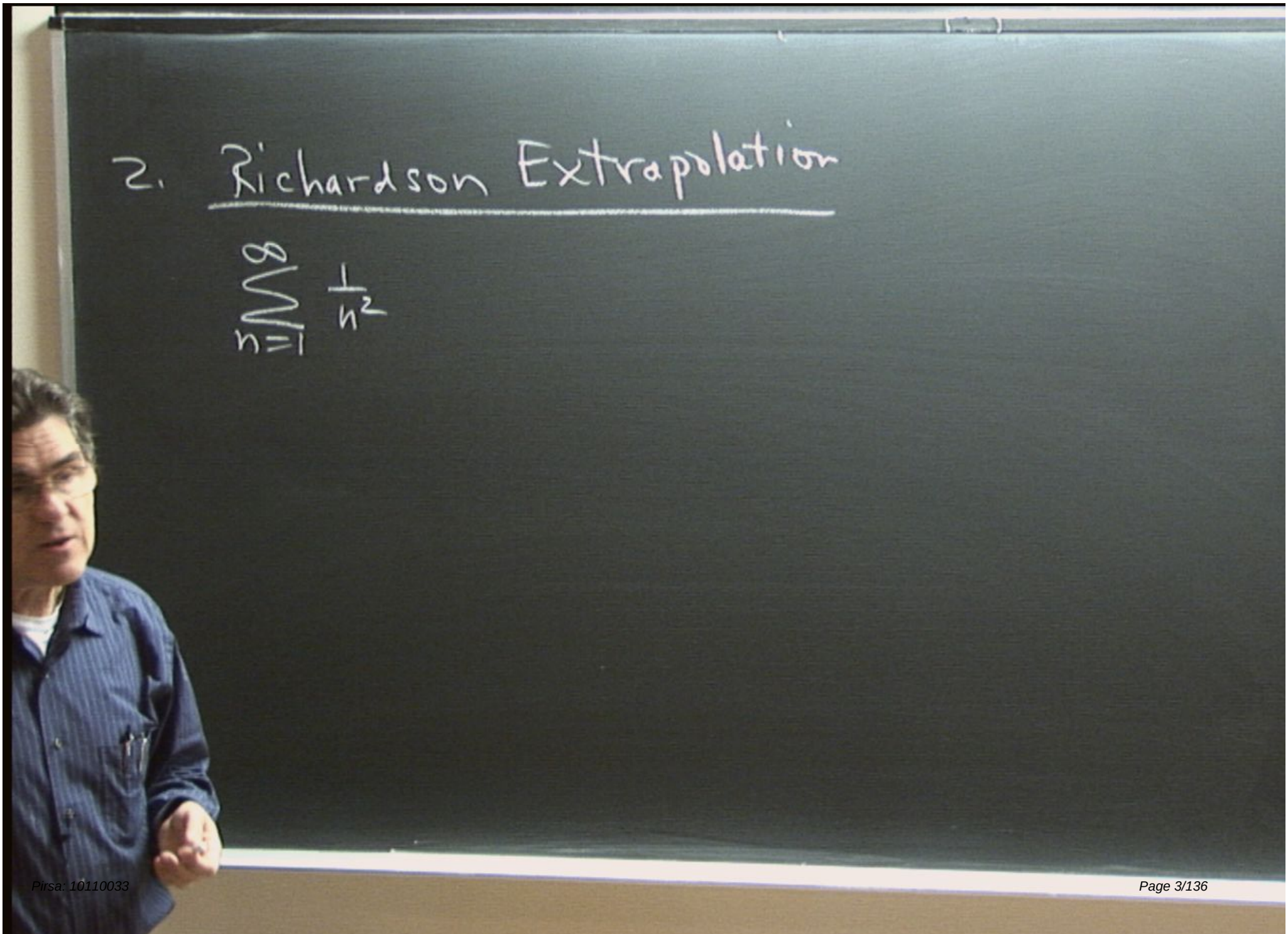
Date: Nov 25, 2010 10:50 AM

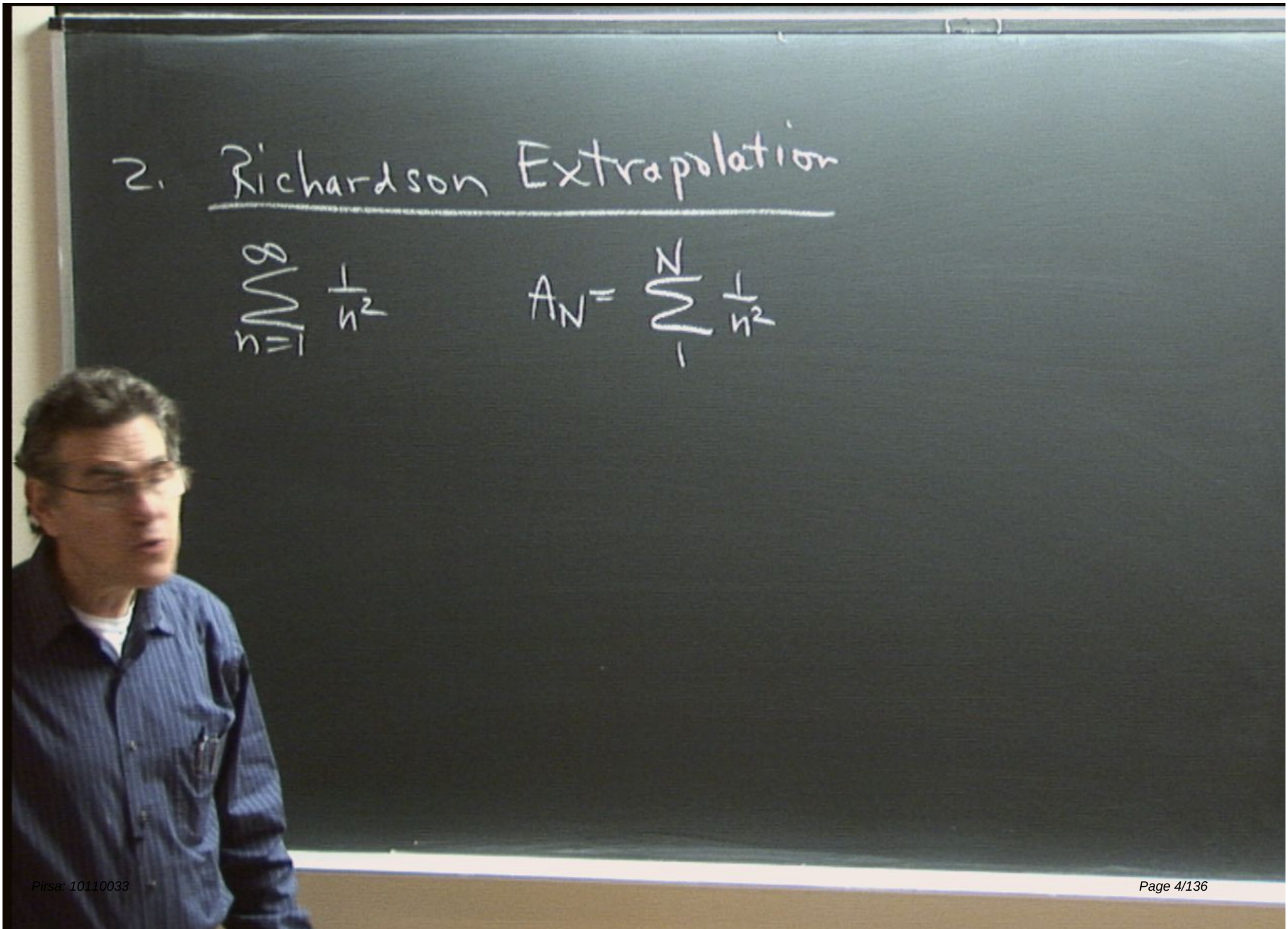
URL: http://pirsa.org/10110033

Abstract:

perimeter scholars

international

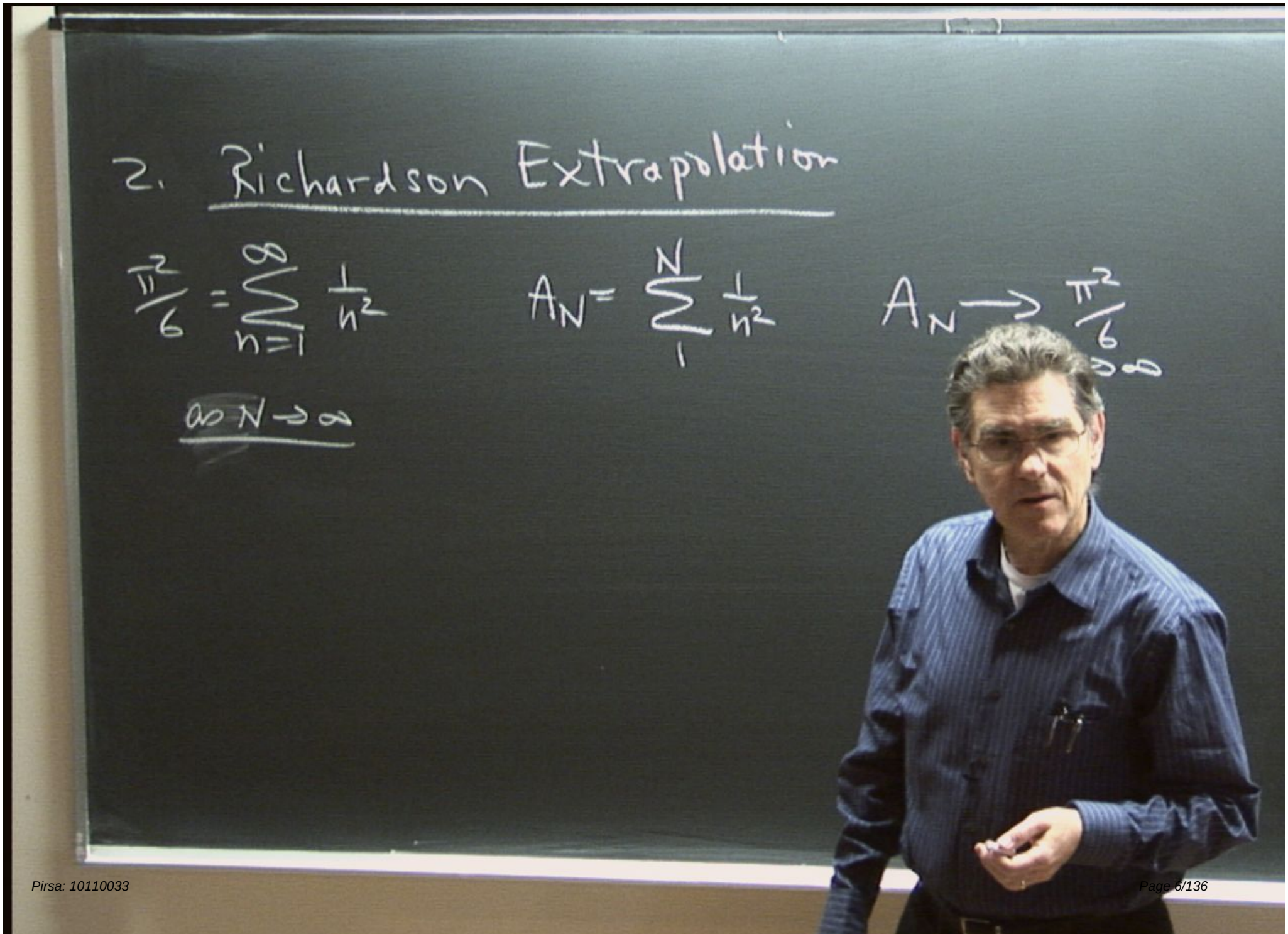

2. Richardson Extrapolation
$\sum_{n=1}^{\infty} \frac{1}{n^2}$

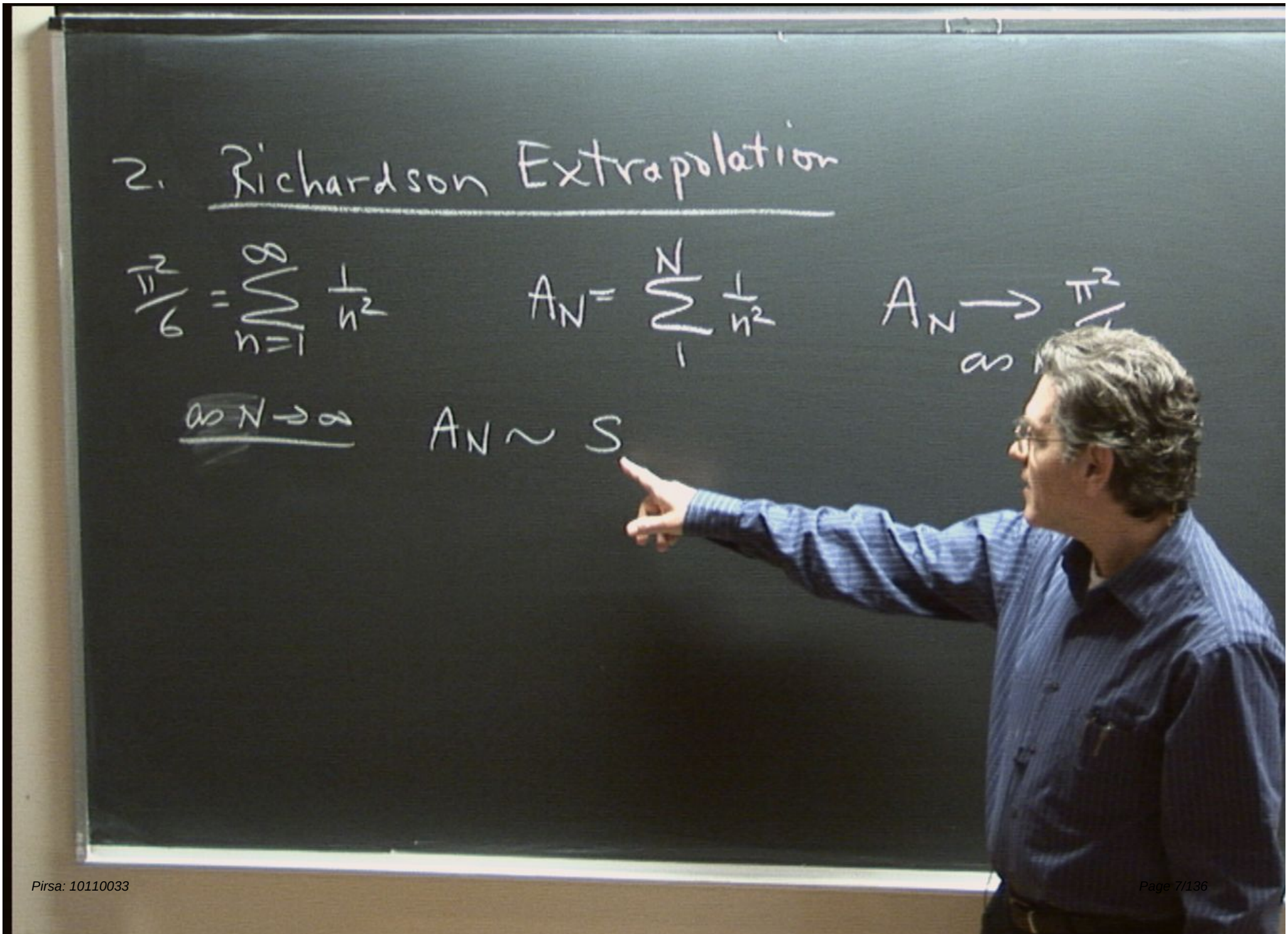

2. Richardson Extrapolation
$\sum_{n=1}^{\infty} \frac{1}{n^2}$
$A_N = \sum_{1}^{N} \frac{1}{n^2}$

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \longrightarrow \frac{\pi^2}{6}$$

as $N \to \infty$


2. Richardson Extrapolation
$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
$A_N = \sum_{1}^{N} \frac{1}{n^2}$
$A_N \longrightarrow \frac{\pi^2}{6}$
$N \to \infty$
as $N \to \infty$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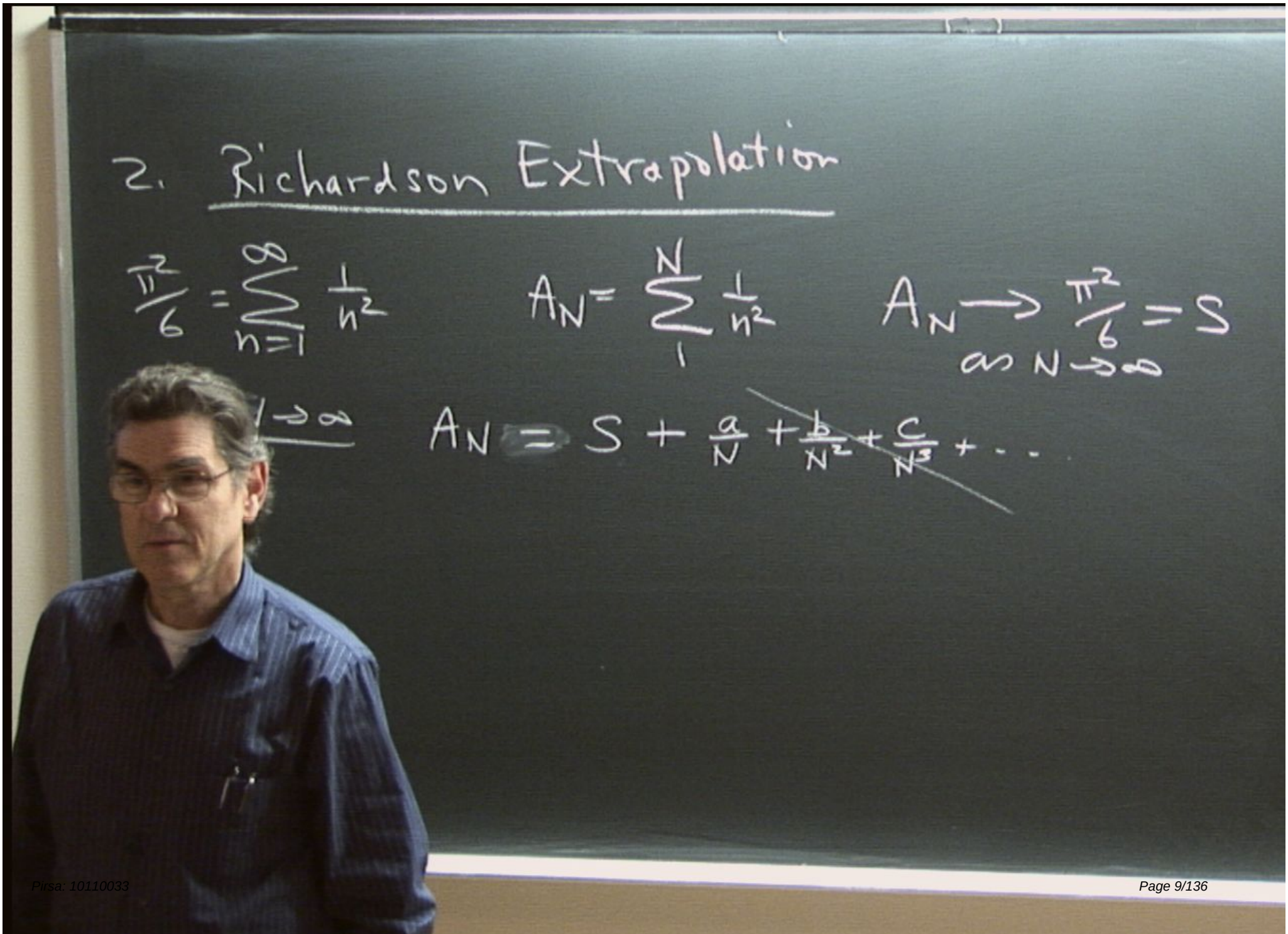
$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
$A_N = \sum_{1}^{N} \frac{1}{n^2}$
$A_N \longrightarrow \frac{\pi^2}{6}$
as
as $N \to \infty$
$A_N \sim S$

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \longrightarrow \frac{\pi^2}{6} = S$$

as $N \to \infty$

as $N \to \infty$ $\quad A_N \sim S + \frac{a}{N} + \frac{b}{N^2} + \frac{c}{N^3} + \cdots$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$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2}$
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$A_N \longrightarrow \frac{\pi^2}{6} = S$
as $N \to \infty$
$A_N = S + \frac{a}{N} + \frac{b}{N^2} + \frac{c}{N^3} + \cdots$

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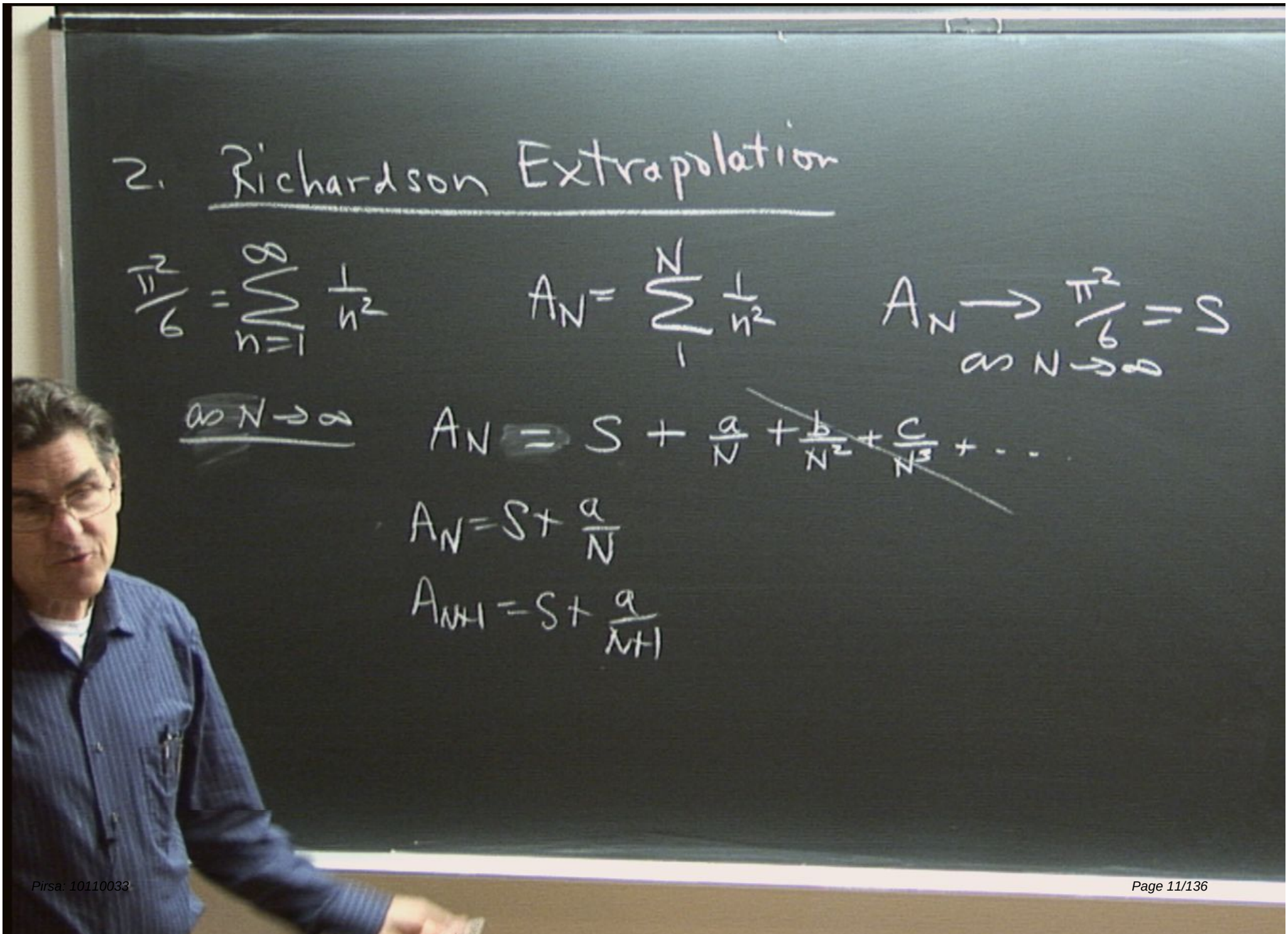
as $N \to \infty$

$\to \infty$

$$A_N = S + \frac{a}{N} + \cancel{\frac{b}{N^2} + \frac{c}{N^3}} + \cdots$$

$$A_N = S + \frac{a}{N}$$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$A_{N+1} = S + \frac{a}{N+1}$

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \longrightarrow \frac{\pi^2}{6} = S \quad \text{as } N \to \infty$$

$N \to \infty$

$$A_N = S + \frac{a}{N} + \cancel{\frac{b}{N^2} + \frac{c}{N^3}} + \cdots$$

$$A_N = S + \frac{a}{N} \rightarrow N A_N = NS + a$$

$$A_{N+1} = S + \frac{a}{N+1} \rightarrow (N+1) A_{N+1} = (N+1)S + a$$

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \longrightarrow \frac{\pi^2}{6} = S$$

as $N \to \infty$

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$$(N+1) A_{N+1} - N A_N = S$$

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \longrightarrow \frac{\pi^2}{6} = S$$

as $N \to \infty$

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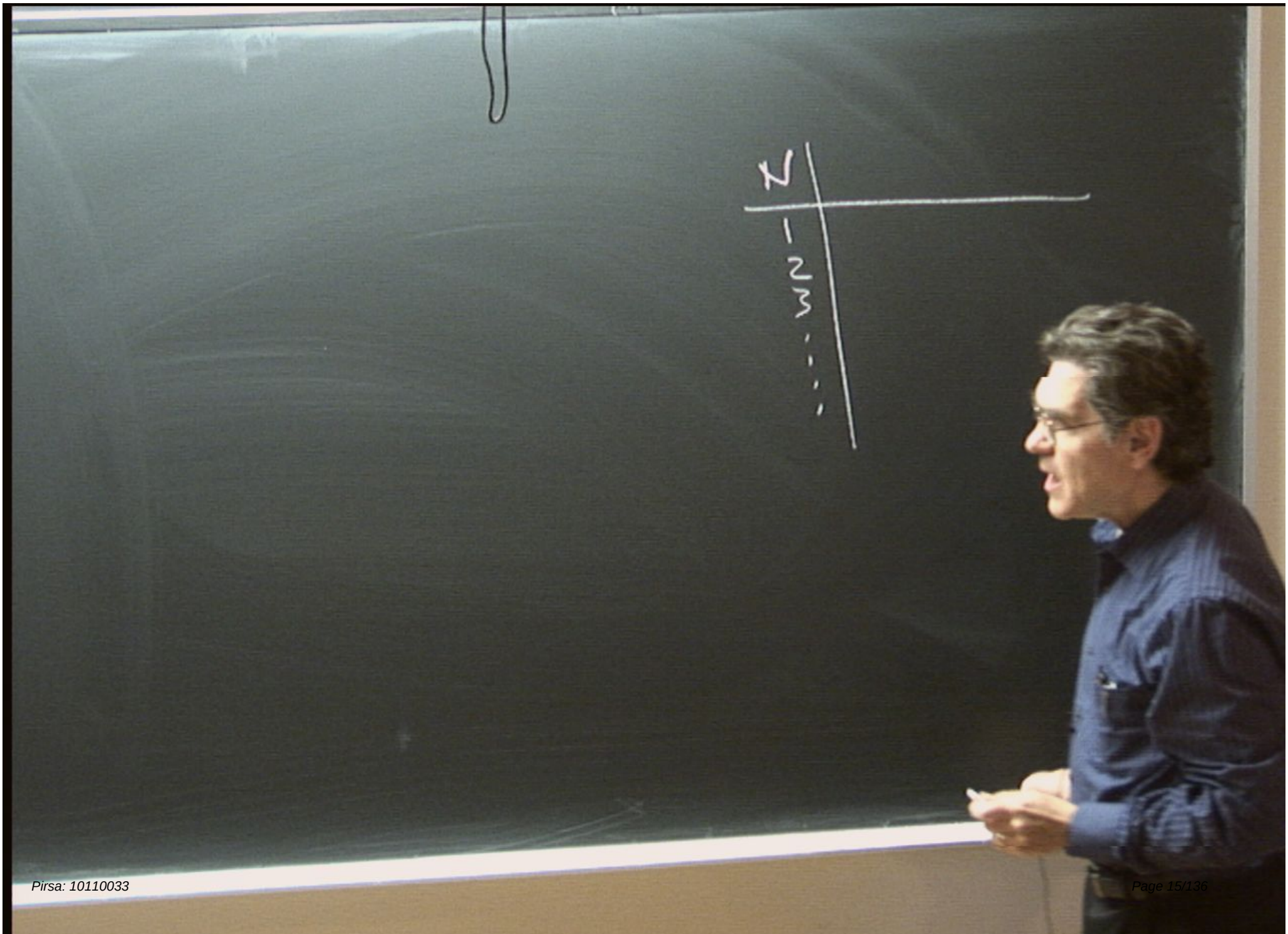
N
1 2 3 . . . .

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \longrightarrow \frac{\pi^2}{6} = S$$

as N

as $N \to \infty$

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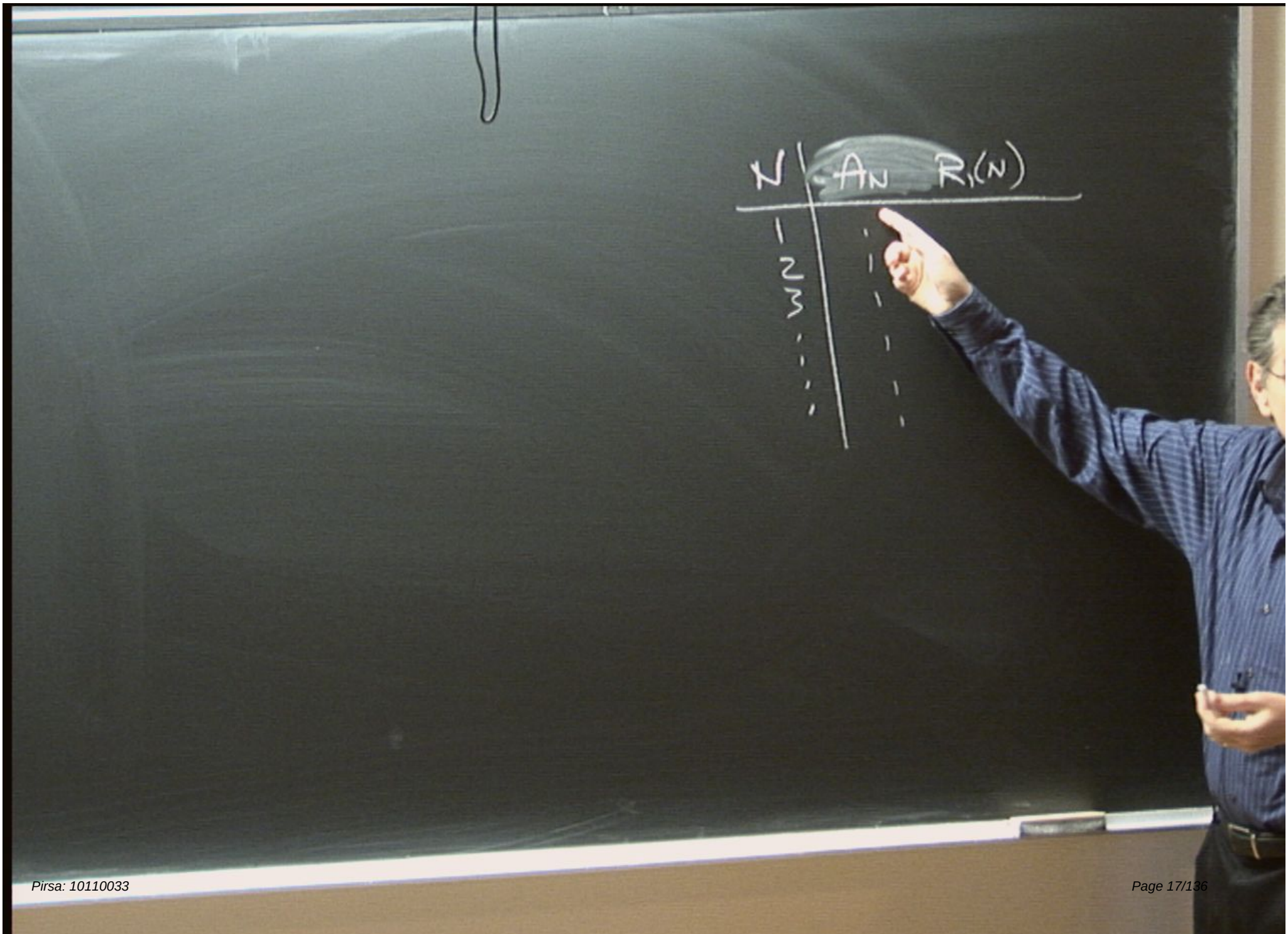
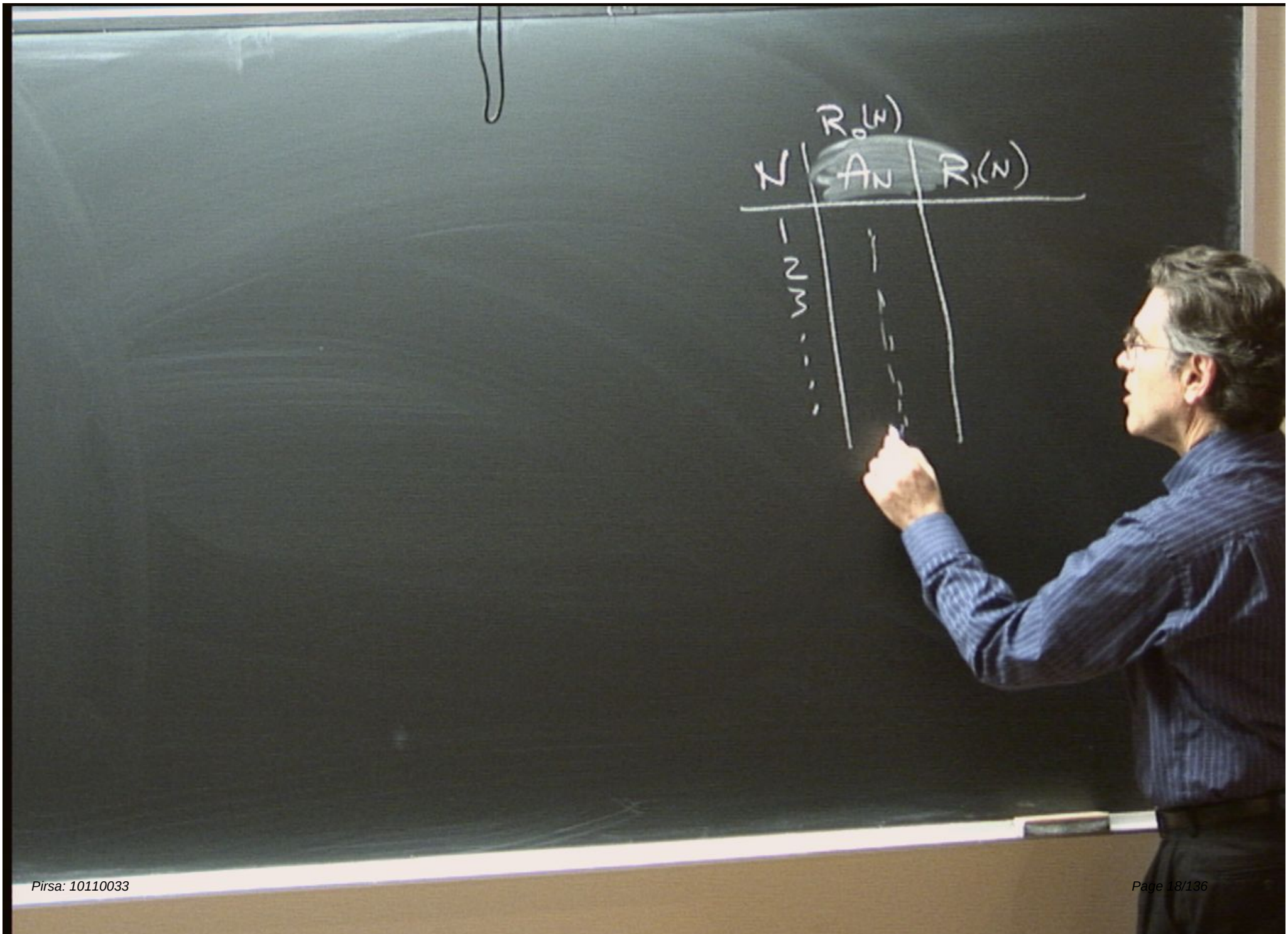
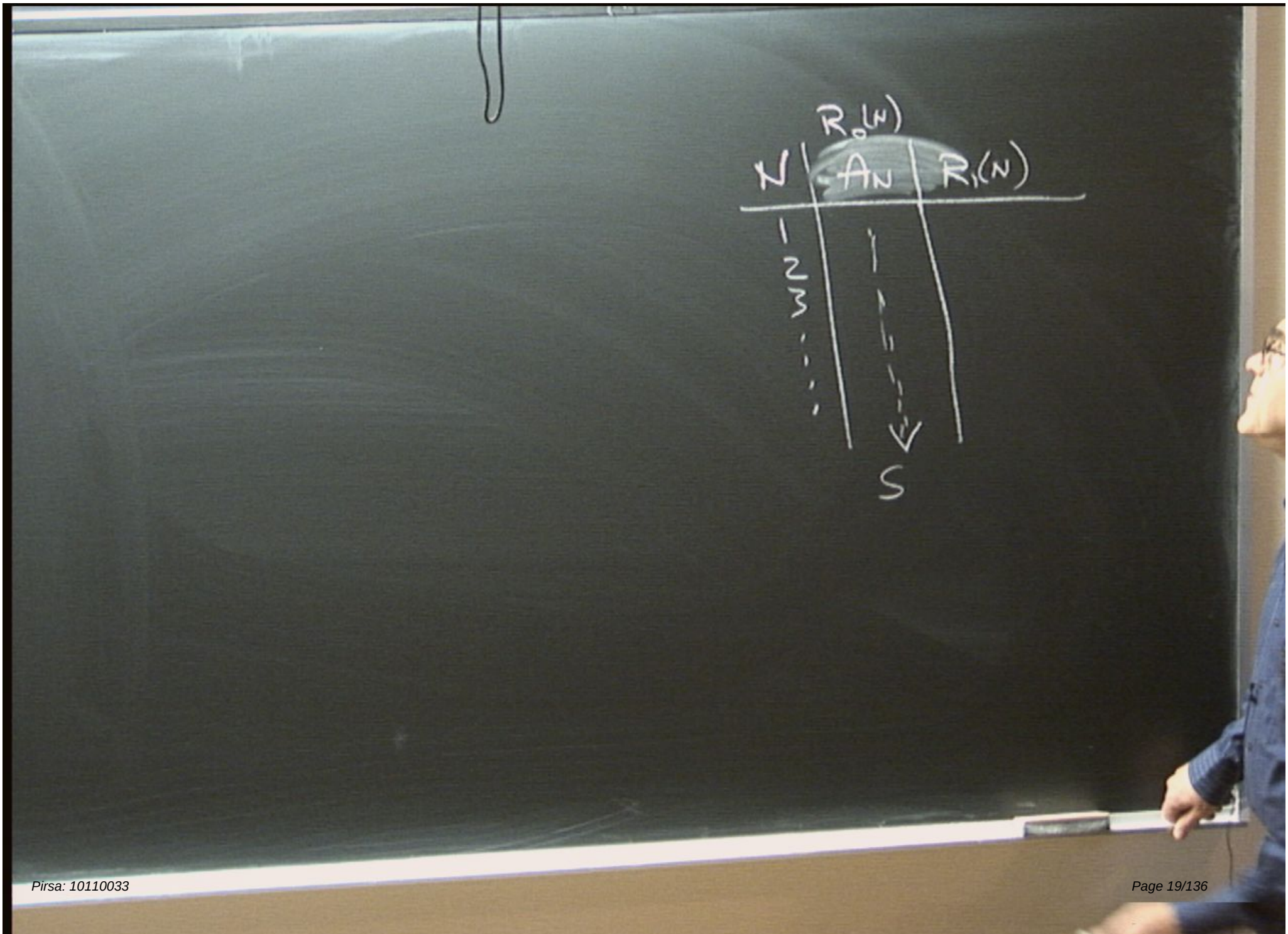
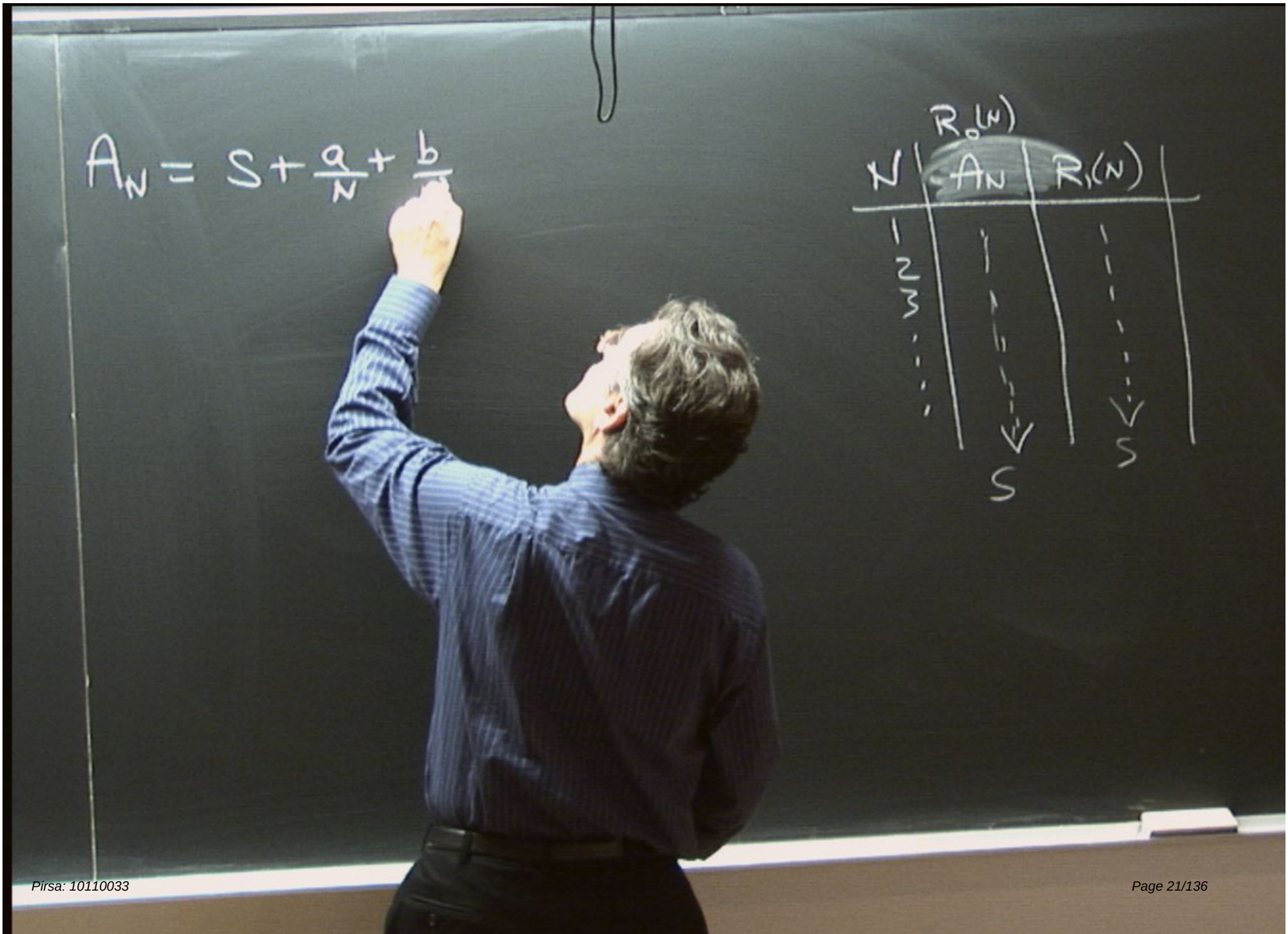
$$R_1 = (N+1) A_{N+1} - N A_N = S$$

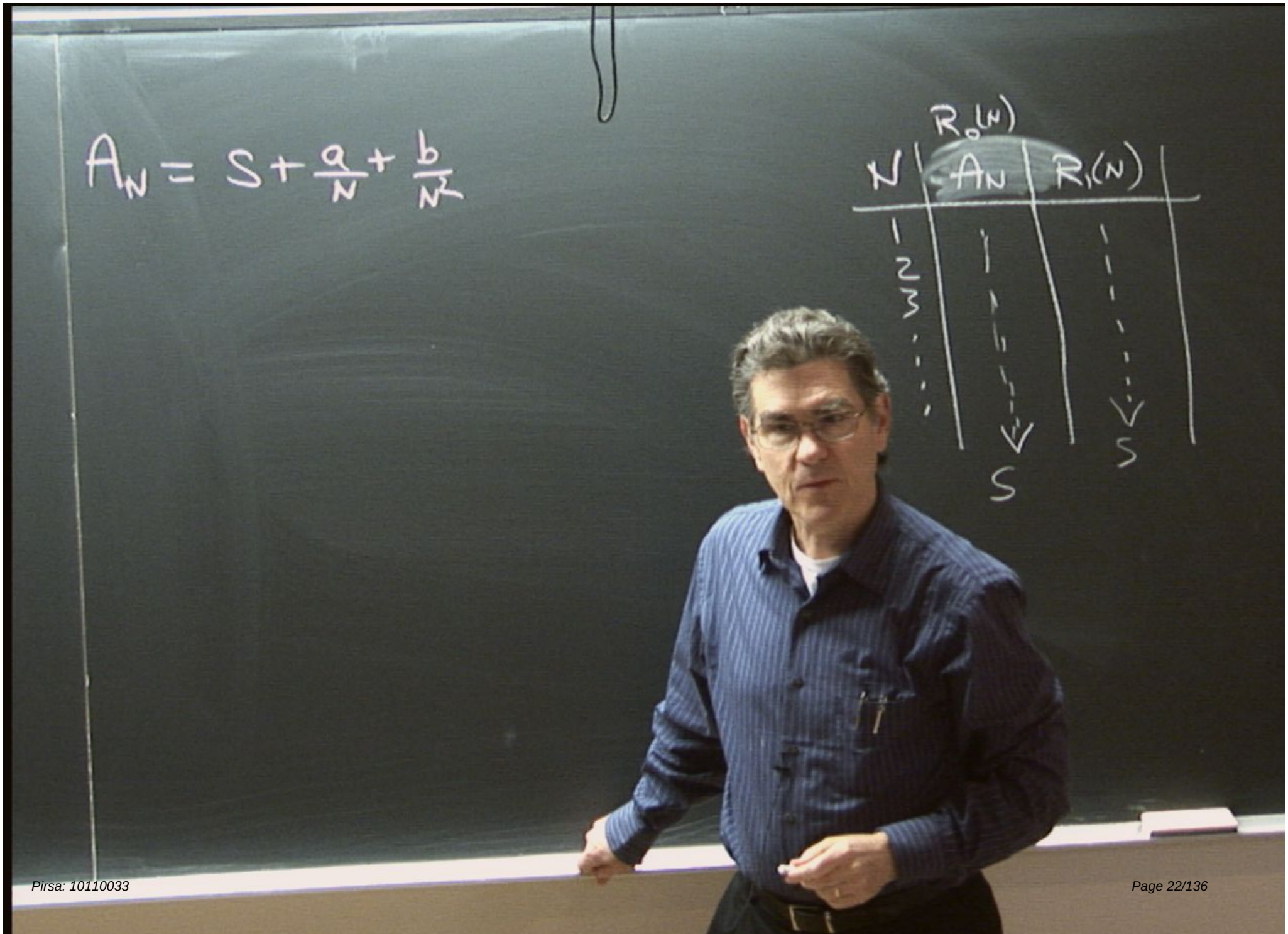

N
A_N $R_1(N)$
1
2
3

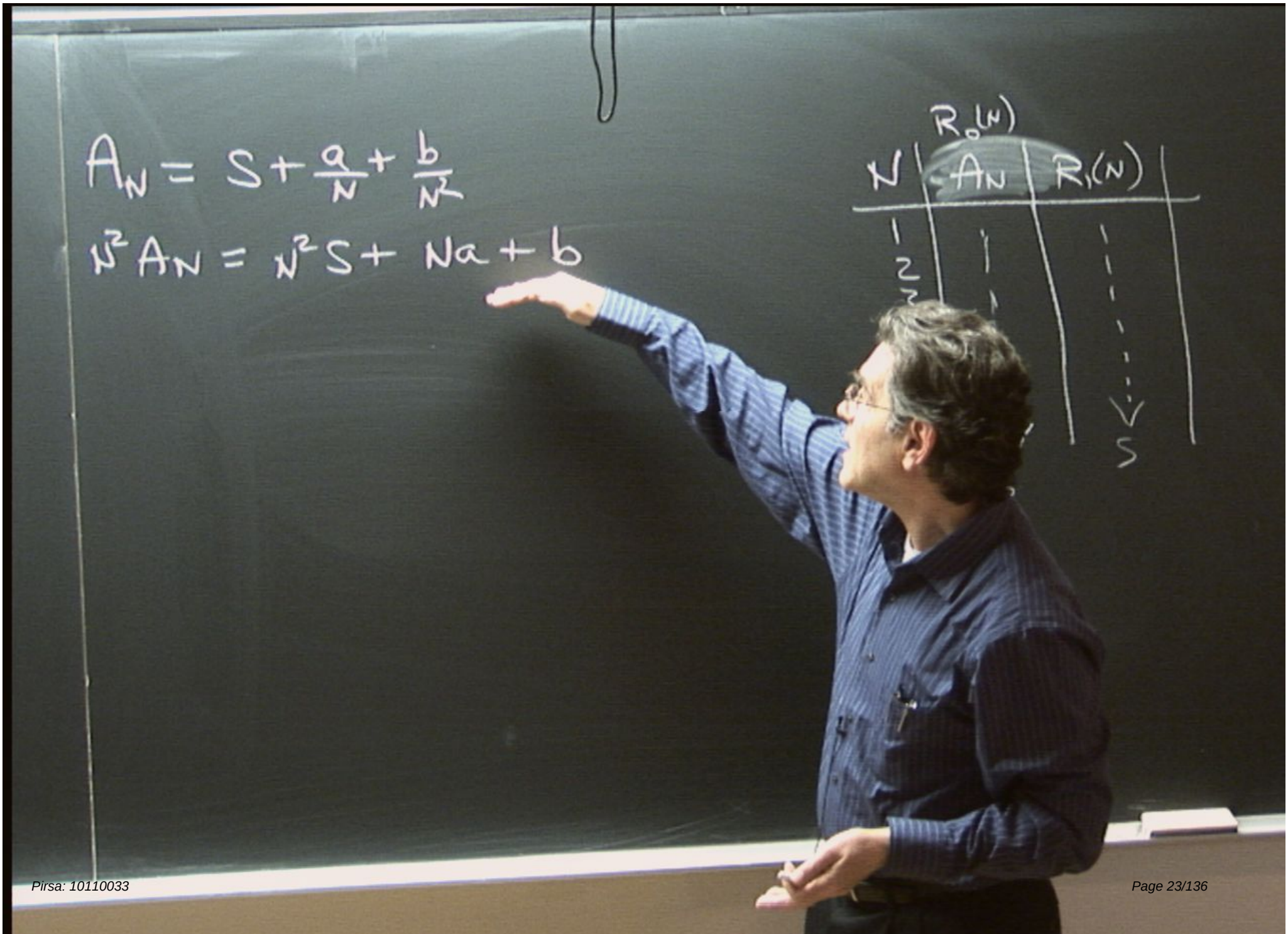

$R_0(N)$
N
A_N
$R_1(N)$
1
2
3

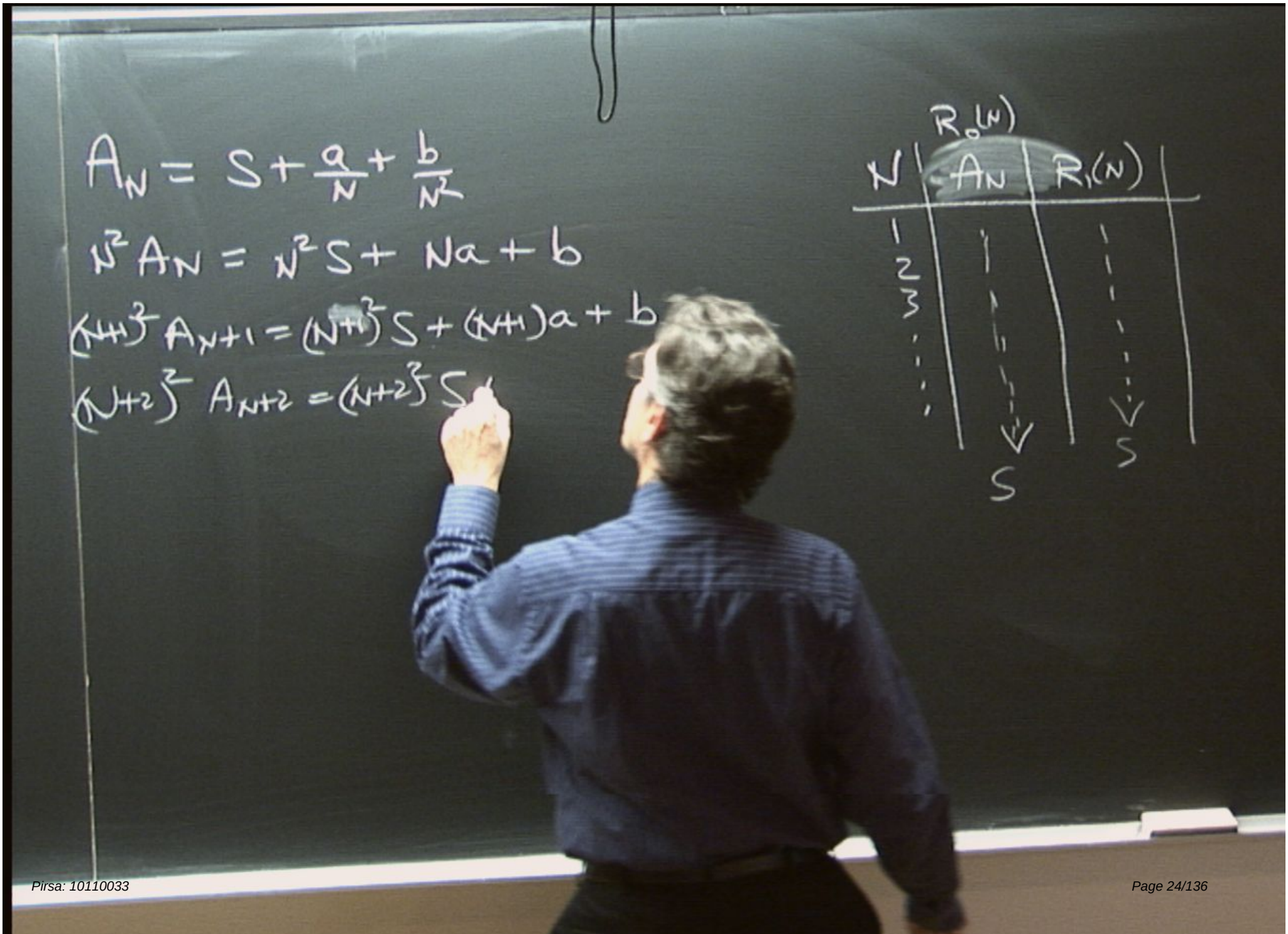

$R_0(N)$
N
A_N
$R_1(N)$
1
2
3
S

N	$R_0(N)$	$R_1(N)$
1	A_N	
2		
3		
⋮	↓	↓
	S	S


$A_N = S + \frac{a}{N} + \frac{b}{\ }$
$R_0(N)$
N
A_N
$R_1(N)$
1
2
3
S
S


$A_N = S + \frac{a}{N} + \frac{b}{N^2}$
$R_0(N)$
N
A_N
$R_1(N)$
1
2
3
S
S


$A_N = S + \frac{a}{N} + \frac{b}{N^2}$
$N^2 A_N = N^2 S + Na + b$
$R_0(N)$
N
A_N
$R_1(N)$
1
2
S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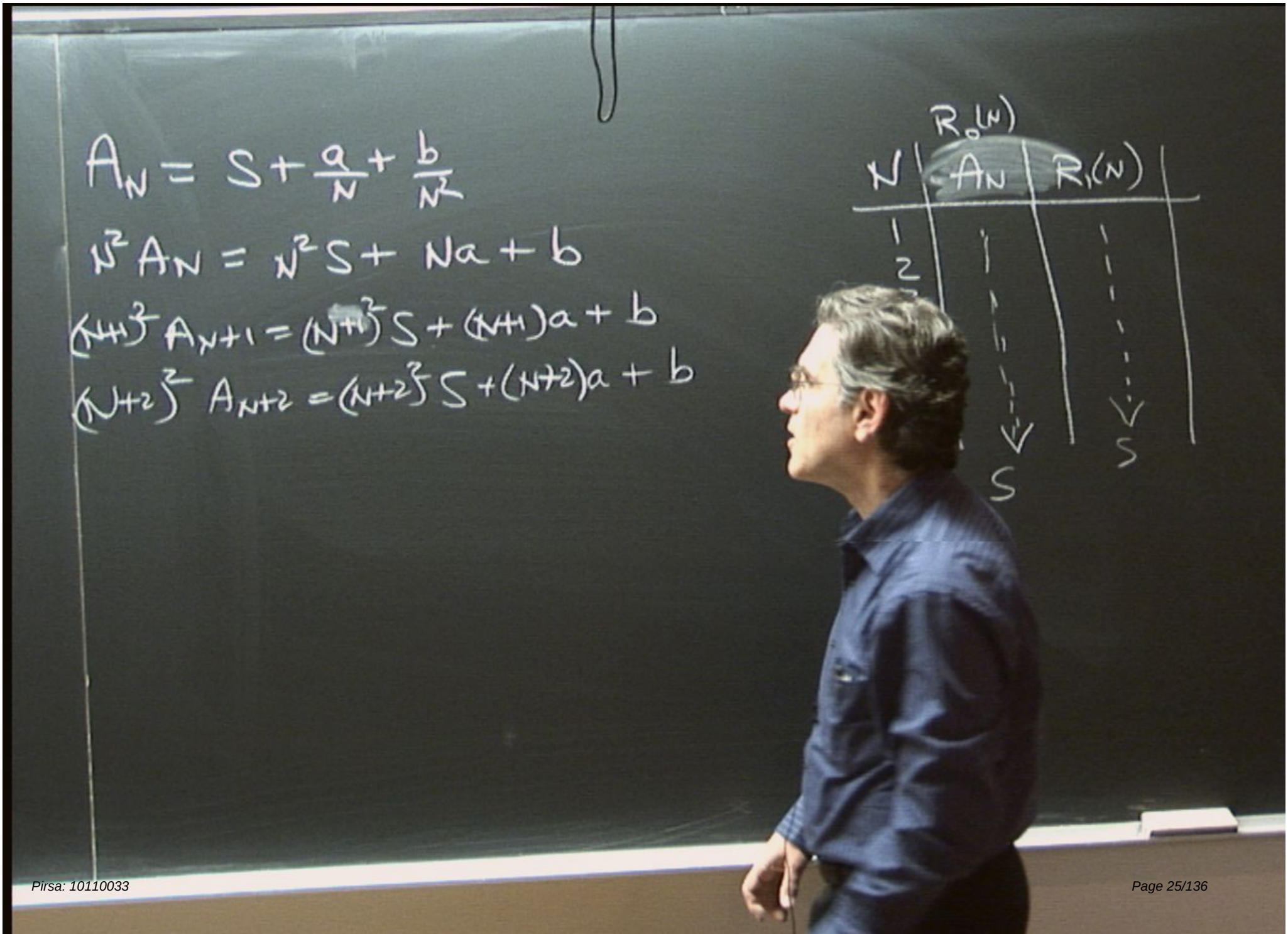
$N^2 A_N = N^2 S + Na + b$
$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b$
$(N+2)^2 A_{N+2} = (N+2)^2 S$
$R_0(N)$
N
A_N
$R_1(N)$
1
2
3
S
S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$R_0(N)$
N
A_N
$R_1(N)$
1
2
S
S

2. Richardson Extrapolation

$$\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^2} \qquad A_N = \sum_{1}^{N} \frac{1}{n^2} \qquad A_N \xrightarrow[\text{as } N \to \infty]{} \frac{\pi^2}{6} = S$$

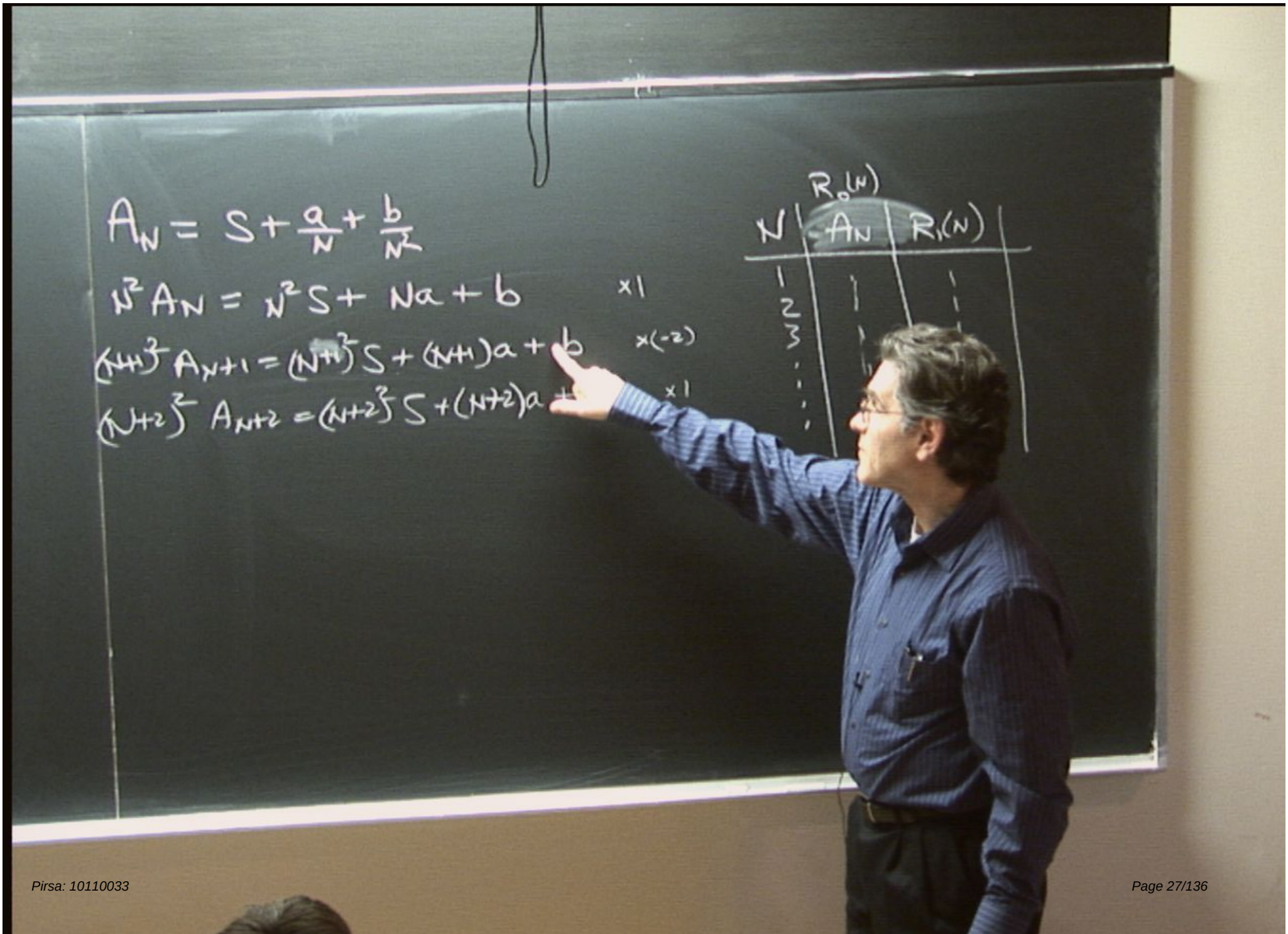
as $N \to \infty$

$$A_N = S + \frac{a}{N} + \cancel{\frac{b}{N^2} + \frac{c}{N^3} + \cdots}$$

$$A_N = S + \frac{a}{N} \Rightarrow N A_N = N S + a$$

$$A_{N+1} = S + \frac{a}{N+1} \rightarrow (N+1) A_{N+1} = (N+1) S + a$$

$$\boxed{R_1 = (N+1) A_{N+1} - N A_N} = S$$


$A_N = S + \frac{a}{N} + \frac{b}{N^2}$
$N^2 A_N = N^2 S + Na + b$ ×1
$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b$ ×(-2)
$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a +$ ×1
$R_0(N)$
N
$-A_N$
$R_1(N)$
1
2
3

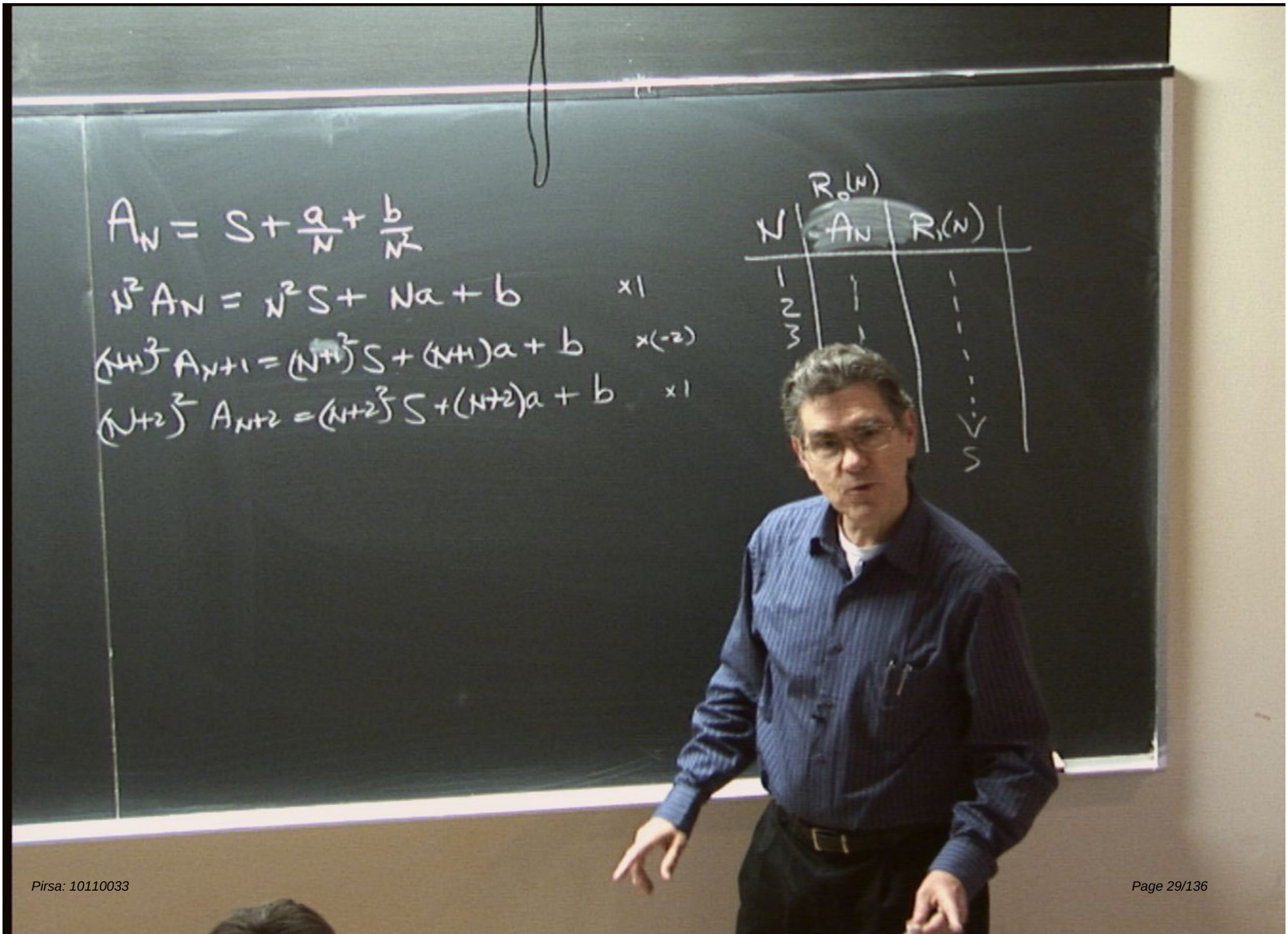
$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

N	$R_0(N)$ $-A_N$	$R_1(N)$
1		
2		
3		
⋮		
	S	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$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b$ ×1
$R_0(N)$
N
$-A_N$
$R_1(N)$
1
2
3
S

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

$$\frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

N	$R_0(N)$ $-A_N$	$R_1(N)$
1		
2		
3		
⋮		

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

$$R_2 = \frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

N	$R_0(N)$ $-A_N$	$R_1(N)$	
1			
2			
3			
⋮		⋮	
	S	S	

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

$$R_2 = \frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

$$R_3 =$$

N	$R_0(N)$ $-A_N$	$R_1(N)$
1		
2		
3		
⋮	↓	↓
	S	S

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

$$R_2 : \frac{A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

$$\frac{(N+3)^3 A_{N+3} - 3(N+2)^3 A_{N+2} + 3(N+1)^3 A_{N+1} - N^3 A_N}{}$$

N	$R_0(N)$: $-A_N$	$R_1(N)$
1		
2		
3		
⋮	⋮	⋮
	$\downarrow$ S	$\downarrow$ S

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

$$R_2 = \frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

$$R_3 = \frac{(N+3)^3 A_{N+3} - 3(N+2)^3 A_{N+2} + 3(N+1)^3 A_{N+1} - N^3 A_N}{6}$$

N	$R_0(N)$ $-A_N$	$R_1(N)$
1		
2		
3		
⋮	⋮	⋮
	S	S

α_n $\quad \dfrac{\alpha_{n+1} - \alpha_n}{\alpha_{n+2} - 2\alpha_{n+1} + \alpha_n}$

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

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$$R_2 = \frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

$$R_3 = \frac{(N+3)^3 A_{N+3} - 3(N+2)^3 A_{N+2} + 3(N+1)^3 A_{N+1} - N^3 A_N}{6}$$

N	$R_0(N)$ A_N	$R_1(N)$
1		
2		
3		
⋮		

$$A_N = S + \frac{a}{N} + \frac{b}{N^2}$$

$$N^2 A_N = N^2 S + Na + b \qquad \times 1$$

$$(N+1)^2 A_{N+1} = (N+1)^2 S + (N+1)a + b \qquad \times(-2)$$

$$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b \qquad \times 1$$

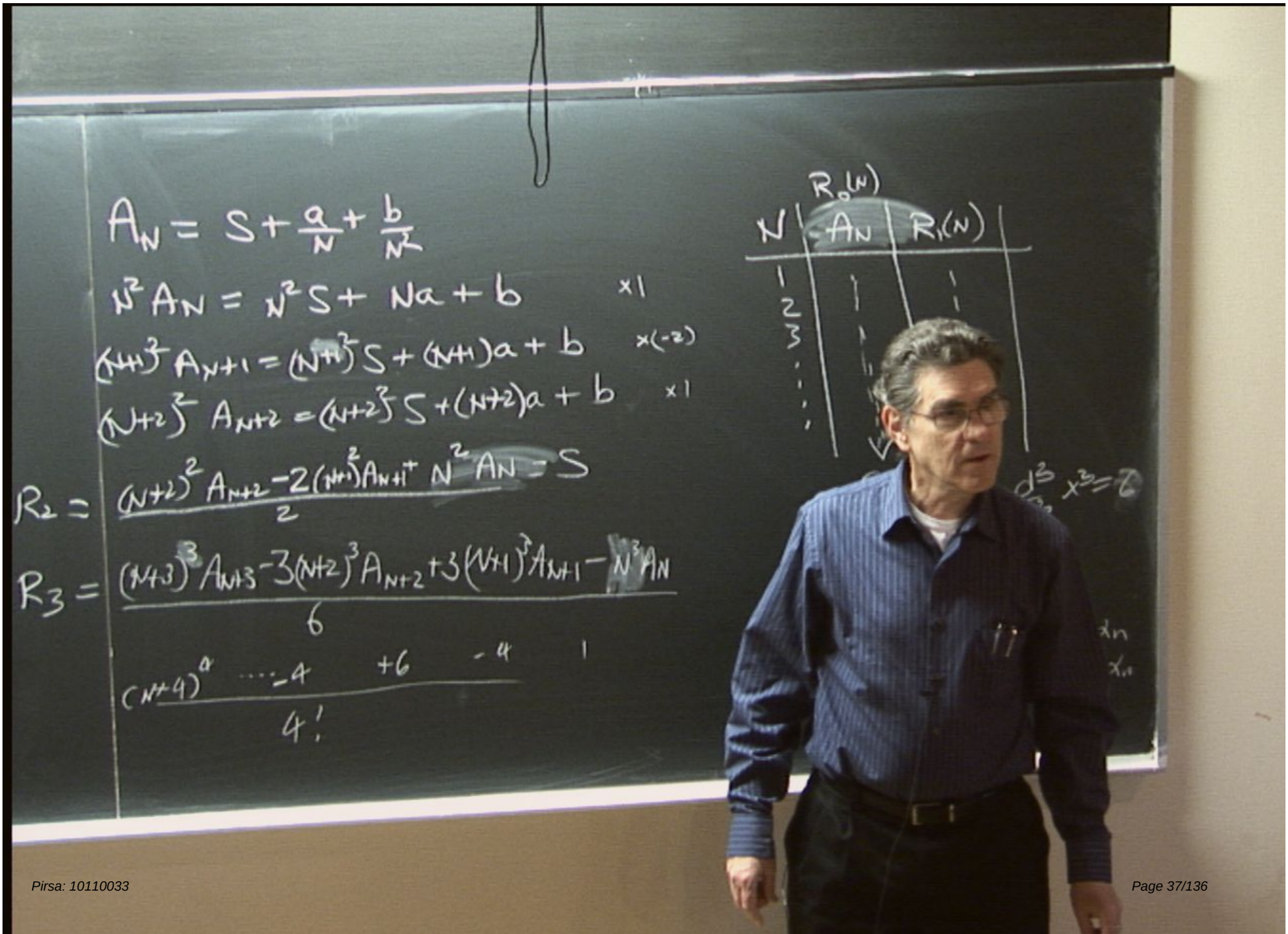
$$R_2 = \frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$$

$$R_3 = \frac{(N+3)^3 A_{N+3} - 3(N+2)^3 A_{N+2} + 3(N+1)^3 A_{N+1} - N^3 A_N}{6}$$

N	$R_0(N)$ $-A_N$	$R_1(N)$	
1	⋮	⋮	
2	⋮	⋮	
3	⋮	⋮	
⋮	⋮	⋮	
	S	S	

$$\alpha_n \qquad \frac{\alpha_{n+1} - \alpha}{\alpha_{n+2} - 2\alpha_{n+1} +}$$

$$\alpha_{n+3} - 3\alpha_{n+2} + 3\alpha_{n+1} -$$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$(N+2)^2 A_{N+2} = (N+2)^2 S + (N+2)a + b$ ×1
$R_2 = \frac{(N+2)^2 A_{N+2} - 2(N+1)^2 A_{N+1} + N^2 A_N}{2} = S$
$R_3 = \frac{(N+3)^3 A_{N+3} - 3(N+2)^3 A_{N+2} + 3(N+1)^3 A_{N+1} - N^3 A_N}{6}$
$\frac{(N+4)^4 \cdots -4 \quad +6 \quad -4 \quad 1}{4!}$
$R_0(N)$
N
$-A_N$
$R_1(N)$
1
2
3

Accelerating the convergence of a convergent series…

1. Richardson extrapolation
2. Shanks transformation
3. Fourier series

Accelerating the convergence of a convergent series…

1. Richardson extrapolation
2. Shanks transformation
3. Fourier series

Shanks transformation

Shanks transform of log 2=1-1/2+1/3-1/4…=0.693 147 2…

n	A	S(A)	S(S(A))	S(S(S(A)))
1	**1.000 000 0**			
2	**0.500 000 0**	**0.700 000 0**		
3	**0.833 333 3**	**0.690 476 2**	**0.693 277 3**	
4	**0.583 333 3**	**0.694 444 4**	**0.693 105 8**	**0.693 148 9**
5	**0.783 333 3**	**0.692 424 2**	**0.693 163 3**	**0.693 146 7**
6	**0.616 666 7**	**0.693 589 7**	**0.693 139 9**	
7	**0.759 523 8**	**0.692 857 1**		
8	**0.634 523 8**			

Shanks transform of $1-1/\sqrt{2}+1/\sqrt{3}-1/\sqrt{4}\ldots=0.604\ 898\ 6\ldots$

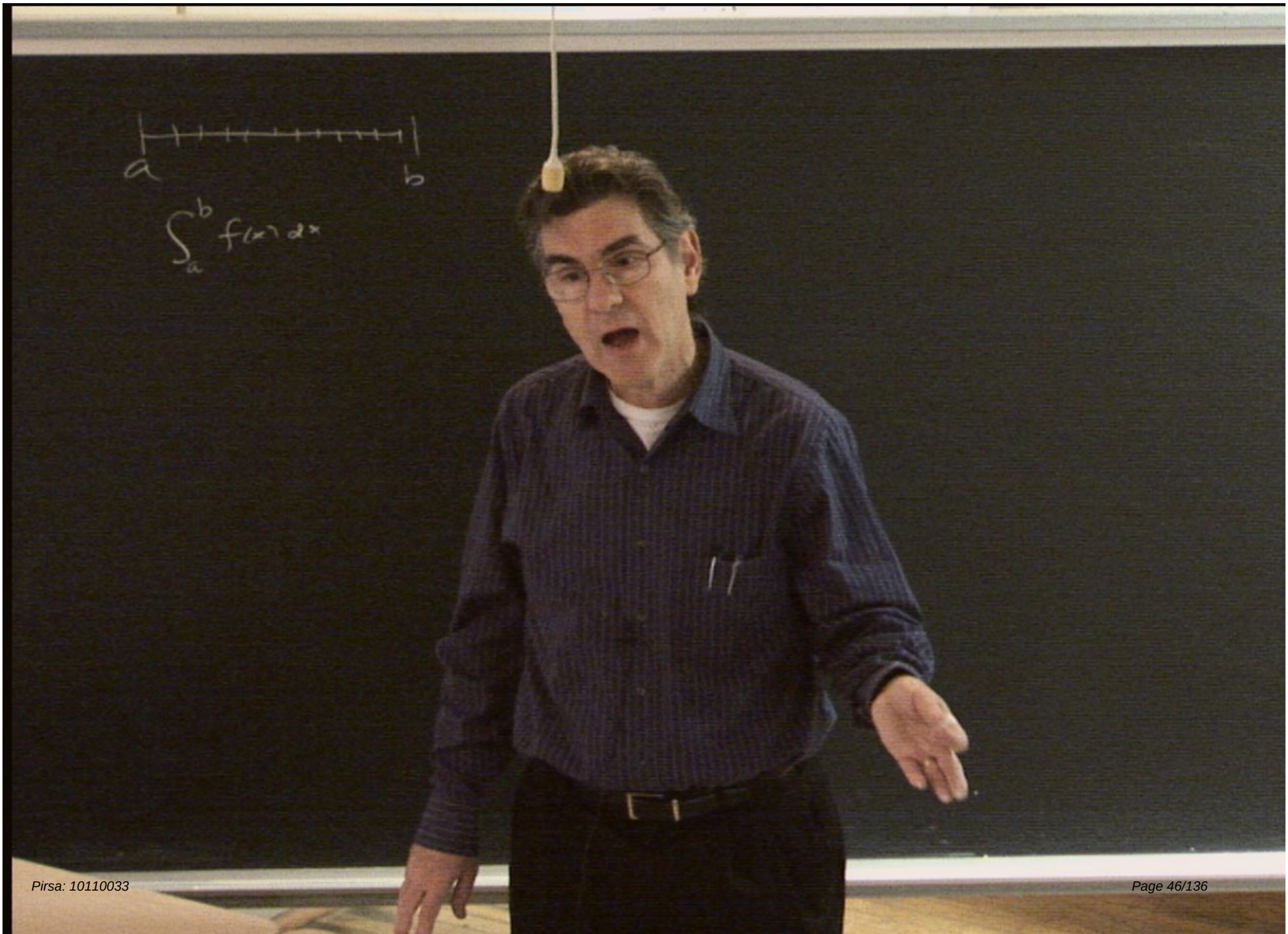
n	A	S(A)	S(S(A))	S(S(S(A)))
1	**1.000 000 0**			
2	**0.292 893 2**	**0.610 730 5**		
3	**0.870 243 5**	**0.602 294 3**	**0.605 015 6**	
4	**0.370 243 5**	**0.606 311 5**	**0.604 858 5**	**0.604 900 3**
5	**0.817 457 1**	**0.604 035 3**	**0.604 915 5**	
6	**0.409 208 8**	**0.605 470 4**		
7	**0.787 173 3**			

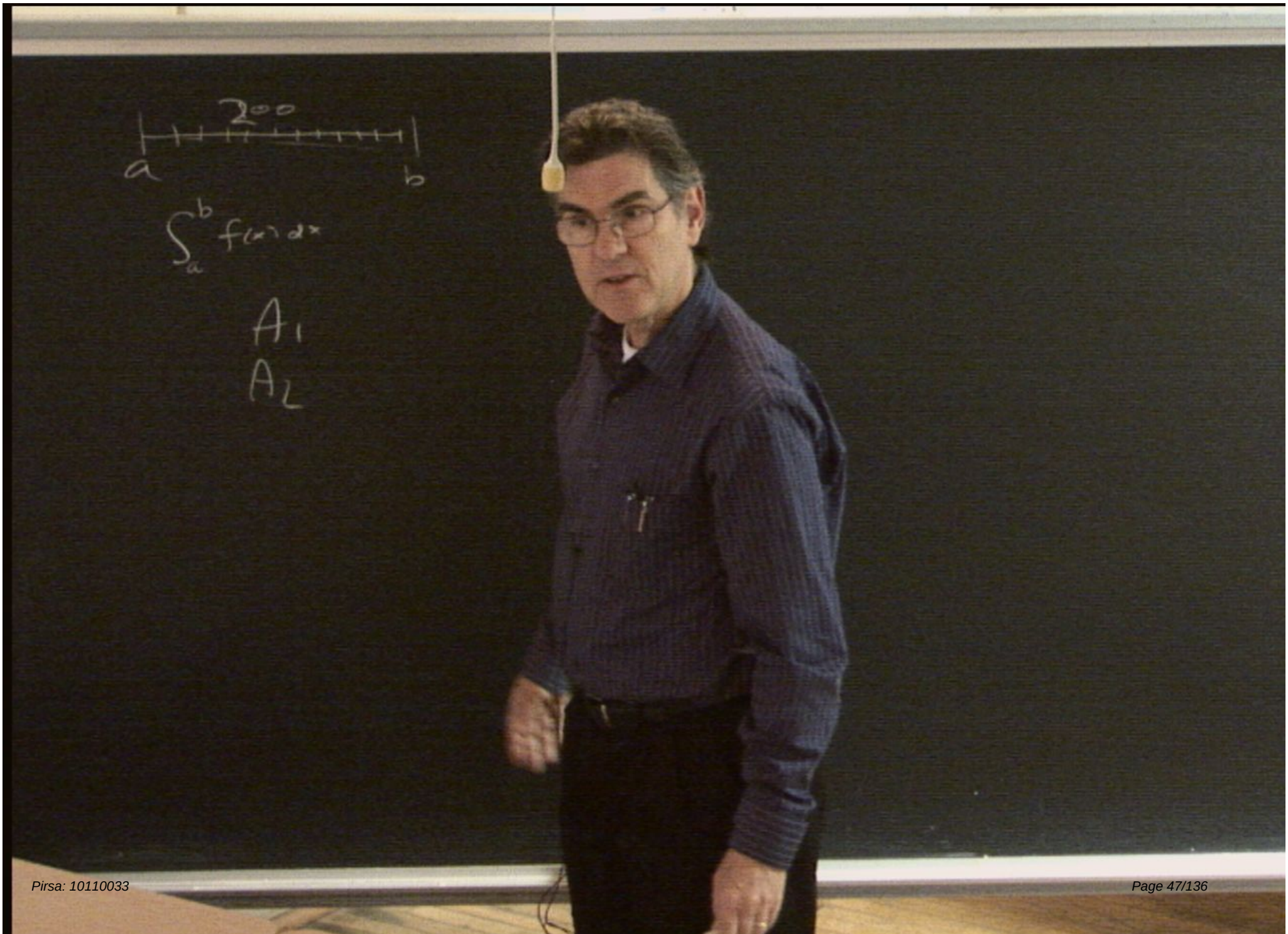
Richardson extrapolation

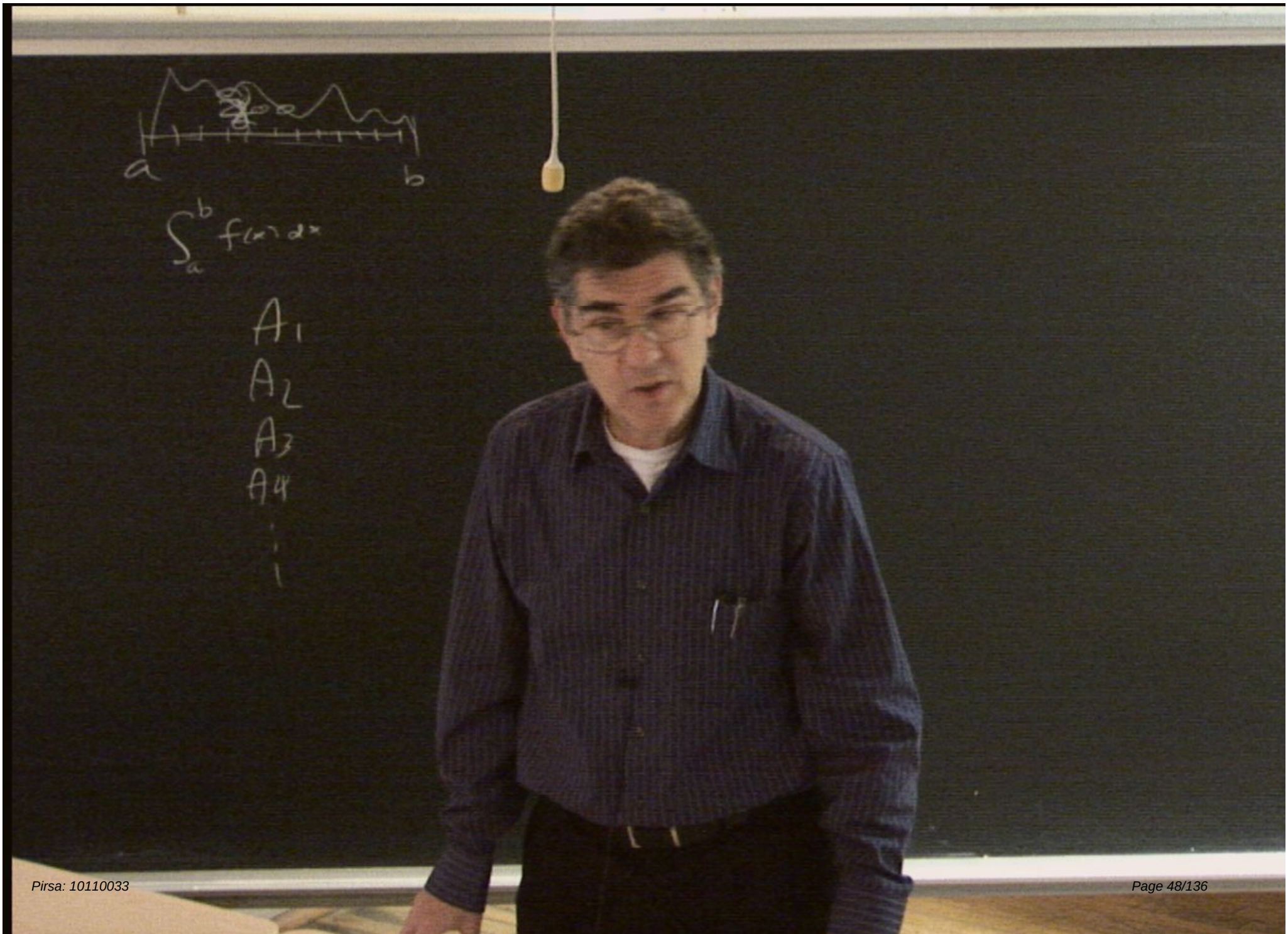
Richardson extrapolation of $1+1/4+1/9+1/16+\ldots=1.644\,934\,066\,848$

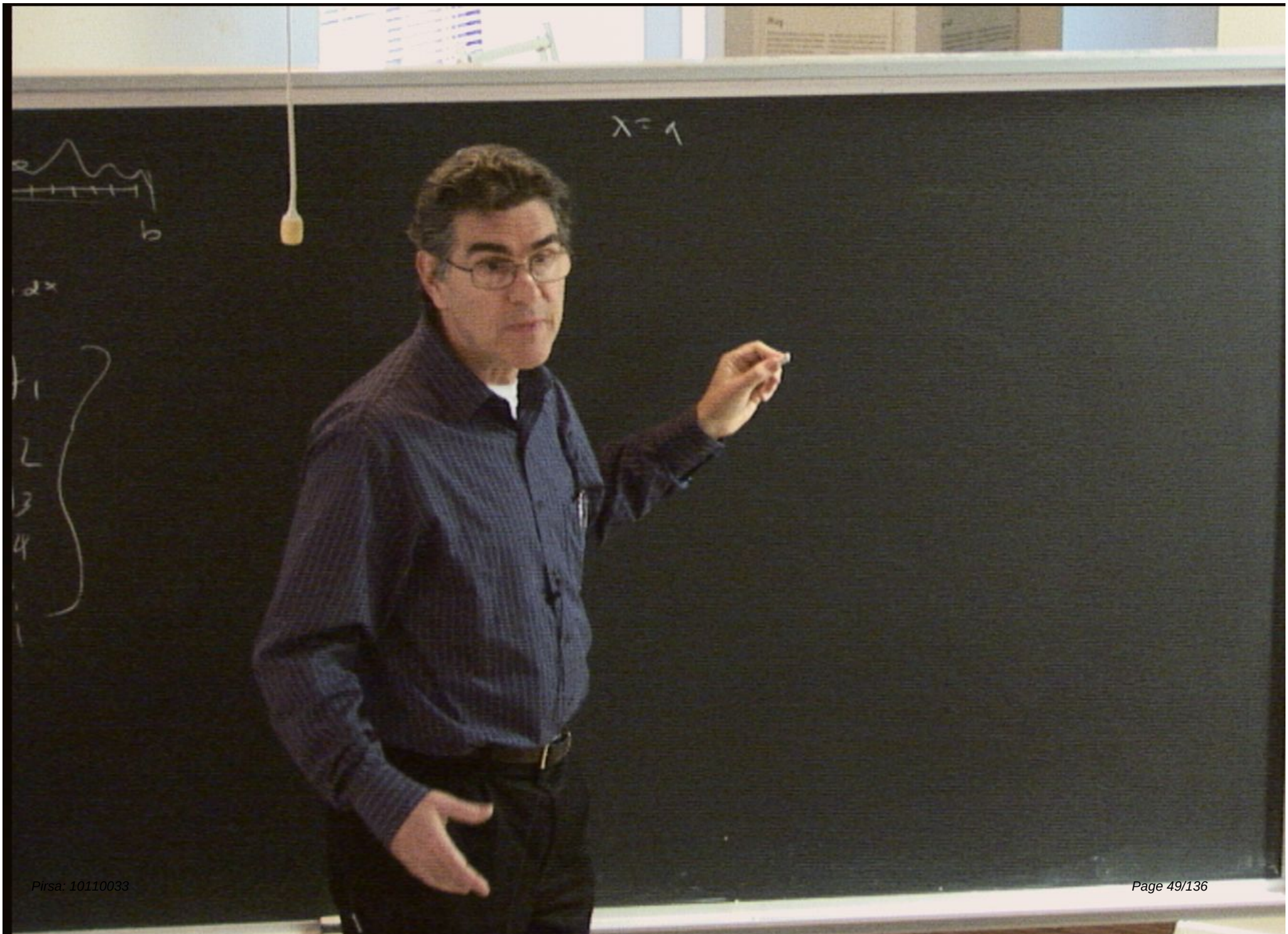
n	N=0	N=2	N=4	N=6
1	**1.000**	**1.625 000**	**1.644 965 278**	**1.644 935 185 185**
5	**1.464**	**1.644 167**	**1.644 935 811**	**1.644 934 060 147**
10	**1.550**	**1.644 809**	**1.644 934 195**	**1.644 934 066 526**
15	**1.580**	**1.644 893**	**1.644 934 090**	**1.644 934 066 812**
20	**1.596**	**1.644 916**	**1.644 934 073**	**1.644 934 066 842**
25	**1.606**	**1.644 924**	**1.644 934 069**	**1.644 934 066 847**

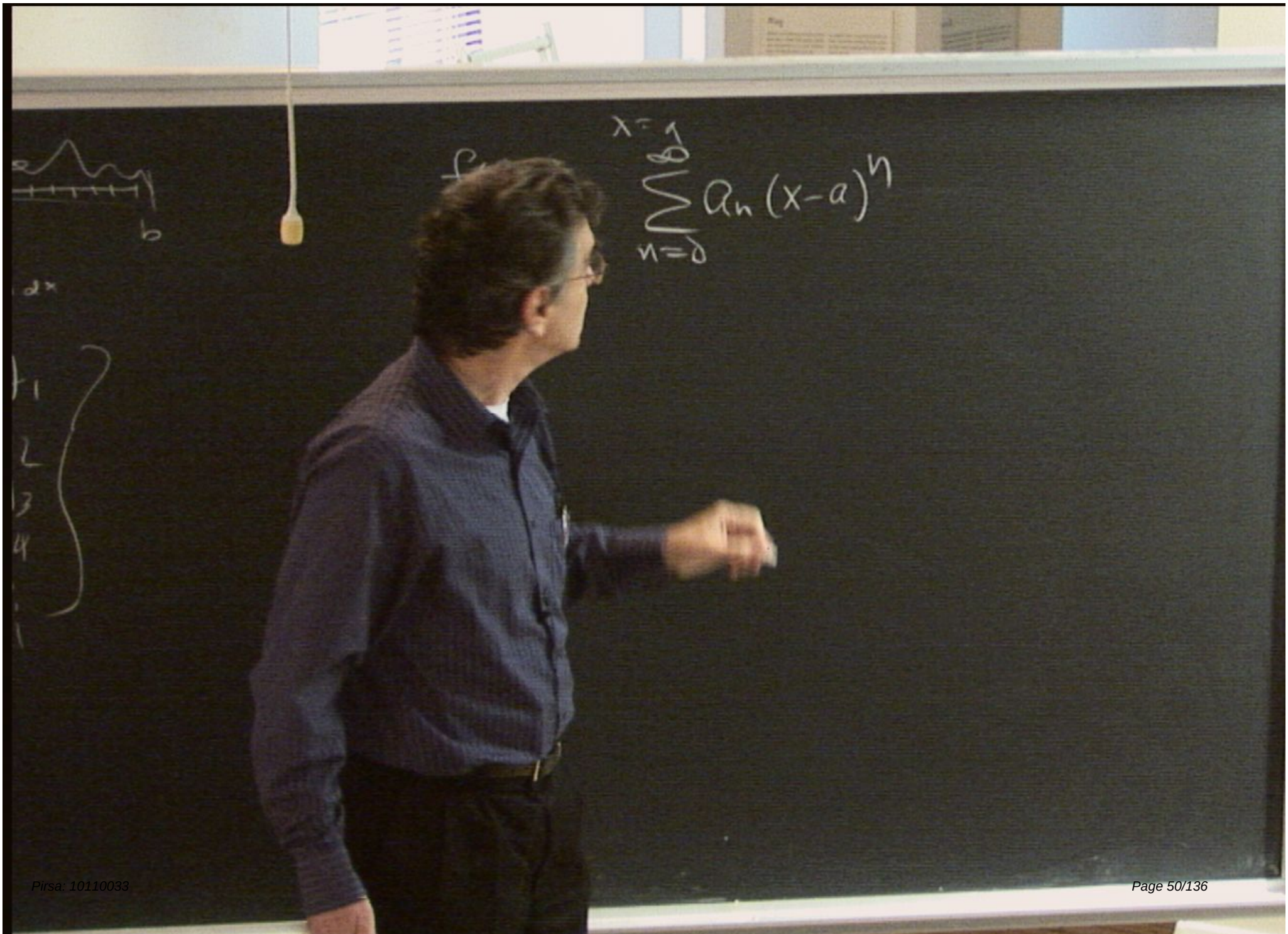
Microsoft Windows xp Professional

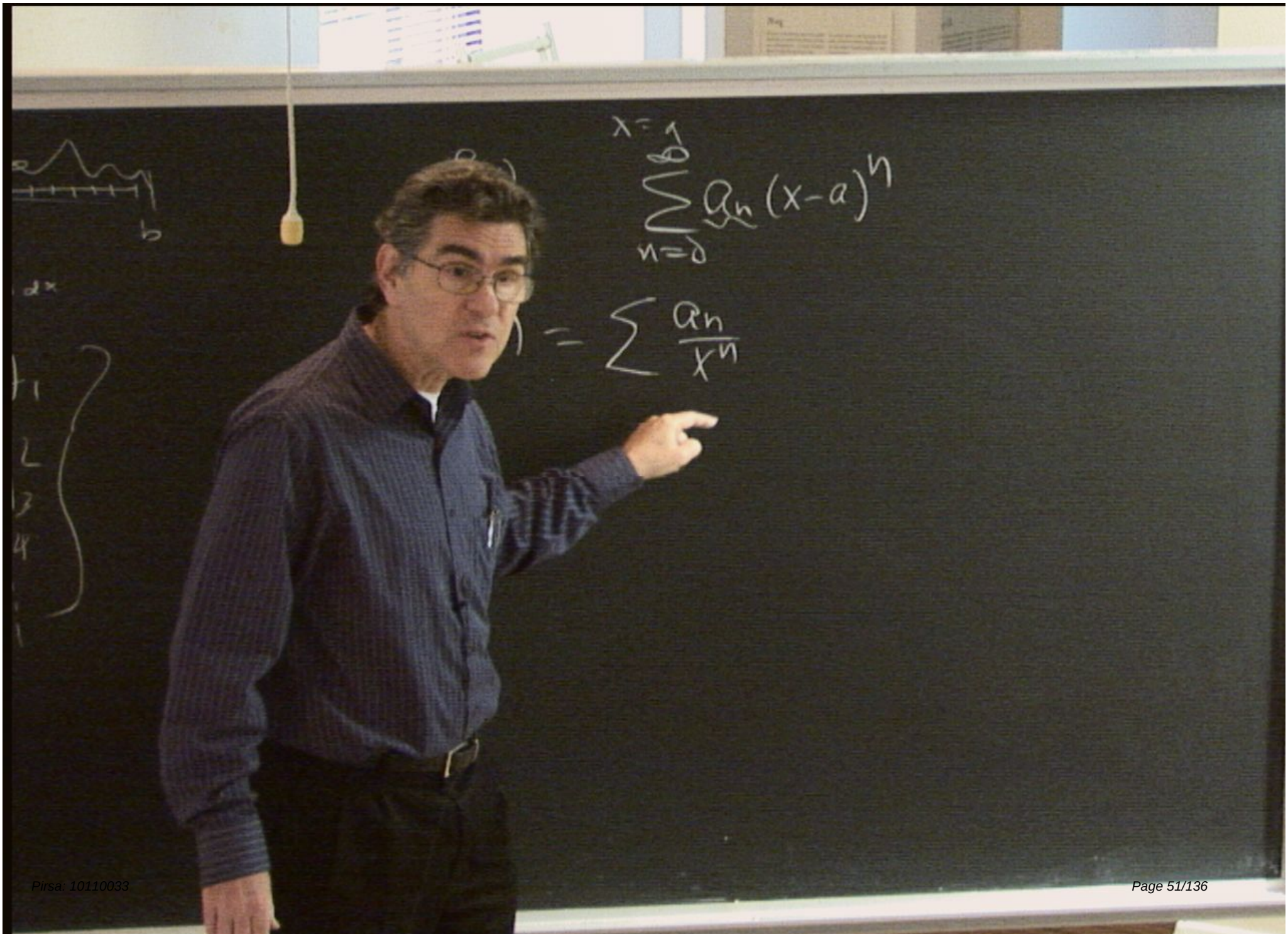

a
b
$\int_a^b f(x)\,dx$

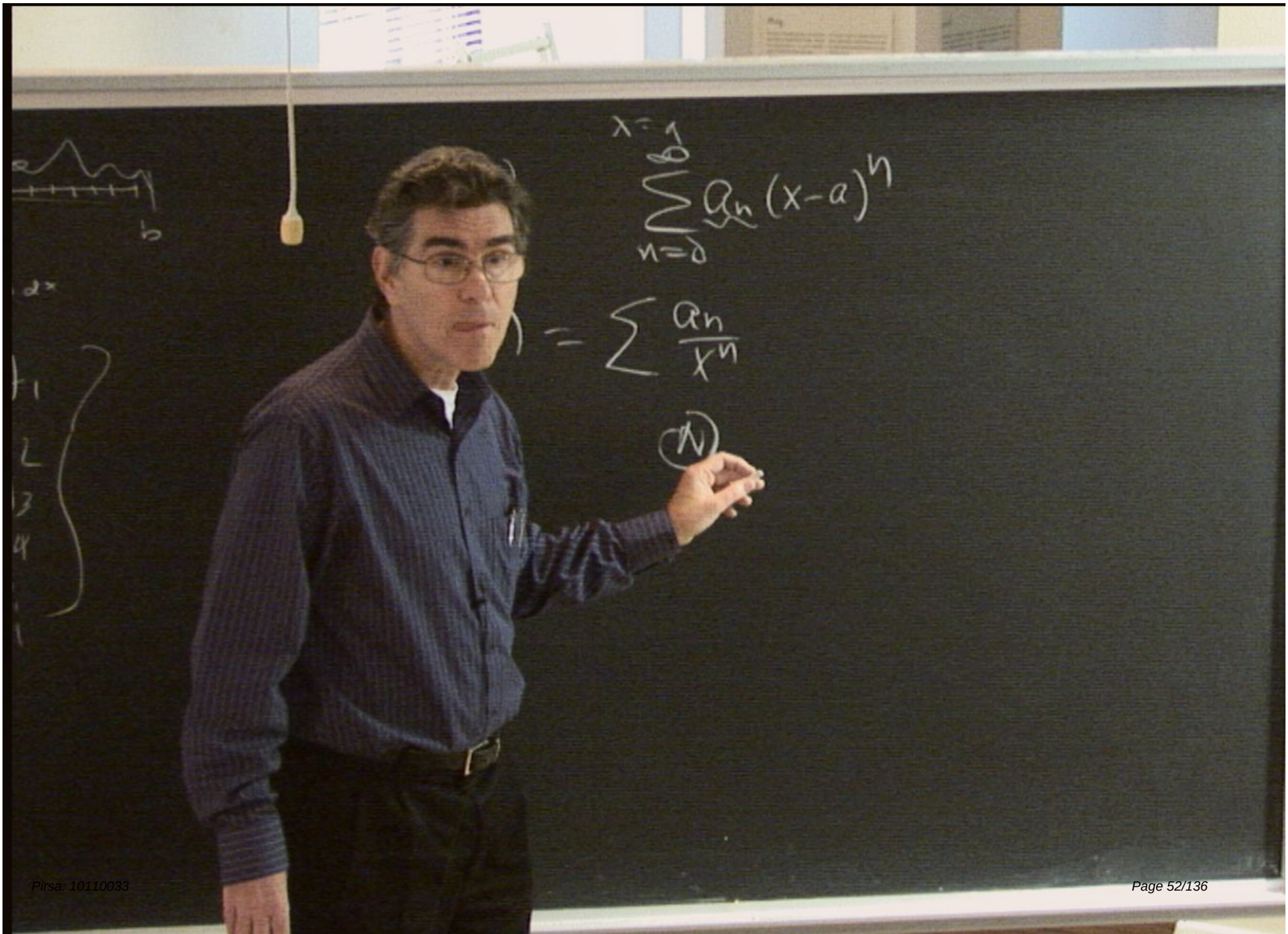

200
a
b
$\int_a^b f(x)\,dx$
A_1
A_2

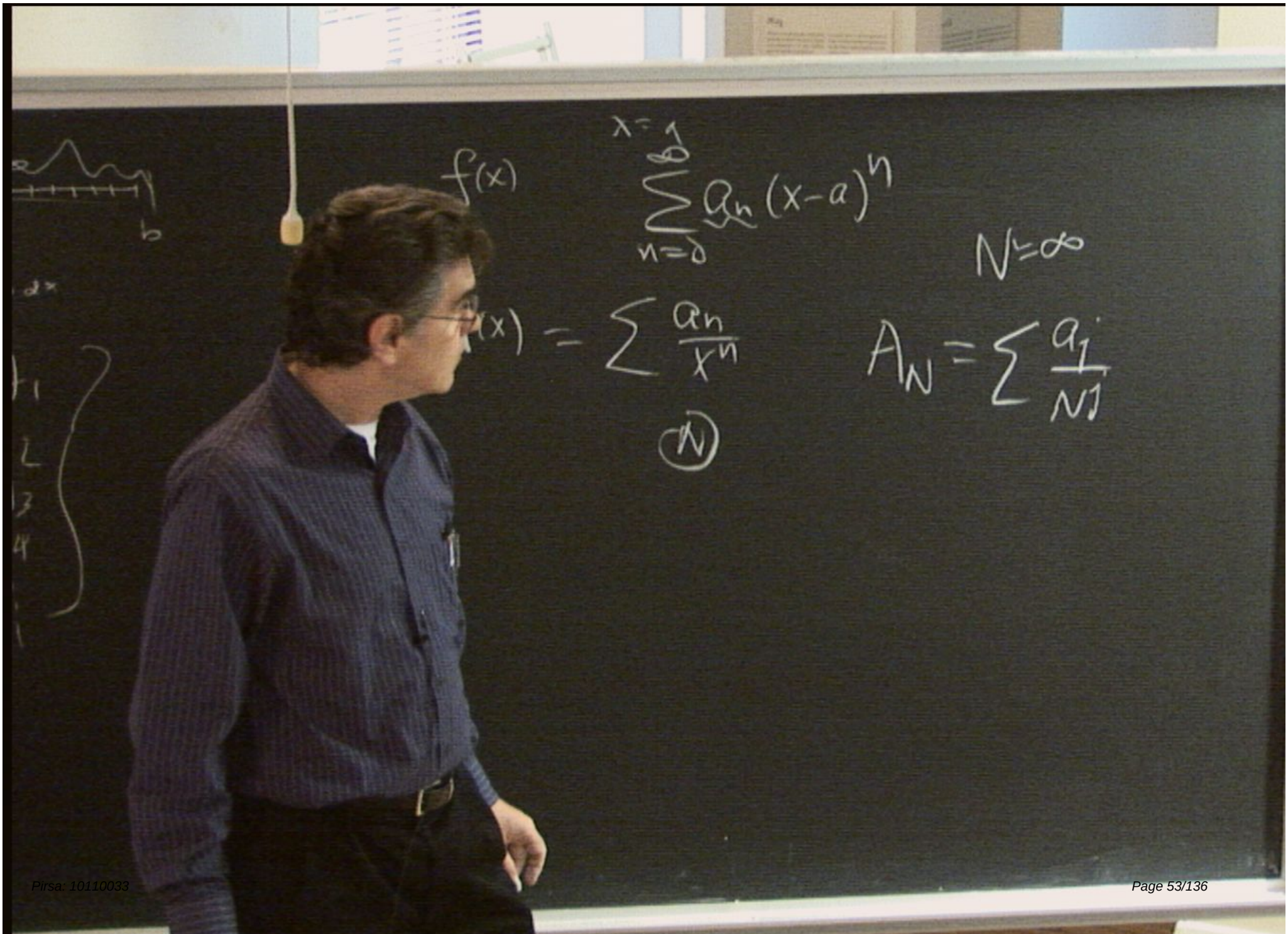

a
b
$\int_a^b f(x)\,dx$
A_1
A_2
A_3
A_4

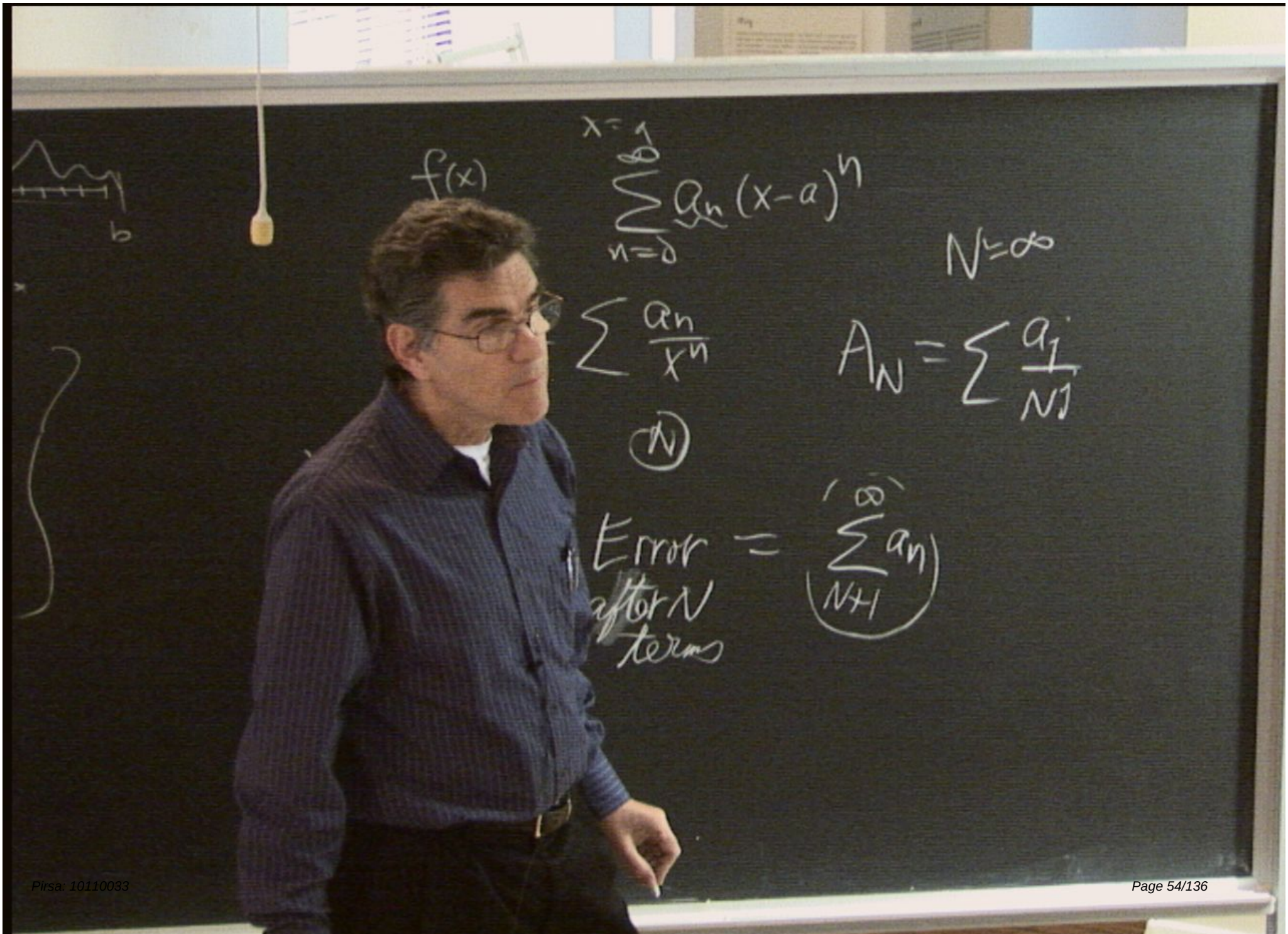

λ = 1

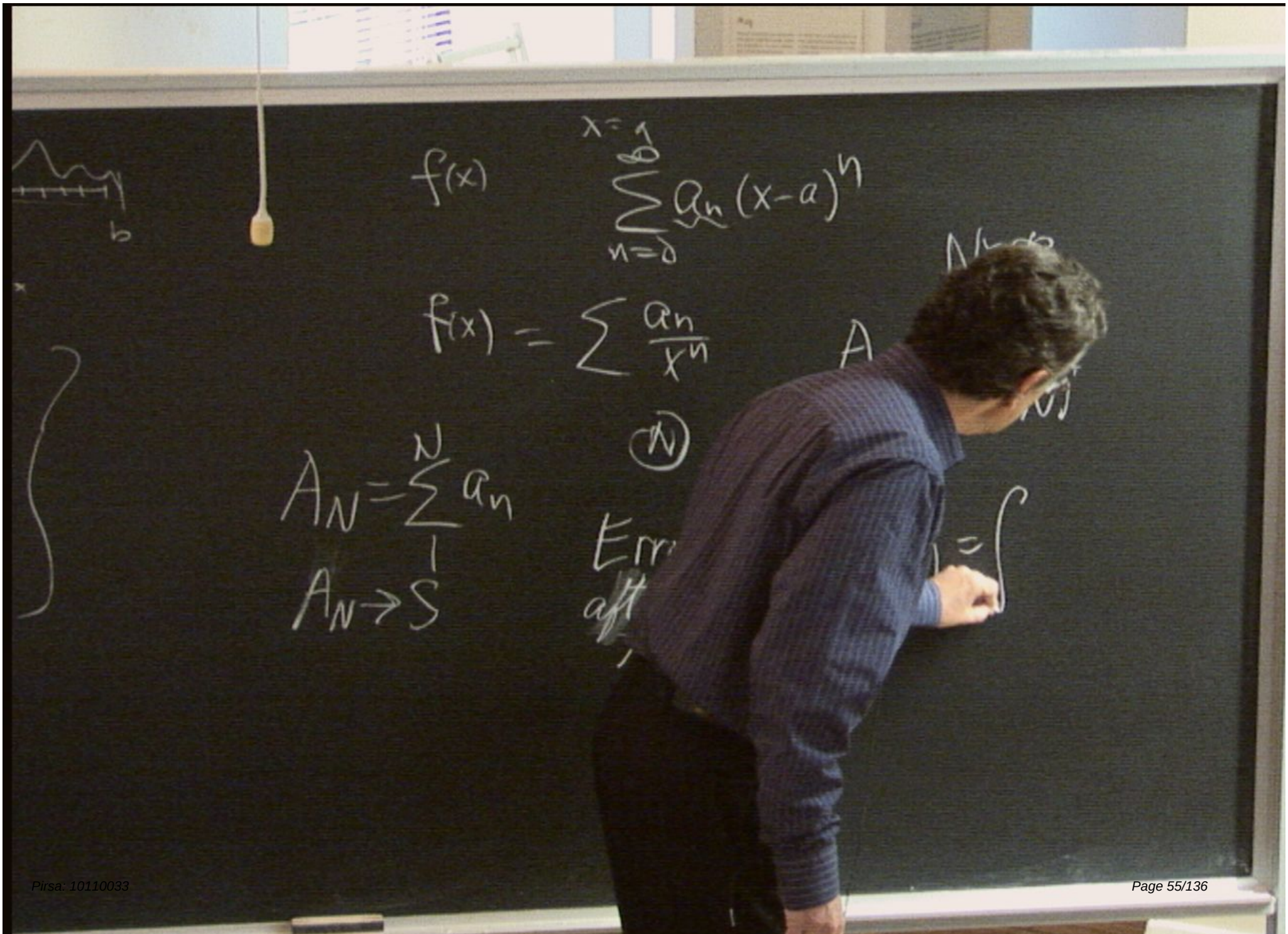

$\sum_{n=0}^{\infty} a_n (x-a)^n$

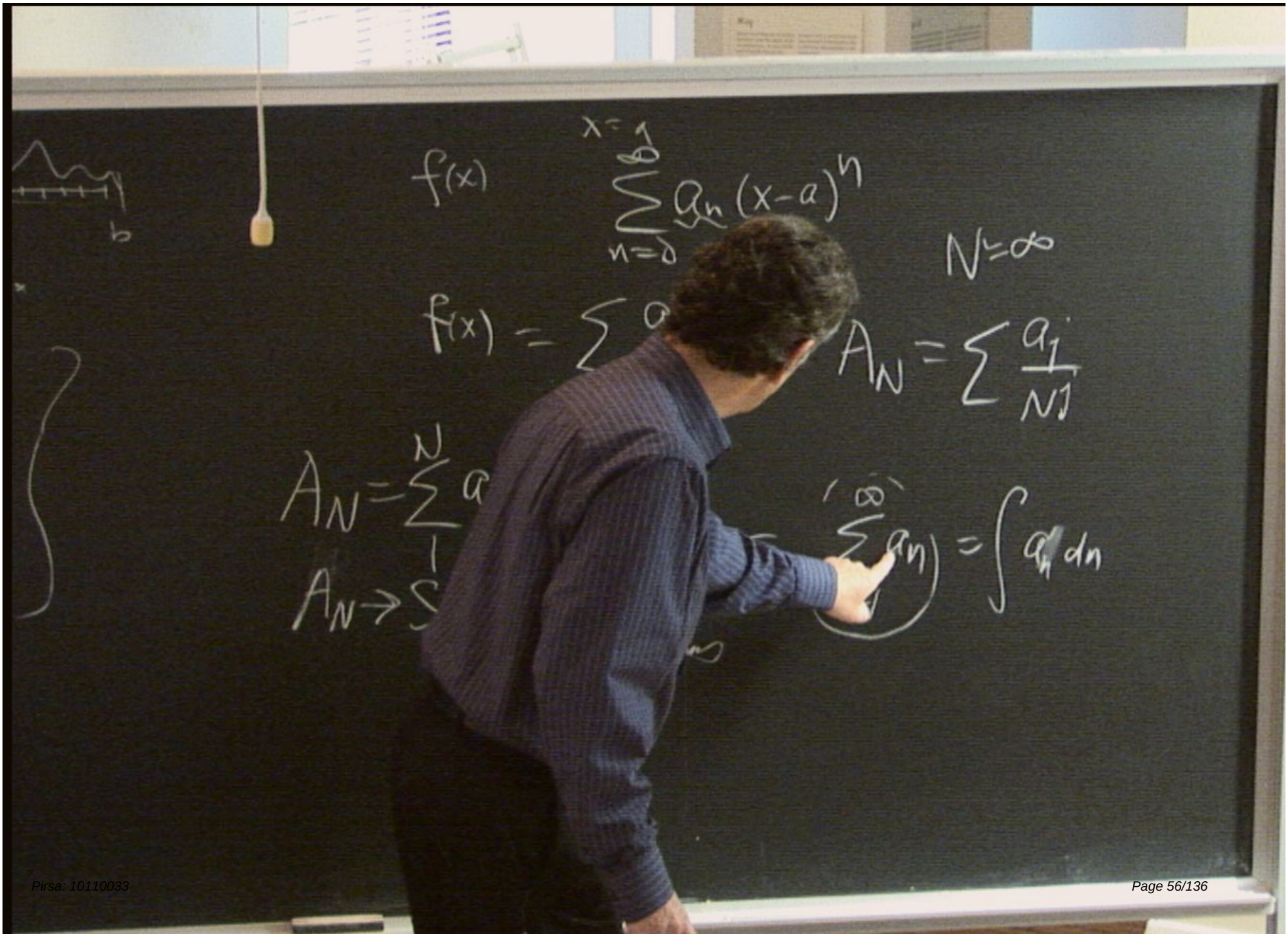

$\sum_{n=0}^{\infty} a_n (x-a)^n$
$= \sum \frac{a_n}{x^n}$

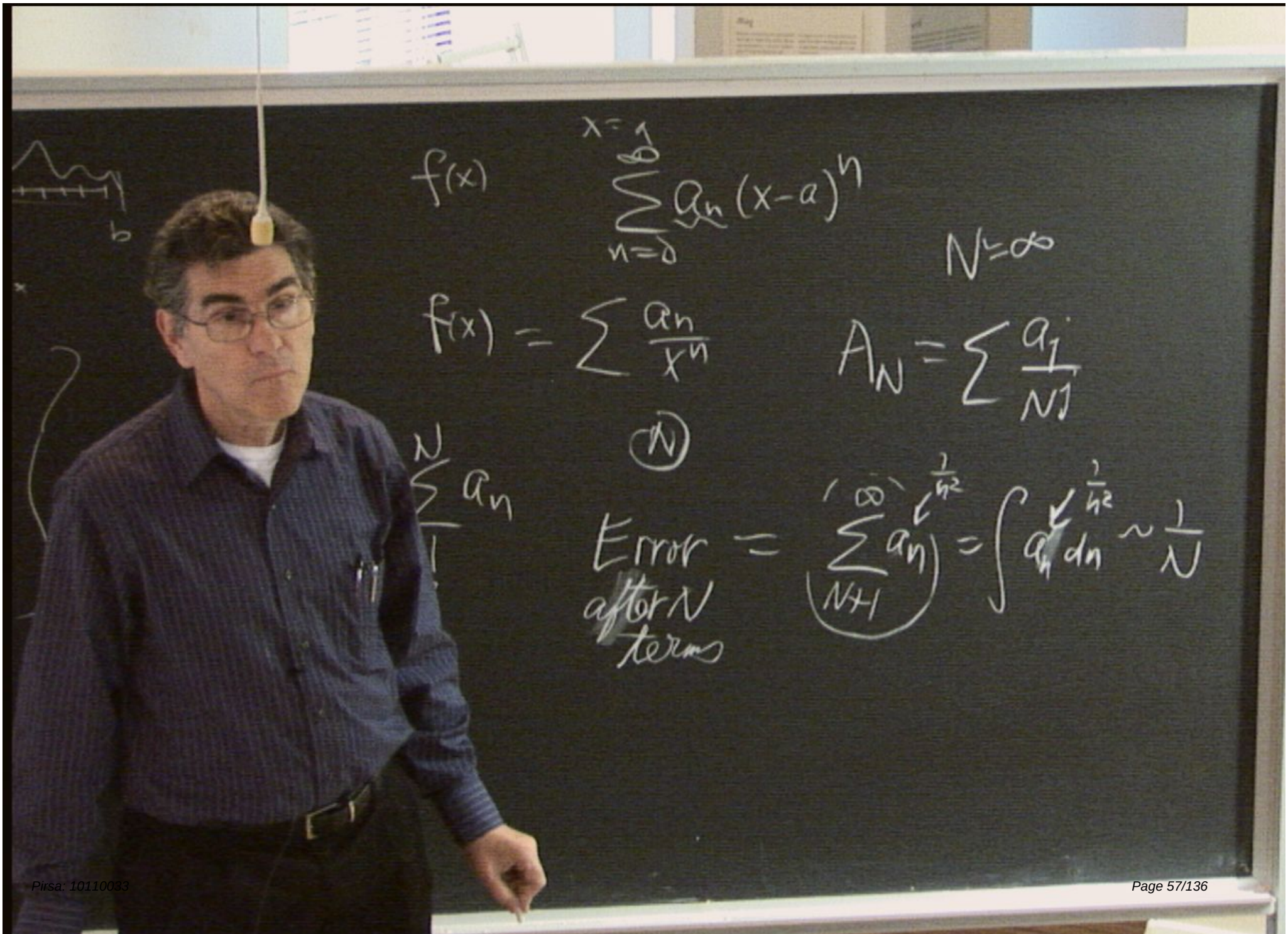

$\sum_{n=0}^{\infty} a_n (x-a)^n$
$= \sum \frac{a_n}{x^n}$
N

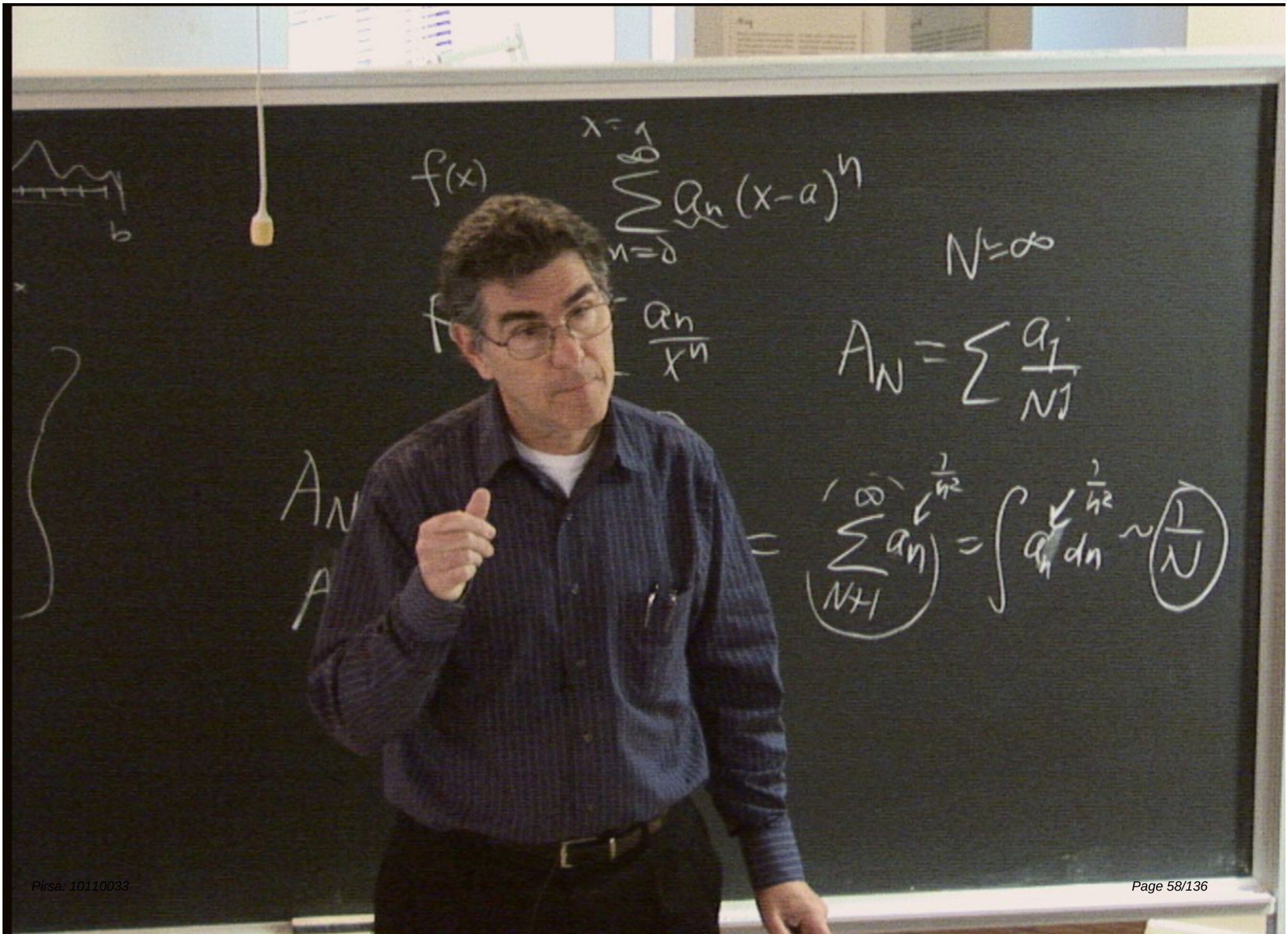

f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
$N=\infty$
$f(x) = \sum \frac{a_n}{x^n}$
$A_N = \sum \frac{a_j}{N^j}$
N

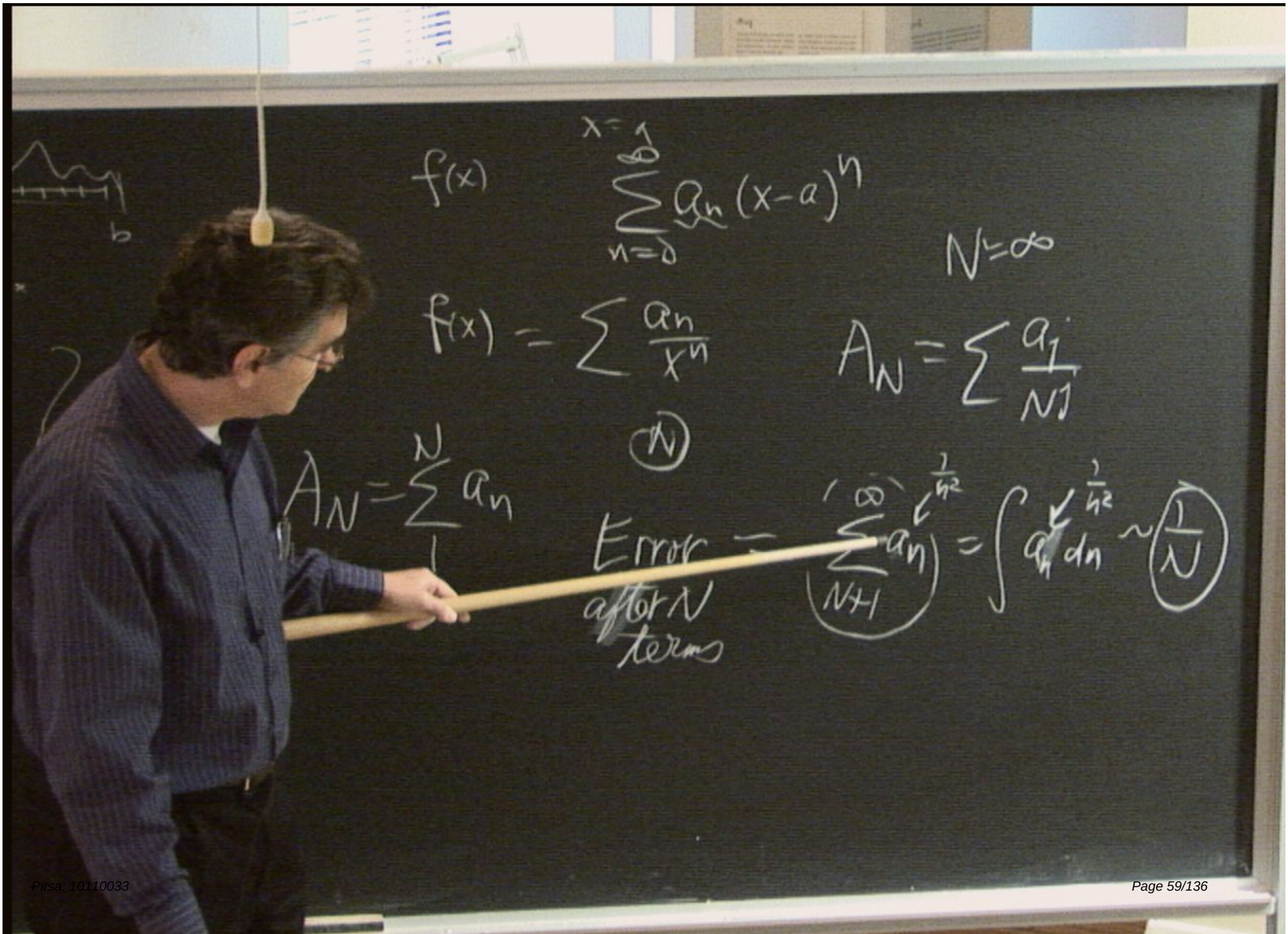

f(x)
b
$\sum_{n=0}^{\infty} a_n (x-a)^n$
$N=\infty$
$\sum \frac{a_n}{x^n}$
$A_N = \sum \frac{a_j}{N!}$
N
Error after N terms = $\sum_{N+1}^{\infty} a_n$

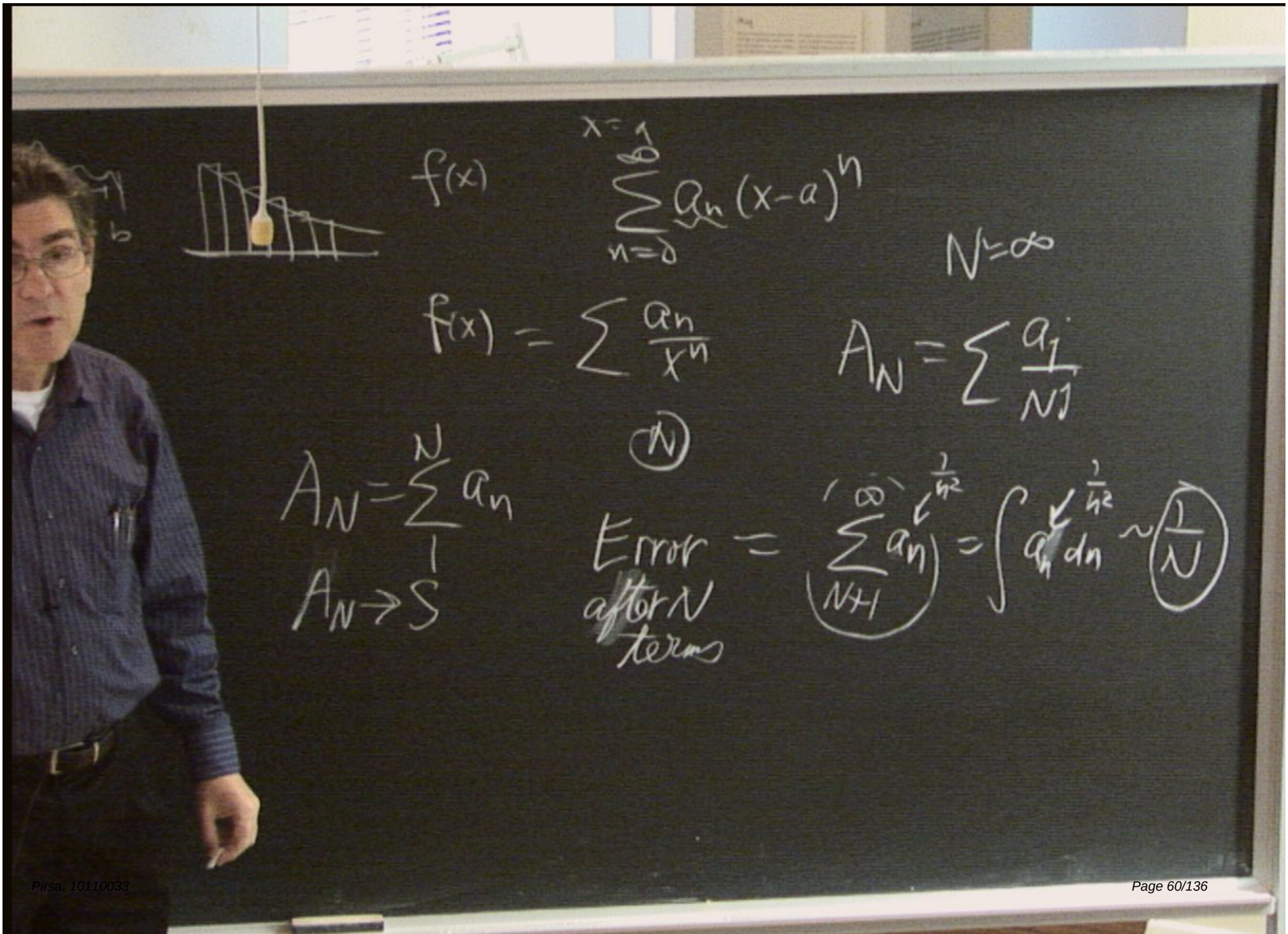

f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
$f(x) = \sum \frac{a_n}{x^n}$
$A_N = \sum_1^N a_n$
$A_N \to S$


f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
N=∞
$A_N = \sum \frac{a_j}{N!}$
$A_N = \sum_1^N a$
$A_N \to S$
$\int a_n \, dn$


f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
$N=\infty$
$f(x) = \sum \frac{a_n}{x^n}$
$A_N = \sum \frac{a_j}{N^j}$
N
$\sum^{N} a_n$
Error after N terms
$= \sum_{N+1}^{\infty} a_n \frac{1}{n^2} = \int a_n \frac{1}{n^2} dn \sim \frac{1}{N}$


f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
$N=\infty$
$A_N = \sum \frac{a_j}{N j}$
$\frac{a_n}{x^n}$
$\sum_{N+1}^{\infty} a_n = \int a_n dn \sim \frac{1}{N}$


f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
N=∞
$f(x) = \sum \frac{a_n}{x^n}$
$A_N = \sum \frac{a_j}{N^j}$
N
$A_N = \sum_1^N a_n$
Error after N terms
$= \sum_{N+1}^{\infty} a_n = \int a_n \, dn \sim \frac{1}{N}$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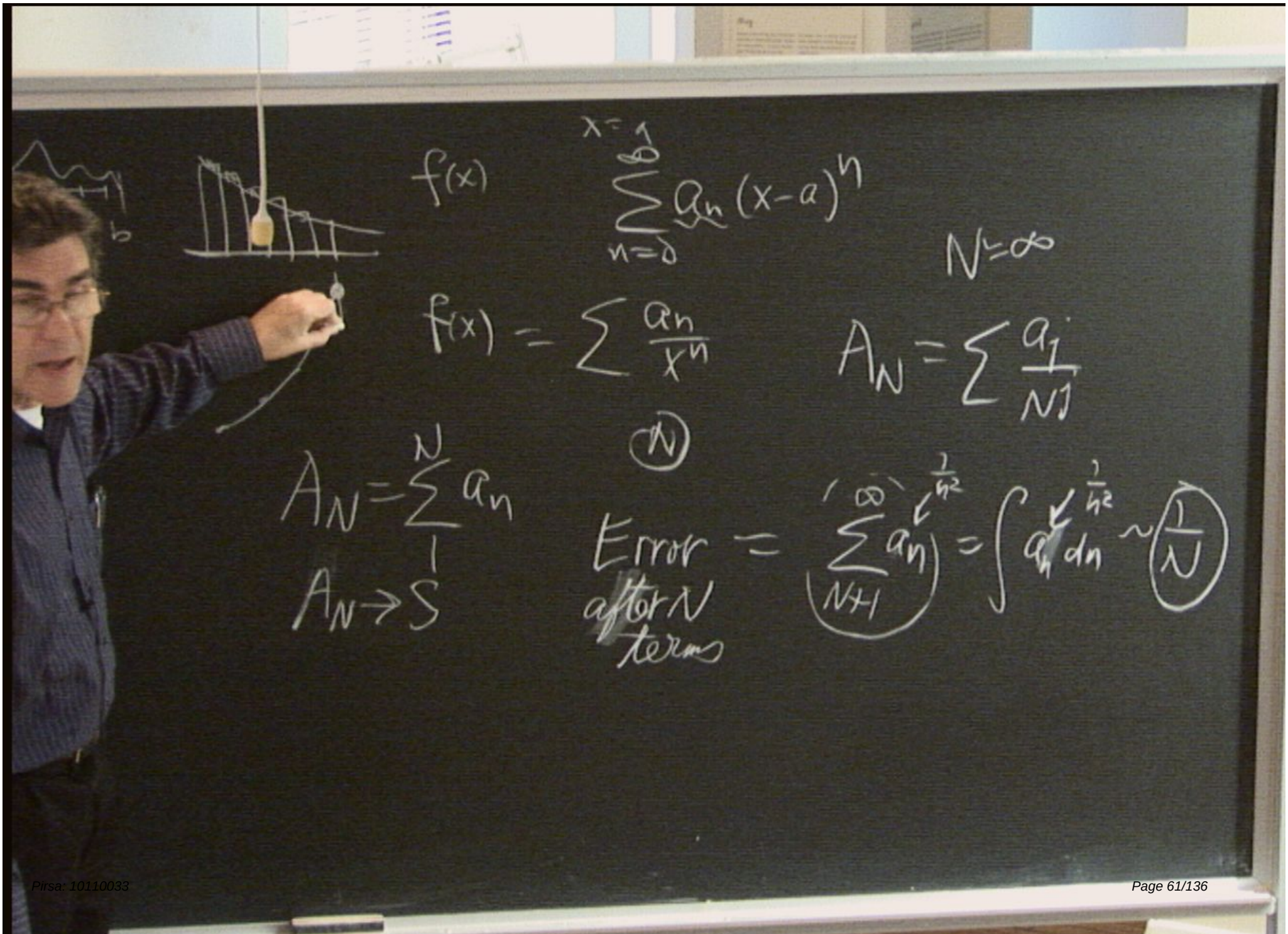
$N=\infty$
$f(x) = \sum \frac{a_n}{x^n}$
$A_N = \sum \frac{a_j}{N^j}$
N
$A_N = \sum_1^N a_n$
$A_N \to S$
Error after N terms
$= \sum_{N+1}^{\infty} a_n = \int a_n \, dn \sim \frac{1}{N}$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$f(x) = \sum \frac{a_n}{x^n}$
$N=\infty$
$A_N = \sum \frac{a_j}{N^j}$
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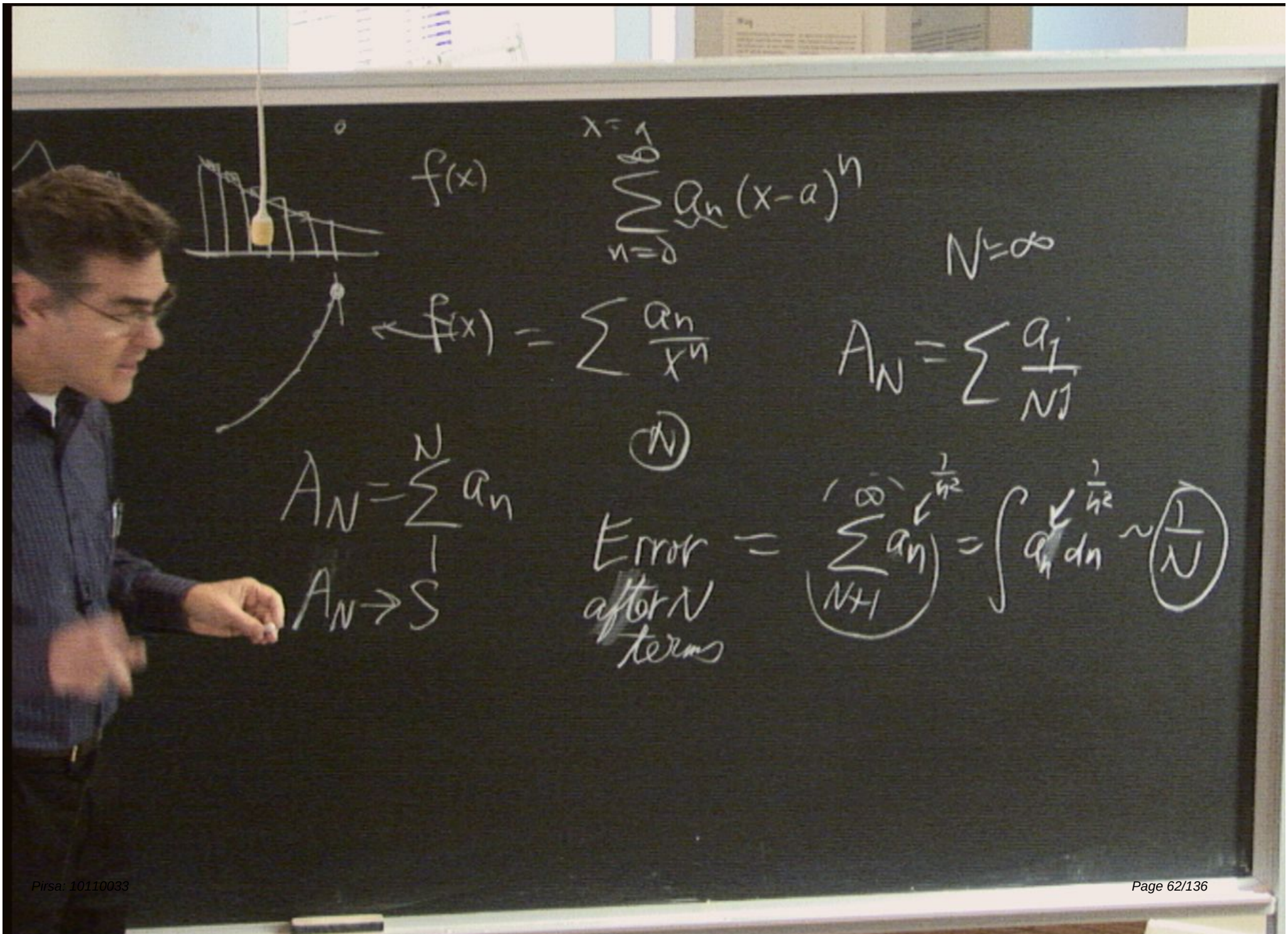
$A_N = \sum_1^N a_n$
$A_N \to S$
Error after N terms
$= \sum_{N+1}^{\infty} a_n = \int a_n \, dn \sim \frac{1}{N}$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$\sum_{n=0}^{\infty} a_n (x-a)^n$
$N=\infty$
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Error after N terms
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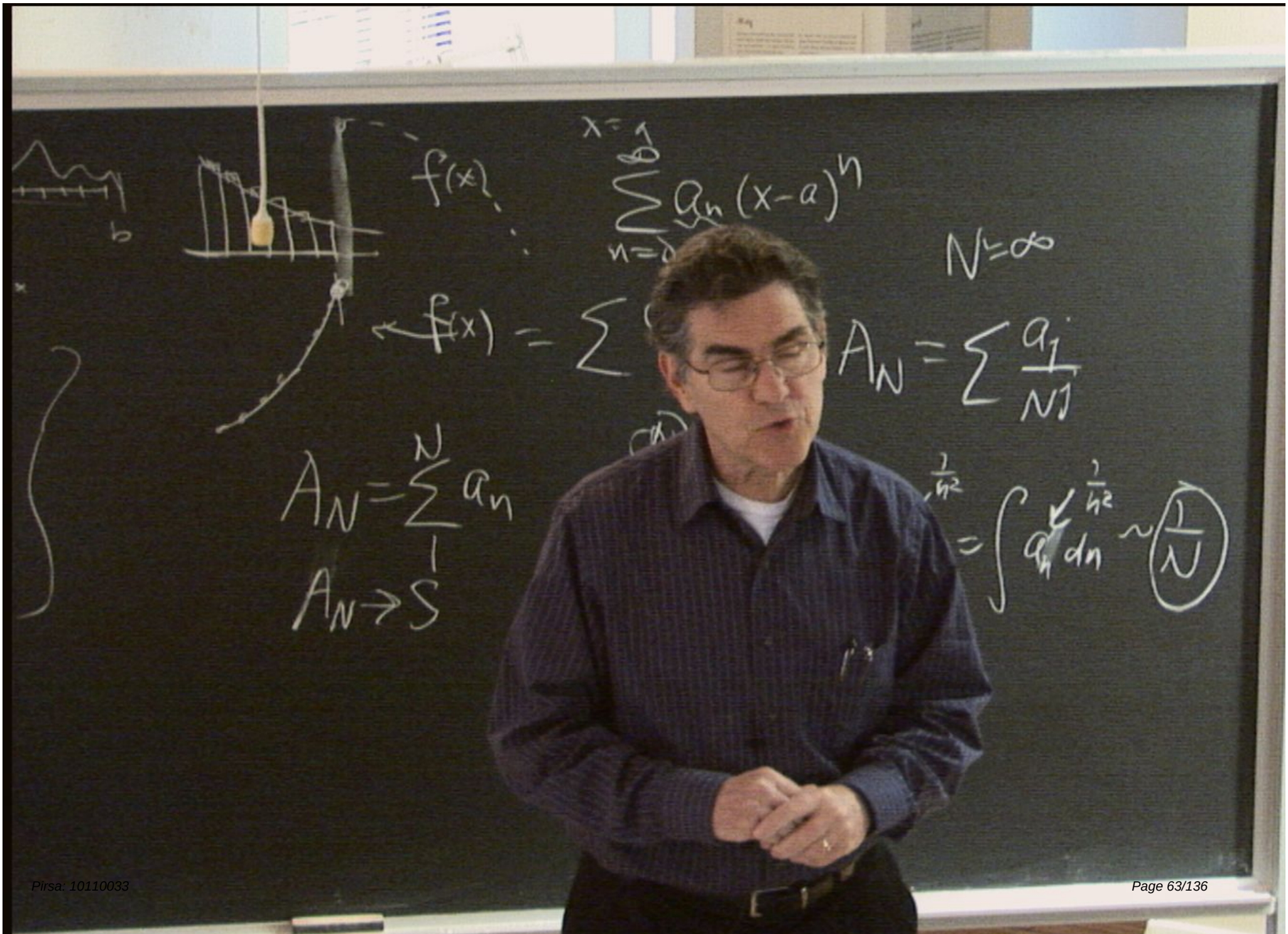
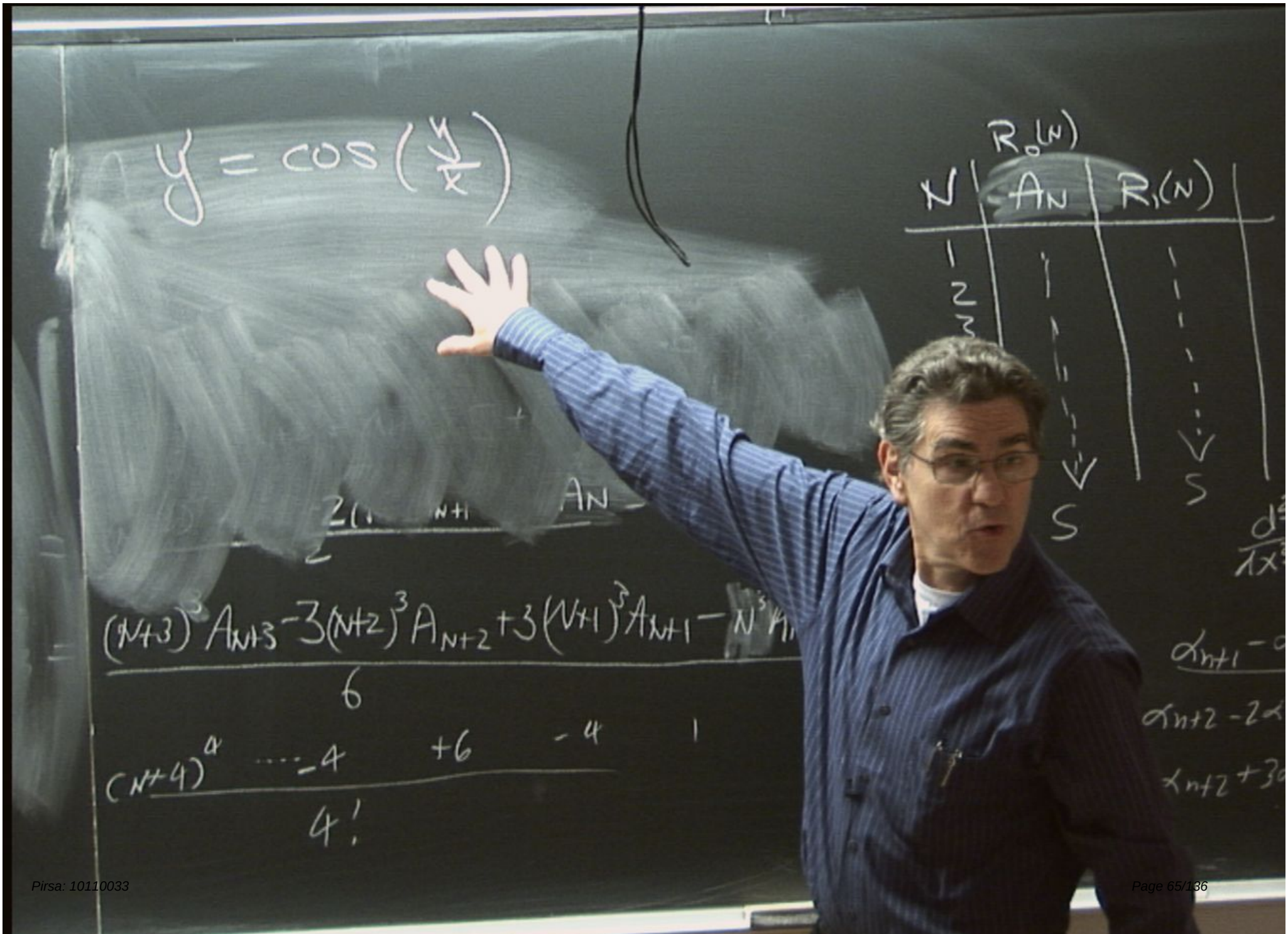
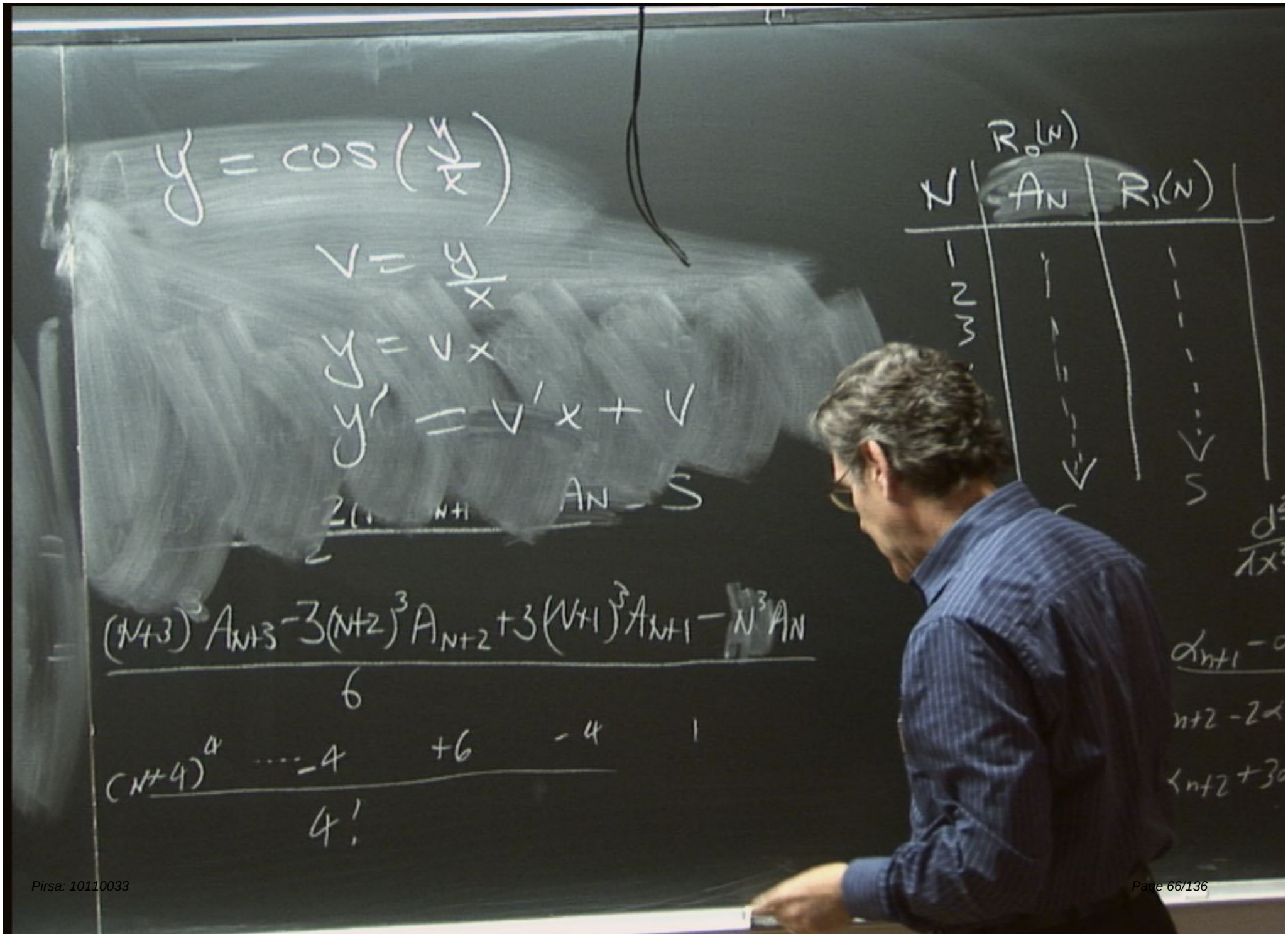

f(x)
$\sum_{n=0}^{\infty} a_n (x-a)^n$
$N=\infty$
$f(x) = \sum$
$A_N = \sum \frac{a_j}{N^j}$
$A_N = \sum_1^N a_n$
$A_N \to S$
$= \int a_n^{\frac{1}{n^2}} dn \sim \frac{1}{N}$

$$y'(x) = \cos(xy)$$


y = cos(y/x)
R₀(N)
N
A_N
R₁(N)
(N+3)³ A_{N+3} − 3(N+2)³ A_{N+2} + 3(N+1)³ A_{N+1} − N³ A
6
(N+4)⁴ ... −4 +6 −4 1
4!


y = cos(y/x)
V = y/x
y = Vx
y' = V'x + V
R₀(N)
N
AN
R₁(N)
(N+3)³ AN+3 − 3(N+2)³ AN+2 + 3(N+1)³ AN+1 − N³ AN
6
(N+4)⁴ ... −4 +6 −4 1
4!

$$y = \cos\left(\frac{y}{x}\right)$$

$$v = \frac{y}{x}$$

$$y = vx$$

$$y' = v'x + v$$

$$v'x + v = \cos v$$

N	$R_0(N)$ A_N	$R_1(N)$
1		
2		
3		
⋮		
	v	
	S	

$$y = \cos\left(\frac{y}{x}\right)$$

$$v = \frac{y}{x}$$

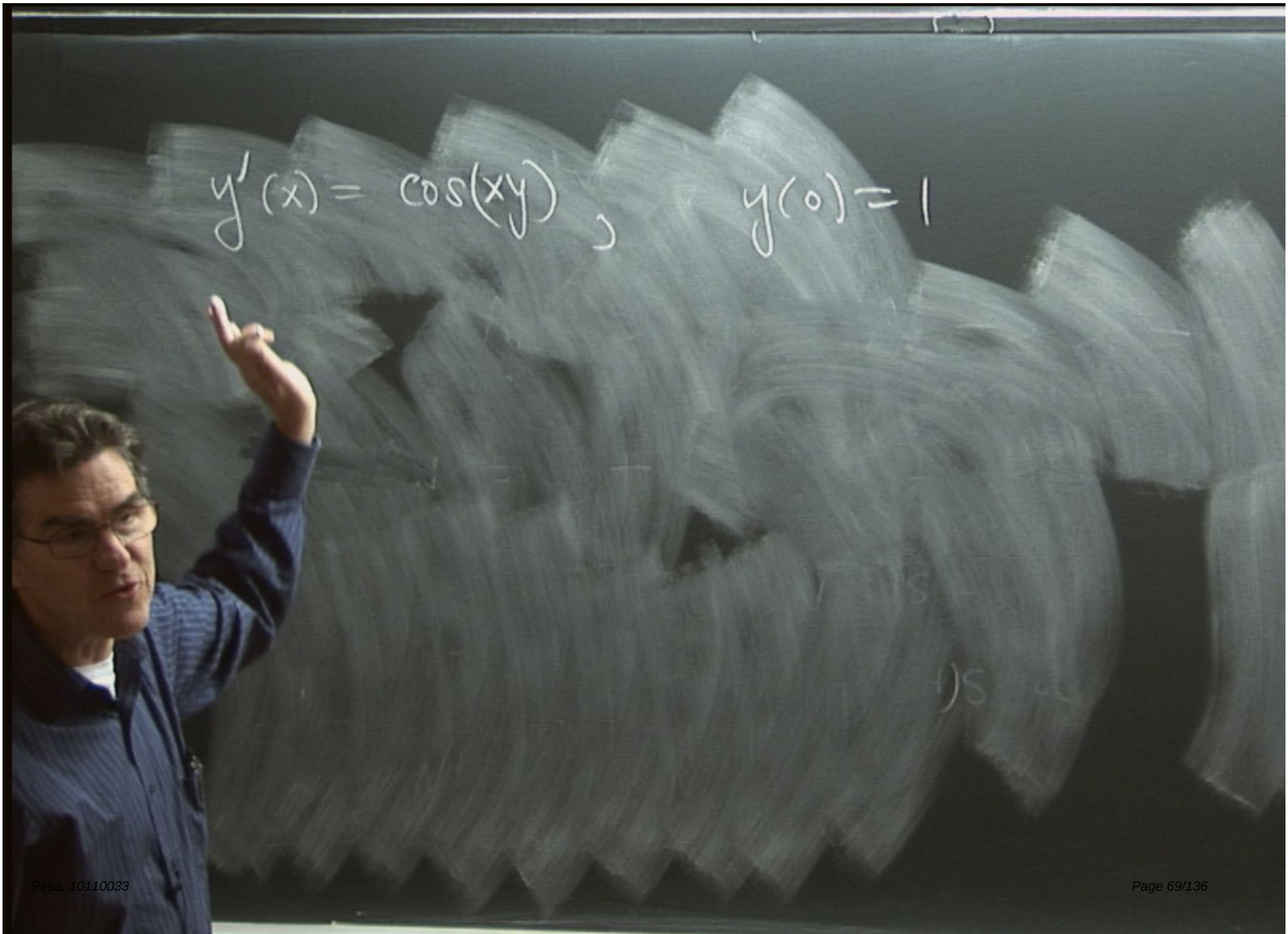
$$y = vx$$

$$y' = v'x + v$$

$$v'x \quad = \cos v \quad - v$$

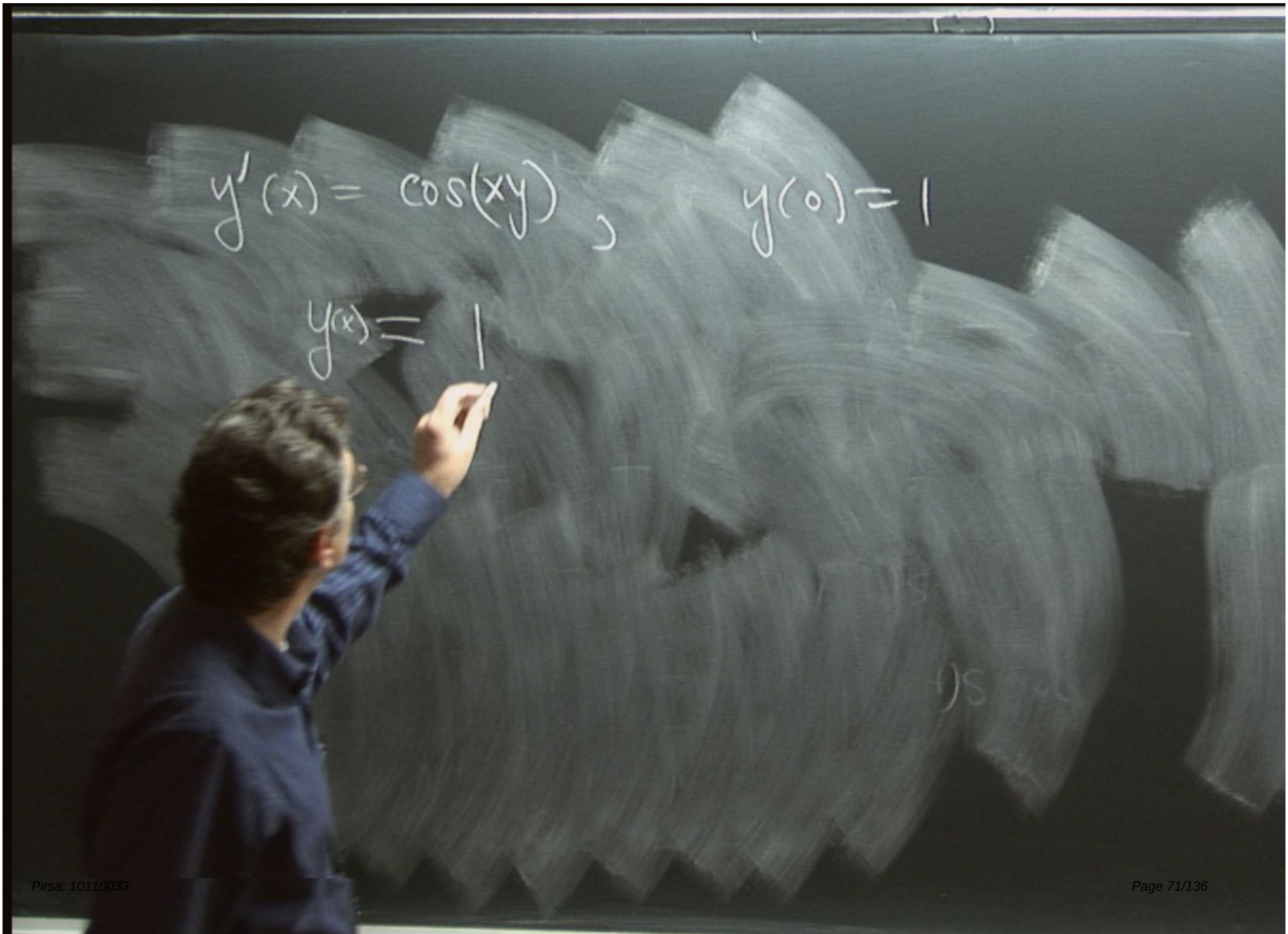
N	$R_0(N)$ A_N	$R_1(N)$
1		
2		
3		
⋮	⋮	⋮
	V	
	S	

$\alpha_{n+3} - 2$


y'(x) = cos(xy) , y(0) = 1

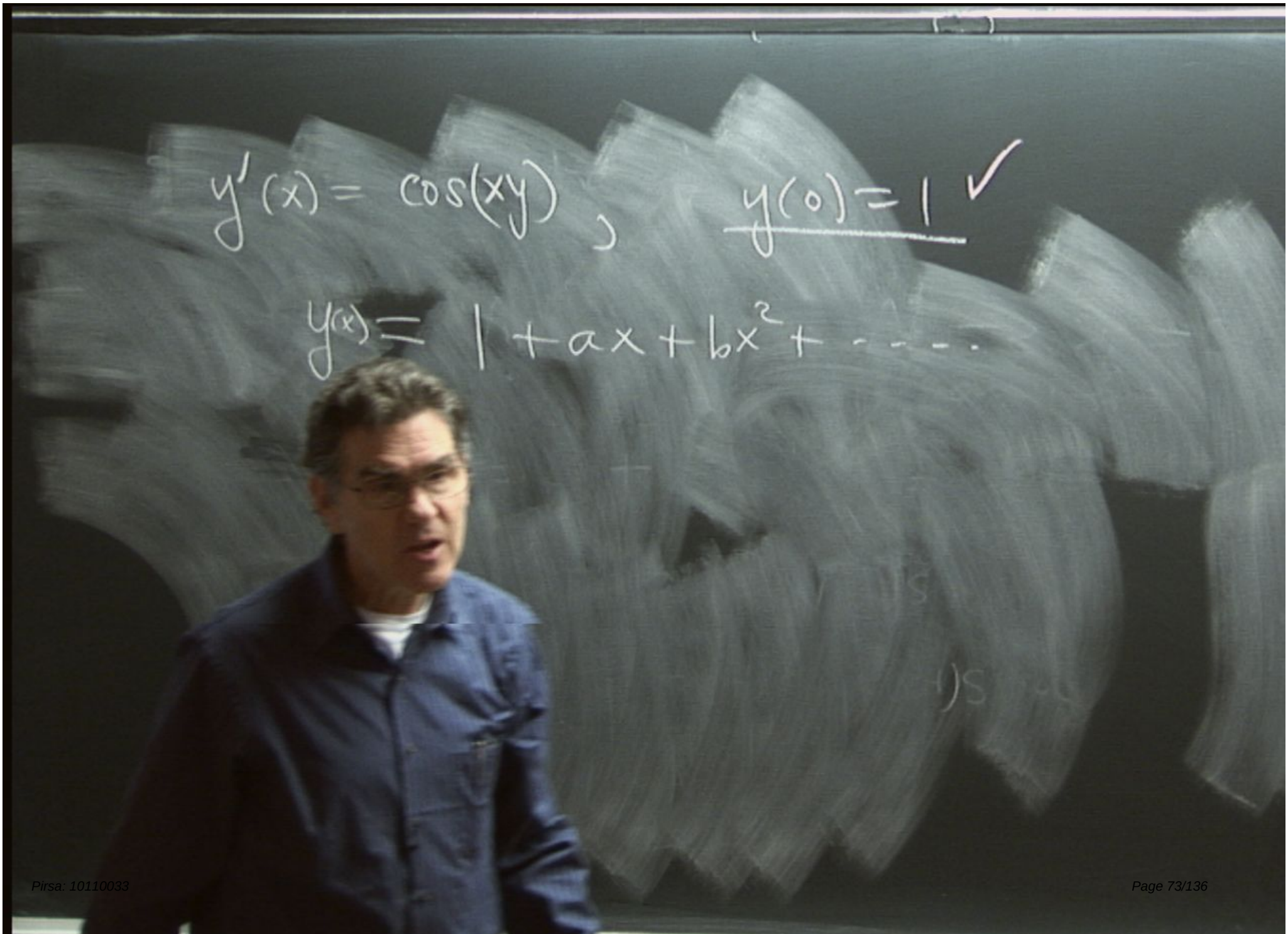
$$y'(x) = \cos(xy), \quad y(0) = 1$$

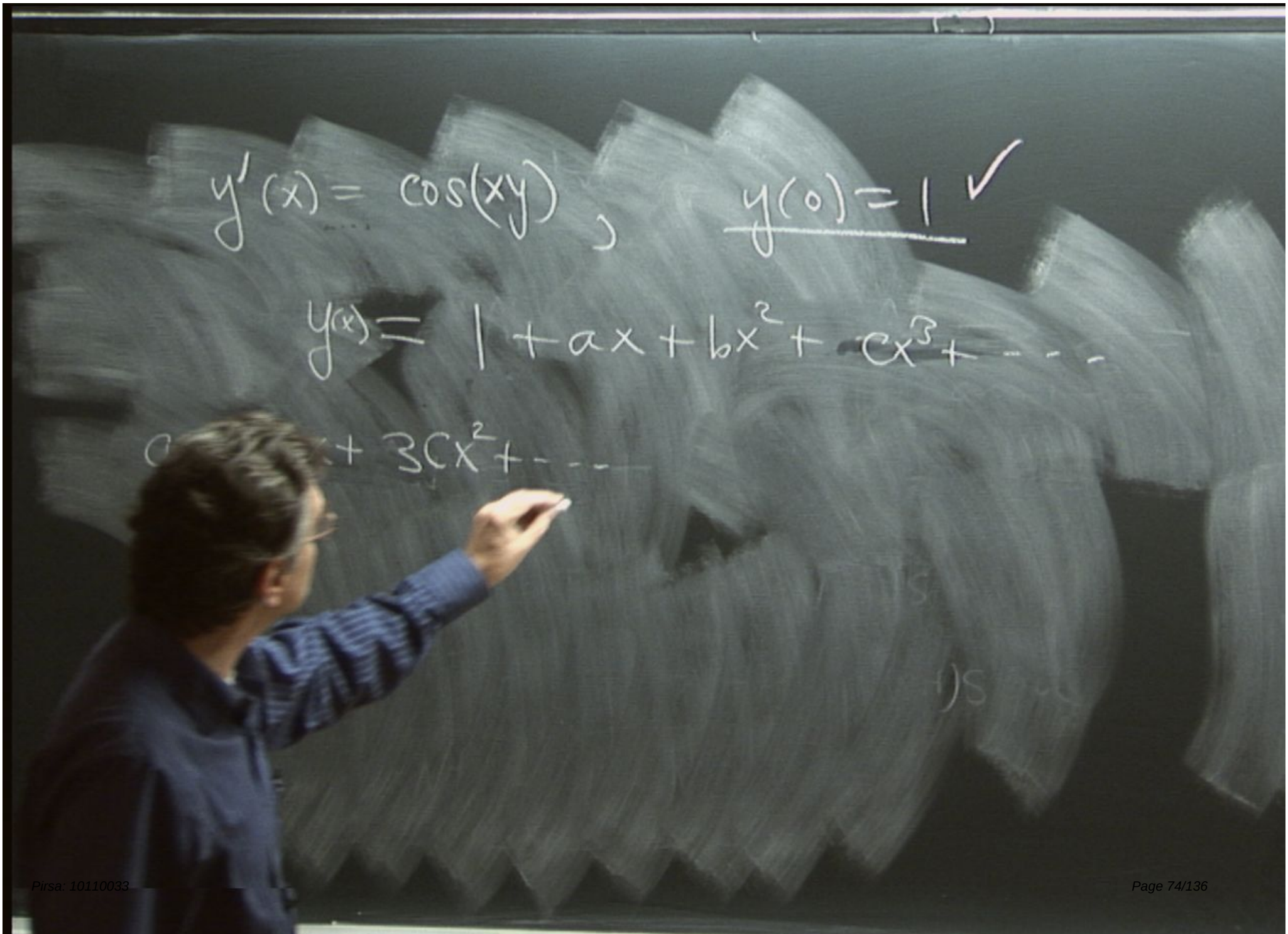
$$y =$$


y'(x) = cos(xy) , y(0) = 1
y(x) = 1

$$y'(x) = \cos(xy), \quad y(0) = 1$$

$$y(x) = 1 + ax + bx^2 + \cdots$$


y'(x) = cos(xy) , y(0) = 1
y(x) = 1 + ax + bx^2 + ...


y'(x) = cos(xy) , y(0) = 1
y(x) = 1 + ax + bx^2 + cx^3 + ...
+ 3cx^2 + ...

$$y'(x) = \cos(xy), \quad \underline{y(0) = 1} \checkmark$$

$$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$$

$$a + 2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$$

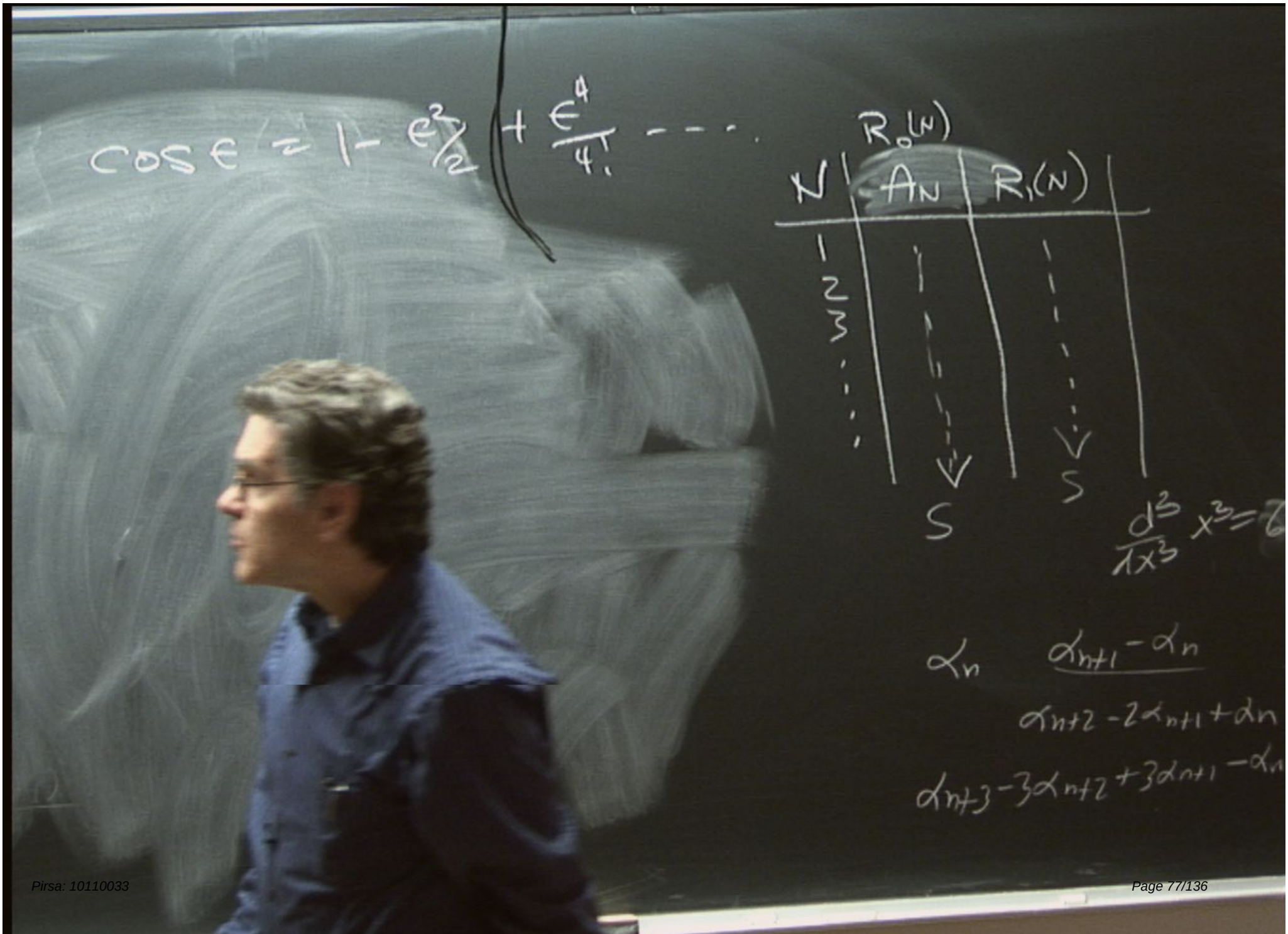
$x \ll 1$

$$y'(x) = \cos(xy), \quad y(0) = 1 \checkmark$$

$$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$$

$$a + 2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$$

$$=$$


cos ε = 1 − ε²/2 + ε⁴/4! − − − −
R₀(N)
N
A_N
R₁(N)
1
2
3
S
S
d³/dx³ x³ = 6
α_n
α_{n+1} − α_n
α_{n+2} − 2α_{n+1} + α_n
α_{n+3} − 3α_{n+2} + 3α_{n+1} − α_n

$x \ll 1$

$y'(x) = \cos(xy)$, $y(0) = 1$ ✓

$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$

$a + 2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$

$= 1 - \frac{1}{2}x^2 + \cdots$

$x \ll 1$

$y'(x) = \cos(xy)$, $y(0) = 1$ ✓

$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$

$a + 2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$

$a = 1$ $\qquad = 1 - \frac{1}{2}x^2 + \cdots$

$x \ll 1$

$y'(x) = \cos(xy)$, $y(0) = 1$ ✓

$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$

$a + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$

$= 1 - \frac{1}{2}x^2 + \cdots$

$= 1$

$b = 0$

$3c = -\frac{1}{2}$

$x \ll 1$

$y'(x) = \cos(xy)$, $y(0) = 1$ ✓

$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$

$2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$

$= 1 - \frac{1}{2}x^2 + \cdots$

$a = 1$

$b = 0$

$c = -\frac{1}{2}$ $c = -\frac{1}{6}$

$x \ll 1$

$y'(x) = \cos(xy)$, $y(0) = 1$ ✓

$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$

$a + 2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$

$= 1 - \frac{1}{2}x^2 + \cdots$

$a = 1$

$b = 0$

$3c = -\frac{1}{2}$ $c = -\frac{1}{6}$

$x \ll 1$

$y'(x) = \cos(xy)$, $y(0) = 1$ ✓

$y(x) = 1 + ax + bx^2 + cx^3 + \cdots$

$a + 2bx + 3cx^2 + \cdots = \cos(x + ax^2 + \cdots)$

$= 1 - \frac{1}{2}x^2 + \cdots$

$a = 1$

$b = 0$

$3c = -\frac{1}{2}$ $c = -\frac{1}{6}$

$y(x) = 1 + x - \frac{1}{6}x^3 \cdots$

$$u_t + c u_x = 0$$

$$u_t + c u_x = 0 \qquad u_t = \frac{\partial u}{\partial t}, \quad u_x = \frac{\partial u}{\partial x}$$

$$u(x,t) = f(x - ct)$$

arb.

$$u_t + c u_x = 0 \qquad u_t = \frac{\partial u}{\partial t}, \quad u_x = \frac{\partial u}{\partial x}$$

$$u(x,t) = f(x - ct)$$

arb.

Rt going wave of unch. shape, speed c.

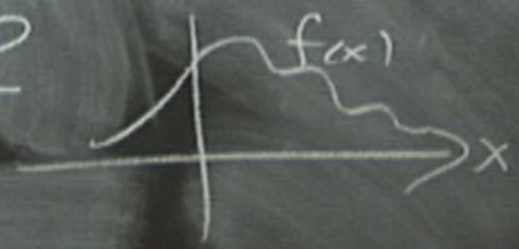
$\underline{t=0}$

$u_t + c u_x = 0$ $\qquad$ $u_t = \frac{\partial u}{\partial t}$, $u_x = \frac{\partial u}{\partial x}$

$u(x,t) = f(x - ct)$, Rt going wave of unch. shape, speed c.

arb.

$\underline{t=0}$



$\underline{t=1}$

$$u_t + c u_x = 0 \qquad u_t = \frac{\partial u}{\partial t}, \quad u_x = \frac{\partial u}{\partial x}$$

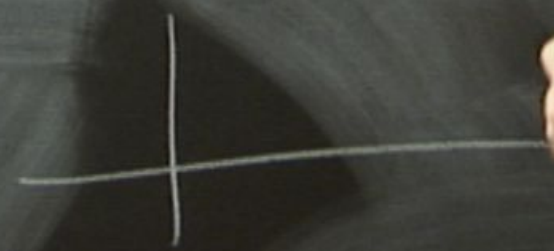
$$u(x,t) = f(x - ct)$$

arb.

Rt going wave of unch. shape, speed c.

$\underline{t=0}$

$\underline{t=1}$



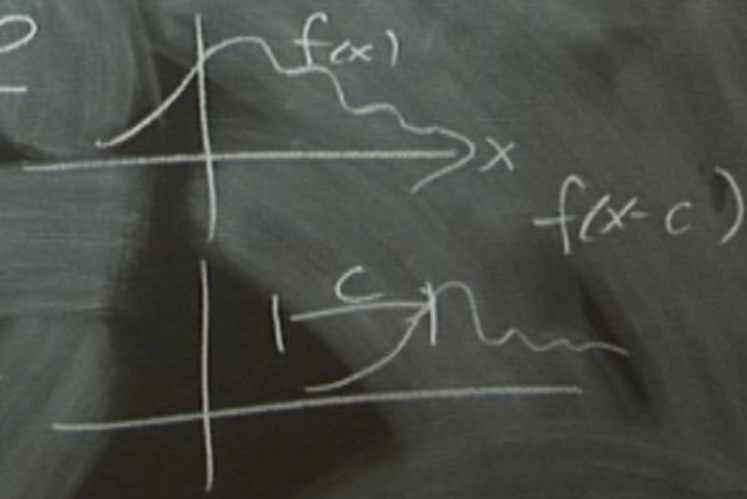
$$u_t + c u_x = 0 \qquad u_t = \frac{\partial u}{\partial t}, \quad u_x = \frac{\partial u}{\partial x}$$

$$u(x,t) = f(x - ct)$$

arb.

Rt going wave of unch. shape, speed c.

$\underline{t=0}$



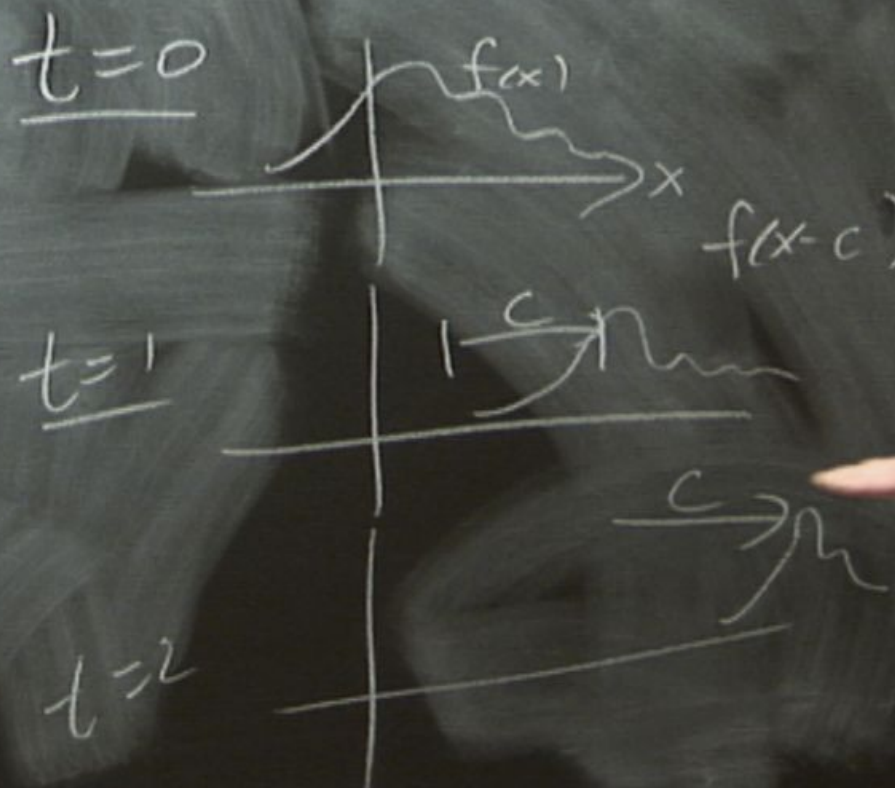
$\underline{t=1}$

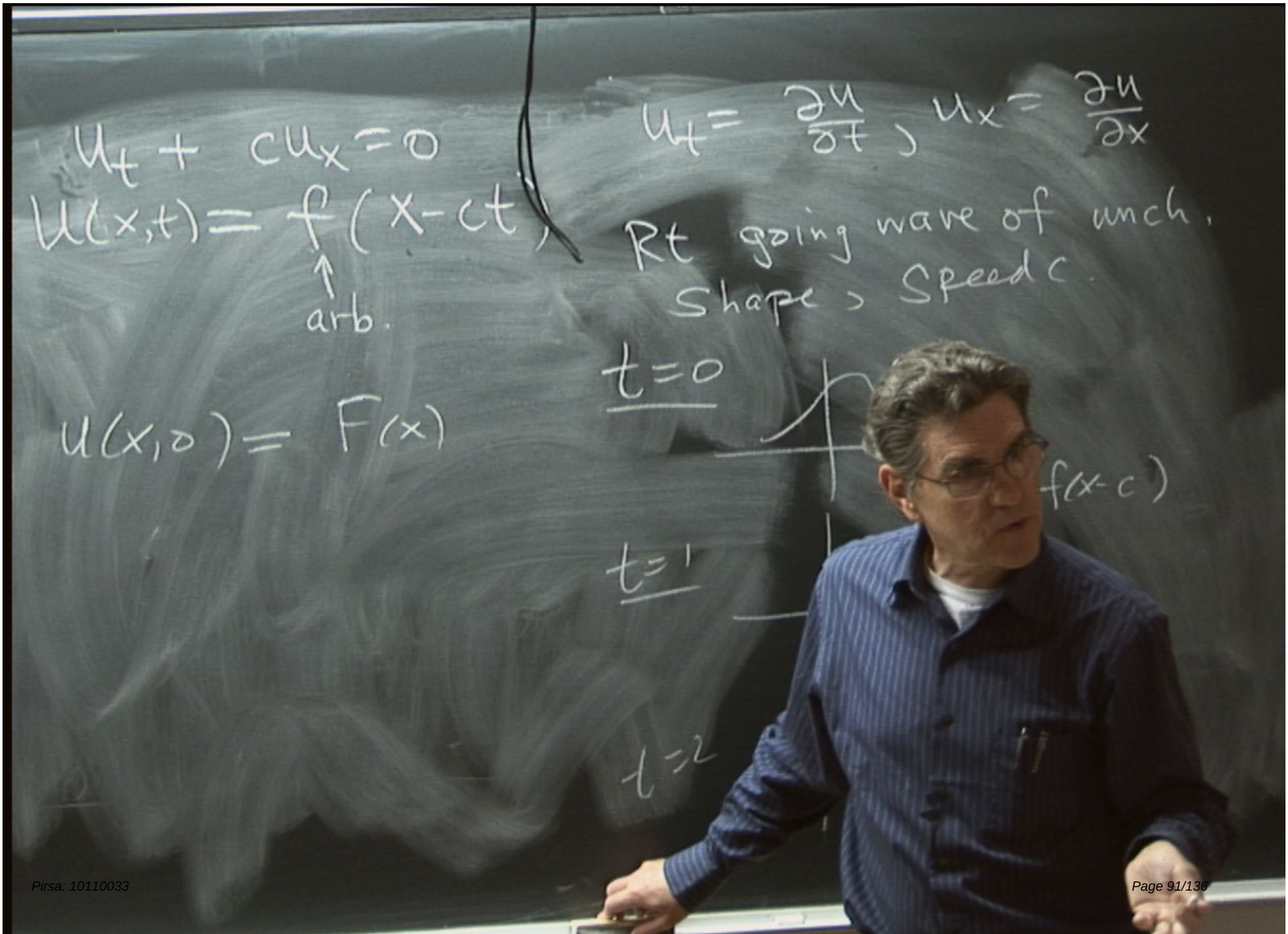
$$u_t + c u_x = 0 \qquad u_t = \frac{\partial u}{\partial t}, \quad u_x = \frac{\partial u}{\partial x}$$

$$u(x,t) = f(x - ct)$$

arb.

Rt going wave of unch. shape, speed c.




$u_t + c u_x = 0$
$u_t = \frac{\partial u}{\partial t}$, $u_x = \frac{\partial u}{\partial x}$
$u(x,t) = f(x-ct)$
arb.
Rt going wave of unch. Shape, Speed c.
$u(x,0) = F(x)$
t=0
f(x-c)
t=1
t=2

$$u_t + c u_x = 0 \qquad u_t = \frac{\partial u}{\partial t}, \quad u_x = \frac{\partial u}{\partial x}$$

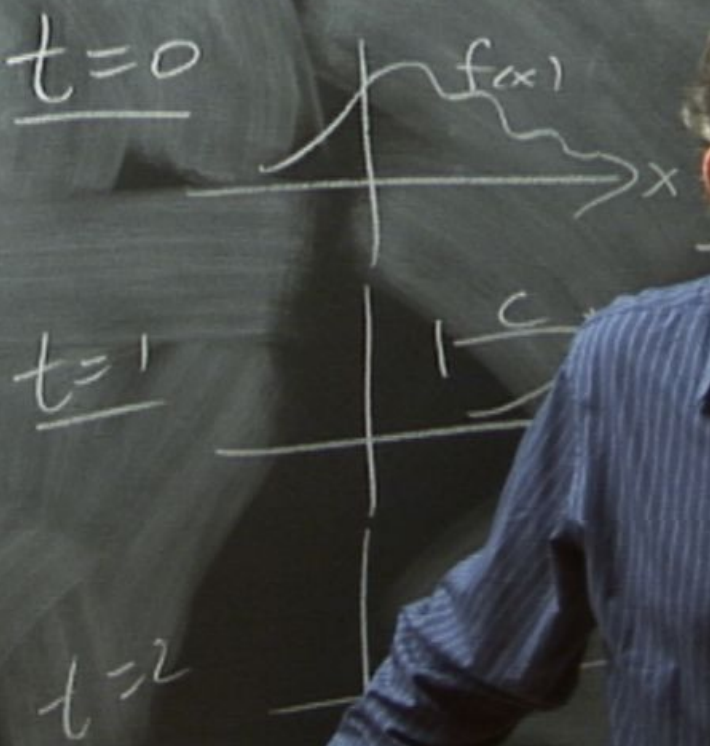
$$u(x,t) = f(x - ct)$$

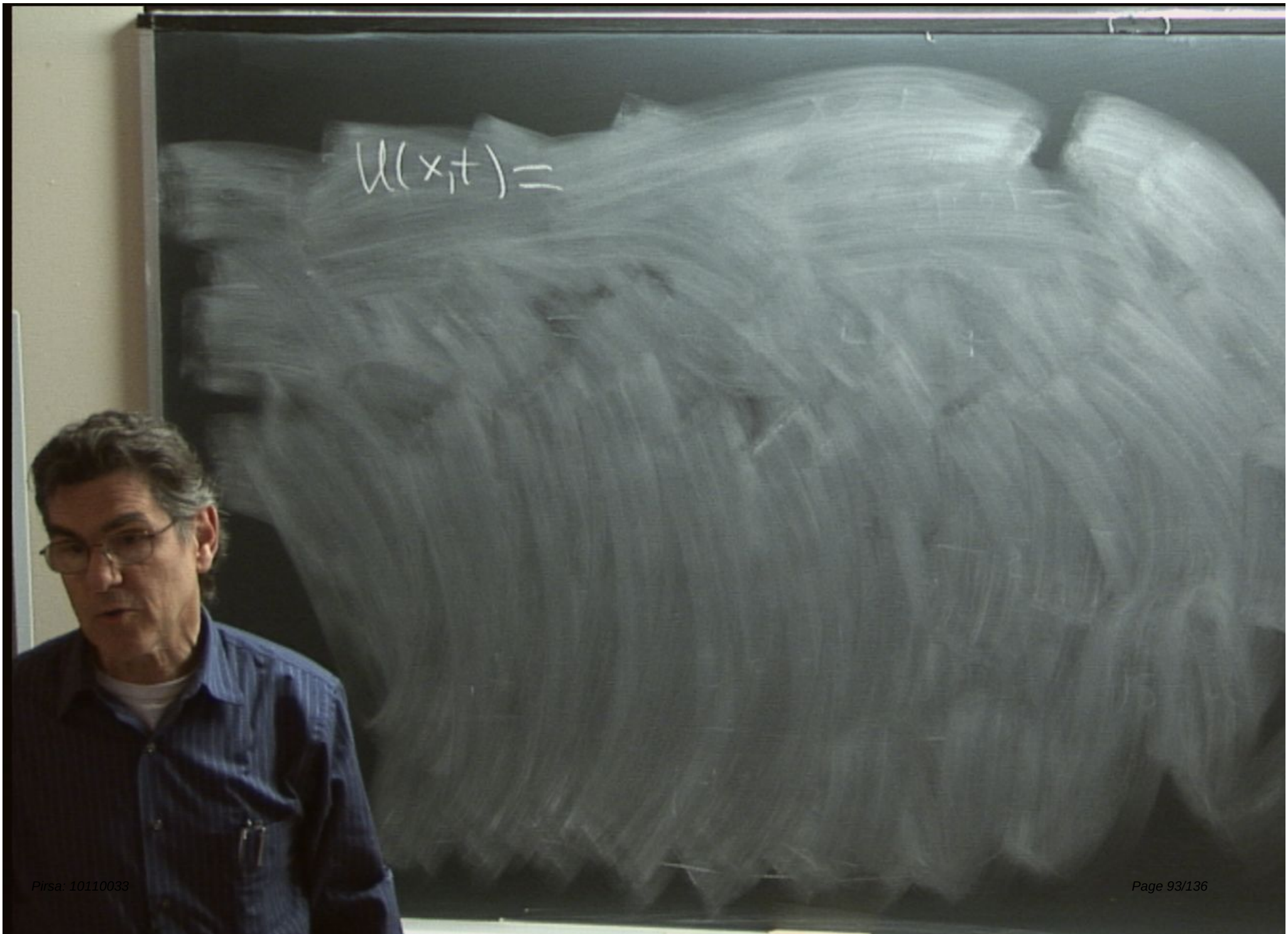
arb.

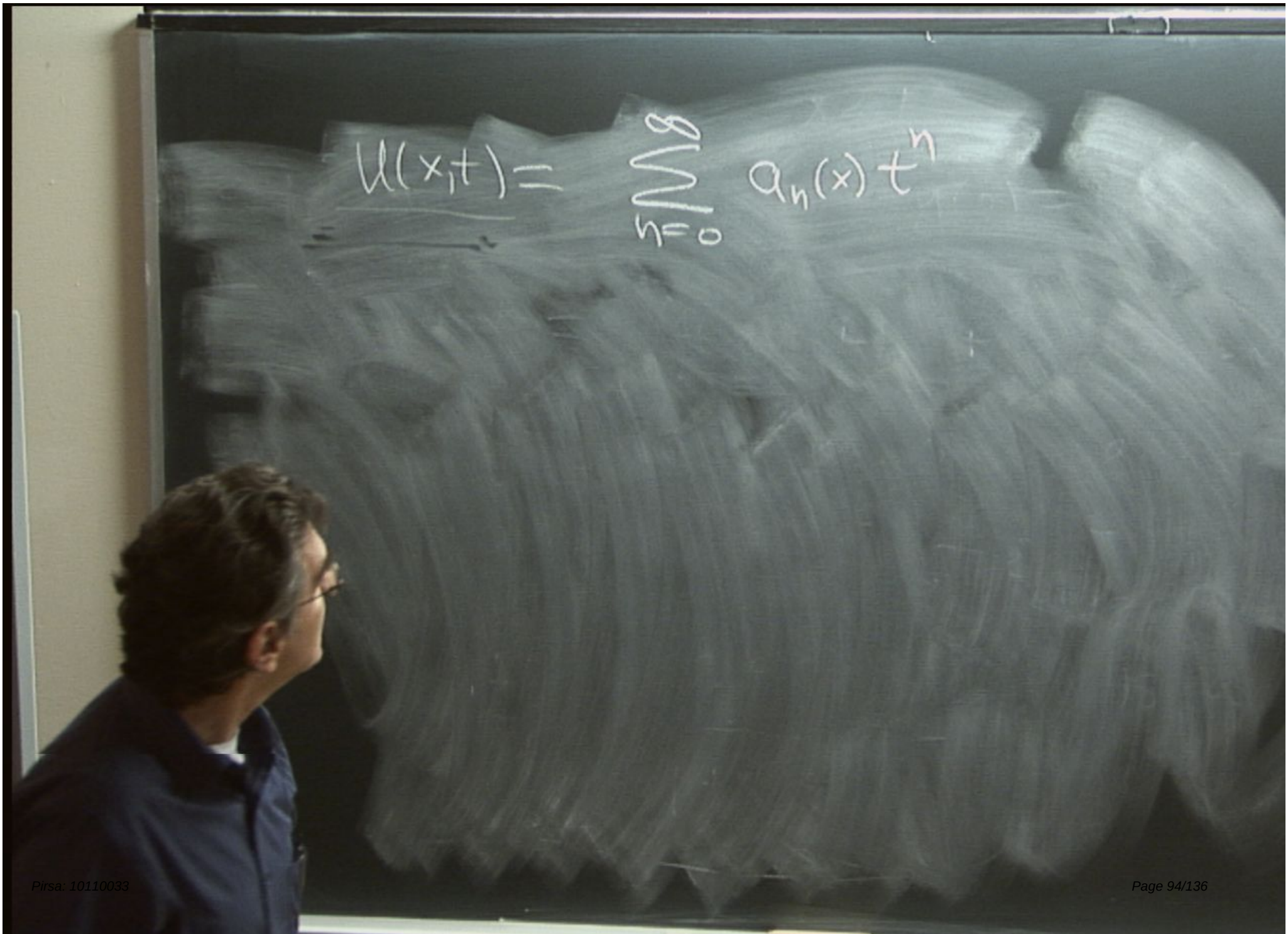
Rt going wave of unch. shape, speed c.

$$u(x,0) = F(x)$$

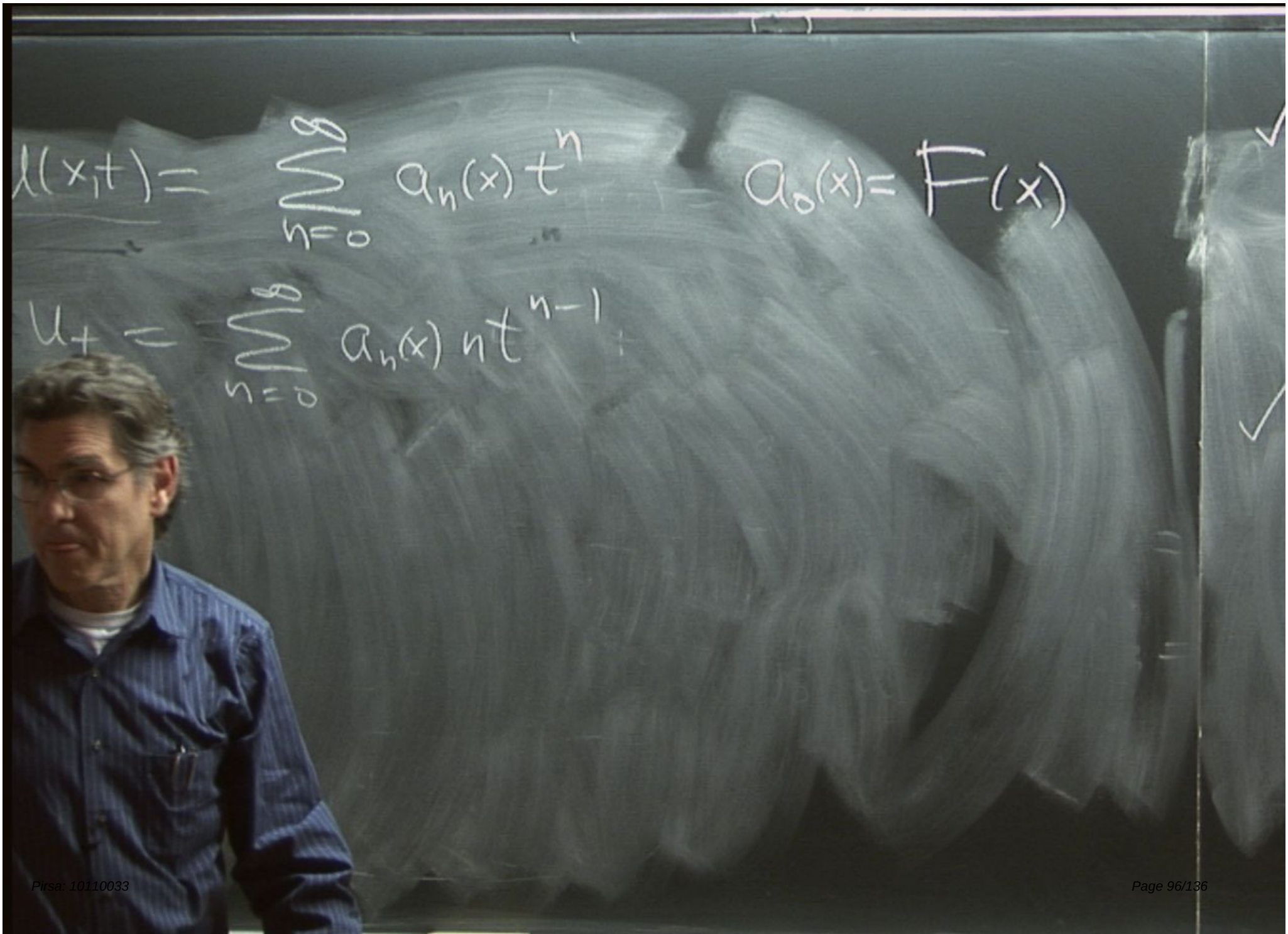
$$u(x,0) = F(x - ct)$$

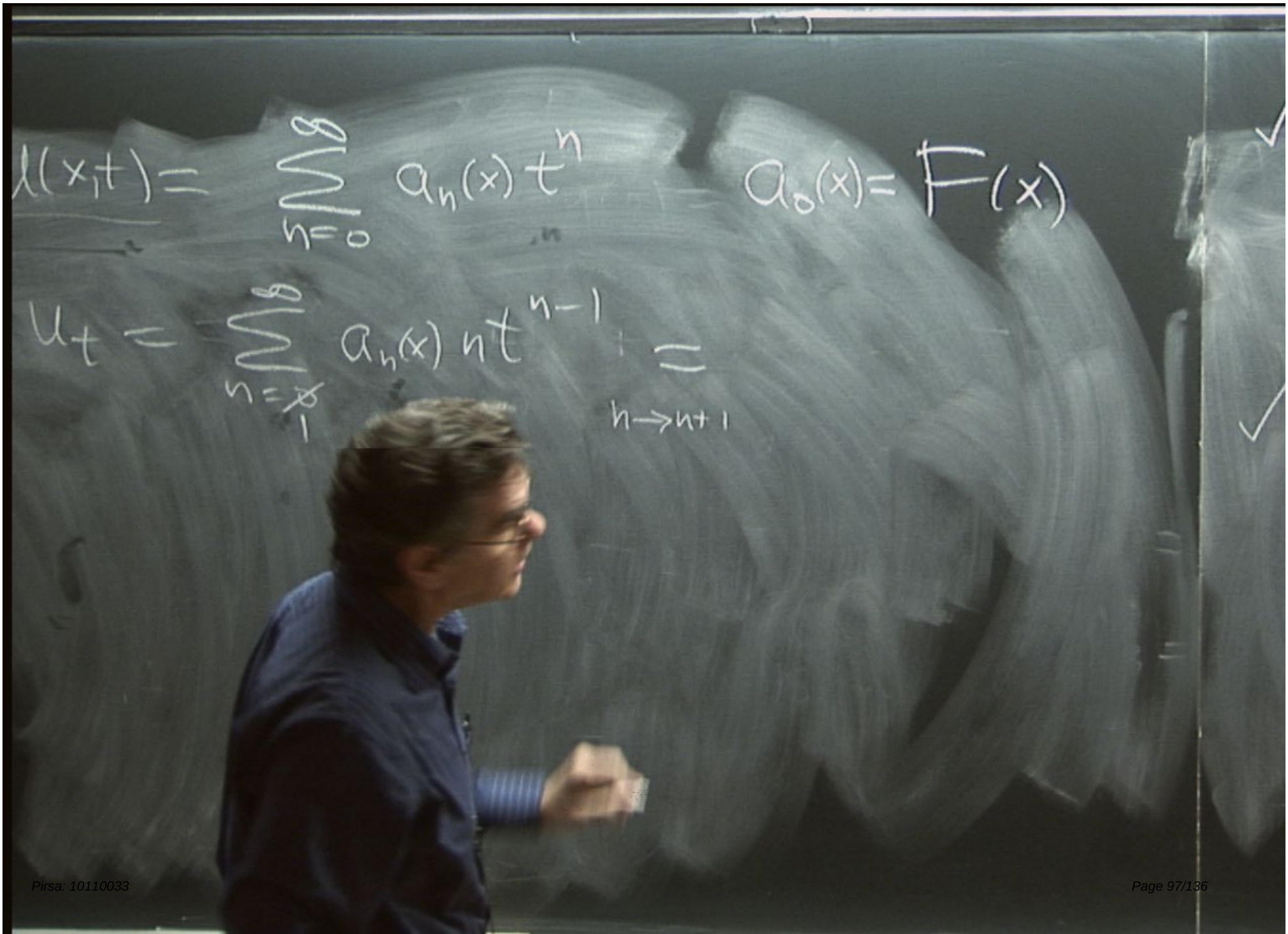



U(x,t) =


$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n \qquad a_0(x) = F(x)$$


$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n$
$a_0(x) = F(x)$
$u_t = \sum_{n=0}^{\infty} a_n(x)\, n t^{n-1}$


$u(x,t) = \sum_{n=0}^{\infty} a_n(x) t^n$
$a_0(x) = F(x)$
$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} =$
$n \to n+1$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n \qquad a_0(x) = F(x)$$

$$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} \underset{n \to n+1}{=} \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n \qquad a_0(x) = F(x)$$

$$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} \underset{n \to n+1}{=} \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$$c\, u_x = \sum_{n=0}^{\infty} c\, a_n'(x)\, t^n$$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n \qquad a_0(x) = F(x)$$

$$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} \underset{n \to n+1}{=} \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$$c\, u_x = \sum_{n=0}^{\infty} c\, a_n'(x)\, t^n$$

$$\sum_{n=0}^{\infty} t^n \left[c\, a_n'(x) + (n+1)\, a_{n+1}(x) \right] = 0$$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n \qquad a_0(x) = F(x)$$

$$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} \underset{n \to n+1}{=} \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$$c\, u_x = \sum_{n=0}^{\infty} c\, a_n'(x)\, t^n$$

$$\sum_{n=0}^{\infty} t^n \left[c\, a_n'(x) + (n+1)\, a_{n+1}(x) \right] = 0$$

$$a_{n+1}(x) = \frac{-c\, a_n'(x)}{n+1}$$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n \qquad a_0(x) = F(x)$$

$$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} \underset{n \to n+1}{=} \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$$c\, u_x = \sum_{n=0}^{\infty} c\, a_n'(x)\, t^n$$

$$\sum_{n=0}^{\infty} t^n \left[c\, a_n'(x) + (n+1)\, a_{n+1}(x) \right] = 0$$

$$a_{n+1}(x) = \frac{-c\, a_n'(x)}{n+1}$$

$$u_t + c u_x = 0$$

$$u(x,t) = f(x - ct)$$

arb.

$u_t = \frac{\partial u}{\partial t}$, $u_x =$

Rt going wave of shape, speed c

at $t=0$

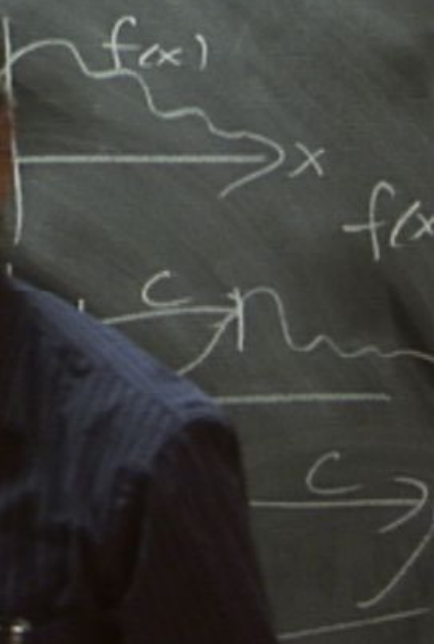
$$u(x,0) = F(x)$$

$$u(x,0) = F(x-ct)$$

$$a_1 = -c F'(x)$$

$$a_2 = \frac{c^2 F''(x)}{2}$$

$$a_3 = -\frac{c^3 F'''(x)}{3!}$$



$$u_t + c u_x = 0$$

$$u(x,t) = f(x - ct)$$

arb.

at $t=0$

$$u(x,0) = F(x)$$

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$$a_1 = -c F'(x)$$

$$a_2 = \frac{c^2 F''(x)}{2}$$

$$a_3 = -\frac{c^3 F'''(x)}{3!}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{1}{n!} (-1)^n$$

$= 0$

$x - ct)$

(x)

$(x - ct)$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{c^n}{n!} t^n F^{(n)}(x)$$

$= 0$

$x - ct)$

(x)

$(x - ct)$

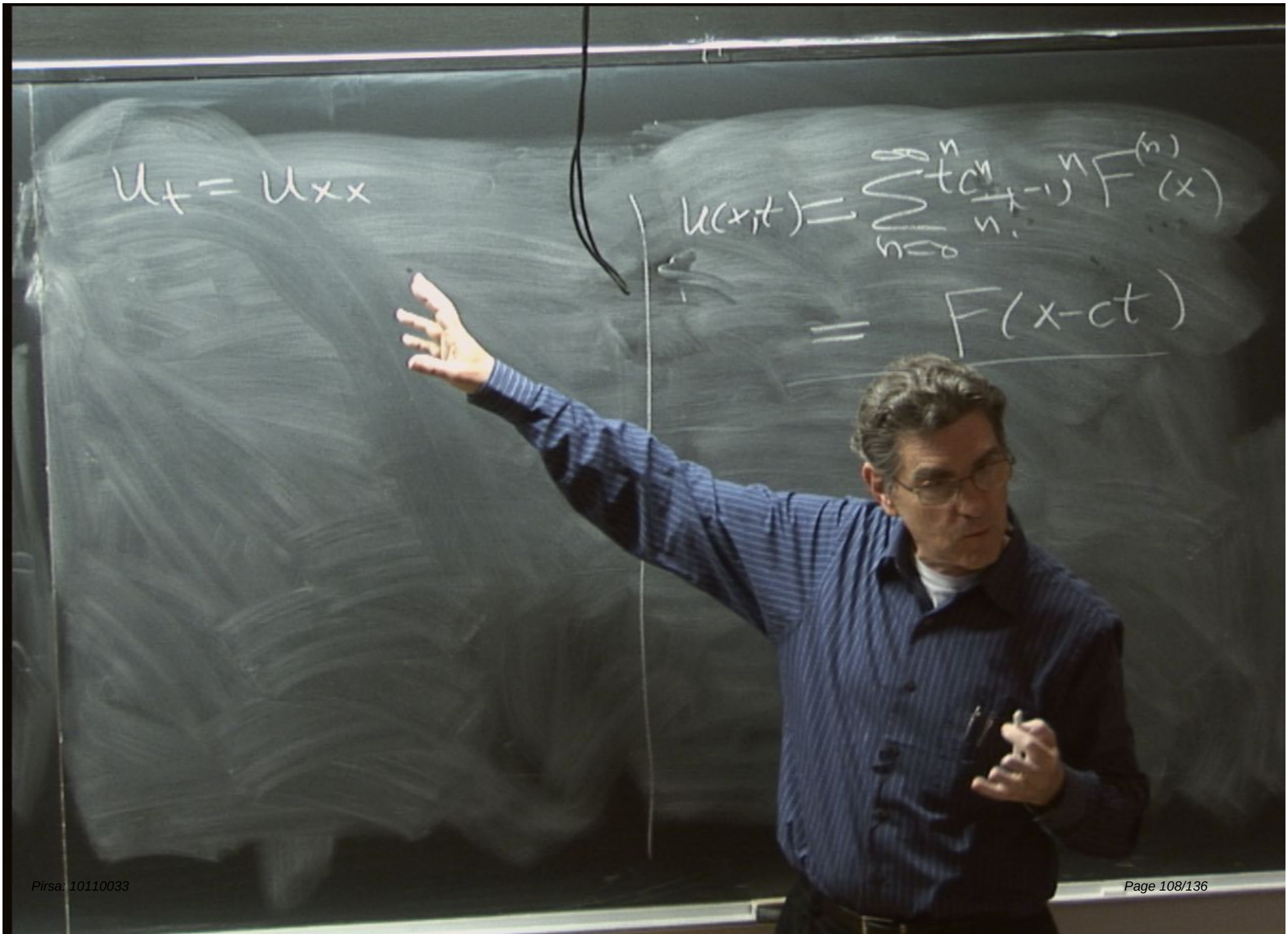
$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n c^n (-1)^n}{n!} F^{(n)}(x)$$

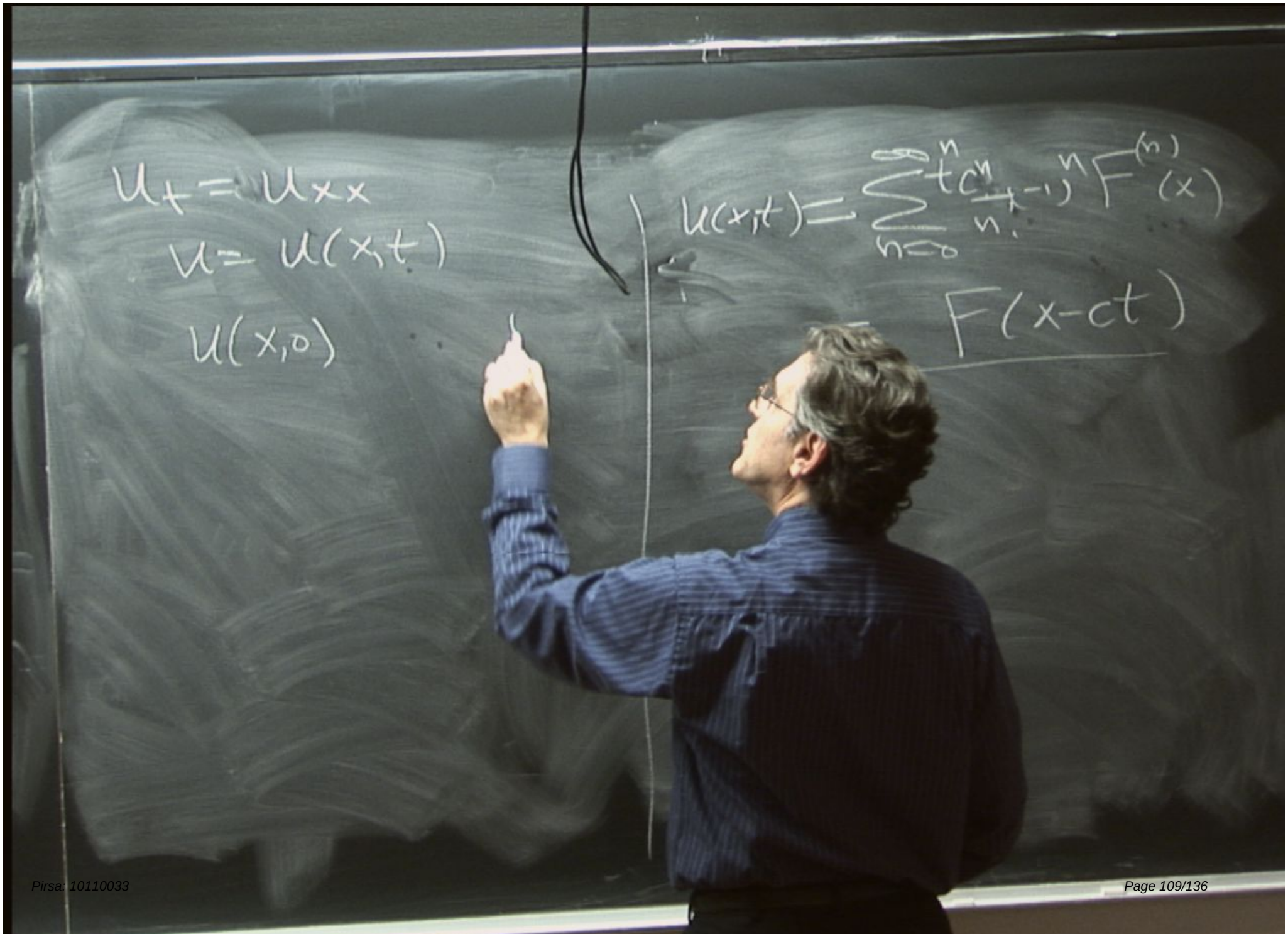
$$= F(x - ct)$$

$$= u_{xx}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n c^n (-1)^n}{n!} F^{(n)}(x)$$

$$= F(x-ct)$$


$u_t = u_{xx}$
$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n c^n}{n!} (-1)^n F^{(n)}(x)$
$= F(x-ct)$


$u_t = u_{xx}$
$u = u(x,t)$
$u(x,0)$
$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n c^n}{n!} (-1)^n F^{(n)}(x)$
$= F(x-ct)$

$$u_t = u_{xx}$$

$$u = u(x,t)$$

$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n c^n}{n!} (-1)^n F^{(n)}(x)$$

$$= F(x-ct)$$

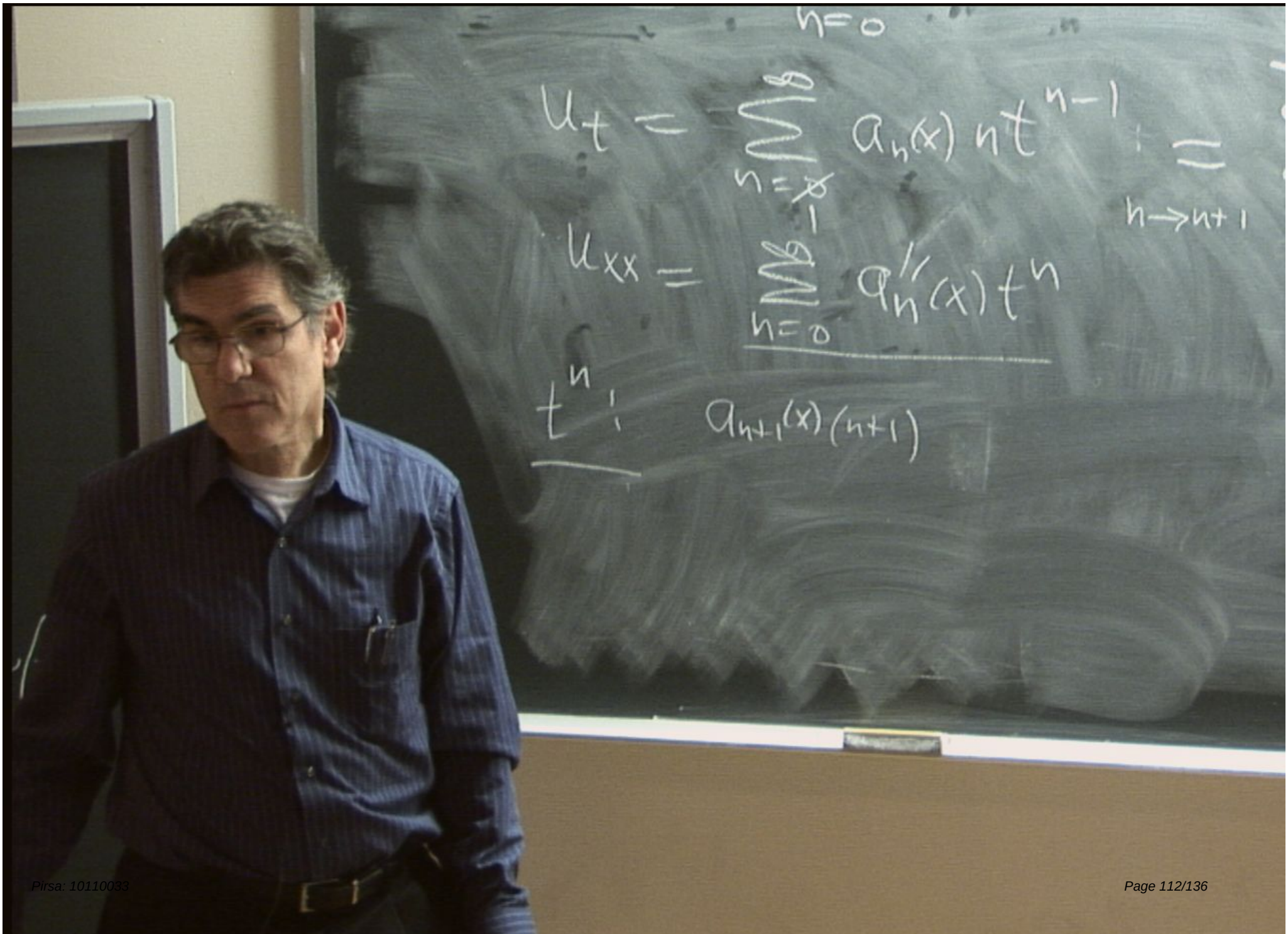
$$u_t = u_{xx}$$

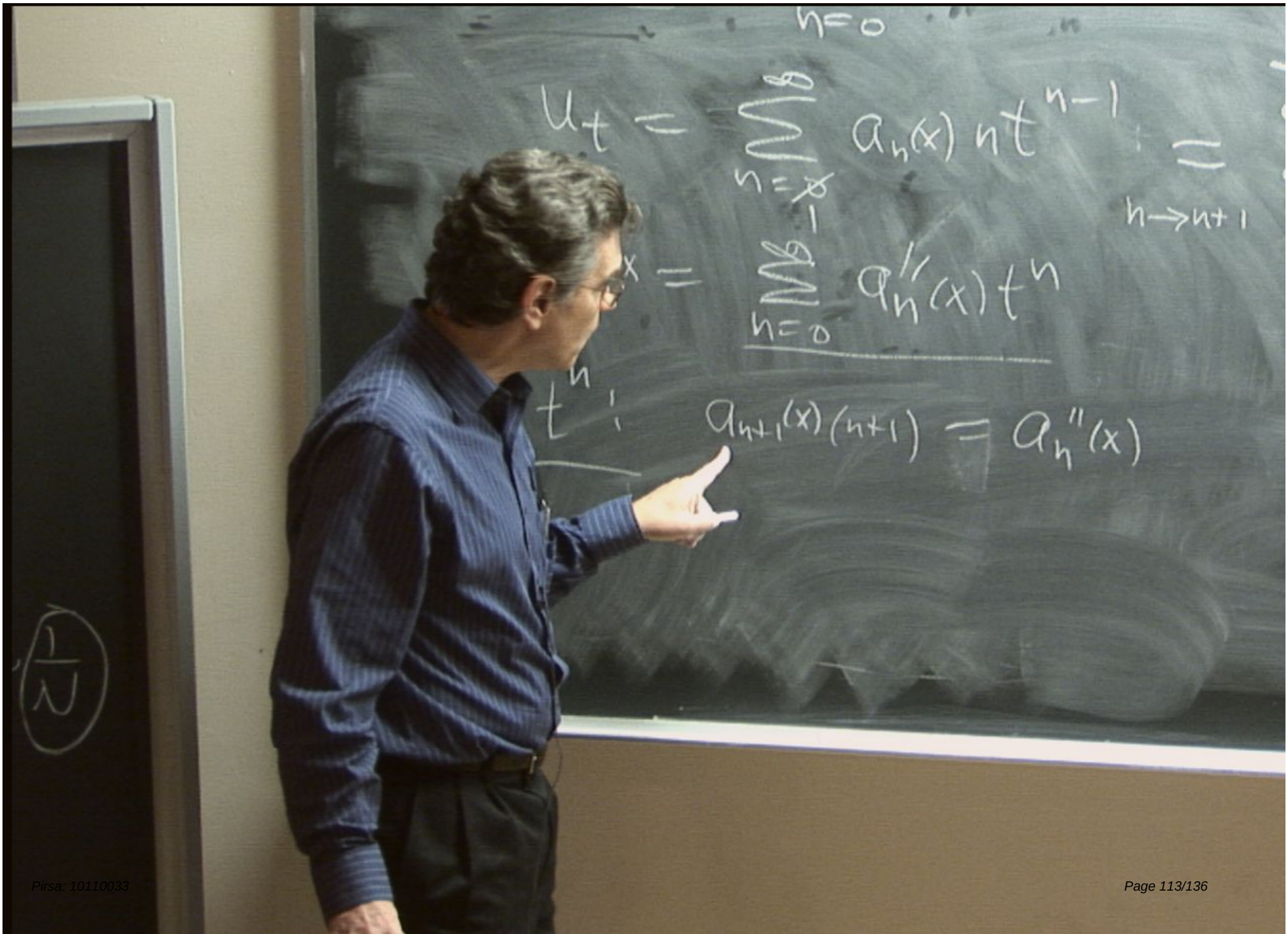
$$u = u(x,t)$$

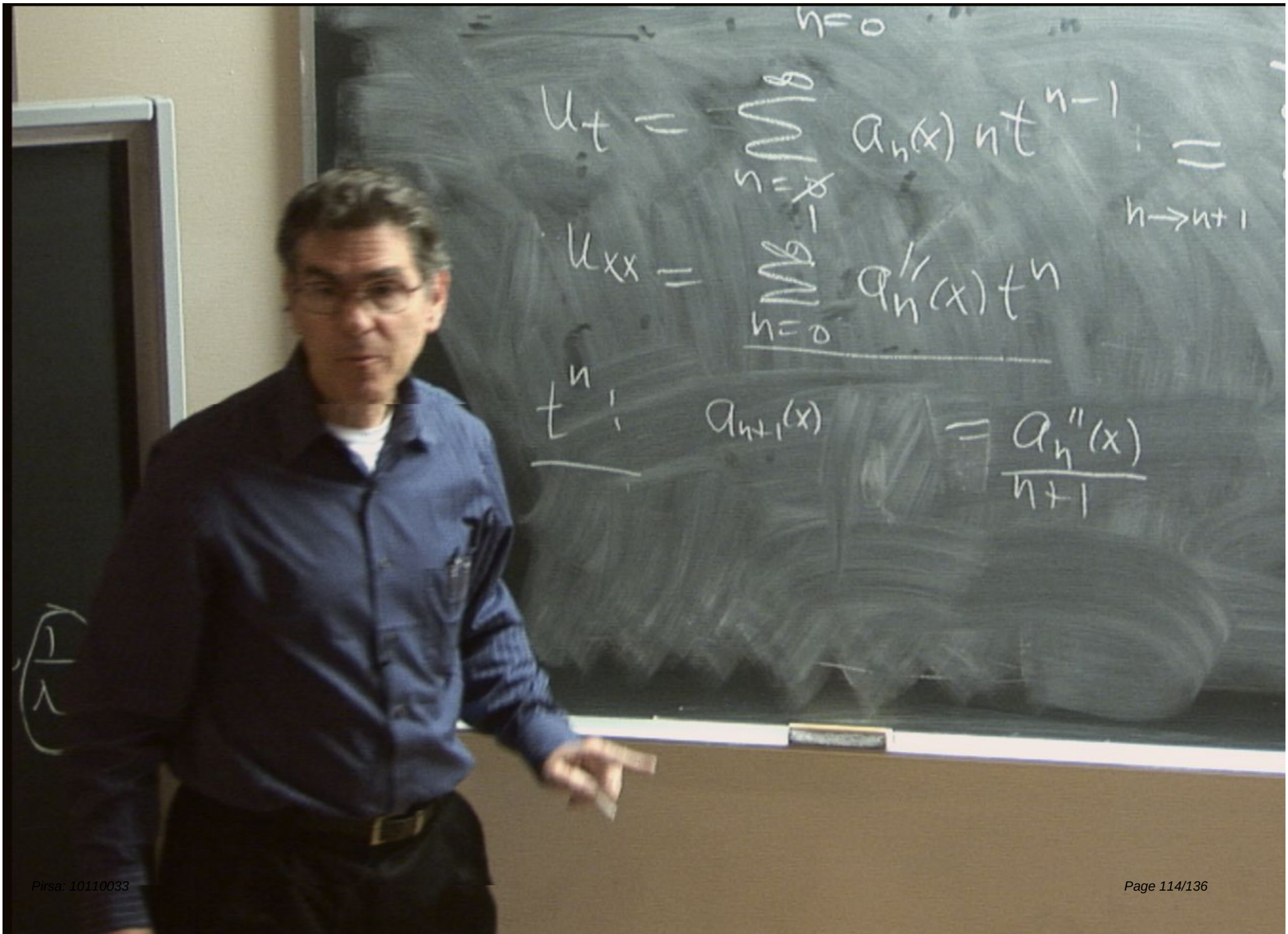
$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

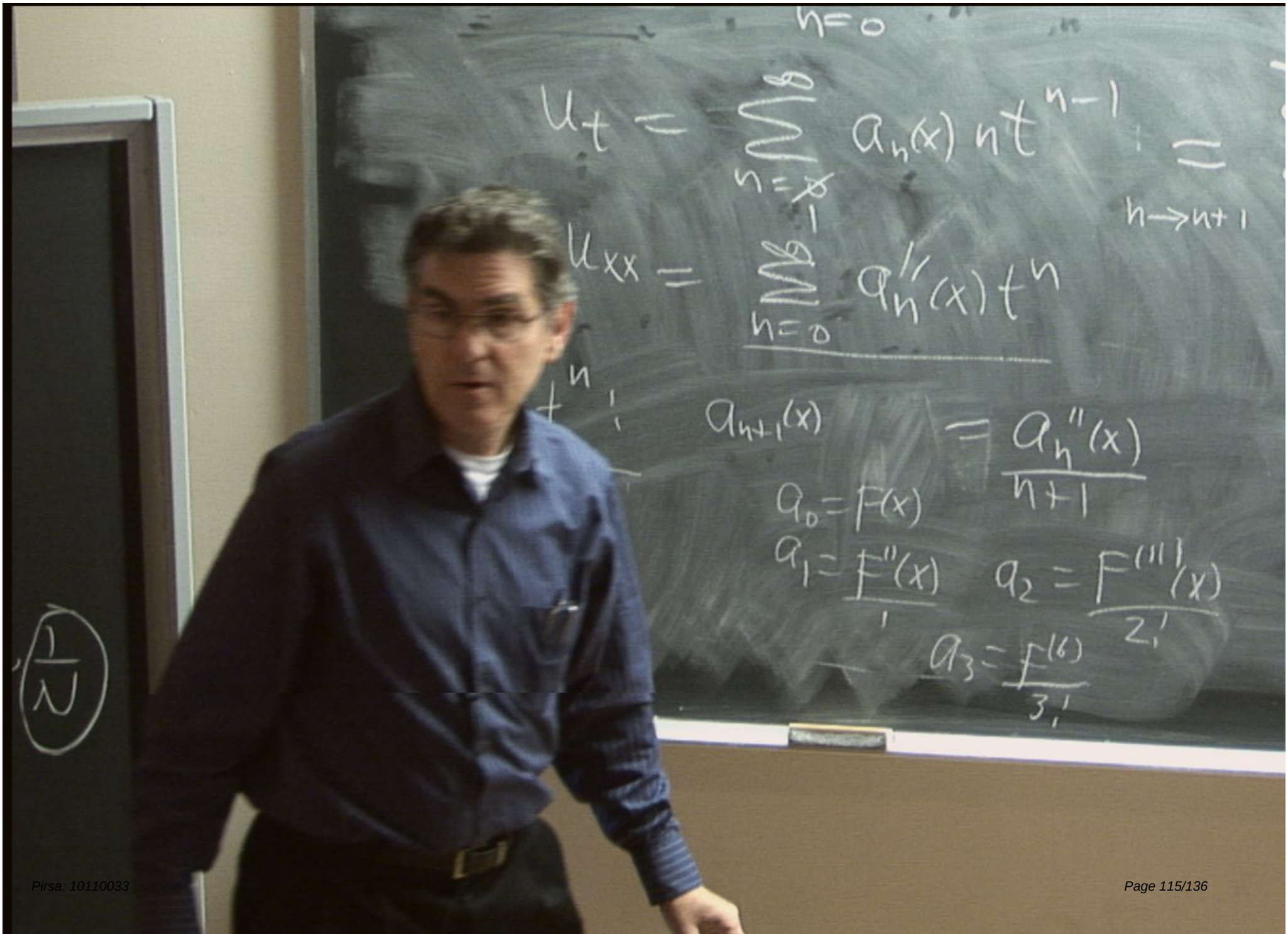
$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n c^n}{n!} (-1)^n F^{(n)}(x)$$

$$= \underline{F(x-ct)}$$


n=0
$u_t = \sum_{n=1}^{\infty} a_n(x)\, n t^{n-1} =$
$n \to n+1$
$u_{xx} = \sum_{n=0}^{\infty} a_n''(x)\, t^n$
t^n : $a_{n+1}(x)(n+1)$


n=0
$u_t = \sum_{n=1}^{\infty} a_n(x)\, n t^{n-1} =$
$n \to n+1$
$x = \sum_{n=0}^{\infty} a_n''(x) t^n$
t^n : $a_{n+1}(x)(n+1) = a_n''(x)$


n=0
$u_t = \sum_{n=\cancel{0}\,1}^{\infty} a_n(x)\, n t^{n-1} =$
$n \to n+1$
$u_{xx} = \sum_{n=0}^{\infty} a_n''(x)\, t^n$
t^n: $a_{n+1}(x) = \frac{a_n''(x)}{n+1}$
$\frac{1}{\lambda}$


n=0
$u_t = \sum_{n=1}^{\infty} a_n(x)\, n t^{n-1} =$
n→n+1
$u_{xx} = \sum_{n=0}^{\infty} a_n''(x) t^n$
$a_{n+1}(x) = \frac{a_n''(x)}{n+1}$
$a_0 = F(x)$
$a_1 = \frac{F''(x)}{1}$
$a_2 = \frac{F^{(IV)}(x)}{2!}$
$a_3 = \frac{F^{(6)}}{3!}$
$\frac{1}{N}$

$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$t) = \sum_{n=0}^{\infty} t^n \frac{F^{(2n)}(x)}{n!}$$

$$= F(x-ct)$$

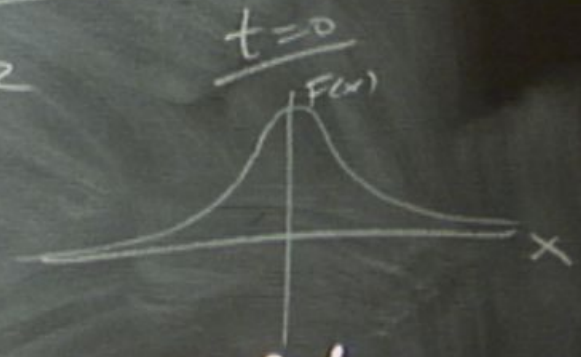
$$F(x) = e^{-x^2}$$

$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

$$= F(x-ct)$$

$$F(x) = e^{-x^2}$$



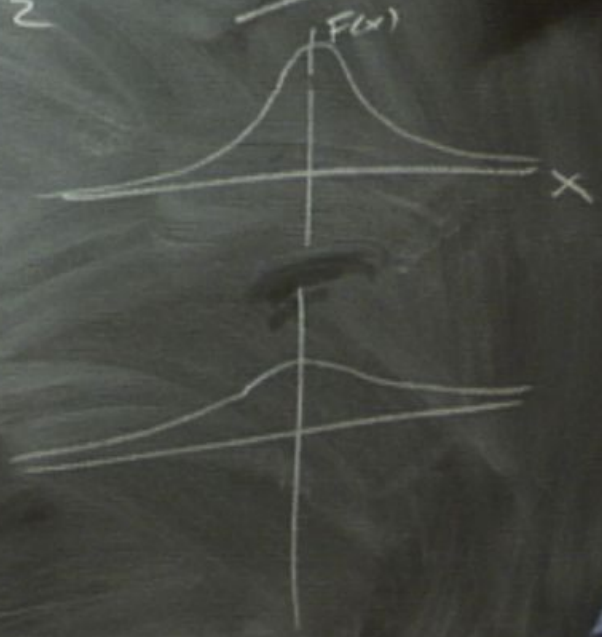
$$u(x,0) = F(x) \quad \underline{IC}$$

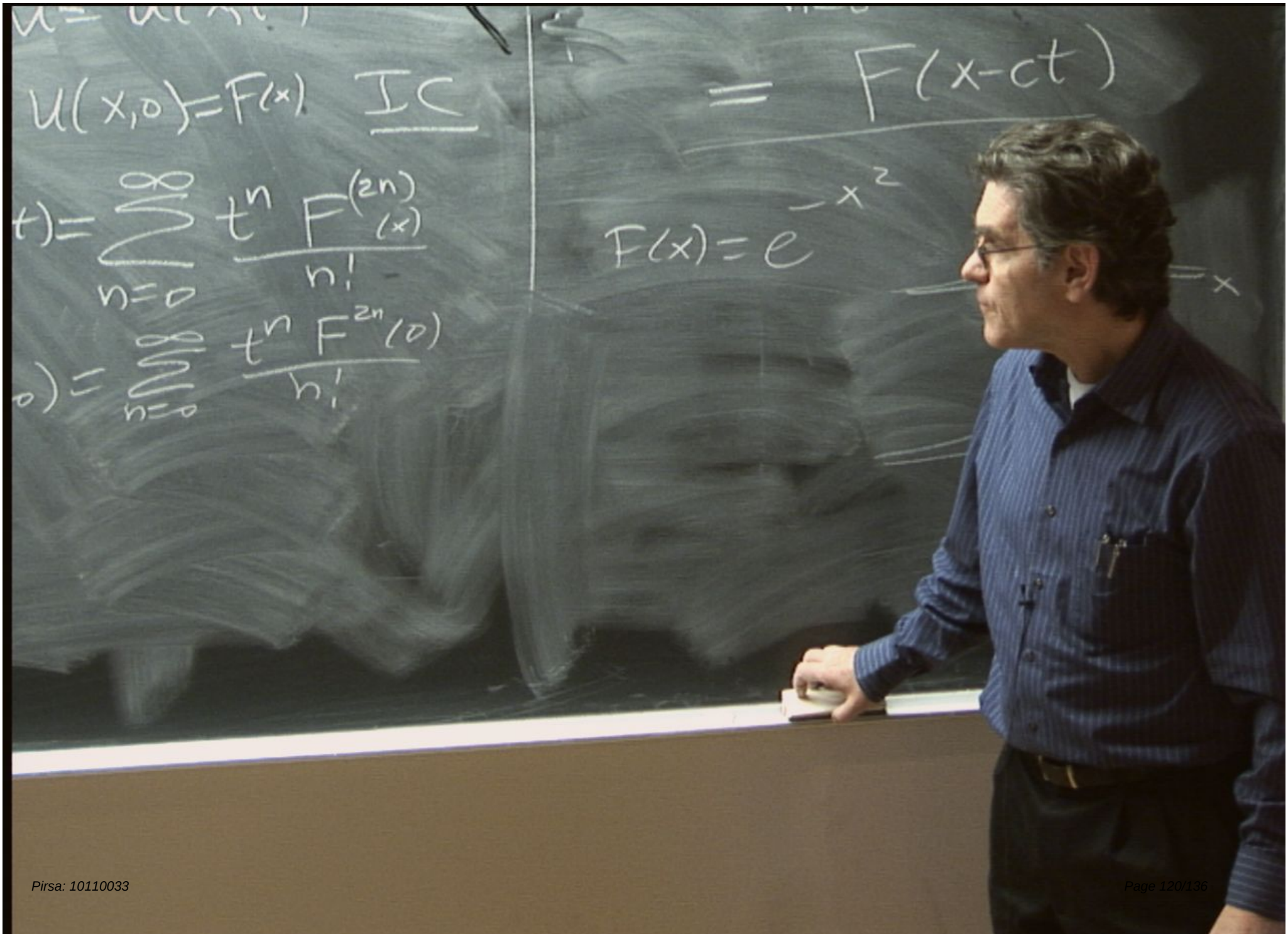
$$t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

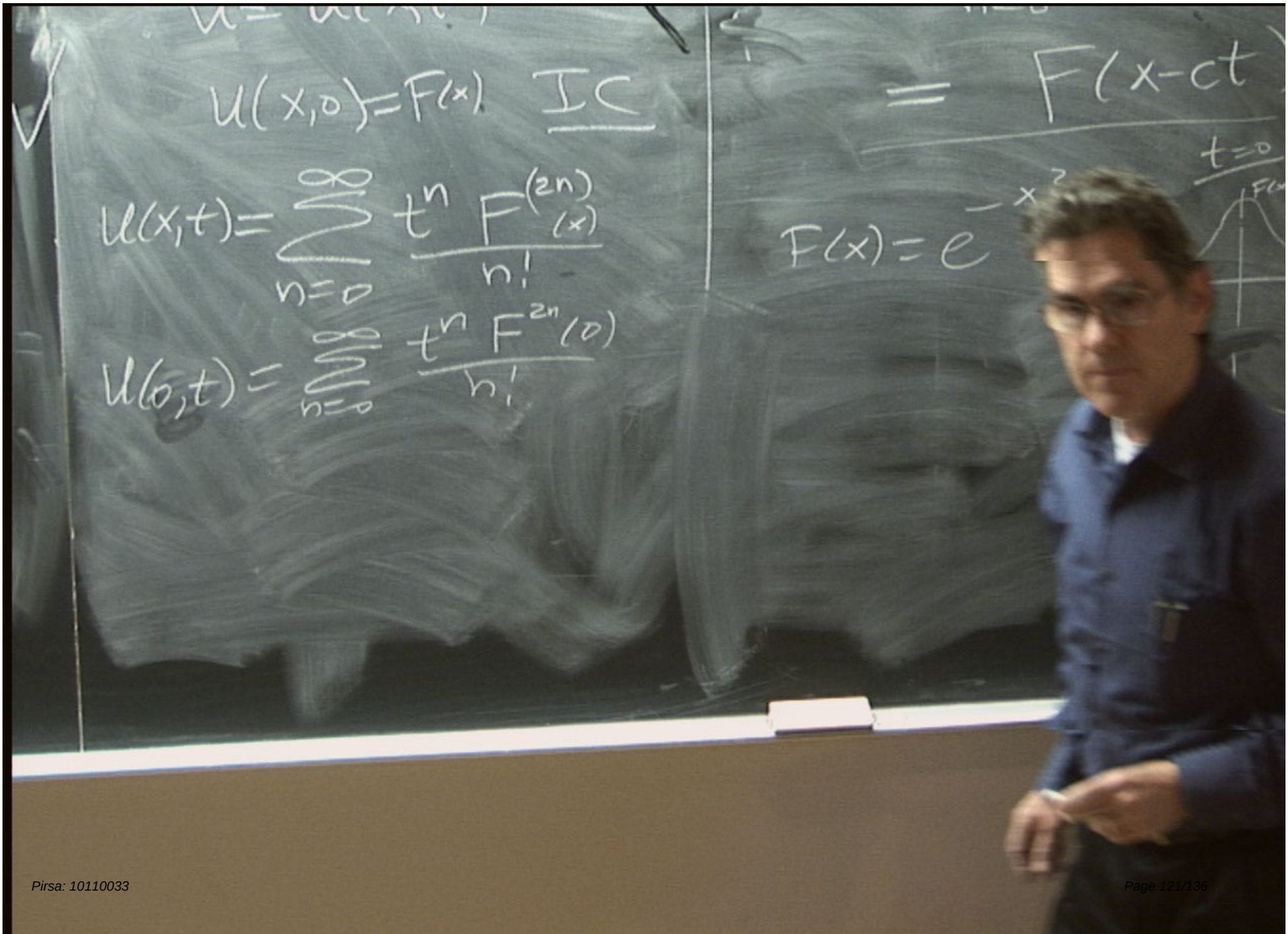
$$= F(x-ct)$$

$$F(x) = e^{-x^2}$$

$\underline{t=0}$




U(x,0)=F(x) IC
t)= Σ_{n=0}^{∞} t^n F^{(2n)}(x) / n!
o)= Σ_{n=0}^{∞} t^n F^{2n}(0) / n!
= F(x-ct)
F(x)=e^{-x^2}


u(x,0)=F(x) IC
u(x,t)=Σ_{n=0}^{∞} t^n F^{(2n)}(x)/n!
u(0,t)=Σ_{n=0}^{∞} t^n F^{2n}(0)/n!
= F(x-ct
t=0
F(x)=e^{-x^2}

$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

$$u(0,t) = \sum_{n=0}^{\infty} \frac{t^n F^{2n}(0)}{n!}$$

$$= F(x-ct)$$

$$F(x) = e^{-x^2}$$

$$= \sum_{n=0}^{\infty} (-x^2)^n$$

$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

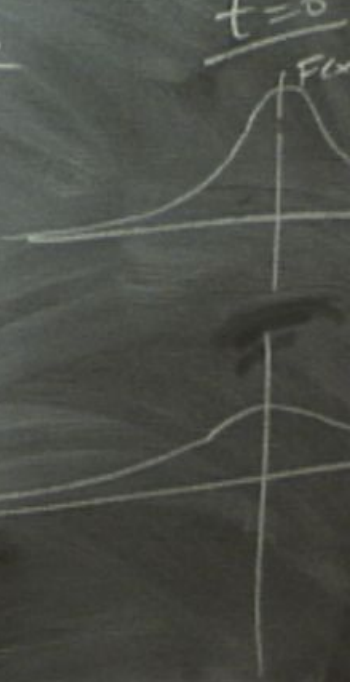
$$u(0,t) = \sum_{n=0}^{\infty} \frac{t^n F^{2n}(0)}{n!}$$

$$= F(x-ct)$$

$$F(x) = e^{-x^2}$$

$$= \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$$

$$F^{(2n)}(0) = \frac{(-1)^n (2n)!}{n!}$$



$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

$$u(0,t) = \sum_{n=0}^{\infty} \frac{t^n F^{2n}(0)}{n!}$$

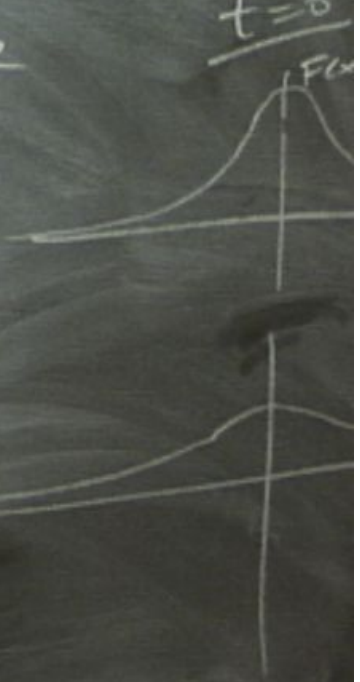
$$= \sum_{n=0}^{\infty} \frac{t^n (2n)!(-1)^n}{n!\,n!}$$

$$= F(x-ct)$$

$$F(x) = e^{-x^2}$$

$$= \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$$

$$F^{(2n)}(0) = \frac{(-1)^n (2n)!}{n!}$$



$$u(x,0) = F(x) \quad \underline{\text{IC}}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

$$u(0,t) = \sum_{n=0}^{\infty} \frac{t^n F^{2n}(0)}{n!}$$

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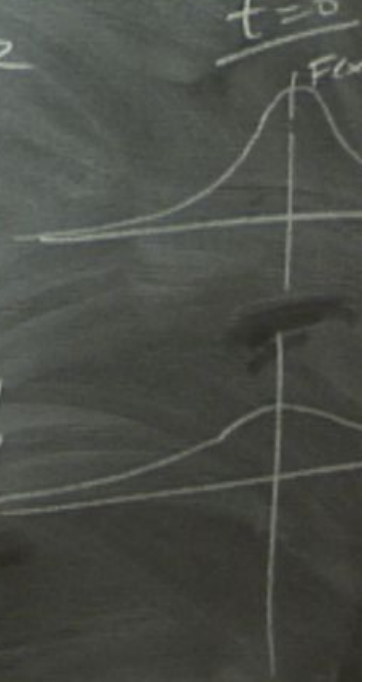
$$F(x) = e^{-x^2}$$

$$= \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$$

$$F^{(2n)}(0) = \frac{(-1)^n (2n)!}{n!}$$



$$u(x,0) = F(x) \quad \underline{IC}$$

$$u(x,t) = \sum_{n=0}^{\infty} \frac{t^n F^{(2n)}(x)}{n!}$$

$$u(0,t) = \sum_{n=0}^{\infty} \frac{t^n F^{2n}(0)}{n!}$$

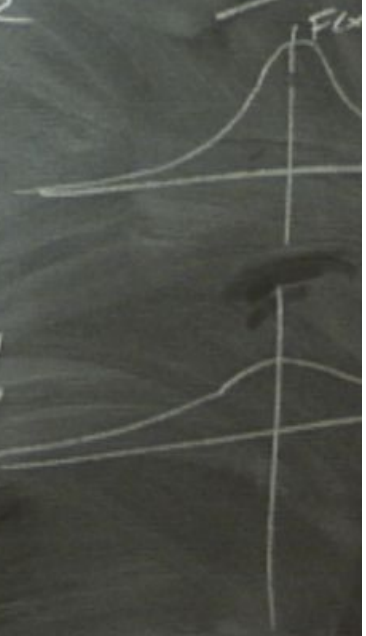
$$= \sum_{n=0}^{\infty} \frac{t^n (2n)! (-1)^n}{n!\, n!}$$

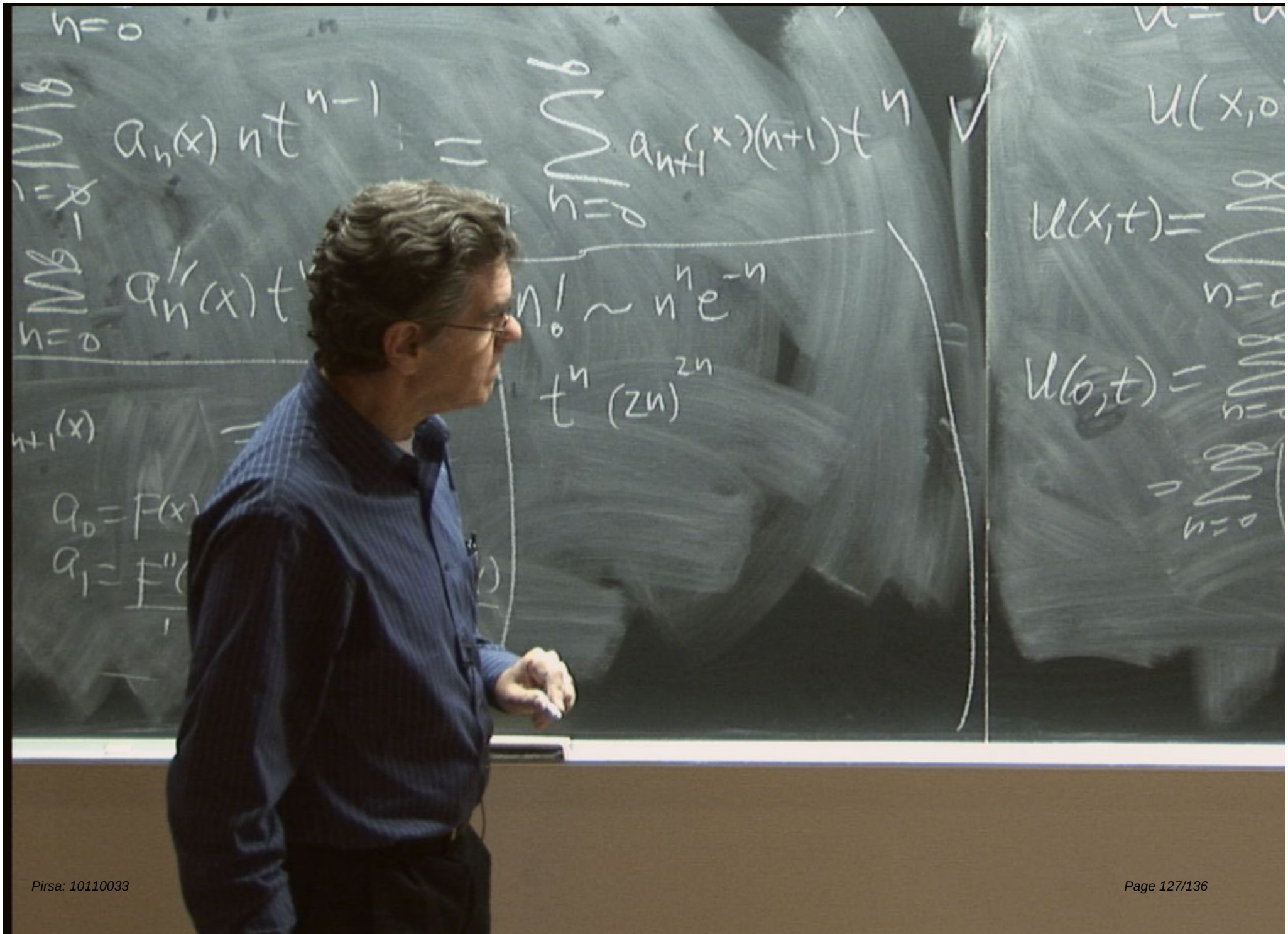
$$= F(x-ct)$$

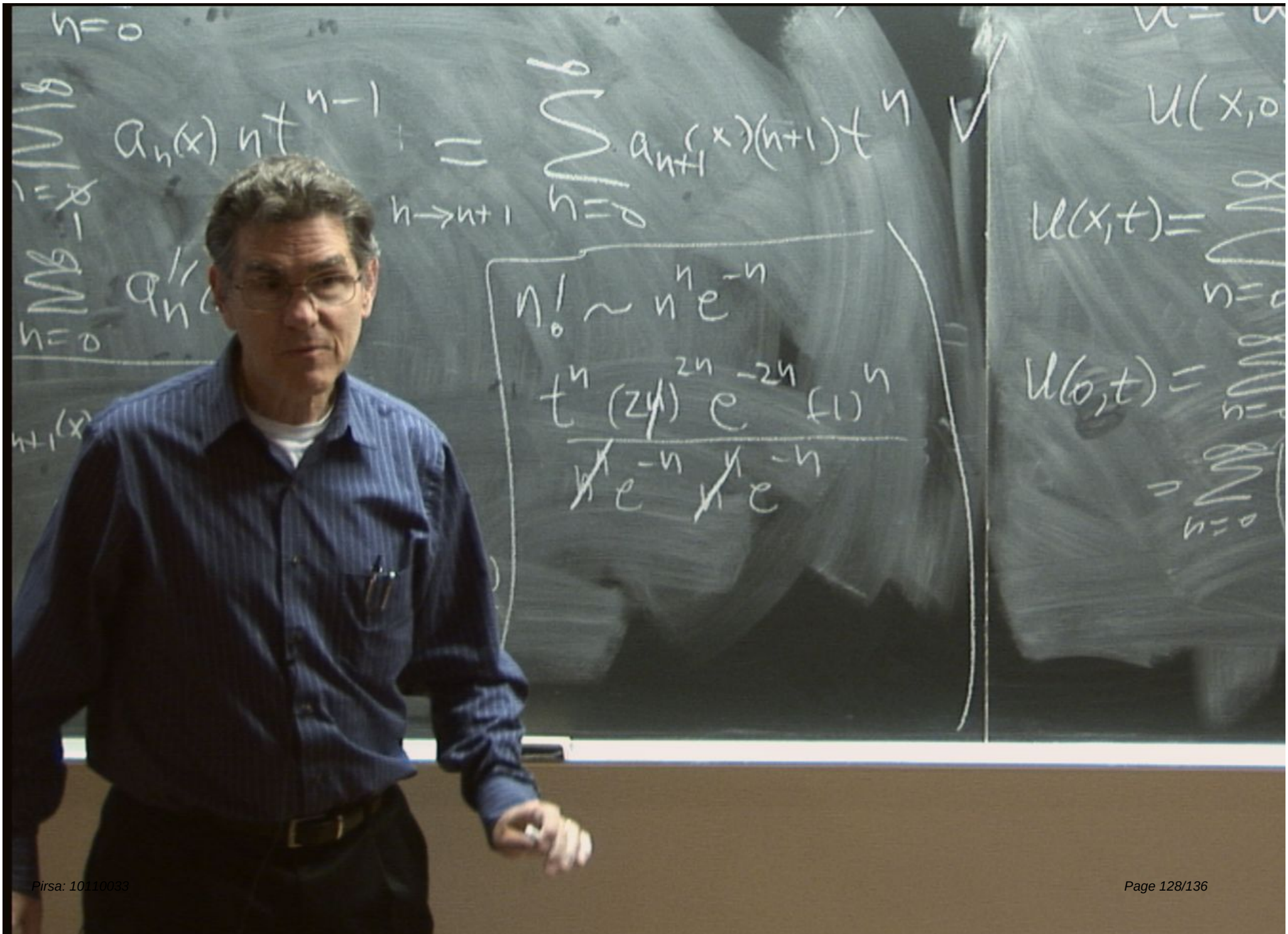
$$F(x) = e^{-x^2}$$

$$= \sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$$

$$F^{(2n)}(0) = \frac{(-1)^n (2n)!}{n!}$$




n=0
a_n(x) n t^{n-1} = Σ_{n=0}^{∞} a_{n+1}(x)(n+1) t^n
n! ~ n^n e^{-n}
t^n (2n)^{2n}
a_0 = F(x)
u(x,t) =
u(0,t) =


n=0
n→n+1
n! ~ n^n e^-n
u(x,t)=
u(0,t)=

$$\sum_{n=0}^{\infty} a_n(x)\, n t^{n-1} = \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$n \to n+1$

$$\sum_{n=0}^{\infty} a_n''(x) t^n$$

$$a_{n+1}(x) = \frac{a_n''(x)}{n+1}$$

$$a_0 = F(x)$$

$$a_1 = \frac{F''(x)}{1} \qquad a_2 = \frac{F^{(IV)}(x)}{2!}$$

$$a_3 = \frac{F^{(6)}}{3!}$$

$$n! \sim n^n e^{-n}$$

$$\frac{t^n (2n)^{2n} e^{-2n} (-1)^n}{n^n e^{-n} n^n e^{-n}}$$

$$\sim (-1)^n (4t)^n$$

$$u(x,0$$

$$u(x,t) = \sum_{n=0}$$

$$u(0,t) = \sum_{n=0}$$

$$= \sum_{n=0}$$

$$\sum_{n=0}^{\infty} a_n(x)\, n t^{n-1} \underset{n \to n+1}{=} \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$$\sum_{n=0}^{\infty} a_n''(x) t^n$$

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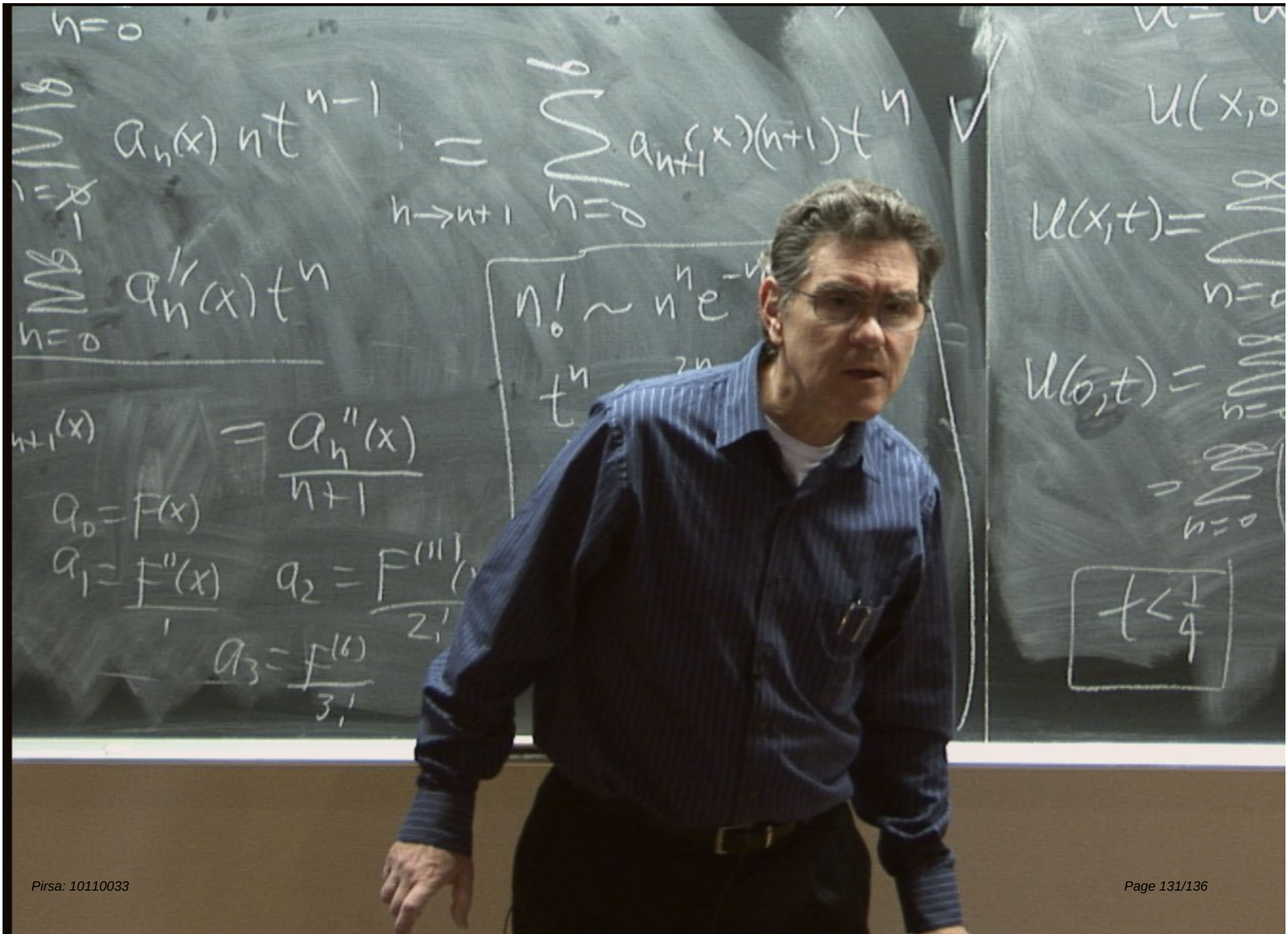

n=0
n→n+1
n! ~ n^n e^-n
t<1/4

$n=0$

$$\sum_{n=1}^{\infty} a_n(x)\, n t^{n-1} = \sum_{n=0}^{\infty} a_{n+1}(x)(n+1) t^n$$

$n \to n+1$

$$\sum_{n=0}^{\infty} a_n''(x) t^n$$

$$a_{n+1}(x) = \frac{a_n''(x)}{n+1}$$

$a_0 = F(x)$

$a_1 = \frac{F''(x)}{1}$ $\quad a_2 = \frac{F'''(x)}{2!}$

$a_3 = \frac{F^{(6)}}{3!}$

$$n! \sim n^n e^{-n}$$

$$\frac{t^n (2n)^{2n} \cancel{e^{-2n}} (-1)^n}{\cancel{n^n} \cancel{e^{-n}} \cancel{n^n} \cancel{e^{-n}}}$$

$$\sim (-1)^n (4t)^n$$

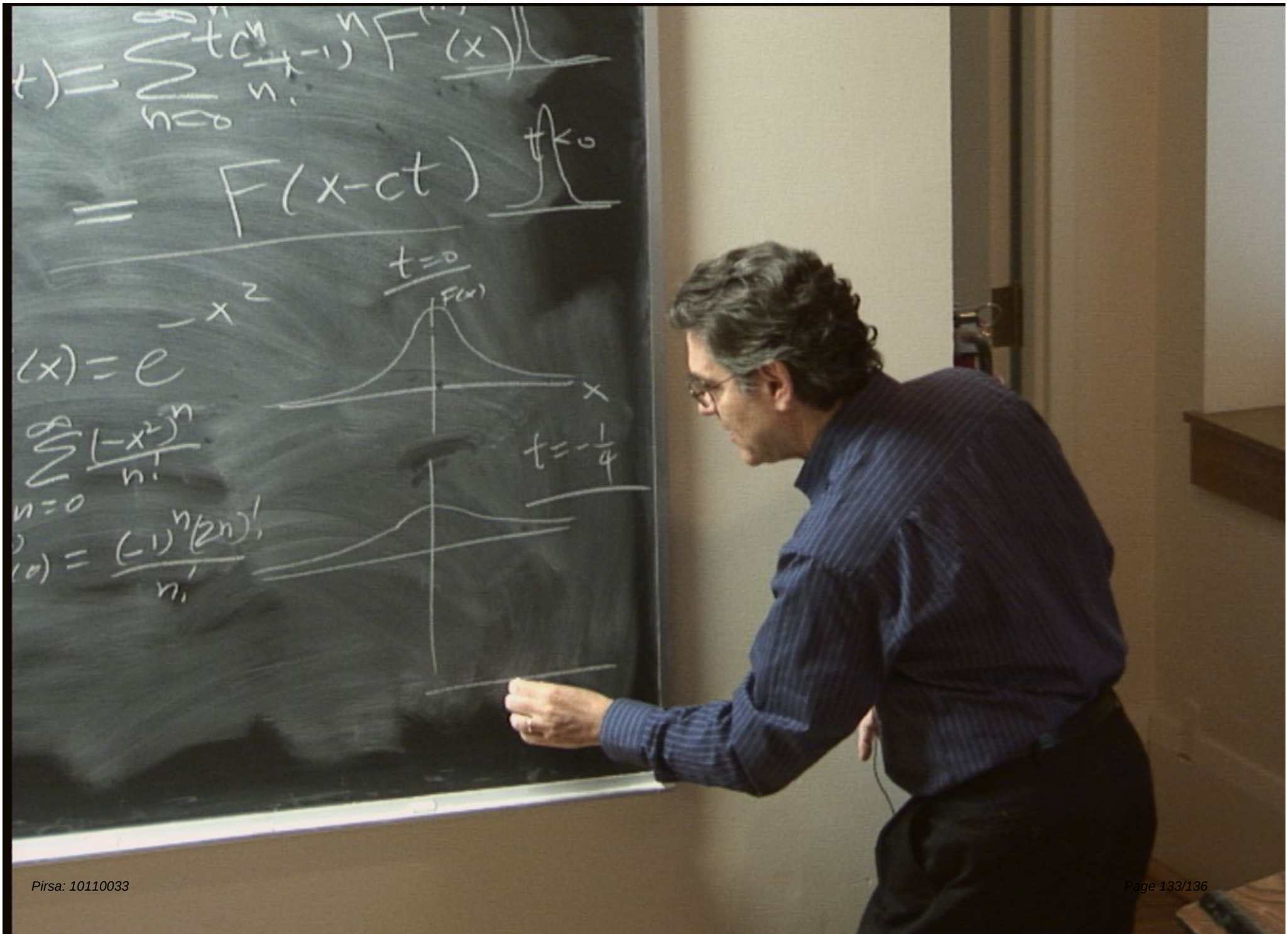
$u(x,0$

$u(x,t) = \sum_{n=0}$

$u(0,t) = \sum_{n=0}$

$= \sum_{n=0}$

$$\boxed{t < \frac{1}{4}}$$


= F(x-ct)
t=0
F(x)
x
t=-1/4

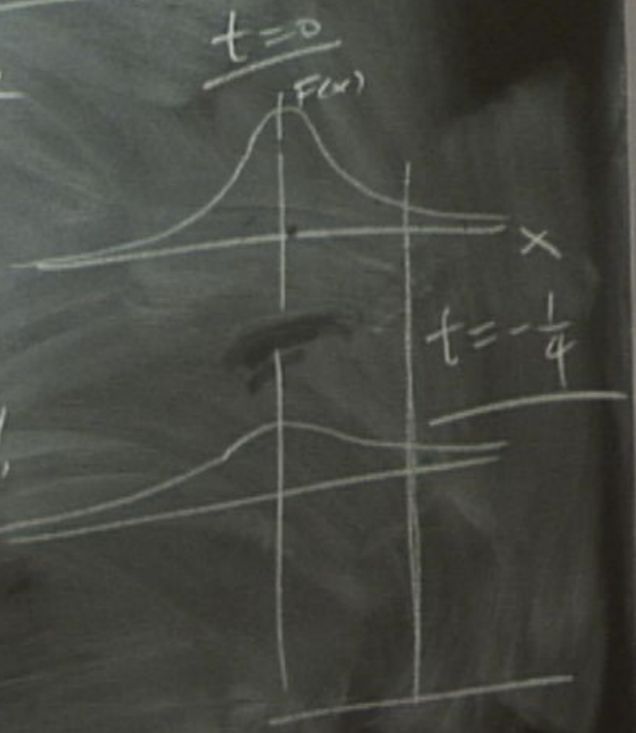
$$t) = \sum_{n=0}^{\infty} \frac{t^n c^n}{n!} (-1)^n F^{(n)}(x)$$

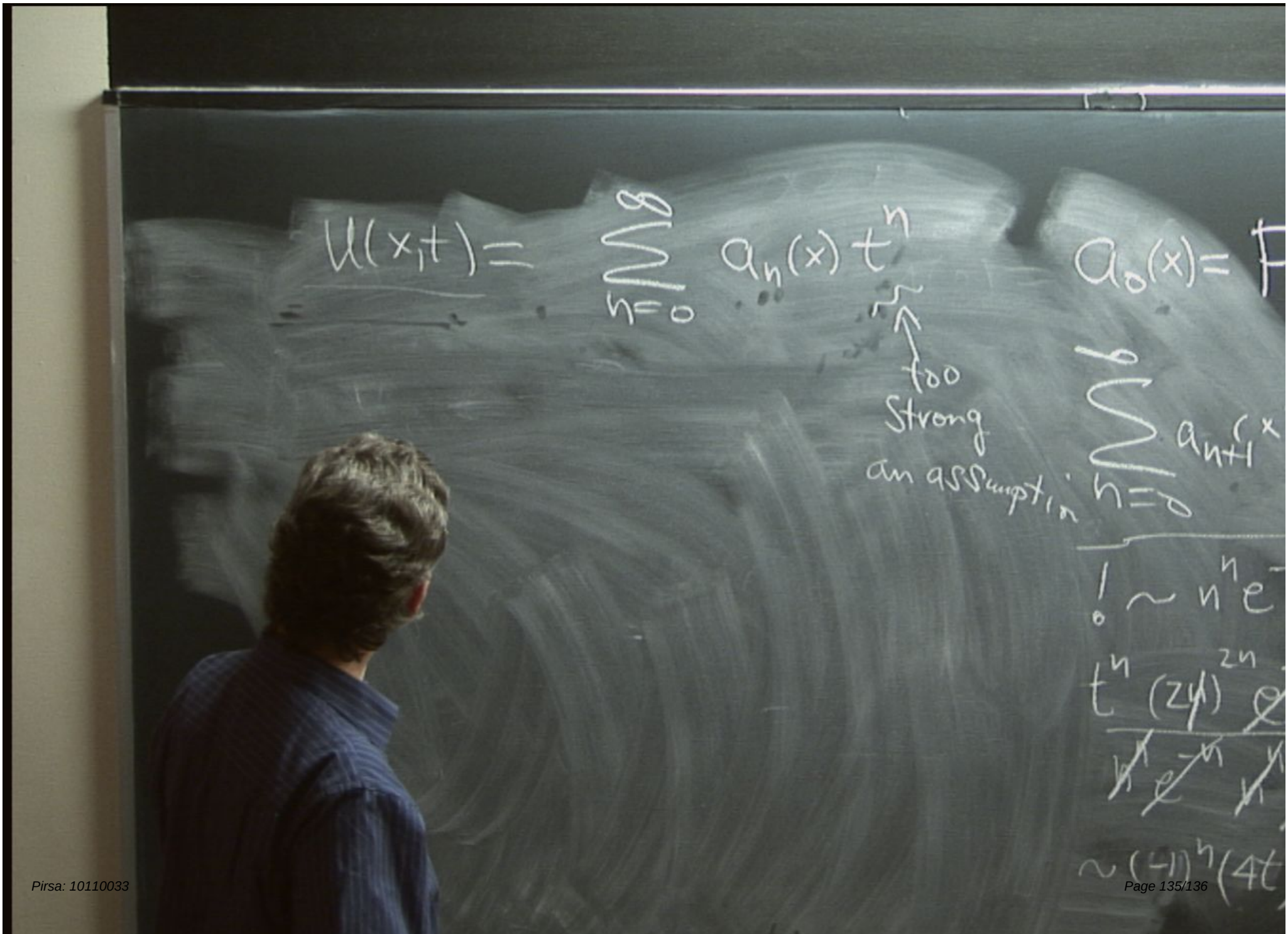
$$= F(x-ct) \quad t<0$$

$$(x) = e^{-x^2}$$

$$\sum_{n=0}^{\infty} \frac{(-x^2)^n}{n!}$$

$$(0) = \frac{(-1)^n (2n)!}{n!}$$




$U(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n$
too
Strong
an assumption
$a_0(x) =$
$\sum_{n=0}^{\infty} a_{n+1}(x)$
$! \sim n^n e^{-}$
$t^n (2n)^{2n}$
$\sim (-1)^n (4t$

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, t^n$$

too Strong an assumption

$$u(x,t) = \sum_{n=0}^{\infty} a_n(x)\, b_n(t)$$

Sep. Var. Series

$$a_0(x) = F$$

$$\sum_{n=0}^{\infty} a_{n+1}(x$$

$$! \sim n^n e^{-}$$

$$\frac{t^n (2n)^{2n} \cancel{e}}{\cancel{n^n} \cancel{e^{-n}} \cancel{n^n}}$$

$$\sim (-1)^n (4t$$