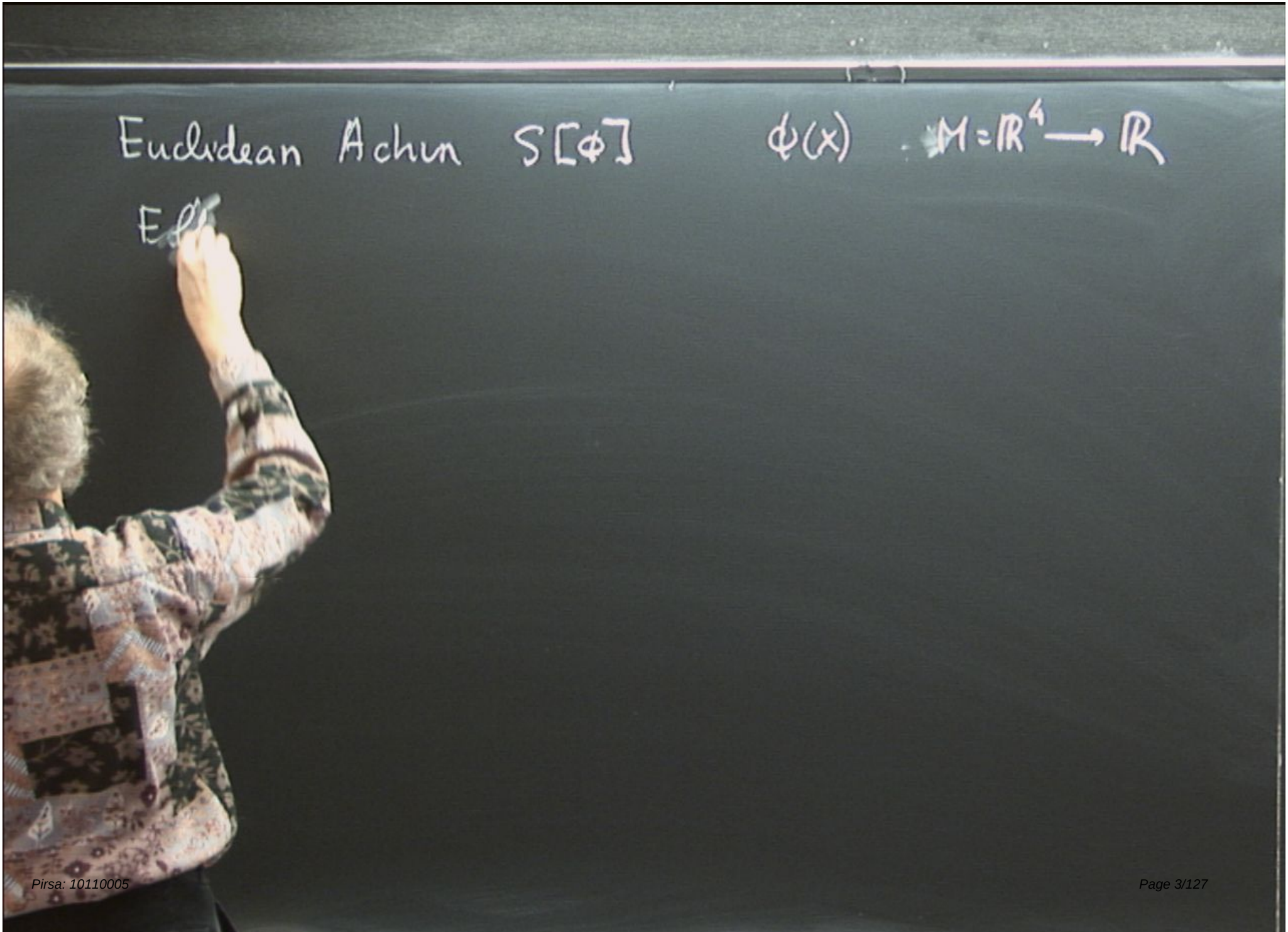
Title: Quantum Field Theory II (PHYS 603) - Lecture 6

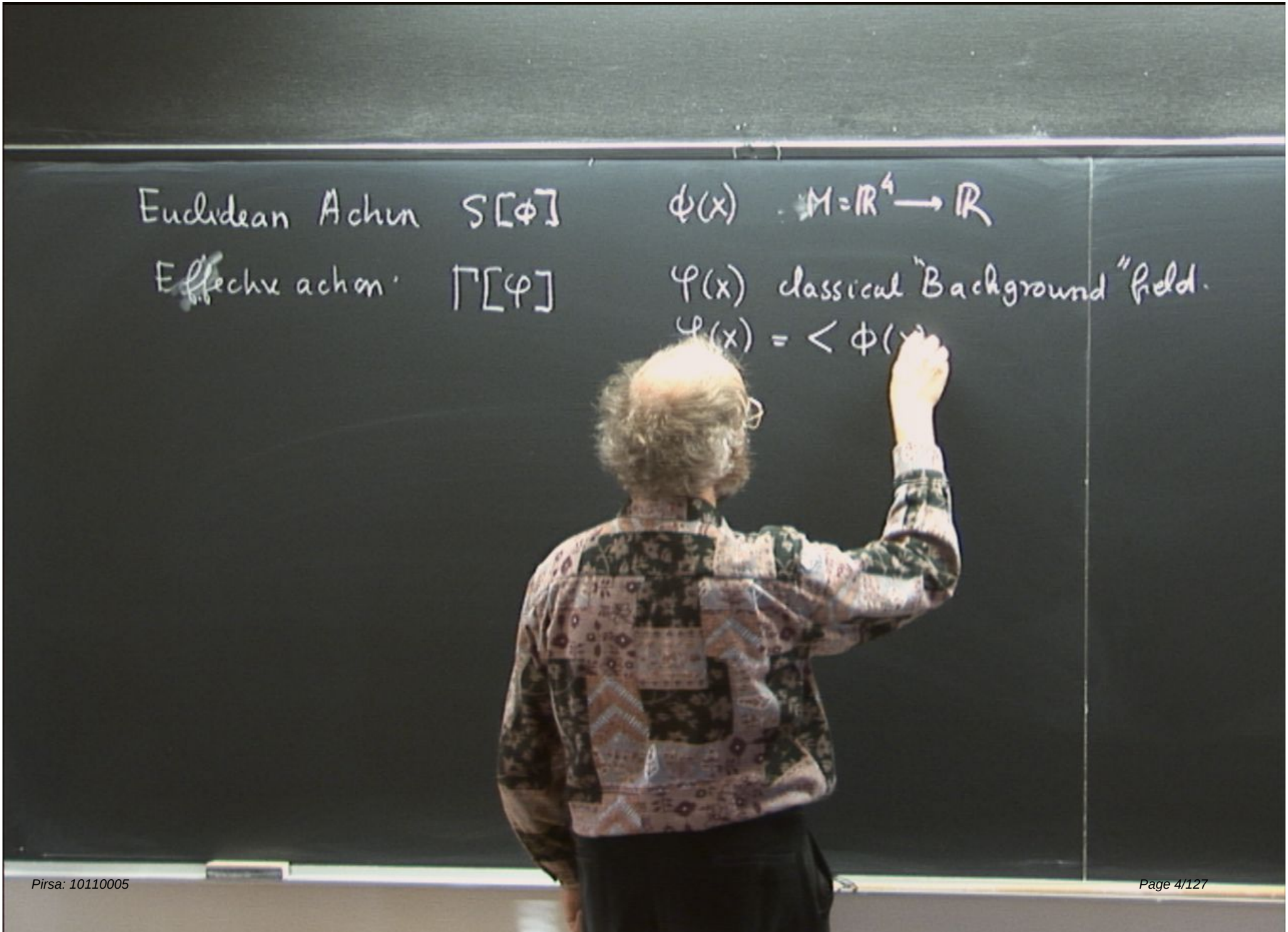
Date: Nov 08, 2010 09:00 AM

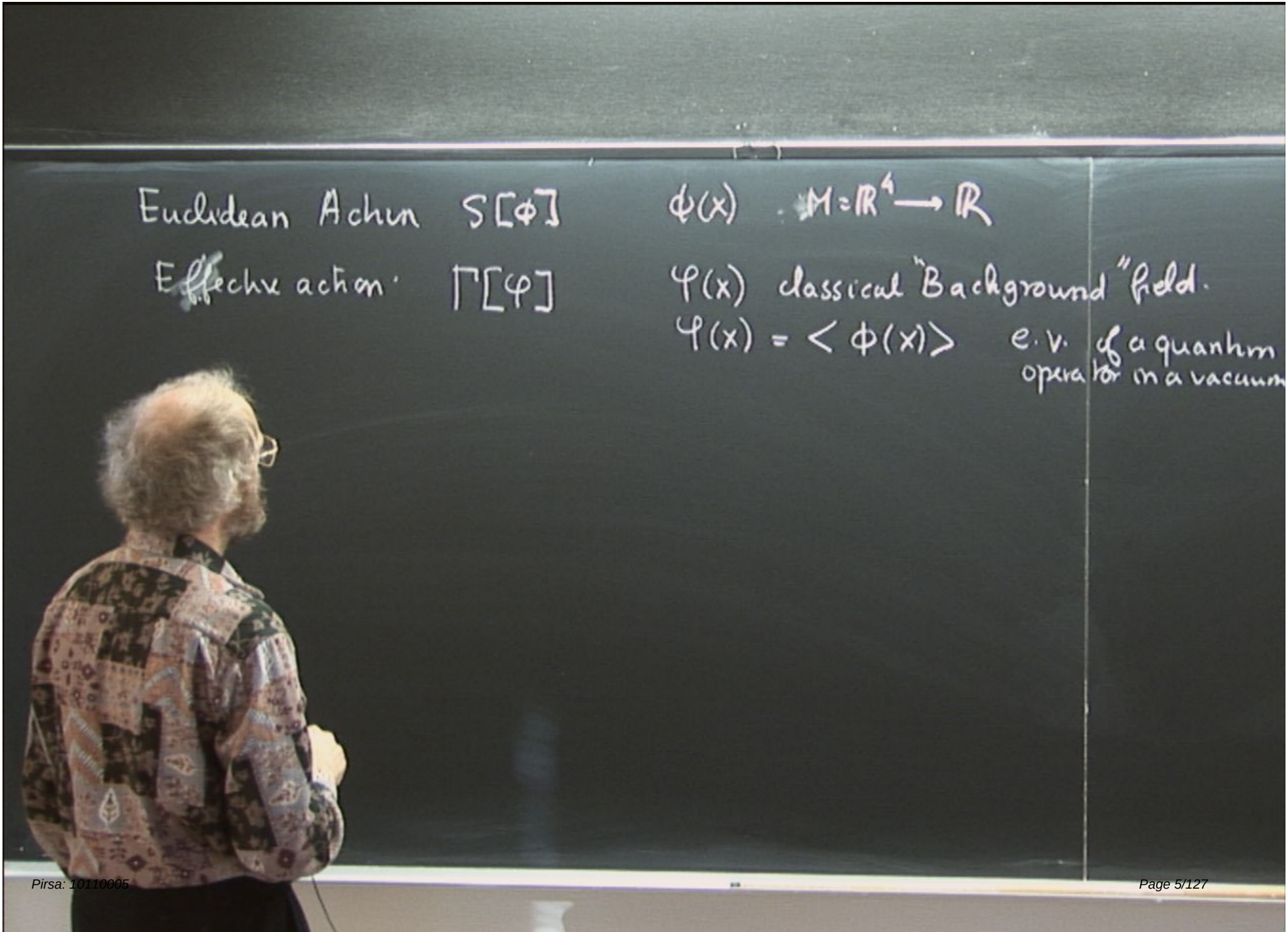
URL: http://pirsa.org/10110005

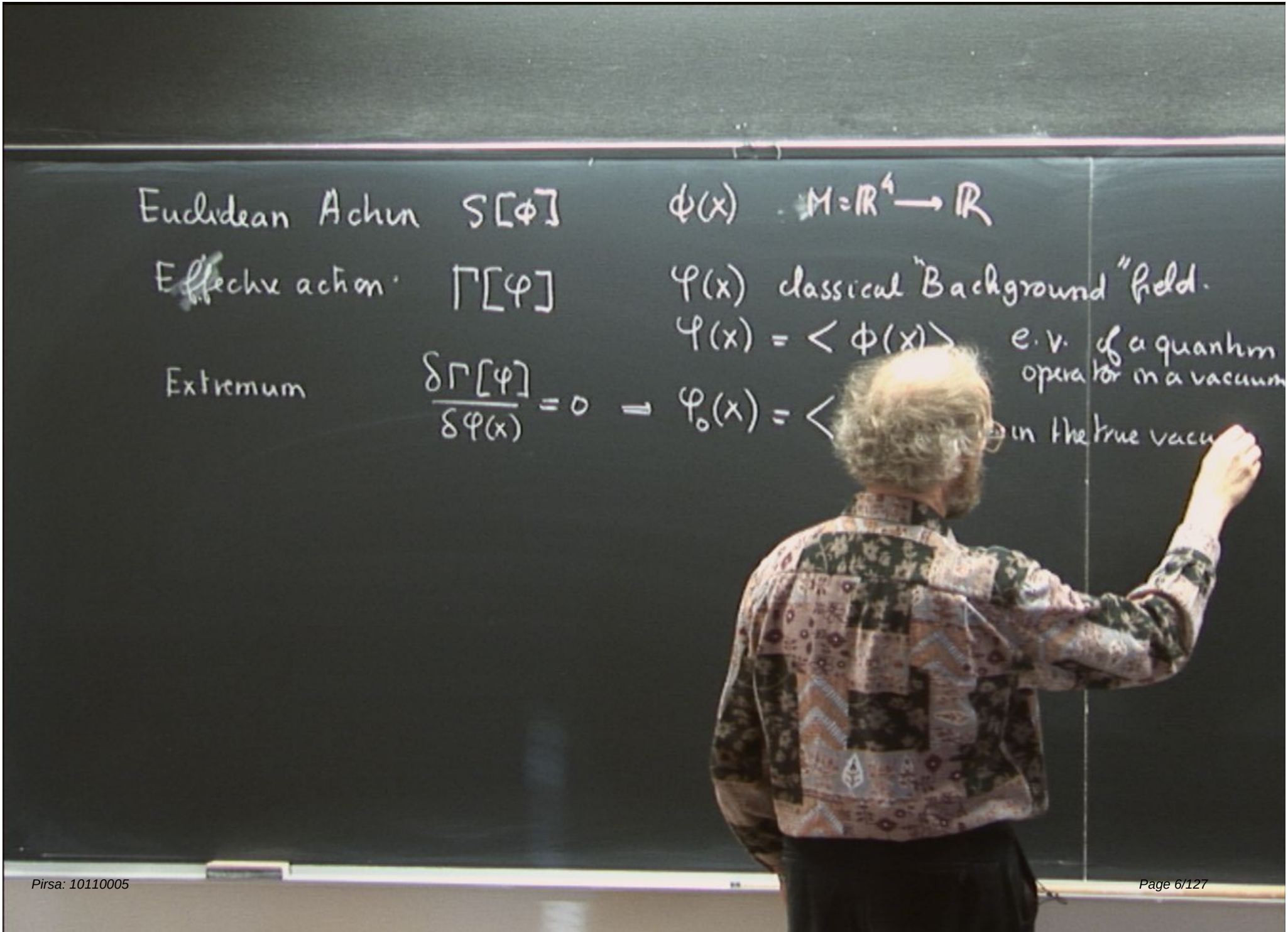
Abstract:

PERIMETER
INSTITUTE FOR THEORETICAL PHYSICS


Euclidean Achun S[φ] φ(x) M = ℝ⁴ → ℝ


Euclidean Action S[φ]
φ(x) . M = ℝ⁴ ⟶ ℝ
Effective action Γ[φ]
φ(x) classical "Background" field.
φ(x) = < φ(


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.
$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.
$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum
Extremum $\frac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \rightarrow \varphi_0(x) = \langle$ in the true vacu

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.

$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum.

Extremum $\dfrac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \Rightarrow \varphi_0(x) = \langle \phi(x) \rangle$ ← in the true vacuum state $|\Omega\rangle$ of the theory

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

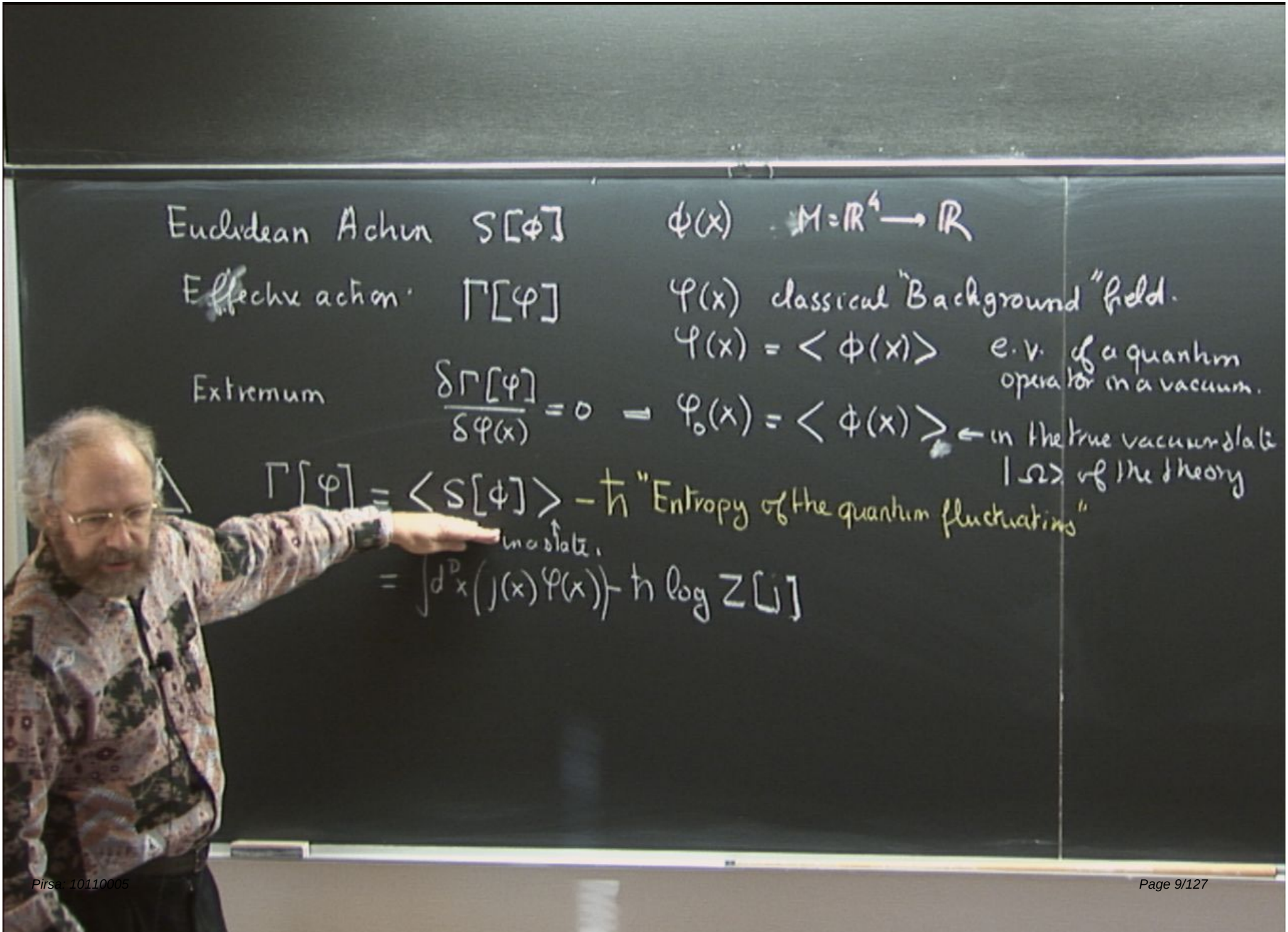
Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.

$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum.

Extremum $\dfrac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \rightharpoonup \varphi_0(x) = \langle \phi(x) \rangle$ ← in the true vacuum/ground $|\Omega\rangle$ of the theory

⚠ $\Gamma[\varphi] = \langle S[\phi] \rangle$

in a state.


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.
$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum.
Extremum $\frac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \rightarrow \varphi_0(x) = \langle \phi(x) \rangle$ ← in the true vacuum state $|\Omega\rangle$ of the theory
$\Gamma[\varphi] = \langle S[\phi] \rangle - \hbar$ "Entropy of the quantum fluctuations"
in a state.
$= \int d^D x \left(J(x) \varphi(x) \right) - \hbar \log Z[J]$

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.

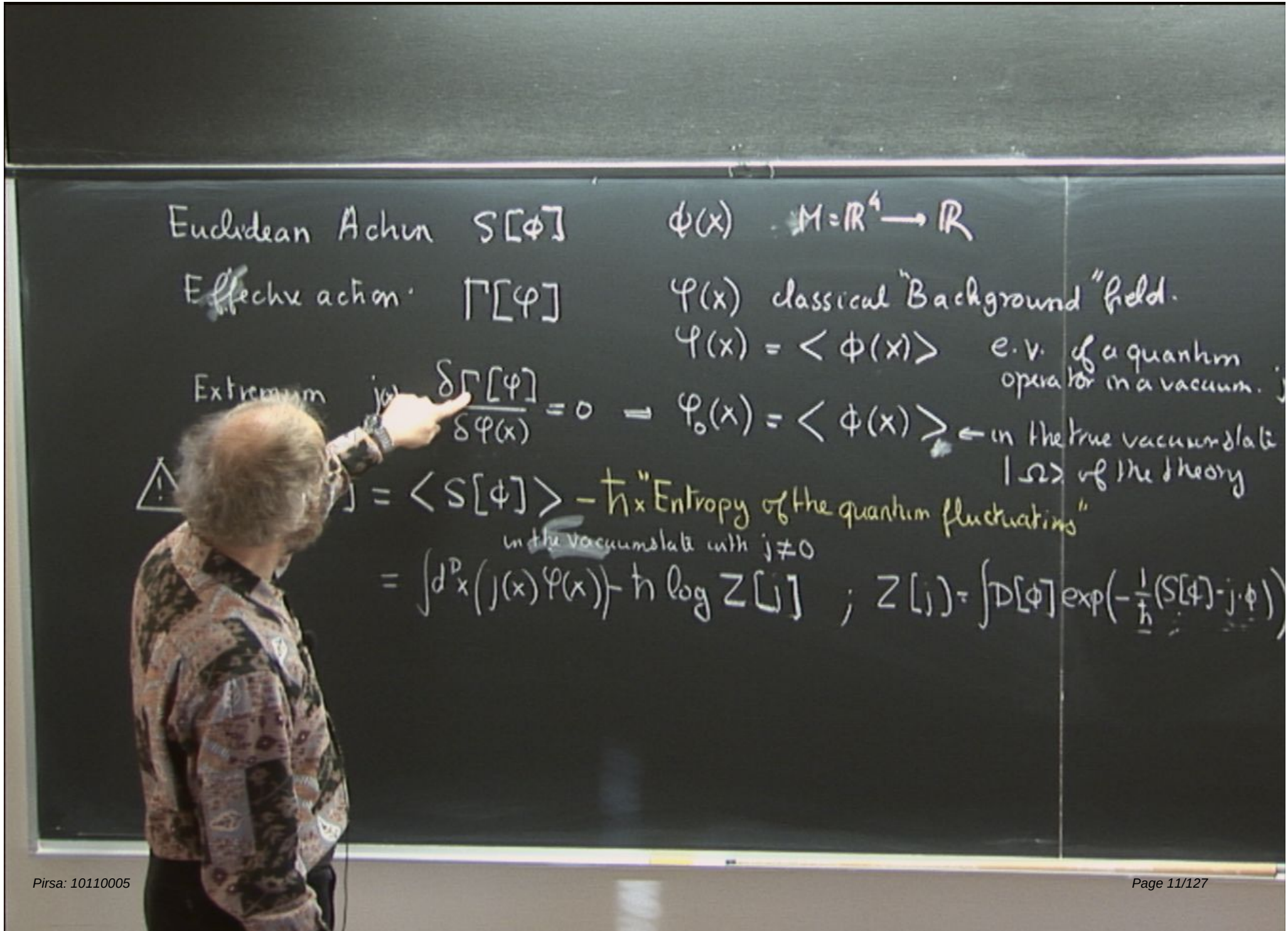
$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum.

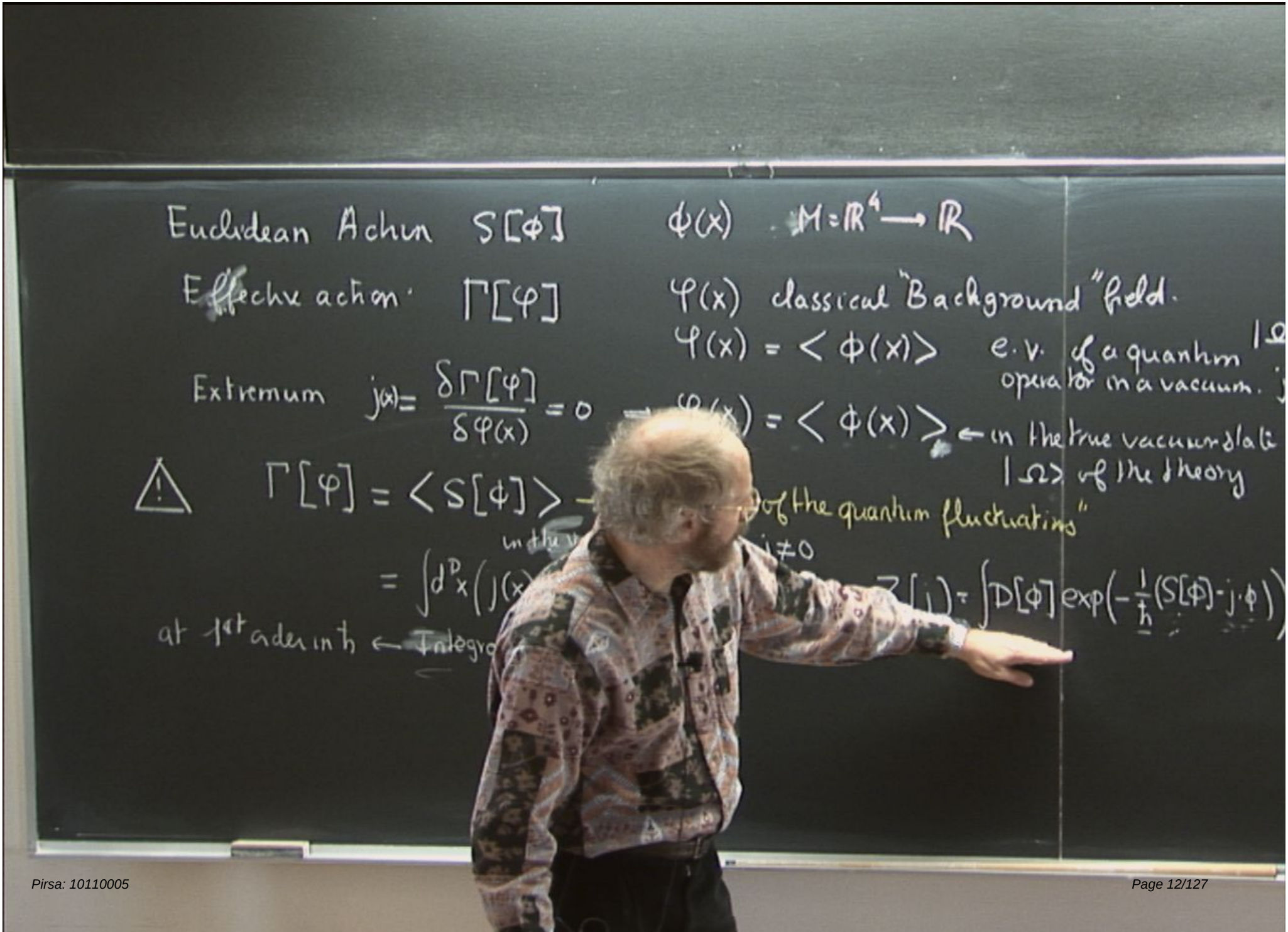
Extremum $\dfrac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \rightharpoonup \varphi_0(x) = \langle \phi(x) \rangle$ ← in the true vacuum state $|\Omega\rangle$ of the theory

$\Gamma[\varphi] = \langle S[\phi] \rangle - \hbar \times$ "Entropy of the quantum fluctuations"

in a state.

$= \int d^D x \, (J(x)\varphi(x)) - \hbar \log Z[J]$; $Z[J] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - J \cdot \phi)\right)$


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.
$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum.
Extremum $\frac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \rightarrow \varphi_0(x) = \langle \phi(x) \rangle$ ← in the true vacuum state $|\Omega\rangle$ of the theory
$= \langle S[\phi] \rangle - \hbar$ "Entropy of the quantum fluctuations"
in the vacuumstate with $j \neq 0$
$= \int d^D x \left(j(x) \varphi(x) \right) - \hbar \log Z[j]$; $Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} (S[\phi] - j \cdot \phi) \right)$


Euclidean Action S[φ]
φ(x) M = ℝ⁴ → ℝ
Effective action: Γ[φ]
φ(x) classical "Background" field.
φ(x) = < φ(x) > e.v. of a quantum operator in a vacuum.
Extremum j(x) = δΓ[φ] / δφ(x) = 0
φ(x) = < φ(x) > ← in the true vacuum state |Ω> of the theory
Γ[φ] = < S[φ] >
of the quantum fluctuations"
j ≠ 0
at 1st order in ħ ← Integro

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

Effective action: $\Gamma[\varphi]$ $\varphi(x)$ classical "Background" field.

$\varphi(x) = \langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle$

Extremum $j(x) = \dfrac{\delta \Gamma[\varphi]}{\delta \varphi(x)} = 0 \rightarrow \varphi_0(x) = \langle \phi(x) \rangle$ ← in the true vacuum state $|\Omega\rangle$ of the theory

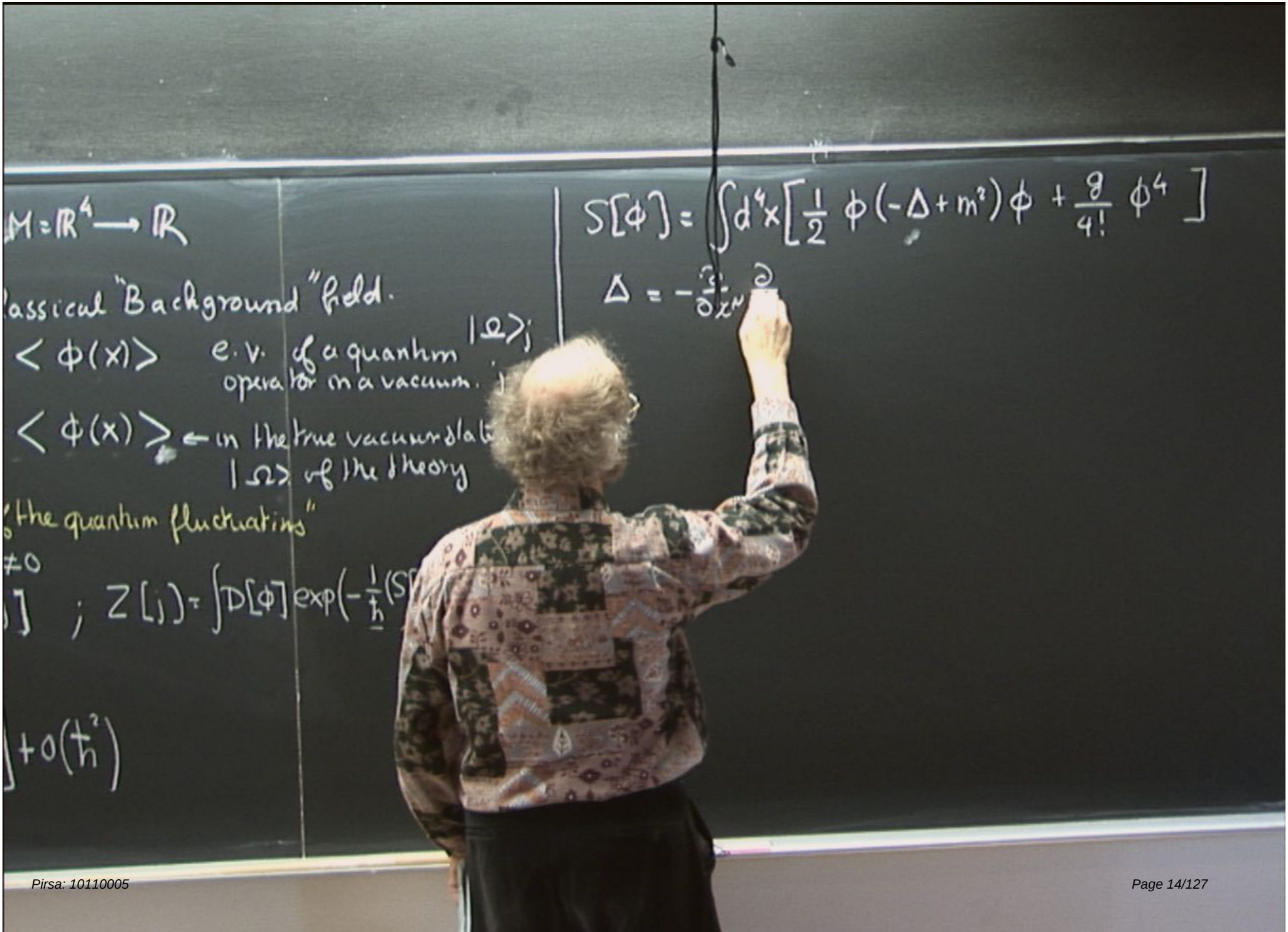
⚠ $\Gamma[\varphi] = \langle S[\phi] \rangle - \hbar \times$ "Entropy of the quantum fluctuations"

in the vacuum state with $j \neq 0$

$$= \int d^D x \left(j(x) \varphi(x) \right) - \hbar \log Z[j] \quad ; \quad Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar} (S[\phi] - j \cdot \phi) \right)$$

at 1st order in $\hbar$ ← Integration over $\phi(x)$

$$\Gamma[\varphi] = S[\varphi] + \frac{\hbar}{2} \mathrm{Tr}\left[\mathrm{Log}\left(S''[\varphi] \right) \right] + O(\hbar^2)$$


$M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
lassical "Background" field.
$\langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle$;
$\langle \phi(x) \rangle$ ← in the true vacuum/ground $|\Omega\rangle$ of the theory
the quantum fluctuating"
$\neq 0$
$S[\phi] = \int d^4x \left[\frac{1}{2} \phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4 \right]$
$\Delta = -\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial}$
$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}(S[$
$+ O(\hbar^2)$

$M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

lassical "Background" field.

$\langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle_j$ $j \neq 0$

$\langle \phi(x) \rangle \leftarrow$ in the true vacuum state $|\Omega\rangle$ of the theory

f the quantum fluctuations"

$\neq 0$

$] \quad ; \quad Z[j] = \int D[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right)$

$] + O(\hbar^2)$

$$S[\phi] = \int d^4x \left[\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4\right]$$

$$\Delta = -\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\mu}$$

$M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

lassical "Background" field.

$\langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle_j$ $j \neq 0$

$\langle \phi(x) \rangle$ ← in the true vacuum/state $|\Omega\rangle$ of the theory

he quantum fluctuations"

$$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right)$$

$+ O(\hbar^2)$

$$S[\phi] = \int d^4x \left[\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4\right]$$

$$\Delta = -\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\mu}$$

$S''[\varphi]$ = operator whose integral kernel is

$$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S''[\varphi]$$

$M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

lassical "Background" field.

$\langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle_j$ $j \neq 0$

$\rangle$ ← in the true vacuum/state $|\Omega\rangle$ of the theory

im fluctuating"

$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right)$

$S[\phi] = \int d^4x \left[\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4\right]$

$\Delta = -\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\mu}$

$S''[\varphi]$ = operator whose integral kernel is

$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S''[\varphi] = S''[\varphi](x,y)$

$S''[\varphi] = (-\Delta + m^2)$

$M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

lassical "Background" field.

$\langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle_j$ $j \neq 0$

$\langle \phi(x) \rangle$ ← in the true vacuum/state $|\Omega\rangle$ of the theory

the quantum fluctuations"

$\neq 0$

$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}(S[\phi] - j\cdot\phi)\right)$

$+ O(\hbar^2)$

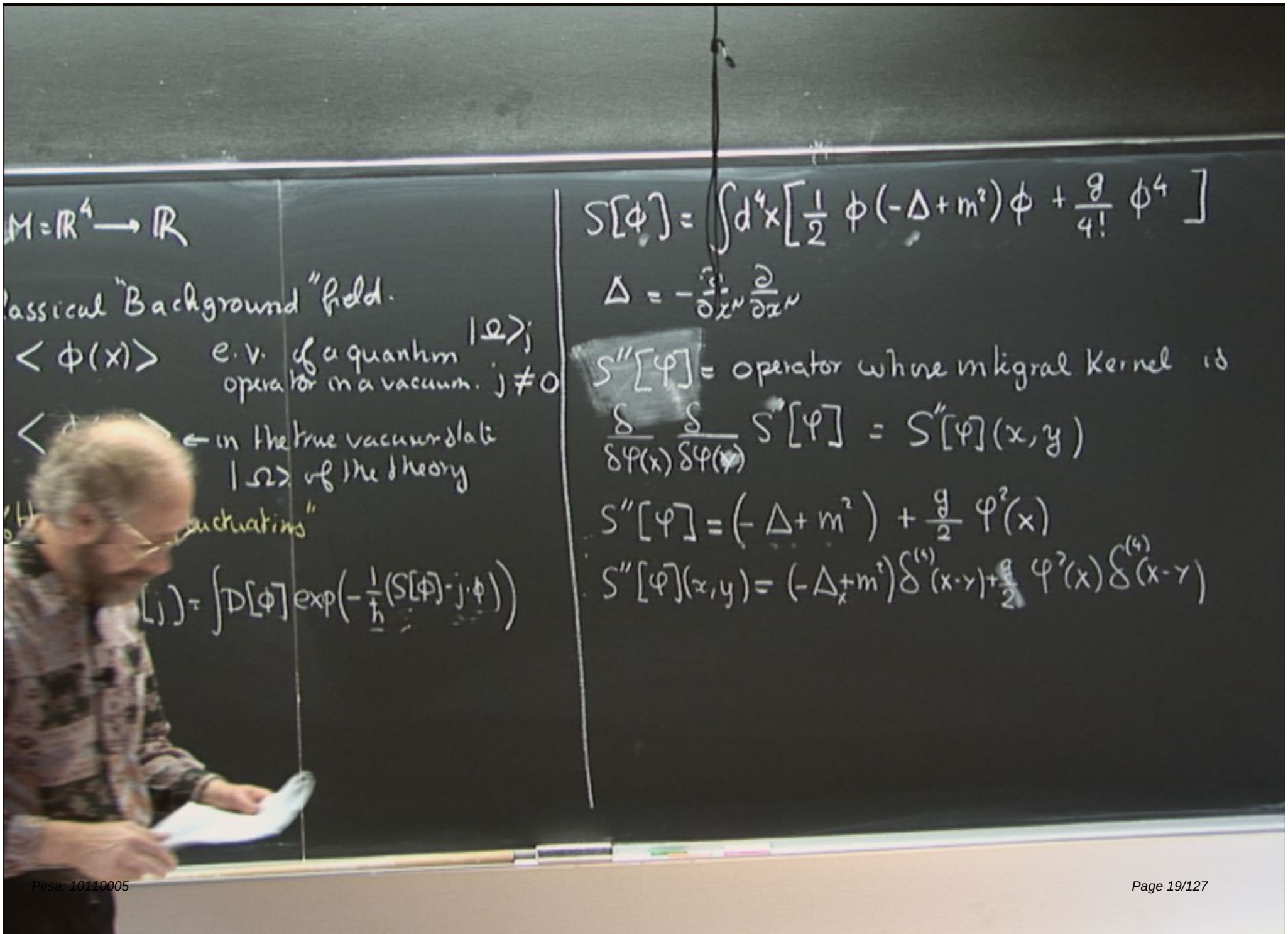
$S[\phi] = \int d^4x \left[\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4\right]$

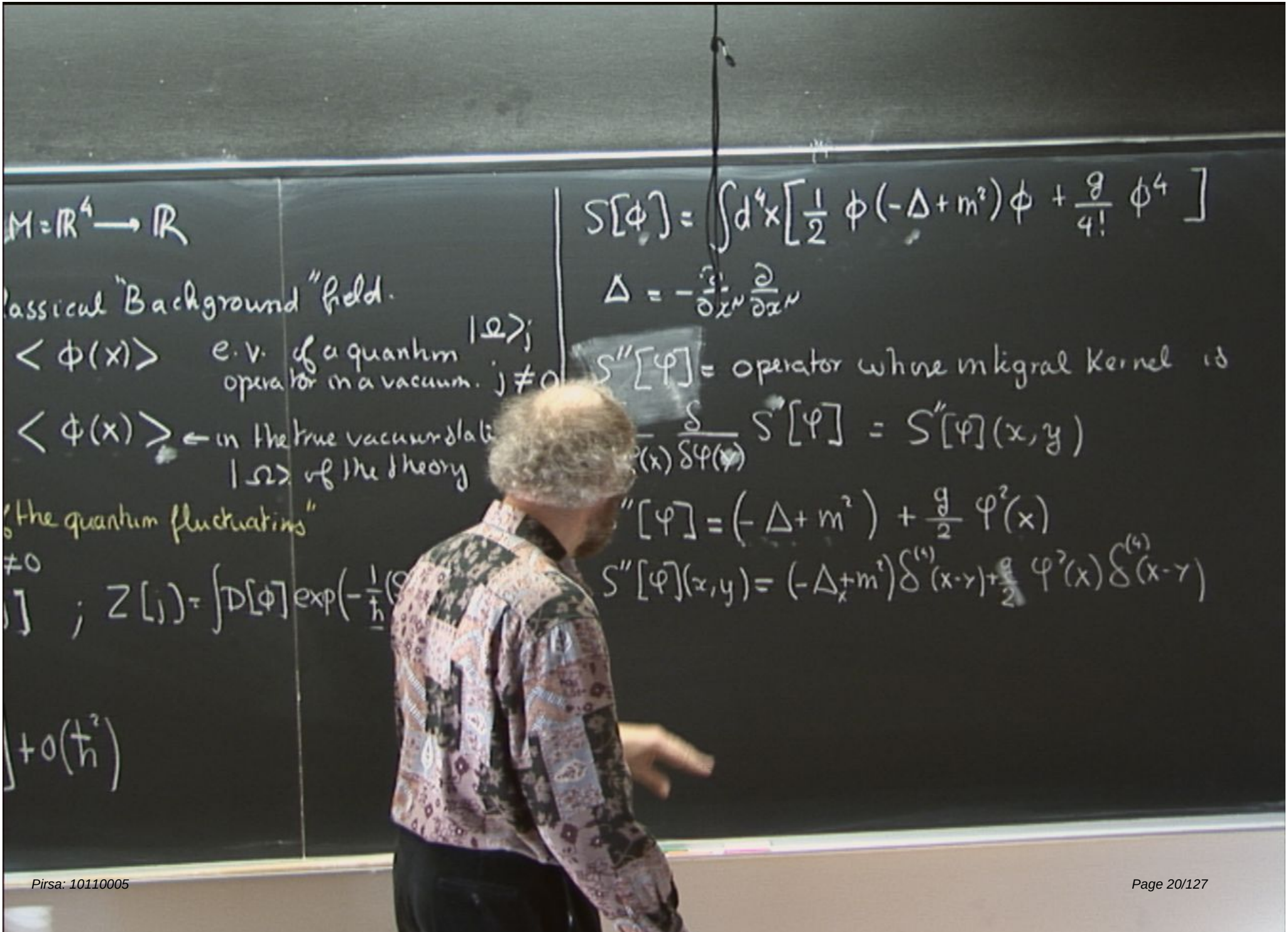
$\Delta = -\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\mu}$

$S''[\varphi]$ = operator whose integral kernel is

$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S''[\varphi] = S''[\varphi](x,y)$

$S''[\varphi] = (-\Delta + m^2)$


M = ℝ⁴ → ℝ
lassical "Background" field.
< φ(x) > e.v. of a quantum operator in a vacuum. |Ω>_j j ≠ 0
← in the true vacuum state |Ω> of the theory
uctuating"
[j] = ∫D[φ] exp(−1/ħ (S[φ] − j·φ))
S[φ] = ∫d⁴x [1/2 φ(−Δ + m²)φ + g/4! φ⁴]
Δ = −∂/∂x^μ ∂/∂x^μ
S''[φ] = operator whose integral kernel is
δ/δφ(x) δ/δφ(y) S''[φ] = S''[φ](x, y)
S''[φ] = (−Δ + m²) + g/2 φ²(x)
S''[φ](x, y) = (−Δ_x + m²) δ⁽⁴⁾(x−y) + g/2 φ²(x) δ⁽⁴⁾(x−y)


$M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
lassical "Background" field.
$\langle \phi(x) \rangle$ e.v. of a quantum operator in a vacuum. $|\Omega\rangle$; $j \neq 0$
$\langle \phi(x) \rangle$ ← in the true vacuum/ground $|\Omega\rangle$ of the theory
the quantum fluctuations"
$\neq 0$
$S[\phi] = \int d^4x \left[\frac{1}{2} \phi(-\Delta + m^2)\phi + \frac{g}{4!}\phi^4 \right]$
$\Delta = -\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial x^\mu}$
$S''[\varphi]$ = operator whose integral kernel is
$\frac{\delta}{\delta\varphi(x)\delta\varphi(y)} S[\varphi] = S''[\varphi](x,y)$
$S''[\varphi] = (-\Delta + m^2) + \frac{g}{2}\varphi^2(x)$
$S''[\varphi](x,y) = (-\Delta_x + m^2)\delta^{(4)}(x-y) + \frac{g}{2}\varphi^2(x)\delta^{(4)}(x-y)$
$Z[j] = \int \mathcal{D}[\phi] \exp\left(-\frac{1}{\hbar}(S$
$+ O(\hbar^2)$

$S''[\varphi]$ = operator whose integral kernel is

$$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S''[\varphi] = S''[\varphi](x,y)$$

$$S''[\varphi] = (-\Delta + m^2) + \frac{g}{2}\varphi^2(x)$$

$$S''[\varphi](x,y) = (-\Delta_x + m^2)\delta^{(4)}(x-y) + \frac{g}{2}\varphi^2(x)\delta^{(4)}(x-y)$$

$$\mathrm{Tr}\log(S''[\varphi]) = \mathrm{Tr}\log\left((-\Delta+m^2)\left[\mathbb{1} + (-\Delta+m^2)^{-1}\right]\right.$$

$S''[\varphi]$ = operator whose integral kernel is

$$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S''[\varphi] = S''[\varphi](x,y)$$

$$S''[\varphi] = (-\Delta + m^2) + \frac{g}{2}\varphi^2(x)$$

$$S''[\varphi](x,y) = (-\Delta_x + m^2)\delta^{(4)}(x-y) + \frac{g}{2}\varphi^2(x)\delta^{(4)}(x-y)$$

$$\mathrm{Tr}\log(S''[\varphi]) = \mathrm{Tr}\left[\log\left((-\Delta+m^2)\left[\mathbb{1} + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right]\right)\right]$$

$S''[\varphi]$ = operator whose integral kernel is

$$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S[\varphi] = S''[\varphi](x,y)$$

$$S''[\varphi] = \left(-\Delta + m^2\right) + \frac{g}{2}\varphi^2(x)$$

$$S''[\varphi](x,y) = \left(-\Delta_x + m^2\right)\delta^{(4)}(x-y) + \frac{g}{2}\varphi^2(x)\,\delta^{(4)}(x-y)$$

$$\mathrm{Tr}\log\left(S''[\varphi]\right) = \mathrm{Tr}\left[\log\left(\left(-\Delta+m^2\right)\left[\mathbb{1} + \left(-\Delta+m^2\right)^{-1}\frac{g}{2}\varphi^2\right]\right)\right]$$

$S''[\varphi]$ = operator whose integral kernel is

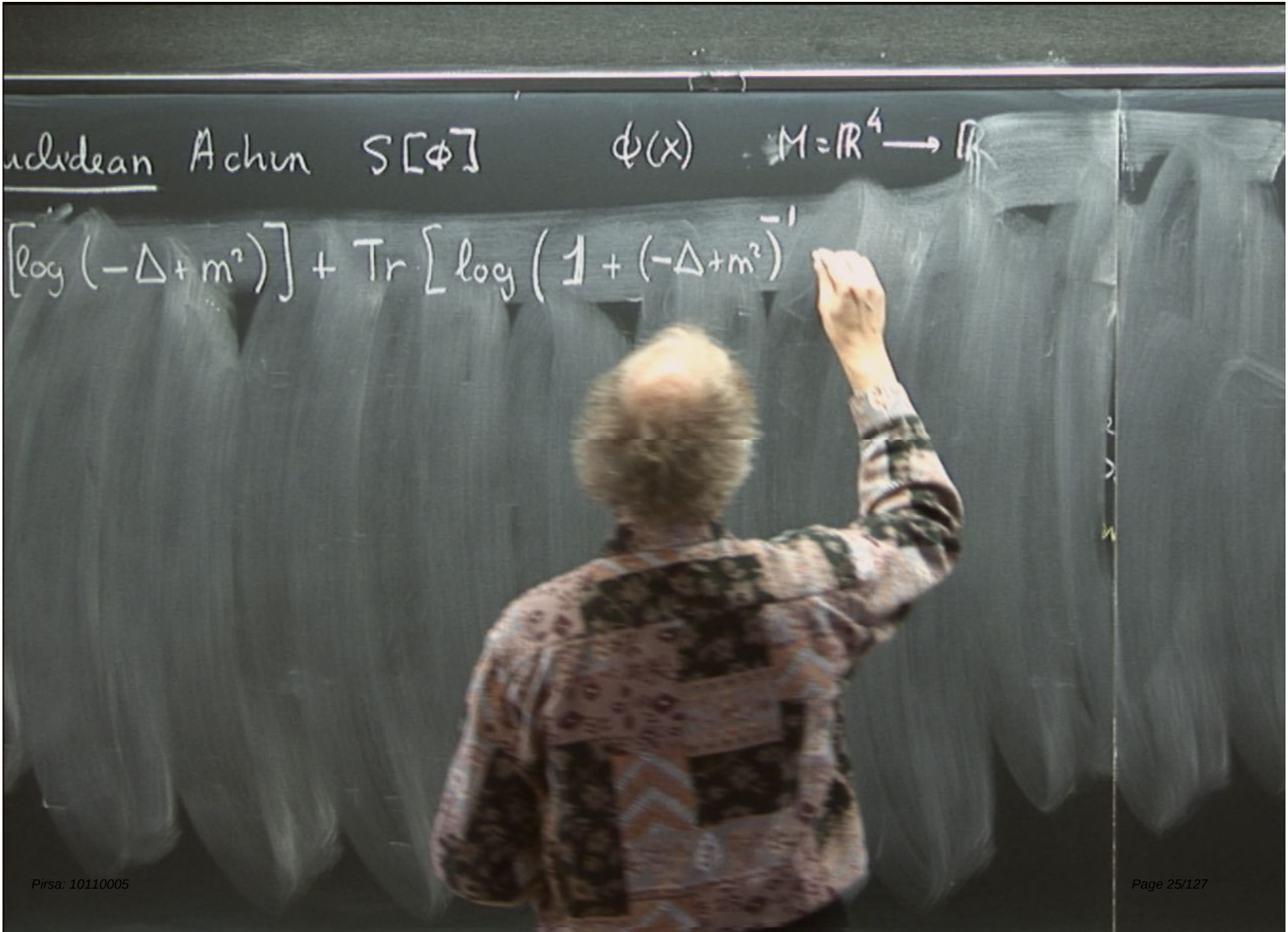
$$\frac{\delta}{\delta\varphi(x)}\frac{\delta}{\delta\varphi(y)} S''[\varphi] = S''[\varphi](x,y)$$

$$S''[\varphi] = (-\Delta + m^2) + \frac{g}{2}\varphi^2(x)$$

$$S''[\varphi](x,y) = (-\Delta_x + m^2)\delta^{(4)}(x-y) + \frac{g}{2}\varphi^2(x)\delta^{(4)}(x-y)$$

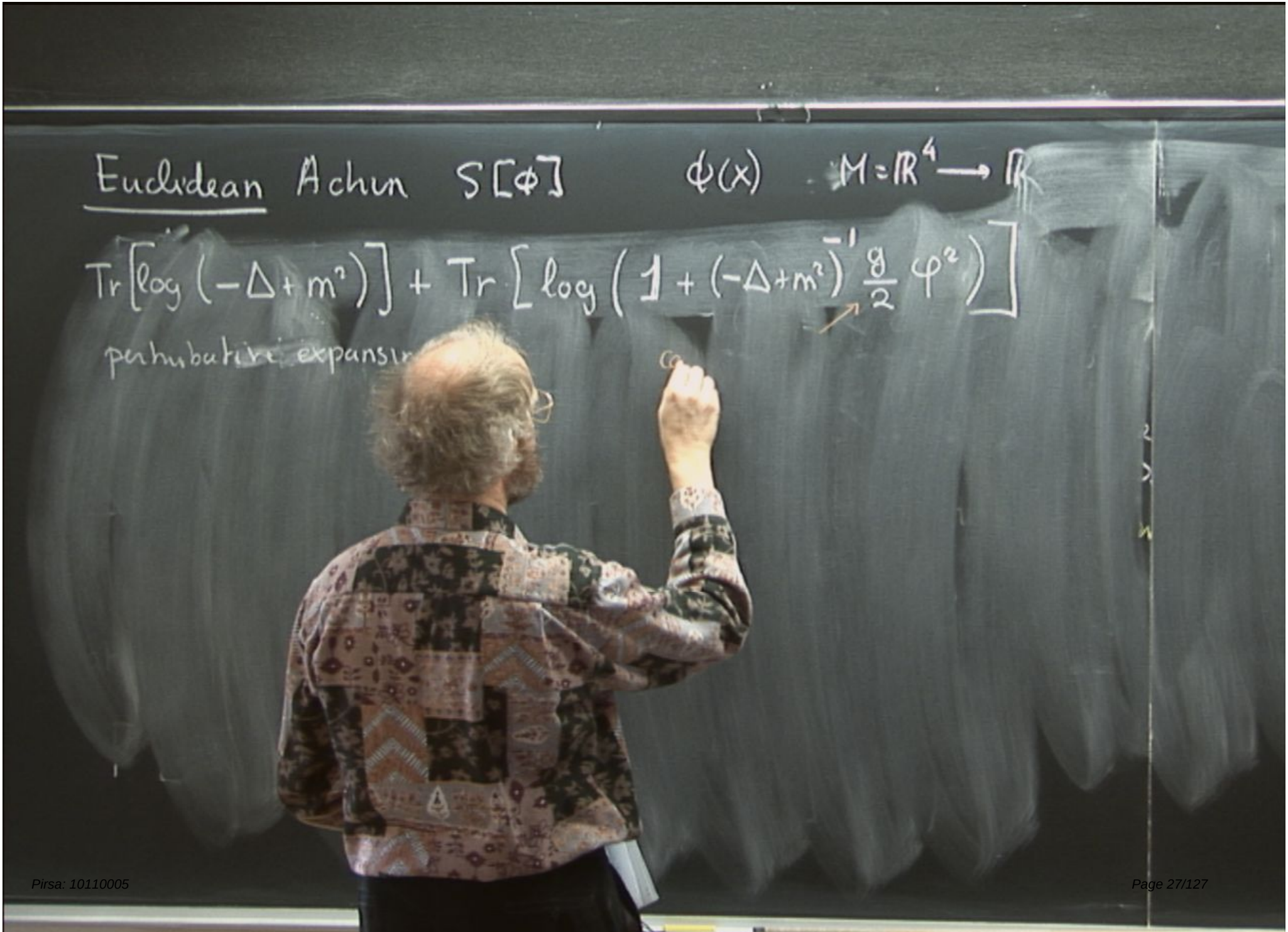
$$\mathrm{Tr}\log(S''[\varphi]) = \mathrm{Tr}\left[\log\left((-\Delta+m^2)\left[\mathbb{1} + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right]\right)\right]$$

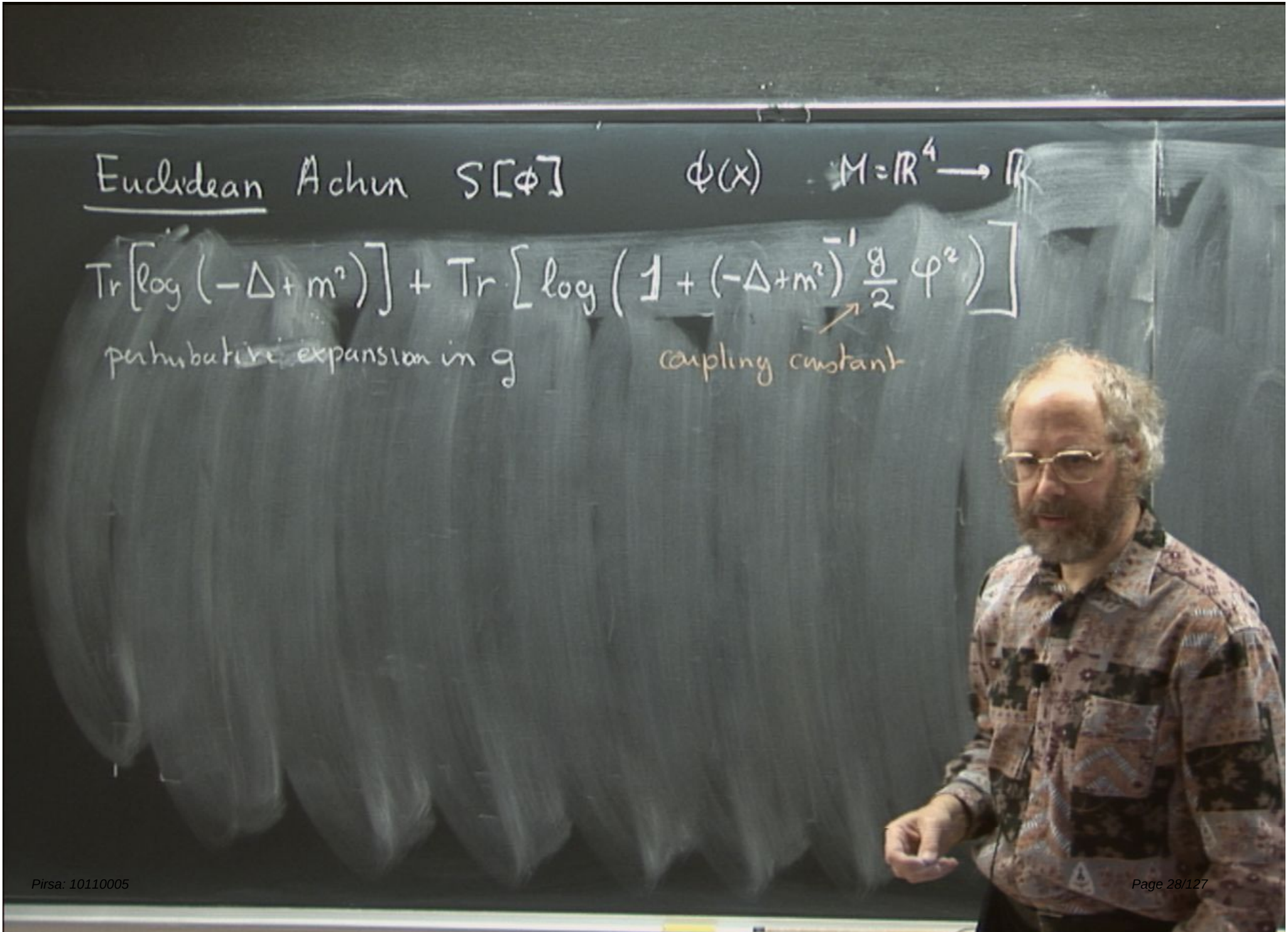
$\mathrm{Tr}\cdot\log = \log\mathrm{Det}$ $\quad \mathrm{Det}(A\cdot B) = \mathrm{Det}(A)\cdot\mathrm{Det}(B)$

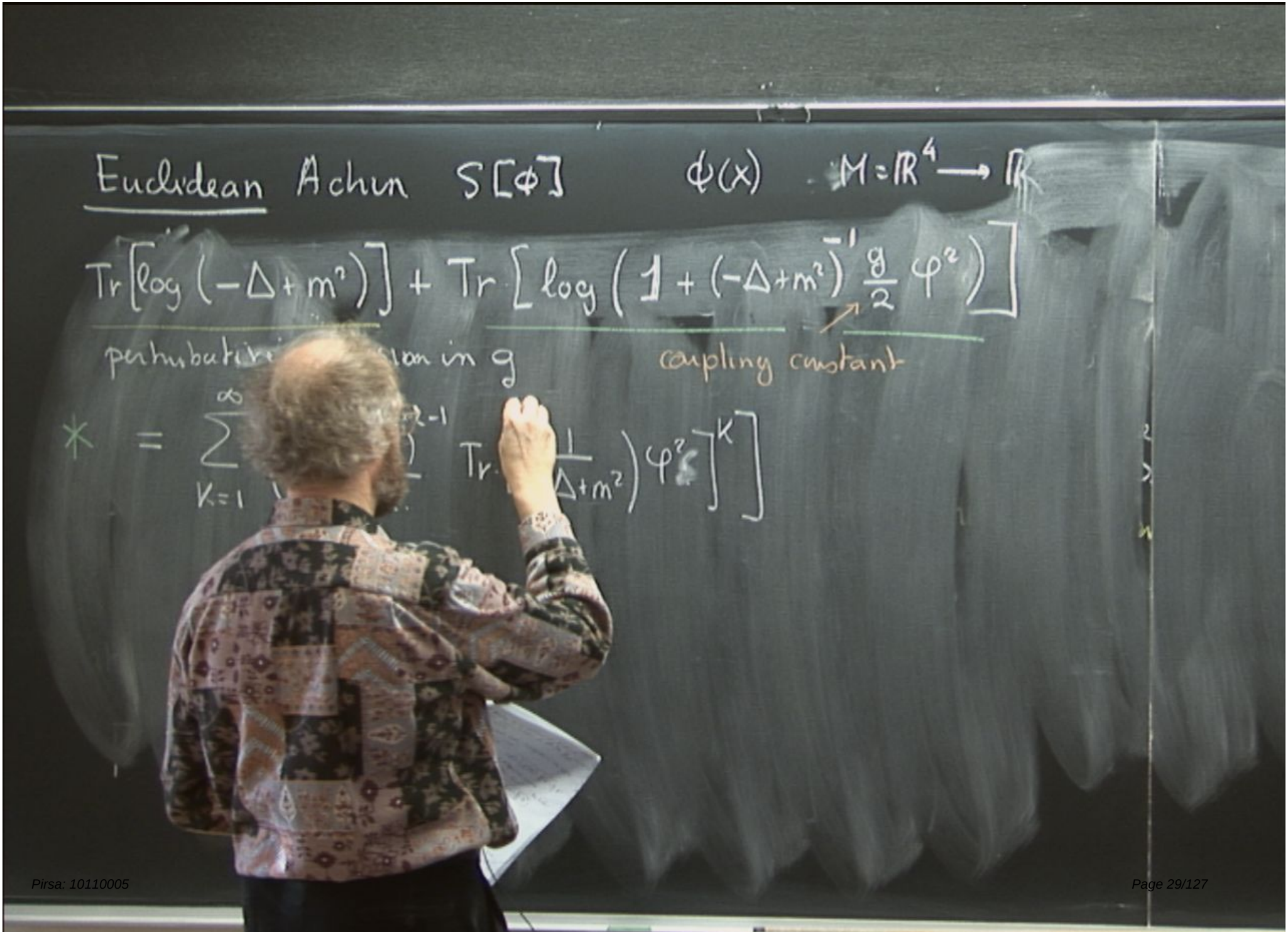

uclidean Achun S[φ] φ(x) M=ℝ⁴ ⟶ ℝ
[log (−Δ+m²)] + Tr [log (1 + (−Δ+m²)⁻¹

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

$$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + \left(-\Delta + m^2\right)^{-1} \frac{g}{2}\varphi^2\right)\right]$$


Euclidean Achun S[φ] φ(x) M = ℝ⁴ → ℝ
Tr[log(−Δ + m²)] + Tr[log(1 + (−Δ+m²)⁻¹ g/2 φ²)]
perturbative expansi


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + \left(-\Delta + m^2\right)^{-1} \frac{g}{2} \varphi^2\right)\right]$
perturbative expansion in g
coupling constant

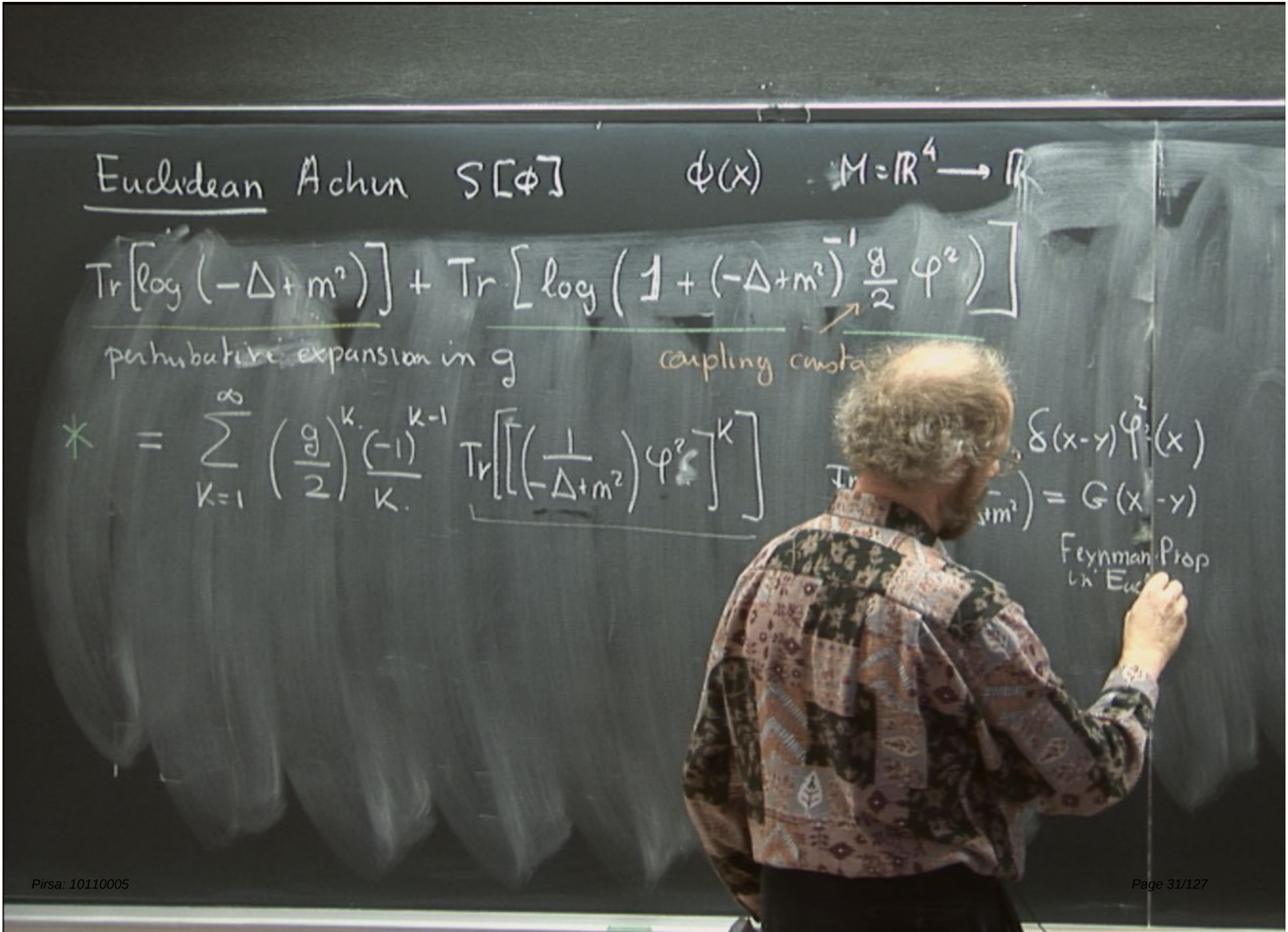

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right]$
coupling constant
$* = \sum_{K=1}^{\infty}$

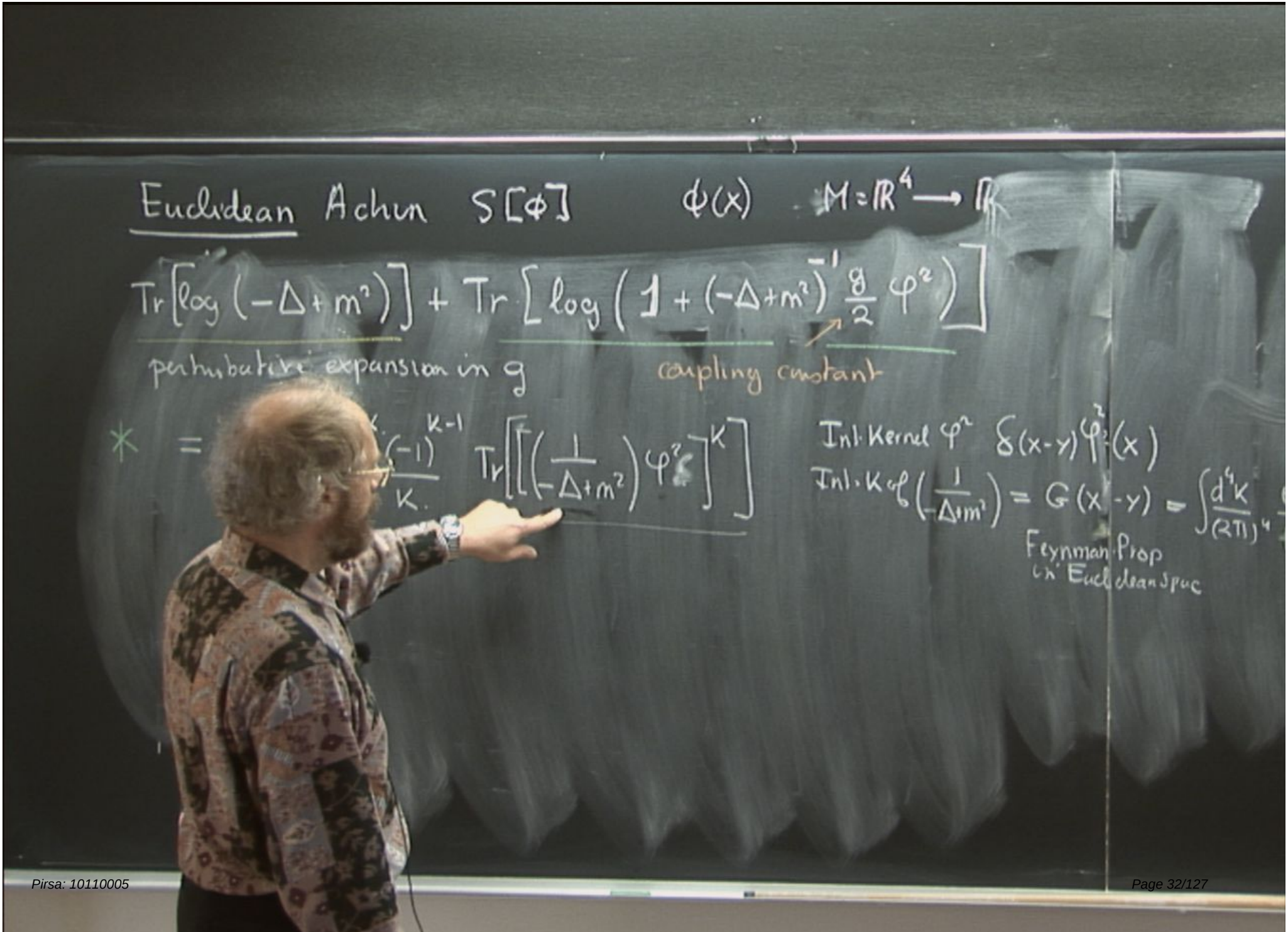
Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

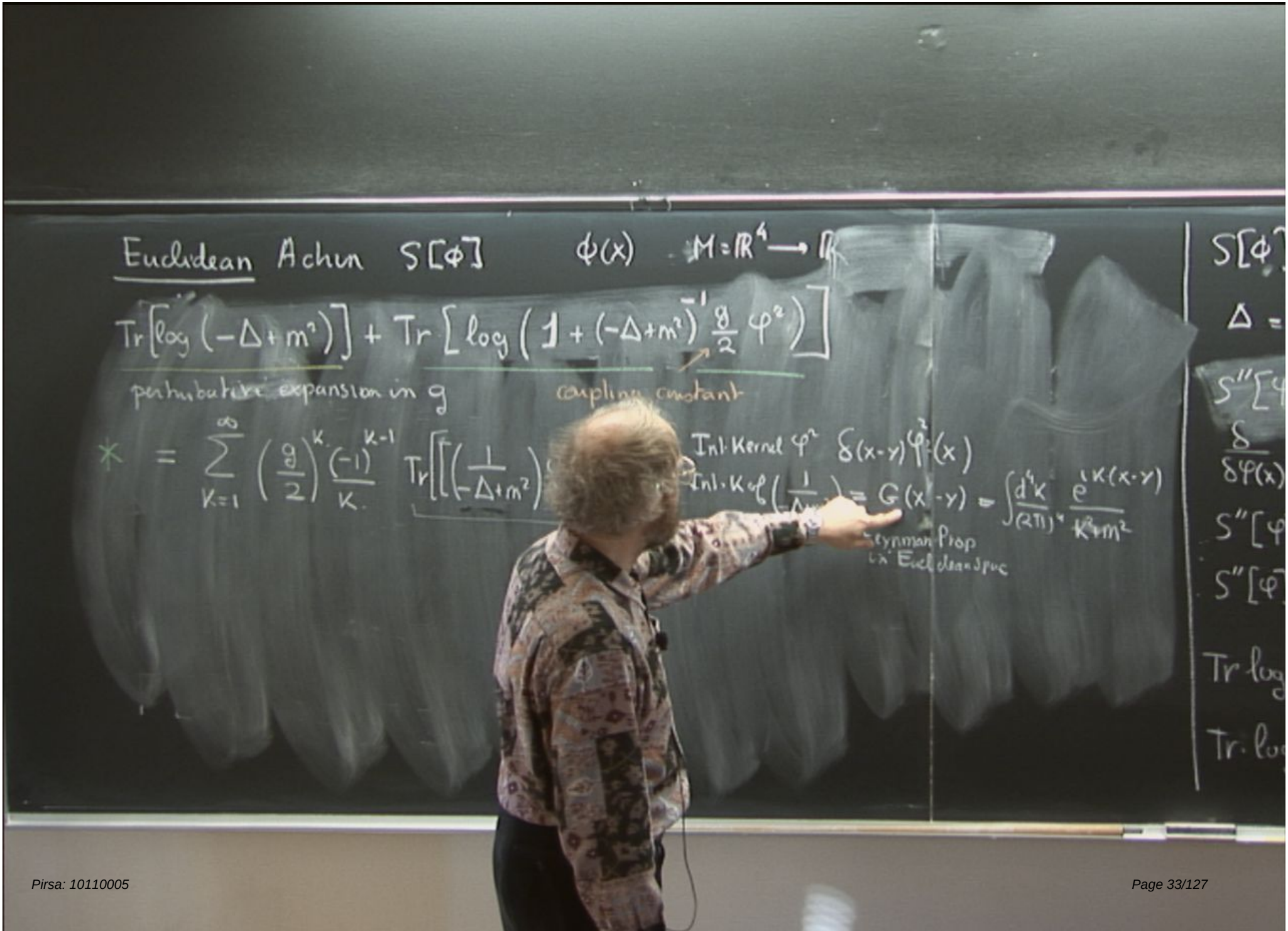
$$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right]$$

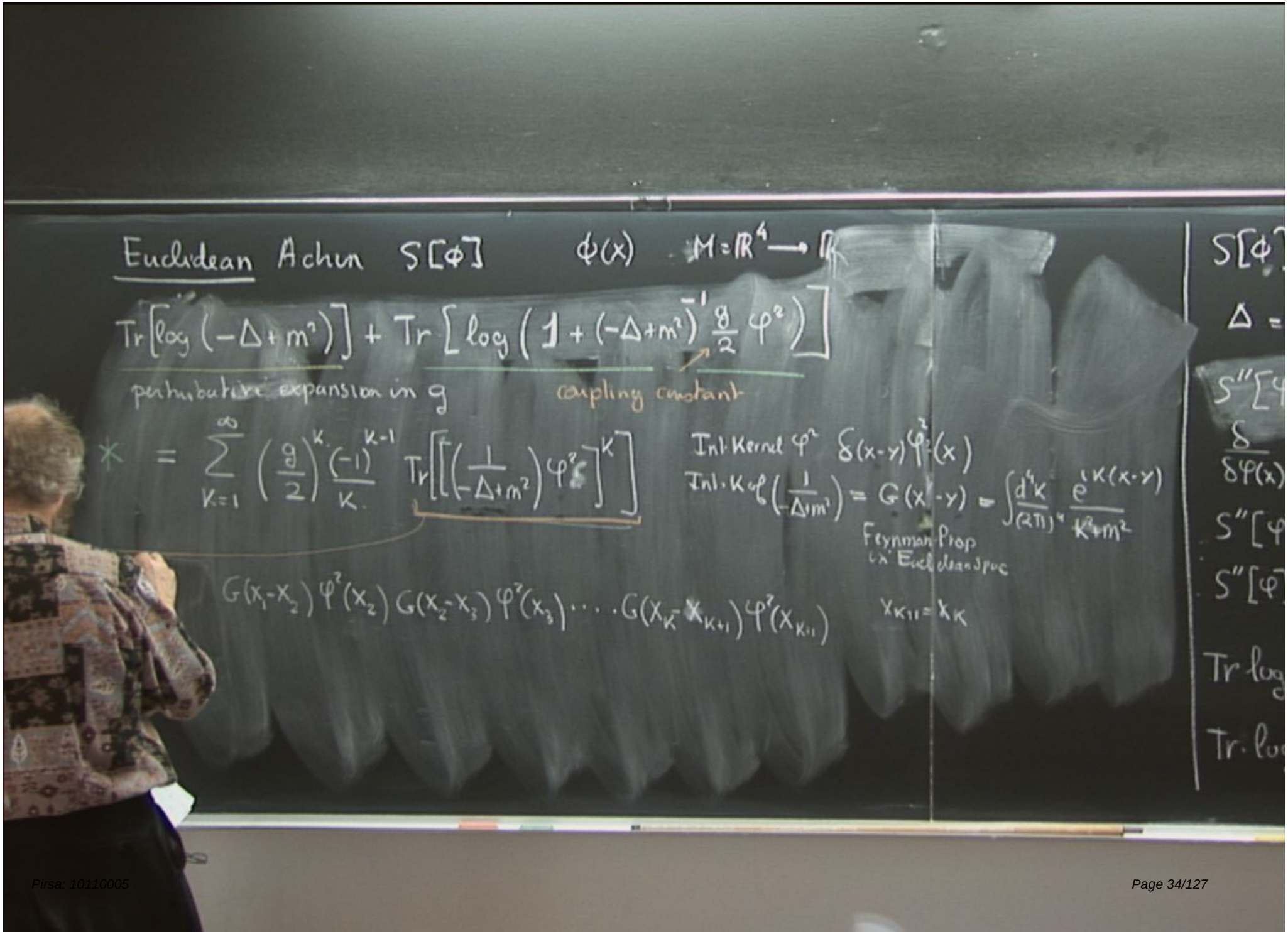
perturbative expansion in g — coupling constant

$$* = \sum_{K=1}^{\infty} \left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K} \mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$$


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right]$
perturbative expansion in g
coupling
$* = \sum_{K=1}^{\infty}\left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K}\,\mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$
$\delta(x-y)\varphi^2(x)$
$= G(x-y)$
Feynman Prop
in Eu


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + \left(-\Delta + m^2\right)^{-1} \frac{g}{2} \varphi^2\right)\right]$
perturbative expansion in g
coupling constant
$* = \sum_{k} \frac{(-1)^{k-1}}{k} \mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta + m^2}\right)\varphi^2\right]^k\right]$
Int. Kernel φ^2 $\delta(x-y)\varphi^2(x)$
Int. Kernel $\left(\frac{1}{-\Delta + m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4}$
Feynman Prop in Euclidean Space


Euclidean Achun S[φ] φ(x) M = ℝ⁴ → ℝ
Tr[log(−Δ+m²)] + Tr[log(1 + (−Δ+m²)⁻¹ g/2 φ²)]
perturbative expansion in g
coupling constant
Inl. Kernel φ² δ(x−y)φ²(x)
Inl. Kernel (1/(−Δ+m²)) = G(x−y) = ∫ d⁴k/(2π)⁴ e^{ik(x−y)}/(k²+m²)
Feynman Prop
in Euclidean Spac
S[φ]
Δ =
δ/δφ(x)
Tr log
Tr. lo


Euclidean Achun $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right]$
perturbative expansion in g
coupling constant
$* = \sum_{K=1}^{\infty}\left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K}\,\mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$
Int. Kernel φ^2 $\delta(x-y)\varphi^2(x)$
Int. Kernel $\left(\frac{1}{-\Delta+m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2+m^2}$
Feynman Prop in Euclidean Spac
$G(x_1-x_2)\varphi^2(x_2)\,G(x_2-x_3)\varphi^2(x_3)\cdots G(x_K-x_{K+1})\varphi^2(x_{K+1})$
$x_{K+1} = x_K$
$S[\phi]$
$\Delta =$
$S''[\varphi]$
$\frac{\delta}{\delta\varphi(x)}$
$S''[\varphi]$
$S''[\varphi]$
Tr log
Tr. log

Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

$$\mathrm{Tr}\left[\log\left(-\Delta + m^2\right)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right]$$

perturbative expansion in g — coupling constant

$$* = \sum_{K=1}^{\infty}\left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K}\,\mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$$

Int. Kernel φ^2 $\delta(x-y)\varphi^2(x)$

Int. Kernel $\left(\frac{1}{-\Delta+m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4}\frac{e^{ik(x-y)}}{k^2+m^2}$

Feynman Prop in Euclidean Space

$$* \int \prod_{i=1}^{K} d^4x_i \left[G(x_1-x_2)\varphi^2(x_2)\,G(x_2-x_3)\varphi^2(x_3)\cdots G(x_K-x_L)\varphi^2(x_L)\right]$$

$x_{K+1} = x_K$

$S[\phi]$

$\Delta =$

$S''[\varphi]$

$\frac{\delta}{\delta\varphi(x)}$

$S''[\varphi]$

$S''[\varphi]$

Tr log

Tr · log

$\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$

$A \cdot B(x,y) = \int d^4z\, A(x,z) B(z,y)$

$\mathrm{Tr} A = \int d^4x\, A(x,x)$

$S[\phi] =$

$+ (-\Delta + m^2)^{-1} \frac{g}{2} \varphi^2 \Big) \Big]$

coupling constant

$\Delta = -\frac{\partial}{\partial}$

$S''[\varphi] =$

$\frac{\delta}{\delta\varphi(x)} \frac{\delta}{\delta\varphi}$

$S''[\varphi] =$

$S''[\varphi](x,$

Infl. Kernel φ^2 $\delta(x-y)\varphi^2(x)$

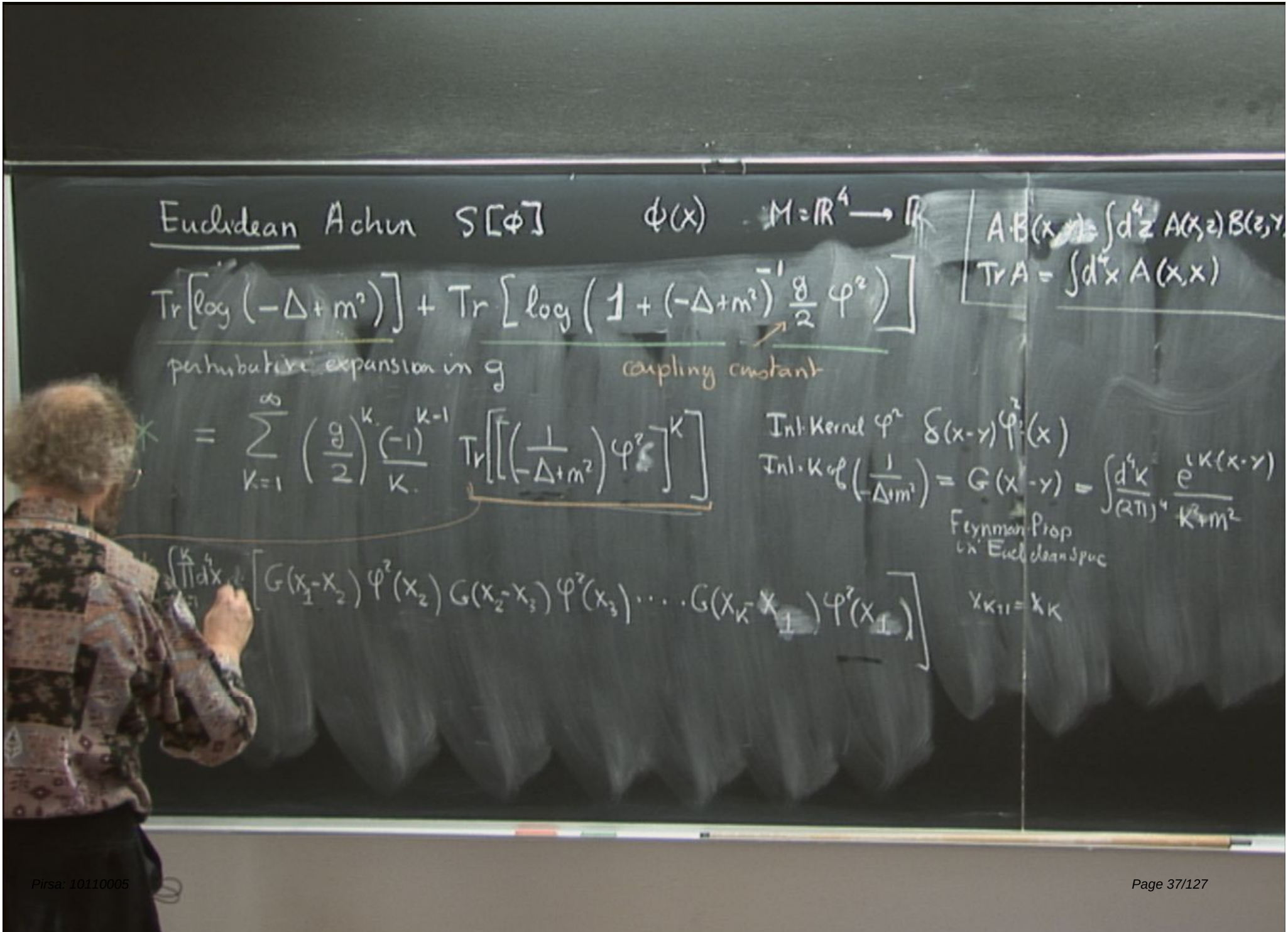
Infl. Kugel $\left(\frac{1}{-\Delta+m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$

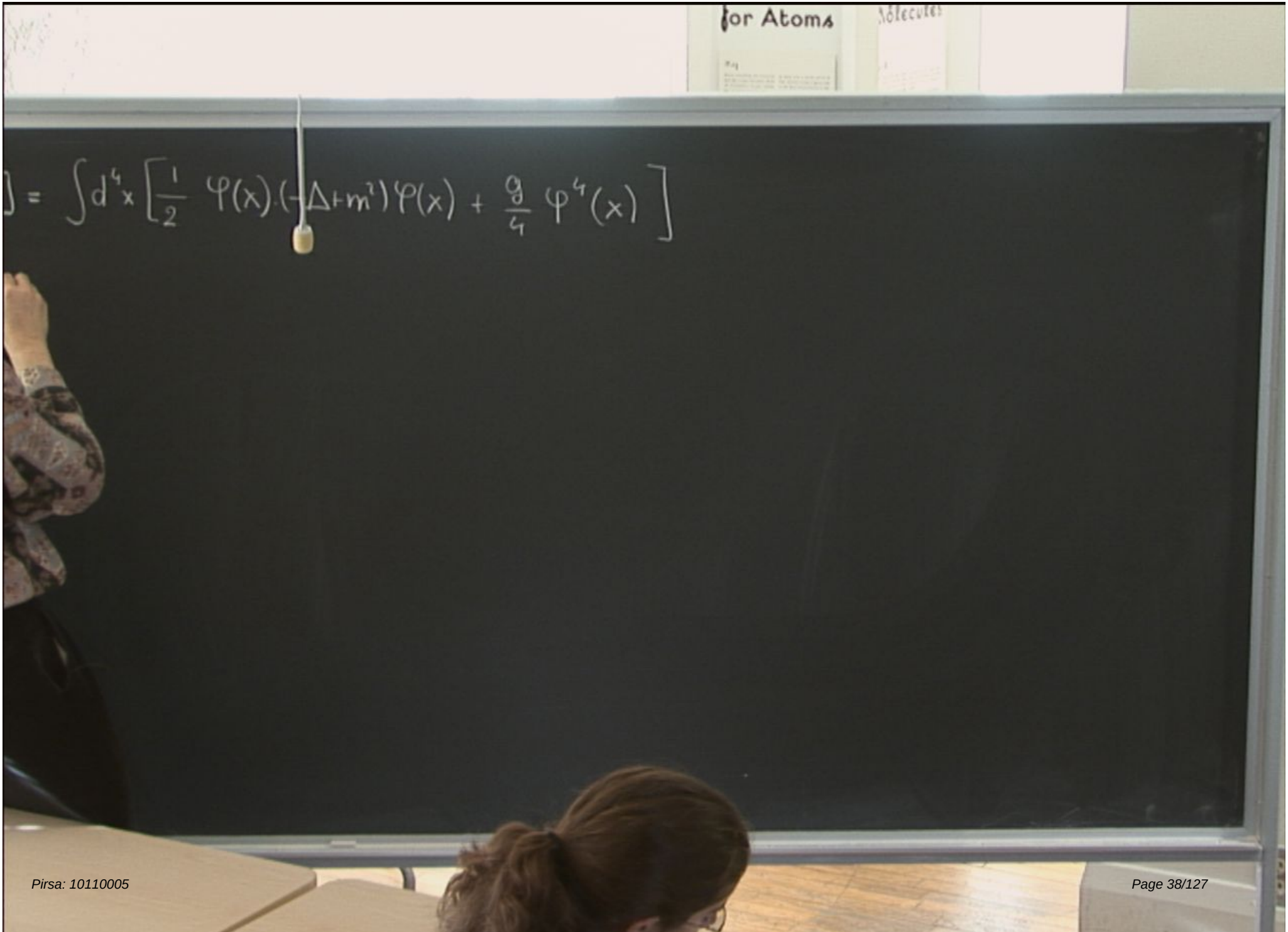
Feynman Prop in Euclidean Spac

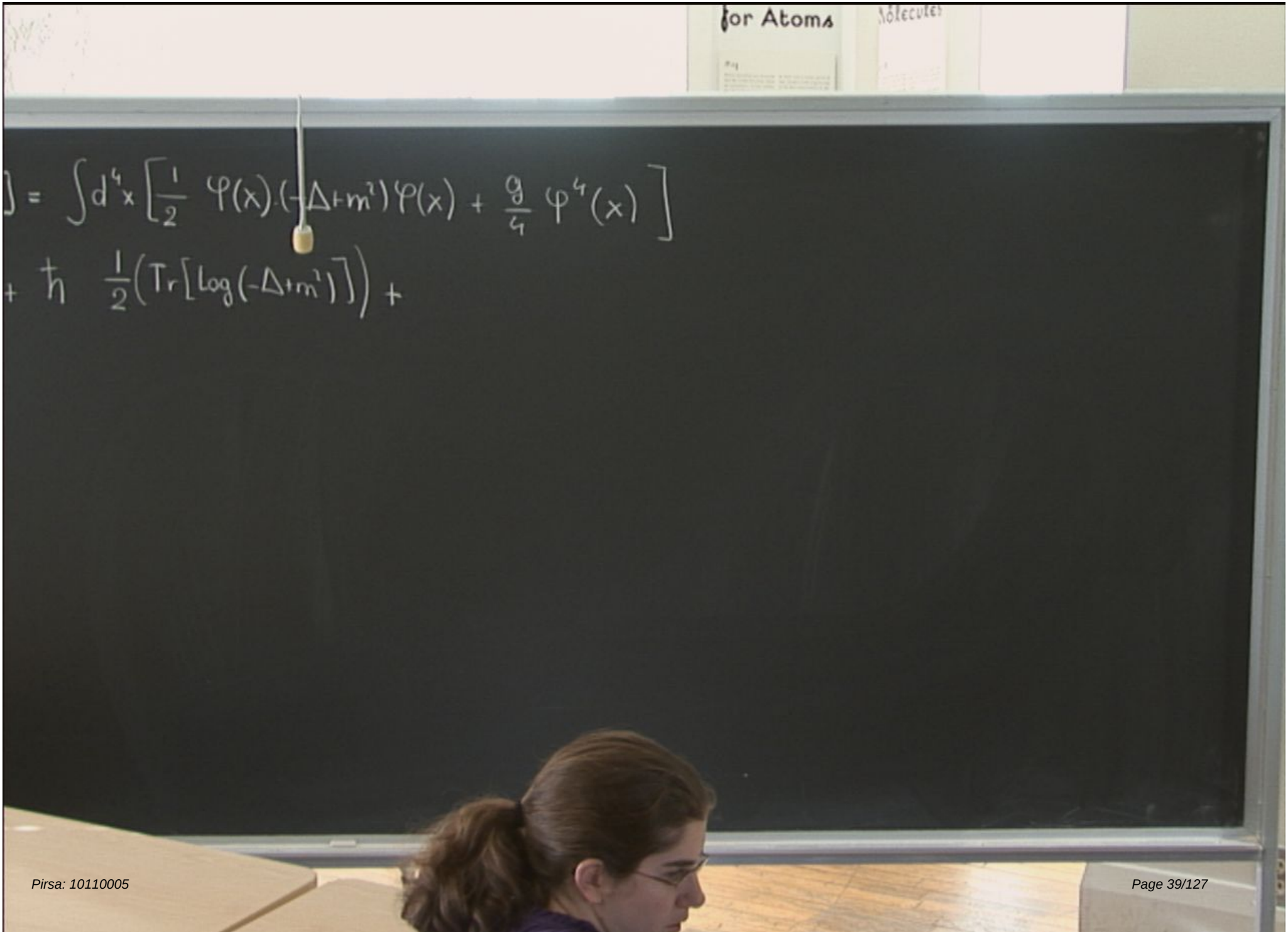
$\ldots G(x_k - x_l) \varphi^2(x_l) \Big]$

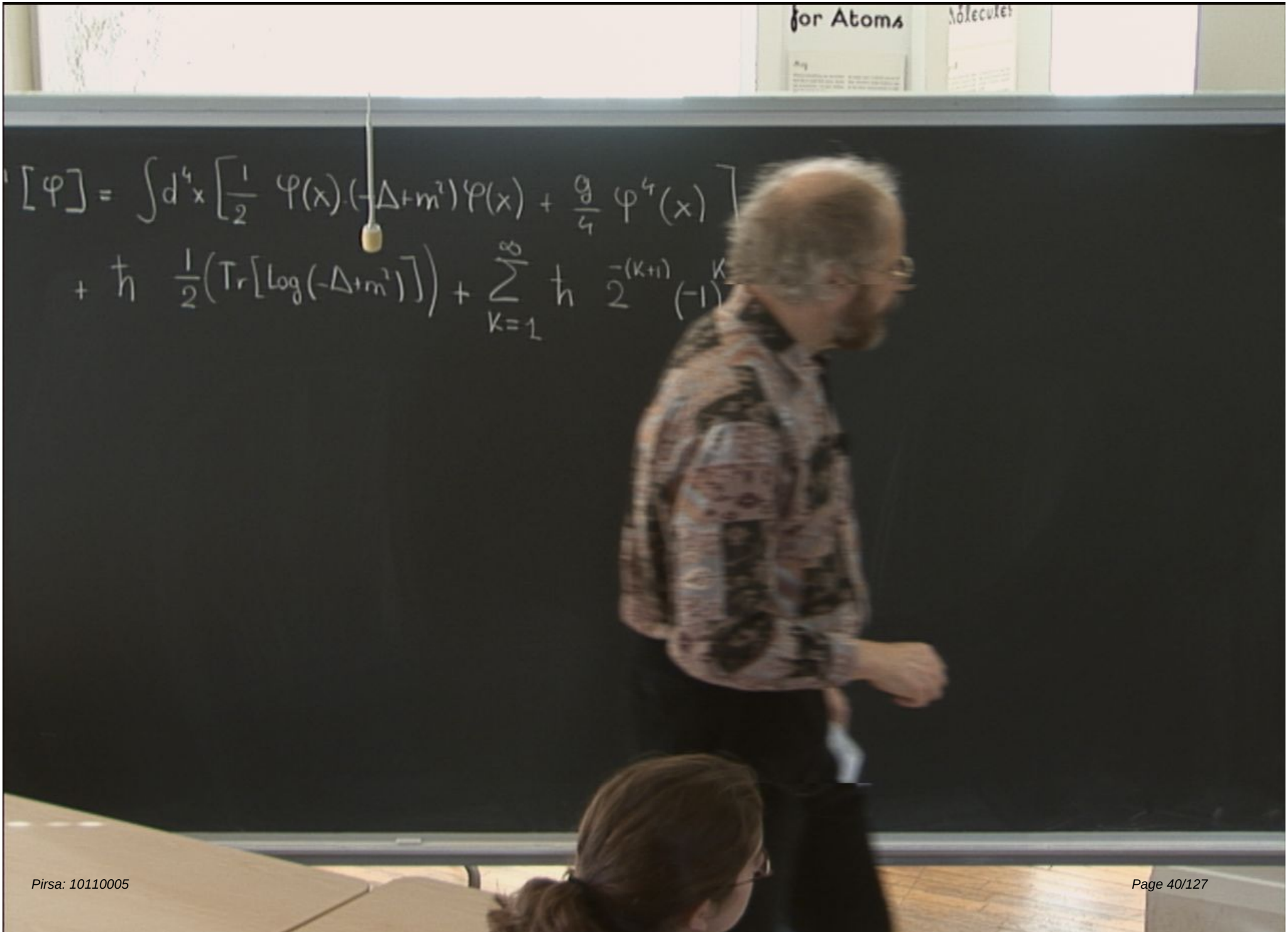
$x_{k+1} = x_k$

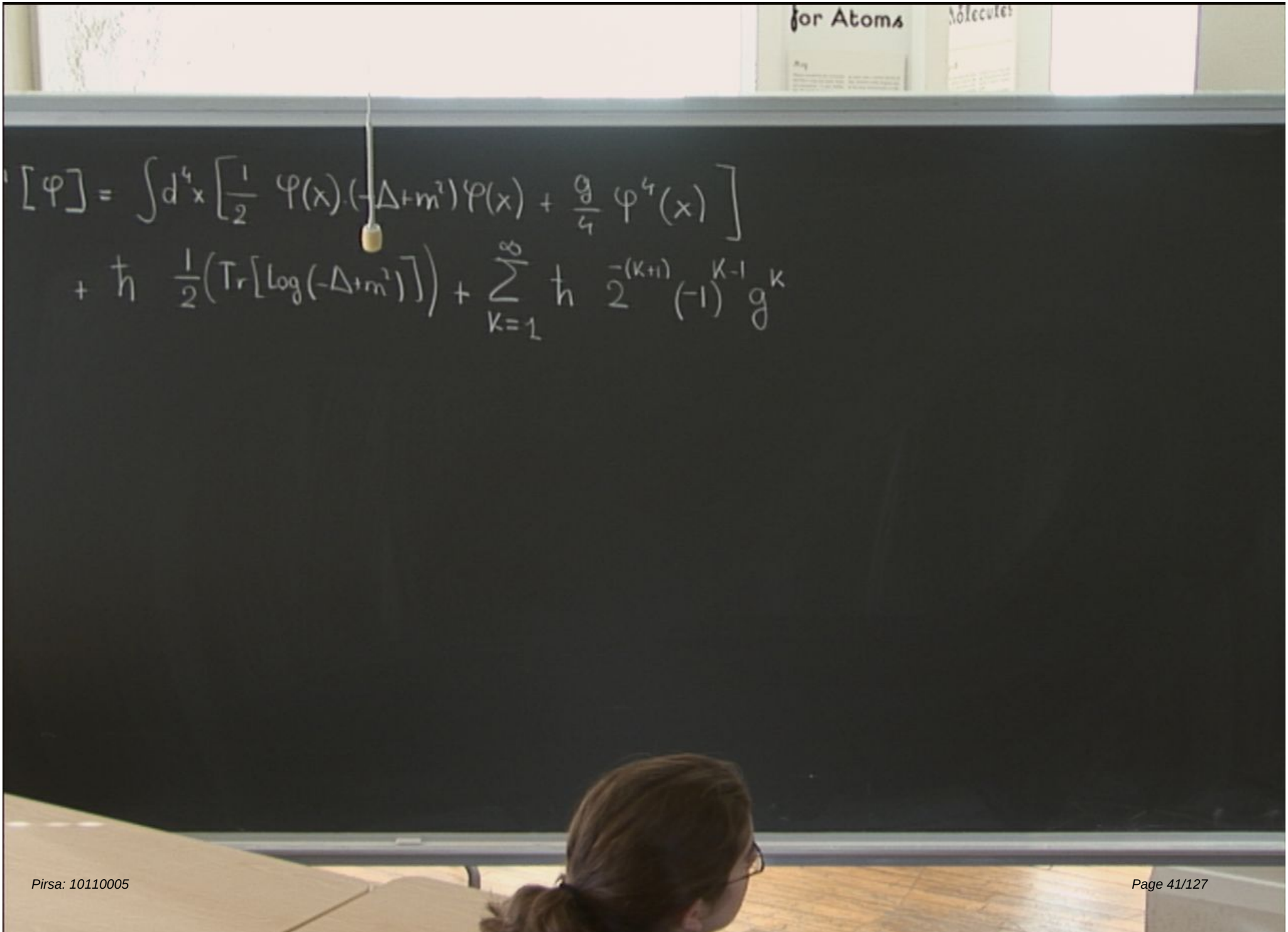
$\mathrm{Tr} \log(S'$

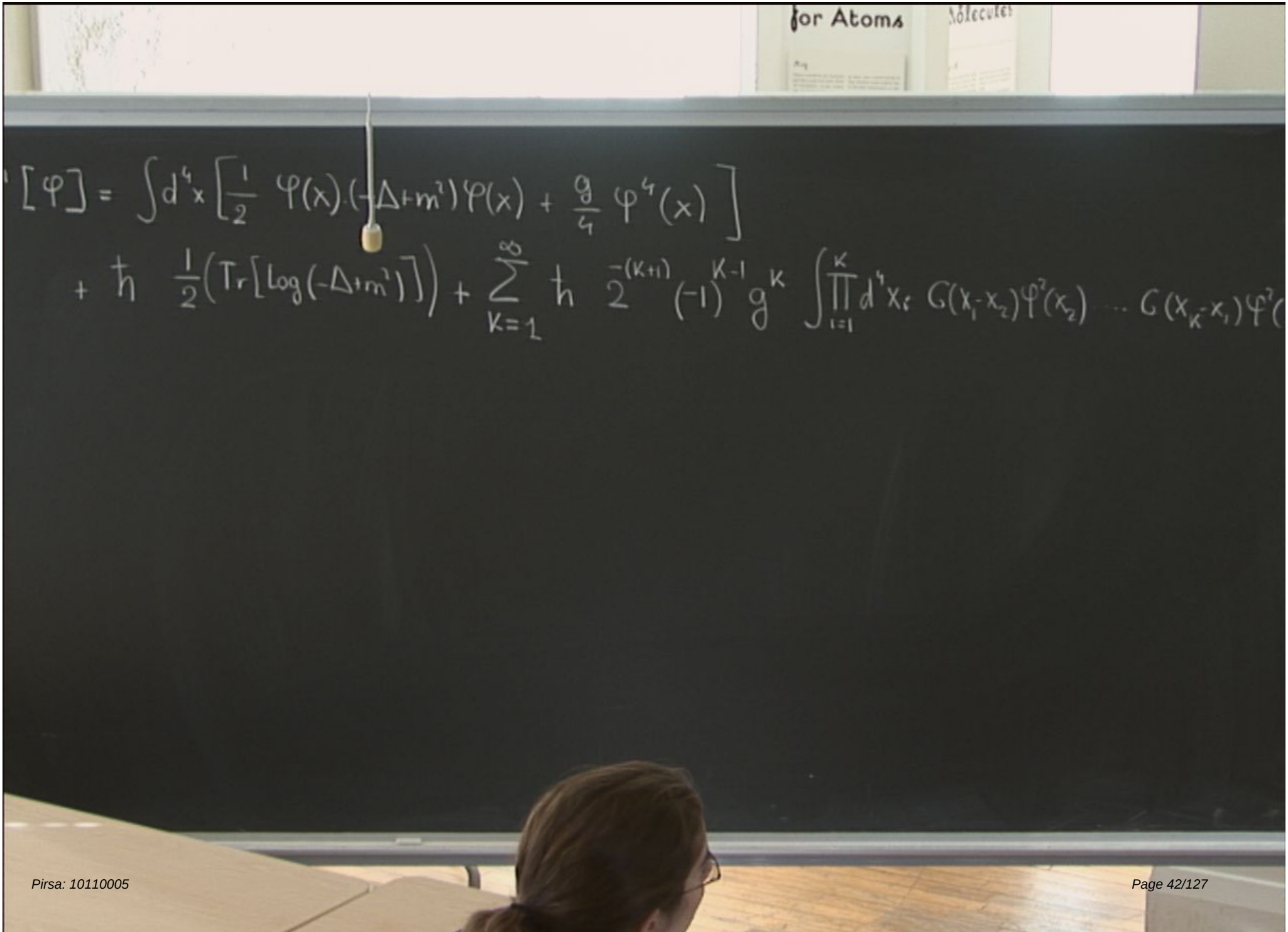

Euclidean Achun $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$A \cdot B(x,y) = \int d^4z\, A(x,z) B(z,y)$
$\mathrm{Tr}\, A = \int d^4x\, A(x,x)$
$\mathrm{Tr}\left[\log(-\Delta + m^2)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1} \frac{g}{2} \varphi^2\right)\right]$
perturbative expansion in g
coupling constant
$= \sum_{K=1}^{\infty} \left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K} \mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$
Int. Kernel φ^2 $\delta(x-y)\varphi^2(x)$
Int. Kernel $\left(\frac{1}{-\Delta+m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2+m^2}$
Feynman Prop in Euclidean Spac
$\int \prod_{i=1}^{K} d^4x_i \left[G(x_1-x_2)\varphi^2(x_2) G(x_2-x_3)\varphi^2(x_3) \cdots G(x_K-x_1)\varphi^2(x_1)\right]$
$x_{K+1} = x_K$

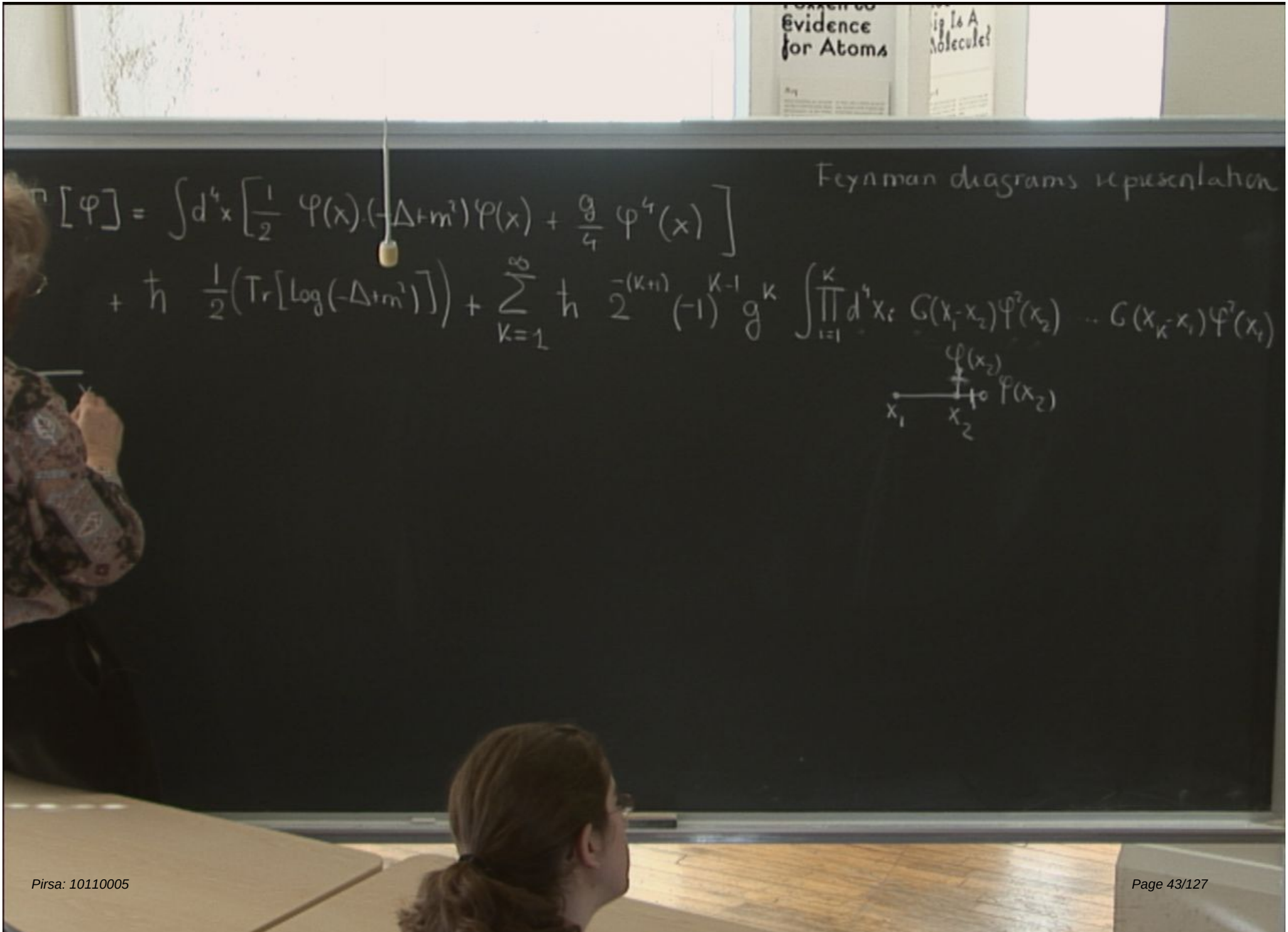

for Atoms
$= \int d^4x \left[\frac{1}{2} \varphi(x) (-\Delta + m^2) \varphi(x) + \frac{g}{4} \varphi^4(x) \right]$

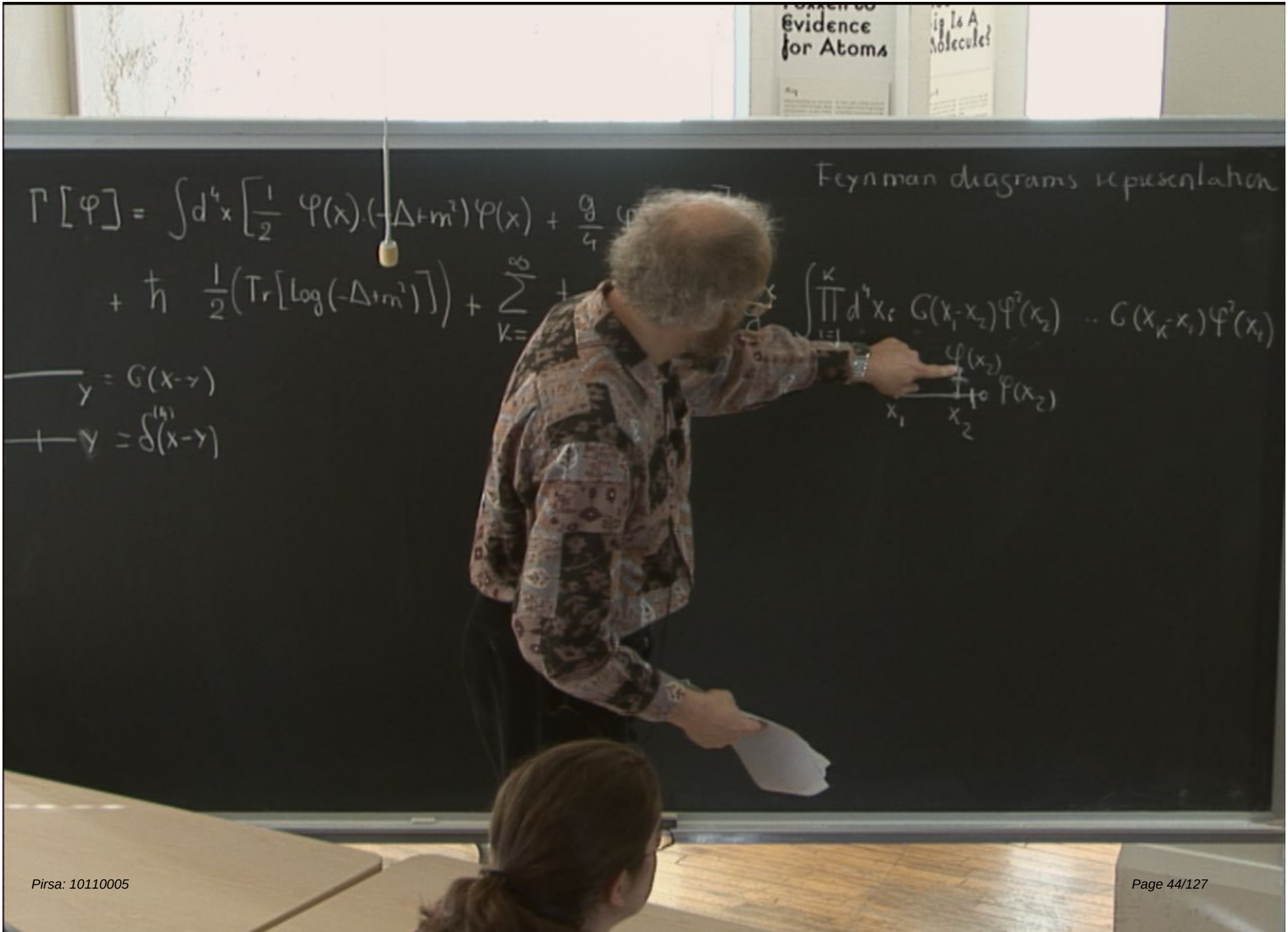

for Atoms
$= \int d^4x \left[\frac{1}{2} \varphi(x) (-\Delta + m^2) \varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2} \left(\mathrm{Tr} \left[\log(-\Delta + m^2) \right] \right) +$

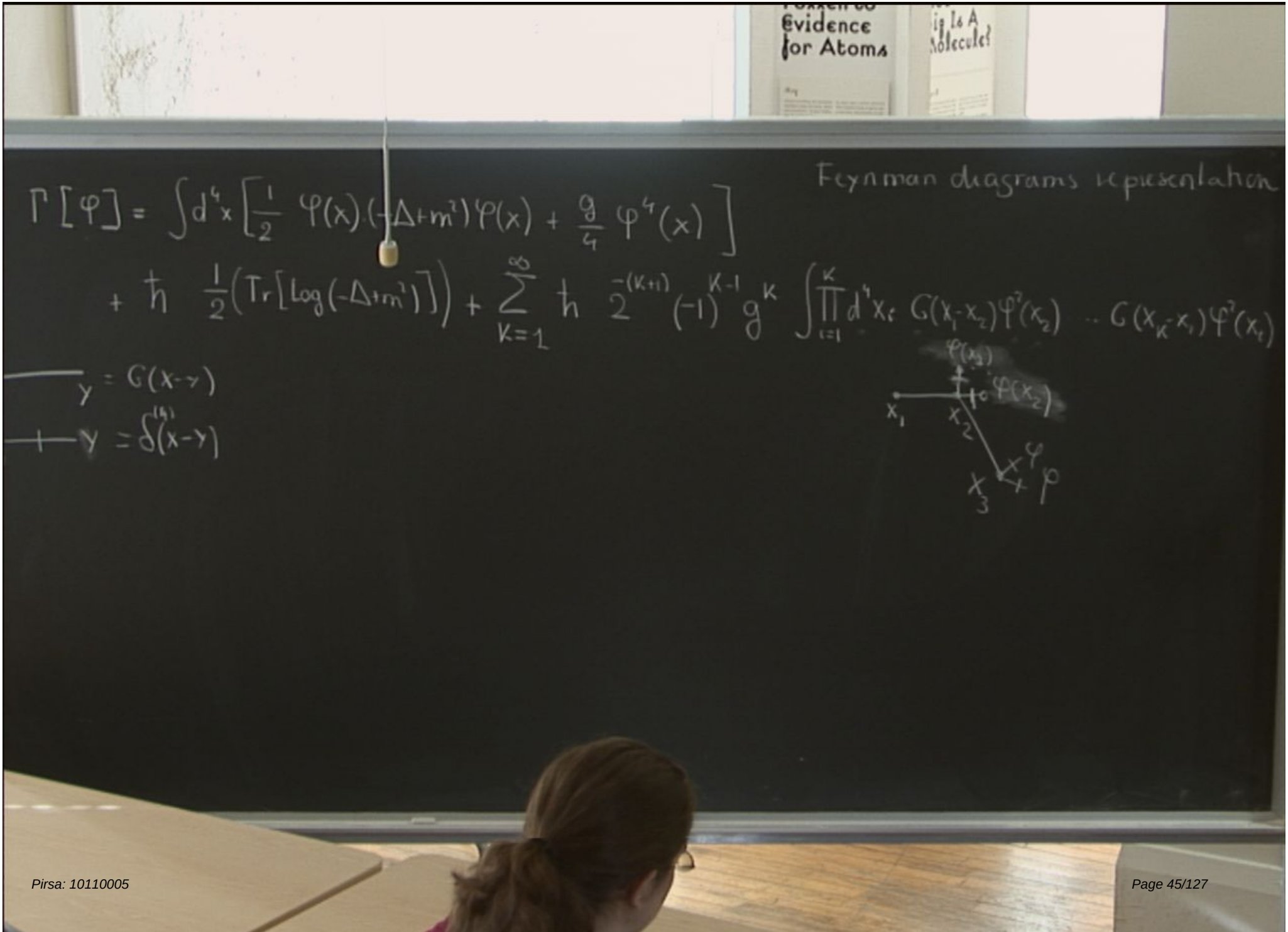

for Atoms
Molecules
$[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}[\log(-\Delta + m^2)]\right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^K$

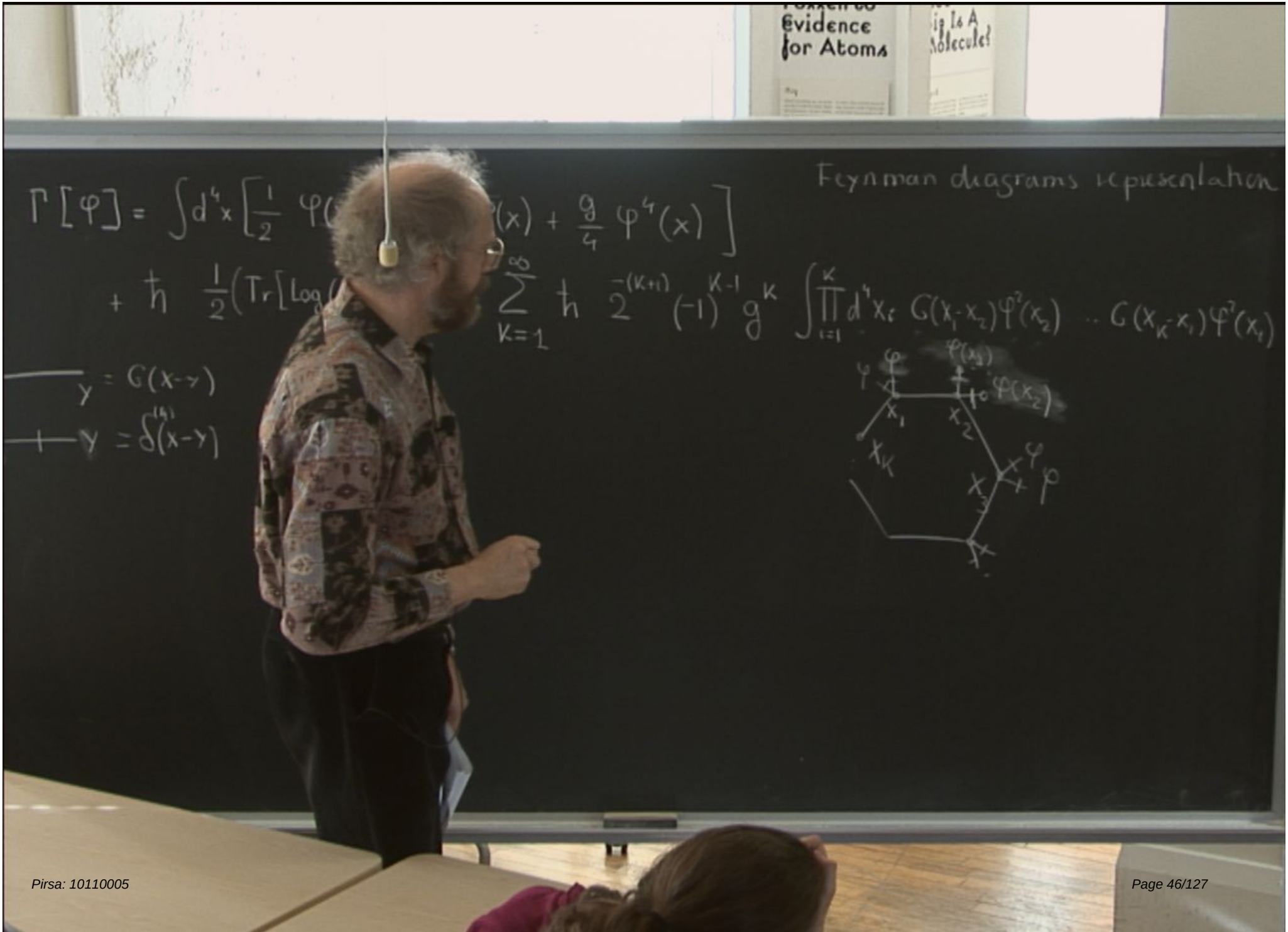

for Atoms
$[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \left(\frac{1}{2} \left(\mathrm{Tr}[\log(-\Delta + m^2)] \right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \right.$

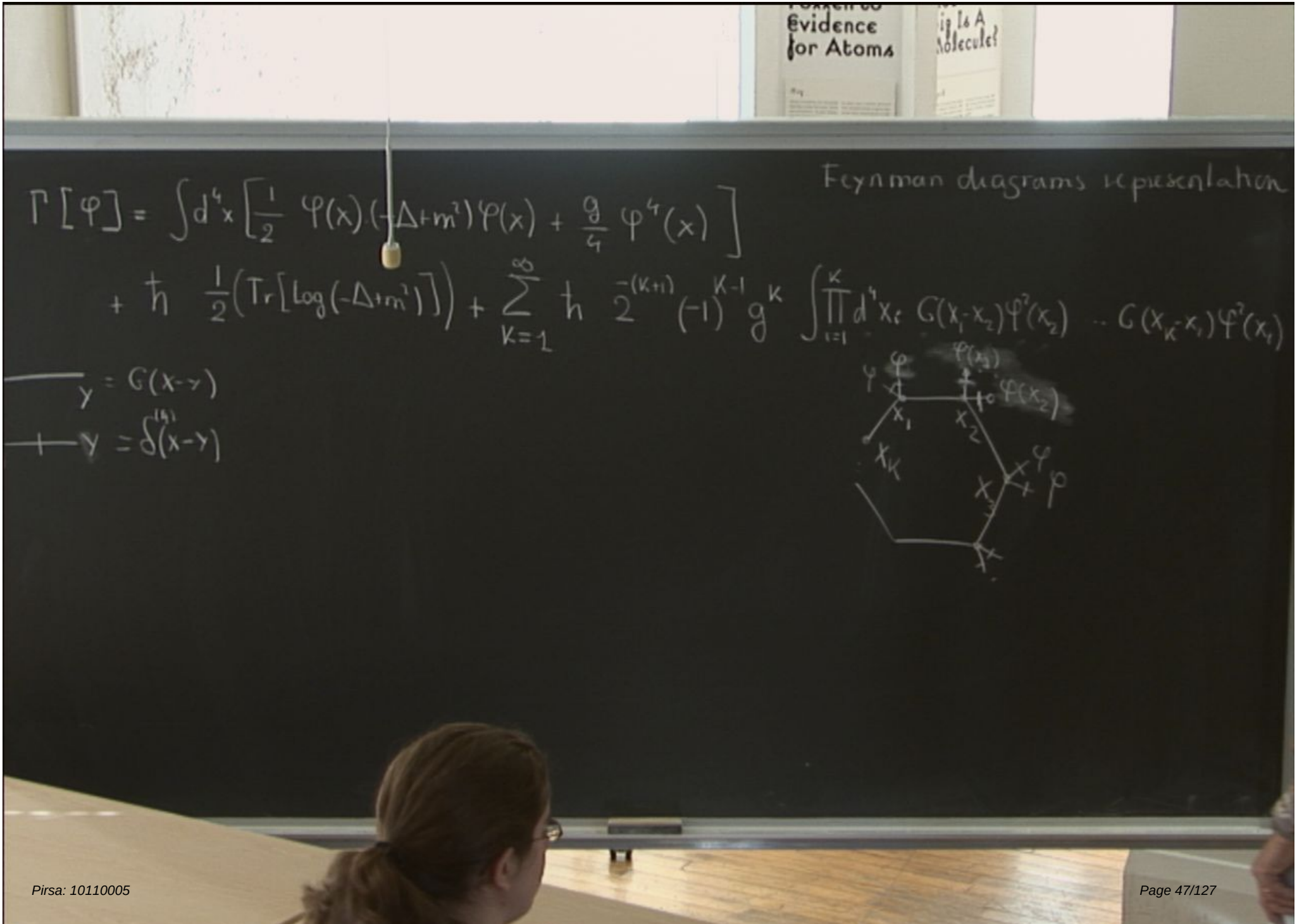

for Atoms
Molecules
$[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}[\log(-\Delta + m^2)]\right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \, G(x_1 - x_2)\varphi^2(x_2) \cdots G(x_K - x_1)\varphi^2($

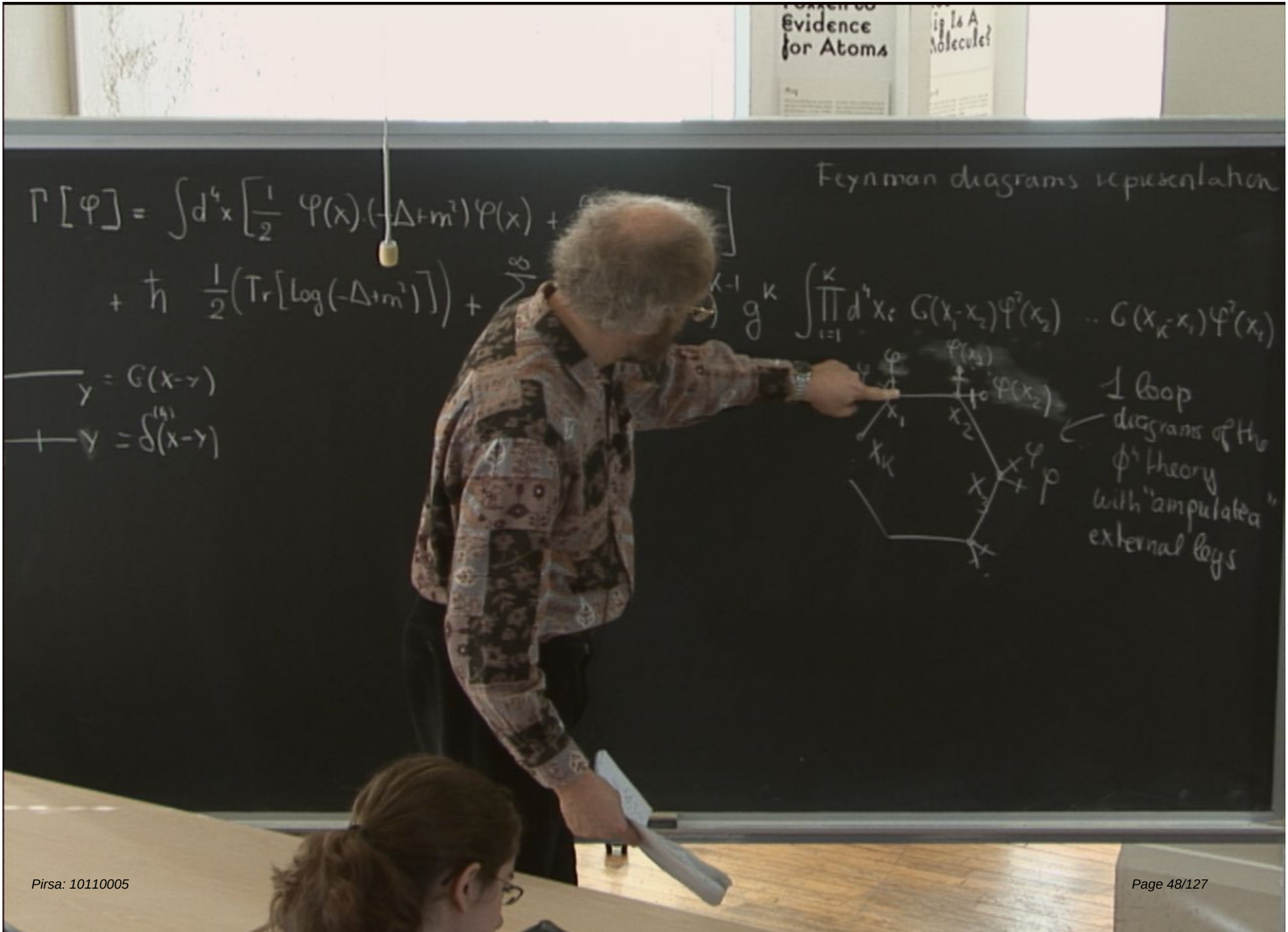

Evidence
for Atoms
Is A
Molecule?
Feynman diagrams representation
$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}[\log(-\Delta + m^2)]\right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \, G(x_1 - x_2)\varphi^2(x_2) \cdots G(x_K - x_1)\varphi^2(x_1)$
$\varphi(x_2)$
$\varphi(x_2)$
x_1
x_2

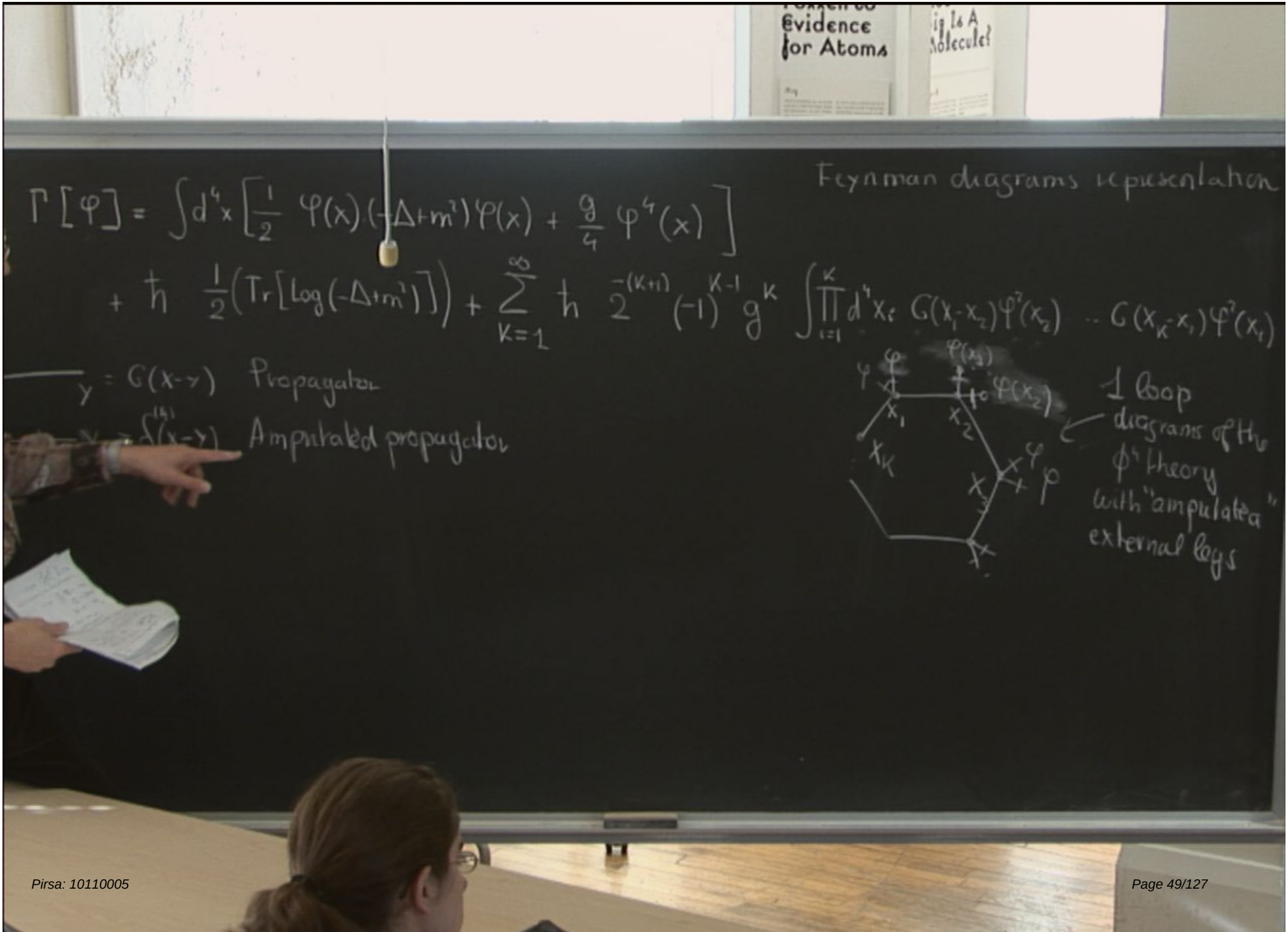

Evidence
for Atoms
Is A
Molecule?
Feynman diagrams representation
$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2}\varphi(x)(-\Delta+m^2)\varphi(x) + \frac{g}{4}\right.$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}\left[\mathrm{Log}(-\Delta+m^2)\right]\right) + \sum_{K=}^{\infty}$
$\int \prod_{i=1}^{K} d^4x_i \; G(x_1-x_2)\varphi^2(x_2) \cdots G(x_K-x_1)\varphi^2(x_1)$
$y = G(x-y)$
$y = \delta(x-y)$
$\varphi(x_2)$
$\varphi(x_2)$
x_1
x_2

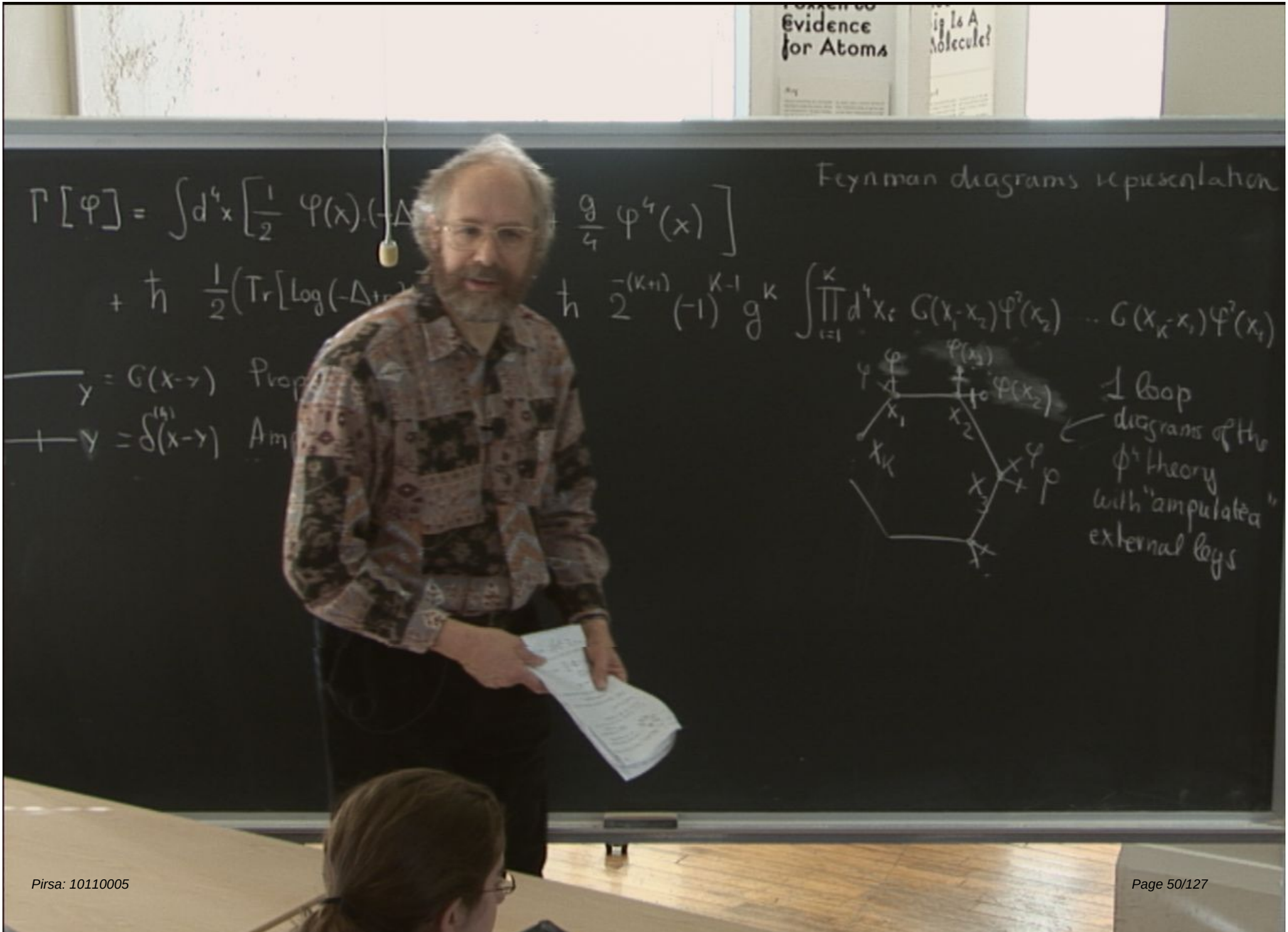

Evidence
for Atoms
Is A
Molecule?
Feynman diagrams representation
$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}\left[\mathrm{Log}(-\Delta + m^2)\right]\right) + \sum_{K=1}^{\infty} \hbar\, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i\, G(x_1 - x_2)\varphi^2(x_2) \cdots G(x_K - x_1)\varphi^2(x_1)$
$= G(x-y)$
$= \delta(x-y)$
$\varphi(x_3)$
$\varphi(x_2)$
x_1
x_2
x_3
x_4
φ
φ


Evidence
for Atoms
Feynman diagrams representation


Evidence
for Atoms
Is A
Molecule?
Feynman diagrams representation
$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}\left[\mathrm{Log}(-\Delta + m^2)\right]\right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \, G(x_1 - x_2)\varphi^2(x_2) \cdots G(x_K - x_1)\varphi^2(x_1)$
$= G(x-y)$
$= \delta(x-y)$
$\varphi(x_3)$
$\varphi(x_2)$
x_1
x_2
x_K


Evidence
for Atoms
Is A
Molecule?
Feynman diagrams representation
$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2}\varphi(x)(-\Delta+m^2)\varphi(x) + \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}\left[\log(-\Delta+m^2)\right]\right) + \sum^{\infty}$
$g^K \int \prod_{i=1}^{K} d^4x_i \; G(x_1-x_2)\varphi^2(x_2) \cdots G(x_K-x_1)\varphi^2(x_1)$
$= G(x-y)$
$= \delta(x-y)$
1 loop
diagrams of the
φ^4 theory
with "amputated"
external legs


Evidence for Atoms
Is A Molecule?
Feynman diagrams representation
$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$
$+ \hbar \frac{1}{2}\left(\mathrm{Tr}\left[\mathrm{Log}(-\Delta + m^2)\right]\right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \, G(x_1 - x_2)\varphi^2(x_2) \cdots G(x_K - x_1)\varphi^2(x_1)$
y = $G(x-y)$ Propagator
Amputated propagator
1 loop diagrams of the φ^4 theory with "amputated" external legs


Evidence
for Atoms
Is A
Molecule?
Feynman diagrams representation
Prop
Am
1 loop
diagrams of the
φ⁴ theory
with "amputated"
external legs

Grains of Pollen to Evidence for Atoms

How is It A Molecule?

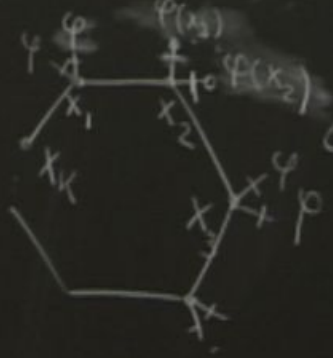
Feynman diagrams representation

$$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x)(-\Delta + m^2)\varphi(x) + \frac{g}{4} \varphi^4(x) \right]$$

$$+ \hbar \frac{1}{2} \left(\mathrm{Tr}\left[\log(-\Delta + m^2) \right] \right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \, G(x_1 - x_2)\varphi^2(x_2) \cdots G(x_K - x_1)\varphi^2(x_1)$$

x —— y $= G(x-y)$ Propagator

x —|— y $= \delta(x-y)$ Amputated propagator



1 loop diagrams of the φ^4 theory with "amputated" external legs

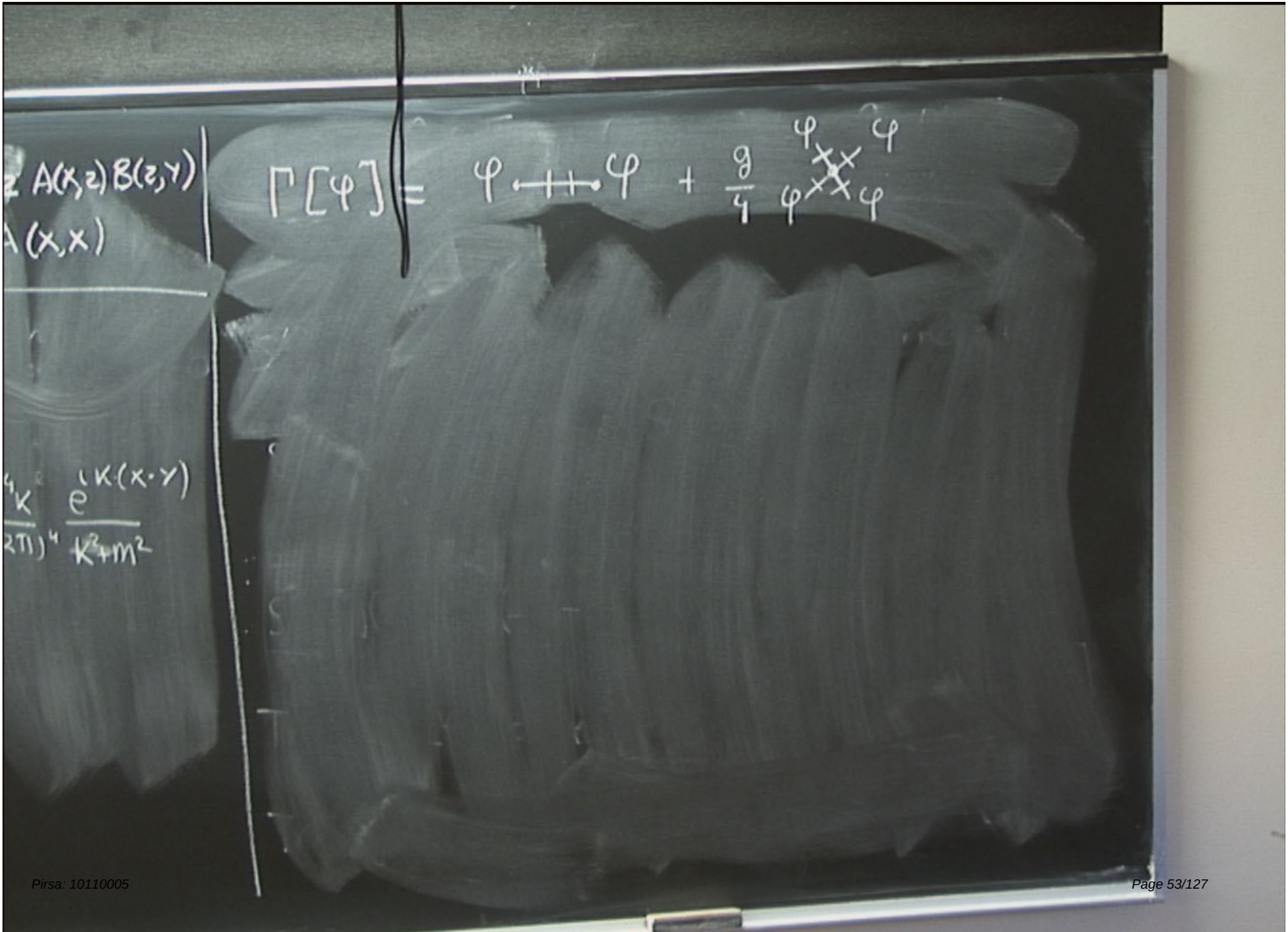
$$\Gamma[\varphi] = \int d^4x \left[\frac{1}{2} \varphi(x) \cdot (-\Delta + m^2) \varphi(x) + \frac{g}{4} \varphi^4(x) \right]$$

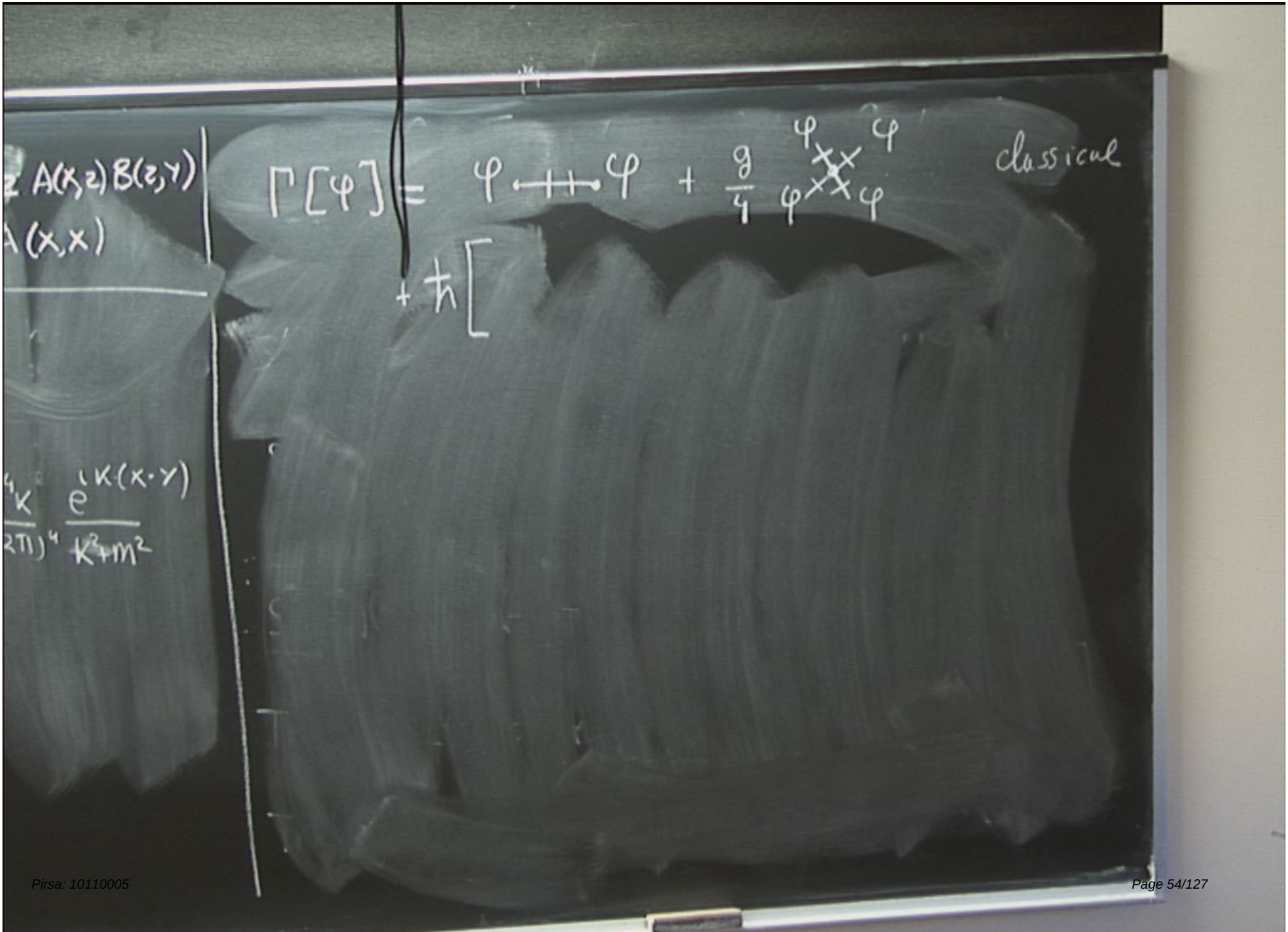
$$+ \hbar \, \frac{1}{2} \left(\mathrm{Tr}\left[\mathrm{Log}(-\Delta + m^2) \right] \right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g$$

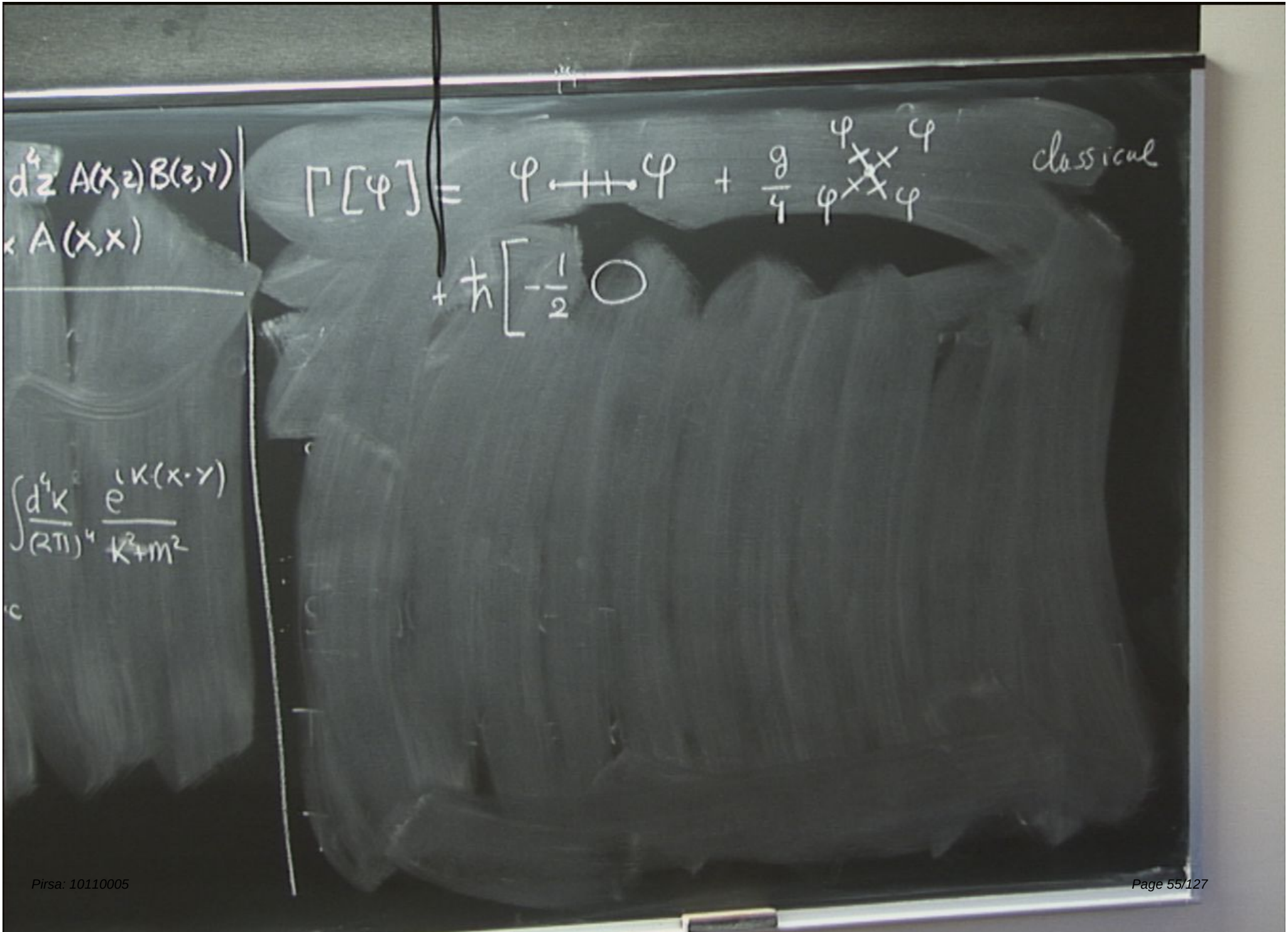
x —— y $= G(x-y)$ Propagator

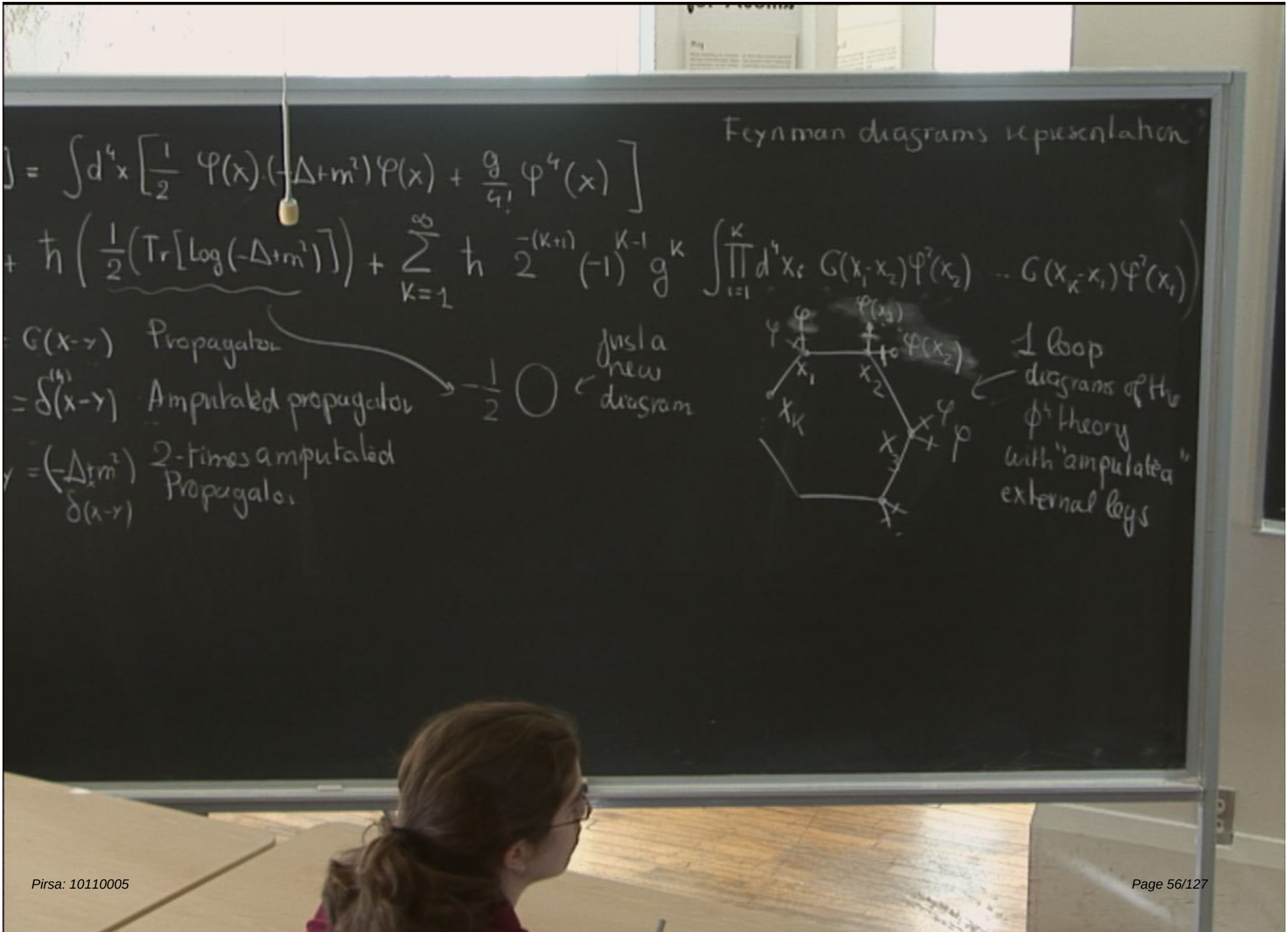
x —|— y $= \delta^{(4)}(x-y)$ Amputated propagator

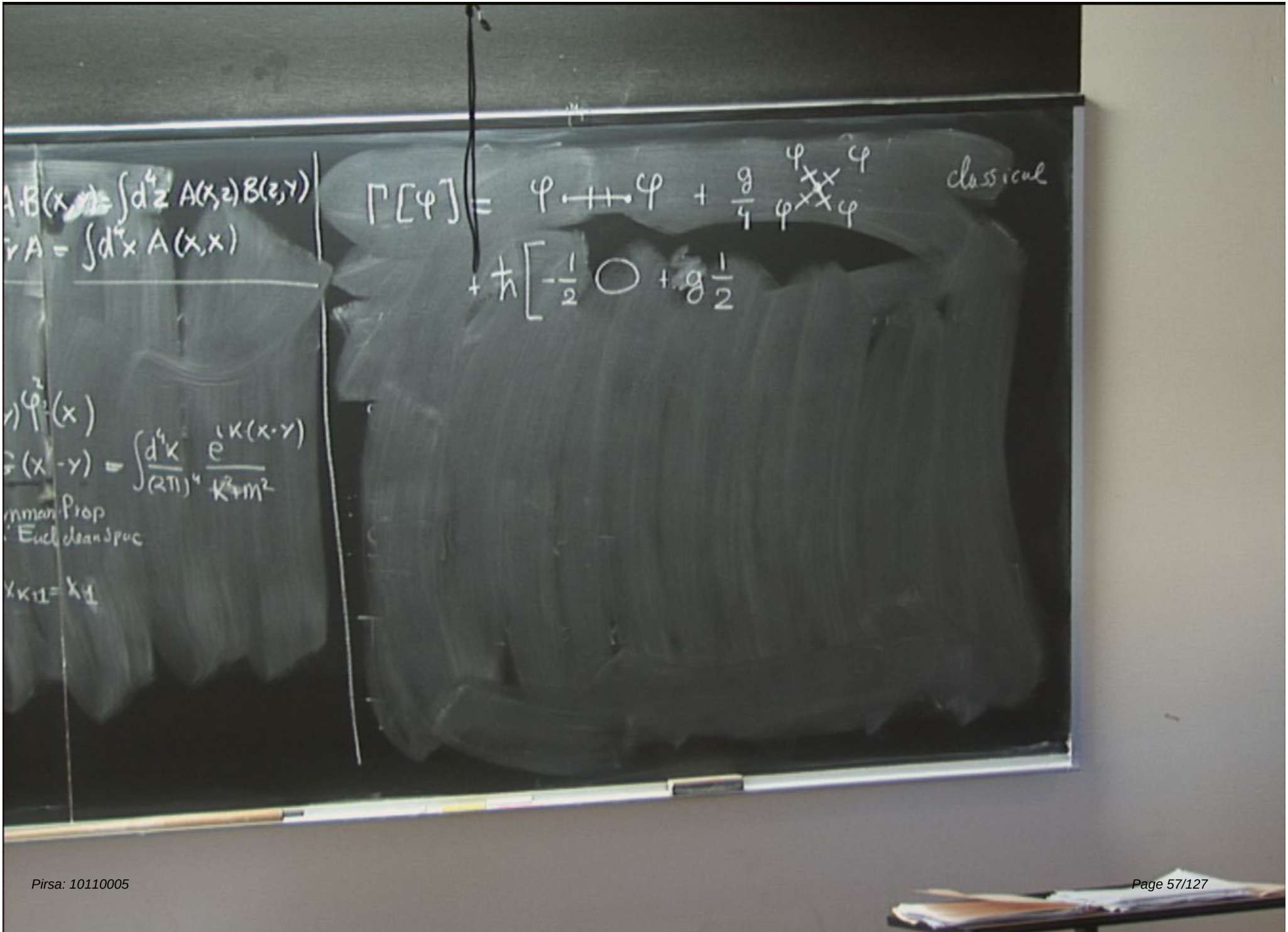
x —||— y $= (-\Delta_x + m^2)\, \delta(x-y)$ 2-times amputated Propagator


A(x,z) B(z,y)
A(x,x)
Γ[φ] = φ φ + g/4 φ φ φ φ
e^{ik·(x-y)}
k²+m²


z A(x,z) B(z,y)
A(x,x)
Γ[φ] = φ φ + g/4 φ φ φ φ
classical
+ ħ[
e^{ik·(x-y)}
k²+m²


$\int d^4z\, A(x,z)B(z,y)$
$= A(x,x)$
$\int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$
$\Gamma[\varphi] =$
φ
φ
$+ \frac{g}{4}$
φ φ φ φ
classical
$+ \hbar \left[-\frac{1}{2} \right.$


Feynman diagrams representation
$= \int d^4x \left[\frac{1}{2} \varphi(x) (-\Delta + m^2) \varphi(x) + \frac{g}{4!} \varphi^4(x) \right]$
$+ \hbar \left(\frac{1}{2} \left(\mathrm{Tr} \left[\log(-\Delta + m^2) \right] \right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \, G(x_1 - x_2) \varphi^2(x_2) \dots G(x_K - x_1) \varphi^2(x_1) \right)$
$G(x-y)$ Propagator
$= \delta^{(4)}(x-y)$ Amputated propagator
$= (-\Delta + m^2) \delta(x-y)$ 2-times amputated Propagator
$-\frac{1}{2}$
just a new diagram
1 loop diagrams of the φ^4 theory with "amputated" external legs


$A \cdot B(x,y) = \int d^4z\, A(x,z) B(z,y)$
$\mathrm{Tr} A = \int d^4x\, A(x,x)$
$\Gamma[\varphi] =$
classical
$+ \hbar \left[-\frac{1}{2} \right.$
$+ g \frac{1}{2}$
$\varphi^2(x)$
$G(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2+m^2}$
Feynman Prop
Euclidean Spac
$x_{k+1} = x_1$

$$\mathrm{Tr}\left[\log\left(-\Delta+m^2\right)\right] + \mathrm{Tr}\left[\log\left(1+\left(-\Delta+m^2\right)\frac{g}{2}\varphi^2\right)\right]$$

perturbative expansion in g — coupling constant

$$* = \sum_{K=1}^{\infty} \left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K} \mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$$

Int. Kernel φ^2 $\delta(x-y)\varphi^2(x)$

Int. $K\varphi\left(\frac{1}{-\Delta+m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4}$

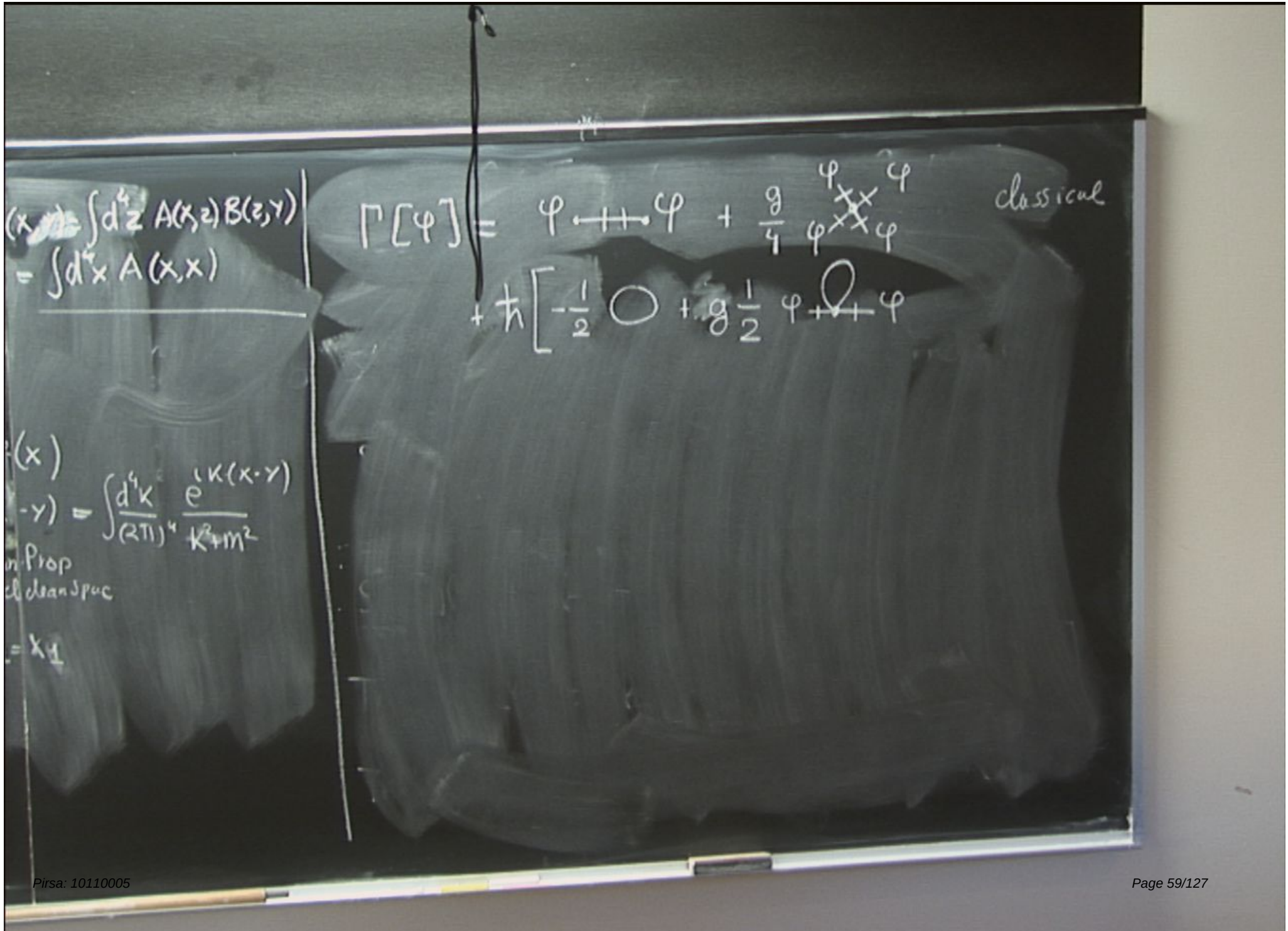
Feynman Prop in Euclidean Spac

$$* \int \prod_{i=1}^{K} d^4x_i \left[G(x_1-x_2)\varphi^2(x_2)\,G(x_2-x_3)\varphi^2(x_3)\cdots G(x_K-x_1)\varphi^2(x_1)\right]$$

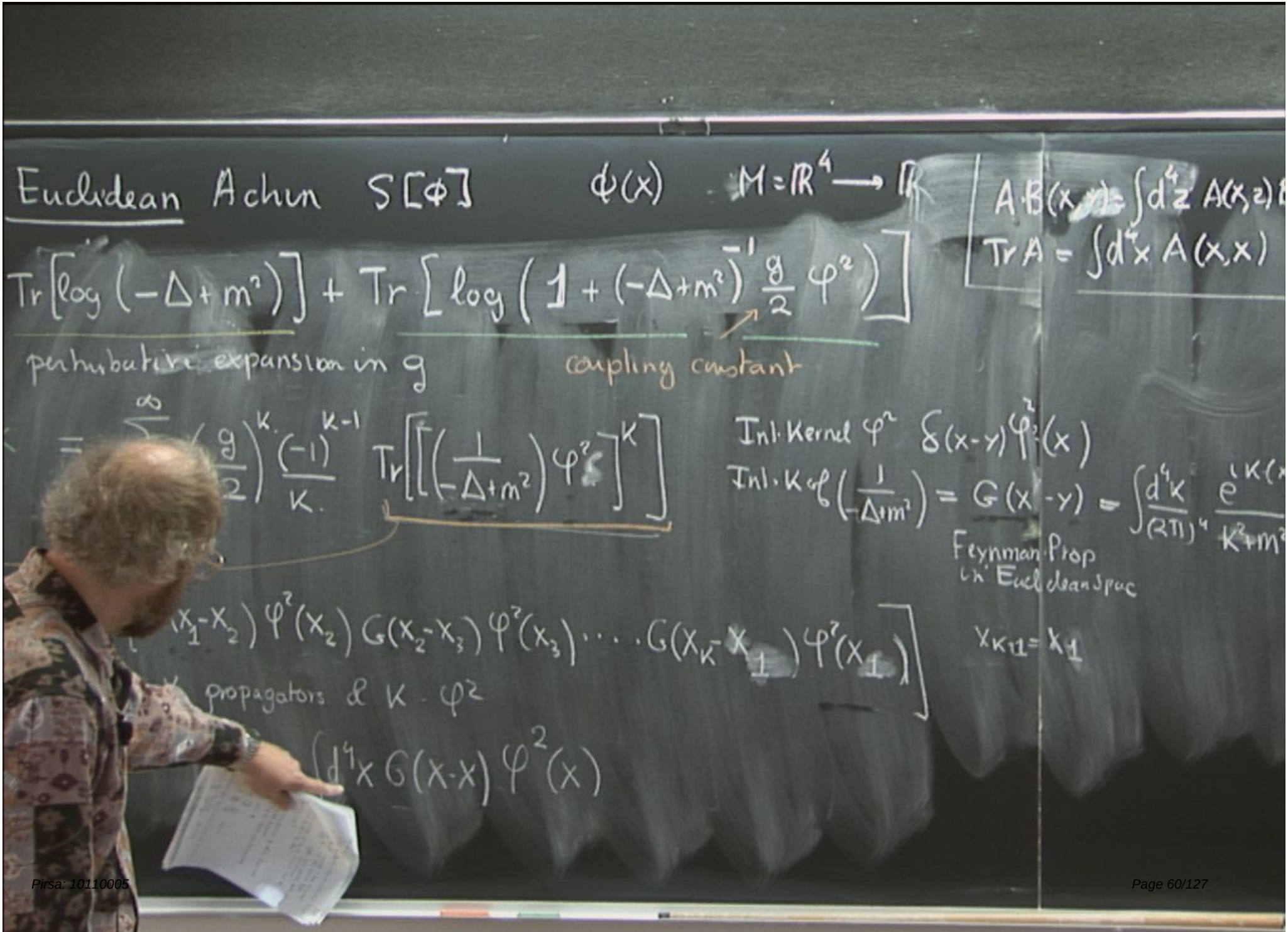
$x_{K+1} = x_1$

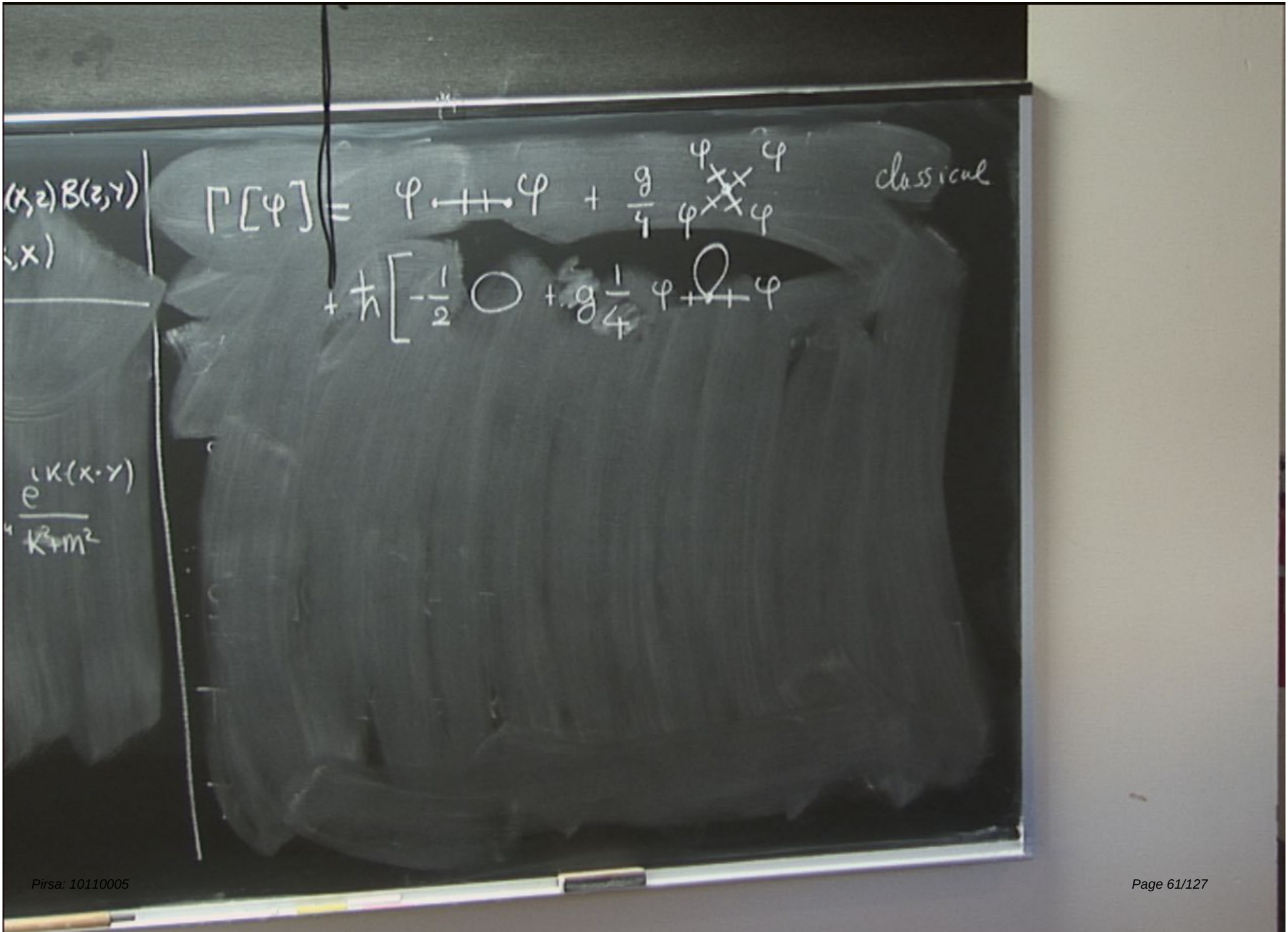
K propagators & K φ^2

$$\int d^4x\, G(x-x)$$



$(x,y) = \int d^4z\, A(x,z)B(z,y)$
$= \int d^4x\, A(x,x)$
(x)
$-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2+m^2}$
Prop
clean spac
$= x_1$
$\Gamma[\varphi] = \varphi \cdots \varphi + \frac{g}{4}$
$+ \hbar\left[-\frac{1}{2} \bigcirc + g\frac{1}{2}\varphi \cdots \varphi\right.$
classical


Euclidean Action $S[\phi]$ $\phi(x)$ $M = \mathbb{R}^4 \longrightarrow \mathbb{R}$
$A \cdot B(x,y) = \int d^4z\, A(x,z)$
$\mathrm{Tr}\, A = \int d^4x\, A(x,x)$
$\mathrm{Tr}\left[\log(-\Delta + m^2)\right] + \mathrm{Tr}\left[\log\left(1 + (-\Delta+m^2)^{-1}\frac{g}{2}\varphi^2\right)\right]$
perturbative expansion in g
coupling constant
$= \sum_{K=1}^{\infty} \left(\frac{g}{2}\right)^K \frac{(-1)^{K-1}}{K} \mathrm{Tr}\left[\left[\left(\frac{1}{-\Delta+m^2}\right)\varphi^2\right]^K\right]$
Int. Kernel φ^2 $\delta(x-y)\varphi^2(x)$
Int. Kernel $\left(\frac{1}{-\Delta+m^2}\right) = G(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x}}{k^2+m^2}$
Feynman Prop in Euclidean Space
$(x_1-x_2)\varphi^2(x_2)\, G(x_2-x_3)\varphi^2(x_3) \cdots G(x_K-x_1)\varphi^2(x_1)$
$x_{K+1} = x_1$
propagators & K φ^2
$\int d^4x\, G(x-x)\, \varphi^2(x)$


(x,z)B(z,y)
,x)
$\Gamma[\varphi] = \varphi \cdots \varphi + \frac{g}{4}$
$+ \hbar\left[-\frac{1}{2} \bigcirc + g\frac{1}{4}\varphi \cdots \varphi\right.$
classical
$\frac{e^{ik(x-y)}}{k^2+m^2}$

$$\int d^4x \left[\frac{1}{2} \varphi(x) (-\Delta + m^2) \varphi(x) + \frac{g}{4!} \varphi^4(x) \right]$$

$$\ln\left(\frac{1}{2} \left(\mathrm{Tr}\left[\log(-\Delta + m^2) \right] \right) \right) + \sum_{K=1}^{\infty} \hbar \, 2^{-(K+1)} (-1)^{K-1} g^K \int \prod_{i=1}^{K} d^4x_i \; G(x_1 - x_2) \varphi^2(x_2) \cdots G(x_K - x_1) \varphi^2(x$$

Feynman diagrams representation

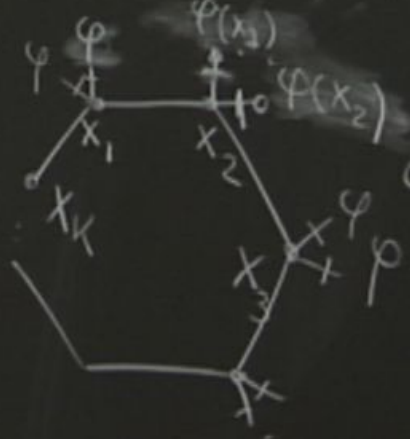
$(x-y)$ Propagator

$(x-y)$ Amputated propagator

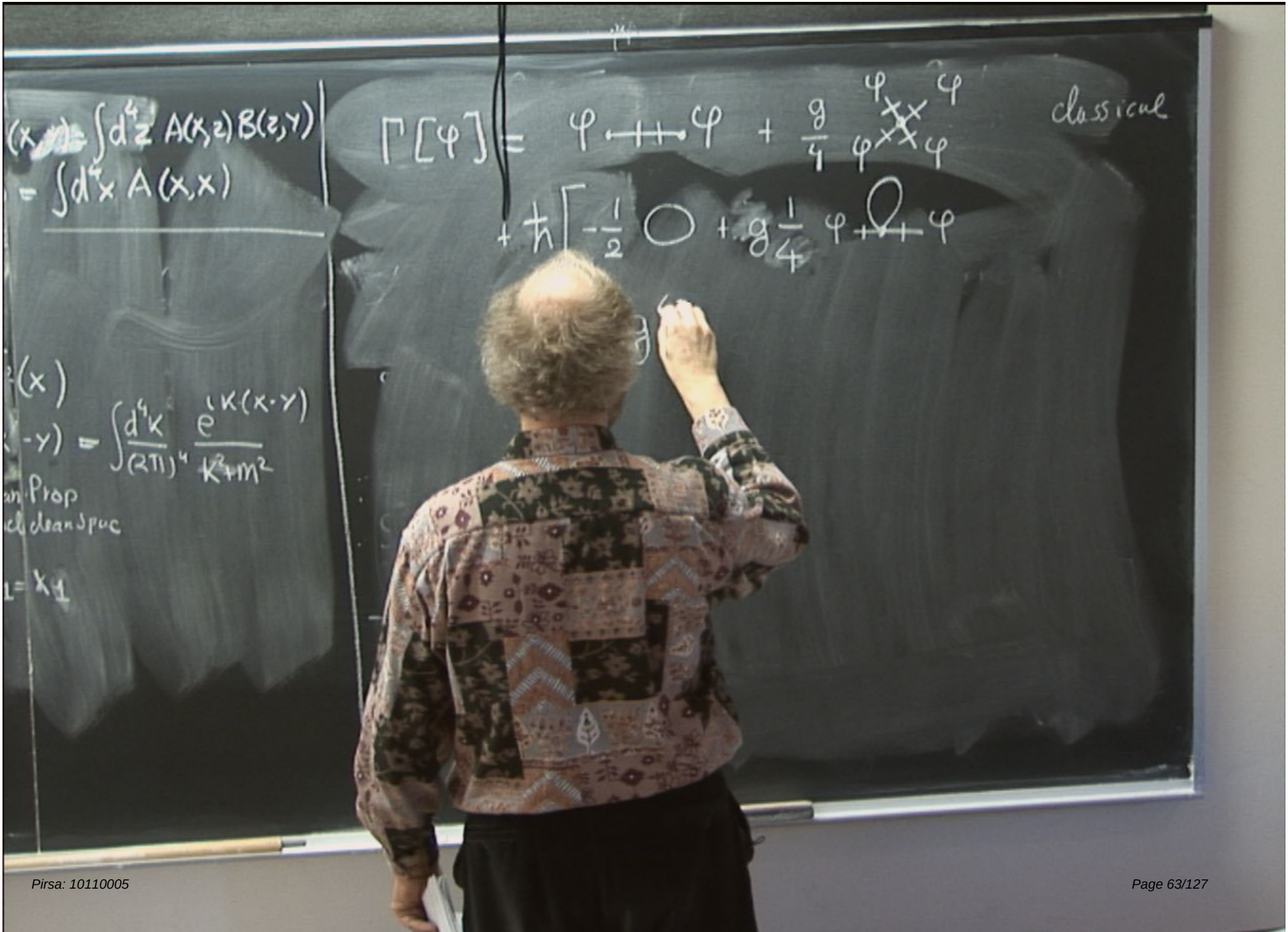
$(-\Delta + m^2)$ 2-times amputated Propagator

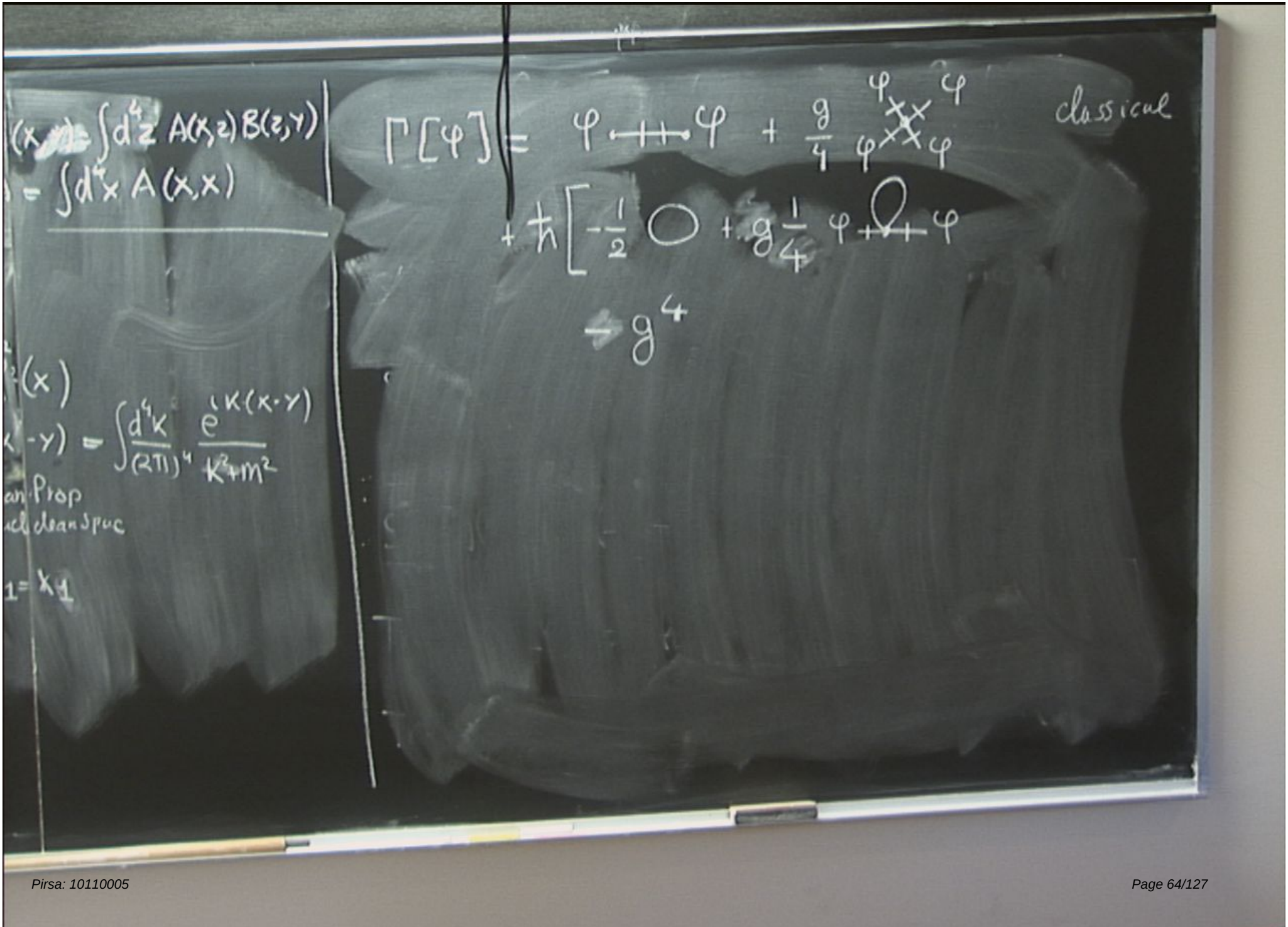
$\delta(x-y)$

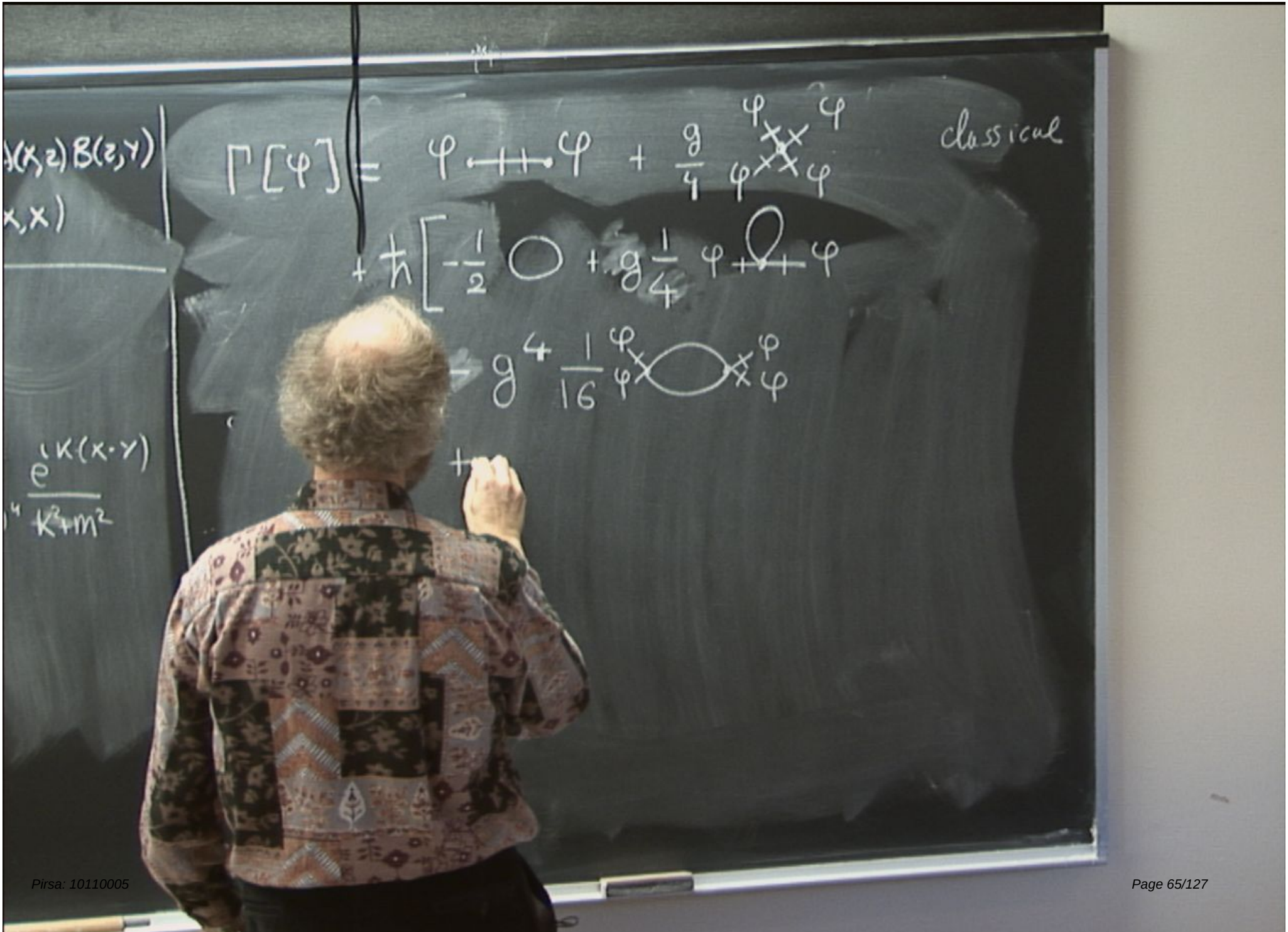
$-\frac{1}{2}$ ○ ← just a new diagram

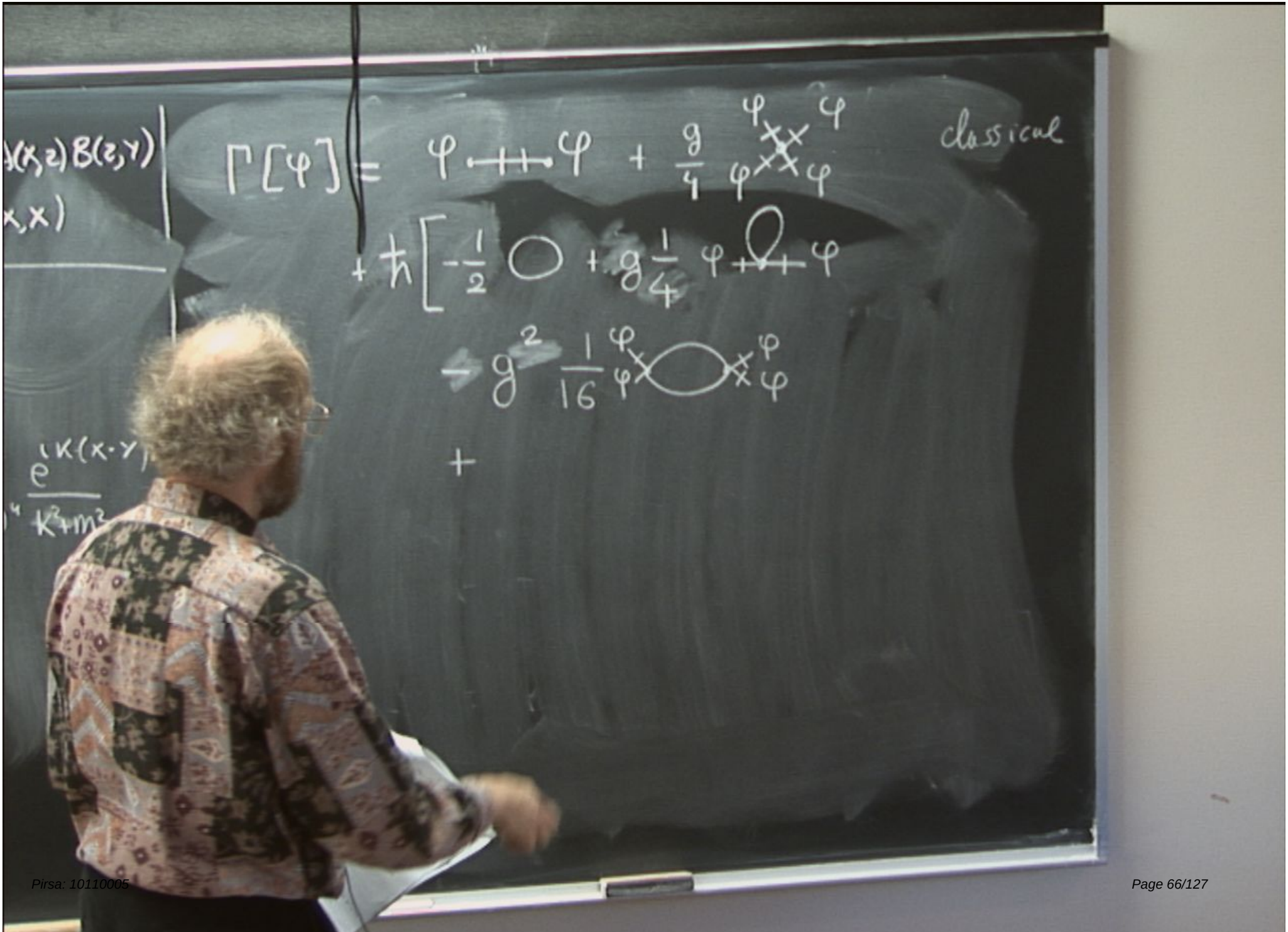


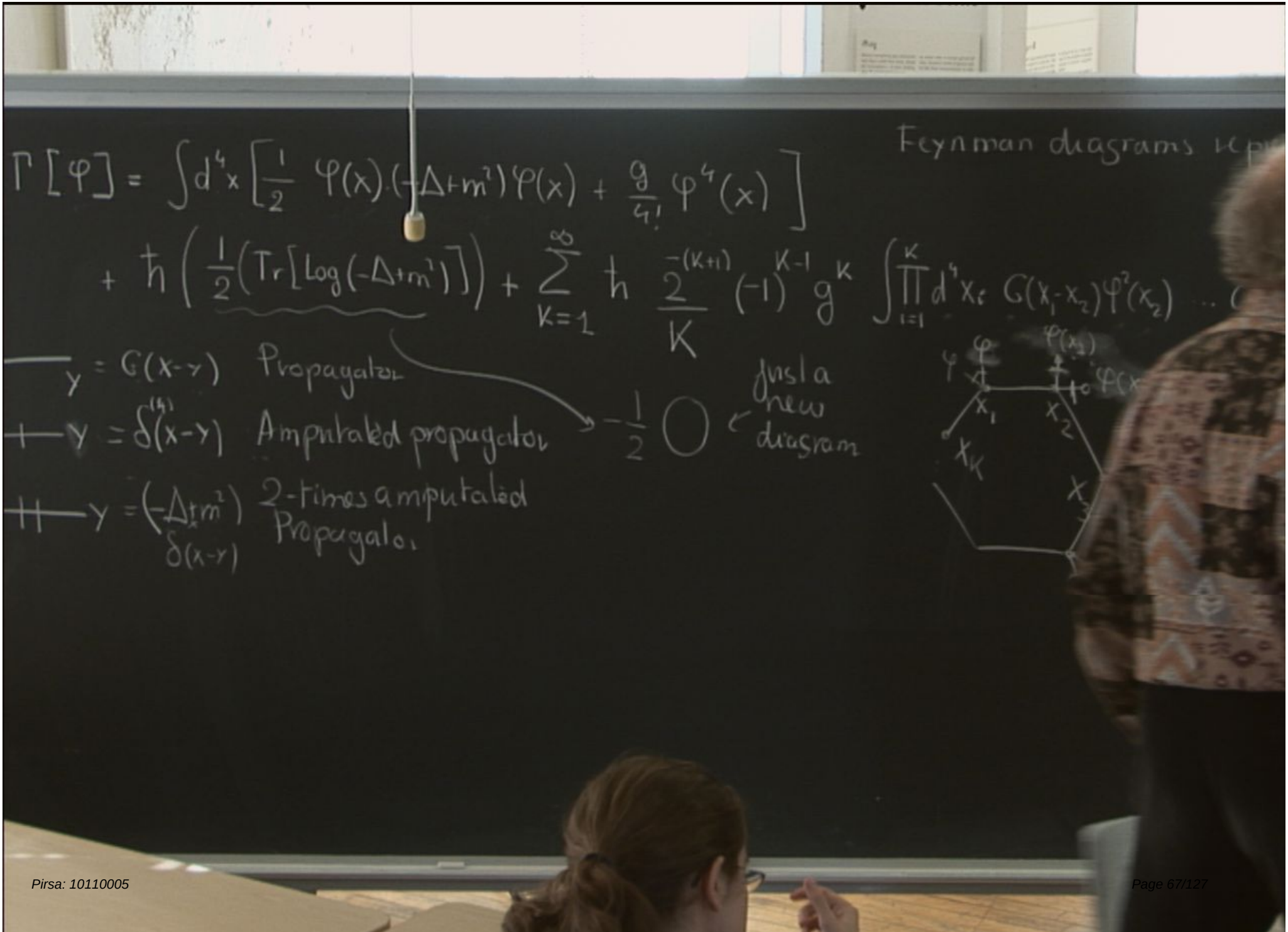
← 1 loop diagrams of the ϕ^4 theory with "amputated" external legs

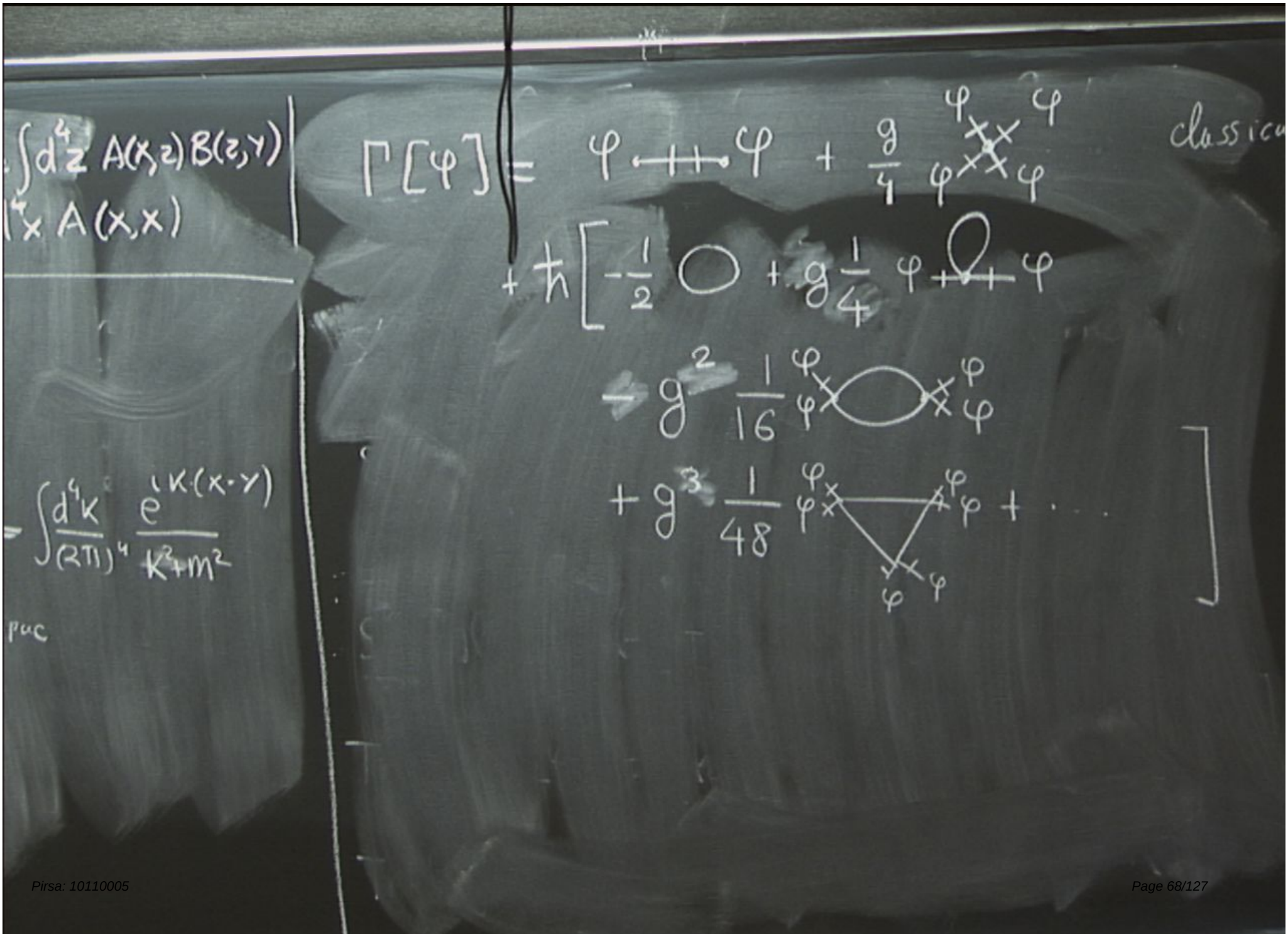

classical

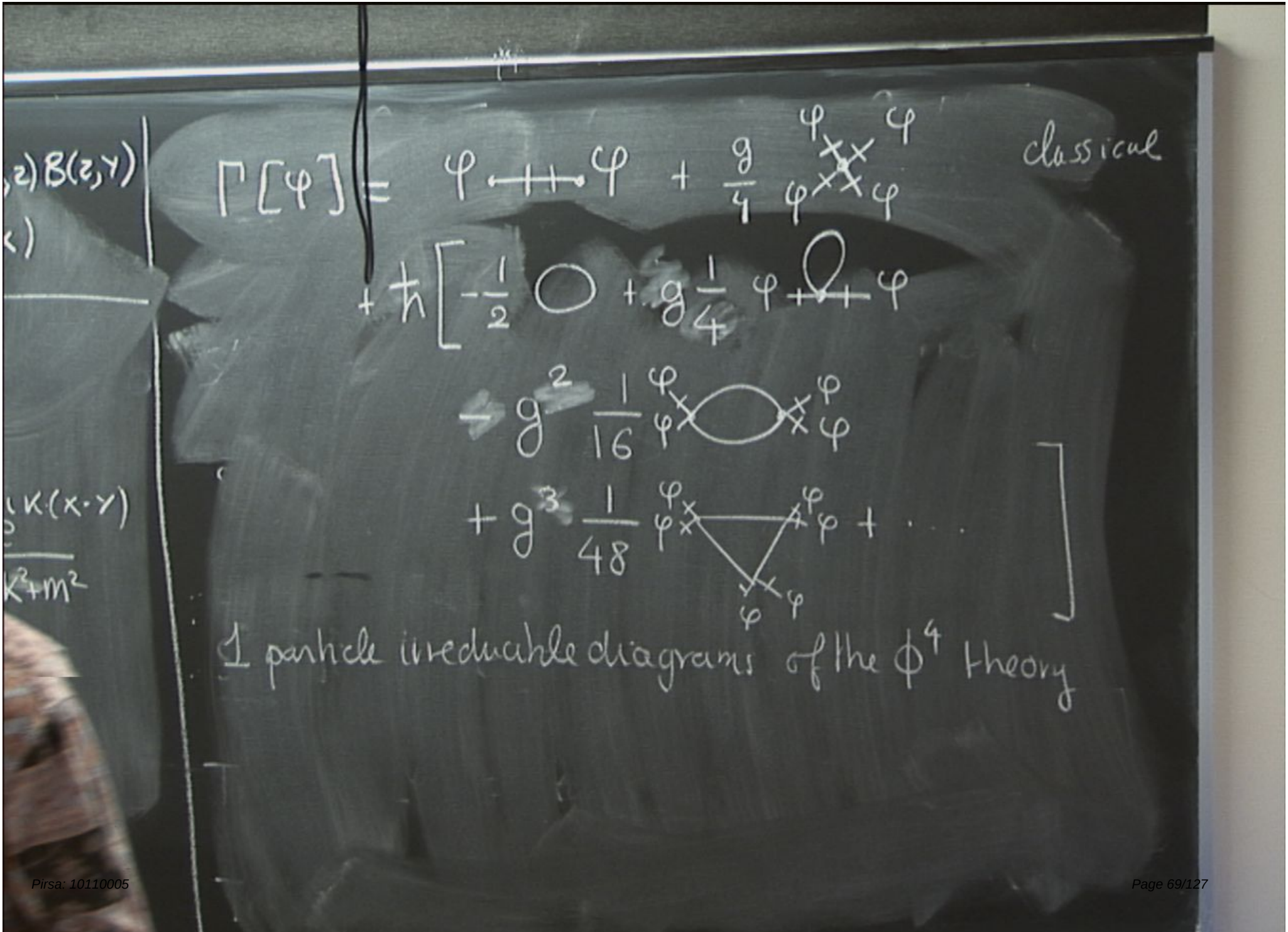

$(x,y) = \int d^4z\, A(x,z)\, B(z,y)$
$= \int d^4x\, A(x,x)$
$\Gamma[\varphi] = \varphi \varphi + \frac{g}{4}$
classical
$+ \hbar\left[-\frac{1}{2}\right.$
$+ g\frac{1}{4}$
$- g^4$
(x)
$-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik(x-y)}}{k^2+m^2}$
Prop
clean Spuc
$1 = x_1$

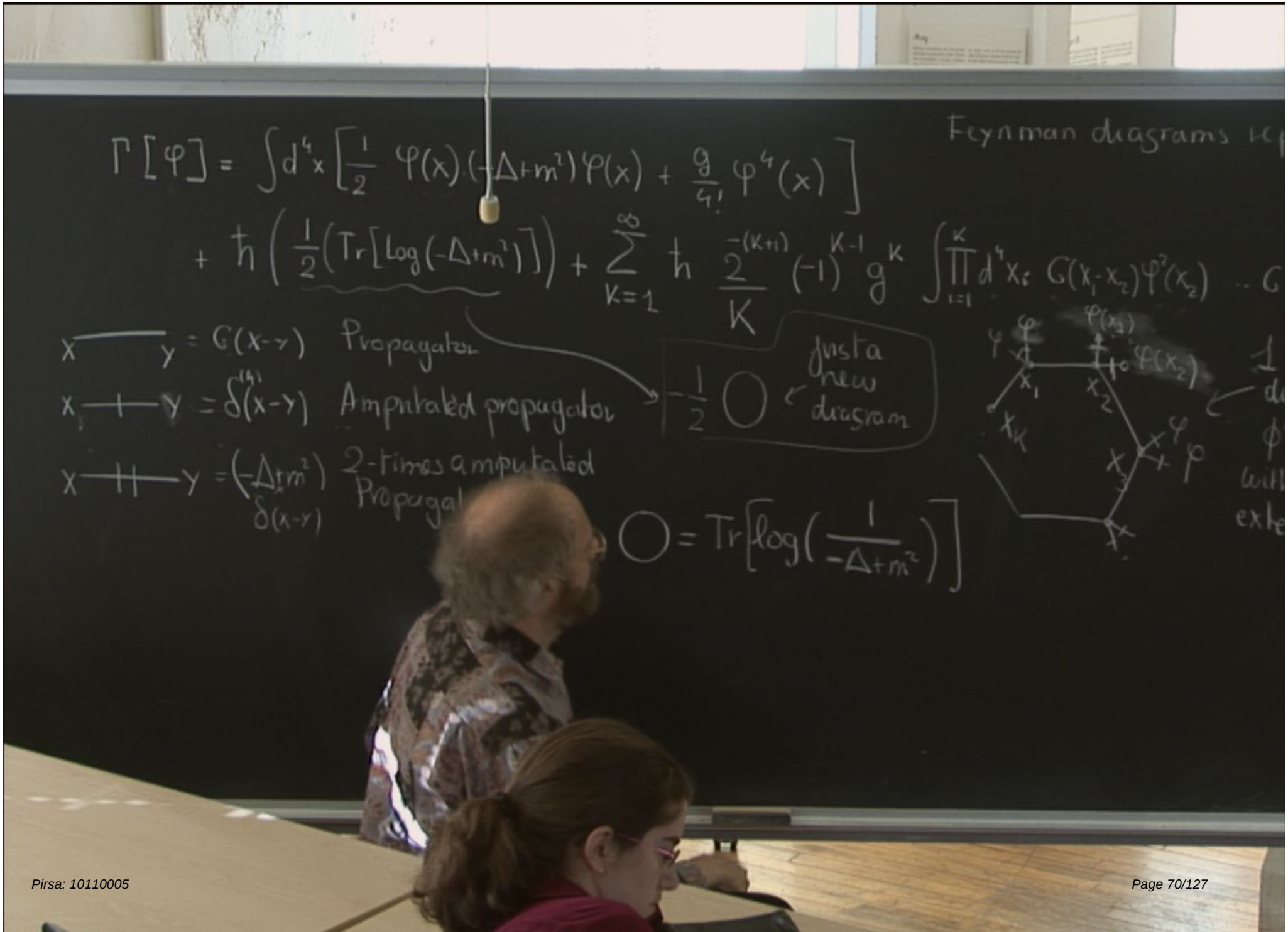

Γ[φ] = φ φ + g/4 φ φ φ φ classical
+ ħ [-1/2 + g 1/4 φ φ
+ g⁴ 1/16 φ φ φ φ
+
e^{ik(x-y)} / k²+m²

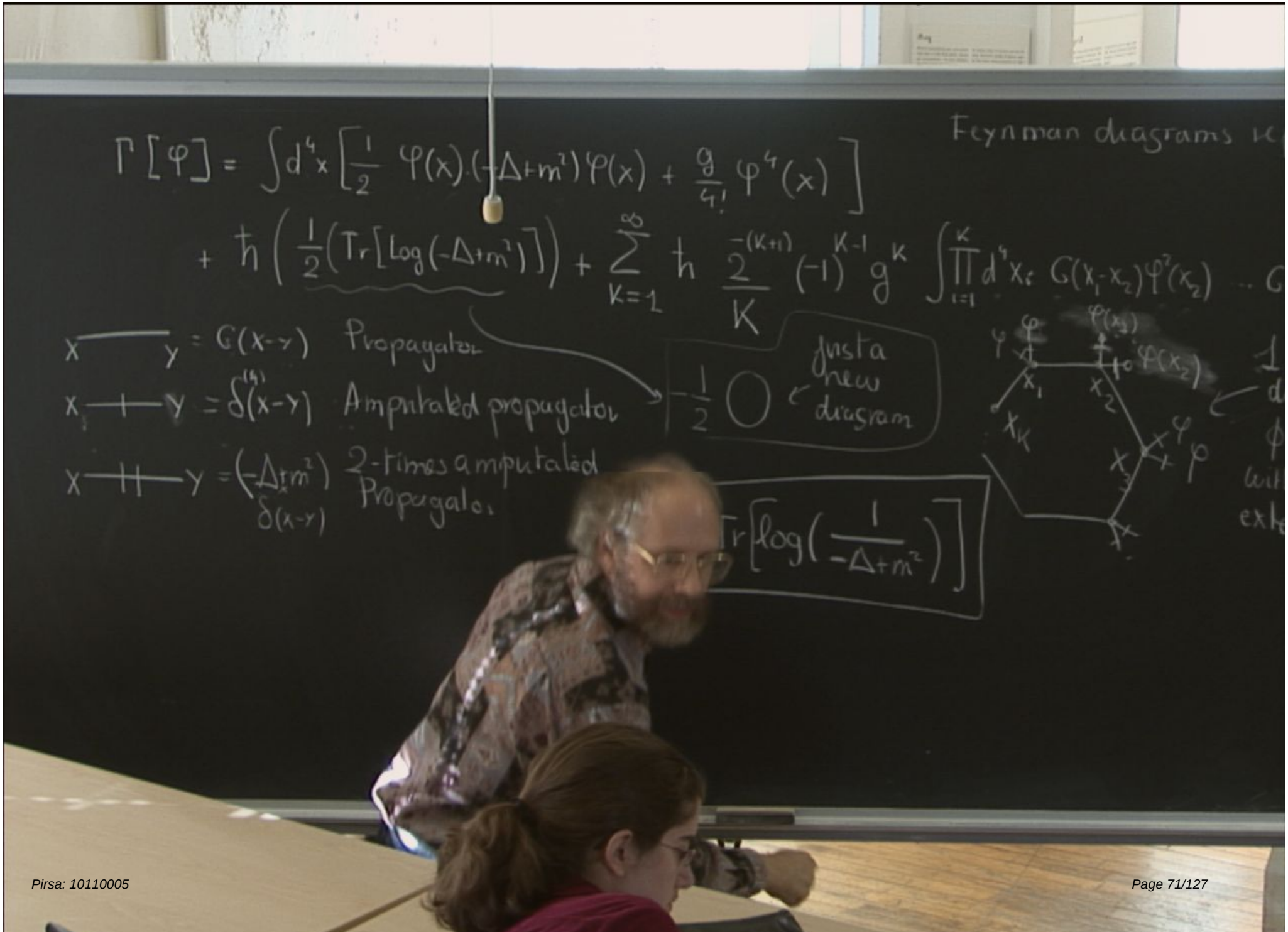

classical

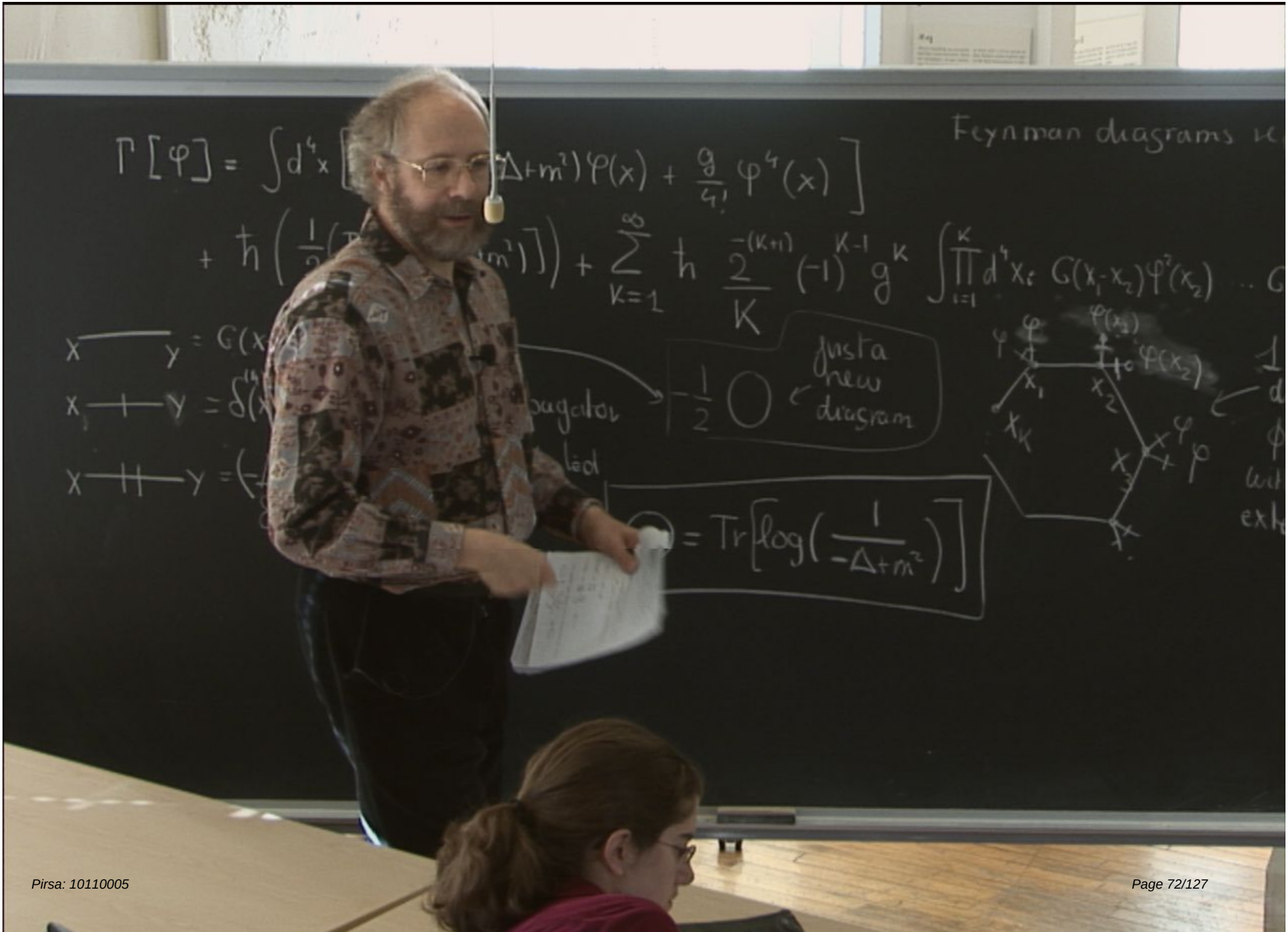

Feynman diagrams
Propagator
Amputated propagator
2-times amputated Propagator
just a new diagram

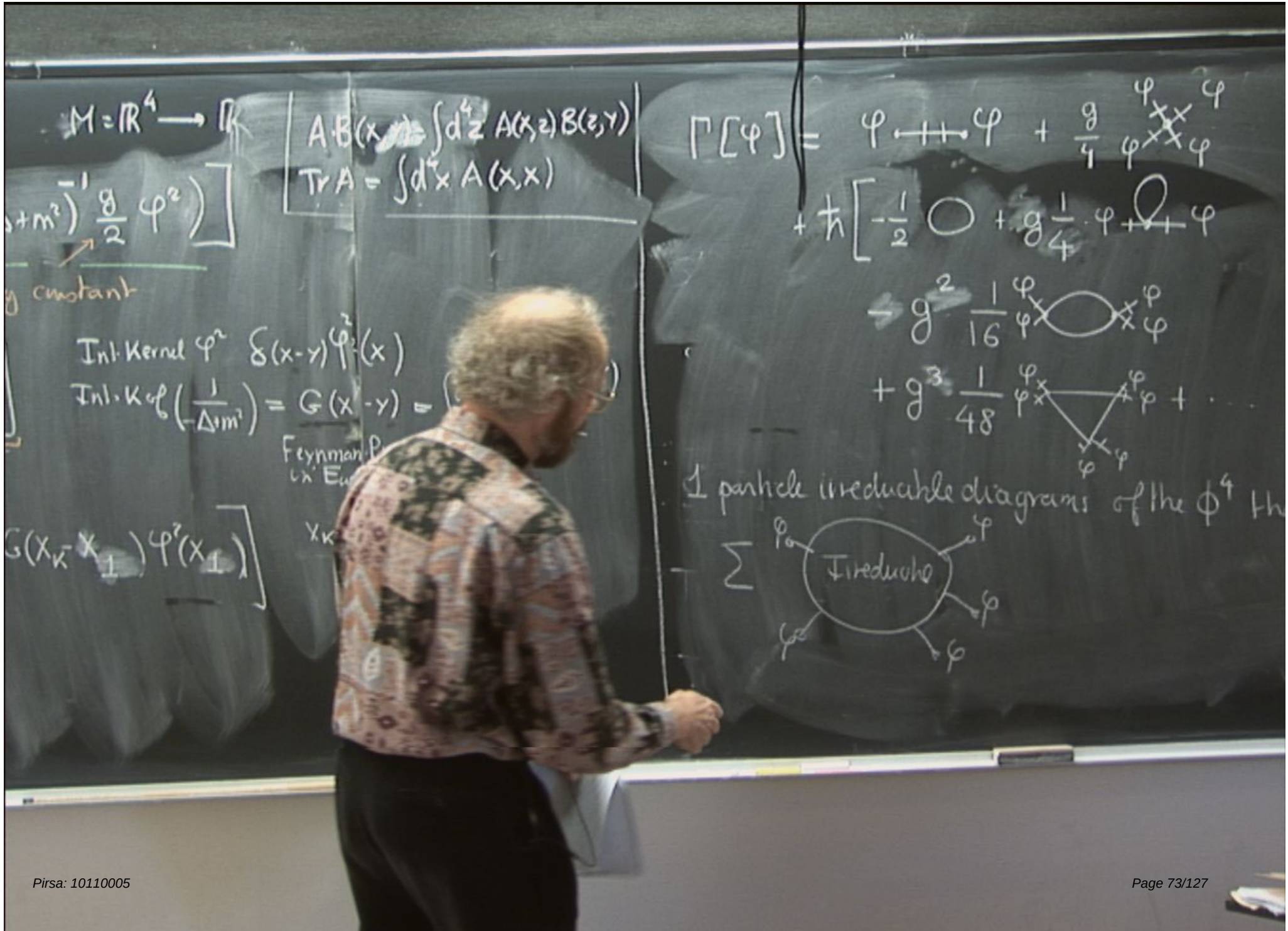

$\int d^4z\, A(x,z)B(z,y)$
$\int d^4x\, A(x,x)$
$\int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$
$\Gamma[\varphi] =$
$+\hbar\left[-\frac{1}{2}\right.$
$+ g\frac{1}{4}$
$- g^2\frac{1}{16}$
$+ g^3\frac{1}{48}$
$+ \cdots \left.\right]$
classica

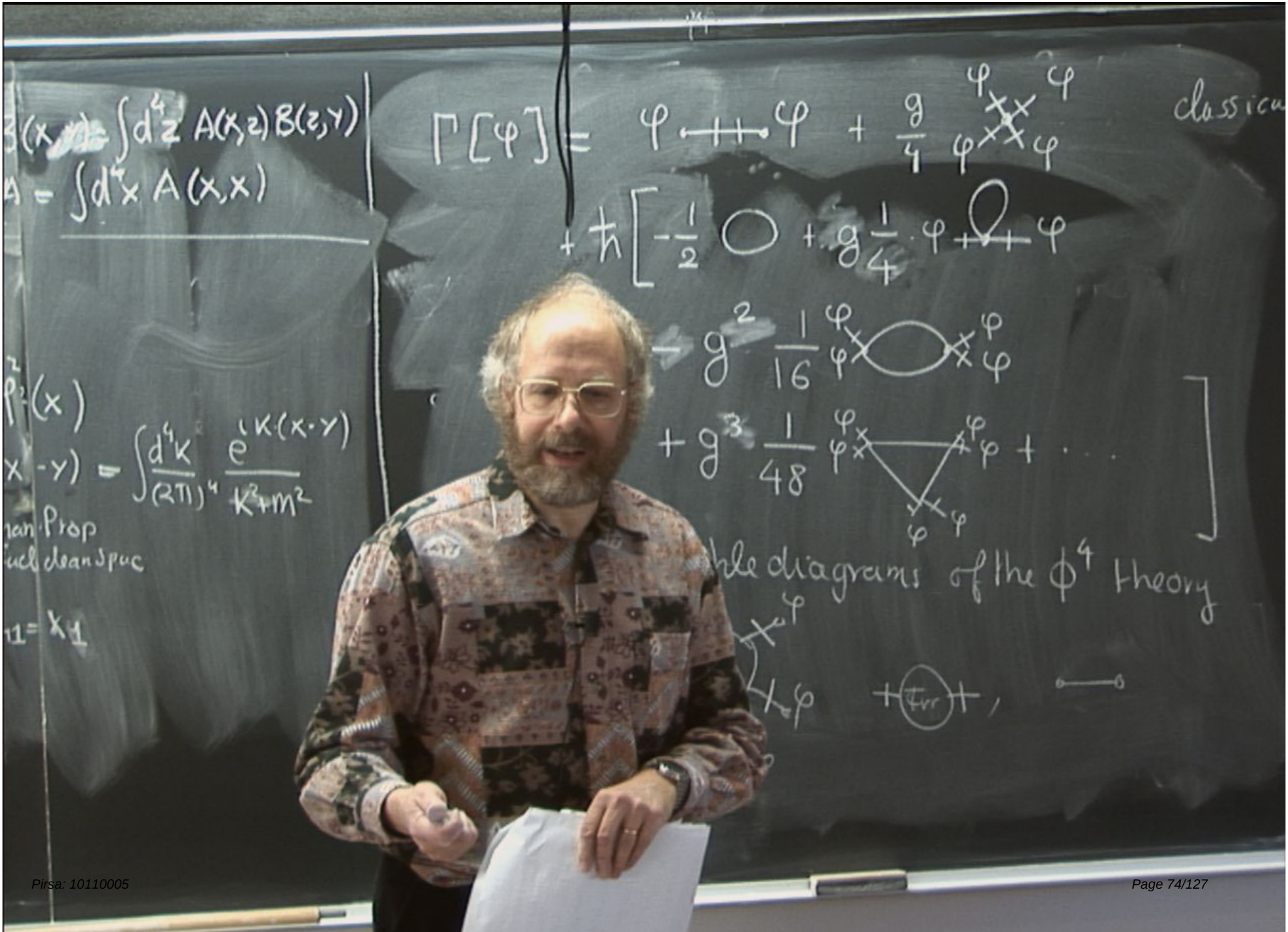

Γ[φ] = φ φ + g/4 φ φ φ φ classical
+ ħ [-1/2 + g 1/4 φ φ
- g² 1/16 φ φ φ φ
+ g³ 1/48 φ φ φ φ φ + . . .
1 particle irreducible diagrams of the φ⁴ theory


Feynman diagrams
Propagator
Amputated propagator
2-times amputated
Just a new diagram

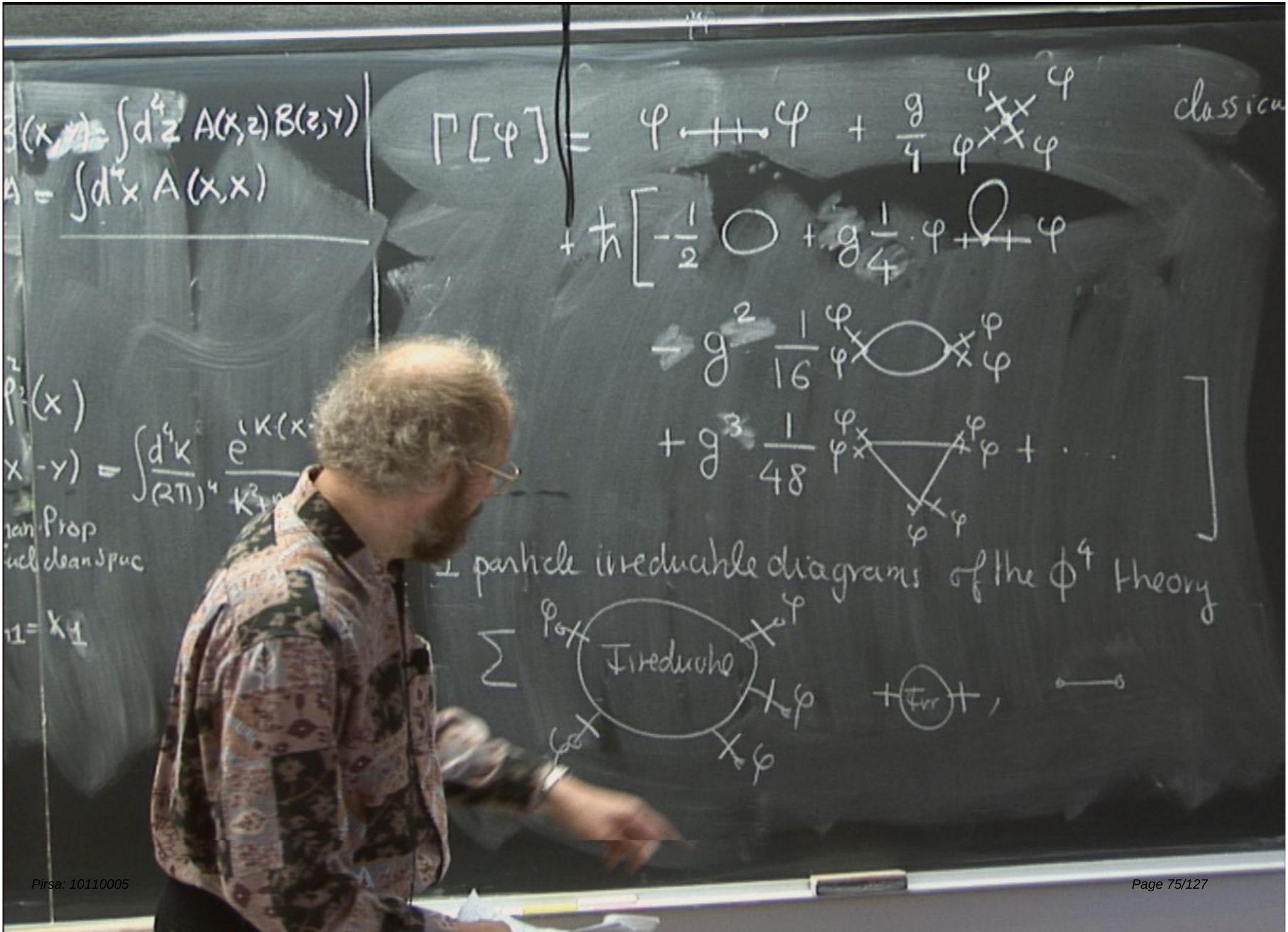

Feynman diagrams
Propagator
Amputated propagator
2-times amputated
Propagator
just a new diagram


Feynman diagrams
Just a new diagram
= Tr[log(1/(-Δ+m²))]

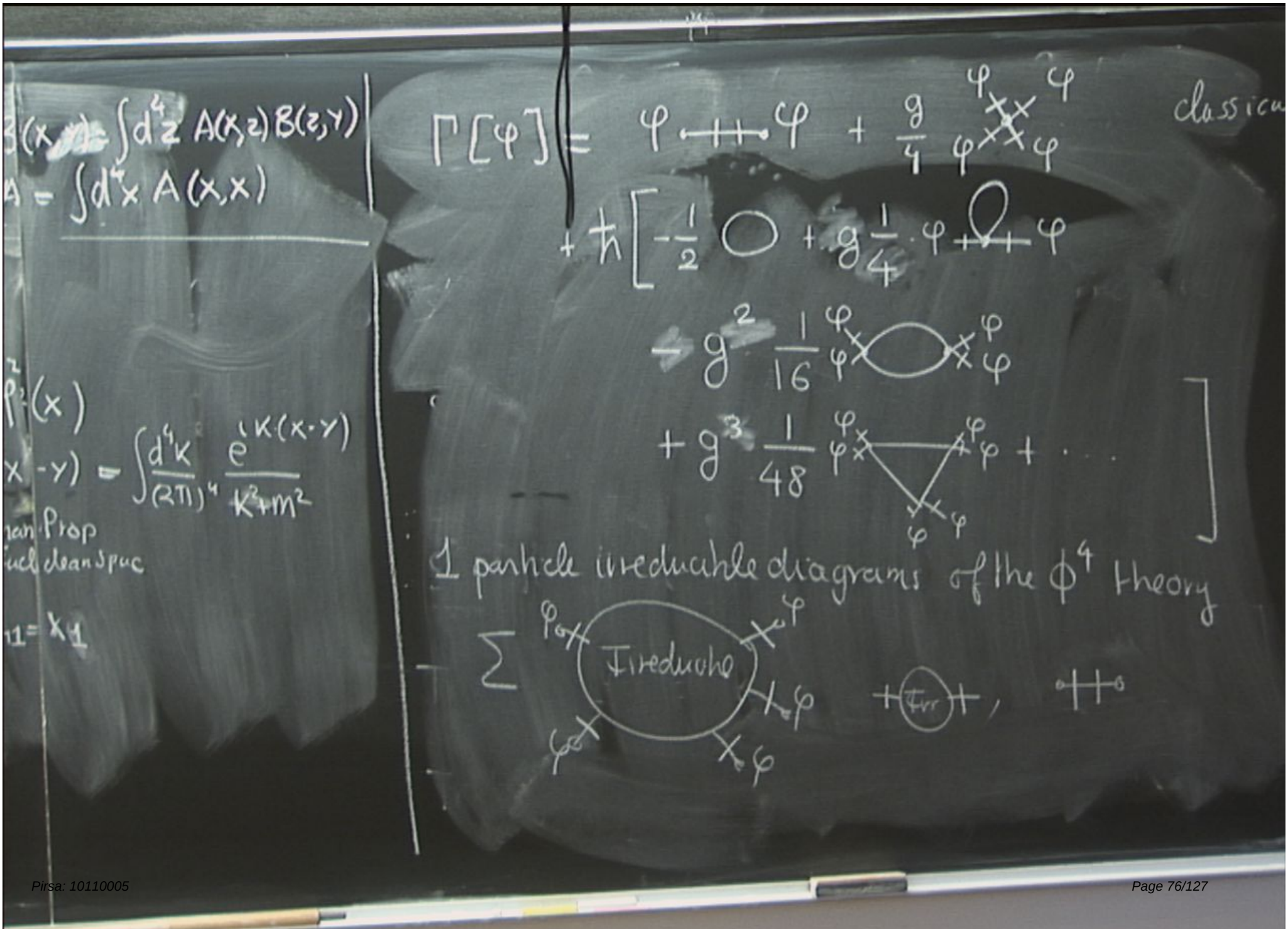

M = ℝ⁴ → ℝ
A·B(x,y) = ∫d⁴z A(x,z)B(z,y)
Tr A = ∫d⁴x A(x,x)
Γ[φ] =
+ ħ[-1/2 + g 1/4
- g² 1/16
+ g³ 1/48 + ...
Int. Kernel φ² δ(x-y)φ²(x)
Int. Kernel (1/(-Δ+m²)) = G(x-y) =
Feynman
1 particle irreducible diagrams of the φ⁴ th
Σ
Irreducible



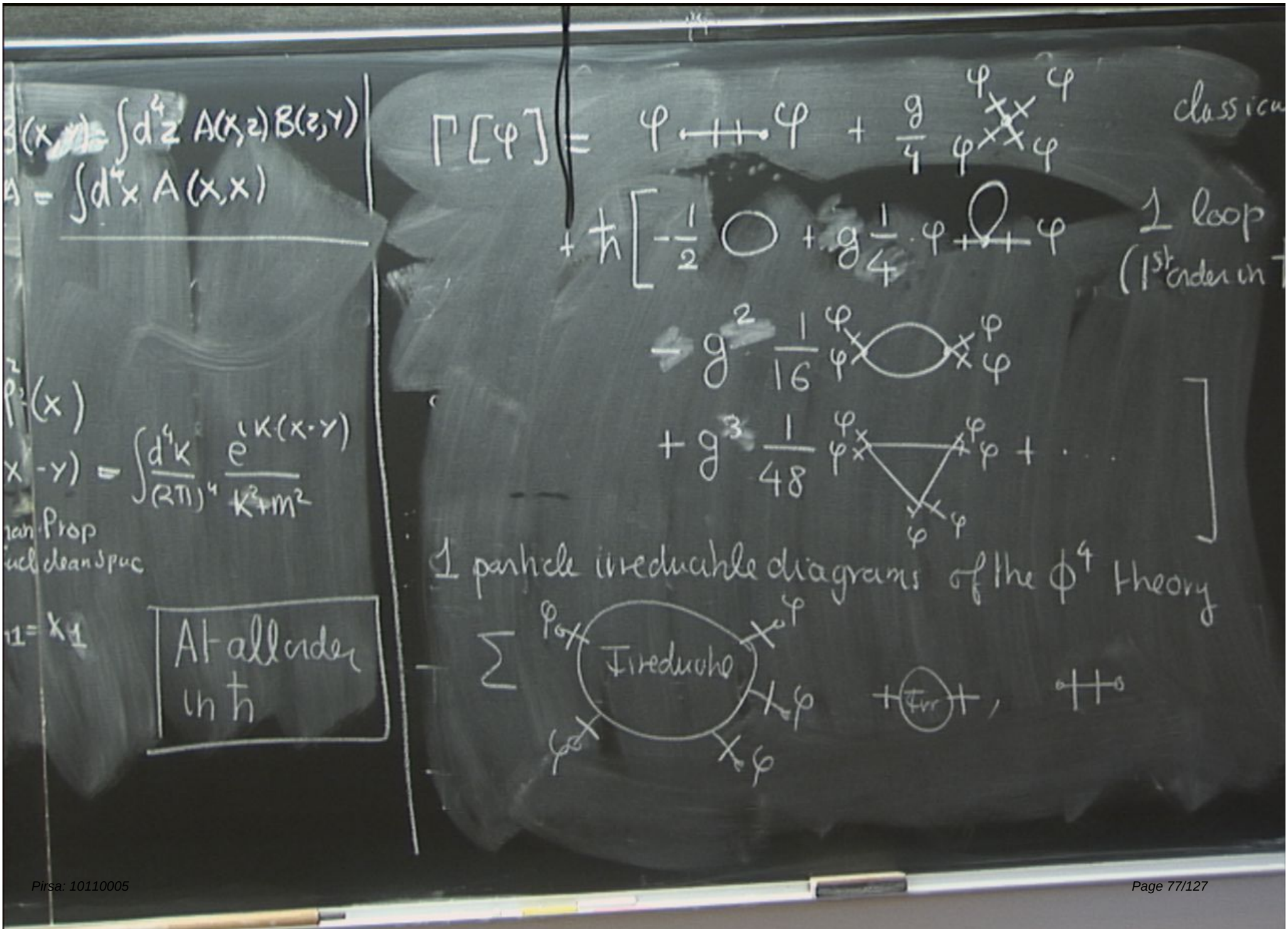
$\int d^4z\, A(x,z)B(z,y)$
$A = \int d^4x\, A(x,x)$
$\Gamma[\varphi] = \varphi \cdots \varphi + \frac{g}{4}$
$+ \hbar\left[-\frac{1}{2} + g\frac{1}{4}\cdot\varphi \cdots \varphi\right.$
$- g^2 \frac{1}{16}$
$+ g^3 \frac{1}{48} + \cdots$
$\int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$
Prop
diagrams of the ϕ^4 theory



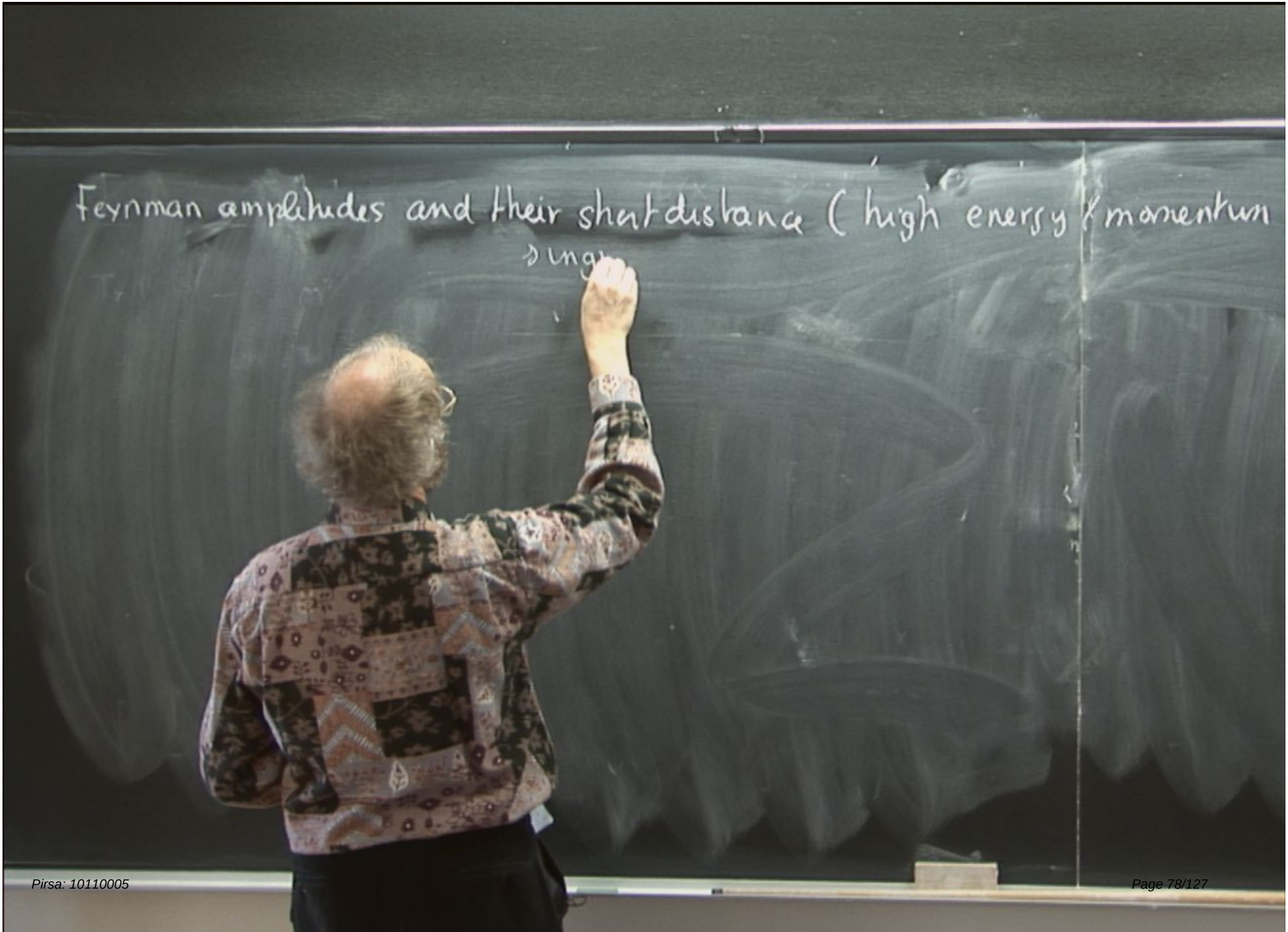
$\int d^4z\, A(x,z)B(z,y)$
$A = \int d^4x\, A(x,x)$
$\Gamma[\varphi] = \varphi \quad \varphi + \frac{g}{4}$
classicu
$+ \hbar\left[-\frac{1}{2} + g\frac{1}{4}\cdot\varphi \quad \varphi\right.$
$- g^2 \frac{1}{16}$
$+ g^3 \frac{1}{48} + \cdots$
particle irreducible diagrams of the ϕ^4 theory
Irreducible
Irr



$B(x,y) = \int d^4z\, A(x,z) B(z,y)$
$A = \int d^4x\, A(x,x)$
$\Gamma[\varphi] = \varphi \quad \varphi + \frac{g}{4}$
classicu
$+ \hbar \left[-\frac{1}{2} \quad + g \frac{1}{4} \cdot \varphi \quad \varphi \right.$
$- g^2 \frac{1}{16}$
$+ g^3 \frac{1}{48} \quad + \cdots \Big]$
$(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$
Prop
clean spac
$= x_1$
1 particle irreducible diagrams of the ϕ^4 theory
$\sum$
Irreducible
Irr



$B(x,y) = \int d^4z\, A(x,z) B(z,y)$
$A = \int d^4x\, A(x,x)$
$\Gamma[\varphi] = \varphi \cdots \varphi + \frac{g}{4}$
classical
$+ \hbar \left[-\frac{1}{2} \bigcirc + g \frac{1}{4} \varphi \cdots \varphi \right.$
1 loop
(1st order in
$- g^2 \frac{1}{16}$
$+ g^3 \frac{1}{48}$
$+ \cdots \Big]$
$(x-y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ik\cdot(x-y)}}{k^2+m^2}$
Prop
clean spac
$= x_1$
At all order in $\hbar$
1 particle irreducible diagrams of the ϕ^4 theory
$\sum$
Irreducible
Irr

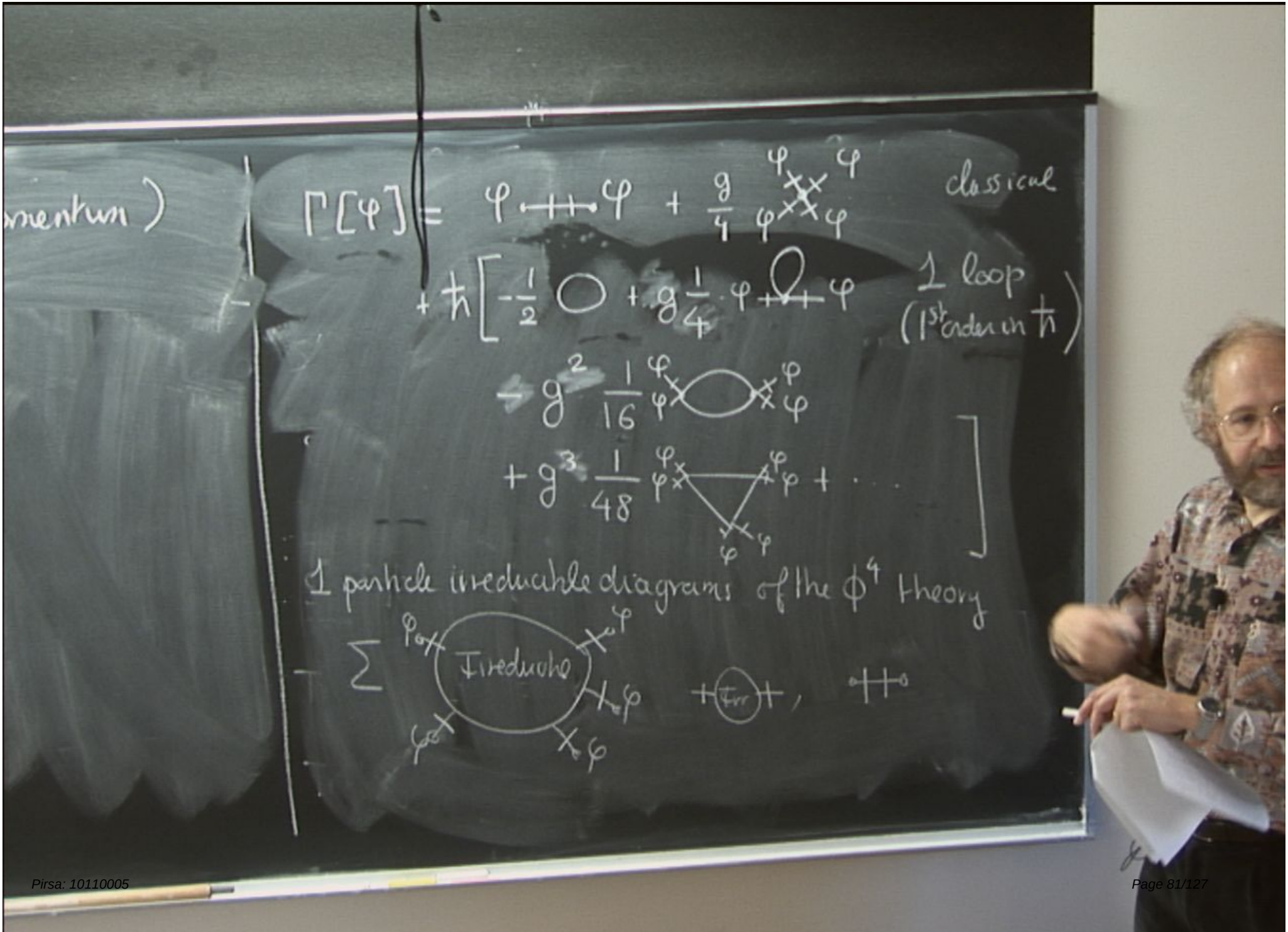

Feynman amplitudes and their short distance (high energy / momentum

Feynman amplitudes and their short distance (high energy / momentum) singularities:

$$G(X_E) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ikx}}{k^2 + m^2}$$

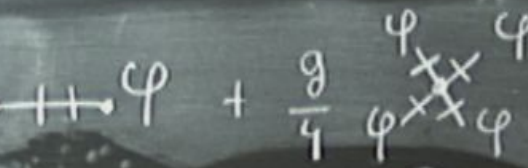
Feynman amplitudes and their short distance (high energy / momentum) singularities:

$$G(X_E) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{ikx}}{k^2 + m^2}$$

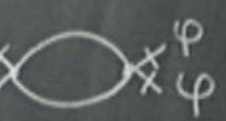


mentum)
Γ[φ] = φ φ + g/4
classical
+ ħ [-1/2 + g 1/4 φ φ
1 loop
(1st order in ħ)
- g² 1/16
+ g³ 1/48 + . . .]
1 particle irreducible diagrams of the φ⁴ theory
Σ
Irreducible
Irr

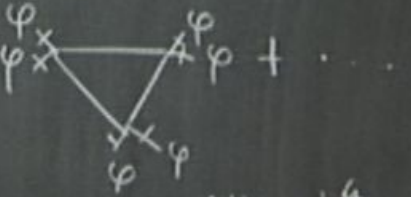
mentum)

$\Gamma[\varphi] =$  classical

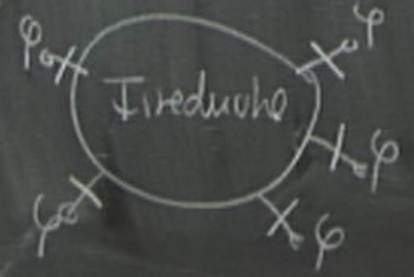
$+\hbar\Big[$ 1 loop (1st order in $\hbar$)

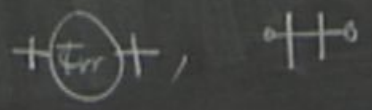
$-g^2\,\frac{1}{16}$ 

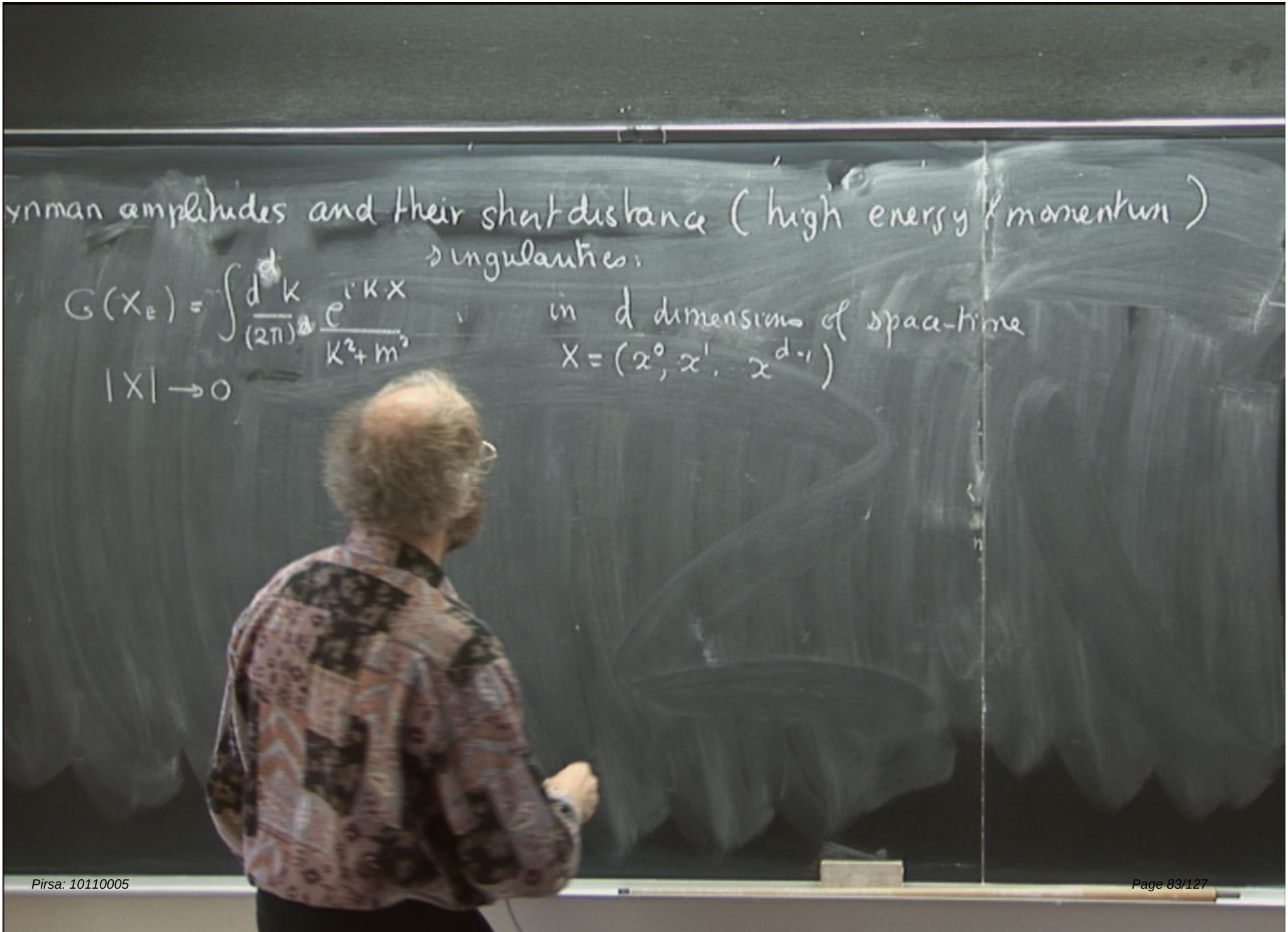
Symmetry factors of the diagram

$+g^3\,\frac{1}{48}$  $+\ \dots\ \Big]$

1 particle irreducible diagrams of the ϕ^4 theory

Σ 




ynman amplitudes and their short distance (high energy / momentum)
singularities.
$G(X_E) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot X}}{k^2 + m^2}$
in d dimensions of space-time
$X = (x^0, x^1, \ldots x^{d-1})$
$|X| \to 0$

ynman amplitudes and their short distance (high energy / momentum)
⇒ singularities:

$$G(X_E) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

$|X| \to 0$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

ynman amplitudes and their short distance (high energy / momentum)
⇒ singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0$ ⇔ $|K| \to \infty$

ynman amplitudes and their short distance (high energy / momentum)
⇒ singularities.

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \iff |K| \to \infty$

$G(X)$

ynman amplitudes and their short distance (high energy / momentum)
⇒ singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \iff |K| \to \infty$

$$G(X) = |X|^{2-d}$$

ynman amplitudes and their short distance (high energy / momentum)
⇒ singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \iff |K| \to \infty$

$$G(X) \sim C_d \cdot |X|^{2-d}$$

ynman amplitudes and their short distance (high energy / momentum)
⇒ singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \iff |K| \to \infty$

$G(X) \sim C_d \cdot |X|^{2-d}$ $\quad d > 2$ its ∞ at $|x| = 0$

ynman amplitudes and their short distance (high energy / momentum) singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \iff |K| \to \infty$

$$G(X) \sim C_d \cdot |X|^{2-d} \qquad d > 2$$

$$\sim \frac{1}{2\pi} \log\left(\frac{1}{|X| m}\right) \qquad d = 2$$

} its ∞ at $|X| = 0$

...tudes and their short distance (high energy / momentum)

singularities:

$$\int \frac{d^d k}{(2\pi)^d} \frac{e^{ik\cdot x}}{k^2+m^2}$$

in d dimensions of space-time

$x = (x^0, x^1, \dots x^{d-1})$

$k = (k_0, k_1, \dots k_{d-1})$

$0 \Leftrightarrow |k| \to \infty$

$\dots_d |x|^{2-d}$ $\quad d > 2$

$\dots_\pi \log\left(\frac{1}{|x| m}\right)$ $\quad d = 2$

$\Big\}$ its ∞ at $|x| = 0$

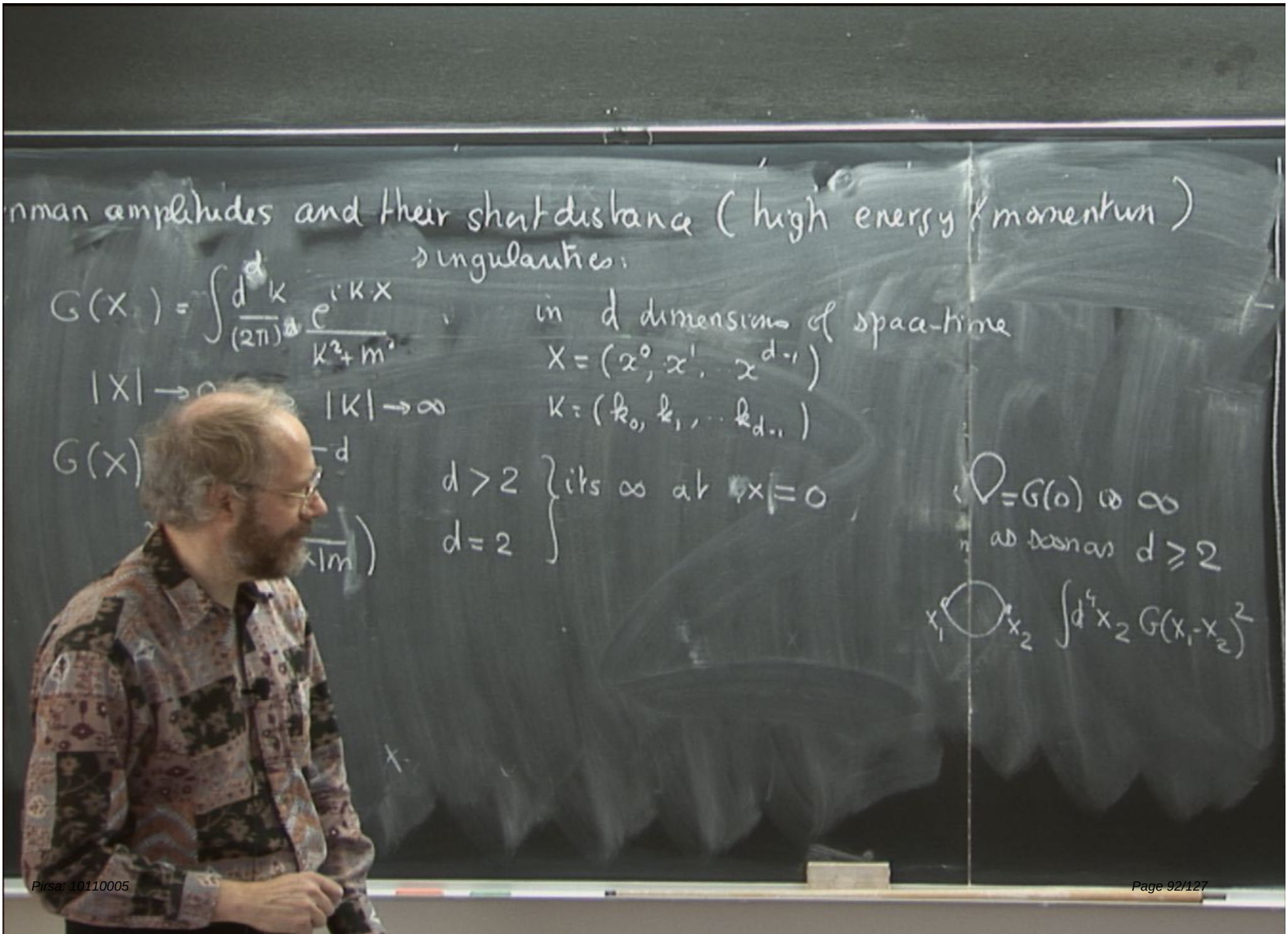
$\bigcirc = G(0) \to \infty$

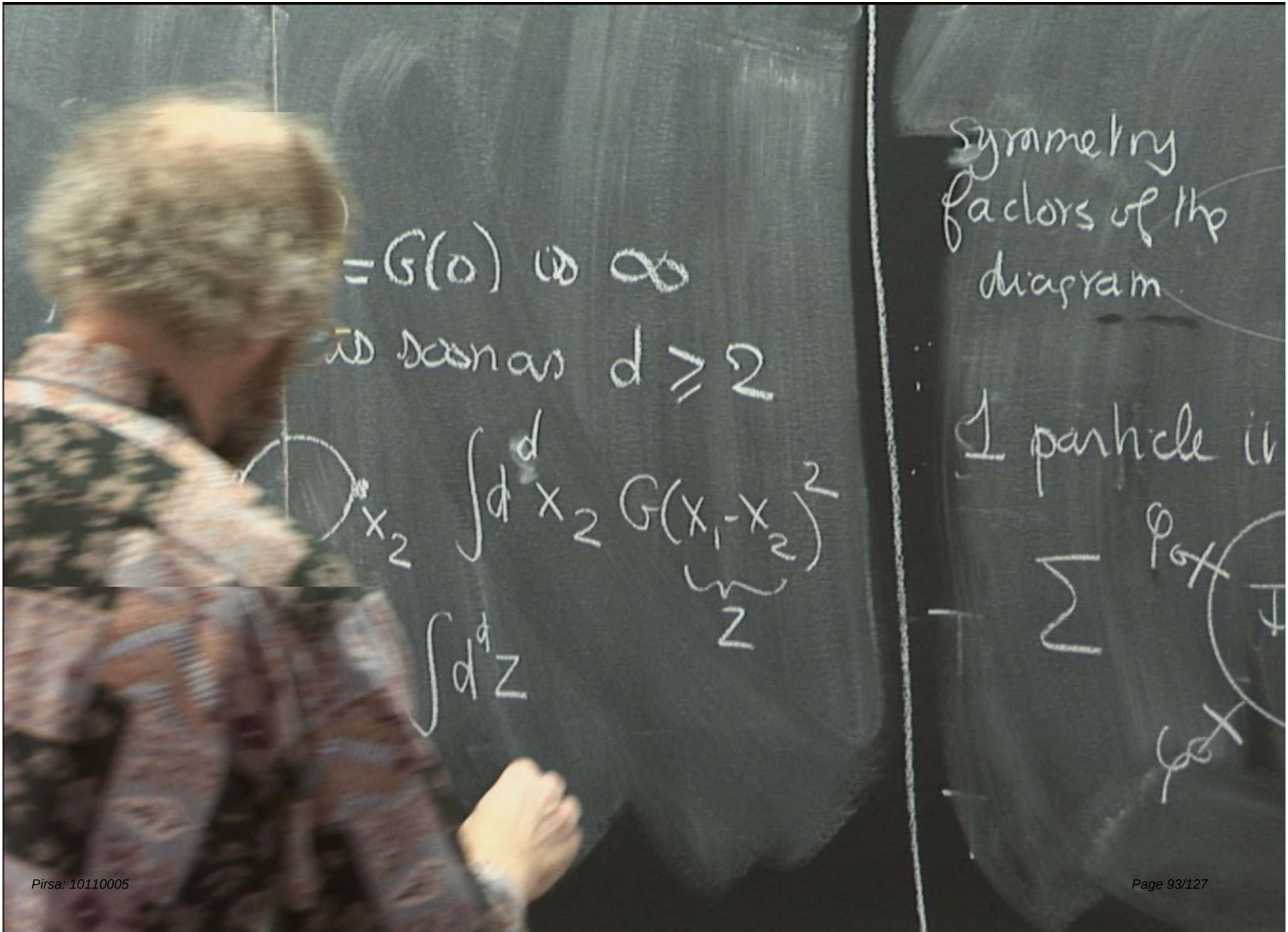
$\Gamma[\varphi] =$

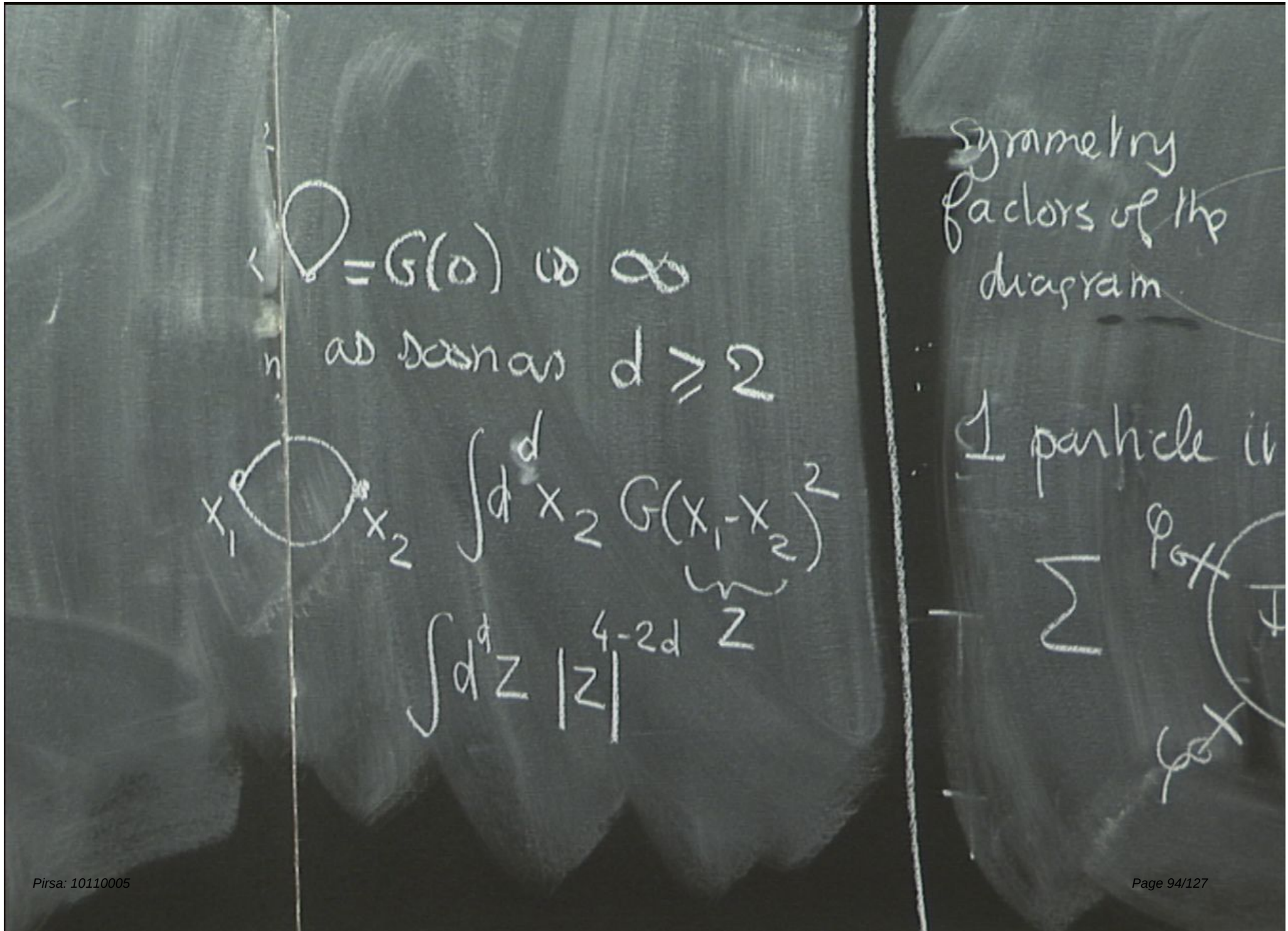
Symmetry factors of the diagram

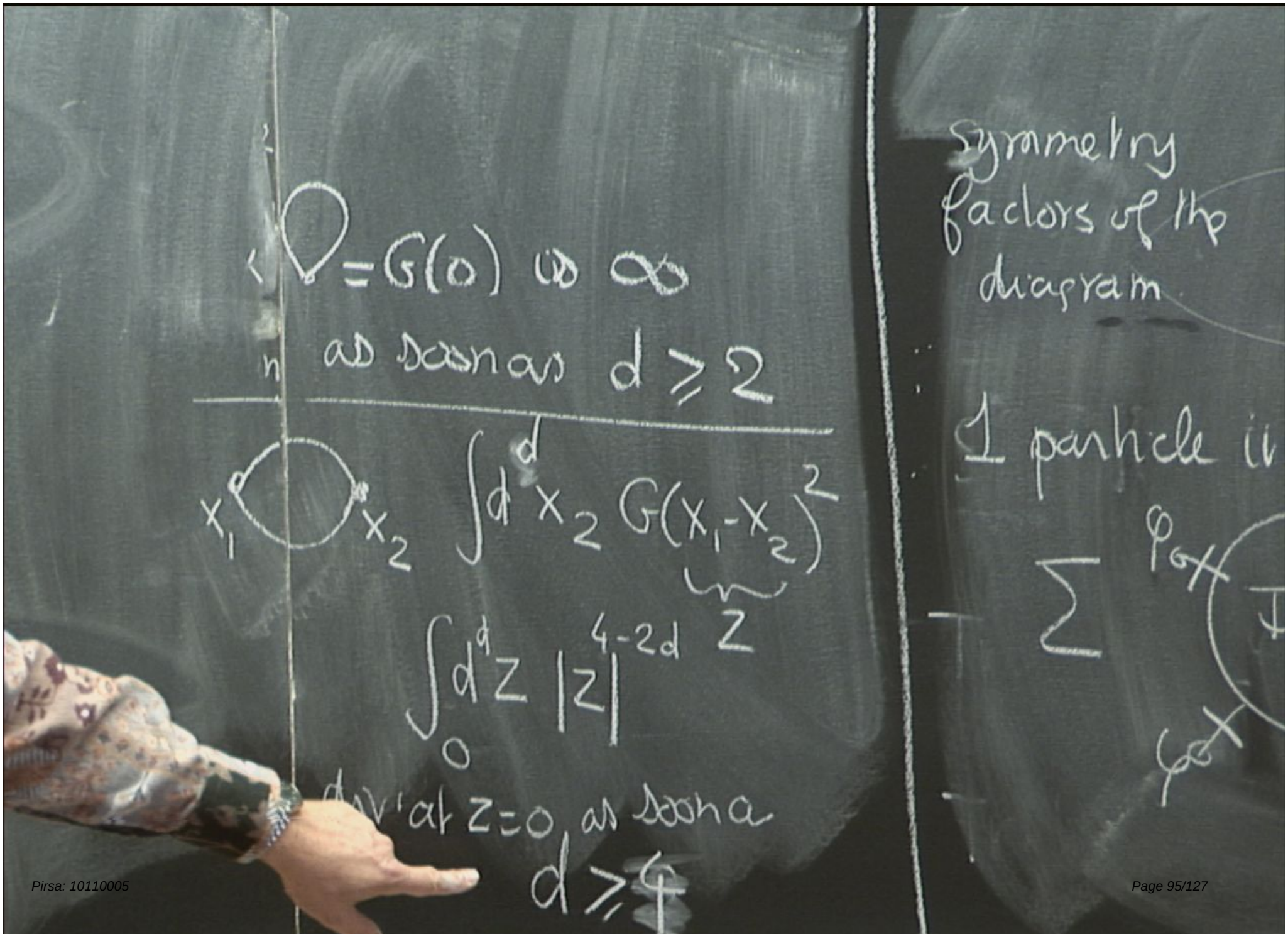
1 particle i...

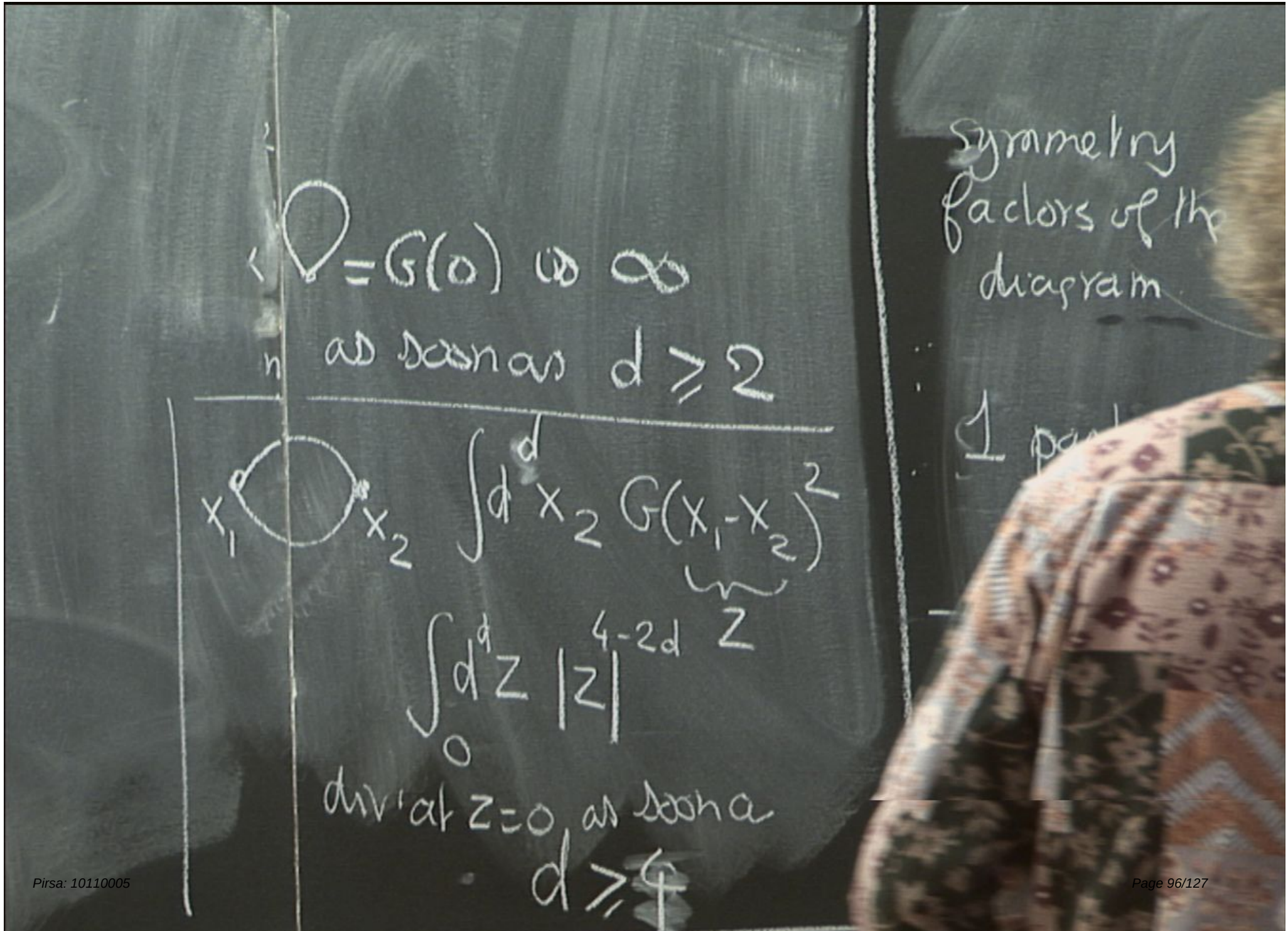
$\sum$

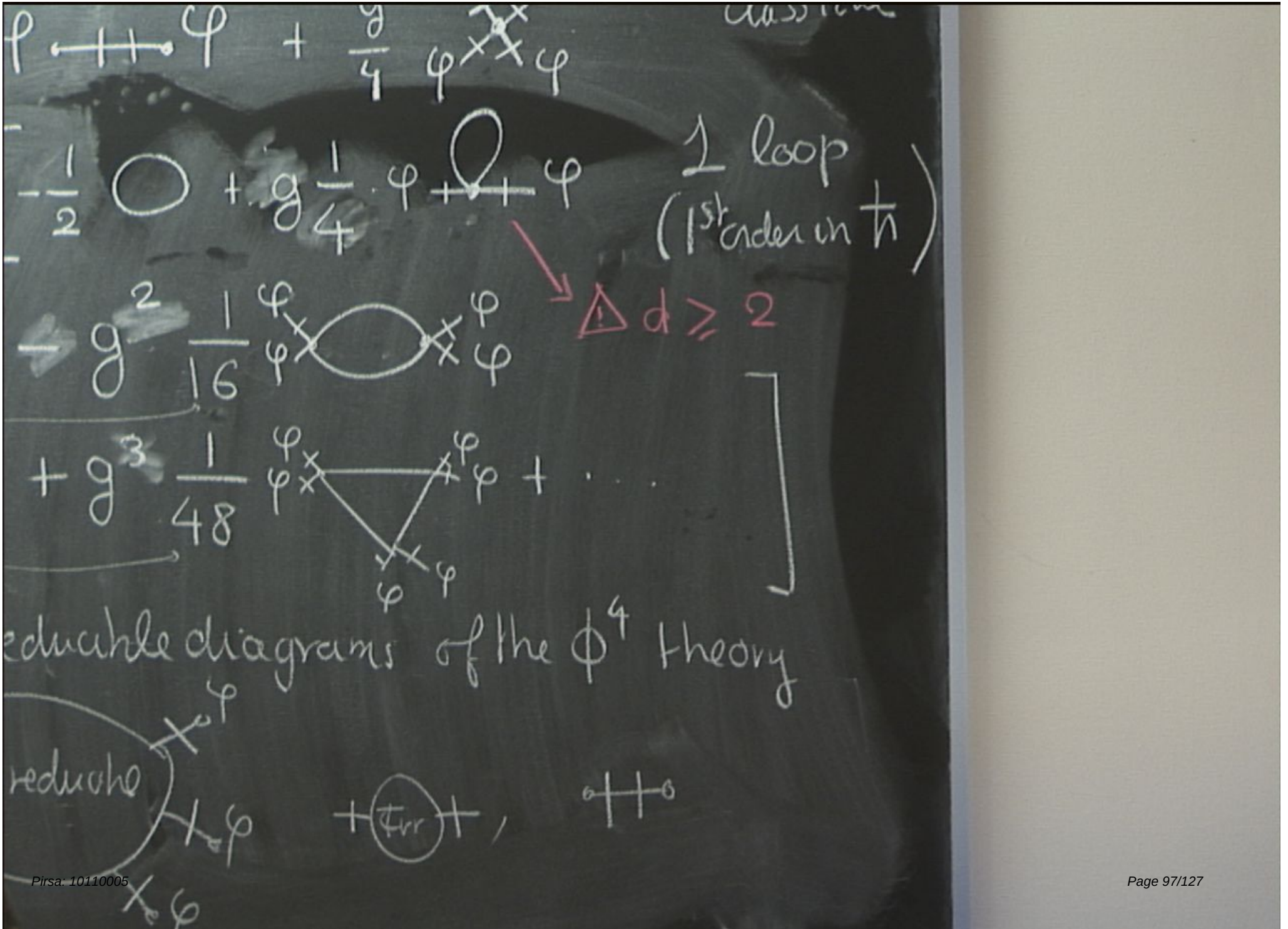

nman amplitudes and their short distance (high energy / momentum)
singularities:
G(X) = ∫ d^d k / (2π)^d e^{ikX} / (k^2 + m^2)
in d dimensions of space-time
X = (x^0, x^1, ... x^{d-1})
k = (k_0, k_1, ... k_{d-1})
|X| → 0
|k| → ∞
G(X)
d > 2
d = 2
its ∞ at x = 0
= G(0) is ∞
as soon as d ≥ 2
x_1
x_2
∫ d^4 x_2 G(x_1 - x_2)^2

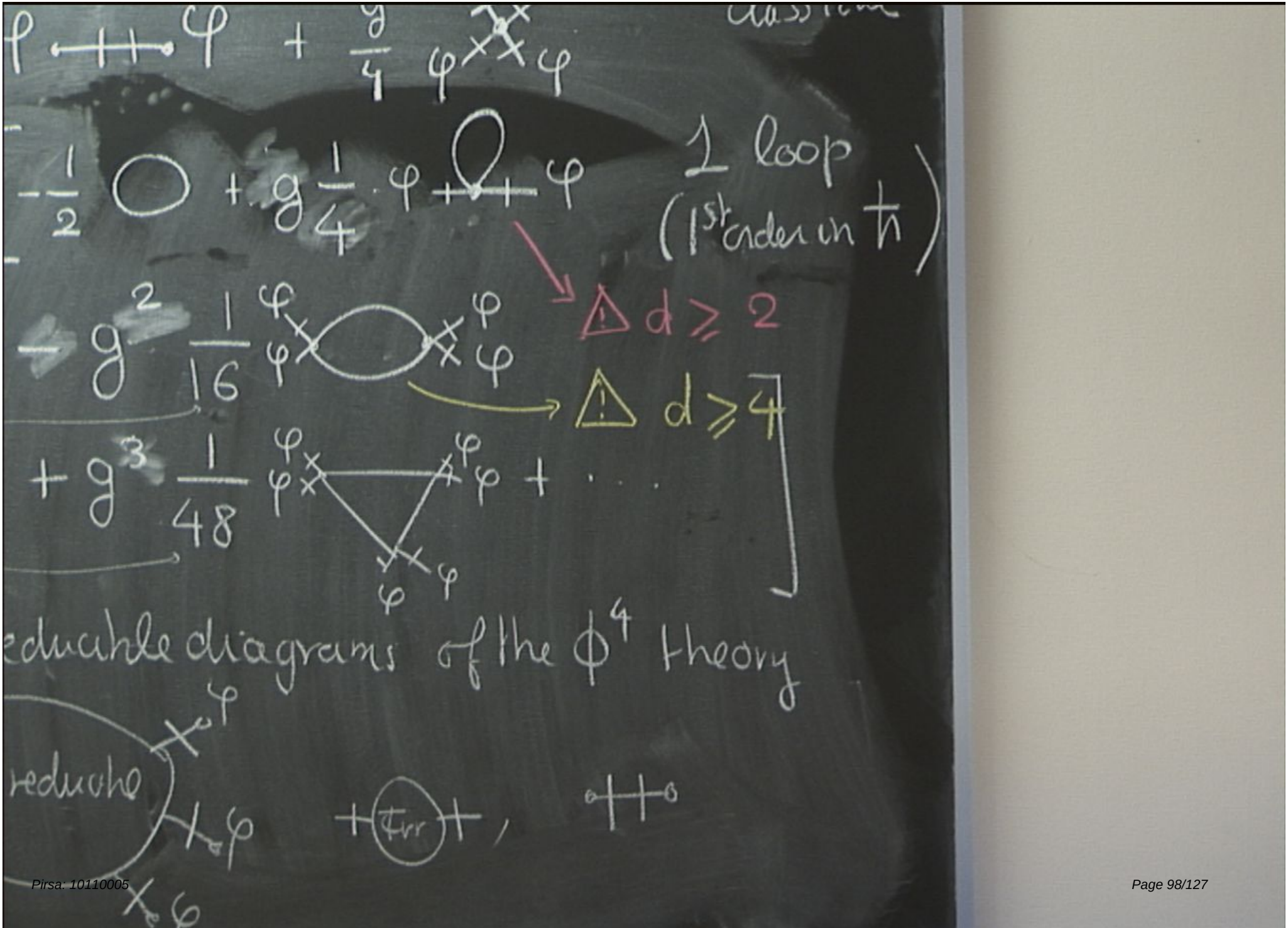

= G(0) is ∞
is soon as d ≥ 2
x2
∫d^d x2 G(x1-x2)^2
z
∫d^d z
Symmetry
factors of the
diagram
1 particle i
Σ
φ0
φ0


= G(0) is ∞
as soon as d ≥ 2
x1
x2
∫d^d x2 G(x1-x2)^2
z
∫d^d z |z|^(4-2d)
Symmetry
factors of the
diagram
1 particle i
Σ
φ
φ


= G(0) is ∞
as soon as d ≥ 2
x1
x2
∫ d^d x2 G(x1-x2)^2
z
∫0 d^d z |z|^4-2d
at z=0 as soon a
d ≥ 4
Symmetry
factors of the
diagram
1 particle i
Σ


= G(0) ∞
as soon as d ≥ 2
x_1
x_2
$\int d^d x_2 \, G(x_1 - x_2)^2$
z
$\int_0^\infty d^q z \, |z|^{4-2d} z$
div at z=0 as soon as
d ≥ 4
symmetry
factors of the
diagram
1


1 loop
(1st order in ħ)
⚠ d ≥ 2
educible diagrams of the φ⁴ theory
reducible


1 loop
(1st order in ħ)
d ≥ 2
d ≥ 4
educible diagrams of the φ⁴ theory
reducible

an amplitudes and their short distance (high energy / momentum) singularities:

$$G(X) = \int \frac{d^d K}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \Leftrightarrow |K| \to \infty$

$$G(X) \sim C_d \cdot |X|^{2-d} \quad d > 2$$

$$\sim \frac{1}{2\pi} \log\left(\frac{1}{|X|m}\right) \quad d = 2$$

} its ∞ at $|X| = 0$

y?

$\langle \phi(X)\phi(Y) \rangle$

$= G(0)$ is ∞ as soon as $d \geq 2$

x_1 x_2 $\int d^d x_2 \, G(x_1 - x_2)^2$

z

$$\int_0 d^d z \, |z|^{4-2d}$$

div. at $z = 0$ as soon as $d \geq 4$

an amplitudes and their short distance (high energy / momentum) singularities:

$$G(X) = \int \frac{d^d K}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

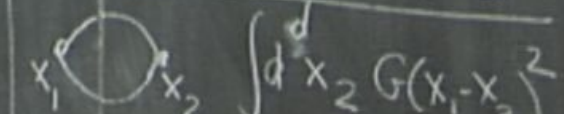
$|X| \to 0 \iff |K| \to \infty$

$$G(X) \sim C_d \cdot |X|^{2-d} \quad d > 2$$

$$\sim \frac{1}{2\pi} \log\left(\frac{1}{|X| m}\right) \quad d = 2$$

} its ∞ at $|X| = 0$

$\bigcirc = G(0)$ is ∞ as soon as $d \geq 2$



$$\int d^d x_2 \, G(x_1 - x_2)^2$$

$$\int_0 d^d z \, |z|^{4-2d} \, z$$

div at $z = 0$ as soon as $d \geq 4$

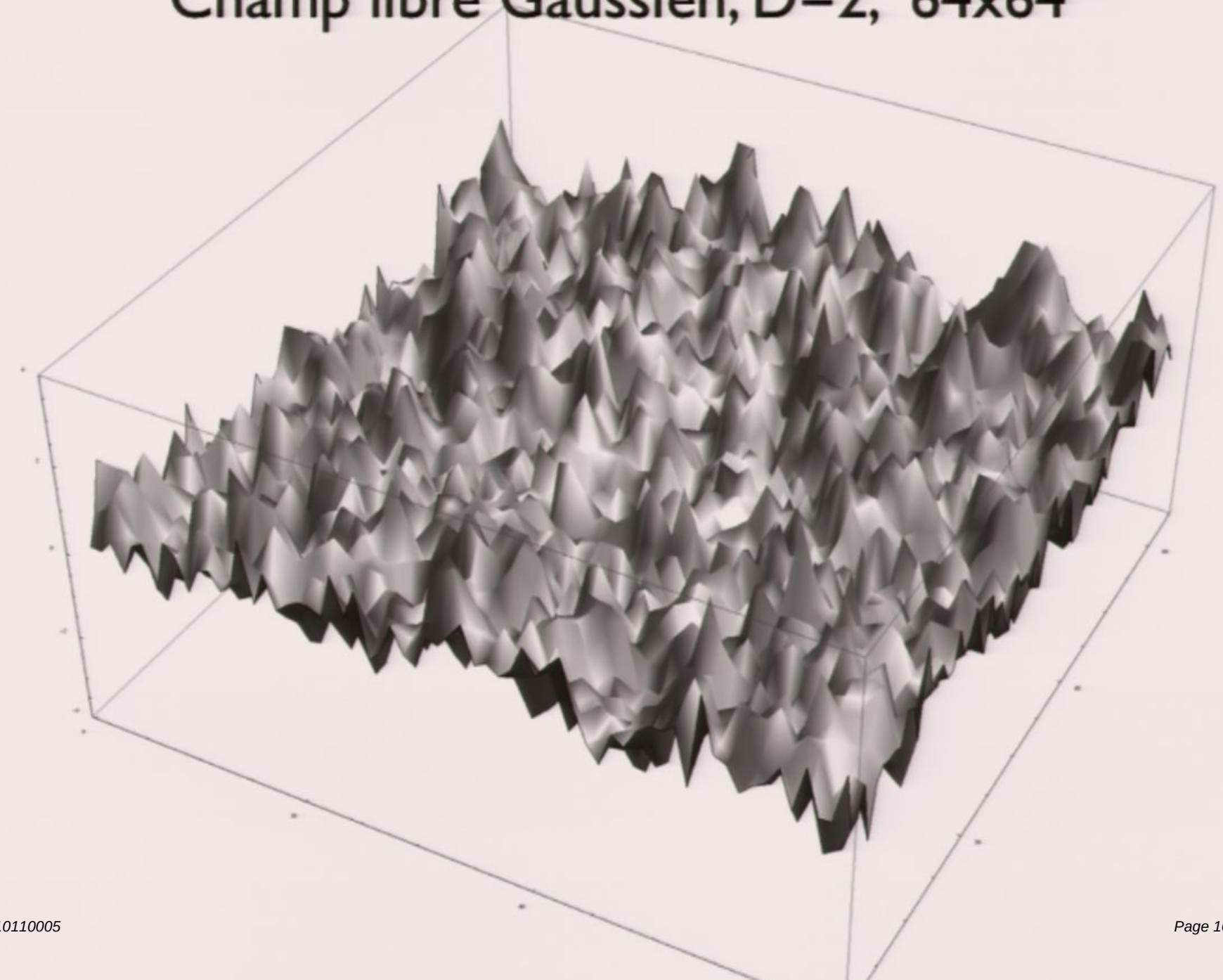
y ?

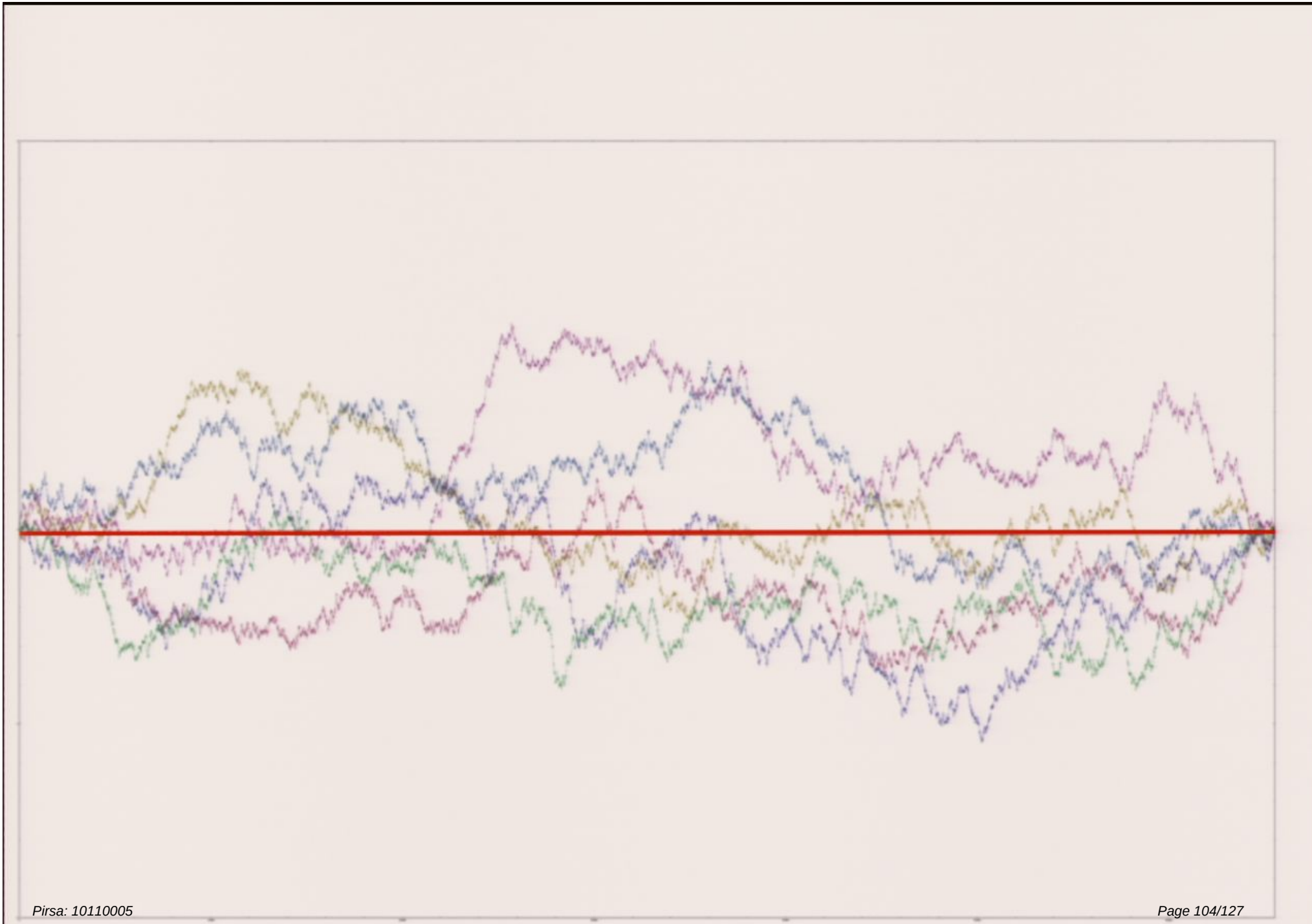
$$\langle \phi(x) \phi(y) \rangle \nearrow \infty \text{ when } X \to Y$$

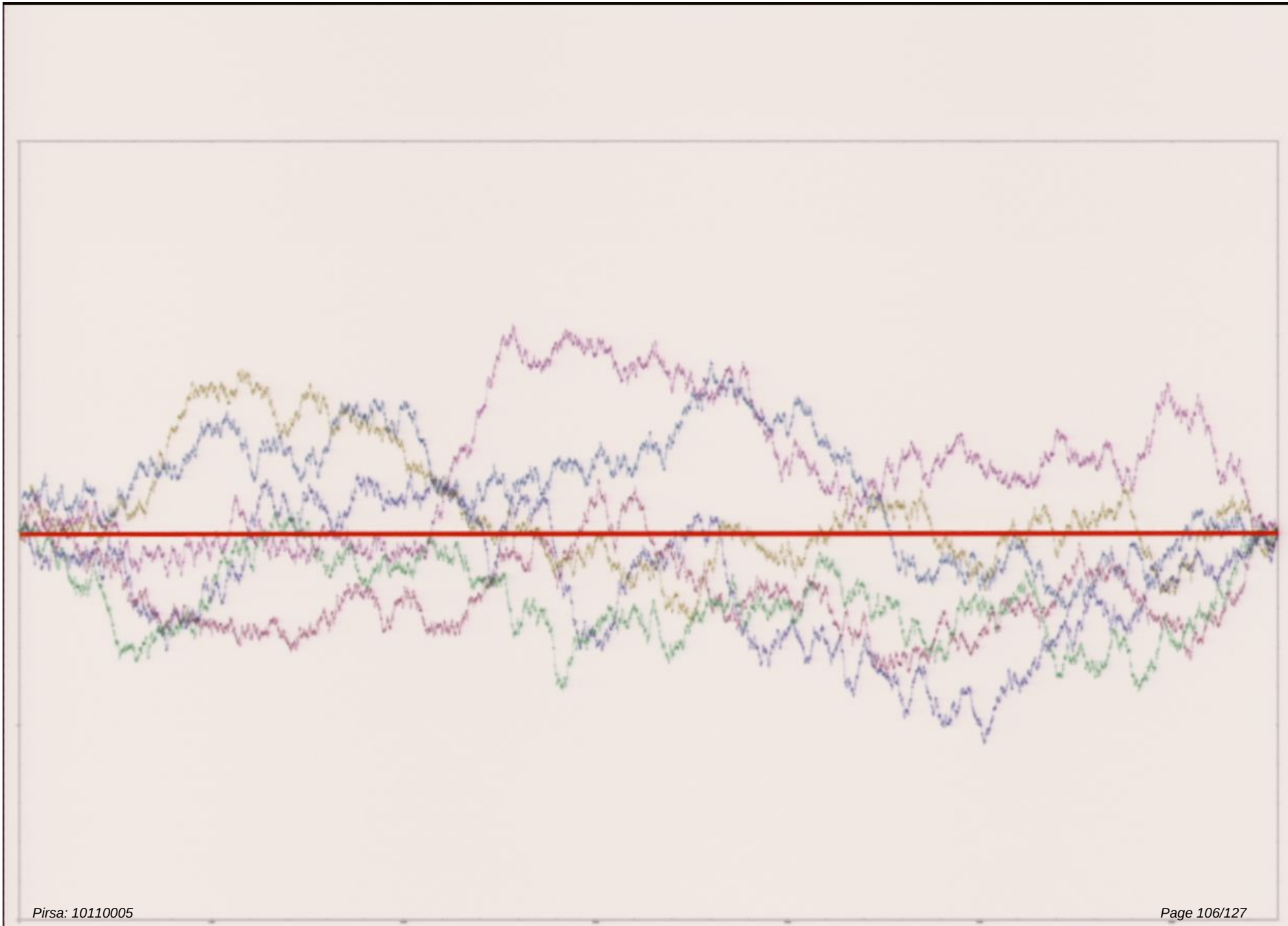
Typical free field configurations

Typical free field configurations

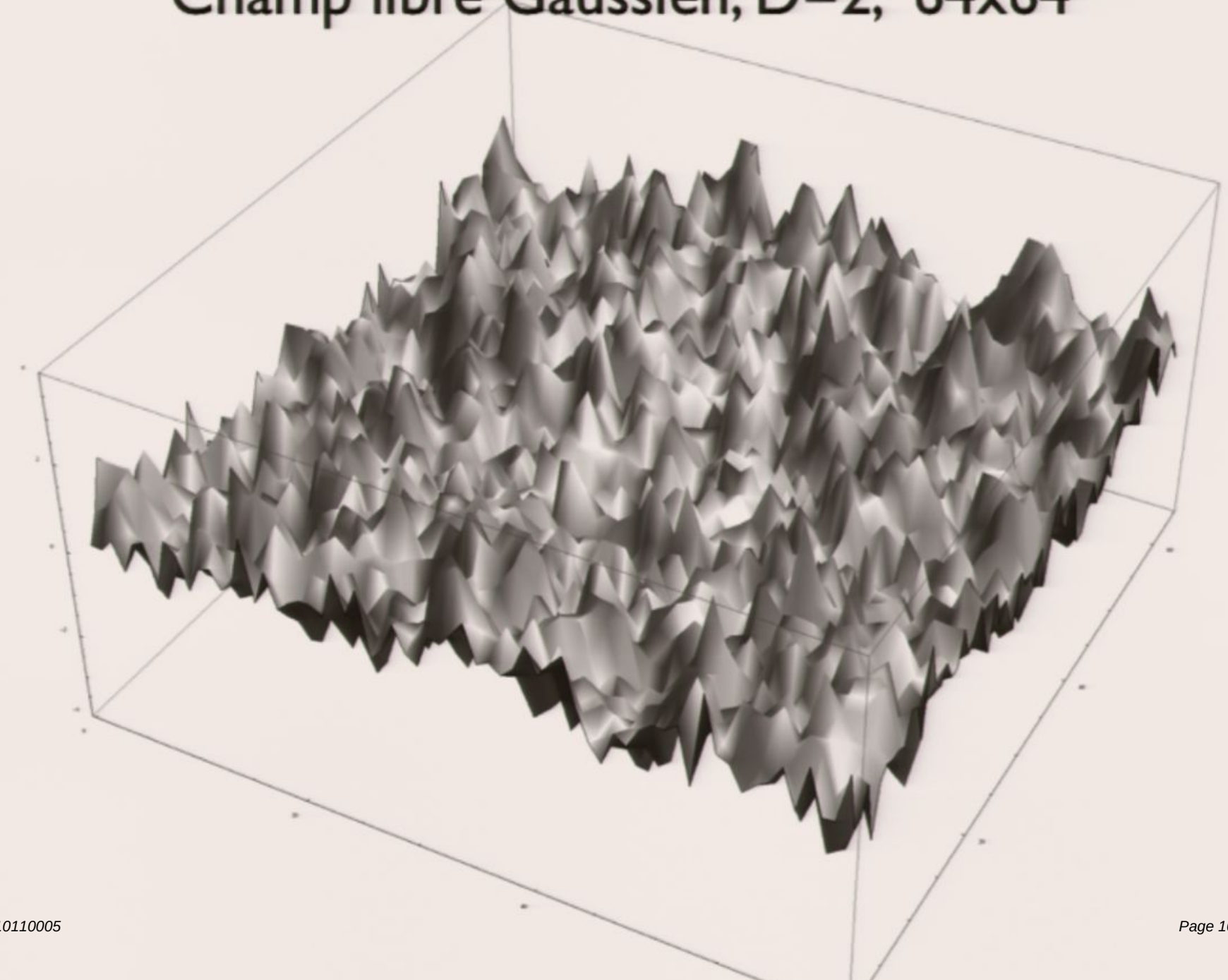
Champ libre Gaussien, D=2, 64x64

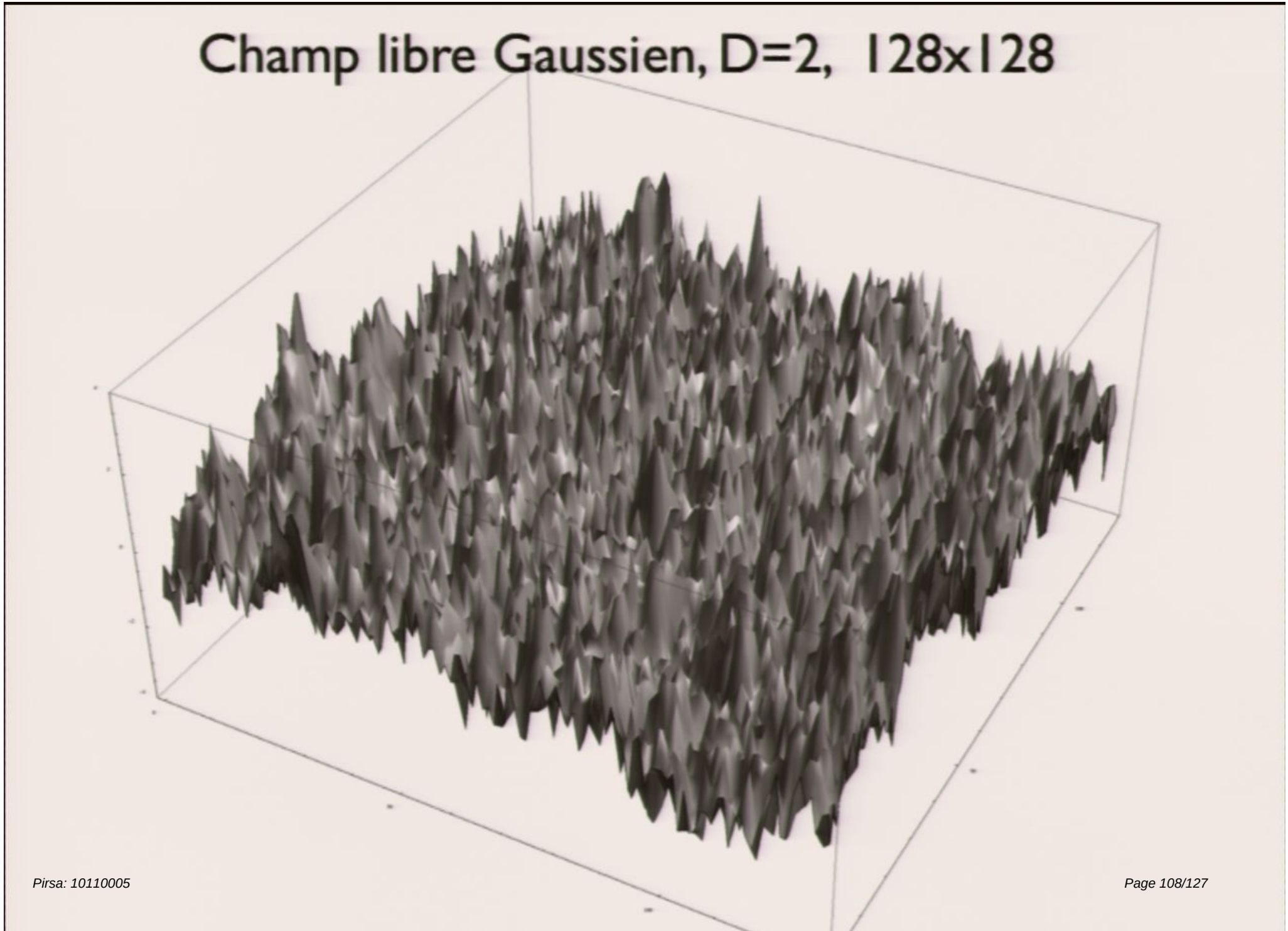


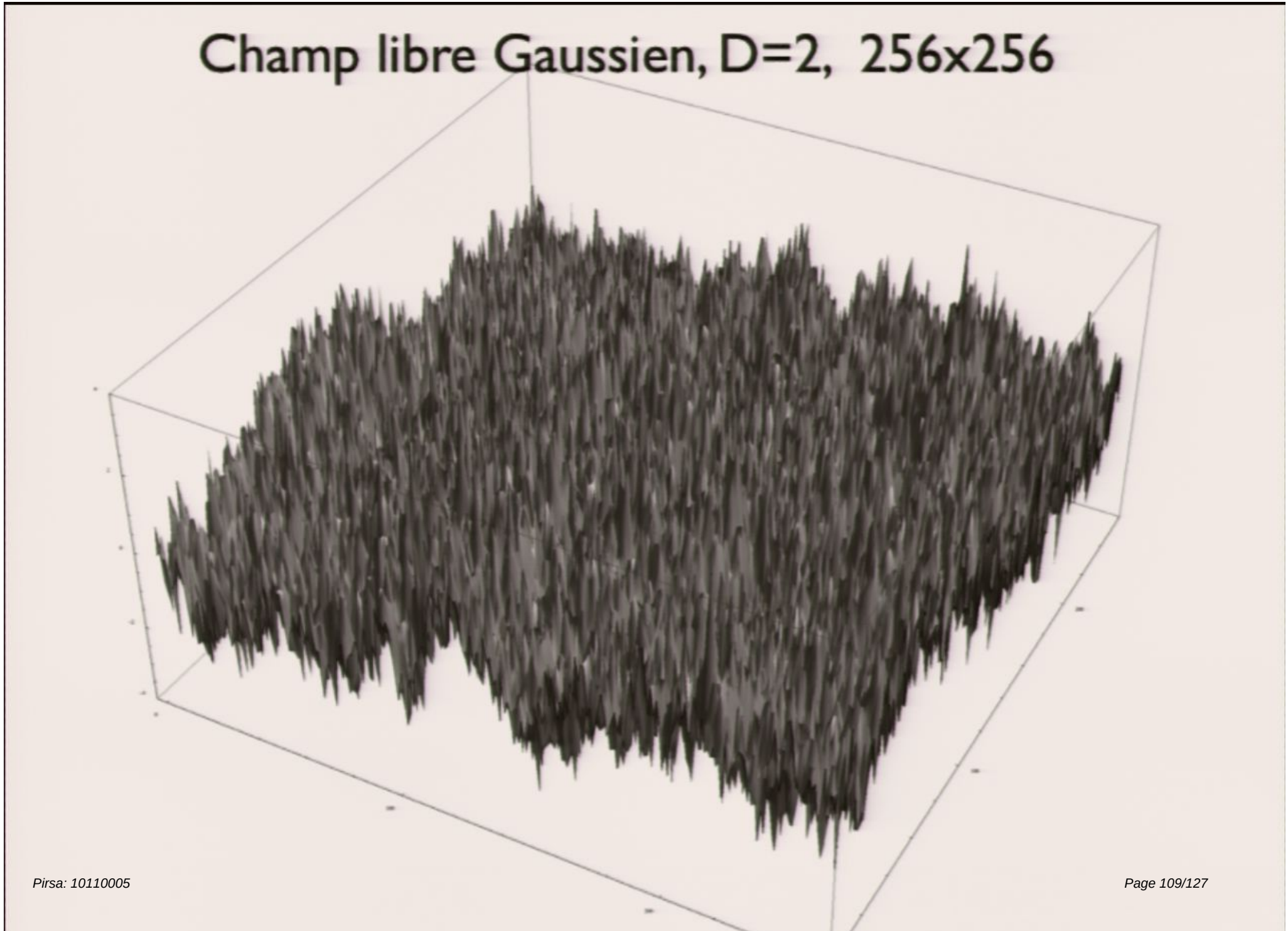




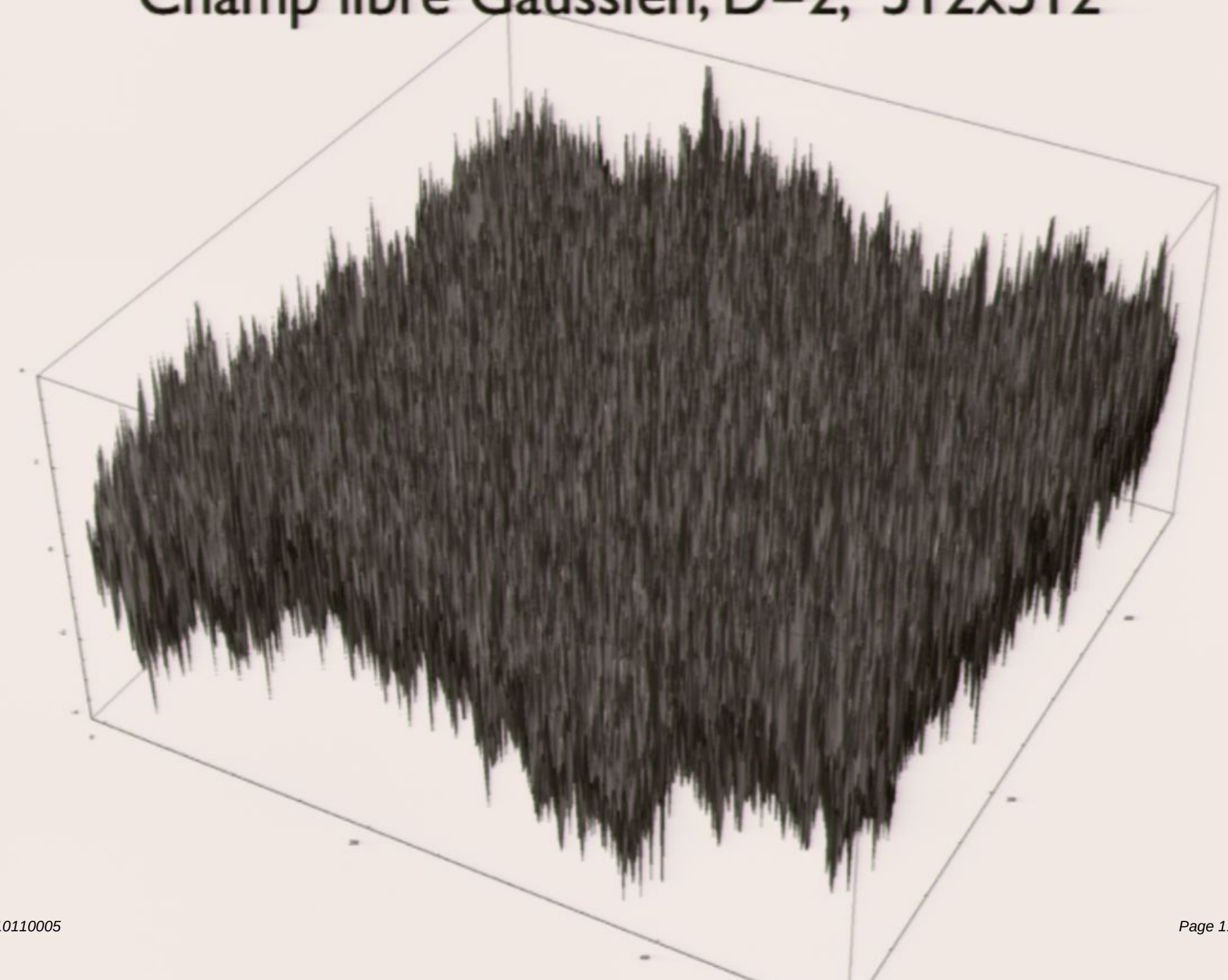
Champ libre Gaussien, D=2, 64x64



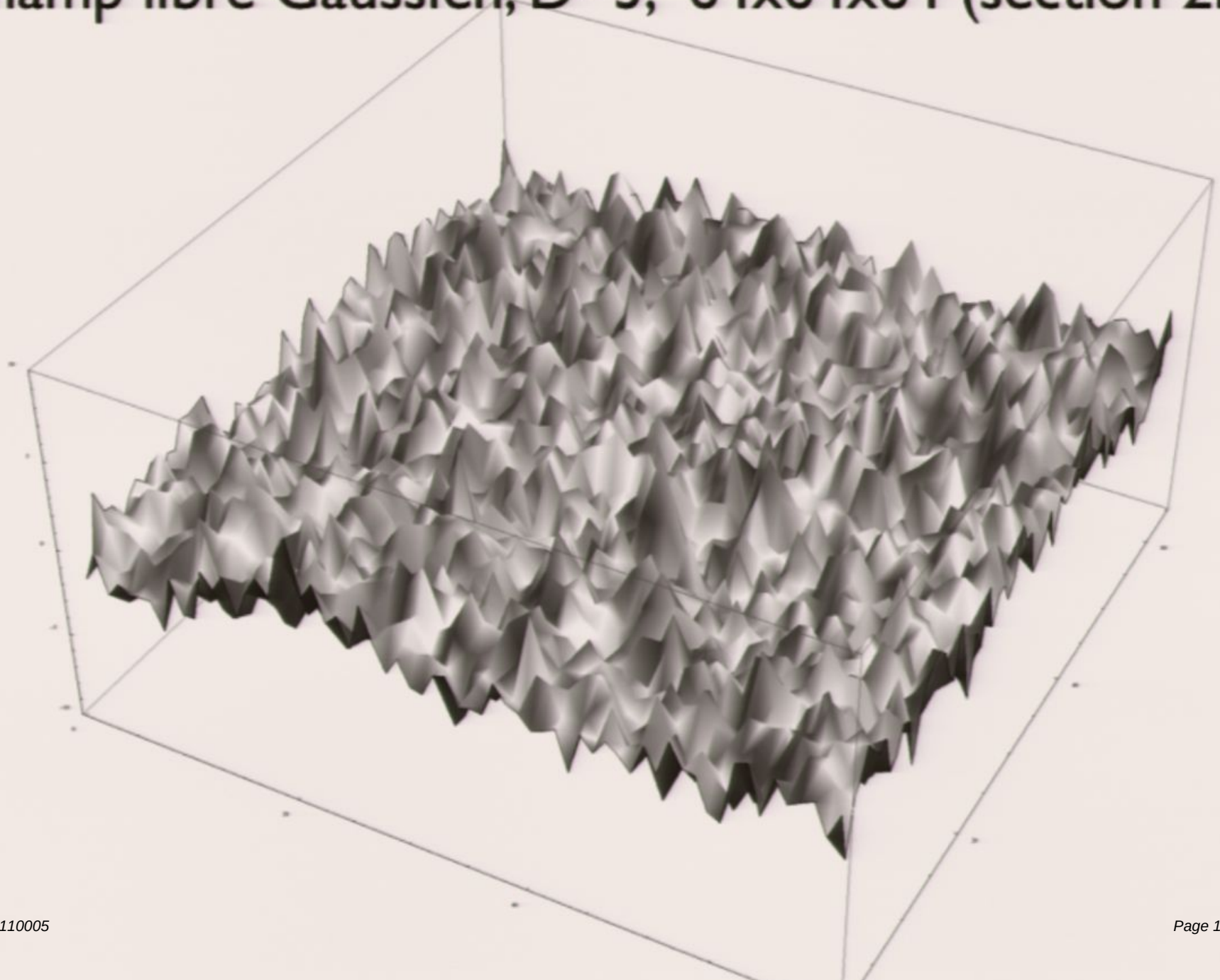

Champ libre Gaussien, D=2, 128x128


Champ libre Gaussien, D=2, 256x256

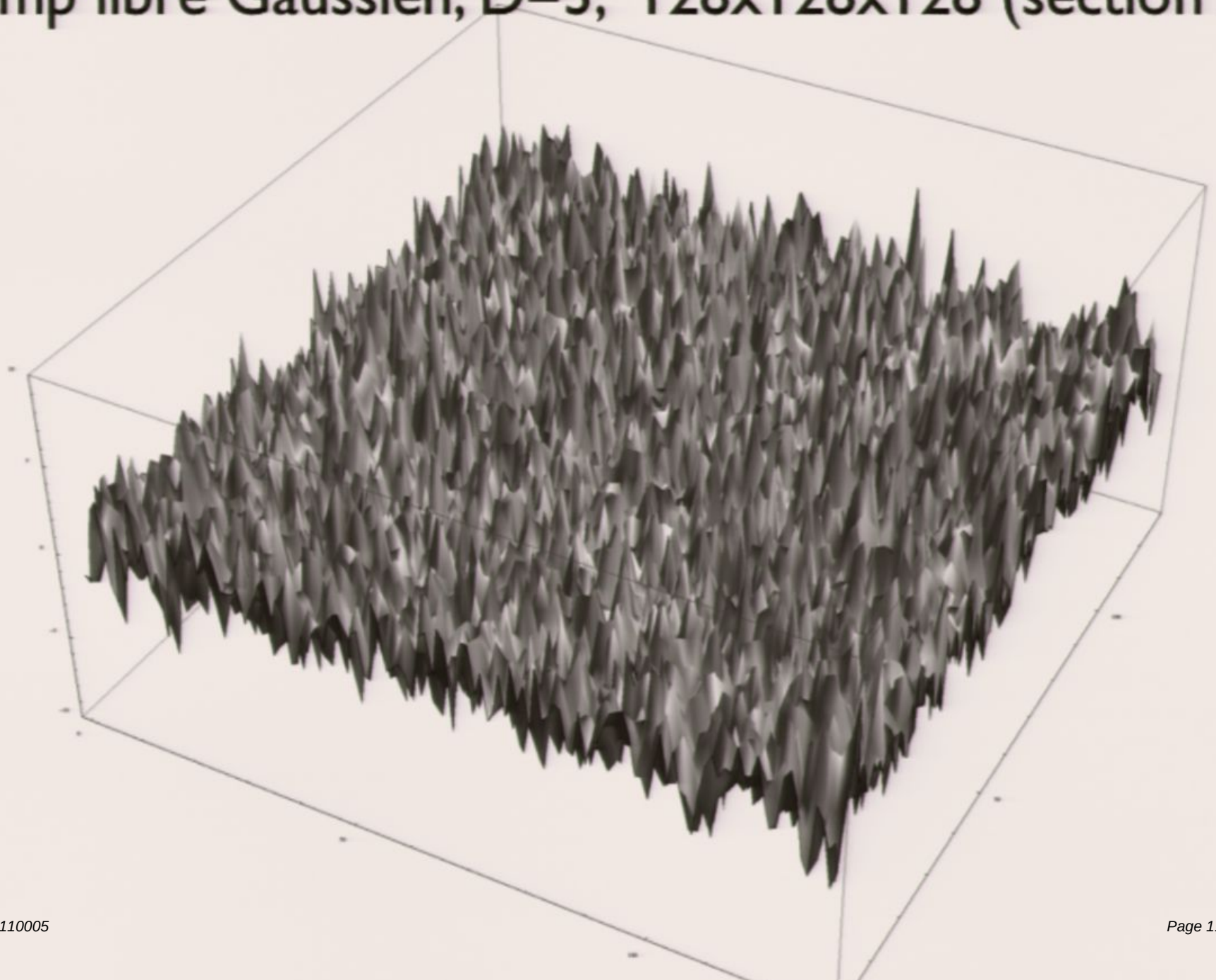
Champ libre Gaussien, D=2, 512x512



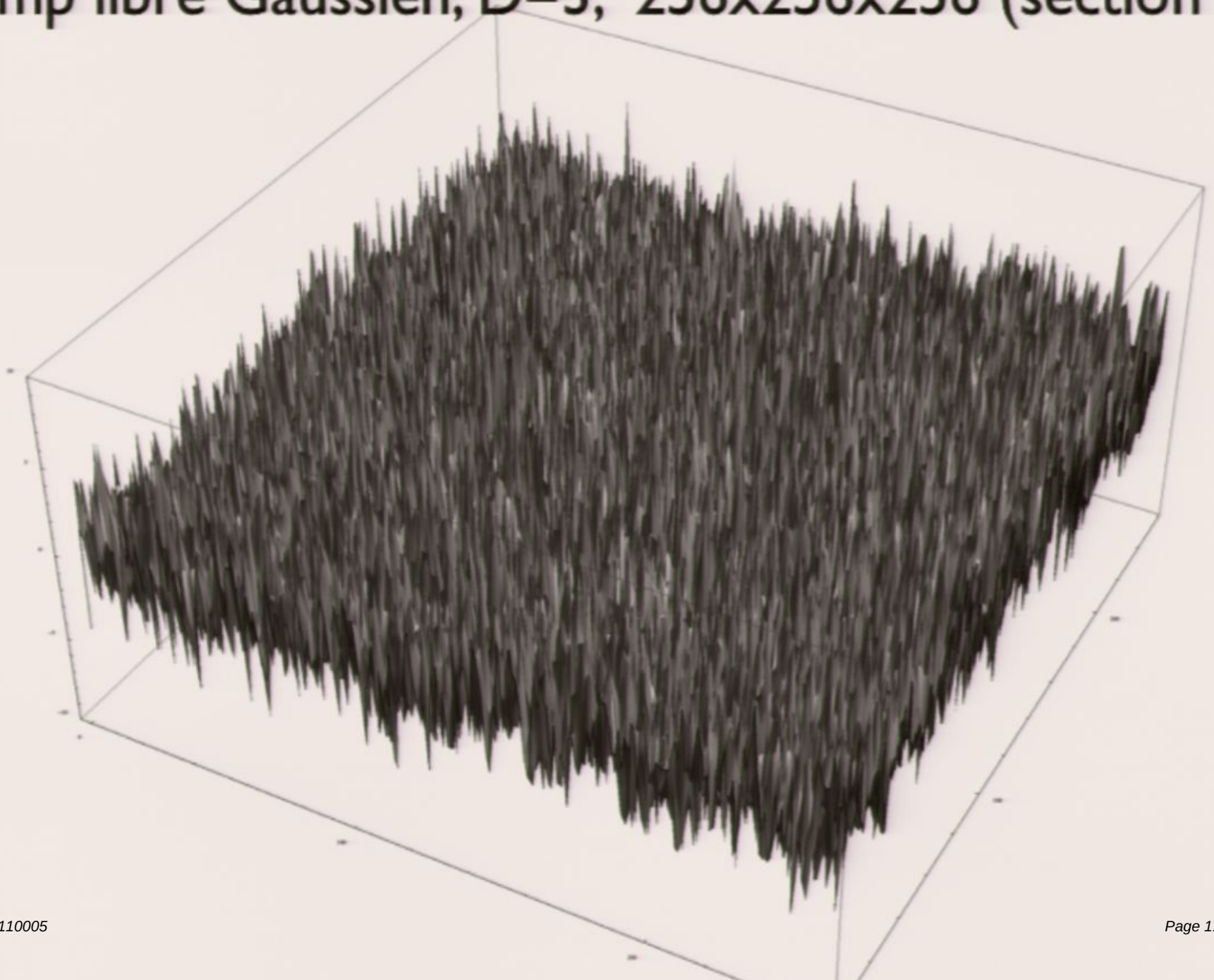
Champ libre Gaussien, D=3, 64x64x64 (section 2D)



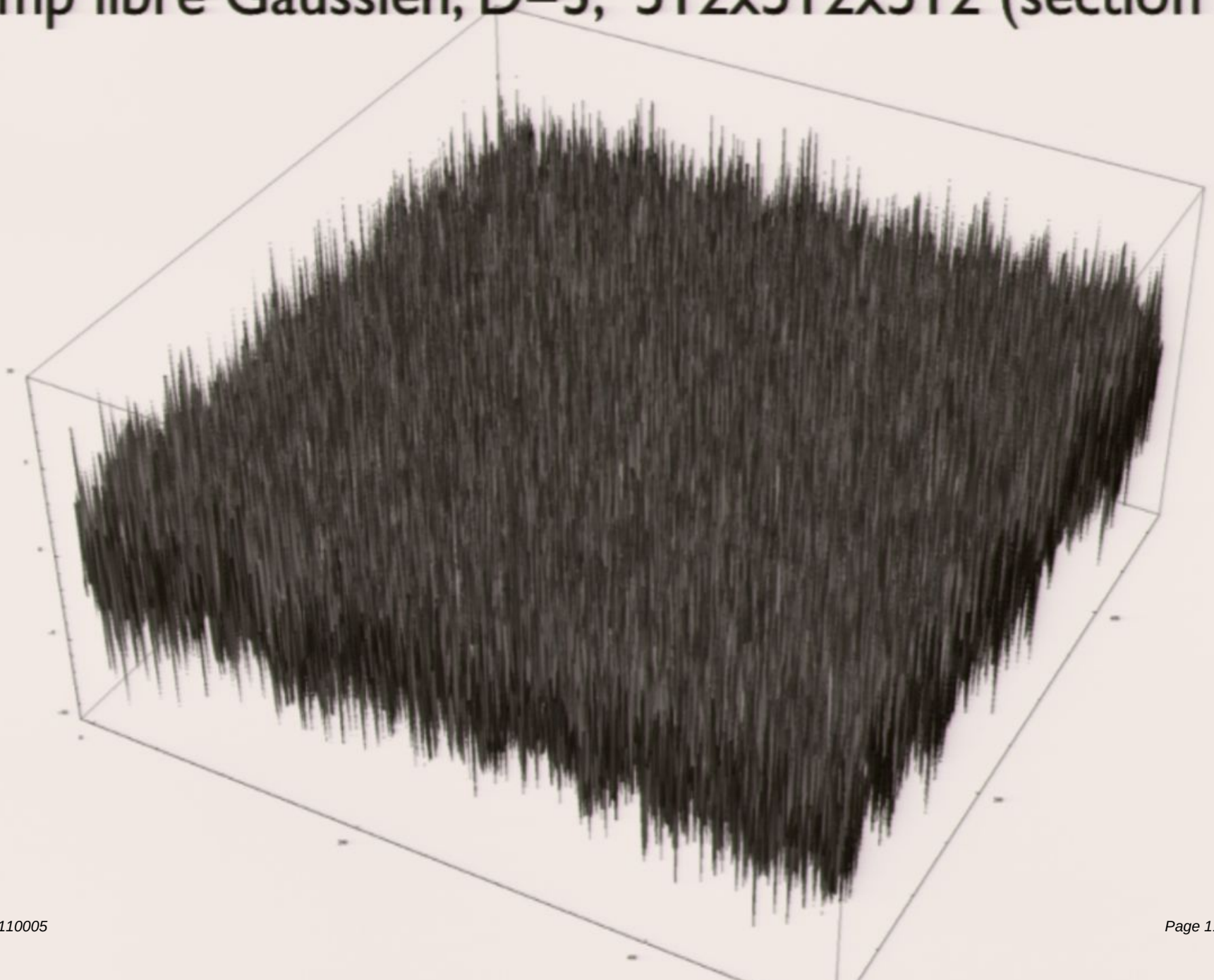
Champ libre Gaussien, D=3, 128x128x128 (section 2D)

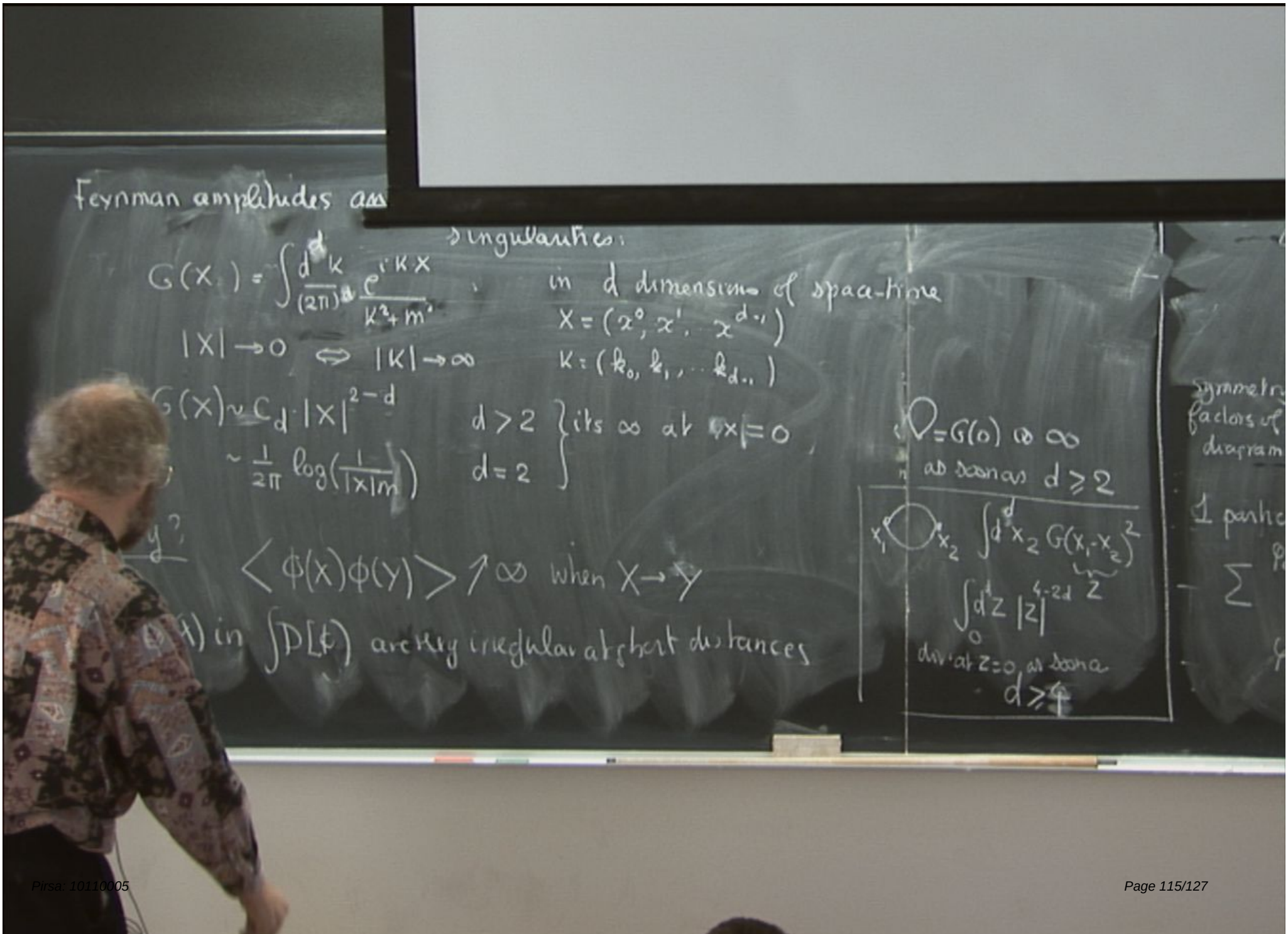


Champ libre Gaussien, D=3, 256x256x256 (section 2D)



Champ libre Gaussien, D=3, 512x512x512 (section 2D)




Feynman amplitudes
singularities:
in d dimensions of space-time
$X = (x^0, x^1, \dots x^{d-1})$
$K = (k_0, k_1, \dots k_{d-1})$
$|X| \to 0 \Leftrightarrow |K| \to \infty$
$G(X) \sim C_d \cdot |X|^{2-d}$
$d > 2$
$d = 2$
its ∞ at $|x| = 0$
$\sim \frac{1}{2\pi} \log\left(\frac{1}{|x| m}\right)$
$\langle \phi(X)\phi(Y) \rangle \nearrow \infty$ when $X \to Y$
in $\int D[\phi]$ are very irregular at short distances
$= G(0) \infty \infty$
as soon as $d \geq 2$
$\int d^d x_2 \, G(x_1 - x_2)^2$
$\int_0^d z \, |z|^{4-2d}$
div. at $z=0$ as soon as
$d \geq 4$
symmetry
factors of
diagram
1 partic

Feynman amplitudes ass

singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

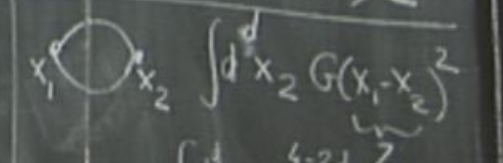
$|X| \to 0 \Leftrightarrow |K| \to \infty$

$$G(X) \sim C_d \cdot |X|^{2-d} \quad d > 2$$

$$\sim \frac{1}{2\pi} \log\left(\frac{1}{|X|m}\right) \quad d = 2$$

} its ∞ at $|X| = 0$

$= G(0)$ is ∞ as soon as $d \geq 2$



$$\int d^d x_2 \, G(x_1 - x_2)^2$$

$$\int_0 d^d z \, |z|^{4-2d} z$$

div. at $z = 0$ as soon as $d \geq 4$

Why?

$$\langle \phi(X) \phi(Y) \rangle \nearrow \infty \text{ when } X \to Y$$

$\phi(x)$ in $\int \mathcal{D}[\phi]$ are very irregular at short distances

the higher d is, the more irregular they are

Feynman amplitudes ass

→ singularities:

$$G(X) = \int \frac{d^d k}{(2\pi)^d} \frac{e^{iKX}}{K^2 + m^2}$$

in d dimensions of space-time

$X = (x^0, x^1, \ldots x^{d-1})$

$K = (k_0, k_1, \ldots k_{d-1})$

$|X| \to 0 \iff |K| \to \infty$

$$G(X) \sim C_d \cdot |X|^{2-d} \quad d > 2$$

$$\sim \frac{1}{2\pi} \log\left(\frac{1}{|X| m}\right) \quad d = 2$$

} its ∞ at $|X| = 0$

$= G(0)$ is ∞ as soon as $d \geq 2$

$$\int_0 d^d z \, |z|^{4-2d}$$

div. at $z = 0$ as soon as $d \geq 4$

symmetry factors of diagram

1 particle

Why?

$$\langle \phi(X) \phi(Y) \rangle \nearrow \infty \text{ when } X \to Y$$

$\phi(X)$ in $\int \mathcal{D}[\phi]$ are very irregular at short distances

the higher d is, the more irregular they are

energy (momentum)

(space-time

$= G(0) \to \infty$

as soon as $d \geq 2$

x_1 x_2 $\int d^d x_2 \, G(x_1 - x_2)^2$

$x_2 - x_1 = z$

$\int_0^1 d^d z \, |z|^{4-2d}$

div. at $z=0$ as soon as

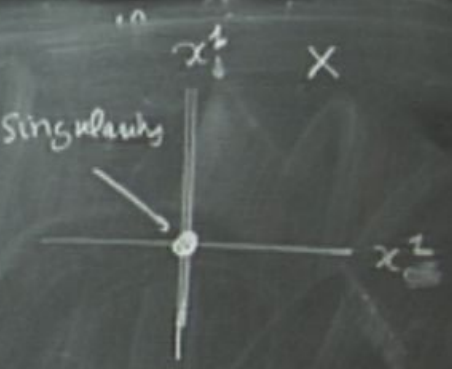
$d \geq 4$

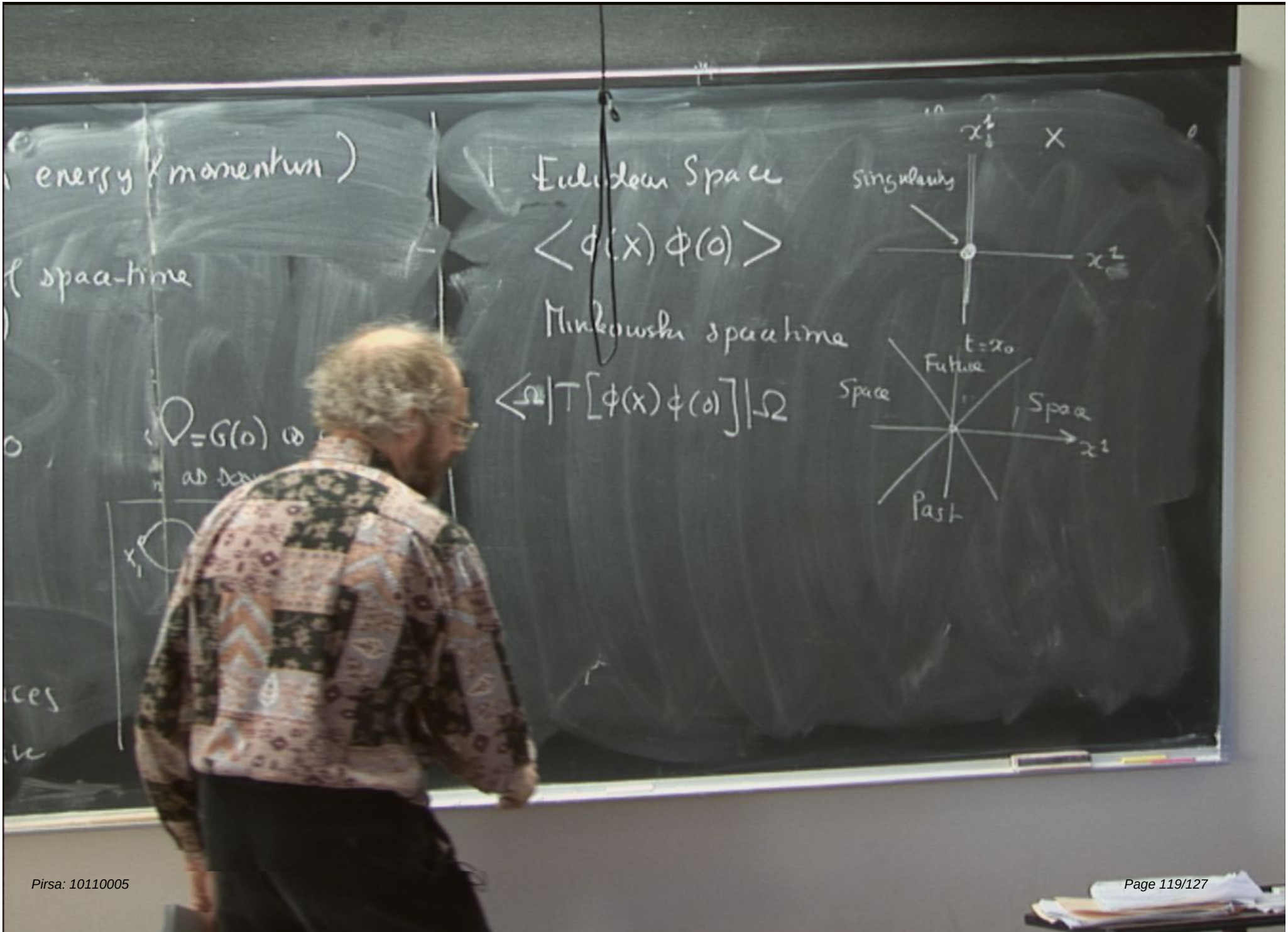
Euclidean Space

$\langle \phi(x) \phi(0) \rangle$

Minkowski space-time

<




energy (momentum)
space-time
Euclidean Space
$\langle \phi(x)\phi(0) \rangle$
Minkowski spacetime
$\langle \Omega | T[\phi(x)\phi(0)] | \Omega$
singularity
x^1
$t = x_0$
Future
Space
Space
Past

energy (momentum)

(space-time

$= G(0) \to \infty$

as soon as $d \geq 2$

x_1 x_2 $\int d^d x_2 \, G(x_1 - x_2)^2$

z

$\int_0^1 d^d z \, |z|^{4-2d}$

div. at $z=0$ as soon as

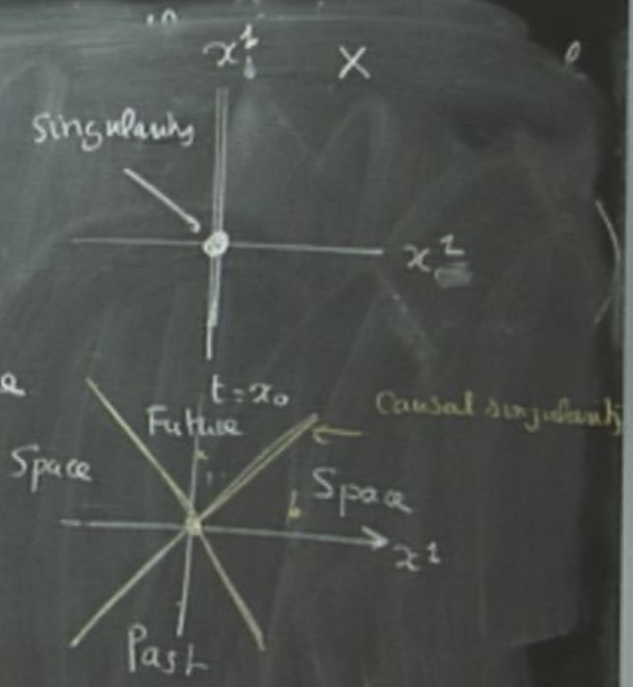
$d \geq 4$

Euclidean Space

$\langle \phi(x) \phi(0) \rangle$

Minkowski spacetime

$\langle \Omega | T[\phi(x) \phi(0)] | \Omega \rangle$



energy (momentum)

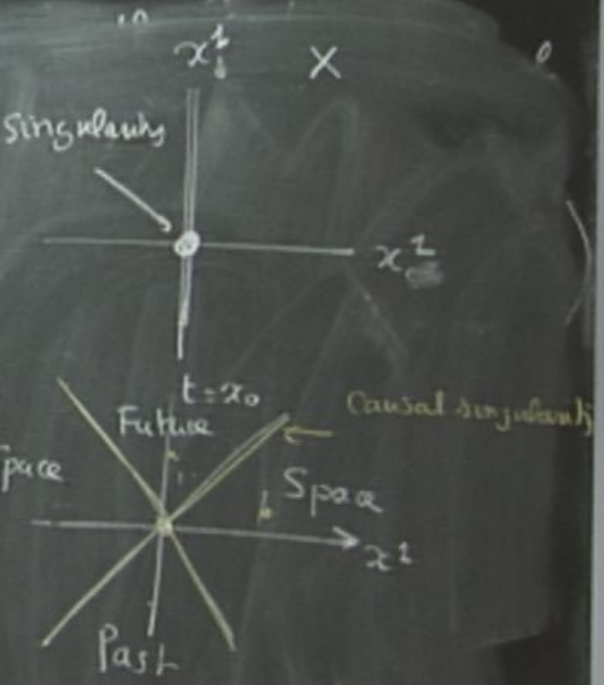
(space-time

Euclidean Space

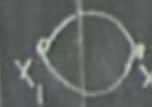
$\langle \phi(x)\,\phi(0) \rangle$

Minkowski spacetime

$\langle \Omega | T[\phi(x)\,\phi(0)] | \Omega$



$= G(0) \to \infty$ as soon as $d \geq 2$

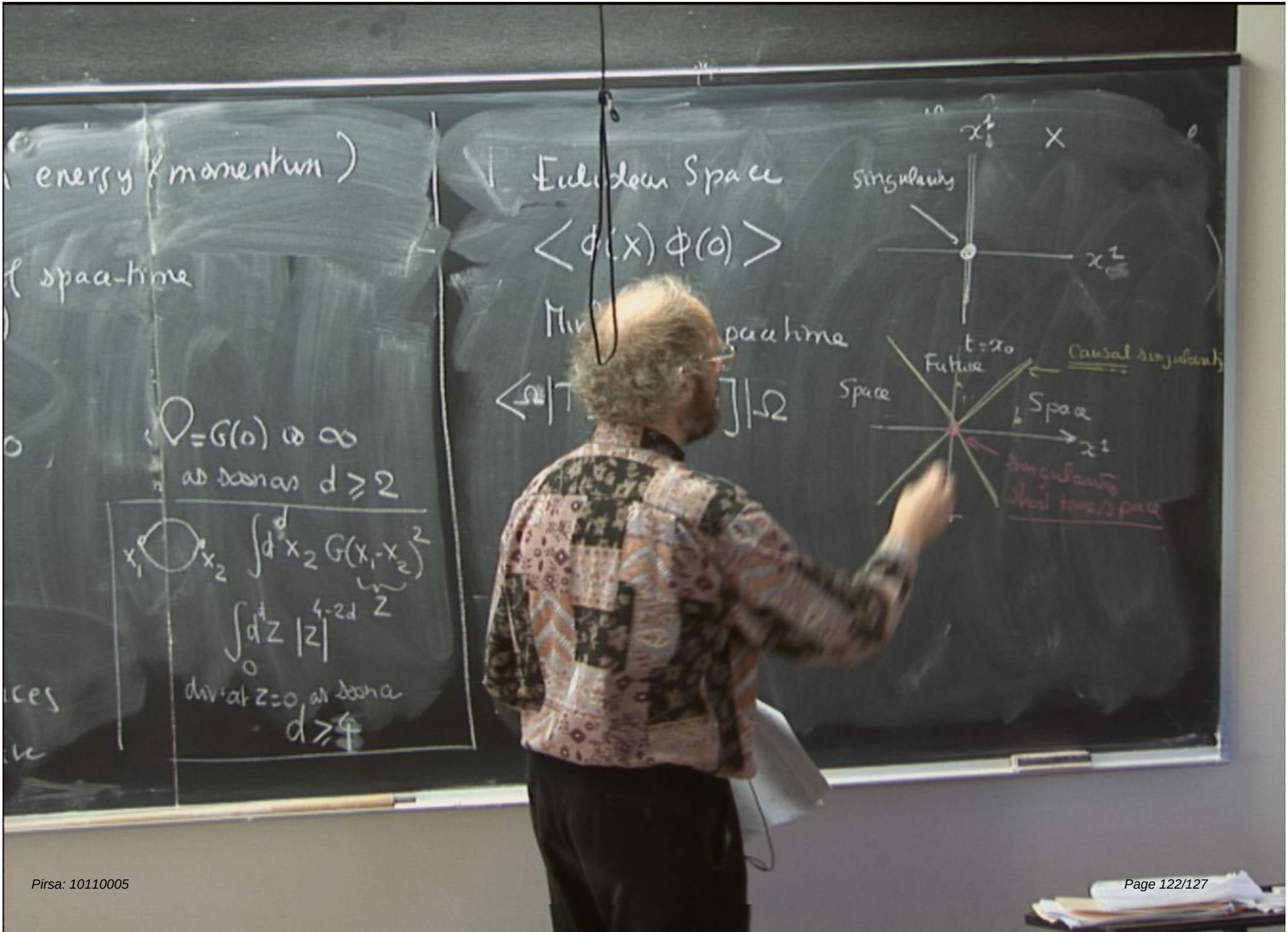


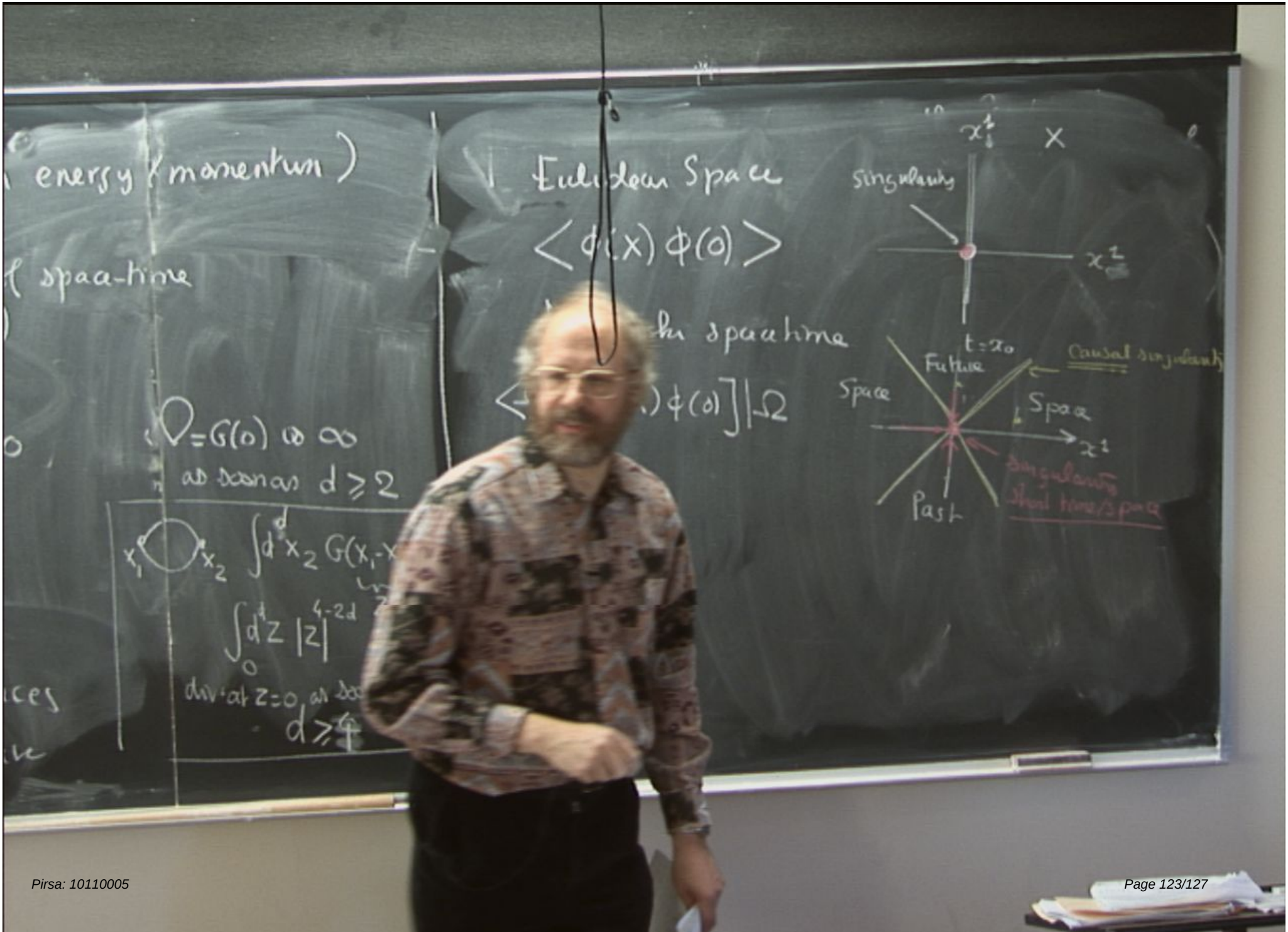
$\int d^d x_2\, G(x_1 - x_2)^2$

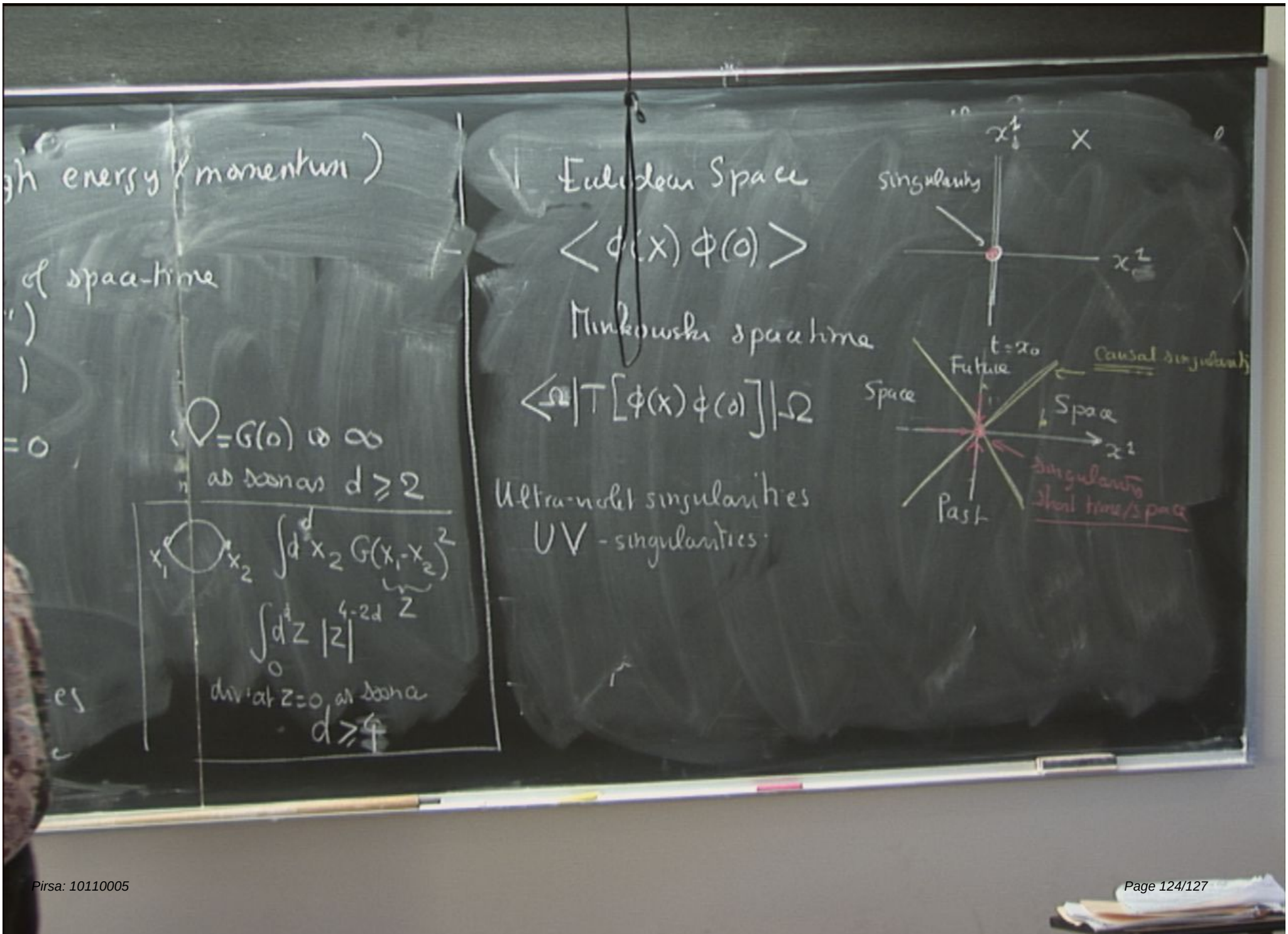
z

$\int_0 d^d z\, |z|^{4-2d}$

divergent $z=0$ as soon as $d \geq 4$


energy (momentum)
space-time
Euclidean Space
singularity
Future
Space
Space
Causal singularity
as soon as d ≥ 2
div. at z=0 as soon as
d ≥ 4


energy (momentum)
space-time
Euclidean Space
$\langle \phi(x)\phi(0) \rangle$
spacetime
$\phi(0)]|\Omega$
$\bigcirc = G(0)$ is ∞
as soon as $d \geq 2$
$\int d^d x_2\, G(x_1 - x$
$\int_0^d d^d z\, |z|^{4-2d}$
div. at $z=0$ as so
$d \geq 4$
singularity
x^1
x
$t = x_0$
Future
Space
Past
Causal singularity
singularity
short time/space



energy (momentum)
of space-time
= 0
= G(0) is ∞
as soon as d ≥ 2
x_1 x_2
$\int d^d x_2 \, G(x_1 - x_2)^2$
z
$\int_0 d^d z \, |z|^{4-2d}$
div at z=0 as soon as
d ≥ 4
Euclidean Space
$\langle \phi(x) \phi(0) \rangle$
Minkowski spacetime
$\langle \Omega | T[\phi(x) \phi(0)] | \Omega$
Ultra-violet singularities
UV - singularities
x^1
X
singularity
x^1
$t = x_0$
Future
Causal singularity
Space
Space
x^1
singularity
short time/space
Past

gh energy (momentum)

of space-time

$= G(0) \to \infty$

as soon as $d \geq 2$

$\int d^d x_2 \, G(x_1 - x_2)^2$

$\int_0 d^i z \, |z|^{4-2d} \, z$

trivial $z=0$ as soon as

$d \geq 4$

Euclidean Space

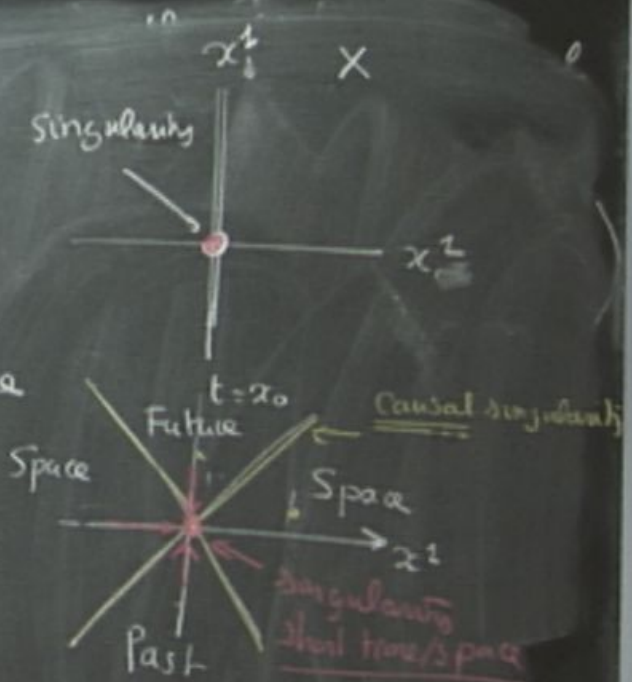
$\langle \phi(x) \phi(0) \rangle$

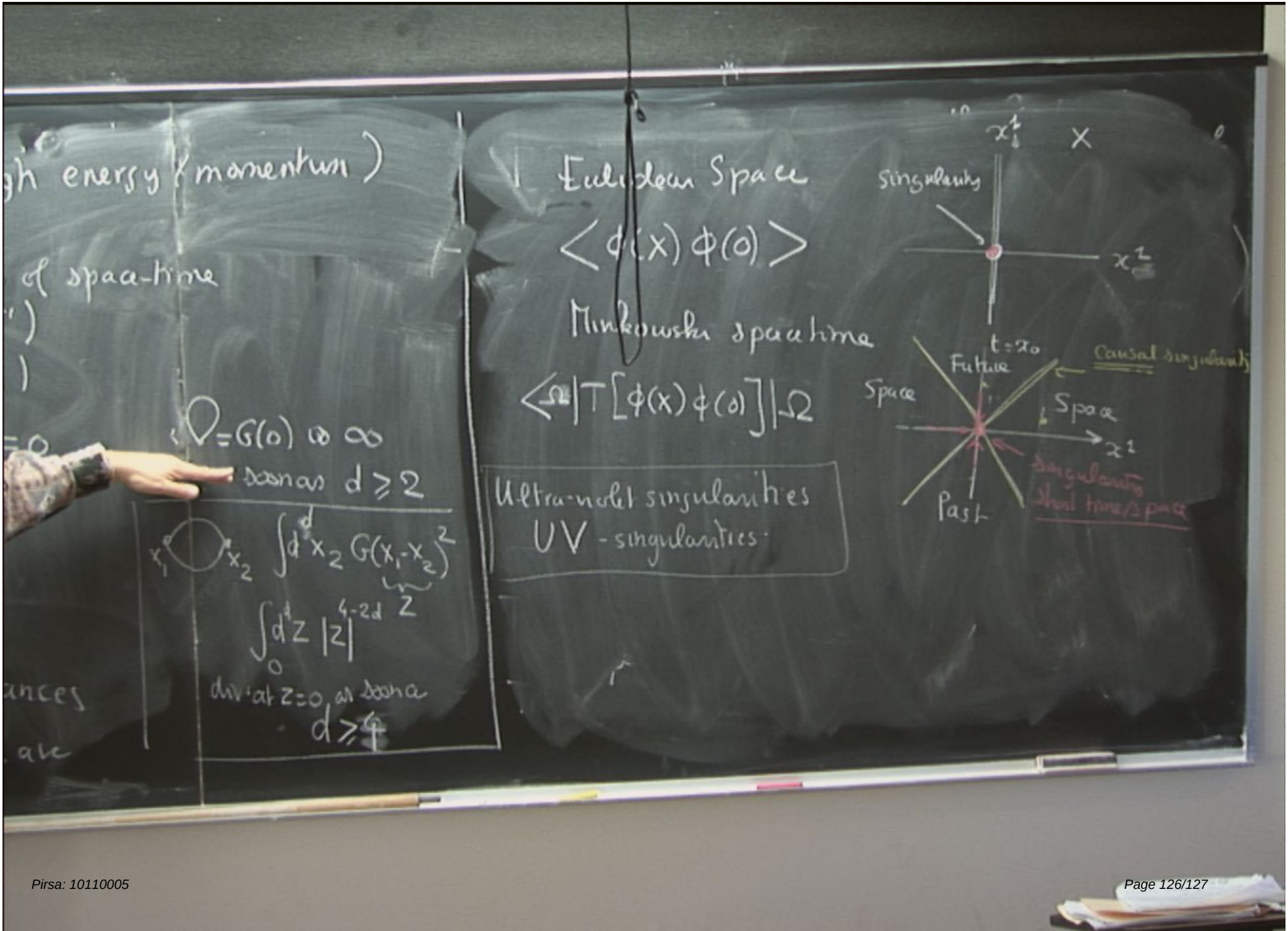
Minkowski spacetime

$\langle \Omega | T[\phi(x) \phi(0)] | \Omega$

Ultra-violet singularities

UV - singularities





energy (momentum)
of space-time
$\mathcal{O} = G(0)$ is ∞
soon as $d \geq 2$
$\int d^d x_2 \, G(x_1 - x_2)^2$
$\int_0 d^d z \, |z|^{4-2d}$
div. at $z=0$ as soon as $d \geq 4$
Euclidean Space
$\langle \phi(x) \phi(0) \rangle$
Minkowski spacetime
$\langle \Omega | T[\phi(x)\phi(0)] | \Omega$
Ultra-violet singularities
UV - singularities
singularity
x^1
$t = x_0$
Future
Space
Past
Causal singularity
singularity short time/space

...nce (high energy / momentum)

... dimensions of space-time
$(x^0, x^1, \ldots x^{d-1})$
$(k_0, k_1, \ldots k_{d-1})$

... at $|x| = 0$

$$\int d|z|\, |z|^{d-1}\, |z|^{4-2d}$$
$$|z|^{3-d}$$

... short distances
... gular they are

$$\bigcirc = G(0) \to \infty$$
as soon as $d \geq 2$

x_1 x_2 $\int d^d x_2\, G(x_1 - x_2)^2$

z

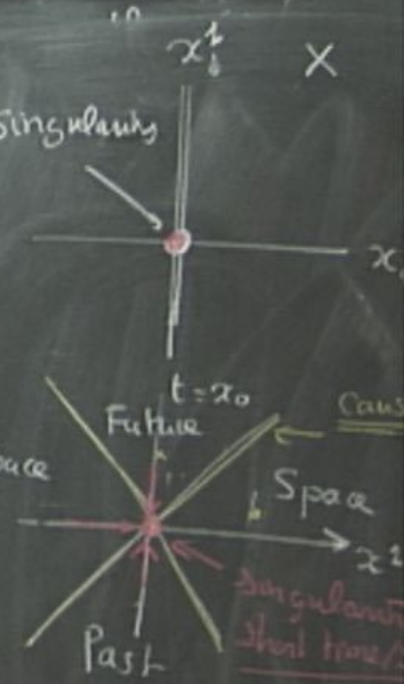
$$\int_0^1 d|z|\, |z|^{4-2d}$$

div. at $z=0$ as soon as $d \geq 4$

Euclidean Space

$$\langle \phi(x)\, \phi(0) \rangle$$

Minkowski spacetime

$$\langle \Omega | T[\phi(x)\, \phi(0)] | \Omega \rangle$$

Ultra-violet singularities
UV - singularities.