

Title: Unitarity and vacuum decay

Date: Oct 28, 2010 02:45 PM

URL: <http://pirsa.org/10100092>

Abstract: The stability of spacetime has been related to the production of particles and also to the imaginary parts of the perturbative series. The unitarity relations of the quantum theory impose relations between these two phenomena.

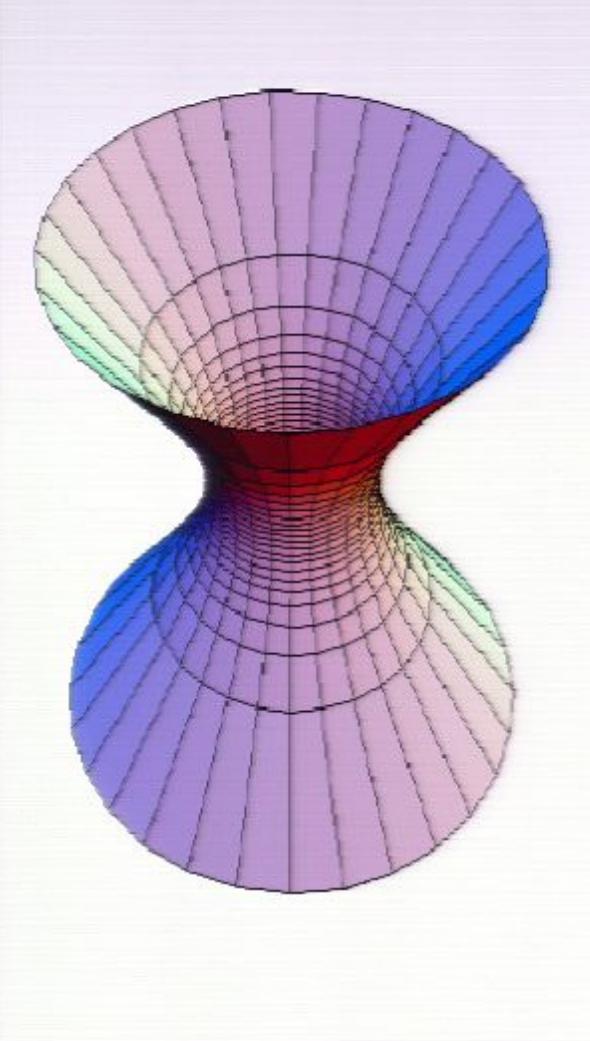
Unitarity and vacuum decay



José Roberto Vidal

UAM (Madrid, Spain)

QFT on de Sitter

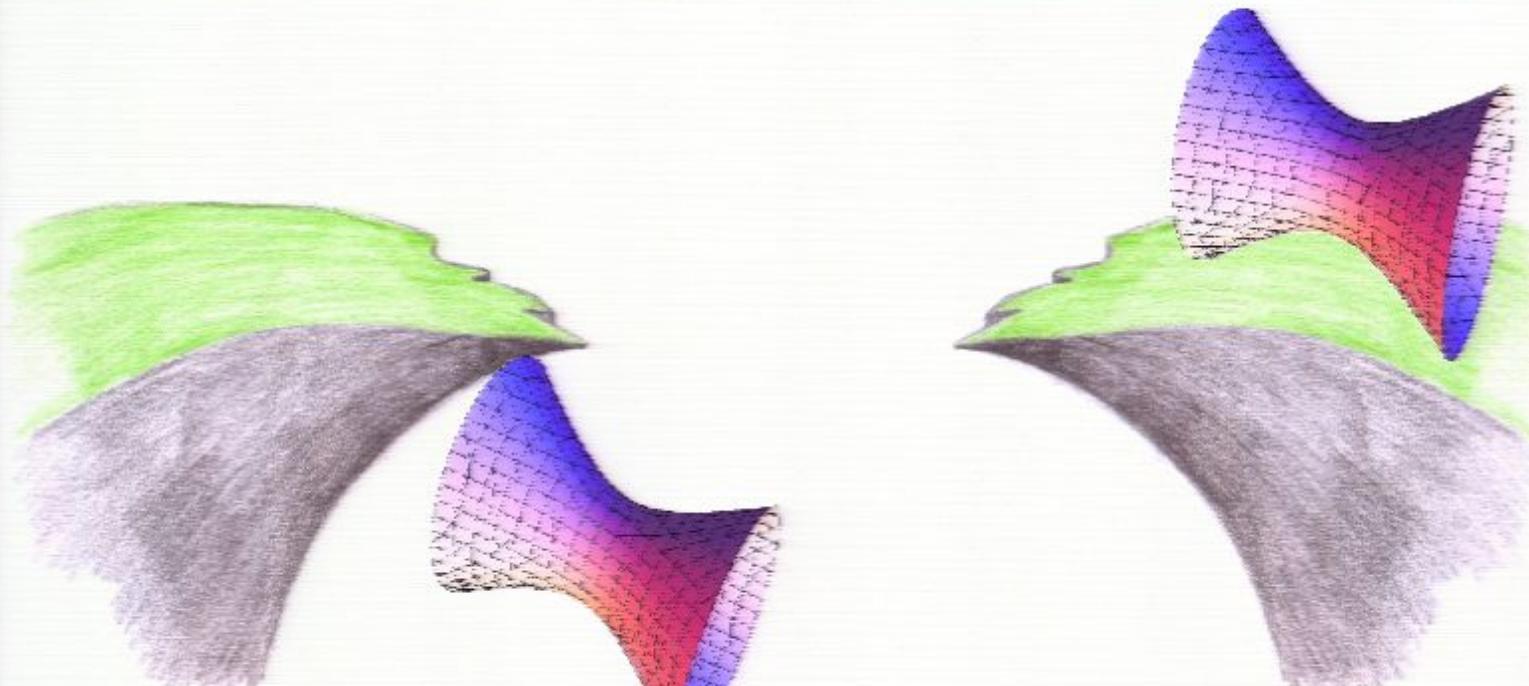


- Does a sensible interacting $|\text{vac}\rangle$ exist over the whole dS?
- Can we perturbate $|\text{vac}\rangle_{\text{BD}}$ to get it?
- Can we do it in a dS invariant way?

QFT on de Sitter

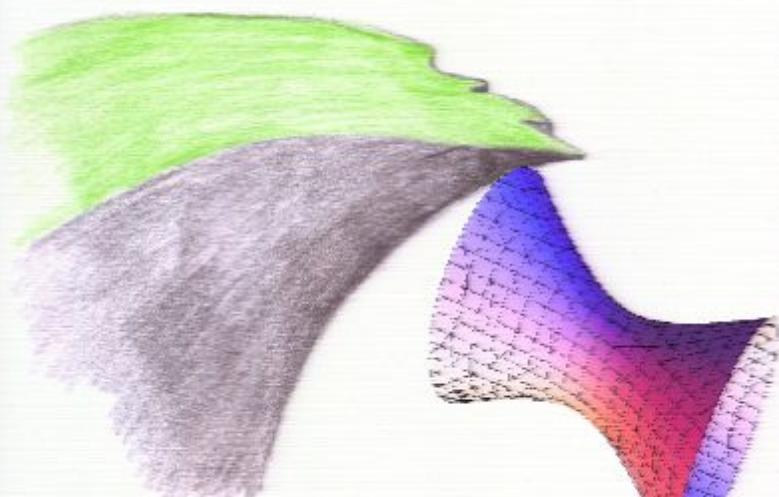
- Vacuum decay
- “Forbidden” particle production

- Euclidean dS FT
- Thermal state



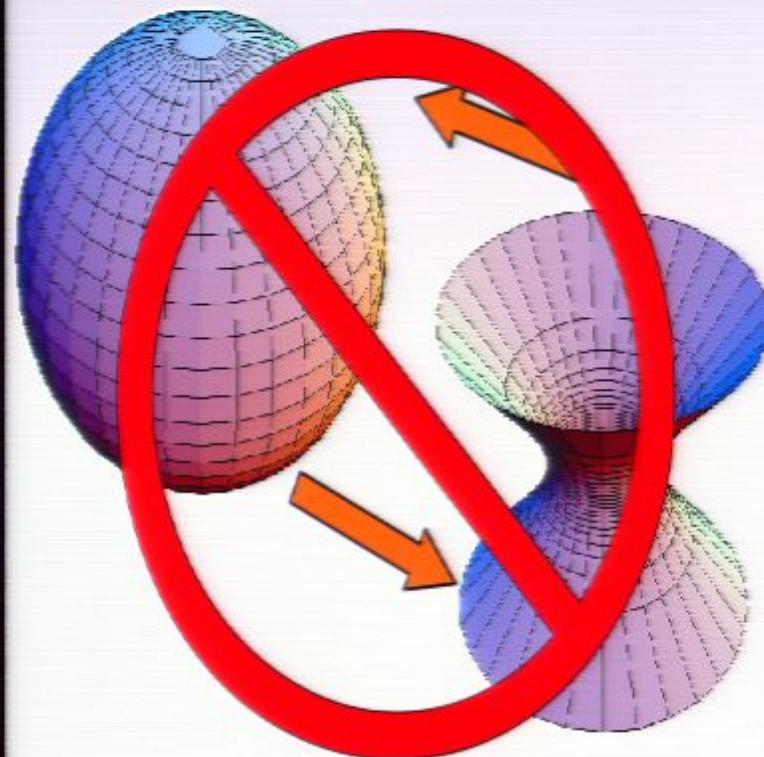
QFT on de Sitter

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Vacuum decay

Polyakov, arXiv:0912.5503

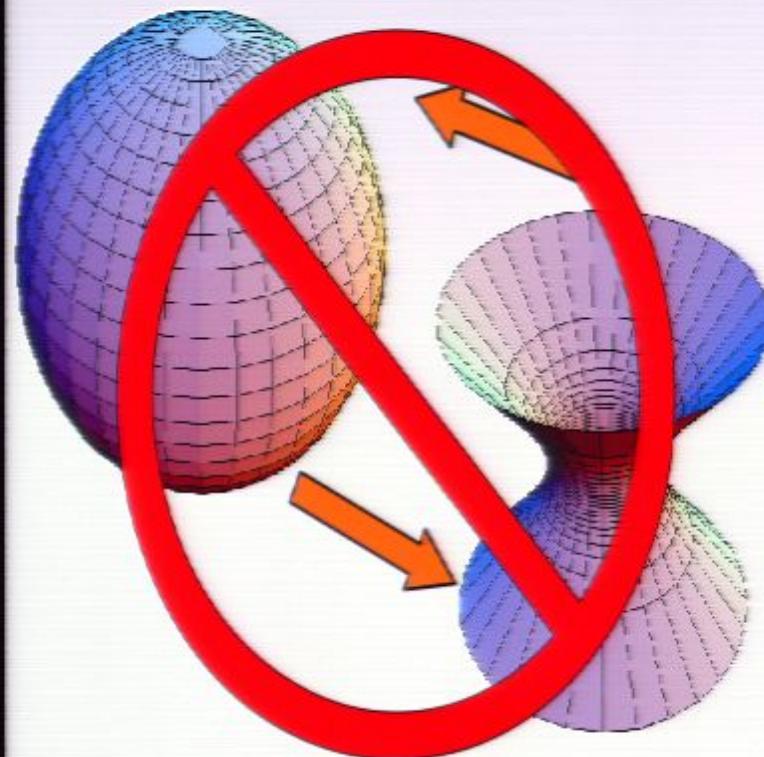


- Imaginary part for



Vacuum decay

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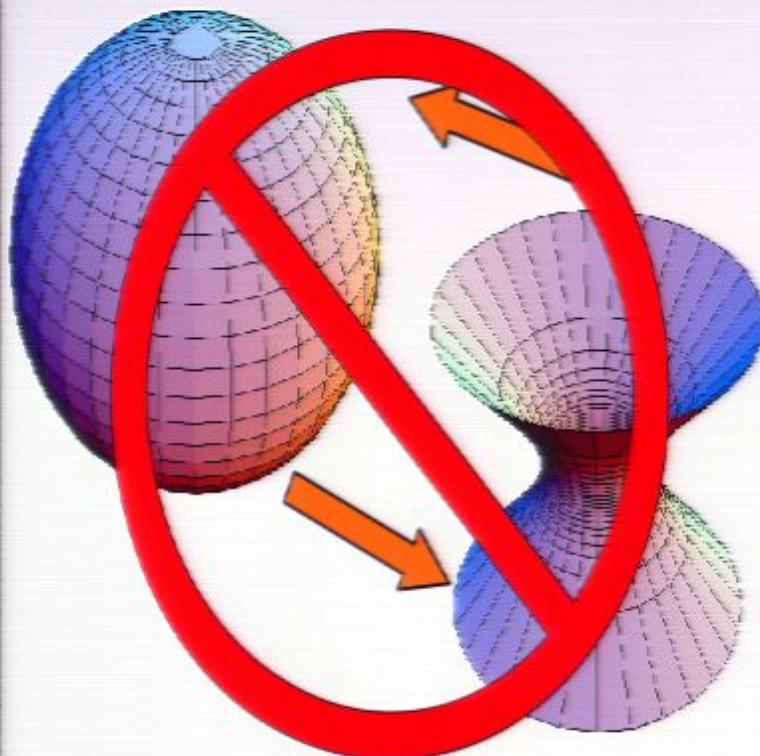
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$$iM \sim \int G(x, x_0)^4 dx$$

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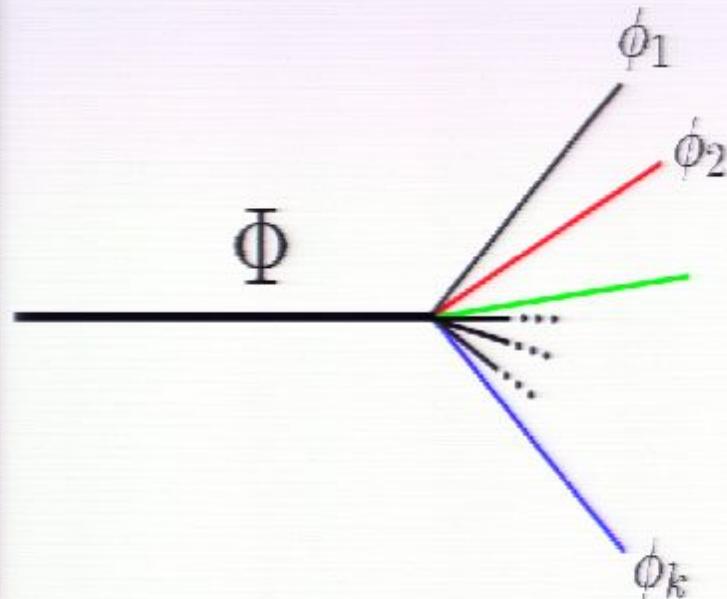
$$\text{Im} M \sim$$

$$\sim \int_1^\infty G(z)^4 \sqrt{z^2 - 1} dz$$

Runaway particle production

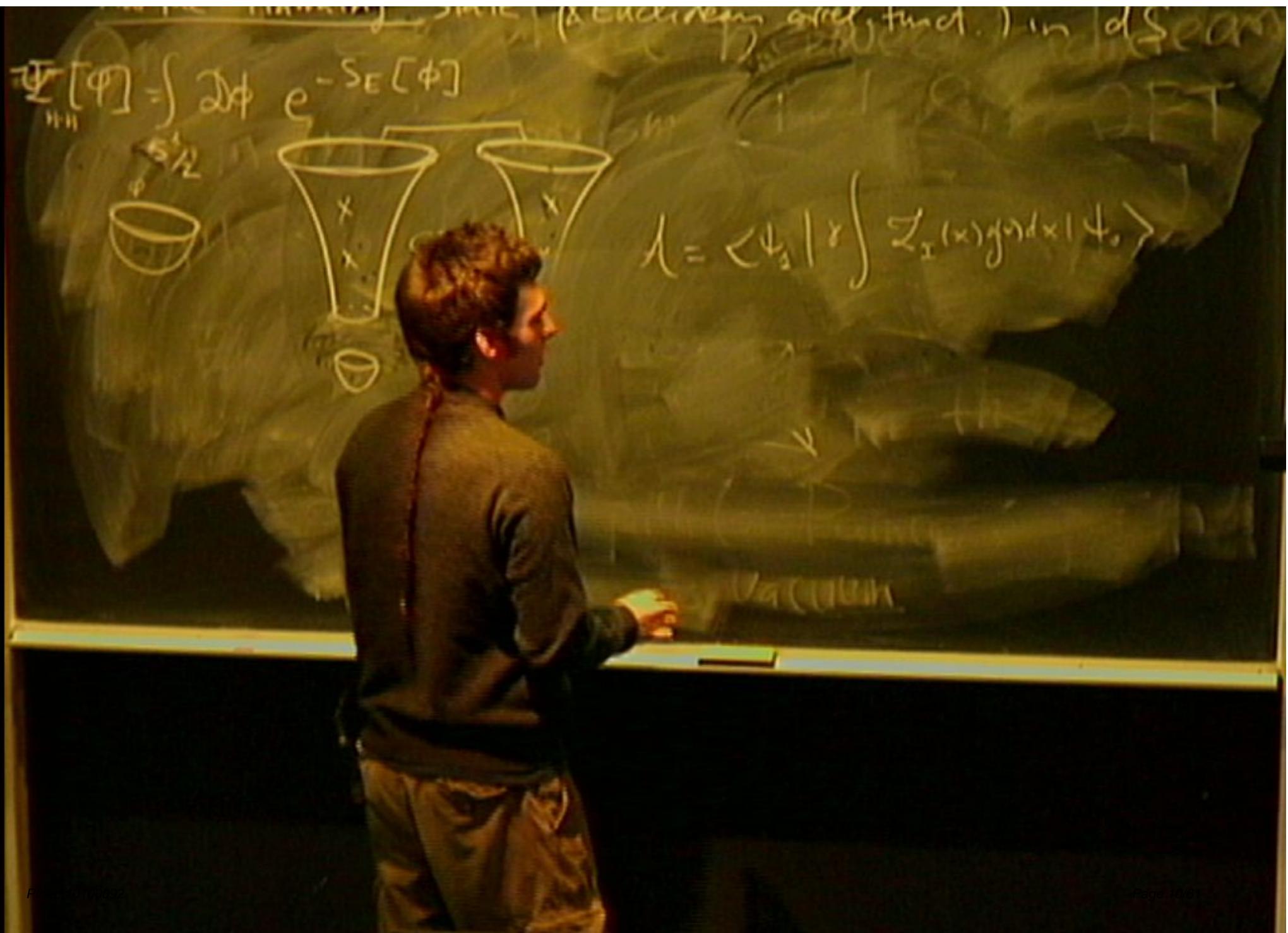
Forbidden processes

Bros, Epstein, Moschella,
arXiv:0812.3513



- General formalism for particle decays
- Bunch-Davies Fock particles
- Decay rate split in two parts

$$\Gamma = L(\psi) \rho(M; m_1, \dots)$$

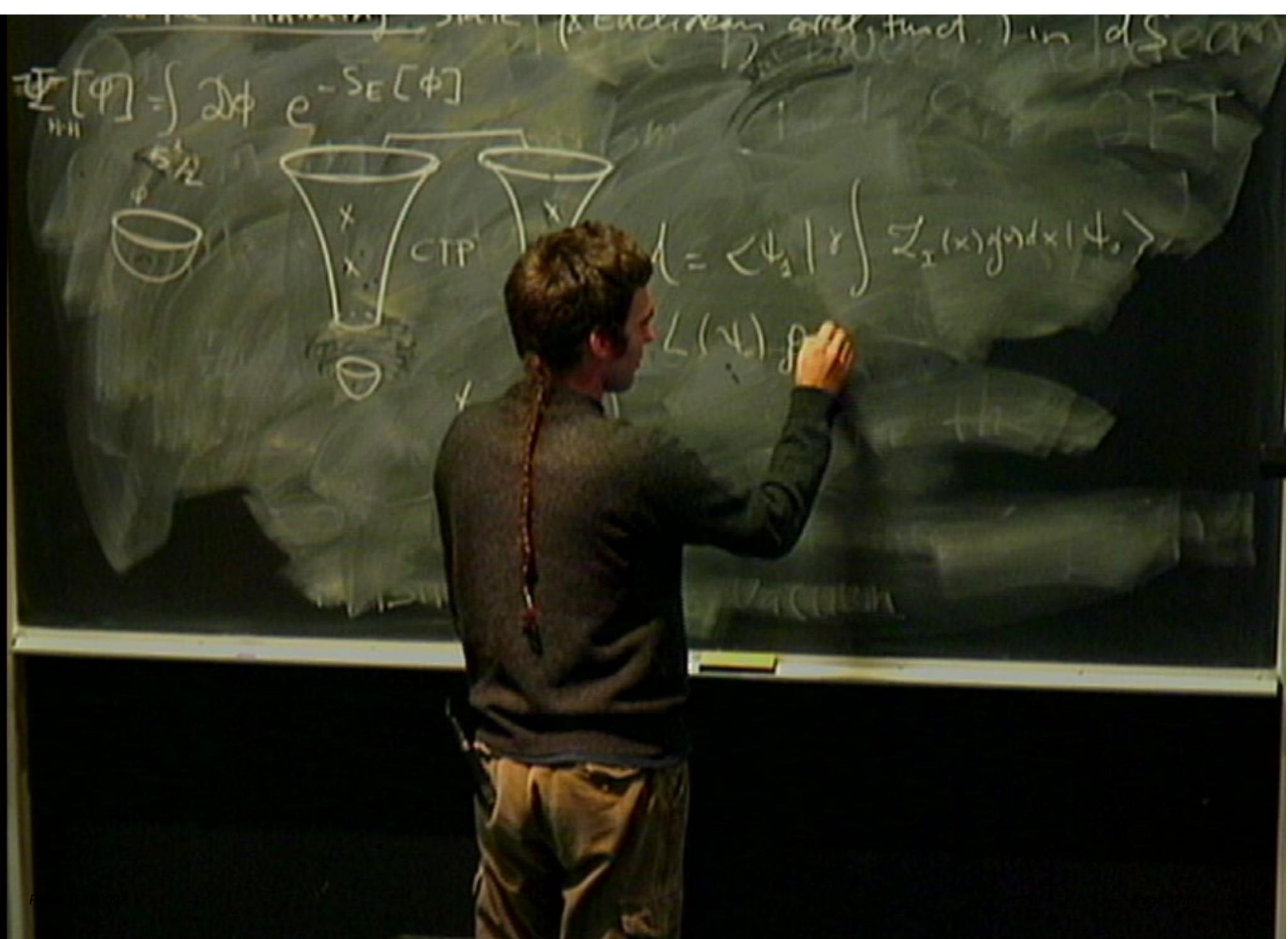


$$\Psi[\phi] = \int d\phi \ e^{-S_E[\phi]}$$

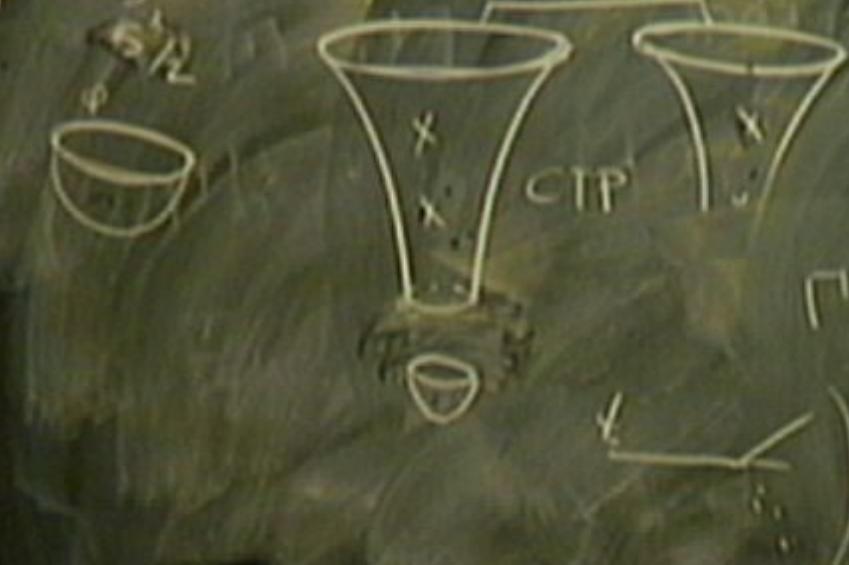


$$\lambda = \langle \psi_s | \gamma \int Z_x(x) g(x) | \psi_s \rangle$$





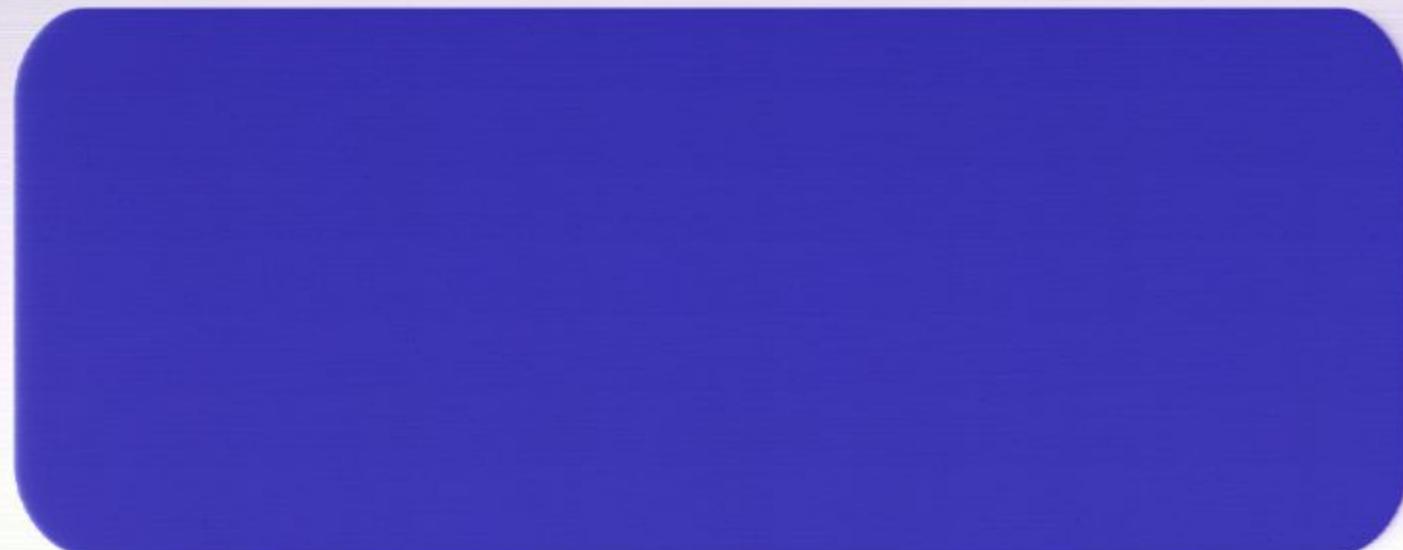
$$\Psi[\phi] = \int d\phi e^{-S_E[\phi]}$$



$$\lambda = \langle \psi_1 | \gamma^5 \int Z_i(x) g(x) dx | \psi_2 \rangle$$

$$G = \langle \psi_1 | \rho | \psi_2 \rangle$$

Unitarity



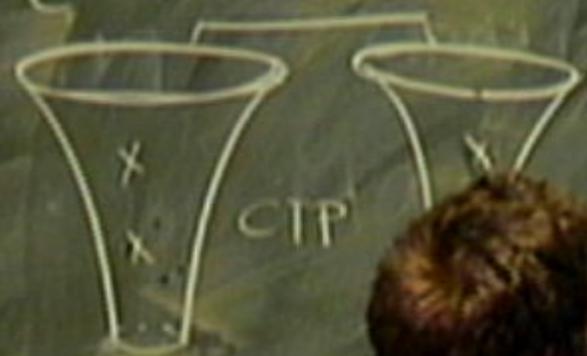
de-Hawking state (& Euclidean correl. funct.) in old Sear

$$d\phi e^{-S_E[\phi]}$$



Le-Hawking state (& Euclidean correl. funct.) in old Sean

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Le - Hawking state (& Euclidean correl. funct.) in old Sean

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(ω_t)



Le-Hawking state (& Euclidean correl. funct.) in old Sear

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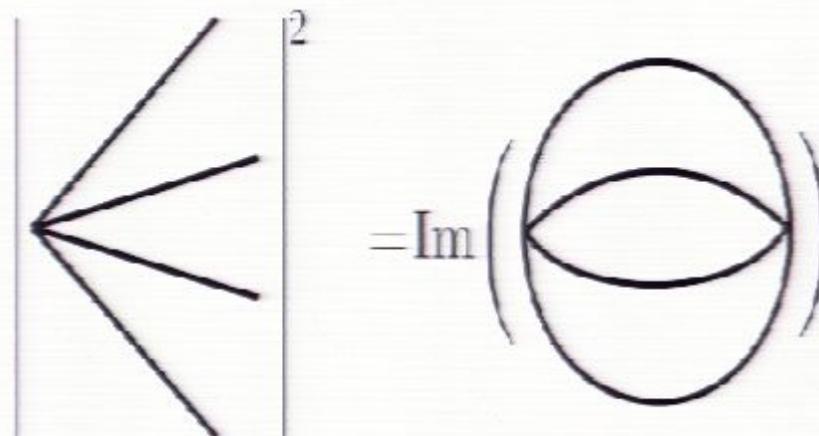
Euclidean Vacuum

Unitarity

$$\left. \begin{array}{l} S^\dagger S = \mathbb{I} \\ S = \mathbb{I} + i\mathcal{T} \end{array} \right\} \Rightarrow 2\text{Im}\langle \alpha | \mathcal{T} | \beta \rangle = \sum_n \mathcal{T}_{\alpha n} \mathcal{T}_{\beta n}^*$$

Unitarity

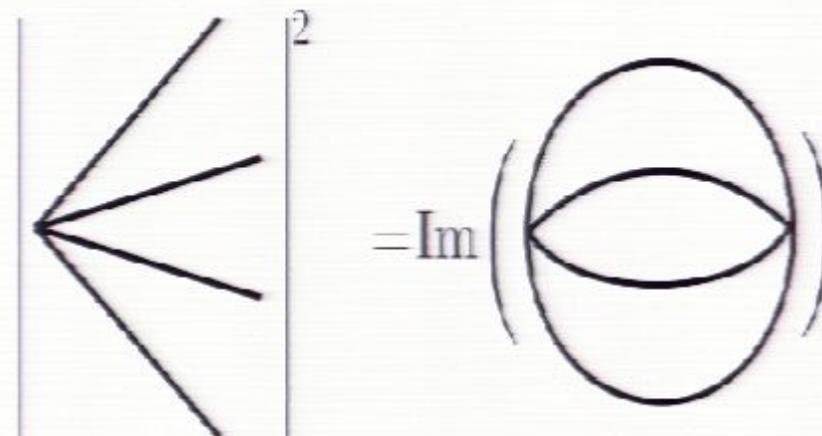
$$\left. \begin{array}{l} S^\dagger S = \mathbb{I} \\ S = \mathbb{I} + i\mathcal{T} \end{array} \right\} \Rightarrow 2\text{Im}\langle\alpha|\mathcal{T}|\beta\rangle = \sum_n \mathcal{T}_{\alpha n} \mathcal{T}_{\beta n}^* \\ 2\text{Im}\langle\text{vac}|\mathcal{T}|\text{vac}\rangle = \Gamma_{\text{vac} \rightarrow \text{stuff}}$$

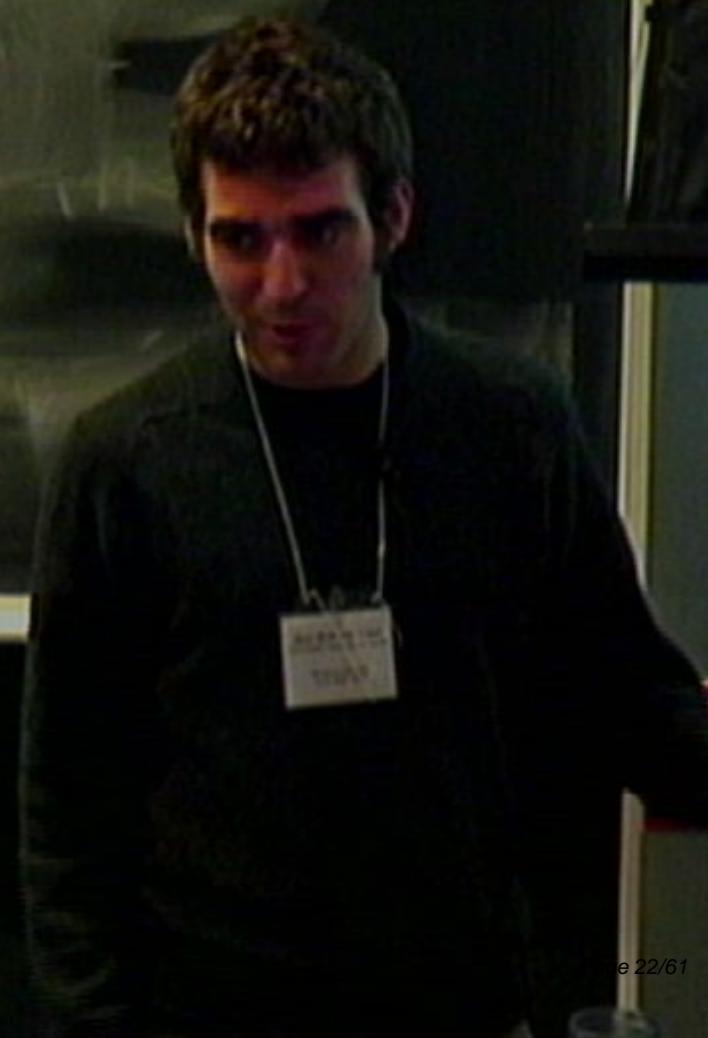
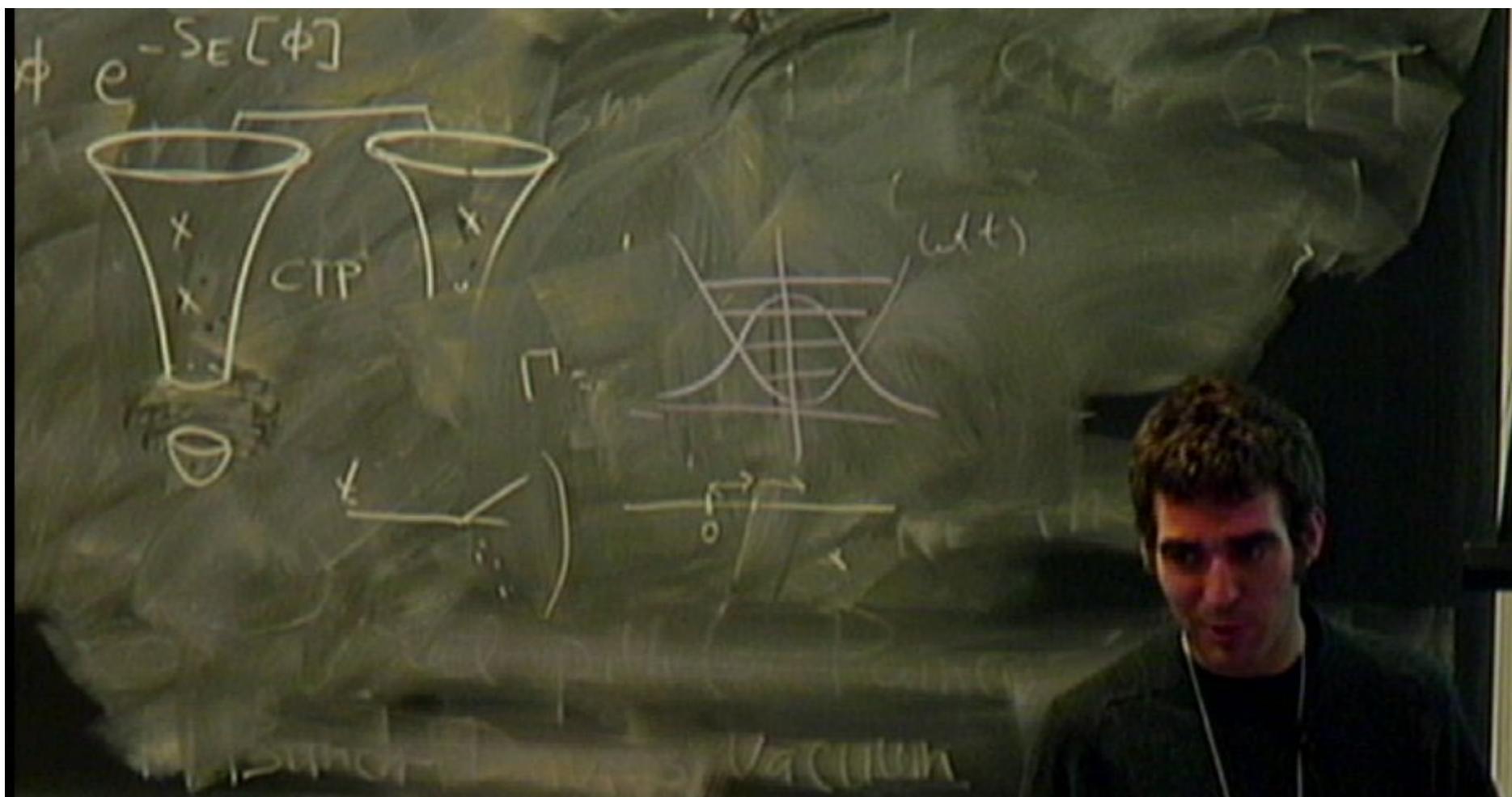


Unitarity

Largest time equation:

$$\text{Re}F(x_1, \dots, x_n) = \sum_{\text{cuttings}} F(x_1, \dots, x_n)$$





Flat space model

$$\mathcal{L} = \frac{1}{2}\partial\phi^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\cos^2\frac{\eta t}{2}\phi^3$$

Flat space model

$$T_{234}(p^2, \{m_i\}) = \\ = \int \frac{d^n k_2 d^n k_3 d^n k_4}{(2\pi)^{3n}} \frac{\delta^n(k_1 + k_2 + k_3 - p)}{(k_2^2 - m_2^2)(k_3^2 - m_3^2)(k_4^2 - m_4^2)}$$

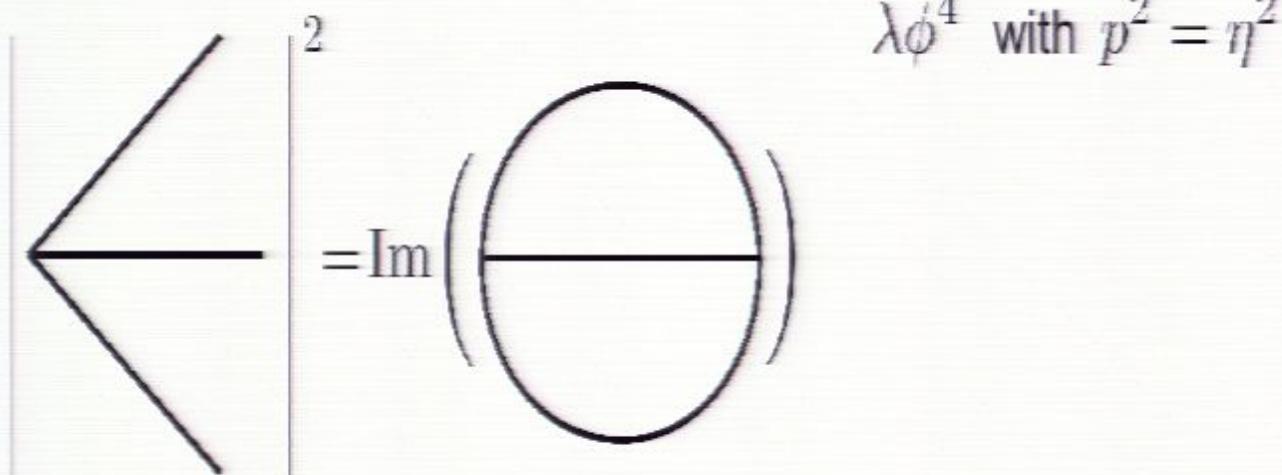
$$\lambda \phi^4 \text{ with } p^2 = \eta^2$$
$$= \text{Im} \left(\text{---} \right)$$

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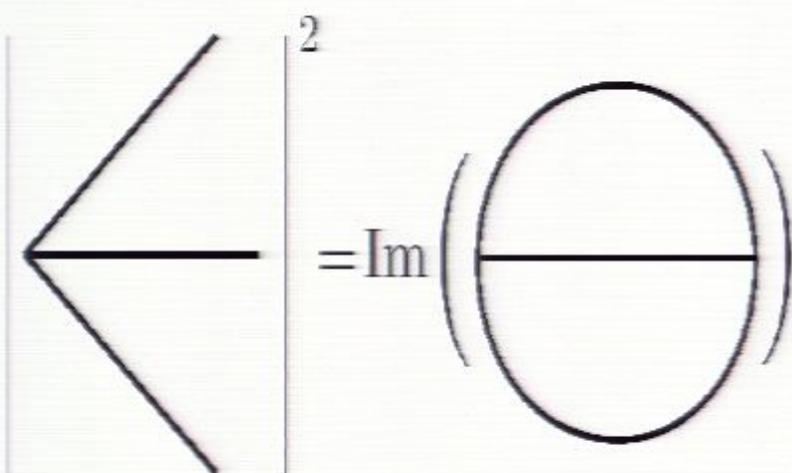
$$\Gamma_{0 \rightarrow 3} \sim \int dE_1 dE_2 \theta_3(E_1, E_2; p^2)$$



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$\lambda\phi^4$ with $p^2 = \eta^2$

$\text{Im} T_{234} = \Gamma_{0 \rightarrow 4}$

Berends, Tausk (1994)

Flat space model

$$T_{\mu\nu}(p^2, \{m_i\}) =$$
$$= \int \frac{d^4 k_2 d^4 k_3 d^4 k_4}{(2\pi)^{12}} \frac{\delta^4(k_1 + k_2 + k_3 - p)}{(k_2^2 - m_2^2)(k_3^2 - m_3^2)(k_4^2 - m_4^2)}$$

$$\Gamma_{0 \rightarrow 4} \sim \int dE_1 dE_2 \theta_3(E_1, E_2; p^2)$$

$$\text{Im} \left(\text{---} \bigcirc \text{---} \bigcirc \right) = \left| \text{---} \bigcirc \text{---} \right|^2 \quad \lambda \phi^4 \text{ with } p^2 = \eta^2$$

Berends, Tausk (1994)

Flat space model

$$-2\text{Re}\langle 0|S^{(2)}|0\rangle - 2\text{Re}\langle 0|S_{\text{tad}}^{(2)}|0\rangle = \Gamma_{0\rightarrow 3} + \Gamma_{0\rightarrow 1}$$

► $-2\text{Re}\langle 0|S_{\text{tad}}^{(2)}|0\rangle = \Gamma_{0\rightarrow 1}$

$$\text{Im} \left(\text{---} \right) = \left| \text{---} \right|^2$$

Flat space model

- Toy Boltzmann equation

$$\dot{n} = C_{0 \rightarrow 3} - C_{3 \rightarrow 0} n^3$$

- Equilibrium state with:

$$|\langle \text{vac} | f \rangle| < 1$$

Flat space model

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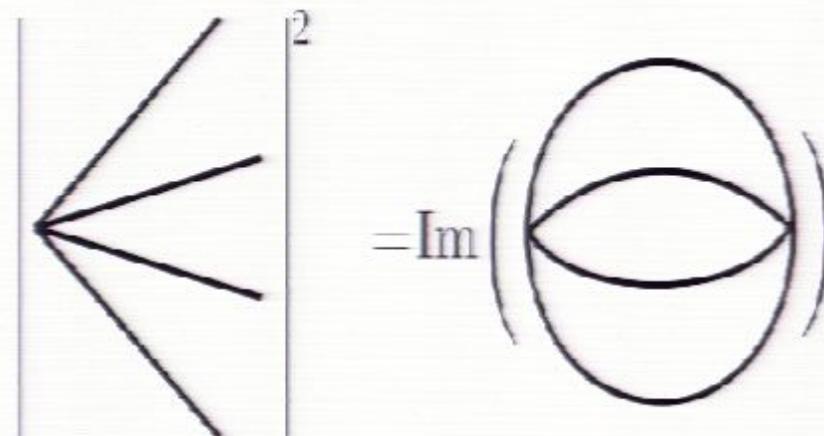
- Asymptotics regions in dS are rather different.

- There is no simple equivalence to a curved lagrangian.

$$\lambda(t), m(t) \sim a(t)$$

Back to dS

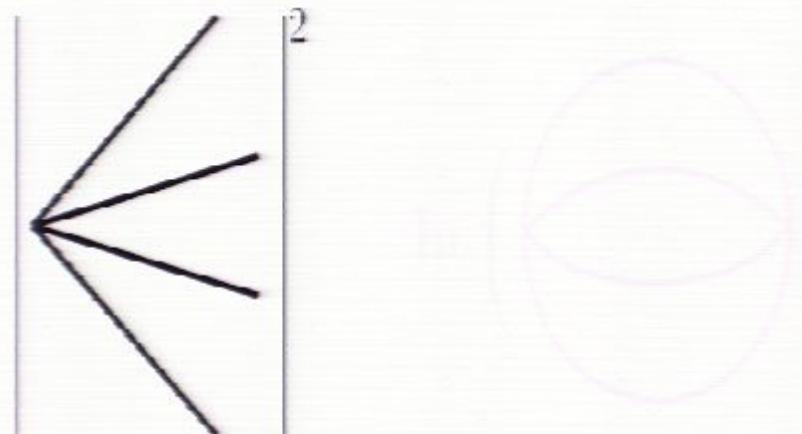
- Massless conformally coupled case: $G(z) = \frac{1}{z - 1 - i\epsilon}$



Back to dS

- Massless conformally coupled case: $G(z) = \frac{1}{z - 1 - i\epsilon}$
- Result in cosmological patch: $\Gamma_{0 \rightarrow 4} \sim \lambda^2 \ell^{-4}$

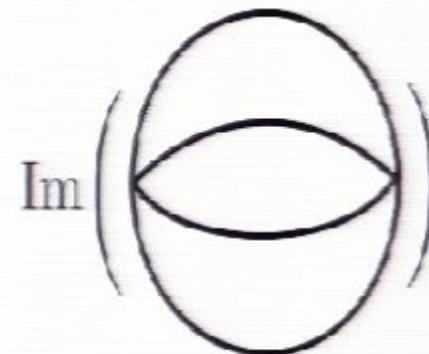
Higuchi, arXiv:0809.1255



Back to dS

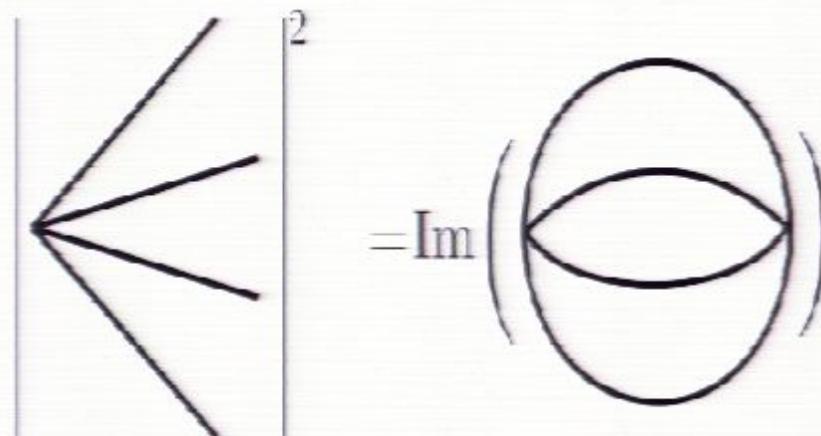
• Global coordinates:

$$M_{0 \rightarrow 0} \propto \int d^4x \frac{1}{(z(x, x_0) - 1 - i\epsilon)^4}$$
$$\propto \int dt d\theta \frac{\cosh^3 t \sin^2 \theta}{(\cosh t \cos \theta - 1 - i\epsilon)^4}$$



Back to dS

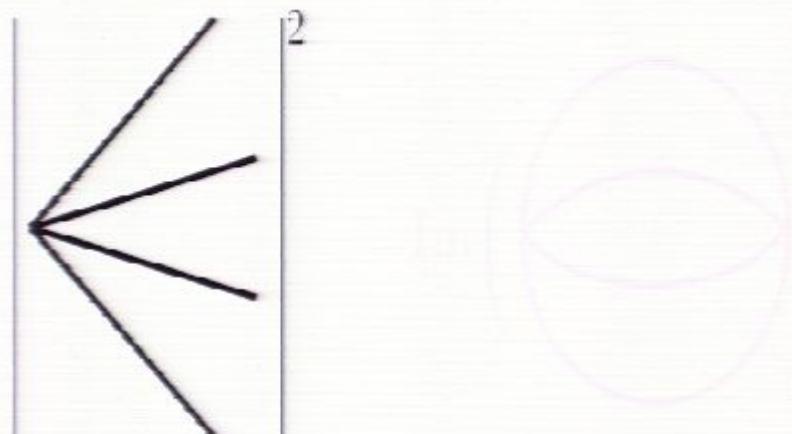
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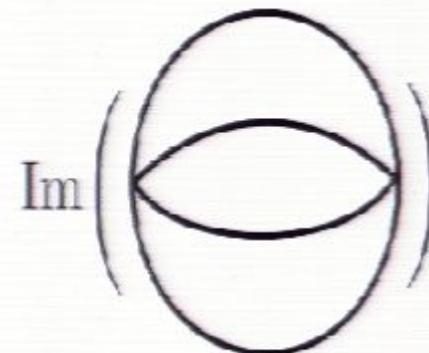
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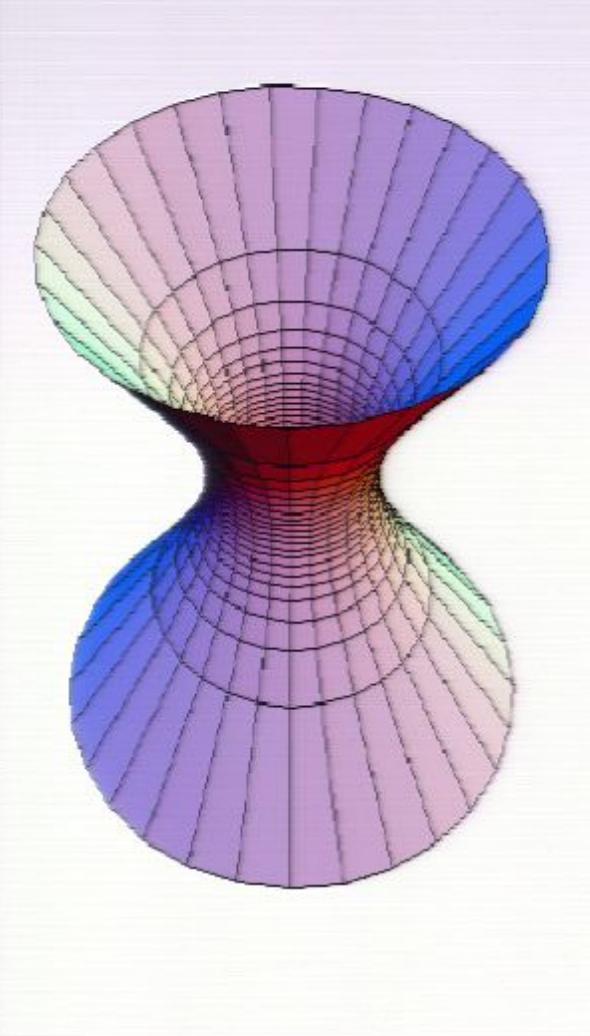
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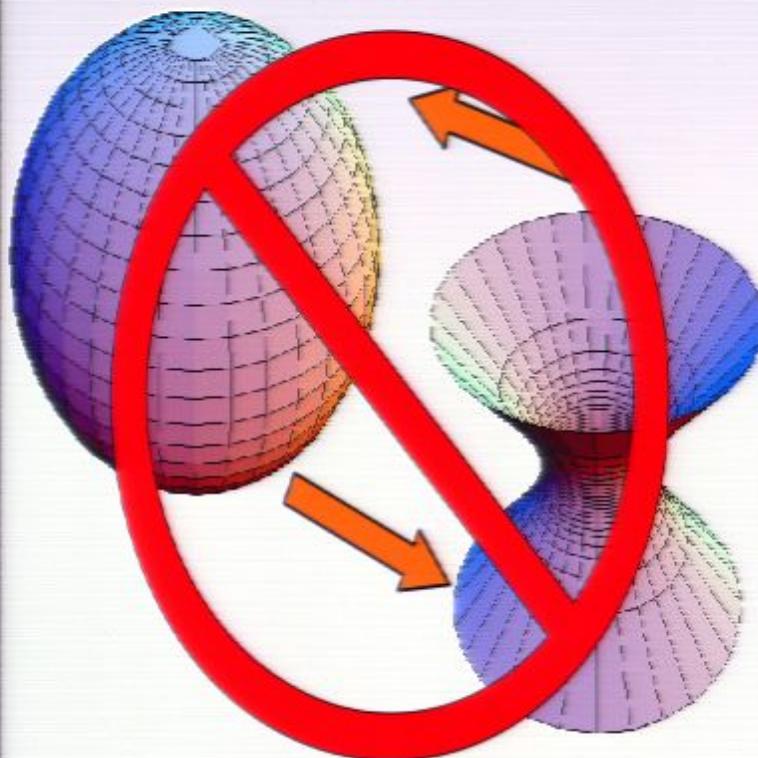
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Runaway particle production

Flat space model

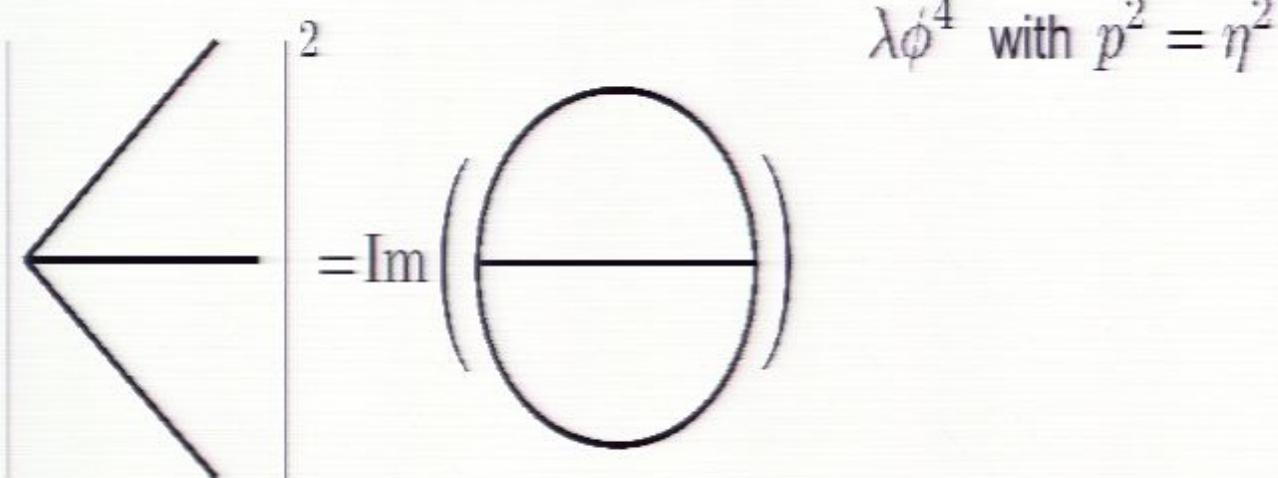
$$\mathcal{L} = \frac{1}{2}\partial\phi^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{3!}\cos^2\frac{\eta t}{2}\phi^3$$

New vertex:

$$\delta(\sum p) \left\{ \delta(\sum p^0) + \delta(\sum p^0 - \eta) + \delta(\sum p^0 + \eta) \right\}$$

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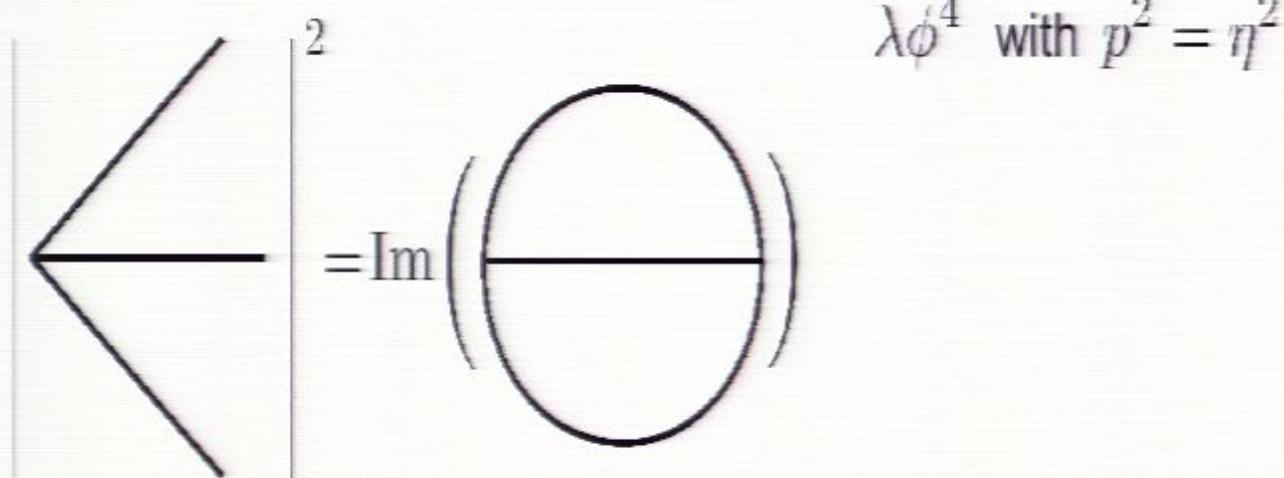


Flat space model

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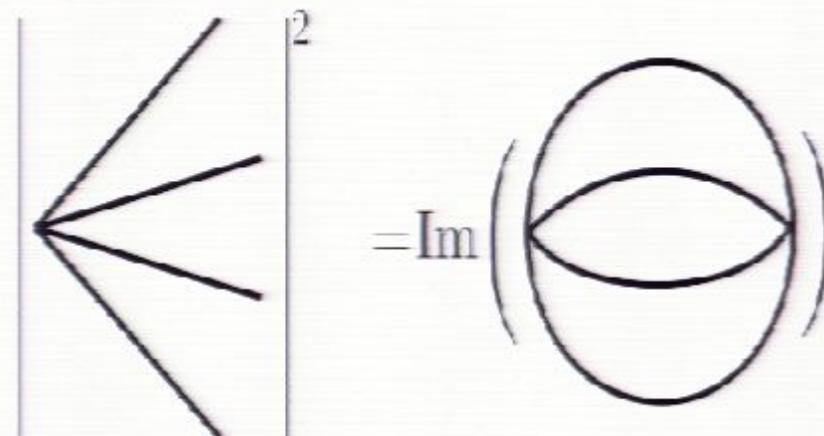
$$= \int \frac{d^n k_2 d^n k_3 d^n k_4}{(2\pi)^{3n}} \frac{\delta^n(k_1 + k_2 + k_3 - p)}{(k_2^2 - m_2^2)(k_3^2 - m_3^2)(k_4^2 - m_4^2)}$$

$$\Gamma_{0 \rightarrow 3} \sim \int dE_1 dE_2 \theta_3(E_1, E_2; p^2)$$



Back to dS

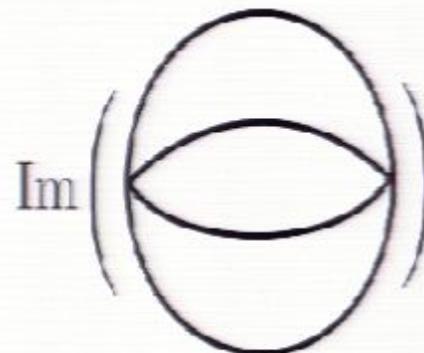
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Back to dS

• Global coordinates: $M_{0 \rightarrow 0} \propto \int d^4x \frac{1}{(z(x, x_0) - 1 - i\epsilon)^4}$

$$\propto \int dt d\theta \frac{\cosh^3 t \sin^2 \theta}{(\cosh t \cos \theta - 1 - i\epsilon)^4} \sim \int dt \frac{1}{\sinh^5 t}$$



Conclusions so far...

- Particle creation and instability are the same phenomena.
- In other words, you should trust or disbelieve them together (for dS).
- Unitarity provides a way to check how catastrophic are these phenomena.

Back to dS

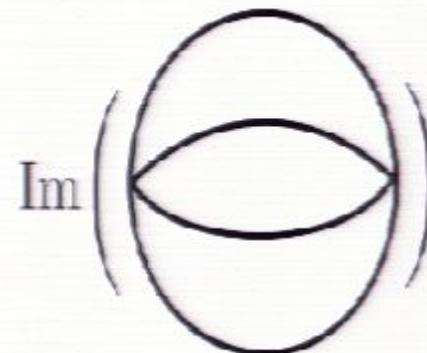
Phase space?

Cosmological patch:

$$V_3 \int dT e^{3HT} \Gamma$$

Global patch:

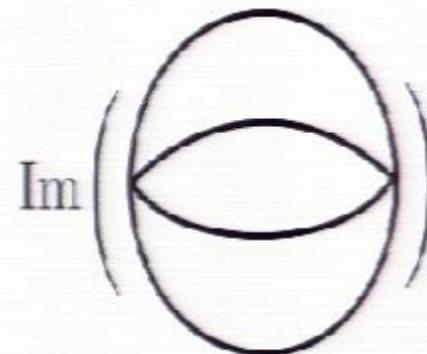
$$V_4 \Gamma$$



Back to dS

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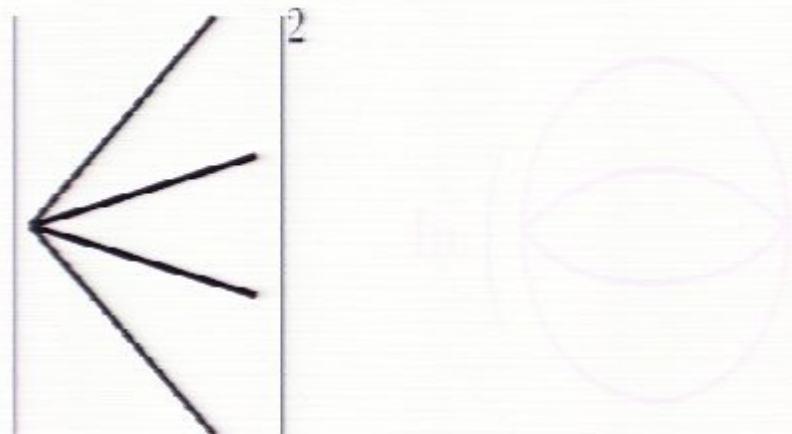
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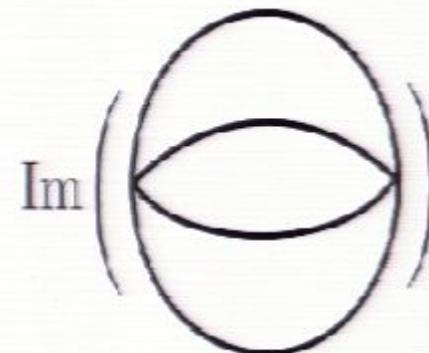


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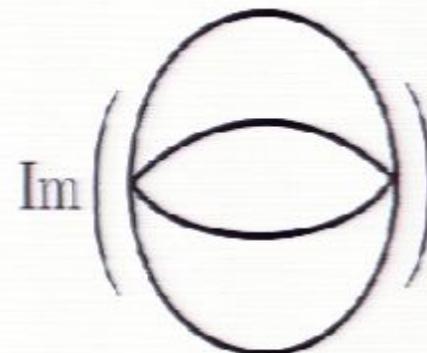
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Back to dS

Phase space?

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Back to dS

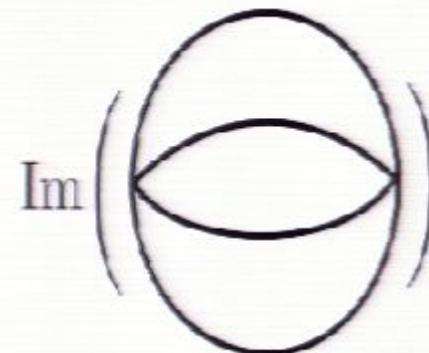
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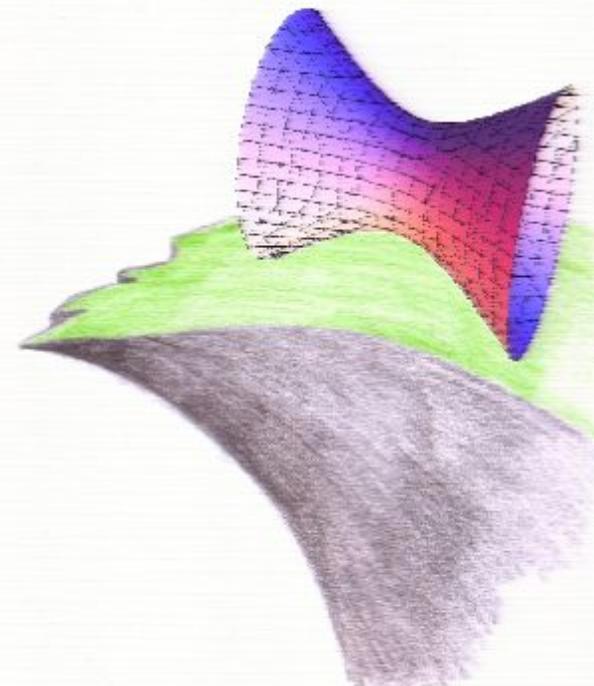


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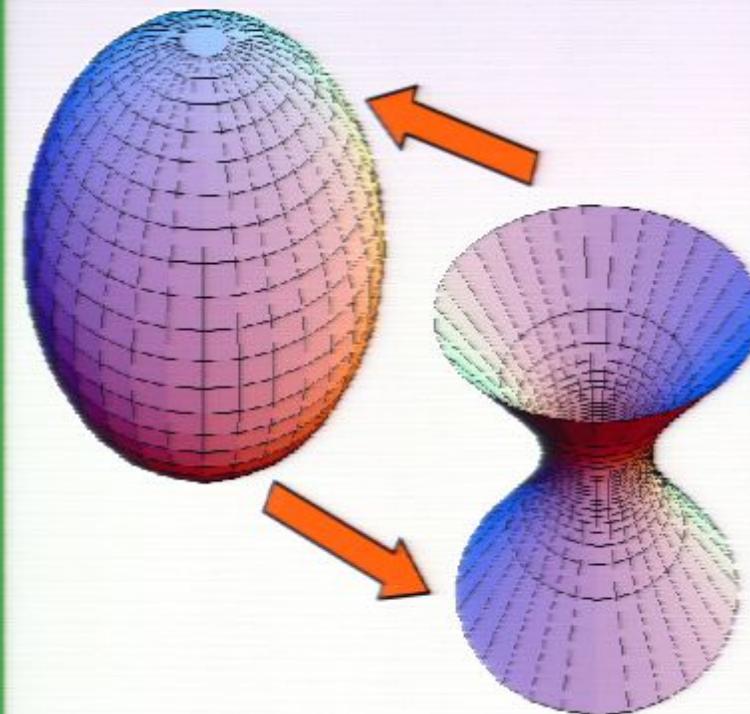
Hold on...

- Euclidean dS FT
- Thermal state



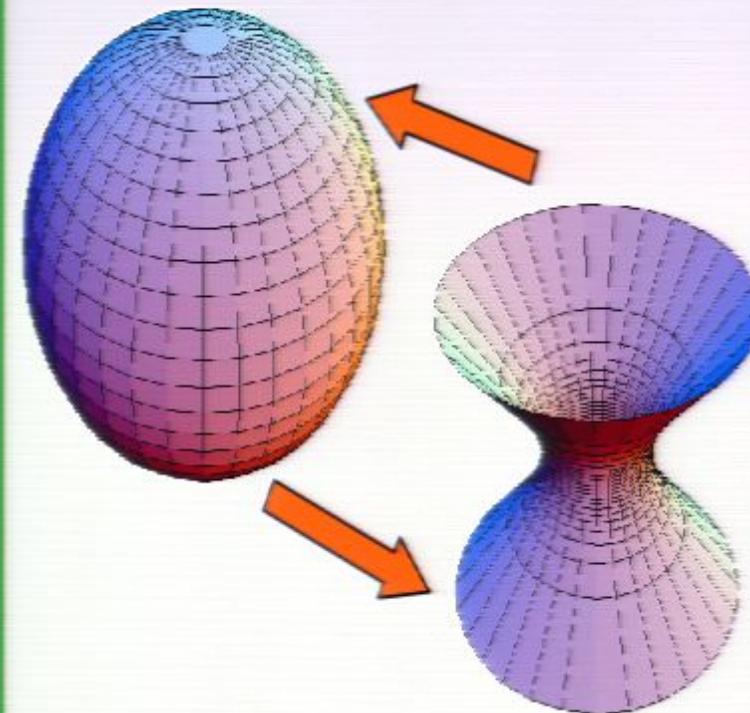
Euclidean dS FT

- The euclidean version of the theory is IR finite.



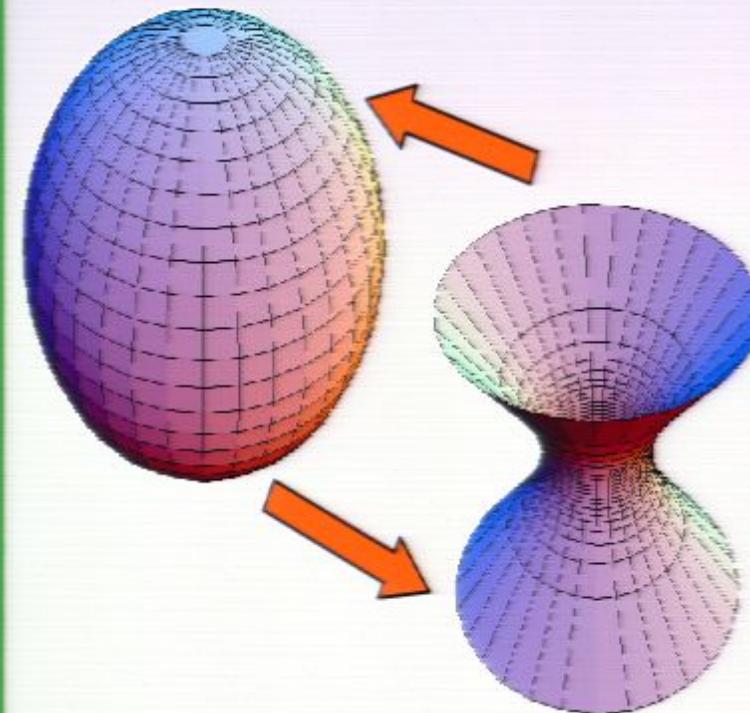
Euclidean dS FT

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Euclidean dS FT

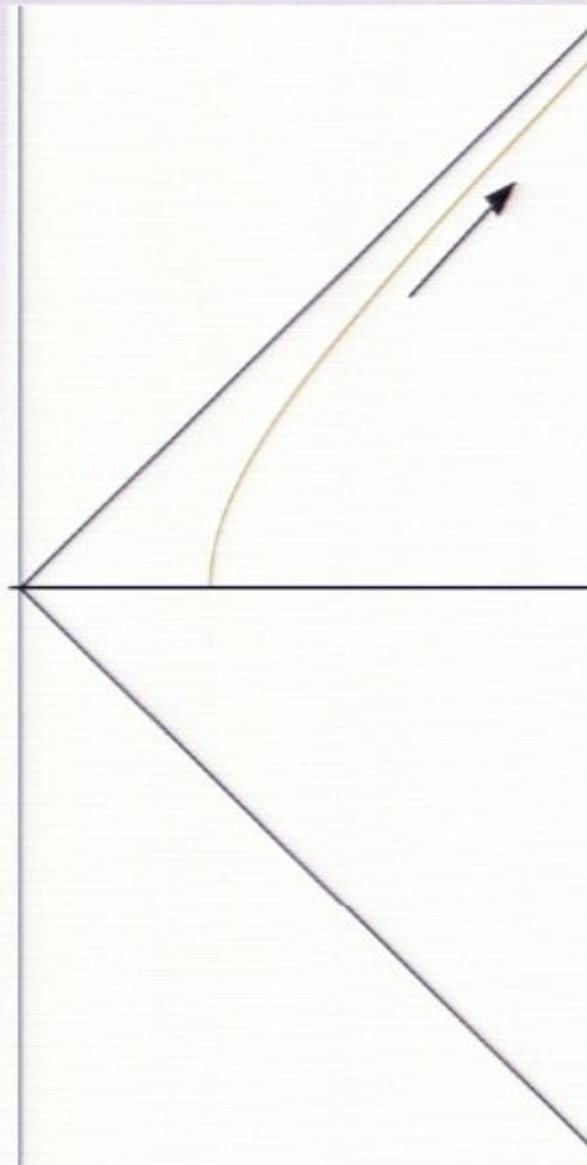
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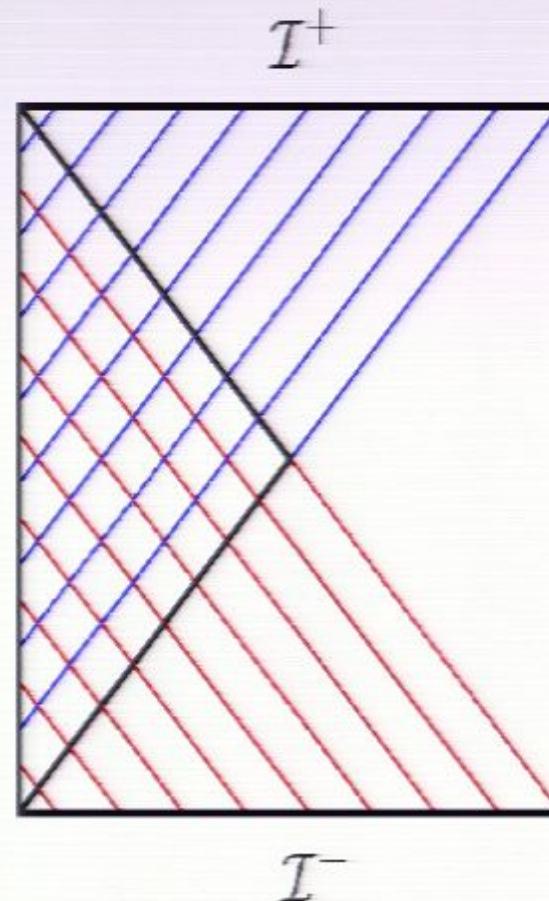
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- Example from Unruh effect.

$$|\text{vac } P\rangle \xrightarrow{\hspace{1cm}} T = \frac{a}{2\pi}$$



Euclidean dS FT

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- Static observer point of view

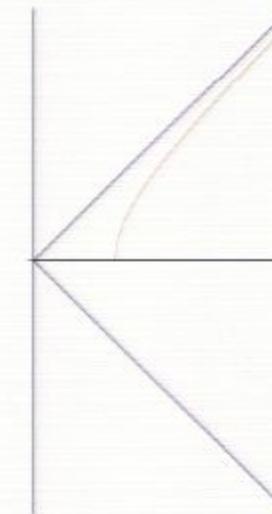
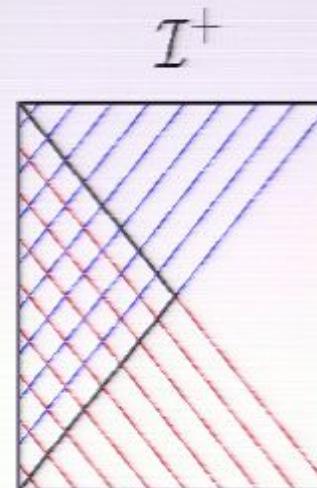


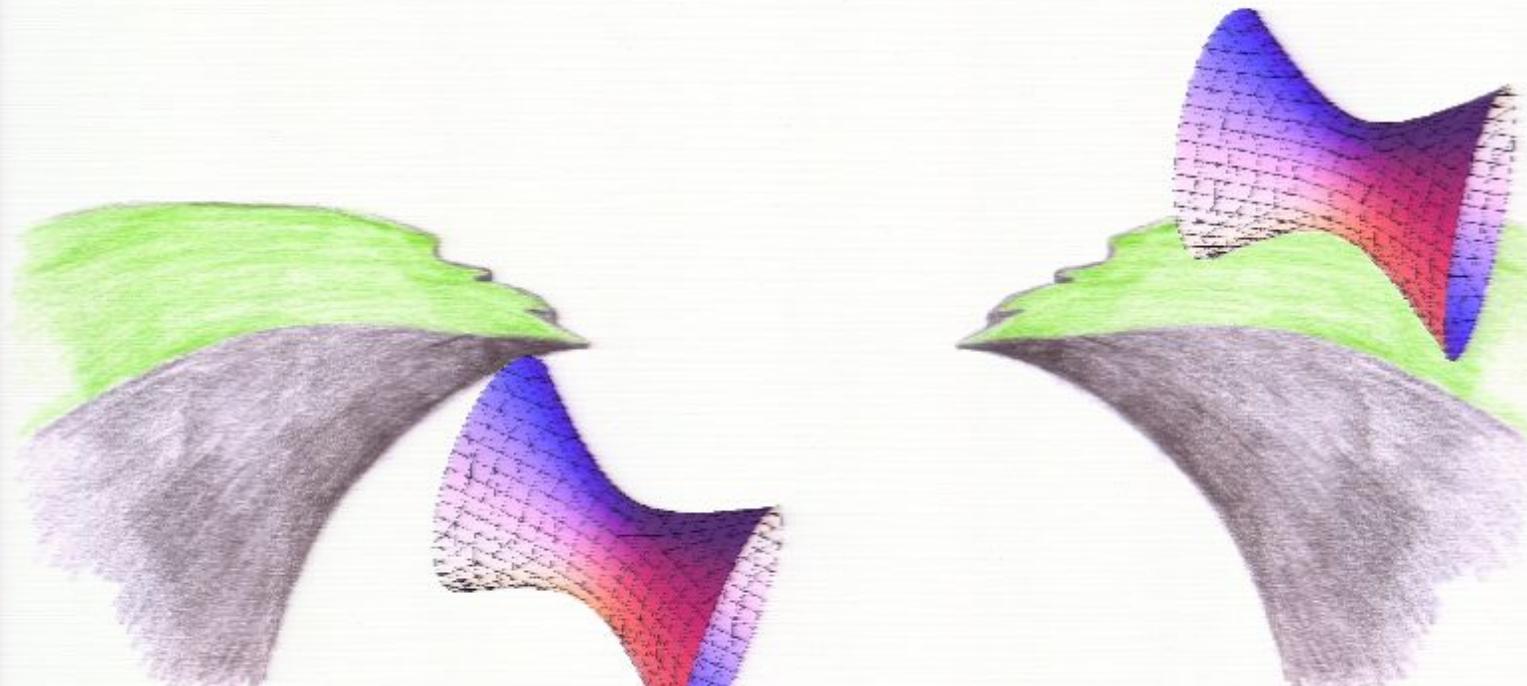
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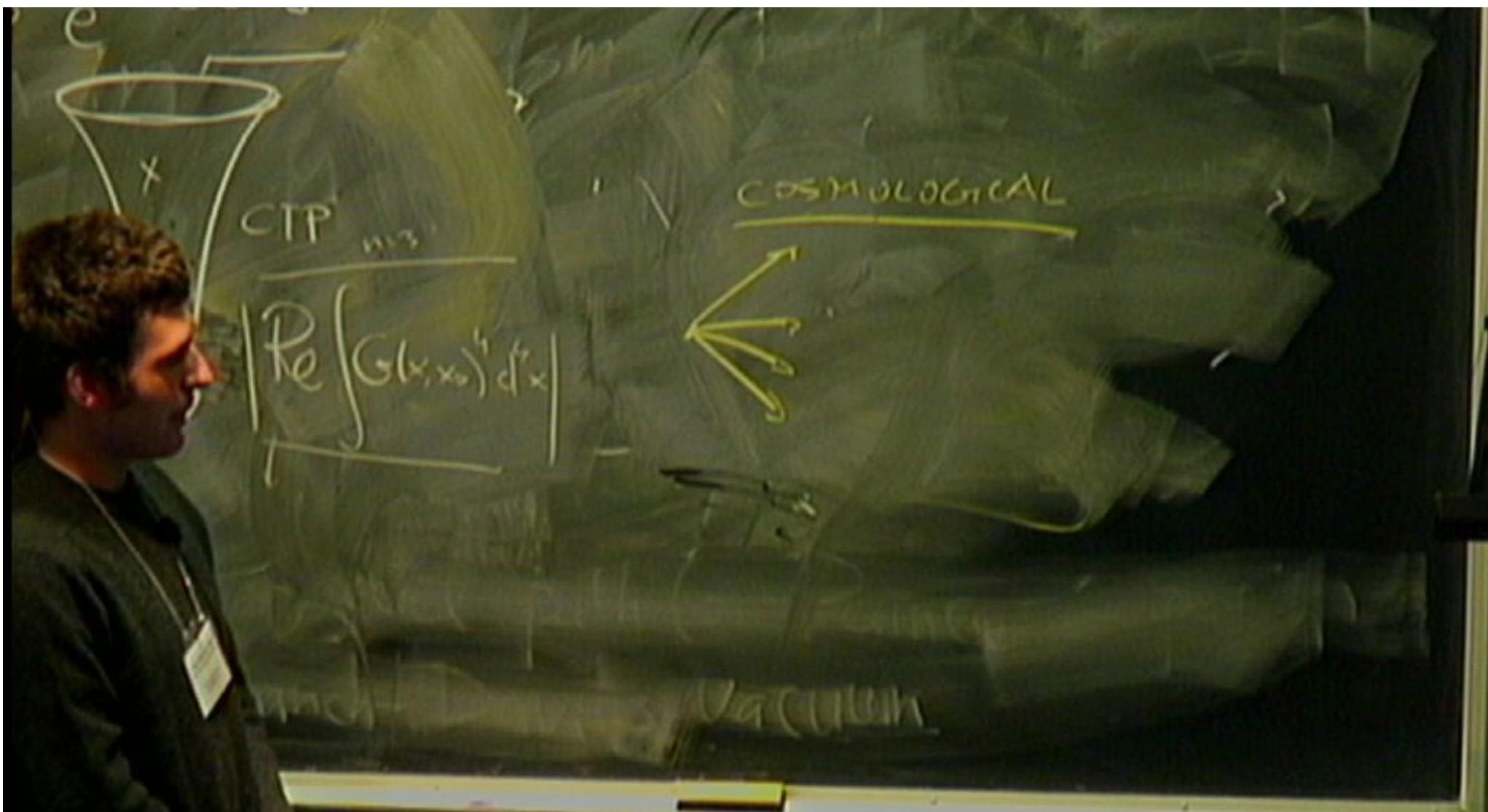
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- Example from Unruh effect.
- Static observer point of view

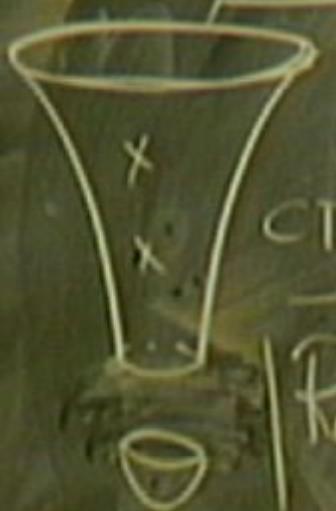
u_ω with $\omega > 0$

$$\langle \omega_{i_1} \dots \omega_{i_k} | H_{\text{int}} | \omega_{j_1} \dots \omega_{j_l} \rangle$$









$$|Re \int G(x, x_0) d^3x|$$

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