

Title: The thermodynamic meaning of negative entropy

Date: Oct 19, 2010 04:00 PM

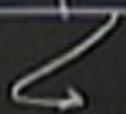
URL: <http://pirsa.org/10100086>

Abstract: Landauer's erasure principle states that there is an inherent work cost associated with all irreversible operations, like the erasure of the data stored in a system. The necessary work is determined by our uncertainty: the more we know about the system, the less it costs to erase it.

Here, we analyse erasure in a general setting where our information about that system can be quantum mechanical. In this scenario, our uncertainty, measured by a conditional entropy, may become negative. We establish a general relation between quantum conditional entropies and a physical quantity, the work cost of erasure. As a consequence, we obtain a thermodynamic interpretation of negative entropies: they quantify the work that can be gained by a quantum observer erasing a system.

(arXiv: 1009.1630)

What does it mean to erase (the data stored in) a system



What does it take to erase (the data stored in) a system
↳ taking system to pure state, $|0\rangle$

What does it take to erase (the data stored in) a system

↳ how much work

(Landauer's pt.)

↳ taking system to pure state, $|0\rangle$

What does it take to erase (the data stored in) a system

↳ how much work

(Landauer's Pt.)

↓
a given observer

↳ taking system to pure state, $|0\rangle$

What does it take to erase (the data stored in) a system

↳ how much
work

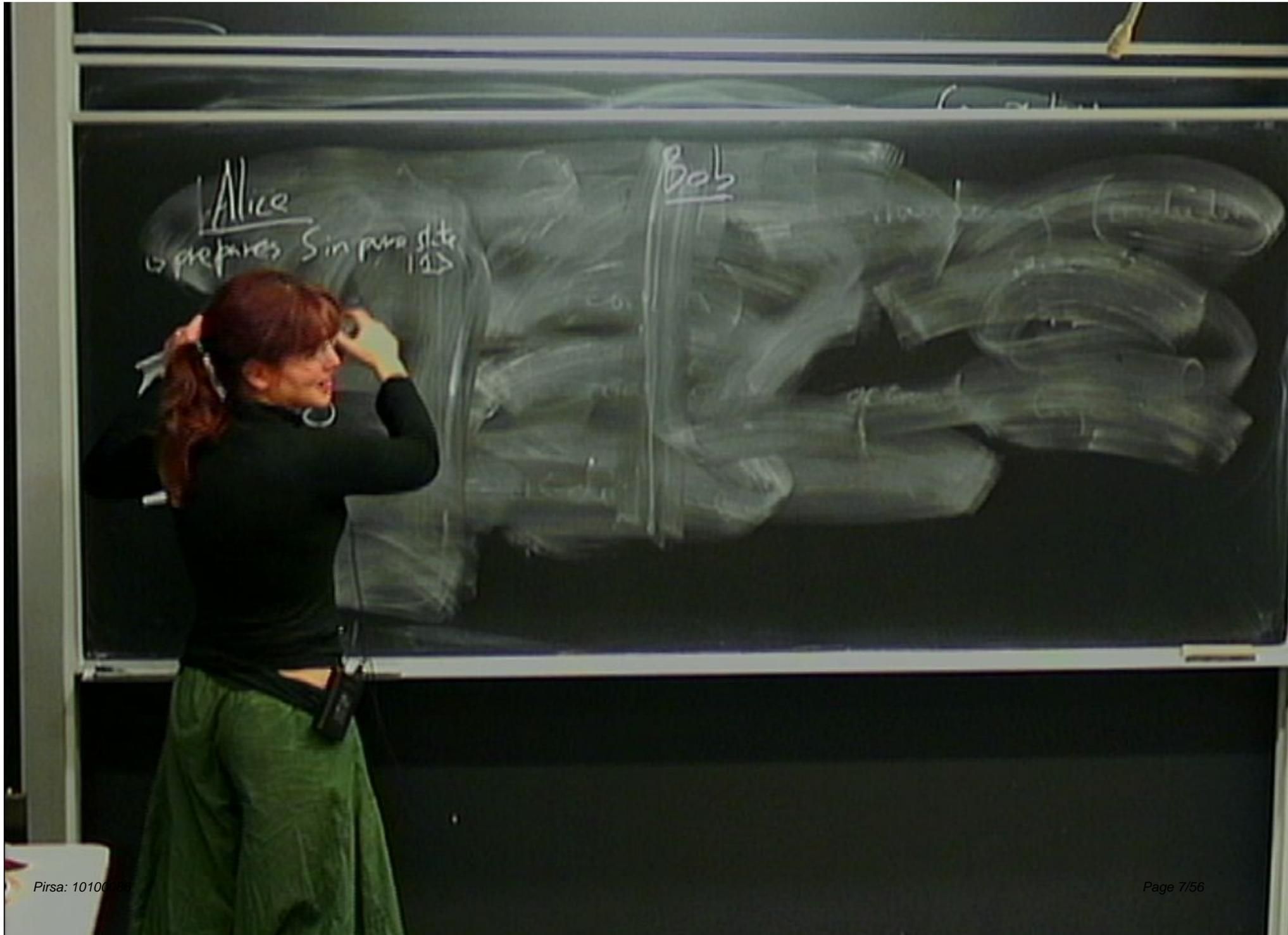
(Landauer's pt.)

↓
a given observer

↳ taking system to pure state, $|0\rangle$

1. Inf. about a system

is n qubits, degenerate.



Alice

prepares S in pure state $|\psi\rangle$

Bob

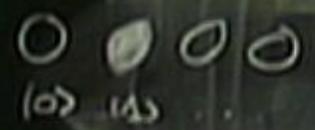
collapses the state

of time

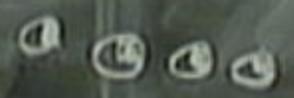
probability

Continuation

Alice
prepares S in pure state $|0\rangle$



Bob
doesn't know state of S



$$f_S = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Conclusion

What does it take to erase (the data stored in) a system

↳ how much work

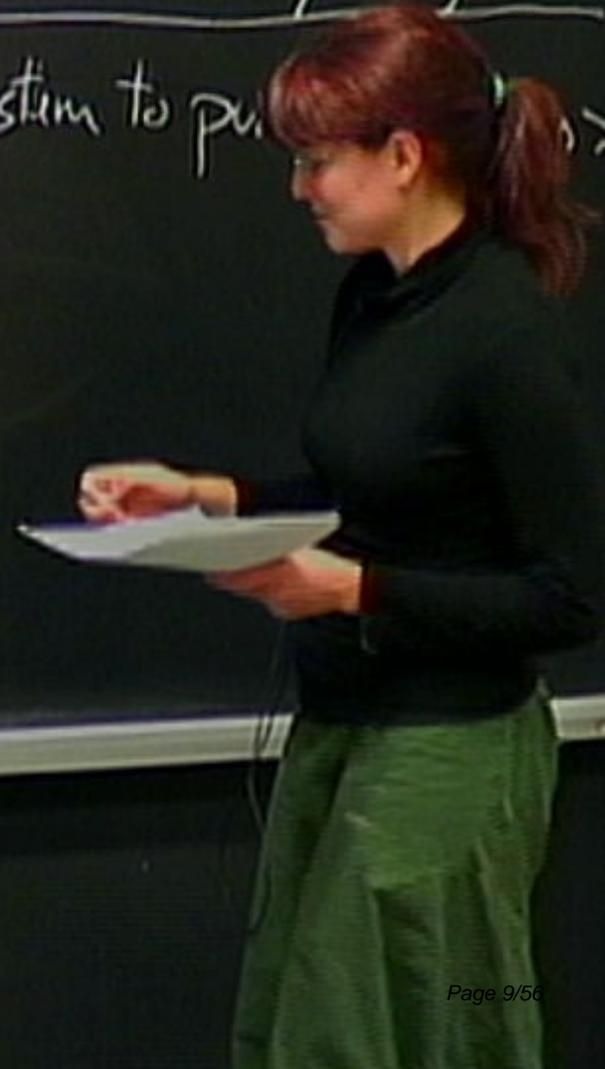
(Landauer's pr.)

↳ a given observer

↳ taking system to put it in a state

1. Inf. about a system \rightarrow entropy:

$\approx n$ qubits, degenerate.



What does it take to erase (the data stored in) a system

↳ how much work

(Landauer's pr.)

↳ a given observer

↳ taking system to pure state, $|0\rangle$

1. Inf. about a system \rightarrow entropy: von Neumann
 $H(S) = -\sum_i p_i \log p_i$
if n qubits, degenerate.

Alice
prepares S in pure state $|0\rangle$

Bob
doesn't know state of S

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0000

$H(S|B) = \ln$

$f_S = \left(\frac{1}{2^n} \right)$

$S|B = 0$

Alice

prepares S in pure state $|s\rangle$

0 0 0 0

$|0\rangle$ $|0\rangle$. . .

$$H(S|A) = 0$$

$$P_{SA} = \frac{1}{2^n} \sum_S |s\rangle \langle s| \otimes |s\rangle \langle s|$$

Bob

doesn't know state of S

1 0 0 0

$$H(S|B) = n$$

$$P_{SB} = \frac{1}{2^n} \sum_S |s\rangle \langle s|$$

$$P_S = \left(\begin{matrix} 1/2 & & & \\ & 1/2 & & \\ & & 1/2 & \\ & & & 1/2 \end{matrix} \right)$$

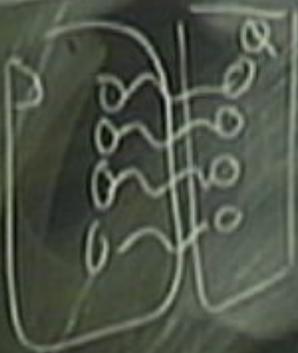
Quasi simaob

ooos

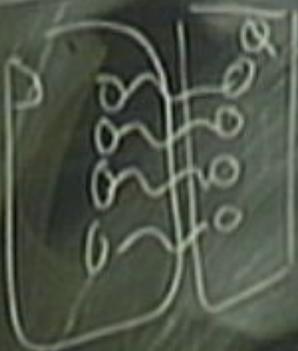
CAUTION

Quasi modo

$$f_{sa} = |\psi\rangle\langle\psi|, \quad f_s = f_a = \frac{1}{2^3}$$



Quasi sima do $\langle \psi | \psi \rangle = 1$, $f_s = f_a = \frac{1}{2}$



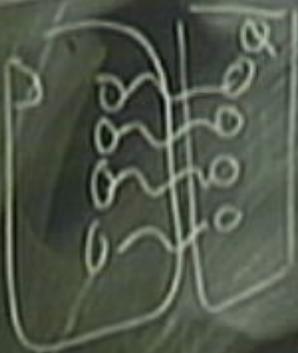
$$H(S|Q) = \underbrace{H(S|Q)}_0 - \underbrace{H(Q)}_h = -h$$

Eq. 16



Quasi-modulo

$$f_{S^1} = |\psi\rangle\langle\psi|, \quad f_S = f_A = \frac{\Delta}{2^2}$$



$$H(S|Q) = \underbrace{H(S)}_0 - \underbrace{H(Q)}_h = -h$$

2. Erasing the system

1007.1630'

2. Erasing the system

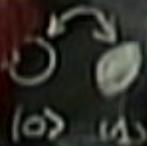
Allow them to manipulate S , ~~the environment~~

- apply unitary op. on S

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Alice

prepares S in pure state $|\Delta\rangle$

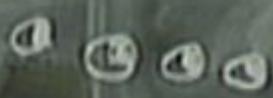


$H(S|A)$

$$|\Delta\rangle = \frac{1}{\sqrt{2}} (|0\rangle \otimes |s\rangle + |1\rangle \otimes |s'\rangle)$$

Bob

doesn't know state of S



$H(S|B) = n$

$$\rho_S = \frac{1}{2} (|0\rangle\langle 0| + |1\rangle\langle 1|)$$

$$\rho_B = \frac{\Delta 1}{2^n} \sum_{s \in \{0,1\}^n} |s\rangle\langle s|$$

Alice

prepares S in pure state $|s\rangle$

$\uparrow E$

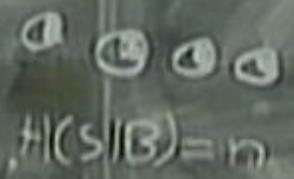


$$H(S|A) = 0 \quad W(S|A) = 0$$

$$\rho_{SA} = \frac{1}{S} \sum_{s \in S} |s\rangle\langle s| \otimes |s\rangle\langle s|$$

Bob

doesn't know state of S



$$H(S|B) = m$$

$$\rho_S = \frac{1}{2^m} \sum_{s \in S} |s\rangle\langle s|$$

$$\rho_{SB} = \frac{1}{2^m} \sum_{s \in S} |s\rangle\langle s|$$

2. Erasing the system

Allow them to manipulate S ~~in various~~

• apply unitary op. on S

• couple system to heat bath at temp T

1007.1630

2. Erasing the system

Allow them to manipulate S , ~~the system~~

• apply unitary op. on S

• couple system to heat bath at temp T

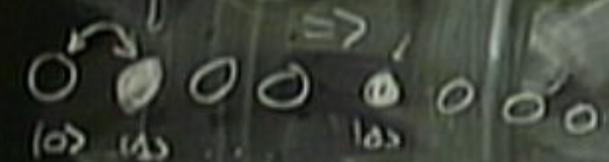
1009 $\rho_S \rightarrow \frac{e^{-\frac{H}{kT}}}{Z}$

• manipulate Hamiltonian (energy levels)

Alice

prepares S in pure state $|A\rangle$

\uparrow
E

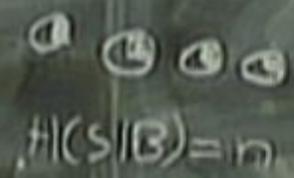


$$H(S|A) = 0 \quad W(S|A) = 0$$

$$\rho_{SA} = \frac{1}{2} (|A\rangle\langle A| \otimes |s\rangle\langle s| + |B\rangle\langle B| \otimes |s\rangle\langle s|)$$

Bob

doesn't know state of S



$$H(S|B) = \ln 2$$

$$\rho_{SB} = \frac{1}{2} (|A\rangle\langle A| + |B\rangle\langle B|)$$

Allow interaction between two systems

- apply unitary op. on S
- couple system to heat bath at temp T

1007

$$\rho_S \rightarrow \frac{e^{-\frac{H_S}{kT}}}{Z}$$

• manipulate Hamiltonian (energy levels)

- → level empty: free
- → level occ: ΔE



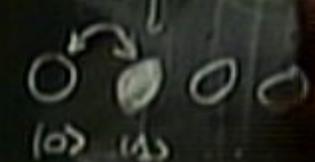
Local Set Hypothesis

G. t Hooft, J. Myrheim &
L. Bombelli, J.-H Lee, D. Major & P. Sen

Alice

prepares S in pure state

\uparrow
 E

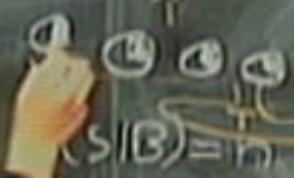


$$H(S|A) = 0$$

$$P_{SA} = |A\rangle\langle A|$$

Bob

doesn't know state of S



$$P_{SB} = \frac{1}{2^n} \sum_{i=1}^n |i\rangle\langle i|$$

Alice

prepares S in pure state $|0\rangle$

\uparrow
E

$0^{\otimes n}$
 $|0\rangle$

$H(S|A)$

$P_{SA} =$

0000

$H(S|A) = 0$

Bob

doesn't know state of S

100
 100

$H(S|B) = \ln 2$

$$P_{SB} = \frac{\Delta 1}{2^n} \otimes |n\rangle\langle n|$$

Alice

prepares S in pure state $|1\rangle$

$\uparrow E$

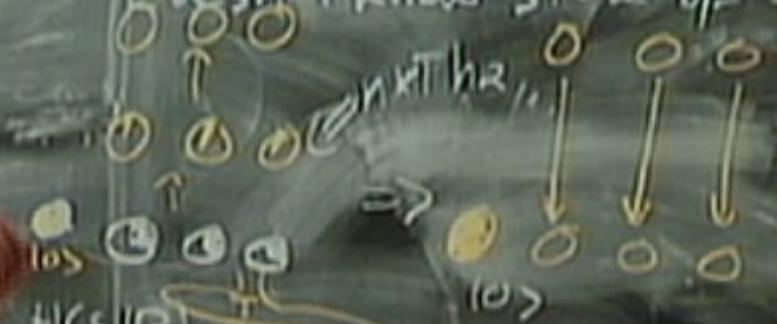


$H(S|A) = 0 \quad W(S|A)$

$\rho_{SA} = |1\rangle\langle 1| \otimes \frac{1}{5}$

Bob

doesn't know state of S



$H(S|B) = \ln 2$

$\rho_{SB} = \frac{1}{2} (|1\rangle\langle 1| + |0\rangle\langle 0|)$

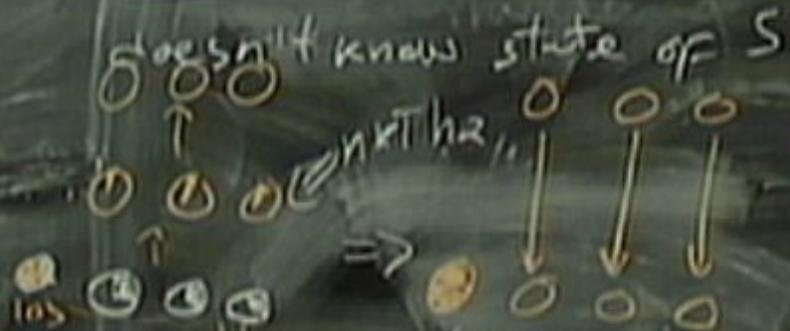
In the continuum a timelike geodesic ^{from 2 toy} maximises proper time

(class. obs \leftarrow)
 $W(S|A) = H(S|A) \times T \ln 2$ Bob



$W(S|A) = 0$

$\sum_{i=1}^n |s_i\rangle \langle s_i|$



$H(S|B) = n$

$W(S|B) = n \times T \ln 2$

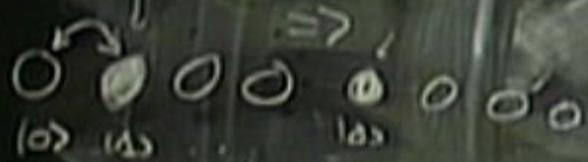
$P_{SC} = \frac{\Delta I}{2^n} \otimes |1^n\rangle \langle 1^n|$

class. obs \leftarrow
 $W(S|C) = H(S|C) \times T \ln 2$

Alice

prepares S in pure state $|0\rangle$

\uparrow E



$H(S|A) = 0 \quad W(S|A) = 0$

$\rho_{SA} = \frac{1}{2^n} \sum_S |S\rangle\langle S| \otimes |S\rangle\langle S|$

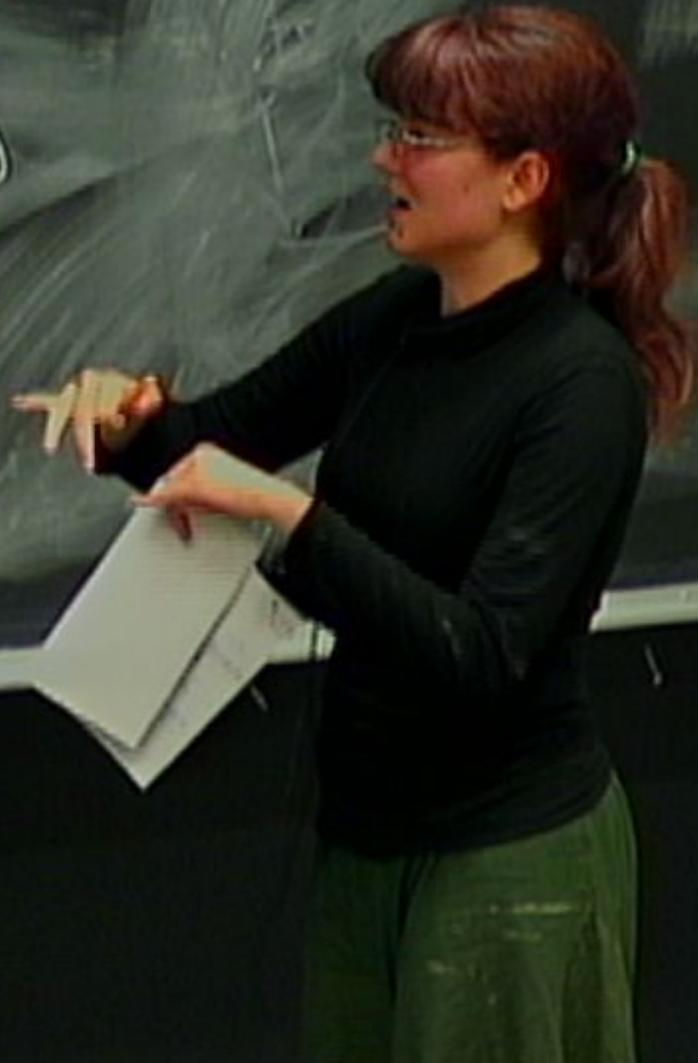
Bob

doesn't know state of S

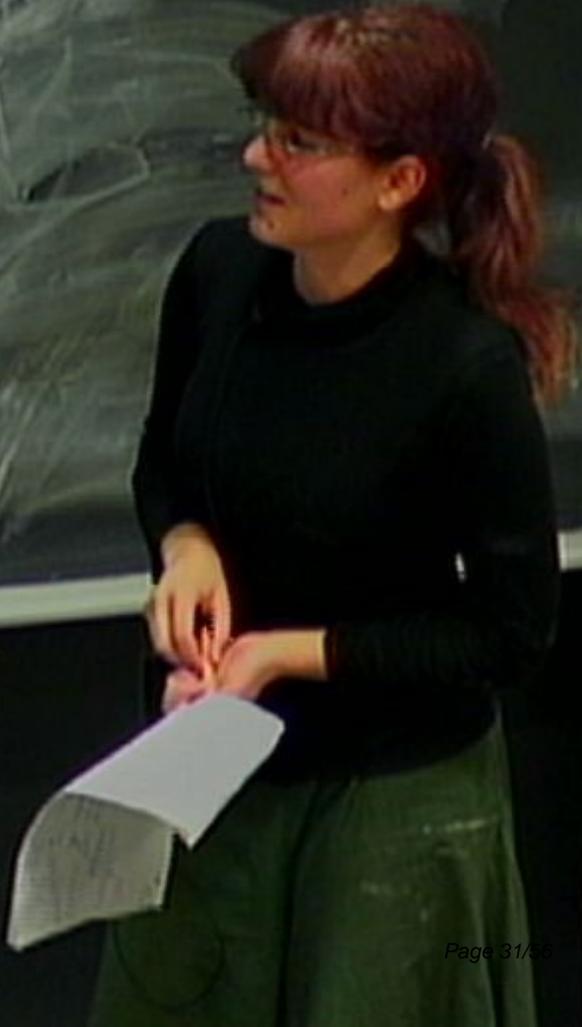


$H(S|B) = n$
 $W(S|B) = n \times T \ln 2$

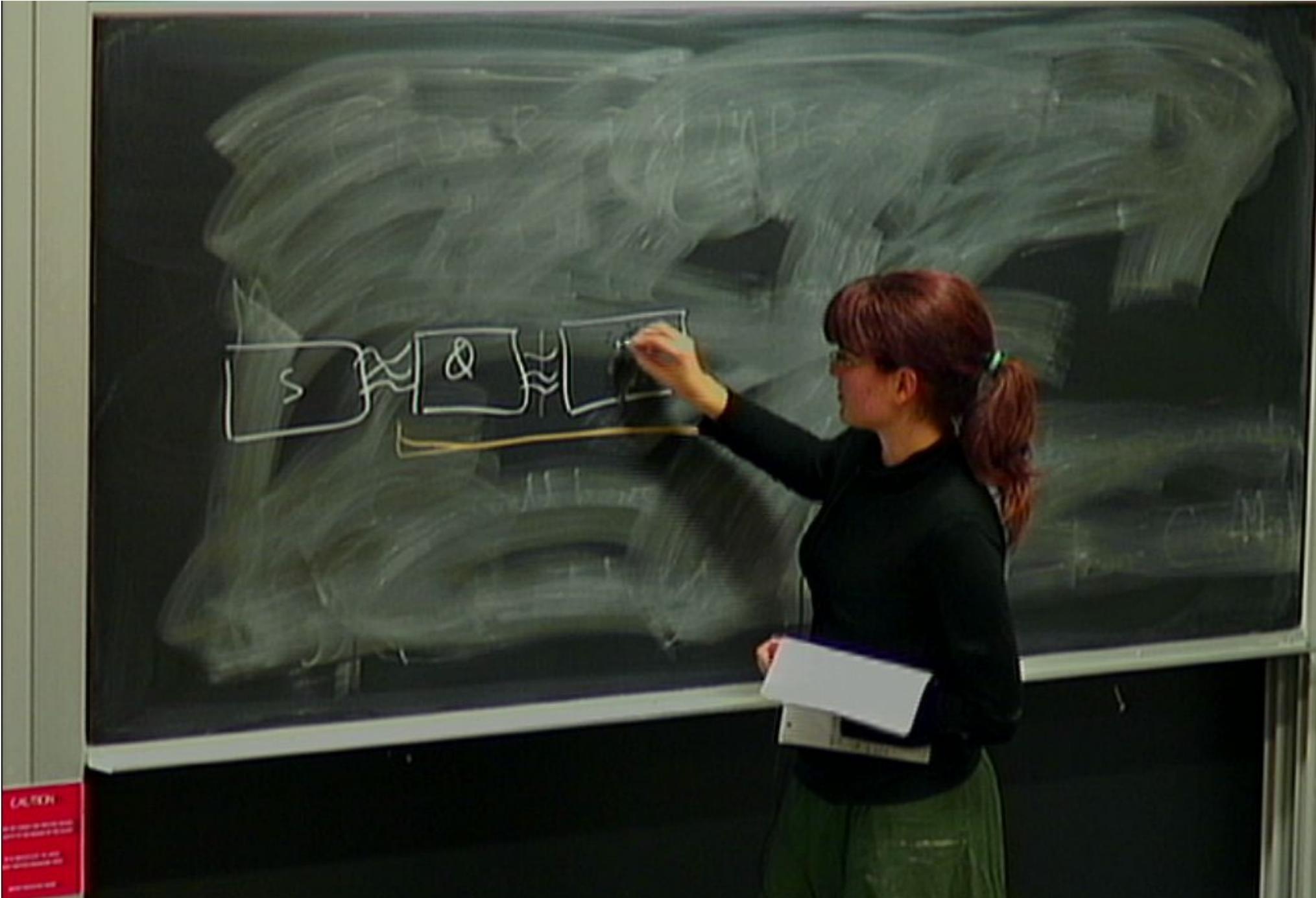
$\rho_B = \frac{1}{2^n} \sum_S |S\rangle\langle S|$



CAUTION



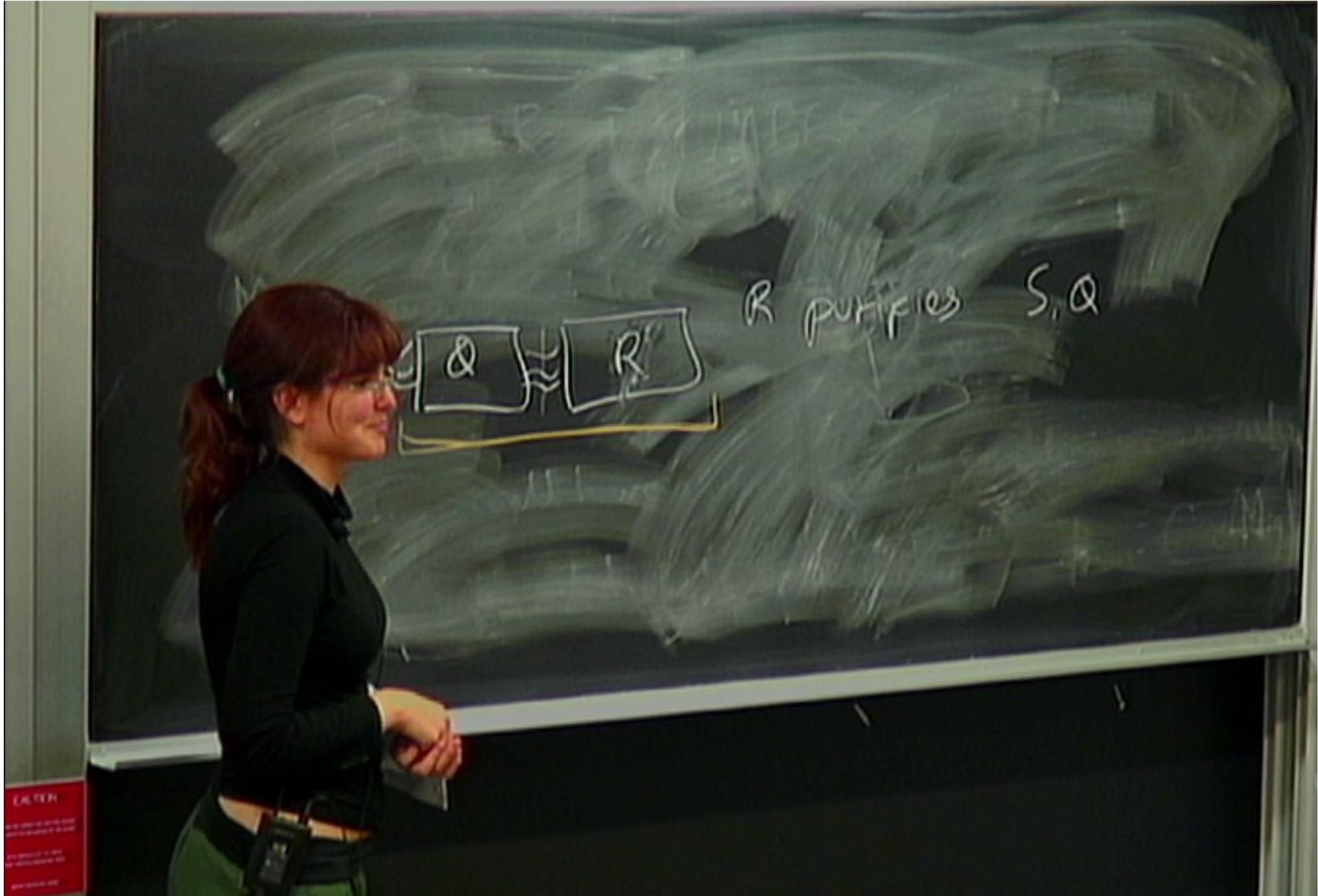
CAUTION



CAUTION
Do not touch the board when it is hot.
Do not touch the board when it is hot.
Do not touch the board when it is hot.



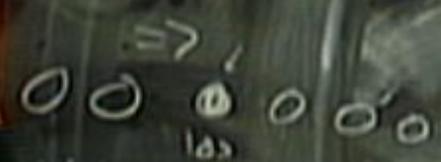
CAUTION
 Please do not touch the board.
 It is not intended for use as a writing surface.



CAUTION
DO NOT TOUCH THE BOARD
OR THE CHALK
OR THE BOARD
OR THE BOARD

class. obs \leftarrow
 $W(S|A) = H(S|A) \times T \ln 2$ Alice Bob

prepares S in pure state $|s\rangle$



$W(S|A) = 0$

$= |s\rangle \langle s| \otimes |s\rangle \langle s|$
 $\sum_{s'} |s'\rangle \langle s'|$

doesn't know state of S

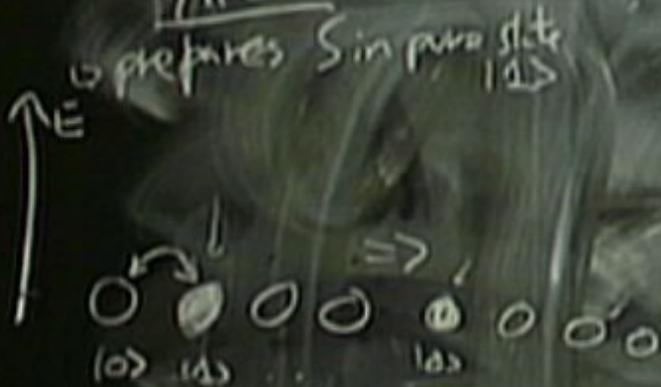


$H(S|B) = n$

$W(S|B) = n \times T \ln 2$

$\rho_B = \frac{1}{2^n} \sum_{s'} |s'\rangle \langle s'|$

class. obs $\left(\begin{array}{l} \text{Alice} \\ \text{Bob} \end{array} \right)$
 $W(S|C) = H(S|C) \times T \ln 2$



$$H(S|A) = 0 \quad W(S|A) = 0$$

$$P_{SA} = \frac{1}{2^n} \sum_S |s\rangle \langle s| \otimes |s\rangle \langle s|$$

doesn't know state of S

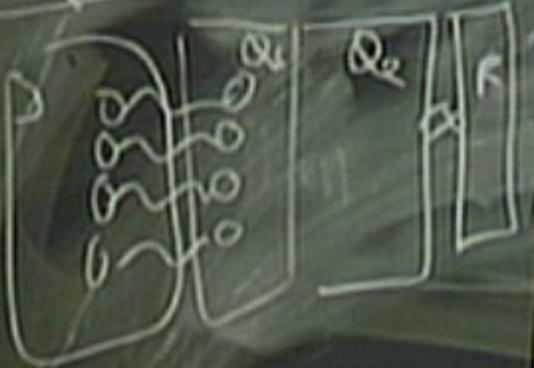


$$H(S|B) = n$$

$$W(S|B) = n \times T \ln 2$$

$$P_{SB} = \frac{1}{2^n} \sum_S |s\rangle \langle s| \otimes |s\rangle \langle s|$$

Quasimodo



$$P_{\text{succ}} = \frac{1}{2} \langle \psi | \sigma_x | \psi \rangle = P_a = \frac{1}{2}$$

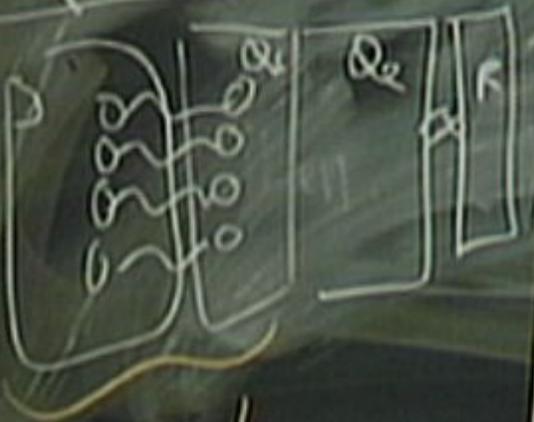
max out

Δ : extracts - work from

$$H(\sin) = \dots$$



Quasimodo

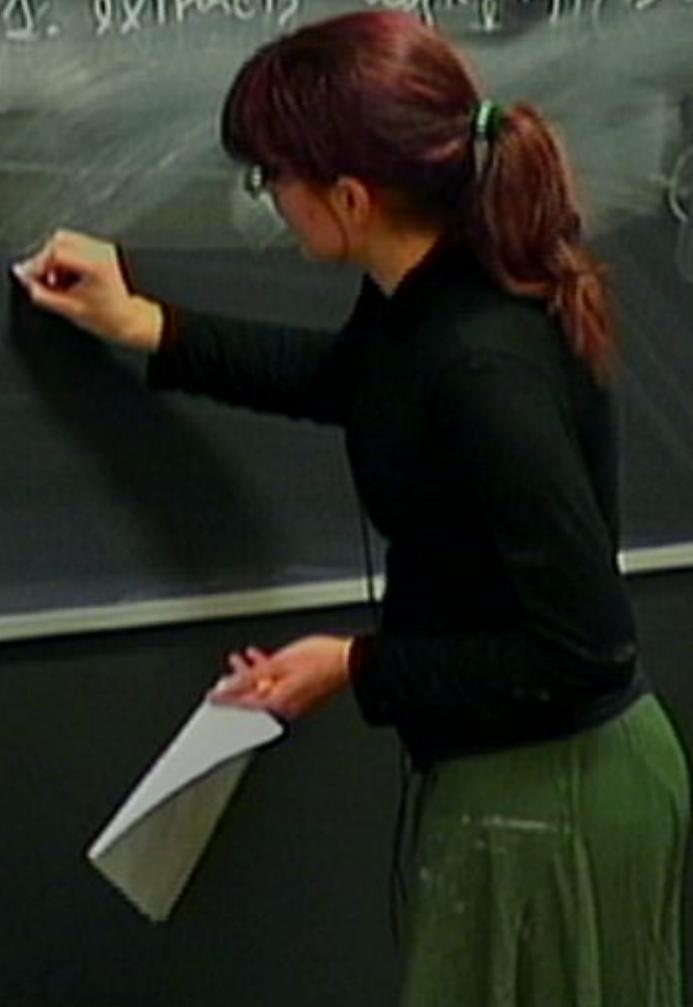


2n qubits

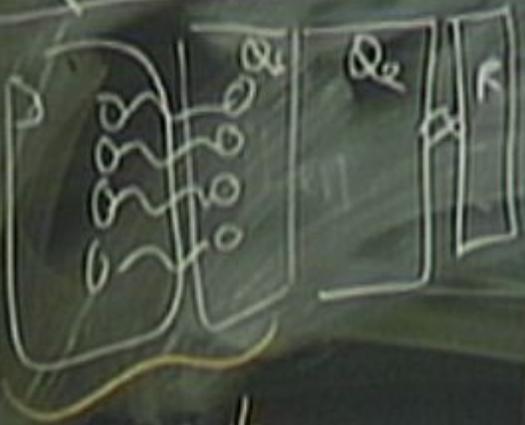
$$\rho_{SQA} = \frac{1}{2^n} \sum_{i,j} |i\rangle\langle j| \otimes \rho_{ij} \quad \rho_{ij} = \frac{1}{2^n} \frac{\Delta_i}{2^n}$$

max ent.

Δ : extracts work from SQA $H(S|Q) = -n$



Quasimodo



2^n sub

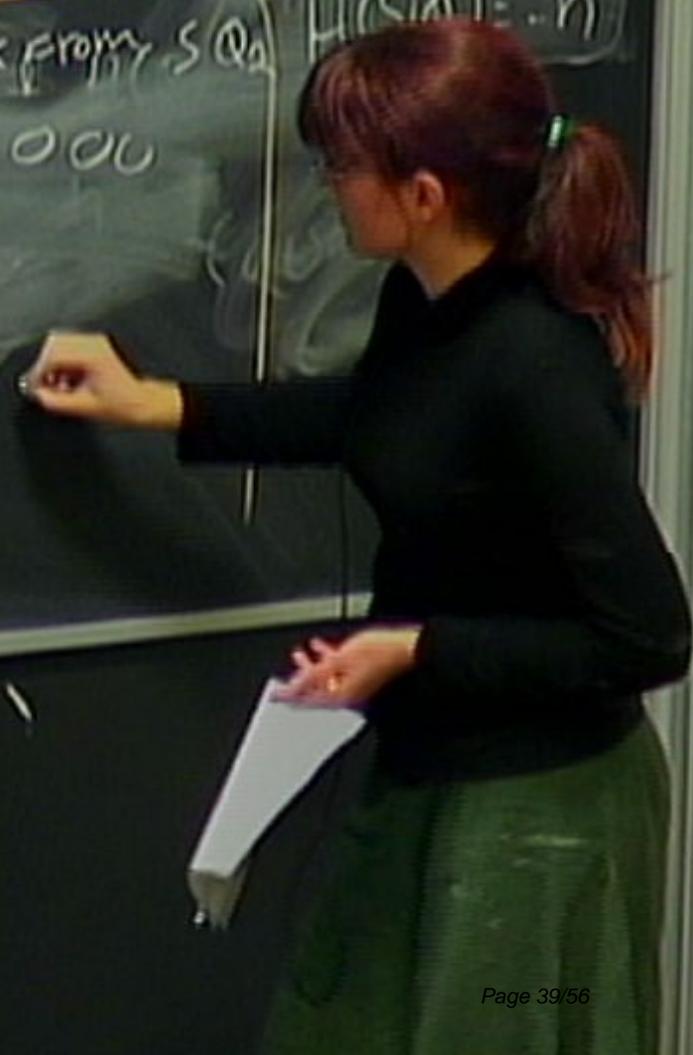
$$P_{SQ} = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle \langle i| \otimes P_S = P_A = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle \langle i|$$

max ent.

Δ : extracts work from SQ

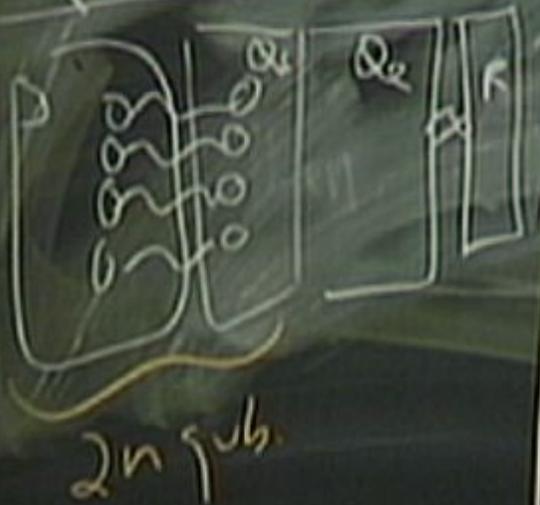


$$H(|S\rangle) = -n$$



CAUTION

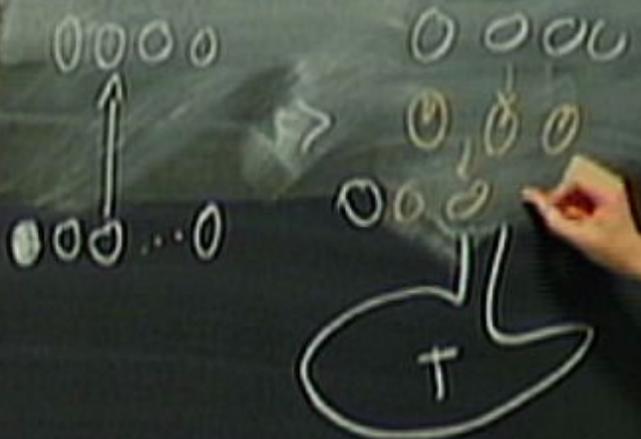
Quasimodo



$$|s_{0n}\rangle = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle |0\rangle$$

max ent.

Δ : extracts work from s_{0n}

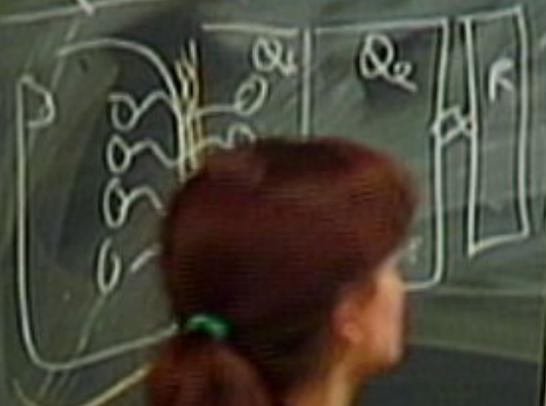


$H(|s_{0n}\rangle) = n$



CAUTION

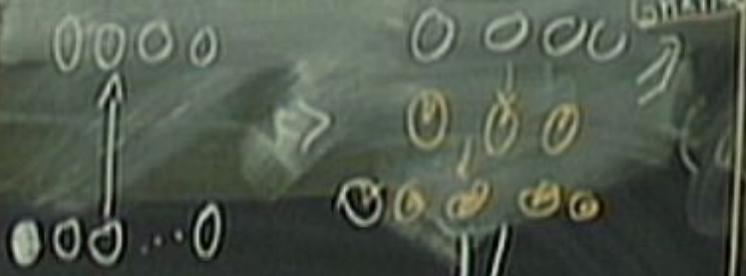
Quasimodo



$$P_{SQ} = \frac{1}{\sqrt{2^n}} \sum_{i=0}^{2^n-1} |i\rangle \langle i| \otimes P_i = P_A = \frac{1}{\sqrt{2^n}}$$

max out.

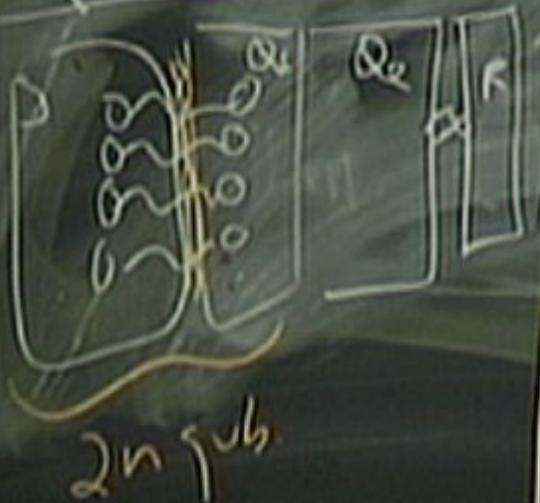
Δ : extracts work from S_{Q_1}



$$P_A = \frac{1}{\sqrt{2^n}}$$

$$H(S|Q) = -n$$

Quasimodo

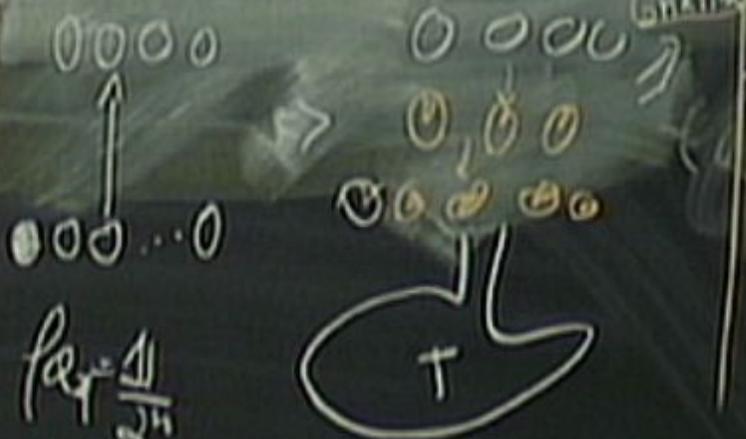


$2n$ sub

$$P_{S|Q} = \frac{|\psi\rangle\langle\psi| \otimes \rho_S}{\rho_S} = P_A = \frac{1}{2^n}$$

max ent.

1. extracts work from $S|Q$

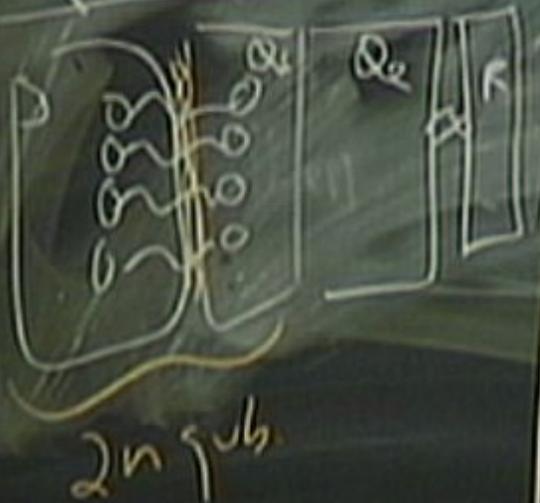


$$P_A = \frac{1}{2^n}$$

$$H(S|Q) = -n$$

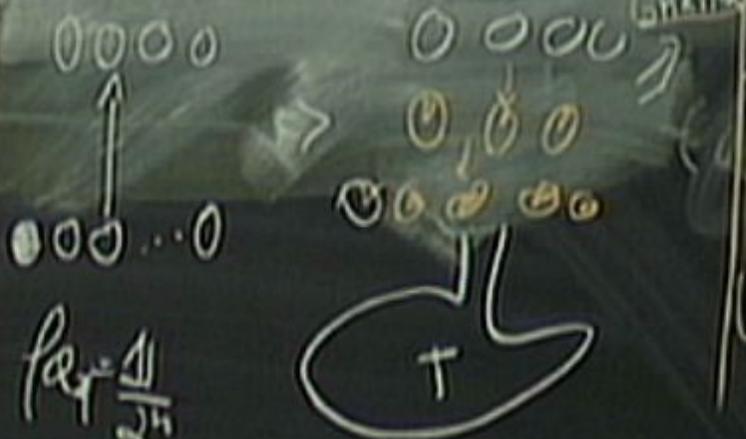
2. erases S
 $\ln n k T$

Quasimacro



$$P_{S|Q} = \frac{|\Psi\rangle\langle\Psi|}{\text{norm out}} \quad P_S = P_Q = \frac{\Delta}{2^n}$$

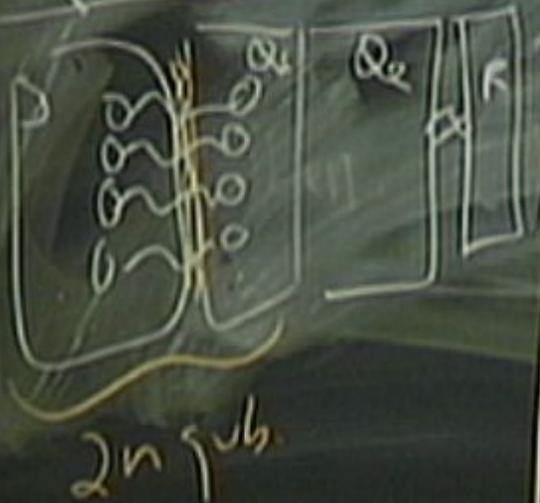
1. extracts work from S (Q)



$$H(S|Q) = -n$$

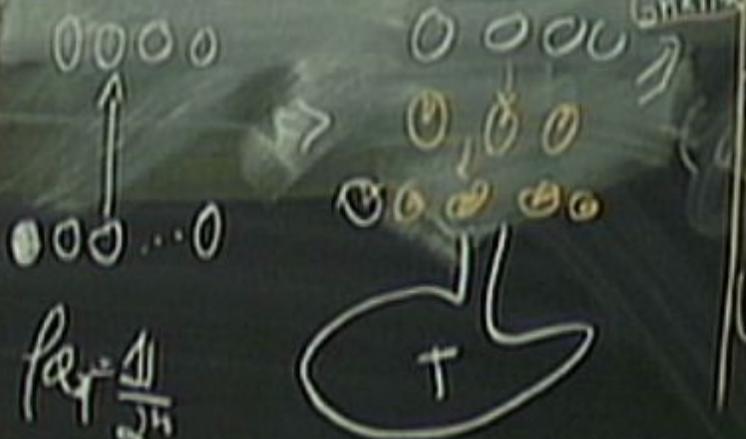
2. erases S
 $\ln n k T \ln 2$
 $W(S|Q) = -n k T \ln 2$

Quasimacro



$$P_{S|Q} = \frac{|\Psi\rangle\langle\Psi|}{\text{norm out.}} \quad P_S = P_Q = \frac{1}{2^n}$$

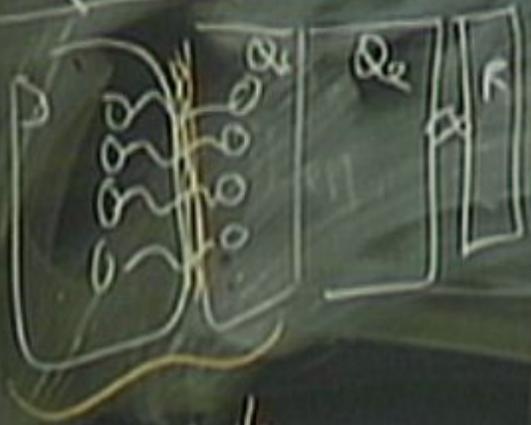
Δ: extracts work from S|Q



$$H(S|Q) = -n$$

2. erases S
 $\ln n k T \ln 2$
 $W(S|Q) = -n k T \ln 2$

Quasimacro



$2n$ sub

$$p_{S|Q} = \frac{|\Psi\rangle\langle\Psi|}{\text{norm out.}} \quad p_S = p_Q = \frac{\Delta}{2^n}$$

1. extracts work from $S|Q$

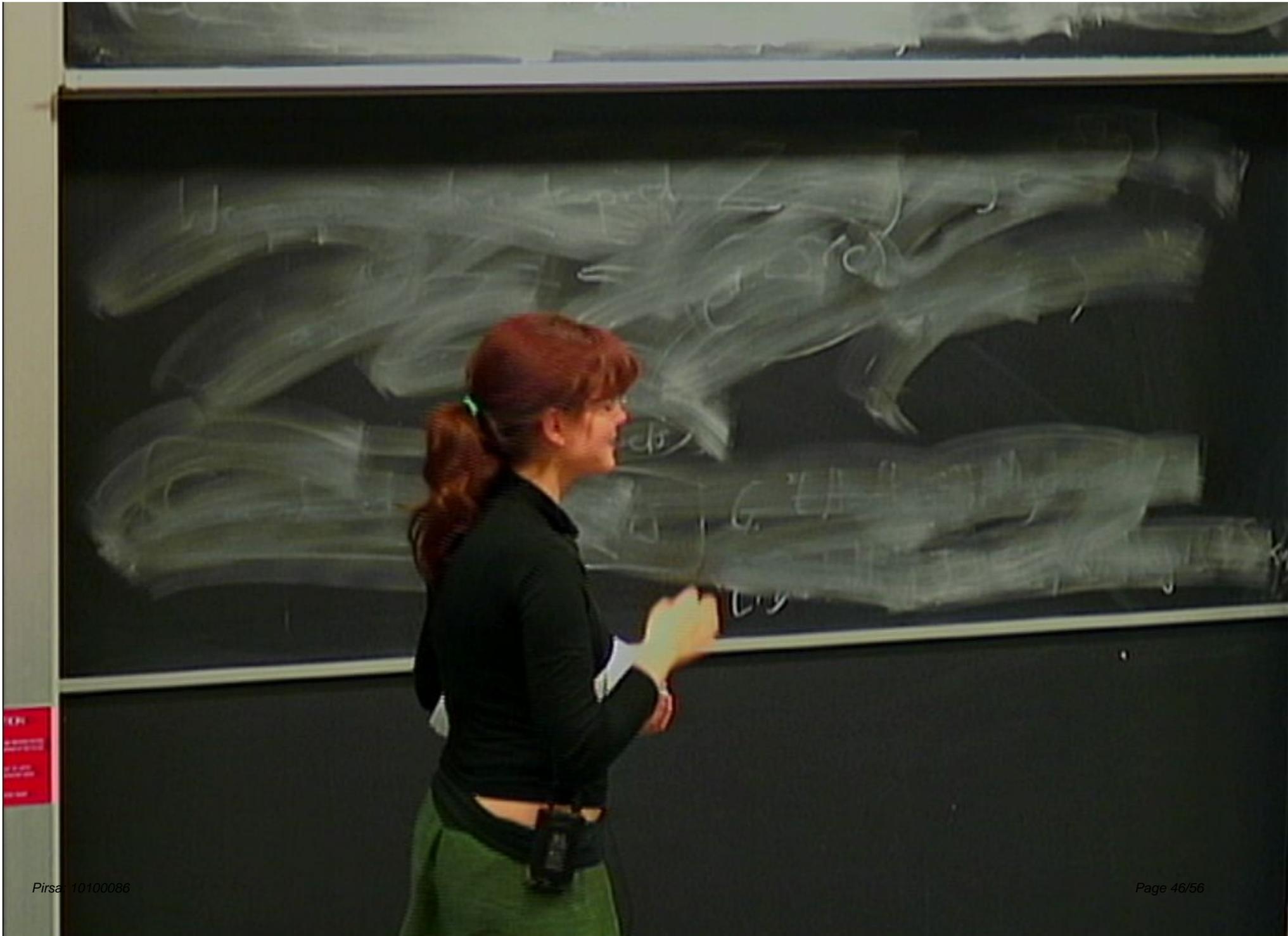


$$p_Q = \frac{\Delta}{2^n}$$

$$H(S|Q) = -n$$

2. erases S
 $\ln n k T \ln 2$

$$W(S|Q) = -n k T \ln 2$$



in general, \mathbb{R}^n

in general, $H(S|Q) \approx \underbrace{H(S|Q)}_{\text{from binary}} kT \ln 2$ (mid limit)

actually

$$H(S|Q) \leq \left[H_{\text{max}}^{\epsilon}(S|Q) + \Delta \right] kT \ln 2,$$

in general, $w(S|Q) \approx \underbrace{H(S|Q)}_{\text{can be neg.}} kT \ln 2$ (iid limit)

actually

$$w(S|Q) \leq \left[\overline{H_{\max}^{\epsilon}}(S|Q) + \Delta \right] kT \ln 2,$$

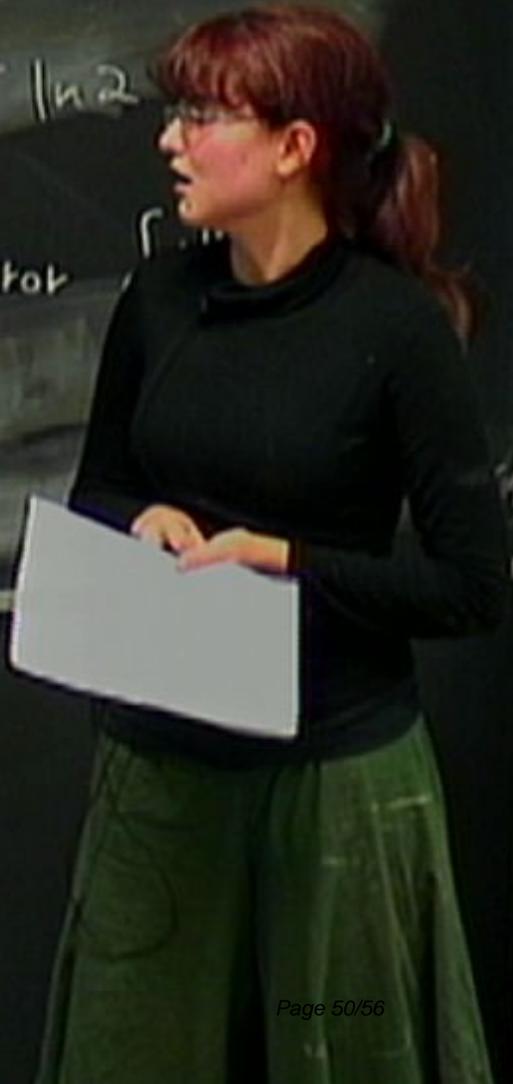
\downarrow (d.s.) \downarrow prob. error $\int \leq \sqrt{2^{-\frac{L\Delta}{kT}} + 12\epsilon}$

in general, $w(S|Q) \approx \underbrace{H(S|Q)}_{\text{entropy}} kT \ln 2$ (mid limit)

actually

$$w(S|Q) \leq \left[\overset{\epsilon}{H_{\max}}(S|Q) + \Delta \right] kT \ln 2$$

↙ prob error



in general, $w(S|Q) \approx \underbrace{H(S|Q)}_{\text{mean message}} kT \ln 2$ (mid limit)

actually

$$w(S|Q) \leq \left[H_{\max}^{(\epsilon)}(S|Q) + \Delta \right] kT \ln 2$$



$\epsilon \rightarrow 0$

prob error $\int \leq \sqrt{2^{-\frac{\Delta}{kT}} + 12\epsilon}$

$\Delta \rightarrow 0$

in general, $w(S|Q) \approx \underbrace{H(S|Q)}_{\text{can be neg.}} kT \ln 2$ (mid limit)

actually

$$w(S|Q) \leq \left[H_{\max}^{(\epsilon)}(S|Q) + \Delta \right] kT \ln 2,$$



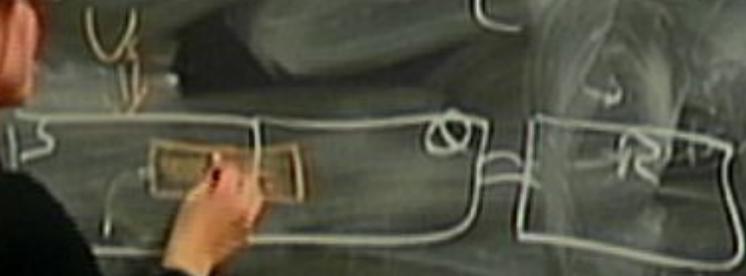
$\epsilon \rightarrow 0$

prob error $\int \leq \sqrt{2^{-\frac{\Delta}{kT}} + 12\epsilon}$

$\Delta \rightarrow \Delta \approx 20$

in general, $w(SIQ) \approx \underbrace{H(SIQ)}_{\text{mean energy}} kT \ln 2$ (mid limit)

more fully $w(SIQ) \approx [H_{max}^{(E)}(SIQ) + \Delta] kT \ln 2$



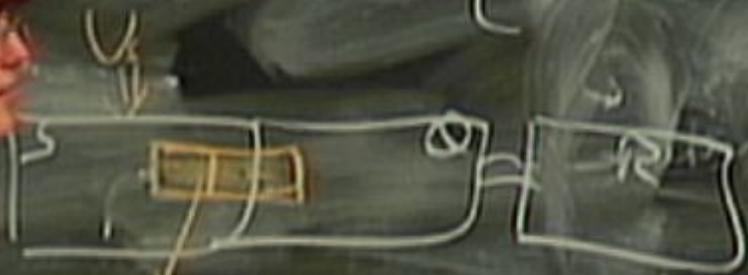
k qubits

$e \leq H - H_{max}^{(E)}(SIQ) - \Delta$

in general, $w(SIQ) \approx \underbrace{H(SIQ)}_{\text{mean energy}} kT \ln 2$ (mid limit)

actually

$$w(SIQ) \leq [H_{\max}^{(\epsilon)}(SIQ) + \Delta] kT \ln 2,$$



x = ℓ qubits

$$e \leq H_{\max}^{(\epsilon)}(SIQ) - \Delta$$

in general, $w(SIQ) \approx \underbrace{H(SIQ)}_{\text{mean energy}} kT \ln 2$ (mid limit)

actually

$$w(SIQ) \leq [H_{\text{max}}^{(\epsilon)}(SIQ) + \Delta] kT \ln 2$$



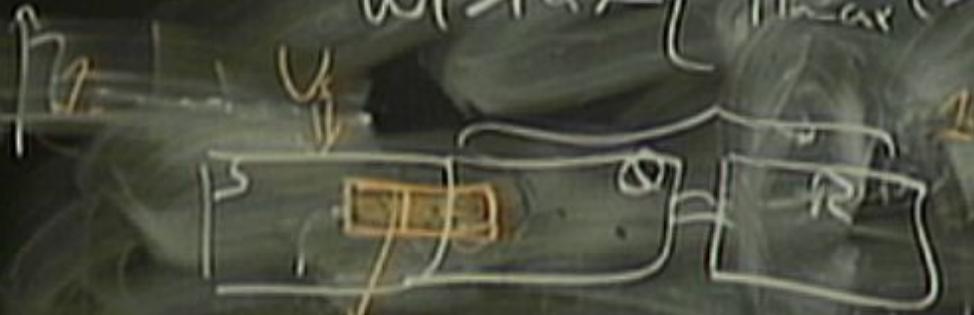
x & l qubits

$$e \leq H - H_{\text{max}}^{(\epsilon)}(SIQ) - \Delta$$

in general, $w(SIQ) \approx \underbrace{H(SIQ)}_{\text{mean energy}} kT \ln 2$ (mid limit)

actually

$$w(SIQ) \leq [H_{\max}^{(\epsilon)}(SIQ) + \Delta] kT \ln 2,$$



$x = \ell$ qubits

$$e \leq H_{\max}^{(\epsilon)}(SIQ) - \Delta$$