Title: On Hydrodynamics of holographic p-wave Superfluids

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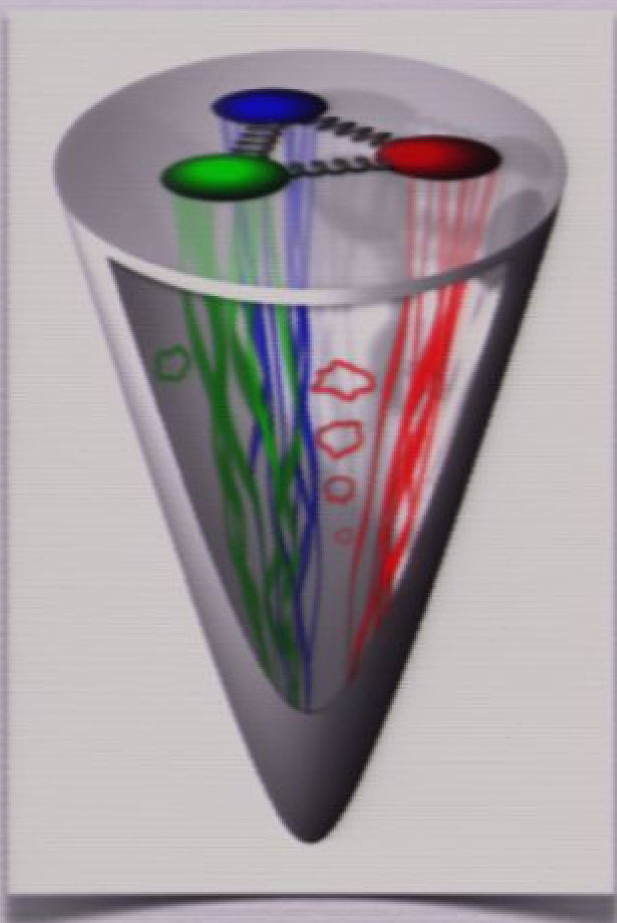
Abstract: We discuss holographic duals of strongly interacting gauge theories which show properties of p-wave superfluids which in addition to an Abelian symmetry also break the spatial rotational symmetry. The gravity duals of these superfluid states are black hole solutions with a vector hair which we construct in a non-Abelian Einstein-Yang-Mills theory and in the D3/D7 brane setup. The latter allows us to identify the dual field theory explicitly. After we constructed the vector hair state we study the conductivity and shear viscosity which is non-universal due to the breaking of the rotational symmetry.

Why is Gauge/Gravity duality useful for Hydrodynamics?



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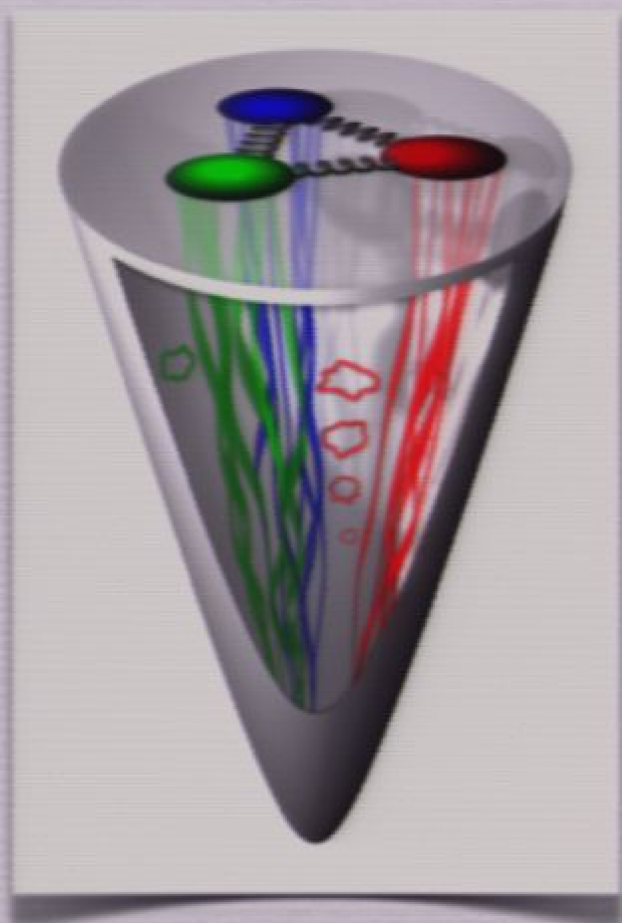

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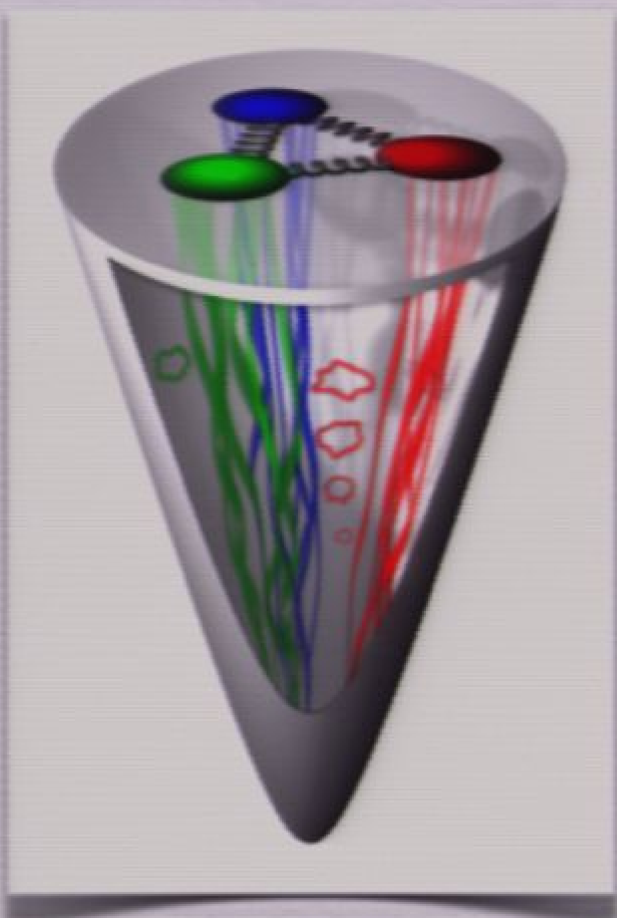

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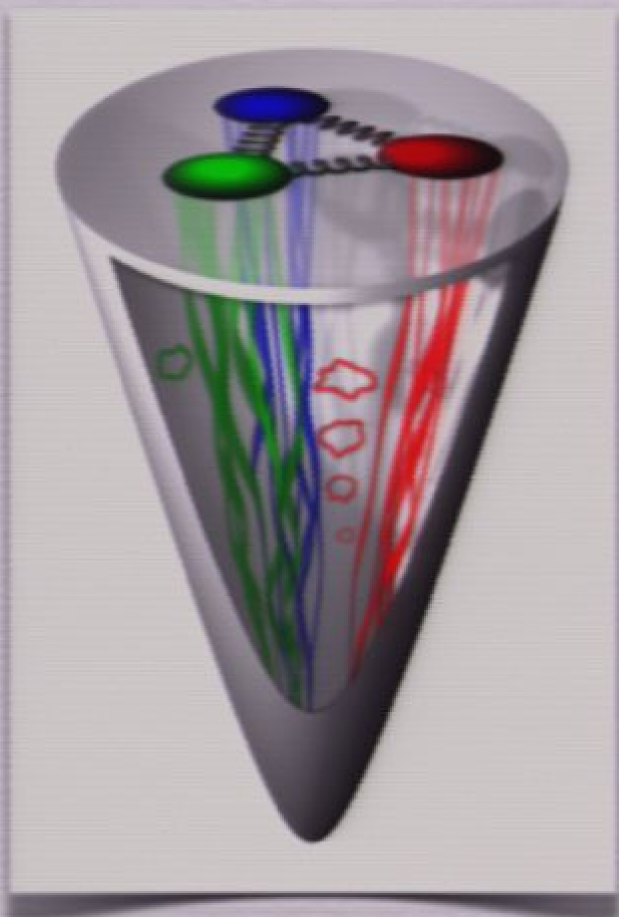

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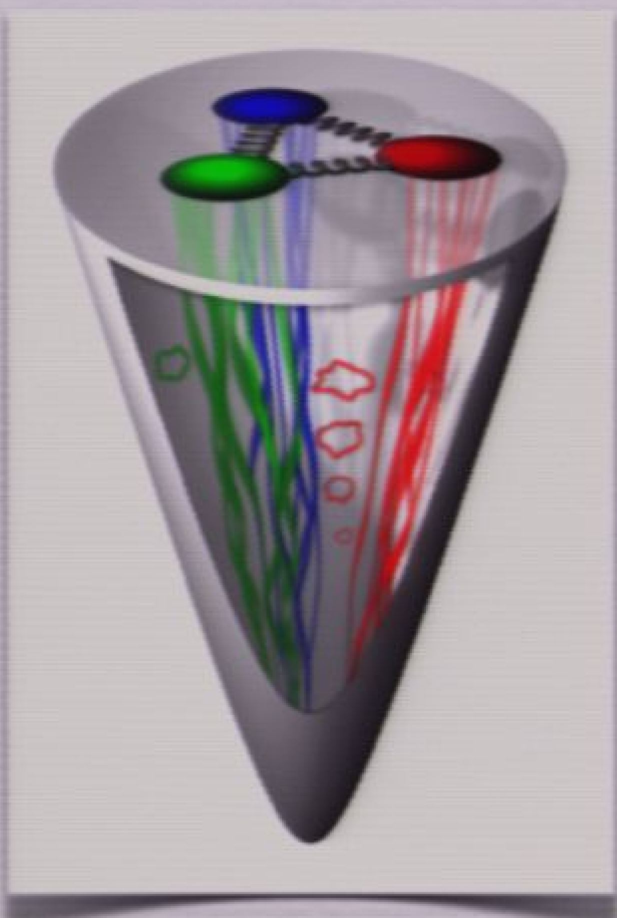

- Allows for simple calculation of real-time correlators [Son, Starinets;...]
- Low-lying poles determine hydrodynamic modes and their dispersion relations
- Unambiguous construction of hydrodynamic description possible
- Discovery of new transport coefficients, e.g. in systems with anomaly [Erdmenger et al.; Son, Su

Why Hydrodynamics of p-wave Superfluids?

- Superfluids in general interesting since new hydrodynamic mode (Goldstone bosons) present, e.g. second and fourth sound [Herzog, Pufu;Yarom; Herzog, Yarom]

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- In p-wave Superfluids rotational symmetry also spontaneously broken $\Rightarrow$ more hydrodynamic modes
- Rotational symmetry is often spontaneously broken in condense matter systems, e.g. liquid crystals, d-wave superconductors

Outline

(1) Motivation (over)

(2) Construct holographic p-wave Superfluid as black hole with vector hair [Ammon, Erdmenger, Grass, P.K., O'Bannon]

(3) Embedding into String Theory [Ammon, Erdmenger, Kaminski, P.K.]

(4) Towards the hydrodynamic description

i. Classification and Decoupling of fluctuations

ii. Conductivity and Shear Viscosity

(5) Summery and Outlook

Gauge/Gravity Duality

- Type IIB SUGRA on $AdS_5 \times X_5$
  is dual to
  Conformal Field Theory at large N_c and large λ
  in the sense

$$Z_{\mathrm{SUGRA}}\left[\phi(x,r)|_{r\to r_{\mathbf{bdy}}} = \phi_0(x)\right] = \left\langle \mathrm{e}^{\int \mathrm{d}^4 x \phi_0(x) \mathcal{O}(x)} \right\rangle$$

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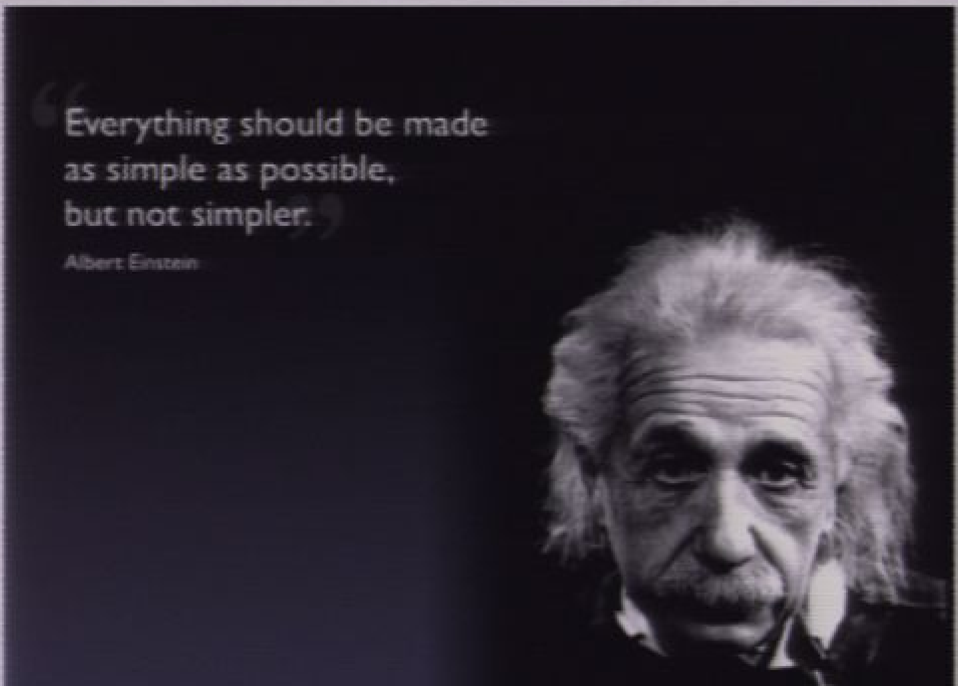
- Black holes correspond to thermal field theories

- Gauge fields A_μ^a are dual to global currents J_a^μ
  especially vevs A_t^3 induce finite chemical potentials μ_I
  (source) and finite densities $\langle J_3^t \rangle$.

Gravity model

- Einstein-Yang-Mills theory with $SU(2)$ gauge group

$$S = \int d^5x\sqrt{-g}\left[\frac{1}{2\kappa_5^2}(R-\Lambda) - \frac{1}{4\hat{g}^2}F^a_{\mu\nu}F^{a\mu\nu}\right] \quad \alpha = \frac{\kappa_5}{\hat{g}}$$



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- Superfluid condensate in addition to chemical potential: Take $\langle J_1^x \rangle$ dual to A_x^1 (only the vev)
- $\langle J_1^x \rangle$ spontaneously breaks $U(1)_3$ down to $\mathbb{Z}_2$ and $SO(3)$ down to $SO(2)$ $\Rightarrow$ p-wave superfluid

Field Theory Interpretation of α

- Holographic calculations of Weyl anomaly:
  $1/\kappa_5^2 \propto c$, c: number of degrees of freedom
- Correlators of $SU(2)$ currents proportional to $1/\hat{g}^2$
  $\Rightarrow 1/\hat{g}^2$ counts degrees of freedom charged under $SU(2)$.

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- Correlators of $SU(2)$ currents proportional to $1/\hat{g}^2$ $\Rightarrow 1/\hat{g}^2$ counts degrees of freedom charged under $SU(2)$.
- Intuitively,

$$\alpha^2 = \frac{\kappa_5^2}{\hat{g}^2} \propto \frac{\#\text{ charged degrees of freedom}}{\#\text{ total degrees of freedom}}$$

Behavior at finite chemical potential

- Reissner-Nordström black hole

$$ds^2 = -N(r)dt^2 + \frac{1}{N(r)}dr^2 + r^2 d\vec{x}^2$$

$$N(r) = r^2 - \frac{2m_0}{r^2} + \frac{2\alpha^2 q^2}{3r^4}$$

$$A = \left(\mu - \frac{q}{r}\right) r^3 dt$$

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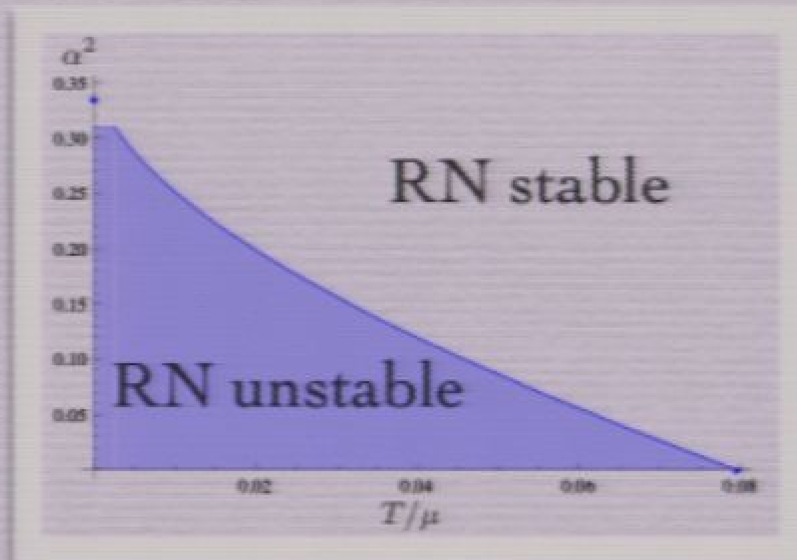

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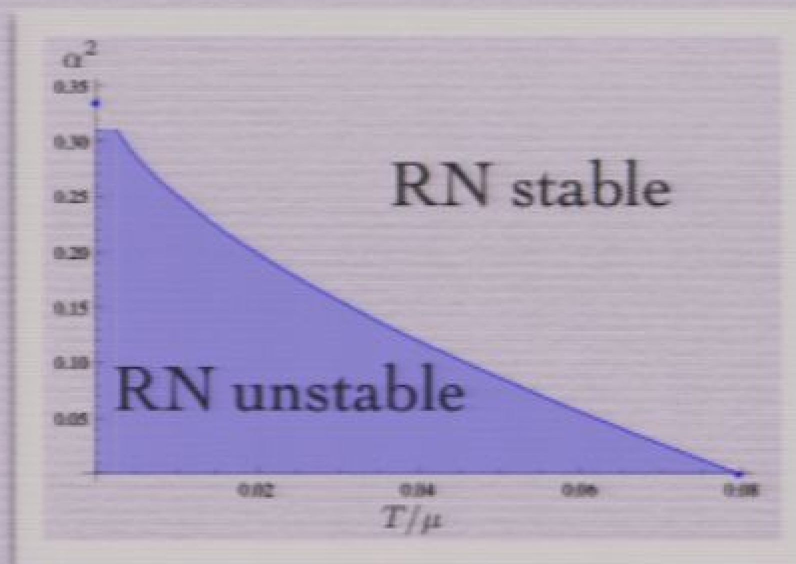

- At low temperature the back hole is unstable against fluctuations in $A^1_x \Rightarrow$ Condensation

Condensation Process

Gubser

- Sketchy action for A^1_x :

$$S \sim \partial_\mu A^1_x \partial^\mu A^1_x + \underbrace{2g^{tt}g^{xx}\left(A^3_t\right)^2}_{=m_{\text{eff}}}\left(A^1_x\right)^2$$

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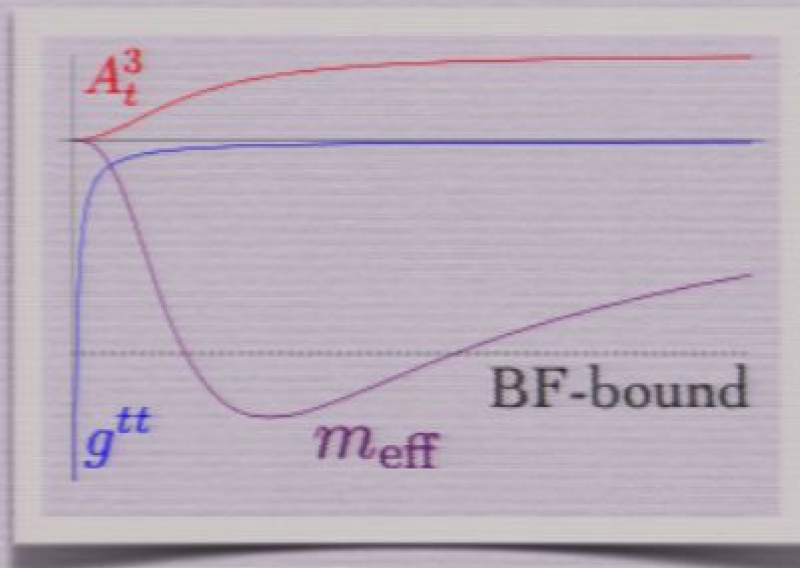

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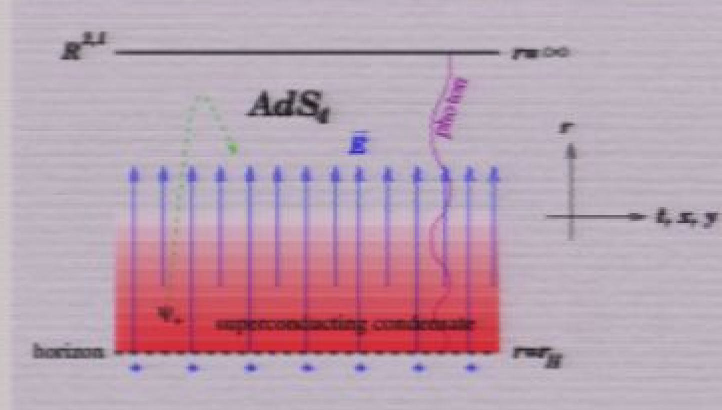

Gubser, Pufu '08

- Hair is stabilized by the equilibrium of electric and gravitational force in AdS space.

Solutions in the broken phase

- We numerically solve the Einstein-Yang-Mills equations for the ansatz

$$\mathrm{d}s^2 = -N(r)\sigma(r)^2\mathrm{d}t^2 + \frac{1}{N(r)}\mathrm{d}r^2$$

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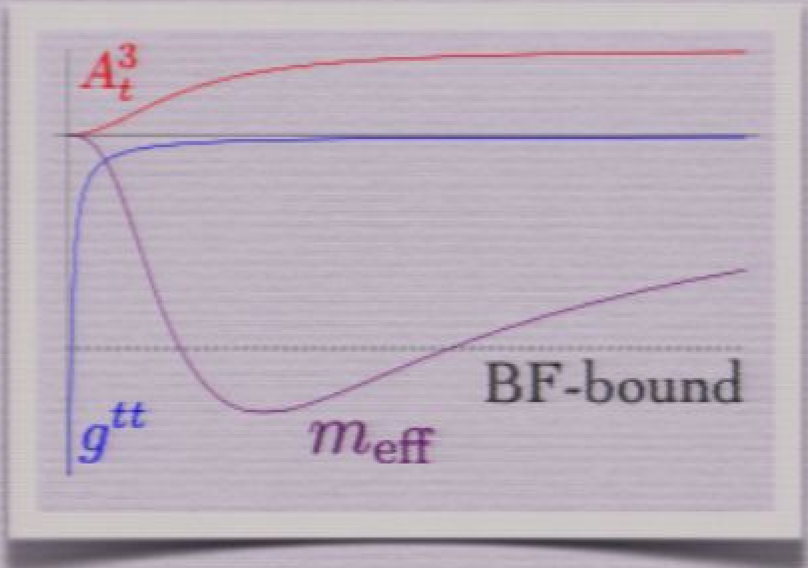

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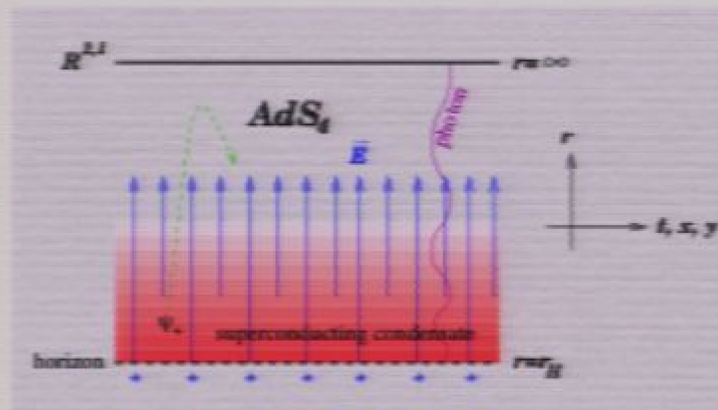

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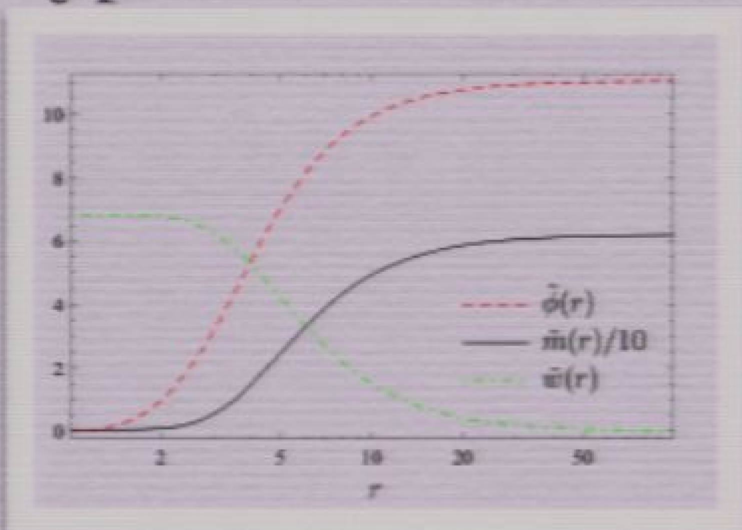

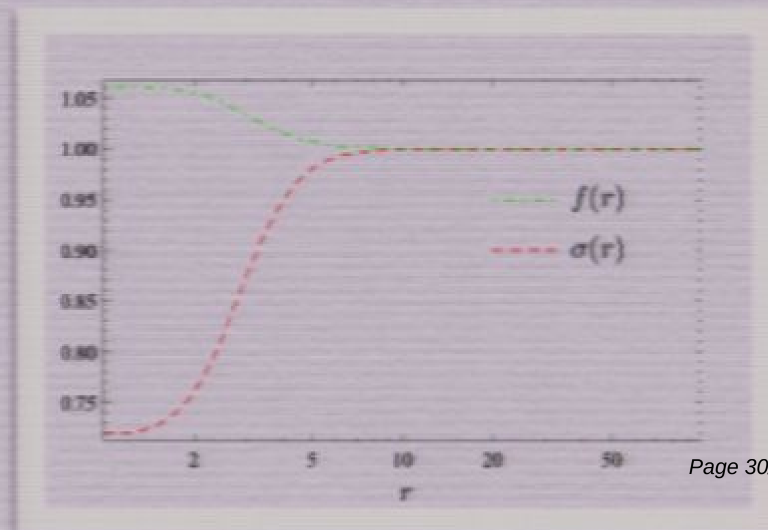

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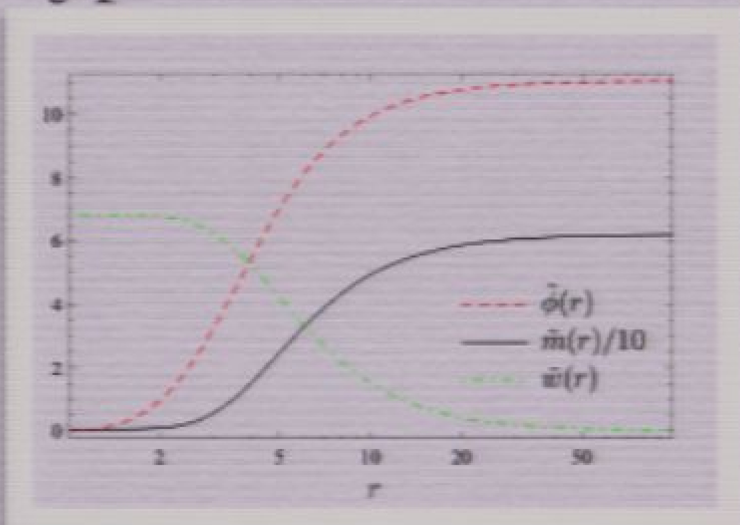
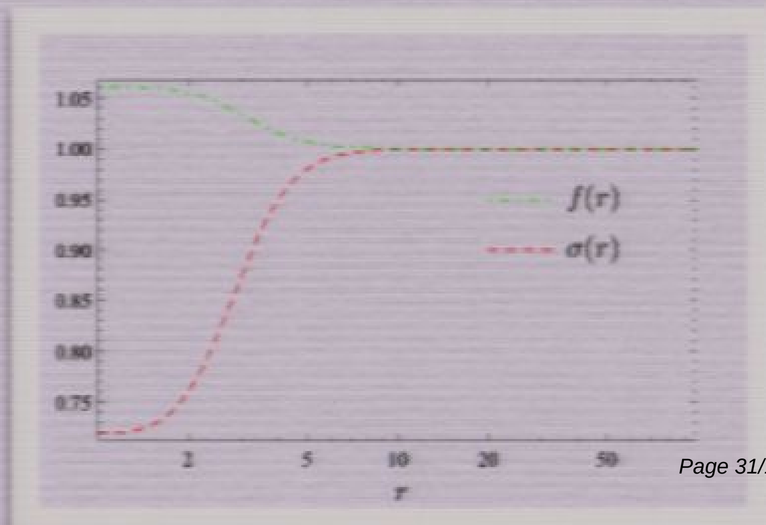
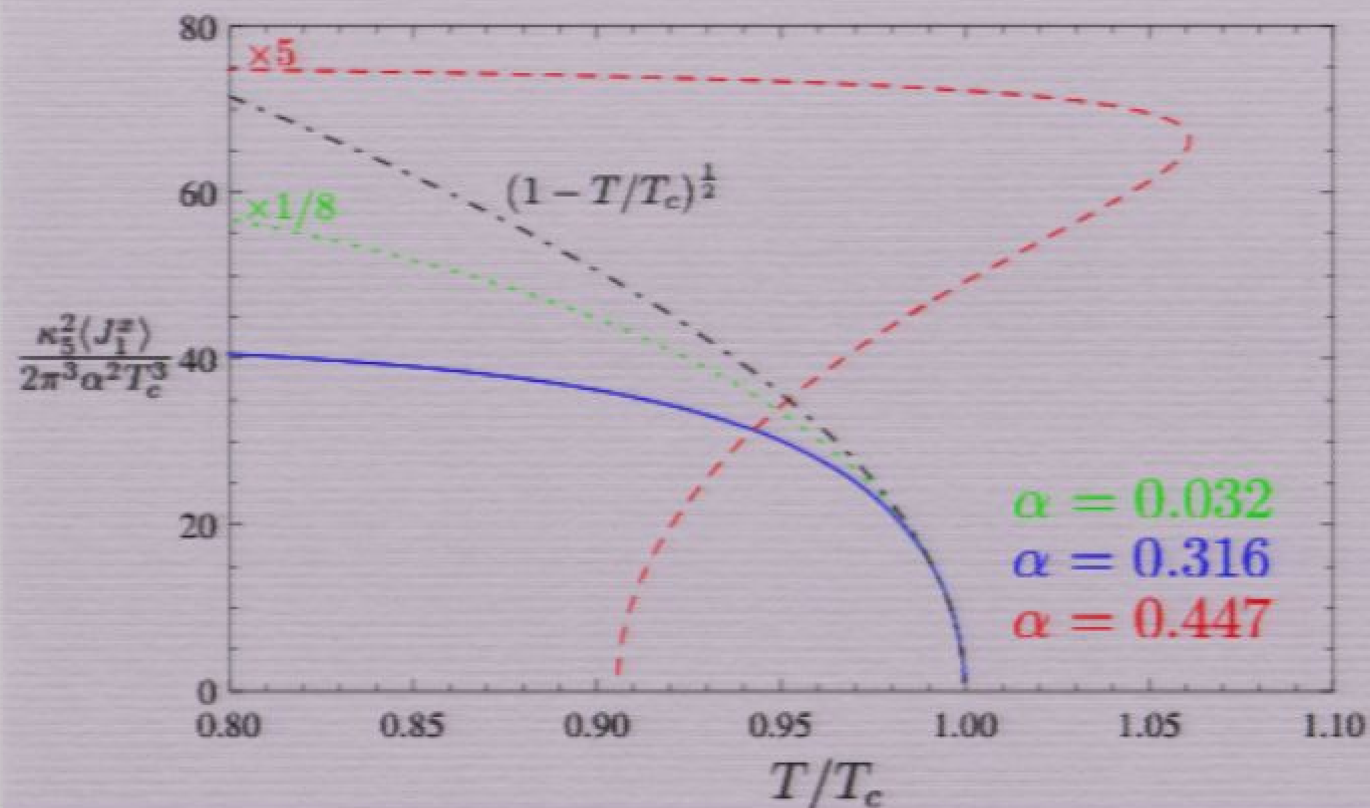




Phase transition

- Order parameter $\langle J_1^x \rangle$ determined by boundary behavior of A_x^1.



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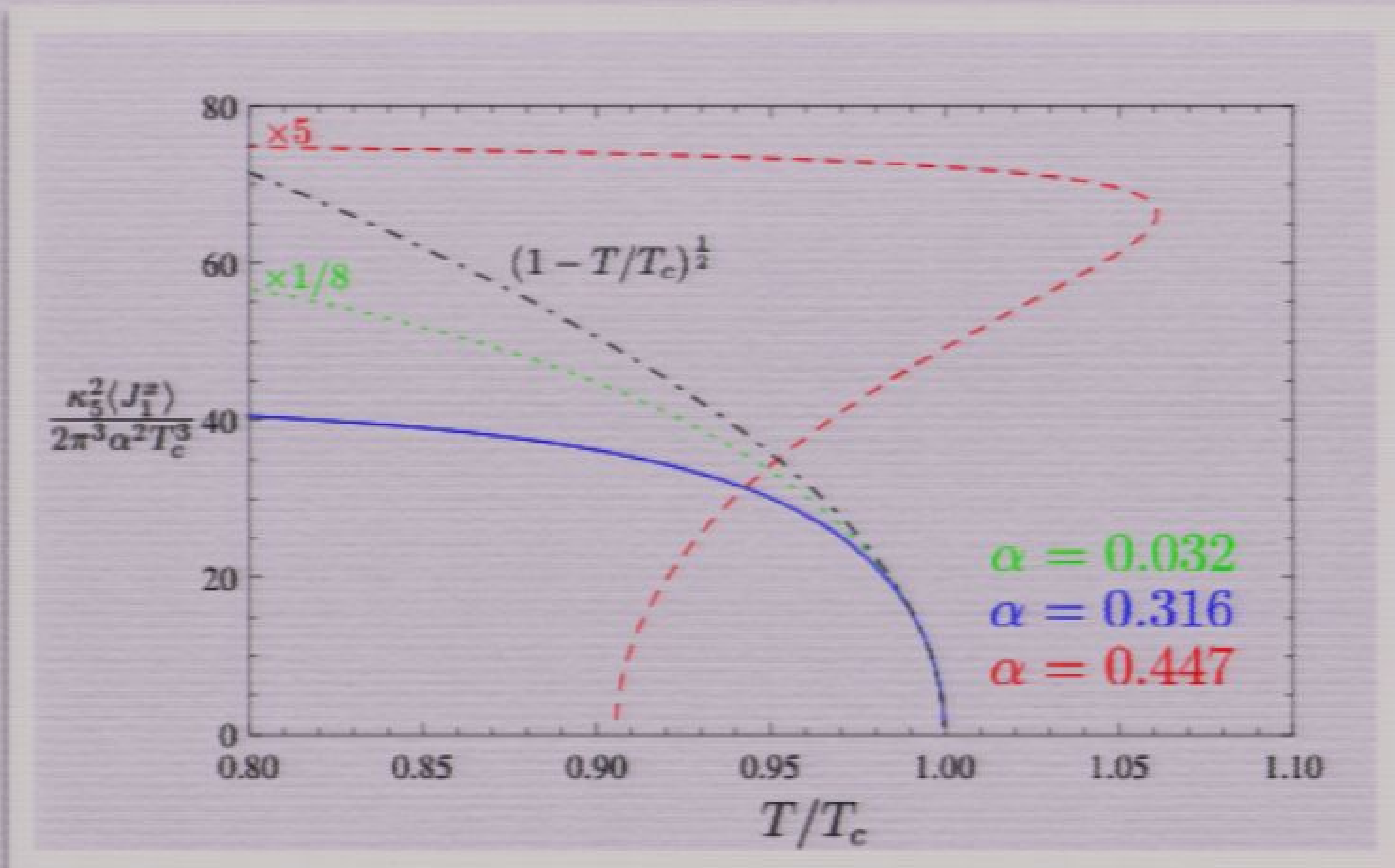
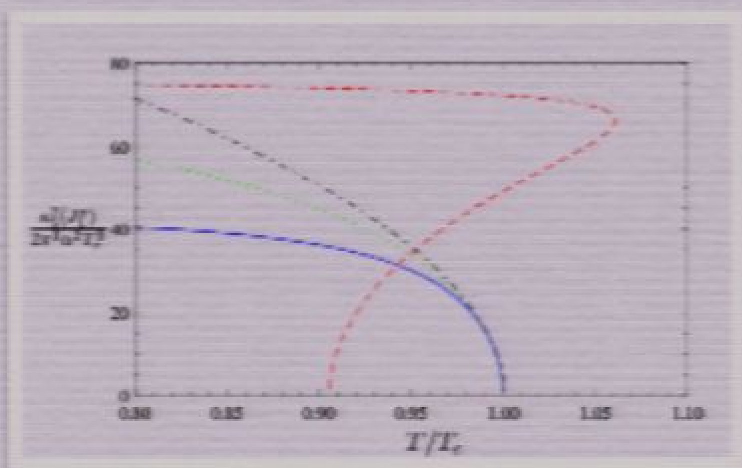


Phase transition

- Order parameter $\langle J_1^x \rangle$ determined by boundary behavior of A_x^1.
- For $\alpha < \alpha_c$ order parameter increases monotonically ⇒ 2nd order transition (mean field)
- For large $\alpha > \alpha_c$ order parameter becomes multivalued ⇒ 1st order transition



Thermodynamics

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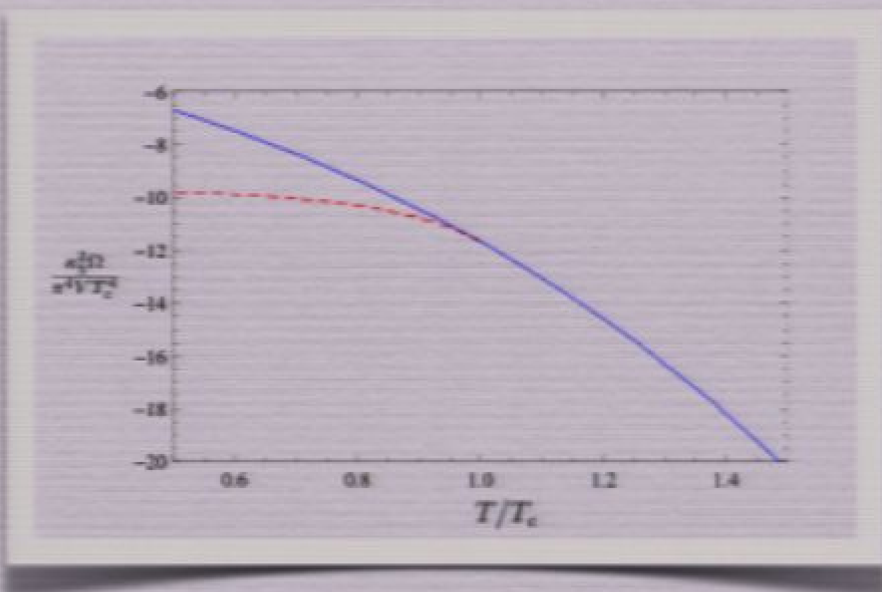

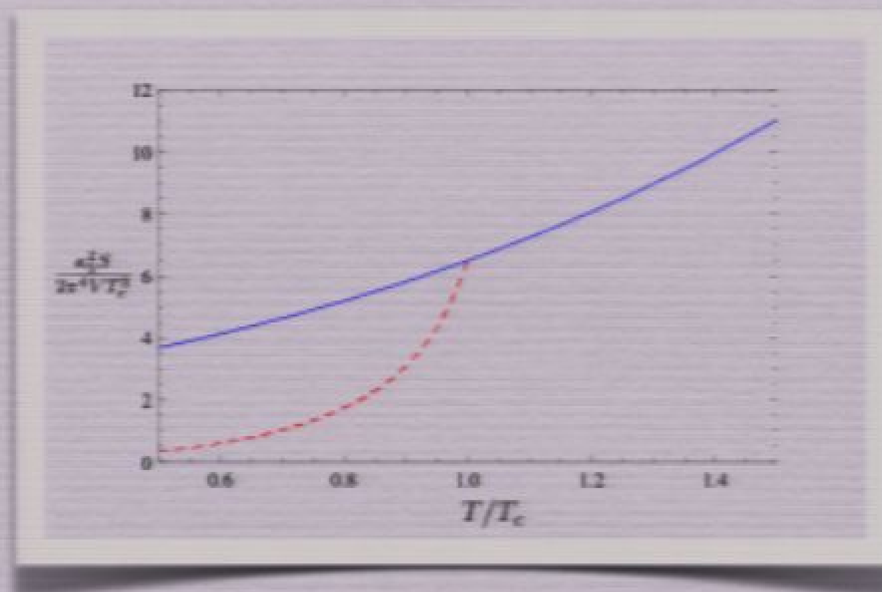

Grand potential

Entropy

normal phase

superfluid phase

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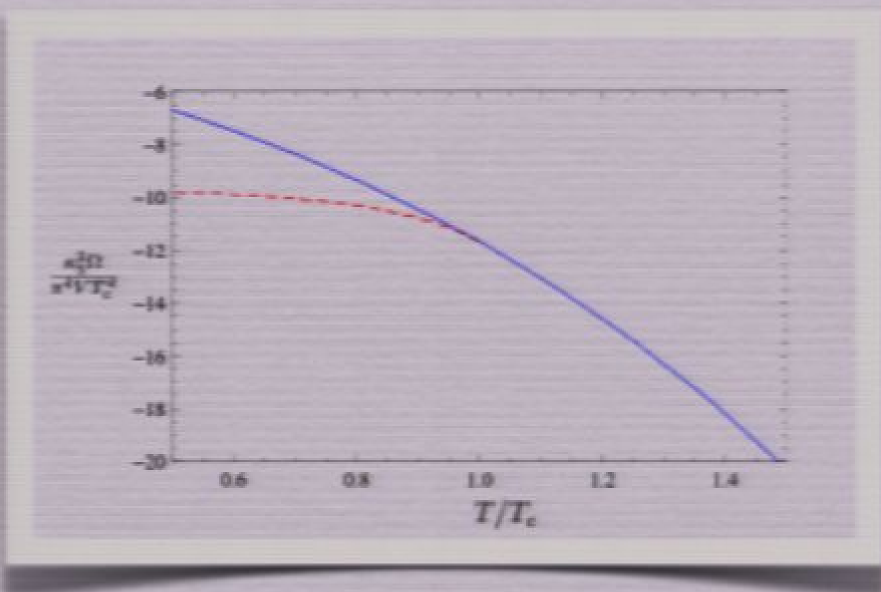

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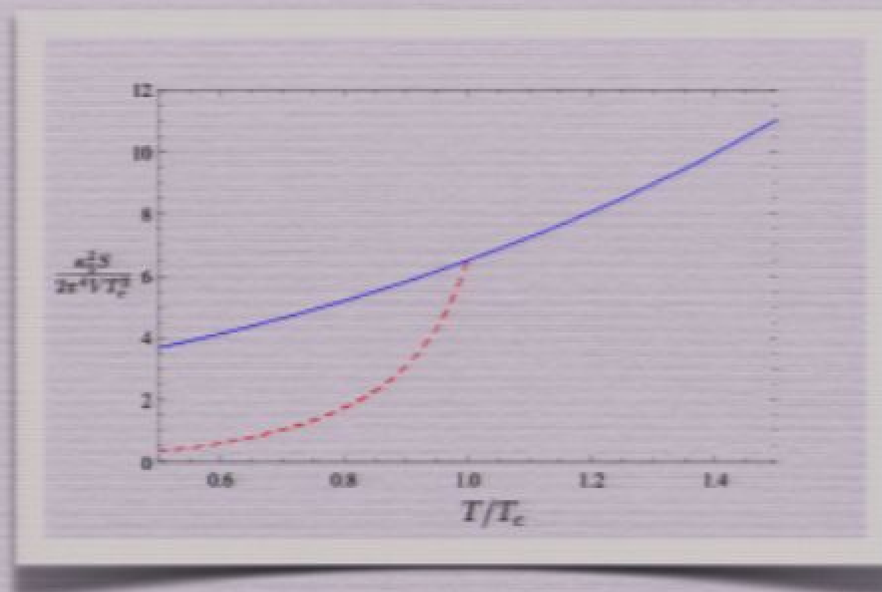

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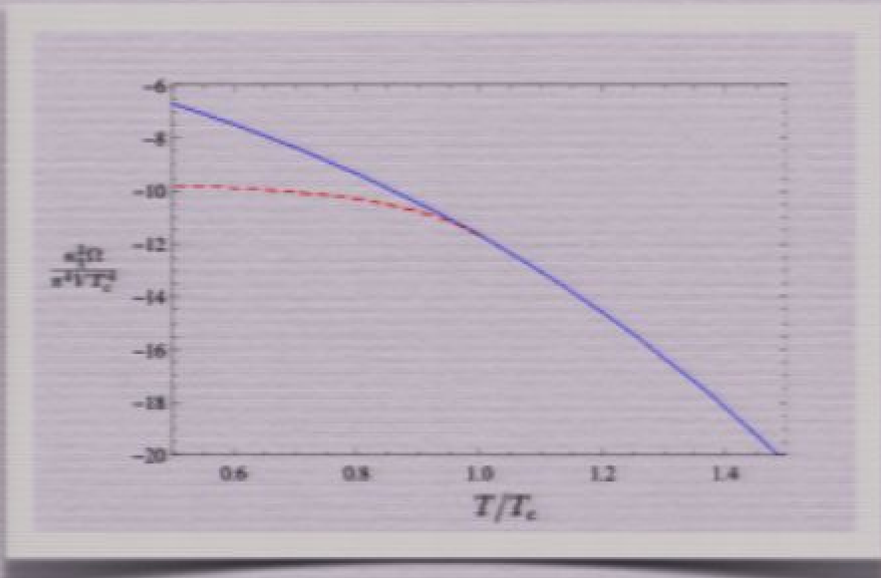

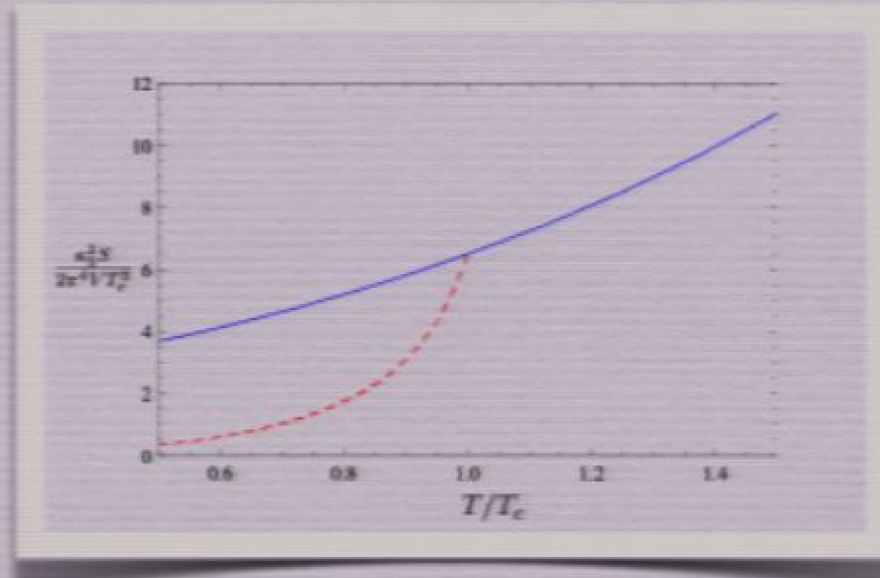

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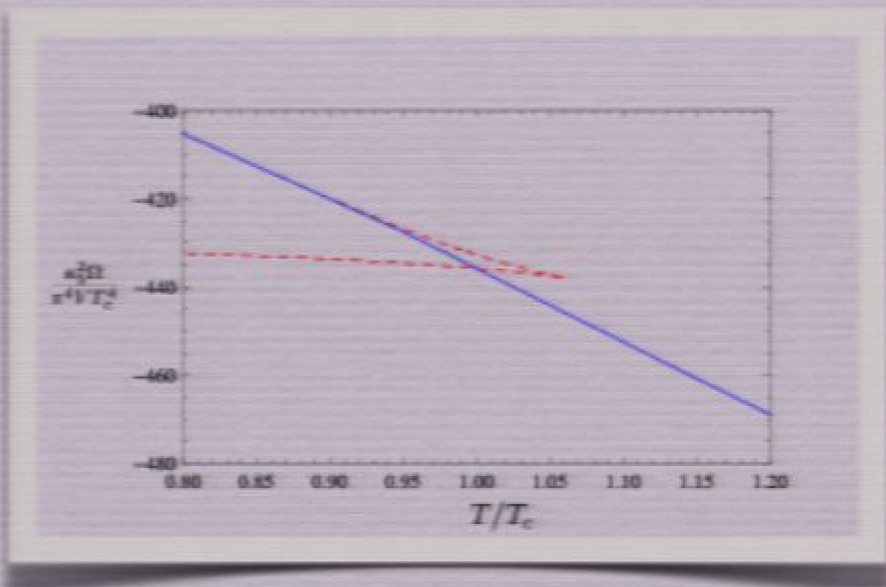

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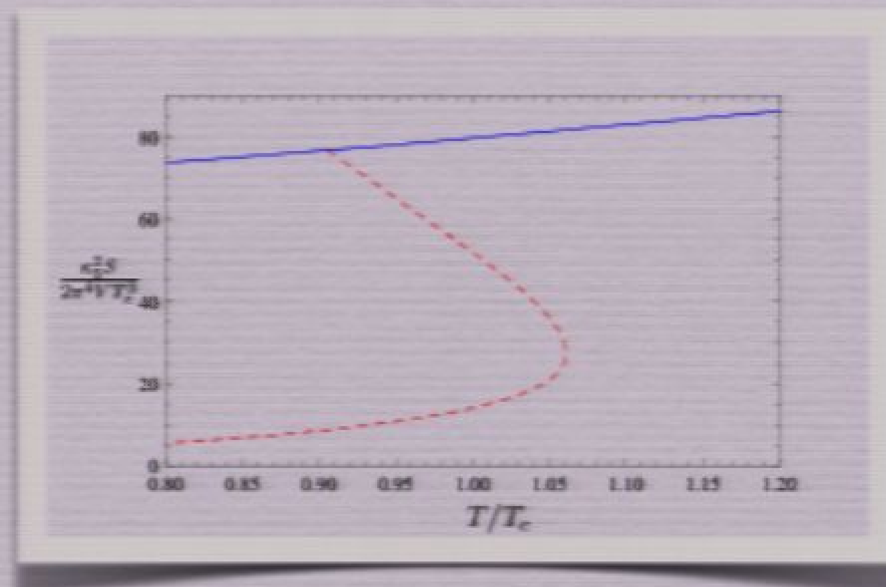

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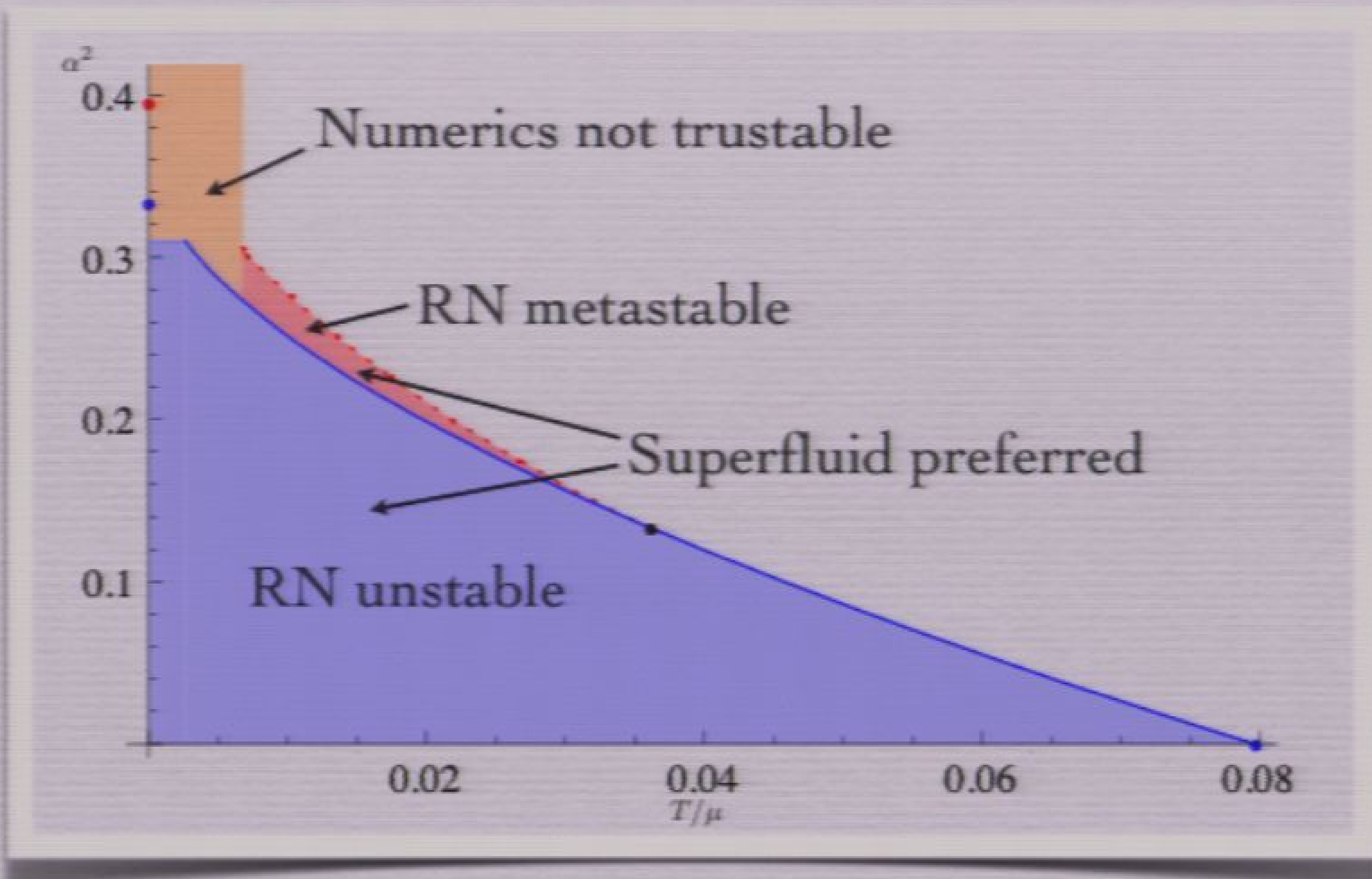
"Phase diagram"



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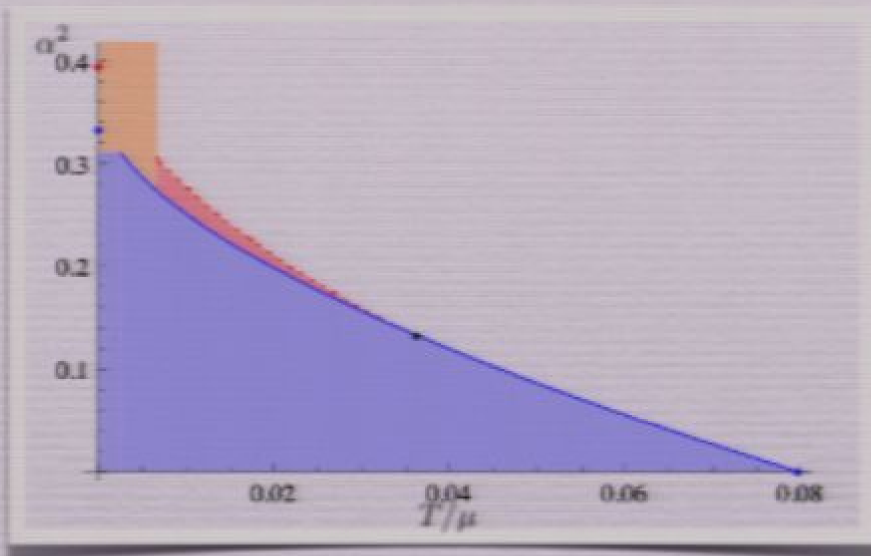

- Superfluid thermodynamically preferred in red and blue region
- Phase transition second order for $\alpha < \alpha_c$ and first order for $\alpha > \alpha_c$ with $\alpha_c = 0.365$
- For $\alpha > 0.628$ no superfluid phase available

Embedding in String Theory

A User's Guide to the Universe

- Gravity model can be embedded into D3/D7 brane setup
- D3/D7 brane setup dual to $\mathcal{N} = 4\ SU(N_c)$ SYM coupled to $\mathcal{N} = 2$ hypermultiplets
- Coincident D7-branes provide a non-Abelian gauge field $\Rightarrow$ Global flavor symmetry

[Ammon, Erdmenger, Kaminski, P.K.]

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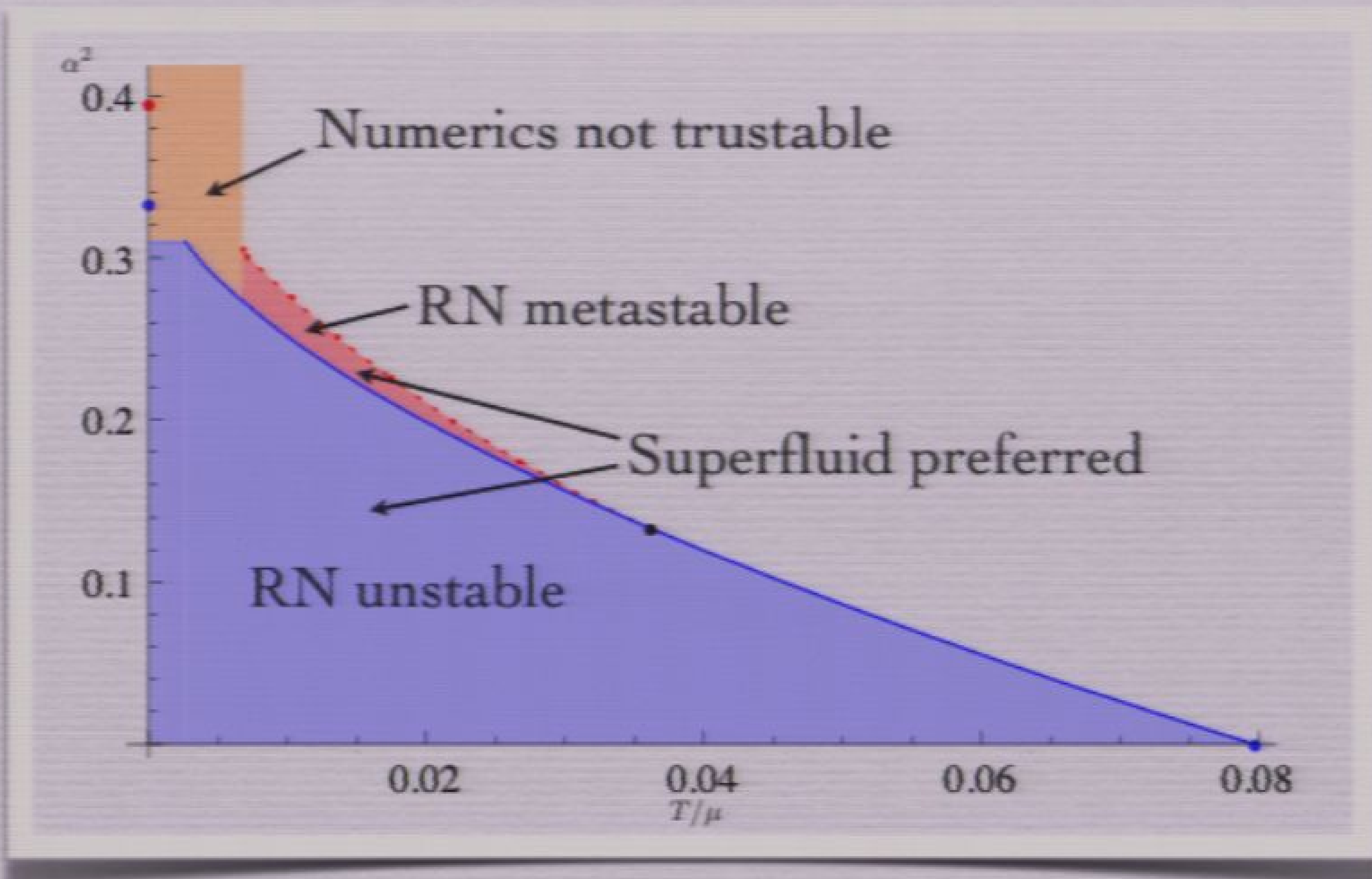

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[Ammon, Erdmenger, Kaminski, P.K.]

Embedding in String Theory non-Abelian DBI action

- Need non-Abelian DBI action (best guess, correct up to $(\alpha')^4$)

$$S_{\mathrm{DBI}} = T_{D7}\,\mathrm{Str}\int \mathrm{d}^8\xi \sqrt{\det Q}\sqrt{\det\left(P_{ab}\left[E_{MN} + E_{Mi}\left(Q^{-1}-\delta\right)^{ij}E_{jN}\right] + 2\pi\alpha' F_{ab}\right)}$$

$$E_{MN} = g_{MN} + B_{MN} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha'\left[\Phi^i,\Phi^k\right]E_{kj} \qquad \begin{array}{l} M,N = 0,\ldots,9 \\ a,b = 0,\ldots,7 \\ i,j = 8,9 \end{array}$$

Embedding in String Theory
non-Abelian DBI action

- Need n… Abelian DBI action (best guess, correct up to (…

???

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- Use symmetries of our setup Erdmenger, Kamiski, PK, Rust 0807.2663

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Looks like Abelian action

String Theory Embedding
Evaluation of Str

- Str prescription only correct up to fourth order
- We use 2 different approaches:

1) expand action to fourth order
+: include maximal number of terms we can trust
-: approximation breaks down in superfluid phase

2) adapt Str prescription: set $\left(\tau^{i}\right)^{2}=\mathbf{1}$ inside Str
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give the same physics!!!

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Comparison to Gravity Model

- D7-branes are probes in D3-brane background. Background determined by Type IIB SUGRA

$$S_{\mathrm{IIB}} \supset N_c^2 \int \mathrm{d}^5 x \sqrt{-g} R$$

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- Action is similar to EYM action $\alpha \propto \sqrt{N_f/N_c} \overset{\text{here}}{=} 0$ with back-reaction dilaton have to considered, too

Hydrodynamics

Fantom XP

- Effective Theory for slowly varying perturbations about the equilibrium $\omega l_{\mathrm{mfp}} \ll 1 \,, q l_{\mathrm{mfp}} \ll 1$
- EOMs: conservation laws for conserved quantities

$$\nabla_\mu T^{\mu\nu} = 0 \,, \quad \nabla_\mu J^\mu = 0$$

- Constitutive equations:
  Dependence on dynamical fields, e.g. velocity u^μ
- Defines transport coefficients, e.g. viscosity
  Determinable by microscopic theory

Fluctuations about the black hole with vector hair

- Consider fluctuations in metric and gauge field

$$\hat{g}_{\mu\nu}(t,x,y,r) = g_{\mu\nu}(r) + h_{\mu\nu}(r)\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}q_x x+\mathrm{i}q_y y}$$

$$\hat{A}^a_\mu(t,x,y,r) = A^a_\mu(r) + a^a_\mu(r)\mathrm{e}^{-\mathrm{i}\omega t+\mathrm{i}q_x x+\mathrm{i}q_y y}$$

- Gauge fixing

$$\hat{g}_{\mu r} = \frac{\delta_{\mu r}}{N(r)} \quad \Rightarrow \quad h_{\mu r} = 0$$

$$\hat{A}^a_r = 0 \quad \Rightarrow \quad a^a_r = 0$$

Classification of Fluctuations

- Finite transverse momentum $q_y \neq 0$
  $\Rightarrow$Rotational symmetry broken down to $\mathbb{Z}_2$
  $\Rightarrow$ all fluctuations couple 10+12 dynamical and 5+3 constraints $\Rightarrow$14 physical modes

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  Helicity 1: $h_{ty},\ h_{xy},\ h_{yr};\ a^a_y$ — 2x4 physical
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total: 14 physical

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Helicity 2 (review)

Kovtun, Son, Starinets; Buchel, Liu; Liu, Iqbal,...

- Effective action for $\phi = h_z^y$ (minimal coupled scalar)

$$S = -\frac{1}{2\kappa^2}\int \mathrm{d}^5x\sqrt{-g}\,\frac{1}{2}(\nabla\phi)^2$$

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So far true for all two derivative theories!!!

Helicity 1 at zero momentum

- The 4 physical modes decouple into two blocks (1+3)

1st block:

$$h'_{ty} = 2h_{ty}\left(\frac{1}{r} + \frac{f'}{f}\right) - 2\alpha^2\phi' a_y^3$$

$$a_y^{3\prime\prime} + a_y^{3\prime}\left(\frac{1}{r} - \frac{2f'}{f} + \frac{N'}{N} + \frac{\sigma'}{\sigma}\right) + a_y^3\left(-\frac{f^4 w^2}{r^2 N} + \frac{\omega^2}{N^2\sigma^2} - \frac{2\alpha^2\phi'^2}{N\sigma^2}\right) = 0$$

2nd block:

$$h''_{xy} = -\frac{2i\alpha^2\omega a_y^2 w\phi}{N^2\sigma^2} + \frac{2\alpha^2 a_y^1 w\phi^2}{N^2\sigma^2} - 2\alpha^2 a_y^{1\prime} w' + h'_{xy}\left(\frac{1}{r} - \frac{2f'}{f} - \frac{N'}{N} - \frac{\sigma'}{\sigma}\right)$$
$$+ h_{xy}\left(-\frac{4}{r^2} + \frac{8}{N} - \frac{\omega^2}{N^2\sigma^2} - \frac{2\alpha^2 f^4 w^2\phi^2}{3r^2N^2\sigma^2} + \frac{4f'}{rf} + \frac{8f'^2}{f^2} + \frac{2\alpha^2 f^4 w'^2}{3r^2} - \frac{2\alpha^2\phi'^2}{3N\sigma^2}\right)$$

$$a_y^{1\prime\prime} = \frac{2i\omega a_y^2\phi}{N^2\sigma^2} + a_y^1\left(-\frac{\omega^2}{N^2\sigma^2} - \frac{\phi^2}{N^2\sigma^2}\right) + \frac{f^4 h'_{xy} w'}{r^2} + h_{xy}\left(-\frac{2f^4 w'}{r^3} - \frac{2f^3 f' w'}{r^2}\right)$$
$$+ a_y^{1\prime}\left(-\frac{1}{r} + \frac{2f'}{f} - \frac{N'}{N} - \frac{\sigma'}{\sigma}\right)$$

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$$a_y^{2''} = -\frac{2i\omega a_y^1\phi}{N^2\sigma^2} + \frac{i\omega f^4 h_{xy} w\phi}{r^2N^2\sigma^2} + a_y^2\left(\frac{f^4 \cdots}{r^2N} \cdots \frac{}{f} - \frac{N'}{N} - \frac{\sigma'}{\sigma}\right)$$

conductivity

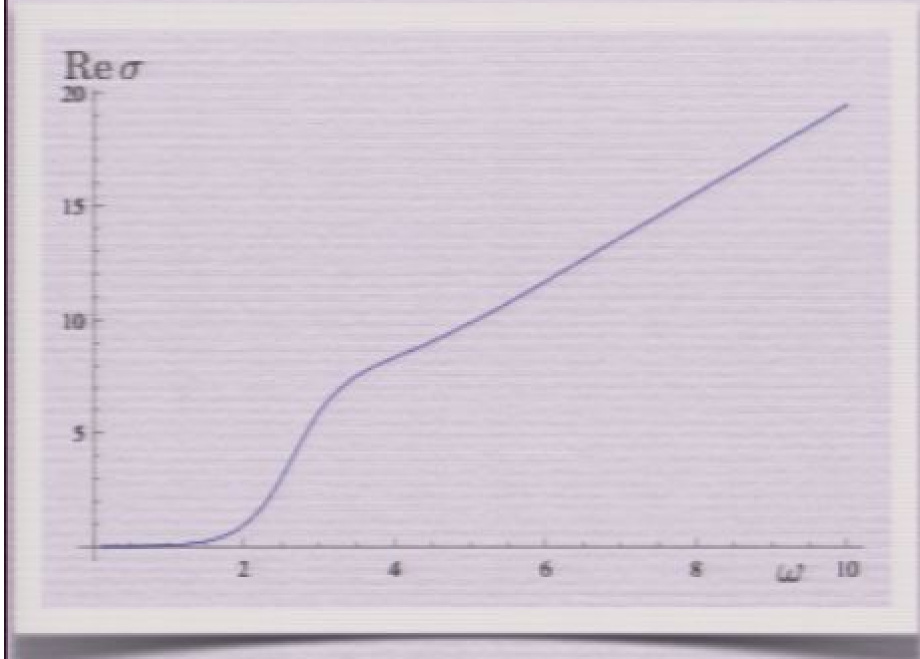
- 1st block $\Rightarrow G^R = \begin{pmatrix} \langle J_3^y J_3^y\rangle & \langle J_3^y T_{ty}\rangle \\ \langle T_{ty} J_3^y\rangle & \langle T_{ty} T_{ty}\rangle \end{pmatrix}$

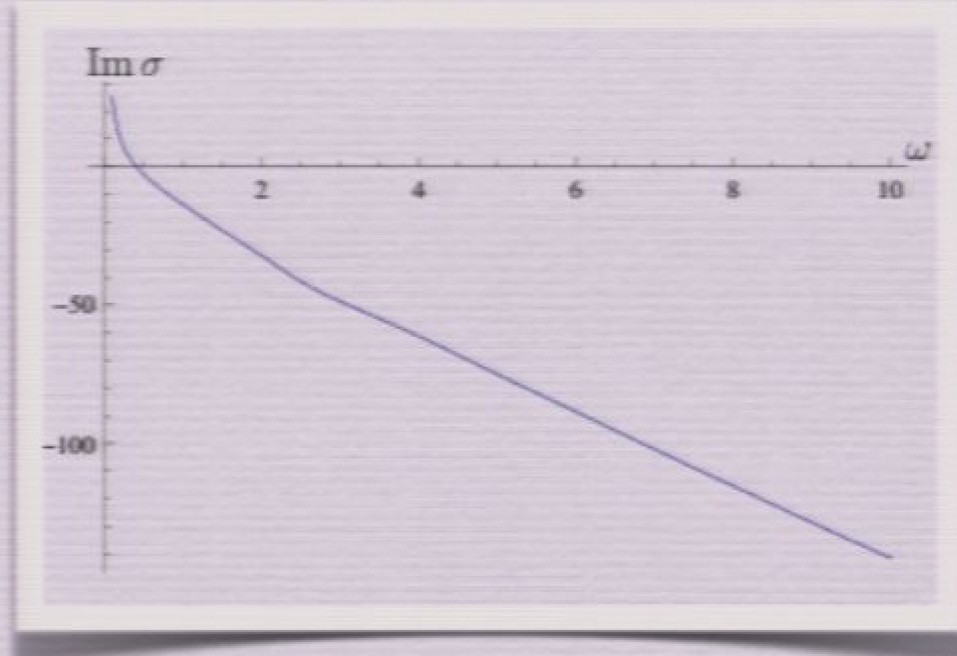
shear viscosity

2nd block $\Rightarrow G^R = \begin{pmatrix} \langle J_1^y J_1^y\rangle & [illegible] & \langle J_1^y T_{xy}\rangle \\ \langle J_2^y J_1^y\rangle & \langle J_2^y J_2^y\rangle & \langle J_2^y T_{xy}\rangle \\ \langle T_{xy} J_1^y\rangle & \langle T_{xy} J_2^y\rangle & \langle T_{xy} T_{xy}\rangle \end{pmatrix}$

Electric Conductivity

- Kubo relation: $\sigma(\omega) = \frac{\mathrm{i}}{\omega}\langle J_3^y J_3^y \rangle$





- Kramers-Kronig relation: $\mathrm{Im}\,\sigma(\omega) = -\frac{1}{\pi}\mathcal{P}\int_{-\infty}^{\infty}\mathrm{d}\omega' \, \frac{\mathrm{Re}\,\sigma(\omega')}{\omega' - \omega}$

Viscosity in general fluids

- Viscosity refers to dissipation due to internal motion

$$T^{ij}_{\text{diss}} = -\eta^{ijkl}\left(\partial_k u_l + \partial_l u_k\right)$$ [Landau, Lifshitz]

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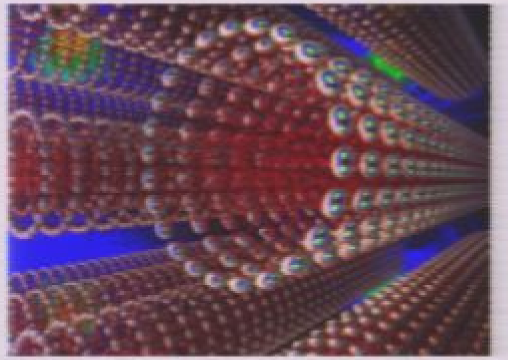
$$\Rightarrow T^{\mu\nu}_{\text{diss}} = -\eta\sigma^{\mu\nu} - \xi\theta P^{\mu\nu}$$

Viscosity in transversely isotropic fluids



ISIS Facility

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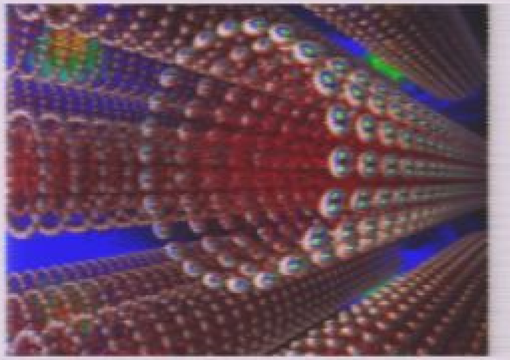

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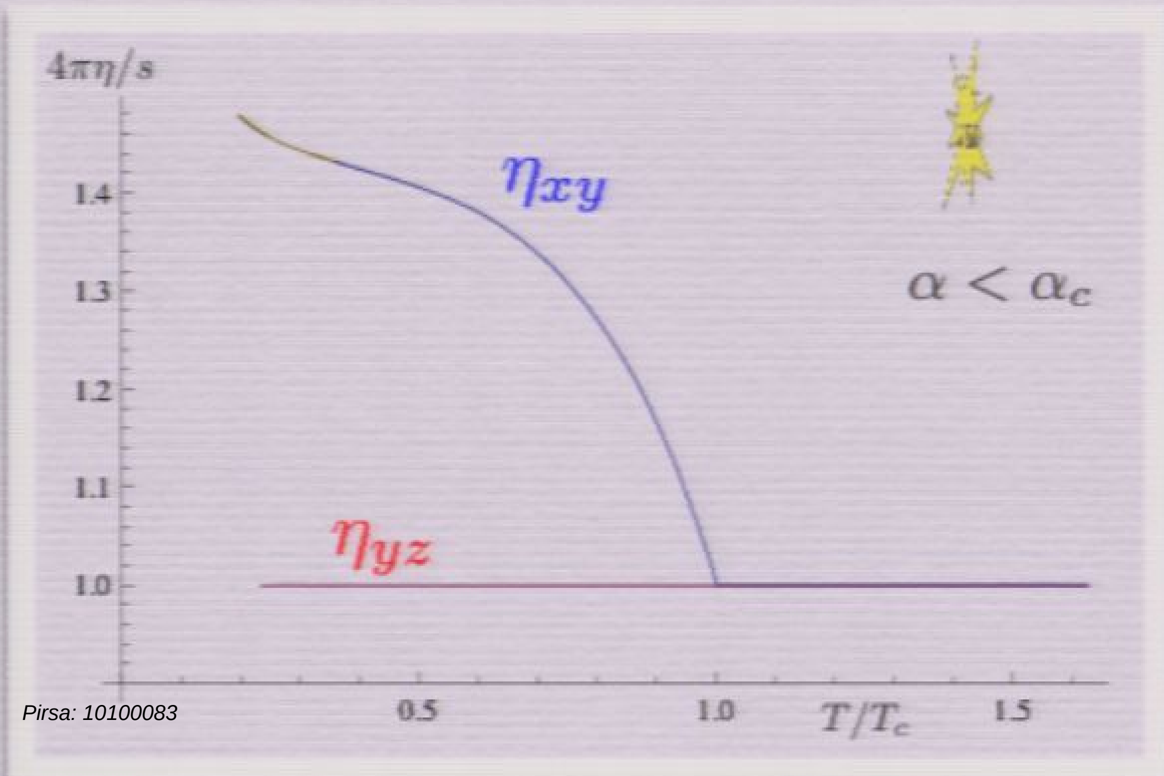

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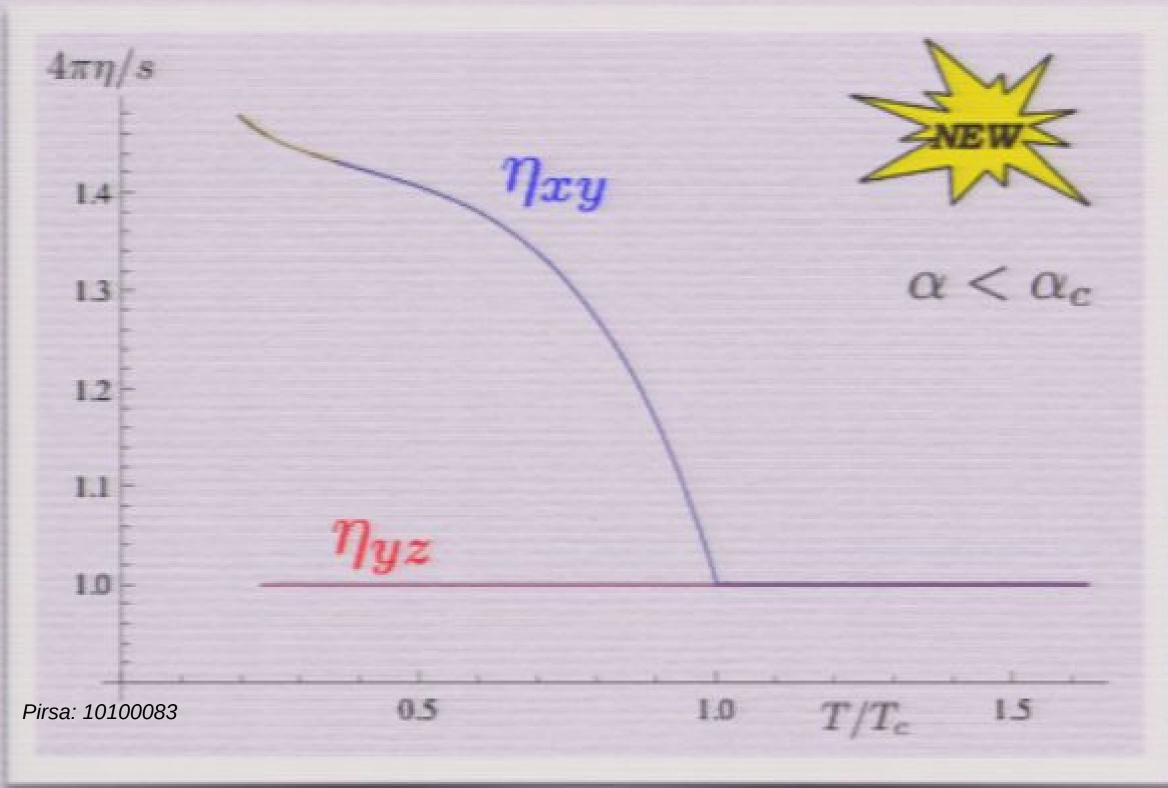

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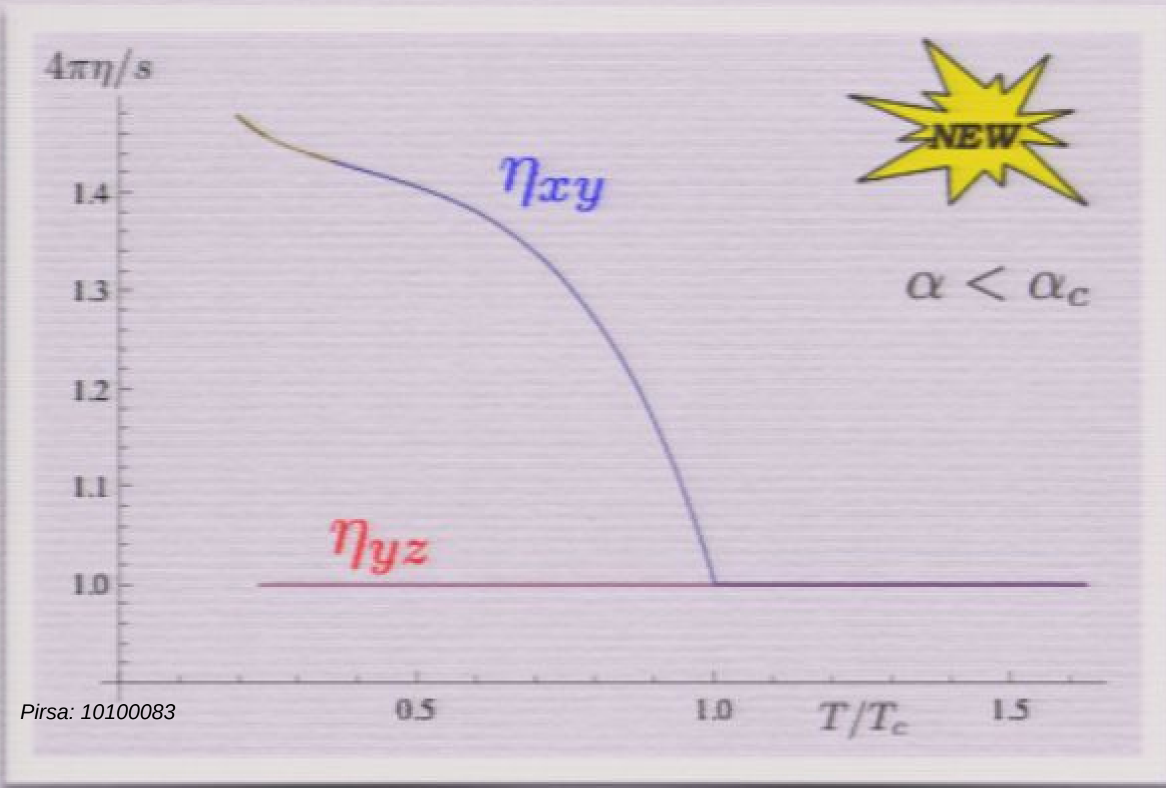

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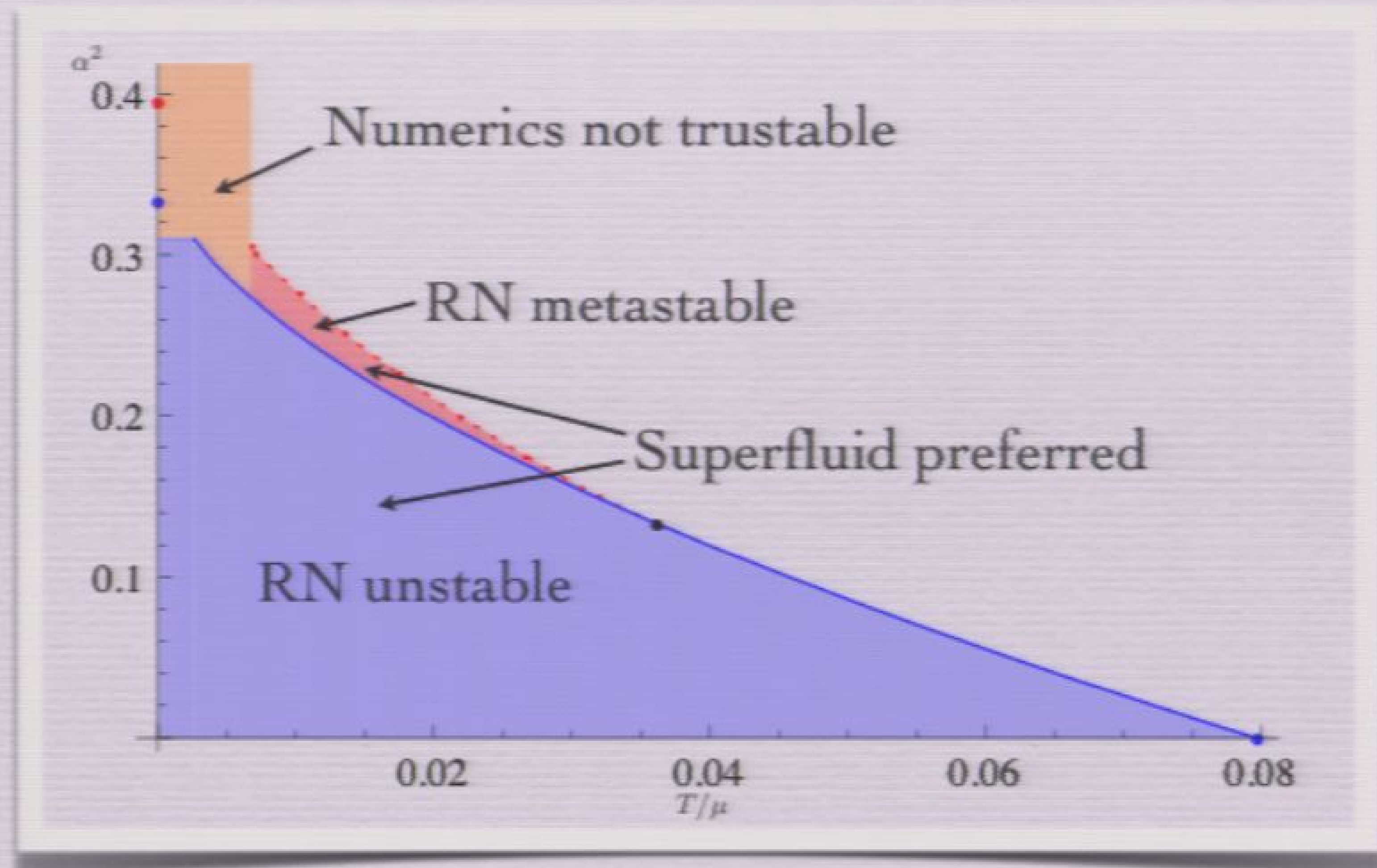
Embedding in String Theory Comparison to Gravity Model

- D7-branes are probes in D3-brane background. Background determined by Type IIB SUGRA

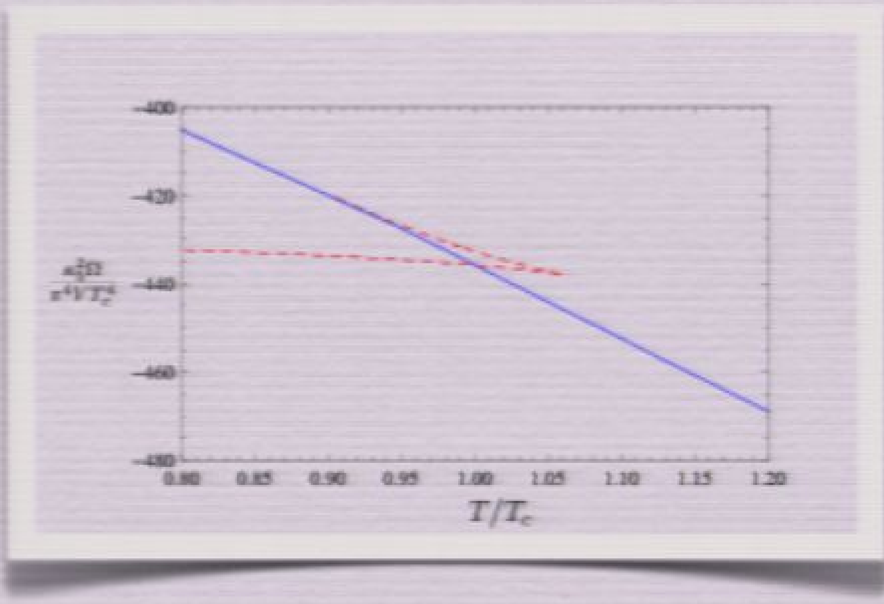
$$S_{\mathrm{IIB}} \supset N_c^2 \int \mathrm{d}^5x\sqrt{-g}R$$

“Phase diagram”

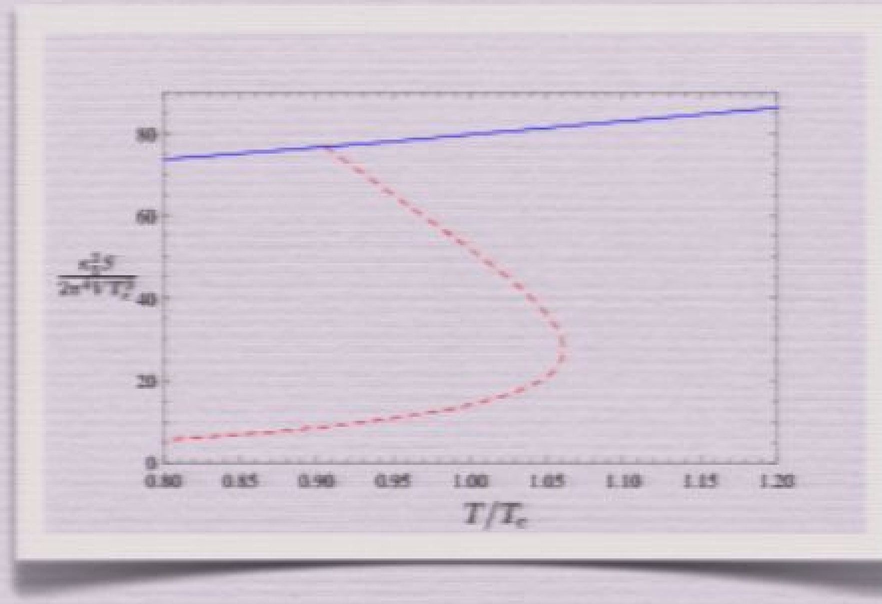


Thermodynamics

$$\alpha > \alpha_c$$



Grand potential



Entropy

normal phase

superfluid phase

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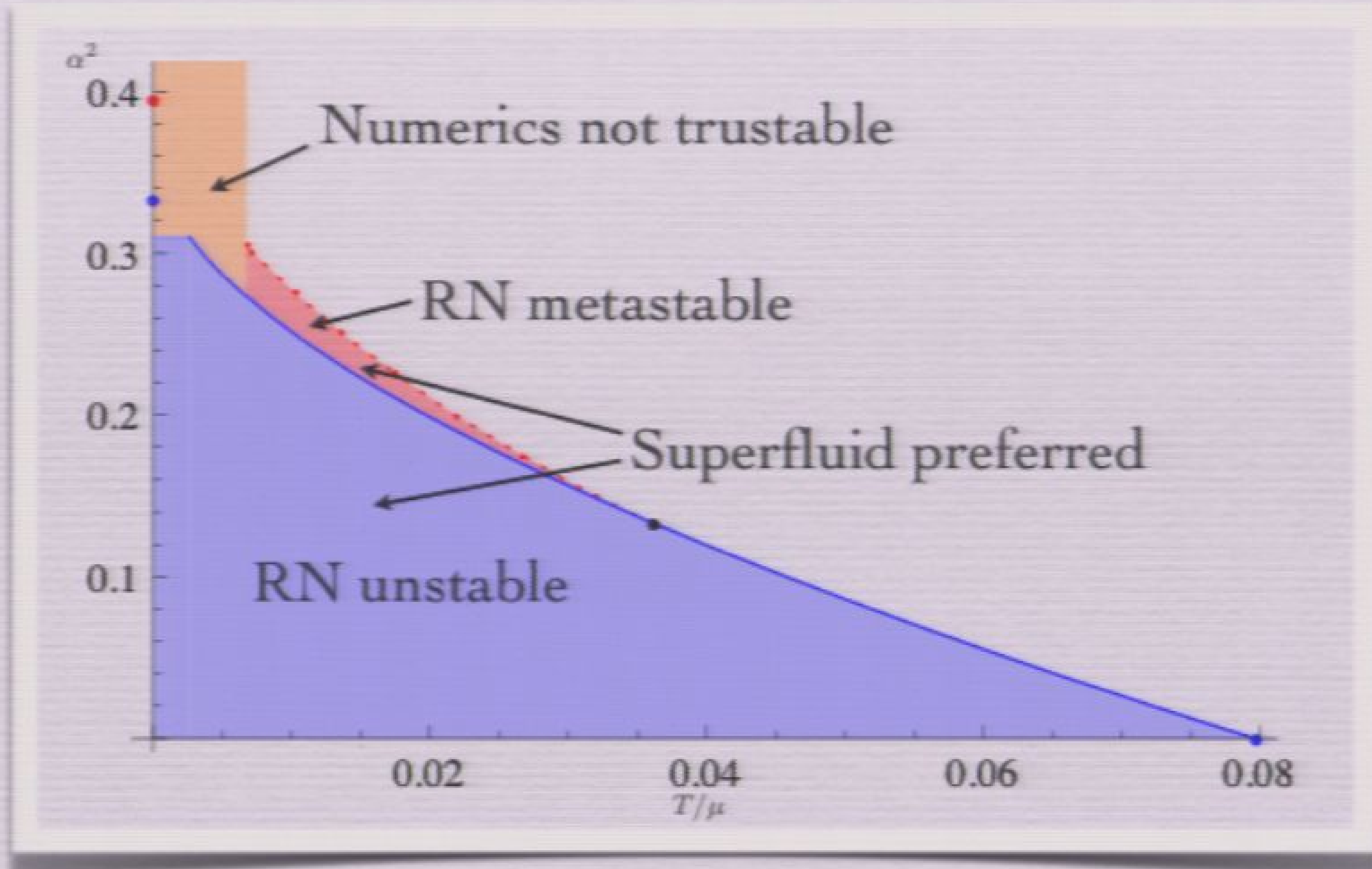

Embedding in String Theory non-Abelian DBI action

- Need n ??? Abelian DBI action (best guess, correct up to (

$$S_{\mathrm{DBI}} = T_D \, \mathrm{Str} \int \mathrm{d}^8\xi \sqrt{\det Q} \sqrt{\det \left(P_{ab} \left[E_{MN} + E_{Mi} \left(Q^{-1} - \delta \right)^{ij} E_{jN} \right] + 2\pi\alpha' F_{ab} \right)}$$

$$E_{MN} = g_{MN} + \cancel{B_{MN}} \qquad Q^i_j = \delta^i_j + \mathrm{i}2\pi\alpha' \left[\Phi^i, \Phi^k \right] E_{kj}$$

$M, N = 0, \ldots, 9$
$a, b = 0, \ldots, 7$
$i, j = 8, 9$

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Classification of Fluctuations

- Finite transverse momentum $q_y \neq 0$
  $\Rightarrow$Rotational symmetry broken down to $\mathbb{Z}_2$
  $\Rightarrow$ all fluctuations couple 10+12 dynamical and 5+3 constraints $\Rightarrow$ 14 physical modes

Complicated! No results yet!

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- Finite longitudinal momentum $q_x \neq 0$ or no
  momentum ⇒ S… rotational symmetry left

  trivial!

  new!

  Helicity 2: $h_{yz},\ h_{yy} - h_{zz}$ 2 physical
  Helicity 1: $h_{ty},\ h_{xy},\ h_{yr};\ a^a_y$ 2x4 physical
  Helicity 0: $h_{tt},\ h_{tx},\ h_{xx},\ h_{yy} + h_{zz},\ h_{tr},\ h_{xr},\ h_{rr};$
  $a^a_t,\ a^a_x,\ a^a_r$ 4 physical

total: 14 physical

Helicity 2 (review)

Kovtun, Son, Starinets;
Buchel, Liu; Liu, Iqbal,...

- Effective action for $\phi = h^y_z$ (minimal coupled scalar)

$$S = -\frac{1}{2\kappa^2}\int \mathrm{d}^5x\sqrt{-g}\,\frac{1}{2}(\nabla\phi)^2$$

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- Viscosity is given by

$$\eta = \lim_{\omega\to 0}\lim_{r\to\infty}\frac{\Pi}{\mathrm{i}\omega\phi} \qquad \Pi = -\frac{1}{2\kappa^2}\sqrt{-g}\,g^{rr}\partial_r\phi$$

- For $\omega \ll 1$ EoMs trivial

$$\partial_r\Pi = 0 + \mathcal{O}(\omega^2\phi) \quad \partial_r(\omega\phi) = 0 + \mathcal{O}(\omega\Pi)$$

- Ingoing boundary condition at horizon

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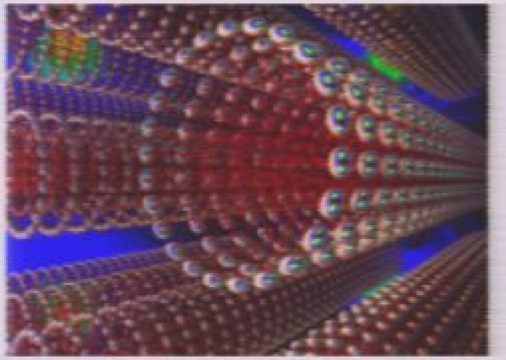

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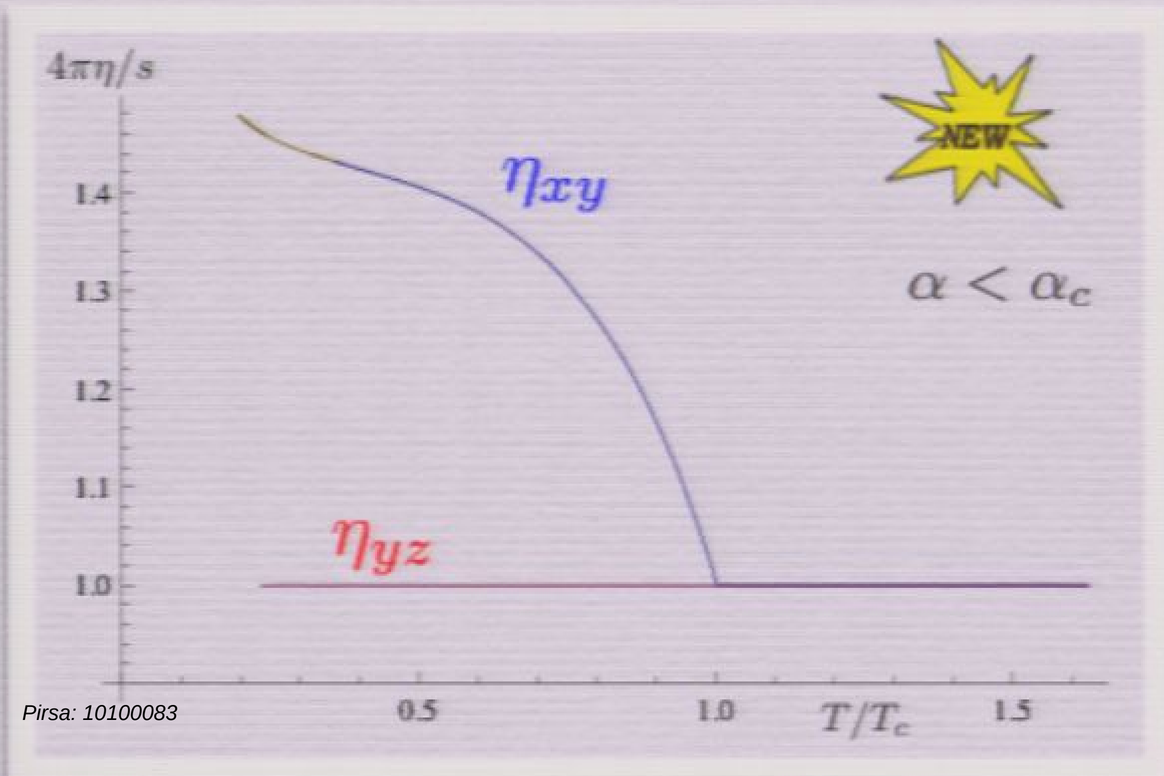

SUMMARY

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- Order of transition depends on number of charged degrees of freedom
- String Theory Embedding D3/D7 brane setup
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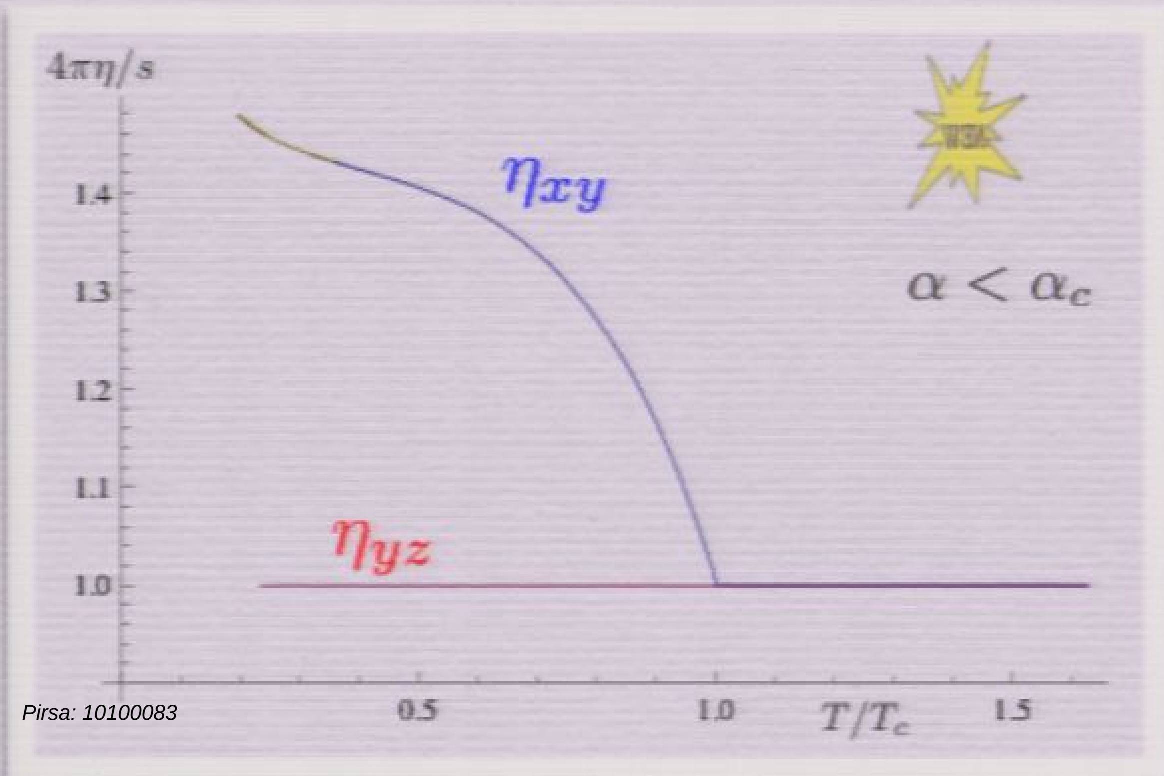

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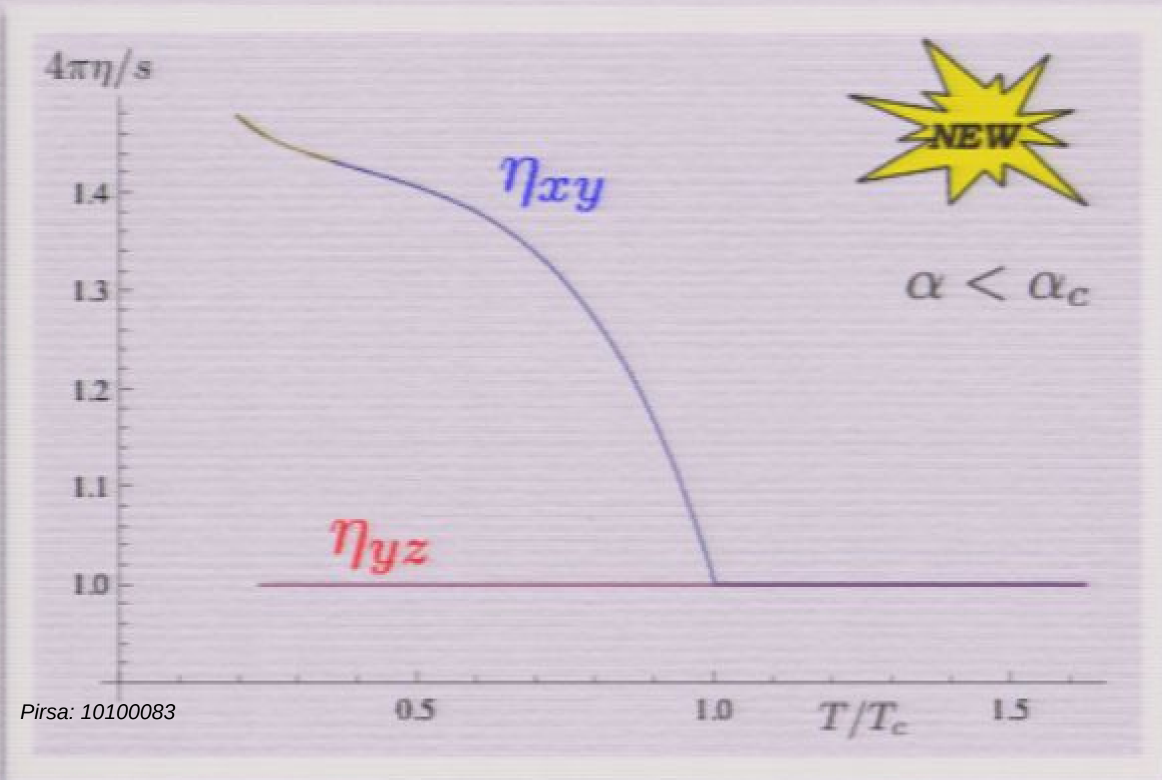

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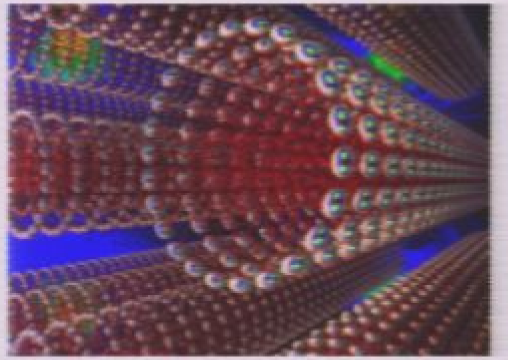

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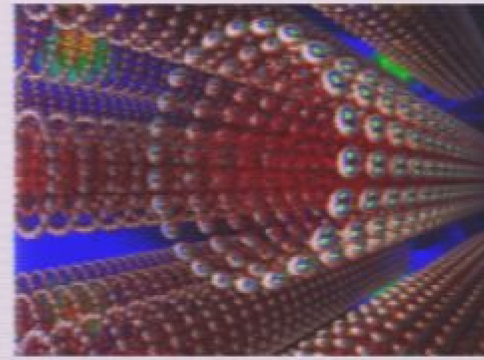

ISIS Facility

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$$\eta_{xxxx} = \xi_x - 2\lambda ,$$

$$\eta_{yyyy} = \eta_{zzzz} = \xi_y - \frac{\lambda}{2} + \eta_{yz} ,$$

$$\eta_{xxyy} = \eta_{xxzz} = \lambda ,$$

$$\eta_{yyzz} = \xi_y - \frac{\lambda}{2} - \eta_{yz} ,$$

$$\eta_{yzyz} = \eta_{yz} ,$$

$$\eta_{xyxy} = \eta_{xzxz} = \eta_{xy} ,$$

$$T^{\text{diss}}_{xy} = -\eta_{xy}(\partial_x u_y + \partial_y u_x)$$
$$T^{\text{diss}}_{xz} = -\eta_{xy}(\partial_x u_z + \partial_z u_x)$$
$$T^{\text{diss}}_{yz} = -\eta_{yz}(\partial_y u_z + \partial_z u_y)$$

Shear Viscosity from Gravity

- Kubo relations:

$$\eta_{xy} = -\lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\,\langle T_{xy}T_{xy}\rangle$$

$$\eta_{yz} = -\lim_{\omega\to 0}\frac{1}{\omega}\text{Im}\,\langle T_{yz}T_{yz}\rangle$$

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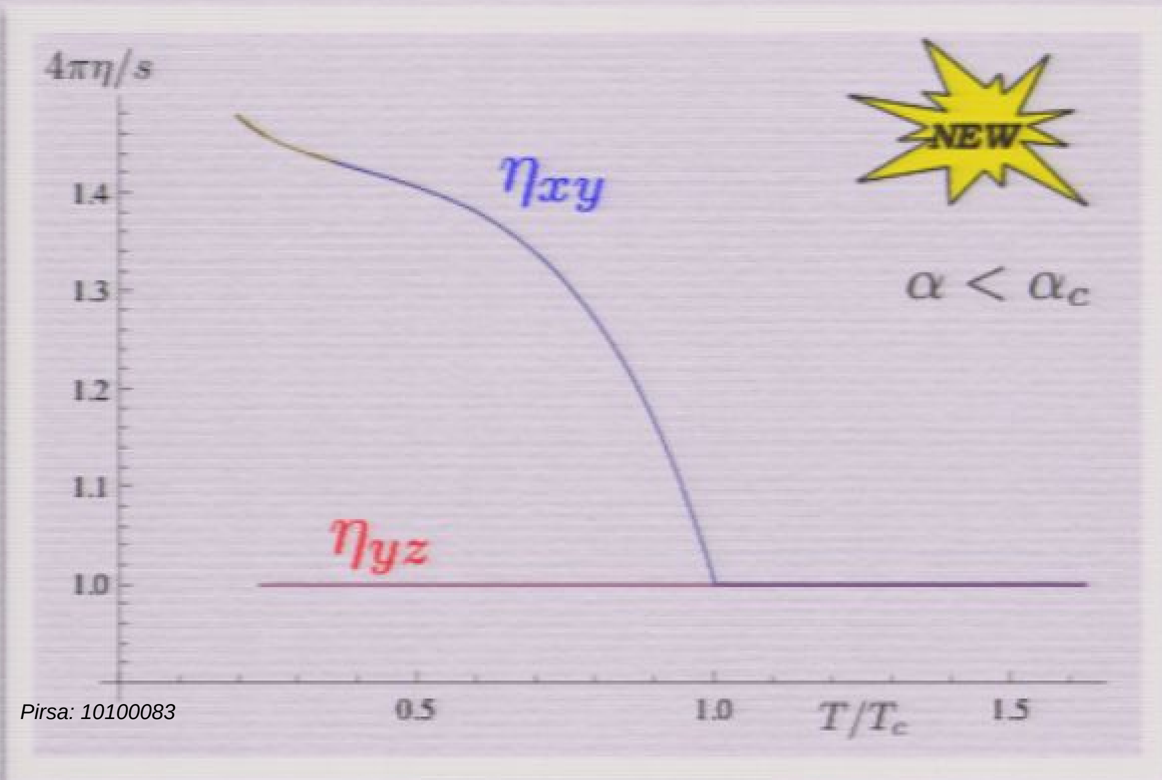

S U M M A R Y

- Realization of holographic p-wave superfluids by black holes with vector hair
- Order of transition depends on number of charged degrees of freedom
- String Theory Embedding D3/D7 brane setup
- Non-universal behavior of shear viscosity NEW

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OUTLOOK

- May shed light on non-conventual superconductors due to QCP
- More transport coefficients to analyze

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Thank you!