

Title: Effects of decoherence on IR divergence

Date: Oct 30, 2010 11:30 AM

URL: <http://pirsa.org/10100082>

Abstract: We propose one way to regularize the fluctuations generated during inflation. We show that, as long as we consider the case that the non-linear interaction acts for a finite duration, observable fluctuations are free from IR divergences not only in the single field models but also in the multi field model. In contrast to the single field model, to discuss observables, we need to take into account the effects of quantum decoherence which pick up a unique history of the universe from various possibilities contained in initial quantum state set naturally in the early stage of the universe.

0904.2415 with Yuko, Urakawa

$$\phi^I$$

$$\phi^0 \text{ is inflat}$$
$$= 0$$

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ϕ^I

ϕ^0 is flat

$\phi^1 = 0$ is curvature

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ϕ^I

ϕ^0 : inflat

$\phi^1 = 0$: isocurvature

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ϕ^I

ϕ^0 : inflat

$\phi^1 = 0$

isocurvature

$\sigma^2 \gtrsim \int_{k_{min}}^{k_{out}}$

$k_{out} = \epsilon_{QH}$

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ϕ^I

ϕ^0 : inflat

$\phi^1 = \sigma$ isocurvature

$$\langle \sigma^2 \rangle \approx \left. \begin{array}{l} k_{\text{cut}} \\ k_{\text{min}} \end{array} \right\}$$

$$k_{\text{cut}} = \epsilon_{\text{QH}}$$

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ϕ^I

ϕ^0 is inflaton

$\phi^1 = 0$

curvature

$$\langle \sigma^2 \rangle \approx \int_{k_{\min}}^{k_{\max}} d^d k$$

large ∞

$k_{\min} \rightarrow 0$
or
 $t \rightarrow \infty$

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ϕ^2

ϕ^0 : inflat

$\phi^1 = 0$ isocurvature

$$\langle \sigma^2 \rangle \approx \int_{k_{\min}}^{k_{\max}} d^3k P_0(k)$$

$$k_{\max} = \epsilon_{\text{QH}}$$

large
 ∞

$k_{\min} \rightarrow 0$
or
 $t \rightarrow \infty$

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ϕ^I

ϕ^0 : inflat

$\phi^1 = 0$: isocurvature

$$\langle \sigma^2 \rangle \approx \int_{k_{\min}}^{k_{\max}} d^3k P_0(k)$$

$$k_{\max} = \epsilon aH$$

large
 ∞

$k_{\min} \rightarrow 0$
or
 $t \rightarrow \infty$

Stock

$$k_{out} = \epsilon a H$$

view

$$\frac{1}{J} = \frac{1}{\epsilon a H}$$

$$k_{out} = \epsilon \alpha T$$

Stochastic view

$$\bar{J} = \int d^3k$$

$$\langle \sigma \rangle \approx \int_{k_{\min}} d^3k P_0(k)$$

large $k_{\min} \rightarrow 0$
 ∞ or $t \rightarrow \infty$

$$k_{\text{cut}} = \epsilon qH$$

Stochastic view

$$\bar{T} = \int d^3k \Theta(k_{\text{cut}} - k) \mathcal{T}_k e^{i\mathbf{k} \cdot \mathbf{x}}$$





k_{min}



k_{min}

$k_{min} \rightarrow 0$

$k_{min} - \text{depen}$



"True observable"

Shu

k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains



"True observable"

Should not depend on k_{min}

k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains



k_{min}

k_{mi}

ance remains

"True observable"

Should not depend on k_{min}

$$\langle O_{1b} O_{1a} \rangle$$



k_{min}

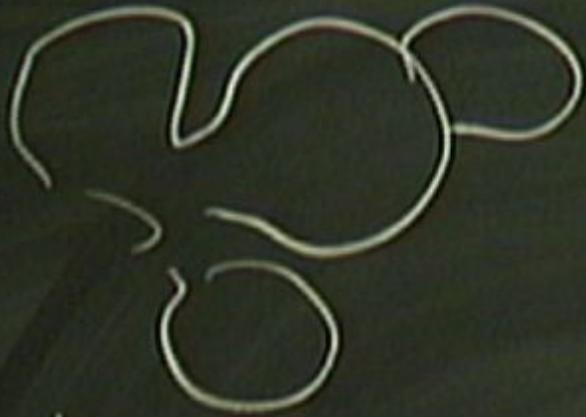
$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$\langle \sigma_{ik} \sigma_{ik} \rangle$



k_{min}

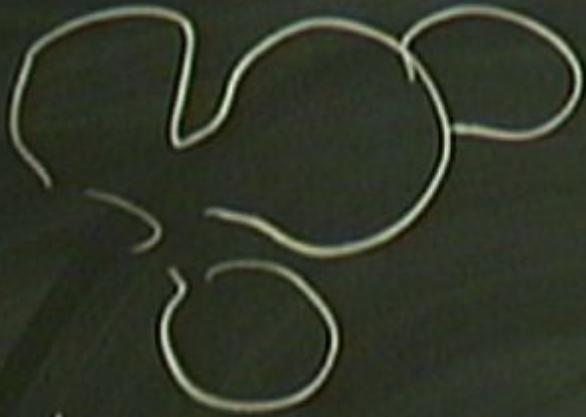
$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$\langle \sigma_{ik} \sigma_{jk} \rangle$



k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$\langle \underline{\sigma}_{lk} \sigma_{lk} \rangle$



k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$\langle \underline{\sigma}_{ik} \sigma_{ik} \rangle$



k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$\langle \underline{\sigma_{ik}} \sigma_{ik} \rangle$



k_{min}

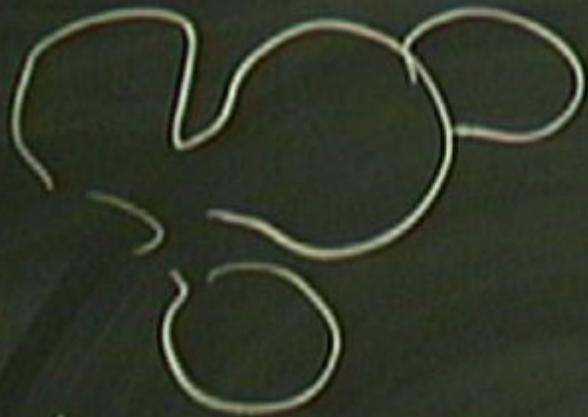
$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$\langle \sigma_{ik} \sigma_{ik} \rangle$



k_{\min}

$k_{\min} \rightarrow 0$

k_{\min} -dependence remains

"True observable"

Should not depend on k_{\min}

$$\langle \underline{\sigma}_{12} \sigma_{12} \rangle$$

$$\langle \sigma(x) \sigma(y) \rangle$$

$$x, y \in \mathbb{Q}$$



k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$$\langle \underline{\sigma}_k \sigma_{-k} \rangle$$

$$\langle \sigma(x) \sigma(y) \rangle$$

$$x, y \in \mathbb{T}$$



k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains

"True observable"

Should not depend on k_{min}

$$\langle \underline{\sigma}_{1k} \sigma_{1k} \rangle$$

$$\langle \sigma(x) \sigma(y) \rangle$$

$x, y \in \mathbb{Z}$
can be divergent



k_{min}

$k_{min} \rightarrow 0$

k_{min} -dependence remains

$$\int = \int (\delta\phi^I)$$

"True observable"

Should not depend on k_{min}

$$\langle \underline{O}_{1k} O_{1k} \rangle$$

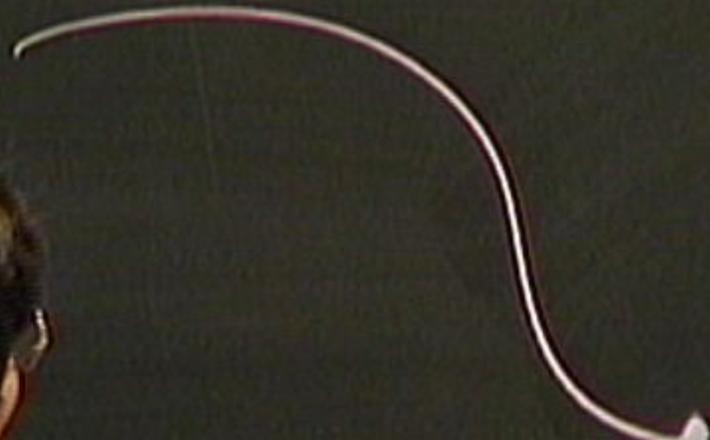
$$\langle \sigma(x) \sigma(y) \rangle$$

$x, y \in \mathbb{D}$
can be divergent

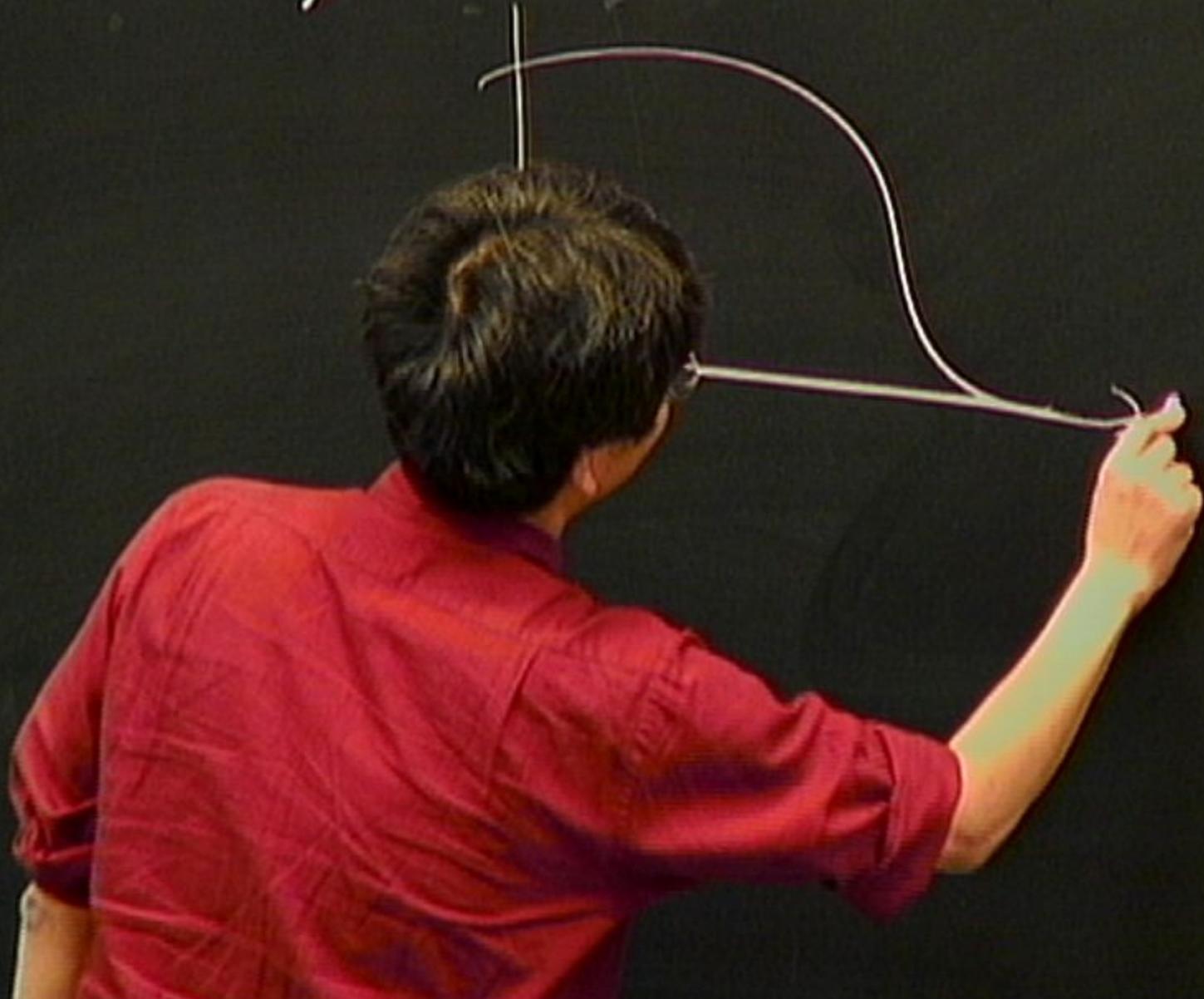
$$\bar{D} = \int d^3x W(x)$$

$$\bar{\sigma} = \int d^3x W(x)$$

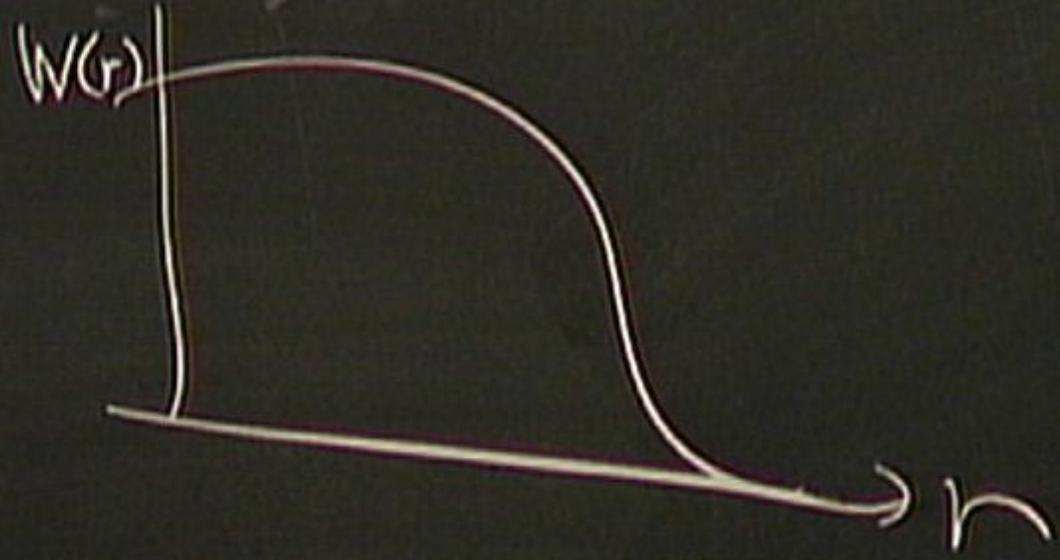
$$\bar{D} = \int d^3x W(x) \sigma(x)$$



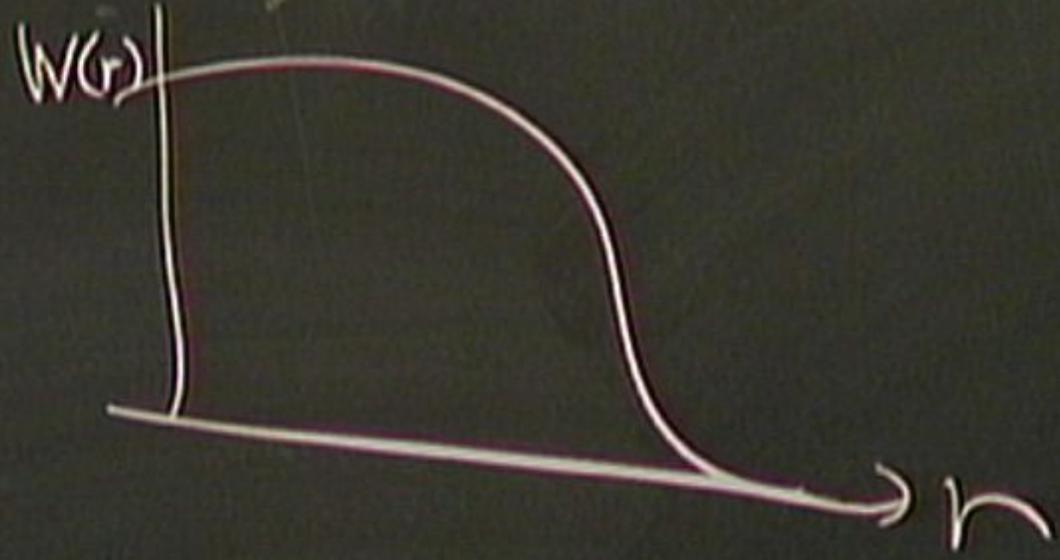
$$\mathbb{D} = \int d^3x W(x) \sigma(x)$$



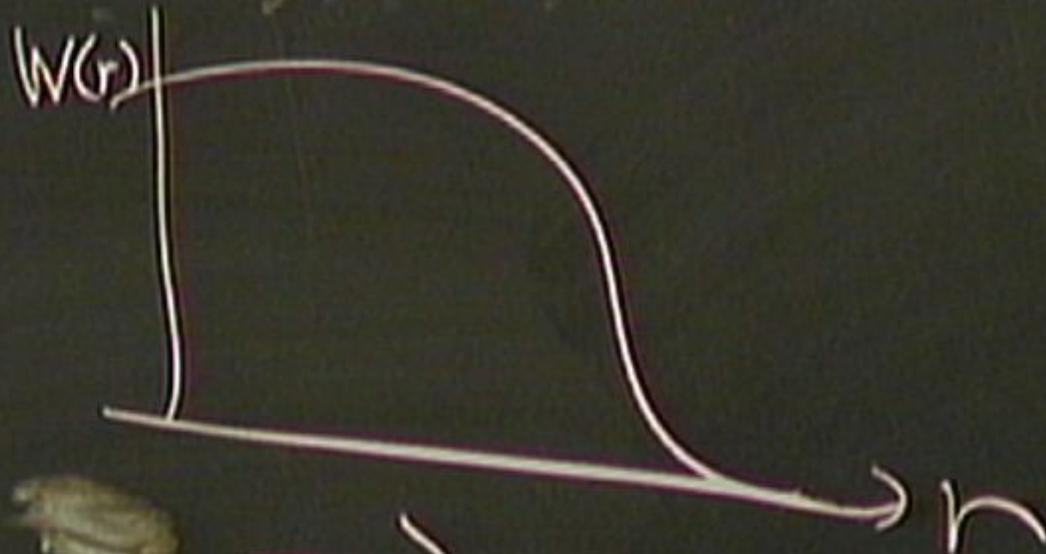
$$D = \int d^3x W(x) \sigma(x)$$



$$D = \int d^3x W(x) \sigma(x)$$



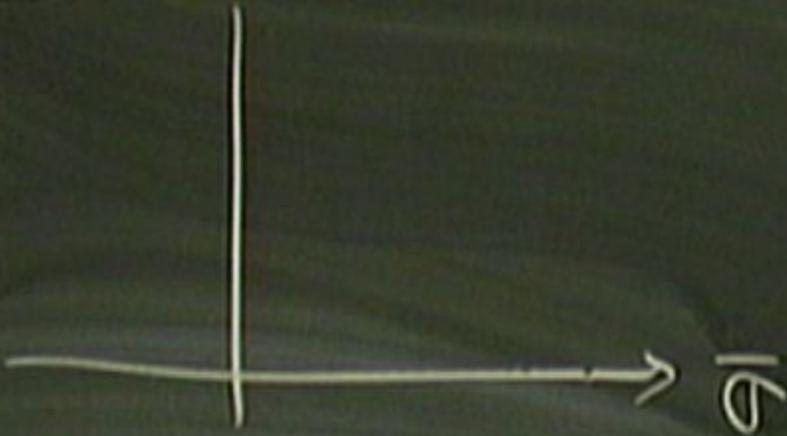
$$D = \int d^3x W(x) \sigma(x)$$



$\left\{ D, \sigma \right\}$
 1 def

Do we really observe large fluctuation of $\bar{\sigma}$

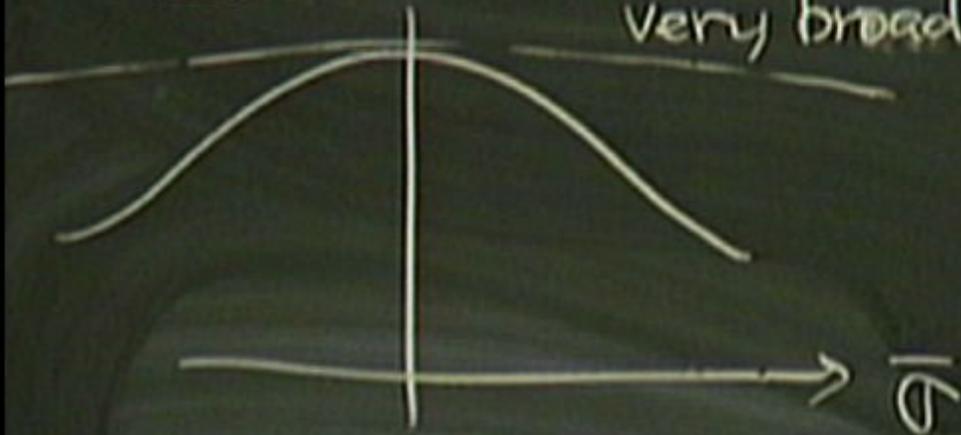
$\Psi(\bar{\sigma})$



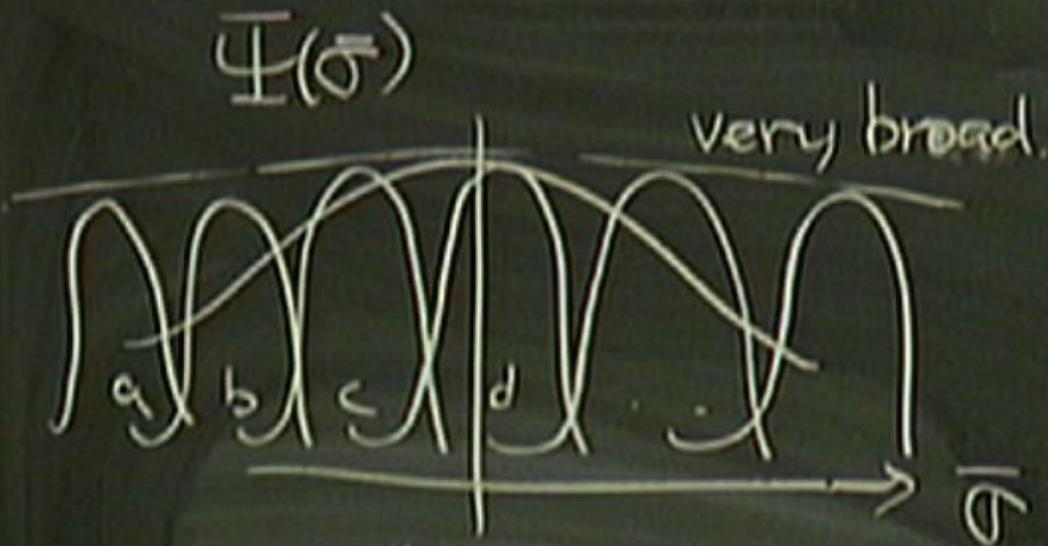
Do we really observe large fluctuation of $\bar{\sigma}$

$\Psi(\bar{\sigma})$

very broad.



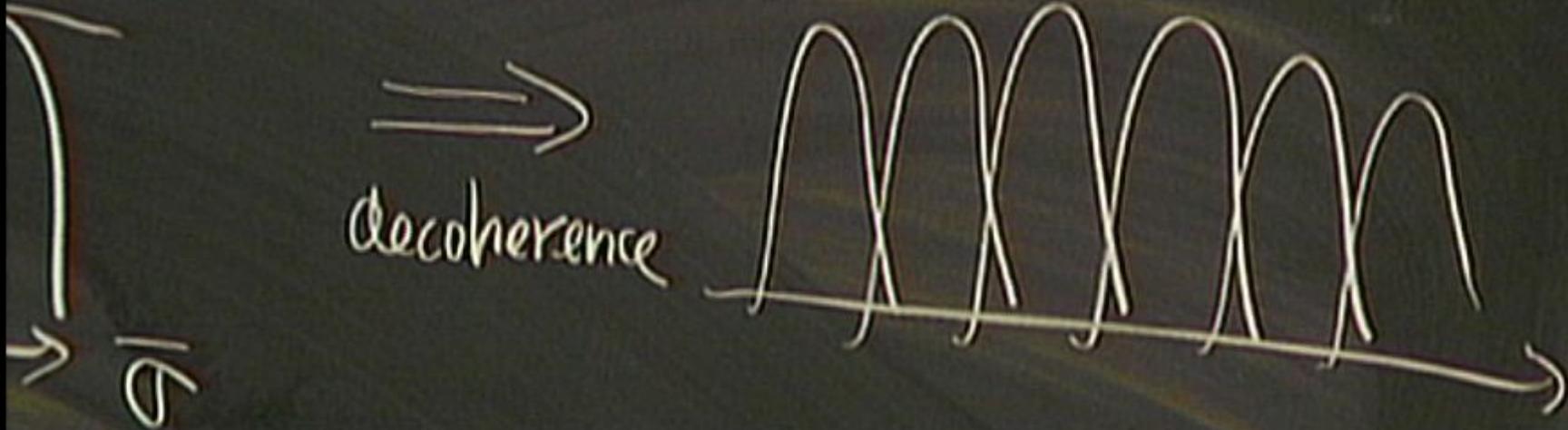
Do we really observe large fluctuation of $\bar{\sigma}$



$$\rho = (\langle a \rangle + \langle b \rangle + \langle c \rangle + \dots) (\langle a \rangle + \langle b \rangle + \langle c \rangle + \dots)$$

observe large fluctuation of $\bar{\sigma}$

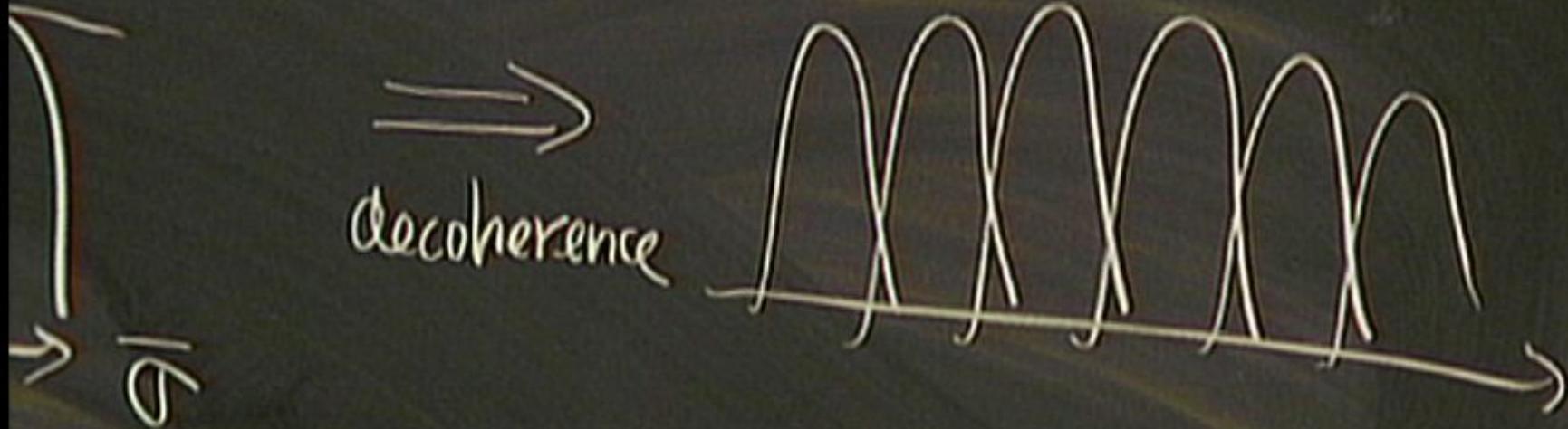
broad.



$$\dots (|a\rangle\langle a| + |b\rangle\langle b| + |c\rangle\langle c| + \dots) \Rightarrow \rho = |a\rangle\langle a| + \boxed{|b\rangle\langle b|} + \dots$$

observe large fluctuation of $\bar{\sigma}$

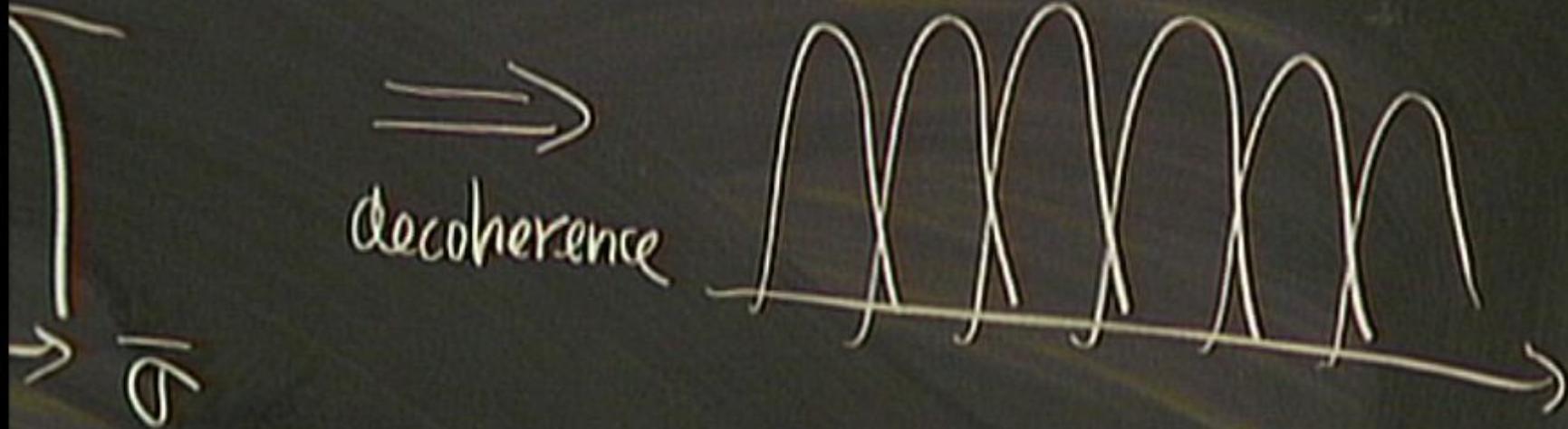
broad.



$$\dots (|a\rangle\langle a| + |b\rangle\langle b| + |c\rangle\langle c| + \dots) \Rightarrow \rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

observe large fluctuation of $\bar{\sigma}$

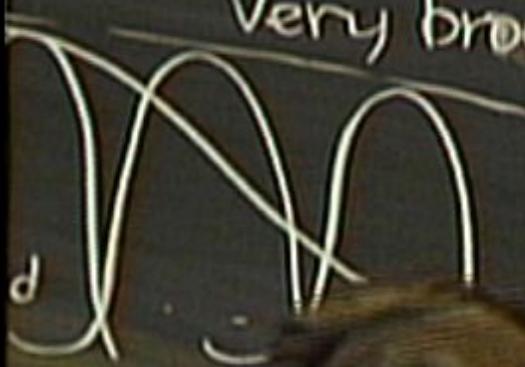
broad.



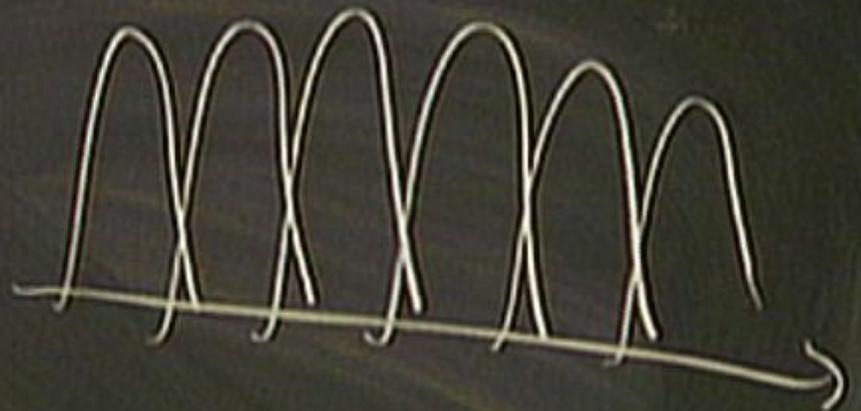
$$\dots(|\langle a| + \langle b| + \langle c| + \dots \rangle) \Rightarrow \rho = |a\rangle\langle a| + \underbrace{|b\rangle\langle b|}_{\substack{+ \dots \\ \uparrow}}$$

usually observe large fluctuation of $\bar{\sigma}$

very broad.



\Rightarrow
decoherence



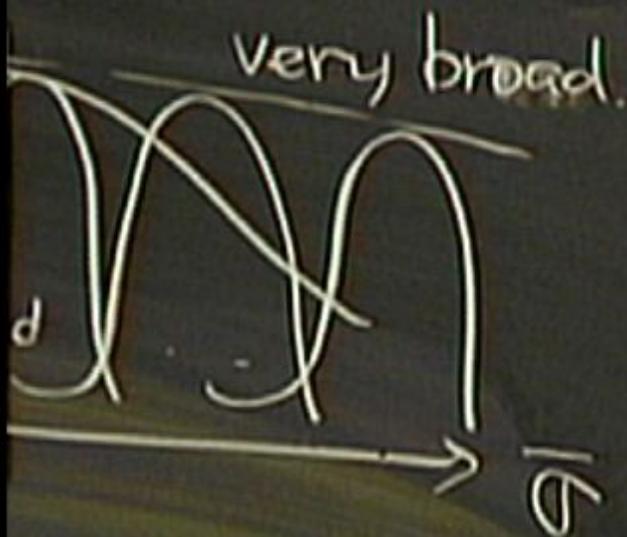
$+ |b\rangle + |c\rangle + \dots$

$(|a\rangle + |b\rangle + |c\rangle + \dots)$

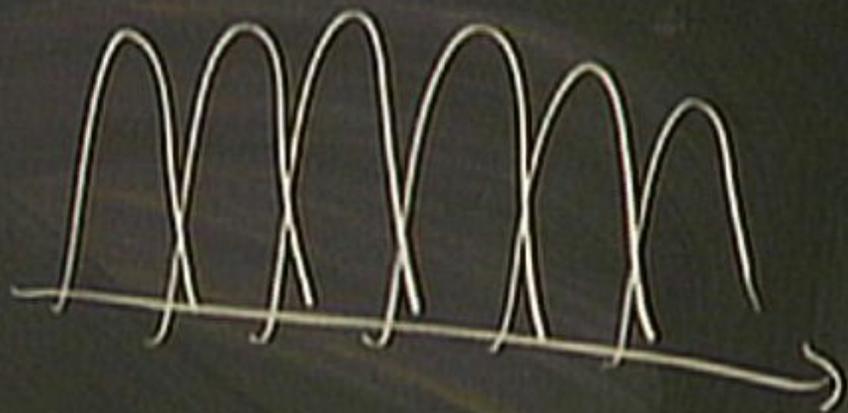
$$\rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

what we observe.

usually observe large fluctuation of $\bar{\sigma}$



\Rightarrow
decoherence

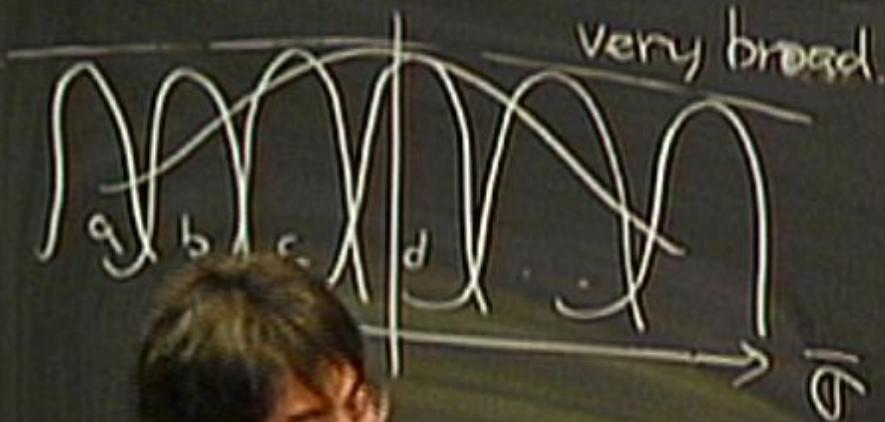


$$(|b\rangle + |c\rangle + \dots)(\langle a| + \langle b| + \langle c| + \dots) \Rightarrow \rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

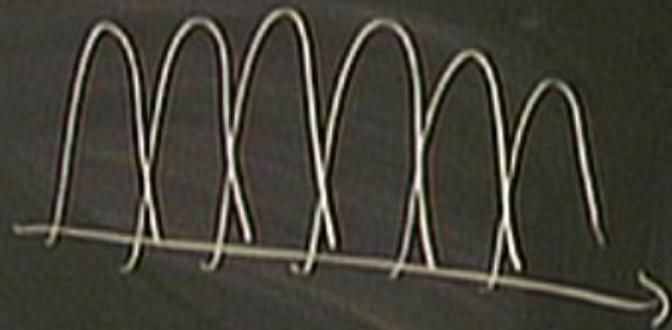
what we observe.

Do we really observe large fluctuation of $\bar{\sigma}$

$\Psi(\bar{\sigma})$



\Rightarrow
decoherence

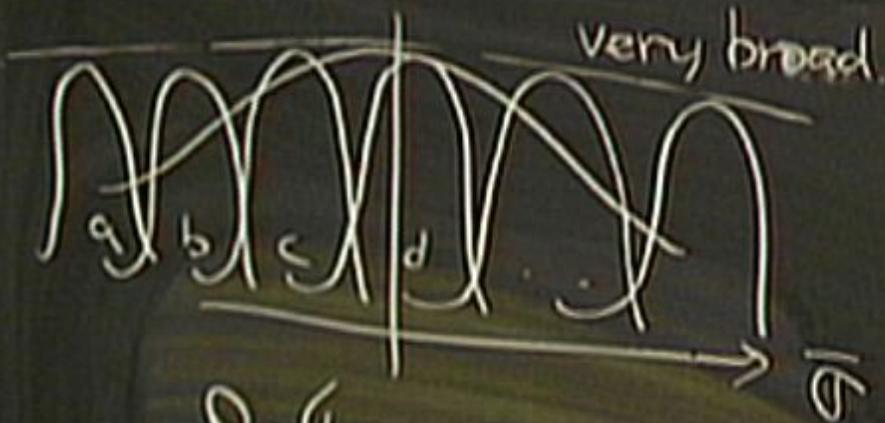


$$(|a\rangle + |b\rangle + |c\rangle + \dots)(\langle a| + \langle b| + \langle c| + \dots) \Rightarrow \rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

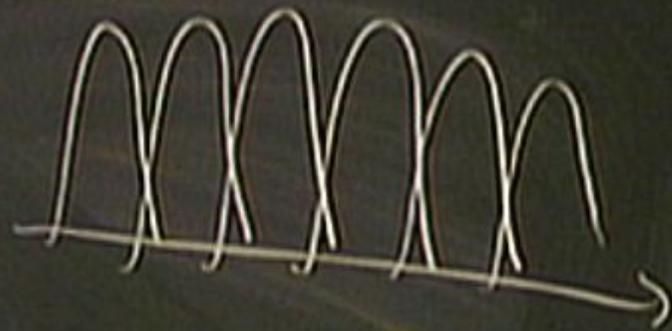
↑
what we observe.

Do we really observe large fluctuation of $\bar{\sigma}$

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\Rightarrow
decoherence



$$\rho = (|a\rangle + |b\rangle + |c\rangle + \dots)(\langle a| + \langle b| + \langle c| + \dots) \Rightarrow \rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

↑
what we observe.

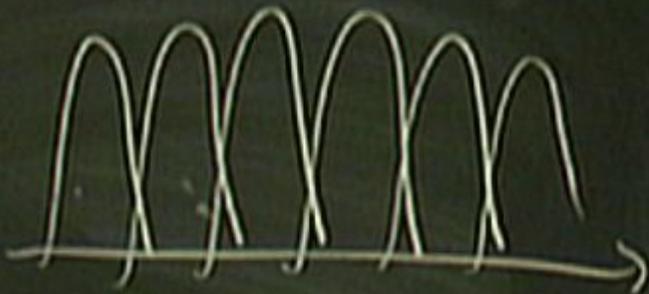
$$(a\psi_k)'' + \left(k^2 - \frac{a''}{a}\right)(a\psi_k) = 0$$

observe large fluctuation of $\bar{\sigma}$

broad.



\Rightarrow
decoherence



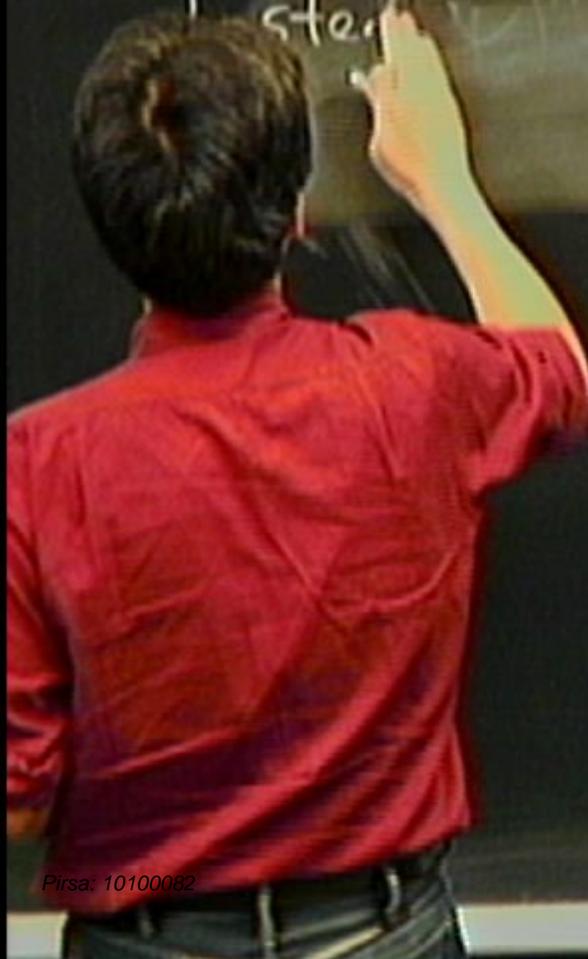
$$\dots \langle a | + \langle b | + \langle c | + \dots \rangle \Rightarrow \rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

what we observe.

$(a| \rho |a)$

Picking up one decohered wave packet. is difficult.

Interfer



Picking up one decohered wave packet. is difficult.

Instead

Picking up one decohered wave packet. is difficult.

Instead



Picking up one decohered wave packet. is difficult.

Instead
$$P = \exp\left(-\frac{\bar{\sigma}^2}{2(\Delta\bar{\sigma})^2}\right)$$

Picking up one decohered wave packet is difficult.

Instead

$$P = \exp\left(-\frac{\sigma^2}{2(\Delta\sigma)^2}\right)$$

$$\langle P \sigma(x) \sigma(y) \rangle$$

Picking up one decohered wave packet. is difficult.

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$$P = \exp\left(-\frac{\sigma^2}{2(\Delta\sigma)^2}\right)$$

$$\langle P \sigma(x) \sigma(y) \rangle$$

Picking up one decohered wave packet. is difficult.

Instead

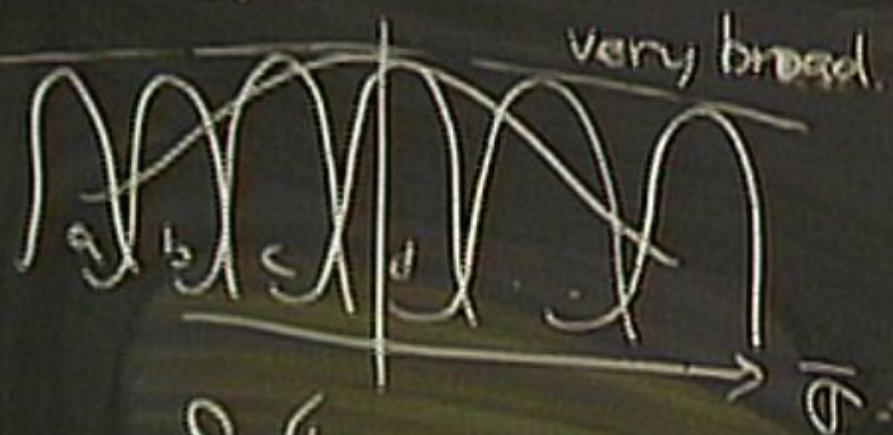
$$P = \exp\left(-\frac{\bar{\sigma}^2}{2(\Delta\bar{\sigma})^2}\right)$$

$$\frac{\langle P \sigma(x) \sigma(y) \rangle}{\langle P \rangle}$$

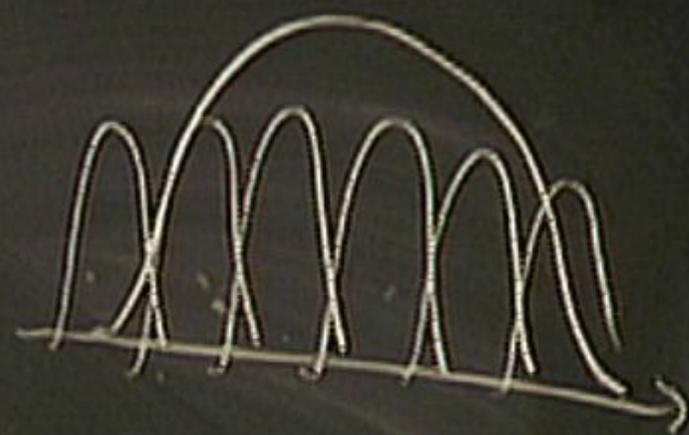
Do we really observe large fluctuation of $\bar{\sigma}$

$\Psi(\sigma)$

very broad



\Rightarrow
decoherence



$$\rho = (|a\rangle + |b\rangle + |c\rangle + \dots)(\langle a| + \langle b| + \langle c| + \dots) \Rightarrow \rho = |a\rangle\langle a| + |b\rangle\langle b| + \dots$$

what we observe.

Picking up one decohered wave packet. is difficult.

Instead

$$P = \exp\left(-\frac{\bar{\sigma}^2}{2(\Delta\bar{\sigma})^2}\right)$$

$$\frac{\langle P \sigma(x) \sigma(y) \rangle}{\langle P \rangle}$$

Is this finite?

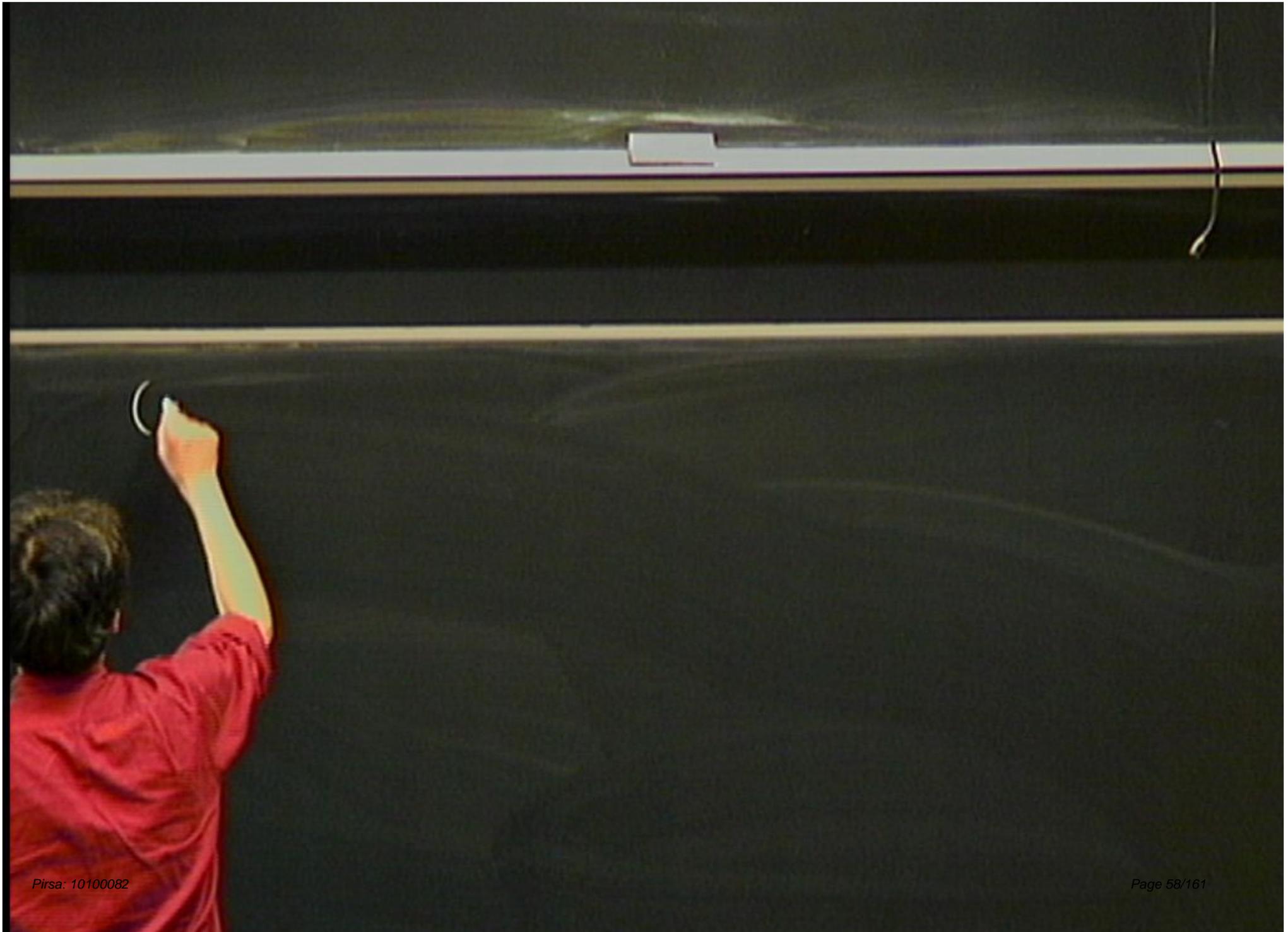
Picking up one decohered wave packet. is difficult.

Instead

$$P = \exp\left(-\frac{\bar{\sigma}^2}{2(\Delta\bar{\sigma})^2}\right)$$

$$\frac{\langle P \sigma(x) \sigma(y) \rangle}{\langle P \rangle}$$

Is this finite?



(u, a)

(u, a)

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \{ u_0 + \Sigma \dots \}$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$? \left\{ u_0 + \sum \dots \right\}$$

?

?

?

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$\left. \begin{array}{l} v_0 \\ v_p \end{array} \right\} u_0 + \sum \dots$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{i p \cdot x} - \frac{W-p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\frac{1}{C(p)} \sum u_p$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{ipx} - \frac{W-p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\frac{1}{C(p)} = \lim_{N \rightarrow \infty} U_p$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{i p \cdot x} - \frac{W-p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\frac{1}{C(p)} = \lim_{\eta \rightarrow 0} U_p$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$\frac{1}{C(p)} = \lim_{\eta \rightarrow 0} U_p$$

$$U_p = U_p e^{i p \cdot x} - \frac{W_p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_P = u_P e^{iP \cdot x} - \frac{W_P}{W_0} \frac{C(0)}{C(P)} u_0$$

$$\int W(x) v_P(x) d^3x = 0$$

$$\frac{1}{C(P)} = \lim_{\eta \rightarrow 0} U_P$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_P = U_P e^{i \int W_0 \mathcal{L}(P)}$$

$$\int W(x) v_P(x) d^3x = 0$$

v_0 contains information

$\lim_{\eta \rightarrow 0} U_P$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{ip \cdot x} - \frac{W-p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\int W(x) v_p(x) d^3x = 0$$

v_0 contains information about

$$\frac{1}{C(p)} =$$



$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\} = \lim_{\eta \rightarrow 0} U_p$$

$$v_p = U_p e^{iP \cdot x} - \frac{W-p}{W} \frac{C(0)}{C(p)} u_0$$

$$\int W(x) v_p(x) dx$$

v_0 contains information about

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{i p \cdot x} - \frac{W_p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\int W(x) v_p(x) d^3x = 0$$

v_0 contains information about

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{i p \cdot x} - \frac{W-p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\int W(x) v_p(x) d^3x = 0$$

v_0 contains information about \vec{J}

$$\frac{1}{C(p)} = \frac{W-p}{W_0} \frac{C(0)}{C(p)}$$

$$(u, a) \xrightarrow{\text{unitary}} (v, b)$$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$v_p = u_p e^{i p \cdot x} - \frac{W_p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\int W(x) v_p(x) d^3x = 0$$

v_0 contains information about \vec{J}

$$\frac{1}{C(p)} = \lim U_p$$

$(u, a) \xrightarrow{\text{unitary}} (v, b)$

$$v_0 = \frac{1}{N} \left\{ u_0 + \sum \dots \right\}$$

$$\frac{1}{C(p)} = \lim_{\eta \rightarrow 0} U_p$$

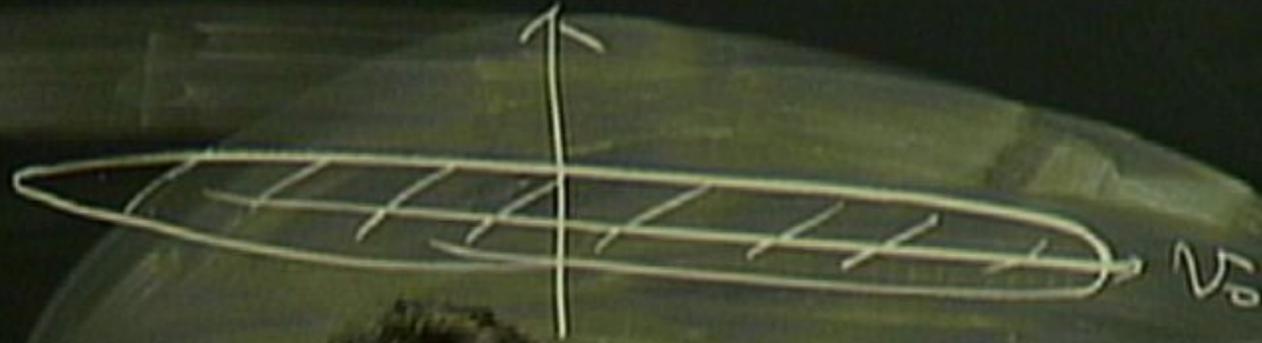
$$v_p = U_p e^{i p \cdot x} - \frac{W-p}{W_0} \frac{C(0)}{C(p)} u_0$$

$$\int W(x) v_p(x) d^3x = 0 \quad p \neq 0$$

v_0 contains information about \vec{J}

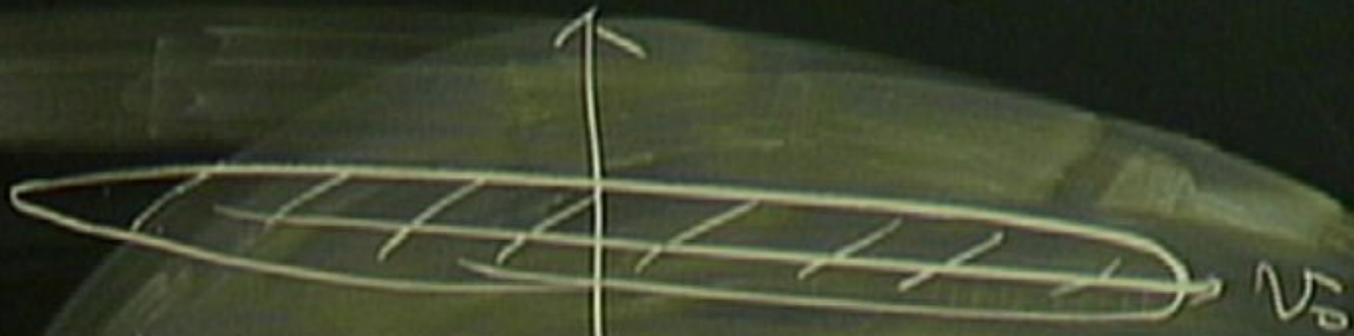
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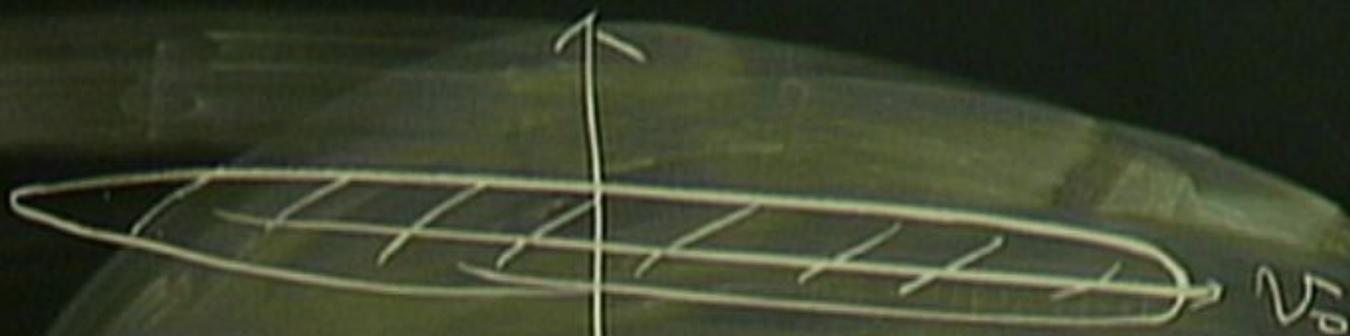
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$$(\tilde{v}, b) \rightarrow (\tilde{v}, \tilde{b})$$

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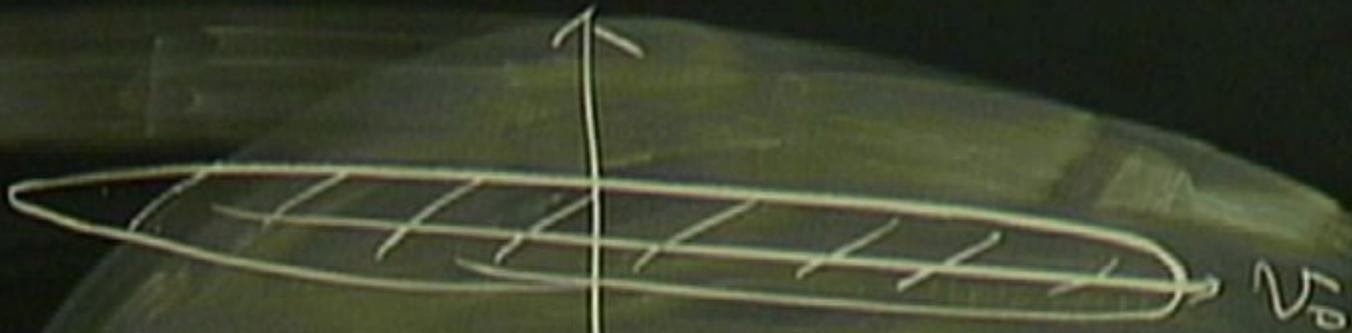


$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

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with Yuko Urakawa

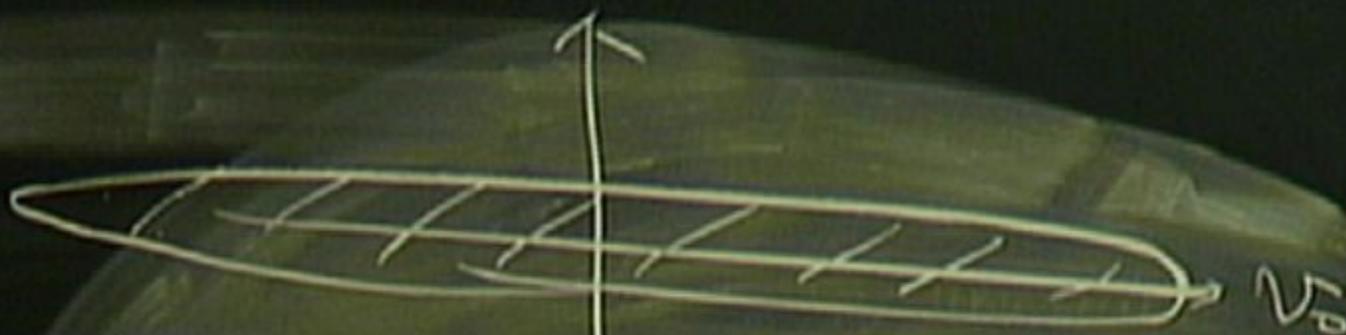


$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

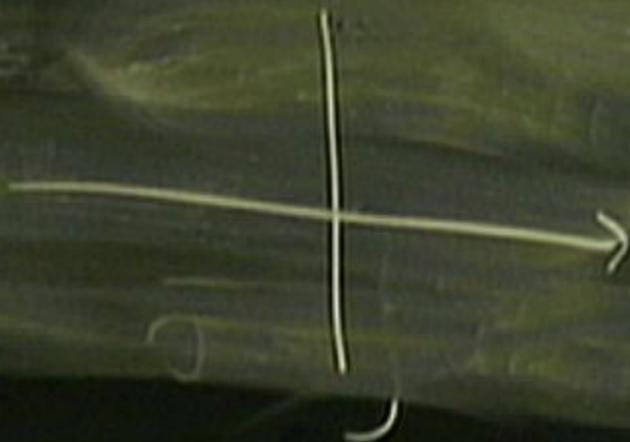
0904.4415

with Yuko, Urakawa



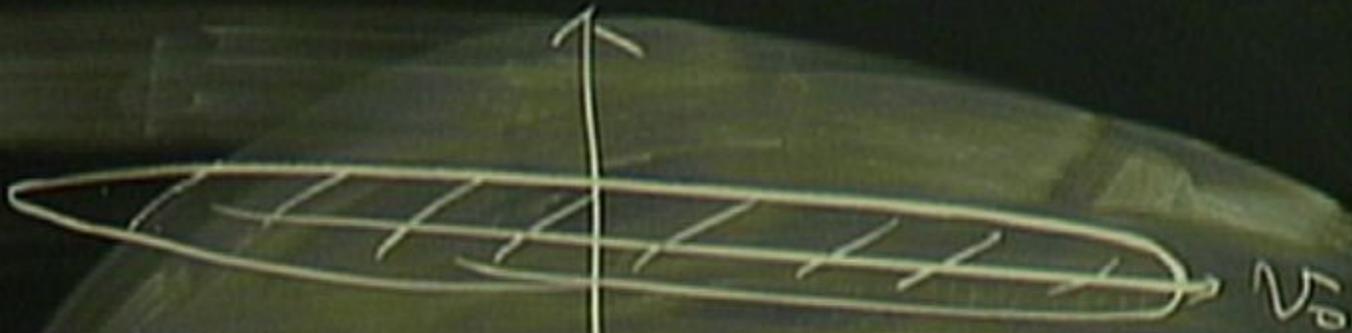
$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$



0904.4415

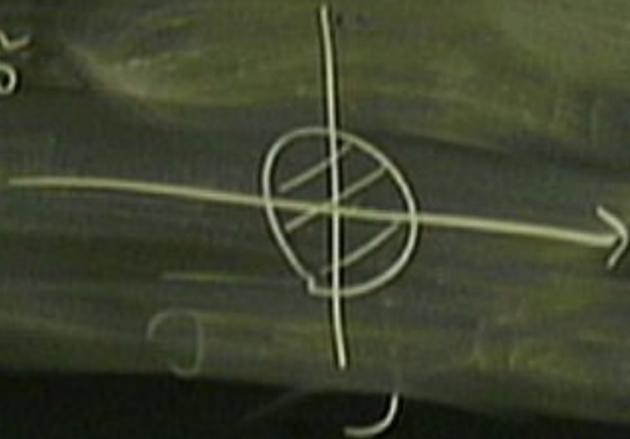
with Yuko Urakawa



$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

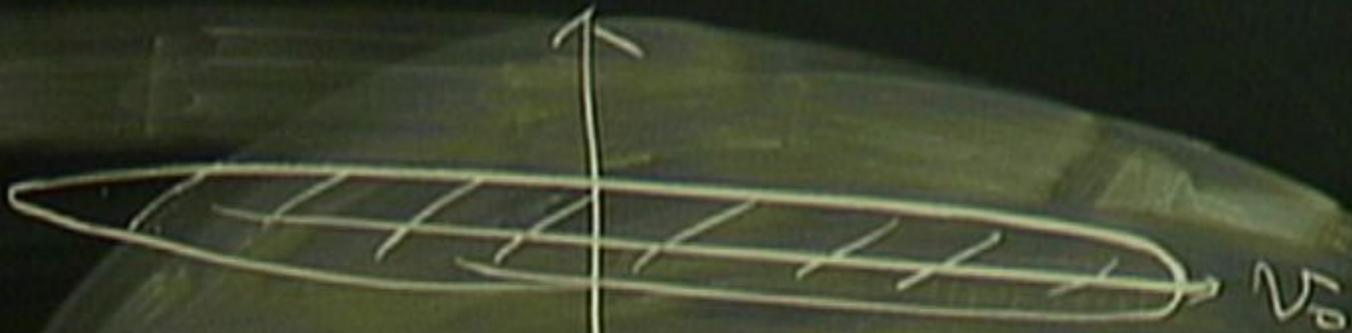
$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$$|0\rangle_b$$



0904.4415

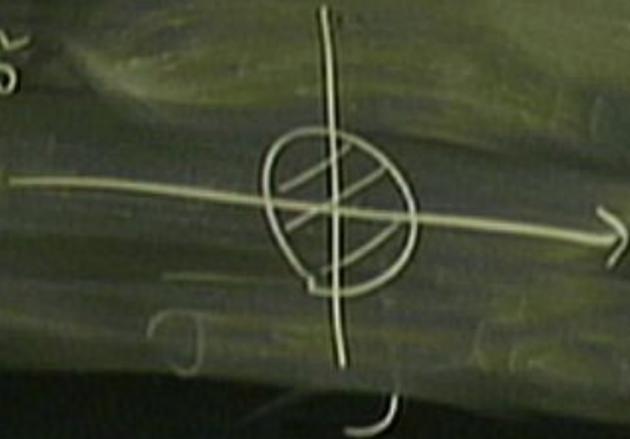
with Yuko Urakawa



$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

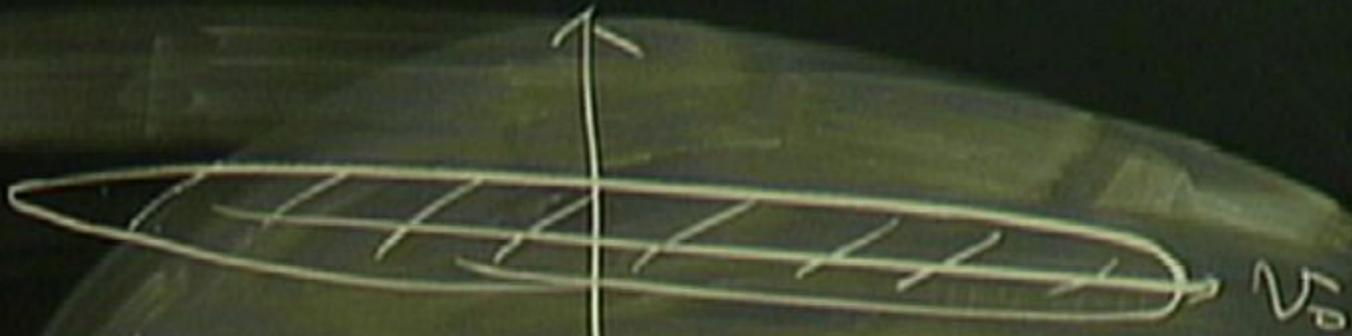
$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$$|0\rangle_b$$



0904.4415

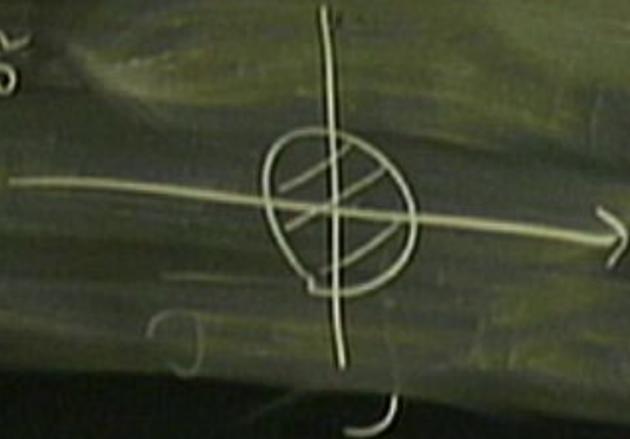
with Yuko Urakawa



$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

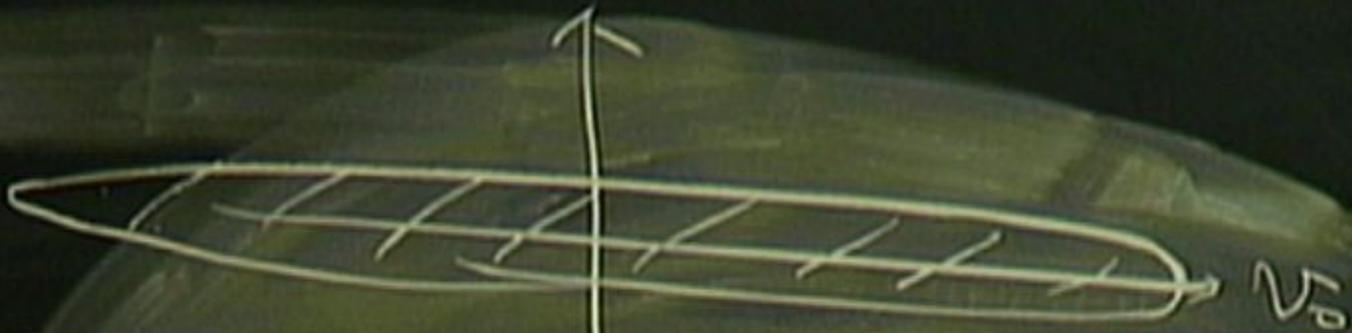
$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$$|0\rangle_{\tilde{b}}$$



0904.4415

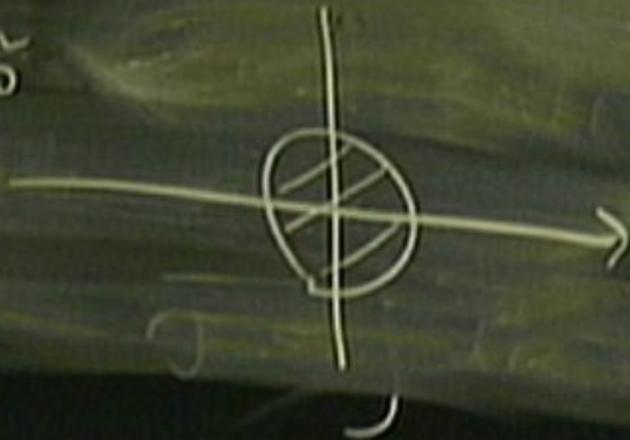
with Yutko, Urakawa



$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

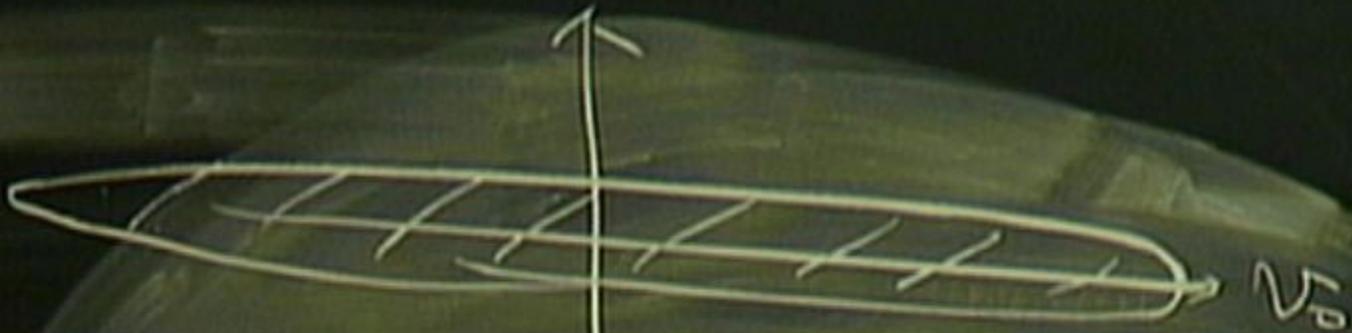
$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$$|0\rangle_b$$



0904.4415

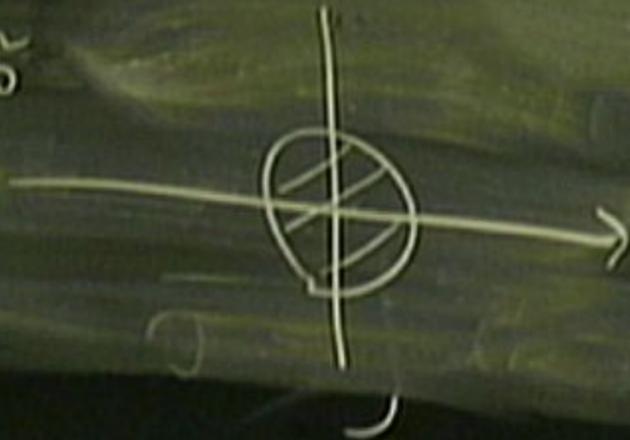
with Yuko, Urakawa



$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$$|0\rangle_b$$



0904.2415

with Yuko, Urakawa

coherent state

$$\tilde{b} |\beta\rangle =$$



$$(\tilde{v}, \tilde{b}) \rightarrow (\tilde{v}, \tilde{b})$$

$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$$|0\rangle_b$$



0904.2415

with Yuko Umekawa

coherent state

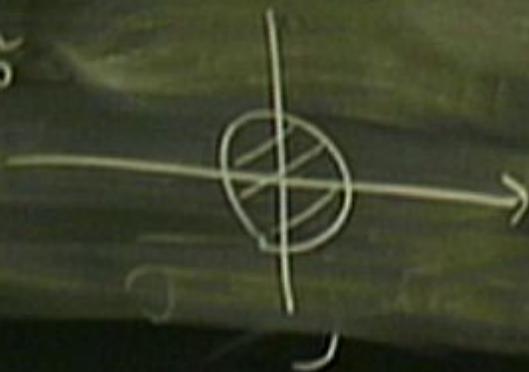
$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$



$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$|0\rangle_b$



$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_b$$

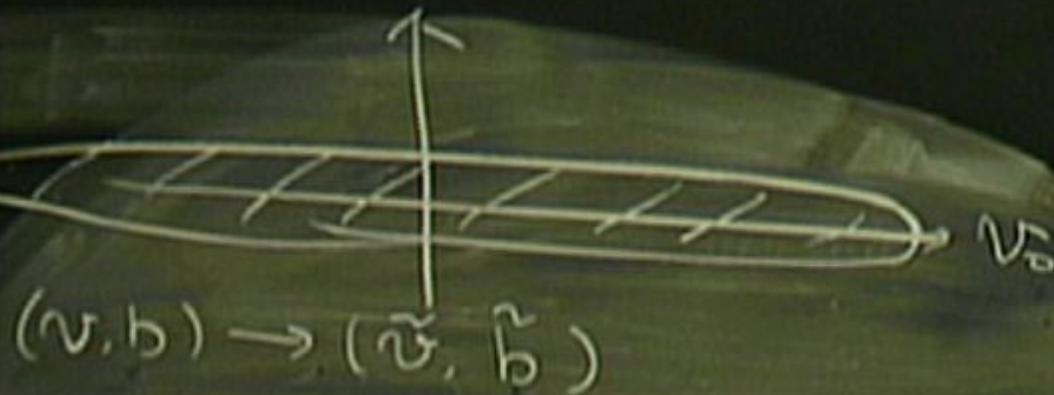


0904.2415

with Yoko Umekawa

coherent state

$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$



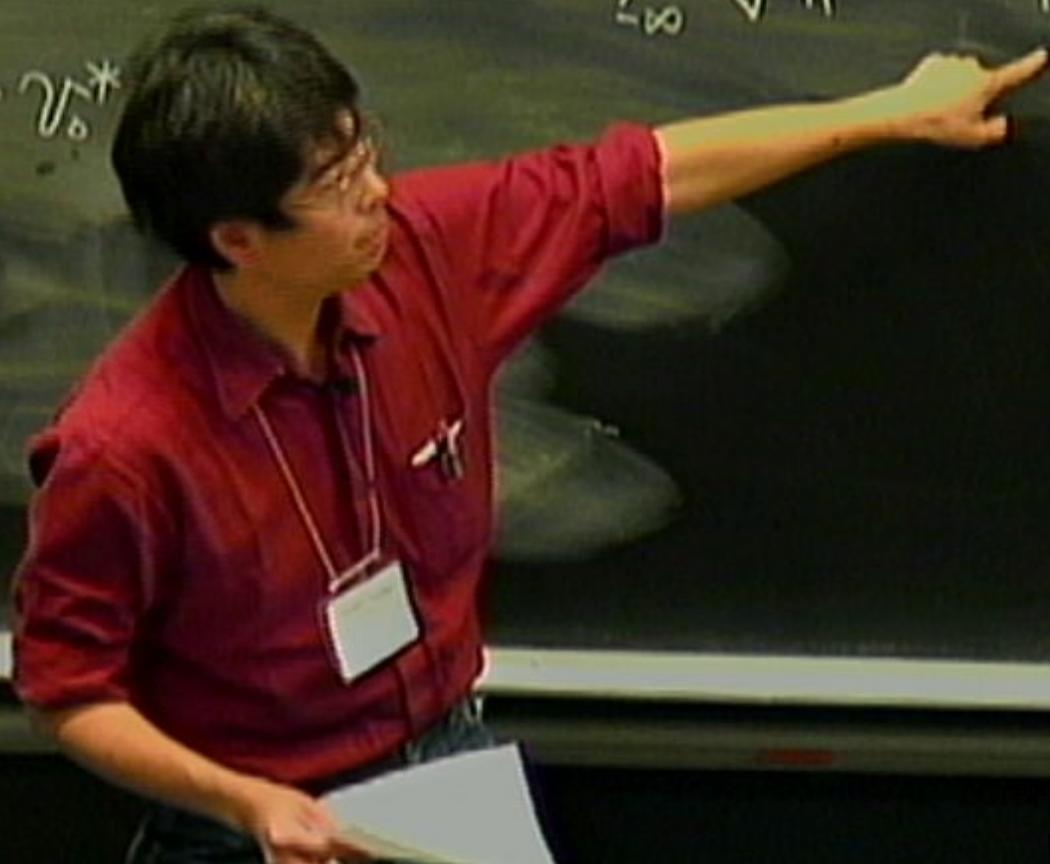
$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$|0\rangle_{\tilde{b}}$



$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_{\tilde{b}}$$

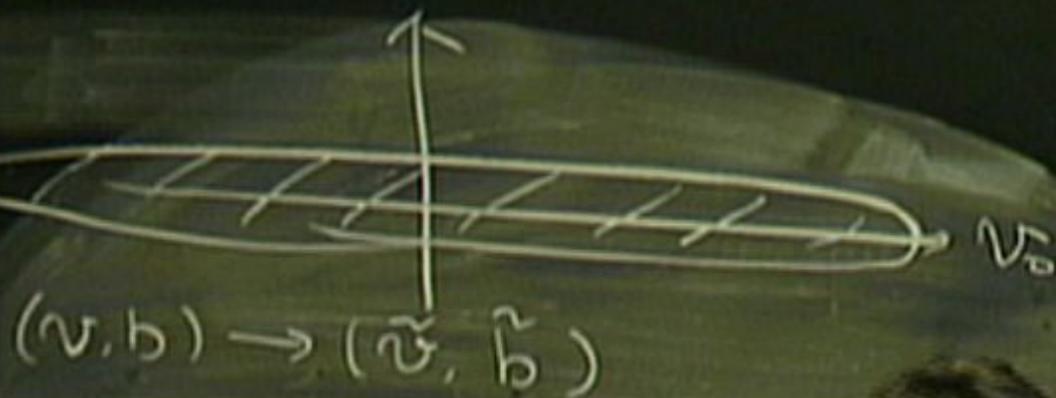


0904.2415

with Yuko Umekawa

coherent state

$$\tilde{b} |\beta\rangle = \beta |\beta\rangle$$



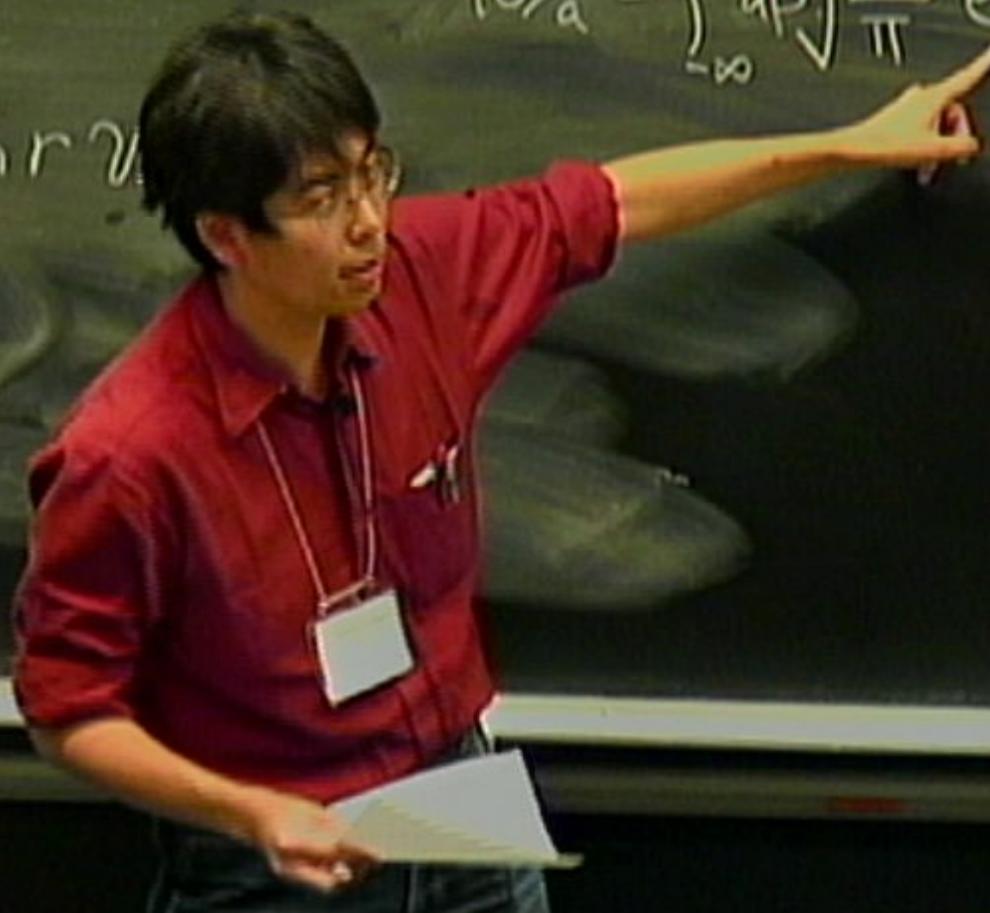
$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_1$$

$|0\rangle_b$



$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_b$$



0904.2415

with Yuko Urakawa

coherent state

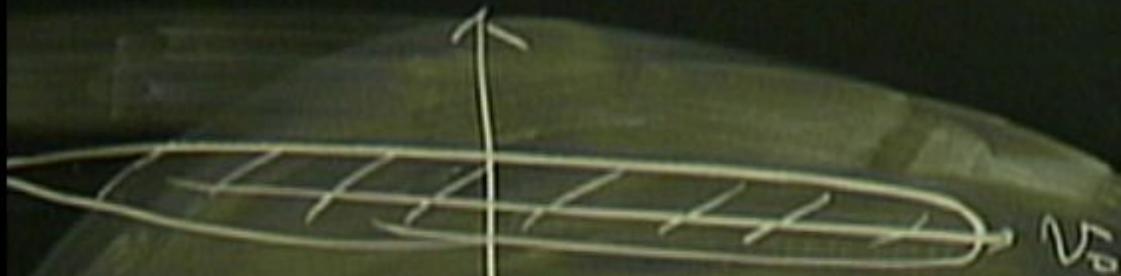
$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_b$$

$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_1$$

$|0\rangle_b$



0904.2415

with Yuko Umekawa

coherent state

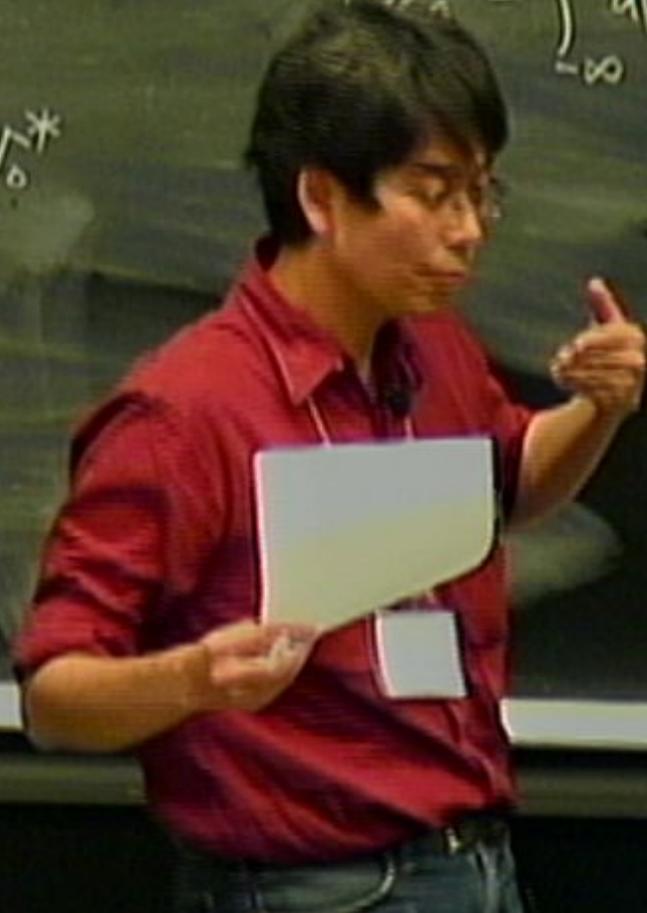
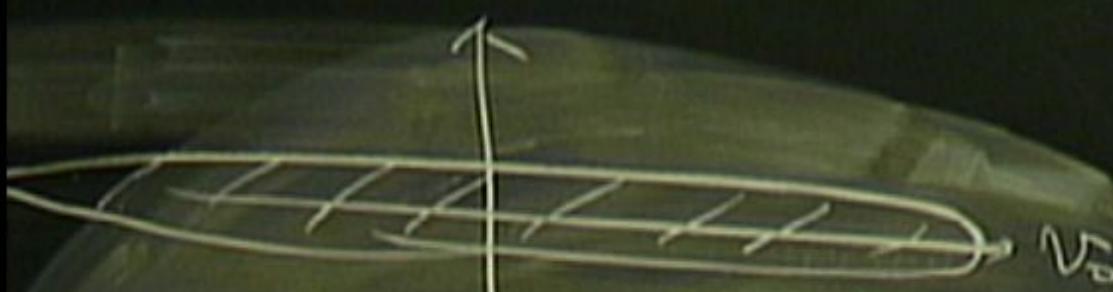
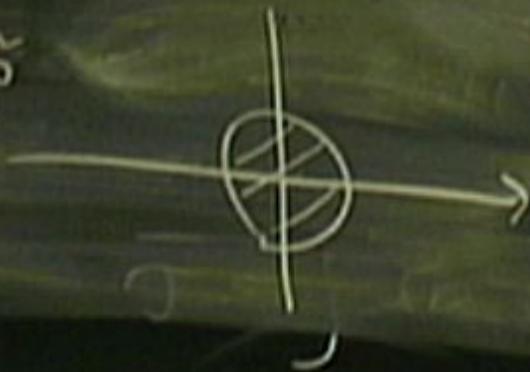
$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

$$|0\rangle_{\tilde{b}} = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-(s\beta)^2} |\beta\rangle_{\tilde{b}}$$

$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$|0\rangle_{\tilde{b}}$



0904.2415

with Yuko Umekawa

coherent state

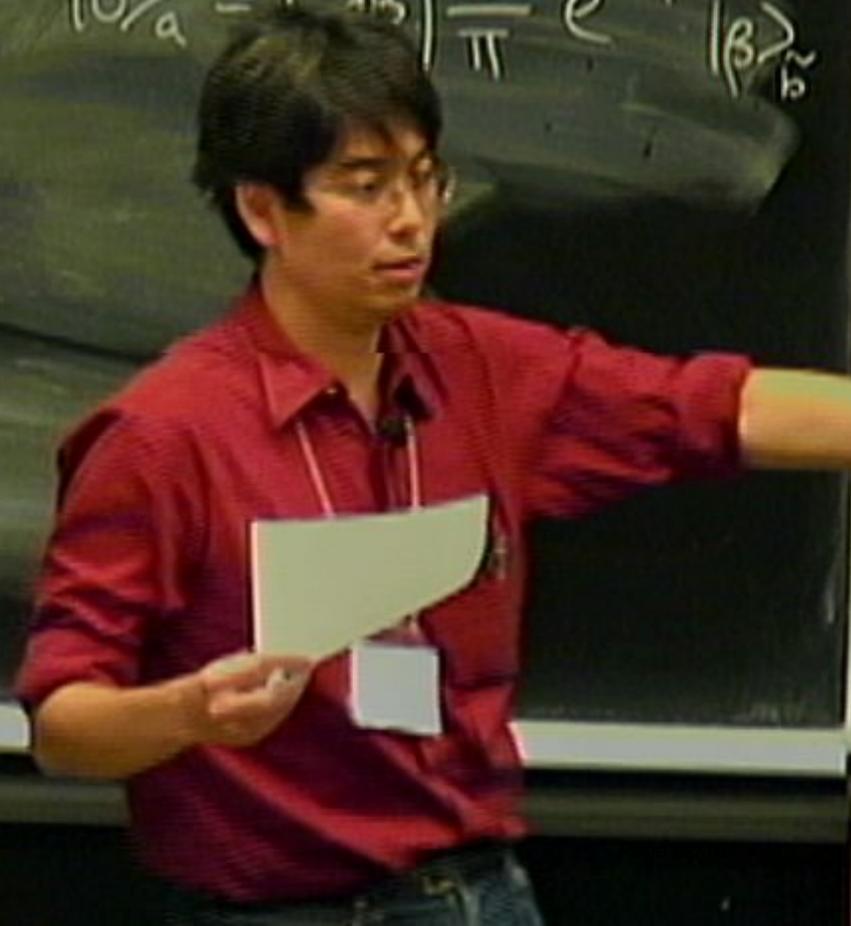
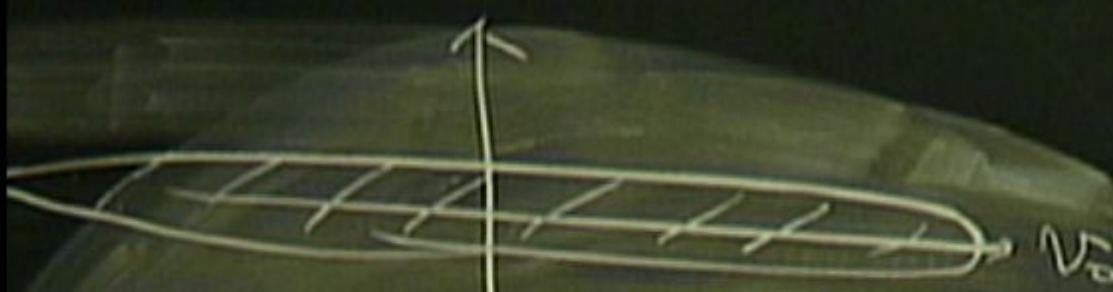
$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

$$|0\rangle_a = \int_{-\infty}^{\infty} \frac{d\beta}{\sqrt{\pi}} e^{-\frac{(\beta)^2}{2}} |\beta\rangle_b$$

$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$|0\rangle_b$



0904.2415

with Yuko Umakawa

coherent state

$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

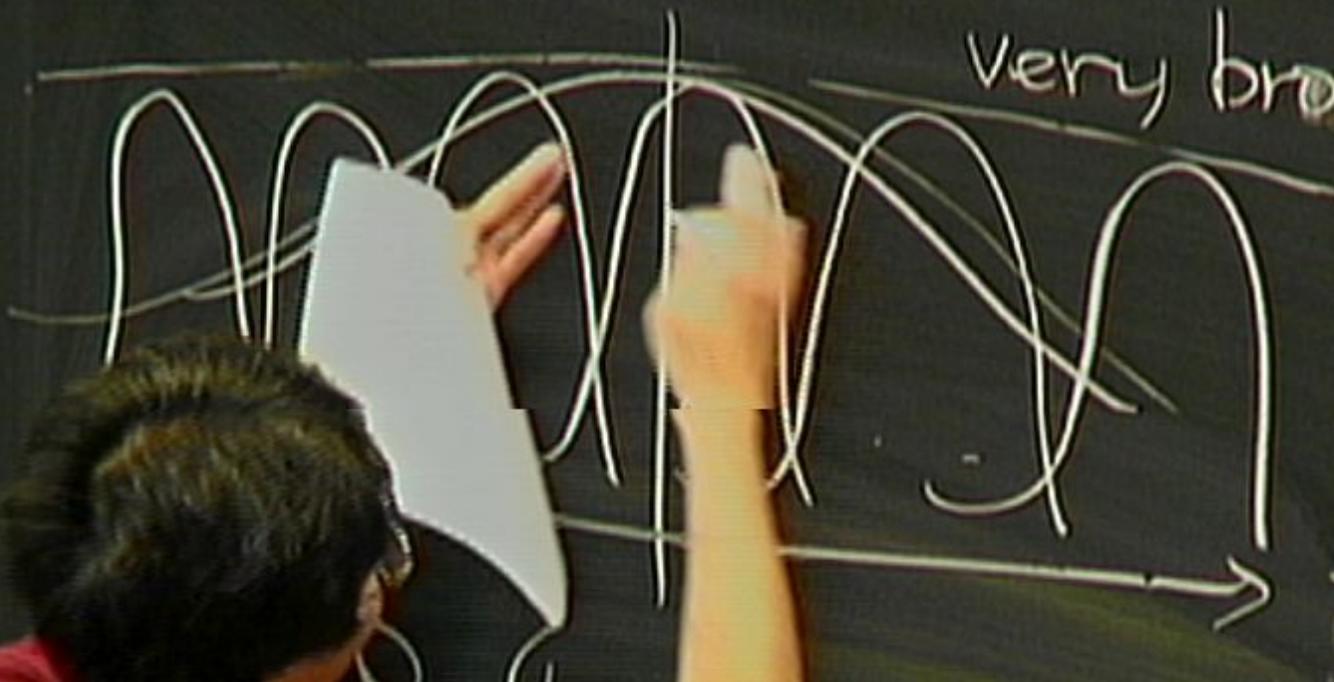
$|0\rangle_b$



$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-(s\beta)^2}$$

$\Psi(x)$

very broad.

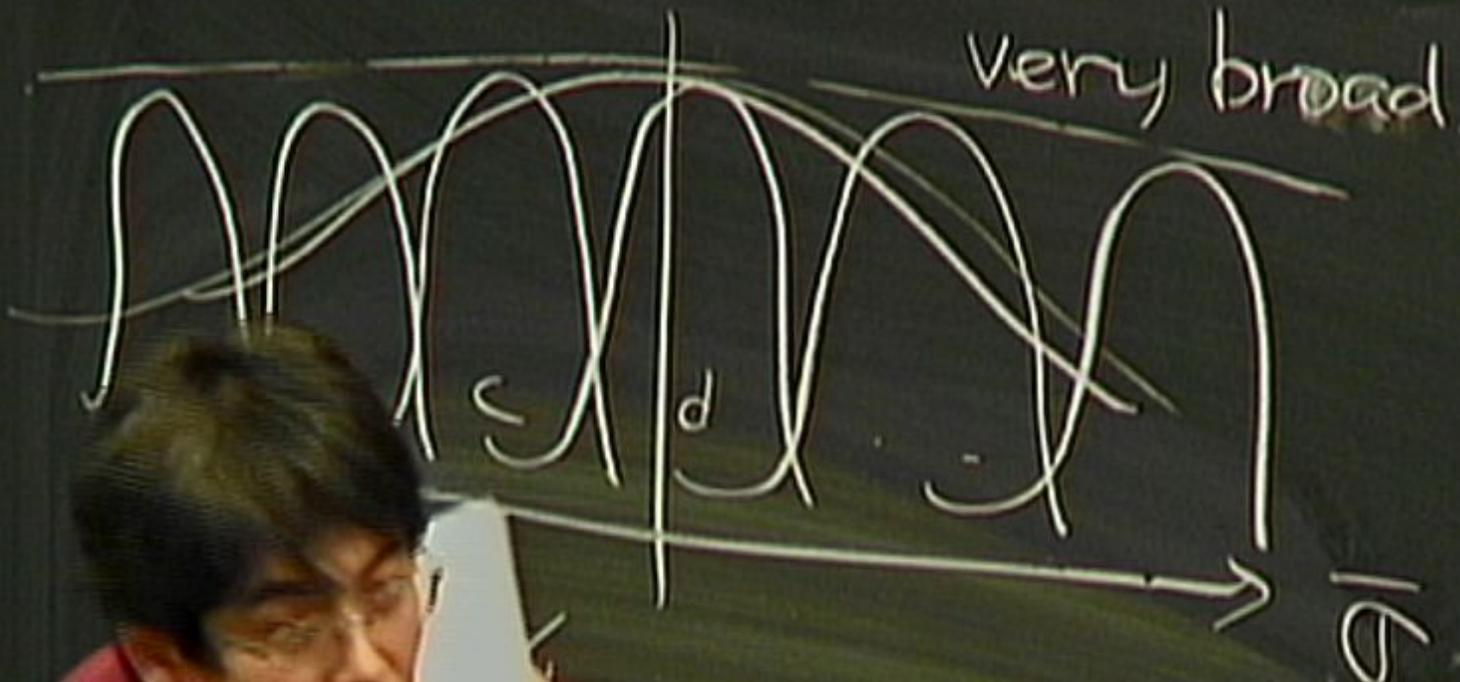


decoh

Ψ

$$|a\rangle + |b\rangle + \dots \rangle (|a\rangle + |b\rangle + |c\rangle + \dots)$$

$\Psi(x)$



$$(|a\rangle + |b\rangle + |c\rangle + \dots)(\langle a| + \langle b| + \langle c| + \dots)$$

0904.2415

with Yuko Umekawa

coherent state

$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

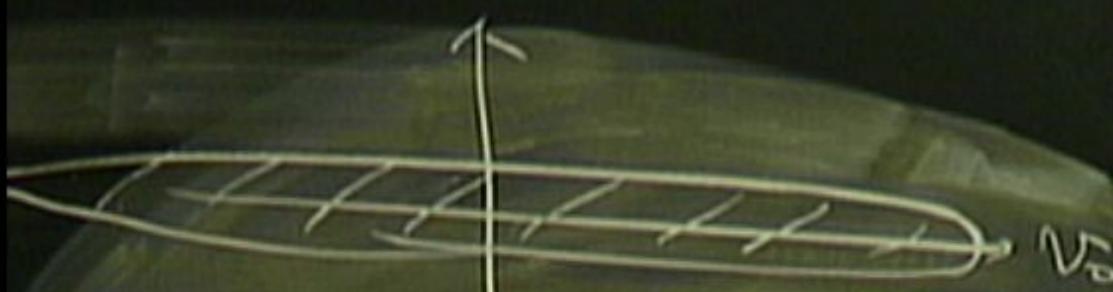
$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-(s\beta)^2} |\beta\rangle_b$$

$$s \rightarrow 0$$

$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = 0 - \text{sh } r v_0^*$$

$$|0\rangle_b$$



0904.2415

with Yuko Umekawa

coherent state

$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

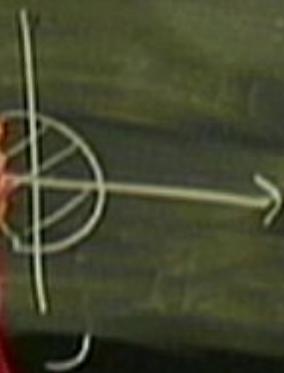
$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

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$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-(s\beta)^2} |\beta\rangle_b$$

$s \rightarrow 0$

$|0\rangle$



0904.2415

with Yuko Umekawa

coherent state

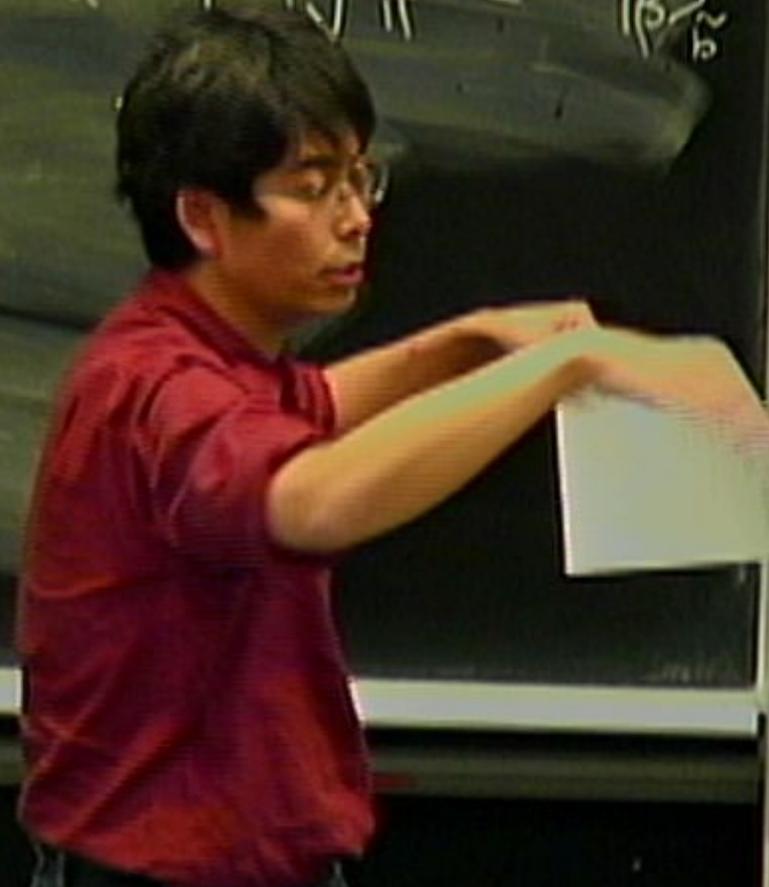
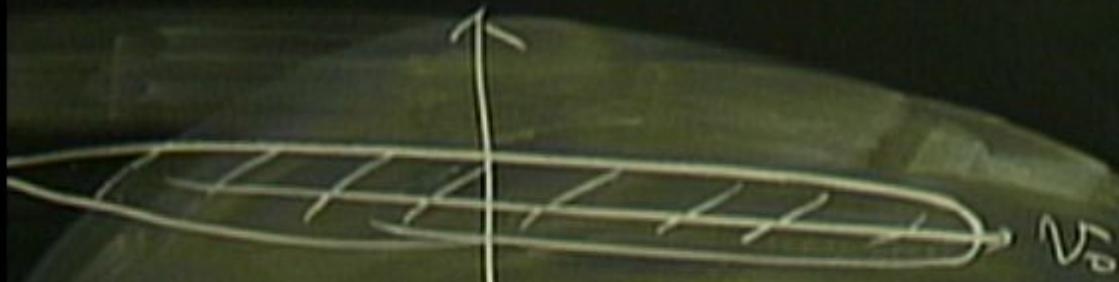
$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

$$|0\rangle_a = \int d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_b$$

$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$|0\rangle_b$



0904.2415

with Yuko Umekawa

coherent state

$$\tilde{b}|\beta\rangle = \beta|\beta\rangle$$

$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_b$$

$$s \rightarrow 0$$

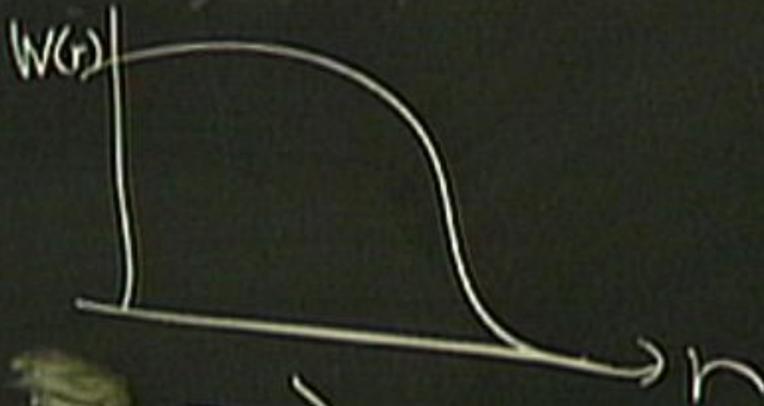
$$(v, b) \rightarrow (\tilde{v}, \tilde{b})$$

$$\tilde{v}_0 = \cosh r v_0 - \sinh r v_0^*$$

$|0\rangle_b$



$$\bar{\sigma} = \int d^3x W(x) \sigma(x)$$



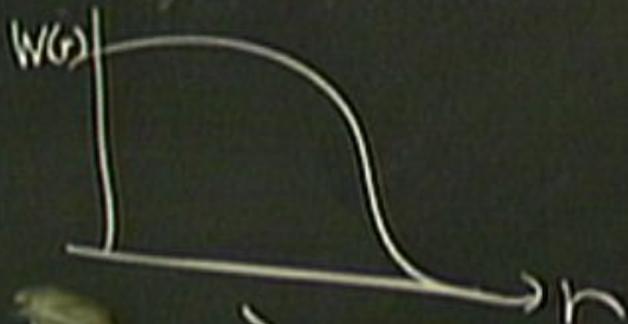
$$\left\{ \bar{\sigma}, \sigma - \bar{\sigma} \right\}$$

1 def

$$\hat{\sigma}_H =$$

what we observe.

$$\bar{\sigma} = \int d^3x W(x) \sigma(x)$$



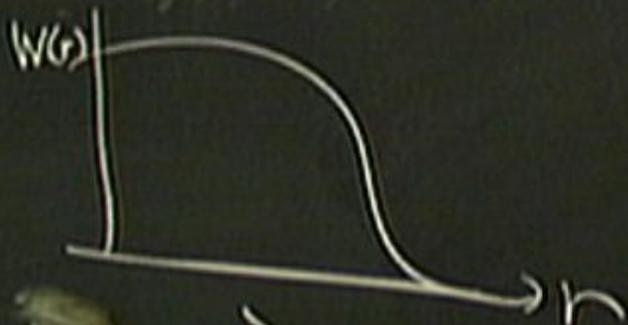
$$\{ \bar{\sigma}, \underline{\sigma - \bar{\sigma}} \}$$

1 def

$$\hat{\sigma}_{HI} = \hat{\sigma}_I + \int G_R(z, x') \Gamma'(x') d^4x'$$

what we observe.

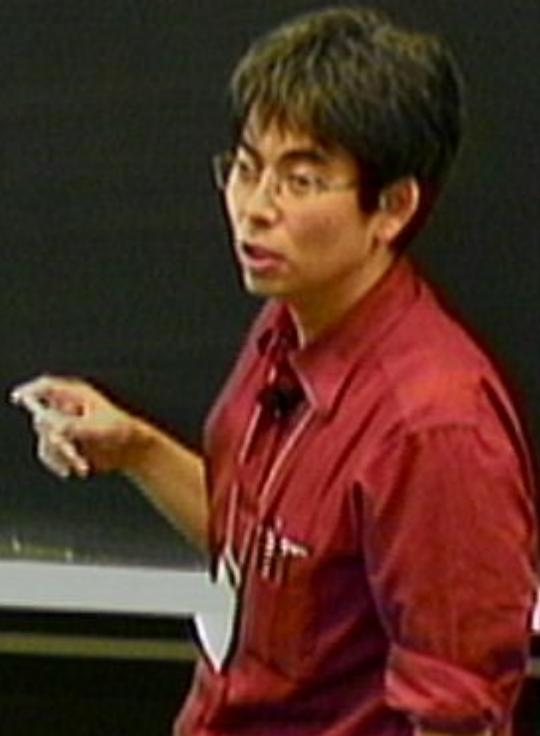
$$\bar{\sigma} = \int d^3x W(x) \sigma(x)$$



$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') d^3x'$$

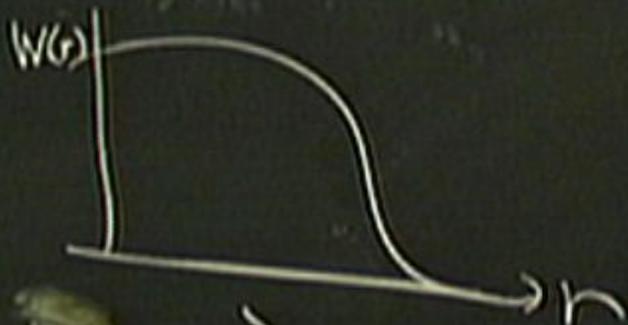
$$\{ \bar{\sigma}, \underline{\sigma} - \bar{\sigma} \}$$

1 def



what we observe.

$$\bar{\sigma} = \int d^3x W(x) \sigma(x)$$



$$\hat{\sigma}_{\text{H.}} = \hat{\sigma}_{\text{I.}} + \int G_{\text{R.}}(z, z') \Gamma'(z') d^3z'$$

$\left\{ \begin{array}{l} \bar{\sigma} \\ \sigma \end{array} \right\}$

W(x)

what we observe.

$$\bar{\sigma} = \int d^3x W(x) \sigma(x)$$



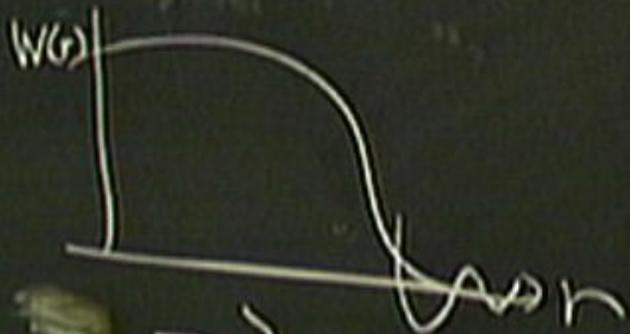
$$\hat{\sigma}_{H.} = \hat{\sigma}_{I.} + \int G_R(z, x') \Gamma'(x') d^4x'$$

{ $\bar{\sigma}$, σ }
1 def

$\sigma = \int$

what we observe.

$$\bar{\sigma} = \int d^3x W(x) \sigma(x)$$

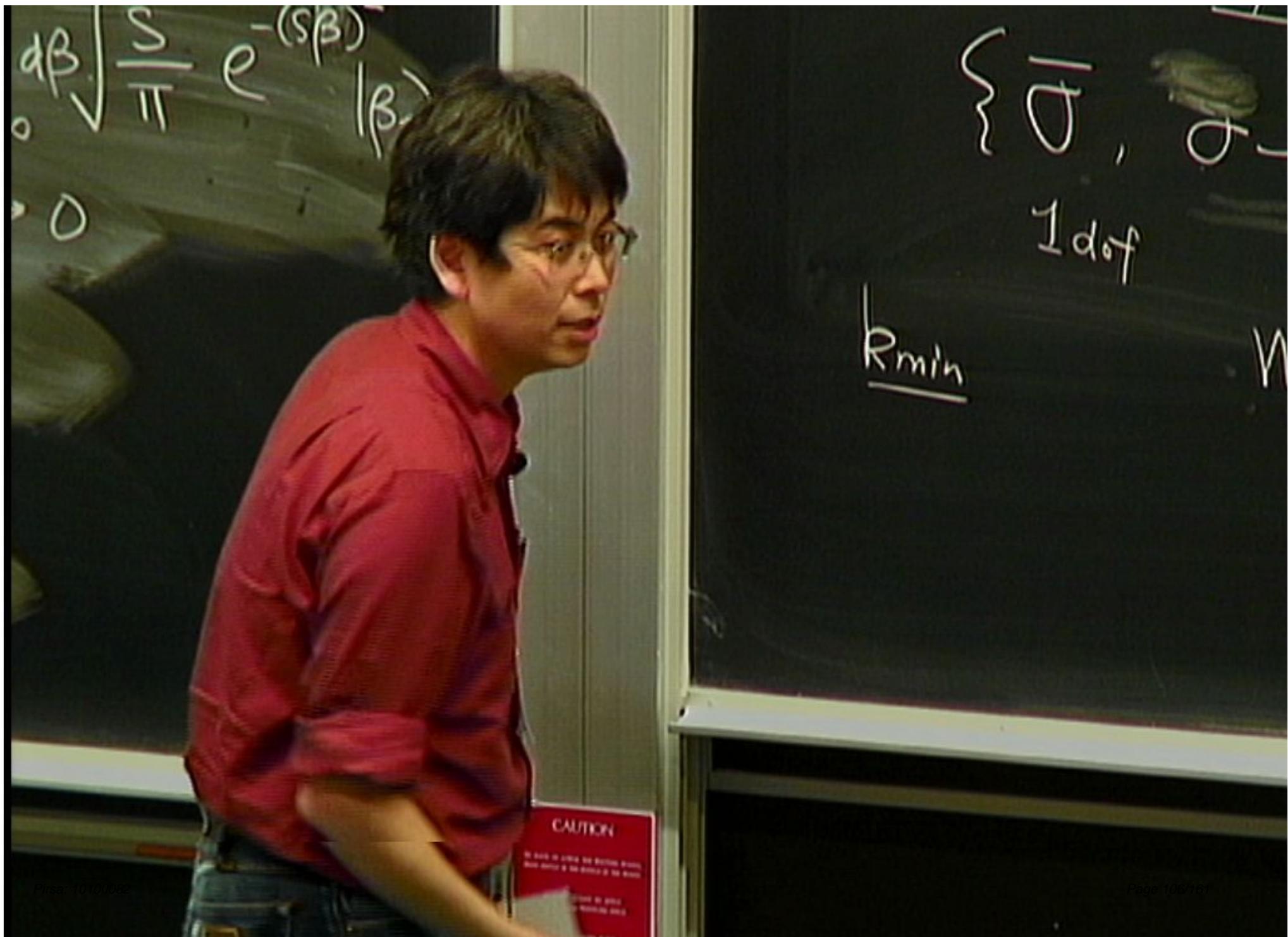


$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') d^3x'$$

$$\{ \bar{\sigma}, \sigma - \bar{\sigma} \}$$

1 def

$$W(x) = \int d^3k e^{ikx} \theta(k_{\text{cut}} - k)$$



$$d\beta \sqrt{\frac{S}{\pi}} e^{-(S\beta)}$$

(β)

$\{D, \sigma\}$
1 dof

R_{min}

CAUTION
BE CAREFUL OF THE HOT SURFACE
DO NOT TOUCH THE HOT SURFACE
OR YOU WILL BE BURNED

what we observe.

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma(x') dx'$$



~~for~~

what we observe.

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \theta | 0 \rangle_a$$

what we observe.

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \theta | 0 \rangle_a$$

what we observe.

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \langle \alpha | P \Theta | \beta \rangle_b$$

what we observe.

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \langle \alpha | P \Theta | \beta \rangle_b$$

what we observe.

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\int d\beta \delta \langle \alpha | P \Theta | \beta \rangle_{\delta^2}$$



x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\int d\beta \langle \alpha | P \Theta | \beta \rangle_{\beta^2}$$



$$\langle \alpha |$$

keith's
Alb

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\int d\beta \langle \alpha | P \Theta | \beta \rangle_{\beta^2}$$

⇓

$$\langle \alpha | \beta \rangle$$

keil... A/b

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\int d\alpha \int d\beta \underbrace{\langle \alpha | P \Theta | \beta \rangle_{\delta^2}}$$

⇓

$$\langle \alpha | \beta \rangle_{\delta^2}$$

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \underbrace{\langle \alpha | P \Theta | \beta \rangle_{\vec{b}}}$$

⇓

$$k e^{ikx} \Theta(k_{cut} - k) \underbrace{\langle \alpha | \beta \rangle_{\vec{b}}} \times \underbrace{\langle 0 | (P \Theta) \rangle_{\vec{b}}}$$

x)

$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

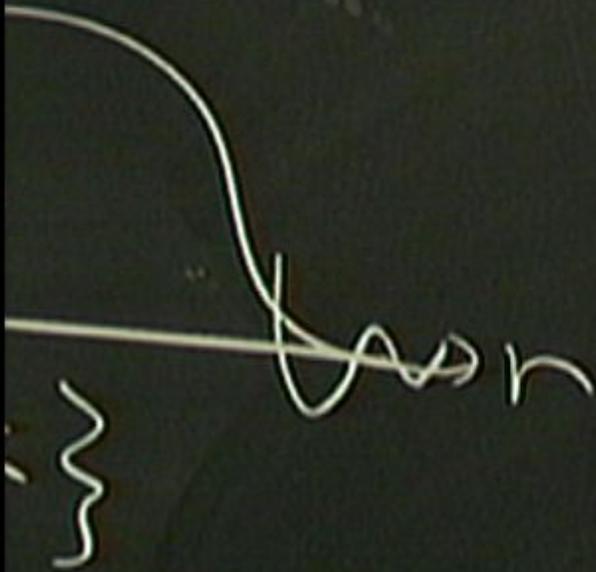
~~for~~

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \underbrace{\langle \alpha | P \Theta | \beta \rangle_{\vec{b}_2}}_{\vec{b}_1}$$

$$k e^{ikx} \Theta(k_{cut} - k) \quad \Downarrow \quad \langle \alpha | \beta \rangle_{\vec{b}_1} \times \langle 0 | (P \Theta)_{\vec{b}_2}$$

$\alpha(x) \theta(x)$



$$\hat{\sigma}_H = \hat{\sigma}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

$$\langle 0 | P \theta | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \langle \alpha | P \theta | \beta \rangle_b$$

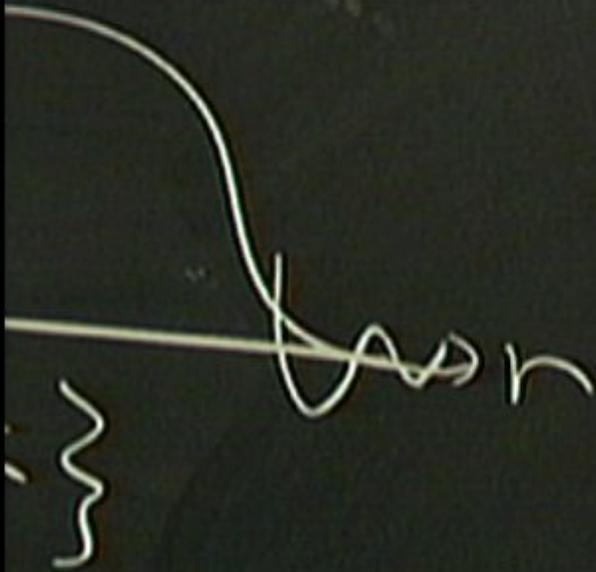
$$\rightarrow \int d^3k e^{i k \cdot x} \theta(k_{out} - k)$$

$$\Downarrow$$

$$\langle \alpha | \beta \rangle_b \times \langle 0 | (P \theta) | 0 \rangle_b$$

$\tilde{b} \rightarrow \tilde{b} + \alpha$
 $\tilde{b}^\dagger \rightarrow \tilde{b}^\dagger + \beta$

$\mathcal{O}(x) \mathcal{O}(x)$



$$\hat{\mathcal{O}}_H = \hat{\mathcal{O}}_I + \int G_R(x, x') \Gamma'(x') dx'$$

$$= \hat{\mathcal{O}}_I + \hat{\mathcal{O}}_I^2 + \dots$$

$$\langle 0 | P \mathcal{O} | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \langle \alpha | P \mathcal{O} | \beta \rangle_b$$

$$\rightarrow \int d^3k e^{ik \cdot x} \theta(k_{\text{cut}} - k) \langle \alpha | \beta \rangle_b \times \langle 0 | (P \mathcal{O}) | 0 \rangle_b$$

\Downarrow
 $\tilde{b} \rightarrow \tilde{b} + \alpha$
 $\tilde{b}^\dagger \rightarrow \tilde{b}^\dagger + \beta$

$$\int d\alpha \int d\beta \int_{\vec{b}} \langle \alpha | P \mathcal{O} | \beta \rangle \sqrt{z}$$



$$\int_{\vec{b}} \langle \alpha | \beta \rangle \times \int_{\vec{b}} \langle 0 | (P \mathcal{O}) | 0 \rangle \sqrt{z}$$

$$\vec{b} \rightarrow \vec{b} + \alpha$$

$$\vec{b}^+ \rightarrow \vec{b}^+ + \beta$$

$\frac{1}{a}$

$$\approx \int d\alpha \int d\beta \underbrace{\langle \alpha | P \theta | \beta \rangle}_{\int_C}$$



at $-k$) $\langle \alpha | \beta \rangle \times \langle 0 | (P \theta) | 0 \rangle$

$\vec{b} \rightarrow \vec{b} + \alpha$
 $\vec{b}^+ \rightarrow \vec{b}^+ + \beta$

$\frac{a}{a}$

$$\approx \int d\alpha \int d\beta \underbrace{\langle \alpha | P \theta | \beta \rangle}_{\int_{\mathbb{R}^2}}$$



at $-k$)

$$\langle \alpha | \beta \rangle \times \langle 0 | (P \theta) | 0 \rangle$$

$\vec{b} \rightarrow \vec{b} + \alpha$
 $\vec{b}^+ \rightarrow \vec{b}^+ + \beta$

Urakawa

coherent state

$$\tilde{b} |\beta\rangle = \beta |\beta\rangle$$

$$|0\rangle_a = \int_{-\infty}^{\infty} d\beta \sqrt{\frac{s}{\pi}} e^{-\frac{(s\beta)^2}{2}} |\beta\rangle_{\tilde{b}}$$

$$s \rightarrow 0$$

$$= \hat{\sigma}_I + \hat{\sigma}_I^2 + \dots$$

$$\langle 0 | P \Theta | 0 \rangle_a$$

$$\approx \int d\alpha \int d\beta \langle \alpha | P \Theta | \beta \rangle_{\tilde{b}}$$



$$\text{cut-}k) \langle \alpha | \beta \rangle_{\tilde{b}} \times \langle 0 | (P \Theta) | 0 \rangle_{\tilde{b}}$$

$$\tilde{b} \rightarrow \tilde{b} + \alpha$$

$$\tilde{b}^\dagger \rightarrow \tilde{b}^\dagger + \beta$$

$$\langle P \Theta \rangle$$

CAUTION
 Do not touch the chalkboard
 as it is very hot and may cause injury.

$$P \propto \exp\left(-\frac{\sigma^2}{2(\Delta\sigma)^2}\right)$$

$$P \propto \exp\left(-\frac{\hat{Q}^2}{2(\Delta\sigma)^2}\right)$$

$$\hat{Q} \sim \hat{b} + \hat{b}^\dagger$$

$$P \propto \exp\left(-\frac{\tilde{\sigma}^2}{2(\Delta\sigma)^2}\right) \rightarrow \exp\left(-\frac{(\alpha+\beta)^2}{2(\Delta\sigma)^2}\right)$$

$$\tilde{\sigma} \rightarrow \tilde{b} + \tilde{b}^* \approx \alpha + \beta$$

$$\langle \alpha | \beta \rangle \propto \exp\left(-\frac{(\alpha-\beta)^2}{2}\right)$$

$$P \propto \exp\left(-\frac{\tilde{\sigma}^2}{2(\Delta\sigma)^2}\right) \rightarrow \exp\left(-\frac{(\alpha+\beta)^2}{2(\Delta\sigma)^2}\right)$$

$$\tilde{\sigma} \rightarrow \tilde{b} + \tilde{b}^* \approx \alpha + \beta$$

$$\langle \alpha | \beta \rangle \propto \exp\left(-\frac{(\alpha-\beta)^2}{2}\right)$$

$$P \propto \exp\left(-\frac{\hat{\sigma}^2}{2(\Delta\sigma)^2}\right) \rightarrow \exp\left(-\frac{(\alpha+\beta)^2}{2(\Delta\sigma)^2}\right)$$

$$\hat{\sigma} \rightarrow \tilde{\sigma} + \hat{\sigma}^+ \approx \alpha + \beta$$

$$\langle \alpha | \beta \rangle \propto \exp\left(-\frac{(\alpha-\beta)^2}{2}\right)$$

$(u, a) \xrightarrow{\text{unitary}} (v, b)$

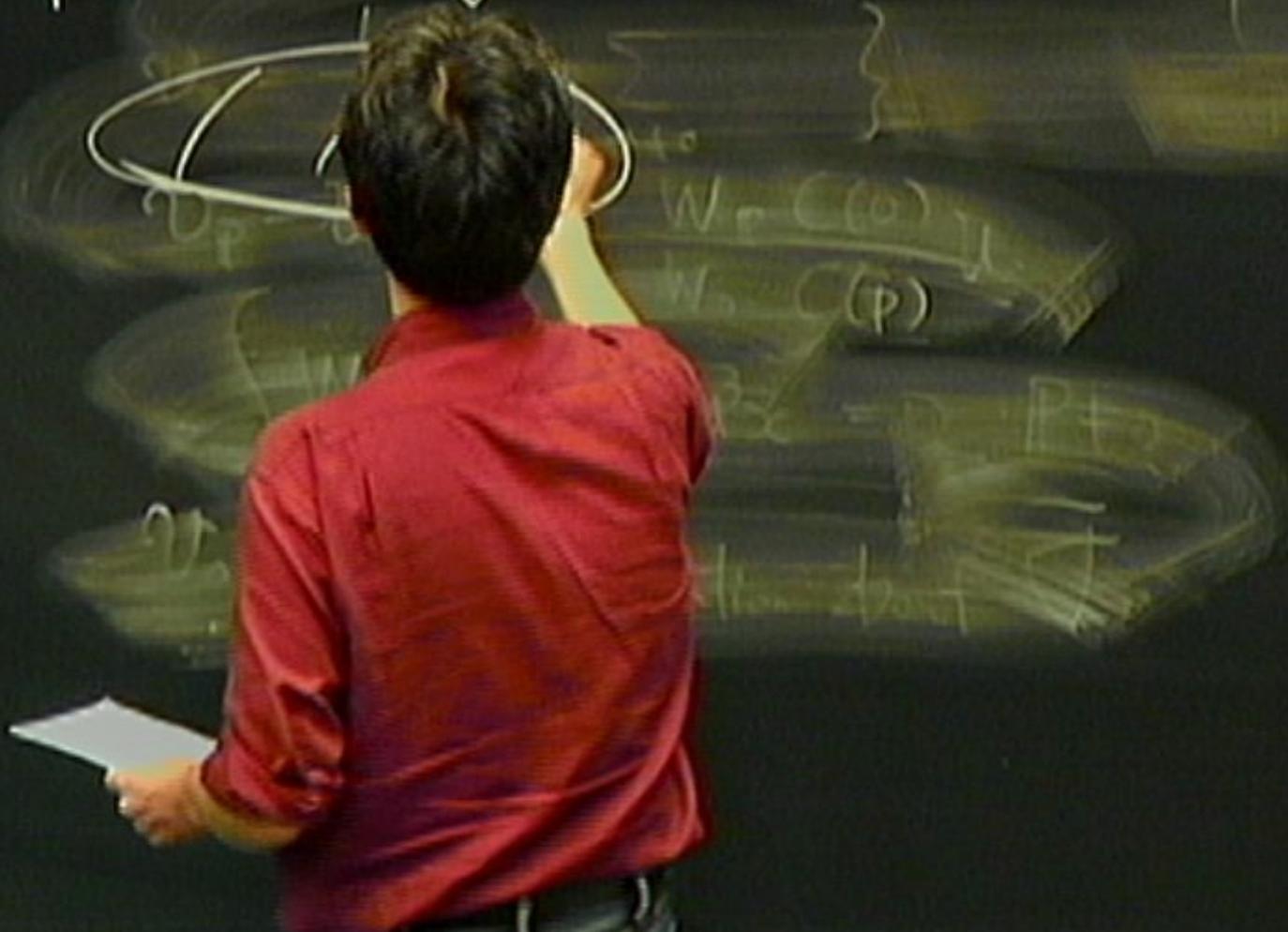
$$v = \left\{ \sum_{p \neq 0} \dots \right\} - \frac{W_p}{W_0} \frac{C(0)}{C(p)} U_0$$

$$\frac{1}{C(p)} = \lim_{\eta \rightarrow 0} U_p$$

$$\int p(x) d^3x = 0 \quad p \neq 0$$

information about \vec{J}

Temporal integral



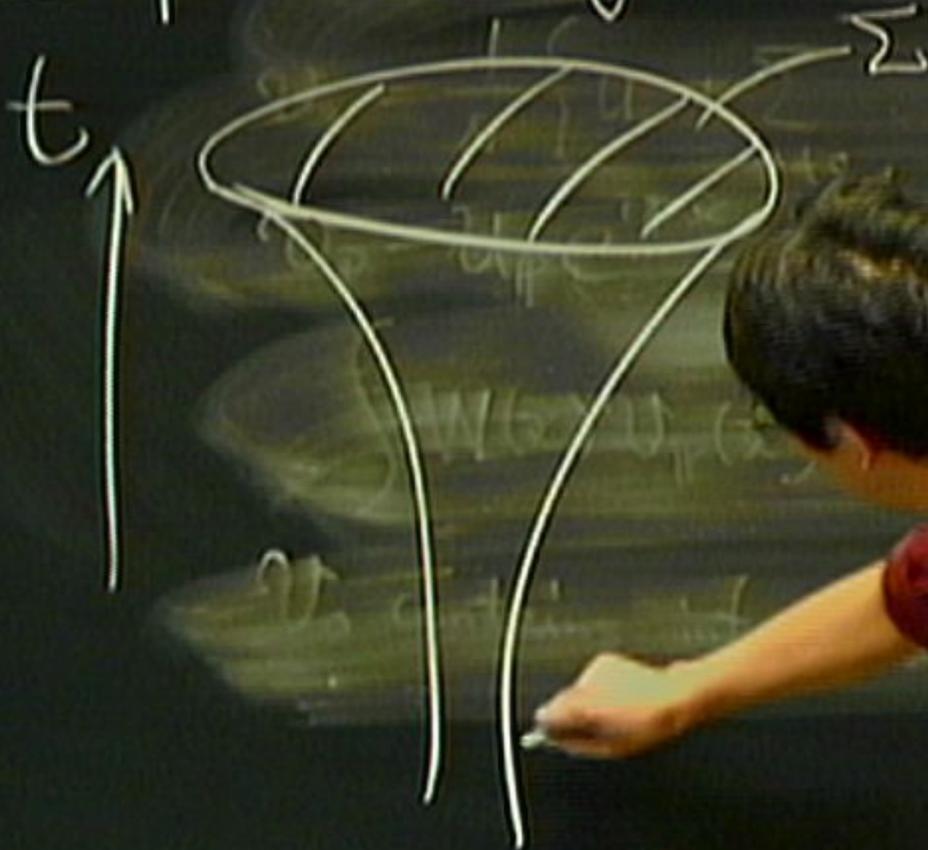
Temporal integral



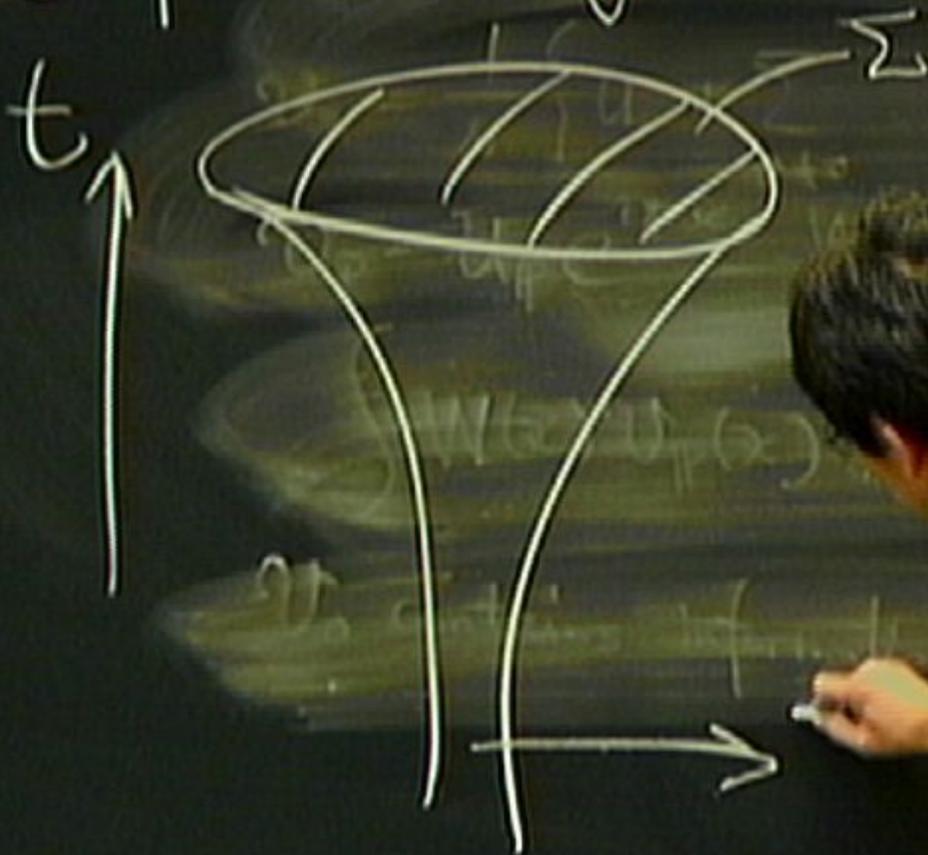
$$\int_{-\infty}^{\infty} W(\omega) U(\omega) d\omega$$

U_s contains

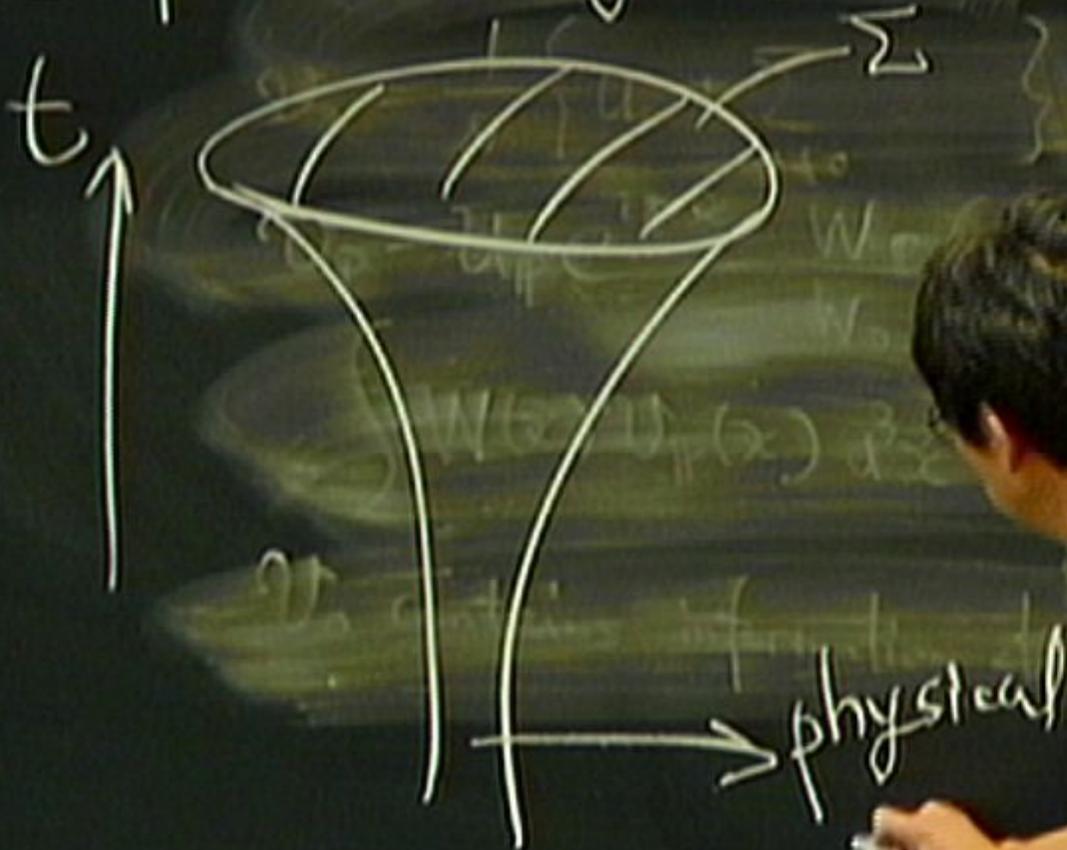
Temporal integral



Temporal integral

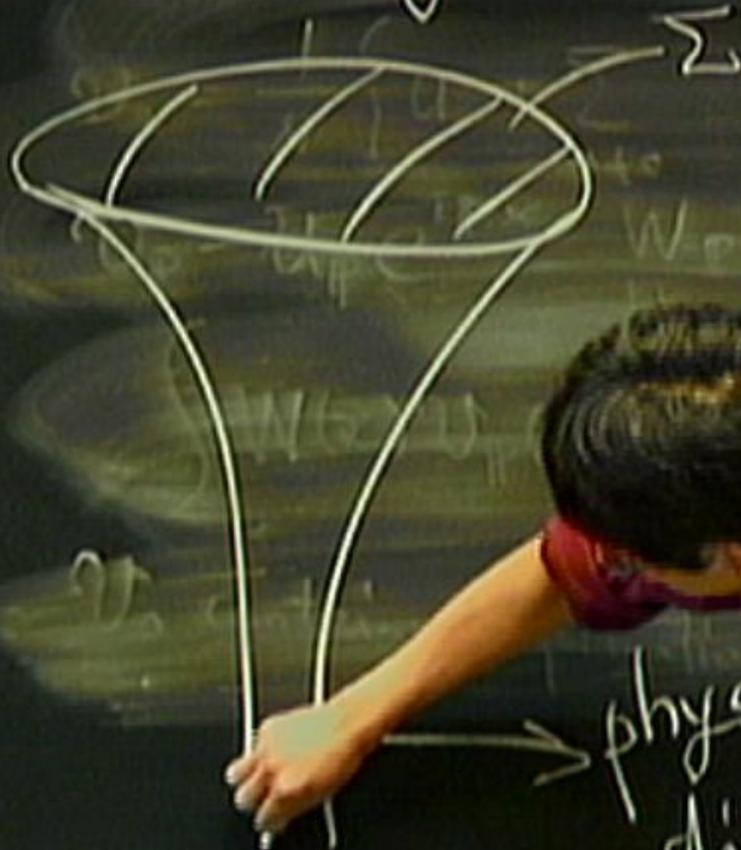


Temporal integral



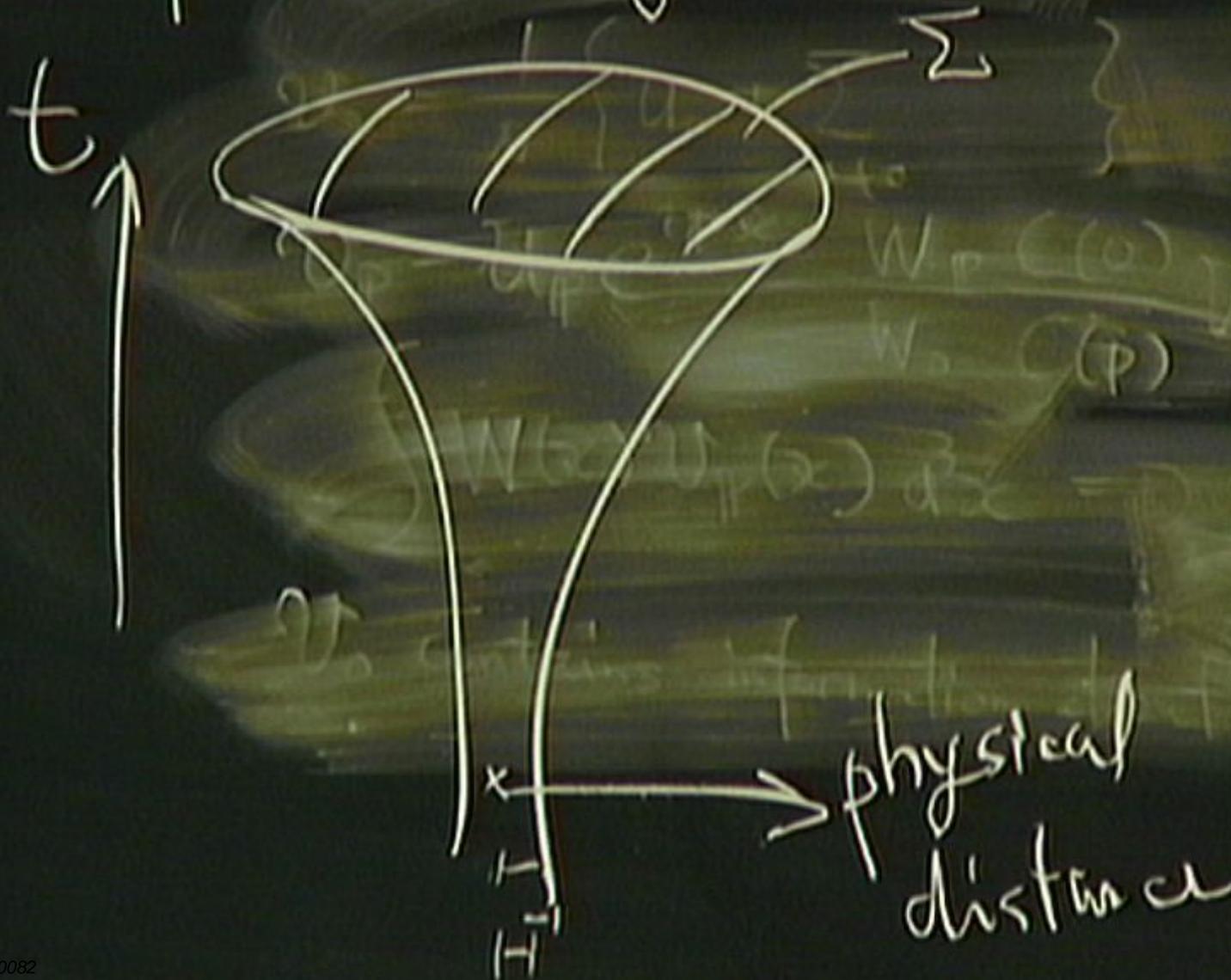
Temporal integral

t ↑

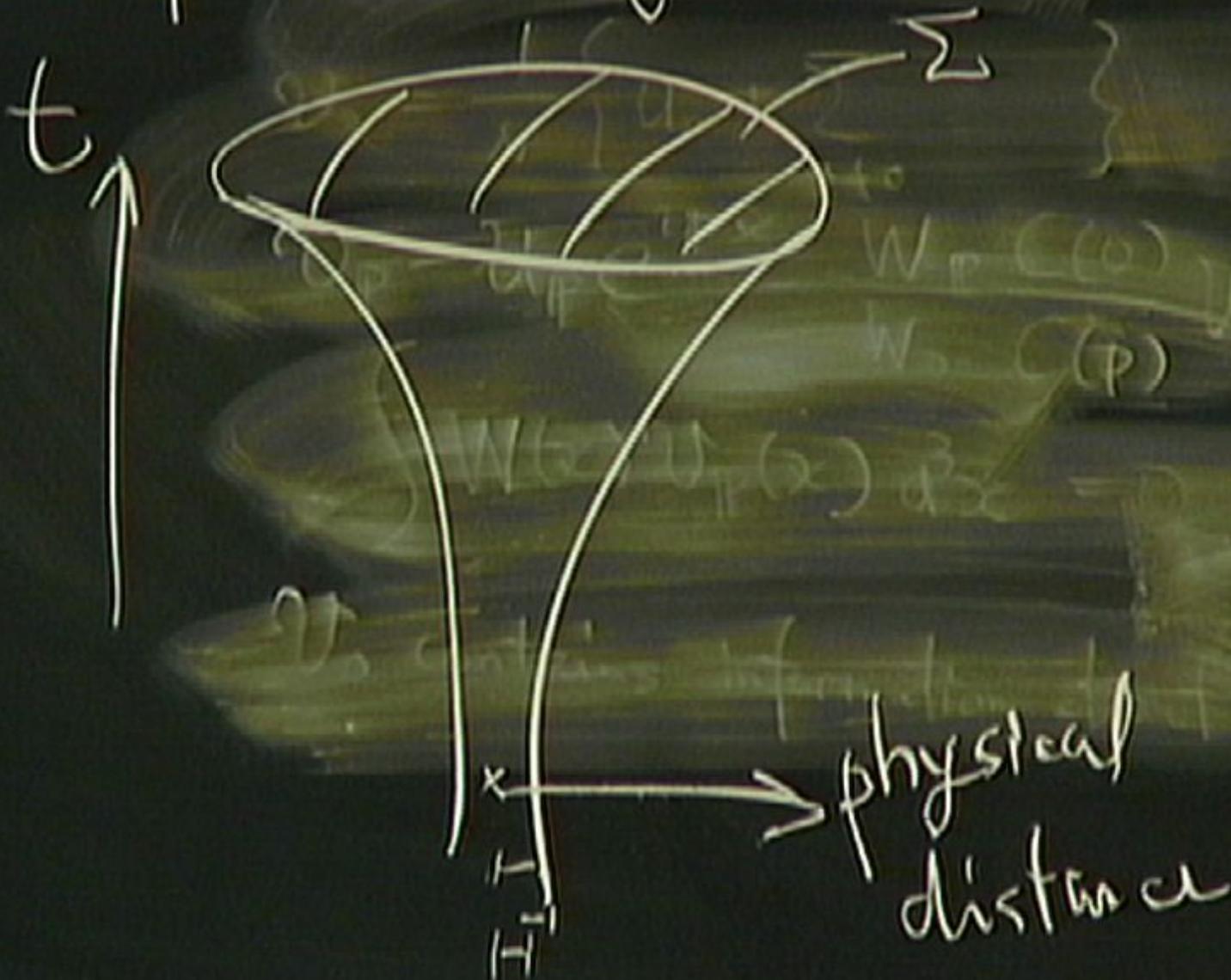


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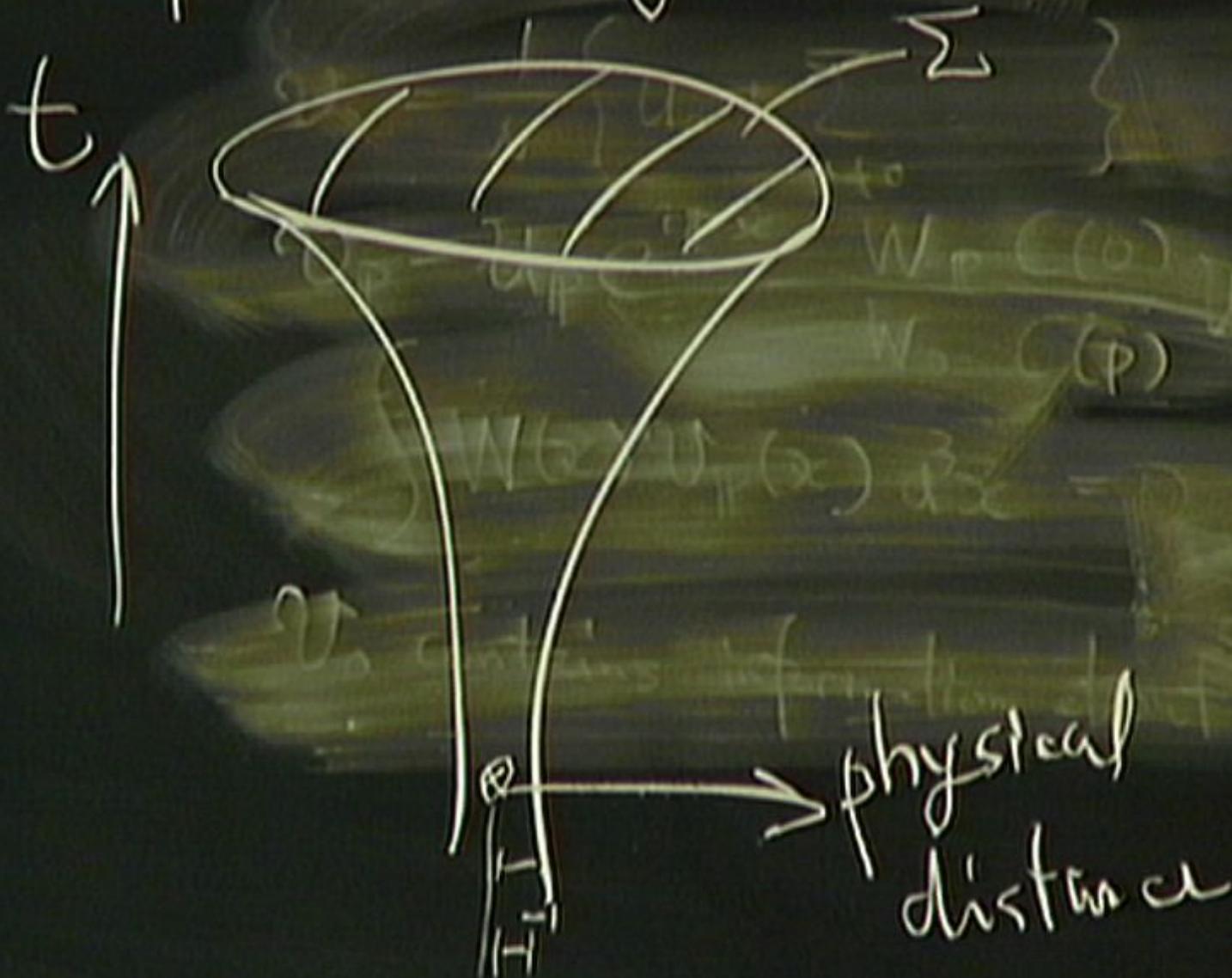
Temporal integral



Temporal integral

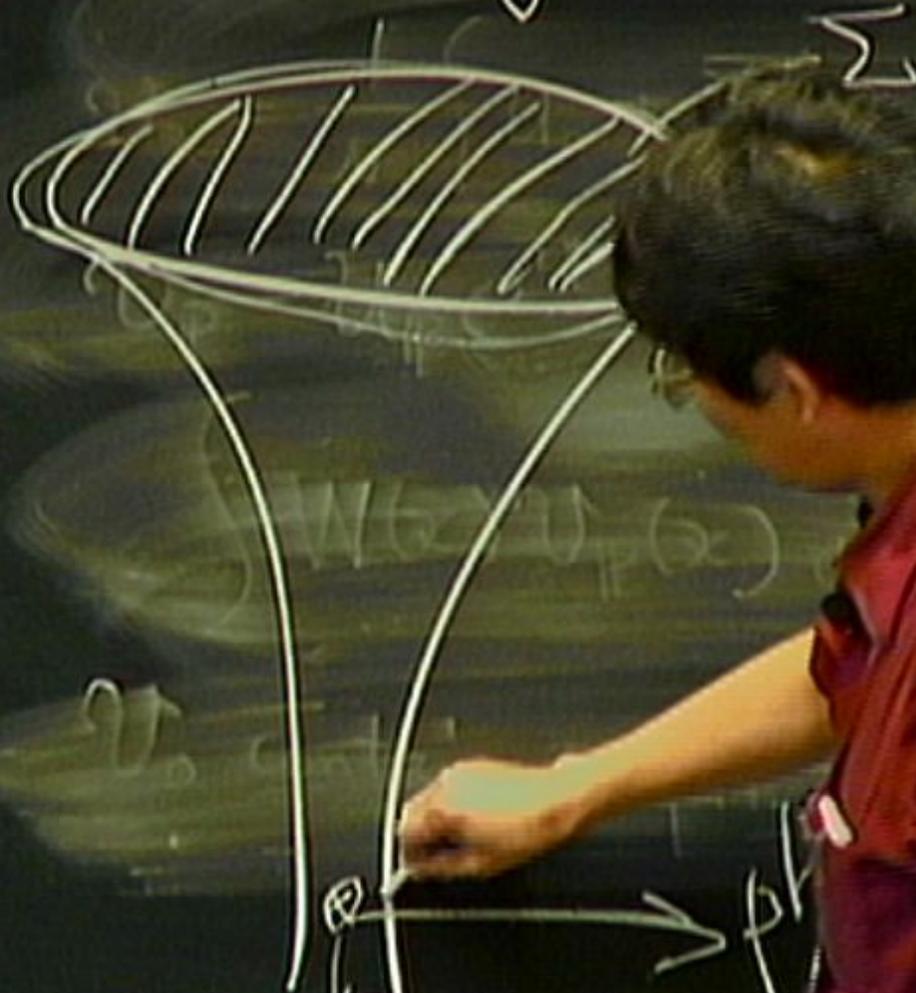


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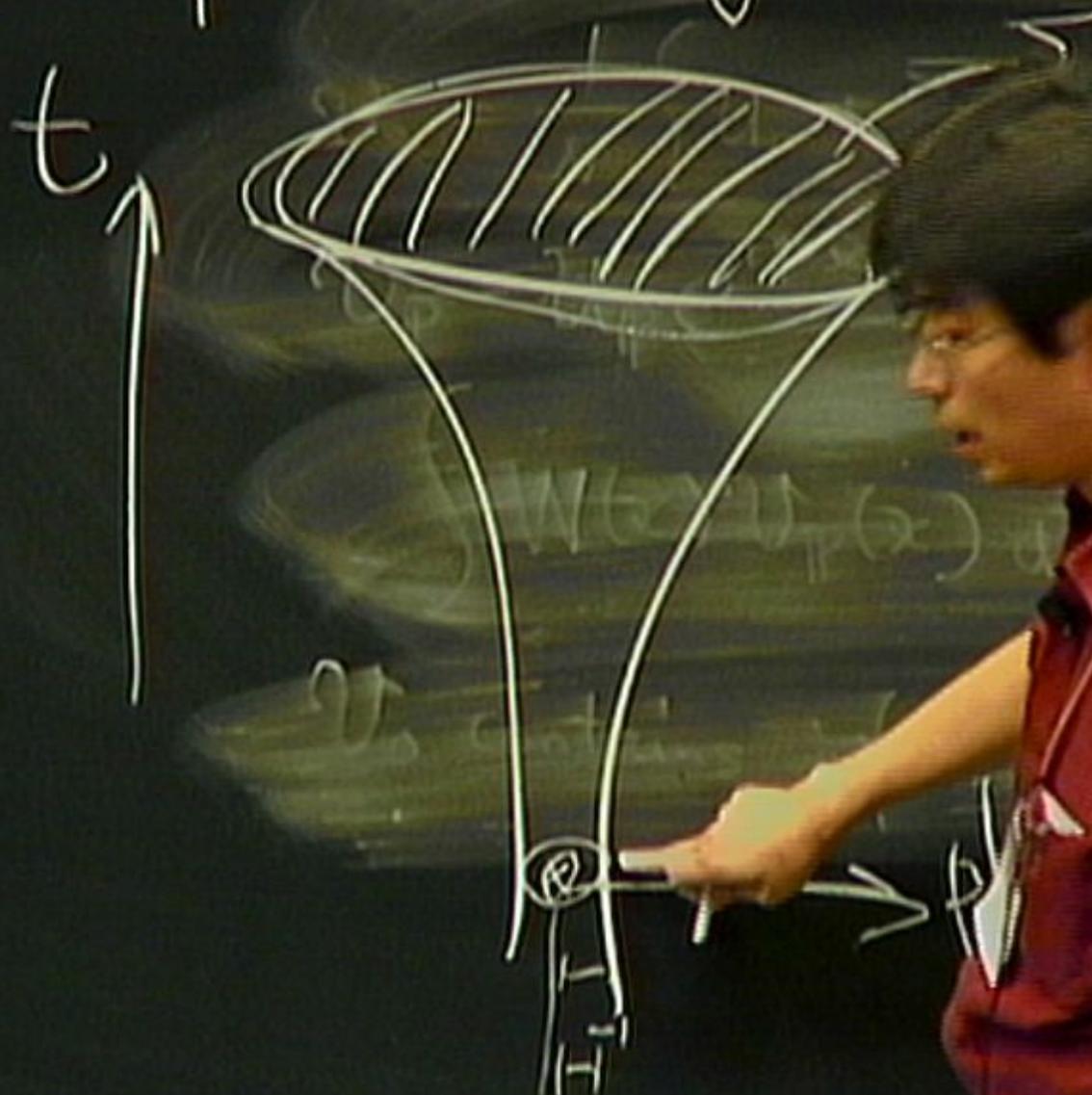


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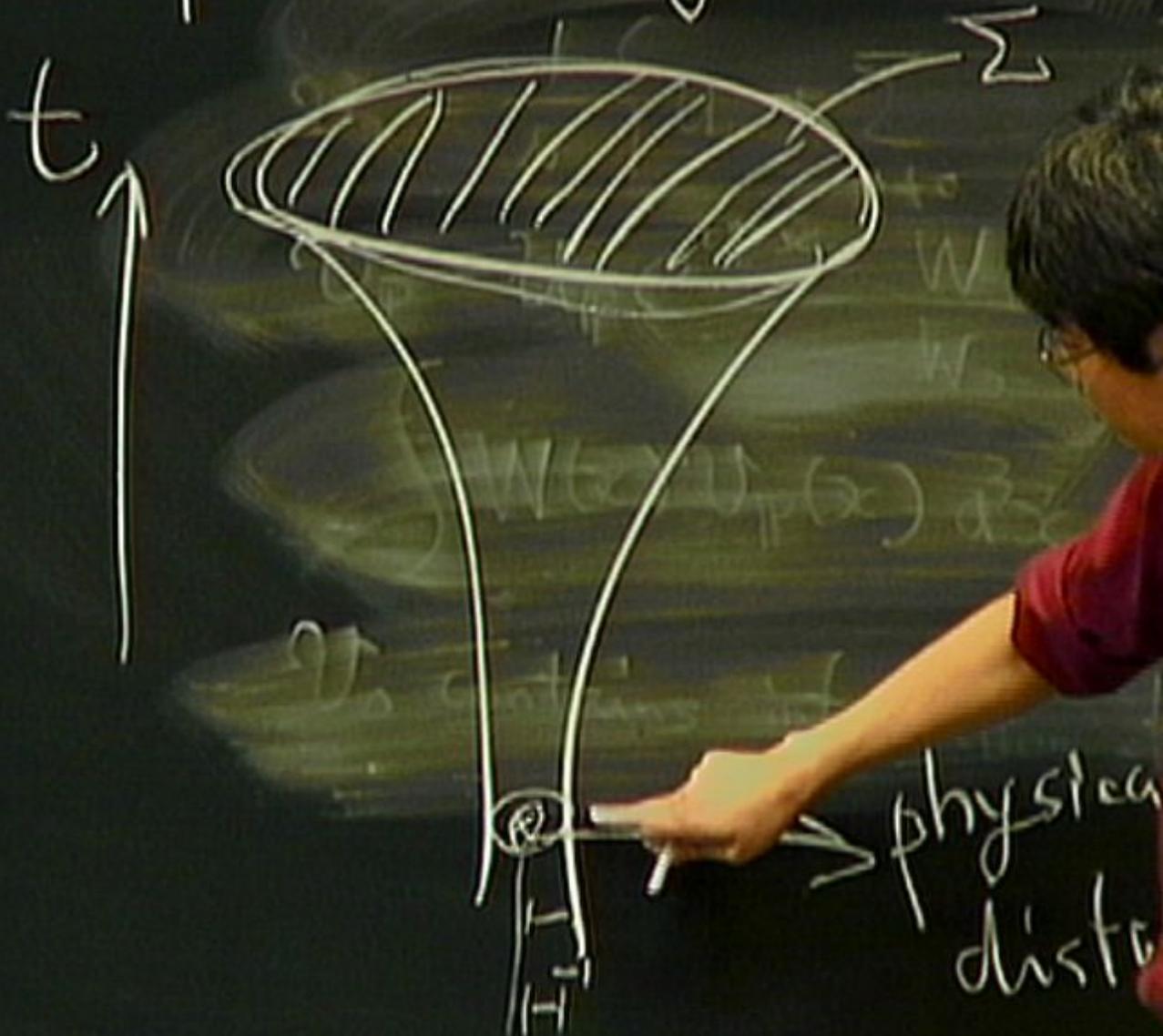
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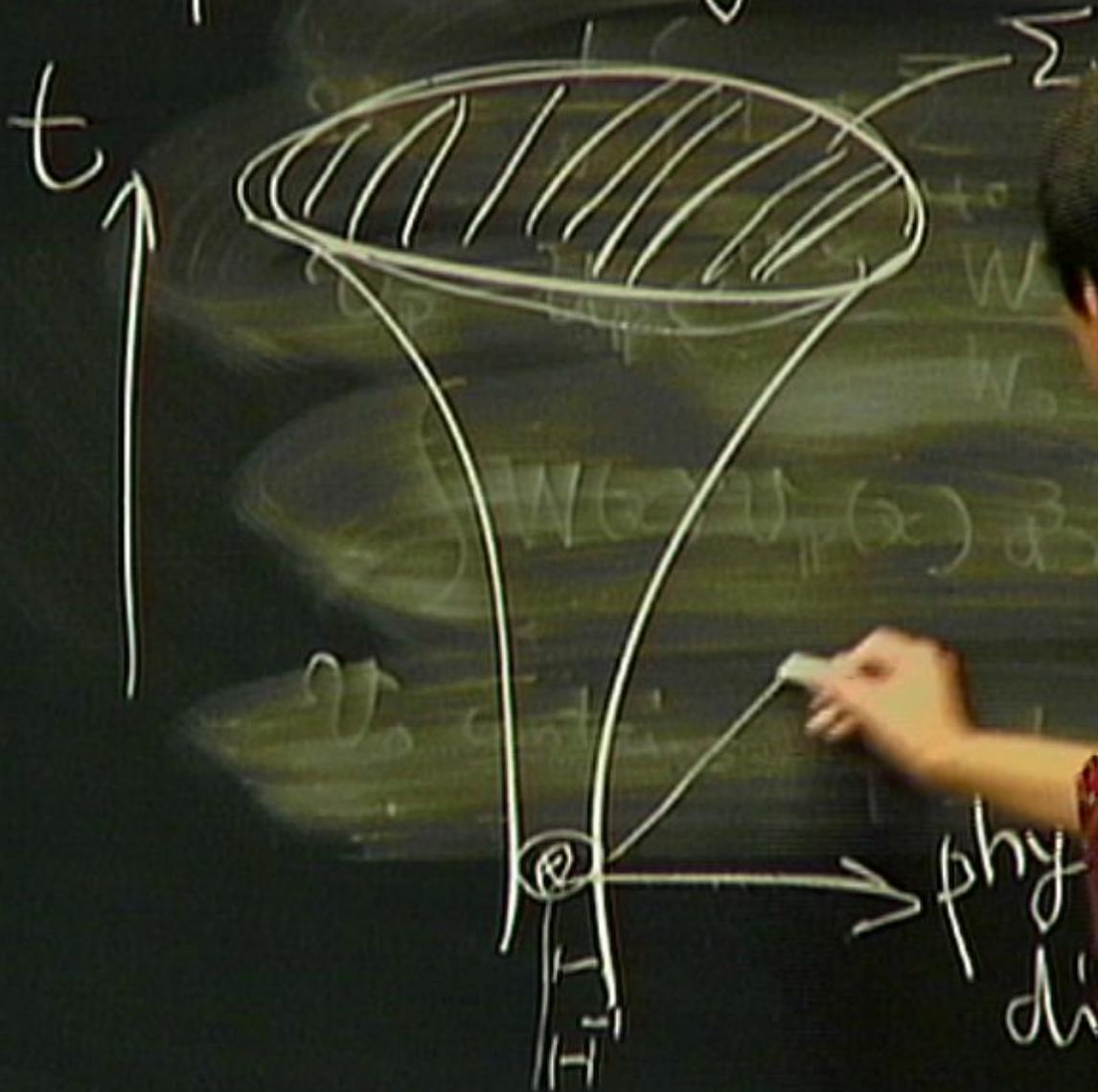
Temporal integral



Temporal integral



Temporal integral

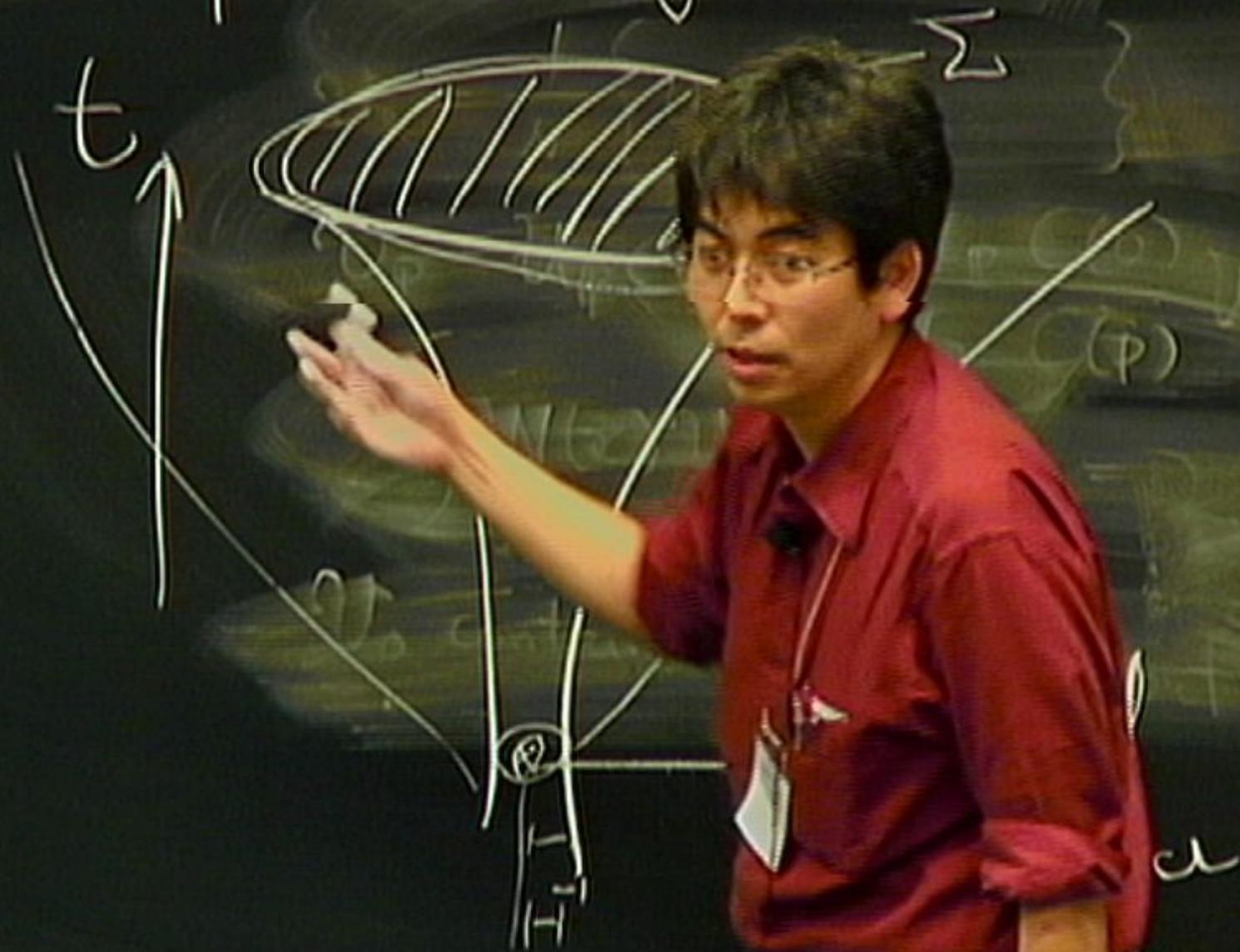


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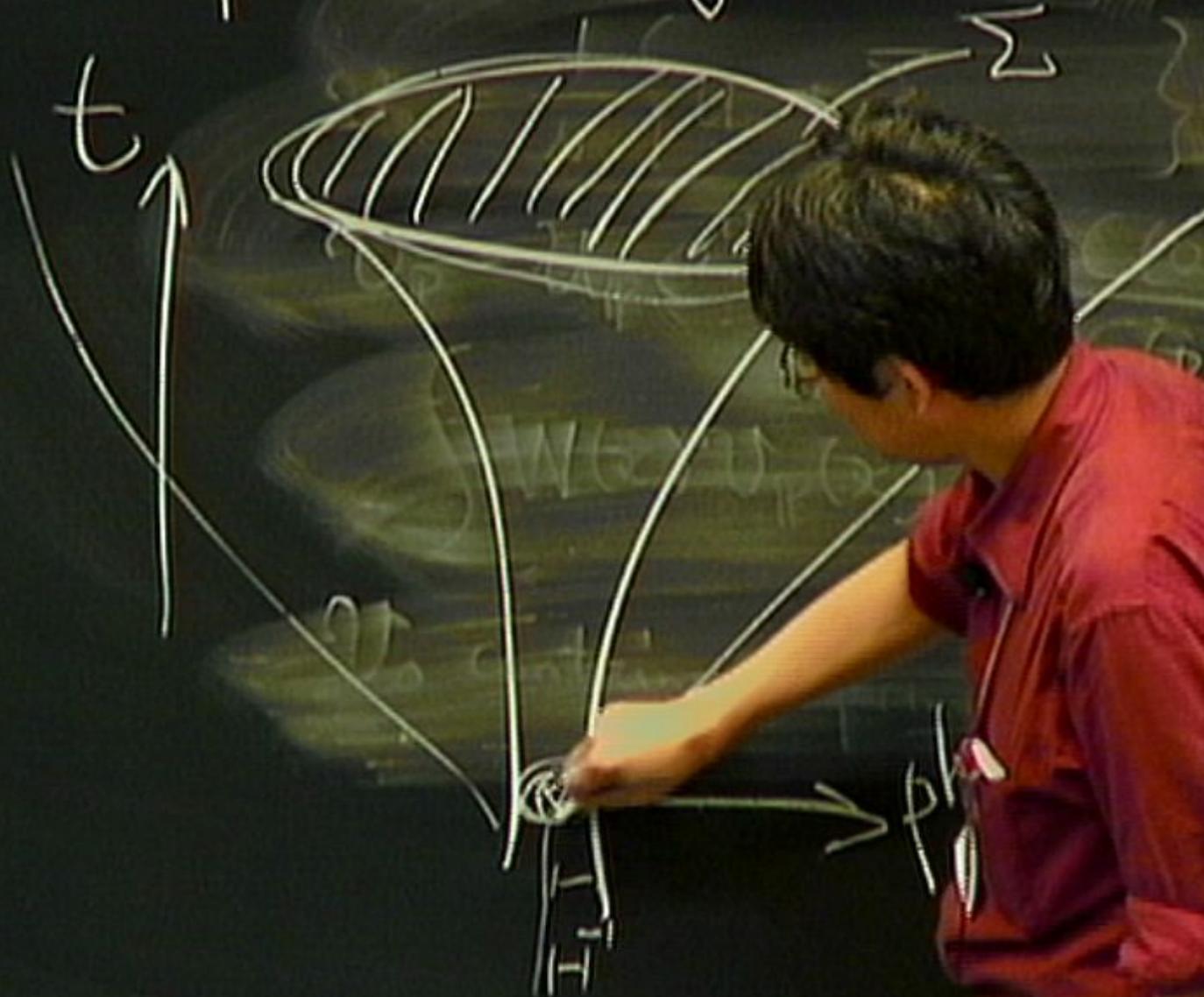
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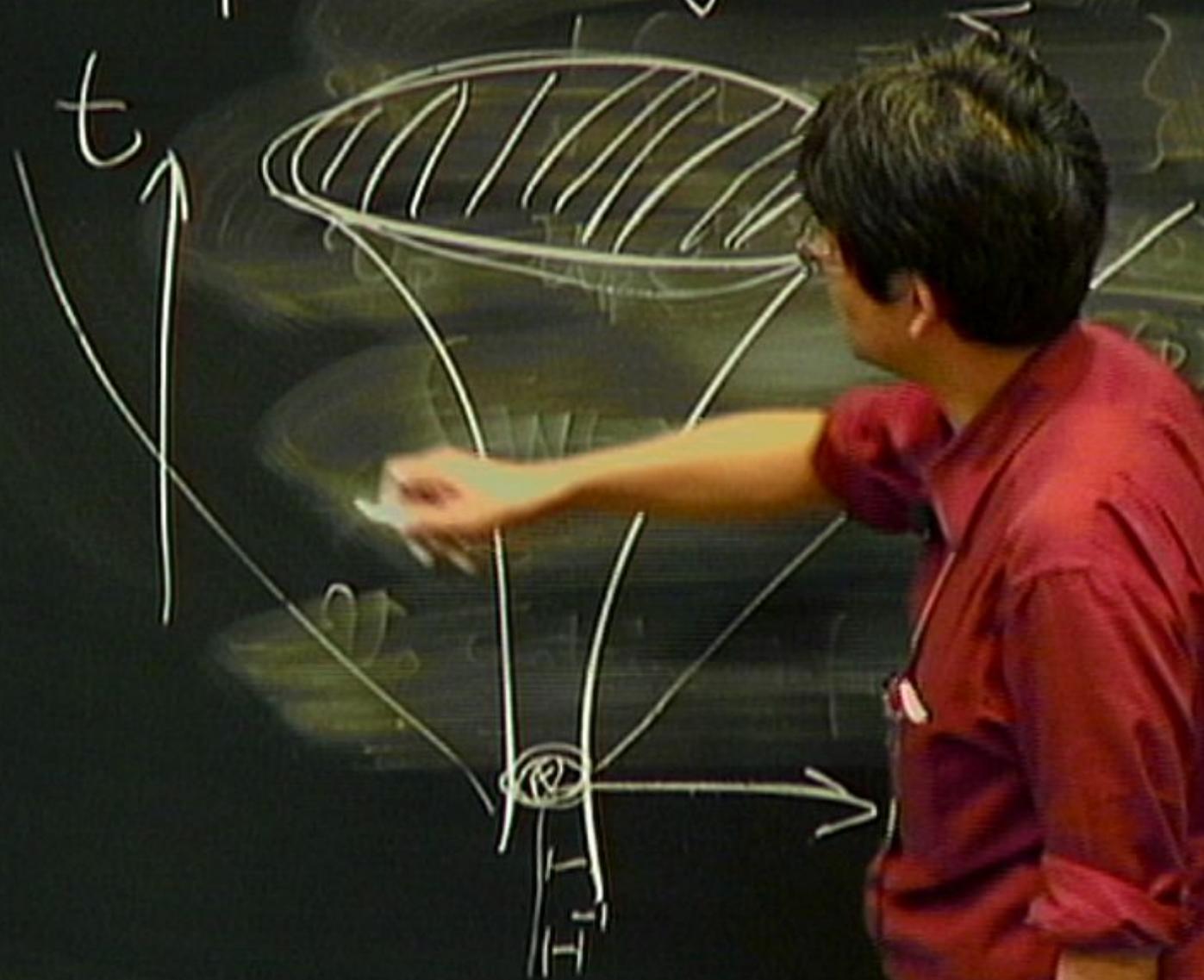
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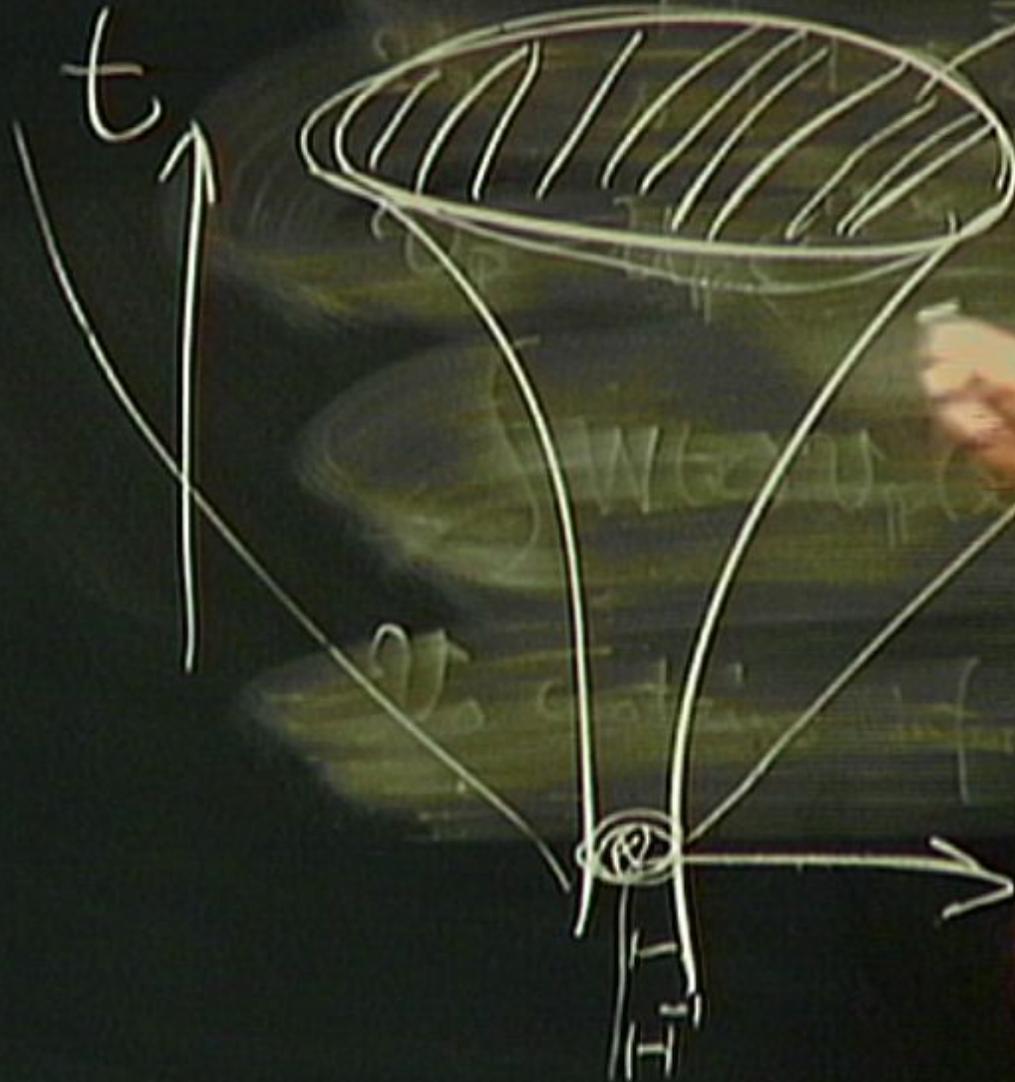
Temporal integral



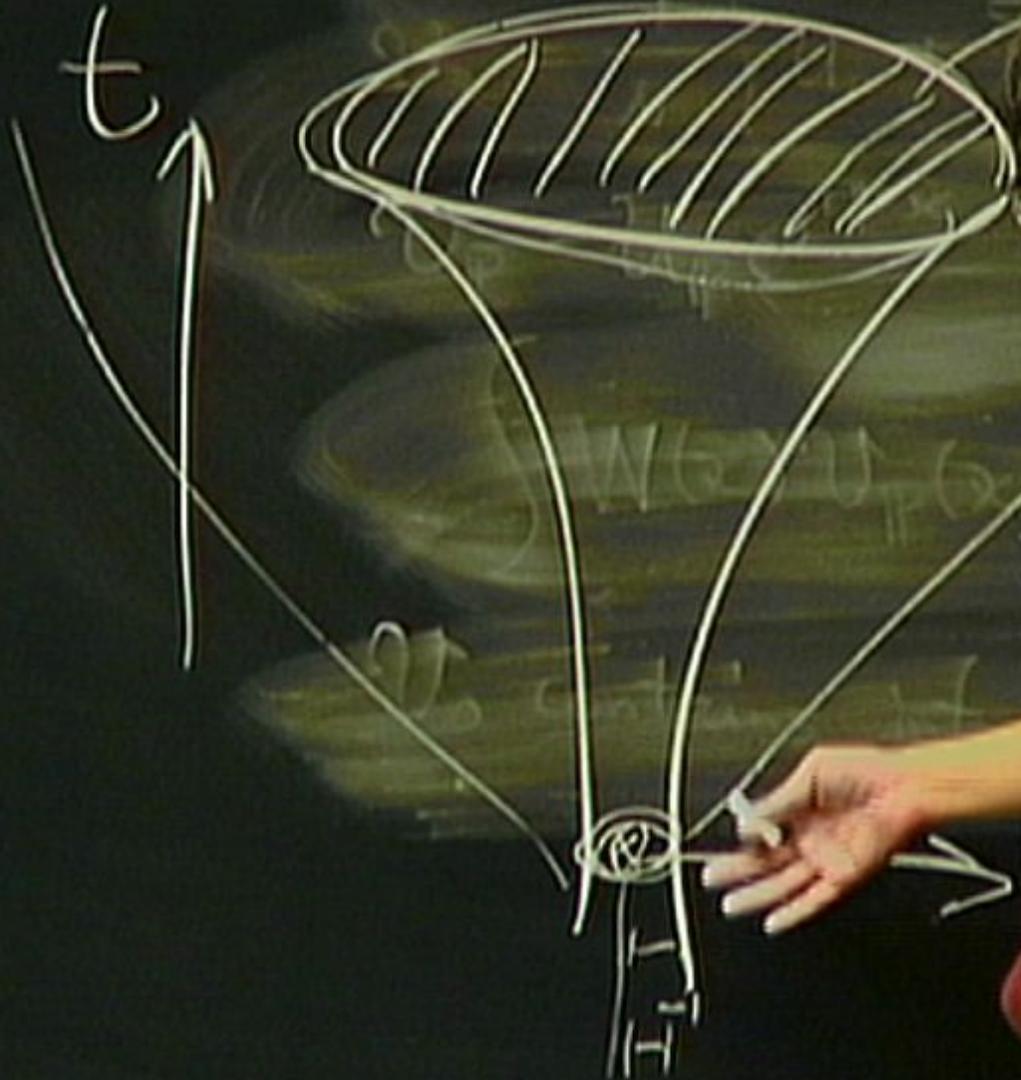
Temporal integral



Temporal integral



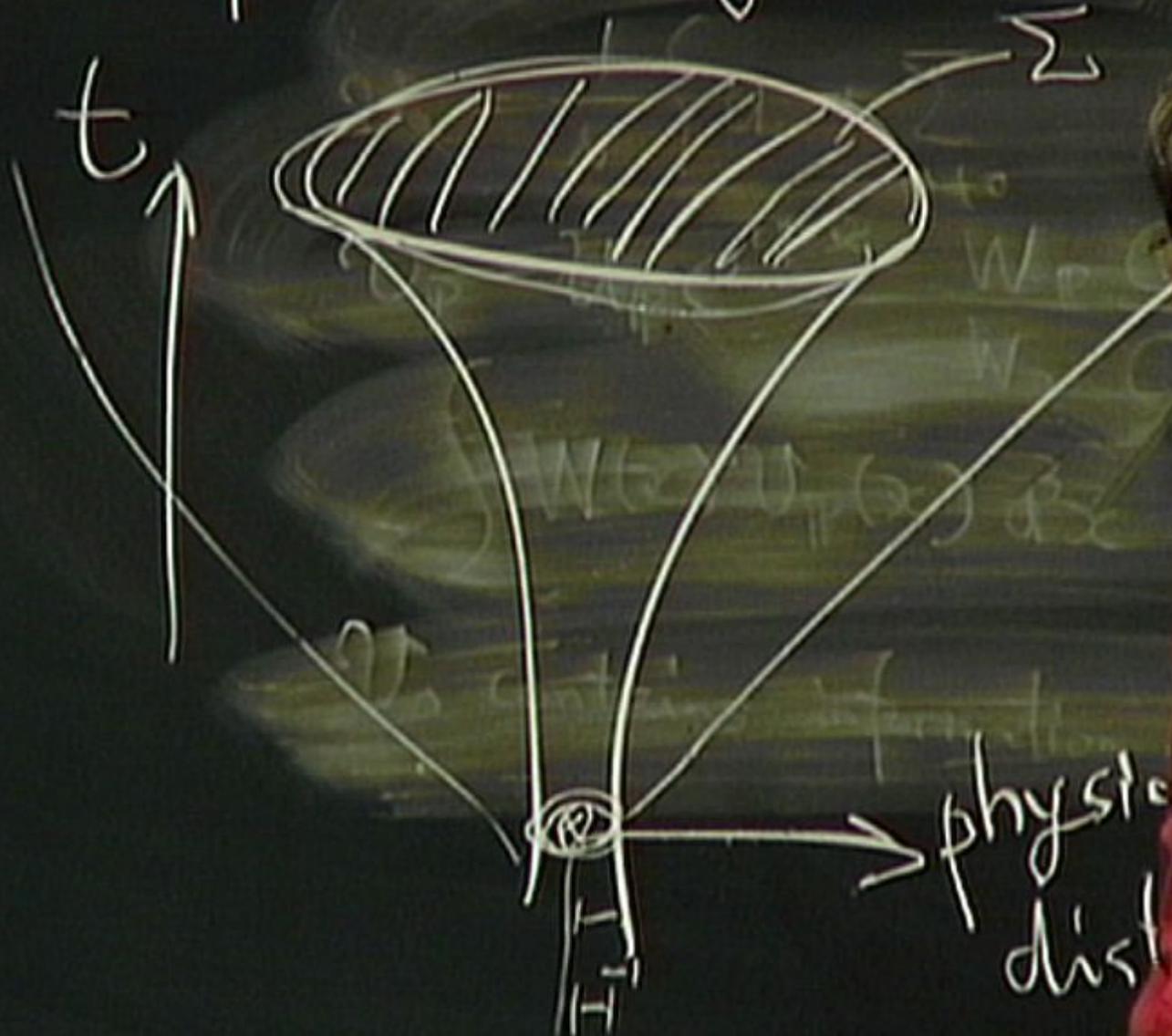
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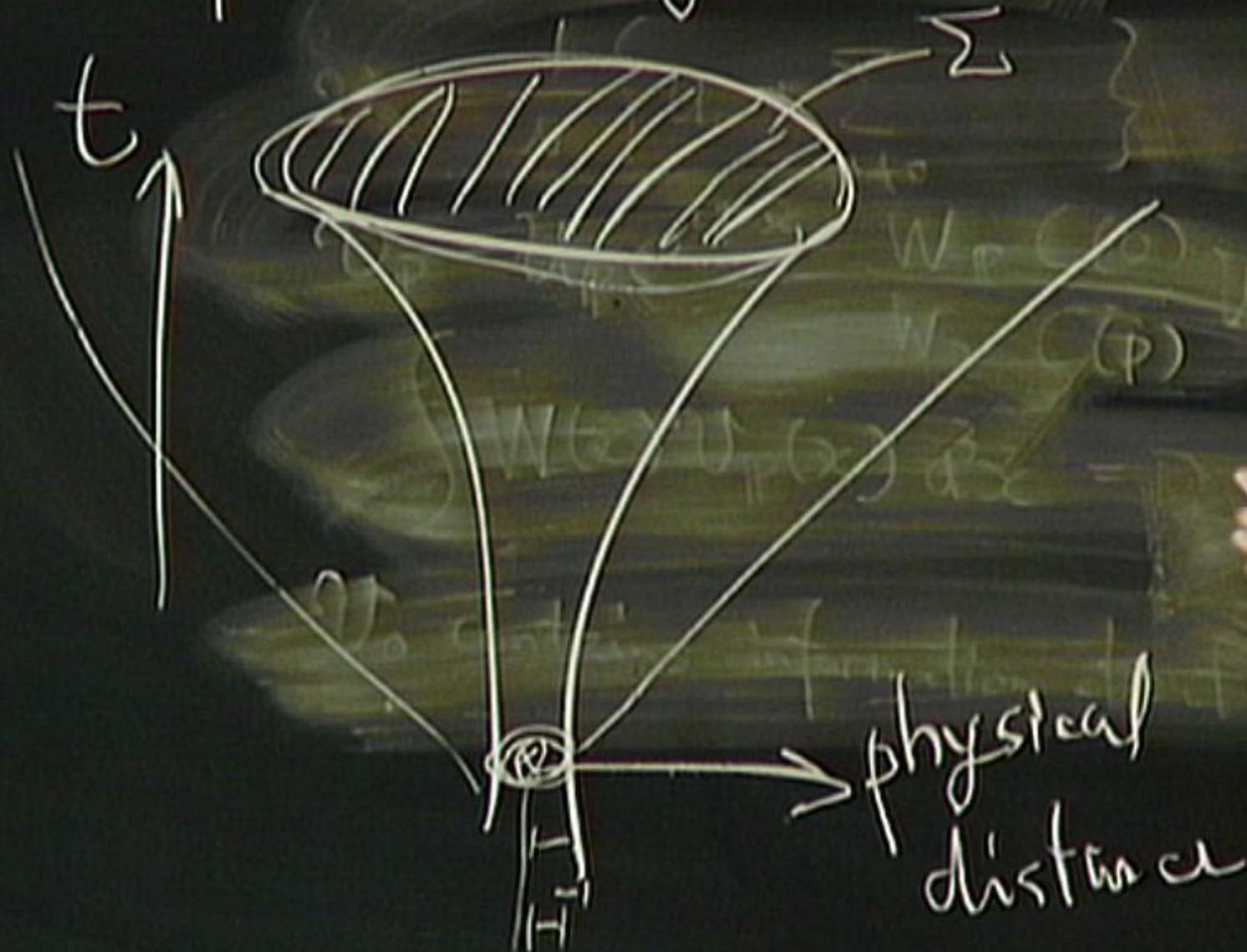
Temporal integral



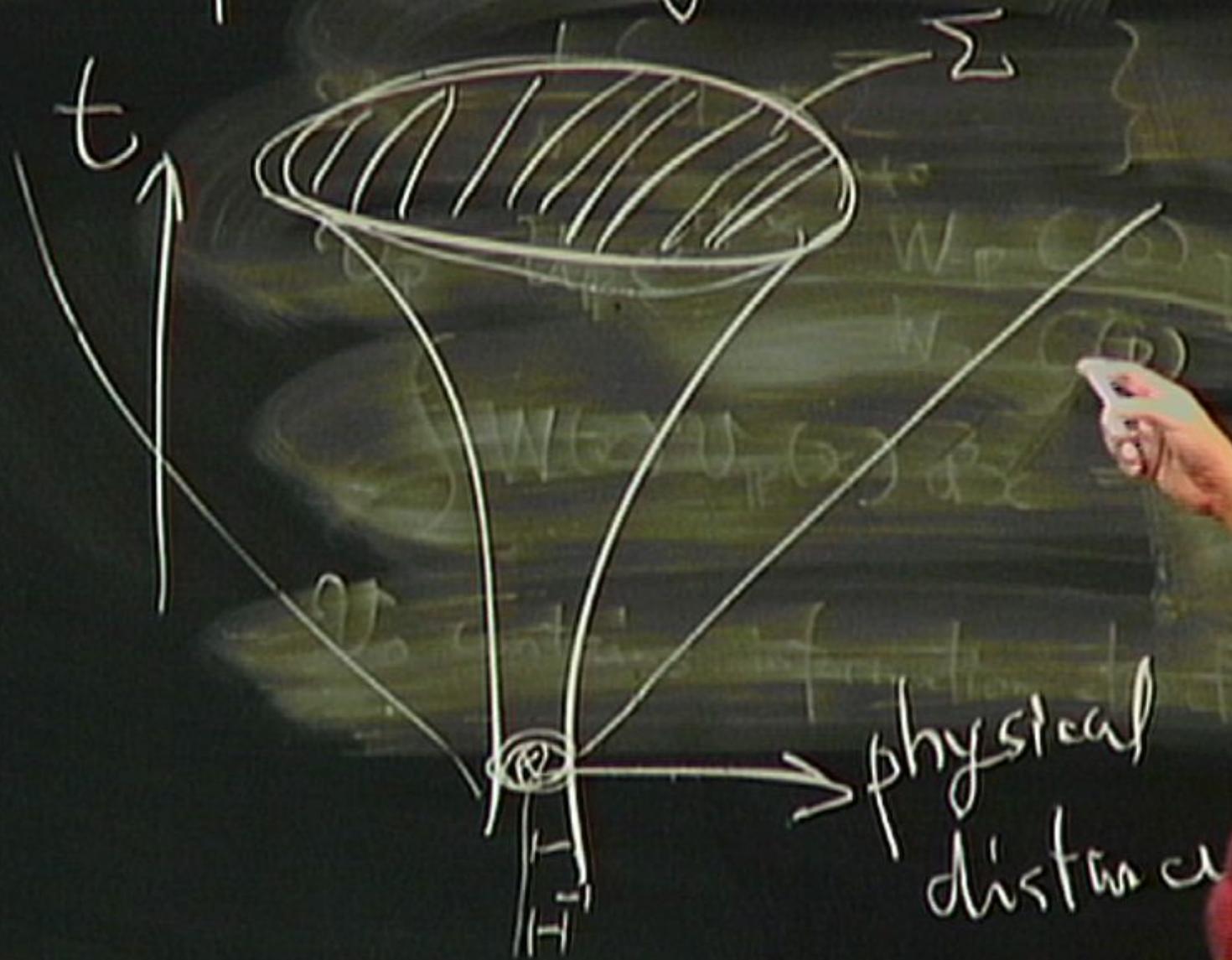
Temporal integral



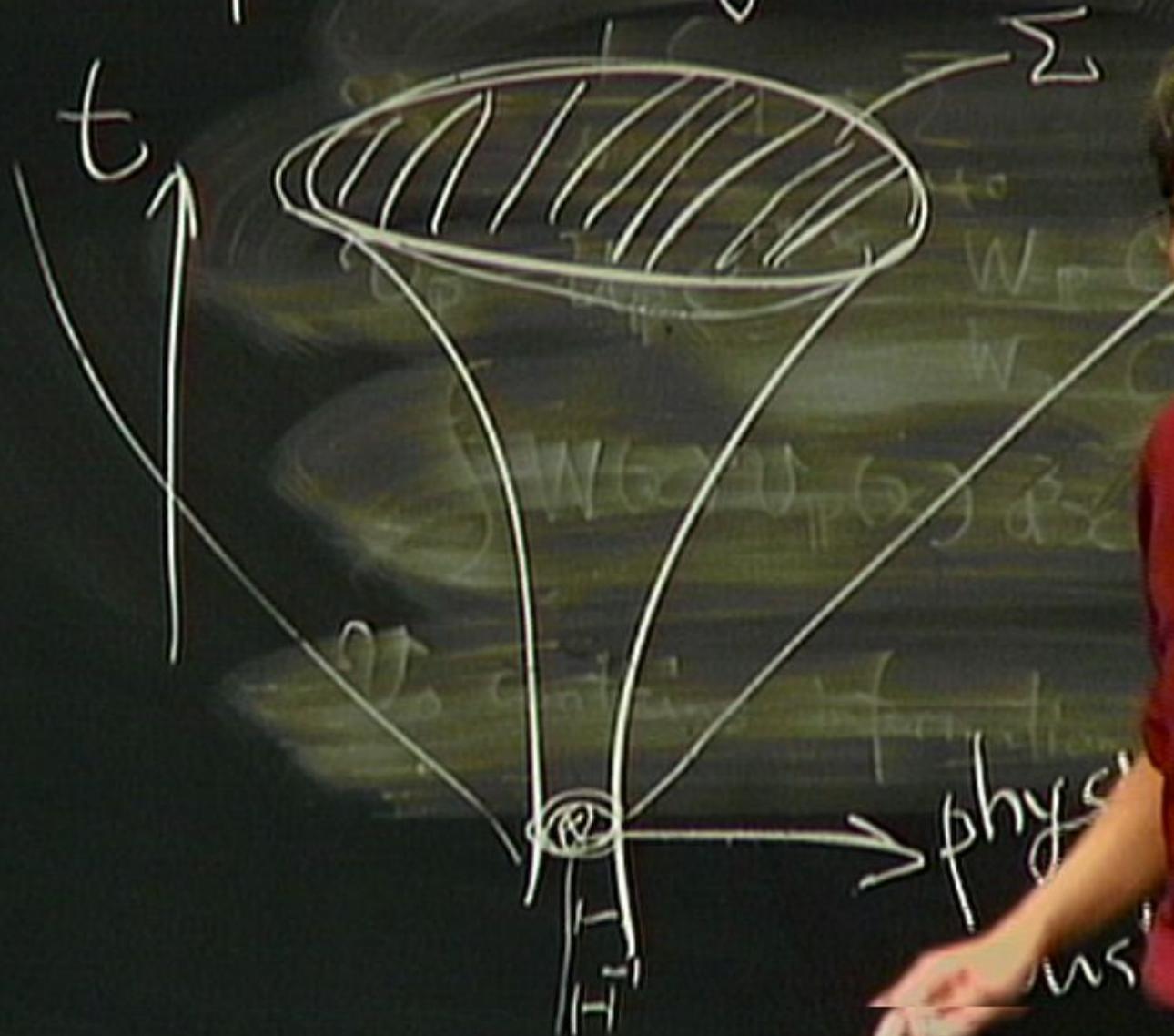
Temporal integral



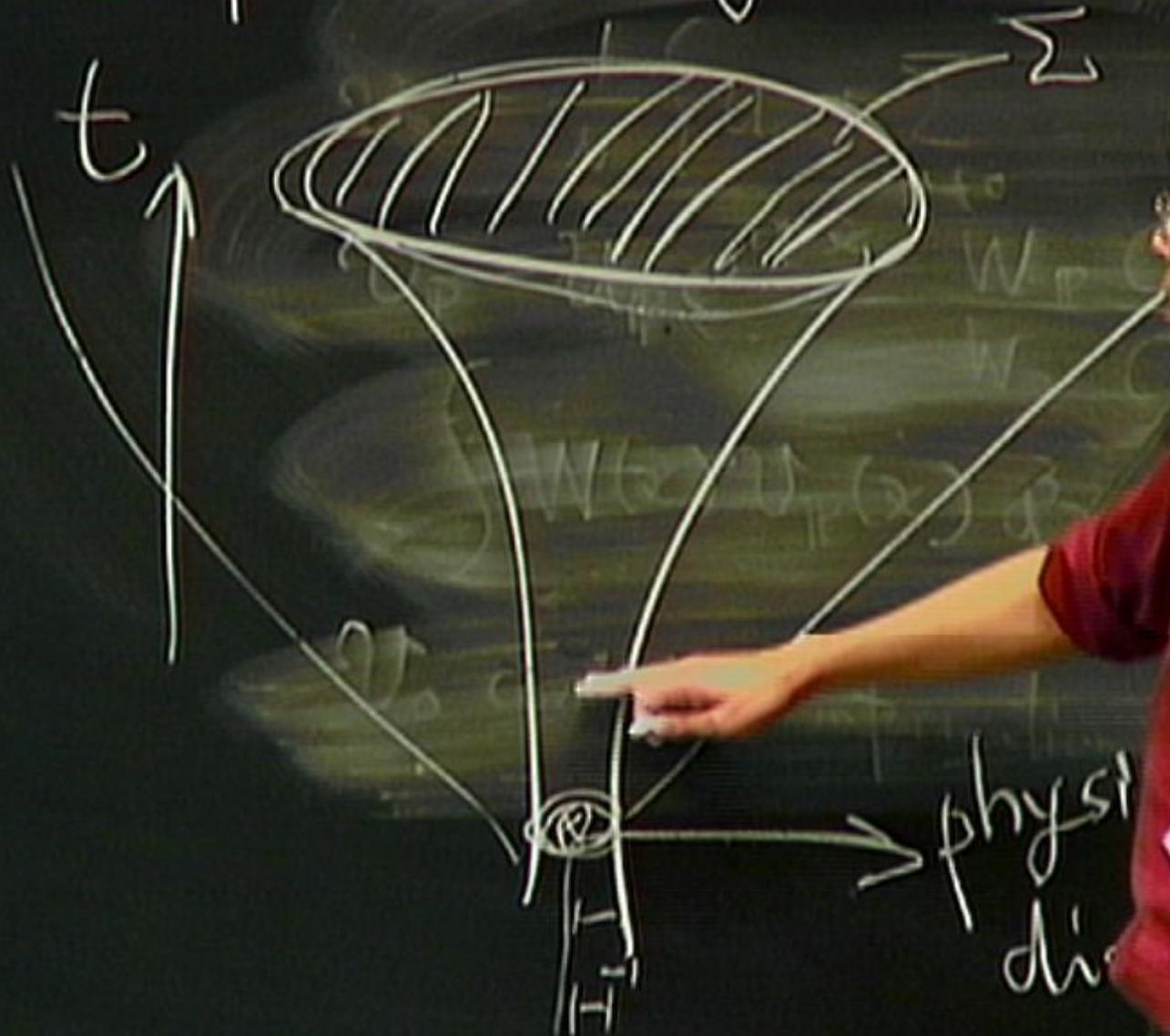
Temporal integral



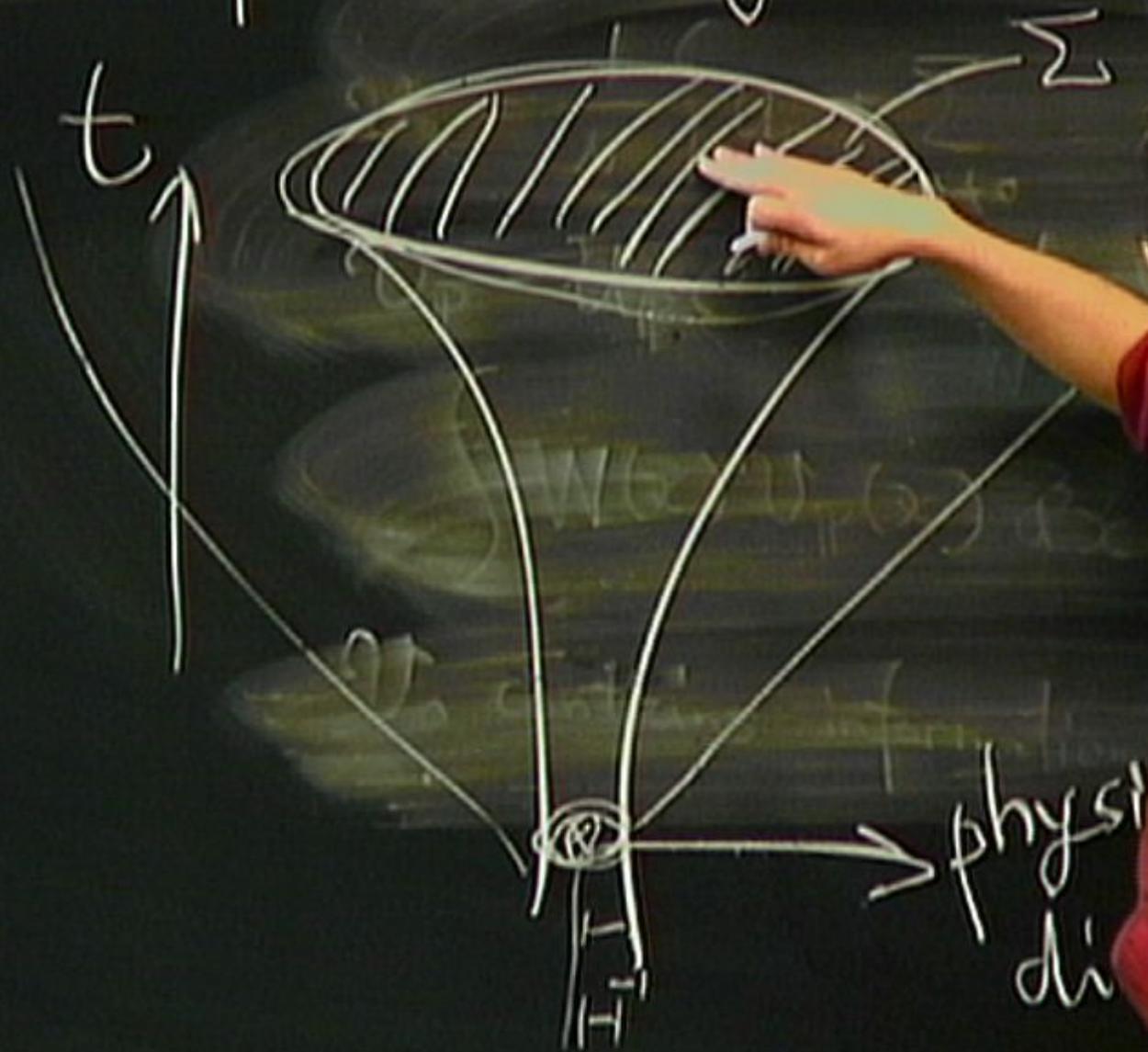
Temporal integral



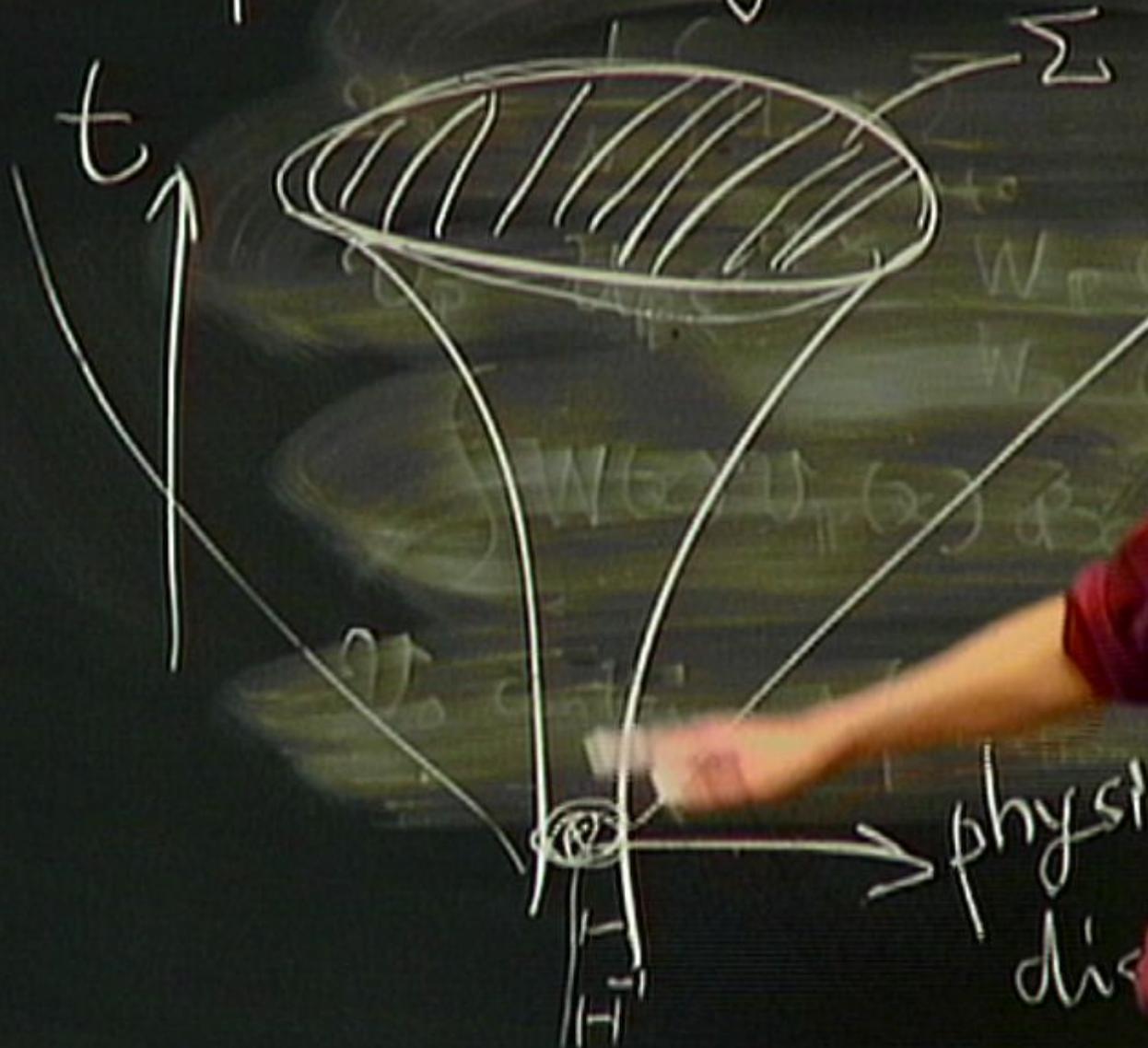
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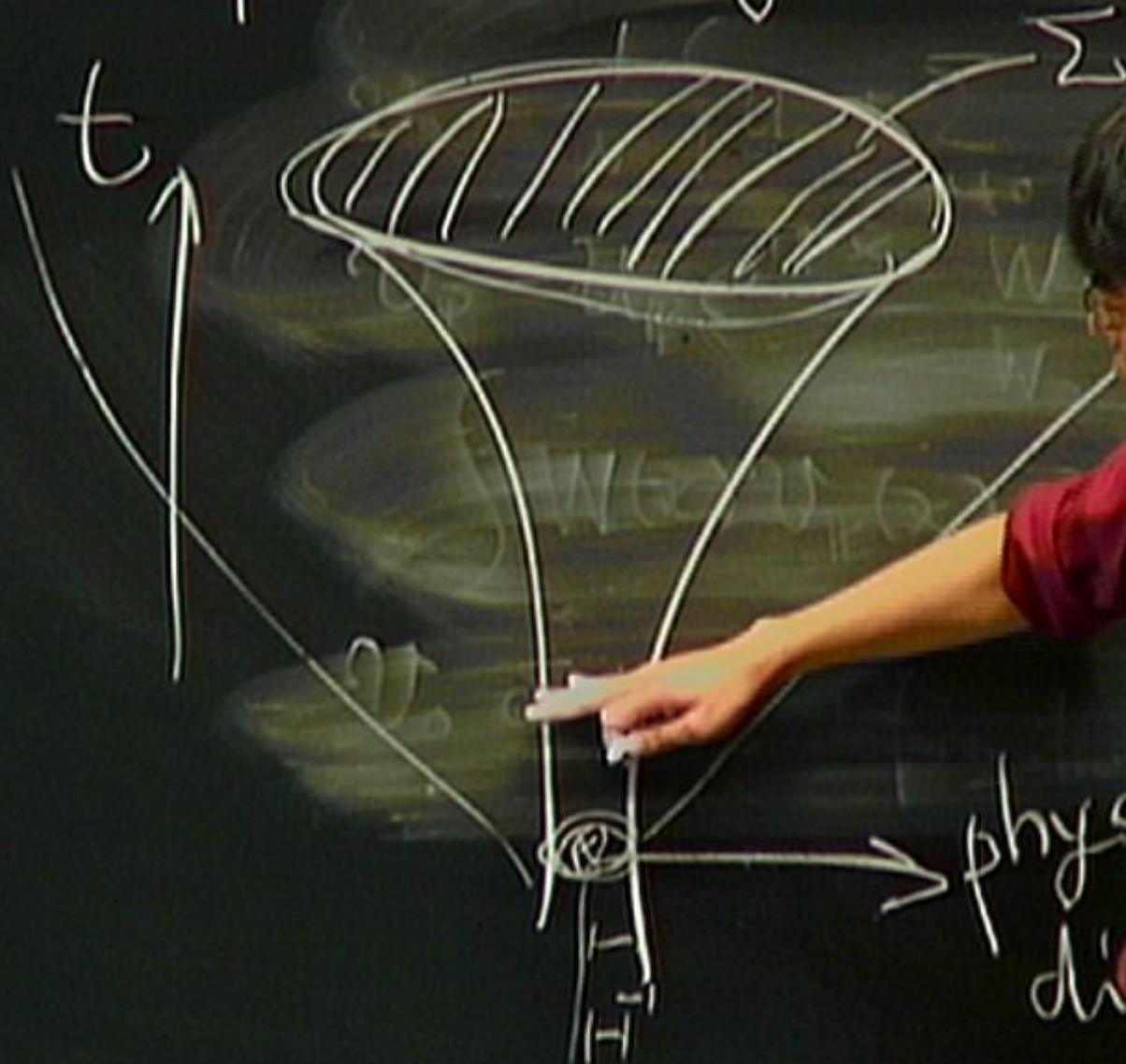
Temporal integral



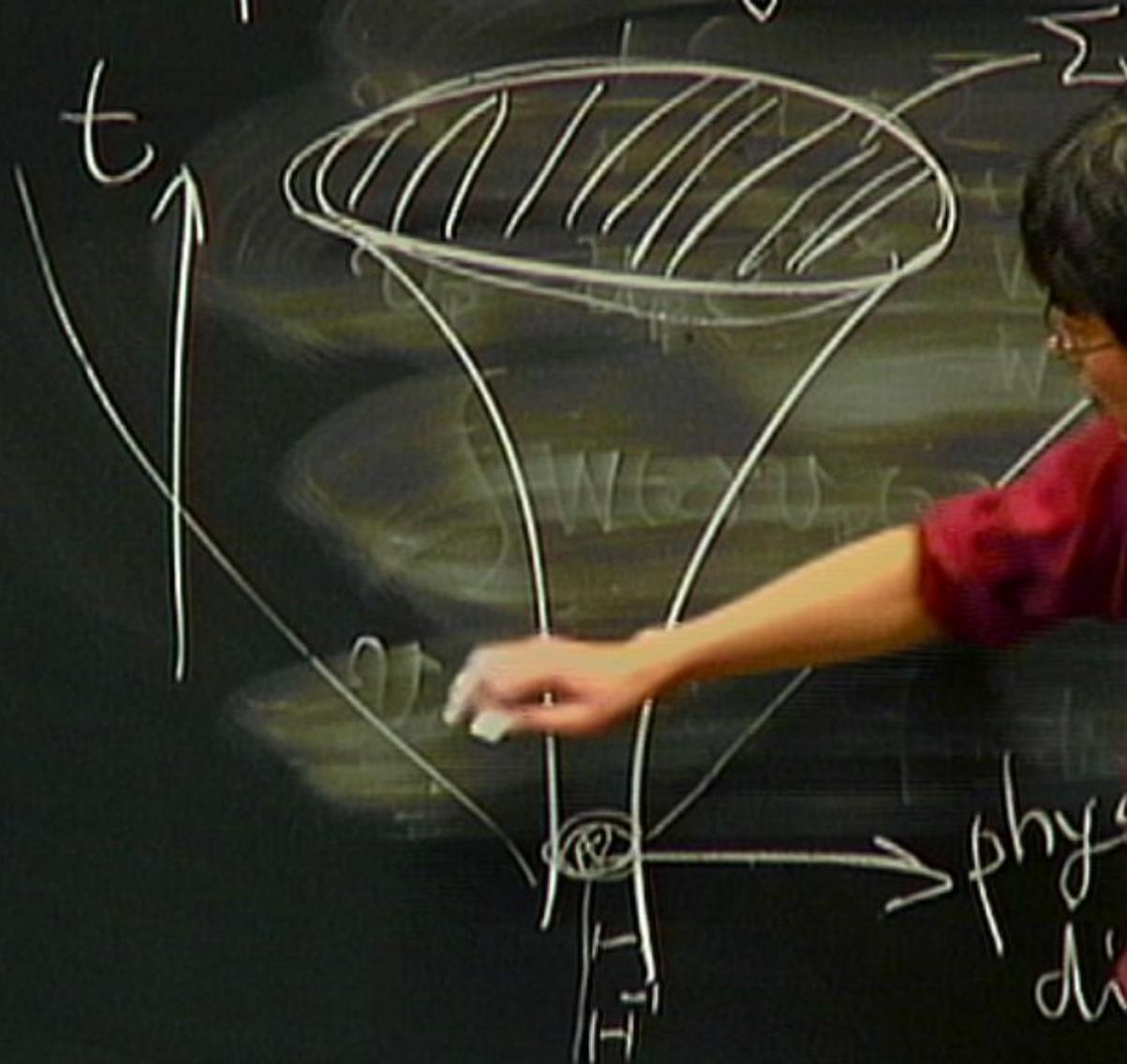
Temporal integral



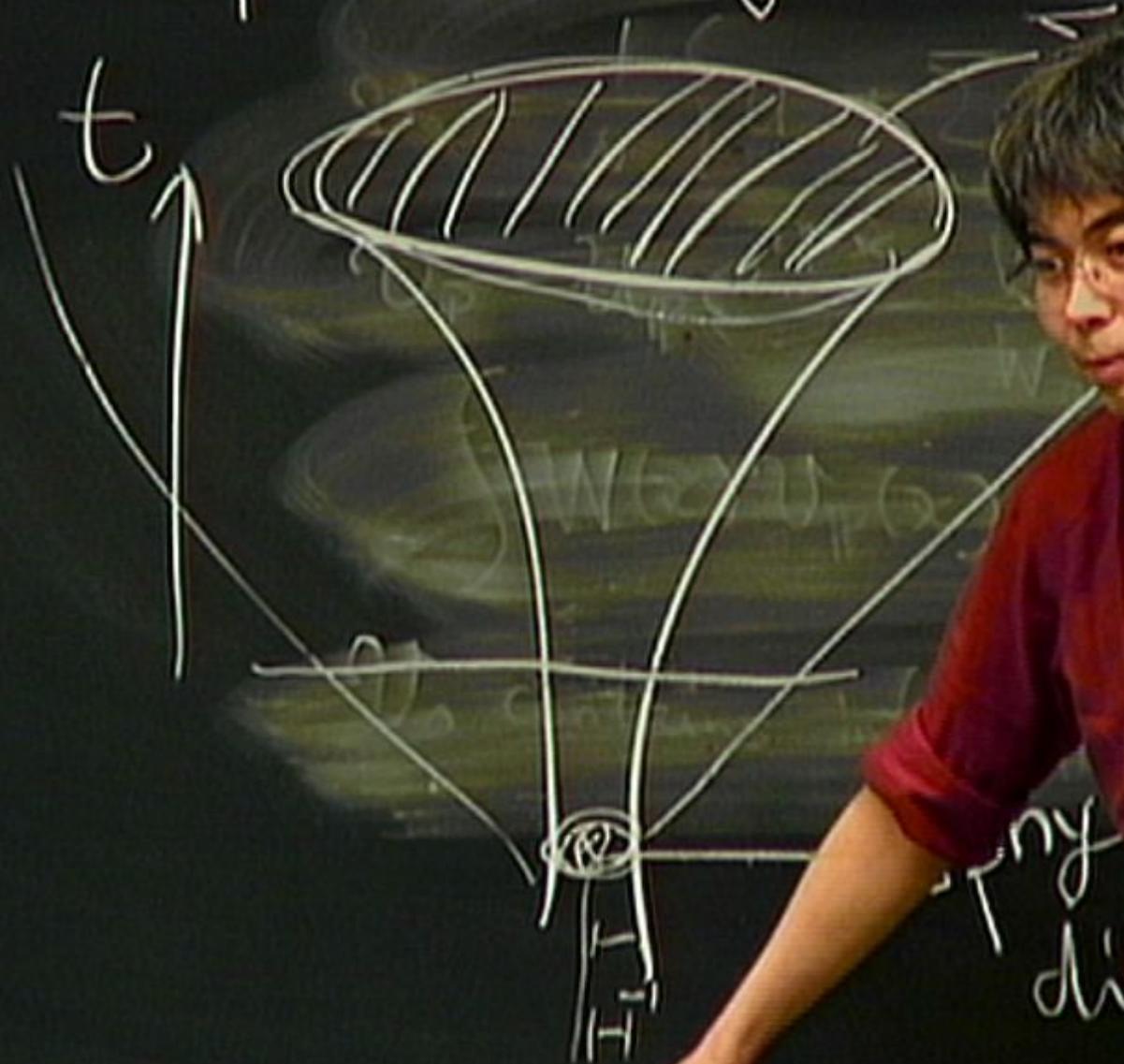
Temporal integral



Temporal integral



Temporal integral



Handwritten notes on a chalkboard:

- Top left: 15
- Diagram of a wave packet moving to the right.
- Equation: $1 + |b| < |b|$ (circled)
- Text: "we observe"

Handwritten notes on a chalkboard:

- Text: "temporal integral"
- Diagram of a funnel with a vertical axis labeled t .
- Diagram of a funnel with a vertical axis labeled t .

