Title: Going beyond de Sitter

Date: Oct 30, 2010 10:45 AM

URL: http://pirsa.org/10100081

Abstract: De Sitter space is a maximally symmetric space, and any (time dependent) backreaction necessarily breaks the symmetry. On the other hand, the backreaction in FLRW spaces (which, from the cosmological perspective, are more realistic spaces) respects the symmetries of the background space. One could therefore argue that it is more natural to study the backreaction on quasi-de Sitter spaces, and more generally on FLRW spaces. As examples, I will present our results on the one loop (Hubble) effective potential calculation, and the one loop stress energy calculation. We find that the backreaction from infrared modes can be important if a massless scalar couples nonminimally to the Ricci with a negative coupling, and if the Universe expands faster than exponentially.

GOING BEYOND DE SITTER

Tomislav Prokopec, ITP & Spinoza Institute, Utrecht University

Based on:

Tomas Janssen & Tomislav Prokopec, arXiv:0906.0666 & 0707.3919 [gr-qc]

Tomas Janssen, Shun-Pei Miao & Tomislav Prokopec, Richard Woodard, arXiv:0808.2449 [gr-qc], Class. Quant. Grav. 25: 245013 (2008); 0904.1151 [gr-qc] JCAP (2009)

Tomas Janssen & Tomislav Prokopec, arXiv:0807.0477 (2008), Annals Phys. (2010)

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Tomi Koivisto, Tomislav Prokopec, 1009.5510 [gr-qc]

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PLEASE BEWARE: HURRICANE Thomas!

OUTLINE

- Hubble effective potential: the effective potential for cosmology

  Miao, Janssen, Prokopec, Woodard 2009

- Sudden radiation-inflation matching as the infrared regulator

  Miao, Janssen, Prokopec, Woodard 2009

  - non-minimally coupled scalars
  - gravitons (in a comoving box)

  Janssen, Prokopec 2009

- Smooth radiation-inflation matching as the infrared regulator

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- FUTURE
  - cosmological observable and Hubble effective potential
  - include backreaction self-consistently, when large
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HUBBLE EFFECTIVE POTENTIAL

Janssen, Miao, Prokopec & Woodard, 0904.1151 [gr-qc] JCAP (2009)

- **SCALAR : BACKGROUND + FLUCTUATIONS:** $\varphi=\Phi(x)+\phi(x)$
- **EFFECTIVE ACTION:** $\Phi(x) \to \Phi_0$, $\Gamma_{eff} = V_{eff}[\Phi_0]$
- **IN COSMOLOGY: THE BACKGROUND IS EVOLVING!**

  $\Rightarrow$ **HUBBLE EFFECTIVE POTENTIAL:** $\Phi(x) \to \Phi_0$ $H(t)$, $\Gamma_{eff} \to V_{hub}[\Phi,H]$

- **Classical equation of motion:**

$$\ddot{\Phi} + 3H\dot{\Phi} + m^2\Phi + \xi R\Phi + \frac{\lambda}{6}\Phi^3 = 0$$

- in constant $\varepsilon=-(dH/dt)/H^2$ (flat FLRW) spaces this becomes :

$$2\varepsilon^2 - 3\varepsilon + 6\xi(2-\varepsilon) + \frac{\lambda}{6}\Phi_0^2 = 0 \quad (m=0)$$

$$\Rightarrow \Phi_0 = \Phi_0(\varepsilon,\lambda) = \text{const}$$

$\Rightarrow$ **ONE LOOP HUBBLE EFFECTIVE POTENTIAL:** $V_{hub}[\Phi]$

1 LOOP HUBBLE EFFECTIVE POTENTIAL

- **USE IR REGULATED PROPAGATOR (comoving box) FOR CONSTANT SPACES**

Janssen, Miao, Prokopec, Woodard (2008)

$$V'_{\rm hub}(\Phi,H)=\frac{\lambda}{96\pi^2}\left(1-5\varepsilon+3\varepsilon^2\right)H^2\Phi+\left\{\xi+\frac{\lambda(\xi-1/6)}{32\pi^2}\left[\ln\left(\frac{(1-\varepsilon)^2H^2}{\mu^2}\right)+\psi\left(\frac{1}{2}+\nu\right)\right]+\psi\left(N+\frac{5}{2}+\nu\right)\right\}R\Phi$$

$$+\left\{\frac{\lambda}{6}+\frac{\lambda^2}{64\pi^2}\left[\ln\left(\frac{(1-\varepsilon)^2H^2}{\mu^2}\right)+\psi\left(\frac{1}{2}+\nu\right)\right]+\psi\left(N+\frac{5}{2}+\nu\right)\right\}\Phi^3$$

$$+\frac{\left(1-\varepsilon^2\right)\lambda}{32\pi^2}\sum_{n=-1}^{N}\frac{1}{\nu-n-3/2}\left\{\frac{\Gamma(2\nu-n)\Gamma(2\nu-2n)}{2\Gamma(n+1)\Gamma^2(\nu-n+1/2)}\left(4k_0^2\eta^2\right)^{n-\nu+3/2}-\left(\nu^2-\frac{1}{4}\right)\right\}H^2\Phi+O(\hbar^2)$$

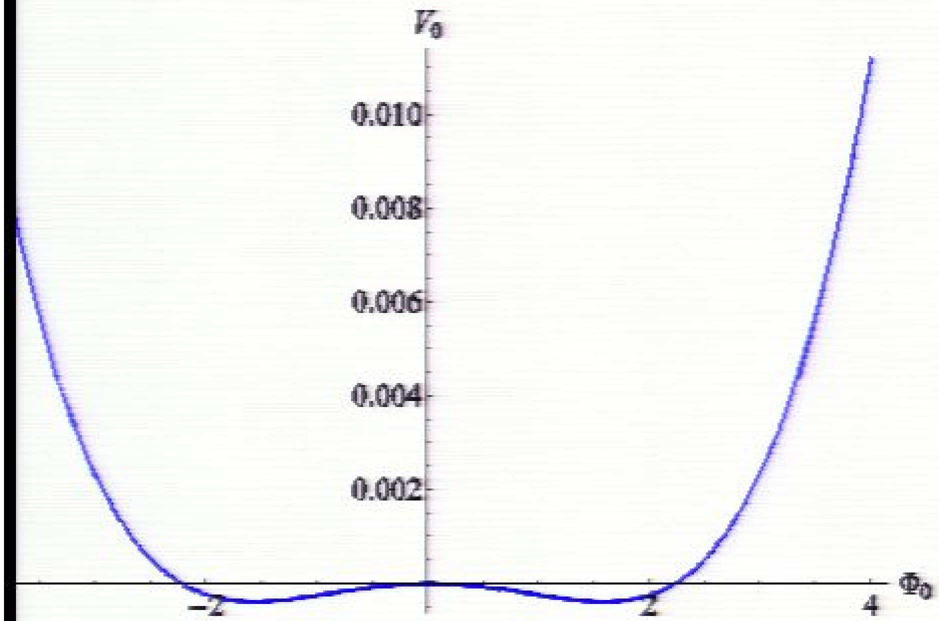
- **where:**

$$\nu_{D=4}^2=\frac{1}{4}-\frac{(\xi-1/6)R+\lambda\Phi^2/2}{(1-\varepsilon)^2H^2},\quad k_0^2\eta^2=\frac{k_0^2}{(1-\varepsilon)^2a^2H^2}$$

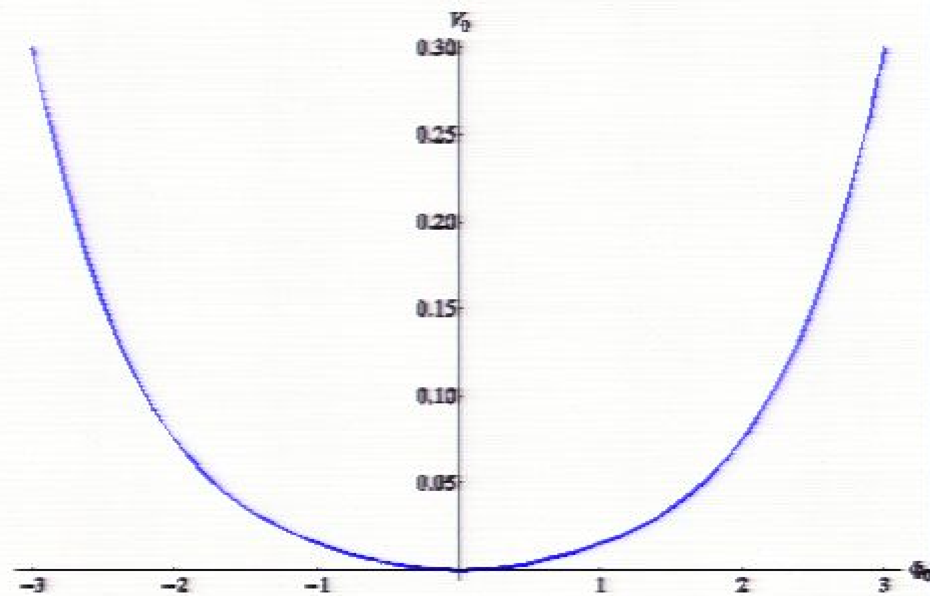
Q: What can we learn from this complicated potential?

- **In the limit when Φ>>H$\to$0, it reproduces the Coleman-Weinberg eff. potential**
- **In the de Sitter limit when $\varepsilon\to$0, it reproduces the result from** Bilandzic & TP (2006)

EVOLUTION OF V_{eff} WITH TIME: SYMMETRY RESTAURATION



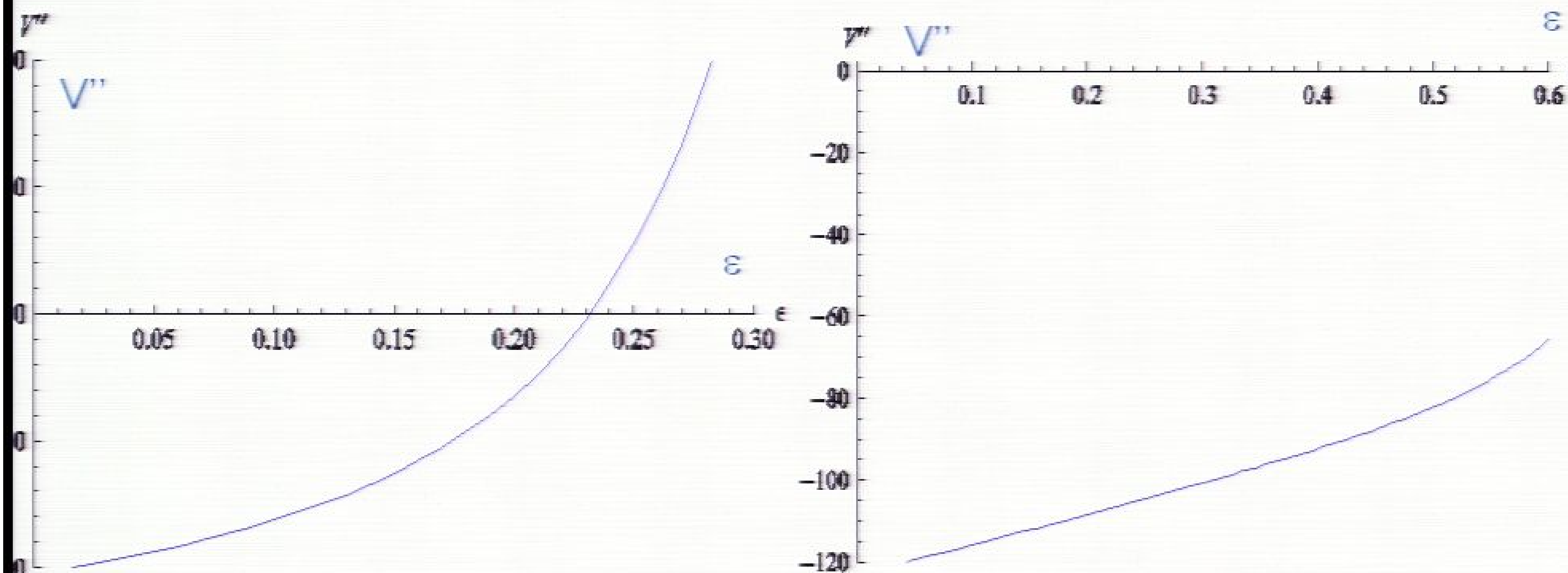
$\Rightarrow$



$\Rightarrow$ early times $\Leftarrow$ $\Rightarrow$ late times (13 efolds) $\Leftarrow$

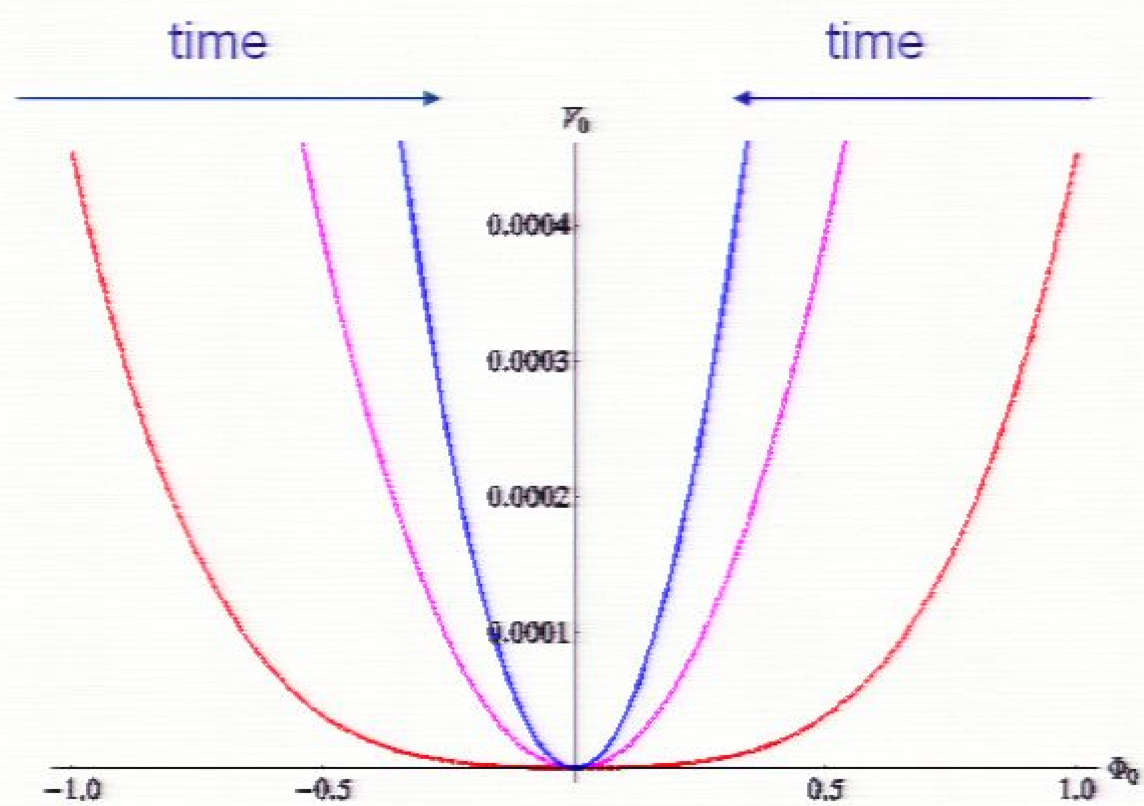
- if ε=-(dH/dt)/H² is larger symmetry gets restored earlier
- here ε=0.1, sym restauration at 9 e-foldings after beginning

SYMMETRY RESTAURATION



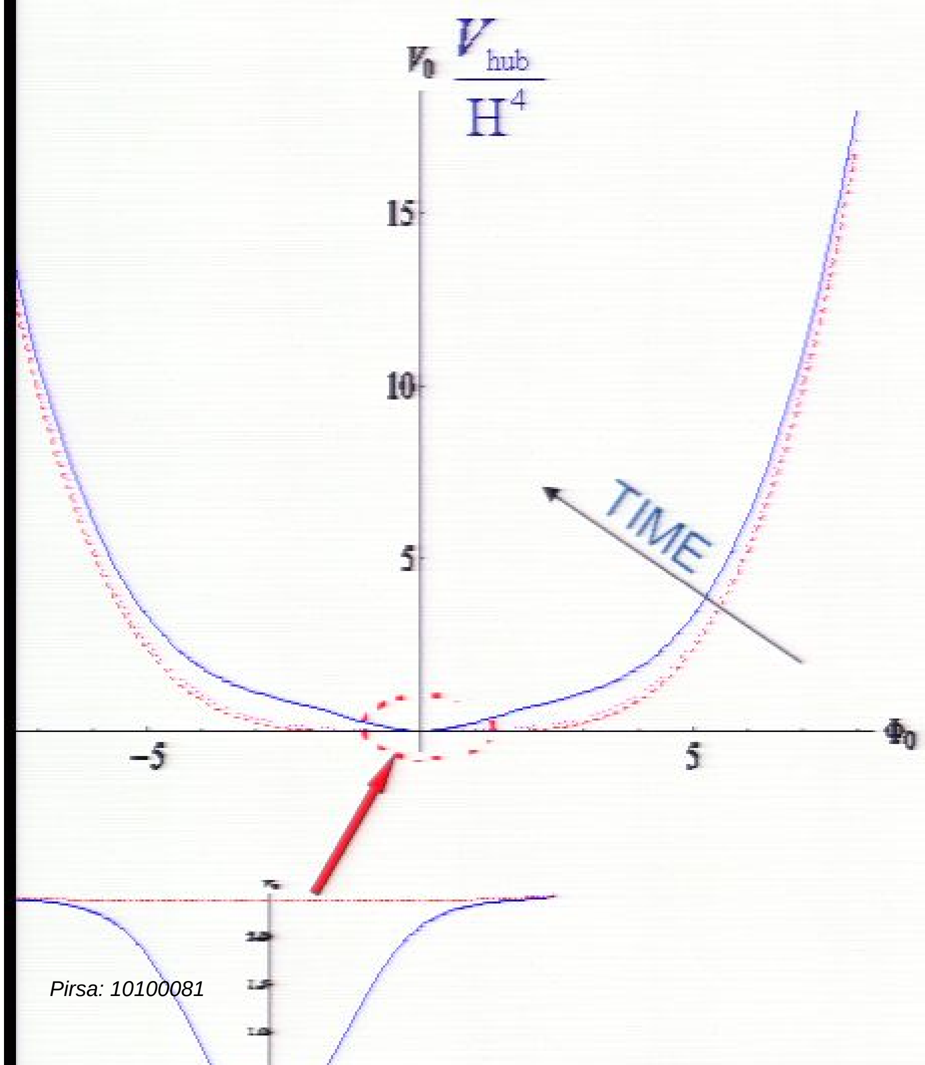
► [here $\xi=0$] symmetry restauration happens earlier for larger ε

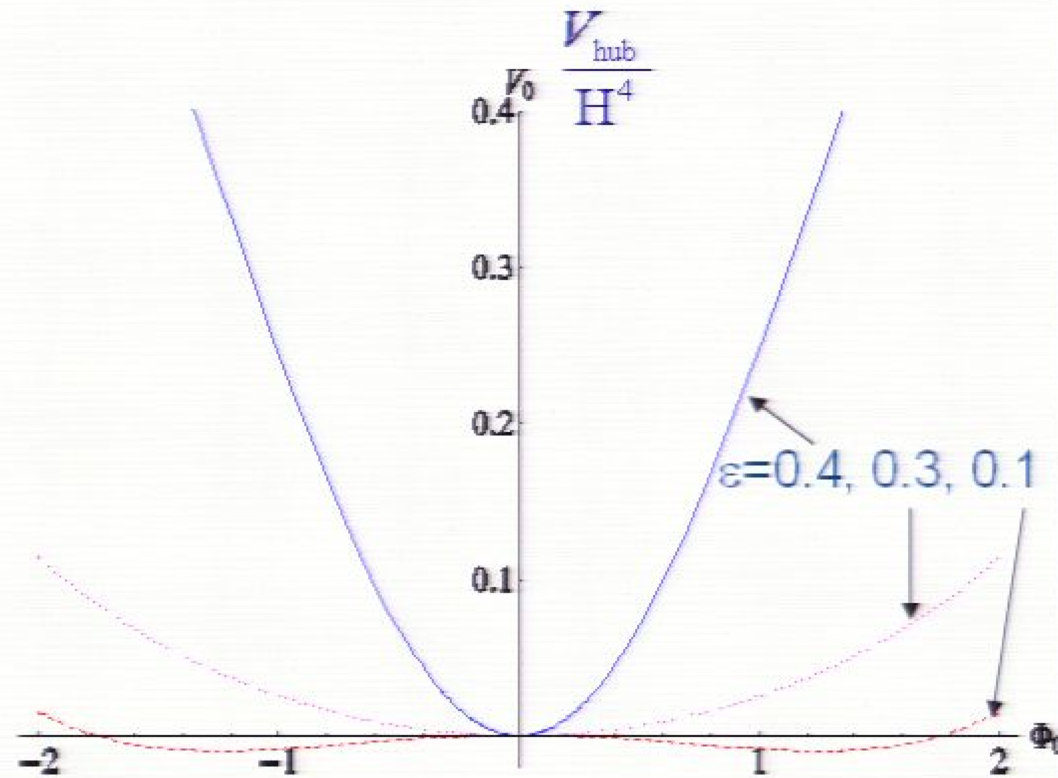
EVOLUTION OF V_{eff} WITH TIME II



- curvature of the effective potential around the origin: strengthens with time
- relevant for preheating: makes production of massive particles more efficient
- suggest breakdown of perturbation theory at late times in inflation (also an important – and recognized -- issue in Higgs inflation where ξ~-50000)

DEPENDENCE Vhub ON TIME & EPSILON





SUMMARY AND DISCUSSION, Part I

- Symmetry restauration in inflation can lead e.g. to particle production during inflation, if nonadiabatic (fermions: TP+Garbrecht, PRD [gr-qc/0602011]; scalars; photons?)

- Changes in the potential affect cosmological perturbations

  ► amplitude and spectral index of scalar and tensor perturbations
  (e.g.: one has to reexamine models such as Albrecht-Steinhardt new inflation)

- Preheating is more dramatic (recall dependence on ε)

WHAT IS (QUANTUM) BACKREACTION?

- Einstein's Equations

$$G_{\mu\nu}(\underbrace{g^b_{\alpha\beta}}_{BACKGD.} + \underbrace{\delta g_{\alpha\beta}}_{FLUCT.}) = 8\pi G T_{\mu\nu}(g^b_{\alpha\beta} + \delta g_{\alpha\beta}, \psi^b_i + \delta\psi_i)$$

→ $g^b_{\alpha\beta}, \delta g_{\alpha\beta}$: background gravitational fields & corresp. (quantum) fluctuations

→ $\psi^b_i, \delta\psi_i$: background matter fields & corresponding (quantum) fluctuations

- Classical Equations:

$$G^b_{\mu\nu} = 8\pi G T^b_{\mu\nu}, \quad G^b_{\mu\nu} = G_{\mu\nu}(g^b_{\alpha\beta})$$

are not correct in presence of strong backreaction from (quantum) fluctuations

- Quantum Einstein Equations:

$$G^b_{\mu\nu} + G^q_{\mu\nu} = 8\pi G(T^b_{\mu\nu} + T^q_{\mu\nu}),$$ ($|\Omega\rangle$: physical state)

▷ $G^q_{\mu\nu} = \langle\Omega|(\hat{G}_{\mu\nu} - G^b_{\mu\nu})|\Omega\rangle$: includes (conserved) contribution from graviton fluctuations

▷ $T^q_{\mu\nu} = \langle\Omega|(\hat{T}_{\mu\nu} - T^b_{\mu\nu})|\Omega\rangle$: includes (conserved) contribution from quantum matter fluct's

NB: Since gravitons couple to matter, it is better to write:

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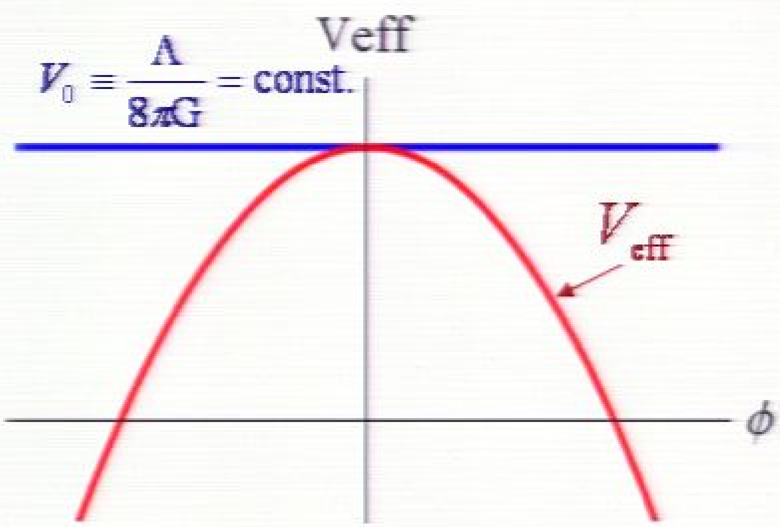
NB: Since gravitons couple to matter, it is better to write: $T^q - G^q/8\pi G \rightarrow T^q$

A FEW WORDS ON THE CCP PROBLEM

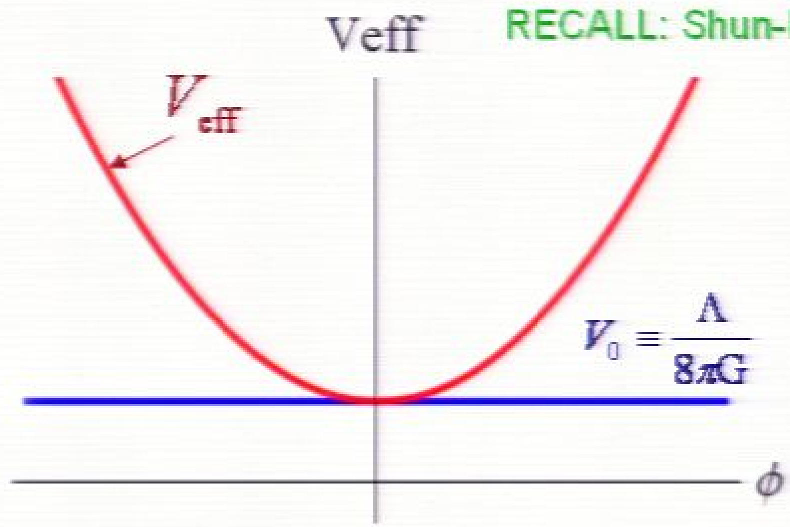
- Quantum corrections in scalar QED, scalar theories ($\lambda\phi$^4) are positive
- Quantum corrections from integrating fermions (QED, yukawa) are negative

FERMIONS + YUKAWA



SCALARS + VECTORS/GRAVITONS

RECALL: Shun-Pei M.



$1000000 Q: Can solving for V_{eff} self consistently with the Friedmann equation stop the Universe from collapsing into a negative energy (`anti-de Sitter') universe?

JFK+TP, in progress

The Universe can also be stabilised by adding a sufficiently many vectors and scalars

NB: An effective potential of the form $V_{eff} \sim -\lambda(\phi^4)\ln(\phi^2/H^2)$ would solve the CCP problem (the dynamics of ϕ would drive $\Lambda \to 0$). But, how to get it?

THE PROBLEM(S) WITH BACKREACTION

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$\rightarrow T^q_{\mu\nu}$ has to be determined by solving dynamical equations for matter and graviton matter perturbations in the expanding Universe setting. Hopelessly hard!

▫ **Statements**: Dark energy can be (perhaps) explained by the backreaction of small scale gravitational + matter perturbations onto the background space time

- Hard to (dis-)prove. Naïve argument against: grav. potential is small: $\phi \sim 10^{-5}$
- Maybe too naïve, because of secular (growing) terms generated by perts

HERE: I will discuss the simplest (!?) possible TOY MODEL:

- a homogeneous universe with constant deceleration parameter q=ε-1, ε=-(dH/dt)/H²

- a massless dynamical scalar ϕ but gravity is non-dynamical

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- a homogeneous universe with constant deceleration parameter q=ε-1, ε=-(dH/dt)/H²

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THE PROBLEM(S) WITH BACKREACTION

- Quantum Einstein's Equations

$$G^b_{\mu\nu} = 8\pi G(T^b_{\mu\nu} + T^q_{\mu\nu}), \qquad \triangleright T^q_{\mu\nu} = \langle\Omega|(\hat{T}_{\mu\nu} - T^b_{\mu\nu})|\Omega\rangle :$$

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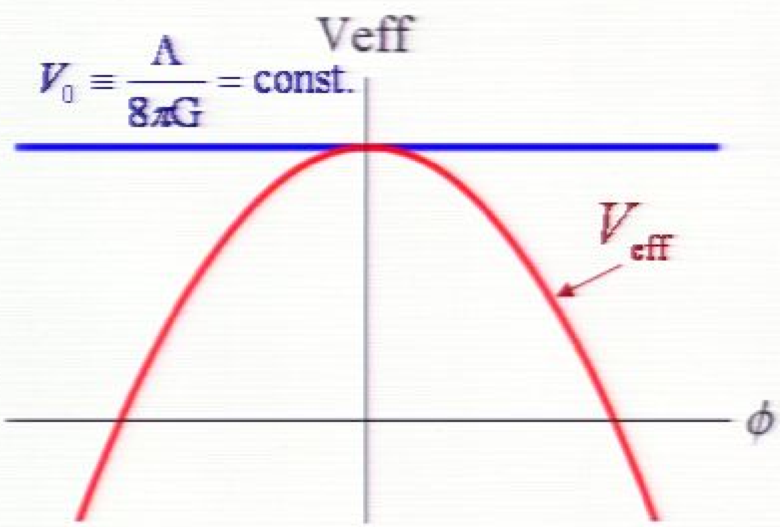
A FEW WORDS ON THE CCP PROBLEM

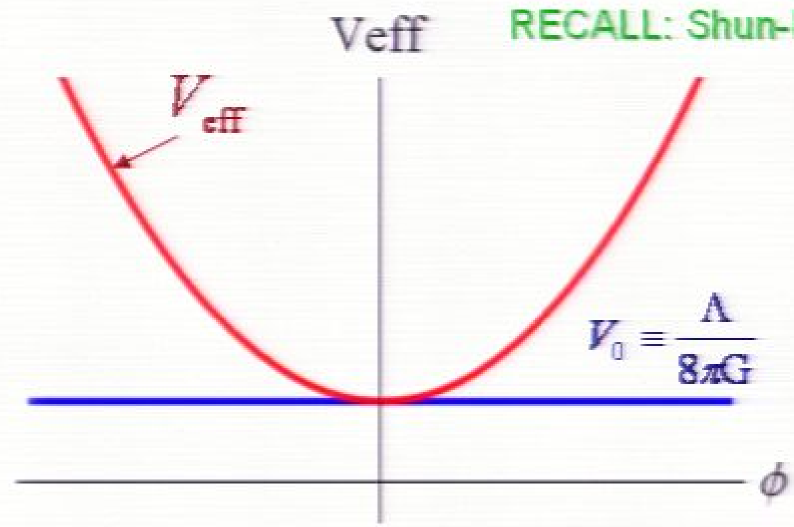
- Quantum corrections in scalar QED, scalar theories ($\lambda\phi$^4) are positive
- Quantum corrections from integrating fermions (QED, yukawa) are negative

FERMIONS + YUKAWA



SCALARS + VECTORS/GRAVITONS

RECALL: Shun-Pei M.



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JFK+TP, in progress

The Universe can also be stabilised by adding a sufficiently many vectors and scalars

NB: An effective potential of the form $V_{eff} \sim -\lambda(\phi^4)\ln(\phi^2/H^2)$ would solve the CCP problem (the dynamics of ϕ would drive $\Lambda \to 0$). But, how to get it?

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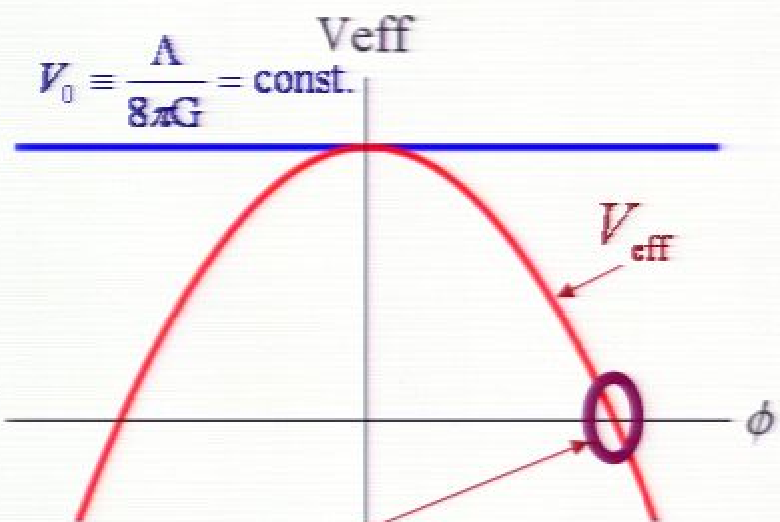

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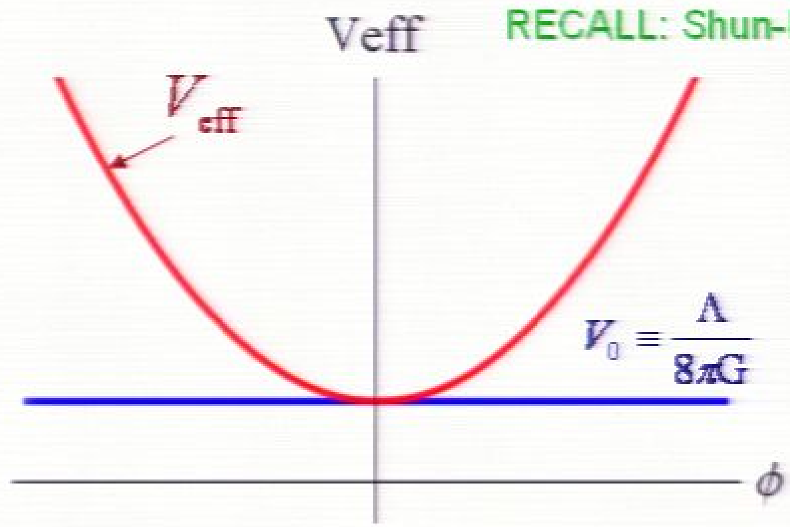

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BACKGROUND SPACE TIME

- LINE ELEMENT (METRIC TENSOR):

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2 \quad or \quad g_{\mu\nu} = a^2(\eta)\eta_{\mu\nu}, \quad \eta_{\mu\nu} = diag(-1, \underbrace{1,1,..}_{D-1})$$

- FRIEDMANN (FLRW) EQUATIONS (Λ=0):

$$H^2 = \frac{8\pi G}{3}\rho_b, \qquad \dot{H} = -4\pi G(\rho_b + p_b)$$

$$\Rightarrow \varepsilon = -\frac{\dot{H}}{H^2} = \frac{3}{2}(1 + w_b) = \text{const.}, \quad w_b = \frac{p_b}{\rho_b}$$

- for power law expansion the scale factor reads:

$$\text{a} = \left(\frac{t}{t_0}\right)^{1/\varepsilon} = \left[-(1-\varepsilon)H_0\eta\right]^{-\frac{1}{1-\varepsilon}}, \quad H = H_0 a^{-\varepsilon}$$

SCALAR 1 LOOP STRESS ENERGY

♦ **CENTRAL Q: Under what conditions can the backreaction from the one loop fluctuations become so large to change evolution of the (background) Universe**

▫ **QUANTUM STRESS ENERGY TRACE:**

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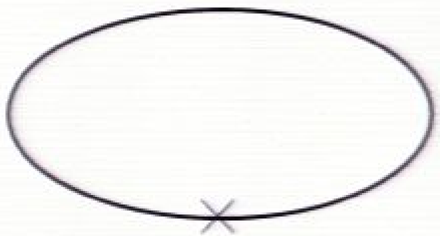

$T^{\mu\nu}(x)$

→ iΔ(x;x): scalar propagator at coincidence

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SCALAR THEORY

- MASSLESS SCALAR FIELD ACTION ($V\to 0$, ξR plays role of a `mass')

$$S_\varphi = \int d^D x \sqrt{-g}\left(-\frac{1}{2} g^{\mu\nu}\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2}\xi R \varphi^2 - V(\varphi)\right)$$

What is ϕ?

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$$\nabla_\mu \nabla^\mu \varphi - \xi R \varphi - V'(\varphi) = \frac{1}{a^2}\left(\partial^2 - (D-2)\frac{a'}{a}\partial_0 - \xi R\right)\varphi - V'(\varphi) = 0, \quad \partial^2 = \eta^{\mu\nu}\partial_\mu \partial_\nu = -\partial_0^2 + \vec{\partial}^2$$

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$$\hat{\varphi}(x) = \int \frac{d^{D-1}k}{(2\pi)^{D-1}}\left(\varphi_k(\eta) e^{i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}} + \varphi_k^*(\eta) e^{-i\vec{k}\cdot\vec{x}} \hat{a}_{\vec{k}}^+\right), \quad \hat{a}_{\vec{k}}|0\rangle = 0$$

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$$(|\alpha_k|^2 - |\beta_k|^2 = 1)$$

- PROPAGATOR EQUATION

$$\left[\nabla_\mu \nabla^\mu - \xi R^b - V''(\phi^b)\right] i\Delta(x;x') = i\delta^D(x-x'),$$

SCALAR PROPAGATOR IN FLRW SPACES in D dimensions

- **MMC SCALAR FIELD PROPAGATOR (V''=0, ε=const)** Janssen & Prokopec 2009, 2007; Janssen, Miao & Prokopec 2008

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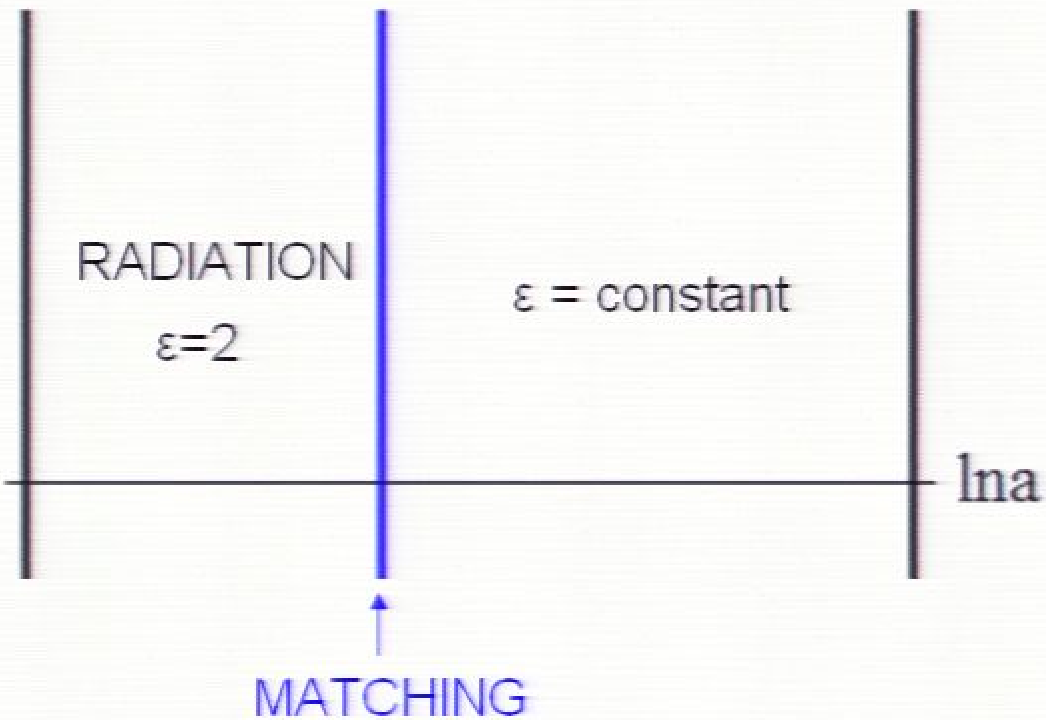

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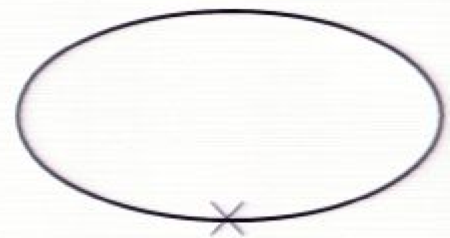

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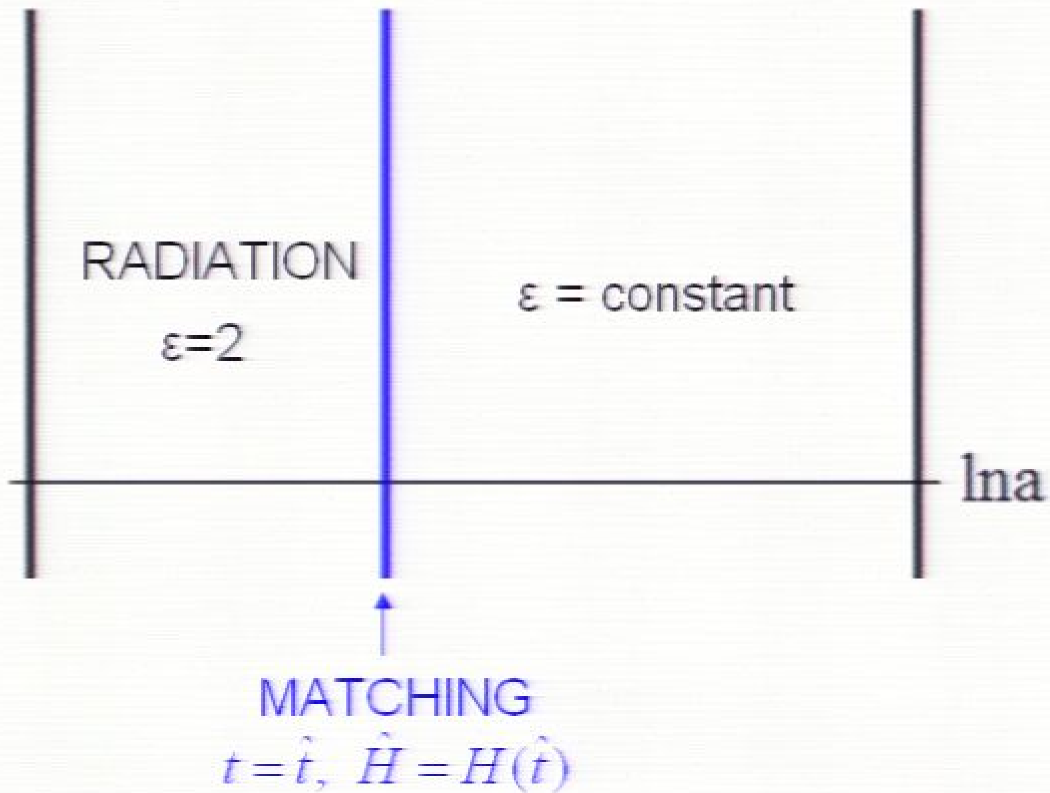

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IR SINGULARITY IN DE SITTER SPACE

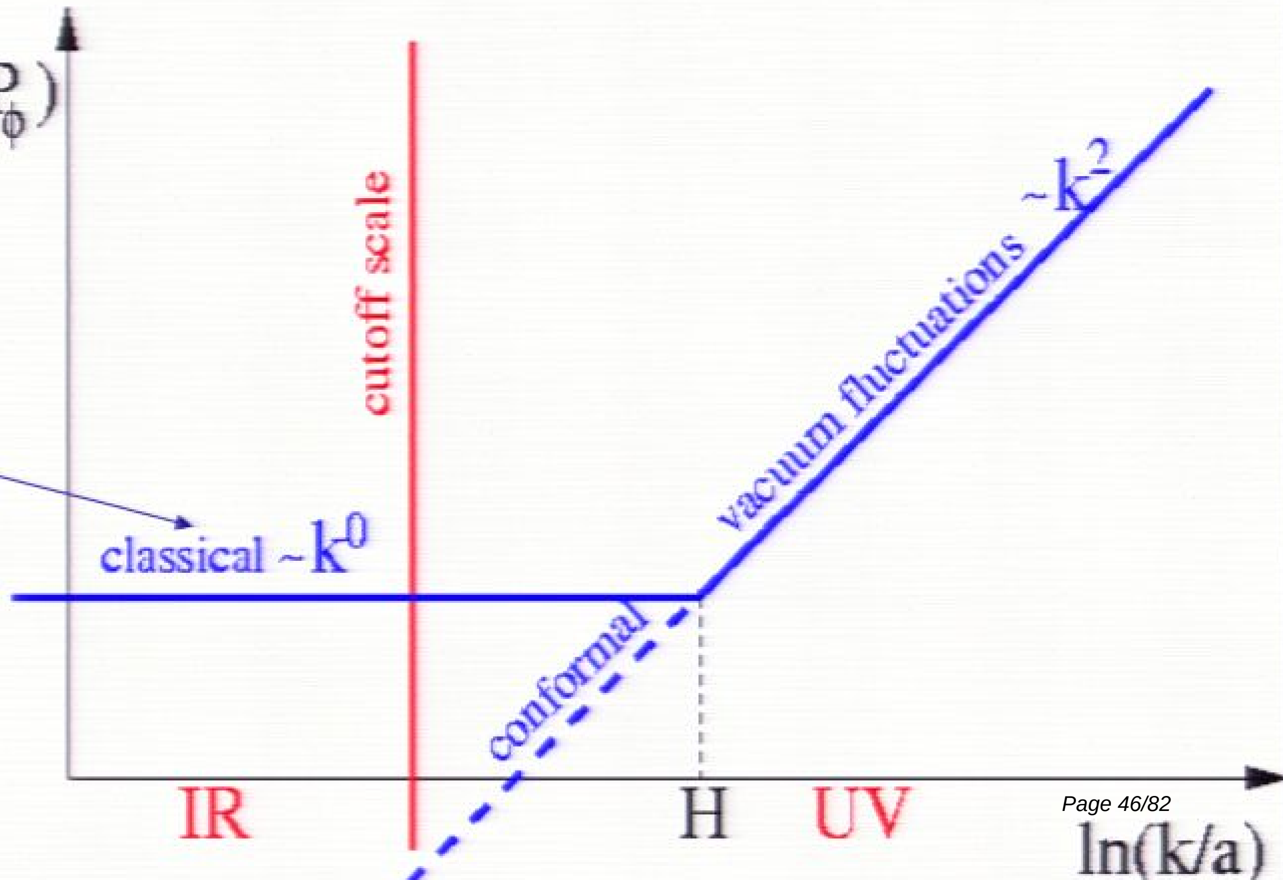
- Scalar field spectrum P_φ in de Sitter (ν=3/2, ε=0)

$$\langle 0|\hat{\varphi}(\vec{x},\eta)\hat{\varphi}(\vec{x}',\eta)|0\rangle = \int \frac{dk}{k} P_\varphi(k,\eta)\frac{\sin(k\Delta x)}{k\Delta x}, \quad \Delta x = \|\vec{x}-\vec{x}'\|$$

$$P_\varphi(k,\eta) = \frac{H^2}{4\pi^2}\left(1+\frac{k^2}{(aH)^2}\right)$$

Source of scalar cosmological perturbations

- IR log SINGULAR
- UV quadratically SINGULAR



SCALAR THEORY: IR SINGULARITIES

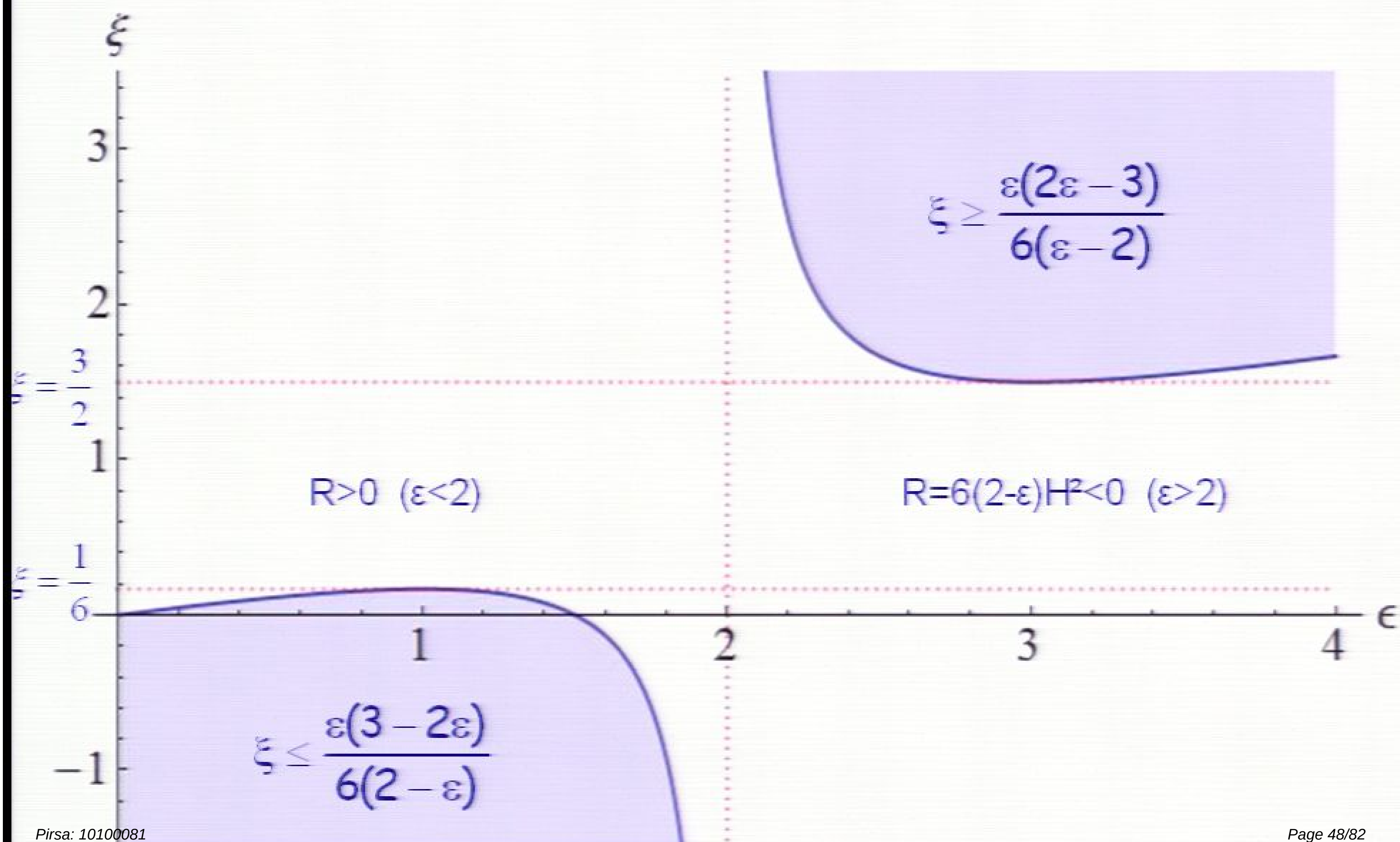
- the IR singularity of a coincident propagator:

$$\langle 0|\varphi(x)^2|0\rangle_{IR} = \int_{k_{\min}}^{k_{\max}} \frac{d^3k}{(2\pi)^3} |\varphi_k|^2 \propto \int_{k_{\min}}^{k_{\max}} \frac{dk}{k} k^{D-1-2\nu} \qquad \nu^2|_{D=4} = \frac{1}{4} + \frac{(1-6\xi)(2-\varepsilon)}{(1-\varepsilon)^2}$$

  - BD vacuum is IR singular (in D=4) for $0 \le \varepsilon \le 3/2$, when $\xi=0$ $\Rightarrow$ large quantum backreaction expected

- When $\xi \neq 0$ the coincident propagator is IR singular in the shaded regions:

SCALAR THEORY: IR OF BD VACUUM



QUANTUM & CLASSICAL ENERGY SCALING

- **SCALINGS:**

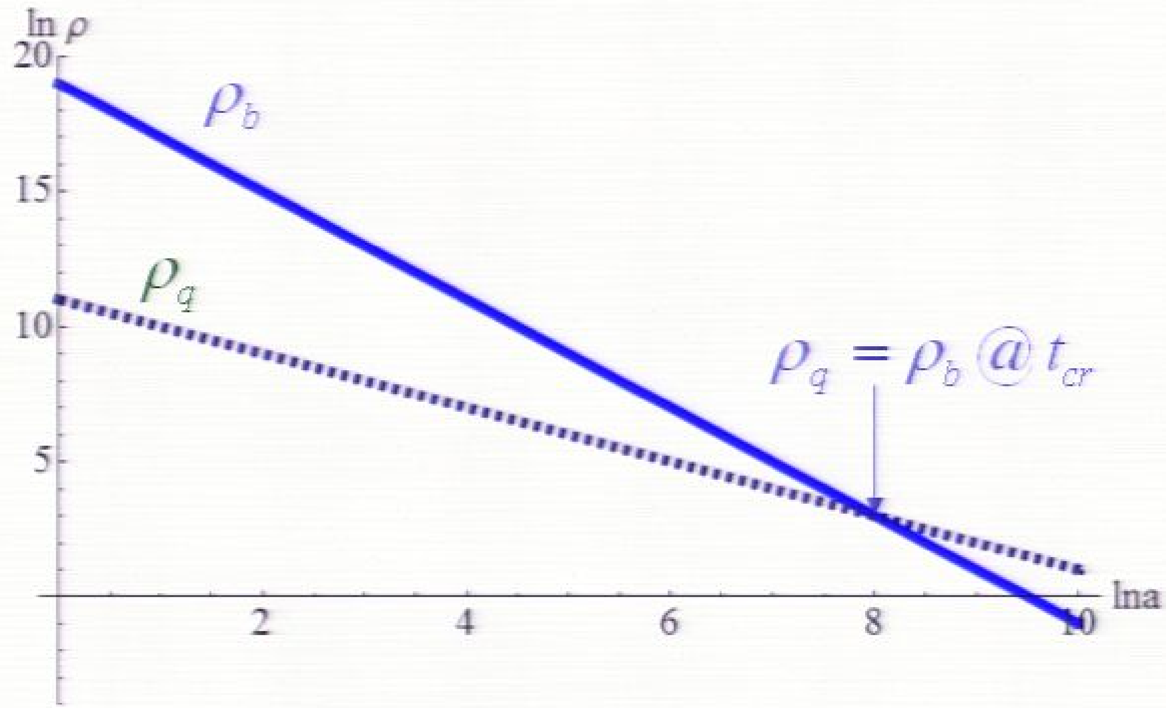
$$\rho_b = \frac{\hat{\rho}_b}{a^{3(1+w_b)}}, \quad \rho_q = \frac{\hat{\rho}_q}{a^{3(1+w_q)}}, \quad \hat{\rho}_b = \rho_b(\hat{t}), \quad \hat{\rho}_q \sim \frac{\hat{H}^2}{M_{Pl}^2}, \quad M_{Pl}^2 = \frac{1}{8\pi G}, \quad w_b = \frac{p_b}{\rho_b}, \; w_q = \frac{p_q}{\rho_q}$$

- **SCALING:**

$$\frac{\rho_q}{\rho_b} \sim \frac{\hat{H}^2}{M_{Pl}^2} a^{3(w_b - w_q)}$$

- **IF : $W_q < W_b$**

$$\Rightarrow \rho_q \sim \rho_b \; @ \; t_{cr} \sim \hat{t} \left(\frac{\hat{H}}{M_{Pl}} \right)^{\frac{1+w_b}{w_b - w_q}}$$



- **TYPICALLY:** $t_{cr} >> \hat{t}$

Q: What is the self-consistent evolution for $t > t_{cr}$, when $\rho_q \geq \rho_b$?

Q2: Can ρ_q play the role of dark energy ?

- AFTER A LOT OF WORK ..

we obtain ρ_q & p_q , i.e. how they scale with scale factor a

- INTERMEDIATE RESULT: **COINCIDENT SCALAR PROPAGATOR:**

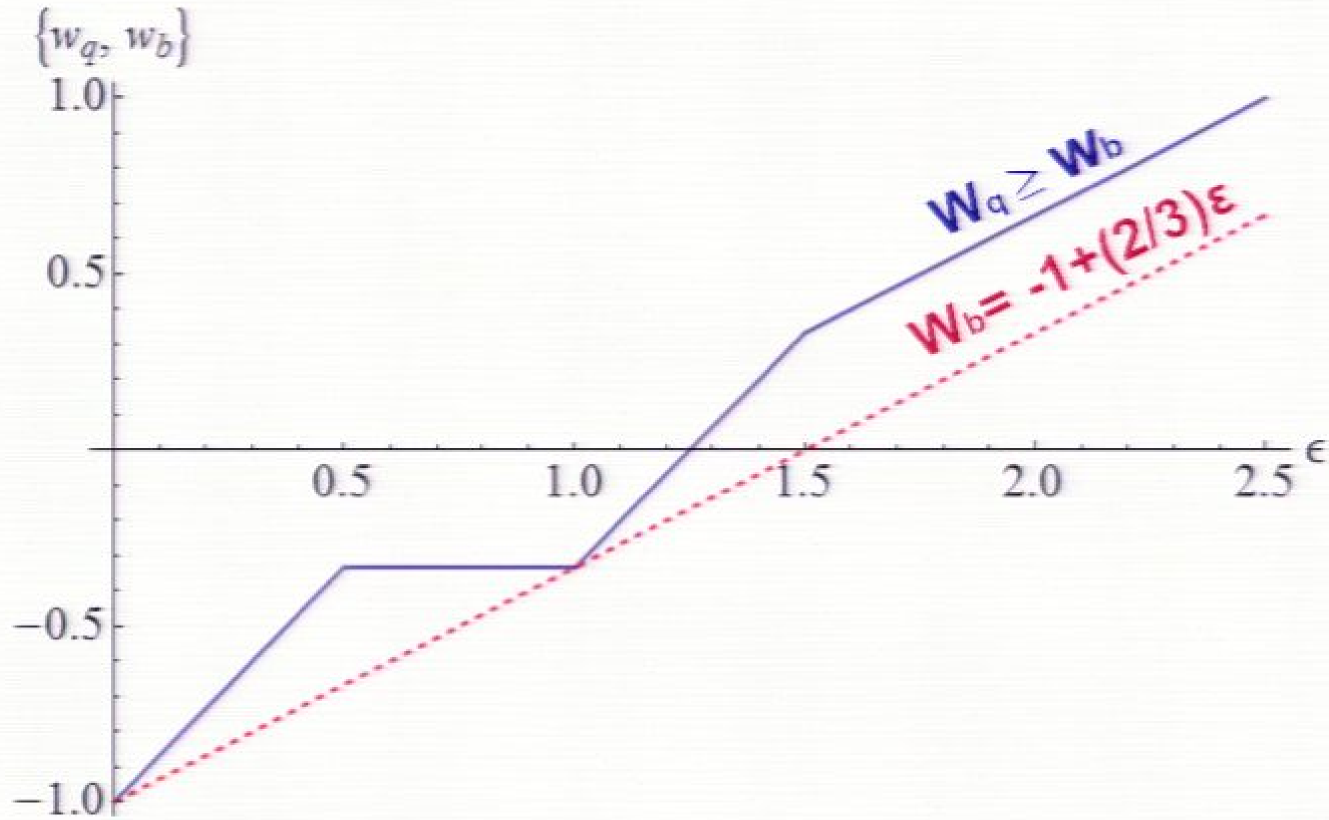
$$i\Delta(x;x)=\frac{(1-\varepsilon)^2H^2}{16\pi^2}\left(\nu^2-\frac{1}{4}\right)\left[\begin{array}{l}-\frac{2\mu^{D-4}}{D-4}-c_\nu+\ln\left(\frac{4\pi\mu^2}{(1-\varepsilon)^2H^2}(1-1/\zeta)^2\right)+2\zeta-\zeta^2-\ln\left(1-\frac{1}{\zeta^2}\right)\\-\frac{\zeta^{1-2\nu}}{1-2\nu}\times{}_2F_1\left(1,\nu-\frac{1}{2};\nu+\frac{1}{2};\frac{1}{\zeta^2}\right)-\ln\left(1-\zeta^2\right)-\frac{\zeta^{3+2\nu}}{3+2\nu}\times{}_2F_1\left(1,\nu+\frac{3}{2};\nu+\frac{5}{2};\zeta^2\right)\end{array}\right]$$

$$\zeta=\frac{(aH)_{\rm match}}{aH},\quad c_\nu=\text{undetermined constant}$$

- can be expanded in ζ (late times, accelerated space); $1/\zeta$ (decelerated: ε>1)

CLASSICAL vs. QUANTUM DYNAMICS

▫ QUANTUM & CLASSICAL ENERGY DENSITY SCALING: w_q vs w_b for $\xi=0$



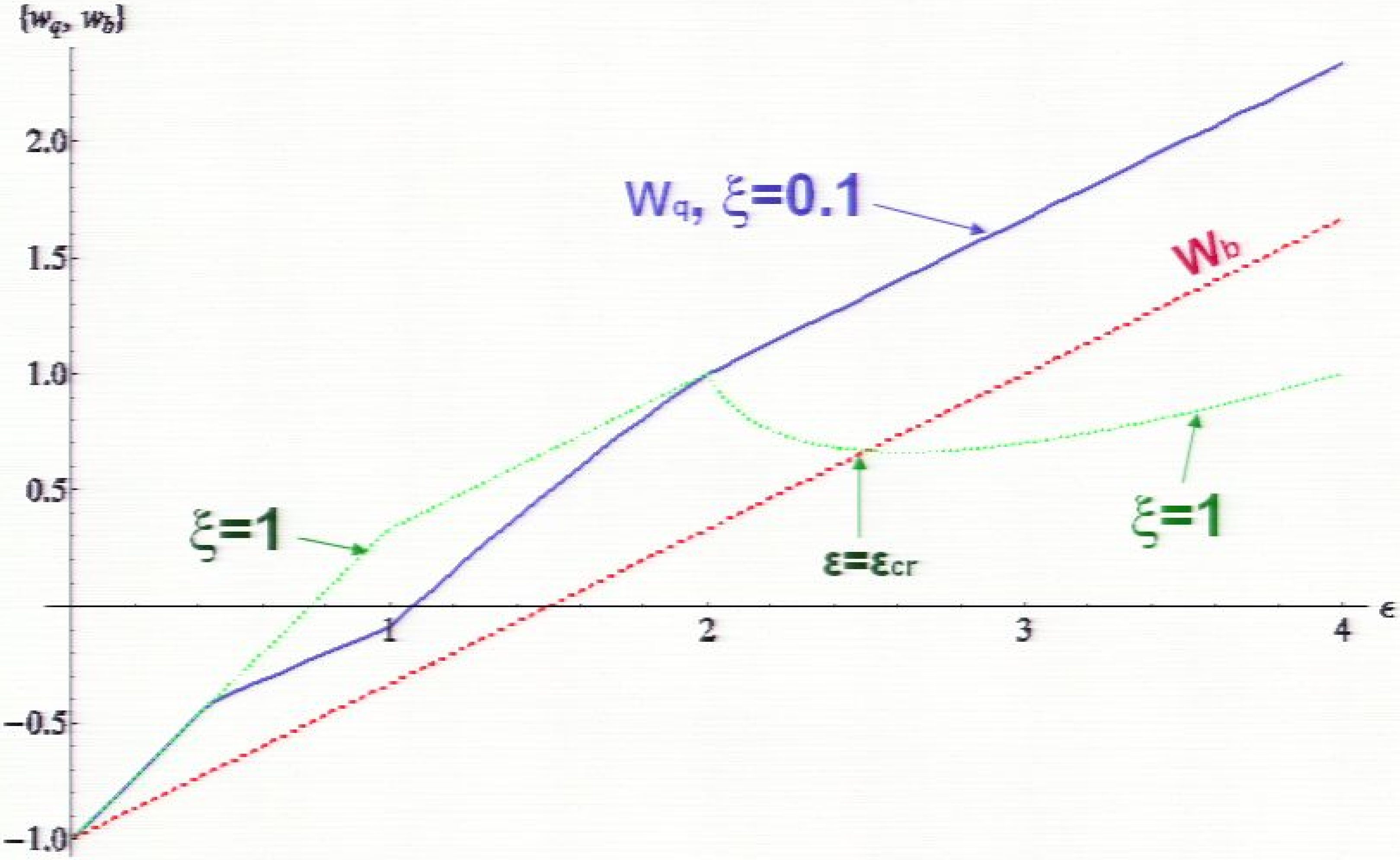
NB: We expect that for ε<1 the graviton undergoes the same scaling:

$$w_q = -1 + \frac{4}{3}\varepsilon \quad (0<\varepsilon<1/2), \quad w_q = -\frac{1}{3} \quad (1/2<\varepsilon<1)$$

- **for ε>1 we expect the ξ=0 scaling:** $w_q = -\frac{1}{3} + \frac{4}{3}(\varepsilon-1) \quad (1<\varepsilon<2), \quad w_q = 1 + \frac{2}{3}(\varepsilon-2) \quad (\varepsilon>2)$

SCALAR QUANTUM EOS PARAMETER : $\xi>0$

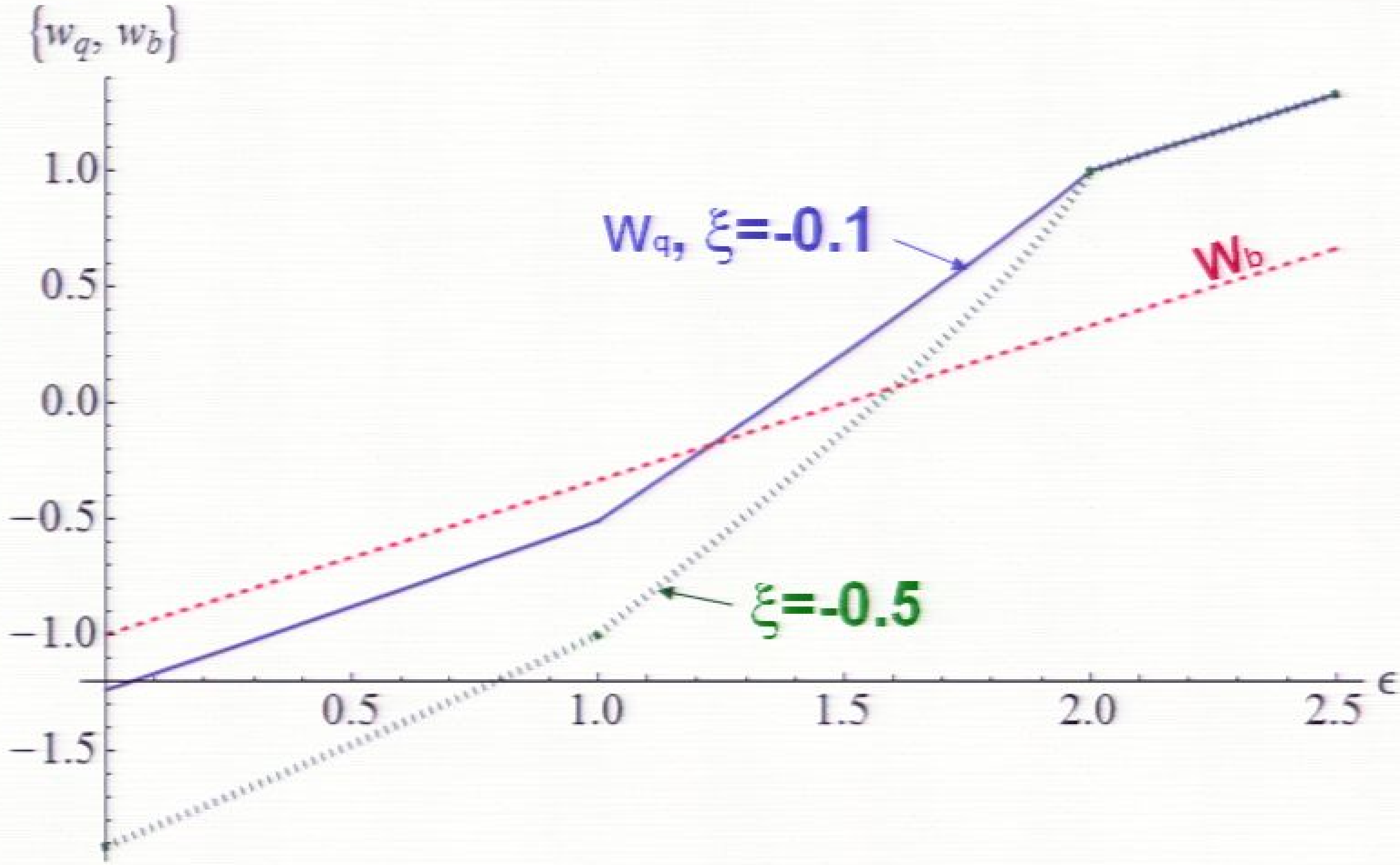
- **QUANTUM ENERGY DENSITY & PRESSURE: $\xi>0$ ($m_{eff}^2>0$)**



NB: For small positive ξ: $w_q>w_b$ $\forall\varepsilon$; for large $\xi>1/6$, $w_q<w_b$ if $\varepsilon>\varepsilon_{cr}>2$

SCALAR QUANTUM EOS PARAMETER: $\xi<0$

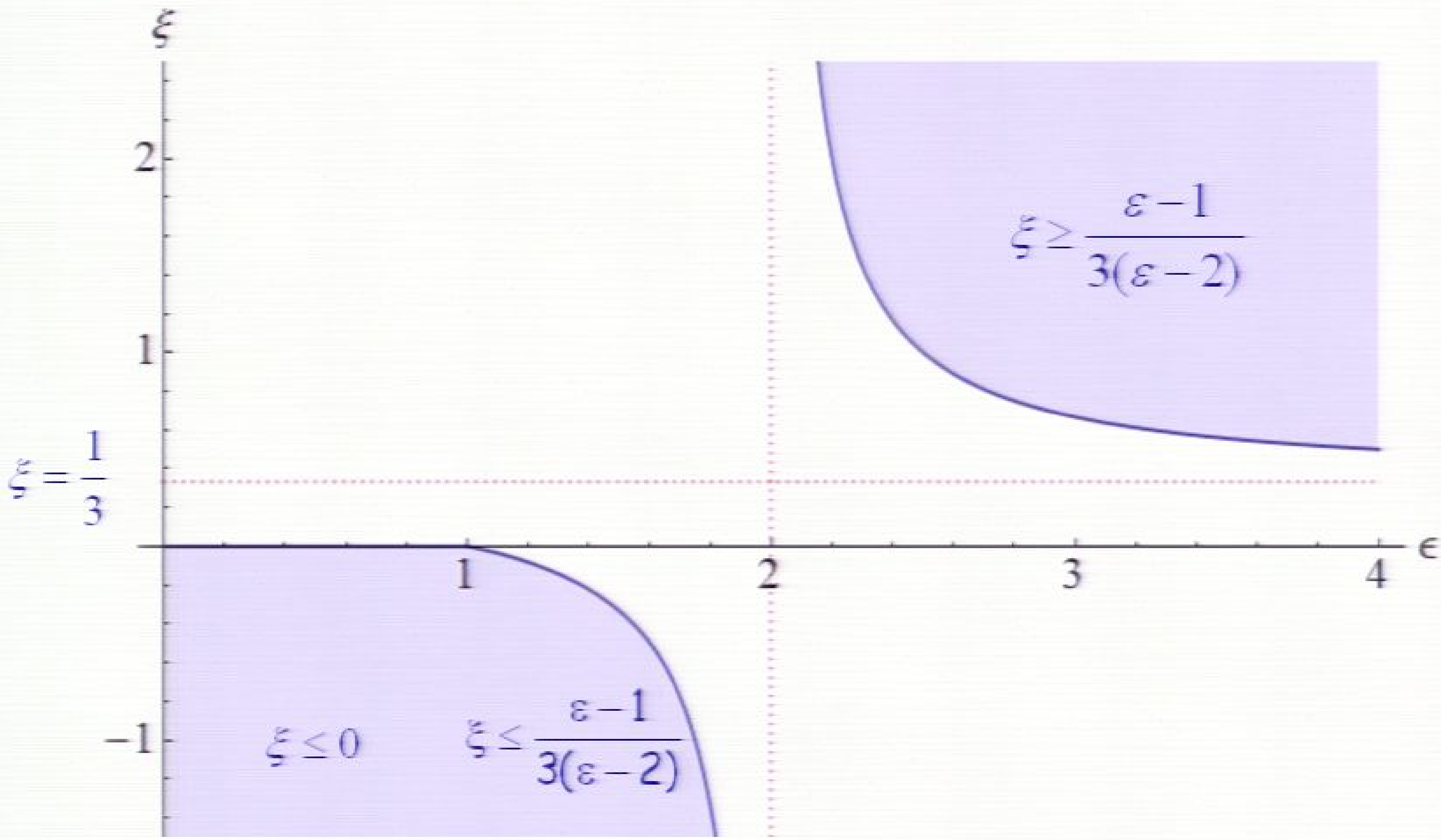
- **QUANTUM ENERGY DENSITY & PRESSURE: $\xi>0$ ($meff^2>0$)**



NB: For negative ξ: $w_q<w_b$ $\forall\, \varepsilon<\varepsilon_{cr}$; $1<\varepsilon_{cr}<2$

THE (ξ,ε) REGIONS WHERE $w_q<w_b$

- **QUANTUM ENERGY DENSITY & PRESSURE**



SHADED REGIONS: $w_q<w_b$: after some time, ρ_q will become dominant over ρ_b

CAN QUANTUM FLUCTUATIONS BE DARK ENERGY?

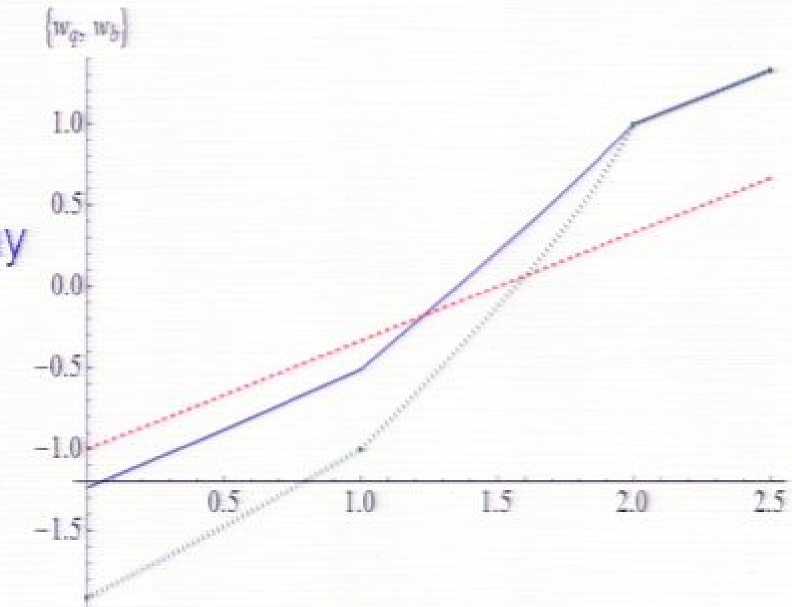
▫ **SIMPLE ESTIMATE: IMAGINE that quantum fluctuations generated at matter-radiation equality (z~3200) are responsible for dark energy**

$$\xi \sim -280, \quad \nu \sim 58, \quad w_q \sim -28$$

→ Ups! It does not work! A much earlier transition is needed!

→ But, from: $t_{cr} \sim \hat{t}\left(\frac{\hat{H}}{M_{Pl}}\right)^{\frac{1+w_b}{w_b-w_q}}$

we have learned that typically a large time delay occurs between the transition and $\rho_q \sim \rho_b$



- NB: It does not work for radiation ε=2

Q: What is the self-consistent evolution for $t > t_{cr}$, when $\rho_q \geq \rho_b$?

Q2: Can ρ_q play the role of dark energy ?

SUMMARY AND DISCUSSION, Part II

- The quantum backreaction from massless scalars in ε=const spaces can become large at 1 loop, provided conformal coupling ξ<0 (ε<2).

OPEN QUESTIONS:

- What about other IR regularisations: (scalar) mass, positive curvature, finite box
- What about the backreaction from scalars/gravitons at higher loop order, non-constant ε FLRW spaces, inhomogeneous spaces, .. ?
- The backreaction from fermions is large, and distabilises the Universe, driving it to a negative energy Universe: can that be stabilised?

Koksma & Prokopec 2009

► What is the effect of $d\varepsilon/dt \neq 0$ (mode mixing)?

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GRAVITONS

- Graviton: lagrangian to second order in $h_{\mu\nu}$

► PERTURBATIONS $\hat{g}_{\mu\nu}(x) = \underbrace{g_{\mu\nu}(\eta)}_{a^2\eta_{\mu\nu}} + \underbrace{\delta g_{\mu\nu}}_{a^2\sqrt{\kappa}\psi_{\mu\nu}}$, $\kappa = 16\pi G_N$, $\hat{\varphi}(x) = \Phi(\eta) + \delta\varphi(x)$

► GRAVITON-SCALAR MIXING

$$L^{(2)} \subset -a^{D-2}\psi_{00}\sqrt{\kappa}\Phi''\varphi - \frac{1}{2}a^{D-2}\sqrt{\kappa}\eta^{\mu\nu}\psi_{\mu\nu}\varphi\underbrace{\left[\Phi'' + (D-2)aH\Phi' + a^2\partial_\varphi V(\Phi)\right]}_{=0 \text{ on shell}}$$

- lagrangian must be diagonalized w.r.t. the scalar fields ψ_{00} & φ

► GAUGE: graviton propagator in exact gauge is not known.
We added a gauge fixing term (Woodard,Tsamis):

$$L_{GF} = -\frac{1}{2}\sqrt{-g}g^{\mu\nu}F_\mu F_\nu, \quad F_\mu = a^2\nabla_\rho\left(\psi^\rho_\mu - \frac{1}{2}\delta^\rho_\mu g^{\lambda\zeta}\psi_{\lambda\zeta}\right) - \varphi\Phi'\sqrt{\kappa}\delta^0_\mu$$

- upon a suitable rotation tensor, vector and 2 scalar fields decouple on shell

GRAVITON PROPAGATOR IN FLRW SPACES

Janssen & Prokopec 2009

- **EOM (symbolic)** $$D\cdot i\Delta = i\delta^D$$
  - **VECTOR DOFs:** $D^{ij}_{\text{vector}} = -D_1\delta^{ij}, \quad i\left[{}_j\Delta^l\right]_{\text{vector}} = -\delta^l_j i\Delta_1$
  - **GHOST DOFs:** $D^{\text{ghost}}_{\mu\nu}\Big|_{\text{on shell}} = \bar{\eta}_{\mu\nu}D_0 - \delta^0_\mu\delta^0_\nu D_1, \quad i\left[{}_\alpha\Delta^\beta\right]_{\text{ghost}} = \bar{\delta}^\beta_\alpha i\Delta_0 + \delta^0_\alpha\delta^0_\beta i\Delta_1, \quad \bar{\eta}_{\mu\nu} = \eta_{\mu\nu} + \delta^0_\mu\delta^0_\nu$

$$D_n = \partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu - \sqrt{-g}\,n\left(D-n-1-\frac{n(n-1)}{2}\varepsilon\right)(1-\varepsilon)H^2, \quad (n=0,1,2)$$

$$D_n i\Delta_n = i\delta^D$$

- **GRAVITON PROPAGATORS**

$$i\Delta_n(x;x') = |1-\varepsilon|^{D-2}\frac{(HH')^{\frac{D}{2}-1}}{(4\pi)^{D/2}}\frac{\Gamma\left(\frac{D-1}{2}+\nu_{D,n}\right)\Gamma\left(\frac{D-1}{2}-\nu_{D,n}\right)}{\Gamma(D/2)}\,{}_2F_1\left(\frac{D-1}{2}+\nu_{D,n},\frac{D-1}{2}-\nu_{D,n};\frac{D}{2};1-\frac{y}{4}\right), \quad H = H_0 a^{-\varepsilon}$$

$$\nu^2_{D,n} = \left(\frac{D-1}{2}\right)^2 - \frac{n\left(D-n-1-\frac{n(n-1)}{2}\varepsilon\right)(1-\varepsilon) - \frac{(D-1)(D-2)}{2}\varepsilon + \frac{D(D-2)}{4}\varepsilon^2}{(1-\varepsilon)^2},$$

DIAGONALISATION & GRAVITON SCALING

- **SCALAR AND TENSOR DOFs (G=3x3 operator matrix):**

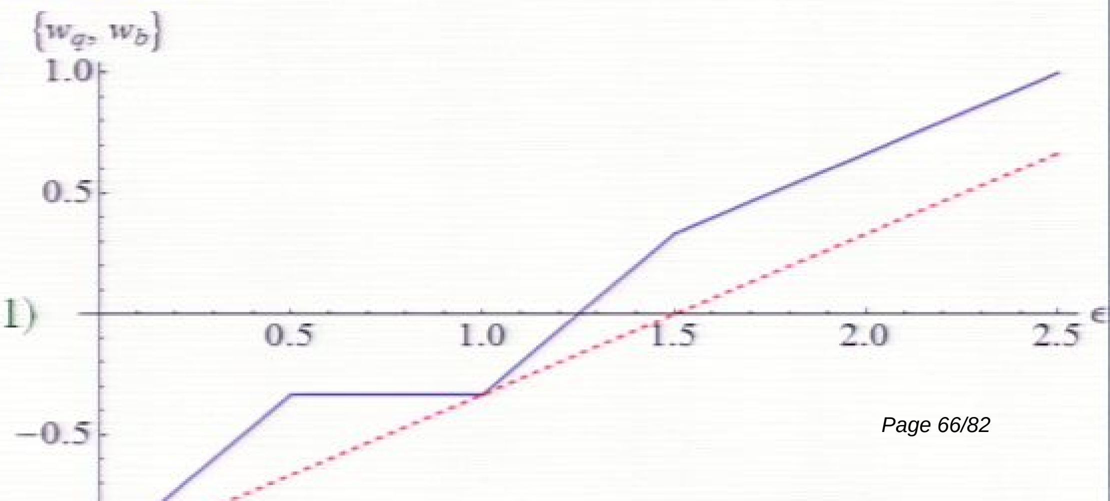
$$G\cdot iM = iI\delta^D$$

$$iM = \begin{pmatrix} i[{}_{rs}\Delta_{kl}] & 0 & 0 \\ 0 & \lambda^2(c_\theta^2 i\Delta_0 + s_\theta^2 i\Delta_2) & \lambda c_\theta s_\theta(i\Delta_0 - i\Delta_2) \\ 0 & \lambda c_\theta s_\theta(i\Delta_0 - i\Delta_2) & (c_\theta^2 i\Delta_0 + s_\theta^2 i\Delta_2) \end{pmatrix}, \quad i[{}_{rs}\Delta_{kl}] = \left(2\delta_{r(k}\delta_{l)s} - \frac{2}{D-3}\delta_{rs}\delta_{kl}\right)i\Delta_0$$

$$\lambda = \sqrt{\frac{2(D-3)}{D-2}}, \quad \tan(2\theta) = \frac{2\sqrt{(D-3)\varepsilon}}{D-3-\varepsilon}, \quad c_\theta \equiv \cos(\theta), \quad s_\theta \equiv \sin(\theta)$$

- **RESULTS (FOR ACCELERATING SPACES)**
- **REGULARISE IN COMOVING BOX (at initial time R_{box}=1/H)**

$$w_q = -1 + \frac{4}{3}\varepsilon \quad (0<\varepsilon<1/2), \quad w_q = -\frac{1}{3} \quad (1/2<\varepsilon<1)$$



NEED TO GO BEYOND SUDDEN MATCHING

- ► SUDDEN MATCHING REQUIRES AN ADDITIONAL C.T.
  + INFINITE RADIATION ENERGY AT MATCHING (removed by a c.t.)
  + A MILD [log(η)] DIVERGENCE AT THE MATCHING:

$\rightarrow$ NOT SATISFACTORY

- ► HENCE: NEED FOR SMOOTH MATCHING. ADVANTAGES:
  - ONLY ONE (STANDARD) UV C.T.
  - IR FULLY FINITE BEFORE AND DURING INFLATION

A MORE GENERAL FLRW

ε not constant!

Koivisto, Prokopec, 1009.5510 [gr-qc]

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$$\frac{a''}{a} = -(1-6\xi)\left[\gamma^2 + \frac{\nu^2 - 1/4}{\eta^2}\right] \Rightarrow \qquad a(\eta) = (-1)^{\nu - 1/2}\sqrt{\pi\nu\eta}\left[c_1 J_\nu(\gamma\eta) + c_2 Y_\nu(\gamma\eta)\right]$$

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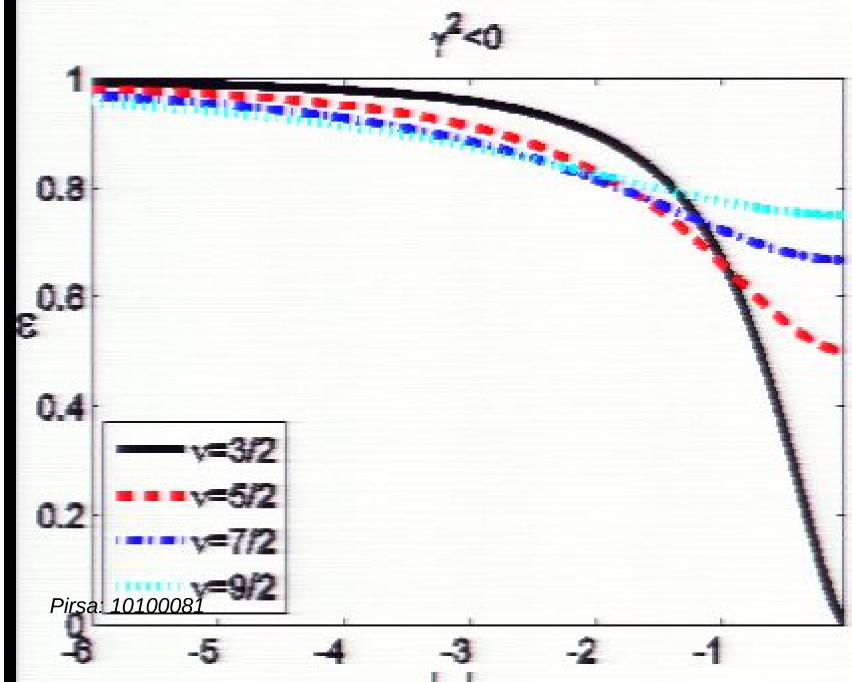
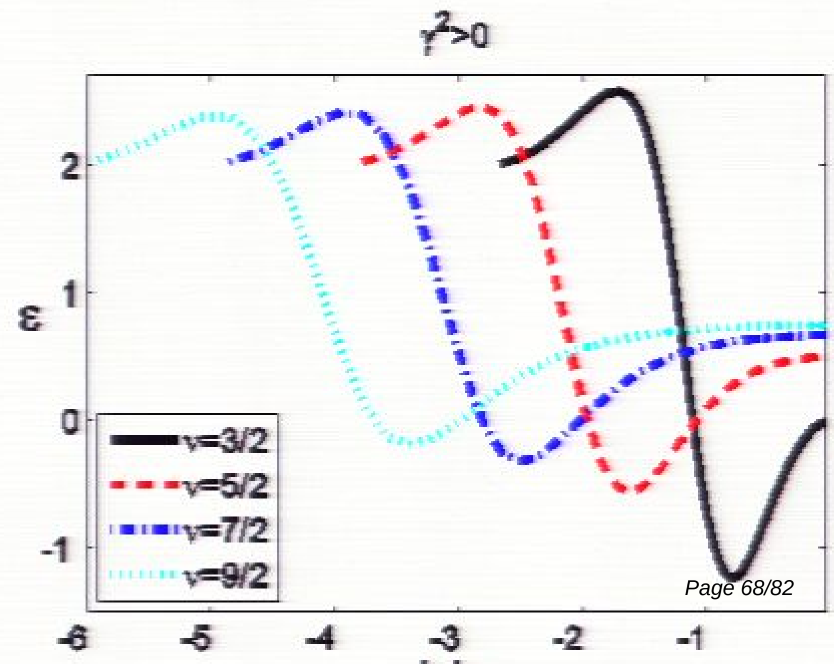
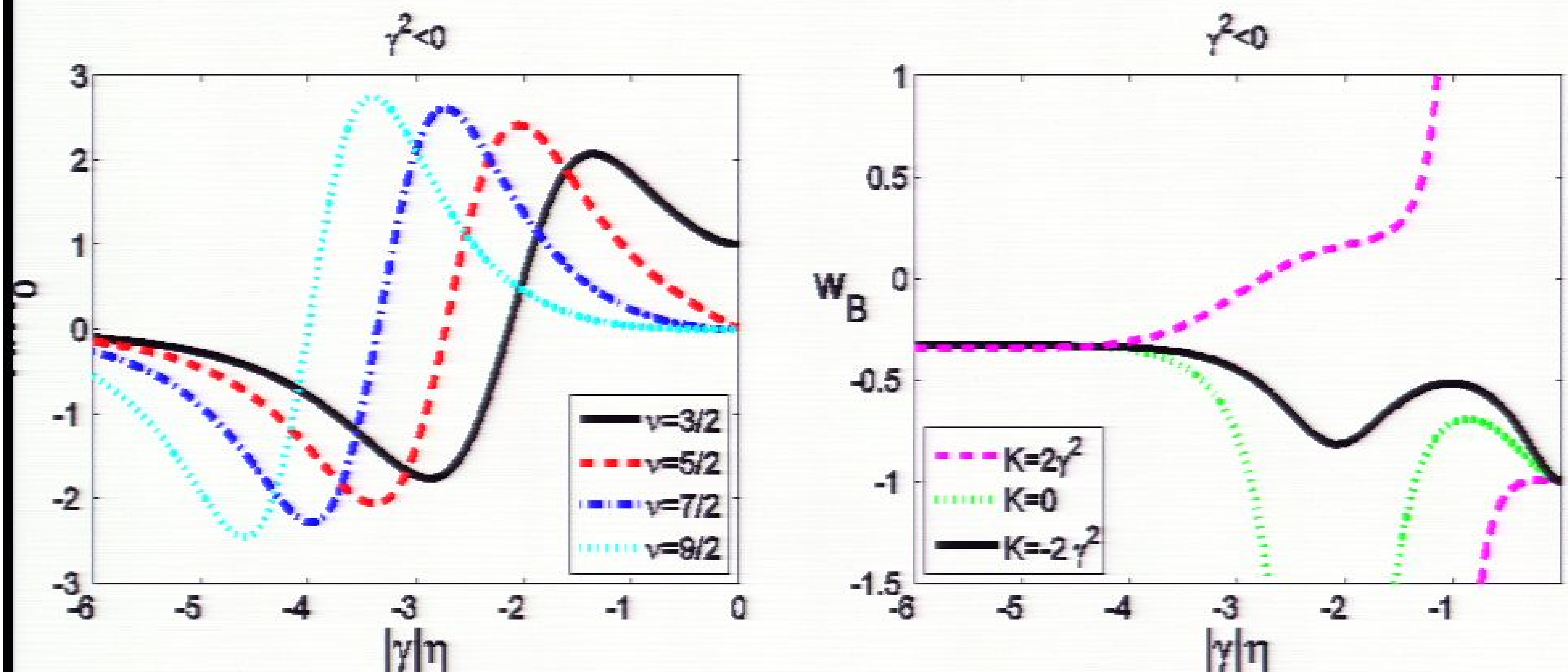




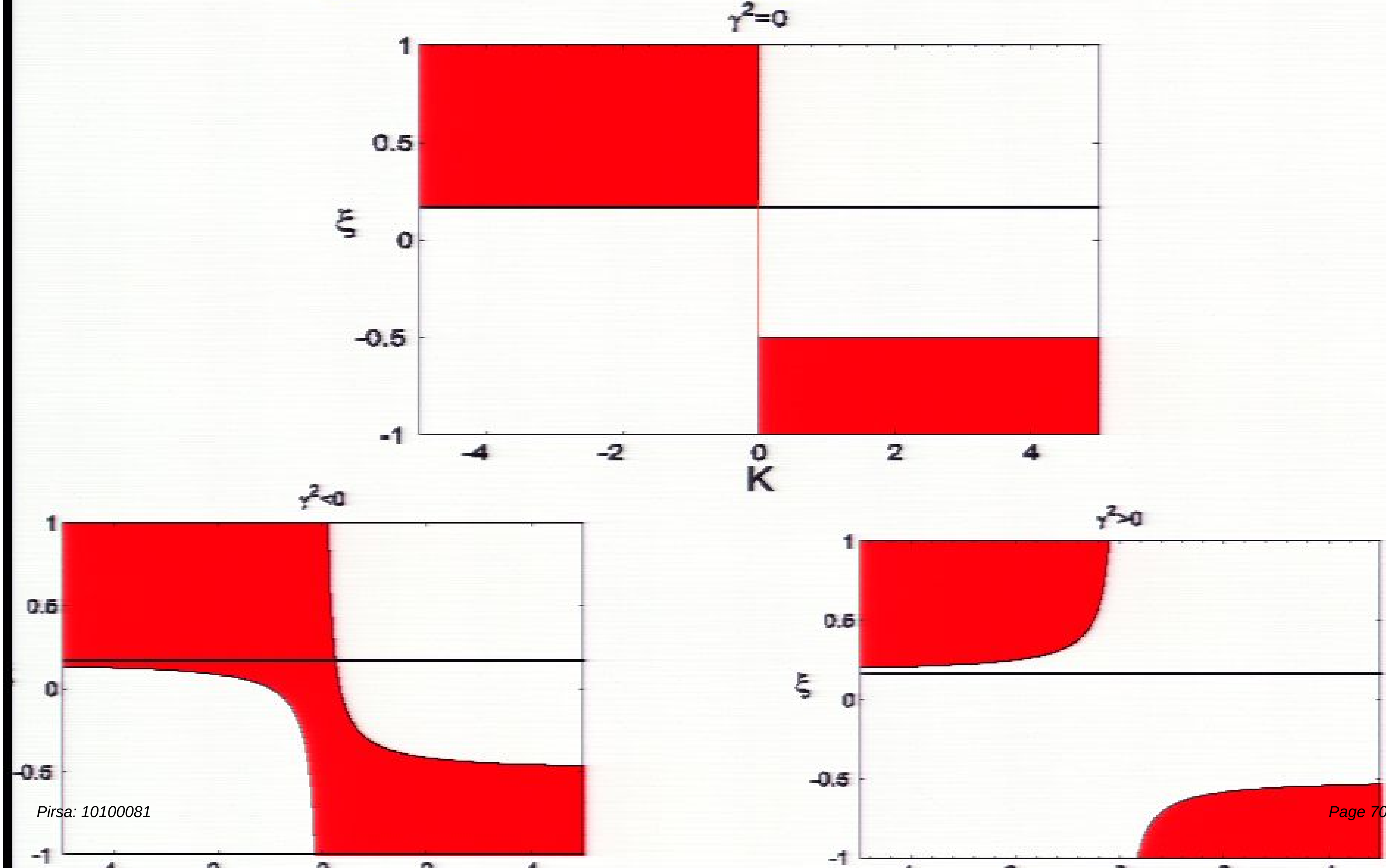
.. ONE CAN GET A BOUNCE



NB: BOTH BOUNCE (BELOW) AND NON-BOUNCE SOLUTIONS (CANNOT BE EXTENDED TO t=-∞)

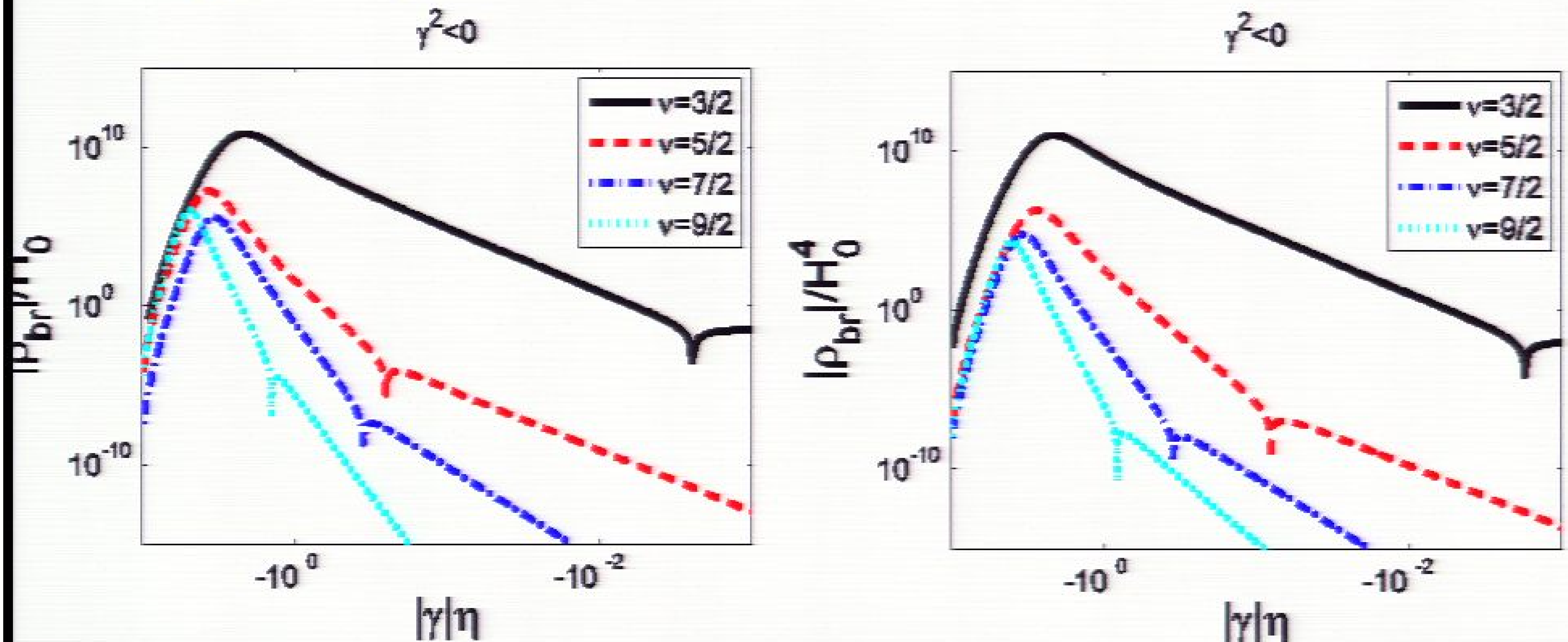
IR DIVERGENCES: NONMIN. COUPLING

- Coloured regions contain IR divergences



ONE LOOP BACKREACTION

- **Energy density**



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regulates IR

- CAUTION: WE CAN DO ANALYSIS ONLY FOR HALF INTEGER

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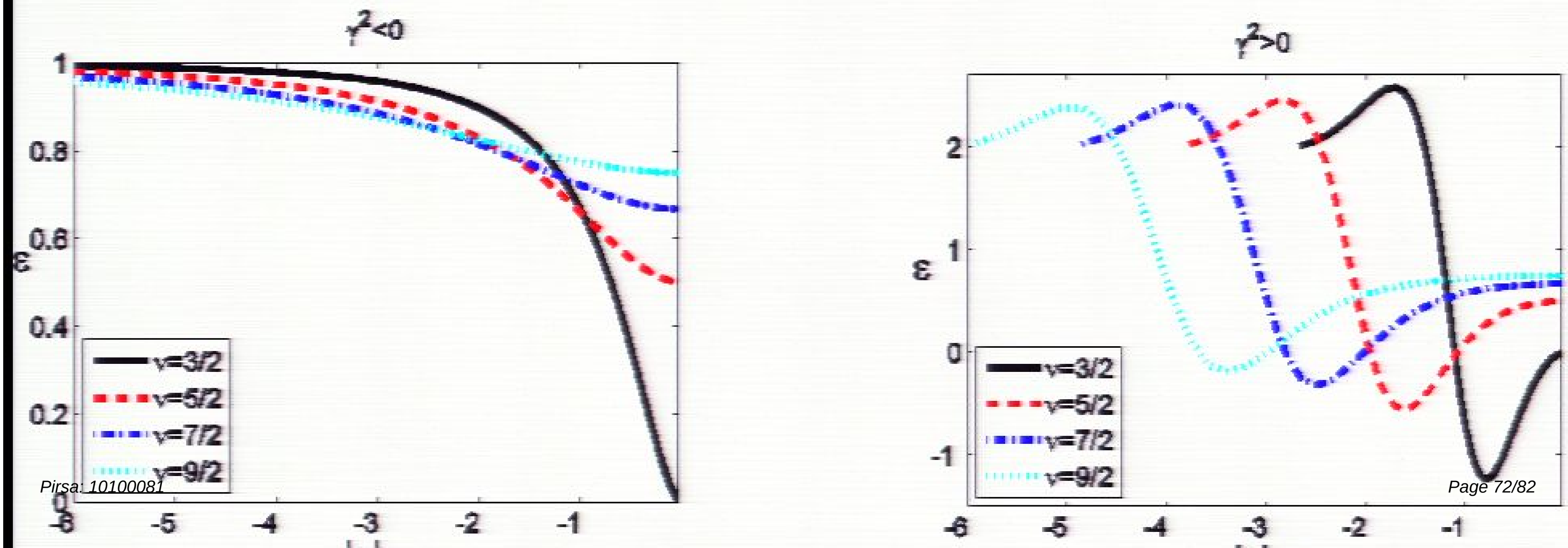

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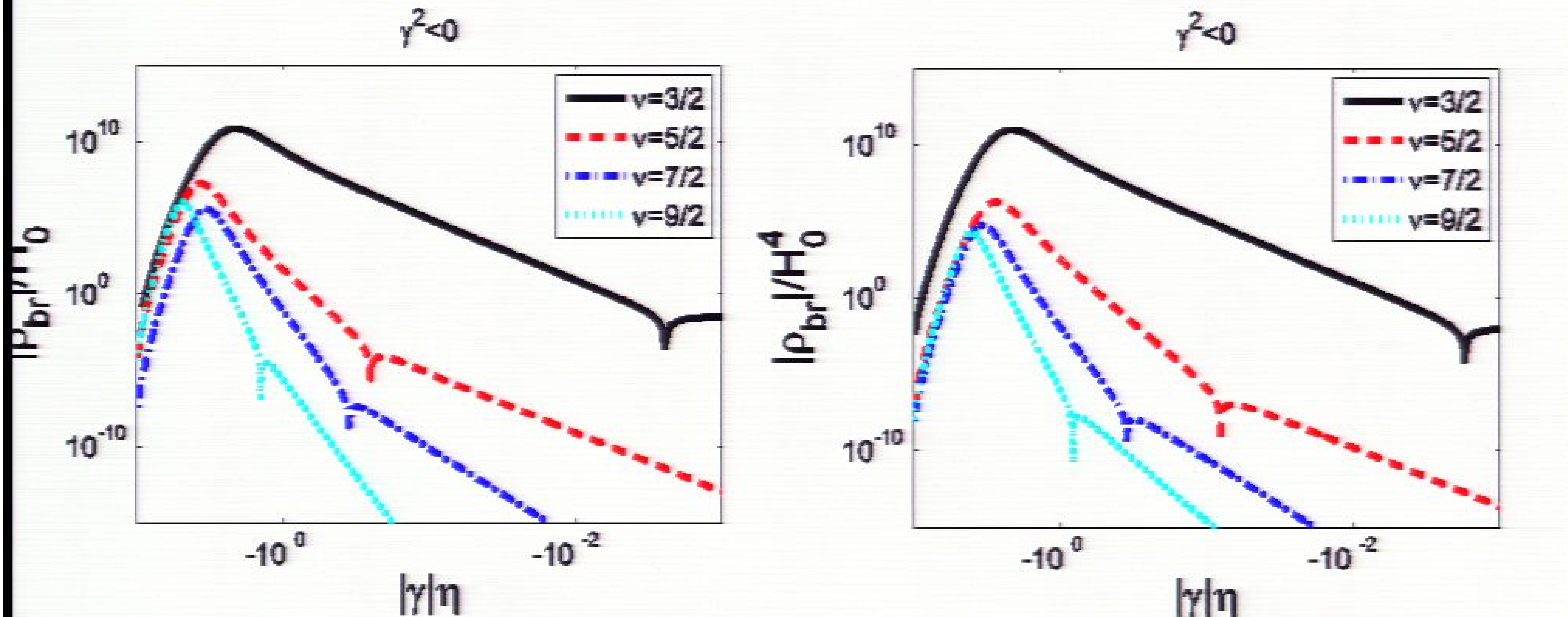

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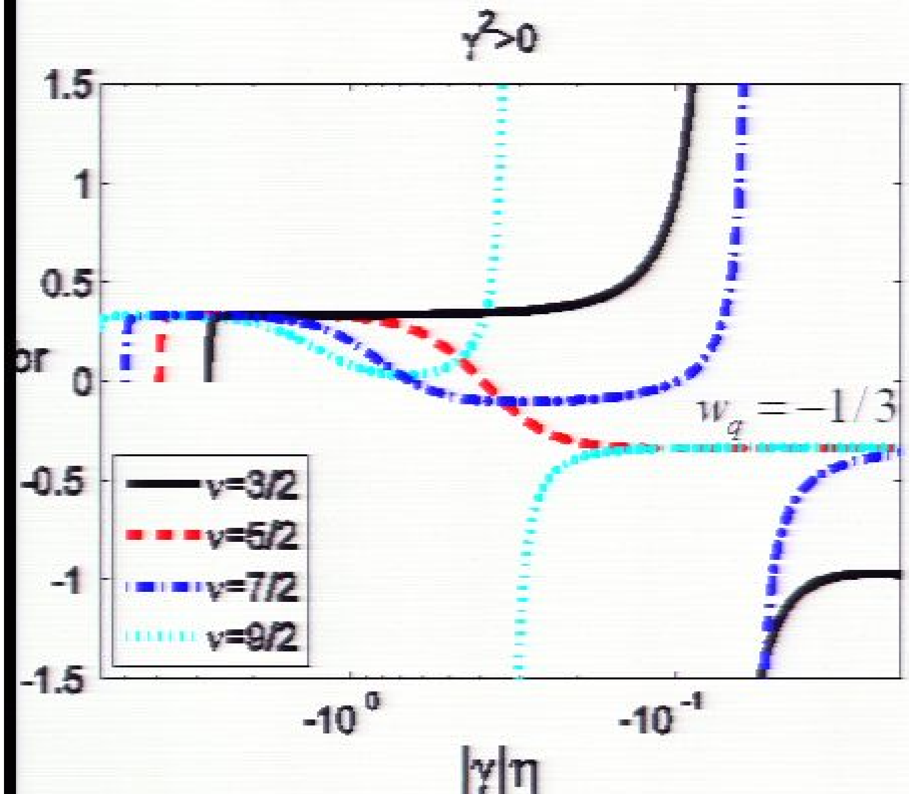
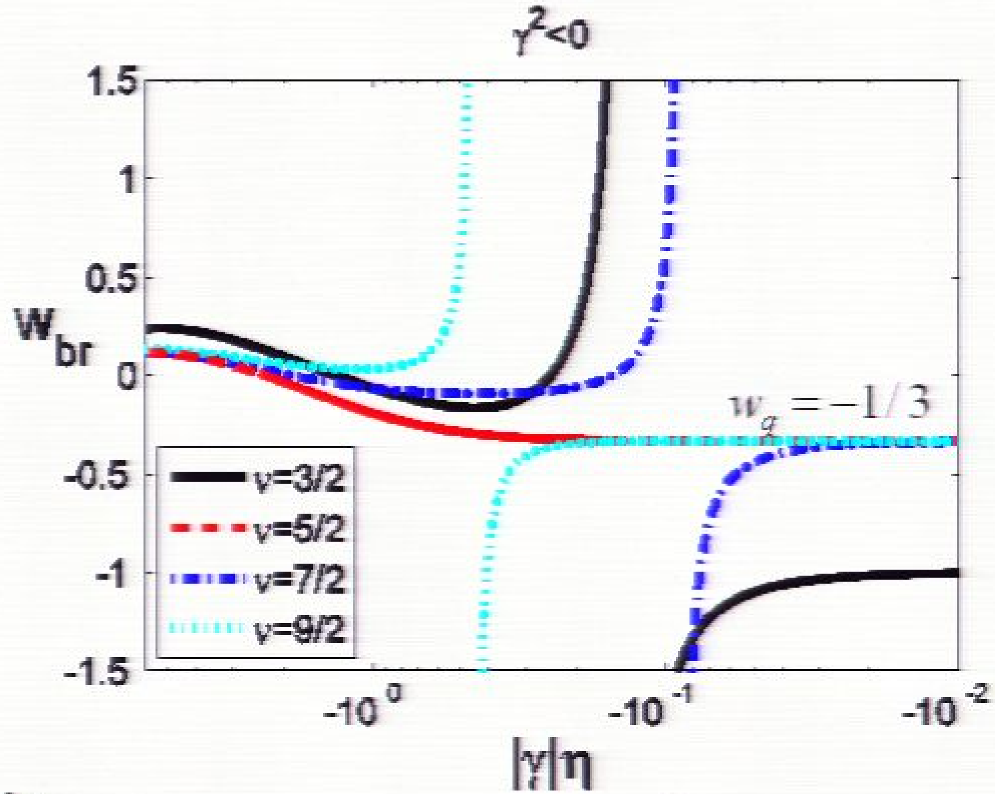
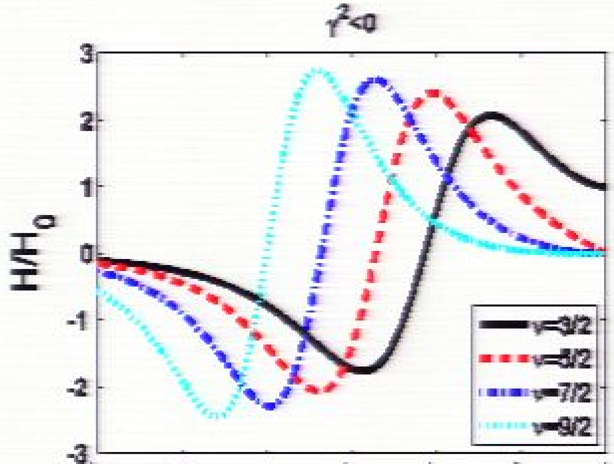
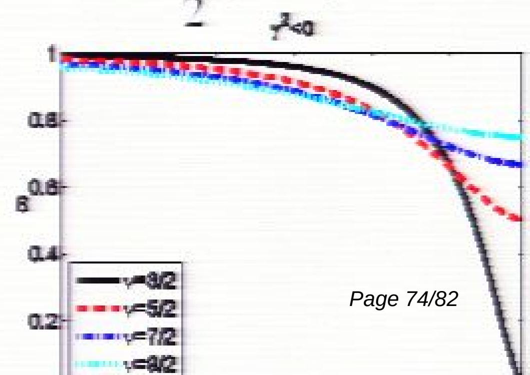




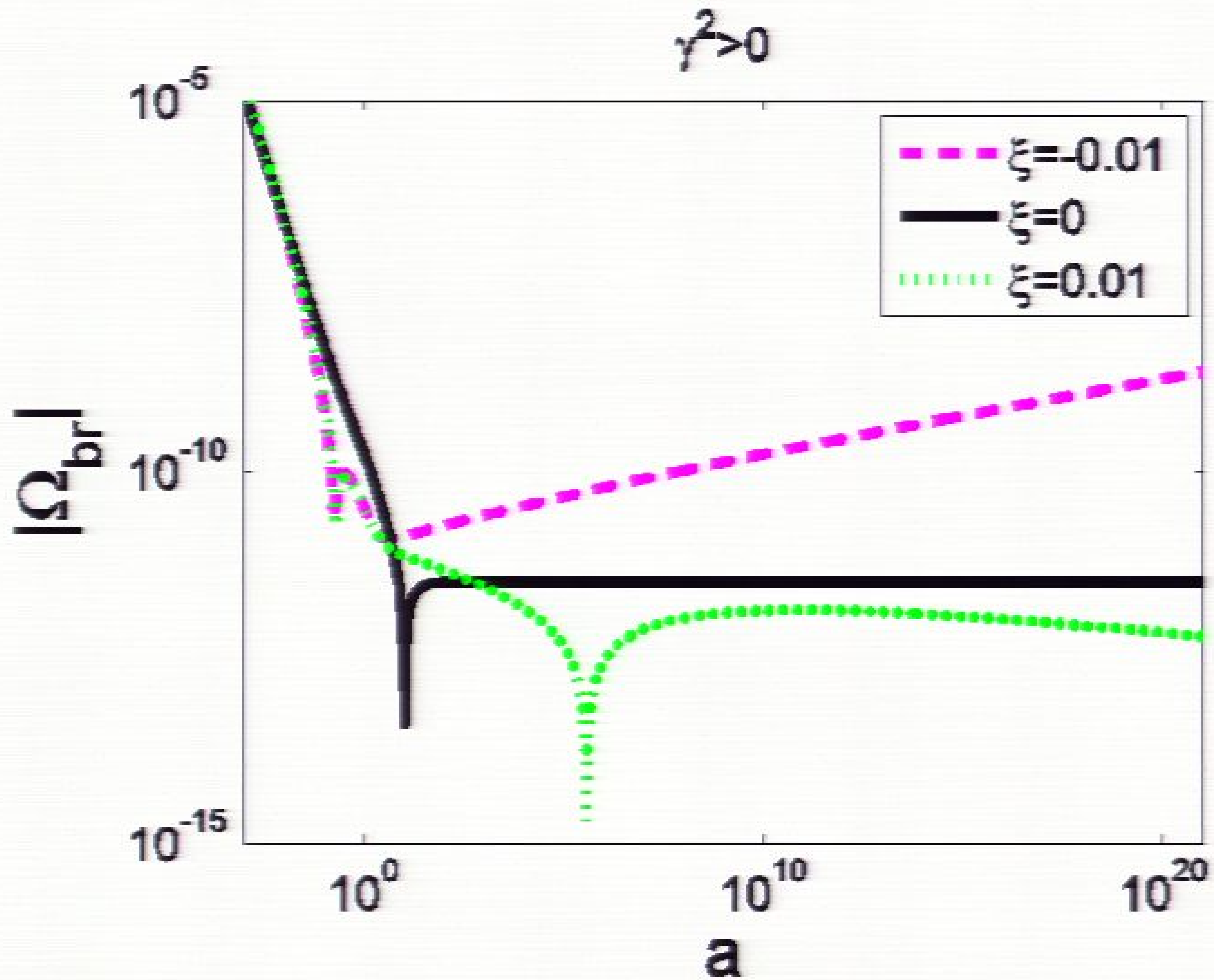
• recall →



$$\varepsilon = \frac{3}{2}(1+w)$$



ONE LOOP BACKREACTION: Ω_q



.. CONCLUSION: BACKREACTION GROWS WHEN $\xi<0$ ($\gamma>0$).

OR SMALL $\xi<0$: takes long time before quantum fluctuations dominate $\rightarrow$ DE?

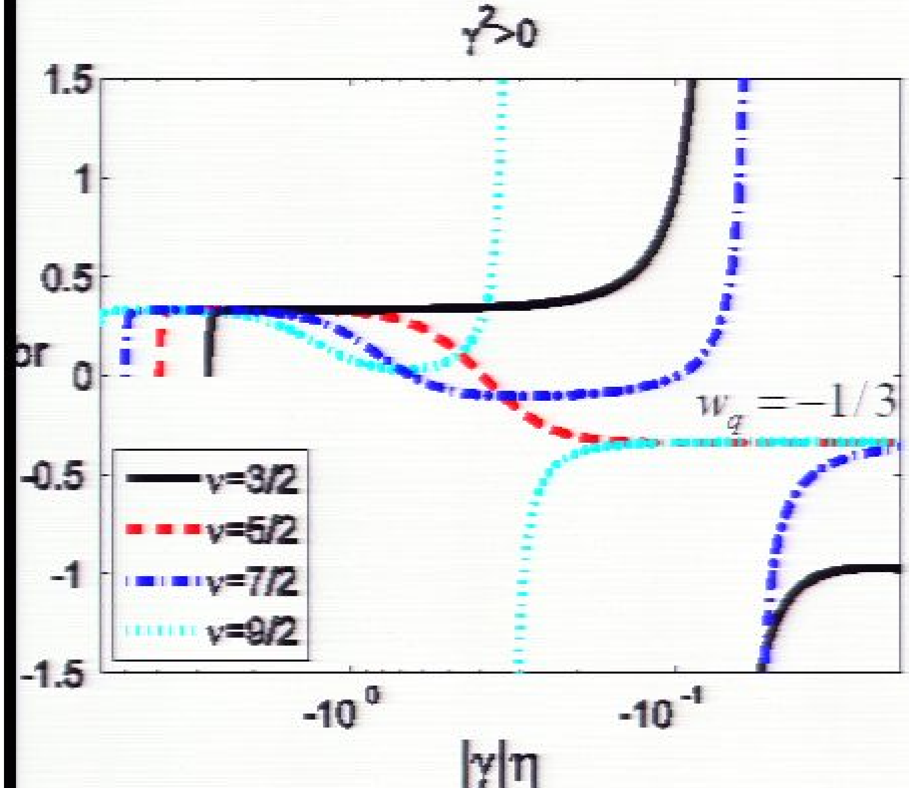
CONCLUSIONS, Part III

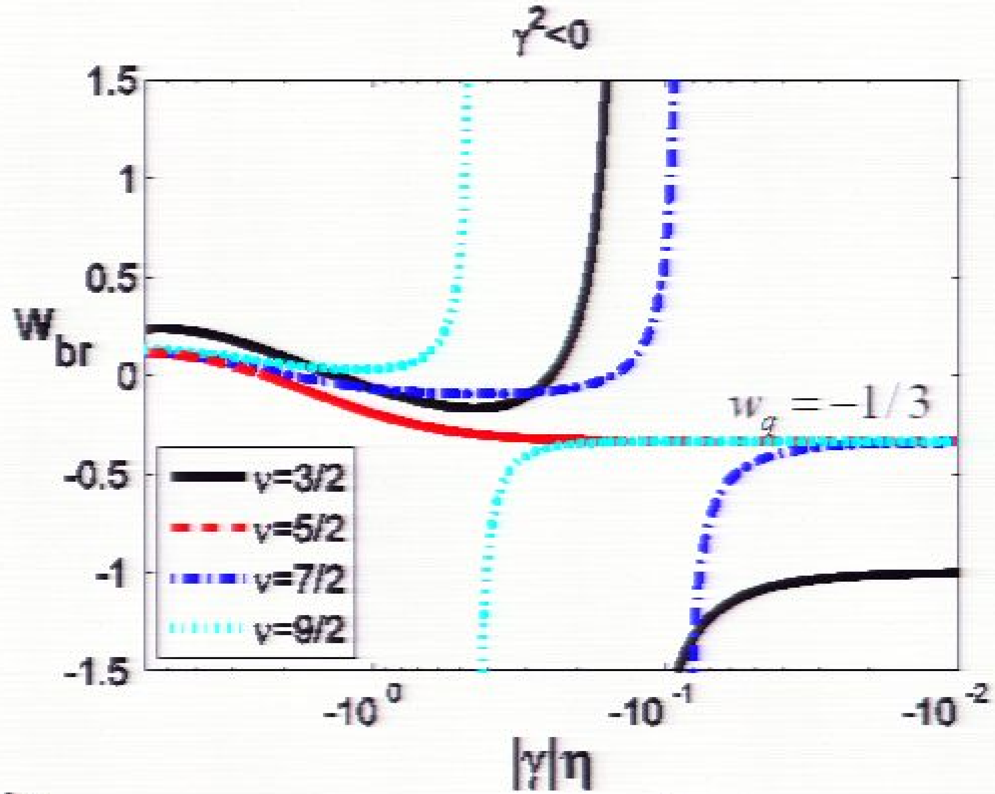
- BACKREACTION GROWS AND CAN DOMINATE IF:
  (a) $\xi<0$ **[non-minimal coupling]**
  (b) $\varepsilon<0$ [superinflation]

- Growth is powerlaw (not logarithmic) [similar as DRG].

- One would like to include gravitational fluctuations, 2 loops, etc.

ONE LOOP BACKREACTION: w_q





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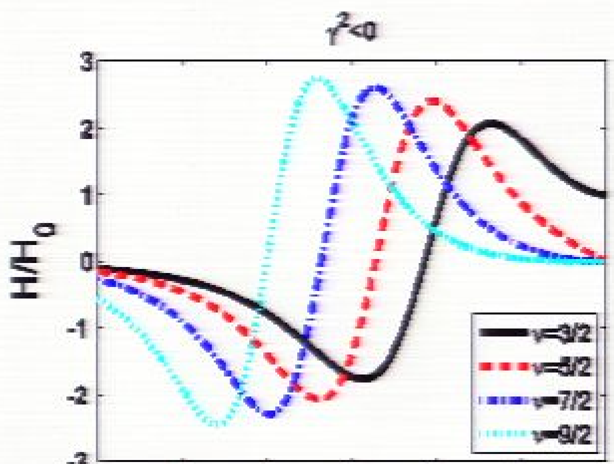

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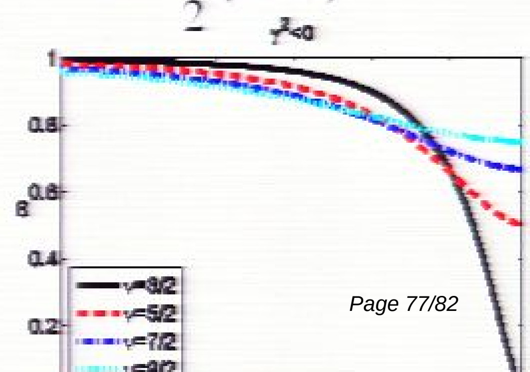

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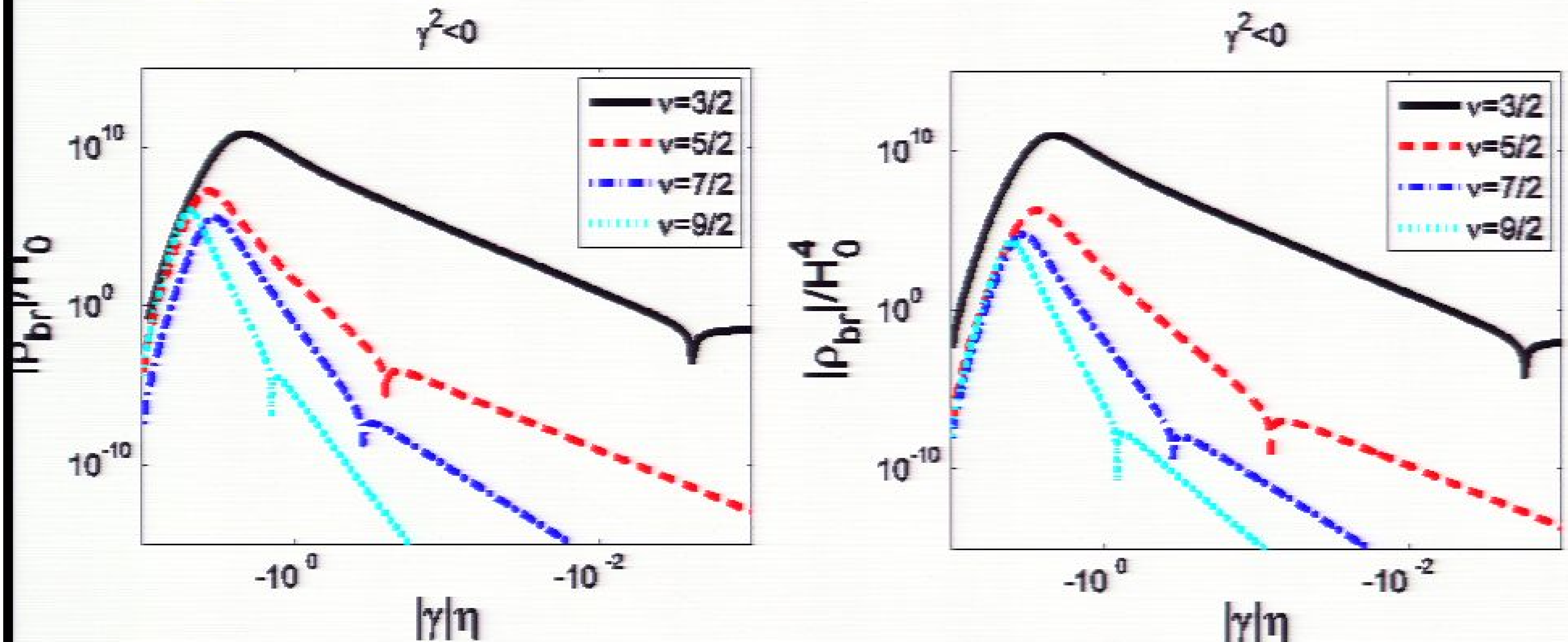

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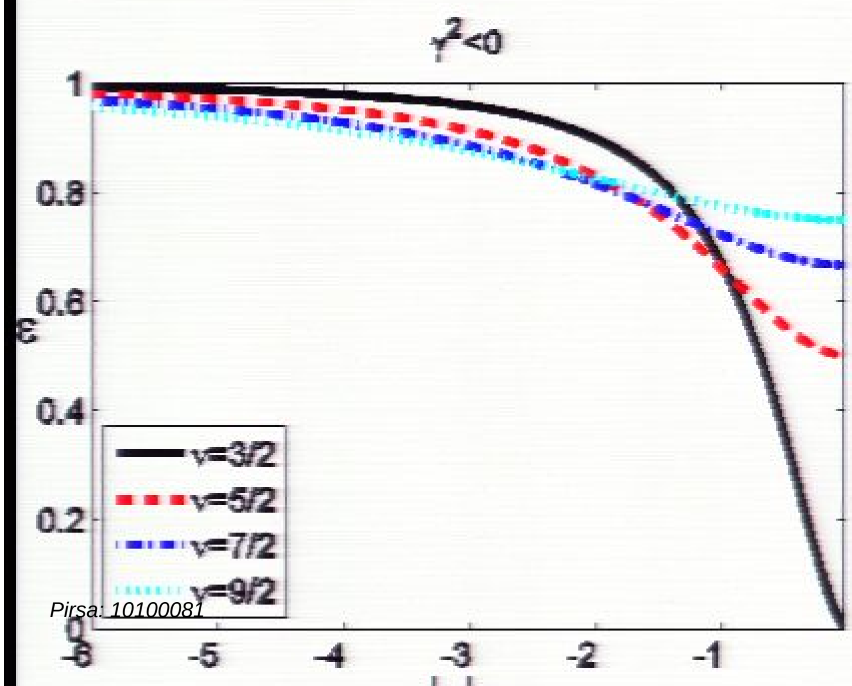

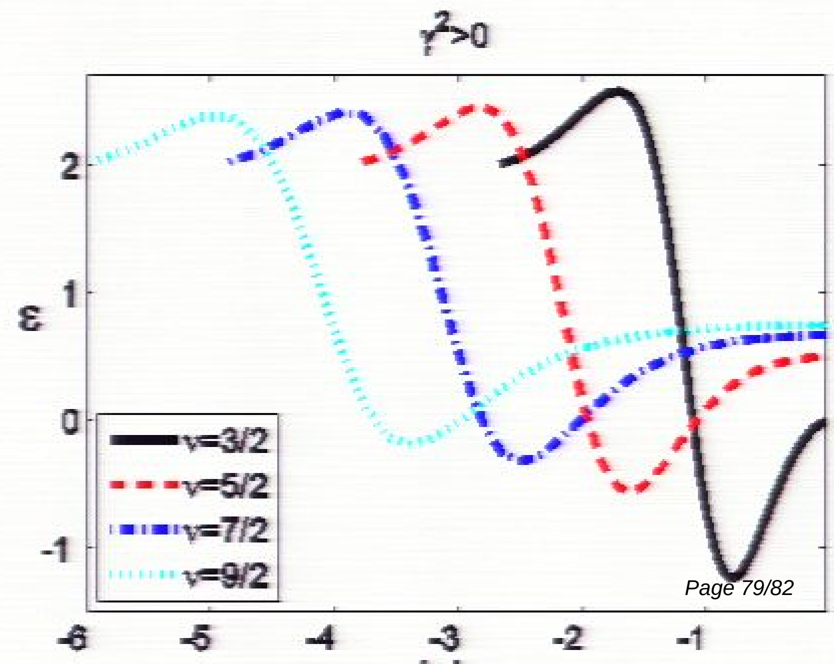

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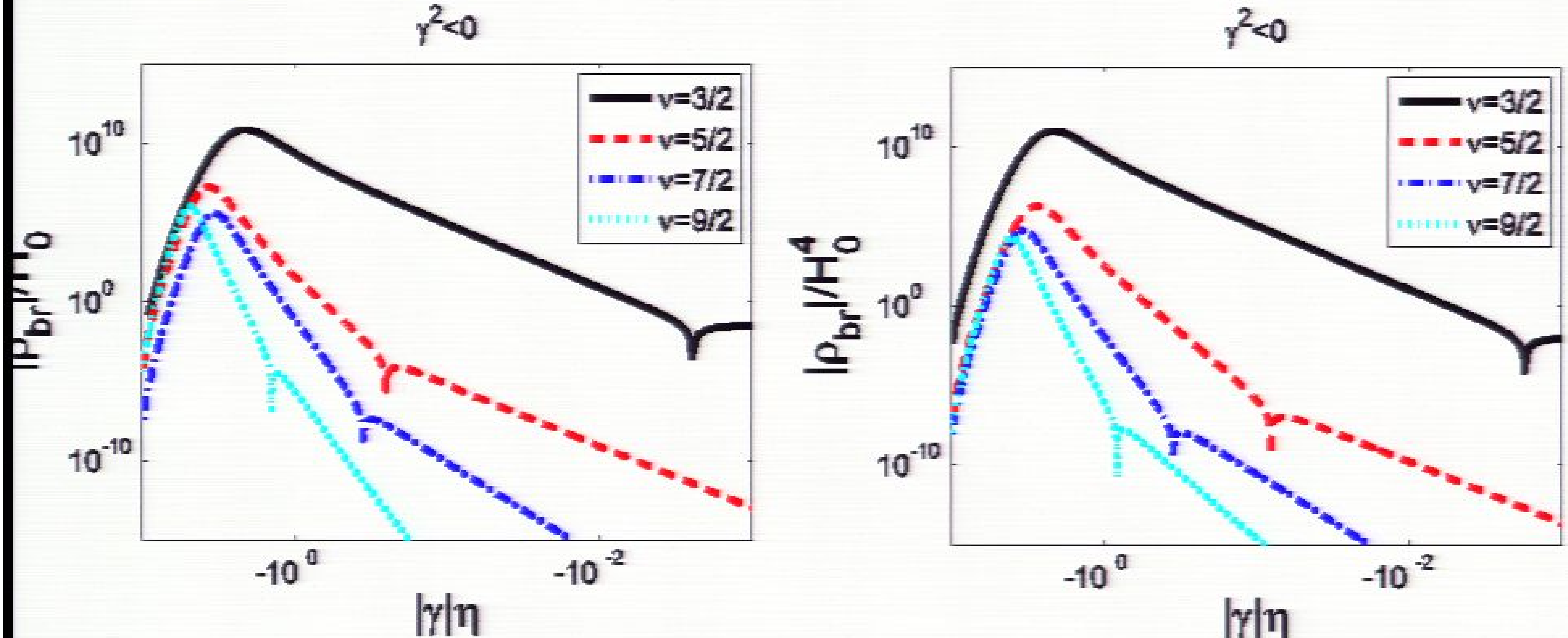

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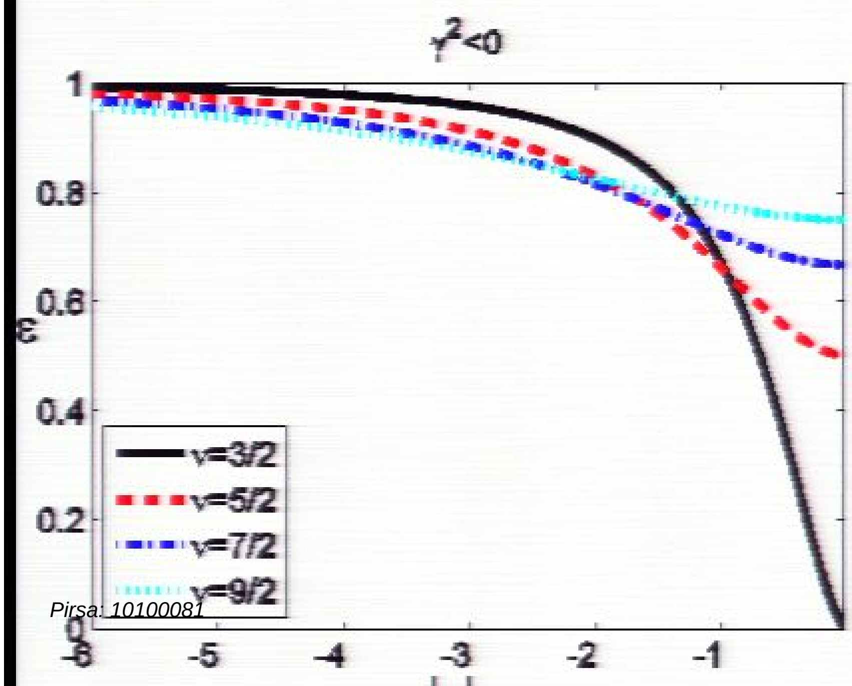

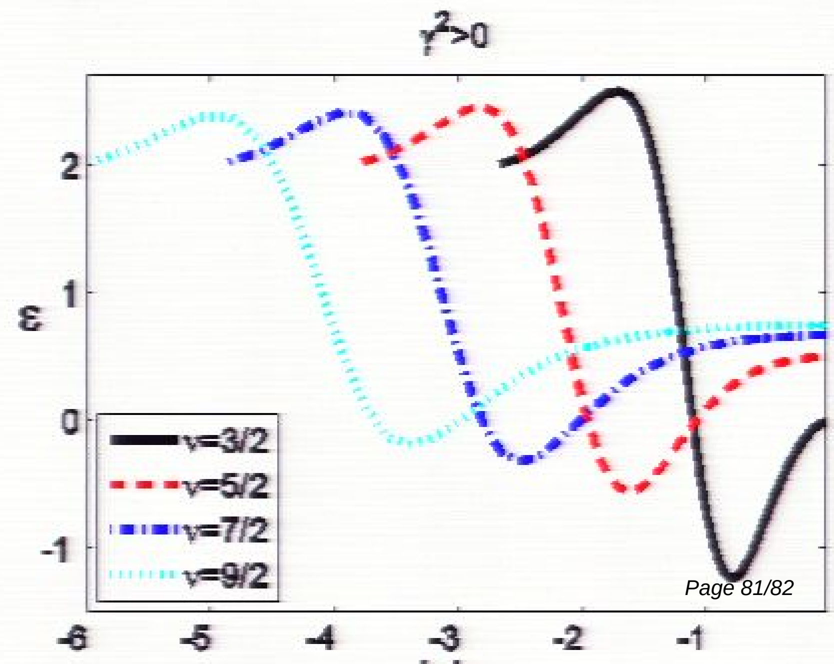

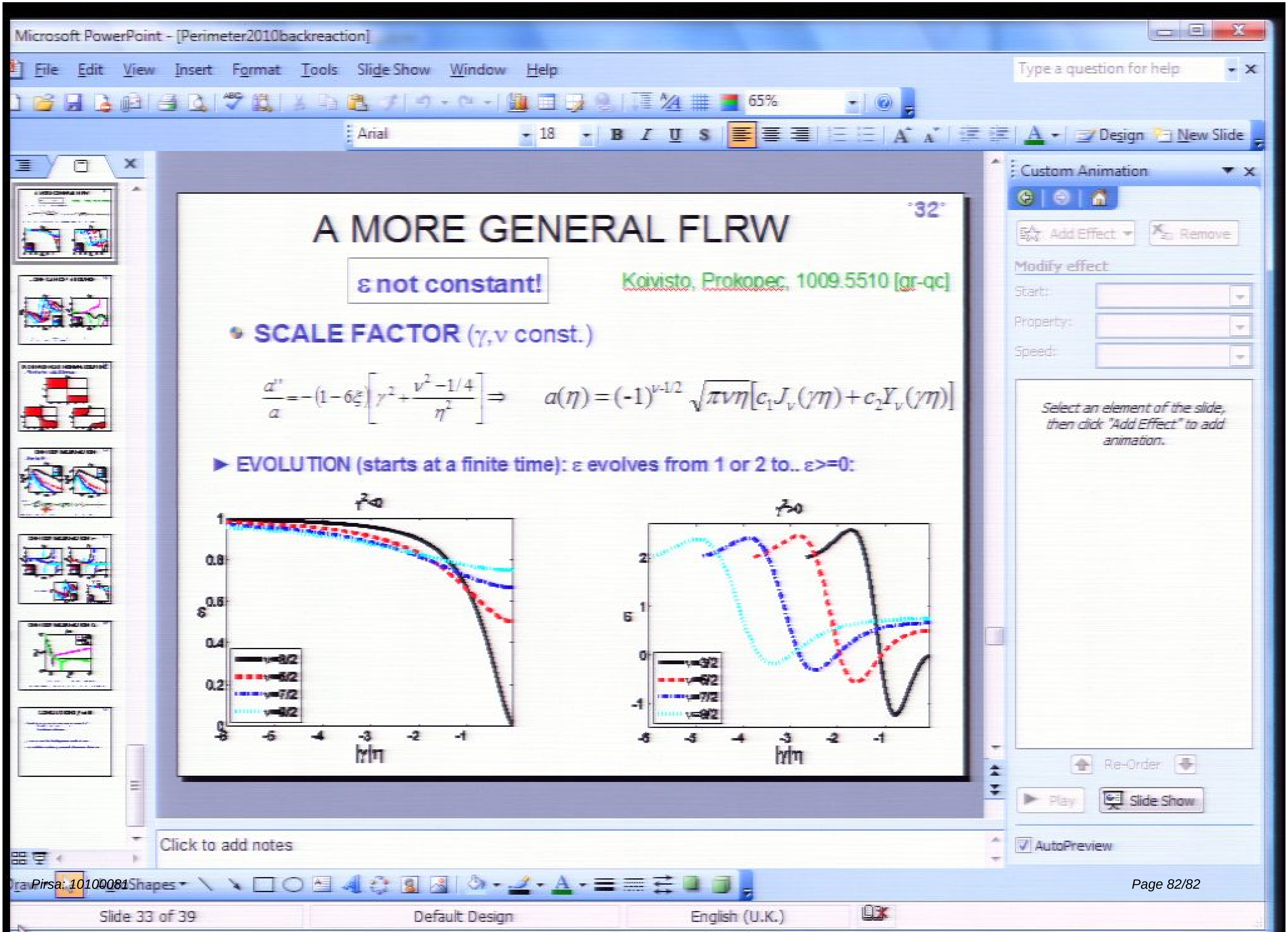
A MORE GENERAL FLRW
32
ε not constant!
Koivisto, Prokopec, 1009.5510 [gr-qc]
SCALE FACTOR (γ,ν const.)
$\frac{a''}{a} = -(1-6\xi)\left[\gamma^2 + \frac{\nu^2 - 1/4}{\eta^2}\right] \Rightarrow \quad a(\eta) = (-1)^{\nu-1/2}\sqrt{\pi\nu\eta}\left[c_1 J_\nu(\gamma\eta) + c_2 Y_\nu(\gamma\eta)\right]$
EVOLUTION (starts at a finite time): ε evolves from 1 or 2 to.. ε>=0:
|γ|η
ε
Microsoft PowerPoint - [Perimeter2010backreaction]
Click to add notes
Slide 33 of 39
Default Design
English (U.K.)
Custom Animation