Title: Ultraviolet Divergences in Cosmological Correlations

Date: Oct 30, 2010 09:30 AM

URL: http://pirsa.org/10100080

Abstract: A method is developed for dealing with ultraviolet divergences in calculations of cosmological correlations, which does not depend on dimensional regularization. An extended version of the WKB approximation is used to analyze the divergences in these calculations, and these divergences are controlled by the introduction of Pauli--Villars regulator fields. This approach is illustrated in the theory of a scalar field with arbitrary self-interactions in a fixed flat-space Robertson--Walker metric with arbitrary scale factor $a(t)$. Explicit formulas are given for the counterterms needed to cancel all dependence on the regulator properties, and an explicit prescription is given for calculating finite correlation functions.

[redacted]

PROBLEMS WITH DIMENSIONAL REGULARIZATION

- Difficult to use when wave functions appearing in interaction-picture fields are not known analytically, so mostly limited to de Sitter cosmology or slow roll inflation.
- Tricky even when wave functions are explicitly known:
  Senatore & Zaldarriaga 2009, citing Weinberg 2006; Adshead, Easther & Lim 2009; Chaicherdsakul 2007; Seery 2007; Dimastrogiovanni & Bartolo 2008; Gao & Xu 2009; Campo 2009.

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THE MODEL

$$\mathcal{L} = \sqrt{-\mathrm{Det} g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] ,$$

$$g_{00} = -1 , \quad g_{0i} = 0 , \quad g_{ij} = a^2(t)\,\delta_{ij} ,$$

$$\ddot{\varphi} + 3H\dot{\varphi} - a^{-2}\nabla^2\varphi + V'(\varphi) = 0 ,$$

$$H \equiv \dot{a}/a$$

$$\varphi(\mathbf{x}, t) = \bar{\varphi}(t) + \delta\varphi(\mathbf{x}, t) ,$$

$$\ddot{\overline{\varphi}} + 3H\dot{\overline{\varphi}} + V'(\overline{\varphi}) = 0 .$$

INTERACTION PICTURE

$$\delta\ddot{\varphi}+3H\delta\dot{\varphi}-a^{-2}\nabla^2\delta\varphi+V''(\overline{\varphi})\delta\varphi=0\,.$$

$$[\delta\varphi(\mathbf{x},t),\delta\dot{\varphi}(\mathbf{y},t)]=ia^{-3}(t)\delta^3(\mathbf{x}-\mathbf{y})\,,$$

$$[\delta\varphi(\mathbf{x},t),\delta\varphi(\mathbf{y},t)]=[\delta\dot{\varphi}(\mathbf{x},t),\delta\dot{\varphi}(\mathbf{y},t)]=0\,.$$

$$\delta\varphi(\mathbf{x},t)=\int d^3q\,\left[e^{i\mathbf{q}\cdot\mathbf{x}}\alpha(\mathbf{q})u_q(t)+\text{c.c.}\right]\,,$$

$$[\alpha(\mathbf{q}),\alpha^\dagger(\mathbf{q}')]=\delta^3(\mathbf{q}-\mathbf{q}')\,,\;\;[\alpha(\mathbf{q}),\alpha(\mathbf{q}')]=0\,,$$

$$\ddot{u}_q+3H\dot{u}_q+a^{-2}q^2u_q+V''(\overline{\varphi})u_q=0$$

$$u_q(t)\to\frac{1}{(2\pi)^{3/2}a(t)\sqrt{2q}}\exp\left[-iq\int_{\mathcal{T}}^t\frac{dt'}{a(t')}\right]$$

"IN-IN" FORMALISM

$$\langle \mathcal{O}_H(t)\rangle_{\text{VAC}} = \left\langle \bar{T}\exp\left(i\int_{-\infty}^{t} H_I'(t')dt'\right)\mathcal{O}_I(t)\right.$$
$$\left.\times\, T\exp\left(-i\int_{-\infty}^{t} H_I'(t')dt'\right)\right\rangle_0$$

$$\langle\cdots\rangle_0 \equiv (\Phi_0,\cdots\Phi_0)$$
$$\alpha(\mathbf{q})\Phi_0 = 0$$

$$H_I' \equiv a^3\int d^3x\,\left[\frac{1}{6}V'''(\overline{\varphi})\delta\varphi^3\right.$$
$$\left.+\frac{1}{24}V''''(\overline{\varphi})\delta\varphi^4+\dots\right]$$

LOOP EXPANSION

$$V(\varphi) \equiv g^{-2} F(g\varphi)$$

$$V'''(\overline{\varphi}) \propto g\,, \;\; V''''(\overline{\varphi}) \propto g^2\,, \cdots$$

$$\langle \delta\varphi(\mathbf{x}_1, t_1) \cdots \delta\varphi(\mathbf{x}_N, t_N) \rangle_{\rm VAC}^{L \text{ loops}}$$

$$\propto \; g^{2L-2+N}$$

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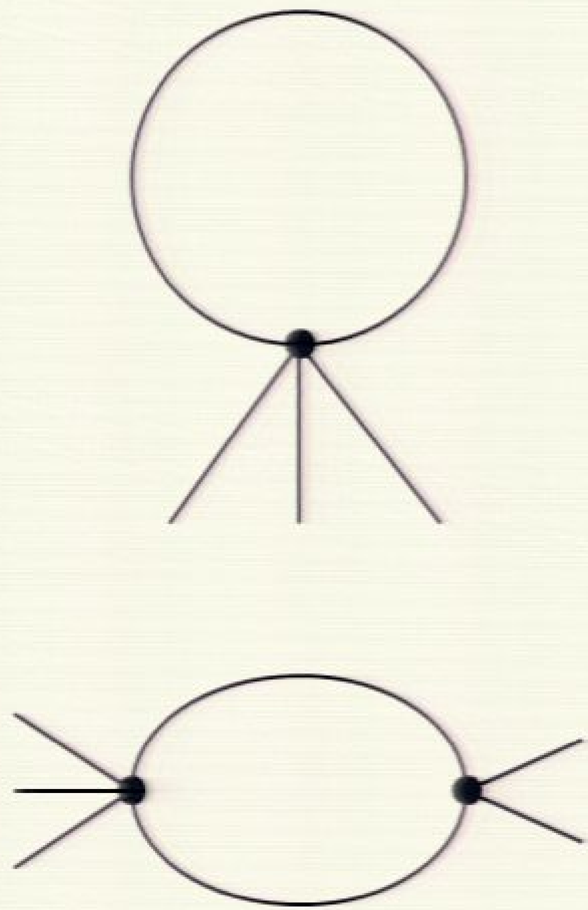
1PI ONE-LOOP DIVERGENCES



ONE-LOOP COUNTERTERMS

$$\mathcal{L}_{\infty}^{1\,\text{loop}} = \sqrt{-\text{Det}g}\Big[A\,V''(\varphi) + B[V''(\varphi)]^2 + C\,R\,V''(\varphi)\Big]\,.$$

$$A \propto \Lambda^2\,, \quad B\,\&\,C \propto \ln\Lambda$$

$$\mathcal{L}_{\infty}^{1\,\text{loop}} = a^3\Bigg[A\left(V'''(\overline{\varphi})\delta\varphi + \frac{1}{2}V''''(\overline{\varphi})\delta\varphi^2 + \dots\right) + B\left(2V''(\overline{\varphi})V'''(\overline{\varphi})\delta\varphi + [V'''^2(\overline{\varphi}) + V''(\overline{\varphi})V''''(\overline{\varphi})]\delta\varphi^2 + \dots\right) - C\,(6\dot{H} + 12H^2)\left(V'''(\varphi)\delta\varphi + \frac{1}{2}V''''(\overline{\varphi})\delta\varphi^2 + \dots\right)\Bigg]\,.$$

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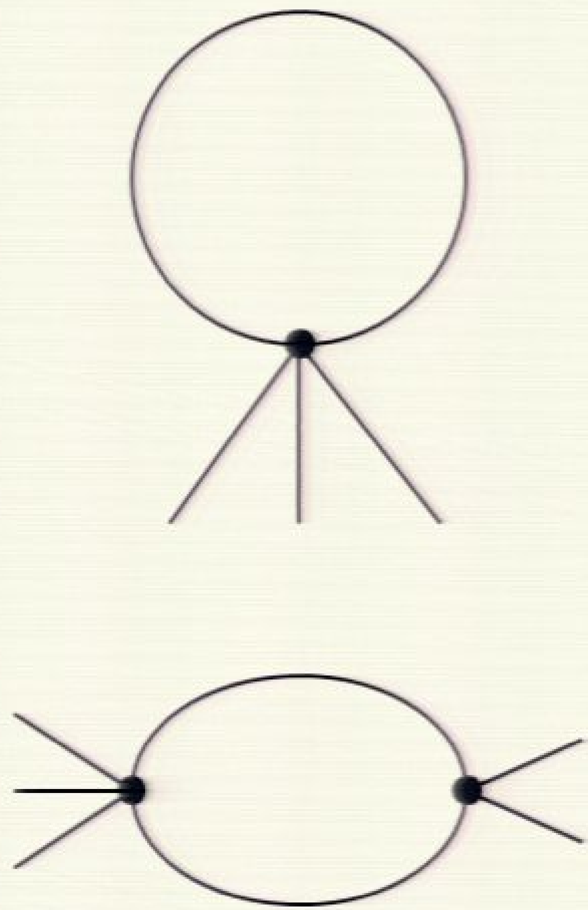

REGULATORS

Pauli & Villars 1949; Bernard & Duncan 1977; Vilenkin 1977

$$\mathcal{L} = \sqrt{-\mathrm{Det} g}\Bigg[-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi$$

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$$-V\left(\varphi + \sum_n \chi_n\right)\Bigg] .$$

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$$+ [V'''^2(\overline{\varphi}) + V''(\overline{\varphi})V''''(\overline{\varphi})]\delta\varphi^2 + \ldots\Big)$$
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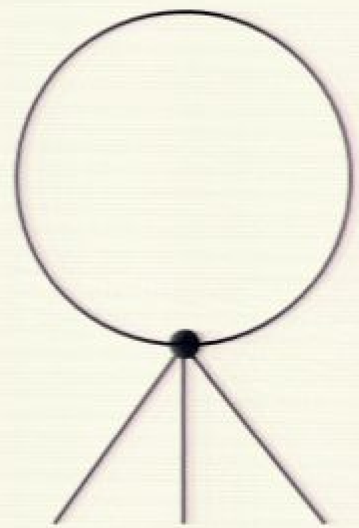

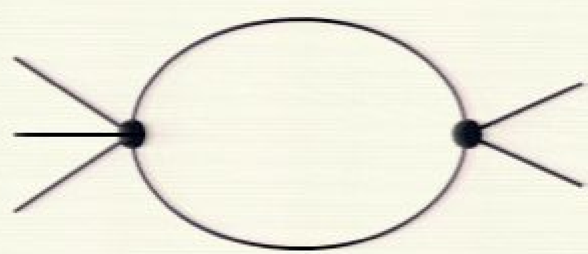

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In one-loop graphs we integrate over a single co-moving wave number q.

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In integral over $q < Q$, ignore regulators. In integral over $q > Q$, use WKB approximation, drop terms that are convergent even before cancellations.

EXTENDED WKB
APPROXIMATION
(to zeroth order in $V''(\overline{\varphi})$)

For $q^2/a^2 \gg H^2$ and $\dot{H}$

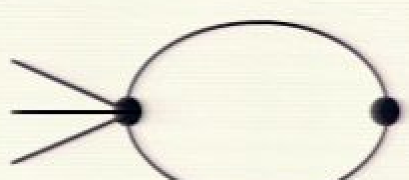
$$|u_q|^2 \rightarrow \frac{1}{2qa^2(2\pi)^3}\left[1 + \frac{\dot{H} + 2H^2}{2q^2/a^2}\right]$$

For q^2/a^2 and $M_n^2 \quad \gg \quad H^2$ and $\dot{H}$

$$|u_{nq}|^2 \rightarrow \frac{1}{2\kappa_{nq}a^3(2\pi)^3}\left[1 + \frac{\dot{H} + 2H^2}{2\kappa_{nq}^2}\right.$$
$$\left. + \frac{(\dot{H} + 3H^2)M_n^2}{4\kappa_{nq}^4} - \frac{5H^2M_n^4}{8\kappa_{nq}^6}\right],$$

$$\kappa_{nq}^2(t) = \Big(q/a(t)\Big)^2 + M_n^2$$

To include $V''(\overline{\varphi})$, treat it as a first-order perturbation:



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ONE-LOOP COUNTERTERMS

$$\mathcal{L}^{1\,\mathrm{loop}}_{\infty} = \sqrt{-\mathrm{Det}g}\Big[A V''(\varphi) + B[V''(\varphi)]^2 + C R V''(\varphi)\Big] ,$$

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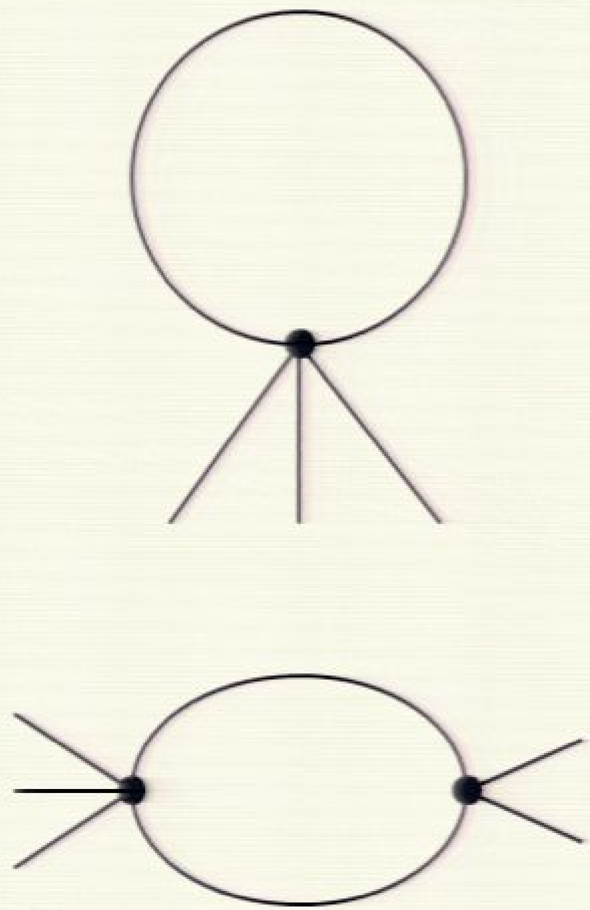

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EXTENDED WKB APPROXIMATION

(to zeroth order in $V''(\overline{\varphi})$)

For $q^2/a^2 \gg H^2$ and $\dot{H}$

$$|u_q|^2 \to \frac{1}{2qa^2(2\pi)^3}\left[1 + \frac{\dot{H} + 2H^2}{2q^2/a^2}\right]$$

For q^2/a^2 and $M_n^2 \;\gg\; H^2$ and $\dot{H}$

$$|u_{nq}|^2 \to \frac{1}{2\kappa_{nq}a^3(2\pi)^3}\left[1 + \frac{\dot{H} + 2H^2}{2\kappa_{nq}^2} + \frac{(\dot{H} + 3H^2)M_n^2}{4\kappa_{nq}^4} - \frac{5H^2M_n^4}{8\kappa_{nq}^6}\right],$$

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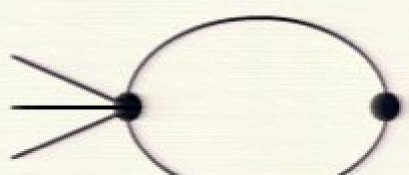

COUNTERTERMS

To cancel dependence on regulators

$$A = -\frac{1}{16\pi^2}\left[\sum_n Z_n^{-1} M_n^2 \ln M_n + \mu_A^2\right]$$

$$B = -\frac{1}{32\pi^2}\left[2\sum_n Z_n^{-1}\ln(M_n/\mu_B)\right.$$

$$\left.+\sum_{nm} Z_n^{-1} Z_m^{-1}\left(\frac{M_n^2 \ln\left(\frac{M_n}{\mu_B}\right) - M_m^2 \ln\left(\frac{M_m}{\mu_B}\right)}{M_n^2 - M_m^2}\right)\right]$$

$$C = \frac{1}{48\pi^2}\left(\frac{5}{6} - \sum_n Z_n^{-1} \ln\left(\frac{M_n}{\mu_C}\right)\right) .$$

μ_A, μ_B, μ_C unknown finite *physical* parameters.

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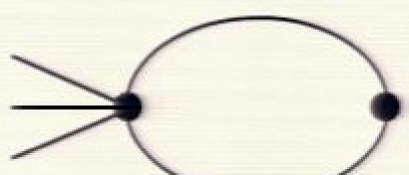

COUNTERTERMS

To cancel dependence on regulators

$$A = -\frac{1}{16\pi^2}\left[\sum_n Z_n^{-1} M_n^2 \ln M_n + \mu_A^2\right]$$

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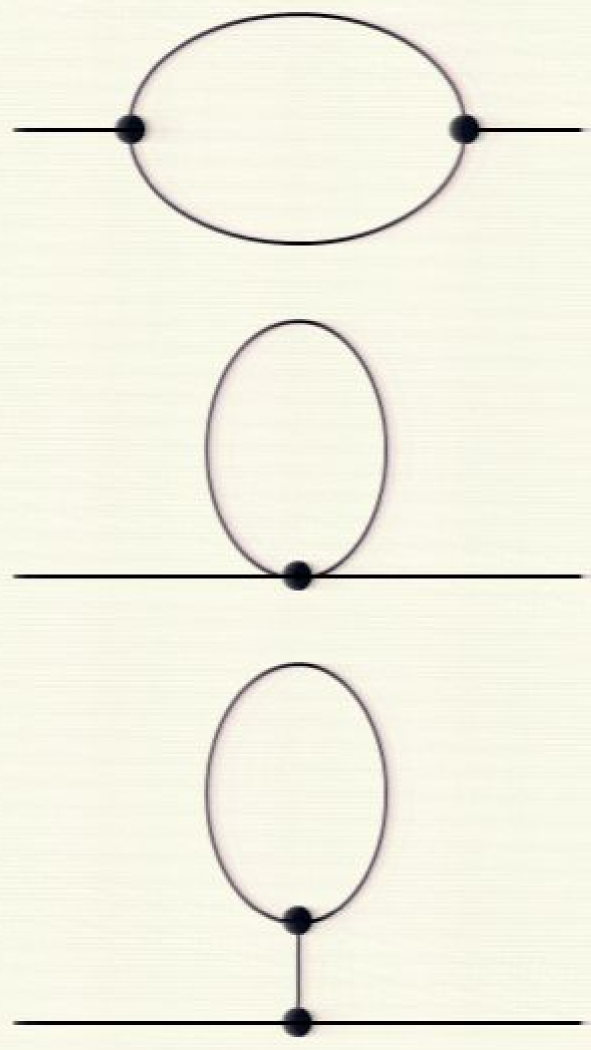
μ_A, μ_B, μ_C unknown finite *physical* parameters.

TWO-POINT FUNCTION



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\begin{aligned}
&\langle \delta\varphi_H(\mathbf{y},t)\,\delta\varphi_H(\mathbf{z},t)\rangle_{\mathrm{VAC}} \\
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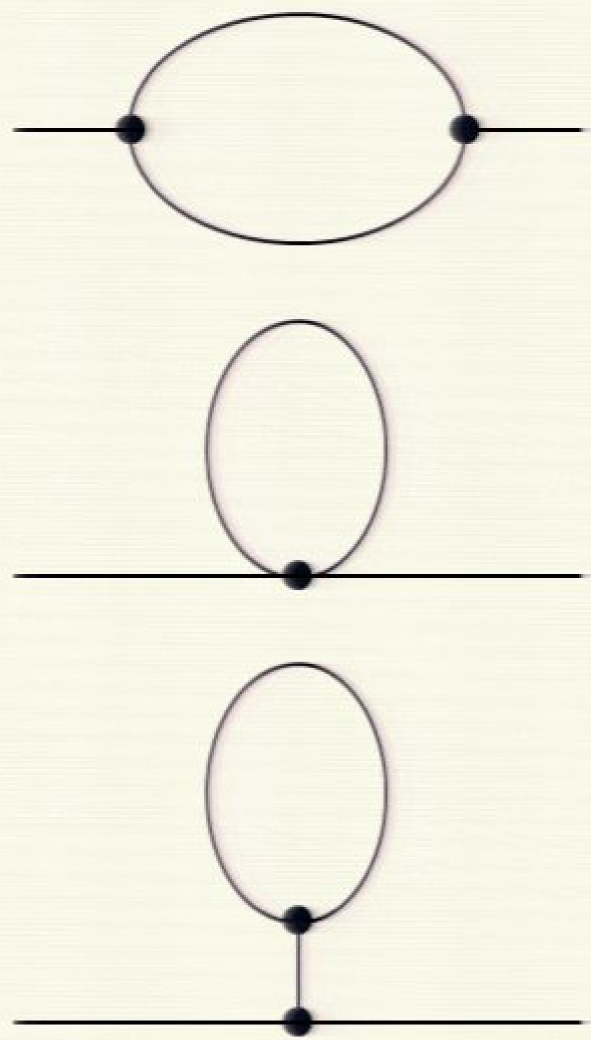

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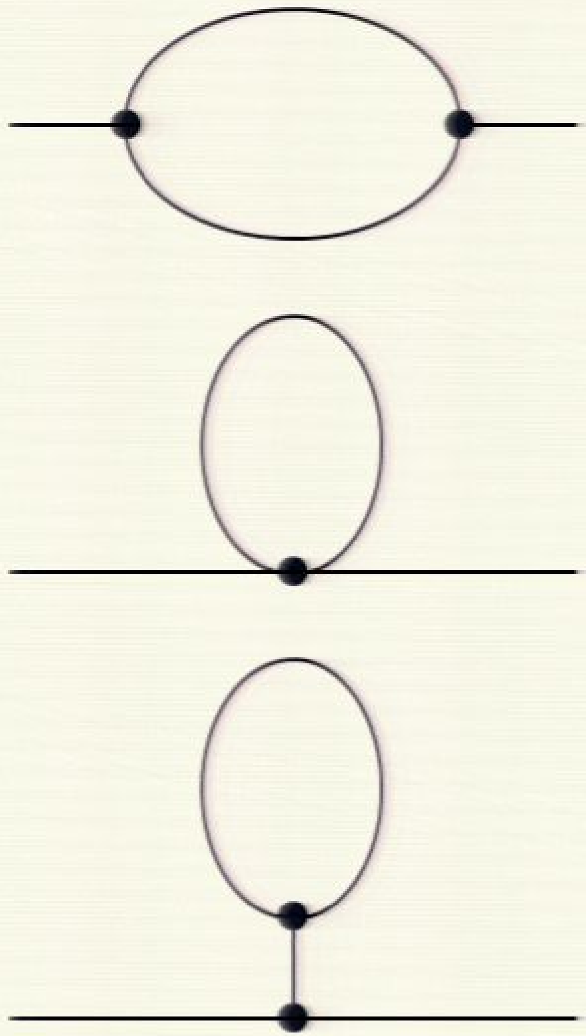

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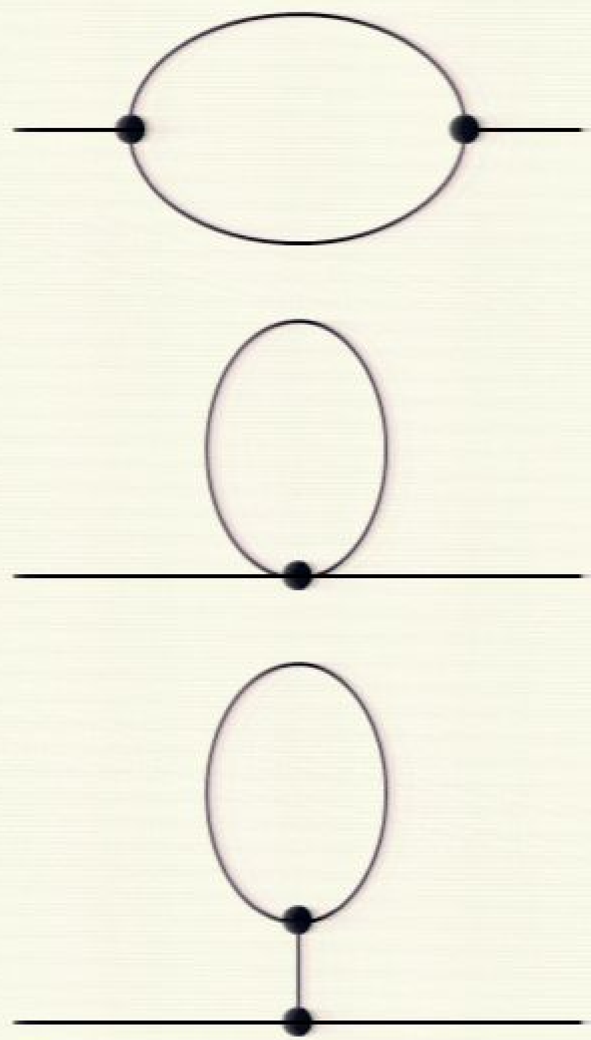

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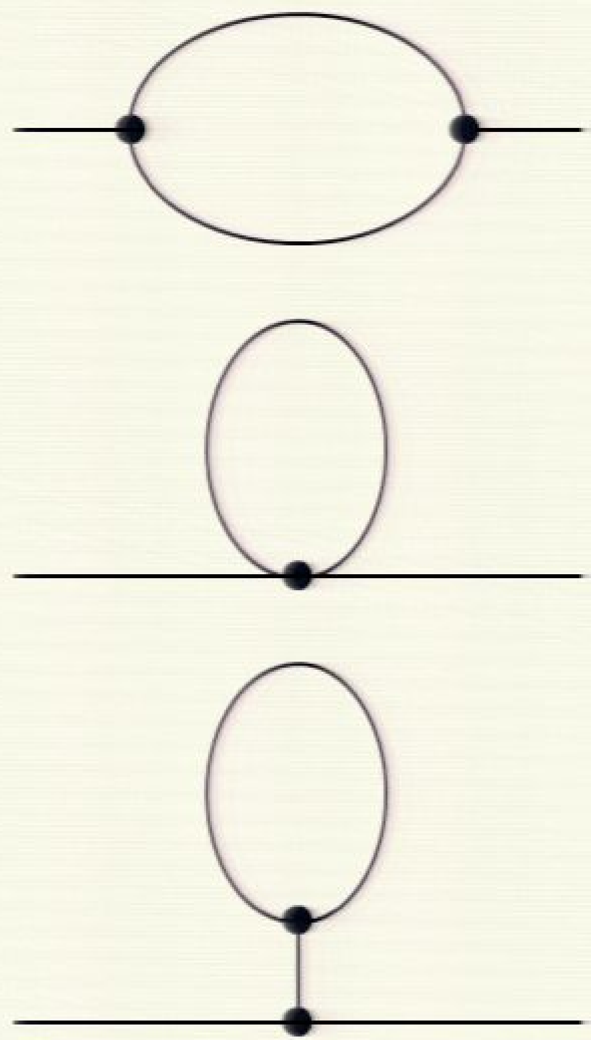

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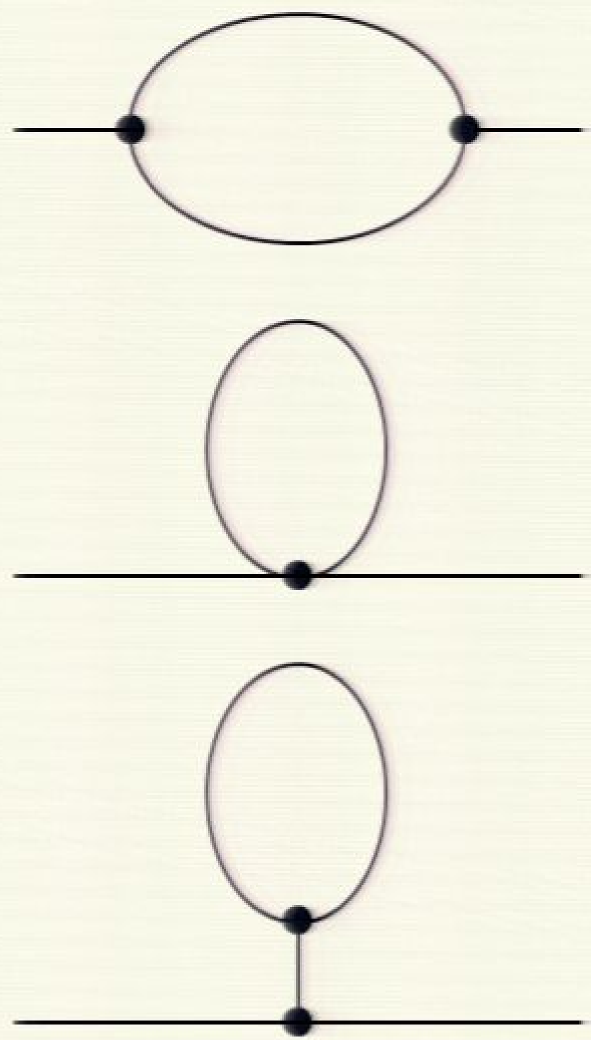

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EXTENDED WKB APPROXIMATION
(to zeroth order in $V''(\overline{\varphi})$)

For $q^2/a^2 \gg H^2$ and $\dot{H}$

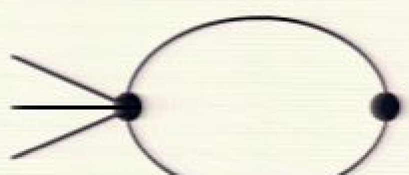
$$|u_q|^2 \to \frac{1}{2qa^2(2\pi)^3}\left[1+\frac{\dot{H}+2H^2}{2q^2/a^2}\right]$$

For q^2/a^2 and $M_n^2 \quad \gg \quad H^2$ and $\dot{H}$

$$|u_{nq}|^2 \to \frac{1}{2\kappa_{nq}a^3(2\pi)^3}\left[1+\frac{\dot{H}+2H^2}{2\kappa_{nq}^2}\right.$$
$$\left.+\frac{(\dot{H}+3H^2)M_n^2}{4\kappa_{nq}^4}-\frac{5H^2M_n^4}{8\kappa_{nq}^6}\right],$$

$$\kappa_{nq}^2(t)=\Big(q/a(t)\Big)^2+M_n^2$$

To include $V''(\overline{\varphi})$, treat it as a first-order perturbation:



$$M_n^2 \gg H^2,\ \dot{H},\ |V''(\overline{\varphi})|,\ p^2/a^2(t)$$

In one-loop graphs we integrate over a single co-moving wave number q.

- For $q^2/a^2(t) \ll M_n^2$, ignore regulators.
- For $q^2/a^2(t) \gg H^2,\ \dot{H},\ |V''(\overline{\varphi})|,\ p^2/a^2(t)$, use WKB approximation.

These ranges overlap. Choose Q in the overlap range:

$$M_n^2 \gg Q^2/a^2(t) \gg H^2,\ \dot{H},\ |V''(\overline{\varphi})|,\ p^2/a^2(t)$$

In integral over $q < Q$, ignore regulators. In integral over $q > Q$, use WKB approximation, drop terms that are convergent even before cancellations.

REGULATORS

Pauli & Villars 1949; Bernard & Duncan 1977; Vilenkin 1977

$$\mathcal{L} = \sqrt{-\mathrm{Det} g}\Bigg[-\frac{1}{2} g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$$

$$-\frac{1}{2}\sum_n Z_n \left(g^{\mu\nu}\partial_\mu\chi_n\partial_\nu\chi_n + M_n^2\chi_n^2 \right)$$

$$-V\left(\varphi + \sum_n \chi_n\right)\Bigg] .$$

$$\sum_n Z_n^{-1} = -1 , \quad \sum_n Z_n^{-1} M_n^2 = 0 ,$$

$$\sum_n Z_n^{-1} M_n^4 = 0 , \quad \ldots , \quad \sum_n Z_n^{-1} M_n^D = 0 .$$

ONE-LOOP COUNTERTERMS

$$\mathcal{L}_{\infty}^{1\,\text{loop}} = \sqrt{-\text{Det}g}\Big[A\,V''(\varphi) + B[V''(\varphi)]^2$$
$$+ C\,R\,V''(\varphi)\Big]\,,$$
$$A \propto \Lambda^2\,, \quad B\;\&\;C \propto \ln\Lambda$$

$$\mathcal{L}_{\infty}^{1\,\text{loop}} = a^3\Bigg[A\left(V'''(\overline{\varphi})\delta\varphi + \frac{1}{2}V''''(\overline{\varphi})\delta\varphi^2 + \ldots\right)$$
$$+ B\Big(2V''(\overline{\varphi})V'''(\overline{\varphi})\delta\varphi$$
$$+ [V'''^2(\overline{\varphi}) + V''(\overline{\varphi})V''''(\overline{\varphi})]\delta\varphi^2 + \ldots\Big)$$
$$- C\,(6\dot{H} + 12H^2)\Big(V'''(\varphi)\delta\varphi$$
$$+ \frac{1}{2}V''''(\overline{\varphi})\delta\varphi^2 + \ldots\Big)\Bigg]\,.$$

$$
\begin{aligned}
&\langle \delta\varphi_H(\mathbf{y},t)\, \delta\varphi_H(\mathbf{z},t)\rangle_{\mathrm{VAC}} \\
&= \int d^3p \, \exp\Big(i\mathbf{p}\cdot(\mathbf{y}-\mathbf{z})\Big) \\
&\times \Bigg[-2(2\pi)^6 \int_{-\infty}^{t} dt_1 \, a^3(t_1) V'''\Big(\overline{\varphi}(t_1)\Big) \\
&\quad \times \int_{-\infty}^{t_1} dt_2 \, a^3(t_2) V'''\Big(\overline{\varphi}(t_2)\Big) \\
&\quad \times \mathrm{Re}\Big\{ u_p^2(t) u_p^*(t_1) u_p^*(t_2) \\
&\quad\; \times \int_{q<Q} d^3q \, u_q(t_1) u_q^*(t_2) u_{q'}(t_1) u_{q'}^*(t_2) \Big\} \\
&+\pi \int_{-\infty}^{t} a^3(t_1) V'''\Big(\overline{\varphi}(t_1)\Big)^2 \mathrm{Im}\{u_p^2(t) u_p^{*2}(t_1)\} \\
&\quad \times \ln\left(\frac{Q}{a(t_1)\mu_B}\right)
\end{aligned}
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PROBLEMS

1. How do UV divergences know about general covariance? (Why $\dot{H}+2H^2$?)
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\begin{aligned}
&-(2\pi)^3 \int_{-\infty}^{t} dt_1 a^3(t_1)\, V'''\Big(\overline{\varphi}(t_1)\Big)\, \mathrm{Im}\{u_p^2(t) u_k^{*2}(t_1)\} \\
&\times \int_{-\infty}^{t_1} dt_2\, G(t_1, t_2) a^3(t_2) V'''\Big(\overline{\varphi}(t_2)\Big) \\
&\times \Bigg\{ \int_{q<Q} d^3q\, |u_q(t_2)|^2 \\
&+ \frac{1}{8\pi^2}\Bigg(-\frac{Q^2}{a^2(t_2)} + V''\Big(\overline{\varphi}(t_2)\Big) \ln\left(\frac{Q}{a(t_2)\mu_B}\right) \\
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where

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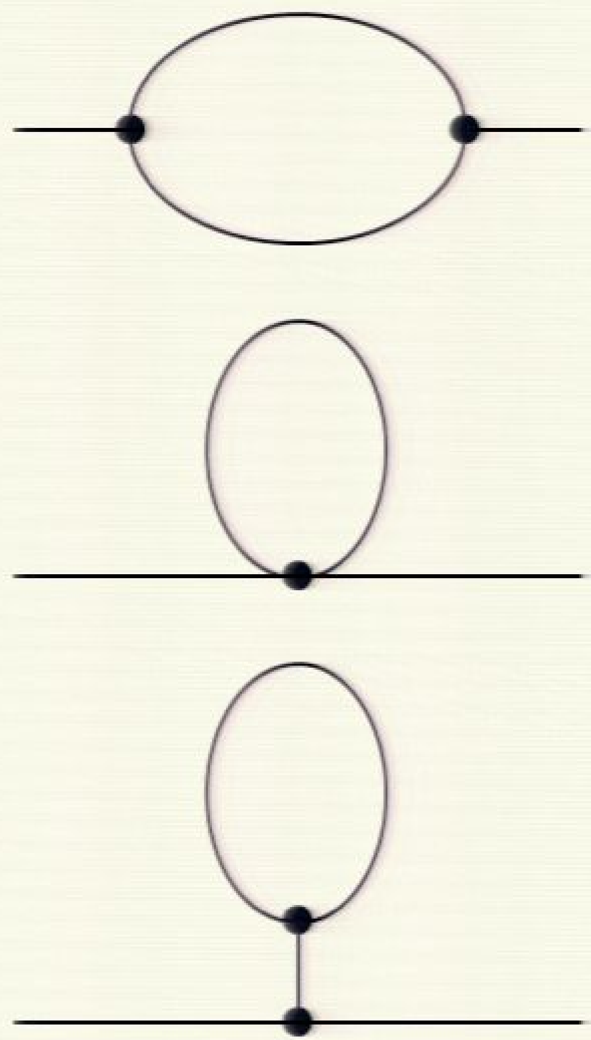
G(t_1, t_2) = u(t_1)\, u(t_2) \int_{t_2}^{t_1} \frac{dt}{a^3(t)\, u^2(t)} ,
$$

$$
\ddot{u} + 3H\dot{u} + V''(\overline{\varphi})u = 0 ,
$$

TWO-POINT FUNCTION



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\begin{aligned}
&\langle \delta\varphi_H(\mathbf{y},t)\,\delta\varphi_H(\mathbf{z},t)\rangle_{\mathrm{VAC}} \\
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&+(2\pi)^3 \int_{-\infty}^{t} a^3(t_1) V''''\Big(\overline{\varphi}(t_1)\Big) \mathrm{Im}\Big\{ u_p^2(t) u_p^{2*}(t_1) \Big\} \\
&\quad \times \Bigg\{ \int_{q<Q} d^3q\, |u_q(t_1)|^2 \\
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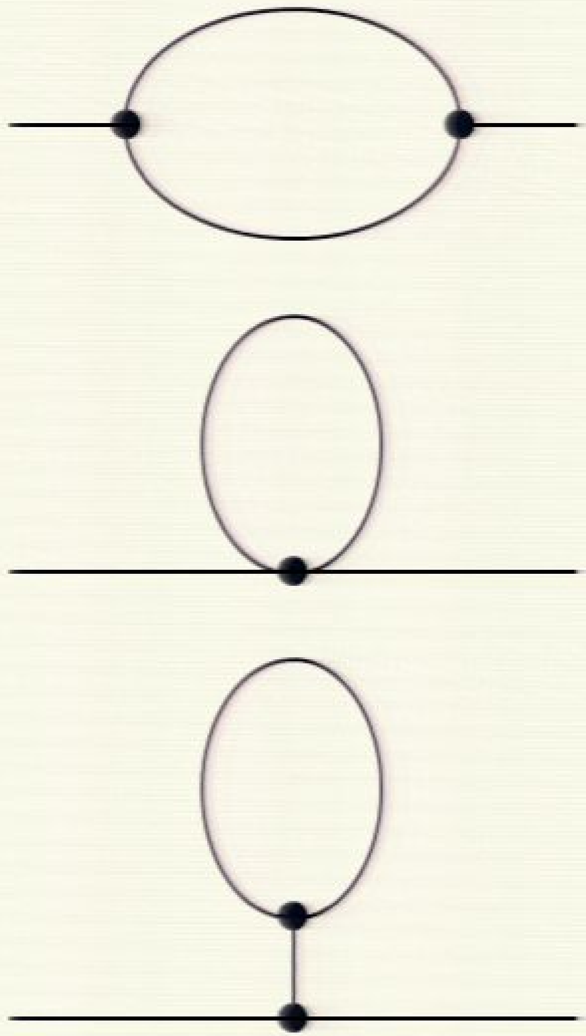
TWO-POINT FUNCTION



EXTENDED WKB
APPROXIMATION
(to zeroth order in $V''(\overline{\varphi})$)

For $q^2/a^2 \gg H^2$ and $\dot{H}$

$$|u_q|^2 \to \frac{1}{2qa^2(2\pi)^3}\left[1+\frac{\dot{H}+2H^2}{2q^2/a^2}\right]$$

For q^2/a^2 and $M_n^2 \;\gg\; H^2$ and $\dot{H}$

$$|u_{nq}|^2 \to \frac{1}{2\kappa_{nq}a^3(2\pi)^3}\left[1+\frac{\dot{H}+2H^2}{2\kappa_{nq}^2}\right.$$
$$\left.+\frac{(\dot{H}+3H^2)M_n^2}{4\kappa_{nq}^4}-\frac{5H^2M_n^4}{8\kappa_{nq}^6}\right],$$

$$\kappa_{nq}^2(t) = \Big(q/a(t)\Big)^2 + M_n^2$$

To include $V''(\overline{\varphi})$, treat it as a first-order perturbation:

LOOP COUNTERTERMS

$$= \sqrt{-\mathrm{Det} g}\Big[A V''(\varphi) + B[V''(\varphi)]^2 + C R V''(\varphi)\Big] ,$$

$$A \propto \Lambda^2 , \quad B \,\&\, C \propto \ln \Lambda$$

$$\mathcal{L}_{\infty}^{1\ \mathrm{loop}} = a^3 \Bigg[A \left(V'''(\overline{\varphi})\delta\varphi + \frac{1}{2} V''''(\overline{\varphi})\delta\varphi^2 + \ldots \right)$$
$$+ B \Big(2V''(\overline{\varphi})V'''(\overline{\varphi})\delta\varphi$$
$$+ [V'''^2(\overline{\varphi}) + V''(\overline{\varphi})V''''(\overline{\varphi})]\delta\varphi^2 + \ldots \Big)$$
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ONE-LOOP COUNTERTERMS

$$\mathcal{L}_{\infty}^{1\text{ loop}} = \sqrt{-\mathrm{Det}g}\Big[A\,V''(\varphi) + B[V''(\varphi)]^2 + C\,R\,V''(\varphi)\Big] .$$

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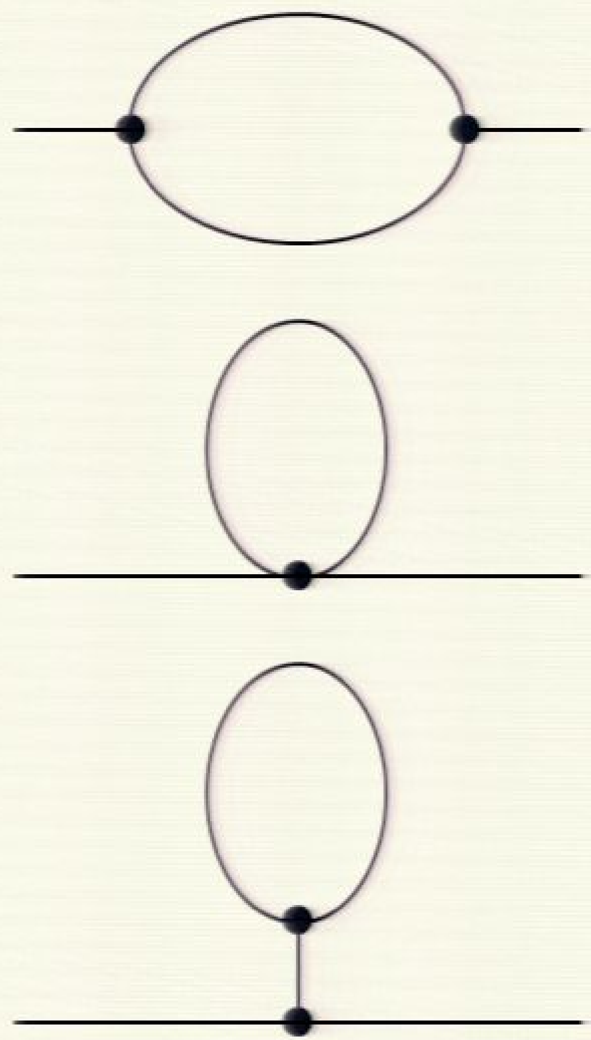
$$\mathcal{L}_{\infty}^{1\text{ loop}} = a^3 \Bigg[A\left(V'''(\overline{\varphi})\delta\varphi + \frac{1}{2}V''''(\overline{\varphi})\delta\varphi^2 + \dots\right) + B\left(2V''(\overline{\varphi})V'''(\overline{\varphi})\delta\varphi + [V'''^2(\overline{\varphi}) + V''(\overline{\varphi})V''''(\overline{\varphi})]\delta\varphi^2 + \dots\right) - C\,(6\dot{H} + 12H^2)\left(V'''(\varphi)\delta\varphi + \frac{1}{2}V''''(\overline{\varphi})\delta\varphi^2 + \dots\right)\Bigg] .$$

TWO-POINT FUNCTION



$$
\begin{aligned}
&+(2\pi)^6 \int_{-\infty}^{t} dt_1\, a^3(t_1) V'''\Big(\overline{\varphi}(t_1)\Big) \\
&\quad \times \int_{-\infty}^{t} dt_2\, a^3(t_2) V'''\Big(\overline{\varphi}(t_2)\Big) |u_p(t)|^2 \\
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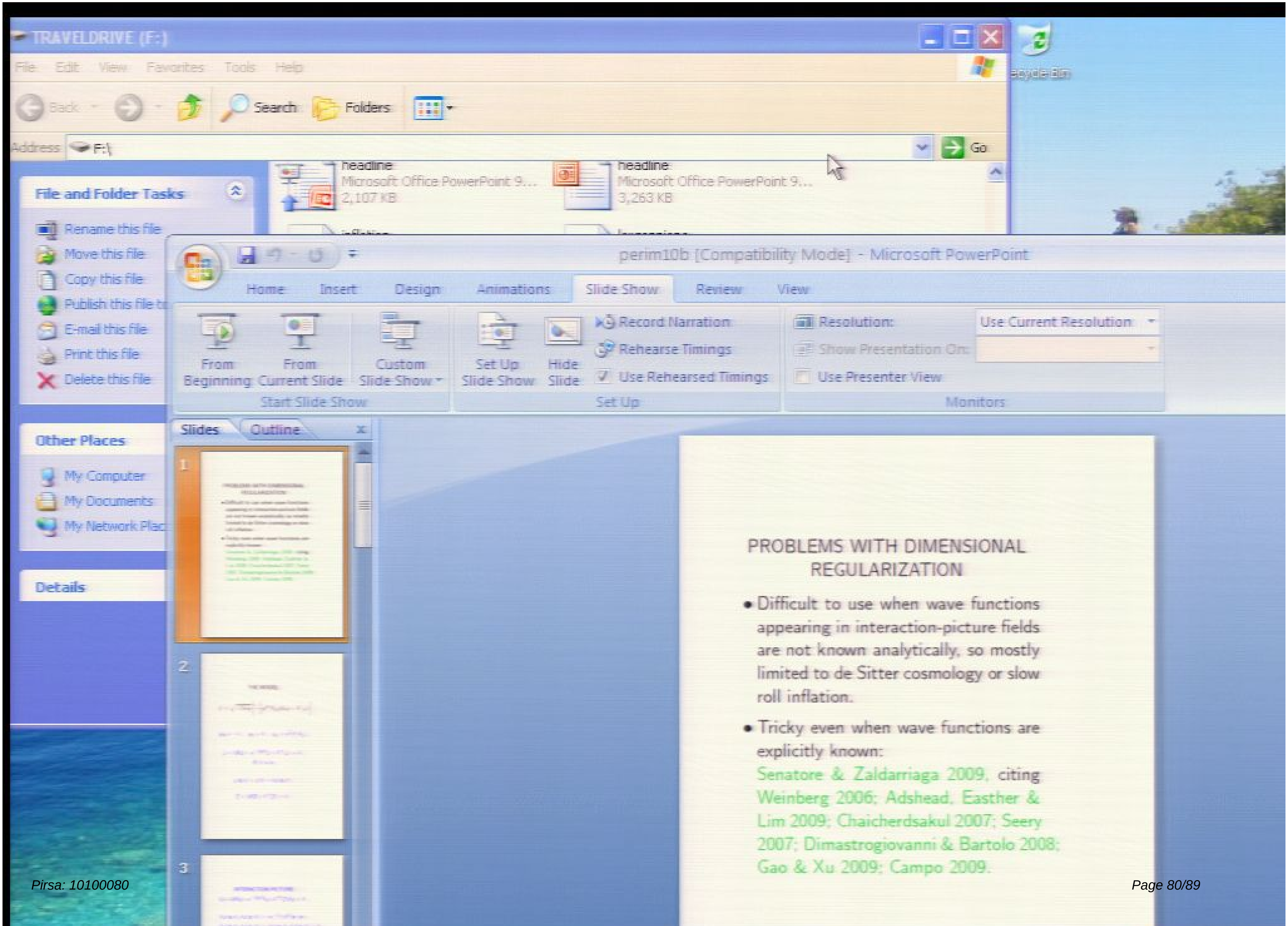
TRAVELDRIVE (F:)
perim10b [Compatibility Mode] - Microsoft PowerPoint
PROBLEMS WITH DIMENSIONAL REGULARIZATION
• Difficult to use when wave functions appearing in interaction-picture fields are not known analytically, so mostly limited to de Sitter cosmology or slow roll inflation.
• Tricky even when wave functions are explicitly known:
Senatore & Zaldarriaga 2009, citing Weinberg 2006; Adshead, Easther & Lim 2009; Chaicherdsakul 2007; Seery 2007; Dimastrogiovanni & Bartolo 2008; Gao & Xu 2009; Campo 2009.

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THE MODEL

$$\mathcal{L} = \sqrt{-\mathrm{Det} g}\left[-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - V(\varphi)\right] \; .$$

$$g_{00} = -1 \; , \quad g_{0i} = 0 \; , \quad g_{ij} = a^2(t)\,\delta_{ij} \; .$$

$$\ddot{\varphi} + 3H\dot{\varphi} - a^{-2}\nabla^2\varphi + V'(\varphi) = 0 \; ,$$

$$H \equiv \dot{a}/a$$

$$\varphi(\mathbf{x}, t) = \bar{\varphi}(t) + \delta\varphi(\mathbf{x}, t) \; ,$$

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\end{aligned}
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