Title: On IR effects in single field inflation

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Abstract: TBA

Leonardo Senatore (Stanford)

On Loops in Single Field Inflation

with M. Zaldarriaga **0912.2734** [hep-th]
in completion

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Outline

- Introduction
- IR effecs in Single Field Inflation
  - Log running $\log(H/\mu)$
  - ζ is not time dependent
  - Zero effect from $\log(kL)$
  - One true IR effect (already resumed)
- Organizing principle:
  - projection effects
  - dynamical effects (null)

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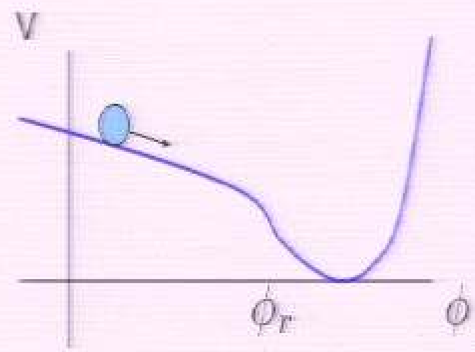
Who cares?



- Tiny Effect

$$\langle \delta\phi_k^2 \rangle_{\rm tree} \sim \frac{H^2}{k^3}$$

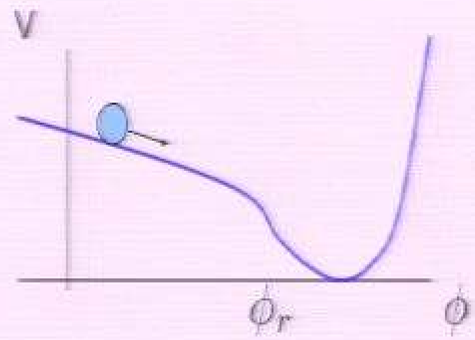
$$\langle \zeta_k^2 \rangle_{\rm tree} \sim \frac{H^2}{\epsilon M_{\rm Pl}^2}\frac{1}{k^3} \sim 10^{-10} \quad \Rightarrow \quad \langle \delta\phi_k^2 \rangle_{1-\rm loop} \sim \frac{H^2}{k^3}\frac{H^2}{M_{\rm Pl}^2} \sim 10^{-10}\, \langle \delta\phi_k^2 \rangle_{\rm tree}$$

- We have more interacting theories (large non-Gaussianities! but still small)
- Weinberg cares: understand prediction of your theory S.Weinberg **PRD72:2005**
  - These are the quantum corrections to the predictions of Inflation.
- dS is a puzzling spacetime, and inflation is a regularization
- Let us elaborate on this...

Eternal Inflation

- If $\langle \delta\phi_k^2 \rangle \sim \dfrac{H^2}{k^3} \quad \Rightarrow$

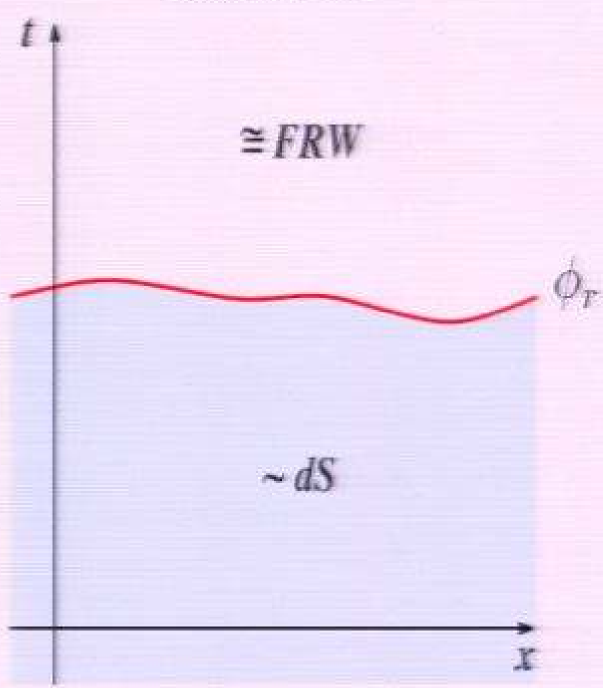
With Creminelli, Dubovsky, Nicolis and Zaldarriaga
JHEP0809:036,2008



$$\langle \delta\phi(x,t)^2 \rangle = \int^{\Lambda a[t]} d^3k \frac{H^2}{k^3} \sim H^2 \log(a) \sim H^3 t + \text{ const. },$$

- With this you can prove that slow roll eternal inflation exists

Standard Infl.



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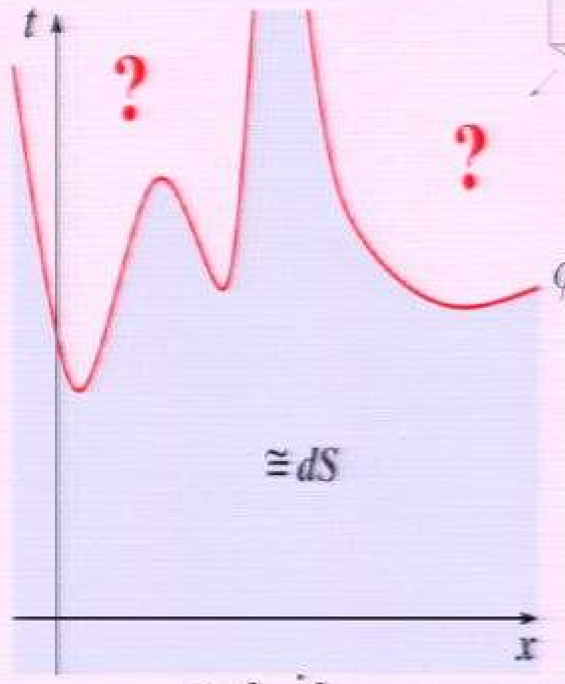

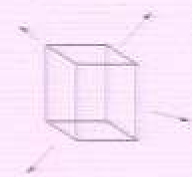

- Sharp phase transition: $P(V=\infty) \neq 0$ for $\Omega \equiv \dfrac{2\pi^2}{3} \dfrac{\dot\phi^2}{H^4} < 1$

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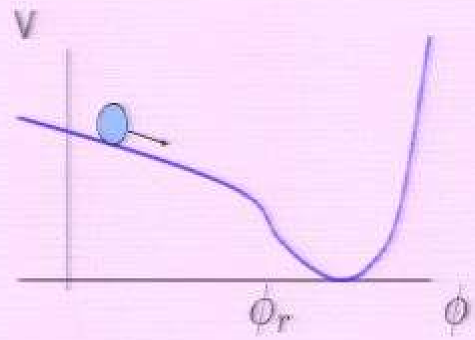

$$\langle \delta\phi(x,t)^2 \rangle = \int^{\Lambda a[t]} d^3k \frac{H^2}{k^3} \sim H^2 \log(a) \sim H^3 t + \text{ const.} \, ,$$

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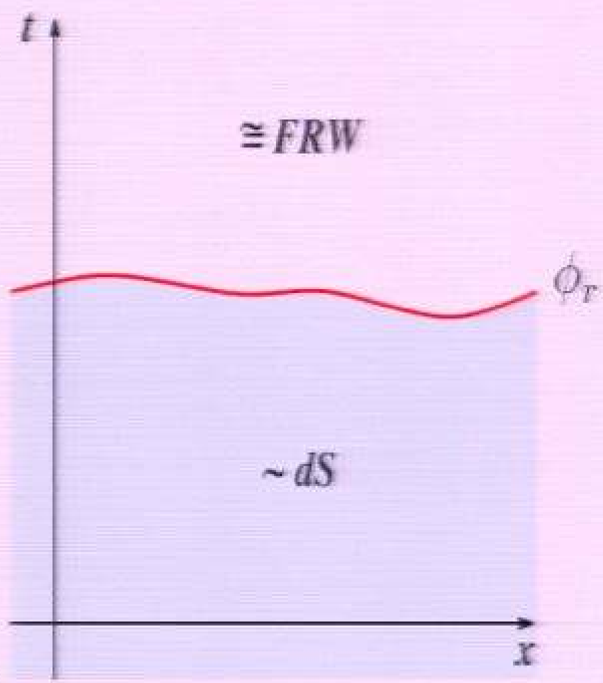

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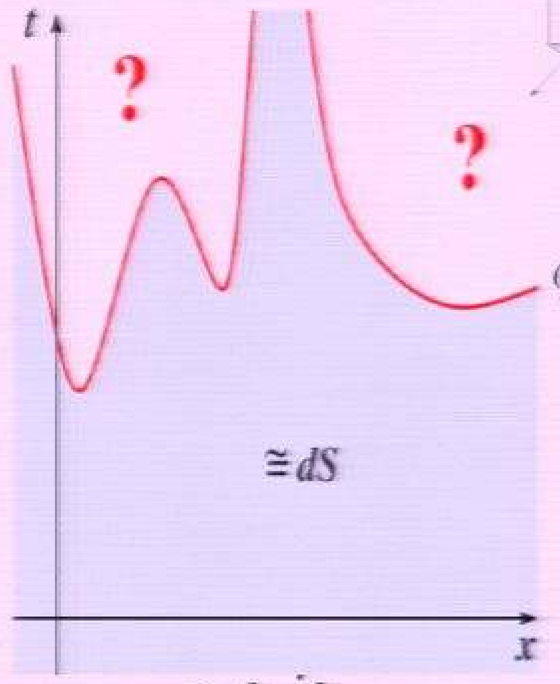

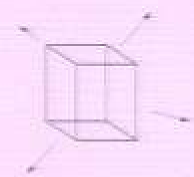

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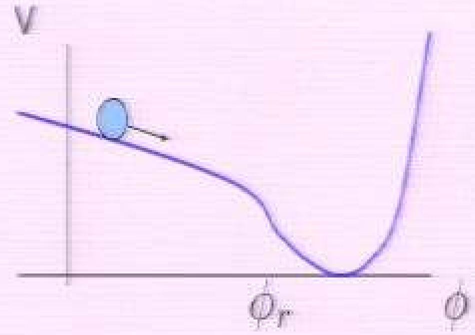
$P(V=\infty) \neq 0$

Eternal Inflation



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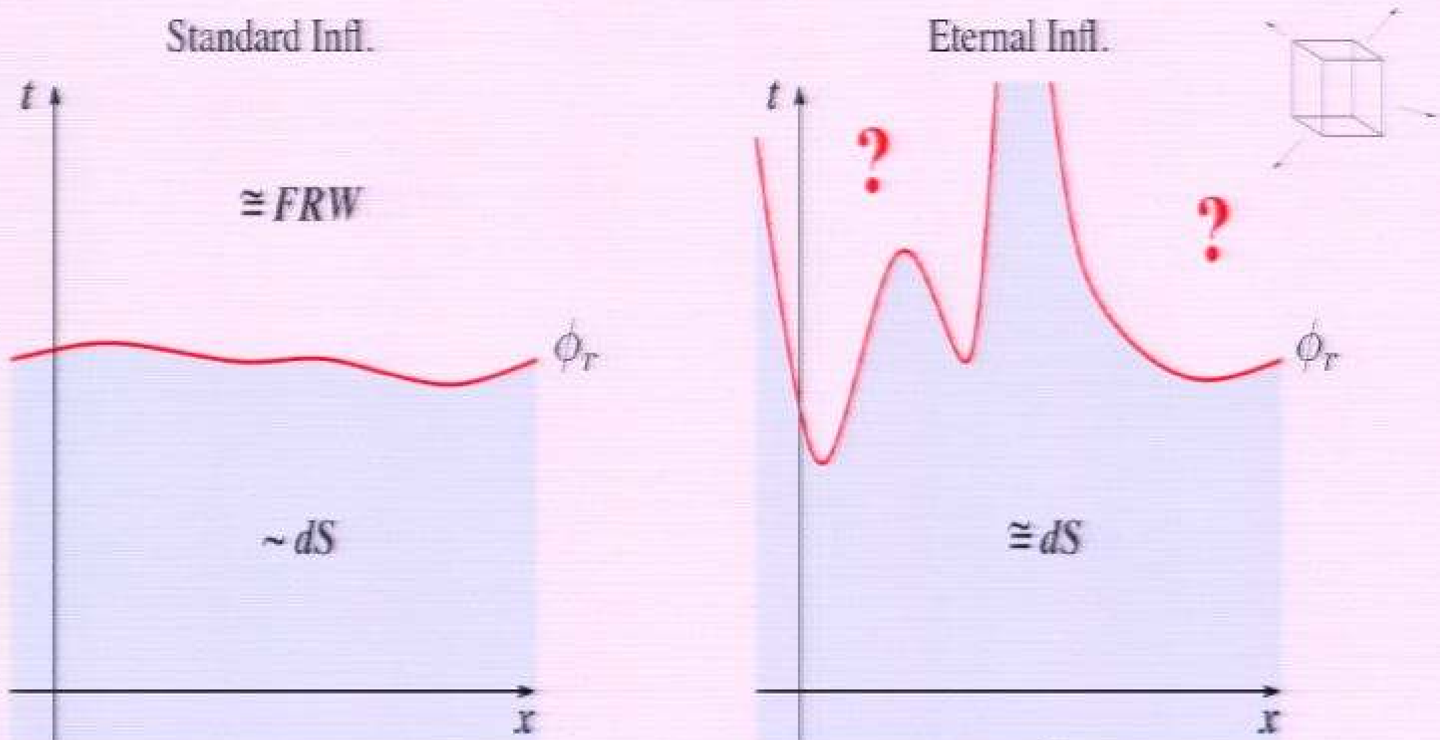

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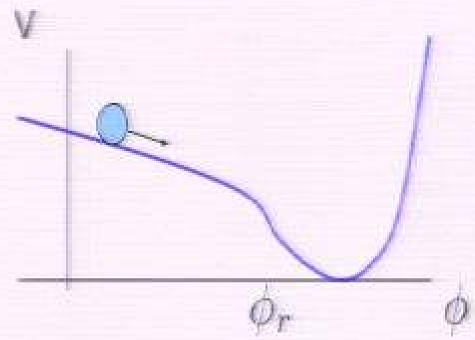

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With Creminelli, Dubovsky, Nicolis and Zaldarriaga **JHEP0809:036,2008**

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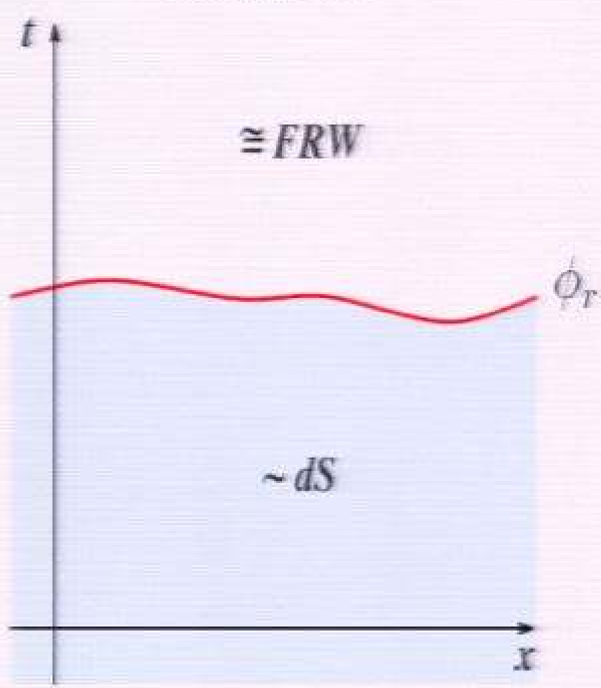

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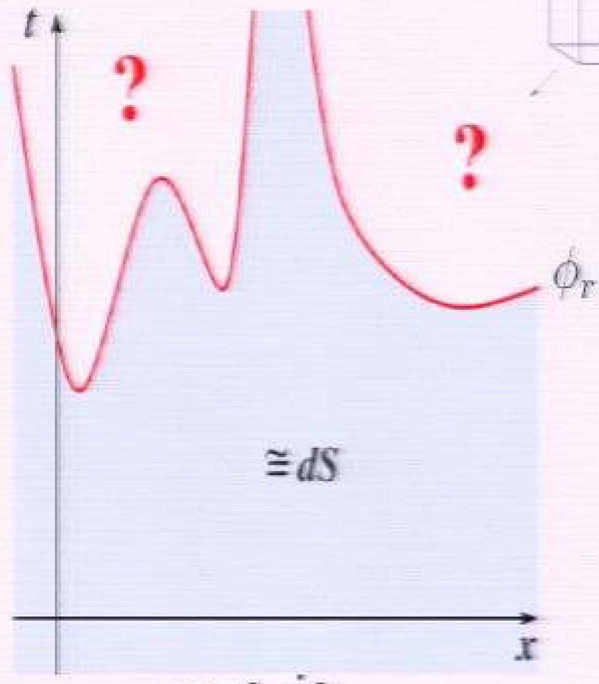
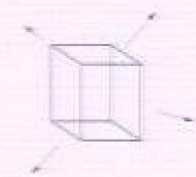
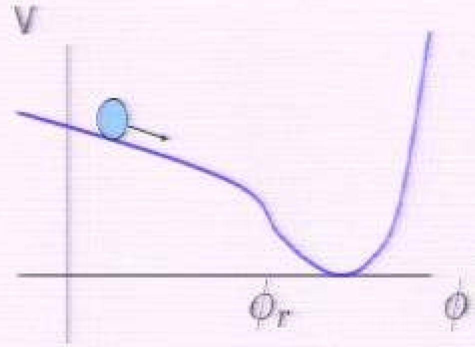




- Sharp phase transition: $P(V = \infty) \neq 0$ for $\Omega \equiv \dfrac{2\pi^2}{9} \dfrac{\dot{\phi}^2}{H^4} < 1$

Eternal Inflation

With Dubovsky and Villadoro
JHEP0904:118,2009
generalization of
Arkani-Hamed *et al.*
JHEP0705:055,2007



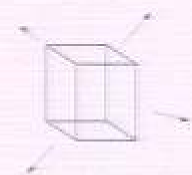
- With quite more work:

$$P\left(V > e^{\frac{S_{\rm ds}}{2}}\right) < e^{-\alpha S_{\rm ds}} \qquad V_{\rm Finite\ Realization} < e^{\frac{S_{\rm ds}}{2}}$$

- All of this fails if

$$\langle \delta\phi_k^2 \rangle_{1-{\rm loop}} \sim \frac{H^2}{k^3}\frac{H^2}{M_{\rm Pl}^2} \times [\log(k) \text{ or } \log(a(t))]$$

$$\langle \delta\phi^2(x,t) \rangle_{1-{\rm loop}} \sim t^2$$



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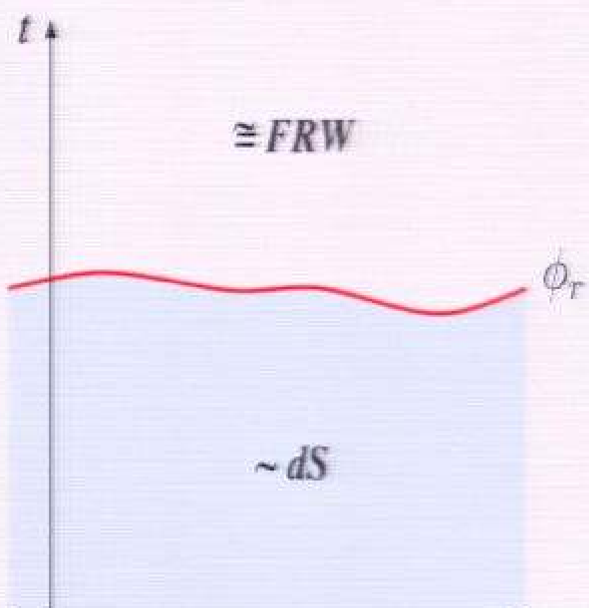

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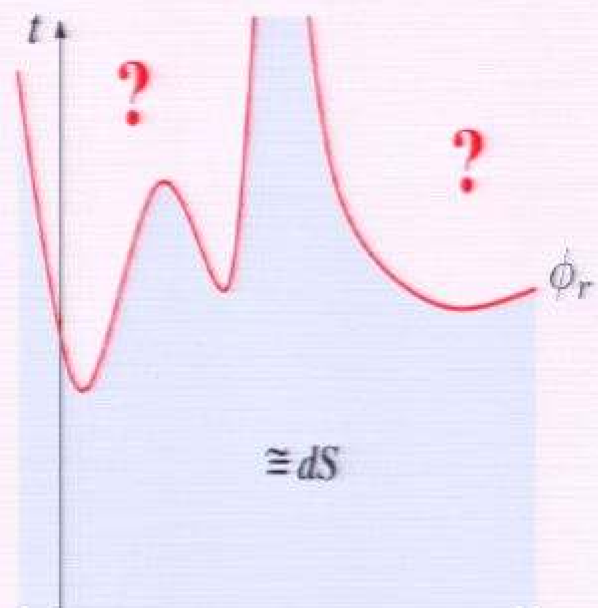

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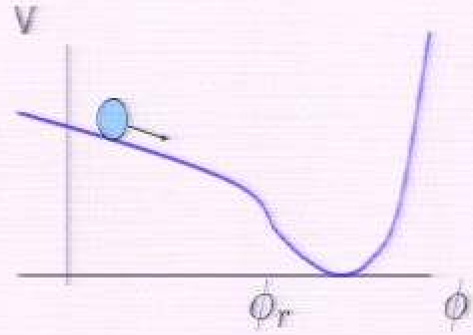

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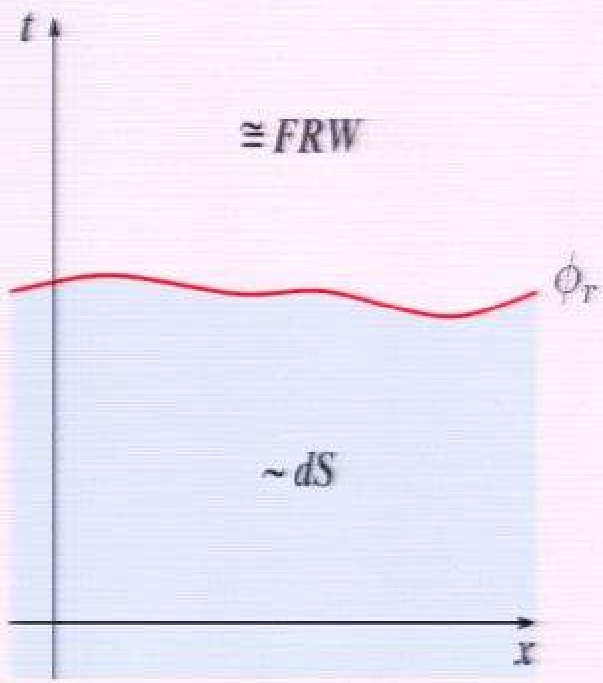

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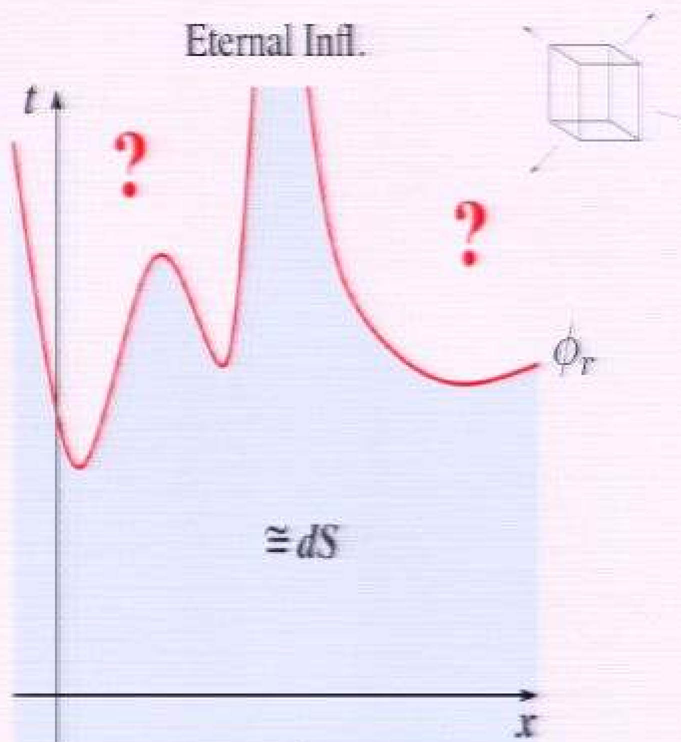

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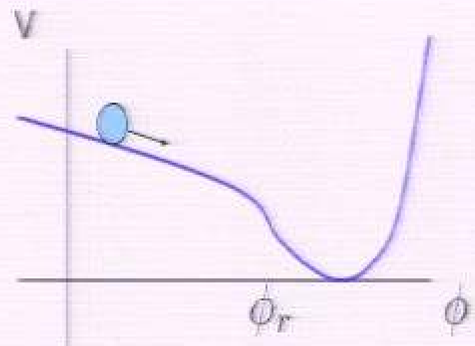

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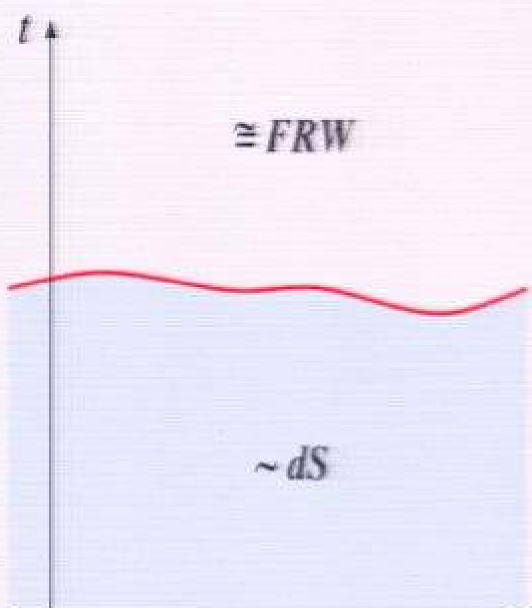

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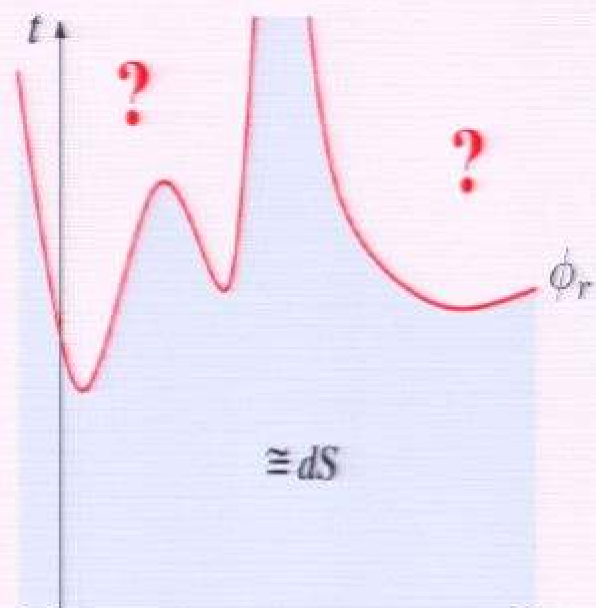

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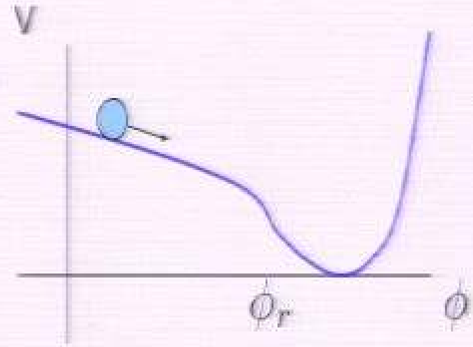
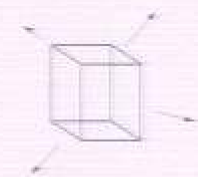
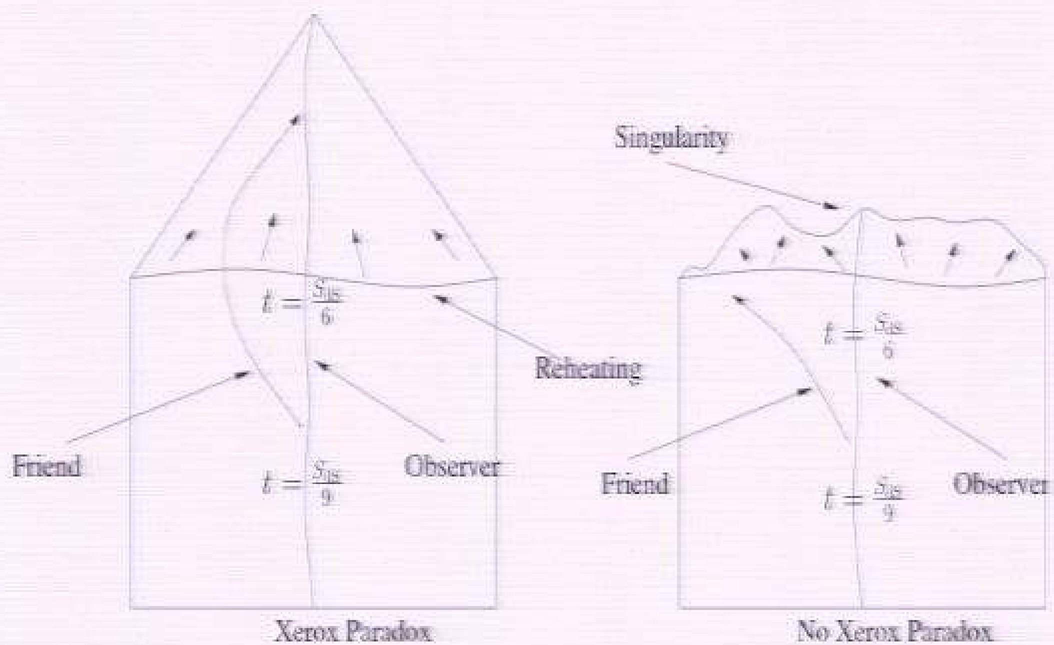


- A consistency check for Holography

$$P(V > e^{\frac{S_{ds}}{2}}) < e^{-\alpha S_{ds}} \qquad V_{\text{Finite Realization}} < e^{\frac{S_{ds}}{2}}$$

- A possible xerox paradox





Eternal Inflation

With Dubovsky and Villadoro
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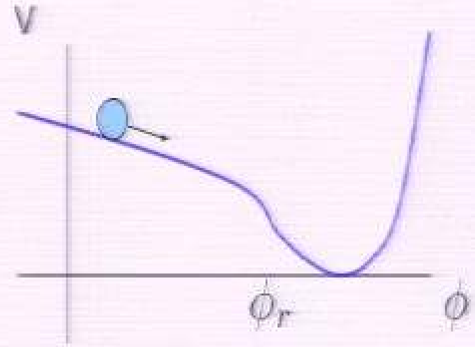

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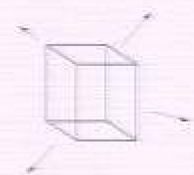

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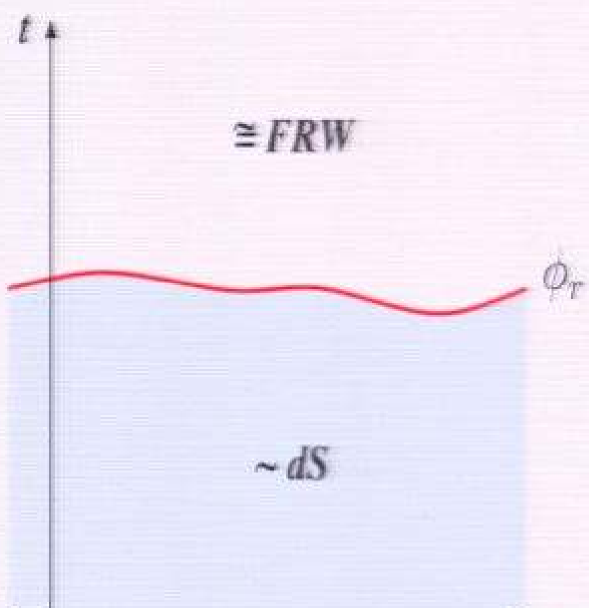

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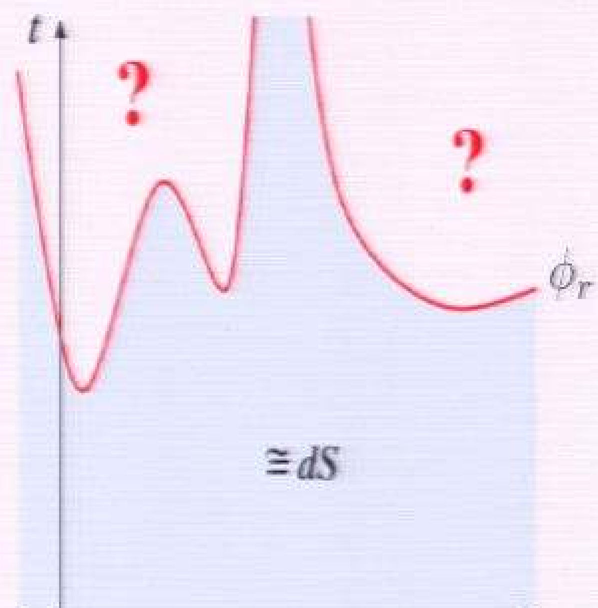

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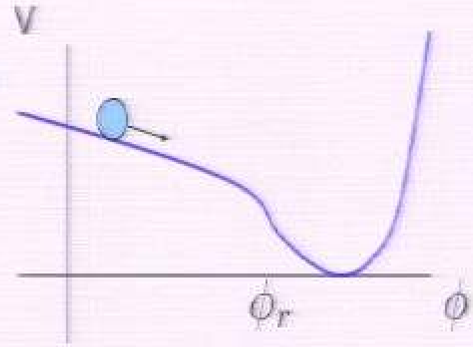

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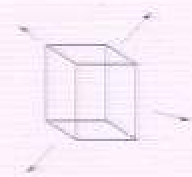
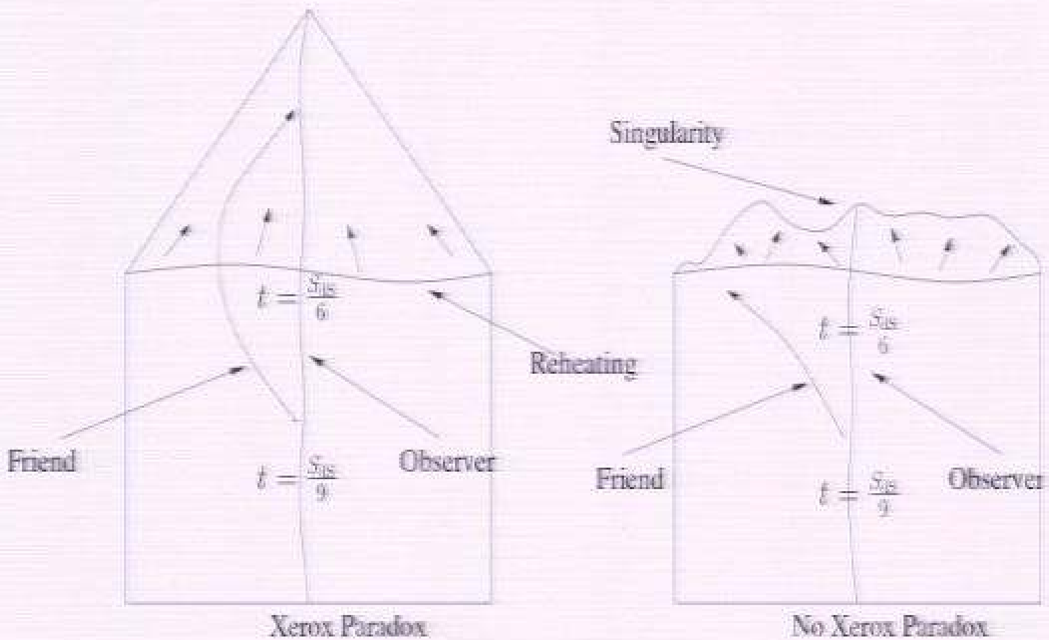
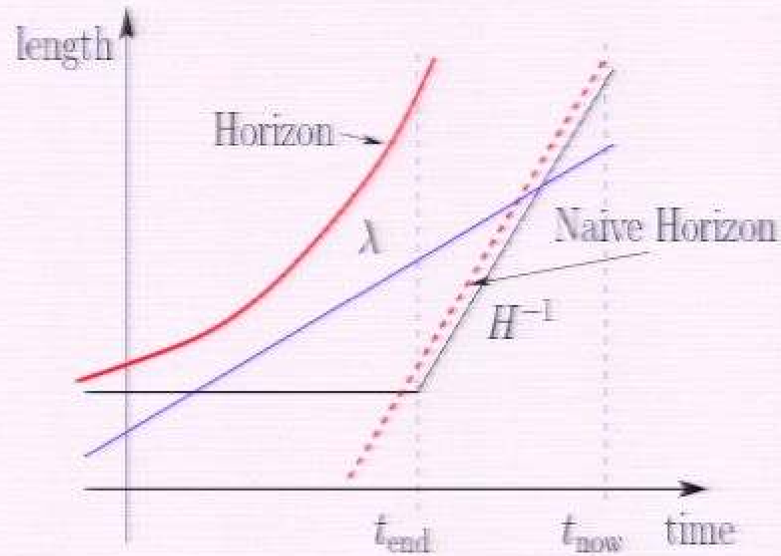




Predictivity of Inflation

- If $\langle\zeta_k^2\rangle_{1-\text{loop}} \sim \langle\zeta_k^2\rangle_{\text{tree}} \frac{H^4}{\dot{H} M_{\text{Pl}}^2} \log(a(t))$ when $k/a \ll H$
  - if ζ is time in-dependent: ok



  - if ζ is time-dependent, we need to know the history.
  - this is a weakly coupled version of what could happen at other epochs

Log Running

Log Running

- Weinberg's result $\langle \zeta_k^2 \rangle_{1-\text{loop}} \sim \langle \zeta_k^2 \rangle_{\text{tree}} \log(k/\mu)$

S.Weinberg **PRD72:2005**
and others thereafter

- Gives you all these troubles (Eternal Inflation, Predictivity of Inflation)
- But problem with gauge symmetry $a \to \lambda a\,, \quad x \to x/\lambda\,, \quad k \to \lambda k$
- Study this finding the simplest possible theory

$$S = \int d^4x\, a^3 \left[-\dot{H} M_{\rm Pl}^2 \left(\dot{\pi}^2 - \frac{1}{a^2} (\partial_i \pi)^2 \right) + \frac{2}{3} c_3 M^4 \left(2\dot{\pi}^3 + 3\dot{\pi}^4 - 3\frac{1}{a^2} \dot{\pi}^2 (\partial_i \pi)^2 \right) \right]$$

Based on EFT of Inflation
with C. Cheung, P. Creminelli,
L. Fitzpatrick, J. Kaplan
JHEP 0803:014,2008

- Technical problem in implementing the regularization
- $\langle \zeta_k^2 \rangle_{1-\text{loop}} \propto H^6 \log(H/\mu)$

with Zaldarriaga **0912:2734** [hep-th]

- Effect in the IR much larger than in Minkowsky space

$$\langle \zeta_k^2 \rangle_{1-\text{loop}} \propto k^6 \log(k/\mu)$$

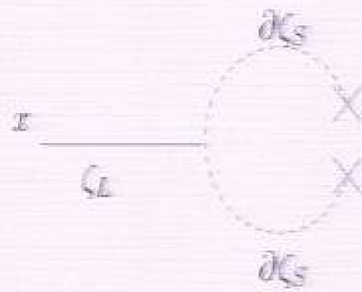
- Analogy with particle physics

Tadpole Diagrams
(and no time-dependent ζ)

Tadpole Diagrams

with Zaldarriaga **0912:2734** [hep-th]

- .



- You need to renormalize the history
- and define ζ accordingly

$$3M_{\rm Pl}^2 H^2 = \frac{1}{2}\dot\phi(t)^2 + V(\phi(t)) + \langle\rho_\sigma(t)\rangle_0 \, ,$$

$$M_{\rm Pl}^2\left(3H^2 + 2\dot H\right) = -\frac{1}{2}\dot\phi(t)^2 + V(\phi(t)) - \langle p_\sigma(t)\rangle_0$$

$$ds^2 = -dt^2 + a(t)_B^2 e^{2\zeta} dx^2$$

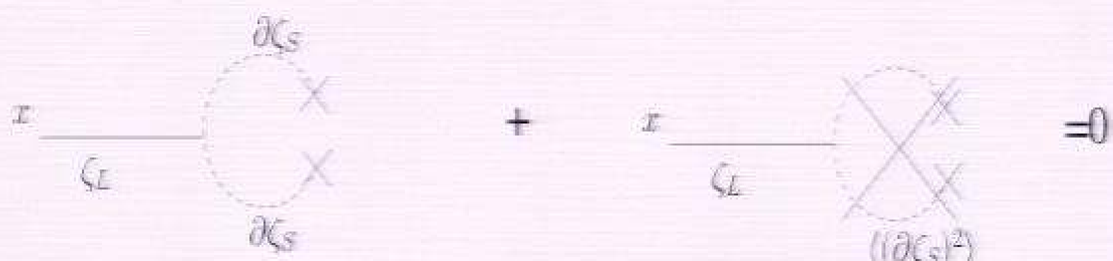
- In practice:

$$S_m = \int d^4x \sqrt{-g} \quad \left[\dot H_{\rm tree} M_{\rm Pl}^2\, \delta g^{00} + \left(3H_{\rm tree}^2 + \dot H_{\rm tree}\right) M_{\rm Pl}^2\right] =$$

$$= \int d^4x \sqrt{-g} \quad \left[\left(\dot H_{\rm tree} + \delta\dot H\right) M_{\rm Pl}^2\, \delta g^{00} + \left(3\left(H_{\rm tree} + \delta H\right)^2 + \left(\dot H_{\rm tree} + \delta\dot H\right)\right) M_{\rm Pl}^2 \right.$$

$$\left. -\delta\dot H M_{\rm Pl}^2\, \delta g^{00} - \left(3H_{\rm tree}\delta H + \delta\dot H\right) M_{\rm Pl}^2\right] \, .$$

- Tadpole cancellation:



- Define ζ here.

Woodard's claim

- ζ s time-dependent

$$\langle \zeta_k^2 \rangle_{1-\mathrm{loop}} \sim \langle \zeta_k^2 \rangle_{\mathrm{tree}} \frac{H^4}{\dot{H} M_{\mathrm{Pl}}^2} \log\left(a(t)\right)$$

E. O. Kahya,[1,*] V. K. Onemli,[2,†] and R. P. Woodard[3,‡]

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ent of Physics, Istanbul Technical University, Maslak, Istanbul 34469,
epartment of Physics, University of Florida, Gainesville, FL 32611, U

nt on the recent arguments by Senatore and Zaldarriaga that loop corr
cannot grow with time after first horizon crossing. We first emphasiz

- .... Forgotten to renormalize the unperturbed history...
- Indeed if you do not...
- Suppose you have a different cc that what you expected
  - $H_{\mathrm{true}}^2 = H_{\mathrm{tree}}^2 + \frac{\delta\Lambda}{M_{\mathrm{Pl}}^2}$

$$a_{\mathrm{true}} = e^{\rho_{\mathrm{true}}} = e^{\rho_{\mathrm{tree}} + \zeta} \quad \Rightarrow \quad H_{\mathrm{true}} = H_{\mathrm{tree}} + \dot{\zeta} \quad \Rightarrow \quad \dot{\zeta} \sim \frac{\delta\Lambda}{M_{\mathrm{Pl}}^2 H}$$

Tadpole!

- Effect on correlation function

Woodard's claim

$$\langle \zeta_k^2 \rangle_{1-\text{loop}} \sim \langle \zeta_k^2 \rangle_{\text{tree}} \frac{H^4}{\dot{H} M_{\text{Pl}}^2} \log\left(a(t)\right)$$

- Effect of correlation function

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...partment of Physics, University of Florida, Gainesville, FL 32611, ...

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...cannot grow with time after first horizon crossing. We first emphasiz...

  - Find mass term for ζ

$$m^2 \sim \frac{\delta\Lambda}{M_{\text{Pl}}^2} \quad \Rightarrow \quad H\dot{\zeta} \sim \frac{\delta\Lambda}{M_{\text{Pl}}^2}\zeta \quad \Rightarrow \quad \dot{\zeta} \sim \frac{\delta\Lambda}{M_{\text{Pl}}^2 H}\zeta \quad \Rightarrow \quad \langle \zeta_k^2 \rangle_{1-\text{loop}} \sim \langle \zeta_k^2 \rangle_{\text{tree}} \frac{H^2}{M_{\text{Pl}}^2} \log\left(a(t\right.$$

$$\delta\Lambda \sim H^4$$

- Second Woodard's mistake:
  - you can not neglect ζ time derivatives.
  - $$\int d^3x \sqrt{-g}\,\delta\Lambda \quad \Rightarrow \quad \int d^3x a^3 e^{3\zeta}(1+\delta N)\delta\Lambda \sim \int d^3x a^3 \left(3\zeta + \frac{\dot{\zeta}}{H}\right)\delta\Lambda \sim \int d^3x \frac{\partial(a^3\zeta)}{\partial t}\frac{\delta\Lambda}{H}$$

Woodard has only this

coupling
slow roll suppressed

- Summary
  - take into account of time derivatives, then take into account of tadpoles, result $\langle \zeta_k^2 \rangle_{1-\text{loop}} \sim 0$

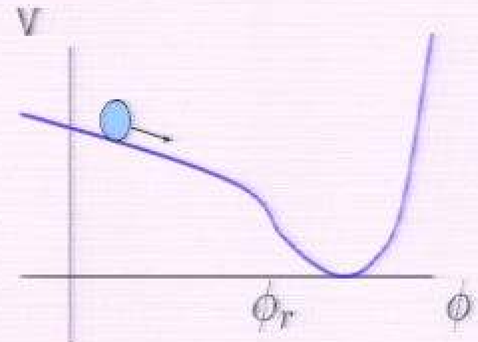
IR logs: just projection effects

$\log(kL)$ IR logs

and their zero physical effect

Large IR logs $\log(kL)$

- Single Field Slow-Roll Inflation (assumption on dynamics)



$$\langle\zeta_k^2\rangle \sim \frac{H^4}{\dot{\phi}^2}\Big|_{t_{h.c.}} \quad \text{where} \quad \frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$$

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  - Modes emitted earlier can change the position on the potential at horizon crossing

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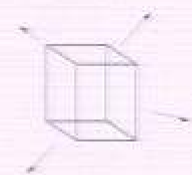
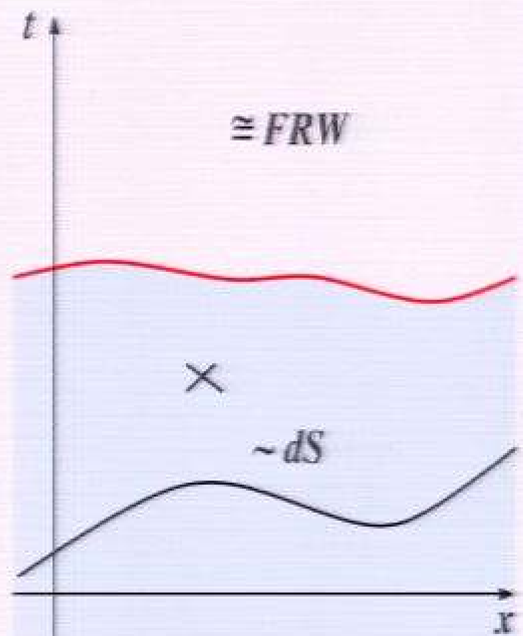


  - But this is all of its effect on the dynamics as:

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta_B} dx^2$$

$$\langle\zeta(\vec{x}_1,t)\zeta(\vec{x}_2,t)\rangle_B = \langle\zeta(e^{-\zeta_B}\vec{x}_1,t)\zeta(e^{-\zeta_B}\vec{x}_2,t)\rangle_0$$



  - By Taylor expanding

$$\langle\zeta_k\rangle_B = \langle\zeta_B^2\rangle \frac{\partial^2\left[k^3\langle\zeta_k^2\rangle\right]}{\partial\log(k)^2} = \widetilde{\langle\zeta\rangle}^2 N_{\text{beginning}}\left((n_s-1)^2+\alpha\right)\langle\zeta_k\rangle^2$$

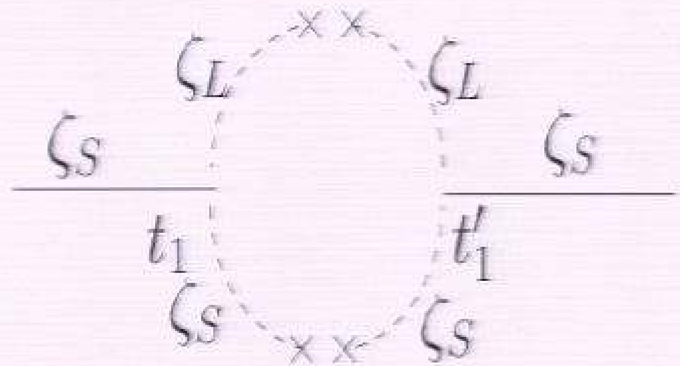
$$\widetilde{\langle\zeta^2\rangle} \sim 10^{-10}$$

$\log(kL)$ Technical point: missing running diagram

– Cut-In-the-Middle diagrams: Each mode interacts once

$$\langle \zeta^{(2)} \zeta^{(2)} \rangle_{1-\mathrm{loop}}$$

$$\zeta^{(2)}(t) = G(t-t')\zeta^{(1)}(t')^2$$



- This gives the tilt squared $(n_s-1)^2$

– Cut-In-the-Side diagrams: One mode interacts twice

$$\langle \zeta^{(3)} \zeta^{(1)} \rangle_{1-\mathrm{loop}}$$

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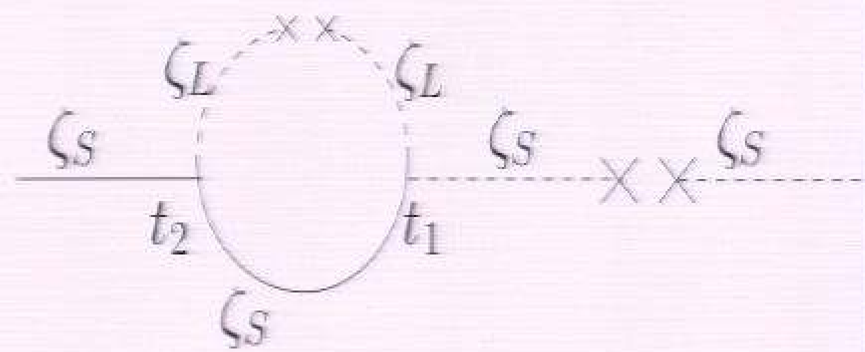

- This gives the running α

Large IR logs $\log(kL)$ =0, physically

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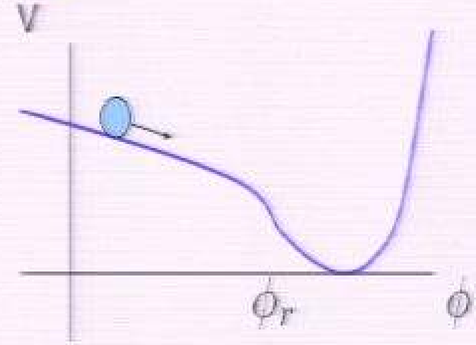

$$\langle\zeta_k^2\rangle \sim \left.\frac{H^4}{\dot\phi^2}\right|_{t_{h.c.}} \quad \text{where} \quad \frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$$

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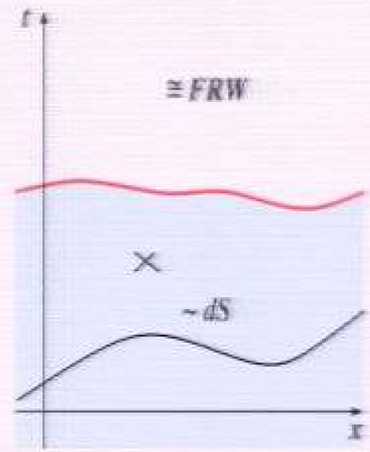
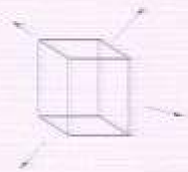


- Let us concentrate on what an observer measures:



  - measures a distance $\Delta r(t_{\rm rh})$
  - ask when mode came out of the horizon:
    - Since a background mode is equal to a rescaling of scale factor

$$ds^2 = -dt^2 + a(t)^2 e^{2\zeta_B} dx^2$$

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$$\Delta r(t) = \frac{e^{\zeta(x,t)}a(t)}{e^{\zeta(x,t_{\rm rh})}a(t_{\rm rh})}\Delta r(t_{\rm rh}) = e^{\zeta(x,t)-\zeta(x,t_{\rm rh})}\frac{a(t)}{a(t_{\rm rh})}\Delta r(t_{\rm rh})$$

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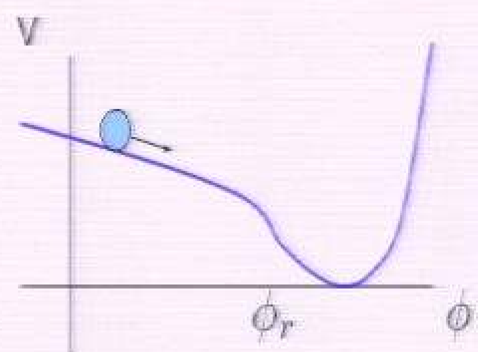

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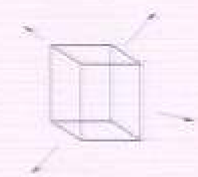

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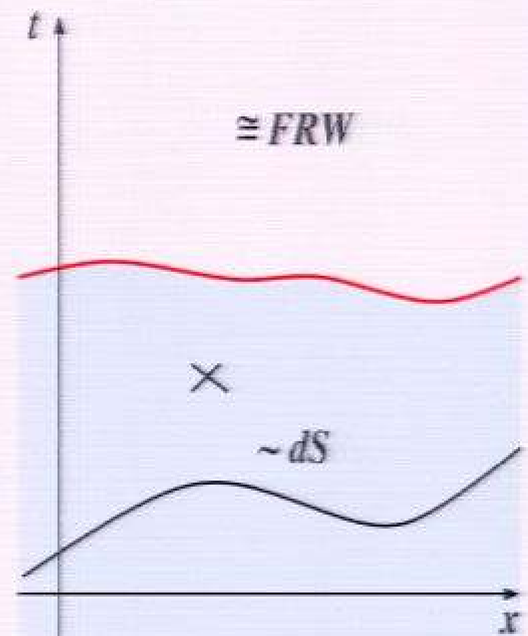

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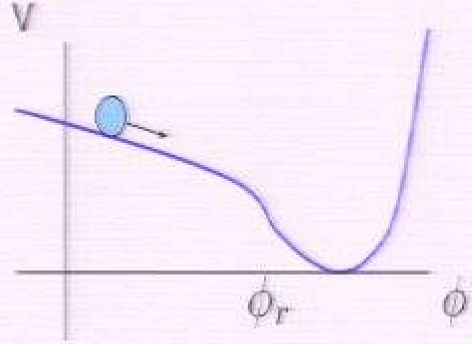

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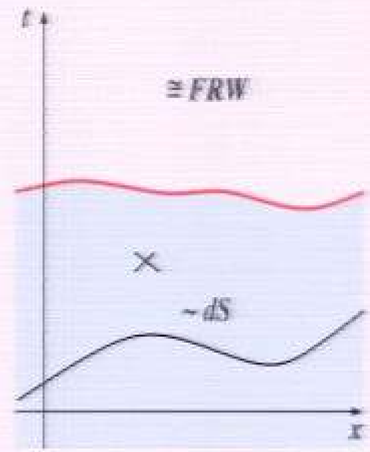

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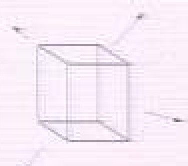

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Second projection effect:
a Physical IR one

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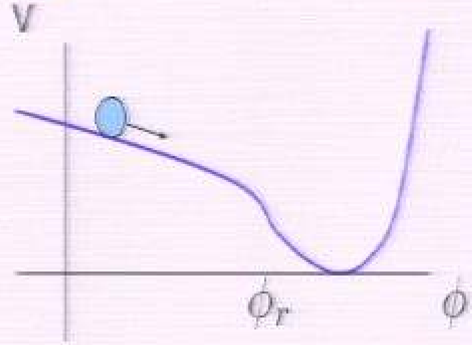

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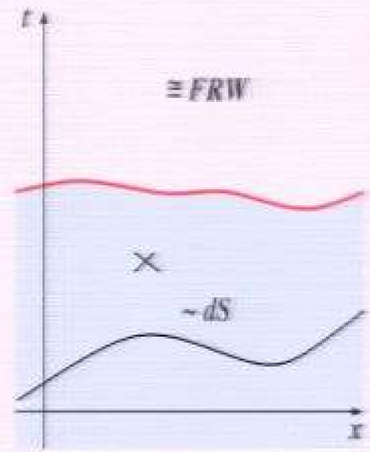

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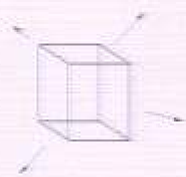

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Second projection effect:
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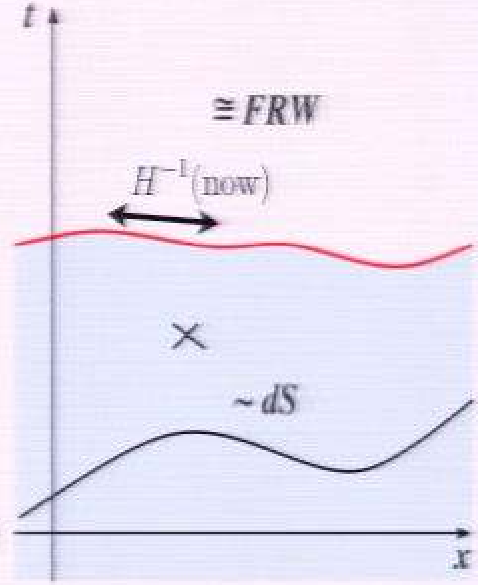
A Physical IR effect

- If we do not see the gradients of ζ_B , we do not observe ζ_B

$$\langle \zeta_k^2 \rangle \sim \frac{H^4}{\dot\phi^2}\bigg|_{t_{h.c.}} \quad \text{where} \quad \frac{k}{a(t_{h.c})} \sim H(t_{h.c.})$$



- Since $k^{us}_{phys.} = \dfrac{k}{a(t_{reh.})e^{\zeta_B(t_{reh})}}$

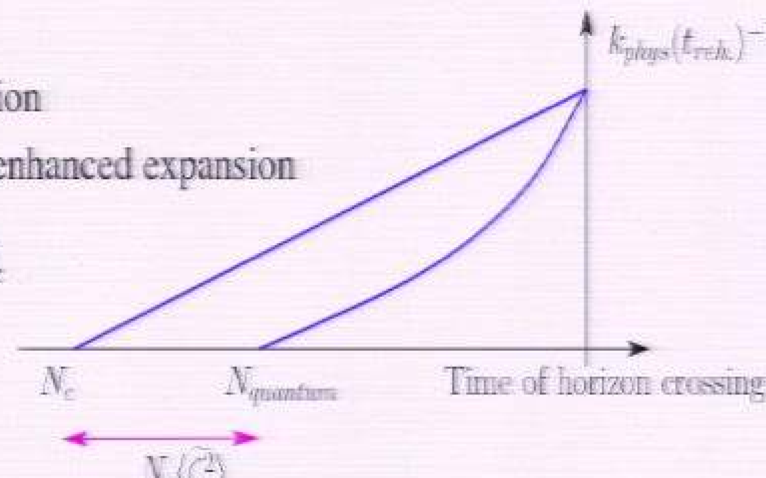
$$\frac{k}{a(t_{h.c.})e^{\zeta_B(t_{h.c.})}} \cdot \left[a(t_{reh.})e^{\zeta_B(t_{reh.})}\right] \sim H(t_{h.c.})$$

- Let us massage the new time of horizon-crossing

$$\frac{a(t_{(reh.)})}{a(t_{h.c.})} \sim e^{N_c}\,, \quad N_c \sim 60$$ classical expansion

enhanced expansion

$$\langle e^{\zeta_B(t_{reh.})-\zeta_B(t_{h.c.})}\rangle \sim 1 + \langle \zeta(x)^2 \rangle^{t_{h.c.}}_{t_{reh.}} \sim 1 + \widetilde{\langle \zeta^2 \rangle} N_c$$



$$\widetilde{\langle \zeta^2 \rangle} \sim 10^{-10}$$

- We have

$$k\, e^{N_c(1+\widetilde{\langle \zeta^2 \rangle})} = H(t_{h.c.}) \quad \Rightarrow \quad \delta N \sim \widetilde{\langle \zeta^2 \rangle} N_c$$

- True IR (tinv) effect:

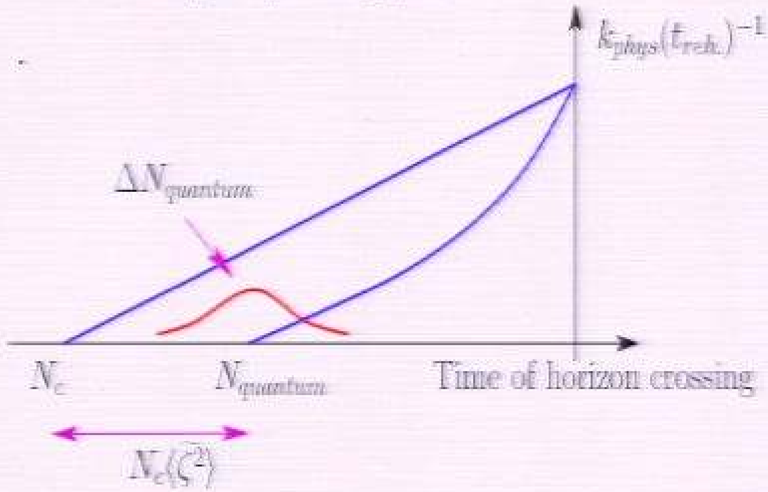
$$\langle \zeta_k^2 \rangle \sim \frac{H^4}{\dot\phi^2}\bigg| \left(1 + (n_s - 1) N_c \widetilde{\langle \zeta^2 \rangle}\right)$$

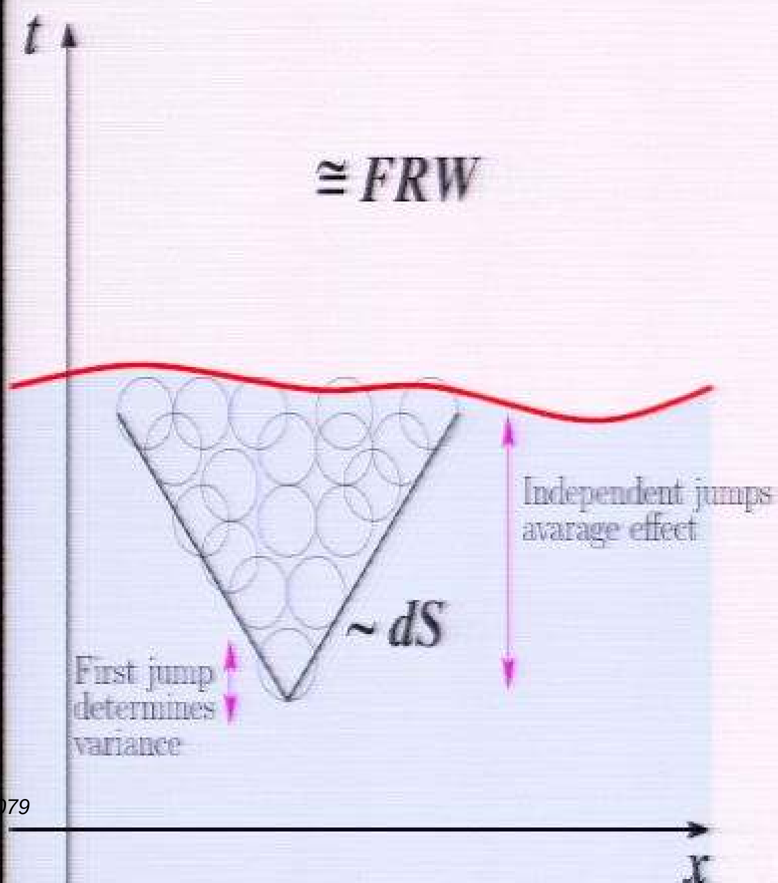
Different tilt and N dependence

First issue

- Why did we took the average of the enhanced expansion? $\langle \delta N \rangle \sim \widetilde{\langle \zeta \rangle} N_c$
  - Small variance:

$$\langle \Delta N^2_{quantum} \rangle^{1/2} \sim \widetilde{\langle \zeta^2 \rangle}^{1/2}$$





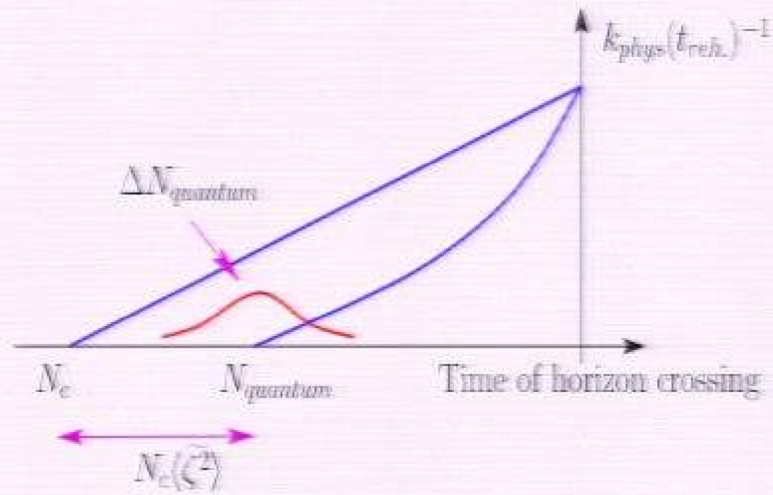
Second issue

- Enhanced expansion $\delta N_{quantum} \sim N_c \widetilde{\langle \zeta^2 \rangle}$
  - What happens for $\widetilde{\langle \zeta^2 \rangle} \sim 1$ (Close to Eternal Inflation)? or very large N_c ?

- Non-perturbative treatment necessary
  - Already done! in With Dubovsky and Villadoro **JHEP0904:118,2009**

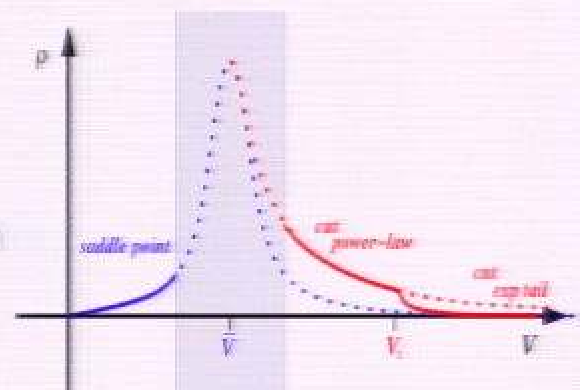
$$\langle V \rangle = \langle e^{3\zeta_B} \rangle \simeq e^{3N_c \frac{2}{1+\sqrt{1-1/\Omega}}}$$

$$\Omega \equiv \frac{2\pi^2}{3} \frac{\dot{\phi}^2}{H^4} \sim 1/\widetilde{\langle \zeta^2 \rangle}$$



  - Maximum enhancement: $\delta N^{max}_{quantum} \simeq N_c/2$
  - Small variance $\langle \Delta N^2_{quantum} \rangle^{1/2} \sim \widetilde{\langle \zeta^2 \rangle}^{1/2}$
  - Probability distribution known (also within eternal inflation)

$$\rho(V,\tau) \approx \mathcal{N} e^{-\Omega\left[\frac{3N}{2}\left(1+\sqrt{1-\frac{1}{\Omega}}\right)-3N_c\right]^2}$$



Dynamical effects

(it matters only the period of horizon crossing)

Summary

On Loops In Inflation

with Zaldarriaga, **0912.2734** [hep-th]
in completion

- We have learnt how to compute quantum corrections to inflationary observables.
- UV divergency
  - The logarithmic running is of the form $\log\left(\frac{H}{\mu}\right)$
- IR divergency in Single Field Inflaton:
  - need to renormalize the unperturbed history
  - need to take into account projection effects:
    - There is no time dependence
    - and no non-trivial scale dependence $\log(kL)$
    - There is a physical enhanced expansion $\delta N_{quantum} \sim N_e \widetilde{\langle\zeta^2\rangle}$
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