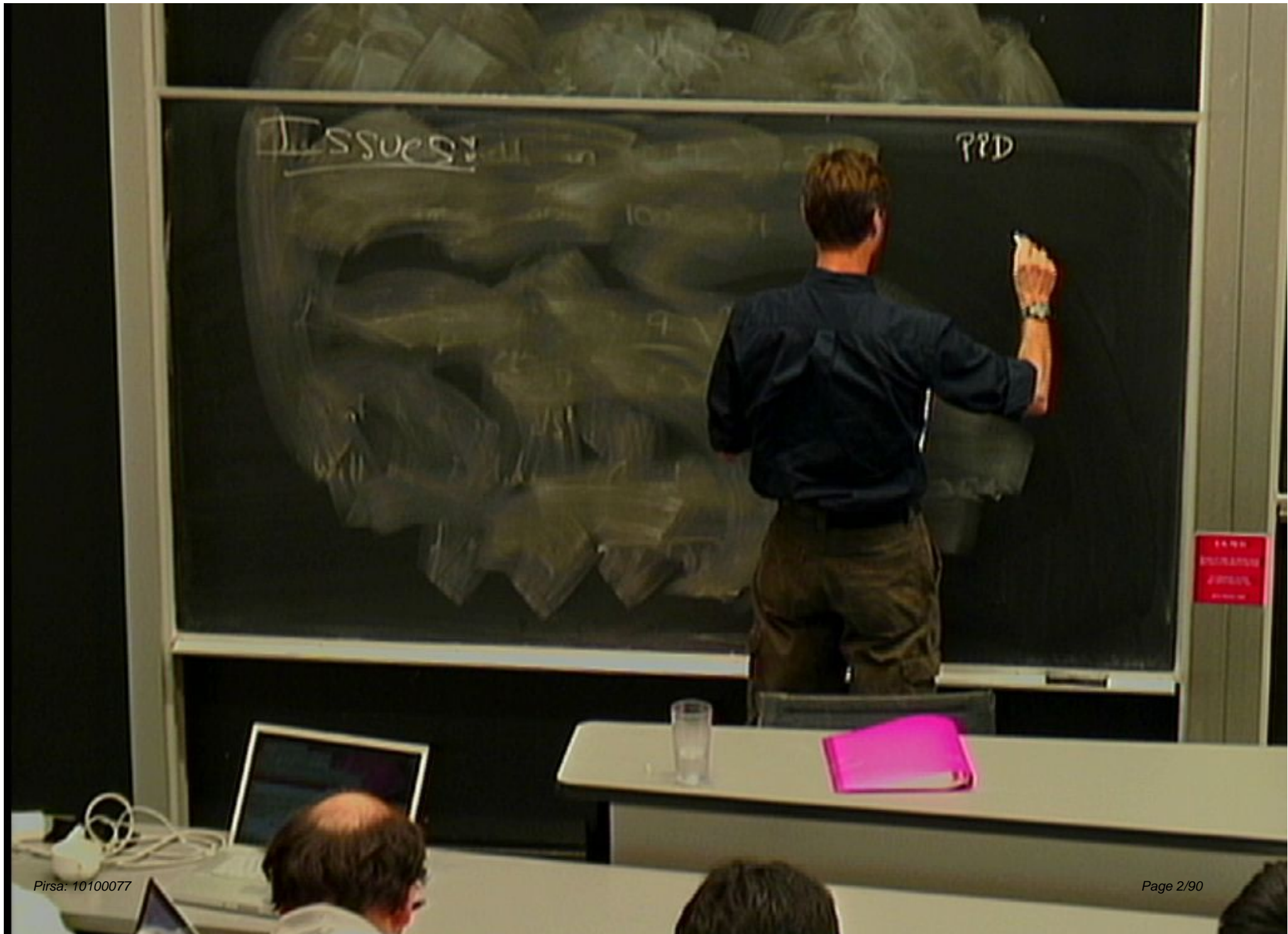


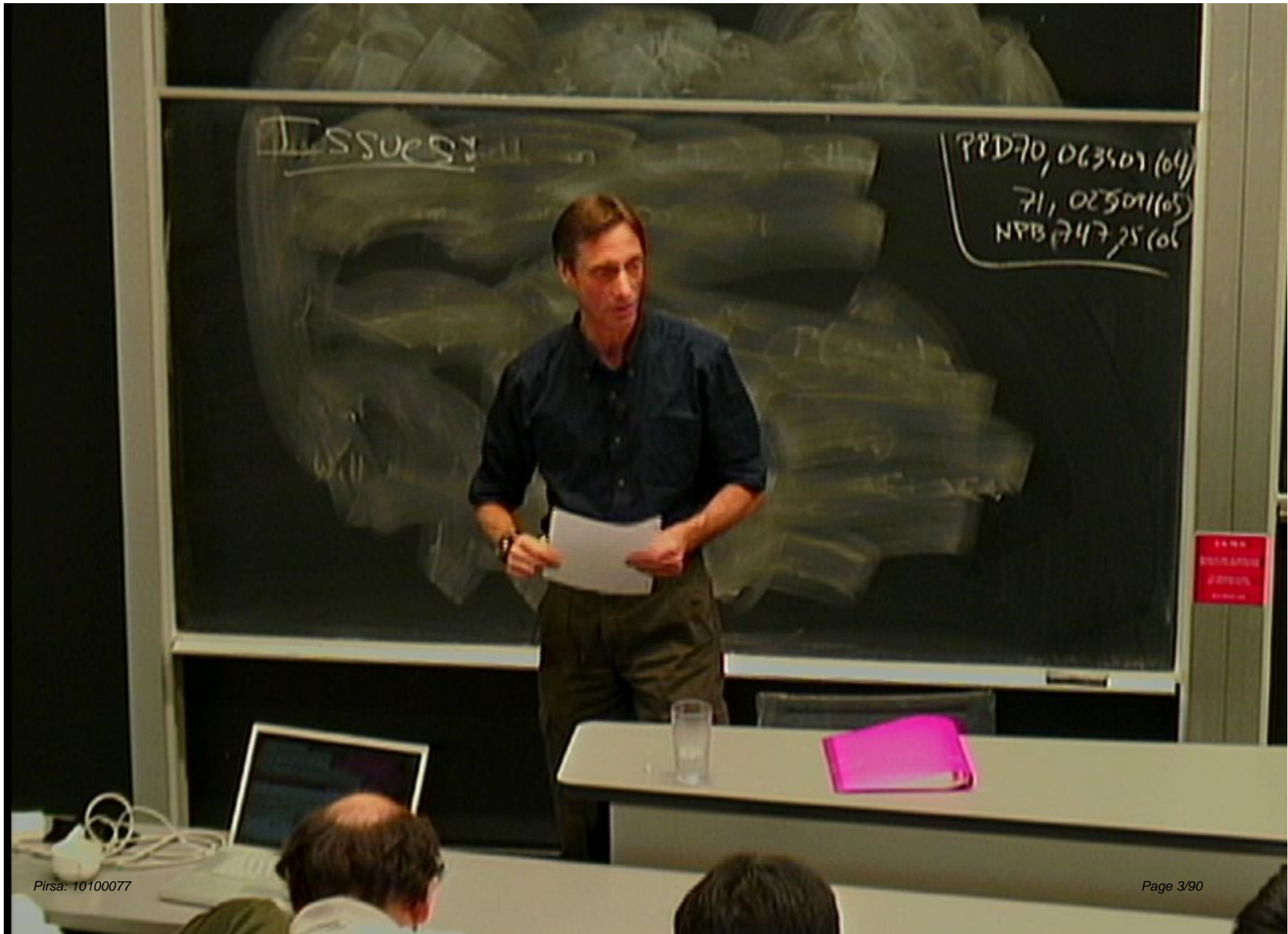
Title: Anomalous scaling dimensions and particle decay during inflation

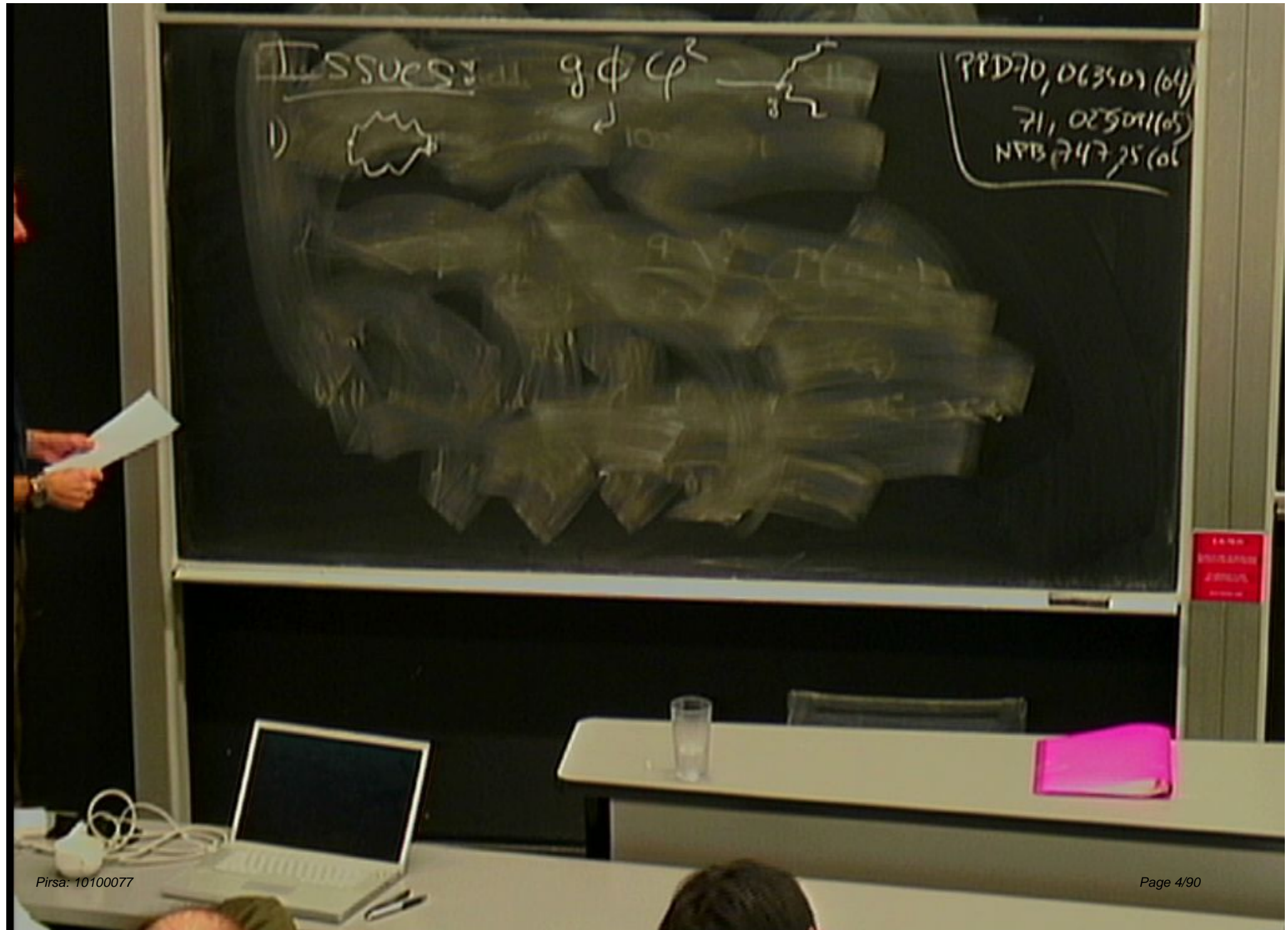
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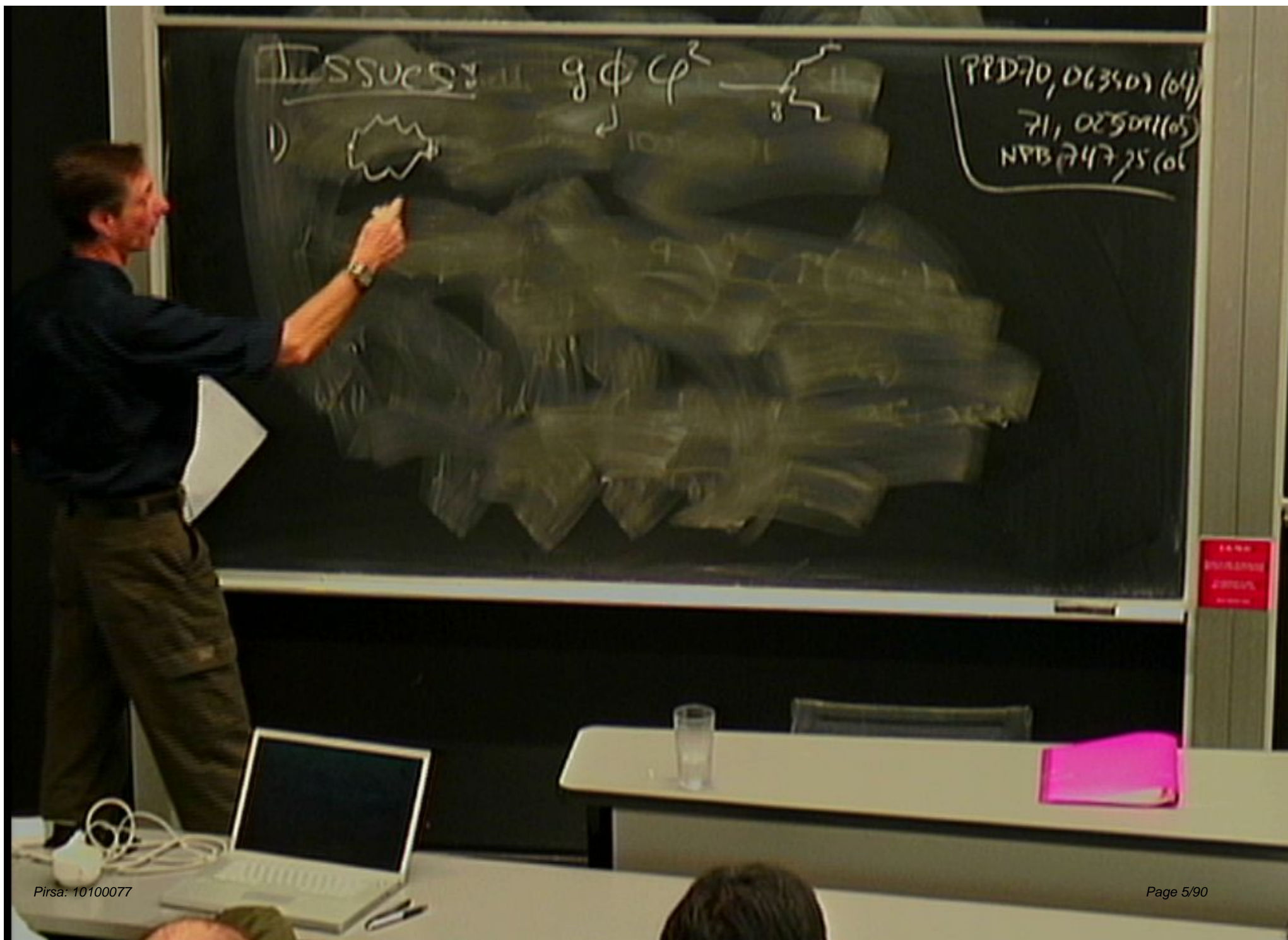
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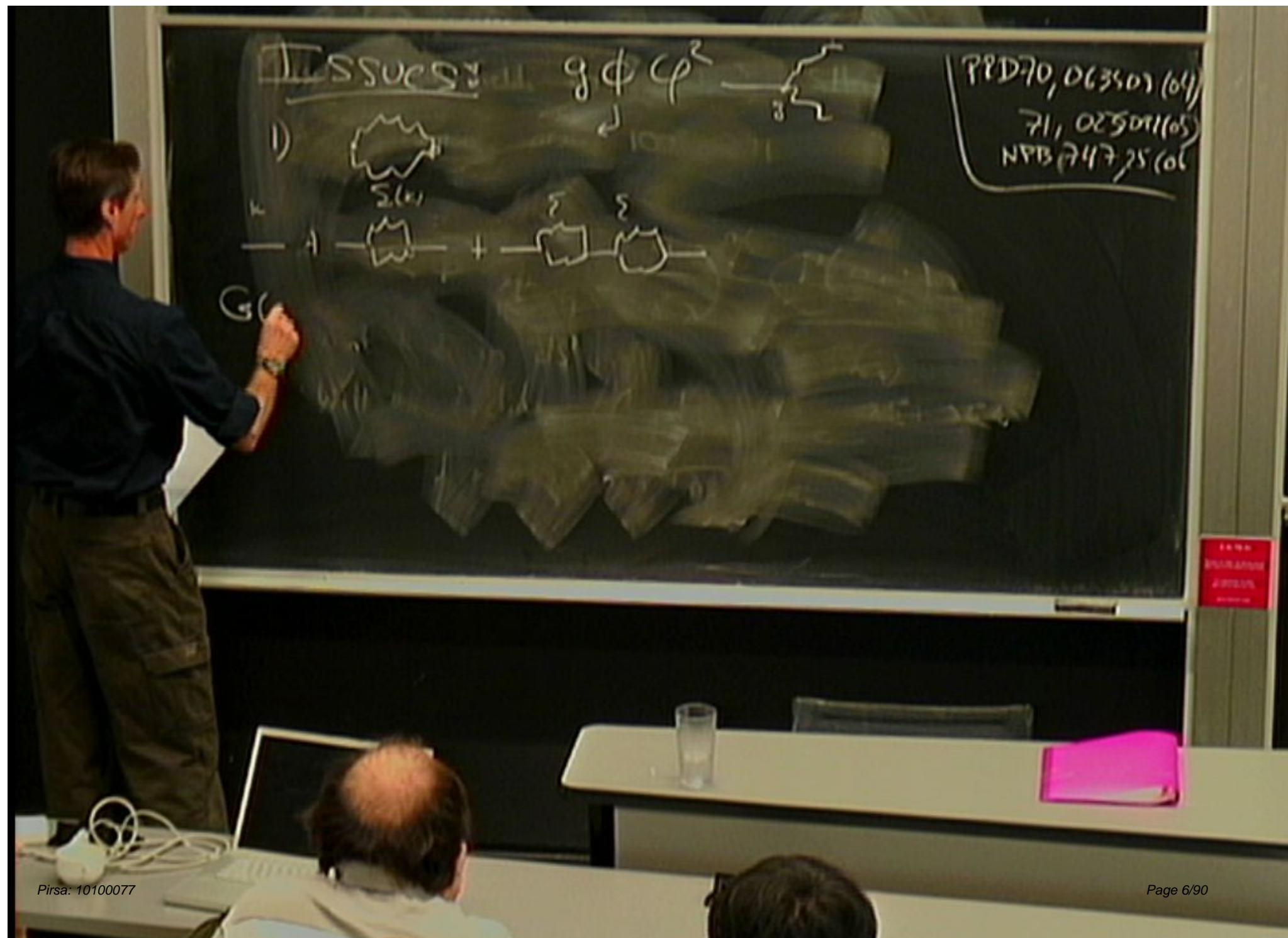
Abstract: I will discuss the emergence of anomalous scaling dimensions for superhorizon fluctuations and the main ideas and concepts of particle decay during inflation, via the resummation of secular terms with the dynamical renormalization group. There are loops, IR effects and (lots of...) issues.





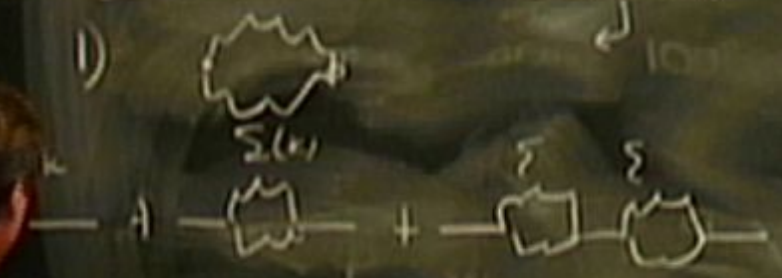






ISSUES: $g\phi\phi^2$

PPD70, 063501 (04)
 71, 025011 (05)
 NPB 747, 25 (06)



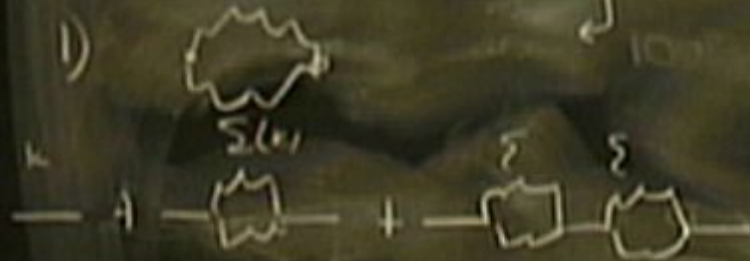
$$G(k) = \frac{1}{k^2 - m^2 - \Sigma(k)}$$

Issues:

$$g\phi\phi^2$$



PPD70, 063501 (04)
 71, 025011 (05)
 NPB 747, 25 (06)



$$G(k) = \frac{1}{k^2 - m^2 - \Sigma(k)}$$

pole $\Sigma_P(M) + i\Sigma_I(M)$

F.T $\int \frac{d^4k}{(2\pi)^4} e^{-ikx}$ $G_R(k) \rightarrow e^{-i\omega t}$

(2) $P = 1 \leftarrow P^2 \sim \frac{\partial^2}{\partial x^2} (x^3)$

$$S_0 = \int (\partial\phi)^2 + m^2 \phi^2 = \sum_{\vec{l}} [L(L+1) + m^2] \phi_{\vec{l}}^2$$

$$S_p = \int \lambda \phi^4$$

$$\langle \phi(x) \phi(y) \rangle_0 = \frac{\int \mathcal{D}\phi \phi(x) \phi(y) e^{-S_0}}{\int \mathcal{D}\phi e^{-S_0}} = \frac{\gamma_L^*(x) \gamma_L(y)}{L(L+1) + m^2}$$

$$\langle \phi_{\vec{l}} \phi_{\vec{l}} \rangle = \frac{1}{L(L+1) + m^2}$$

3)

$$3) \Rightarrow T_{fi}$$

$$3.) \Rightarrow \overline{T}_{fi} = \int_{t_i}^{t_f} \langle \chi_f | \chi_{f-v} | e^{iH_0 t} H_1 e^{-iH_0 t} | \chi_i \rangle dt$$

$$3.) \Rightarrow \bar{T}_{pi} = \int_{t_i}^{t_f} \langle 1\chi_p | \chi_{p-v} | e^{iH_0 t} H_1 e^{-iH_0 t} | \chi_v \rangle dt$$

$$3.) \Rightarrow \bar{T}_{fi} = \int_{t_i}^{t_f} \langle 1\chi_f | 1\chi_{f-v} | e^{iH_0 t} H_1 e^{-iH_0 t} | 1\chi_v \rangle dt$$

$$= \frac{g}{\sqrt{2\epsilon_x} \sqrt{2\epsilon_y} \sqrt{2\epsilon_z}} \sin[(E_f - E_i)t]$$

$$\begin{aligned}
 3) \Rightarrow \bar{T}_{fi} &= \int_{t_i}^{t_f} \langle \chi_f | \chi_{f-k} | e^{iH_0 t} H_1 e^{iH_0 t} | \chi_i \rangle dt \\
 &= \frac{g}{\sqrt{2\epsilon_k} \sqrt{2\epsilon_k} \sqrt{2\epsilon_k}} \frac{\sin[(E_k^u - E_{f-k}^u - E_i^u)(t_f - t_i)]}{[E - E - E]}
 \end{aligned}$$

$$\begin{aligned}
 3.) \Rightarrow \bar{T}_{fi} &= \int_{t_i}^{t_f} \langle \chi_f | \chi_{f-k} | e^{iH_0 t} H_1 e^{-iH_0 t} | \chi_i \rangle dt \\
 &= \frac{g}{\sqrt{2\epsilon_k} \sqrt{2\epsilon_k} \sqrt{2\epsilon_k}} \frac{\sin[(E_k - E_{f-k} - E_i)(t_f - t_i)]}{[E - E - E]}
 \end{aligned}$$

$$3.) \Rightarrow T_{fi} = \int_{t_i}^{t_f} \langle \chi_f | \chi_{f-k} | e^{iH_0 t} H_I e^{-iH_0 t} | \chi_i \rangle dt$$

$$= \frac{g}{\sqrt{2\epsilon_k} \sqrt{2\epsilon_k} \sqrt{2\epsilon_\lambda}} \frac{\sin[(E_k - E_{f-k} - E_i)(t_f - t_i)]}{[E - E - E]}$$

$$|T_{fi}|^2 = \frac{g^2}{\epsilon} \frac{\sin^2[(E - E - E)\pi]}{(E - E - E)^2} \rightarrow 2\pi$$

$$3) \Rightarrow T_{fi} = \int_{t_i}^{t_f} \langle \chi_f | \chi_{p-v} | e^{iH_0 t} H_1 e^{-iH_0 t} | \chi_i \rangle dt$$

$$= \frac{g}{\sqrt{2\epsilon_k} \sqrt{2\epsilon_v} \sqrt{2\epsilon_f}} \frac{\sin[(E_k - E_{p-v} - E_f)(t_f - t_i)]}{[E - E - E]}$$

$$|T_{fi}|^2$$

$$\frac{\sin^2[(E - E - E)\pi]}{(E - E - E)^2} \rightarrow 2\pi\pi \delta(E - E - E)$$

$$3) \Rightarrow T_{fi} = \int_{L_i}^{L_f} \langle 1\chi_f | 1\chi_{f-k} | e^{iH_0 t} H_1 e^{-iH_0 t} | 1\chi_i \rangle dt$$

$$= \frac{g}{\sqrt{2\epsilon_k} \sqrt{2\epsilon_k} \sqrt{2\epsilon_k}} \frac{\sin[(E_k - E_{f-k} - E_i)(t_f - L_i)]}{[E - E - E]}$$

$$|T_{fi}|^2 = \frac{g^2}{\epsilon} \frac{\sin^2[(E - E - E)\pi]}{(E - E - E)^2} \rightarrow 2\pi\pi \delta(E - E - E)$$

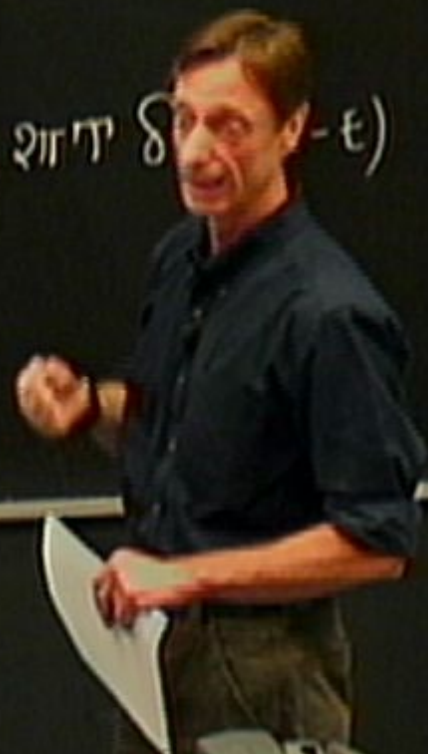
$$\frac{dP_{fi}}{dt} = 1$$

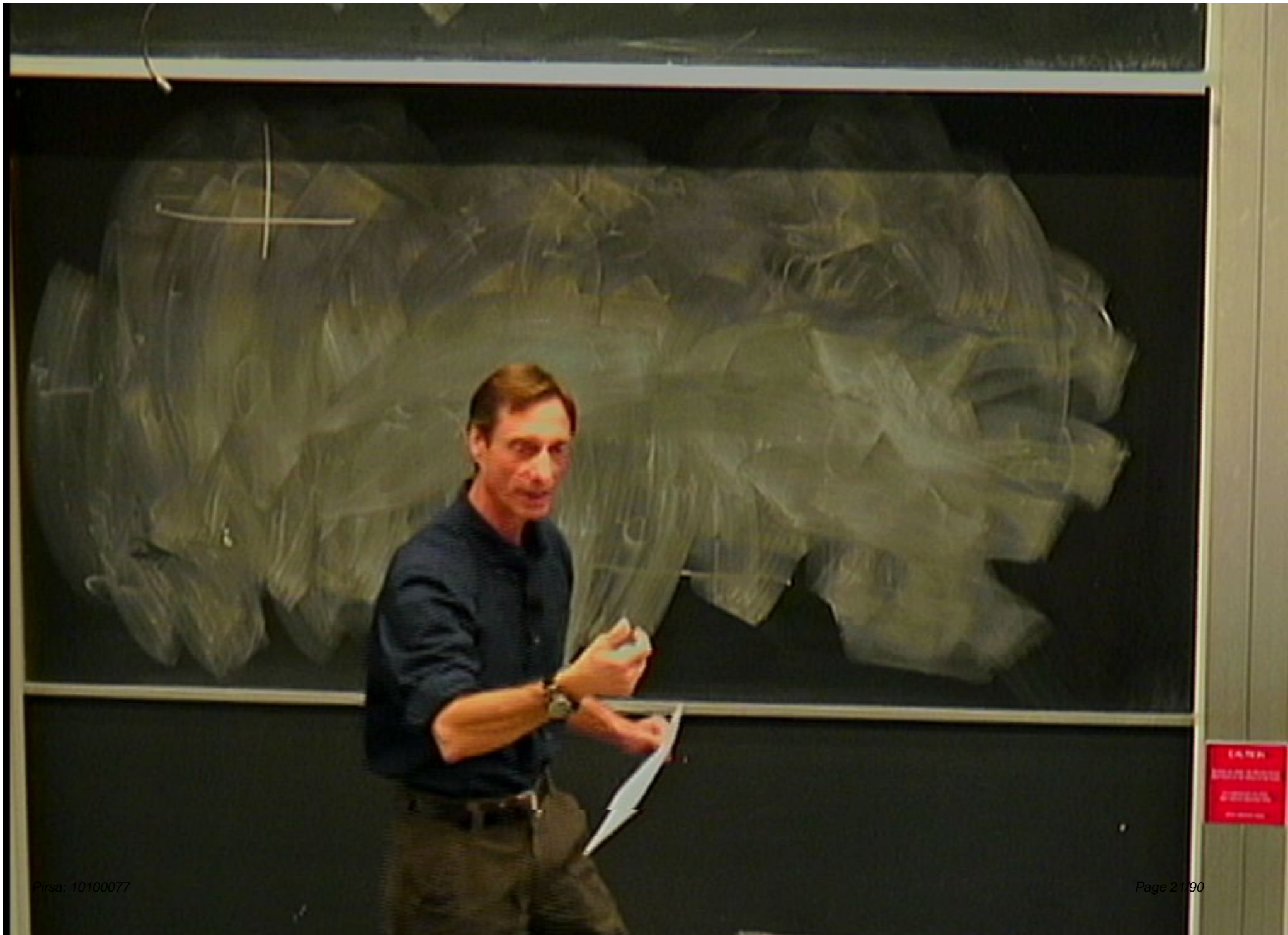
$$3) \Rightarrow T_{fi} = \int_{t_i}^{t_f} \langle \chi_f | \chi_{f-v} | e^{iH_0 t} H_1 e^{-iH_0 t} | \chi_i \rangle dt$$

$$= \frac{g}{\sqrt{2\epsilon_k} \sqrt{2\epsilon_k} \sqrt{2\epsilon_k}} \frac{\sin[(E_k - E_{f-v} - E_i)(t_f - t_i)]}{[E - E - E]}$$

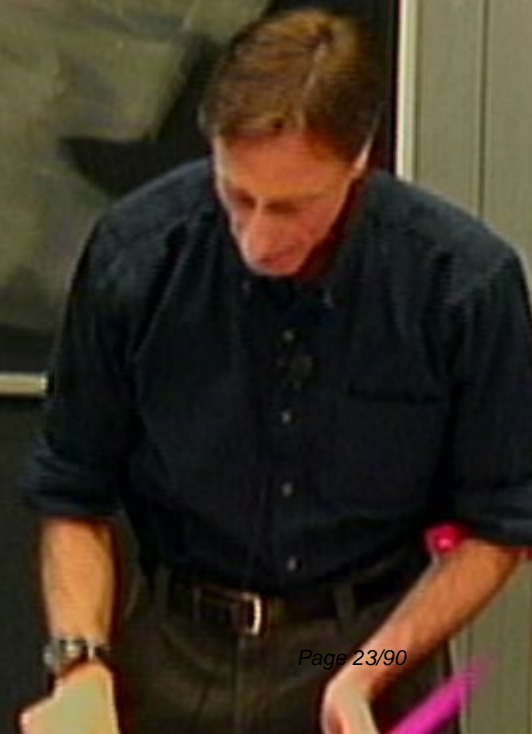
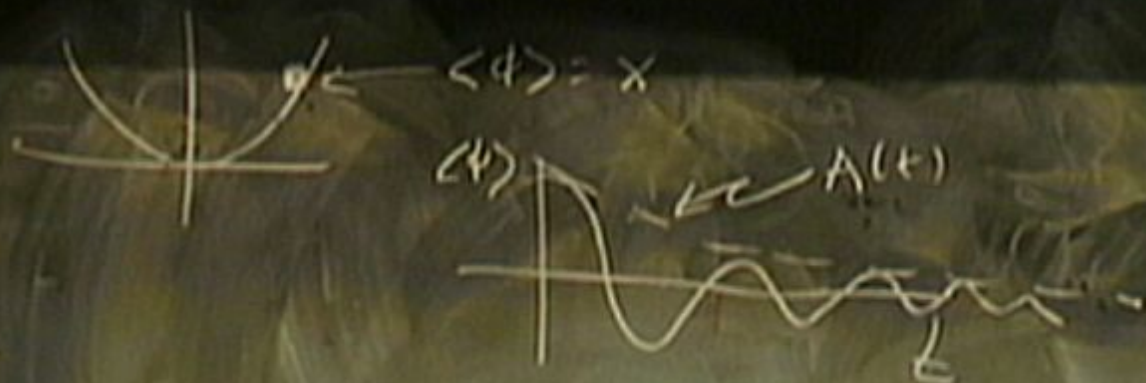
$$|T_{fi}|^2 = \frac{g^2}{\epsilon} \frac{\sin^2[(E - E - E)\pi]}{(E - E - E)^2} \rightarrow 2\pi\pi \delta(E - E)$$

$$\frac{dP_{fi}}{dt} = 1$$









$$\langle \phi = \sum_{\langle q \rangle} \psi$$

$$\phi = \frac{\chi}{\langle \phi \rangle} + \psi ; \quad \frac{1}{2} (\partial_\mu \phi)^2 - \frac{M^2}{2} \phi^2 - g \phi \psi^2, \dots$$

$$\langle \phi = \frac{x}{\langle \phi \rangle} + \psi \rangle, \quad \frac{1}{2} (\partial_\mu \phi)^2 - \frac{M^2}{2} \phi^2 - g \phi \psi^2, \dots$$

$$- (\partial^\mu \partial_\mu X + M^2 X) \psi - g X \psi^2 - g \psi^4$$

$$\phi = \frac{X}{\langle \phi \rangle} + \psi, \quad \frac{1}{2} (\partial_\mu \phi)^2 - \frac{M^2}{2} \phi^2 - g \phi \psi^2, \dots$$

$$= (\partial^\mu \partial_\mu X + M^2 X) \psi - g X \psi^2 - g \psi^3$$

$$\text{Tr } X S(t) \rightarrow U(t) S(0) U^\dagger(t)$$

Tr S

$$\phi = \frac{X}{\langle \phi \rangle} + \psi ; \quad \frac{1}{2} (\partial_\mu \phi)^2 - \frac{M^2}{2} \phi^2 - g \phi \psi^2, \dots$$

$$- (\partial^\mu \partial_\mu X + M^2 X) \psi - g X \psi^2 - g \psi^3$$

$$\underbrace{\text{Tr } X S(t)}_{\text{Tr } S} \rightarrow U(t) S(0) U^\dagger(t)$$

$\langle q \rangle$

$$- (\partial^\mu \partial_\mu \chi + m^2 \chi) \varphi - g \chi \varphi^2 - g \varphi^4$$

$$\text{Tr } X \mathcal{S}(t) \rightarrow U(t) \mathcal{S}(0) U^\dagger(t)$$

$\text{Tr } \mathcal{S}$

$\langle 4 \rangle$

$$- (\partial^\mu \partial_\mu \chi + m^2 \chi) \varphi - g \chi \varphi^2 - g \varphi^4$$

$$\text{Tr } X \mathcal{S}(t) \rightarrow U(t) \mathcal{S}(0) U^\dagger(t)$$

$\text{Tr } \mathcal{S}$

$$\ddot{X}_k(t) + \omega_k^2 X_k(t) + \int_{-b}^t \sum_R (k, t-t') X_k(t') = 0$$

$\int_{-b}^t dt'$

$$\langle \phi \rangle$$

$$- (\partial^\mu \partial_\mu \chi + m^2 \chi) \phi - g \chi \phi^2 - g \phi^4$$

$$\text{Tr } X \mathcal{P}(t) \rightarrow U(t) \mathcal{P}(0) U^\dagger(t)$$

Tr S

$$\ddot{X}_k(t) + \omega_k^2 X_k(t) + \int_{-b}^t \sum_R (k, t-t') X_k(t') dt' = 0$$

\tilde{X}

$\langle \varphi \rangle$

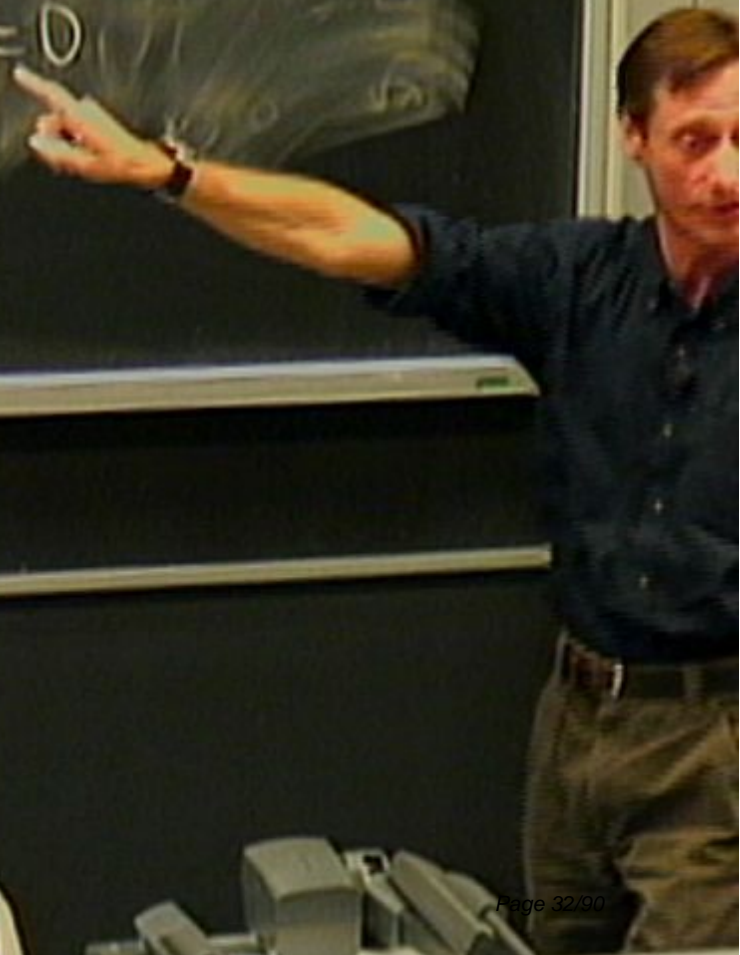
$$- (\partial^\mu \partial_\mu \chi + m^2 \chi) \varphi - g \chi \varphi^2 - g \varphi^4$$

$$\text{Tr } X \mathcal{S}(t) \rightarrow U(t) \mathcal{S}(0) U^\dagger(t)$$

Tr-S

$$\ddot{X}_k(t) + \omega_k^2 X_k(t) + \int_{-t_0}^t \sum_R (k, t-t') X_k(t') dt' = 0$$

$$\tilde{X}_k(s) = \frac{X_k(0)}{s^2 + \omega_k^2 + \sum_R (k, s)}$$



$\langle 4 \rangle$

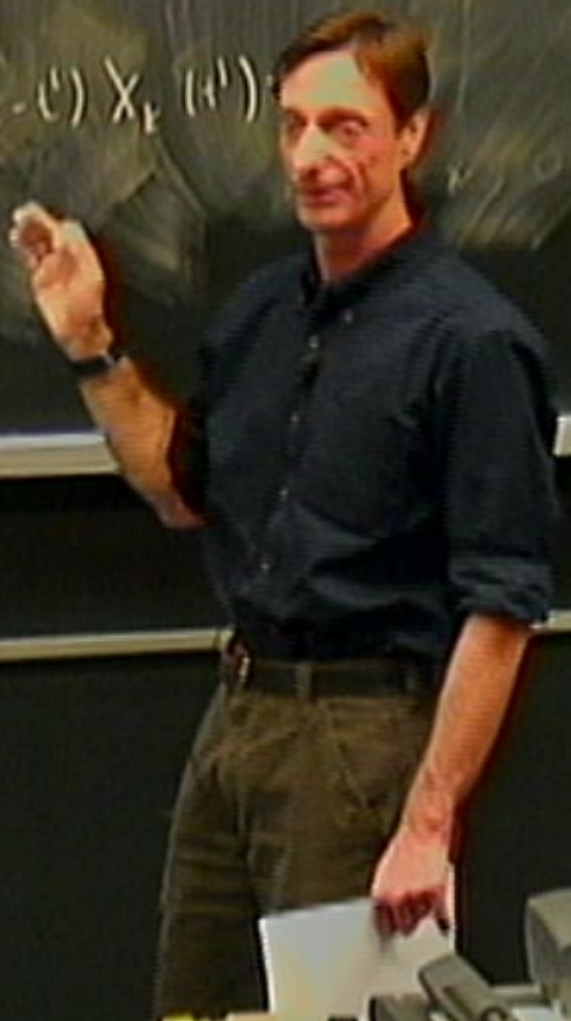
$$- (\partial^\mu \partial_\mu \chi + m^2 \chi) \varphi - g \chi \varphi^2 - g \varphi^4$$

$$\text{Tr } X \mathcal{S}(t) \rightarrow U(t) \mathcal{S}(0) U^\dagger(t)$$

Tr S

$$\ddot{X}_k(t) + \omega_k^2 X_k(t) + \int_{-t_0}^t \Sigma_R(k, t-t') X_k(t')$$

$$\tilde{X}_k(s) = \frac{X_k(0)}{s^2 + \omega_k^2 + \Sigma_R(k, s)}$$



$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{i\omega_k t} + B e^{-i\omega_k t}$$

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$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \Sigma(t-t') \dot{X}^0(t') dt'$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{i\omega_k t} + B e^{-i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \underbrace{\Sigma(t-t')} X^0(t') dt'$$

$$X^1(t) = \int_0^t g(t-t') X^0(t') dt'$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{i\omega_k t} + B e^{-i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \underbrace{\int \Sigma(t-t') X^0(t') dt'}_{\text{source term}}$$

$$X^{(1)} = \int_0^t g(t-t') \mathcal{R}(t')$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{-i\omega_k t} + B e^{i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \Sigma(t-t') X^0(t') dt'$$

$$X^{(1)} = \int_0^t g(t-t') X(t')$$

$$X^1(t) = A e^{-i\omega t}$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{i\omega_k t} + B e^{-i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \Sigma(t-t') X^0(t') dt'$$

$$X^1(t) = \int_0^t g(t-t') X^0(t') dt'$$

$$X^1(t) = A e^{-i\omega_k t} [\alpha t + \dots] + B e^{i\omega_k t} [\dots]$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{i\omega_k t} + B e^{-i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \Sigma(t-t') X^0(t') dt'$$

$$X^1(t) = \int_0^t g(t-t') \mathcal{R}(t')$$

$$X^1(t) = A e^{-i\omega_k t} [\alpha t + \dots] + B e^{i\omega_k t} [\beta t + \dots]$$

$$-i \Sigma_R(\omega) - \Sigma_I(\omega)$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{i\omega_k t} + B e^{-i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \Sigma(t-t') X^0(t') dt'$$

$$X^1(t) = \int_0^t g(t-t') R(t') dt'$$

$$X^1(t) = A e^{-i\omega_k t} [\dots + \dots]$$

$$-i Z_R(t') = \dots$$

$$1 - \Sigma_I(\omega)$$

$$X = X^0 + g^2 X^1 + \dots$$

$$\ddot{X}^0 + \omega_k^2 X^0 = 0 \Rightarrow A e^{-i\omega_k t} + B e^{i\omega_k t}$$

$$\ddot{X}^1 + \omega_k^2 X^1 = - \int \Sigma(t-t') X^0(t') dt'$$

$$X^1(t) = \int_0^t g(t-t') \mathcal{R}(t')$$

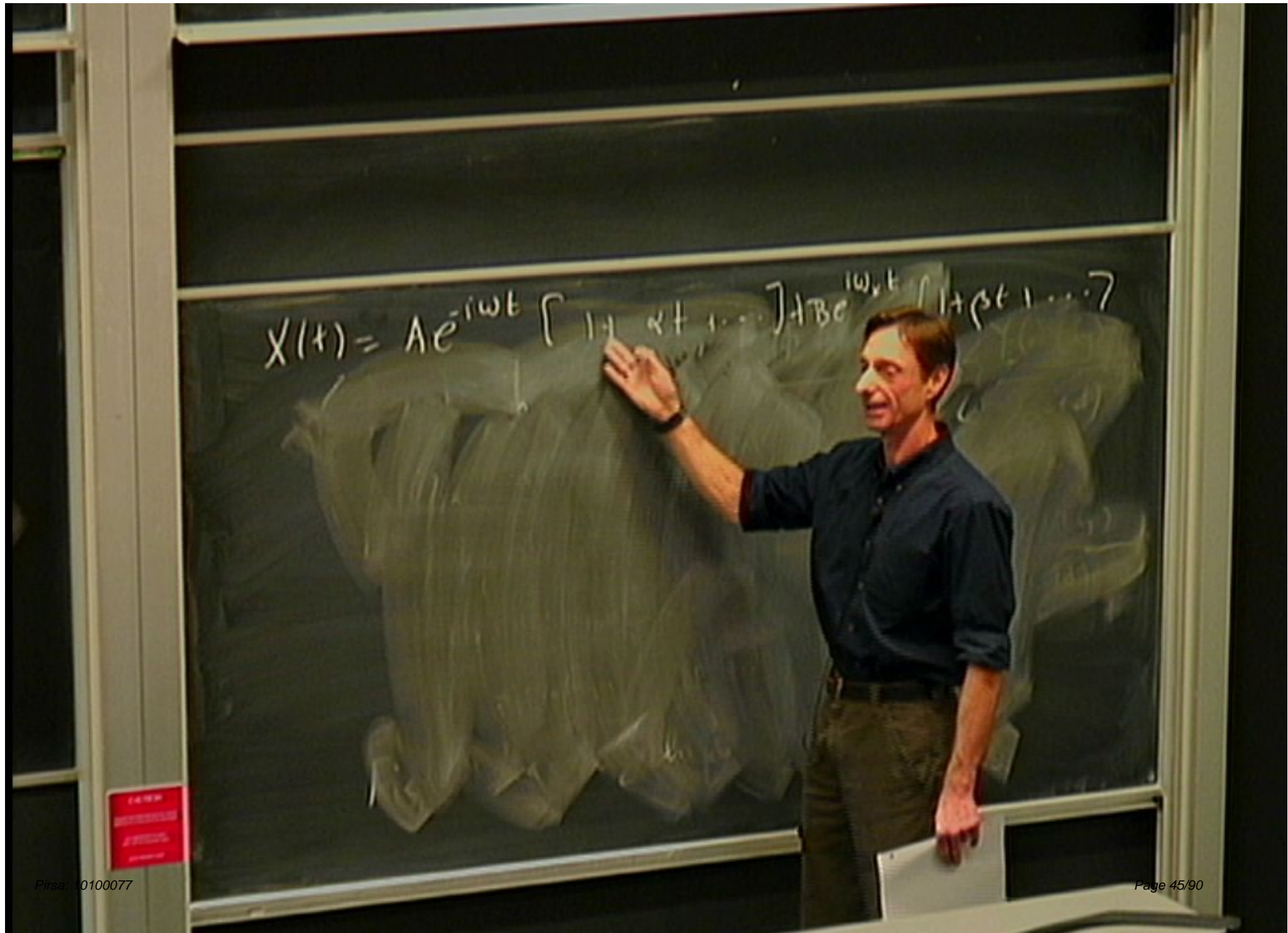
$$X^1(t) = A e^{-i\omega_k t} [\omega t + \dots] + B e^{i\omega_k t} [\omega t + \dots]$$

$$-i \Sigma_R(\omega) - \Sigma_I(\omega)$$

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$$X(t) = Ae^{-i\omega t} [1 + \alpha t + \dots] + Be^{i\omega t} [1 + \beta t + \dots]$$

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$$x(t) = Ae^{-i\omega t} [1 + \alpha t + \dots] + Be^{i\omega t} [1 + \beta t + \dots]$$

$$X(t) = A e^{-i\omega t} [1 + \alpha t + \dots] + B e^{i\omega t} [1 + \beta t + \dots]$$

$$A = A(\tau) Z^A(\tau), \quad B = B(\tau) Z^B(\tau)$$

$$Z^A = 1 + g^2 \delta_A + \dots \quad Z^B = 1 + g^2 \delta_B + \dots$$

$$X(t) = A e^{-i\omega t} [1 + \alpha t + \dots] + B e^{i\omega t} [1 + \beta t + \dots]$$

$$A = A(\tau) z^A(\tau), \quad B = B(\tau) z^B(\tau)$$

$$z^A = 1 + g^A \tau + \dots \quad z^B = 1 + g^B \tau + \dots$$

$$A = A(\tau) Z^A(\tau); \quad B = B(\tau) Z^B(\tau)$$

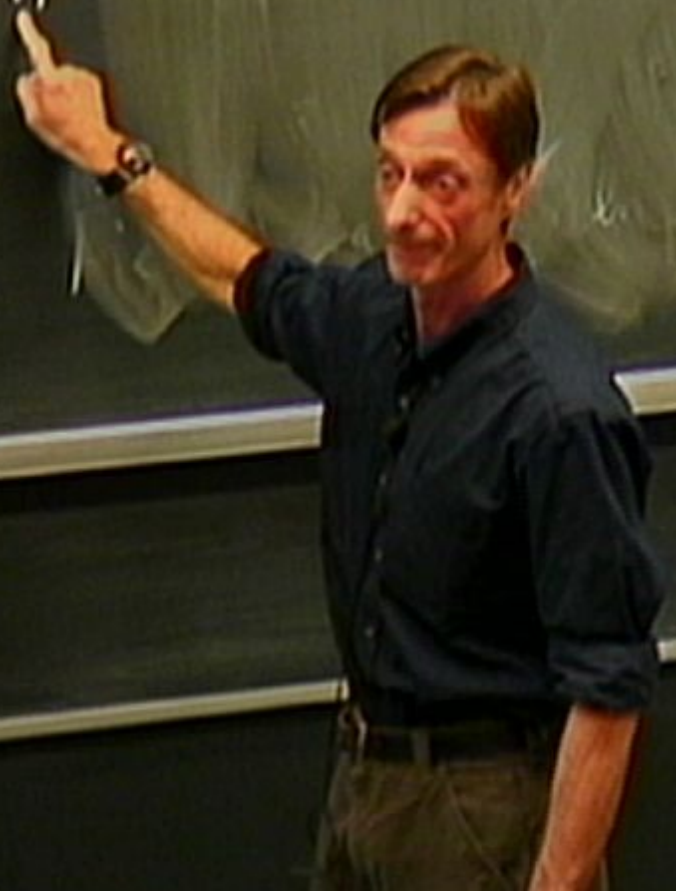
$$Z^A = 1 + g^2 Z_A + \dots \quad Z^B = 1 + g^2 Z_B + \dots$$

$$X(t) = A(\tau) e^{-i\omega_\nu t} [1 + \alpha(1-\tau)]$$

$$A = A(\tau) Z^A(\tau) ; B = B(\tau) Z^B(\tau)$$

$$Z^A = 1 + g^2 Z_A + \dots \quad Z^B = 1 + g^2 Z_B + \dots$$

$$X(t) = A(\tau) e^{-i\omega_v t} [1 + \alpha(1-\tau) + \dots] + B(\tau) e^{-i\omega_v t} [1 + \beta(1-\tau) + \dots]$$



$$A = A(\tau) Z^A(\tau), \quad B = B(\tau) Z^B(\tau)$$

$$Z^A = 1 + g^2 Z_A + \dots \quad Z^B = 1 + g^2 Z_B + \dots$$

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$$\frac{dX}{d\tau} = 0$$



$$A = A(\tau) Z^A(\tau), \quad B = B(\tau) Z^B(\tau)$$

$$Z^A = 1 + g^2 Z_A + \dots \quad Z^B = 1 + g^2 Z_B + \dots$$

$$X(t) = A(\tau) e^{-i\omega_v t} [1 + \alpha(1-\tau) + \dots] + B(\tau) e^{-i\omega_v t} [1 + \beta(1-\tau) + \dots]$$

$$\frac{dX}{d\tau} = 0$$

$$A = A(\tau) Z^A(\tau), \quad B = B(\tau) Z^B(\tau)$$

$$Z^A = 1 + g^2 Z_A + \dots \quad Z^B = 1 + g^2 Z_B + \dots$$

$$X(t) = A(\tau) e^{-i\omega_v t} [1 + \alpha(1-\tau) + \dots] + B(\tau) e^{-i\omega_v t} [1 + \beta(1-\tau) + \dots]$$

$$\frac{dX}{d\tau} = 0$$

$$\frac{dA(\tau)}{d\tau} = \alpha A(\tau) \quad ; \quad \frac{dB(\tau)}{d\tau} = \beta B(\tau)$$

$$Z^A = 1 + g^2 Z_A + \dots \quad Z^B = 1 + g^2 Z_B + \dots$$

$$X(t) = A(\tau) e^{-i\omega_v t} [1 + \alpha(1-\tau) + \dots] + B(\tau) e^{-i\omega_v t} [1 + \beta(1-\tau) + \dots]$$

$$\frac{dX}{d\tau} = 0 \quad \frac{dA(\tau)}{d\tau} = \alpha A(\tau) \quad ; \quad \frac{dB(\tau)}{d\tau} = -\beta B(\tau) = 0$$

$$x(t) = A_0 e^{-\pi t} e^{i(\omega_0 + \delta_R)t}$$

$$\dot{x}(t) = A_0 e^{-\pi t} e^{i(\omega_0 + \delta_R)t} + B_0 e^{\pi t} e^{i(\omega_0 + \delta_R)t}$$

$$X(t) = A_0 e^{-\gamma t} e^{i(\omega_0 + \delta_R)t} + B_0 e^{-\gamma t} e^{i(\omega_0 + \delta_R)t}$$

$$ds^2 = \frac{1}{(f(r))^2} (dt^2 + d\vec{x}^2)$$

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\int

$$ds^2 = \frac{1}{(4\pi)^2} (dn^2 + d\vec{x}^2) \quad \int dn d^3x \mathcal{L}[\chi]$$

$$\mathcal{L}[\chi] = \frac{\dot{\chi}^2}{2} + \frac{(\nabla\chi)^2}{2} - \frac{M^2(m)}{2} \chi^2 - \frac{1}{2} \delta M^2(m) \chi^2 - \frac{1}{4\pi} \tilde{g}(m) \chi^3$$

$$ds^2 = \frac{1}{(\hbar m)^2} (dn^2 + d\vec{x}^2)$$

$$\int dn d^3x \mathcal{L}[\chi]$$

$$\mathcal{L}[\chi] = \frac{\dot{\chi}^2}{2} + \frac{(\nabla \chi)^2}{2} - \frac{M^2(n)}{2} \chi^2 - \frac{1}{2} \delta M^2(n) \chi^2 - \frac{1}{\hbar m} \tilde{g}(n) \chi^3 - \lambda \chi^4$$

$$M^2(n) = c^2 M^2 - \frac{c''}{c} = \frac{1}{\hbar^2} (\dot{\chi}^2 - 1/4)$$



$$ds^2 = \frac{1}{(f(n))^2} (dn^2 + dx^2)$$

$$\int dn d^3x \mathcal{L}[X]$$

$$\mathcal{L}[X] = \frac{\dot{X}^2}{2} + \frac{(X')^2}{2} - \frac{M^2(n)}{2} X^2 - \frac{1}{2} \delta M^2(n) X^2 - \underbrace{\frac{1}{4n}}_{\sim} g(n) X^3 - \lambda X^4$$

$$M^2(n) = c^2 M^2 - \frac{c'}{c} \frac{1}{12} (v^2 - 1/4)$$

$$\Delta = \frac{3}{2} - \nu$$

$$ds^2 = \frac{1}{(4\pi)^2} [dn^2 + d\vec{x}^2]$$

$$\int dn d^3x \mathcal{L}[\chi]$$

$$\mathcal{L}[\chi] = \frac{\dot{\chi}^2}{2} + \left(\frac{\nabla \chi}{2}\right)^2 - \frac{M^2(n)}{2} \chi^2 - \frac{1}{2} \delta M^2(n) \chi^2 - \frac{1}{4\pi} \tilde{g}(n) \chi^3 - \lambda \chi^4$$

$$M^2(n) = c^2 M^2 - \frac{c''}{c} = \frac{1}{4} (v^2 - 1/4)$$

$$\Delta = \frac{3}{2} - \nu \Rightarrow \text{IR}$$

$$ds^2 = \frac{1}{(4m)^2} (dn^2 + d\vec{x}^2) \quad \int dn d^3x \mathcal{L}[\chi]$$

$$\mathcal{L}[\chi] = \frac{\dot{\chi}^2}{2} + \frac{(\nabla\chi)^2}{2} - \frac{M^2(n)}{2} \chi^2 - \frac{1}{2} \delta M^2(n) \chi^2 - \underbrace{\frac{1}{4m}}_{\sim} g(n) \chi^3 - \lambda \chi^4$$

$$M^2(n) = c^2 M^2 - \frac{c''}{c} = \frac{1}{4} (\dot{\chi}^2 - 1/4)$$

$$\Delta = \frac{3}{2} - \nu \Rightarrow \text{IR} \approx \frac{M^2}{H^2} < 1$$

$$ds^2 = \frac{1}{(4m)^2} (dn^2 + d\vec{x}^2) \quad \int dn d^3x \mathcal{L}[\chi]$$

$$\mathcal{L}[\chi] = \frac{\dot{\chi}^2}{2} + \frac{(\nabla \chi)^2}{2} - \frac{M^2(n)}{2} \chi^2 - \frac{1}{2} \delta M^2(n) \chi^2 - \underbrace{\frac{1}{4m}}_{\sim} g(n) \chi^3 - \lambda \chi^4$$

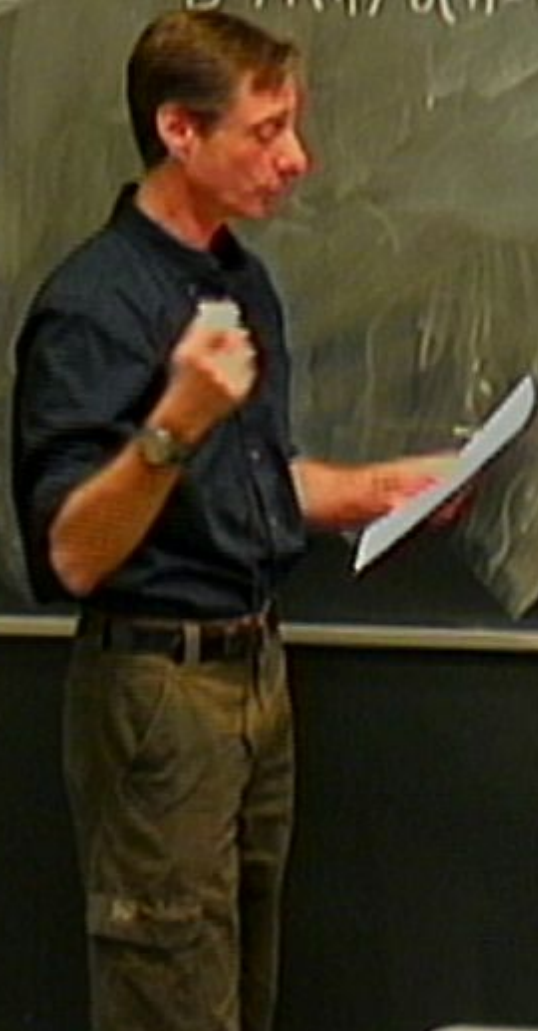
$$M^2(n) = c^2 M^2 - \frac{c''}{c} = \frac{1}{4} (v^2 - 1/4)$$

$$\Delta = \frac{3}{2} - \nu \Rightarrow \text{IR} \approx \frac{M^2}{H^2} < 1$$

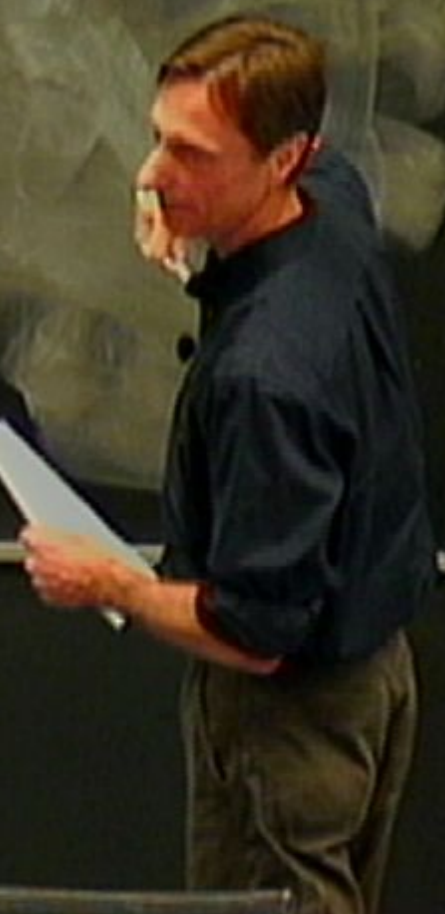
$$X''_k + \left(k^2 - \frac{V^2 - \frac{1}{4}}{\eta^2} \right) X_k + \int_{\eta_0}^{\eta} \Sigma(\epsilon, \eta, \eta') X_k(\eta') d\eta' = 0$$

$$\chi''_k + \left(k^2 - \left(\frac{V^2}{4} \right) \right) \chi_k + \int_{\eta_0}^{\eta} \Sigma(k; \eta, \eta') \chi_k(\eta') d\eta' = 0$$

$$\Sigma = S M^2(\eta) \delta(\eta - \eta') + Q$$



$$\begin{aligned}
 & \chi''_k + \left(k^2 - \left(\frac{N^2}{4} \right) \right) \chi_k + \int_{\eta_0}^{\eta} \Sigma(\epsilon, \eta, \eta') \chi_k(\eta') d\eta' = 0 \\
 & \Sigma = S M^2(\eta) \delta(\eta - \eta') + \text{loop} + \text{loop}
 \end{aligned}$$



$$\chi''_k + \left(k^2 - \frac{N^2}{4}\right) \chi_k + \int_{\eta_0}^{\eta} \Sigma(k; \eta, \eta') \chi_k(\eta') d\eta' = 0$$

$$\Sigma = S M^2(\eta) \delta(\eta - \eta') + \text{loop} \delta(\eta - \eta') + \text{bubble}$$

$$\text{loop} = \frac{d^2}{d\eta^2} \left[\frac{1}{k^2} + \dots \right]$$

$$\chi''_k + \left(k^2 - \left(N^2 - \frac{1}{4}\right)\right) \chi_k + \int_{\eta_0}^{\eta} \Sigma(k; \eta, \eta') \chi_k(\eta') d\eta' = 0$$

$$\Sigma = S M^2(\eta) \delta(\eta - \eta') + \text{loop} \delta(\eta - \eta') + \text{self-energy}$$

$$\text{loop} = \frac{d}{d\eta} \left(\frac{1}{H^2 \eta^2} \right)$$

$$\chi''_k + \left(k^2 - \frac{N^2}{4} \right) \chi_k + \int_{\eta_0}^{\eta} \Sigma(k; \eta, \eta') \chi_k(\eta') d\eta' = 0$$

$$\Sigma = S M^2(\eta) \delta(\eta - \eta') + \text{loop} \delta(\eta - \eta') + \text{self-energy}$$

$$\text{loop} \frac{d}{d\eta} \left(\frac{1}{H^2 \eta^2} \right) [\Lambda^2 + \ln \Lambda] + \frac{1}{\Delta}$$

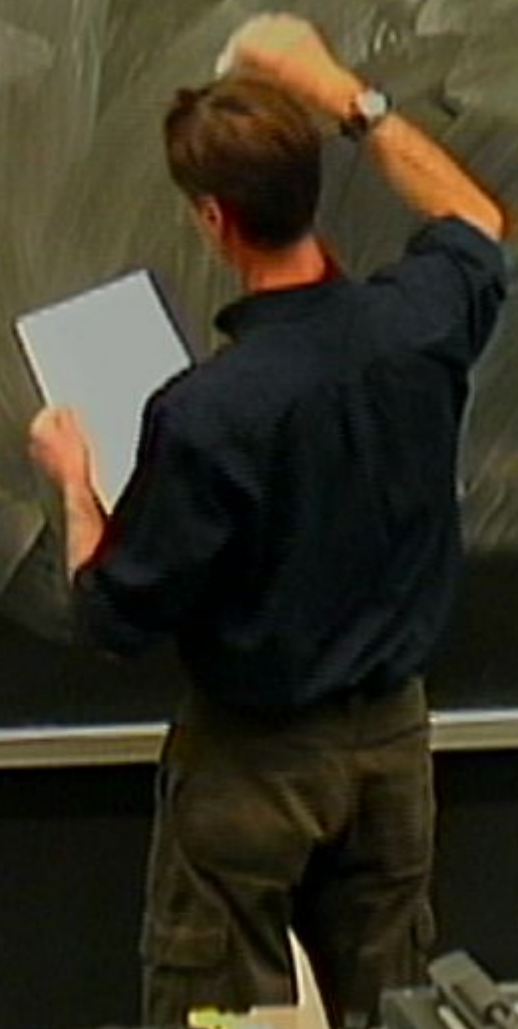
$$\chi_k'' + \left(k^2 - \frac{V^2}{4} \right) \chi_k + \int_{\eta_0}^{\eta} \Sigma(\epsilon, \eta, \eta') \chi_k(\eta') d\eta' = 0$$

$$\Sigma = S M^2(\eta) \delta(\eta - \eta') + \text{loop} \delta(\eta - \eta') + \text{self-energy}$$

$$\text{loop} \sim \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} [\Lambda^2 + \ln \Lambda] + \frac{1}{\Delta} + (\text{g.p.} + \text{v.v. fin.})$$

$$k \rightarrow 0$$

$$X_i = A \gamma_i^{\beta_1} d^+ \ln\left(\frac{\gamma_i}{n_0}\right) - B \gamma_i^{\beta_2}$$



$$X_1 = A \eta^{\beta_1} d^+ \ln\left(\frac{\eta}{n_0}\right) - B \eta^{\beta_2} d^- \ln\left(\frac{\eta}{n_0}\right)$$

$$p^{\pm} = \frac{1}{2} \pm v$$

$$X_1 = A \gamma^{(3)} d^+ \ln\left(\frac{\gamma}{n_0}\right) - B \gamma^{(3)} d^- \ln\left(\frac{\gamma}{n_0}\right)$$

$$p^{\pm} = \frac{1}{2} \pm v$$

$$d^+ \approx \frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln(4\lambda\Delta)} \frac{g^2}{c} \right]$$

$$d^- = -\frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln(4\lambda\Delta)} \frac{g^2}{c} \right]$$

$$X_1 = A \eta^{\beta_1} d^+ \ln\left(\frac{\eta}{n_0}\right) - B \eta^{\beta_2} d^- \ln\left(\frac{\eta}{n_0}\right)$$

$$\rho^{\pm} = \frac{1}{2} \pm v$$

$$d^+ \approx \frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{2} \frac{\omega^2}{\Delta} \right]$$

$$d^- = -\frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{2} \frac{\omega^2}{\Delta} \right]$$

$$X_0 = A \eta^{\beta_1} + B \eta^{\beta_2}$$

$$X_1 = A \eta^{\beta_1} d^+ \ln \left(\frac{\eta}{\eta_0} \right) - B \eta^{\beta_2} d^- \ln \left(\frac{\eta}{\eta_0} \right)$$

$$\rho^{\pm} = \frac{1}{2} \pm v$$

$$d^+ \approx \frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln^2(1/\Delta)} \right]$$

$$d^- = -\frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln^2(1/\Delta)} \right]$$

$$X_0 = A \eta^{\beta_1} + B \eta^{\beta_2}$$



$$X_1 = A \eta^{\beta_1} d^+ \ln\left(\frac{\eta}{\eta_0}\right) - B \eta^{\beta_2} d^- \ln\left(\frac{\eta}{\eta_0}\right)$$

$$p^{\pm} = \frac{1}{2} \pm v$$

$$d^+ \approx \frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln(11\lambda)} \right]$$

$$d^- = -\frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{2v} \frac{g^2}{C} \right]$$

$$X_0 = A \eta^{\beta_1} + B \eta^{\beta_2}$$

$$X_1 = A \eta^{\beta_1} d^+ \ln\left(\frac{\eta}{n_0}\right) - B \eta^{\beta_2} d^- \ln\left(\frac{\eta}{n_0}\right)$$

$$p^{\pm} = \frac{1}{2} \pm v$$

$$d^+ \approx \frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln(11\Delta)} \frac{g^2}{c} \right]$$

$$d^- = -\frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{\ln(11\Delta)} \frac{g^2}{c} \right]$$

$$X_0 = A \eta^{\beta_1} + B \eta^{\beta_2}$$

$$X(\eta) = \left(\frac{\eta}{n_0}\right)^p \left[A_0 \left(\frac{\eta}{n_0}\right)^{\beta_2} + B \right]$$

$$X_1 = A \eta^{\beta_+} d^+ \ln\left(\frac{\eta}{\eta_0}\right) - B \eta^{\beta_-} d^- \ln\left(\frac{\eta}{\eta_0}\right)$$

$$\rho^{\pm} = \frac{1}{2} \pm v$$

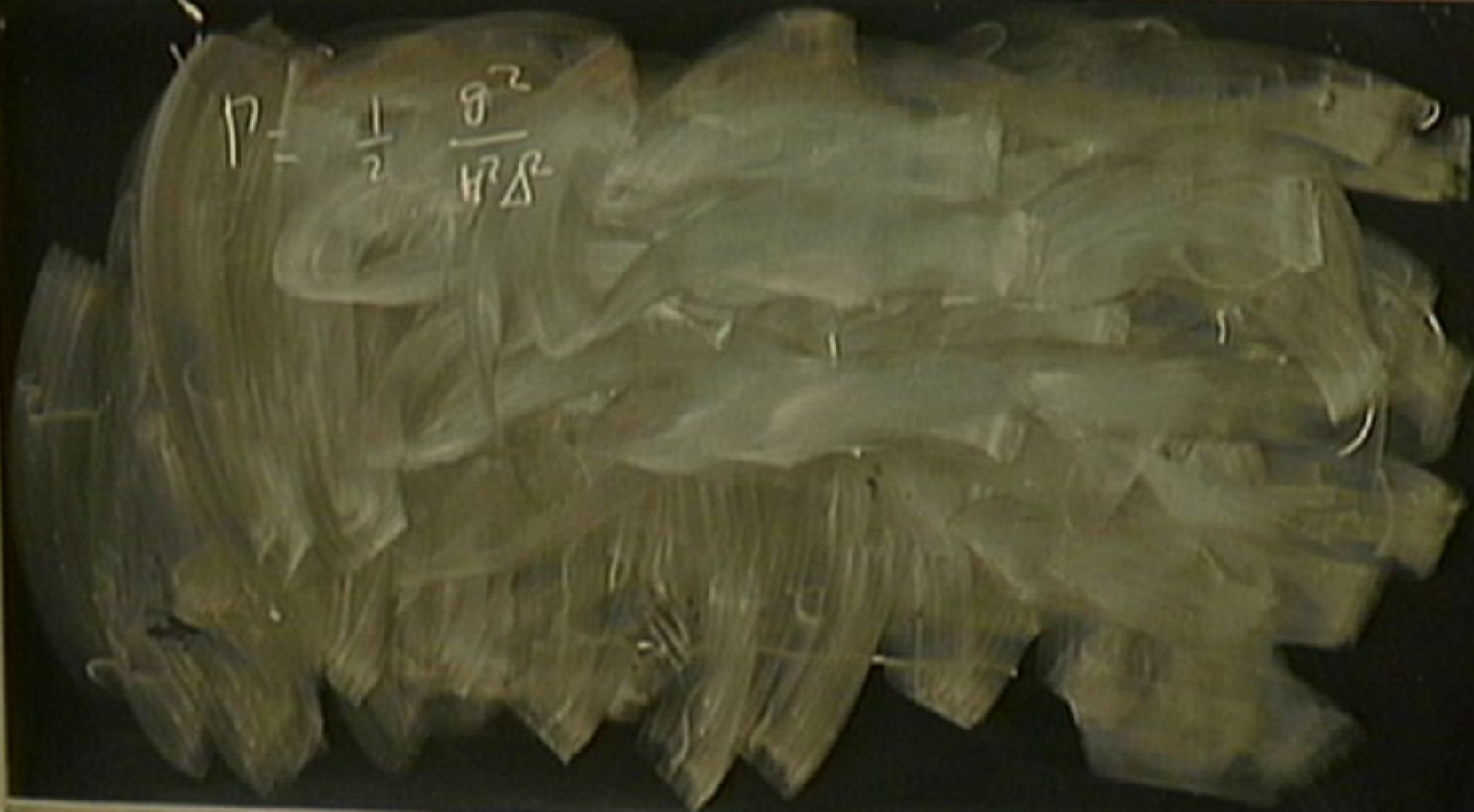
$$d^+ \approx \frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{6\pi^2} \frac{g^2}{(H\Delta)} \right]$$

$$d^- = -\frac{1}{2v} \left[3 \frac{\lambda}{\Delta} + \frac{1}{6\pi^2} \frac{g^2}{(H)} \right]$$

$$X_0 = A \eta^{\beta_+} + B \eta^{\beta_-}$$

$$X(\eta) = \left(\frac{\eta}{\eta_0}\right)^{\beta_+} \left[A_0 \left(\frac{\eta}{\eta_0}\right)^{\beta_-} + B_0 \left(\frac{\eta}{\eta_0}\right)^{\beta_+} \right]$$

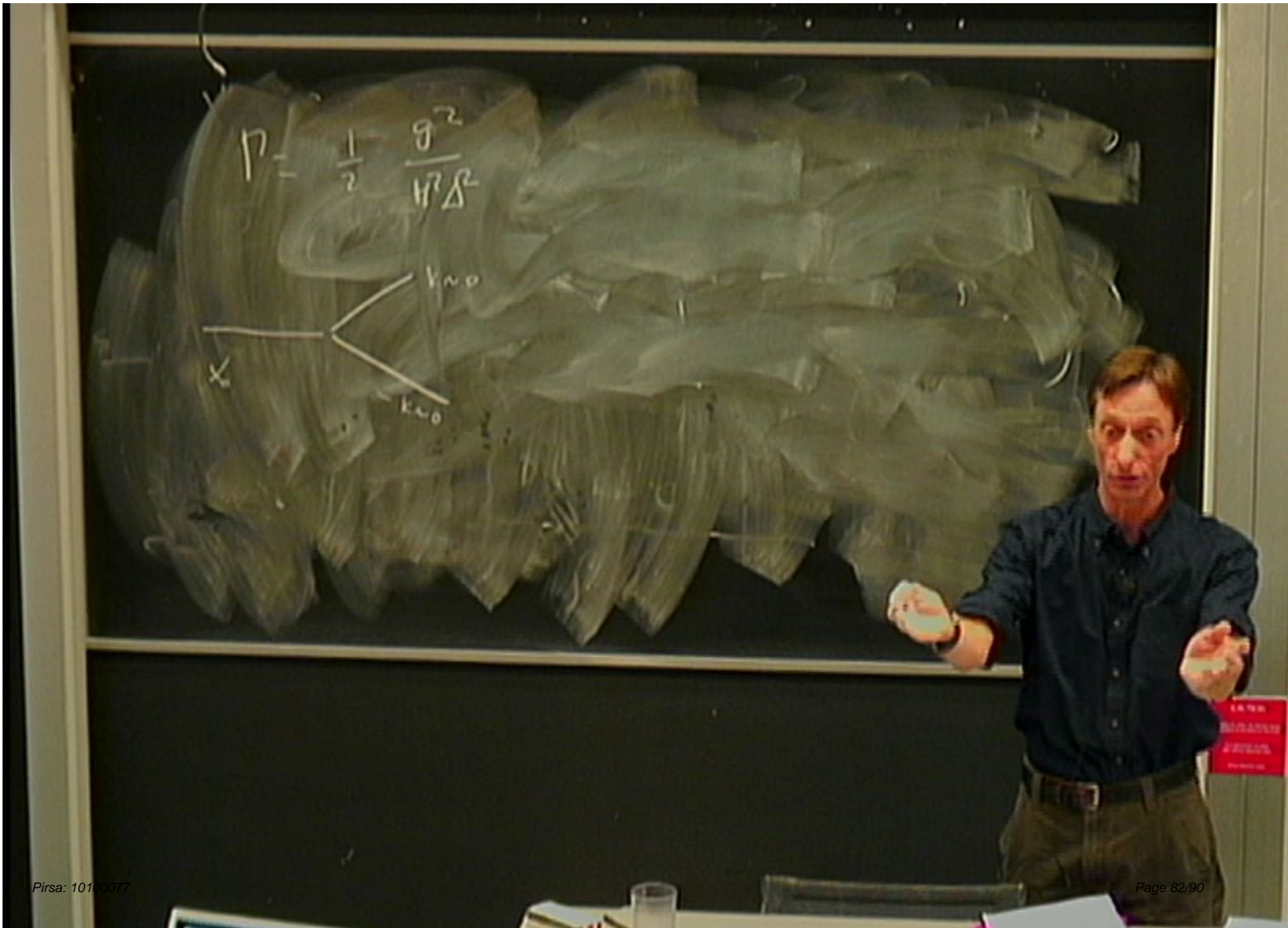




$$p = \frac{1}{2} \frac{g^2}{4\pi R^2}$$

$$\eta = \frac{1}{2} \frac{g^2}{H^2 \Delta^2}$$

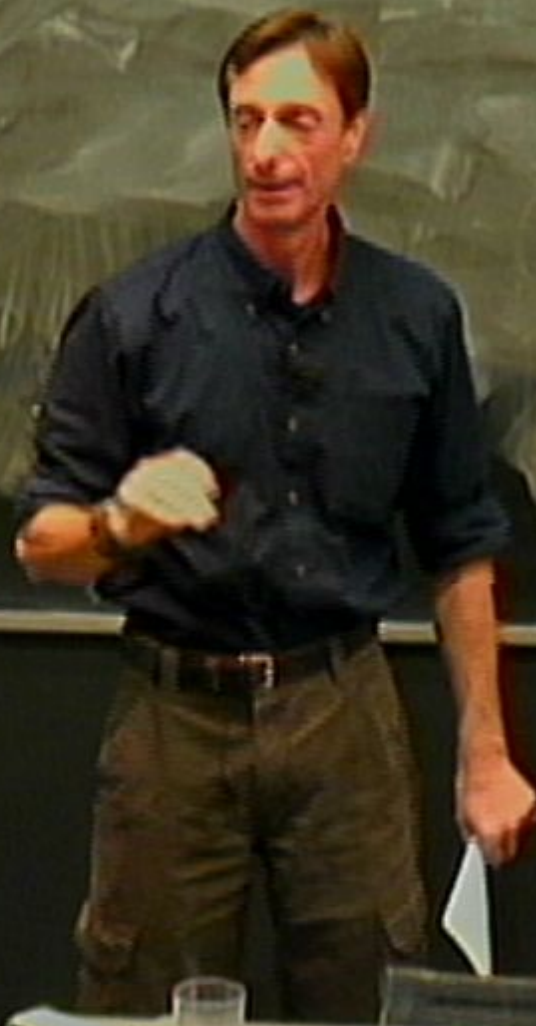




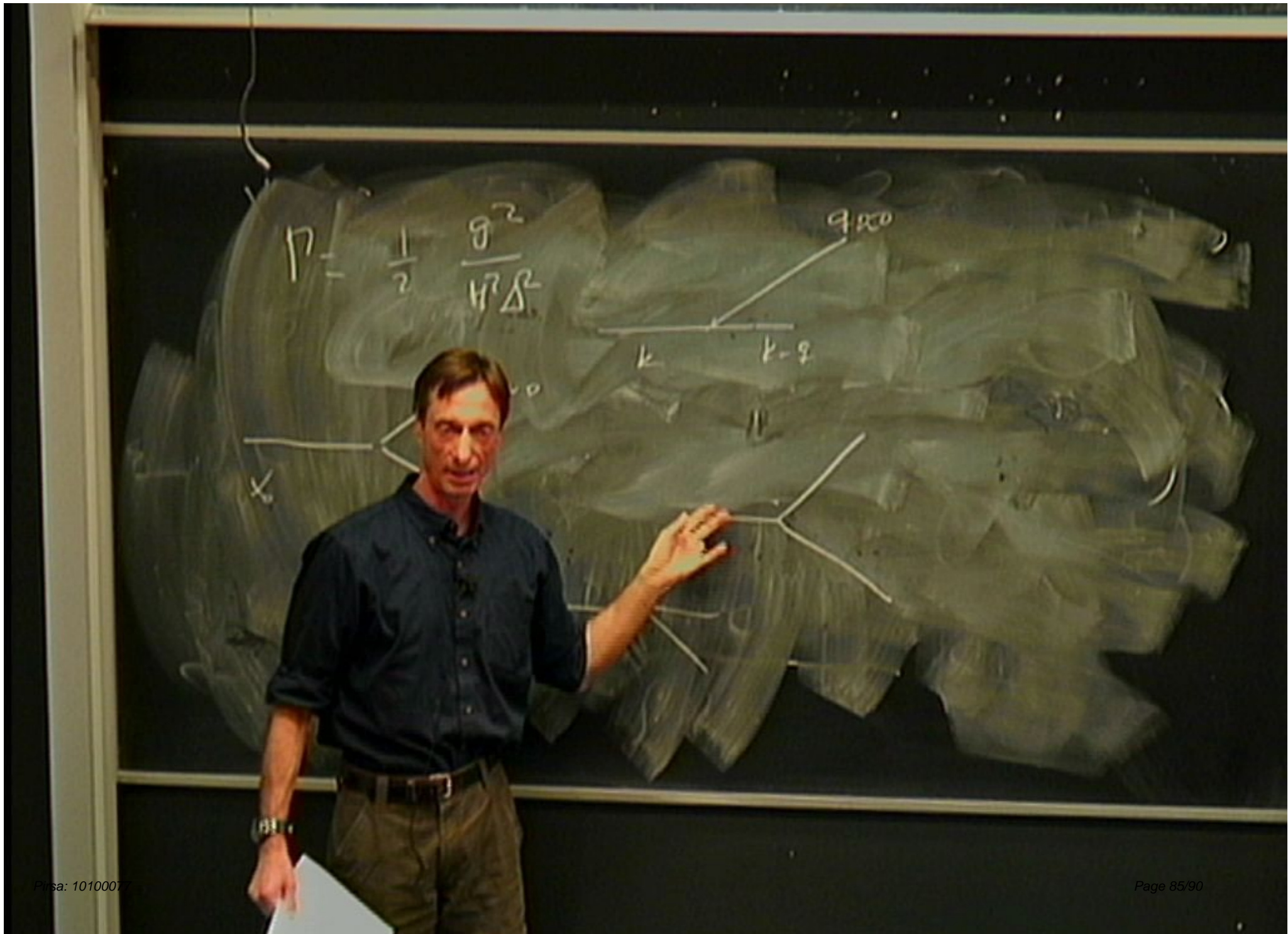
$$\rho = \frac{1}{2} \frac{g^2}{4\pi^2 \Delta^2}$$



$$\Gamma = \frac{1}{2} \frac{g^2}{4\pi^2 \Delta^2}$$







$$p = \frac{1}{2} \frac{g^2}{4\pi\Delta^2}$$



$$P = \frac{1}{2} \frac{g^2}{4\pi\Delta^2}$$

$$Q \sim \frac{1}{\Delta}$$

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$$\lambda \phi^4 \rightarrow \lambda \phi^2 \langle \phi^2 \rangle + \lambda (\phi^4 - \langle \phi^2 \rangle^2)$$

$$P = \frac{1}{2} \frac{g^2}{4\pi\Delta^2}$$

$$O \sim \frac{1}{\Delta}$$

$$\lambda \phi^4 \rightarrow \lambda \phi^2 \langle \phi^2 \rangle + \lambda (\phi^4 - \langle \phi^2 \rangle \phi^2)$$

$$\langle \phi^2 \rangle = 0 = \frac{1}{M}$$

$$\langle \phi^2 \rangle = \frac{1}{K} \quad 1 \langle \phi^2 \rangle$$

$$P = \frac{1}{2} \frac{g^2}{4\pi\Delta^2}$$

$$Q \sim \frac{1}{\Delta}$$

$$\lambda \phi^4 \rightarrow \lambda \phi^2 \langle \phi^2 \rangle + \lambda (\phi^4 - \langle \phi^2 \rangle \phi^2)$$

$$\langle \phi^2 \rangle = 0 = \frac{1}{M}$$

$$\langle \phi^2 \rangle = \frac{1}{15}$$