

Title: Massless fields in Euclidean de Sitter space

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URL: <http://pirsa.org/10100076>

Abstract: In theories with light or massless fields, loop diagrams can develop infrared divergences. We demonstrate how these can be resolved for a theory of massless interacting scalar fields in Euclidean de Sitter space. We also comment on applications to the in-in formalism in Lorentzian de Sitter space.





Rajaraman, arxiv: 1008.1271



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Massless scalar fields in (Euclidean) de Sitter

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$$\mathcal{L} = \partial\phi^2$$

Massless scalar fields in (Euclidean) de Sitter

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$$= (\partial\phi)^2 - m^2$$

Massless scalar fields in (Euclidean) de Sitter

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$$= (\partial\phi)^2 - m^2\phi^2$$

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$$\mathcal{L} = (\partial\phi)^2 - m^2\phi^2$$



Massless scalar fields in (Euclidean) de Sitter

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$$\mathcal{L} = (\partial\phi)^2 - m^2\phi^2 - \lambda\phi^4$$

$$m^2 = 0$$

Want to do in dS.



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$$\mathcal{L} = (\partial\phi)^2 - m^2\phi^2 - \lambda\phi^4$$

$$m^2 = 0$$

to do in  $dS^D$

Will use Euclidean continuation to  $S^D$



$$m^2 = 0$$

Want to do in  $dS^D$

Will use Euclidean continuation to  $S^D$

$$\mathcal{L} = (\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4$$



Expand in spherical harmonics

$$\phi(x) = \sum_{l=0}^{\infty} \phi_l Y_l(x)$$

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$$\phi(x) = \sum_L \phi_L Y_L(x)$$

$$\nabla^2 Y_L = -L(L+d) Y_L \quad d = D-1$$

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Find correlation functions

$$\langle \phi(x) \phi(y) \rangle = \frac{\int D\phi (\phi(x) \phi(y)) e^{-S}}{\int D\phi e^{-S}}$$

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$$\langle \phi(x) \phi(y) \rangle = \frac{\int D\phi (\phi(x) \phi(y) e^{-S}}{\int D\phi e^{-S}}$$

Break up action into leading + perturbation

$$S_0 = \int (\partial\phi)^2 + m^2\phi^2$$

$$S_p = \int \lambda\phi^4$$

Break up action into leading + perturbation.

$$S_0 = \int (\partial\phi)^2 + m^2\phi^2 = \sum_{\vec{l}} L(L+d)\phi_{\vec{l}}^2$$

$$S_p = \int \lambda\phi^4$$

Break up action into leading + perturbation.

$$S_0 = \int (\partial\phi)^2 + m^2\phi^2 = \sum_{\vec{l}} [L(L+d) + m^2] \phi_{\vec{l}}^2$$

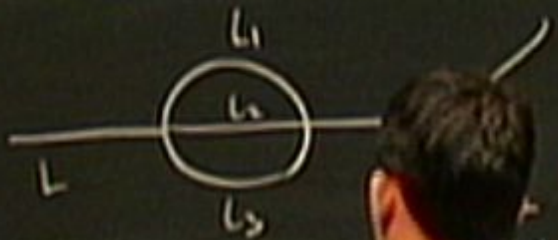
$$S_p = \int \lambda\phi^4$$

$$\langle \phi(x) \phi(y) \rangle_0 = \frac{\int \mathcal{D}\phi \phi(x) \phi(y) e^{-S}}{\int \mathcal{D}\phi e^{-S}} = \frac{Y_L^m(x) Y_L(y)}{L(L+d) + m^2}$$

$$\langle \phi(y) \rangle_0 = \frac{\int \mathcal{D}\phi \phi(x) \phi(y) e^{-S_0}}{\int \mathcal{D}\phi e^{-S_0}} = \frac{Y_L}{L}$$

$$\langle \phi_L \phi_L \rangle = \frac{1}{L(L+d) + m^2}$$





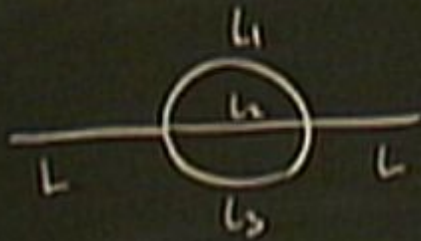
$$\frac{1}{L_1(L_1+d)+m^2} \cdot \frac{1}{L_2(L_2+d)+m^2} \cdot \frac{1}{L_3(L_3+d)+m^2}$$

SAFETY  
 INFORMATION  
 WARNING

SAFETY  
 INFORMATION  
 WARNING

$$\sum_{L_1, L_2, L_3} \frac{1}{L_1 + d + m^2} + \frac{1}{d + m^2} + \frac{1}{L_3(L_3 + d) + m^2}$$

$$\text{If } m^2 \rightarrow 0$$

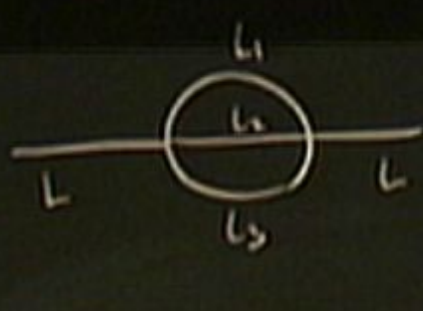


$$\sum_{L_1, L_2, L_3} \frac{1/\lambda^2}{L_1(L_1+d)+m^2} \cdot \frac{1}{L_2(L_2+d)+m^2} \cdot \frac{1}{L_3(L_3+d)+m^2}$$

If  $m^2 \rightarrow 0$  . divergent due to  $\tilde{L}=0$

$$\int \delta p e^{-S_0} \quad L((L+d) + m^2)$$

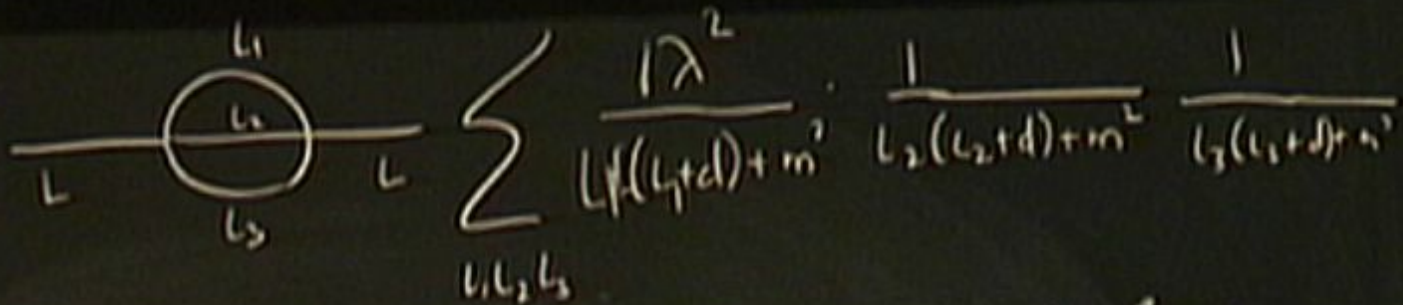
$$\langle \phi_L \phi_L \rangle = \frac{1}{L((L+d) + m^2)}$$



$$\sum_{L_1, L_2, L_3} \frac{1/\lambda^2}{L_1((L_1+d) + m^2)} \cdot \frac{1}{L_2((L_2+d) + m^2)} \cdot \frac{1}{L_3((L_3+d) + m^2)}$$

If  $m^2 \rightarrow 0$  . divergent due to  $L$

$$\langle \phi_L | \phi_L \rangle = \frac{\int \phi e^{-\dots}}{L(L+d)+m^2}$$



The diagram shows a circle divided into three segments labeled  $L_1$ ,  $L_2$ , and  $L_3$ . A horizontal line with segments labeled  $L$  passes through the center of the circle. To the right of the diagram is a summation formula:

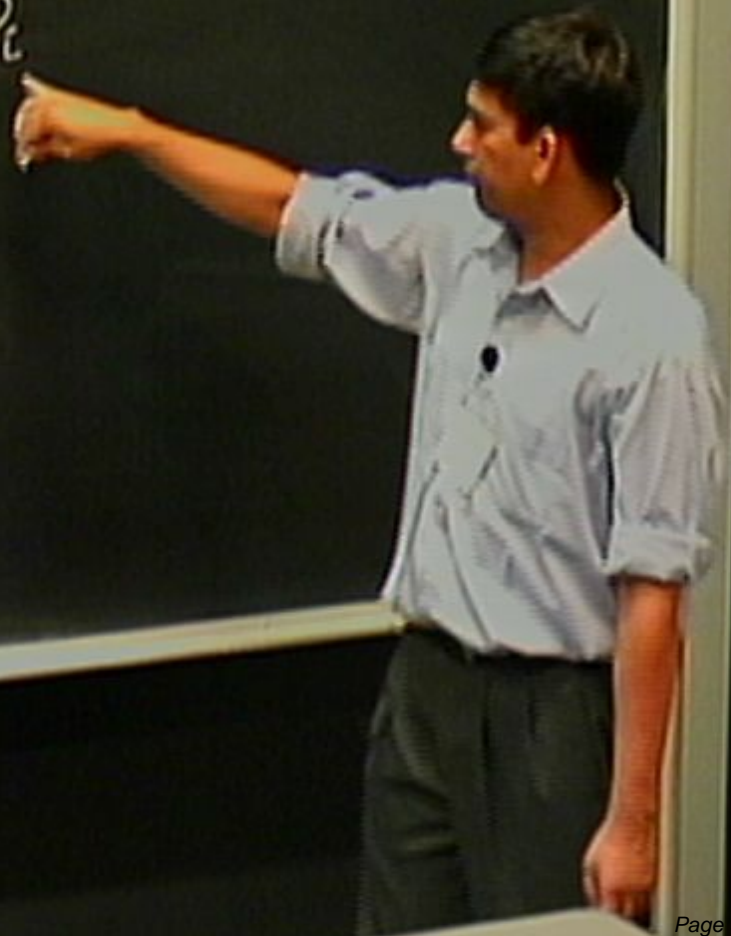
$$\sum_{L_1, L_2, L_3} \frac{1/\lambda^2}{L_1(L_1+d)+m^2} \cdot \frac{1}{L_2(L_2+d)+m^2} \cdot \frac{1}{L_3(L_3+d)+m^2}$$

If  $m^2 \rightarrow 0$  . divergent due to  $\tilde{L}=0$

Reason:  $\phi_i$  no longer appears in  $S_i$

Need to include some interaction terms in  $S_0$

$$S_0 = \sum L(L+d) \phi_c^2$$



Need to include some interaction terms in  $S_0$

$$S_0 = \sum L(L+d) \phi_L^2 + \lambda \phi_0^4$$



Need to include some interaction terms in  $S_0$

$$S_0 = \sum L(L+d) \phi_L^2 + \lambda \phi_0^4$$

$$S_p = \lambda \phi_0^2 \phi_L^2 + \lambda \phi_0 \phi_L^3 + \lambda \phi_L^4$$



Need to include some interaction terms in  $S_0$

$$S_0 = \sum L(L+d) \phi_L^2 + \lambda \phi_0^4$$

$$S_p = \lambda \phi_0^2 \phi_L^2 + \lambda \phi_0 \phi_L^3 + \lambda \phi_0^4$$

$$\langle \phi(x) \phi(y) \rangle = \frac{\int \mathcal{D}\phi \phi(x) \phi(y) e^{-S}}{\int \mathcal{D}\phi e^{-S}}$$

$$= \gamma_0(\omega)\gamma_0(y) \frac{\int \mathcal{D}\phi_0 \phi_0^2 e^{-\lambda \phi_0^4}}{\int \mathcal{D}\phi_0 e^{-\lambda \phi_0^4}} + \sum_{l \geq 0} \frac{\gamma_l(\omega)\gamma_l(y)}{L(l+d)}$$

$$= Y_0'(x)Y_0(y) \frac{\int D\phi_0 \phi_0^2 e^{-\lambda \phi_0^4}}{\int D\phi_0 e^{-\lambda \phi_0^4}} + \sum_{l=2}^{\infty} \frac{Y_l(x)Y_l(y)}{l(l+d)}$$

$$= \frac{C_2}{\sqrt{\lambda}} Y_0'(x)Y_0(y) + \sum_{l=2}^{\infty} \frac{Y_l(x)Y_l(y)}{l(l+d)}$$

$$\int d^4x e^{-\lambda \phi^2} + \sum_{l=0}^{\infty} \frac{\gamma_l(x)\gamma_l(y)}{l(l+1)}$$

$$= \frac{c_2}{\sqrt{\lambda}} \gamma_0(x)\gamma_0(y) + \sum_{l=0}^{\infty} \frac{\gamma_l(x)\gamma_l(y)}{l(l+1)}$$

Field has develop

$$\text{mass}^2 = \frac{\sqrt{\lambda}}{c_2}$$





+



resums to a mass for  $\phi_0$



resums to a mass for  $\phi_0$

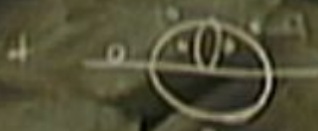


resurms to a mass for  $\phi_0$

What if we started with a finite mass for  $\phi_0$ ?

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EDUCATION  
#201504





Resumming a mass for  $\phi_0$

What if (with a finite mass for  $\phi$ )

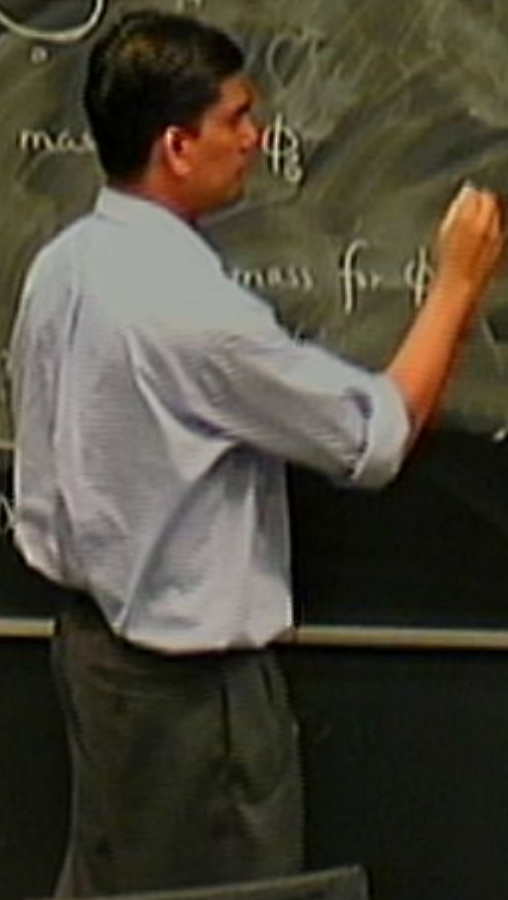
$$\langle \phi \rangle = \frac{\int \mathcal{D}\phi \phi^n e^{-\int d^4x (\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 + \lambda \phi^4)}}{\int \mathcal{D}\phi e^{-\int d^4x (\frac{1}{2} \partial_\mu \phi \partial^\mu \phi + m^2 \phi^2 + \lambda \phi^4)}}$$



resums to a mass  $\phi_0$

What if we started with mass for  $\phi$

$$\langle \phi_i | \phi_0 \rangle = \frac{\int D\phi \phi_i}{\int D\phi}$$



EXIT  
BUSINESS  
FLOOR



resums to a mass for  $\phi_0$

if we started with a finite mass for  $\phi \rightarrow$

$$\frac{\int D\phi \phi^2 e^{-\int (m^2 \phi^2 - \lambda \phi^4)}}{\int D\phi \dots}$$



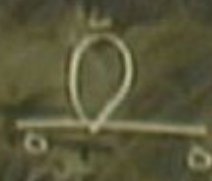
Resums a mass for  $\phi_0$

What if we add a finite mass for  $\phi$ ?

$$\langle \phi_1 | \phi_2 \rangle = \int \mathcal{D}\phi \exp(i \int d^4x \mathcal{L}(\phi))$$



CAUTION  
DO NOT TOUCH  
EQUIPMENT



resums to a mass for  $\phi_0$

What if we started with a finite mass for  $\phi$ ?

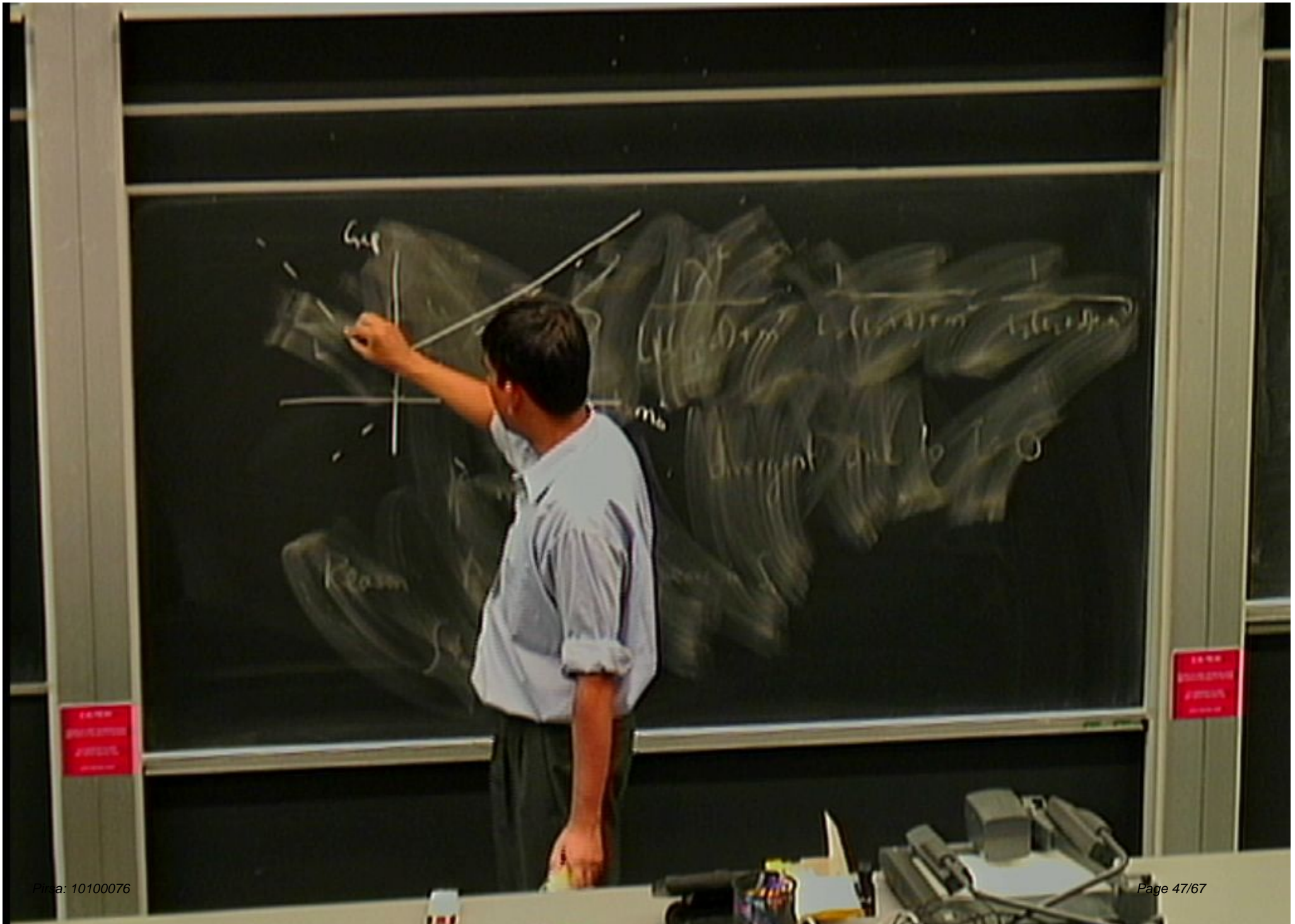
$$\langle \phi_0 \phi_0 \rangle = \frac{\int D\phi \phi_0^2 e^{-m^2 \phi^2 - \lambda \phi^4}}{\int D\phi \dots} \langle \delta \dots \rangle$$

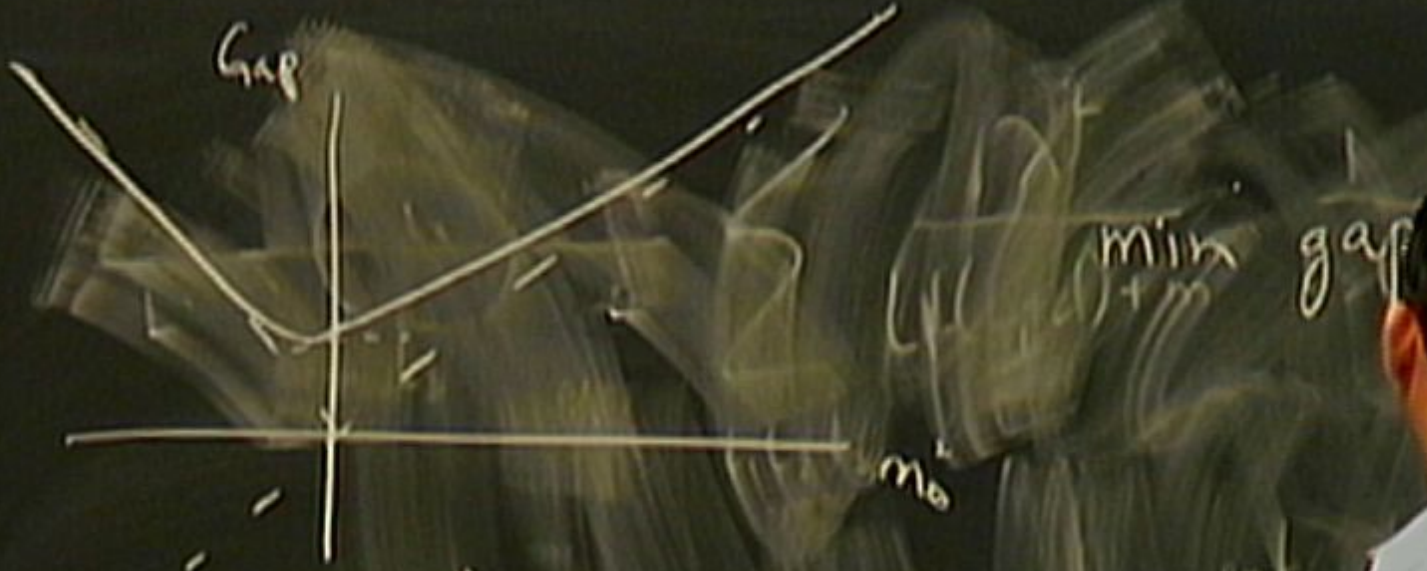


resurms to a mass for  $\phi_0$

What if we started with a finite mass for  $\phi$ ?

$$\langle \phi_i \phi_0 \rangle = \frac{\int D\phi \phi_i \phi_0 e^{i \int (m^2 \phi^2 - \lambda \phi^4)}}{\int D\phi \dots} \quad \langle \delta \rangle$$



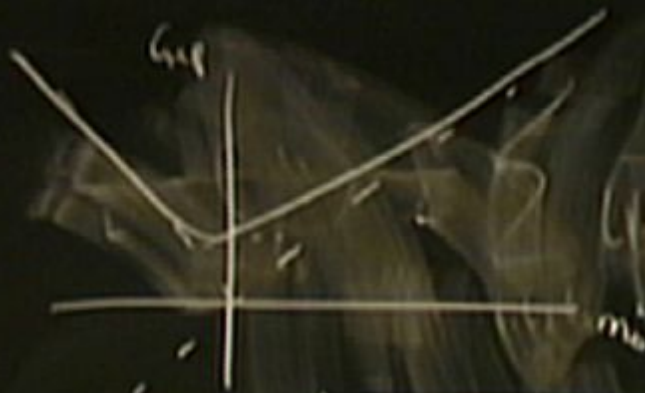


Reason

no

divergent

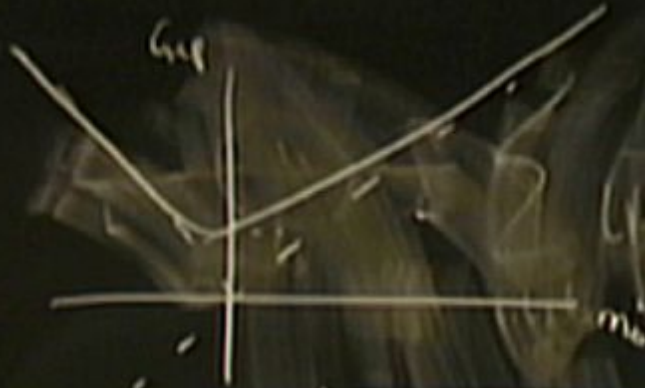




$$\min \text{ gap} \sim O(\lambda^{1/4})$$

$$\langle \phi^2 \rangle_{\text{EOM}} = \left( \langle \phi(x) \phi(y) \rangle_{\text{flat}} \Big|_{x=y=0} \right)$$



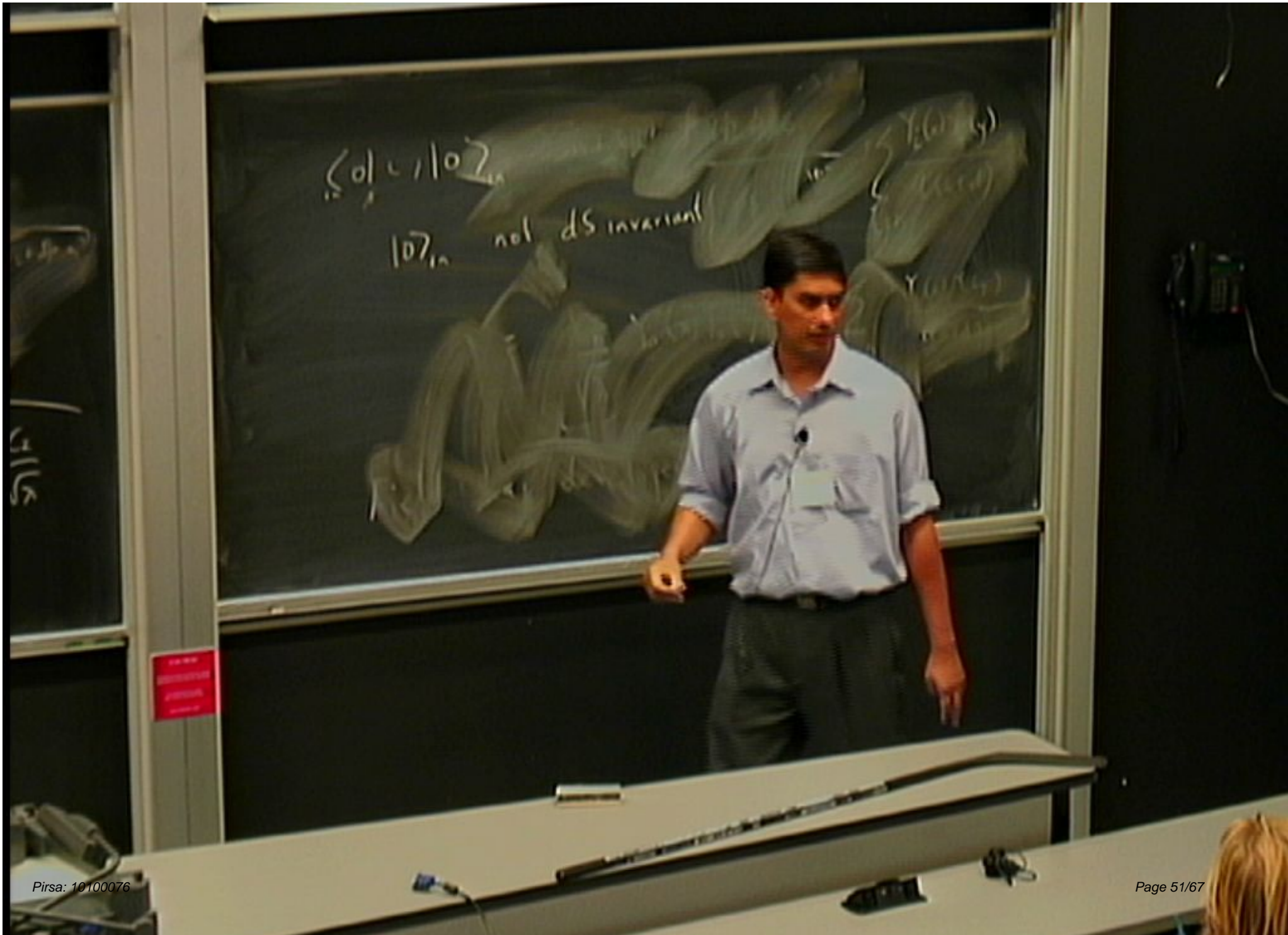


$$\min_{\text{gap}} \sim O(\lambda^{1/4})$$

$$\langle \phi^2 \rangle = \left( \langle \phi(x) \phi(y) \rangle_{\text{flat}} \right) \Big|_{x=y=0} \sim \frac{1}{\sqrt{\lambda}}$$

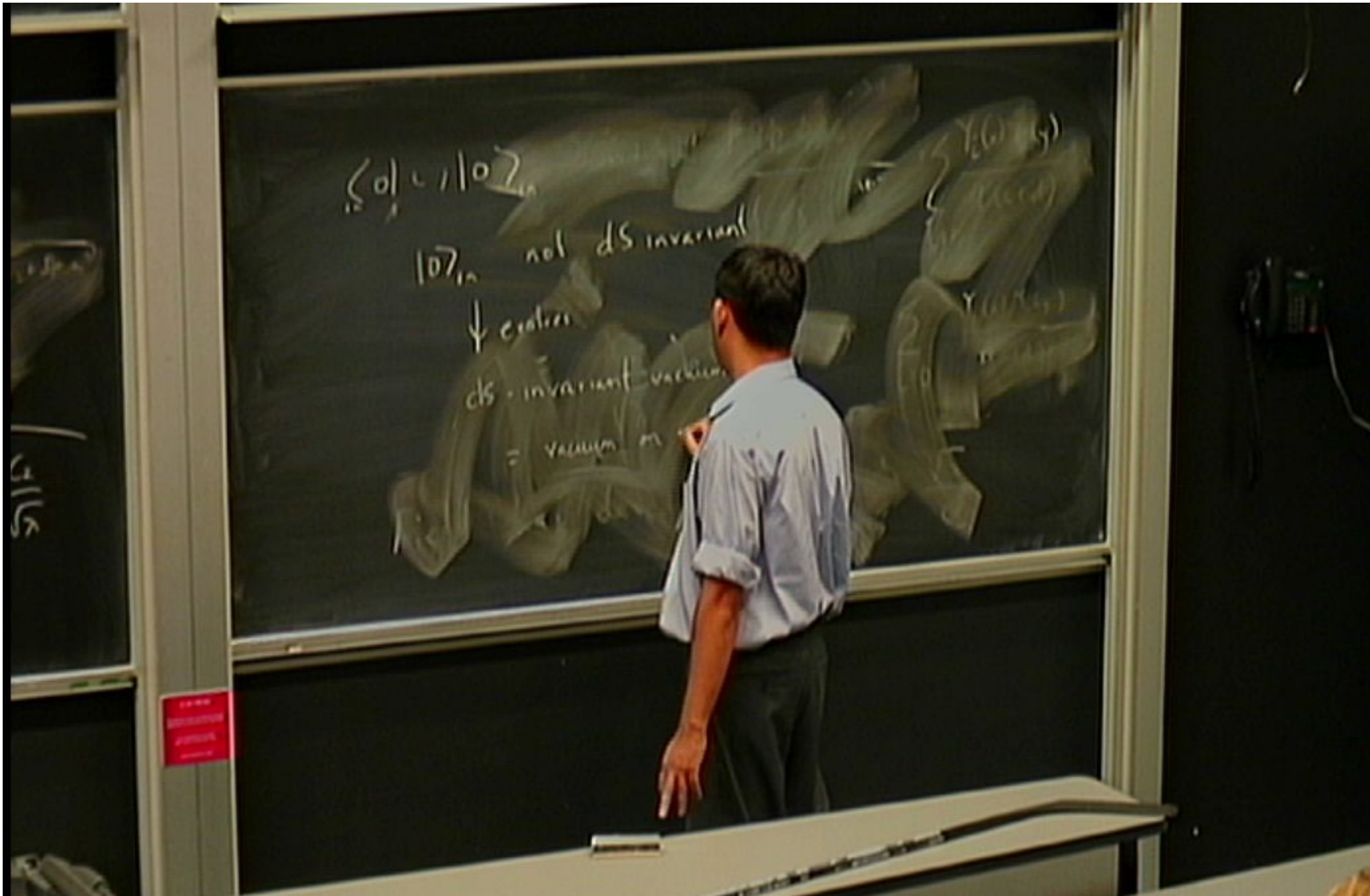
Red warning sticker on the left window frame.

Red warning sticker on the right window frame.



$\{0, 1, 10\}_{in}$

$107, in$  not dS invariant



$$\langle \sigma_1, \dots, \sigma_n \rangle$$

not ds invariant

↓ criteria

ds-invariant vacuum

= vacuum m

$$\langle \sigma_i | \sigma_j | \sigma_k \rangle$$

$|\sigma_i\rangle$  not dS invariant

↓ creates

dS-invariant vacuum

= vacuum on sphere

$$\langle \sigma_i | \sigma_j | \sigma_k \rangle$$

$|\sigma_i\rangle$  not dS invariant

↓ center

dS-invariant vacuum

= vacuum on sphere

$$\rightarrow \int \ln d$$

$$\langle \sigma_i | \psi \rangle$$

$\langle \sigma_i | \psi \rangle$  not dS invariant

↓ criteria

dS invariant vacuum

= vacuum on sphere

→  $\int \ln d$  should match  $\psi_{loc}$

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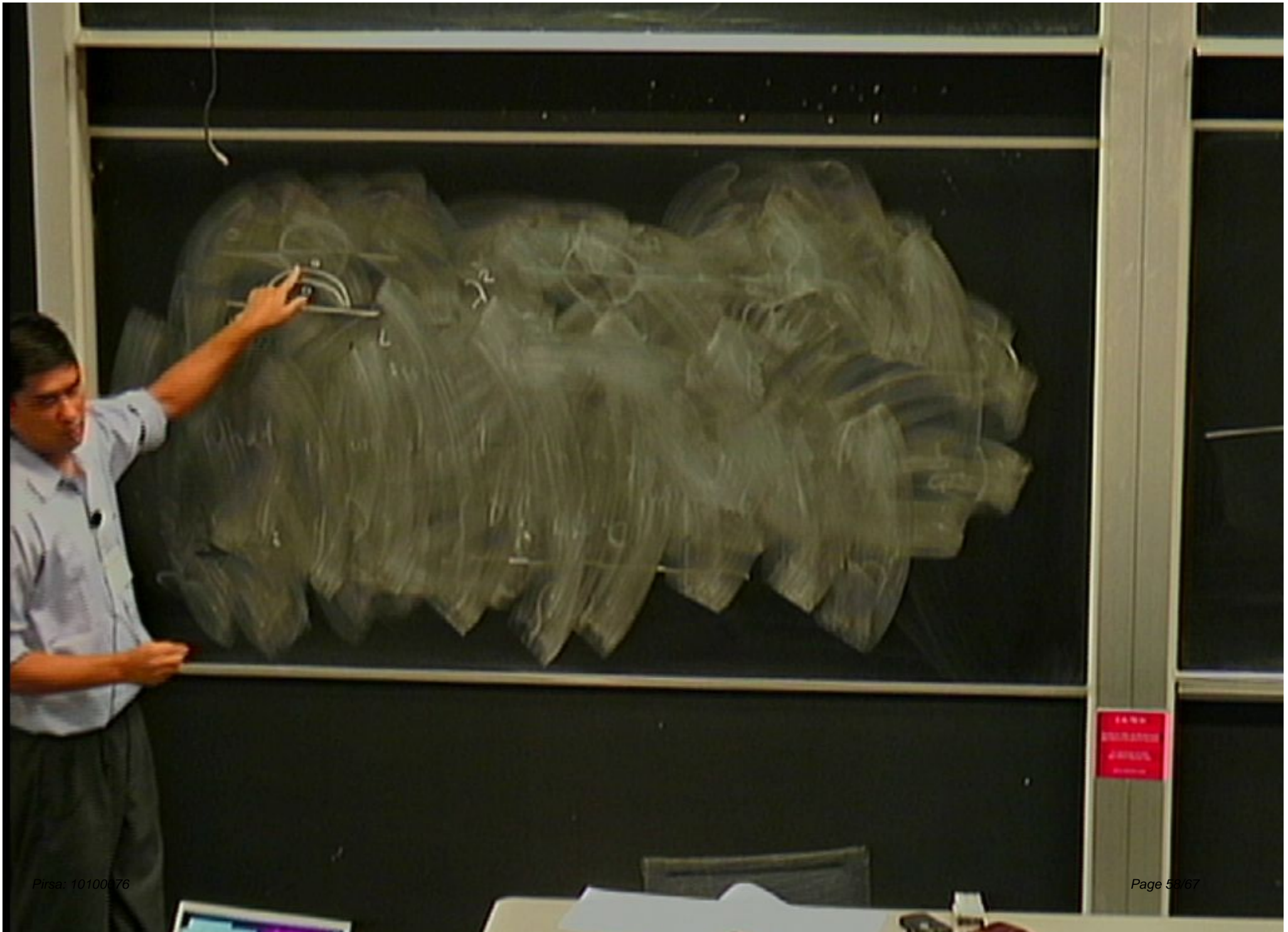
$$\langle \phi^{2n} \rangle_{\text{sym}} = \left( \frac{9}{\pi^2 \lambda} \right)^n \frac{\Gamma^2\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$$

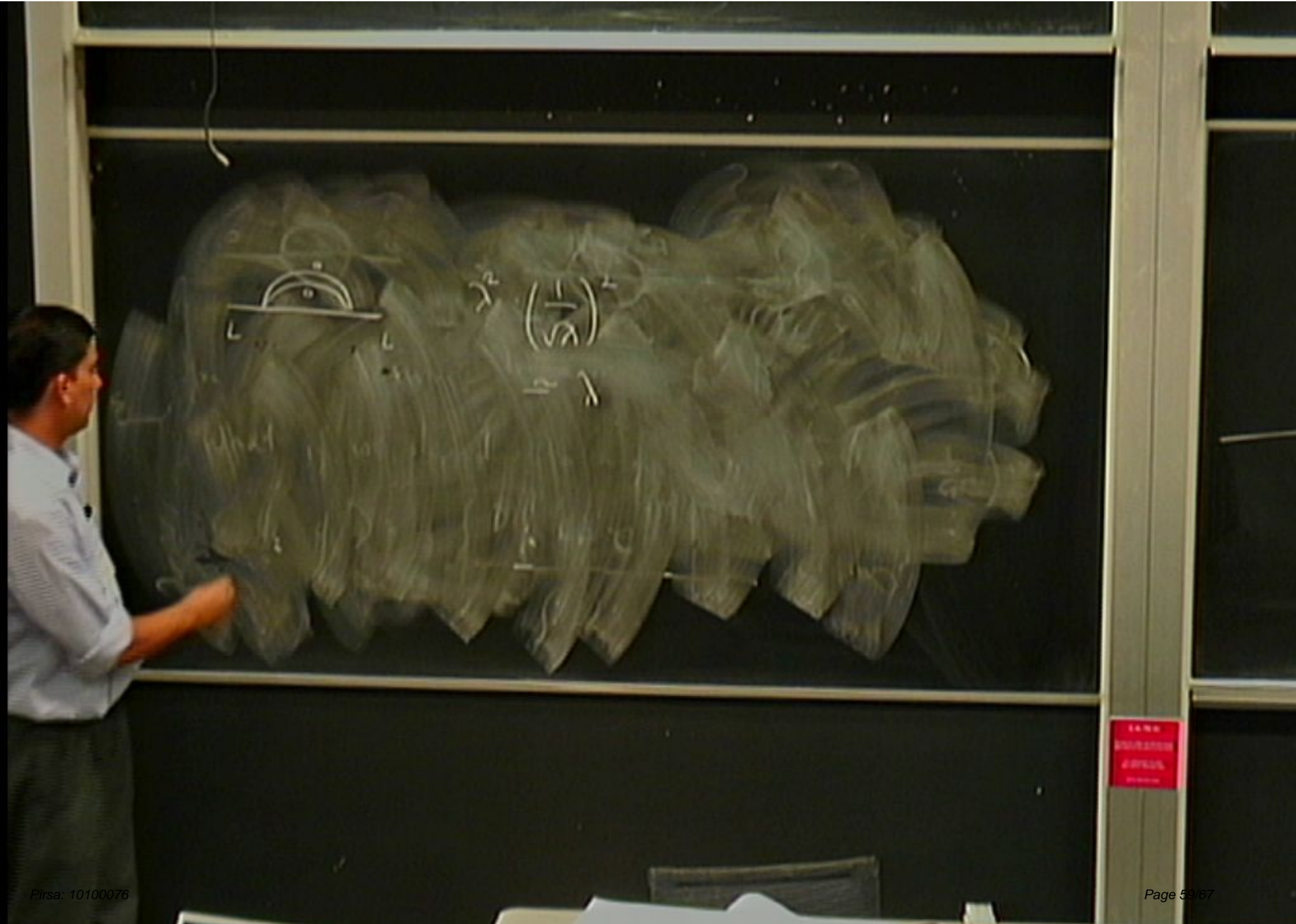


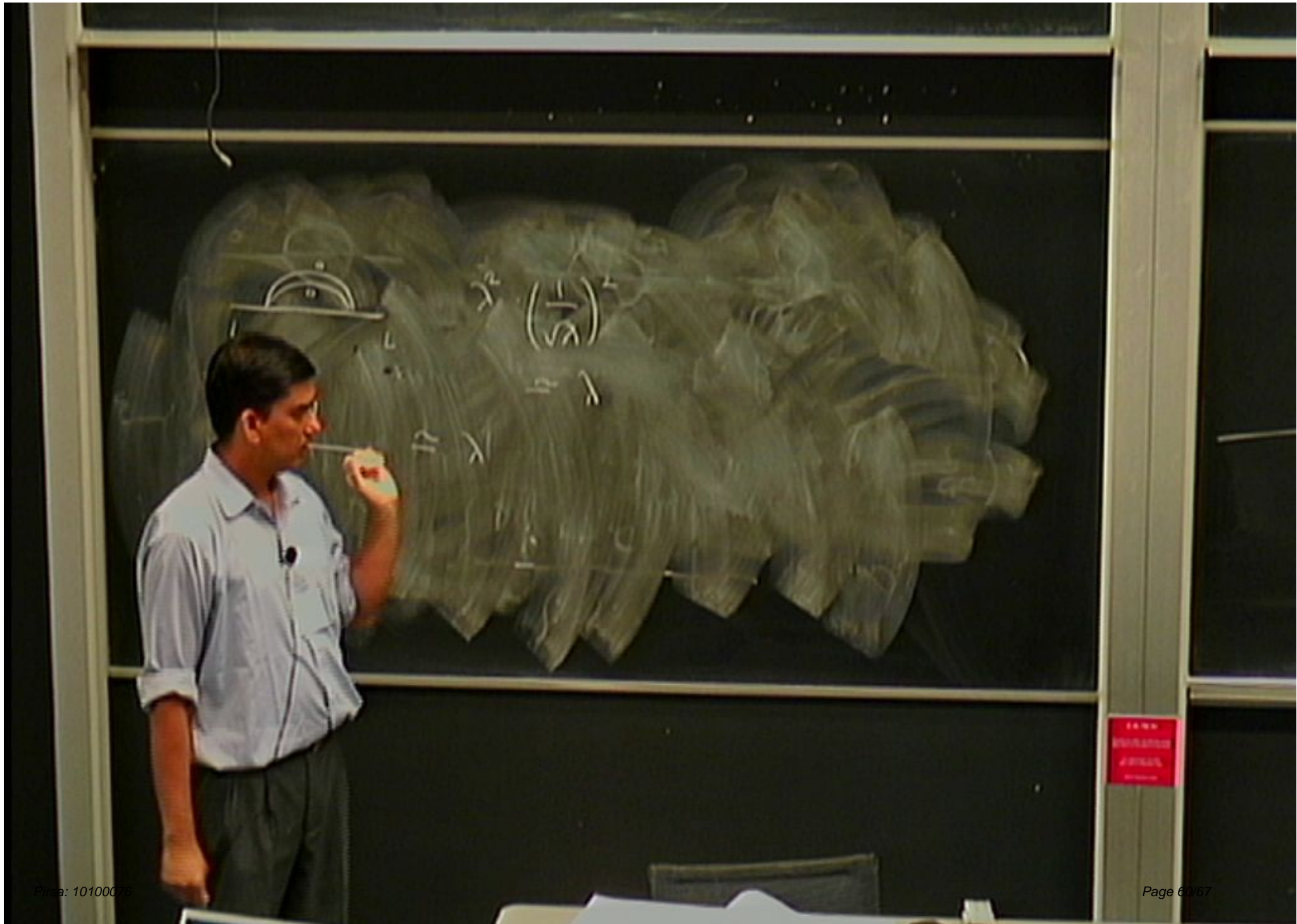
Massless scalar fields in (Euclidean) de Sitter

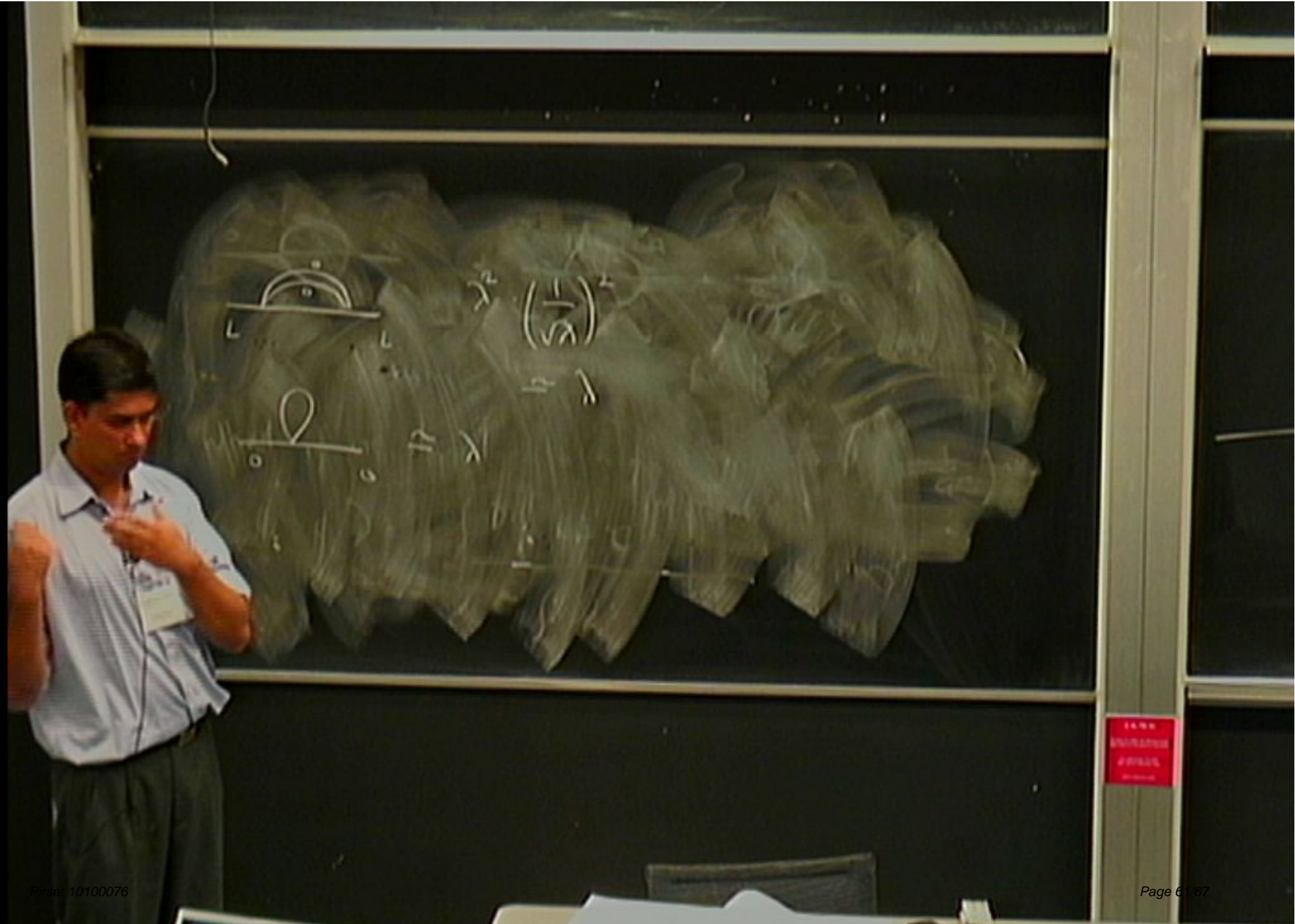
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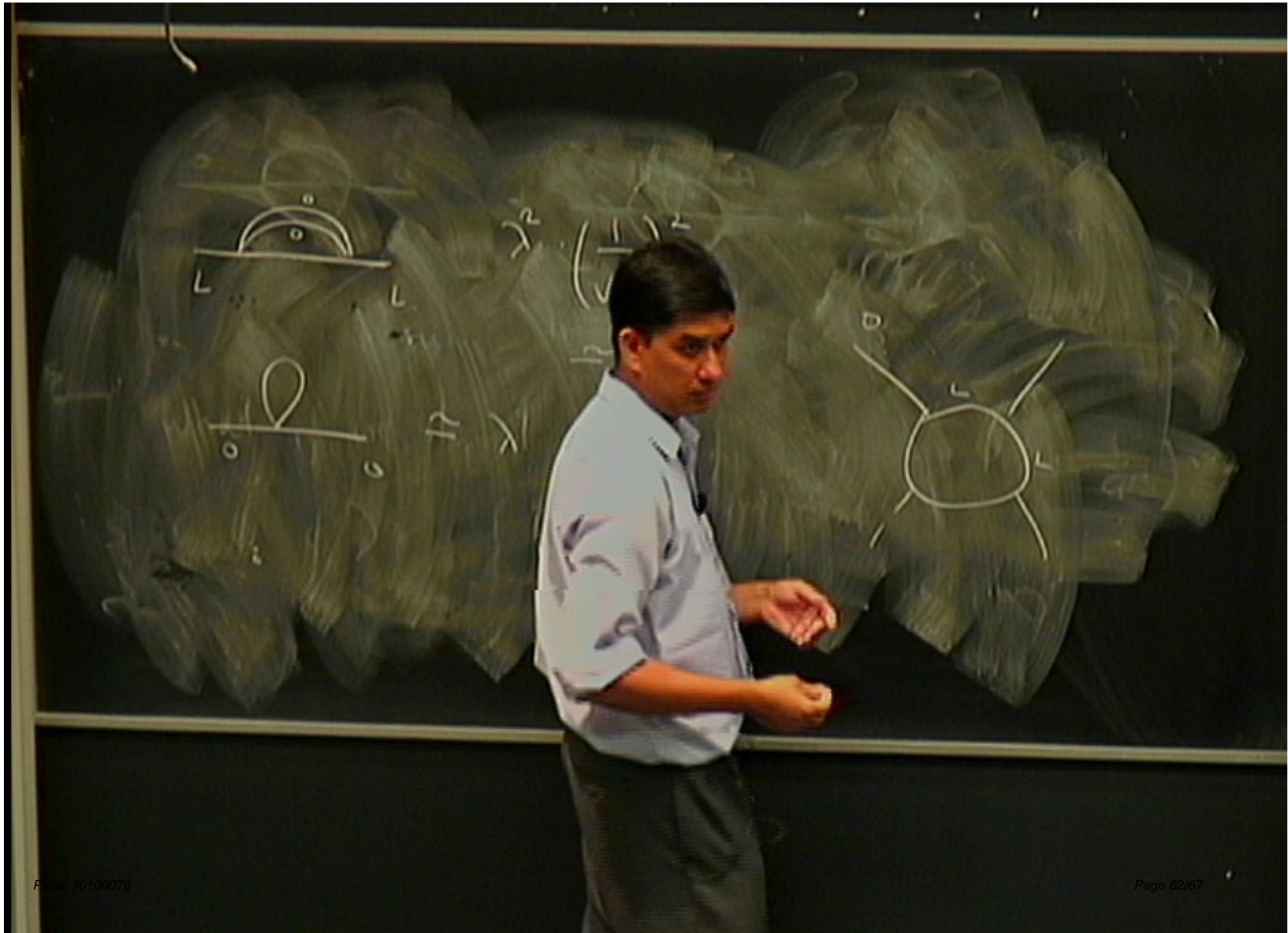
$$\langle \phi^{2n} \rangle_{\text{spn}} = \left( \frac{9}{\pi^2 \lambda} \right)^{n/2} \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$$











$$\langle \phi^{2n} \rangle_{\text{spk}} = \left( \frac{g}{\pi^2 \lambda} \right)^{n/2} \frac{\Gamma\left(\frac{n}{2} + \frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}\right)}$$

Will we include

$$\int \phi(x) \phi(x) \dots e^{-S(\phi)}$$

$\int \phi$

$$\langle \phi^{2n} \rangle_{\text{spk}} = \left( \frac{q}{\pi^2 \lambda} \right)^{n/2} \frac{\Gamma\left(\frac{n}{2} + \frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

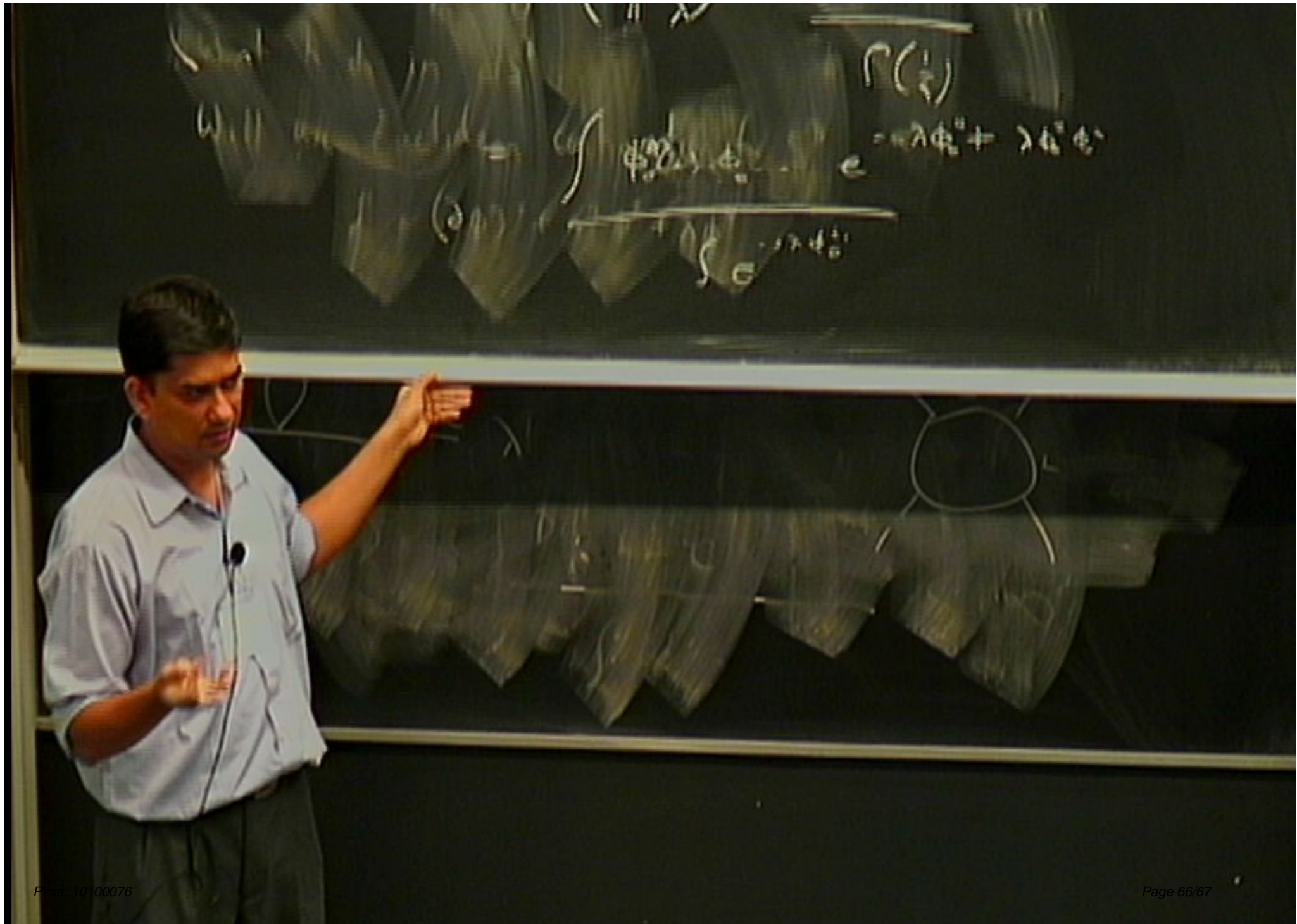
$$= \frac{\int \phi(x) \phi(x) \dots e^{-\lambda \phi^2} dx}{\int e^{-\lambda \phi^2} dx}$$





$$\langle \phi^{2n} \rangle_{\text{spk}} = \left( \frac{q}{\pi^2 \lambda} \right)^{n/2} \frac{\Gamma\left(\frac{n}{2} + \frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}\right)}$$

$$= \frac{\int \phi_0^{2n} e^{-\lambda \phi_0^4} d\phi_0}{\int e^{-\lambda \phi_0^4} d\phi_0}$$



$$\langle 0 | \psi | 0 \rangle_{in}$$

$|0\rangle_{in}$  not dS invariant

↓ evolves

dS invariant vacuum

= vacuum on sphere

$a \rightarrow \int \ln a$  should match

