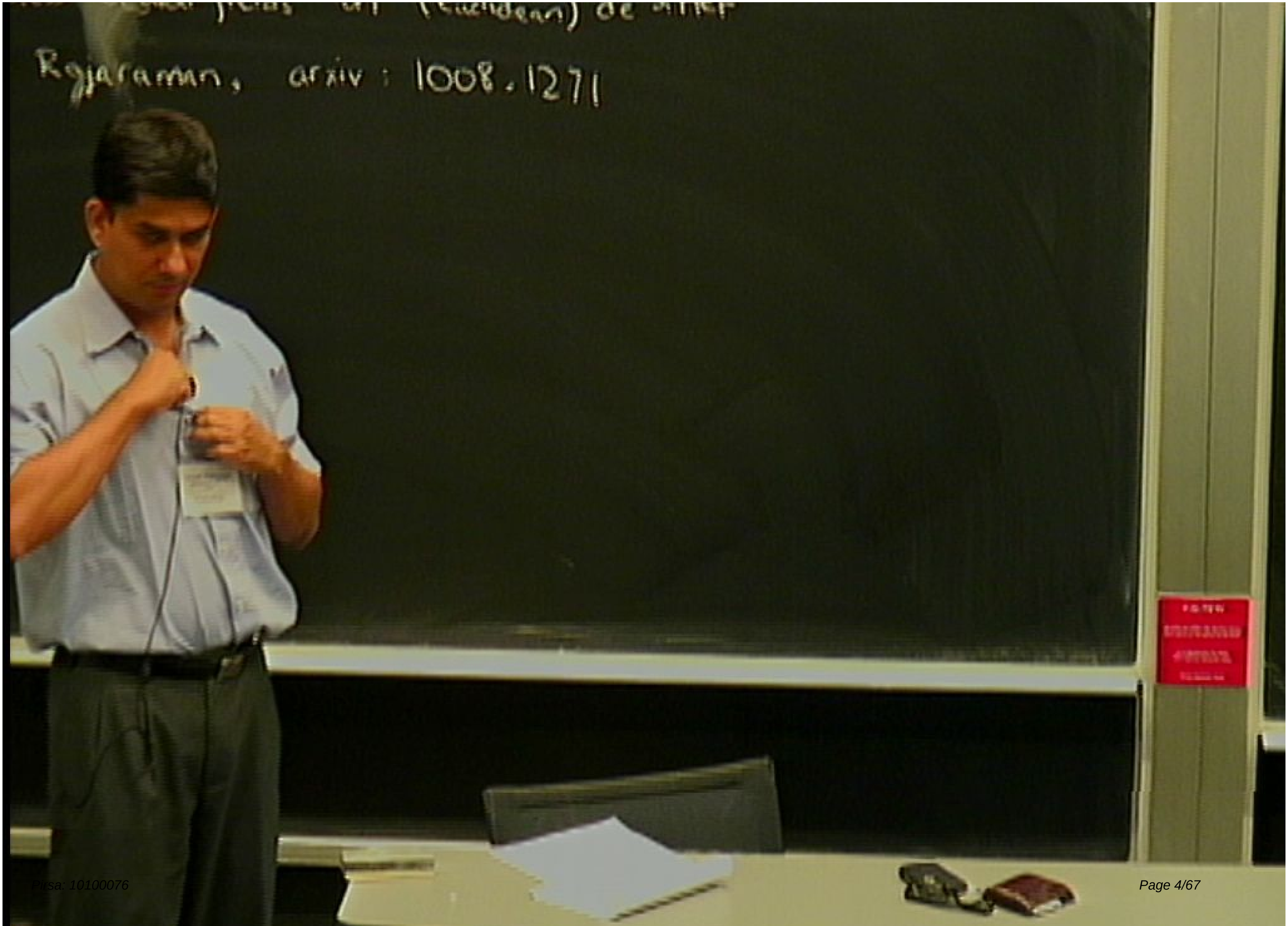
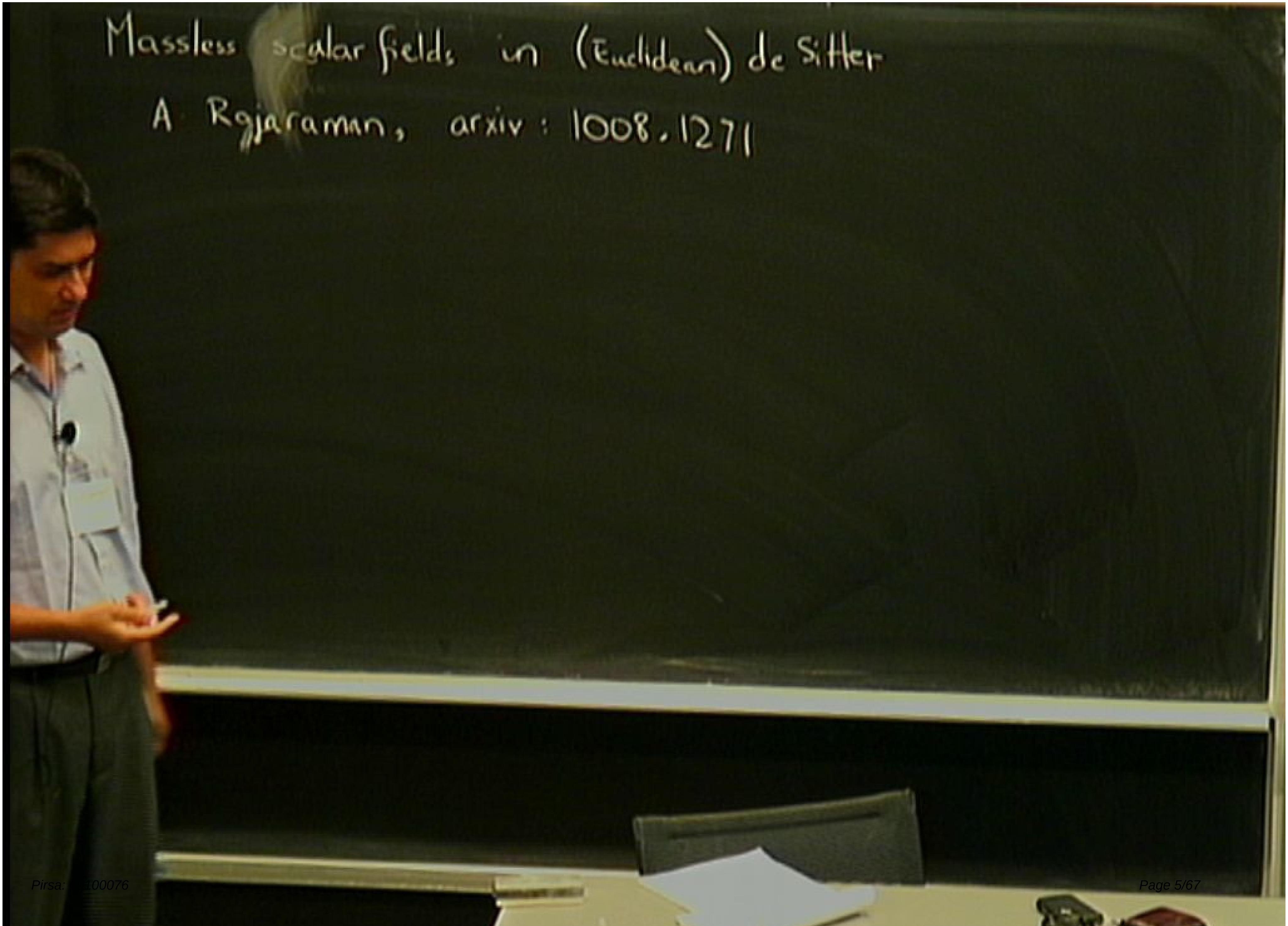
Title: Massless fields in Euclidean de Sitter space

Date: Oct 29, 2010 02:00 PM

URL: http://pirsa.org/10100076

Abstract: In theories with light or massless fields, loop diagrams can develop infrared divergences. We demonstrate how these can be resolved for a theory of massless interacting scalar fields in Euclidean de Sitter space. We also comment on applications to the in-in formalism in Lorentzian de Sitter space.


Rajaraman, arxiv : 1008.1271


Massless scalar fields in (Euclidean) de Sitter
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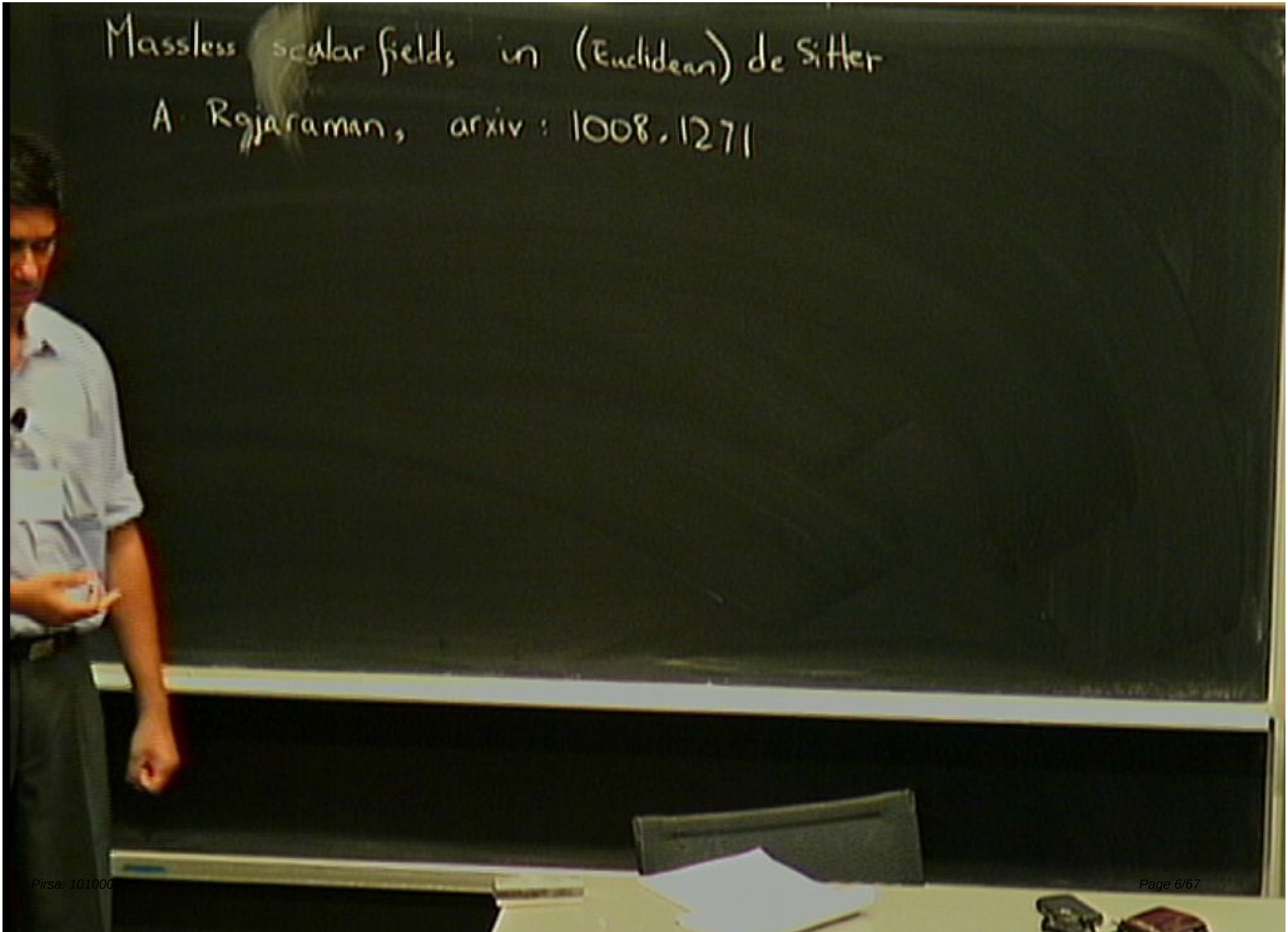
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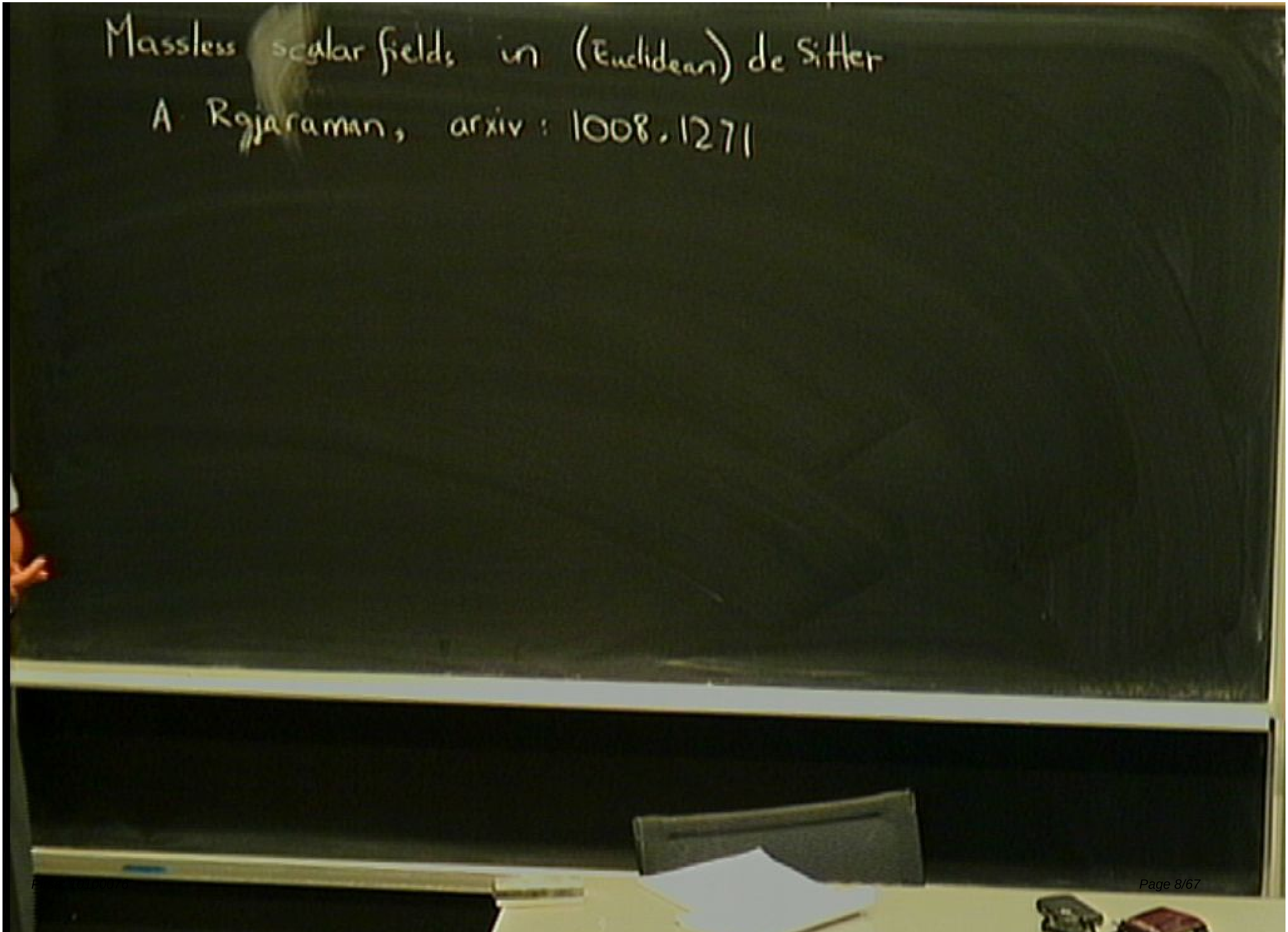
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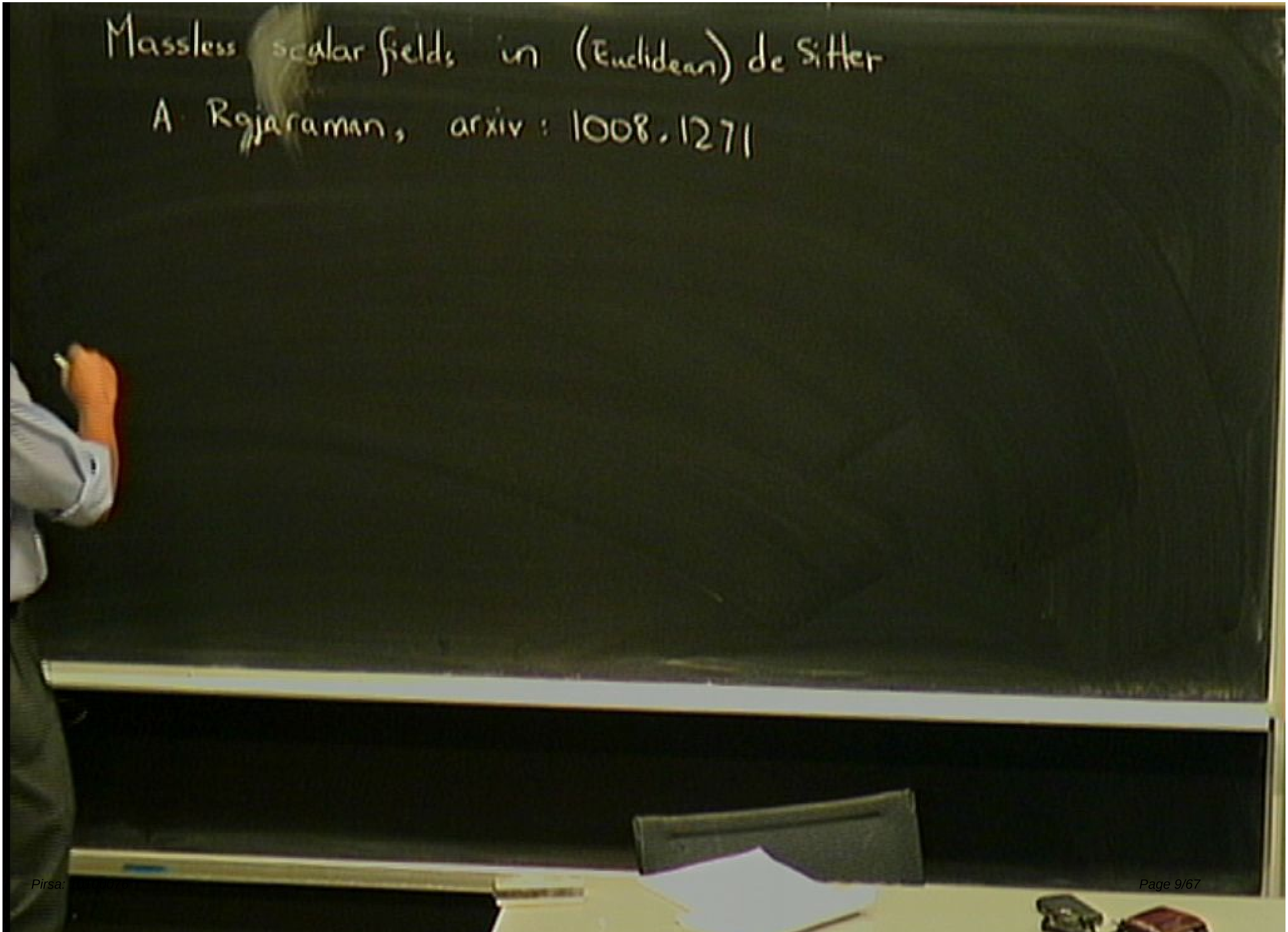
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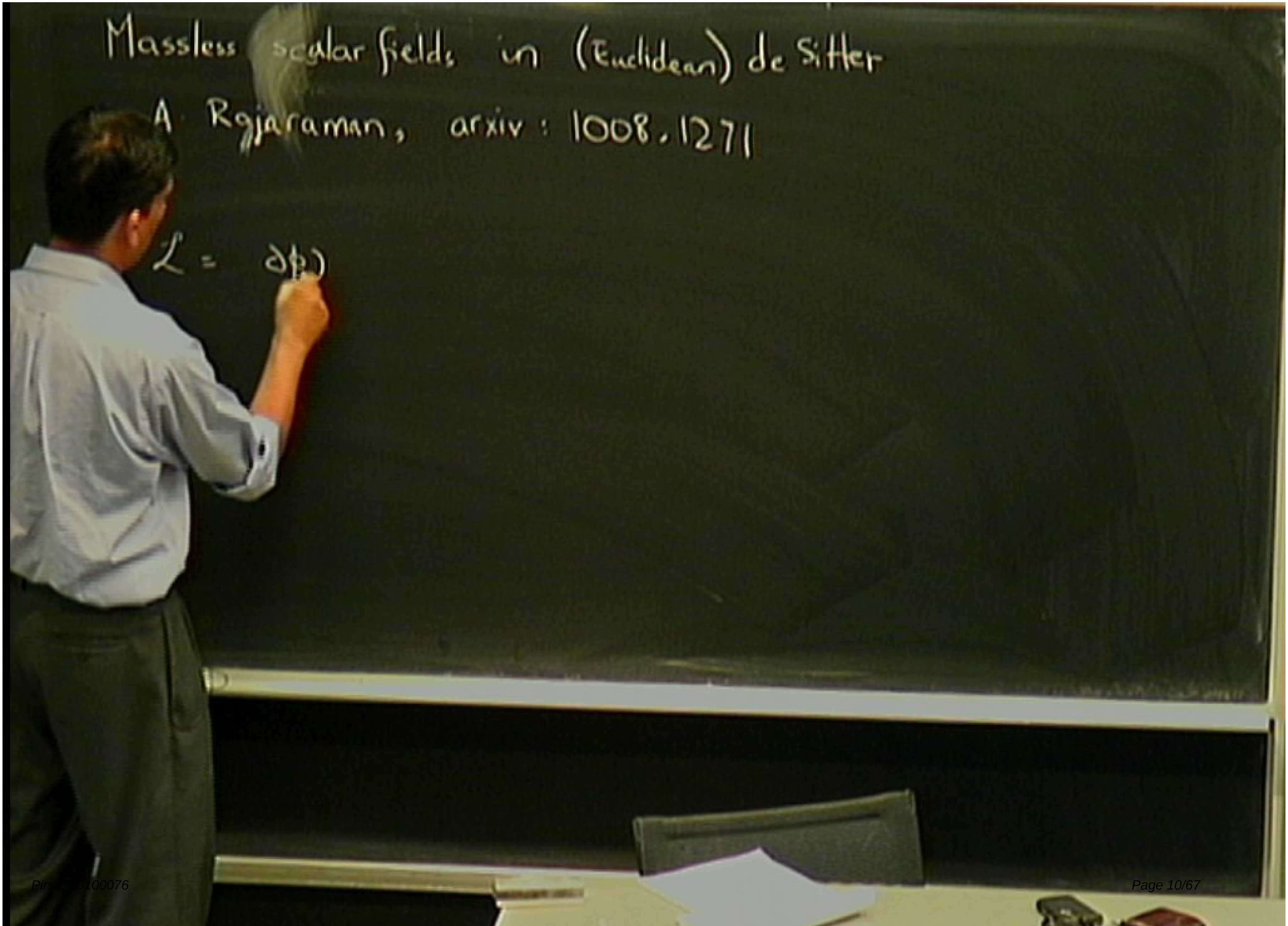
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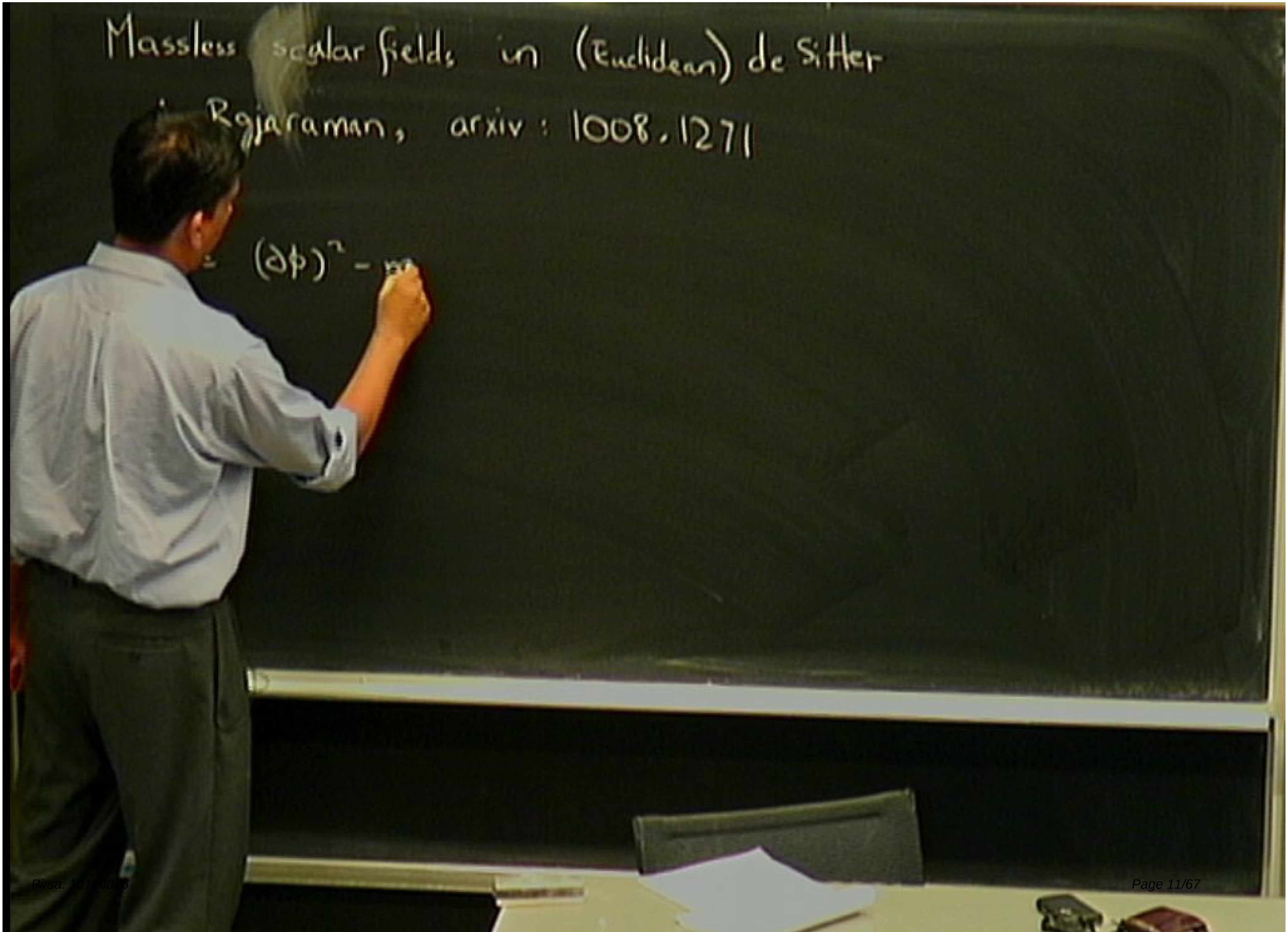
$\mathcal{L} = \partial\phi)$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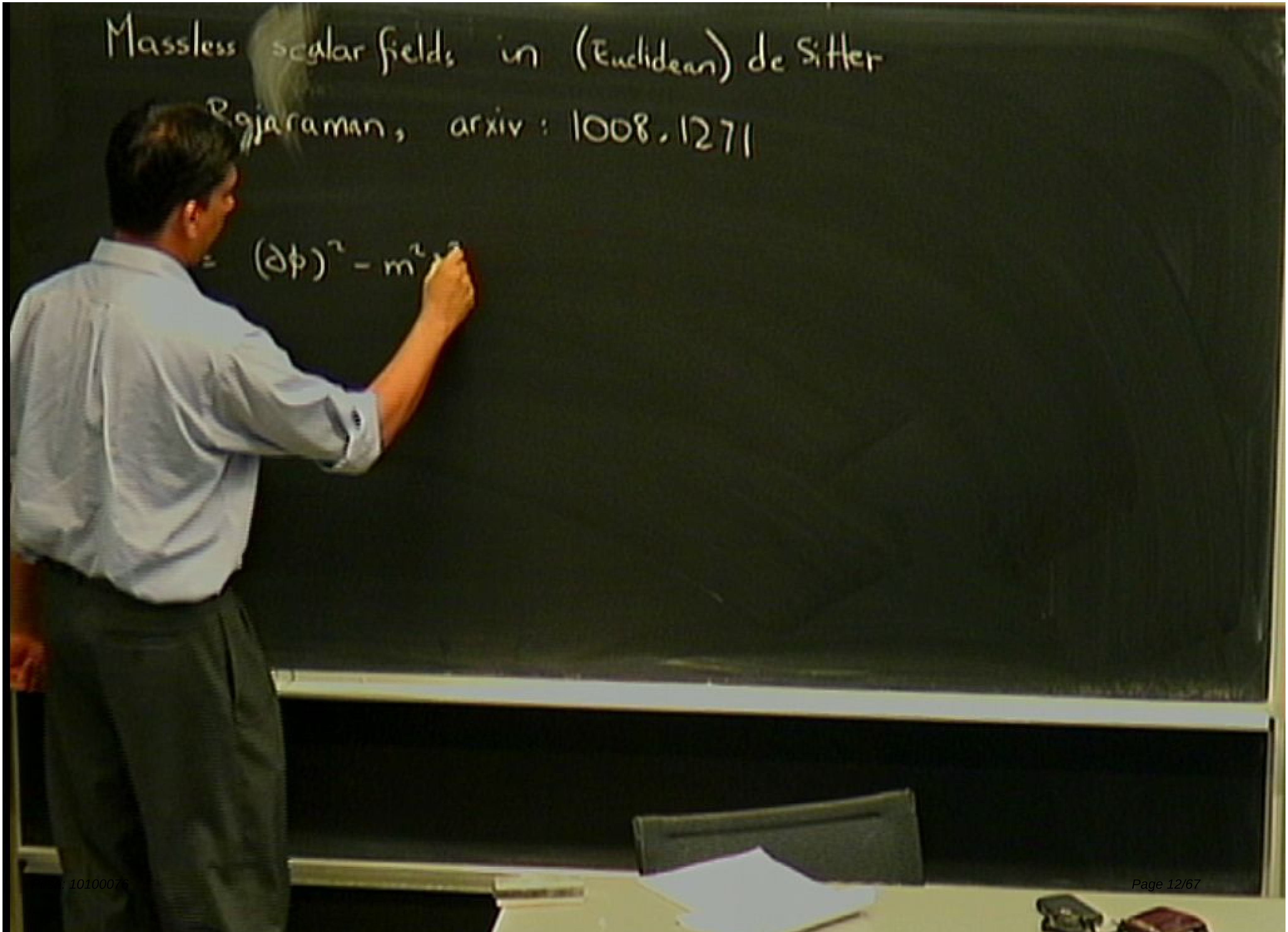
$(\partial\phi)^2 -$


Massless scalar fields in (Euclidean) de Sitter
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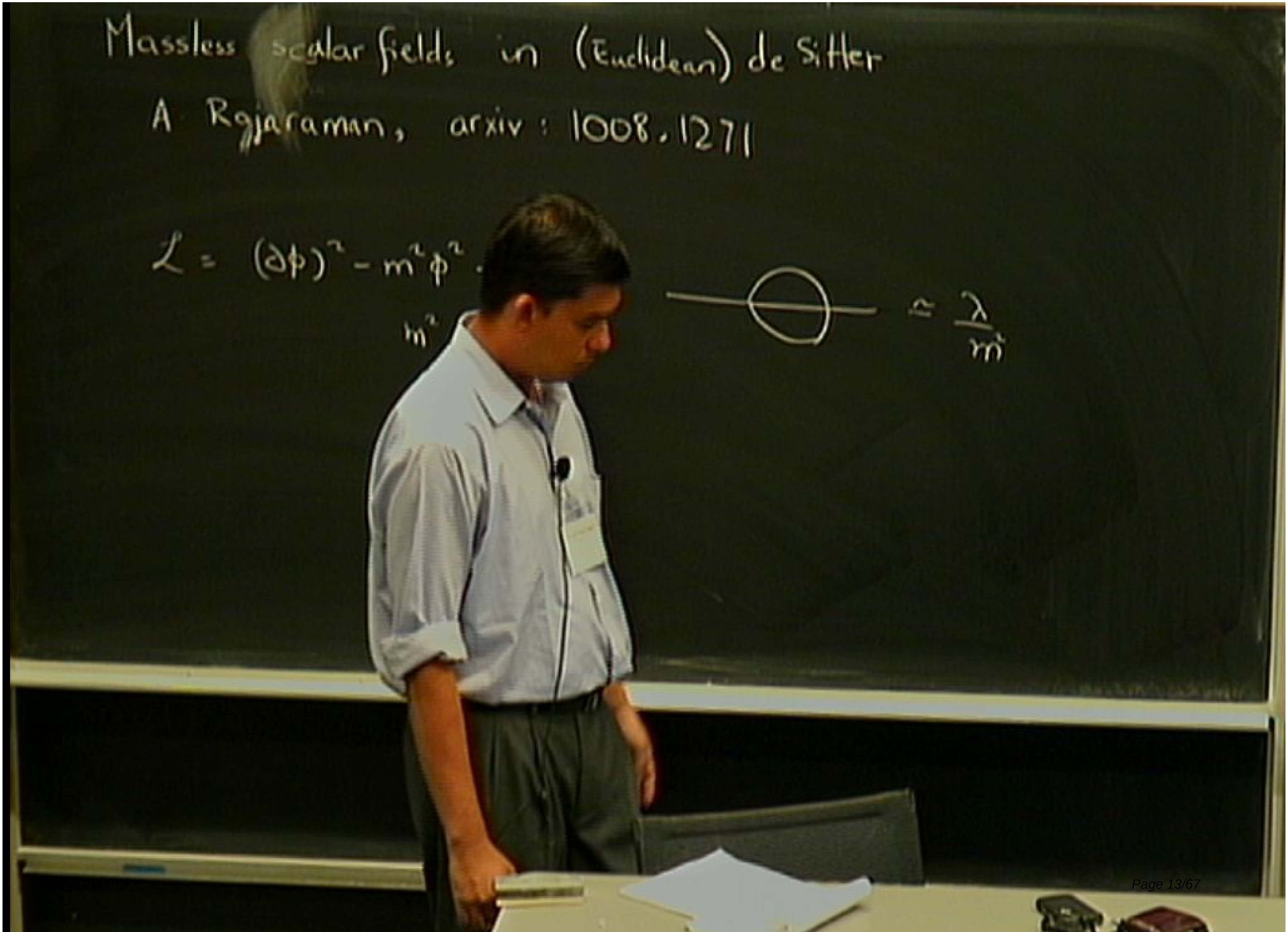
= (∂φ)² − m²


Massless scalar fields in (Euclidean) de Sitter
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$\mathcal{L} = (\partial\phi)^2 - m^2\phi^2$
m^2
$\simeq \frac{\lambda}{m^2}$

Massless scalar fields in (Euclidean) de Sitter

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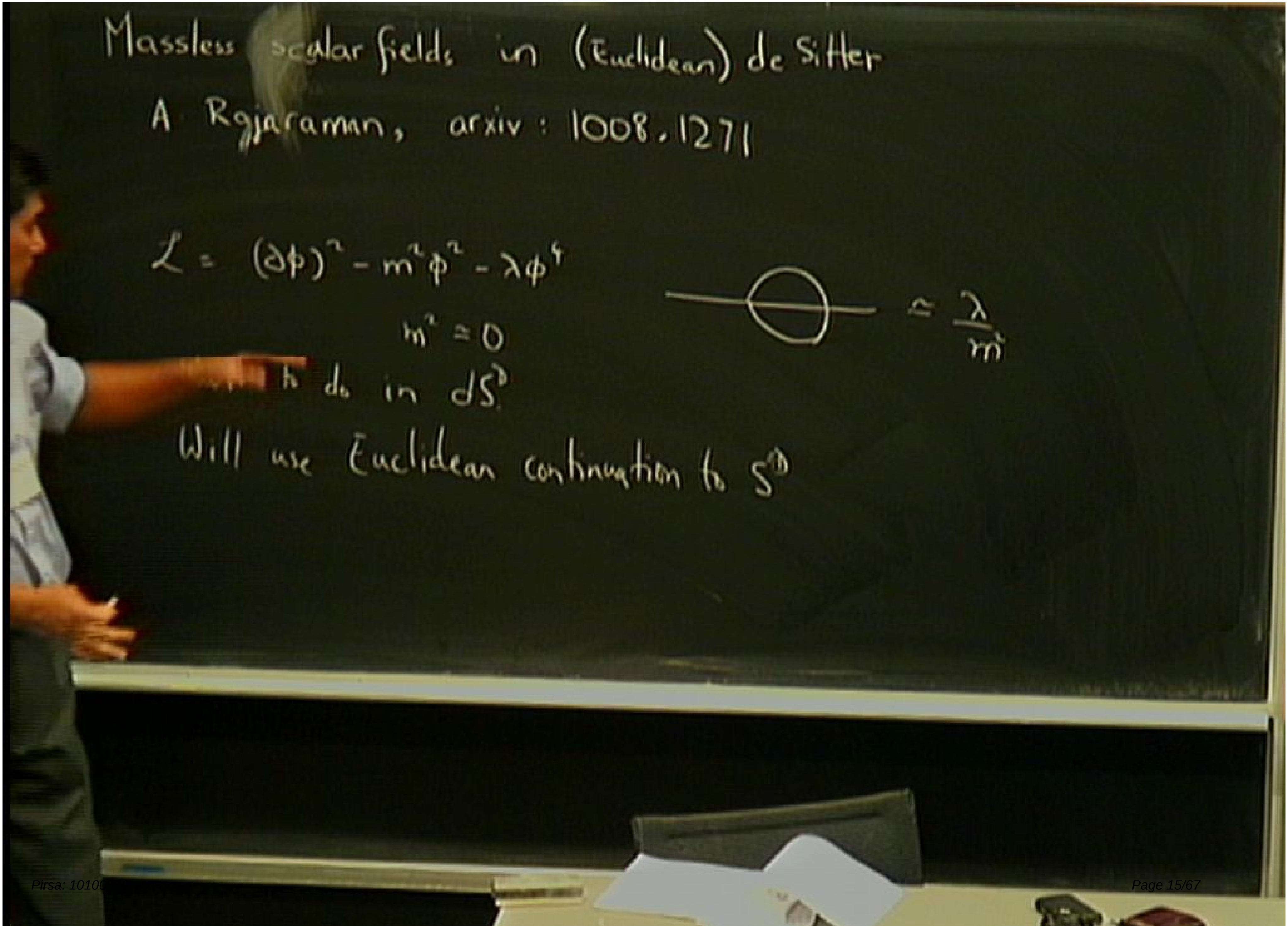
$$\mathcal{L} = (\partial\phi)^2 - m^2\phi^2 - \lambda\phi^4$$

 $\simeq \frac{\lambda}{m^2}$

$$m^2 = 0$$

Want to do in dS


Massless scalar fields in (Euclidean) de Sitter
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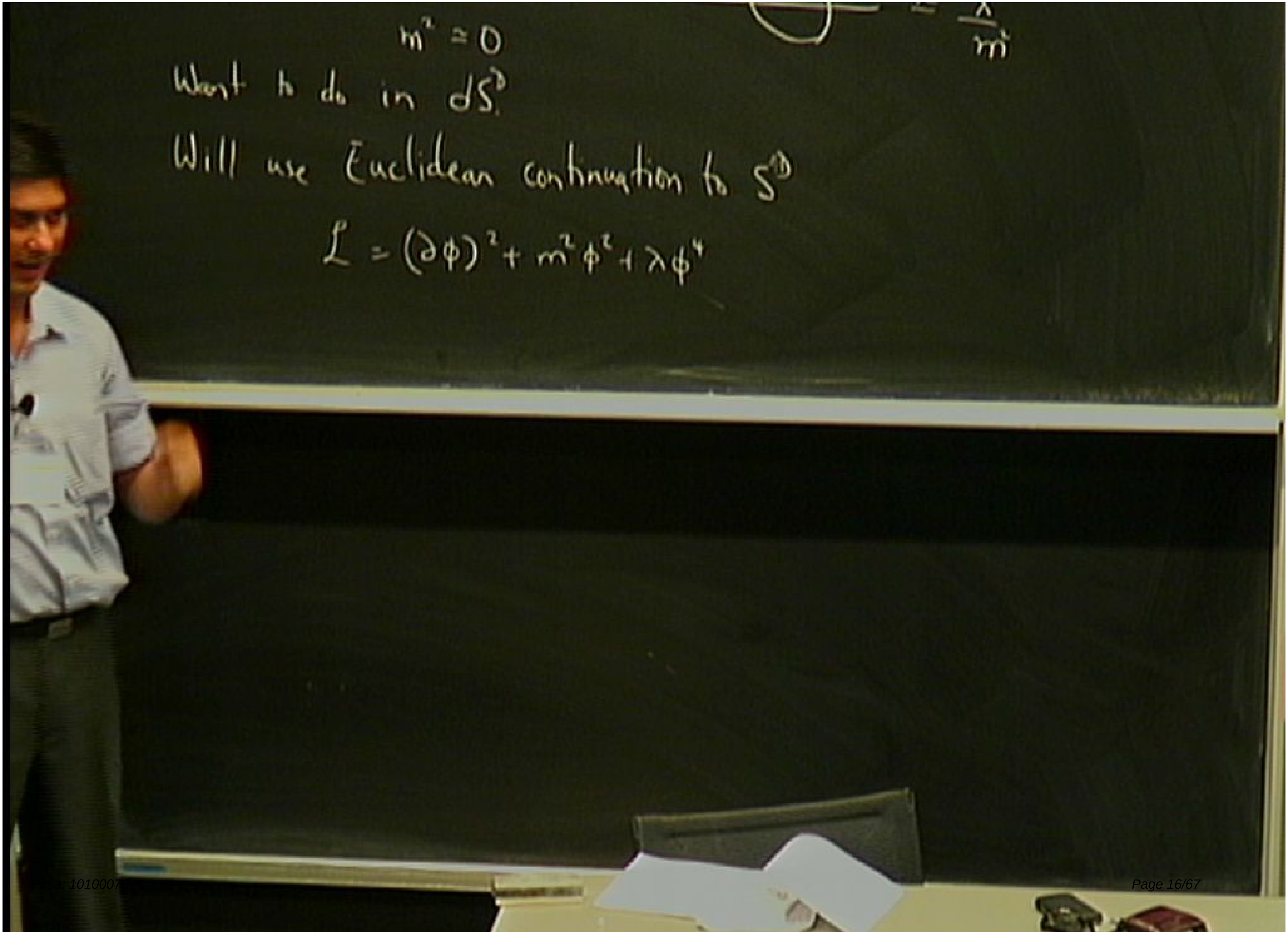
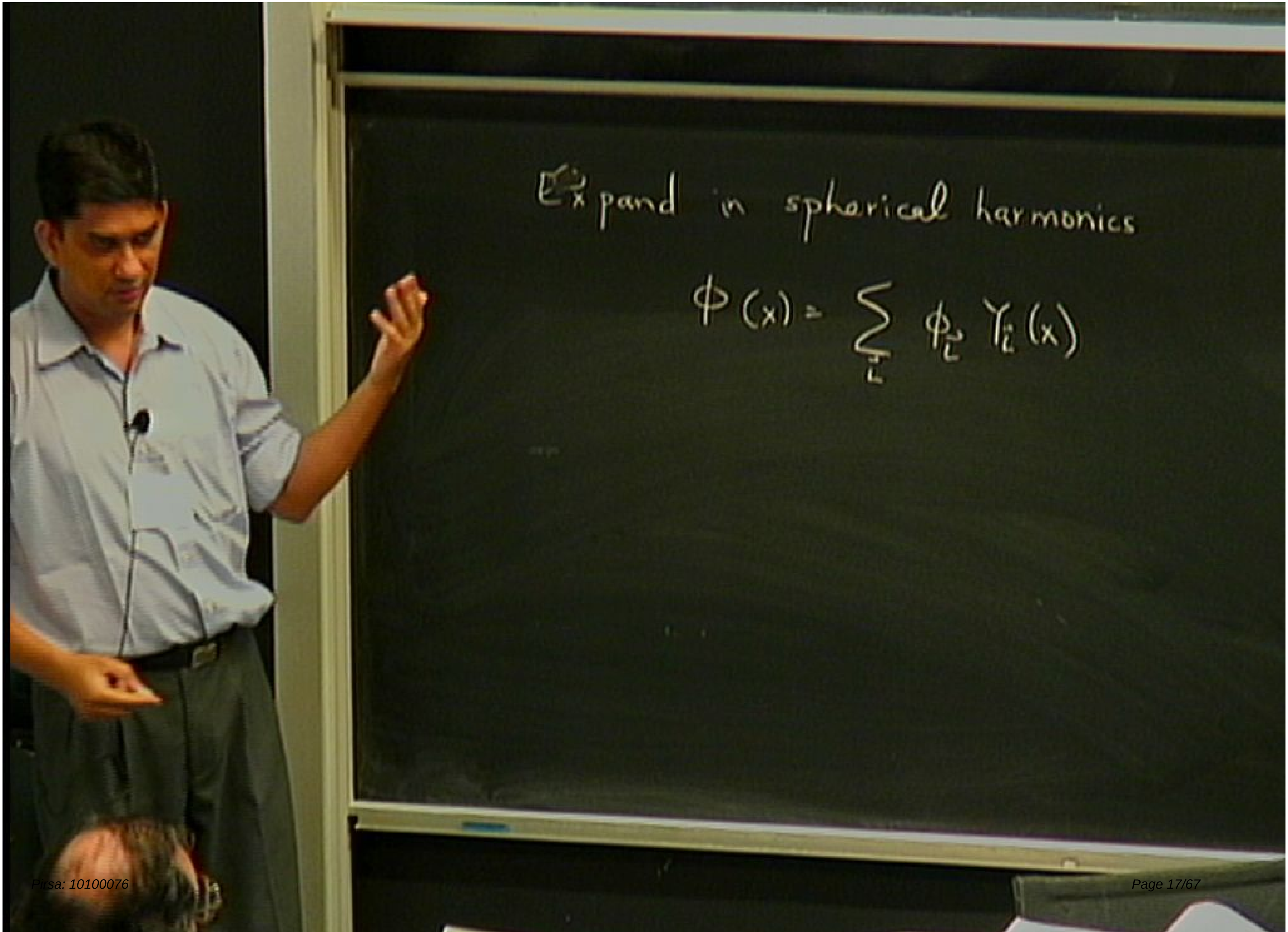
L = (∂φ)² − m²φ² − λφ⁴
m² = 0
to do in dS^D
Will use Euclidean continuation to S^D
≃ λ/m²


$m^2 \simeq 0$
Want to do in dS^D
Will use Euclidean continuation to S^D
$\mathcal{L} = (\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4$


Expand in spherical harmonics
$\phi(x) = \sum_{l} \phi_{l} Y_{l}(x)$

Expand in spherical harmonics

$$\phi(x) = \sum_{L} \phi_L Y_L(x)$$

$$\nabla^2 Y_L = -L(L+d')Y_L \qquad d = D-1$$

Expand in spherical harmonics

$$\phi(x) = \sum_{\vec{L}} \phi_{\vec{L}} Y_{\vec{L}}(x)$$

$$\nabla^2 Y_{\vec{L}} = -L(L+d) Y_{\vec{L}} \qquad d = D-1$$

Find correlation functions

$$\langle \phi(x)\phi(y) \rangle = \frac{\int D\phi \, \phi(x)\phi(y) e^{-S}}{\int D\phi \, e^{-S}}$$

Expand in spherical harmonics

$$\phi(x) = \sum_{\vec{L}} \phi_{\vec{L}} \, Y_{\vec{L}}(x)$$

$$\nabla^2 Y_{\vec{L}} = -L(L+d) \, Y_{\vec{L}} \qquad d = D-1$$

Find correlation functions

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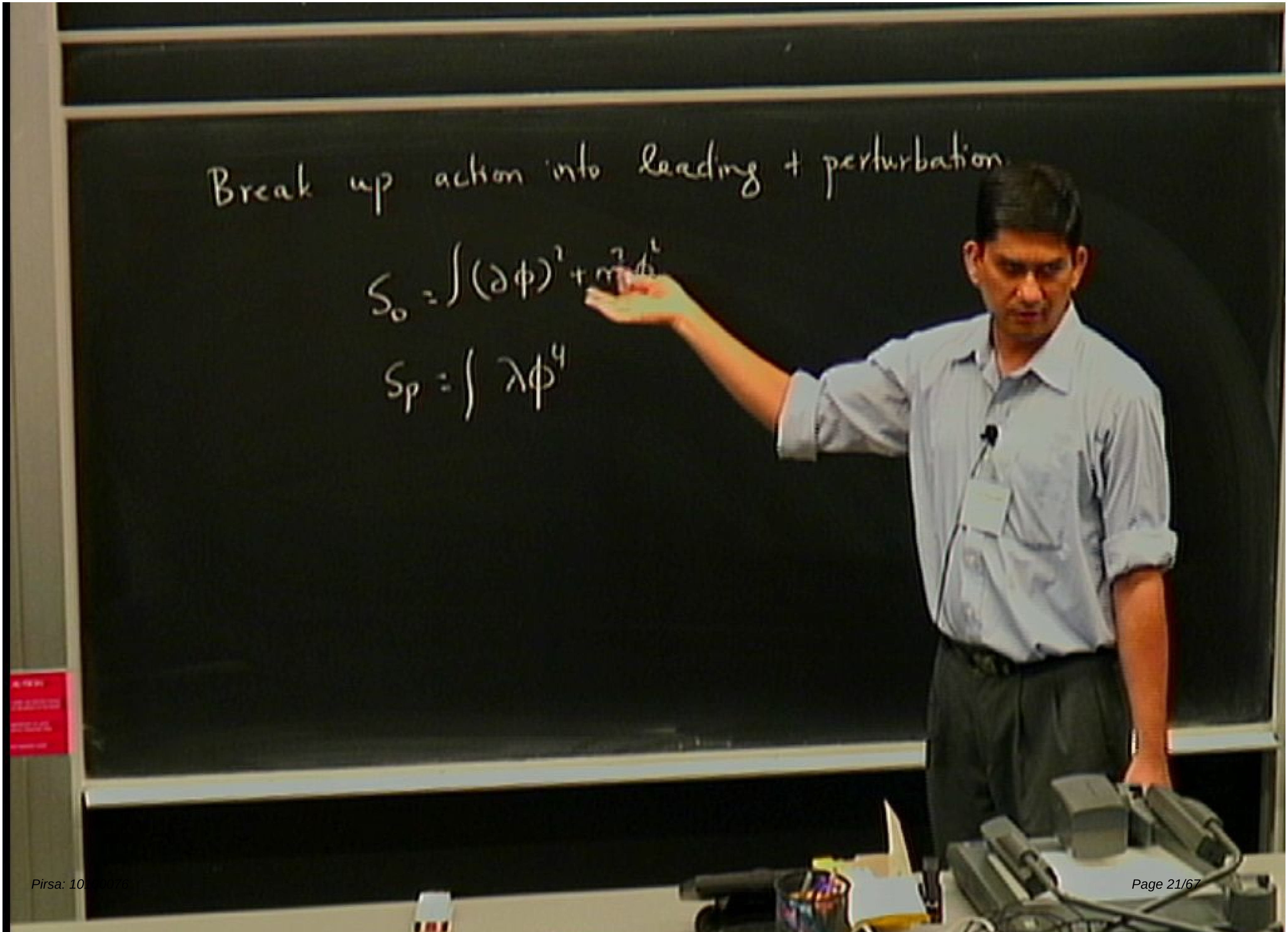

Break up action into leading + perturbation
$S_0 = \int (\partial\phi)^2 + m^2\phi^2$
$S_p = \int \lambda\phi^4$

Break up action into leading + perturbation.

$$S_0 = \int (\partial\phi)^2 + m^2\phi^2 = \sum_{\vec{L}} L(L+d)\,\phi_L^2$$

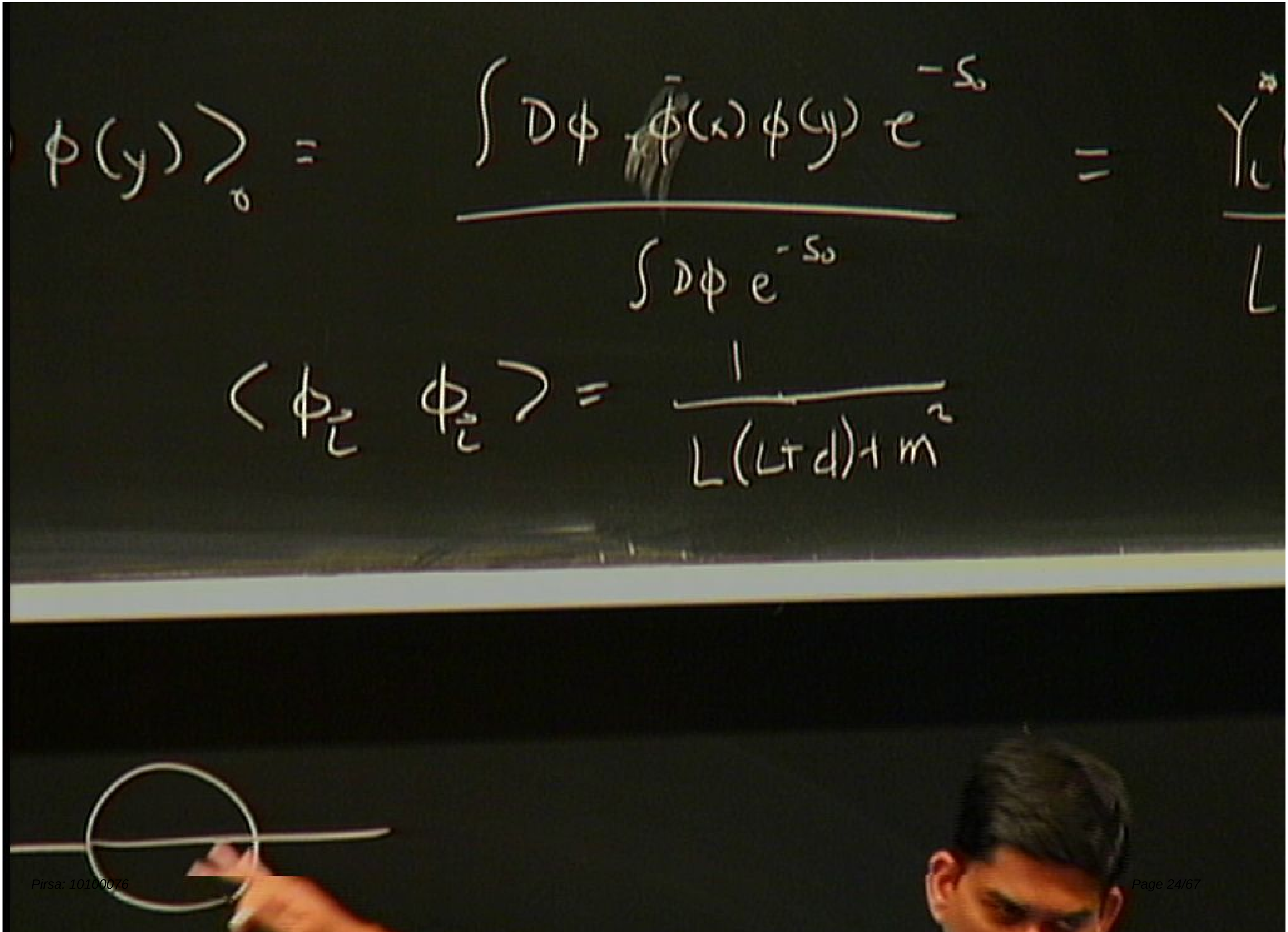
$$S_p = \int \lambda\phi^4$$

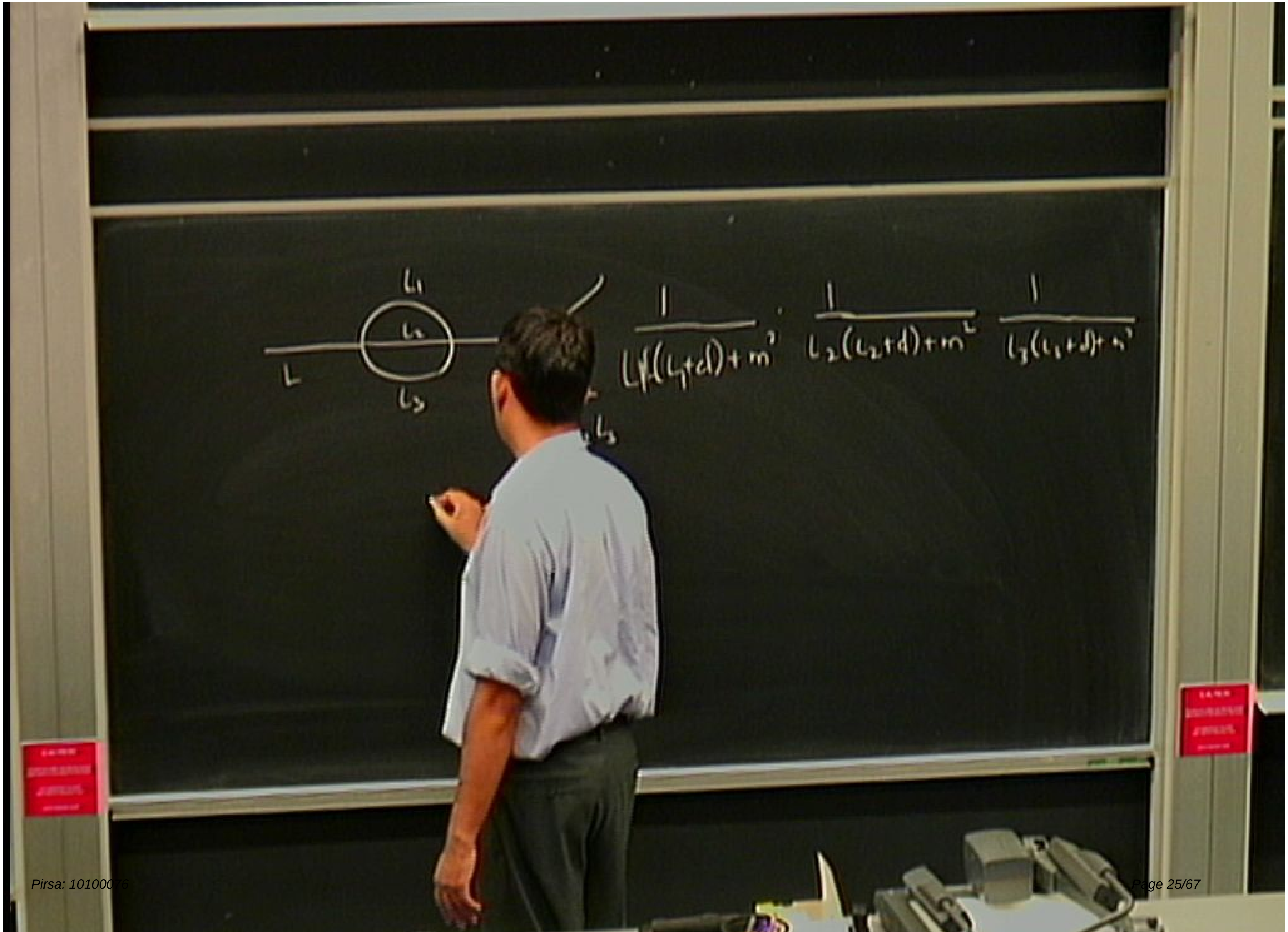
Break up action into leading + perturbation

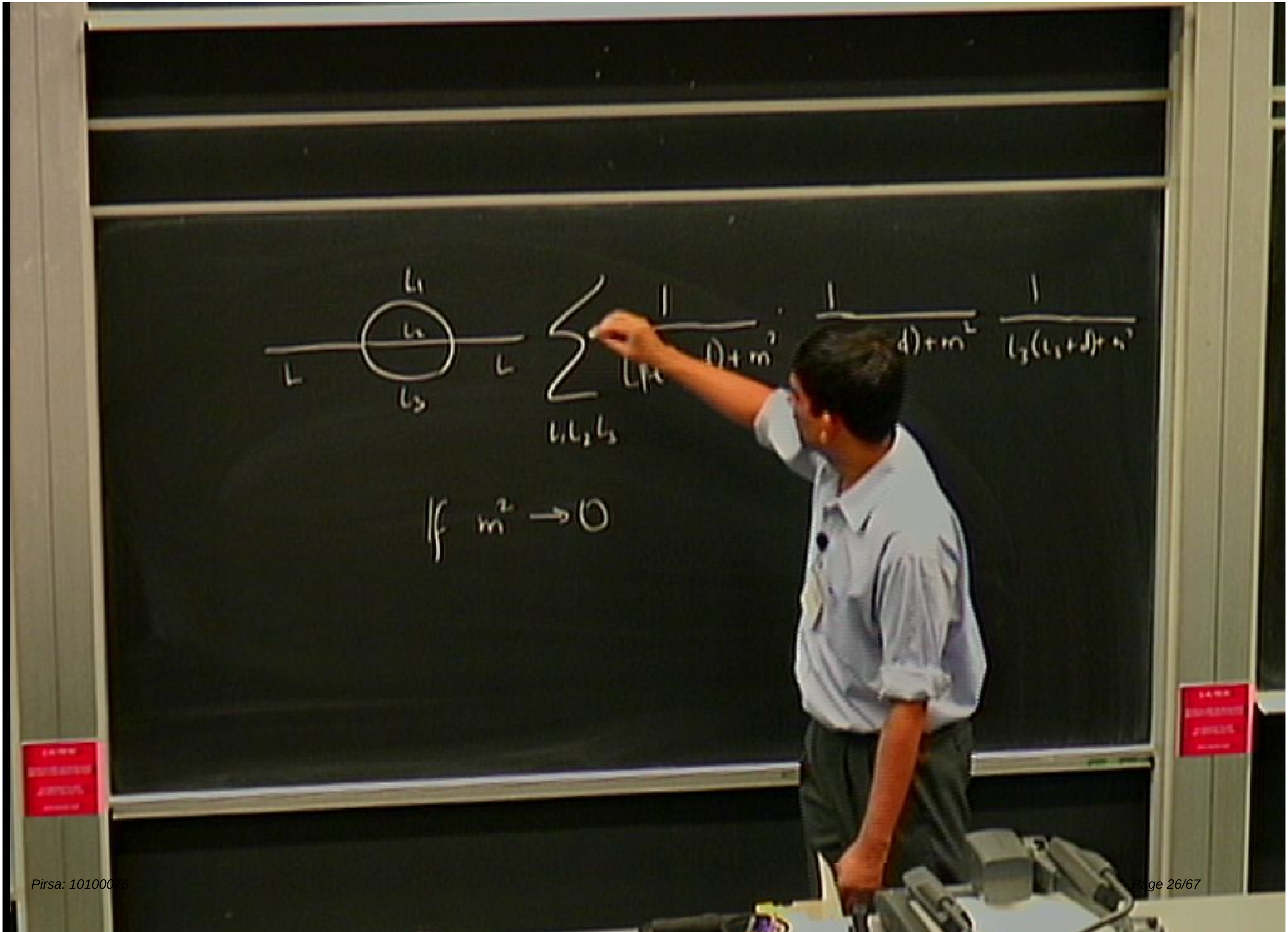
$$S_0 = \int (\partial\phi)^2 + m^2\phi^2 = \sum_{\vec{L}} \left[L(L+d) + m^2 \right] \phi_{\vec{L}}^2$$

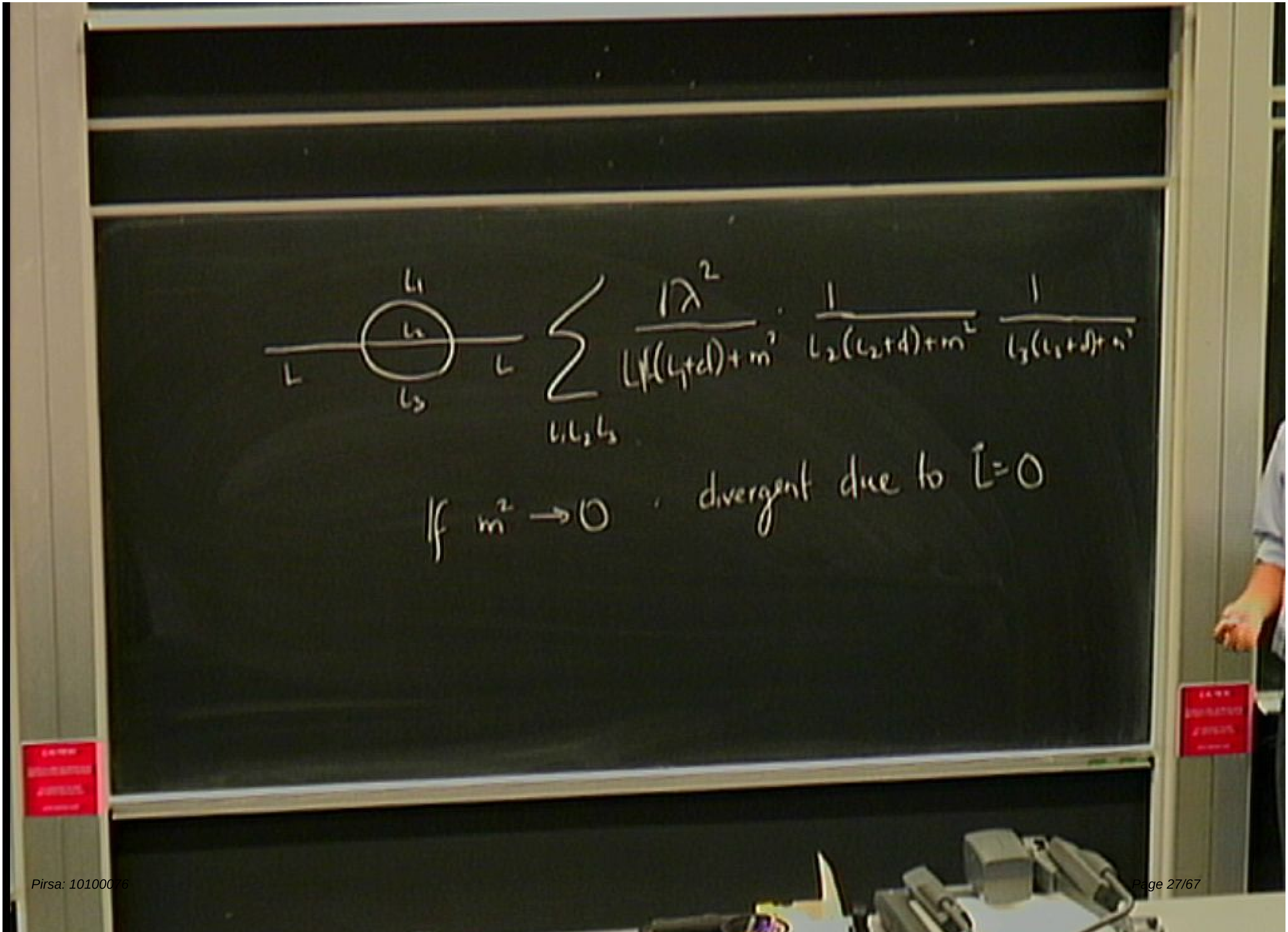
$$S_p = \int \lambda \phi^4$$

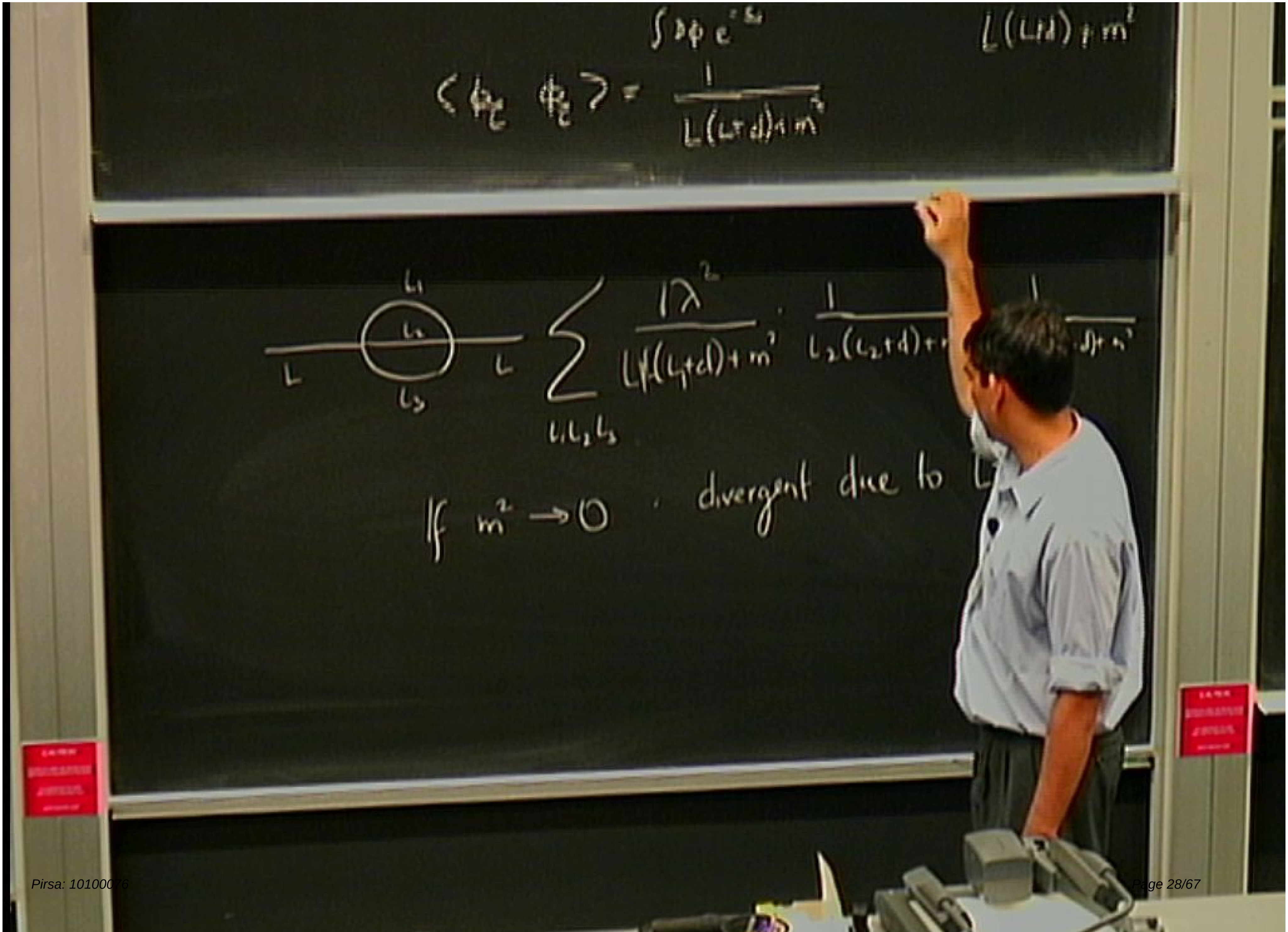
$$\langle \phi(x)\, \phi(y) \rangle_0 = \frac{\int D\phi\, \phi(x)\, \phi(y)\, e^{-S_0}}{\int D\phi\, e^{-S_0}} = \frac{Y_L^*(x)\, Y_L(y)}{L(L+d) + m^2}$$

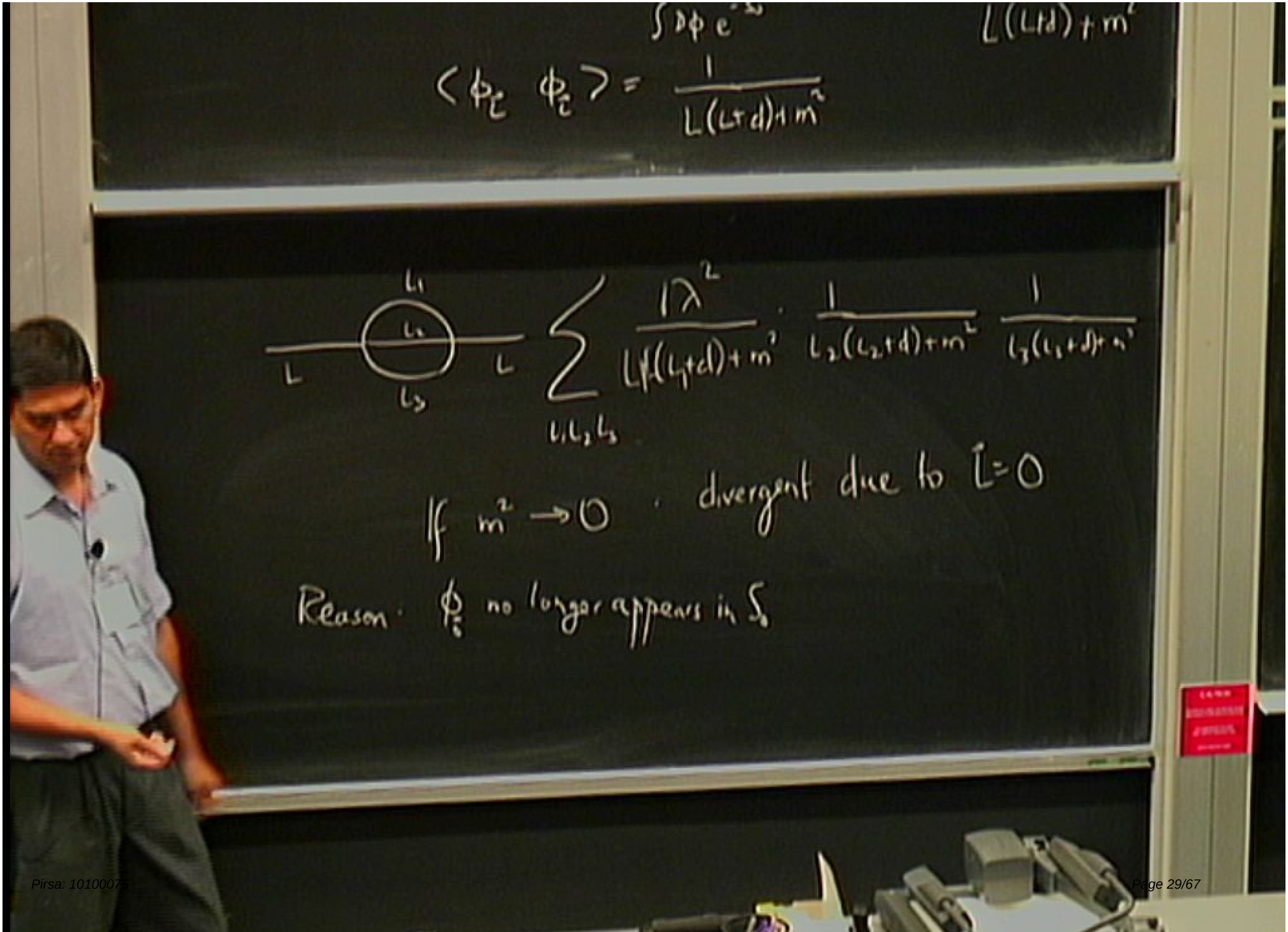

$\phi(y)\rangle_0 = \frac{\int D\phi\, \phi(x)\phi(y)\, e^{-S_0}}{\int D\phi\, e^{-S_0}} = \frac{Y_l}{L}$
$\langle \phi_{\vec{l}}\, \phi_{\vec{l}} \rangle = \frac{1}{L(L+d)+m^2}$

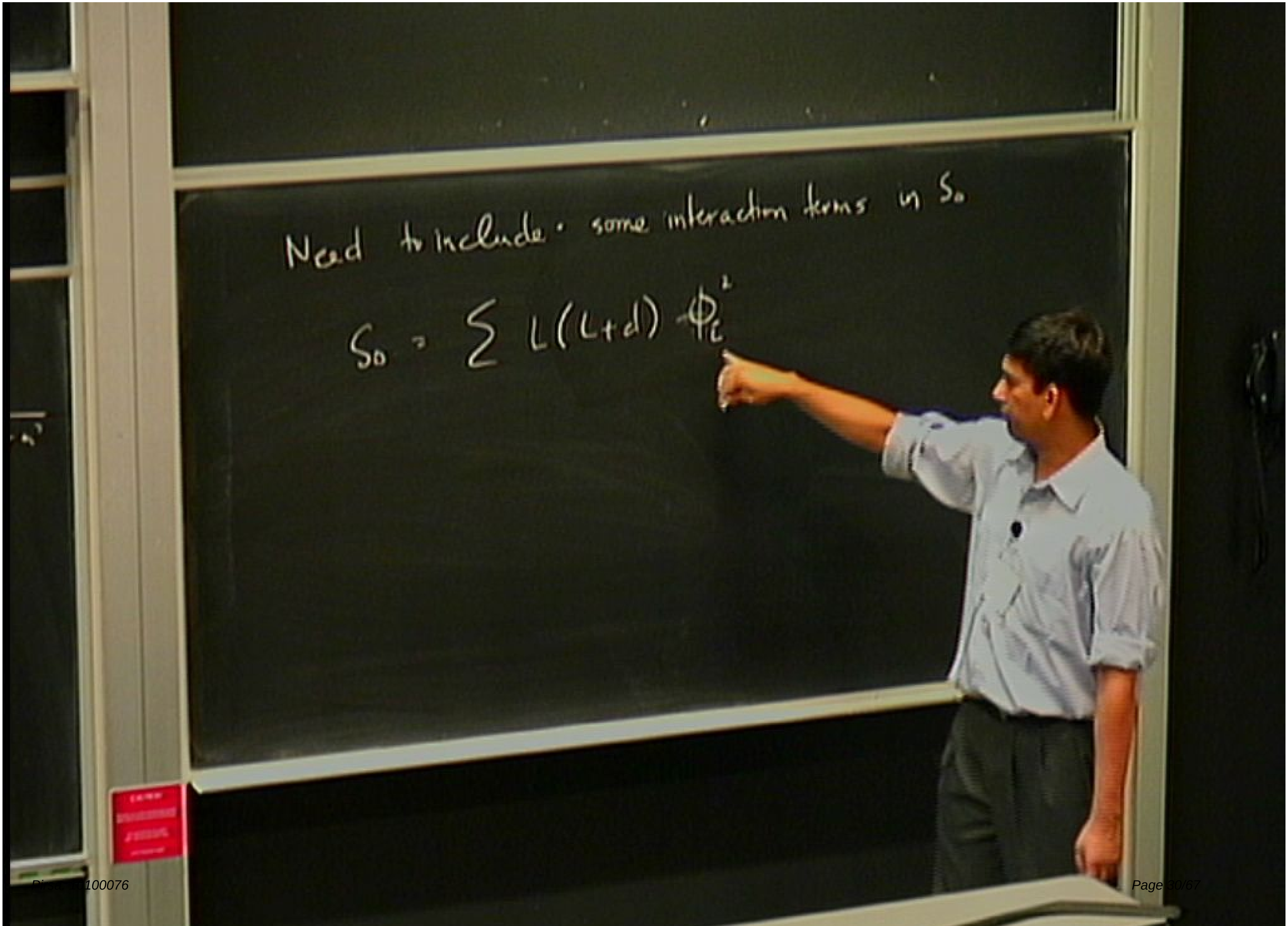


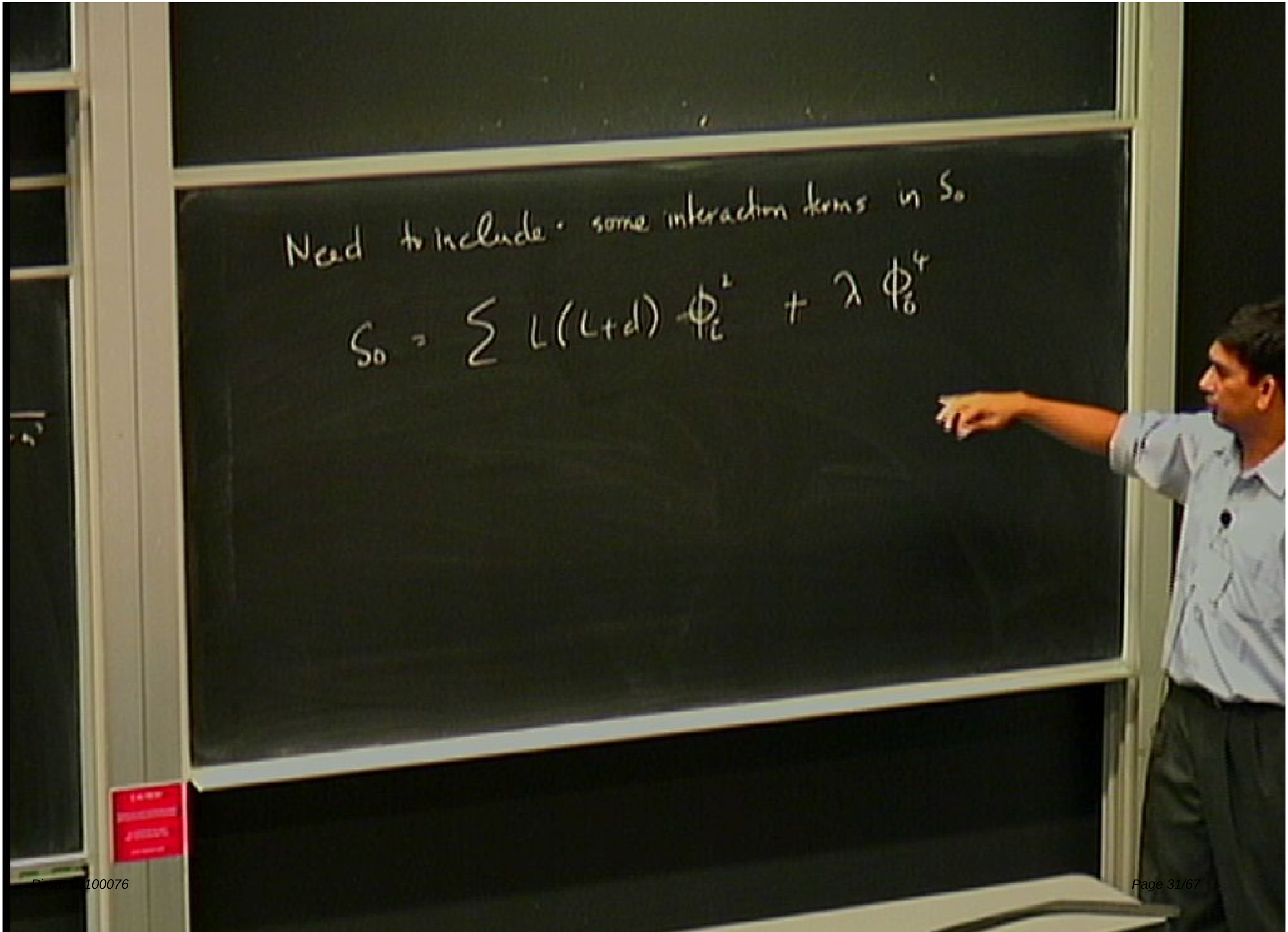

l1
l2
l3
L
L
If m² → 0

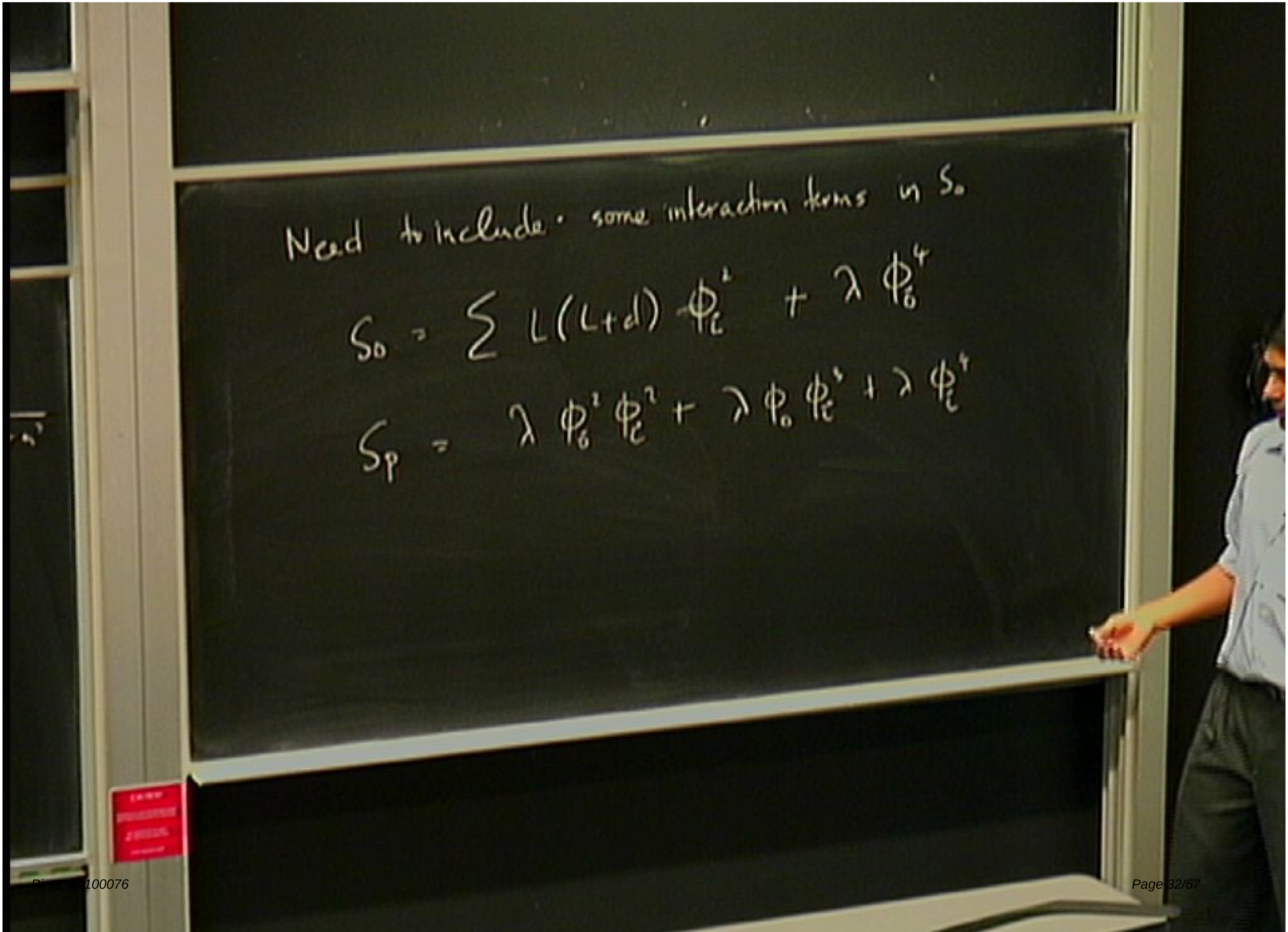

l_1
l_2
l_3
L
L
$\sum_{l_1 l_2 l_3} \frac{|\lambda|^2}{l_1(l_1+d)+m^2} \cdot \frac{1}{l_2(l_2+d)+m^2} \frac{1}{l_3(l_3+d)+m^2}$
If $m^2 \to 0$. divergent due to $\vec{l}=0$

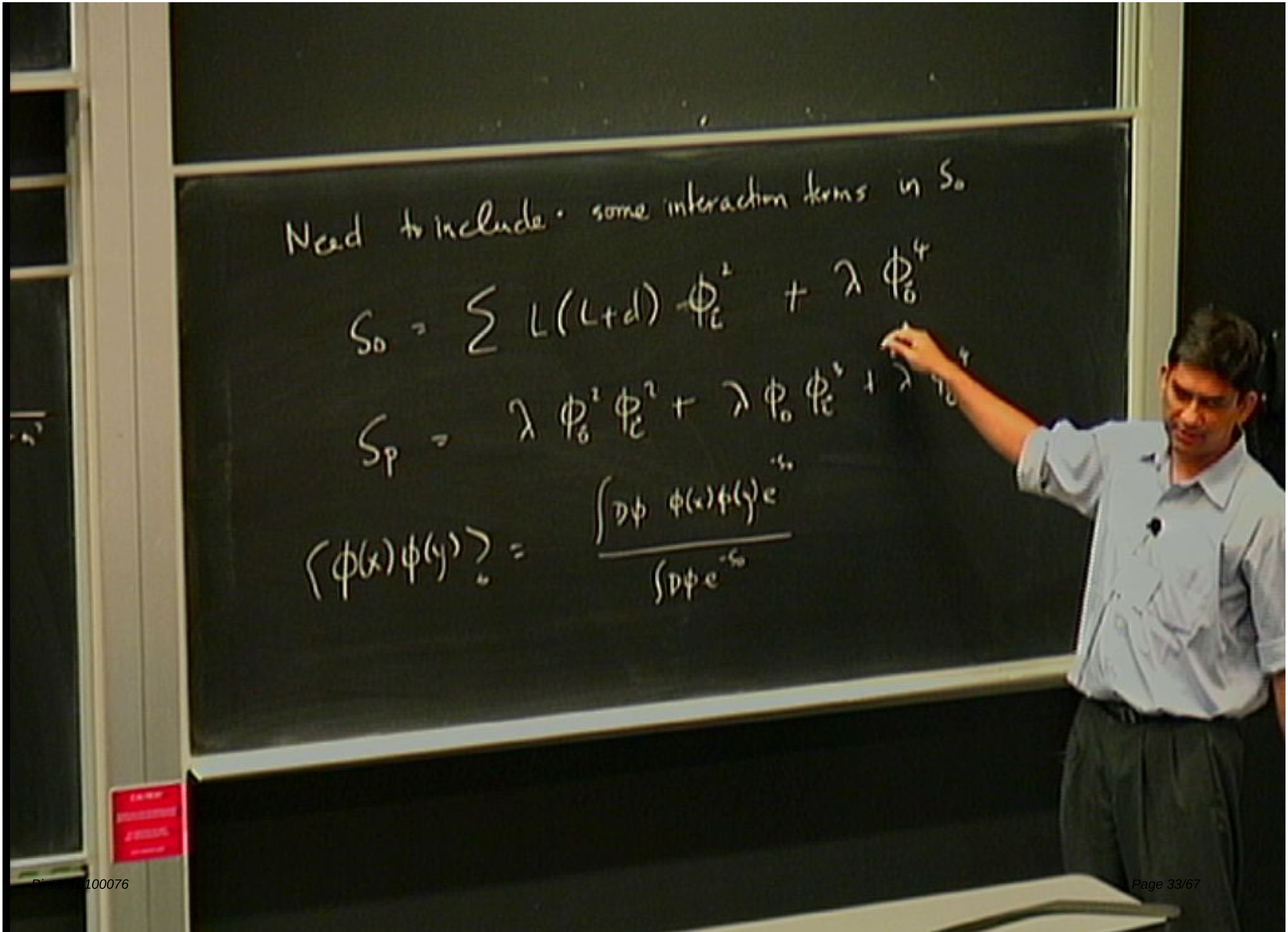

⟨φ φ⟩ = 1 / L(L+d) + m²
L(L+d) + m²
divergent due to
If m² → 0


$\int D\phi\, e^{-S_0}$
$L(L+d)+m^2$
$\langle \phi_{\bar{L}} \; \phi_{\bar{L}} \rangle = \frac{1}{L(L+d)+m^2}$
L_1
L_2
L
L
L_3
$\sum_{L_1 L_2 L_3} \frac{|\lambda|^2}{L_1(L_1+d)+m^2} \cdot \frac{1}{L_2(L_2+d)+m^2} \frac{1}{L_3(L_3+d)+m^2}$
If $m^2 \to 0$. divergent due to $\bar{L}=0$
Reason: ϕ_0 no longer appears in S_0


Need to include some interaction terms in S_0
$S_0 = \sum L(L+d)\,\phi_L^2$


Need to include some interaction terms in S_0
$S_0 = \sum L(L+d)\,\phi_L^2 + \lambda\,\phi_L^4$

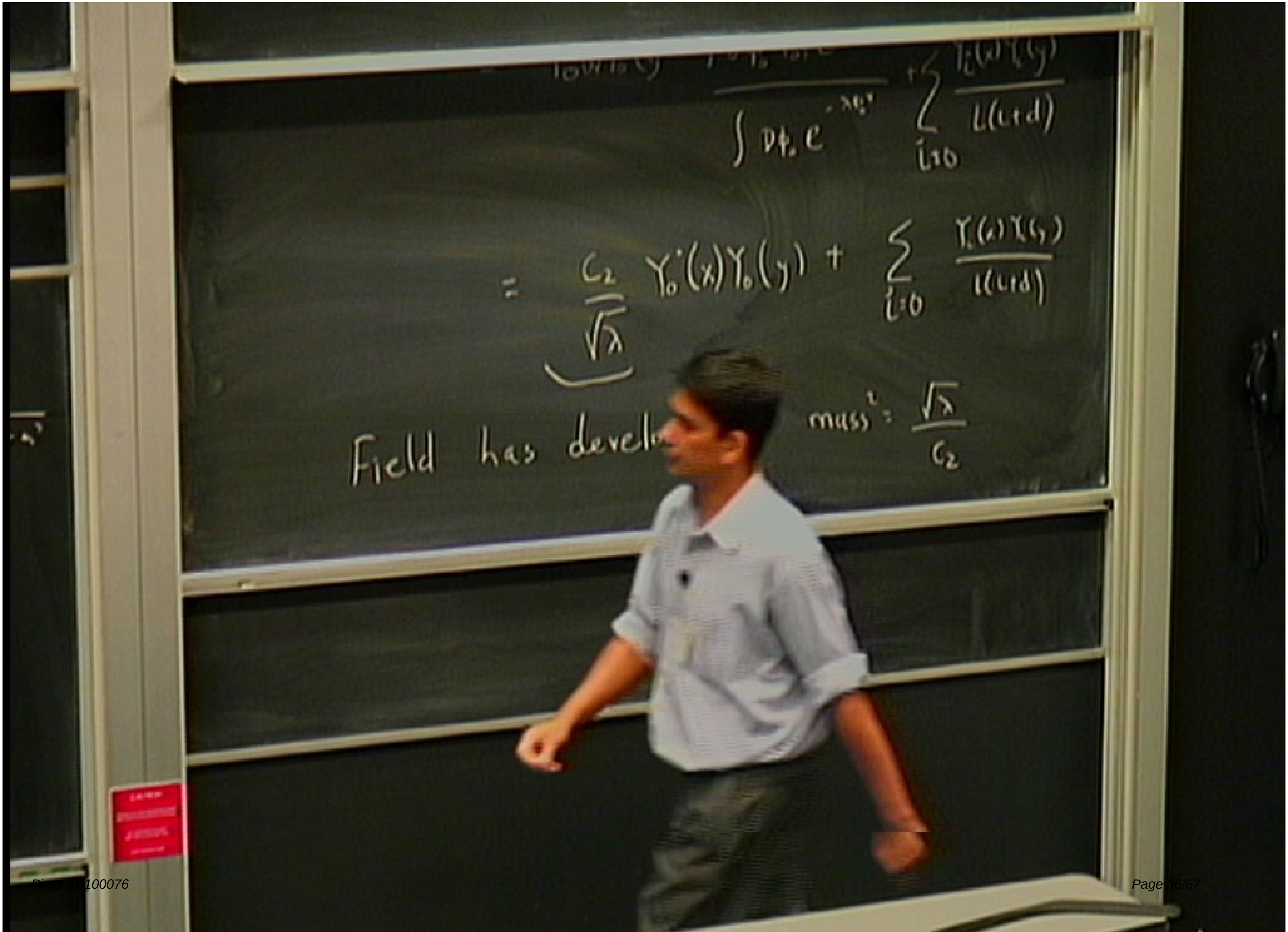

Need to include some interaction terms in S_0
$S_0 = \sum L(L+d)\, \phi_L^2 + \lambda \phi_0^4$
$S_p = \lambda \phi_0^2 \phi_L^2 + \lambda \phi_0 \phi_L^3 + \lambda \phi_L^4$

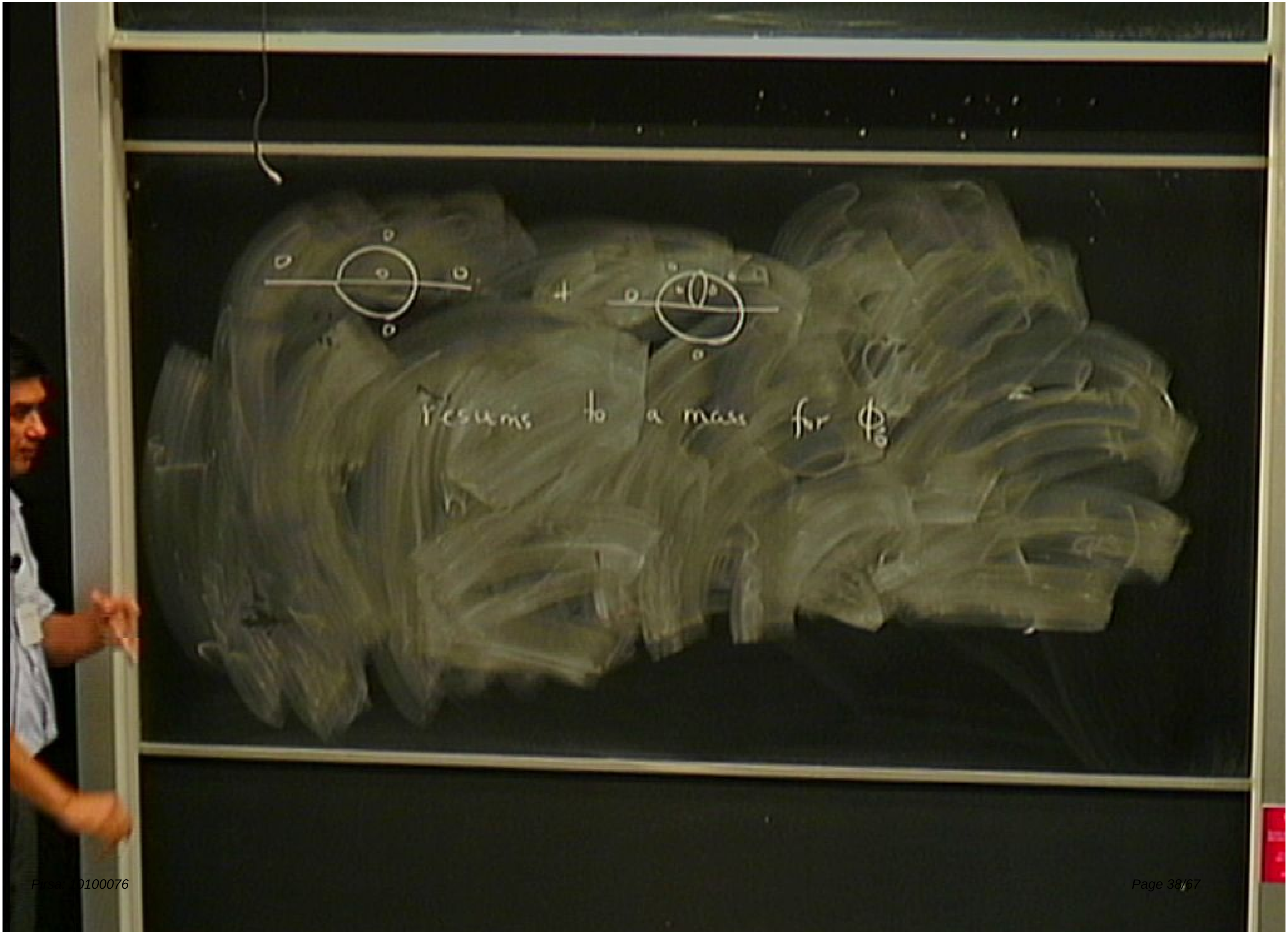

Need to include some interaction terms in S_0
$S_0 = \sum L(L+d) \phi_L^2 + \lambda \phi_0^4$
$S_p = \lambda \phi_0^2 \phi_L^2 + \lambda \phi_0 \phi_L^3 + \lambda \phi_L^4$
$\langle \phi(x)\phi(y) \rangle_0 = \frac{\int D\phi \, \phi(x)\phi(y) e^{-S_0}}{\int D\phi \, e^{-S_0}}$

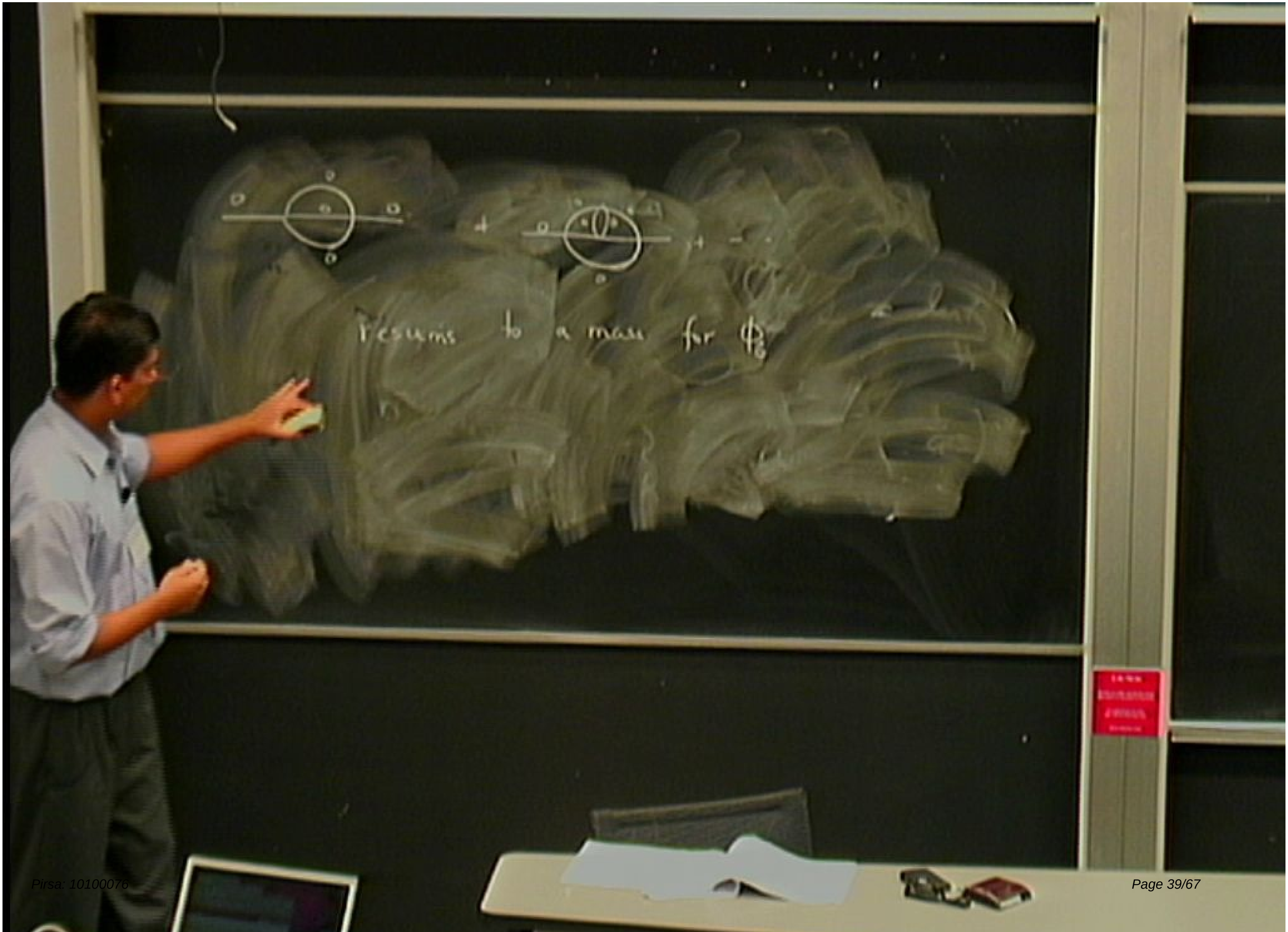
$$= Y_0^*(x)Y_0(y)\,\frac{\int D\phi_0\,\phi_0^2\,e^{-\lambda\phi_0^4}}{\int D\phi_0\,e^{-\lambda\phi_0^4}} + \sum_{l\neq 0}\frac{Y_l^*(x)Y_l(y)}{l(l+d)}$$

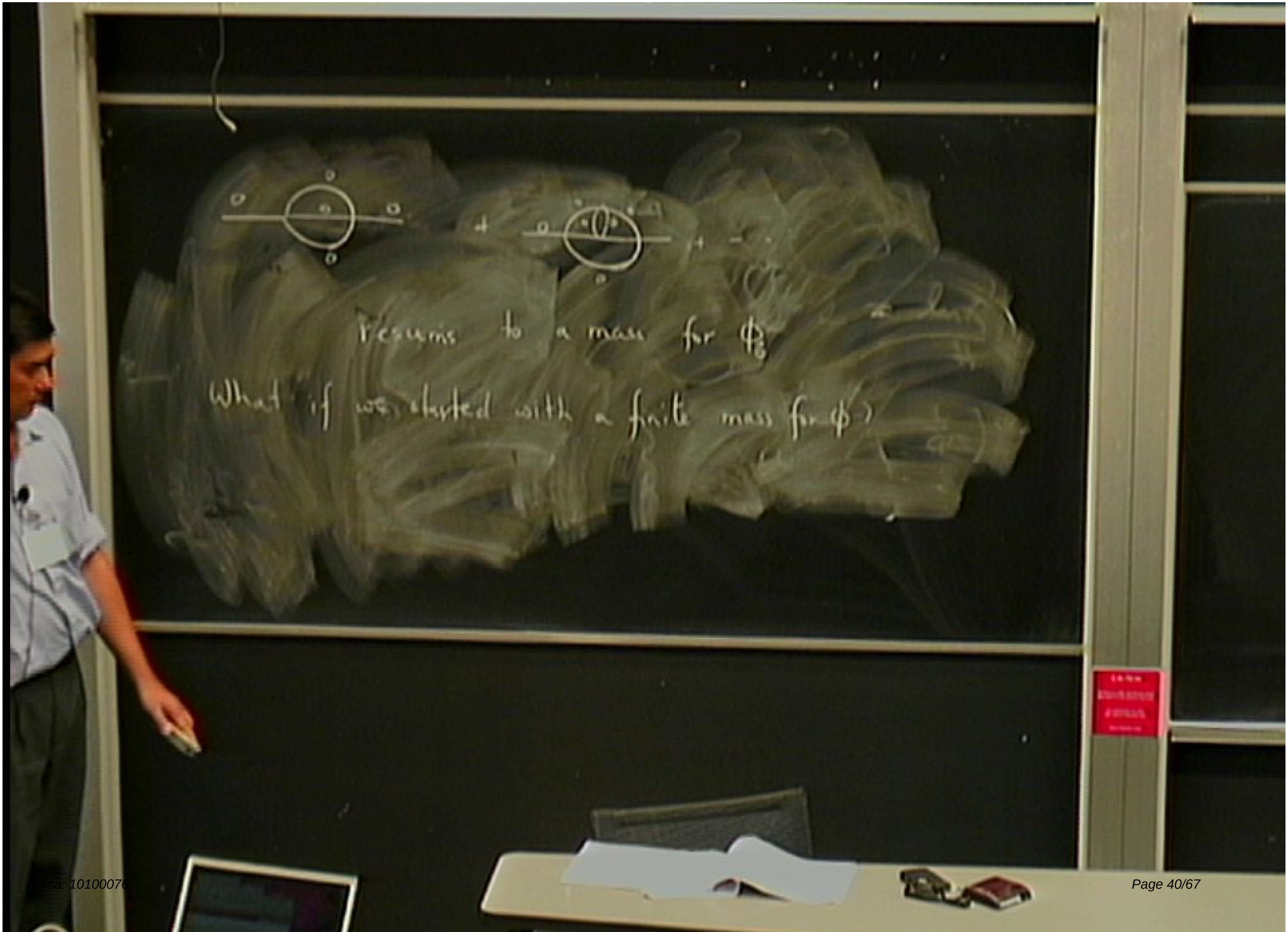
$$= Y_0^*(x) Y_0(y) \frac{\int D\phi_0\, \phi_0^2\, e^{-\lambda\phi_0^4}}{\int D\phi_0\, e^{-\lambda\phi_0^4}} + \sum_{l\neq 0} \frac{Y_l^*(x) Y_l(y)}{l(l+d)}$$

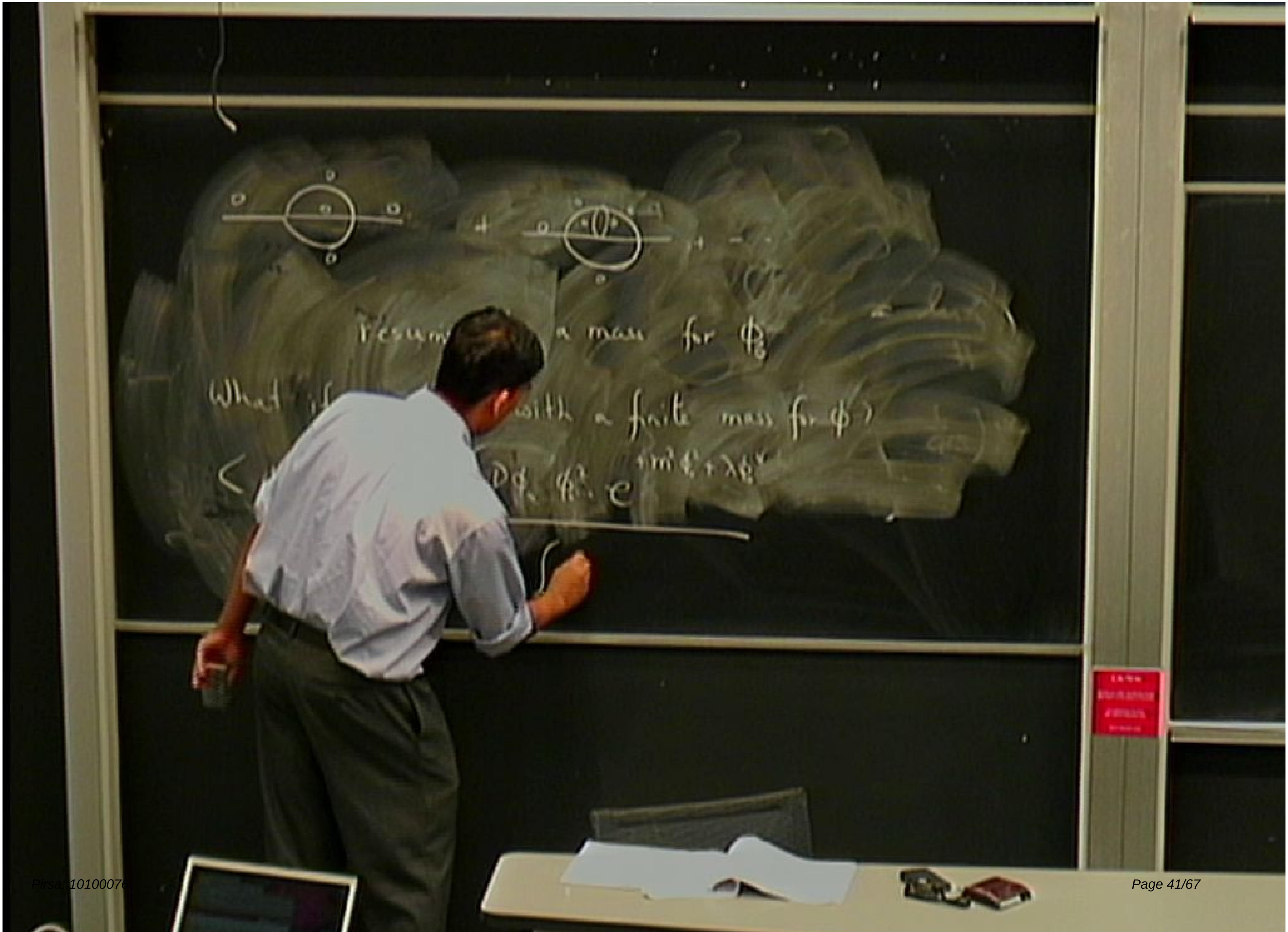
$$= \frac{c_2}{\sqrt{\lambda}}\, Y_0^*(x) Y_0(y) + \sum_{l\neq 0} \frac{Y_l^*(x) Y_l(y)}{l(l+d)}$$

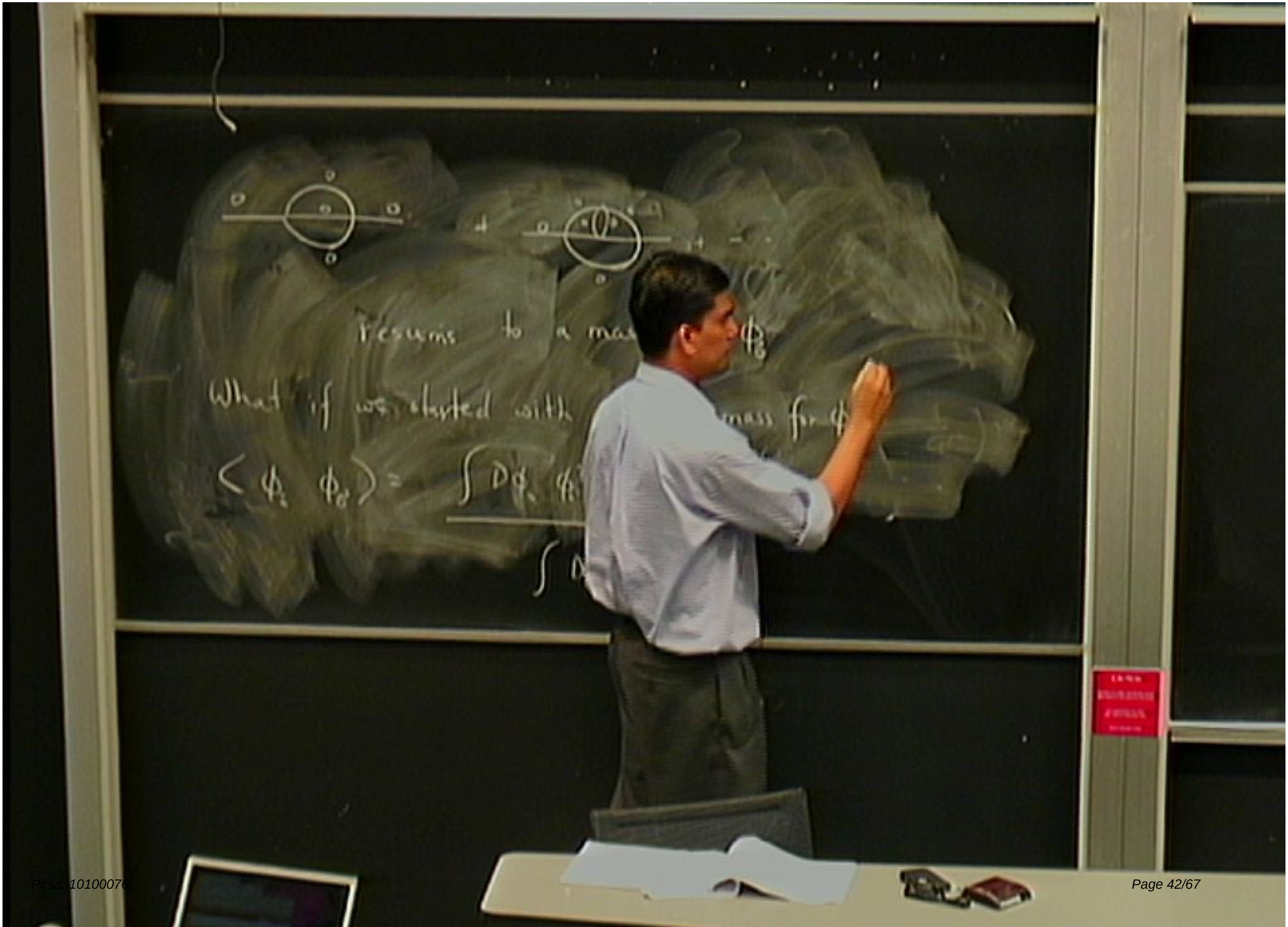

$\int D\phi_0 \, e^{-\lambda\phi_0^4}$
$+\sum_{l\neq 0} \frac{Y_l(x)Y_l(y)}{l(l+d)}$
$= \frac{C_2}{\sqrt{\lambda}} Y_0^*(x)Y_0(y) + \sum_{l\neq 0} \frac{Y_l(x)Y_l(y)}{l(l+d)}$
Field has devel
$\text{mass}^2 = \frac{\sqrt{\lambda}}{C_2}$

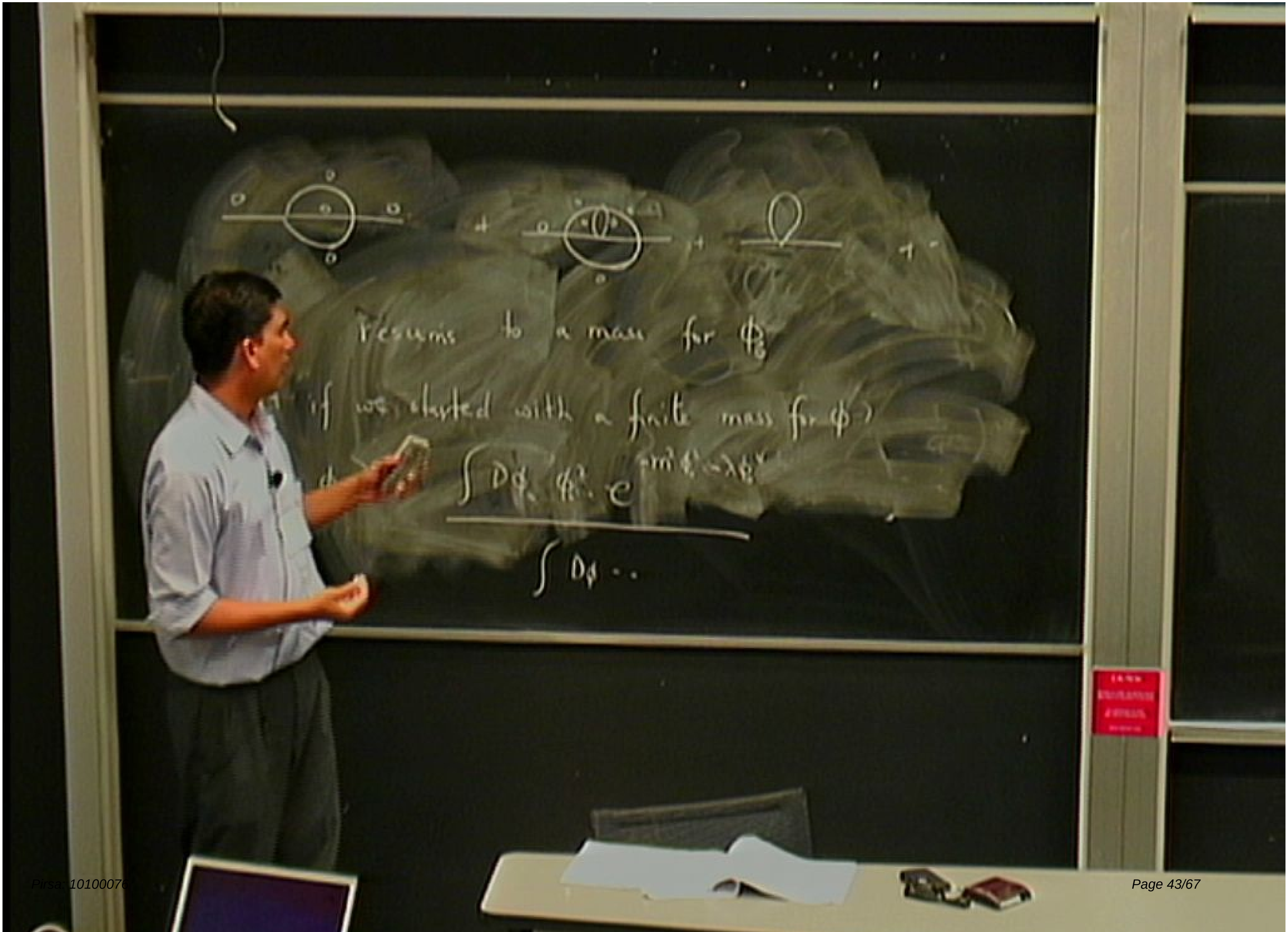

resums to a mass for ϕ_0

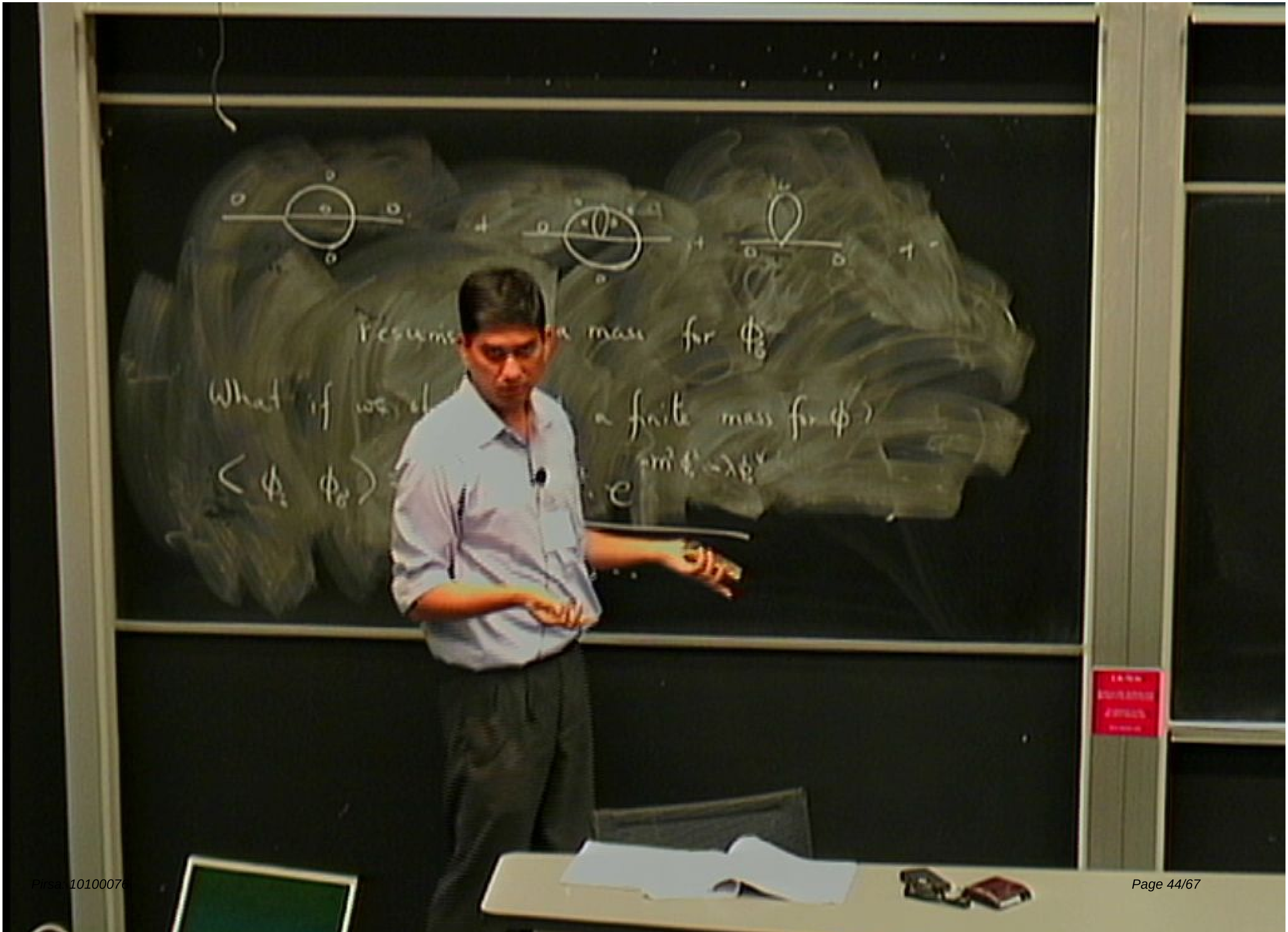

resums to a mass for

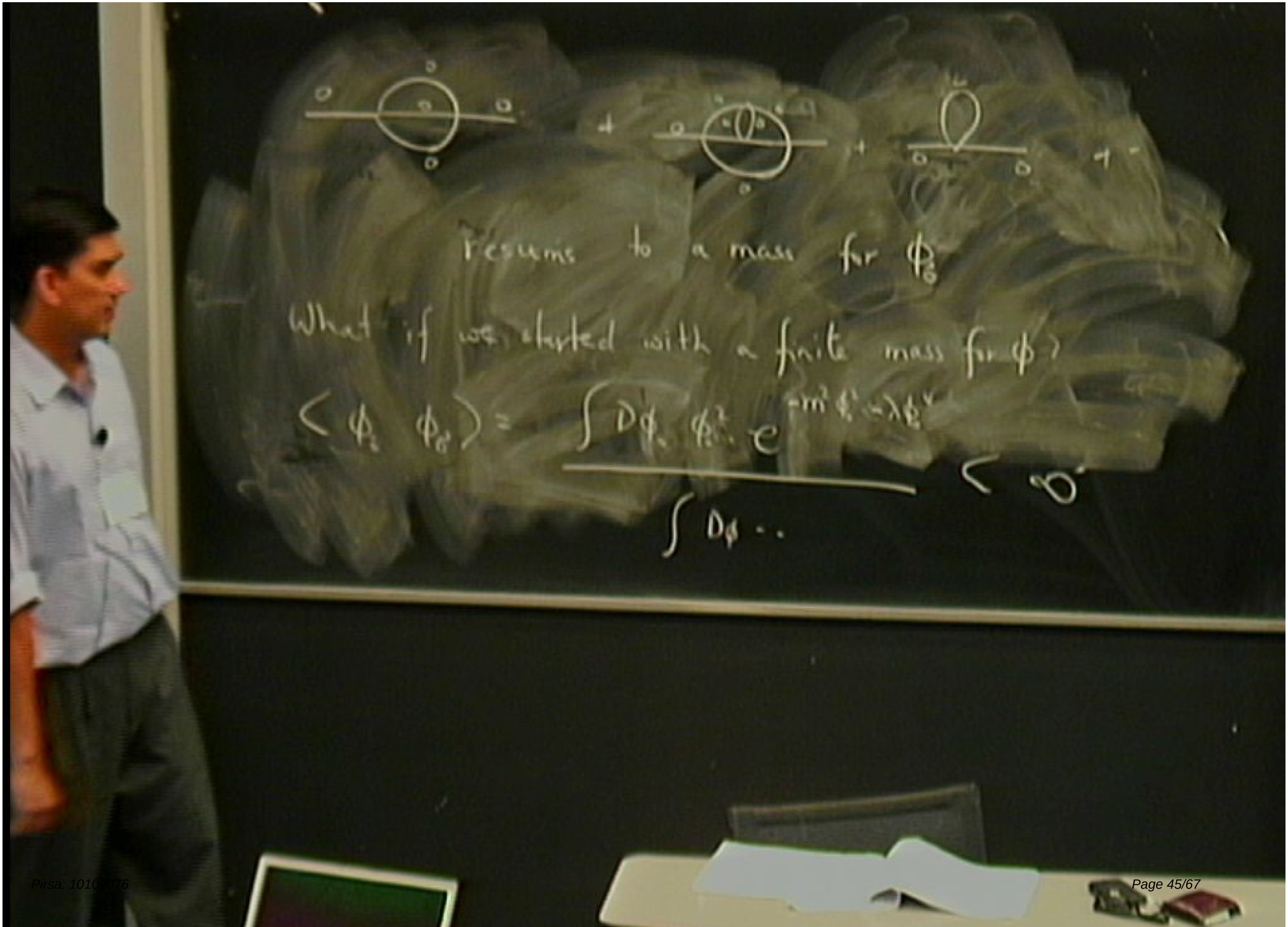

Resums to a mass for φ
What if we started with a finite mass for φ?

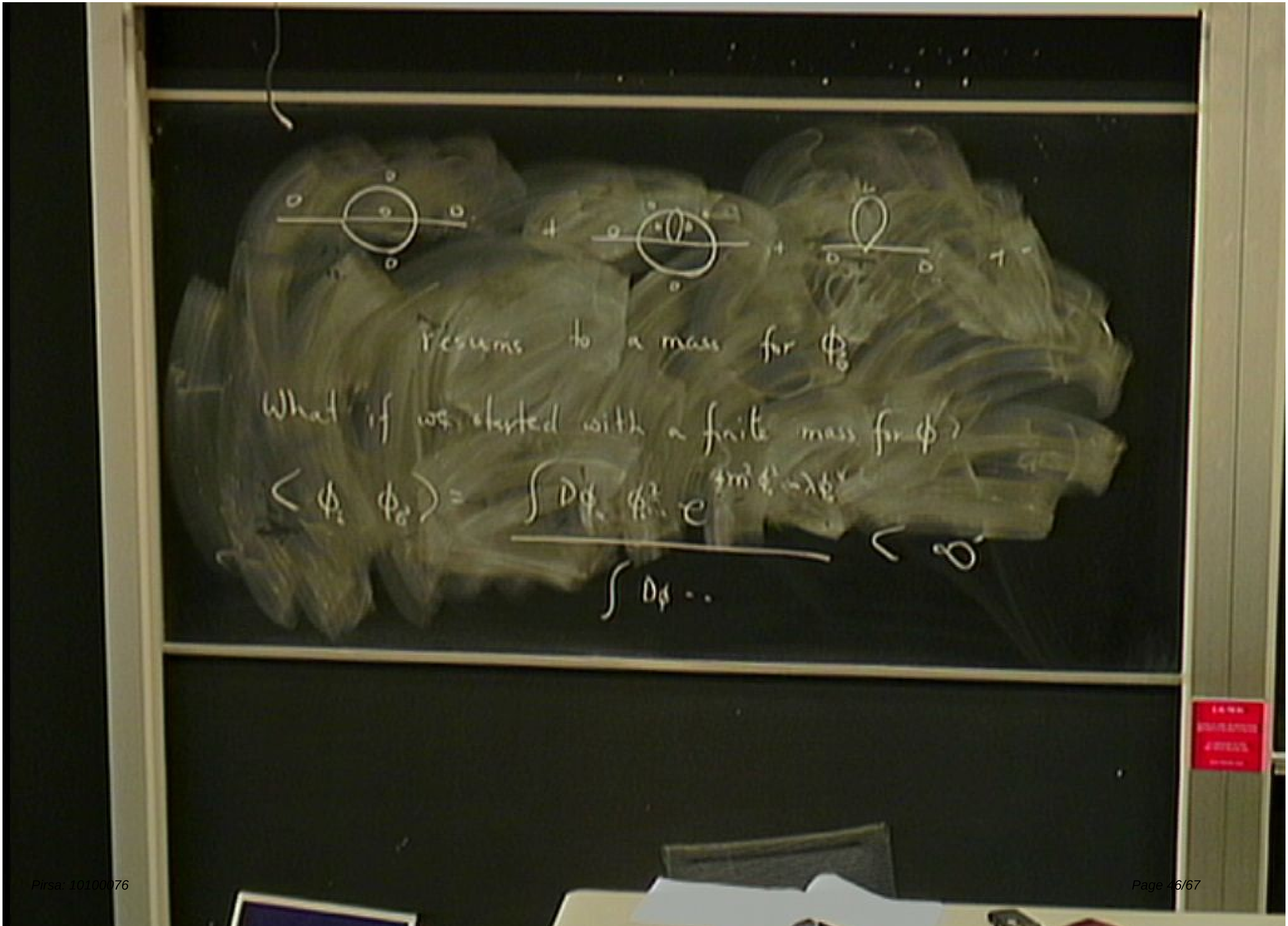

a mass for φ
What
with a finite mass for φ?

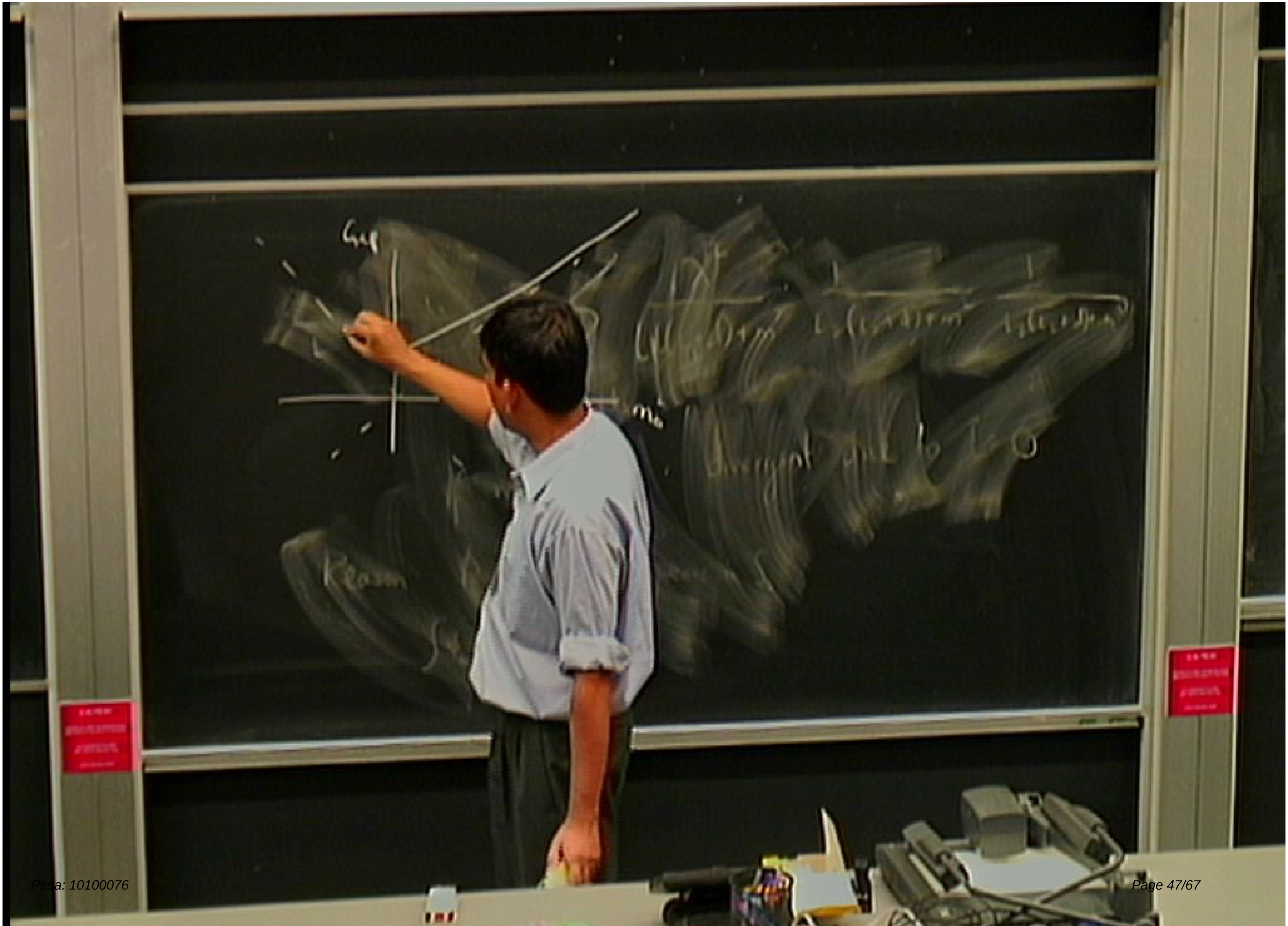

Resums to a
What if we started with

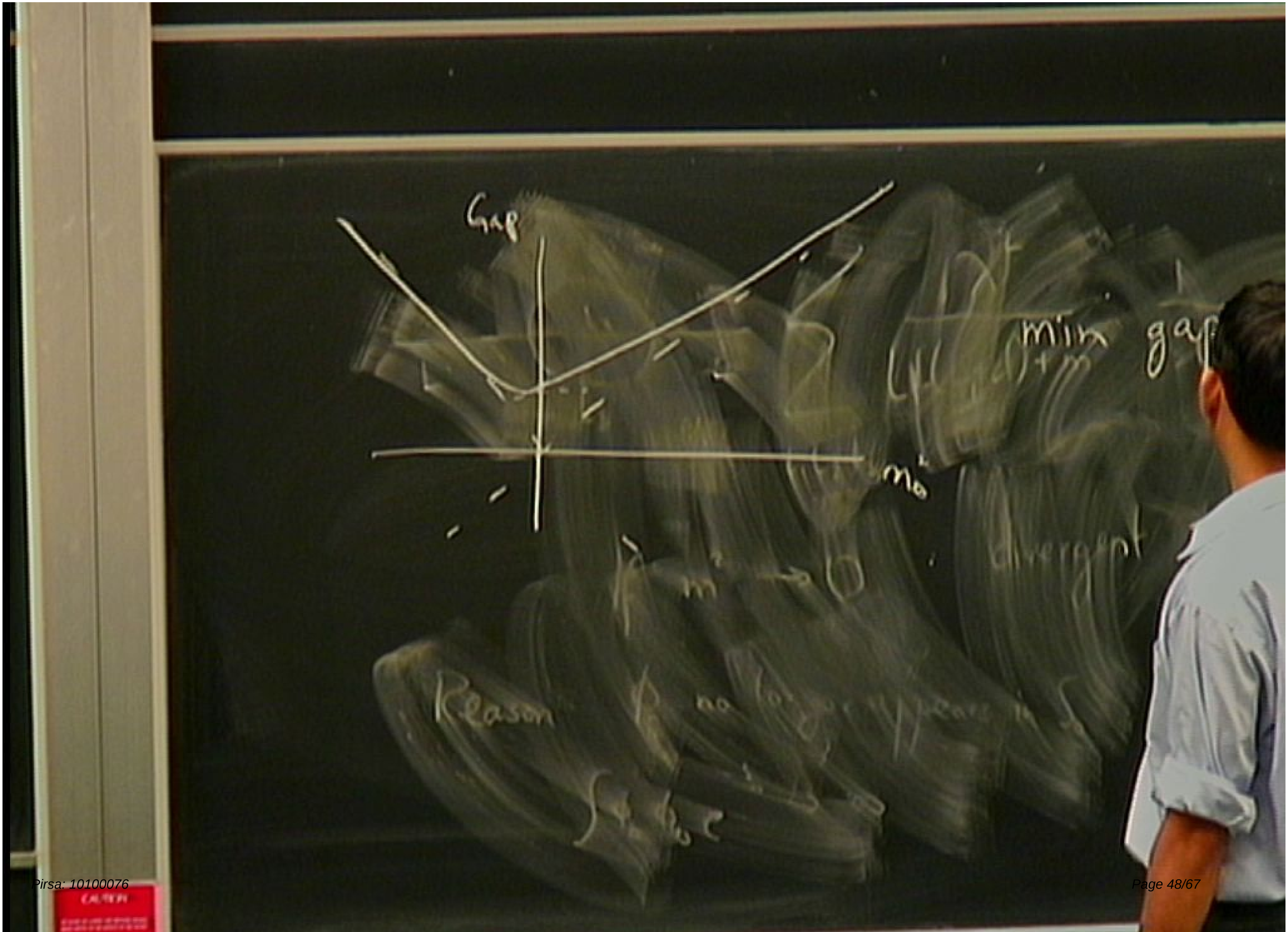

Resums to a mass for
if we started with a finite mass for

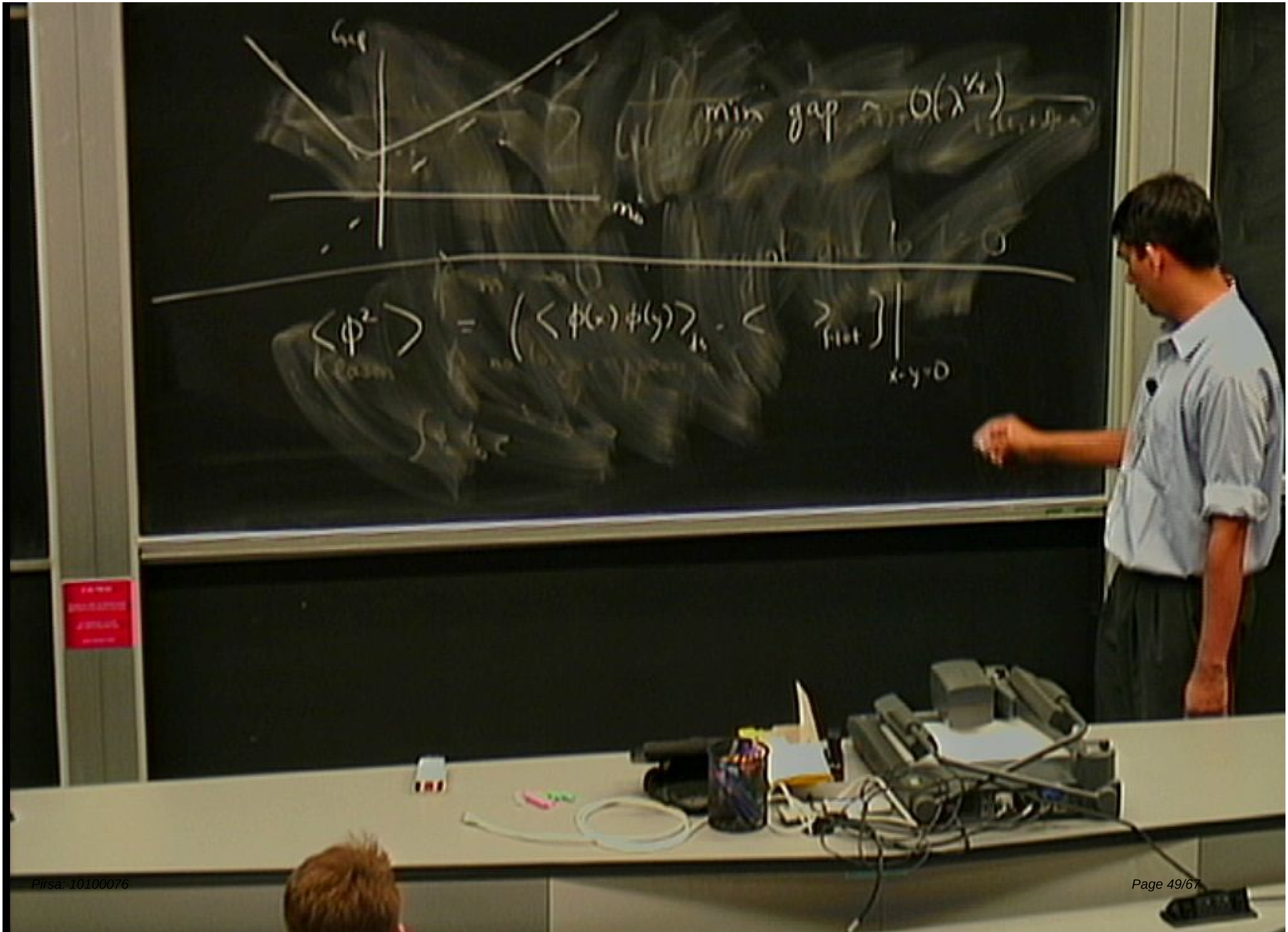

mass for
a finite mass for

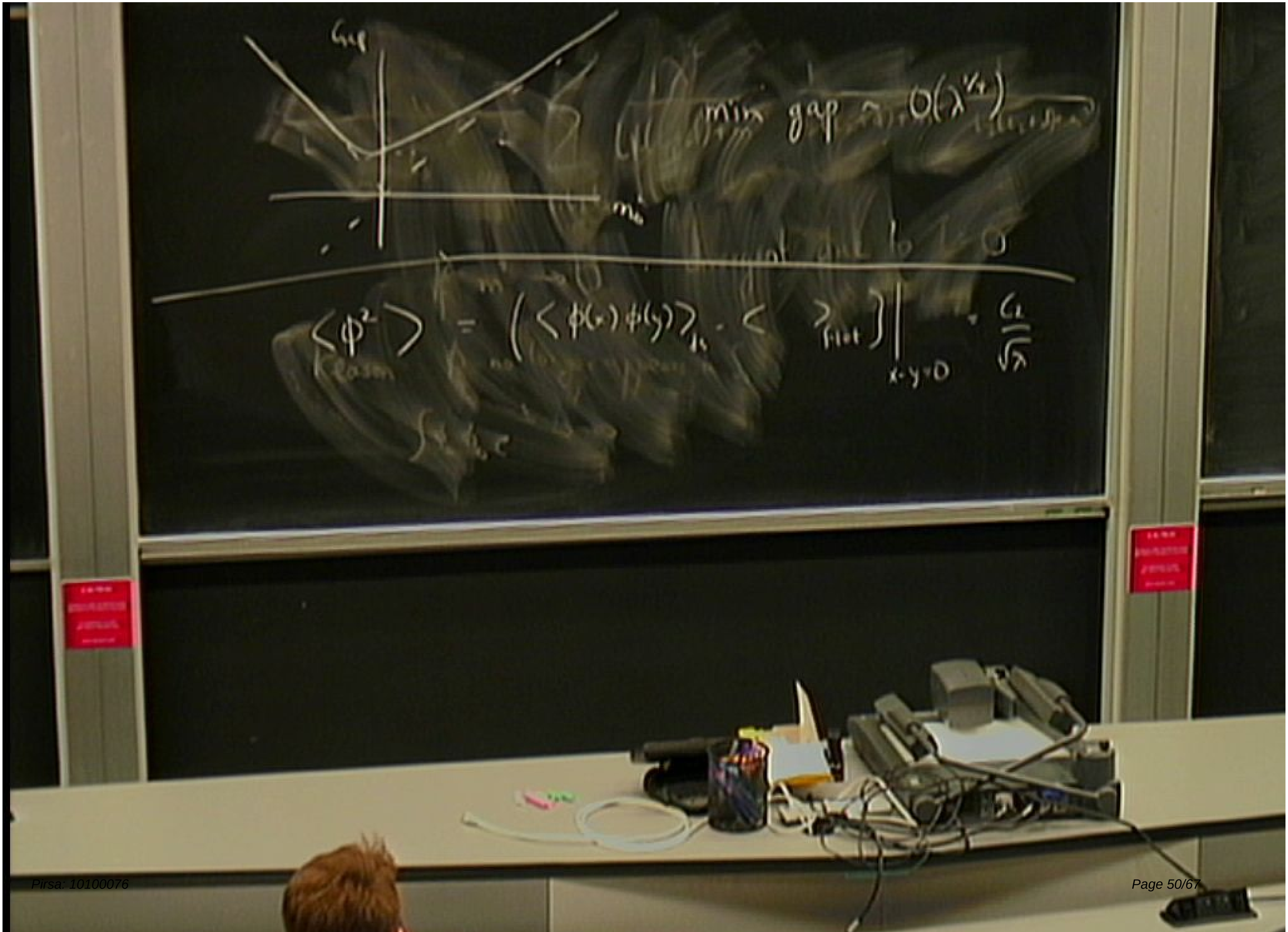

resums to a mass for ϕ_0
What if we started with a finite mass for ϕ?
$\langle \phi_0 \, \phi_0 \rangle = \dfrac{\int D\phi_0 \, \phi_0^2 \, e^{-m^2\phi_0^2 - \lambda\phi_0^4}}{\int D\phi \ldots}$

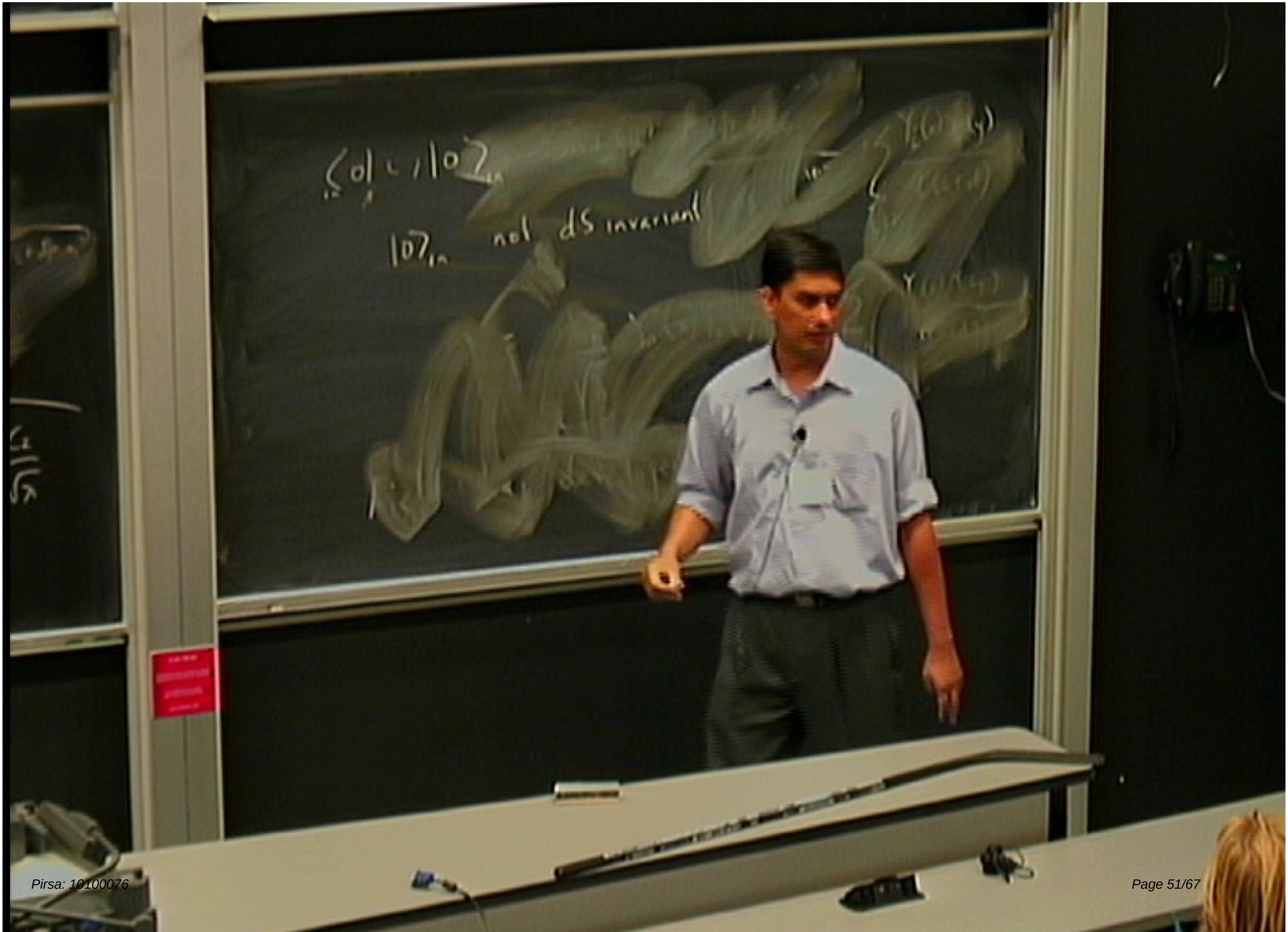

Resums to a mass for φ0
What if we started with a finite mass for φ?
∫Dφ ..



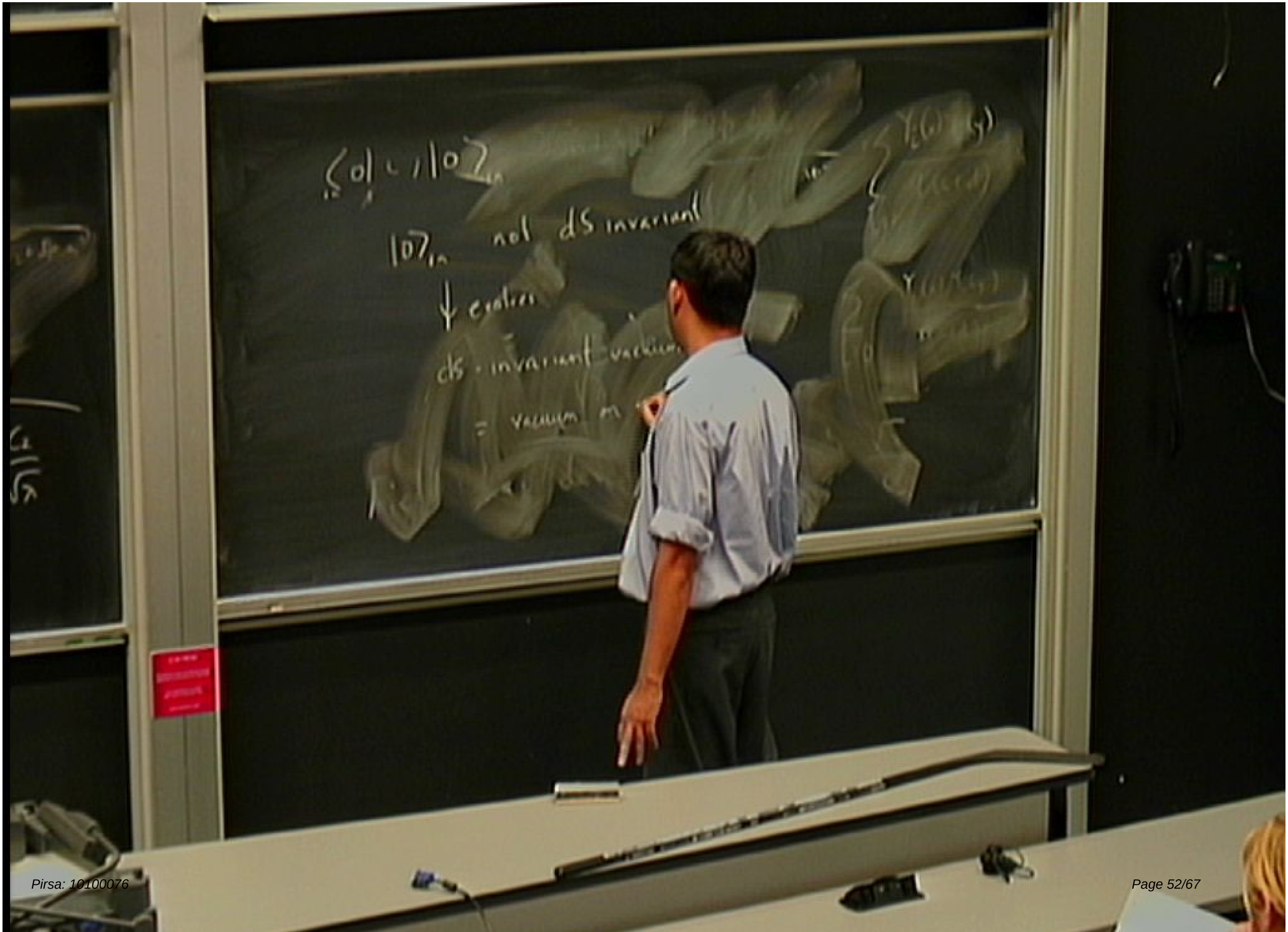

Gap
min gap
Reason

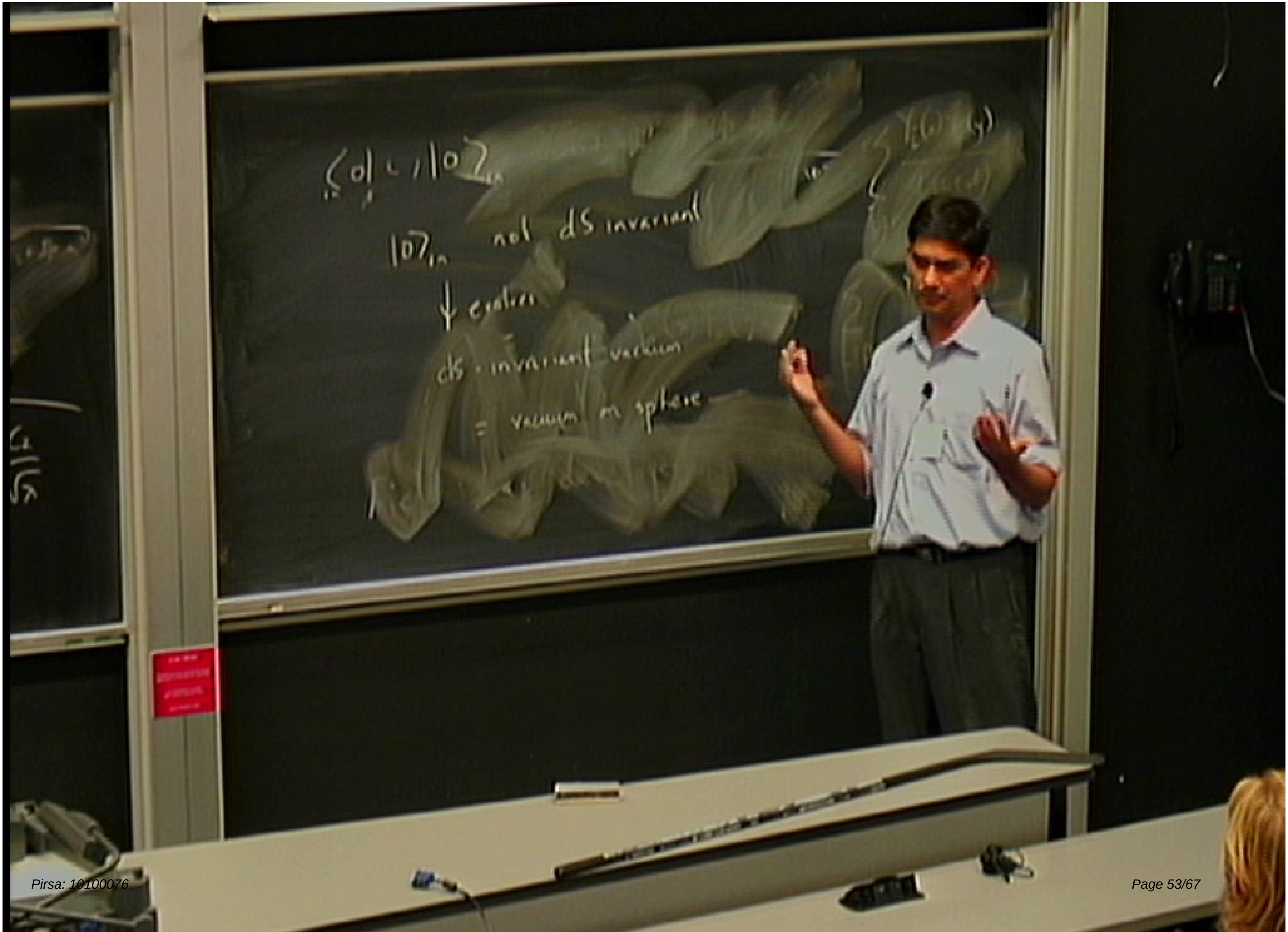

gap
min gap ~ O(λ^{1/4})
⟨φ²⟩ = (⟨φ(x) φ(y)⟩_{th} - ⟨ ⟩_{init}) |
x-y=0


min gap

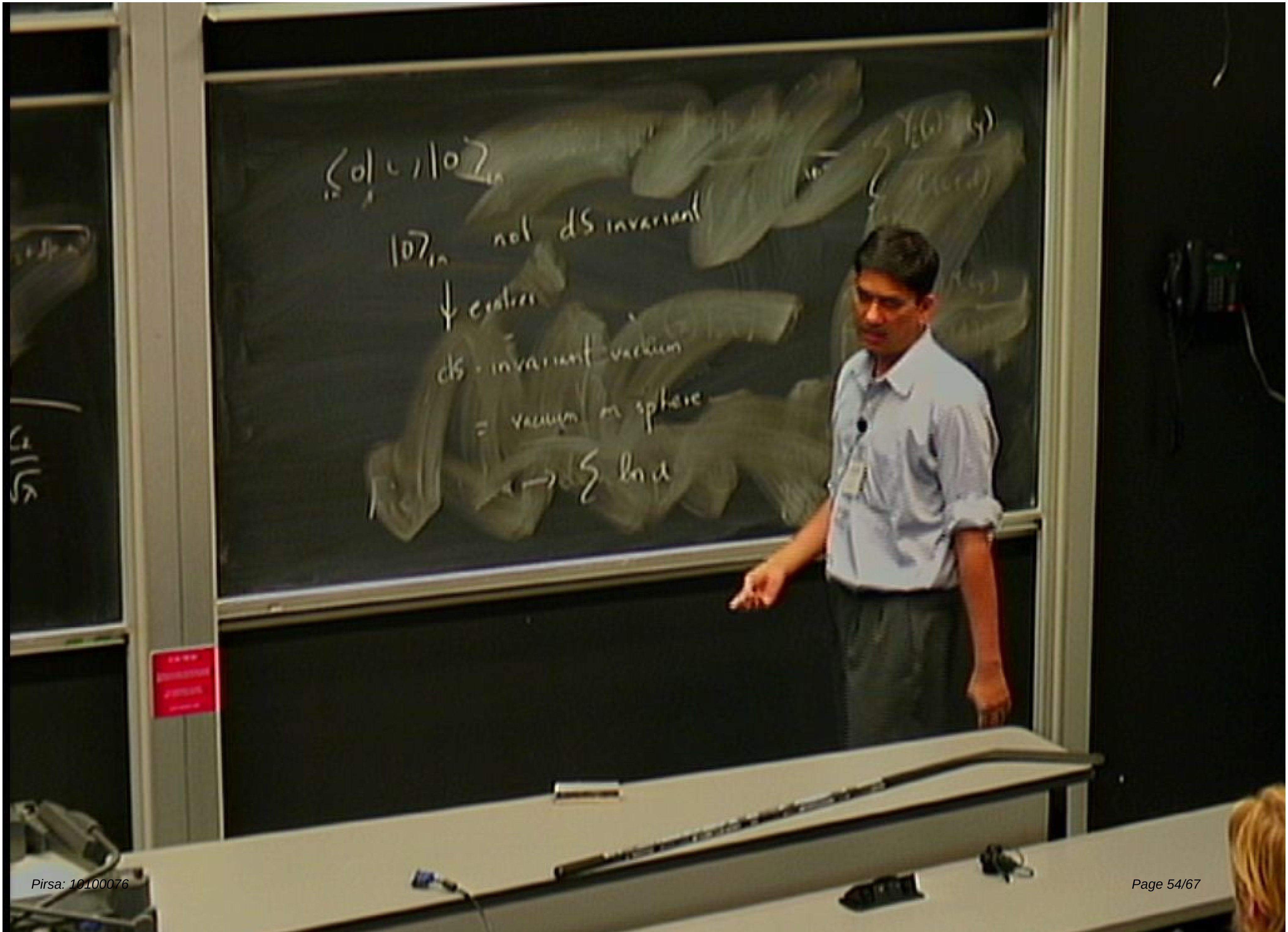


not dS invariant

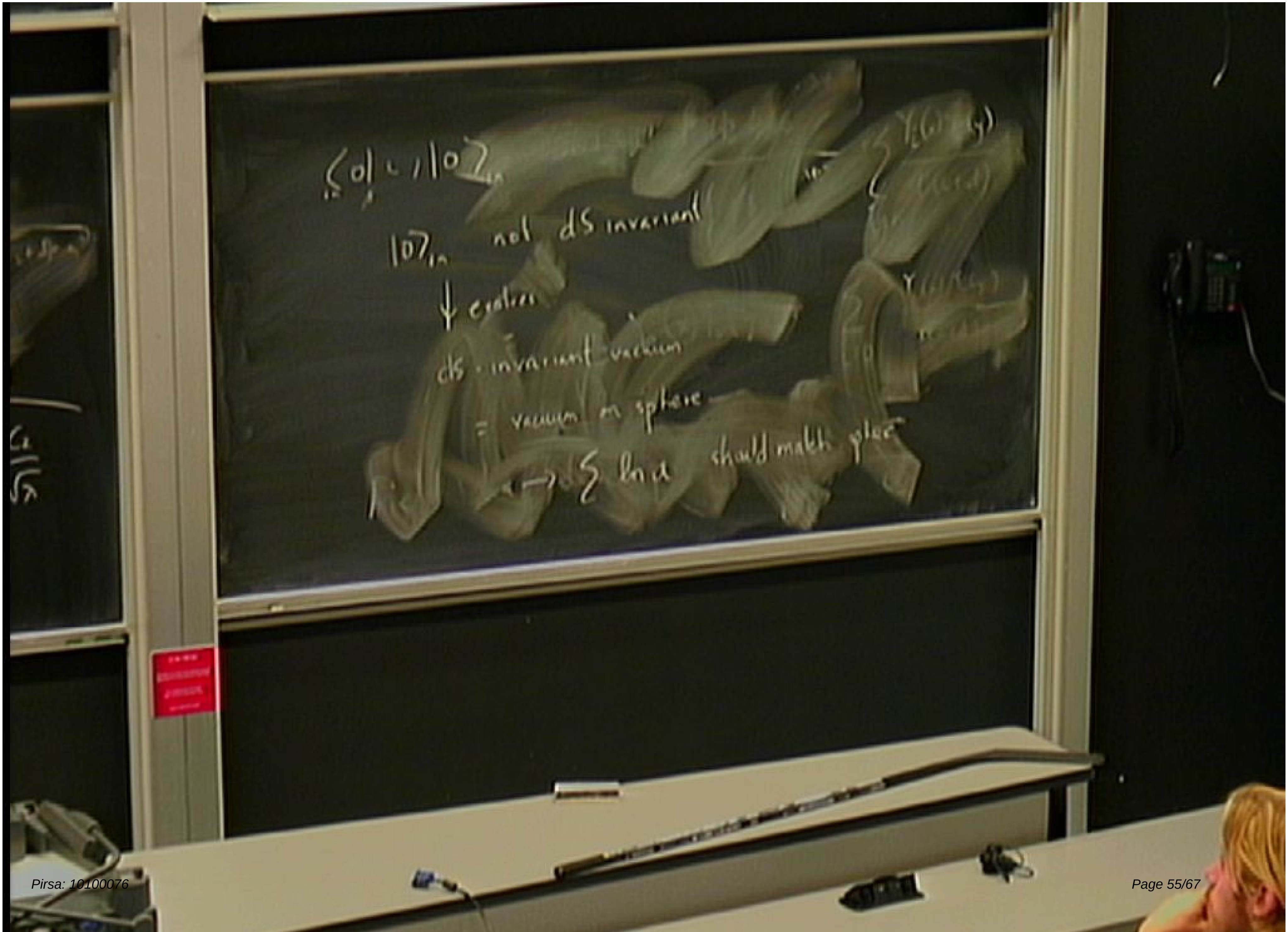

not dS invariant
evolve
dS - invariant
= vacuum

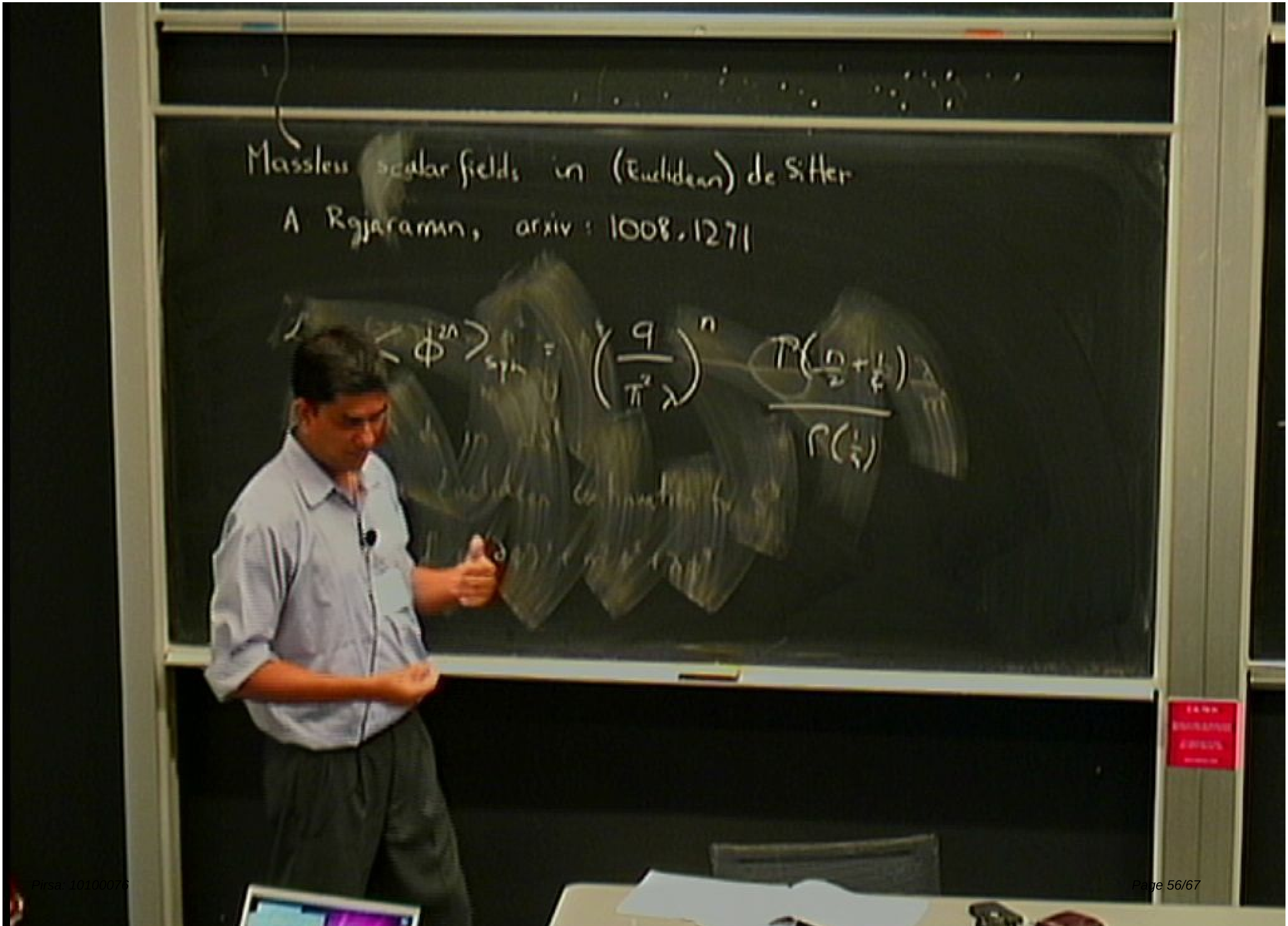


not dS invariant
evolve
dS - invariant vacuum
= vacuum on sphere



not dS invariant
dS - invariant vacuum
= vacuum on sphere


not dS invariant
dS - invariant vacuum
= vacuum on sphere
should match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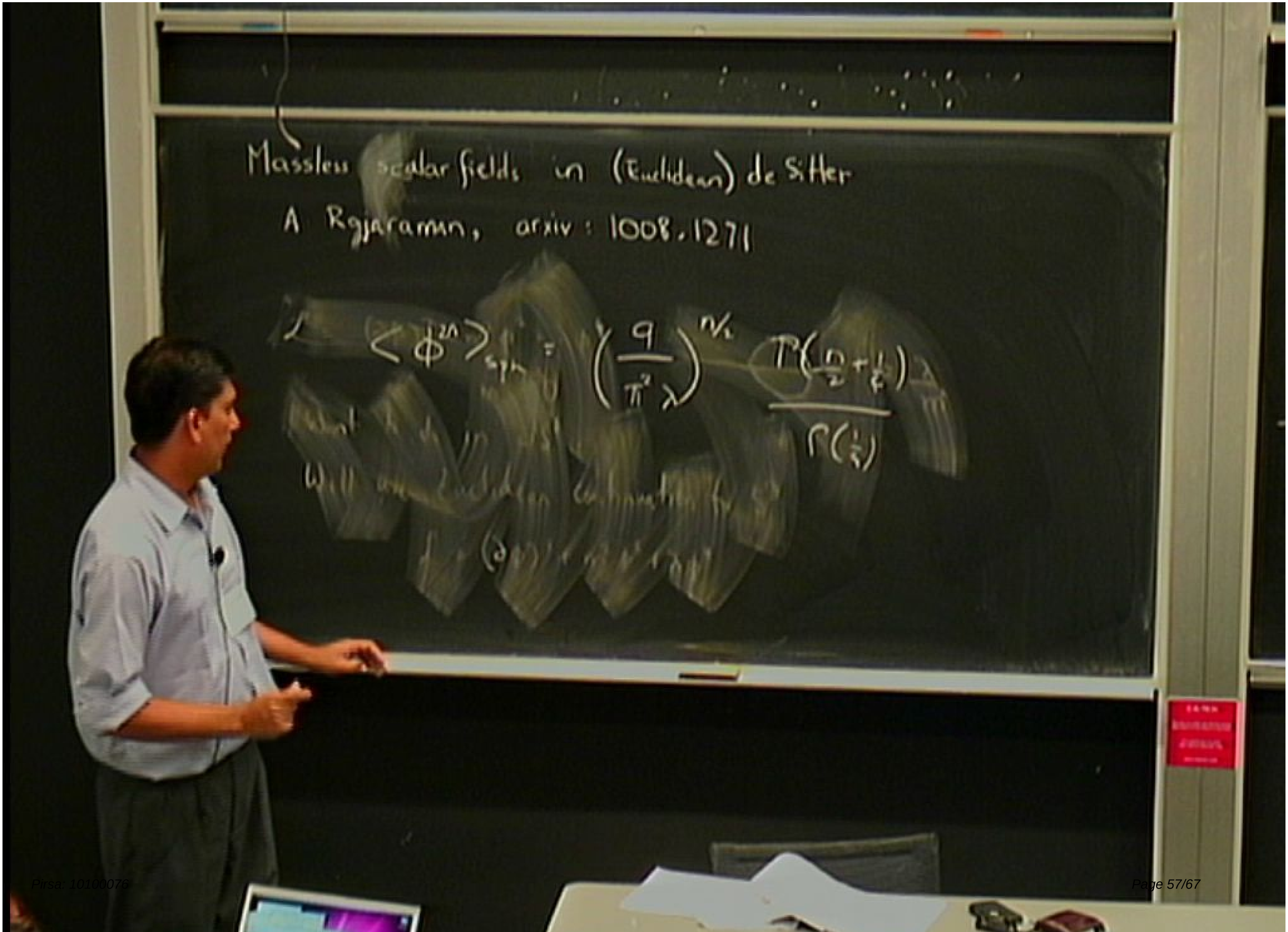
$\langle \phi^{2n} \rangle_{sym} = \left(\frac{9}{\pi^2 \lambda}\right)^n \frac{\Gamma(\frac{n}{2}+\frac{1}{4})}{\Gamma(\frac{1}{4})}$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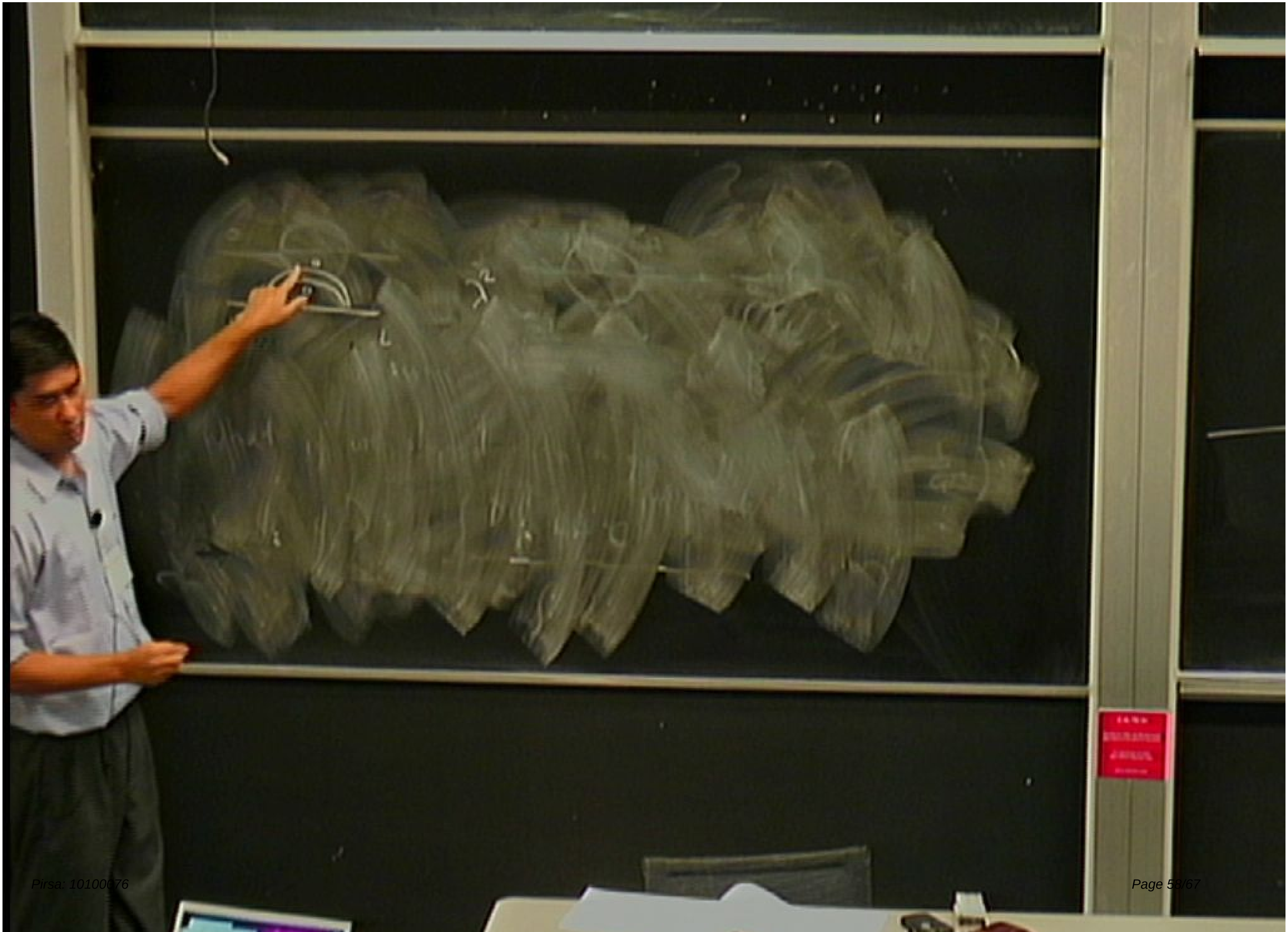
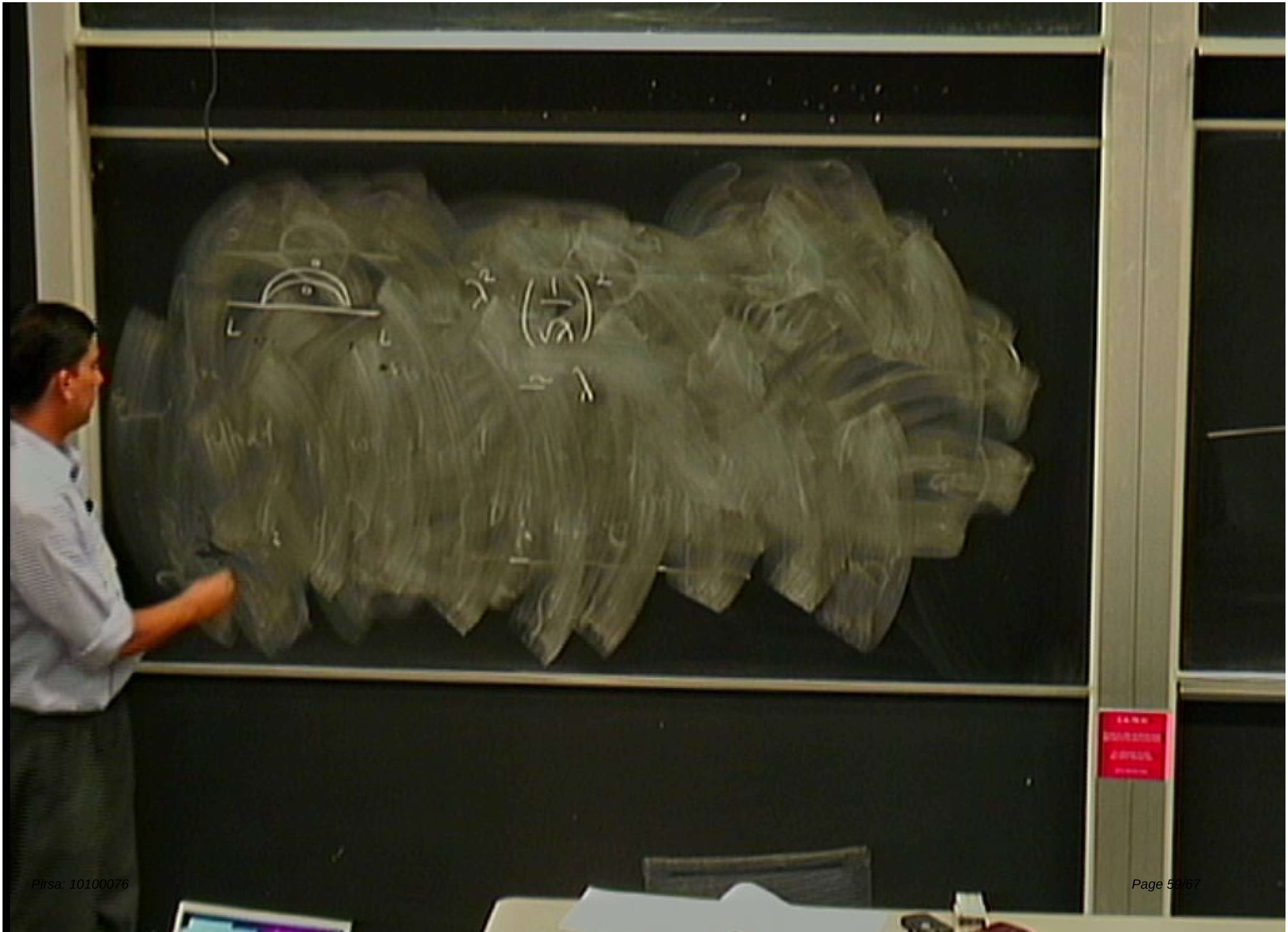
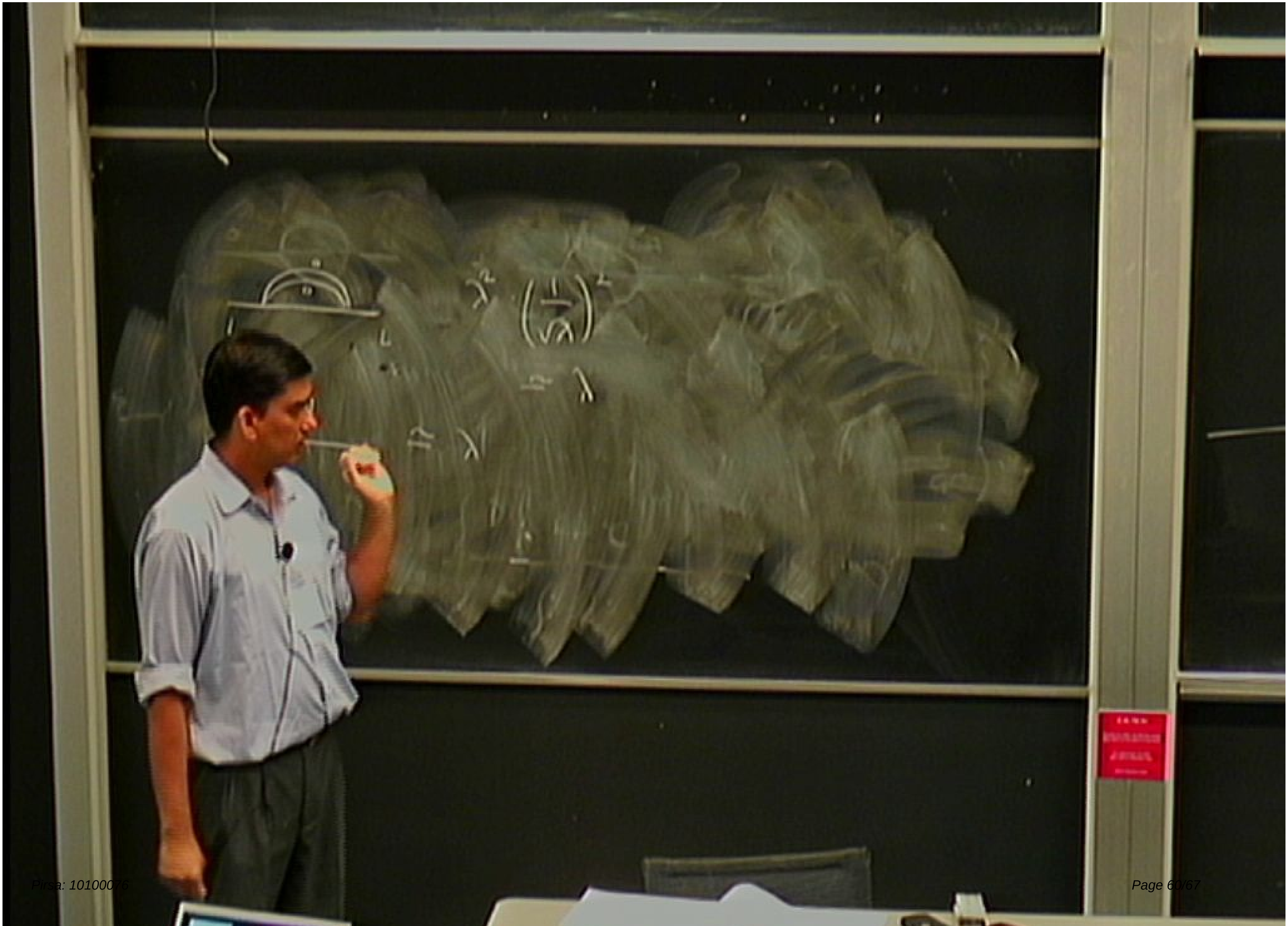
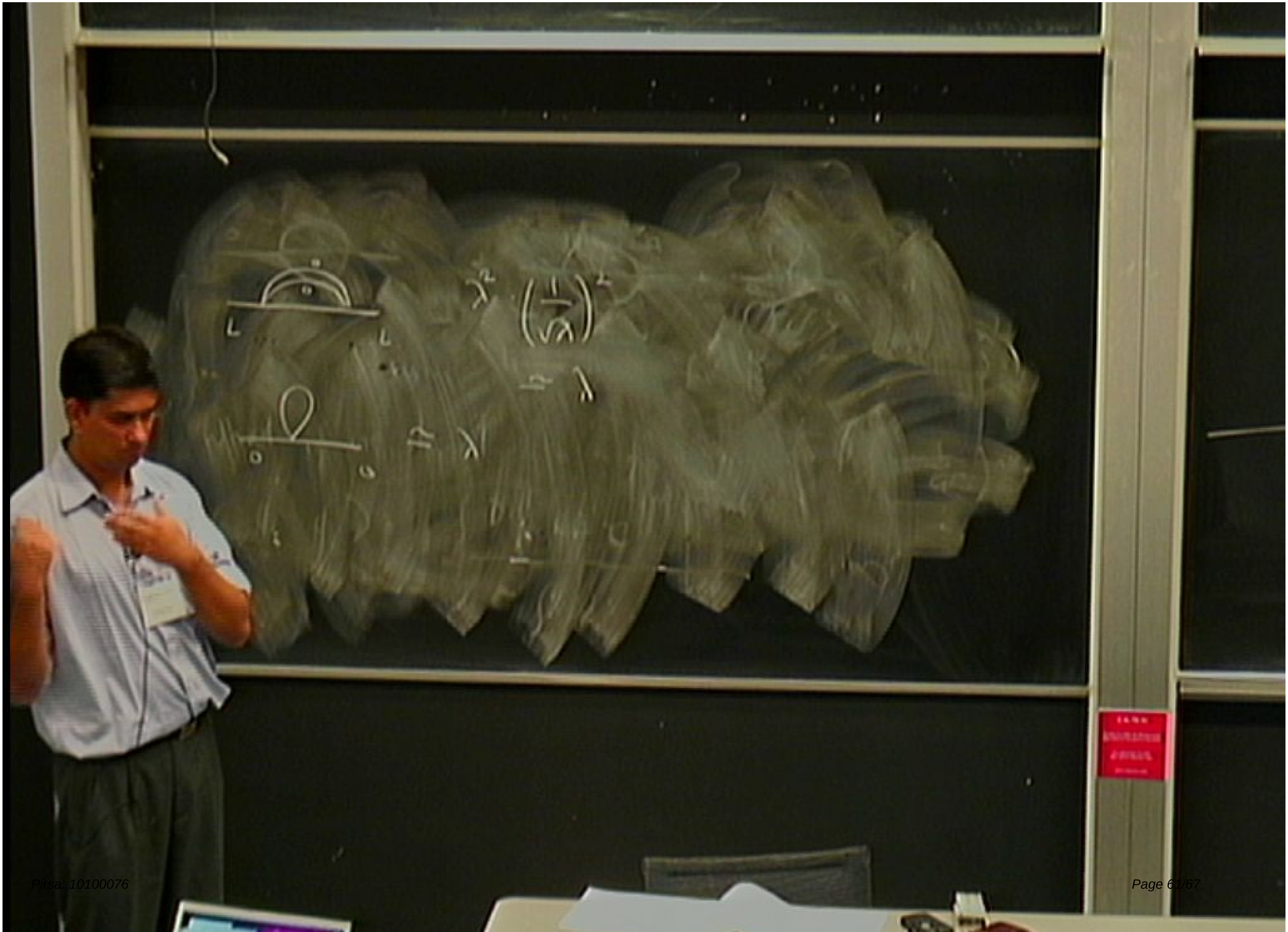
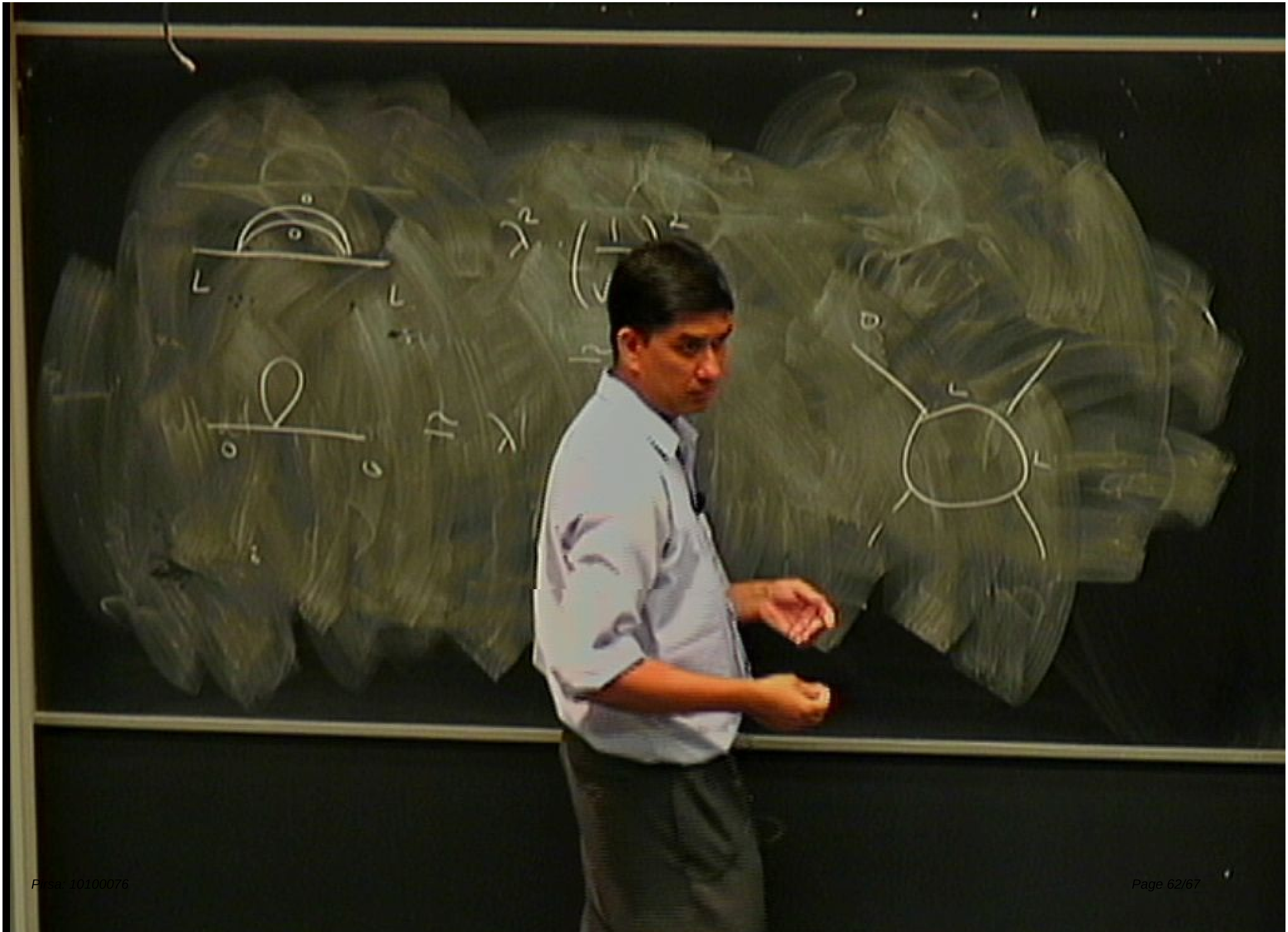
A Rajaraman, arxiv: 1008.1271
$\langle \phi^{2n} \rangle_{sph} = \left(\frac{9}{\pi^2 \lambda}\right)^{n/2} \frac{\Gamma(\frac{n}{2}+\frac{1}{4})}{\Gamma(\frac{1}{4})}$

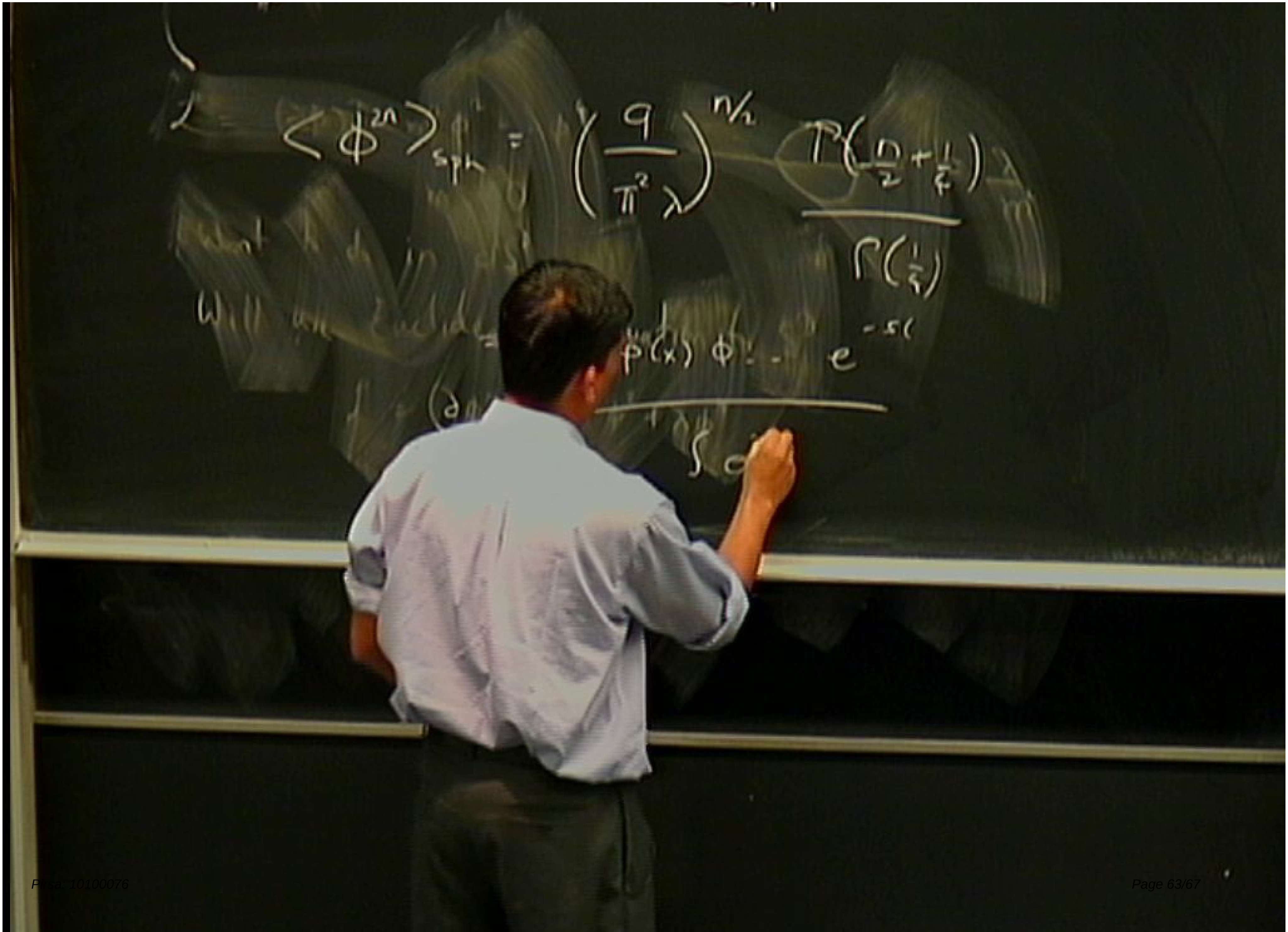


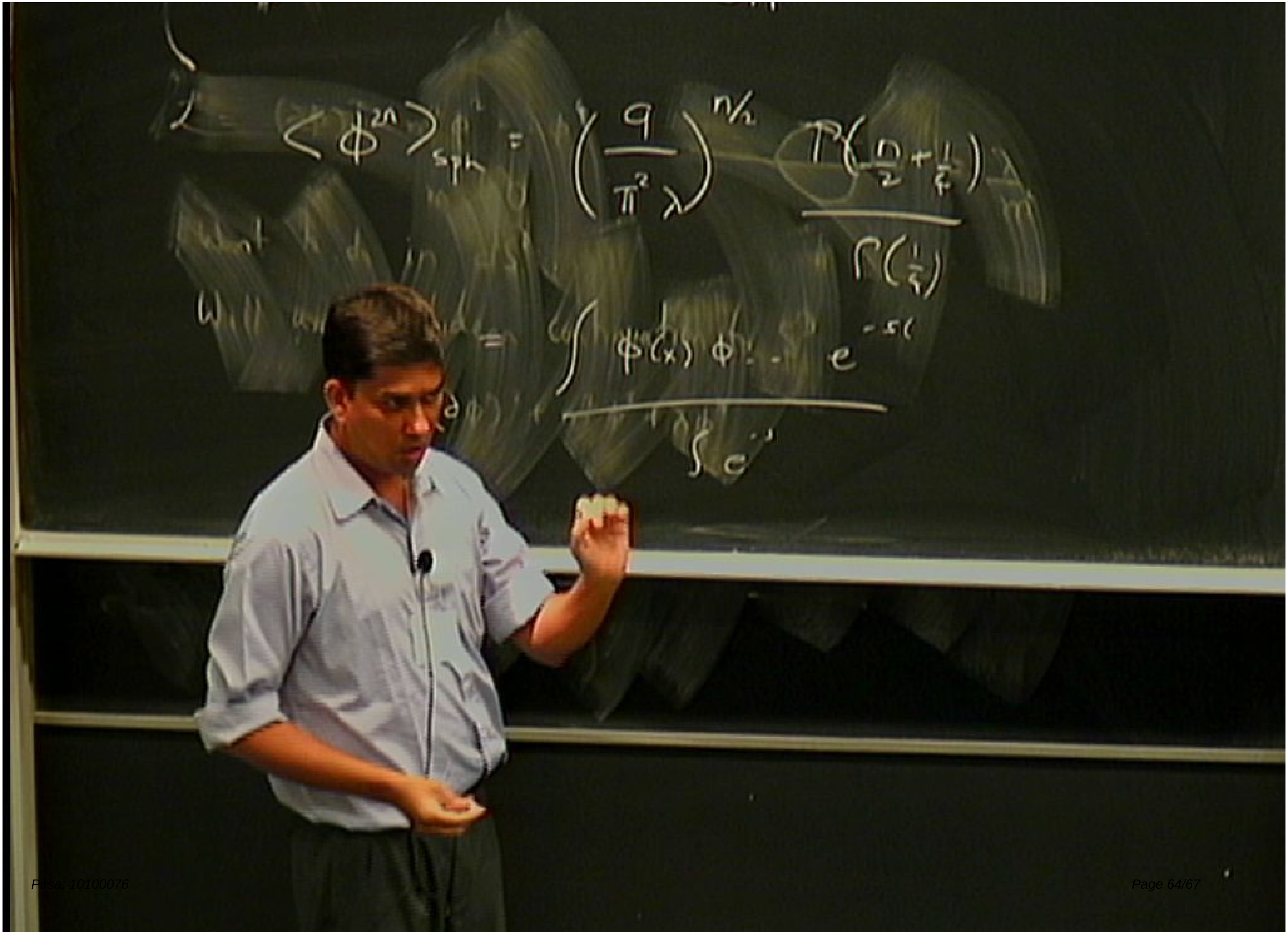


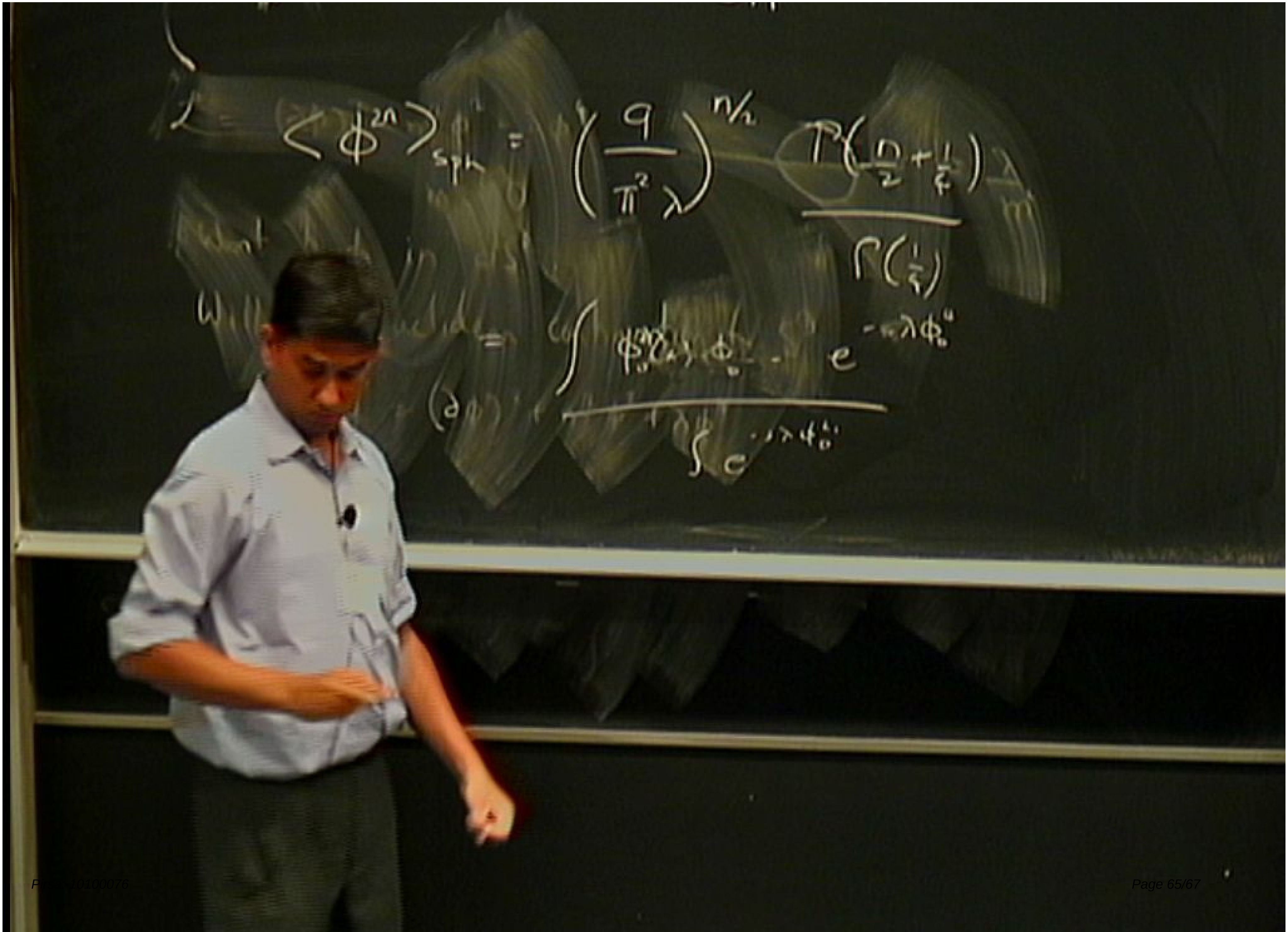







$\langle \phi^{2n} \rangle_{sph} = \left(\frac{9}{\pi^2 \lambda} \right)^{n/2} \frac{\Gamma(\frac{n}{2}+\frac{1}{4})}{\Gamma(\frac{1}{4})}$


$\langle \phi^{2n} \rangle_{sph} = \left(\frac{9}{\pi^2 \lambda}\right)^{n/2} \frac{\Gamma(\frac{n}{2}+\frac{1}{4})}{\Gamma(\frac{1}{4})}$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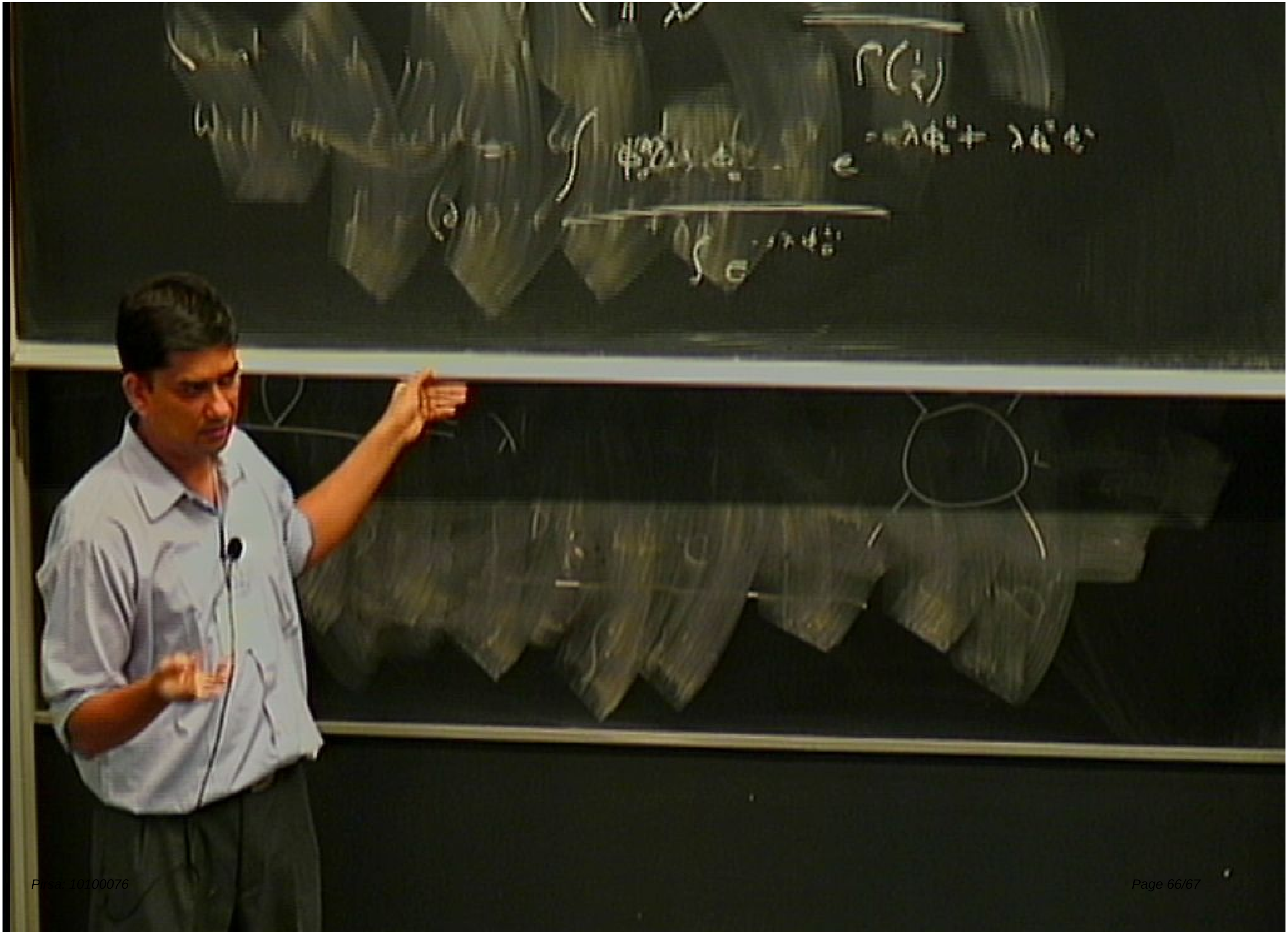

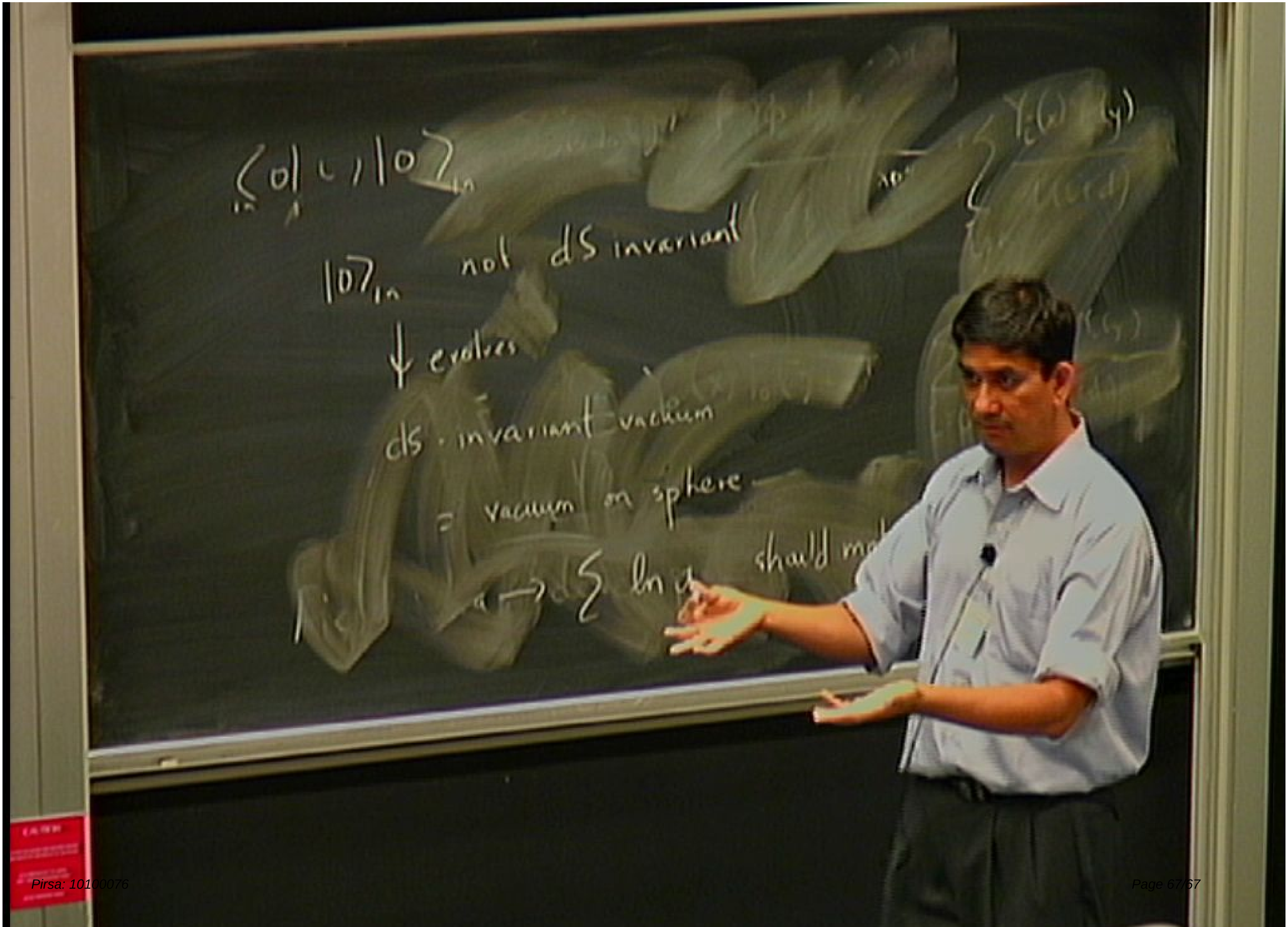
not dS invariant
evolves
dS invariant vacuum
vacuum on sphere