Title: Semiclassical approaches to IR issues in quasi de Sitter universes

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Abstract: Using simple semiclassical relations it is possible to show that the conventional cosmological correlation functions are affected by significant IR corrections in quasi de Sitter space-times when averaged over very large volumes (in the "large box"). The IR effects apparently imply a breakdown of perturbation theory in the large box on sufficiently long time scales, for example the time between self-reproduction and reheating in chaotic inflation. An interpretation of the apparent breakdown of the perturbative expansion of gravity will also be briefly discussed.

Semiclassical approaches to IR issues in quasi de Sitter universes

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Based on arXiv:1005.1056 and arXiv:1005.3287 w. Steve Giddings

Motivations

- Stability of de Sitter/Cosmological constant problem?
- Eternal inflation?
- Possible effects on inflationary observables?
- Relation to black holes?
- Systematic understanding

➡ New insights/techniques

Stability of de Sitter

- The two-point function of a massless scalar field in de Sitter is IR divergent

$$\langle \phi^2(x) \rangle = U.V. + \frac{H^2}{4\pi^2} \ln(aH/L)$$

- The IR divergency has prompted many to believe that de Sitter could by itself be unstable
- Relaxation of the cosmological constant...?

[Polyakov, 1982, 2007; Tsamis and Woodard, 1993;...]

Eternal Chaotic Inflation

- In chaotic inflation our inflating volume is typically described by a huge total number of e-foldings

➡ Correlation functions in principle plagued by large IR loop contributions

➡ We have to understand how to deal with IR loop contributions in cosmology!

NEW INSIGHTS?

- A proper understanding of the IR issues may bring about additional new insights

Examples:

1. Relation to Black Hole information paradox [Giddings, 2007,2009; Arkani-Hamed et al., 2007]
2. Systematics of n-point correlation functions of non-Gaussianity to all orders in n [Jarnhus and Sloth, 2008]

Outline

1. Tensor fluctuations in de Sitter: a test probe of IR issues
2. Quasi de Sitter: Slow-roll inflation
3. Interpretations

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Tensor fluctuations in de Sitter

- Consider the two-point function of a free massless test scalar field in de Sitter

$$\langle \sigma_{k_1} \sigma_{k_2} \rangle$$

- With the metric on the form

$$ds^2 = -dt^2 + a^2(t)\left(e^{\gamma}\right)_{ij} dx^i dx^j$$

- The effect of a long wavelength γ_{ij} mode is to shift the momentum

$$k^2 \rightarrow k_i \left(e^{-\gamma}\right)_{ij} k_j = k_i k_i - \gamma_{ij} k_i k_j + \frac{1}{2}\gamma_{il}\gamma_{lj} k_i k_j + \ldots$$

Tensor fluctuations in de Sitter

- Taylor expanding the correlation function in the shifted momentum:

$$\begin{aligned}\langle \sigma_{k_1}\sigma_{k_2}\rangle_\gamma &= \langle \sigma_{k_1}\sigma_{k_2}\rangle_0 \\ &+ \left(-\gamma_{ij}k_ik_j + \frac{1}{2}\gamma_{il}\gamma_{lj}k_ik_j\right)\frac{\partial}{\partial k^2}\langle \sigma_{k_1}\sigma_{k_2}\rangle\Big|_0 + \frac{1}{2}(\gamma_{ij}k_ik_j)^2\left(\frac{\partial}{\partial k^2}\right)^2\langle \sigma_{k_1}\sigma_{k_2}\rangle\Big|_0 + \dots\end{aligned}$$

- Averaging over all soft graviton modes:

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➡ Equivalent to averaging over a *"large box"*

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Tensor fluctuations in de Sitter

- In order to evaluate the average over soft gravitons, expand

$$\gamma_{ij}(x) = \sum_{s=+,\times} \int \frac{d^3k}{(2\pi)^3} \left[b^s_{\mathbf{k}} \epsilon^s_{ij}(\mathbf{k}) \gamma_k(t) + b^{s\dagger}_{-\mathbf{k}} \epsilon^{s*}_{ij}(-\mathbf{k}) \gamma^*_k(t) \right] e^{i\mathbf{k}\cdot\mathbf{x}}$$

- The mode function is similar to the one of a free massless scalar in de Sitter

$$\gamma_k(\eta) = \frac{H}{\sqrt{k^3}}(1 + ik\eta)e^{-ik\eta}$$

- and the variance is similarly IR divergent

$$\langle \gamma^2(x) \rangle = \frac{1}{4} \langle \gamma_{ij}(x)\gamma_{ij}(x) \rangle \approx 2\left(\frac{H}{2\pi}\right)^2 \int_{a_i H}^{a_* H} \frac{dq}{q} = -2\left(\frac{H}{2\pi}\right)^2 \log(\Lambda_{IR}/a_* H)$$

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- Using

$$\frac{k_i k_j}{k^2}\langle \gamma_{il}\gamma_{lj}\rangle = \frac{4}{3}\langle \gamma^2(x)\rangle \qquad \frac{k_i k_j}{k^2}\frac{k_k k_l}{k^2}\langle \gamma_{ij}\gamma_{kl}\rangle = \frac{8}{15}\langle \gamma^2(x)\rangle$$

- The average becomes

$$\left\langle \langle \sigma_{k_1}\sigma_{k_2}\rangle_\gamma \right\rangle = \left\{ 1 + \frac{2}{3}\langle \gamma^2(x)\rangle_* \left[\frac{2}{5}k^4\left(\frac{\partial}{\partial k^2}\right)^2 + k^2\frac{\partial}{\partial k^2} \right] \right\} \langle \sigma_{k_1}\sigma_{k_2}\rangle$$

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Tensor fluctuations in de Sitter

Verdict:

- No breaking of scale invariance

➡ No IR effects

Example:

- Consider non-scale invariant two-point function

$$\langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle \approx (2\pi)^3 \delta^3(k_1 + k_2) \frac{H^4 \eta^4}{2} k$$

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$$\left\langle \langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle_\gamma \right\rangle = \langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle \left[1 + \frac{4}{15} \langle \gamma^2(x) \rangle_* \right]$$

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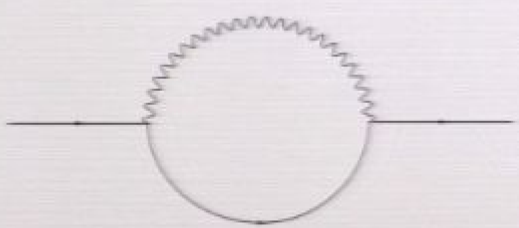
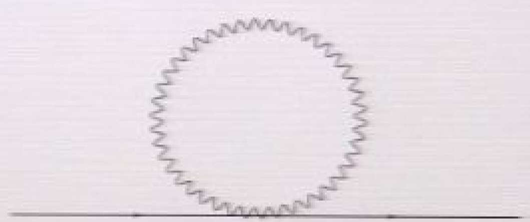
Consistency Check

Full *in-in* calculation:

- As a check, we can calculate $\langle \sigma_{k_1} \sigma_{k_2} \rangle$ and $\langle \dot{\sigma}_{k_1} \dot{\sigma}_{k_2} \rangle$ using the full *in-in* QFT approach in the *"large box"*
- The contributing diagrams are





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[arXiv:1005.3287]

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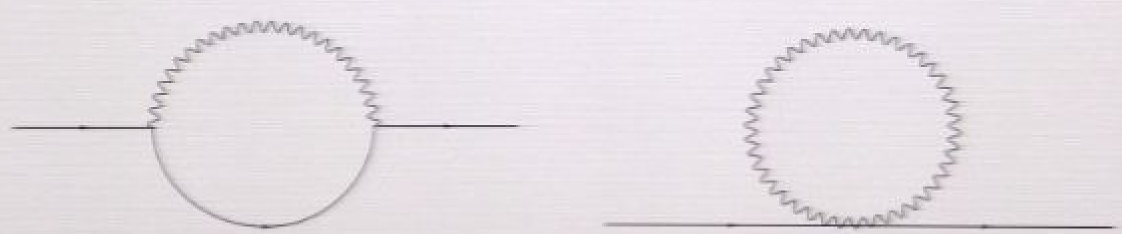

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TENSOR FLUCTUATIONS IN DE SITTER

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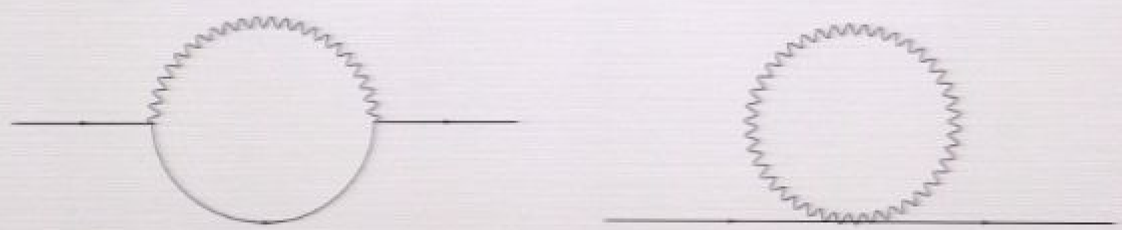

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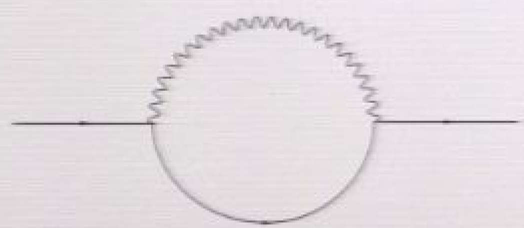

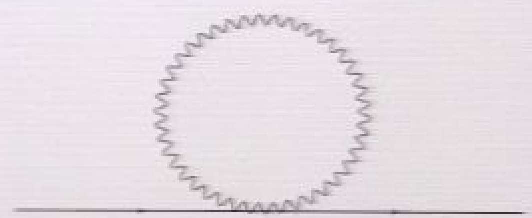

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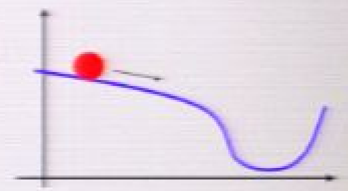
II

Slow-roll

- Quasi de Sitter: time-translation invariance broken by the slow-rolling of a scalar



$$S = \frac{1}{2}\int \sqrt{-g}\,[R - \partial_\mu\phi\partial^\mu\phi - 2V(\phi)]$$

- Symmetry breaking parametrized by slow-roll parameters

$$\epsilon = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \quad , \quad \eta = \frac{V''}{V}$$

➡ Spectrum of perturbations not scale invariant

$$\langle \delta\phi_{k_1}\delta\phi_{k_2}\rangle = \delta(\mathbf{k}_1 + \mathbf{k}_2)\frac{H^2}{2k^3}\left(\frac{k}{aH}\right)^{n_s - 1} \quad , \qquad n_s - 1 = 2\eta - 6\epsilon$$

➡ Large IR effects...?

Gauge considerations

- Let's consider scalar perturbations in the comoving gauge

$$\phi \equiv \phi(t)\ , \qquad ds^2 = -dt^2 + a^2(t)e^{2\zeta}(e^{\gamma})_{ij}dx^i dx^j$$

- As opposed to the uniform curvature gauge

$$\phi \equiv \phi(t) + \delta\phi(\mathbf{x}, t)\ , \qquad ds^2 = -dt^2 + a^2(t)(e^{\gamma})_{ij}dx^i dx^j$$

- ζ is the conserved curvature perturbation on large scales
- It is related by a gauge transformation to the field fluctuation

$$\zeta = \frac{\dot{\phi}}{H}\delta\phi$$

Tensor loops

- Expanding again on a background of soft gravitons

$$\langle\zeta_{k_1}\zeta_{k_2}\rangle_\gamma = \langle\zeta_{k_1}\zeta_{k_2}\rangle_0 + \left(-\gamma_{ij}k_ik_j + \frac{1}{2}\gamma_{il}\gamma_{lj}k_ik_j\right)\frac{\partial}{\partial k^2}\langle\zeta_{k_1}\zeta_{k_2}\rangle\Big|_0 + \frac{1}{2}(\gamma_{ij}k_ik_j)^2\left(\frac{\partial}{\partial k^2}\right)^2\langle\zeta_{k_1}\zeta_{k_2}\rangle\Big|_0 + \dots$$

- and averaging over all modes in the "large box"

$$\left\langle\langle\zeta_{k_1}\zeta_{k_2}\rangle_\gamma\right\rangle = \left\{1+\frac{2}{3}\langle\gamma^2(x)\rangle_*\left[\frac{2}{5}k^4\left(\frac{\partial}{\partial k^2}\right)^2 + k^2\frac{\partial}{\partial k^2}\right]\right\}\langle\zeta_{k_1}\zeta_{k_2}\rangle$$

- we now obtain a IR divergent result

$$\left\langle\langle\zeta_{k_1}\zeta_{k_2}\rangle_\gamma\right\rangle = \left[1+\frac{n_s-4}{3}\frac{n_s-1}{5}\langle\gamma^2(x)\rangle_*\right]\langle\zeta_{k_1}\zeta_{k_2}\rangle$$

- where again

$$\langle\gamma^2(x)\rangle \approx 2\left(\frac{H}{2\pi}\right)^2\int_{a_iH}^{a_*H}\frac{dq}{q} = -2\left(\frac{H}{2\pi}\right)^2\log(\Lambda_{IR}/a_*H)$$

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$$\left\langle \langle \zeta_{k_1}\zeta_{k_2}\rangle_\gamma \right\rangle = \left[1 + \frac{n_s-4}{3}\frac{n_s-1}{5}\langle \gamma^2(x)\rangle_*\right]\langle \zeta_{k_1}\zeta_{k_2}\rangle$$

- where again

$$\langle \gamma^2(x)\rangle \approx 2\left(\frac{H}{2\pi}\right)^2\int_{a_iH}^{a_*H}\frac{dq}{q} = -2\left(\frac{H}{2\pi}\right)^2\log(\Lambda_{IR}/a_*H)$$

Tensor loops

- Similarly one can easily compute the IR effect of tensors on tensors

$$\left\langle \langle \gamma_{k_1} \gamma_{k_2} \rangle_\gamma \right\rangle = \left[1 + \frac{n_t - 3}{3} \frac{n_t}{5} \left\langle \gamma^2(x) \right\rangle_* \right] \langle \gamma_{k_1} \gamma_{k_2} \rangle$$

Scalar loops

- Long wavelength background scalar mode, $\bar{\zeta}$, shifts the momentum $k^2 \to k_{\bar{\zeta}}^2 = (e^{-\bar{\zeta}} k)^2$
- Expanding on the shifted momentum yields

$$\begin{aligned} \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\bar{\zeta}} &= \left[1 + \bar{\zeta} \frac{\partial}{\partial \zeta} + \frac{1}{2} \bar{\zeta}^2 \frac{\partial^2}{\partial \zeta^2} + \cdots \right] \left[e^{-6\bar{\zeta}} \langle \zeta(e^{-\bar{\zeta}} k_1) \zeta(e^{-\bar{\zeta}} k_2) \rangle \right] \\ &= \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 - (n_s - 1)\, \bar{\zeta} \, \langle \zeta_{k_1} \zeta_{k_2} \rangle \big|_0 + \left(\frac{1}{2}(n_s - 1)^2 + \alpha_s \right) \bar{\zeta}\bar{\zeta} \, \langle \zeta_{k_1} \zeta_{k_2} \rangle \big|_0 + \cdots \end{aligned}$$

$\alpha_s = dn_s / d\ln(k)$

- and averaging over soft scalar modes in the "large box" gives

$$\left\langle \langle \zeta_{k_1} \zeta_{k_2} \rangle_{\bar{\zeta}} \right\rangle \simeq \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 + \left(\frac{1}{2}(n_s - 1)^2 + \alpha_s \right) \langle \zeta_{k_1} \zeta_{k_2} \rangle_0 \, \langle \zeta^2(x) \rangle_* .$$

- where as usual, the variance diverge in the IR

$$\langle \zeta^2(x) \rangle_* \approx \frac{1}{2\epsilon} \frac{H^2}{(2\pi)^2} \int_{a_i H}^{a_* H} \frac{dq}{q} = -\frac{1}{2\epsilon} \frac{H^2}{(2\pi)^2} \log(\Lambda_{IR} / a_* H)$$

Scalar loops

- Similar the scalar IR loop correction to the tensors becomes

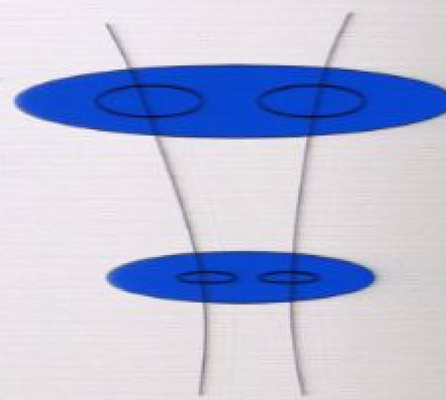
$$\left\langle \langle \gamma_{k_1} \gamma_{k_2} \rangle_{\zeta} \right\rangle \simeq \langle \gamma_{k_1} \gamma_{k_2} \rangle_0 + \left(\frac{1}{2}(n_t)^2 + \alpha_t\right) \langle \gamma_{k_1} \gamma_{k_2} \rangle_0 \left\langle \zeta^2(x) \right\rangle_*$$

$$n_t = -2\epsilon \qquad \alpha_t = dn_t/d\ln(k)$$

Comparing with the δN approach

- The curvature perturbation on super-horizon scales can be described by the relative evolution of separate unperturbed universes

$$\zeta = \delta N = N' \delta\phi|_{hor.cros.} + \frac{1}{2} N'' \delta\phi^2|_{hor.cros.} + \dots$$



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δN continued...

A related example:

- In single field slow-roll inflation non-Gaussianity is created at horizon crossing

➡ The δN approach fails if the initial non-Gaussianity at the horizon crossing is neglected

[Seery, Lidsey, Sloth, 2006; Seery, Sloth, Vernizzi, 2008]

Observables in the "Large Box"

Tensor-scalar relation

- In single field slow-roll inflation there is a definite prediction for the tensor-scalar ratio at tree-level

$$r = 4\frac{\langle|\gamma_k|^2\rangle}{\langle|\zeta_k|^2\rangle} = 16\epsilon$$

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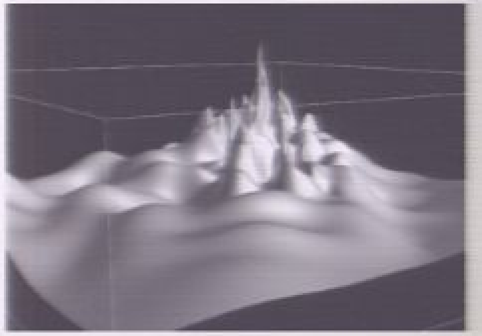

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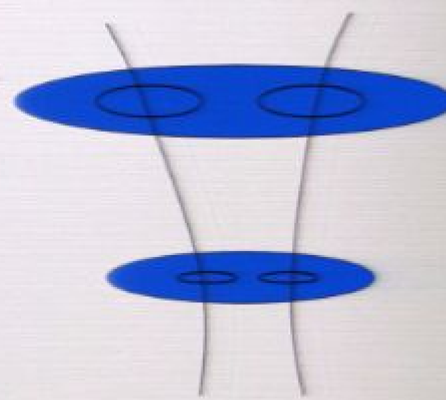

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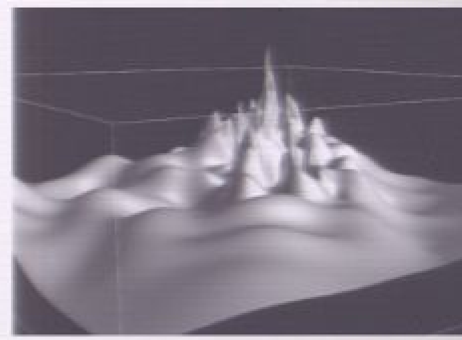

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Issues with the perturbative expansion

- Do we have any other indications of a perturbative break down of gravity?
- If we drop something into a black hole, its information appears to be lost, but it must be entangled in the Hawking radiation to preserve unitarity
- On the other hand, nothing special happens to the observer falling through the horizon, so if the information carried by him is also radiated out through the horizon, there is a problem with locality, since information at spatial separated points must be independent [For a more careful formulation using nice slices, see Steve's talk]

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- This is the time scale at which information needs to start to coming out of the black hole

➡ Identifying a source for a perturbative breakdown would indicate no information paradox, only information problem [Giddings, 2007, 2009]

- This appears parallel to the breakdown of the perturbative approach in de Sitter on a time scale

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Local interpretation of scalar perturbations

- In comoving gauge the metric takes the form (neglecting for now TT-tensor pert.)

$$ds^2 = -dt^2 + a^2 e^{2\zeta} dx^2$$

- then writing the effect of long wave scalar modes as

$$\zeta \to \tilde{\zeta} = \zeta + \zeta_L$$

- Locally we can hide the effect of ζ_L by locally taking

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[Unruh, and others...]

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$$\mathcal{P}_\zeta(k) = \frac{1}{2\epsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{n_s-1}$$

- So neglecting for simplicity the running of n_s, the effect of

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Possible outcome (under investigation):

- For local observers: the effects may be resummed / absorbed to eliminate large effects

- but globally, effects can plausibly not be eliminated

SUMMARY

- We found simple semiclassical relations for deriving the IR loop effects during inflation
- The semiclassical relations provide a leading log series
- We checked the semiclassical relations with exact *in-in* calculations, which matches the results diagram for diagram
- We developed new *"Cosmological Diagrammatic rules"* that makes the *in-in* calculation more efficient.
- The IR effects can become large in the total inflated volume ("large vol.") in realistic models of inflation
- The time scale for corrections to become large in the "large vol." is $t \sim RS$, which coincides with the time scale on which one expects a breakdown of perturbative physics in the black hole context.

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$$\langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \equiv (2\pi)^3\delta(\sum_a \mathbf{k}_a)B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3)$$

- which can parametrized in terms the non-linearity parameter

$$B_\zeta \equiv -\frac{6}{5}f_{NL}[P_\zeta(k_1)P_\zeta(k_2) + 2 \text{ permutations}]$$

- In single field slow-roll inflation at tree-level in the squeezed limit ($k_1 << k_2, k_3$)

$$f_{NL} = \frac{5}{12}(n_s - 1)$$

- At one-loop in the squeezed limit, the semiclassical relations imply

$$f_{NL} = \frac{5}{12}(n_s - 1)\left[1 + ((n_s - 1)^2 - 2\alpha_s)\langle \zeta^2(x)\rangle_*\right]$$

Observables in the "large box"

Tensor-scalar relation

- In single field slow-roll inflation there is a definite prediction for the tensor-scalar ratio at tree-level

$$r = 4\frac{\langle|\gamma_k|^2\rangle}{\langle|\zeta_k|^2\rangle} = 16\epsilon$$

- The the one loop corrected tensor-scalar relation becomes

$$r = 16\epsilon\left[1 + \frac{n_t^2 + 2\alpha_t - ((n_s-1)^2 + 2\alpha_s)}{2}\langle\zeta^2(x)\rangle_* + \frac{(n_t-3)n_t - (n_s-4)(n_s-1)}{15}\langle\gamma^2(x)\rangle_*\right]$$

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OBSERVABLES IN THE "LARGE BOX"

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δN continued...

A related example:

- In single field slow-roll inflation non-Gaussianity is created at horizon crossing

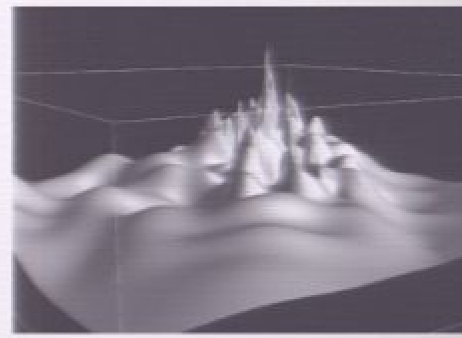
➡ The δN approach fails if the initial non-Gaussianity at the horizon crossing is neglected

[Seery, Lidsey, Sloth, 2006; Seery, Sloth, Vernizzi, 2008]

Example: Chaotic inflation

- Inflation starts at the self-reproduction regime when

$$\delta\phi \sim \dot{\phi}_c \Delta t$$



➡ A vast total number of e-foldings

➡ Large IR effects in the "large box"

- The fractional IR corrections of scalar and tensor loops are given by

$$g_{IR} \equiv \left(\frac{1}{2}(n_s-1)^2+\alpha_s\right)\left\langle\zeta^2(x)\right\rangle_*\ ,\ h_{IR}=(n_s-1)\left\langle\gamma^2(x)\right\rangle_*$$

Issues with the perturbative expansion

- Do we have any other indications of a perturbative break down of gravity?
- If we drop something into a black hole, its information appears to be lost, but it must be entangled in the Hawking radiation to preserve unitarity
- On the other hand, nothing special happens to the observer falling through the horizon, so if the information carried by him is also radiated out through the horizon, there is a problem with locality, since information at spatial separated points must be independent [For a more careful formulation using nice slices, see Steve's talk]

➡ Black hole information paradox

Issues with the perturbative expansion

- It is an indication that the perturbative approach must fail on a time scale of order the black hole evaporation time

$$t_{ev} \sim R_{bh} S_{bh} \sim M^3$$

- This is the time scale at which information needs to start to coming out of the black hole

➡ Identifying a source for a perturbative breakdown would indicate no information paradox, only information problem [Giddings, 2007, 2009]

- This appears parallel to the breakdown of the perturbative approach in de Sitter on a time scale

$$t_{ds} \sim R_{ds} S_{ds} \sim 1/H^3$$

[Giddings, 2007; Arkani-Hamed et al., 2007]

Local interpretation of scalar perturbations

- In comoving gauge the metric takes the form (neglecting for now TT-tensor pert.)

$$ds^2 = -dt^2 + a^2 e^{2\zeta} dx^2$$

- then writing the effect of long wave scalar modes as

$$\zeta \rightarrow \tilde{\zeta} = \zeta + \zeta_L$$

- Locally we can hide the effect of ζ_L by locally taking

$$a \rightarrow \tilde{a} = a e^{\zeta_L}$$

[Unruh, and others...]

- Globally ζ_L is not homogenous, and can not be absorbed in the scale factor in the "large box"

- The power spectrum scale as

$$\mathcal{P}_\zeta(k) = \frac{1}{2\epsilon}\left(\frac{H}{2\pi}\right)^2\left(\frac{k}{aH}\right)^{n_s-1}$$

- So neglecting for simplicity the running of n_s, the effect of

$$a \to \tilde{a} = ae^{\bar{\zeta}}$$

is

$$\mathcal{P}_\zeta(k) \to \mathcal{P}_\zeta(k)e^{-(n_s-1)\bar{\zeta}} = \mathcal{P}_\zeta(k)(1-(n_s-1)\bar{\zeta} + \frac{1}{2}(n_s-1)^2\bar{\zeta}^2 + \ldots)$$

- This is locally equivalent to the effect we derived before from

$$k^2 \to k_\zeta^2 = (e^{-\bar{\zeta}}k)^2$$

since the functional dependence of the spectrum on a and k is the same and we can always write

$$d\log(k) = d\log(a)\ , \qquad \text{with} \qquad k = aH$$

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