Title: Interacting Quantum Fields in de Sitter Space

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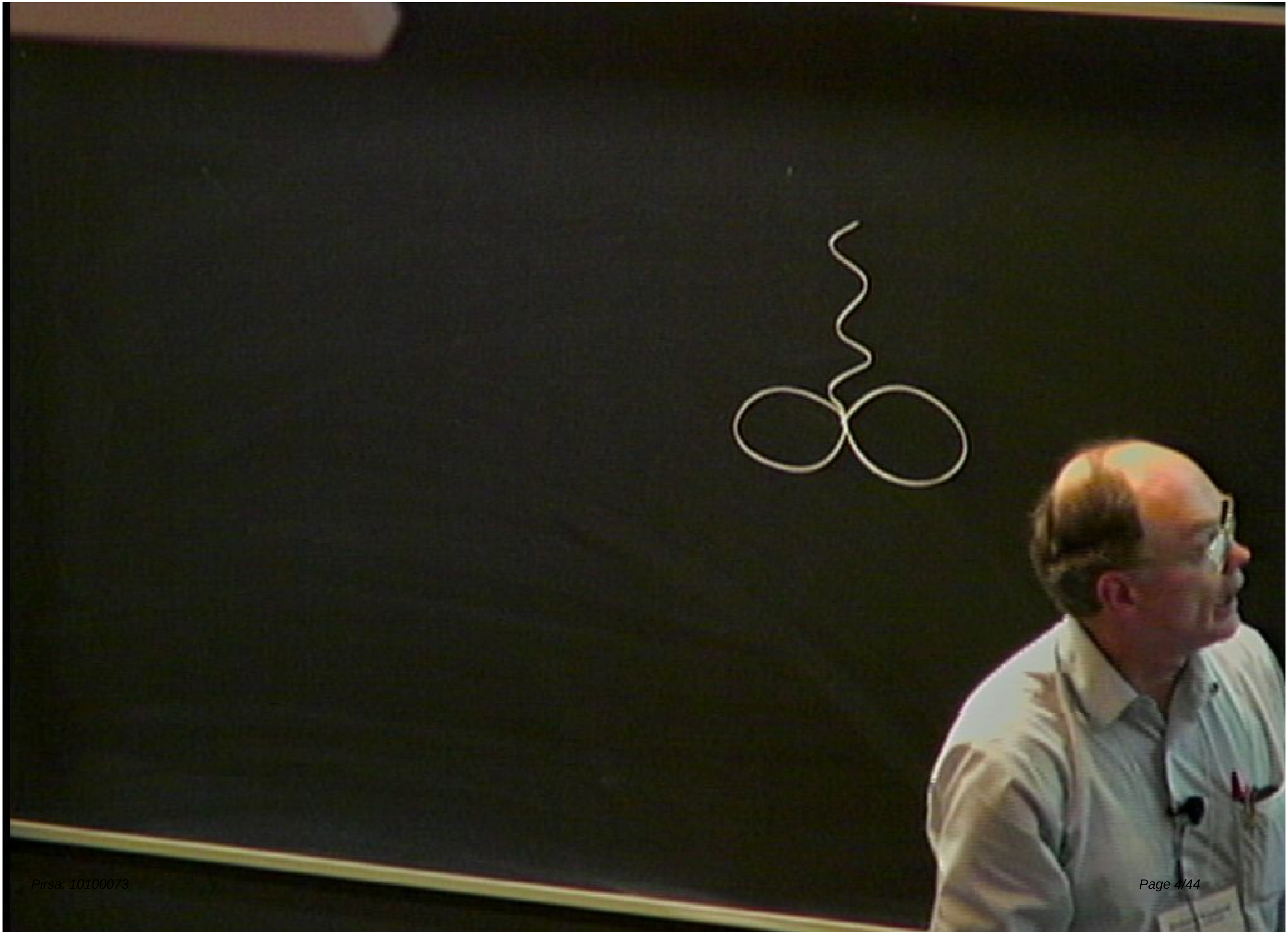
Abstract: Infrared logarithms are factors of the logarithm of the inflationary scale factor which arise in quantum field theoretic loop corrections that involve either massless, minimally coupled scalars or gravitons. They have been found by myself and collaborators in 1PI functions and by Steven Weinberg in the power spectrum of primordial perturbations. Because the inflationary scale factor grows so rapidly, infrared logarithms enhance loop corrections far beyond expectations based upon the coupling constant. They also inject time dependence into what are usually static results. For a very long period of inflation this enhancement can grow so large that weak field perturbation theory breaks down. In this talk I explain the physical reasons why infrared logarithms occur, I review the computations in which they have been seen and I describe Starobinskii's technique for evolving past the breakdown of perturbation theory. I also comment on the potential of infrared logarithms to change our understanding of inflationary cosmology.

Spacetime Exp. Strengthens QFT

- Why?
  - Loops → classical physics of virtuals
  - Expansion → holds virtuals apart longer
- Maximum Effect for
  - Inflation
  - M=0
  - No conformal invariance (classically)
- Two Particles
  - MMC scalars
  - gravitons

Infrared Logarithms

1. WHAT: factors of $\ln(a) = Ht$ in QFT results
2. EG: $\lambda\varphi^4$
   - $p = \lambda H^4[-2 \ln^2(a) - 4/3 \ln(a)]/(4\pi)^4 + \ldots$
3. WHY:
   - $\langle\Omega|\varphi^2|\Omega\rangle = H^2 [\ln(a) + UV]/(4\pi^2)$
     (Ford & Vilenkin, Linde, Starobinsky, 1982)
   - $\int^t dt'\, 1 = t = \ln(a)/H$
4. NB: ln(a)'s even in Power Spectrum
   - Weinberg, hep-th/0605244
   - Giddings+Sloth, Hebecker et al., Kayha+Onemli



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How many ln(a)'s per coupling constant?

- $\Delta\mathcal{L}$ = (c.c.)$(\varphi, h_{\mu\nu})^N$ x (other fields)
- → up to N ln(a)'s for each $(c.c.)^2$

1. $\lambda\varphi$ → $\ln^2(a)$ for each λ
2. SQED ($e\varphi^*\partial\varphi A$ & $e^2\varphi^*\varphi AA$)
   → ln(a) for each e^2
3. Yukawa ($g\varphi\psi\psi$) → ln(a) for each g^2
4. QG ($\kappa h\partial h\partial h$) → ln(a) for each GH^2

The Perturbative Conundrum

1. When lowest ln(a) loops go order 1 …
   - $\lambda\varphi^4 \rightarrow \ln(a) \approx 1/\sqrt{\lambda}$
   - SQED $\rightarrow \ln(a) \approx 1/e^2$
   - Yukawa $\rightarrow \ln(a) \approx 1/g^2$
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2. … ALL ln(a) loops go order 1!
3. And perturbation theory breaks down.

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Leading Log Approximation

(1) $\frac{\rho}{H^4} \sim \lambda \ln^2(a) + \ldots + \lambda^{\ell-1}\Big[\ln^{2\ell-2}(a) + \ldots + \ln^2(a)\Big] + \ldots$

(2) Breaks down for $\ln(a) \simeq \frac{1}{\sqrt{\lambda}}$

(3) Subdominant logs still small

- $\lambda^{\ell-1}\ln^{2\ell-3}(a) \simeq \sqrt{\lambda}$

(4) IGNORE THEM!

- $\Big(\frac{\rho}{H^4}\Big)_{\text{leading}} = \sum_{\ell=2}^{\infty} a_\ell \Big(\lambda \ln^2(a)\Big)^{\ell-1}$

Starobinsky's Rules (astro-ph/9407016)

(1) Simplify: $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + \frac{\lambda}{6}\varphi^3 = 0$

$\Longrightarrow 3H(\dot{\varphi} - f) + \frac{\lambda}{6}\varphi^3 = 0$

- $f(t,\vec{x}) \equiv \int \frac{d^3k}{(2\pi)^3}\delta(k - Ha)\frac{H^2}{\sqrt{2k}}\left\{e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\right\}$

(2) Langevin Eqn: $\dot{\varphi} = f - \frac{\lambda}{18H}\varphi^3$

- $\varphi(t,\vec{x})$ a stochastic field driven by noise $f(t,\vec{x})$
- $\left\langle\Omega\middle|f(t,\vec{x})f(t',\vec{x})\middle|\Omega\right\rangle = \frac{H^3}{4\pi^2}\delta(t-t')$

(3) Fokker–Planck Eqn:

- $\left\langle\Omega\middle|F\Big(\varphi(t,\vec{x})\Big)\middle|\Omega\right\rangle = \int d\varphi\,\rho(t,\varphi)F(\varphi)$
- $\dot{\rho} = \frac{1}{2}\frac{\partial^2}{\partial\varphi^2}\Big[\frac{H^3}{4\pi^2}\rho\Big] - \frac{\partial}{\partial\varphi}\Big[-\frac{\lambda}{18H}\varphi^3\rho\Big]$

(4) Reproduces leading IR logs

- but why?
- what about the uncertainty principle?
- where did $f(t,\vec{x})$ come from?

Proof (with Tsamis, gr-qc/0505115)

(1) Heisenberg Field Eqn: $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + \frac{\lambda}{6}\varphi^3 = 0$

(2) Yang – Feldman Eqn: $\varphi(t,\vec{x}) = \varphi_0(t,\vec{x}) - \frac{\lambda}{6}\int_0^t dt' a'^3 \int d^3x' G(x;x')\varphi^3(x')$

- $\varphi_0(t,\vec{x}) \equiv \int \frac{d^3k}{(2\pi)^3}\Big\{u(t,k)e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + u^*(t,k)e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\Big\}$
- $u(t,k) = \frac{H}{\sqrt{2k^3}}\Big[1 - \frac{ik}{Ha}\Big]\exp\Big[\frac{ik}{Ha}\Big]$
- $G(x;x') = i\theta(\Delta t)\int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\Delta\vec{x}}\Big\{u(t,k)u^*(t',k) - u^*(t,k)u(t',k)\Big\}$

Generic Free Field Expansion

- $\varphi = \varphi_0 + \lambda V \varphi^3$

  $= \varphi_0 + \lambda V \varphi_0{}^3 + \lambda^2 V^2 \varphi_0{}^5 + \ldots$

  $= \varphi_0 \, [1 + \lambda V \varphi_0{}^2 + (\lambda V \varphi_0{}^2)^2 + \ldots]$
- $\varphi^N = \varphi_0{}^N \, [1 + \lambda V \varphi_0{}^2 + \ldots]$
- Hence each extra λ brings $V \, \varphi_0{}^2$
  - One ln(a) from V
  - One ln(a) from $\varphi_0{}^2$

Infrared Truncations

(1) Long Wavelength Expansion:

- $u(t,k) = \frac{H}{\sqrt{2k^3}}\Big\{1 + \frac{1}{2}\Big(\frac{k}{Ha}\Big)^2 + \frac{i}{3}\Big(\frac{k}{Ha}\Big)^3 + \dots\Big\}$

(2) IR Logs from φ_0 require only

- $\varphi_0(t,\vec{x}) \longrightarrow \int \frac{d^3k}{(2\pi)^3}\theta(Ha-k)\frac{H}{\sqrt{2k^3}}\Big\{e^{i\vec{k}\cdot\vec{x}}\alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\Big\} \equiv \Phi_0(t,\vec{x})$

(3) IR Logs from $G(x;x')$ require only

- $$G(x;x') \longrightarrow i\theta(\Delta t)\int \frac{d^3k}{(2\pi)^3}e^{i\vec{k}\cdot\Delta\vec{x}}\frac{H^2}{k^3}\Big\{\frac{i}{3}\Big(\frac{k}{Ha}\Big)^3 - \frac{i}{3}\Big(\frac{k}{Ha'}\Big)^3\Big\}$$
$$= \frac{\theta(t-t')}{3H}\Big[\frac{1}{a'^3} - \frac{1}{a^3}\Big]\delta^3(\vec{x}-\vec{x}')$$

- Can even drop $\frac{1}{a^3}$.

IR Truncated Yang-Feldman

(1) Heisenberg Field Eqn: $\ddot{\varphi} + 3H\dot{\varphi} - \frac{\nabla^2}{a^2}\varphi + \frac{\lambda}{6}\varphi^3 = 0$

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(3) IR Truncated Eqn: $\Phi(t,\vec{x}) = \Phi_0(t,\vec{x}) - \frac{\lambda}{18H}\int_0^t dt'\, \Phi^3(t',\vec{x})$

(4) Differentiate: $\dot{\Phi}(t,\vec{x}) = \dot{\Phi}_0(t,\vec{x}) - \frac{\lambda}{18H}\Phi^3(t,\vec{x})$

- $\dot{\Phi}_0(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\delta(Ha - k)\frac{H^2}{\sqrt{2k}}\Big\{e^{i\vec{k}\cdot\vec{k}}\alpha(\vec{k}) + e^{-i\vec{k}\cdot\vec{x}}\alpha^\dagger(\vec{k})\Big\} = f(t,\vec{x})$

(5) Starobinskii was right to all orders!

Non-Perturbative Solution

- Physics of competition
  - Inflationary particle production pushes φ up
  - Classical force pushes it back down
- Expect a static limit for ρ(t,φ)

$$\dot{\rho} \to 0 = \frac{\partial^2}{\partial\varphi^2}\Big[\frac{H^2}{8\pi^2}\rho_\infty\Big] + \frac{\partial}{\partial\varphi}\Big[\frac{\lambda}{18H}\varphi^3\rho_\infty\Big]$$

- $\frac{\partial}{\partial\varphi}\rho_\infty(\varphi) = -\frac{4\pi^2\lambda}{9H}\Big(\frac{\varphi}{H}\Big)^3\rho_\infty(\varphi)$
- $\rho_\infty(\varphi) = \frac{2}{\Gamma(\frac{1}{4})}\Big(\frac{\pi^2\lambda}{9H^4}\Big)^{\frac{1}{4}}\exp\Big[-\frac{\pi^2\lambda}{9}\Big(\frac{\varphi}{H}\Big)^4\Big]$
- $\lim_{t\to\infty}\big\langle\Omega\big|\varphi^{2n}(t,\vec{x})\big|\Omega\big\rangle = \frac{\Gamma(\frac{n+1}{2})}{\Gamma(\frac{1}{4})}\Big(\frac{9H^4}{\pi^2\lambda}\Big)^{\frac{n}{2}}$

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$$= \frac{\theta(t-t')}{3H}\Big[\frac{1}{a'^3} - \frac{1}{a^3}\Big]\delta^3(\vec{x}-\vec{x}')$$

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More General Theories

(1) Starobinskii′s technique works for

- $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$

(2) Other models with IR logs

- $\mathcal{L}_{\rm SQED} = -\frac{1}{4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} - \Big(\partial_\mu - ieA_\mu\Big)\varphi^*\Big(\partial_\nu + ieA_\nu\Big)\varphi g^{\mu\nu}\sqrt{-g}$
  $$-\delta\xi\varphi^*\varphi\sqrt{-g} - \frac{1}{4}\delta\lambda(\varphi^*\varphi)^2\sqrt{-g}$$
- $\mathcal{L}_{\rm Yukawa} = -\frac{1}{2}\partial_\alpha\varphi\partial_\beta\varphi g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}\delta\xi\varphi^2 R\sqrt{-g} - \frac{1}{4!}\delta\lambda\varphi^4\sqrt{-g}$
  $$+i\overline{\psi}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi\sqrt{-g} - f\varphi\overline{\psi}\psi\sqrt{-g}$$
- $\mathcal{L}_{\rm QG} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g} + \text{BPHZ counterterms}$
- $\mathcal{L}_{\rm QG+Dirac} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g} + i\overline{\psi}e^\beta{}_b\gamma^b\mathcal{D}_\beta\psi\sqrt{-g} + \text{BPHZ counterterms}$

(3) Two kinds of fields

- Active ($\varphi \quad h^{tt}_{ij}$) $\Longrightarrow$ cause IR logs
- Passive ($\psi \quad A_\mu$) $\Longrightarrow$ don′t cause IRlogs

Dealing with Passive Fields

- Can't ignore them
  - They carry IR logs (even if they don't cause)
  - They mediate new ints between actives
- Can't ignore IR ($H<k<Ha$) vs. UV ($Ha<k$)
  - IR logs only from IR of actives
  - Effective ints from IR+UV of passives
- What to do?
  - Integrate out passive
    - Complicated Effective Action!
  - IR truncate and simplify
    - Reduces to Effective Potential

Results for SQED (with Prokopec & Tsamis)

Perturbative:

- 1 loop vacuum polarization
- 1 loop scalar self-mass-squared
- 2 loop scalar & photon bilinears
- 2 loop $\langle T_{\mu\nu} \rangle$

Nonperturbative:

- $\langle \varphi^* \varphi \rangle \approx 1.6495\, H^2/e^2$
- $M^2_\gamma \approx 3.32133\, H^2$
- $M^2_\varphi \approx .8961 \cdot 3e^2H^2/8\pi^2$
- $\rho_{vac} \approx -.6551 \cdot 3H^4/8\pi^2$

Problems with Derivative Interactions

- Similar to passive fields
  - Cf. $R^{\alpha}{}_{\beta\rho\sigma} \sim [1+h+h^2+\ldots][\partial^2 h + \partial h \partial h]$
  - $\partial^2 h$ & $\partial h \partial h$ can't give IR logs but ≠ 0
  - Each h^2 gives up to 1 ln(a) for each GH^2
  - UV and IR equal in $\partial^2 h$ & $\partial h \partial h$
- But the SAME operator in h and ∂h!
  - How do we integrate out ∂h & not h?
  - Which free field will acquire the ∂?
- Frustrates IR truncating Yang-Feldman

Spin versus Kinetic Energy

- Kahya (0709.0536 & 0710.5281)
  - QG induces no ln(a) in 1 loop φ
- Miao (gr-qc/0506056 & 051140)
  - QG induces GH^2 ln(a) in 1 loop ψ
- Miao (0803.2377)
  - Effect entirely from spin connection
  - KE redshifts but spin doesn't
  - NOTE: effect from $h \partial h \psi$ terms!

Admonitions from 1006.3999 with Kahya and Onemli

- Don't neglect $\Delta\mathcal{L} \sim \varepsilon = -\dot{H}/H^2$
  - ζ-ζ propagator ~ $1/\varepsilon$
  - 1 loop: 2 verts & 4 props → $1/\varepsilon^2$
- Crucial which lines get ∂'s
  - No 1 loop ln(a) from $\zeta\partial\sigma\partial\sigma$
  - Do get 1 loop ln(a) from $\zeta\partial\zeta\partial\zeta$
- Avoid unknown approx. with ΔN
- Folly of $GH^2 \sim 10^{-10}$ vs $\ln(a/a_k) \sim 60$
  - Constants are observable too (Λ, G, . . .)
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   - Giddings+Sloth, Hebecker et al., Kayha+Onemli

More General Theories

(1) Starobinskii's technique works for

- $\mathcal{L} = -\frac{1}{2}\partial_\mu\varphi\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} - V(\varphi)\sqrt{-g}$

(2) Other models with IR logs

- $\mathcal{L}_{\rm SQED} = -\frac{1}{4}F_{\mu\nu}F_{\rho\sigma}g^{\mu\rho}g^{\nu\sigma}\sqrt{-g} - \Big(\partial_\mu - ieA_\mu\Big)\varphi^*\Big(\partial_\nu + ieA_\nu\Big)\varphi g^{\mu\nu}\sqrt{-g}$
  $\qquad -\delta\xi\varphi^*\varphi\sqrt{-g} - \frac{1}{4}\delta\lambda(\varphi^*\varphi)^2\sqrt{-g}$
- $\mathcal{L}_{\rm Yukawa} = -\frac{1}{2}\partial_\alpha\varphi\partial_\beta\varphi g^{\alpha\beta}\sqrt{-g} - \frac{1}{2}\delta\xi\varphi^2 R\sqrt{-g} - \frac{1}{4!}\delta\lambda\varphi^4\sqrt{-g}$
  $\qquad + i\overline{\psi}e^{\beta}{}_{b}\gamma^b\mathcal{D}_\beta\psi\sqrt{-g} - f\varphi\overline{\psi}\psi\sqrt{-g}$
- $\mathcal{L}_{\rm QG} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g}$ + BPHZ counterterms
- $\mathcal{L}_{\rm QG+Dirac} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g} + i\overline{\psi}e^{\beta}{}_{b}\gamma^b\mathcal{D}_\beta\psi\sqrt{-g}$ + BPHZ counterterms

(3) Two kinds of fields

- Active ($\varphi \quad h_{ij}^{tt}$) $\Longrightarrow$ cause IR logs
- Passive ($\psi \quad A_\mu$) $\Longrightarrow$ don't cause IRlogs

Problems with Derivative Interactions

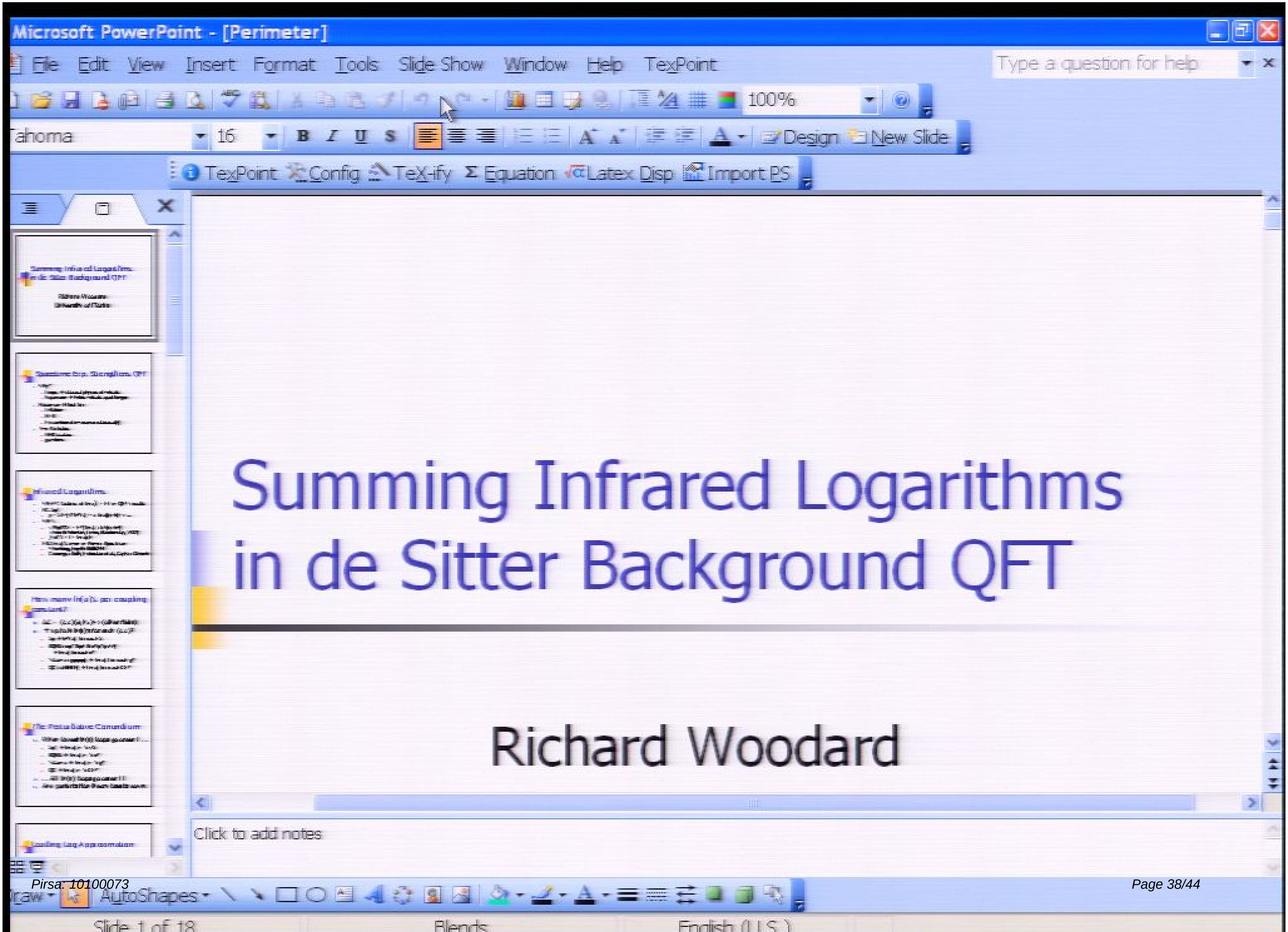
- Similar to passive fields
  - Cf. $R^{\alpha}{}_{\beta\rho\sigma} \sim [1+h+h^2+\ldots][\partial^2 h + \partial h \partial h]$
  - $\partial^2 h$ & $\partial h \partial h$ can't give IR logs but $\neq 0$
  - Each h^2 gives up to 1 ln(a) for each GH^2
  - UV and IR equal in $\partial^2 h$ & $\partial h \partial h$
- But the SAME operator in h and ∂h!
  - How do we integrate out ∂h & not h?
  - Which free field will acquire the ∂?
- Frustrates IR truncating Yang-Feldman

Spin versus Kinetic Energy

- Kahya (0709.0536 & 0710.5281)
  - QG induces no ln(a) in 1 loop φ
- Miao (gr-qc/0506056 & 0511140)
  - QG induces GH^2 ln(a) in 1 loop ψ
- Miao (0803.2377)
  - Effect entirely from spin connection
  - KE redshifts but spin doesn't
  - NOTE: effect from $h\partial h\psi$ terms!

Admonitions from 1006.3999 with Kahya and Onemli

- Don't neglect $\Delta\mathcal{L} \sim \varepsilon = -\dot{H}/H^2$
  - ζ-ζ propagator $\sim 1/\varepsilon$
  - 1 loop: 2 verts & 4 props → $1/\varepsilon^2$
- Crucial which lines get ∂'s
  - No 1 loop ln(a) from $\zeta\partial\sigma\partial\sigma$
  - Do get 1 loop ln(a) from $\zeta\partial\zeta\partial\zeta$
- Avoid unknown approx. with ΔN
- Folly of $GH^2 \sim 10^{-10}$ vs $\ln(a/a_k) \sim 60$
  - Constants are observable too (Λ, G, . . .)
  - Can have really BIG IR log corrections!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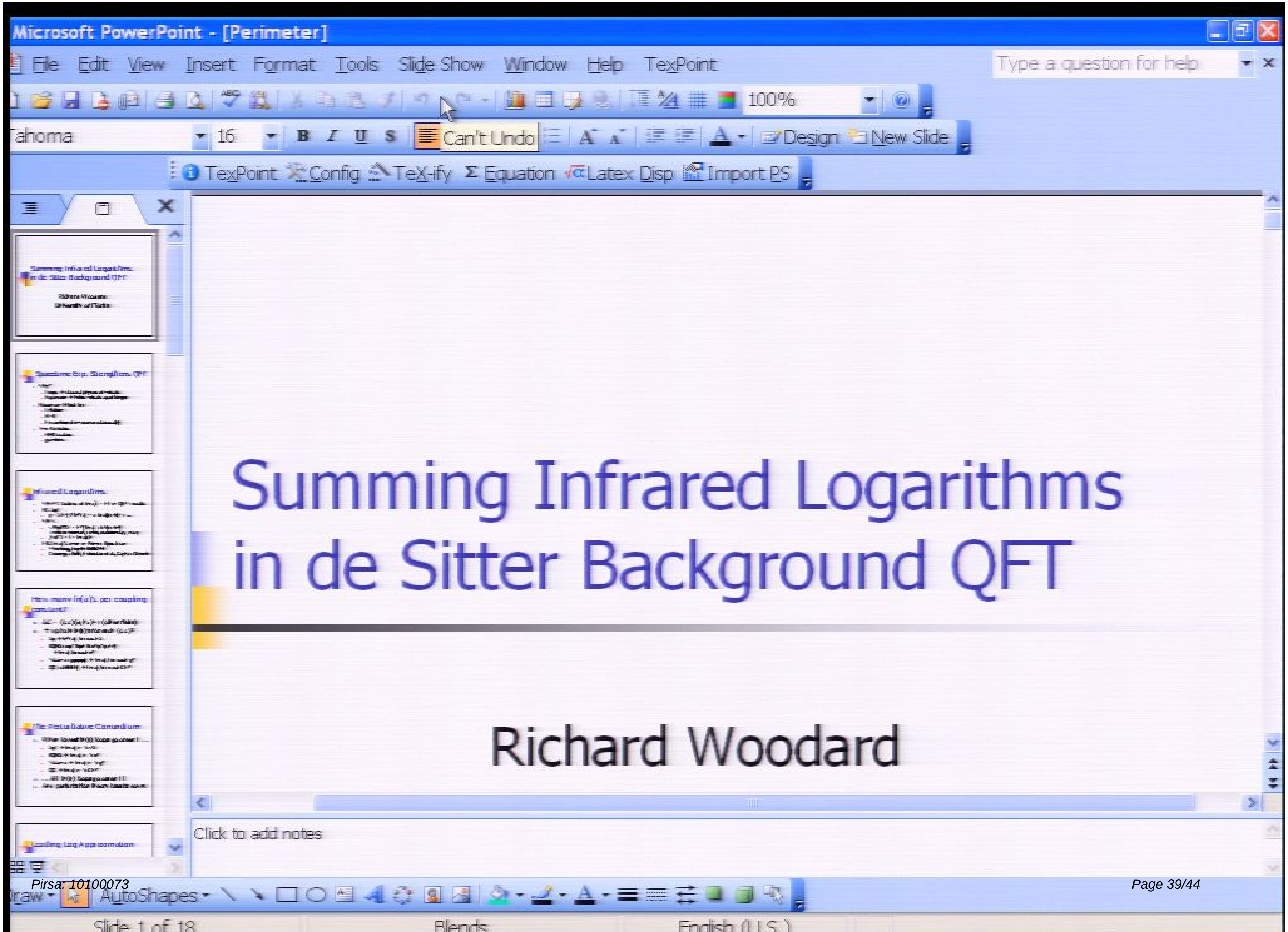
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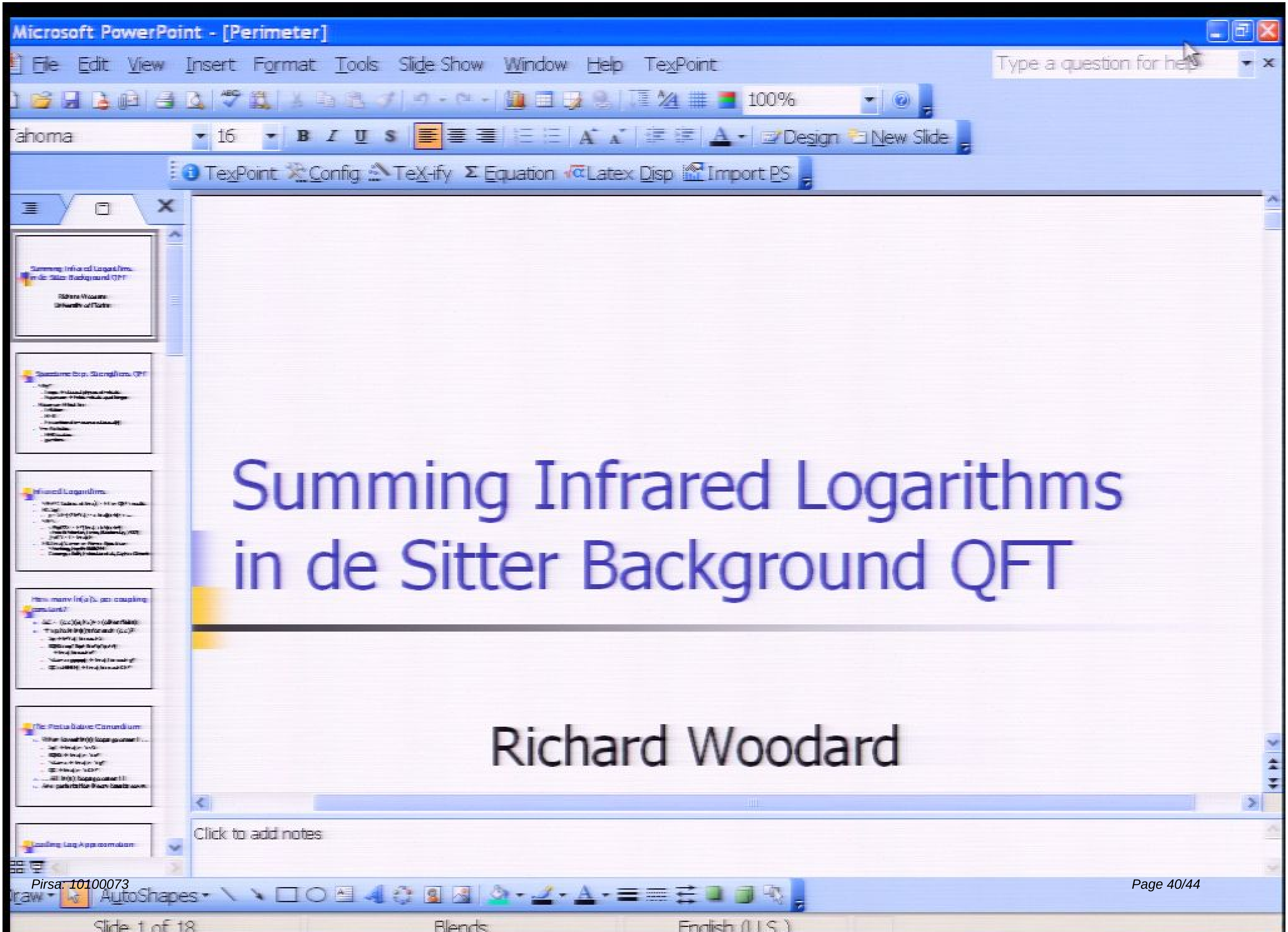
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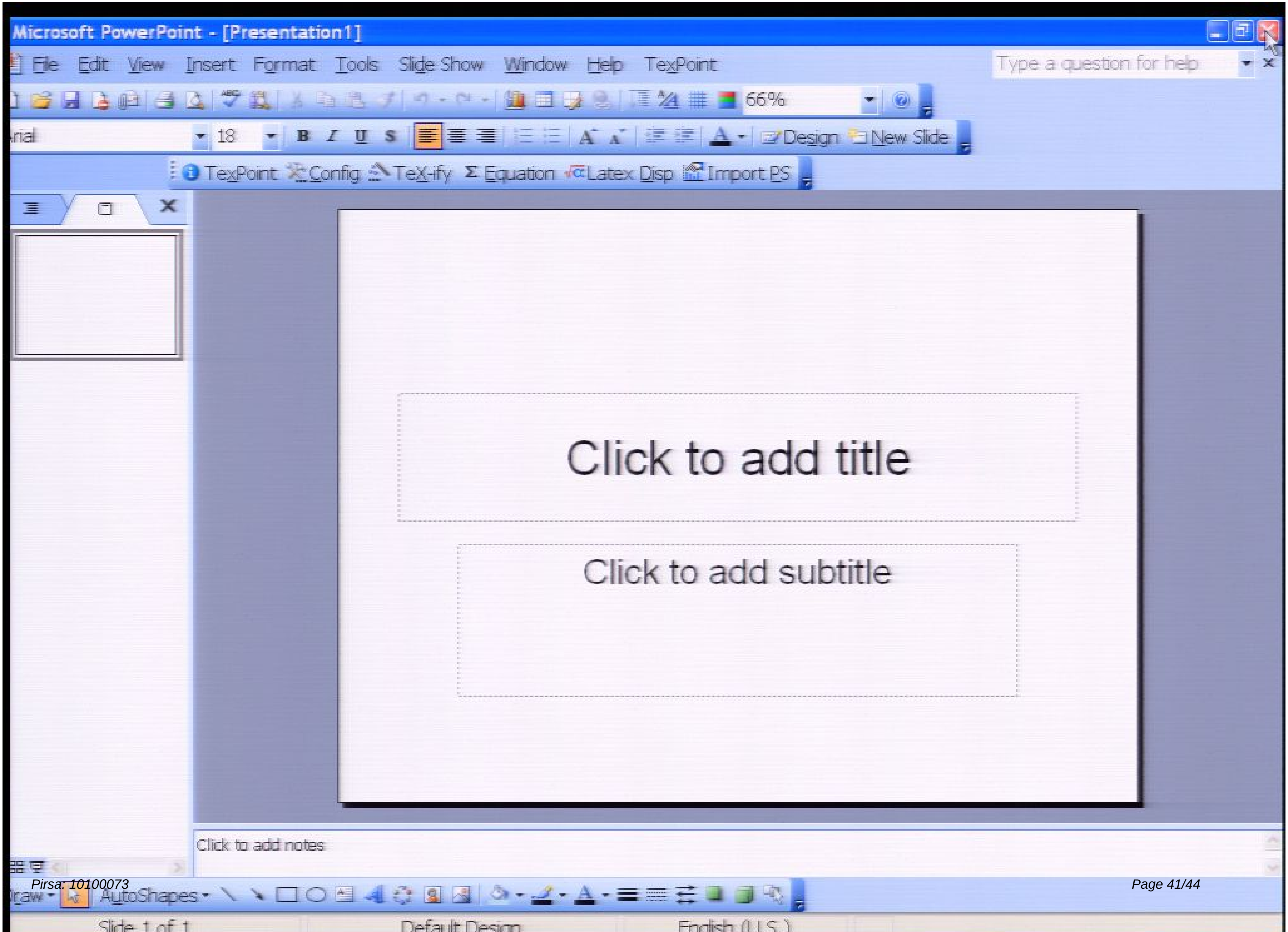
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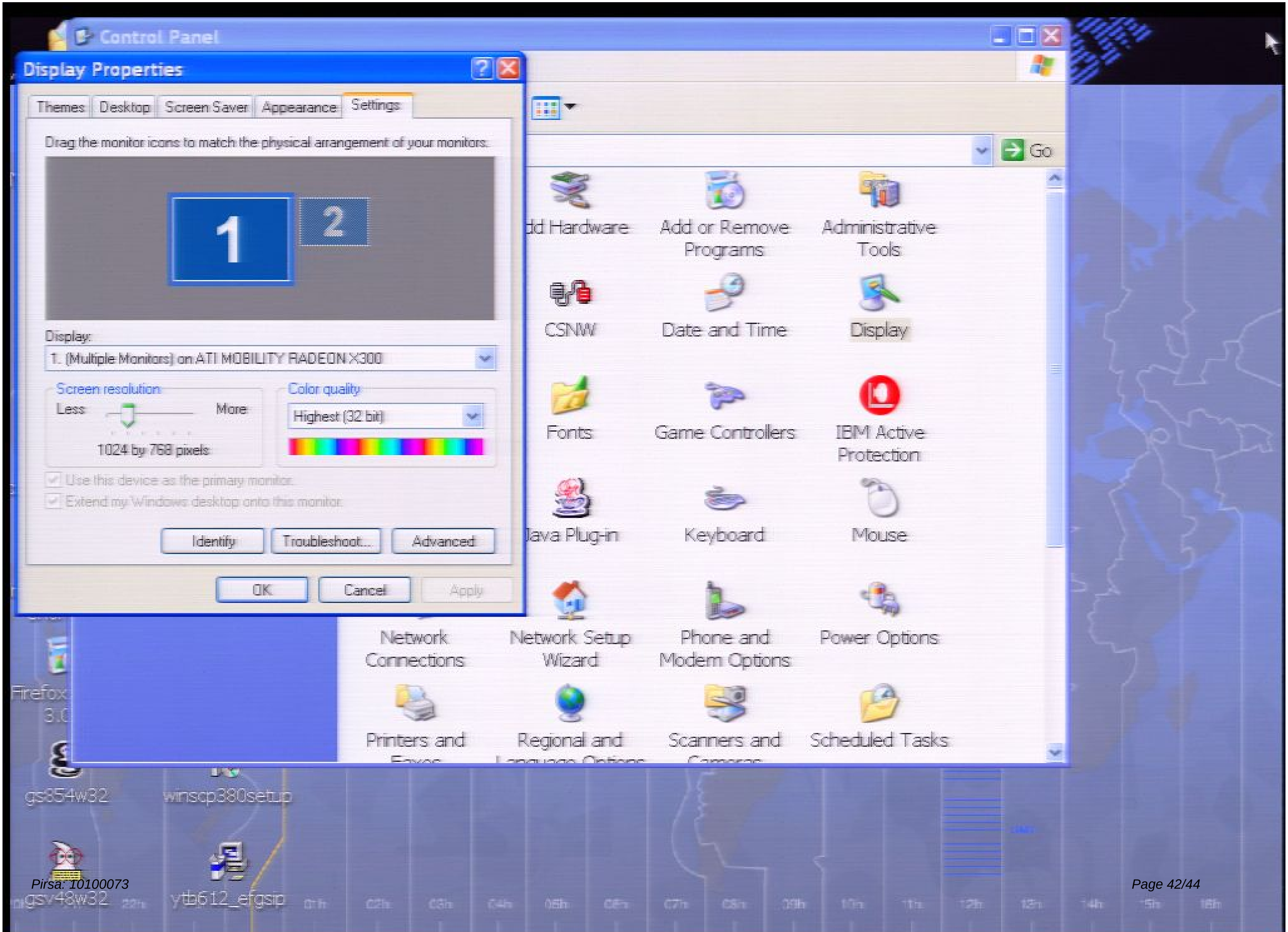
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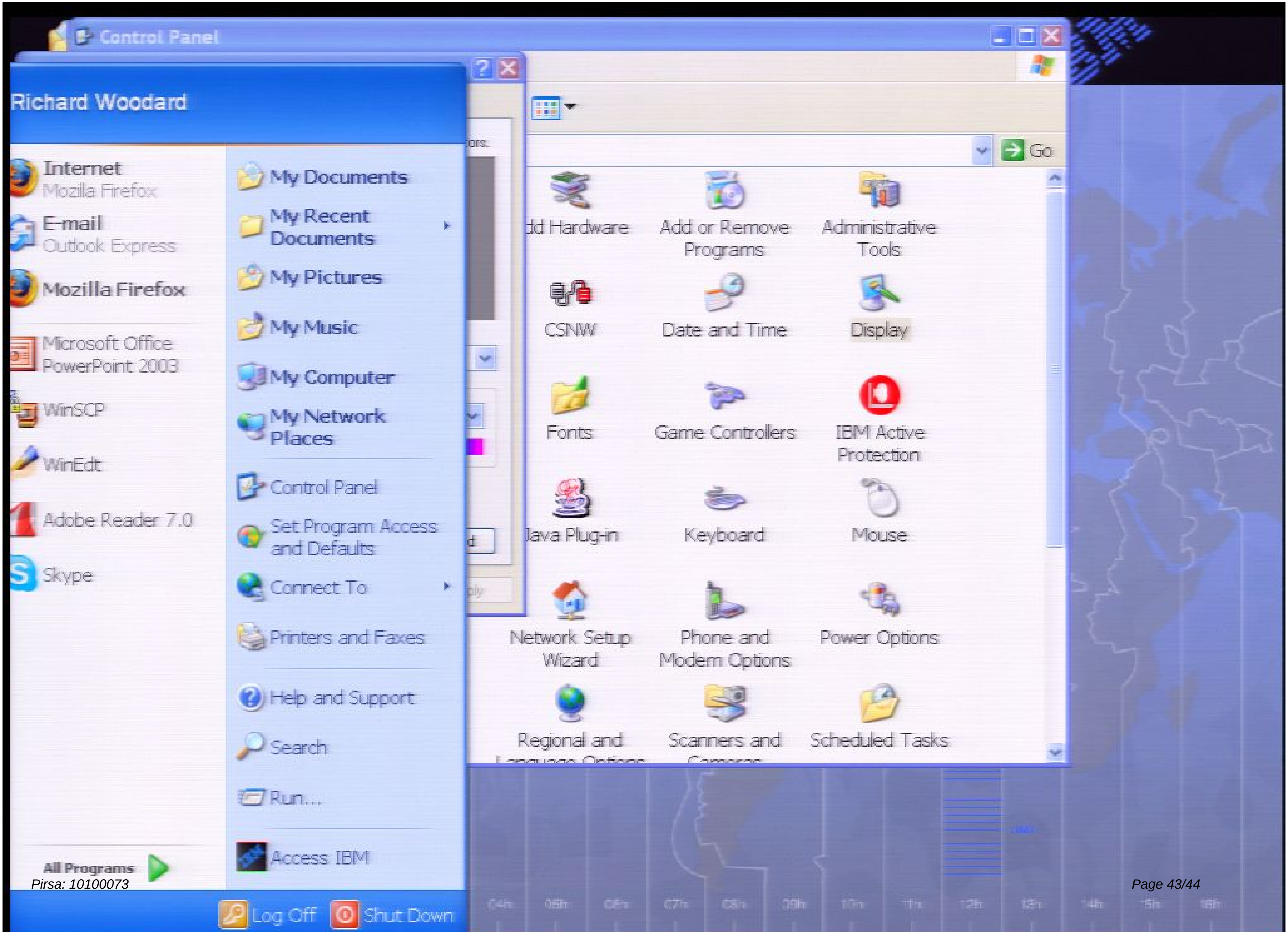
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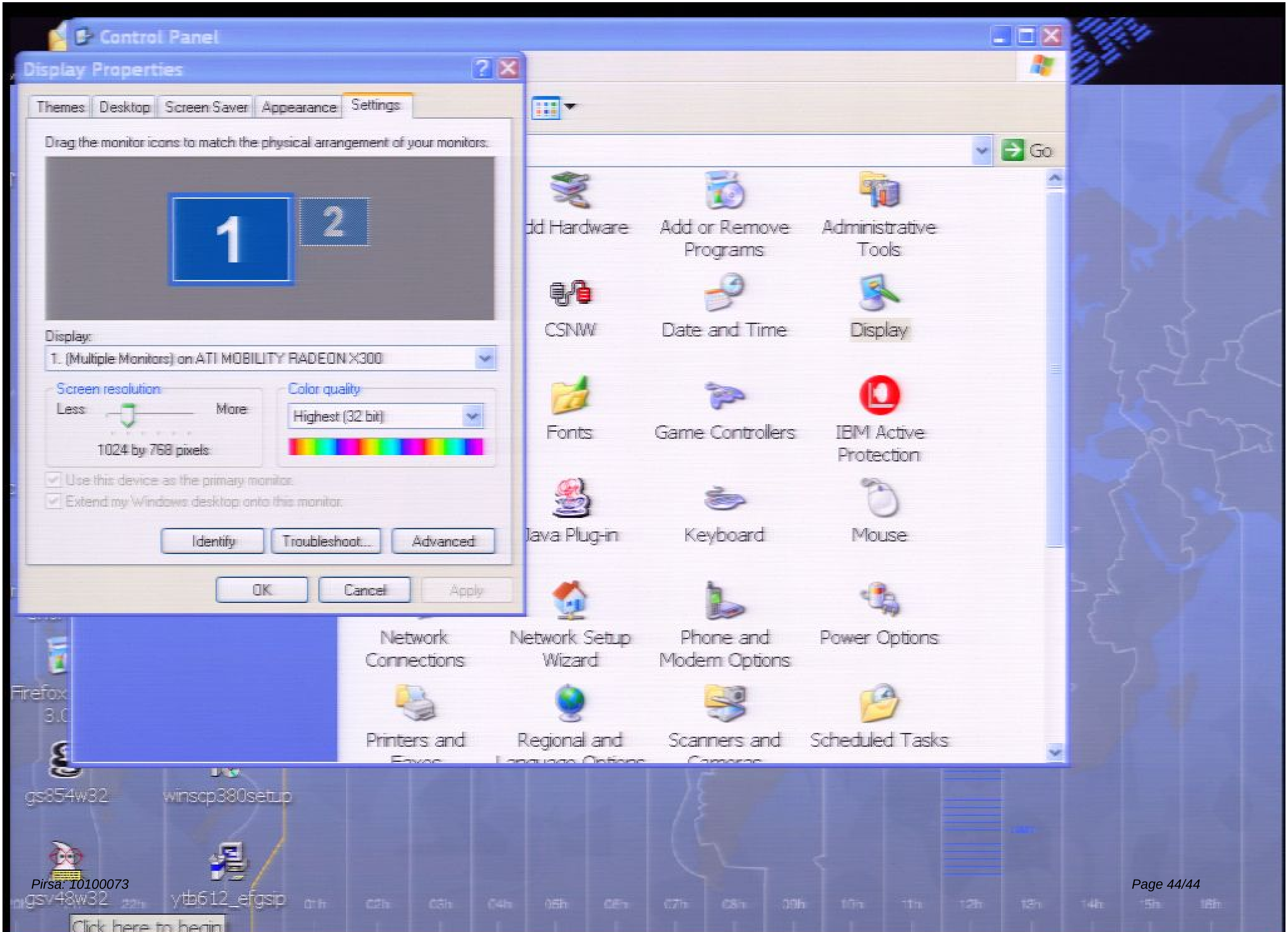
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