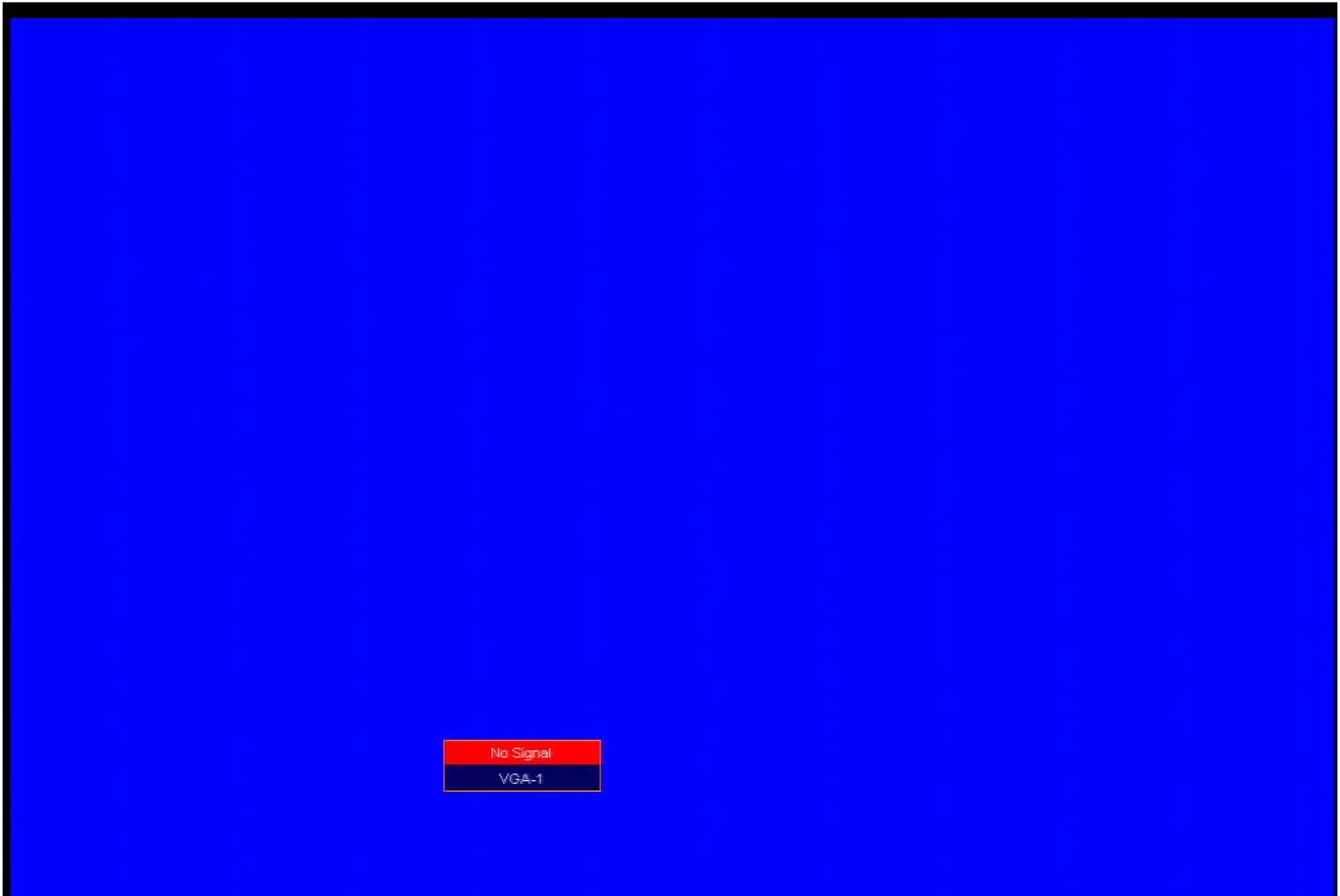
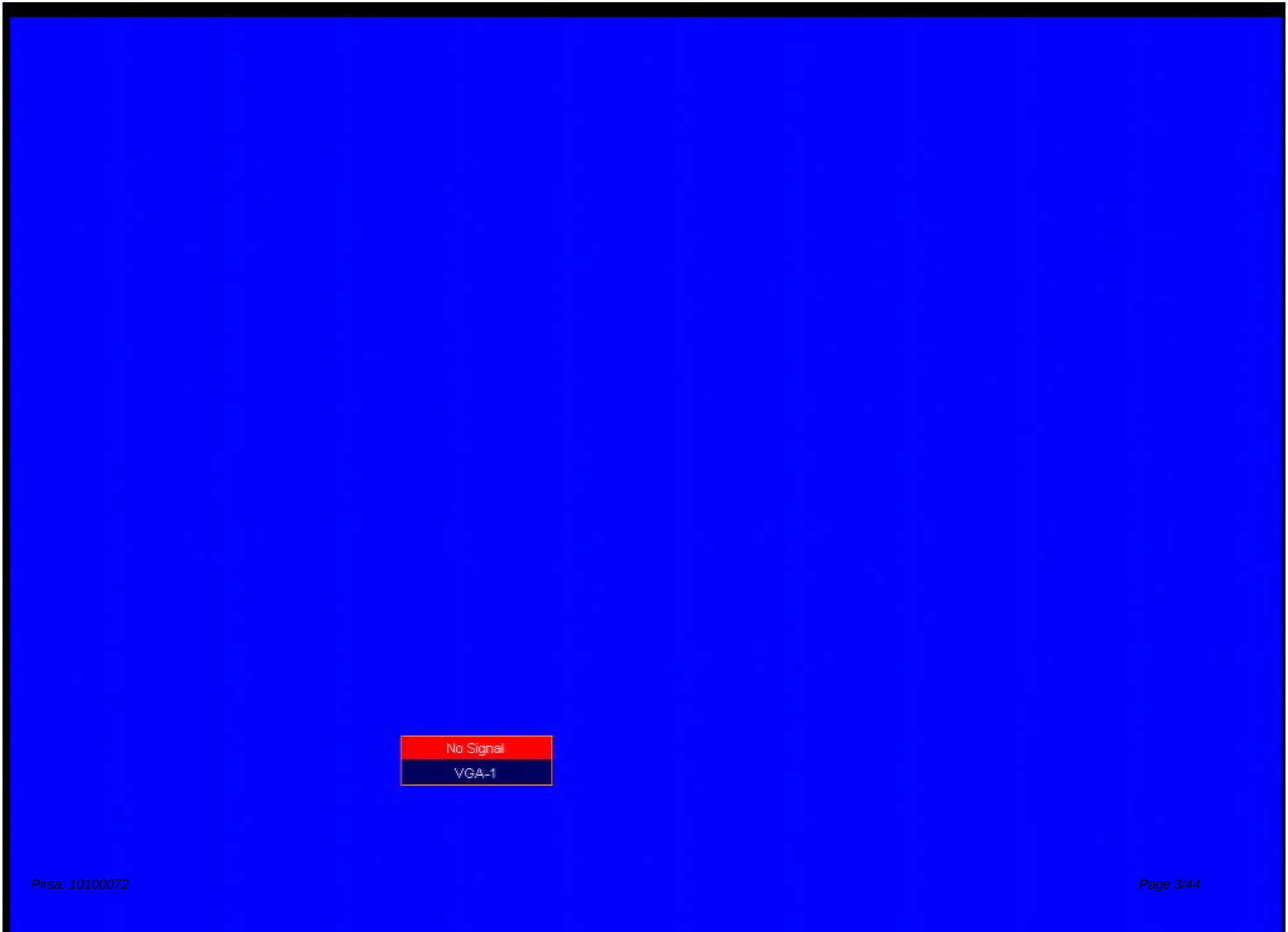
Title: On radiation and IR divergences in dS space

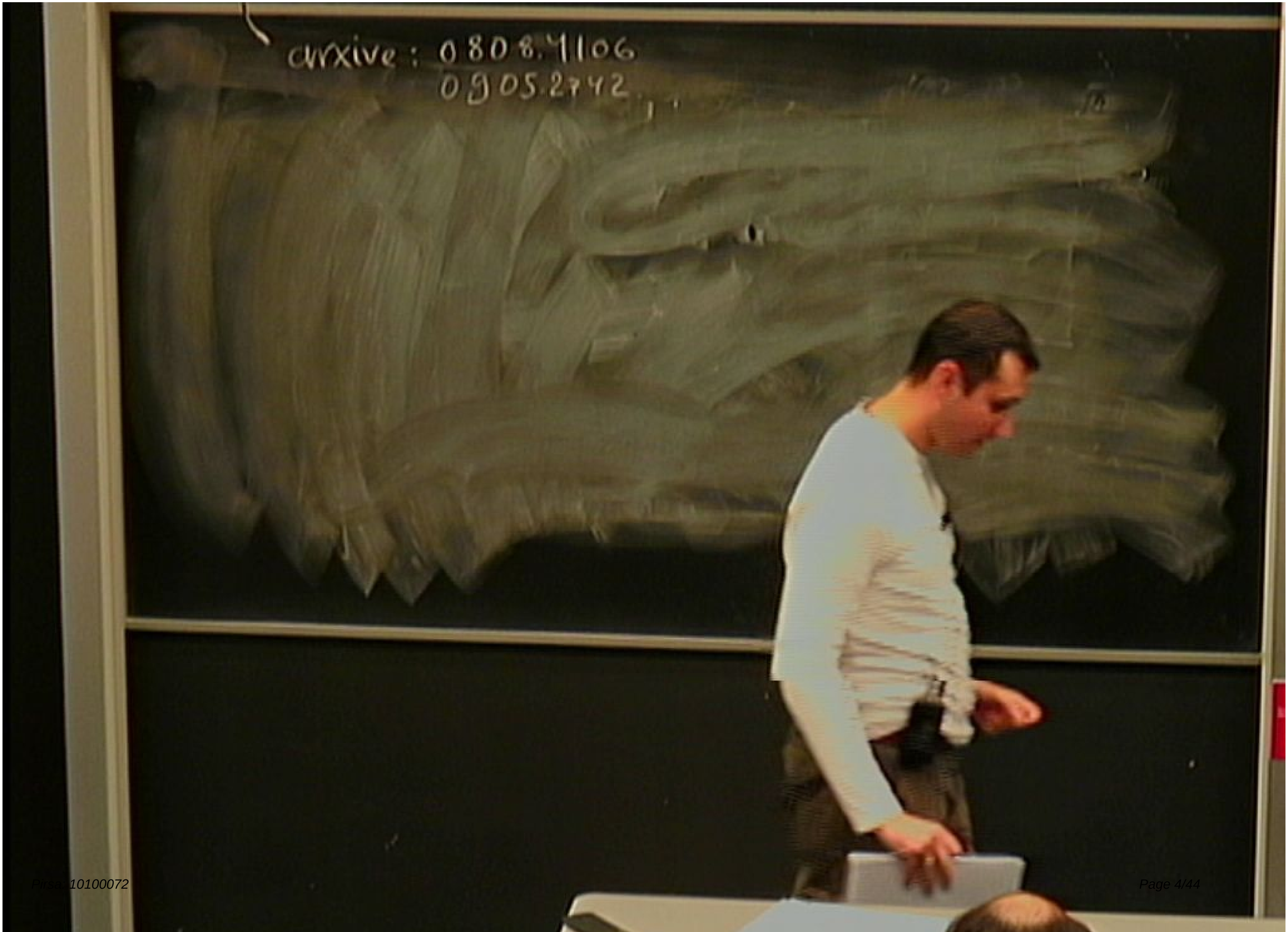
Date: Oct 28, 2010 04:45 PM

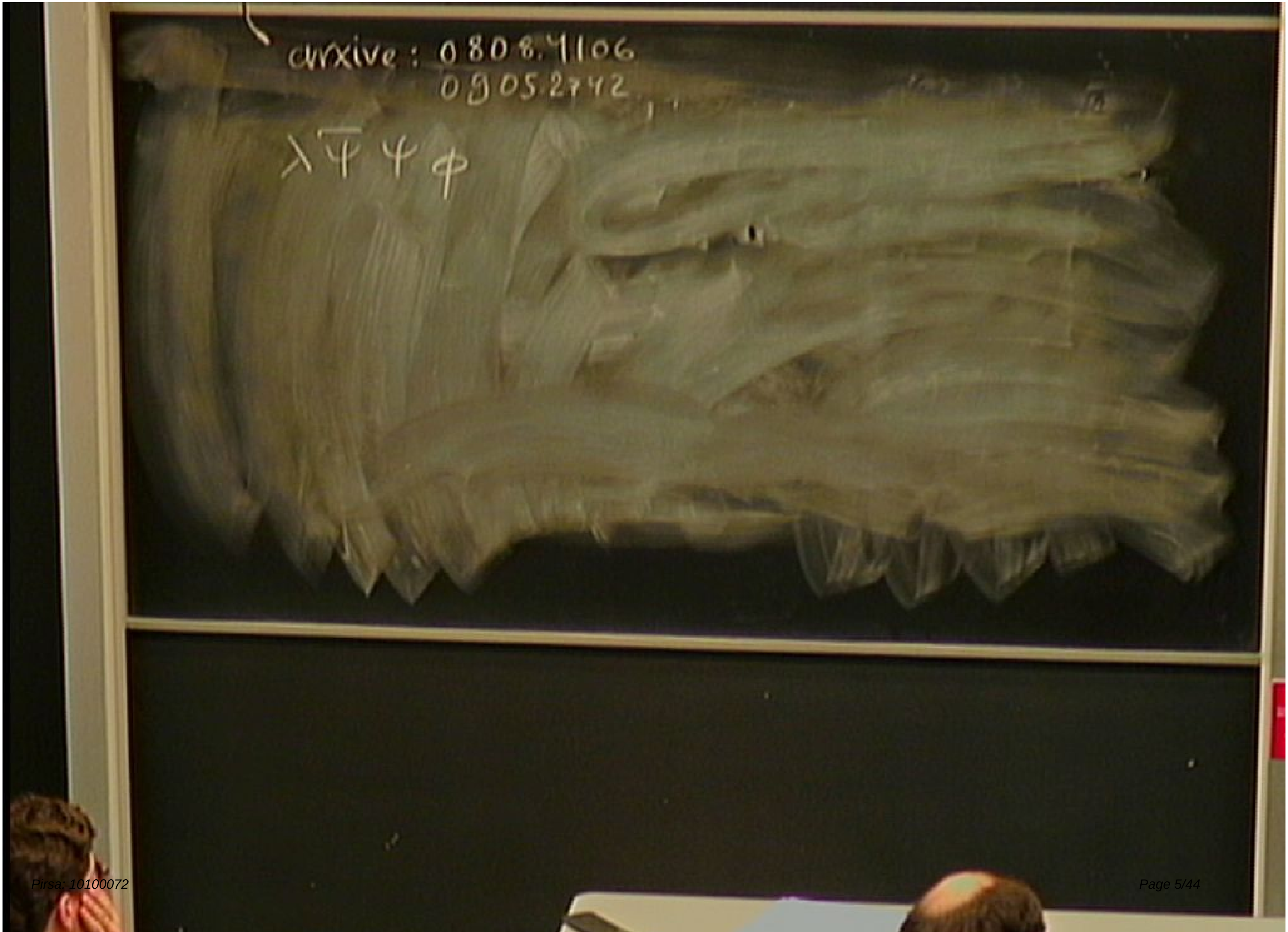
URL: http://pirsa.org/10100072

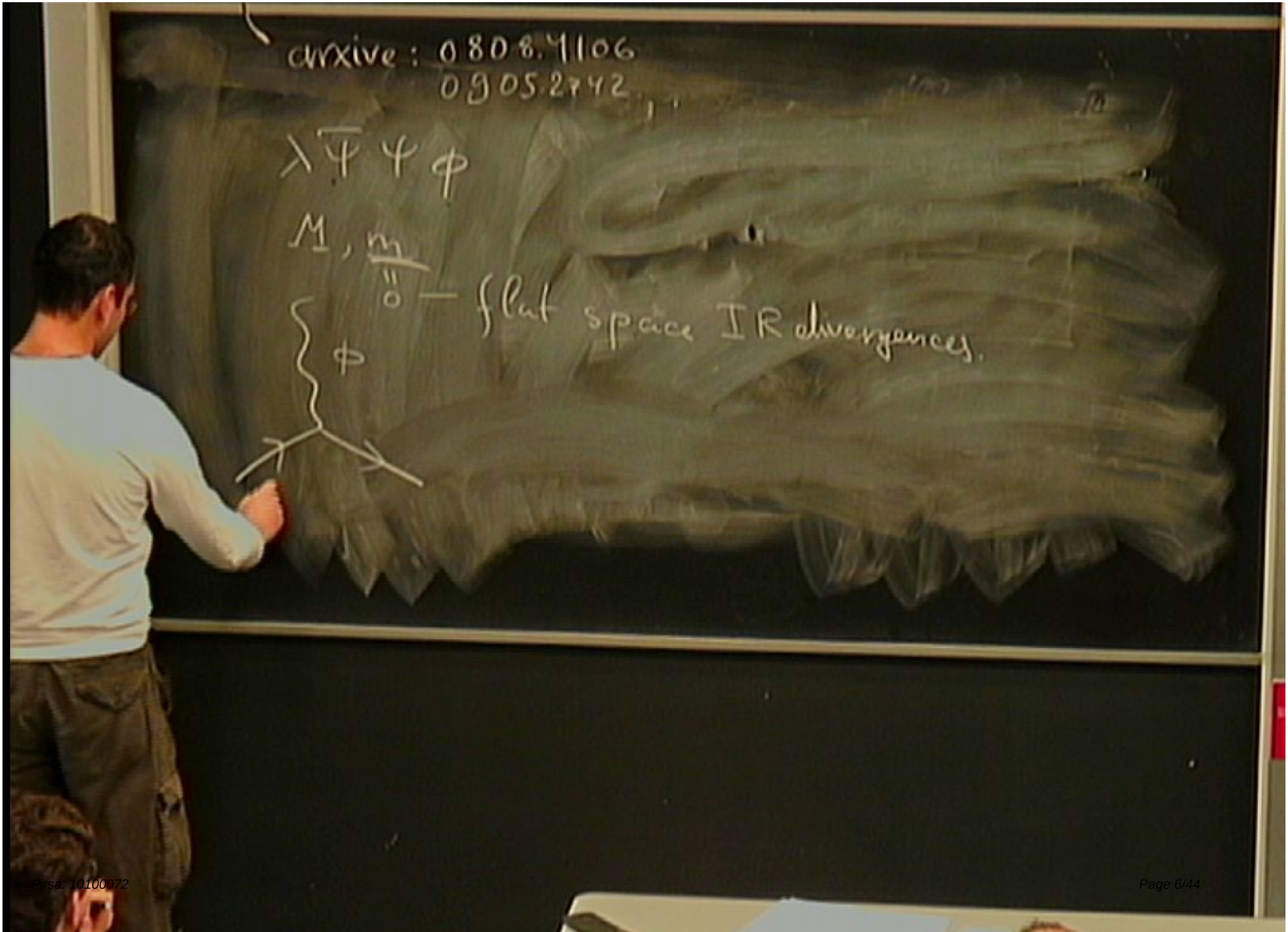
Abstract: It is well known that there should be a total cancellation of the IR divergences in closed systems described by interacting quantum field theories, such as QED and gravity. I am going to show that such a cancellation does not happen in de Sitter space.

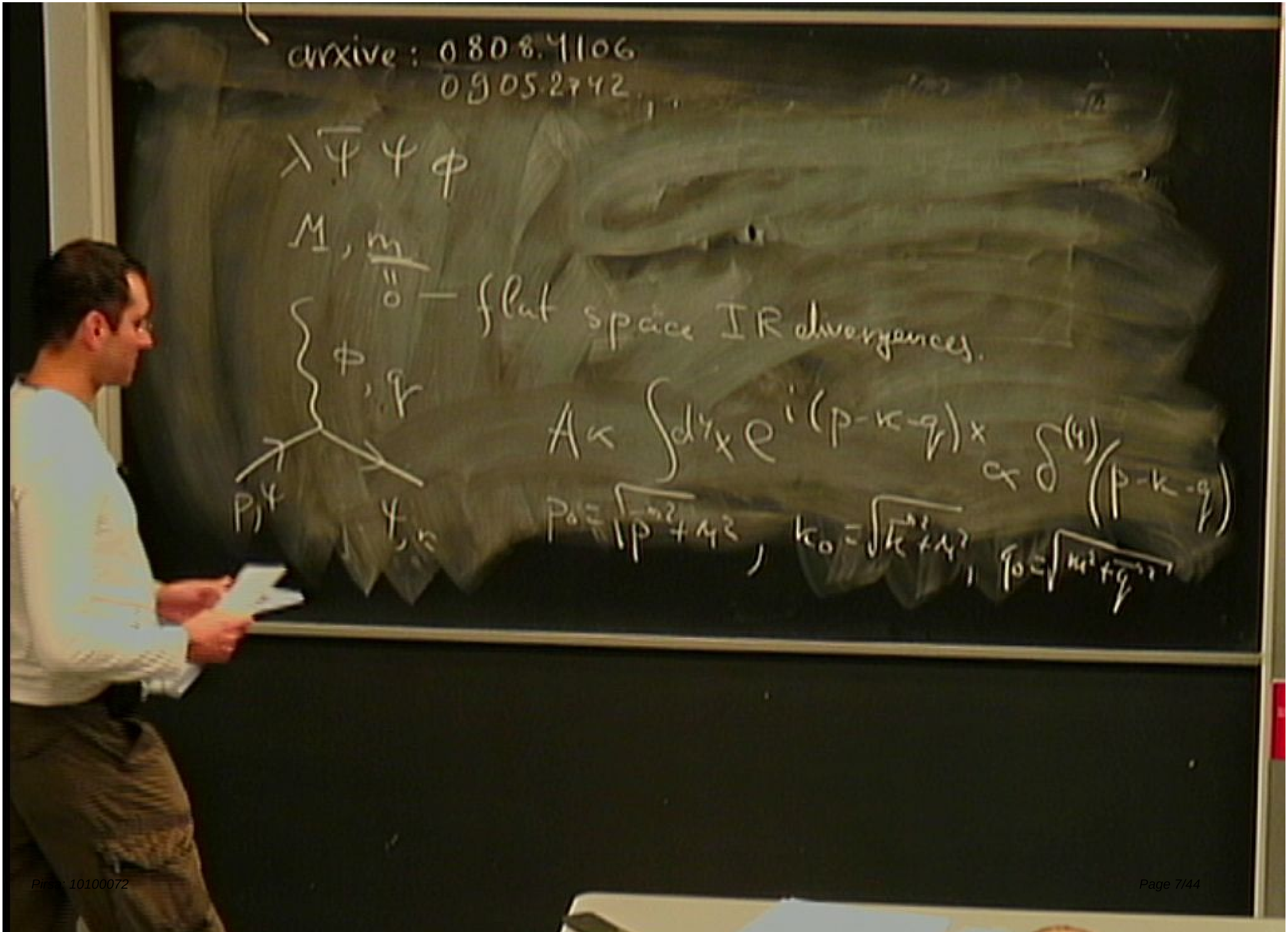

No Signal
VGA-1

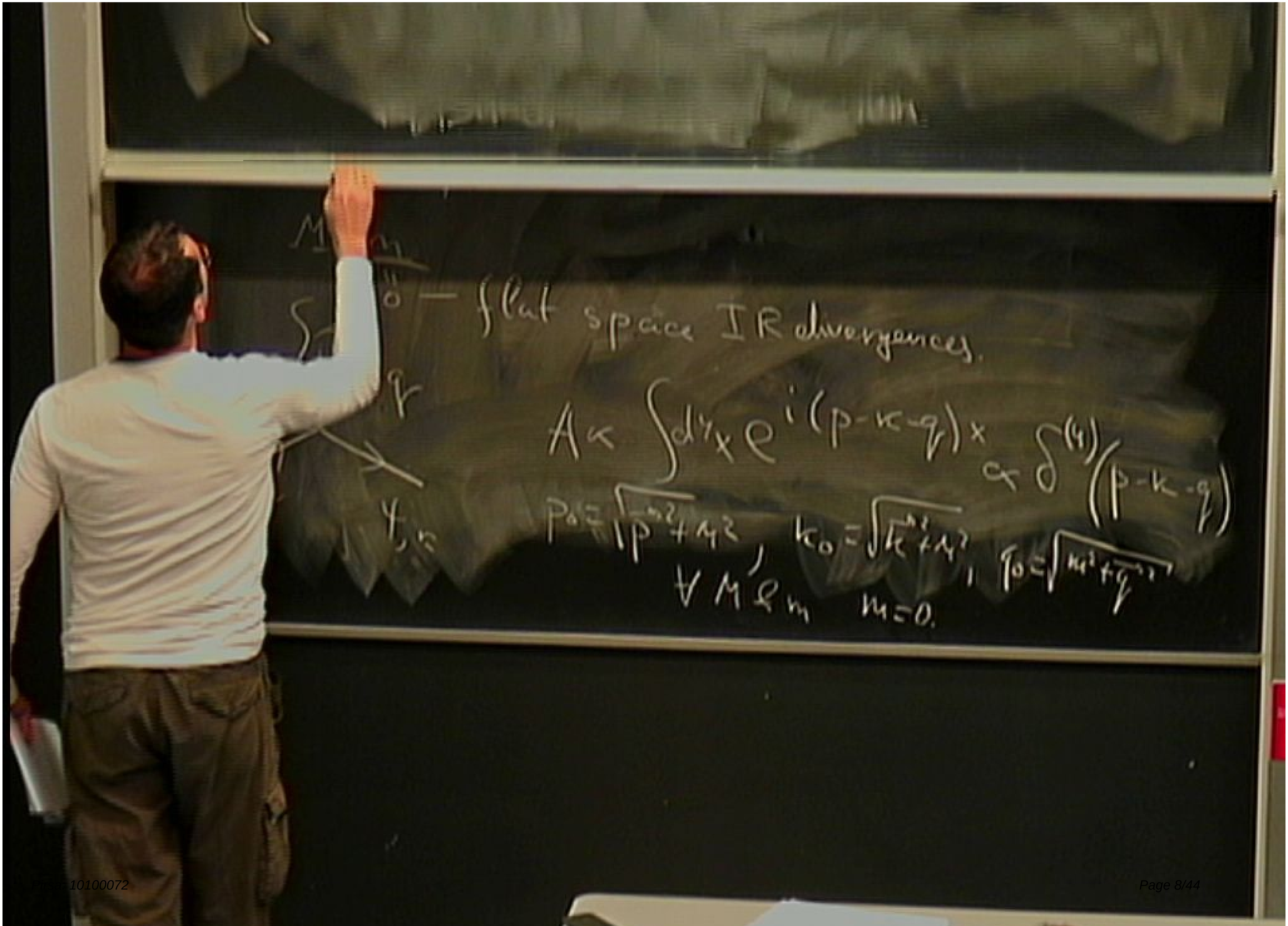

No Signal
VGA-1

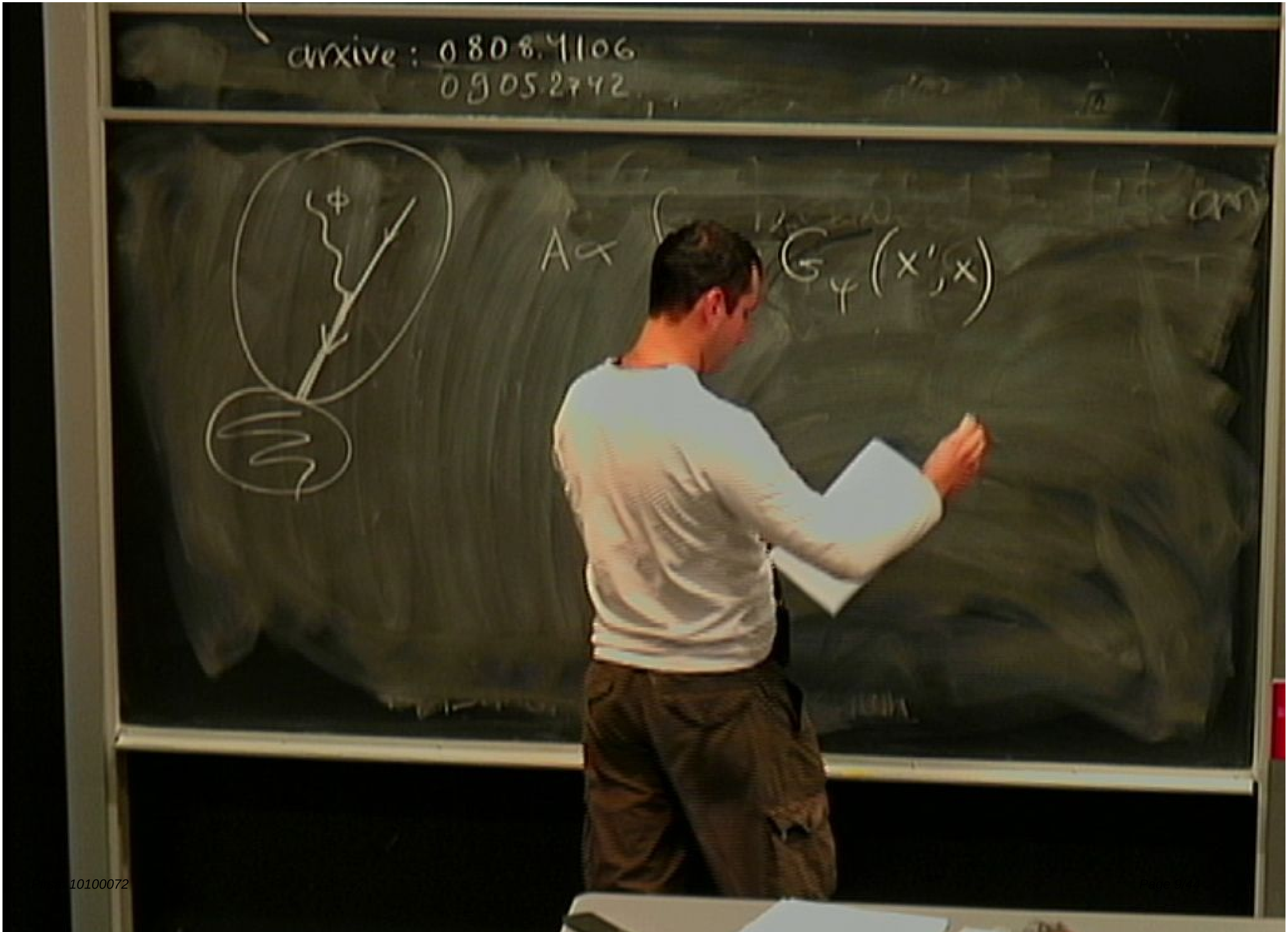

arxive : 0808.4106
0905.2742

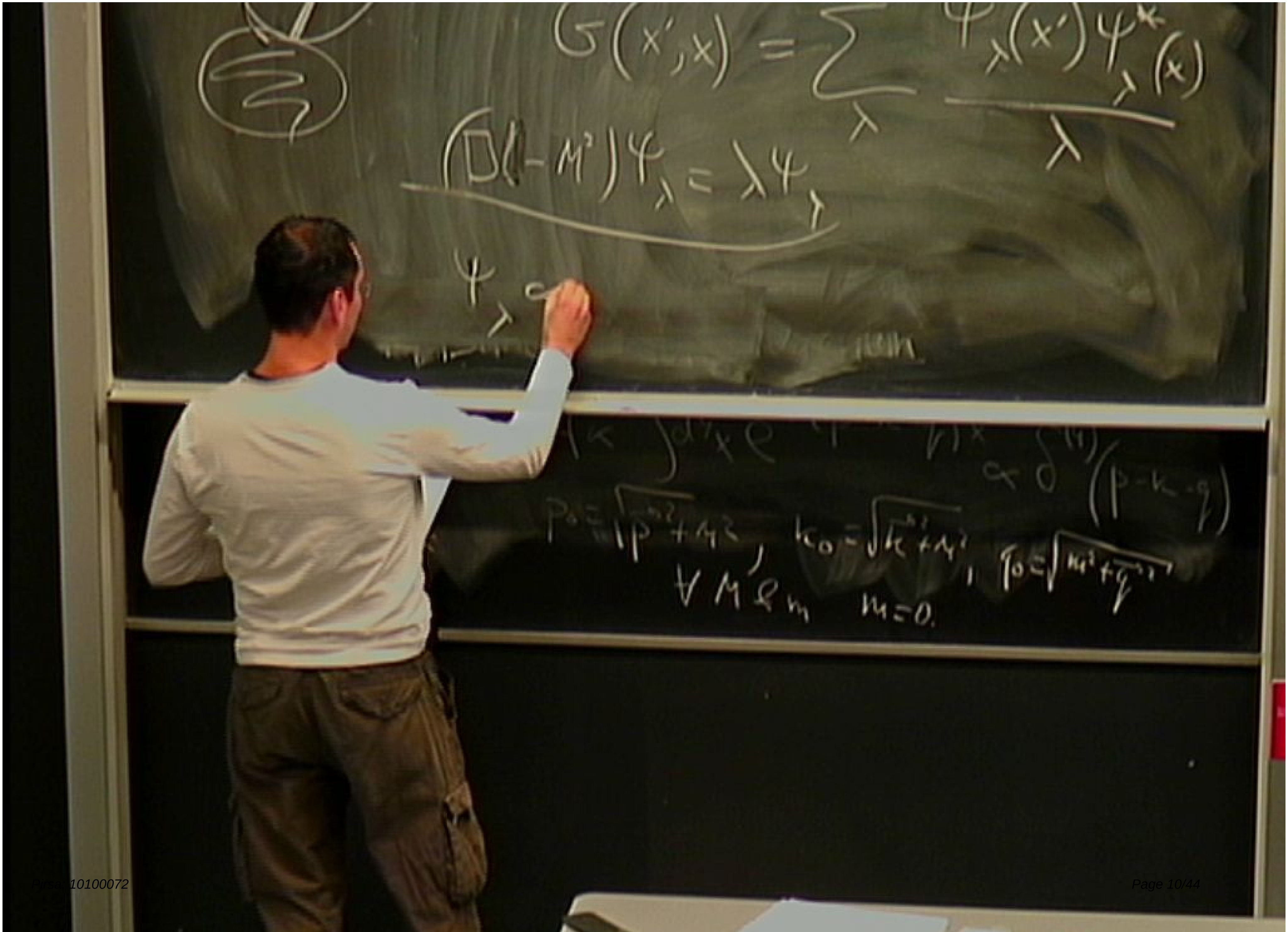

arxive : 0808.4106
0905.2742
$\lambda \bar{\Psi} \psi \phi$

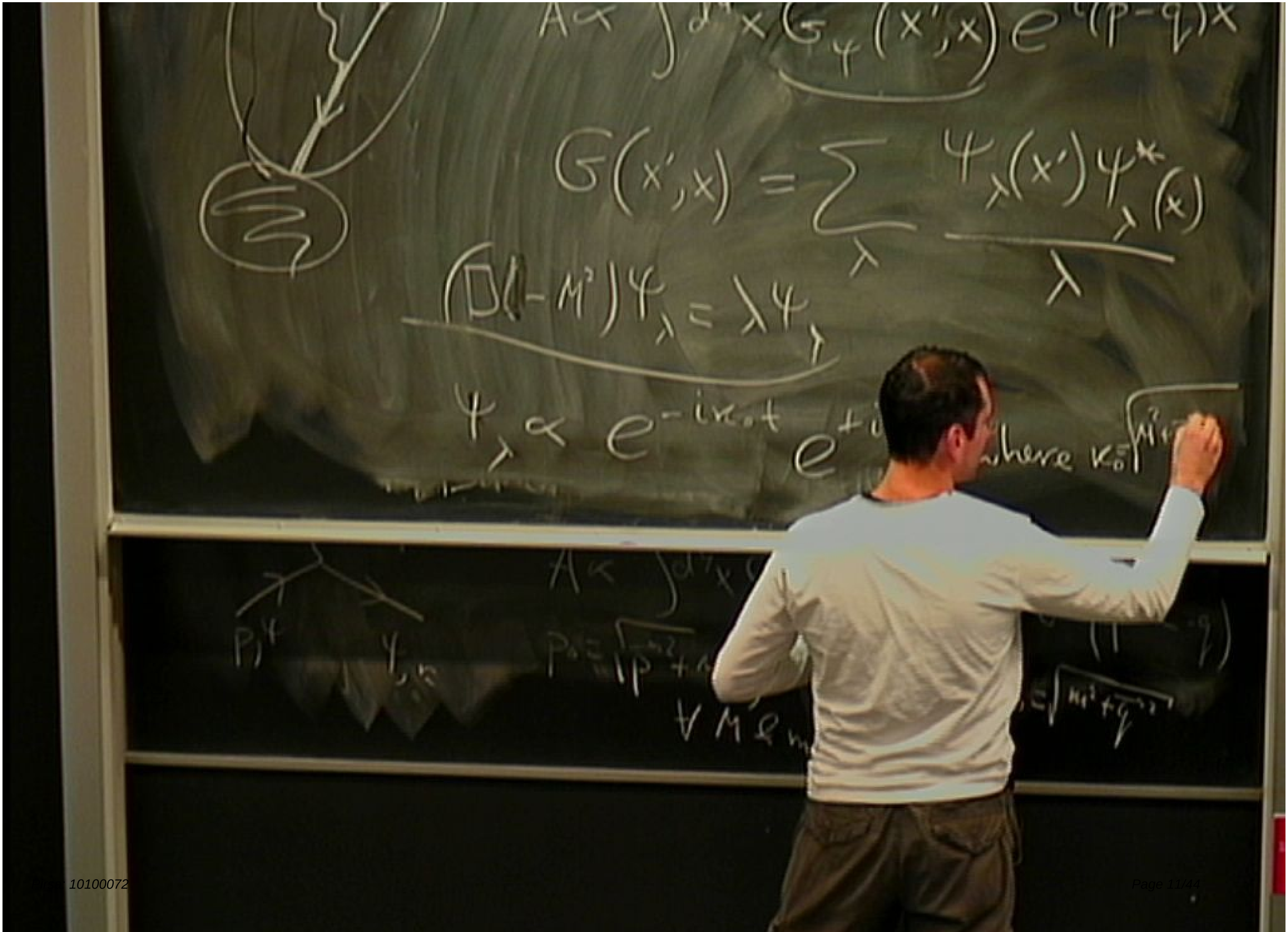

arxive : 0808.4106
0905.2742
λ Ψ̄ Ψ φ
M, m
0 — flat space IR divergences.
φ

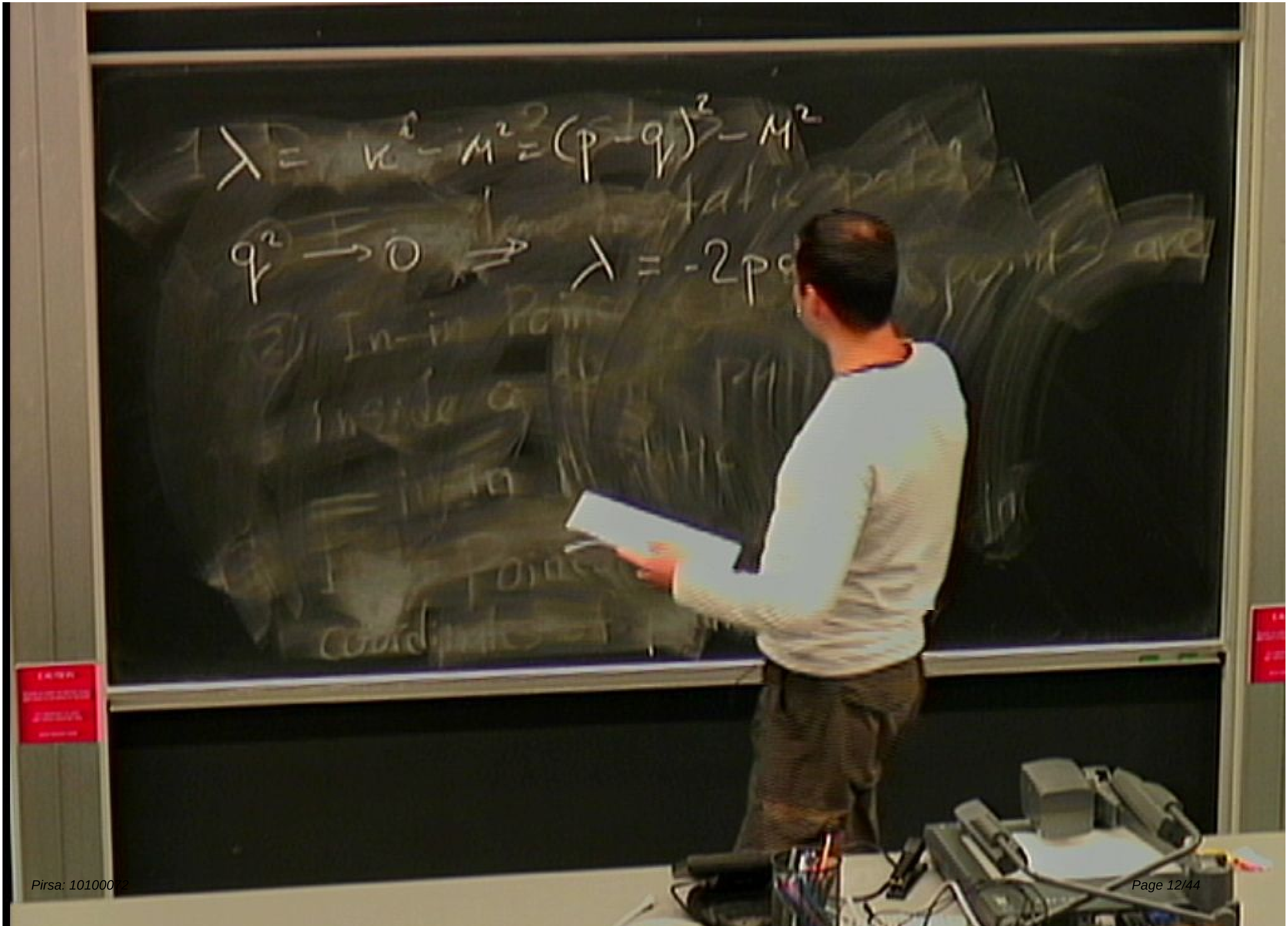

arxive : 0808.4106
0905.2742
λ Ψ̄ Ψ φ
M, m = 0 — flat space IR divergences.
φ, q
p, Ψ
Ψ, k
A ∝ ∫d⁴x e^{i(p-k-q)x} ∝ δ^{(4)}(p-k-q)
p₀ = √(p⃗² + M²), k₀ = √(k⃗² + M²), q₀ = √(m² + q⃗²)

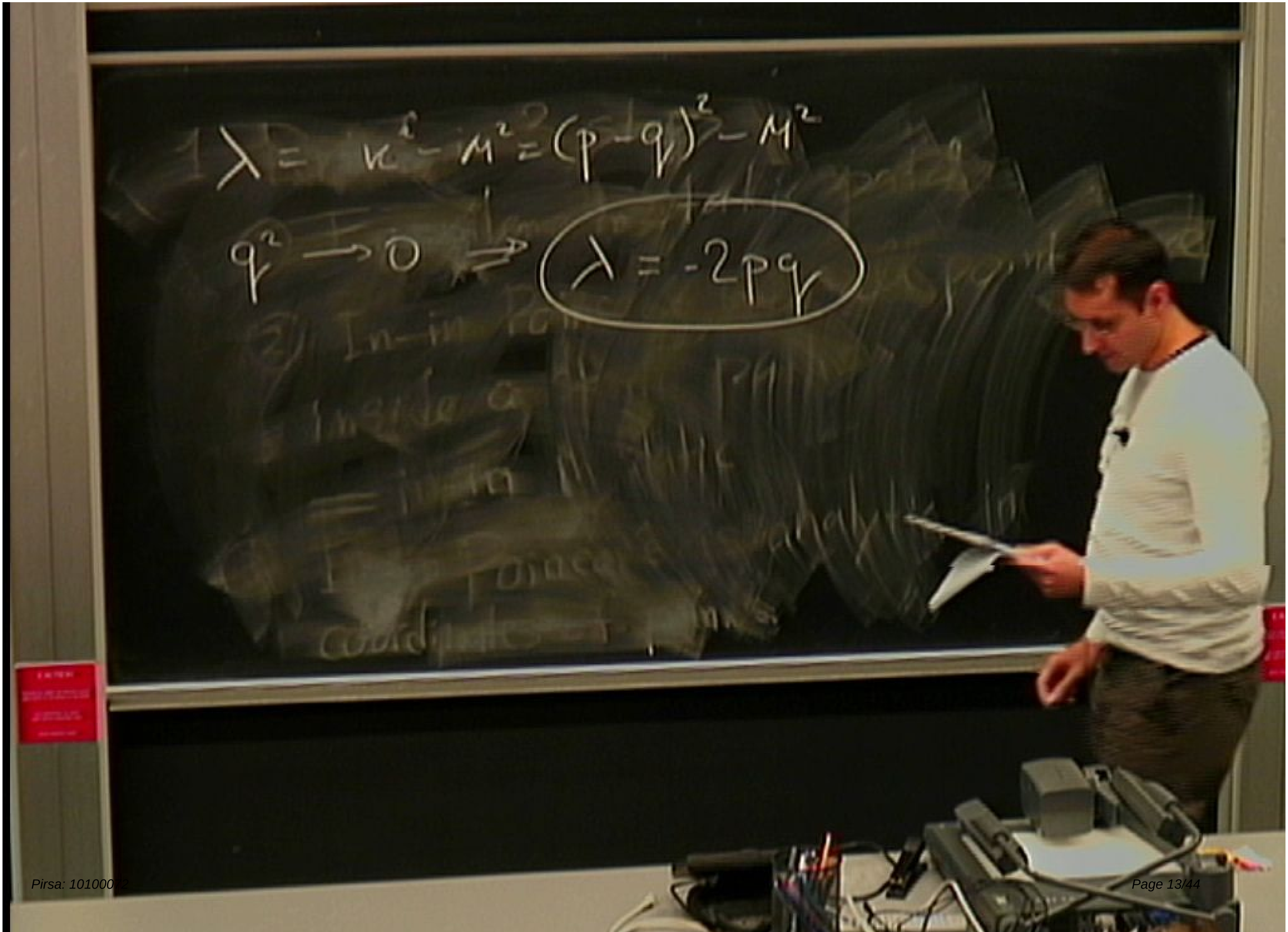

flat space IR divergences.
A ∝ ∫d⁴x e^{i(p-k-q)x} ∝ δ^{(4)}(p-k-q)
p₀ = √(p² + M²), k₀ = √(k² + M²), q₀ = √(m² + q²)
∀ M & m   m = 0.

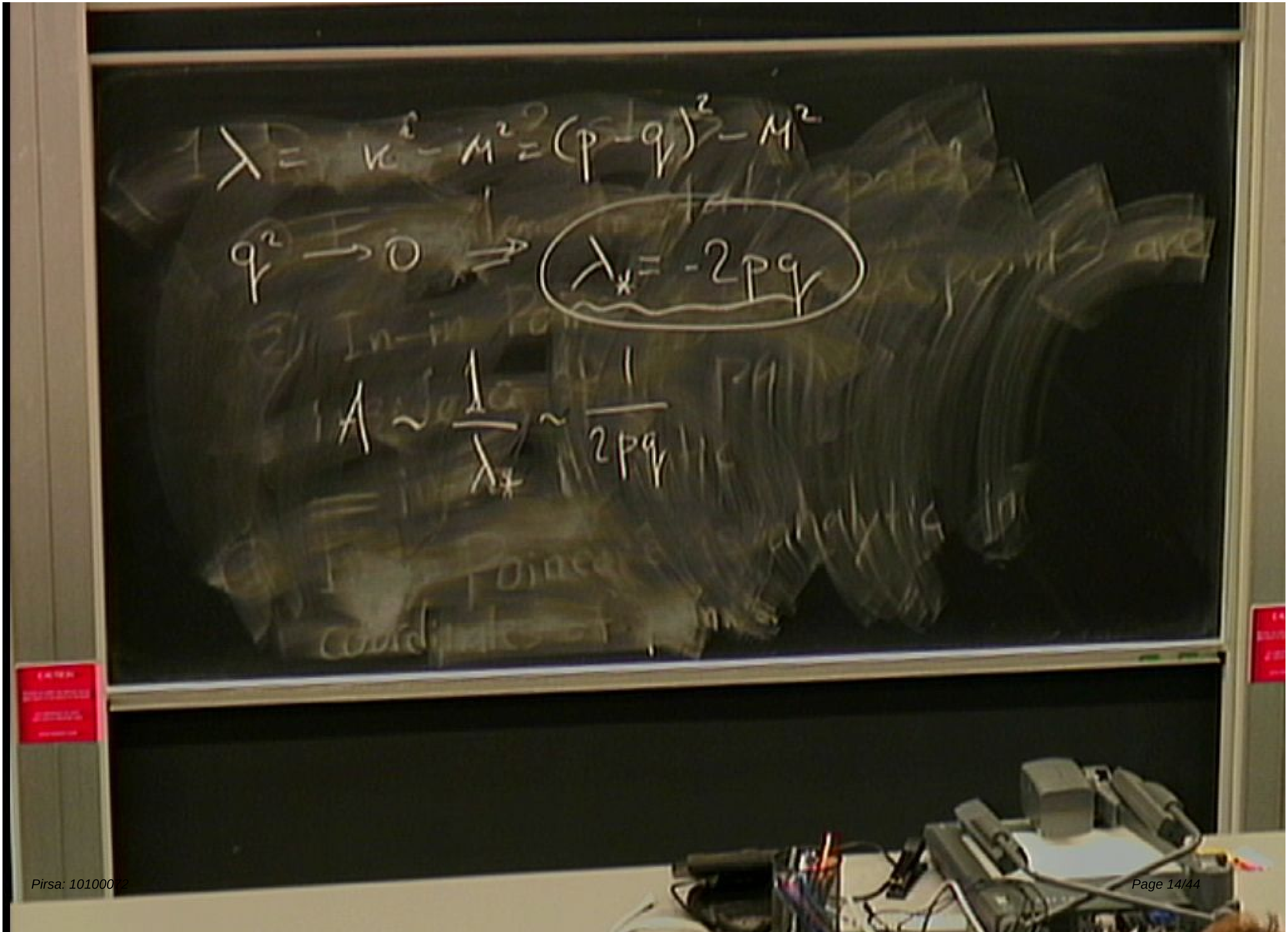

arxive : 0808.4106
0905.2742
ϕ
A ∝ Gψ(x',x)
10100072

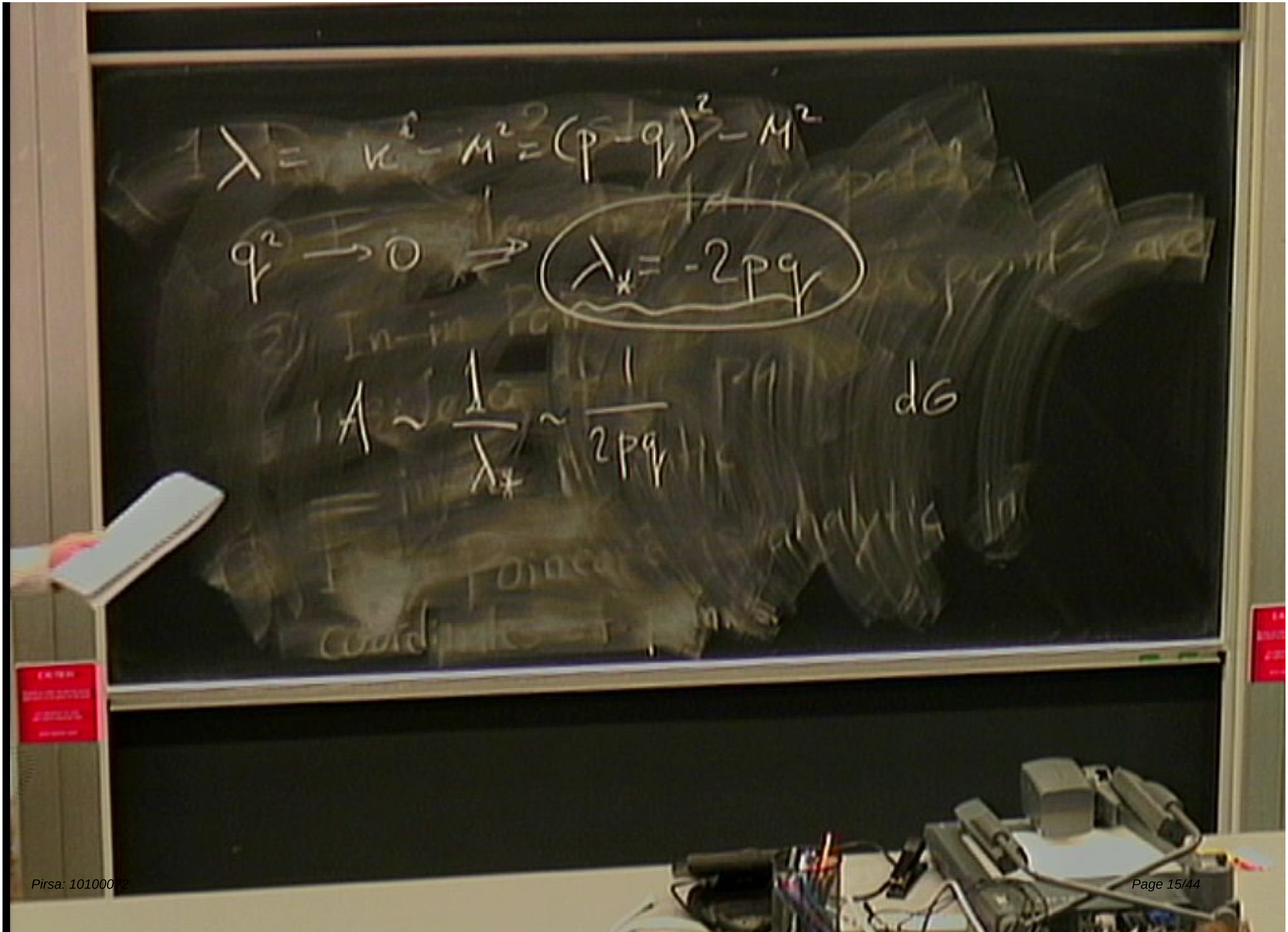

G(x';x) = Σ_λ ψ_λ(x') ψ*_λ(x) / λ
(□ - M²) ψ_λ = λ ψ_λ
∀ M & m   m=0.


$G(x',x) = \sum_\lambda \frac{\psi_\lambda(x')\,\psi^*_\lambda(x)}{\lambda}$
$(\Box - M^2)\psi_\lambda = \lambda\psi_\lambda$
where

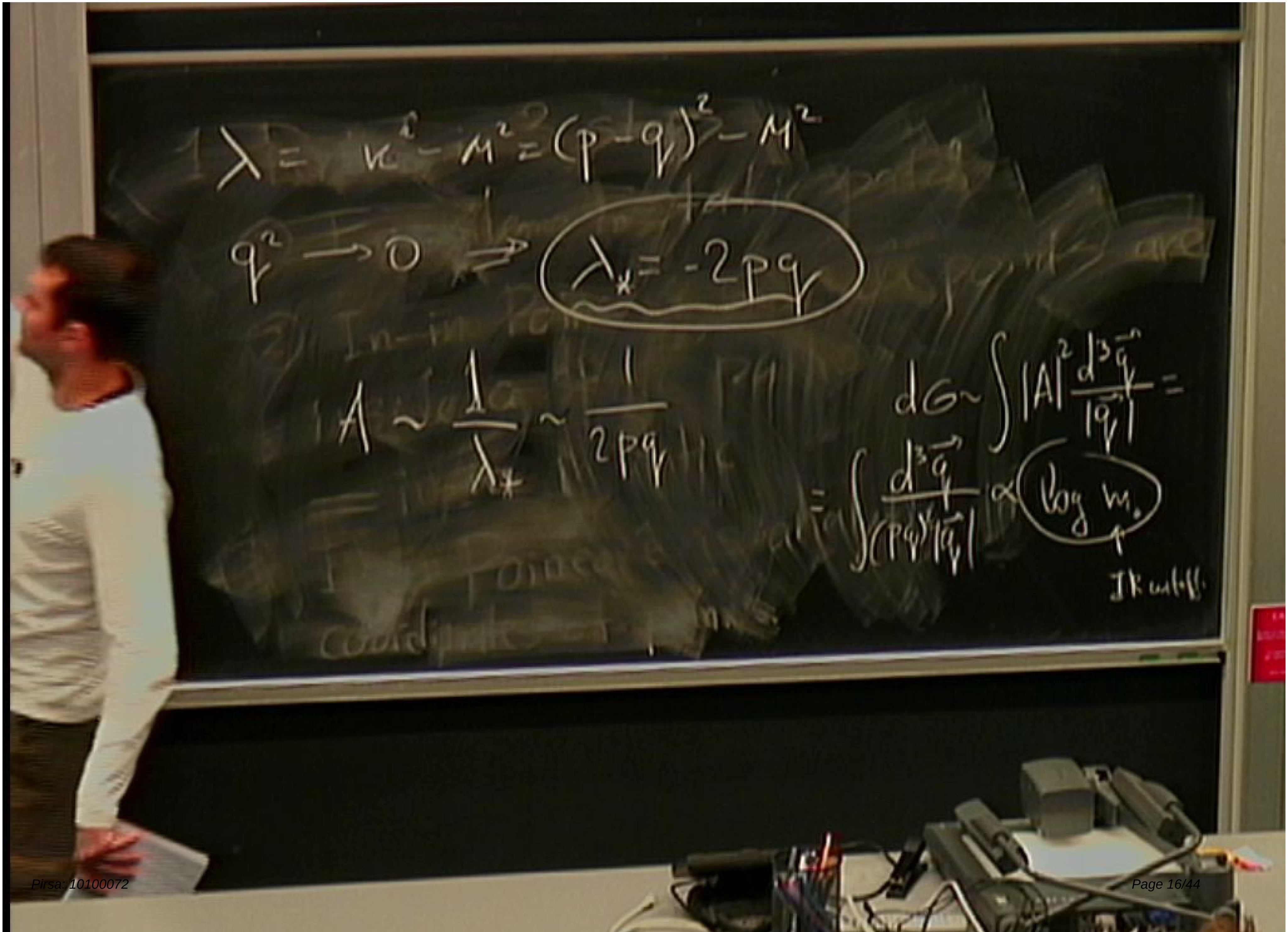

$\lambda = k^2 - M^2 = (p-q)^2 - M^2$
$q^2 \to 0 \Rightarrow \lambda = -2pq$


λ = k² − M² = (p − q)² − M²
q² → 0 ⇒ λ = −2pq

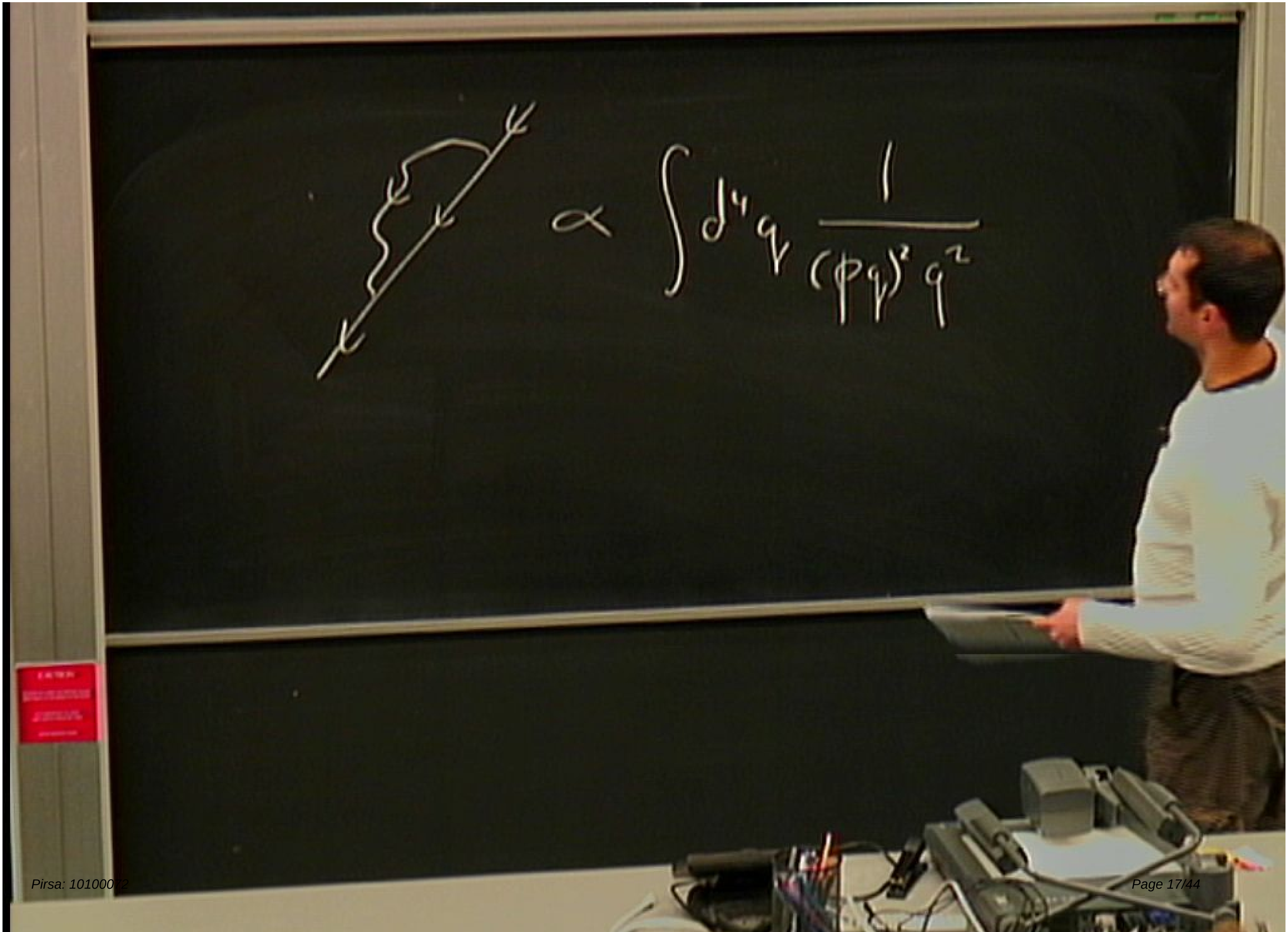

$\lambda = k^2 - M^2 = (p-q)^2 - M^2$
$q^2 \to 0 \Rightarrow \lambda_* = -2pq$
$A \sim \frac{1}{\lambda_*} \sim \frac{1}{2pq}$

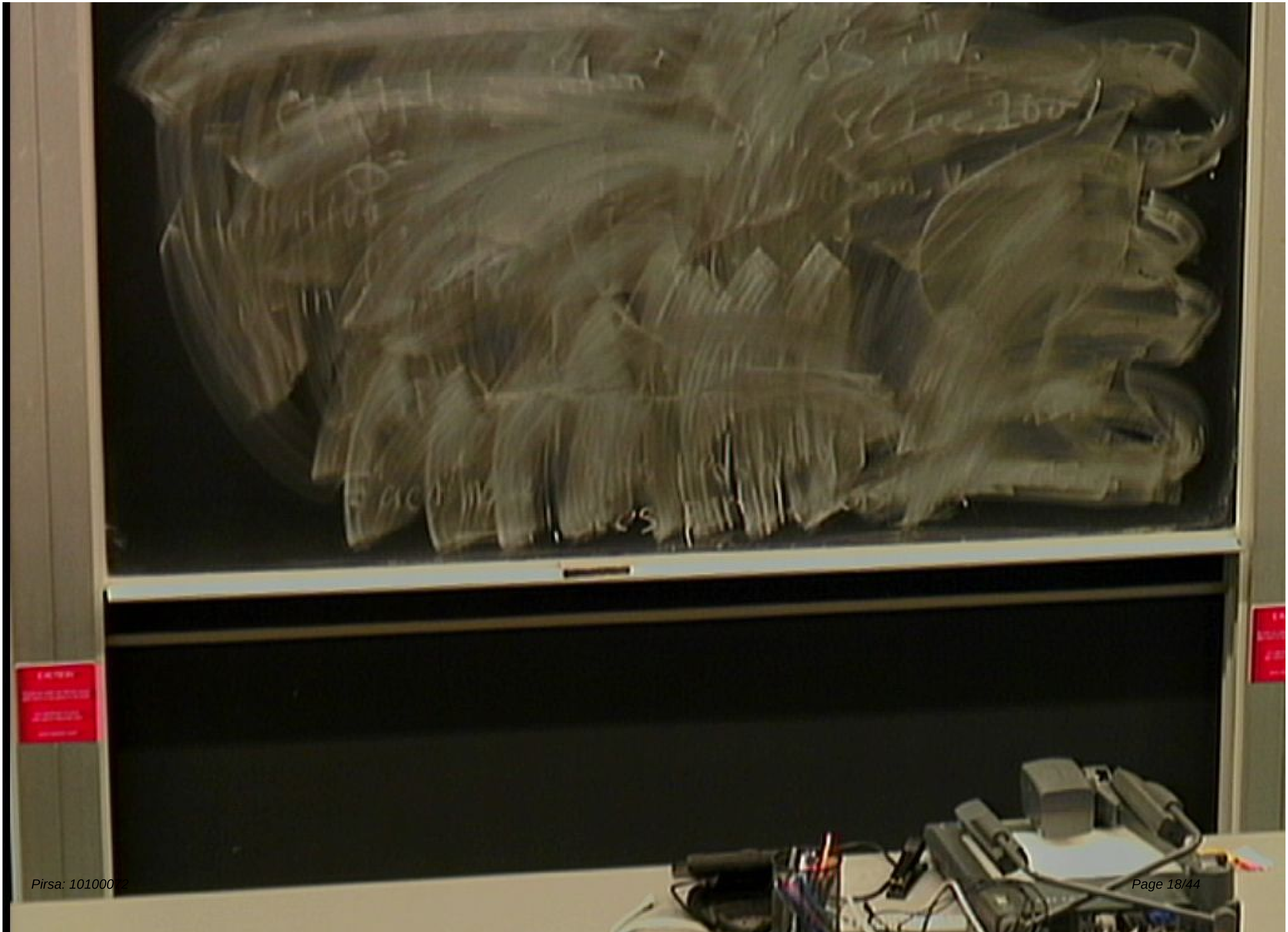


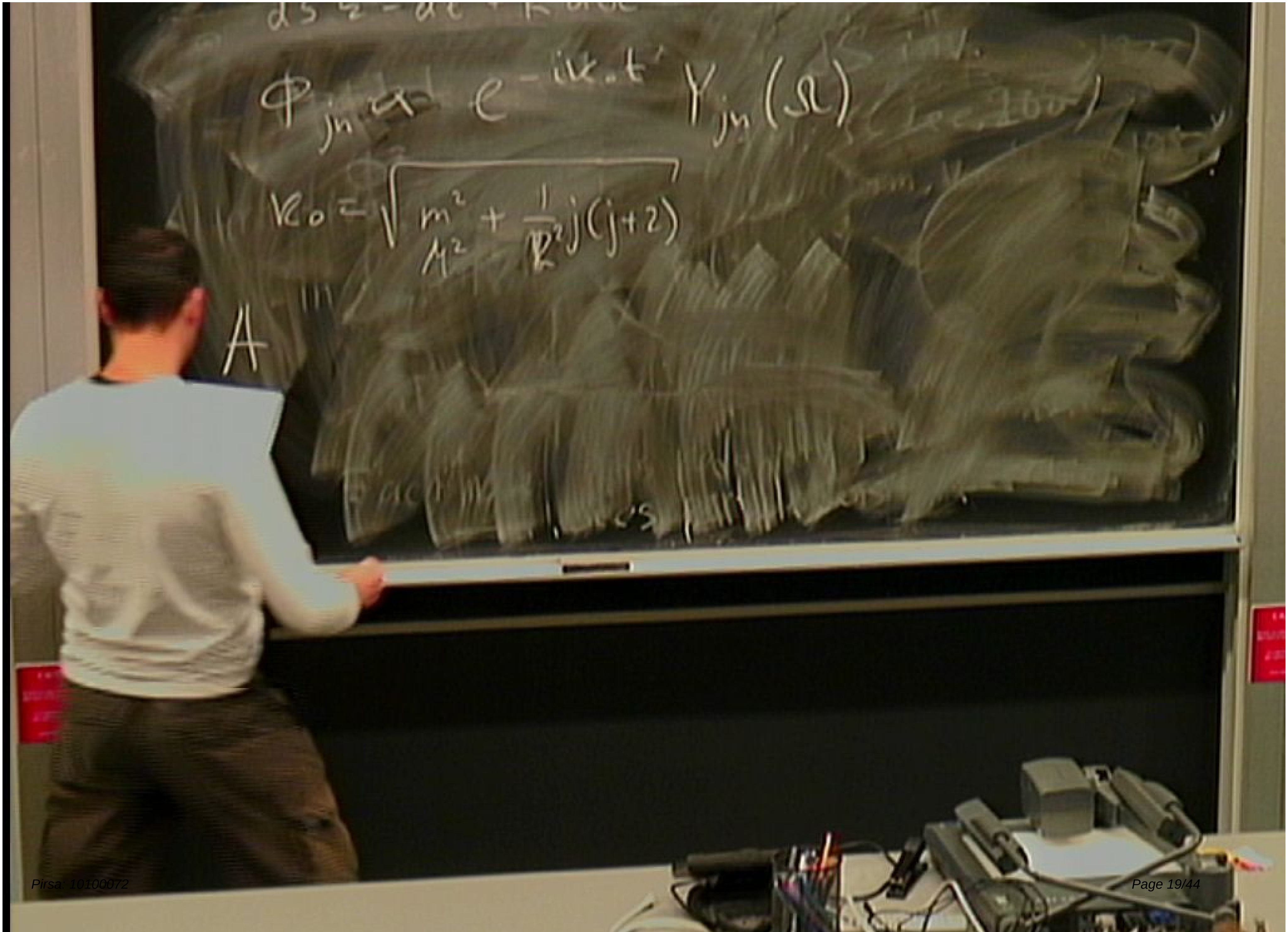
$\lambda = k^2 - M^2 = (p-q)^2 - M^2$
$q^2 \to 0 \Rightarrow \lambda_* = -2pq$
$A \sim \frac{1}{\lambda_*} \sim \frac{1}{2pq}$
dG



$\lambda = k^2 - M^2 = (p-q)^2 - M^2$
$q^2 \to 0 \Rightarrow \lambda_* = -2pq$
$A \sim \frac{1}{\lambda_*} \sim \frac{1}{2pq}$
$d\sigma \sim \int |A|^2 \frac{d^3\vec{q}}{|\vec{q}|} =$
$= \int \frac{d^3\vec{q}}{(pq)^2|\vec{q}|} \propto \log m_0$
IR cutoff


$\propto \int d^4q \frac{1}{(pq)^2 q^2}$





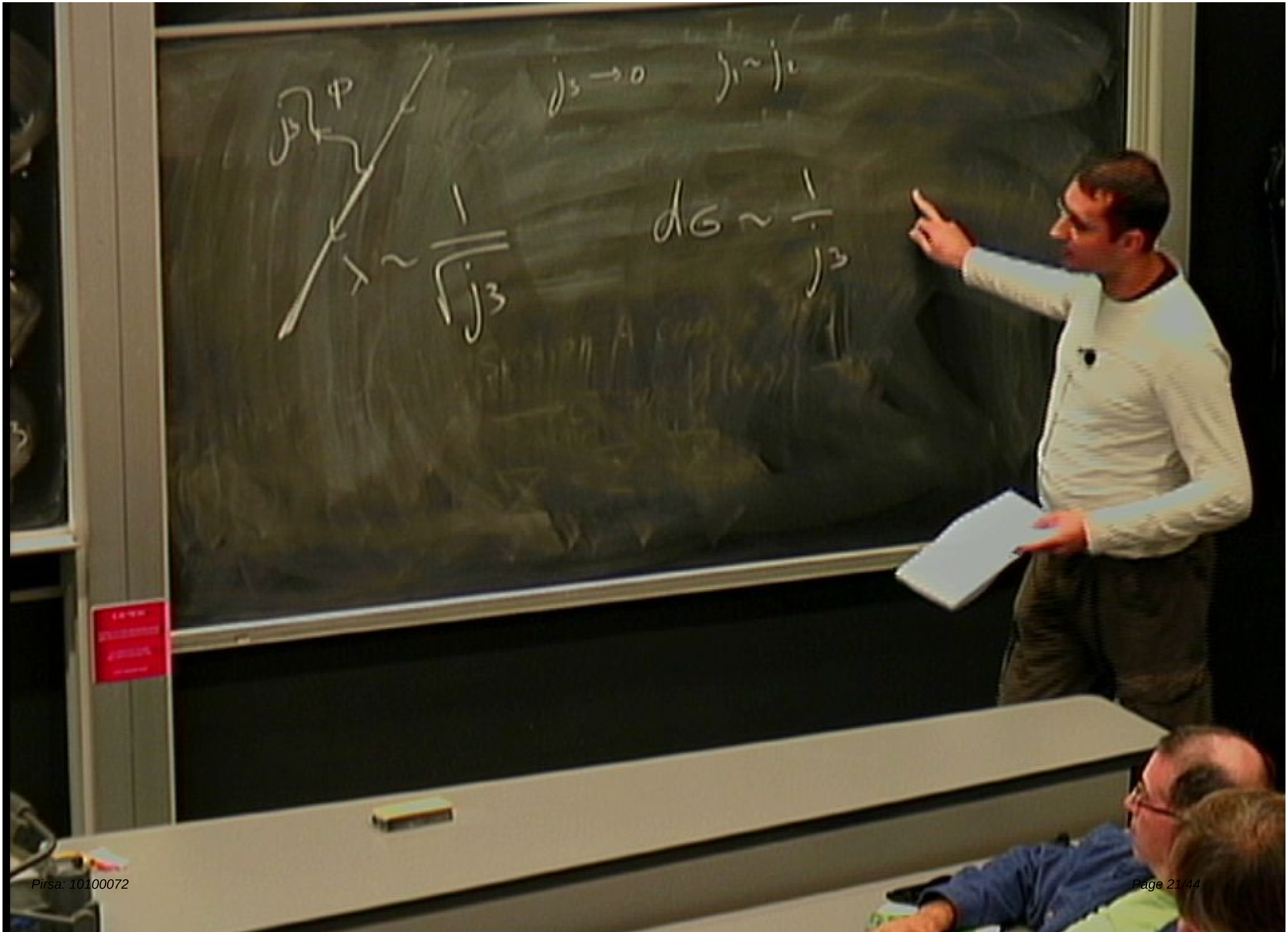
$\Phi_{jm} \propto e^{-ik_0 t} Y_{jm}(\Omega)$
$k_0 = \sqrt{\frac{m^2}{M^2} + \frac{1}{R^2} j(j+2)}$
A

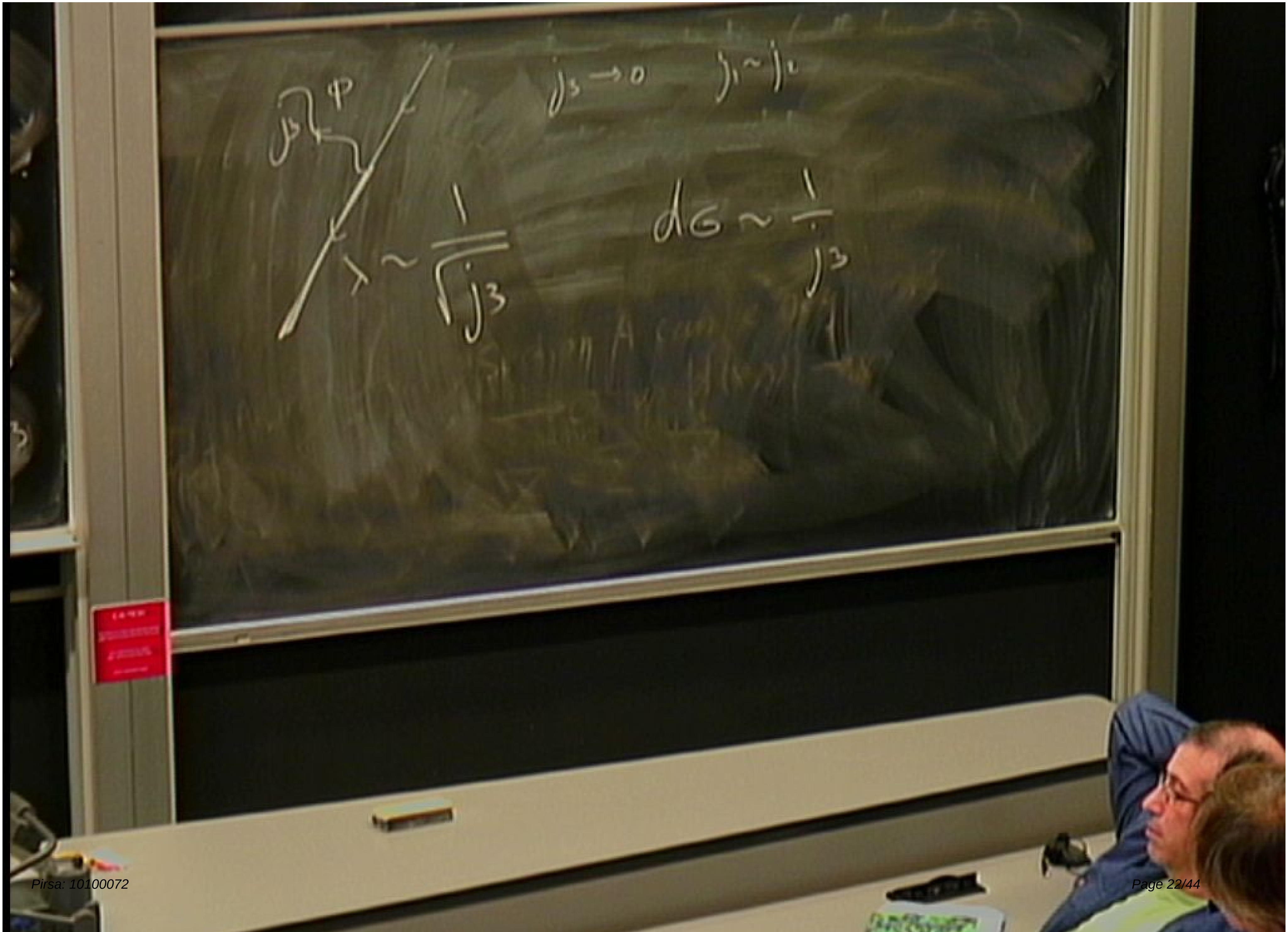
$$\Phi_{jn} \propto e^{-ik_0 t'} Y_{jn}(\Omega)$$

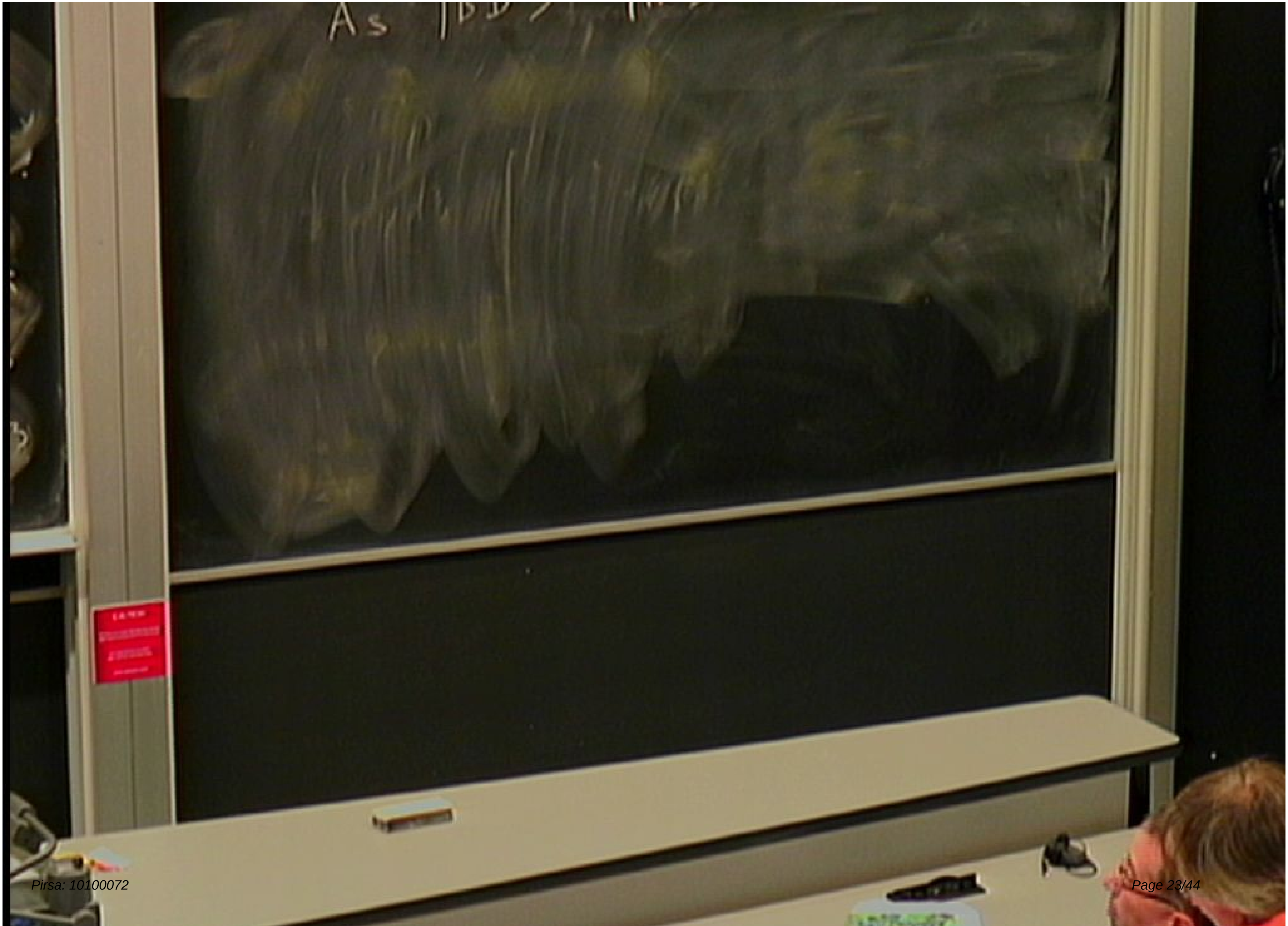
$$k_0 = \sqrt{\frac{m^2}{A^2} + \frac{1}{R^2} j(j+2)}$$

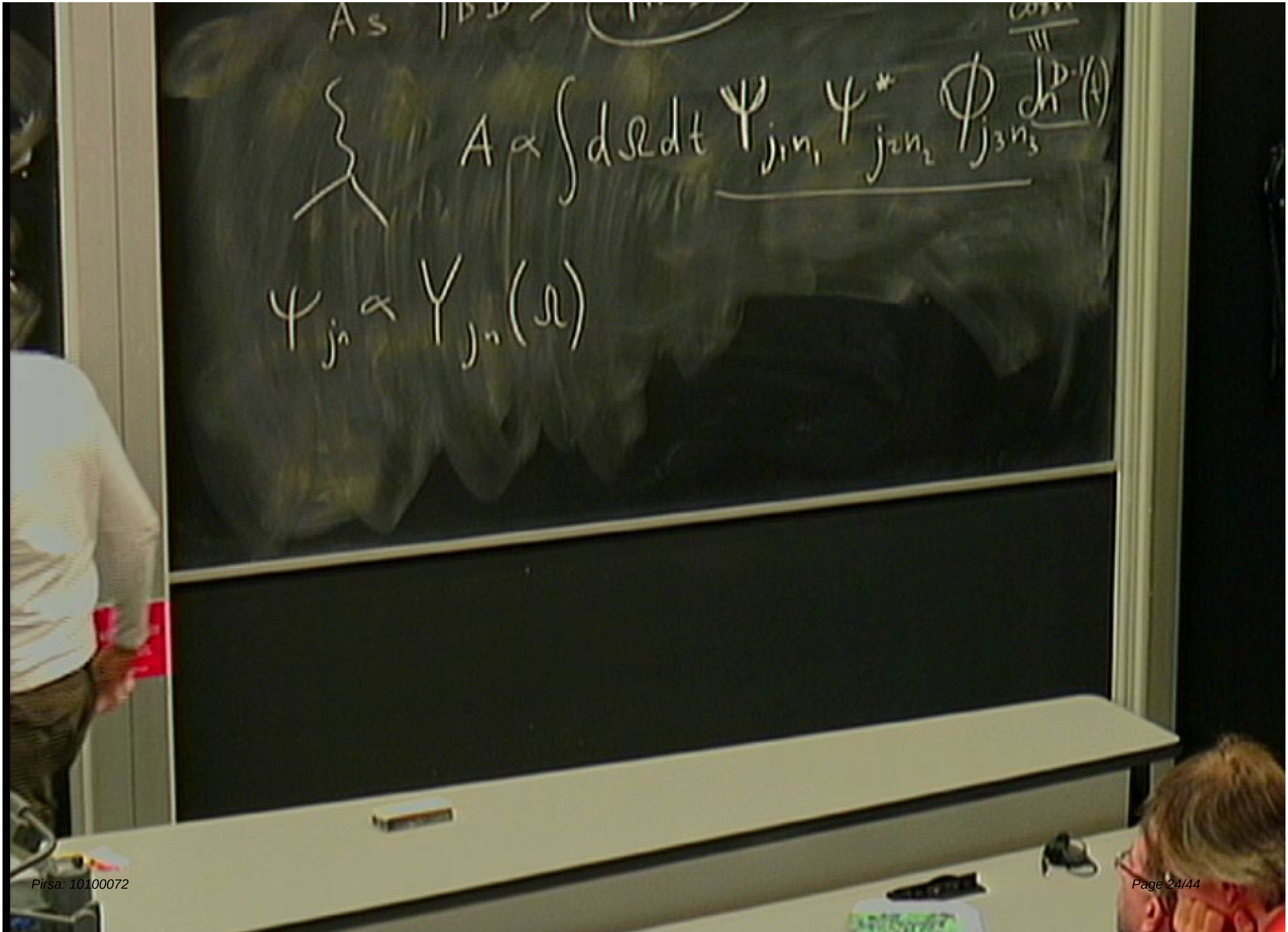
$$A \propto \boxed{\int_{-\infty}^{+\infty} dt\, e^{i(p_0 - k_0 - q_0)t}} \int d\Omega\, Y_{j_1} Y^*_{j_2} Y^*_{j_3}$$

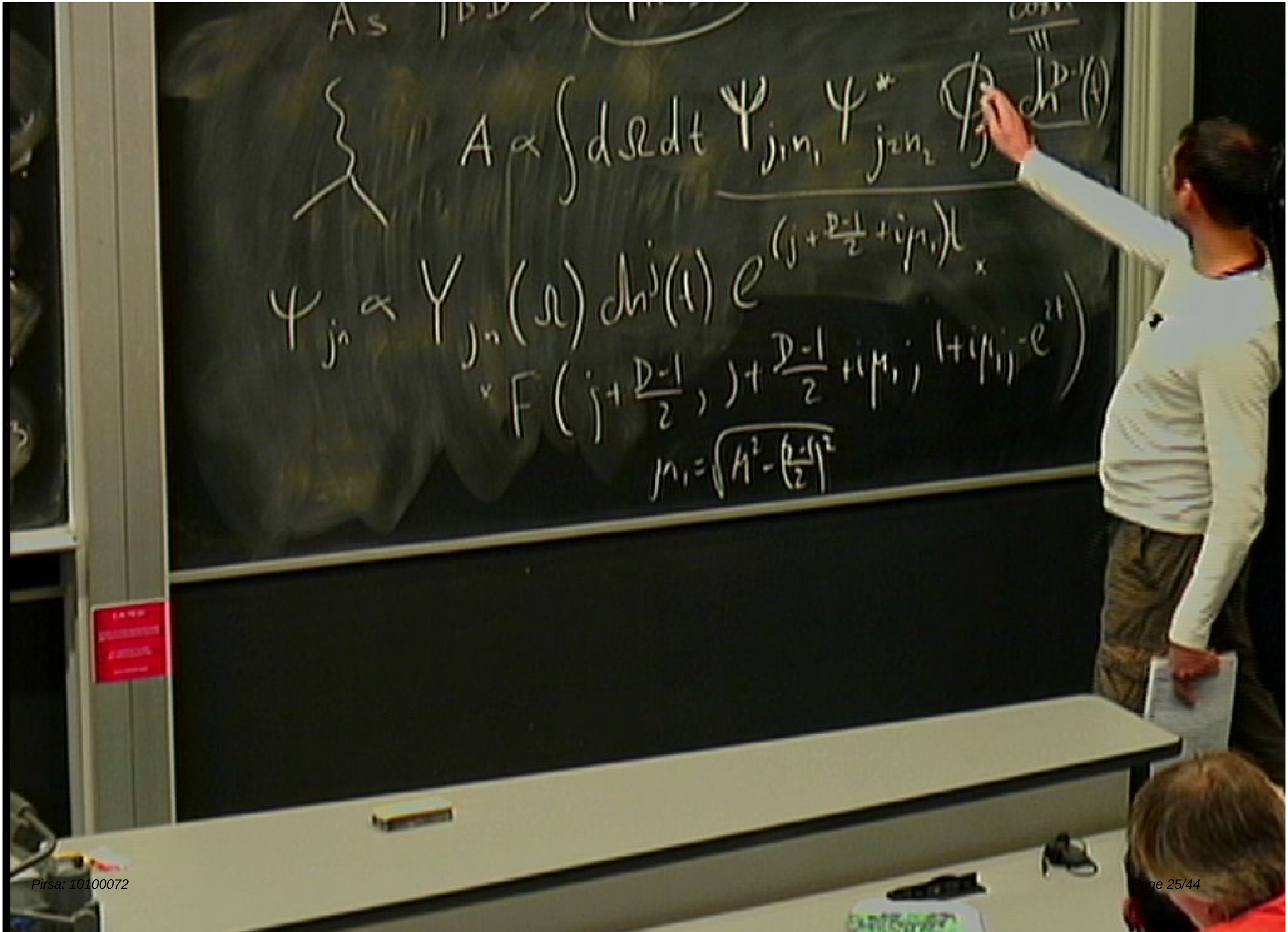
$$|j_2 - j_3| \le j_1 \le j_2 + j_3$$

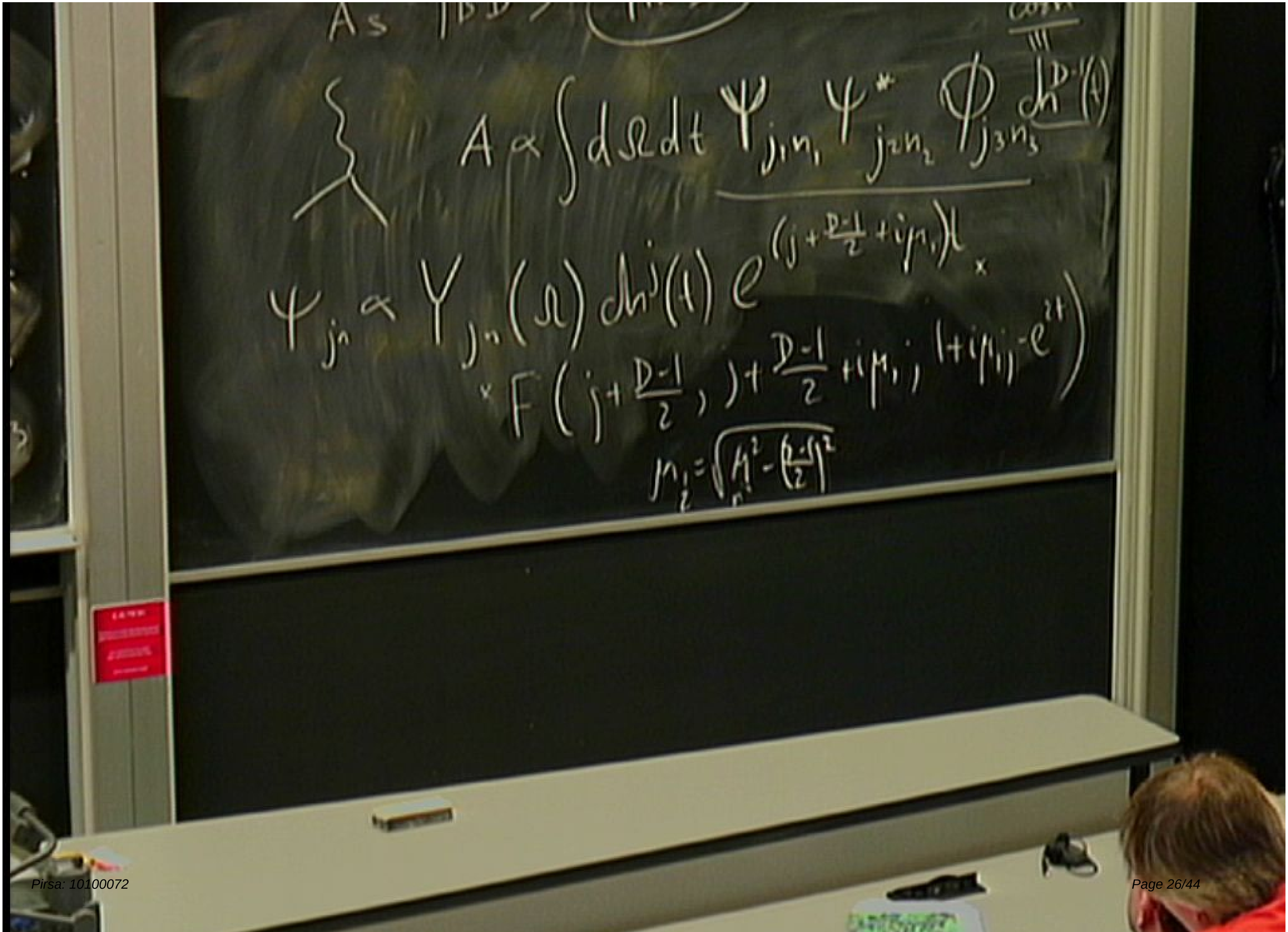

$j_3 \to 0$
$j_1 \sim j_2$
$\lambda \sim \frac{1}{\sqrt{j_3}}$
$d\sigma \sim \frac{1}{j_3}$

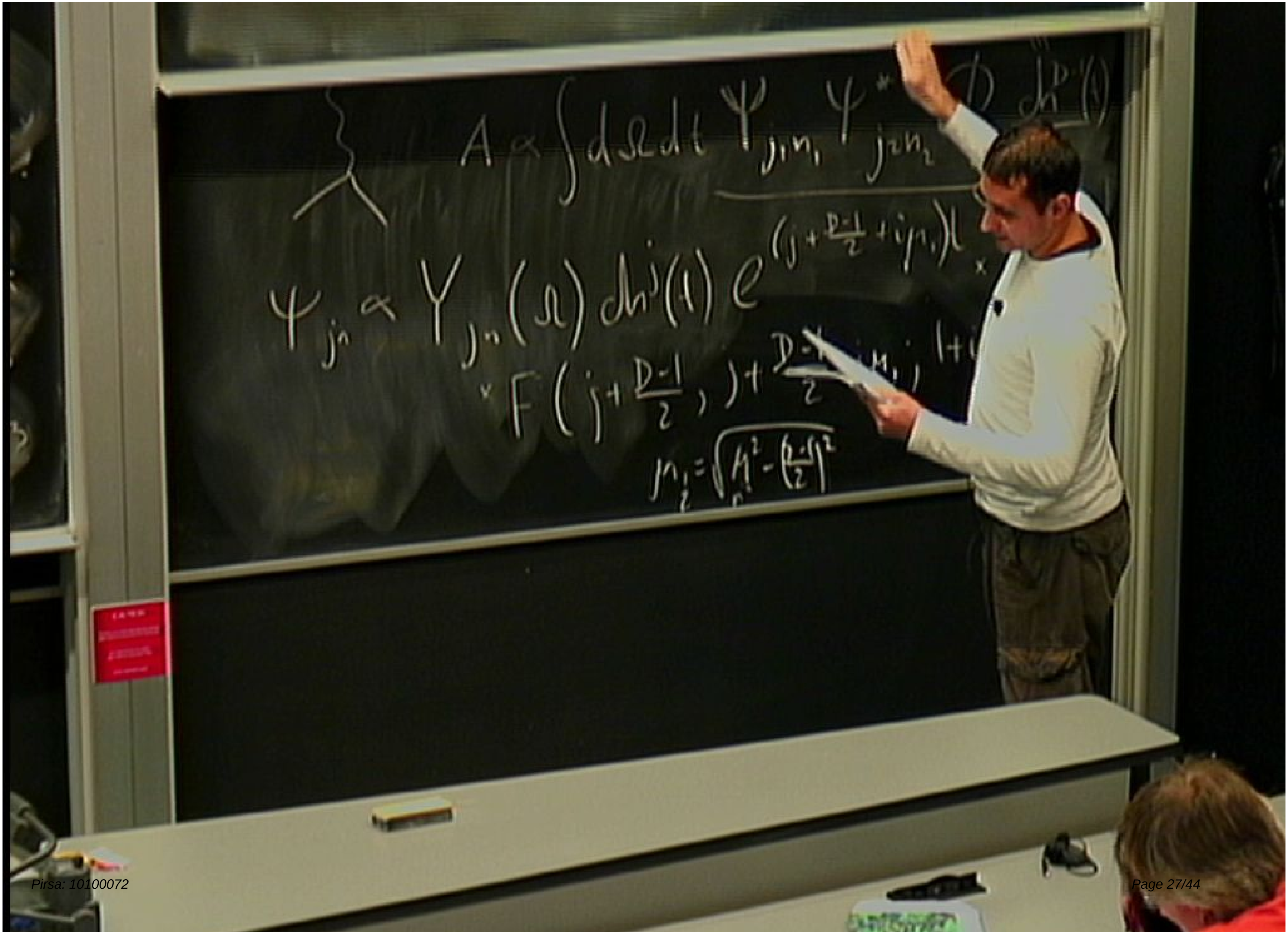

j3 φ
j3 → 0
j1 ~ j2
λ ~ 1/√j3
dσ ~ 1/j3



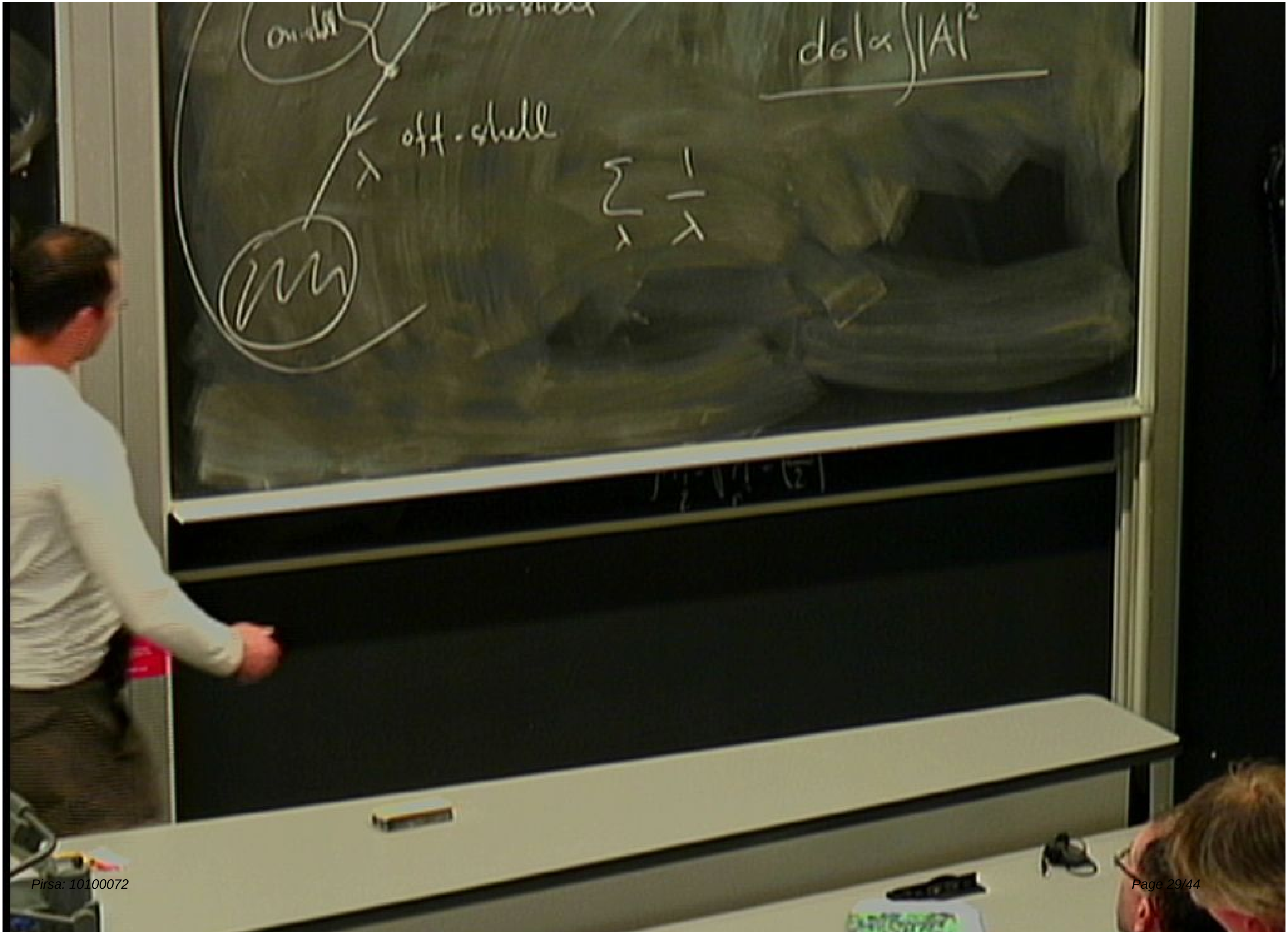

$A \propto \int d\Omega\, dt\, \Psi_{j_1 n_1} \Psi^*_{j_2 n_2} \phi_{j_3 n_3}$
$\Psi_{jn} \propto Y_{jn}(\Omega)$

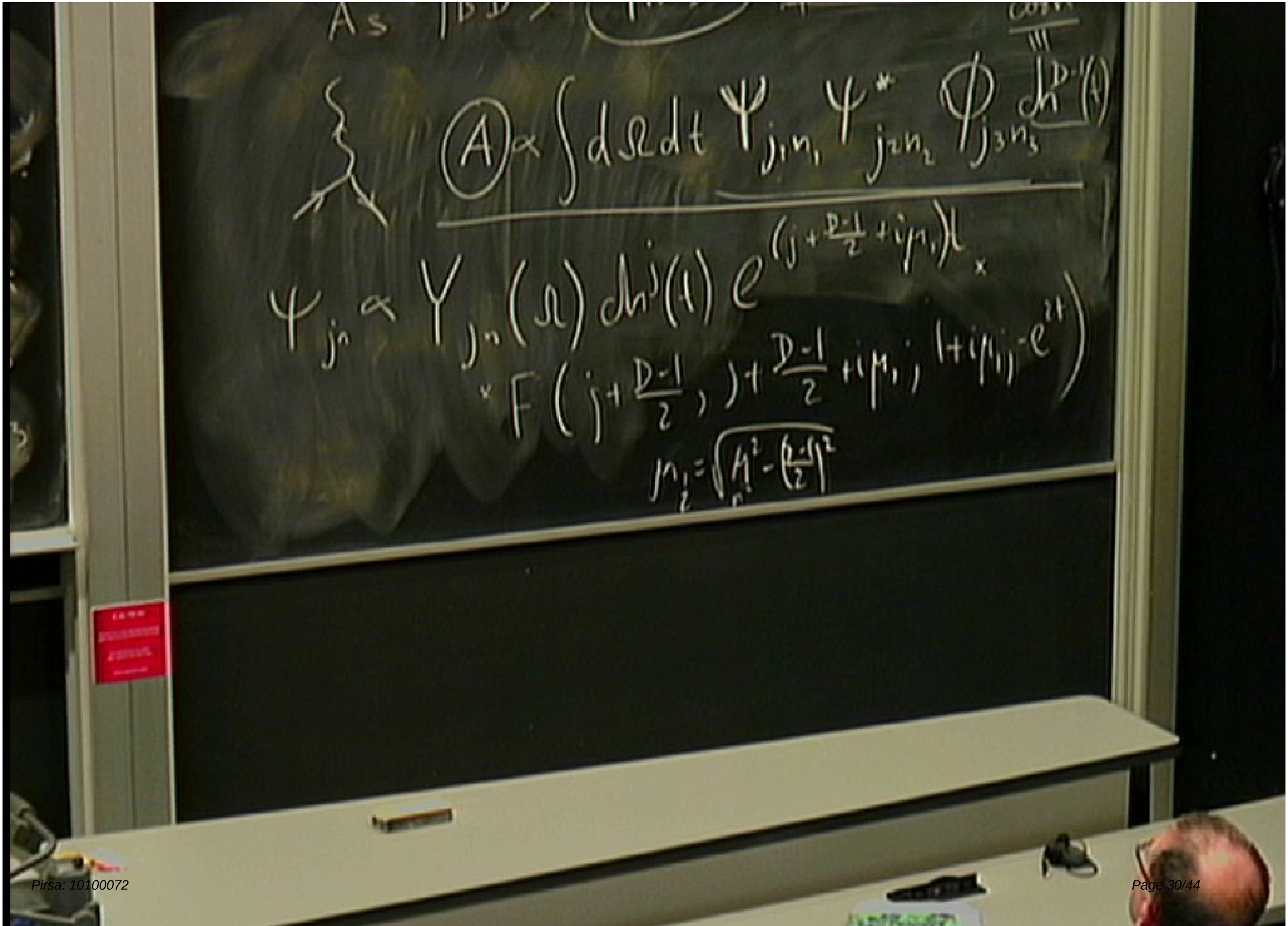

A ∝ ∫ dΩ dt Ψ_{j1 n1} Ψ*_{j2 n2} φ_j ch^{D-1}(t)
Ψ_{jn} ∝ Y_{jn}(Ω) ch^j(t) e^{(j + (D-1)/2 + iμ1)t} ×
× F(j + (D-1)/2 , j + (D-1)/2 + iμ1 ; 1 + iμ1 ; -e^{2t})
μ1 = √(M² - ((D-1)/2)²)


A ∝ ∫ dΩ dt Ψ_{j1 n1} Ψ*_{j2 n2} Φ_{j3 n3} ch^{D-1}(t)
Ψ_{jn} ∝ Y_{jn}(Ω) ch^j(t) e^{(j + (D-1)/2 + iμ_1)t} ×
× F(j + (D-1)/2, j + (D-1)/2 + iμ_1; 1 + iμ_1; -e^{2t})

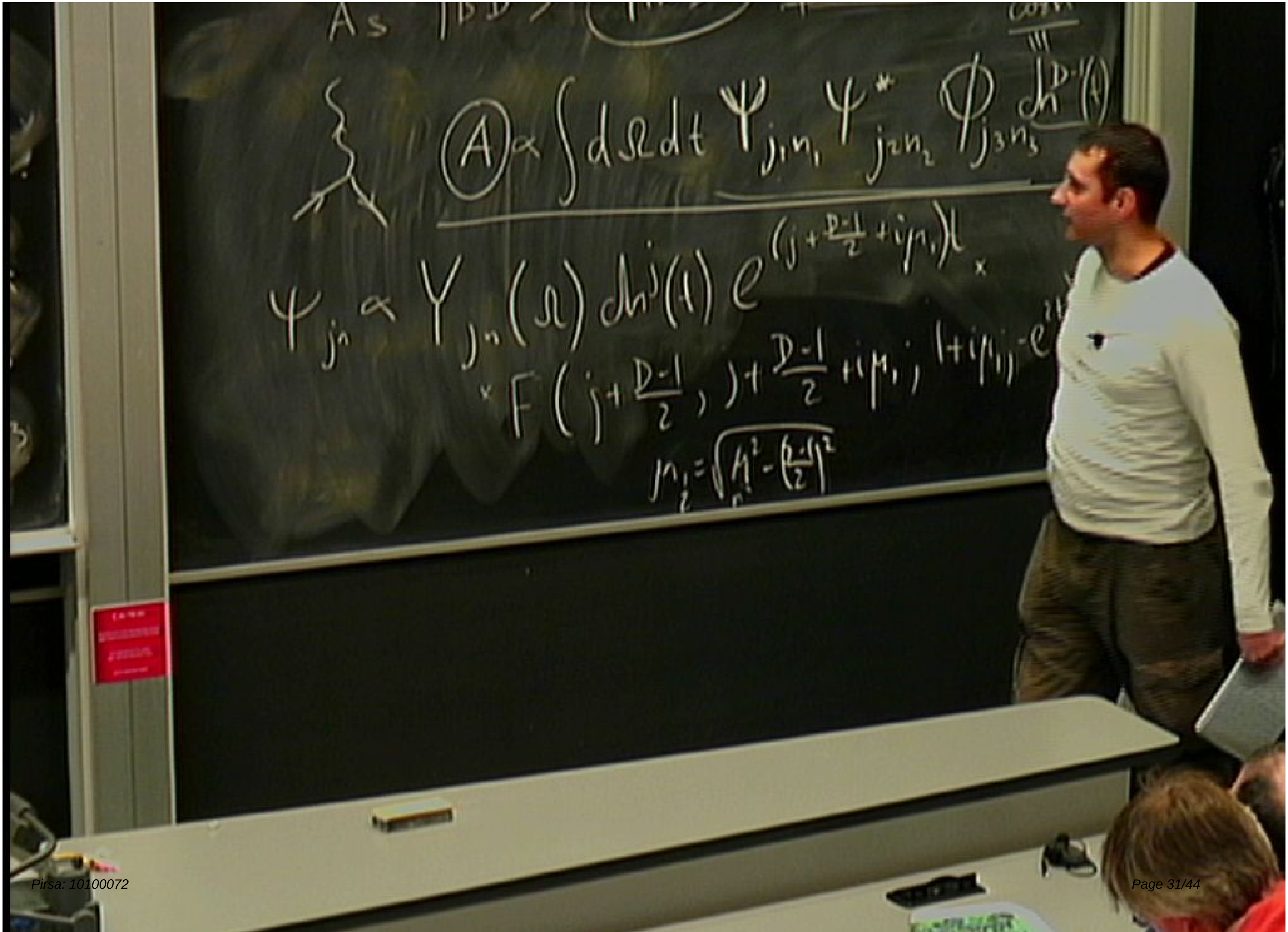


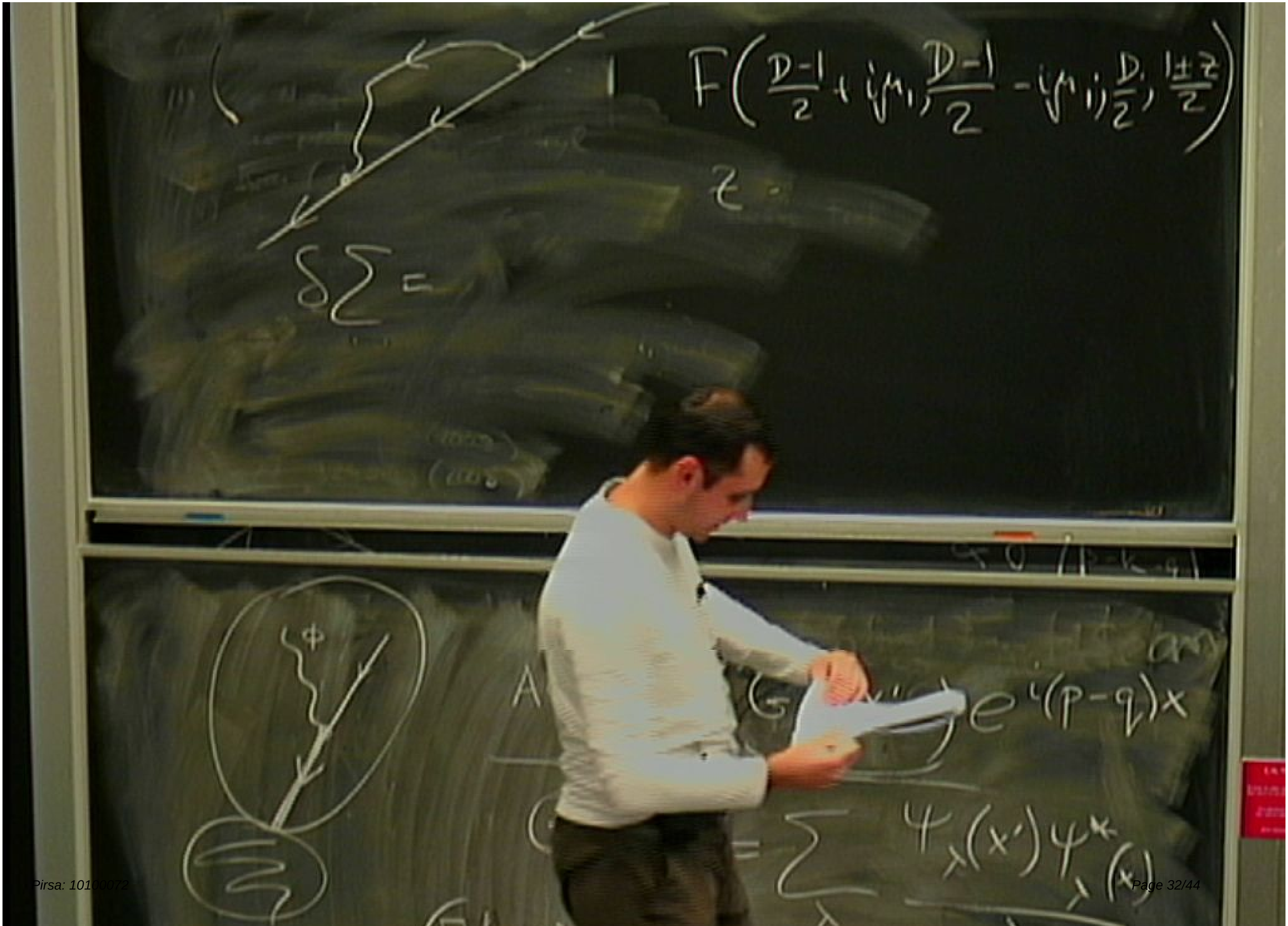

on-shell
on-shell
off-shell

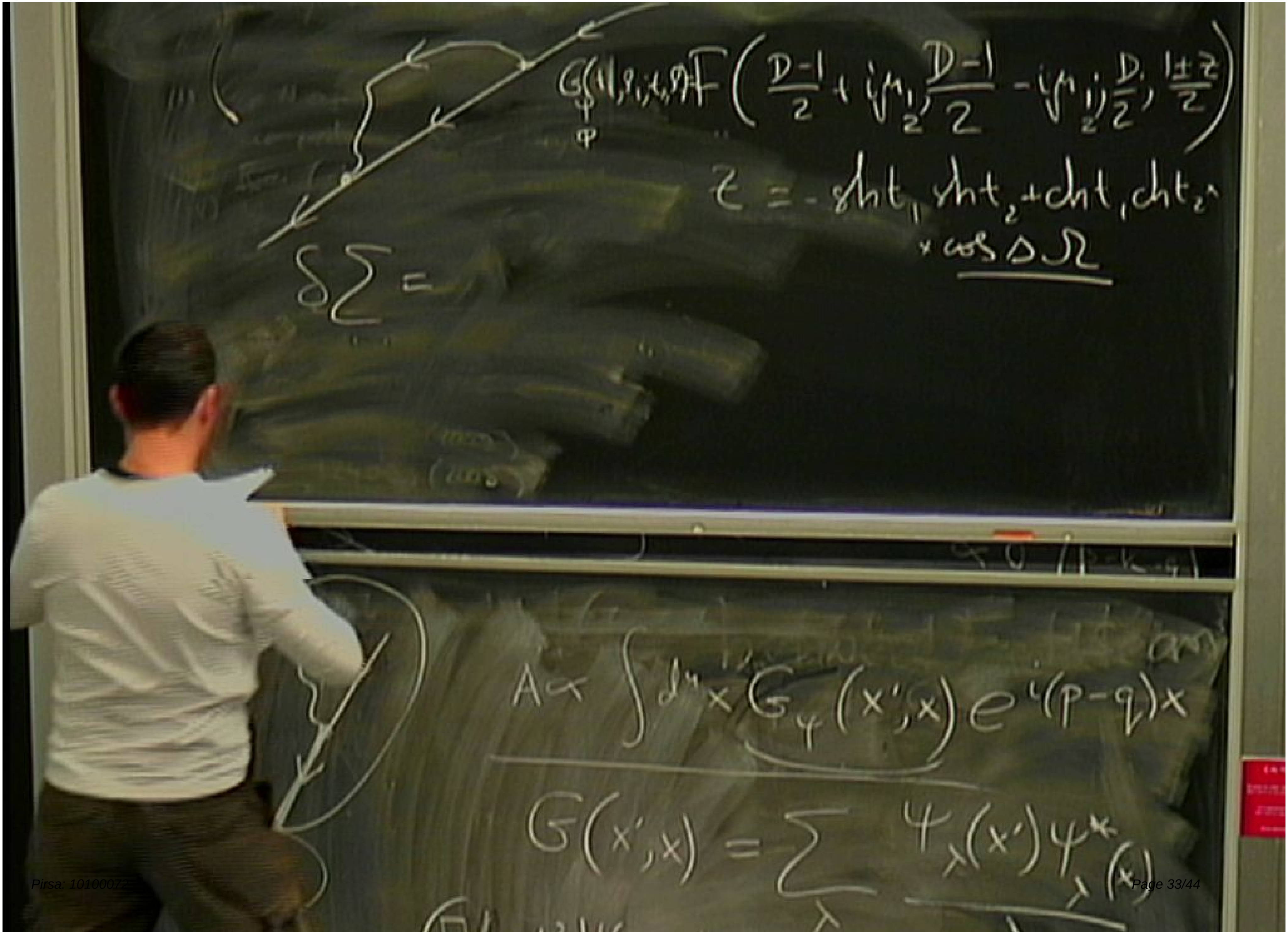

on-shell
off-shell
$d\sigma \propto \int |A|^2$
$\sum_\lambda \frac{1}{\lambda}$



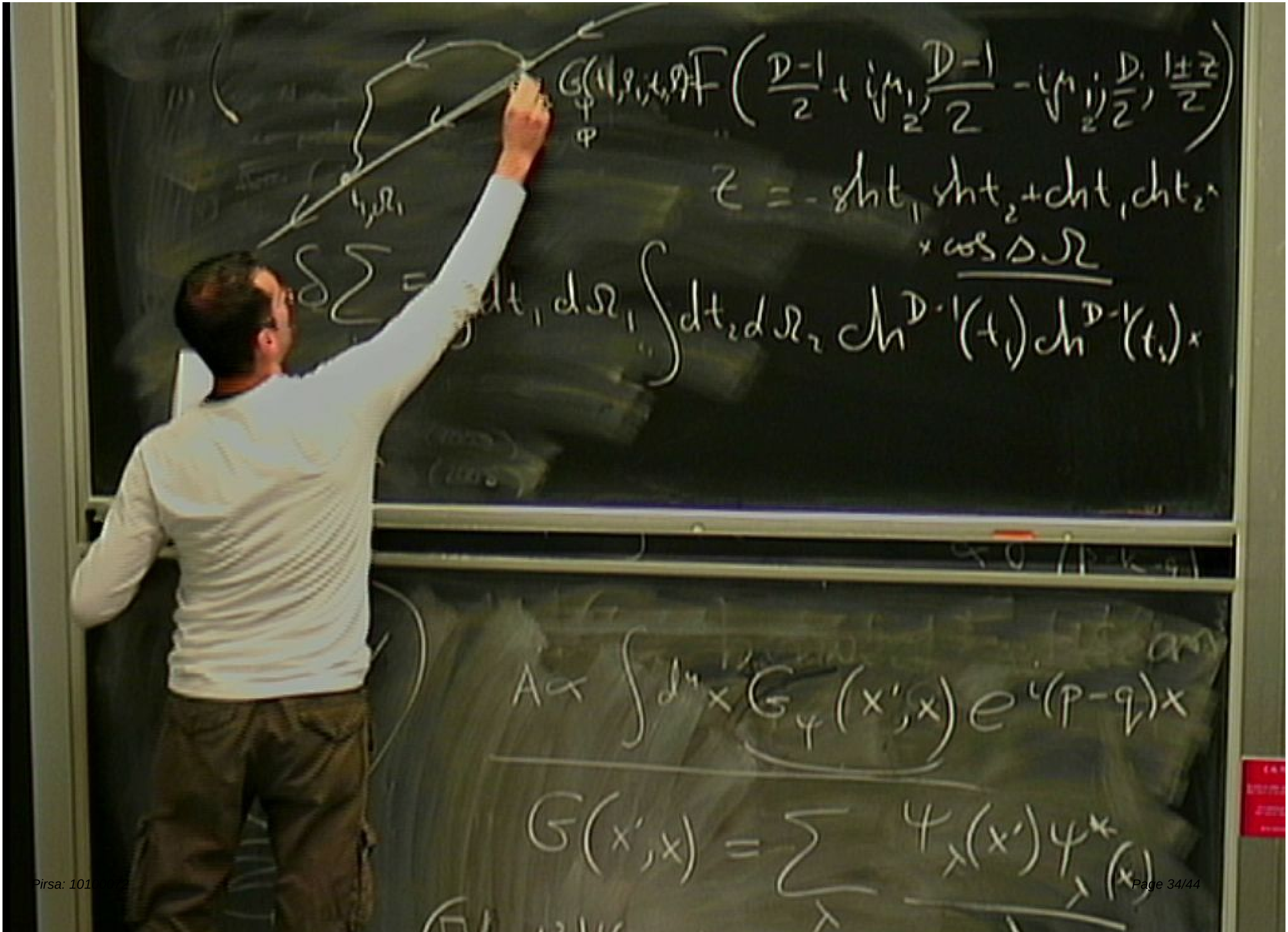
$A \propto \int d\Omega\, dt\, \Psi_{j_1 n_1} \Psi^*_{j_2 n_2} \Phi_{j_3 n_3}\, ch^{D-1}(t)$
$\Psi_{jn} \propto Y_{jn}(\Omega)\, ch^j(t)\, e^{(j+\frac{D-1}{2}+i\mu_1)t} \times$
$\times F\left(j+\frac{D-1}{2},\, j+\frac{D-1}{2}+i\mu_1;\, 1+i\mu_1;\, -e^{2t}\right)$
$\mu_1 = \sqrt{M_n^2 - \left(\frac{D-1}{2}\right)^2}$

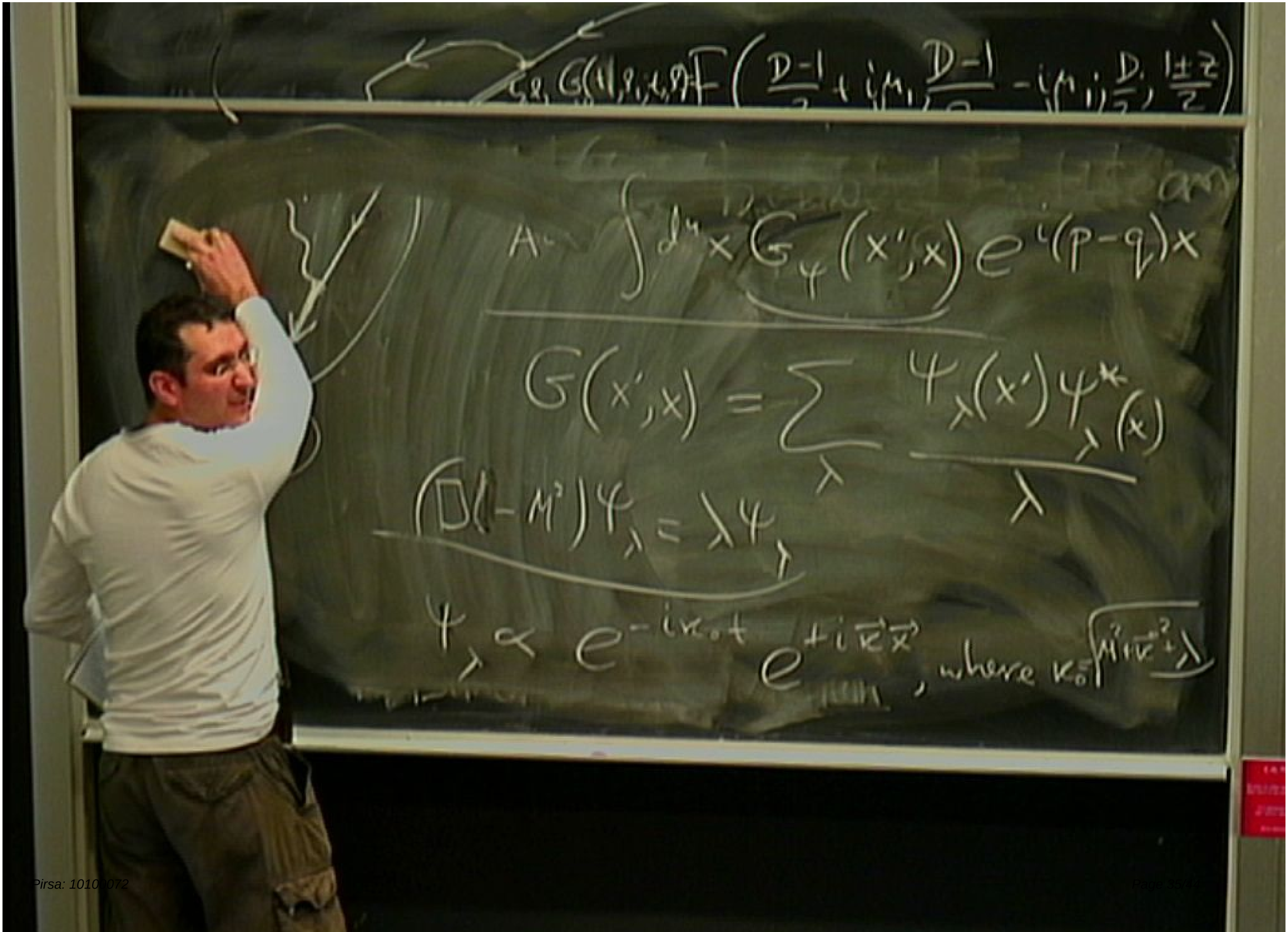


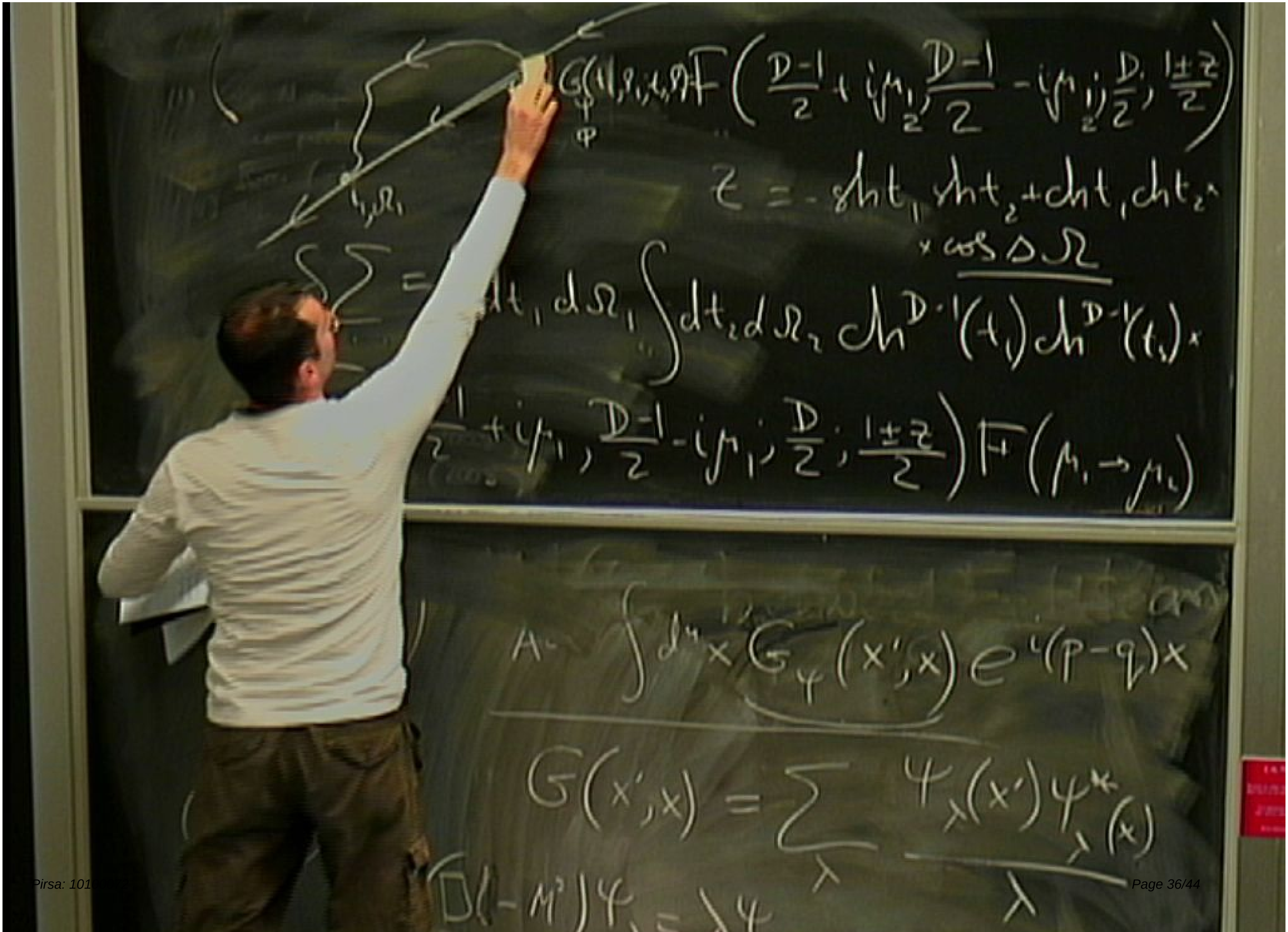

F(D-1/2 + iμ, D-1/2 - iμ ; D/2 ; 1±z/2)
z =
δΣ =
e^{i(p-q)x}
ψ_λ(x') ψ*_λ(x)



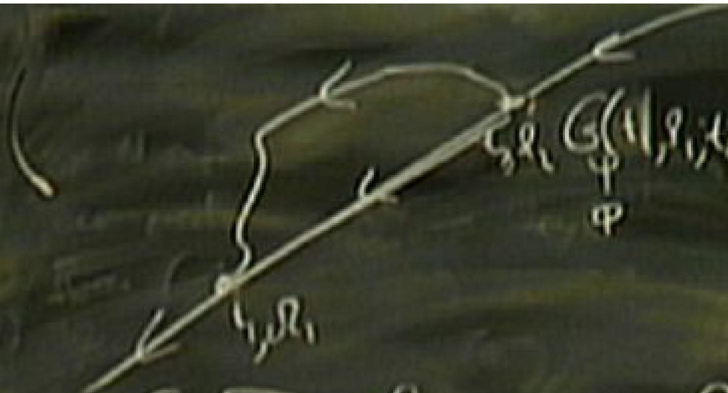
$F\left(\frac{D-1}{2}+i\mu, \frac{D-1}{2}-i\mu; \frac{D}{2}; \frac{1\pm z}{2}\right)$
$z = -\operatorname{sh} t_1 \operatorname{sh} t_2 + \operatorname{ch} t_1 \operatorname{ch} t_2 \times \cos \Delta\Omega$
$\delta\Sigma =$
$A \propto \int d^n x\, G_\psi(x',x)\, e^{i(p-q)x}$
$G(x',x) = \sum_\lambda \psi_\lambda(x')\psi^*_\lambda(x)$


z = - sht1 sht2 + cht1 cht2 × cos ΔΩ
A ∝ ∫d4x Gψ(x',x) ei(p-q)x
G(x',x) = Σ ψλ(x')ψ*λ(x)


$A = \int d^n x \, G_\psi(x',x) \, e^{i(p-q)x}$
$G(x',x) = \sum_\lambda \frac{\psi_\lambda(x')\psi^*_\lambda(x)}{\lambda}$
$(\Box - M^2)\psi_\lambda = \lambda \psi_\lambda$
$\psi_\lambda \propto e^{-ik_0 t} e^{+i\vec{k}\vec{x}}$, where $k_0 = \sqrt{M^2 + \vec{k}^2 + \lambda}$



z = - sh t1 sh t2 + ch t1 ch t2 × cos ΔΩ
G(x', x) = Σλ ψλ(x') ψ*λ(x) / λ



t_1,Ω_1

$t_2\Omega_2\; G_\varphi(t_1,\Omega_1;t_2,\Omega_2)\, F\left(\frac{D-1}{2}+i\mu_{1,2},\ \frac{D-1}{2}-i\mu_{1,2};\ \frac{D}{2};\ \frac{1\pm z}{2}\right)$

$z = -\mathrm{sh}\,t_1\,\mathrm{sh}\,t_2 + \mathrm{ch}\,t_1\,\mathrm{ch}\,t_2 \times \underline{\cos\Delta\Omega}$

$$\delta\Sigma = \int dt_1\, d\Omega_1 \int dt_2\, d\Omega_2\, \mathrm{ch}^{D-1}(t_1)\,\mathrm{ch}^{D-1}(t_2)\,\times$$

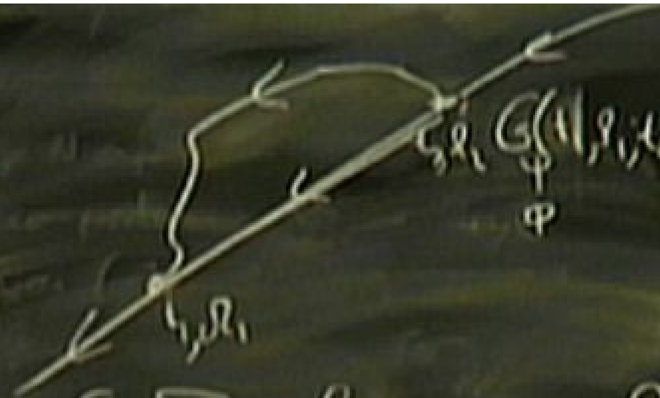
$$\times F\left(\frac{D-1}{2}+i\mu_1,\ \frac{D-1}{2}-i\mu_1;\ \frac{D}{2};\ \frac{1\pm z}{2}\right) F(\mu_1\to\mu_2)$$

$\int G(x,y)\,D(y,x)$

$$A \sim \int d^4x\, \underline{G_\varphi(x',x)}\, e^{i(p-q)x}$$

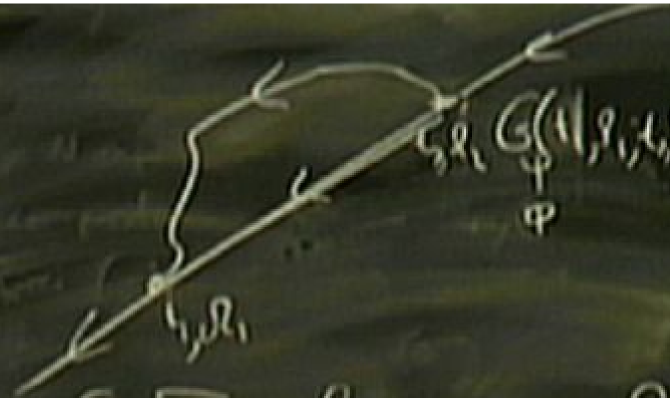
$$G(x',x) = \sum_\lambda \psi_\lambda(x')\,\psi^*_\lambda(x)$$

$$\left(\sum_{t_2, \Omega_2} G(t_1, \Omega_1; t_2, \Omega_2)\, F\!\left(\frac{D-1}{2} + i\mu, \frac{D-1}{2} - i\mu; \frac{D}{2}; \frac{1 \pm z}{2}\right) \right)$$



$$z = -\operatorname{sh} t_1 \operatorname{sh} t_2 + \operatorname{ch} t_1 \operatorname{ch} t_2 \times \cos \Delta\Omega$$

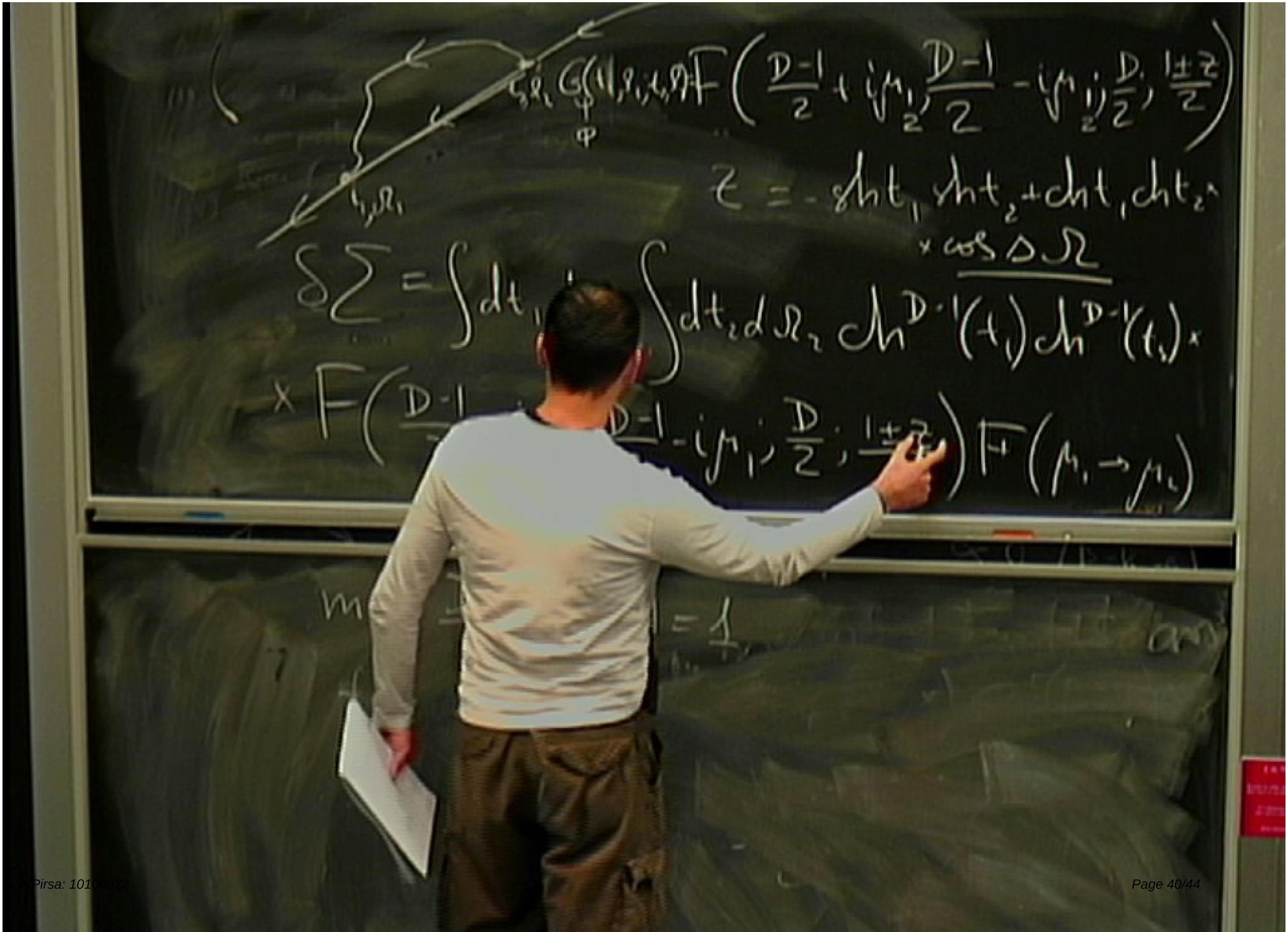
$$\delta \Sigma = \int \ldots \int \ldots$$

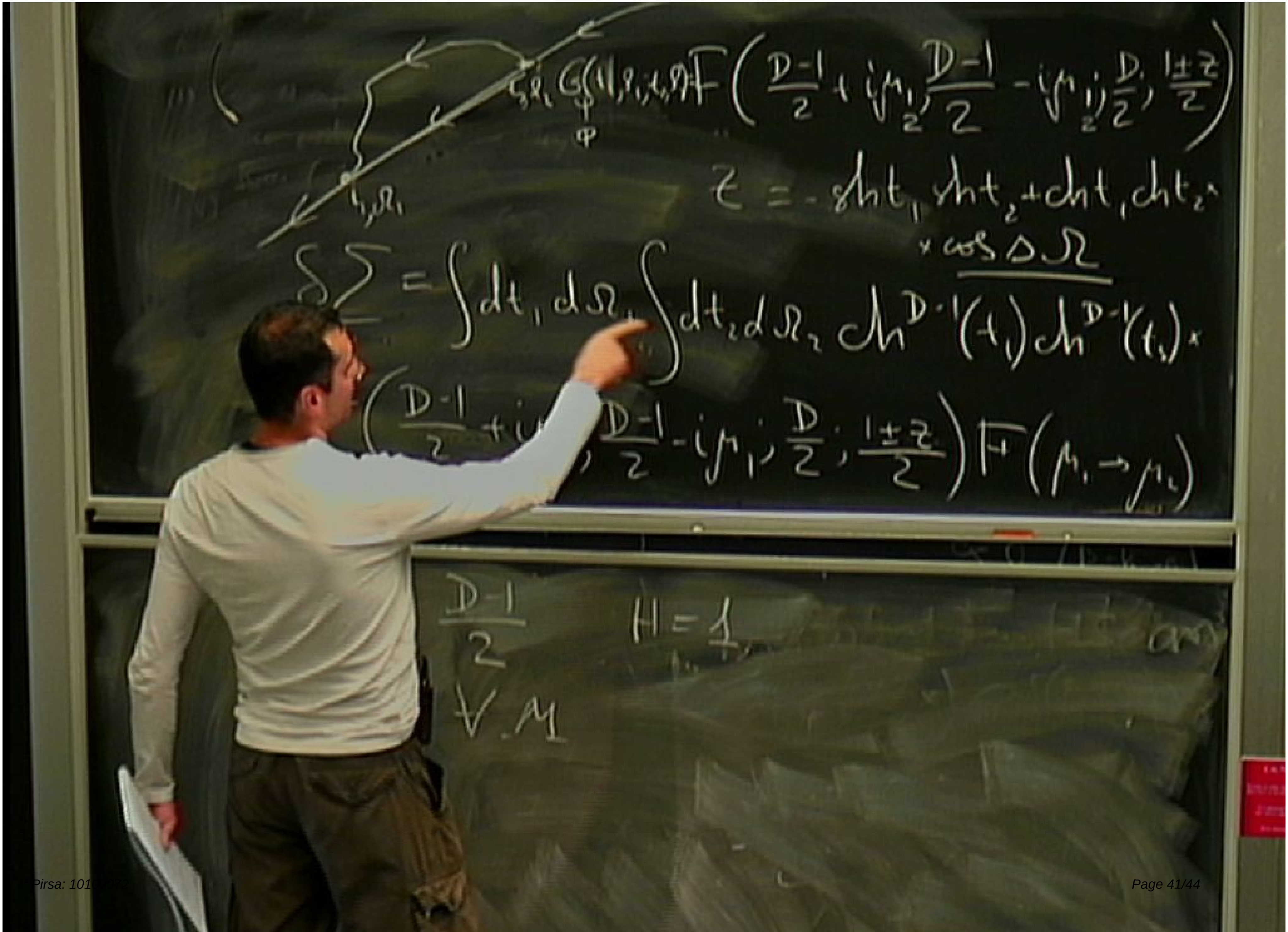


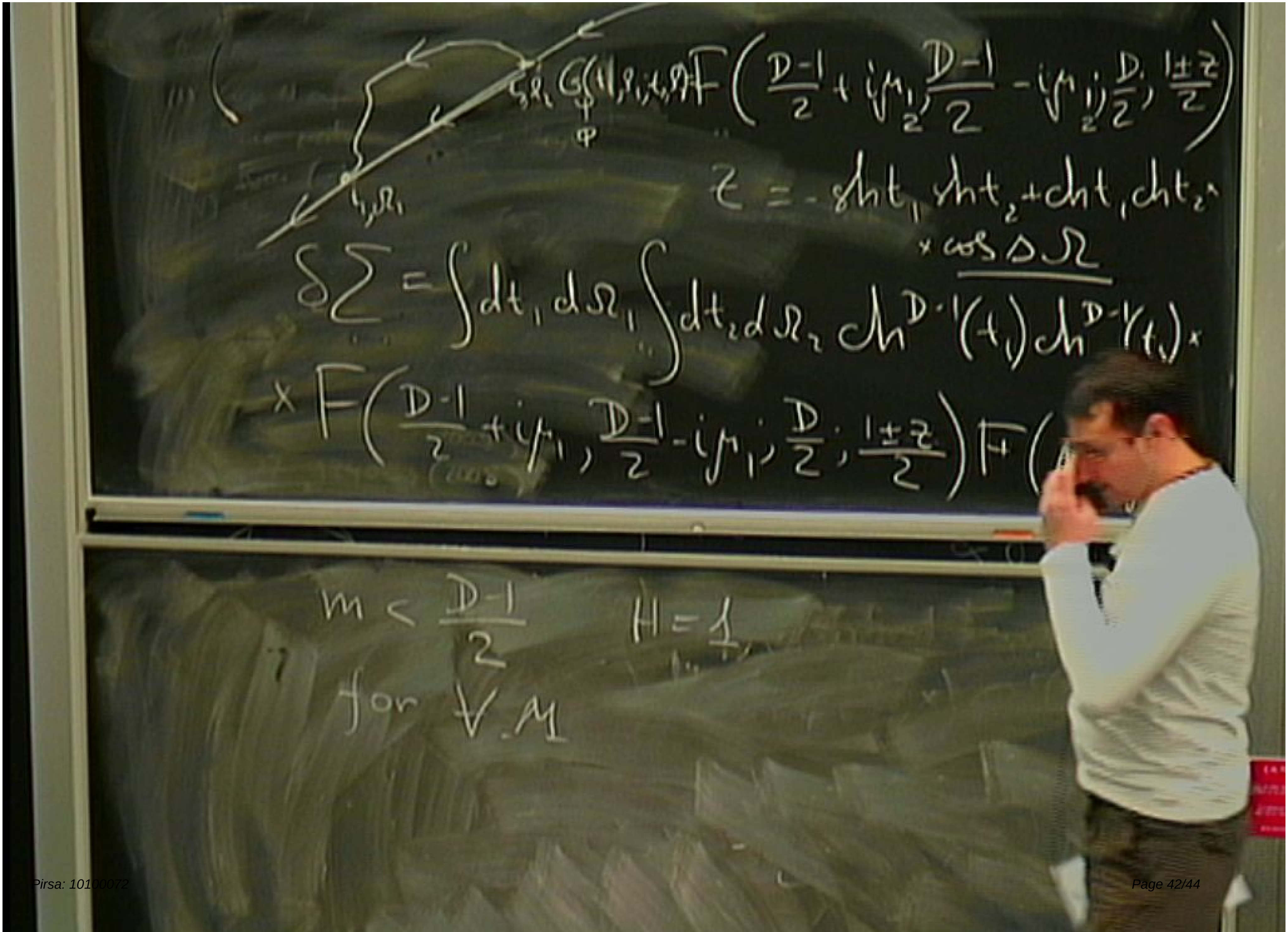
$$\Big(\quad G(t_1,\Omega_1; t_2,\Omega_2)\, F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1; \frac{D}{2}; \frac{1 \pm z}{2} \right) \Big)$$

$$z = -\operatorname{sh} t_1 \operatorname{sh} t_2 + \operatorname{ch} t_1 \operatorname{ch} t_2 \times \cos \Delta\Omega$$

$$\delta\Sigma = \int dt_1\, d\Omega_1 \int dt \; \Big(D \quad \Big) \Big(\frac{D-1}{2} \Big) \Big(\frac{D-1}{2} \Big)$$


z = - sh t1 sh t2 + ch t1 ch t2 × cos ΔΩ
δΣ = ∫dt1 ∫dt2 dΩ2 ch^{D-1}(t1) ch^{D-1}(t2) ×
F(μ1 → μ2)


z = - sh t1 sh t2 + ch t1 ch t2 × cos ΔΩ
H = 1


z = - sh t1 sh t2 + ch t1 ch t2 × cos ΔΩ
δΣ = ∫dt1 dΩ1 ∫dt2 dΩ2 ch^(D-1)(t1) ch^(D-1)(t2) ×
m < (D-1)/2
H = 1
for ∀ m

$$\int_{t_2,\Omega_2} G(t_1,\Omega_1;t_2,\Omega_2)\, F\left(\frac{D-1}{2}+i\mu_2, \frac{D-1}{2}-i\mu_2; \frac{D}{2}; \frac{1\pm z}{2}\right)$$

φ

t_1,Ω_1

$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos\Delta\Omega$$

$$\delta\Sigma = \int dt_1\, d\Omega_1 \int dt_2\, d\Omega_2\, \cosh^{D-1}(t_1)\cosh^{D-1}(t_2)\, \times$$

$$\times F\left(\frac{D-1}{2}+i\mu_1, \frac{D-1}{2}-i\mu_1; \frac{D}{2}; \frac{1\pm z}{2}\right) F\left(\mu_1 \to \mu_2\right)$$

$m < \frac{D-1}{2}$ $\quad H = 1$

for $\forall\, \mu$

$$\left(\ldots \right. \quad G_\varphi(t_1,\Omega_1;t_2,\Omega_2)\, F\left(\frac{D-1}{2}+i\mu_1,\frac{D-1}{2}-i\mu_1;\frac{D}{2};\frac{1\pm z}{2}\right)$$

$$z = -\operatorname{sh} t_1 \operatorname{sh} t_2 + \operatorname{ch} t_1 \operatorname{ch} t_2 \times \cos\Delta\Omega$$

$$\delta\Sigma = \int dt_1\, d\Omega_1 \int dt_2\, d\Omega_2\, \operatorname{ch}^{D-1}(t_1)\operatorname{ch}^{D-1}(t_2) \times$$

$$\times F\left(\frac{D-1}{2}+i\mu_1,\frac{D-1}{2}-i\mu_1;\frac{D}{2};\frac{1\pm z}{2}\right) F\left(\mu_1 \to \mu_2\right)$$

$$m < \frac{D-1}{2} \qquad H = 1$$

for $\forall\, M$