

Title: On radiation and IR divergences in dS space

Date: Oct 28, 2010 04:45 PM

URL: <http://pirsa.org/10100072>

Abstract: It is well known that there should be a total cancellation of the IR divergences in closed systems described by interacting quantum field theories, such as QED and gravity. I am going to show that such a cancellation does not happen in de Sitter space.

No Signal
VGA-1

No Signal
VGA-1

arXiv: 0808.4106
0905.2742



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09052742

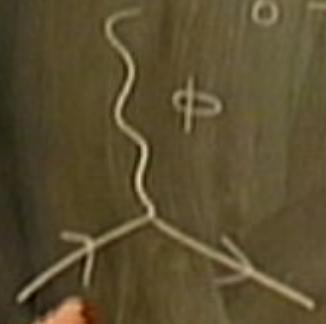
$\lambda \bar{\psi} \psi \phi$

arXiv: 0808.4106
09052742

$$\lambda \bar{\Psi} \Psi \phi$$

$$M, \underline{m} = 0$$

flat space IR divergences.

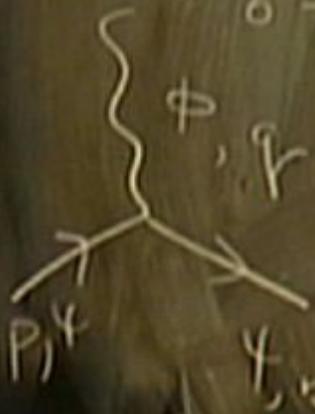


arXiv: 0808.4106
09052742

$$\lambda \bar{\Psi} \Psi \phi$$

$$M, m=0$$

flat space IR divergences.



$$A \propto \int d^4x e^{i(p-k-q)x} \propto \delta^{(4)}(p-k-q)$$
$$p_0 = \sqrt{p^2 + M^2}, \quad k_0 = \sqrt{k^2 + M^2}, \quad q_0 = \sqrt{q^2 + m^2}$$



flat space IR divergences.

$$A \propto \int d^4x e^{i(p-k-q)x} \propto \delta^{(4)}(p-k-q)$$

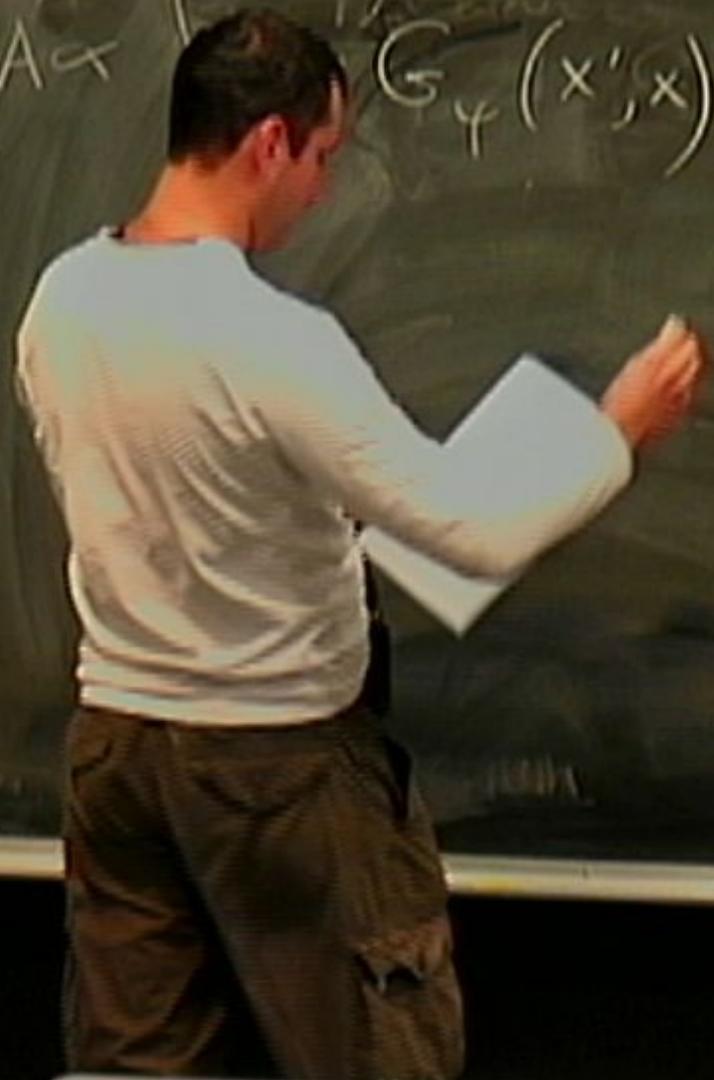
$$p_0 = \sqrt{p^2 + M^2}, \quad k_0 = \sqrt{k^2 + M^2}, \quad l_0 = \sqrt{l^2 + M^2}$$

$\begin{matrix} \text{M} \\ \text{A} \end{matrix} \quad \begin{matrix} \text{M} \\ \text{A} \end{matrix} \quad \begin{matrix} \text{M} \\ \text{A} \end{matrix}$

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$A \times$ $G \times ((x', x))$



$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{\lambda}$$

$$(\square(-M^2))\psi_{\lambda} = \lambda\psi_{\lambda}$$

$$\psi_{\lambda}$$

$$p_0 = \sqrt{p^2 + M^2}, \quad k_0 = \sqrt{k^2 + M^2}, \quad \Gamma_0 = \sqrt{m^2 + \omega^2}$$

$\psi_{\lambda} = e^{i(p \cdot x - E t)}$

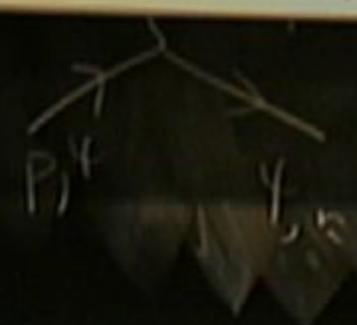
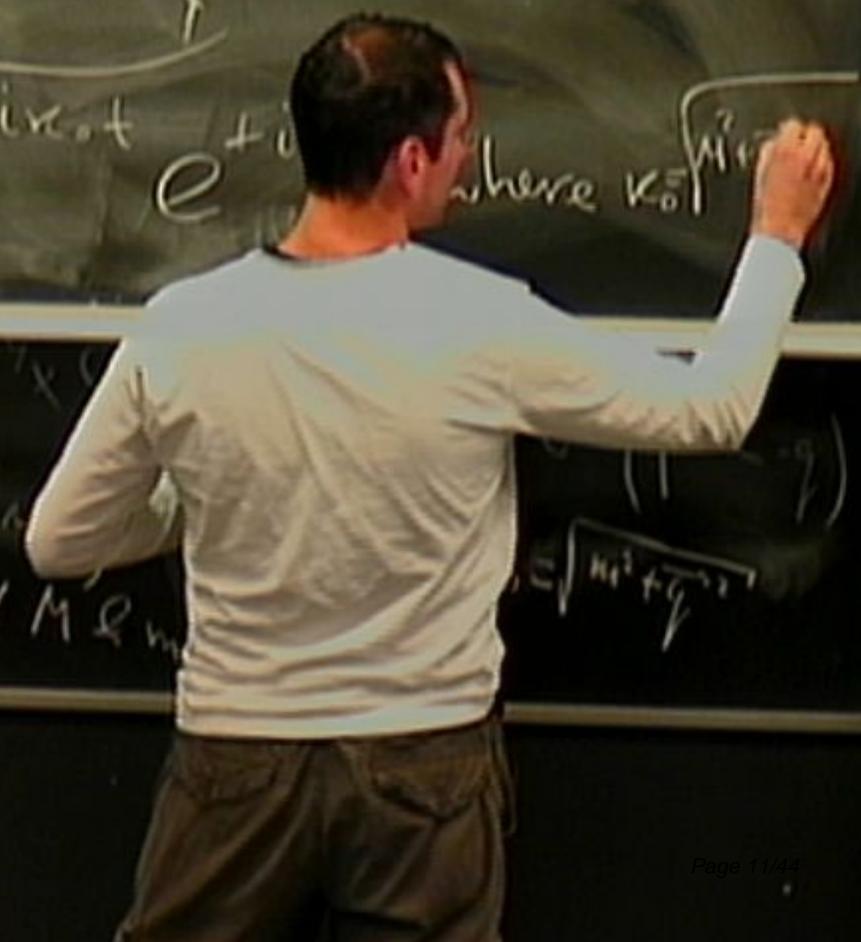


$$A \propto \int dx' G_{\psi}(x', x) e^{i(p-q)x}$$

$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{\lambda}$$

$$(\square - M^2) \psi_{\lambda} = \lambda \psi_{\lambda}$$

$$\psi_{\lambda} \propto e^{-i\kappa_0 t} e^{+i\kappa_0 x} \text{ where } \kappa_0 = \sqrt{M^2 - \dots}$$



$$A \propto \int dx' G_{\psi}(x', x) e^{i(p-q)x}$$

$$p = \sqrt{m^2 + \dots}$$

$$y = \sqrt{m^2 + \dots}$$

$$\lambda = k^2 - M^2 = (p - q)^2 - M^2$$

$$q^2 \rightarrow 0 \Rightarrow \lambda = -2pq$$

② In-in Points

Inside a

= in-in

In-out Points

Coordinates

$$\lambda = \kappa^2 - M^2 = (p - q_V)^2 - M^2$$

$$q^2 \rightarrow 0 \Rightarrow \lambda = -2pq_V$$

② In-in Poisson

Imagine a

= in-in

In-in Poisson

coordinates

$$\lambda = k^2 - M^2 = (p - q)^2 - M^2$$

$$q^2 \rightarrow 0 \Rightarrow \lambda_* = -2pq$$

$$A \sim \frac{1}{\lambda_*} \sim \frac{1}{2pq}$$

$$\lambda = k^2 - M^2 = (p - q)^2 - M^2$$

$$q^2 \rightarrow 0 \Rightarrow \lambda_* = -2pq$$

$$A \sim \frac{1}{\lambda_*} \sim \frac{1}{2pq}$$

dG

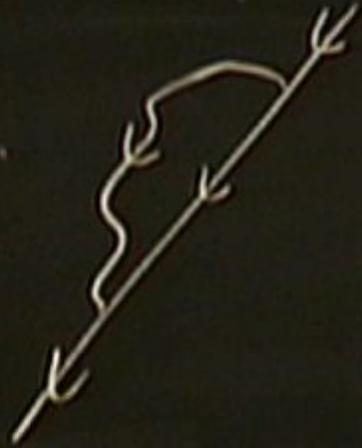
$$\lambda = k^2 - M^2 = (p - q)^2 - M^2$$

$$q^2 \rightarrow 0 \Rightarrow \lambda_* = -2pq$$

$$A \sim \frac{1}{\lambda_*} \sim \frac{1}{2pq}$$

$$dG \sim \int |A|^2 \frac{d^3 \vec{q}}{|\vec{q}|} = \int \frac{d^3 \vec{q}}{(pq)|\vec{q}|} \propto \log m_0$$

JK unit!



$$\propto \int d^4 q \frac{1}{(p-q)^2 q^2}$$



$$ds^2 = -dt^2 + r^2 d\Omega^2$$

$$\Phi_{j,m} \propto e^{-i\omega t} Y_{j,m}(\Omega)$$

$$k_0 = \sqrt{\frac{m^2}{r^2} + \frac{1}{r^2} j(j+2)}$$

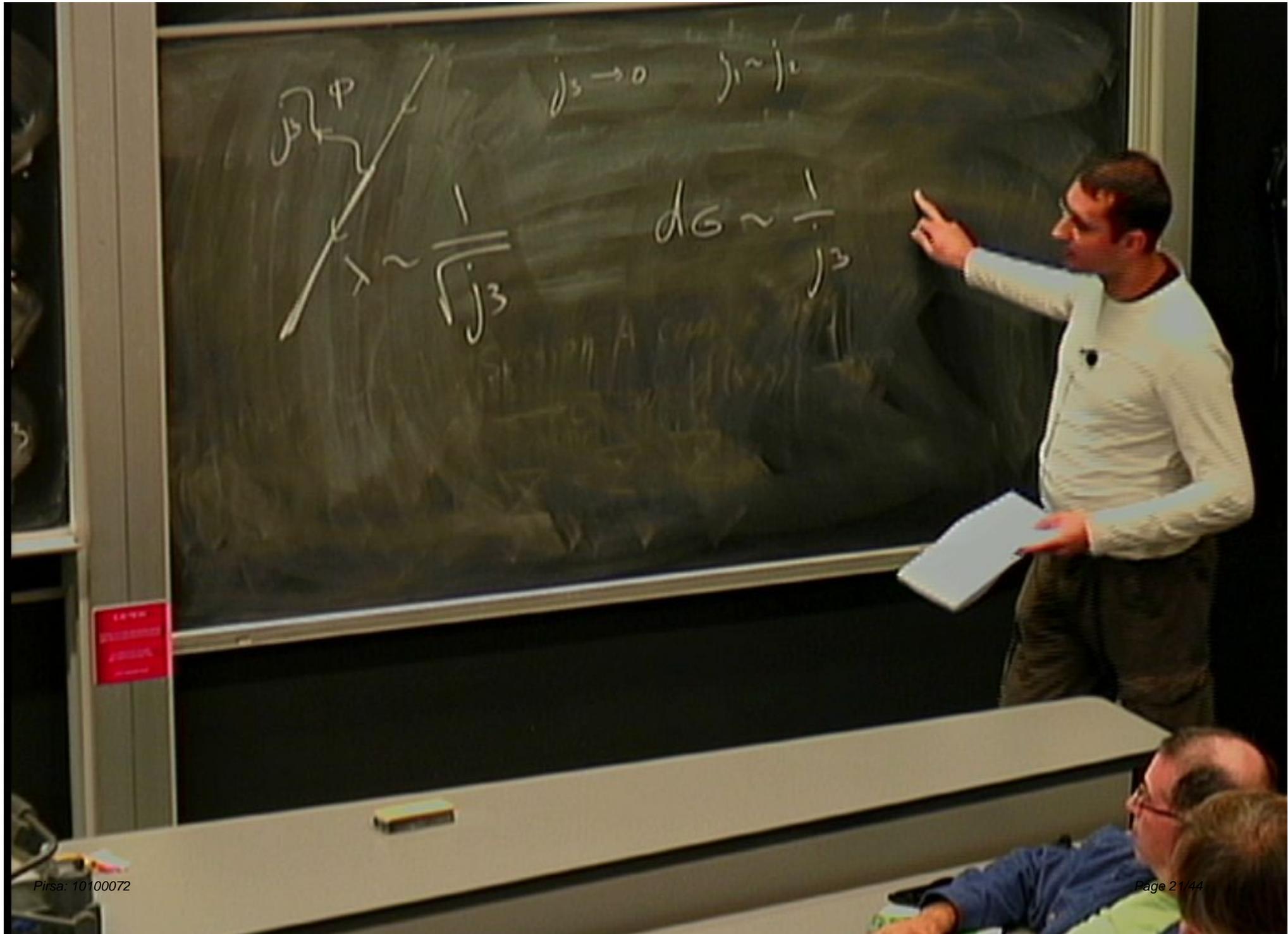
A

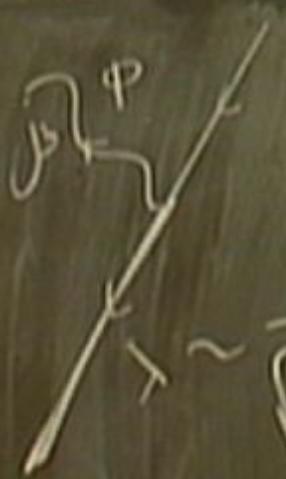
$$\Phi_{jm} \propto e^{-i\kappa_0 t} Y_{jm}(\Omega)$$

$$\kappa_0 = \sqrt{m^2 + \frac{1}{R^2} j(j+2)}$$

$$A \propto \int_{-\infty}^{+\infty} dt e^{i(p_0 - \kappa_0 - q_0)t} \int d\Omega Y_{j_1} Y_{j_2}^* Y_{j_3}^*$$

$|j_2 - j_3| \leq j_1 \leq j_2 + j_3$



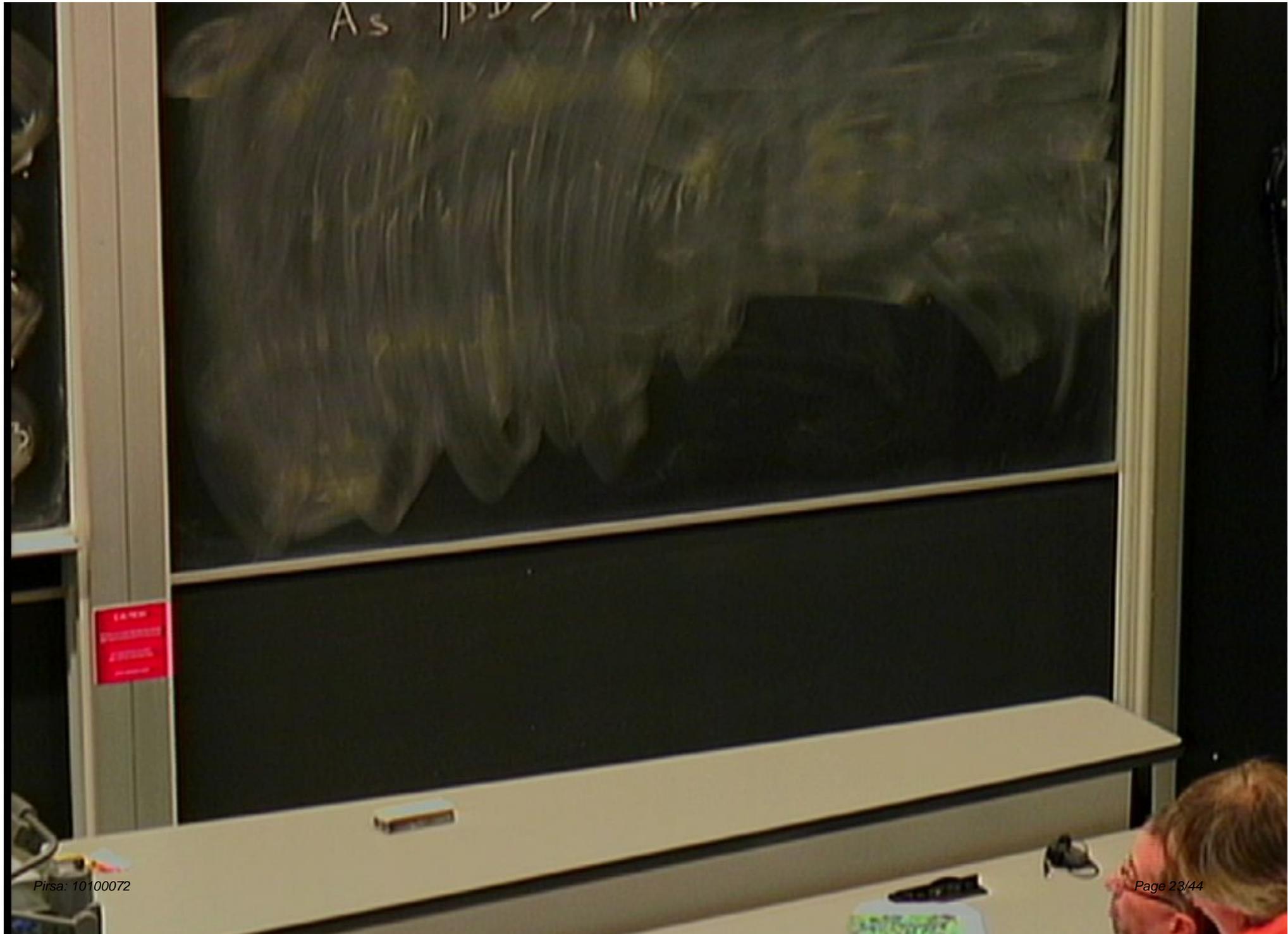


$$j\omega \rightarrow 0 \quad |j\omega| \sim |j\omega|$$

$$dG \sim \frac{1}{j^3}$$

$$\sim \frac{1}{j^3}$$

CAUTION
 WARNING
 DANGER



As $\int_{\mathbb{D}^D} \dots$

$$A \propto \int d\Omega dt \Psi_{j_1 n_1} \Psi_{j_2 n_2}^* \Phi_{j_3 n_3} \underbrace{\int_{\mathbb{D}^{D-1}} \dots}_{\text{const}}$$

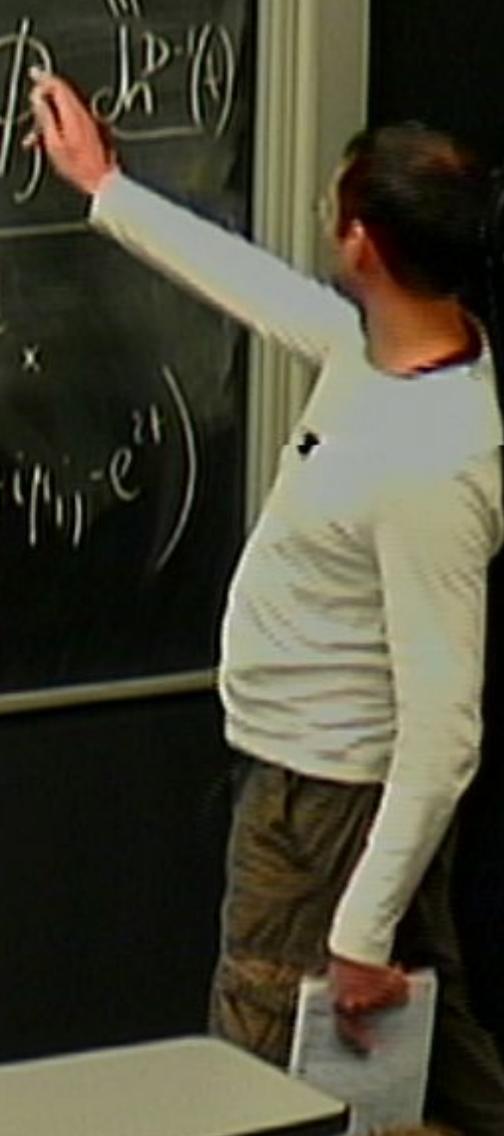
$$\Psi_{j_n} \propto Y_{j_n}(\Omega)$$

As $\int d\Omega dt \Psi_{j_1 m_1} \Psi_{j_2 m_2}^* \Phi$

$$\Psi_{j_1 m_1} \propto Y_{j_1 m_1}(\Omega) \chi_{j_1 m_1}(t) e^{(j_1 + \frac{D-1}{2} + i\mu_1)t}$$

$$\times F\left(j_1 + \frac{D-1}{2}, j_1 + \frac{D-1}{2} + i\mu_1, 1 + i\mu_1, -e^{2t}\right)$$

$$m_1 = \sqrt{A^2 - \left(\frac{D-1}{2}\right)^2}$$



CAUTION
 UNIVERSITÄT
 WÜRZBURG

As $\int d\Omega dt \Psi_{j_1 n_1} \Psi_{j_2 n_2}^* \Phi_{j_3 n_3} \int d\Omega \dots$

$$\Psi_{j_1 n_1} \propto Y_{j_1 n_1}(\Omega) \chi_{j_1}^{n_1}(t) e^{(j_1 + \frac{D-1}{2} + i\mu_1)t}$$

$$\times F\left(j_1 + \frac{D-1}{2}, j_1 + \frac{D-1}{2} + i\mu_1, 1 + i\mu_1, -e^{2t}\right)$$

$$m_1 = \sqrt{A^2 - \left(\frac{D-1}{2}\right)^2}$$

CAUTION
 DO NOT TOUCH
 THE BOARD

$$A \propto \int d\Omega dt \Psi_{j_1 m_1} \Psi_{j_2 m_2}^* \Phi \mathcal{R}(\Phi)$$

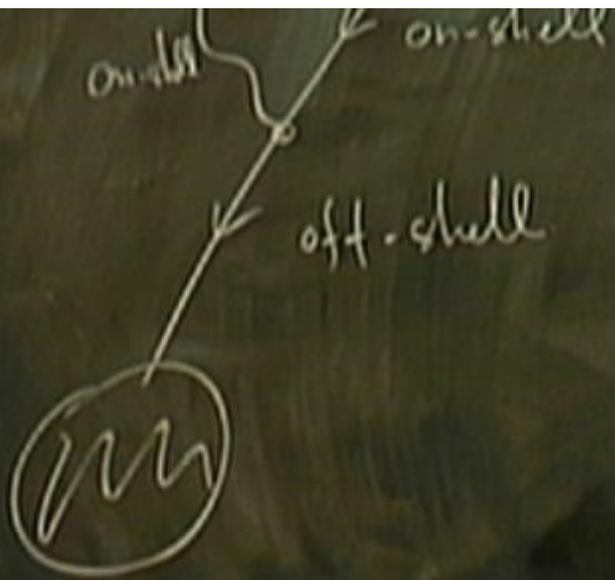
$$\Psi_{j_1} \propto Y_{j_1 m_1}(\Omega) \chi_{j_1}^{m_1}(r) e^{(j_1 + \frac{D-1}{2} + i\eta_1)l}$$

$$\times F(j_1 + \frac{D-1}{2}, \dots + \frac{D-1}{2}, \dots, 1+i)$$

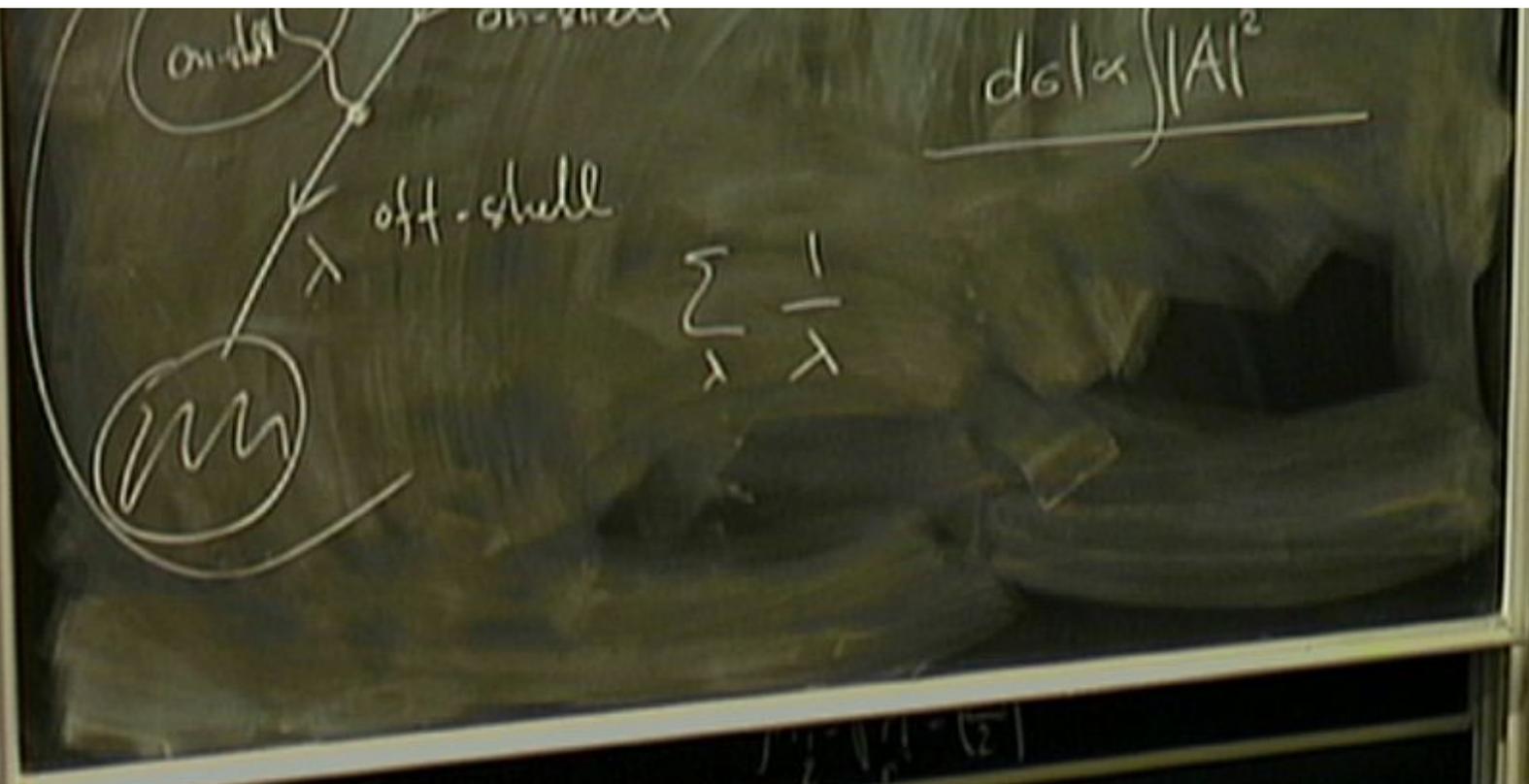
$$m_1 = \sqrt{M^2 - \left(\frac{D-1}{2}\right)^2}$$



CAUTION
 No smoking
 No mobile phones



CAUTION
DO NOT TOUCH
EQUIPMENT



As $\int d\Omega dt$ $\Psi_{j_1 m_1} \Psi_{j_2 m_2}^* \Phi_{j_3 m_3} \frac{1}{h^{D-1}}$

$$\textcircled{A} \int d\Omega dt \Psi_{j_1 m_1} \Psi_{j_2 m_2}^* \Phi_{j_3 m_3} \frac{1}{h^{D-1}}$$

$$\Psi_{j m} \propto Y_{j m}(\Omega) \chi^j(t) e^{(j + \frac{D-1}{2} + i\mu_1)t} \times F\left(j + \frac{D-1}{2}, j + \frac{D-1}{2} + i\mu_1, 1 + i\mu_1, -e^{2t}\right)$$

$$m_1 = \sqrt{A^2 - \left(\frac{D-1}{2}\right)^2}$$



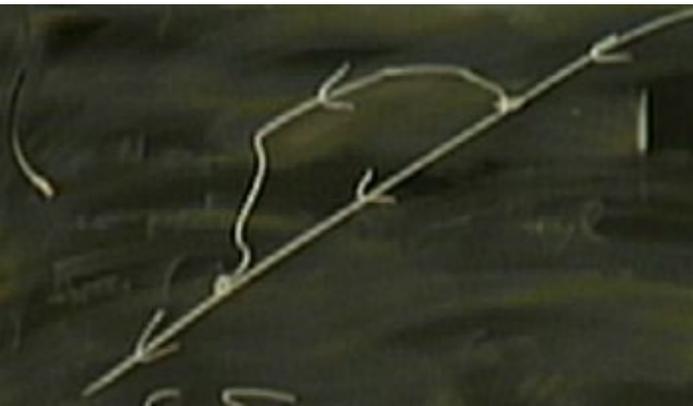
As $\int d\Omega dt \Psi_{j_1 n_1} \Psi_{j_2 n_2}^* \Phi_{j_3 n_3} \frac{d}{dt} \dots$

$\Psi_{j_1 n_1} \propto Y_{j_1 n_1}(\Omega) \chi_{j_1}^{(1)}(t) e^{(j_1 + \frac{D-1}{2} + i\mu_1)t}$

$\times F(j_1 + \frac{D-1}{2}, j_1 + \frac{D-1}{2} + i\mu_1, 1 + i\mu_1, -e^{2it})$

$\mu_1 = \sqrt{A^2 - \left(\frac{D-1}{2}\right)^2}$





$$F\left(\frac{D-1}{2} + i\eta, \frac{D-1}{2} - i\eta, \frac{D}{2}, \frac{1+z}{2}\right)$$

z

$$\delta\Sigma =$$



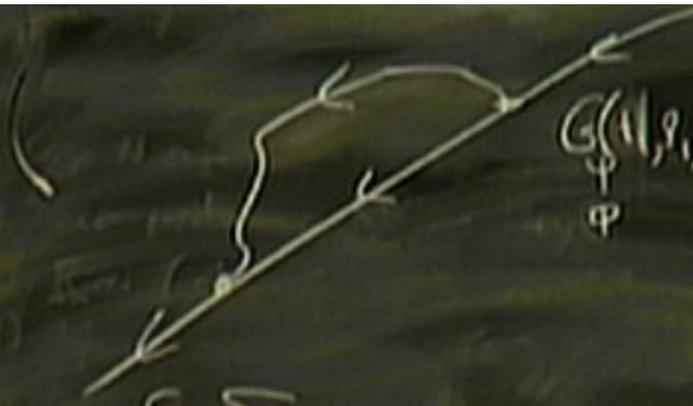
$$G \rightarrow e^{i(p-q)x}$$

$$= \sum_x \psi_x(x) \psi_x^*(x)$$

$$G_{\frac{D}{2}}(t_1, t_2, \Omega) \Gamma \left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1, \frac{D}{2}, \frac{1 \pm z}{2} \right)$$

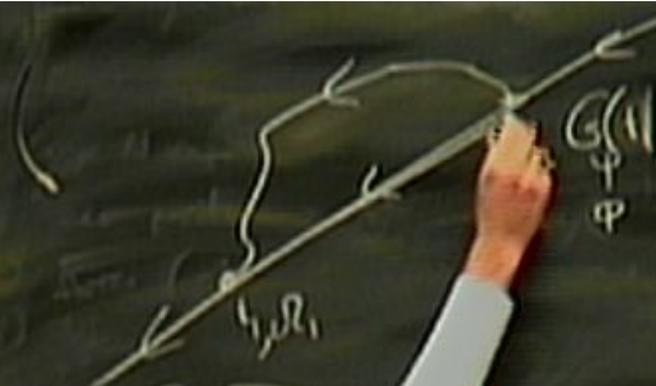
$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos \Delta \Omega$$

$$\delta \Sigma =$$



$$A \propto \int d^4 x G_{\frac{D}{2}}(x', x) e^{i(p-q)x}$$

$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{\dots}$$



$$G_{\frac{D}{2}}(t_1, \Omega_1, t_2, \Omega_2) \Gamma \left(\frac{D-1}{2} + i\nu_1, \frac{D-1}{2} - i\nu_1, \frac{D-1}{2}, \frac{1+z}{2} \right)$$

$$z = -sht_1 sht_2 + cht_1 cht_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \det^{D-1}(t_1) \det^{D-1}(t_2) \times$$

$$A \propto \int d^D x G_{\psi}(x', x) e^{i(p-q) \cdot x}$$

$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{\psi_{\lambda}(x) \psi_{\lambda}^*(x)}$$

$$G(x', x) = \int \frac{d^D p}{(2\pi)^D} e^{i(p-q)x} \left(\frac{D-1}{2} + i\epsilon, \frac{D-1}{2} - i\epsilon; \frac{D}{2}, \frac{1 \pm z}{2} \right)$$

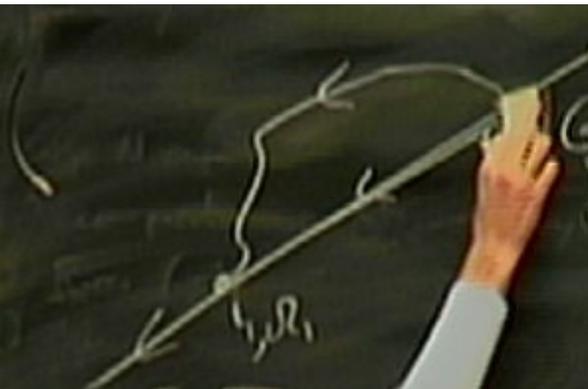
$$A = \int d^4 x G_{\psi}(x', x) e^{i(p-q)x}$$

$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{\lambda}$$

$$(\square - M^2) \psi_{\lambda} = \lambda \psi_{\lambda}$$

$$\psi_{\lambda} \propto e^{-i\kappa_0 t} e^{+i\vec{\kappa} \cdot \vec{x}}, \text{ where } \kappa_0 = \sqrt{M^2 + \vec{\kappa}^2}$$





$$G_{\mu}(\mu_1, \mu_2, \Omega) F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1, \frac{D}{2}, \frac{1 \pm z}{2}\right)$$

$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos \Delta \Omega$$

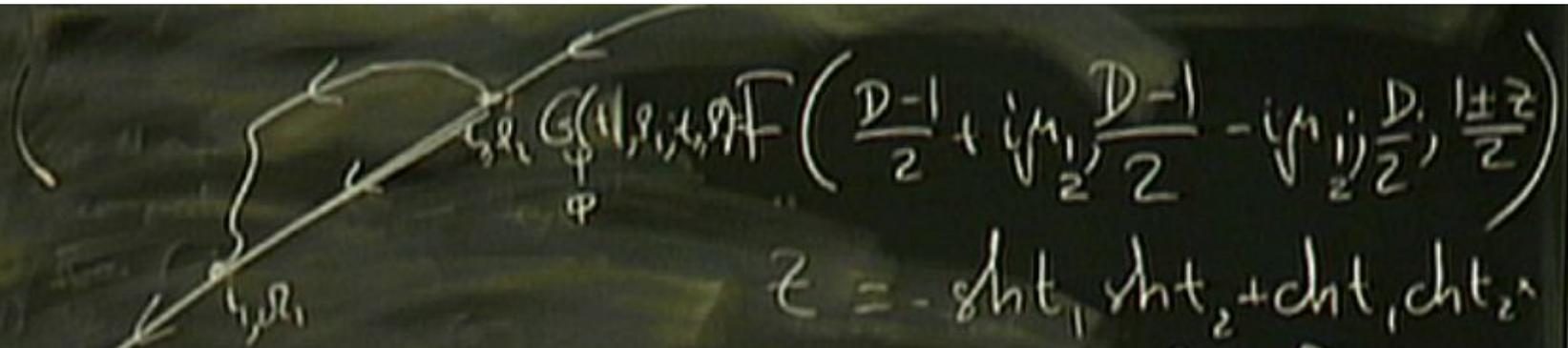
$$\Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \chi^{D-1}(t_1) \chi^{D-1}(t_2) \times$$

$$\chi^{\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1, \frac{D}{2}, \frac{1 \pm z}{2}} F(\mu_1 \rightarrow \mu_2)$$

$$A = \int d^4 x' G_{\mu}(x', x) e^{i(p-q)x}$$

$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{\lambda}$$

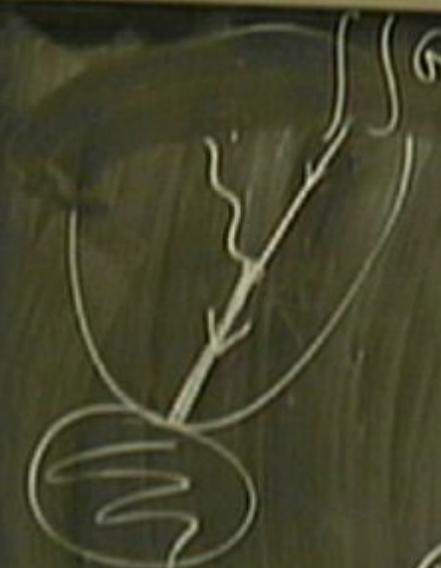
$$D(-M^2) \psi_{\lambda} = \lambda \psi_{\lambda}$$



$$G(t_1, \Omega_1, t_2, \Omega_2) F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1; \frac{D}{2}, \frac{1+z}{2}\right)$$

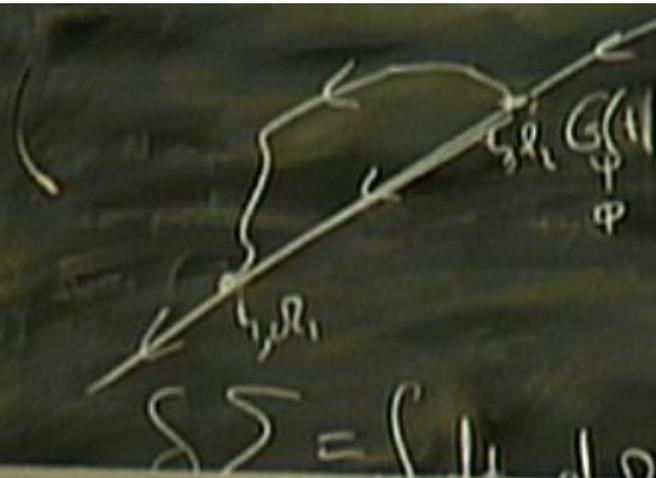
$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \det^{D-1}(t_1) \det^{D-1}(t_2) \times F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1; \frac{D}{2}, \frac{1+z}{2}\right) F(\mu_1 \rightarrow \mu_2)$$



$$A = \int d^4x G(x, y) D(y, x) \int d^4x' G_y(x', x) e^{i(p-q)x}$$

$$G(x', x) = \sum_{\lambda} \frac{\psi_{\lambda}(x') \psi_{\lambda}^*(x)}{E_{\lambda}}$$



$$G(s, t, \varphi) \Gamma\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1, \frac{D}{2}, \frac{1+z}{2}\right)$$

$$z = -\operatorname{sh}t_1 \operatorname{sh}t_2 + \operatorname{ch}t_1 \operatorname{ch}t_2$$

$$\times \cos \Delta \Omega$$

$$\mathcal{D}\mathcal{D} = \int_{r_1}^{r_2} \dots \int_{r_1}^{r_2} \dots$$

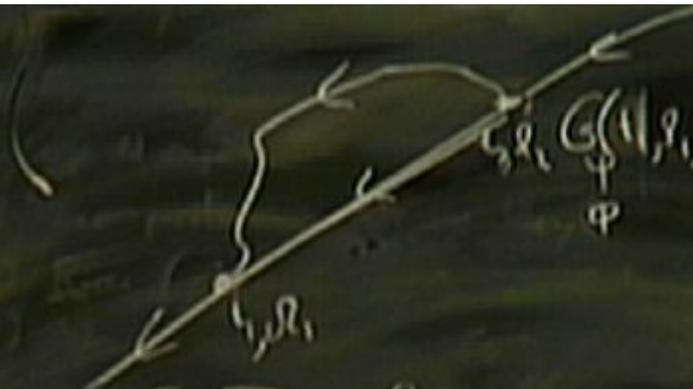
$\int_{r_1}^{r_2} G(z) dz$

$$F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1, \frac{D}{2}, \frac{1+z}{2}\right)$$

$$z = -\operatorname{sh}t_1, \operatorname{sh}t_2 + \operatorname{ch}t_1, \operatorname{ch}t_2$$

$$\times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 \dots \int dt_{D-1} d\Omega_{D-1}$$

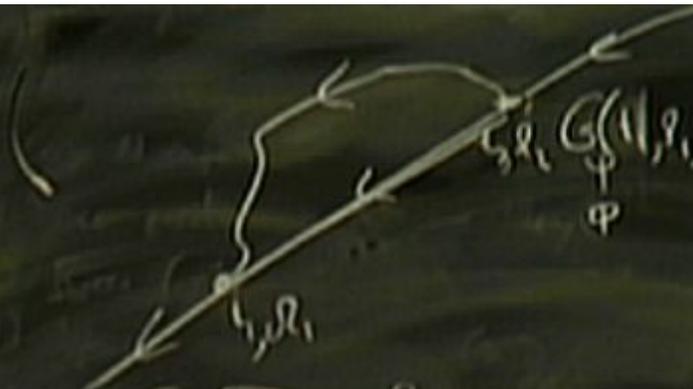


$$G(t_1, \Omega_1, t_2, \Omega_2) = F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_2, \frac{D}{2}, \frac{1+z}{2}\right)$$

$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 \int dt_2 d\Omega_2 \text{ch}^{D-1}(t_1) \text{ch}^{D-1}(t_2) \times F\left(\frac{D-1}{2} - i\mu_1, \frac{D-1}{2}, \frac{1+z}{2}\right) F(\mu_1 \rightarrow \mu_2)$$

$$m = \frac{1}{2}$$



$$G(t_1, R_1, t_2, R_2) \Gamma\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_2, \frac{D}{2}, \frac{1+z}{2}\right)$$

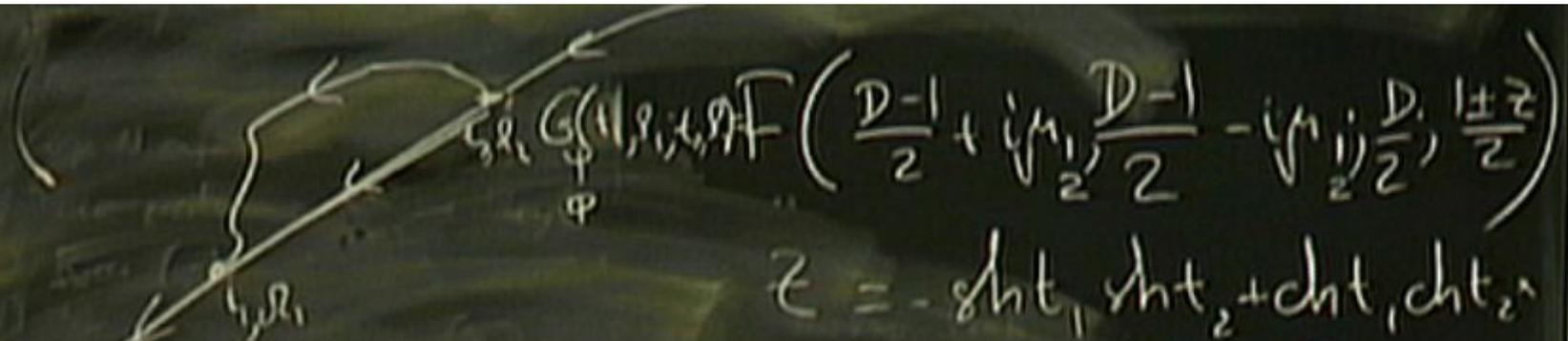
$$z = -sht_1 sht_2 + cht_1 cht_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \text{ch}^{D-1}(t_1) \text{ch}^{D-1}(t_2) \times$$

$$\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_2, \frac{D}{2}, \frac{1+z}{2}\right) \Gamma(\mu_1 \rightarrow \mu_2)$$

$\frac{D-1}{2}$
 V_M

$H = \frac{1}{2}$



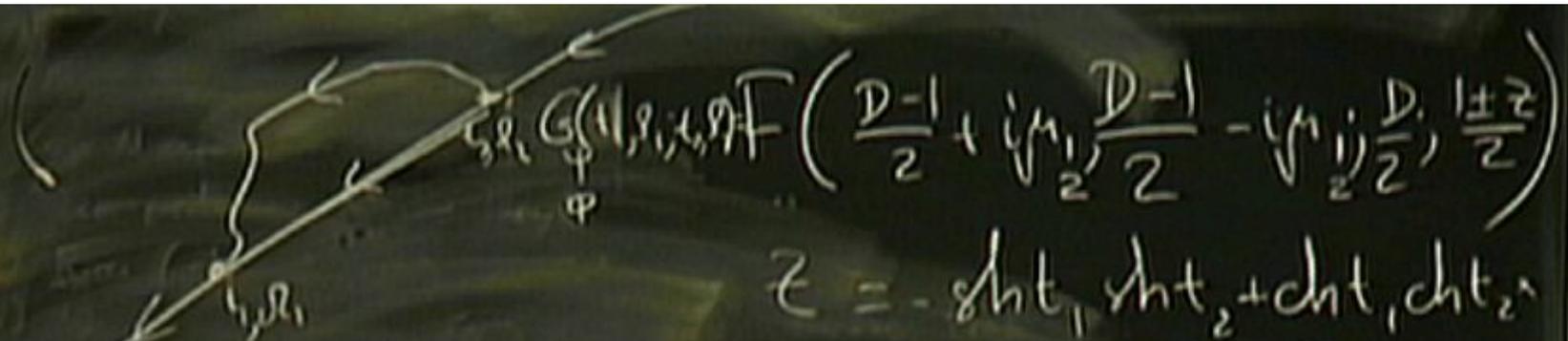
$$F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_2, \frac{D}{2}, \frac{1+z}{2}\right)$$

$$z = -sht_1 sht_2 + cht_1 cht_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \text{ch}^{D-1}(t_1) \text{ch}^{D-1}(t_2) \times F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_2, \frac{D}{2}, \frac{1+z}{2}\right) F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_2, \frac{D}{2}, \frac{1+z}{2}\right)$$

$m < \frac{D-1}{2}$ $H=1$
for $\forall M$



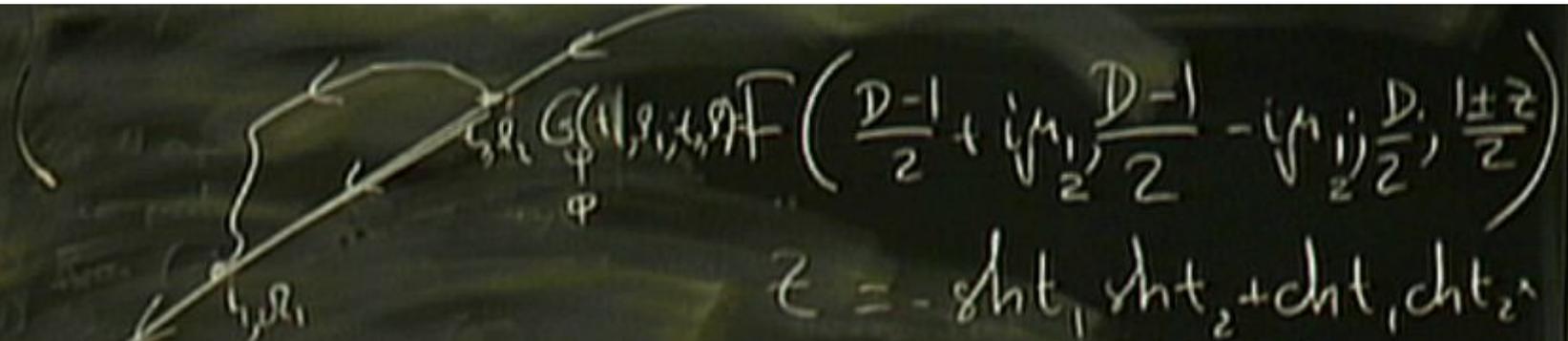


$$G(t_1, \Omega_1, t_2, \Omega_2) = F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1; \frac{D}{2}, \frac{1+z}{2}\right)$$

$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \det^{D-1}(t_1) \det^{D-1}(t_2) \times F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1; \frac{D}{2}, \frac{1+z-i}{2}\right) F(\mu_1 \rightarrow \mu_2)$$

$m < \frac{D-1}{2}$ $H=1$
 for $\forall M$



$$z = -\sinh t_1 \sinh t_2 + \cosh t_1 \cosh t_2 \times \cos \Delta \Omega$$

$$\delta \Sigma = \int dt_1 d\Omega_1 \int dt_2 d\Omega_2 \cosh^{D-1}(t_1) \cosh^{D-1}(t_2) \times F\left(\frac{D-1}{2} + i\mu_1, \frac{D-1}{2} - i\mu_1, \frac{D}{2}, \frac{1+z}{2}\right) F(\mu_1 \rightarrow \mu_2)$$

$m < \frac{D-1}{2}$ $H=1$
for $\forall M$