Title: Infrared instability of massless scalars in De Sitter space-time.

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Abstract: Vacuum expectation value of the square of the quantum field operator of massless or light scalar field is calculated in the De Sitter space-time. The suggested method of calculation is different from the standard one used in the 80th. The calculations are heavily based on the De Sitter covarinace of the relevant quantities. The found result is significanlty

different for the old one for the massless field but coinsides with the classical result for light massive field. Possible explanation of the dicrepancy by a spontaneous breaking of De Sitter invariance or by finite duration of the (quasi) De Sitter stage is discussed.

INFRARED INSTABILITY OF MASSLESS SCALARS IN DE SITTER SPACE-TIME.

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IR Issues and Loops
in De Sitter Space
PI, Waterloo, Ontario

October 27-30, 2010.

Content of the talk:

Calculations of $\langle\varphi^2\rangle$ in the DeSitter space-time by two different methods give coinciding results for small but non-zero mass, while they have the same magnitude but differ by sign for $m = 0$.

This may point to breaking of DS invariance by quantum effects or to problems with UV renormalization.

A scalar field with the action:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}\,(\nabla_a\varphi\nabla^a\varphi - -m^2\varphi^2 - \xi R\varphi^2).$$

Equation of motion in FRW metric:

$$\ddot{\varphi} - \Delta\varphi/a^2 + 3H\dot{\varphi} + m^2\varphi + \xi R\varphi = 0\,.$$

Energy-momentum tensor operator:

$$T^{ab} = -\frac{1}{2} g^{ab} \left(\nabla_c \varphi \nabla^c \varphi - m^2 \varphi^2 \right)$$
$$+ \nabla^a \varphi \nabla^b \varphi - \xi \left(R^{ab} - \frac{1}{2} g^{ab} R \right) \varphi^2$$
$$- \xi \left(\nabla^a \nabla^b - g^{ab} \nabla_c \nabla^c \right) \varphi^2 .$$

NB: T_a^b is covariantly conserved:

$$\nabla_b T_a^b = 0 \, .$$

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Vacuum expectation value of the energy momentum tensor of φ in DS is proportional to metric tensor:

$$\langle T_{ab}\rangle_{ren} = \frac{g_{ab}}{64\pi^2}\left\{m^2\left[m^2 - \left(\xi - \frac{1}{6}\right) R\right.\right.$$

$$\left[\psi\left(\frac{3}{2}+\nu\right) + \psi\left(\frac{3}{2}-\nu\right) - ln\left(\frac{12m^2}{|R|}\right)\right.$$

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For $m \to 0$ and $\xi = 0$, the energy-momentum tensor becomes:

$$T_{ab} = \frac{g_{ab} H^4}{\pi^2} \left(\frac{3}{32} + \frac{1}{960} - \frac{1}{32} \right).$$

The first term is the standard non-anomalous term in the energy density at $m = 0$. The other two terms are anomalous. The second term survives in the conformal limit, while the third disappears only for $\xi = 1/6$ and does not vanish for $m = 0$ and $\xi = 0$.

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Quantum average value of φ^2 in De Sitter space-time is singular when $m \to 0$. In particular, for $\xi = 0$ and $m^2 \ll H^2$:

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The usual (old) result is derived by straightforward quantization:

$$\varphi(t,x) = \int \frac{d^3p}{(2\pi)^{3/2}}\left[f_p(t)e^{ipx}a_p^\dagger + h.c.\right]$$

where the mode functions are:

$$f_p(t) = c_1(p)H^{(1)}_{3/2}(p\eta) + c_2(p)H^{(2)}_{3/2}(p\eta)$$

where η is the conformal time.
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$$f_p(t) = \frac{iH}{\sqrt{2p^3}}\left(1 + \frac{k}{iH}\right) e^{(ik/H)},$$

where $k = p\exp(-Ht)$.

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$$\langle\varphi^2\rangle = \int \frac{d^3k}{2(2\pi)^3 k}\left(1 + \frac{H^2}{k^2}\right).$$

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Long wave modes, UV and IR cutoff:

$$\langle\varphi^2\rangle = \frac{H^2}{16\pi^3}\int_{H\exp(-Ht)}^{H}\frac{d^3k}{k^3} = \frac{H^3t}{4\pi^2}.$$

In terms of physical momentum $p = k\exp(-Ht)$:

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Alternative derivation using equation of motion for $f \equiv \langle\varphi\rangle^2$ (AD, Pelliccia, 2005):

$$\nabla_a \nabla^a \varphi^2 = 2\varphi \nabla_a \nabla^a \varphi + 2\nabla_a \varphi \nabla^a \varphi.$$

Hence

$$(\nabla_a \nabla^a + m^2 + \xi R) f = 2 \langle \nabla_a \varphi \nabla^a \varphi \rangle - m^2 f - \xi R f.$$

Almost closed equation, except for $(\nabla\varphi)^2$ term.

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Express the first term in the r.h.s. through the energy density of φ, assuming that its energy-momentum has DeSitter covariant form:

$$(1-6\,\xi)\nabla_a\nabla^a f - 2\,m^2 f = \\ = -2\langle T^a{}_a\rangle = -8\,\langle\varrho\rangle\,.$$

In homogenous case:

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formally the same as for $\xi = 1/6$ but remember that $\langle T^a_a \rangle = T^a_a(\xi, m)$.

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If only non-anomalous terms in T_{ab} are taken into account, i.e. $T_{ab} = 3g_{ab}H^4/32\pi^2$, we obtain the classical result:

$$\langle\varphi^2\rangle = \frac{3H^4}{8\pi^2 m^2}$$

With trace anomaly included the coefficient is different:

$$\langle\varphi^2\rangle \approx \frac{61H^4}{240\pi^2 m^2}.$$

WHY???

If $m = 0$ from the very beginning the solution is (for $\xi = 0$):

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The same solution can be obtained from the general one in the limit of $m \to 0$ under condition that the solution is not singular at $m = 0$. To this end C_2 should be singular:
$C_2 = C_{20} - 4\langle \varrho \rangle / m^2$, where C_{20} is a non-singular constant.

A problem with the equation in the conformal limit $m = 0$ and $\xi = 1/6$:

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Anomalous contribution should be take[n]
into account in the l.h.s.
Analogy to the Maxwell equations in DS with anomaly (AD, 1981):

$$\partial_\nu F^\nu_\mu + \frac{2\alpha N_f}{3\pi}(\partial_\nu \ln a) F^\nu_\mu = 0\,,$$

where a is the cosmological scale factor. The equation describes massless photon production in DS.
In fact the equation is nonlocal but in DS it can be essentially reduced to

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Analogy to the Maxwell equations in DS with anomaly (AD, 1981):

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For free scalar field in curved space-time one has to calculate its Green's function and inverse of it would give the equation of motion with anomalous corrections.

It may have infrared problems.

Conclusion

I. Alternative calculations of $\langle\varphi^2\rangle$ free of infrared problems give the same result for non-zero mass as the standard one.

II. The same calculations for $m = 0$ differ by sign from the the previously found results in large time limit.

III. Possible explanations:

a. Modification of the mode functions at low momenta.

b. UV renormalization (NT, RW).

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d. Breakdown of DS symmetry.

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e. Nothing of the above.

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For small mass, $m^2/H^2 \ll 1$, and short time, $t < 3H/2m^2$, the solution is

$$f = C_1 e^{-3Ht} + C_2\left(1 + \frac{2m^2}{3H}t\right) + \frac{4\langle\varrho\rangle}{m^2}.$$

If $C_{1,2}$ are not singular at $m = 0$, the dominant term for $m \to 0$ is

$$\langle\varphi^2\rangle = 4\langle\varrho\rangle/m^2,$$

formally the same as for $\xi = 1/6$ but remember that $\langle T^a_a\rangle = T^a_a(\xi, m)$.

Alternative derivation using equation of motion for $f \equiv \langle\varphi\rangle^2$ (AD, Pelliccia, 2005):

$$\nabla_a \nabla^a \varphi^2 = 2\varphi \nabla_a \nabla^a \varphi + 2\nabla_a \varphi \nabla^a \varphi.$$

Hence

$$(\nabla_a \nabla^a + m^2 + \xi R) f = 2 \langle \nabla_a \varphi \nabla^a \varphi \rangle - m^2 f - \xi R f.$$

Almost closed equation, except for $(\nabla \varphi)^2$ term.

Long wave modes, UV and IR cutoff:

$$\langle\varphi^2\rangle = \frac{H^2}{16\pi^3}\int_{H\exp(-Ht)}^{H}\frac{d^3k}{k^3} = \frac{H^3t}{4\pi^2}.$$

In terms of physical momentum $p = k\exp(-Ht)$:

$$\langle\varphi^2\rangle = \frac{H^2}{4\pi^2}\int_0^{Ht} d\,ln\left(\frac{p}{H}\right) = \frac{H^3t}{4\pi^2}.$$

The result sits on small p and/or k, for which c_1 may be nonzero and c_2 different from its flat space limit.

Vacuum expectation value of the energy momentum tensor of φ in DS is proportional to metric tensor:

$$\langle T_{ab}\rangle_{ren} = \frac{g_{ab}}{64\pi^2}\left\{m^2\left[m^2 - \left(\xi - \frac{1}{6}\right)R\right.\right.$$

$$\left[\psi\left(\frac{3}{2}+\nu\right) + \psi\left(\frac{3}{2}-\nu\right) - ln\left(\frac{12m^2}{|R|}\right)\right.$$

$$-\frac{1}{2}\left(\xi - \frac{1}{6}\right)^2 R^2 + \frac{R^2}{2160}$$

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$$m^2\left(\xi - \frac{1}{6}\right)R - \frac{m^2 R}{18}\right\}$$

(Dowker, Critchley, 1976; Bunch, Davie

Express the first term in the r.h.s. through the energy density of φ, assuming that its energy-momentum has DeSitter covariant form:

$$(1-6\,\xi)\nabla_a\nabla^a f - 2\,m^2 f = \\ = -2\langle T^a{}_a\rangle = -8\,\langle\varrho\rangle\,.$$

In homogenous case:

$$(1-6\xi)\left(\ddot{f} + 3\,H\,\dot{f}\right) - 2\,m^2 f = -8\,\langle\varrho\rangle$$

NB: ρ well behaves in infrared

A problem with the equation in the conformal limit $m = 0$ and $\xi = 1/6$:

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