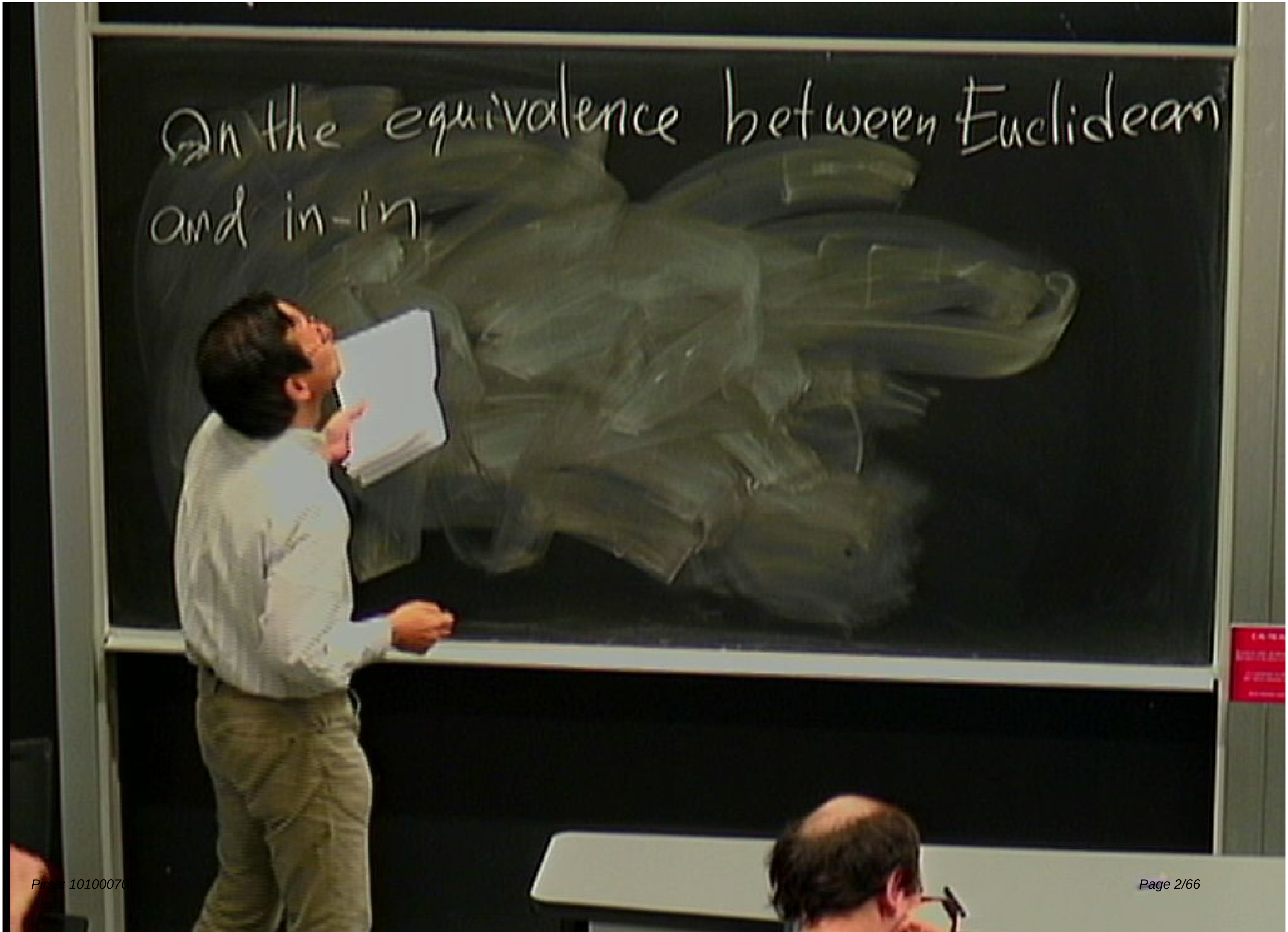
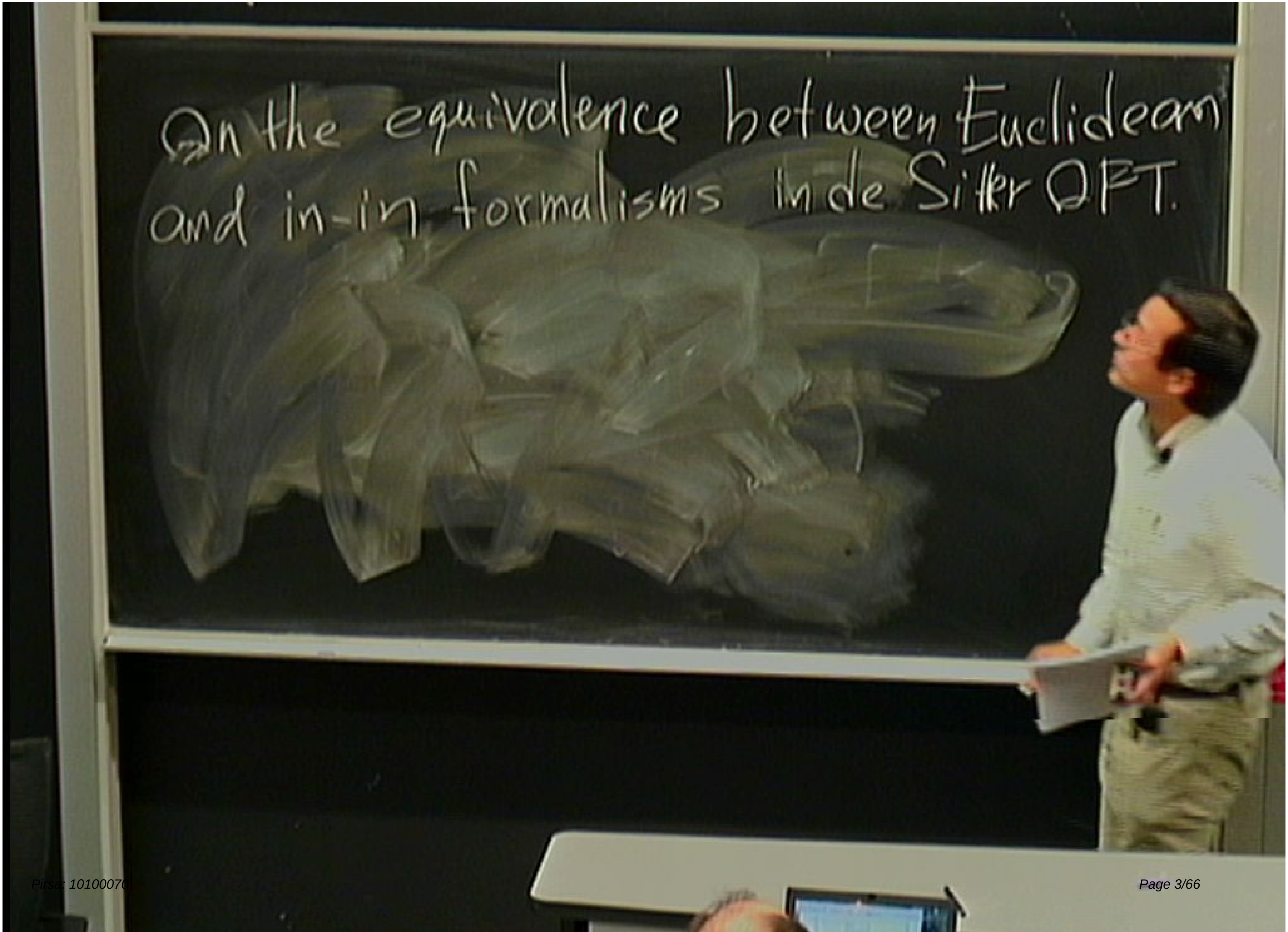
Title: On the Equivalence between Euclidean and In-In Formalisms in de Sitter QFT

Date: Oct 28, 2010 11:30 AM

URL: http://pirsa.org/10100070

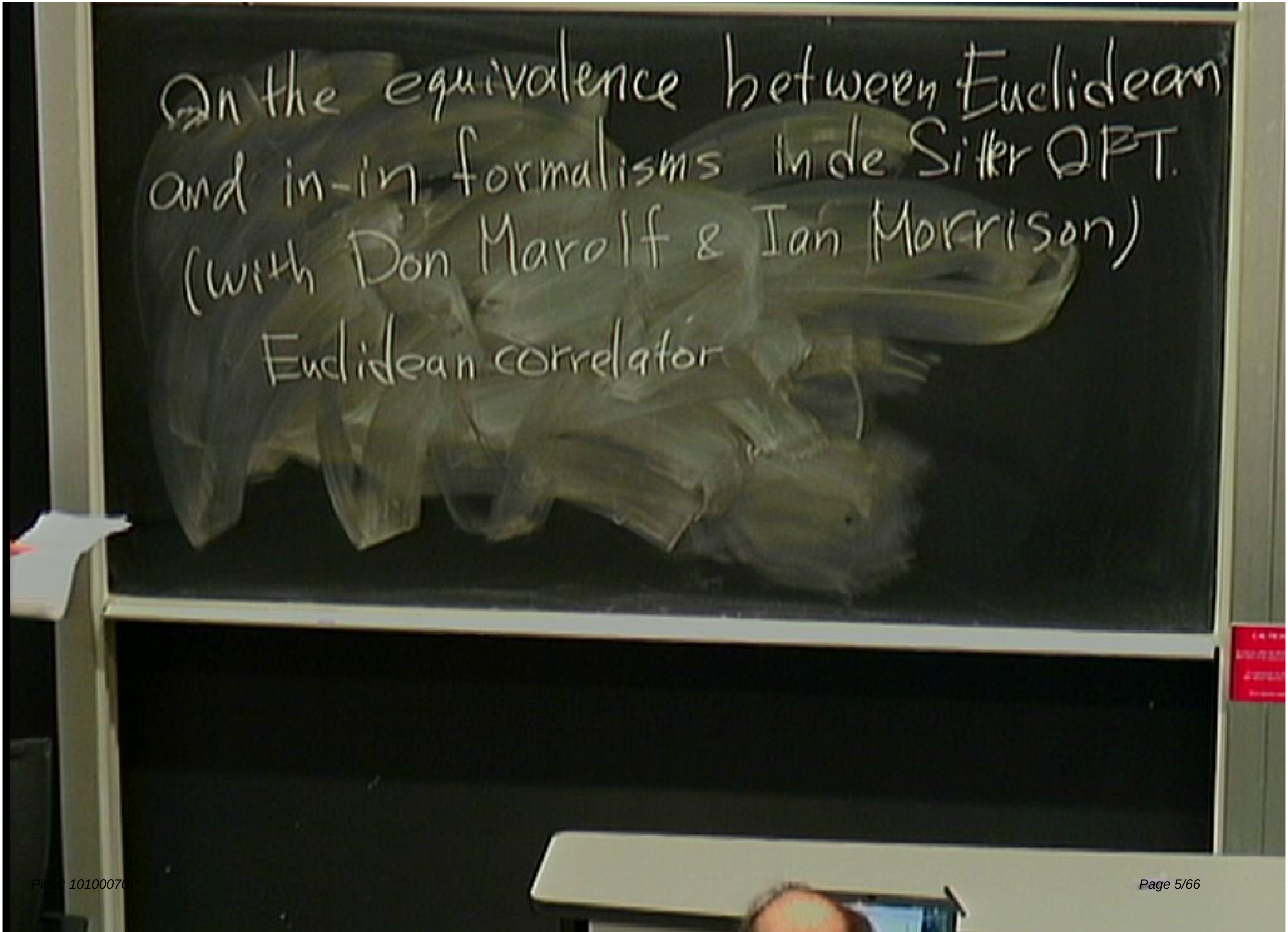
Abstract: We study the relation between two sets of correlators in interacting quantum field theory on de Sitter space. The first are correlators computed using in-in perturbation theory in the region of de Sitter space to the future of a cosmological horizon (also known as the expanding cosmological patch, the conformal patch, or the Poincare patch), and for which the free propagators are taken to be those of the free Euclidean vacuum. The second are correlators obtained by analytic continuation from interacting QFT on Euclidean de Sitter; i.e., they are correlators in the Hartle-Hawking vacuum. We give an analytic argument that these correlators coincide for interacting massive scalar fields with any positive mass. We also verify this result via direct analytical and numerical calculation in two simple examples. The correspondence holds diagram by diagram, and at any finite value of a Pauli-Villars regulator mass M. Along the way, we note interesting connections between various prescriptions for perturbation theory in general static spacetimes with bifurcate Killing horizons.

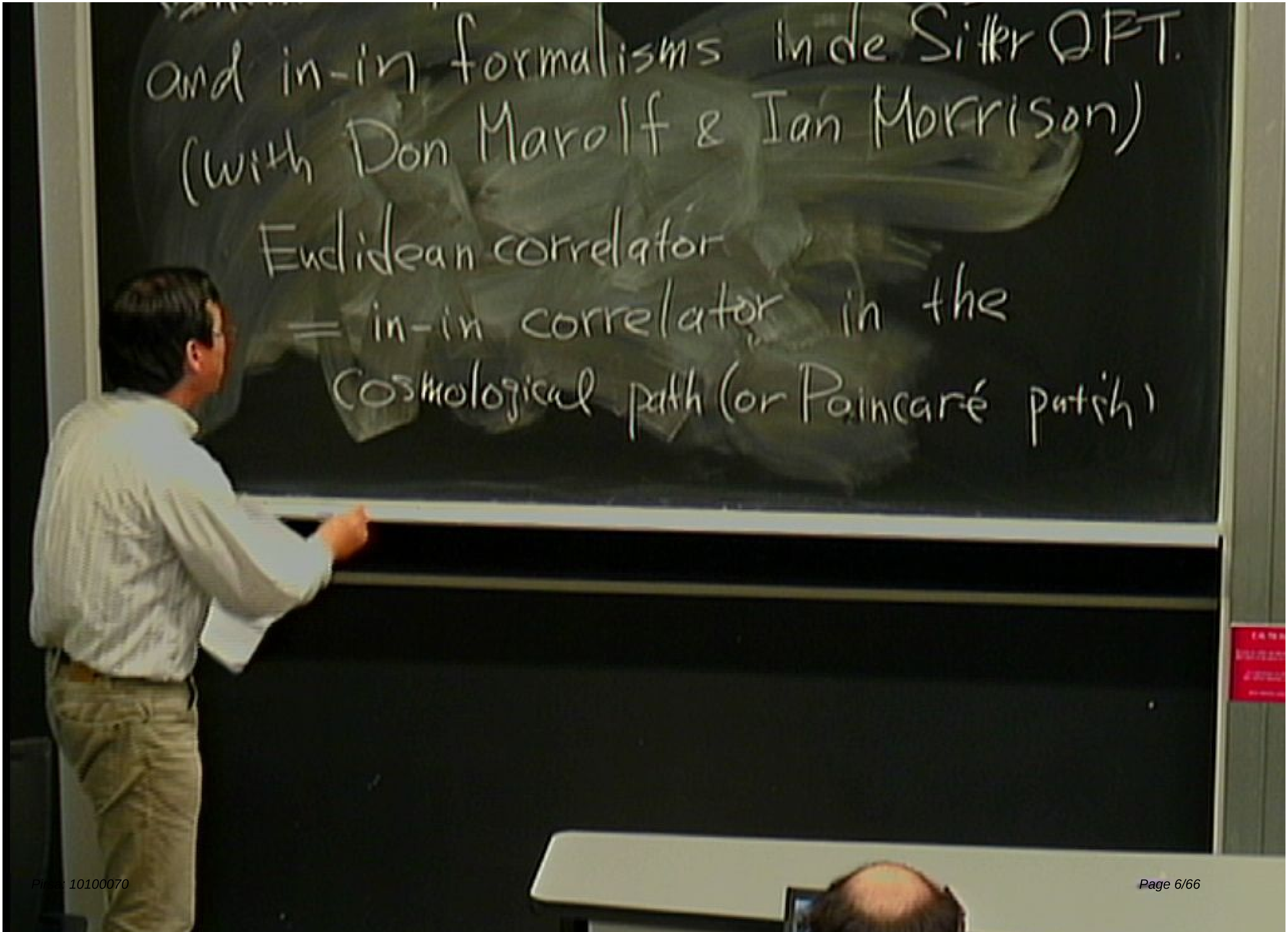

On the equivalence between Euclidean
and in-in

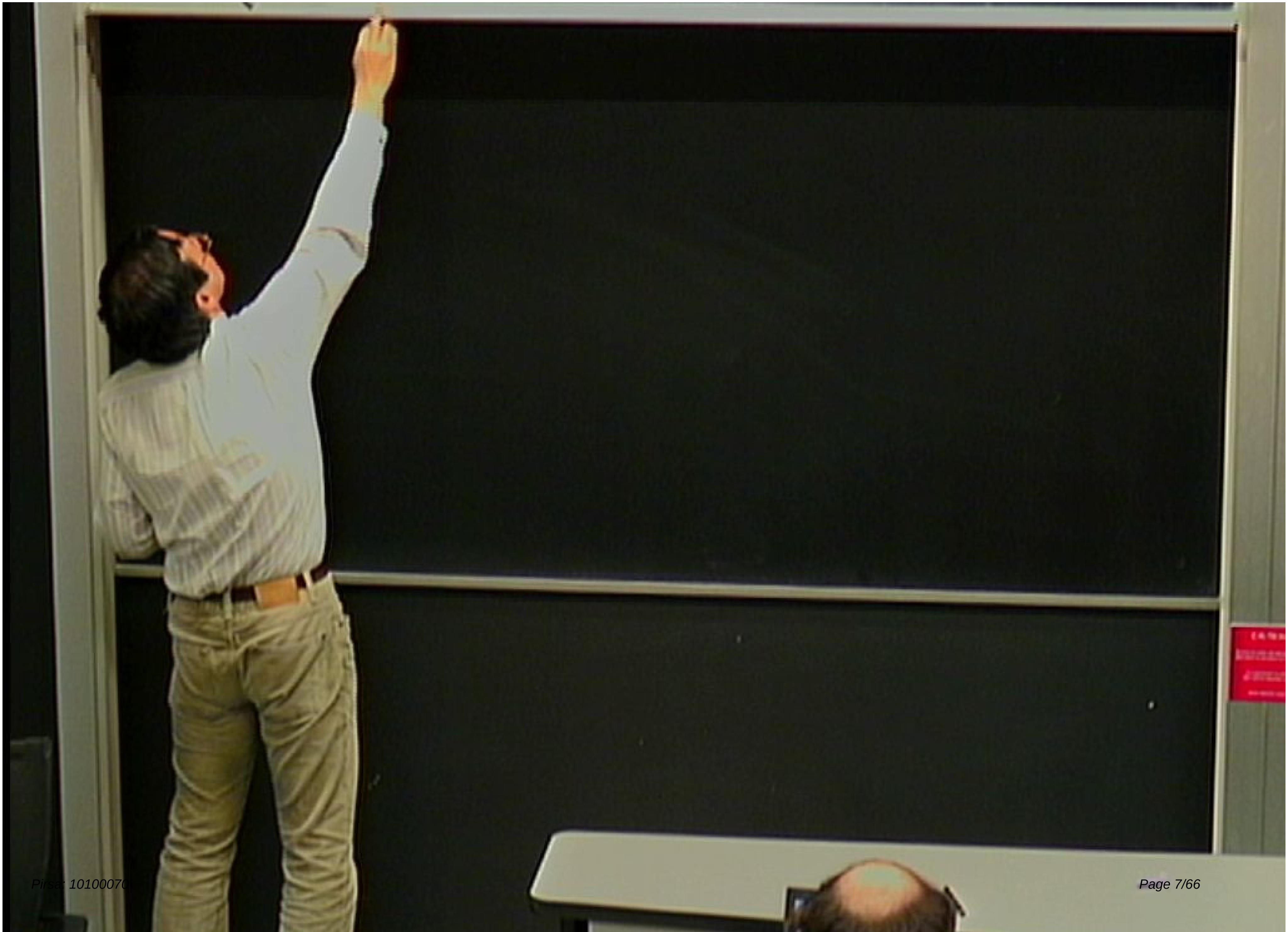

On the equivalence between Euclidean
and in-in formalisms in de Sitter QFT.

On the equivalence between Euclidean and in-in formalisms in de Sitter QFT.

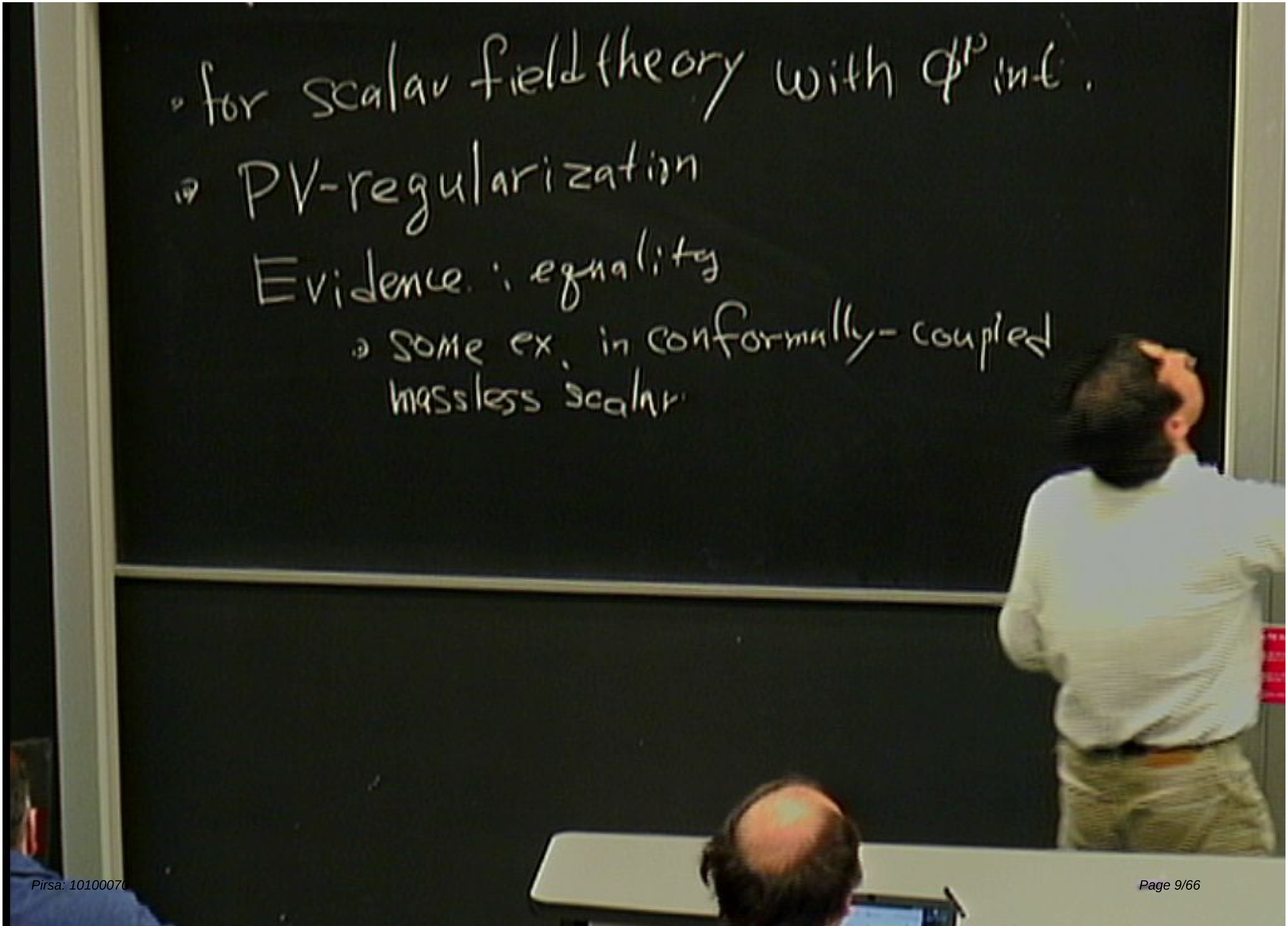
(with Don Marolf & Ian Morrison)

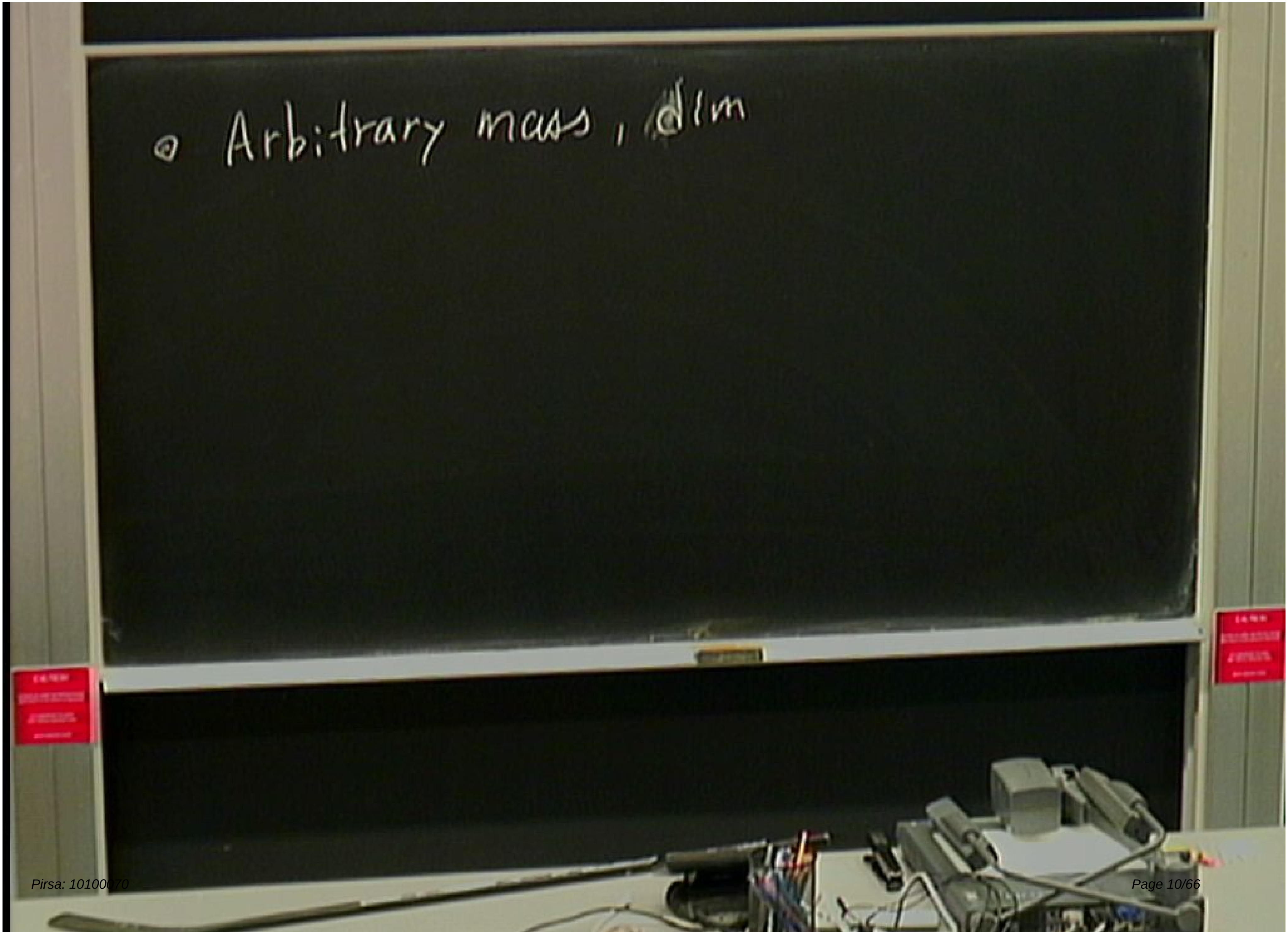

On the equivalence between Euclidean
and in-in formalisms in de Sitter QFT.
(with Don Marolf & Ian Morrison)
Euclidean correlator


and in-in formalisms in de Sitter QFT.
(with Don Marolf & Ian Morrison)
Euclidean correlator
= in-in correlator in the
cosmological path (or Poincaré patch)



- for scalar field theory with ϕ^p int.
- PV-regularization

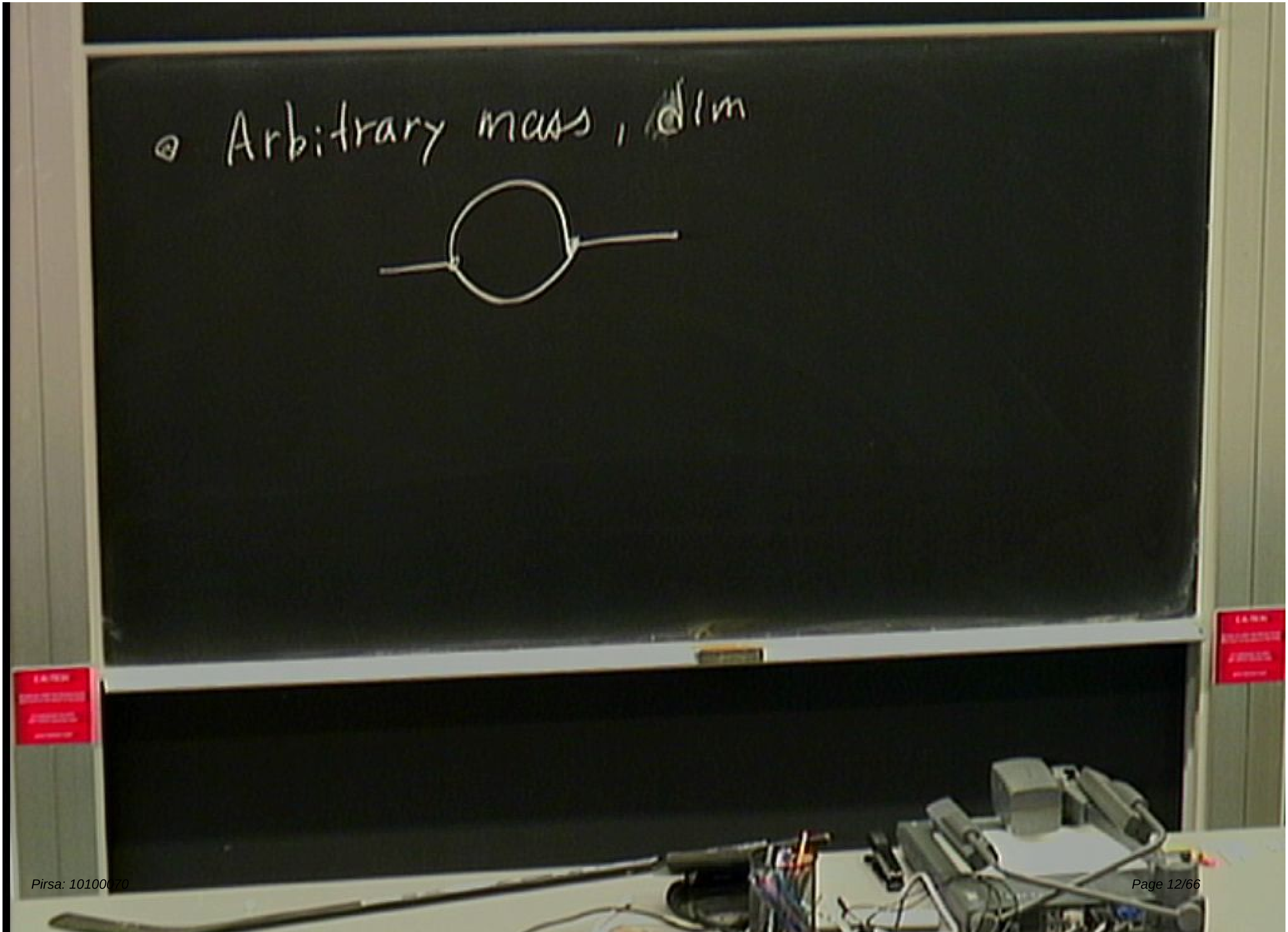

for scalar field theory with ϕ^p int.
PV-regularization
Evidence: equality
some ex. in conformally-coupled
massless scalar


• Arbitrary mass, dim

- for scalar field theory with ϕ^p int.
- PV-regularization $(m^2>0)$

Evidence: equality

- some ex. in conformally-coupled massless scalar


• Arbitrary mass, dim

1. Proof in 3 steps

1

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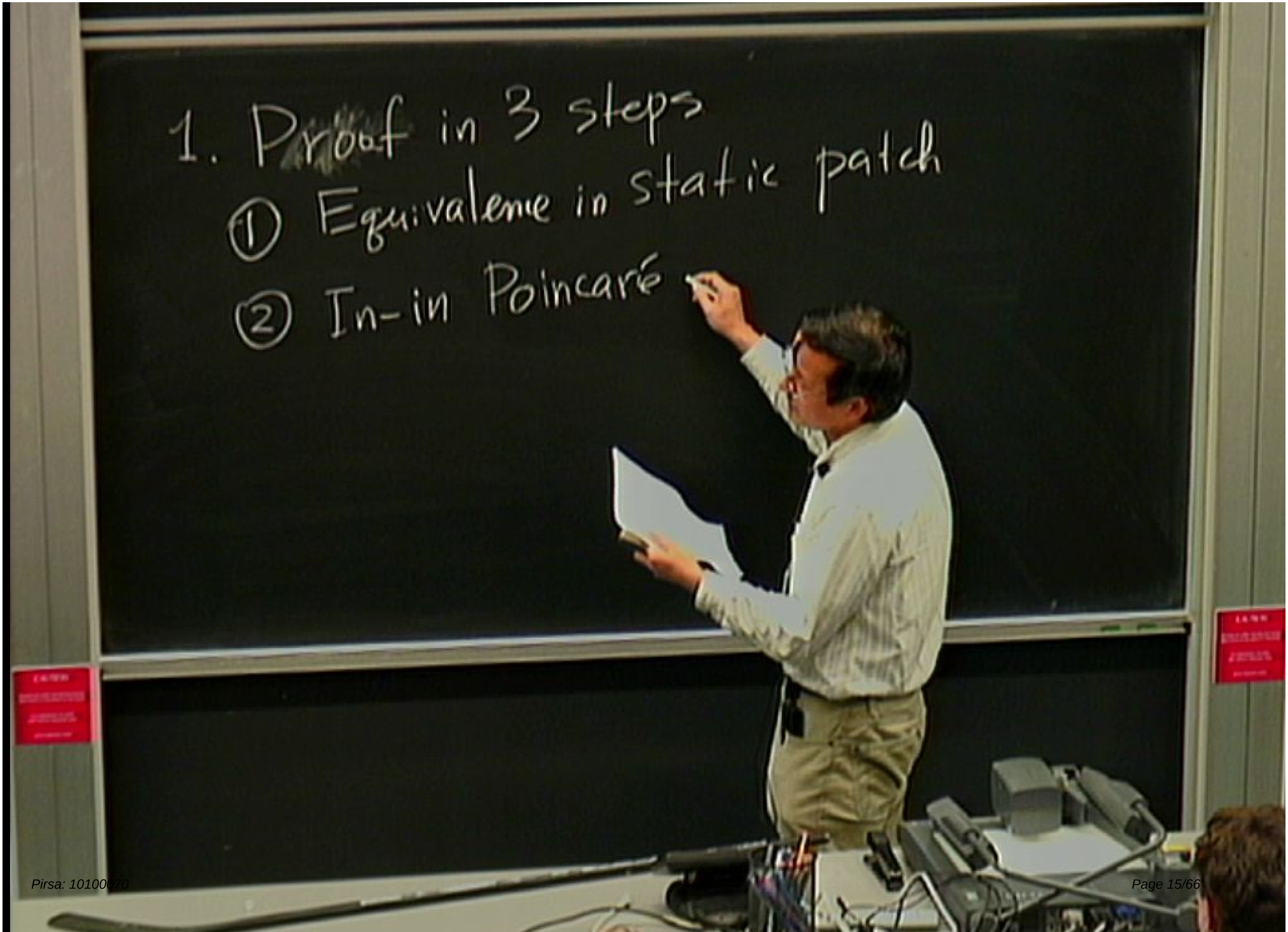
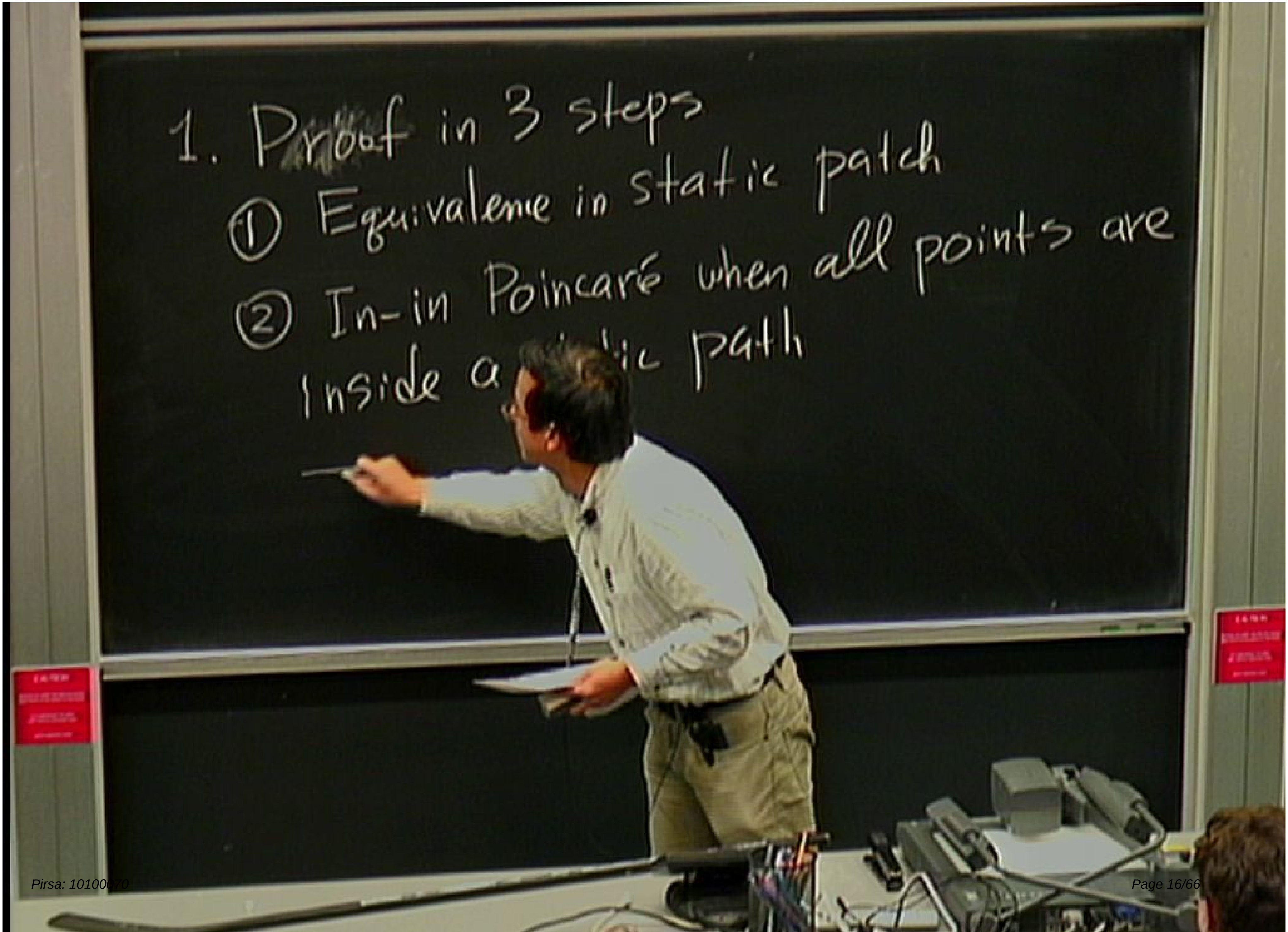
(1) Equivalene in static patch


1. Proof in 3 steps
① Equivalence in static patch
② In-in Poincaré

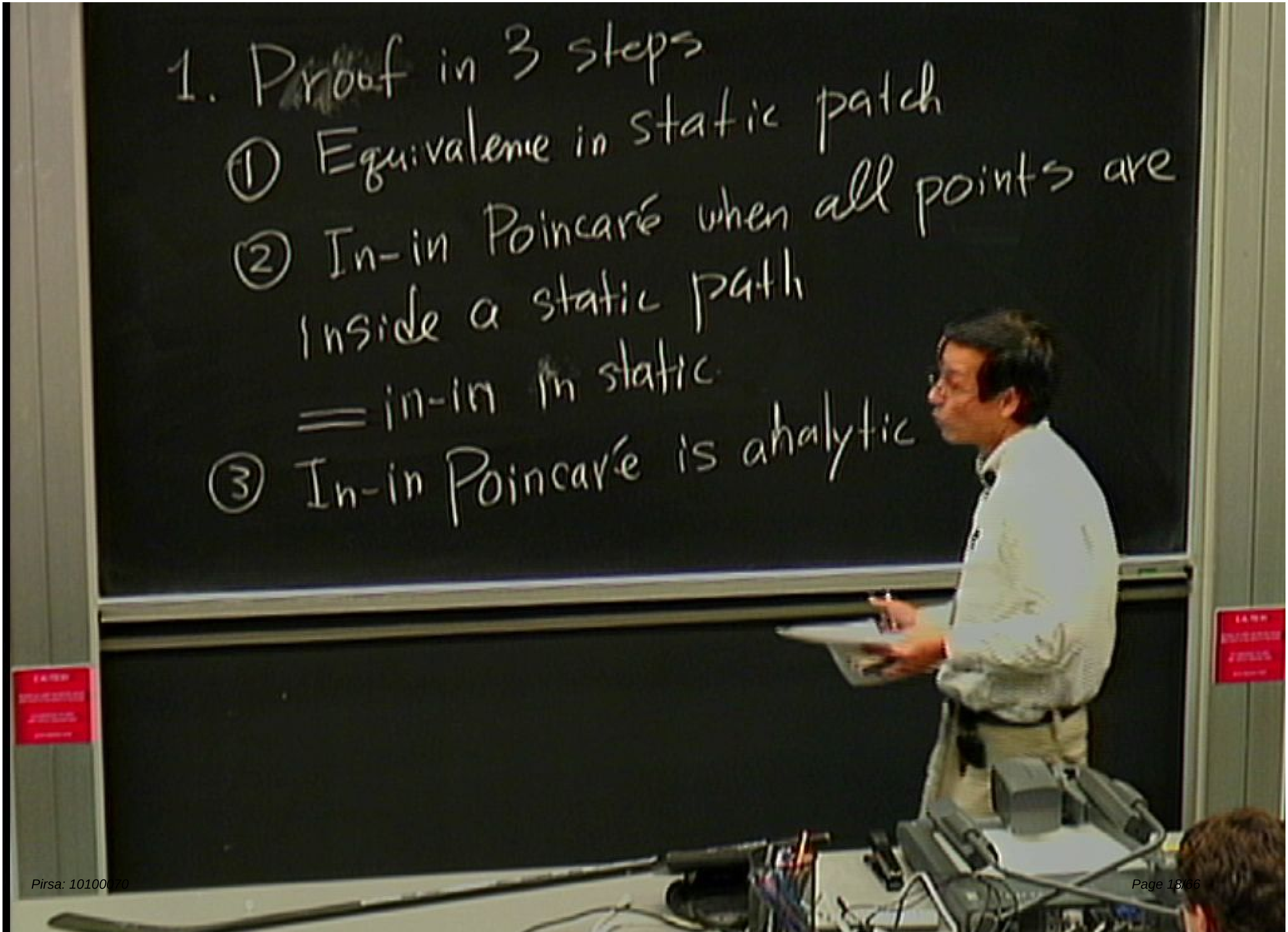

1. Proof in 3 steps
① Equivalence in static patch
② In-in Poincaré when all points are
inside a
ic path

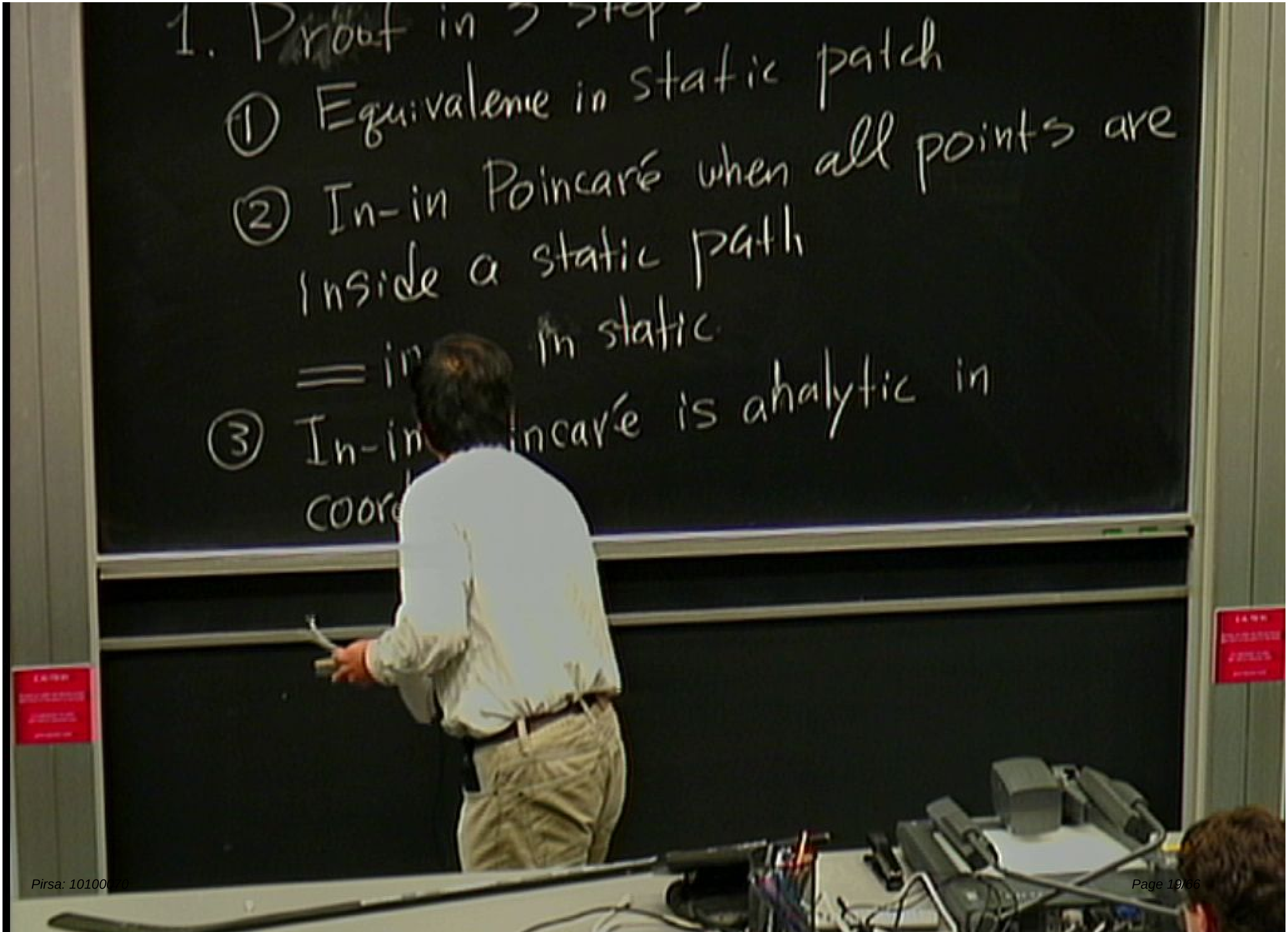
1. Proof in 3 steps

① Equivalence in static patch

② In-in Poincaré when all points are inside a static path

$=$ in-in in static


1. Proof in 3 steps
① Equivalene in static patch
② In-in Poincaré when all points are
inside a static path
= in-in in static
③ In-in Poincaré is analytic

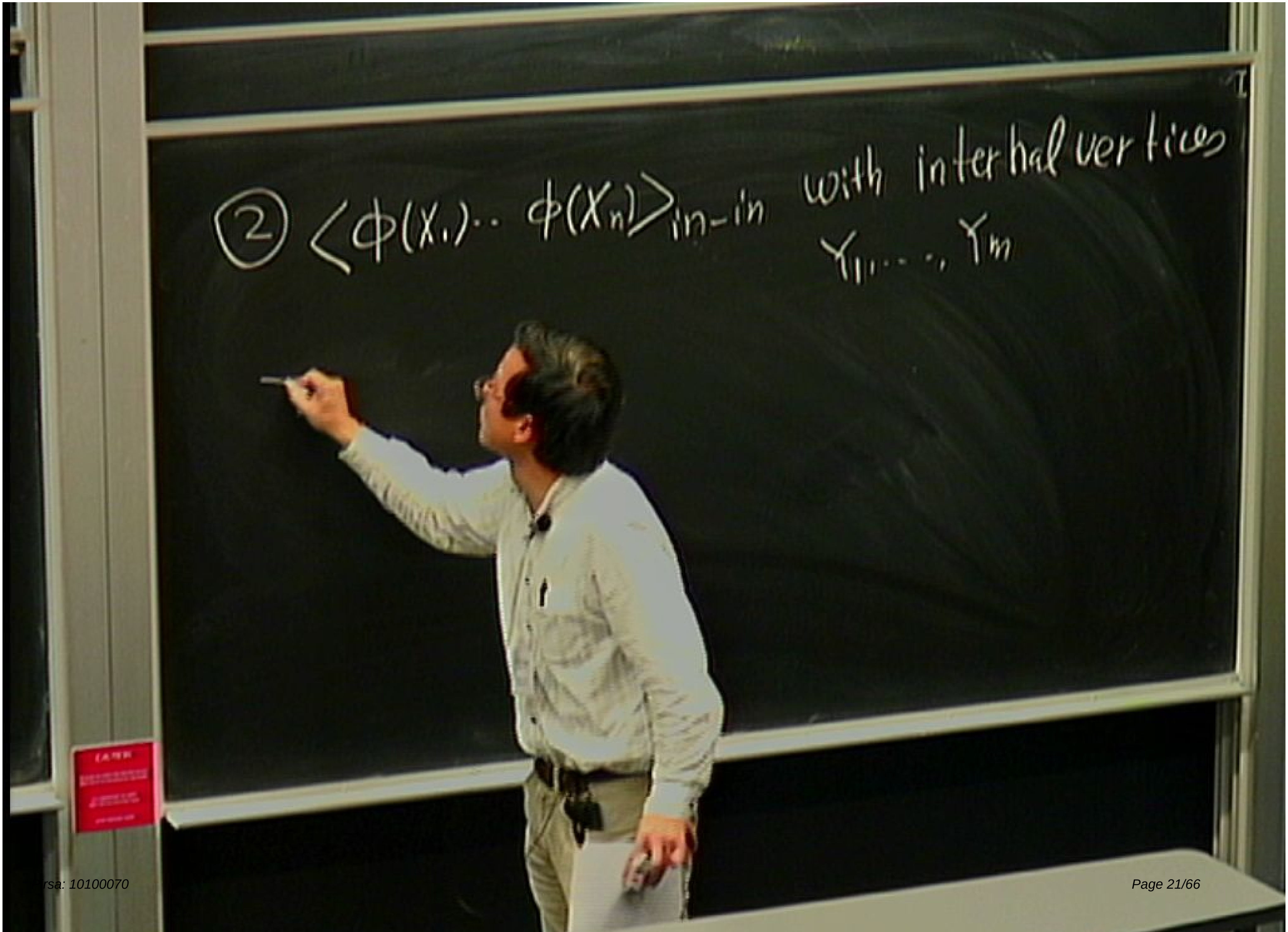

1. Proof in 3
1 Equivalence in static patch
2 In-in Poincaré when all points are
inside a static path
= in
in static
3 In-in
ncaré is analytic in
coor

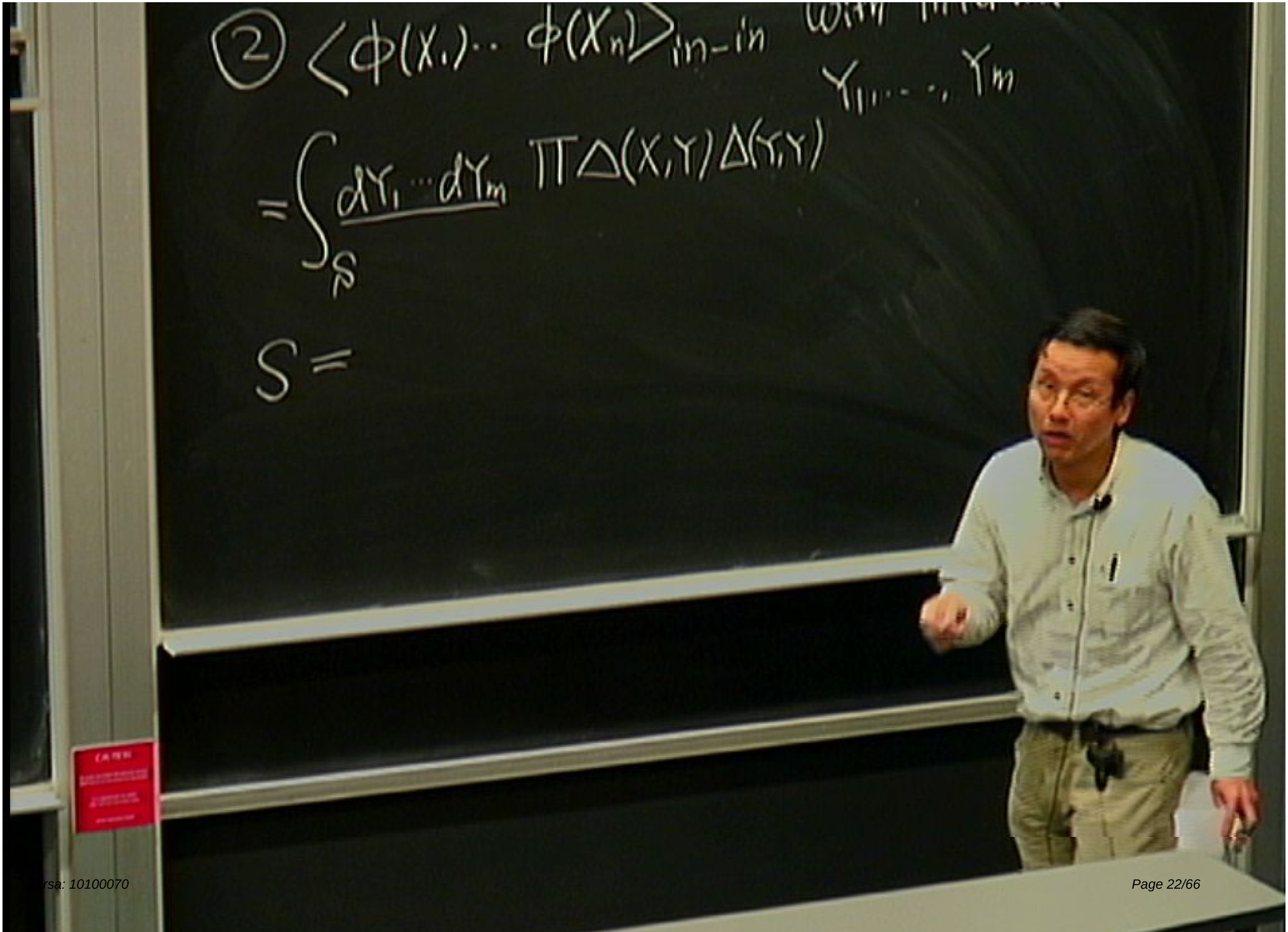
1. Proof in 3 steps

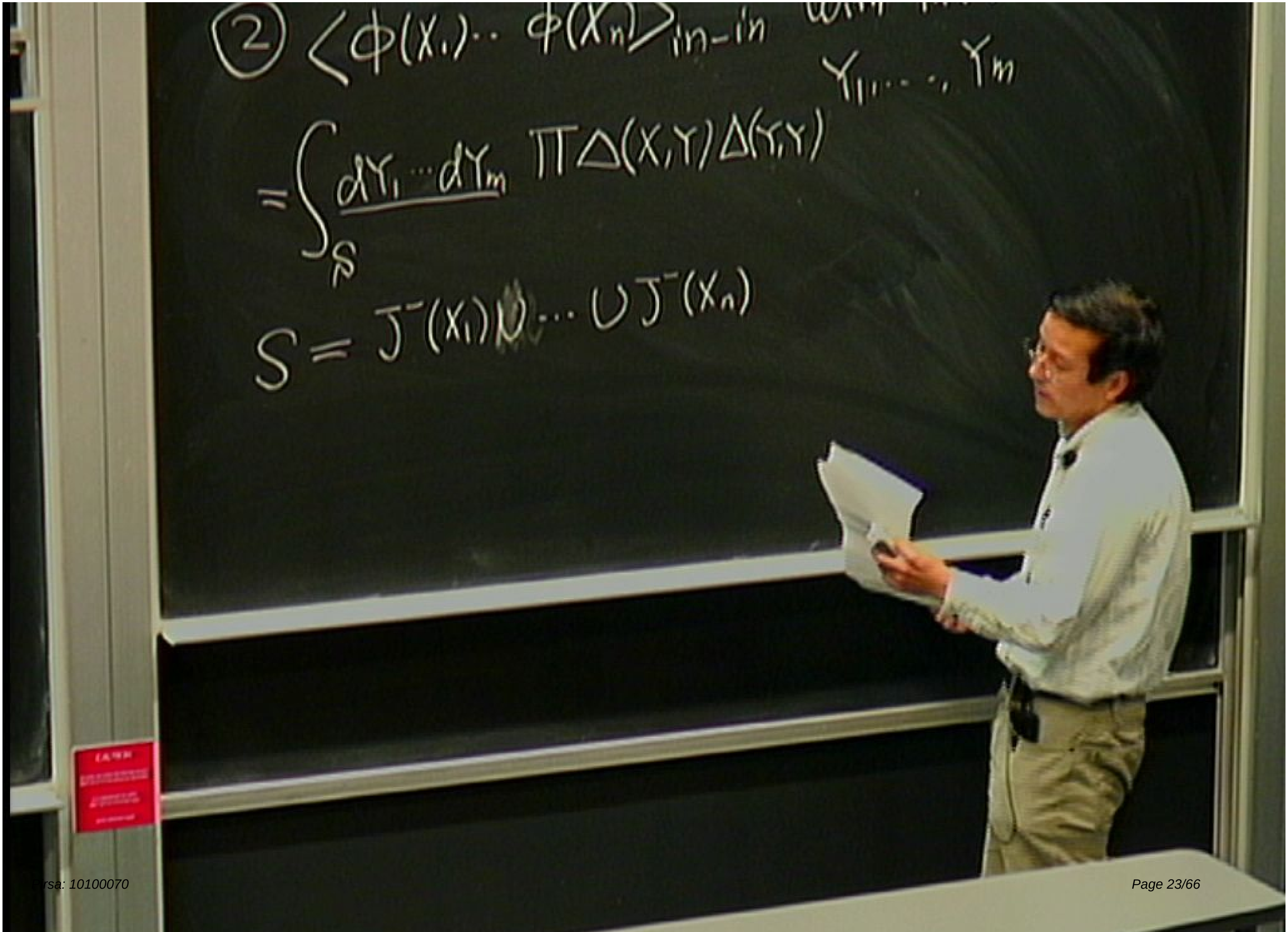
① Equivalence in static patch

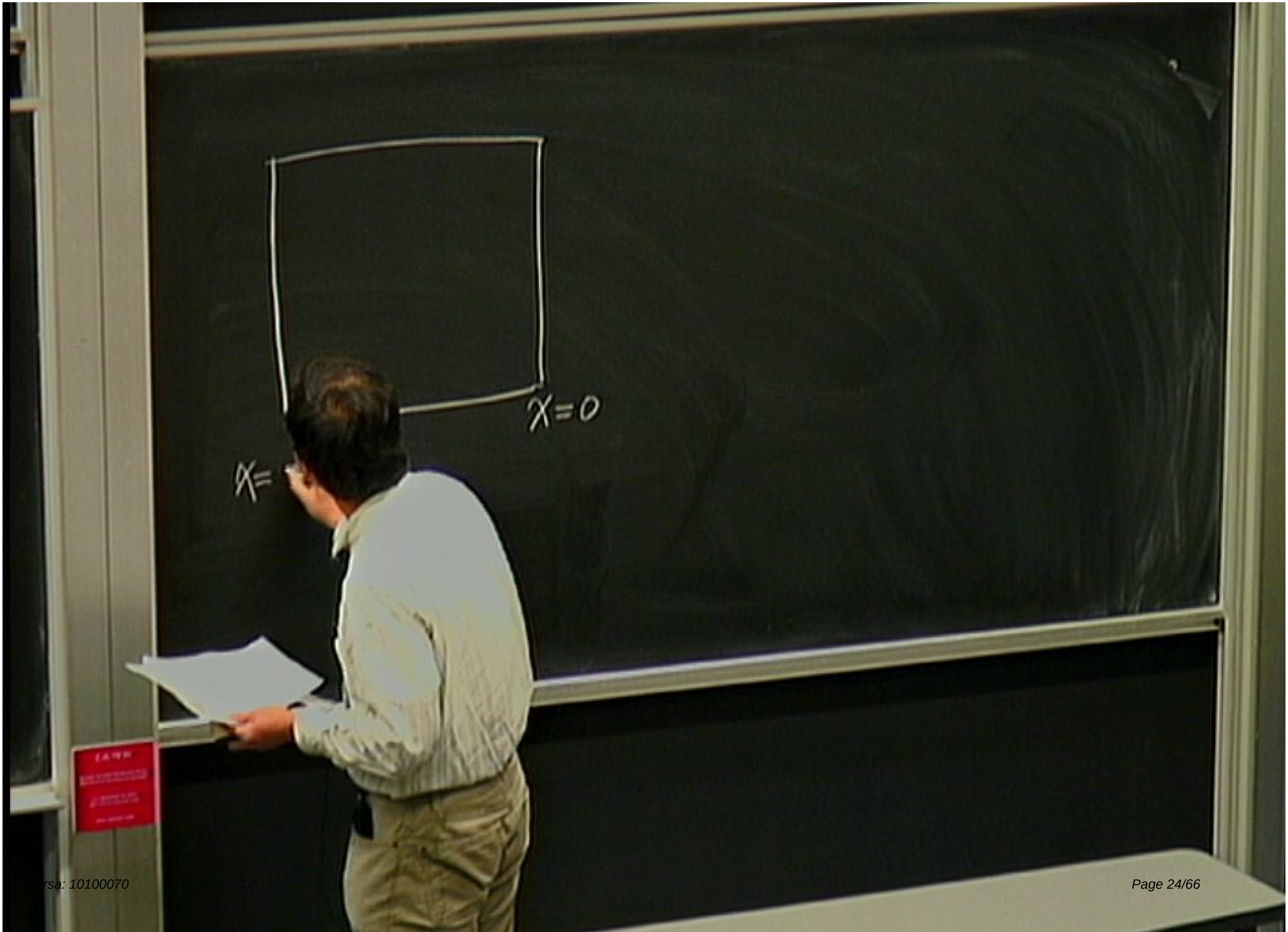
② In-in Poincaré when all points are inside a static path
= in-in in static

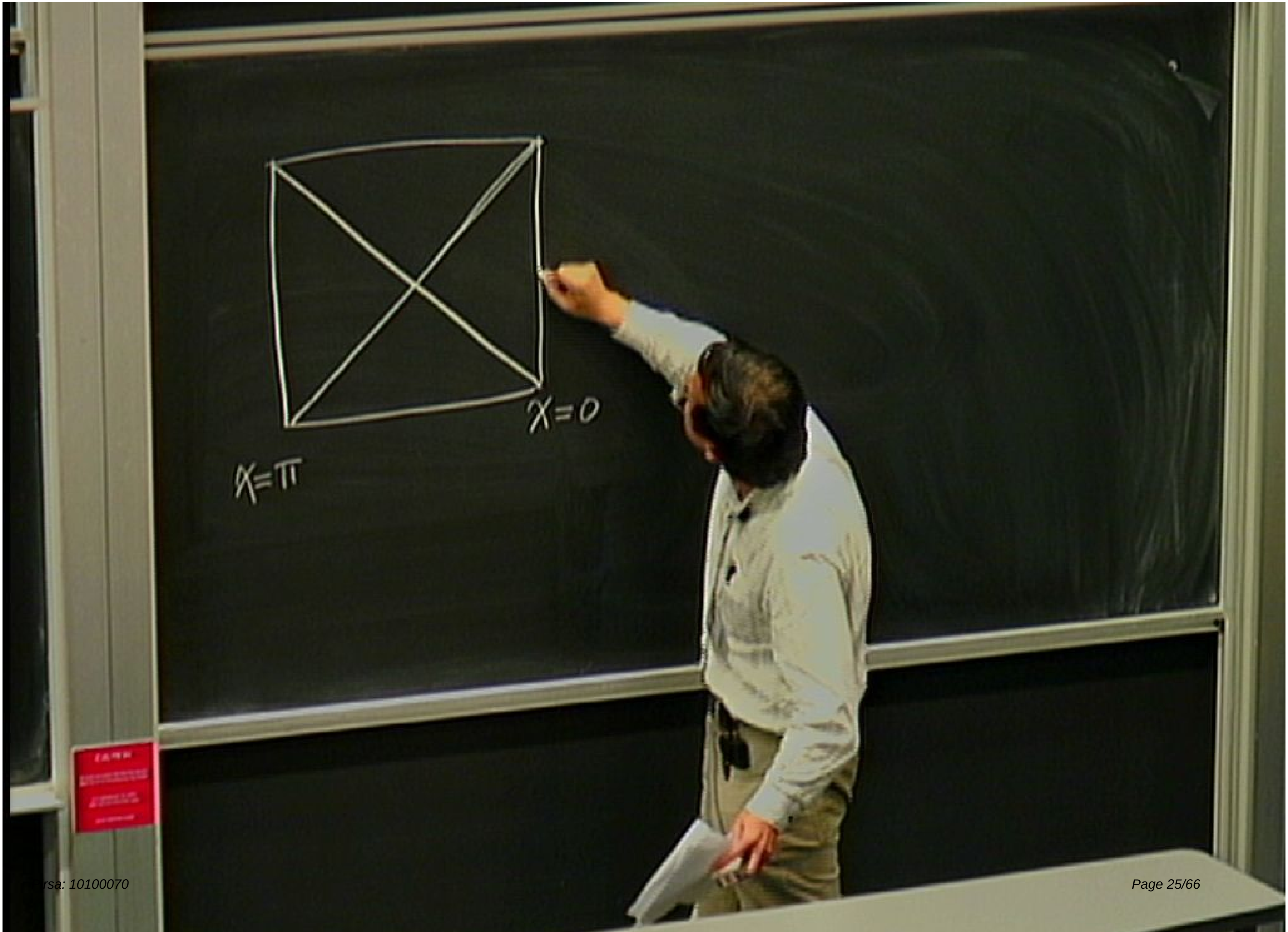
③ In-in Poincaré is analytic in coordinates of points.

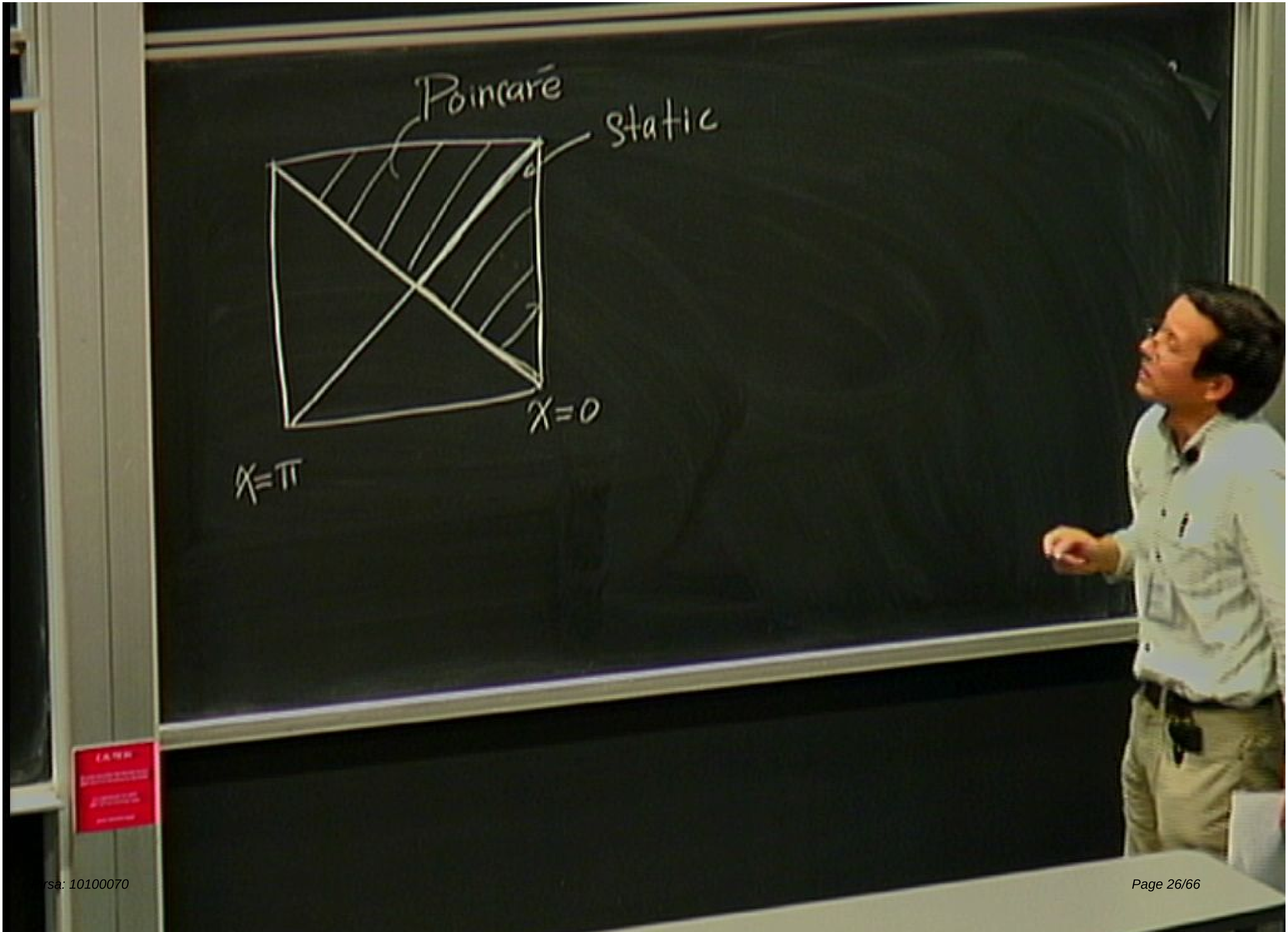

② $\langle \phi(x_1) \cdots \phi(x_n) \rangle_{in-in}$ with internal vertices
$y_1, \ldots, y_m$

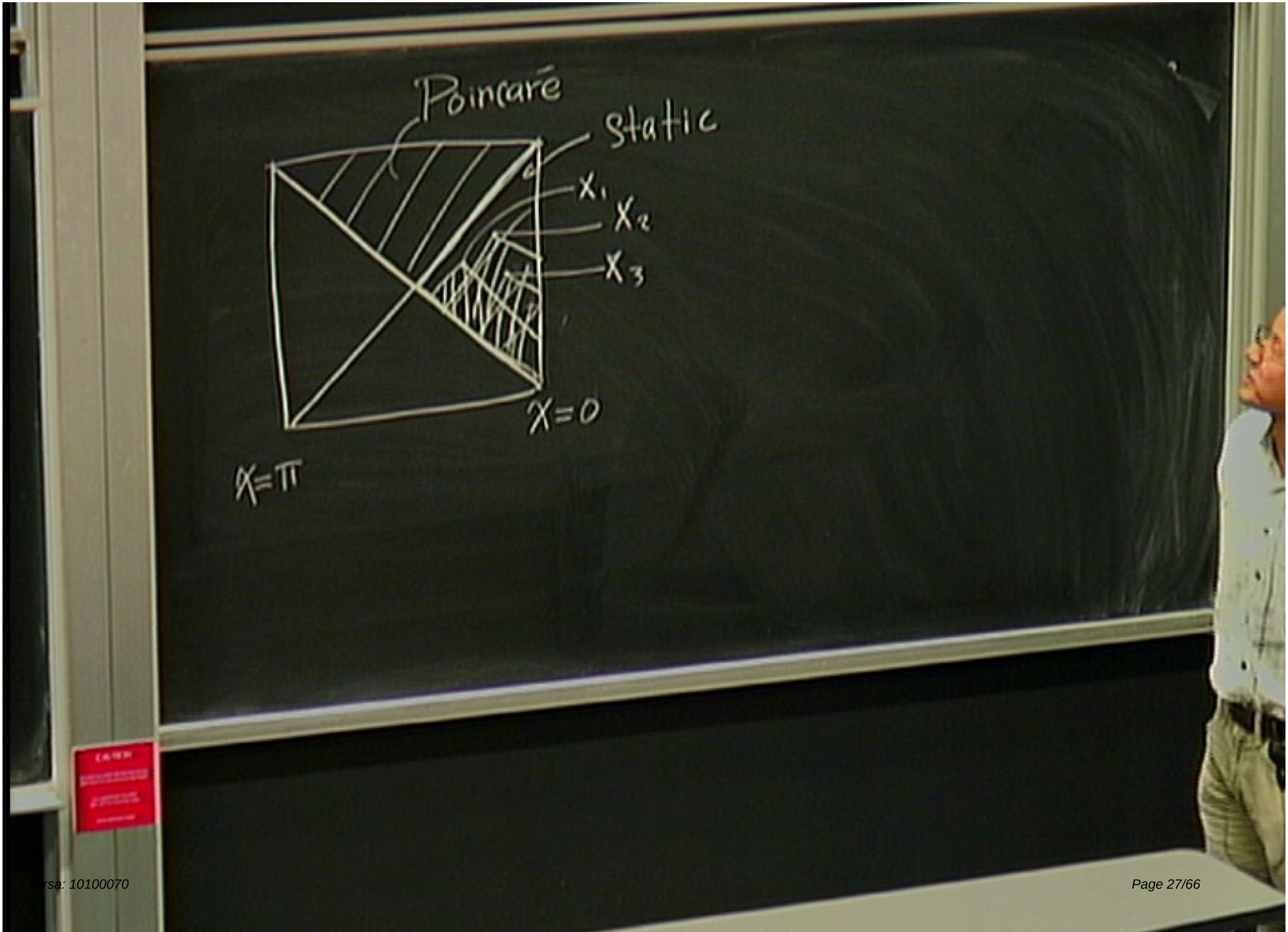

② $\langle \phi(X_1)\cdots\phi(X_n)\rangle_{in-in}$
$Y_1,\ldots,Y_m$
$=\int_S \underline{dY_1\cdots dY_m}\ \prod \Delta(X,Y)\Delta(Y,Y)$
$S=$

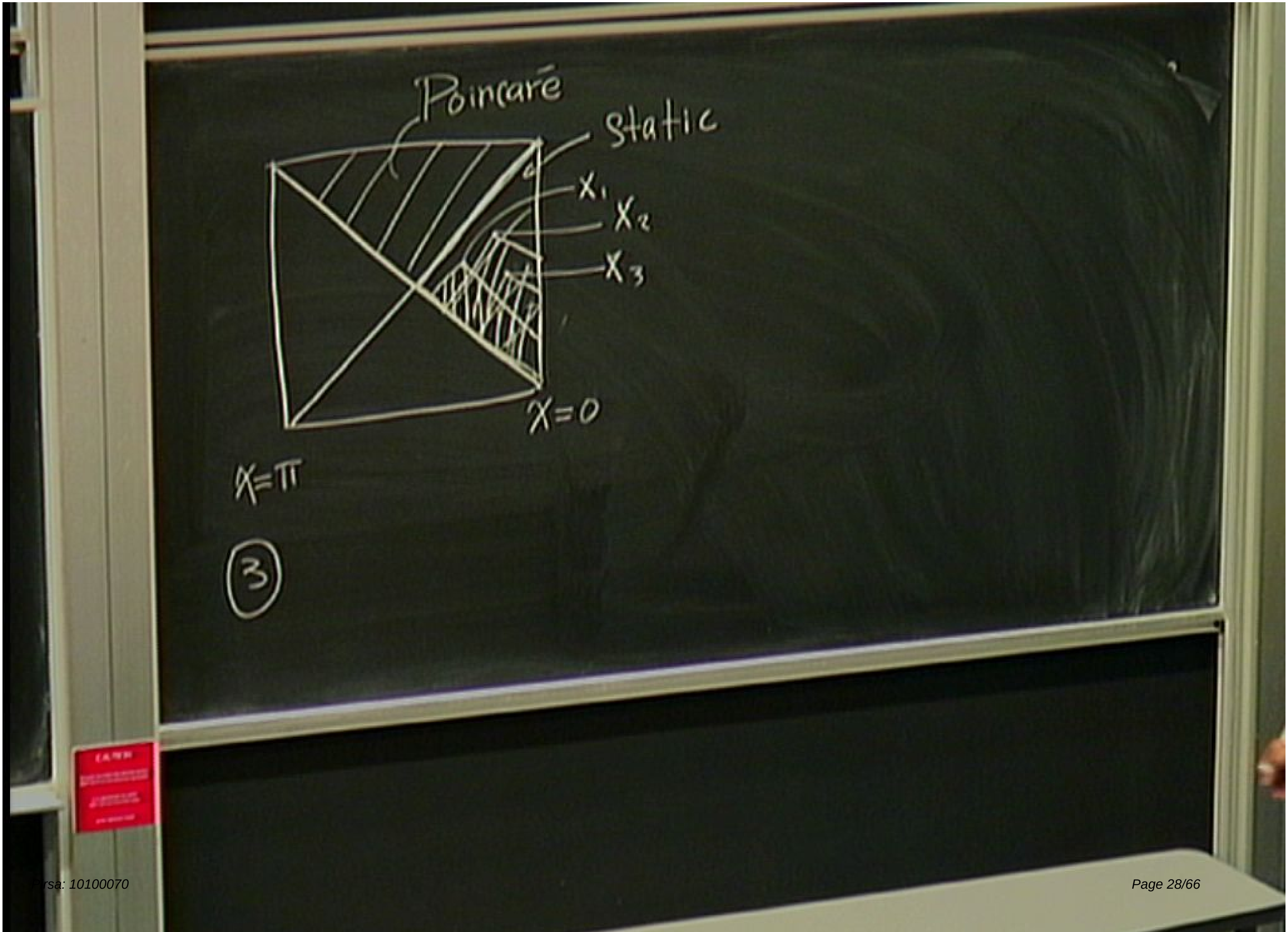

$② \ \langle \phi(X_1)\cdots\phi(X_n)\rangle_{in-in}$
$Y_1, \ldots, Y_m$
$= \int_S dY_1 \cdots dY_m \prod \Delta(X,Y)\Delta(Y,Y)$
$S = J^-(X_1) \cup \cdots \cup J^-(X_n)$

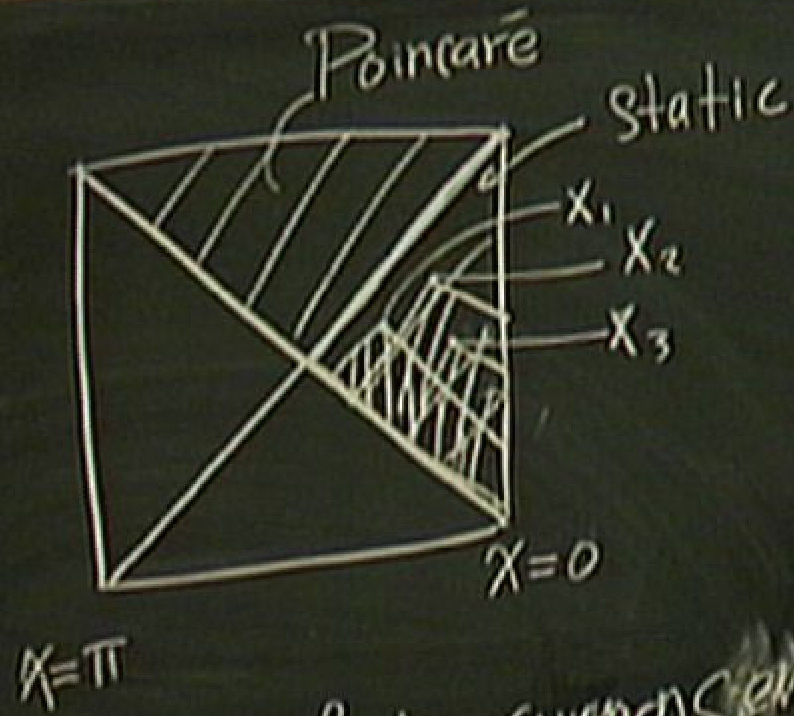

x=0
x=


$x=0$
$x=\pi$


Poincaré
static
χ=0
χ=π


Poincaré
static
X_1
X_2
X_3
$\chi=0$
$\chi=\pi$

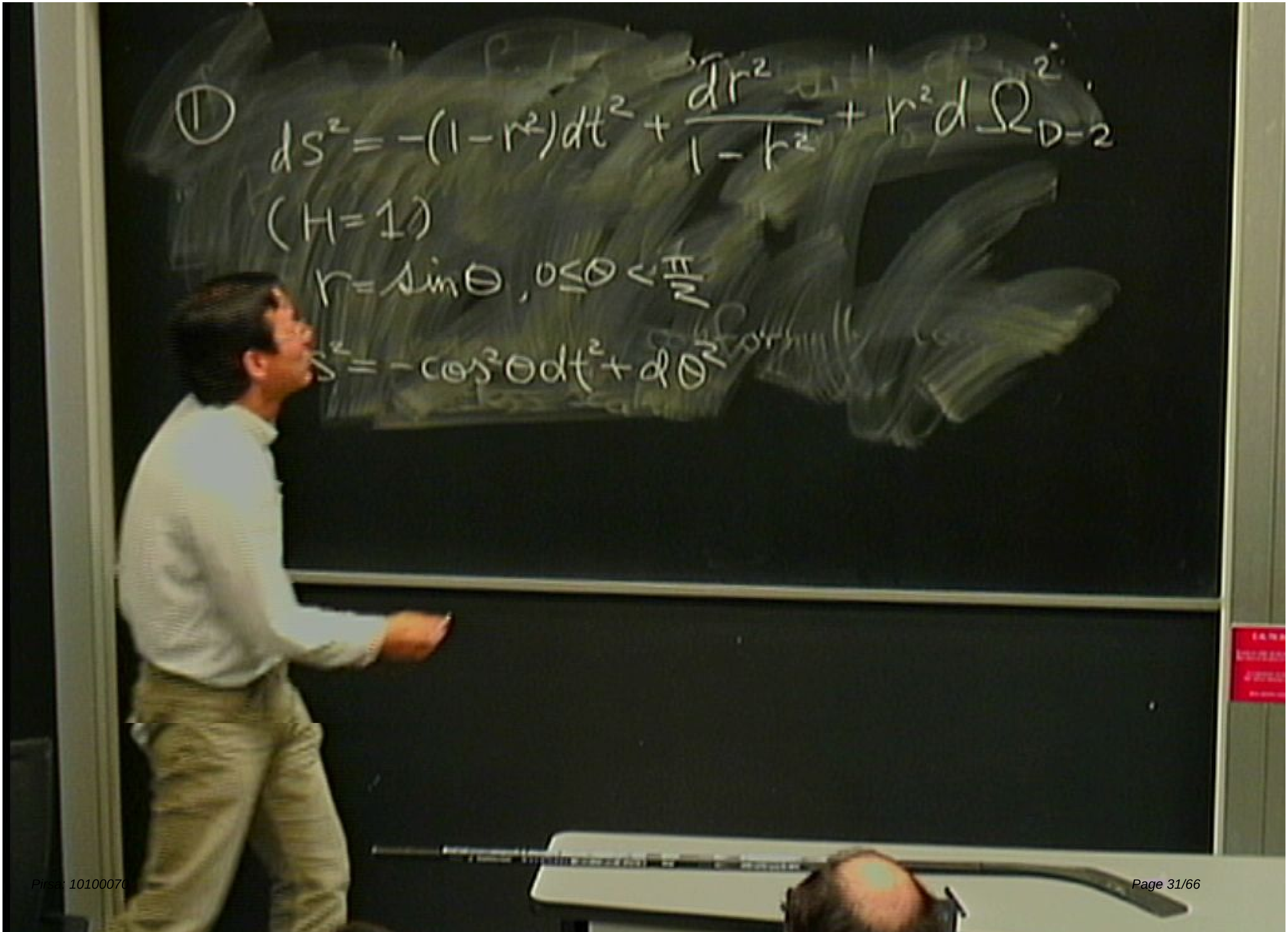

Poincaré
static
X_1
X_2
X_3
$\chi=0$
$\chi=\pi$
3



③ Proof by commonsense.

① $ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega^2_{D-2}$

$(H=1)$


① $ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega^2_{D-2}$
$(H=1)$
$r = \sin\theta, \ 0 \le \theta < \frac{\pi}{2}$
$ds^2 = -\cos^2\theta \, dt^2 + d\theta^2$

① $$ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega^2_{D-2}$$

$(H=1)$

$$r = \sin\theta, \quad 0 \le \theta < \frac{\pi}{2}$$

$$ds^2 = -\cos^2\theta\, dt^2 + d\theta^2 + \sin^2\theta\, d\Omega^2_{D-2}$$

① $$ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_{D-2}^2$$

$(H=1)$

$$r = \sin\theta, \quad 0 \le \theta < \frac{\pi}{2}$$

$$ds^2 = -\cos^2\theta\, dt^2 + d\theta^2 + \sin^2\theta\, d\Omega_{D-2}^2$$

$(t = i\tau$

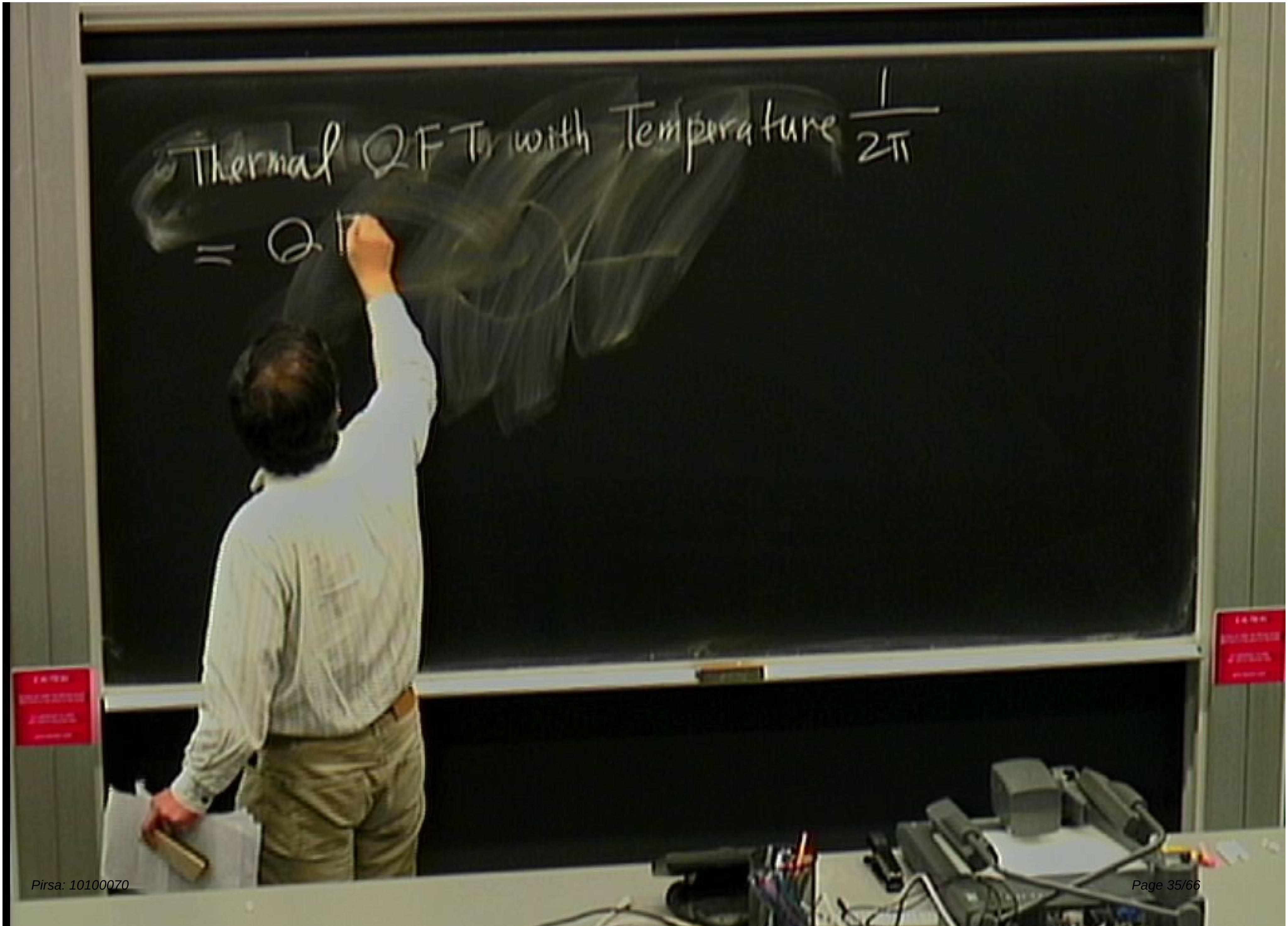
① $ds^2 = -(1-r^2)dt^2 + \frac{dr^2}{1-r^2} + r^2 d\Omega_{D-2}^2$

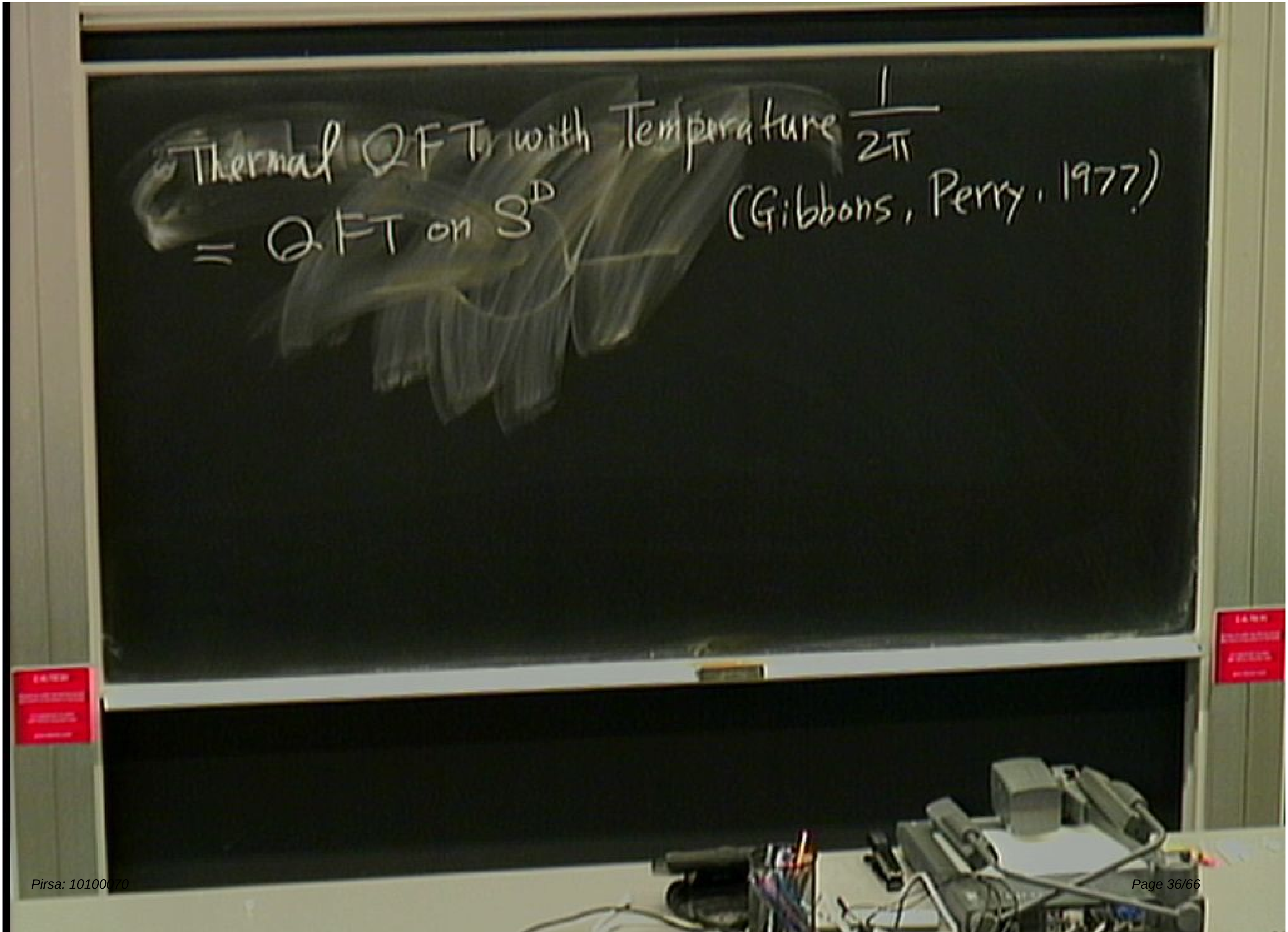
$(H=1)$

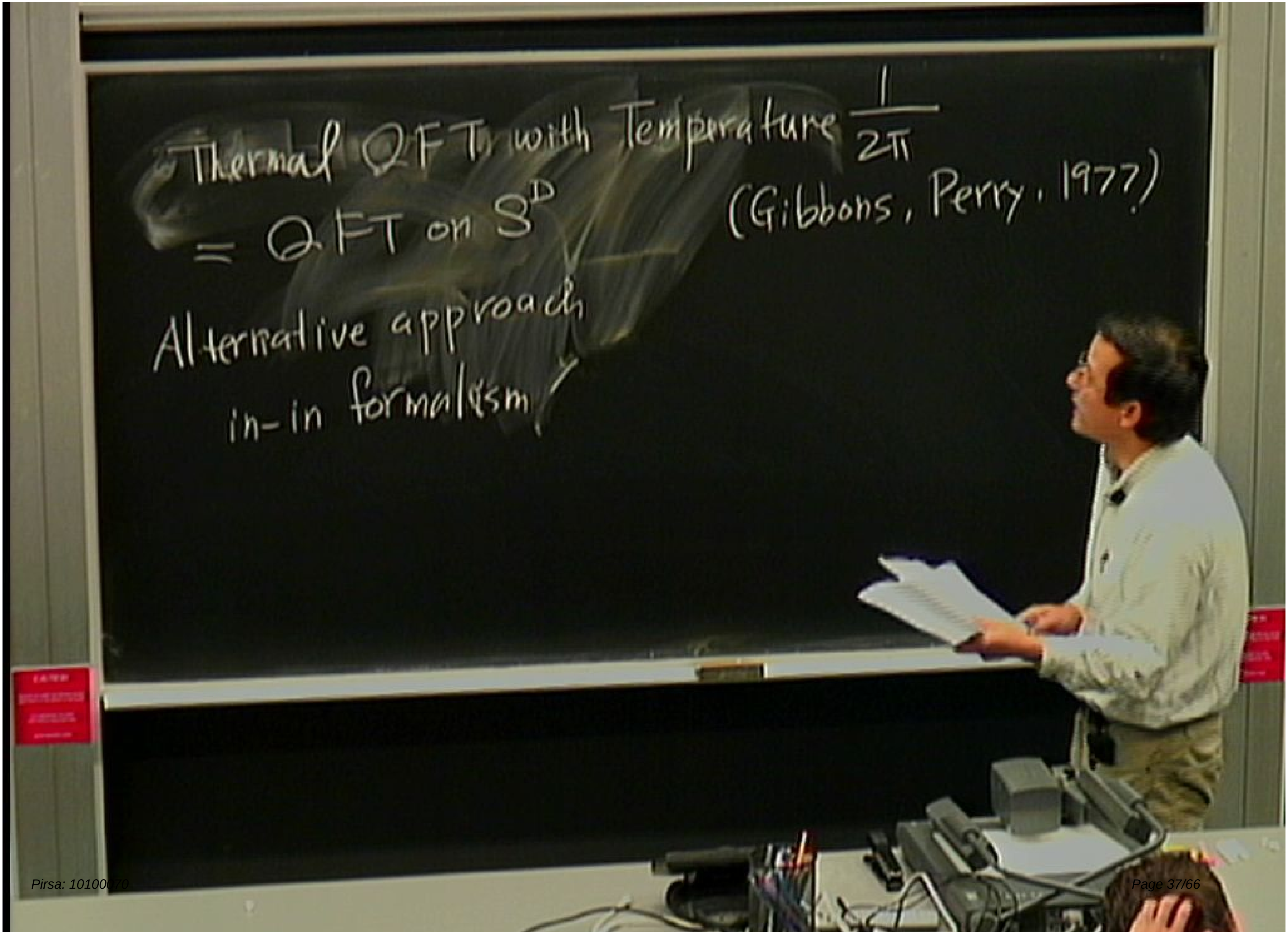
$r = \sin\theta, \; 0 \le \theta < \frac{\pi}{2}$

$ds^2 = +\cos^2\theta\, dt^2 + d\theta^2 + \sin^2\theta\, d\Omega_{D-2}^2$

$(t = i\tau, \; \tau \sim \tau + 2\pi) \longrightarrow S^D$


Thermal QFT with Temprature 1/2π
= Q


Thermal QFT with Temperature $\frac{1}{2\pi}$
$= $ QFT on S^D
(Gibbons, Perry, 1977)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(Gibbons, Perry, 1977)
Alternative approach
in-in formalism

Thermal QFT with Temperature $\frac{1}{2\pi}$

$= $ QFT on S^D (Gibbons, Perry, 1977)

Alternative approach (Landsman, Van Weert, 1987)

in-in formalism

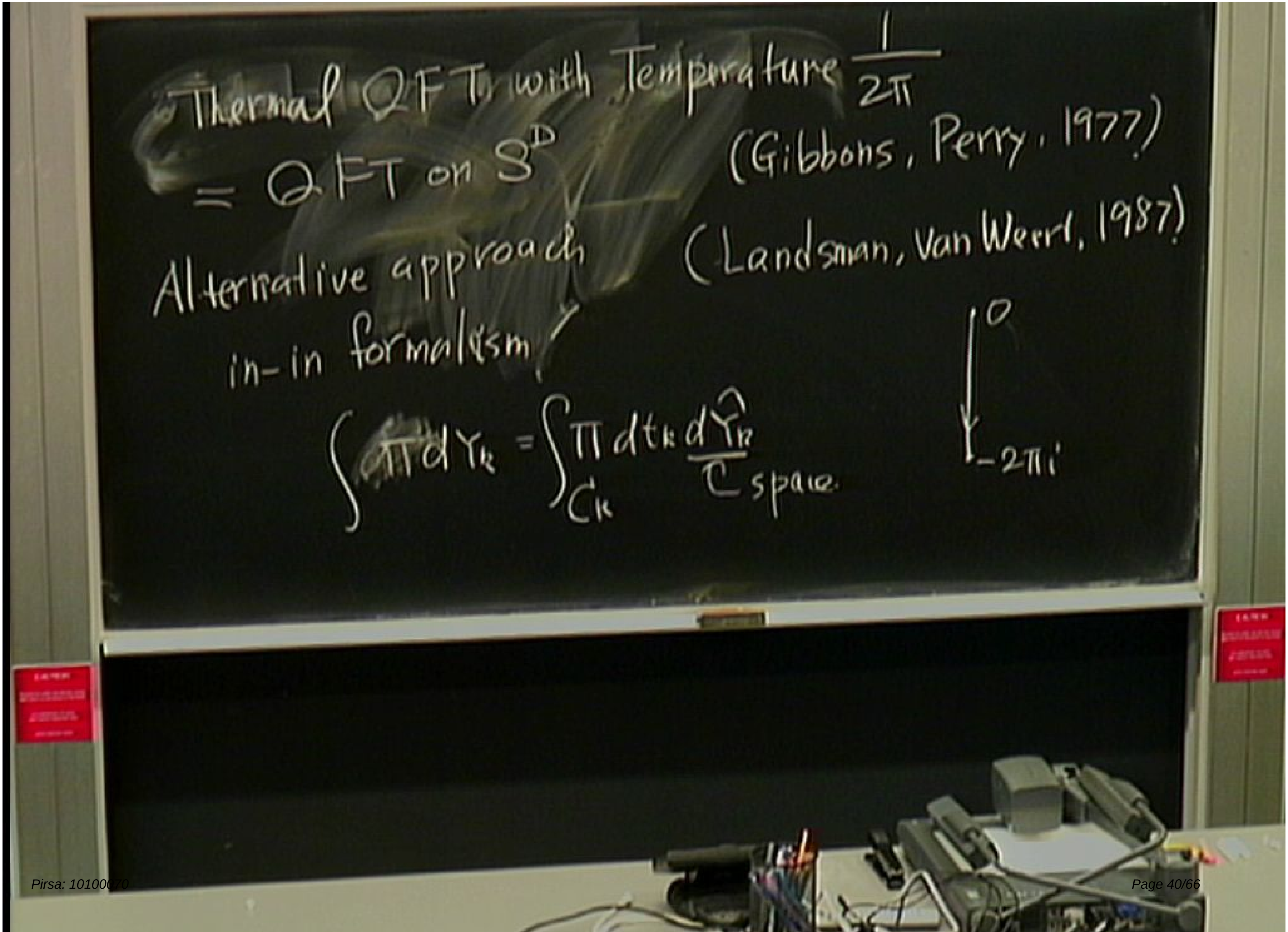
Thermal QFT with Temperature $\frac{1}{2\pi}$

$=$ QFT on S^D (Gibbons, Perry, 1977)

Alternative approach (Landsman, van Weert, 1987)

in-in formalism

$$\int \prod dY_k = \int \prod dt_k \frac{d\hat{Y}_k}{\mathbb{C}_{\text{space}}}$$



Thermal QFT with Temperature $\frac{1}{2\pi}$
$= QFT$ on S^D
(Gibbons, Perry, 1977)
Alternative approach
in-in formalism
(Landsman, van Weert, 1987)
$\int \prod d\gamma_k = \int_{C_k} \prod dt_k \frac{d\hat{\gamma}_k}{C_{space}}$
$\int_{-2\pi i}^{0}$

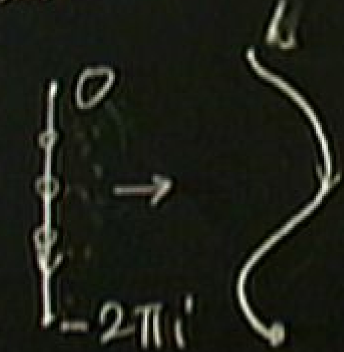
Thermal QFT with Temperature $\frac{1}{2\pi}$

$=$ QFT on S^D (Gibbons, Perry, 1977)

Alternative approach (Landsman, van Weert, 1987)

in-in formalism

$Y_k = (t_k, \hat{Y}_k)$



$$\int \prod dY_k = \int_{C_k} \prod dt_k \, \underbrace{d\hat{Y}_k}_{\mathcal{C}\text{ space}}$$

Thermal QFT with Temperature $\frac{1}{2\pi}$

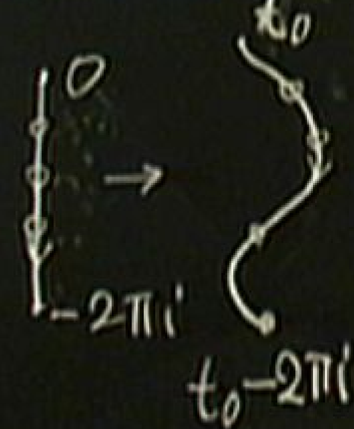
$= $ QFT on S^D (Gibbons, Perry, 1977)

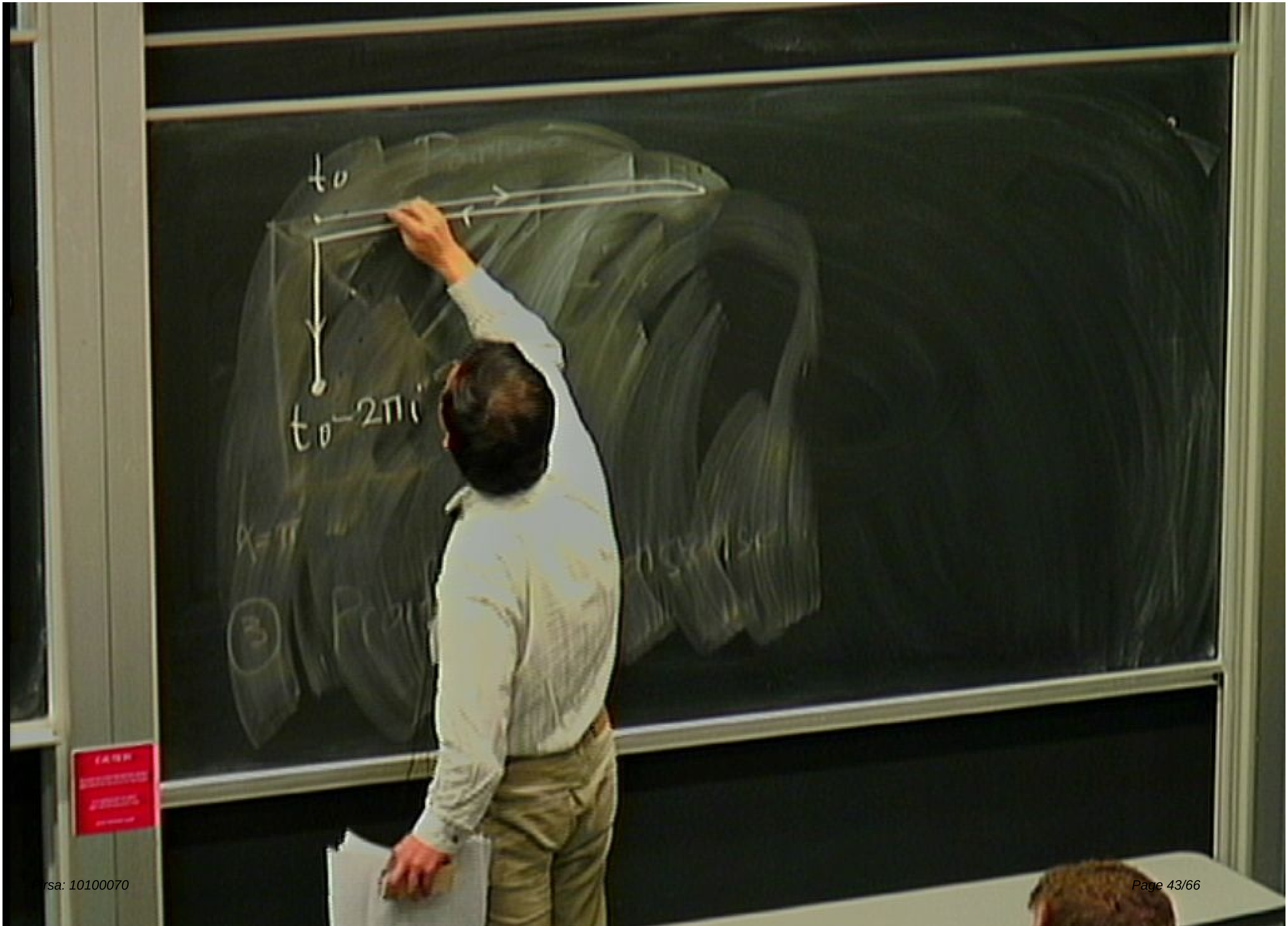
Alternative approach (Landsman, van Weert, 1987)

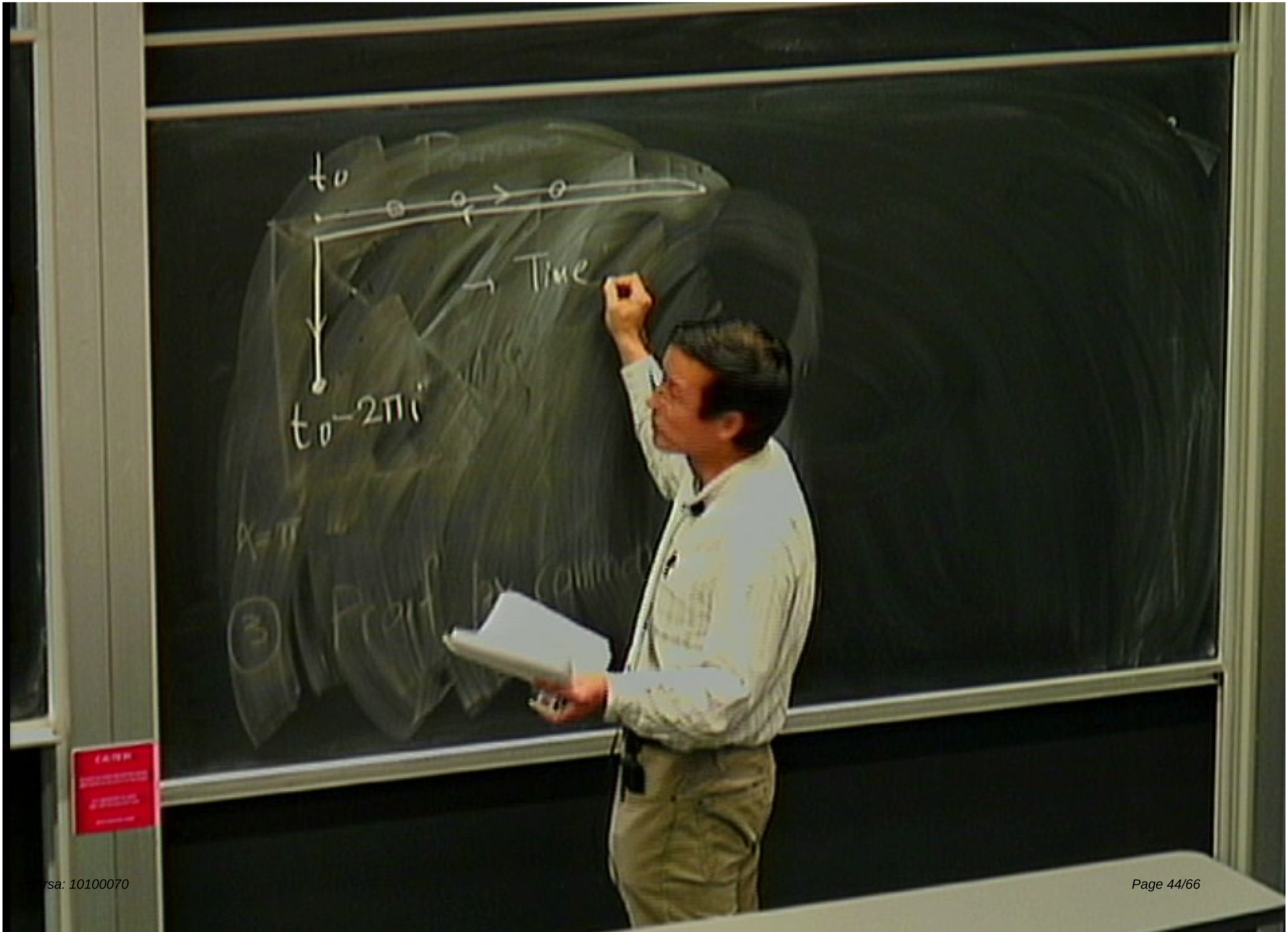
in-in formalism

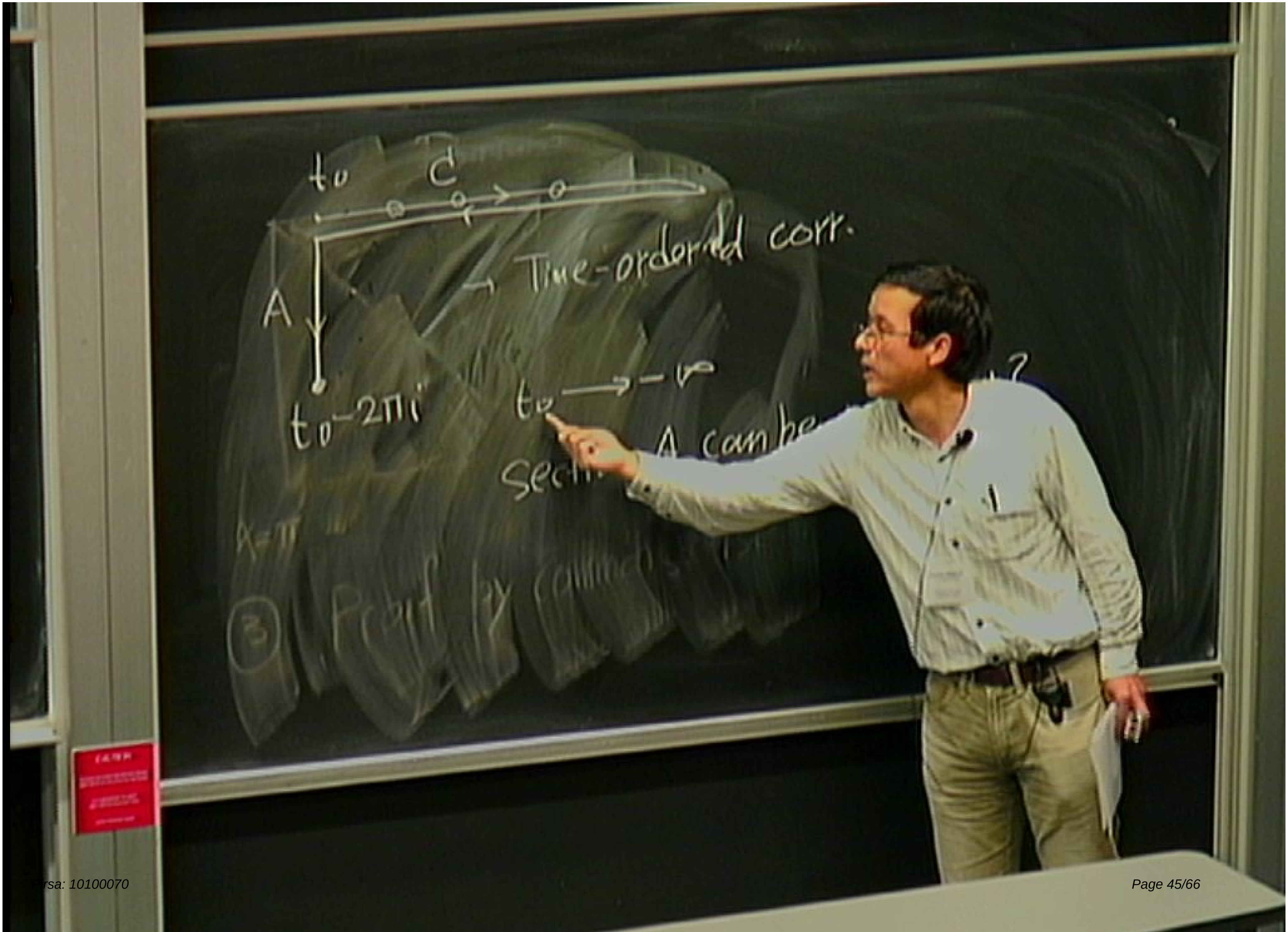
$\gamma_k = (t_k, \hat{\gamma}_k)$

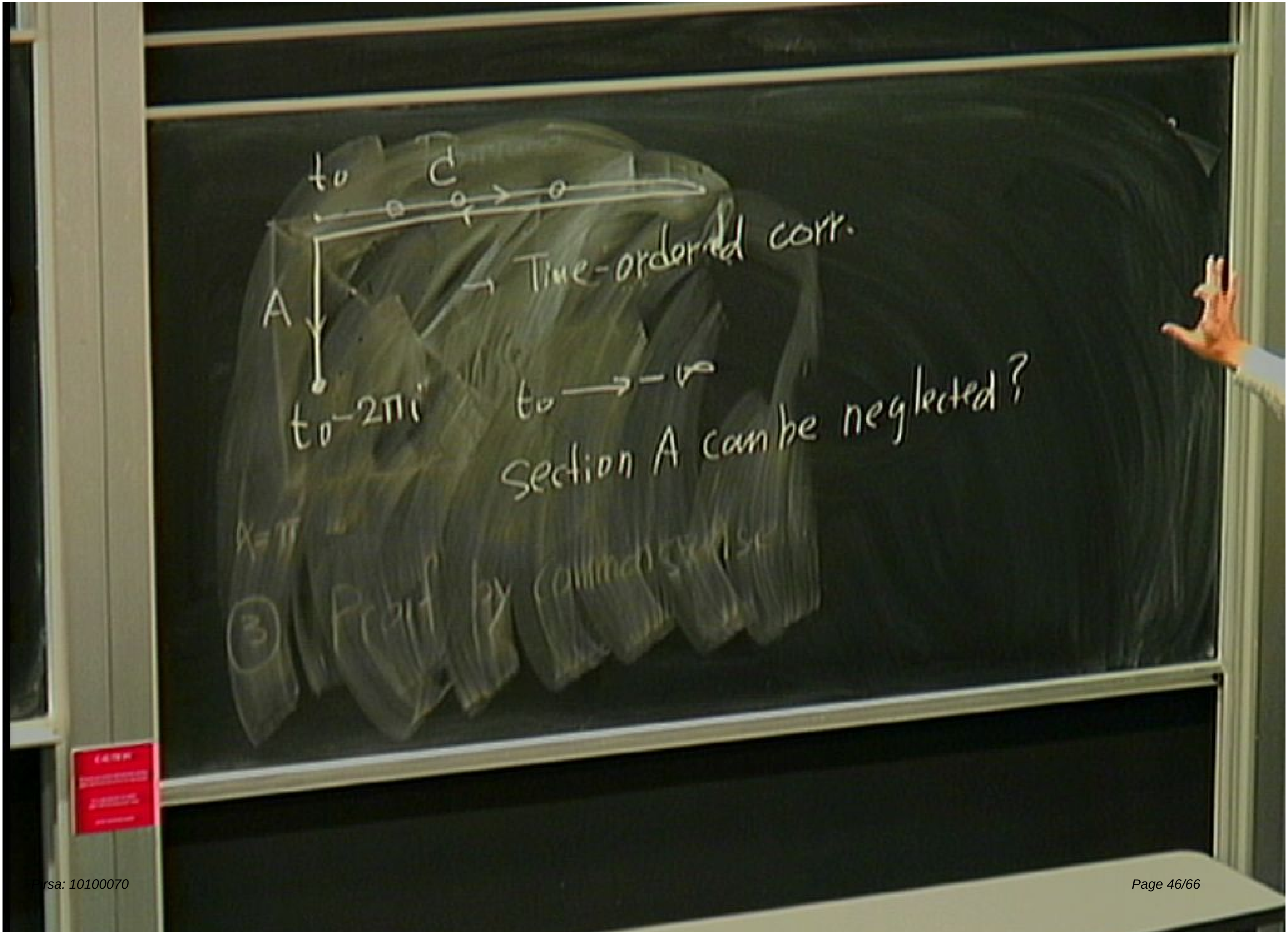
$$\int \prod d\gamma_k = \int_{C_k} \prod dt_k \frac{d\hat{\gamma}_k}{\mathbb{C}\,\text{space}}$$

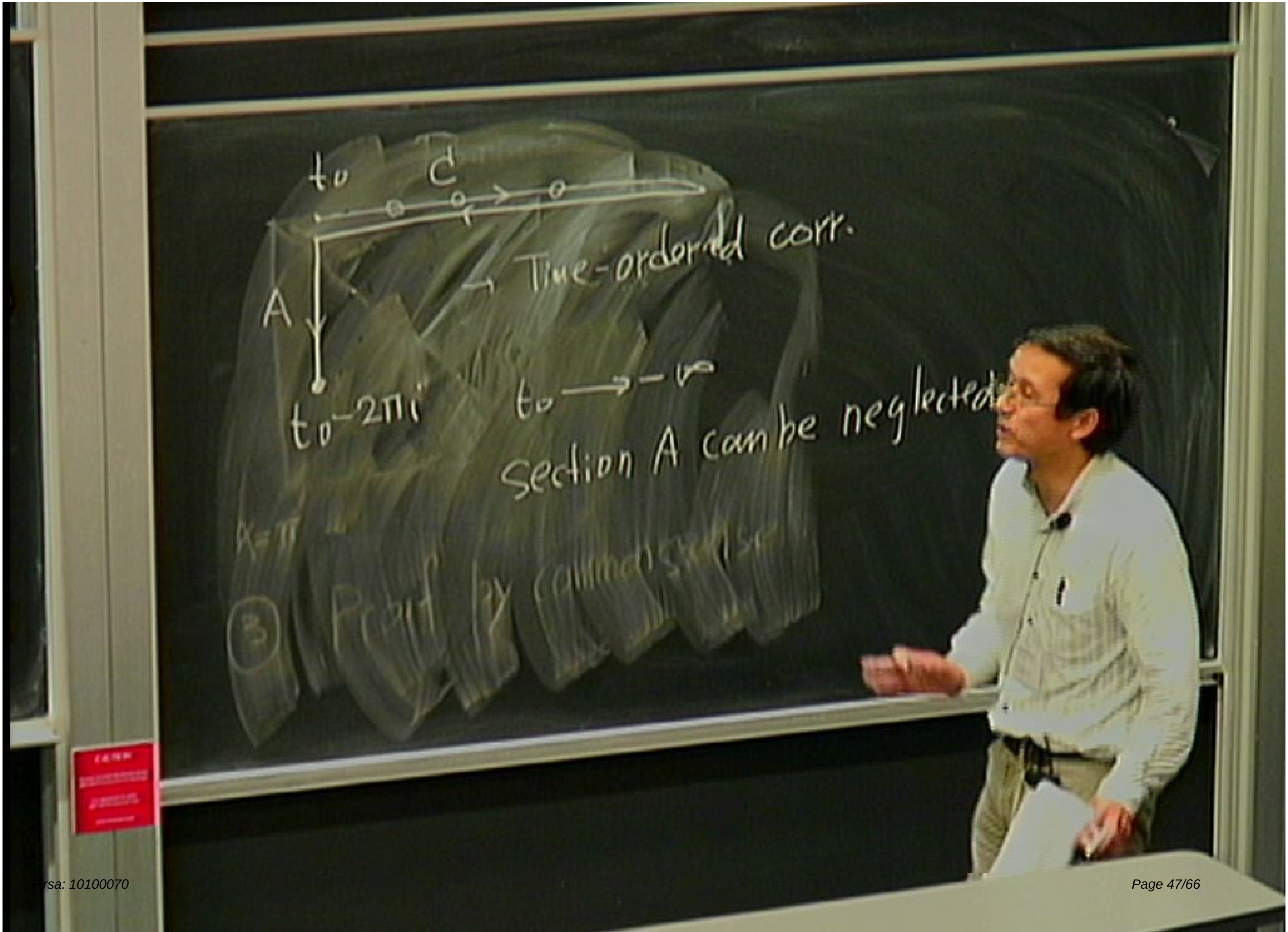


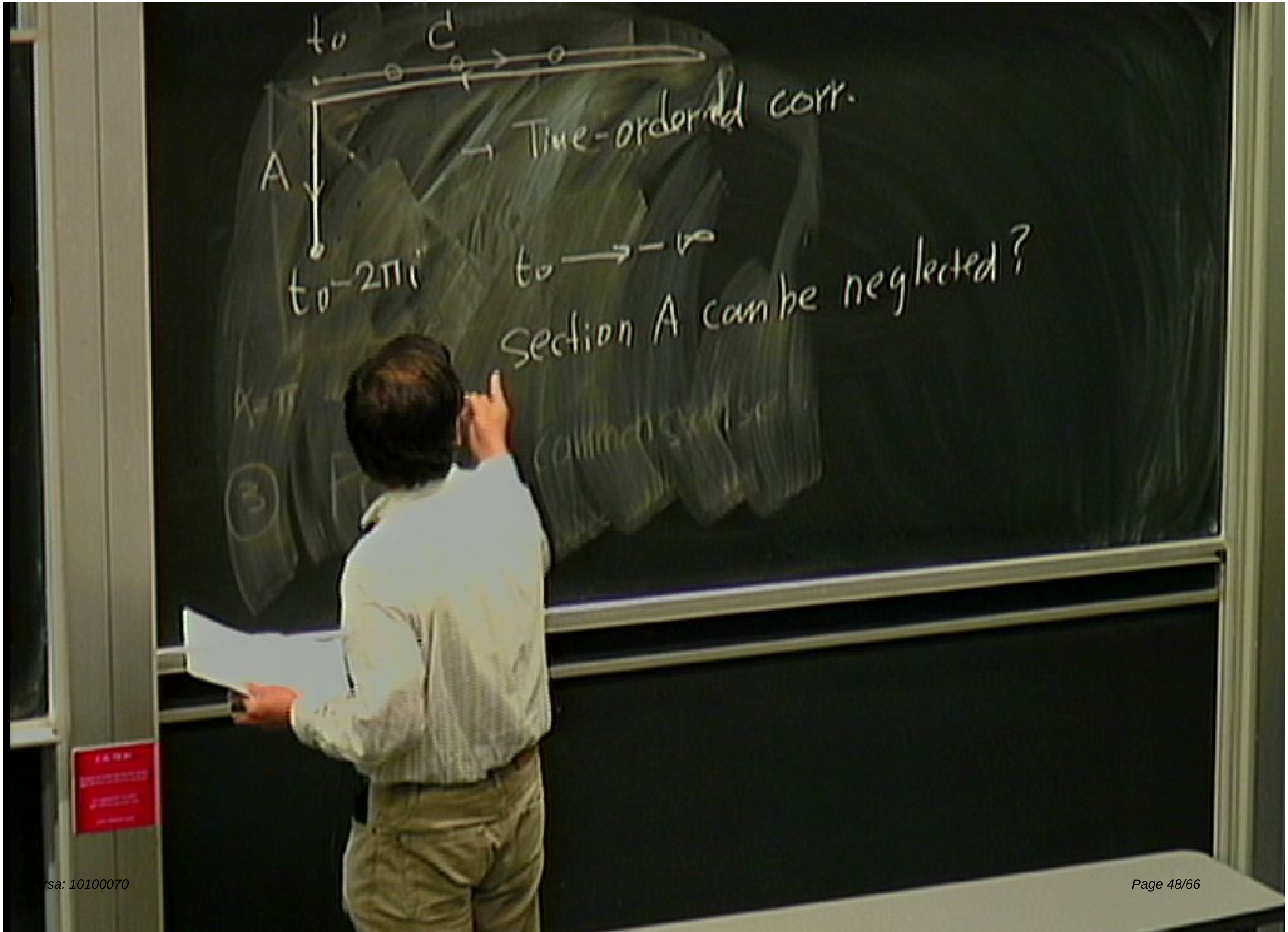

t_0
$t_0 - 2\pi i$

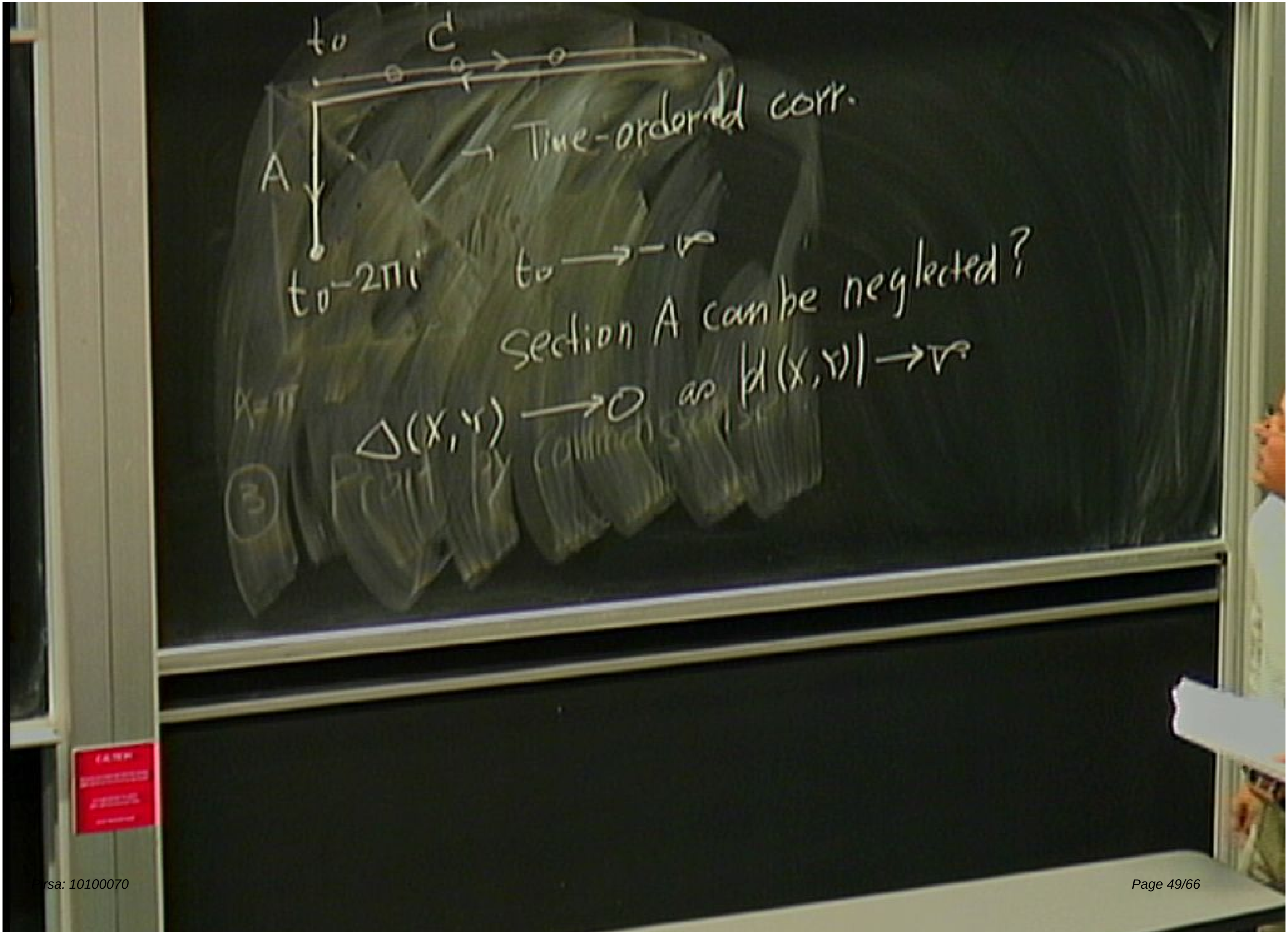

t0
Time
t0−2πi

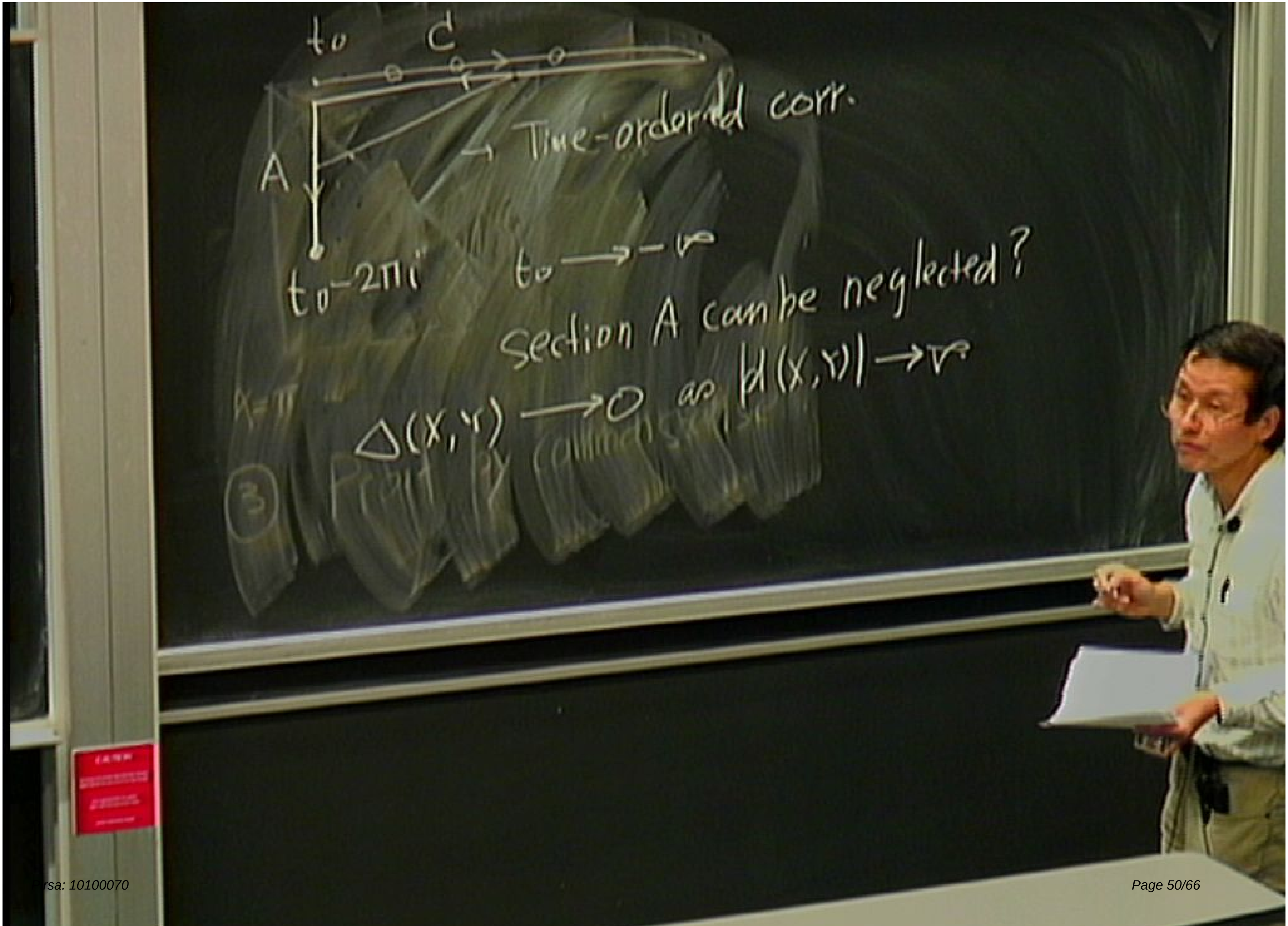

t0
C
A
Time-ordered corr.
t0-2πi

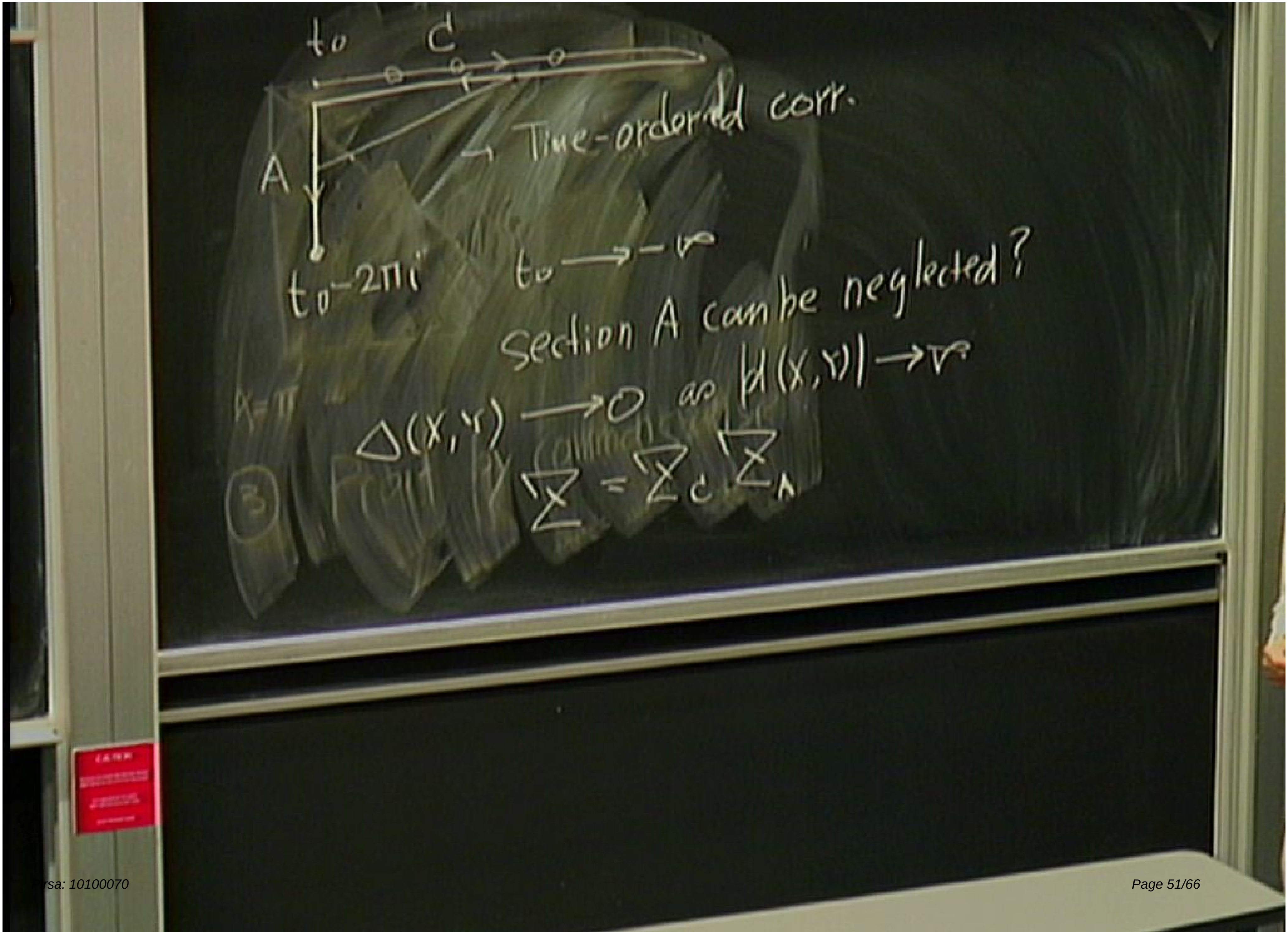

t0
C
Time-ordered corr.
A
t0−2πi
t0 → −∞
Section A can be neglected?


t0
C
A
Time-ordered corr.
t0-2πi
t0 → -∞
section A can be neglected

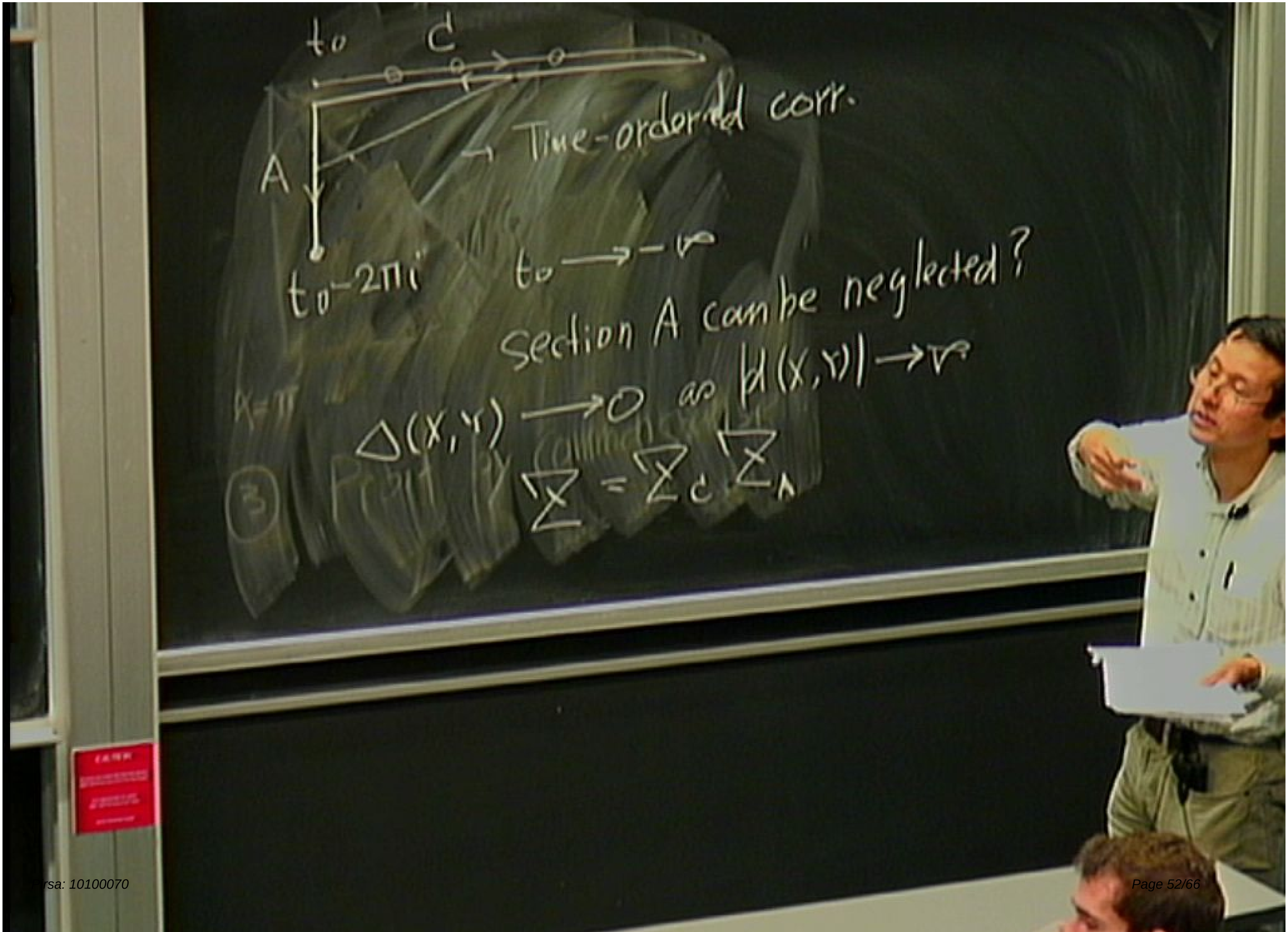

t0
C
Time-ordered corr.
A
t0-2πi
t0 → -∞
Section A can be neglected?


t0
C
Time-ordered corr.
A
t0-2πi
t0 → -∞
Section A can be neglected?
Δ(x,y) → 0 as |d(x,y)| → ∞


t0
C
Time-ordered corr.
A
t0 − 2πi
t0 → −∞
section A can be neglected?
Δ(x,τ) → 0 as |x(x,τ)| → ∞

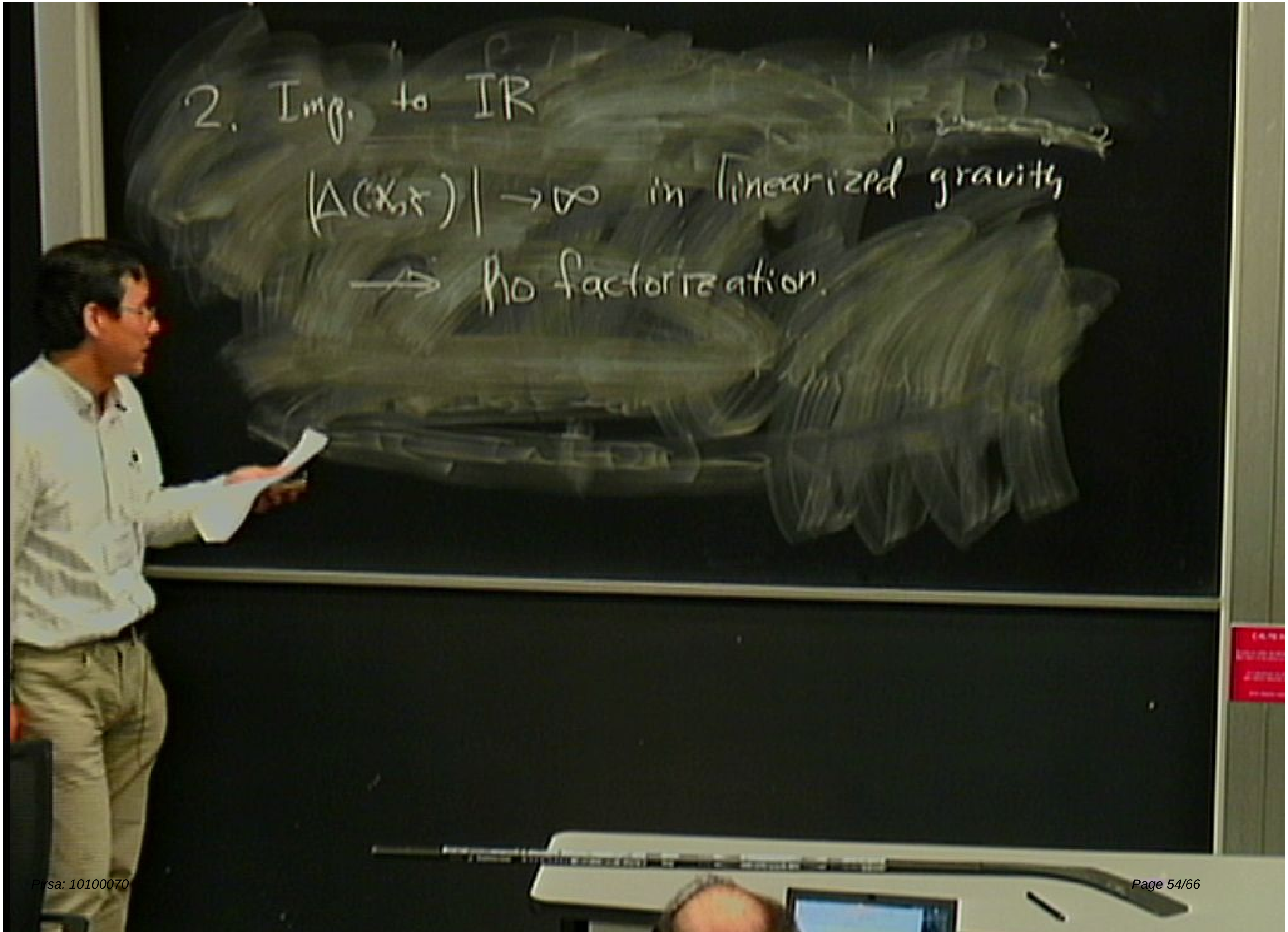


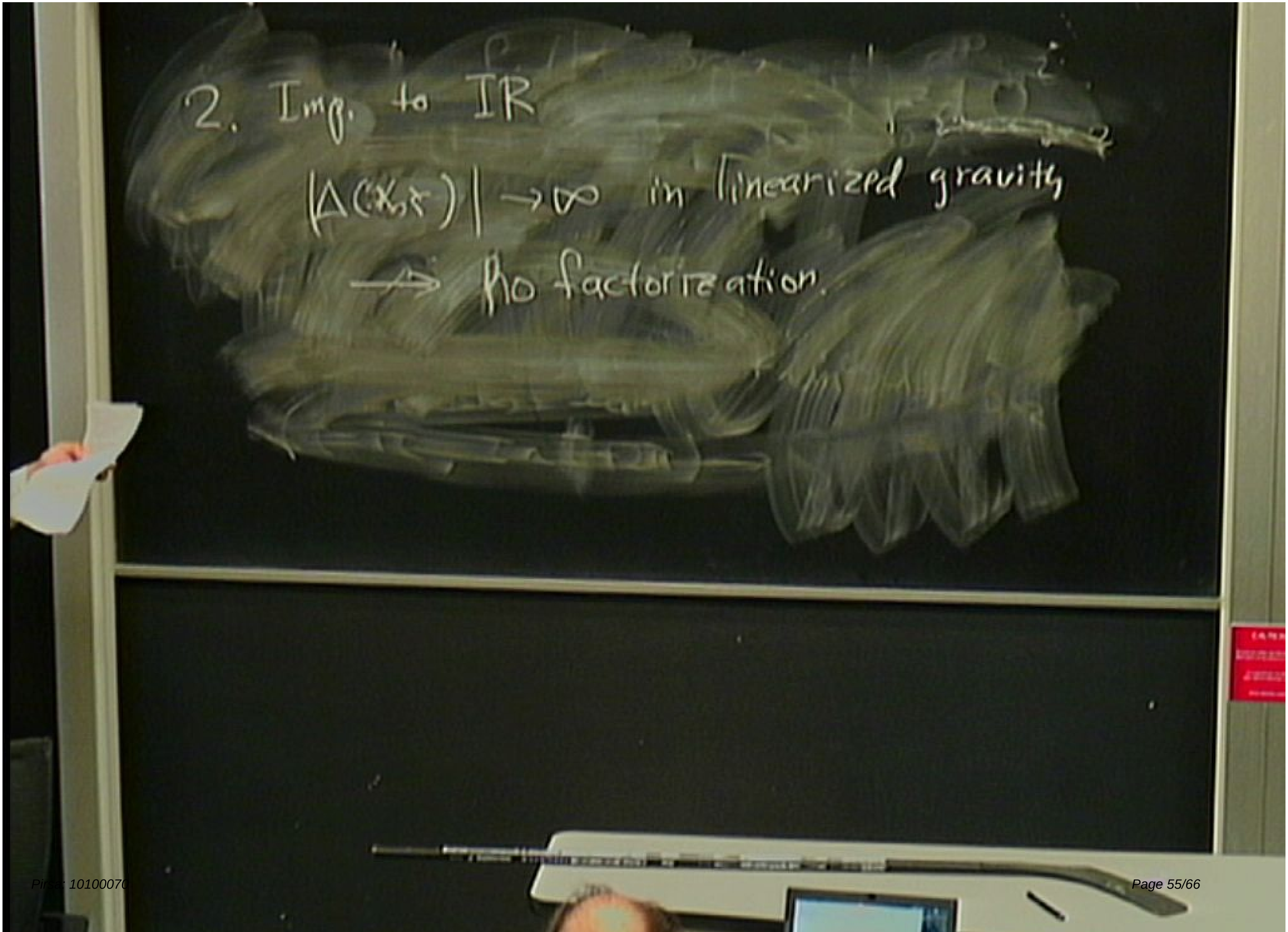
t_0
C
A
Time-ordered corr.
$t_0 - 2\pi i$
$t_0 \to -\infty$
Section A can be neglected?
$\Delta(x,y) \to 0$ as $|d(x,y)| \to \infty$
$\Sigma = \Sigma_C \Sigma_A$

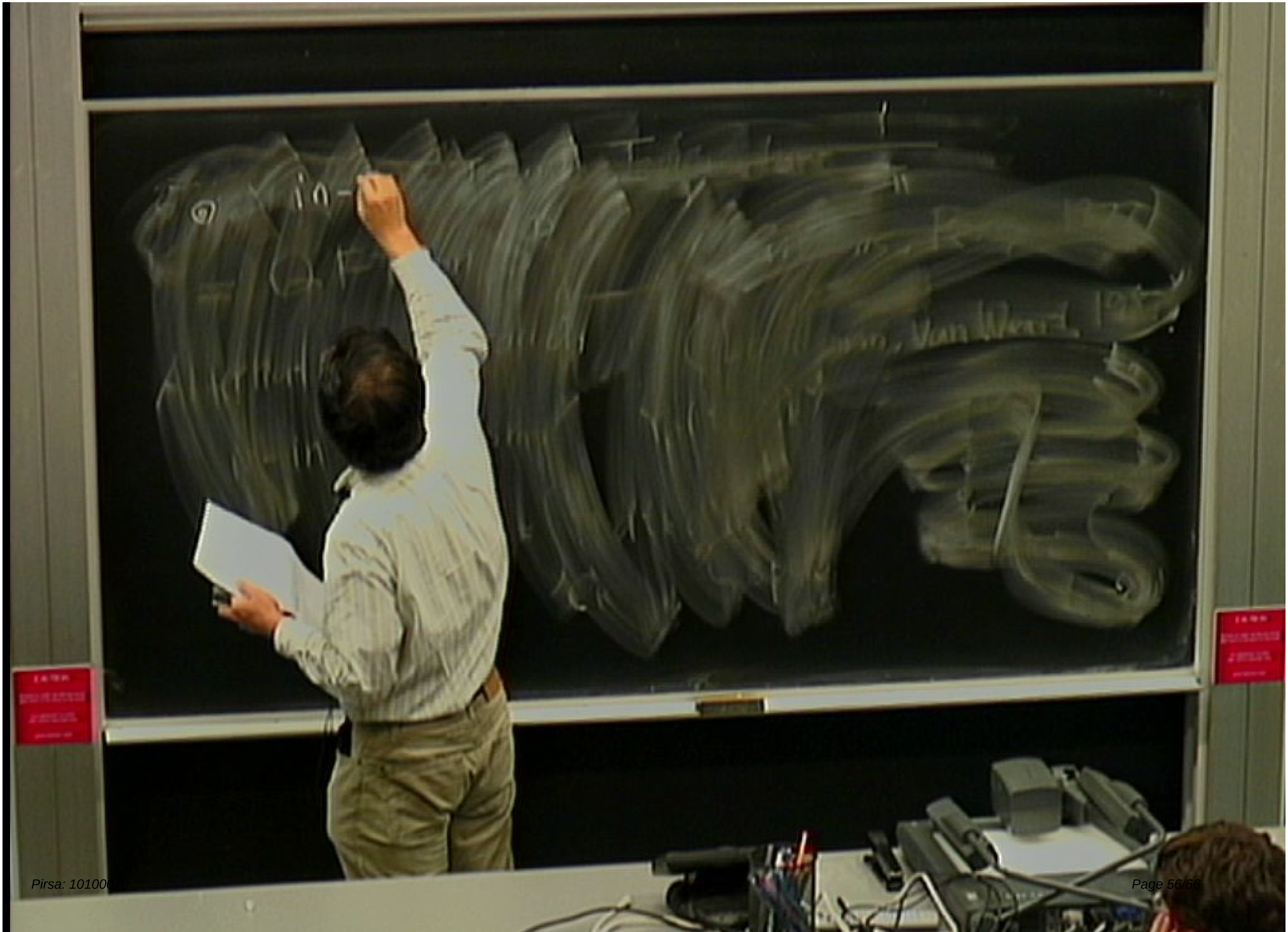

t0
C
Time-ordered corr.
A
t0 - 2πi
t0 → -∞
section A can be neglected?
Δ(x,τ) → 0 as |d(x,τ)| → ∞
Σ = ΣC ΣA

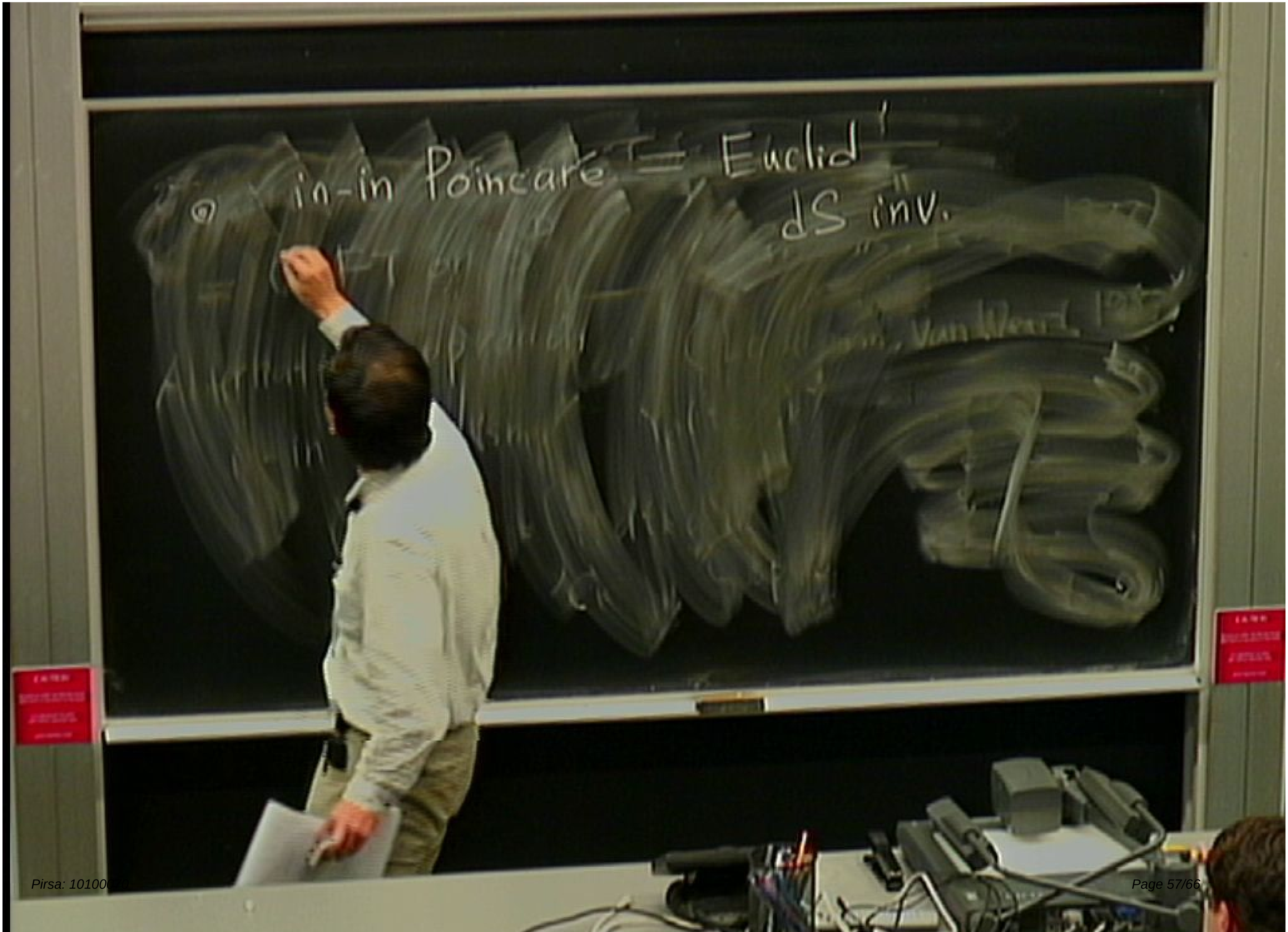
2. Img. to IR

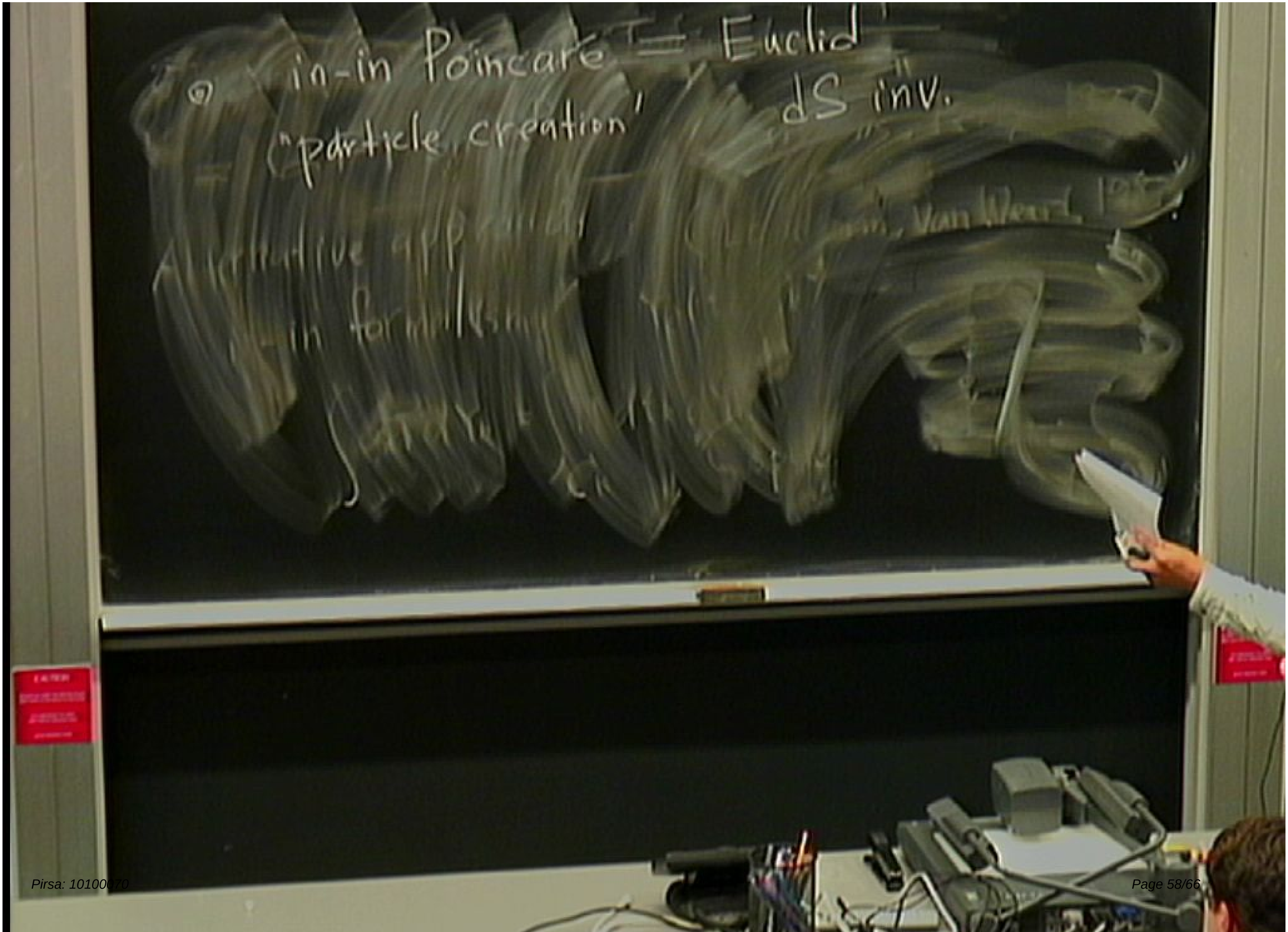
$d\Omega_{D-2}$

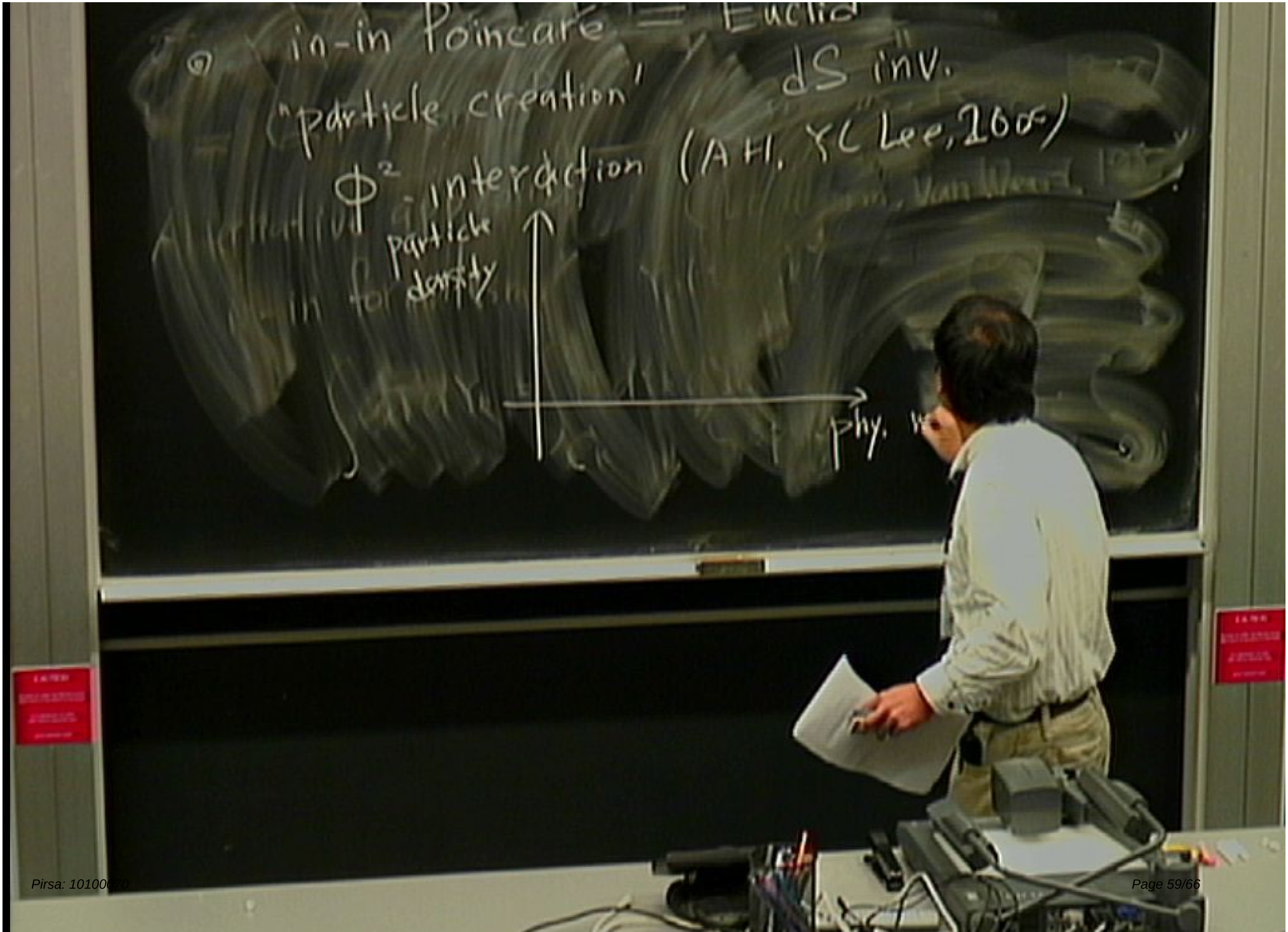

2. Imp. to IR
|Δ(x,τ)| → ∞ in linearized gravity
⟹ no factorization.

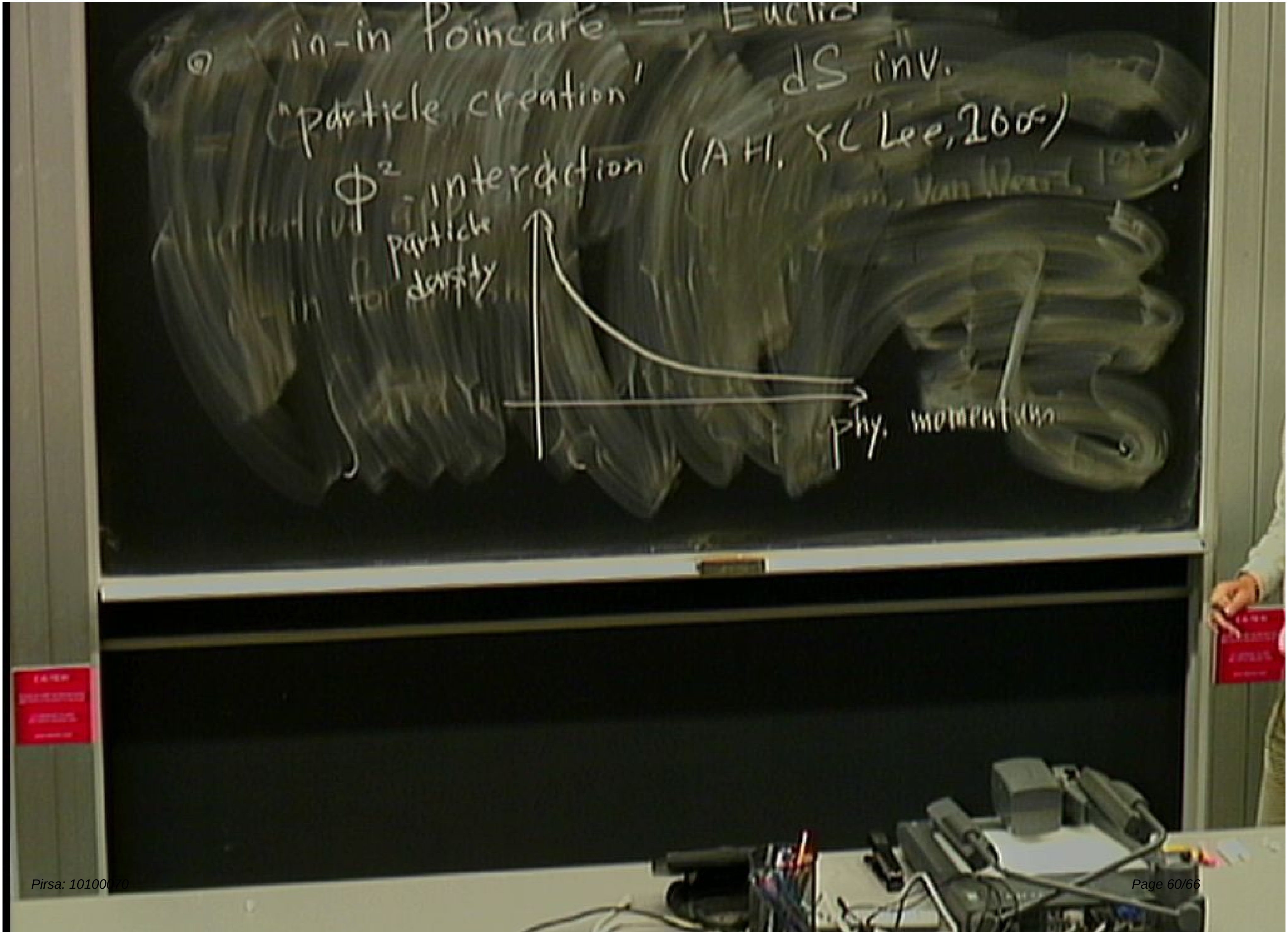

2. Imp. to IR
|Δ(x,x')| → ∞ in linearized gravity
⟹ no factorization.

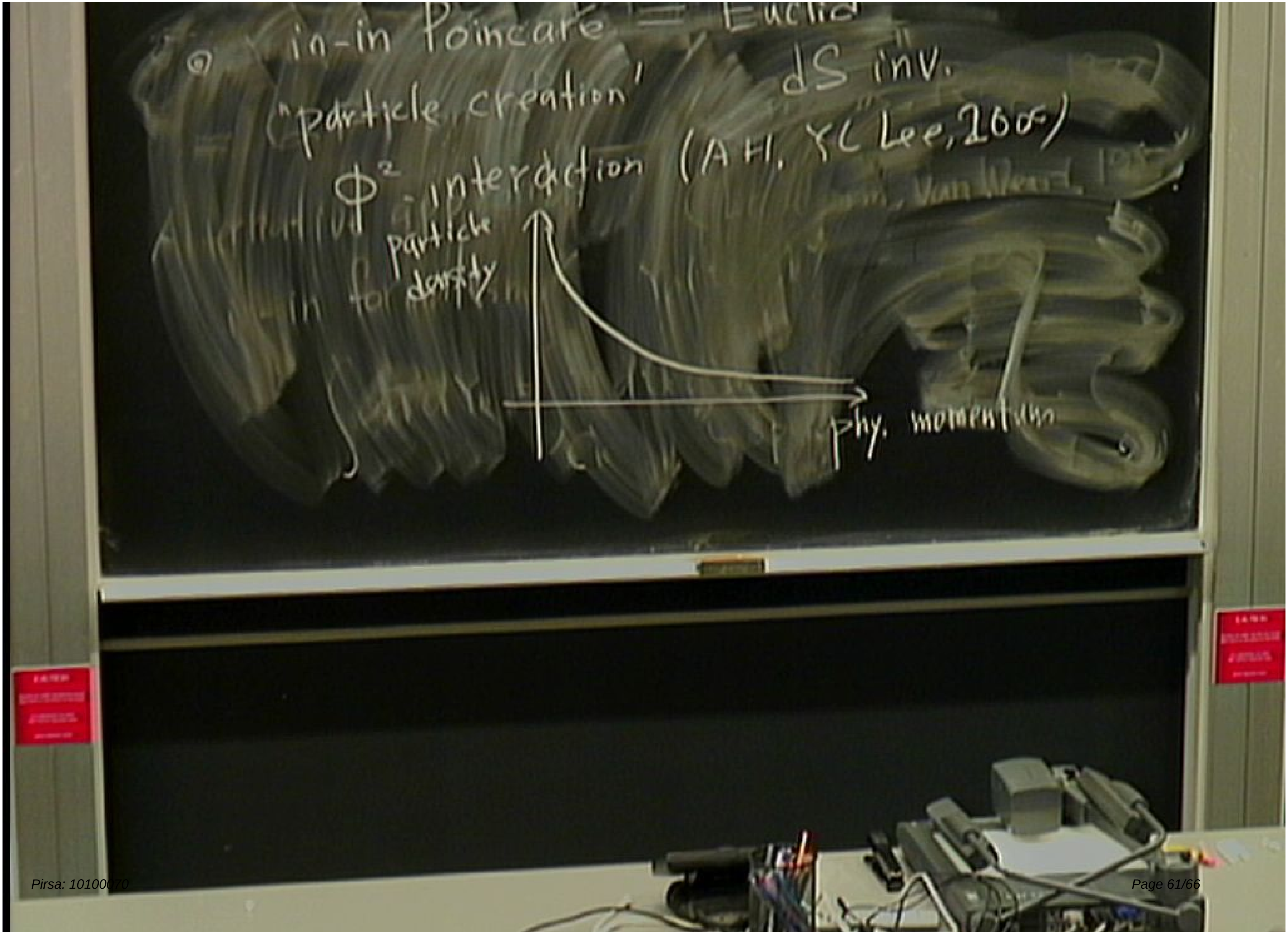



in-in Poincaré = Euclid'
dS inv.


in-in Poincaré = Euclid
dS inv.
"particle creation"


in-in Poincaré = Euclid
dS inv.
"particle creation"
φ² interaction (A H, YC Lee,
particle density
phy.


in-in Poincaré
dS inv.
"particle creation"
φ² interaction (A H, Y C Lee, 200
particle density
phy. momentum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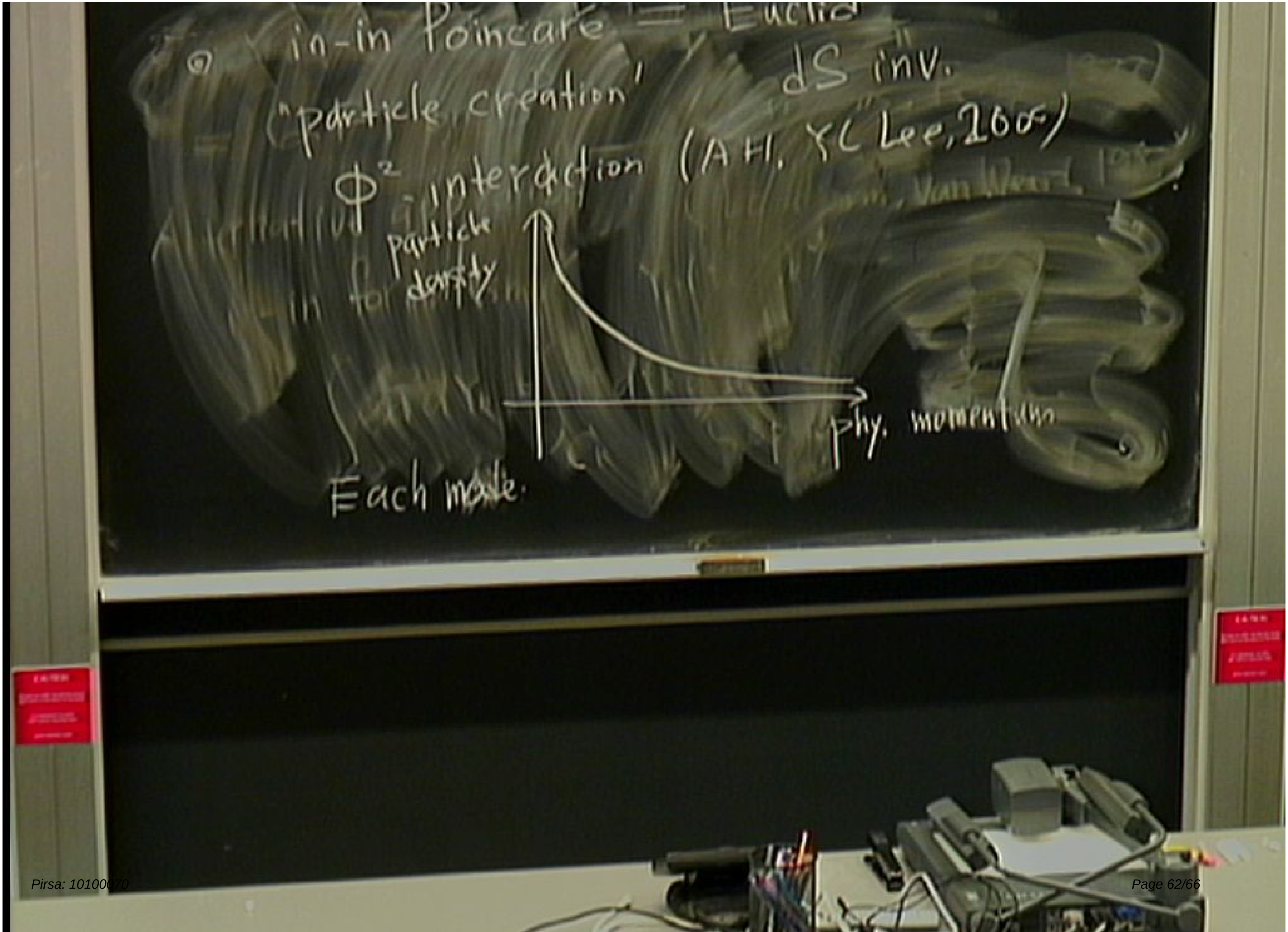
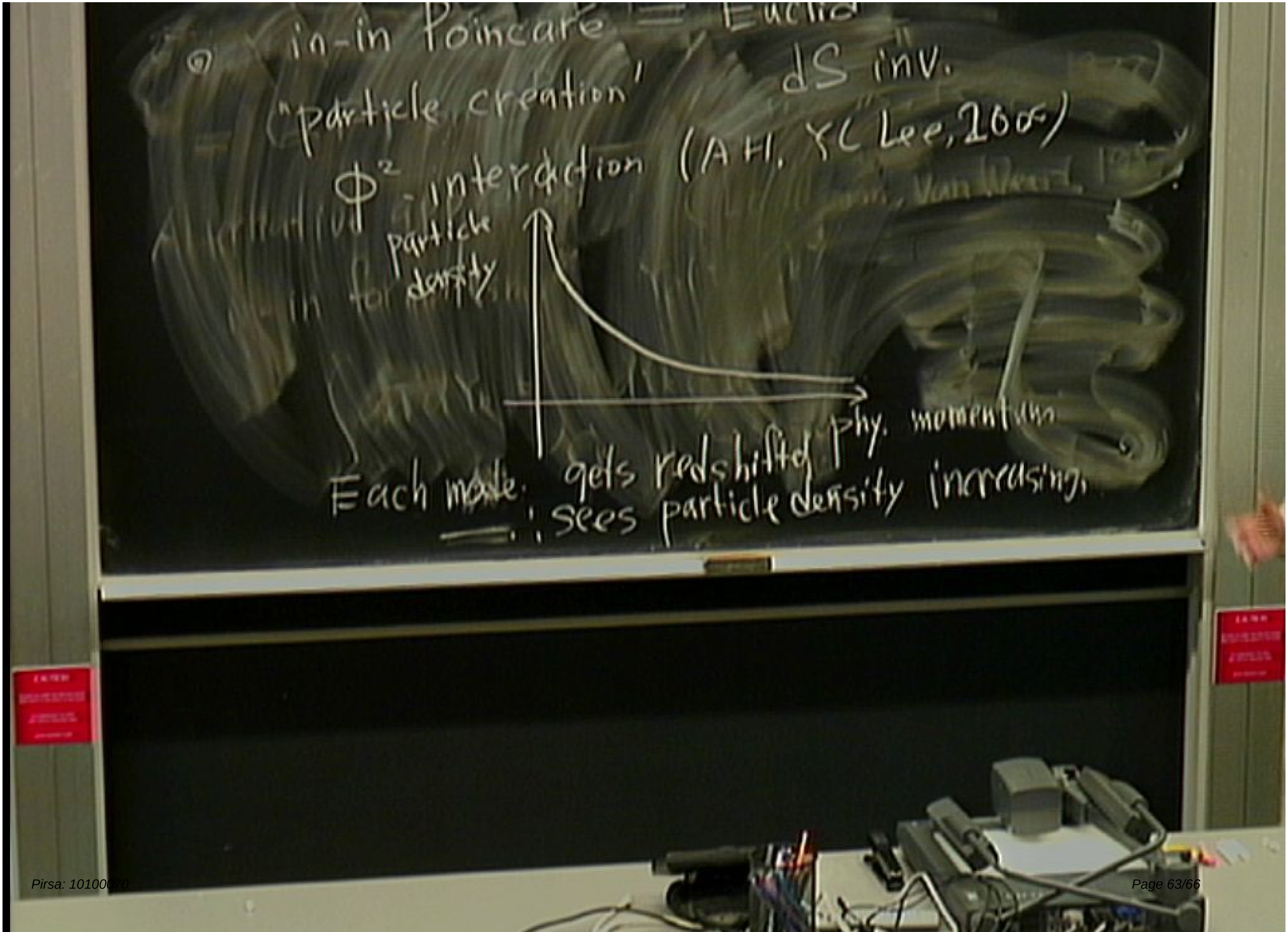
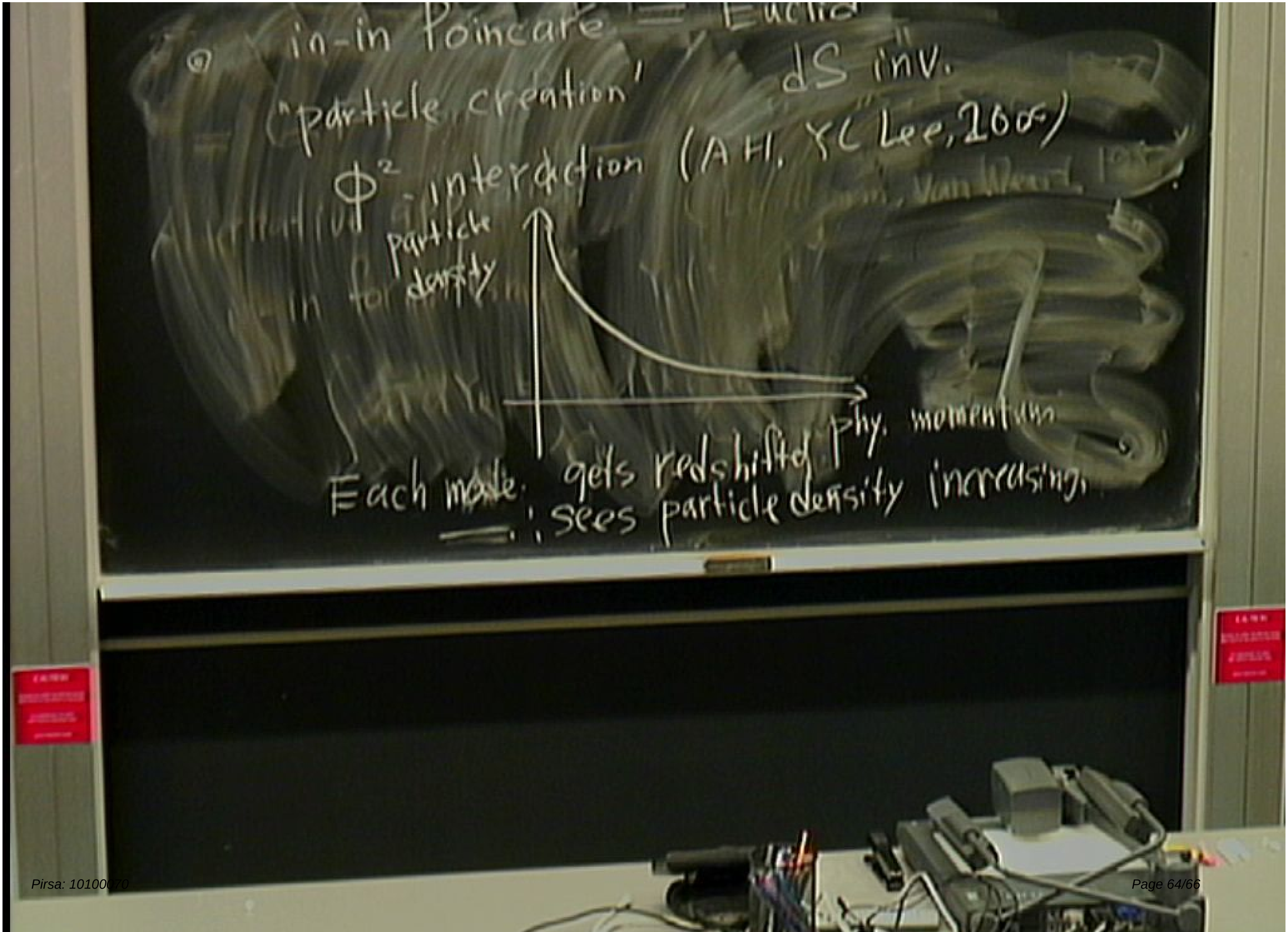


in-in Poincaré
dS inv.
"particle creation"
φ² interaction (A.H., Y.C. Lee,
particle density
phy. momentum
Each mode.


in-in Poincare
dS inv.
"particle creation"
ϕ^2 interaction
particle density
phy. momentum
Each mode gets redshifted
sees particle density increasing.


in-in Poincaré = Euclid
dS inv.
"particle creation"
ϕ^2 interaction (AH, YC Lee, 2006)
particle density
phy. momentum
Each mode gets redshifted
∴ sees particle density increasing.

in-in Poincaré = Euclid

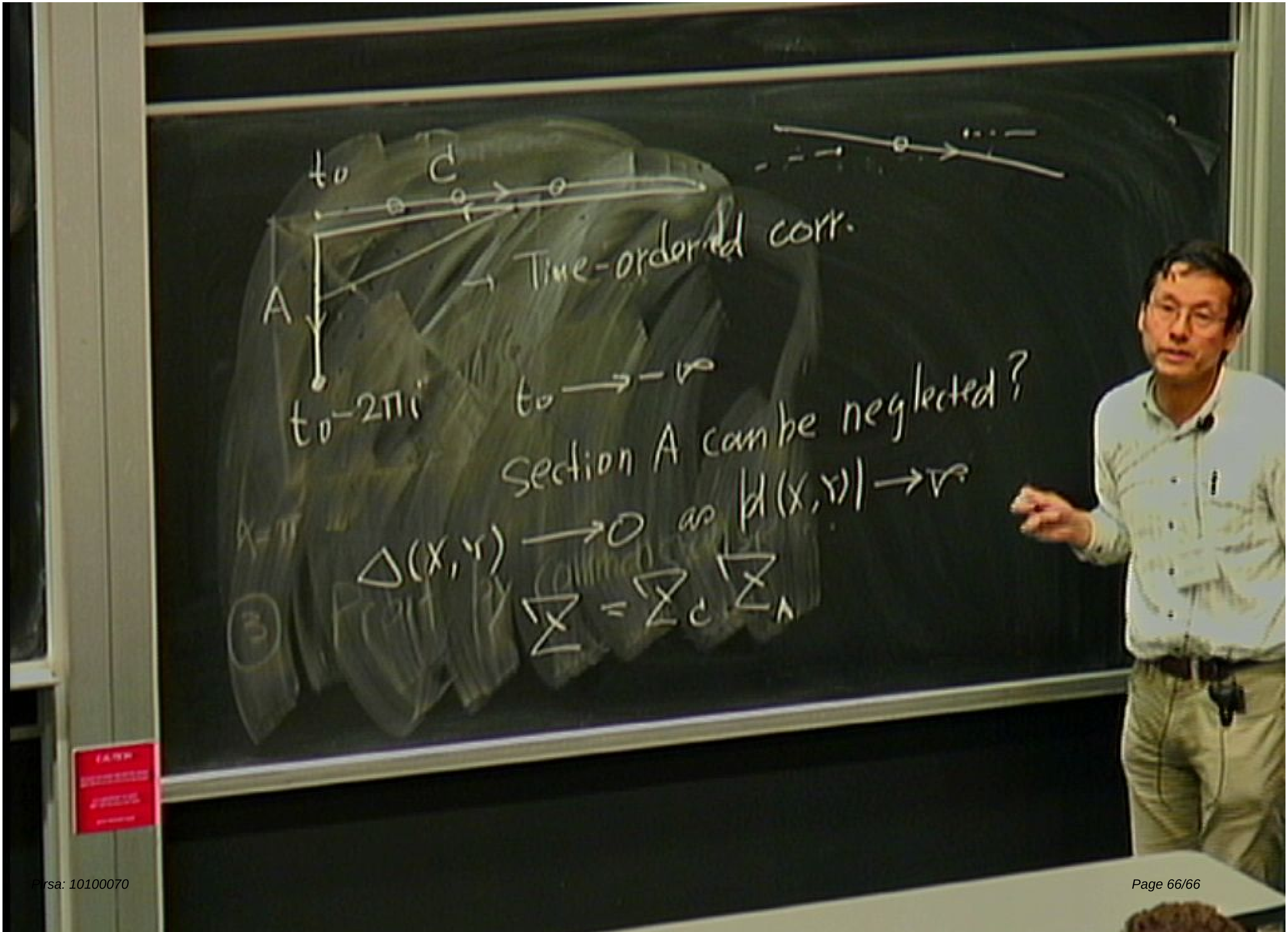
◎ dS inv.

"particle creation"

ϕ^2 interaction (A.H., Y.C. Lee, 200?)

Each mode gets redshifted

∴ sees particle density increasing.


t0
C
Time-ordered corr.
A
t0-2πi
t0 → -∞
section A can be neglected?
Δ(X,Y) → 0 as |d(X,Y)| → ∞
Σ = Σc ΣA