Title: Particle decay in the de Sitter universe

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Abstract: We study particle decay in the de Sitter spacetime as given by first order perturbation theory in an interacting quantum field theory.
We discuss first a general construction of bosonic two-point functions, including a recently discovered class of tachyonic theories that do exist in the de Sitter spacetime at discrete negative values of the squared mass parameter and have no Minkowskian counterpart.
We show then that for fields with masses above a critical mass m_c there is no such thing as particle stability, so that decays forbidden in flat space-time do occur there.
The lifetime of such a particle also turns out to be independent of its velocity when that lifetime is comparable with de Sitter radius.
For particles with lower mass is yet not completely solved. We show however that the masses of their decay products should obey quantification rules.

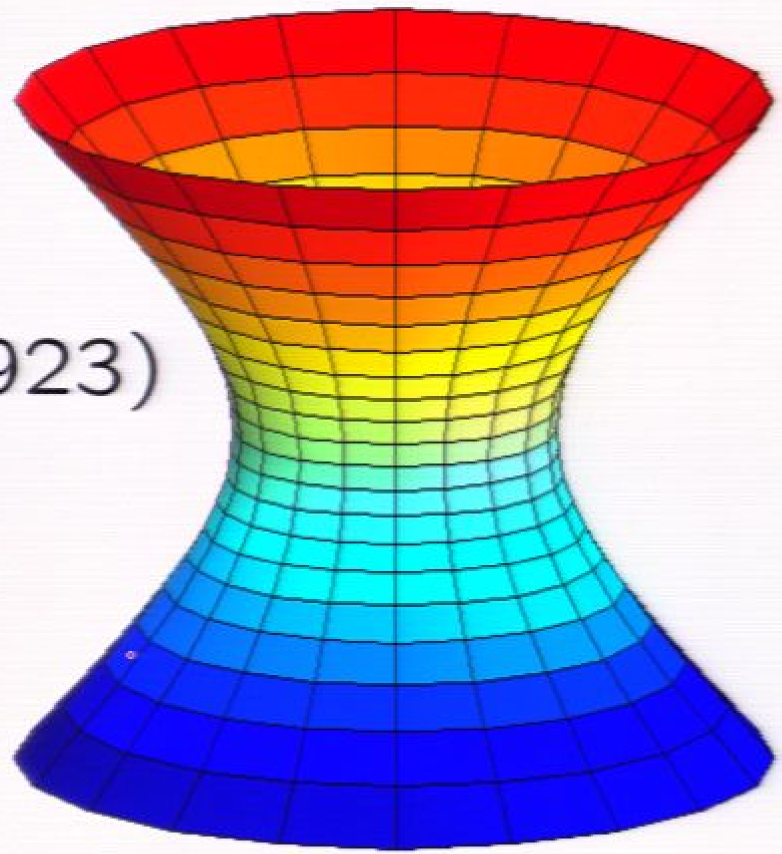
Particle decay in the de Sitter Universe

Ugo Moschella
Università dell'Insubria
IR issues and loops in de Sitter space
Perimeter Institute, October 29, 2010

The de Sitter manifold

$M^{(d+1)}$ Minkowski in $d+1$ dimensions,

$\eta^{\mu\nu} = diag(1, -1, \ldots, -1)$



The de Sitter
hyperboloid world (Weyl 1923)

$\{X_0^2 - X_1^2 - \ldots X_d^2 = -R^2\}$

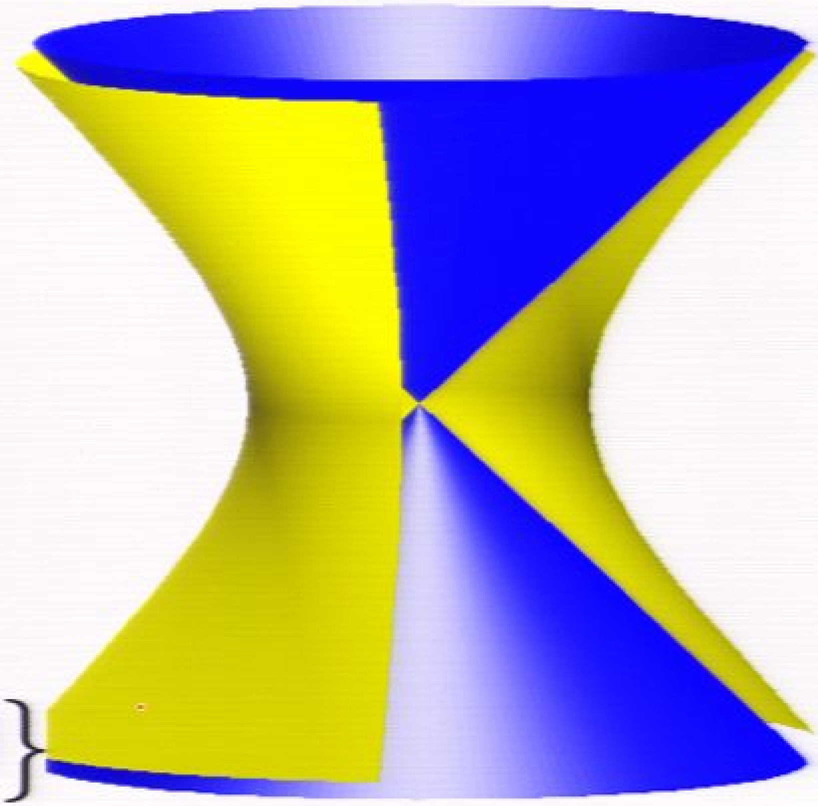
Relativity group
$G = SO(1, d)$

The asymptotic cone

$M^{(d+1)}$

$\eta^{\mu\nu} = diag(1, -1, \ldots, -1)$

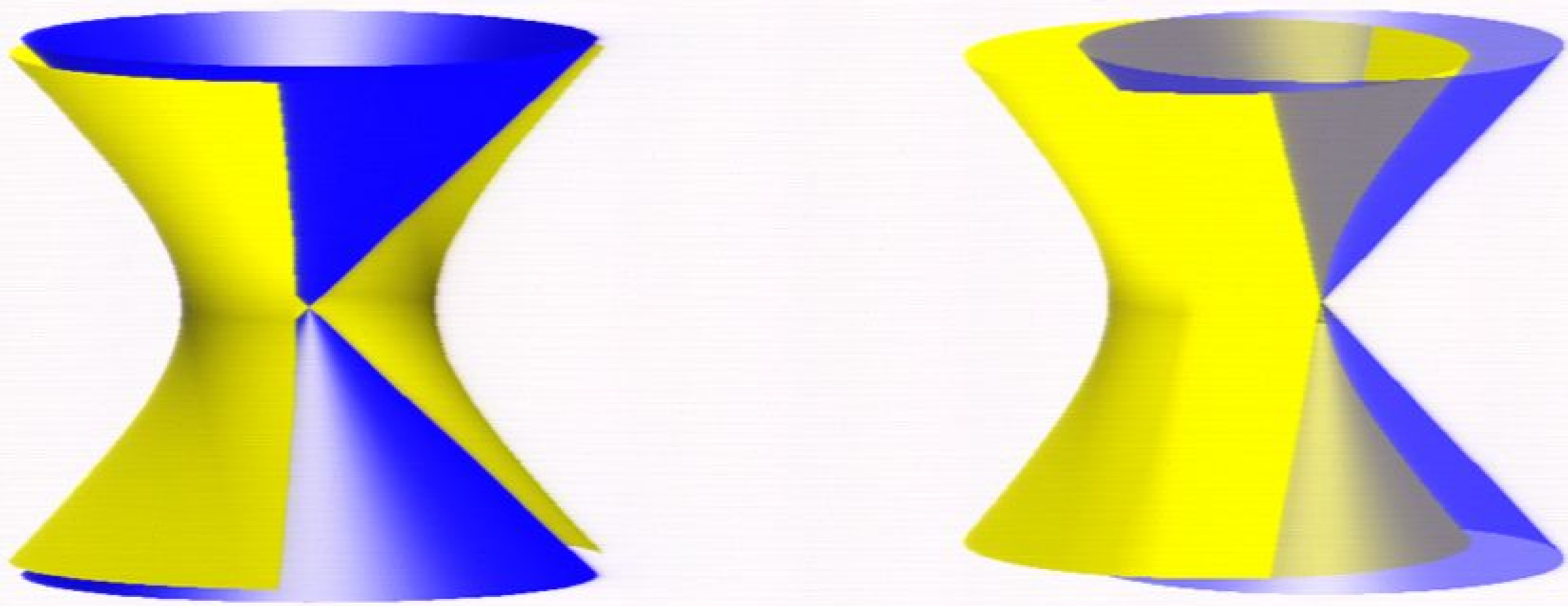
The asymptotic cone is the lightcone of the ambient spacetime



$$\{\xi_0^2 - \xi_1^2 - \ldots - \xi_d^2 = 0\}$$

$$\{X_0^2 - X_1^2 - \ldots X_d^2 = -R^2\}$$

The asymptotic cone: causal structure



X, Y are spacelike separated:

$$(X-Y)^2 = X^2 + Y^2 - 2X\cdot Y = -2R^2 - 2X\cdot Y < 0$$

$Y - X$ is outside the cone whose tip is X

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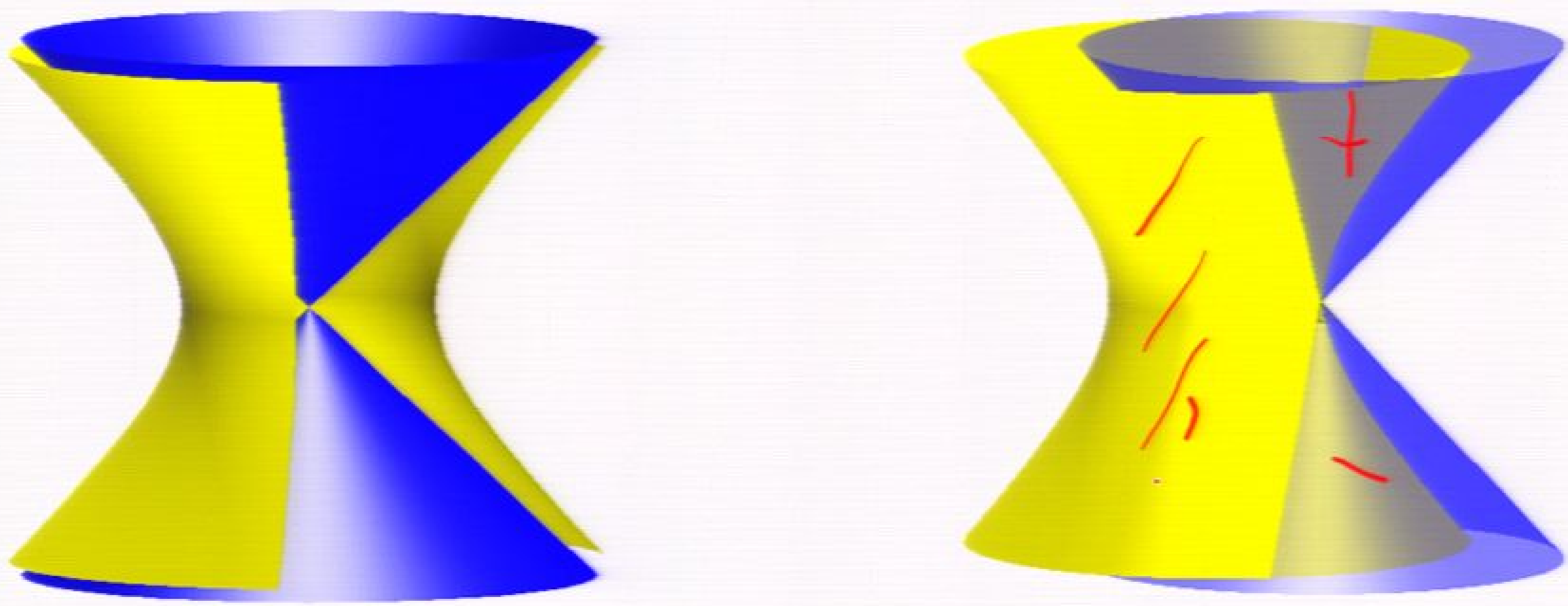

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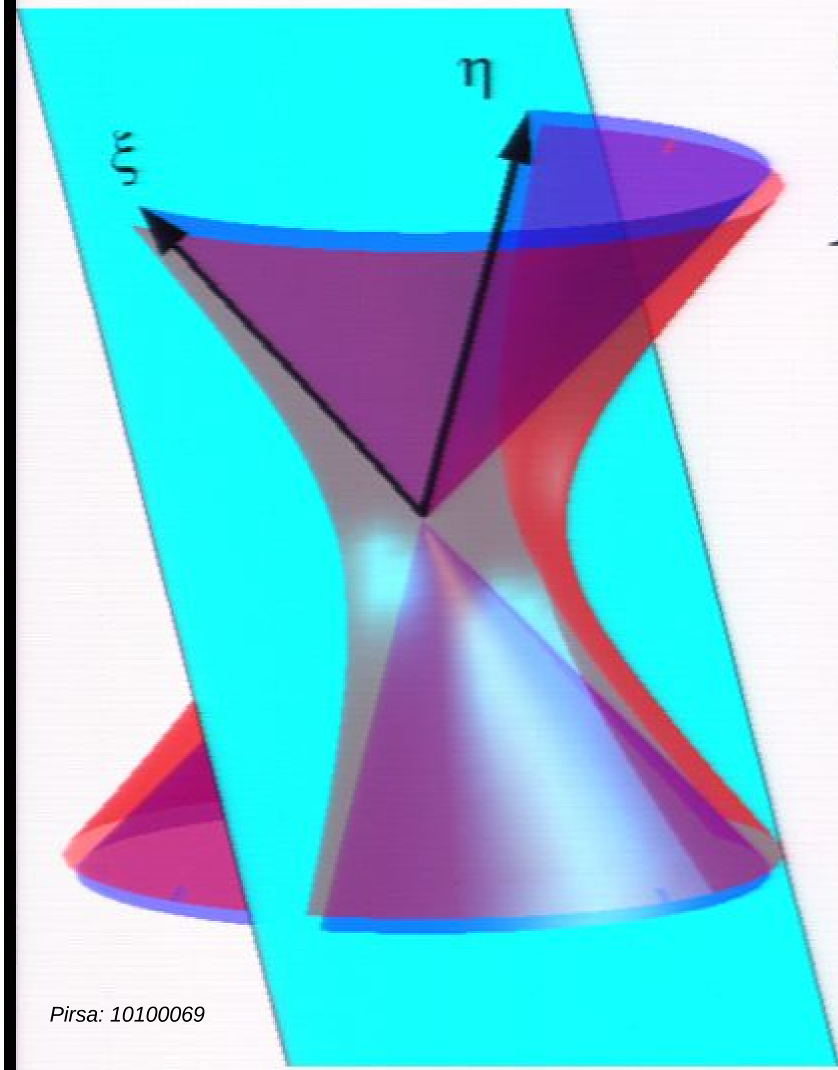
The asymptotic cone as the de Sitter momentum space



Geodesics: de Sitter

$$X_\mu(\tau) = \frac{R}{\sqrt{2\xi \cdot \eta}} \left(\xi_\mu e^{\frac{\tau}{R}} - \eta_\mu e^{-\frac{\tau}{R}} \right)$$

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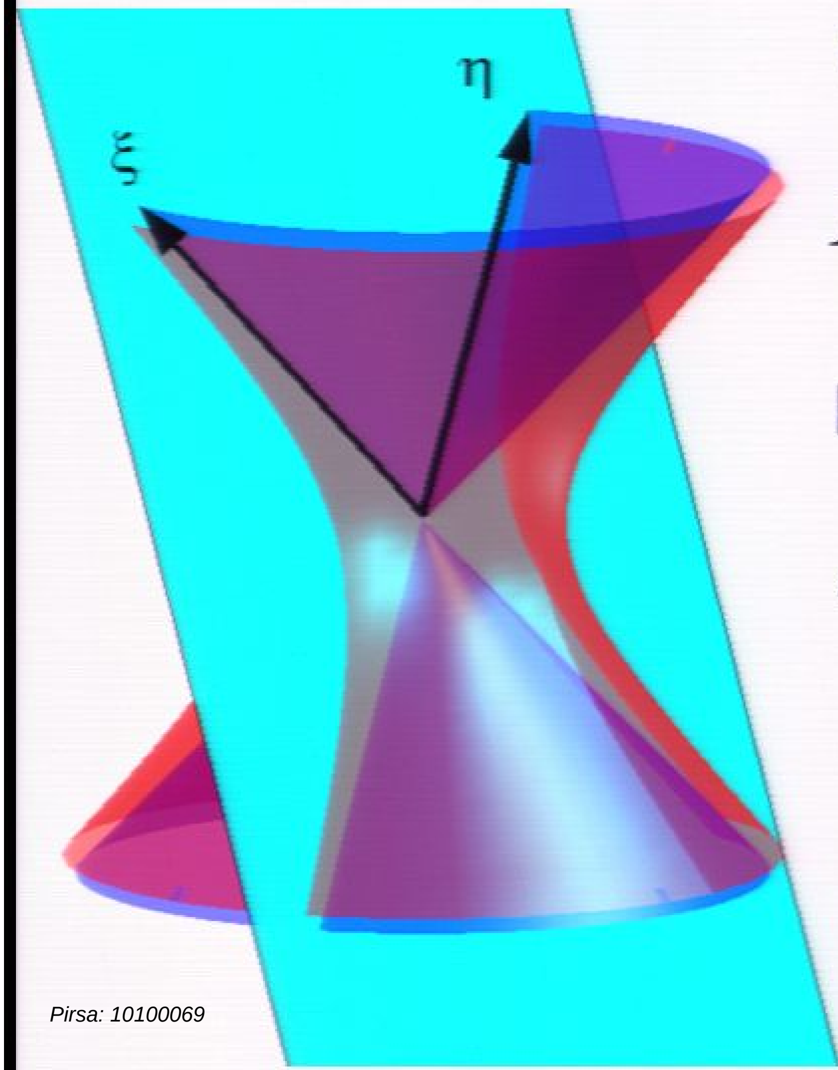

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Minkowski

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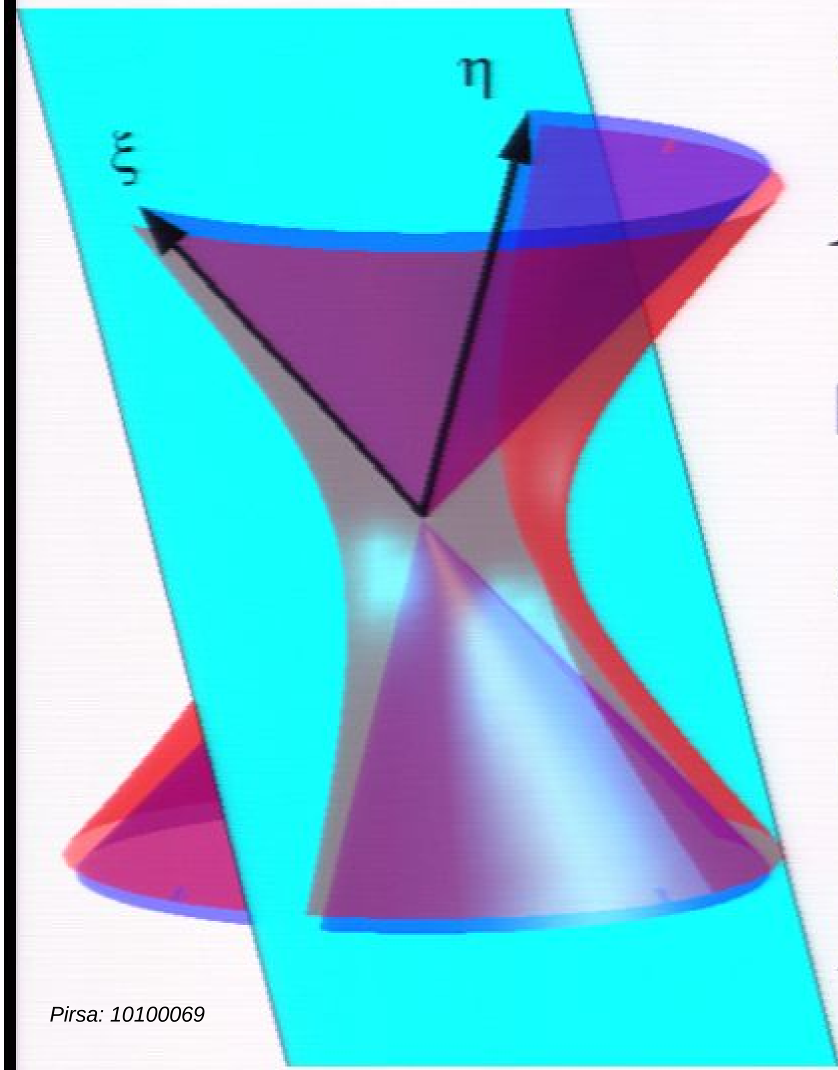

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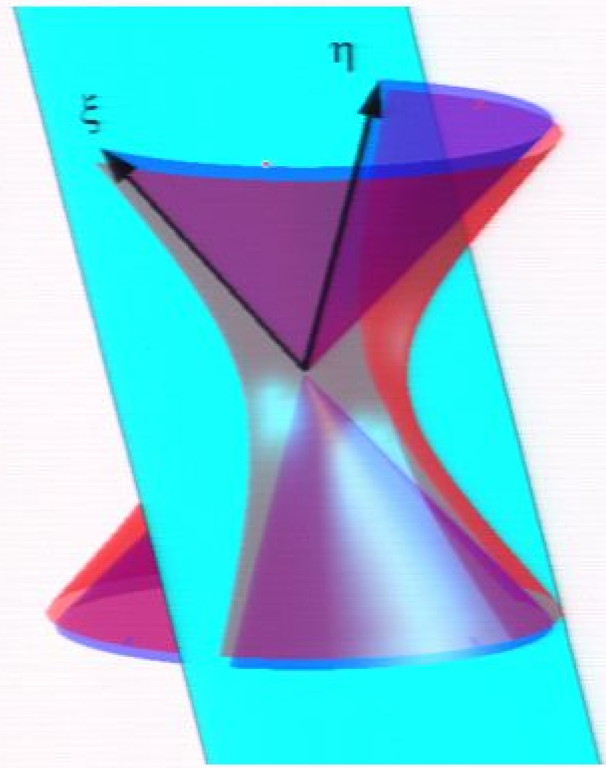
Note that

$$X_\mu(0) = \frac{R}{\sqrt{2\xi\cdot\eta}}(\xi_\mu - \eta_\mu)$$

Conserved quantities

$$X_\mu(\tau) = \frac{R}{\sqrt{2\xi\cdot\eta}}\left(\xi_\mu e^{\frac{c\tau}{R}} - \eta_\mu e^{-\frac{c\tau}{R}}\right)$$

$$K_{\xi,\eta} = mc\frac{\xi\wedge\eta}{\xi\cdot\eta}$$



Classical scattering

$$b_1 + b_2 \longrightarrow c_1 + \ldots + c_N$$

Problem: find the outgoing momenta (ξ_f, η_f) given the ingoing ones (χ_i, ζ_i):

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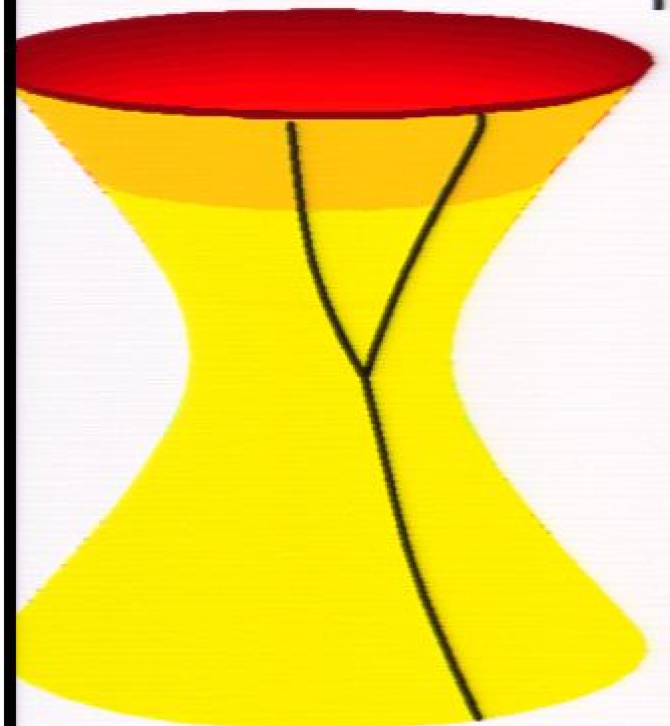
Example: particle decay



$$m_1 \longrightarrow \mu_1 + \mu_2 \,,$$

$$\xi = \frac{1}{\sqrt{2}}\left(\frac{m_1^2+\mu_1^2-\mu_2^2}{2m_1\mu_1}, \mp\frac{\sqrt{(m_1^2-\mu_1^2-\mu_2^2)^2-4\mu_1^2\mu_2^2}}{2m_1\mu_1}, 1\right)$$

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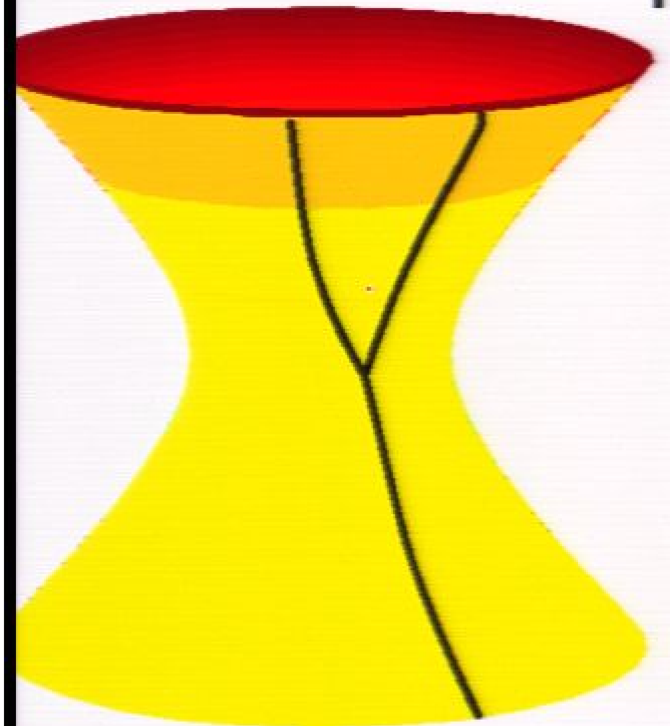

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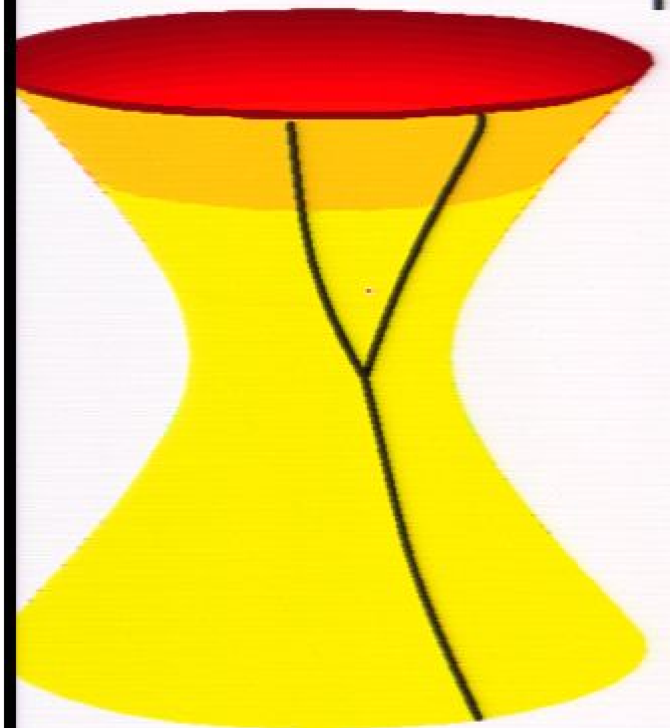

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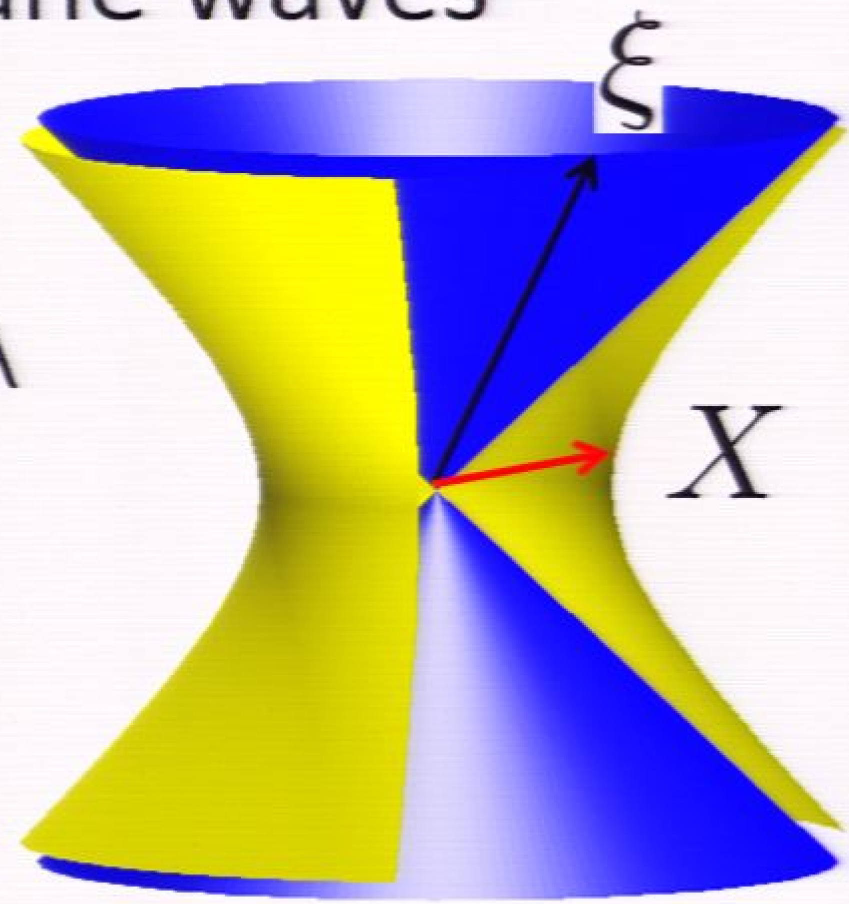
Mass subadditivity

will not hold in the quantum description

de Sitter plane waves

$$\psi_\lambda(X,\xi) = (X \cdot \xi)^\lambda$$

$$\lambda \in \mathbf{C}, \quad \xi^2 = 0$$



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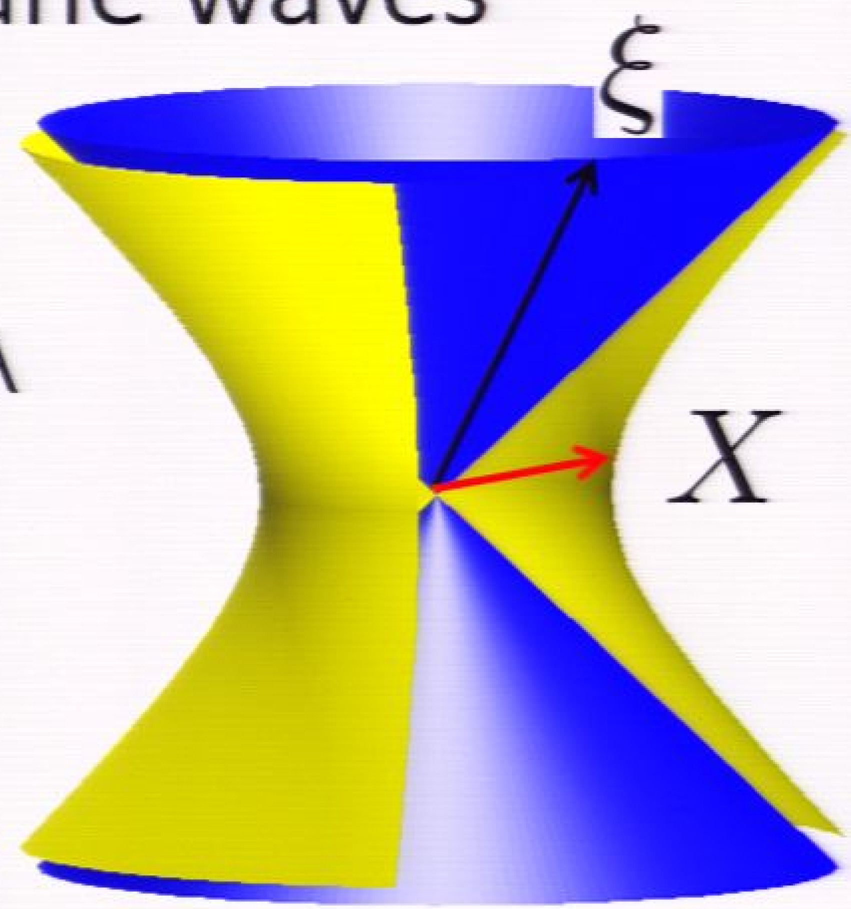

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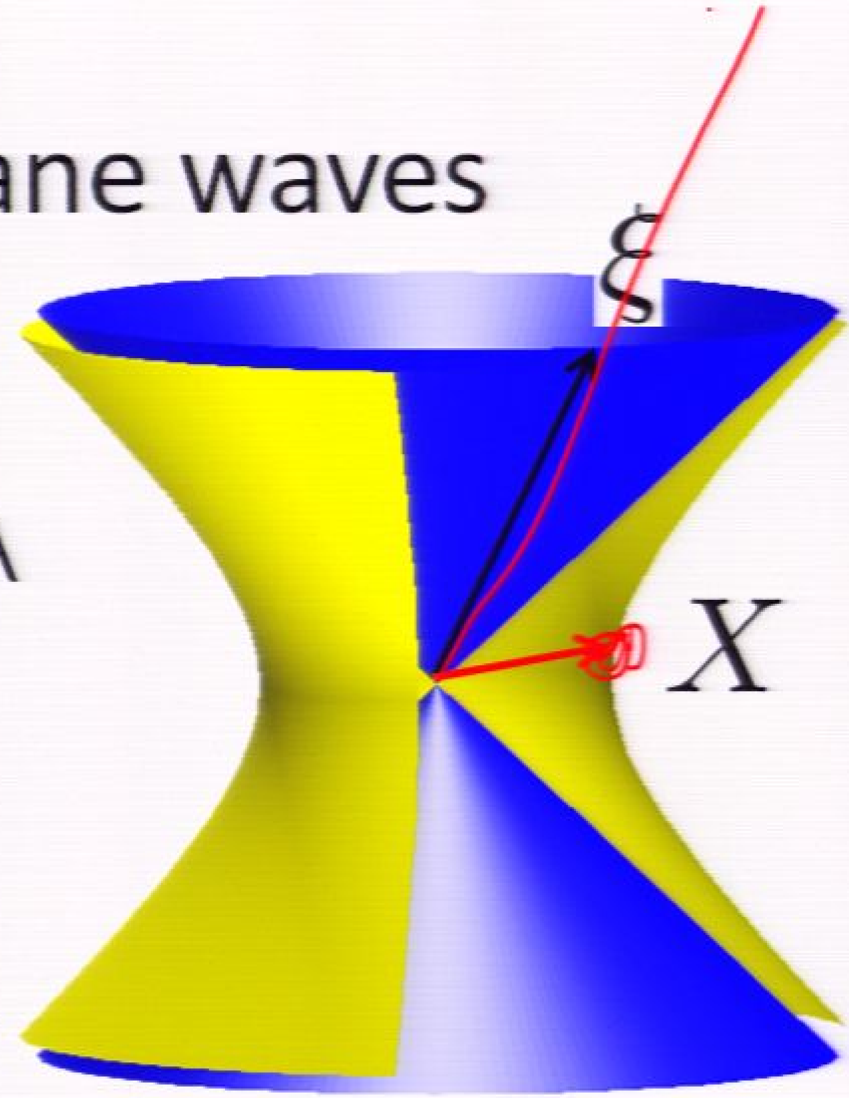

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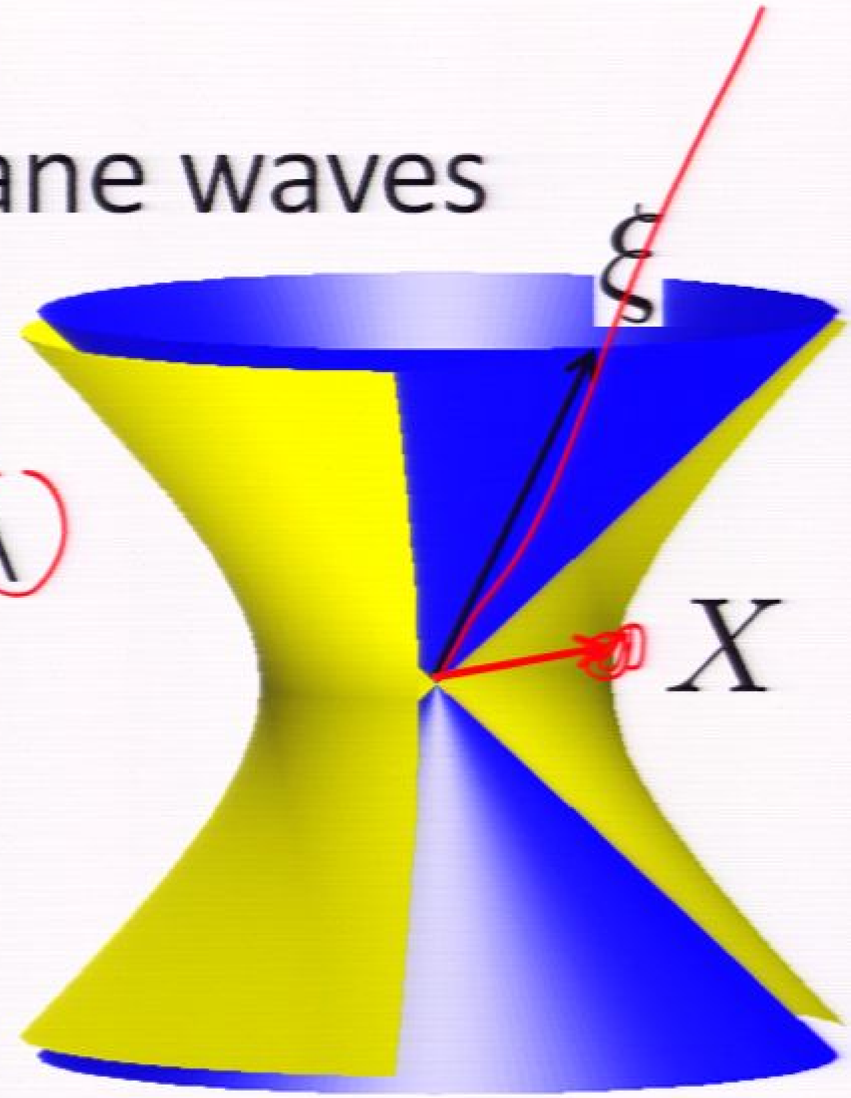

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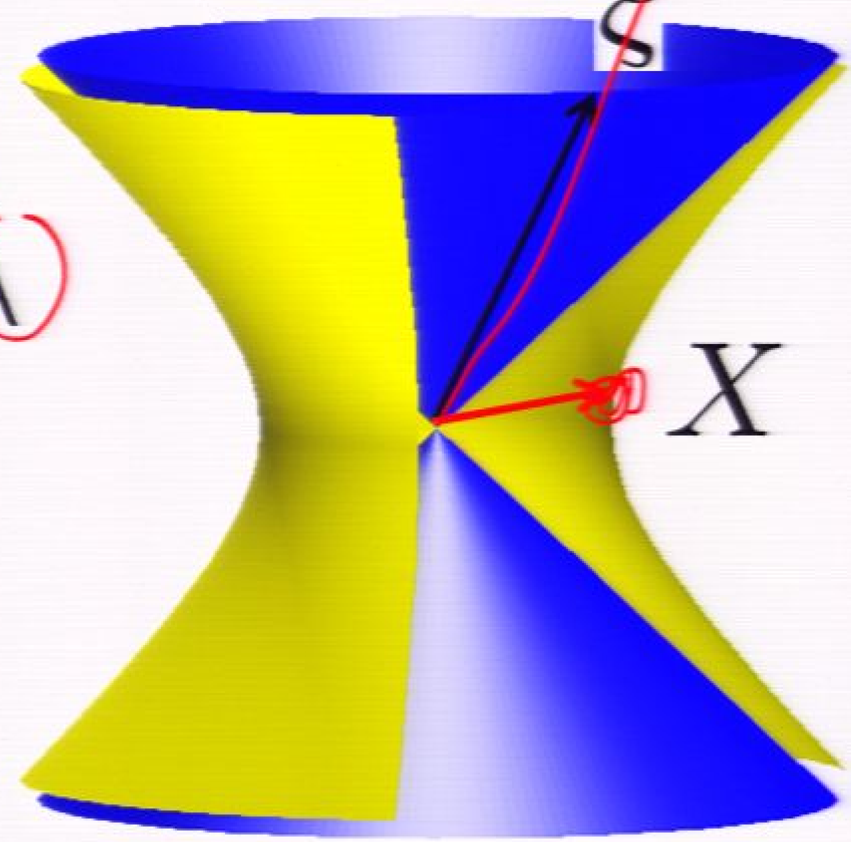
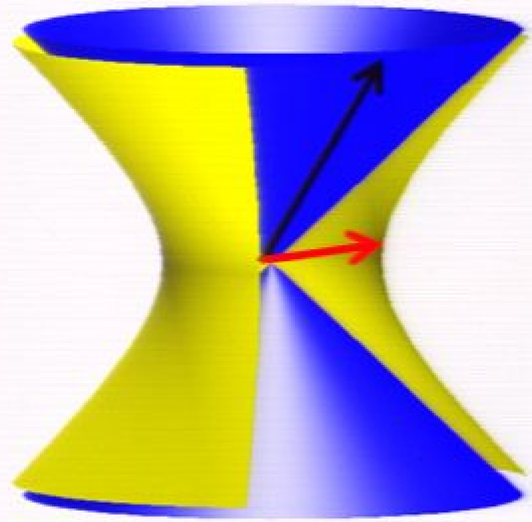


de Sitter plane waves

$$\Box(X \cdot \xi)^{\lambda} = \lambda(\lambda + d - 1)(X \cdot \xi)^{\lambda}$$



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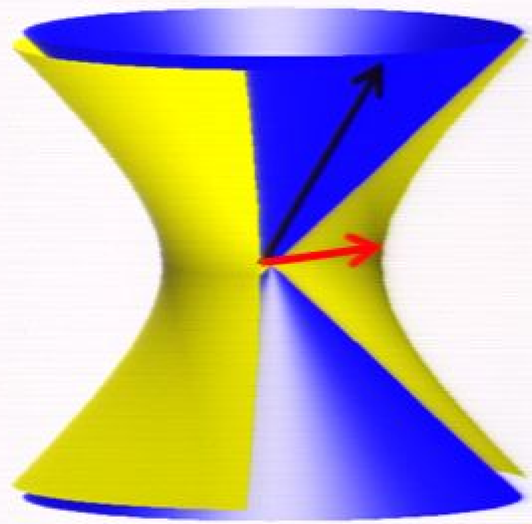

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Involution:

$$\lambda \longrightarrow \bar{\lambda} = -\lambda - (d-1)$$

$$\lambda + \bar{\lambda} = -(d-1)$$

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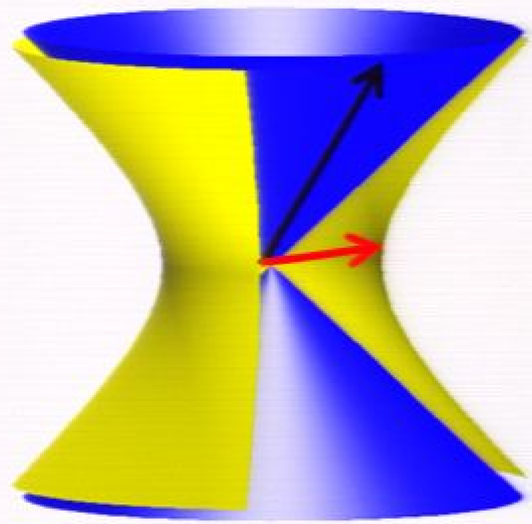

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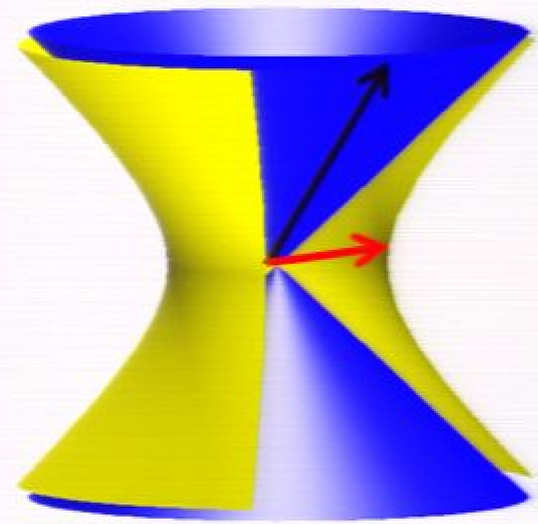

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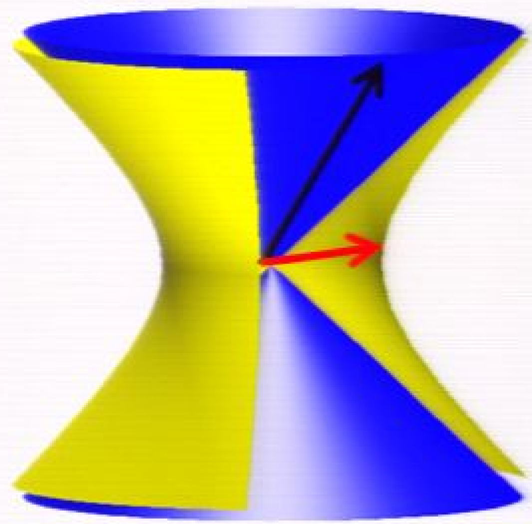

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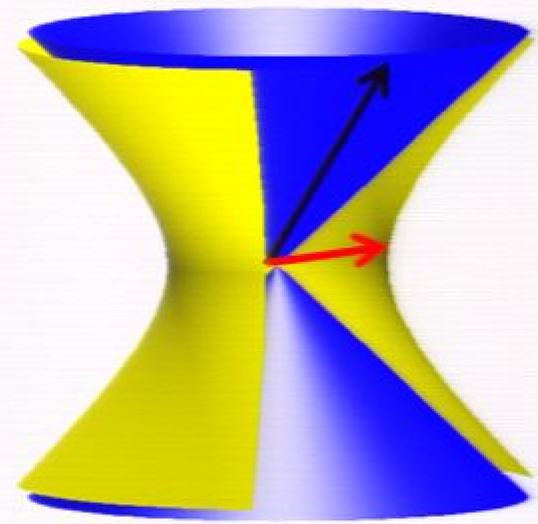

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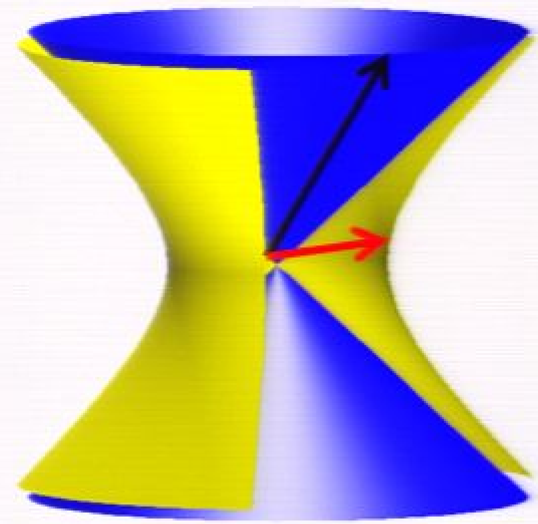
Scalar waves with (complex) squared mass:

$m^2 = \lambda\bar{\lambda}$

$\left(\Box + \lambda\bar{\lambda}\right)(X\cdot\xi)^{\lambda} = 0, \qquad \left(\Box + \lambda\bar{\lambda}\right)(X\cdot\xi)^{\bar{\lambda}} = 0$

Principal de Sitter waves



$$\lambda = -\frac{d-1}{2} + i\nu\,, \quad \nu \in \mathbf{R}$$

$$\psi_\lambda(X,\xi) = (X\cdot\xi)^{-\frac{d-1}{2}+i\nu}$$

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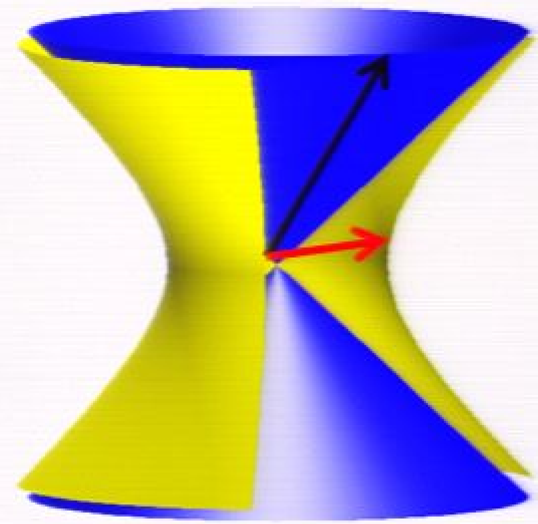

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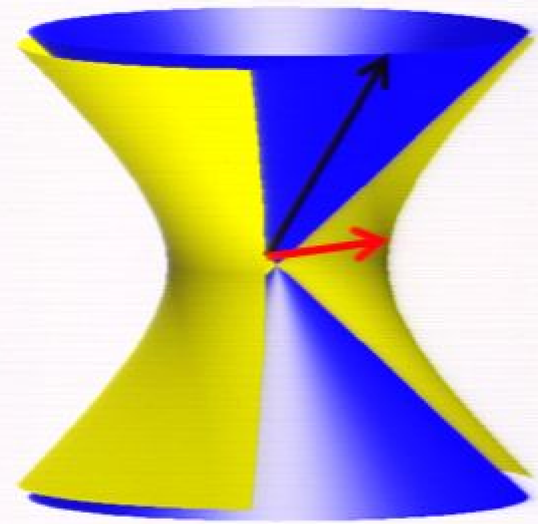

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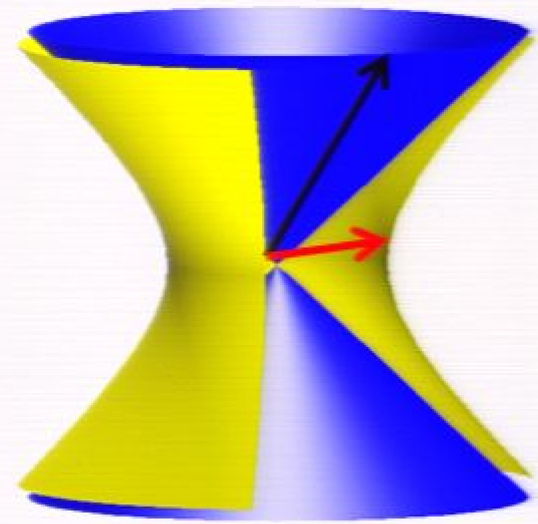

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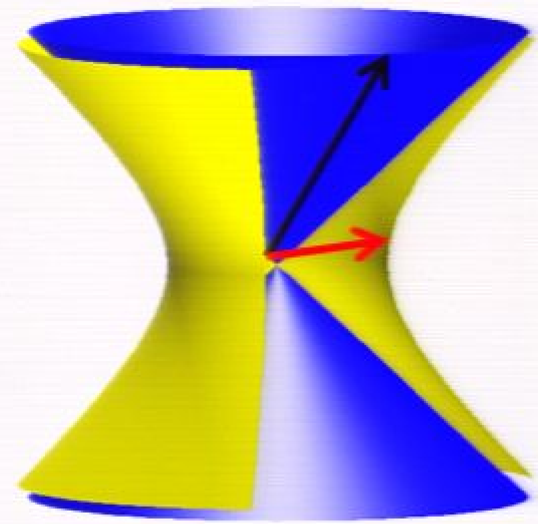

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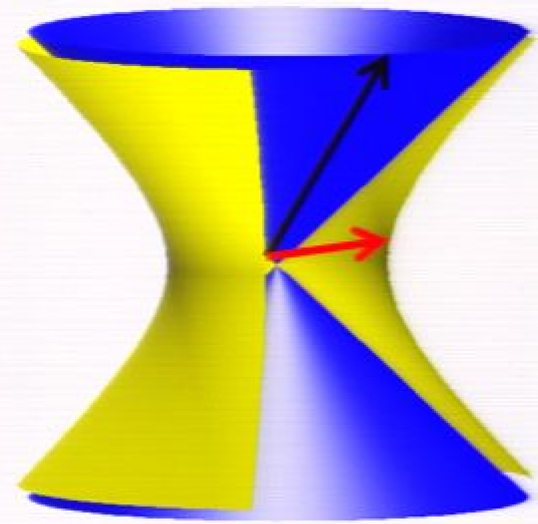

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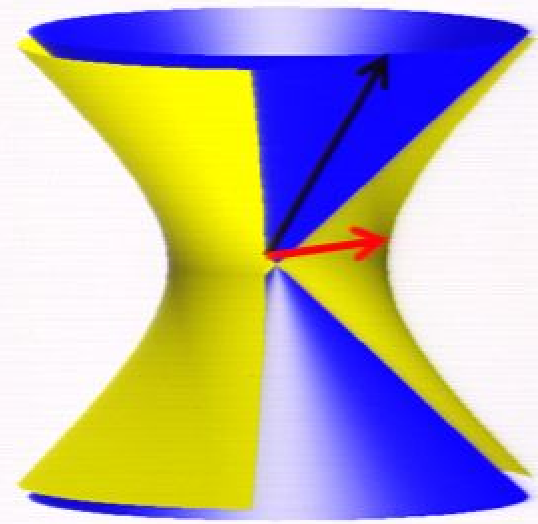
Complementary de Sitter waves



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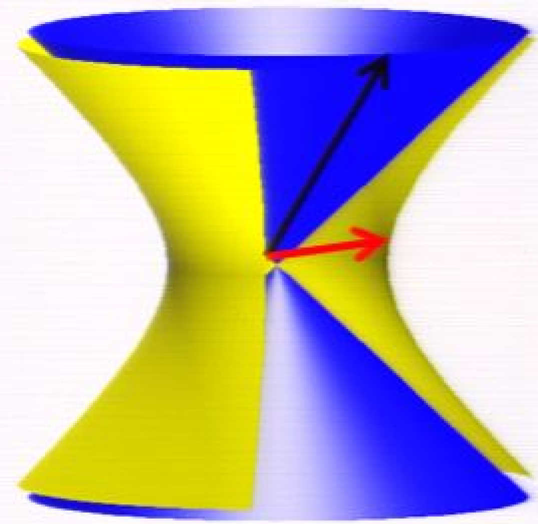

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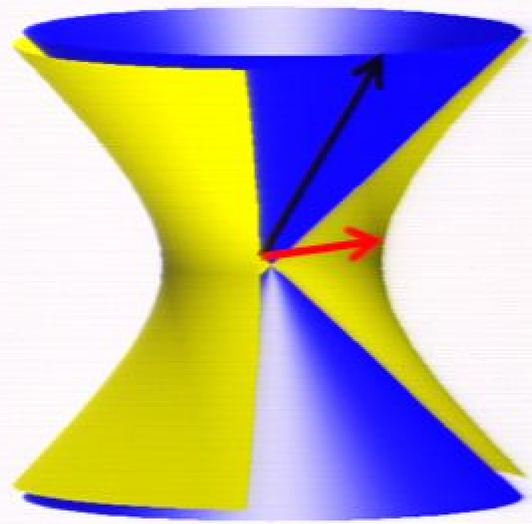

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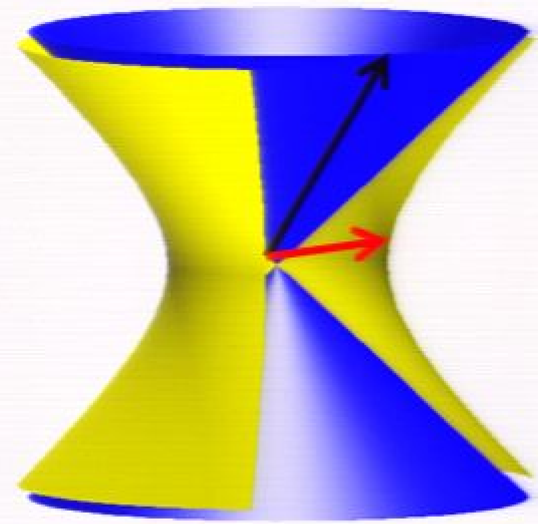

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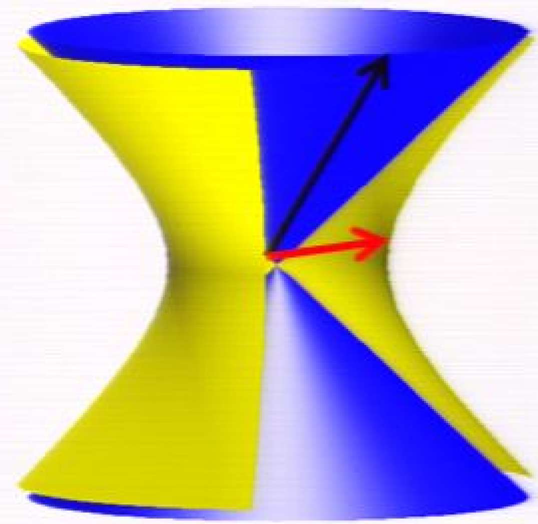

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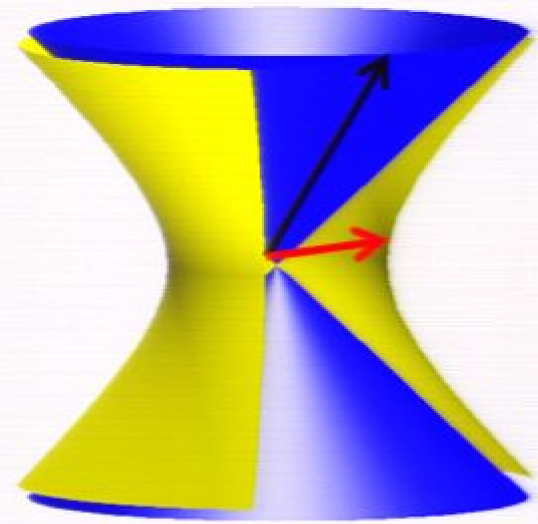

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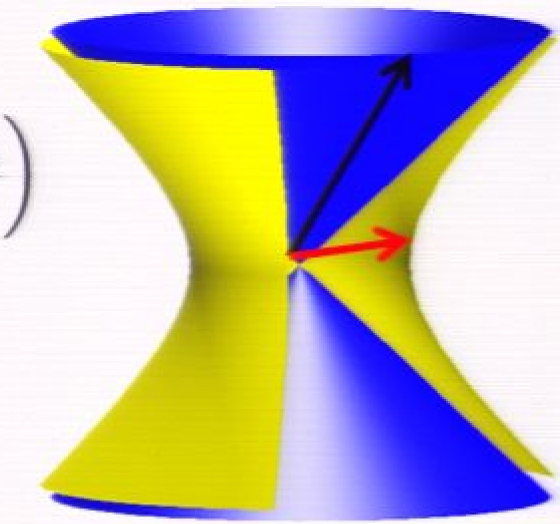
Discrete de Sitter waves



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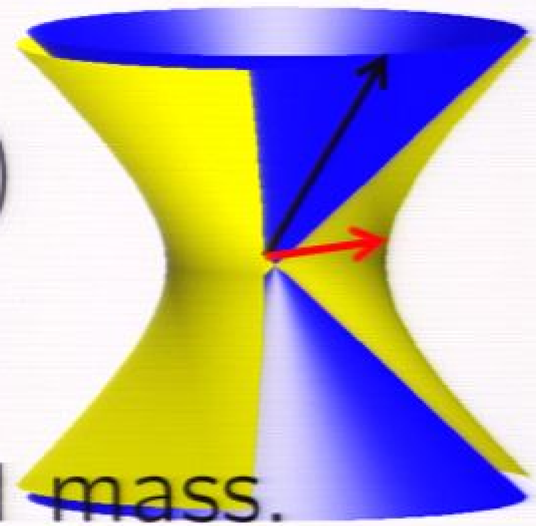

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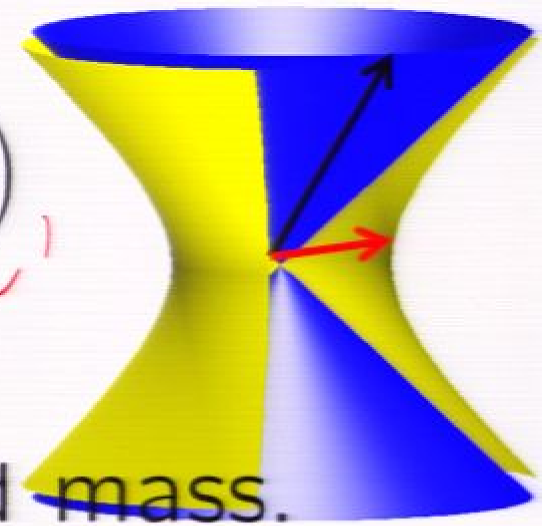

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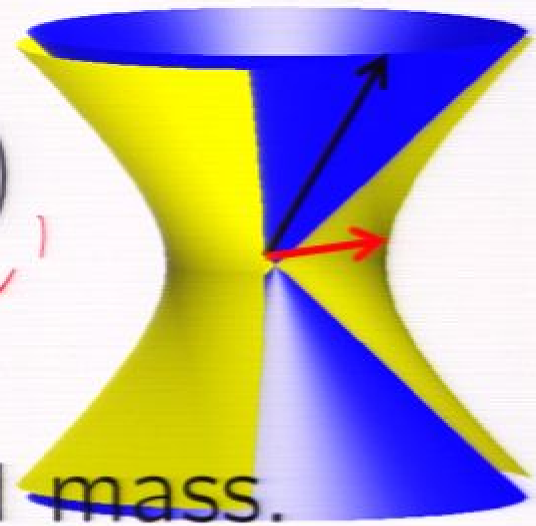

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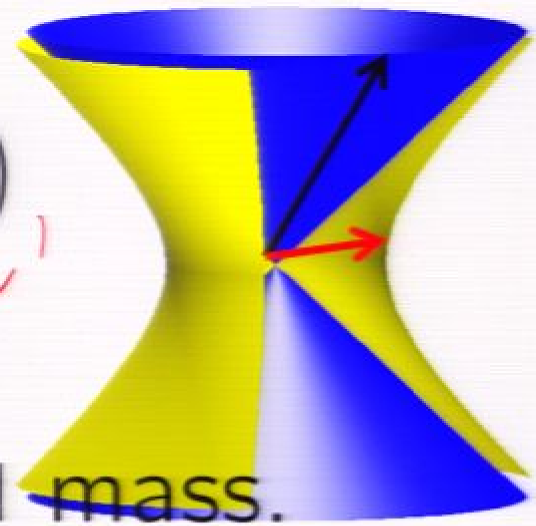

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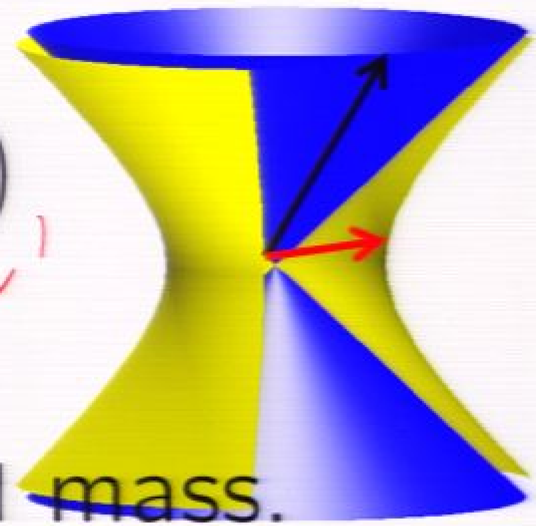

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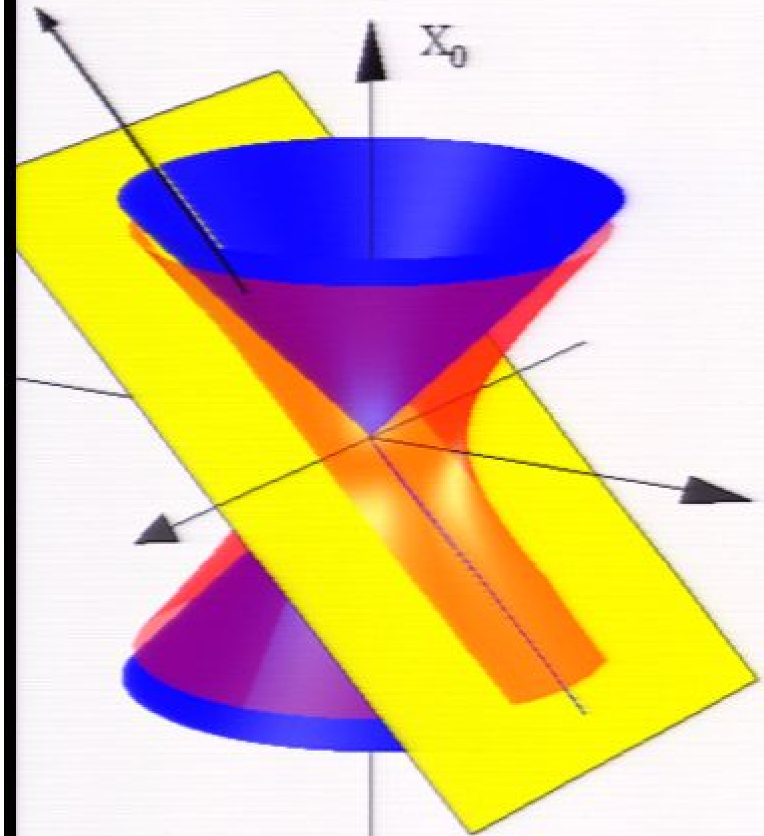

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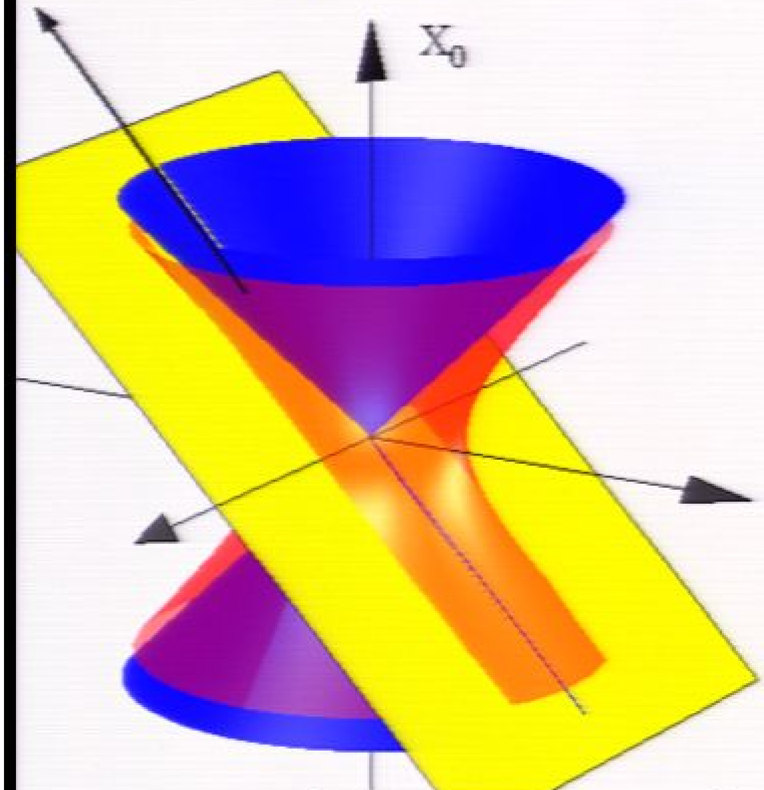

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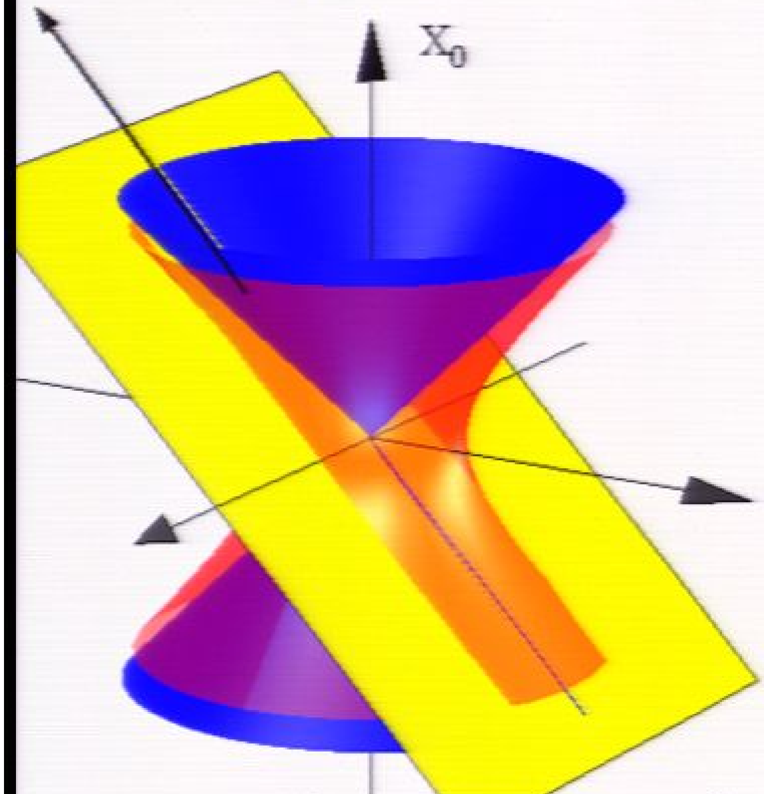

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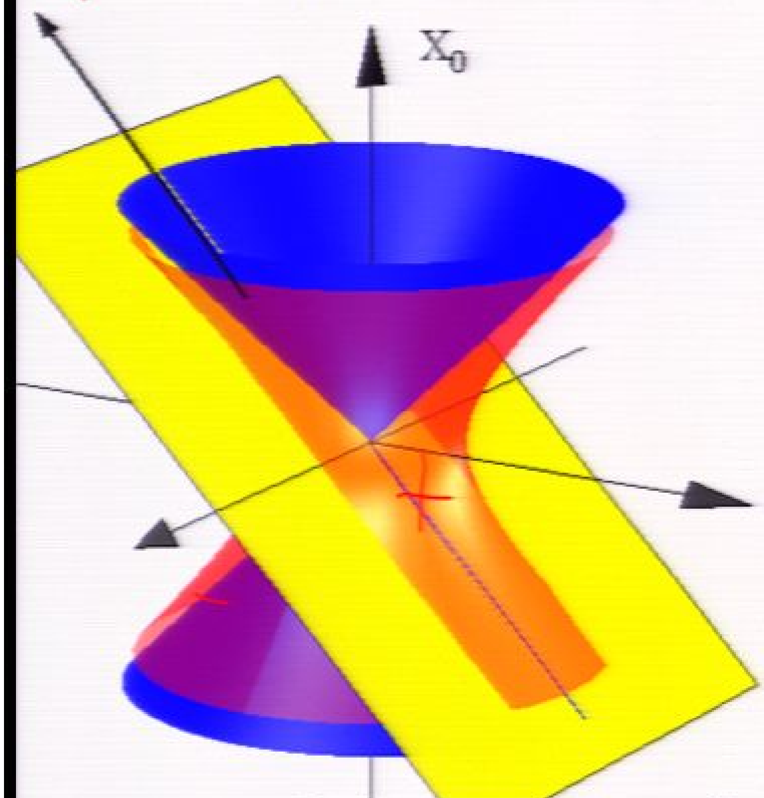

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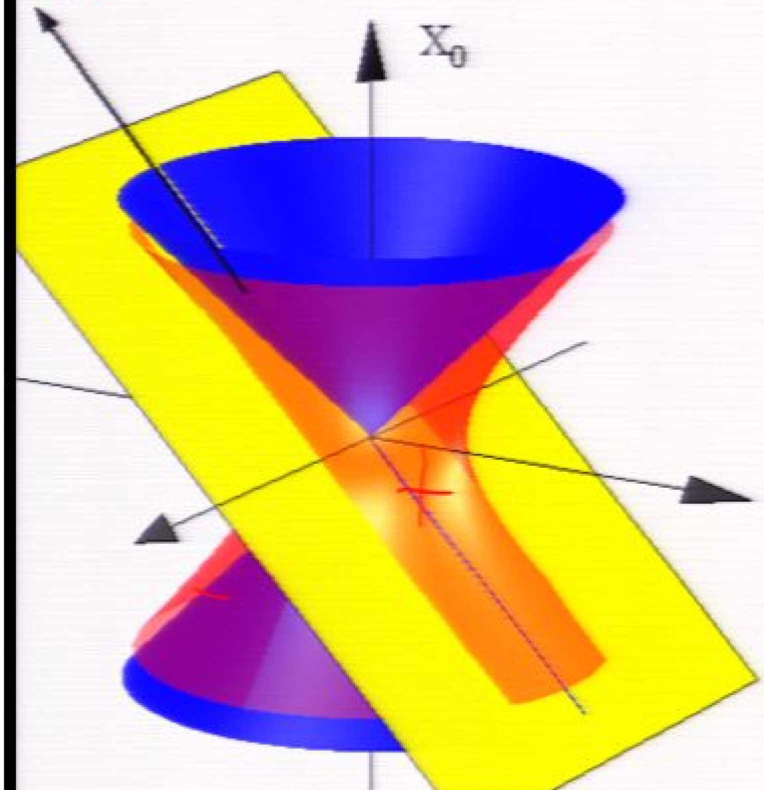

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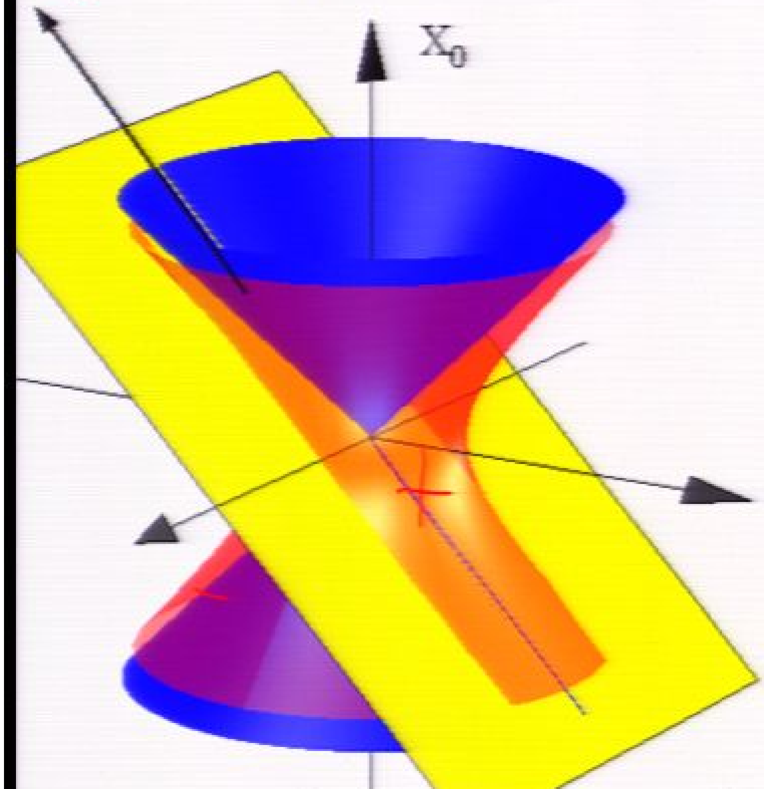

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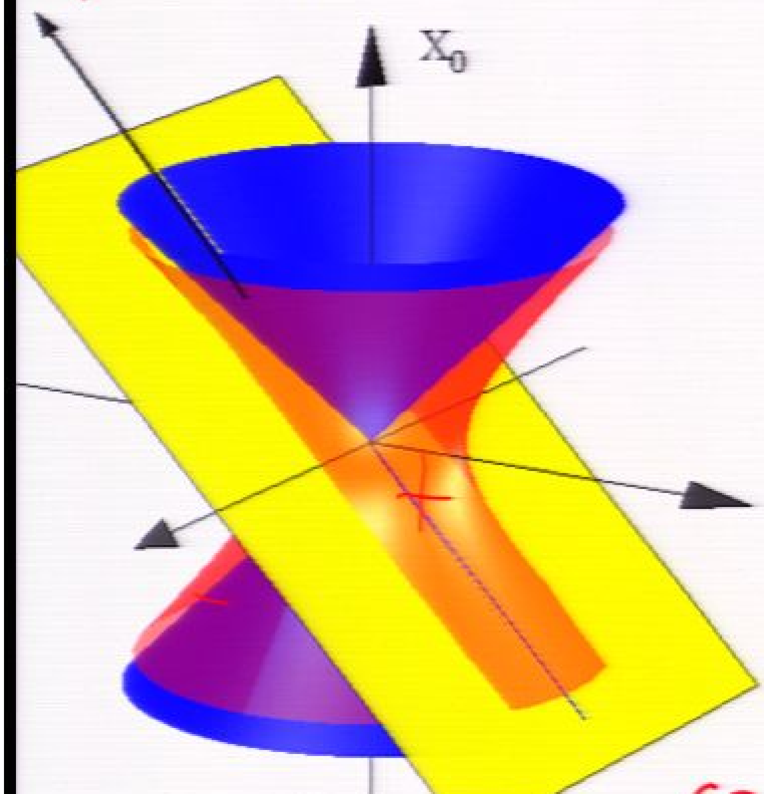

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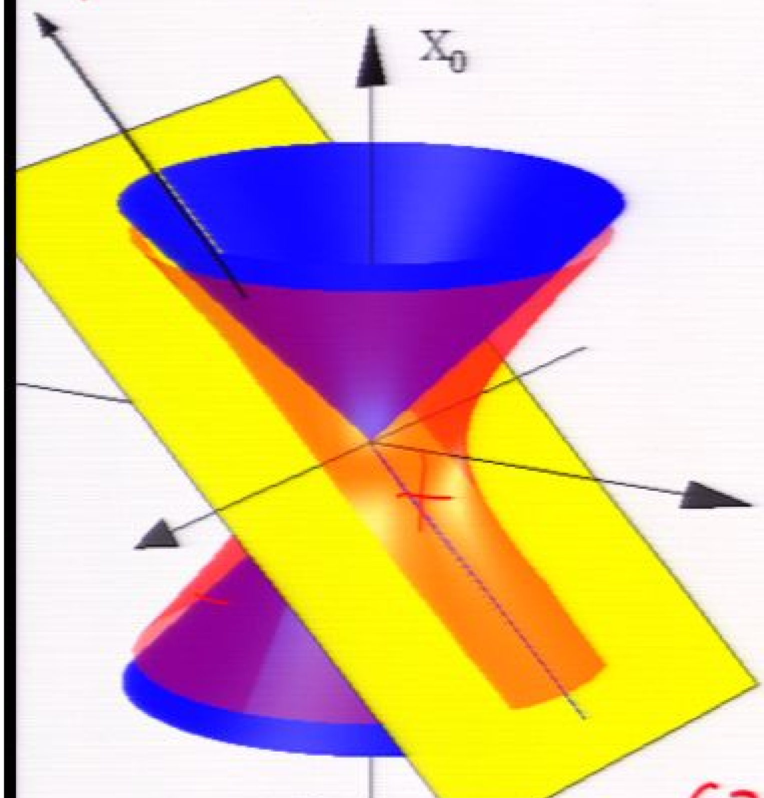

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Choice of a and b: go to QFT!

QFT: commutators

- For linear fields the algebraic structure (i.e. the degrees of freedom) is provided by the commutator function: a bidistribution that vanishes at spacelike separated pairs

$$[\phi(x_1), \phi(x_2)] = C(x_1, x_2)1$$

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- If the manifold is globally hyperbolic the equations of motion plus canonical initial conditions uniquely determine the commutator

$$(\Box_{x_i} + m^2)C(x_1, x_2) = 0, \quad i = 1, 2$$

$$C(x_1, x_2)|_{x_1^0 = x_2^0} = 0,$$

$$\frac{\partial}{\partial x_2^0} C(x_1, x_2)|_{x_1^0 = x_2^0} = i\hbar\delta(\mathbf{x_1} - \mathbf{x_2}) = 0$$

Quantization

Quantizing the theory means representing the commutation rules in a Hilbert space.

$$\phi(x) \to \hat{\phi}(x) \in \mathcal{T}'(\mathcal{M}, Op(\mathcal{H}))$$

$$[\hat{\phi}(x_1), \hat{\phi}(x_2)] = C(x_1, x_2)\mathbf{1}_{\mathcal{H}}$$

Quantization

- A Hilbert space representation is associated to any two-point function that solves the equations of motion
$$(\Box_{x_1} + m^2)W(x_1, x_2) = (\Box_{x_2} + m^2)W(x_1, x_2) = 0$$

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- For Minkowski or dS or other simmetries if unbroken: invariance:

$$W(gx_1, gx_2) = W(x_1, x_2)$$

Construction of two-point functions

$$W(x_1, x_2) = \int e^{-ip\cdot x_1} e^{ip\cdot x_2} \theta(p^0)\delta(p^2 - m^2)dp$$

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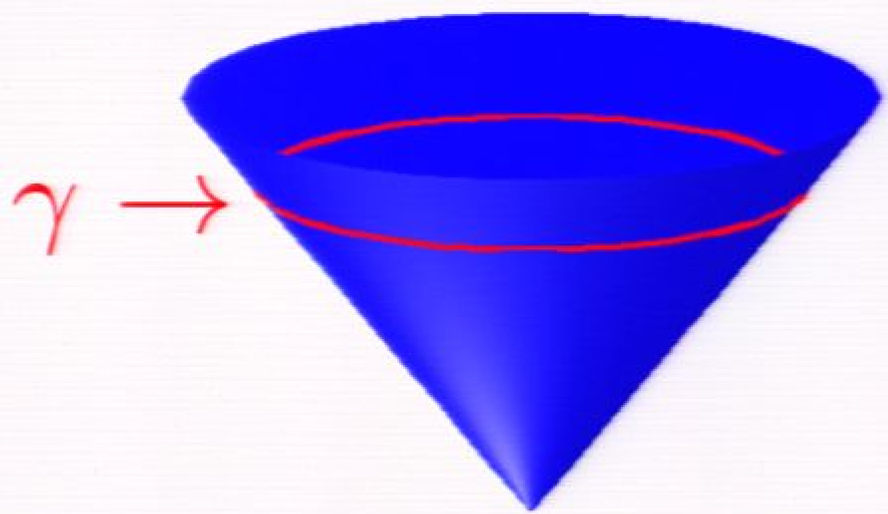

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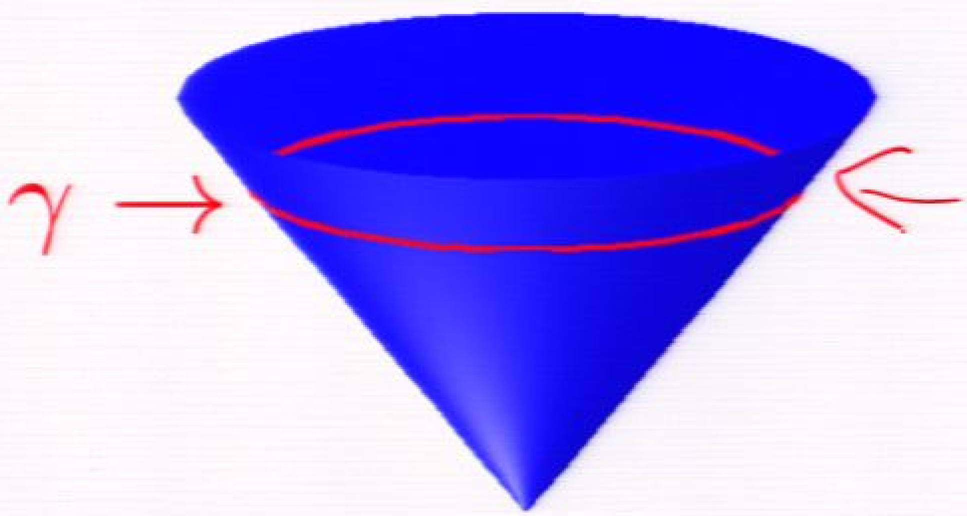

$$\left(\Box_{X_i} + \lambda\bar{\lambda}\right) F_{\lambda}(X_1, X_2) = 0,$$

They are dS invariant

Plane waves are homogeneous functions of ξ

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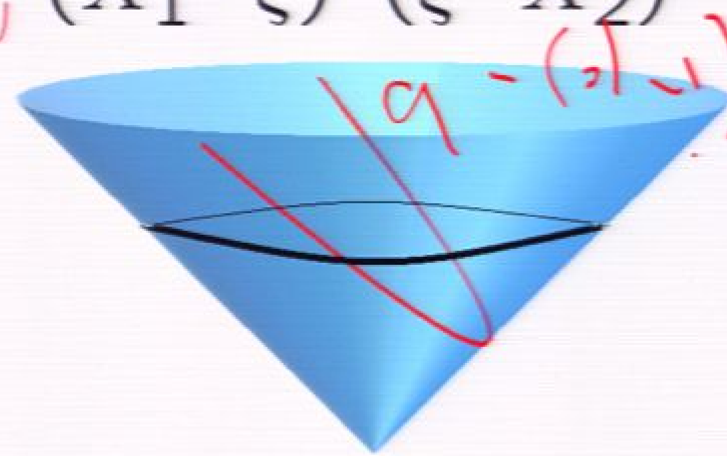

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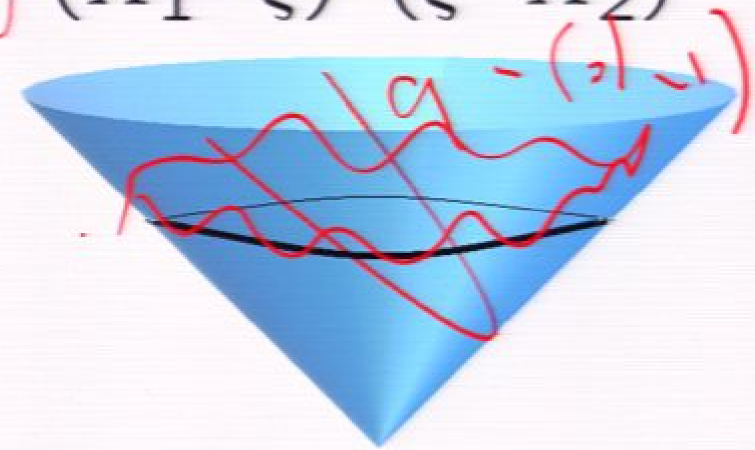

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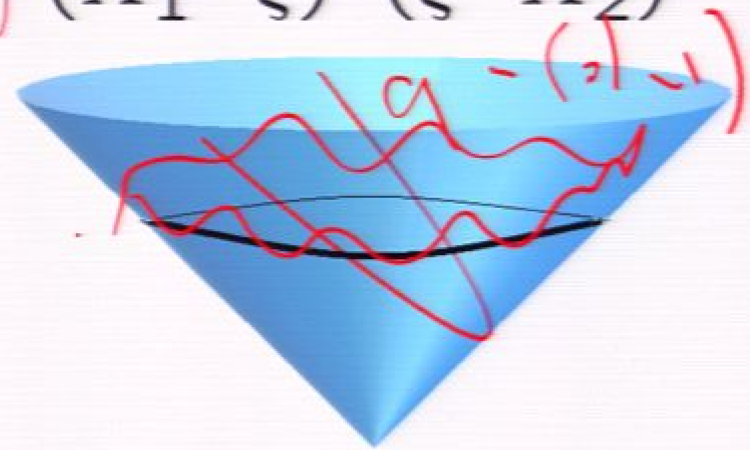

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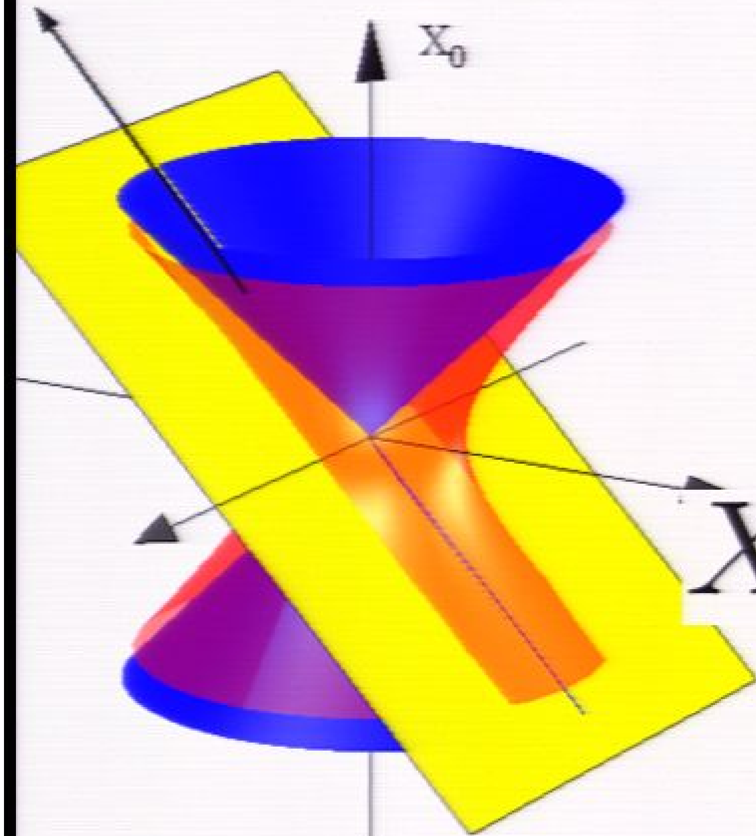
How to chose the right coefficients?



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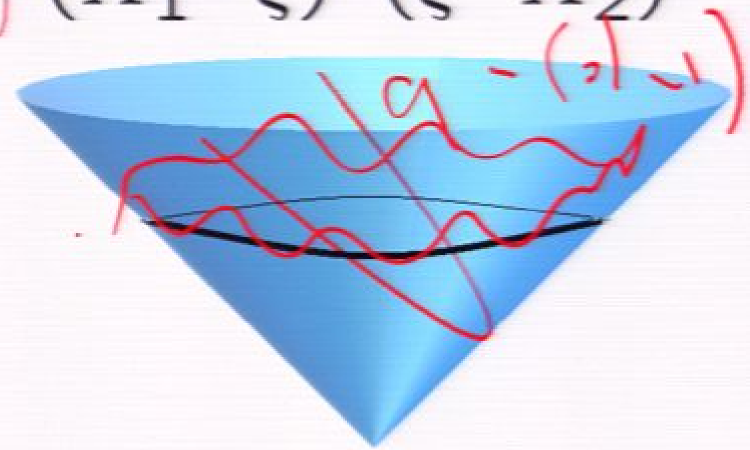

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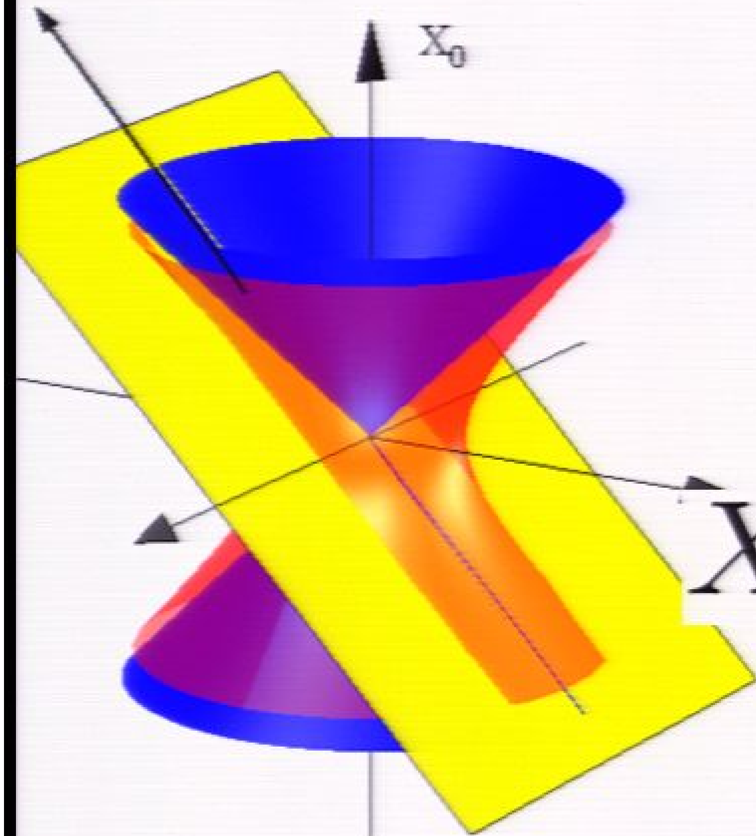

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Recall: spectral property of standard textbook QFT

There exists a complete set of nonegative energy states
(The energy-momentum spectrum is in the closed future cone)

equivalent to

The n point-function $W(x_1,\ldots\ldots, x_n)$ is the boundary value of a function $W(z_1,\ldots\ldots, z_n)$ holomorphic in a "tube" of the complex Minkowski spacetime
(tube = {Im $(z_{k+1} - z_k)$ contained in the closed future cone})

Consequences

Unique determination of the vacuum. All the common wisdom of perturbative renormalizable local and covariant QFT follows.

Spectral condition → the Fourier representation of the two-point function is meaningful in a domain of the complex Minkowski spacetime

$$W(x - x') = \int e^{-ip\cdot x} e^{ip\cdot x'} \theta(p^0)\delta(p^2 - m^2)$$

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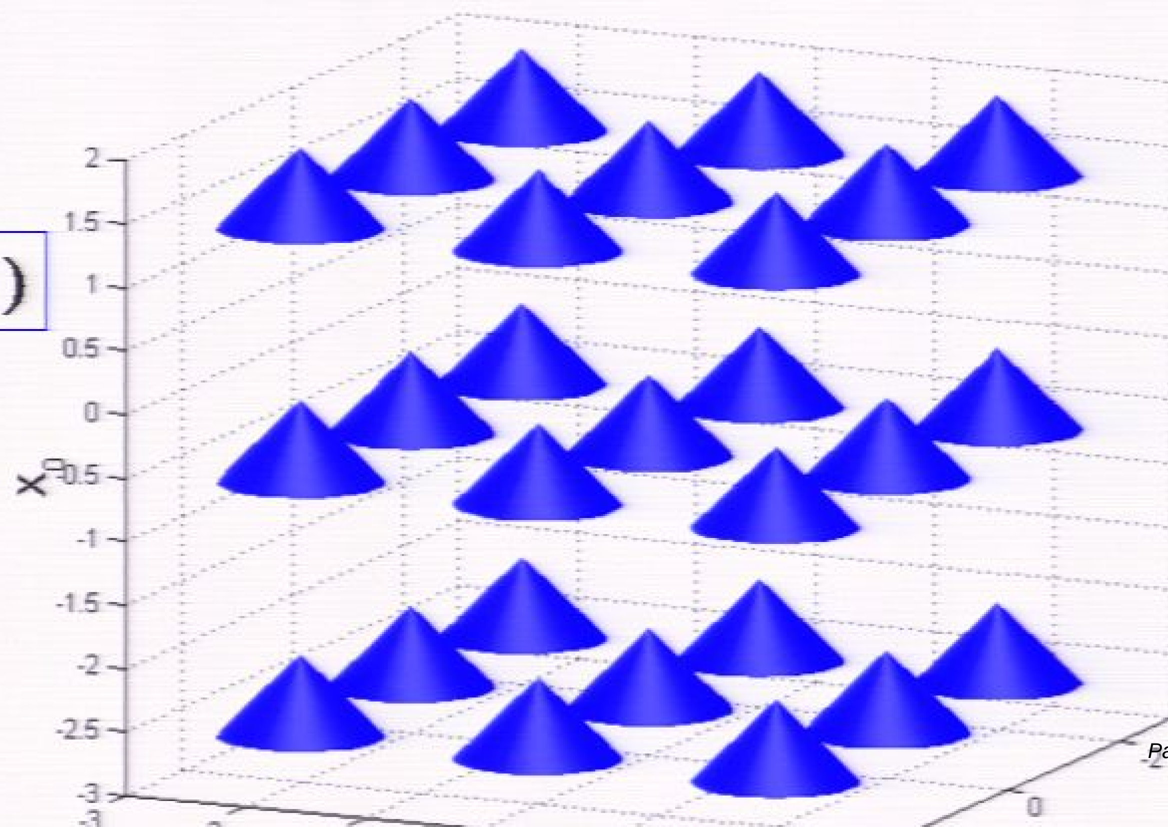

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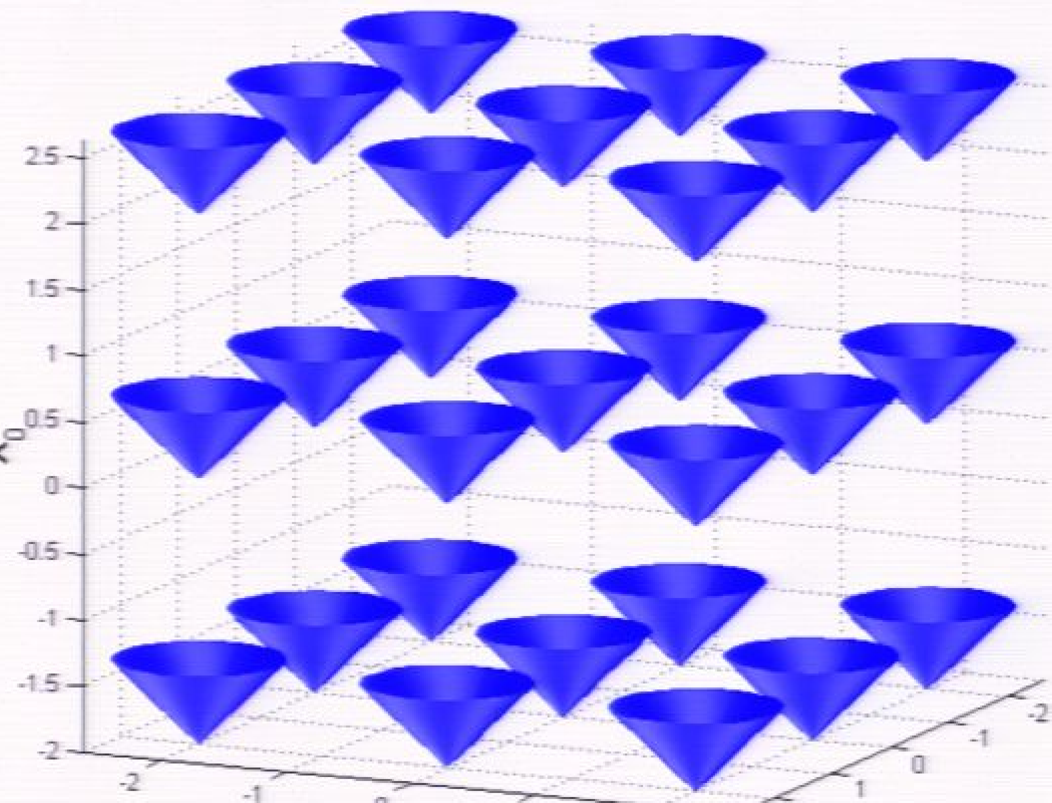
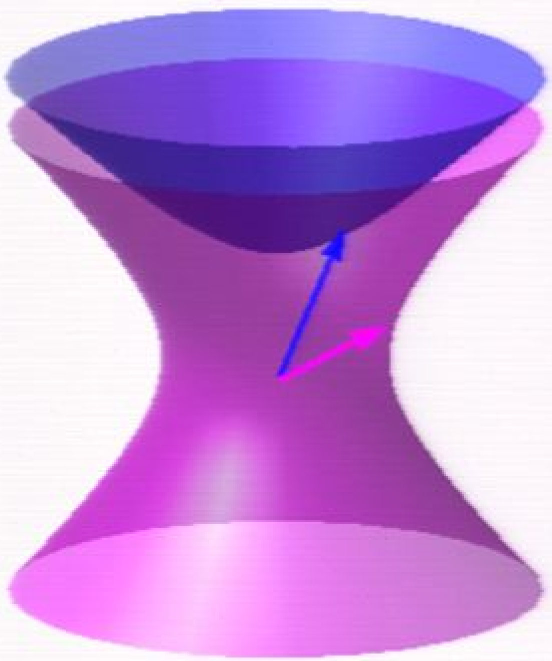


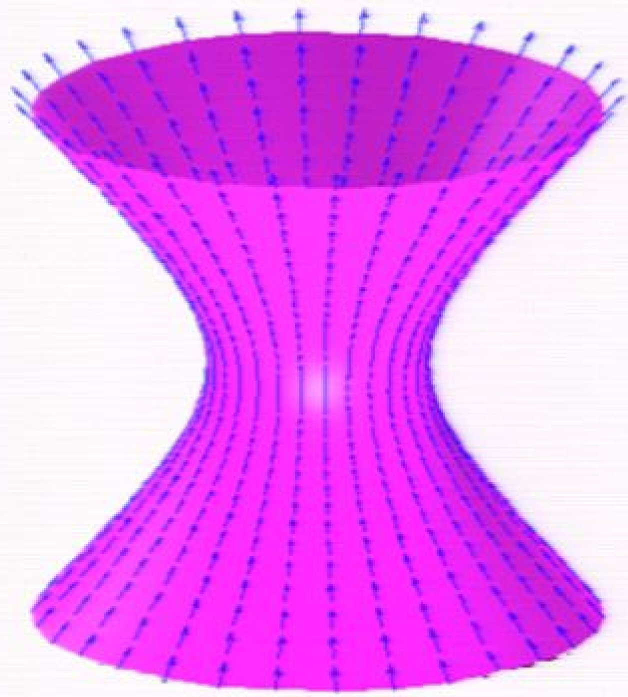
de Sitter tubes

$$dS^c = Z_0^2 - Z_1^2 - \dots Z_d^2 = -R^2$$

$$Z = X + iY, \quad X^2 - Y^2 = -R^2 \quad X \cdot Y = 0$$

$\mathcal{T}^+ = Y$ in the forward cone.





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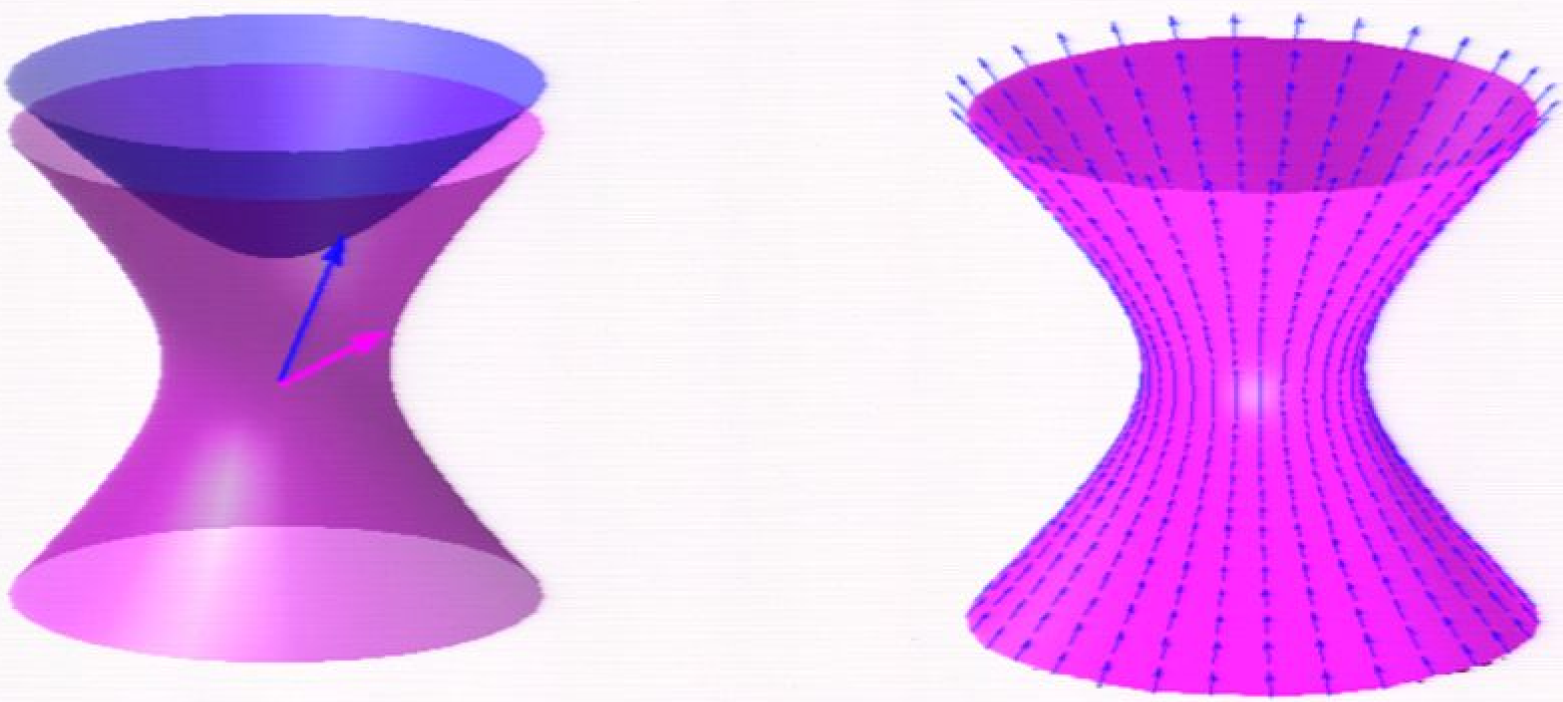

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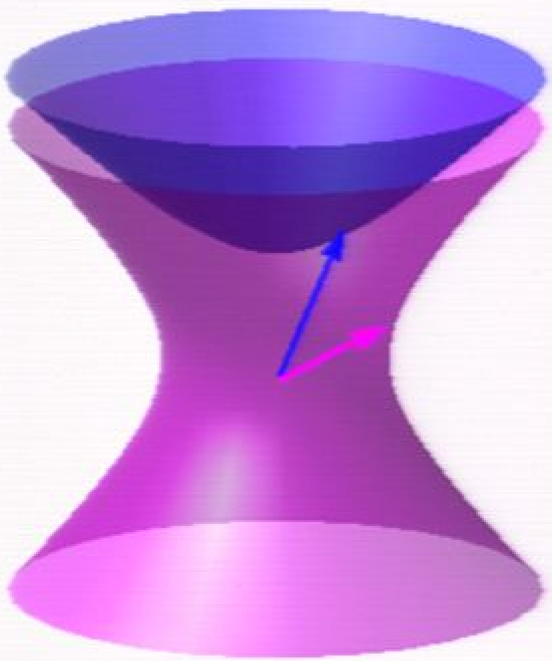
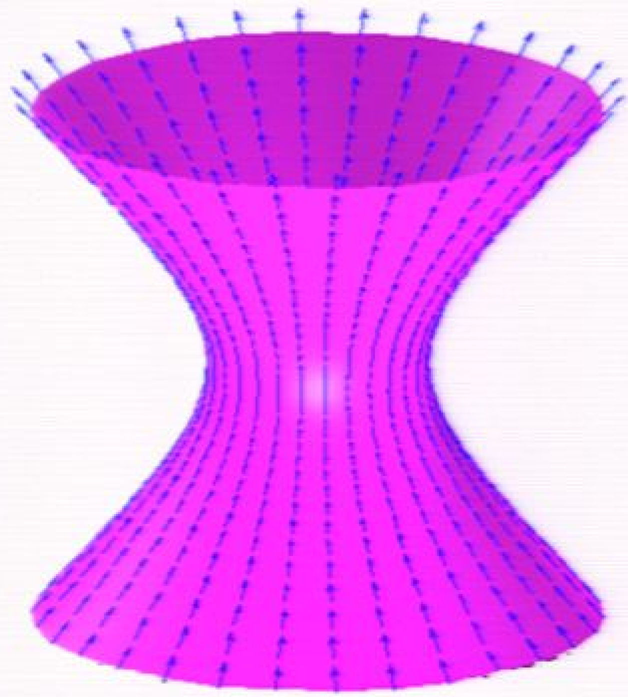
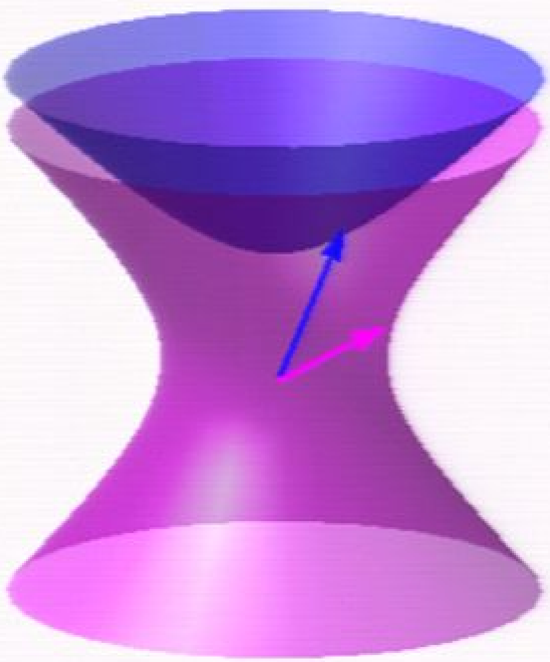
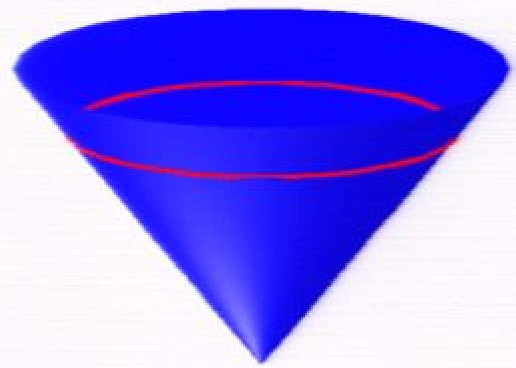




$\mathcal{T}^- = Y$ in the backward cone.

$\psi_\lambda(Z,\xi) = (Z \cdot \xi)^\lambda$ is globally well defined in both the past and future tubes because the imaginary part $Y \cdot \xi$ is always positive (negative) for $Z \in \mathcal{T}^+$ (alternatively $Z \in \mathcal{T}^+$)





Boundary values on the reals:

$$(X \cdot \xi)_\pm^\lambda \to |X \cdot \xi|^\lambda \left(\theta(X \cdot \xi) + e^{\pm i\pi\lambda}\theta(-X \cdot \xi)\right)$$

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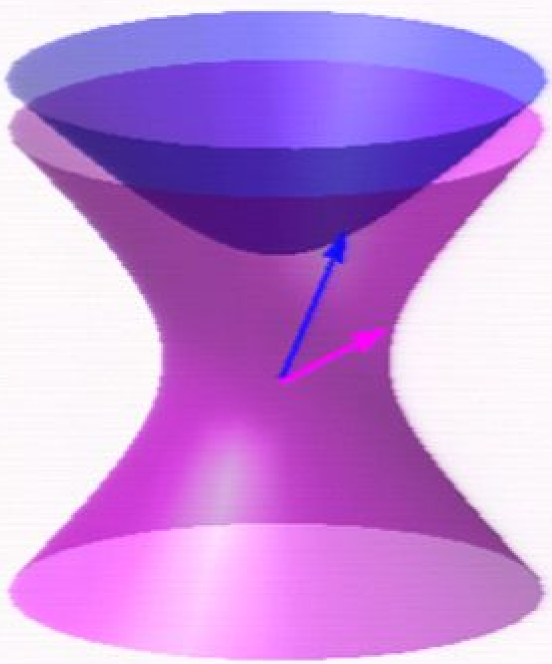

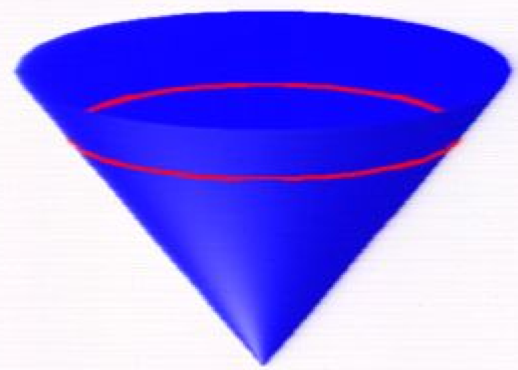

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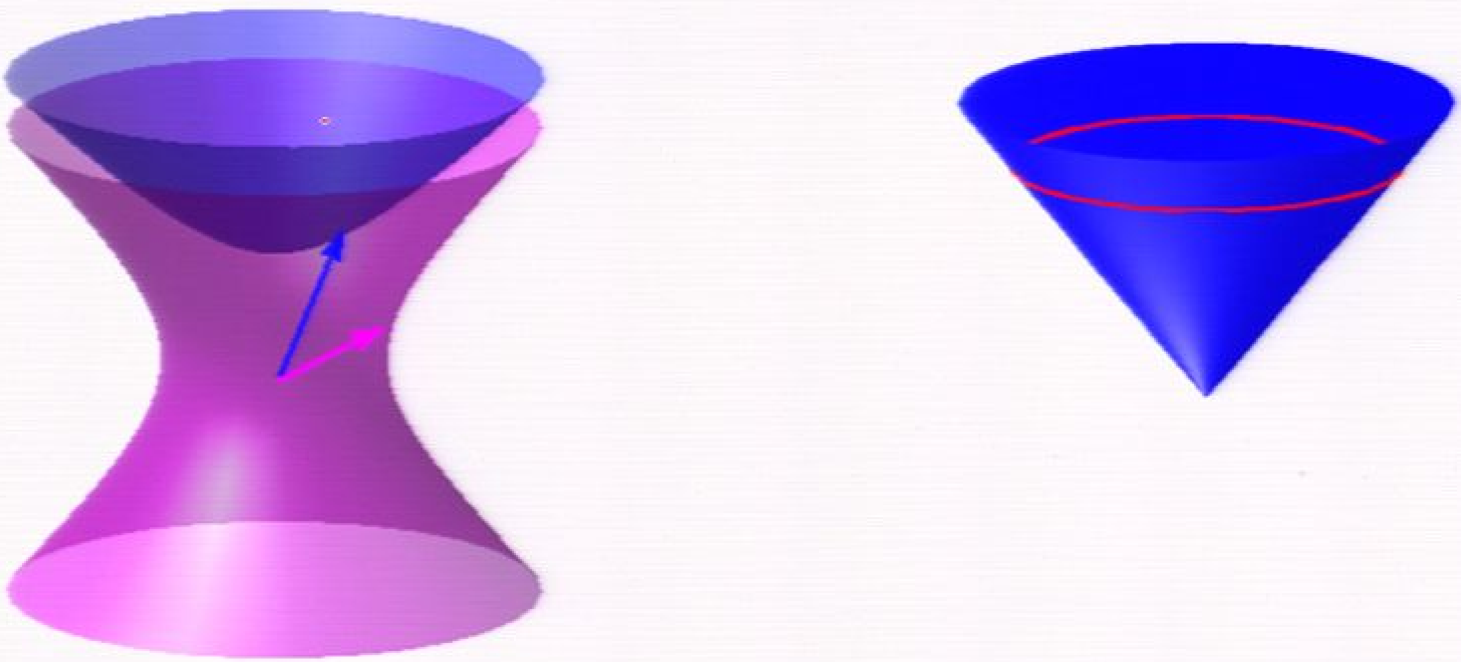

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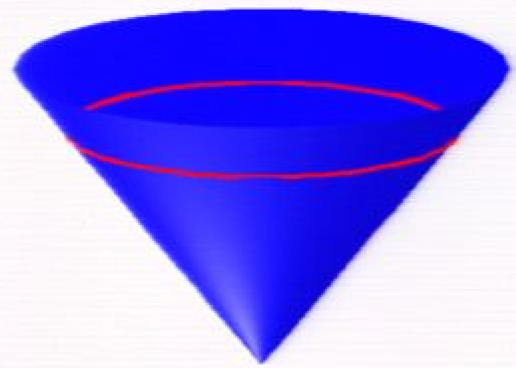

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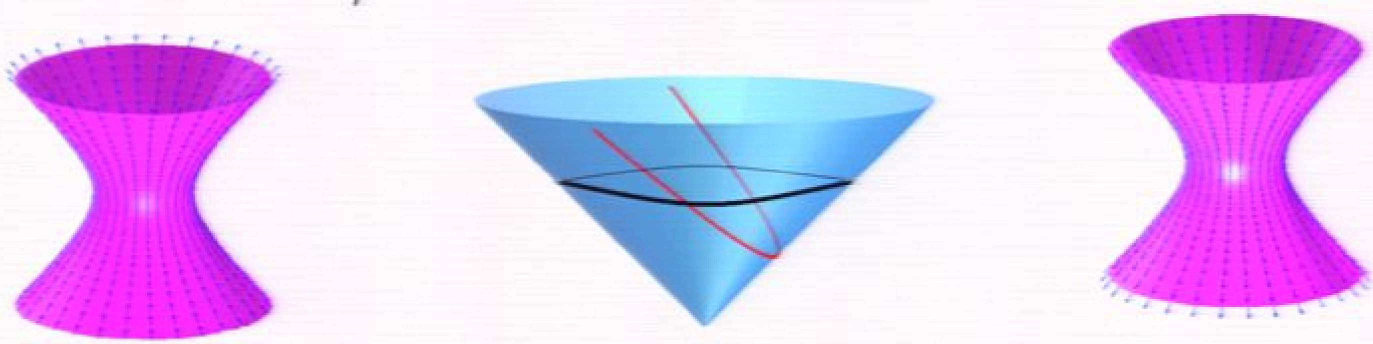
Fourier representation for Bunch-Davies aka Euclidean aka ……two-point functions

For $Z_1 \in \mathcal{T}^-$ e $Z_2 \in \mathcal{T}^+$

$$W_\lambda(Z_1, Z_2) = \int_\gamma (Z_1 \cdot \xi)^\lambda \ (\xi \cdot Z_2)^{-\lambda-(d-1)} d\mu(\xi)$$



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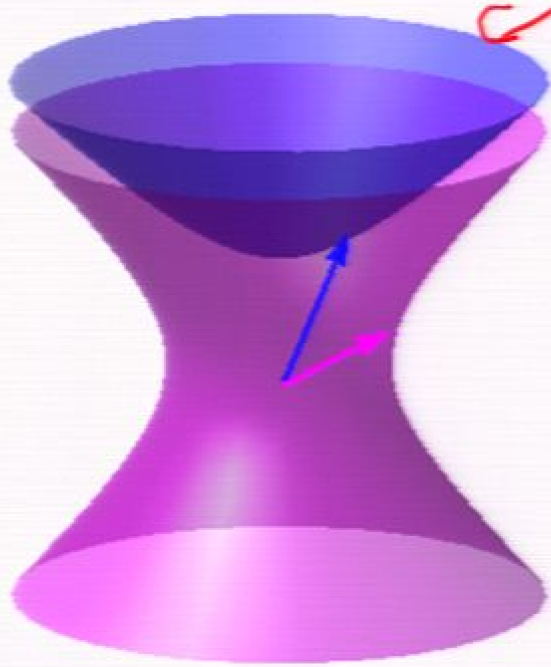

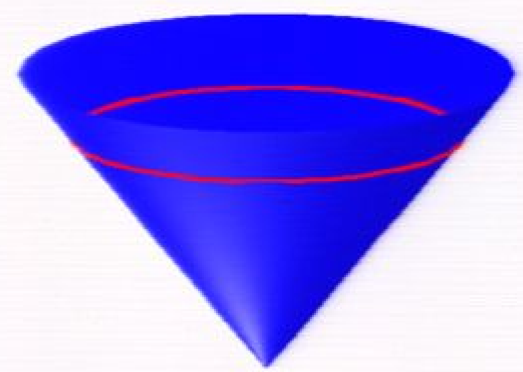

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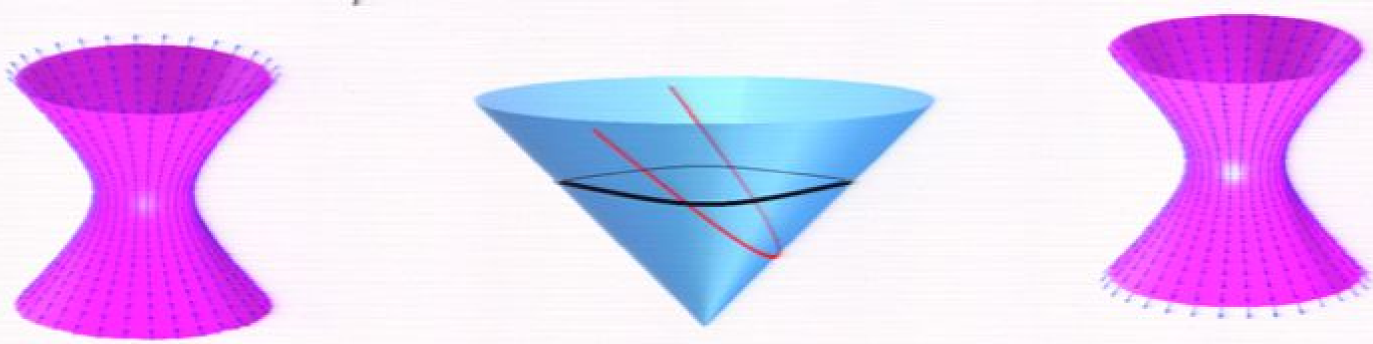


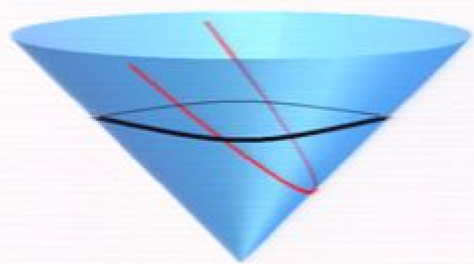
To be compared with the standard flat case:

$$W(z_1 - z_2) = \int e^{-ip\cdot z_1} e^{ip\cdot z_2} \theta(p^0)\delta(p^2 - m^2) d^4p$$

$z_1 \in T^-$ $z_2 \in T^+$

Fourier representation for Bunch-Davies aka Euclidean on the real manifold

$$W_\lambda(X_1, X_2) = \int_\gamma |X_1 \cdot \xi|^\lambda \, |X_2 \cdot \xi|^{\bar{\lambda}} \times$$
$$\times \left(\theta(X_1 \cdot \xi) + e^{-i\pi\lambda}\theta(-X_1 \cdot \xi)\right) \left(\theta(X_2 \cdot \xi) + e^{i\pi\bar{\lambda}}\theta(-X_2 \cdot \xi)\right) d\mu(\xi)$$



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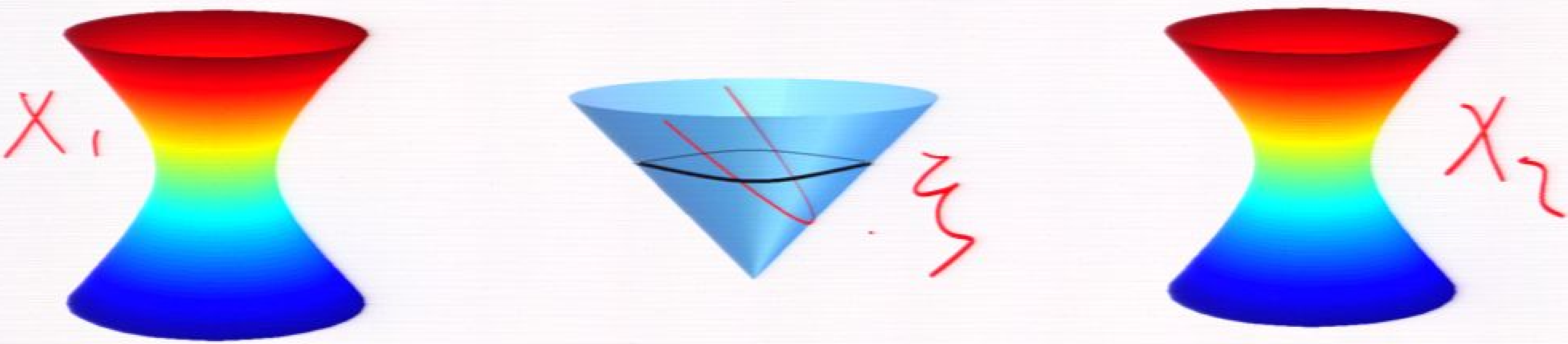

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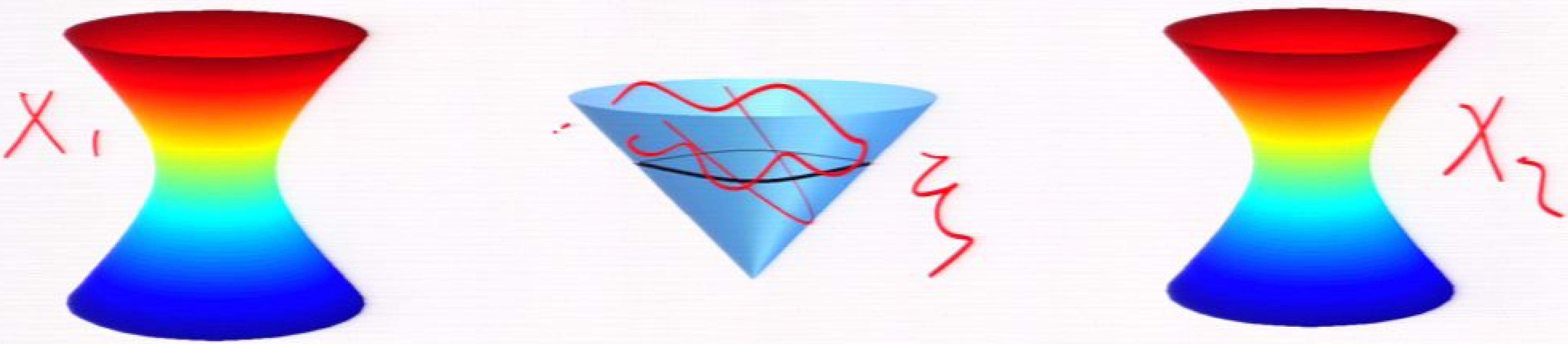

$$
\begin{aligned}
W_\lambda(Z_1,\ Z_2) &= \int_\gamma (Z_1\cdot\xi)^\lambda\ (\xi\cdot Z_2)^{-\lambda-(d-1)} d\mu(\xi) \\
&= \frac{\Gamma(-\lambda)\Gamma(\lambda+d-1)}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}\right)}\ {}_2F_1\left(-\lambda,\ \lambda+d-1;\ \frac{d}{2};\ \frac{1-\zeta}{2}\right)\ . \\
&= \frac{\Gamma(-\lambda)\,\Gamma(\lambda+d-1)}{2(2\pi)^{\frac{d}{2}}}\ (\zeta^2-1)^{-\frac{d-2}{4}} P^{-\frac{d-2}{2}}_{-\lambda-\frac{d}{2}}(\zeta)
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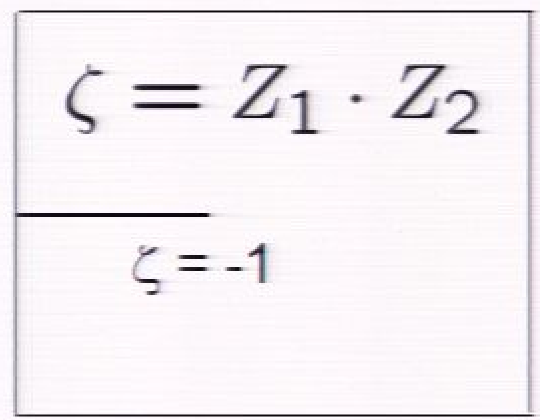
a) $W(Z_2, Z_2)$ is invariant under the complex de Sitter group

$W(Z_1, Z_2)$ is maximally analytic.

The cut reflects causality



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$$\zeta = Z_1 \cdot Z_2$$

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b) $W(X_1, X_2)$ is b.v.of $W(Z_1, Z_2)$ from $\mathcal{T}^- \times \mathcal{T}^+$

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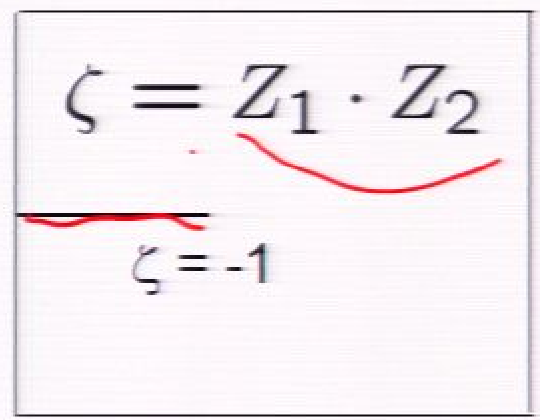

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The permuted function $W(X_2, X_1)$
is b.v.of the same $W(Z_1, Z_2)$ from $\mathcal{T}^+\times\mathcal{T}^-$

What about positivity?

$$\int W_\lambda(X_1, X_2)\bar{f}(X_1)f(X_2)dX_1dX_2 \geq 0$$

For principal fields positivity is obvious

$$W_\nu(Z_1, Z_2) = \int_\gamma (Z_1{\cdot}\xi)^{-\frac{d-1}{2}+i\nu}(\xi{\cdot}Z_2)^{-\frac{d-1}{2}-i\nu}d\mu(\xi)$$

dS-Fourier transform

$$\tilde{f}(\xi,\nu) = \int (\xi \cdot X)_+^{-\frac{d-1}{2}-i\nu} d\mu(\xi) f(x)dx$$

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Complementary fields (inflation)

$$W_{\nu'}(Z_1, Z_2) = \int_\gamma (Z_2 \cdot \xi)^{-\frac{d-1}{2}+\nu'} (\xi \cdot Z_2)^{-\frac{d-1}{2}-\nu'} d\mu(\xi)$$

The proof is a little more difficult;

based on the following relation valid for $\nu > 0$:

$$[(X \pm iY) \cdot \xi']^{-\frac{d-1}{2}+\nu} =$$

$$= \frac{e^{\pm i\pi\nu}\Gamma\left(\frac{d-1}{2}+\nu\right)}{(2\pi)^{\frac{d-1}{2}} 2^\nu \Gamma(\nu)} \int (\xi \cdot \xi')^{-\frac{d-1}{2}+\nu} [(X \pm iY) \cdot \xi']^{-\frac{d-1}{2}-\nu} d\mu(\xi')$$

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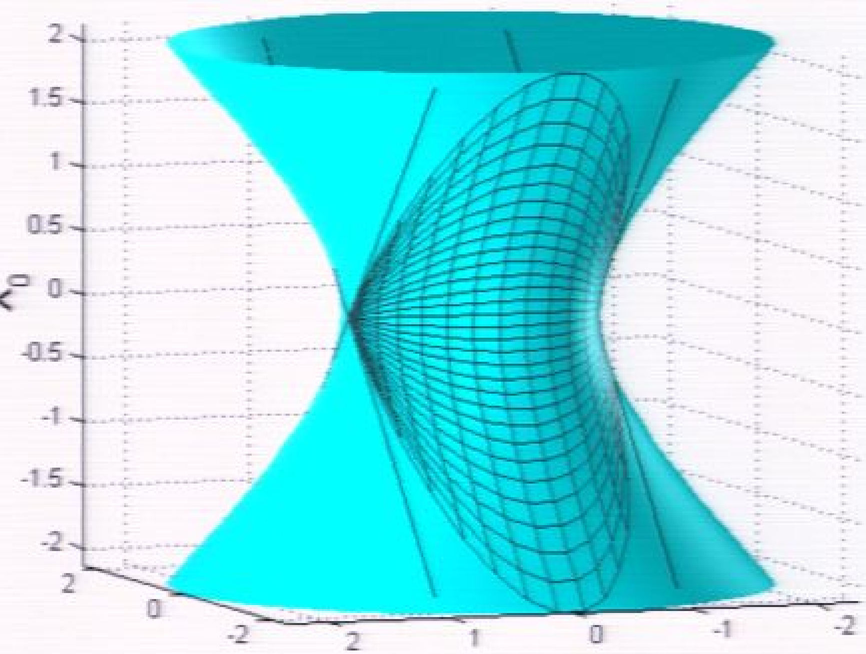
Physical interpretation: temperature

$$X(t,\vec{r}) = \begin{cases} X_0 &= R \sinh \frac{t}{R}\sqrt{R^2 - |\vec{r}|^2} \\ X_i &= r_i \quad (|\vec{r}|^2 \le R^2) \\ X_d &= R \cosh \frac{t}{R}\sqrt{R^2 - |\vec{r}|^2} \end{cases}$$



Time translations:

$$\alpha(s)X(t,\vec{r}) = X(t+s,\vec{r}) \equiv X(s)$$

Maximal analyticity ☛ KMS condition

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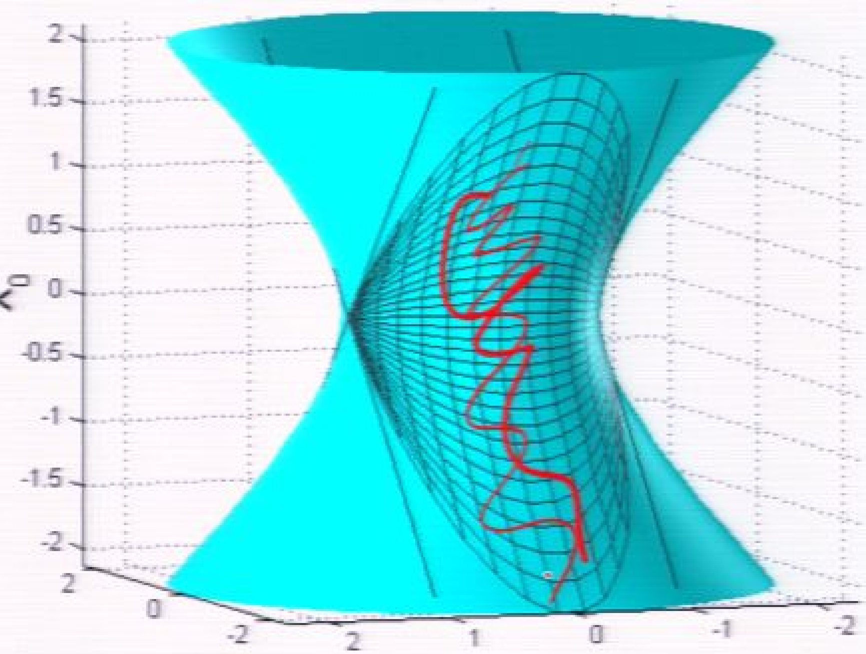

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De Sitter Tachyons

$$W_\lambda(Z_1,\ Z_2) = \Gamma(-\lambda)\, G_\lambda(\zeta)\ , \quad \zeta = Z_1 \cdot Z_2\,,$$

$$G_\lambda(\zeta) = \frac{\Gamma(\lambda + d - 1)}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}\right)}\, {}_2F_1\left(-\lambda,\ \lambda + d - 1;\ \frac{d}{2};\ \frac{1-\zeta}{2}\right)\ .$$

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$$G_\lambda(\zeta) = \frac{\Gamma(\lambda + d - 1)}{(4\pi)^{d/2}\Gamma\left(\frac{d}{2}\right)}\, {}_2F_1\left(-\lambda,\ \lambda + d - 1;\ \frac{d}{2};\ \frac{1-\zeta}{2}\right)\ .$$

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$\widehat{W}_n$ has the right commutator:

$$\widehat{W}_n(X_1, X_2) - \widehat{W}_n(X_2, X_1) = C_n(X_1, X_2)$$

The field equation gets an anomaly

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$$Q_n^- \,|phys\rangle = 0$$

Fourier representation

$$\widehat{W}_n(z_1, z_2) = W_n(z_1, z_2) - F_n^1(z_1, z_2) - F_n^2(z_1, z_2) + G_n(z_1,\ z_2).$$

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Remarks

For $d \geq 2$ is an even integer $z = -(z_1 - z_2)^2/4 = (1+\zeta)/2$.

$$\widehat{w}_n = z^{1-\frac{d}{2}} A(z,\, n,\, d) - \log(z) B(z,\, n,\, d) + C(z,\, n,\, d),$$

$A,\ B,\ C$ are polynomials in z
The most singular term is locally Hadamard(CCR)
$(4\pi)^{-\frac{d}{2}} \Gamma\left(\frac{d}{2} - 1\right) z^{1-\frac{d}{2}}$.

A fully positive de Sitter non-invariant Allen-Folacci type quantization does not exist for $m \neq 0$. Note that the Allen-Folacci two-point function does not coincide with $\widehat{w}_n$ on the physical space.

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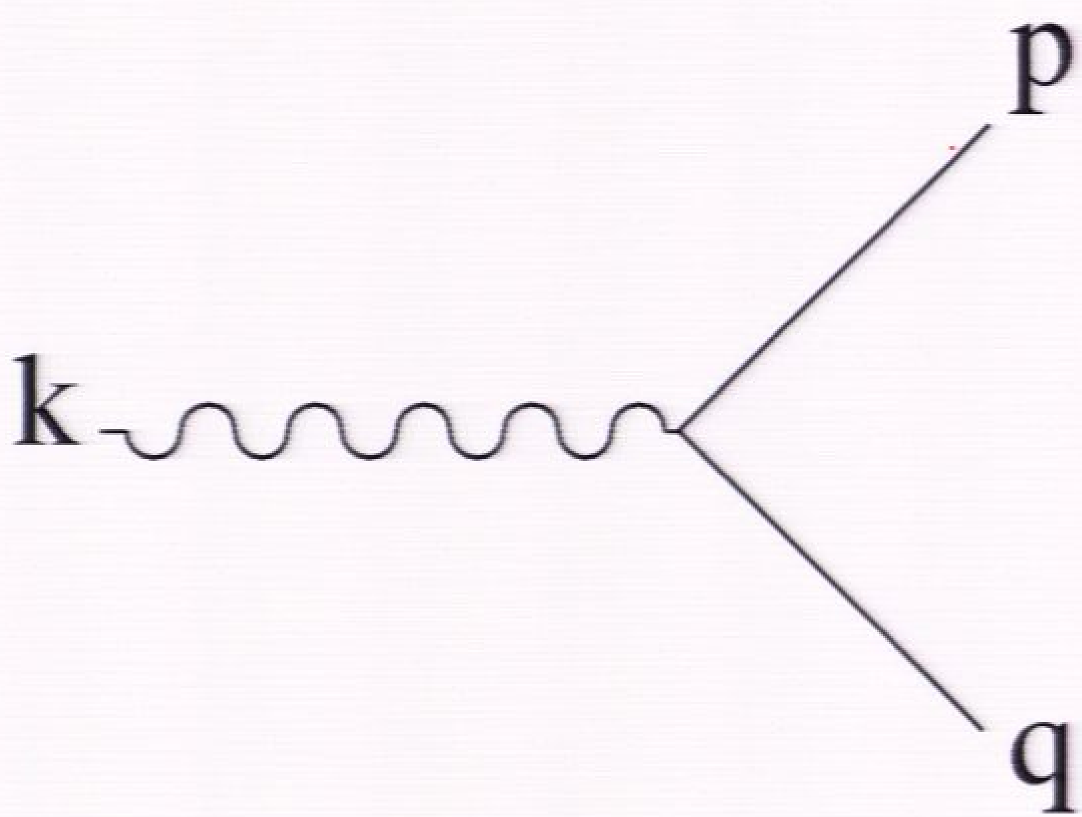
Unstable Particles



$$\langle pq|S|k\rangle \simeq \frac{2gi(2\pi)^4}{\sqrt{8p_0q_0k_0}}\delta_4(k-p-q)$$

Lifetime

Sum over all final states

$$\Gamma(1,2) = \frac{1}{T}\int dq \int dp |\langle pq|S|k\rangle|^2$$
$$\rightarrow \frac{g^2}{8\pi M}\sqrt{M^2 - 4m^2}\ \ \theta(M - 2m)$$

In the comoving system

$$\tau_0 = \frac{1}{\Gamma(1,2)}$$

Lifetime dilation

$$\tau = \frac{\tau_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

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Configuration space

Three scalar fields

$\phi_0,\ \phi_1,\ \phi_2$ masses $m_0,\ m_1,\ m_2$

The fields are independent

$$(\Omega,\ \phi_j(x)\,\phi_k(y)\Omega) = \delta_{jk}\,\mathcal{W}_{m_j}(x,\ y)$$

Fock space

$$\mathcal{H} = \mathcal{F}^{(0)} \otimes \mathcal{F}^{(1)} \otimes \mathcal{F}^{(2)}$$

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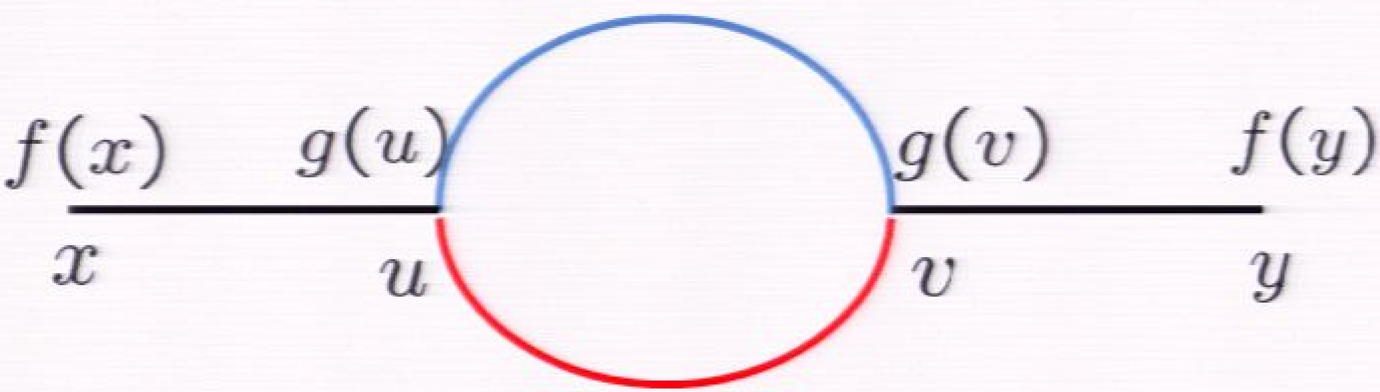
Transition probability

Sum over all final states:

$$\Gamma = \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\ u)\, [\mathcal{W}_{m_1}(u,\ v)\mathcal{W}_{m_2}(u,\ v)]\, \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\ .$$



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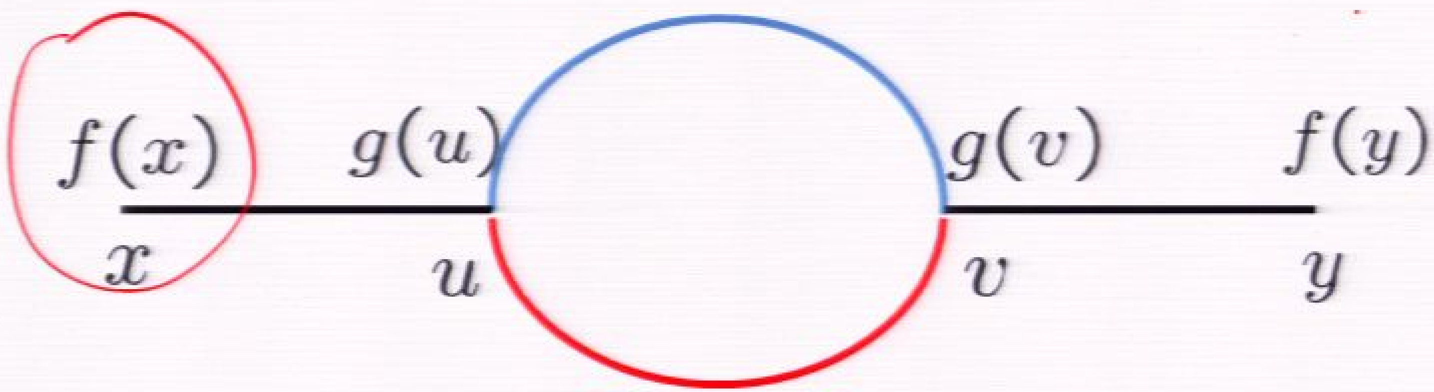

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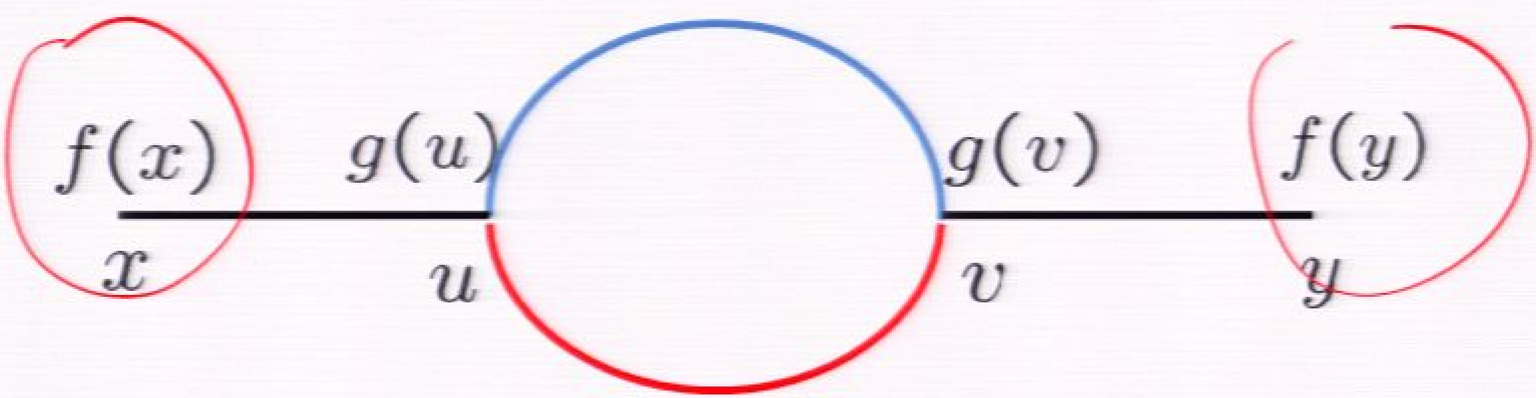

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Perturbations

The transition amplitude between 2 normalized states ψ_0 and ψ_1 is given by the scalar product

$$(\psi_0,\ S(\gamma g)\psi_1),$$

$$S(\gamma g) = \sum_{n=0}^{\infty} \frac{i^n \gamma^n}{n!} \int_{X^n} g(x_1)\, dx_1 \cdots g(x_n)\, dx_n\, T(\mathcal{L}(x_1) \ldots \mathcal{L}(x_n))$$

At first order, for orthogonal states

$$(\psi_0,\ S(\gamma g)\psi_1) = i(\psi_0, \int \gamma g(x)\, \mathcal{L}(x)\, dx\ \psi_1)$$

Interaction

Switch an interaction

$$\int \gamma\, g(x)\, \mathcal{L}(x)\, dx$$

$$\mathcal{L}(x) = \, : \phi_0(x) \phi_1(x)^{n_1} \phi_2(x)^{n_2} :$$

g(x) is an infrared cutoff . *g(x)* -> 1 in the end (adiabatic limit).

Special case $\mathcal{L}(x) = \, : \phi^n(x) :$

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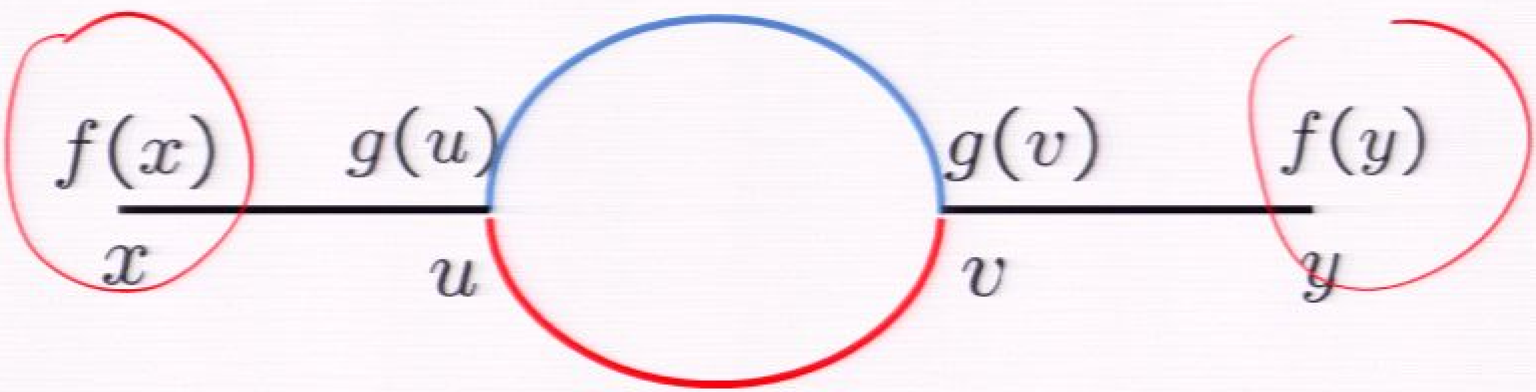
Sum over all final states:

$$\Gamma = \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\ u)\ [\mathcal{W}_{m_1}(u,\ v)\mathcal{W}_{m_2}(u,\ v)]\ \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\ .$$



Infinite volume limit $\qquad g(u) \to 1 \qquad g(v) \to 1.$

Final formula

$$\Gamma \;=\; \frac{\gamma^2}{(\psi_0,\ \psi_0)} \int \overline{f_0(x)}\, f_0(y)\, g(u)\, g(v) \times$$

$$\times \quad \mathcal{W}_{m_0}(x,\ u)\ [\mathcal{W}_{m_1}(u,\ v)\mathcal{W}_{m_2}(u,\ v)]\, \mathcal{W}_{m_0}(v,\ y)\, dx\, du\, dv\, dy\ .$$

$$\Gamma = \gamma^2 C(m_0, d)\ \rho(m_0^2; m_1, m_2)\ \frac{\int |F_0(v)|^2 g(v)\, dv}{\int \overline{f_0(x)}\, \mathcal{W}_{m_0}(x,\ y)\, f_0(y) dx\, dy}$$

$$\int \mathcal{W}_{m_0}(x,\ u)\, \mathcal{W}_a(u,\ v)\, du = C(m_0,\ d)\delta(m_0^2 - a^2)\mathcal{W}_{m_0}(x,\ v).$$

$$\mathcal{W}_{m_1}(u,\ v)\mathcal{W}_{m_2}(u,\ v) = \int \rho(a^2; m_1, m_2)\, \mathcal{W}_a(u,\ v)\, da^2$$

$$F_0(v) = \int \mathcal{W}_{m_0}(v,\ x)\, f_0(x) dx$$

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Projector identity

- Proiector identity: non trivial holds only for the pricipal series

$$\int w_\nu(z,\, x)\, w_{\nu'}(x,\, y)\, dx = 2\pi \coth \pi\nu\delta(\nu^2 - \nu'^2) w_\nu(z,\, y)$$

- Need integration over the whole de Sitter manifold
- Badly diverging for complementary fields

KL weight: non trivial

Evaluate the Mehler-Fock transform

$$h_d(\kappa,\nu,\lambda) = \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(u)\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\lambda}(u)(u^2-1)^{-\frac{d-2}{4}}\, du$$

which provides the Kallen-Lehmann weight

$$\rho(\kappa^2,\nu,\lambda) \ = \ \frac{\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)\Gamma\left(\frac{d-1}{2}+i\lambda\right)\Gamma\left(\frac{d-1}{2}-i\lambda\right)}{2(2\pi)^{1+\frac{d}{2}}}\sinh(\pi\kappa)\ h_d(\kappa,\nu,\lambda),$$

Equal masses

$$W_\nu^2(z,z') = \int_0^\infty d\kappa^2 \; \rho(\kappa^2,\nu) W_\kappa(z,z')$$

Mehler-Fock transform of the squared 2-point function:

$$\rho(\kappa^2,\nu) = \frac{\left(\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)\right)^2 \sinh \pi\kappa}{2(2\pi)^{1+\frac{d}{2}} R^{d-2}} \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(x)\,[P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(x)]^2 (x^2-1)^{-\frac{d-2}{4}}\, dx.$$

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Formula 19, p. 535 of Prudnikov et al. Vol. III:

$${}_3F_2\left(\begin{matrix} a,\ b,\ c;\ 1 \\ a+b+\frac{1}{2},\ \ c+\frac{1}{2}\end{matrix}\right) \stackrel{?}{=} \sqrt{\pi}\,\Gamma\left[\begin{matrix} a+b+\frac{1}{2},\ \ c+\frac{1}{2},\ \ \frac{1}{2}+c-a-b \\ a+\frac{1}{2},\ \ b+\frac{1}{2},\ \ \frac{1}{2}+c-a,\ \ \frac{1}{2}+c-b\end{matrix}\right]$$

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By using Mellin transform techniques we got

$$\rho(\kappa^2,\nu) = \frac{R^{2-d}\sinh \pi\kappa}{(4\pi)^{\frac{d+2}{2}}\sqrt{\pi}\Gamma(\frac{d-1}{2})} \frac{\Gamma\left(\frac{d-1}{4}+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}-\frac{i\kappa}{2}\right)}{\Gamma\left(\frac{d+1}{4}+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d+1}{4}-\frac{i\kappa}{2}\right)}$$
$$\times\, \Gamma\left(\frac{d-1}{4}+i\nu+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}-i\nu+\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}+i\nu-\frac{i\kappa}{2}\right)\Gamma\left(\frac{d-1}{4}-i\nu-\frac{i\kappa}{2}\right)$$

KL weight: unequal masses

Evaluate the Mehler-Fock transform

$$h_d(\kappa,\nu,\lambda) = \int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(u)\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\lambda}(u)(u^2-1)^{-\frac{d-2}{4}}\, du$$

Mellin's techniques fail. One needs a vectorial Fourier transform adapted to the de Sitter geometry

Another integral representation

Let us evaluate the 2 pt function at purely imaginary events of the past e future tubes

$$\mathcal{W}_\nu(-iy, iy') = \frac{\Gamma(\frac{d-1}{2}+i\nu)\Gamma(\frac{d-1}{2}-i\nu)}{2^{d+1}\pi^d}\int_\gamma (y\cdot\xi)^{-\frac{d-1}{2}+i\nu}\left(\xi\cdot y'\right)^{-\frac{d-1}{2}-i\nu} d\mu_\gamma(\xi) =$$

$$= \frac{\Gamma\left(\frac{d-1}{2}+i\nu\right)\Gamma\left(\frac{d-1}{2}-i\nu\right)}{2(2\pi)^{\frac{d}{2}}}\left(\left(y\cdot y'\right)^2-1\right)^{-\frac{d-2}{4}} P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}\left(y\cdot y'\right)$$

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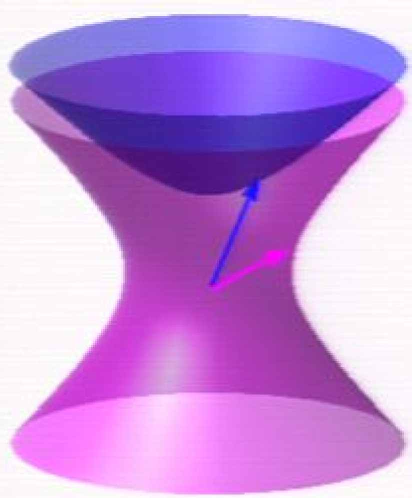

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$$y = (u, \sqrt{u^2-1}, \vec{\omega})$$
$$\xi = (1, \vec{\Omega})$$
$$y' = (1, 0, \ldots, 0)$$
$$y\cdot y' = y^0 = u \geq 1$$
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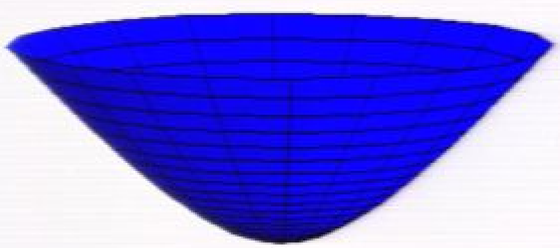

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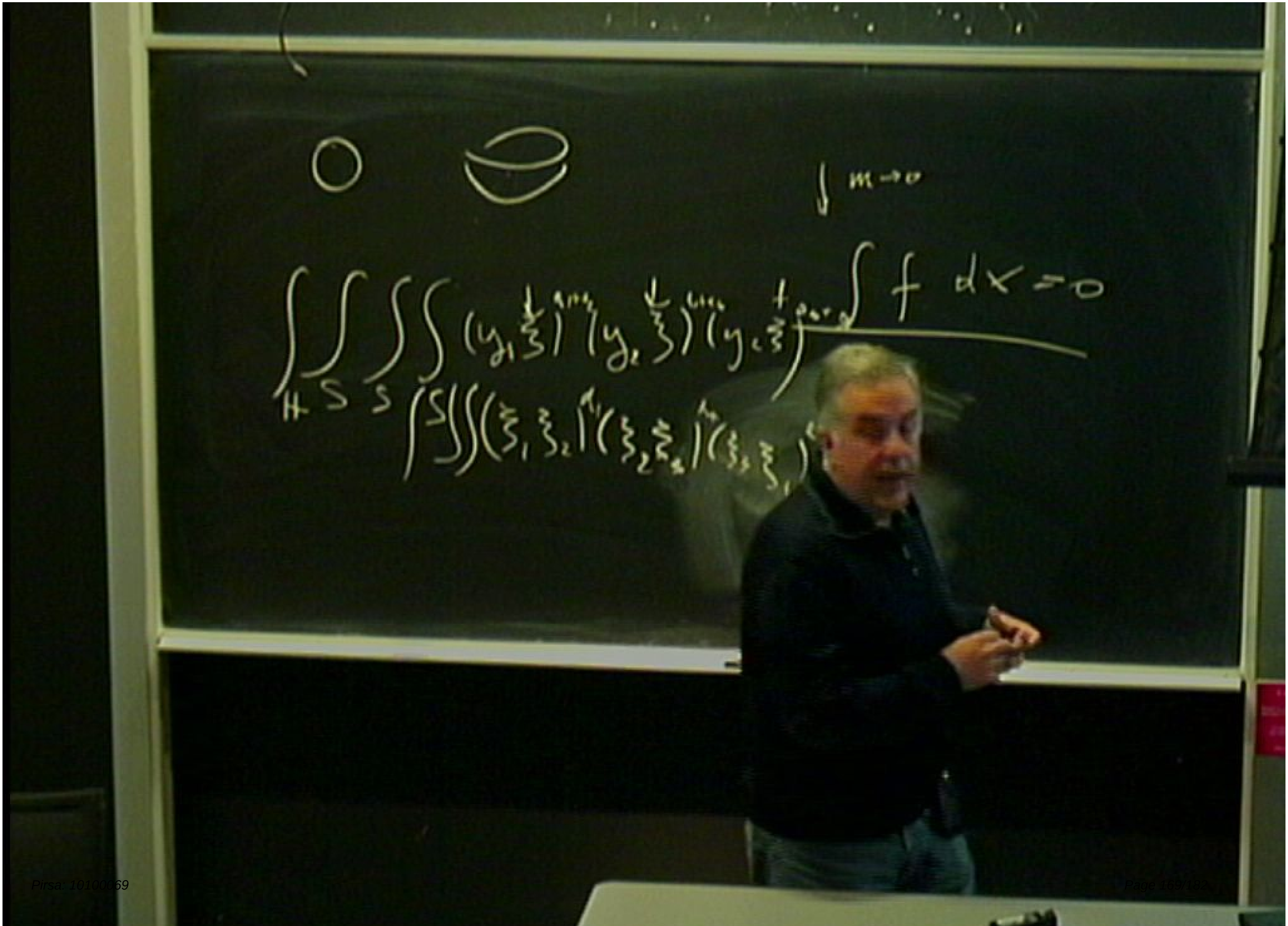
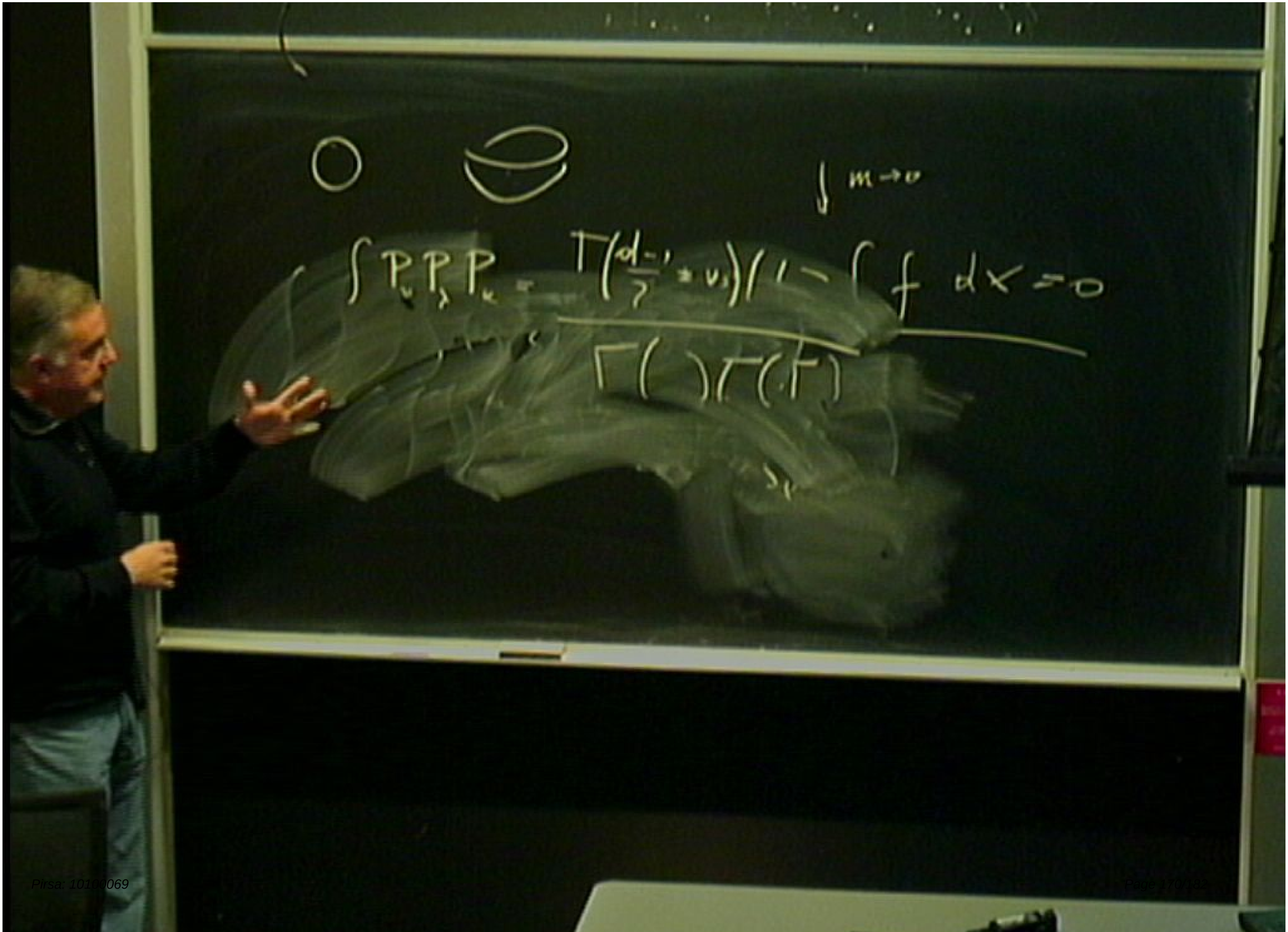
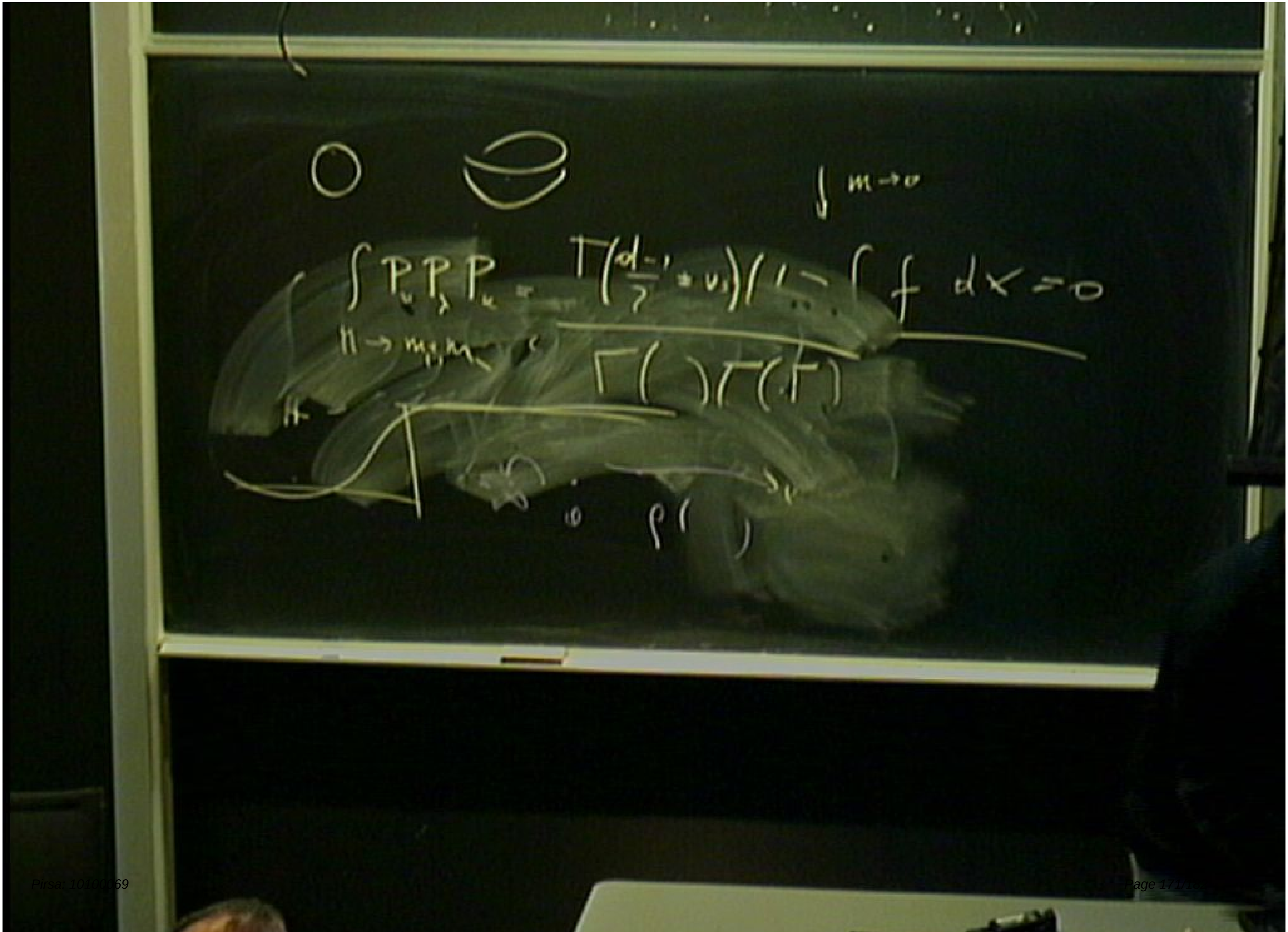
$$\xi = (1, \vec{\Omega})$$
$$y' = (1, 0, \ldots, 0)$$
$$y\cdot y' = y^0 = u \geq 1$$
$$y'\cdot\xi = 1$$

$$\left(u^2-1\right)^{-\frac{d-2}{4}} P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) = \frac{1}{(2\pi)^{\frac{d}{2}}}\int_{\gamma_0} (y\cdot\xi)^{-\frac{d-1}{2}-i\nu} d\mu_\gamma(\xi)$$


m → 0
f dx = 0


m → 0
f dx = 0



Integrate the triangle

2) Triple integral on the sphere

$$J = \int_{S^3_{d-1}} (\xi_1 \cdot \xi_2)^{a_3} (\xi_2 \cdot \xi_3)^{a_1} (\xi_3 \cdot \xi_1)^{a_2} \, d\Omega_1 \, d\Omega_2 \, d\Omega_3,$$

A Beautiful formula

$$\int_1^\infty P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\kappa}(u)\, P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\nu}(u) P^{-\frac{d-2}{2}}_{-\frac{1}{2}+i\lambda}(u)(u^2-1)^{-\frac{d-2}{4}}\, du =$$

$$= \frac{\prod_{\epsilon,\epsilon',\epsilon''=\pm 1}\Gamma\left(\frac{d-1}{4}+\frac{i\epsilon\kappa+i\epsilon'\nu+i\epsilon''\lambda}{2}\right)}{\left[\prod_{\epsilon=\pm 1}\Gamma\left(\frac{d-1}{2}+i\epsilon\kappa\right)\right]\left[\prod_{\epsilon'=\pm 1}\Gamma\left(\frac{d-1}{2}+i\epsilon'\nu\right)\right]\left[\prod_{\epsilon''=\pm 1}\Gamma\left(\frac{d-1}{2}+i\epsilon''\lambda\right)\right]}$$

ρ never vanishes.
For $m > m_c = (d-1)/2R$ decays into heavier particeles are always possible

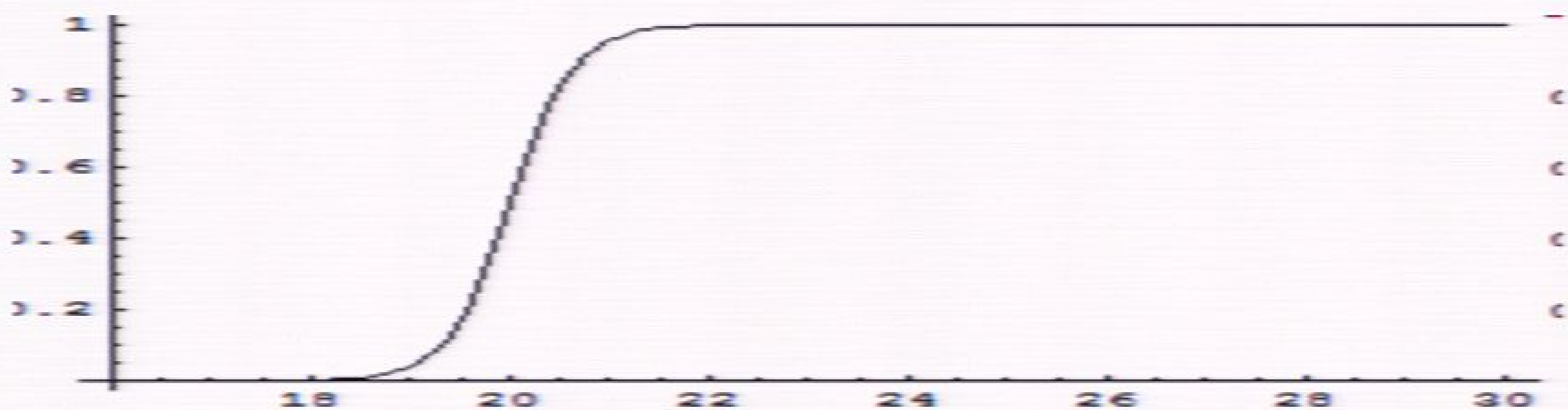
Surprisingly (for me) the Minkowskian result is recovered in the zero curvature limit by posing $\kappa = MR$, $\nu = mR$ $\lambda = m'R$:

$$\lim_{R\to\infty} \rho(\kappa^2;\nu,\lambda)\ d\kappa^2 = \rho(M^2; m, m')\ dM^2.$$

Dimension d=3

$$W_\nu(x,y)^2 = \frac{1}{4\pi}\int_0^\infty \frac{\tanh(\frac{\pi\kappa}{2})}{(\cosh\pi\kappa + \cosh 2\pi\nu)}\,\frac{\sinh\pi\kappa}{\kappa} W_\kappa(x,y)\,d\kappa^2$$

$$\Gamma(1,2) = \frac{R}{4\nu^2}\frac{\tanh(\frac{\pi\nu}{2})}{\left(1+\frac{\cosh 2\pi\kappa}{\cosh\pi\nu}\right)} \to \frac{1}{4M^2}\theta(M-2m)$$



Off

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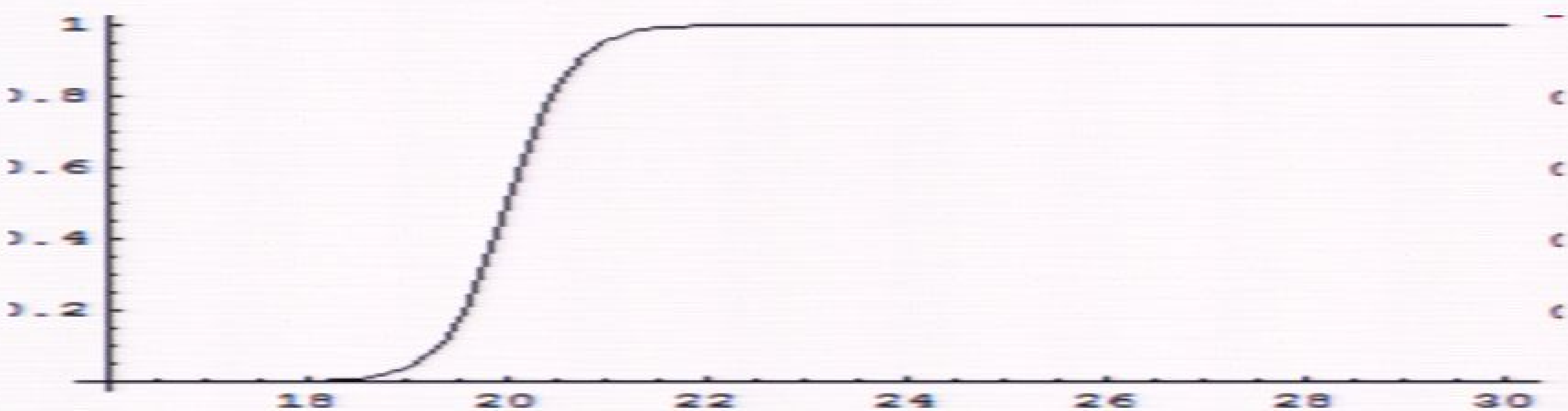

Complementary fields. Inflation

$$W_\nu^2(z,z') = \int_{-\infty}^{\infty} \kappa d\kappa \;\rho(\kappa^2,\nu) W_\kappa(z,z')$$

$$W_\nu^2 = \int_{-\infty}^{\infty} \kappa\, d\kappa \;\rho_\nu(\kappa) W_\kappa \;+ \sum_{n=0}^{N-1} A_n(\nu)\, W_{i(\mu+2i\nu+2n)}$$

$$A_n(\nu) \;=\; \frac{8\pi(-1)^n}{n!2^d\pi^{\frac{1+d}{2}} R^{d-2}\Gamma(\mu)} \frac{\Gamma(\mu+2i\nu+n)\Gamma(-2i\nu-n)}{\Gamma(\mu+2i\nu+2n)\Gamma(-\mu-2i\nu-2n)} \times \frac{\Gamma(\mu+n)\Gamma(-i\nu-n)\Gamma(\mu+i\nu+n)}{\Gamma(-i\nu-n+\frac{1}{2})\Gamma(\mu+i\nu+n+\frac{1}{2})}$$

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$$W_\nu^2(z,z') = \int_{-\infty}^{\infty} \kappa d\kappa \; \rho(\kappa^2,\nu) W_\kappa(z,z')$$

$$W_\nu^2 = \int_{-\infty}^{\infty} \kappa\, d\kappa \; \rho_\nu(\kappa) W_\kappa \; + \sum_{n=0}^{N-1} A_n(\nu)\, W_{i(\mu+2i\nu+2n)}$$

$$A_n(\nu) \;=\; \frac{8\pi(-1)^n}{n! 2^d \pi^{\frac{1+d}{2}} R^{d-2} \Gamma(\mu)} \frac{\Gamma(\mu+2i\nu+n)\Gamma(-2i\nu-n)}{\Gamma(\mu+2i\nu+2n)\Gamma(-\mu-2i\nu-2n)} \times \frac{\Gamma(\mu+n)\Gamma(-i\nu-n)\Gamma(\mu+i\nu+n)}{\Gamma(-i\nu-n+\frac{1}{2})\Gamma(\mu+i\nu+n+\frac{1}{2})}$$

$$\mu = (d-1)/2$$

Complementary fields. Inflation

$$W^2_\nu(z,z') = \int_{-\infty}^{\infty} \kappa d\kappa \; \rho(\kappa^2,\nu) W_\kappa(z,z')$$

$$W^2_\nu = \int_{-\infty}^{\infty} \kappa \, d\kappa \; \rho_\nu(\kappa) W_\kappa \; + \sum_{n=0}^{N-1} A_n(\nu) \, W_{i(\mu+2i\nu+2n)}$$

$$A_n(\nu) \;=\; \frac{8\pi(-1)^n}{n! 2^d \pi^{\frac{1+d}{2}} R^{d-2} \Gamma(\mu)} \frac{\Gamma(\mu+2i\nu+n)\Gamma(-2i\nu-n)}{\Gamma(\mu+2i\nu+2n)\Gamma(-\mu-2i\nu-2n)} \times \frac{\Gamma(\mu+n)\Gamma(-i\nu-n)\Gamma(\mu+i\nu+n)}{\Gamma(-i\nu-n+\frac{1}{2})\Gamma(\mu+i\nu+n+\frac{1}{2})}$$

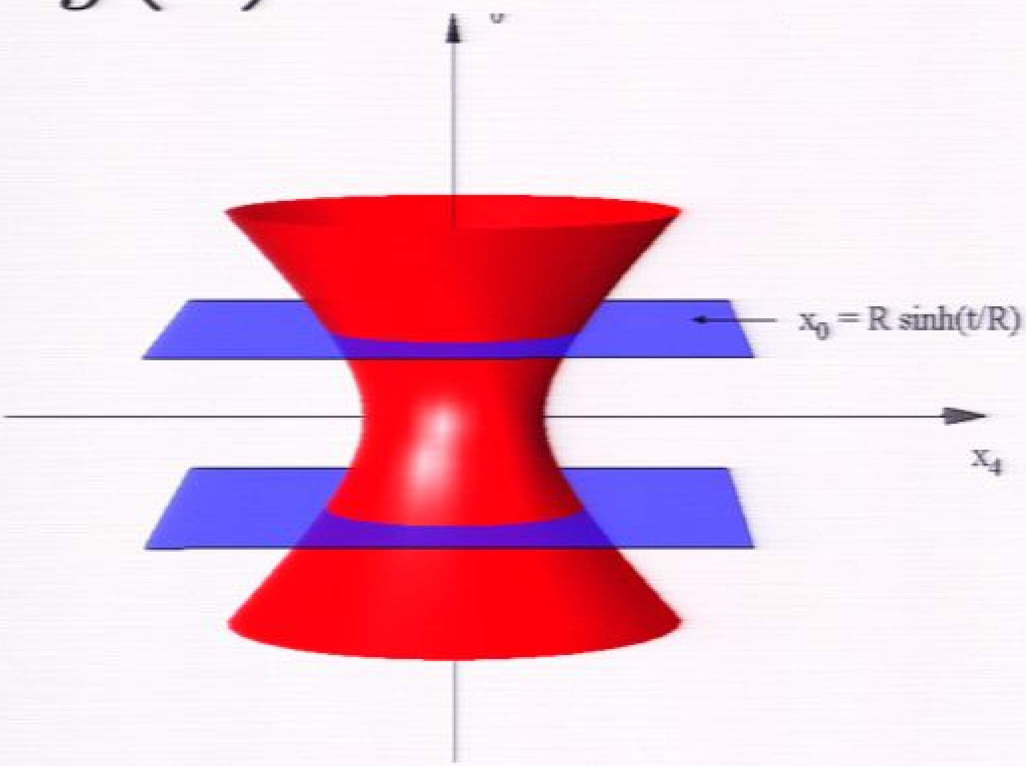
$$\mu = (d-1)/2$$

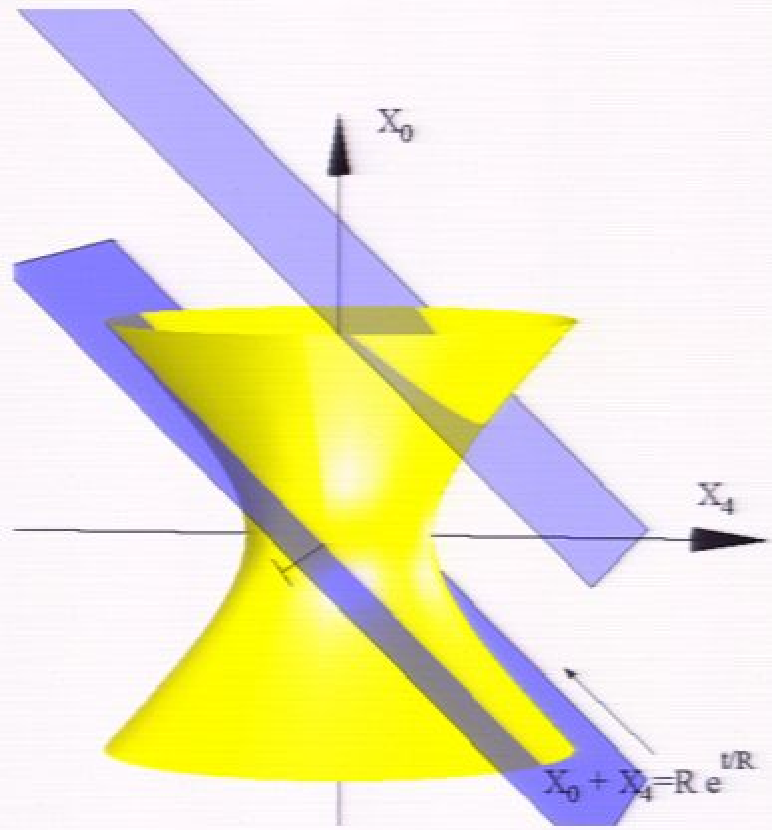
The number of discrete terms is the largest N satisfying $N < 1 + |\Im\nu| - \mu/2$, or 0 if this is negative
A particle of the complementary series with parameter $\kappa = i\beta$ can only decay into two particles with parameter $\nu = \frac{i}{2}(|\beta| + \mu + 2n)$, where n is any integer such that $0 \le 2n < \mu - |\beta|$, and the decay is instantaneous.

A particle with mass $M \ll m_c$ can only decay into two particles of mass $m \sim M/\sqrt{2}$

Infrared limit

$$g(x) \to 1$$





$$\lim_{T\to\infty} \frac{2\lambda^2 C(\kappa)\rho(\kappa^2,\nu)\ \int g(x)\,|F(x)|^2\,dx}{T\int \overline{f_0(x)}W_\kappa(x,\ y)\,f_0(y)\,dx\,dy} = \rho(\kappa^2,\nu)\frac{2\lambda^2\pi\coth(\pi\kappa)^2}{|\kappa|}$$

Perspective, friends and coworkers

- A lot to be done
- Lifetime of a de Sitter Particle - dS QFT and thermal properties -Tachyons *Jacques Bros, Vincent Pasquier, Michel Gaudin (SPhT-Cea-Saclay) Henri Epstein (IHES-Bures sur Yvette)* (PRL CMP) JCAP CMP (1995-1998-2008-2010)
- Symmetries and conservation laws on the de Sitter universe. *Vittorio Gorini, Sergio Cacciatori (Insubria) Alexander Kamenshchik (Bologna)* CQG (2008)
- Generalized Bogoliubov Transformations *R. Schaeffer* JCAP 2008