Title: IR divergence problem in single-field models of inflation

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Abstract: We clarify the origin of IR divergence in single-field models of inflation and provide the correct way to calculate the observable fluctuations. First, we show the presence of gauge degrees of freedom in the frequently used gauges such as the comoving gauge and the flat gauge. These gauge degrees of freedom are responsible for the IR divergences that appear in loop corrections of primordial perturbations. We propose, in this talk, one simple but explicit example of gauge-invariant quantities. Then, we explicitly calculate such a quantity to find that the IR divergence is absent in the slow-roll approximation. In this formalism, we revisit the consistency relation that connects the three-point function in the squeezed limit with the spectral index.

IR divergence problem in single field models of inflation

Yuko Urakawa (Waseda U. → U. Barcelona)

with Takahiro Tanaka (Yukawa institute, Kyoto U.)

Y.U. and T. Tanaka 0902.3209[hep-th], P.T.P.122:779-803,2009

Y.U. and T. Tanaka 1007.0468[hep-th],

Y.U. and T. Tanaka 1009.2947[hep-th],

Primordial perturbation

Important tools to study models of inflation

Curvature perturbation ζ

- 2-point fn.

$$\langle\zeta\zeta\rangle = \langle\zeta\zeta\rangle_{\rm tree} + \langle\zeta\zeta\rangle_{\rm 1loop} + \langle\zeta\zeta\rangle_{\rm 2loop} + \cdots$$

- 3-point fn.

$$\langle\zeta\zeta\zeta\rangle = \langle\zeta\zeta\zeta\rangle_{\rm tree} + \langle\zeta\zeta\zeta\rangle_{\rm 1loop} + \langle\zeta\zeta\zeta\rangle_{\rm 2loop} + \cdots$$

Leading — Consistent with CMB

Sub-leading — Divergent due to IR mode

→ Break down of perturbation theory??

Infrared(IR) divergence

- Two point function $\langle \zeta_k \zeta_{k'} \rangle$

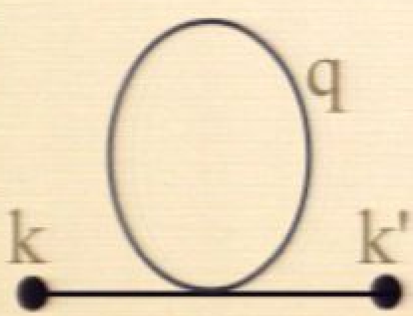
$\mathcal{L}_{\rm int} \propto \zeta^4$ $\qquad\qquad$ ζ: mass-less field

■ Leading order

$$\langle \zeta_k \zeta_{k'} \rangle = |\zeta_k|^2 \propto k^{-3}$$

Scale-invariant

■ Next to leading order



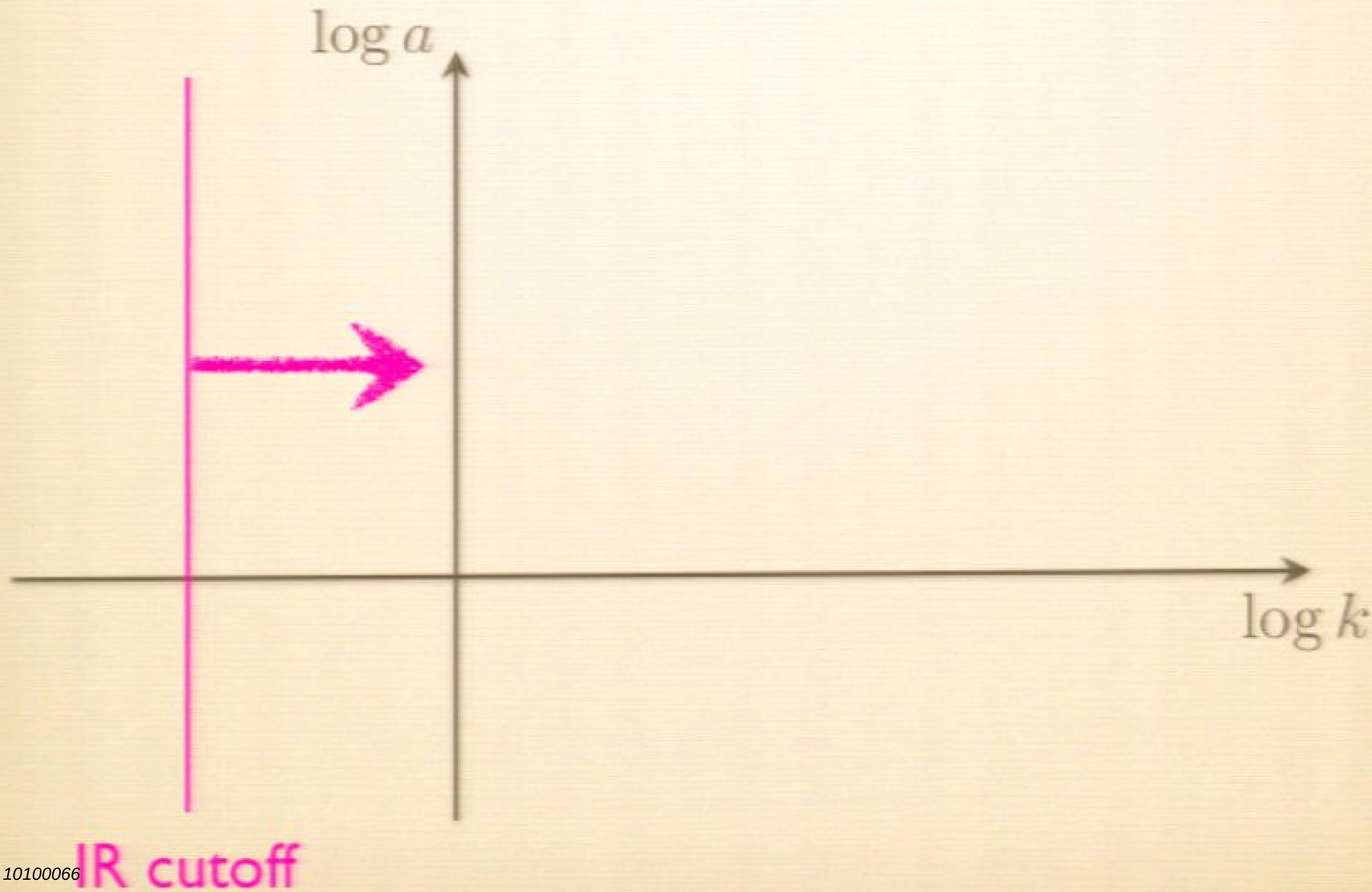
Momentum (Loop)integral

$$\int \mathrm{d}^3 q \, |\zeta_q|^2 = \int \mathrm{d}^3 q / q^3$$

Logarithmic divergence

Introduction of IR cutoff

Which modes participate in loop corrections?



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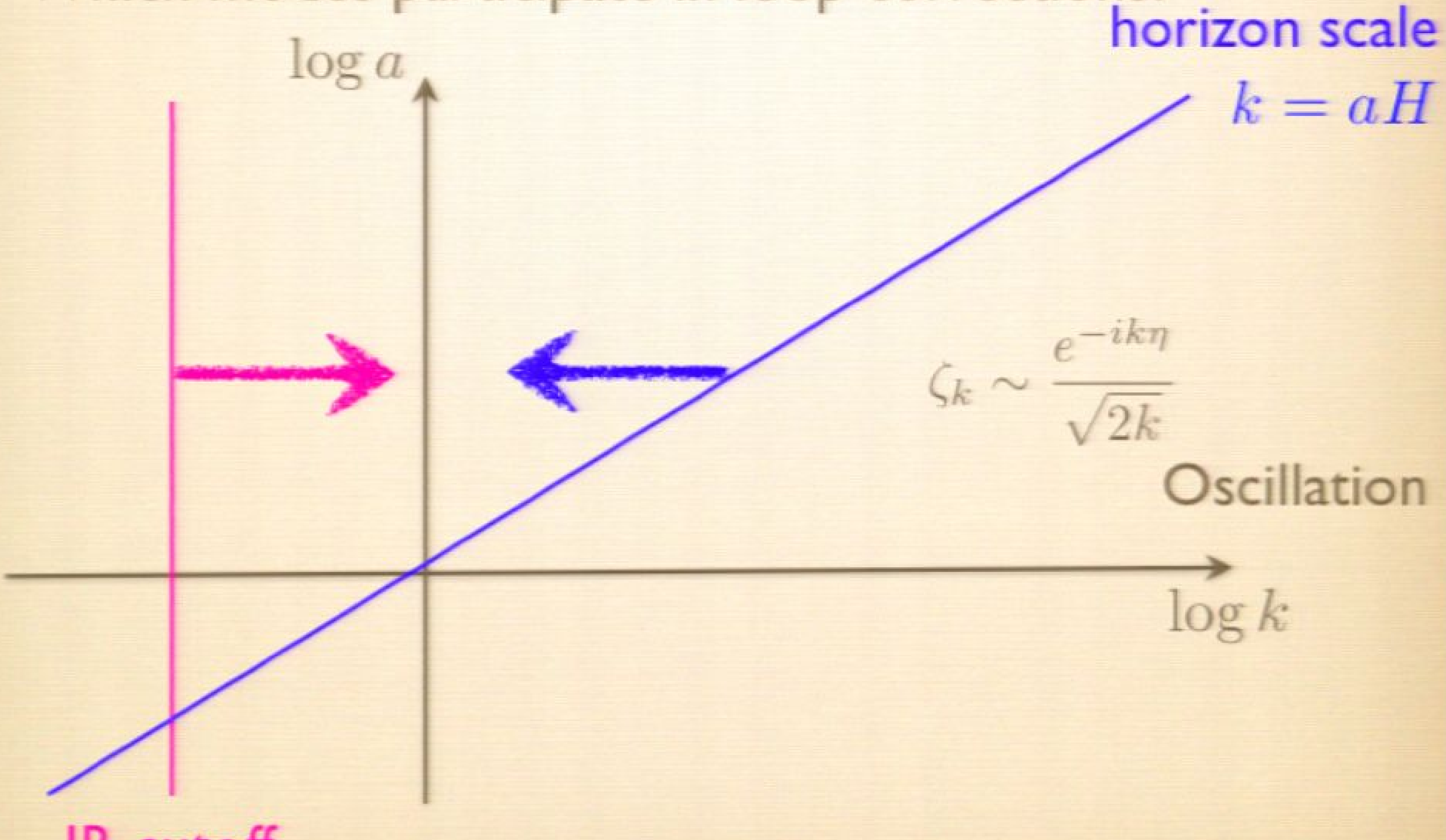

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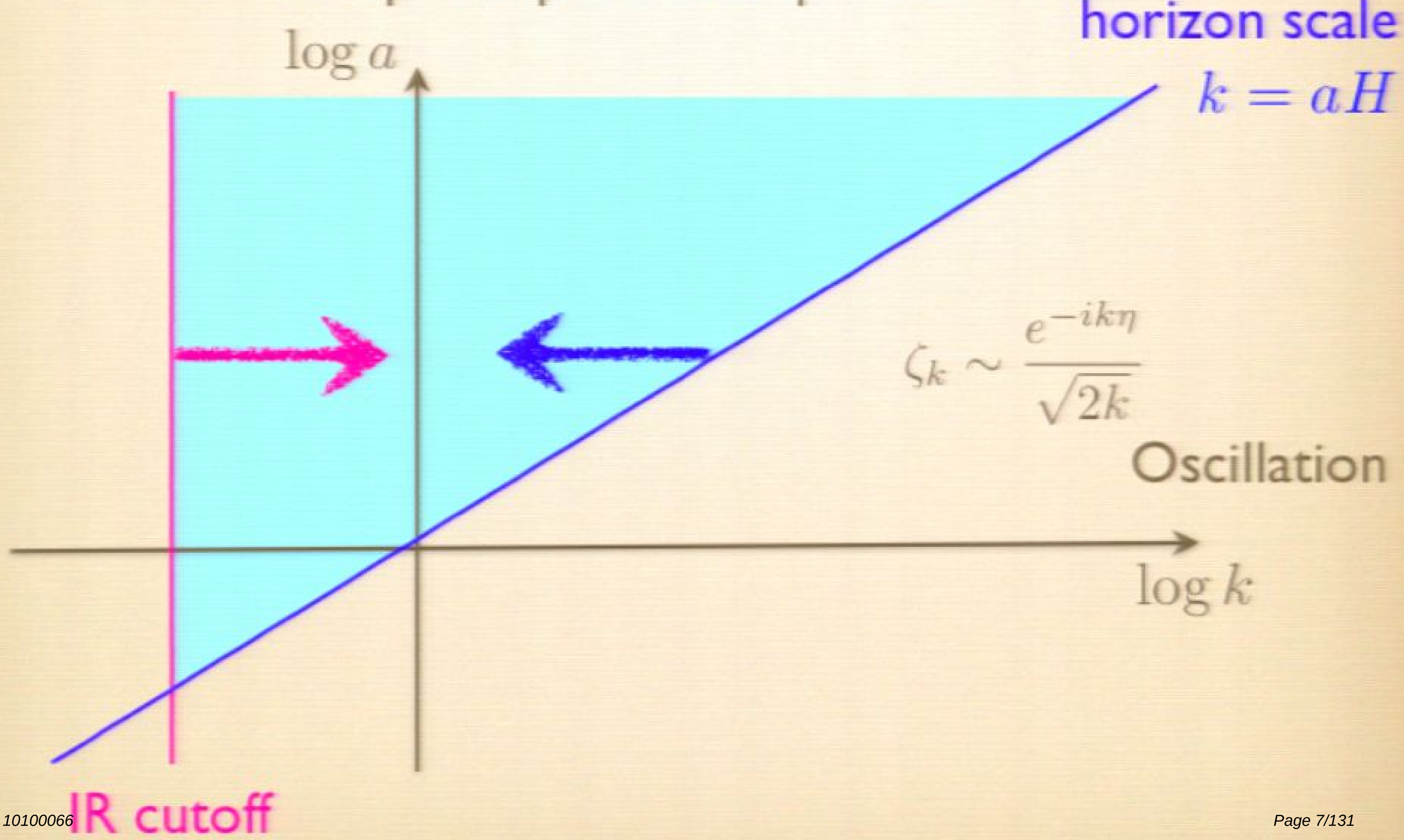

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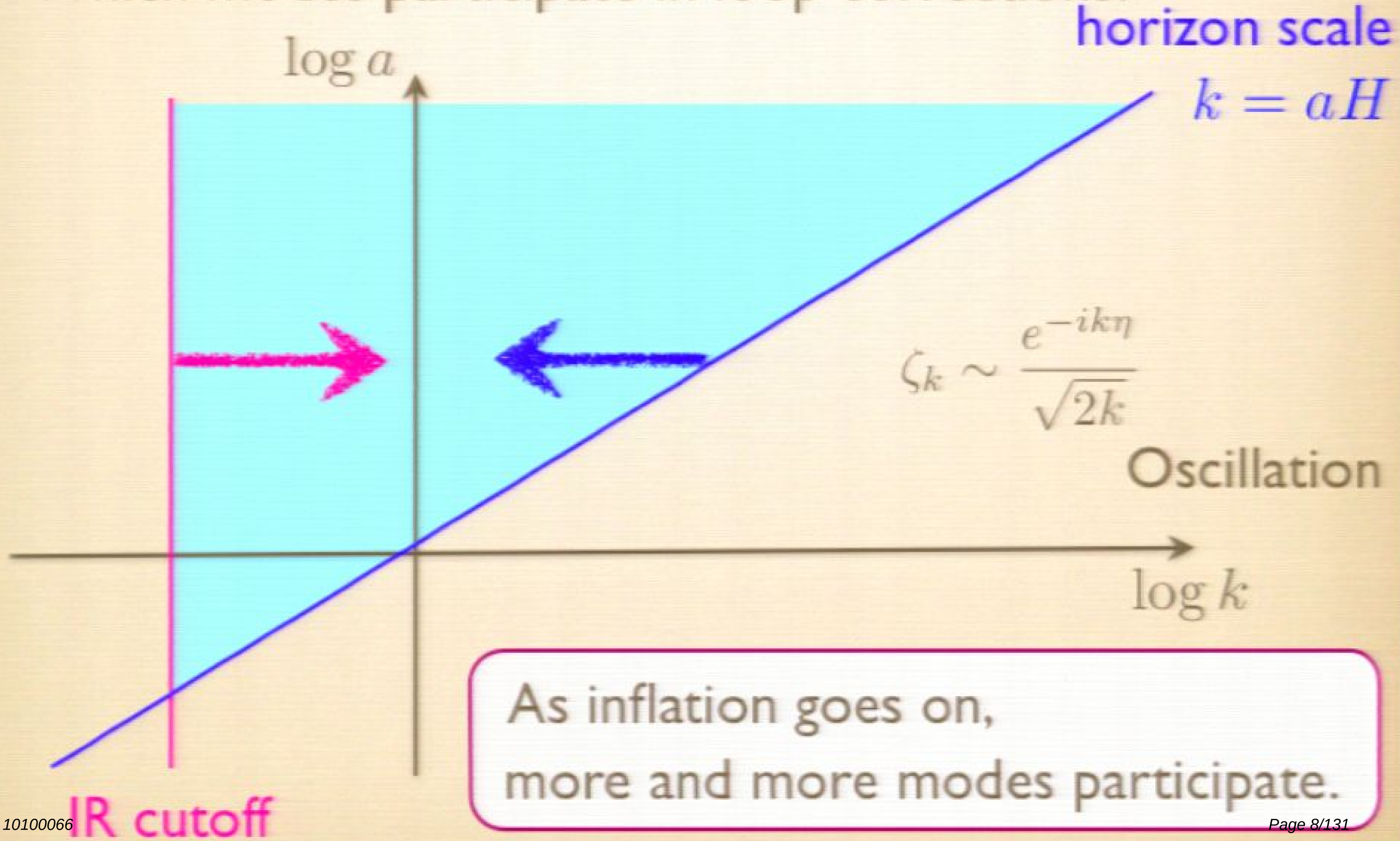
Which modes participate in loop corrections?



As inflation goes on,
more and more modes participate.

In this talk, ...

"IR divergence is physical or not?"

in single field models of inflation

- Contents

1. Origin of Infrared divergence

2. Two ways of regularization

ADM formalism

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

Comoving gauge

Maldacena (2002)

$$\delta\phi = 0 \qquad h_{ij} = e^{2(\rho+\zeta)}\left(\delta_{ij} + \delta\gamma_{ij}\right)$$

$$\gamma^{ij}\delta\gamma_{ij} = 0 \qquad \partial^i \delta\gamma_{ij} = 0$$

e^ρ: scale factor

$$S = S_{\rm EH} + S_\phi = S[N, N_i, \zeta, \delta\gamma_{ij}]$$

- Lagrange multiplier N / N_i

Hamiltonian constraint $\partial\mathcal{L}/\partial N = 0$

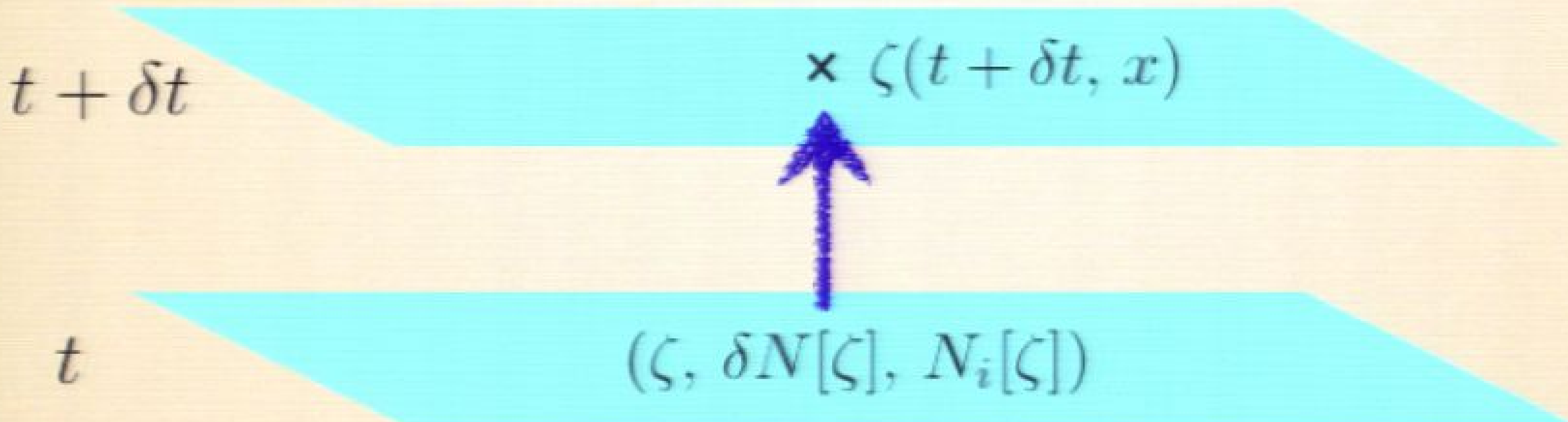
Momentum constraints $\partial\mathcal{L}/\partial N^i = 0$

→ $N = N[\zeta]$, $N_i = N_i[\zeta]$

$$S = S[N, N^i, \zeta] = S[\zeta]$$

DOFs in boundary conditions

$\delta\phi = 0$ Fix the temporal gauge



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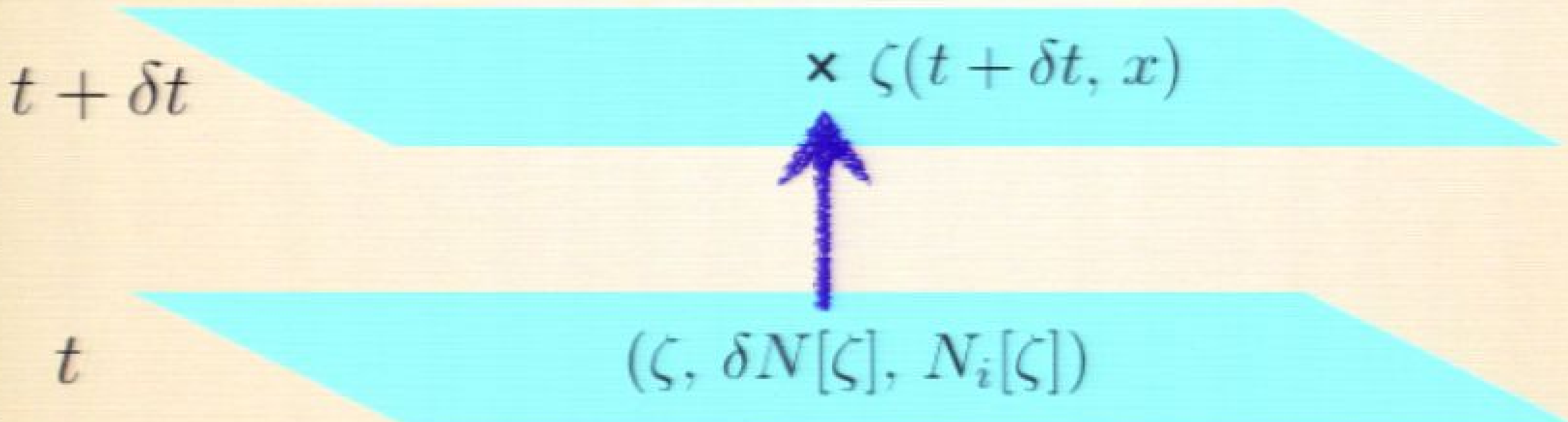

Constraint eqs.

$\partial^2 \delta N = \mathcal{S}[\zeta] \qquad \partial^2 N_i = \mathcal{S}_i[\zeta] \qquad \partial^2 = \partial_i \partial^i$

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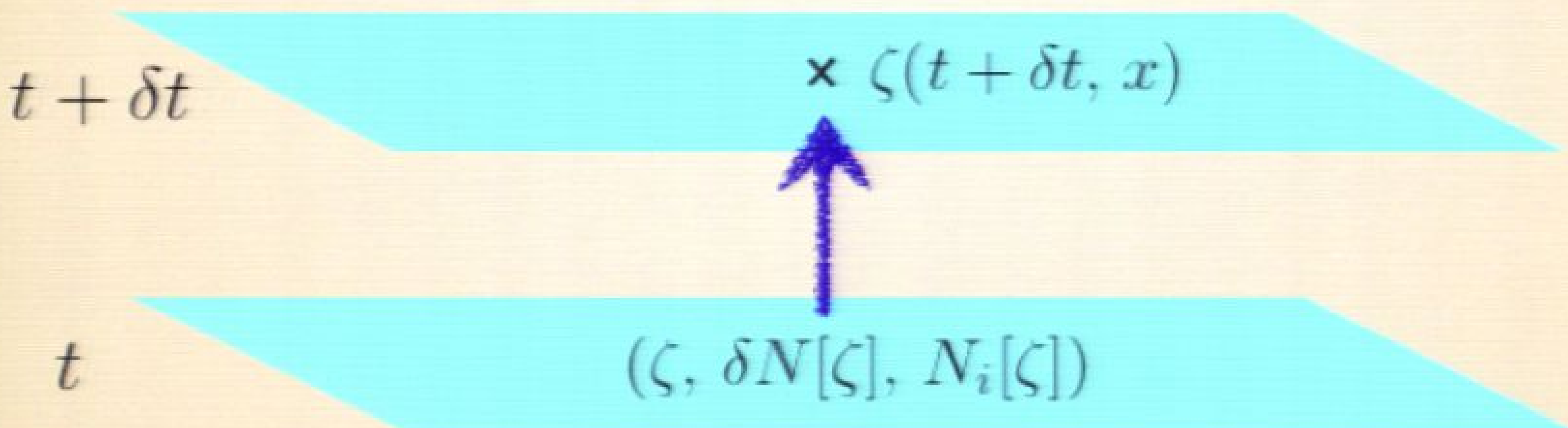

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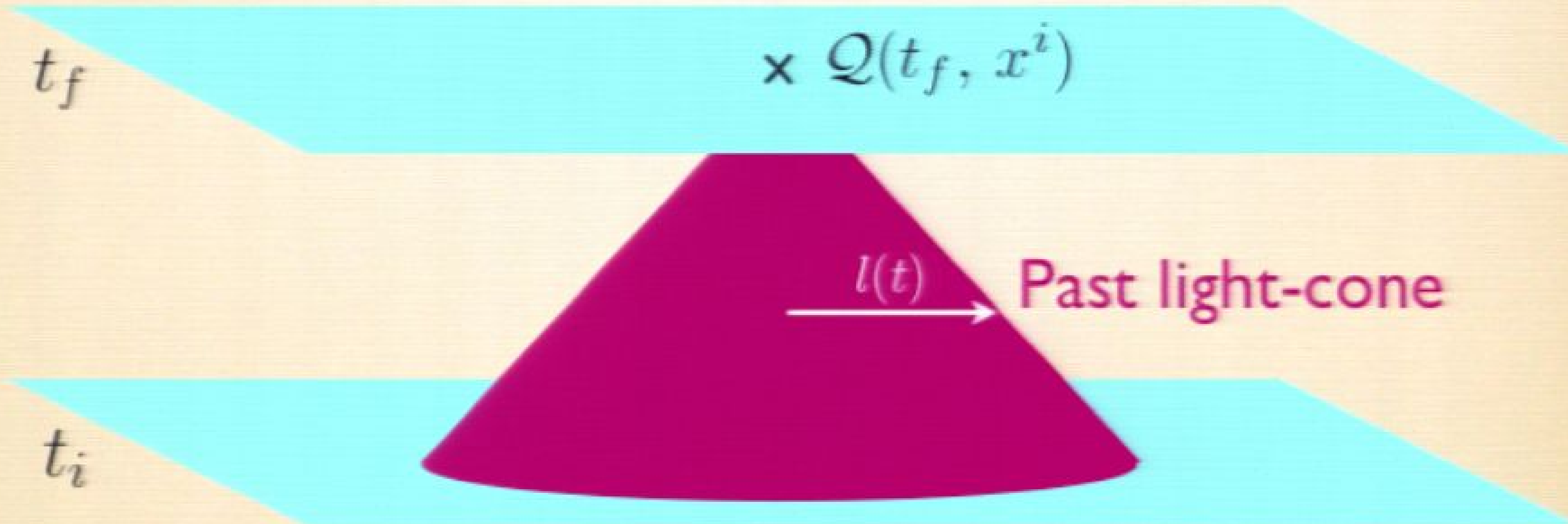
Elliptic-type eqs. → DOFs in boundary conditions

Change the evolution of ζ

DOFs in boundary conditions 2

Time evolution from t_i to t_f

- Hyperbolic system



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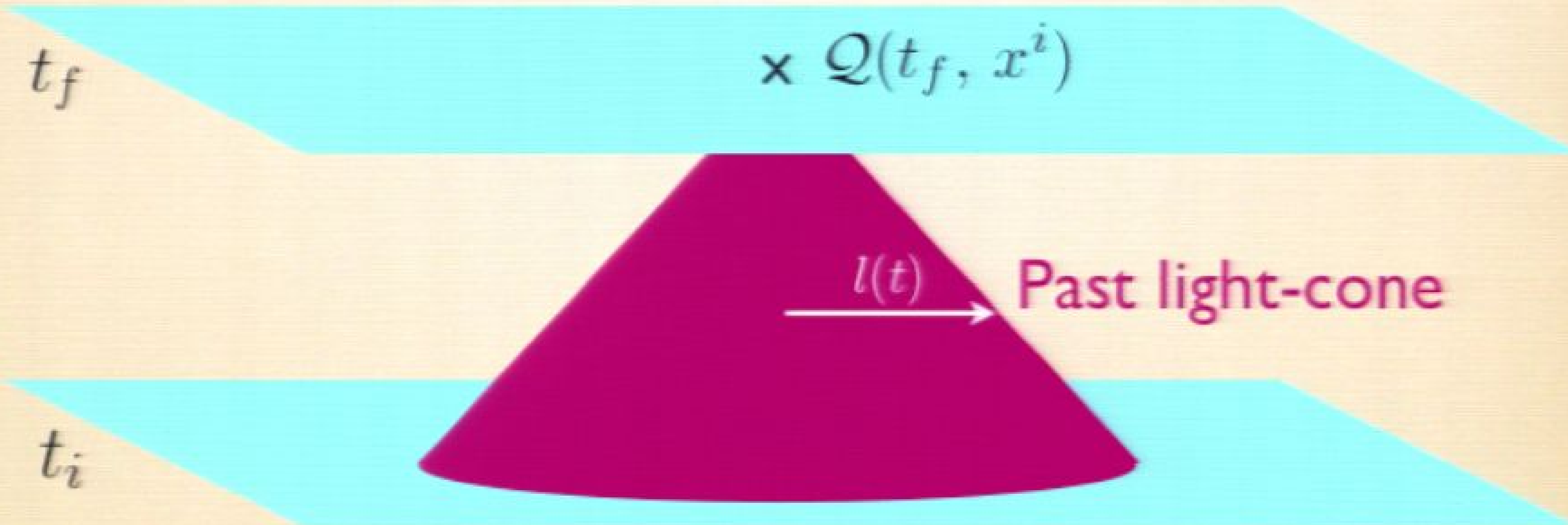

Vertexes in $\mathcal{Q}(t_f, x^i)$ $\int_{t_i}^{t_f} \mathrm{d}t \int_{|x| \le l(t)} \mathrm{d}^3 x$

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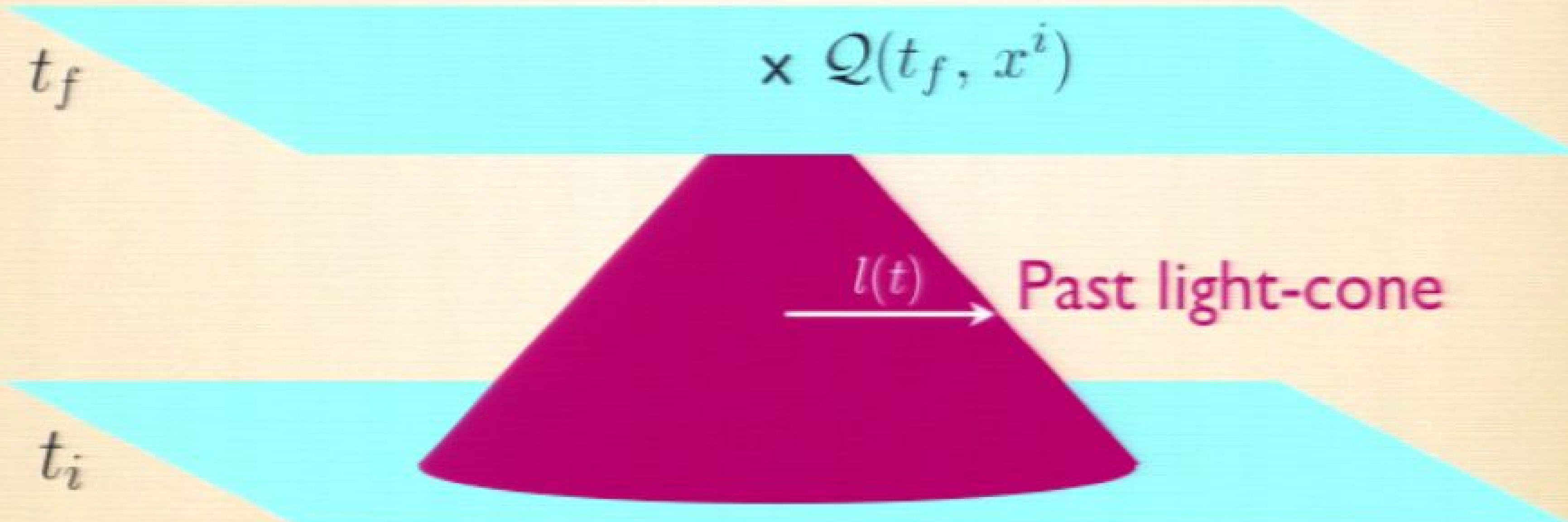

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Wavelengths of fluctuation that affect $\zeta(t_f, x)$

→ Bounded by $k > 1/l(t)$

DOFs in boundary conditions 3

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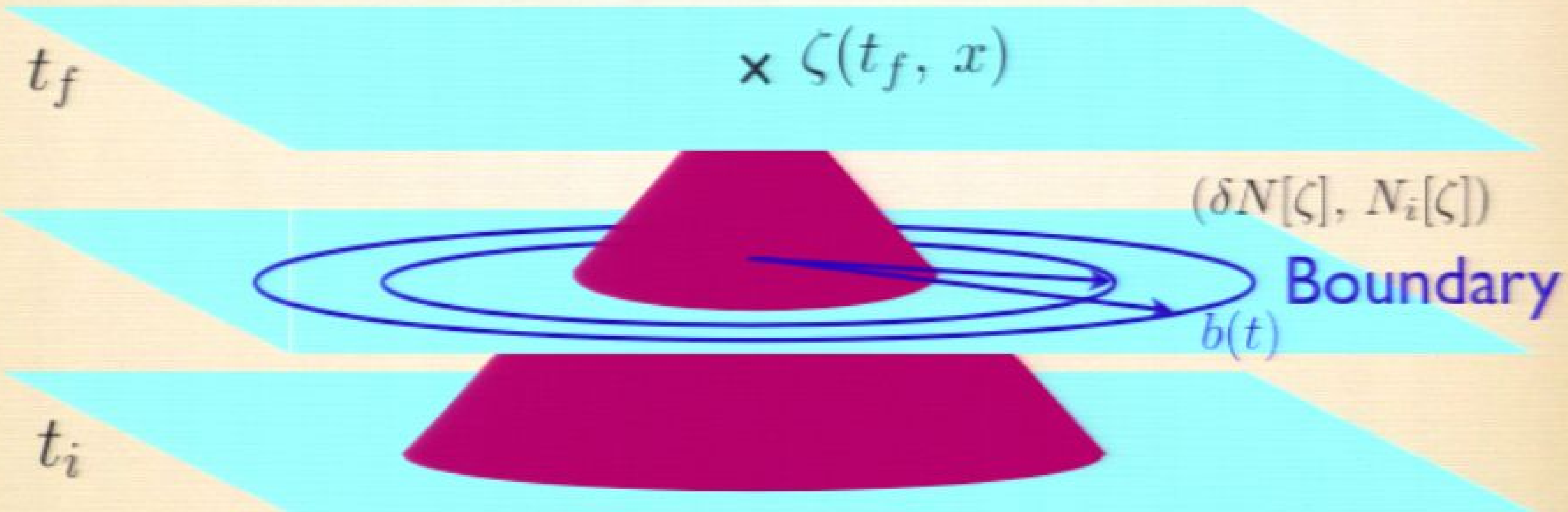

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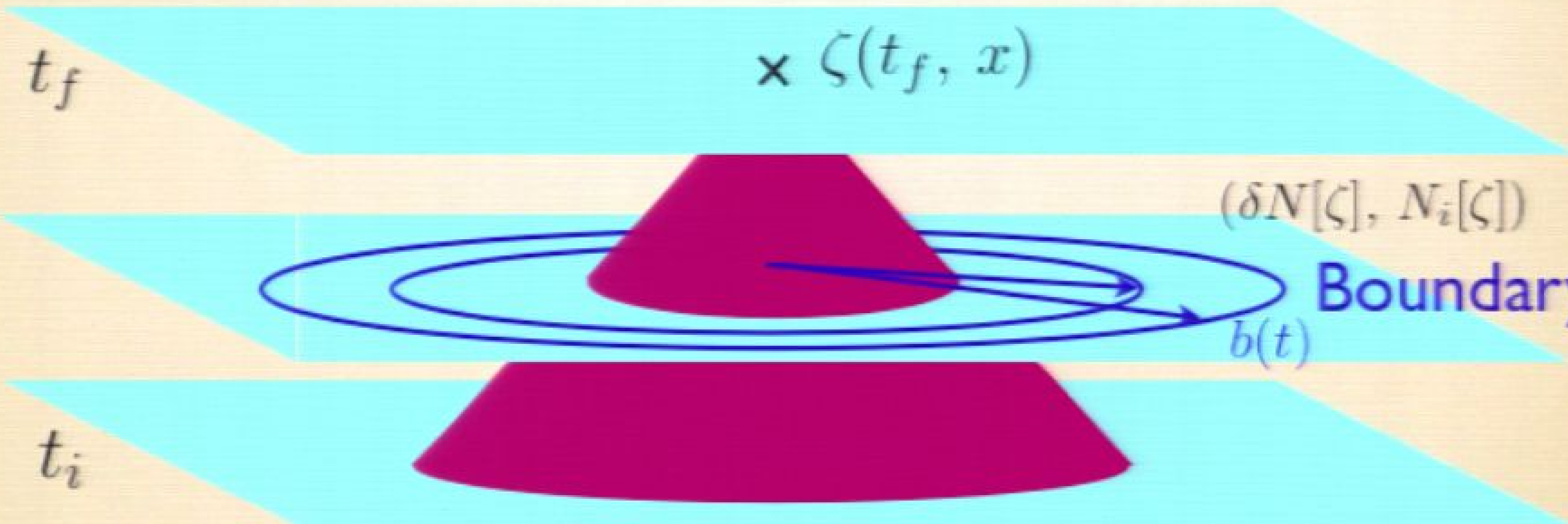

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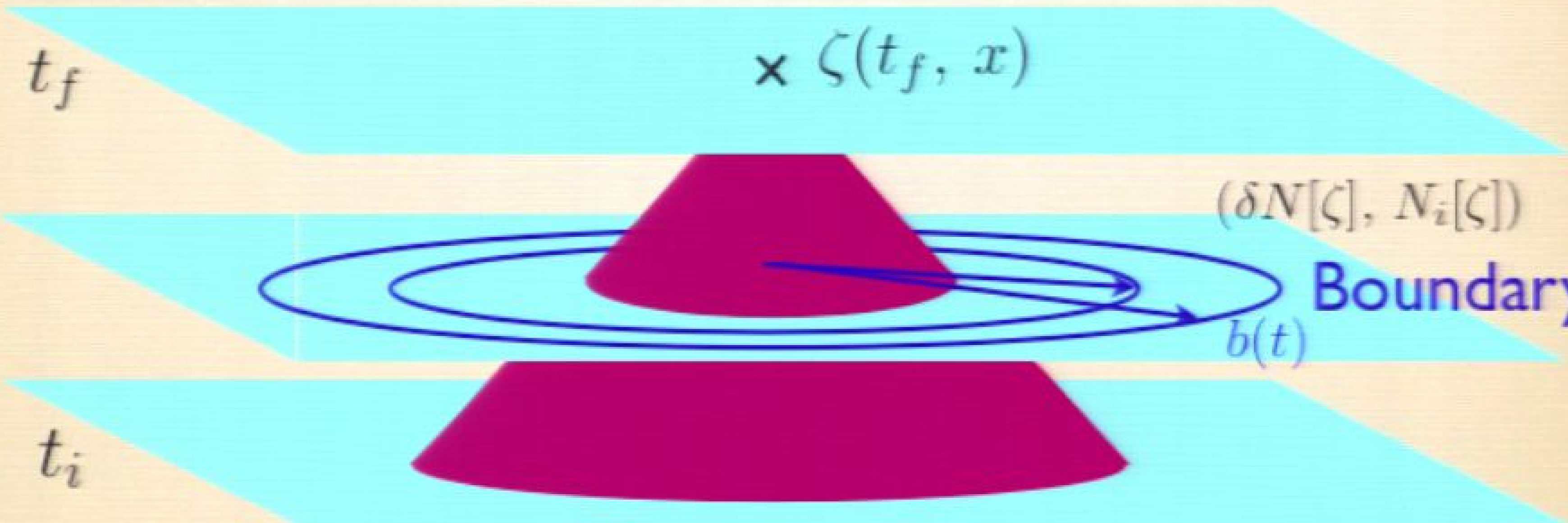

Vertexes in $\zeta(t_f, x)$ $\int \mathrm{d}t \int_{|x|\leq b(t)} \mathrm{d}^3x$

Wavelengths of fluctuation that affect $\zeta(t_f, x)$

→ Unbounded. For $b(t) \to \infty$, IR div. appear

Residual gauge modes

Single field inflation

$$S_\phi = -\frac{1}{2}\int \sqrt{-g}\,[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)]\mathrm{d}^4x$$

- General solutions of δN, $\check{N}_i = e^{-\rho}N_i$

From Hamiltonian&Momentum constraints at 1st order

$$\delta N_1(x) = \frac{1}{\rho'}\left(\zeta_1'(x) - \frac{1}{4}\partial^i G_{i,1}(x)\right) \qquad \partial^2 G_{i,1}(x) = 0$$

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DOFs in δN & N_i → Residual gauge DOFs

Residual gauge modes 2

$\left(\delta N,\, \check{N}_i\right)$ for $G_i = 0 \longrightarrow \left(\delta \tilde{N},\, \tilde{\check{N}}_i\right)$ for $G_i \neq 0$

- Gauge transformation: $(t,\, x^i) \to (t + \delta t,\, x^i + \delta x^i)$
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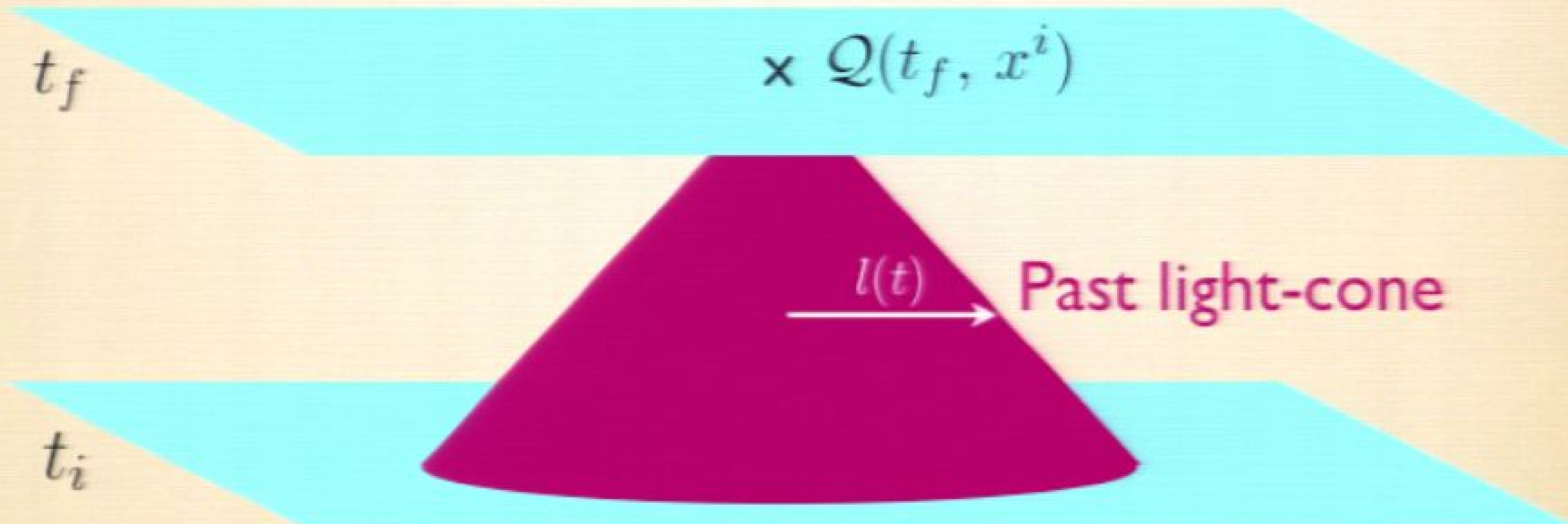

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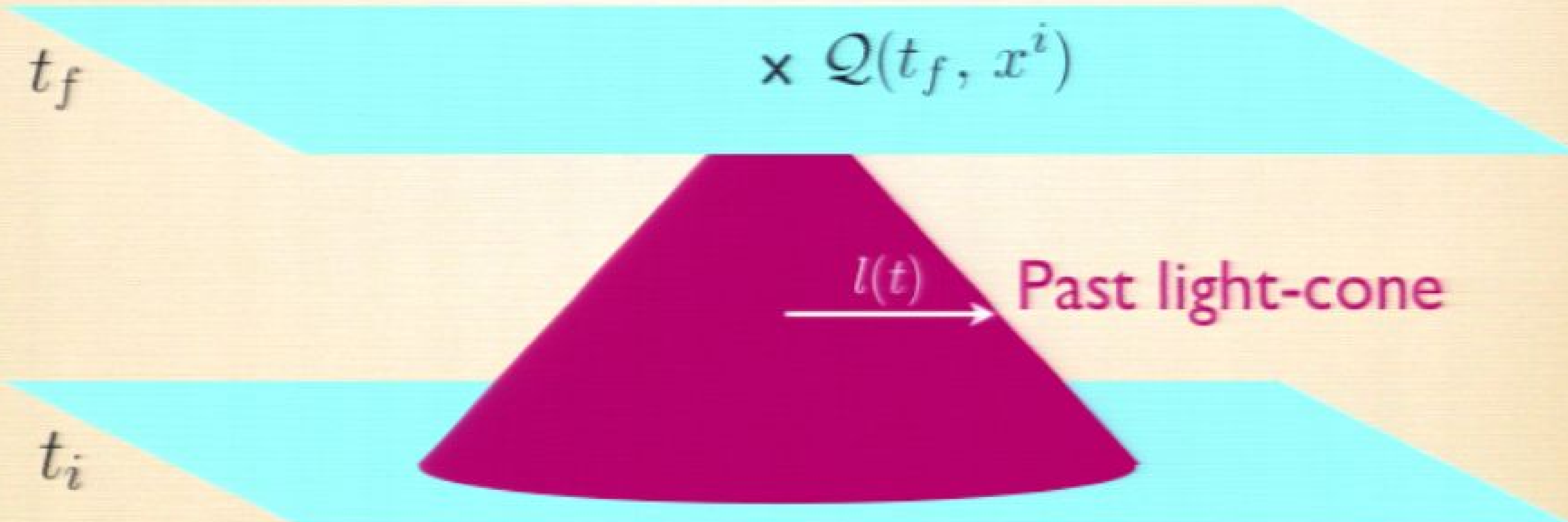

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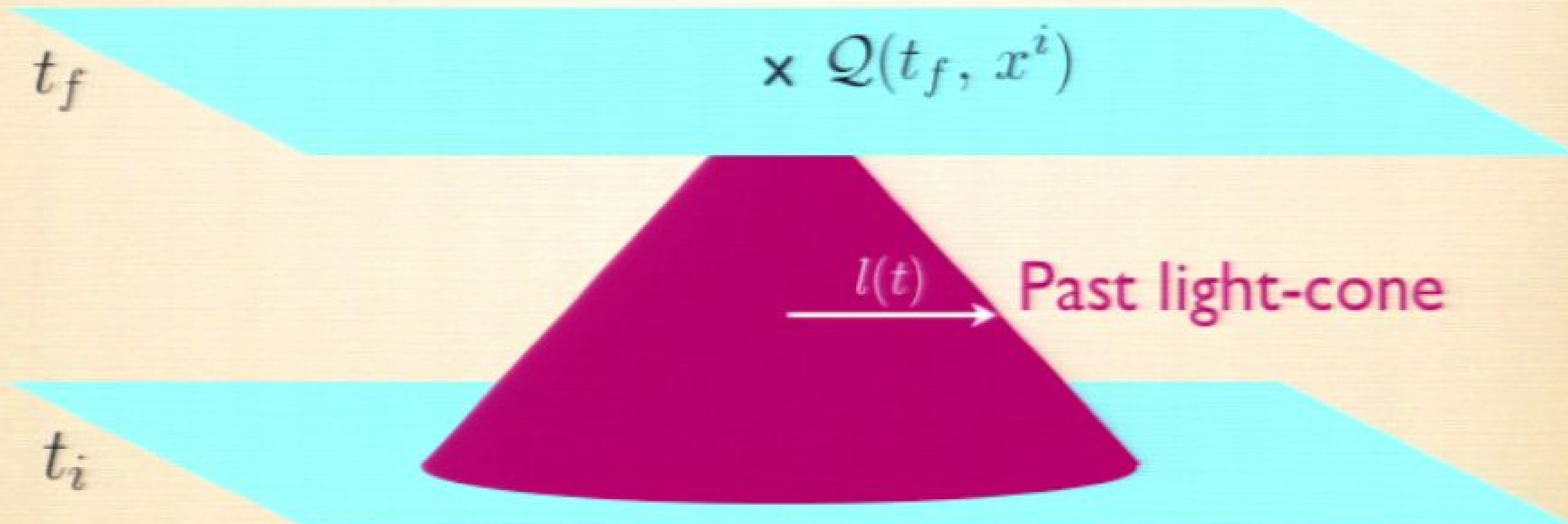

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Proposal of IR regularization

IR corrections, that yield divergence, are changed by the residual gauge DOFs.

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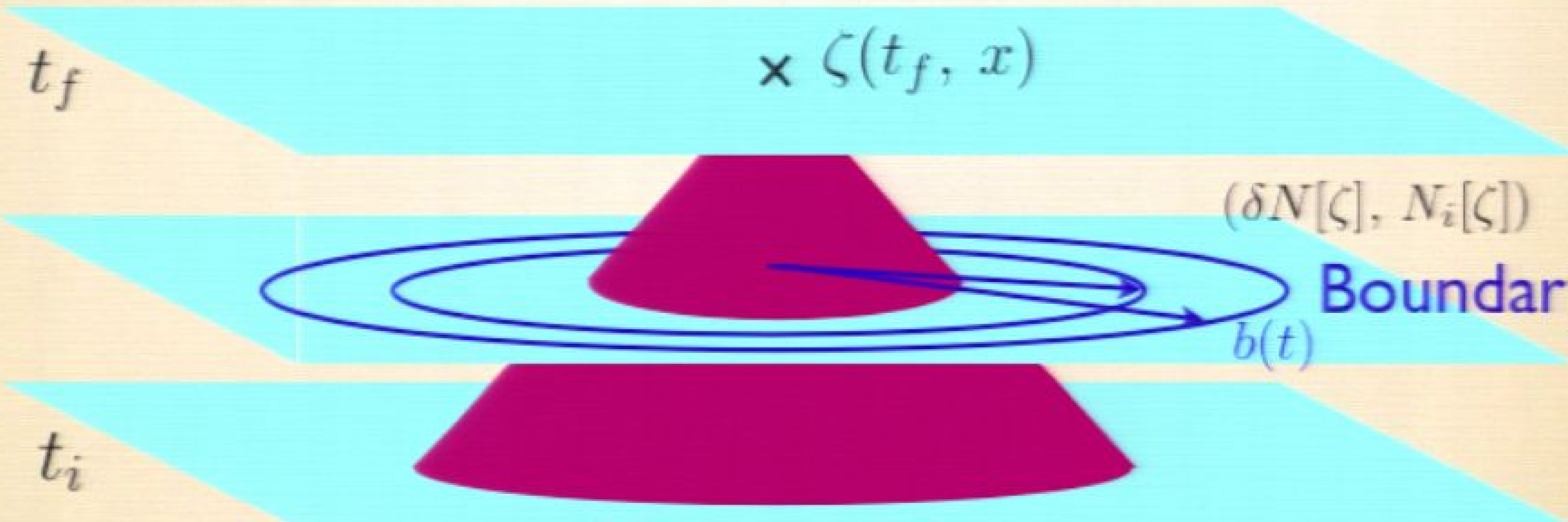

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DOFs in δN & N_i → Residual gauge DOFs

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Y.U.&T.Tanaka(09)

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Local gauge condition

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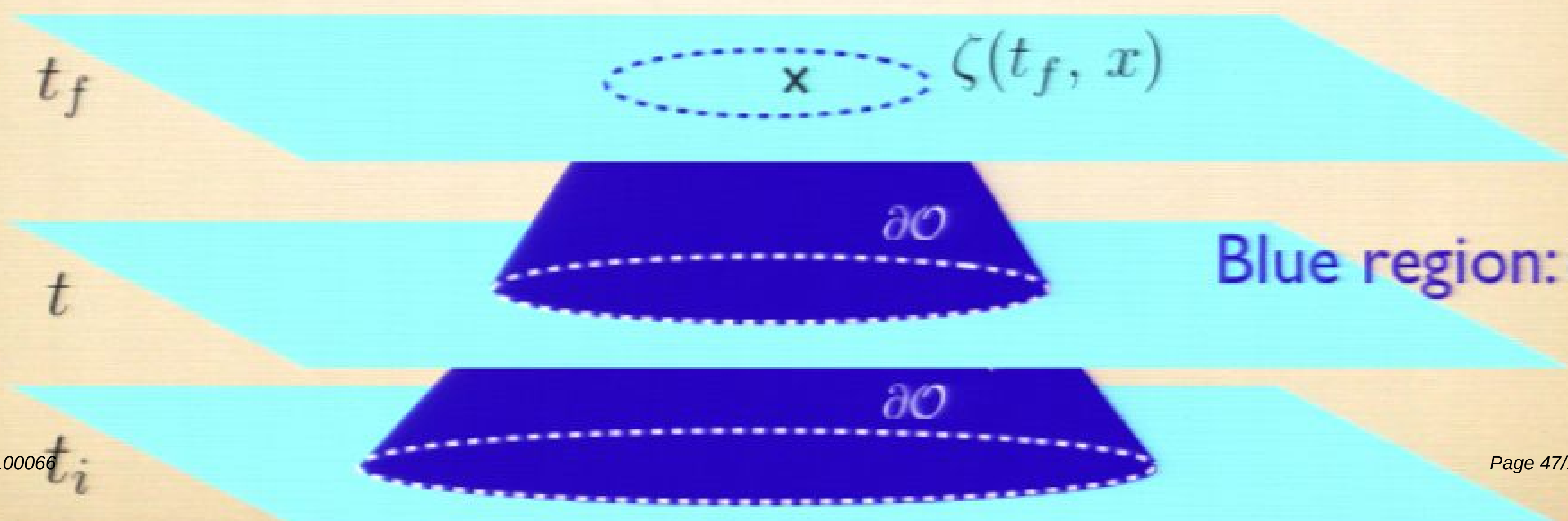
Remove gauge DOFs associated with boundary cond.

- Boundary conditions for δN & N_i

$\mathcal{O}$: Observable region = Causally connected region

Boundary conditions at $\partial\mathcal{O}$
Fluctuations within $\mathcal{O}$ → Not affected by outside of $\mathcal{O}$



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Y.U.&T.Tanaka(09)

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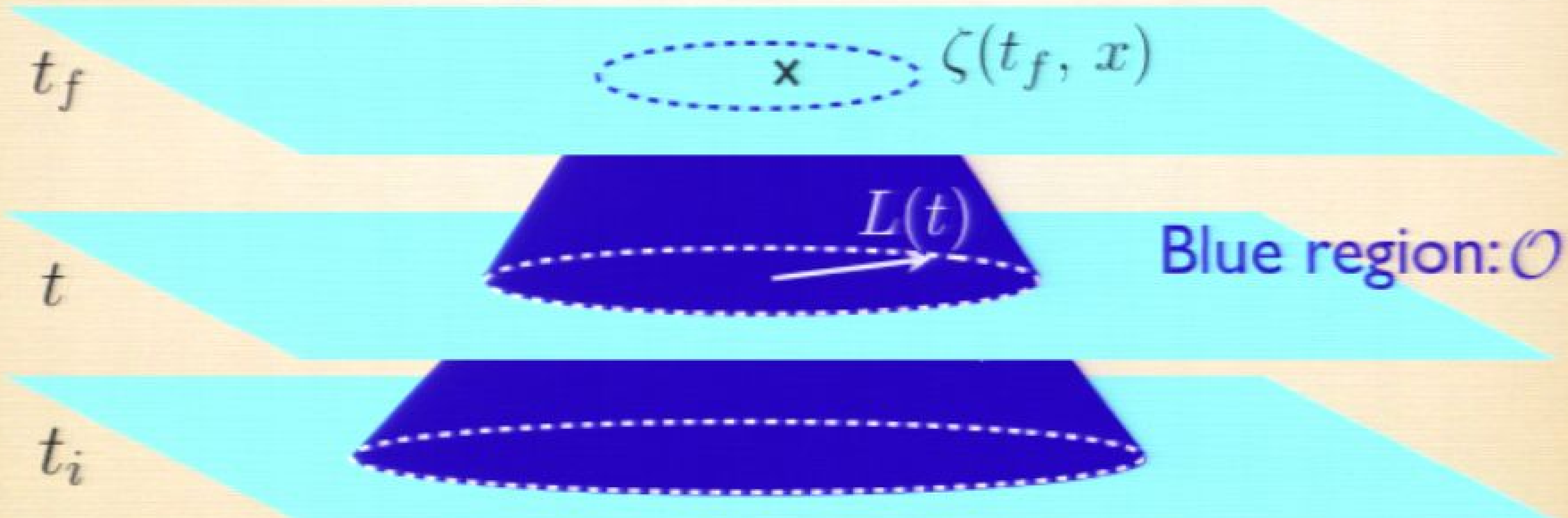

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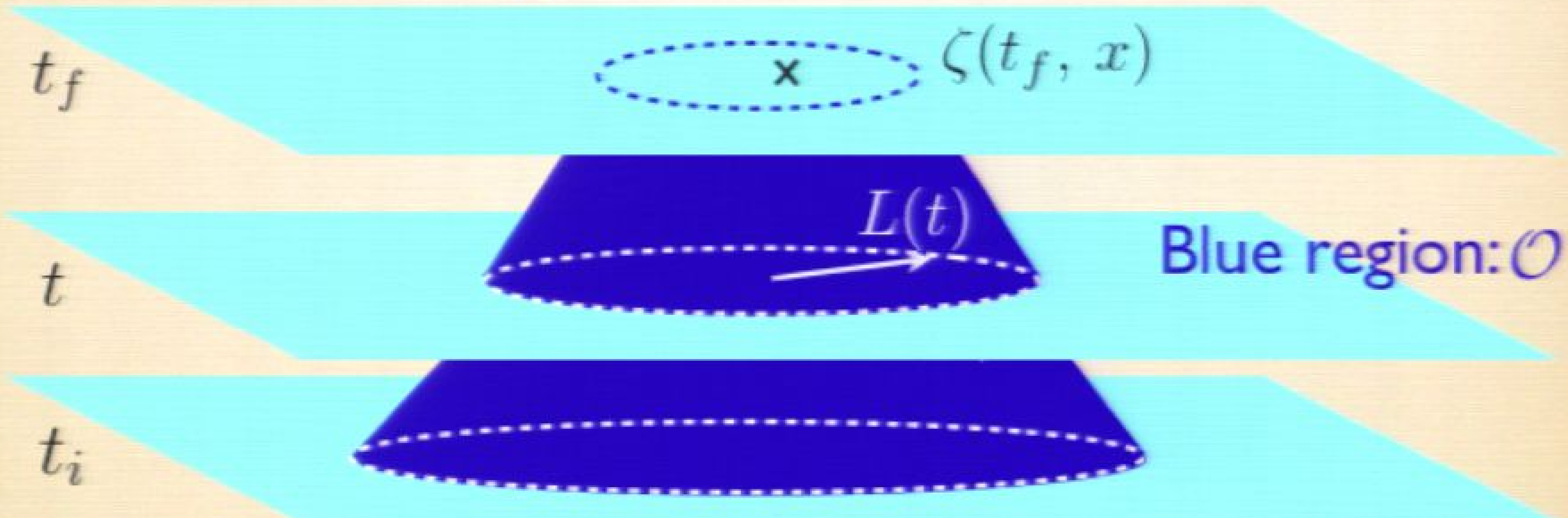

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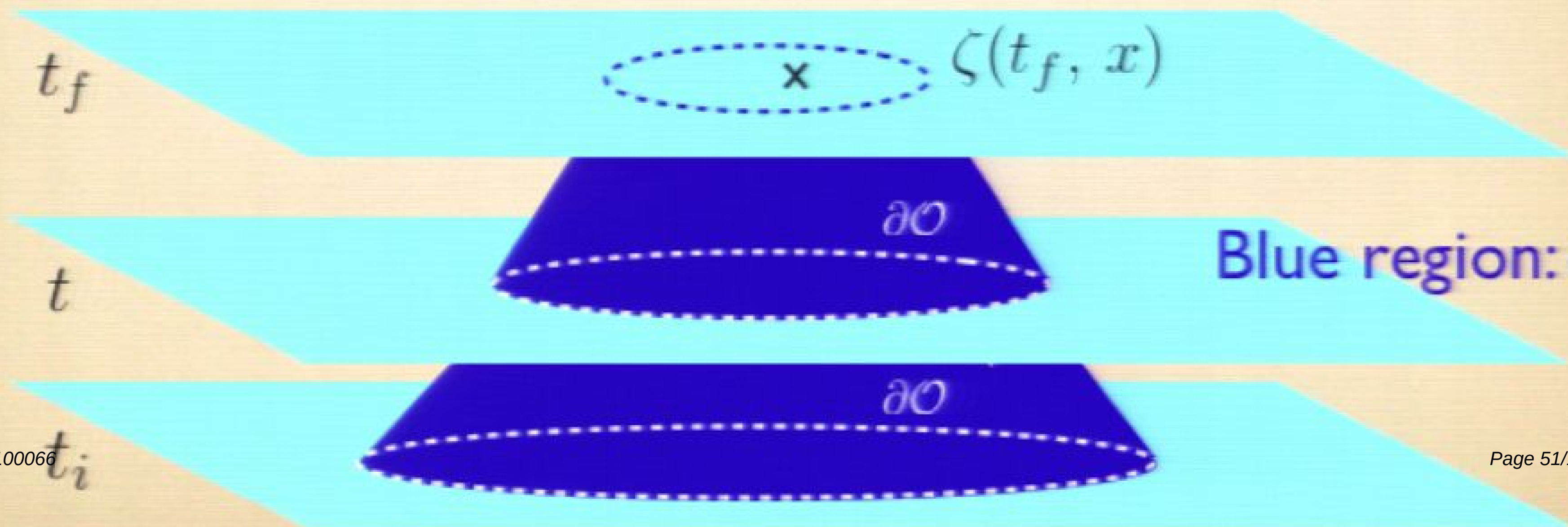

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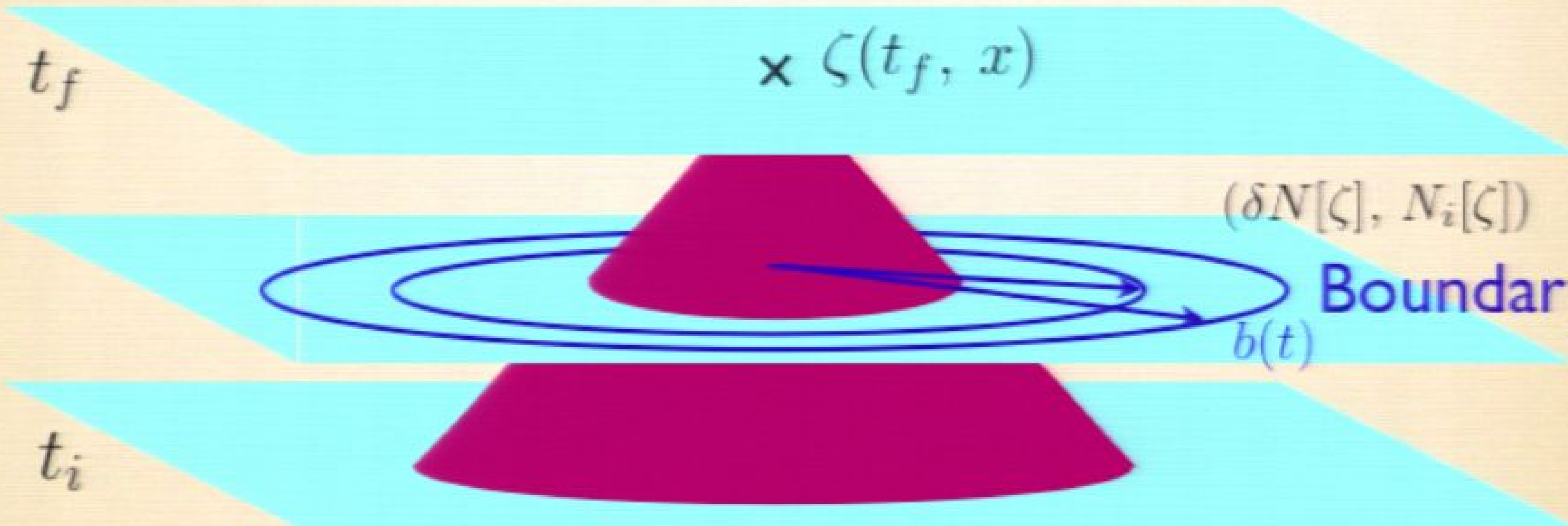

Residual gauge modes

Single field inflation

$$S_\phi = -\frac{1}{2}\int \sqrt{-g}\,[g^{\mu\nu}\phi_{,\mu}\phi_{,\nu} + 2V(\phi)]\mathrm{d}^4x$$

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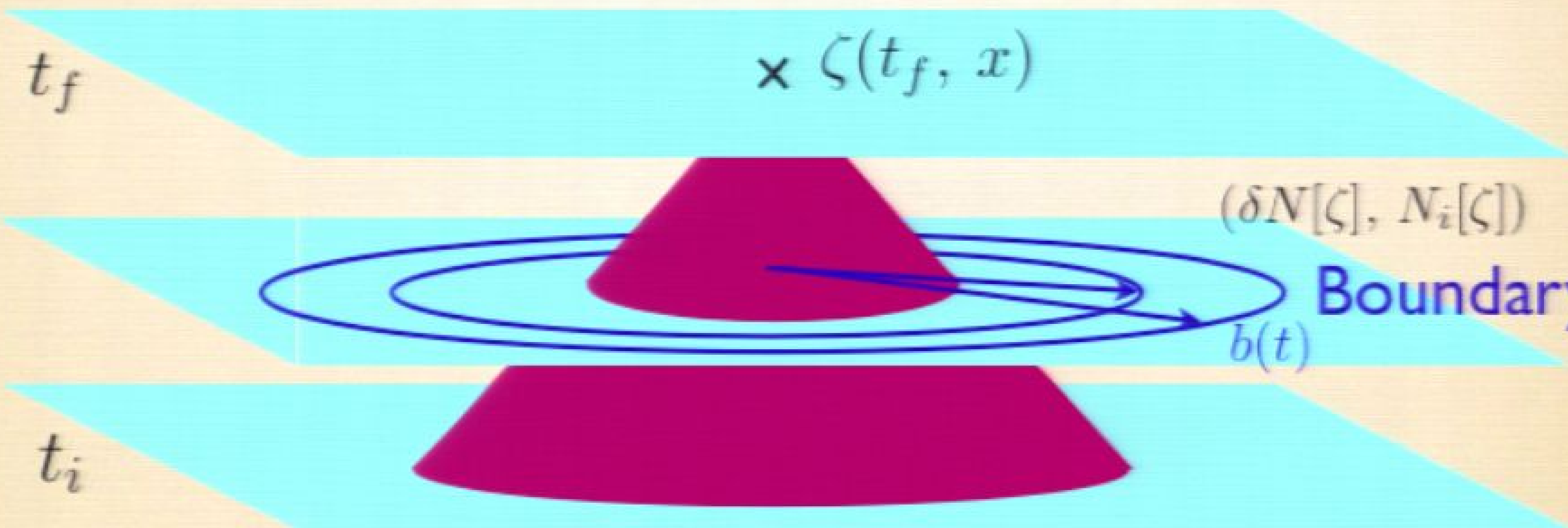

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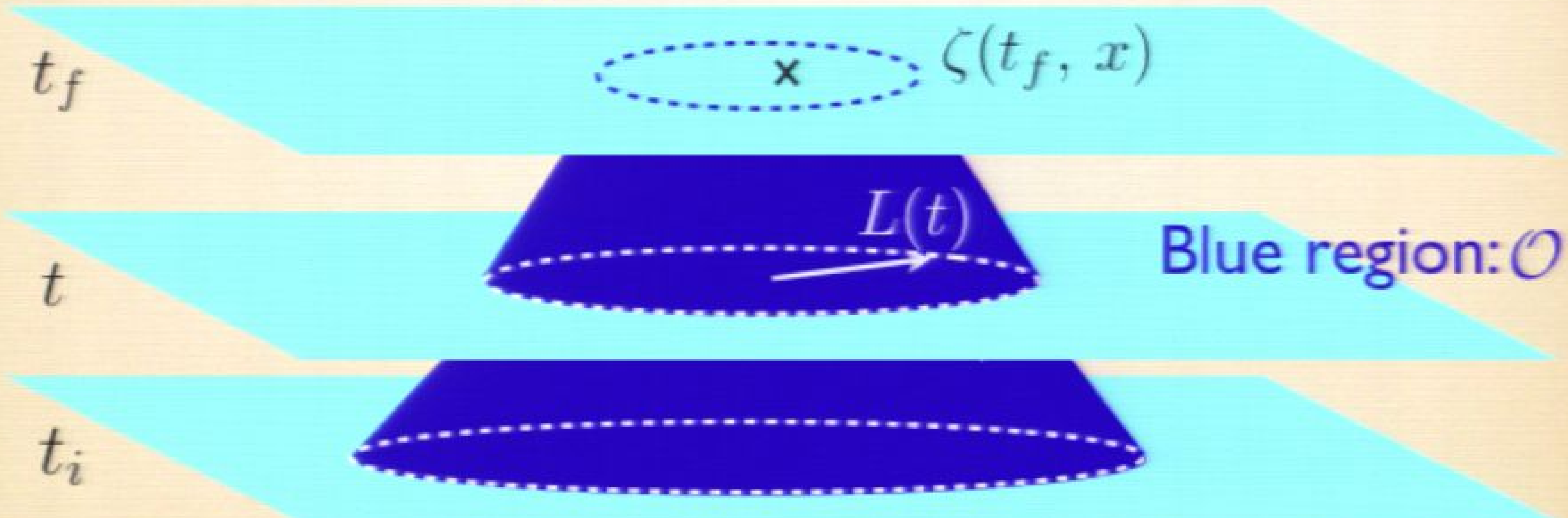

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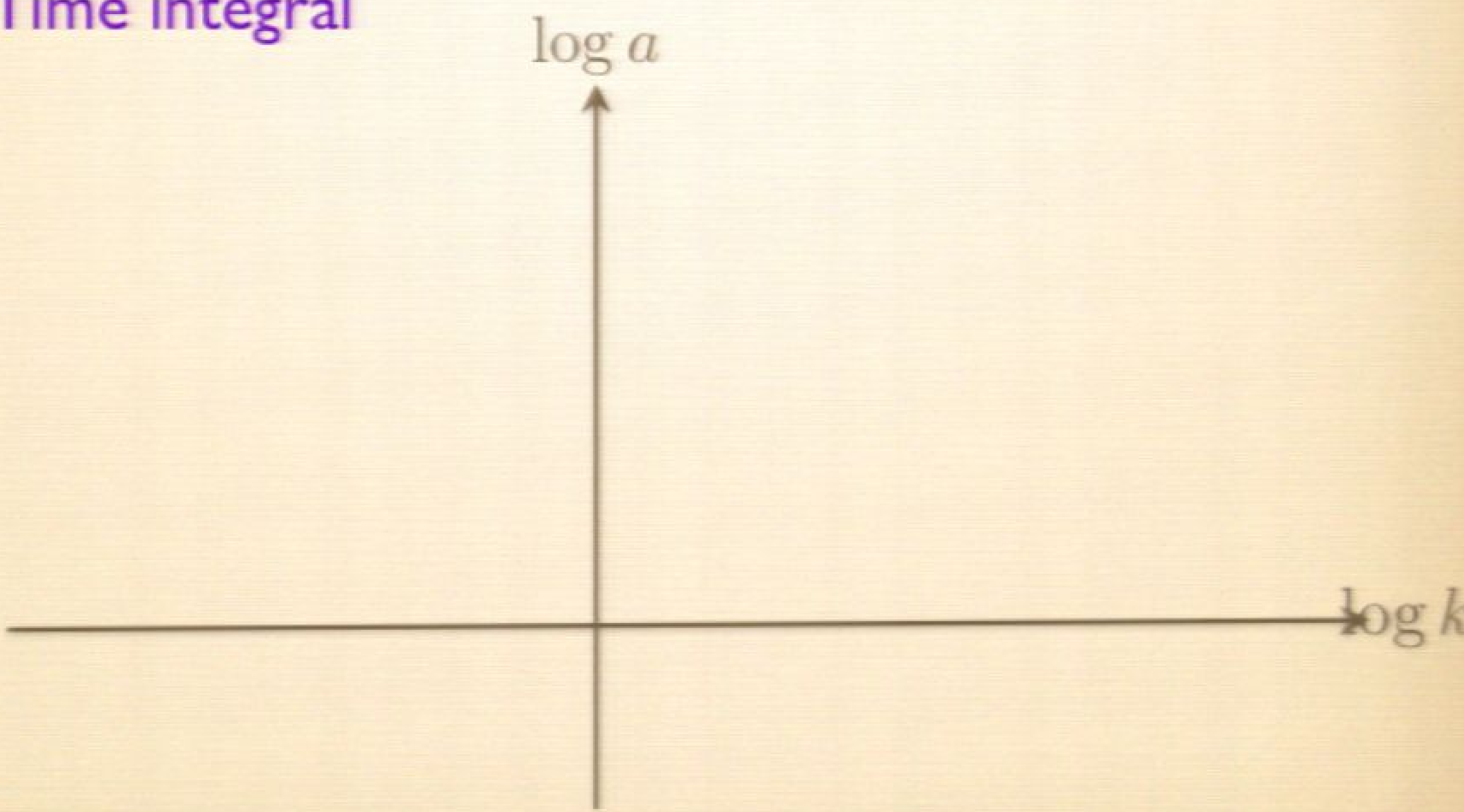

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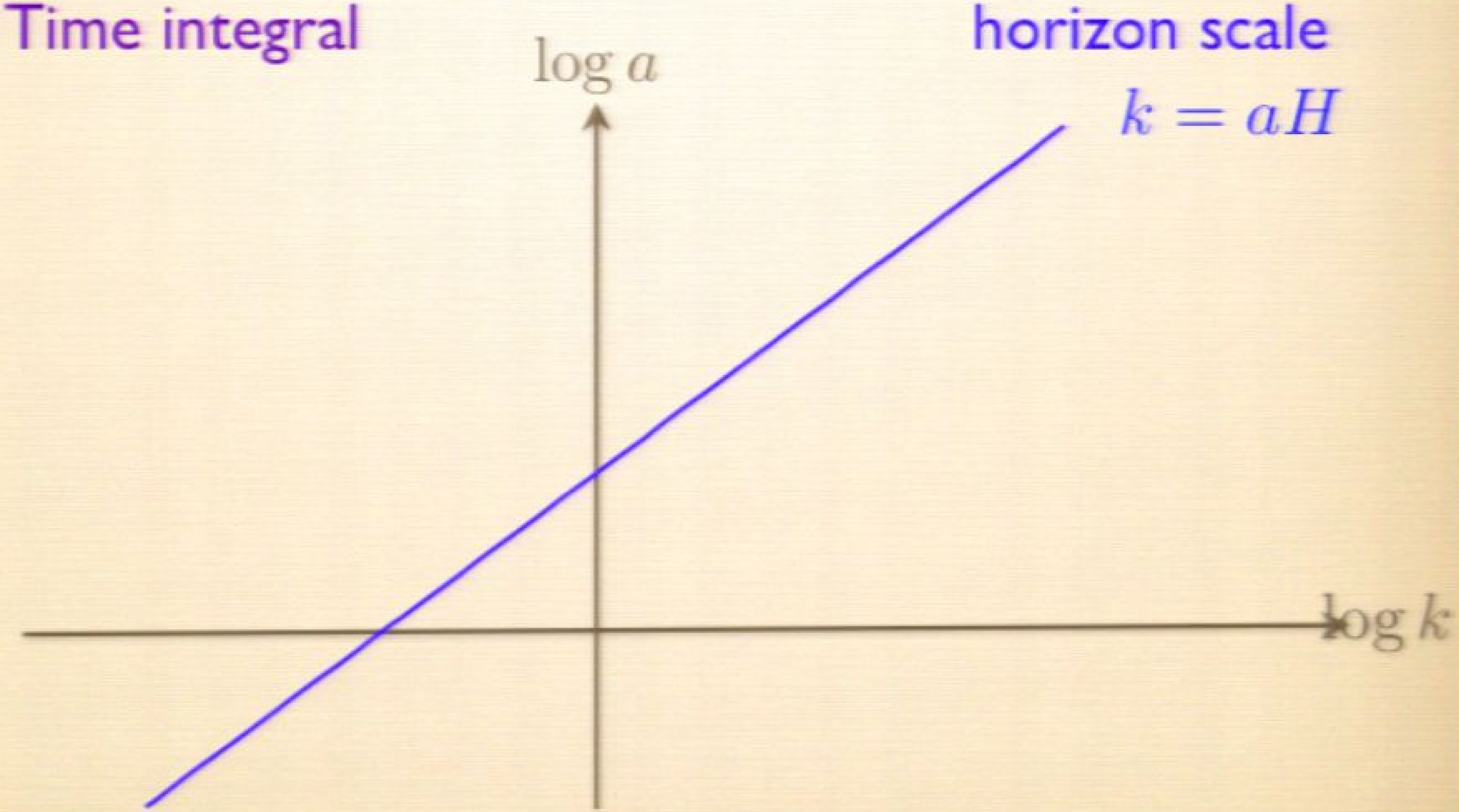

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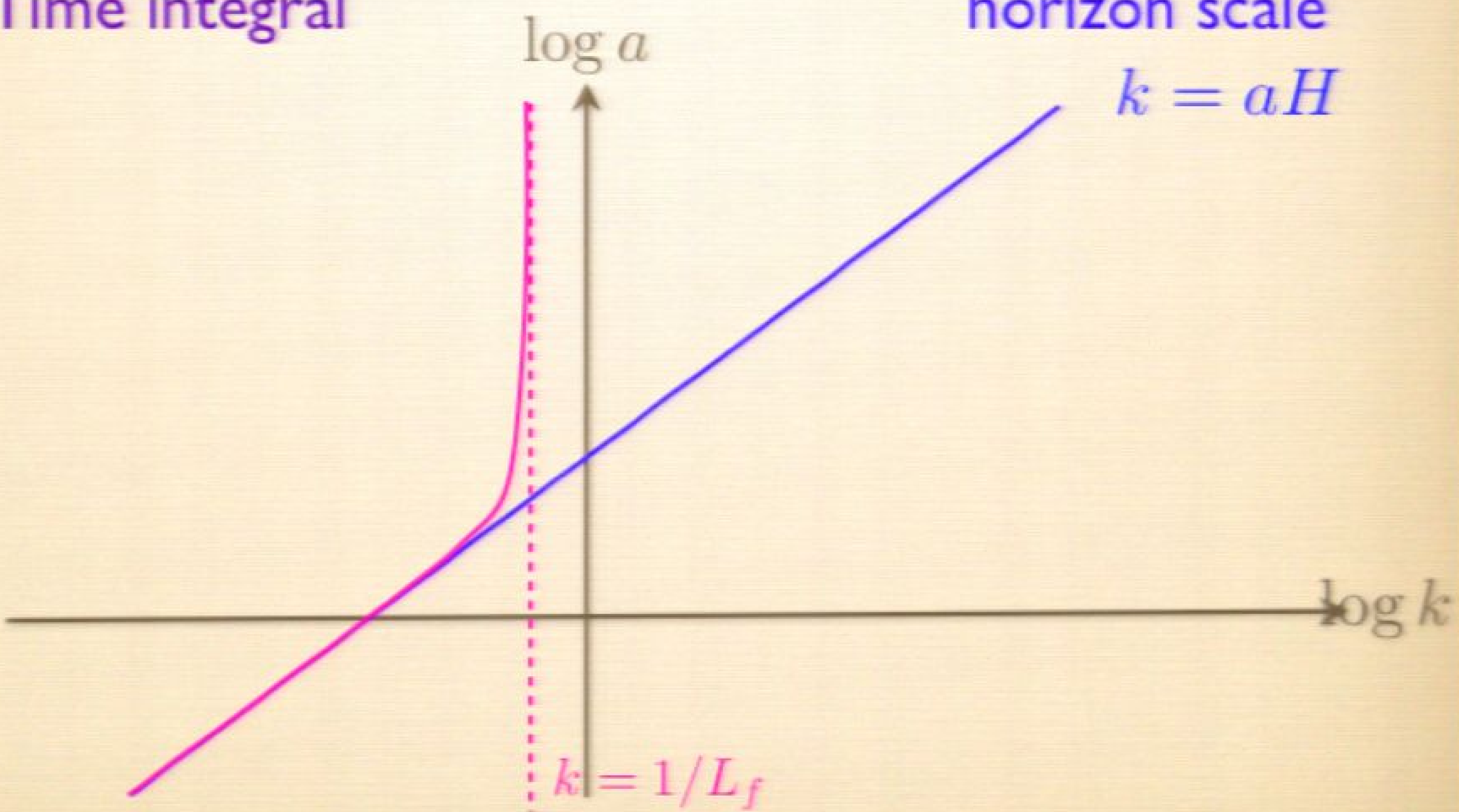

Effective cutoff $L(t) \sim L_f + (aH)^{-1}$

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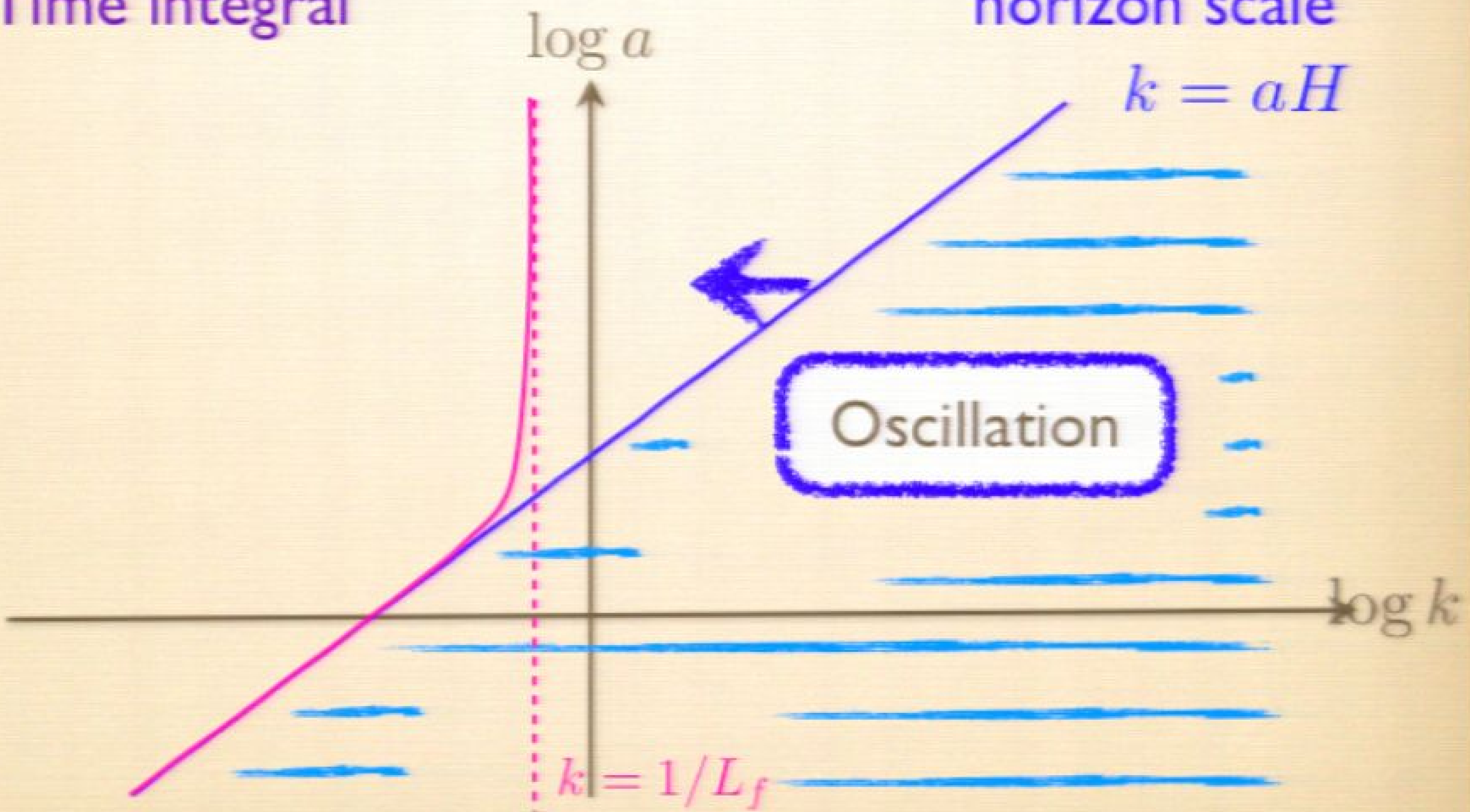

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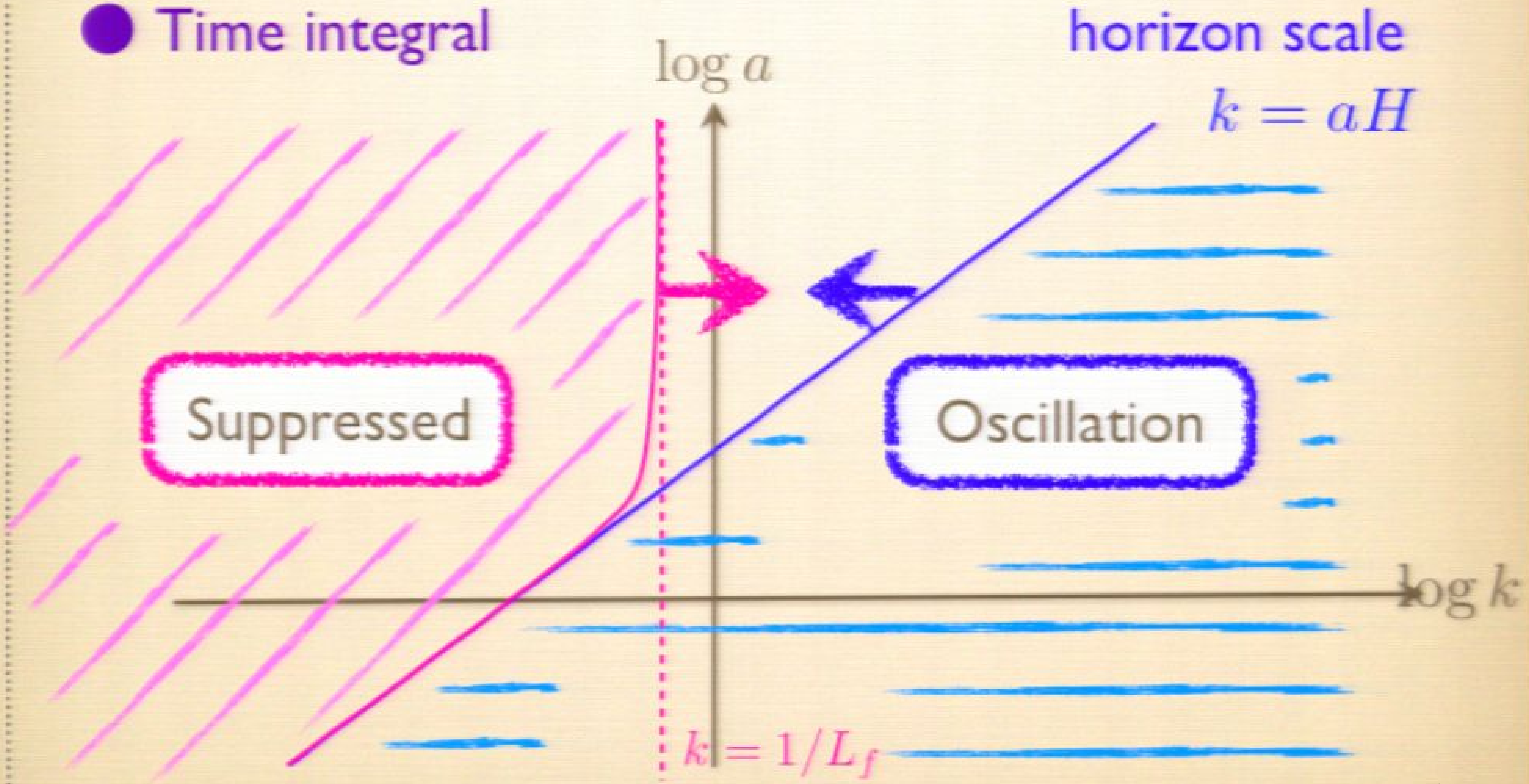

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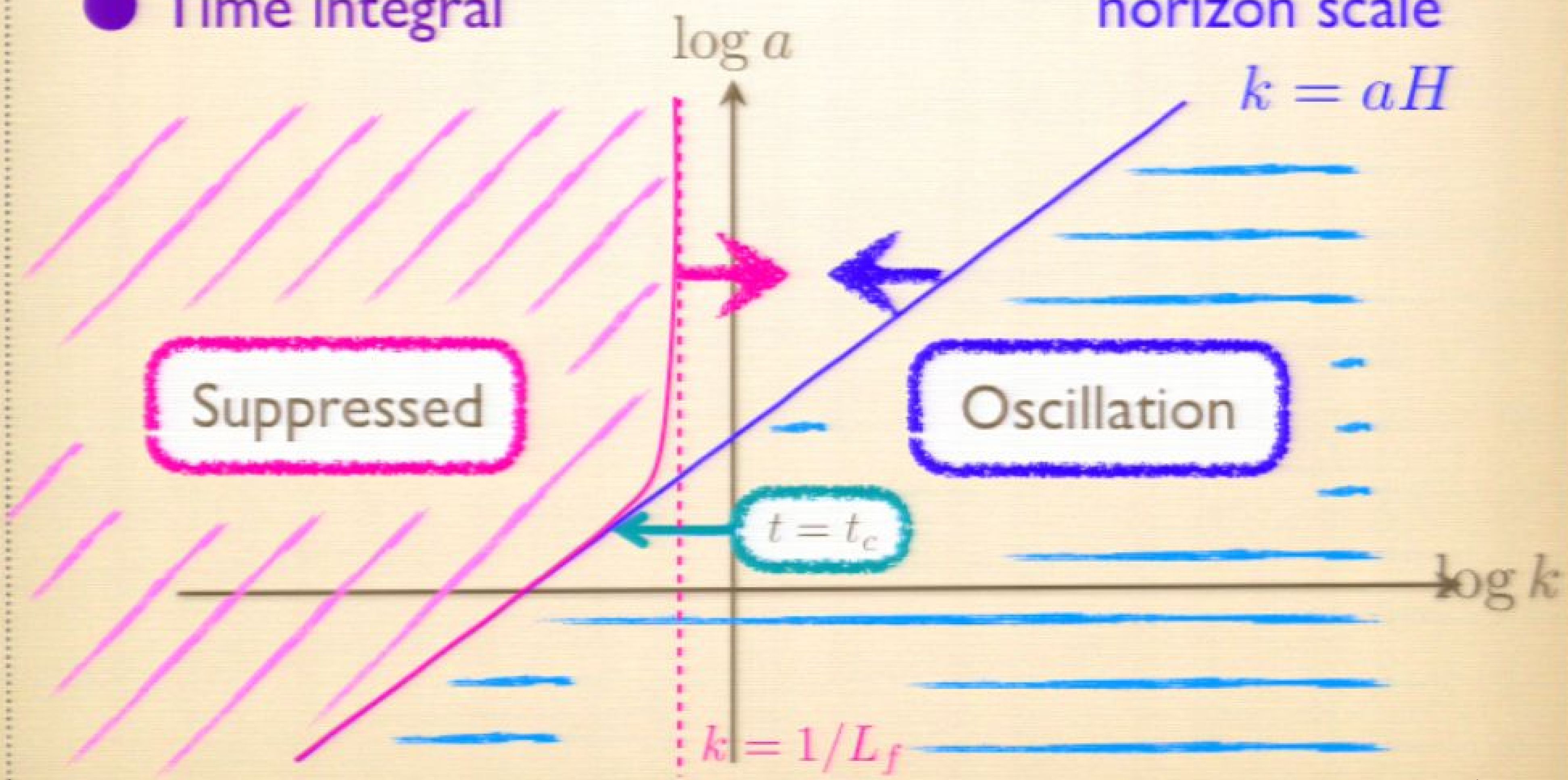

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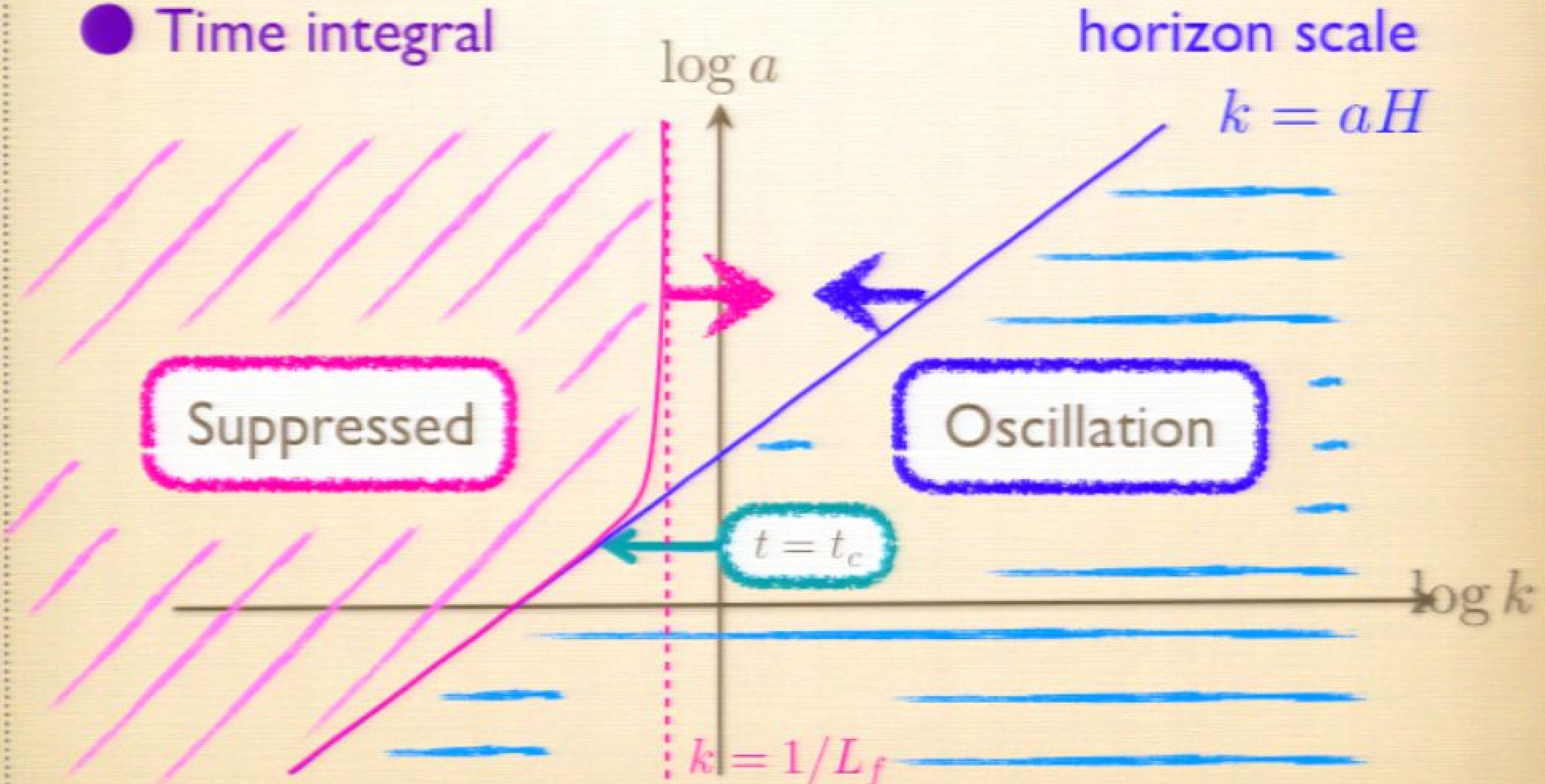

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Upper bound on secular growth

Assumption

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- Upper bound

Correlation fns. $\langle \zeta\zeta\zeta\dots\rangle$

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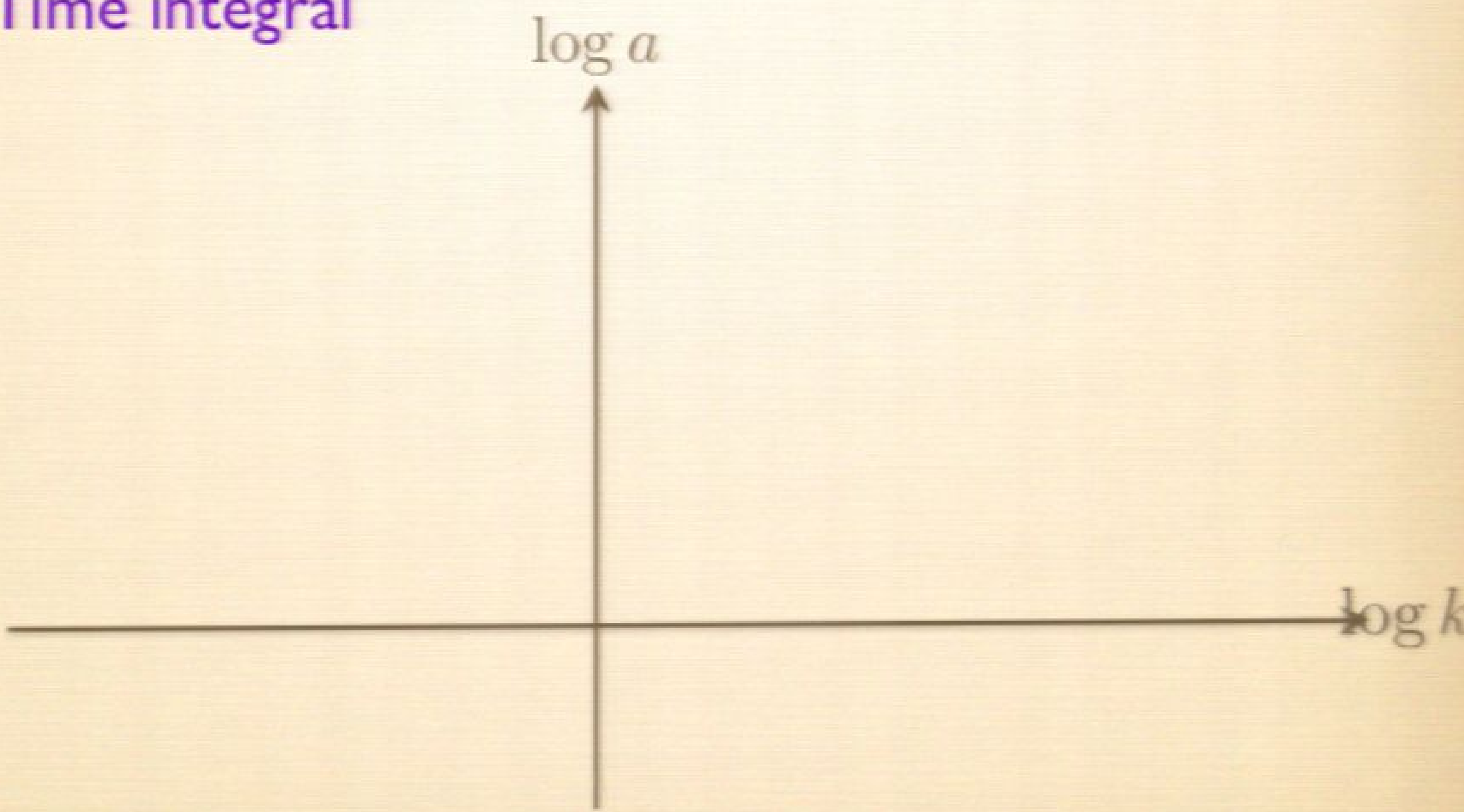

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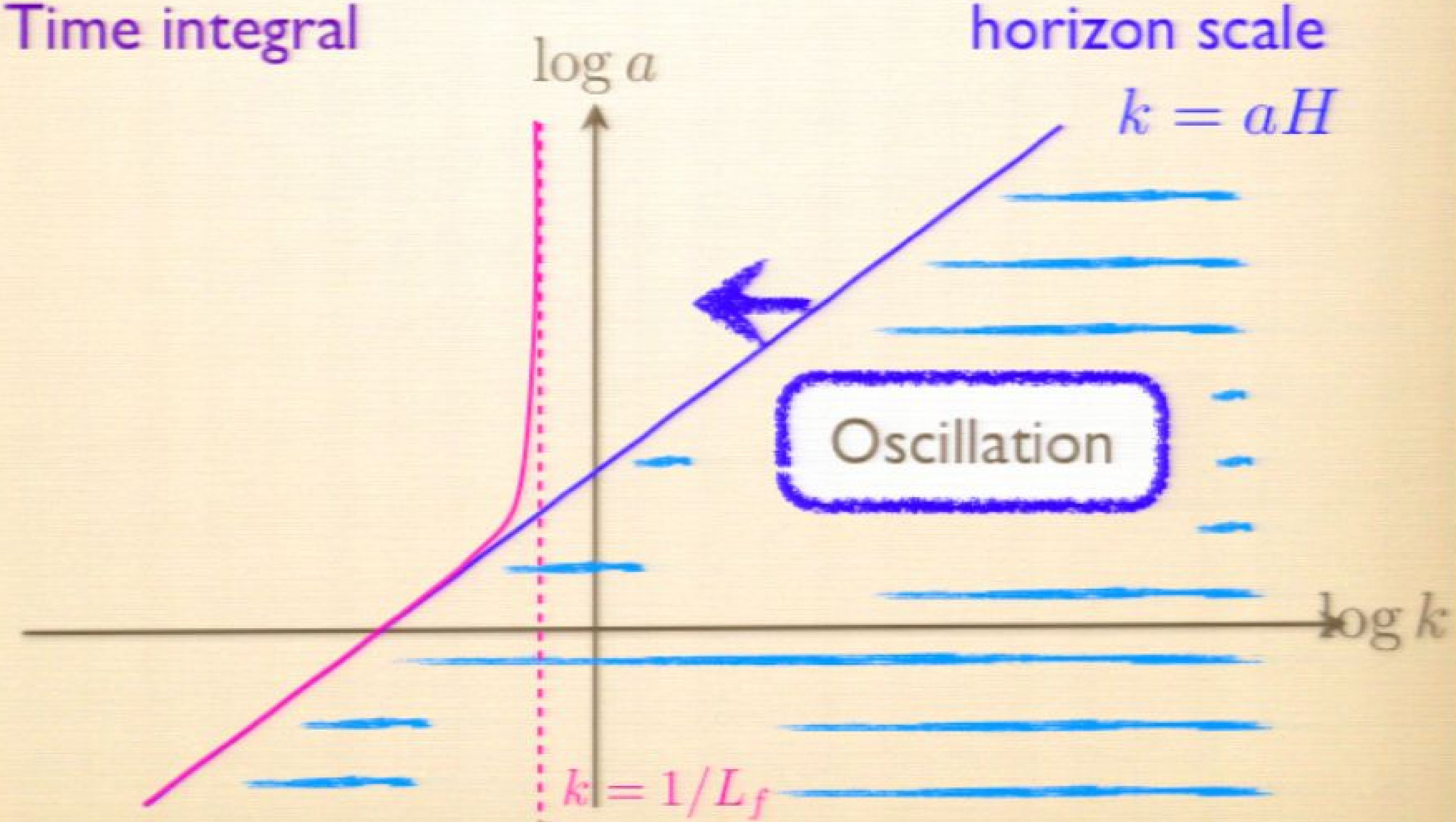

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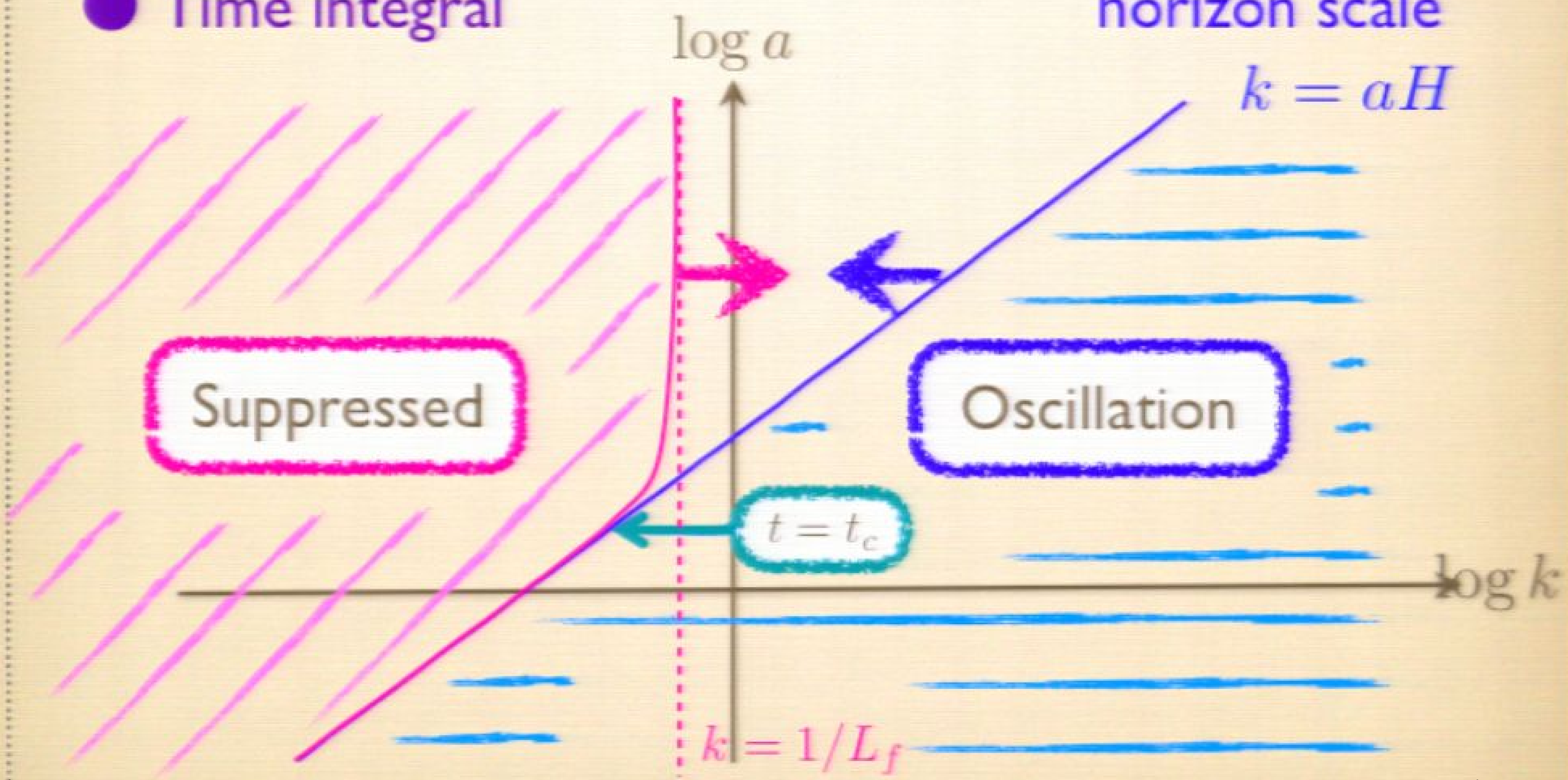

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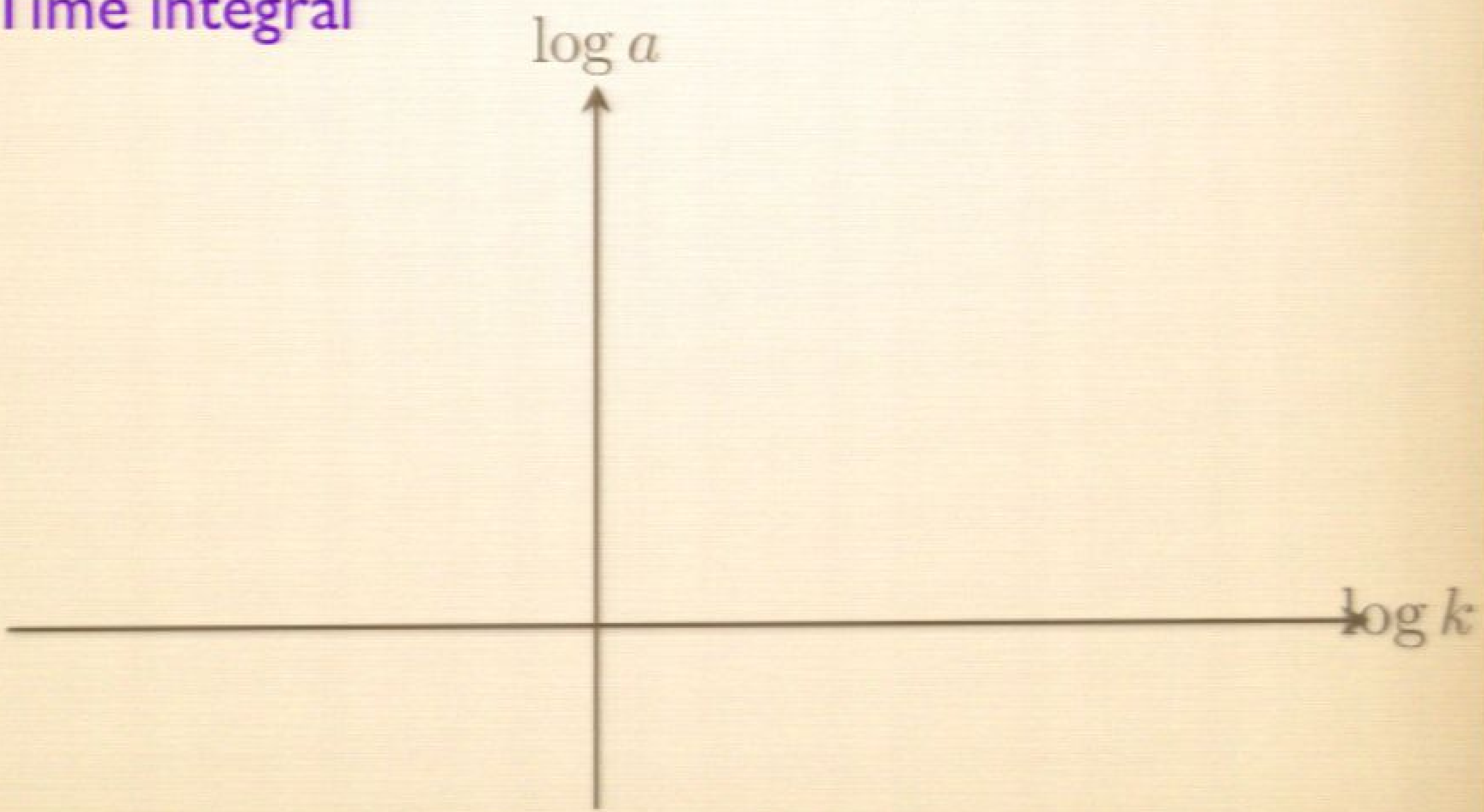

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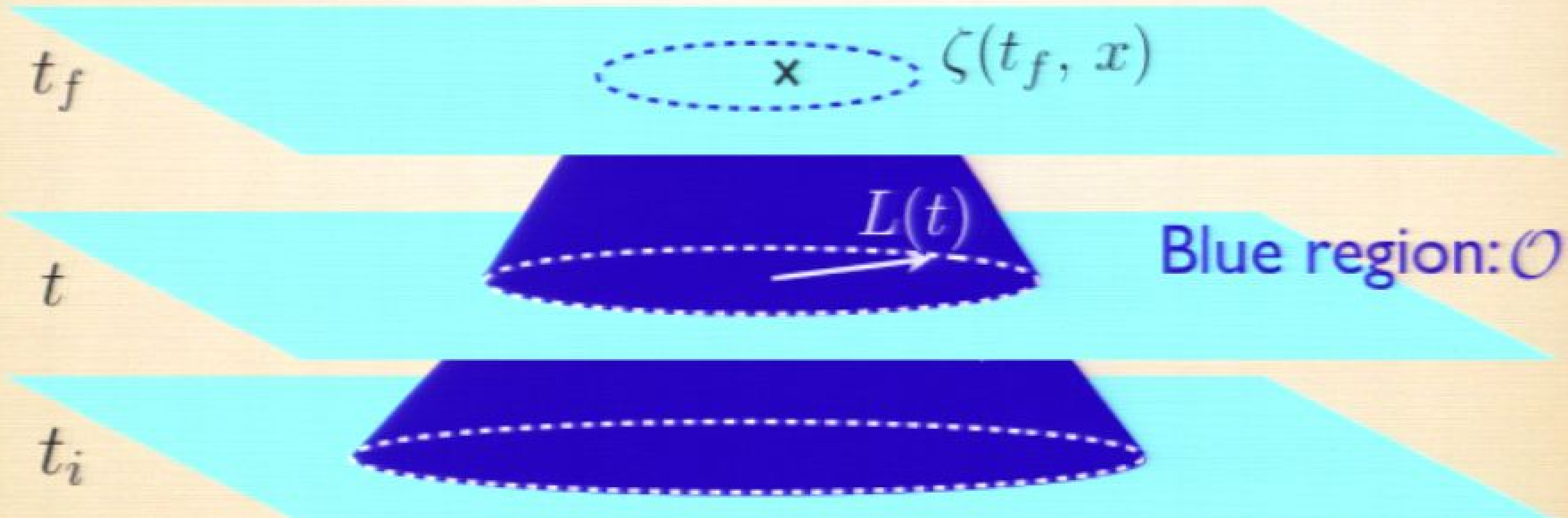

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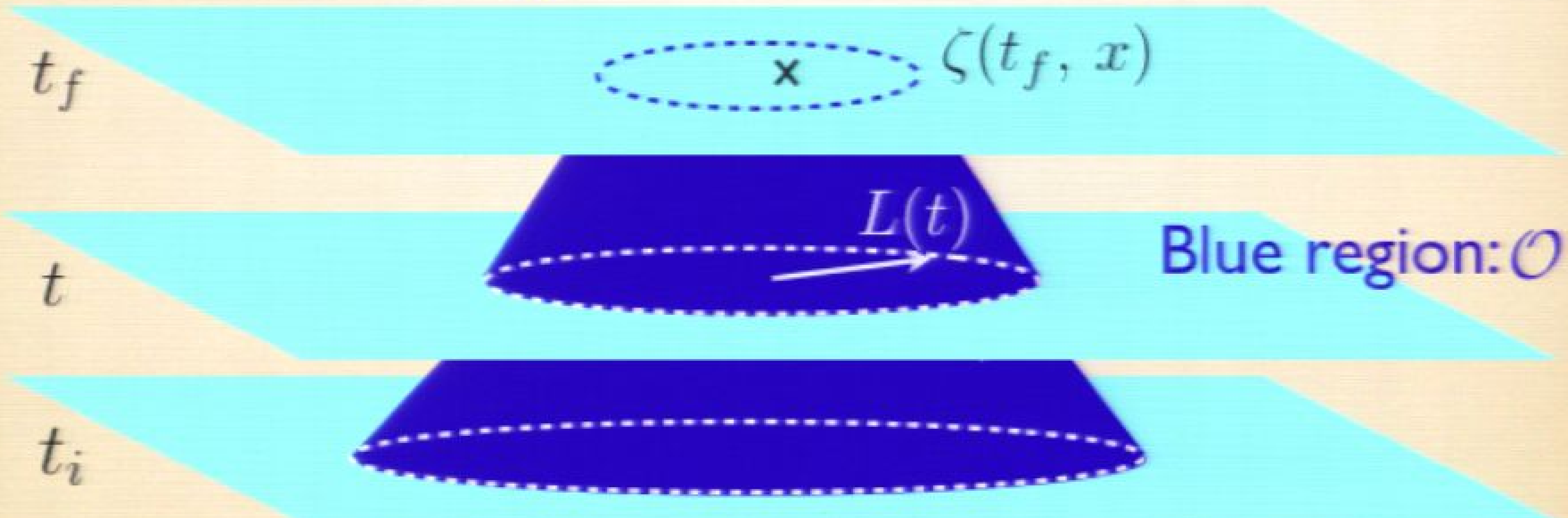

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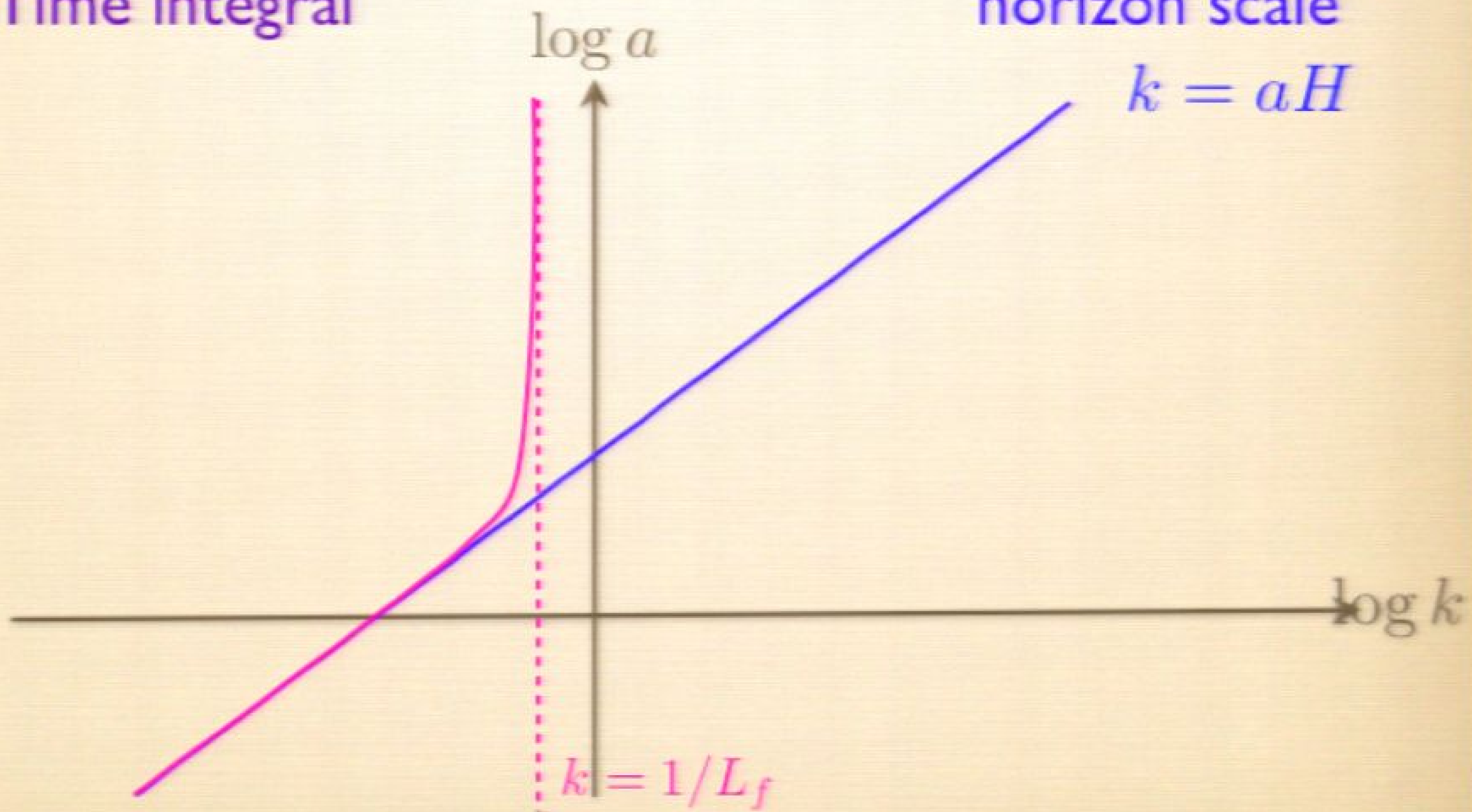

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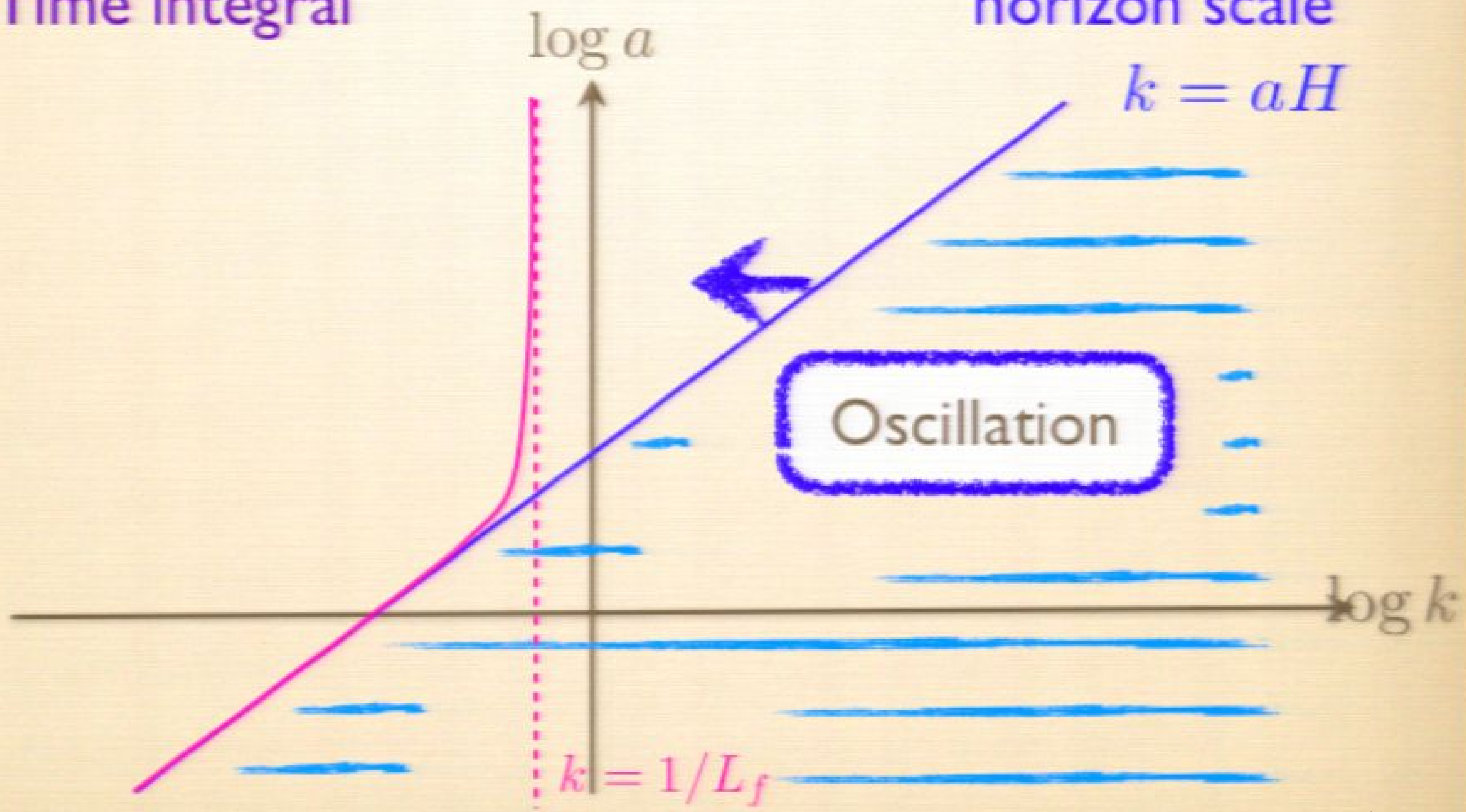

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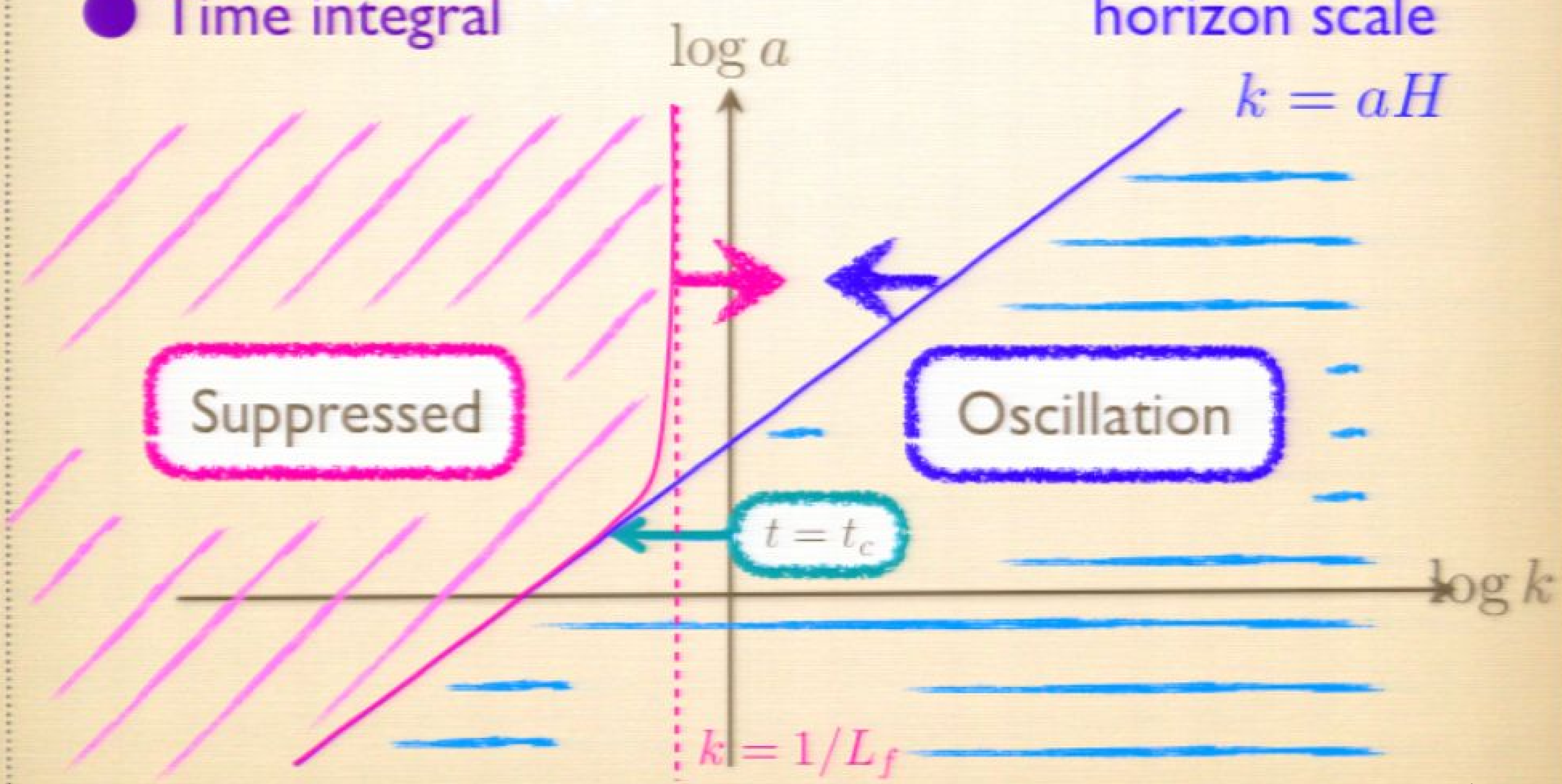

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$$n_c := \varepsilon^{-1} - 1 = \mathcal{O}(100)$$

Upper bound on secular growth

<u>Assumption</u>

UV renormalization is safely performed.

- Upper bound

Correlation fns. $\langle \zeta\zeta\zeta \cdots \rangle$

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Summary of the local gauge

Comoving gauge	Local comoving gauge
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$\delta\gamma_{ii} = 0 \quad \partial^i \delta\gamma_{ij} = 0$	$\int_{\mathcal{O}} \mathrm{d}^3 x\, \tilde{\zeta}(t, x^i) = 0$ etc
ζ with all k	$\tilde{\zeta}$ with $k \geq 1/L(t)$

+ Locality

Initial condition

ζ_I:Adiabatic vacuum → Gauge trans. $\tilde{\zeta}_I(x) = \zeta_I(x) - \int_{\mathcal{O}} \mathrm{d}^3 x\, \zeta_I(x)$

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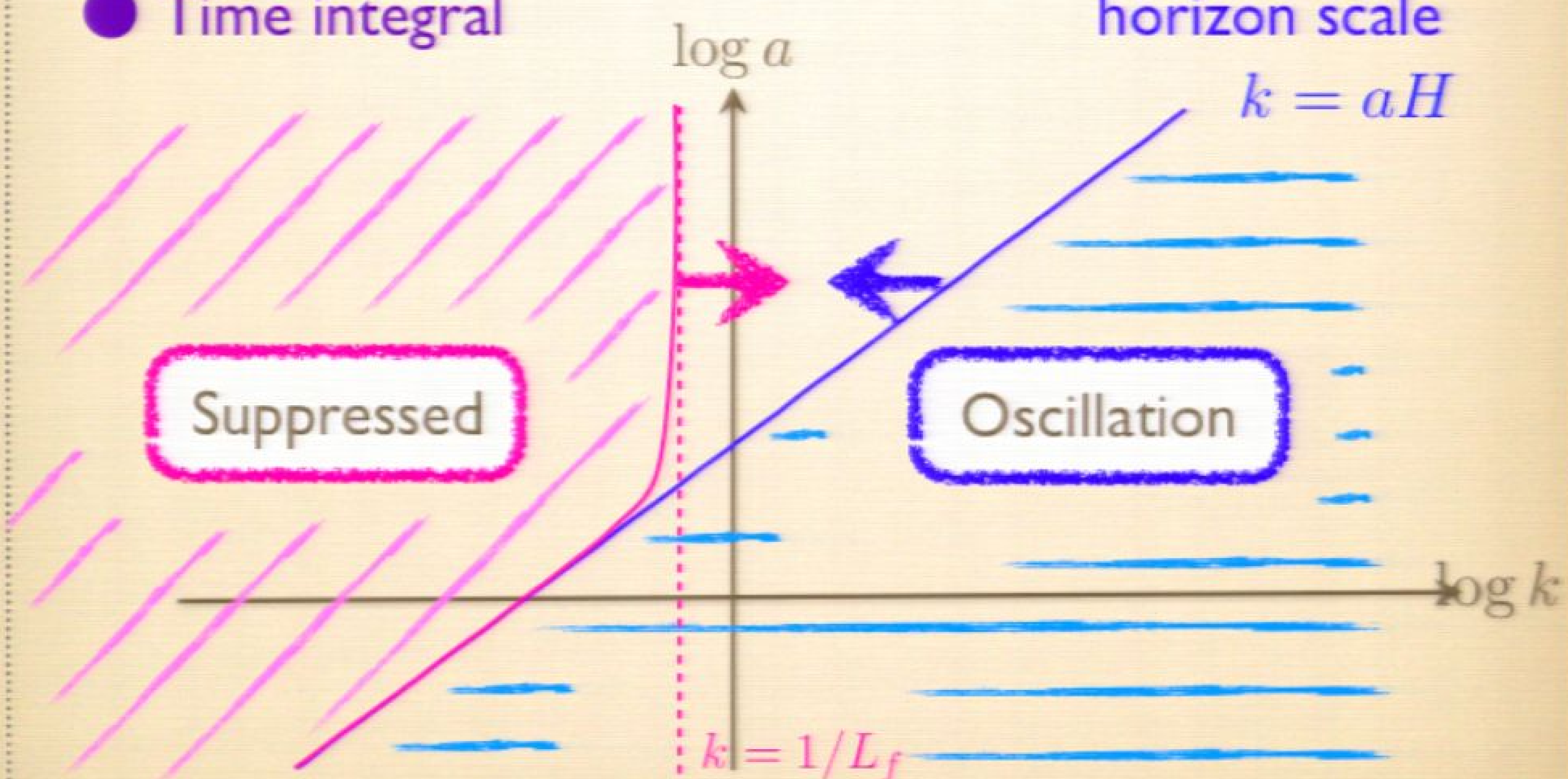
Vertex integral $\int \mathrm{d}t \int_{1/L(t)} \mathrm{d}^3k$

- Time integral



Effective cutoff $L(t) \sim L_f + (aH)^{-1}$

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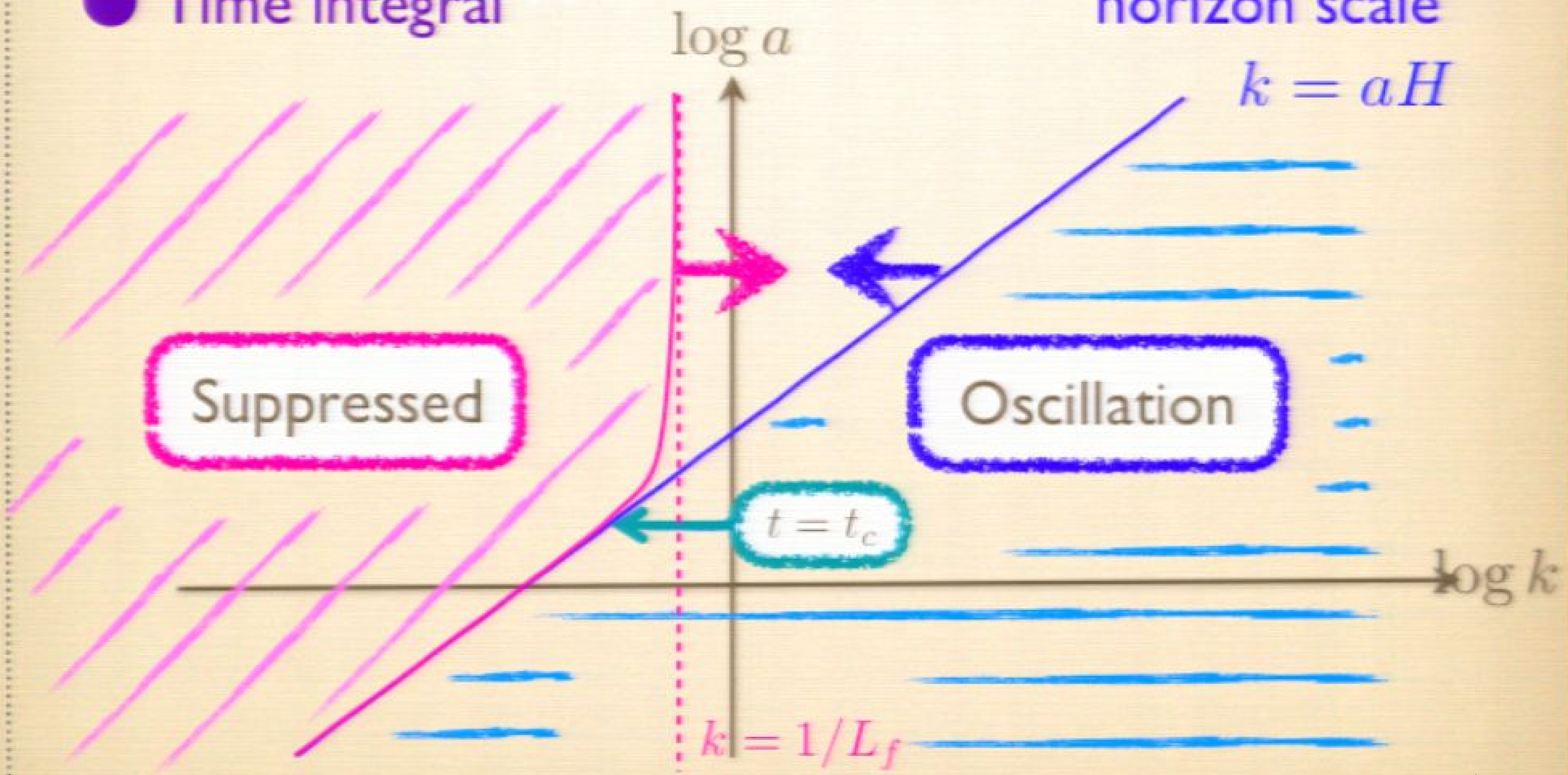

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Proposal of IR regularization

IR corrections, that yield divergence, are changed by the residual gauge DOFs.

> Why don't you perform gauge-inv. perturbations?

1. Complete gauge fixing

   Y.U.&T.Tanaka(09)

2. Construction of gauge-invariant variables

   Y.U.&T.Tanaka(10^1,10^2)

Genuine gauge–inv. quantities

It's only necessary to evaluate the gauge-inv. quantities.

- Genuine gauge-inv. quantities Y.U.&T.Tanaka(10)

Gauge invariance regarding $x^i \to \tilde{x}^i = x^i + \delta x^i$

Scalar quantity, labeled by the gauge-invariant argument

${}^sR$: 3D scalar curvature ${}^sR \propto \partial^2\zeta$

$${}^sR(x) \to {}^sR(x) - \underline{\delta x^i \partial_i {}^sR(x)}$$

Due to change of the argument

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Due to change of the argument

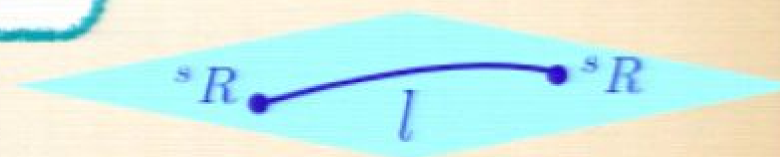
Not appear if we specify the arguments of ${}^sR$ by gauge-invariant quantity

Genuine gauge-inv. quantities 2

1. Geodesic normal coordinate

$\langle {}^sR {}^sR \rangle(l),\ \langle {}^sR {}^sR {}^sR \rangle(l_1, l_2), \ldots$

l_m: Geodesic distance



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2. Gauge-invariant initial state

Quantization { Physical DOFs / Gauge DOFs }

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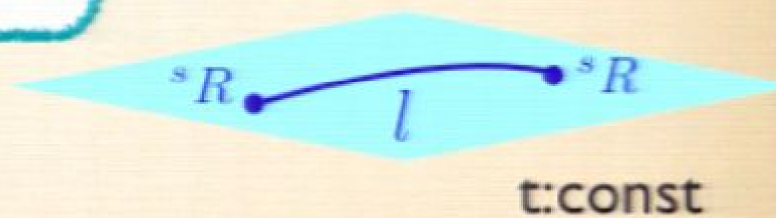

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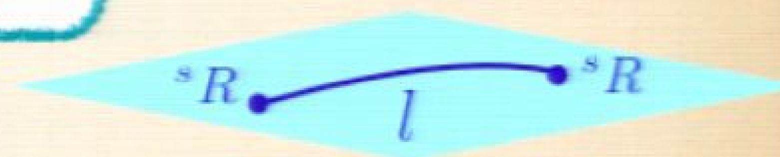

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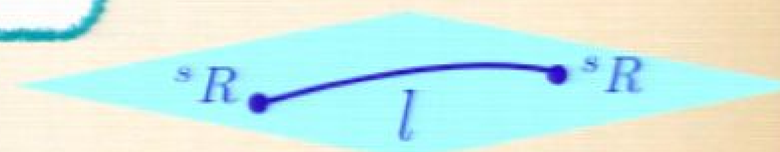

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Quantization
- Physical DOFs
- ~~Gauge DOFs~~

Initial state $|\Psi\rangle$: Need to restrict to the physical state

Gauge-invariant initial state

- Initial condition in interaction picture

ζ : Heisenberg picture field   Y.U.&T.Tanaka (10)

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1. $\zeta(t_i) = \zeta\left[\zeta_I(t_i)\right]$
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■ One-loop corrections

Total derivatives

$$\langle {}^sR {}^sR \rangle(l) \simeq \int \mathrm{d}(\log k)\partial_{\log k}(\ldots)$$

+ (Divergent terms) + (Regular terms)

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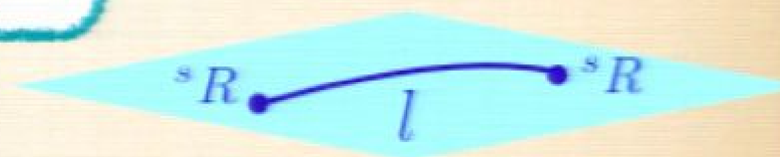

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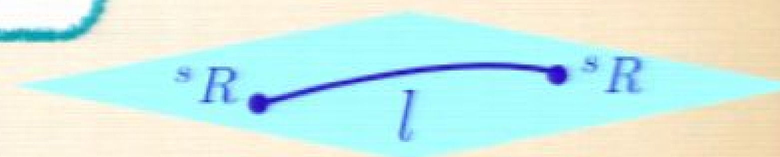

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Necessary condition for gauge-invariance

Gauge-invariant initial state 2

1. $\zeta(t_i) = \zeta\,[\zeta_I(t_i)]$

Y.U.&T.Tanaka(10)

Heisenberg eq. $\mathcal{L}\zeta = \mathcal{S}[\zeta]$

$$\zeta = \Sigma_i a_i F[\zeta_I] + \mathcal{L}^{-1}\mathcal{S}$$

$\mathcal{L}$: Derivative op.

homogeneous solution

$\mathcal{L}F[\zeta_I] = 0$

Conditions on a_i → (C1)

2. Positive frequency fn. for ζ_I

ρ:e-folding

$$(1+\varepsilon)\,\partial_\rho\zeta_k - x^i\partial_i\zeta_k + \varepsilon\zeta_k + \ldots = -\left(\partial_{\log k} + 3/2\right)\zeta_k$$

(C2)

Remarks

- Slow-roll approximation
  - Leading order $\mathcal{O}(\varepsilon^0)$

    Bunch-Davies vacuum (C1), (C2) OK!
  - Higher orders

    Adiabatic vacuum & $\zeta_H(t_i) = \zeta_I(t_i)$

    → (C1), (C2) are not satisfied

- Canonical commutation relations

  Choosing the appropriate a_i

  → Commutation relations can be compatible with the gauge-invariance condition

Concluding this talk, ...

1. Origin of Infrared divergence

   → Presence of non-local gauge DOFs

2. Two ways of regularization

   → Gauge inv. perturbations

   (1) Gauge fixing

   Momentum integrals are regularized.
   No secular growth for $n < n_c = \mathcal{O}(100)$

   (2) Construct genuine gauge inv. variables

   a. Geodesic normal coordinate

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3. Implications on observable fluctuations, such as NGs

Origins of IR divergences

	Single field (Adiabatic)	Multi field (Isocurvature)
Momentum integral	Gauge artifacts → This talk → Arthur's talk	Absence of decoherence → Takahiro's talk
Time integral		Controversial

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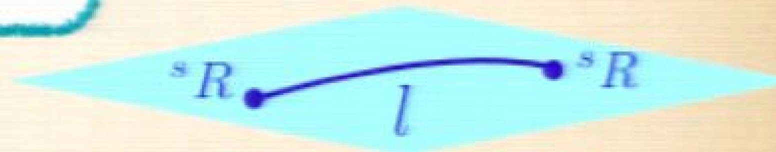

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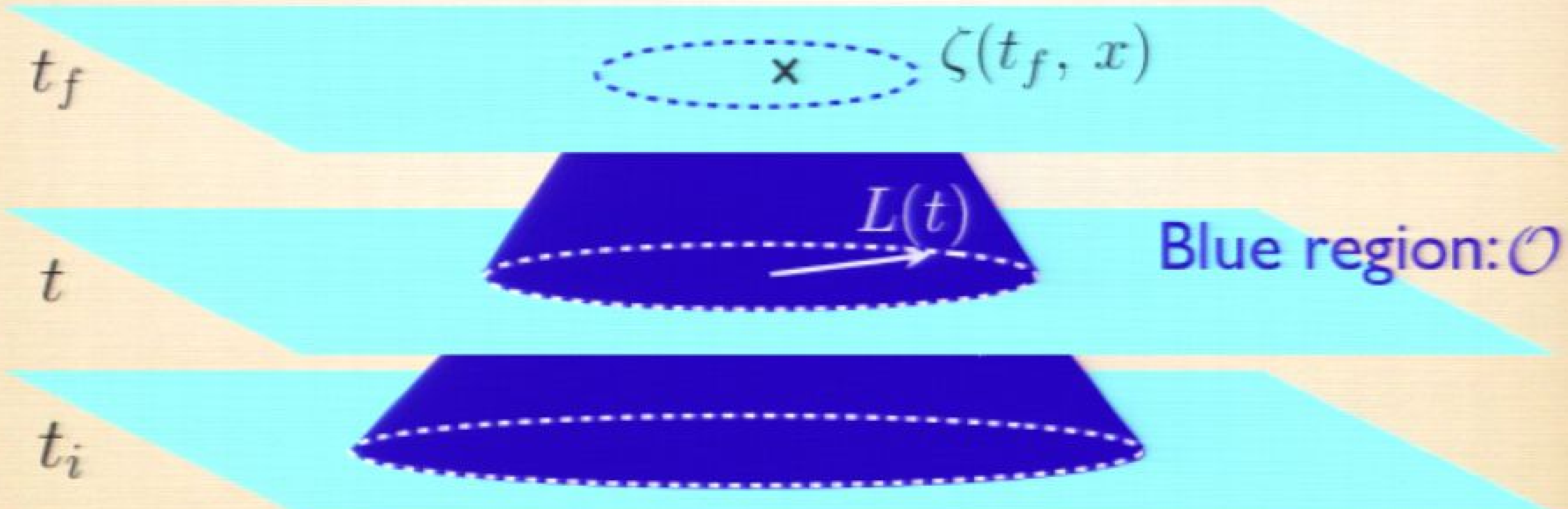
Y.U.&T.Tanaka(09)

Vertex integral $\int \mathrm{d}t \int \ \mathrm{d}^3k$

- Momentum integral



Vertexes in $\tilde{\zeta}(t_f, x^i)$ $\int \mathrm{d}t \int_{|x| \le L(t)} \mathrm{d}^3x$

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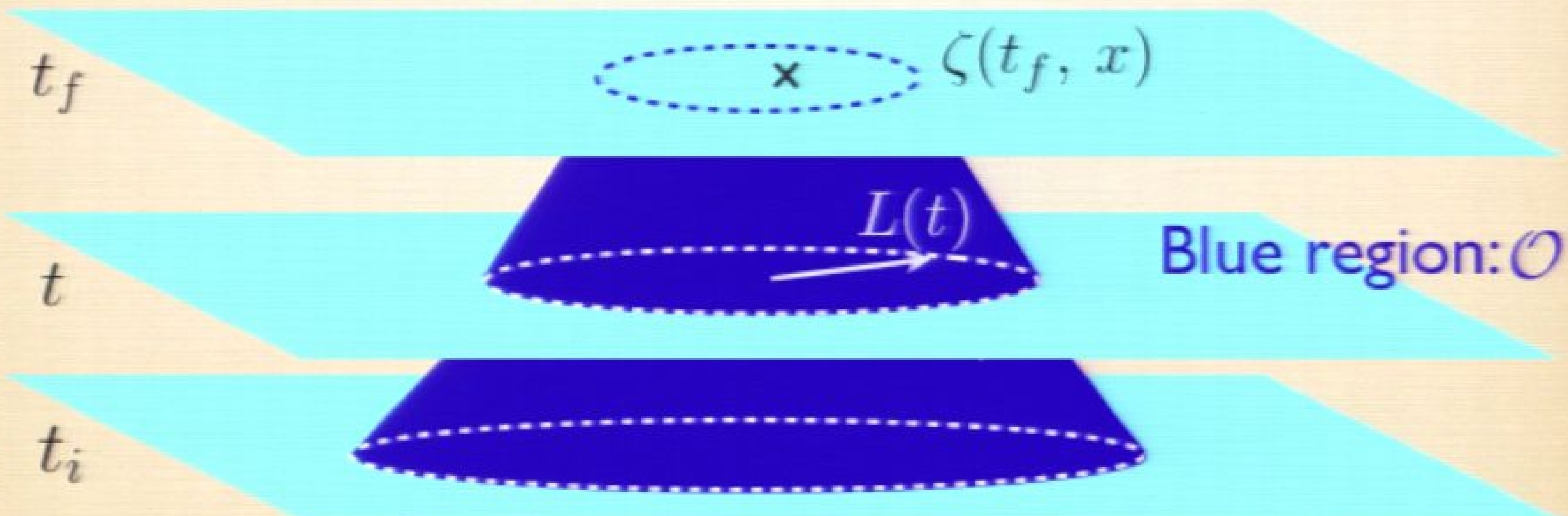

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⟶ Effective cutoff at $L(t) = L_f + \int_t^{t_f} \frac{\mathrm{d}t}{a(t)} \simeq L_f + \frac{1}{a(t)H}$

Regularization scheme 2

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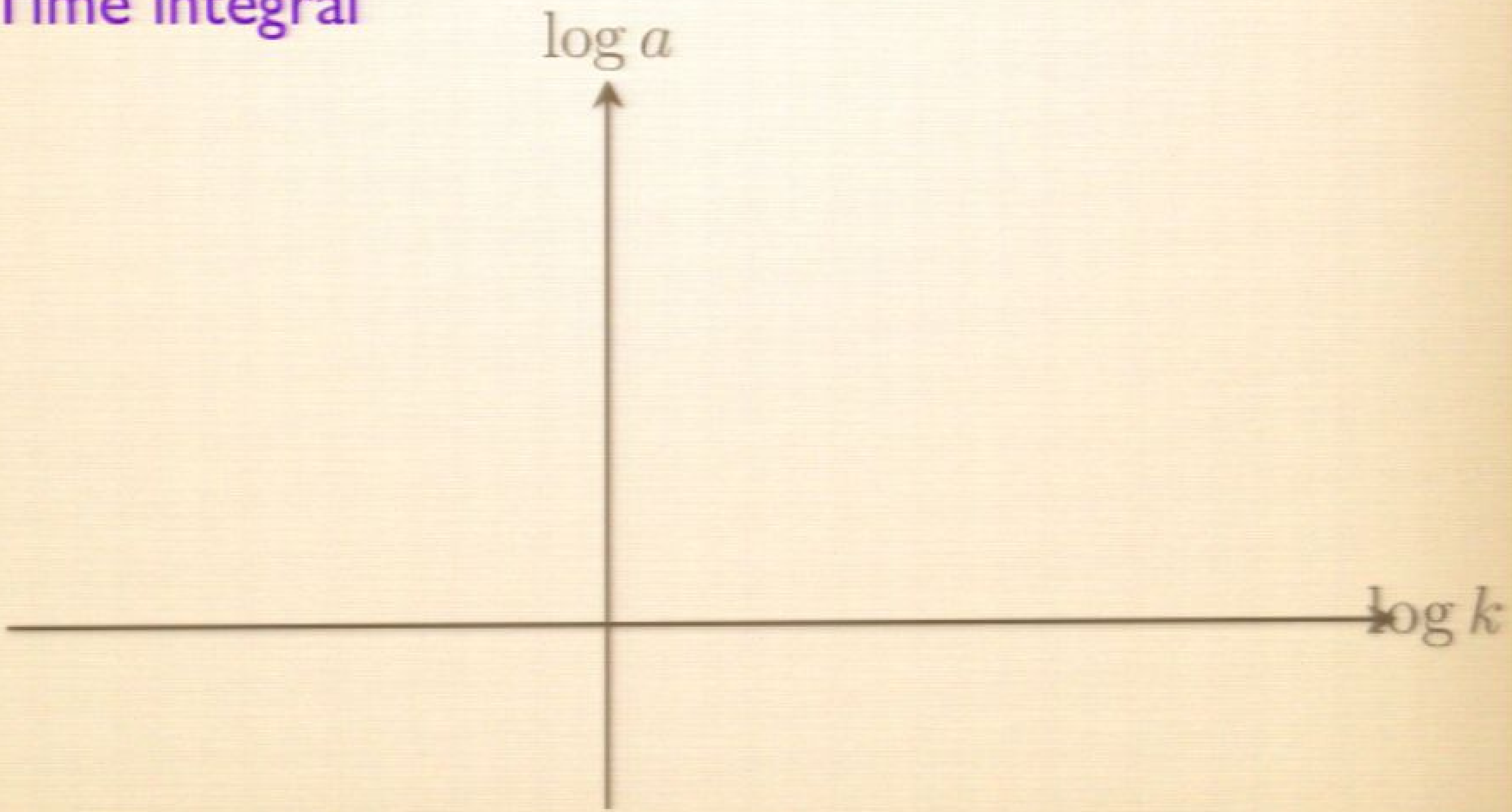

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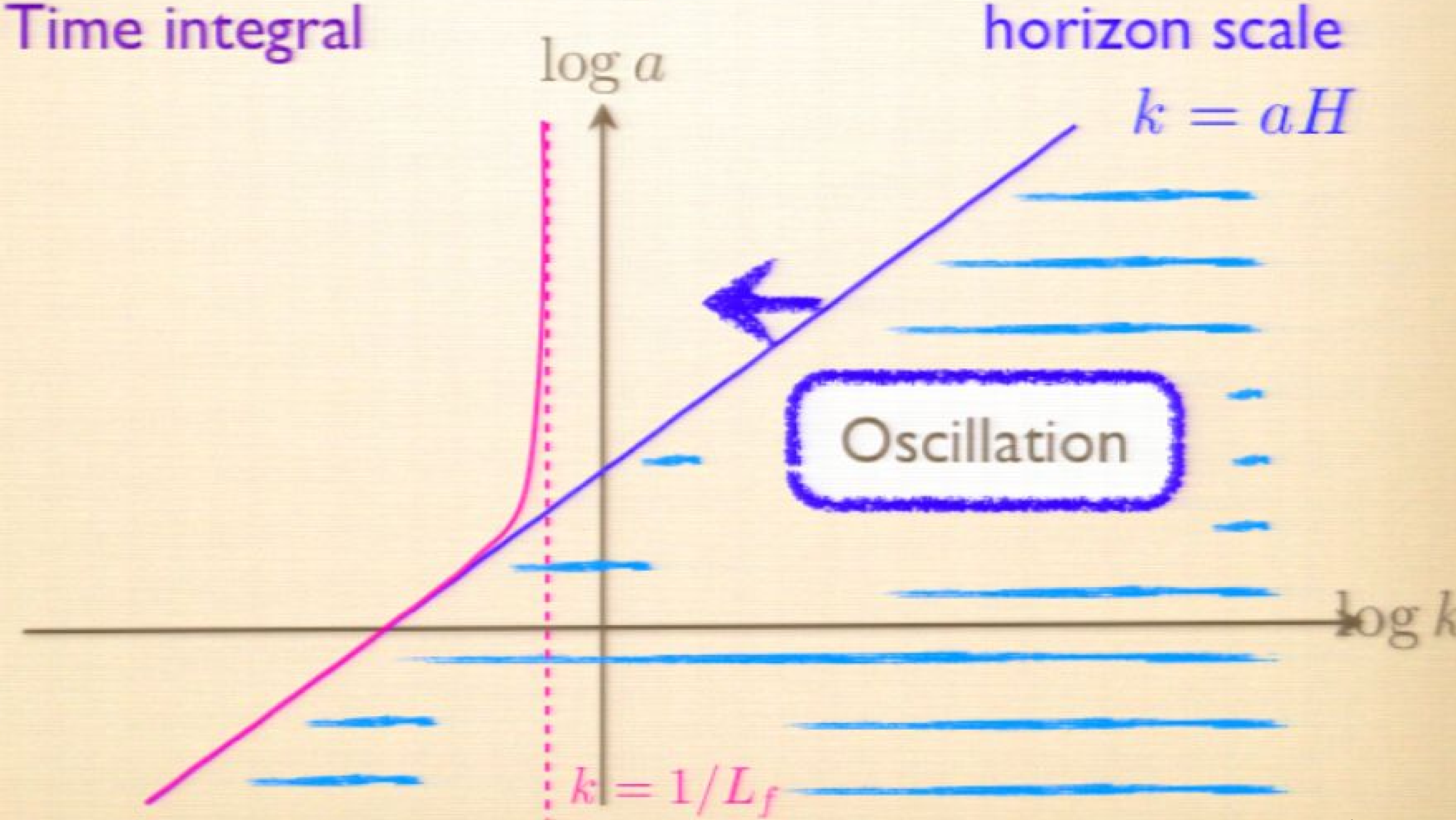

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Regularization scheme 2

Vertex integral $\int_{t_i} dt \int_{1/L(t)} d^3k$

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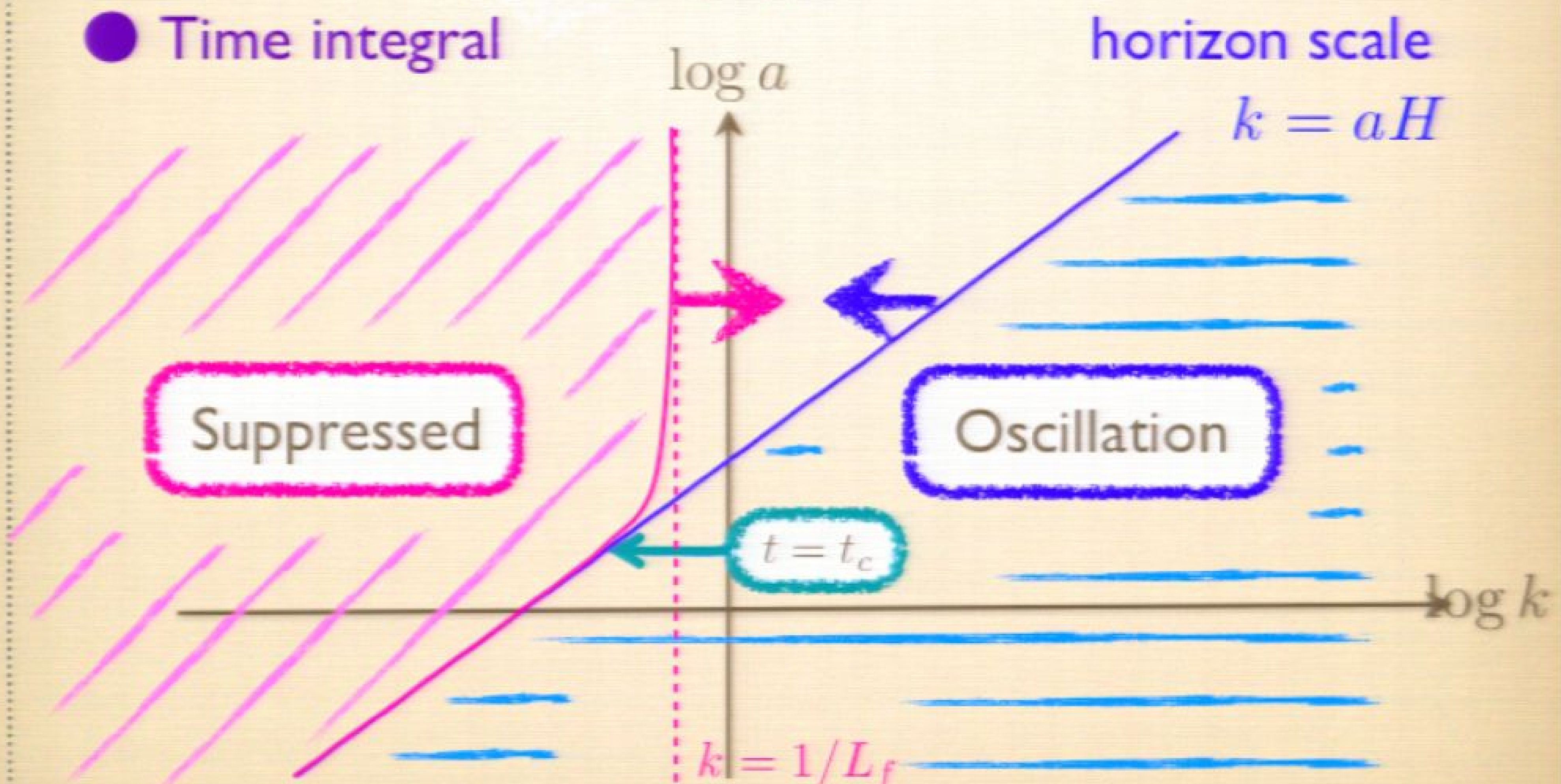

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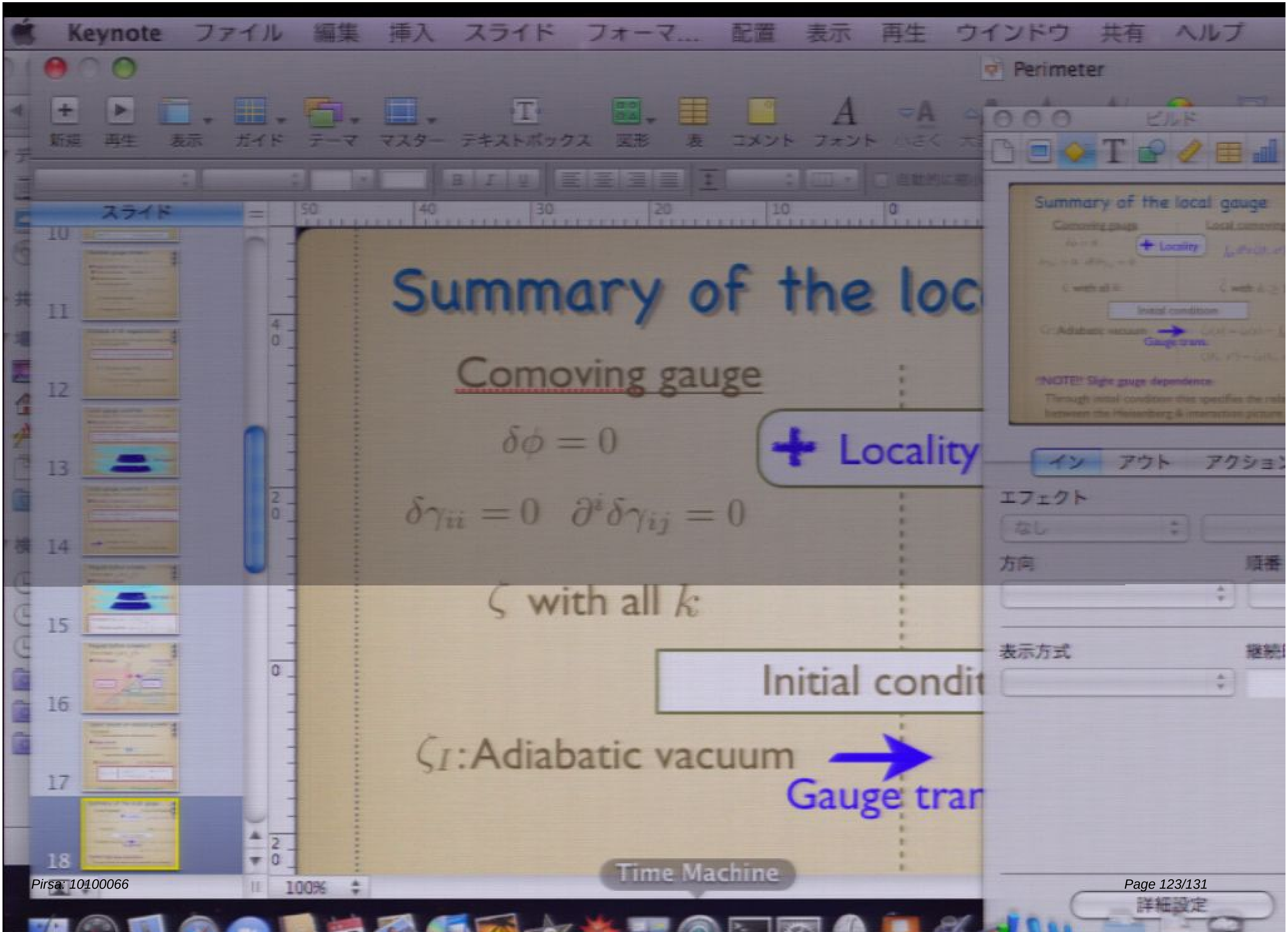
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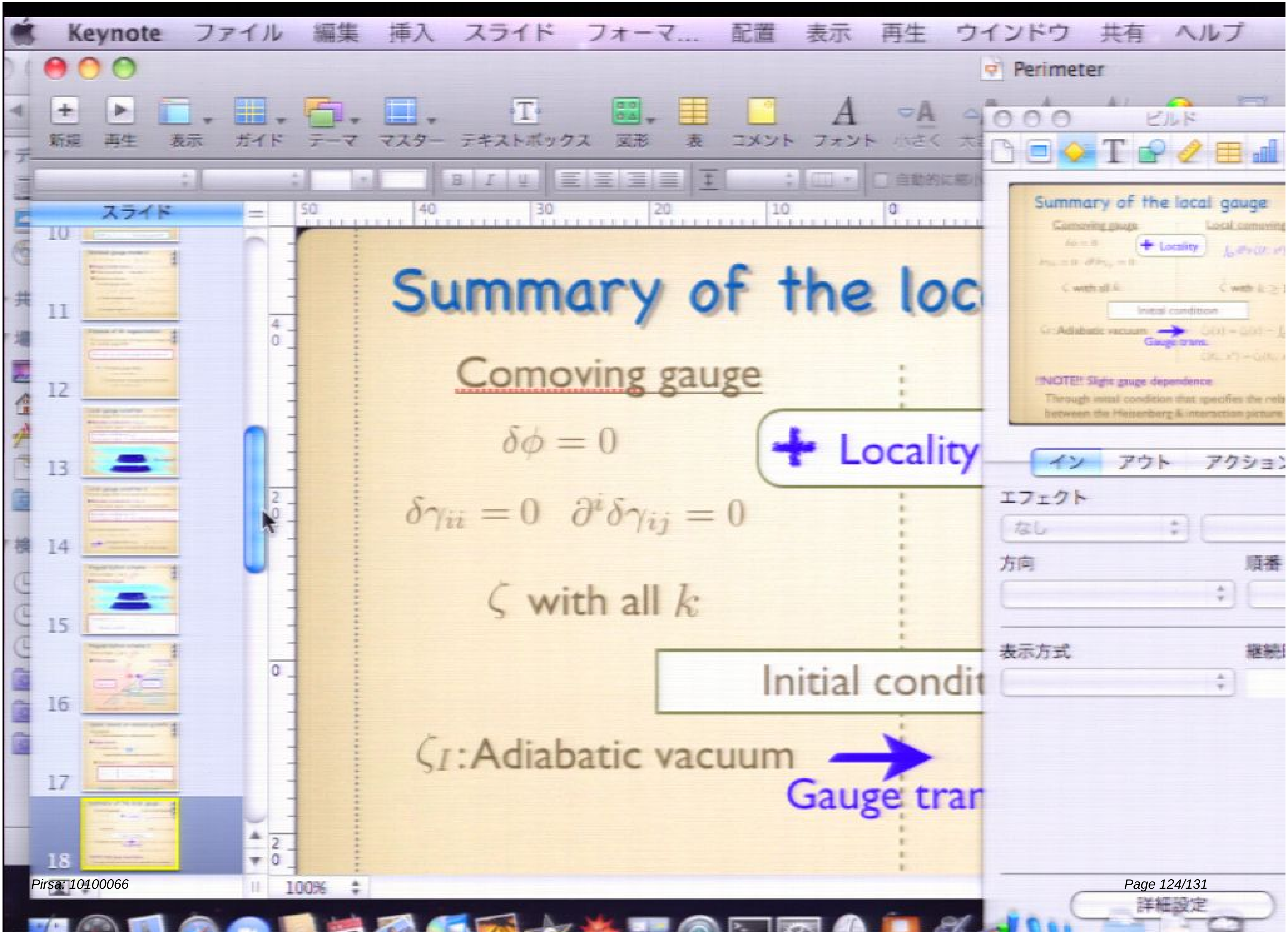
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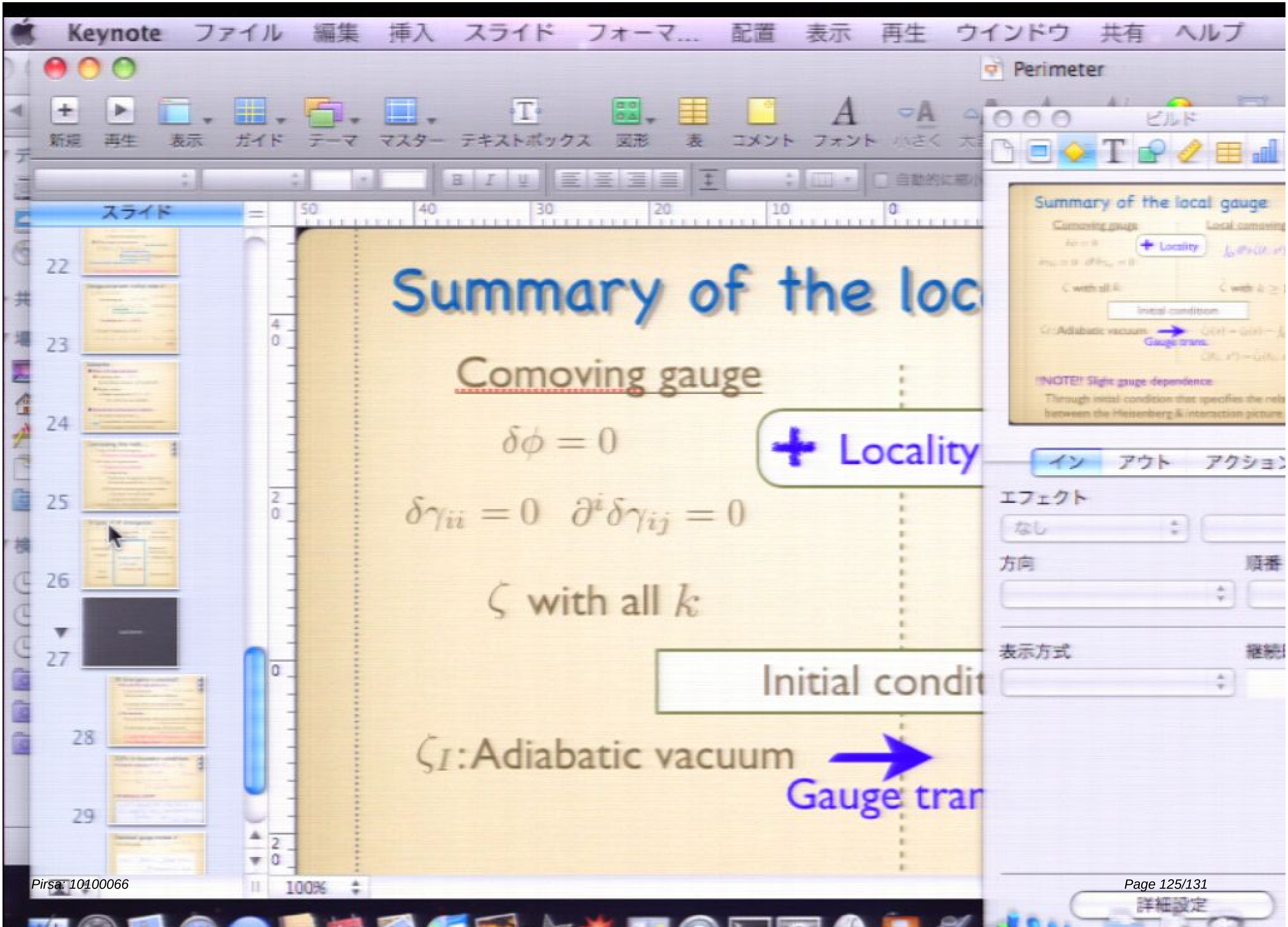
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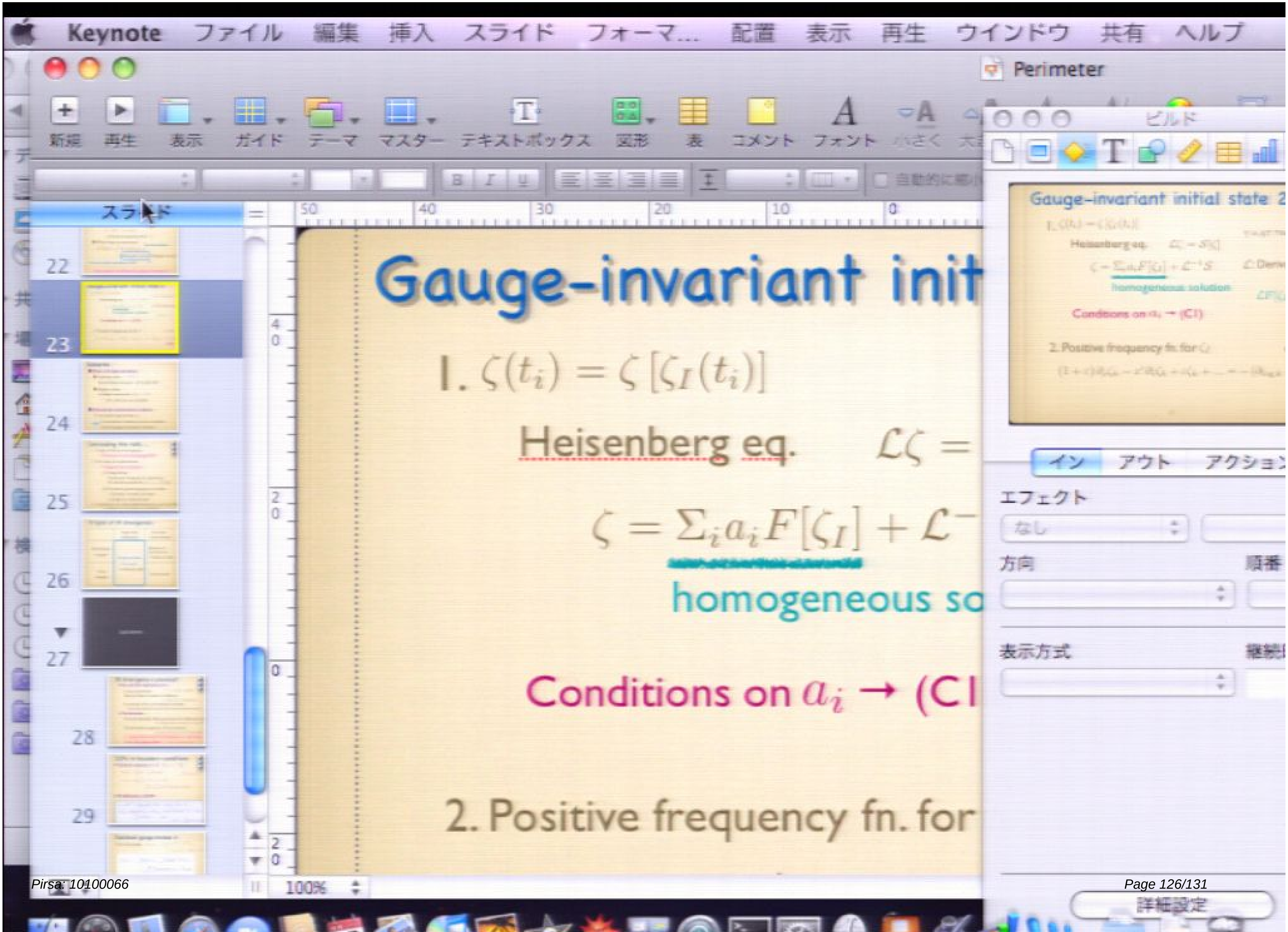
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$\zeta = \Sigma_i a_i F[\zeta_I] + \mathcal{L}^{-}$
homogeneous so
Conditions on a_i → (CI
2. Positive frequency fn. for
Gauge-invariant initial state
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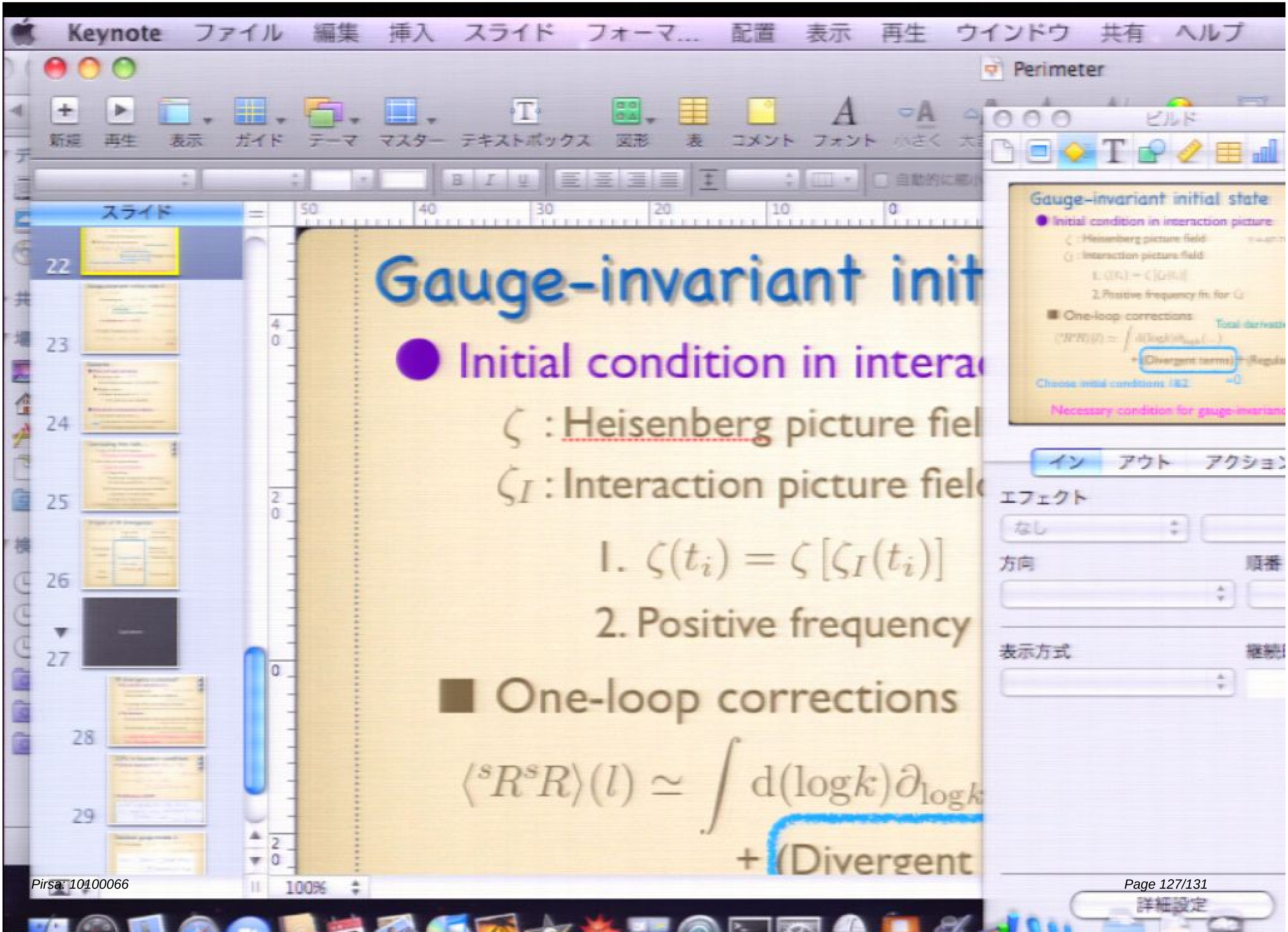
詳細設定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スライド
ビルド
Gauge-invariant init
Initial condition in interac
ζ : Heisenberg picture fiel
ζ_I : Interaction picture fiel
1. ζ(t_i) = ζ[ζ_I(t_i)]
2. Positive frequency
One-loop corrections
⟨^sR^sR⟩(l) ≃ ∫ d(log k)∂_log k
+ (Divergent
Gauge-invariant initial state
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100%
詳細設定

Gauge-invariant initial state

- Initial condition in interaction picture

ζ : Heisenberg picture field — Y.U.&T.Tanaka(10)

ζ_I : Interaction picture field

1. $\zeta(t_i) = \zeta\left[\zeta_I(t_i)\right]$

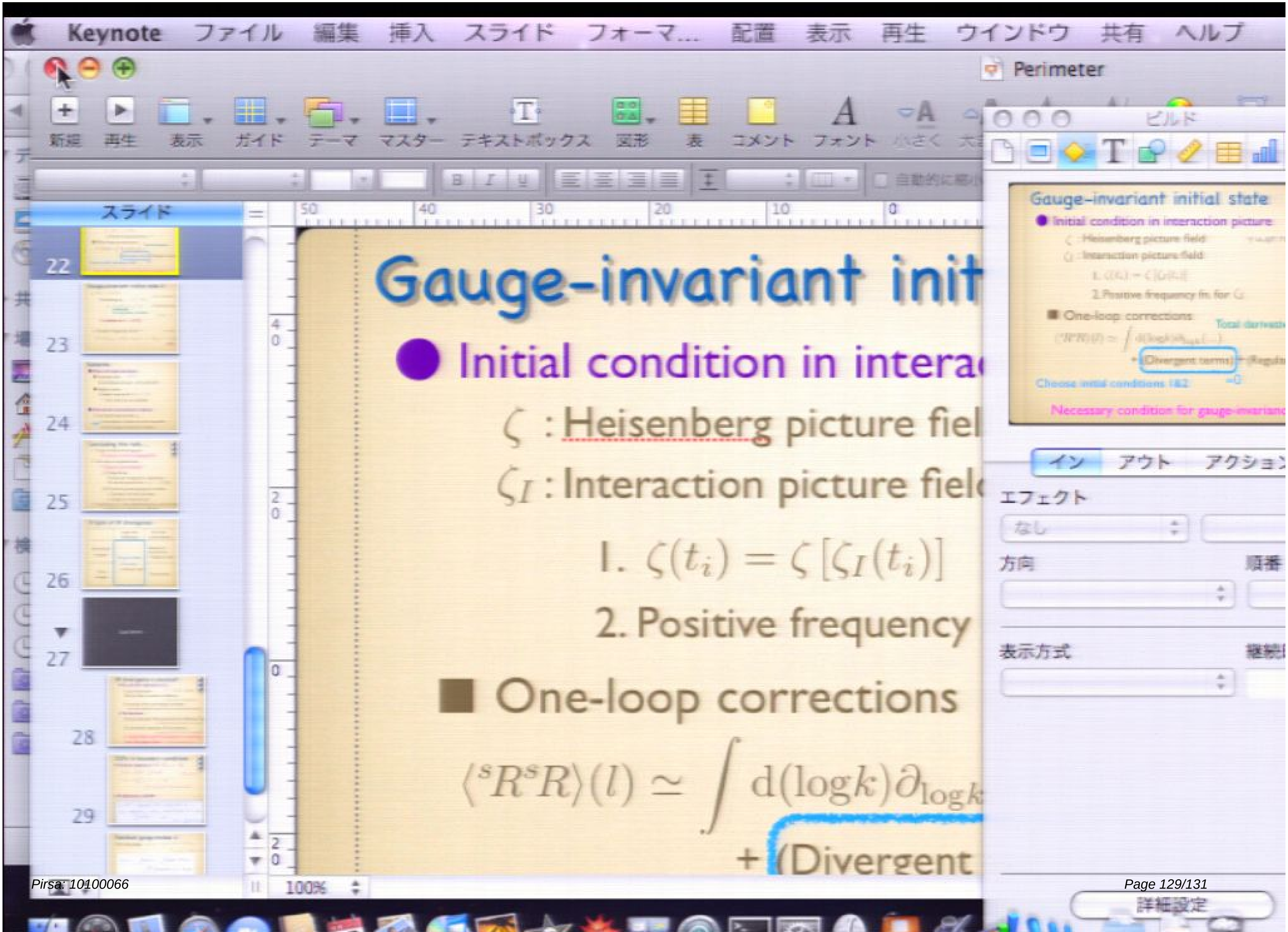
2. Positive frequency fn. for ζ_I

■ One-loop corrections

Total derivatives

$$\langle {}^sR^sR\rangle(l) \simeq \int \mathrm{d}(\log k)\partial_{\log k}(\ldots)$$

+ (Divergent terms) + (Regular terms)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スライド
Gauge-invariant init
Initial condition in interac
ζ : Heisenberg picture fiel
ζ_I : Interaction picture fiel
1. $\zeta(t_i) = \zeta[\zeta_I(t_i)]$
2. Positive frequency
One-loop corrections
$\langle {}^sR^sR\rangle(l) \simeq \int d(\log k)\partial_{\log k}$
+ (Divergent
Gauge-invariant initial state
Initial condition in interaction picture
One-loop corrections
Choose initial conditions 1&2
Necessary condition for gauge-invarianc
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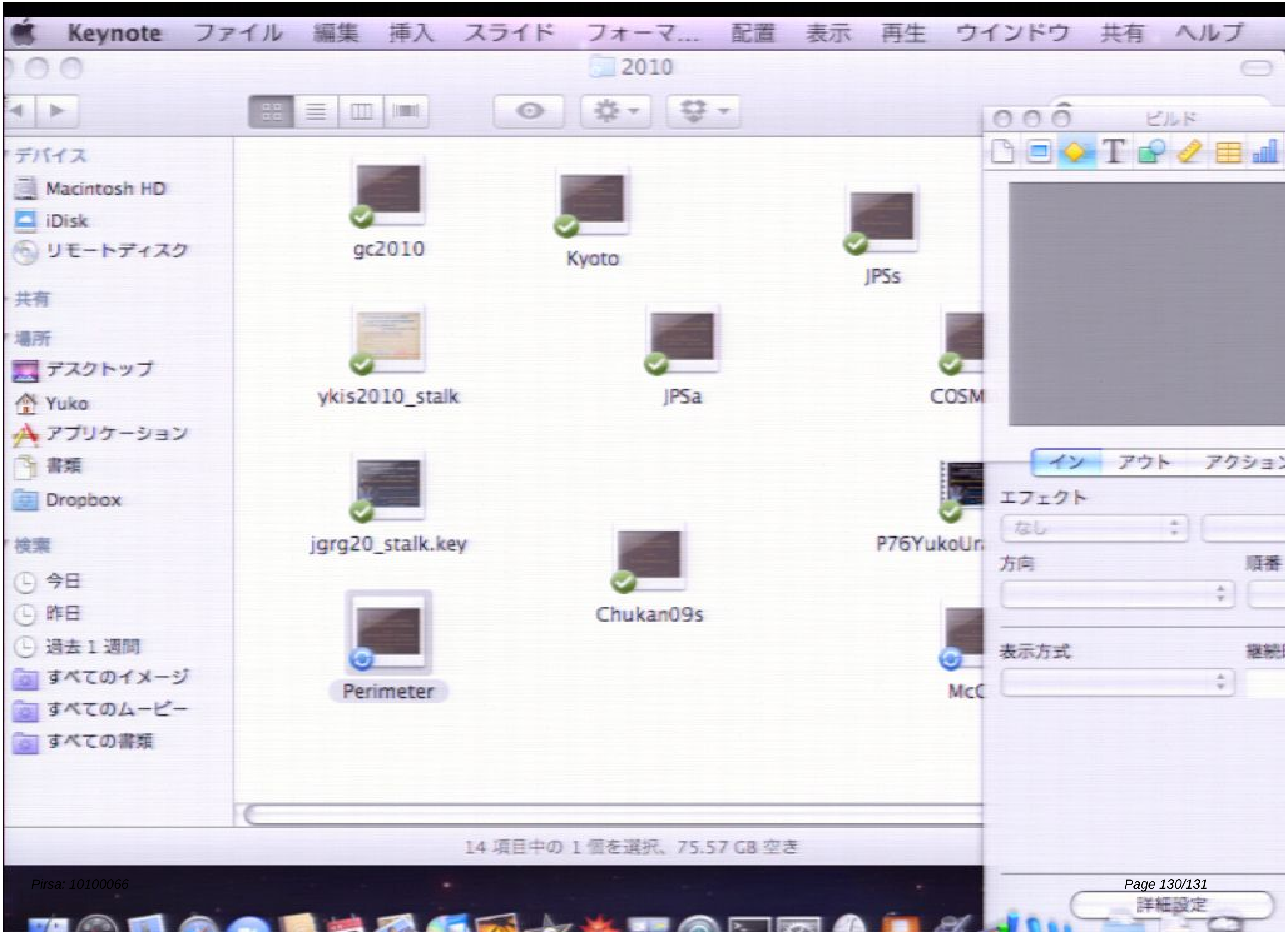
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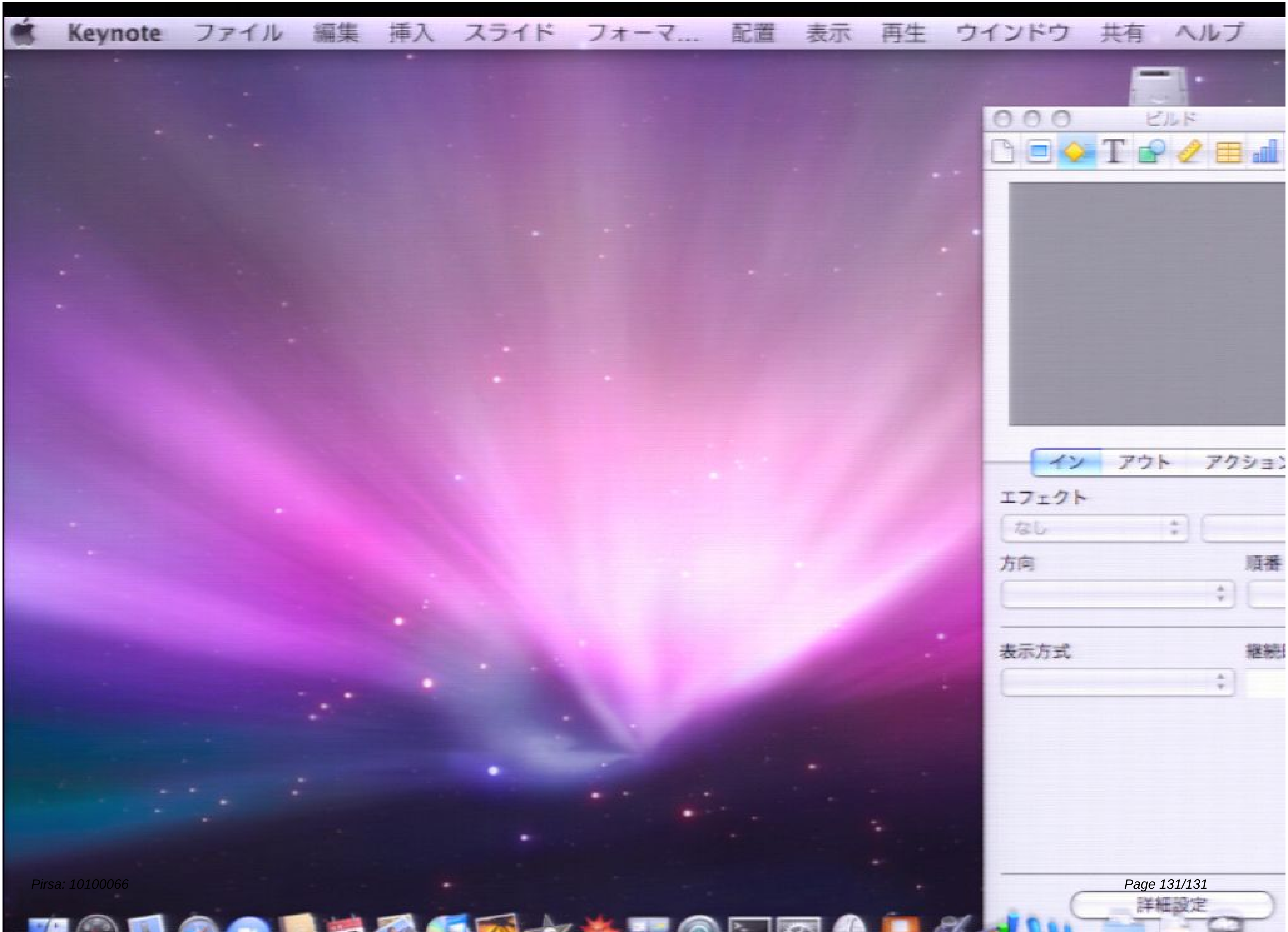
表示方式
詳細設定
14 項目中の 1 個を選択、75.57 GB 空き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