Title: The Feynman Propagator and Correlation Functions in an Inflating Spacetime

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Abstract: We discuss the definition of the Feynman propagator in de Sitter space. We show that the ambiguities in the propagator zero-mode can be used to make sense of the behavior of low-momentum modes in an inflating space-time. We use this tool to calculate loop corrections to non-Gaussian correlation functions, and show that there are limits where the loop terms dominate. These models can be probed with the Planck satellite.

The Feynman Propagator and Correlation Functions in an Inflating Space-Time

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IR behavior of propagator

- basic issue:
- typical de Sitter behavior of scalar correlator
  - $\langle \phi(k)\, \phi(-k) \rangle \sim H^2 / 2k^3$
- corresponding spatial behavior
  - $\langle \phi(0)\, \phi(\Delta x) \rangle \sim \int d^3k\, (H^2 / 2k^3)\, e^{ik\Delta x} \sim \ln (\Delta x)$
- correlator blows up at long distance
  - not a problem at tree-level, since we measure differences of correlators
  - problem when propagator appears in loops
- result: not sure how to make sense of scalar correlators in de Sitter space....
- basic question: what to do about the IR modes in the integral?

possibilities....

- scalar field gets a mass through quantum corrections
  - mass cuts off IR divergence
- physical cutoff to the size of the universe
  - lower bound on k
- spacetime might not inflate eternally
- either way, still left with confusion....
  - correlator depends on cutoff parameter which is becoming large, even though not infinite
  - even if not diverging, perturbation theory becomes questionable
  - besides field could really be massless , inflation might be eternal, there might not be a physical cutoff....

- how do we think about the IR behavior?

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another way to think about it....

- denote by L the size of the observable universe
- modes with $k < L^{-1}$ are roughly indistinguishable from a zero mode
- the diverging behavior arises from modes which are effectively zero modes
- should a constant mode really give you a physical divergence?
- analogy to field theory....
  - divergences from unobservable small wavelengths can be cancelled by counterterms
  - can something similar be true for unobservable large wavelengths?

our plan....

- first, describe treatment of the Feynman propagator which makes clear how one can safely treat the very low momentum modes....

- second, describe how one can use this in clear calculations....

- caution
  - some will think the following is either completely obvious, or obviously wrong
  - I'll try to convince you it's not wrong
  - as for being obvious... maybe it is (or should be)
  - still, it might pay to look at it from this point of view....

defining the Feynman propagator...

- just to remind you of things you already know....
  - $\langle \phi(x)\, \phi(x') \rangle = G_0(x-x')$
  - $\langle [\phi(x), \phi(x')] \rangle = G_R(x-x')$ $\quad [\, x^0 > x'^0 \,]$
  - $\langle T\{ \phi(x)\, \phi(x') \} \rangle = G_F(x-x')$
- $G_0(x-x')$ = 2-pt. correlator
  - satisfies homogeneous wave equation
- $G_R(x-x')$ = retarded propagator
- $G_F(x-x')$ = Feynman propagator
  - both satisfy inhomogeneous wave equation
- any two propagators differ only by a homogeneous solution
  - equivalently, by a choice of boundary conditions

boundary conditions

- retarded propagator
  - boundary conditions set by causality (vanish outside light-cone)
  - same in de Sitter as in Minkowski
- Feynman propagator
  - boundary conditions set by frequency
    - positive freq. modes propagate forward in time
  - ambiguous for the zero-mode
  - can shift G_F by an arbitrary constant

- this amounts to shifting the 2-pt correlator
  - $\langle\phi\phi\rangle \rightarrow \langle\phi\phi\rangle + C$
  - G_R is unchanged
  - G_F shifts by the same constant

cutoff point of view....

- easiest way to think about variance is in terms of a cutoff....
- consider a hard IR cutoff μ in the limit $\mu \to 0$

$$\langle \phi\phi \rangle \approx \lim_{\mu \to 0} \int_{\mu} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3} + C$$

- if we parameterize the shift as $C = -(H/2\pi)^2 \ln (k_{IR} / \mu)$, then after shift we get

$$\langle \phi\phi \rangle \approx \int_{k_{IR}} \frac{d^3k}{(2\pi)^3} \frac{H^2}{2k^3}$$

- so we take real cutoff $\mu \to 0$, but correlator is similar to what we would get with cutoff scale k_{IR}
  - get higher order corrections to C, but form of correlator is fixed

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what does this mean...?

- this ambiguity is fixed in Minkowski space by cluster decomposition
  - massless scalar field acts like a modulus
- ambiguity is not fixed in de Sitter
  - no scattering amplitudes, so no cluster decomposition constraint
  - massless field behavior stochastic, not like moduli
- have to fix by hand
  - amounts to setting the variance by hand
  - determined by measurement
  - not set by direct measurements, since we typically measure differences of correlators
  - but variance will affect non-linear corrections

de Sitter invariance

- setting a finite variance by hand amounts to breaking de Sitter invariance by the vacuum
- a de Sitter invariant vacuum should have infinite variance of the zero mode

- makes sense from stochastic point of view
  - as modes cross the horizon, they freeze out and give the field a "kick" ($\propto$ H) with random phase
  - if variance is finite at some finite time, it will be larger at some later time
  - would violate de Sitter invariance

- so this amounts to considering a (semi-)eternally inflating spacetime, but with a finite variance at fixed time

cutoff point of view....

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- nothing discontinuous about having an arbitrary ambiguity in the zero-mode, with no ambiguity in any mode with finite wavenumber
- only thing we can observe is a "practical" zero-mode which comes from averaging over our observable universe
  - includes both the exact zero-mode and modes with arbitrarily small wavenumber (divergences between them cancel)
- our propagator modification is like establishing a cutoff (k_{IR}), but is not precisely this
  - we in fact sum over all modes down to $\mu \to 0$
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application to a loop calculation....

- now that we have a technique, we can apply it to a calculation....
- regulates both G_F and G_0 (of course not G_R)
- we'll consider a loop calculation of non-Gaussianity
- brief review of set-up
  - consider a function $N(\phi)=N_0+N_1\phi + \frac{1}{2}N_2\phi^2 + \ldots$
  - the N_i are phenomenological parameters of the model, ϕ is a fundamental Gaussian field
  - non-linear dependence on ϕ induces non-Gaussianity in N correlations
  - (prototypical example → N = number of efolds)
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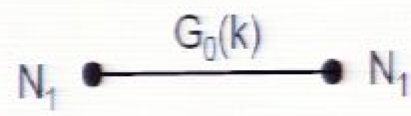
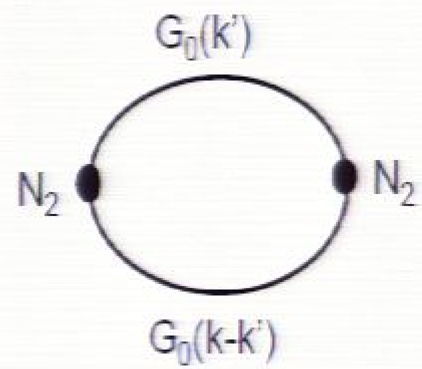
- compute ⟨ N(k) N(-k) ⟩
- linear term = $N_1^2 \, G_0(k)$
- first nonlinear term



$$N_2^2 \int \frac{d^3k'}{(2\pi)^3} G_0(\vec{k}') G_0\left(\vec{k} - \vec{k}'\right)$$



- potential divergence at low wavenumber at $k' \approx 0, k$
- resolved by using the zero-point modified correlator

$$\sim N_2^2 G_{ZMM}(k) \int_{k_{IR}}^{k} \frac{d^3k'}{(2\pi)^3} G_{ZMM}(\vec{k}') \sim N_2^2 G_{ZMM}(k) \left(\frac{H}{2\pi}\right)^2 \ln \frac{k}{k_{IR}}$$

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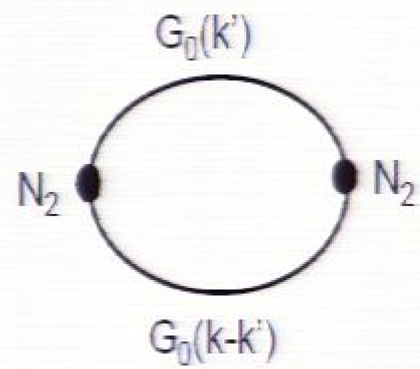

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next case → 3pt. correlator

- consider $\langle N(k_1)\, N(k_2)\, N(k_3) \rangle$
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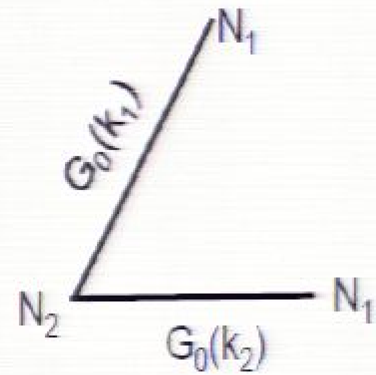
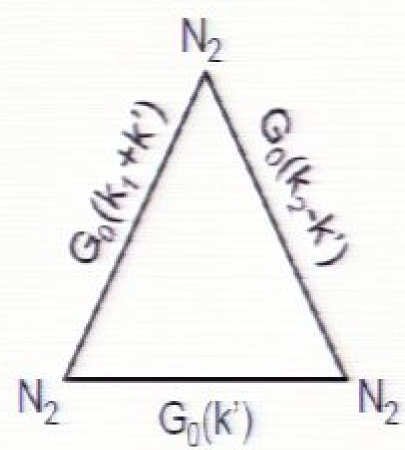
- let's just worry about loop integral now





$$I(k_1,k_2,k_3)=\int\frac{d^3k'}{(2\pi)^3}\frac{H^6}{8\left|\vec{k}_1+\vec{k}'\right|^3\left|\vec{k}_2-\vec{k}'\right|^3 k'^3}+perms.$$

regulating loop integral....

- divergences potentially arise when one of the loop correlator momenta becomes small
- replace the correlator with zero-mode modified version

$$I(k_1,k_2,k_3) \approx \frac{H^2}{2\vec{k}_1^3}\frac{H^2}{2\vec{k}_2^3}\int\frac{d^3k'}{(2\pi)^3}\frac{H^2}{2k'^3} + perms. \rightarrow \frac{H^2}{2\vec{k}_1^3}\frac{H^2}{2\vec{k}_2^3}\int_{k_{IR}}\frac{d^3k'}{(2\pi)^3}\frac{H^2}{2k'^3}$$

- result now depends on k_{IR}
- log behavior cut-off at high momentum when $k' \sim k_1$, k_2

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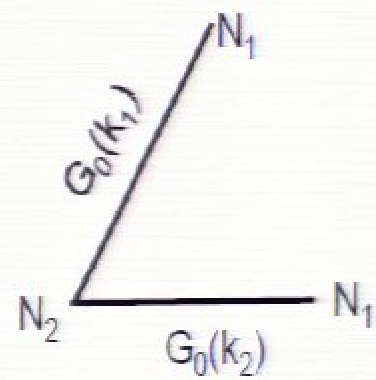

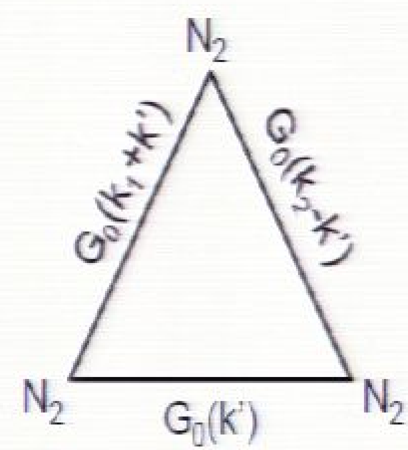

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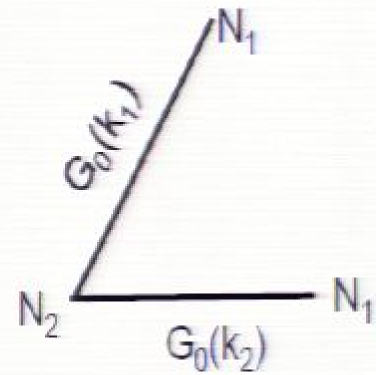

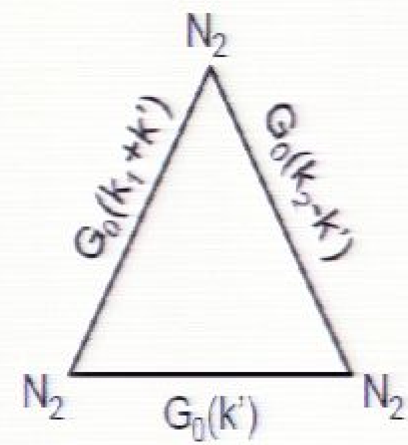

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next case → 3pt. correlator

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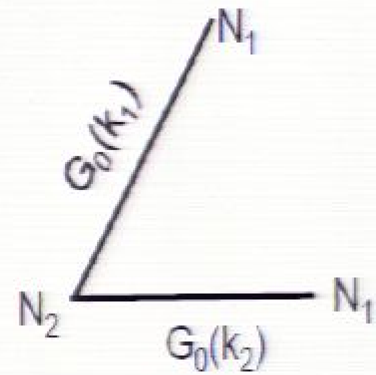
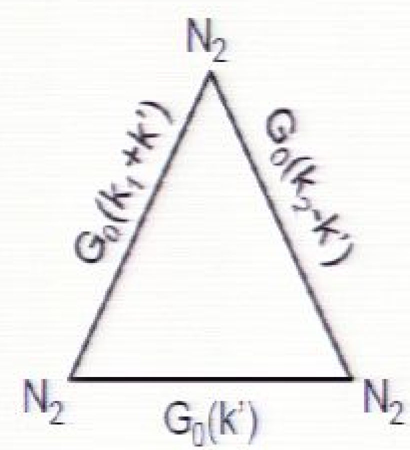
- let's just worry about loop integral now





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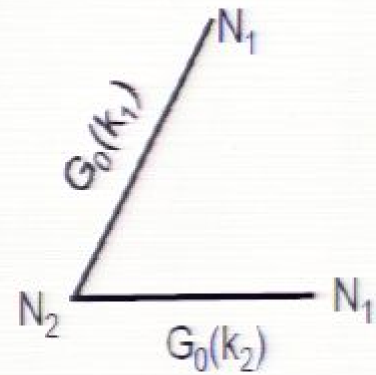

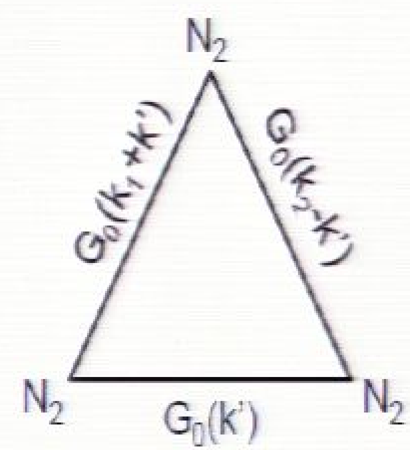

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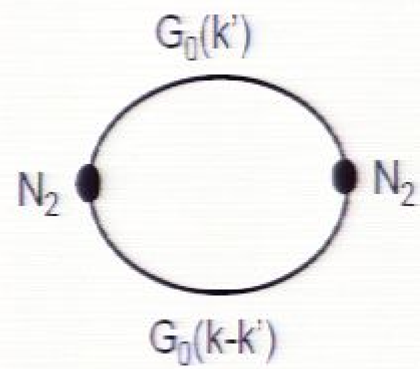
first case → 2pt. correlator

- compute $\langle N(k)\, N(-k) \rangle$
- linear term = $N_1^2\, G_0(k)$

- first nonlinear term

$$N_2^2 \int \frac{d^3k'}{(2\pi)^3} G_0(\vec{k}')\, G_0\left(\vec{k} - \vec{k}'\right)$$



- potential divergence at low wavenumber at $k' \approx 0, k$
- resolved by using the zero-point modified correlator

$$\sim N_2^2 G_{ZMM}(k) \int_{k_{IR}}^{k} \frac{d^3k'}{(2\pi)^3} G_{ZMM}(\vec{k}') \sim N_2^2 G_{ZMM}(k) \left(\frac{H}{2\pi}\right)^2 \ln \frac{k}{k_{IR}}$$

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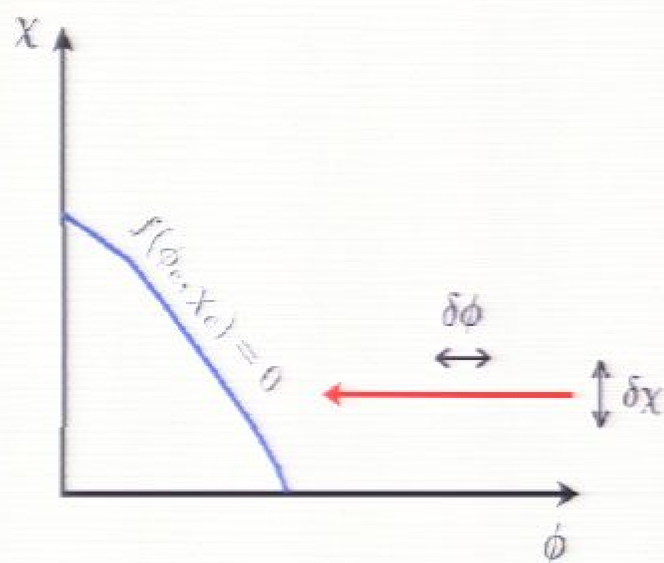
let's use δN formalism....

- model: inflaton ϕ and another scalar χ (take hybrid inflation)
- N = # of efolds
- end of inflation determined by reheating surface, where slow-roll conditions no longer satisfied
  - depends on ϕ and on fluctuations of χ
- $\delta N \approx N_\phi\, \phi - N_\phi\, \gamma_1 \chi - ½\, N_\phi\, \gamma_2 \chi^2 + \ldots$



non-Gaussianity....

- curvature 2pt. gets tree-level contributions from ϕ and χ, and loop contribution from χ
  - $\langle\, \delta N(k)\, \delta N(k)\, \rangle \propto N_\phi{}^2\, P\, [1 + \gamma_1{}^2 + \gamma_2{}^2\, P \ln(kL)]$

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- non-Gaussianity in 3-pt. function local, measured in terms of f_{NL}
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$$f_{NL} \approx -\frac{5}{6}\frac{\gamma_1^2\gamma_2}{N_\phi}\left[1+\frac{\gamma_2^2 P}{\gamma_1^2}\ln(kL)\right]$$

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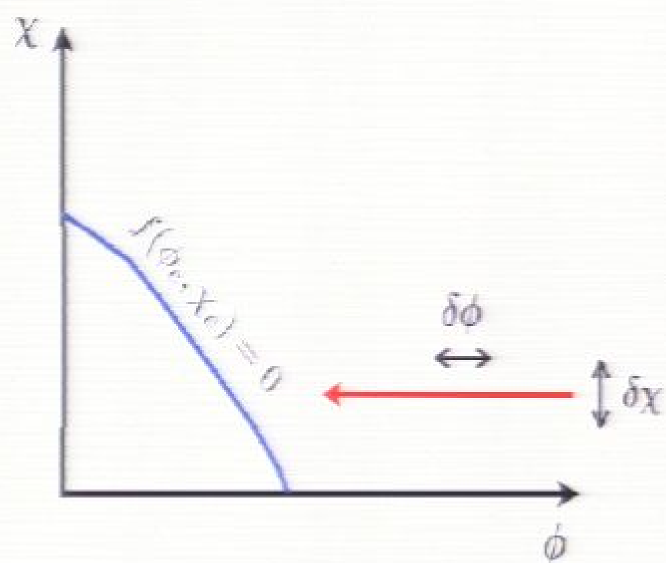

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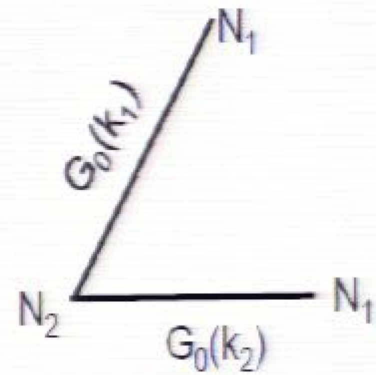
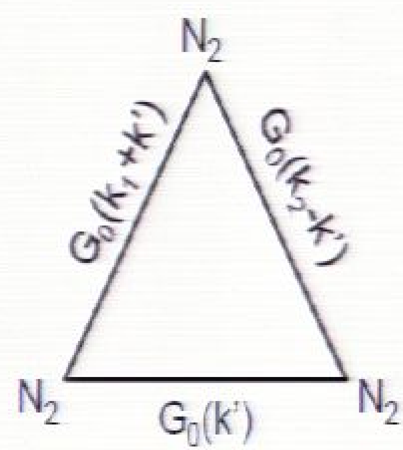
next case → 3pt. correlator

- consider $\langle N(k_1)\, N(k_2)\, N(k_3) \rangle$
- non-Gaussianity induced by non-linear relation between ϕ and N
- includes both tree-level and loop contributions
- let's just worry about loop integral now





$$I(k_1,k_2,k_3)=\int\frac{d^3k'}{(2\pi)^3}\frac{H^6}{8\left|\vec{k}_1+\vec{k}'\right|^3\left|\vec{k}_2-\vec{k}'\right|^3 k'^3}+perms.$$

what is k_{IR}?

- good time to get to the physical meaning of k_{IR}
- basic point can be seen from the stochastic point of view
  - observed zero mode gets contributions from every wavenumber between k_{IR} and 1/L (inverse size of the observable universe)
  - phase is random, but cancels out is variance

$$\langle \phi\phi \rangle \propto \int_{k_{IR}}^{1/L} \frac{d^3k}{(2\pi)^3}\frac{1}{k^3} \propto \ln(k_{IR}L)$$

- the variance of the zero-mode we "observe" (either directly or through loop effects) is set by $k_{IR}L$
- in the loop integral, we should integrate from $L^{-1} \rightarrow \infty$
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regulating loop integral....

- divergences potentially arise when one of the loop correlator momenta becomes small
- replace the correlator with zero-mode modified version

$$I(k_1,k_2,k_3) \approx \frac{H^2}{2\vec{k}_1^3}\frac{H^2}{2\vec{k}_2^3}\int\frac{d^3k'}{(2\pi)^3}\frac{H^2}{2k'^3} + perms. \rightarrow \frac{H^2}{2\vec{k}_1^3}\frac{H^2}{2\vec{k}_2^3}\int_{k_{IR}}\frac{d^3k'}{(2\pi)^3}\frac{H^2}{2k'^3}$$

- result now depends on k_{IR}
- log behavior cut-off at high momentum when $k' \sim k_1$, k_2

$$I(k_1,k_2,k_3) \approx \frac{H^6 \ln(k_{1,2}/k_{IR})}{16\pi^2 \vec{k}_1^3 \vec{k}_2^3} \approx G_0(k_1)G_0(k_2)P\ln\left(\frac{k_{1,2}}{k_{IR}}\right) \qquad P = \left(\frac{H}{2\pi}\right)^2$$

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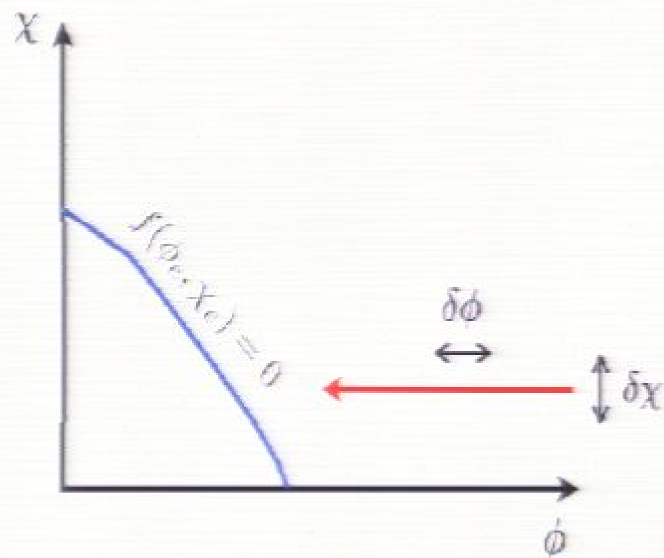

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f_{NL}

- constraints
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  - loop contribution does dominate the 3pt. correlator
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4pt. correlator

- as expected, tree and loop have same shape
- $\tau_{NL} \rightarrow (k_1 k_{13} k_2)^{-3}$ + perms
  - $k_{ij}=|k_i+k_j|$
- as expected, if f_{NL} is loop-dominated, then so is τ_{NL}
- $\tau_{NL} \sim (P\gamma_2^{\,2} / P^{1/2} N_\phi)^2 \ln(kL)$
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- bounds
  - WMAP $|\tau_{NL}| < 10^8$
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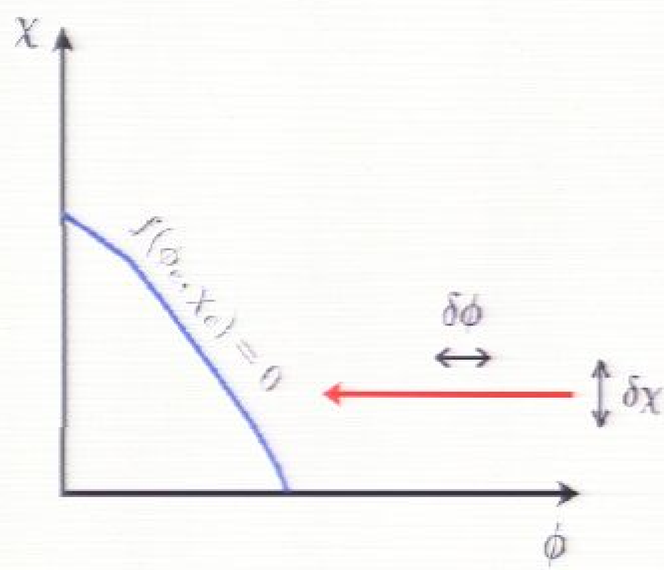

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dependence on L

- distinction between γ_1 and γ_2 depends on choice of L
- equivalently → separation between tree-level and loop contribution depends on L
- "loop-dominated" is a statement about the split between zero-mode and non-zero-mode as we would observe it in today's universe

- changing L changes γ_1 and γ_2
  - f_{NL} unchanged, but changes whether or not loop-term dominates

- for small δL → n_{NG} changes little
- for large δL → no longer loop-dominated

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application to a loop calculation....

- now that we have a technique, we can apply it to a calculation....
- regulates both G_F and G_0 (of course not G_R)
- we'll consider a loop calculation of non-Gaussianity
- brief review of set-up
  - consider a function $N(\phi)=N_0+N_1\phi + \frac{1}{2}N_2\phi^2 + ...$
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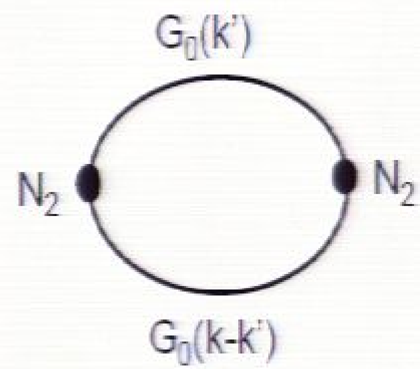
first case → 2pt. correlator

- compute $\langle N(k)\, N(-k) \rangle$
- linear term = $N_1^2\, G_0(k)$
- first nonlinear term

$$N_2^2\int\frac{d^3k'}{(2\pi)^3}G_0(\vec{k}')G_0\left(\vec{k}-\vec{k}'\right)$$



- potential divergence at low wavenumber at $k' \approx 0, k$
- resolved by using the zero-point modified correlator

$$\sim N_2^2 G_{ZMM}(k)\int_{k_{IR}}^{k}\frac{d^3k'}{(2\pi)^3}G_{ZMM}(\vec{k}')\sim N_2^2 G_{ZMM}(k)\left(\frac{H}{2\pi}\right)^2\ln\frac{k}{k_{IR}}$$

next case → 3pt. correlator

- consider $\langle N(k_1)\, N(k_2)\, N(k_3) \rangle$
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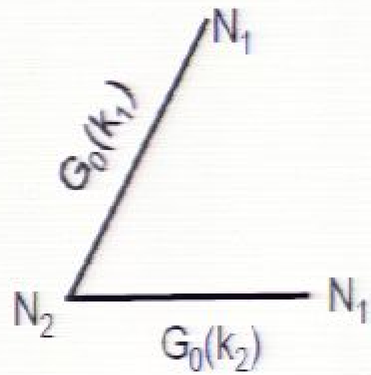
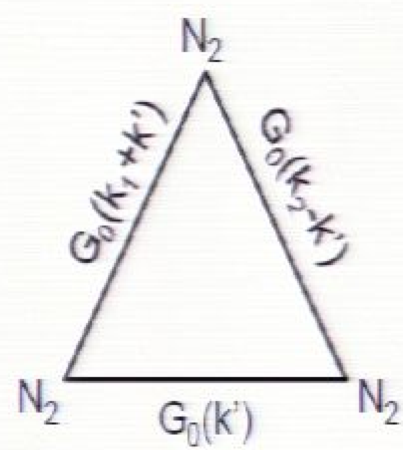
- includes both tree-level and loop contributions
- let's just worry about loop integral now





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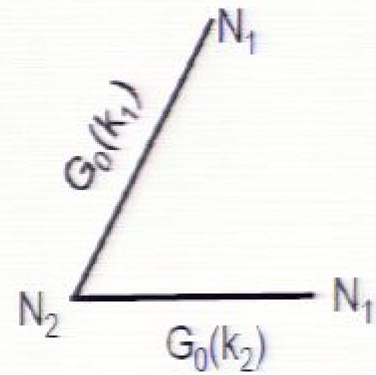

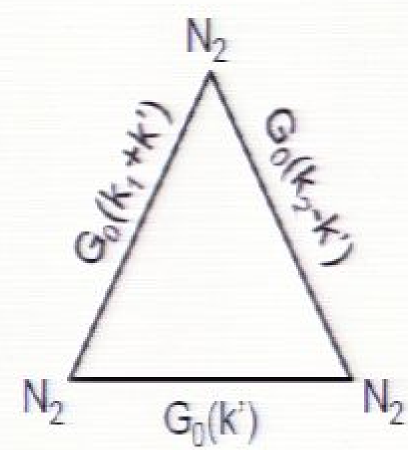

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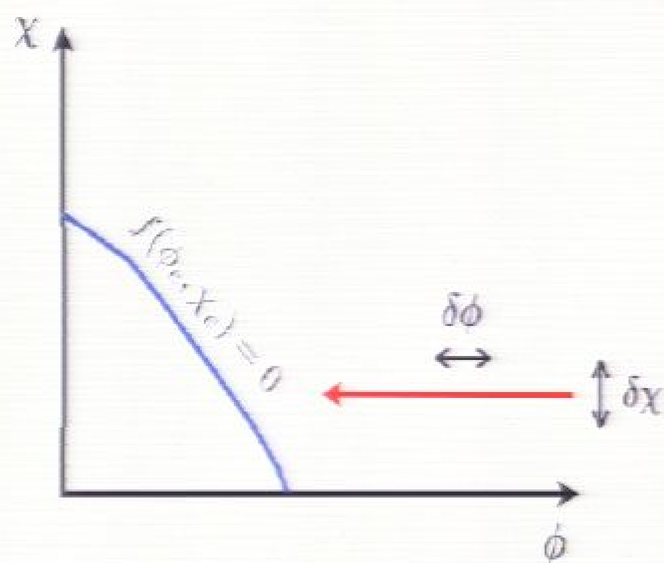

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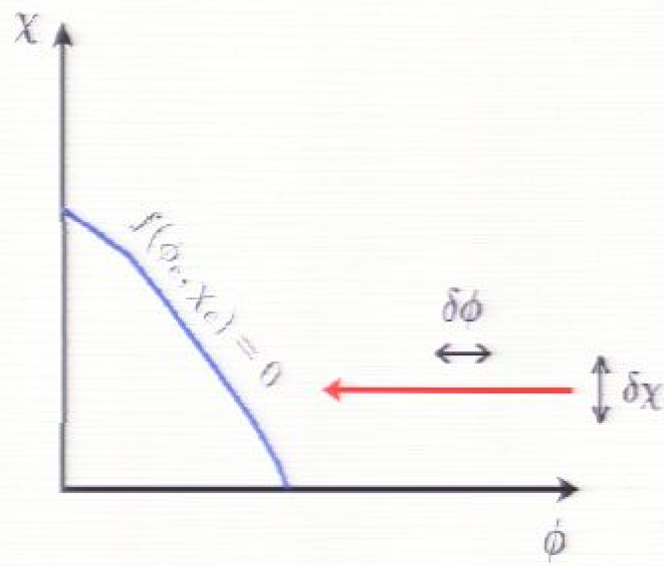

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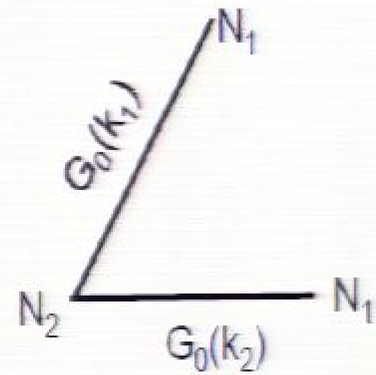

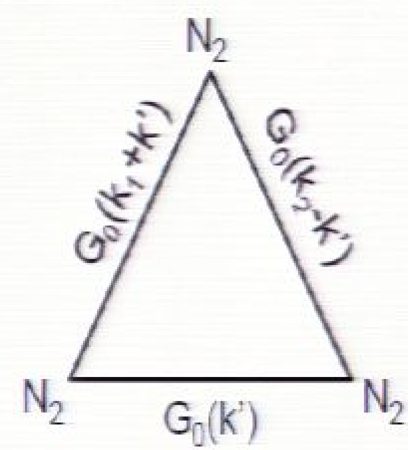

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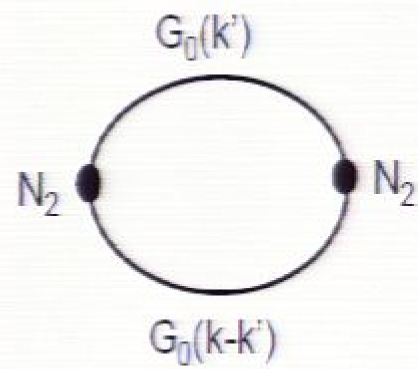

- potential divergence at low wavenumber at $k' \approx 0, k$
- resolved by using the zero-point modified correlator

$$\sim N_2^2 G_{ZMM}(k) \int_{k_{IR}}^{k} \frac{d^3k'}{(2\pi)^3} G_{ZMM}(\vec{k}') \sim N_2^2 G_{ZMM}(k) \left(\frac{H}{2\pi}\right)^2 \ln \frac{k}{k_{IR}}$$

first case→ 2pt. correlator

- compute ⟨ N(k) N(-k) ⟩
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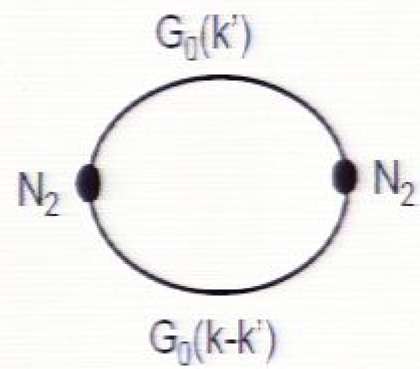

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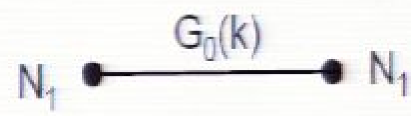

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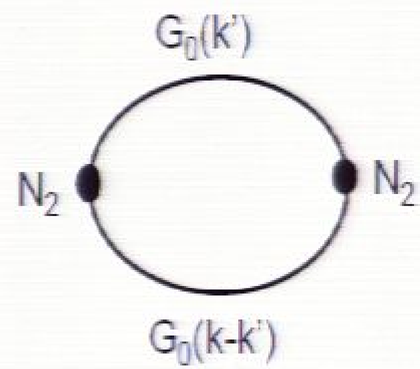

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regulating loop integral....

- divergences potentially arise when one of the loop correlator momenta becomes small
- replace the correlator with zero-mode modified version

$$I(k_1,k_2,k_3) \approx \frac{H^2}{2\vec{k}_1^3}\frac{H^2}{2\vec{k}_2^3}\int \frac{d^3k'}{(2\pi)^3}\frac{H^2}{2k'^3} + perms. \rightarrow \frac{H^2}{2\vec{k}_1^3}\frac{H^2}{2\vec{k}_2^3}\int_{k_{IR}} \frac{d^3k'}{(2\pi)^3}\frac{H^2}{2k'^3}$$

- result now depends on k_{IR}
- log behavior cut-off at high momentum when $k' \sim k_1$, k_2

$$I(k_1,k_2,k_3) \approx \frac{H^6 \ln(k_{1,2}/k_{IR})}{16\pi^2 \vec{k}_1^3 \vec{k}_2^3} \approx G_0(k_1)G_0(k_2)P\ln\left(\frac{k_{1,2}}{k_{IR}}\right) \qquad P = \left(\frac{H}{2\pi}\right)^2$$

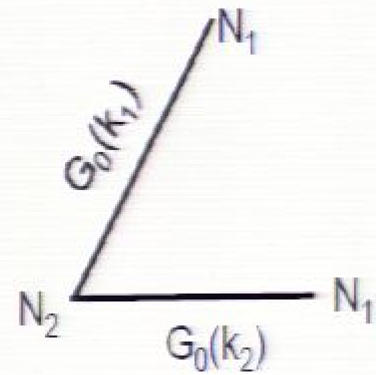
non-Gaussianity....

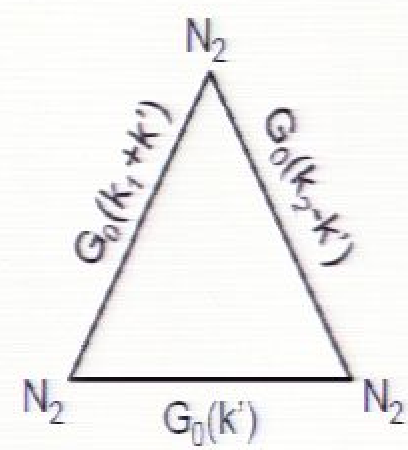
- curvature 2pt. gets tree-level contributions from ϕ and χ, and loop contribution from χ
  - $\langle \delta N(k)\, \delta N(k) \rangle \propto N_\phi^2 P\, [1 + \gamma_1^2 + \gamma_2^2 P \ln(kL)]$
- curvature 3pt. gets contributions from χ (tree-level and loop)
  - $\langle \delta N(k_1)\, \delta N(k_2)\, \delta N(k_3) \rangle \propto N_\phi^3 P^2 [\gamma_1^2 \gamma_2 + \gamma_2^3 P \ln(kL)] \times [\, \Sigma k_i^3 / (k_1^3 k_2^3 k_3^3)\,]$
- non-Gaussianity in 3-pt. function local, measured in terms of f_{NL}
  - $\langle \delta N(k_1)\, \delta N(k_2)\, \delta N(k_3) \rangle \sim f_{NL} (P N_\phi^2)^2 \times [\, \Sigma k_i^3 / (k_1^3 k_2^3 k_3^3)\,]$

$$f_{NL} \approx -\frac{5}{6} \frac{\gamma_1^2 \gamma_2}{N_\phi} \left[1 + \frac{\gamma_2^2 P}{\gamma_1^2} \ln(kL) \right]$$

next case → 3pt. correlator

- consider $\langle N(k_1)\, N(k_2)\, N(k_3) \rangle$
- non-Gaussianity induced by non-linear relation between ϕ and N
- includes both tree-level and loop contributions
- let's just worry about loop integral now





$$I(k_1,k_2,k_3) = \int \frac{d^3k'}{(2\pi)^3} \frac{H^6}{8\left|\vec{k}_1+\vec{k}'\right|^3 \left|\vec{k}_2-\vec{k}'\right|^3 k'^3} + perms.$$

application to a loop calculation....

- now that we have a technique, we can apply it to a calculation....
- regulates both G_F and G_0 (of course not G_R)
- we'll consider a loop calculation of non-Gaussianity
- brief review of set-up
  - consider a function $N(\phi)=N_0+N_1\phi + \frac{1}{2}N_2\phi^2 + \ldots$
  - the N_i are phenomenological parameters of the model, ϕ is a fundamental Gaussian field
  - non-linear dependence on ϕ induces non-Gaussianity in N correlations
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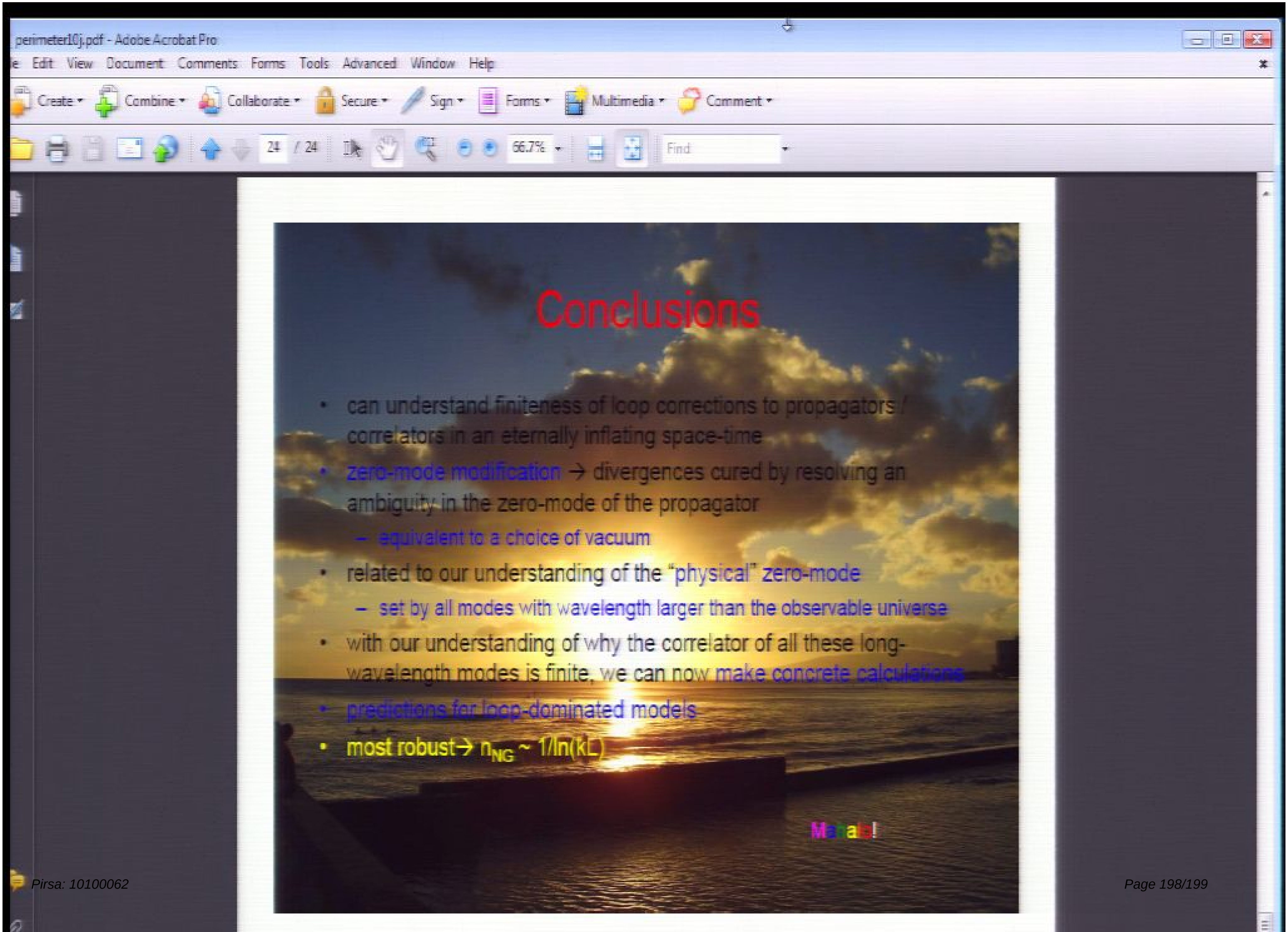
- zero-mode modification → divergences cured by resolving an ambiguity in the zero-mode of the propagator
  - equivalent to a choice of vacuum
- related to our understanding of the "physical" zero-mode
  - set by all modes with wavelength larger than the observable universe
- with our understanding of why the correlator of all these long-wavelength modes is finite, we can now make concrete calculations
- predictions for loop-dominated models
- most robust → $n_{NG} \sim 1/\ln(kL)$

Mahalo!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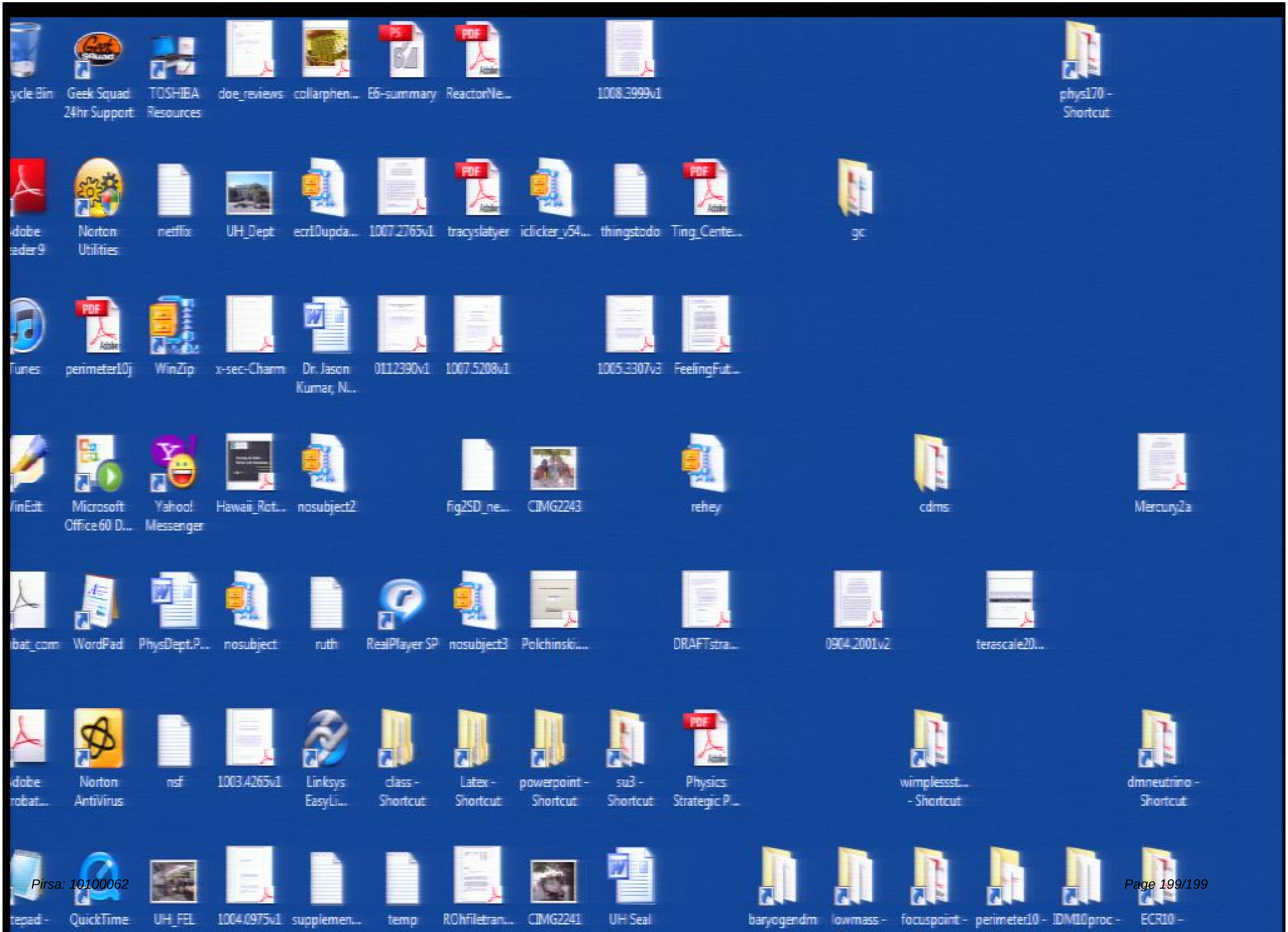
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E6-summary
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UH_Dept
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