Title: New Horizons in Cosmology: The Trace Anomaly, Cosmological Horizon Modes and Dynamical Dark Energy

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Abstract: General Relativity receives quantum corrections relevant at macroscopic distance scales and near event horizons. These arise from the conformal scalar degrees of freedom in the extended effective field theory of gravity generated by the trace anomaly of massless quantum fields in curved space. Linearized perturbations of the Bunch-Davies state in de Sitter space show that these new scalar degrees of freedom are associated with macroscopic changes of state on the cosmological horizon scale, with potentially large stress tensors that can lead to substantial backreaction effects in cosmology. In the extended effective theory the cosmological ``constant" is a state dependent condensate whose value is scale dependent and which possesses an infrared stable conformal fixed point at zero. These considerations suggest that the observed dark energy of our universe may be a macroscopic finite size effect whose value depends not upon Planck scale physics but upon extreme infrared physics on the cosmological horizon scale.

New Horizons in Cosmology

The Trace Anomaly, Cosmological Horizon Modes, & Dynamical Dark Energy

E. Mottola, LANL

Review: Acta Phys. Pol. B 41, 2031 (2010)

w. M. Giannotti, *Phys. Rev. D* 79, 045014 (2009)

w. P. Anderson & C. Molina-Paris, *Phys. Rev. D* 80, 084005 (2009)

w. P. Anderson & R. Vaulin, *Phys. Rev. D* 76, 024018 (2007)

Review Article: w. I. Antoniadis & Mazur, *N. Jour. Phys.* 9, 11 (2007)

w. P. O. Mazur, *Proc. Natl. Acad. Sci.* 101, 9545 (2004)

Outline

- Cosmological Vacuum Energy
  - Microscopic & Macroscopic (Infrared) Features
  - Quantum Effects in de Sitter Space
- Effective Field Theory --The Role of Anomalies
  - The Axial Anomaly in QCD
  - The Trace/Conformal Anomaly in 2D Gravity
  - Massless Scalar Poles in 4D Anomalous Amplitudes
- Effective Theory of Low Energy Gravity
  - New Scalar Degrees of Freedom from the Trace Anomaly
  - Conformal Phase and *IR* Running of Λ_{eff}
- Linear Response in de Sitter Space
  - New Scalar Cosmological Horizon Modes from the Anomaly
- Conformal Invariance and the CMB
  - Conformal Invariance on the de Sitter horizon
  - Non-Gaussian signatures of the conformal phase of gravity
- Cosmological Term as Finite Size Effect, Macroscopic Condensate

The New Cosmology

- Non-Luminous (Dark) Matter, presumed Non-Baryonic is **24%** of the Universe
- Relativistic Dark Energy with *negative* pressure,

$$p \approx -\rho < 0$$

is **72%** of all the energy in the observable universe
- Ordinary Baryonic Matter is only a few percent
- Since $\rho + 3p < 0$, the expansion is *accelerating*
- Tiniest pure number in Nature: (note involves $\hbar$ and **G**)

$$\hbar G \Lambda_{obs} / c^3 \cong 3.6 \times 10^{-122}$$

We live in a de Sitter-like Universe dominated by Vacuum Dark Energy

No apparent explanation in the Standard Model

Quantum Effects in Gravity: Microscopic or Macroscopic ?

We deal with UV divergences by **Renormalization**, and now understand most (all?) QFT's as *Effective Field Theories*

Λ_{eff} is a free parameter of the **Low Energy Effective Theory** which mixes with the renormalization of $\langle T^a{}_b \rangle$ and can run with scale

The Standard Model has **Spontaneous Symmetry Breaking**

When the ground state changes, so does its energy –

so we should expect generically $\Lambda_{eff} > 0$ now

Λ can be dynamical, run, and be vacuum state-dependent

Very High Energy (UV) Scales and IR physics generally decouple

Like the Casimir effect, this is an issue of fixing the boundary conditions of the **Quantum Vacuum State of Macroscopic Gravity**

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Just because something is infinite does not necessarily mean that it is zero – W. Pauli

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Classical de Sitter Spacetime

- Maximally Symmetric Soln. to Einstein's Eqs. with a Positive Cosmological Constant (Vacuum Energy)

$$G_{ab} + \Lambda\, g_{ab} = 0$$

- Symmetry Group is $O(4,1)$: Hyperboloid of Revolution in D=5 flat spacetime

$$ds^2 = -dT^2 + dW^2 + dX^2 + dY^2 + dZ^2 \qquad \text{with fixed 'radius'}$$

$$-T^2 + W^2 + X^2 + Y^2 + Z^2 = H^{-2} \qquad H^2 = \Lambda/3$$

- Line Element in with closed S^3 spatial sections: globally complete

$$ds^2 = H^{-2} \sec^2\eta\, (-d\eta^2 + d\chi^2 + \sin^2\chi\, d\Omega^2)$$

- Line Element in FLRW form (flat spatial sections, proper time): inflation

$$ds^2 = -d\tau^2 + e^{2H\tau}(dx^2 + dy^2 + dz^2) \qquad \text{Scale Factor: } a(\tau) = e^{H\tau}$$

- Line Element in Static Coordinates: de Sitter's original form

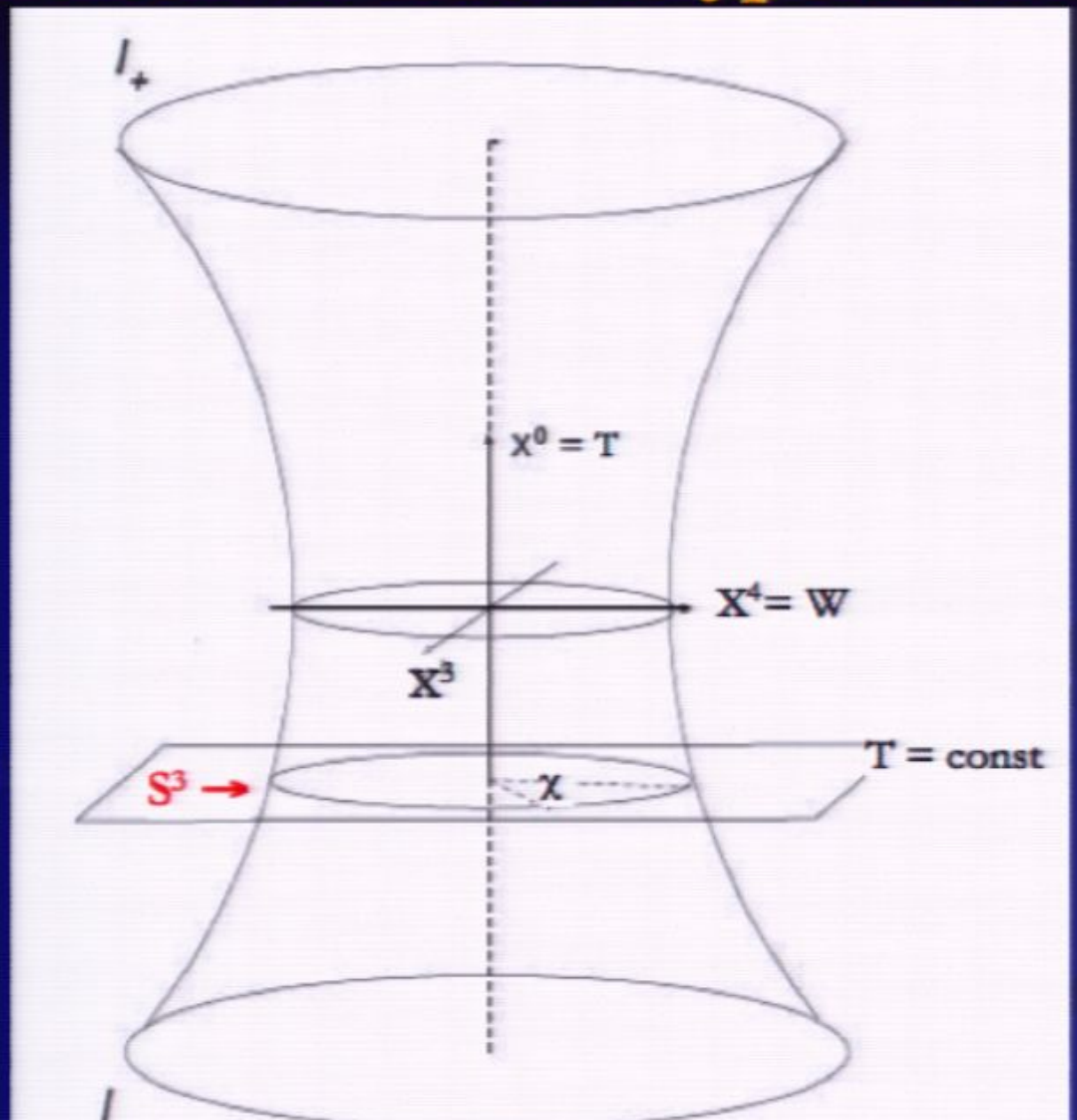
$$ds^2 = -(1 - H^2r^2)\, dt^2 + (1 - H^2r^2)^{-1}\, dr^2 + r^2\, d\Omega^2$$

$r_H = 1/H$ is the Hubble-de Sitter horizon scale

All the same de Sitter spacetime (or parts thereof) described in different coordinates

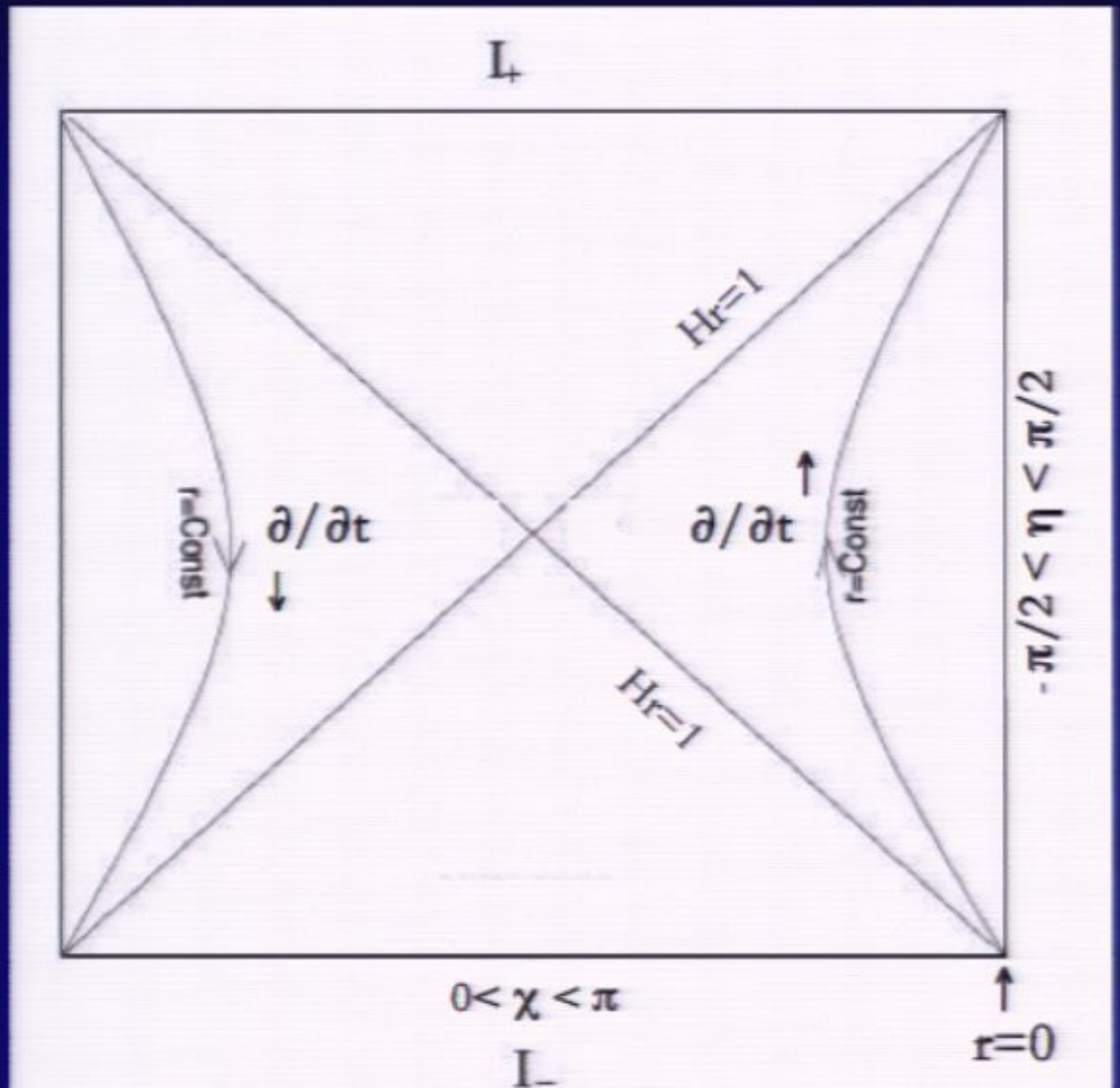
Classical de Sitter Hyperboloid

S^3 sections



Classical de Sitter Spacetime: Carter-Penrose Conformal Diagram

Static time Killing field $\partial/\partial t$ changes sign



Quantum Effects in de Sitter Space

- Quantum 'Vacuum' is non-trivial

  Spontaneous Particle Creation PRD 31, 754 (1985)

  Decay Rate: $\Gamma \sim H^4 \exp(-m/T_H)$ $T_H = H/2\pi$

  for massive fields
- Compare to Schwinger Effect: 'Shorting' the vacuum

  $\Gamma \sim (eE)^2 \exp(-m^2/eE)$

$$\frac{dE}{dt} = -j$$

- Backreaction should decrease H

$$\frac{dH}{dt} = -\frac{4\pi G}{c^2}(\rho + p)$$

- Maximally Symmetric *O(4,1)* Bunch-Davies State has exact time reversal symmetry -- thermodynamic equilibrium ?

  but negative heat capacity ('85)

  unstable to thermodynamic fluctuations

  (compare to black hole case)

$$T_H = \frac{\hbar H}{2\pi k_B} \propto \left(\frac{c^5}{2GH}\right)^{-1} = E_H^{-1}$$

Quantum Effects in de Sitter Space

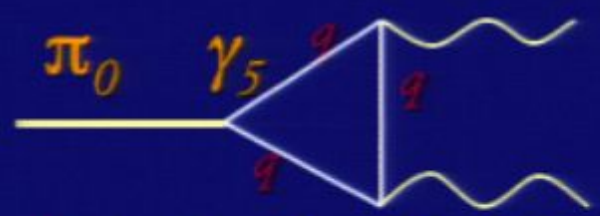
- Temperature Fluctuations lead to divergent stress tensor
  on the horizon: $\langle T^a{}_b \rangle \sim (T^4 - T_H{}^4)/(1-H^2r^2)^2$
- Infinite BlueShift
- No *O(4,1)* Bunch-Davies Thermal State at all
  for massless, minimally coupled fields or gravitons
- Graviton Propagator grows logarithmically with distance
  No Cluster Decomposition, S-Matrix
- Global Symmetry Restoration: No Goldstone Bosons
  Similar to Massless Scalar Theory in D=2
- Non-trivial Infrared Properties
- Infrared Relevant Operator Missing in EFT of Gravity

Effective Field Theory & Quantum Anomalies

- EFT = Expansion of Effective Action in *Local* Invariants
- Assumes Decoupling of Short (*UV*) from Long Distance (*IR*)
- But *Massless* Modes do not decouple
- Massless Chiral, Conformal Symmetries are *Anomalous*
- Macroscopic Effects of Short Distance physics
- Special Non-Local Terms Must be Added to Low Energy EFT
- *IR* Sensitivity to *UV* degrees of freedom
- Important on horizons because of large blueshift/redshift

Axial Anomaly in QCD

- QCD with N_f massless quarks has an apparent $U(N_f) \otimes U_{ch}(N_f)$ Symmetry
- But $U_{ch}(1)$ Symmetry is Anomalous
- Effective Lagrangian in Chiral Limit has $N_f^2 - 1$ (*not* N_f^2) massless pions at low energies
- Low Energy $\pi_0 \to 2\gamma$ dominated by the anomaly



$$\partial_\mu j^{\mu 5} = e^2 N_c F_{\mu\nu} \widetilde{F}^{\mu\nu} / 16\pi^2$$

- No Local Action in chiral limit in terms of $F_{\mu\nu}$ but Non-local IR Relevant Operator that violates naïve decoupling of UV
- Measured decay rate verifies $N_c = 3$ in QCD

Anomaly Matching of IR $\leftrightarrow$ UV

2D Gravity

$$S_{cl}[g] = \int d^2x \sqrt{g}(\gamma R - 2\lambda)$$

has no local degrees of freedom in 2D, since

$$g_{ab} = \exp(2\sigma)\bar{g}_{ab} \to \exp(2\sigma)\eta_{ab}$$

(all metrics conformally flat) and

$$\sqrt{g}R = \sqrt{\bar{g}}\bar{R} - 2\sqrt{\bar{g}}\bar{\Box}\sigma$$

gives a total derivative in S_{cl}

Quantum Trace or Conformal Anomaly

$$\langle T^a{}_a \rangle = -\frac{c_m}{24\pi}R$$

$c_m = N_S + N_F$ for massless scalars or fermions

Linearity in σ in the variational eq.

$$\frac{\delta \Gamma_{WZ}}{\delta \sigma} = \sqrt{g}\,\langle T^a{}_a \rangle$$

determines the Wess-Zumino Action by *inspection*

Quantum Effects of 2D Anomaly Action

- The stress-energy tensor of the 2D anomaly action is

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$

$$\frac{c}{\quad}\left[\nabla_a\nabla_b\varphi - g_{ab}\Box\varphi + \frac{1}{\quad}\nabla_a\varphi\nabla_b\varphi - \frac{g_{ab}}{\quad}\nabla_c\varphi\nabla^c\varphi\right]$$

dynamical & itself fluctuates freely

- Gravitational Dressing of critical exponents

static Schwarzschild & de Sitter: $ds^2 = f(-dt^2 + dr^{*2})$

- Non-perturbative/non-classical conformal fixed

$$T_t{}^t = \frac{cH^2}{\quad}\left\{-\frac{1}{\quad}(p^2+q^2-1)+1\right\}$$

- Quantum stress tensor fully determined from the anomaly

- Additional non-local Infrared Relevant Operator in S_{EFT}

$$24\pi\left[f\ \quad\right]$$

- Finite if $p = 0$, $q = \pm 1$ (B-D)
- q is a kind of topological charge

Quantum Trace Anomaly in 4D Flat Space

Eg. QED in an External EM Field A_μ

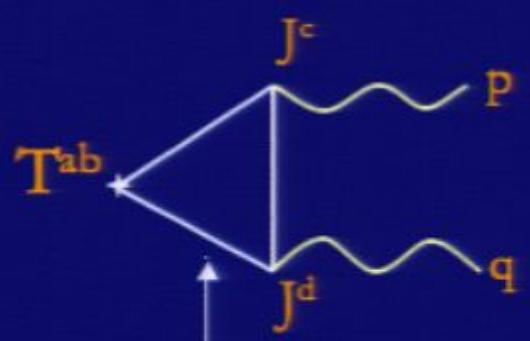
$$\langle T^\mu_\mu \rangle = \frac{e^2}{24\pi^2} F^{\mu\nu} F_{\mu\nu}$$

Triangle One-Loop Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd} p\cdot q - q^c p^d) F_1(k^2) + \text{(traceless terms)}$$

In the limit of massless fermions, $F_1(k^2)$ must have a massless pole:

$k = p + q$



$$F_1(k^2) = \frac{e^2}{18\pi^2 k^2}$$

$$\rho_T(s) \to \frac{e^2}{6\pi^2}\delta(s)$$

Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

<TJJ> Triangle Amplitude in QED

Determining the Amplitude by Symmetries and Its Finite Parts

M. Giannotti & E. M. *Phys. Rev. D* **79, 045014 (2009)**

$$\Gamma^{abcd}(p,q) = \int d^4x \int d^4y\, e^{ip\cdot x + iq\cdot y} \left.\frac{\delta^2 \langle T^{ab}(0)\rangle_A}{\delta A_c(x)\delta A_d(y)}\right|_{A=0}$$

Γ^{abcd}: Mass Dimension 2

Use low energy symmetries:

1. By Lorentz invariance, can be expanded in a complete set of 13 tensors $t_i^{abcd}(p,q)$, $i = 1, \ldots 13$:

$$\Gamma^{abcd}(p,q) = \Sigma_i F_i\, t_i^{abcd}(p,q)$$

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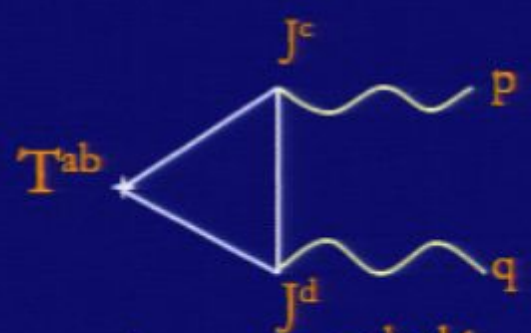

2. By current conservation: $p_c t_i^{abcd}(p,q) = 0 = q_d t_i^{abcd}(p,q)$

All (but one) of these 13 tensors are dimension ≥ 4, so $\dim(F_i) \leq -2$ so these scalar $F_i(k^2; p^2, q^2)$ are completely UV Convergent

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Ward Identities

3. By stress tensor conservation Ward Identity: $\partial_b \langle T^{ab} \rangle_A = eF^{ab} \langle J_b \rangle \Rightarrow$

$$k_b \Gamma^{abcd}(p,q) = (g^{ac} p_b - \delta^c_b p^a)\Pi^{bd}(q) + (g^{ad} q_b - \delta^d_b q^a)\Pi^{bc}(p)$$

4. Bose exchange symmetry: $\Gamma^{abcd}(p,q) = \Gamma^{abdc}(q,p)$

Finally all 13 scalar functions $F_i(k^2; p^2, q^2)$ can be found in terms of

finite (IR) Feynman parameter integrals and the polarization,

$$\Pi^{ab}(p) = (p^2 g^{ab} - p^a p^b)\, \Pi(p^2)$$

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd} p\cdot q - q^c p^d)\, F_1(k^2; p^2, q^2) + \ldots$$

(12 other terms, 11 traceless, and 1 with zero trace when m=0)

Result:

$$F_1(k^2;p^2,q^2) = \frac{e^2}{18\pi^2 k^2}\left\{1 - 3m^2 \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{D}\right\}$$

with $D = (p^2 x + q^2 y)(1-x-y) + xy\, k^2 + m^2$

UV Regularization Independent

<TJJ> Triangle Amplitude in QED

Spectral Representation and Sum Rule

$$F_1(k^2;p^2,q^2) = \frac{1}{3k^2}\int_0^\infty \frac{ds}{k^2+s-i\epsilon}\left[(k^2+s)\rho_T - m^2\rho_m\right]$$

Numerator & Denominator cancel here

Im $F_1(k^2 = -s)$: Non-anomalous,vanishes when m=0

$$\rho_T(s;p^2,q^2) = \frac{e^2}{2\pi^2}\int_0^1 dx\int_0^{1-x} dy\ (1-4xy)\ \delta\left(s - \frac{(p^2x+q^2y)(1-x-y)+m^2}{xy}\right)$$

$$\int_0^\infty ds\,\rho_T(s;p^2,q^2) = \frac{e^2}{6\pi^2}$$

obeys a finite sum rule independent of p^2, q^2, m^2

and as $p^2, q^2, m^2 \to 0^+$

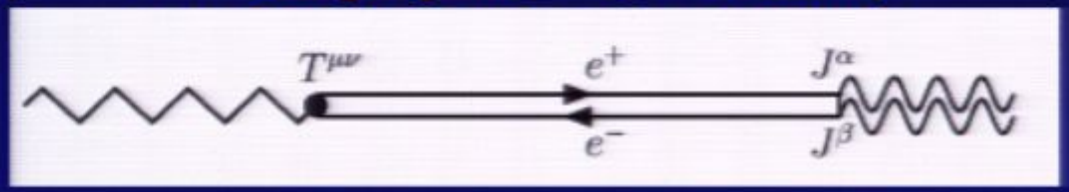
$$\rho_T(s) \to \frac{e^2}{6\pi^2}\delta(s)$$

$$F_1(k^2) \to \frac{e^2}{18\pi^2k^2}$$

Massless scalar intermediate two-particle state analogous to the pion in chiral limit of QCD

Massless Anomaly Pole

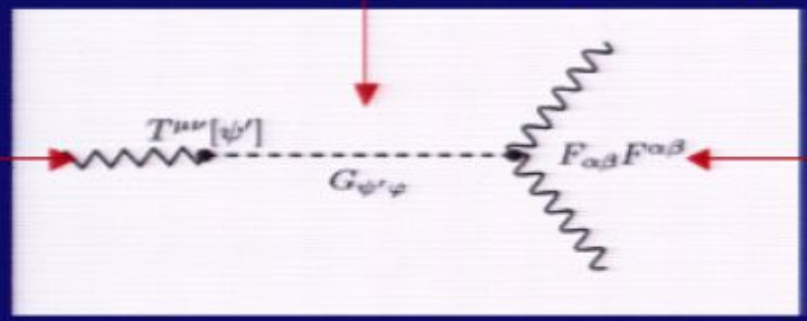
For $p^2 = q^2 = 0$ (both photons on shell) and $m_e = 0$ the pole at $k^2 = 0$ describes a massless $e^+ e^-$ pair moving at $v=c$ collinearly, with opposite helicities in a total spin-0 state (relativistic *Cooper pair* in QFT *vacuum*)



$\Rightarrow$ a massless scalar 0^+ state which couples to gravity

vertex

$h_{\mu\nu}\,(g^{\mu\nu}\,\Box - \partial^\mu\partial^\nu)\psi'$



Effective

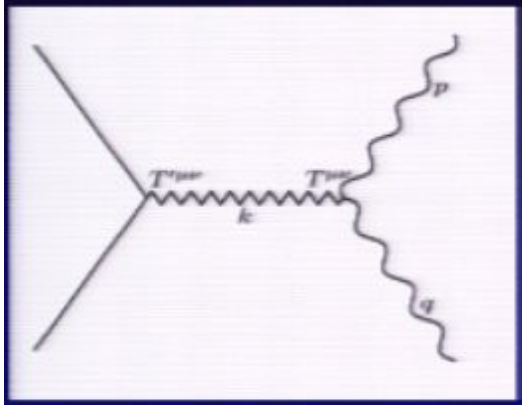
$\varphi\, F^{\alpha\beta}F_{\alpha\beta}$

Effective Action

$$\int d^4x\sqrt{-g}\left\{-\psi'\Box\varphi - \frac{R}{3}\psi' - \frac{e^2}{48\pi^2}\varphi F^{\alpha\beta}F_{\alpha\beta}\right\}$$

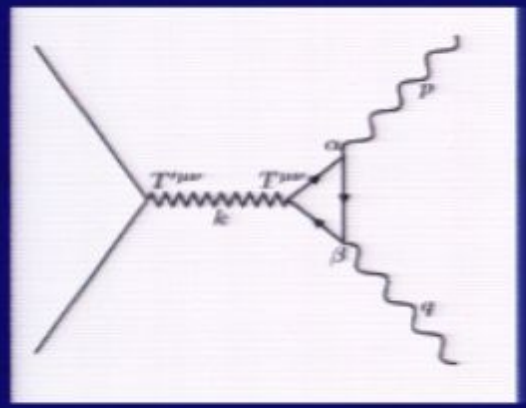
special case of general form

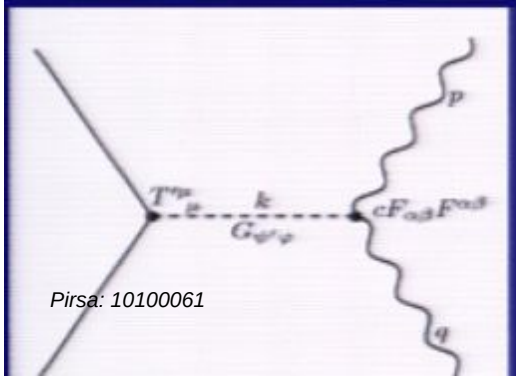
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (spin-2) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)

- But for $m_e = 0$ there is a scalar pole in the $\langle TJJ \rangle$ triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$





- Couples to trace $T'^{\mu}{}_{\mu}$
- $\langle TTT \rangle$ triangle of massless photons has similar pole
- New scalar degrees of freedom in EFT

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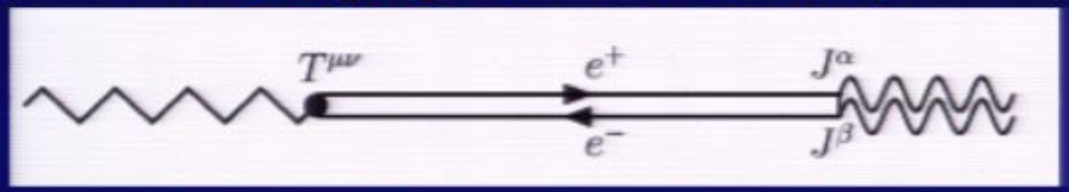

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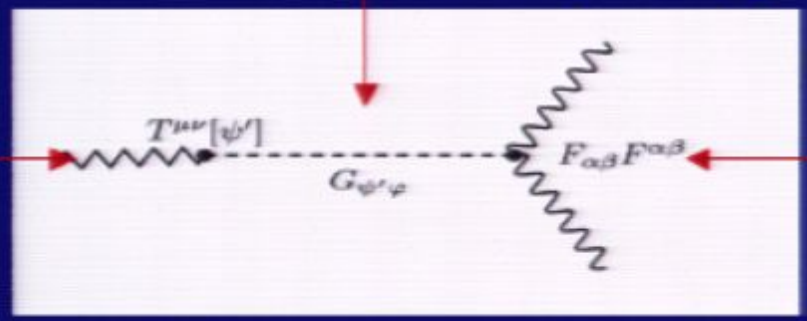

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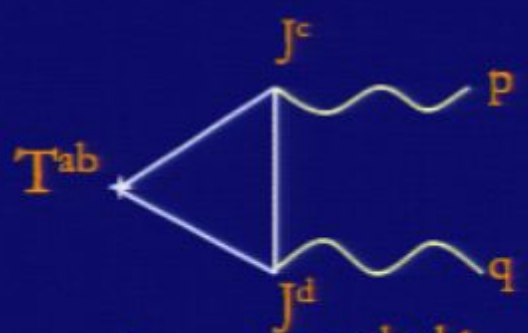

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$$3k^2 \int_0 \quad k^2 + s - i\epsilon$$

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Massless scalar intermediate two-particle state analogous to the pion in chiral limit of QCD

<TJJ> Triangle Amplitude in QED

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M. Giannotti & E. M. *Phys. Rev. D* **79, 045014 (2009)**

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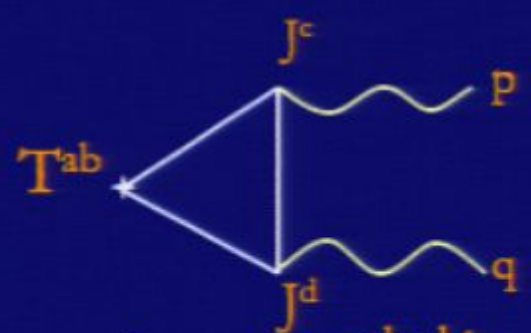

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Spectral Representation and Sum Rule

$$F_1(k^2;p^2,q^2) = \frac{1}{3k^2}\int_0^\infty \frac{ds}{k^2+s-i\epsilon}\left[(k^2+s)\rho_T - m^2\rho_m\right]$$

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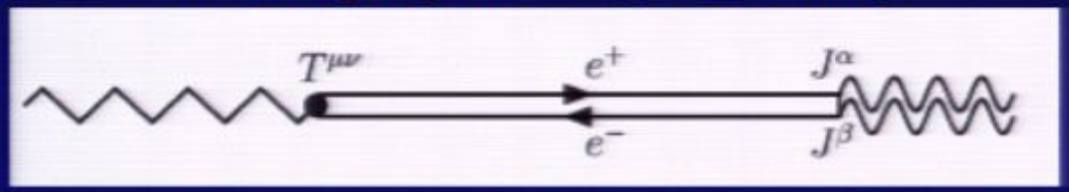
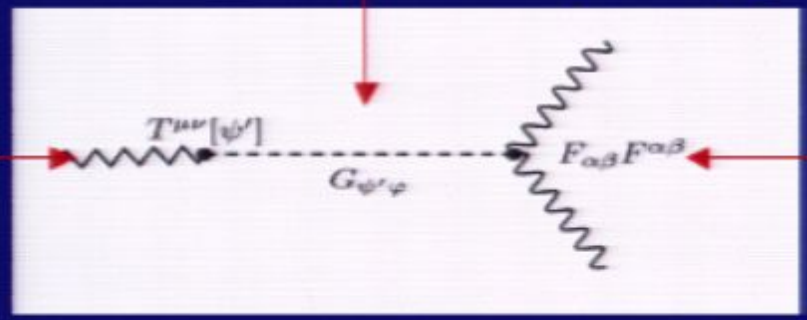


⇒ a massless scalar 0^+ state which couples to gravity

vertex

$h_{\mu\nu}\,(g^{\mu\nu}\,\Box - \partial^\mu\partial^\nu)\psi'$



Effective

$\varphi\, F^{\alpha\beta}F_{\alpha\beta}$

Effective Action

$$\int d^4x\sqrt{-g}\left\{-\psi'\Box\varphi - \frac{R}{3}\psi' - \frac{e^2}{48\pi^2}\varphi F^{\alpha\beta}F_{\alpha\beta}\right\}$$

special case of general form

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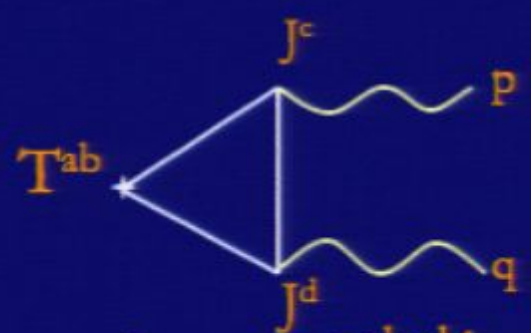

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Quantum Trace Anomaly in 4D Flat Space

Eg. QED in an External EM Field A_μ

$$\langle T^\mu_\mu \rangle = \frac{e^2}{24\pi^2} F^{\mu\nu} F_{\mu\nu}$$

Triangle One-Loop Amplitude as in Chiral Case

$$\Gamma^{abcd}(p,q) = (k^2 g^{ab} - k^a k^b)(g^{cd} p\cdot q - q^c p^d) F_1(k^2) + \text{(traceless terms)}$$

In the limit of massless fermions, $F_1(k^2)$ must have a massless pole:

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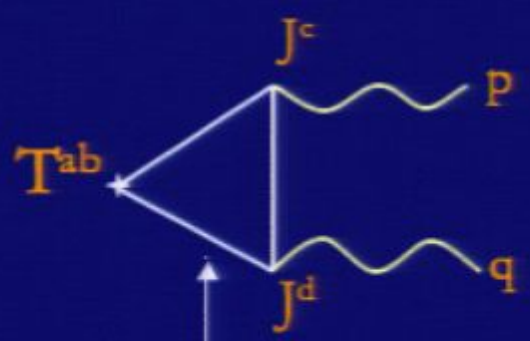

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Corresponding Imag. Part Spectral Fn. has a δ fn
This is a new massless scalar degree of freedom in the two-particle correlated spin-0 state

Quantum Effects of 2D Anomaly Action

- Modification of Classical Theory required by Quantum Fluctuations & Covariant Conservation of $\langle T^a{}_b \rangle$
- Metric conformal factor $e^{2\sigma}$ (was constrained) becomes dynamical & itself fluctuates freely
- Gravitational 'Dressing' of critical exponents: long distance macroscopic physics
- Non-perturbative/non-classical conformal fixed point of 2D gravity: Running of Λ
- Additional non-local Infrared Relevant Operator in S_{EFT}

New Massless Scalar Degree of Freedom at low energies

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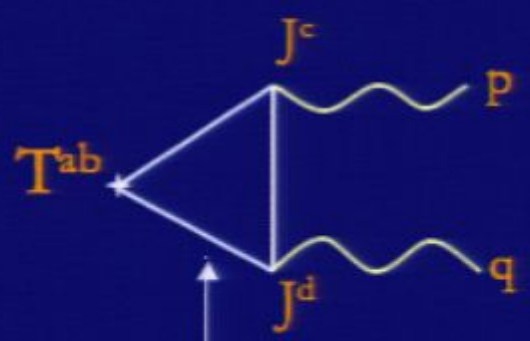

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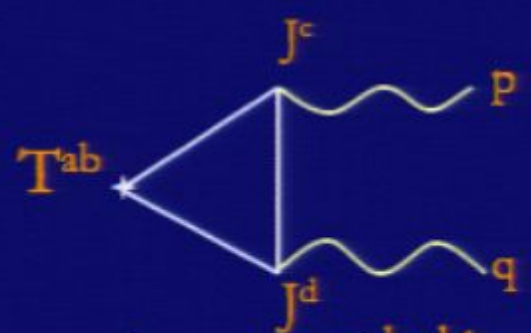

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Scalar Pole in Gravitational Scattering

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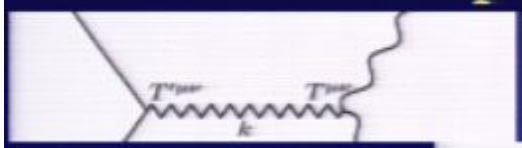
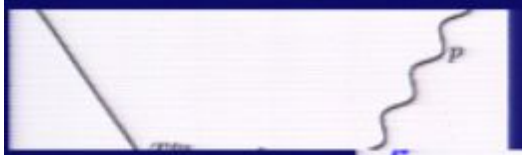


polarized waves (spin-2) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$

- But for $m_e = 0$ there is a scalar pole in the

Effective

- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$



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- $\langle T'T'T \rangle$ triangle of massless photons has similar pole of general form

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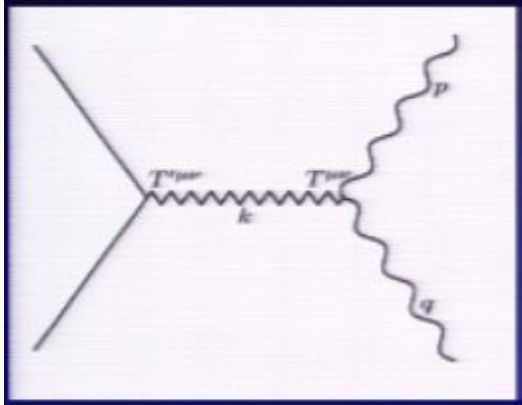
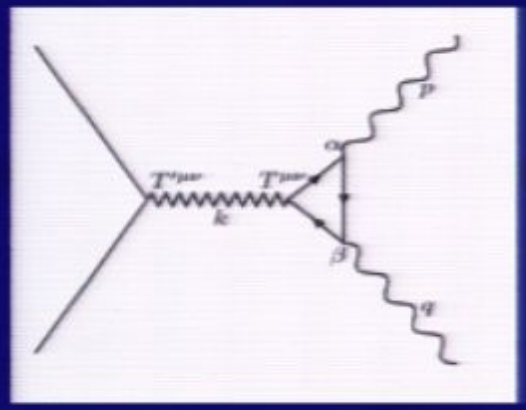
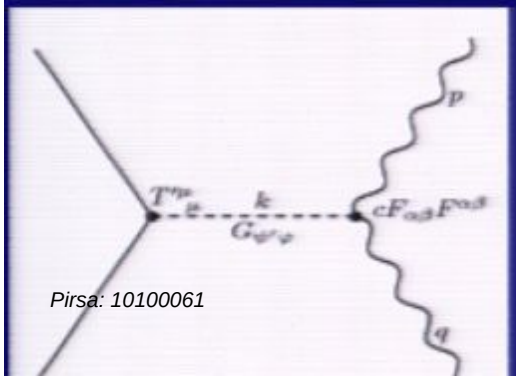
Scalar Pole in Gravitational Scattering



- In Einstein's Theory only transverse, tracefree polarized waves (spin-2) are emitted/absorbed and propagate between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$
- The scalar parts give only non-progagating constrained interaction (like Coulomb field in E&M)

- But for $m_e = 0$ there is a scalar pole in the $\langle TJJ \rangle$ triangle amplitude coupling to photons
- This scalar wave propagates in gravitational scattering between sources $T'^{\mu\nu}$ and $T^{\mu\nu}$





- Couples to trace $T'^{\mu}{}_{\mu}$
- $\langle T'TT \rangle$ triangle of massless photons has similar pole
- New scalar degrees of freedom in EFT