Title: Relativity (PHYS 604) - Lecture 15 - Part 2

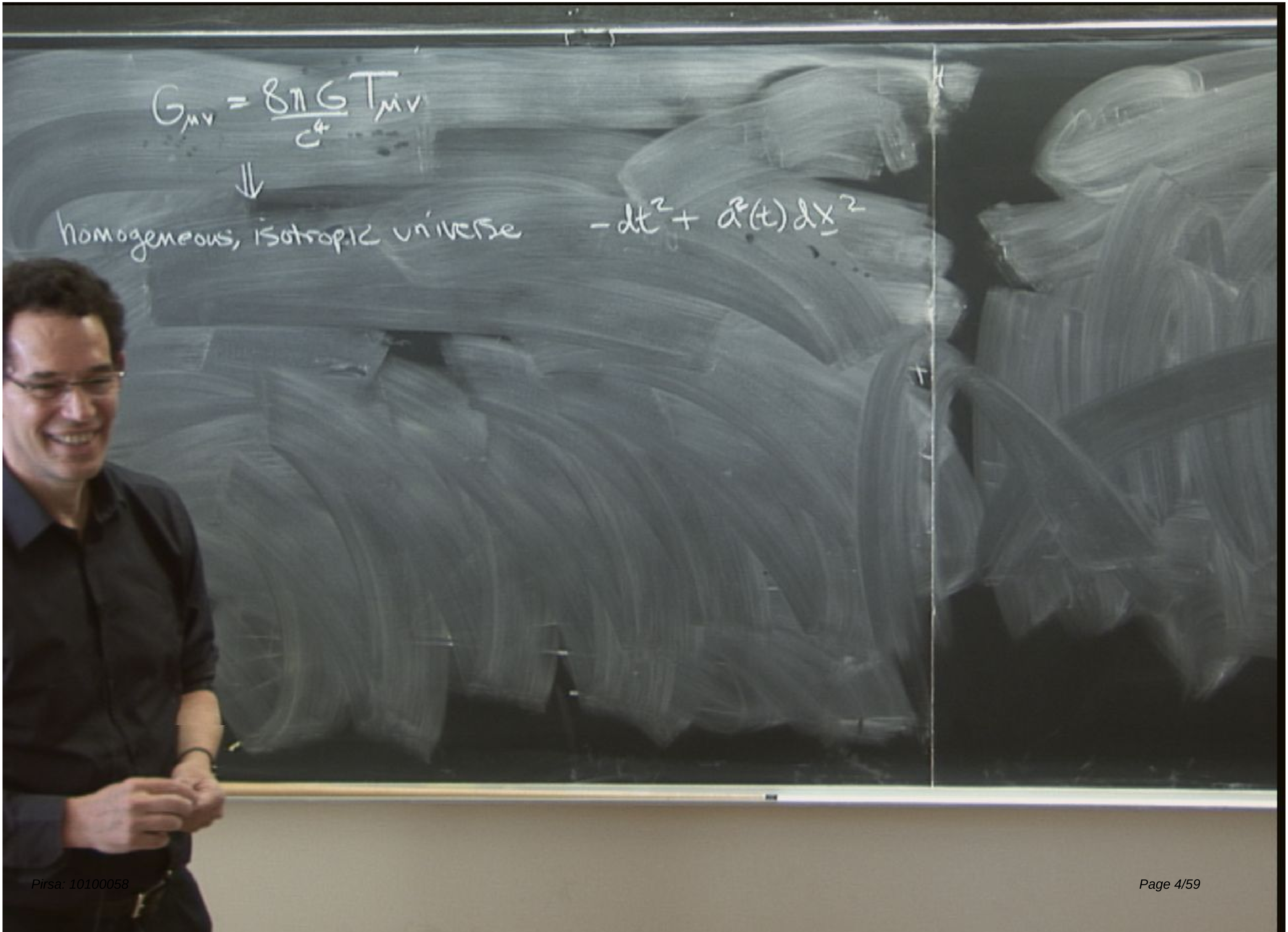
Date: Oct 01, 2010 11:15 AM

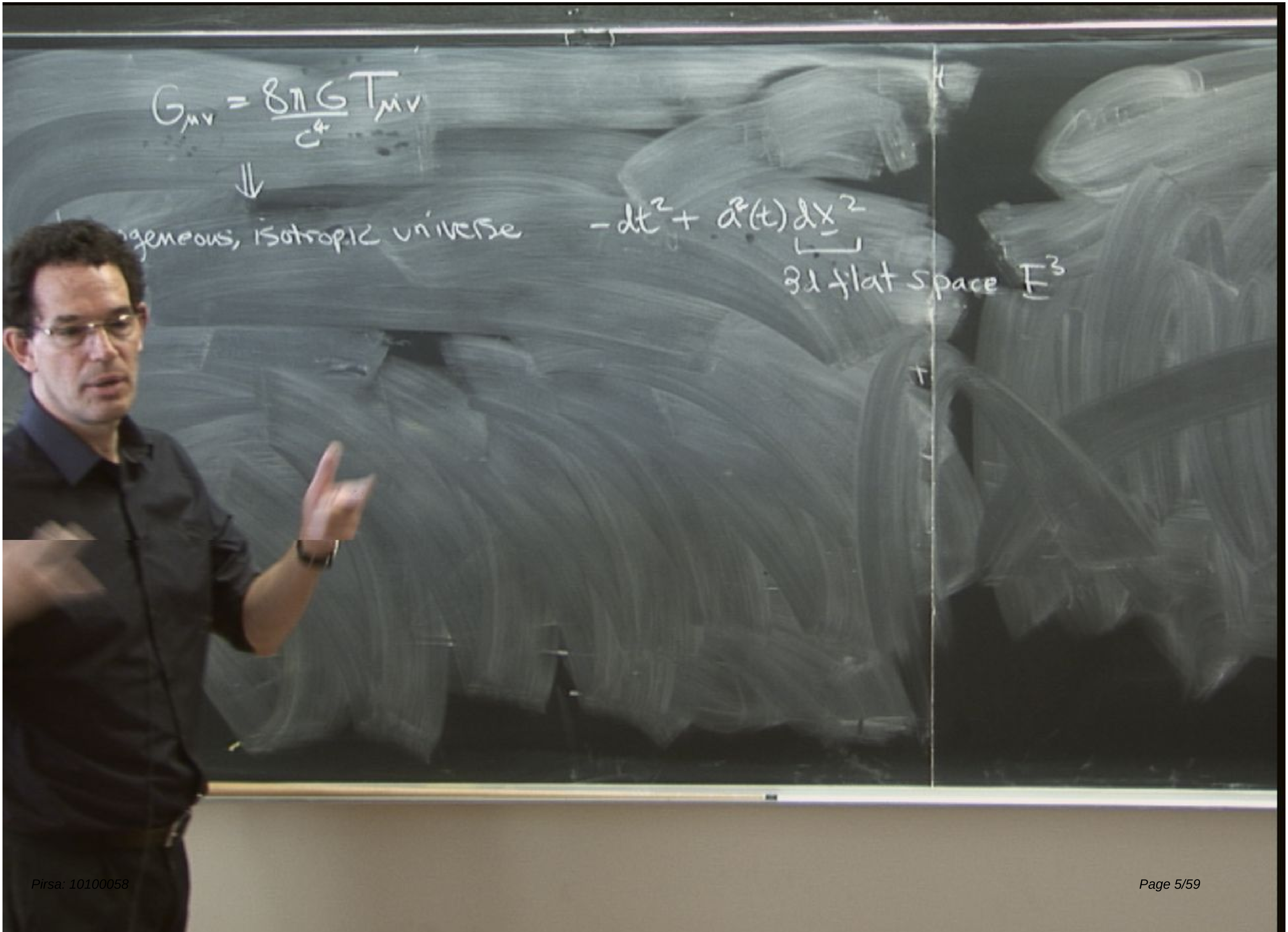
URL: http://pirsa.org/10100058

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS


$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\Downarrow$


$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\Downarrow$
homogeneous, isotropic universe
$-dt^2 + a^2(t)\,d\underline{x}^2$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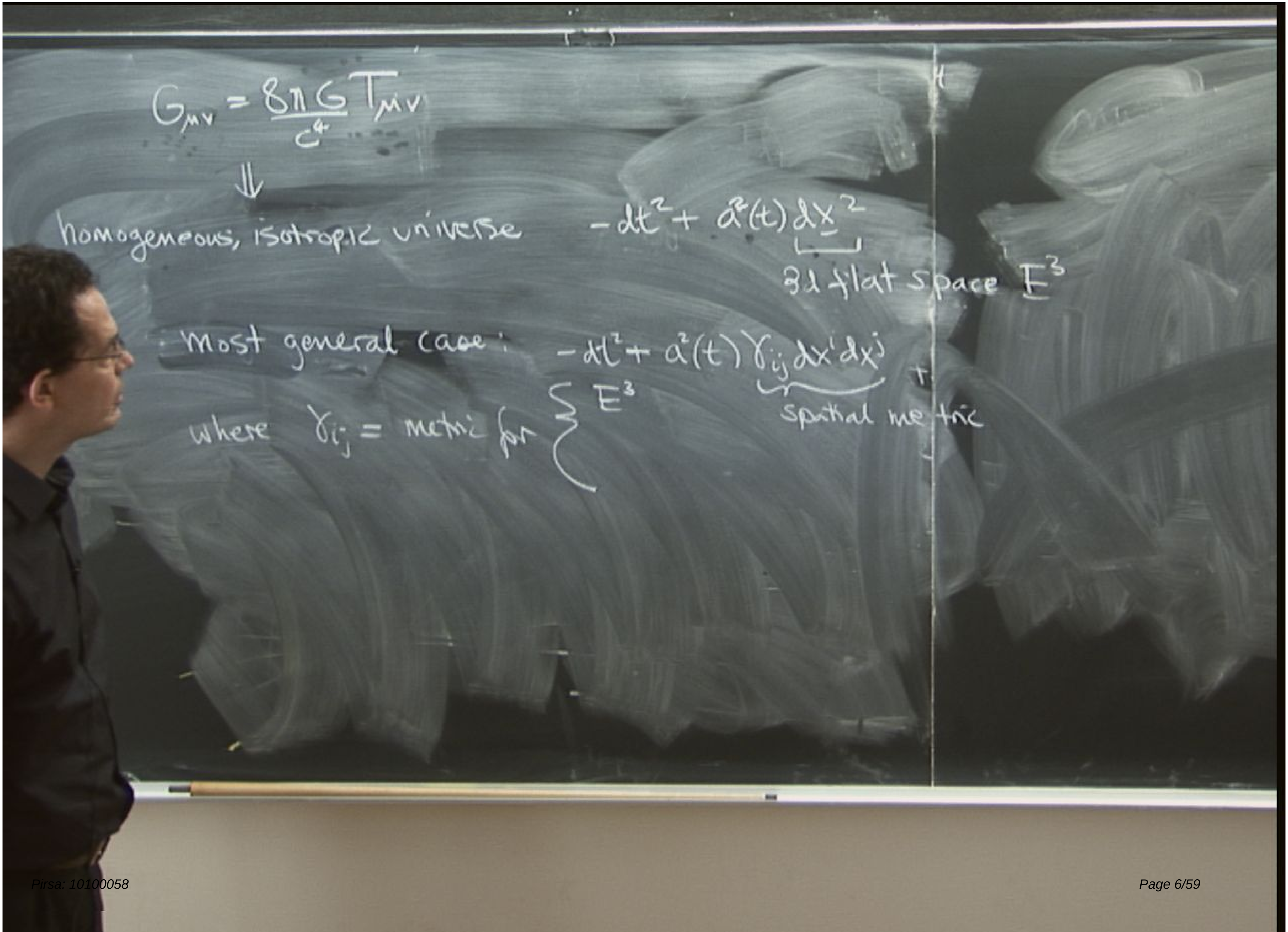
$\Downarrow$
geneous, isotropic universe
$-dt^2 + a^2(t)\,\underbrace{d\underline{x}^2}$
3d flat space E^3


$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$
$\Downarrow$
homogeneous, isotropic universe
$-dt^2 + a^2(t)\,\underbrace{d\underline{x}^2}$
3d flat space E^3
most general case: $-dt^2 + a^2(t)\,\underbrace{\gamma_{ij}\,dx^i dx^j}$
spatial metric
where γ_{ij} = metric for $\left\{ E^3 \right.$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$\Downarrow$

homogeneous, isotropic universe $\quad -dt^2 + a^2(t)\, \underbrace{d\underline{x}^2}_{\text{3d flat space } \mathbb{E}^3}$

most general case: $\quad -dt^2 + a^2(t)\, \underbrace{\gamma_{ij}\, dx^i dx^j}_{\text{spatial metric}}$

where γ_{ij} = metric for $\begin{cases} \mathbb{E}^3 & \\ S^3 & \text{3-sphere} \\ H^3 & \text{3-hyperboloid} \end{cases}$

S^3: $x^2 + y^2 + z^2 + w^2 = 1$ in $\mathbb{E}^4$

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\Downarrow$$

homogeneous, isotropic universe $\quad -dt^2 + a^2(t)\, \underbrace{d\underline{x}^2}_{\text{3d flat space } E^3}$

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S^3: $x^2 + y^2 + z^2 + w^2 = 1$ in E^4

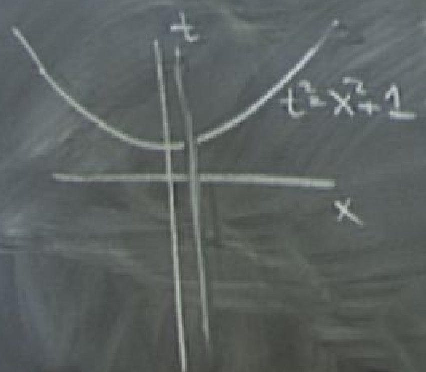
H^3: $t^2 - x^2 - y^2 - z^2 = 1$ in M^4

Minkowski

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

$$\Downarrow$$

homogeneous, isotropic universe $\quad -dt^2 + a^2(t)\, \underbrace{d\underline{x}^2}_{\text{3d flat space } E^3}$



Most general case: $\quad -dt^2 + a^2(t)\, \underbrace{\gamma_{ij}\, dx^i dx^j}_{\text{spatial metric}}$

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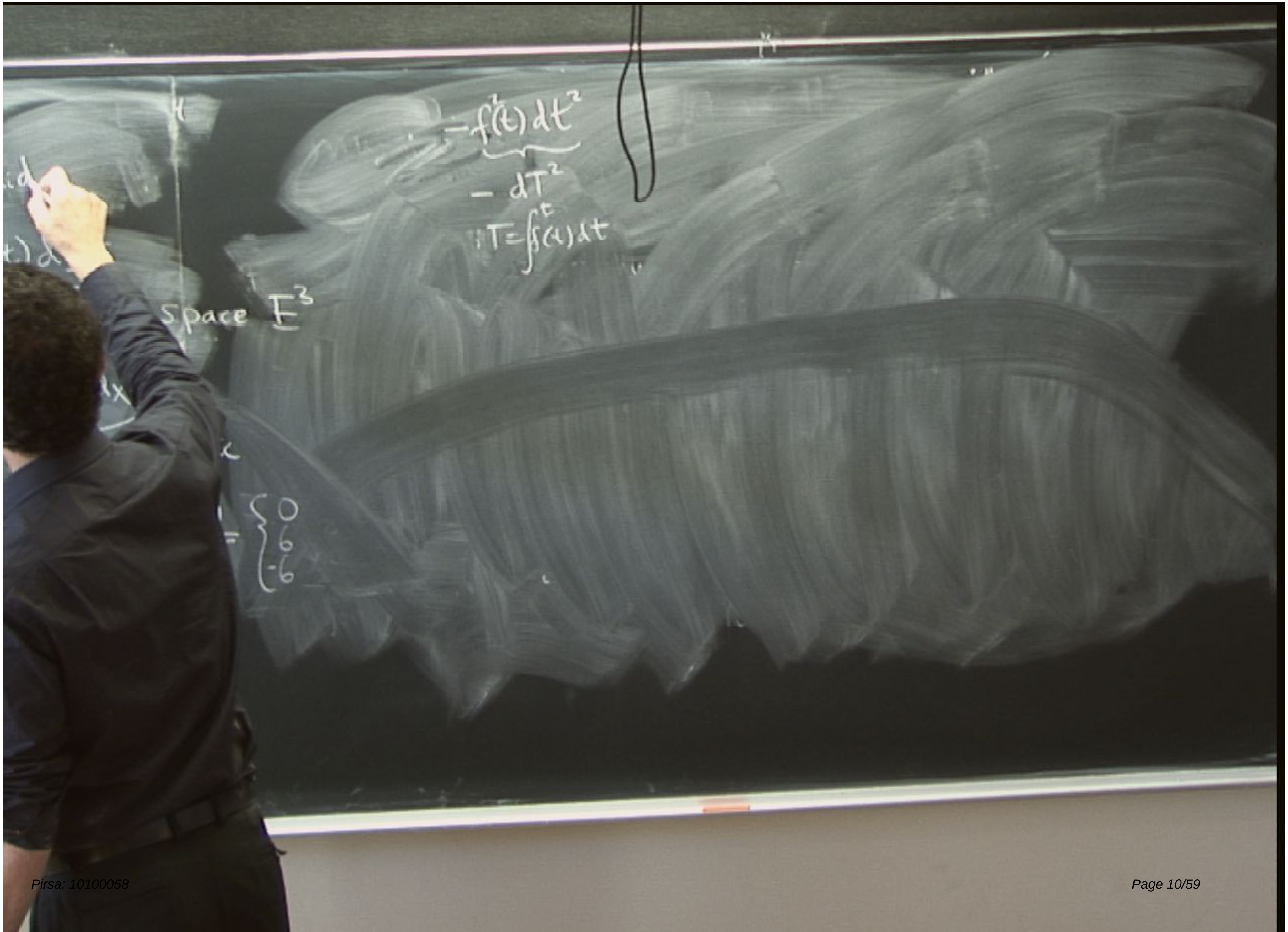
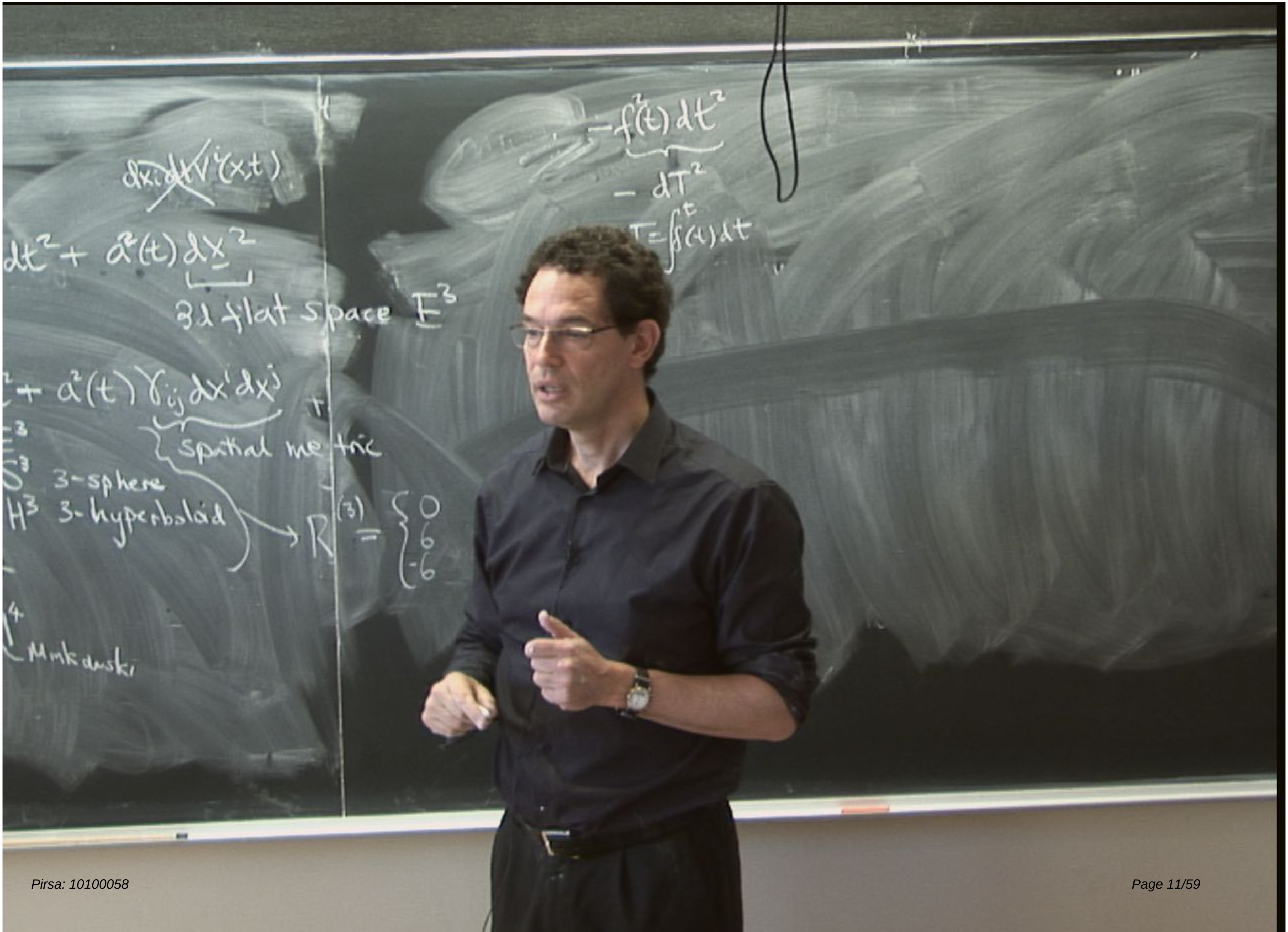
S^3: $x^2+y^2+z^2+w^2=1$ in E^4

H^3: $t^2-x^2-y^2-z^2=1$ in M^4 ← Minkowski


$-f^2(t)\,dt^2$
$-dT^2$
$T=\int^t f(t)\,dt$
Space E^3


$-f^2(t)\,dt^2$
$-dT^2$
$dt^2 + a^2(t)\,dx^2$
3d flat space E^3
$+ a^2(t)\,\gamma_{ij}\,dx^i dx^j$
spatial metric
S^3 3-sphere
H^3 3-hyperboloid
Minkowski

$\cancel{dx_i\, dt}\, V^i(x,t)$

$dt^2 + a^2(t)\, \underbrace{d\underline{x}^2}_{\text{3d flat space } E^3}$

$+ a^2(t)\, \underbrace{\gamma_{ij}\, dx^i dx^j}_{\text{spatial metric}}$

E^3

S^3 3-sphere

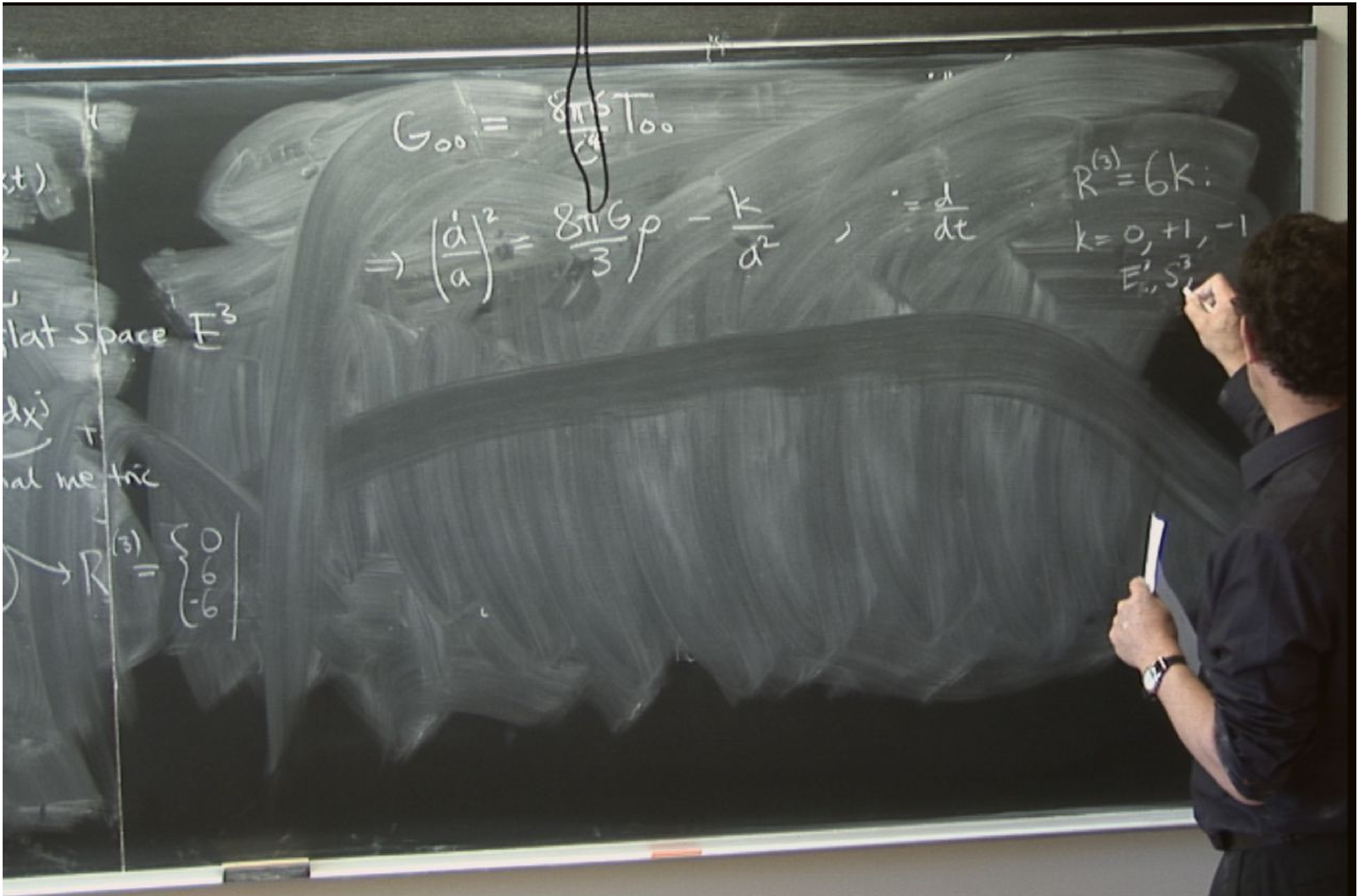
H^3 3-hyperboloid

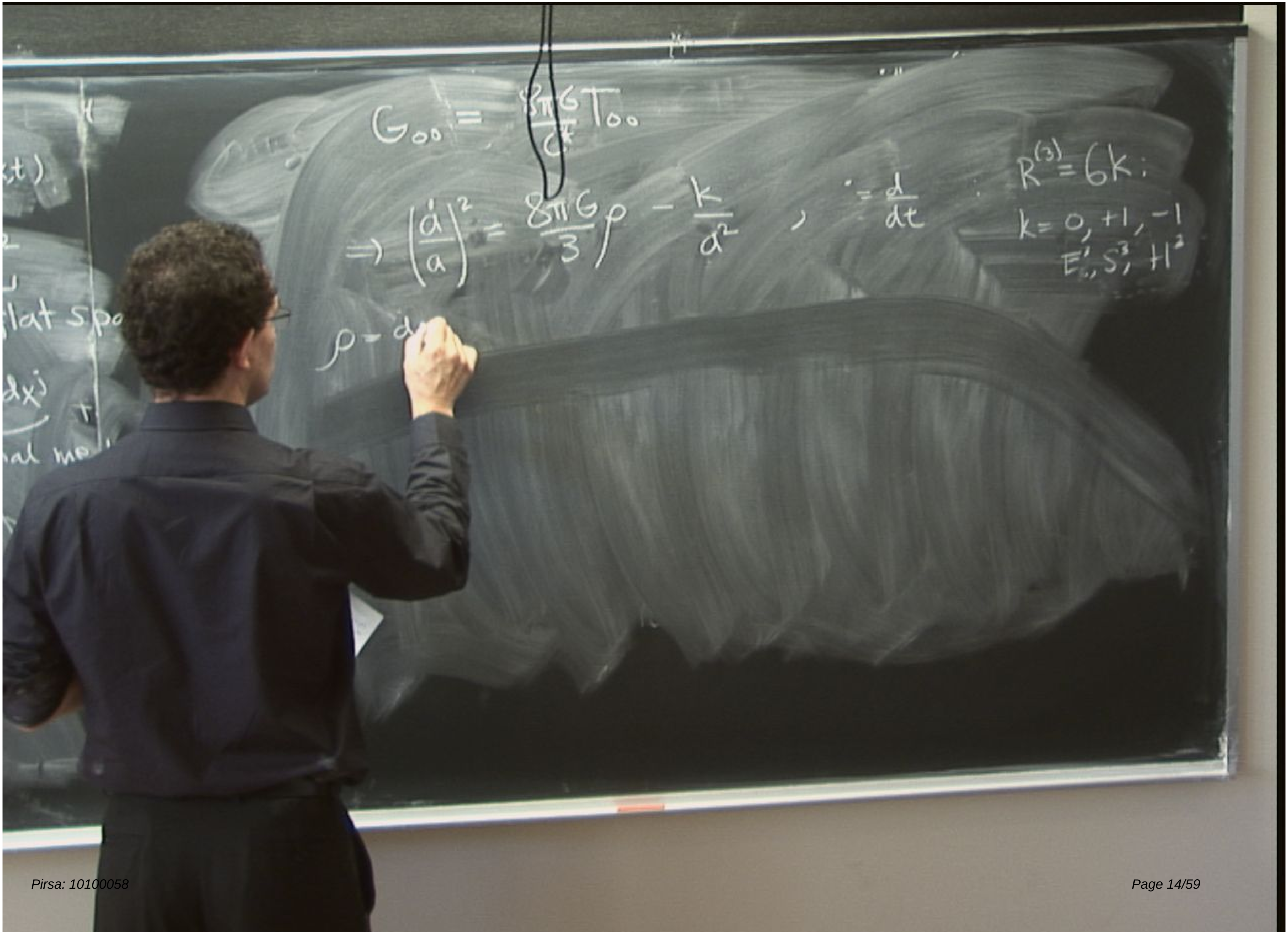
$R^{(3)} = \begin{cases} 0 \\ 6 \\ -6 \end{cases}$

M^4 Minkowski

$G_{00} = \frac{8\pi G}{c^4} T_{00}$

$\Longrightarrow$


$G_{00} = \frac{8\pi G}{c^4} T_{00}$
$\Rightarrow \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$, $\dot{} = \frac{d}{dt}$
$R^{(3)} = 6k$;
$k = 0, +1, -1$
$E^3, S^3,$
flat space E^3
dx^j
al metric
$R^{(3)} = \begin{cases} 0 \\ 6 \\ -6 \end{cases}$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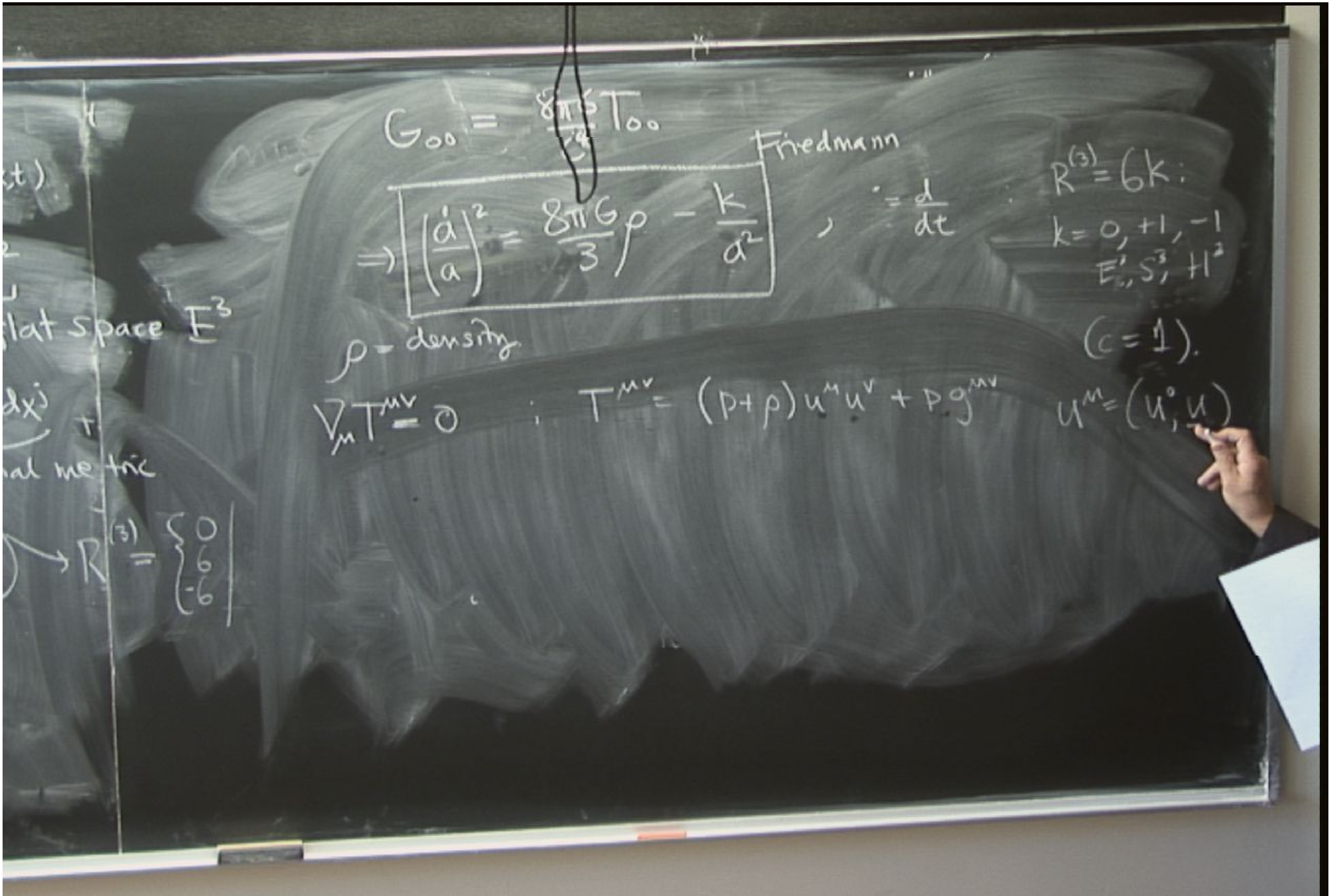
$k = 0, +1, -1$
E^3, S^3, H^3



$G_{00} = \frac{8\pi G}{c^4} T_{00}$
Friedmann
$\Rightarrow \boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$, $\dot{} = \frac{d}{dt}$
$R^{(3)} = 6k$;
$k = 0, +1, -1$
E^3, S^3, H^3
$(c = 1)$.
ρ - density
$\nabla_\mu T^{\mu\nu} = 0$; $T^{\mu\nu} = (p+\rho) u^\mu u^\nu + p g^{\mu\nu}$
$u^\mu = (u^0, \underline{u})$
flat space E^3
al metric
$R^{(3)} = \begin{cases} 0 \\ 6 \\ -6 \end{cases}$

(t)

flat space E^3

dx^j

al metric

$\rightarrow R^{(3)} = \begin{cases} 0 \\ 6 \\ -6 \end{cases}$

$$G_{00} = \frac{8\pi G}{c^4} T_{00}$$

Friedmann

$$\Rightarrow \boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}} \quad , \quad \dot{} = \frac{d}{dt}$$

ρ = density

$$\nabla_\mu T^{\mu\nu} = 0 \quad ; \quad T^{\mu\nu} = (p+\rho) u^\mu u^\nu + p g^{\mu\nu}$$

$R^{(3)} = 6k$;

$k = 0, +1, -1$

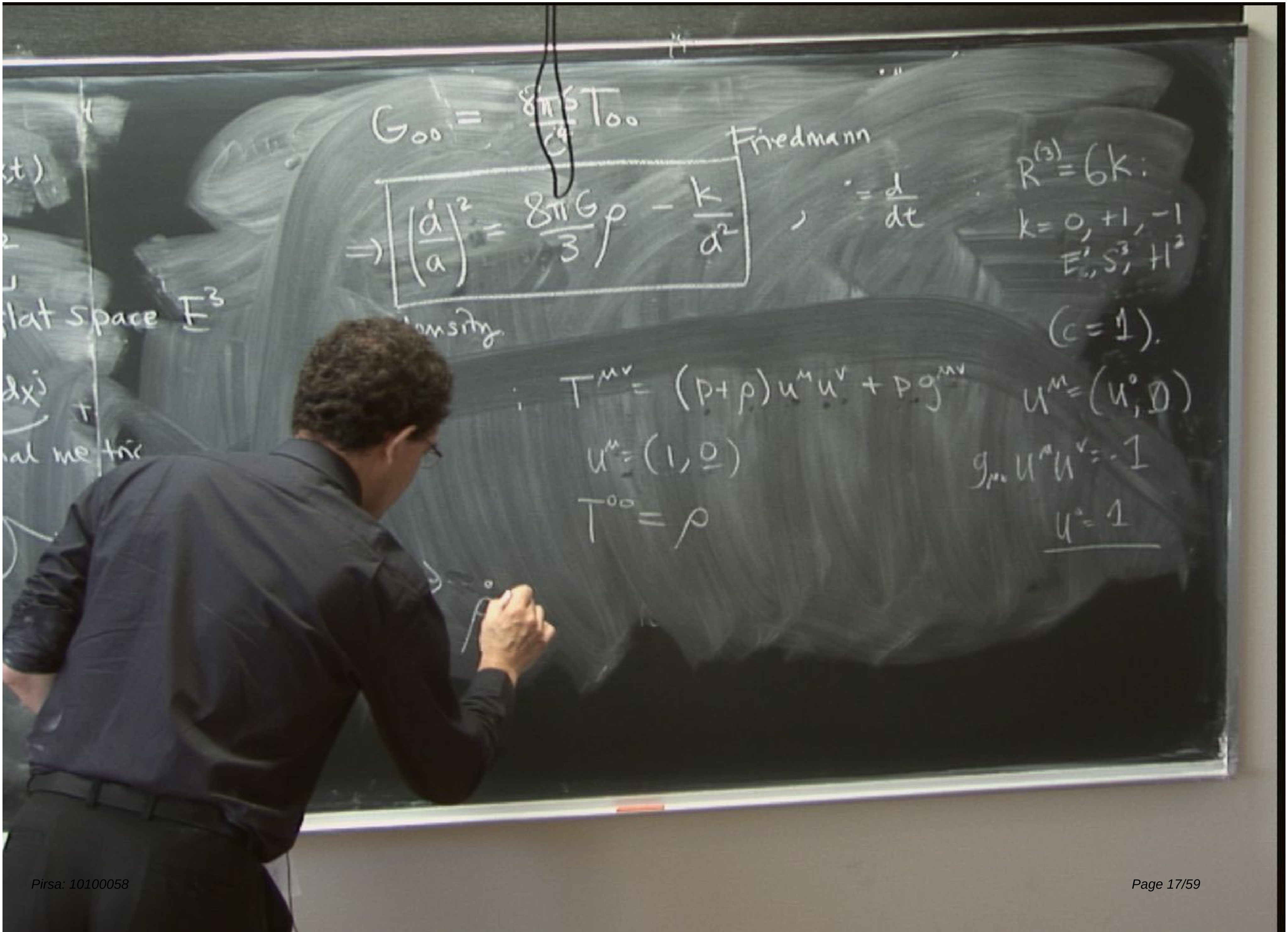
E^3, S^3, H^3

$(c = 1)$.

$u^\mu = (u^0, 0)$

$g_{\mu\nu} u^\mu u^\nu = -1$

$u^0 = 1$


$G_{00} = \frac{8\pi G}{c^4} T_{00}$
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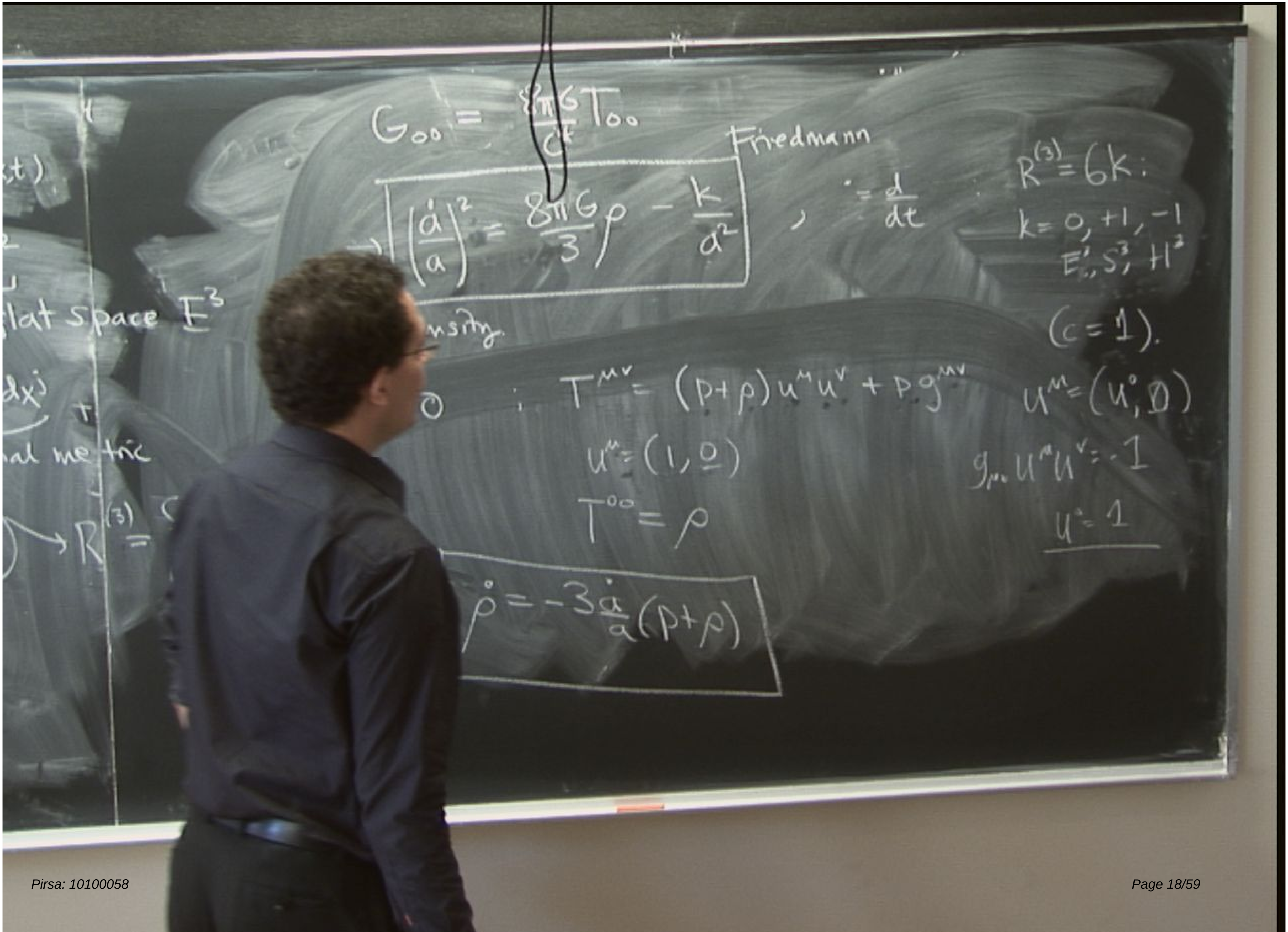
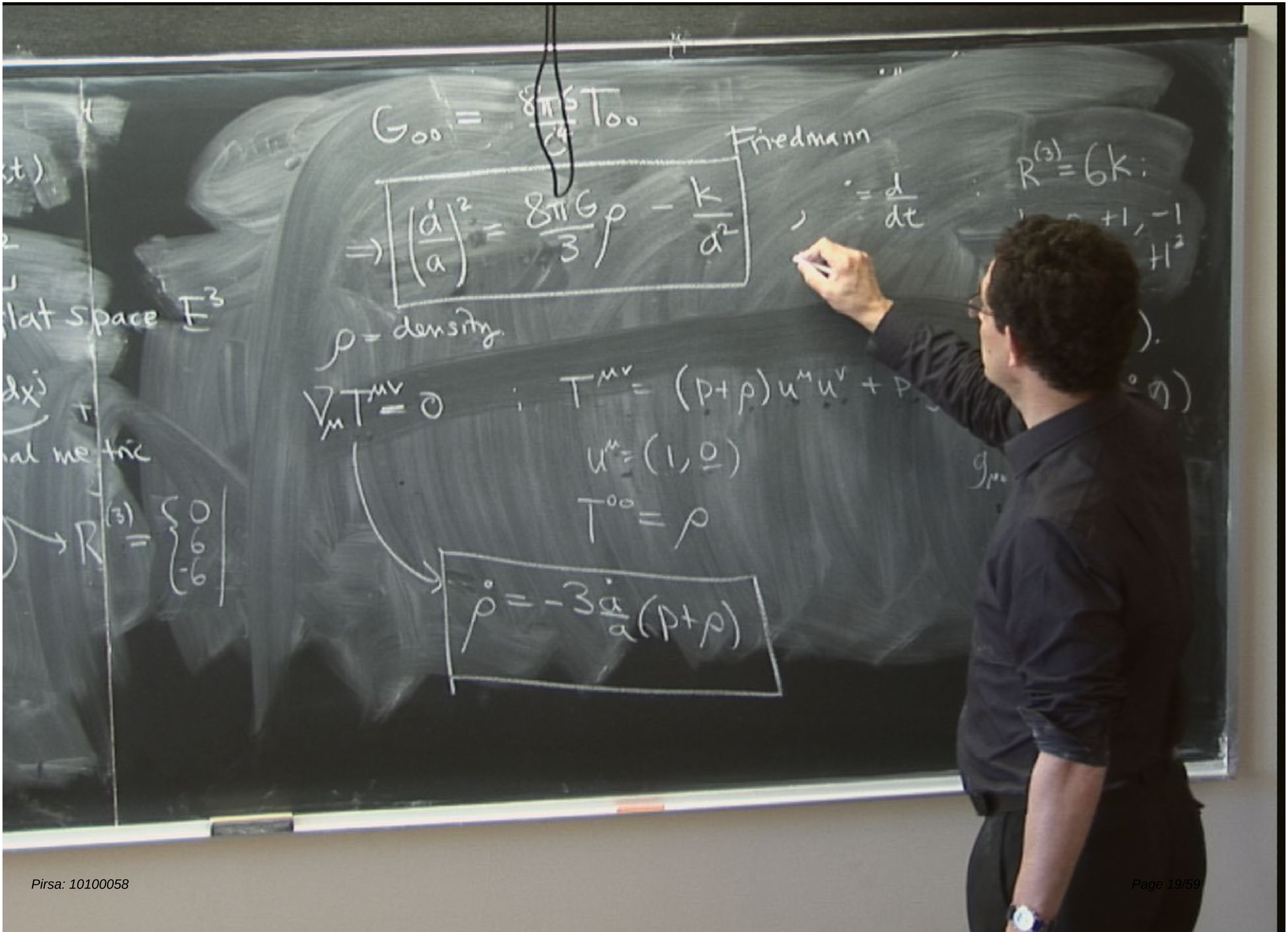
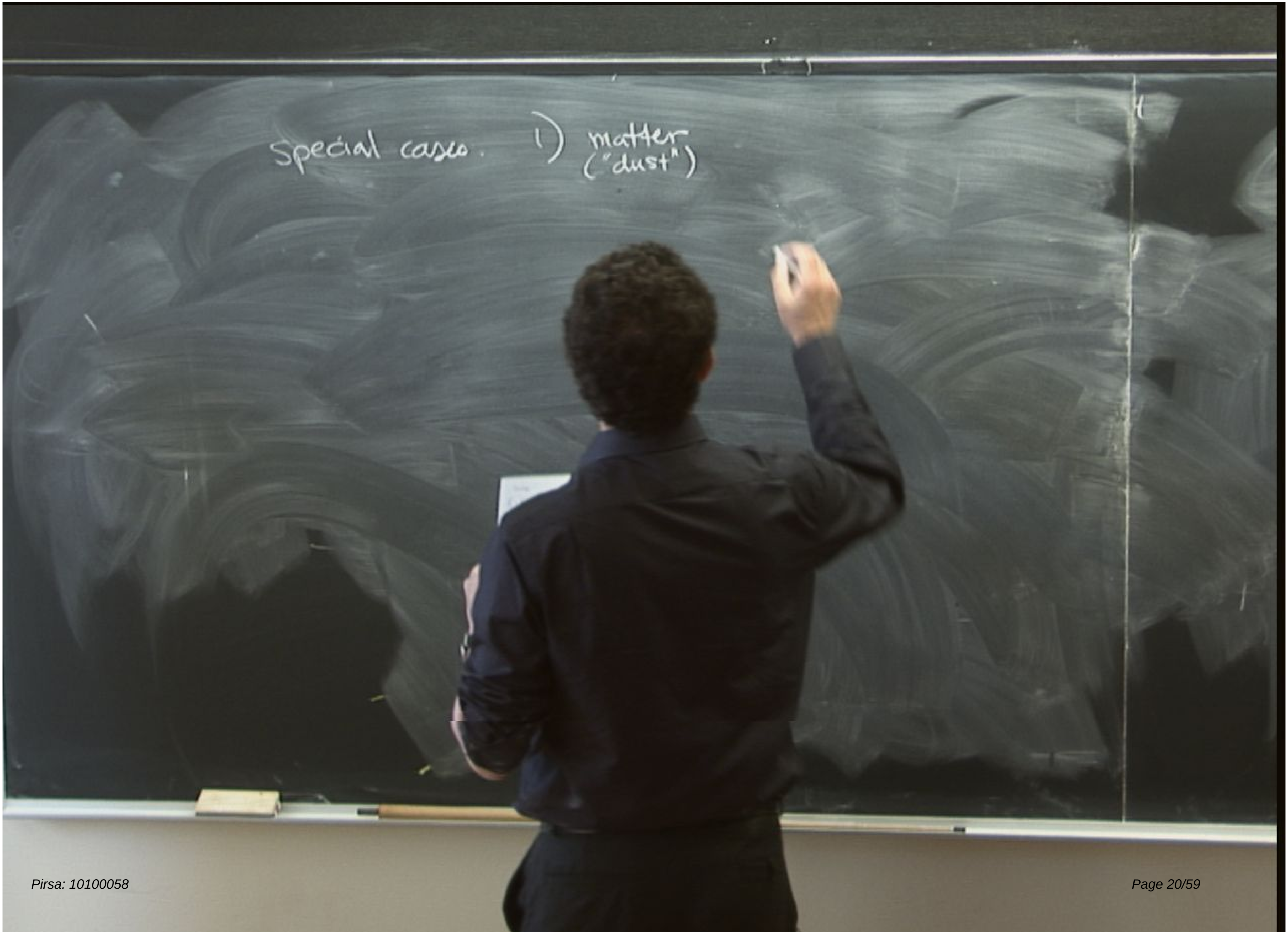
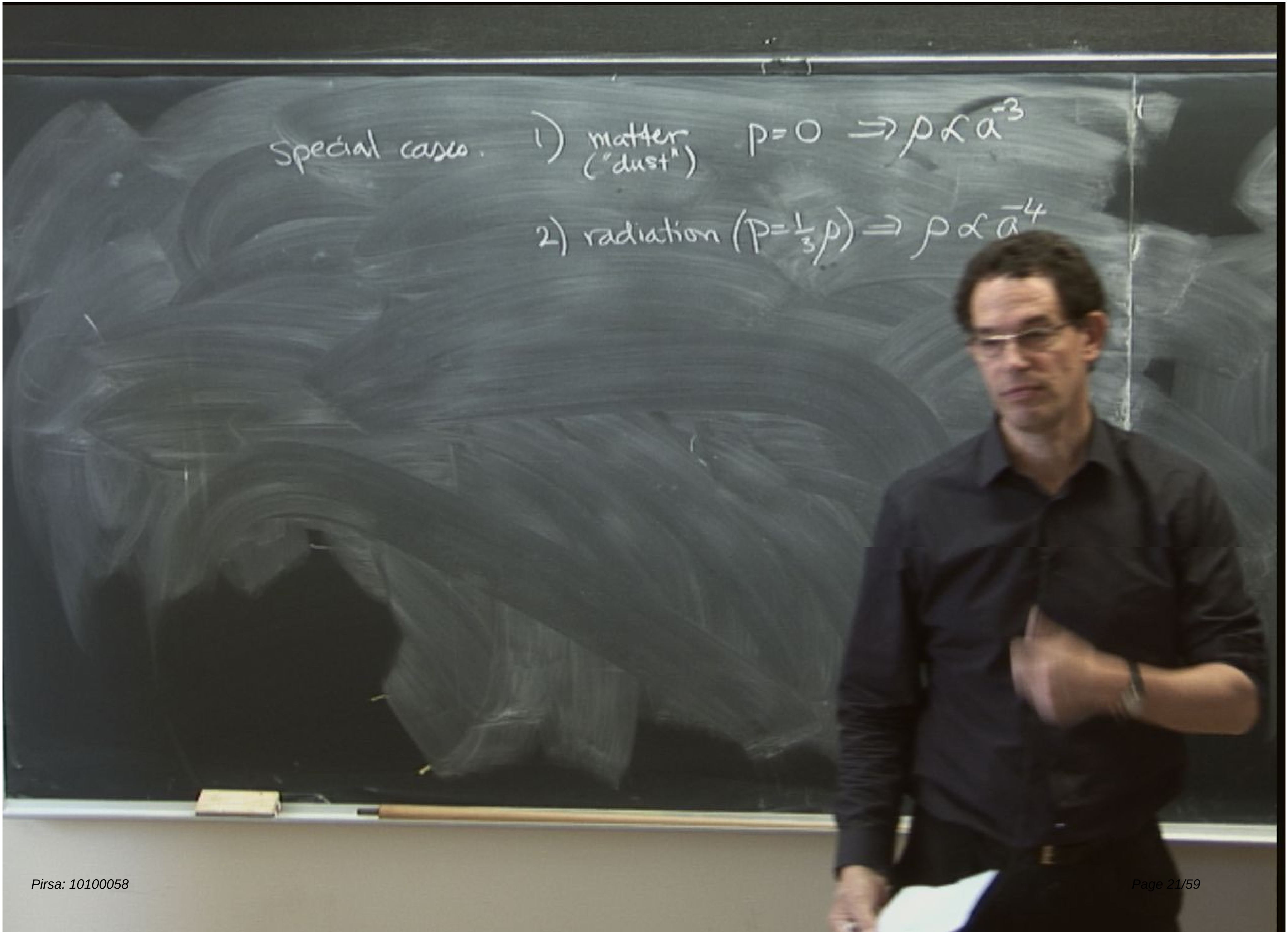
density
$\dot{} = \frac{d}{dt}$
$R^{(3)} = 6k$:
$k = 0, +1, -1$
E^3, S^3, H^3
$(c = 1)$.
flat space E^3
$T^{\mu\nu} = (p+\rho)u^\mu u^\nu + p g^{\mu\nu}$
$u^\mu = (u^0, \underline{0})$
$u^\mu = (1, \underline{0})$
$g_{\mu\nu} u^\mu u^\nu = -1$
$T^{00} = \rho$
$u^0 = 1$

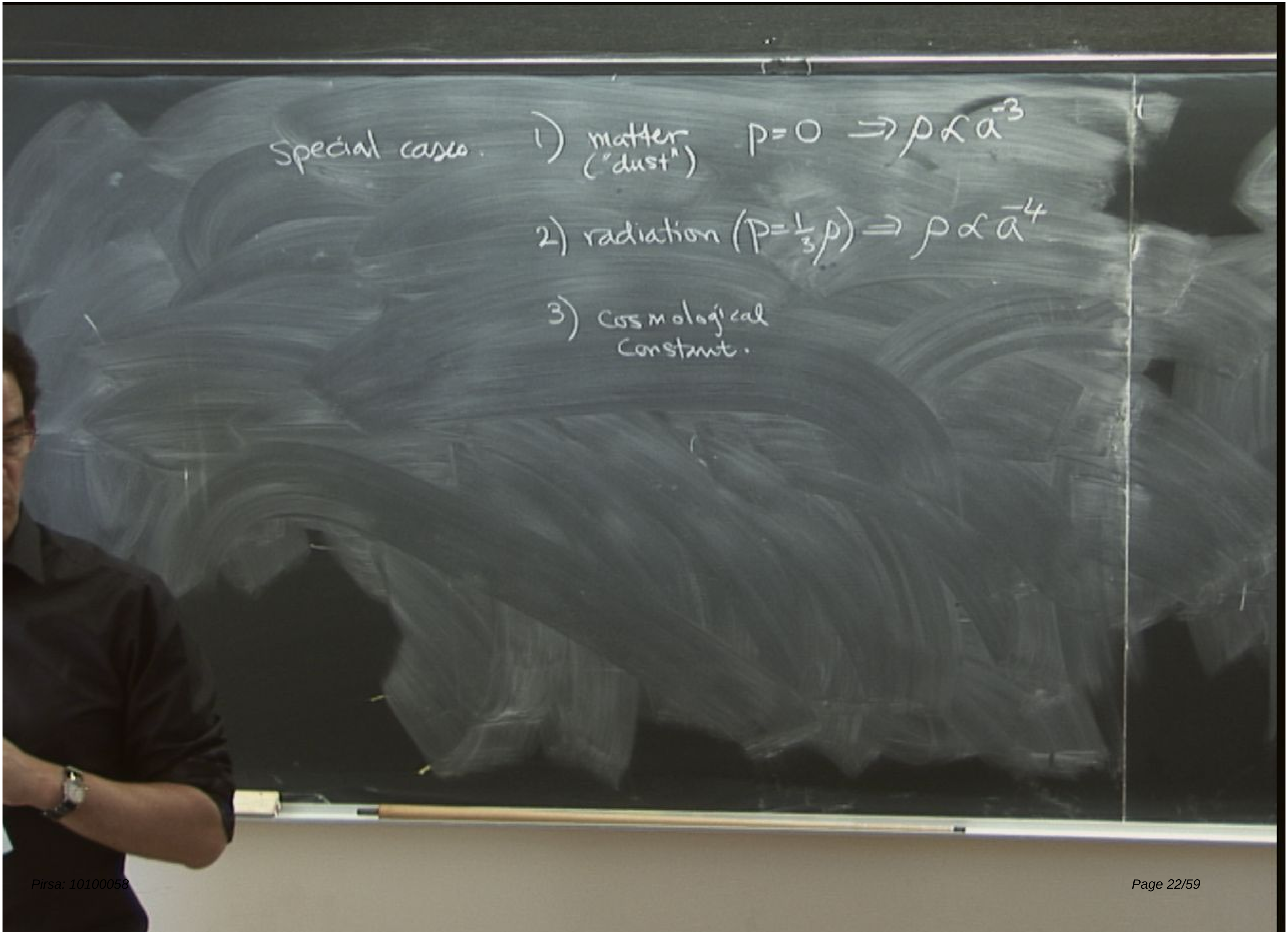


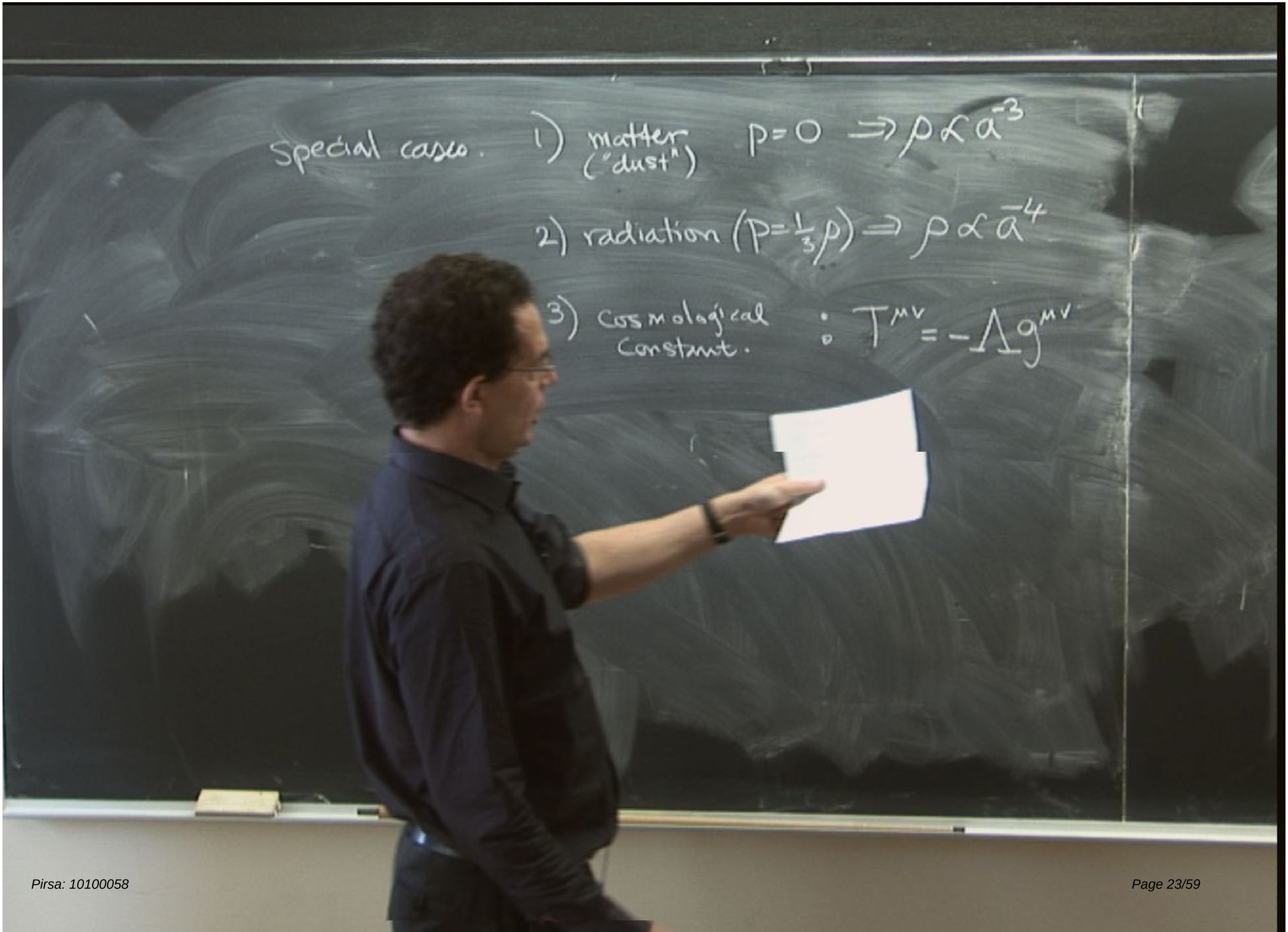
$G_{00} = \frac{8\pi G}{c^4} T_{00}$
Friedmann
$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$
$\dot{} = \frac{d}{dt}$
$R^{(3)} = 6k$
$k = 0, +1, -1$
E^3, S^3, H^3
$(c = 1)$
flat space E^3
density
$T^{\mu\nu} = (p+\rho)u^\mu u^\nu + p g^{\mu\nu}$
$u^\mu = (u^0, 0)$
$u^\mu = (1, 0)$
$g_{\mu\nu} u^\mu u^\nu = -1$
$T^{00} = \rho$
$u^0 = 1$
al metric
$\dot\rho = -3\frac{\dot a}{a}(p+\rho)$

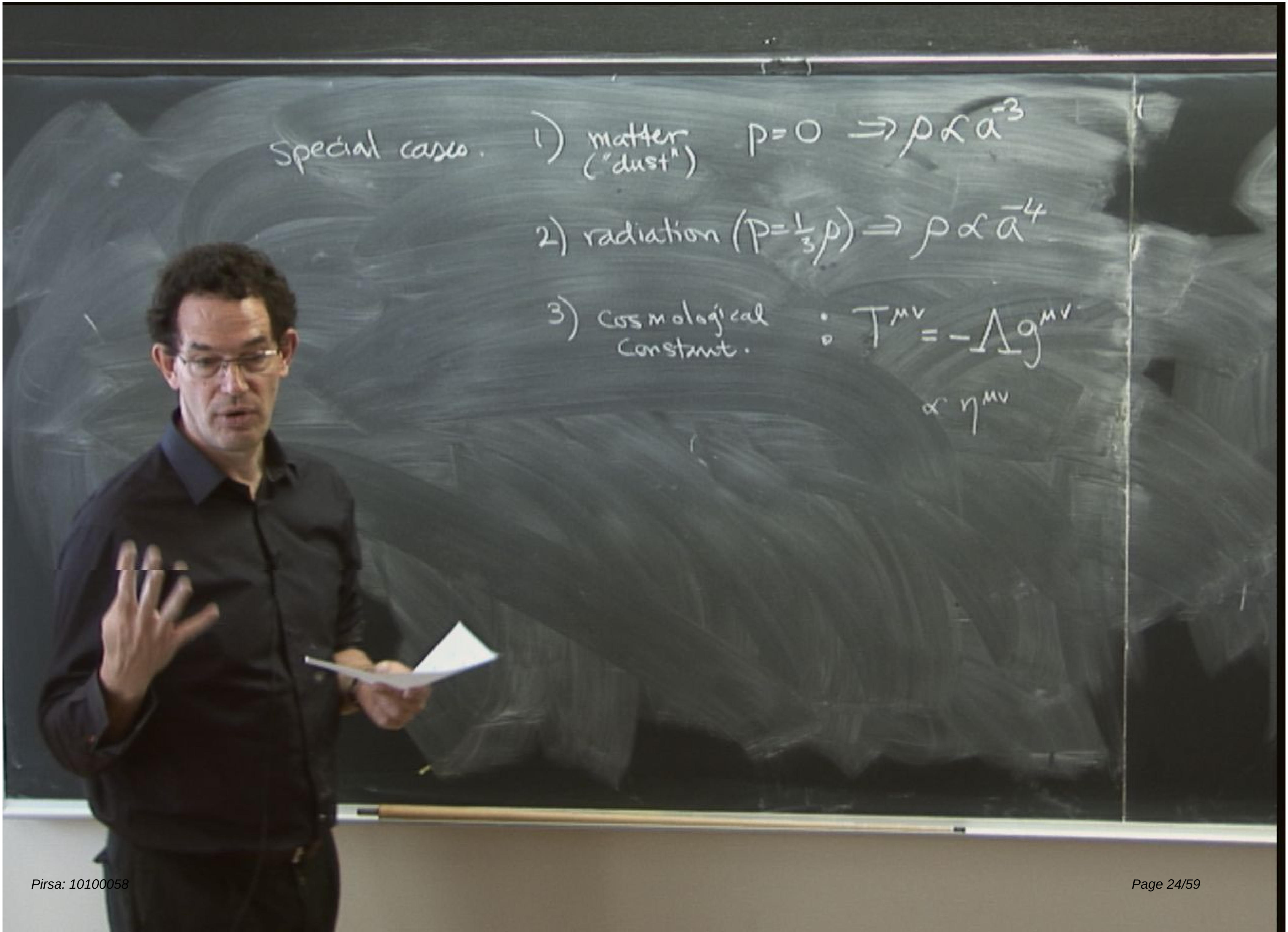

G00 = 8πG/c4 T00
Friedmann
(ȧ/a)² = 8πG/3 ρ − k/a²
˙ = d/dt
R(3) = 6k
ρ = density
∇μ Tμν = 0
Tμν = (p+ρ) uμ uν + p
uμ = (1, 0)
T00 = ρ
ρ̇ = −3 ȧ/a (p+ρ)
flat space E3
al metric


special cases. 1) matter
("dust")


special cases.
1) matter ("dust") p=0 ⇒ ρ ∝ a^-3
2) radiation (p = 1/3 ρ) ⇒ ρ ∝ a^-4


Special cases.
1) matter ("dust") $p=0 \Rightarrow \rho \propto a^{-3}$
2) radiation $(p=\frac{1}{3}\rho) \Rightarrow \rho \propto a^{-4}$
3) cosmological constant.


special cases.
1) matter ("dust") p=0 ⇒ ρ ∝ a^-3
2) radiation (p=1/3 ρ) ⇒ ρ ∝ a^-4
3) cosmological constant. : T^μν = -Λg^μν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3) cosmological constant. : $T^{\mu\nu} = -\Lambda g^{\mu\nu}$
$\propto \eta^{\mu\nu}$

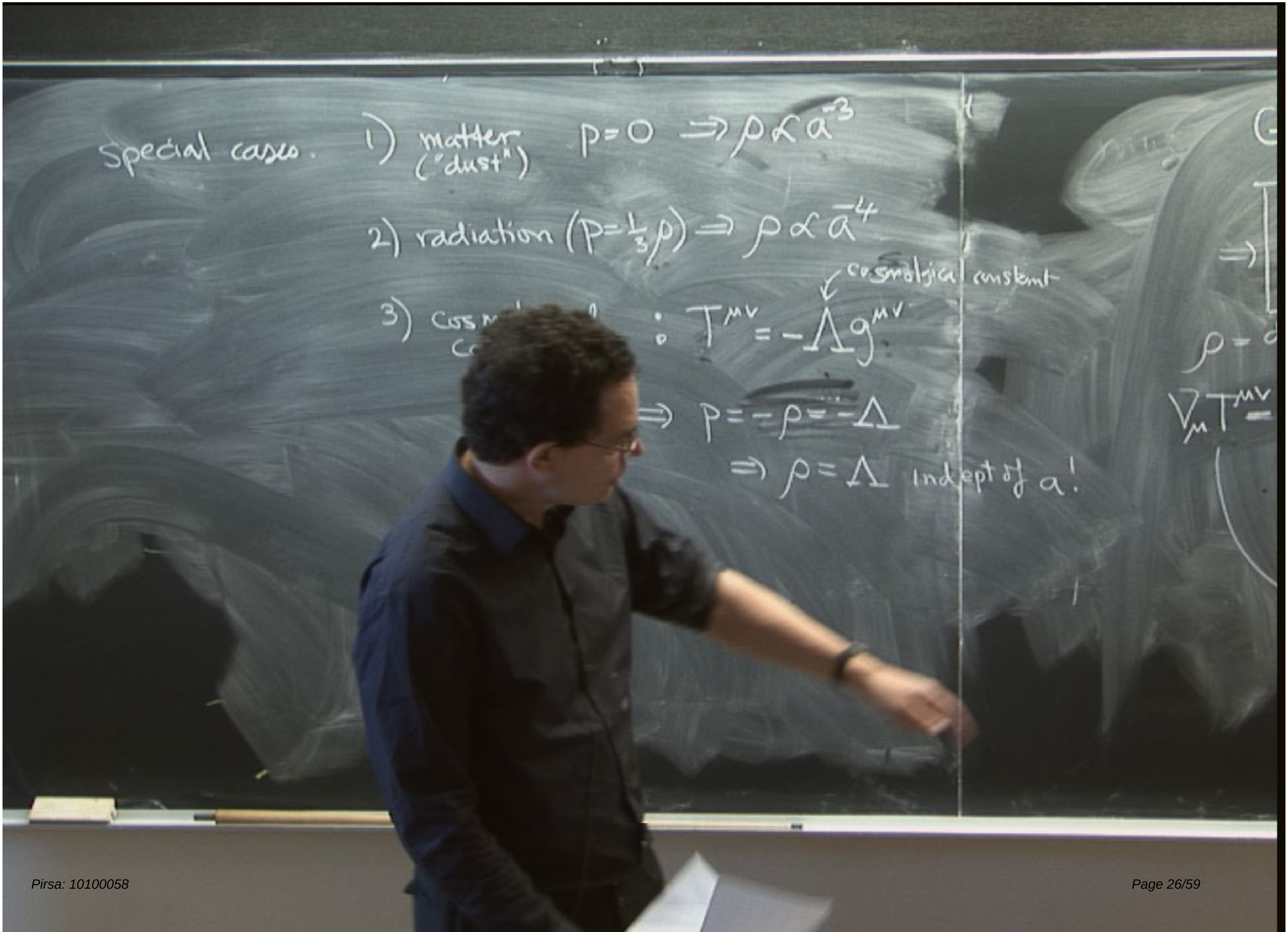
special cases.

1) matter ("dust") $p = 0 \Rightarrow \rho \propto a^{-3}$

2) radiation $(p = \frac{1}{3}\rho) \Rightarrow \rho \propto a^{-4}$

3) cosmological constant : $T^{\mu\nu} = -\Lambda g^{\mu\nu}$

cosmolgical constant


Special cases. 1) matter ("dust") $p=0 \Rightarrow \rho \propto a^{-3}$
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cosmolgical constant
3) cos... c... : $T^{\mu\nu} = -\Lambda g^{\mu\nu}$
$\Rightarrow p = -\rho = -\Lambda$
$\Rightarrow \rho = \Lambda$ indept of a!
$\Rightarrow$
$\rho =$
$\nabla_\mu T^{\mu\nu} =$

Special cases. 1) matter ("dust") $p=0 \Rightarrow \underline{\rho \propto a^{-3}}$

2) radiation $(p=\frac{1}{3}\rho) \Rightarrow \underline{\rho \propto a^{-4}}$

3) cosmological constant. : $T^{\mu\nu} = -\Lambda g^{\mu\nu}$ ← cosmological constant

$\nabla_\mu T^{\mu\nu} = 0$

$\Rightarrow p = -\rho = -\Lambda$

$\Rightarrow \underline{\rho = \Lambda}$ indep't of a!

G_{00}

$\Rightarrow \left(\frac{\dot{a}}{a}\right.$

ρ = dens

$\nabla_\mu T^{\mu\nu} = 0$

special cases. 1) matter ("dust") $p=0 \Rightarrow \rho \propto a^{-3}$

2) radiation $(p=\frac{1}{3}\rho) \Rightarrow \rho \propto a^{-4}$

3) cosmological constant. : $T^{\mu\nu} = -\Lambda g^{\mu\nu}$ cosmological constant $\nabla_\mu T^{\mu\nu} = 0$

$\Rightarrow p = -\rho = -\Lambda$

$\Rightarrow \rho = \Lambda$ indept of a!

$$\rho = \frac{C_m}{a^3} + \frac{C_r}{a^4} + \Lambda$$

G_{00}

$\Rightarrow \left(\frac{\dot{a}}{a}\right.$

ρ = dens

$\nabla_\mu T^{\mu\nu} = 0$

$$G_{00} = \frac{8\pi G}{c^4} T_{00}$$

Friedmann

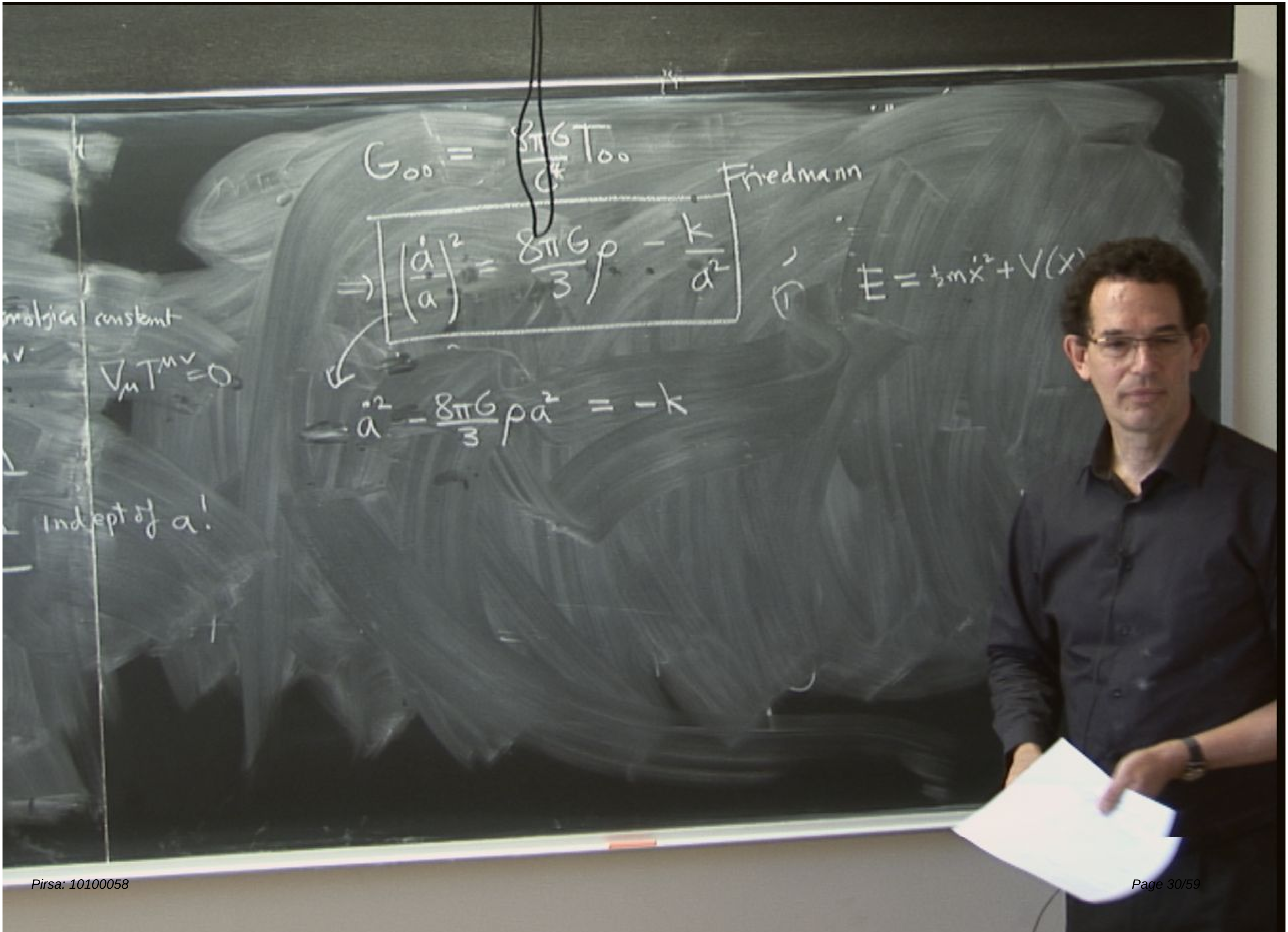
$$\Rightarrow \boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

$$\dot{a}^2 - \frac{8\pi G}{3}\rho a^2 = -k$$

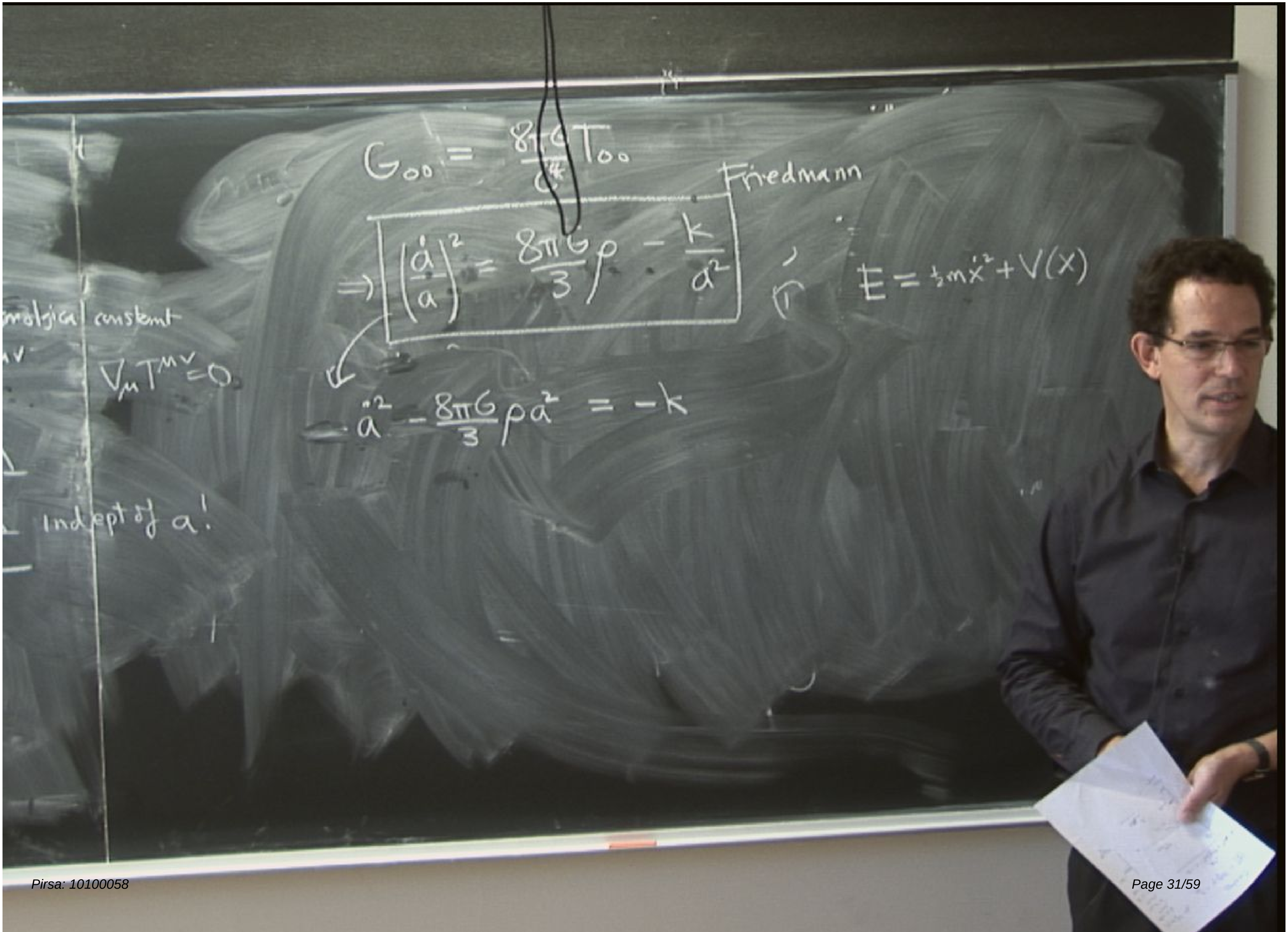
...mological constant

$\nabla_\mu T^{\mu\nu} = 0$

indept of a!



$G_{00} = \frac{8\pi G}{c^4} T_{00}$
Friedmann
$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}$
$E = \frac{1}{2}m\dot{x}^2 + V(x)$
molgical constant
$\nabla_\mu T^{\mu\nu} = 0$
$\dot{a}^2 - \frac{8\pi G}{3}\rho a^2 = -k$
indept of a!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mological constant
$\nabla_\mu T^{\mu\nu} = 0$
$\dot{a}^2 - \frac{8\pi G}{3}\rho a^2 = -k$
indept of a!

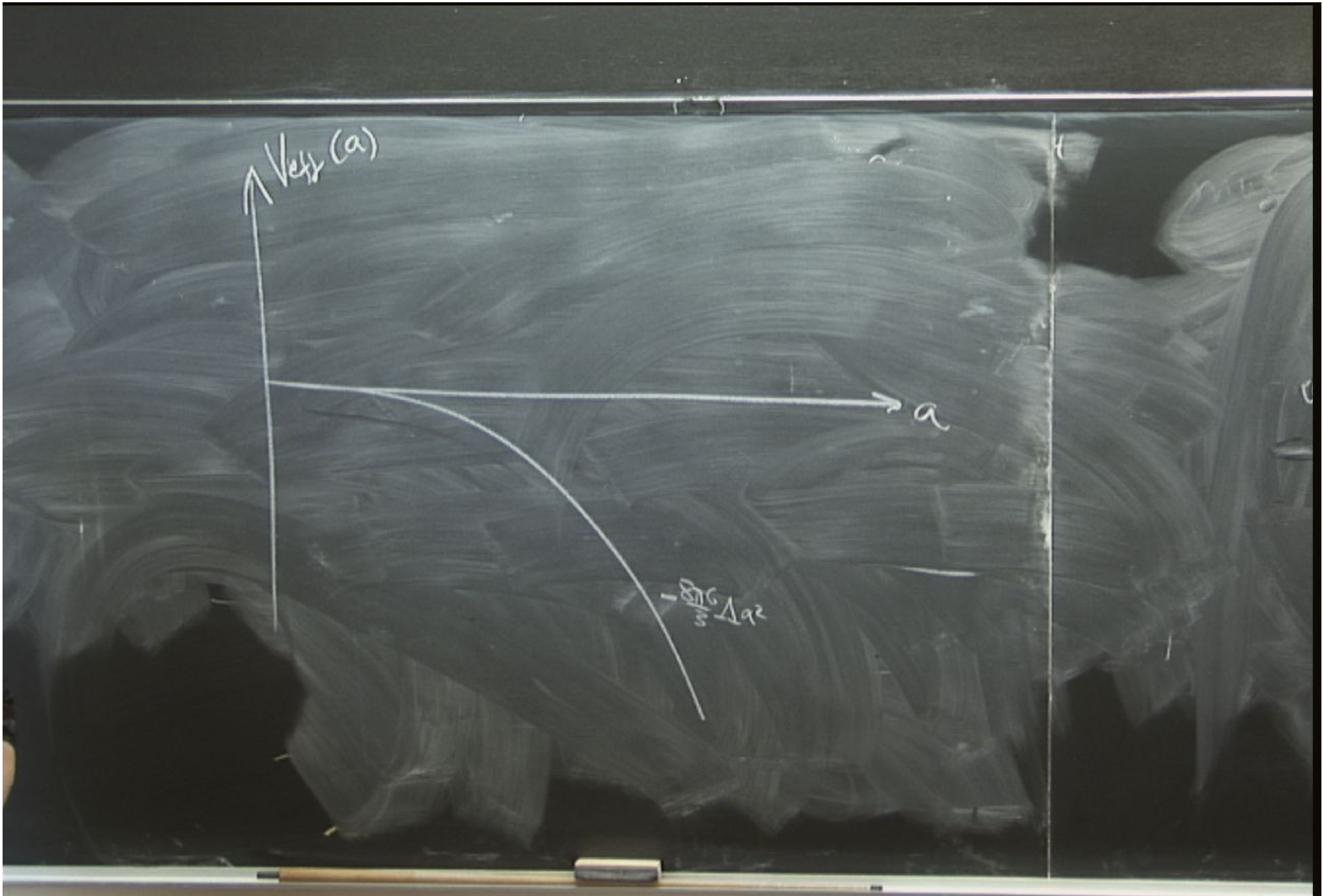
$$G_{00} = \frac{8\pi G}{c^4} T_{00}$$

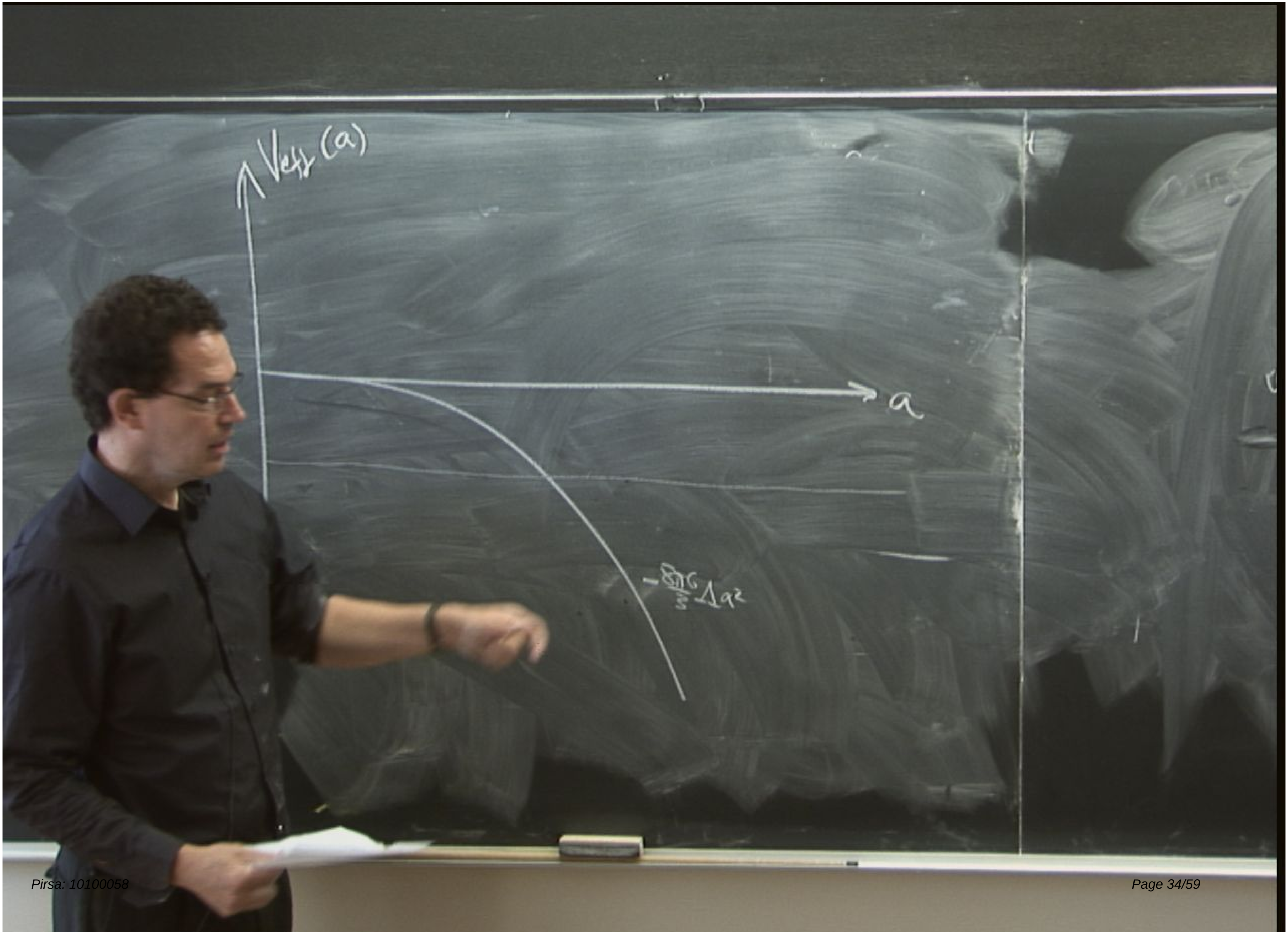
Friedmann

$$\Rightarrow \boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

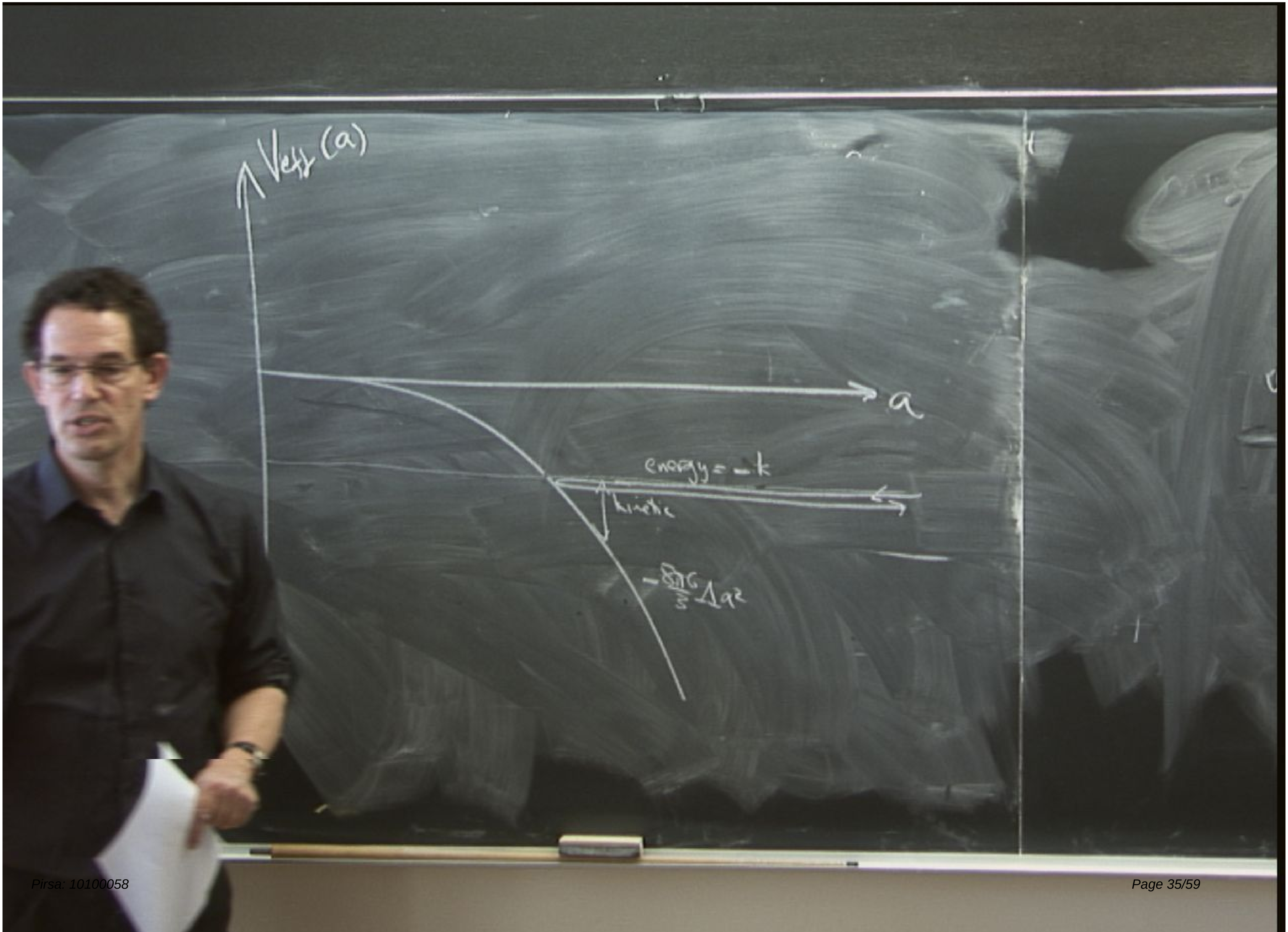
$$E = \tfrac{1}{2} m \dot{x}^2 + V(x)$$

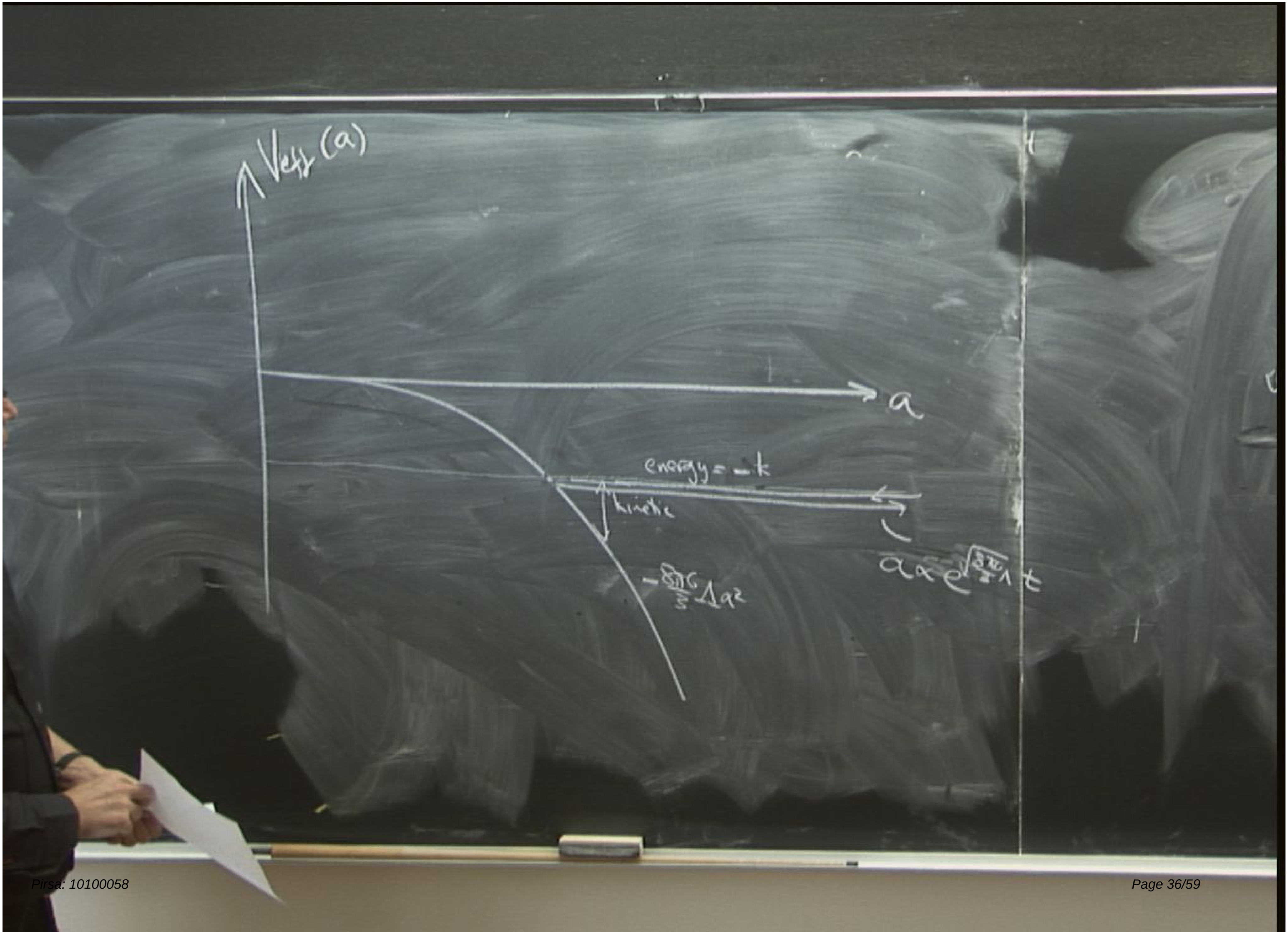
$$\dot{a}^2 \underbrace{- \frac{8\pi G}{3}\rho a^2}_{V_{eff}(a)} = -k$$


$V_{eff}(a)$
a
$-\frac{8\pi G}{3}\Delta a^2$

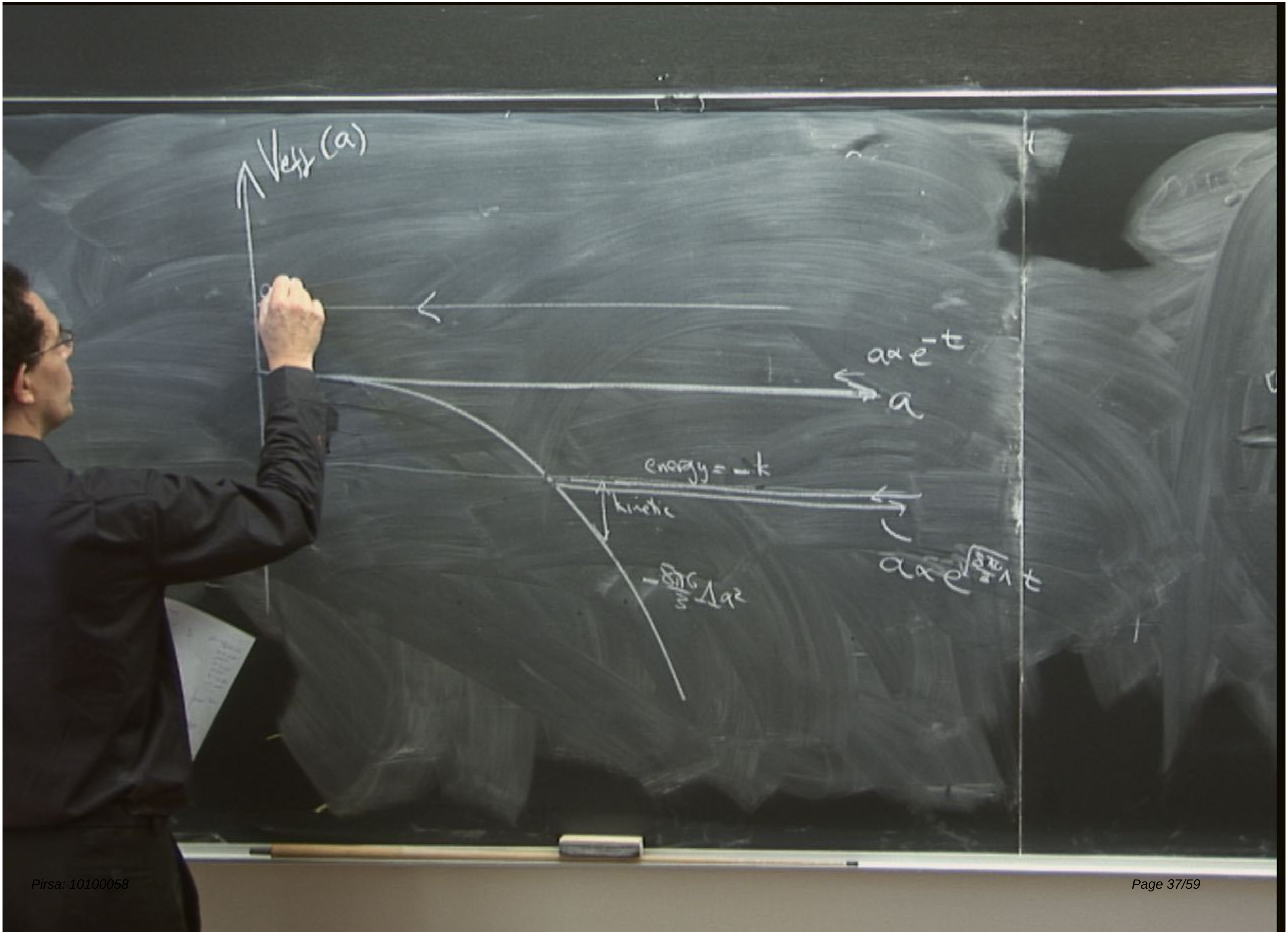


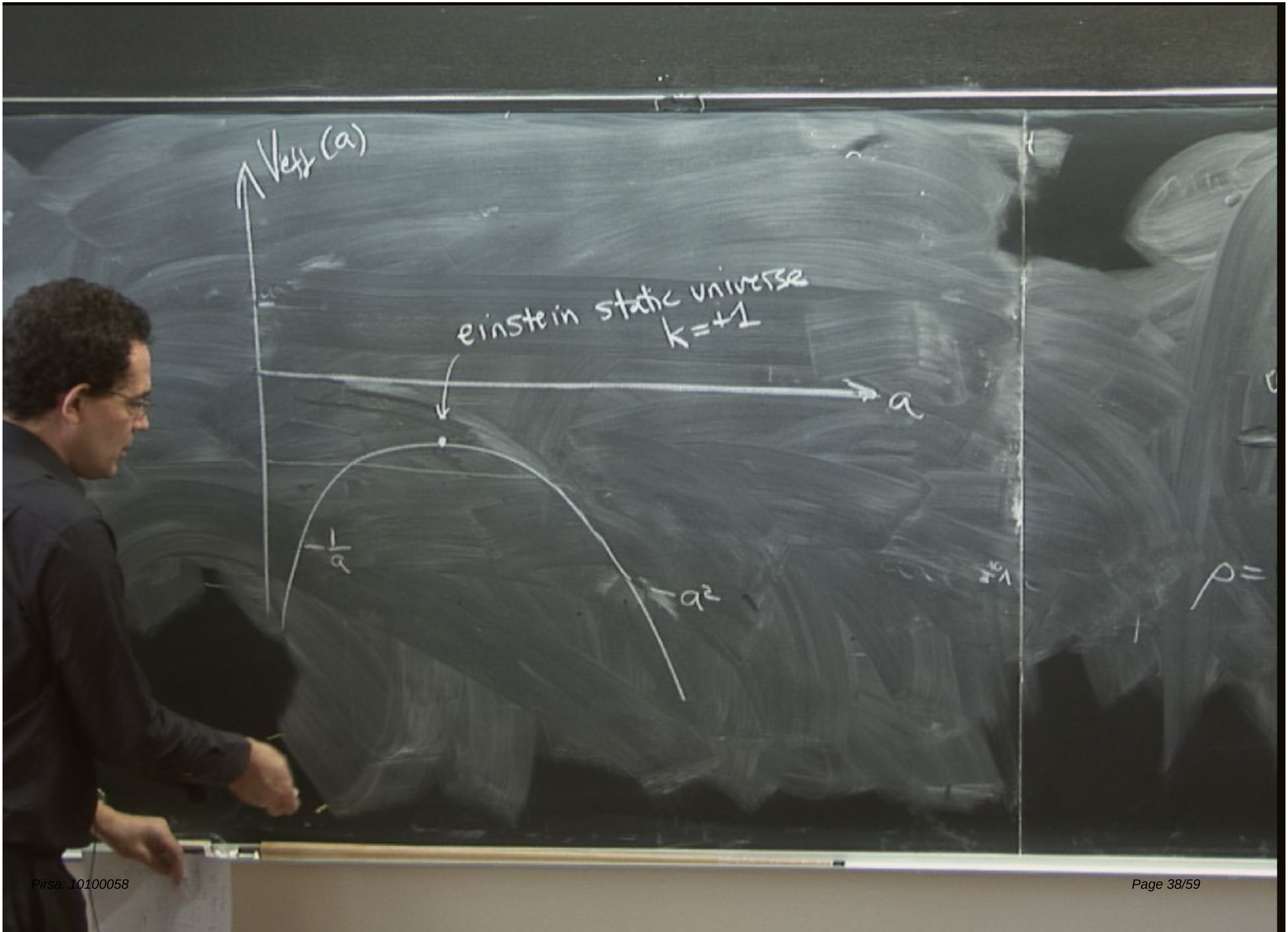
$V_{eff}(a)$
a
$-\frac{8\pi G}{3}\Lambda a^2$


$V_{eff}(a)$
a
energy = −k
kinetic
$-\frac{8\pi G}{3}\Delta a^2$

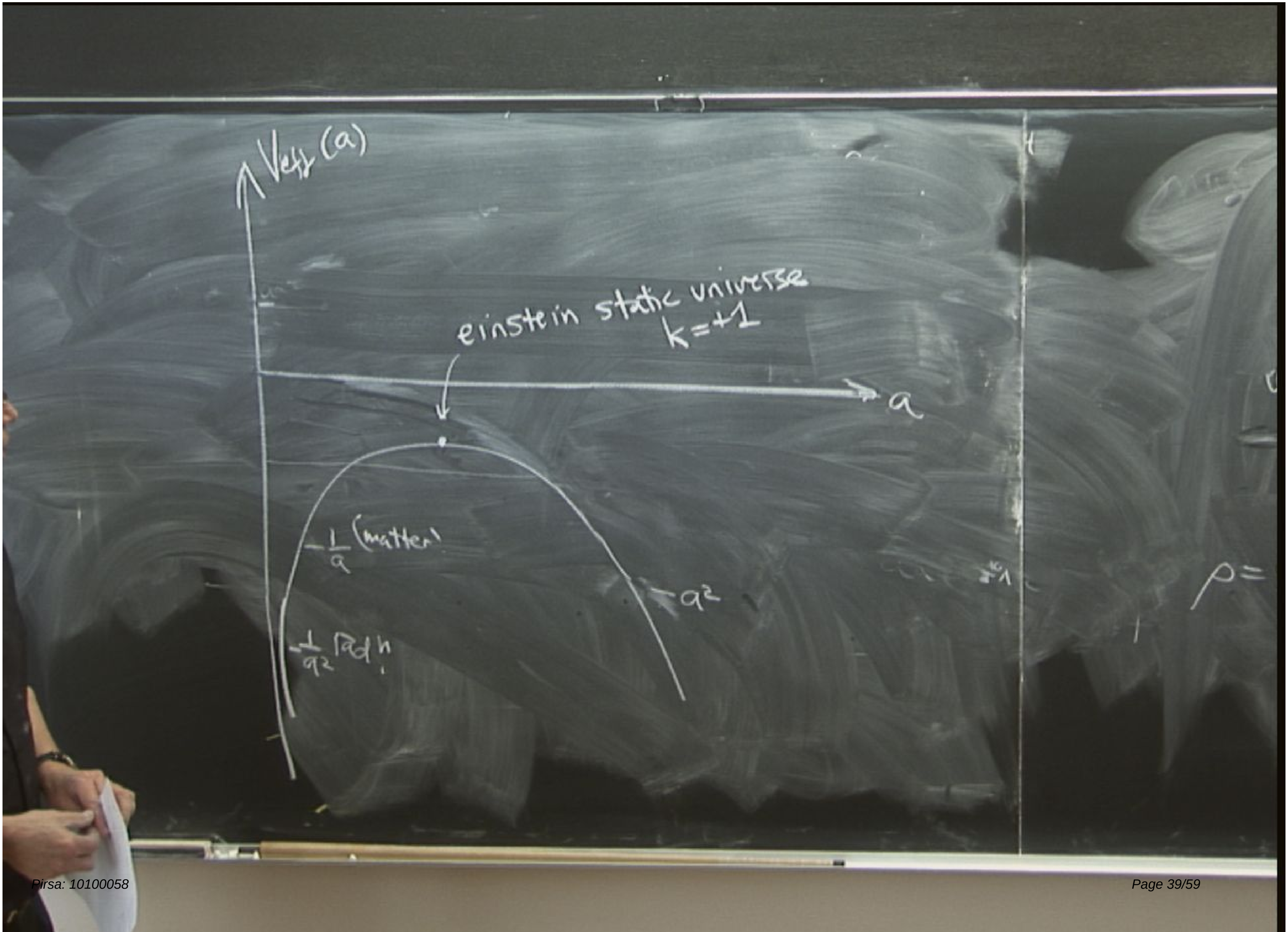


$V_{eff}(a)$
a
energy = −k
kinetic
$-\frac{8\pi G}{3}\Lambda a^2$
$a \propto e^{\sqrt{\frac{8\pi G}{3}\Lambda}\, t}$


$V_{eff}(a)$
$a \propto e^{-t}$
a
energy = $-k$
kinetic
$-\frac{8\pi G}{3}\Lambda a^2$
$a \propto e^{\sqrt{\frac{8\pi}{3}\Lambda}\, t}$
t



$V_{eff}(a)$
einstein static universe
$k=+1$
a
$-\frac{1}{a}$
$-a^2$


Veff (a)
einstein static universe
k=+1
a
-1/a (matter)
-a²
-1/a² rad'n
ρ=

our universe

$\rightarrow a$

$$G_{00} = \frac{8\pi G}{c^4} T_{00}$$

Friedmann

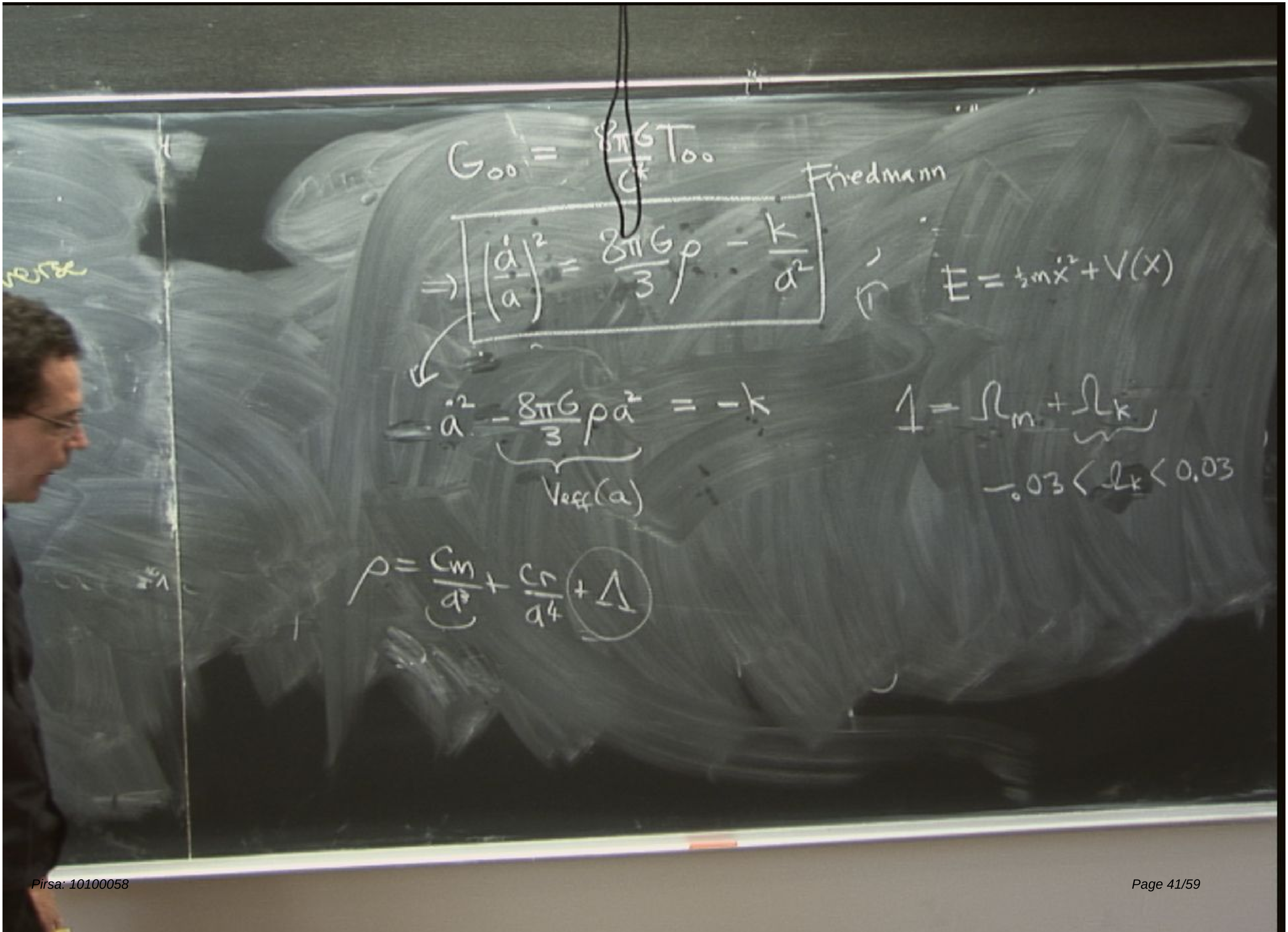
$$\Rightarrow \boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

$$E = \tfrac{1}{2}m\dot x^2 + V(x$$

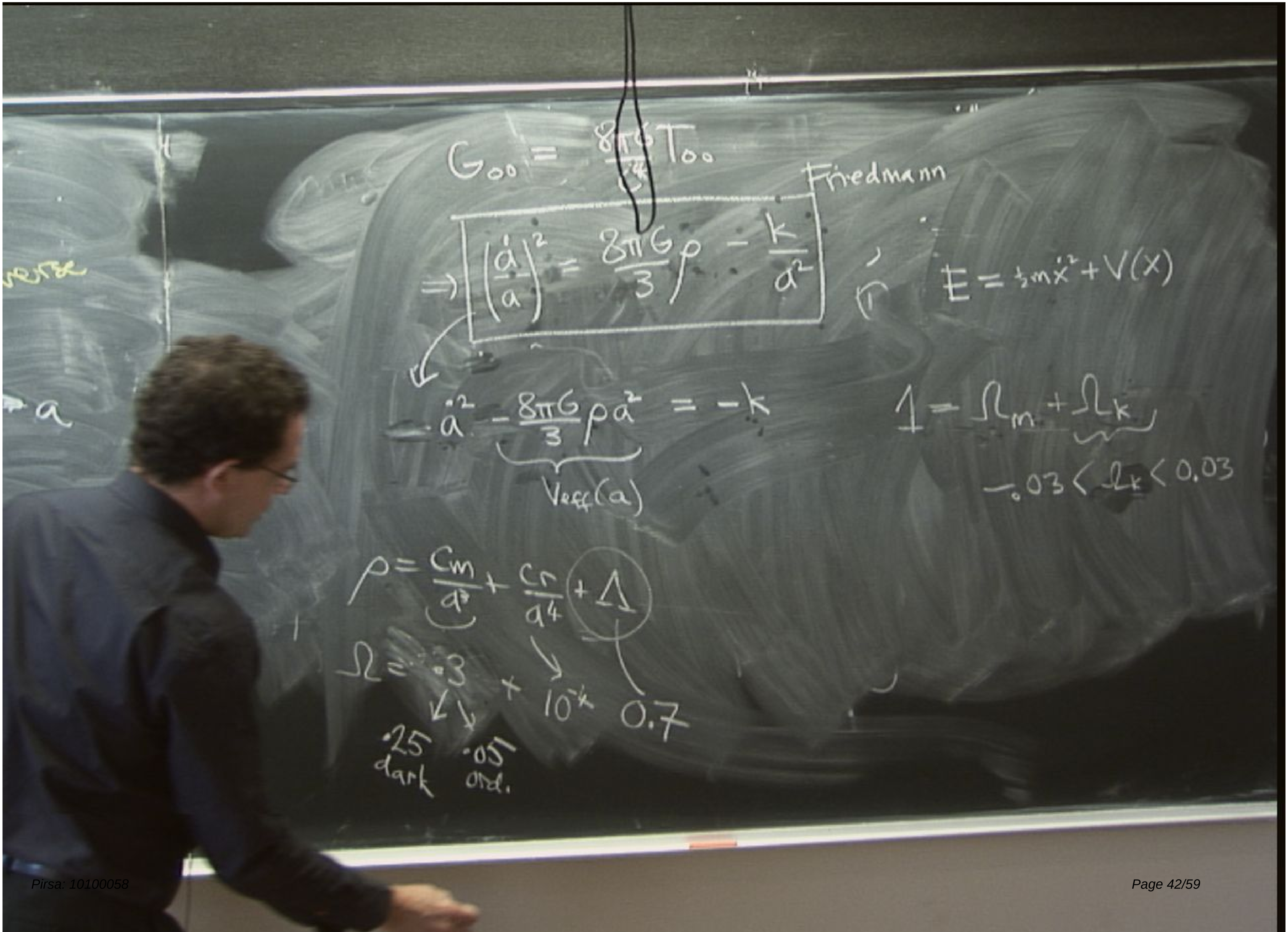
$$\dot a^2 - \underbrace{\frac{8\pi G}{3}\rho a^2}_{V_{eff}(a)} = -k$$

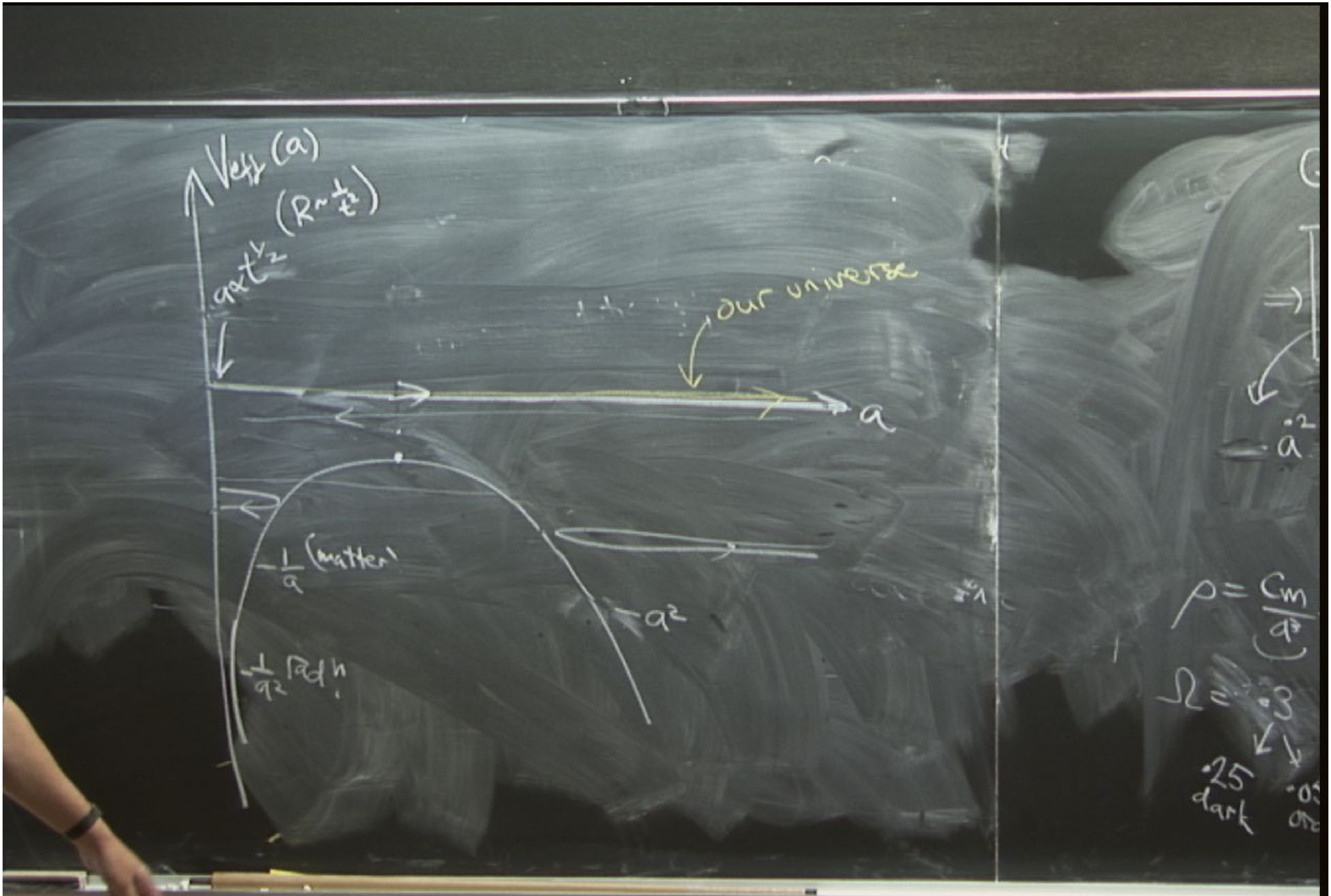
$$1 = \Omega_m + \Omega_k$$

$$\rho = \frac{c_m}{a^3} + \frac{c_r}{a^4} + \Lambda$$

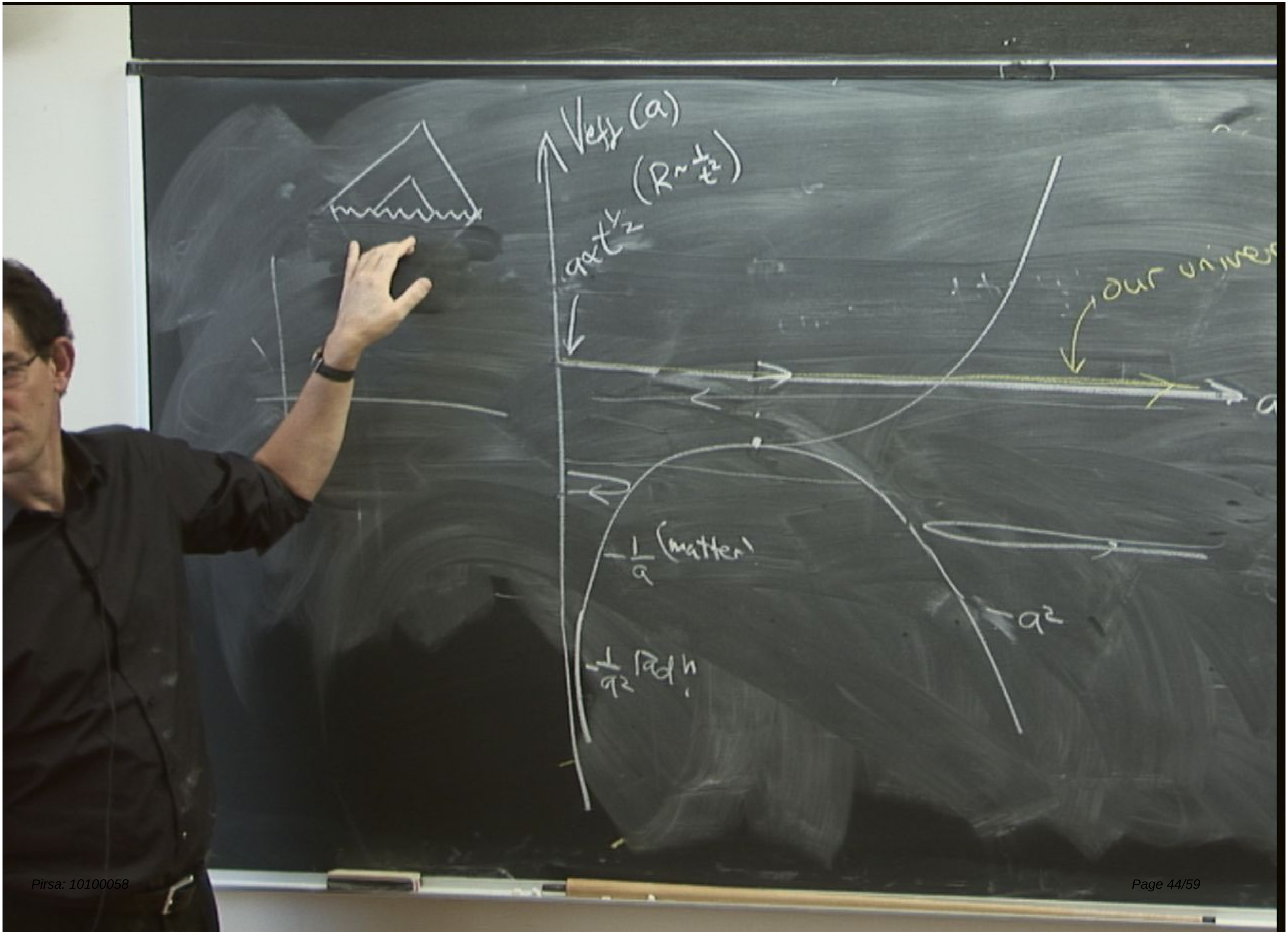


$G_{00} = \frac{8\pi G}{c^4} T_{00}$
Friedmann
$\Rightarrow \boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$
$E = \frac{1}{2}m\dot x^2 + V(x)$
$\underbrace{\dot a^2 - \frac{8\pi G}{3}\rho a^2}_{V_{eff}(a)} = -k$
$1 = \Omega_m + \Omega_k$
$-.03 < \Omega_k < 0.03$
$\rho = \frac{c_m}{a^3} + \frac{c_r}{a^4} + \Lambda$

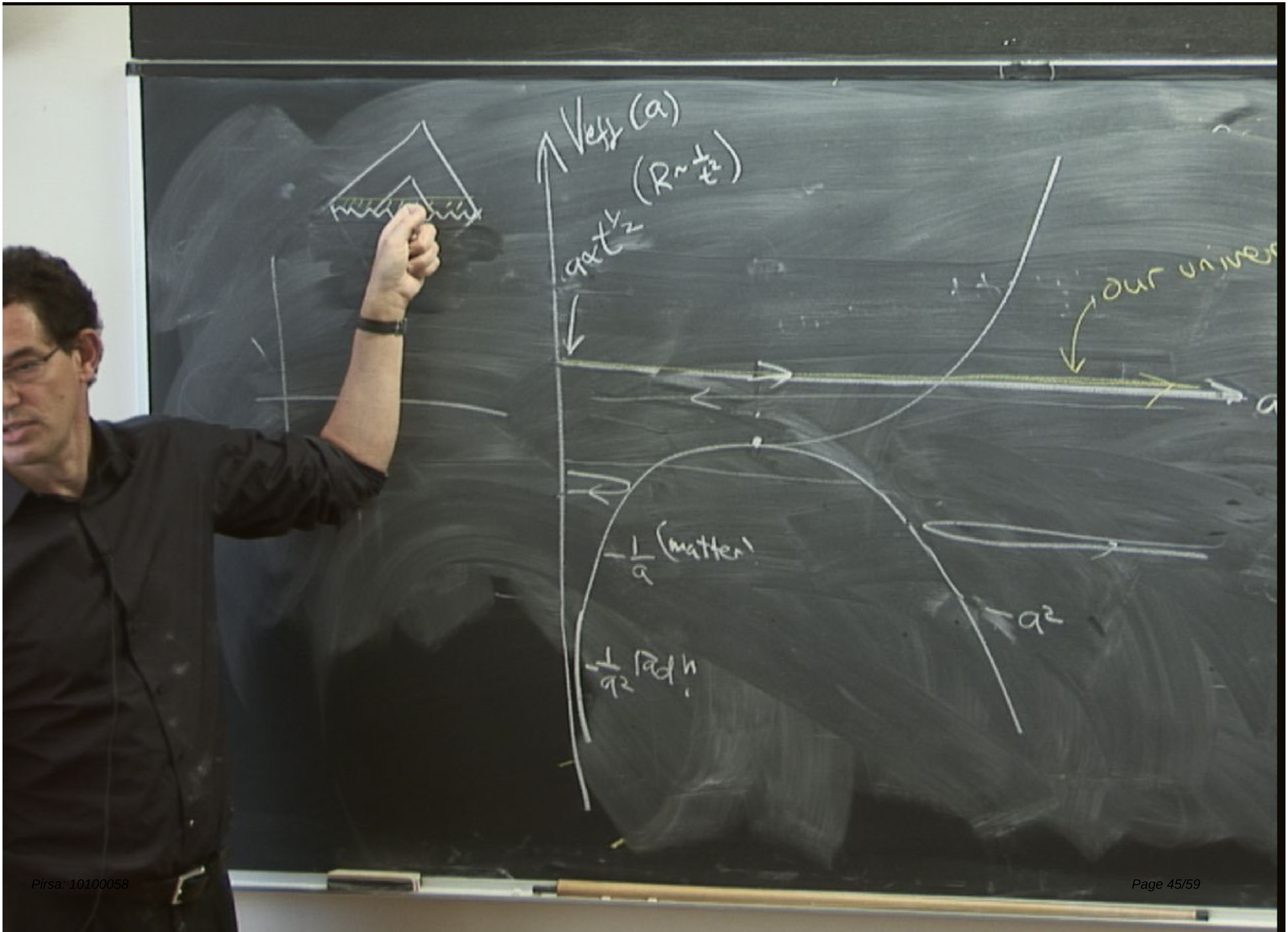

G00 = 8πG T00
Friedmann
(ȧ/a)² = 8πG/3 ρ − k/a²
E = ½mẋ² + V(x)
ȧ² − 8πG/3 ρa² = −k
Veff(a)
1 = Ωm + Ωk
−.03 < Ωk < 0.03
ρ = cm/a³ + cr/a⁴ + Λ
Ω = .3 + 10⁻⁴ 0.7
.25 dark
.05 ord.



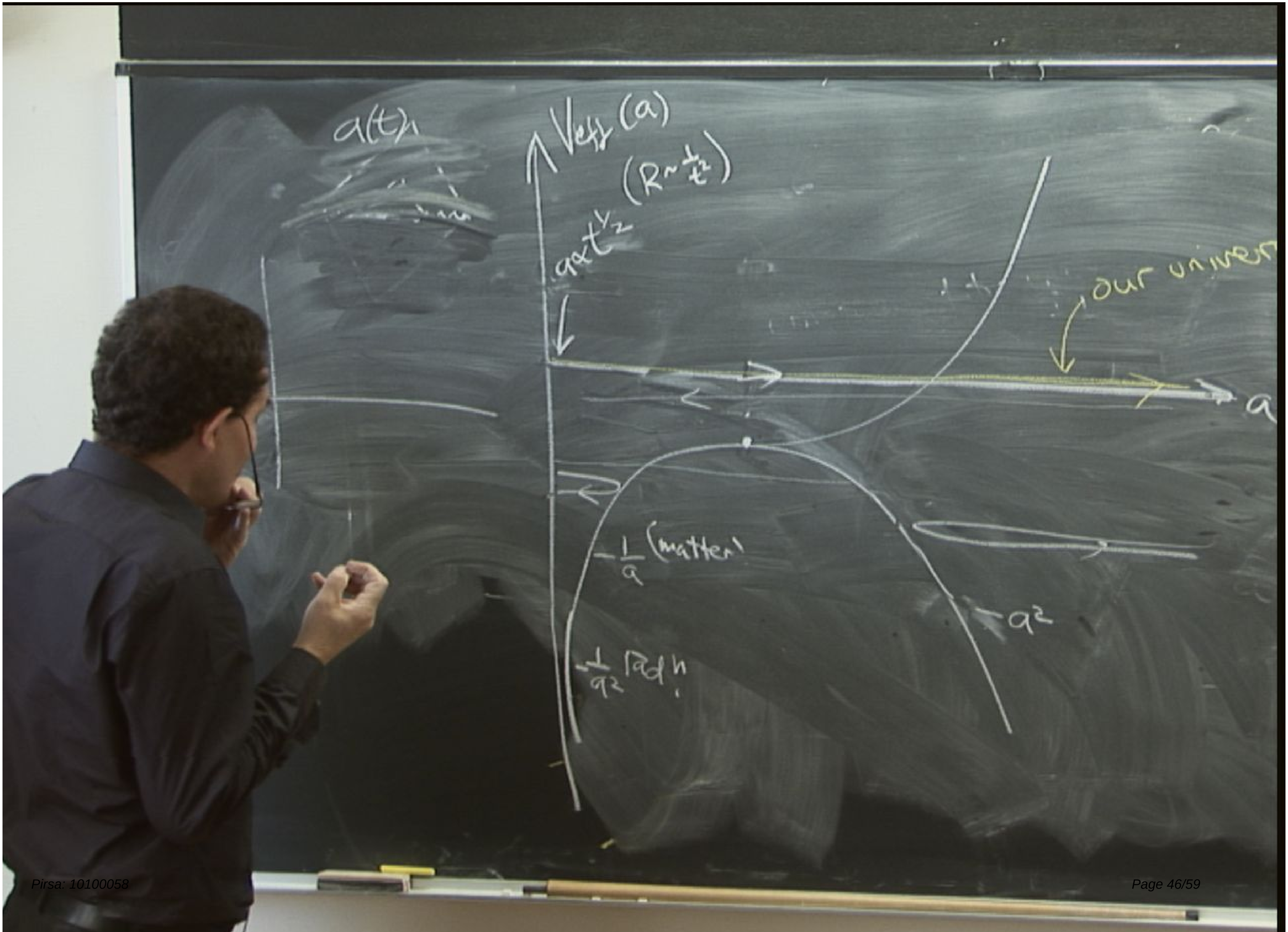
$V_{eff}(a)$
$(R \sim \frac{1}{t^2})$
$a \propto t^{1/2}$
our universe
a
$-\frac{1}{a}$ (matter)
$-a^2$
$-\frac{1}{a^2}$ rad'n
$\rho = \frac{C_m}{a^3}$
$\Omega = .3$
.25
dark



$V_{eff}(a)$
$(R \sim \frac{1}{t^2})$
$a \propto t^{1/2}$
our univer
a
$-\frac{1}{a}$ (matter)
$-a^2$
$-\frac{1}{a^2}$ rad'n



Veff (a)
(R~1/t²)
a∝t^1/2
our univer
a
-1/a (matter)
-1/a² rad'n
a²



a(t)
$V_{eff}(a)$
$(R \sim \frac{1}{t^2})$
$a \propto t^{1/2}$
our univers
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$-\frac{1}{a}$ (matter)
$-a^2$
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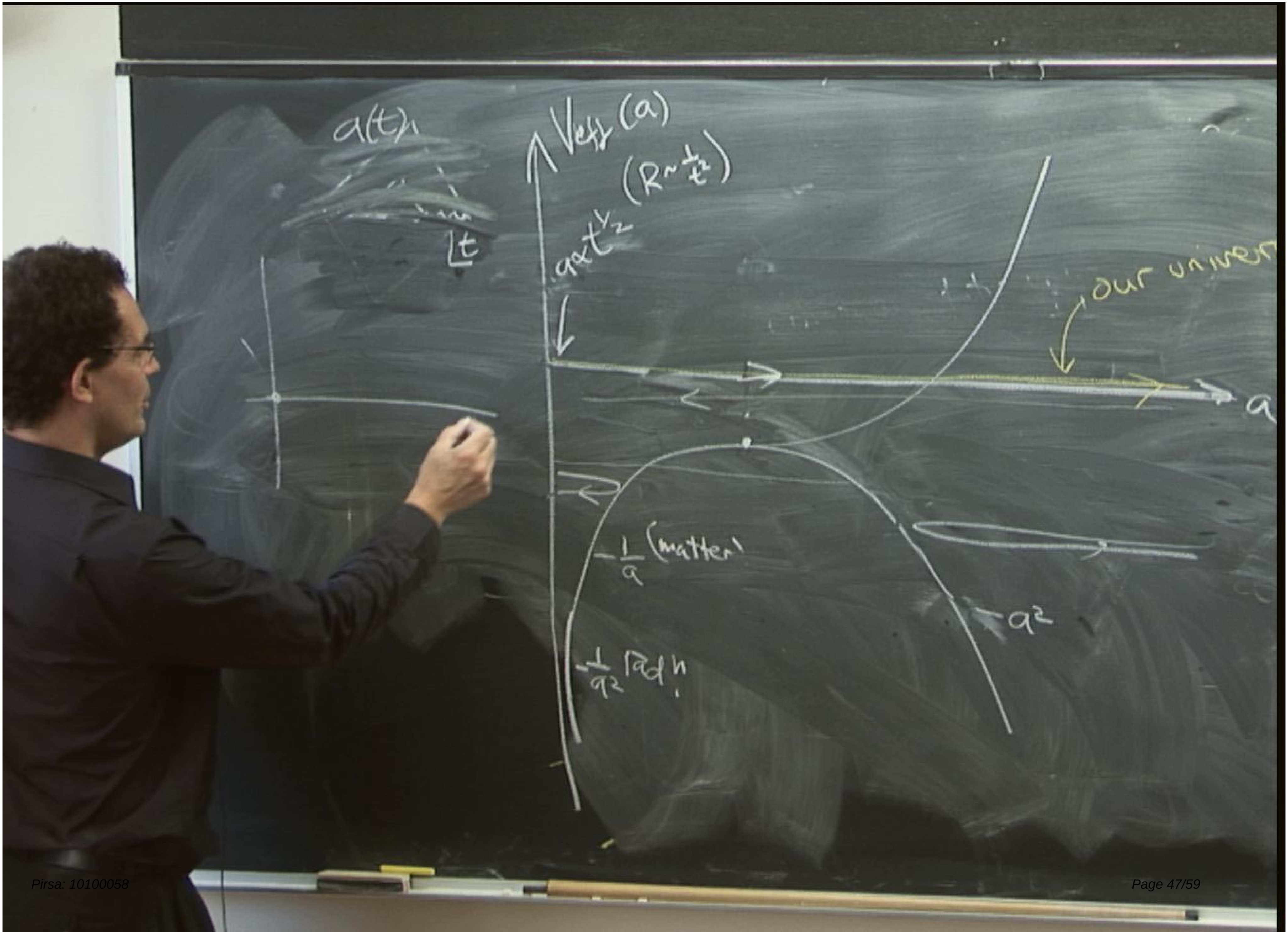
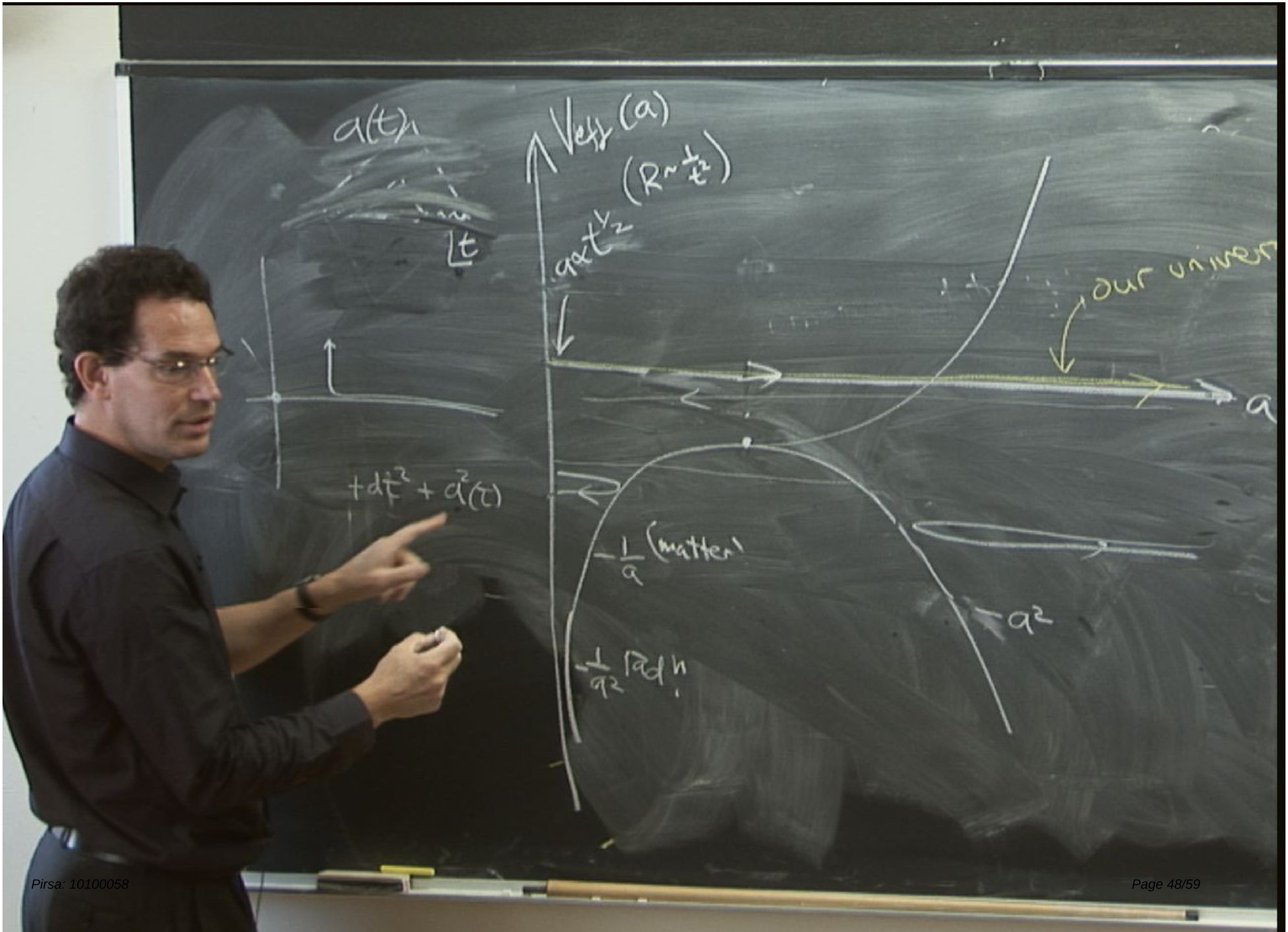
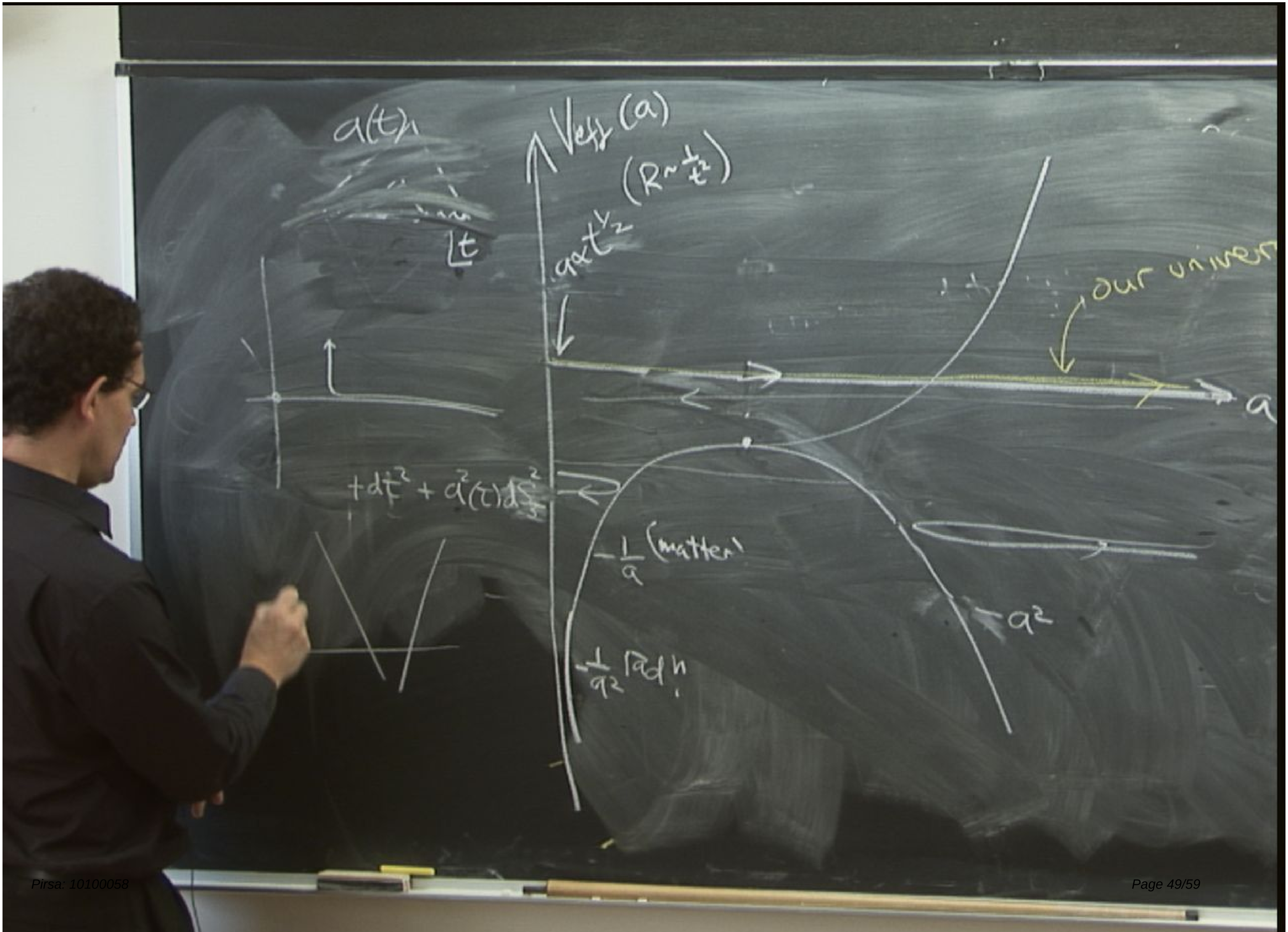
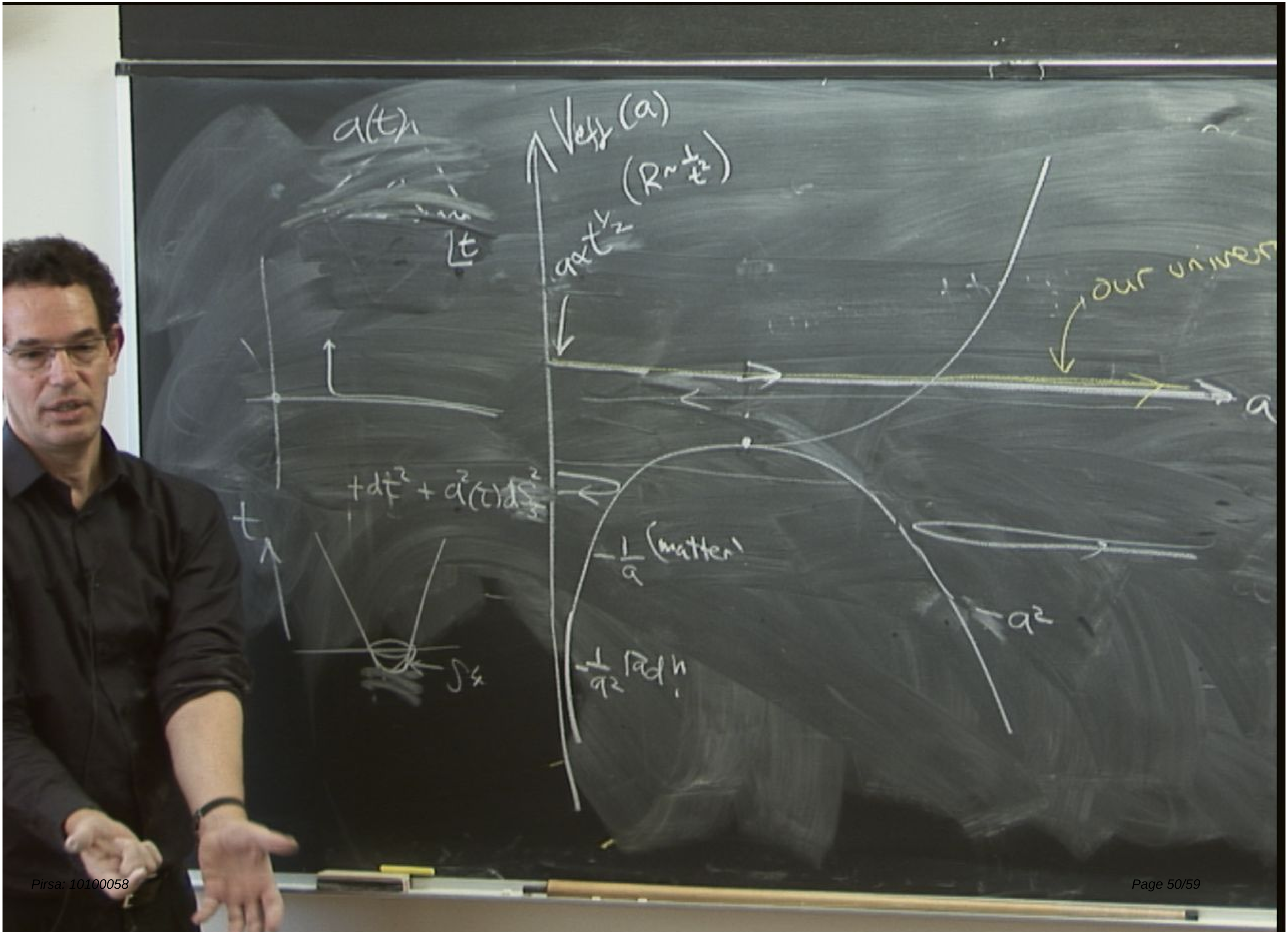

a(t)
Veff (a)
(R ~ 1/t²)
a ∝ t^1/2
t
our univer
a
-1/a (matter
-a²
-1/a² rad'n

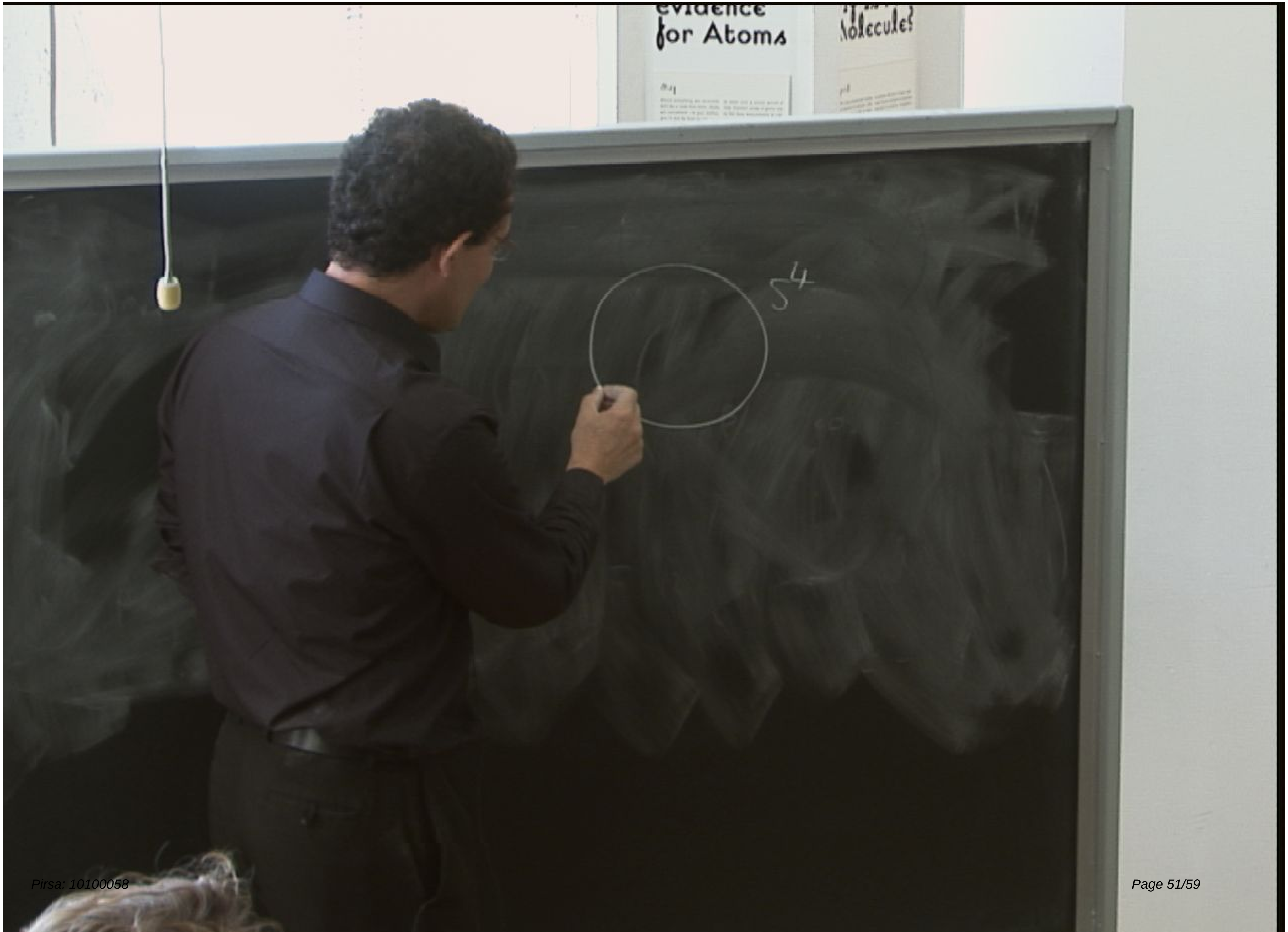


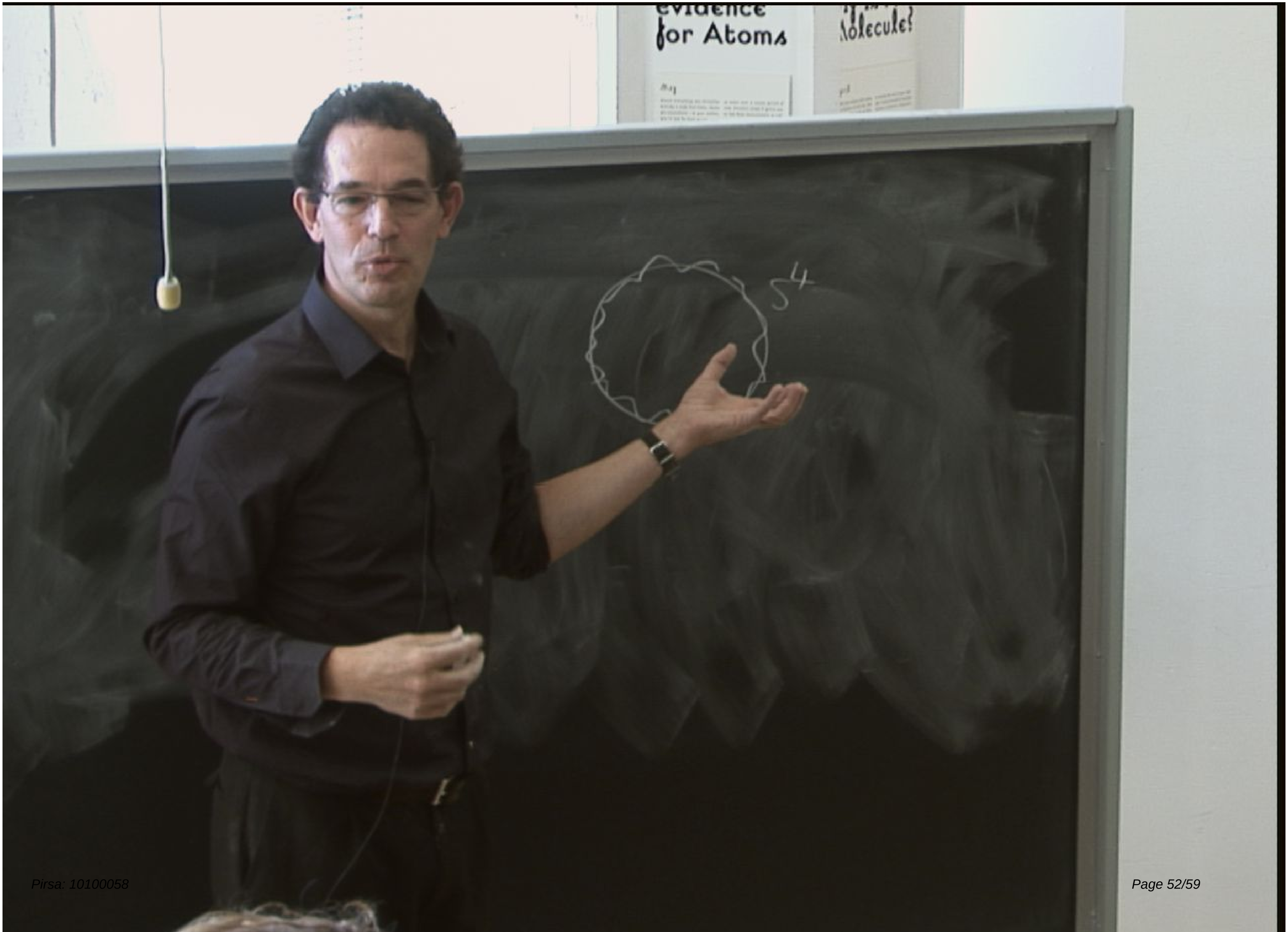
a(t)
t
a ∝ t^1/2
V_eff (a)
(R ~ 1/t²)
our univer
a
+dt² + a²(t)
−1/a (matter)
−1/a² rad'n
−a²

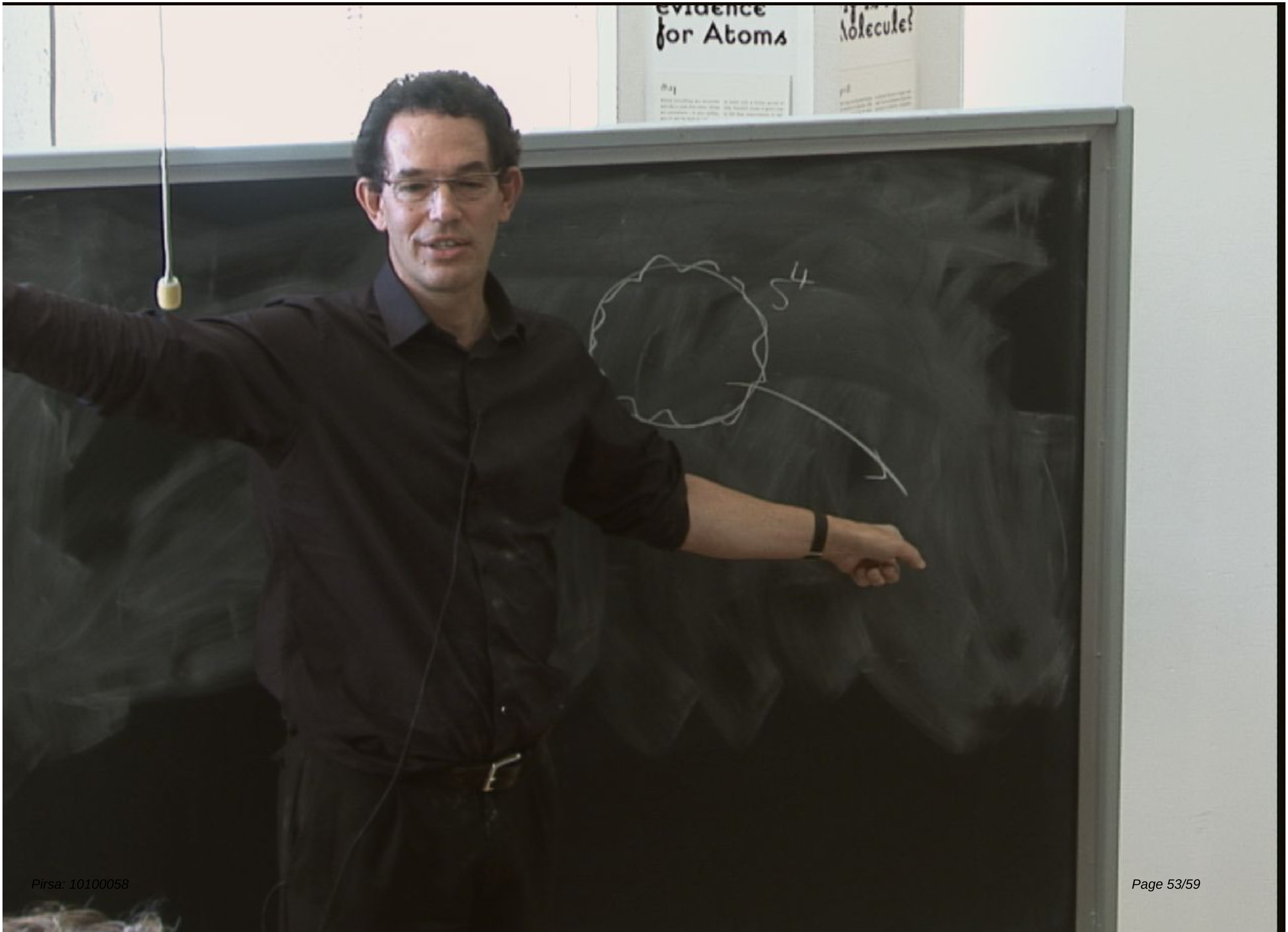

a(t)
t
Veff (a)
(R~1/t²)
a∝t^1/2
our univer
a
+dt² + a²(τ)dS₃²
-1/a (matter)
a²
-1/a² rad'n

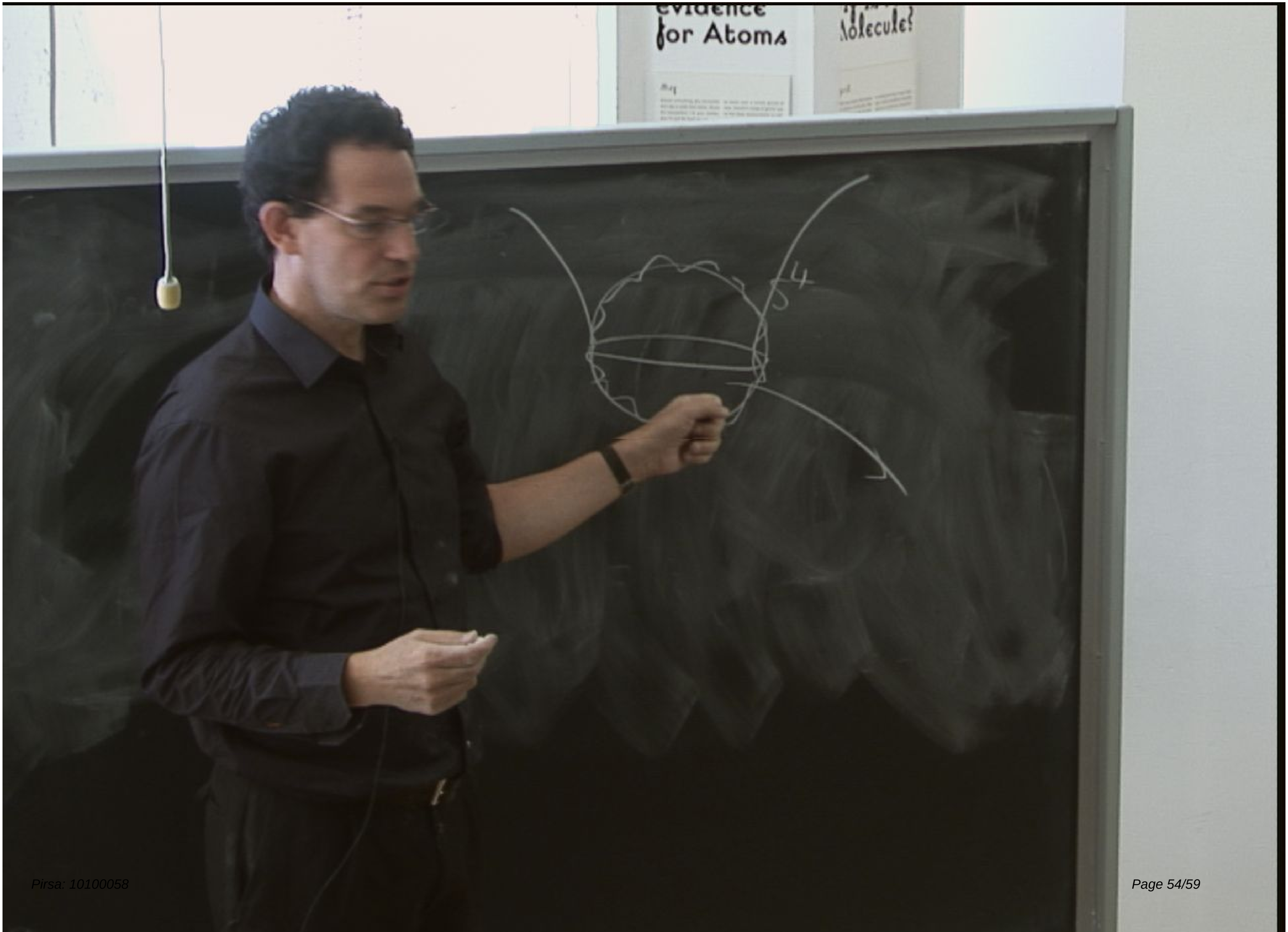


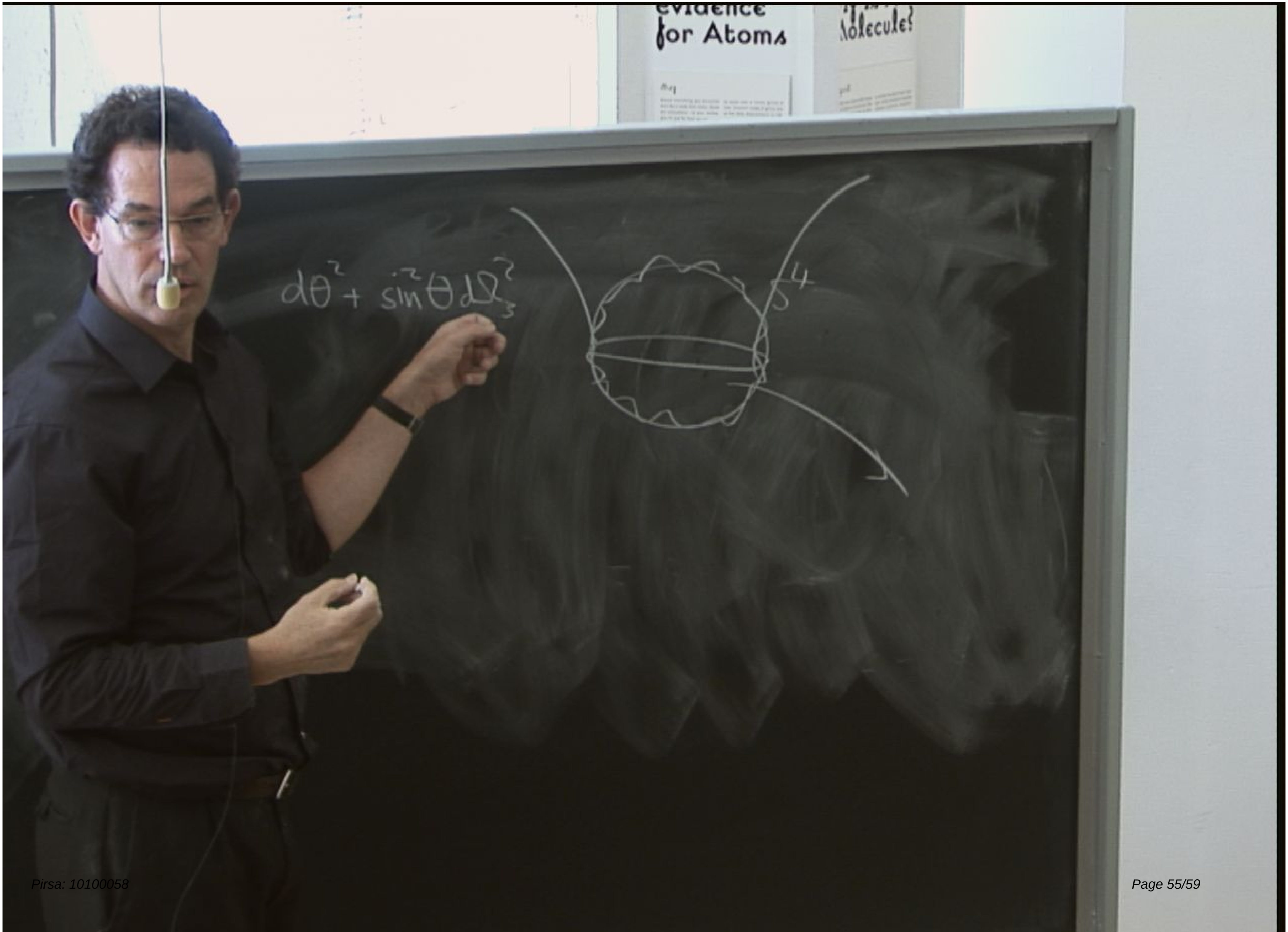
a(t)
t
Veff (a)
(R ~ 1/t²)
a ∝ t^½
our univer
a
+dt² + a²(t)dΩ²
t
−1/a (matter)
−a²
−1/a² rad'n

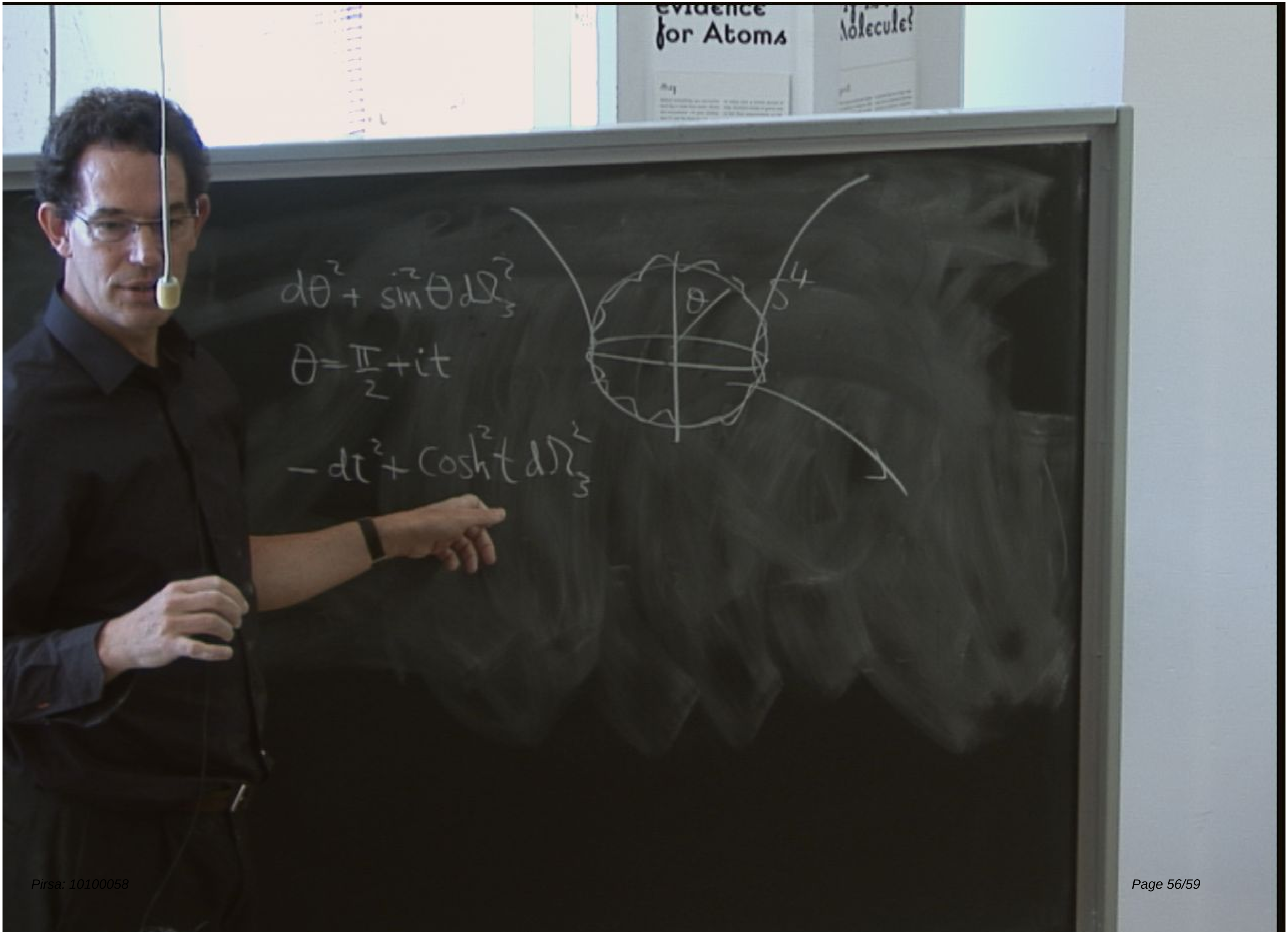

evidence
for Atoms
S^4


evidence
for Atoms
S^4


evidence
for Atoms
S^4


evidence
for Atoms
Molecule?
4

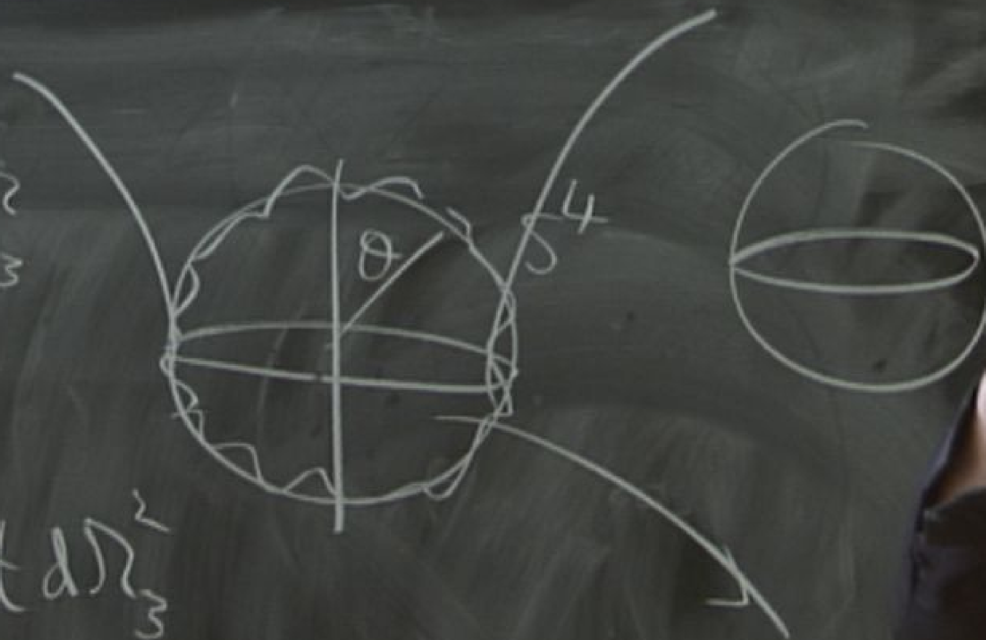

for Atoms
$d\theta^2 + \sin^2\theta \, d\Omega_3^2$
S^4

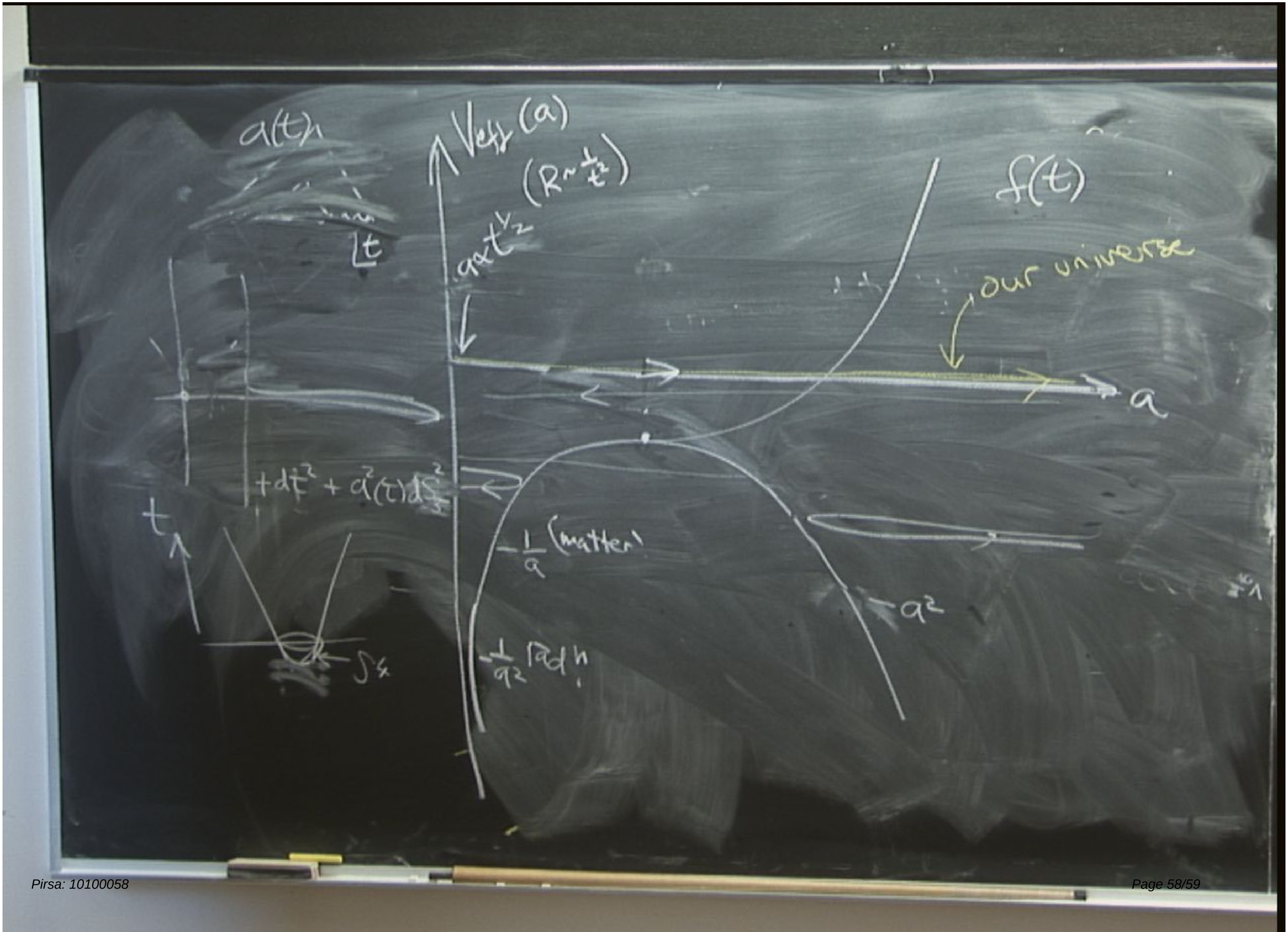

evidence
for Atoms
Molecule?
$d\theta^2 + \sin^2\theta \, d\Omega_3^2$
$\theta = \frac{\pi}{2} + it$
$-dt^2 + \cosh^2 t \, d\Omega_3^2$
θ
S^4

$$d\theta^2 + \sin^2\theta \, d\Omega_3^2$$

$$\theta = \frac{\pi}{2} + it$$

$$-dt^2 + \cosh^2 t \, d\Omega_3^2$$



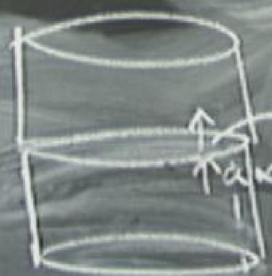

a(t)
Veff (a)
(R ~ 1/t²)
a ∝ t^1/2
f(t)
our universe
a
+dt² + a²(t)dΩ₃²
t
-1/a (matter)
-a²
-1/a² rad'n

$f(t)$

our universe

$a \propto e^{Ht}$

a



$$G_{00} = \frac{8\pi G}{c^4} T_{00}$$

Friedmann

$$\boxed{\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}}$$

$E = \frac{1}{2} m \dot x$

$$\dot a^2 - \underbrace{\frac{8\pi G}{3}\rho a^2}_{V_{eff}(a)} = -k$$

$1 = \Omega_m +$

$.03$

$$\rho = \frac{c_m}{a^3} + \frac{c_r}{a^4} + \Lambda$$

$$\Omega = .3 + 10^{-4} + 0.7$$

.25 dark

.05 ord.