Title: Why is the generalized second law true?

Date: Oct 12, 2010 02:00 PM

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Abstract: The entropy outside of an event horizon can never decrease if one includes a term proportional to the horizon area. For a long time, this astonishing result had only been shown for quantum fields that are in an approximately steady state. I will describe a new proof of the generalized second law for arbitrary slices of semiclassical, rapidly-changing horizons. I will start with the simplest case, Rindler horizons, and then describe how the proof can be adapted to other cases (black holes, de Sitter, etc.) by restricting the field algebra to the horizon. The generalized second law holds because the horizon is invariant under a larger symmetry group than the rest of the spacetime.

Why is the Generalized Second Law True?

by Aron Wall,
Center for Fundamental Physics
University of Maryland

This work was aided by discussions with
Ted Jacobson, Rafael Sorkin, and William Donnelly.

The Second Law of Thermodynamics

$$\frac{dS}{dt} \geq 0$$

for any CLOSED system.

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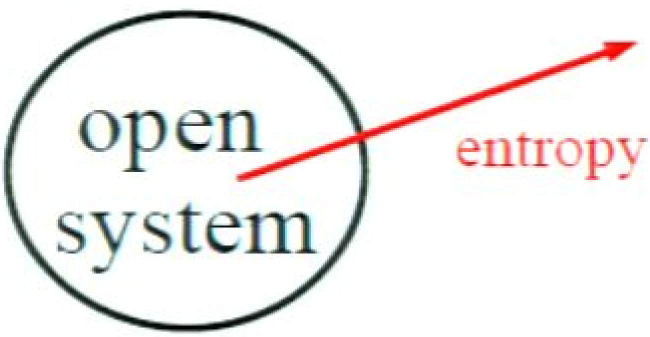

In an OPEN system entropy can normally just exit, lowering S(system)

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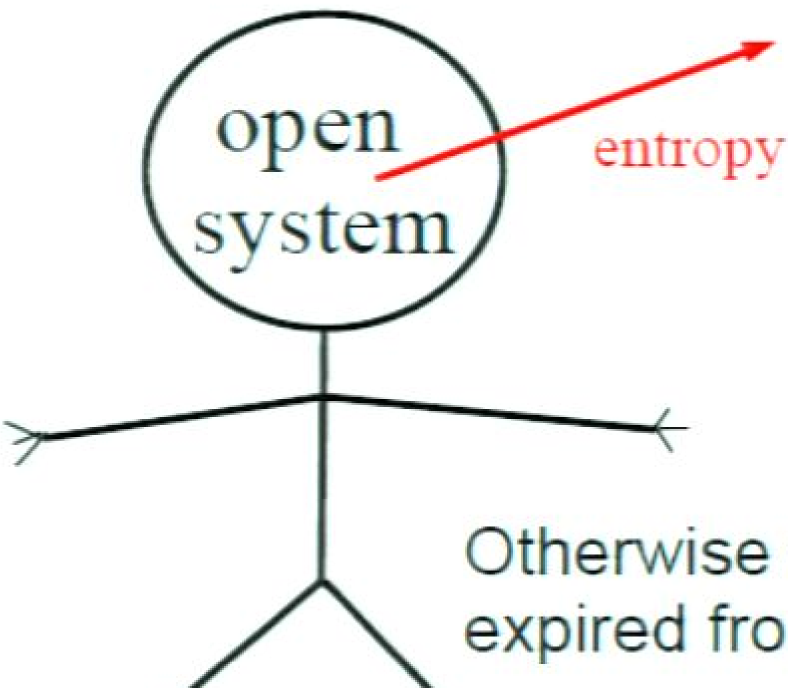


In an OPEN system entropy can normally just exit, lowering S(system)

Otherwise we would already have expired from personal heat death...

Causal Horizons

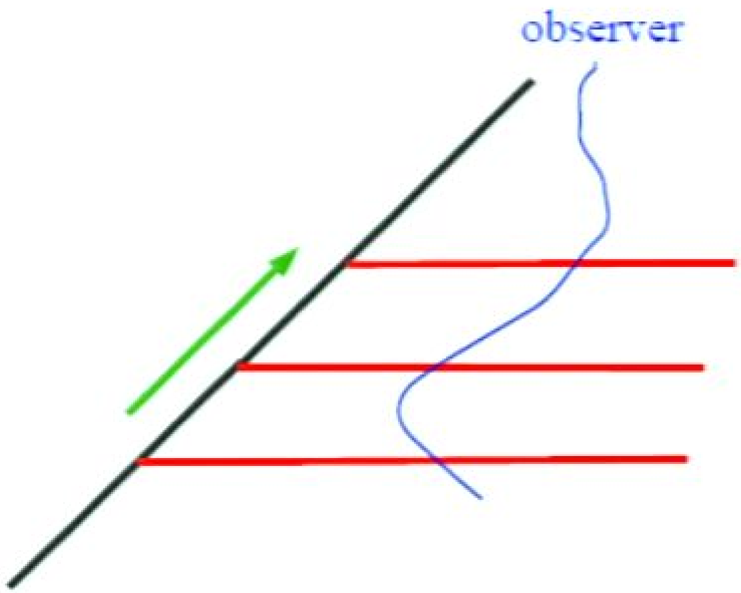
Causal horizon = boundary of past of any future infinite worldline.
(an "observer")

"Outside" means the side with the worldline.

Examples:

Horizon Thermodynamics

The outside of a horizon is an OPEN system—
info can leave (but not enter).



But the generalized entropy

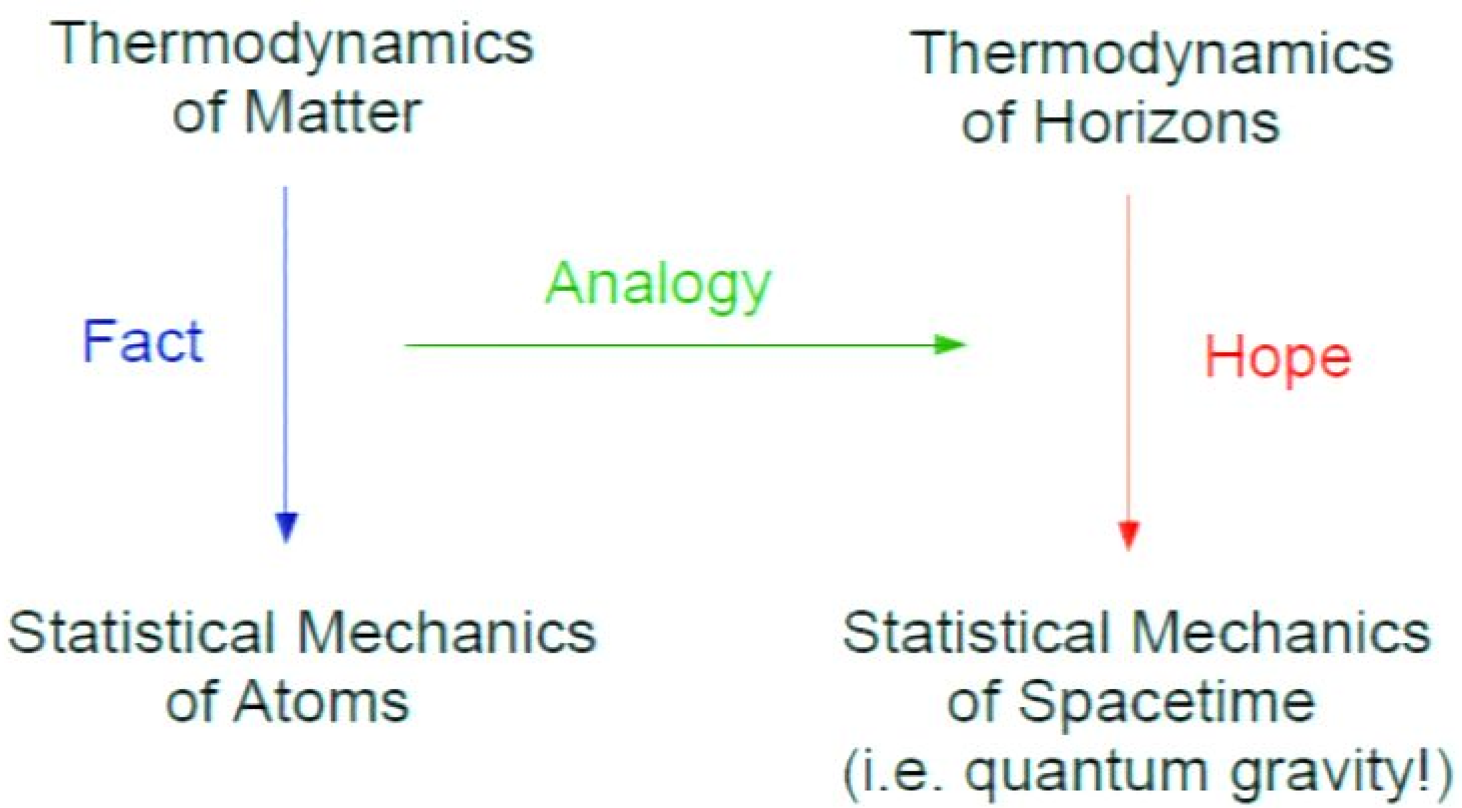
$$S_{\text{gen}} = \frac{A}{4\hbar G} + S_{\text{out}}$$

still increases. Area A is included as boundary term:

$$\frac{dS_{gen}}{dt} \geq 0$$

Generalized Second Law (GSL).

Why study the GSL?



My view: Too early to take sides about correct quantum gravity theory.

I want to find out what horizon thermodynamics implies about micro degrees of freedom without being constrained to a specific model.

Semiclassical approximation

1. Pick a classical background and look at QFT state ρ.

2. Expand out metric in powers of $\hbar$:

$$g_{ab} = g^{0}_{ab} + g^{1/2}_{ab} + g^{1}_{ab} + \mathcal{O}(\hbar^{3/2})$$

classical background metric

quantized gravitons

gravitational fields of matter/gravitons calculated using expectation of Einstein equation

$$\langle G_{ab} \rangle = 8\pi G \langle T_{ab} \rangle$$

3. See if the generalized entropy of the state ρ increases:

$$\frac{dS_{\mathrm{gen}}}{dt} = \frac{d}{dt}\left[\frac{\langle A \rangle}{4\hbar G} - \mathrm{tr}(\rho \ln \rho)\right] \geq 0$$

Entanglement entropy divergence must be renormalized!
This will be done by considering only entropy *differences*

Why try to prove the semiclassical GSL?

1) If we know *why* the GSL is true semiclassically,
may be able to figure out what is required at the Planck scale.

2) We want to know *when* the GSL applies:

* Can it be proven in a way which is local on the horizon?

* Does it hold in a differential form at each horizon point,
or only globally for stationary —> stationary processes?

* What does the GSL assume (if anything) about
matter (e.g. number of fields) and gravity (e.g. derivative couplings)

* Is there a generalization to null surfaces which aren't horizons?

3) We understand QFT and GR better than quantum gravity.

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Proving the Semiclassical GSL

Don't Read This Slide

Instead read "Ten Proofs of the Generalized Second Law" (09):
A.C. Wall, arXiv:1007.1493, JHEP 06 (2009) 021

PROOF BY	REGIME	PERTURB.	EXTRA CONDITIONS / DIFFICULTIES
Hawking	classical	any	null energy condition, cosmic censorship
Zurek & Thorne	semiclassical	quasi-steady*	entropy localization, renormalizability
Wald	hydrodynamic	quasi-steady*	adiabaticity (fixable)
Frolov & Page	semiclassical	quasi-steady*	CPT insufficient for charged BH (fixable)
Sorkin 1	quantum gravity	any	inconsistent assumptions
Sorkin 2	semiclassical	quasi-steady*	thermality undefined, not superradiant
Mukohyama	semiclassical	quasi-steady*	free scalar field, not superradiant
Flanagan et al.	hydrodynamic	any	null energy condition, Bekenstein bound
Bousso et al.	hydrodynamic	any	entropy gradient, isolation assumption
Fiola et al.	semiclassical	any	RST model, large N, apparent horizon

classical: assumes a classical background and neglects the outside entropy term.
hydrodynamic: the entropy is approximated by a fully localizable four-vector.
semiclassical: neglects fluctuations in the metric.
quasi-steady: a small, slowly changing perturbation to a stationary black hole.

*may be a rapidly changing small pertrubation, to only prove entropy increase from start to finish.

Quasi-Steady Approximation

Before my work, the only semiclassical proofs of the GSL

$$\frac{dS_{gen}}{dt} \geq 0$$

were for fields in a (nearly) steady state.

This allows use of the "First Law": $dE = TdS$

but then one can't check whether the GSL holds at each instant of time —need to check this with rapidly changing fields. E.g. throwing a tea-cup into a black hole.

Would like to prove the GSL at every spacetime point on the horizon. Like integral vs. differential form of Maxwell's equations:

$$\oint E \cdot n = Q \quad \rightarrow \quad \nabla \cdot E = \rho$$

Proof of the GSL for rapidly changing fields

1. Applies to rapidly evolving semiclassical perturbations to any stationary background horizon (e.g. Rindler, de Sitter, Kerr).

2. Proves $\Delta S_{\text{gen}} \geq 0$ for arbitrary initial and final slices of the horizon.

3. Works for free fields of any spin. Can also accommodate interactions at the level of naïve perturbation theory (within the Fock space)

start with special case: Rindler horizons.

Rindler GSL proof summary

"A proof of the generalized second law
for rapidly evolving Rindler horizons"
A.C, Wall arXiv:1007.1493 (2010)

Basic ideas:

1. Relate generalized entropy to free (boost) energy in wedge.
2. Relate free energy to a quantity known as "relative entropy"
3. Apply theorem that says relative entropy can't increase.

free boost energy — relative entropy

$$S_{\text{gen}} = (TS_{\text{out}} - K)/T = -S(\rho\,|\,\sigma)$$

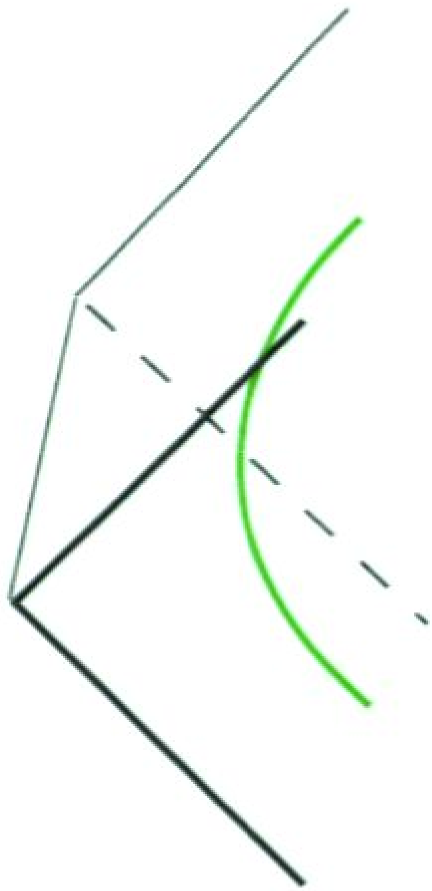
(up to additive constants)

Previous proofs of the GSL implicitly used the concept of relative entropy (Casini 08)

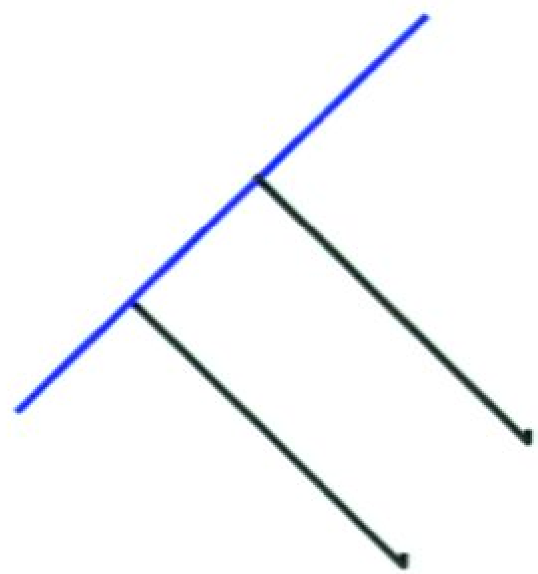
Rindler Wedges

A Rindler wedge is the intersection of the
past & future of uniformly accelerating worldline.

1-parameter family of Rindler wedges
share same future horizon & fit inside each other.



perspective drawing of wedge
& accelerating observer

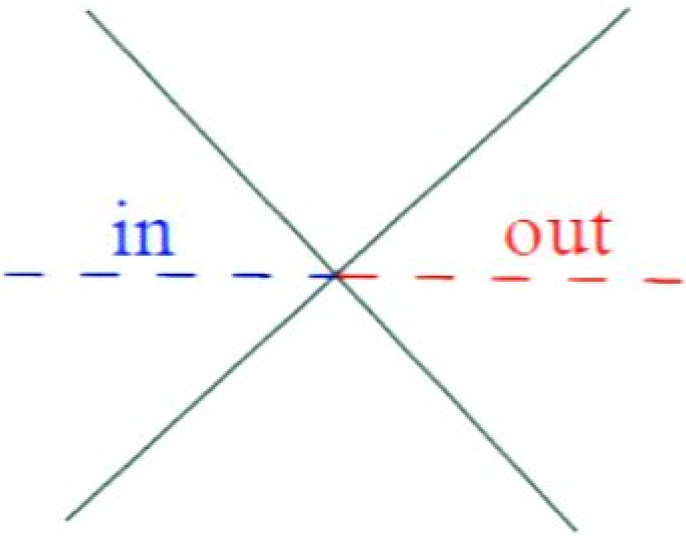


Go from bigger wedge to smaller wedge by restriction..

Rindler Wedges are Thermal

QFT vacuum always KMS (i.e. thermal) in boost energy *K* when restricted to Rindler wedge, at temperature $T = \hbar/2\pi$

Consequence of the wedge's *boost symmetry*.



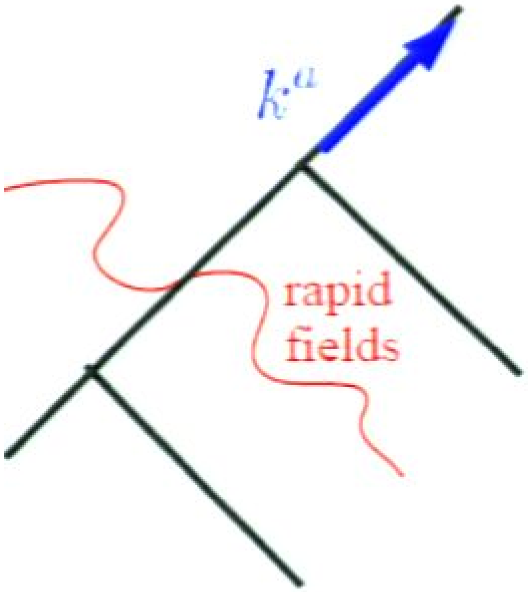
$$\mathrm{tr}_{\mathrm{in}}(|0\rangle\langle 0|) \propto e^{-2\pi K_{\mathrm{out}}/\hbar}$$

(formally) where the boost Killing energy on a slice Σ is:

$$K_{\mathrm{out}} = \int_{\mathrm{out}} T_{ab}\, \xi^a d\Sigma^b$$

So the vacuum is thermal. Next we will perturb it with quantum fields.

Area deficit ~ Boost Energy



Along each horizon generating lightray, the Raychaudhuri & Einstein equations hold:

$$\dot{\theta} = -\theta^2/2 - \sigma_{ab}\sigma^{ab} - 8\pi G T_{kk}$$

where $\theta = (1/A)(dA/d\lambda)$ is the expansion w.r.t. an affine parameter λ.

Linearize and integrate to get expression in terms of $K(\lambda)$, the boost energy of the wedge, up to constants.

$$A(\lambda) = A(\infty) - 8\pi G \int_\lambda^\infty T_{kk}(\lambda' - \lambda) d\lambda' = -8\pi G[K(\lambda) - K_{\text{rad}}] + A(\infty)$$

constants

Horizon area canonically conjugate to boost time: generalizes Carlip & Teitelboim (95), Massar & Parentani (00) to dynamical situations.

Relative Entropy

Information theory property of two mixed states ρ and σ.

$$S(\rho\,|\,\sigma) = \mathrm{tr}(\rho \ln \rho) - \mathrm{tr}(\rho \ln \sigma)$$

(definition can be extended to arbitrary algebras of observables)

Properties:

* Range is $[0, +\infty]$. Finite for nice enough states (no renormalization).

* $S(\rho\,|\,\rho) = 0$

* If σ is a KMS (thermal) state, proportional to free energy difference:

$$S(\rho\,|\,\sigma) = [T^{-1}E - S]_\rho - [T^{-1}E - S]_\sigma$$ (Araki & Sewell 77)

* Monotonicity: Always nonincreasing under restriction to subsystems:

$$S(\rho\,|\,\sigma)_M \geq S(\rho\,|\,\sigma)_{M'}$$ when $M' \subset M$ (Araki 75)

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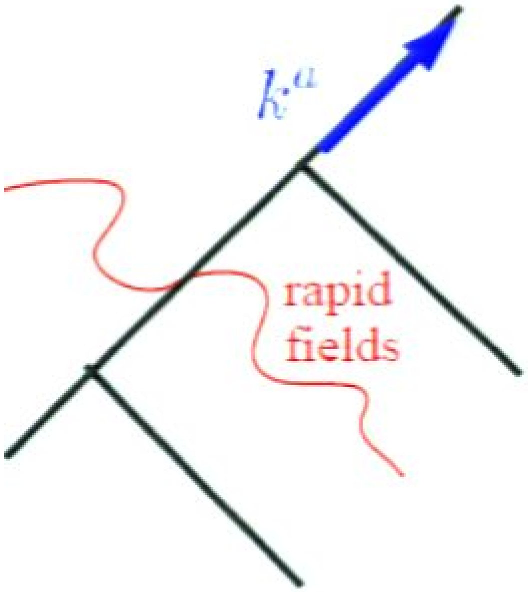

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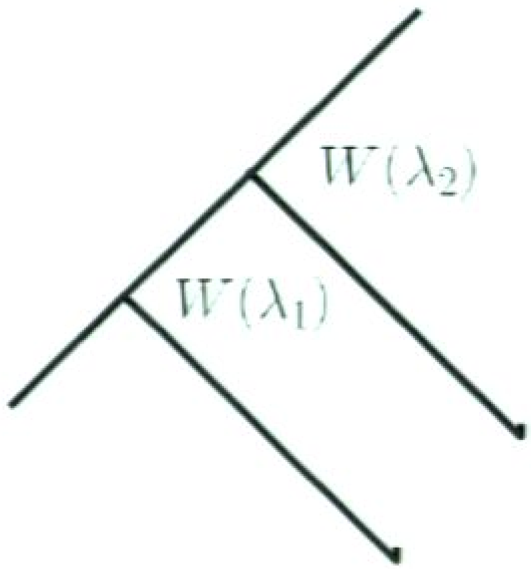
Proof of the Rindler GSL



Let ρ be the state we are interested in proving the GSL for.

Let σ be the Minkowski vacuum state.

Since σ is thermal in each wedge, $S(\rho\,|\,\sigma)$ is the free boost energy up to terms constant in each wedge:

$$S(\rho\,|\,\sigma) = [\tfrac{2\pi}{\hbar}K - S_{\text{out}}]_\rho = -\frac{A}{4\hbar G} - S_{\text{out}} = -S_{\text{gen}}$$

$S_{\text{out}} = S_\rho - S_\sigma$ is the *renormalized* entropy.

Relative entropy is monotonic under restriction, so the GSL holds!

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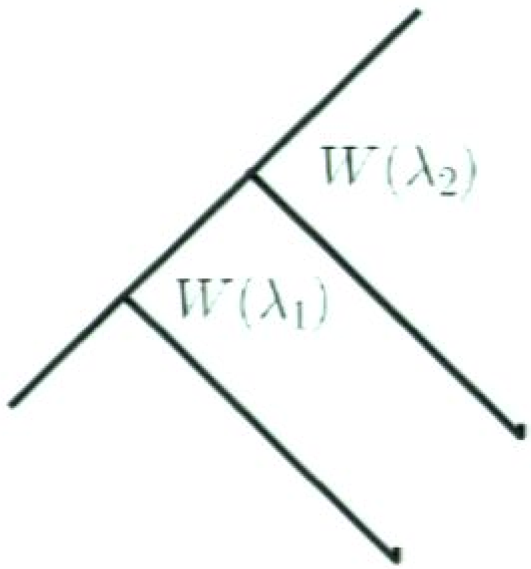

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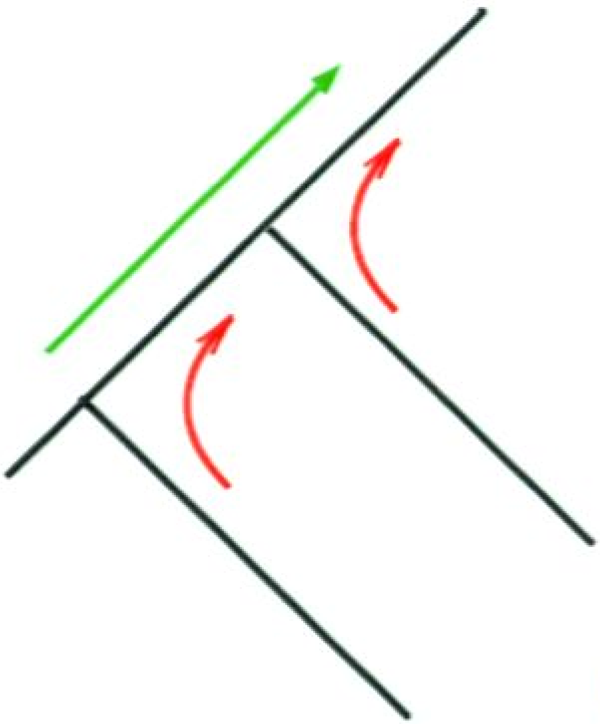
The GSL comes from Horizon Symmetry

Rindler Symmetry

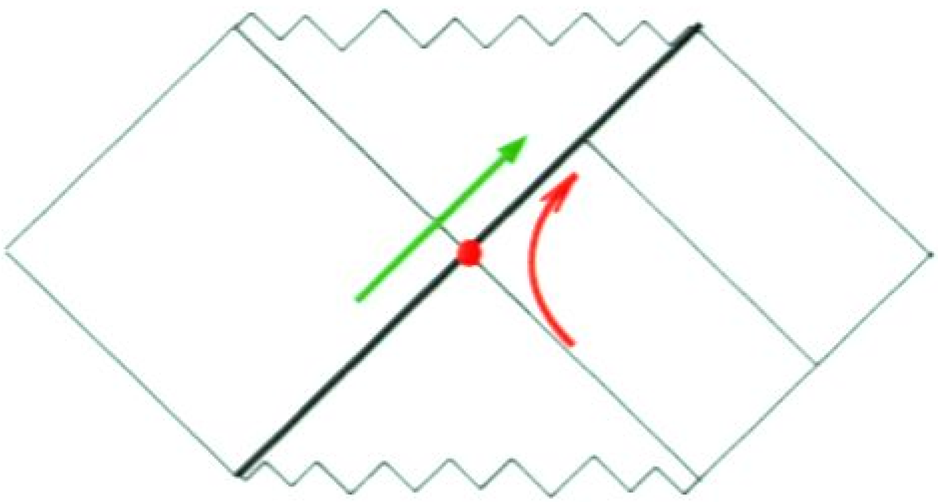


Argument just given requires each wedge to have a boost symmetry so that the vacuum state σ is thermal.

Commutator of two boosts is a null translation symmetry. Vacuum state σ invariant under this too.

Rindler horizon invariant under 2d Lie group.
That's why it works.

Black Holes have less symmetry



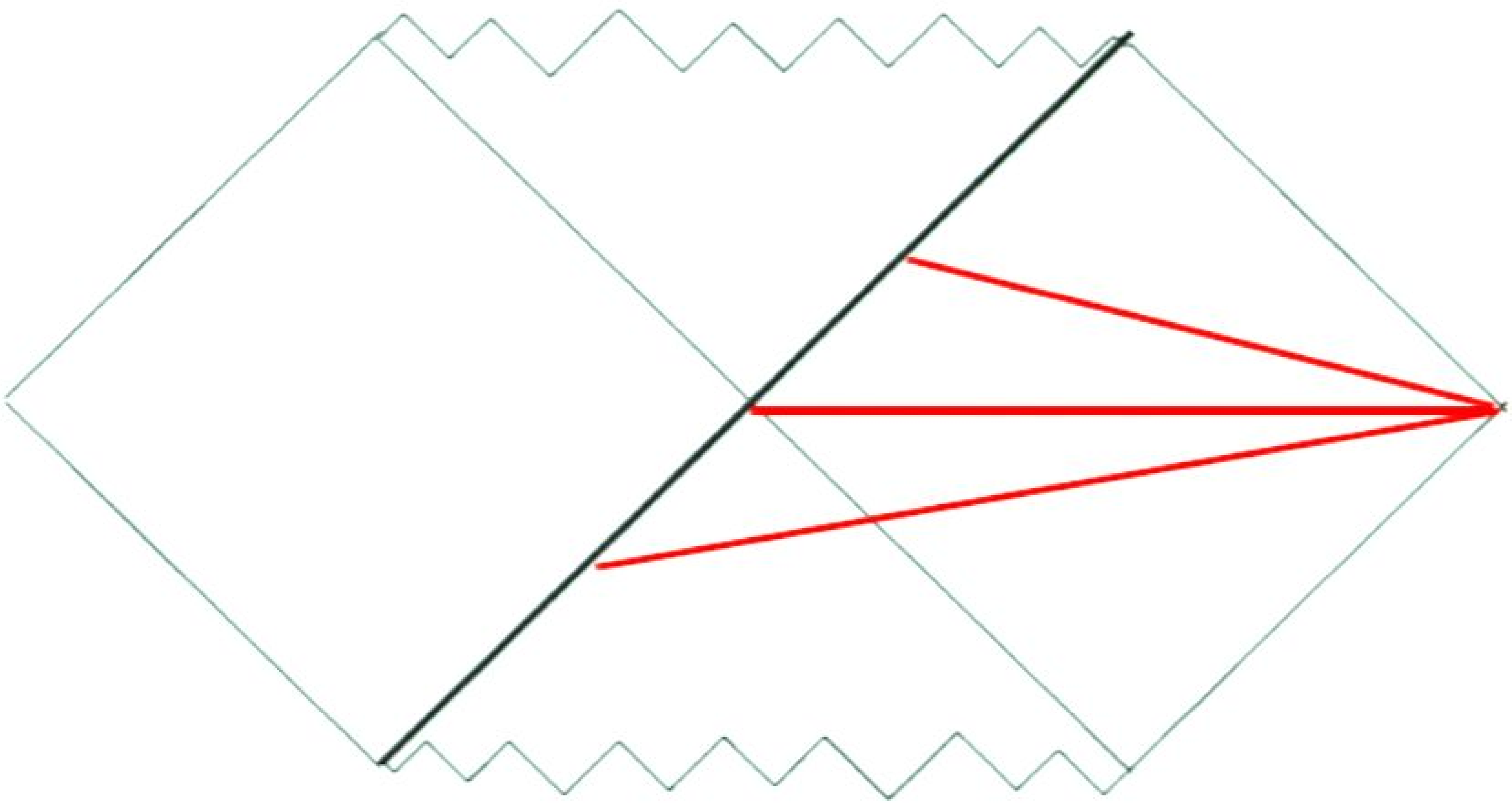
Spacetime has a Killing boost symmetry only about the bifurcation surface.

No null translation Killing symmetry.

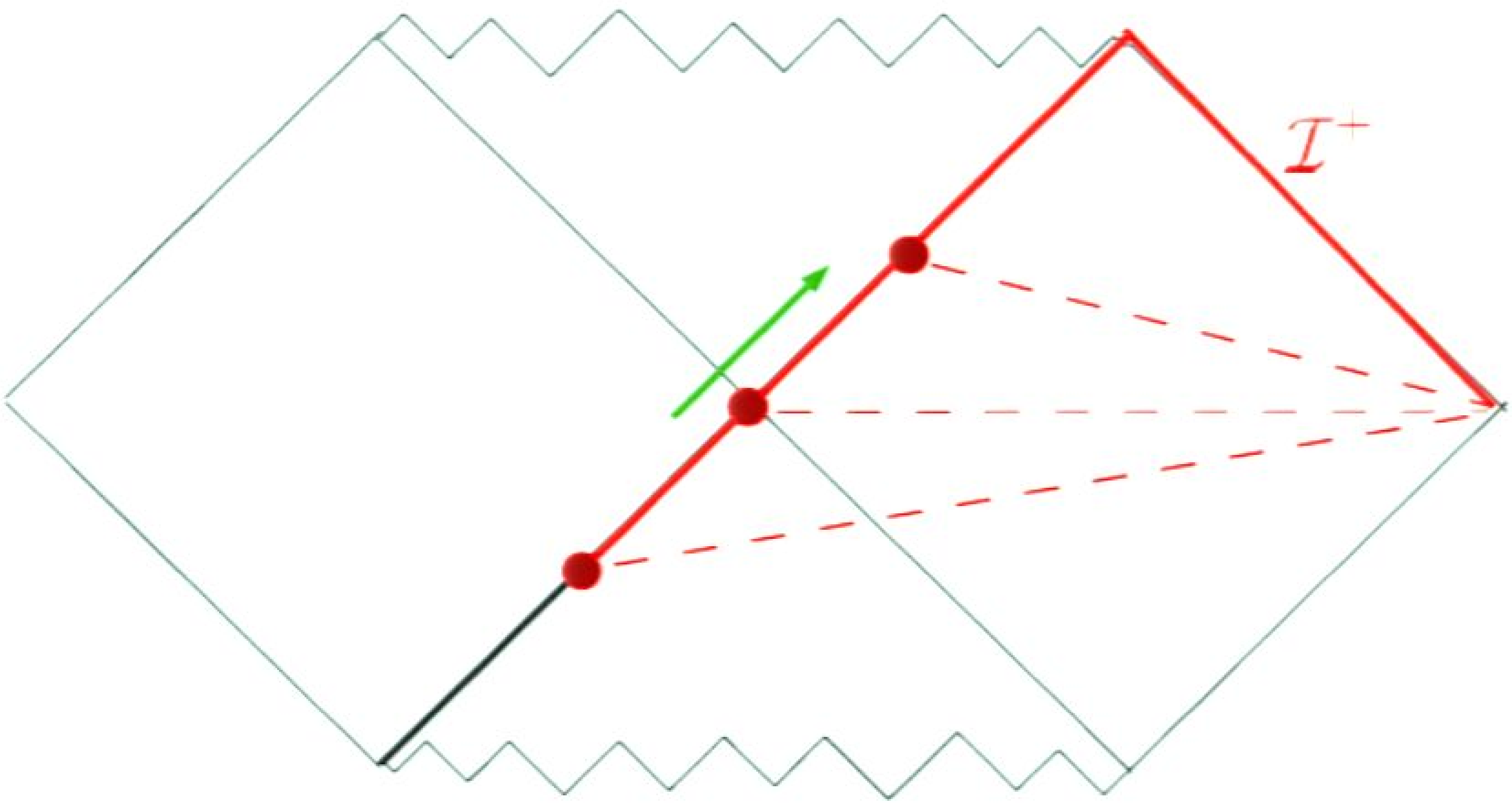
Kerr is even worse because no thermal Hartle-Hawking state exists at all (angular momentum is unbounded below).

Proof does not work—take near-horizon limit?

Instead of using spacelike slices:



Push forward to the horizon itself



The horizon has translation symmetry,
even though the full spacetime does not.

Restrict fields to horizon algebra

Possible to restrict free fields operators to the event horizon itself.
Tricky since fields must be smeared only across the horizon.
One finds that:

* Φ can't be restricted, but $\nabla_k \Phi$ can be. Normally derivatives hurt but in the field is already smeared in the k direction, and null mass shell tells us that $\nabla_+ \Phi \sim p_+ \Phi \sim \frac{1}{p_-} \Phi$ so it actually helps.

* The horizon algebra is ultralocal; each horizon generator is independent.

* There is an infinite dimensional symmetry group:
translations and dilations of each horizon generator *independently*.
(boosts = dilations on the horizon)

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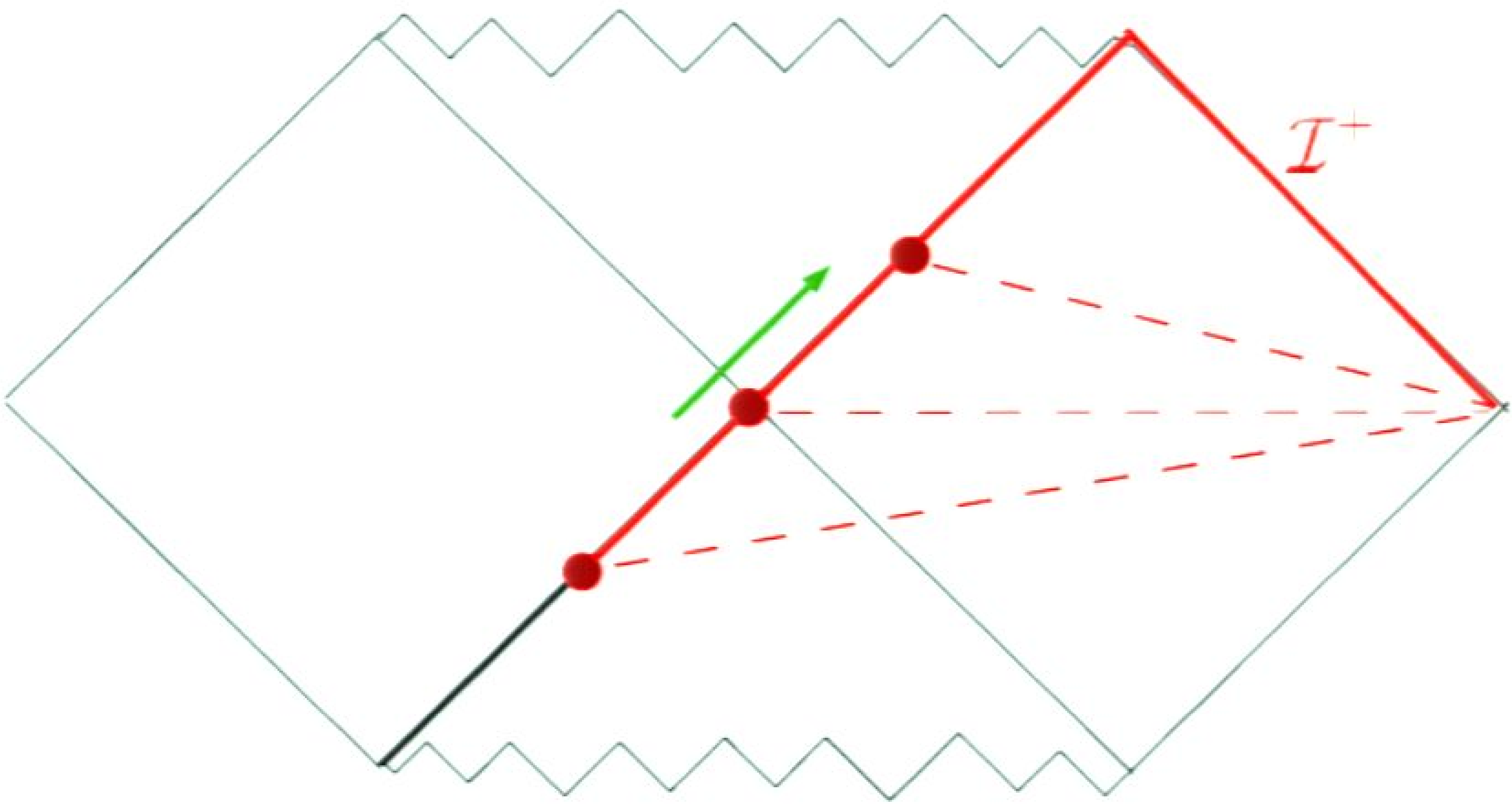

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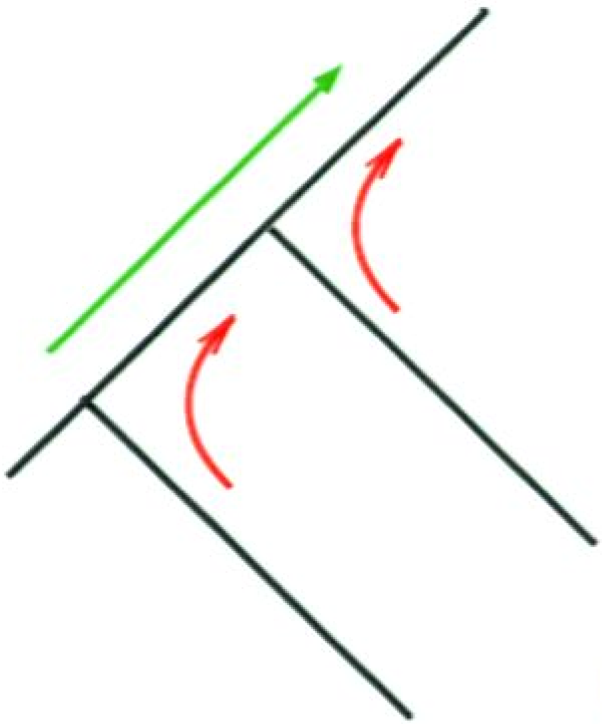

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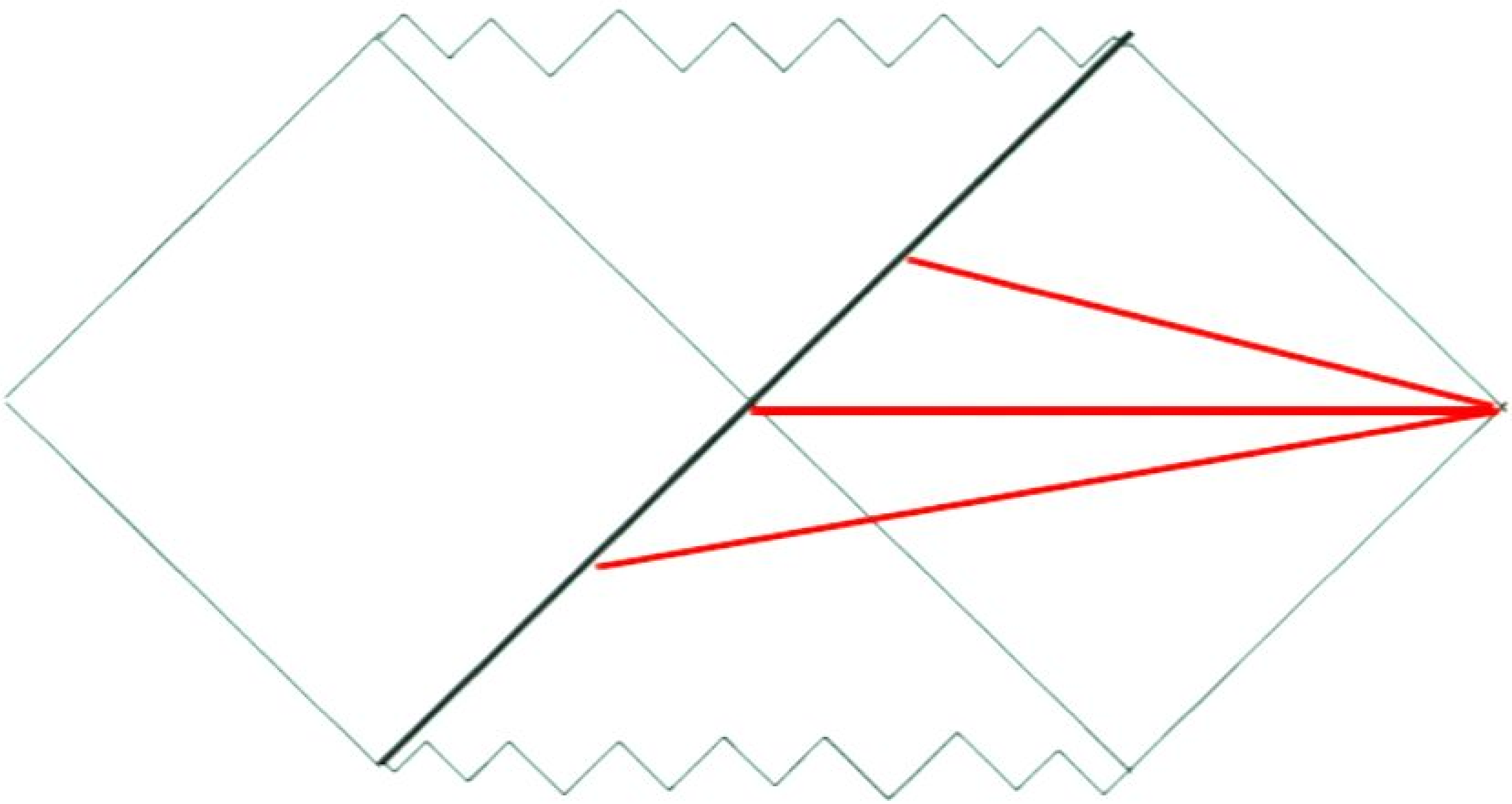

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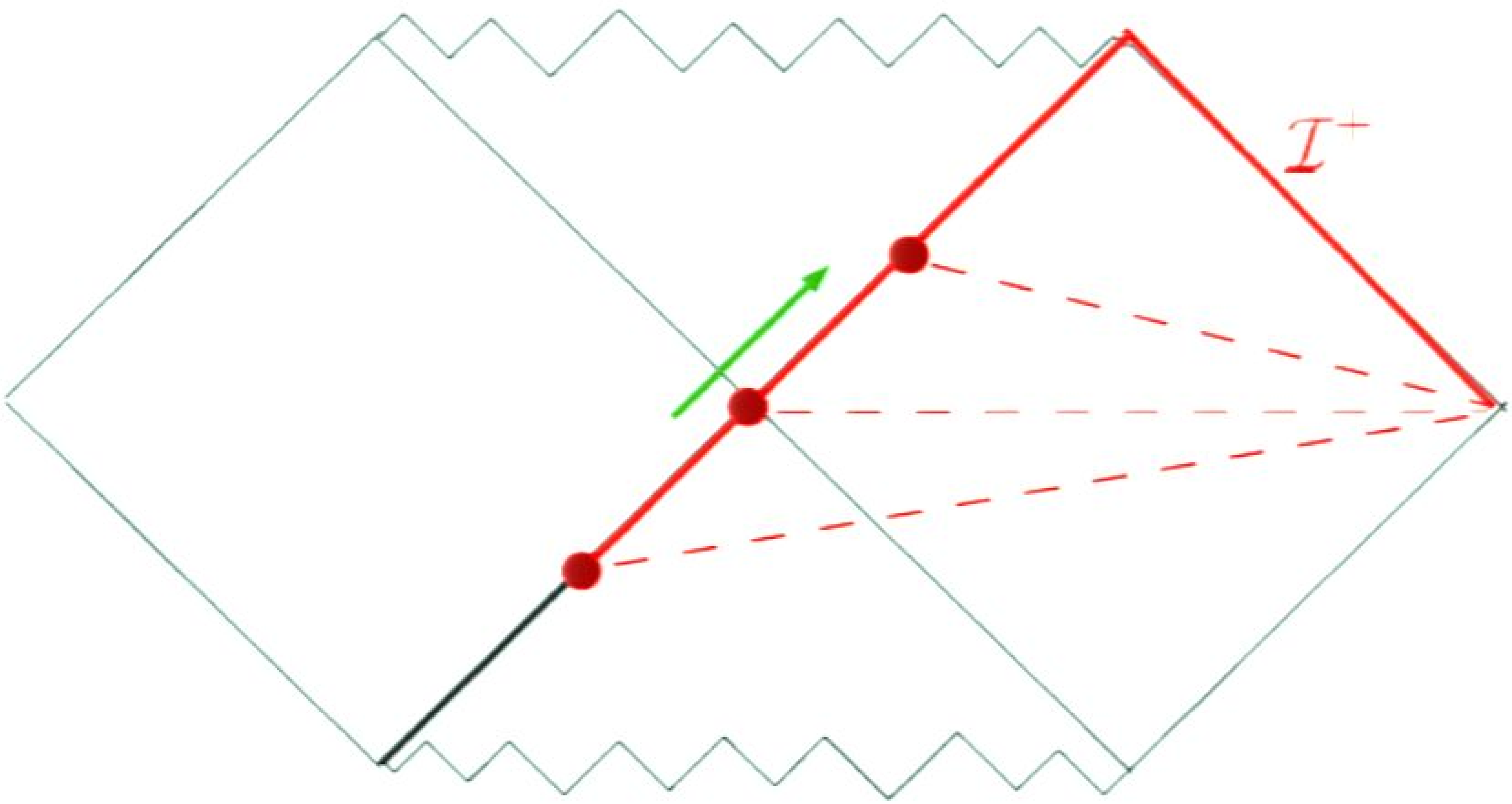

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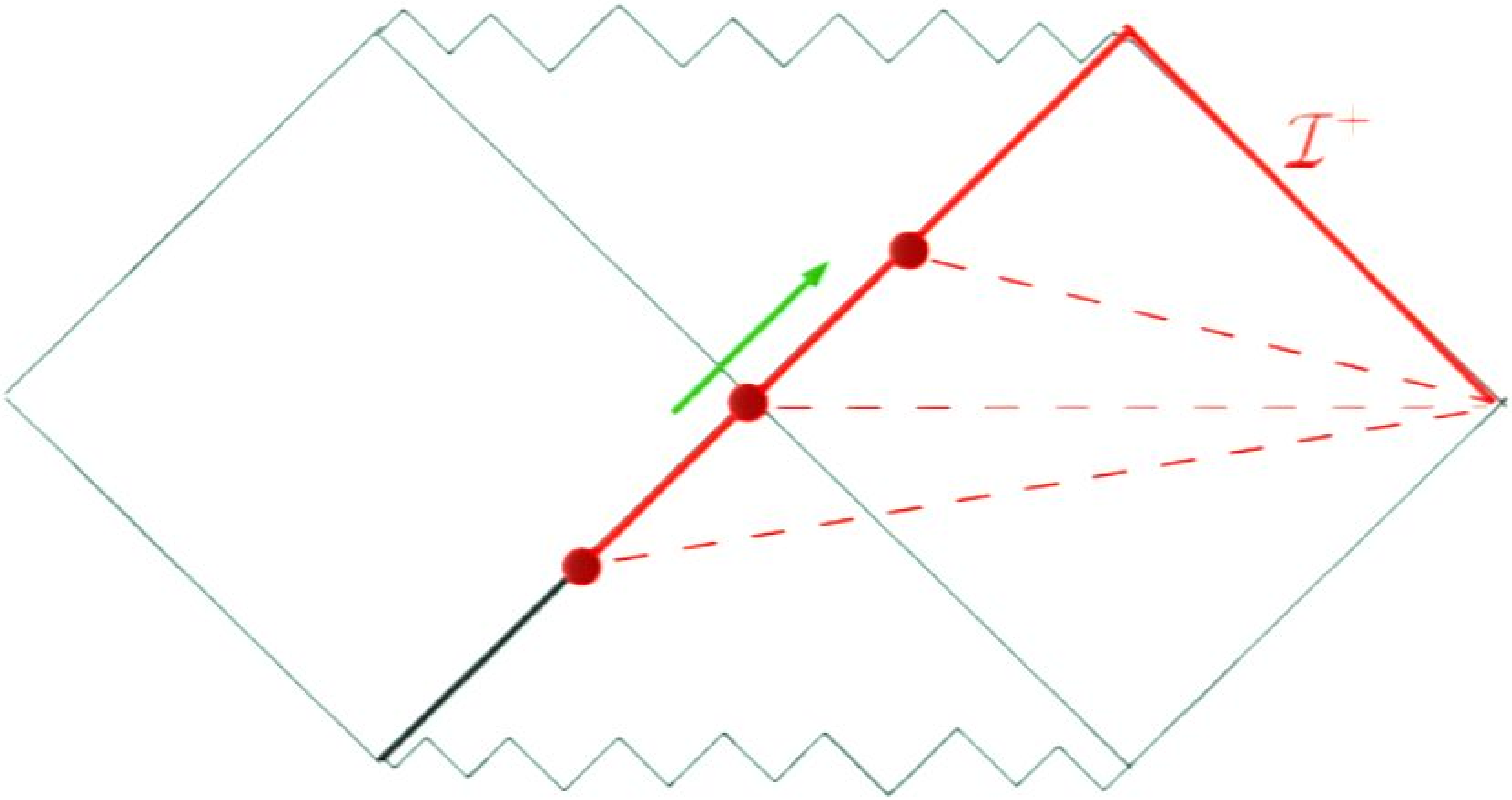

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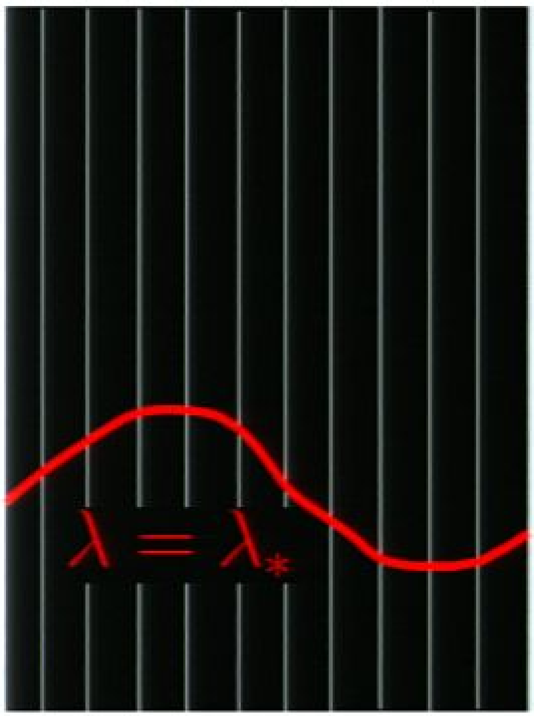
Arbitrary horizons, arbitrary slices:



a piece of a horizon, with grey generators

Because each horizon generator can be independently translated, can translate to wiggly slices.

Can define a canonical vacuum state w.r.t. all null translation symmetries (Sewell 81).

σ is KMS above *any* slice w.r.t. dilations about that slice. Formally,

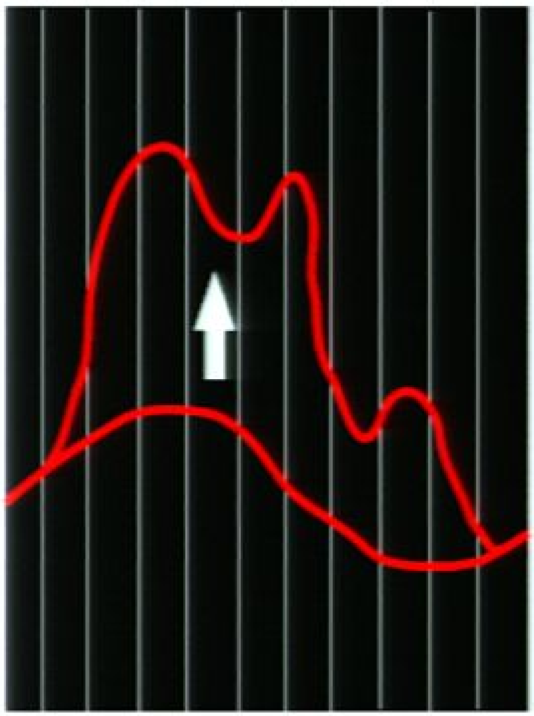
$$\sigma_{\lambda > \lambda_*} = e^{-2\pi K(\lambda_*)/\hbar}$$

where

$$K(\lambda_*) = \int_{\lambda_*}^{\infty} T_{kk}(\lambda - \lambda_*)\, d\lambda$$

This works on any background with a stationary horizon even when no Hartle-Hawking state can be defined on the bulk spacetime (e.g. Kerr).

Adapting the Proof of the GSL



a piece of a horizon, with grey generators

GSL can now be proven analogously to Rindler case for semiclassical perturbations to any stationary horizon:

Let ρ be the bulk state we are interested in,
Let σ be any bulk state which restricts to the vacuum state on the horizon, and is otherwise arbitrary. For each slice:

* σ is thermal with respect to $K(\lambda_*)$, which is proportional to the area A of the slice λ_*.

* $S(\rho\,|\,\sigma) = -S_{\text{gen}}$ up to additive constants.

* Thus the GSL holds by monotonicity of relative entropy.

Should be on the arXiv in a couple weeks

Wild Speculations

1. How does this relate to quantum gravity statistical mechanics?
Two different ways to approach issue:

A) Look for this infinite dimensional symmetry of the horizon in a quantum gravity theory to get the GSL for similar reasons,

Or try to reverse the argument...

B) Find some other way to derive GSL in quantum gravity, and then try to see Lorentz symmetry emerge from it.

2. Can (interacting) gravity be quantized directly on the horizon?
An ultraviolet scaling limit...
Poisson brackets of GR have been analyzed (Reisenberger 08).

Conclusions:

1. The reason why the semiclassical GSL holds is because the horizon has more symmetry than the rest of the spacetime.

2. The proof of this works for arbitrary free fields, maybe with (nonderivative) perturbative interactions, but rigorous interactions may require a more delicate near-horizon limit.

3. Except in the case of Rindler horizons, where arbitrary interactions may be accommodated.

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A) Look for this infinite dimensional symmetry of the horizon in a quantum gravity theory to get the GSL for similar reasons,

Or try to reverse the argument...

B) Find some other way to derive GSL in quantum gravity, and then try to see Lorentz symmetry emerge from it.

2. Can (interacting) gravity be quantized directly on the horizon?
An ultraviolet scaling limit...
Poisson brackets of GR have been analyzed (Reisenberger 08).

Conclusions:

1. The reason why the semiclassical GSL holds is because the horizon has more symmetry than the rest of the spacetime.

2. The proof of this works for arbitrary free fields, maybe with (nonderivative) perturbative interactions, but rigorous interactions may require a more delicate near-horizon limit.

3. Except in the case of Rindler horizons, where arbitrary interactions may be accommodated.

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