Title: There Is No Gravitational Stress-Energy Tensor

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Abstract: The question of the existence of gravitational stress-energy in general relativity has exercised investigators in the field since the very inception of the theory. Folklore has it that no adequate definition of a localized gravitational stress-energetic quantity can be given. Most arguments to that effect invoke one version or another of the Principle of Equivalence. I argue that not only are such arguments of necessity vague and hand-waving but, worse, are beside the point and do not address the heart of the issue. Based on an analysis of what it may mean for one tensor to depend in the proper way on another, I prove that, under certain natural conditions, there can be no tensor whose interpretation could be that it represents gravitational stress-energy in general relativity. It follows that gravitational energy, such as it is in general relativity, is necessarily non-local. Along the way, I prove a result of some interest in own right about the structure of the associated jet bundles of the bundle of Lorentz metrics over spacetime.

On Tensorial Concomitants and the Non-Existence of a Gravitational Stress-Energy Tensor

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October 1, 2010

Outline

1. Energetic Quantities in General Relativity
2. The Principle of Equivalence: A Bad Argument
3. Geometric Fiber Bundles
4. Concomitants
5. There Is No Gravitational Stress-Energy Tensor

1. Energetic Quantities in General Relativity
2. The Principle of Equivalence: A Bad Argument
3. Geometric Fiber Bundles
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5. There Is No Gravitational Stress-Energy Tensor

As soon as the principle of conservation of energy was grasped, the physicist practically made it his definition of energy, so that energy was that something which obeyed the law of conservation. He followed the practice of the pure mathematician, defining energy by the properties he wished it to have, instead of describing how he measured it. This procedure has turned out to be rather unlucky in the light of the new developments.

A. Eddington
The Mathematical Theory of Relativity

Properties of T_{ab}

10 components 6 for the classical stress-tensor; 3 for linear momentum; 1 for scalar energy density

two covariant indices linear mapping from tangent vector of worldline to 4-momentum covector of field; bi-linear mapping from tangent vectors to energy-density—momental phenomena linear in velocity, energetic phenomena quadratic in velocity

symmetric infinitesimally, yields "conservation of angular momentum"; part of relativistic equivalence of momentum flux and energy density

Properties of T_{ab}

covariantly divergence-free infinitesimally, yields "classical energy and linear momentum conservation"; part of relativistic equivalence of momentum flux and energy density

a tensor the localization of ponderable stress-energy and its invariance as a physical quantity (multi-linear map acting only on the tangent plane)

all stress-energy tensors have same physical dimension thermodynamic fungibility of energetic phenomena

The Thermodynamical Fungibility of Stress-Energy

The First Law of Thermodynamics: all forms of stress-energy are in principle ultimately fungible—any form of stress-energy can in principle be transformed into any other form.

Thus all stress-energy tensors must have the "physical dimensions of stress-energy", if $T_{ab} + S_{ab}$ is to make physical sense.

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Physical Dimension

all physical units in relativity can be expressed as a constant multiple of a single fundamental unit, say time (*e.g.*, setting $c = 1$); to multiply the metric by a constant rescales the measure of time in a fixed way, and so all other physical units

it follows that a necessary condition for a quantity to have a given physical dimension is that it transform under constant re-scaling of metric in a particular way

in particular, stress-energy remains fixed under constant re-scaling—one can see this from the Einstein Field-Equation

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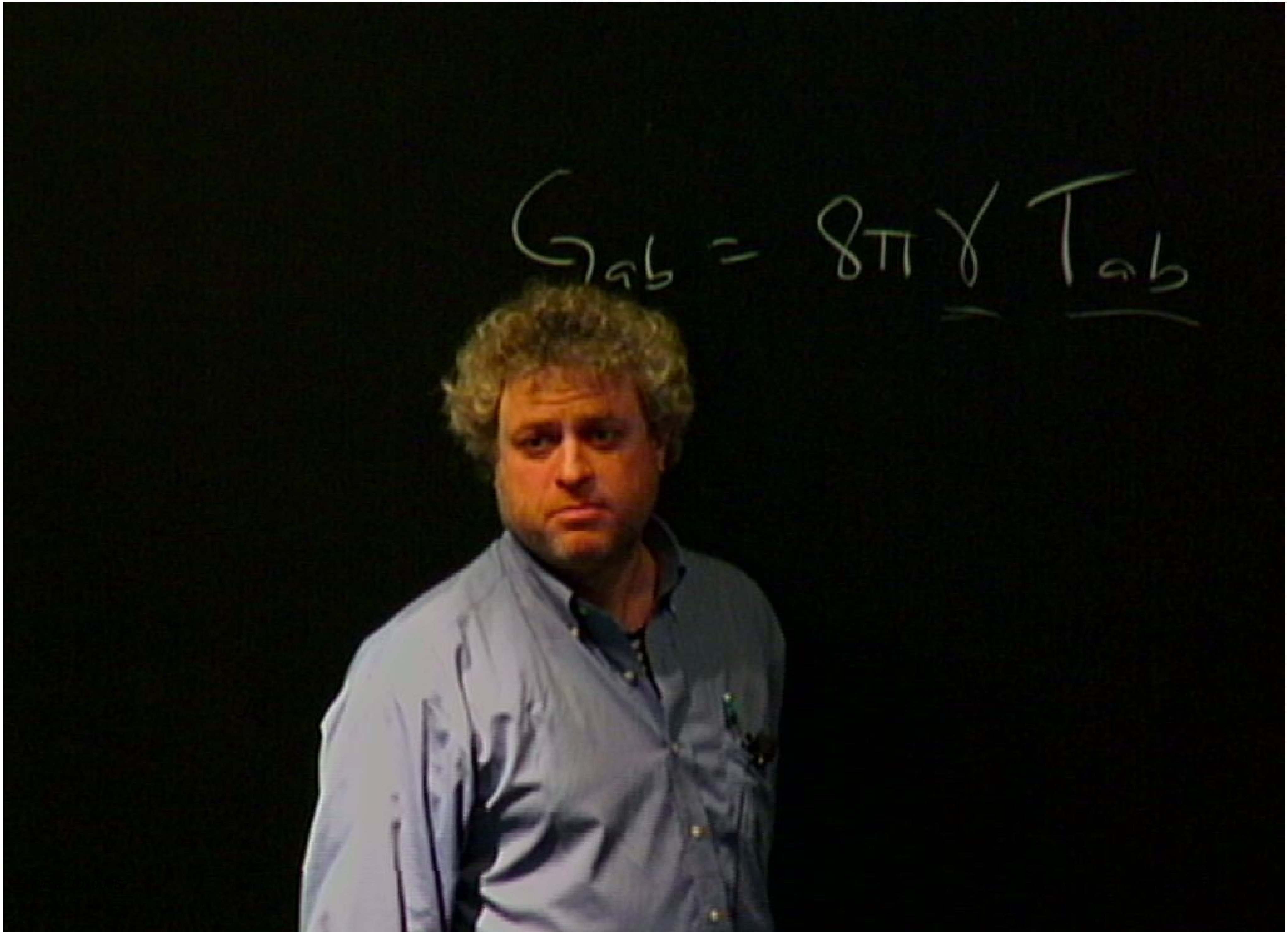
Gab = 8π γ Tab

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S_{ab}: Stress-Energy Tensor of "Gravitational Field"

Can one find such a thing so as to localize gravitational stress-energy?

This 'non local' character of gravitational energy is in fact obvious from a formulation of the equivalence principle which says that the gravitational field appears as non existent to one observer in free fall. It is, mathematically, a consequence of the fact that the pseudo-riemannian connexion which represents the gravitational field can always be made to vanish along a given curve by a change of coordinates.

Y. Choquet-Bruhat
"Two Points of View on Gravitational Energy"
(1983)

Invocation of "Principle of Equivalence"

1. all known field energies quadratic in field intensities
2. based on Newtonian analogy, metric represents gravitational field potential
3. **therefore,** coefficients of affine structure represent "gravitational field intensity"
4. but can always be transformed to zero along arbitrary curve

ERGO: no invariant expression for gravitational energy

IRRELEVANT!

if there is gravitational energy, it must depend on curvature (at the least, non-trivial geodesic deviation)

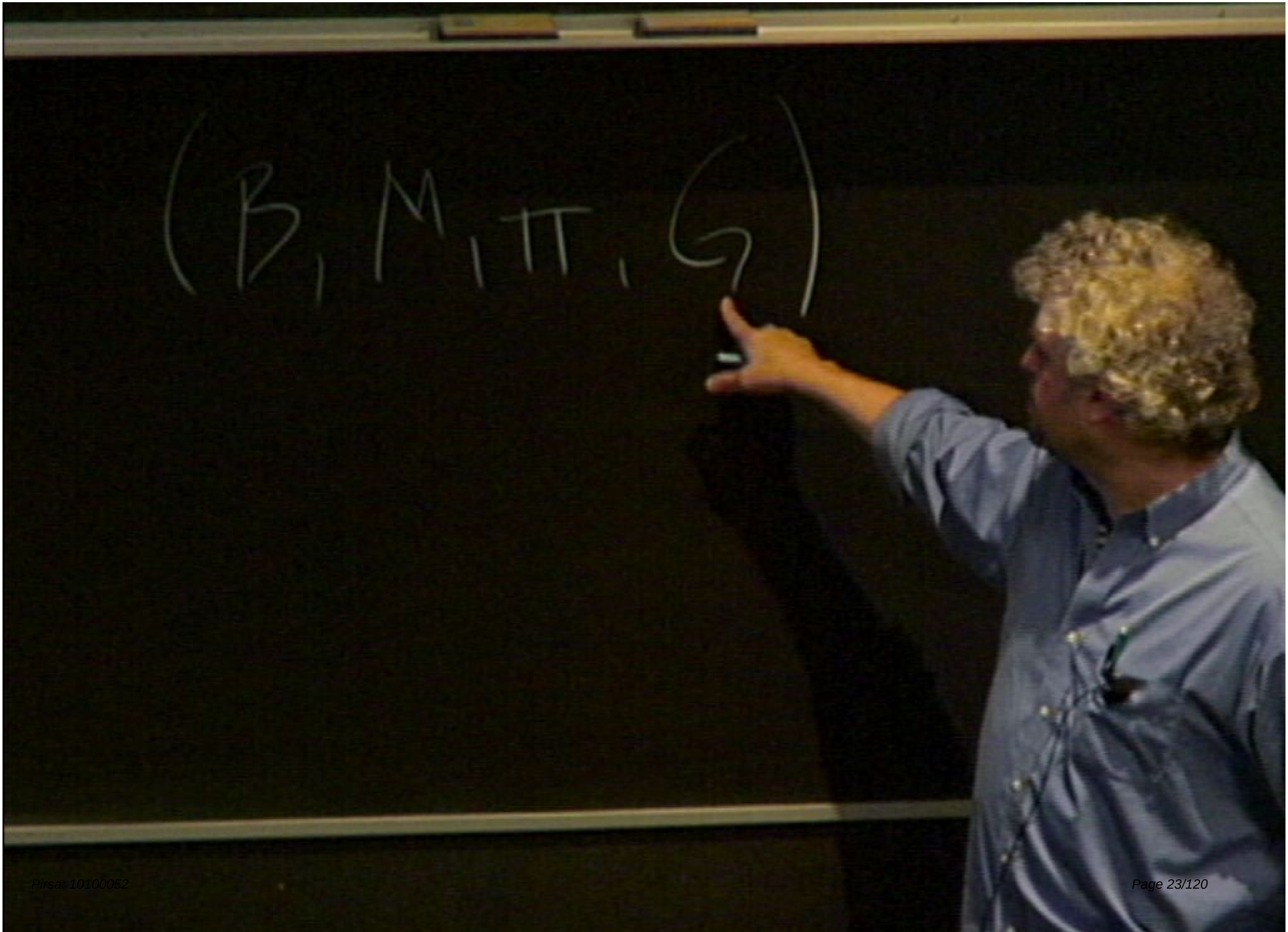
Concomitant: tensor that is "invariant function" of other tensors (mapping from fiber bundle to fiber bundle) $\Rightarrow$ gravitational stress-energy tensor should be concomitant of Riemann tensor, zero when $R^a{}_{bcd} = 0$

Strongly Consistent Diffeomorphisms

Fix a fiber bundle $(\mathcal{B}, \mathcal{M}, \pi)$. A diffeomorphism $\phi^\sharp$ of $\mathcal{B}$ is *consistent* with ϕ, a diffeomorphism of the base space $\mathcal{M}$, if, for all $u \in \mathcal{B}$,

$$\pi(\phi^\sharp(u)) = \phi(\pi(u))$$

$\phi^\sharp$ is *strongly consistent* with ϕ if, when ϕ is the identity inside an open set $O \subset \mathcal{M}$, $\phi^\sharp$ is the identity on $\pi^{-1}[O]$.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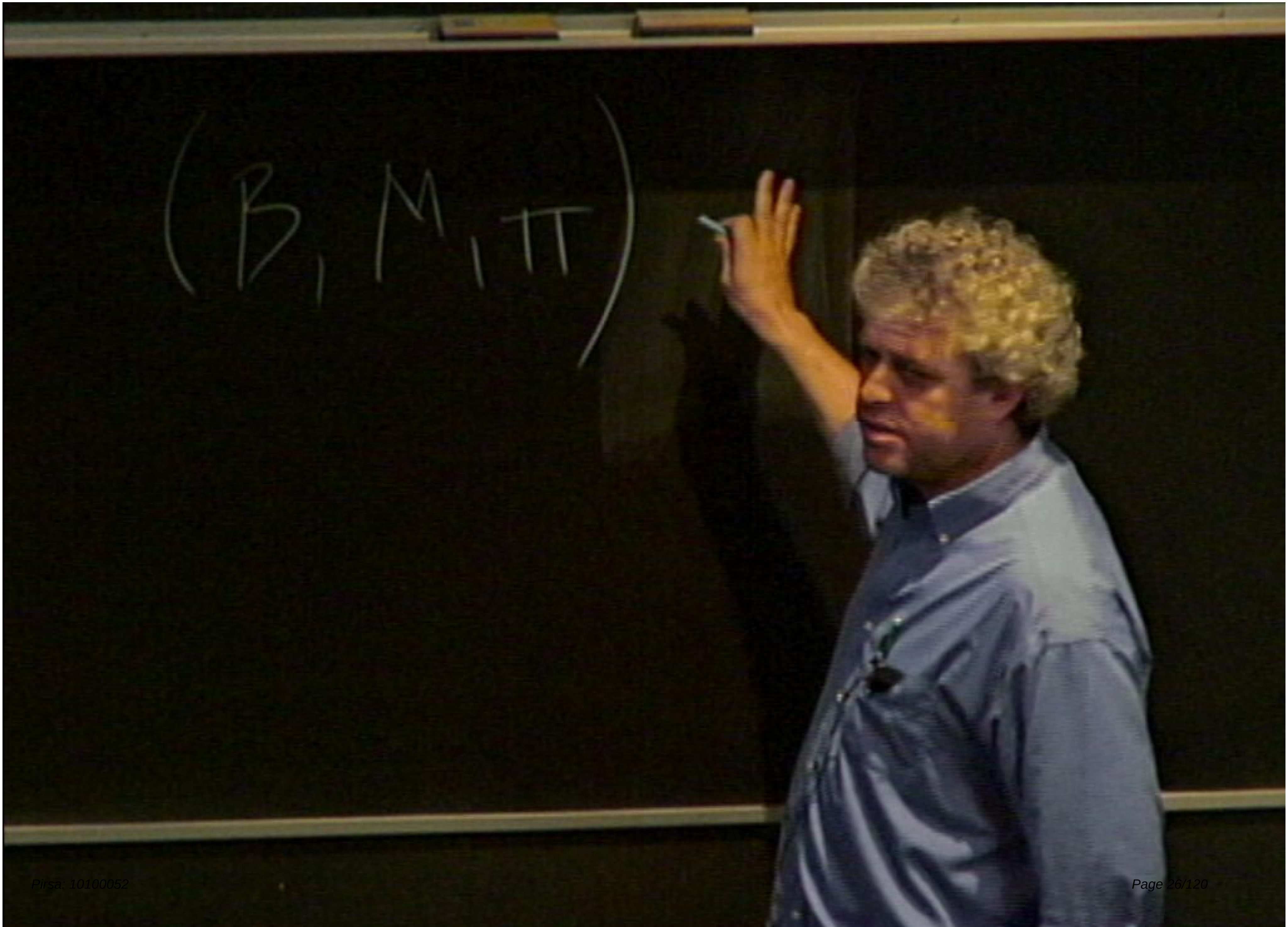
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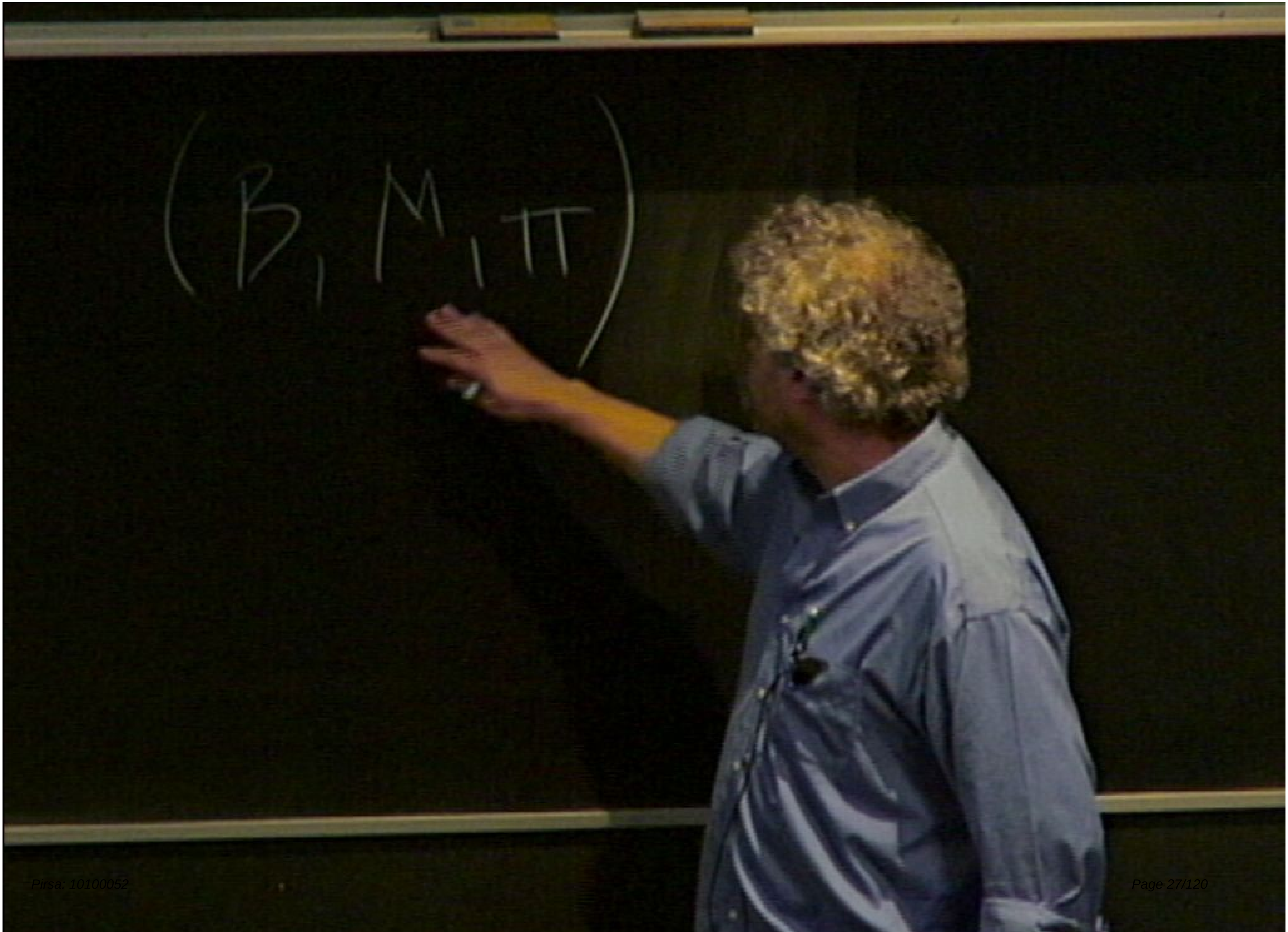
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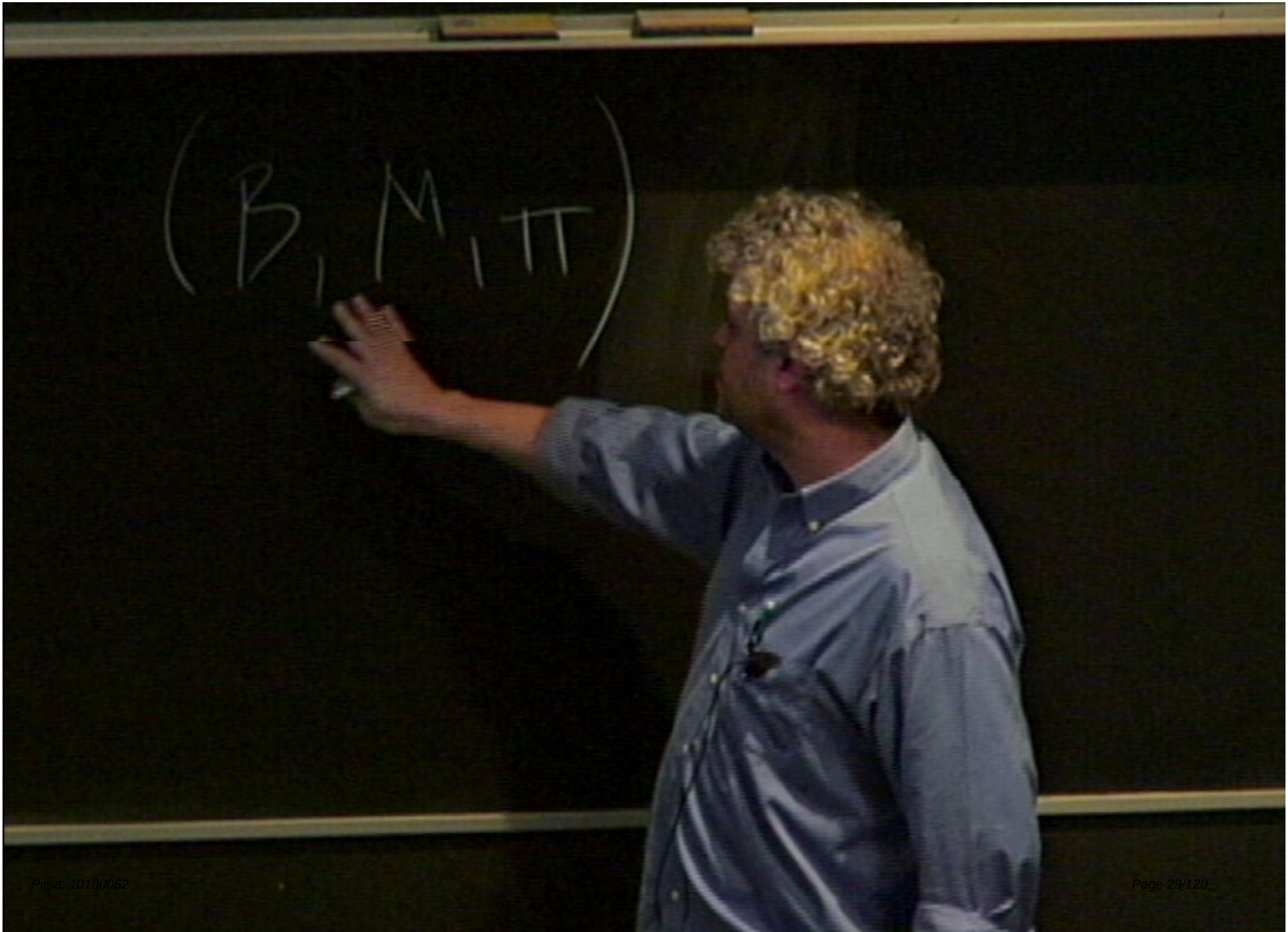
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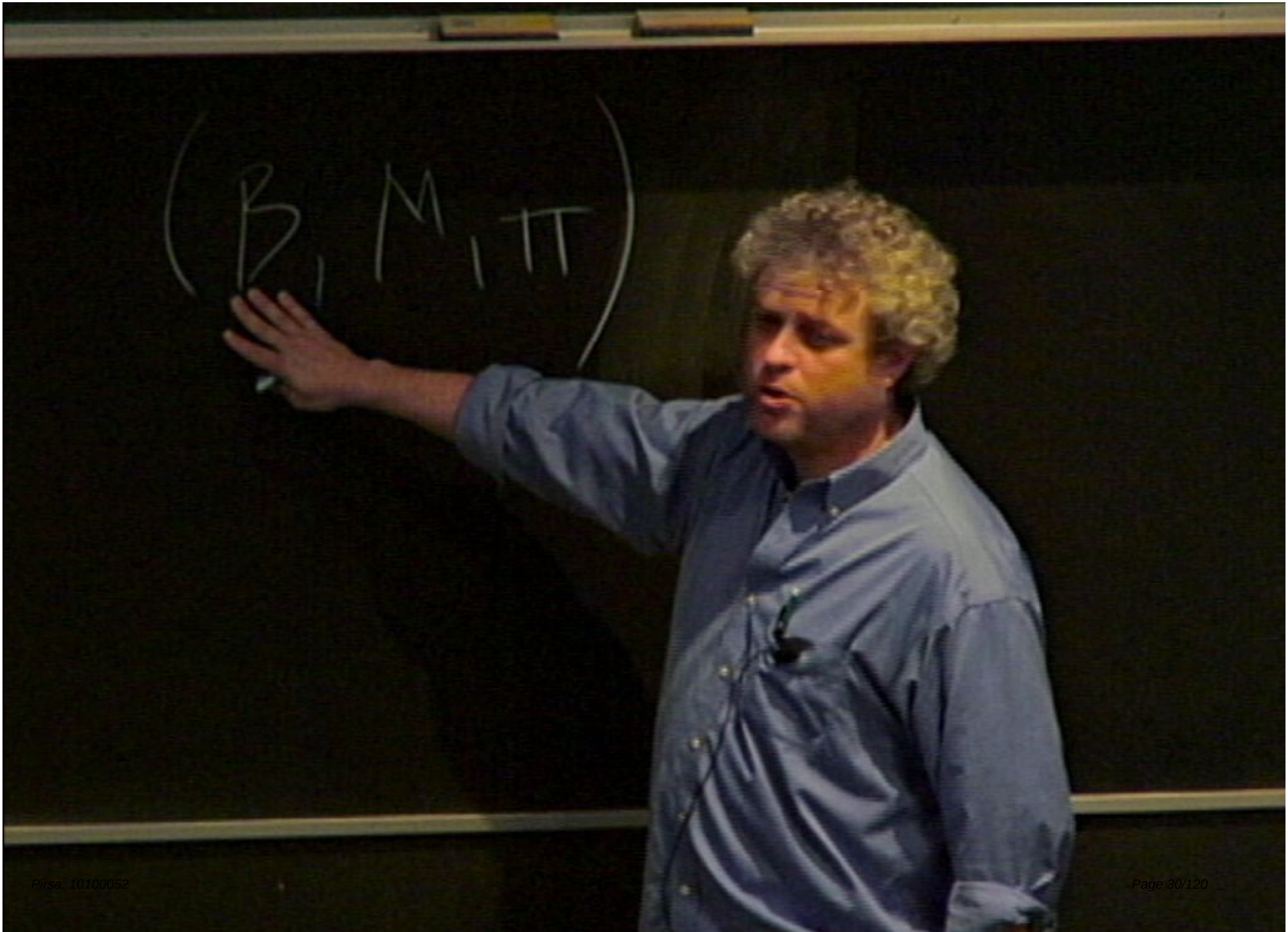
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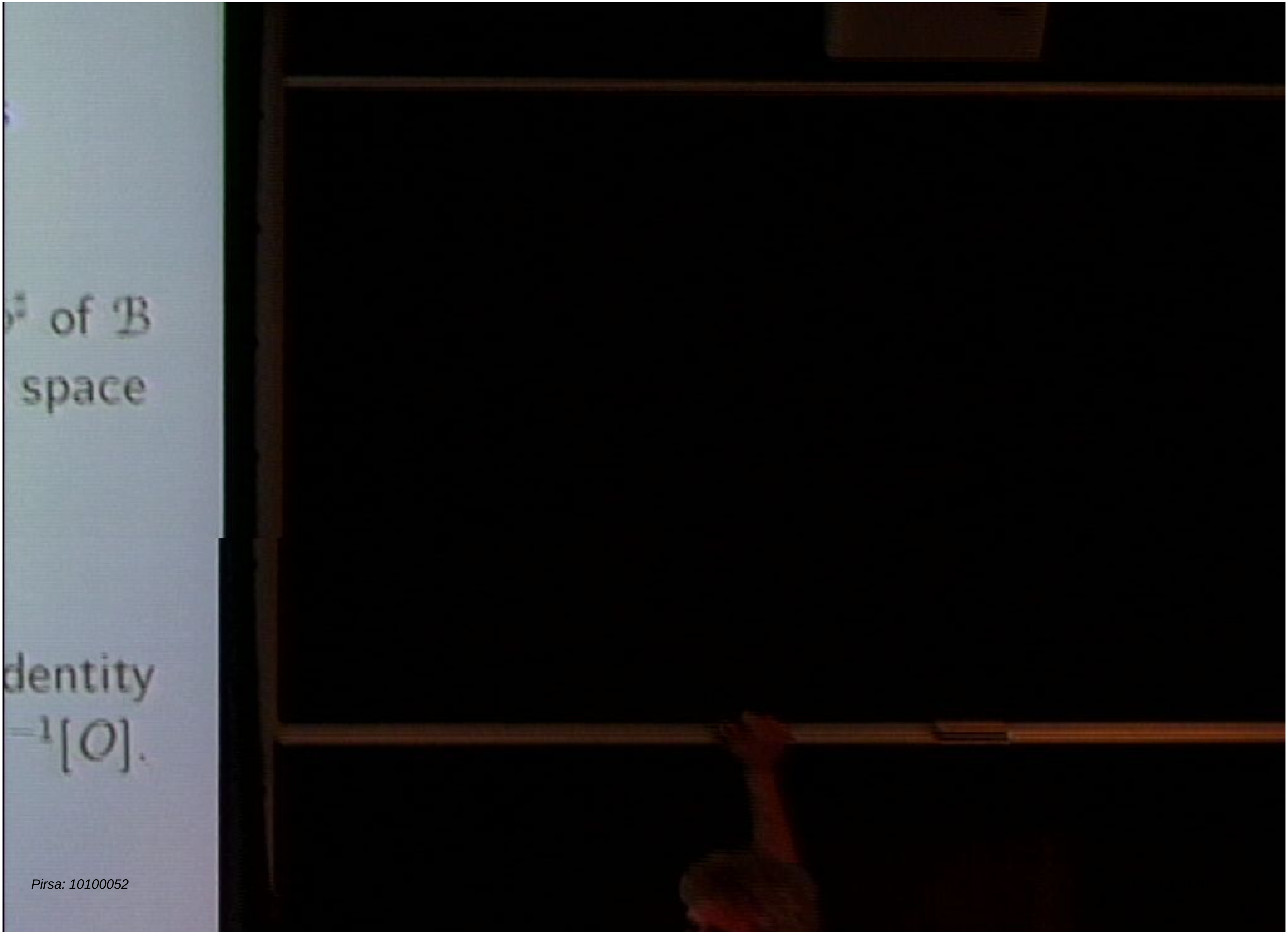
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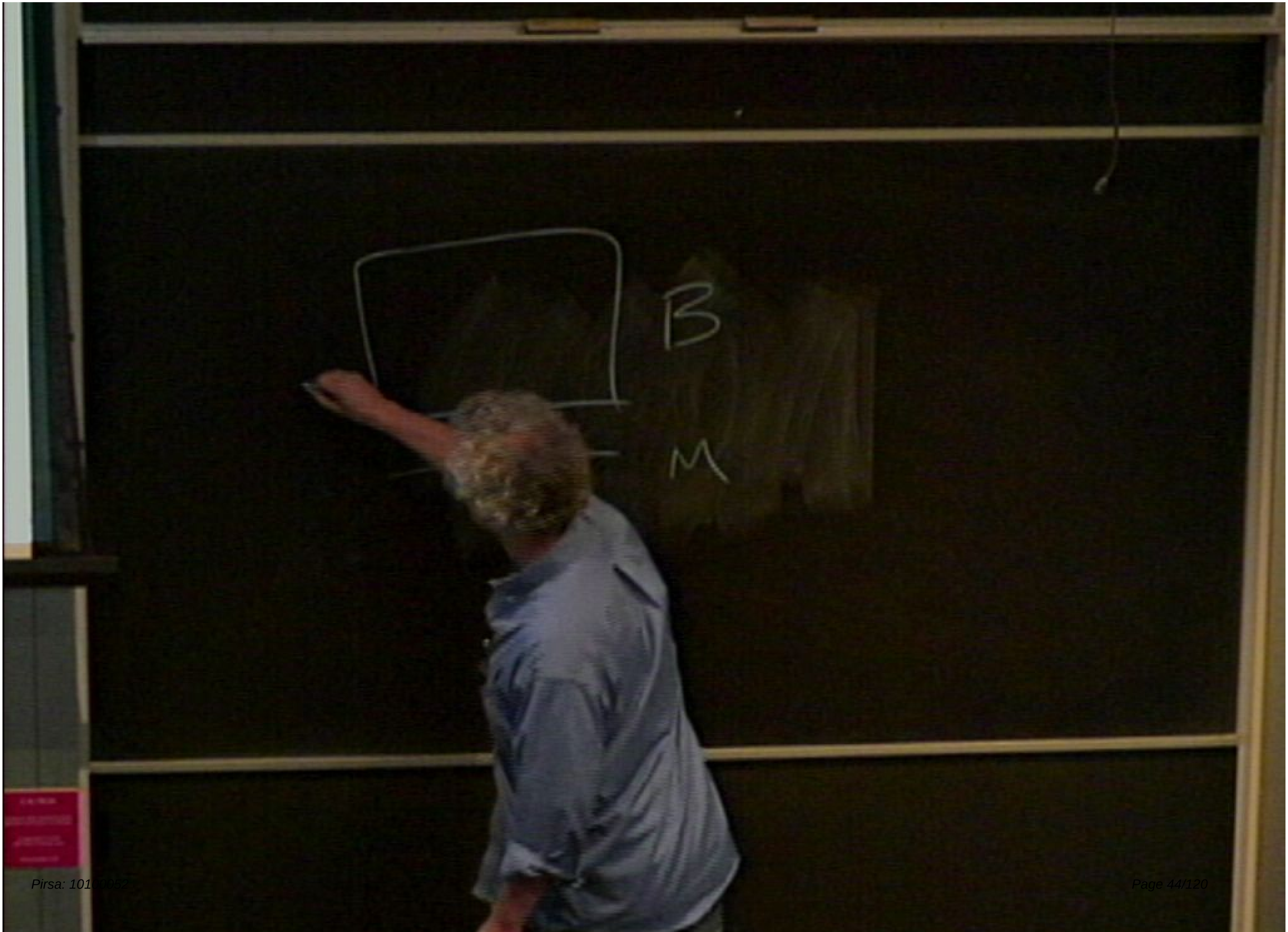
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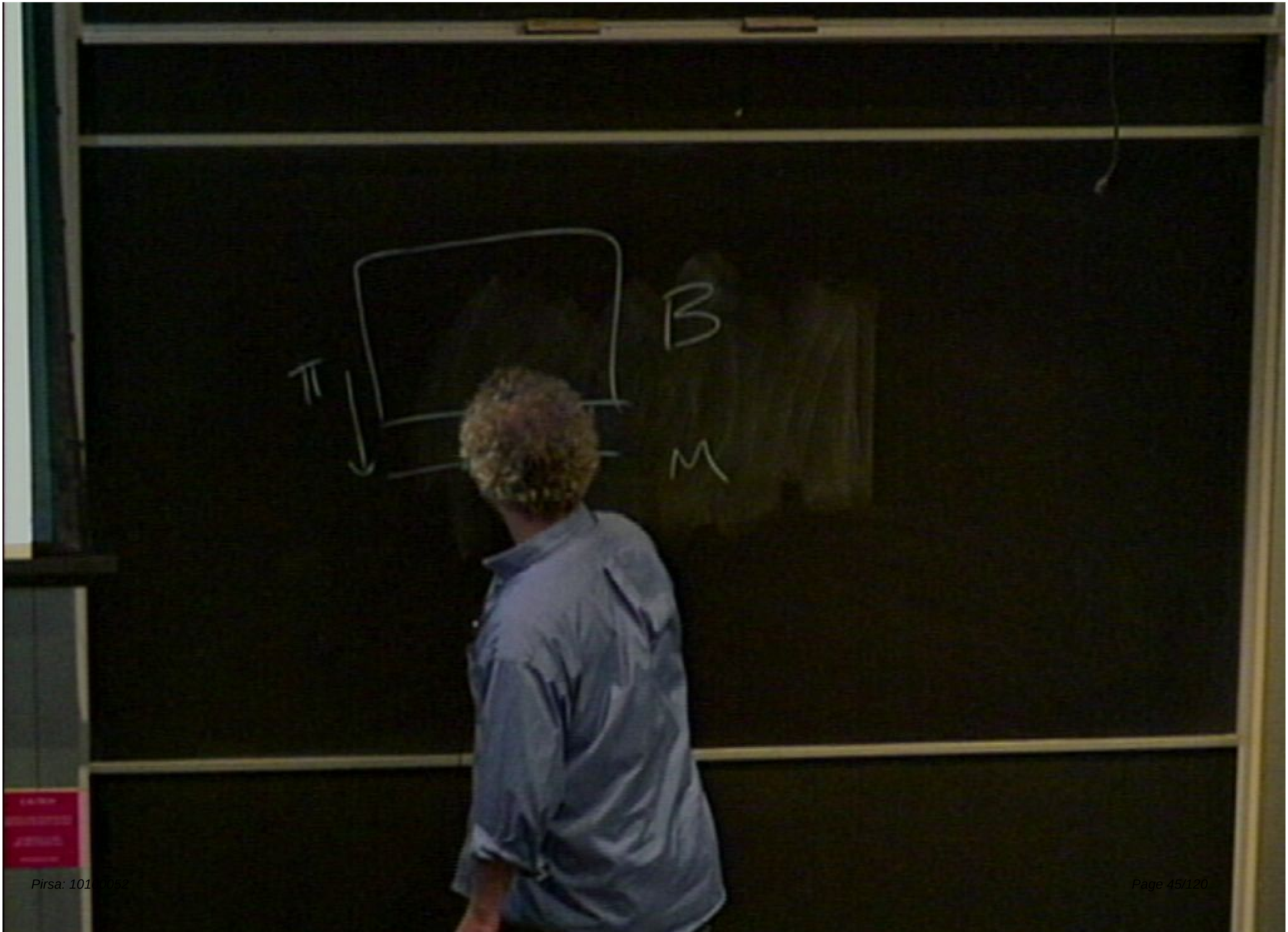
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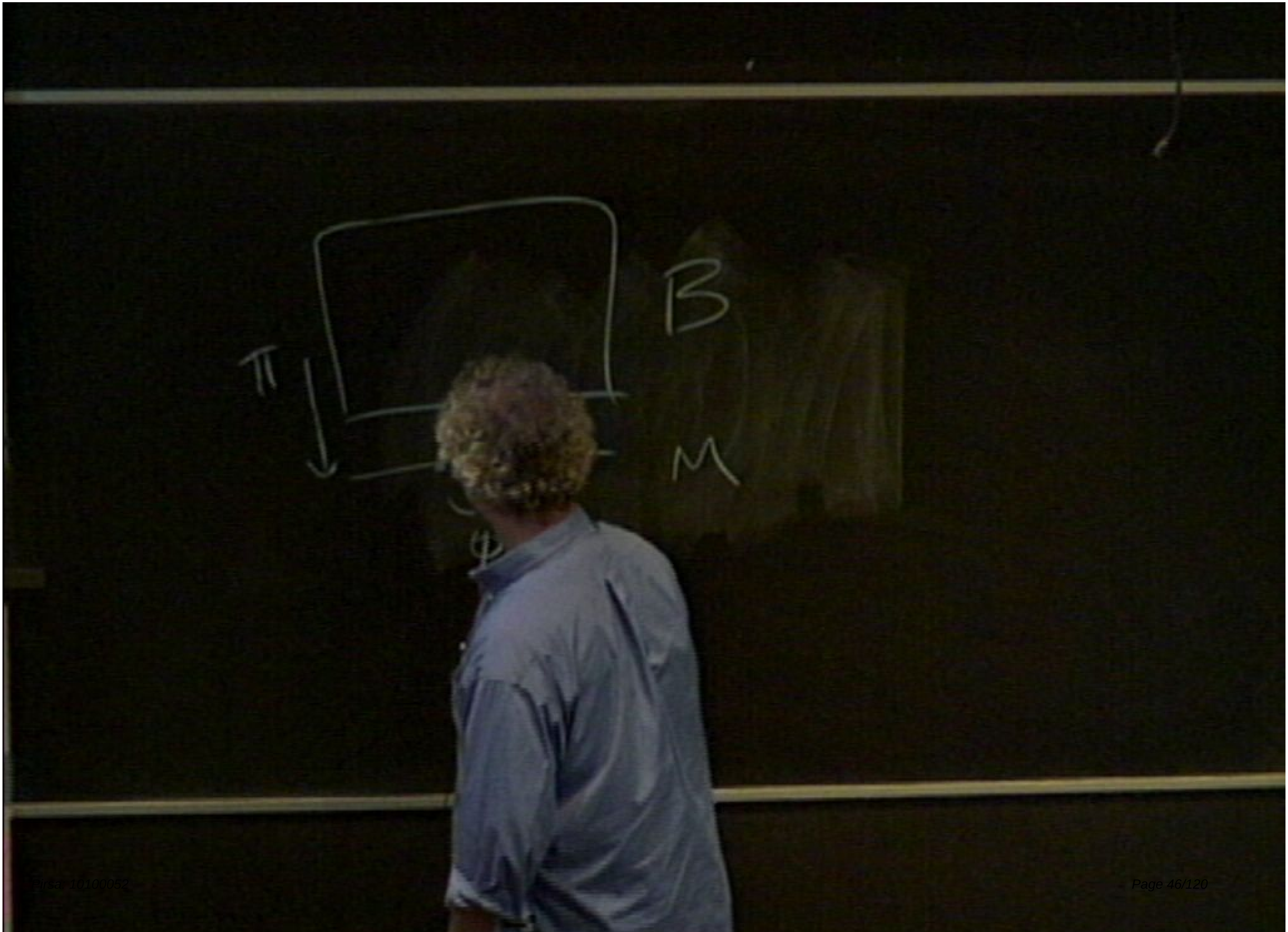
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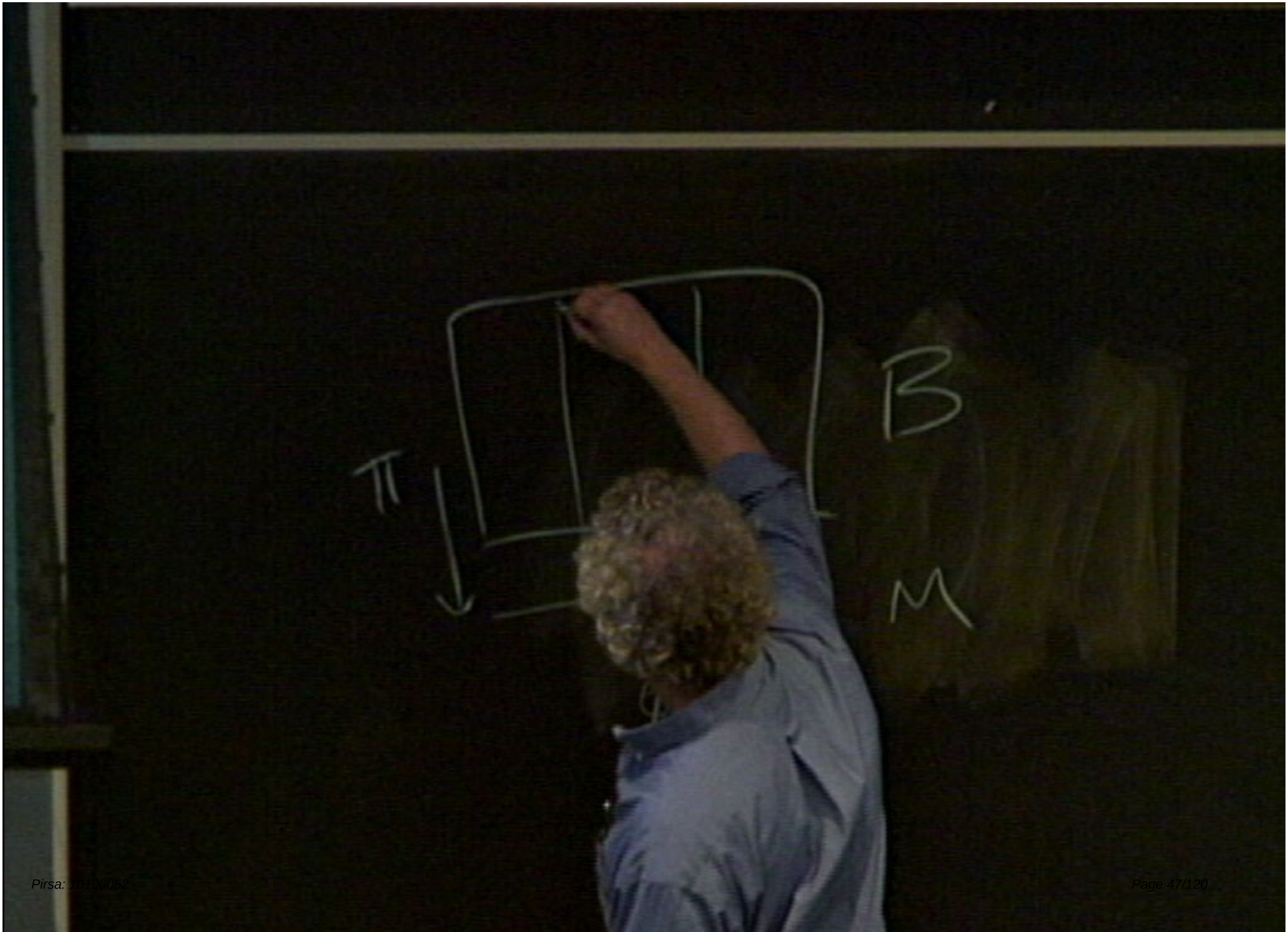
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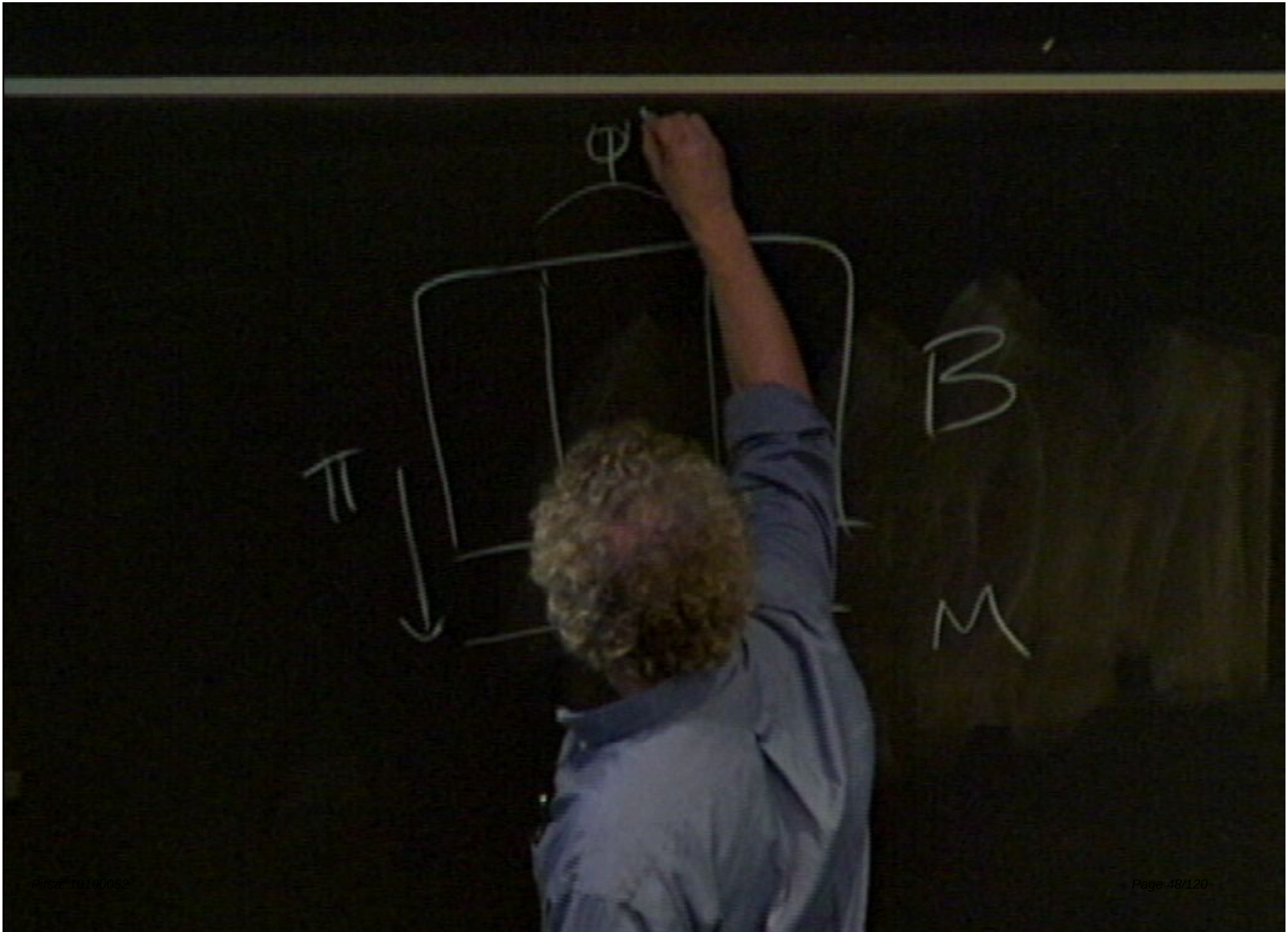
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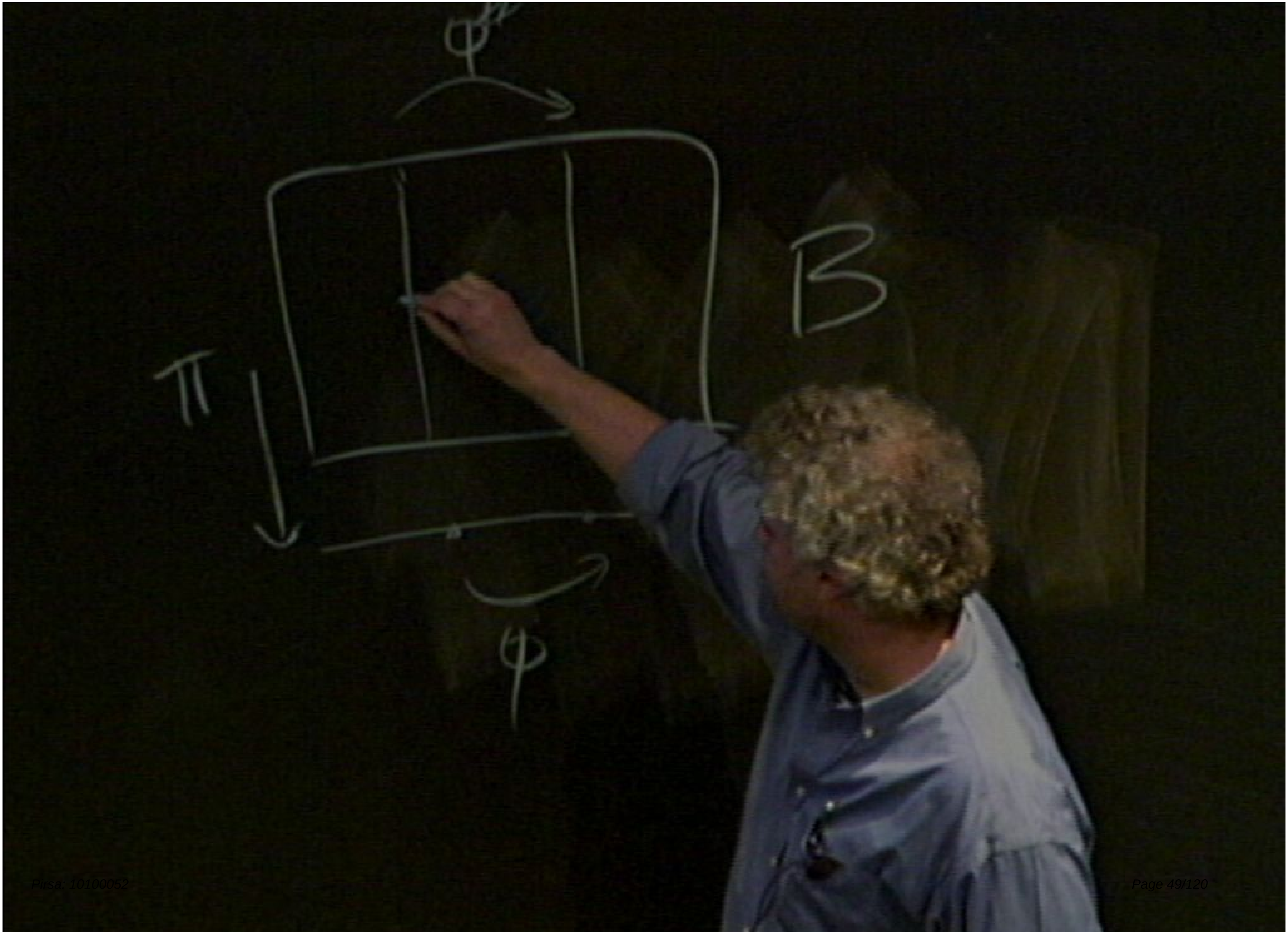
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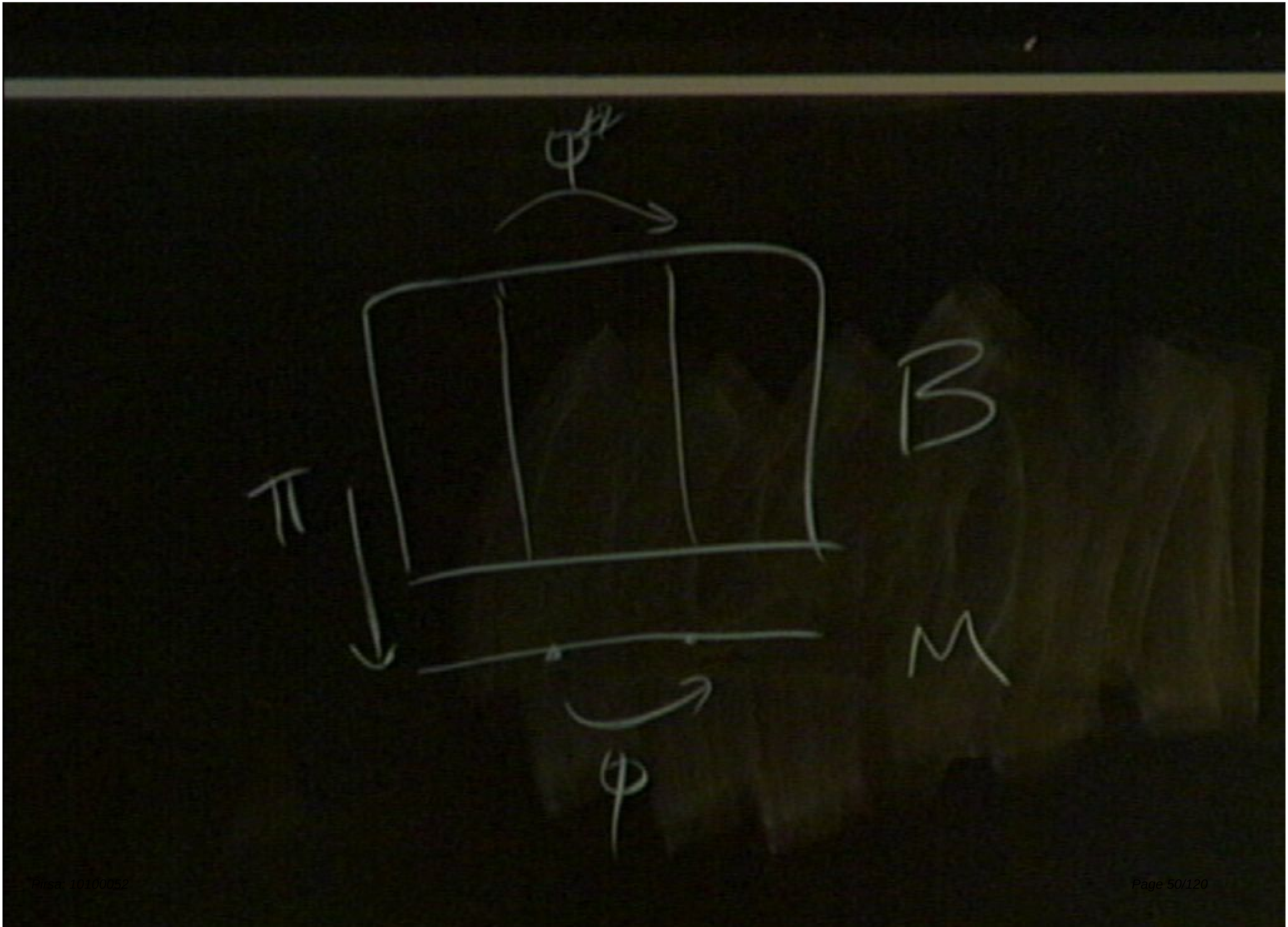
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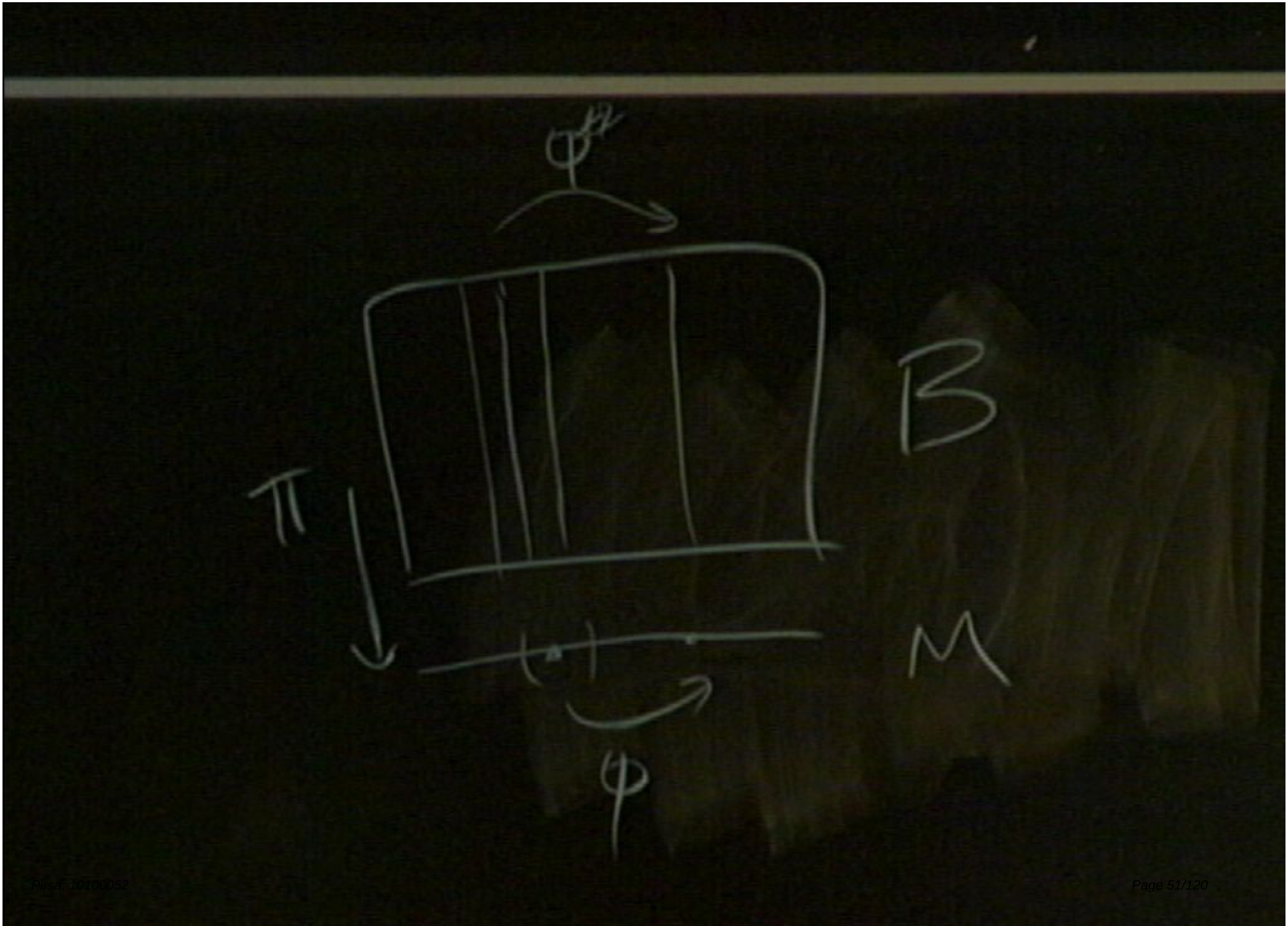
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Inductions

Let $\mathfrak{D}_{\mathcal{M}}$ and $\mathfrak{D}_{\mathcal{B}}$ be, respectively, the groups of diffeomorphisms on $\mathcal{M}$ and $\mathcal{B}$ to themselves, respectively. Define

$$\mathfrak{D}^{\sharp}_{\mathcal{B}} = \{\phi^{\sharp} \in \mathfrak{D}_{\mathcal{B}} : \exists \phi \in \mathfrak{D}_{\mathcal{M}} \text{ such that } \phi^{\sharp} \text{ is strongly consistent with } \phi\}$$

$\mathfrak{D}^{\sharp}_{\mathcal{B}}$ forms a subgroup of $\mathfrak{D}_{\mathcal{B}}$.

An *induction* is an injective homomorphism $\iota : \mathfrak{D}_{\mathcal{M}} \to \mathfrak{D}^{\sharp}_{\mathcal{B}}$.

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Geometric Fiber Bundles

$(\mathcal{B}, \mathcal{M}, \pi, \iota)$ is a *geometric fiber bundle* if $(\mathcal{B}, \mathcal{M}, \pi)$ is a fiber bundle and ι is an induction.

- tensor bundles are natural geometrical fiber bundles
- spinor bundles are not

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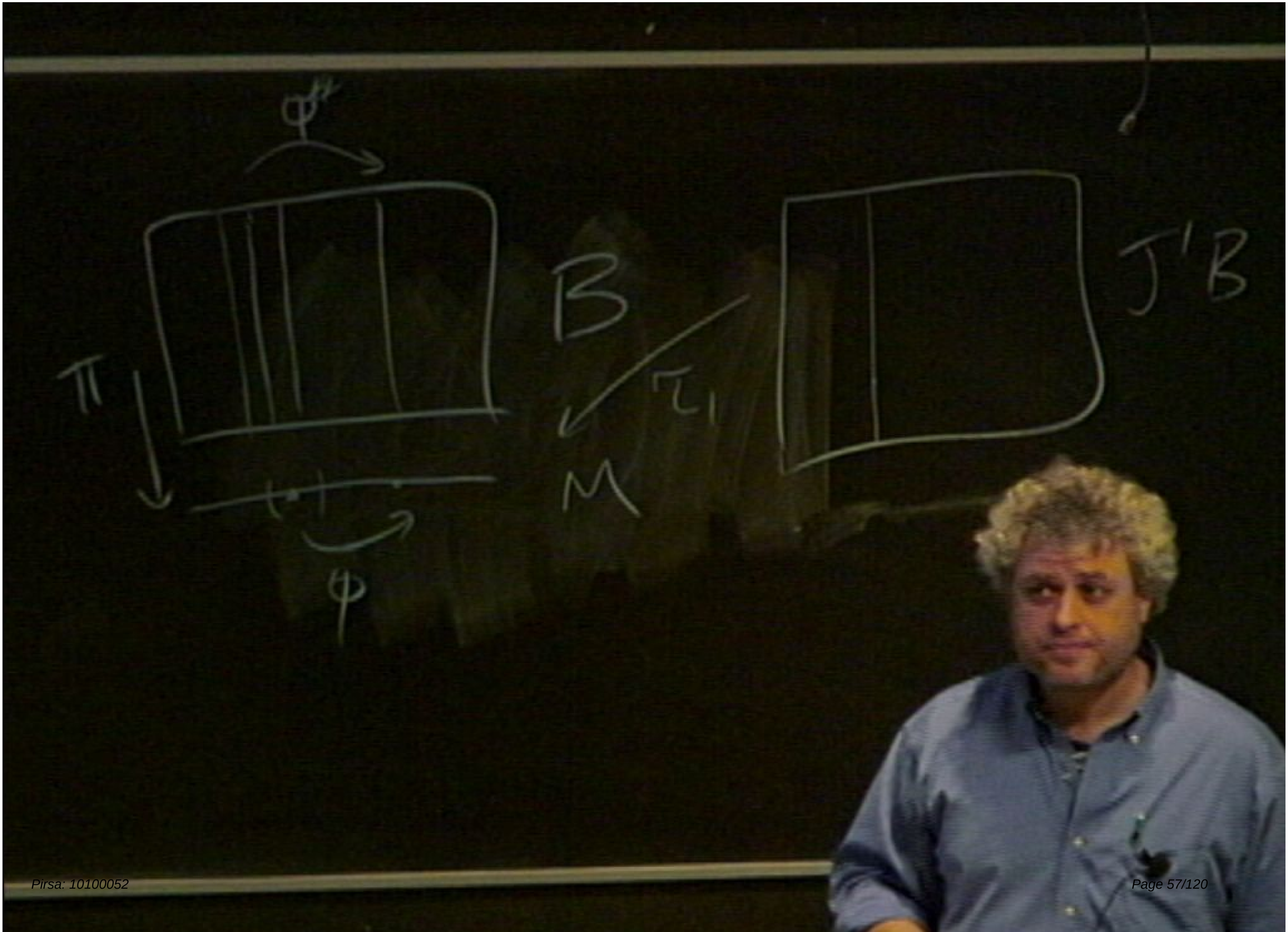
Jet Bundles

A bundle over a primary fiber bundle capturing the idea of a generalized derivative of a geometrical object—"equivalence of Taylor-series expansions to n^{th} order" gives $J^n\mathcal{B}$, n^{th}-order jet bundle over $\mathcal{B}$

A jet bundle over a geometric fiber bundle naturally inherits an induction itself.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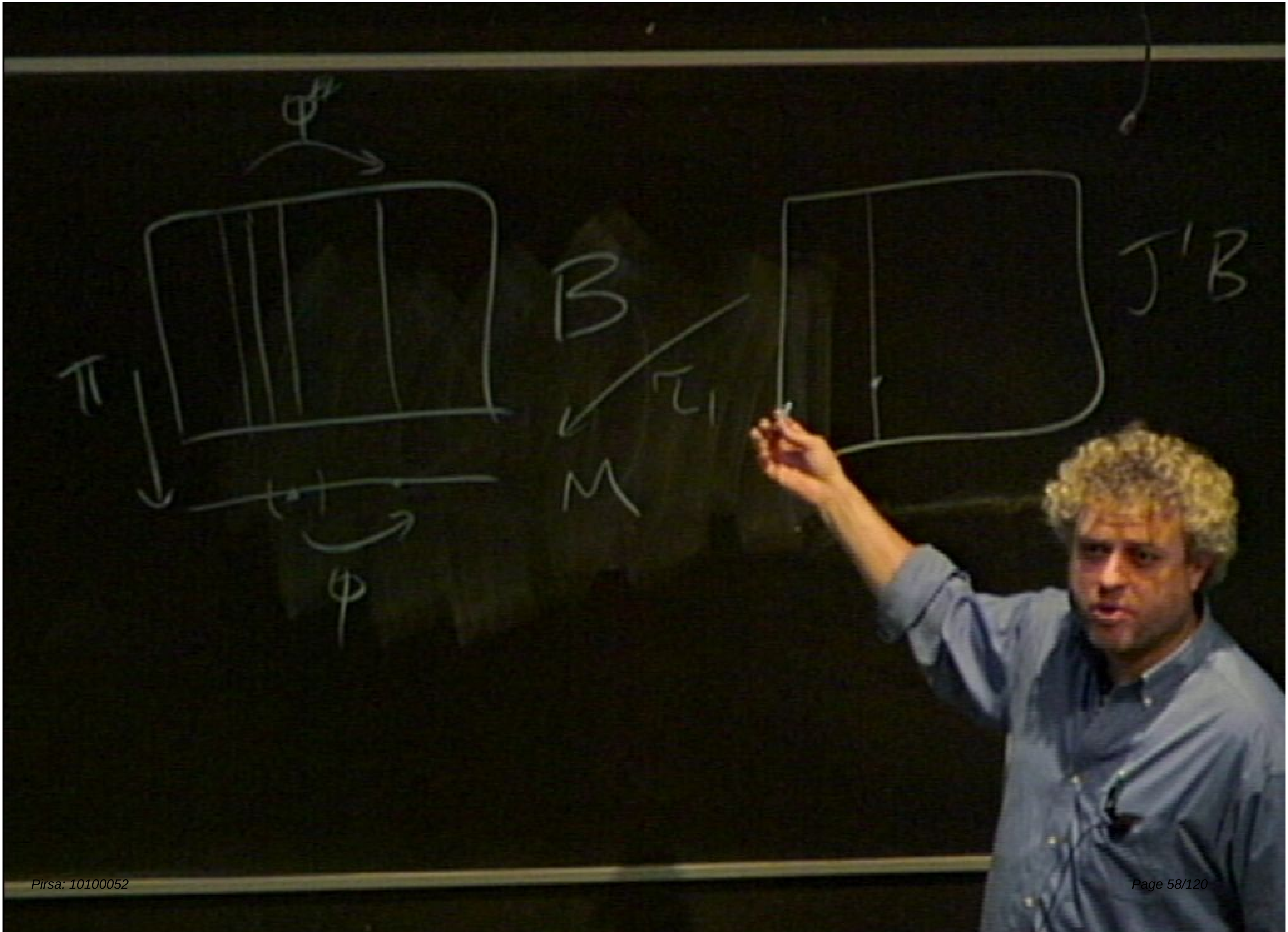
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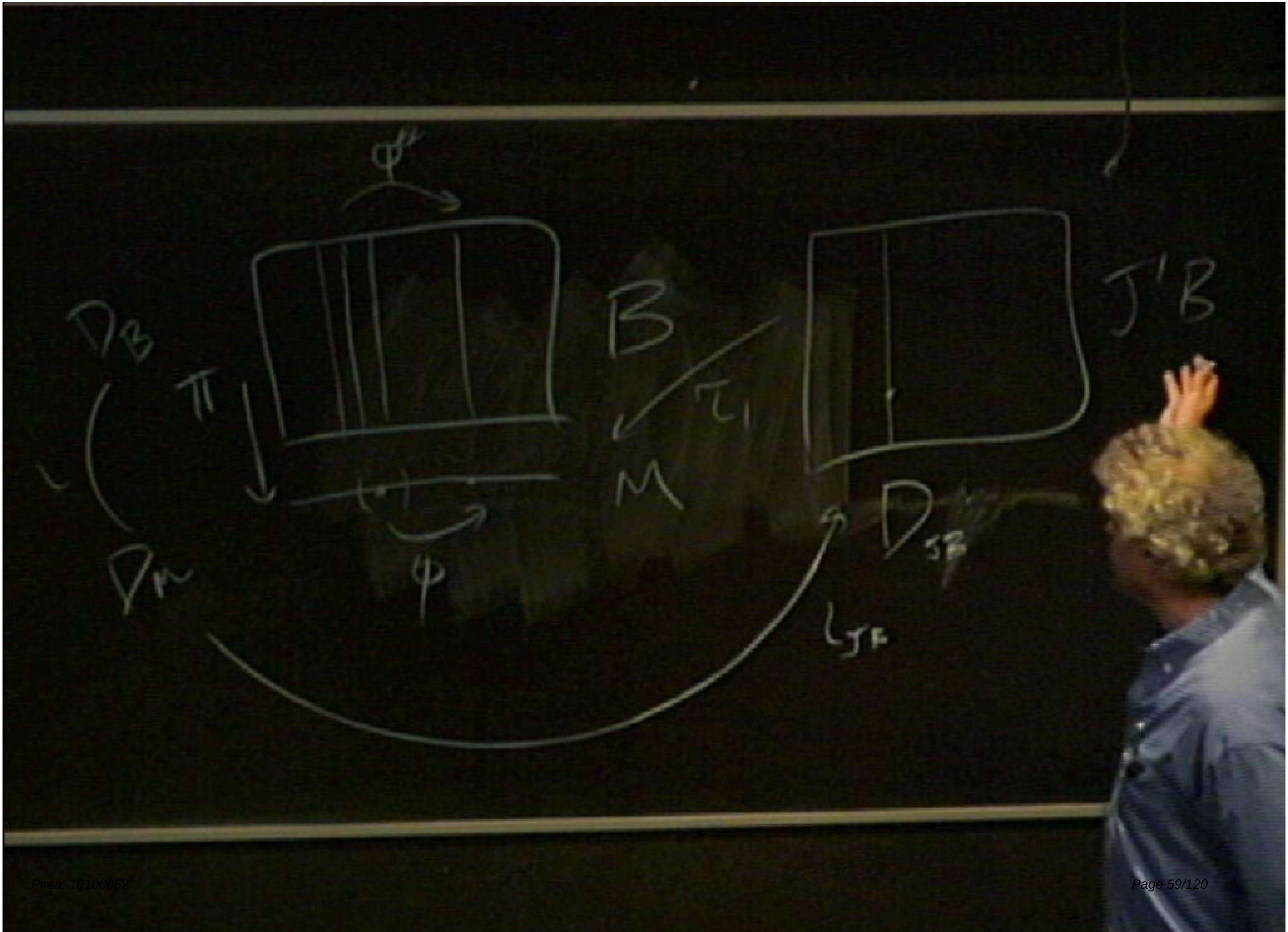
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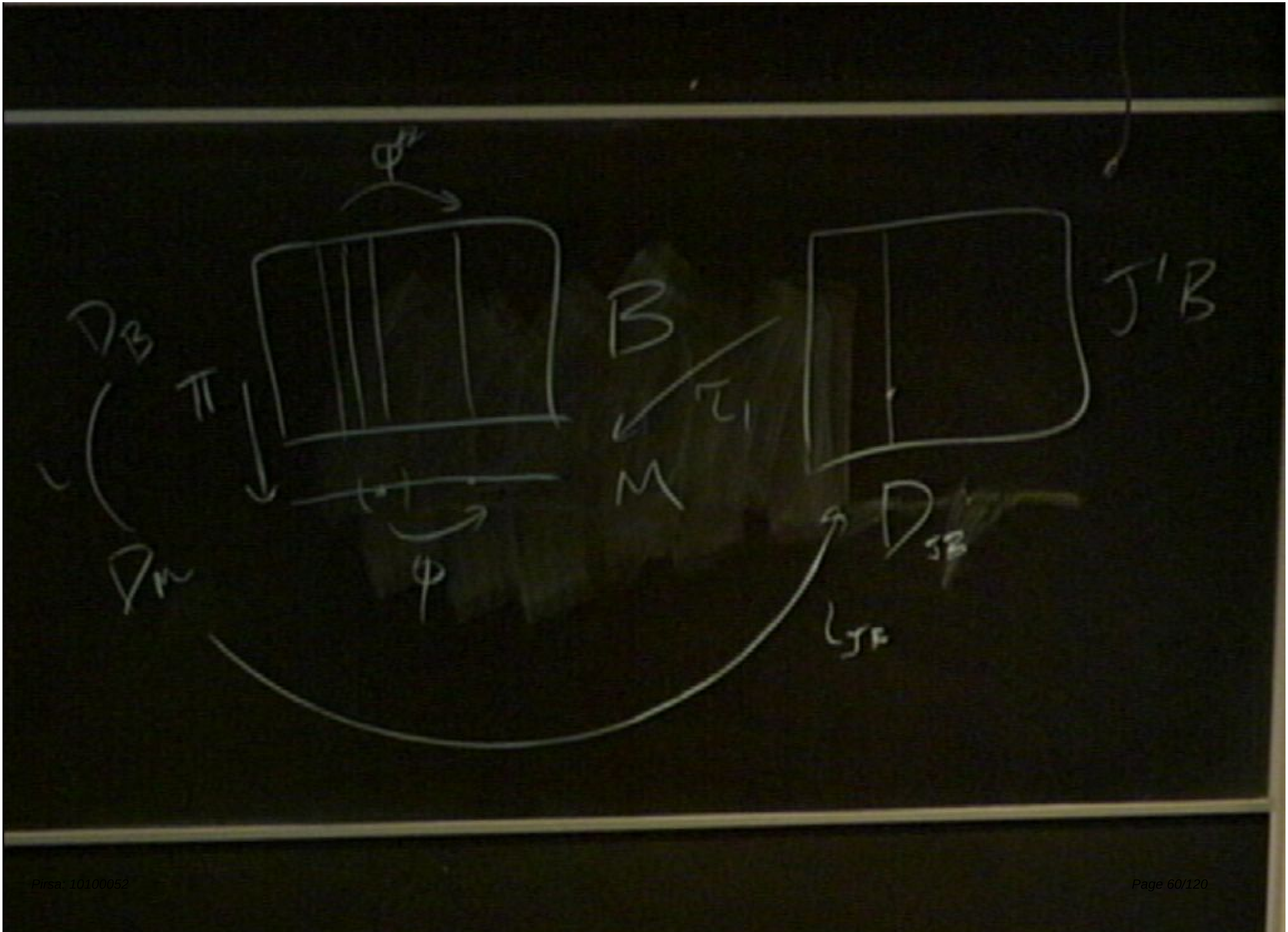

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Truncations

There is a natural projection from $J^2\mathcal{B}$ to $J^1\mathcal{B}$, the *truncation* $\theta^{2,1}$, characterized by "dropping the second-order terms in the Taylor expansion". In general, one has the natural truncation $\theta^{n,m} : J^n\mathcal{B} \to J^m\mathcal{B}$ for all $0 < m < n$.

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Concomitants

Let $(\mathcal{B}_1, \mathcal{M}, \pi_1, \iota)$ and $(\mathcal{B}_2, \mathcal{M}, \pi_2, \jmath)$ be two geometric fiber bundles.

A *zeroth-order concomitant* from $\mathcal{B}_1$ to $\mathcal{B}_2$ is a smooth mapping $\chi : \mathcal{B}_1 \longrightarrow \mathcal{B}_2$ such that

$$(\forall p \in \mathcal{B}_1)(\forall \phi \in \mathfrak{A}_{\mathcal{M}})\{\jmath(\phi)(\chi(p)) = \chi(\iota(\phi)(p))\}$$

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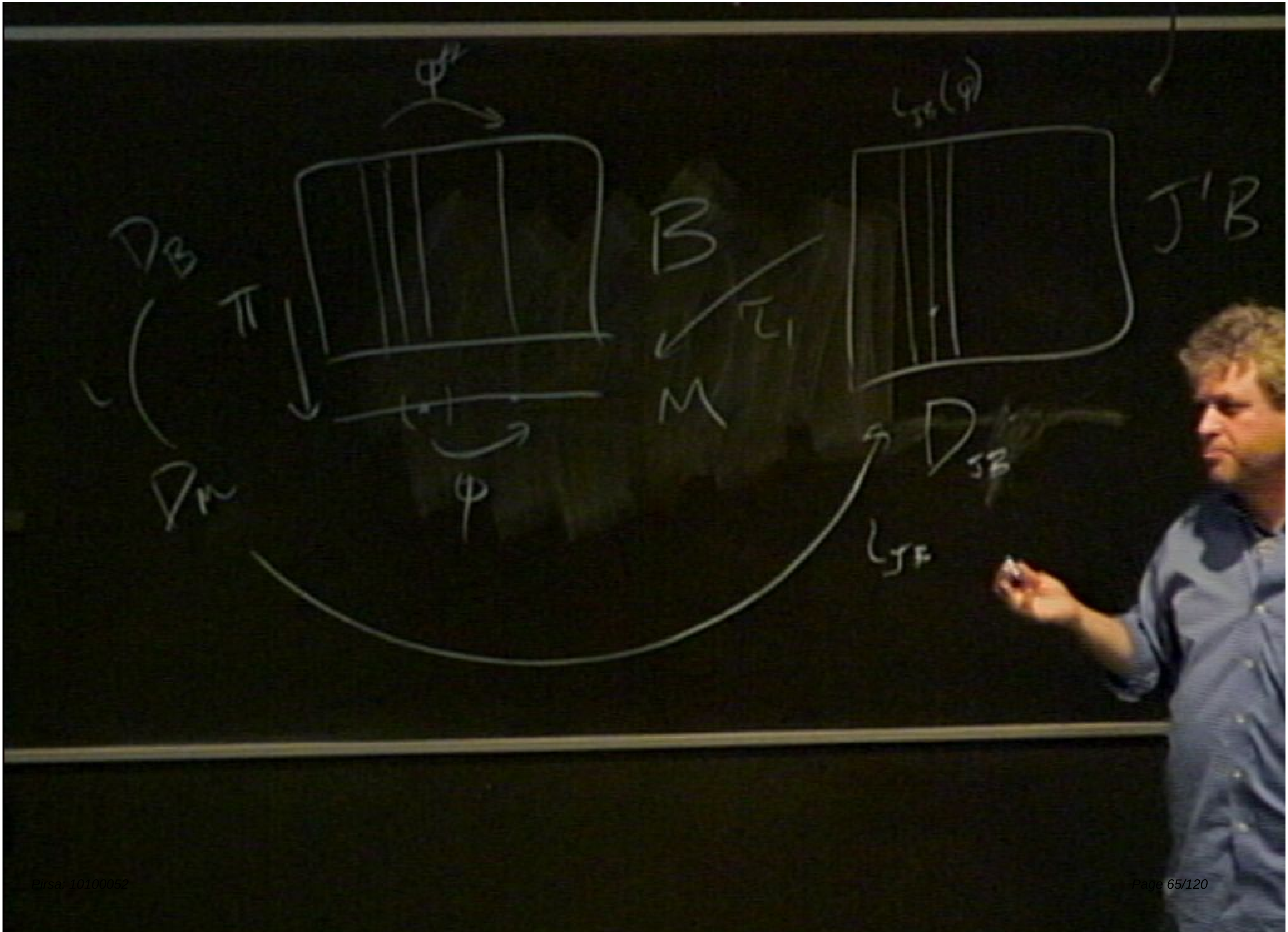

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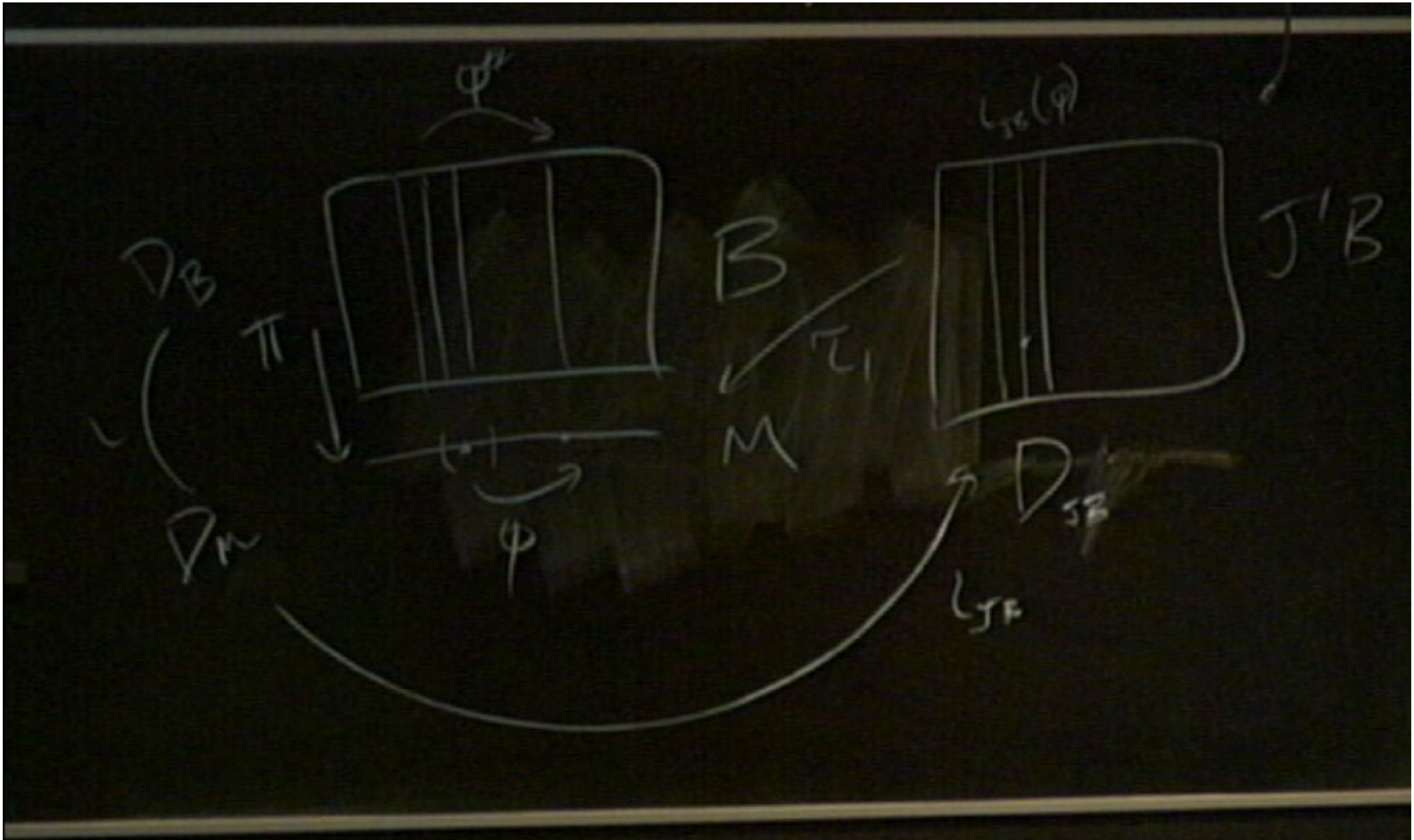

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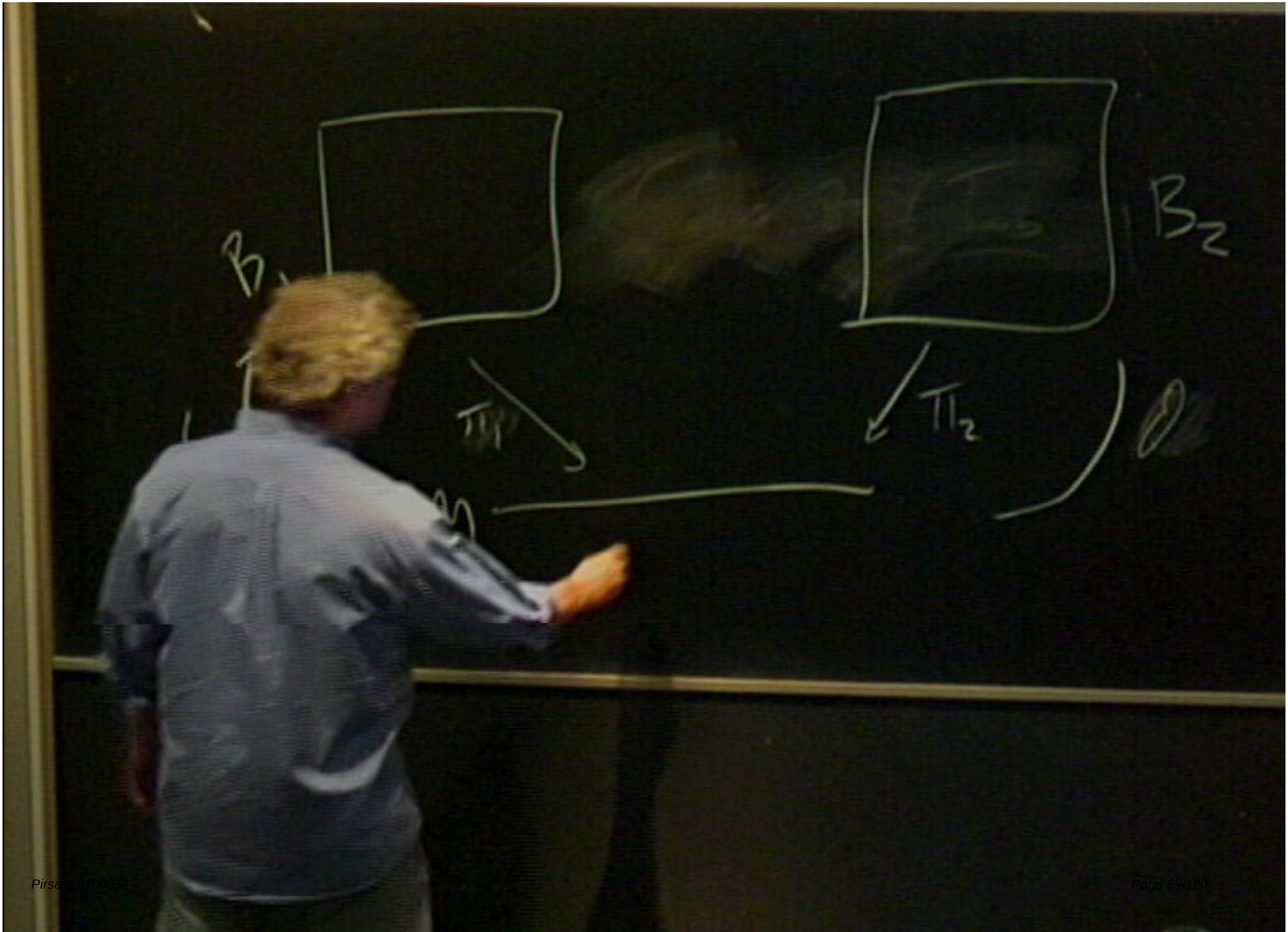
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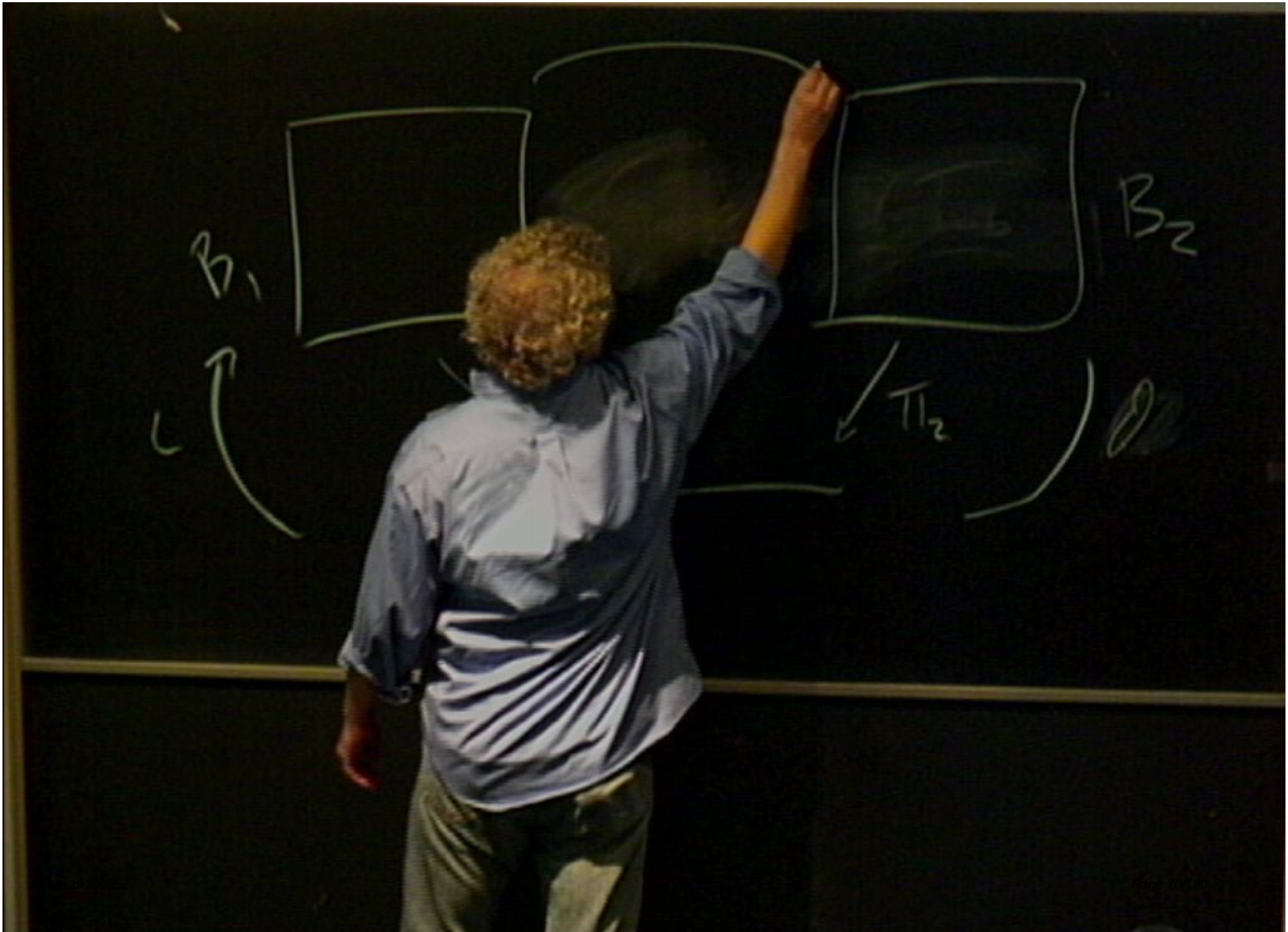
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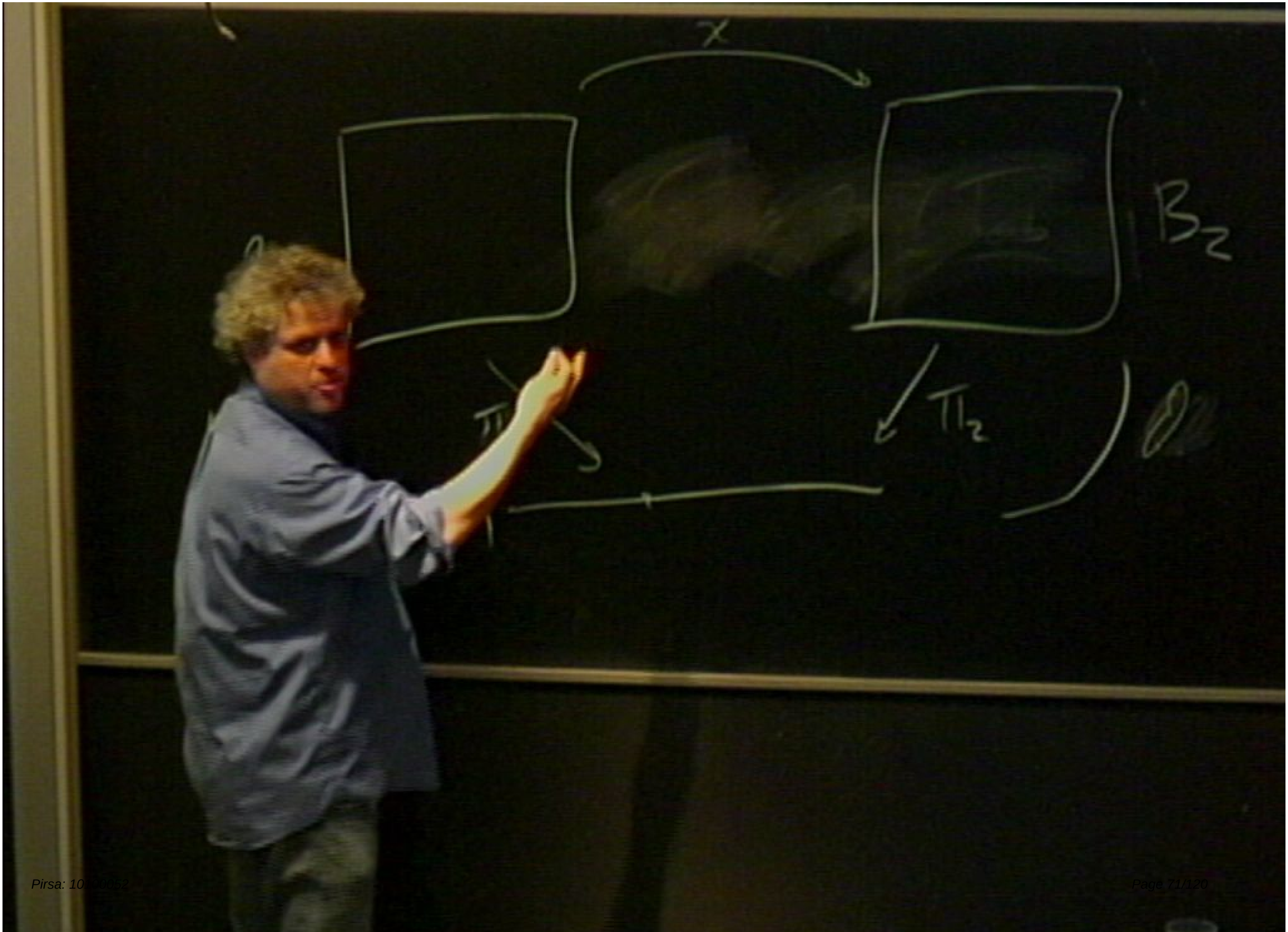
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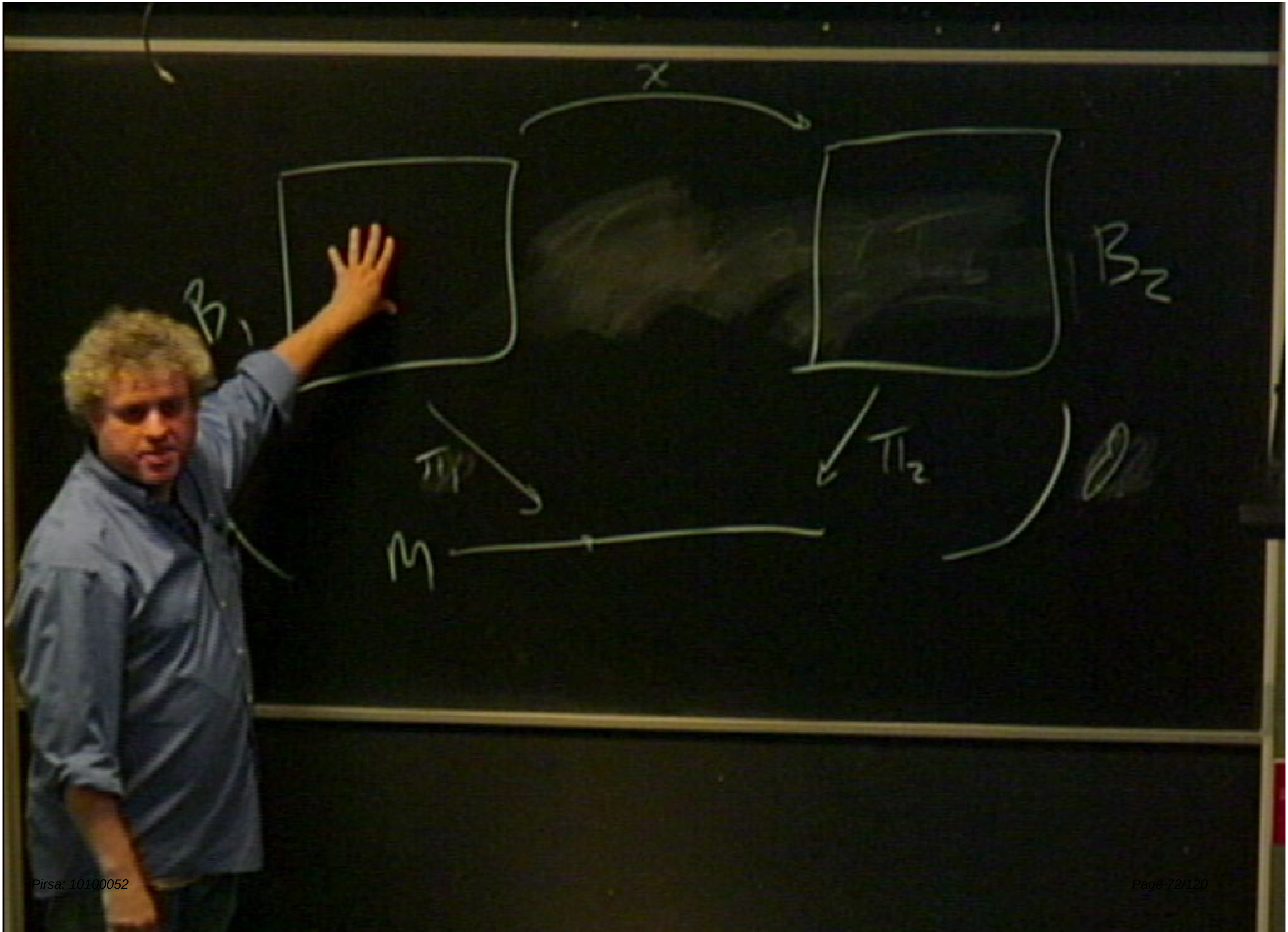
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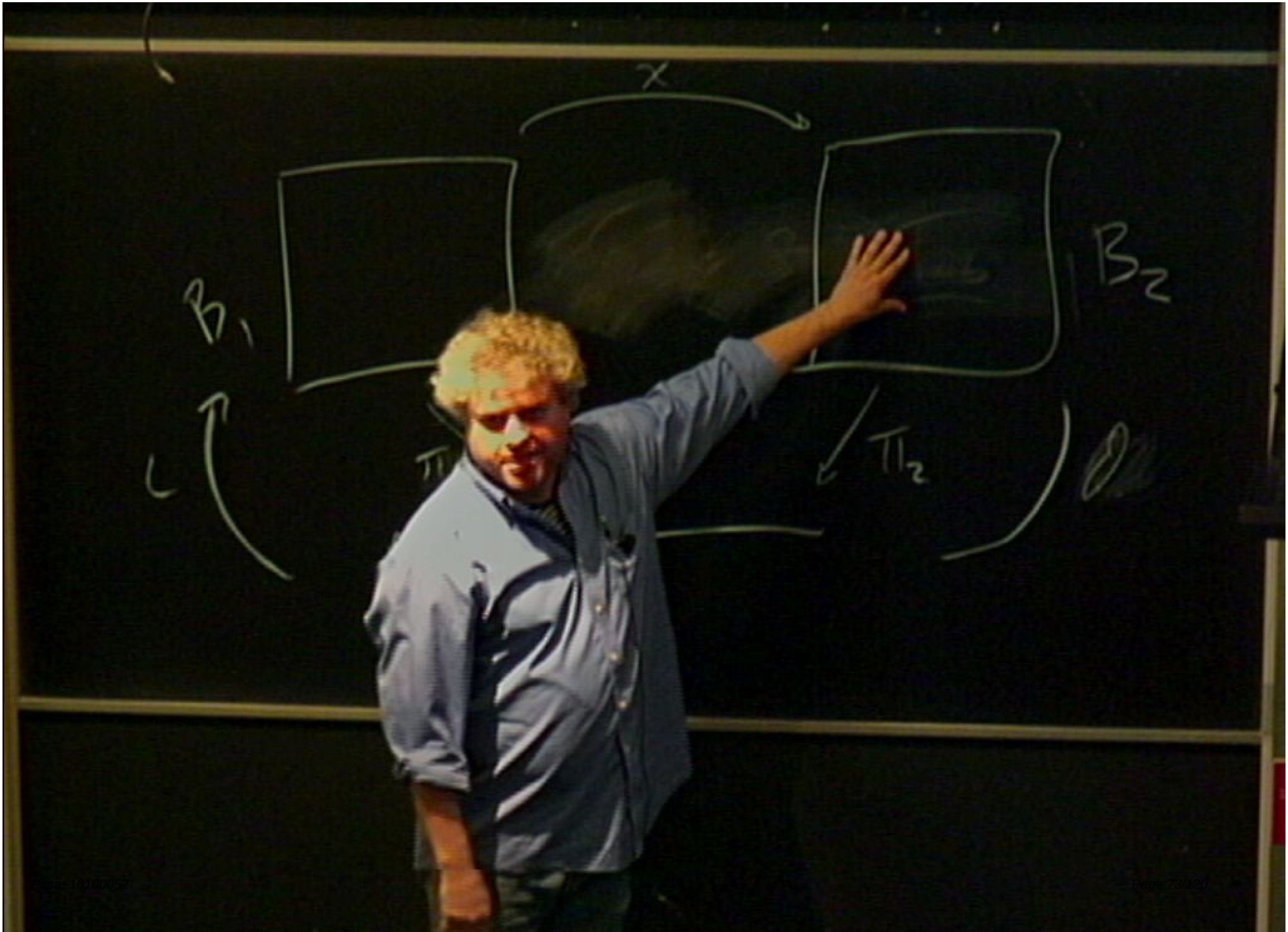
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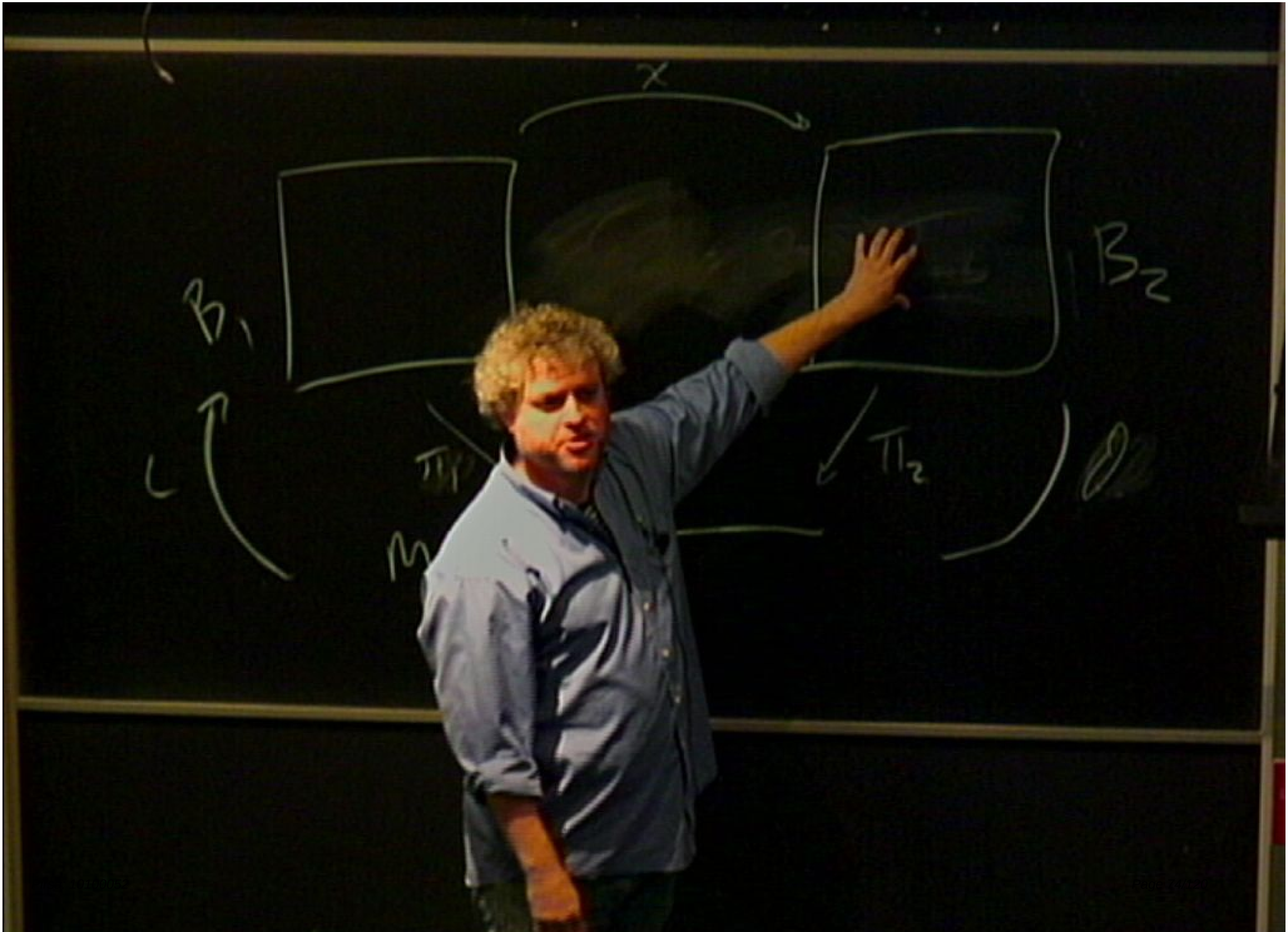
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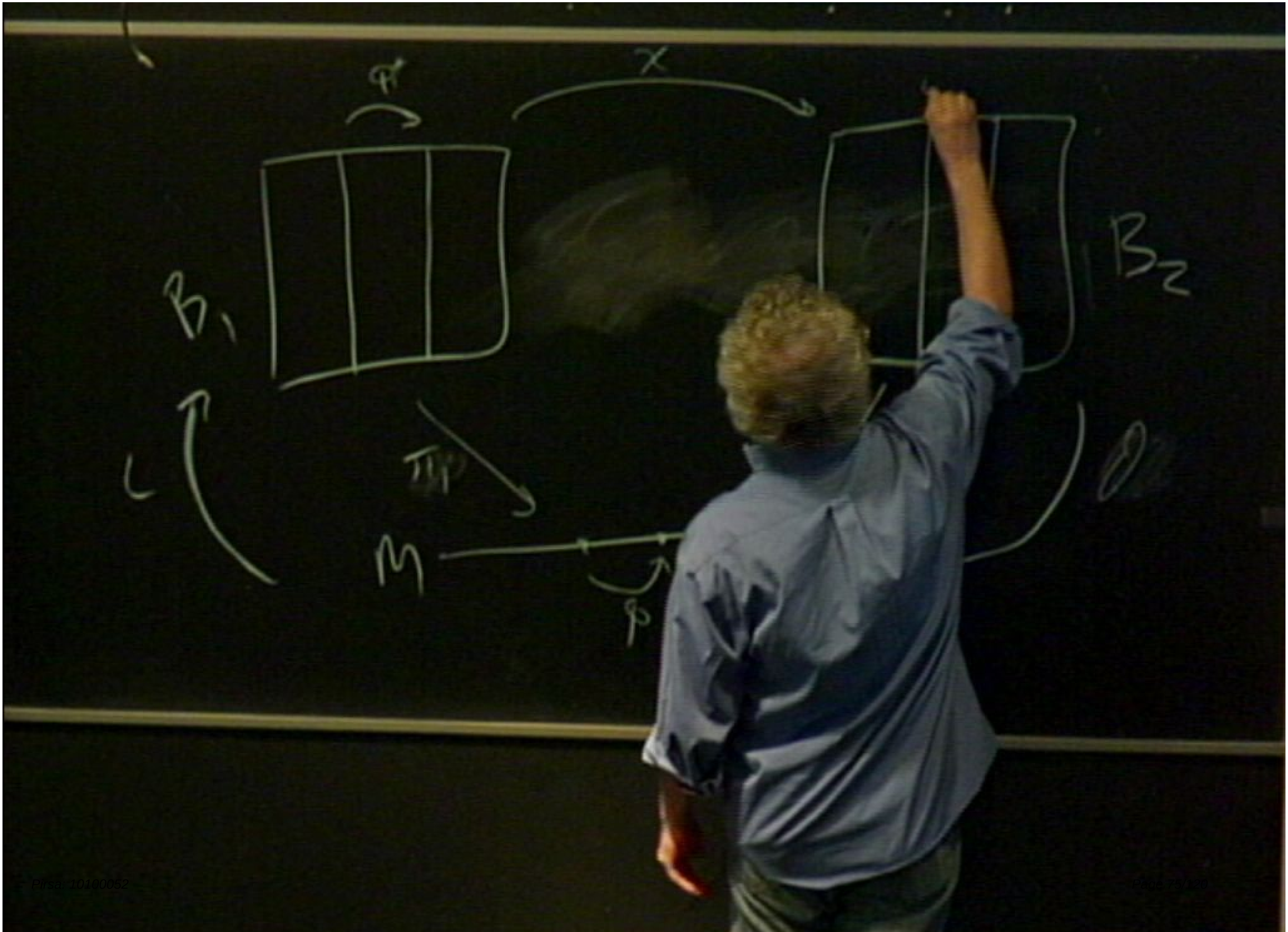
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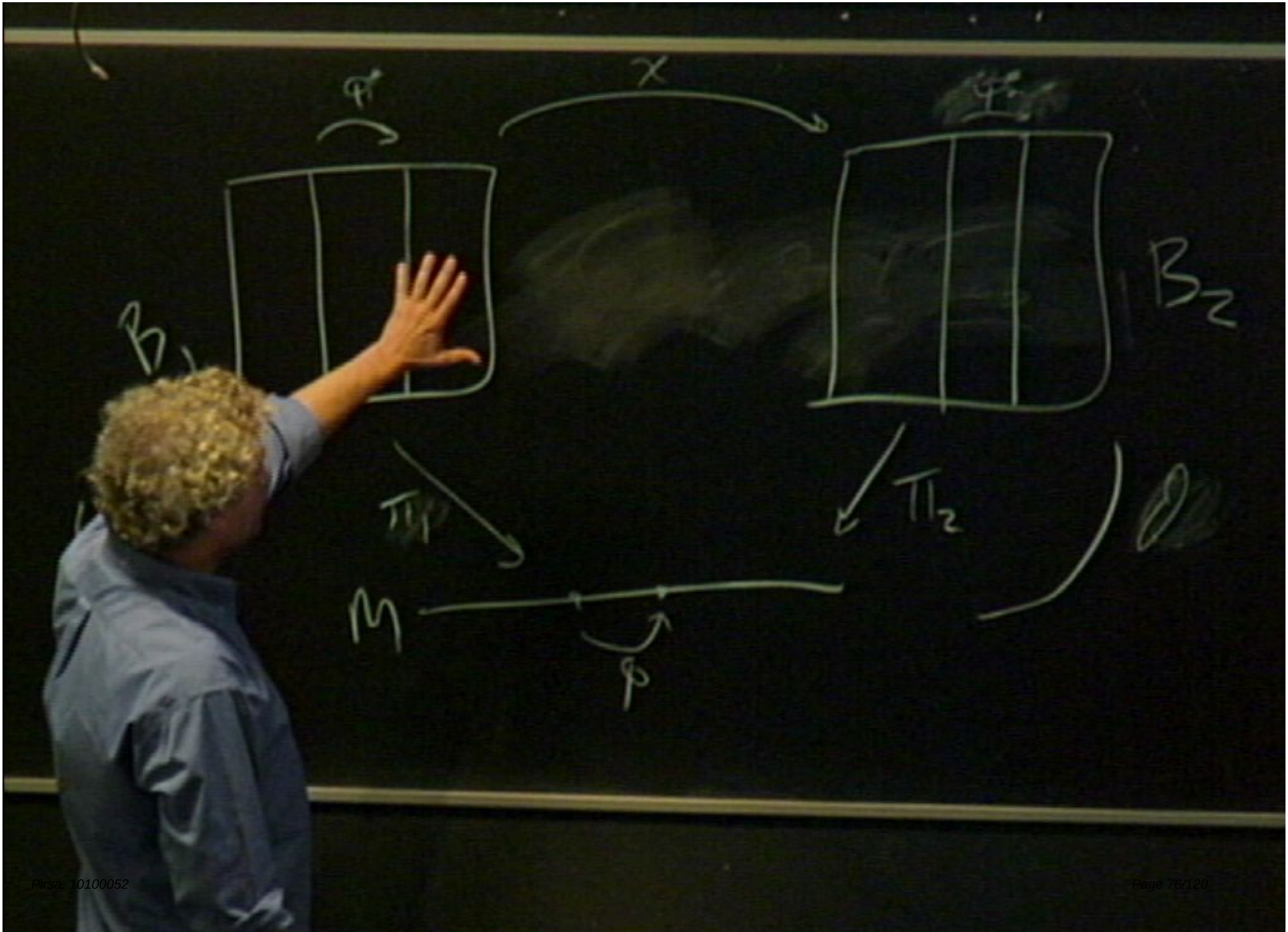
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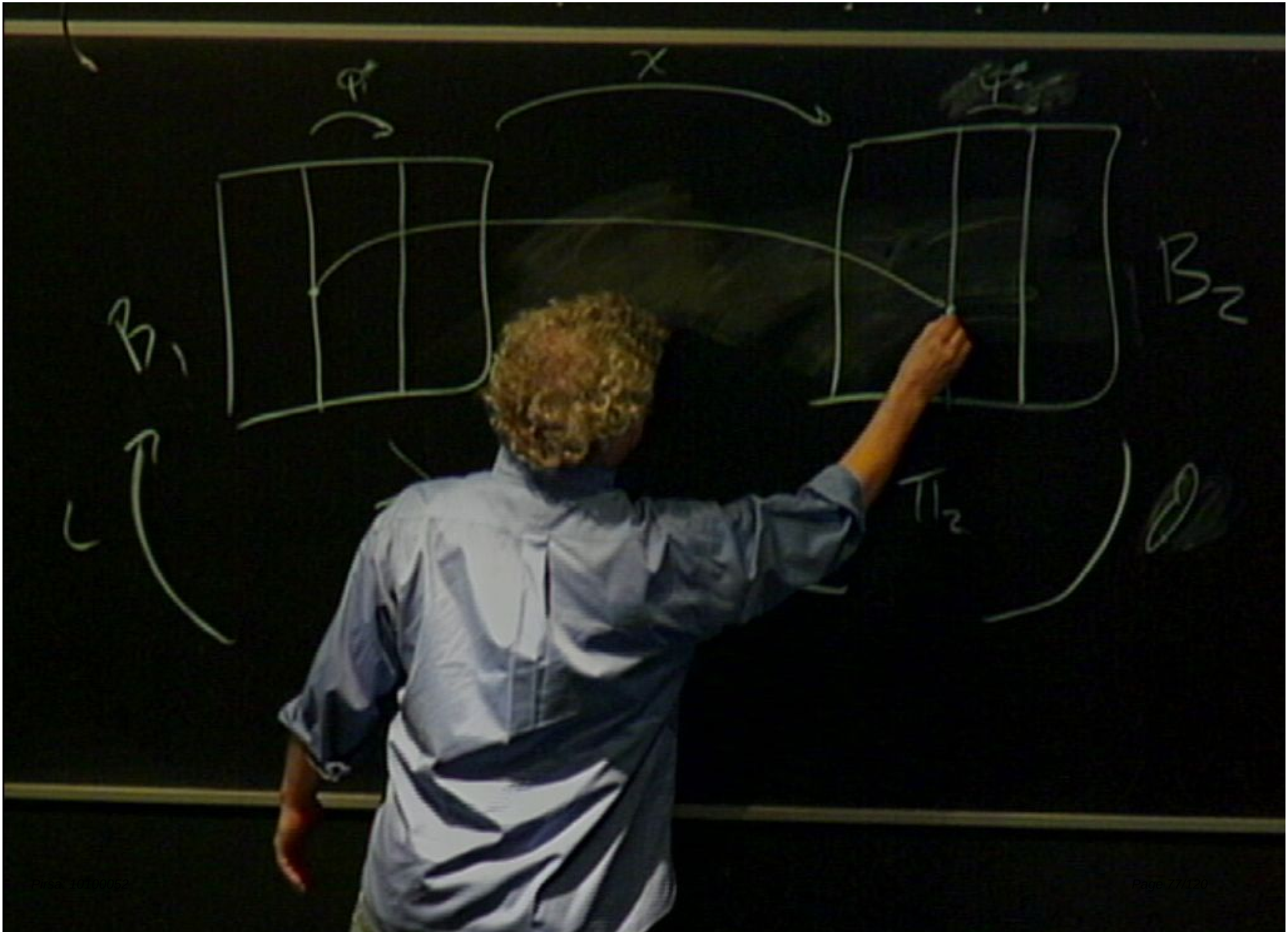
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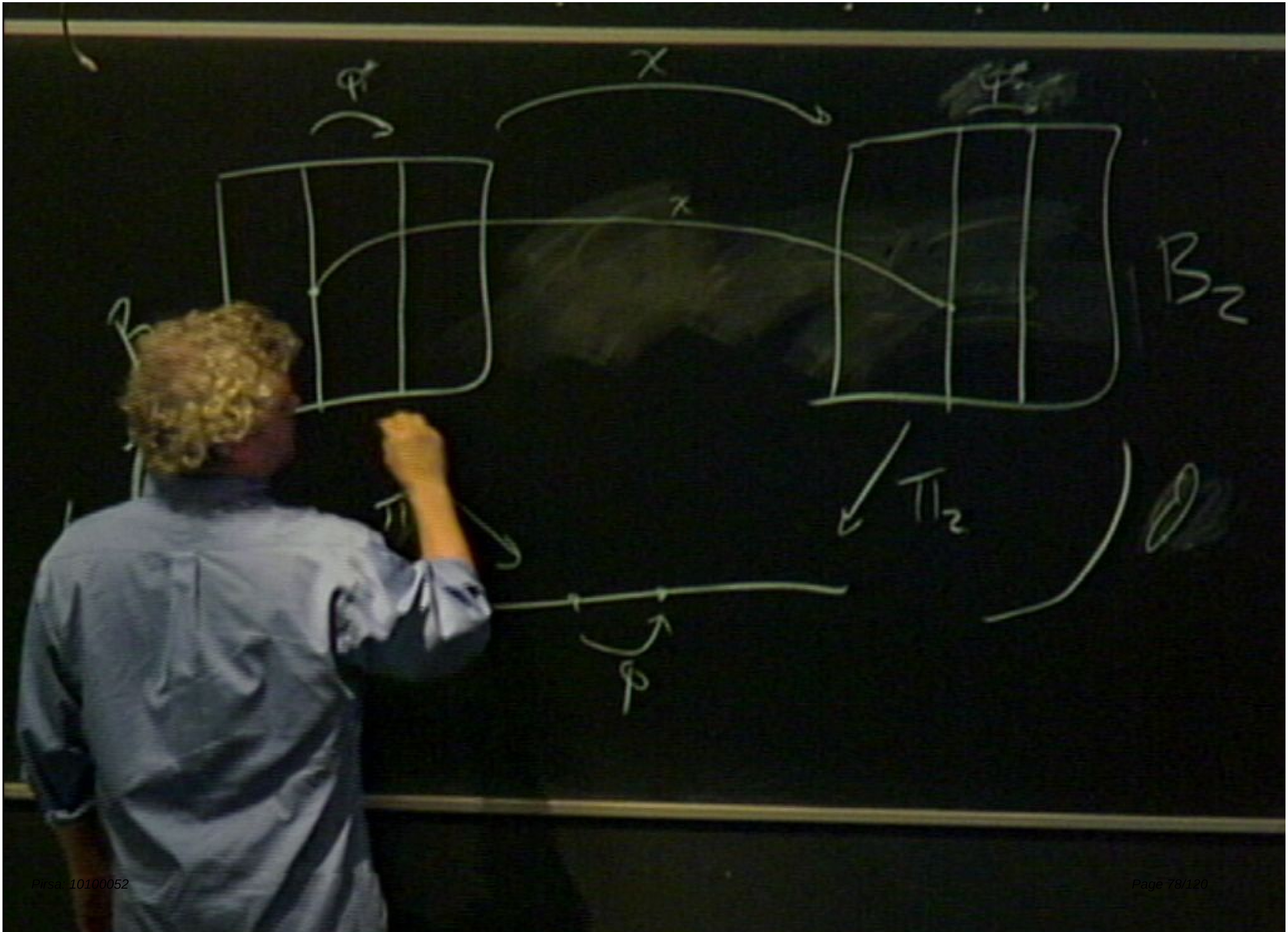
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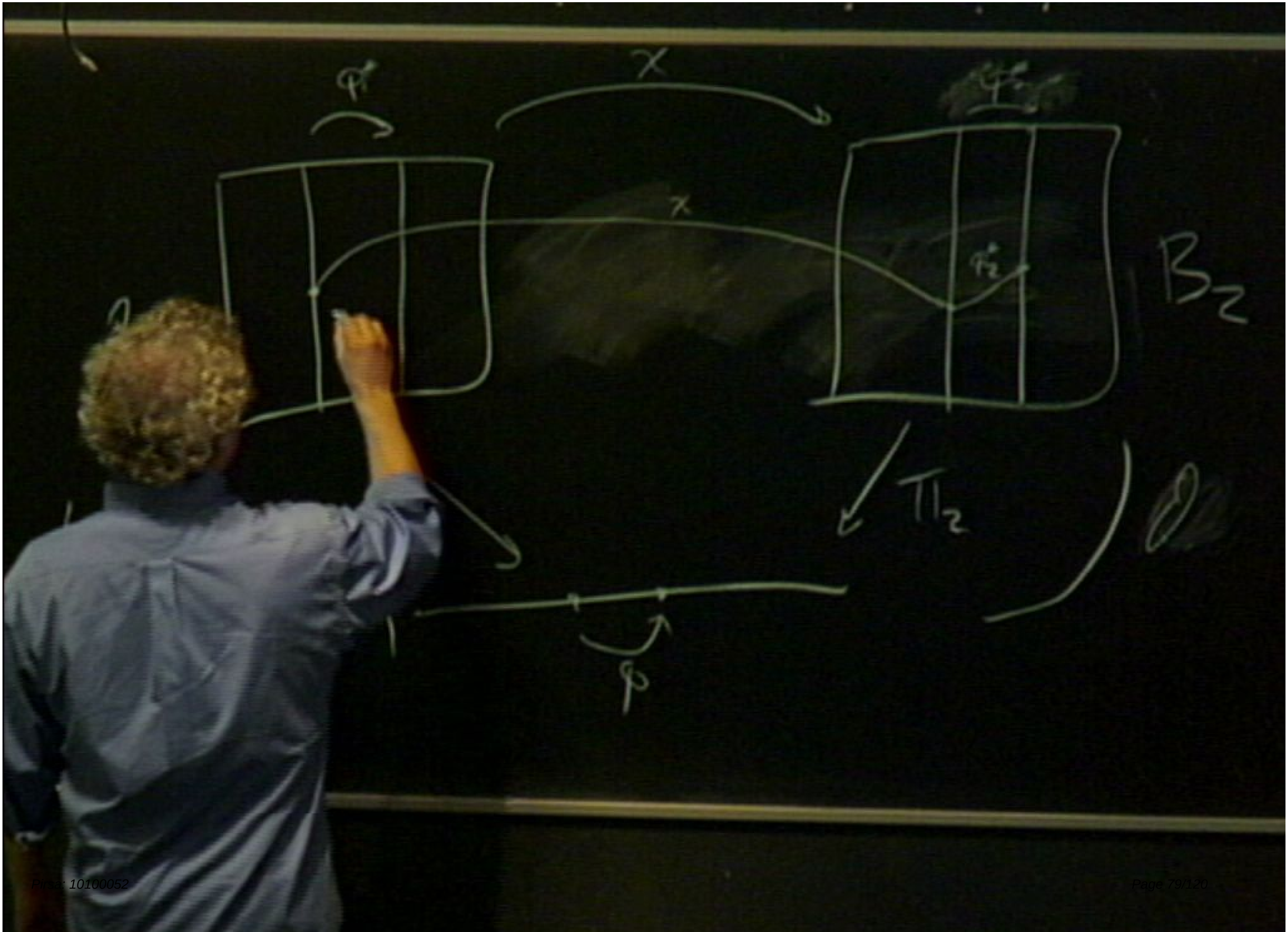
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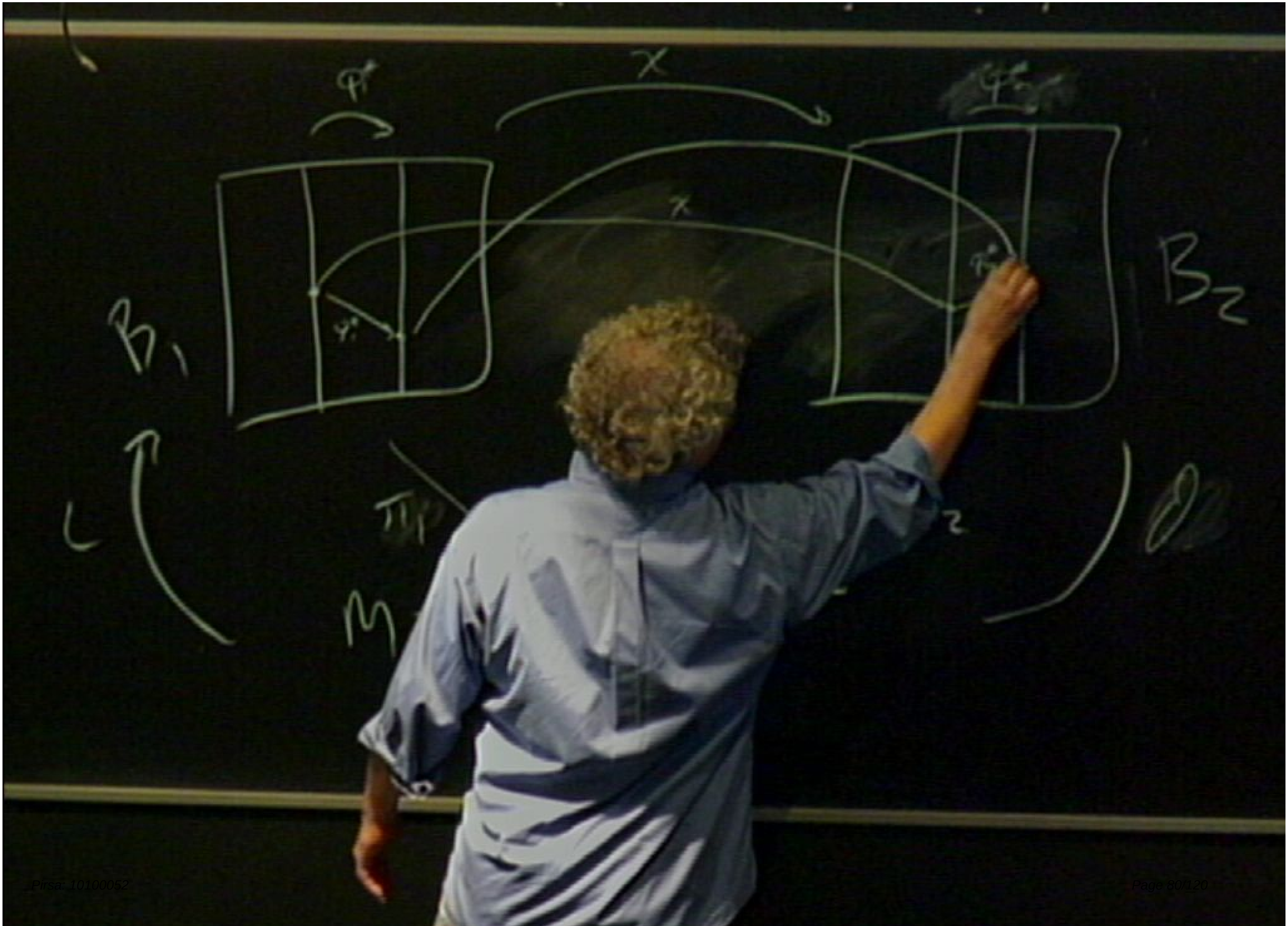
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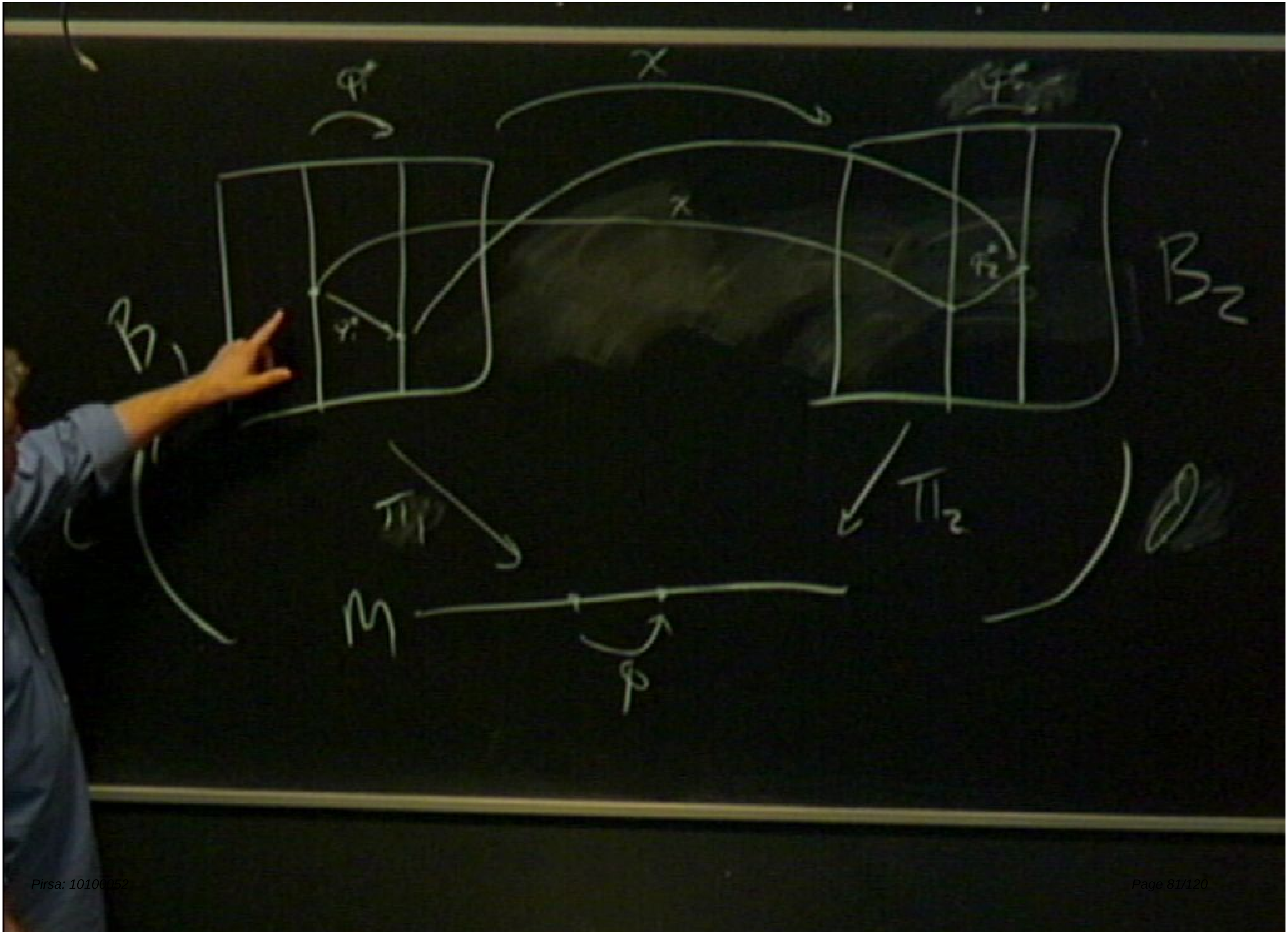
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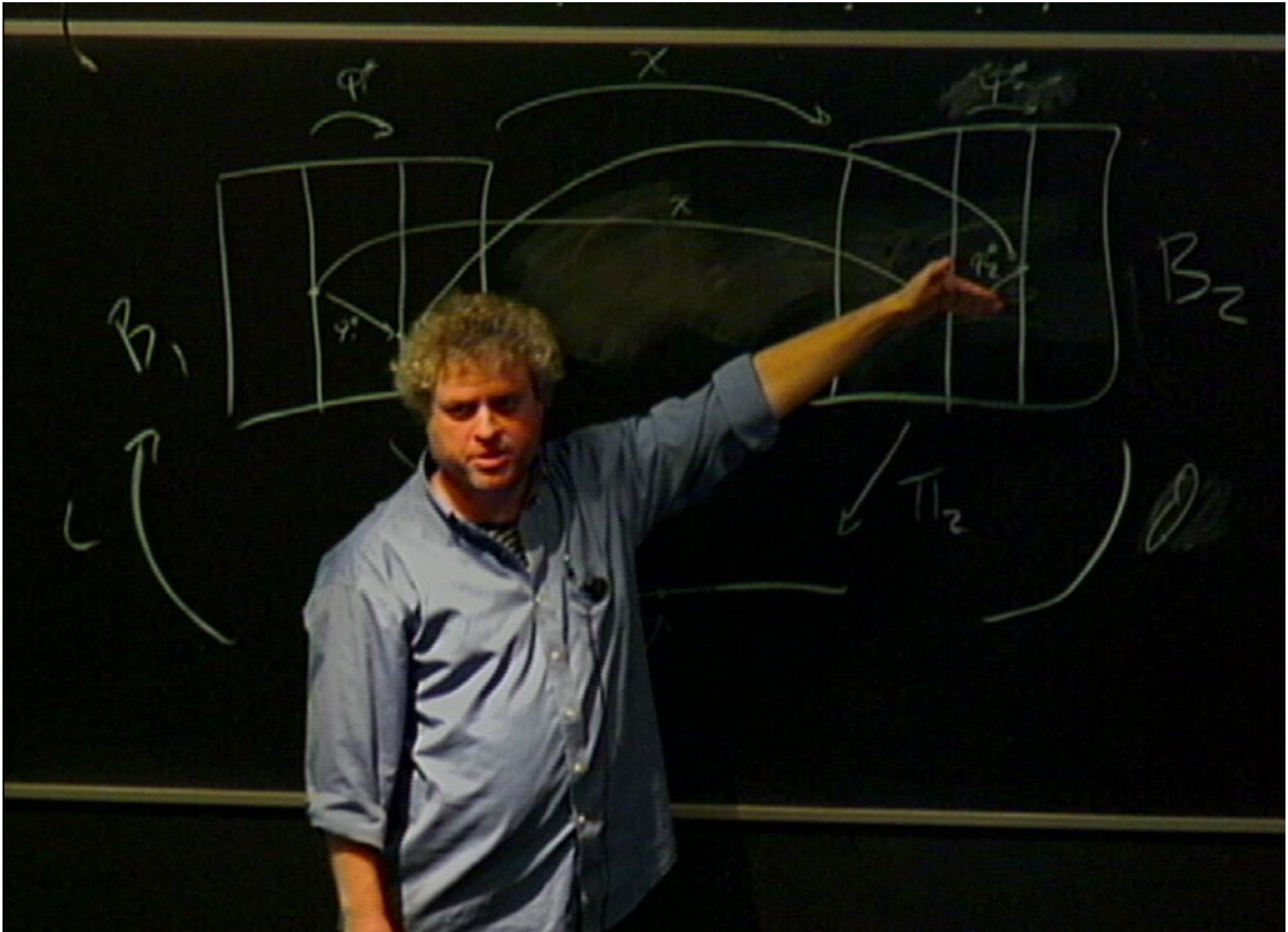
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Concomitants of the Metric

Theorem

First jet bundle of bundle of metrics, $J^1\mathcal{B}_g$, is naturally diffeomorphic to the fiber bundle with fibers consisting of all (g_{ab}, ∇_a); the second jet bundle, $J^2\mathcal{B}_g$, is naturally diffeomorphic to the fiber bundle with fibers consisting of all $(g_{ab}, \nabla_a, R^a{}_{bcd})$.

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1. Energetic Quantities in General Relativity
2. The Principle of Equivalence: A Bad Argument
3. Geometric Fiber Bundles
4. Concomitants
5. There Is No Gravitational Stress-Energy Tensor

Homogeneous Concomitants

A concomitant is *homogeneous of weight w* if for any constant scalar field σ

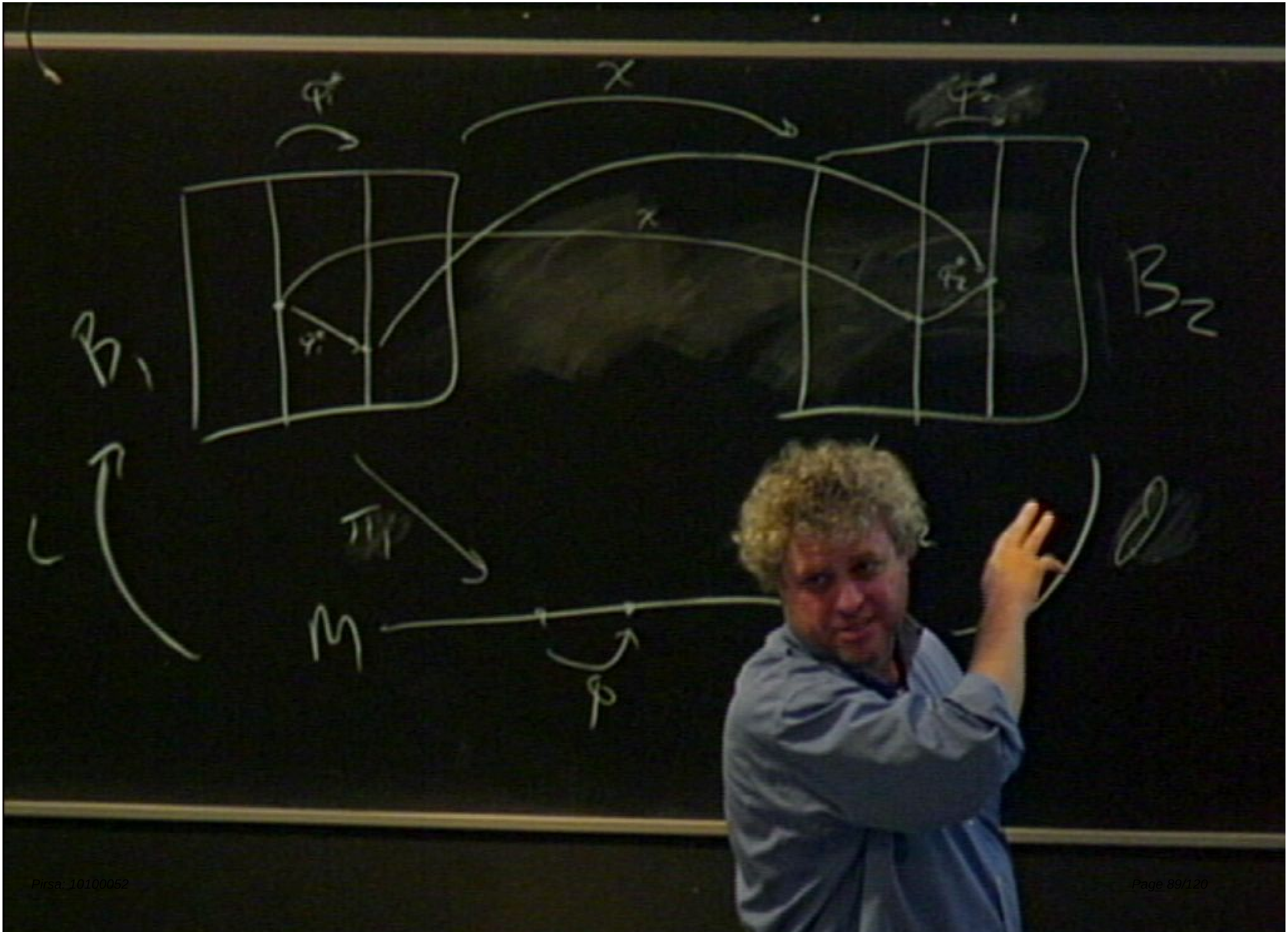
$$\chi(\iota_1[\phi](\sigma u_1)) = \sigma^w \iota_2[\phi](\chi(u_1))$$

Homogeneous Concomitants of the Metric

Lemma

If, for $n \geq 2$, S_{ab} is an n^{th}-order homogeneous concomitant of g_{ab}, then to rescale the metric by the constant real number λ multiplies S_{ab} by λ^{n-2}.

In particular, S_{ab} does not rescale when $g_{ab} \longrightarrow \lambda g_{ab}$ if and only if it is a second-order concomitant of g_{ab}.


B1
B2
M

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Demand Vanishing of Covariant Derivative?

Can the "gravitational field" interact with ponderable matter fields so as to exchange stress-energy? If so, then, presumably, there could be interaction states characterized (in part) by:

1. $\nabla^n(T_{na} + S_{na}) = 0$
2. $\nabla^n T_{na} \neq 0$
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To avoid deep and speculative waters about the way S_{ab} might enter the righthand side of the Einstein Field Equation, safest to demand vanishing covariant divergence only when there is no ponderable matter.

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Necessary Condition for Gravitational Stress-Energy Tensor

To sum up, we have the following necessary condition:

Condition

The only viable candidates for a gravitational stress-energy tensor are two covariant-index, symmetric, second-order, zero-weight homogeneous concomitants of the metric that are not zero when the Riemann tensor is not zero and that have vanishing covariant divergence when the stress-energy tensor of ponderable matter vanishes.

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All Second-Order and Higher, Symmetric, Divergence-Free Concomitants of the Metric

Theorem

Every second-order and higher, symmetric, divergence-free concomitants of the metric of the form S_{ab} is the variation with respect to the metric of some scalar curvature-invariant.

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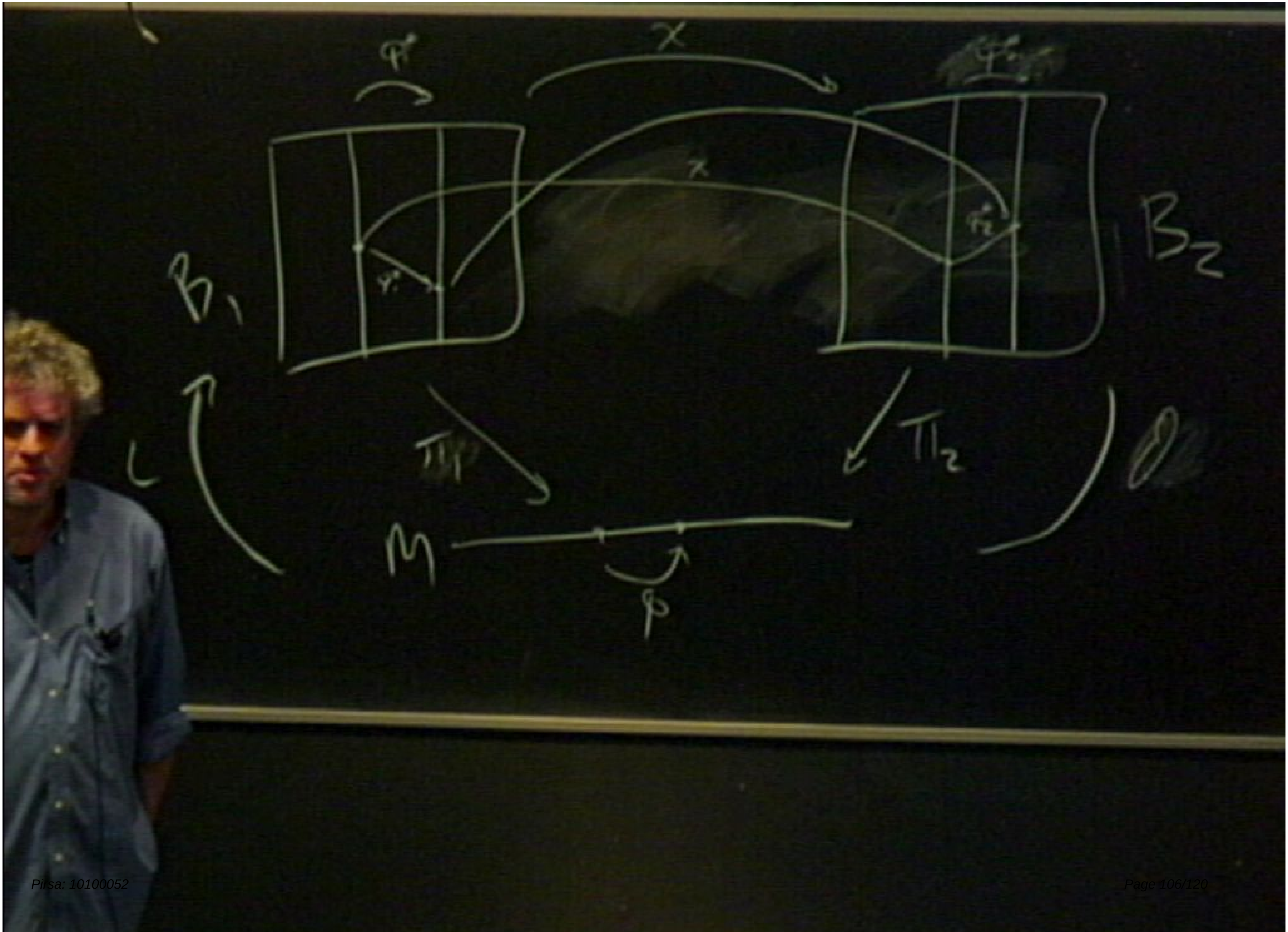
There Is No Gravitational Stress-Energy Tensor

Theorem

The only two covariant-index, symmetric, divergence-free, second-order, zero-weight homogeneous concomitants of the metric are constant multiples of the Einstein tensor.

Corollary

There are no two covariant-index, symmetric, divergence-free, second-order, homogeneous concomitants of the metric that do not vanish when the Riemann tensor does not vanishx.



χ
B_1
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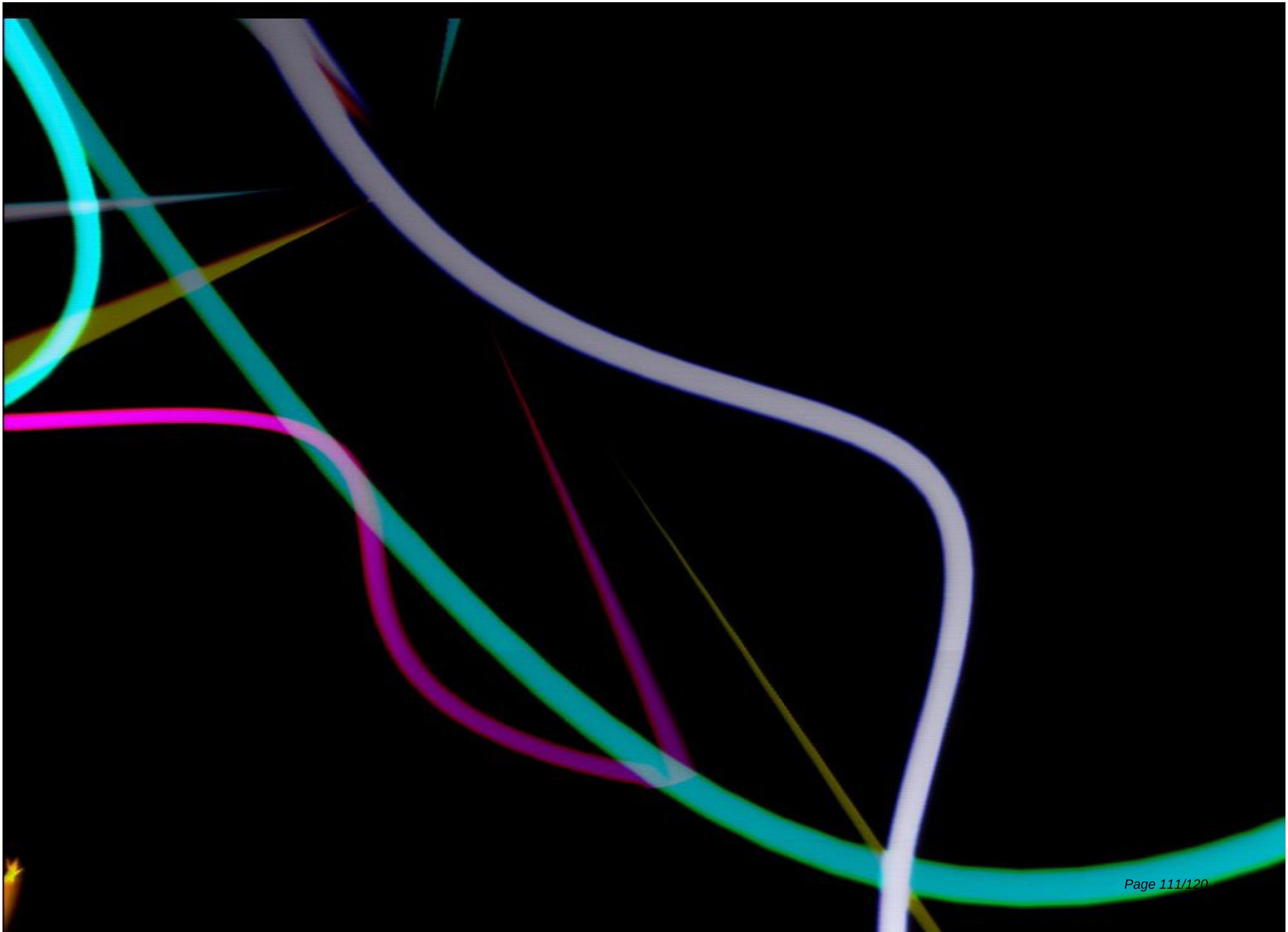

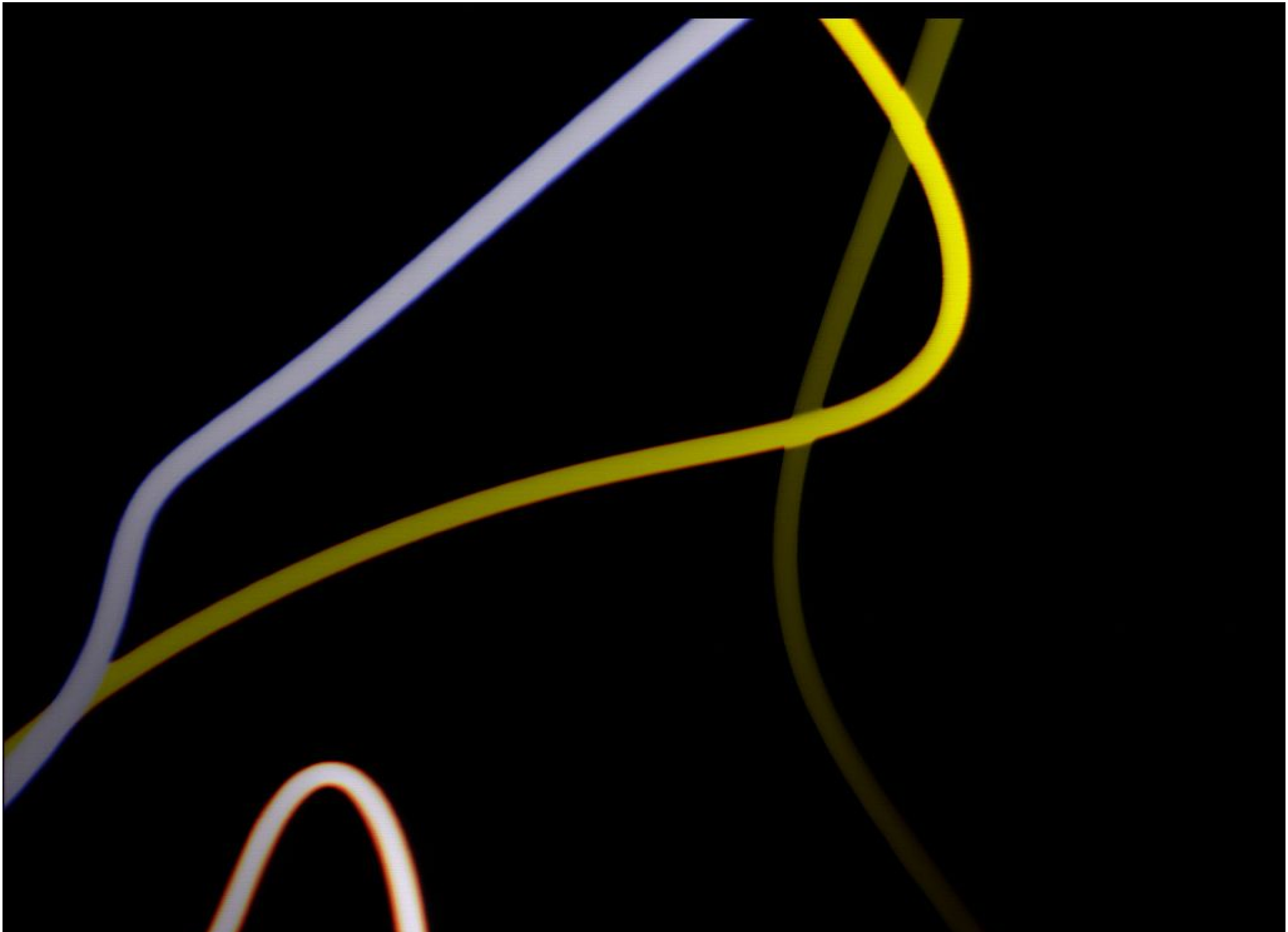

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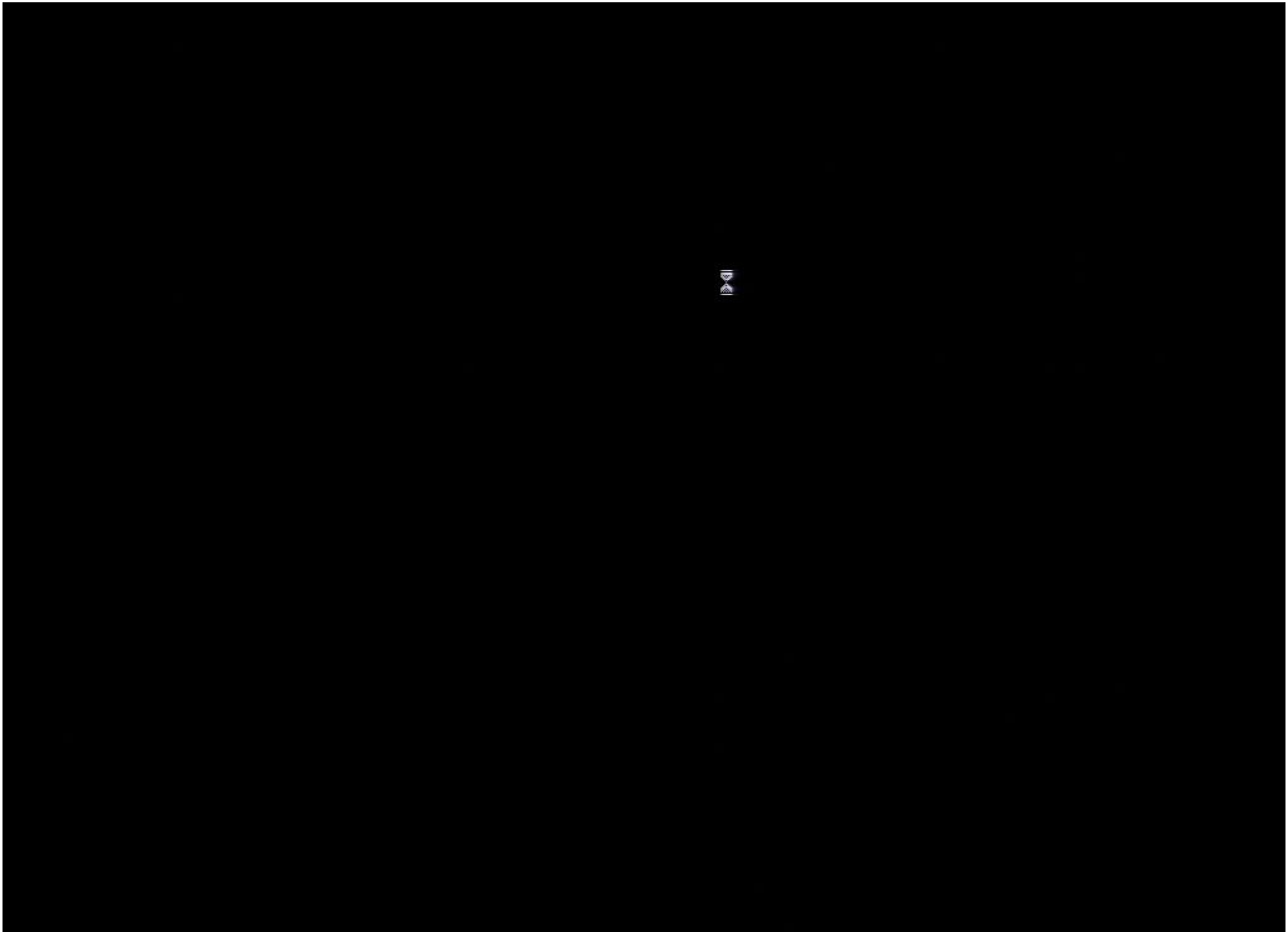

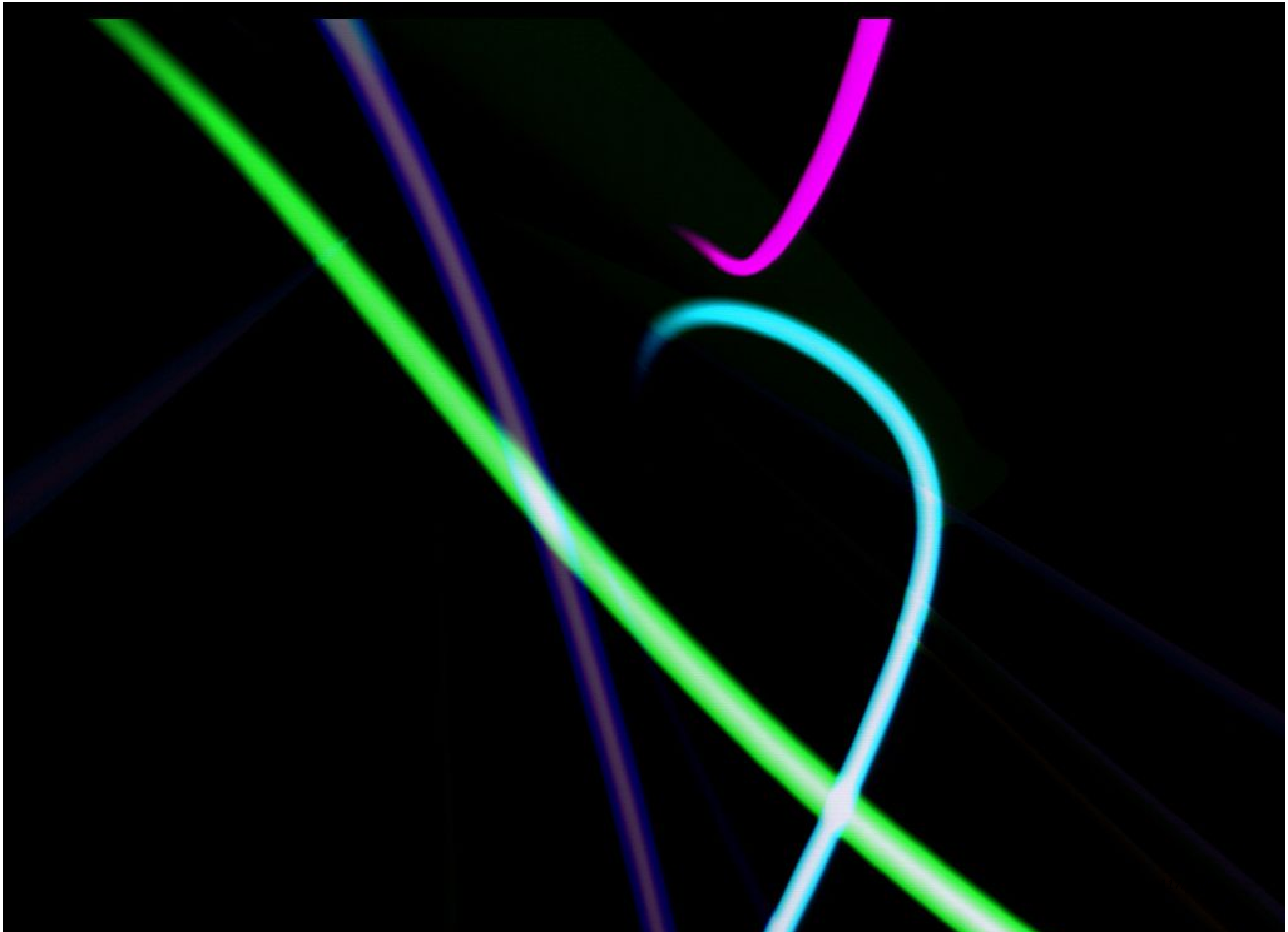

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On Tensorial Concomitants and the Non-Existence of a Gravitational Stress-Energy Tensor

Erik Curiel

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October 1, 2010

As soon as the principle of conservation of energy was grasped, the physicist practically made it his definition of energy, so that energy was that something which obeyed the law of conservation. He followed the practice of the pure mathematician, defining energy by the properties he wished it to have, instead of describing how he measured it. This procedure has turned out to be rather unlucky in the light of the new developments.

A. Eddington
The Mathematical Theory of Relativity