Title: Exact results in the AdS/CFT correspondence

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Abstract: The AdS/CFT correspondence relates large-N, planar quantum gauge theories to string theory on the Anti-de-Sitter background. I will discuss exact results in field theories with AdS duals, which can be obtained with the help of diagram resummations, mapping to quantum spin chains and two-dimensional sigma-models.

Exact Results in the AdS/CFT Correspondence

Konstantin Zarembo
(Nordita)

Perimeter Institute, 13.10.10

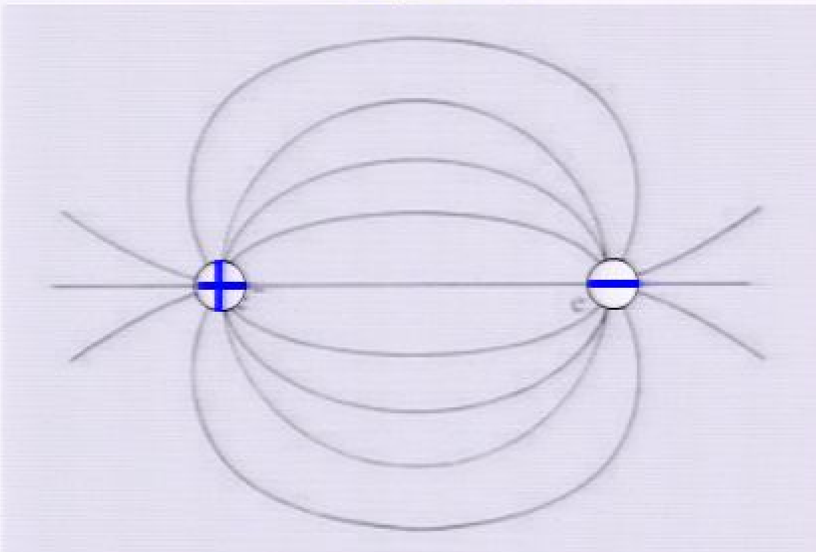
AdS/CFT correspondence:

- method to describe strongly-coupled field theories using:
  - ✓ string theory
  - ✓ classical gravity (Einstein's equations)

General Q's:

- regime of applicability
- accuracy

Coulomb field (QED) vs. Flux tube (QCD)



string carries constant energy per unit length

$$V(r) = -\frac{e^2}{4\pi r}$$

$$V(r) \approx Tr$$

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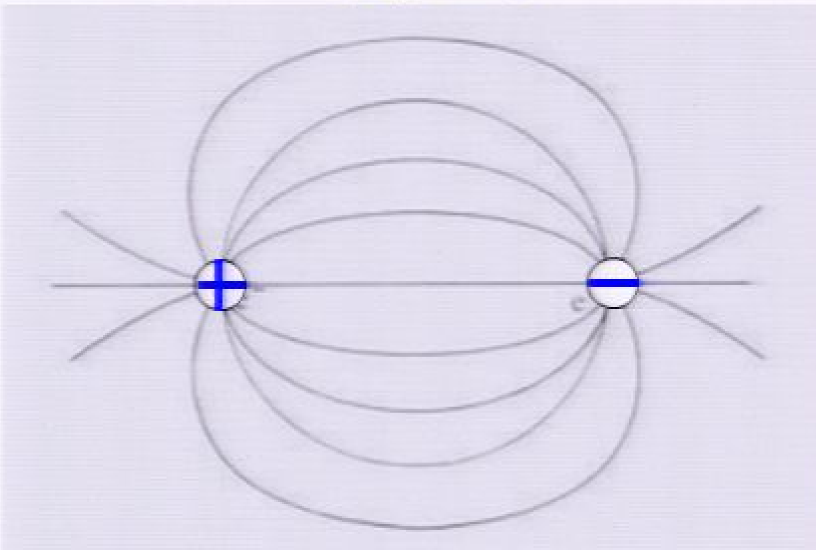

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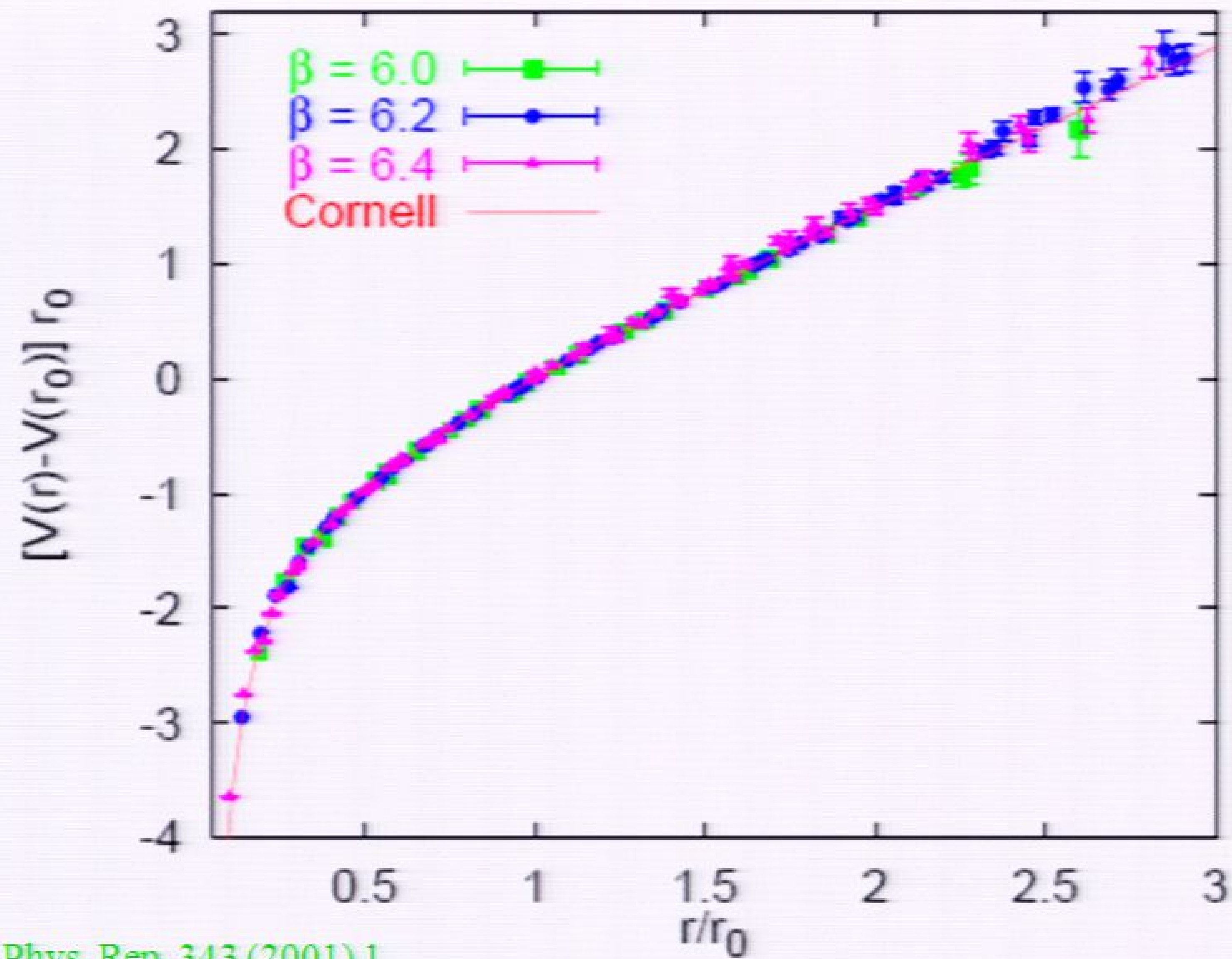

From Bali, Phys. Rep. 343 (2001) 1

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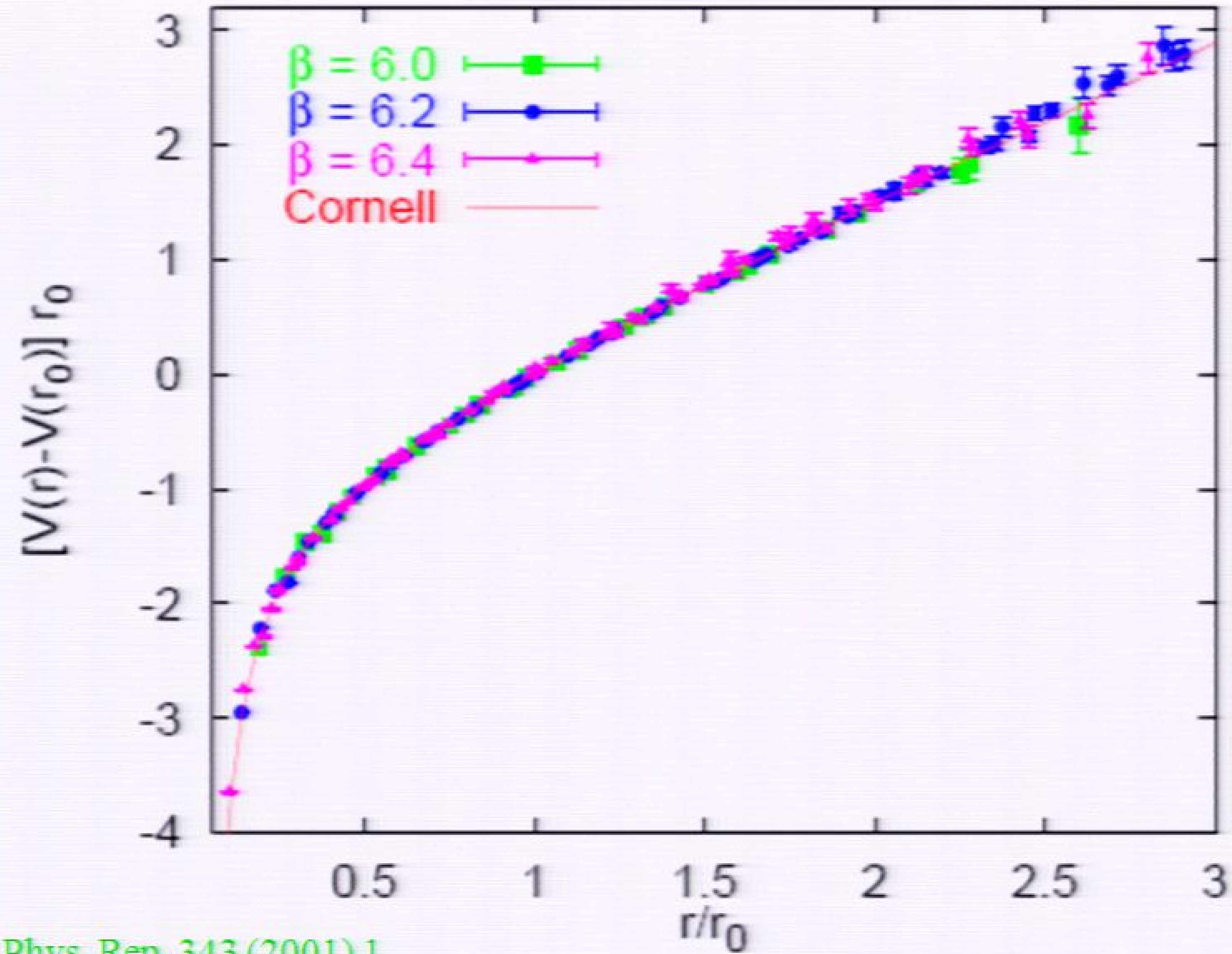
Caselle,Fiore,Gliozzi,Hasenbusch,Provero'97; Caselle,Pepe,Rago'04; …



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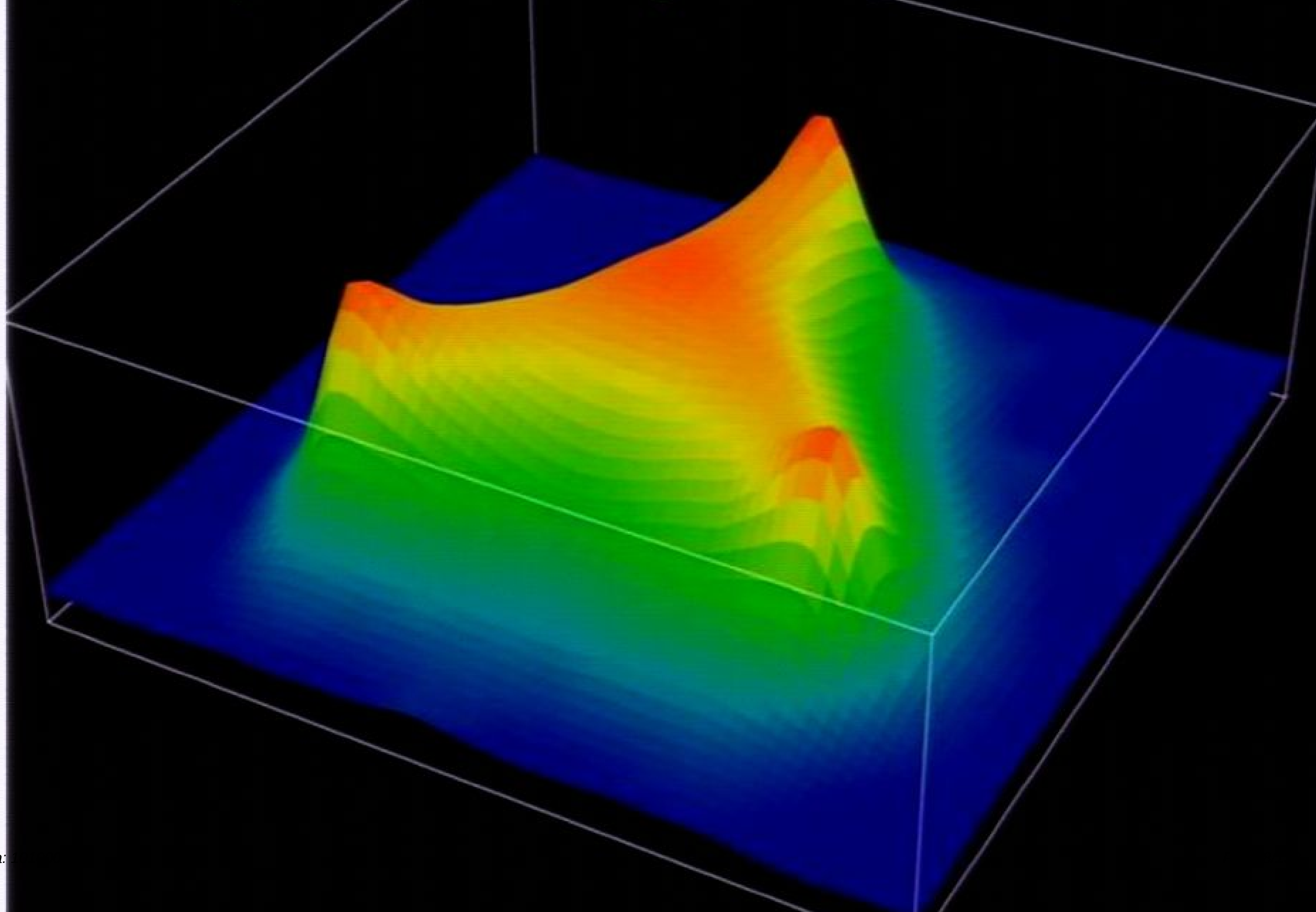
Energy density distribution inside baryon

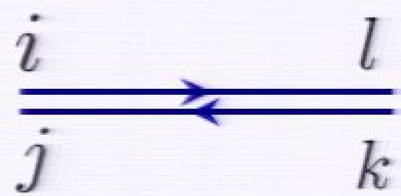
From Bornyakov,Ichie,Mori,Pleiter,Polikarpov,Scierholz,Streuer,Stüben,Suzuki'04



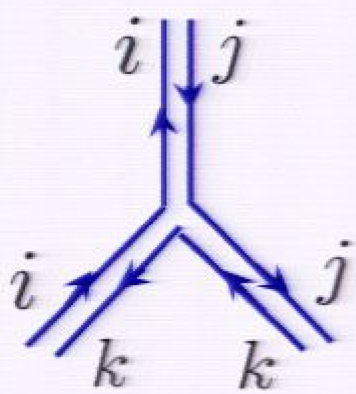
Large-N expansion

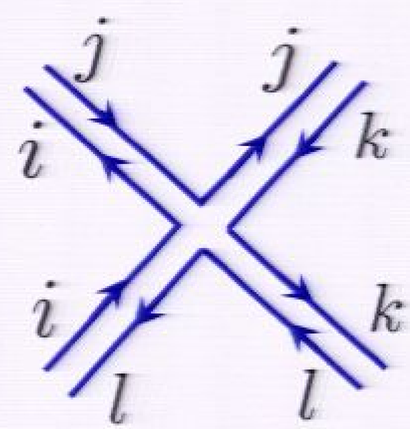
't Hooft'74

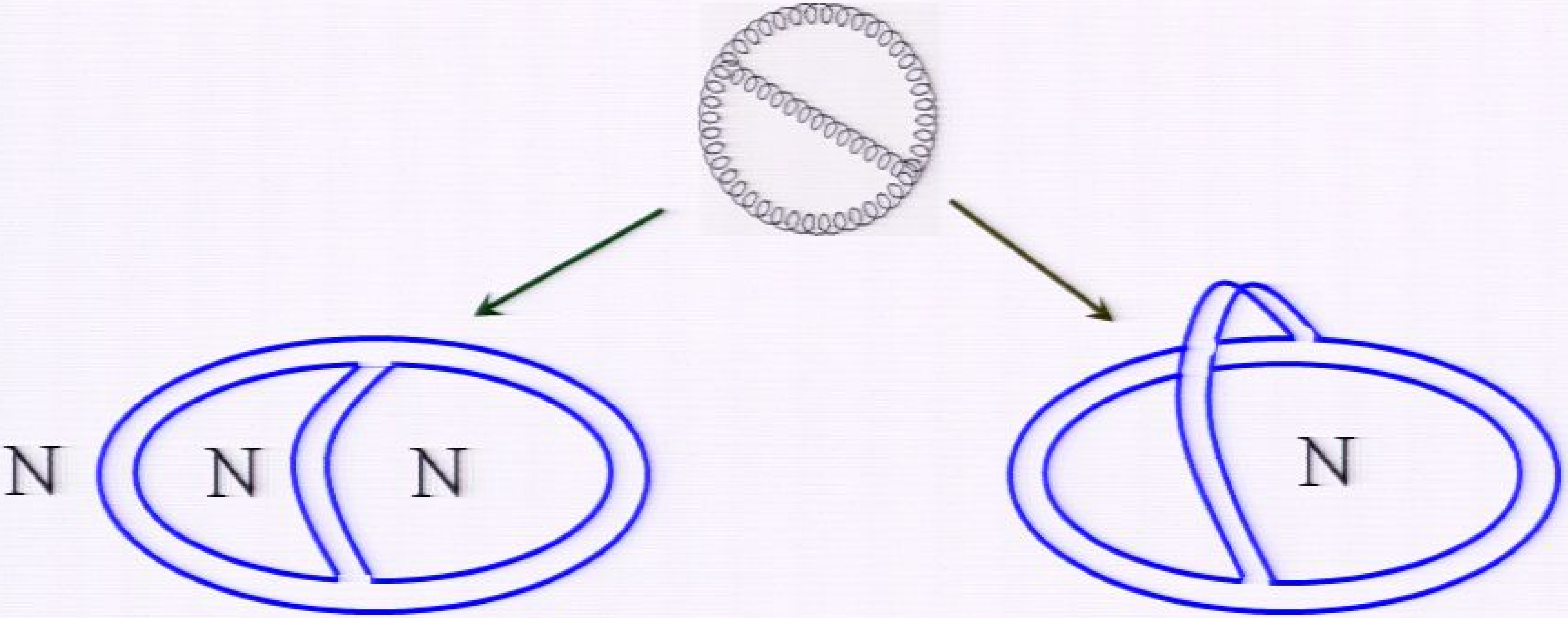
$$\left\langle A_\mu^{ij}(x) A_\nu^{kl}(x) \right\rangle_0 = \delta^{il}\delta^{jk}\, D_{\mu\nu}(x-y) \qquad i,j,k,l = 1\ldots N$$



"Index conservation law":







$g_{YM}^2 N^3 = O(N^2)$ $\qquad$ $g_{YM}^2 N = O(N^0)$

Two expansion parameters:

't Hooft coupling: $\lambda = g^2 N$

string coupling: $\dfrac{1}{N^2}$

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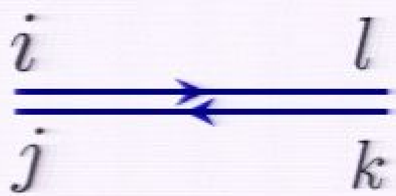

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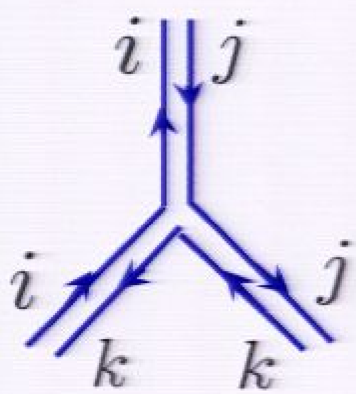

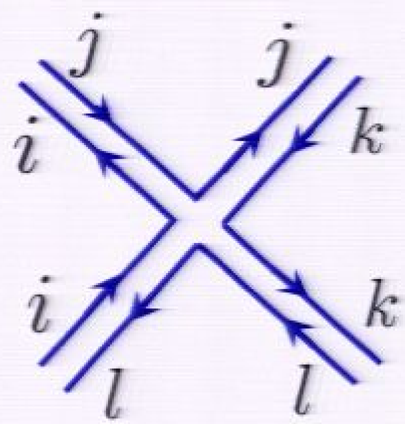

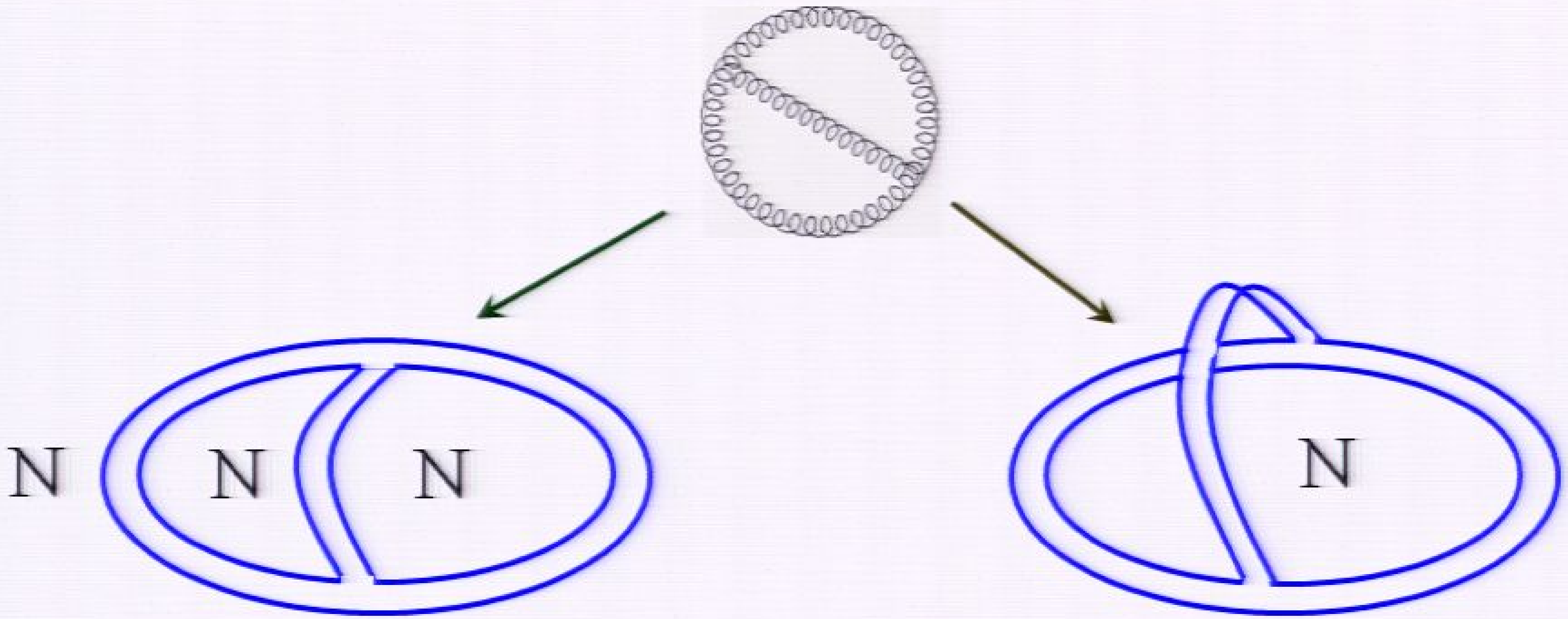

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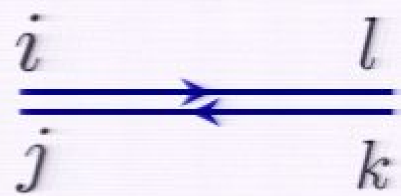

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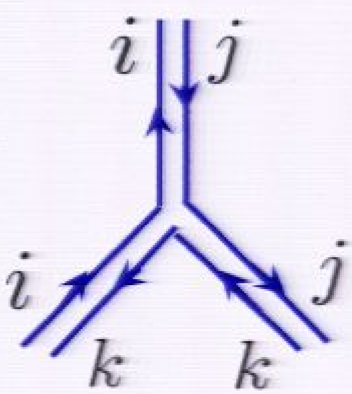

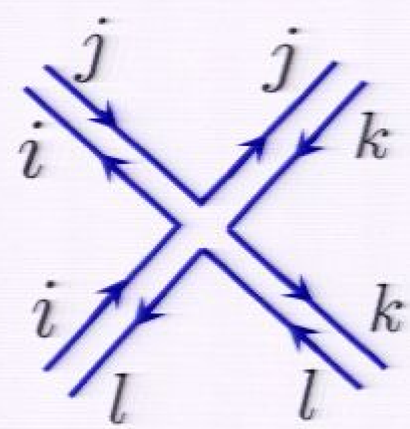

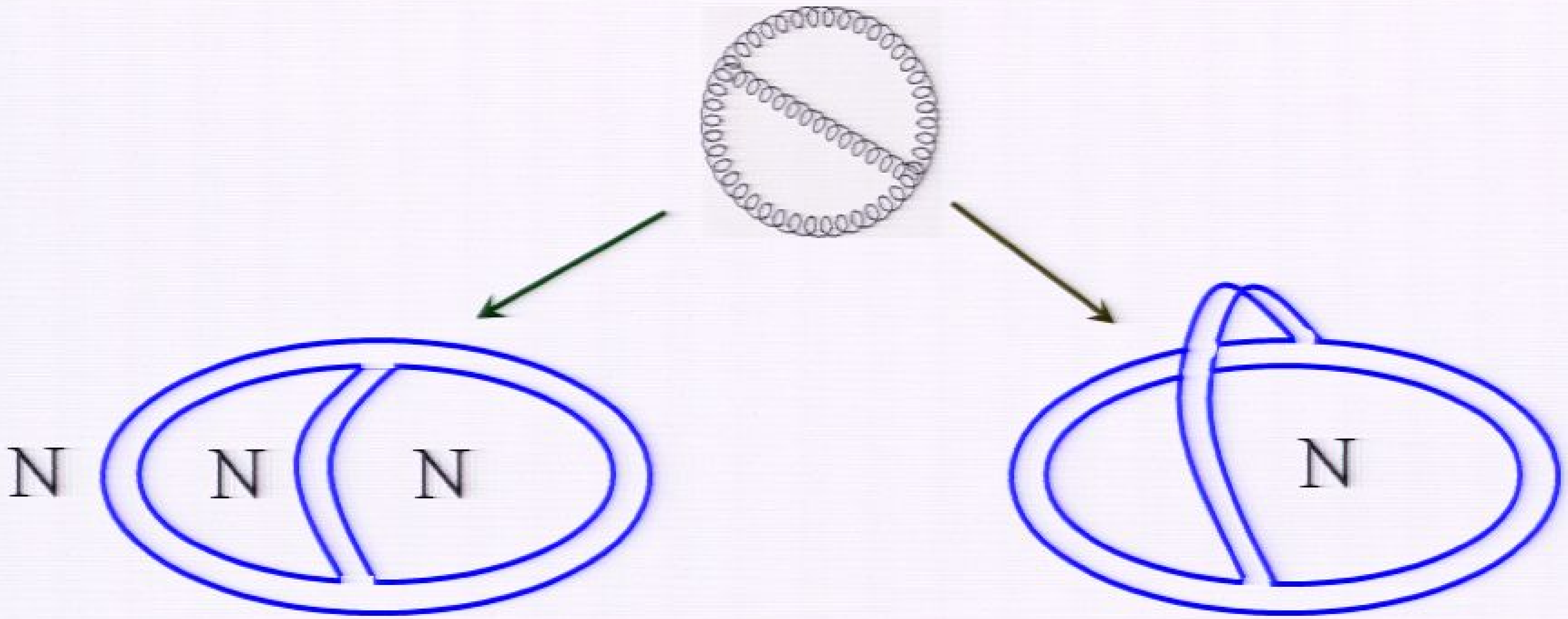

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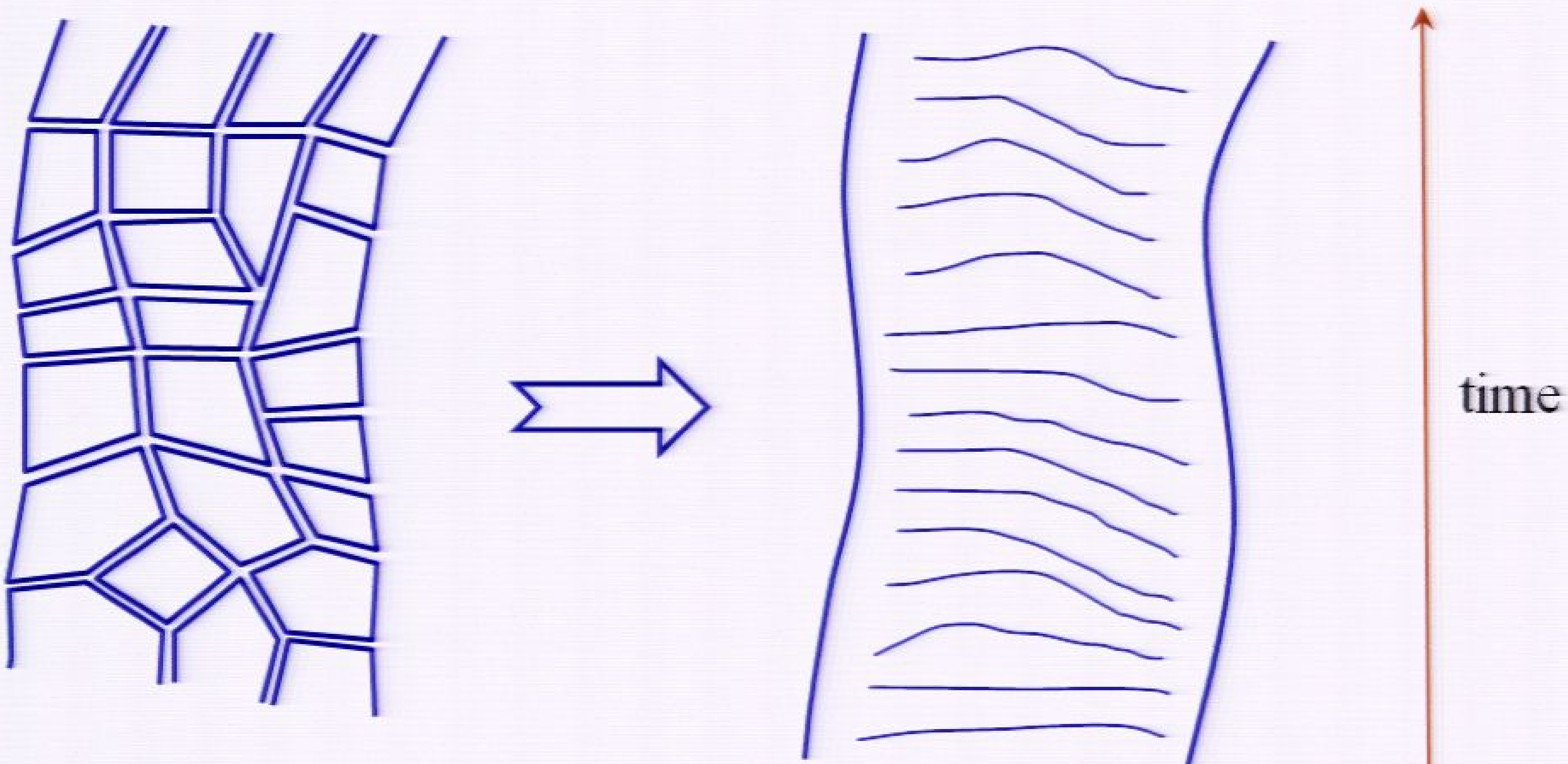
Planar diagrams and strings

Large-N limit: $N \to \infty, \qquad \lambda - \text{fixed}$



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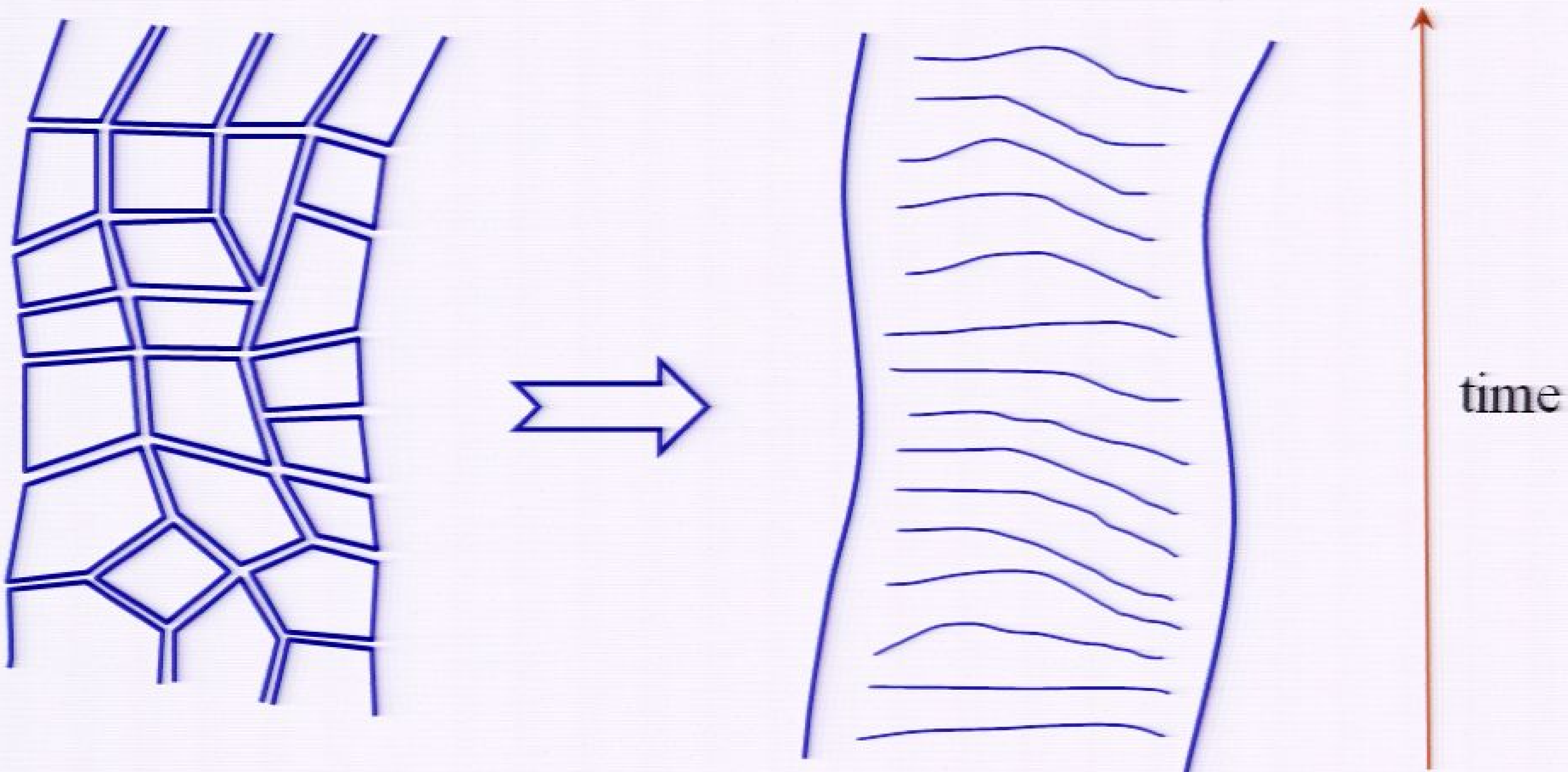

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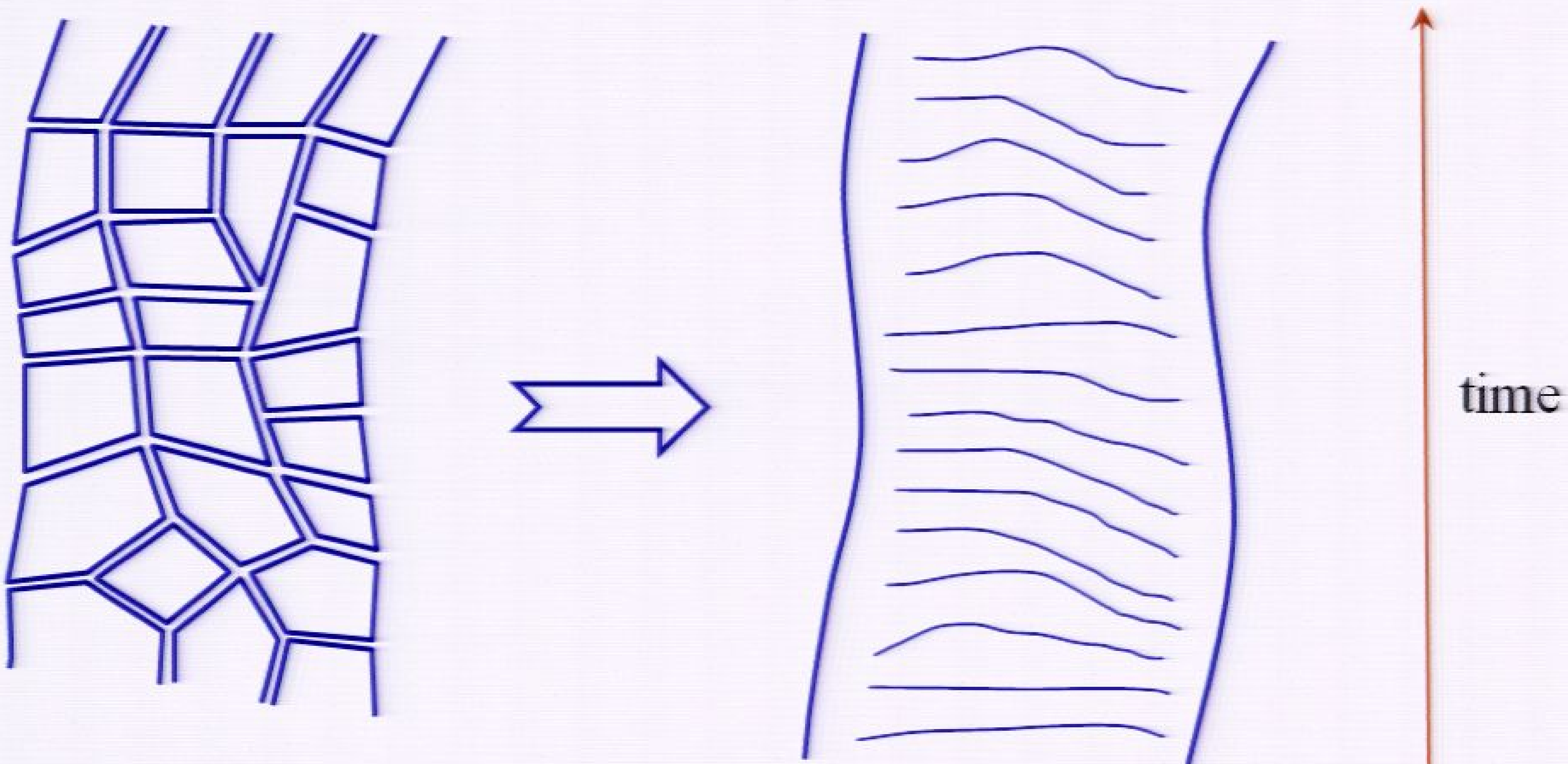

Conceptual problems:

- Closed strings describe gravity. What is graviton in YM?
- String theory is only consistent in ten dimensions.
- How does the string remember that it is made of gluons?

Holography

Maldacena'97

- strings and gauge fields live in space-time of different dimension

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Anti-de-Sitter space (AdS_5)

$$\boxed{ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}}$$

Two-point correlation functions

$$\langle \mathcal{O}(x)\mathcal{O}(y)\rangle$$

Gubser,Klebanov,Polyakov'98

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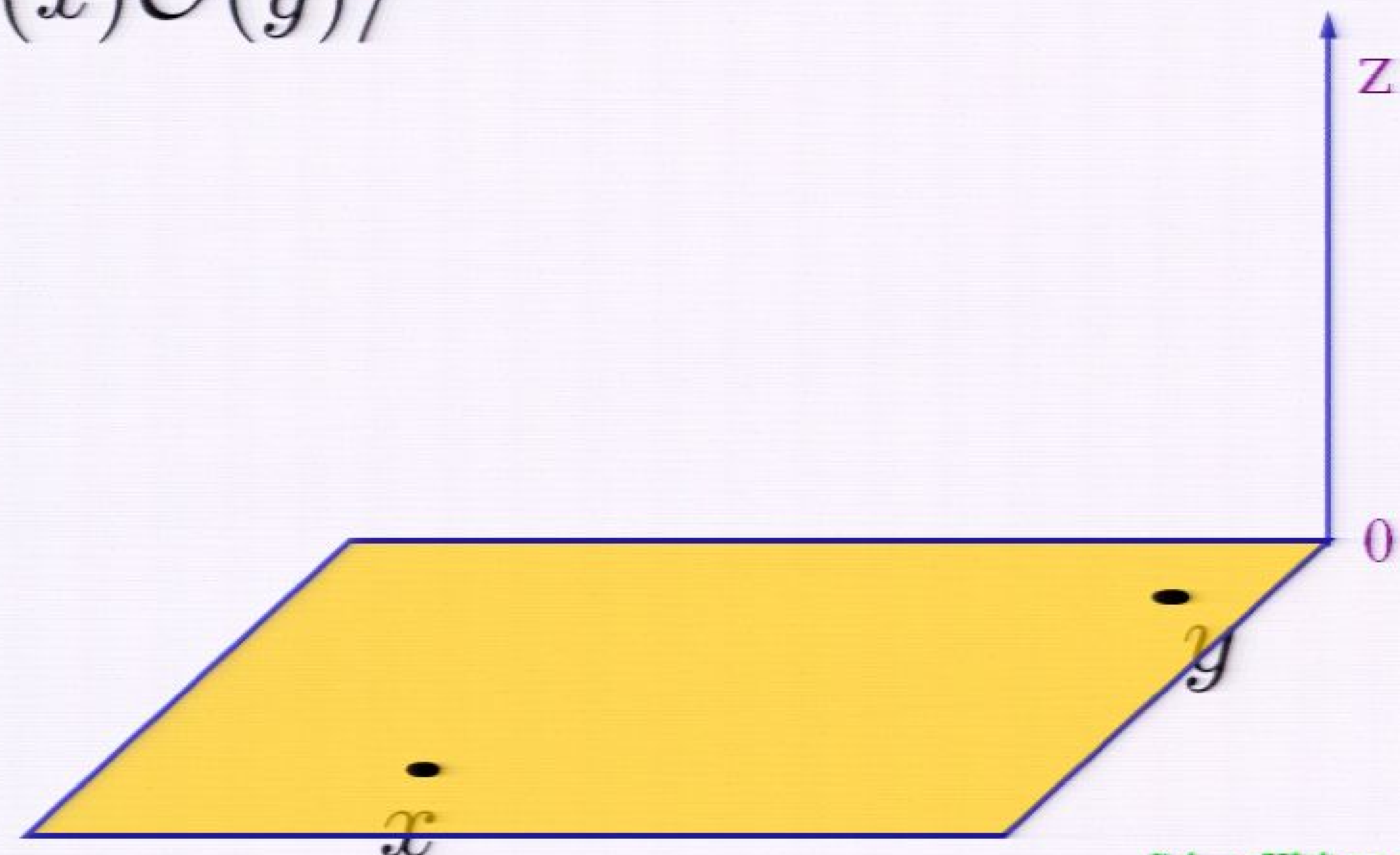

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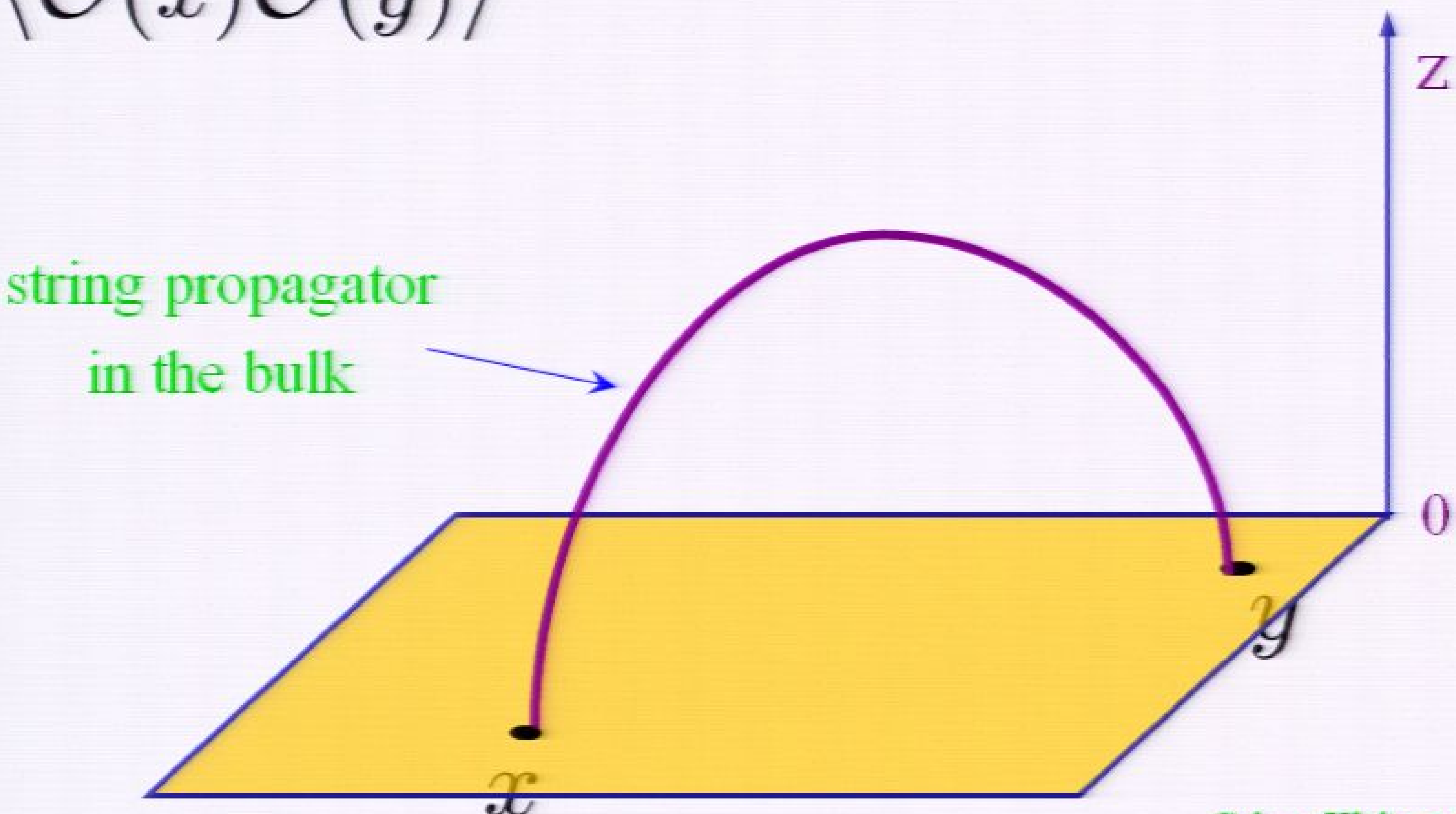

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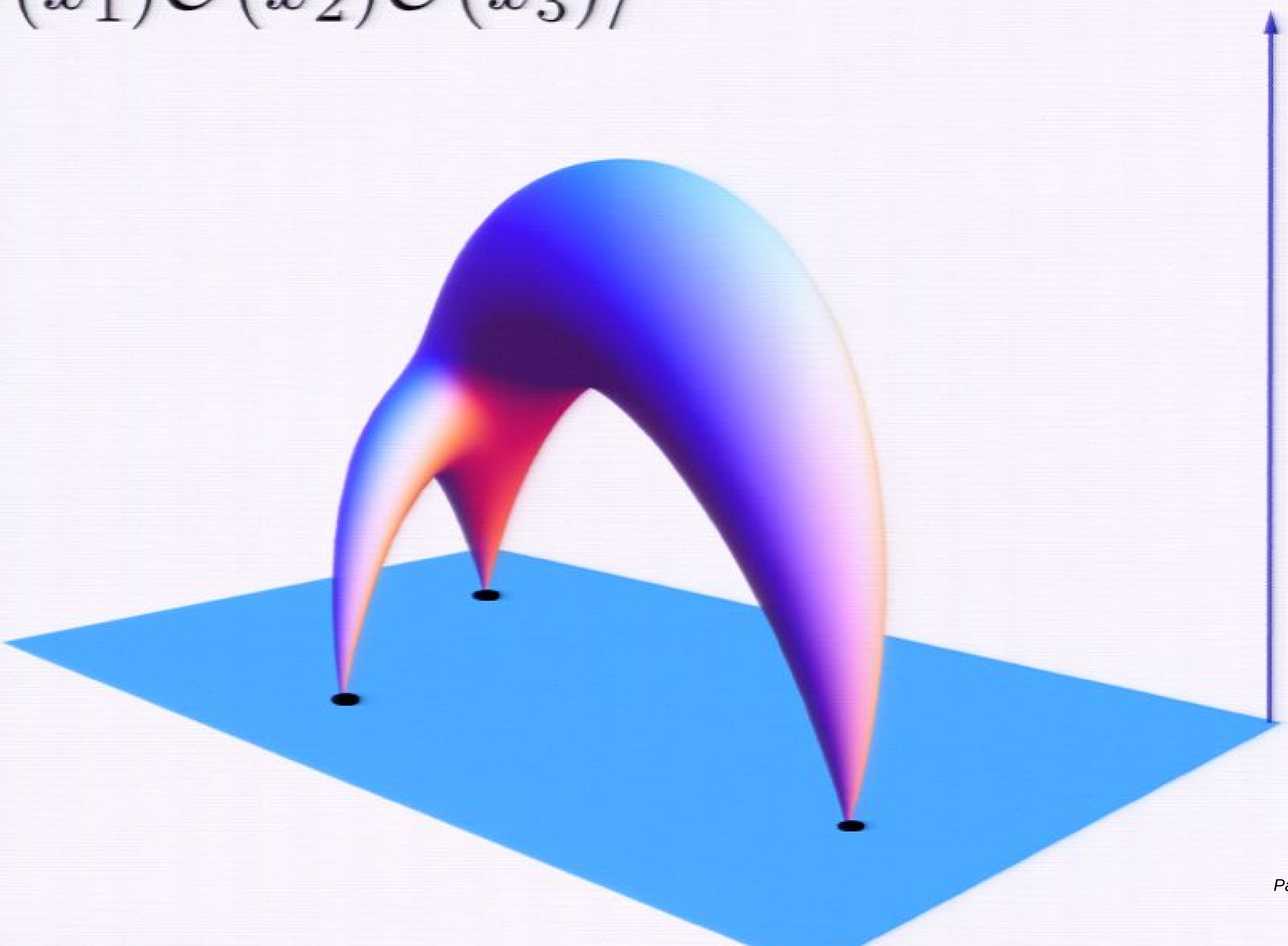
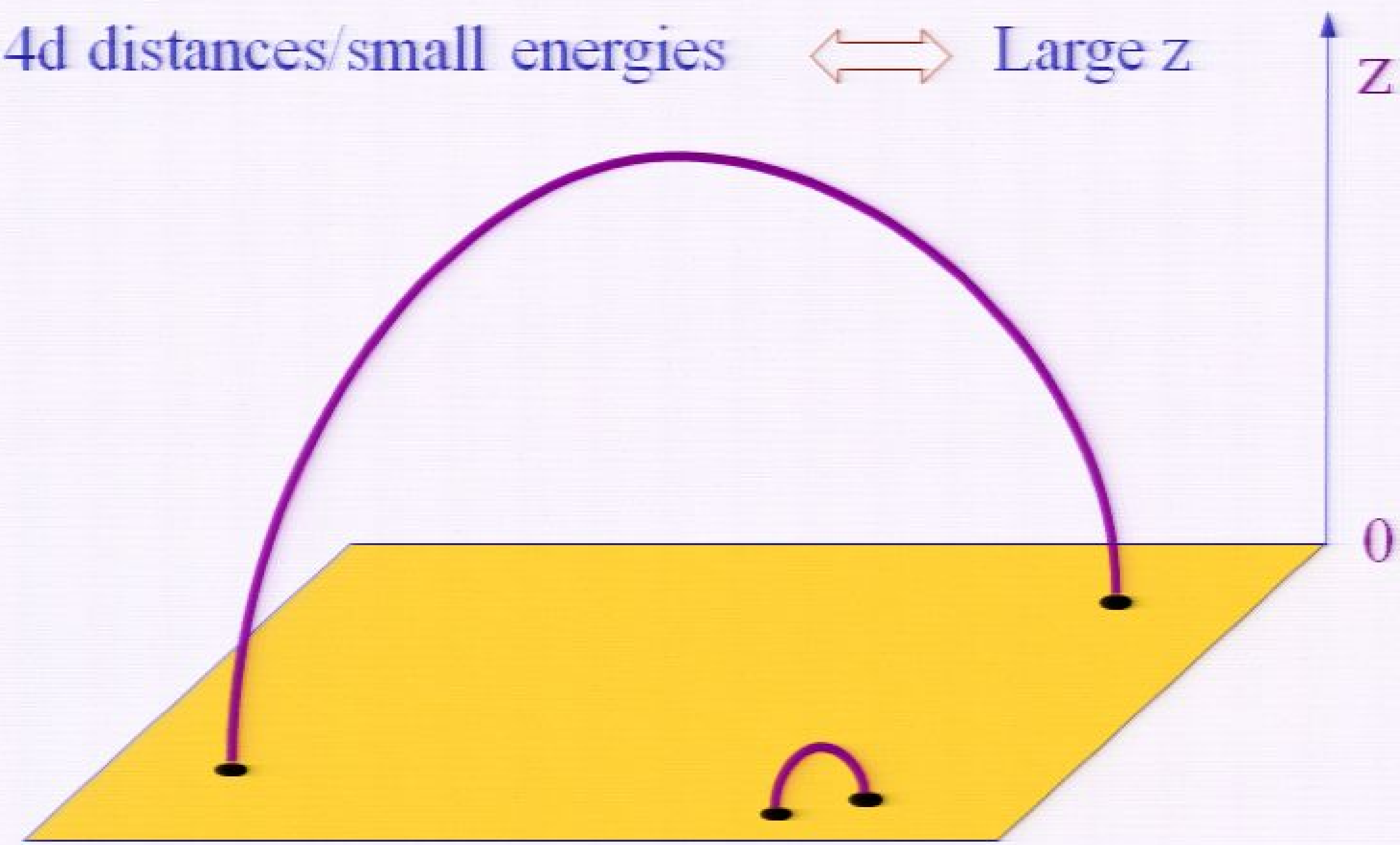


Extra dimension as energy scale

Large 4d distances/small energies $\Longleftrightarrow$ Large z



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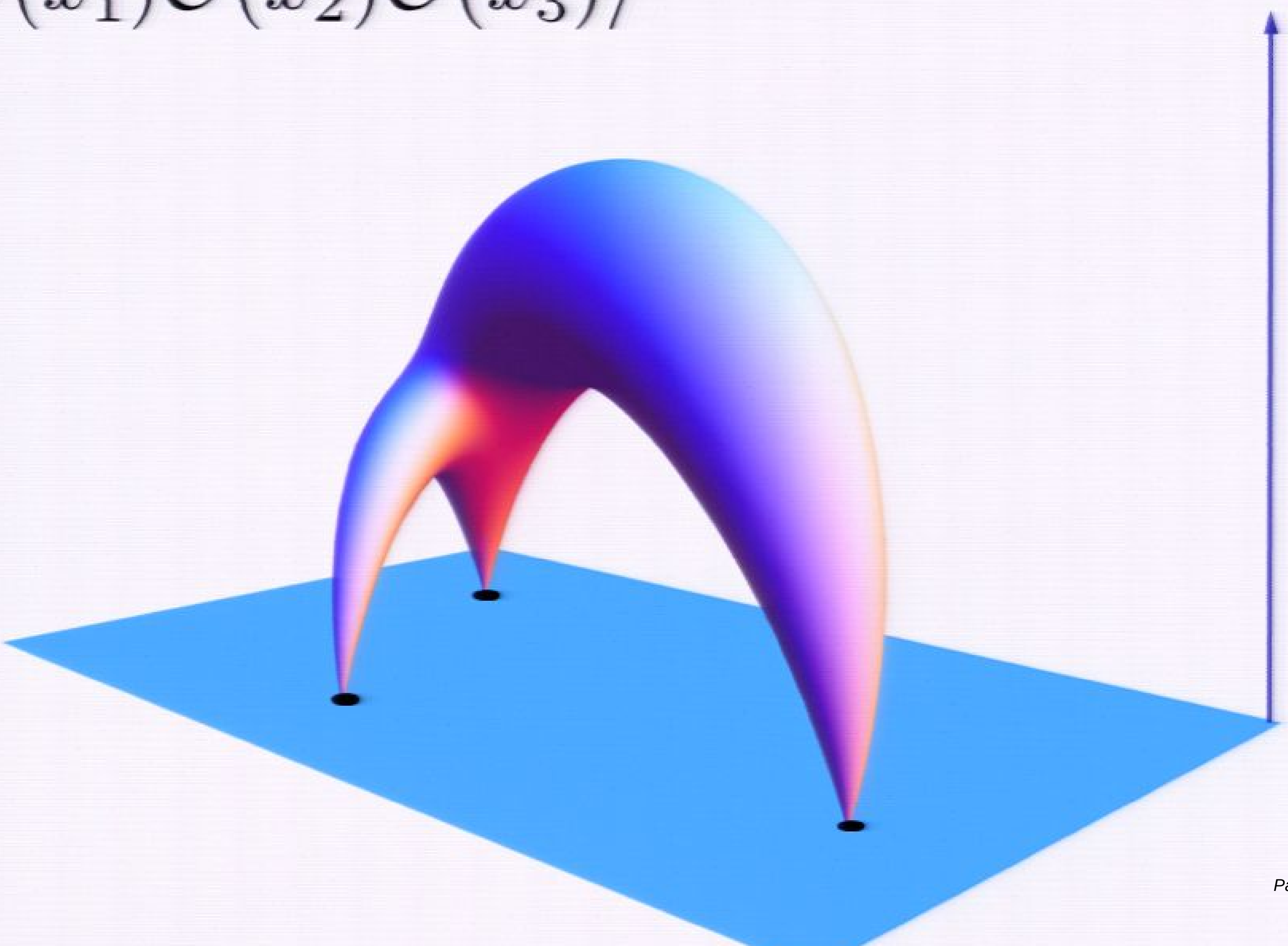

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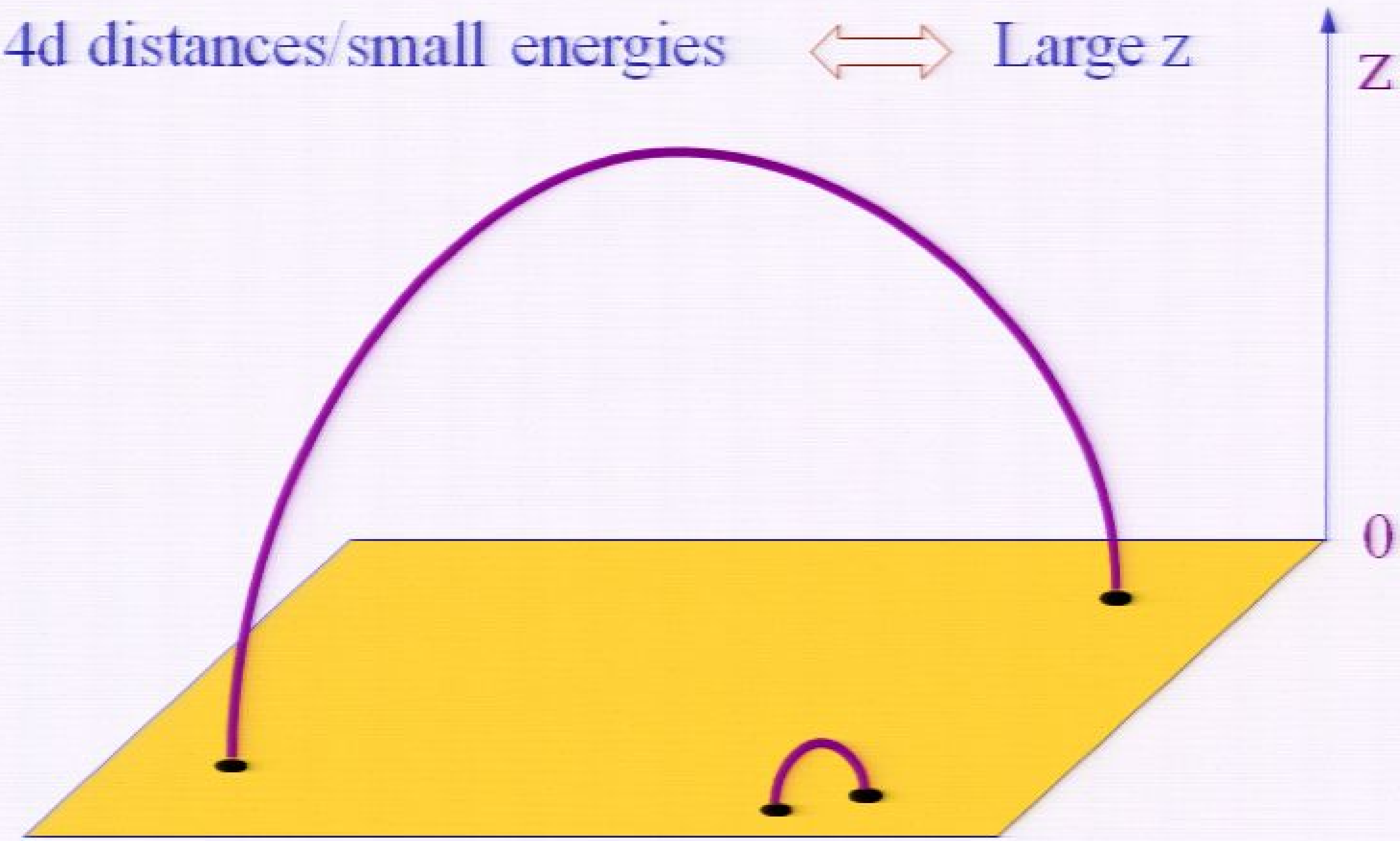

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Scale invariance

$$ds^2 = \frac{dx^\mu dx_\mu + dz^2}{z^2}$$

$$\begin{aligned} x^\mu &\rightarrow ax^\mu \\ z &\rightarrow az \end{aligned}$$

leaves metric invariant

gauge theory dual to strings in AdS_5 is scale invariant (conformal)

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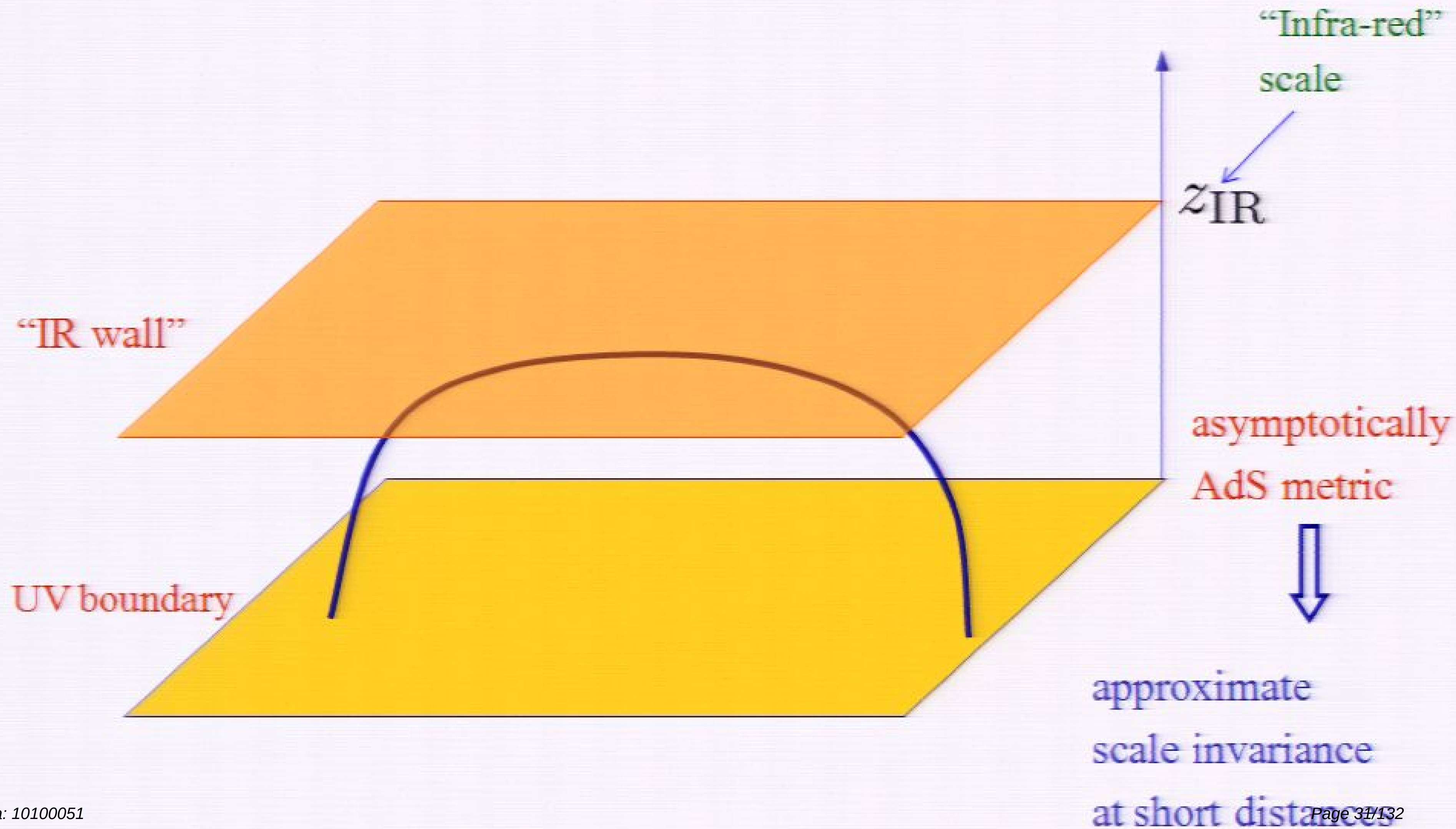
Breaking scale invariance



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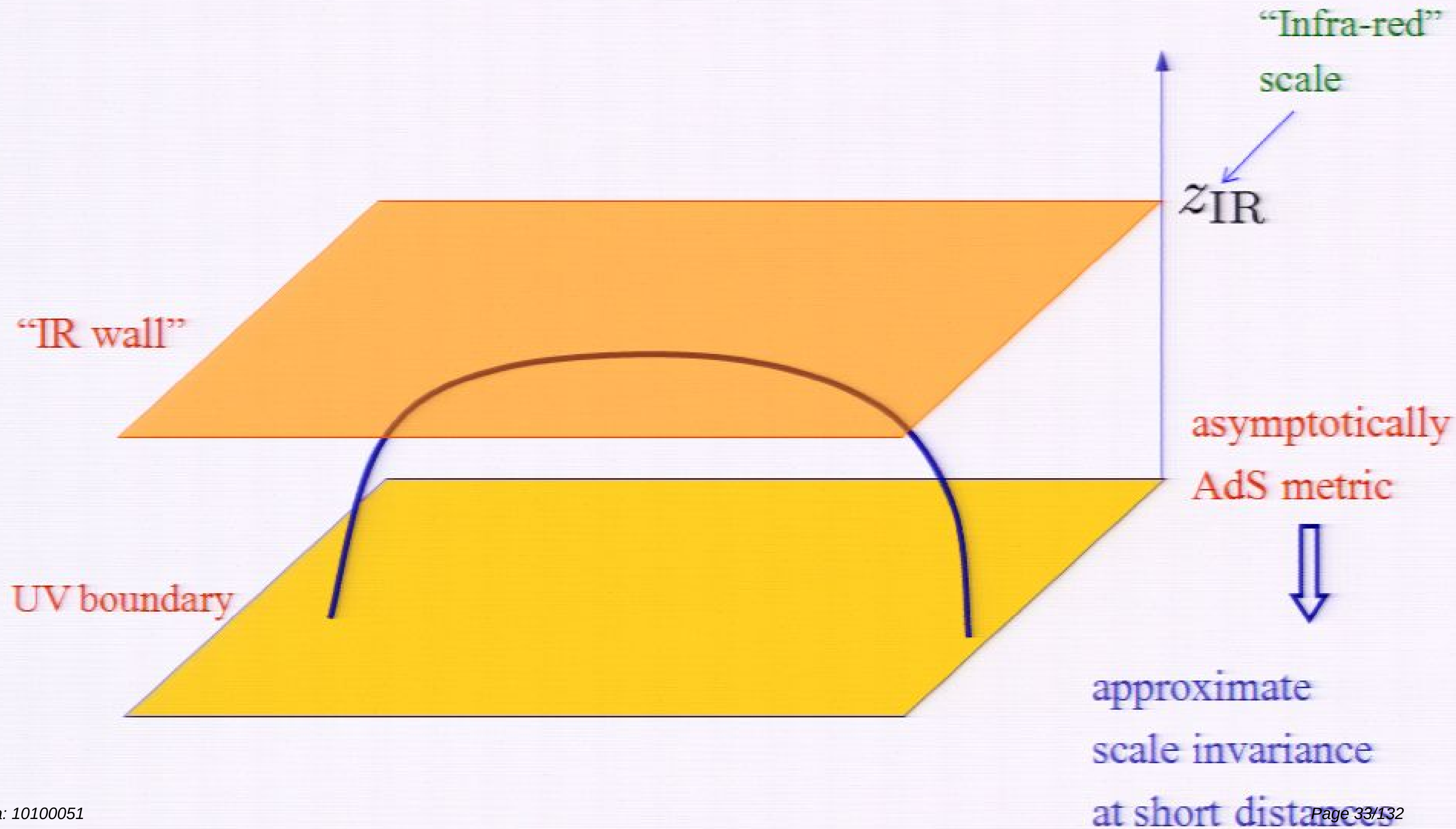

String states $\Longleftrightarrow$ Bound states in QFT (mesons, glueballs)

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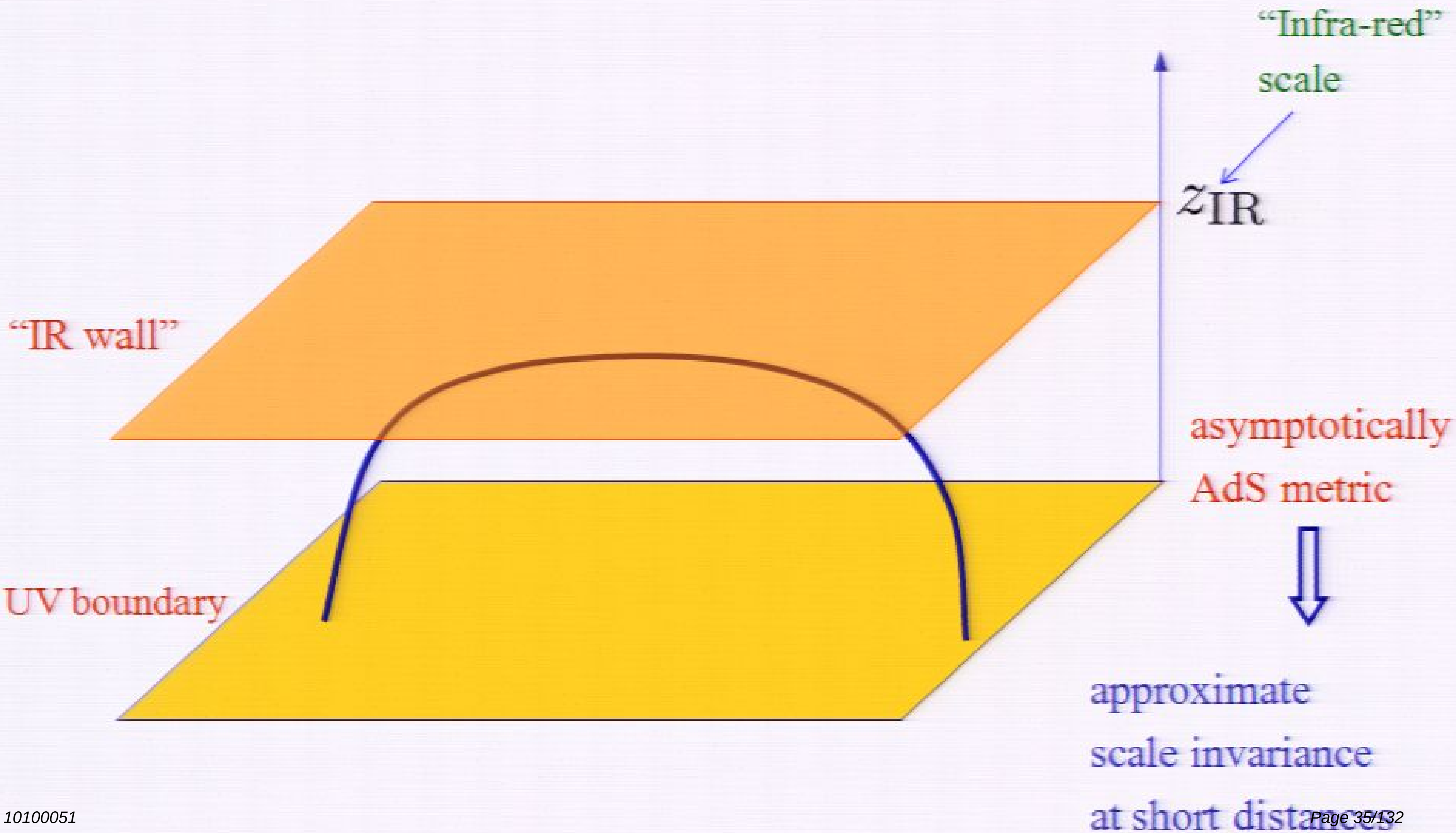

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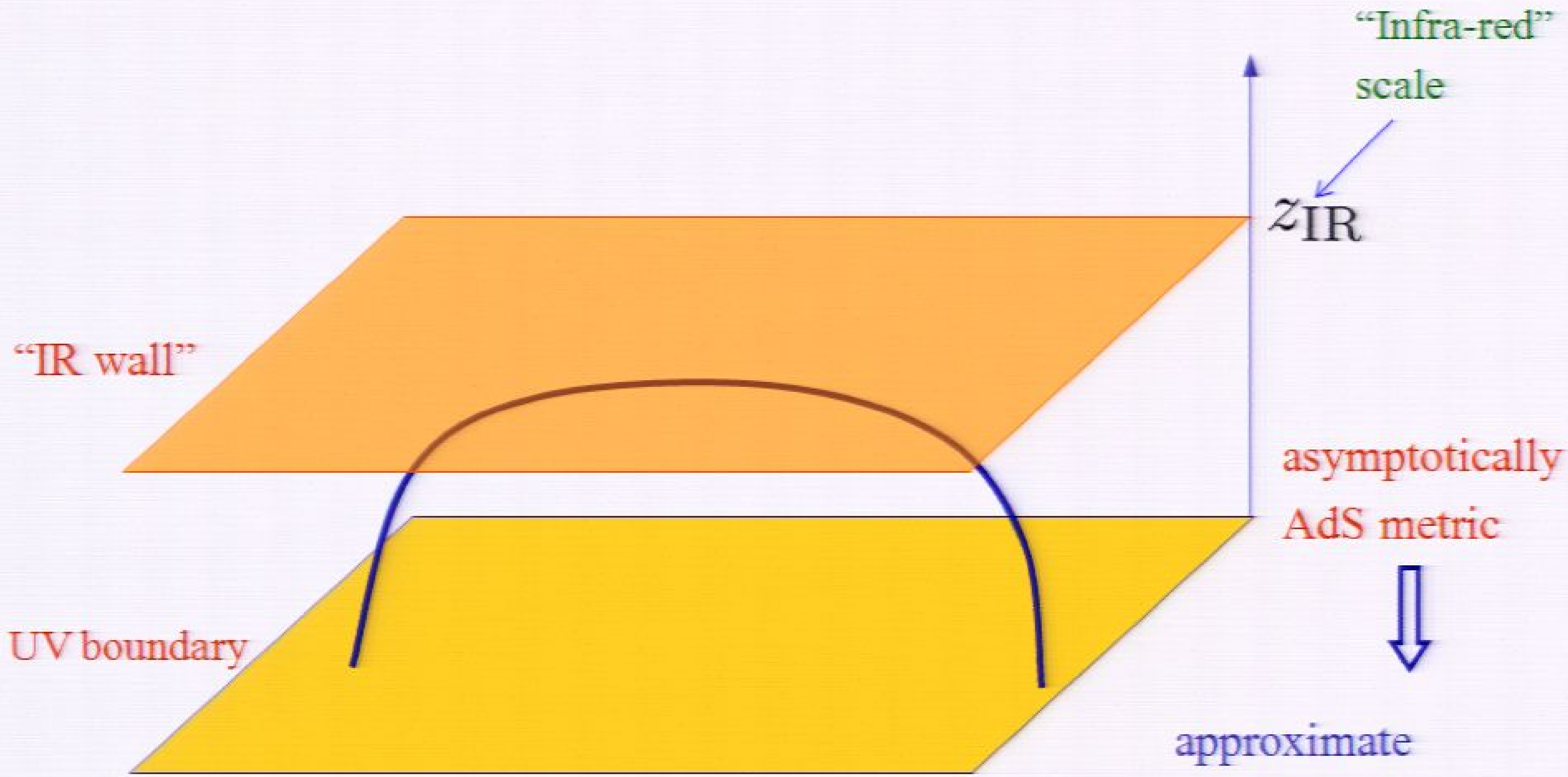

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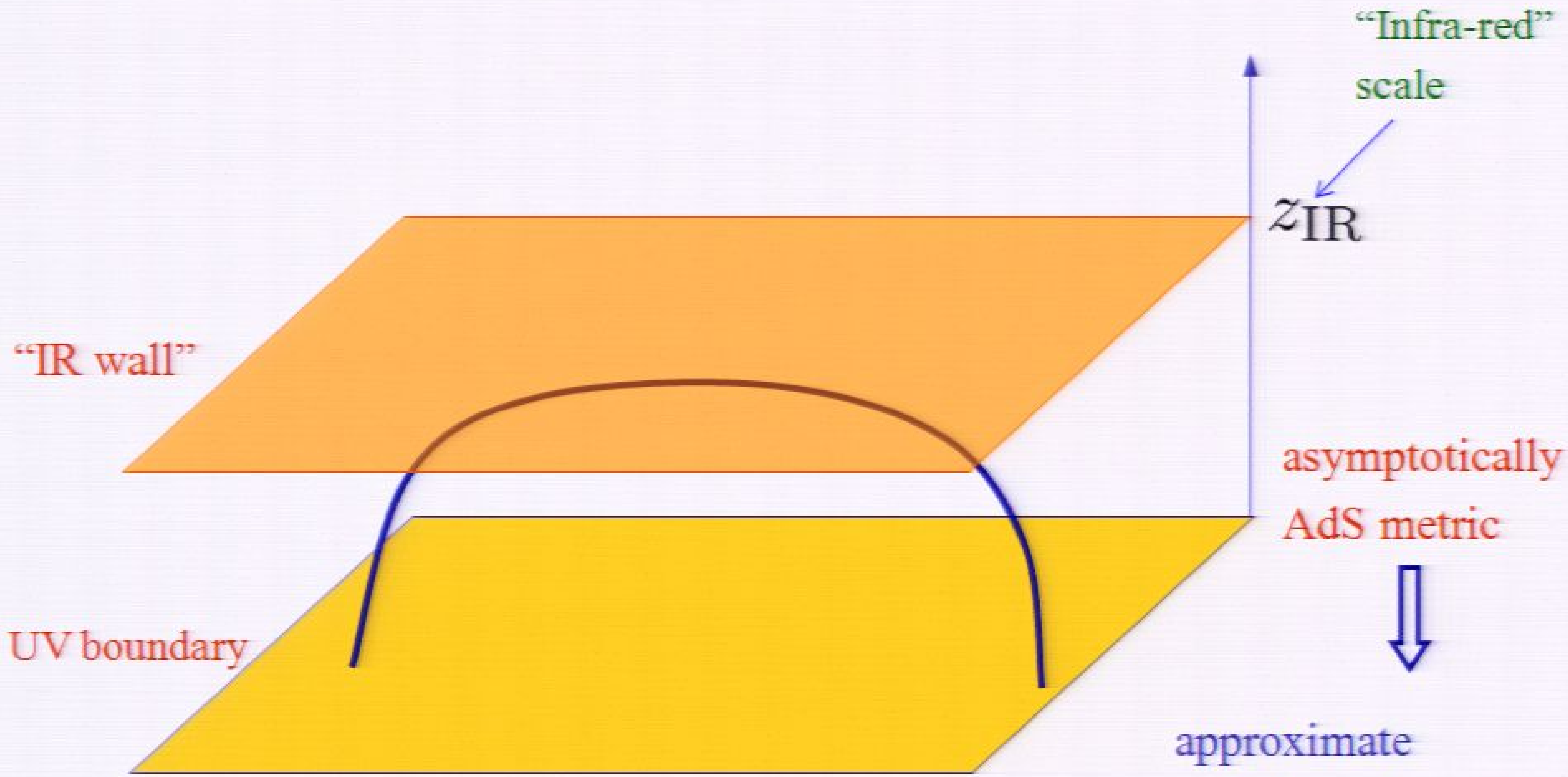

$\Downarrow$

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String states $\Longleftrightarrow$ Local operators

Resolves many puzzles of putative large-N string:

- graviton is not a massless glueball, but is the dual of the energy-momentum tensor $T_{\mu\nu}$
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$$J^a = \bar{\psi} t^a_{\text{flavor}} \gamma^\mu \psi$$

$$\int d^4x \, \mathrm{e}^{iqx} \left\langle J^a_\mu(x) J^b_\nu(0) \right\rangle = \delta^{ab}(q_\mu q_\nu - q^2 \eta_{\mu\nu}) \Pi(-q^2)$$

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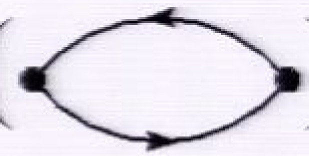
Perturbation theory ()

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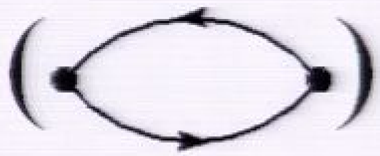

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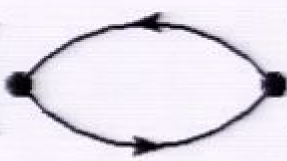
Perturbation theory ()

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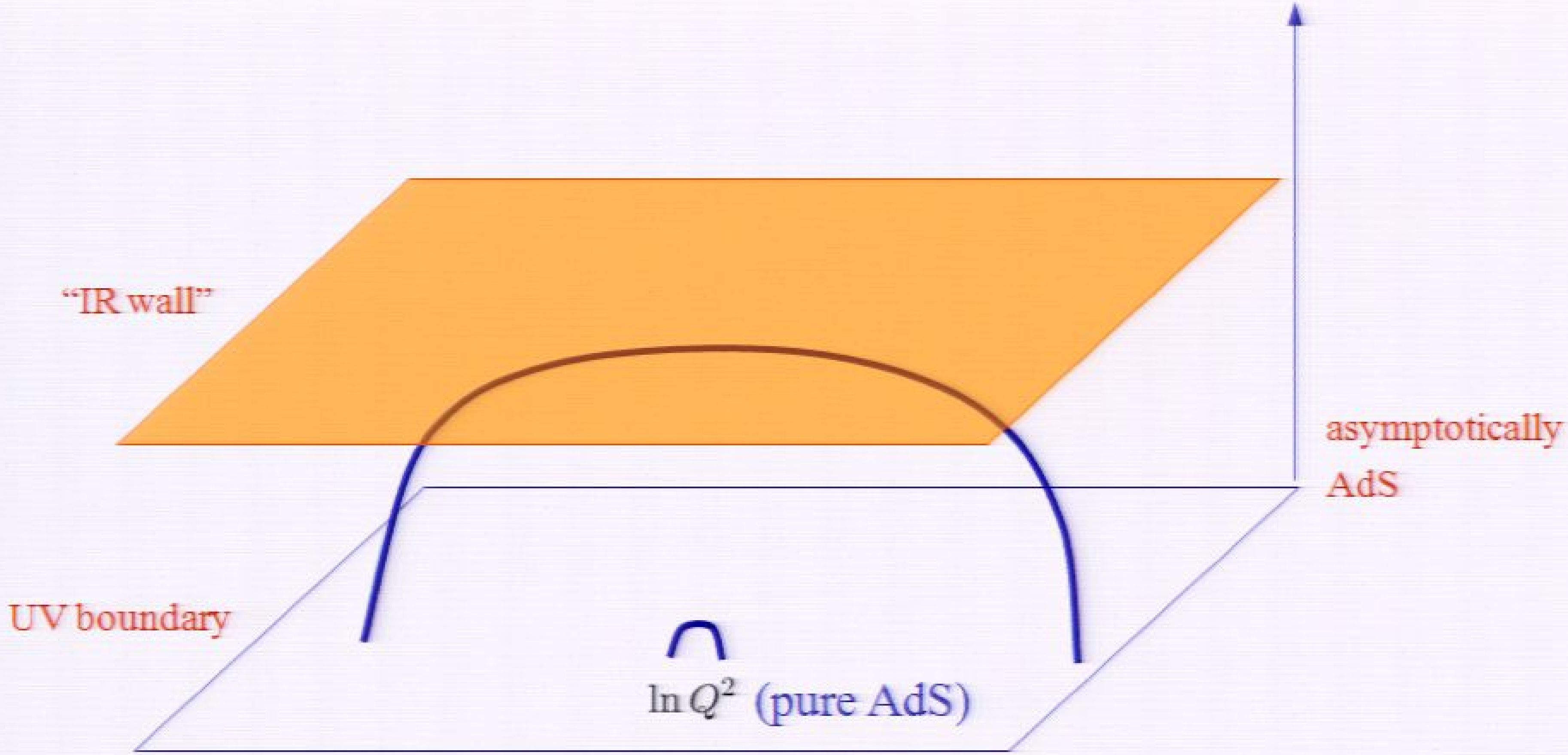

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The simplest phenomenological model describes all data in th

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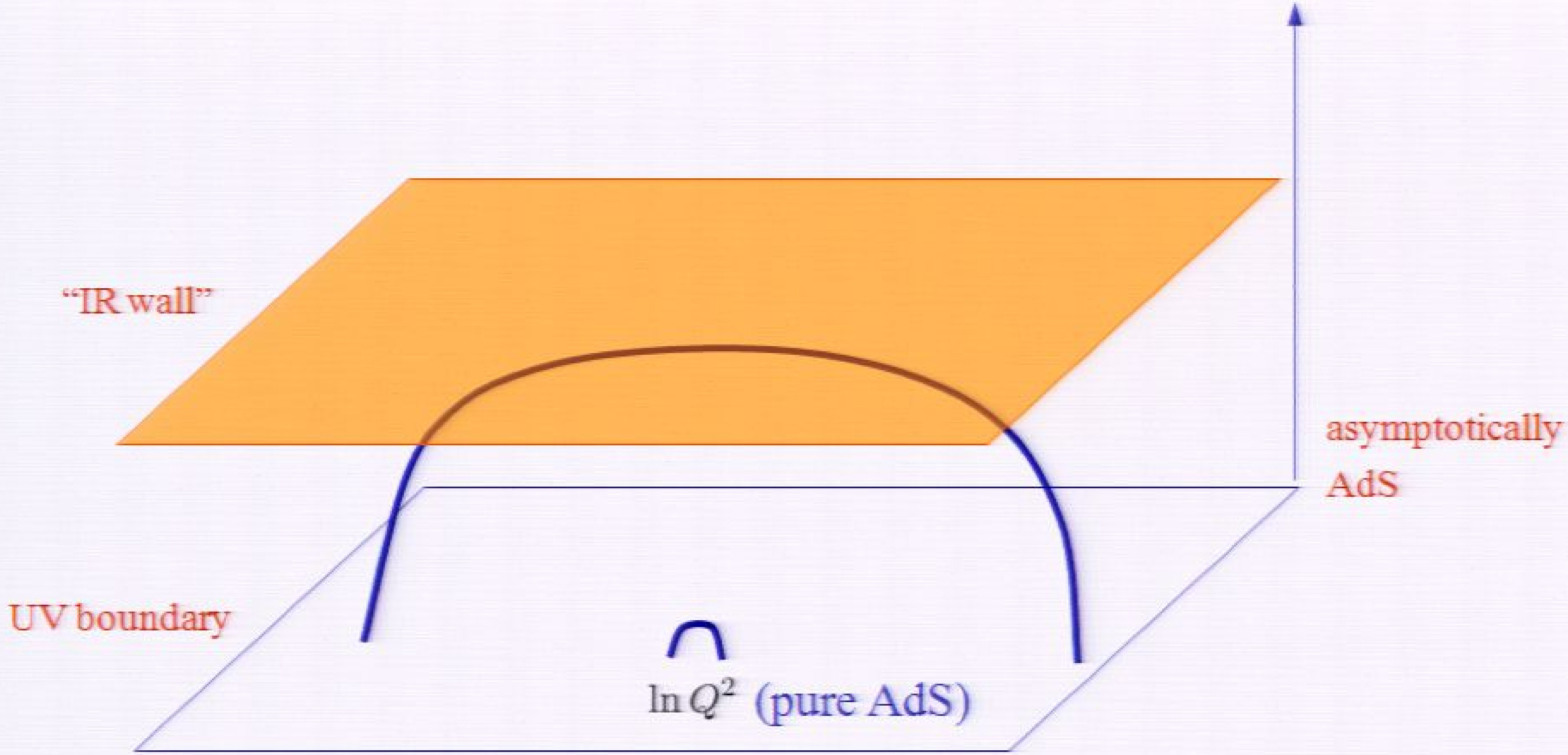

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Yang-Mills theory with

N=4 supersymmetry

Exact equivalence $\Updownarrow$

Maldacena'97

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Strong coupling	Classical strings
Local operators	String states
Scaling dimension: Δ	Energy: E

Gubser,Klebanov,Polyakov'98

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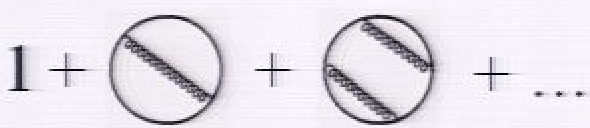
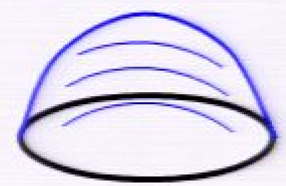
Strong-weak coupling interpolation

0 $\longrightarrow$ λ

SYM perturbation theory

String perturbation theory





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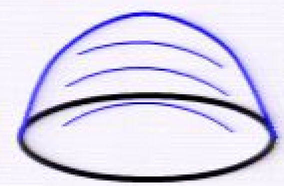

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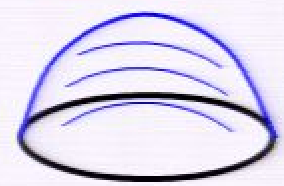

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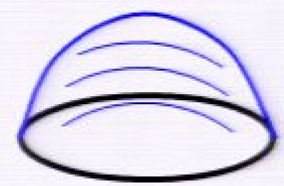


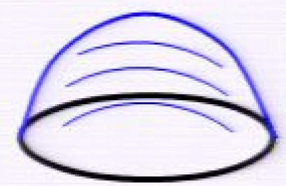
Strong-weak coupling interpolation

0 → λ

SYM perturbation theory

String perturbation theory

$1 +$  $+$  $+ \ldots$



Circular Wilson loop (exact):

$$W(\text{circle}) = \frac{2}{\sqrt{\lambda}} I_1\left(\sqrt{\lambda}\right)$$

Erickson,Semenoff,Zarembo`00
Drukker,Gross`00
Pestun`07

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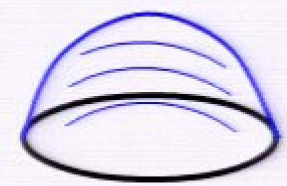

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$\lambda \rightarrow 0$

$$W(\text{circle}) = 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \ldots$$

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0 $\longrightarrow$ λ

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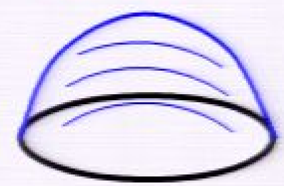

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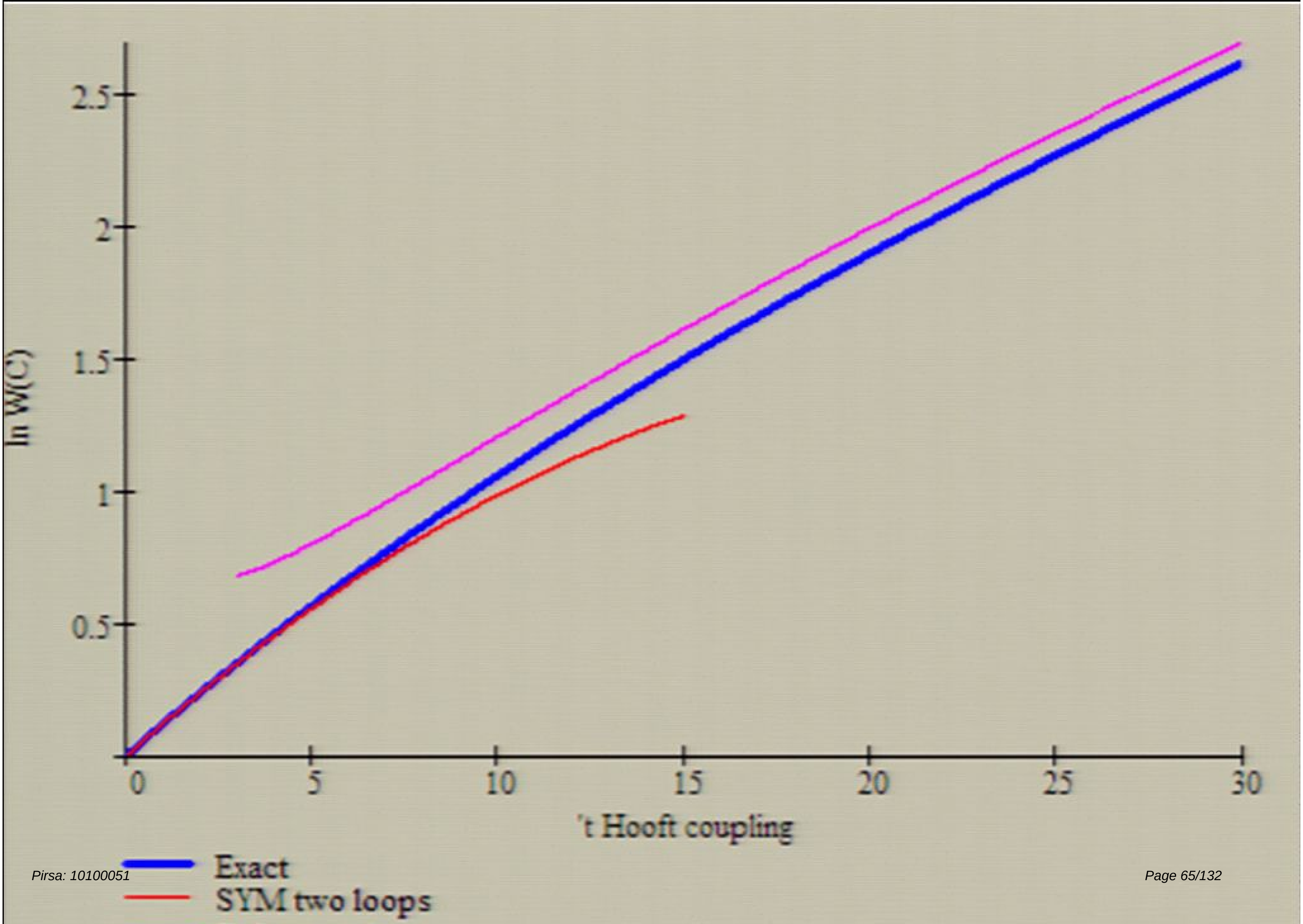
$\lambda \to 0$

$\lambda \to \infty$

$$W(\text{circle}) = 1 + \frac{\lambda}{8} + \frac{\lambda^2}{192} + \ldots$$

$$W(\text{circle}) = \sqrt{\frac{2}{\pi}}\lambda^{-3/4}\,\mathrm{e}^{\sqrt{\lambda}}$$



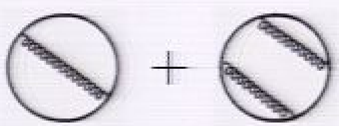
ln W(C)
2.5
2
1.5
1
0.5
0
5
10
15
20
25
30
't Hooft coupling
Exact
SYM two loops

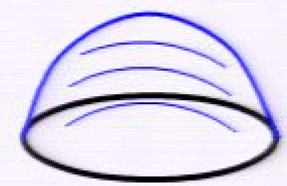
Strong-weak coupling interpolation

0 $\longrightarrow$ λ

SYM perturbation theory

String perturbation theory

$1 +$  $+ \ldots$



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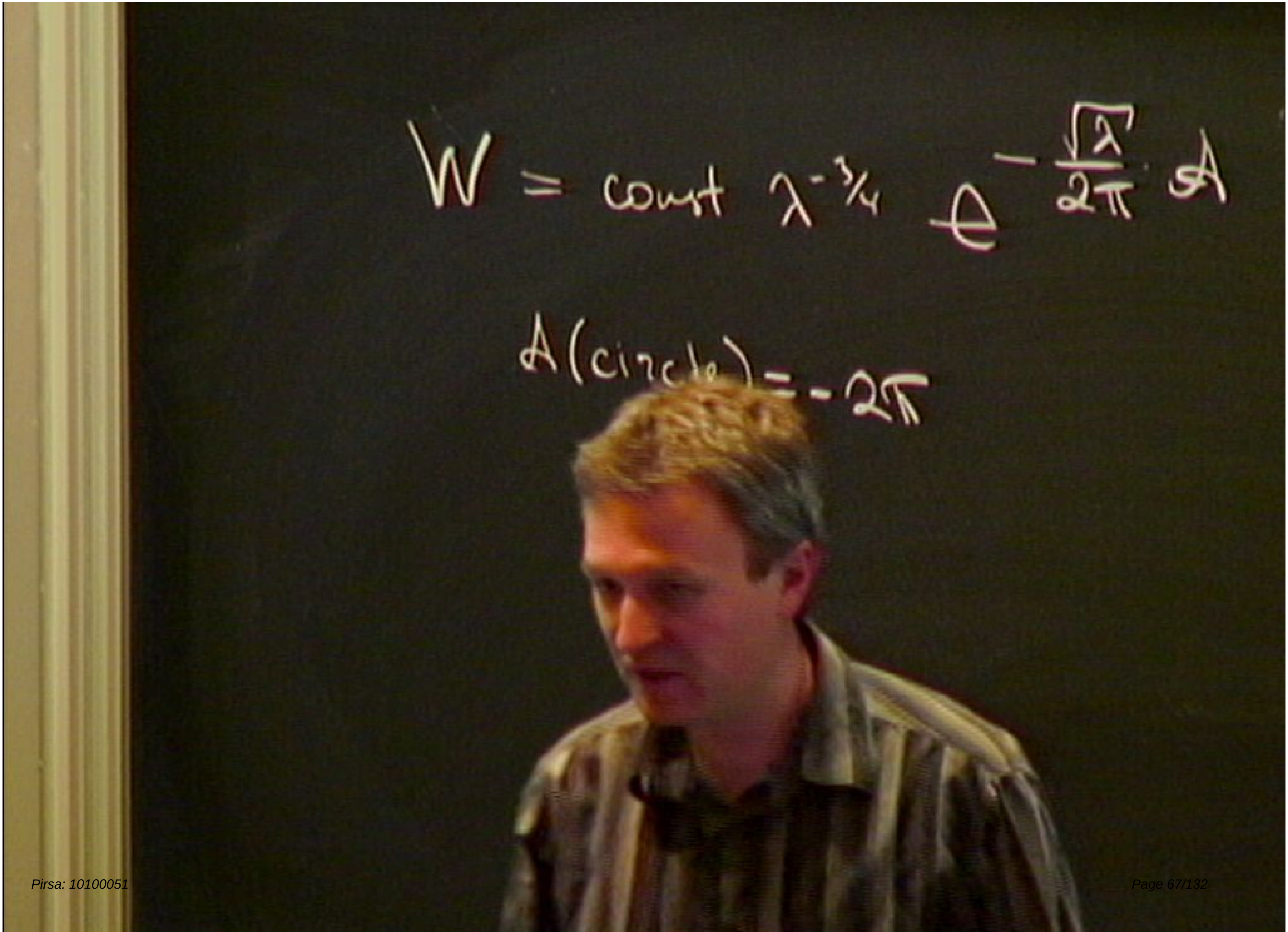
$W = \text{const}\ \lambda^{-3/4}\ e^{-\frac{\sqrt{\lambda}}{2\pi} A}$
$A(\text{circle}) = -2\pi$

$$W = \text{const}\ \lambda^{-3/4}\ e^{-\frac{\sqrt{\lambda}}{2\pi}\cdot \mathcal{A}}$$

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[redacted]

Strong-weak coupling interpolation

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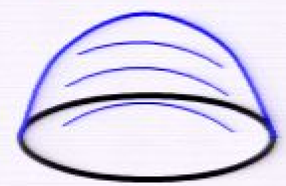

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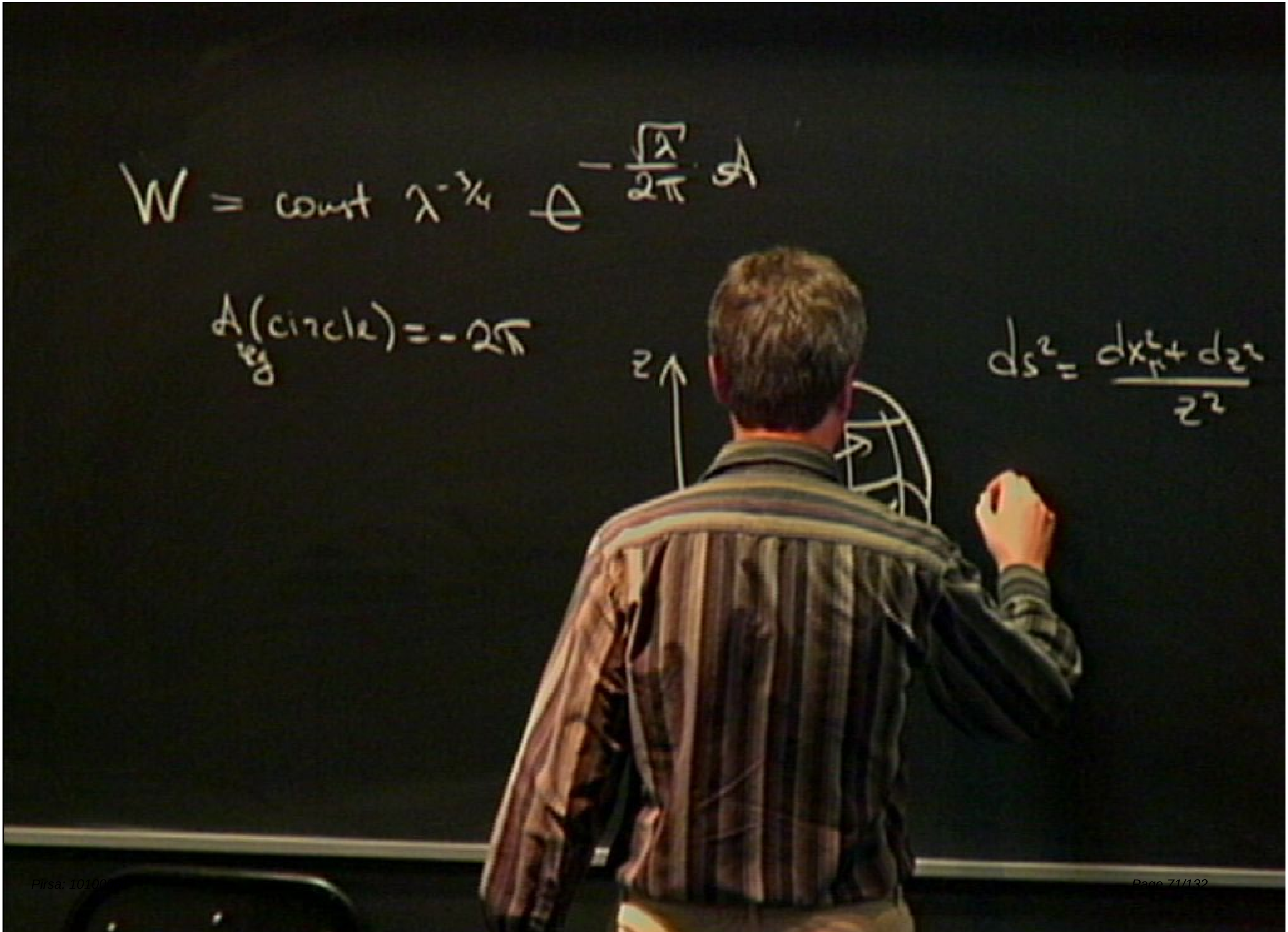

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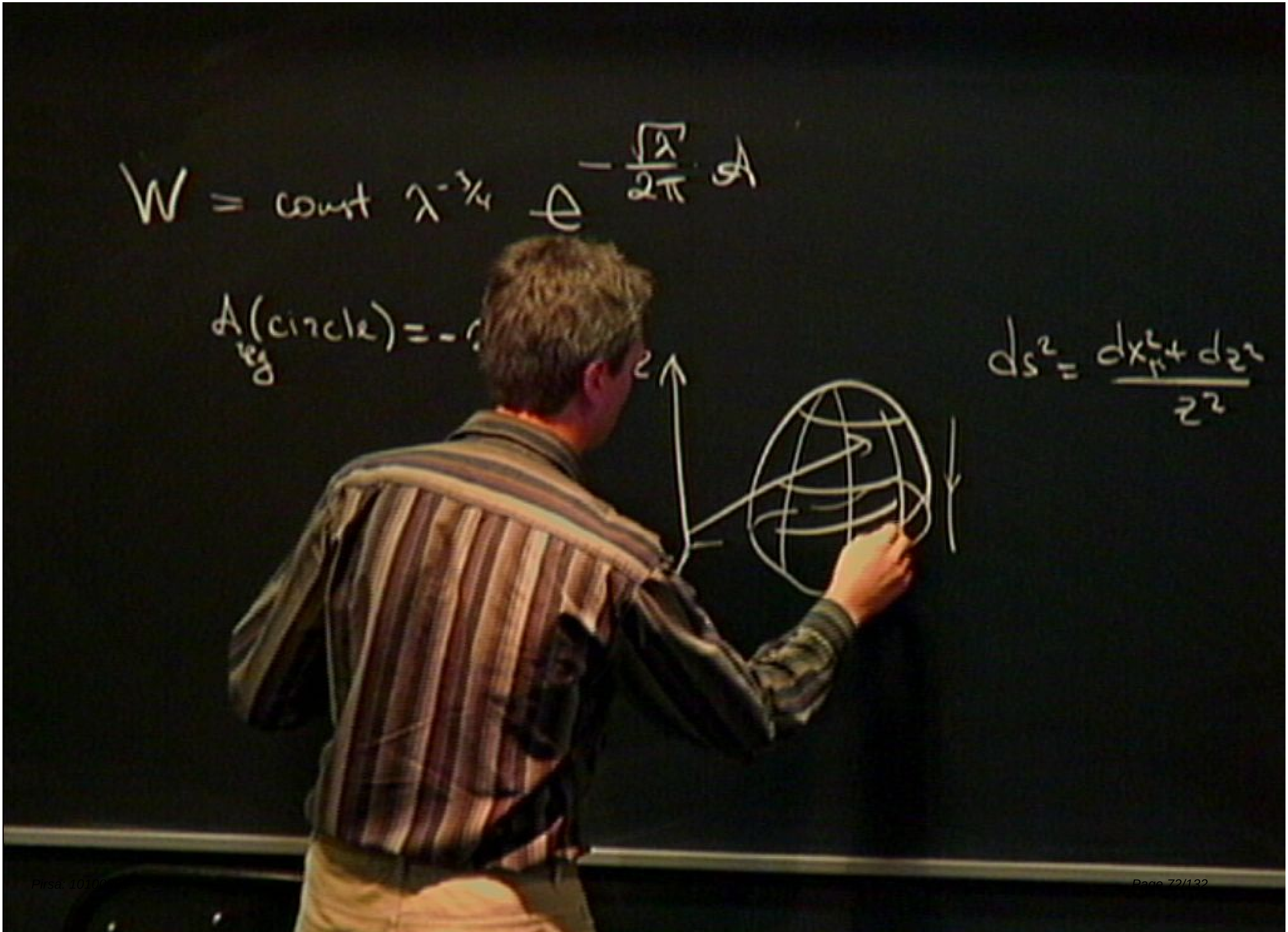
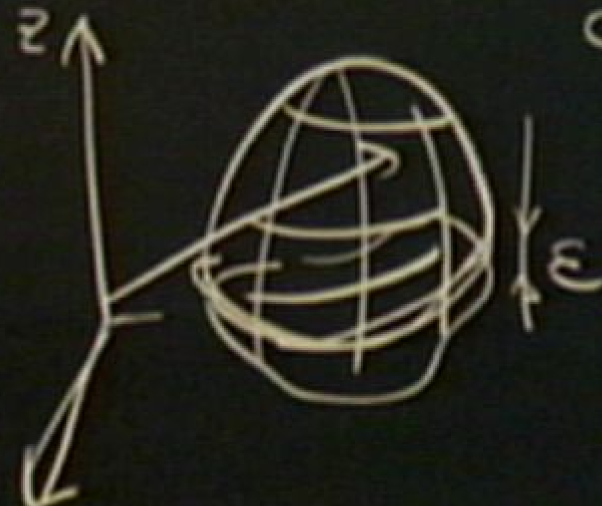
$A_{reg}(\text{circle}) = -2\pi$
z
$ds^2 = \frac{dx_i^2 + dz^2}{z^2}$



W = const λ^{-3/4} e^{-√λ/2π · A}
A_reg(circle) = -
ds² = (dx_μ² + dz²)/z²
z

$$W = \text{const}\ \lambda^{-3/4}\, e^{-\frac{\sqrt{\lambda}}{2\pi} \mathcal{A}}$$

$$\mathcal{A}_{reg}(\text{circle}) = -2\pi$$



$$ds^2 = \frac{dx_r^2 + dz^2}{z^2}$$

$$W = \text{const}\ \lambda^{-3/4}\ e^{-\frac{\sqrt{\lambda}}{2\pi} A_{reg}}$$

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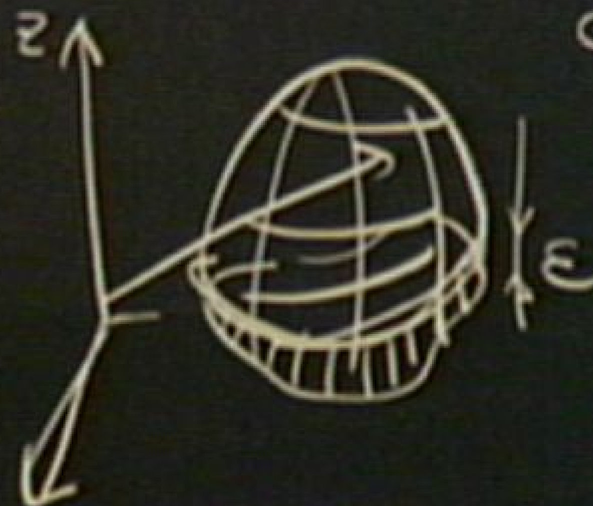

$$ds^2 = \frac{dx_r^2 + dz^2}{z^2}$$

$$A = \frac{L}{\varepsilon} + A_{reg}$$

$$W = \text{const}\, \lambda^{-3/4}\, e^{-\frac{\sqrt{\lambda}}{2\pi} A_{reg}} \left(1 + \frac{c_1}{\sqrt{\lambda}} + \frac{c_2}{\lambda} + \frac{c_3}{\lambda^{3/2}} + \dots\right)$$

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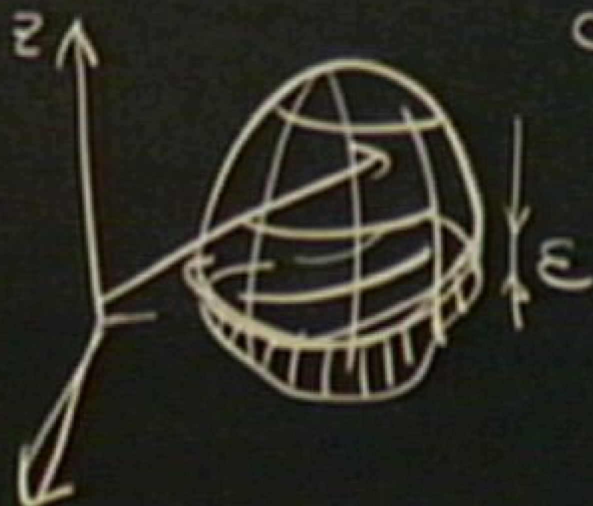

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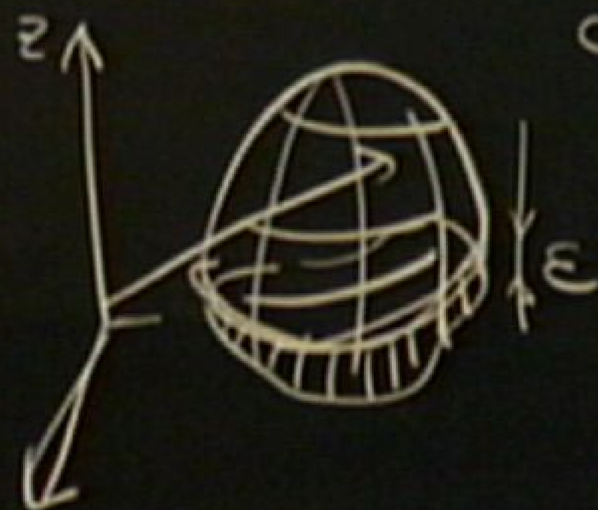
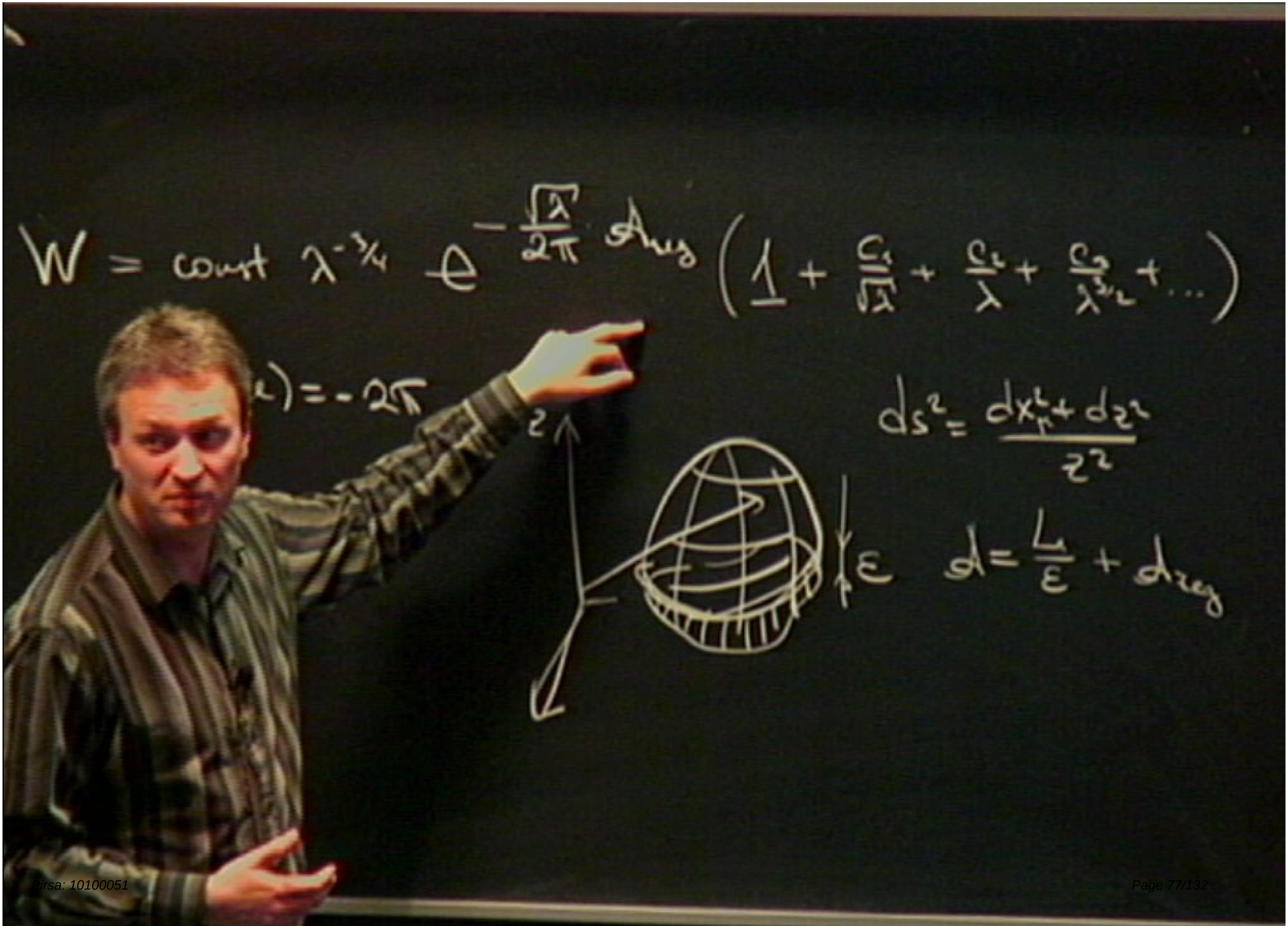
$$ds^2 = \frac{dx_\mu^2 + dz^2}{z^2}$$



$$A = \frac{L}{\varepsilon} + A_{r}$$



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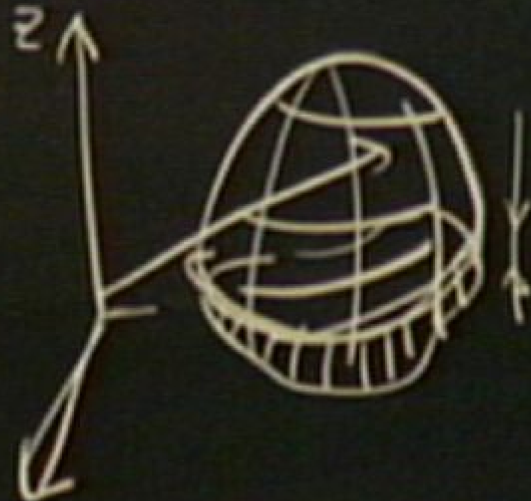

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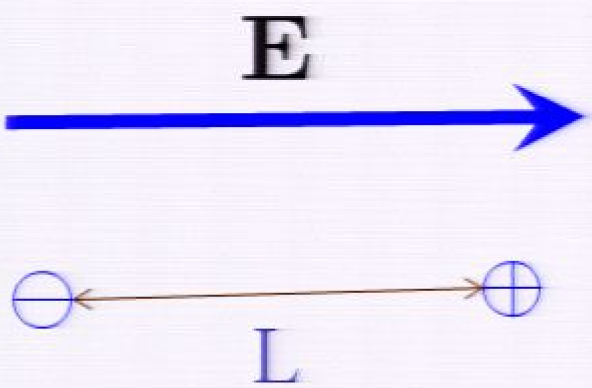
Schwinger pair production

Schwinger'51



Wins energy if $EL>2m$.

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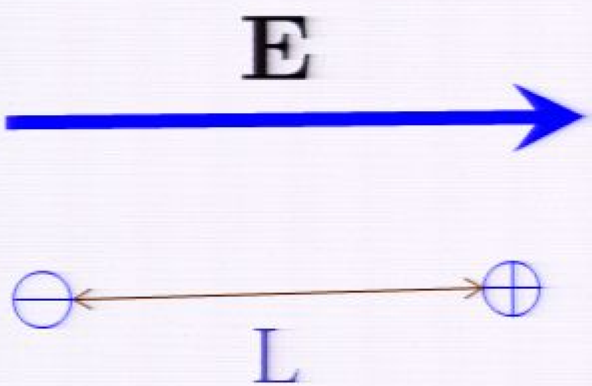

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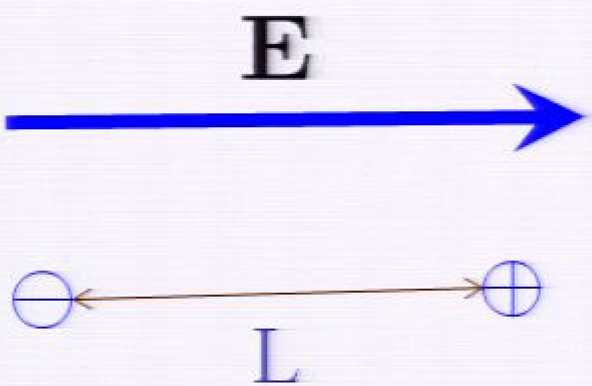

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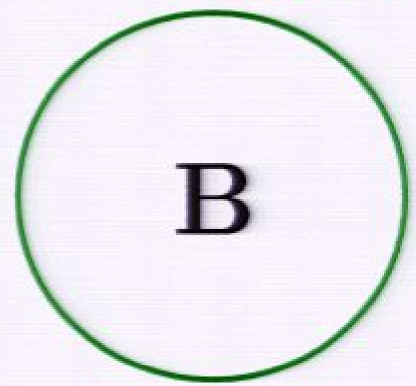
Imaginary-time tunneling picture

Affleck,Alvarez,Manton'82



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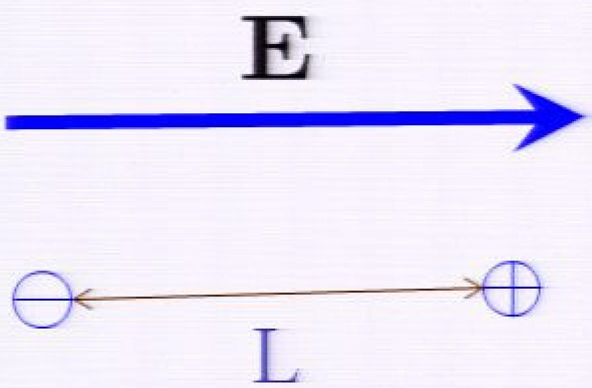

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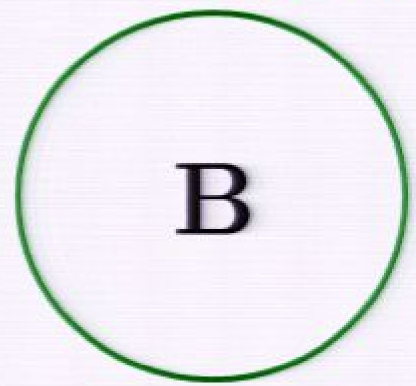

$$\mathcal{E} = 2\pi m R - \pi R^2 E \qquad R_c = \frac{m}{E}$$

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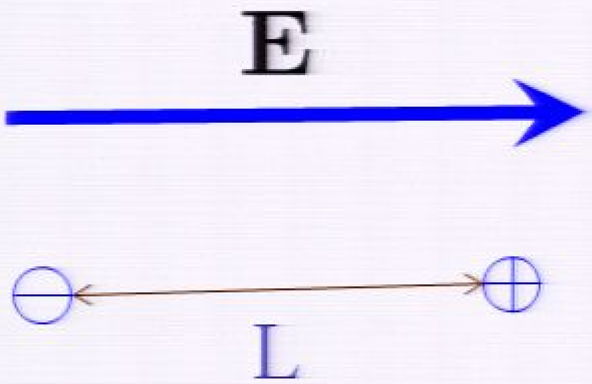

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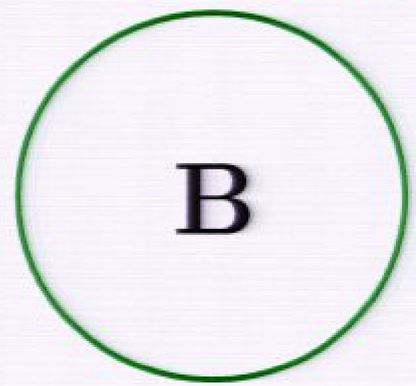

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Pair production rate: $\Gamma \sim \mathrm{e}^{-\frac{\pi m^2}{E}}$

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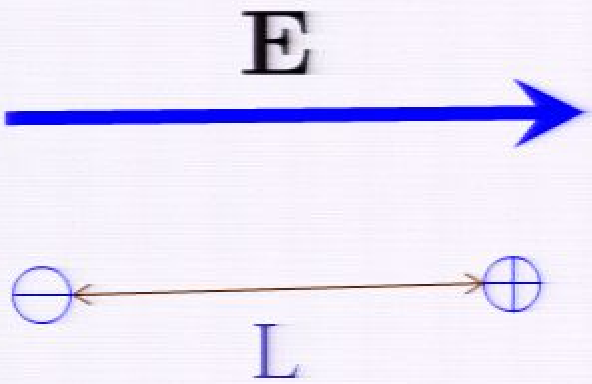

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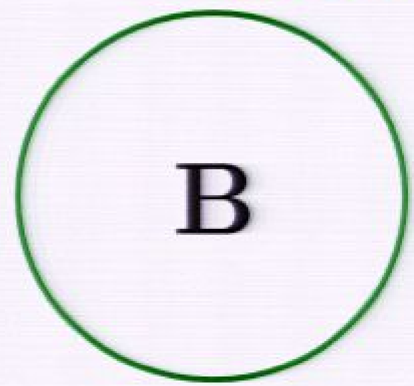

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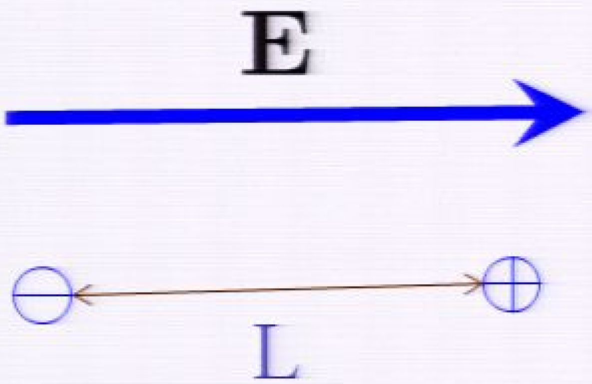

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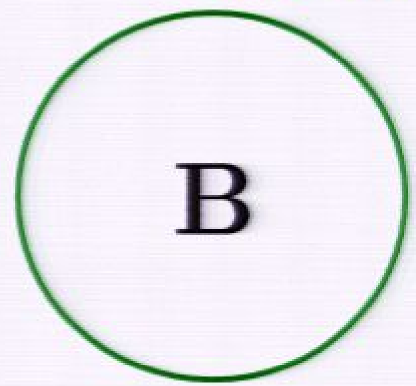

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Holographic calculation: $\Gamma \sim \mathrm{e}^{\sqrt{\lambda} - \frac{\pi m^2}{E}} \qquad E_c = \frac{m^2}{\pi\sqrt{\lambda}}$

Correlation functions

$$\langle \mathcal{O}(x)\,\mathcal{O}(y)\rangle = \frac{1}{|x-y|^{2\Delta}}$$

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Dilatation operator:

$$\hat{D}\, |\mathcal{O}\rangle = \Delta\, |\mathcal{O}\rangle$$

$$\hat{D} = \Delta_0 + \underbrace{\lambda \hat{D}_1 + \lambda^2 \hat{D}_2 + \lambda^3 \hat{D}_3 + \ldots}_{\text{matrix of anomalous dimensions}}$$

Local operators and spin chains

$$Z = \Phi_1 + i\Phi_2$$

$$W = \Phi_3 + i\Phi_4$$

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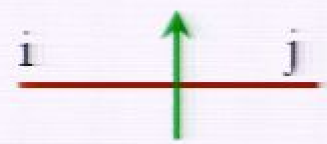
$Z_{ij} \quad \Longleftrightarrow$



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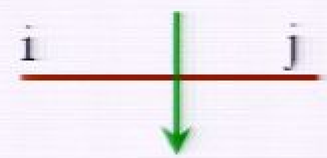

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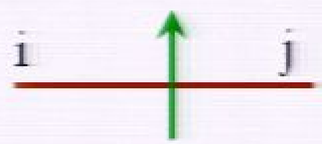
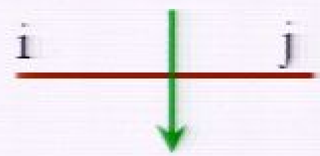
$Z_{ij} \quad \Longleftrightarrow$ 

$W_{ij} \quad \Longleftrightarrow$ 

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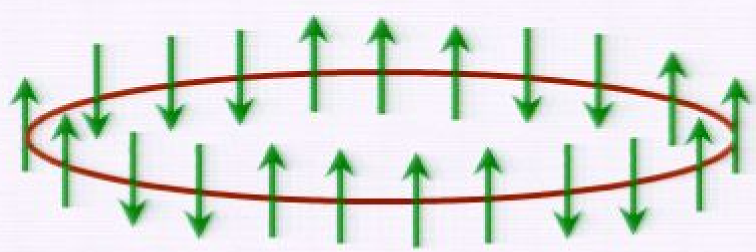
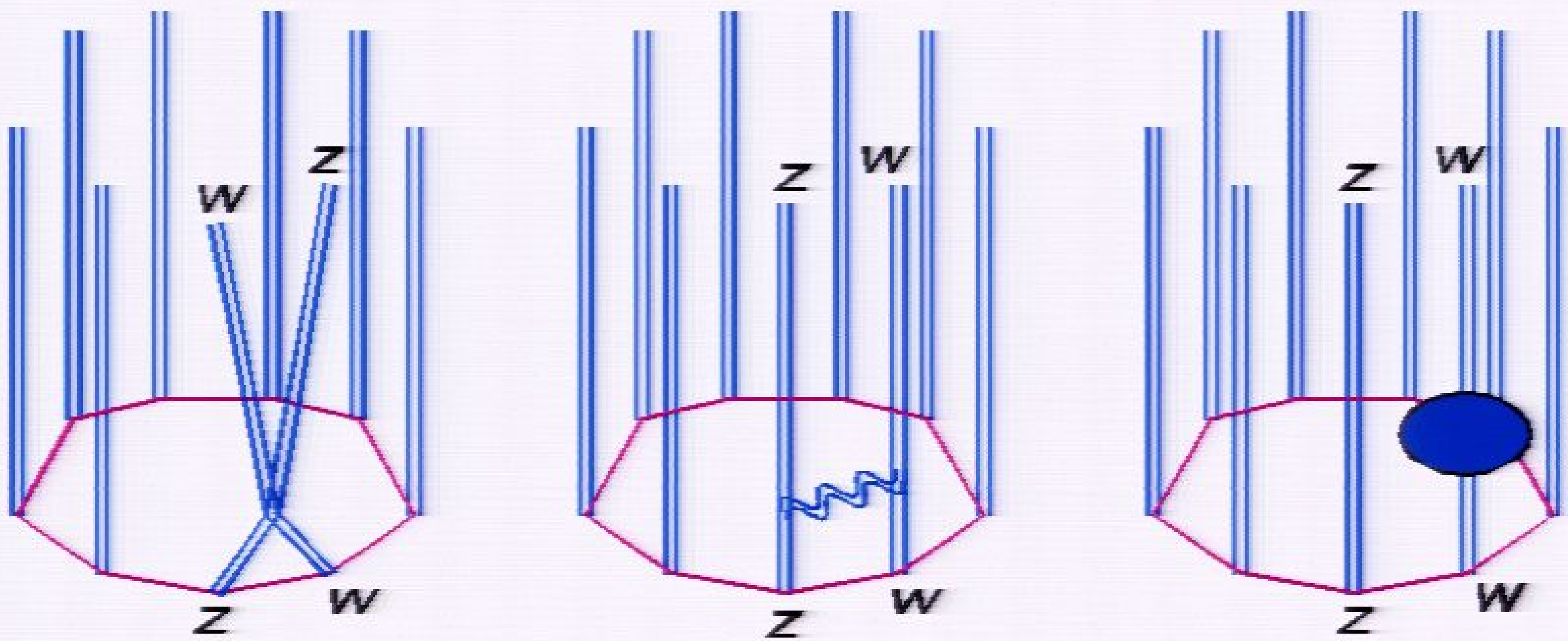


One loop planar ($N \to \infty$) diagrams:



One loop planar dilatation generator:

$$D = L + \frac{\lambda}{16\pi^2} \sum_{l=1}^{L} (1 - \boldsymbol{\sigma}_l \cdot \boldsymbol{\sigma}_{l+1}) + O(\lambda^2)$$

Minahan,Z.'02

Heisenberg Hamiltonian

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Minahan,Z.'02

Heisenberg Hamiltonian

$$Q = \sum_{l=1}^{L} \boldsymbol{\sigma}_l \cdot [\boldsymbol{\sigma}_{l+1} \times \boldsymbol{\sigma}_{l+2}]$$

Integrability!

$$[D, Q] = 0$$

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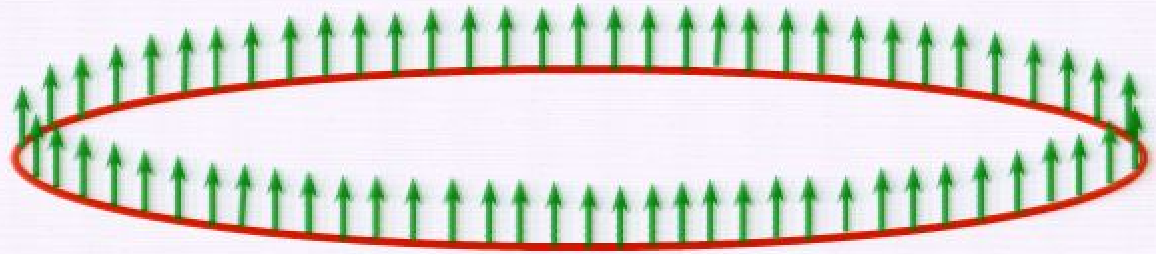
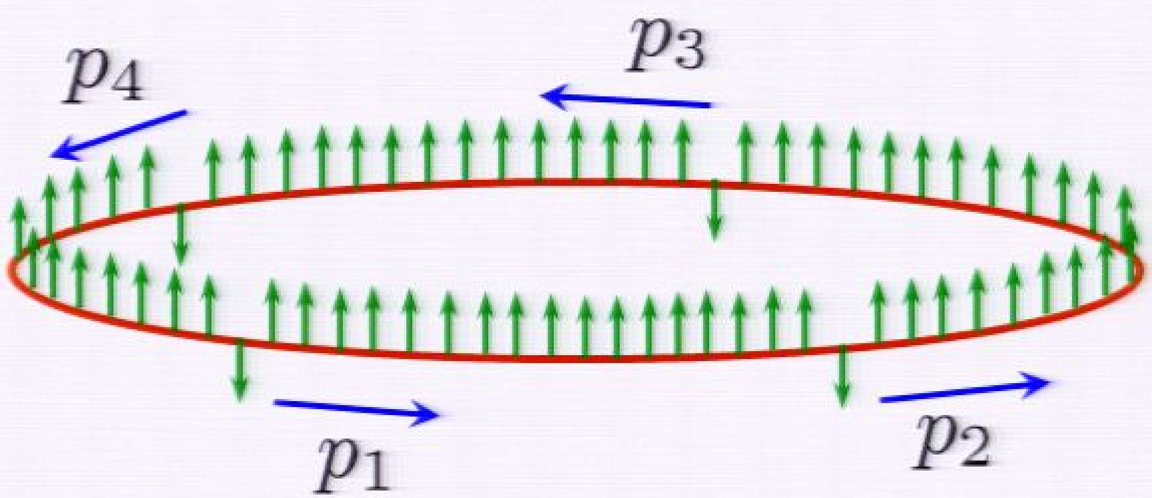
The spectrum

Ground state:



Excited states (magnons):



$$\varepsilon(p) = \frac{\lambda}{2\pi^2}\sin^2\frac{p}{2}$$

The spectrum

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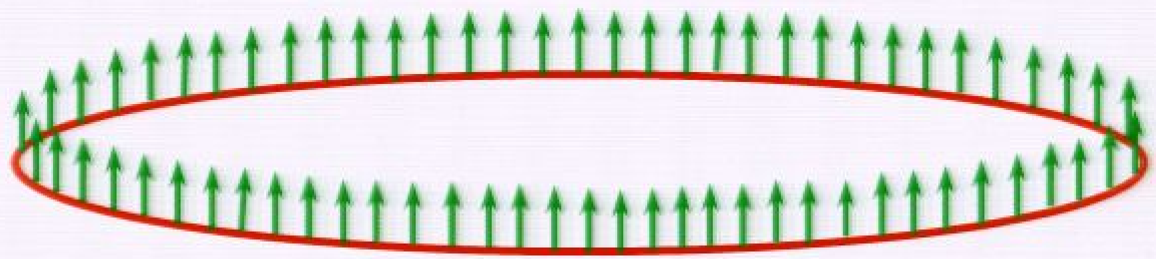

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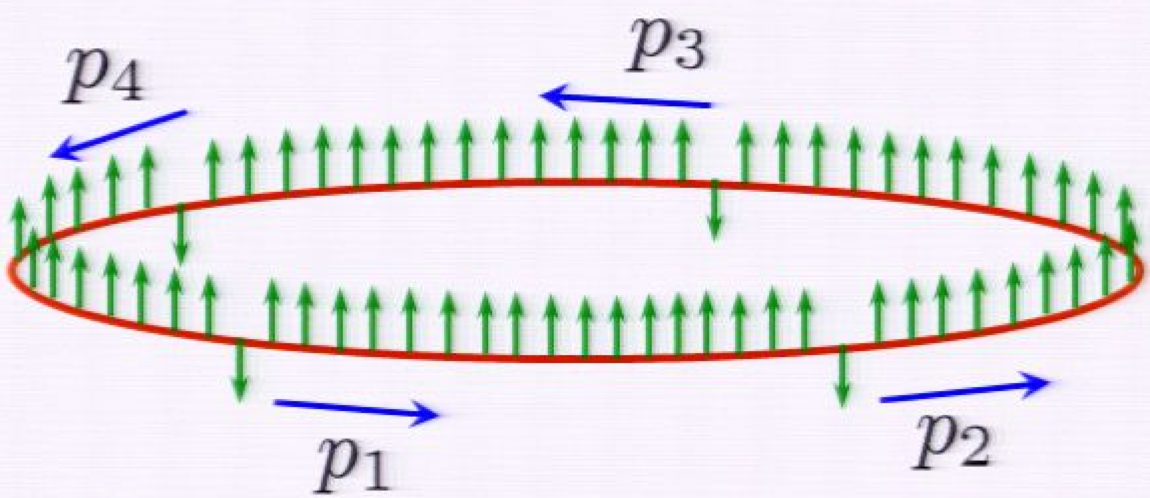

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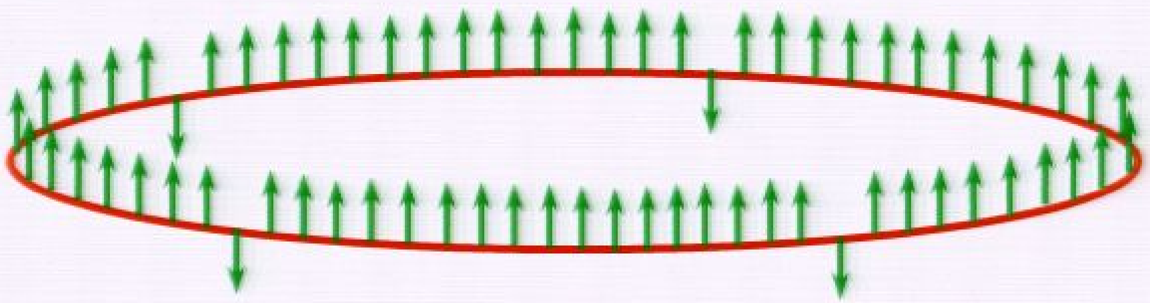
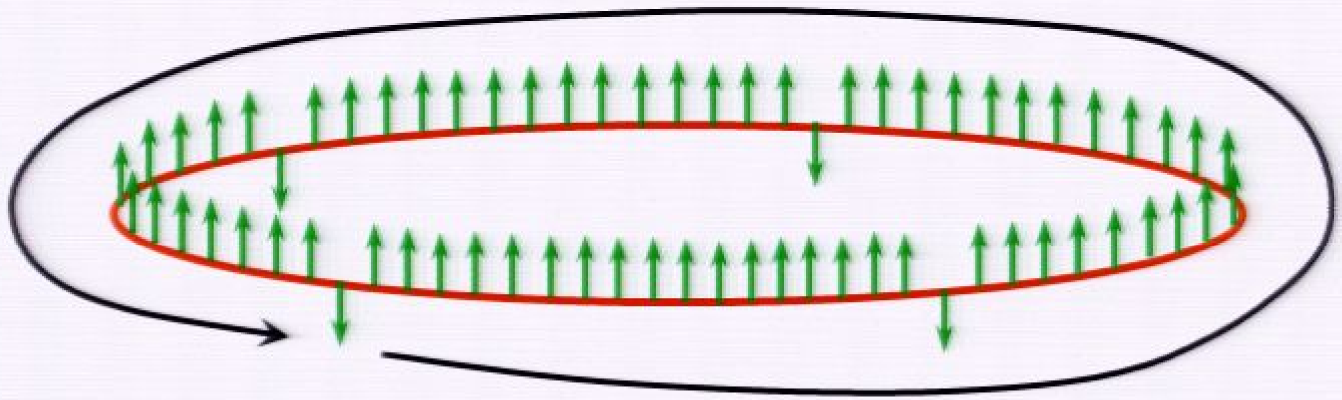
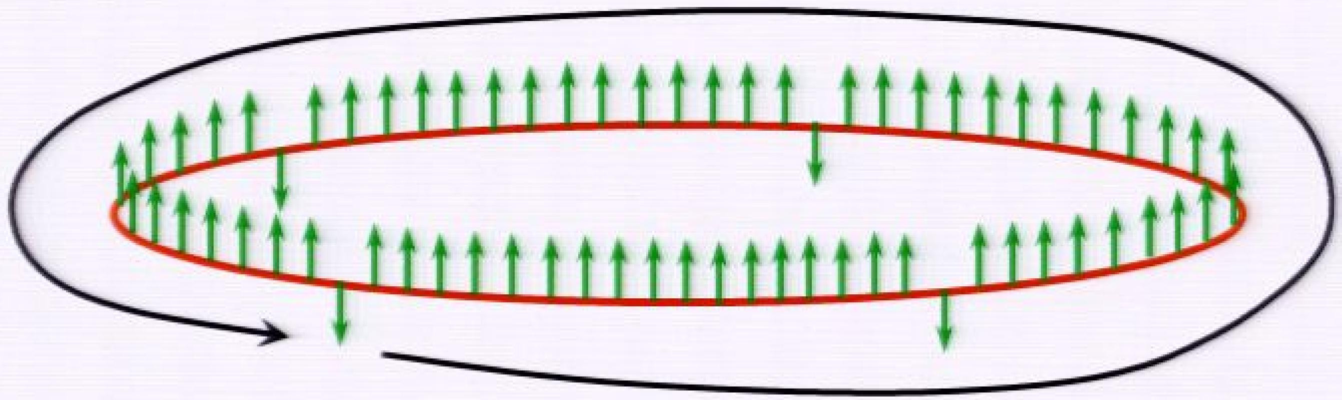
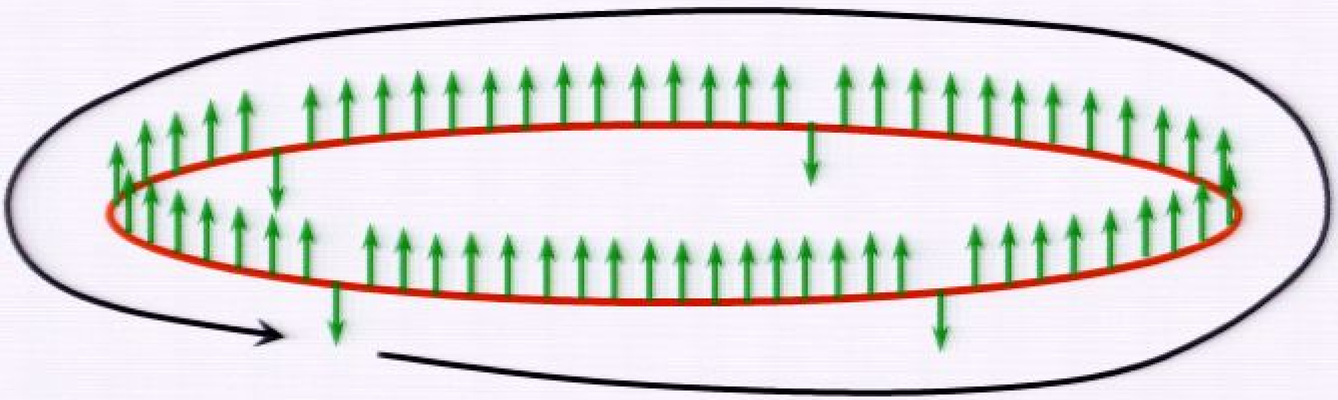






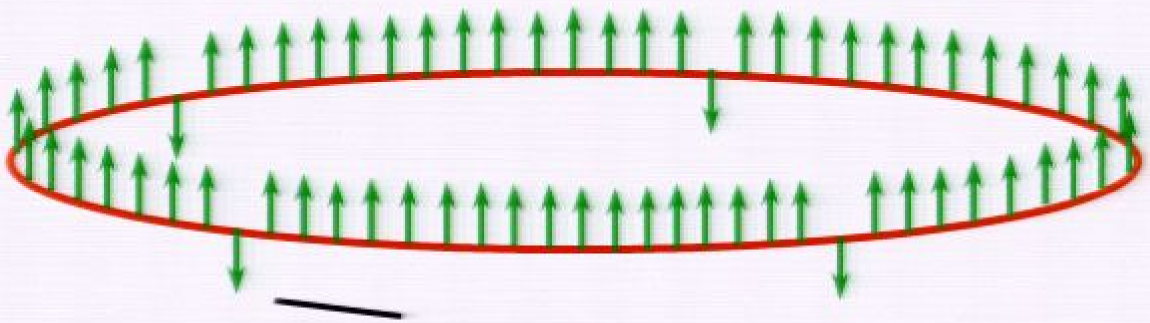
$$e^{\,i\langle \text{phase shift}\rangle} = e^{\,ipL}$$

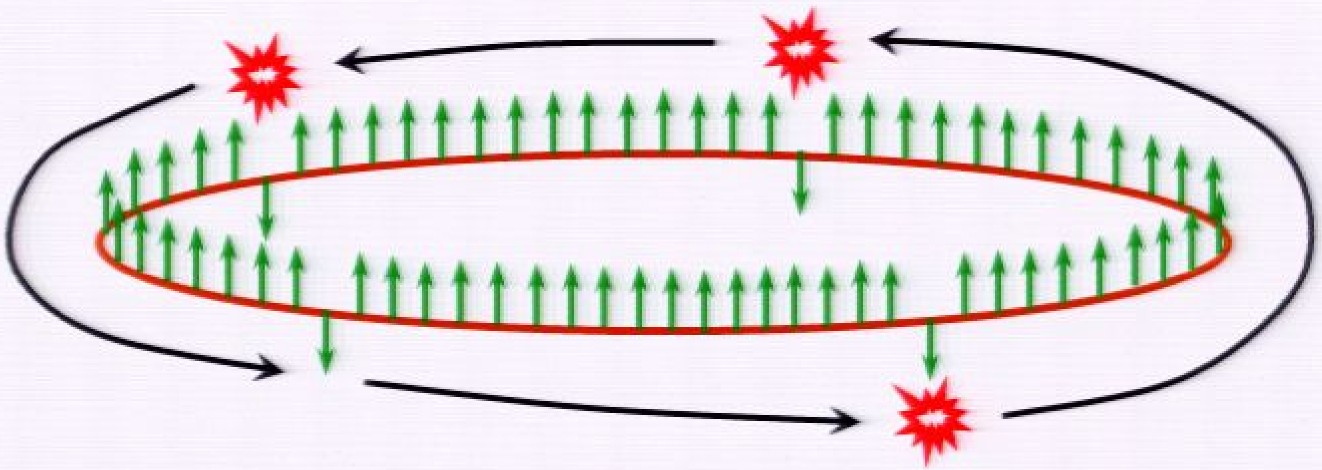


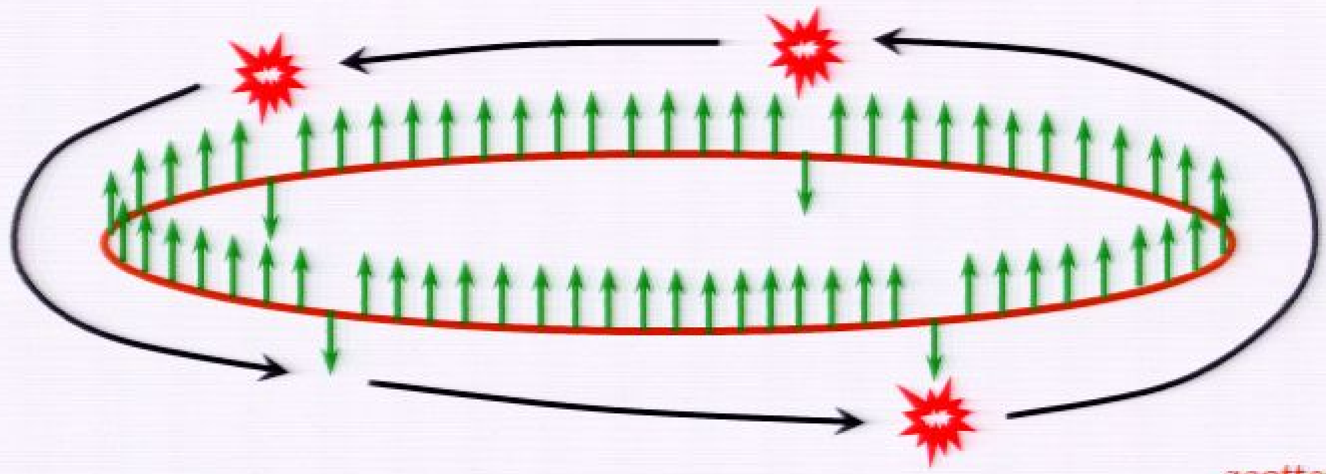
$$e^{\,i\langle \text{phase shift}\rangle} = e^{\,ipL}$$

$\Downarrow$ periodicity of the wave function

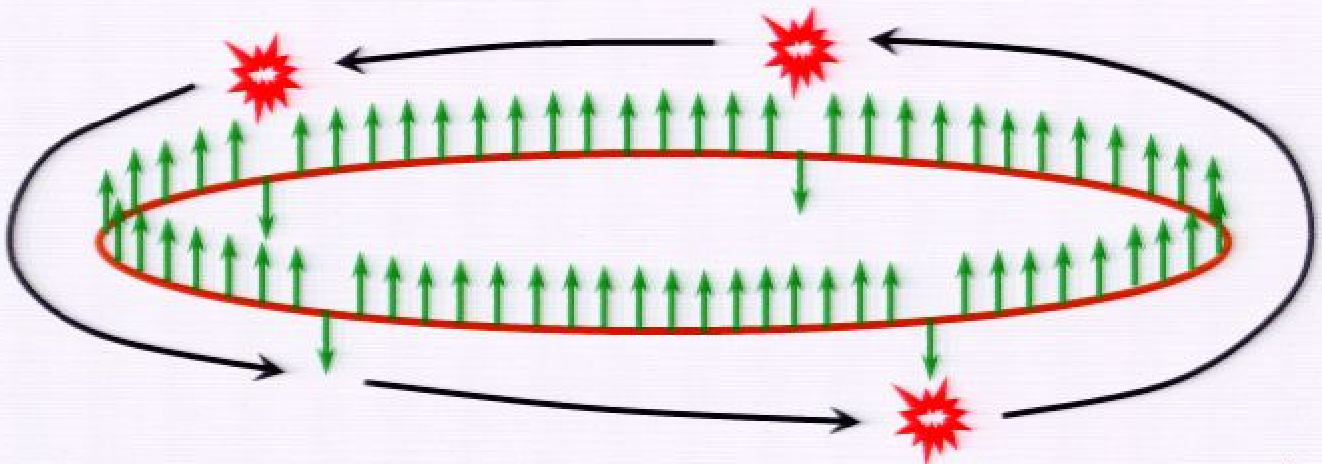
$$e^{\,ipL} = 1$$







$$e^{i\langle\text{total phase shift}\rangle} = e^{ip_j L} \prod_{k\neq j} e^{i\delta(p_j,p_k)}$$



momentum

scattering phase shifts

$$\mathrm{e}^{i\langle\text{total phase shift}\rangle} = \mathrm{e}^{ip_jL} \prod_{k\neq j} \mathrm{e}^{i\delta(p_j,p_k)}$$

Exact periodicity condition:

$$\mathrm{e}^{ip_jL} = \prod_{k\neq j} \mathrm{e}^{-i\delta(p_j,p_k)}$$

Bethe equations for Heisenberg model

Rapidity: $$e^{ip} = \frac{u + i/2}{u - i/2}$$

$$\boxed{\left(\frac{u_k + i/2}{u_k - i/2}\right)^L = \prod_{j \neq k} \frac{u_k - u_j + i}{u_k - u_j - i}}$$

Bethe'31

Anomalous dimension:

$$\Delta - L = \frac{\lambda}{8\pi^2} \sum_{k=1}^{M} \frac{1}{u_k^2 + 1/4}$$

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Full asymptotic BA for Super-Yang-Mills

Beisert,Staudacher'05

Beisert,Eden,Staudacher'06

Beisert,Hernandez,Lopez'06

$$1 = \prod_{j\neq k} \frac{s_k - s_j - i}{s_k - s_j + i} \prod_{l} \frac{s_k - r_l + \frac{i}{2}}{s_k - r_l - \frac{i}{2}}$$

$$1 = \prod_{j} \frac{z_k - x_j^+}{z_k - x_j^-} \prod_{l} \frac{r_k - s_l + \frac{i}{2}}{r_k - s_l - \frac{i}{2}}$$

$$\left(\frac{x_k^+}{x_k^-}\right)^L = \prod_{j\neq k} \mathrm{e}^{\,i\theta(x_k,x_j)} \frac{x_k^+ - x_j^-}{x_k^- - x_j^+} \frac{1 - \frac{1}{x_k^+ x_j^-}}{1 - \frac{1}{x_k^- x_j^+}} \prod_{l} \frac{x_k^- - z_l}{x_k^+ - z_l} \prod_{m} \frac{x_k^- - y_l}{x_k^+ - y_l}$$

$$1 = \prod_{j} \frac{y_k - x_j^+}{y_k - x_j^-} \prod_{l} \frac{v_k - w_l + \frac{i}{2}}{v_k - w_l - \frac{i}{2}}$$

$$\theta(x,x') = \sum_{r,s=\pm} rs\,\chi(x_r, x'_s)$$

$$1 = \prod_{j\neq k} \frac{w_k - w_j - i}{w_k - w_j + i} \prod_{l} \frac{w_k - v_l + \frac{i}{2}}{w_k - v_l - \frac{i}{2}}$$

$$\chi(x,y) = \frac{i}{8\pi^2} \oint_{|z|=1=|w|} \frac{dz}{z} \frac{dw}{w} \frac{1}{xz-1} \frac{1}{yw-1} \ln \frac{\Gamma\left(1 + \frac{i\sqrt{\lambda}}{4\pi}\left(z + \frac{1}{z} - w - \frac{1}{w}\right)\right)}{\Gamma\left(1 - \frac{i\sqrt{\lambda}}{4\pi}\left(z + \frac{1}{z} - w - \frac{1}{w}\right)\right)}$$

$$r = z + \frac{\lambda}{16\pi^2 z} \qquad v = y + \frac{\lambda}{16\pi^2 y} \qquad u = x + \frac{\lambda}{16\pi^2 x} \qquad u \pm \frac{i}{2} = x^{\pm} + \frac{\lambda}{16\pi^2 x^{\pm}}$$

$$\Delta - L = \sum_k \left[i(x_k^- - x_k^+) - 1\right] \qquad \prod_k \frac{x_k^+}{x_k^-} = 1$$

Wrapping/finite size effects

$$\Delta_{\text{exact}} = \Delta_{\text{asymptotic BA}} + O(\mathrm{e}^{-\kappa(\lambda)L})$$

$$\kappa(\lambda) = 2\,\mathrm{arcsinh}\,\frac{\pi}{\sqrt{\lambda}}$$

Ambjørn,Janik,Kristjansen'05

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Ambjørn,Janik,Kristjansen'05

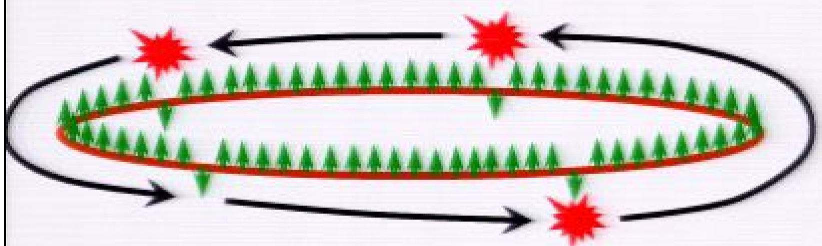
Weak coupling:

$$\Delta = \underbrace{\Delta_0 + \Delta_1\lambda + \ldots}_{\text{captured by ABA}} + O(\lambda^L)$$

wrapping order

Beisert,Kristjansen,Staudacher '03

Spin chain



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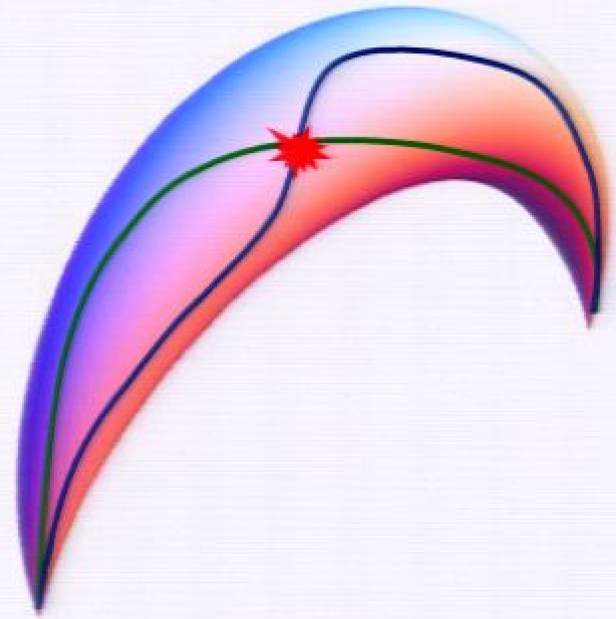

Magnons $\longleftrightarrow$ String vibrations

Exact dispersion relation: $$\varepsilon(p) = \sqrt{1 + \frac{\lambda}{\pi^2}\sin^2\frac{\pi p}{\sqrt{\lambda}}}$$ Beisert,Dippel,Staudacher'04 Beisert'05

Known S-matrix Beisert'05

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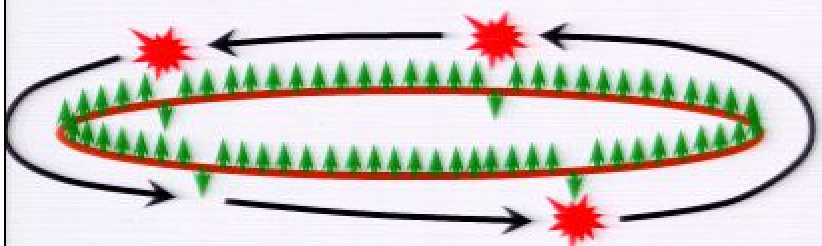

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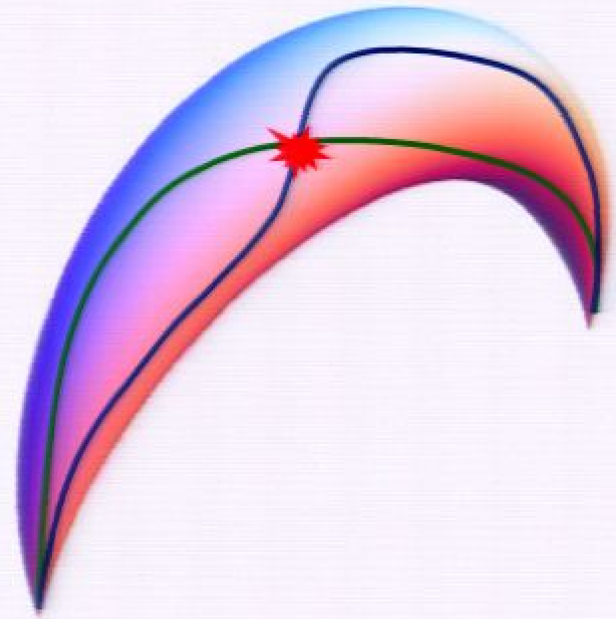

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Known S-matrix Beisert'05

Gromov,Kazakov,Vieira'09

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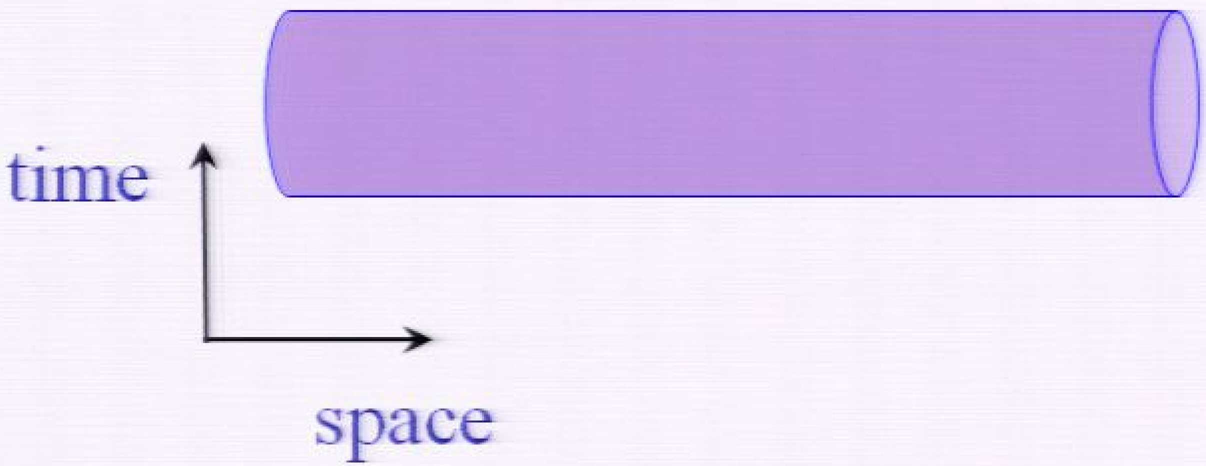
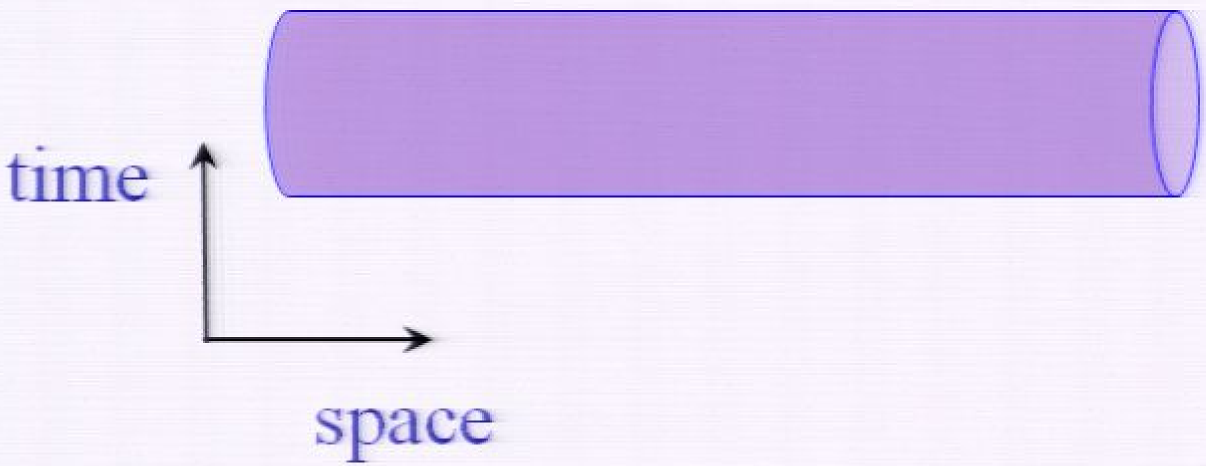


periodic time = thermodynamics

Gromov,Kazakov,Vieira'09



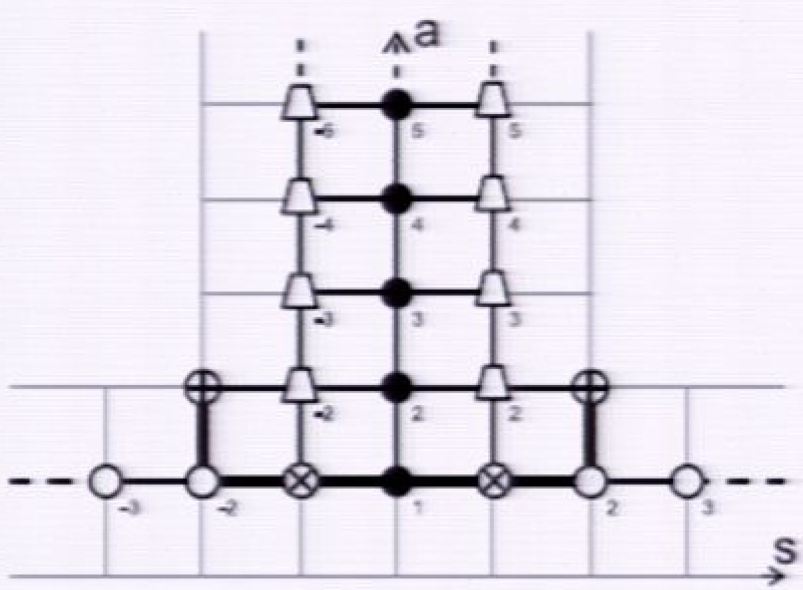
periodic time = thermodynamics

Exact solution

Gromov,Kazakov,Vieira'09
Gromov,Kazakov,Kozak,Vieira'09
Bombardelli,Fioravanti,Tateo'09
Arutyunov,Frolov'09

Y-system of thermodynamic Bethe ansatz:

$$\frac{Y^+_{a,s}Y^-_{a,s}}{Y_{a+1,s}Y_{a-1,s}} = \frac{(1+Y_{a,s+1})(1+Y_{a,s-1})}{(1+Y_{a+1,s})(1+Y_{a-1,s})}.$$



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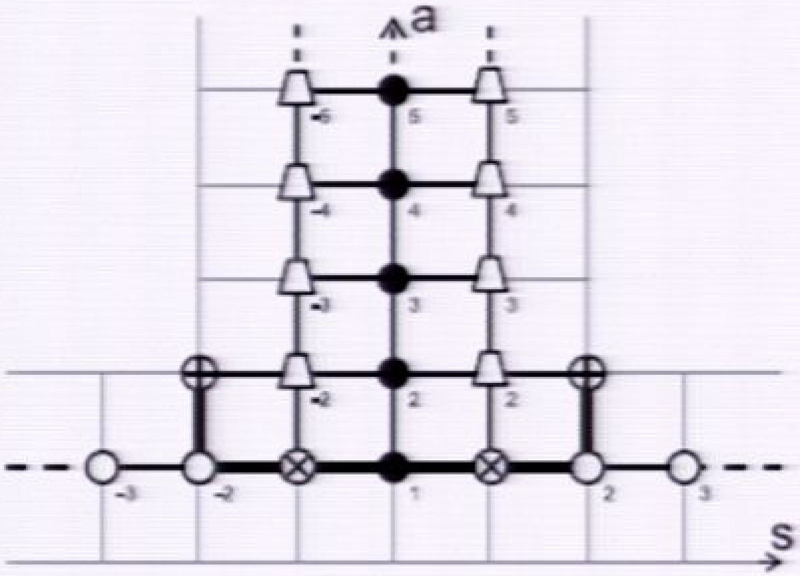
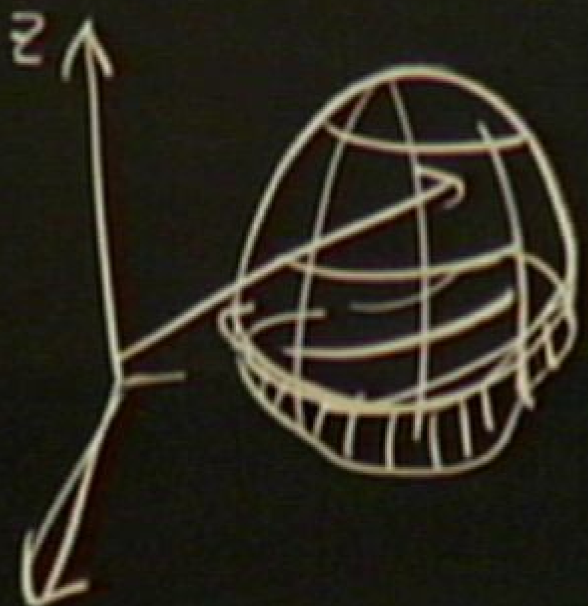


$$e^{-\frac{\sqrt{\lambda}}{2\pi} A_{reg}} \left(1 + \frac{c_1}{\sqrt{\lambda}} + \frac{c_2}{\lambda} + \frac{c_3}{\lambda^{3/2}} + \dots \right)$$

2π

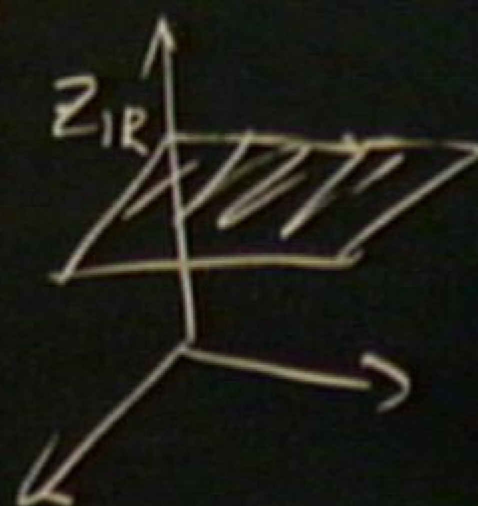


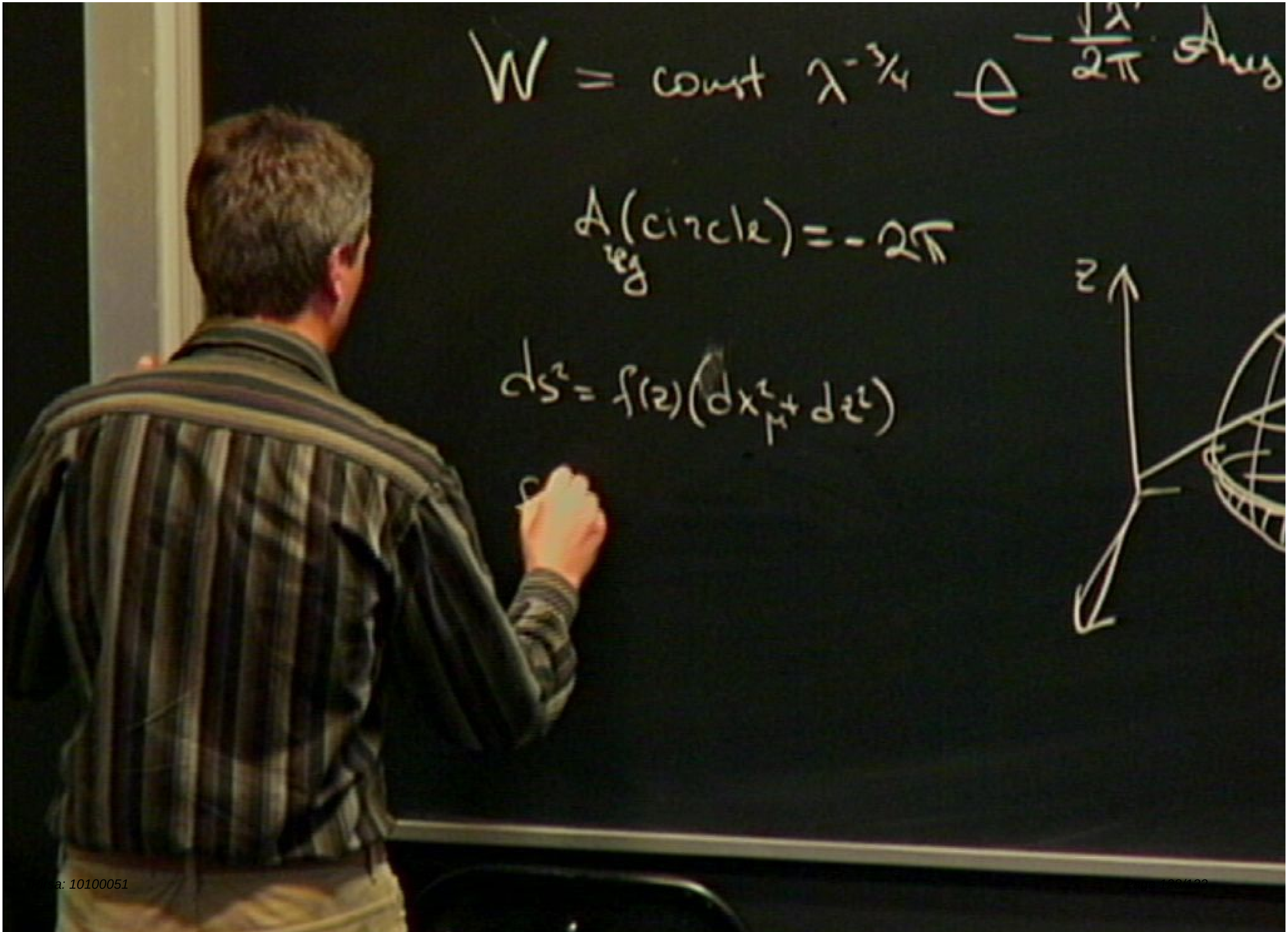
$$\boxed{ds^2 = \frac{dx_\mu^2 + dz^2}{z^2}}$$

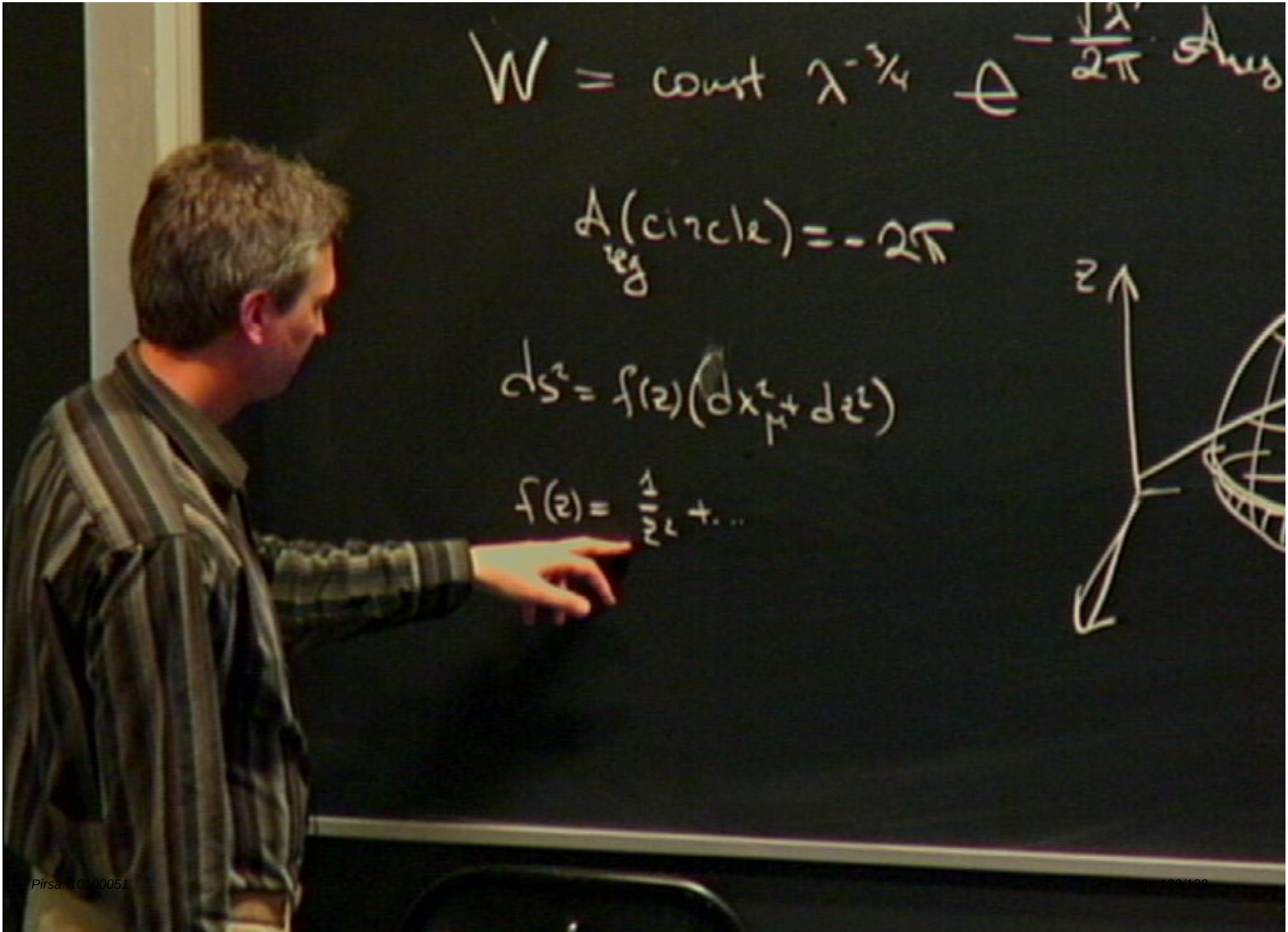
$x_\mu \to a x_\mu$

$z \to a z$

ε $\quad A = \frac{L_1}{\varepsilon} + A_{reg}$




$W = \text{const}\ \lambda^{-3/4}\ e^{-\frac{\sqrt{\lambda}}{2\pi}\cdot A_{reg}}$
$A_{reg}(\text{circle}) = -2\pi$
$ds^2 = f(z)\left(dx_\mu^2 + dz^2\right)$
z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$f(z) = \frac{1}{z^2} + \dots$
z

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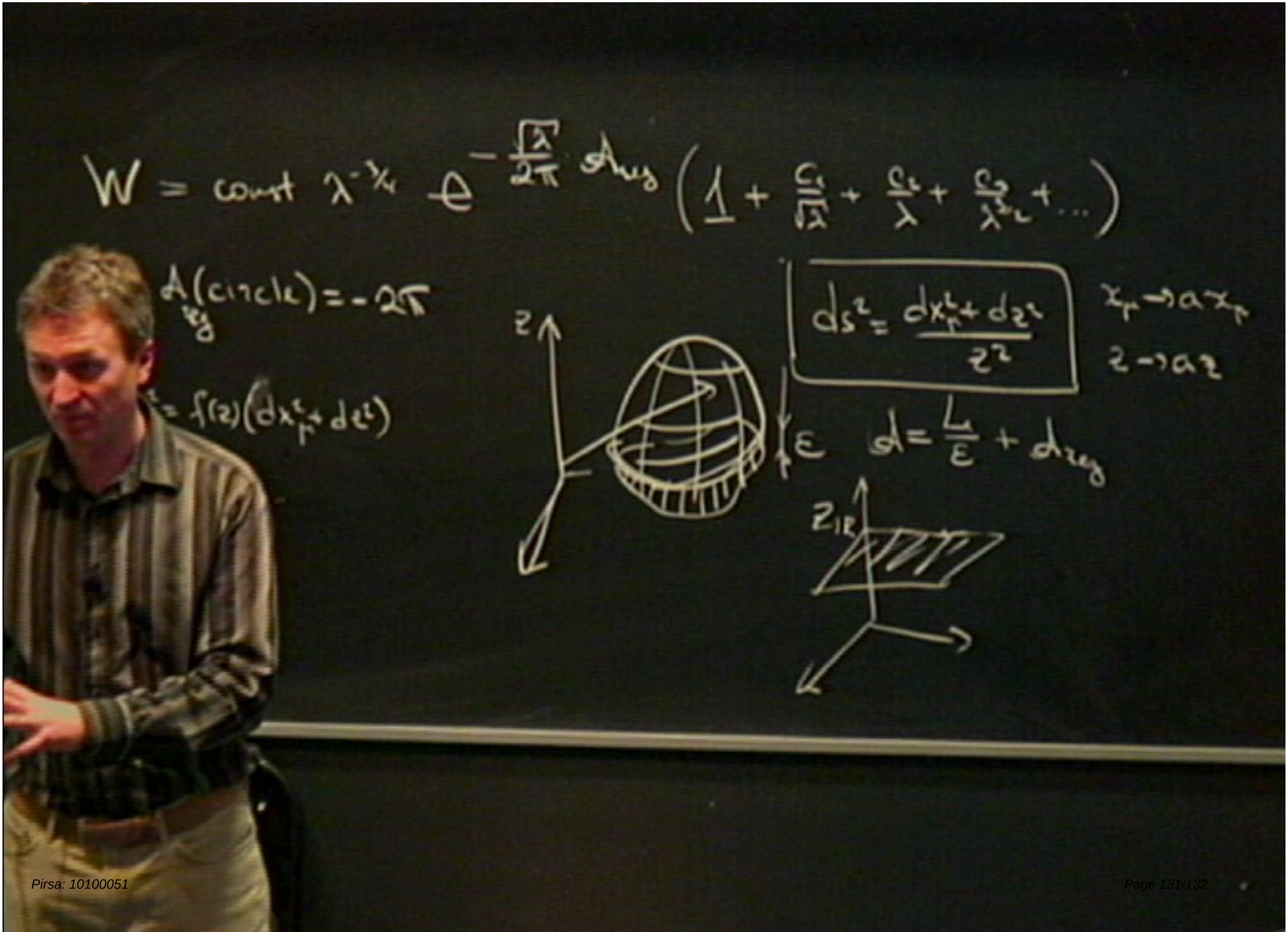
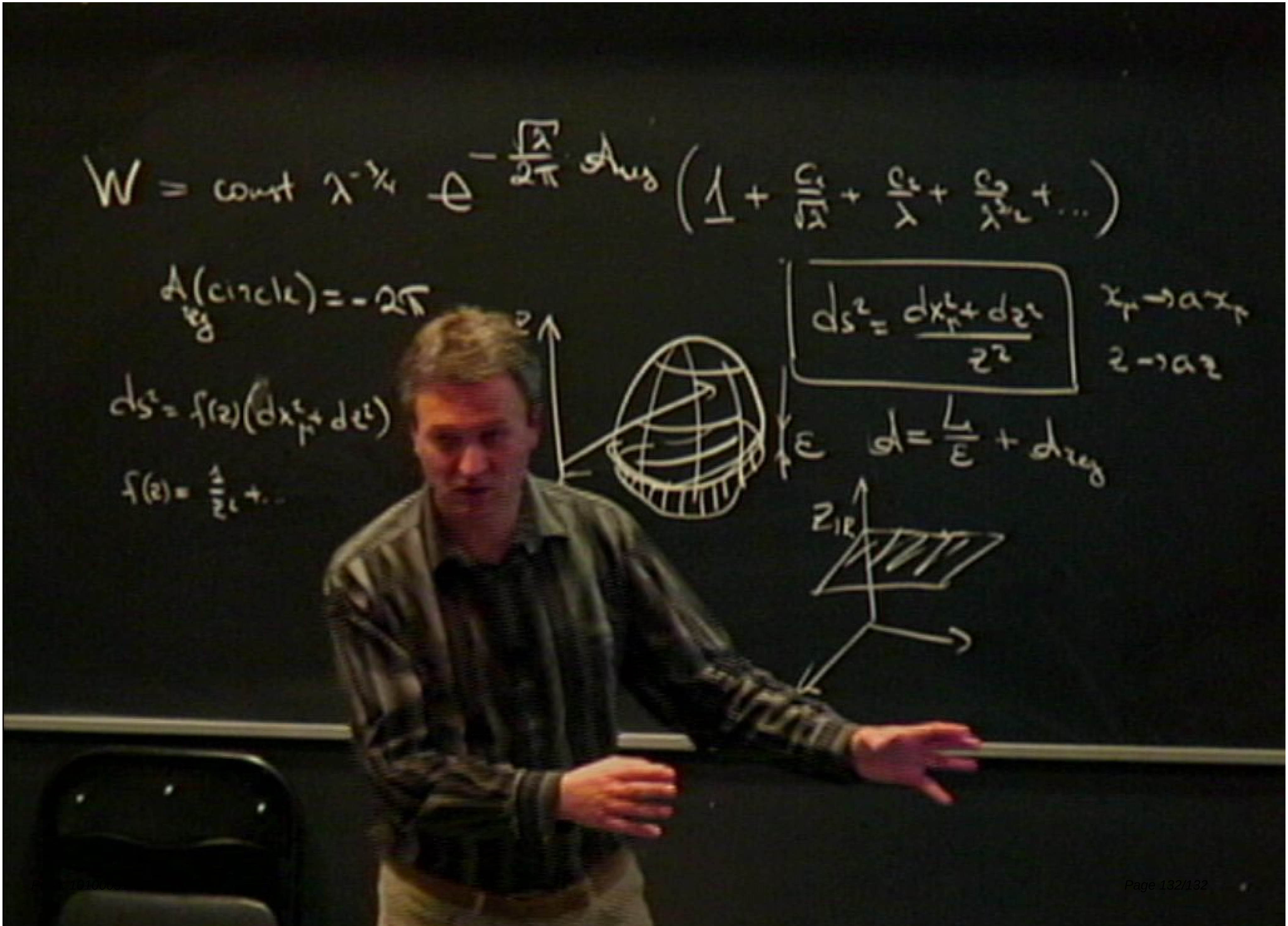

W = const λ^{-3/4} e^{-√λ/2π d_reg} (1 + c_1/√λ + c_2/λ + c_3/λ^{3/2} + …)
A_reg(circle) = -2π
= f(z)(dx_μ^2 + dz^2)
z
ds^2 = (dx_μ^2 + dz^2)/z^2
x_μ → a x_μ
z → a z
ε
d = L/ε + d_reg
z_IR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A_reg(circle) = -2π
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f(z) = 1/z² + ...
ds² = (dx_μ² + dz²)/z²
x_μ → a x_μ
z → a z
z
ε
A = L/ε + A_reg
z_IR