Title: Proposal for Divergence-Free Quantization of Covariant Scalar Fields

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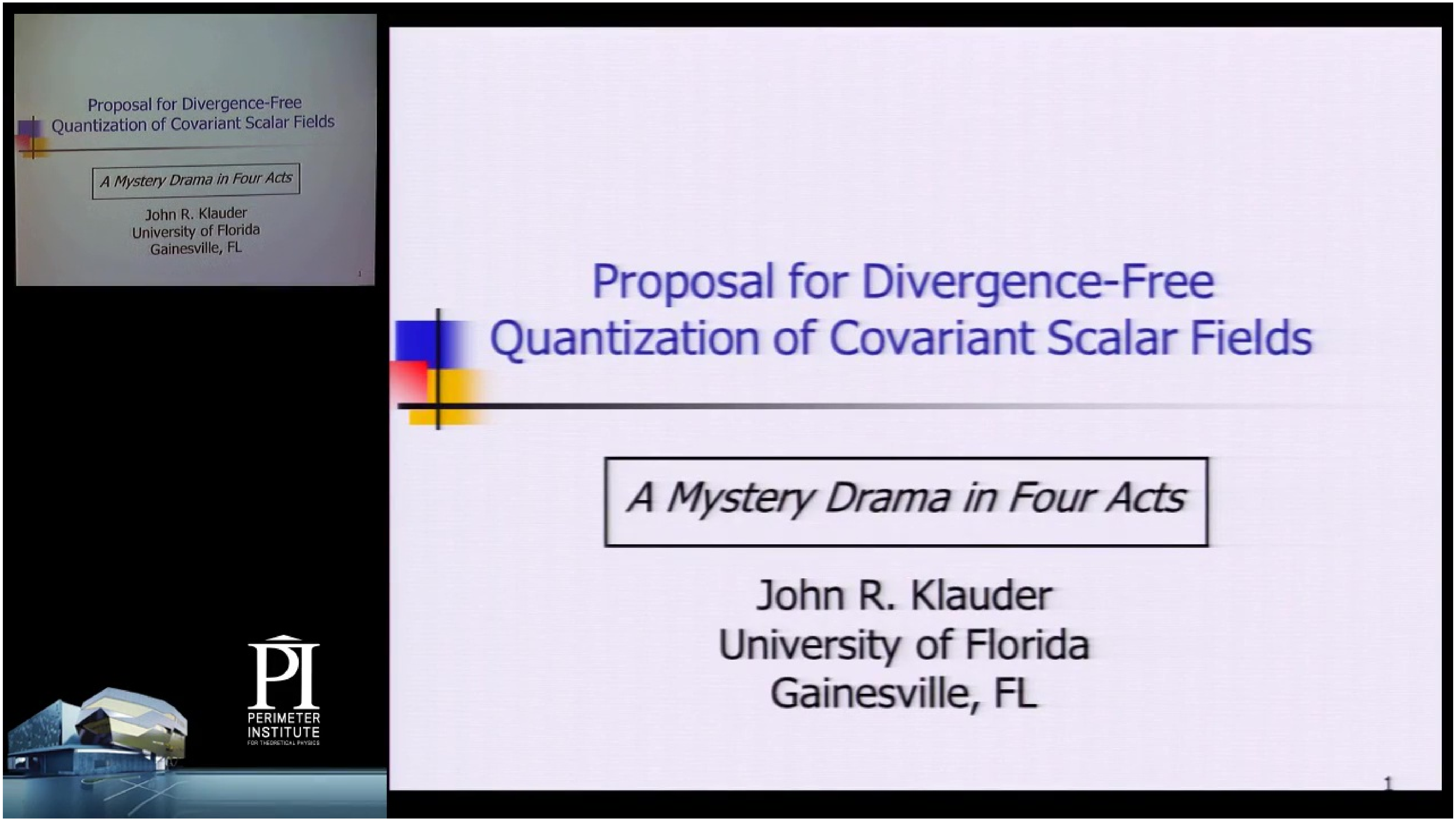
Abstract: Guided by idealized but soluble nonrenormalizable models, a nontraditional proposal for the quantization of covariant scalar field theories is advanced, which achieves a term-by-term, divergence-free perturbation analysis of interacting models expanded about a suitable pseudofree theory [differing from a free theory by an $O(\hbar^2)$ term]. This procedure not only provides acceptable solutions for models for which no acceptable solution currently exists, e.g., φ^4_n, for spacetime dimension $n\ge4$, but offers a new, divergence-free solution, for less-singular models as well, e.g., φ^4_n, for $n=2,3$.


Proposal for Divergence-Free
Quantization of Covariant Scalar Fields
A Mystery Drama in Four Acts
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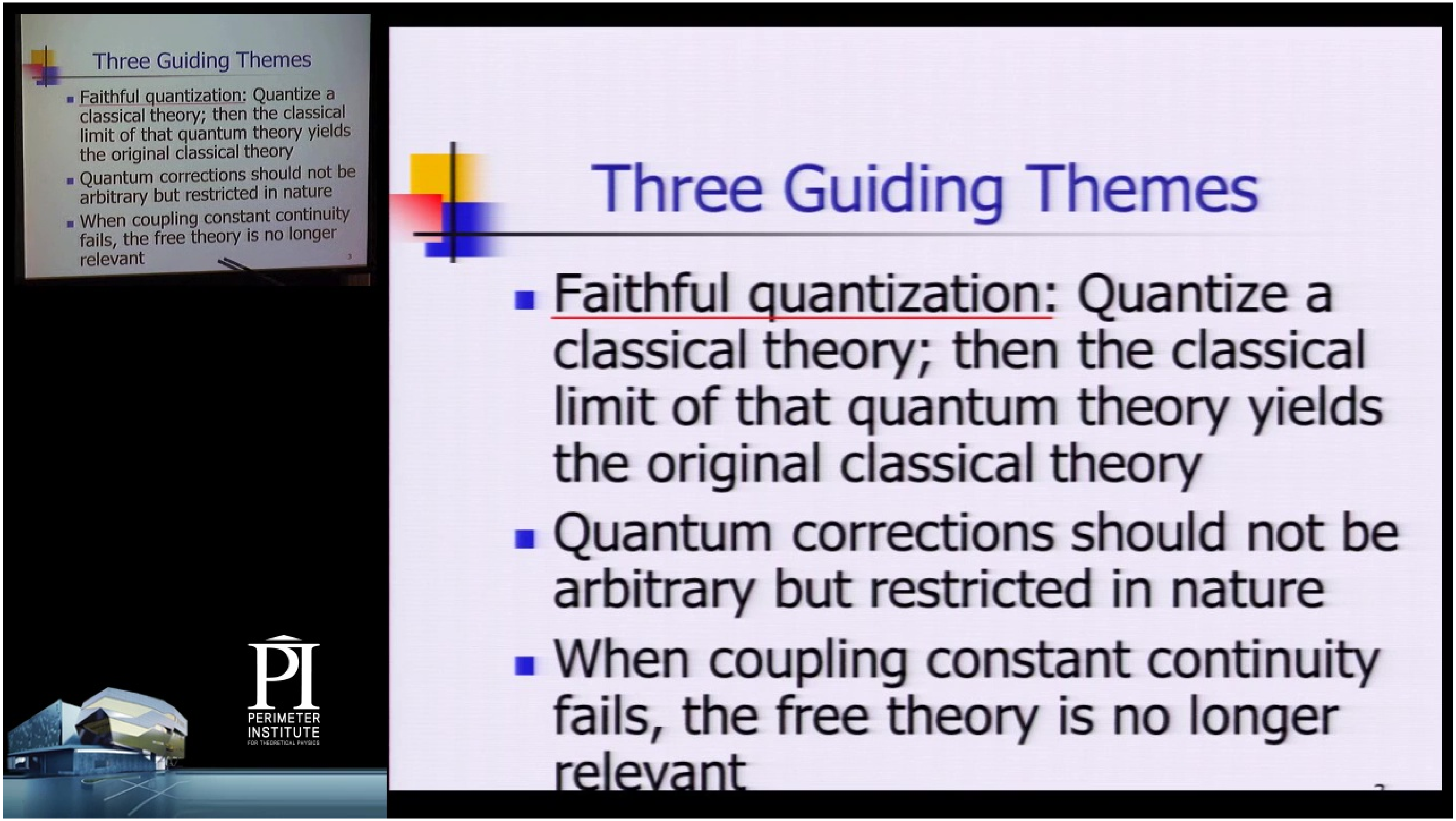
ACT 1: What and How

- Seek "faithful quantization" of interacting scalar fields for all spacetime dimensions
- Solve high dimension models first and then extend to lower dimensions
- Use soluble models as principal guide


Three Guiding Themes
Faithful quantization: Quantize a classical theory; then the classical limit of that quantum theory yields the original classical theory
Quantum corrections should not be arbitrary but restricted in nature
When coupling constant continuity fails, the free theory is no longer relevant
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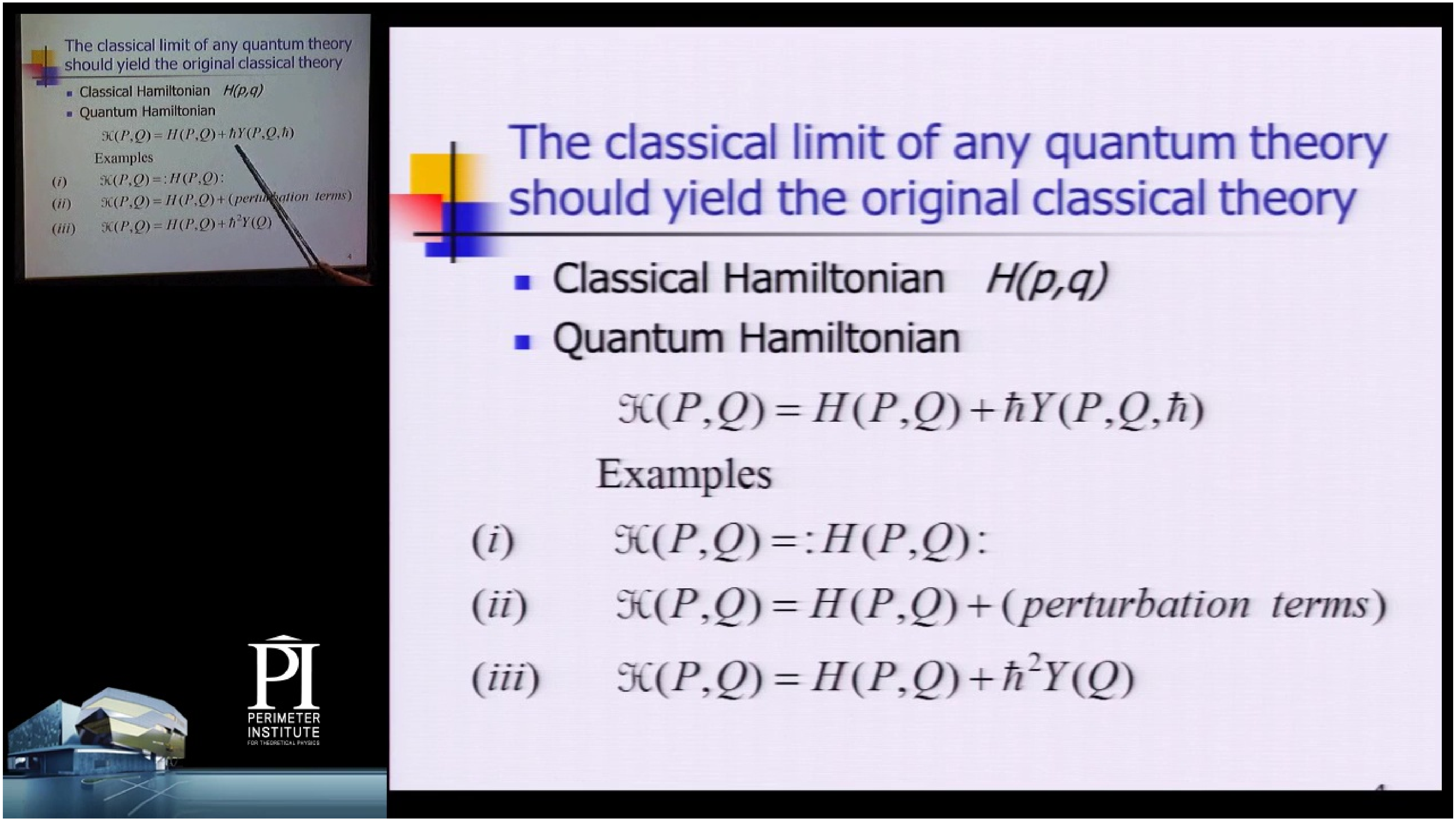
The classical limit of any quantum theory should yield the original classical theory
Classical Hamiltonian $H(p,q)$
Quantum Hamiltonian
$\mathcal{H}(P,Q) = H(P,Q) + \hbar Y(P,Q,\hbar)$
Examples
(i) $\mathcal{H}(P,Q) = :H(P,Q):$
(ii) $\mathcal{H}(P,Q) = H(P,Q) + (perturbation\ \ terms)$
(iii) $\mathcal{H}(P,Q) = H(P,Q) + \hbar^2 Y(Q)$
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Quantum corrections should not be arbitrary but restricted in nature

$$\mathcal{H}(P,Q) = H(P,Q) + \hbar^2 Y(Q)$$

Make a scale transformation :

$$P \to S^{-1}P \ , \ \ Q \to SQ \ , \ \ \ (S > 0)$$

$$\mathcal{H}(S^{-1}P, SQ) = H(S^{-1}P, SQ) + \hbar^2 Y(SQ)$$

$$\mathcal{H}(P,Q) = \tfrac{1}{2}(P^2 + m^2Q^2) + gQ^4 + \hbar^2 \Sigma_\beta c_\beta Q^\beta$$

Favored β: $(\beta \neq 6)$, $\beta = 4$, $\beta = 2$, $\underline{\beta = -2}$

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Coupling constant continuity may fail

$$A_g = \int[\tfrac{1}{2}(\dot{x}^2 - x^2) - gx^{-4}]dt \ \ , \ \ g \geq 0 \qquad \underline{(hc)}$$

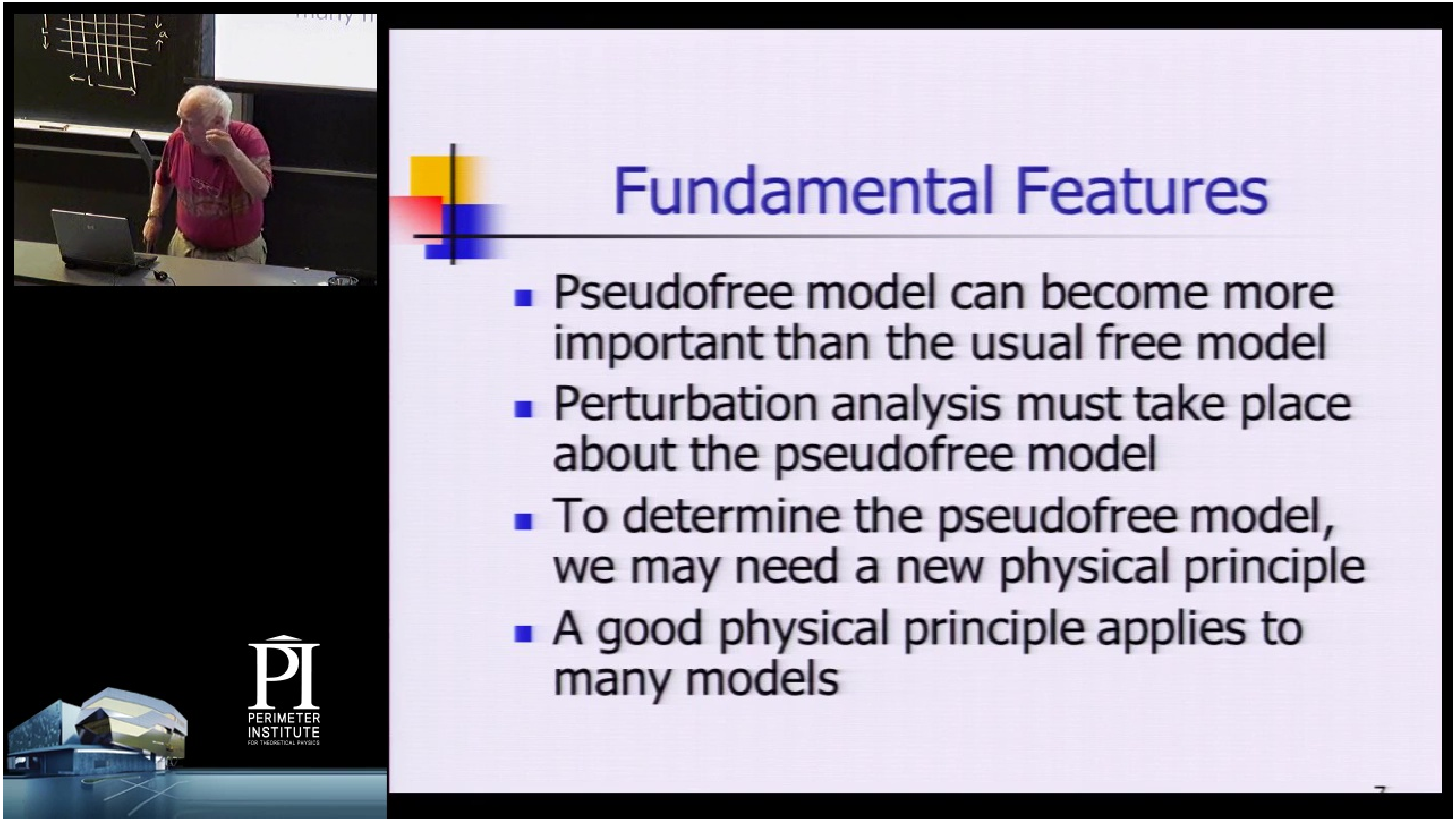
$$W_f\{x = B\cos(t+b)\} \equiv W_{g=0}$$

$$\neq \lim{}_{g\to 0} W_g \equiv W_{pf}\{x = \pm\,|\,B\cos(t+b)\,|\}$$

$$\lim{}_{g\to 0}\int_{x(0)=x'}^{x(T)=x''} \exp\{-\textstyle\int_0^T[\tfrac{1}{2}(\dot{x}^2 + x^2) + gx^{-4}]dt\}\ \ Dx$$

$$= \theta(x''x')\sum_{n=0}^{\infty} h_n(x'')[h_n(x') - h_n(-x')]e^{-(n+1/2)T}, \ \underline{(pf)}$$

$$\neq \sum_{n=0}^{\infty} h_n(x'')h_n(x')e^{-(n+1/2)T}, \ \ \underline{(f)} \qquad ;[\,x^4, \ \exp(x^4)\,]$$



Fundamental Features
Pseudofree model can become more important than the usual free model
Perturbation analysis must take place about the pseudofree model
To determine the pseudofree model, we may need a new physical principle
A good physical principle applies to many models
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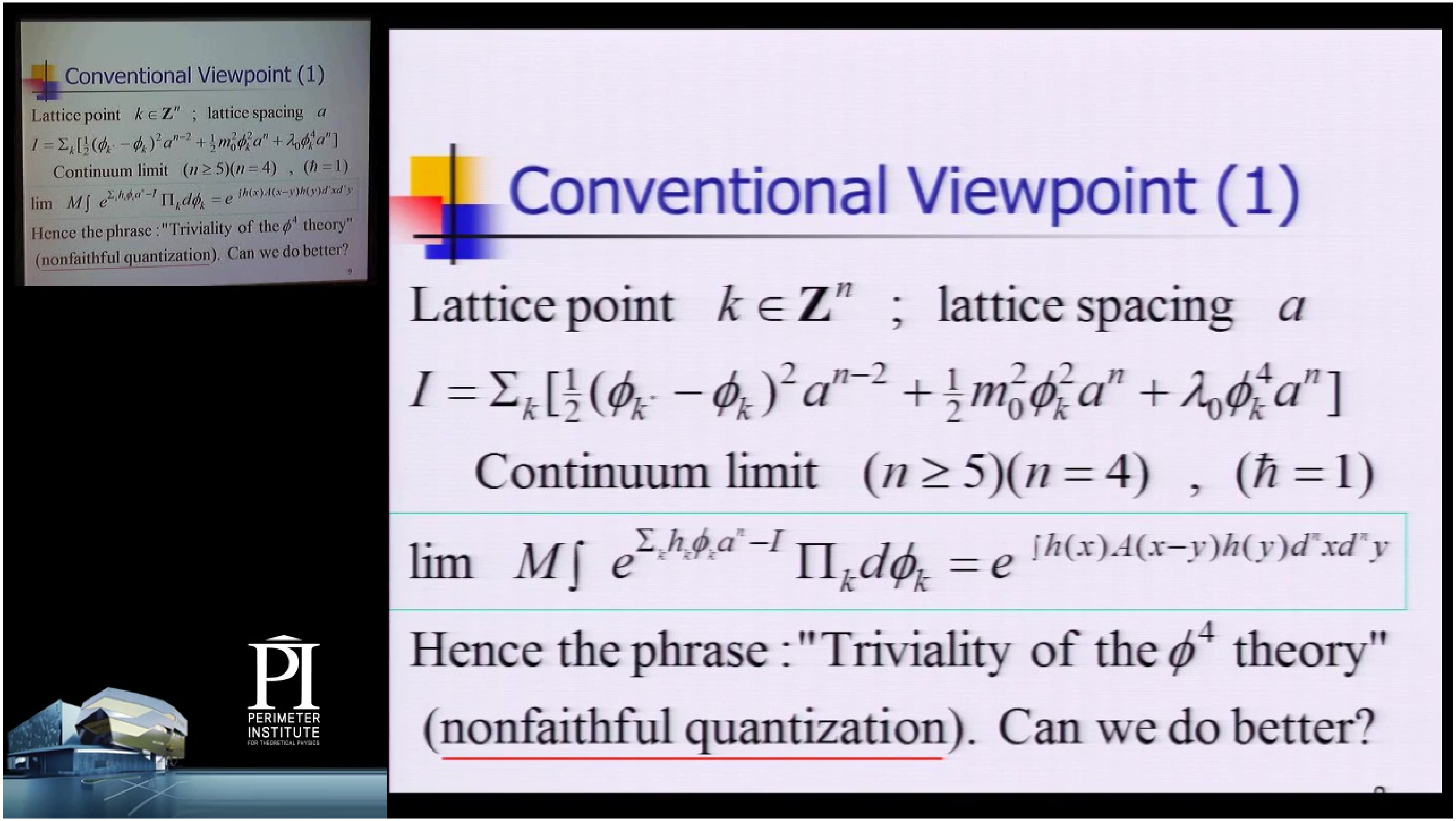
Outline of Talk

- Covariant nonrenormalizable models involve a hard-core interaction
- Solve a nonrenormalizable model
- Discover the main physical principle
- Apply this principle to covariant nonrenormalizable models
- Extend principle to covariant renormalizable models



Conventional Viewpoint (1)
Lattice point $k \in \mathbf{Z}^n$; lattice spacing a
$I = \Sigma_k [\frac{1}{2}(\phi_{k'} - \phi_k)^2 a^{n-2} + \frac{1}{2} m_0^2 \phi_k^2 a^n + \lambda_0 \phi_k^4 a^n]$
Continuum limit $(n \geq 5)(n = 4)$, $(\hbar = 1)$
$\lim \; M \int e^{\Sigma_k h_k \phi_k a^n - I} \Pi_k d\phi_k = e^{\int h(x) A(x-y) h(y) d^n x d^n y}$
Hence the phrase : "Triviality of the ϕ^4 theory"
(nonfaithful quantization). Can we do better?
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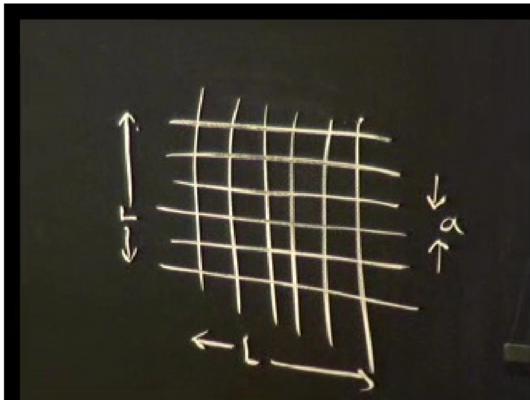
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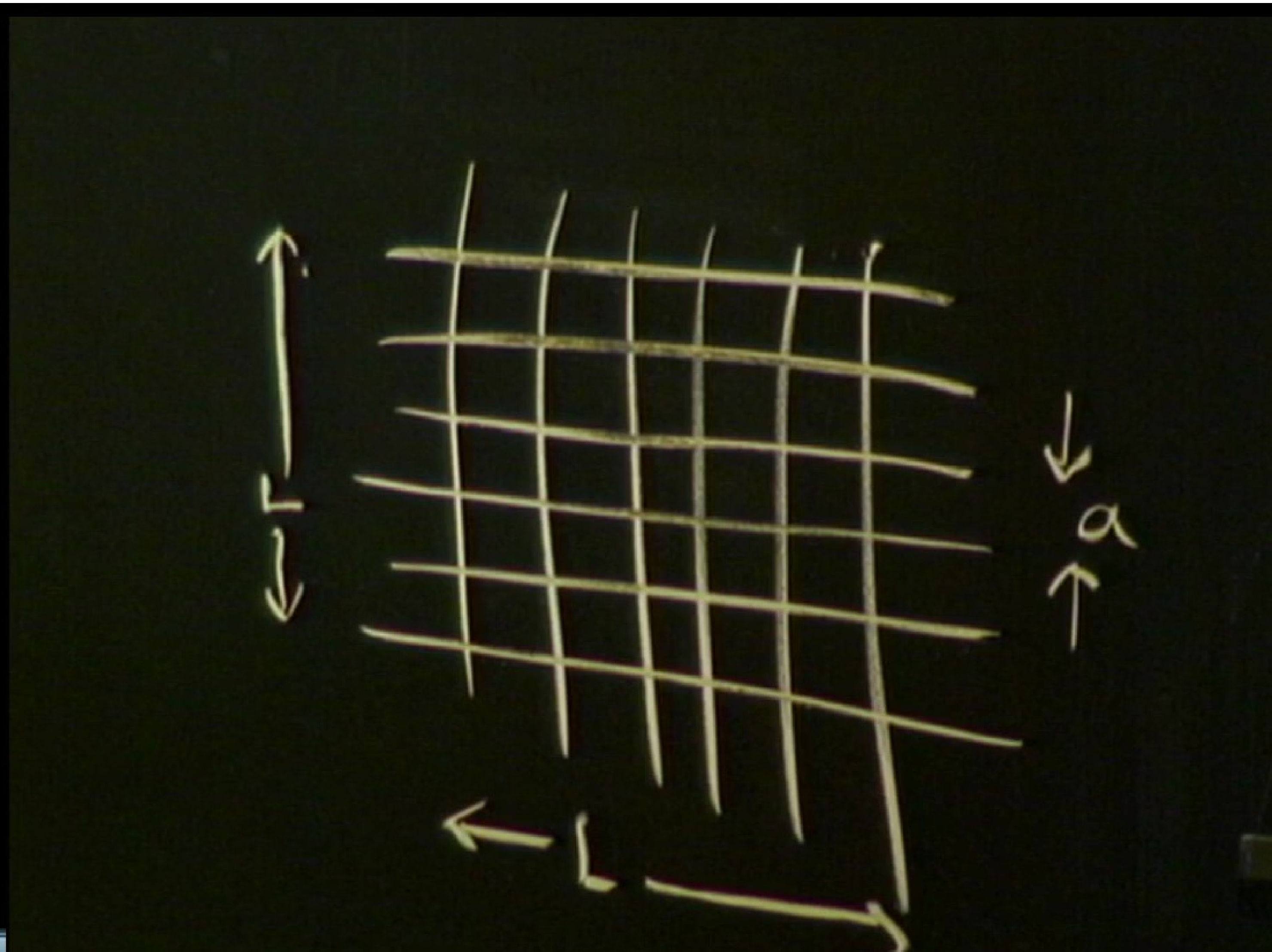
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Conventional Viewpoint (2)
For $n \geq 5$, perturbation theory leads to infinitely many distinct counter terms of an ever increasing degree of divergence. Hence the label: "Nonrenormalizable".
This behavior can be understood as due to a hard-core interaction.

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Hard-core Behavior (1)

Consider the Sobolev - type inequality

$$\{\int \phi(x)^4 d^n x\}^{1/2} \le C \int \{[\nabla \phi(x)]^2 + \phi(x)^2\} d^n x$$

$$\text{if } \underline{n \le 4} \ , \ C = 4/3 \ ; \quad \text{if } \underline{n \ge 5} \ , \ C = \infty$$

Free solutions are strictly limited

Hard-core Behavior (2)
Quantum implications of the same inequality
$$\lim \frac{\{\int \phi(x)^4 d^n x\}^{1/2}}{\int \{[\nabla \phi(x)]^2 + \phi(x)^2\} d^n x} \le C$$
if $n \le 4$, $C = 4/3$; if $n \ge 5$, $C = \infty$
Interaction acts partially as a hard - core; the interacting theory does NOT converge to the free theory as coupling constant goes to zero

Hard-core Behavior (2)

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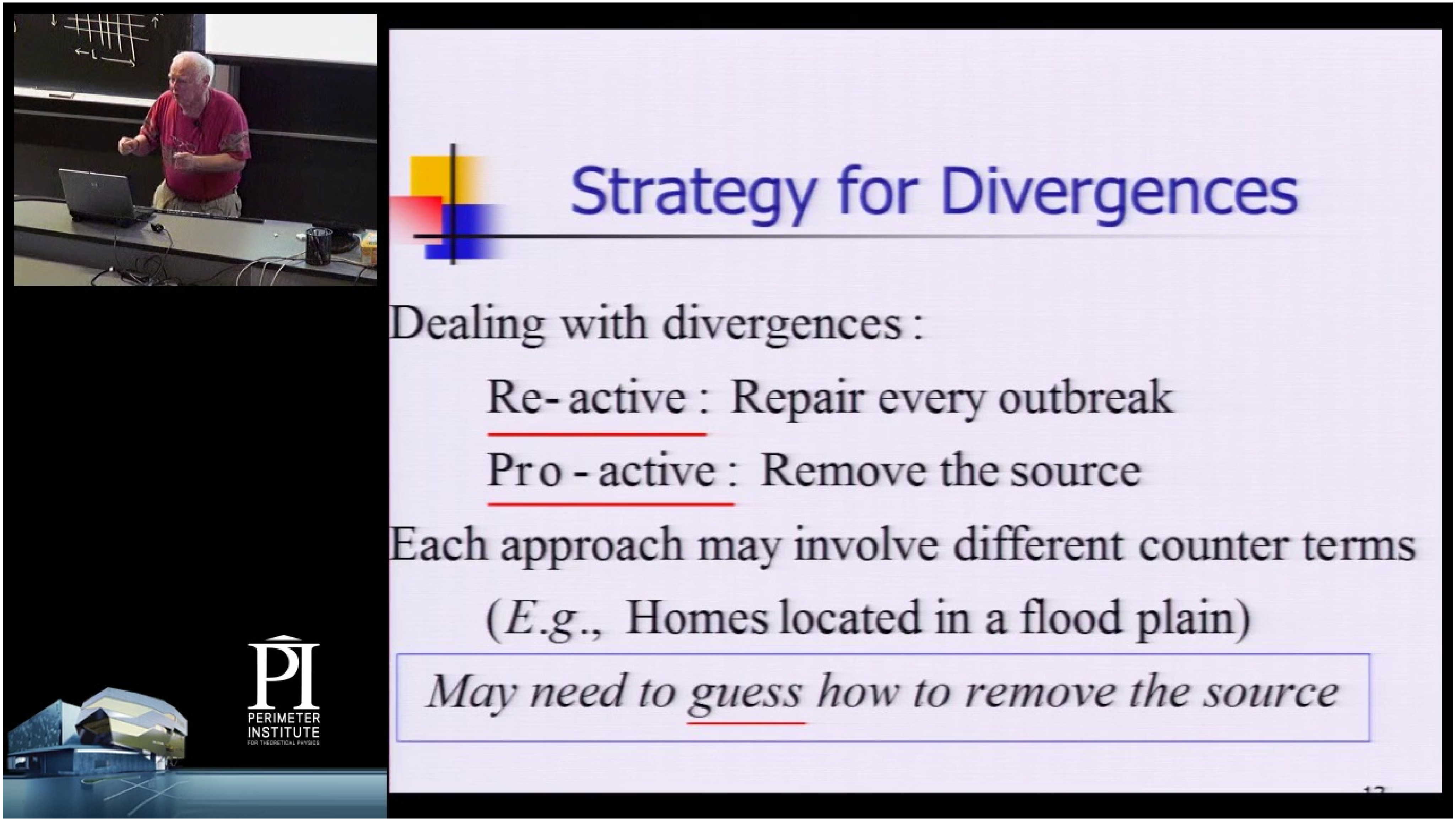
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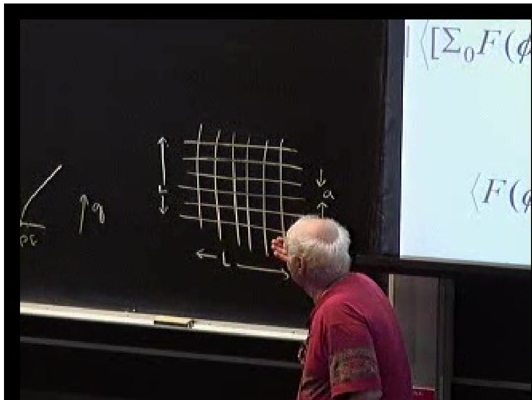
Strategy for Divergences
Dealing with divergences :
Re- active : Repair every outbreak
Pro - active : Remove the source
Each approach may involve different counter terms
(E.g., Homes located in a flood plain)
May need to guess how to remove the source



Bounds on Lattice Averages

$$\left\langle [\Sigma_0 F(\phi,a)a]^p \right\rangle \equiv M \int [\Sigma_0 F(\phi,a)a]^p e^{-I/\hbar} \, \Pi_k d\phi_k$$

$$\left\langle [\Sigma_0 F(\phi,a)a]^p \right\rangle = \widetilde{\Sigma}_0 a^p \left\langle F(\phi_1,a)\cdots F(\phi_p,a) \right\rangle$$

$$\left| \left\langle [\Sigma_0 F(\phi,a)a]^p \right\rangle \right| \le \widetilde{\Sigma}_0 a^p \left| \left\langle F(\phi_1,a)\cdots F(\phi_p,a) \right\rangle \right|$$

$$\le \widetilde{\Sigma}_0 a^p \left| \left\langle F(\phi_1,a)^p \right\rangle \cdots \left\langle F(\phi_p,a)^p \right\rangle \right|^{1/p}$$

$$\left\langle F(\phi,a)^p \right\rangle = \int F(\phi,a)^p \, \Psi(\phi)^2 \, \Pi'_k d\phi_k$$

ACT 2: The Study of a Soluble Nonrenormalizable Scalar Field

- Classical action for the ultralocal scalar quartic interaction is

$$A=\int\{\tfrac{1}{2}[\dot{\phi}^2(t,x)-m_0^2\phi^2(t,x)]-\lambda_0\phi^4(t,x)\}dt\ d^sx$$

- Classical free and pseudofree theories are strictly different
- The absence of a gradient term leads to a nonrenormalizable quantum model for any spatial dimension
- Limited to mass and coupling constant renormalization, quantization is trivial (CLT); this is a nonfaithful quantization

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Ultralocal Scalar Models (1)

Classical Hamiltonian; no $(\nabla\phi)^2$ term; (pf)

$$H = \int\{\tfrac{1}{2}[\pi^2 + m_0^2\phi^2] + \lambda_0\phi^4\}d^s x \quad , \quad \lambda_0 \geq 0$$

Free ground - state distribution $(\lambda_0 = 0)$

$$\Psi_0^2(\phi) = K\exp[-m_0 \int \phi^2 d^s x]$$

Generator for free distribution (G)

$$\boxed{S(h) = K\int e^{\int h\phi d^s x - m_0 \int \phi^2 d^s x}\Pi'_x d\phi(x) \;= e^{\frac{1}{4} m_0^{-1}\int h^2 d^s x}}$$

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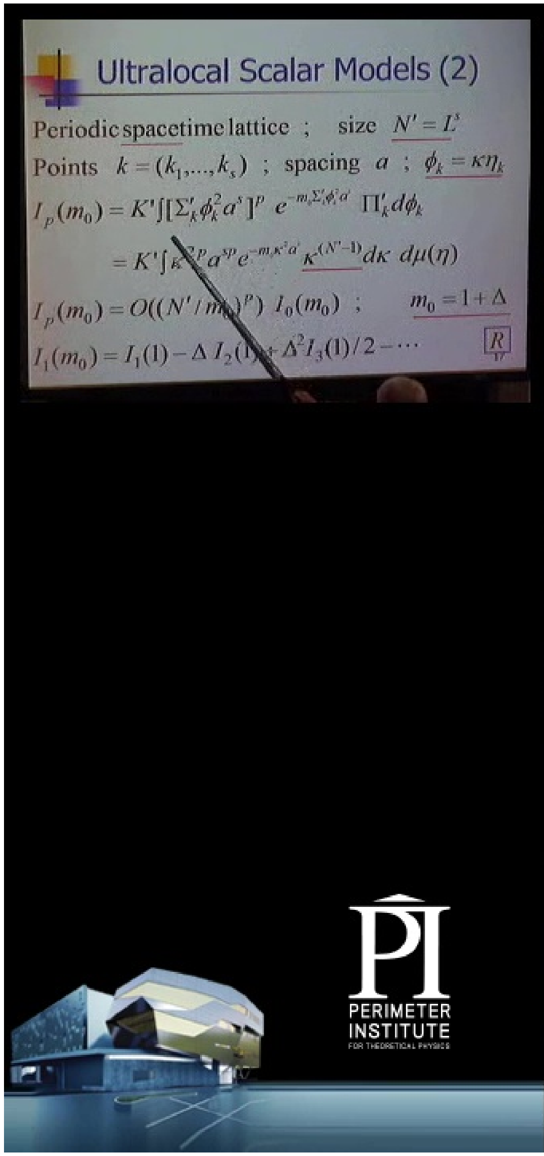

Ultralocal Scalar Models (2)

Periodic spacetime lattice ; size $N' = L^s$

Points $k = (k_1, \ldots, k_s)$; spacing a ; $\phi_k = \kappa \eta_k$

$$I_p(m_0) = K' \int [\Sigma'_k \phi_k^2 a^s]^p \; e^{-m_0 \Sigma'_k \phi_k^2 a^s} \; \Pi'_k d\phi_k$$

$$= K' \int \kappa^{2p} a^{sp} e^{-m_0 \kappa^2 a^s} \, \underline{\kappa^{(N'-1)}} d\kappa \; d\mu(\eta)$$

$I_p(m_0) = O((N'/m_0)^p) \; I_0(m_0)$; $m_0 = 1 + \Delta$

$I_1(m_0) = I_1(1) - \Delta \, I_2(1) + \Delta^2 I_3(1)/2 - \cdots$ $\boxed{R}$

Ultralocal Scalar Models (3)

Free ground state distribution

$$\Psi_G(\phi)^2 \propto \exp[-m_0 \Sigma'_k \phi_k^2 a^s]$$

Pseudofree ground state distribution

$$\Psi_I(\phi)^2 \propto \{\kappa^{-(N'-R)}\} \exp[-m_0 \Sigma'_k \phi_k^2 a^s]$$

$$\kappa^2 = \Sigma'_k \phi_k^2 \quad ; \qquad R = b a^s N' < \infty$$

Mass moments are now finite!

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Ultralocal Scalar Models (4)

$$S_{pf}(h) \qquad \longleftarrow \qquad \boxed{m_0 = ba^s m}$$

$$= \text{"}\widetilde{K} \int e^{\kappa \Sigma'_k h_k \eta_k a^s} e^{-m_0 \kappa^2 a^s} \underline{\kappa^{(ba^s N'-1)}} d\kappa \, d\mu(\eta) \text{"}$$

$$= \Pi'_k \{\tfrac{1}{2} ba^s \int e^{h_k \phi a^s} e^{-m_0 \phi^2 a^s} d\phi / |\phi|^{1-ba^s}\}$$

$$= \Pi'_k \{1 + \tfrac{1}{2} ba^s \int [e^{h_k \phi} - 1] \, e^{-bm\phi^2} d\phi / |\phi|^{1-ba^s}\}$$

$$\rightarrow \exp\{\tfrac{1}{2} b \int d^s x \int [e^{h(x)\phi} - 1] \, e^{-bm\phi^2} d\phi / |\phi|\} \;\; \underline{(P)}$$

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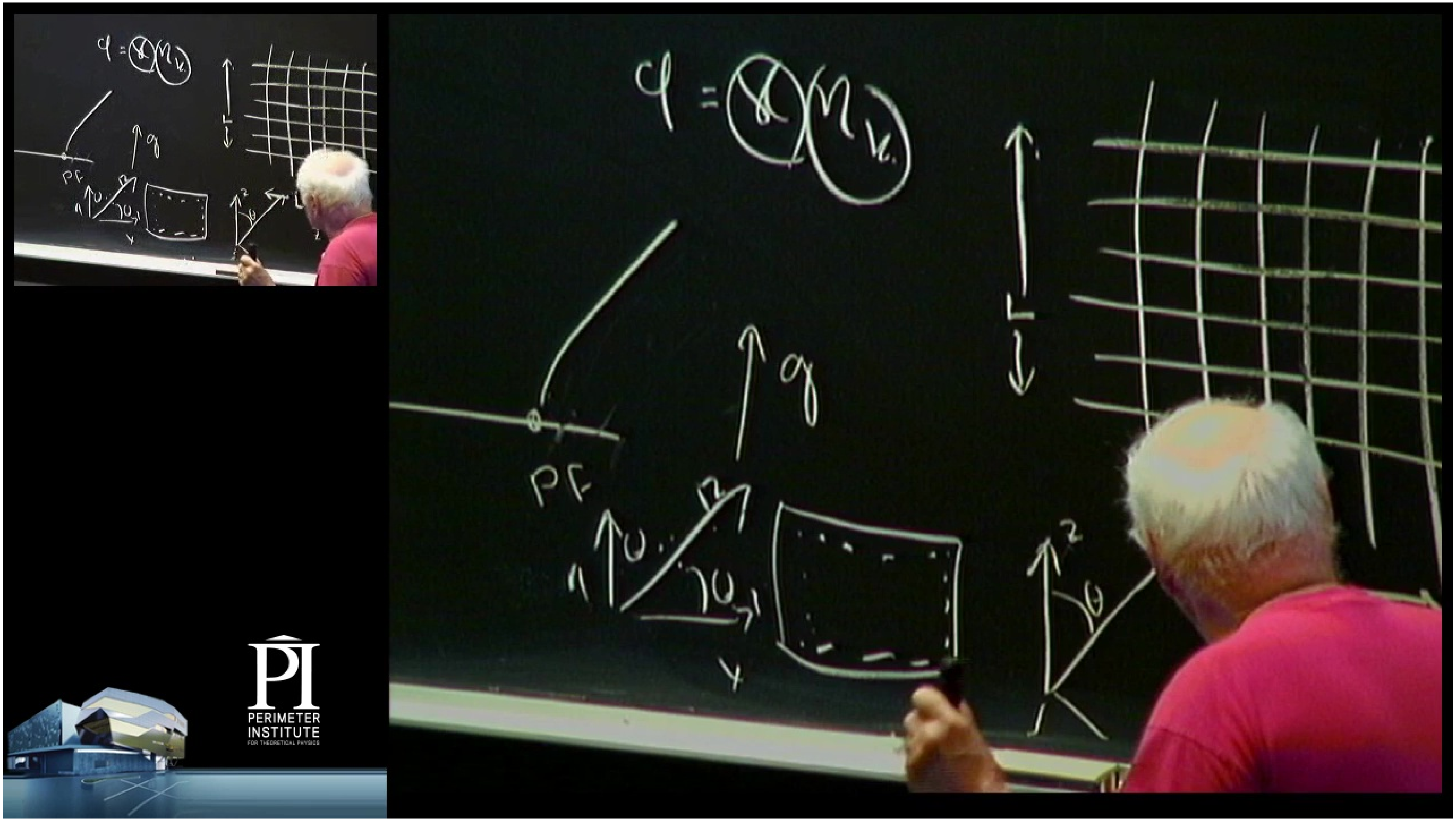
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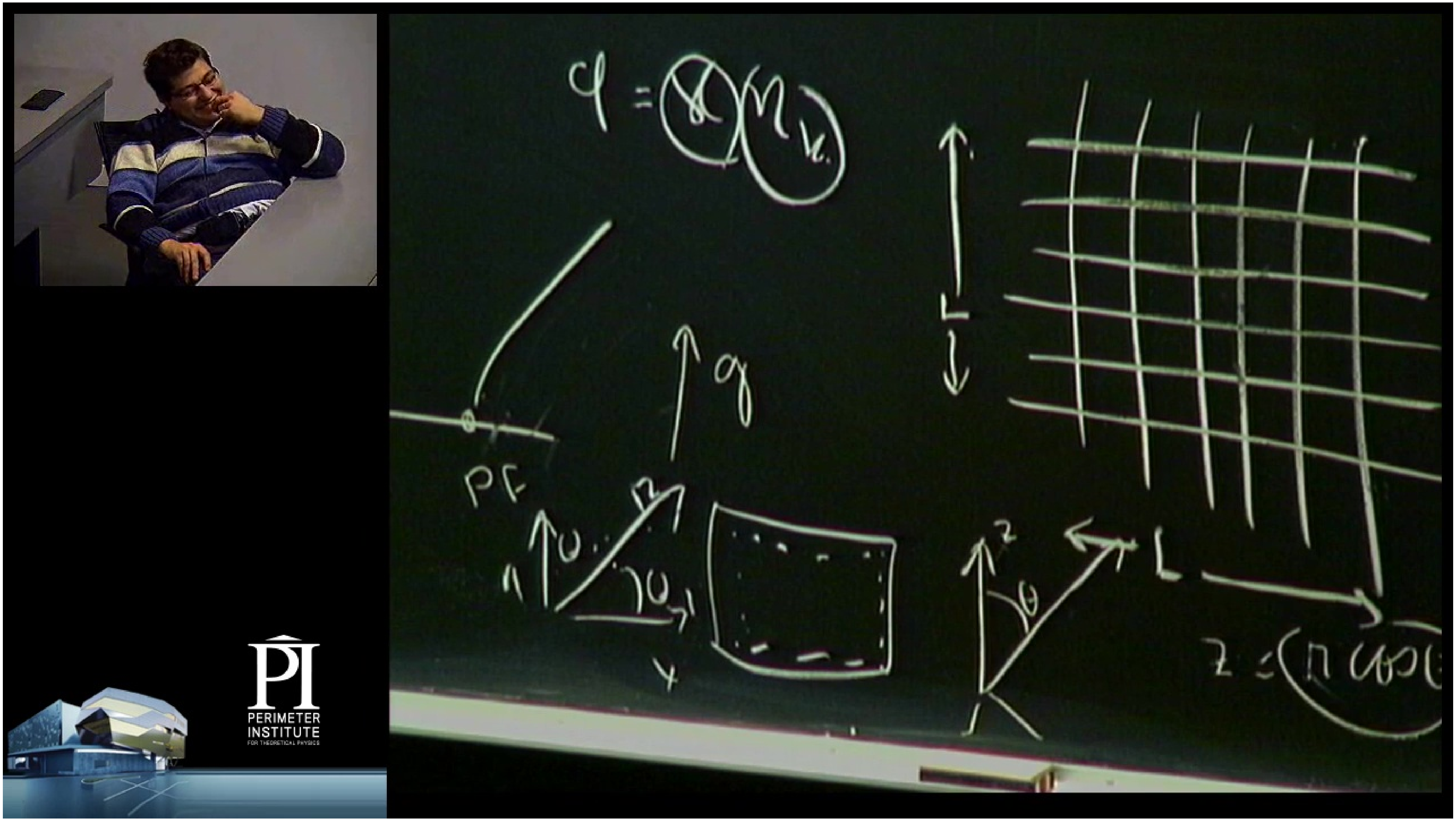
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Ultralocal Scalar Models (5)

Old, interactin g, ground - state distributi on

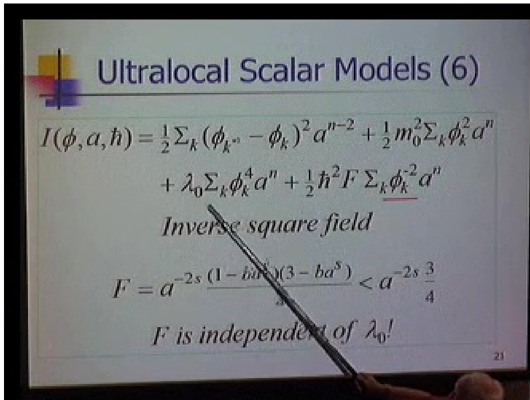
$$\Psi_0(\phi)^2 \propto \exp[-\, U_0(\phi, a, \hbar)]$$

New, interactin g, ground - state distributi on

$$\Psi_{0'}(\phi)^2 \propto \{\kappa^{-(N' - ba^s N')}\} \, \exp[-\, U_0'(\phi, a, \hbar)]$$

$$\boxed{\phi_k = \kappa\, \eta_k \;\; ; \;\; \kappa^2 = \Sigma_k' \phi_k^2 \;\; ; \;\; 1 = \Sigma_k' \eta_k^2}$$

Ultralocal symmetry dictates proper form



Ultralocal Scalar Models (6)

$$I(\phi,a,\hbar) = \tfrac{1}{2}\Sigma_k(\phi_{k^*} - \phi_k)^2 a^{n-2} + \tfrac{1}{2} m_0^2 \Sigma_k \phi_k^2 a^n$$

$$+ \lambda_0 \Sigma_k \phi_k^4 a^n + \tfrac{1}{2}\hbar^2 F\, \Sigma_k \underline{\phi_k^{-2}} a^n$$

Inverse square field

$$F = a^{-2s}\,\frac{(1-ba^s)(3-ba^s)}{4} < a^{-2s}\,\frac{3}{4}$$

F is independent of λ_0*!*

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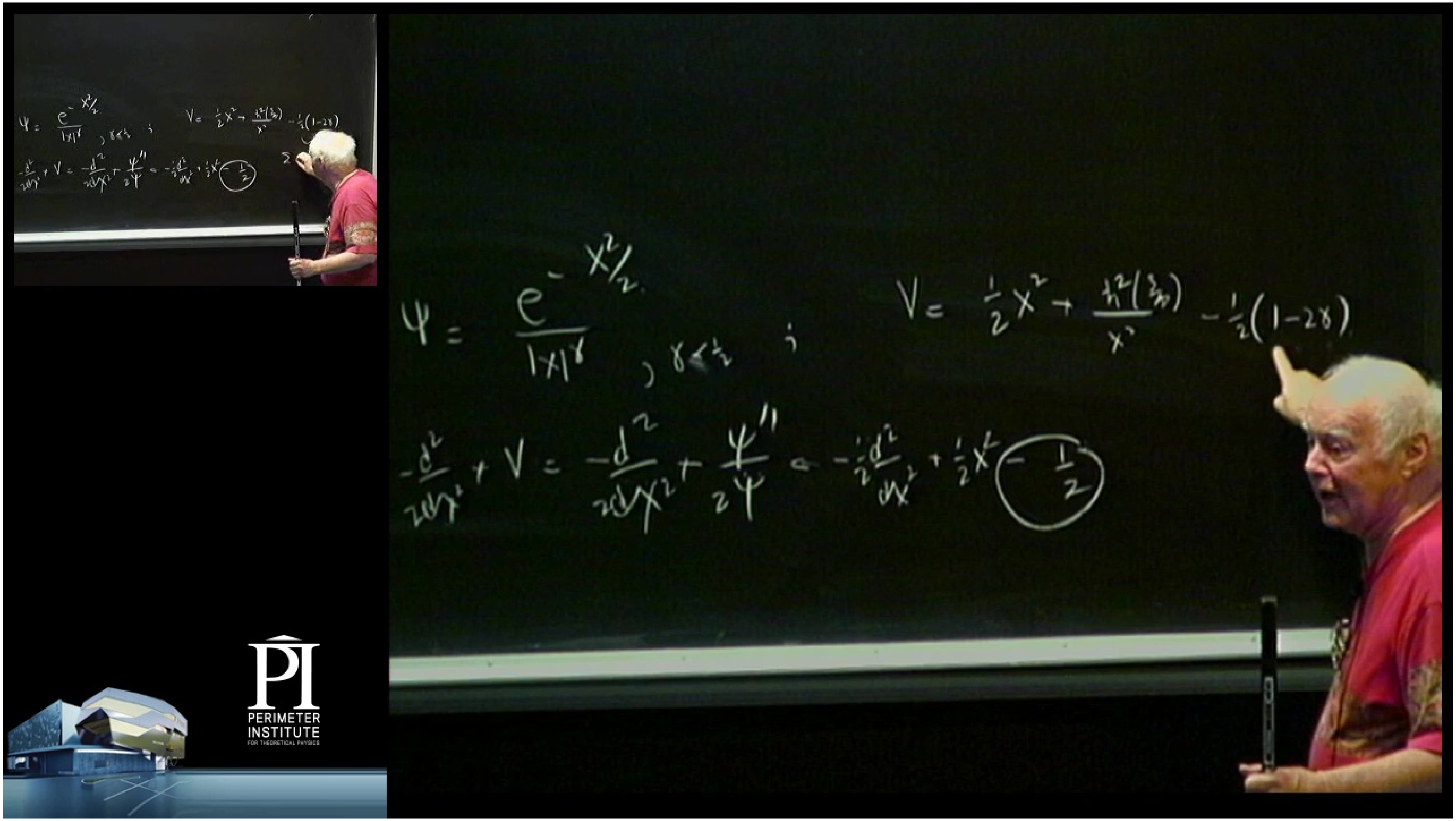
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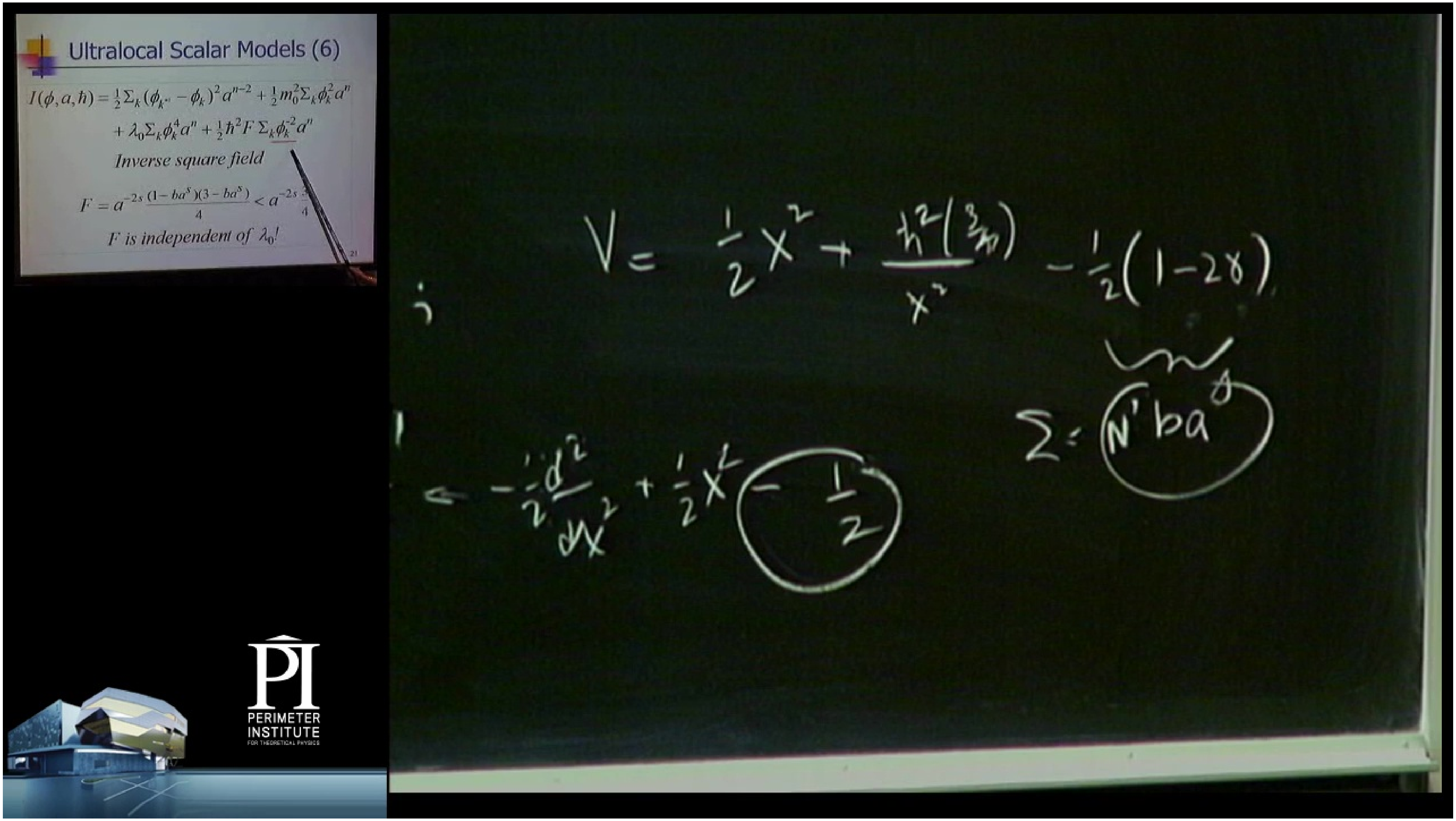
Ultralocal Scalar Models (6)
$I(\phi,a,\hbar)=\frac{1}{2}\Sigma_k(\phi_{k^*}-\phi_k)^2a^{n-2}+\frac{1}{2}m_0^2\Sigma_k\phi_k^2a^n$
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Inverse square field
$F=a^{-2s}\frac{(1-ba^s)(3-ba^s)}{4}<a^{-2s}\frac{3}{4}$
F is independent of λ_0!
21
$V=\frac{1}{2}x^2+\frac{\hbar^2(\frac{3}{\hbar})}{x^2}-\frac{1}{2}(1-2\gamma)$
$\Sigma = N'ba^\delta$
$-\frac{1}{2}\frac{d^2}{dx^2}+\frac{1}{2}x^2-\frac{1}{2}$
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A Simple Remark: Cov 1

For one variable $(I = [-1\ ,\ 1])$

$$\int_I \frac{d\phi}{[\phi^2]^{1/2}} = \infty$$

For several variables $(p > 1)$

$$\int_{I^p} \frac{d^p\phi}{[\phi_1^2 + \cdots + \phi_p^2]^{1/2}} < \infty$$

Covariant models connect neighboring points.

Use that fact to eliminate divergence at $\phi_k = 0$.

Proposed Ground State: Cov 2

$$\phi_k^2 \Rightarrow \tfrac{1}{2s+1}[\phi_k^2 + \Sigma_{l\in\{nn\,\mathrm{to}\,k\}}\,\phi_l^2] \equiv <\phi_k^2>$$

Ground state solution

$$\Psi(\phi) = \frac{e^{-U(\phi,a,\hbar)/2}}{\Pi'_k[<\phi_k^2>]^{(1-ba^s)/4}} \quad ; \quad \underline{R = ba^s N'}$$

Normalization $\qquad s = 2$

$$\int \frac{e^{-U(\phi,a,\hbar)}}{\Pi'_k[<\phi_k^2>]^{(1-ba^s)/2}}\,\Pi'_k d\phi_k = 1$$

Proposed Lattice Action: Cov 3

$$\boxed{\begin{aligned} I(\phi,a,\hbar) &= \tfrac{1}{2}\Sigma_k(\phi_{k^*}-\phi_k)^2 a^{n-2} + \tfrac{1}{2}m_0^2\Sigma_k\phi_k^2 a^n \\ &\quad + \lambda_0\Sigma_k\phi_k^4 a^n + \tfrac{1}{2}\hbar^2\Sigma_k \mathsf{F}_k(\phi)a^n \end{aligned}}$$

$$\mathsf{F}_k = a^{-2s}\mathrm{D}^{-1}\mathrm{D}_{,kk} \quad , \quad \mathrm{D} = \Pi'_k[<\phi_k^2>]^{-\gamma}$$

$$\gamma = (1 - ba^s)/4 \quad , \quad N' = L^s$$

No new parameters; scales as kinetic term

Proposed Lattice Action: Cov 4

$$\mathsf{F}_k(\phi) = \tfrac{1}{4}(1 - ba^s)^2 \left(\Sigma'_t \frac{a^{-s} J_{t,k}\phi_k}{[<\phi_t^2>]}\right)^2$$

$$+ \tfrac{1}{2}(1 - ba^s)\ \Sigma'_t \left\{ 2\frac{a^{-2s} J_{t,k}^2 \phi_k^2}{[<\phi_t^2>]^2} - \frac{a^{-2s} J_{t,k}}{[<\phi_t^2>]} \right\}$$

$$<\phi_k^2> \equiv \Sigma'_l J_{k,l}\phi_l^2 \qquad \underline{(lp)}$$

$$J_{k,l} \equiv \frac{1}{2s+1}\delta_{k,l\in\{k\cup k_{nn}\}} \qquad s = 2$$

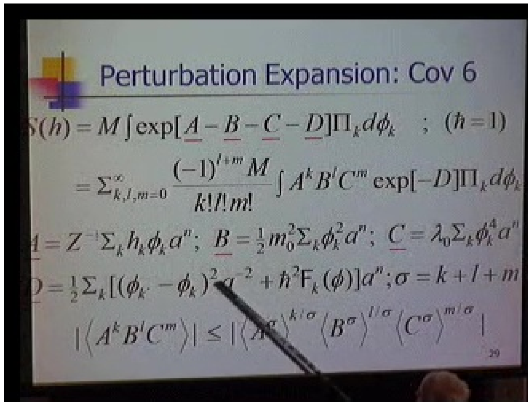
Basic Ingredients: Cov 5

$$\Psi(\phi)=\frac{e^{-U(\phi,a,\hbar)/2}}{\Pi'_k[\Sigma'_l J_{k,l}\phi_l^2]^{(1-ba^s)/4}}\;;\quad \mathcal{H}\Psi(\phi)=0$$

$$\mathcal{H}=-\tfrac{\hbar^2}{2}\Sigma'_k\,\tfrac{\partial^2}{\partial\varphi_k^2}\,a^{-s}+\tfrac{1}{2}\Sigma'_k(\phi_{k^*}-\phi_k)^2a^{s-2}-E_0$$

$$+\tfrac{1}{2}m_0^2\Sigma'_k\phi_k^2a^s+\lambda_0\Sigma'_k\phi_k^4a^s+\tfrac{\hbar^2}{2}\Sigma'_k\mathsf{F}_k(\phi)a^s$$

$$S(h)=M\int e^{Z^{-1/2}\Sigma_k h_k\phi_k a^n/\hbar-I(\phi,a,\hbar)/\hbar}\,\Pi_k d\phi_k\;;\quad \underline{(Z)}$$



Perturbation Expansion: Cov 6

$$S(h) = M\int \exp[\underline{A} - \underline{B} - \underline{C} - \underline{D}]\Pi_k d\phi_k \quad ; \quad (\hbar = 1)$$

$$= \Sigma_{k,l,m=0}^{\infty} \frac{(-1)^{l+m} M}{k!l!m!} \int A^k B^l C^m \exp[-D]\Pi_k d\phi_k$$

$$\underline{A} = Z^{-\frac{1}{2}}\Sigma_k h_k \phi_k a^n; \; \underline{B} = \tfrac{1}{2} m_0^2 \Sigma_k \phi_k^2 a^n; \; \underline{C} = \lambda_0 \Sigma_k \phi_k^4 a^n$$

$$\underline{D} = \tfrac{1}{2}\Sigma_k[(\phi_{k^*} - \phi_k)^2 a^{-2} + \hbar^2 \mathsf{F}_k(\phi)]a^n; \sigma = k + l + m$$

$$|\langle A^k B^l C^m \rangle| \le |\langle A^\sigma \rangle^{k/\sigma} \langle B^\sigma \rangle^{l/\sigma} \langle C^\sigma \rangle^{m/\sigma}|$$

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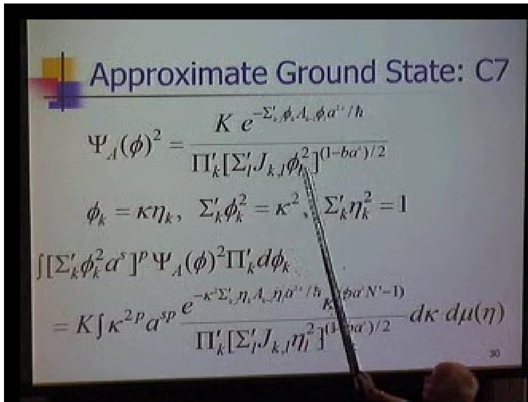


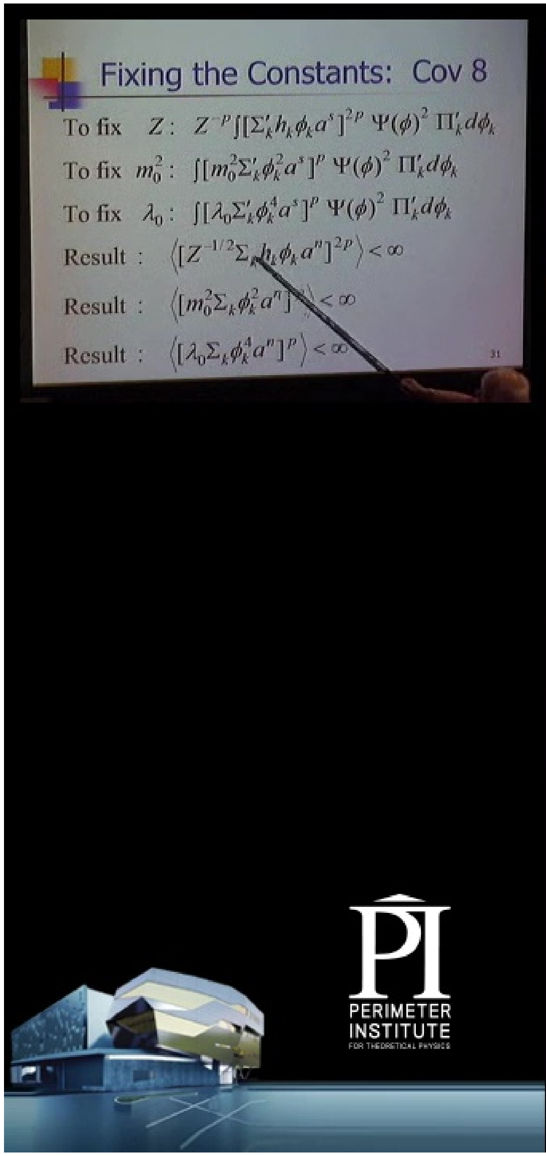
Approximate Ground State: C7

$$\Psi_A(\phi)^2 = \frac{K\, e^{-\Sigma'_{k,l}\phi_k A_{k-l}\phi_l a^{2s}/\hbar}}{\Pi'_k[\Sigma'_l J_{k,l}\phi_l^2]^{(1-ba^s)/2}}$$

$$\phi_k = \kappa\eta_k,\;\; \Sigma'_k\phi_k^2 = \kappa^2,\;\; \Sigma'_k\eta_k^2 = 1$$

$$\int[\Sigma'_k\phi_k^2 a^s]^p\,\Psi_A(\phi)^2\,\Pi'_k d\phi_k$$

$$= K\int \kappa^{2p} a^{sp}\, \frac{e^{-\kappa^2\Sigma'_{k,l}\eta_k A_{k-l}\eta_l a^{2s}/\hbar}\,\kappa^{(ba^s N'-1)}}{\Pi'_k[\Sigma'_l J_{k,l}\eta_l^2]^{(1-ba^s)/2}}\, d\kappa\, d\mu(\eta)$$



Fixing the Constants: Cov 8

To fix $\quad Z: \quad Z^{-p}\int[\Sigma'_k h_k \phi_k a^s]^{2p}\, \Psi(\phi)^2\, \Pi'_k d\phi_k$

To fix $\; m_0^2: \quad \int[m_0^2 \Sigma'_k \phi_k^2 a^s]^p\, \Psi(\phi)^2\, \Pi'_k d\phi_k$

To fix $\quad \lambda_0: \quad \int[\lambda_0 \Sigma'_k \phi_k^4 a^s]^p\, \Psi(\phi)^2\, \Pi'_k d\phi_k$

Result : $\quad \left\langle [Z^{-1/2}\Sigma_k h_k \phi_k a^n]^{2p} \right\rangle < \infty$

Result : $\quad \left\langle [m_0^2 \Sigma_k \phi_k^2 a^n]^p \right\rangle < \infty$

Result : $\quad \left\langle [\lambda_0 \Sigma_k \phi_k^4 a^n]^p \right\rangle < \infty$

Fixing the Constants: Cov 9

$$\int [m_0^2 \Sigma'_k \phi_k^2 a^s]^p \, \Psi_A(\phi)^2 \, \Pi'_k d\phi_k$$

$$= K \int m_0^{2p} a^{sp} \kappa^{2p} \frac{e^{-\kappa^2 \sum'_{k,l} \eta_k A_{k-l} \eta_l a^{2s} / \hbar} \, \kappa^{(ba^s N'-1)}}{\Pi'_k [\Sigma'_l J_{k,l} \eta_l^2]^{(1-ba^s)/2}} d\kappa \, d\mu(\eta)$$

$$\propto m_0^{2p} a^{sp} / (N' a^{(s-1)})^p \qquad ; \quad \underline{m_0^2 = N'(qa)^{-1} m^2}$$

Result : $\left\langle [m_0^2 \Sigma_k \phi_k^2 a^n]^p \right\rangle < \infty$

Fixing the Constants: Cov 10

$$Z^{-p}\int[\Sigma'_k h_k\phi_k a^s]^{2p}\;\Psi(\phi)^2\;\Pi'_k d\phi_k : \underline{Z = N'^{-2}(qa)^{1-s}}$$

$$\int[m_0^2\Sigma'_k\phi_k^2 a^s]^{p}\;\Psi(\phi)^2\;\Pi'_k d\phi_k : \underline{m_0^2 = N'(qa)^{-1}m^2}$$

$$\int[\lambda_0\Sigma'_k\phi_k^4 a^s]^{p}\;\Psi(\phi)^2\;\Pi'_k d\phi_k : \underline{\lambda_0 = N'^3(qa)^{s-2}\lambda}$$

$$\left\langle[m_0^2\Sigma_k\phi_k^2 a^n]^p\right\rangle < \infty \quad ; \quad Zm_0^2 = m^2/[N'(qa)^s]$$

$$\left\langle[\lambda_0\Sigma_k\phi_k^4 a^n]^p\right\rangle < \infty \quad ; \quad Z^2\lambda_0 = \lambda/[N'(qa)^s]$$

Fixing the Constants: Cov 10

$$Z^{-p}\int[\Sigma'_k h_k\phi_k a^s]^{2p}\;\Psi(\phi)^2\;\Pi'_k d\phi_k : \underline{Z = N'^{-2}(qa)^{1-s}}$$

$$\int[m_0^2\Sigma'_k\phi_k^2 a^s]^{p}\;\Psi(\phi)^2\;\Pi'_k d\phi_k : \underline{m_0^2 = N'(qa)^{-1}m^2}$$

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$$\left\langle[m_0^2\Sigma_k\phi_k^2 a^n]^p\right\rangle < \infty \quad ; \quad Zm_0^2 = m^2/[N'(qa)^s]$$

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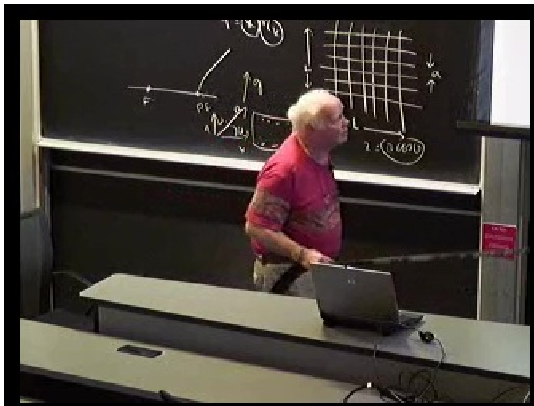

ACT 4: Commentary - Response to Critics

- RG? The continuum limit with suitable inverse square fields is nontrivial
- SUSY and SS? Designed to eliminate traditional divergences; not needed
- Effective theory? Maybe; more likely a nonfaithful quantization
- Fermions? Under investigation

Extension to Other Models

- For n<5 the classical pseudofree and free theories coincide. Use of the counter term is still allowed. Advantages:
- Local products are from the OPE rather than normal ordering
- Recall that $:\phi(x)^2:$ is *not* positive while the OPE square *is* positive; thus the OPE appears to be more physical
- All divergences disappear

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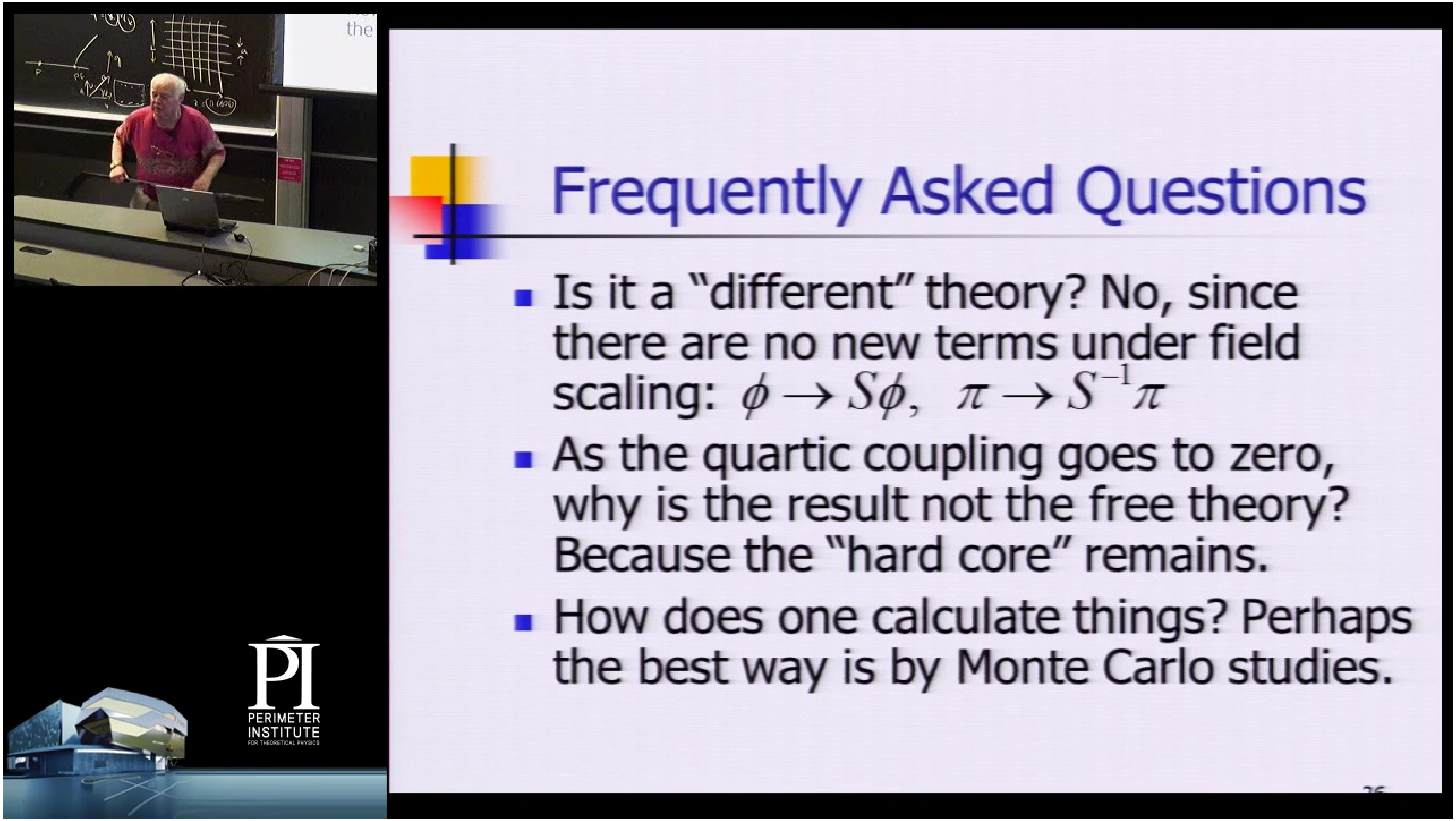

Frequently Asked Questions
Is it a "different" theory? No, since there are no new terms under field scaling: $\phi \to S\phi,\ \ \pi \to S^{-1}\pi$
As the quartic coupling goes to zero, why is the result not the free theory? Because the "hard core" remains.
How does one calculate things? Perhaps the best way is by Monte Carlo studies.
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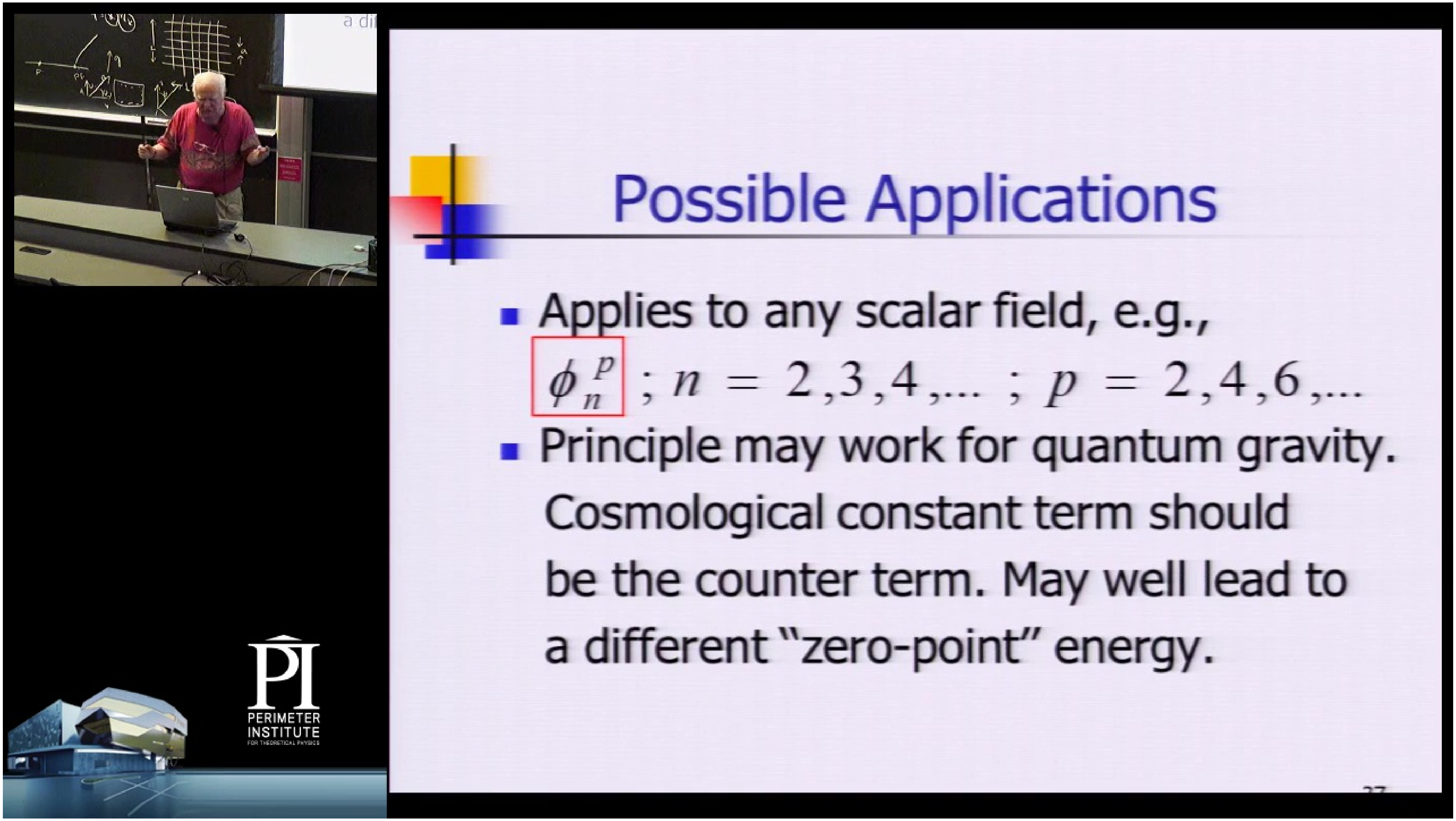
Possible Applications
Applies to any scalar field, e.g.,
$\boxed{\phi_n^{\,p}}$; $n = 2,3,4,\ldots$; $p = 2,4,6,\ldots$
Principle may work for quantum gravity. Cosmological constant term should be the counter term. May well lead to a different "zero-point" energy.
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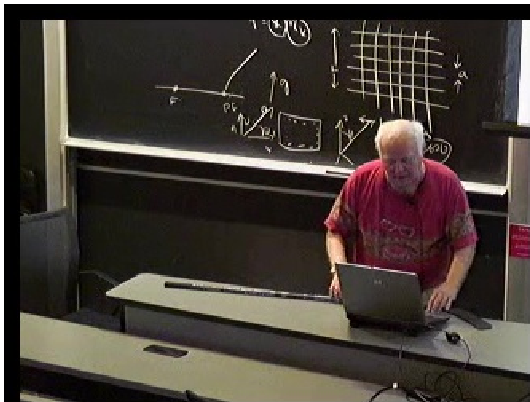
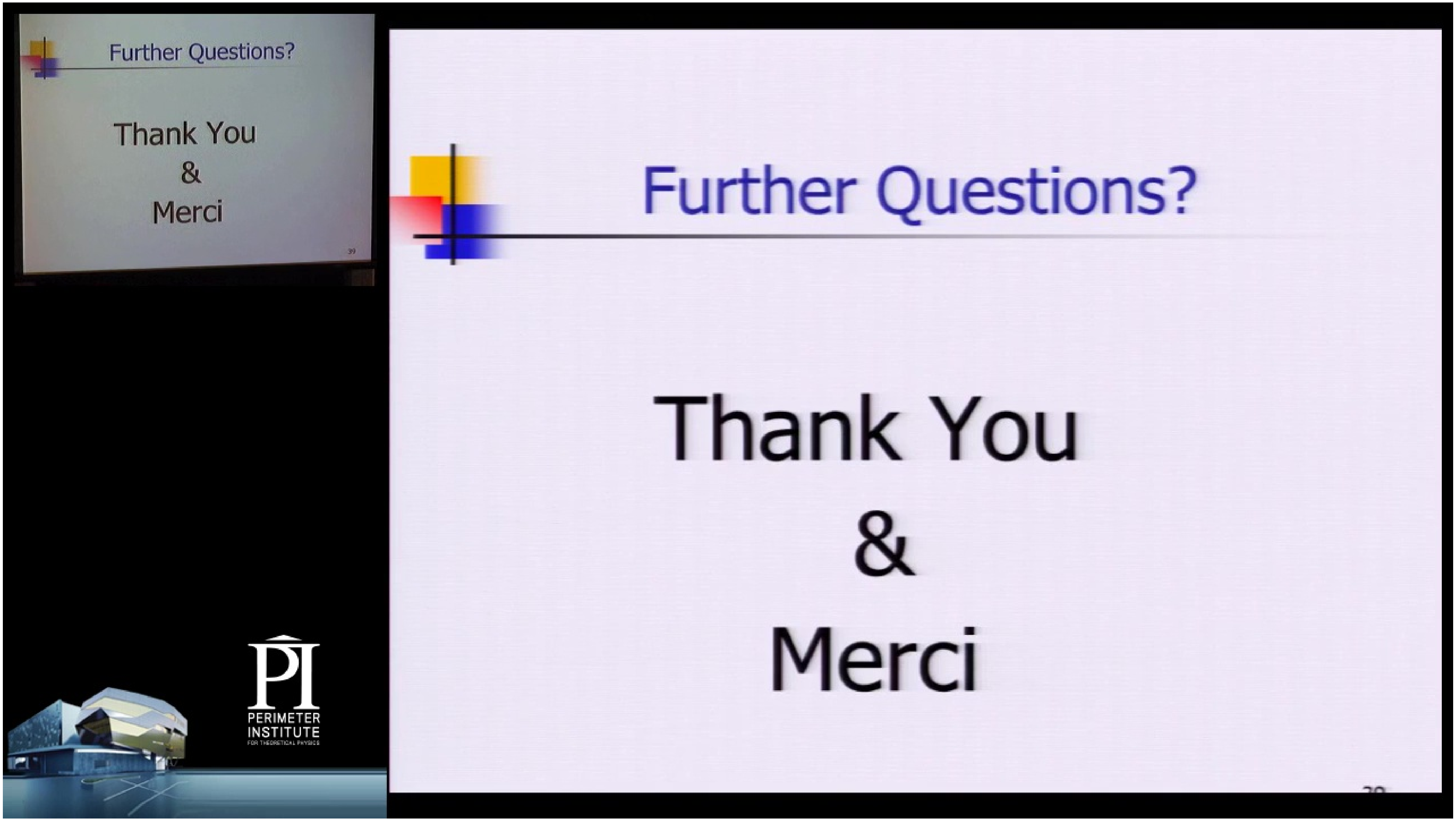


Summary

- Add a fixed counter term that is $\hbar$ dependent and scales as kinetic term
- Determine parameters by self consistency in continuum limit
- Leads to divergence-free perturbative formalism about pseudofree model which contains the counter term


Further Questions?
Thank You
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