Title: Topological Superconductivity

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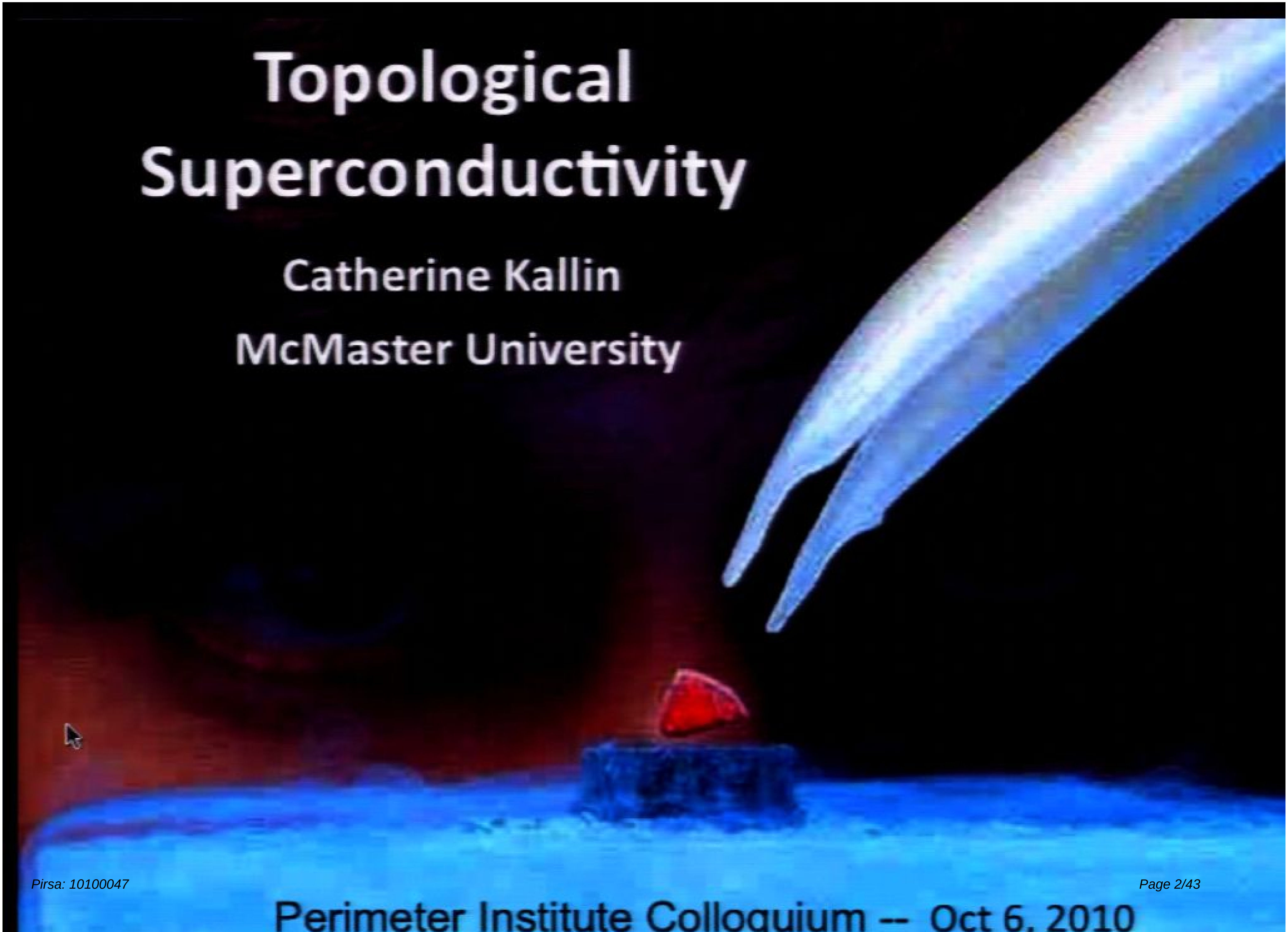
Abstract: &quot;Conventional&quot; superconductivity is one of the most dramatic phenomena in condensed matter physics, and yet by the 1970's it was fully understood - a solved problem much like quantum electrodynamics. The discovery of high temperature superconductivity changed all that and opened the door, not only to higher Tc's, but also to a wealth of even more exotic phenomena, including things like topologically ordered superconductors with factional vortices and non-Abelian statistics. I will describe some of the evolution of the field of exotic superconductivity, with a focus on recent theoretical and experimental work which sheds light on whether strontium ruthenate supports topological chiral superconductivity.



# Topological Superconductivity

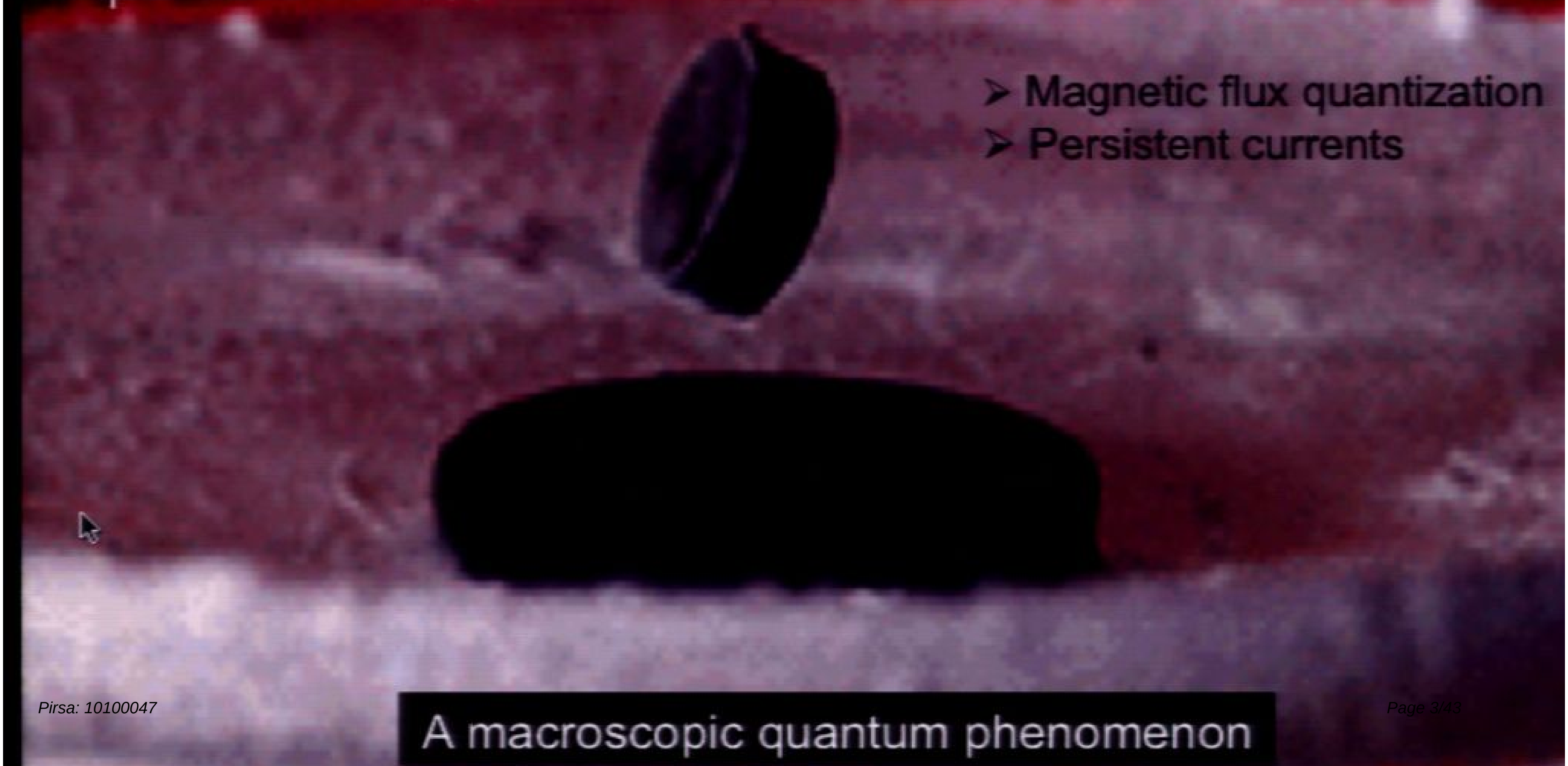
Catherine Kallin

McMaster University

Perimeter Institute Colloquium -- Oct 6, 2010

# Superconductivity

- perfect conductivity – dissipationless supercurrents
- perfect diamagnetism – expels magnetic fields or traps flux in quantized vortices



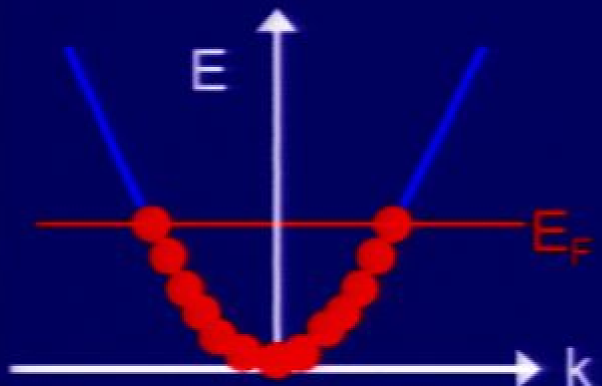
> Magnetic flux quantization

> Persistent currents

A macroscopic quantum phenomenon

Conventional understanding of superconductivity rests on two major paradigms of condensed matter physics:

## 1. Landau Fermi liquid theory



Interacting Fermi liquid adiabatically connected to non-interacting Fermi gas: quasiparticles with same charge/spin.

Describes all conventional metals.

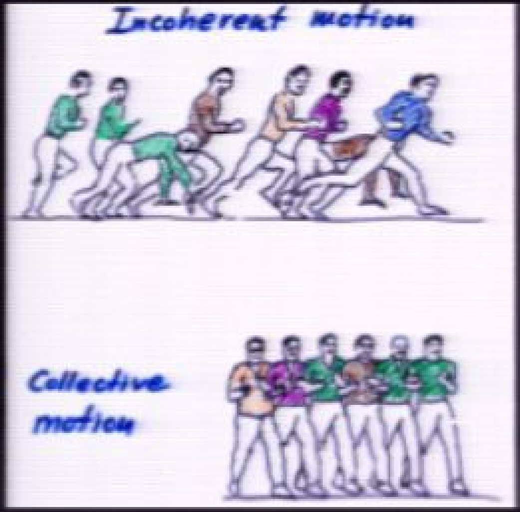
## 2. Spontaneous Symmetry Breaking & Ginzburg-Landau-Wilson Theory

- examples: crystals, charge and spin order
- local order parameter describes state (e.g. $<\mathbf{m}_i>$ for a ferromagnet)

# Superconductivity

BCS 1957

**Bardeen** **Cooper** **Schrieffer**



## Key ideas:

- **Electrons pair (Cooper pairs)**

  - Fermi liquid (metal) unstable to arbitrarily weak attractive interaction

- **Pairs are coherent**

  - Global U(1) symmetry broken → phase rigidity

Wave function characterized by an energy gap.

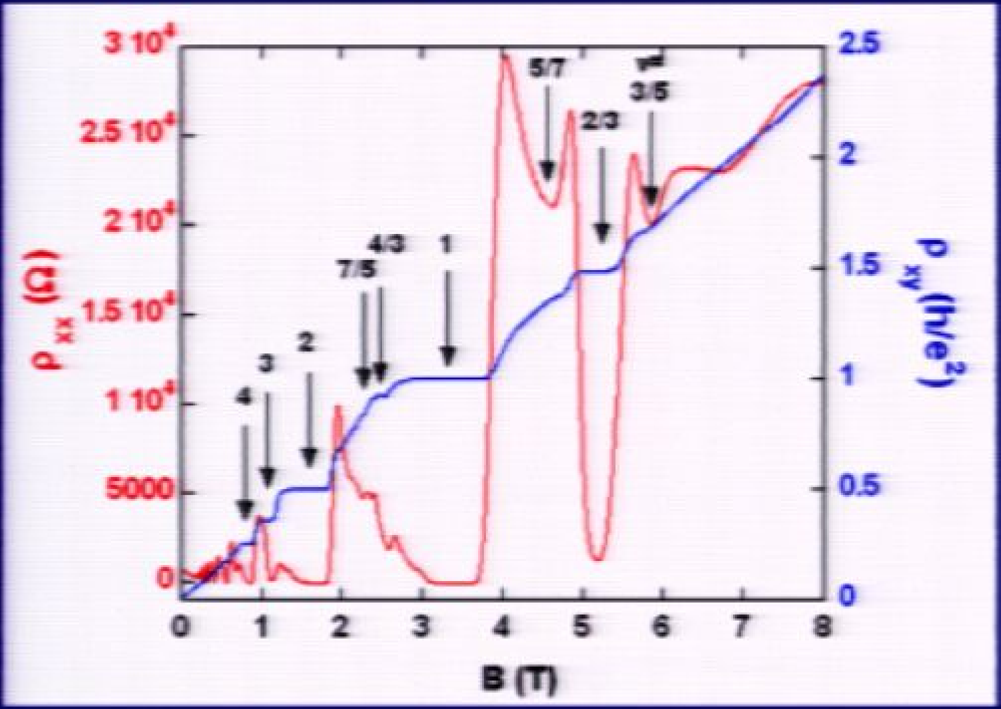
Order parameter: $<\psi_{\uparrow}(r)\psi_{\downarrow}(r)>\sim\Delta(r)$

Revolutionary step in understanding Fermi liquids

FLT, SSB & BCS explained all known electron behaviour in materials pre-19

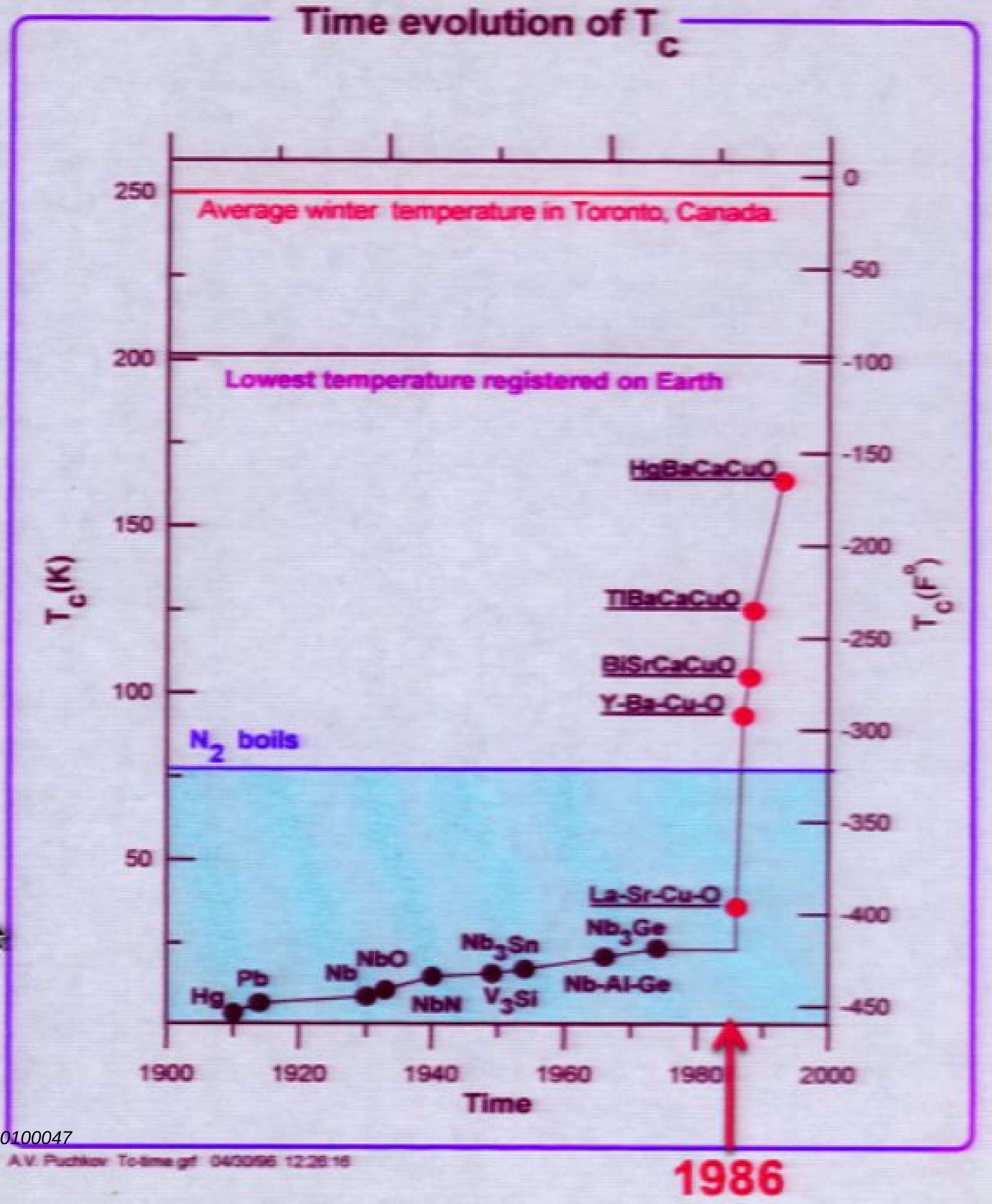
# 1980's: Quantum Hall Effect

Integer QHE discovered in 1980
by von Klitzing



Fractional QHE discovered in 1982
by Tsui, Stormer & Gossard

→ new way of looking at nature of quantum ground states in CM



# High Temperature Superconductivity in the Cuprates

High transition temperatures

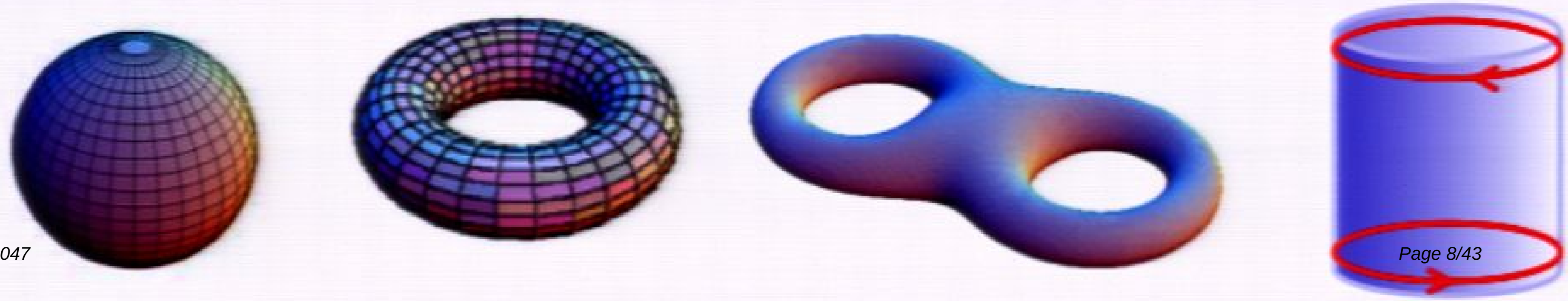
Strong electron correlations

Standard paradigms failed – many new ideas generated

QHE & spin liquids → topological order (Xiao-Gang

# Topological Order

- Not broken symmetry state; not Fermi liquid
- Classified by topological invariant
- Ground state degeneracy depends on topology
- Gapless edge/surface states in open geometry; bulk gap
- May support quasiparticles with new (fractionalized) quantum numbers
- Topological quantum entanglement
- Topological order may be one route to quantum computing



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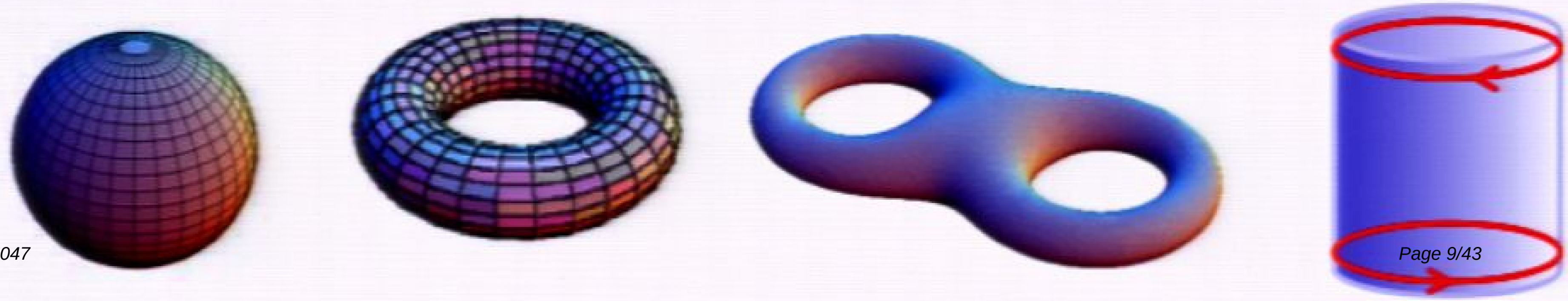
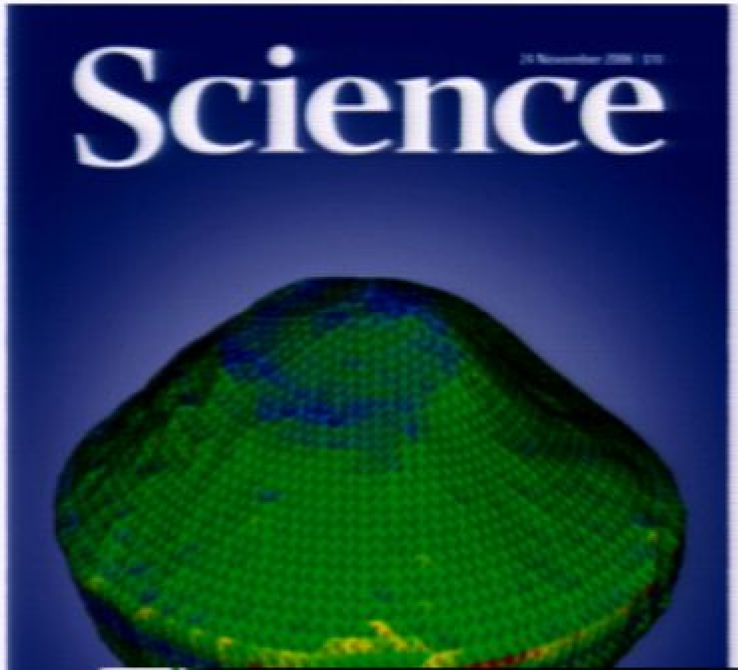




Superconductivity can be understood as topological order, with (gapped) edge states, spin-charge separation, topological degeneracy. [Hansson, Oganesyan, Sondhi, 2004; also Wen.]

PERSPECTIVES

PHYSICS

## Superconductivity with a Twist

**Maurice Rice**

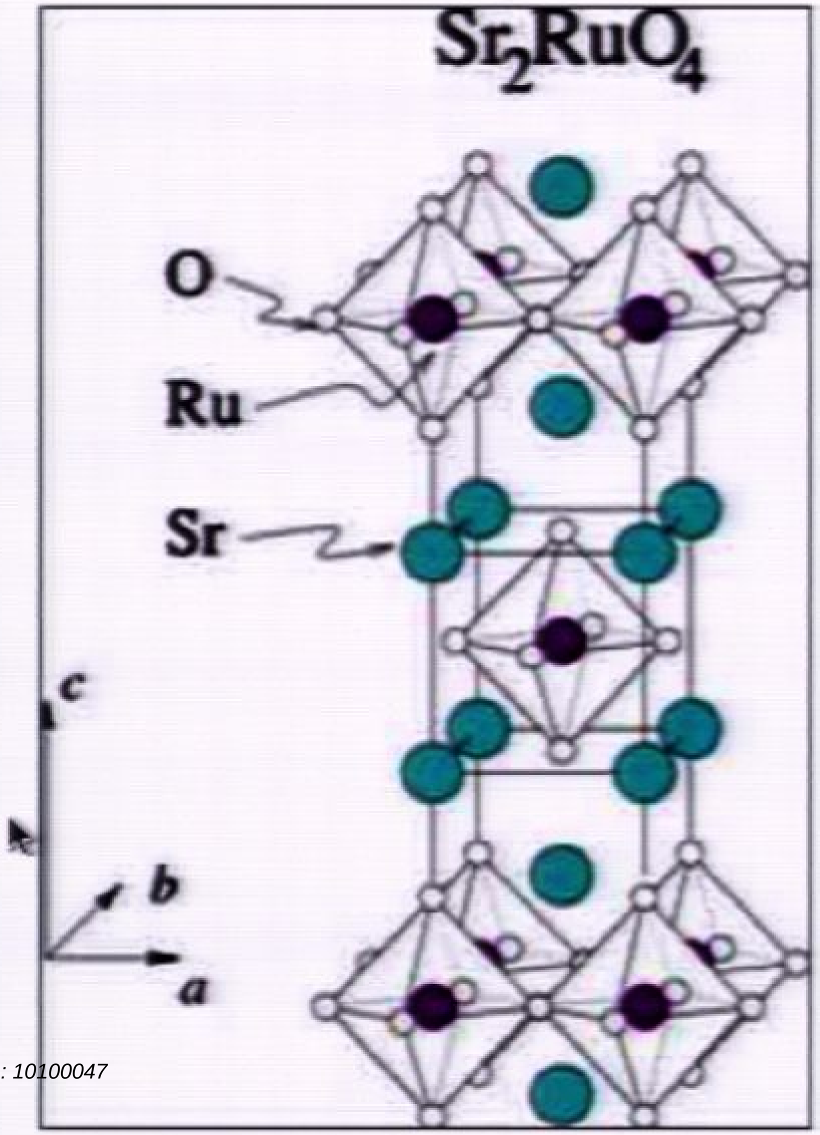
A material related to high-temperature superconductors exhibits mobile regio of twisted symmetry much like ordered magnetic regions in magnets.

Cuprate oxides, which were found to be superconducting at relatively high temperatures about 20 years ago, have several less well-known cousins. One of the most intriguing of these "unconventional" materials, strontium ruthenate ($Sr_2RuO_4$), was observed to be a superconductor in 1994 (*1*). Now, two reports, one by Kidwingira *et al.* on page 1267

Kidwingira *et al.* and Xia *et al.* have directly established the presence of an orbital moment in $Sr_2RuO_4$. They find that this superconductor has some similarities to a magnet—in technical terms, it breaks time-reversal symmetry. What this means quantum mechanically is that its pair wave function is inherently complex (that is, having real and imaginary

s-wave lead superconductor. The crit rent passing through this junction o played the same pattern as conv Josephson junctions with increasi strength. But for other junctions of t type, and even for thermal cycling of t junction, Kidwingira *et al.* found co different and random patterns. They ex

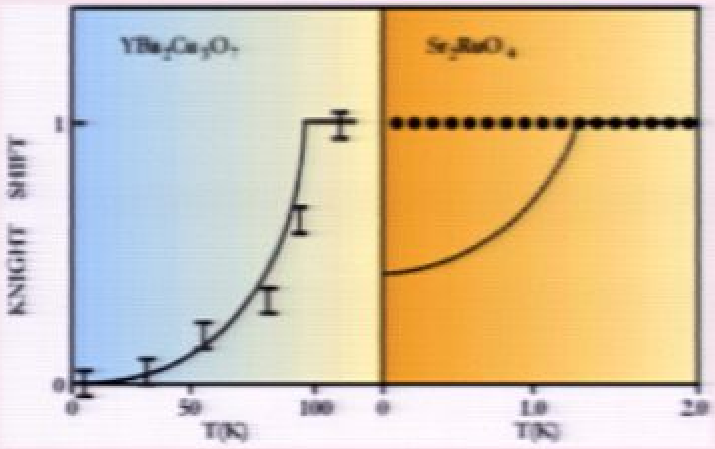
# Strontium Ruthenate



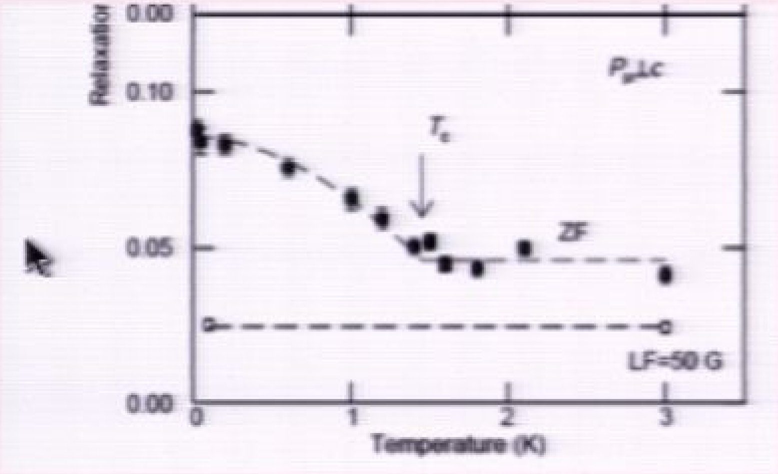
Fermi surface of Sr2RuO4

- Same structure as cuprates
- Quasi-two-dimensional
- Discovered in 1994 by Yoshi Maeno
- $T_c \leq 1.5K$ (disorder dependent)

# Early experiments → spin triplet pairing with broken time reversal symmetry



NMR measurement of spin susceptibility
→ **triplet pairing (most likely p-wave)**
K. Ishida et al., *Nature* **396**, 658 (1998).



MuSR measurement of internal B fields
→ **internal B fields turn on with Tc**
→**broken time reversal symmetry**
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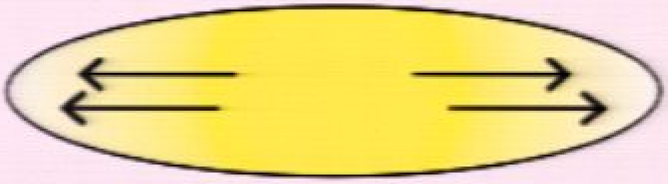
# Chiral p-wave superconductivity

$$\left[\text{General triplet OP}:\ \ \psi = \Delta_{\uparrow\uparrow}(\vec{p})\chi_{\uparrow\uparrow} + \Delta_{\downarrow\downarrow}(\vec{p})\chi_{\downarrow\downarrow} + \Delta_{\uparrow\downarrow}(\vec{p})(\chi_{\uparrow\downarrow} + \chi_{\downarrow\uparrow})\right]$$

$$\boxed{\begin{aligned}\psi &= \Delta_0 \frac{p_x \pm i p_y}{p_F}\chi_{s_c=0}\\ &\equiv \Delta(\vec{p})\chi_{s_c=0}\end{aligned}}$$

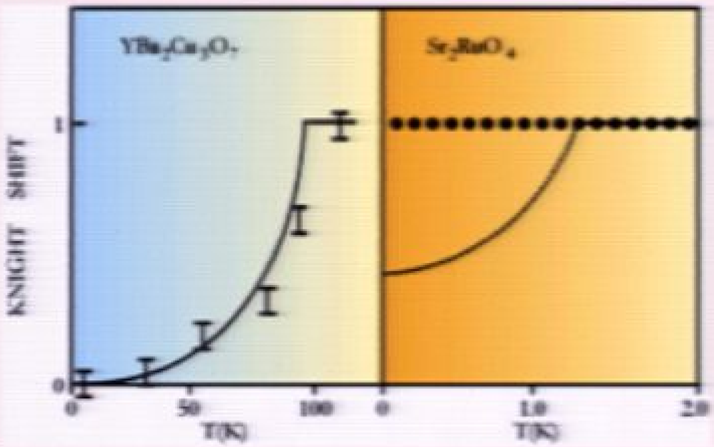
$$\chi_{s_z=0} = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)_z$$

$$= \frac{1}{\sqrt{2}}(-e^{-i\alpha}|\uparrow\uparrow\rangle + e^{i\alpha}|\downarrow\downarrow\rangle)_{\alpha(}$$



Equal spin pairing in ab (xy) plane

# Early experiments → spin triplet pairing with broken time reversal symmetry



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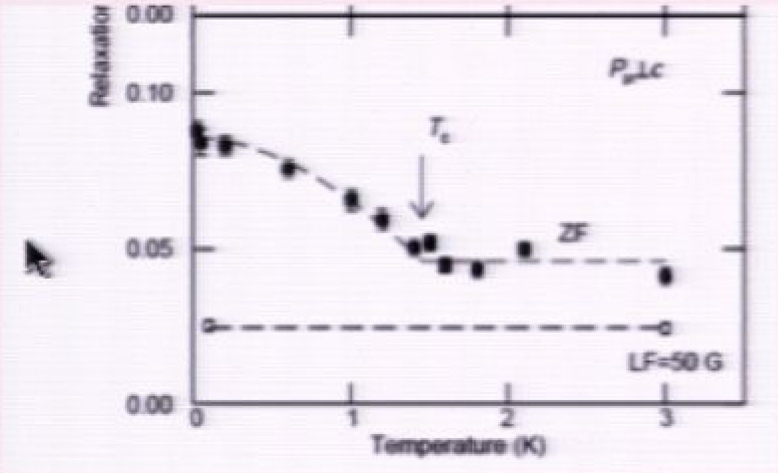

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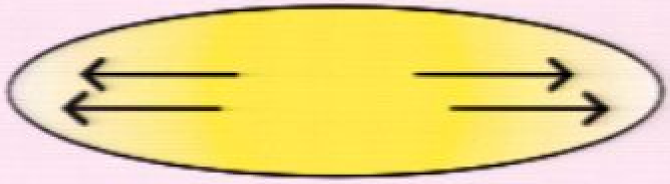
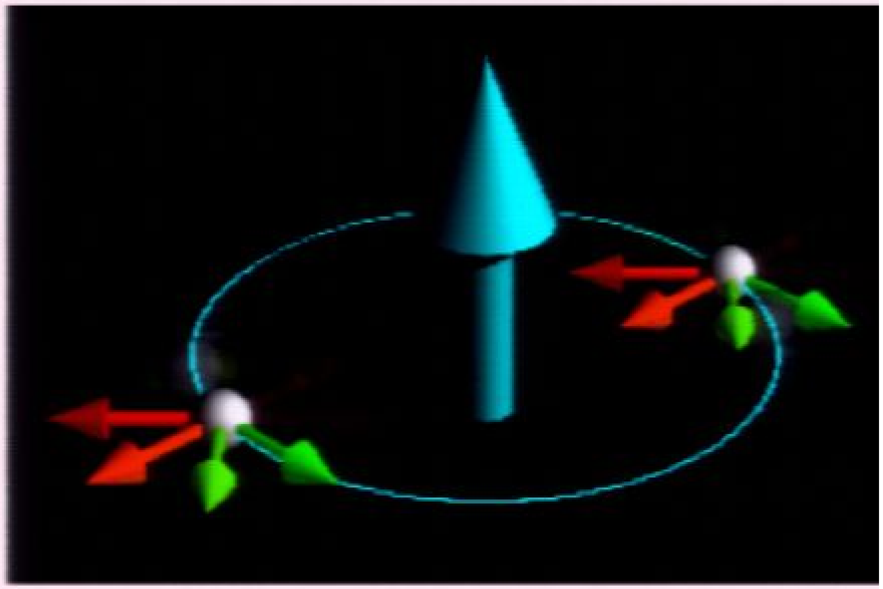


Equal spin pairing in ab (xy) plane

# Chiral p-wave superconductivity

$$\Delta(\vec{p}) = \Delta_0 \frac{p_x \pm i p_y}{p_F}$$

$$\left|\Delta(\vec{p})\right| = \Delta_0$$



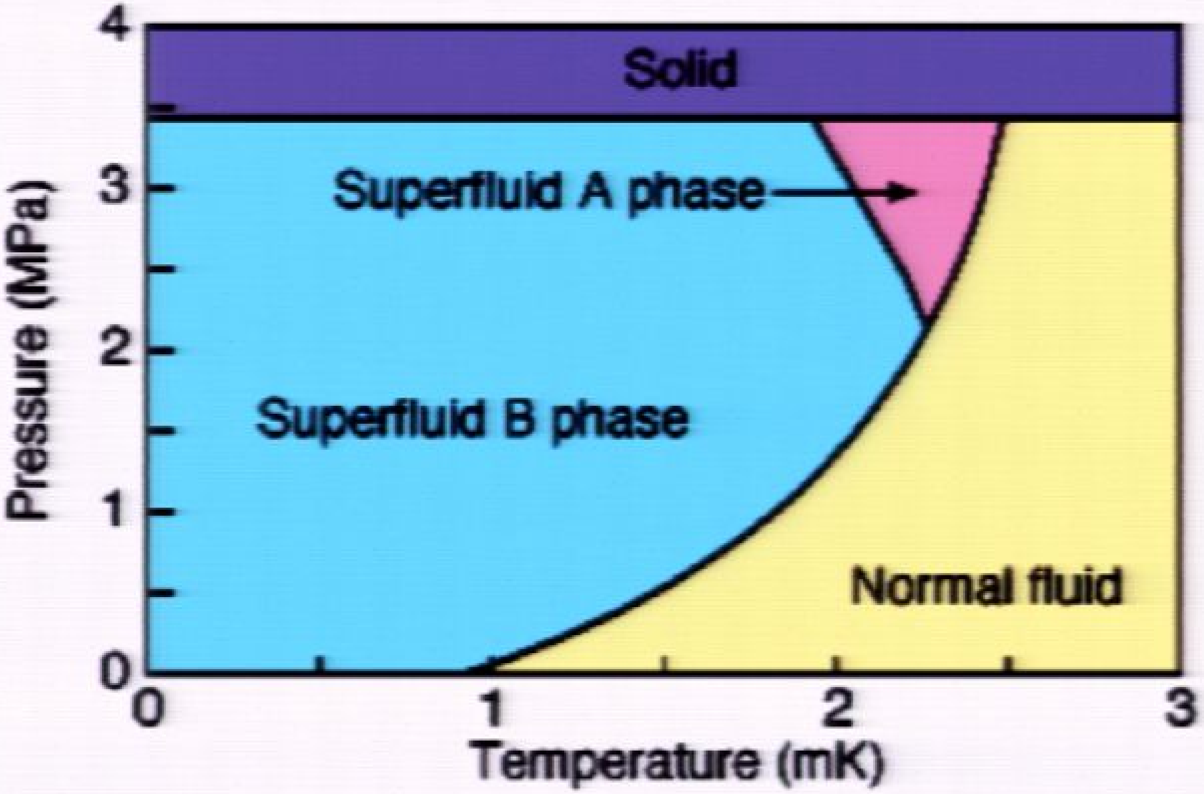
**Breaks time-reversal symmetry**

**Angular momentum $L_z$=±1**
**→ chiral (or twist)**

$p_x+ip_y$ degenerate with $p_x-ip_y$ → can have domains

# Superfluid $^3$He

Lee, Osheroff & Richardson (1971)



Proposals also exist for stabilizing this phase in cold atomic Fermi gases.

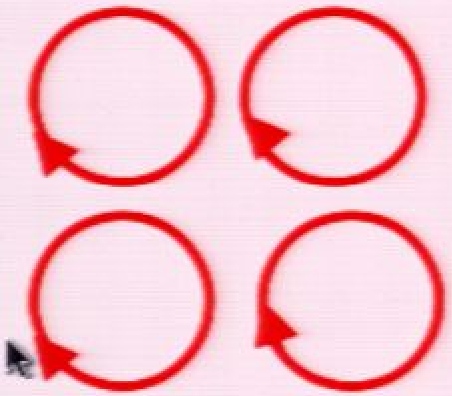
# Angular momentum & edge currents

Each Cooper pair carries angular momentum ħ and the BCS state

$$\psi_{\mathrm{BCS}} = \prod_{k\sigma}\left(|u_k| + |v_k| e^{i\phi_k} a^{+}_{k\sigma} a^{+}_{-k\sigma}\right)|0\rangle$$

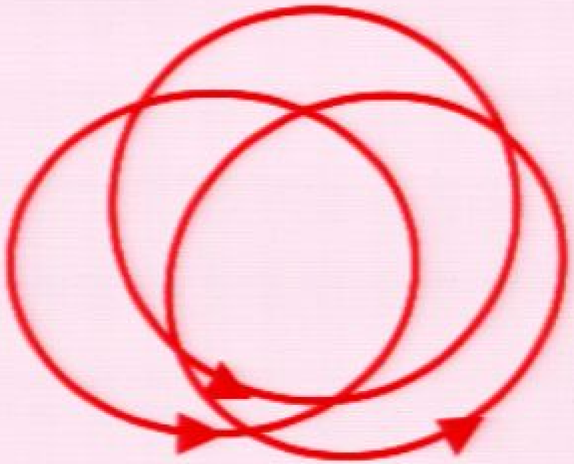
carries angular momentum $\langle L_z \rangle = N\hbar/2$. (Stone & Roy, PRB 2004.)

“Angular momentum paradox”

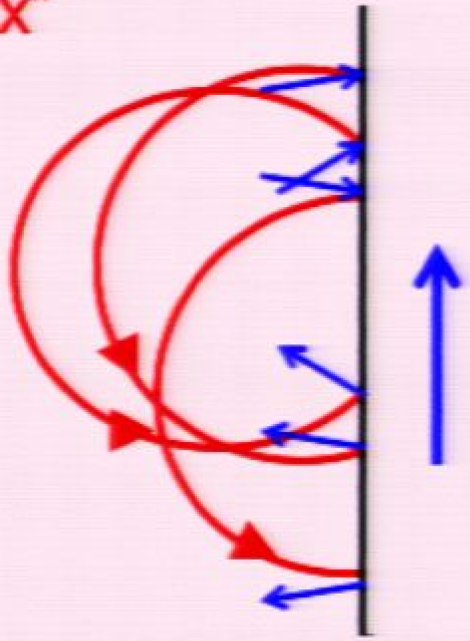


Strong coupling, $\xi < a$, like FM with local moments adding

non-topological



Large Cooper pairs ~ $\xi$
Only $e^-$ within $\Delta$ of $E_F$ are paired. Factor of $(\Delta/E_F)$?



Scattering at edge → edge states (also from top. order and full ang. mom.

# Topology of chiral state connected to winding of Δ

$$H = \begin{pmatrix} \varepsilon(k) - \mu & \Delta(k) \\ \Delta^*(k) & -[\varepsilon(k) - \mu] \end{pmatrix} = \vec{\delta}(\vec{k}) \cdot \vec{\tau} \qquad \Delta(k) = \Delta_0 (k_x \pm i k_y) / k_F$$

$$\delta(\vec{k}) = (\mathrm{Re}(\Delta(\vec{k})), \mathrm{Im}(\Delta(\vec{k})), \epsilon(\vec{k}) - \mu)$$

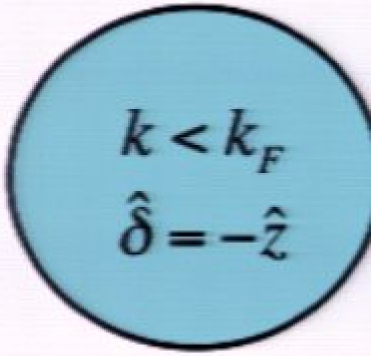
Anderson pseudospin representation of BCS

Anderson pseudospin has nontrivial (skyrmion) form in momentum space for μ>0 (BCS).

[For μ<0 (BEC) δ is trivial and points along +z.]

$$N = \frac{1}{4\pi} \int d^2k \hat{\delta} \cdot \left( \partial_x \hat{\delta} \times \partial_y \hat{\delta} \right)$$

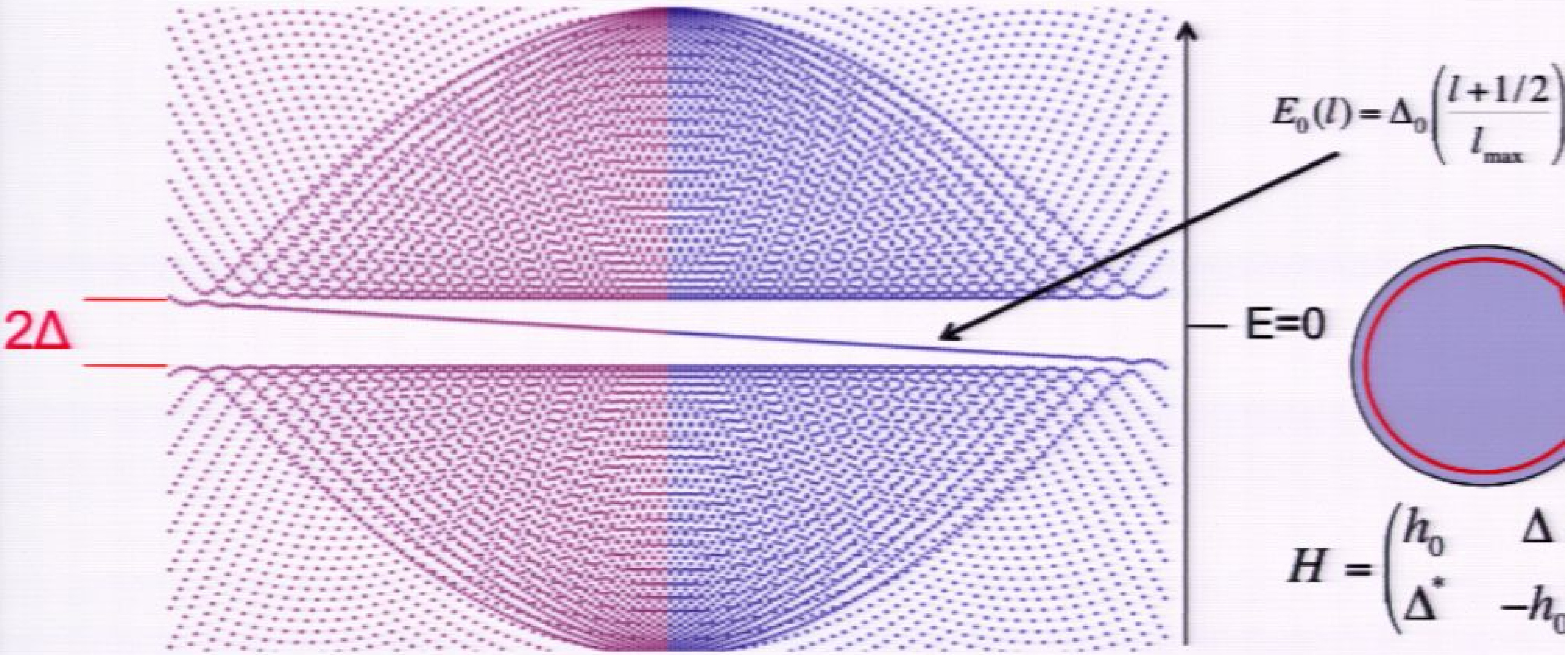
$k > k_F \Rightarrow \hat{\delta} = +$



k=$k_F$ → δ winds b
2π around FS

$\mu > 0$

# Chiral edge states exist in any open geometry



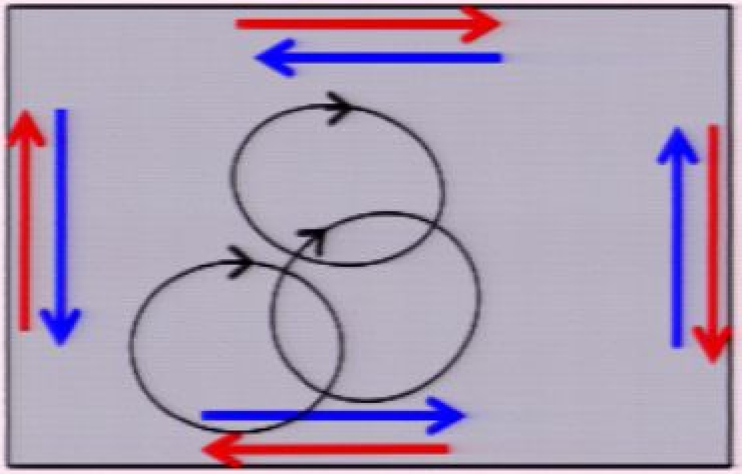
$E_0(l) = \Delta_0\left(\frac{l+1/2}{l_{\max}}\right)$

$H = \left(\begin{matrix} h_0 & \Delta \\ \Delta^* & -h_0 \end{matrix}\right.$

- Part of BdG energy spectrum as function of angular momentum for chiral p-wave in disk geometry.

Chiral p-wave state has topological order, analogous to 5/2 Moore-Read QH state, characterized by Chern number = ±1. (Read & Green 2000)

# Spontaneous supercurrents for chiral p-wave



Equilibrium supercurrent within ξ of surface

Screening current within λ+ξ of surface

→ Magnetic field B~10G within λ of surface and B~20G at domain walls.

Stone and Roy (2004)
Matsumato and Sigrist (1999)

The equilibrium supercurrent is connected to the angular momentum and to the topological order.

# Scanning SQUID microscopy: search for edge and domain currents

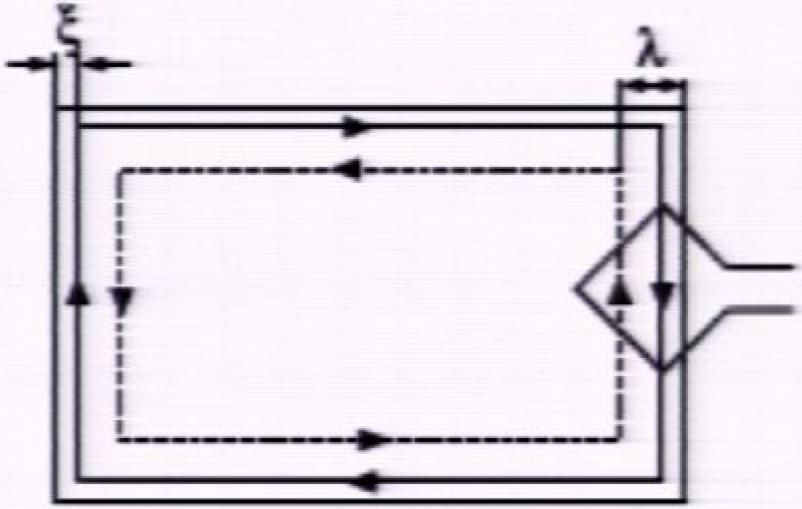
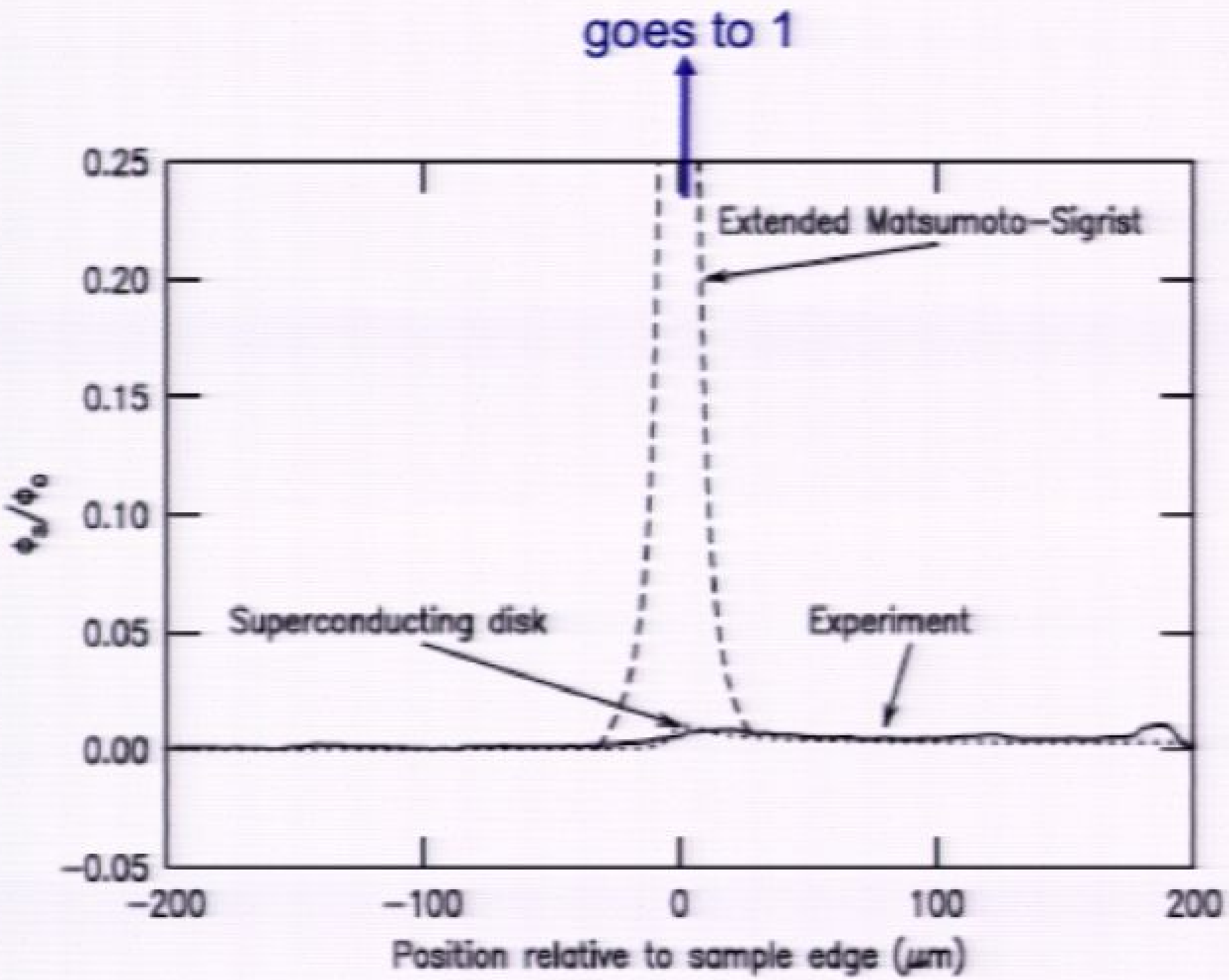
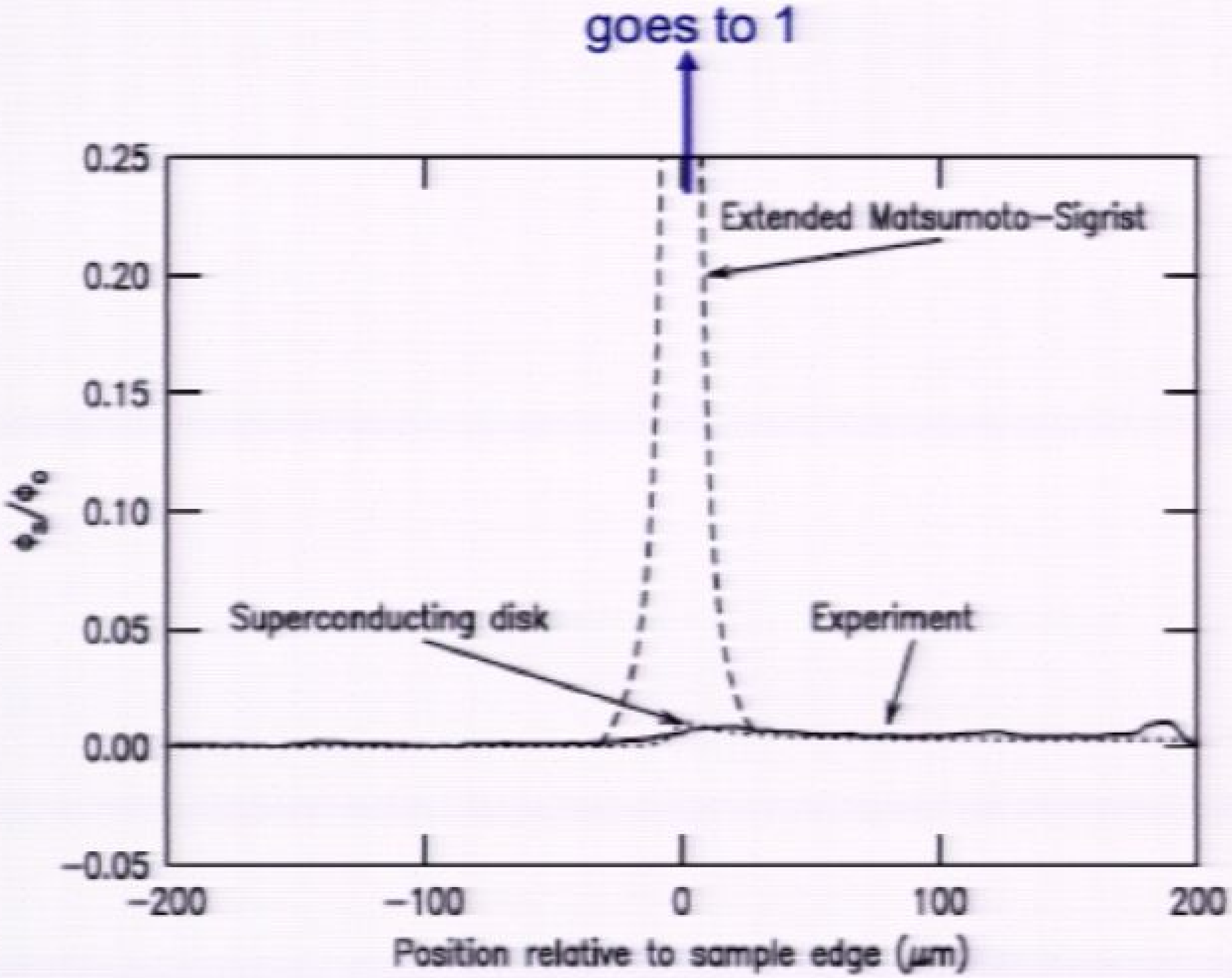


Fig. 5. Electric currents (solid lines) of the chiral edge states along the inner and outer edges of $Sr_2RuO_4$. The dashed lines show superfluid counterflow. The rectangular coil on the right represents a SQUID pickup loop.


goes to 1

He3 scanning SQUID signal across ab face of $Sr_2RuO_4$ single crystal at T=0.27K

J.R. Kirtley, C. Kallin, C. Hicks, E.A. Kim, Y. Liu, K.A. Moler, Y. Maeno, PRB 76, 014526 (2007).



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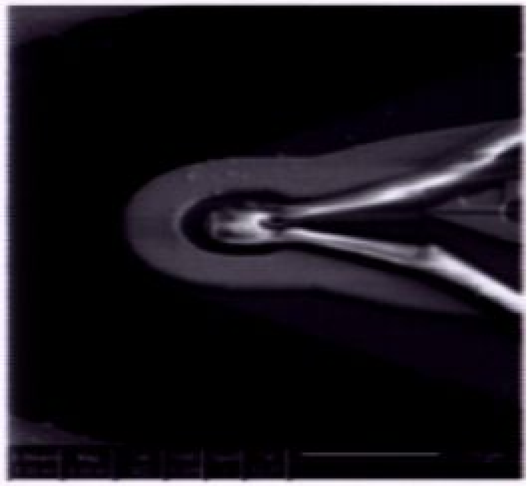
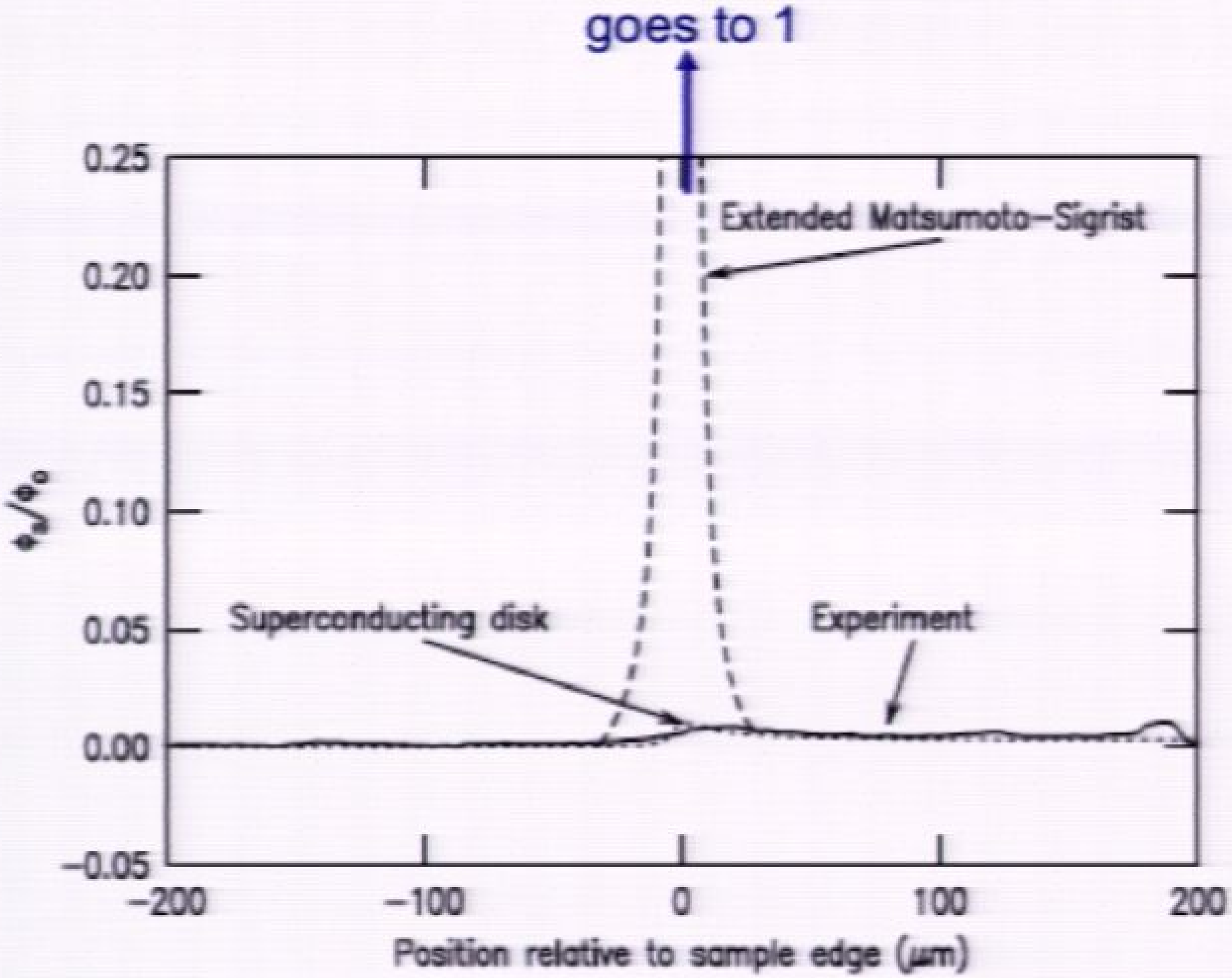


Smaller SQUIDs, Hall b
probes, micron sample
→ still no spontaneous
fields observed



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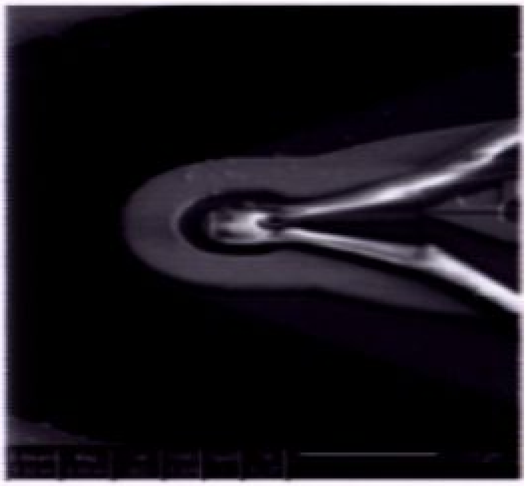


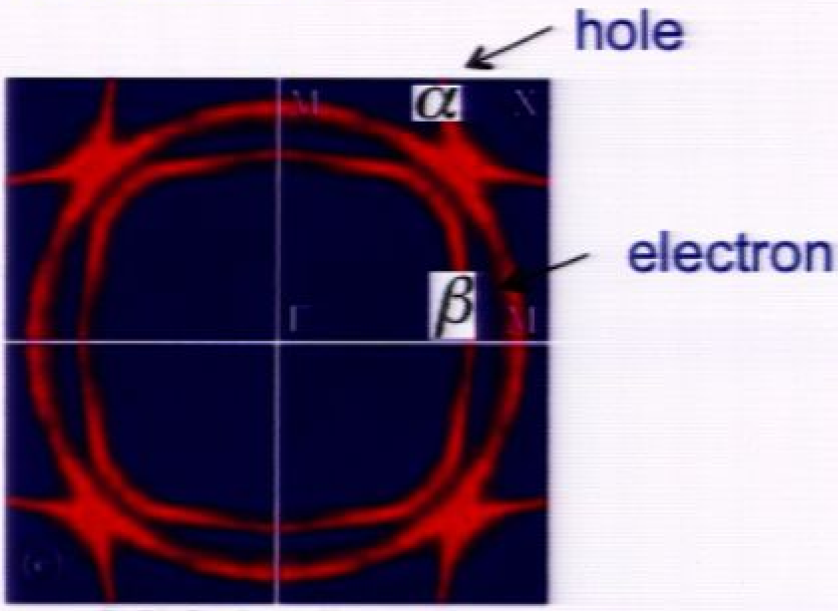
Smaller SQUIDs, Hall
probes, micron sample
→ still no spontaneous
fields observed

Experiments put uppe
bounds on edge
currents which are ~
3 orders of magnitude
smaller than predicted

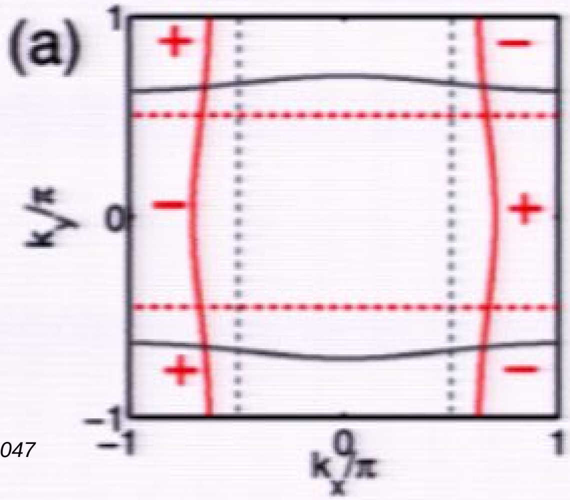
# Time Reversal Symmetry Breaking

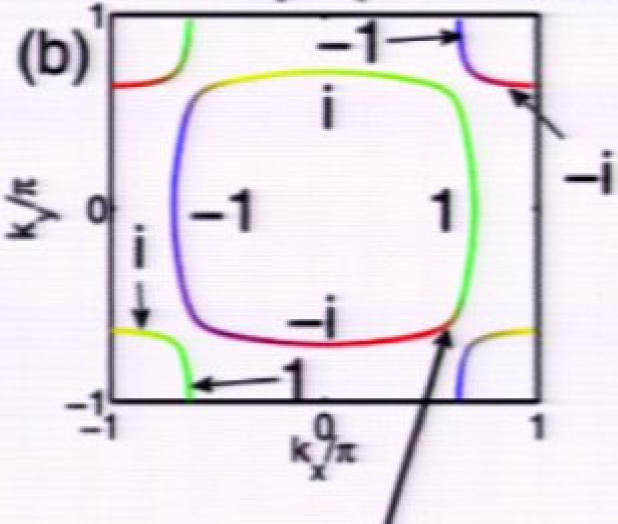
| Experiment | TRSB? | Domain size [limit >0.3μ ] |
|---|---|---|
| muSR (Luke & Ishida) | Yes | < 2 μ |
| SQUID (Kirtley)<br>Nano-SQUID (Hicks) | No<br>No | < 2 μ<br>< 0.5 μ |
| Scanning Hall Probe (Moler) | No | < 1 μ |
| Kerr rotation (Kapitulnik) | Yes | > 50 μ with field cooling<br>~ > 15-20 μ in ZFC |
| Tunneling (van Harlingen) | Maybe | < 1 μ ~0.5 μ dynamic |
| Tunneling (Liu)<br>Corner junctions | Parity<br>No? | >10-50 μ<br>(only seen in one case) |

Raghu, Kapitulnik and Kivelson have proposed chiral p-wave SC on the quasi 1-d bands, arXiv:1003.2266.



$Sr_2RuO_4$ cleaved at 180 K





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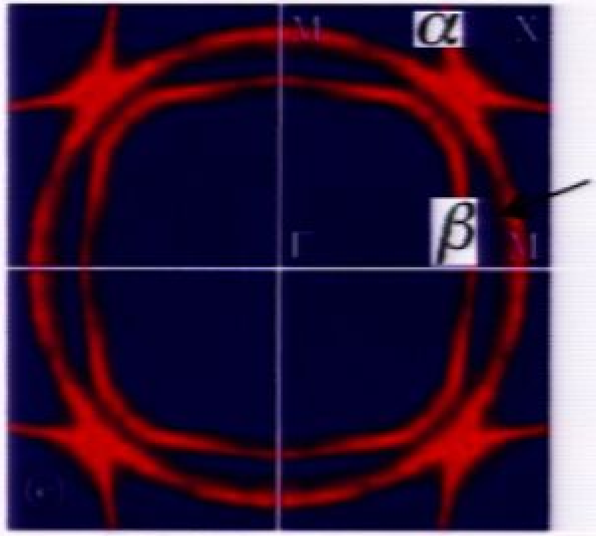

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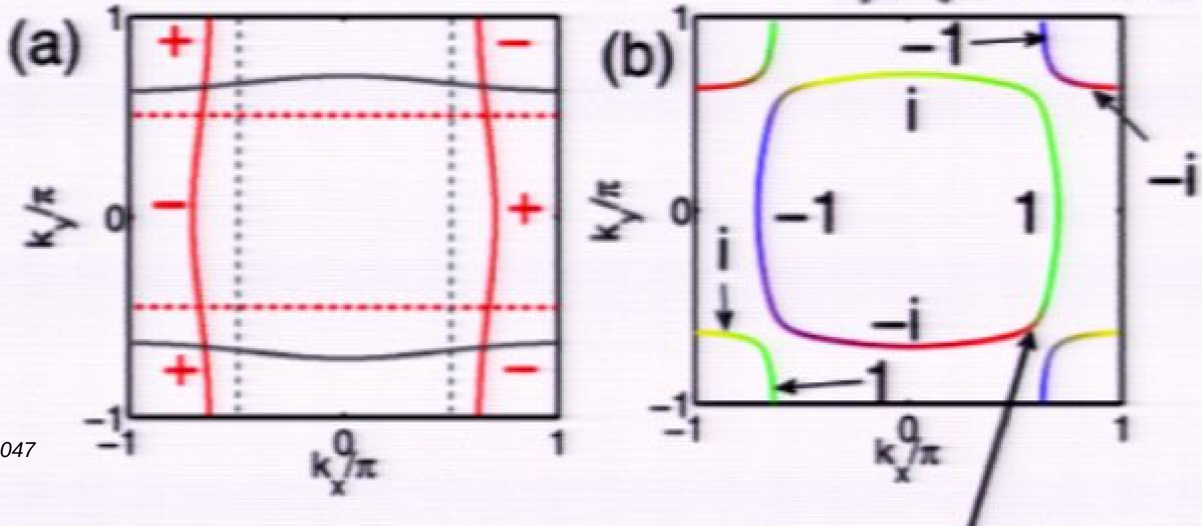
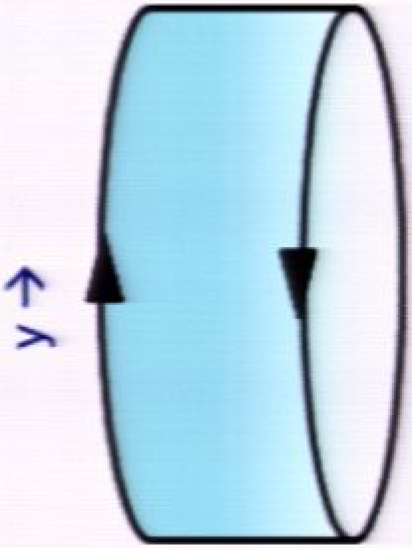
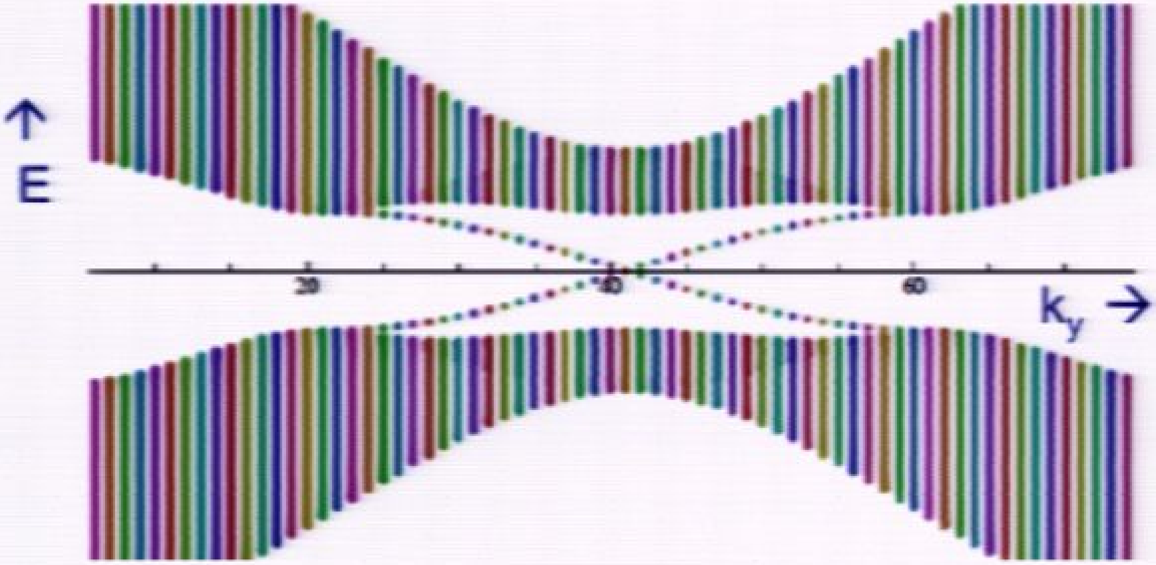
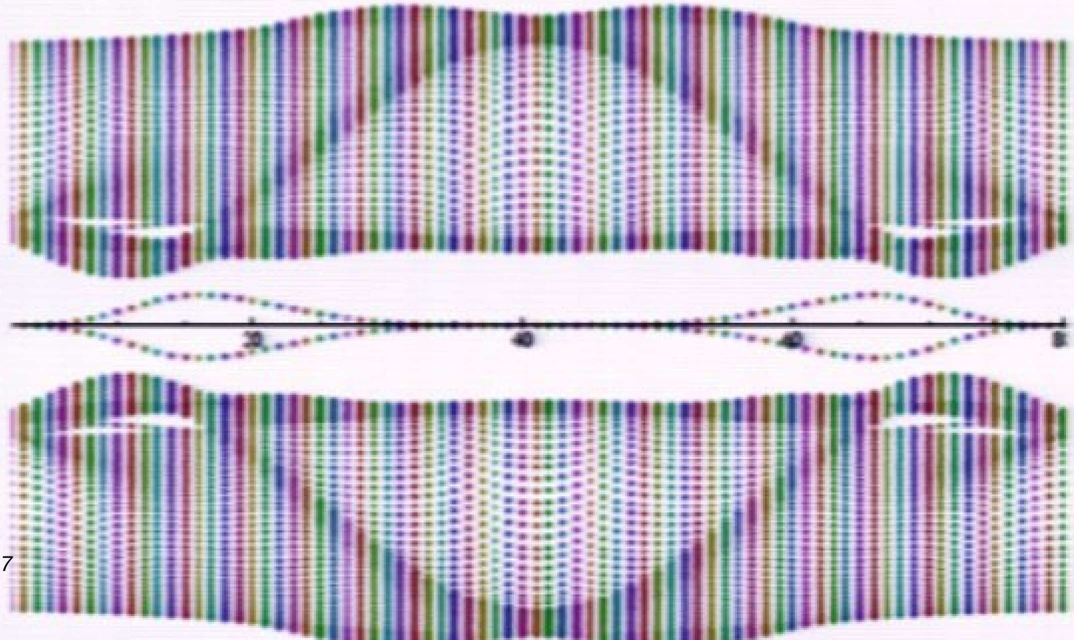




# Energy Spectra for Chiral p-wave SC in Strip Geometry



Chiral p-wave SC on 2d xy band gives one chiral mode at each edge of strip. (Chern number is 1.)



Chiral p-wave SC on 1d xz and yz bands gives one non-chiral mode at each edge of strip.

Hole and electron bands
→ Chern number is 0
→ not topologically protected

# Zero modes in SC and Majorana Fermions

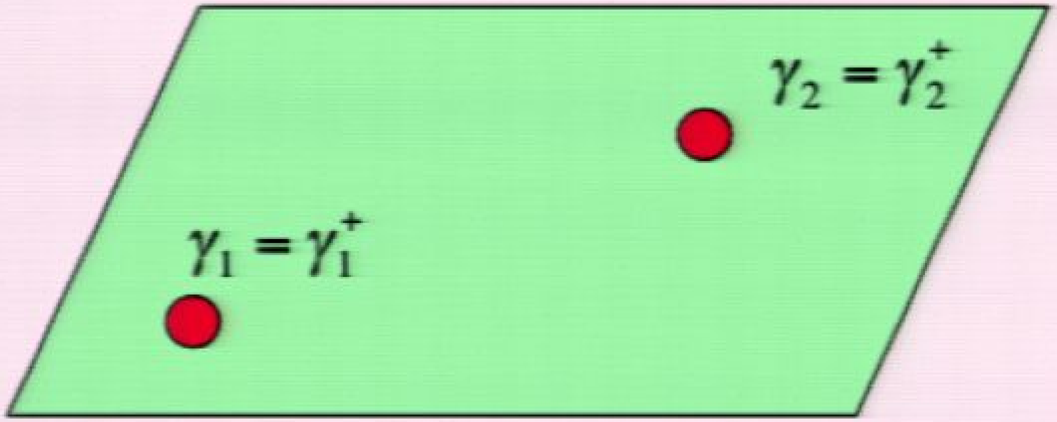
BCS quasiparticles are superposition of electrons and holes

$$\gamma_n^+ = \sum_i (u_n a_i^+ + v_n a_i)$$

$$\mathrm{E} = 0 \;\Rightarrow\; \gamma_0^+ = \sum_i (u_0 a_i^+ + u_0^* a_i) = \gamma_0$$

**Majorana Fermion**

(spinless or one spin only)

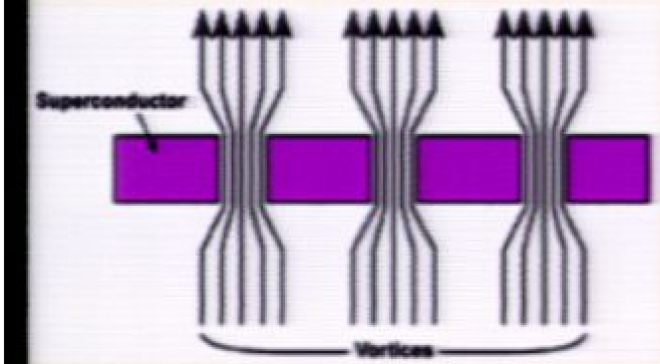


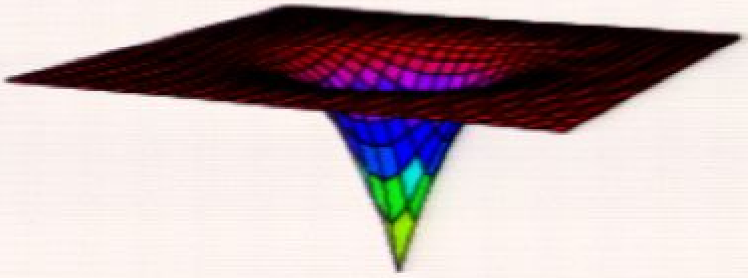
A single γ mode cannot accommodate an electron, need 2!

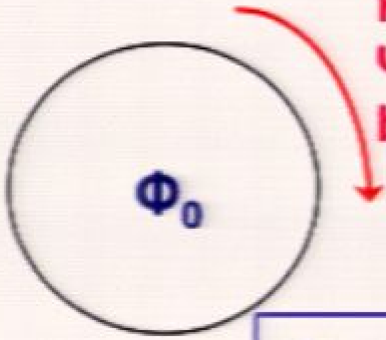
$$\mathrm{c} = \gamma_1 + i\gamma_2 \quad c^+ = \gamma_1 - i\gamma_2$$

Always come in pairs, but can be spatially separated in topological state, and protected by bulk gap. E.g., one Majorana zero mode at vortex core or at edge.

# Vortices in a superconductor







**Phase**
**Ψ wind**
**by 2π**

**$\Phi_0$=hc/2**

# Half-quantum vortices in triplet SC

Both the spin and the orbital parts of Ψ can wind around the vortex

$$\psi = \Delta(\vec{p})\left[-e^{i\alpha}|\uparrow\uparrow\rangle_x + e^{-i\alpha}|\downarrow\downarrow\rangle_x\right]$$

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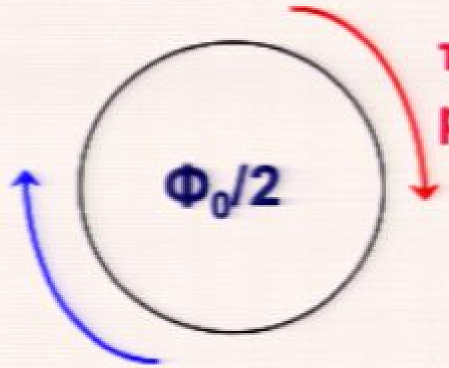

**π from**
**phase**

**π from rotation**
**of spin state**

→ Vortex in only 1 spin component!

# Half-quantum vortices in chiral p-wave

- half-quantum vortex has spectrum $E_n = n\omega_0$, $\omega_0 = \Delta^2/E_F$ → one zero mode → Majorana fermion
- Since BdG qps, γ, pick up phase π when ψ changes by 2π phase, interchanging 2 half-quantum vortices (Ivanov, 2001) →

$$\gamma_i \rightarrow \gamma_{i+1} \qquad \gamma_{i+1} \rightarrow -\gamma_i \qquad \gamma_j \rightarrow \gamma_j$$

- moving HQV (with Majorana fermions) around each other moves one between degenerate gs → non-Abelian braiding statistics → of interest for quantum computing

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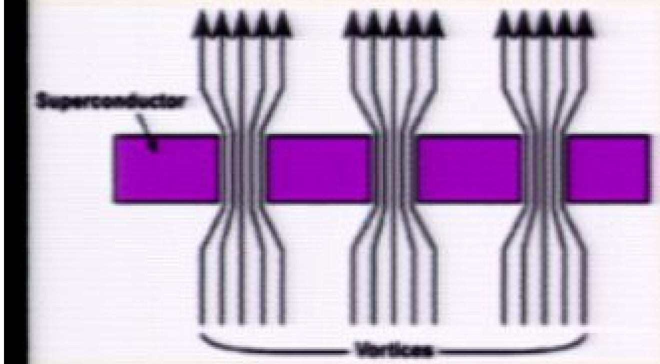

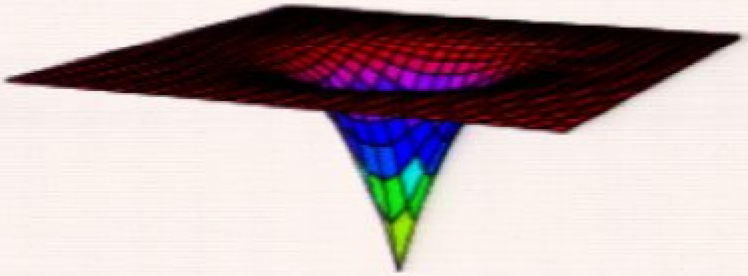

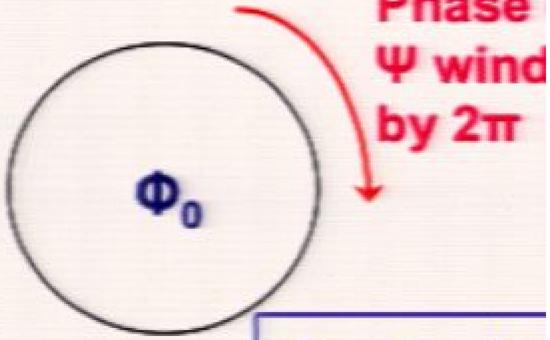

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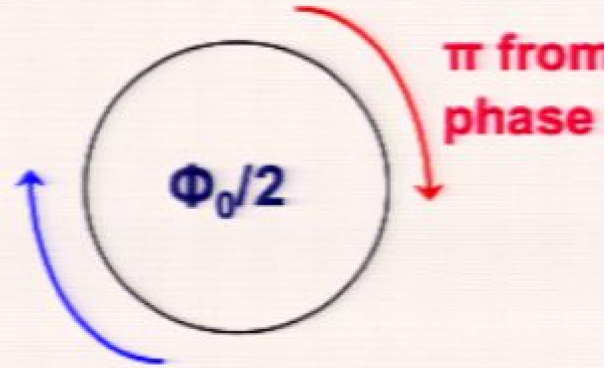

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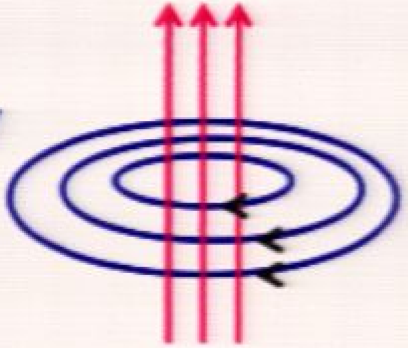
# Are HQV vortices stable?

- Vortex in only one spin component → circulating supercurrents carry spin current → log divergent energy



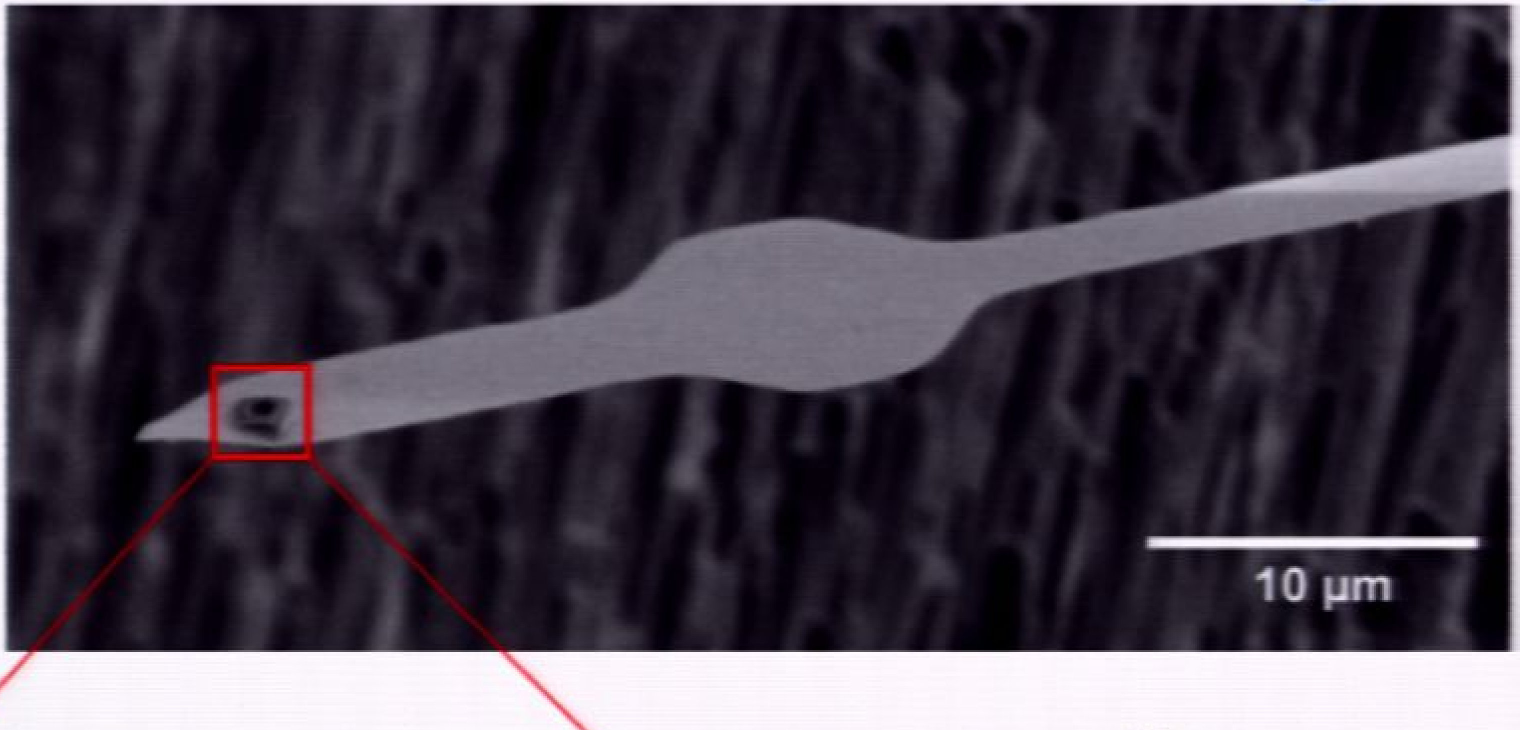
$$f = \frac{1}{2}\left(\frac{\hbar}{2m}\right)\left[\rho_s\left(\nabla\theta - \frac{2e}{\hbar c}A\right)^2 + \rho_{sp}(\nabla\alpha)^2\right] + \frac{1}{8\pi}(\nabla\times A)^2$$

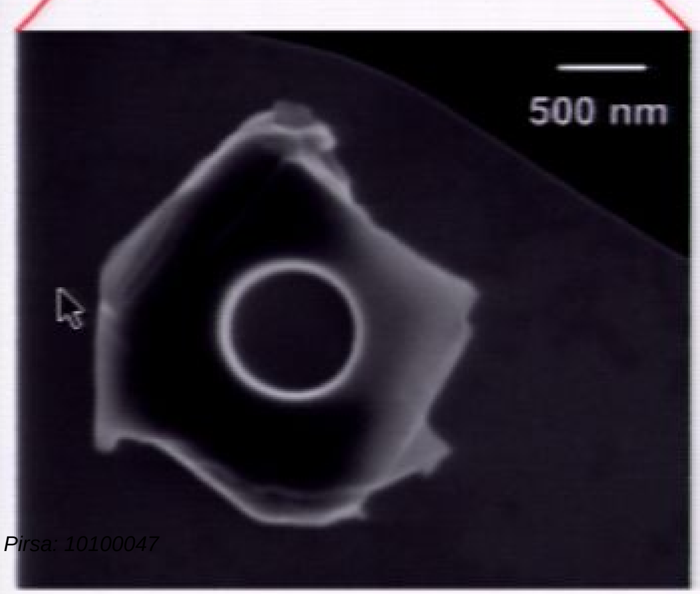
no screening

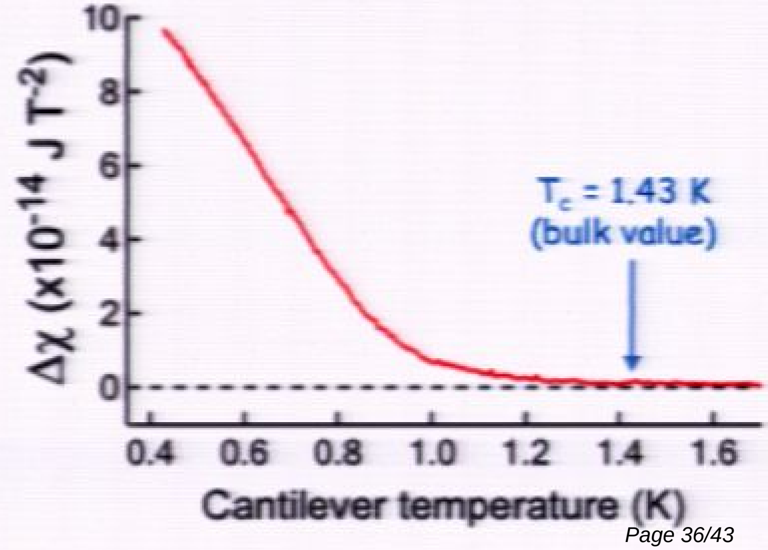
- Look for HQV in mesoscopic samples to minimize this energy cost

- The HQV has a spin polarization because $v_\uparrow \neq v_\downarrow$ [Vakaryuk & Leggett, PRL 2009]. For $S_c$=0 state, this polarization lies in the plane.

# Ultrasensitive Cantilever Magnetometry

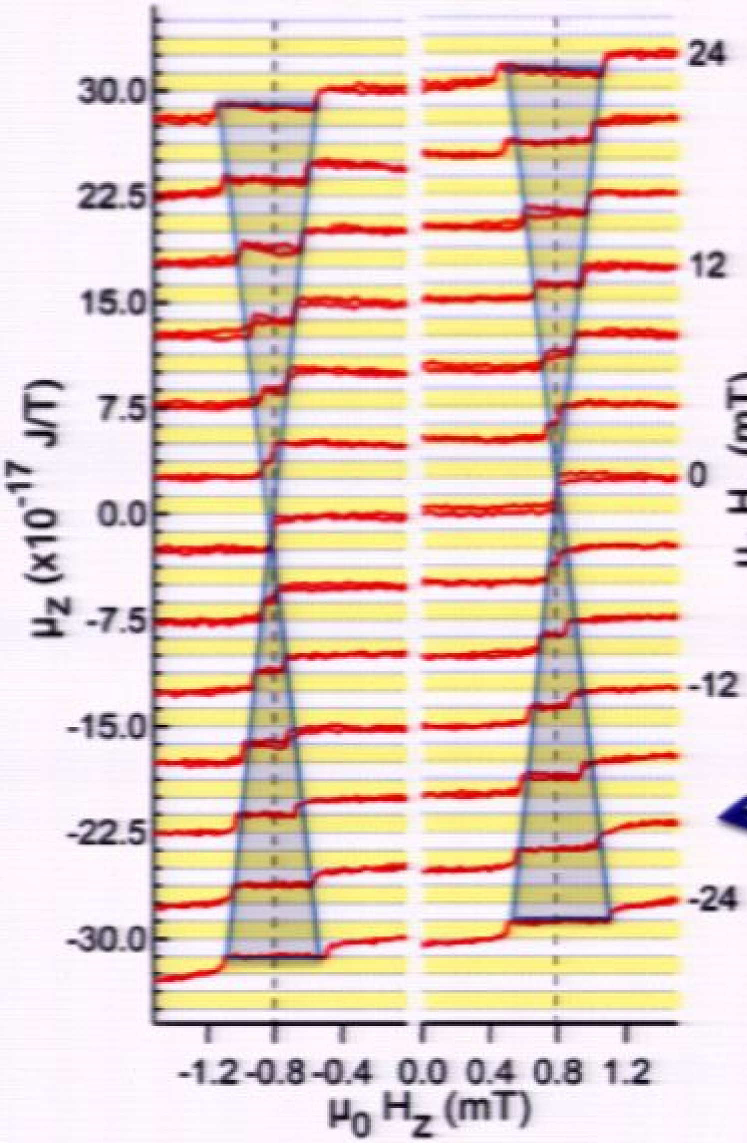






Raffi Budakian (UIUC) unpublishe

# Evolution of the Fractional Fluxoid State



$H_x=0$ → see jumps in magnetization corresponding to integer SC flux quanta

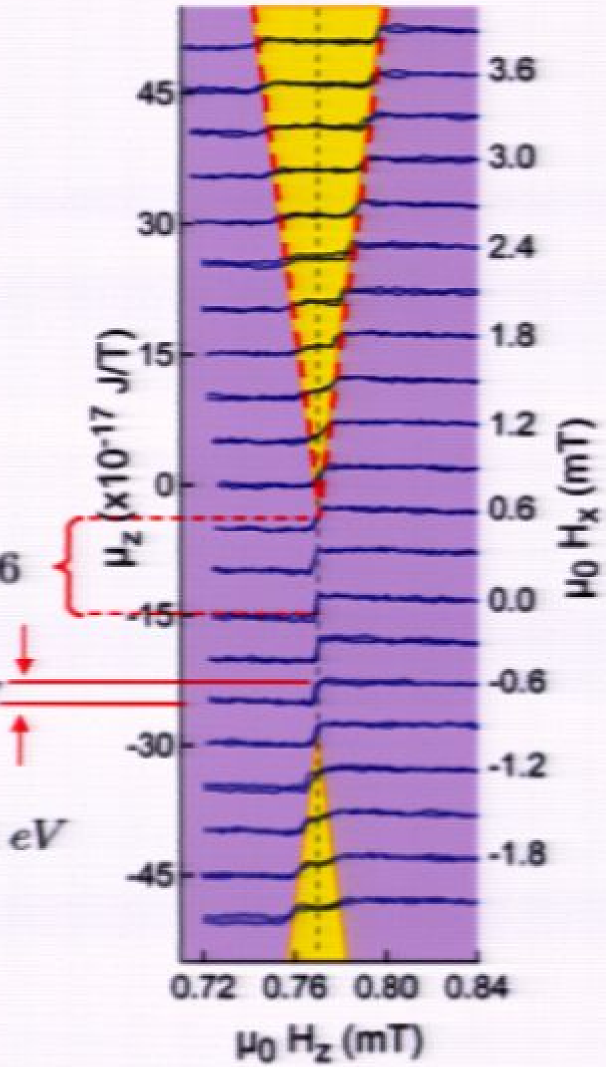
$H_x \neq 0$ → magnetization jumps split into two ½ jumps.

T=0.6 K

Budakian (UIUC) unpublished

$$G = \underbrace{E_0(n - \tfrac{\phi_a}{\phi_0})^2}_{\text{FF state costs higher energy at } H_x=0} + E_{FF} - \vec{\mu}_{FF} \cdot \vec{H}$$

FF stabilized by $H_x$ fro
coupling to spontaneous
polarization of FF state



$E_{FF}/E_0 = 0.26$

$\Delta\mu_z \approx 2 \times 10^{-17} J/T$

$E_0 \approx \frac{\Delta\mu_z \phi_0}{2\pi R^2} \approx 0.1\ eV$

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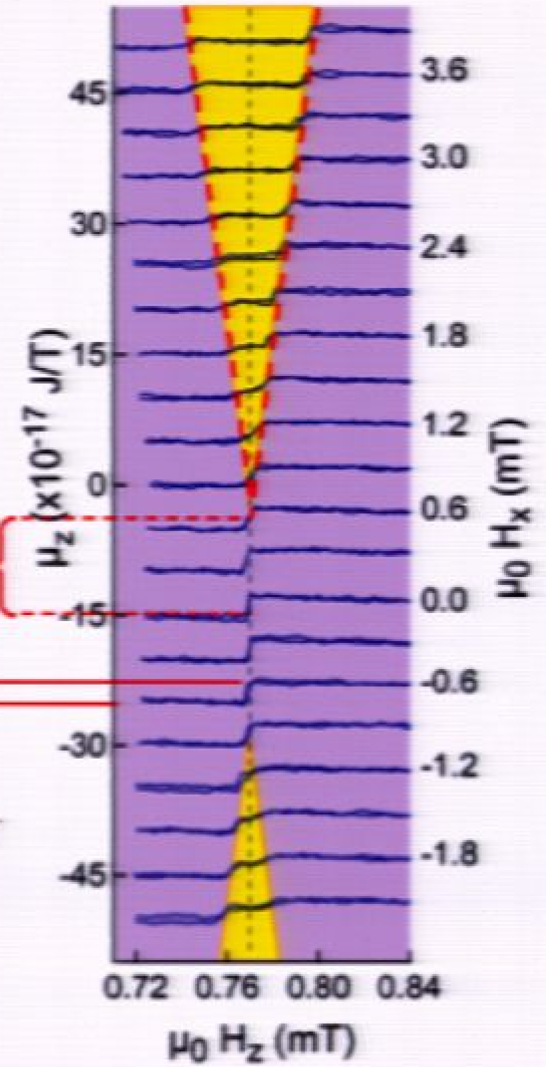

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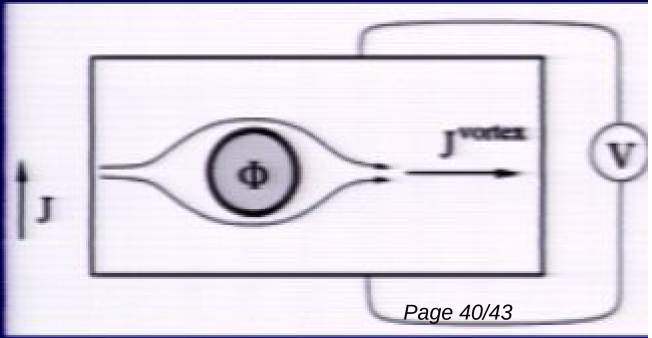
**Spontaneous FF polarization:**

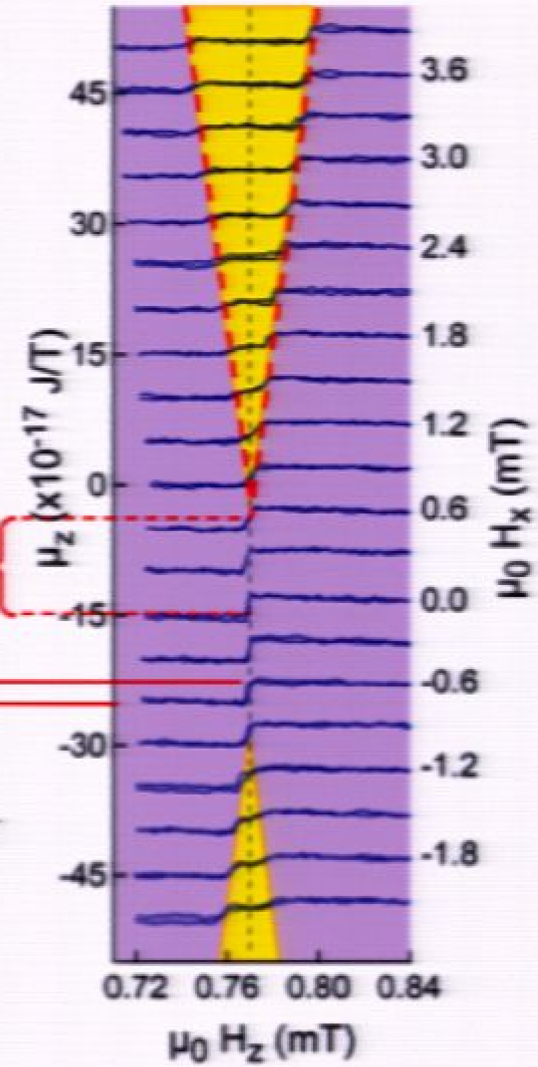
$$\mu_{FF} \approx 10^4\ \mu_B$$

**in agreement with prediction of Vakaryuk & Leggett.**

# In conclusion

- Compelling evidence that $Sr_2RuO_4$ is a triplet SC and several experiments see evidence of broken TR symmetry
- Absence of observable fields at surfaces due to spontaneous supercurrents is puzzling → different pairing symmetry or more to learn about chiral p-wave?
- Direct observation of topological nature is difficult due to screening. Thermal conductivity? Hall viscosity?
- Budakian's experiments suggest how to stabilize HQV for other studies → evidence of Majorana fermions would be direct evidence of topological order.





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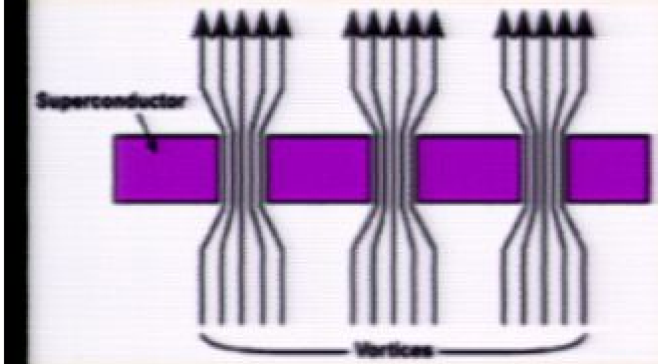

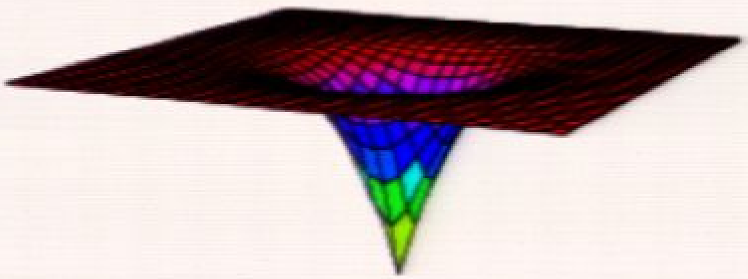

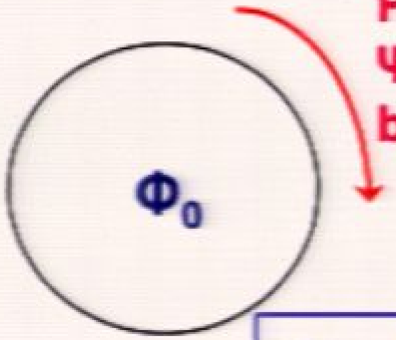

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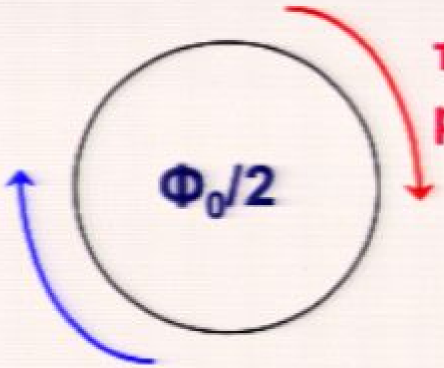

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