Title: Gravitational collapse and far from equilibrium dynamics in holographic CFTs

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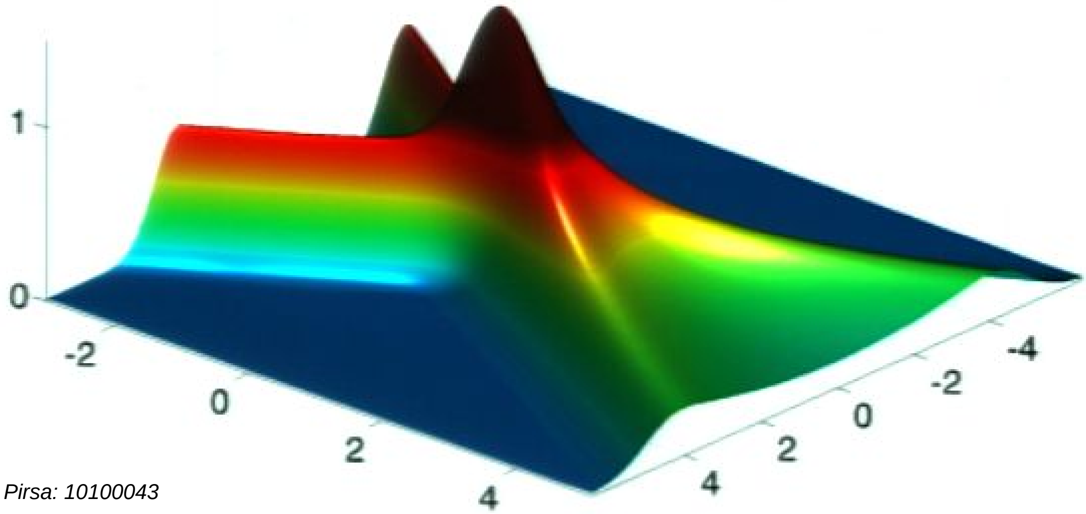
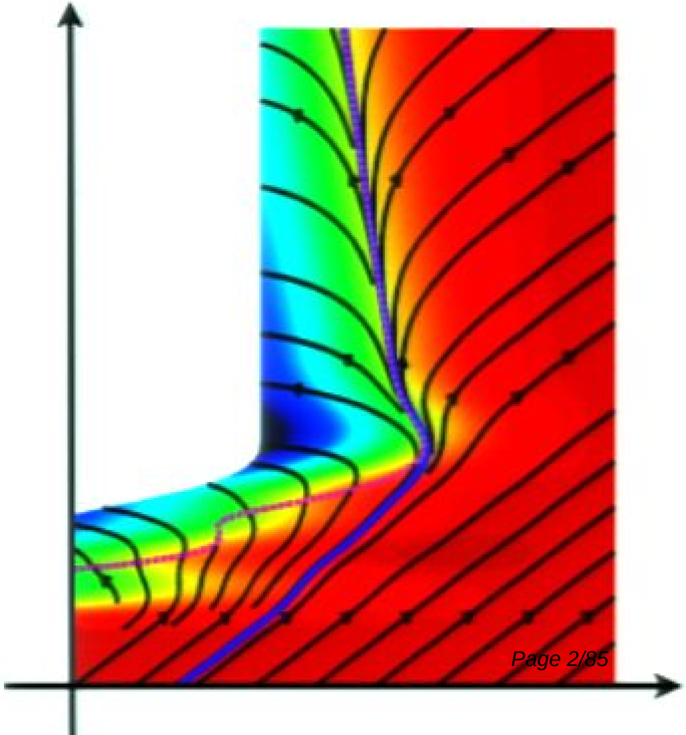
Abstract: A remarkable result from heavy ion collisions at the Relativistic Heavy Ion Collider is that shortly after a collision, the medium produced behaves as a nearly ideal liquid. The system is very dynamic and evolves from a state of two colliding nuclei to a liquid in a time roughly equivalent to the time it takes light to cross a proton. Understanding the mechanisms behind the rapid approach to a liquid state is a challenging task. In recent years holography has emerged as a powerful tool to study non-equilibrium phenomena, mapping the (challenging) dynamics of quantum systems onto the dynamics of classical gravitational systems. The creation of a liquid in a quantum theory maps onto the classical process of gravitational collapse and black hole formation. I will describe how one can use holography to study processes which mimic the dynamics of heavy ion collisions.

Gravitational collapse and far from equilibrium dynamics in holographic CFTs

Paul Chesler

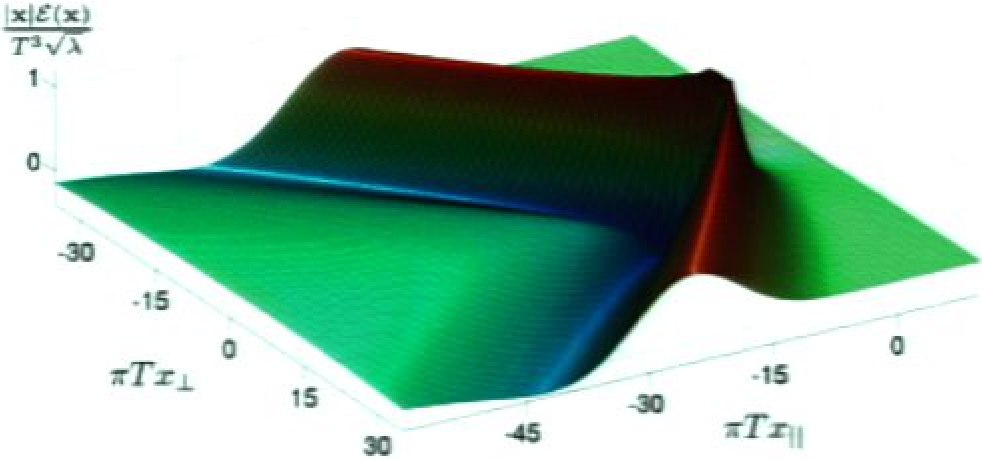
Massachusetts
Institute of
Technology





Real time physics from holography

Near-equilibrium physics



- Radiation & jets.
- Quasinormal modes.
- Hydrodynamics.
- Transport coefficients.

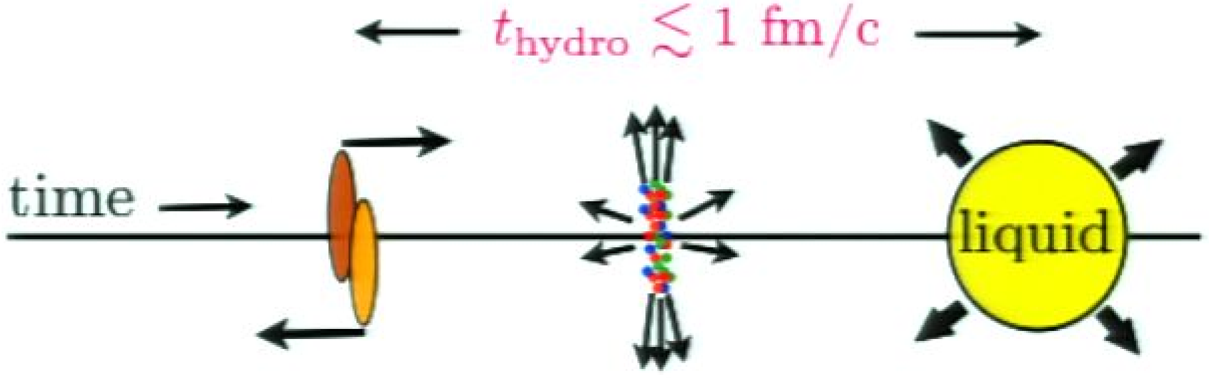
Far-from-equilibrium physics

- QGP formation.
- Thermalization.
- Turbulence.

$\Bigg\}$ non-trivial, non-linear, time dependent bulk geometries

Far-from-equilibrium dynamics in heavy-ion collisions

- Initial state: nuclei.
- "Final" state: QGP.



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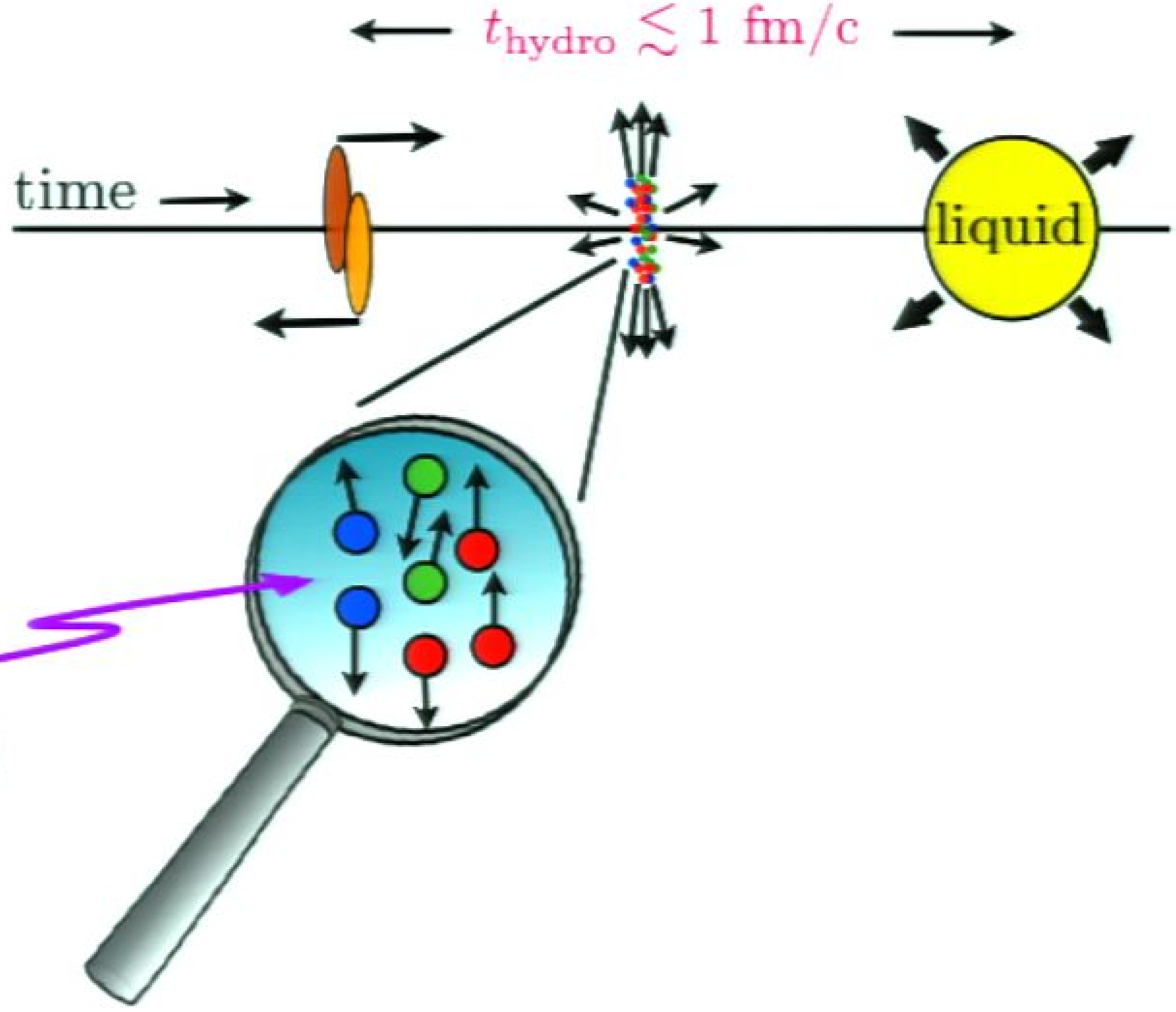

Anisotropic stress

$T^{\mu\nu} \approx \text{diag}(\epsilon, p_\perp, p_\perp, p_{||})$

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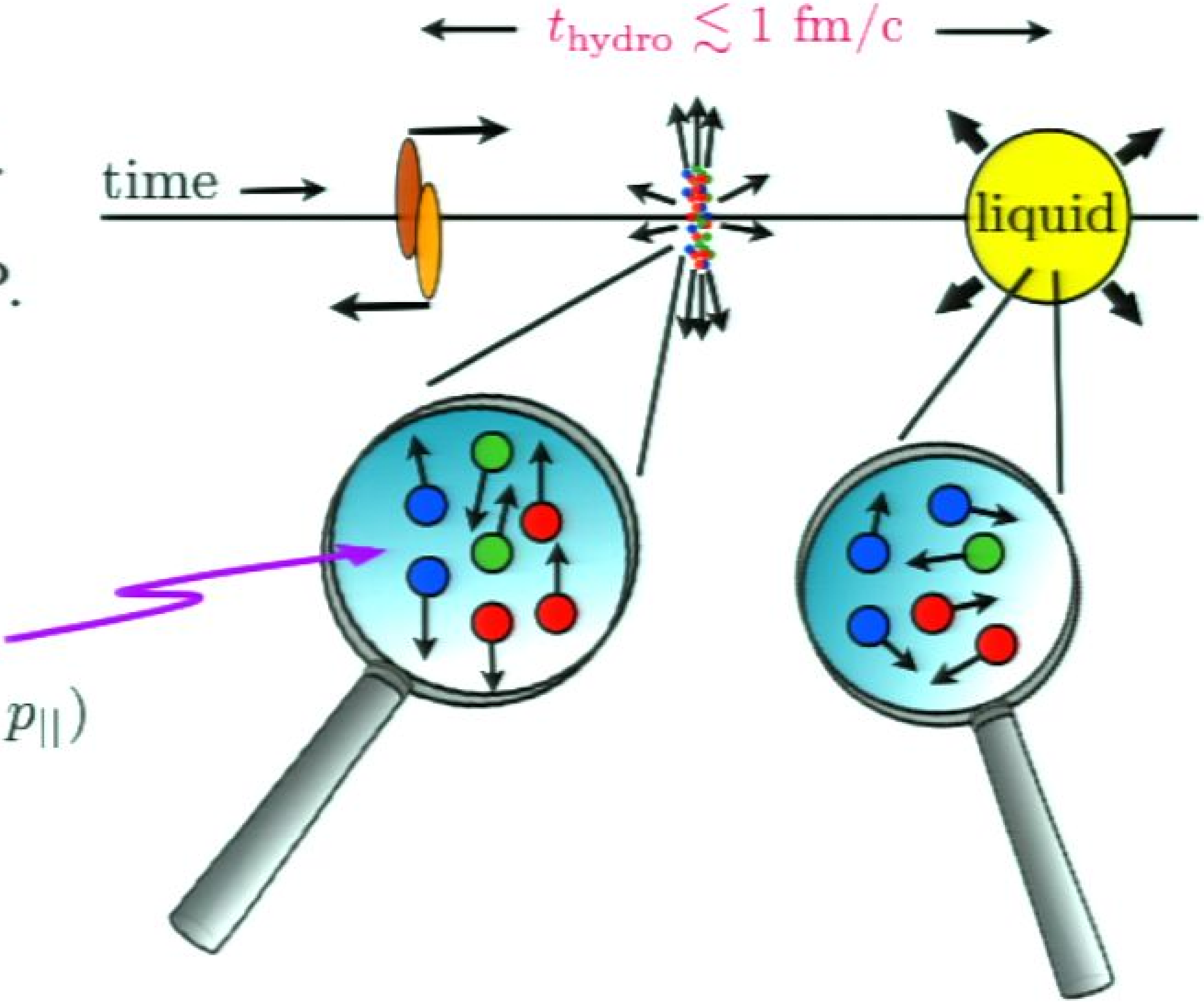

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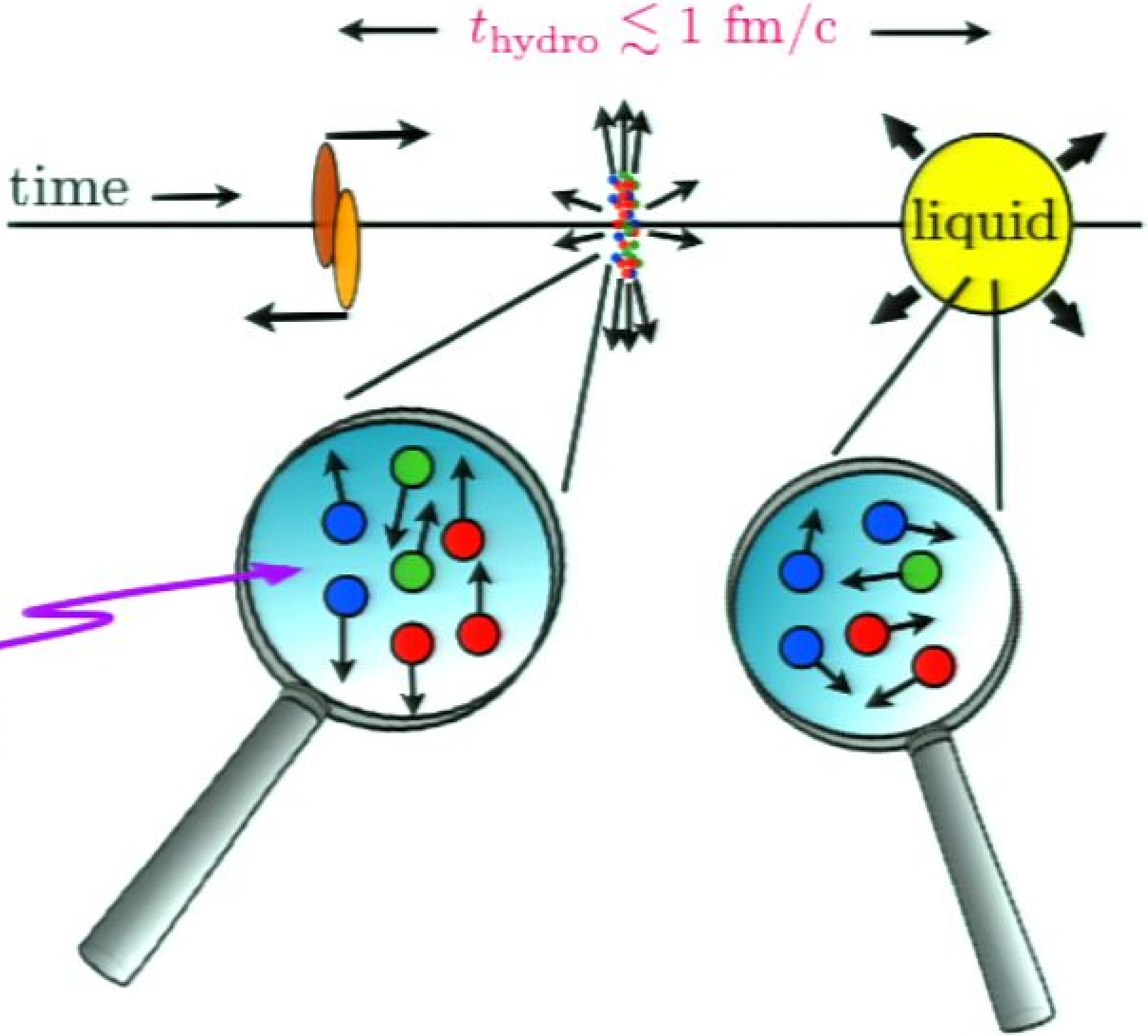

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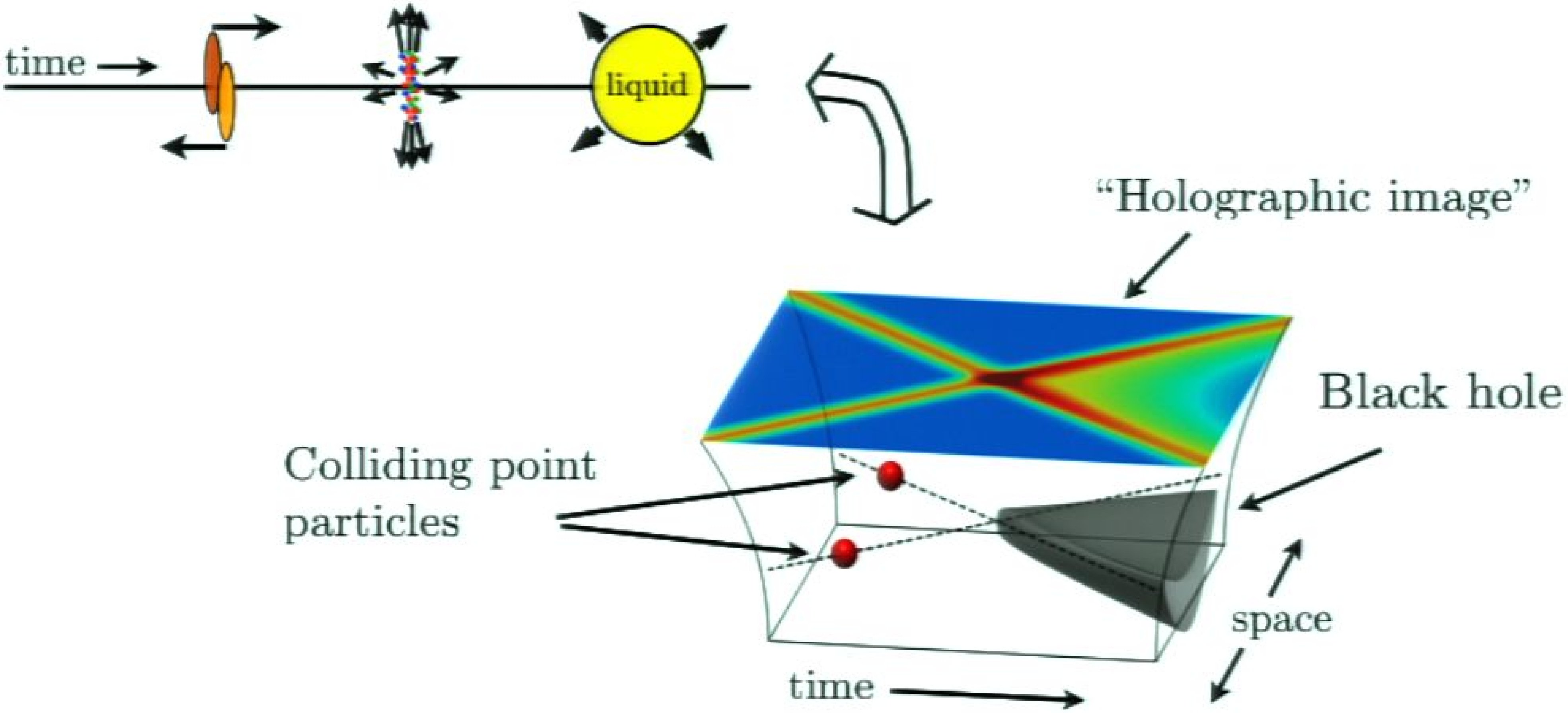
Understanding the physics behind rapid thermalization/isotropization is challenging!

The utility of holography

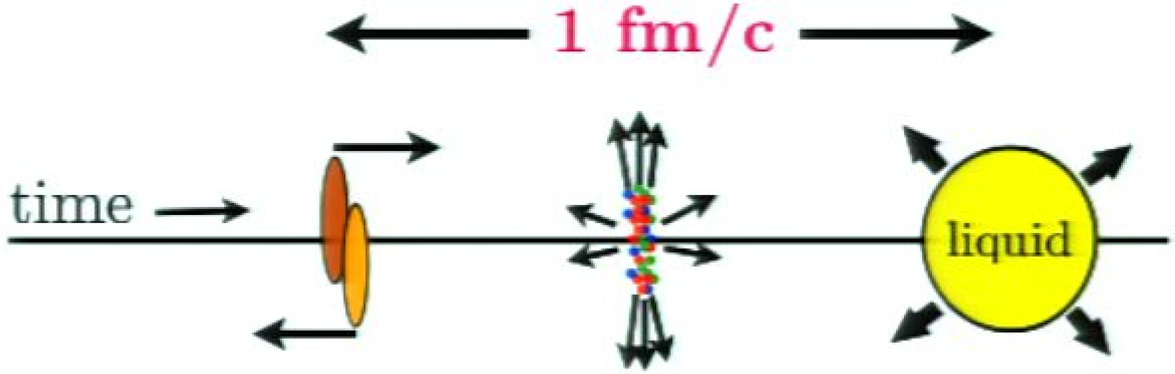
Strongly coupled QFT dynamics ⇔ classical gravity in $5d$.



All physics – from far from equilibrium dynamics to hydrodynamics – is contained in Einstein's equations.

Approachable questions within holography

- What are far from equilibrium relaxation mechanisms and times?



- How are initial conditions imprinted on hydro evolution?

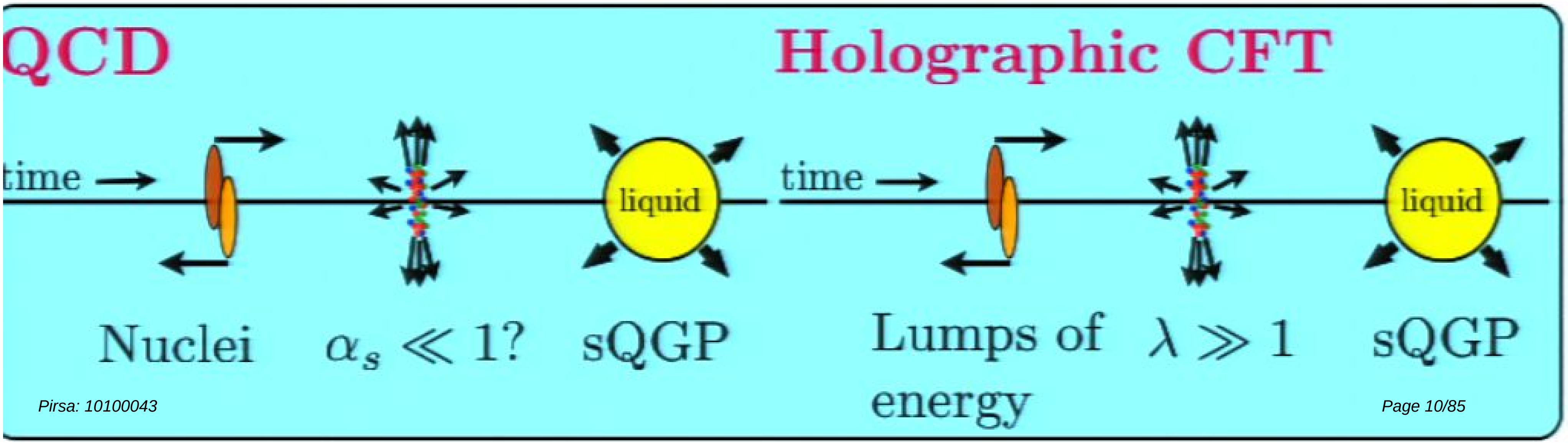
- What does the hydrodynamic evolution look like?

Why lie when you can omit

No known holographic description of QCD.

Best holography can currently offer:

- Theoretical access to QCD-like theories.
  - Mimics dynamics of strongly coupled QGP.
  - Omits asymptotic freedom.
- Simplest example: **Conformal Field Theories.**



QCD

Holographic CFT

The ground state and AdS_5

State $|\psi\rangle$ in QFT $\Leftrightarrow$ classical field configuration $\{\Phi_i\}$ in gravitational theory.

Vacuum

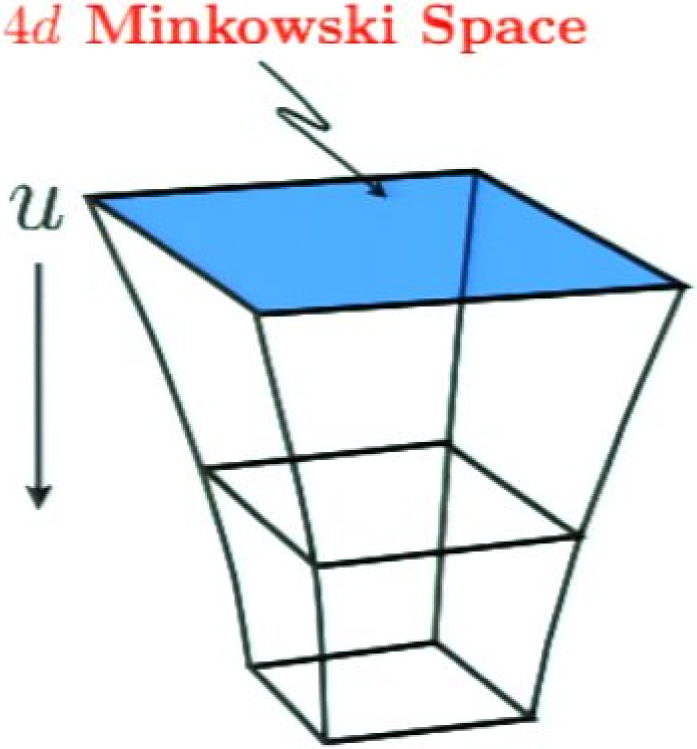
QFT

- $\langle 0|T_{\mu\nu}|0\rangle = 0,\ \langle 0|J^{\mu}|0\rangle = 0, \ldots,$

Gravitational description

- Einstein: $R_{MN} - \frac{1}{2}G_{MN}(R + 2\Lambda) = 0.$
- Most symmetric solution: AdS_5,

$$ds^2 = \frac{L^2}{u^2}\left[-dt^2 + d\boldsymbol{x}^2 + du^2\right].$$



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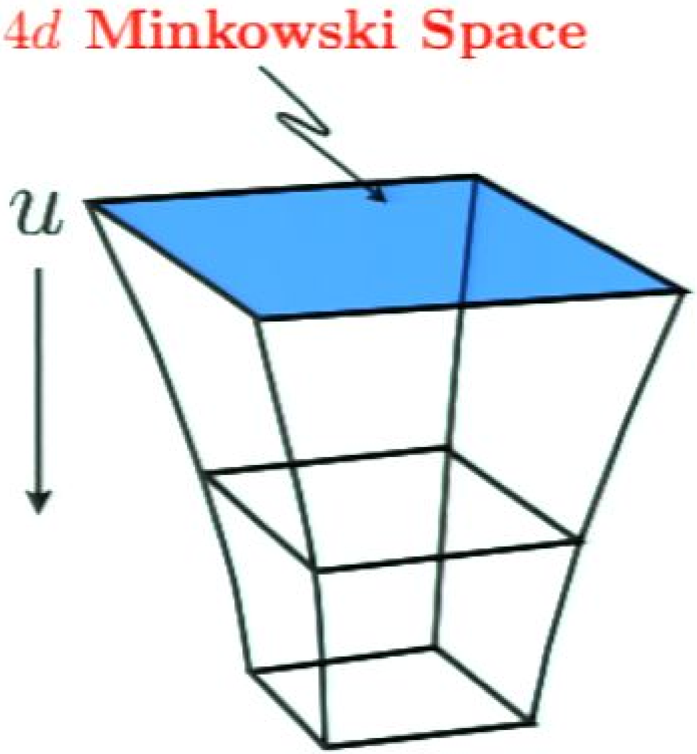
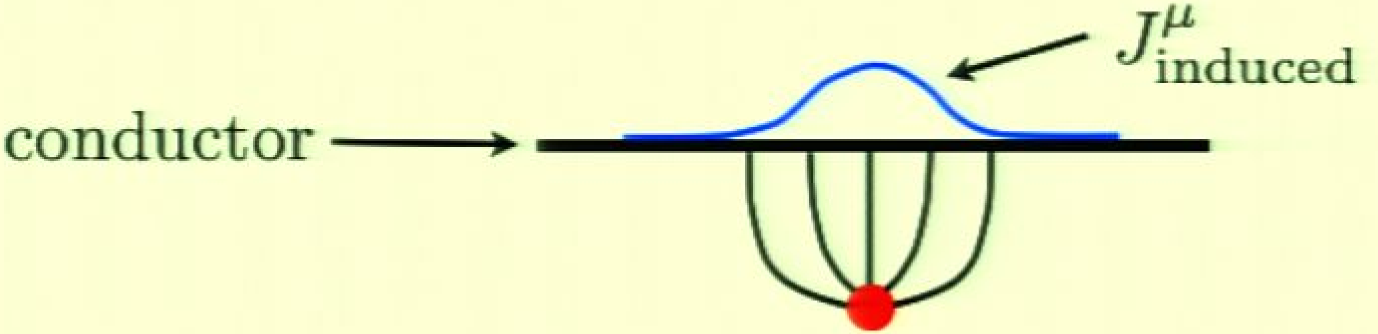


Deviations in geometry from AdS$_5$ ⇔ excited states in QFT

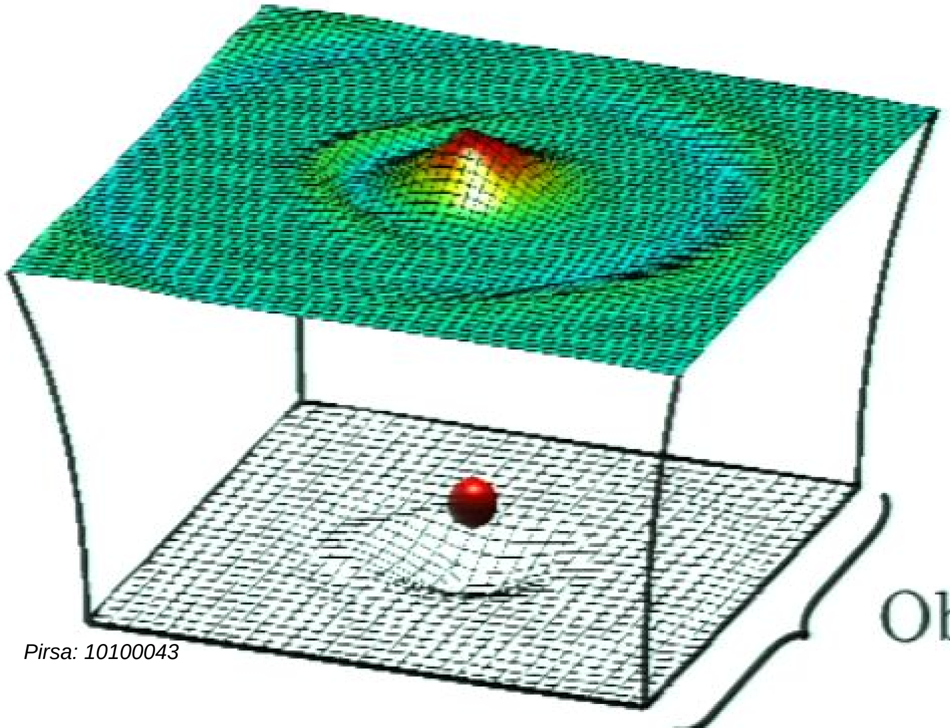
Excited states and expectation values

Image problem from electrodynamics



conductor

J^{μ}_{induced}



“Image” $T^{\mu\nu}$

[de Haro, Solodukhin & Skenderis: hep-th/0002230]

Object deforms $5d$ geometry.

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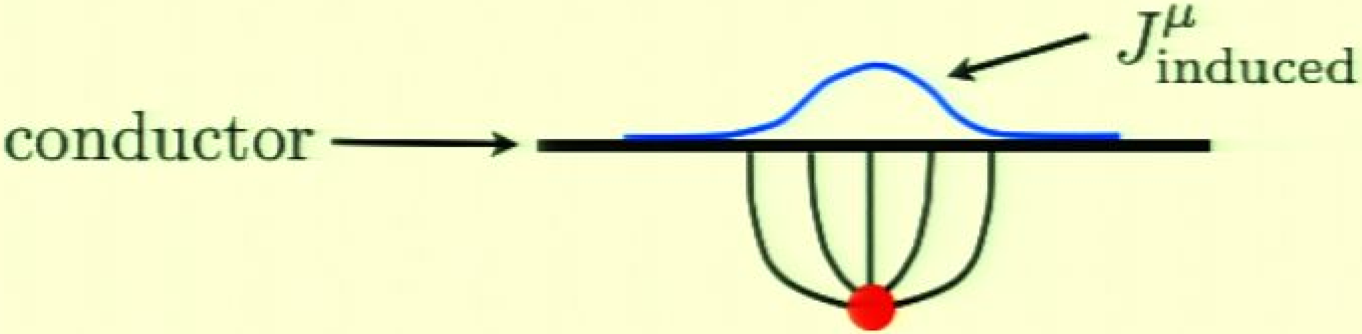

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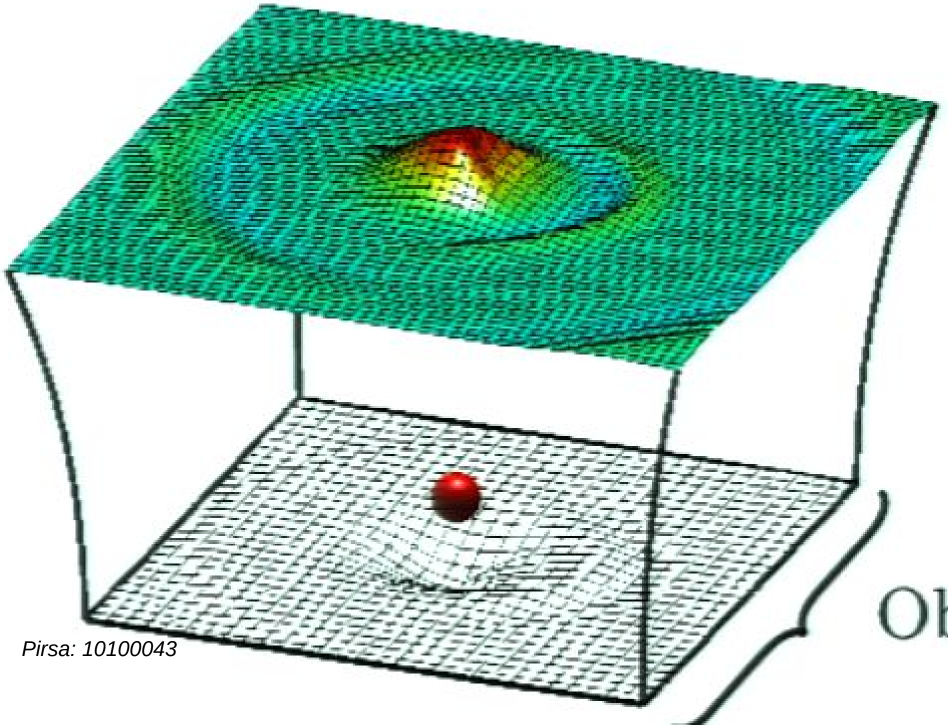

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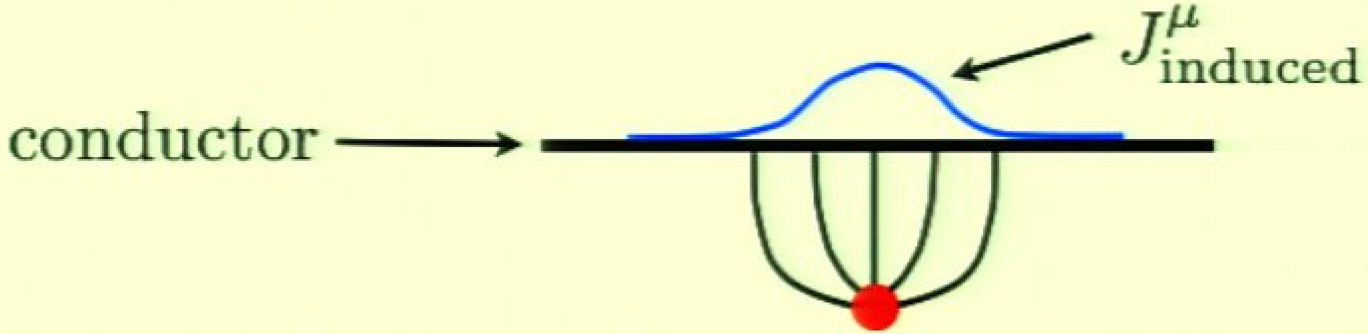

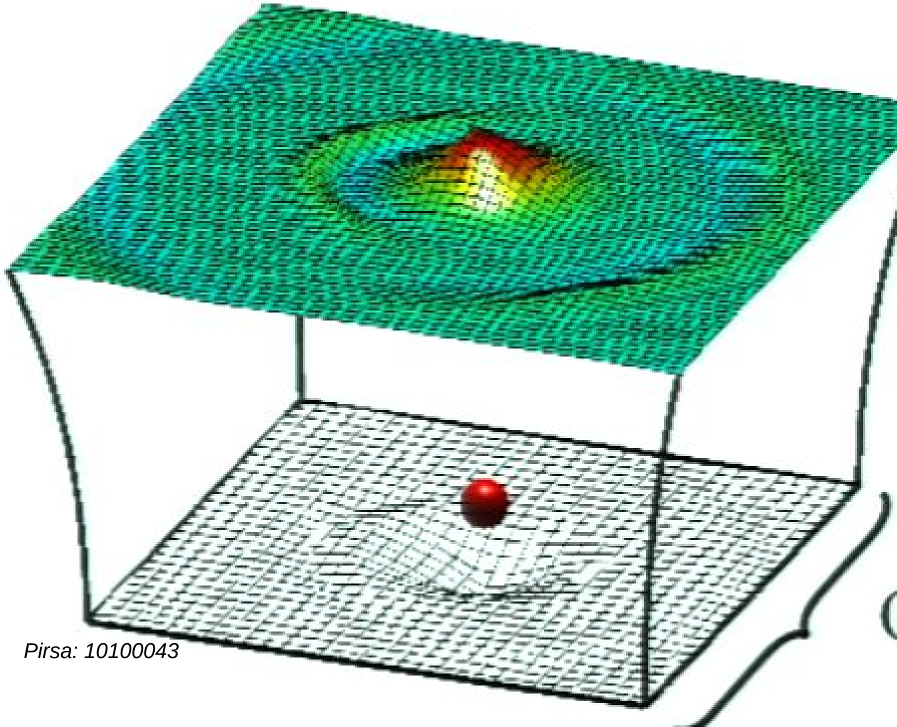

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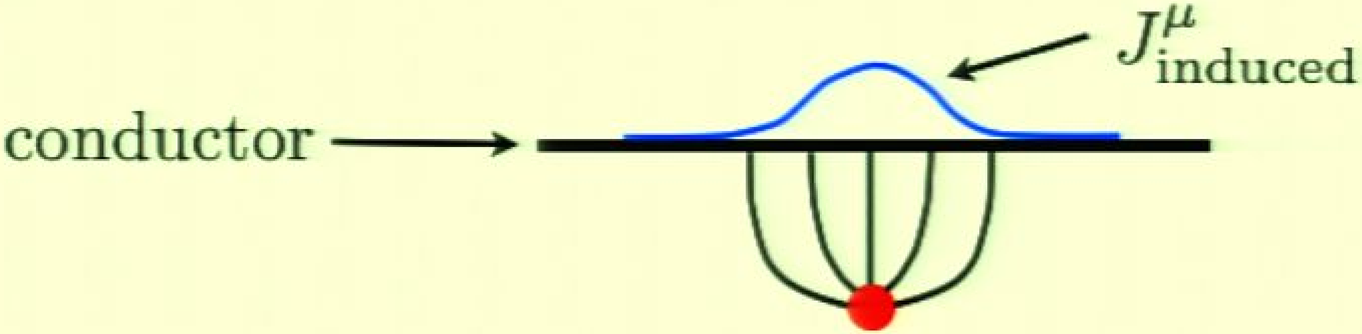

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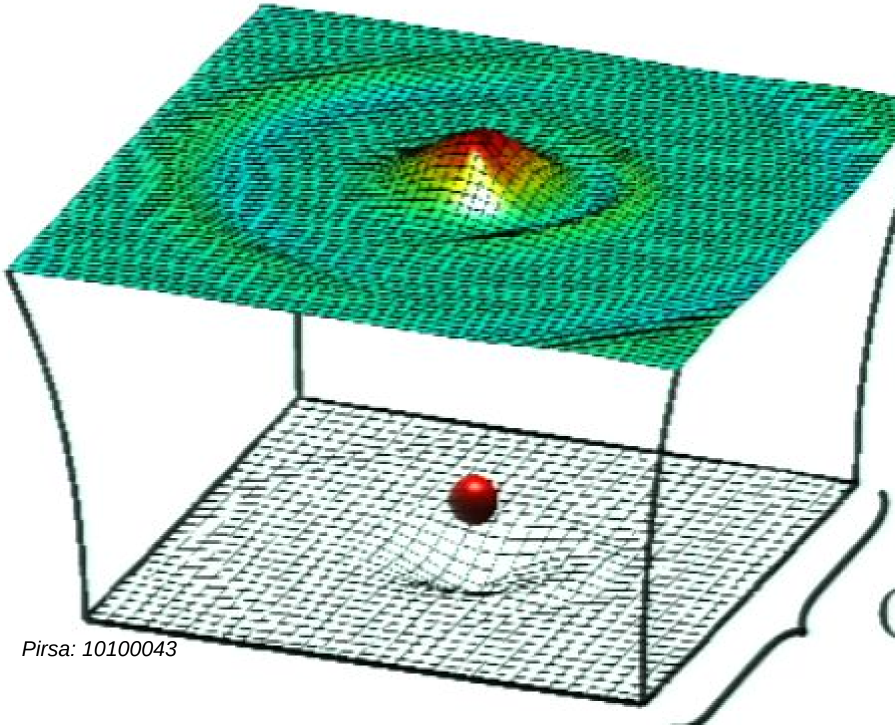

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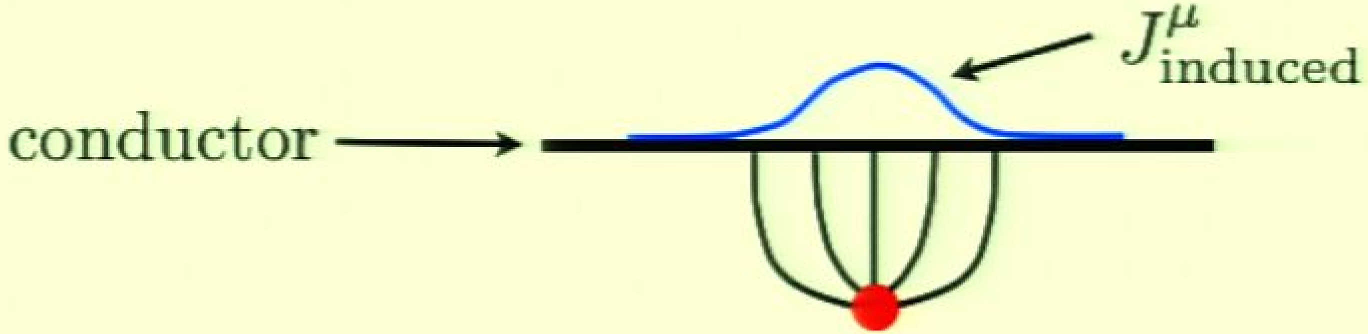

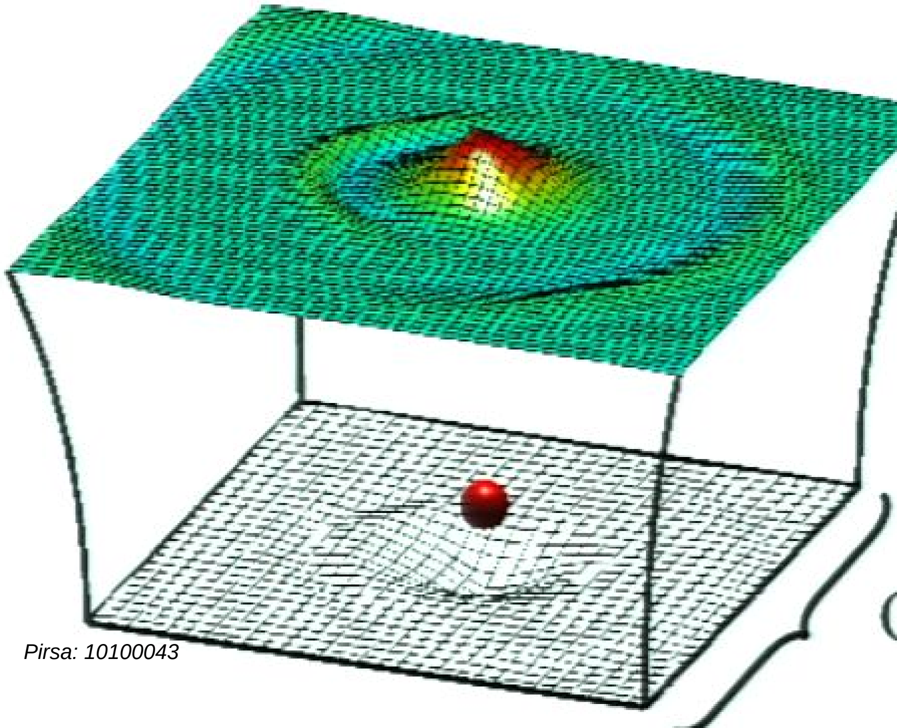

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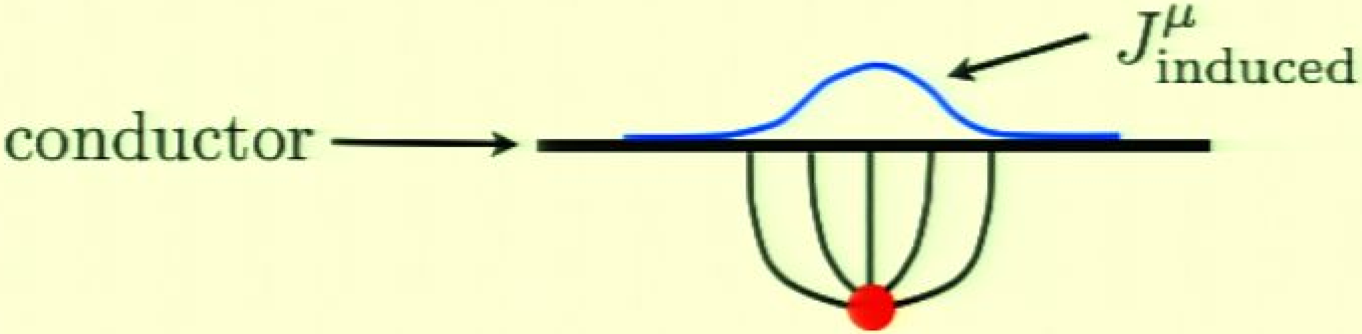

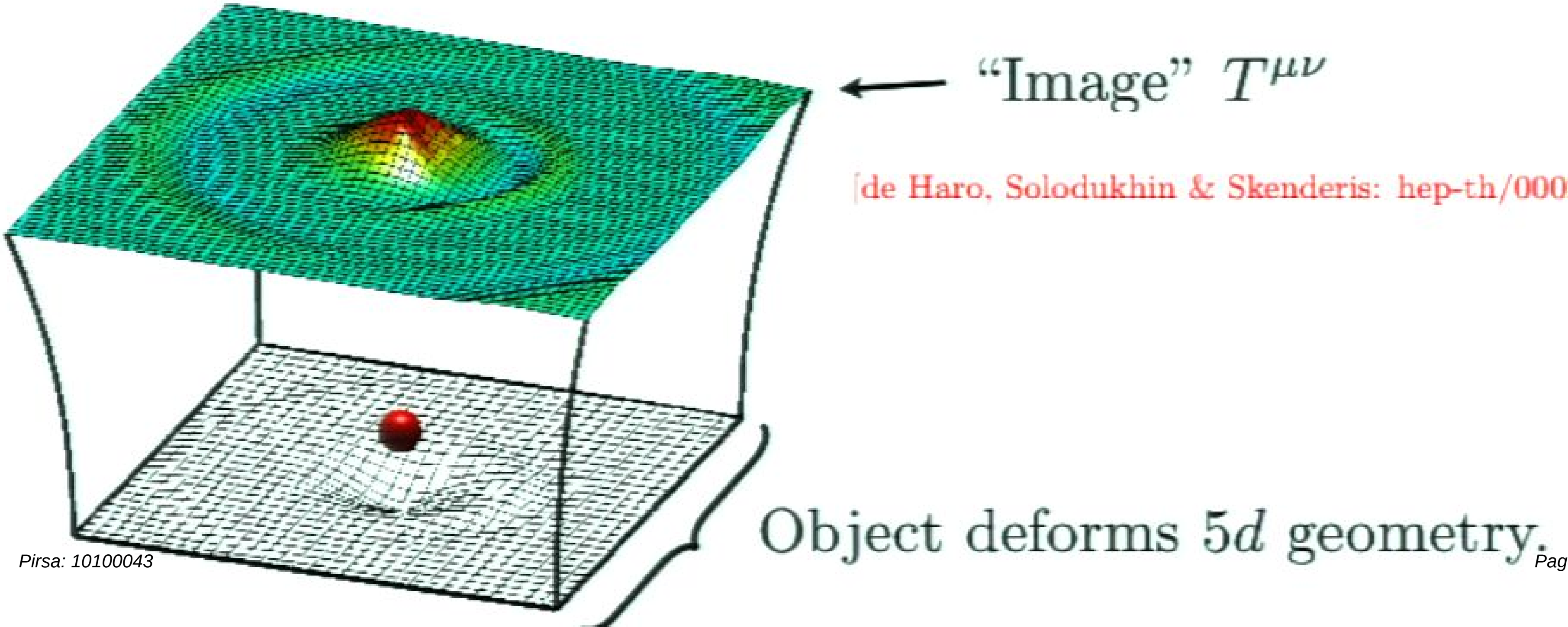

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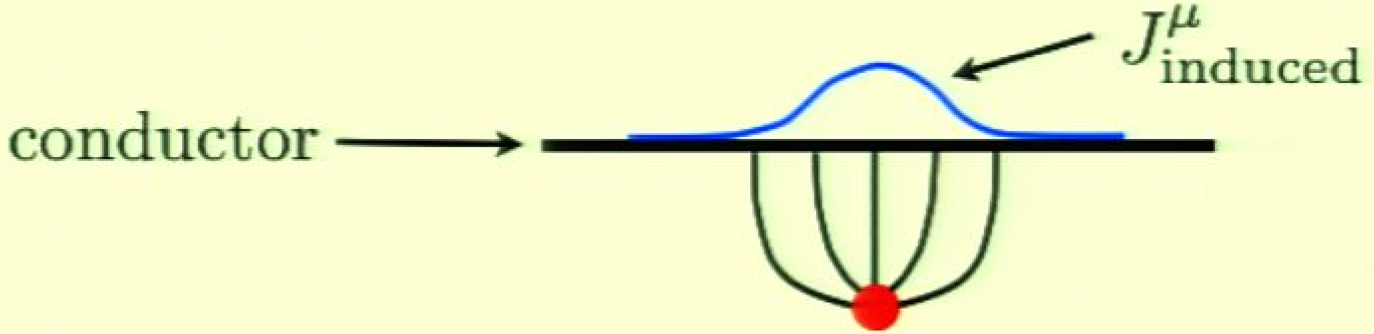

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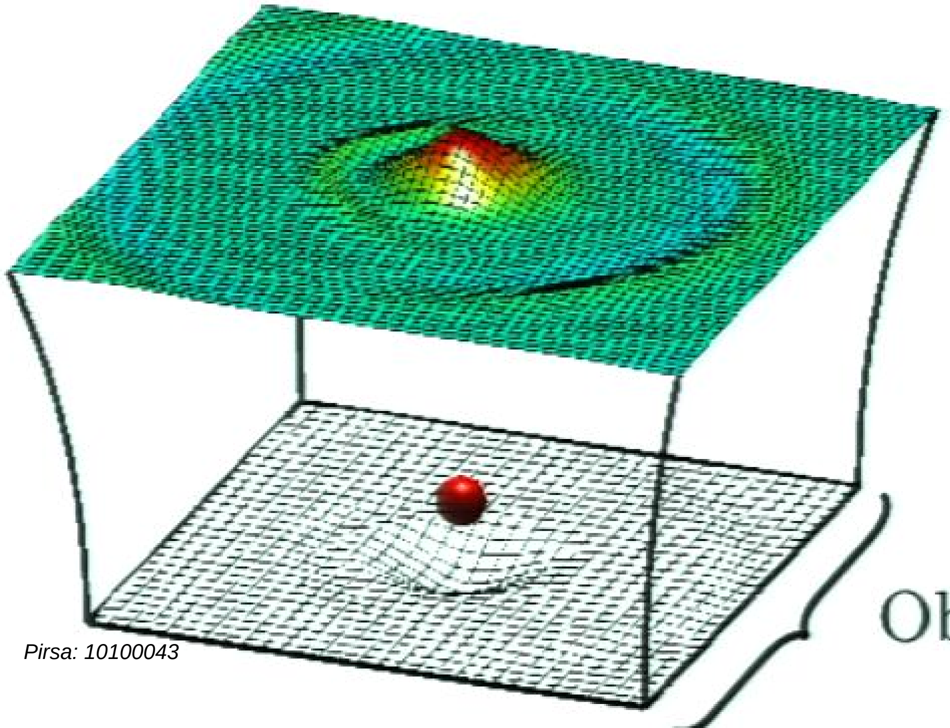

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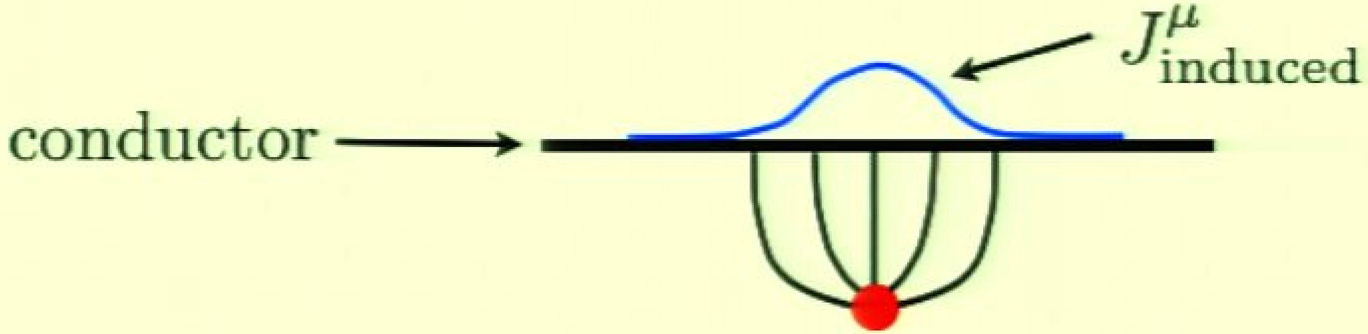

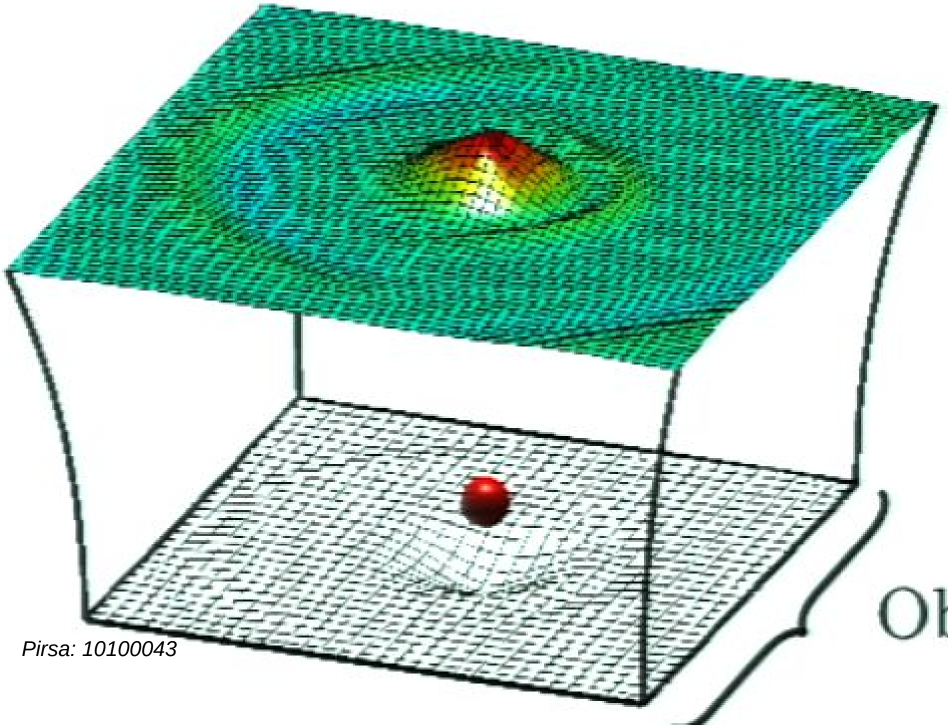

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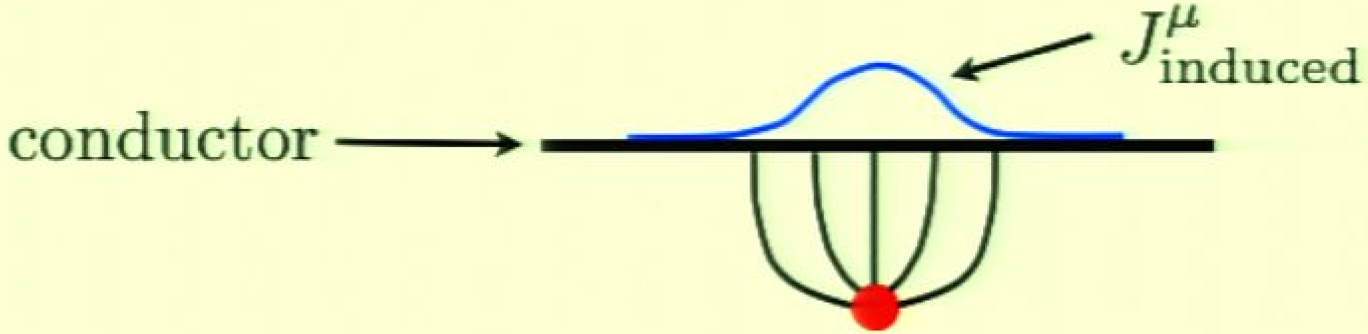

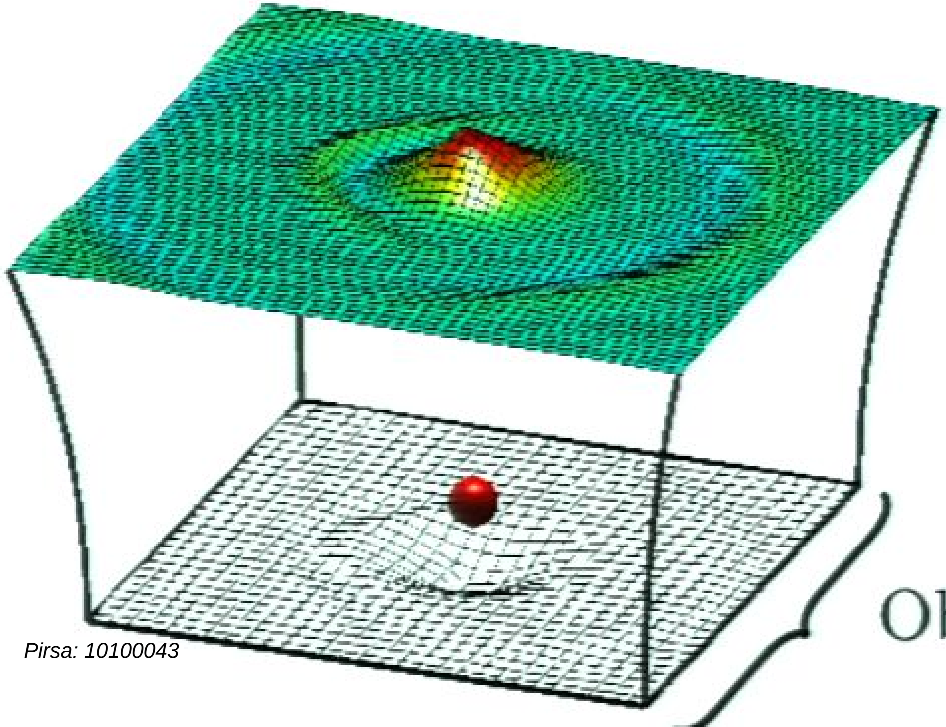

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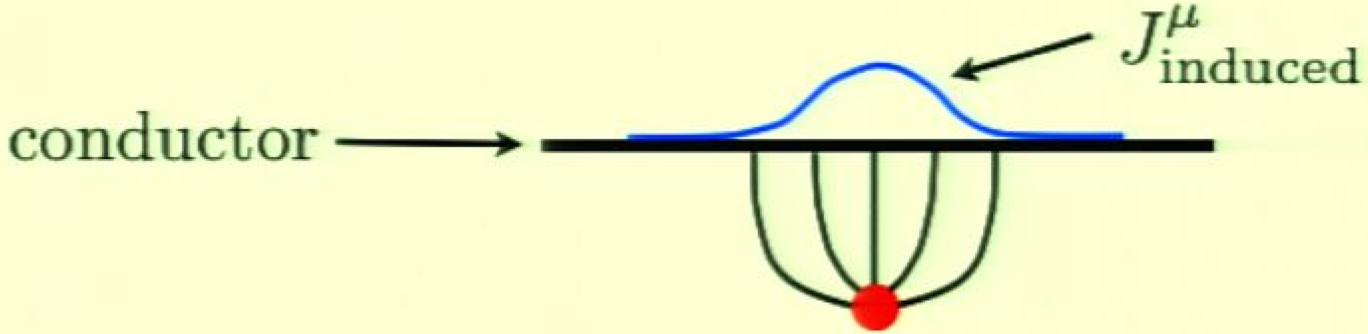

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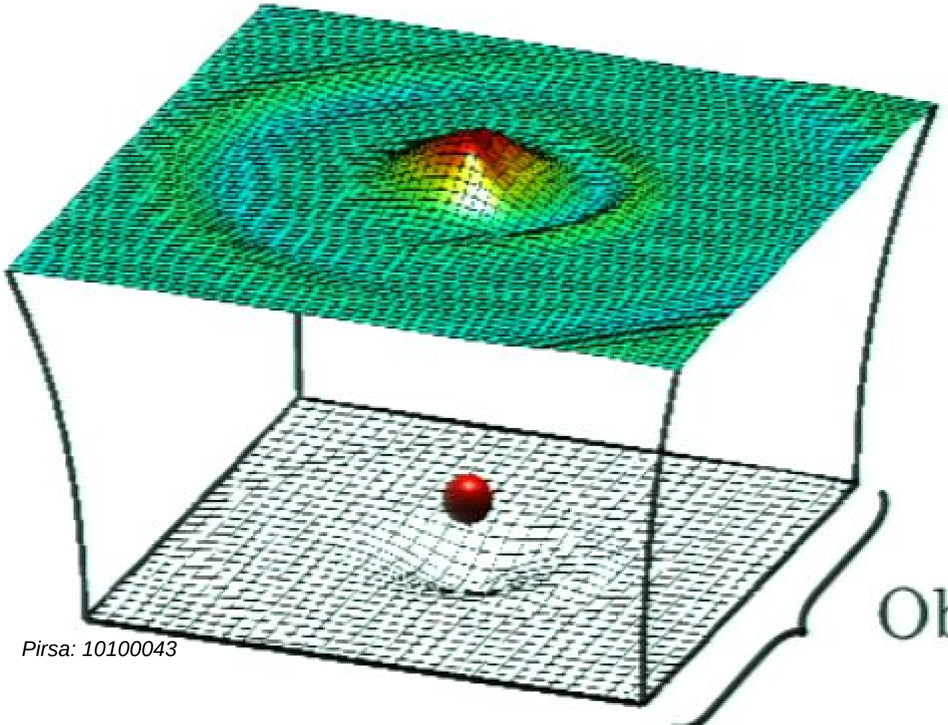

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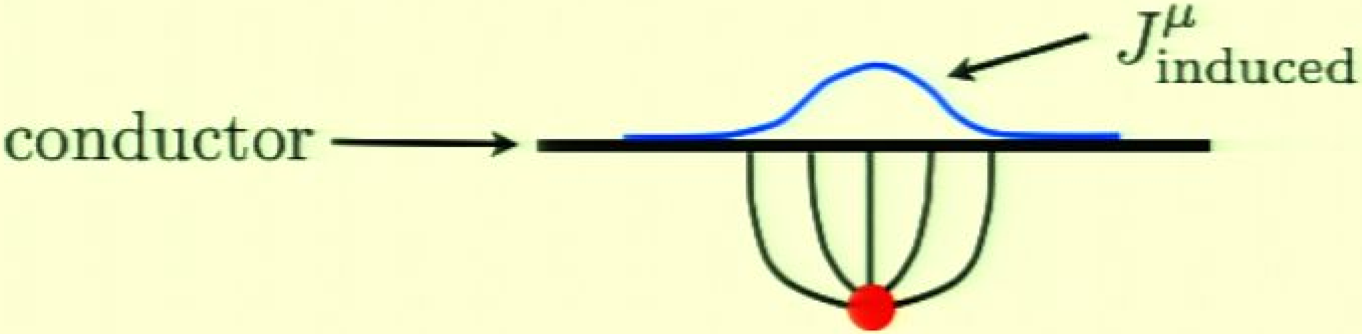

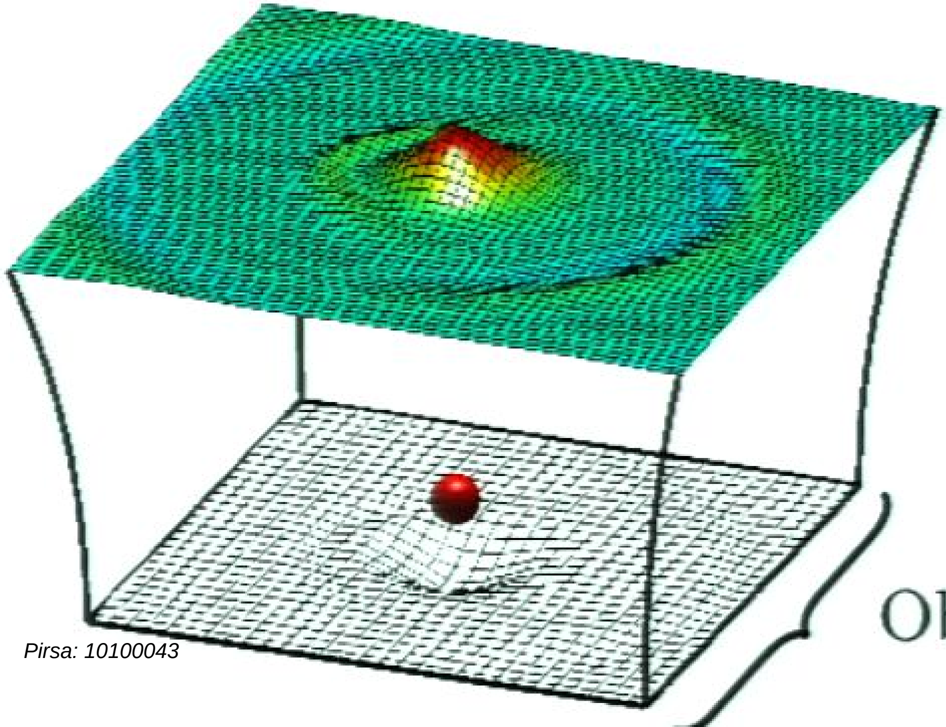

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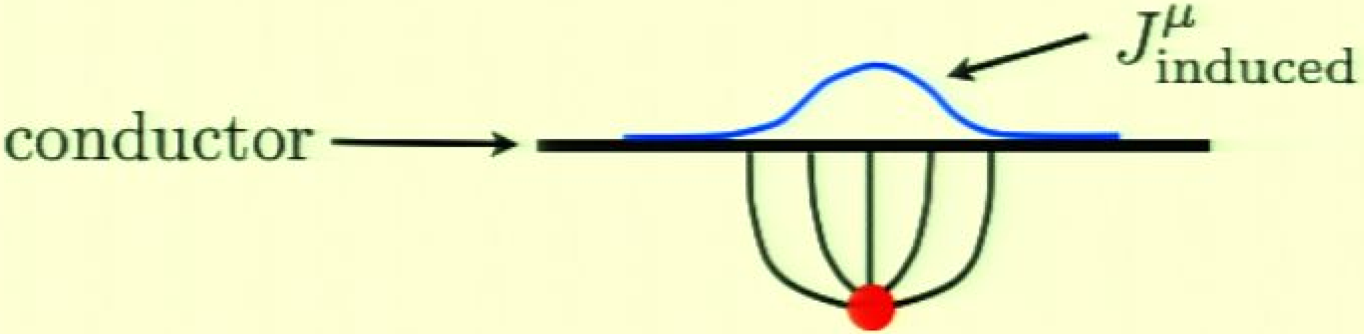

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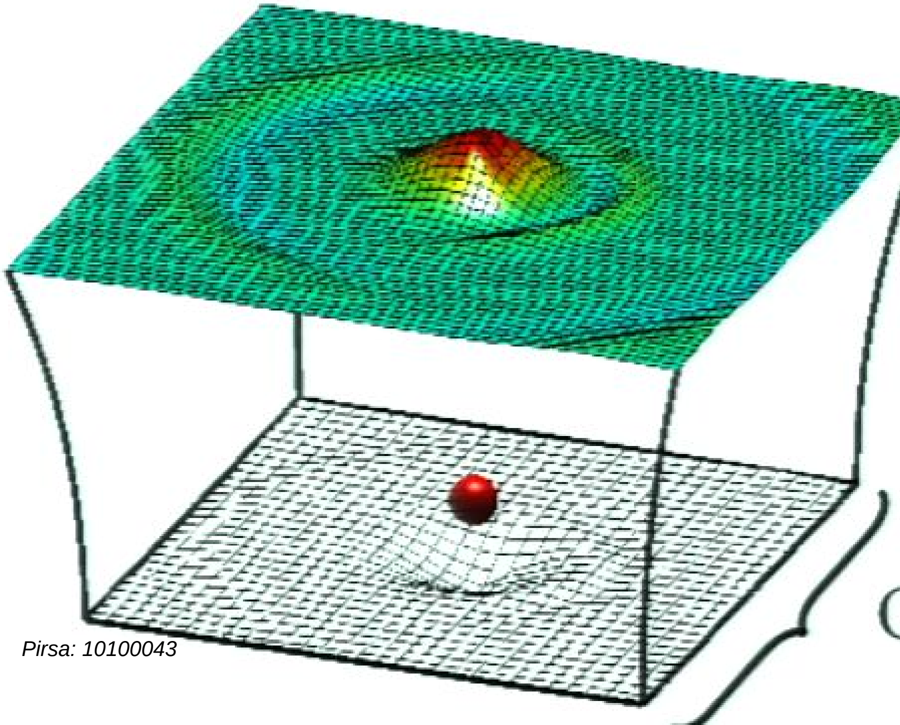

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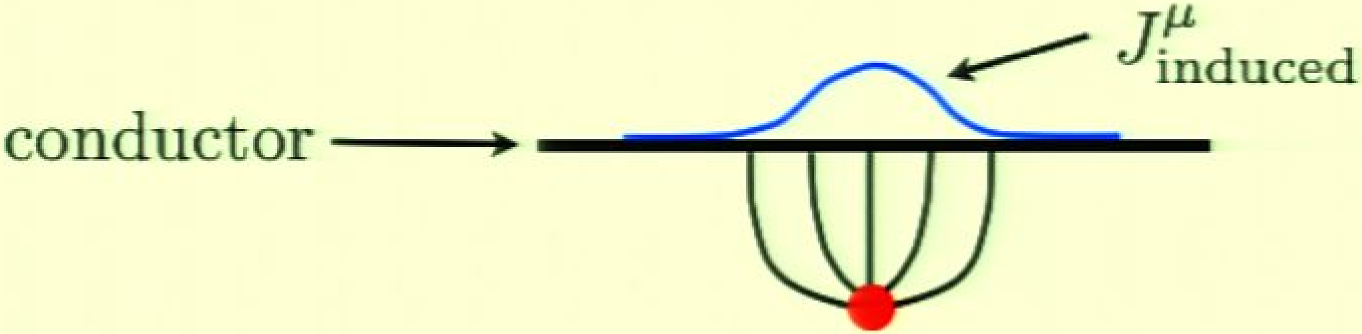

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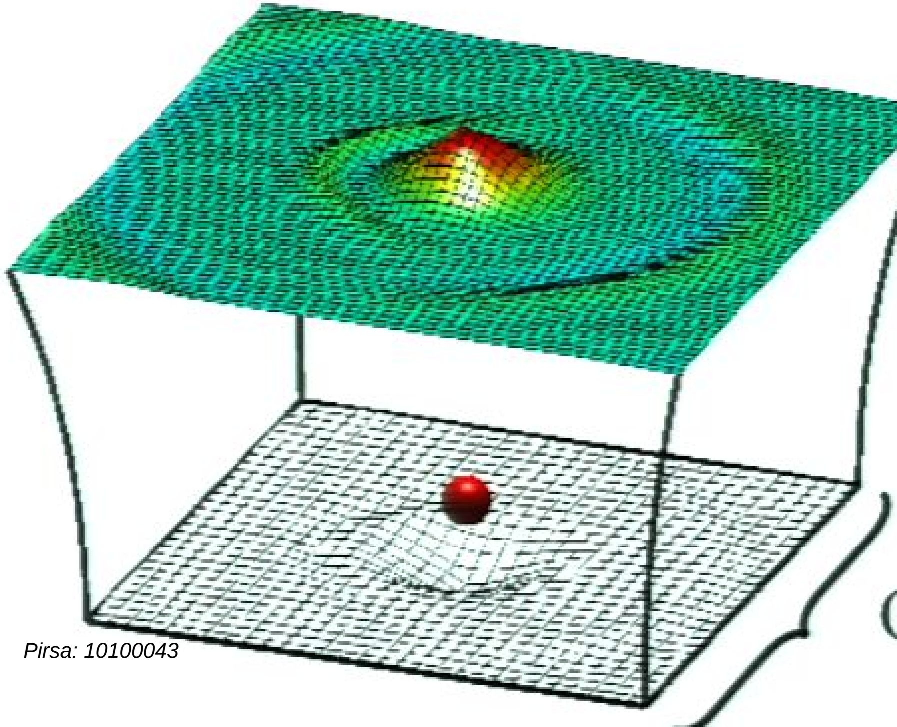

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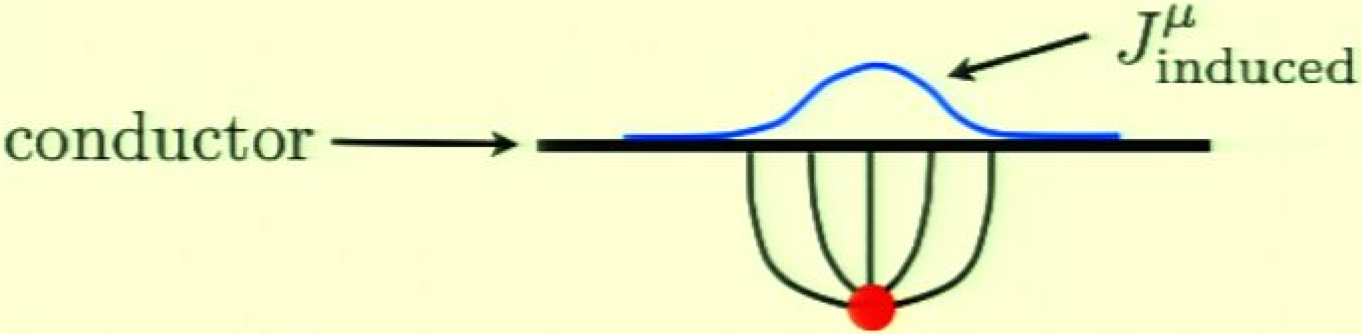

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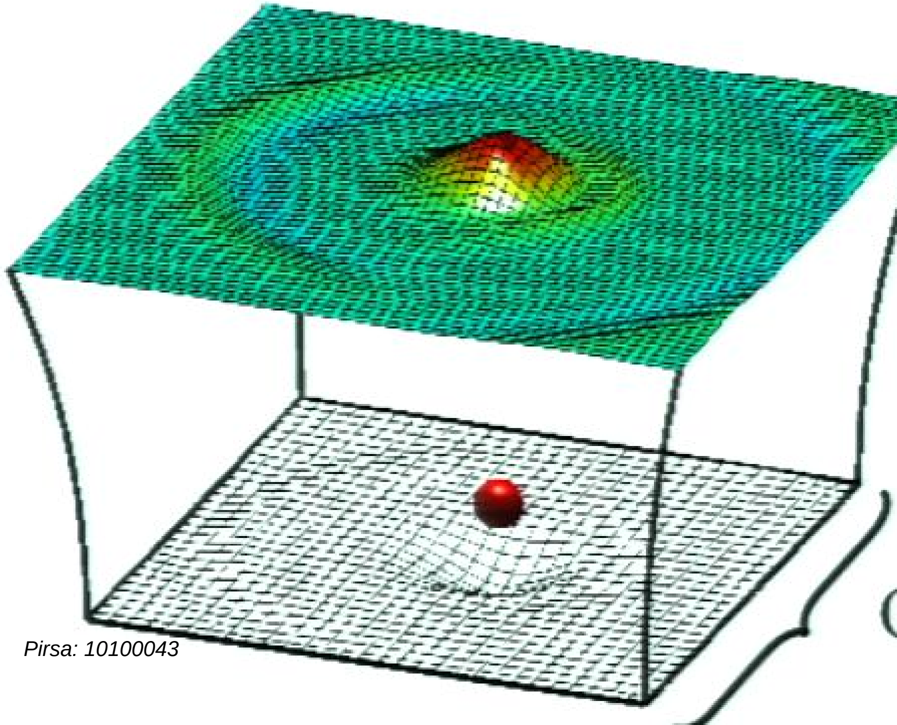

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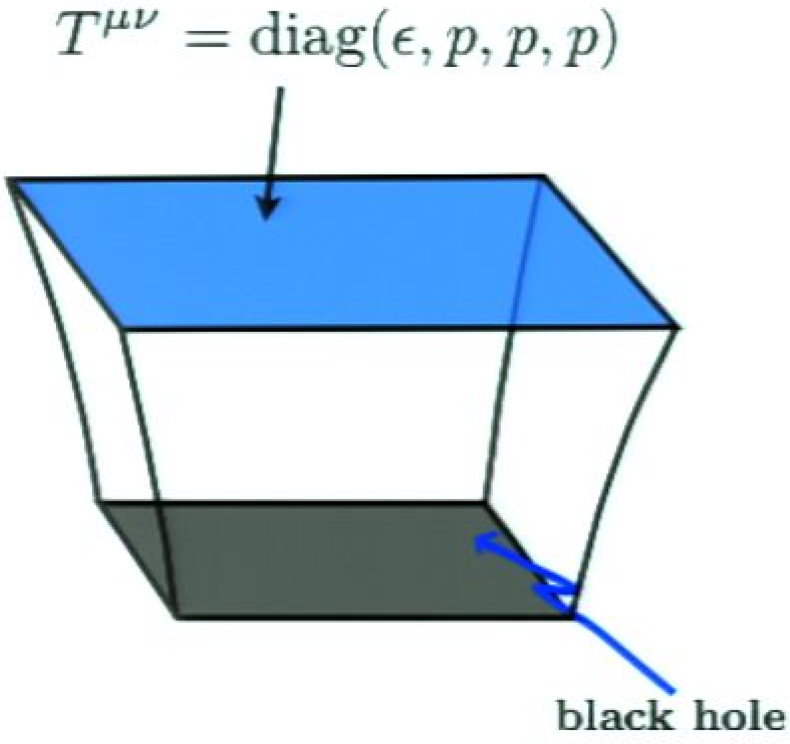
What should the geometry corresponding to a liquid look like?

Metric: AdS-Schwarzchild geometry

$$ds^2 = \frac{L^2}{u^2}\left[-f(u)dt^2 + dx^2 + \tfrac{du^2}{f(u)}\right],$$

where $f(u) = 1 - \frac{u^4}{u_h^4}$ with $u_h = 1/\pi T$.



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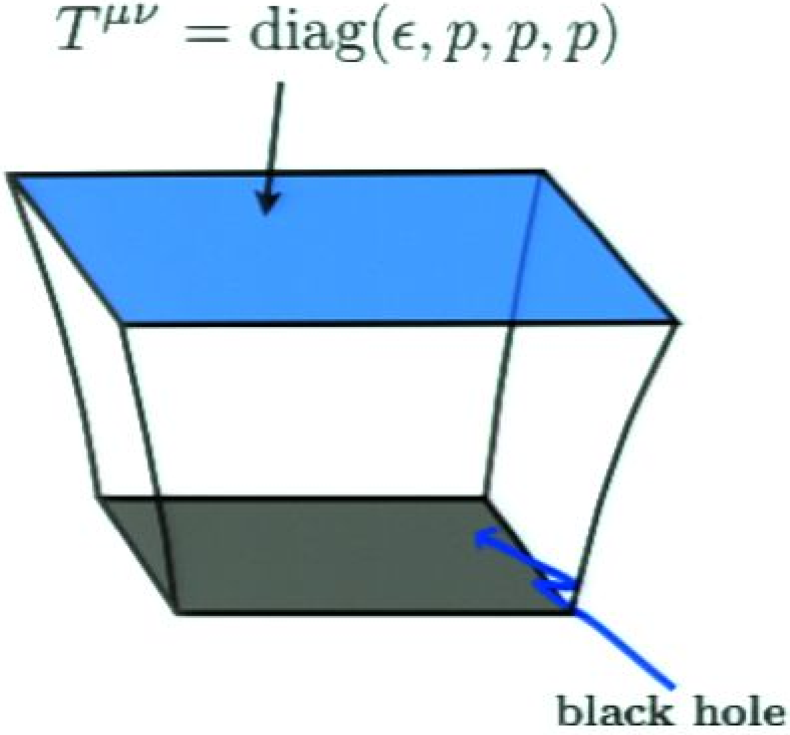

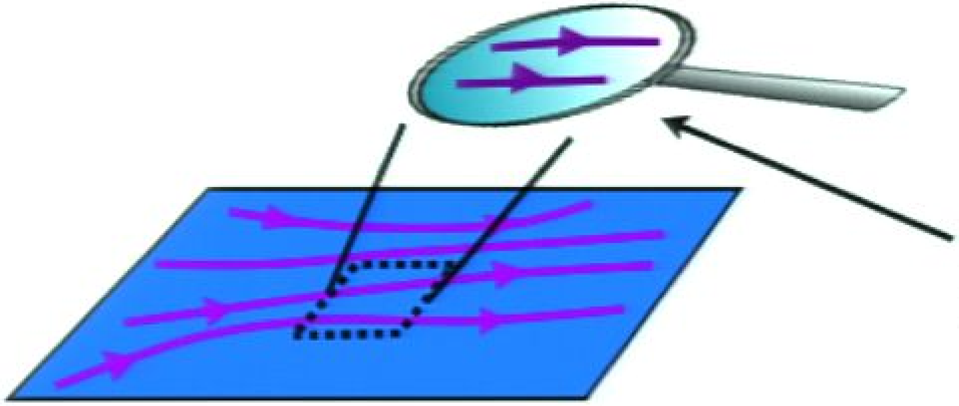

Stress in local rest frame:
$T^{\mu\nu} = \text{diag}(\epsilon, p, p, p)+$ gradients

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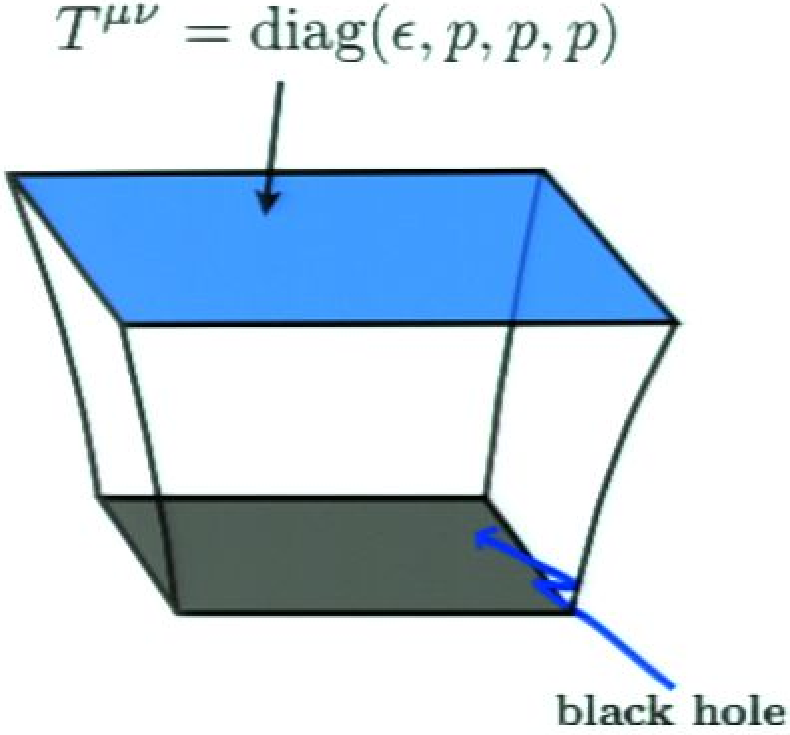
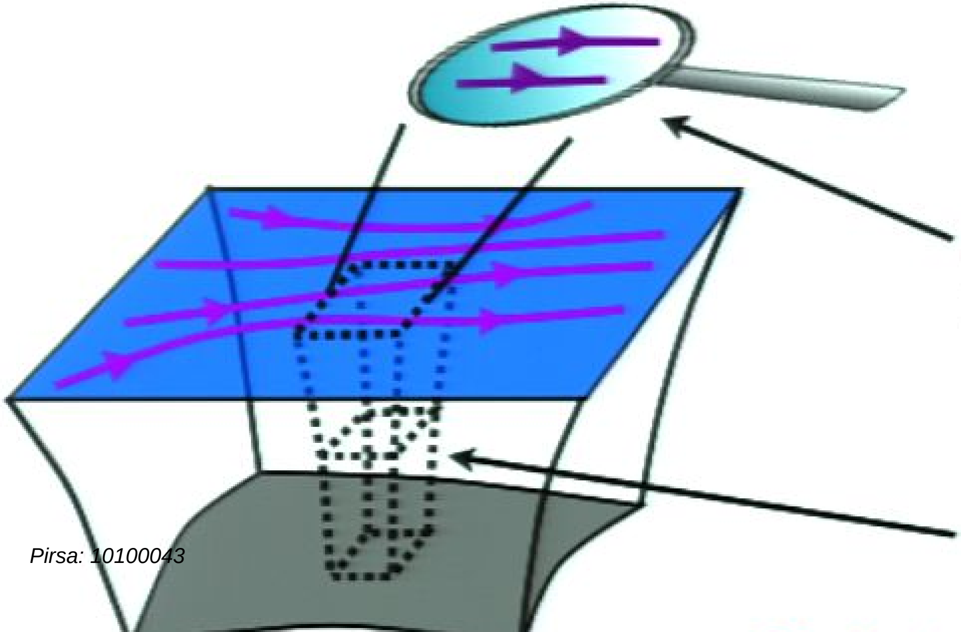
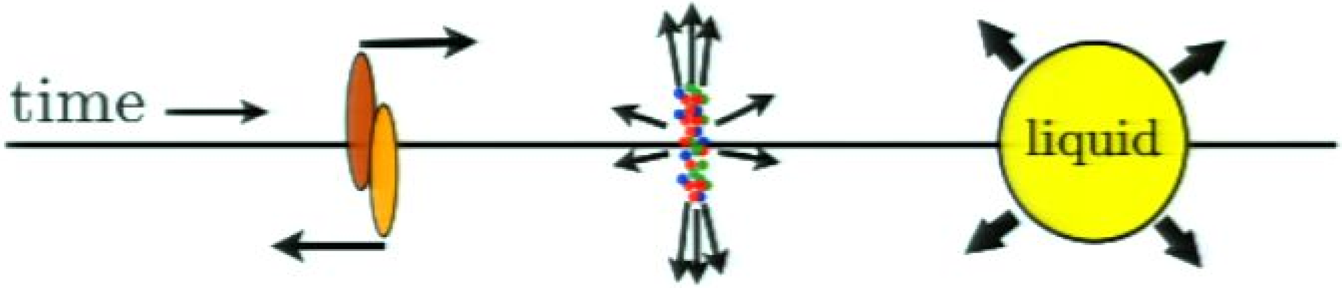




Stress in local rest frame:
$T^{\mu\nu} = \mathrm{diag}(\epsilon, p, p, p) +$ gradients

Geometry in local rest frame:
AdS-Schwarzchild + gradients

The destination: collision of particles in $5d$



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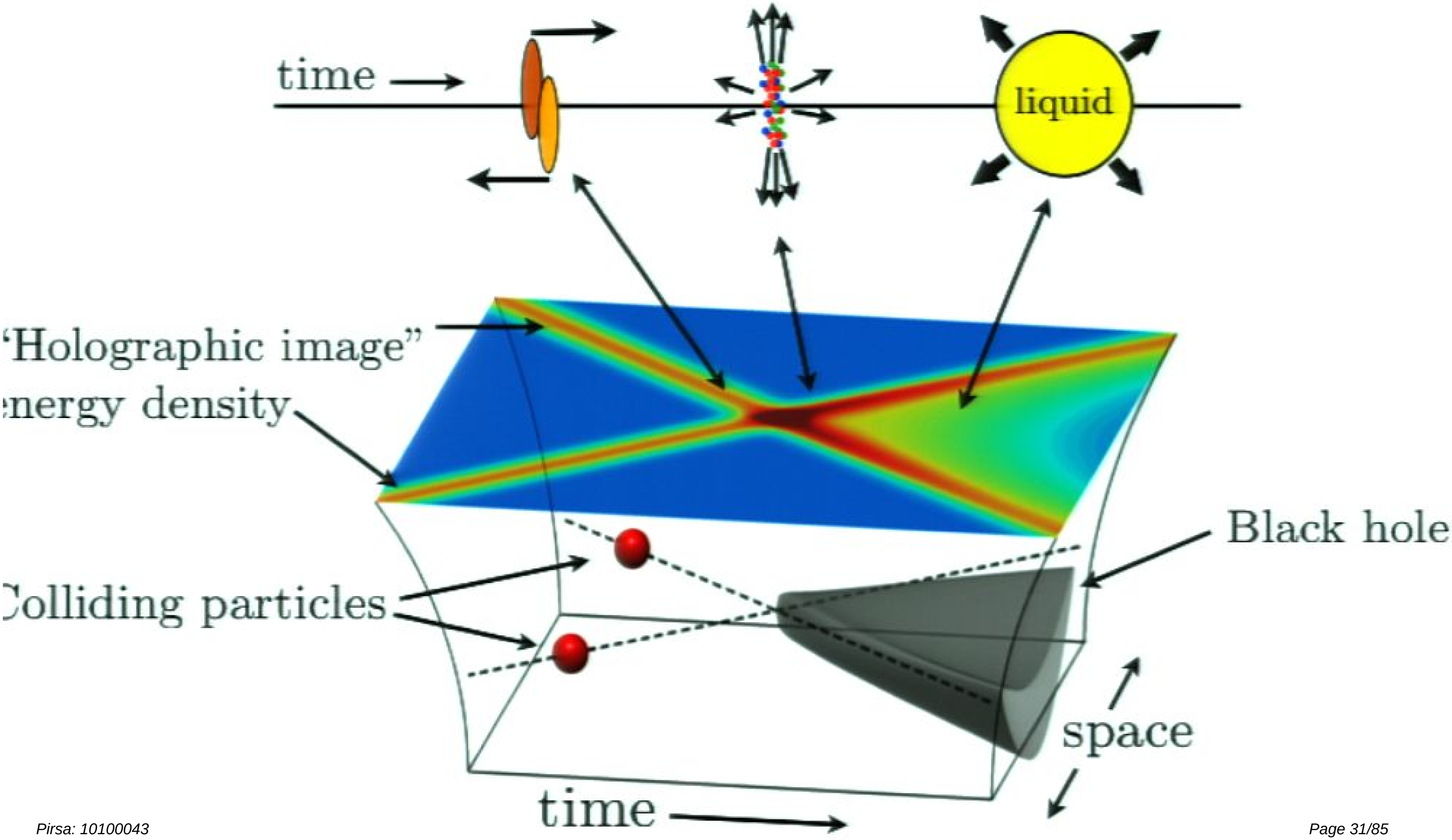

time

liquid

‘Holographic image”

nergy density

olliding particles

Black hole

space

time

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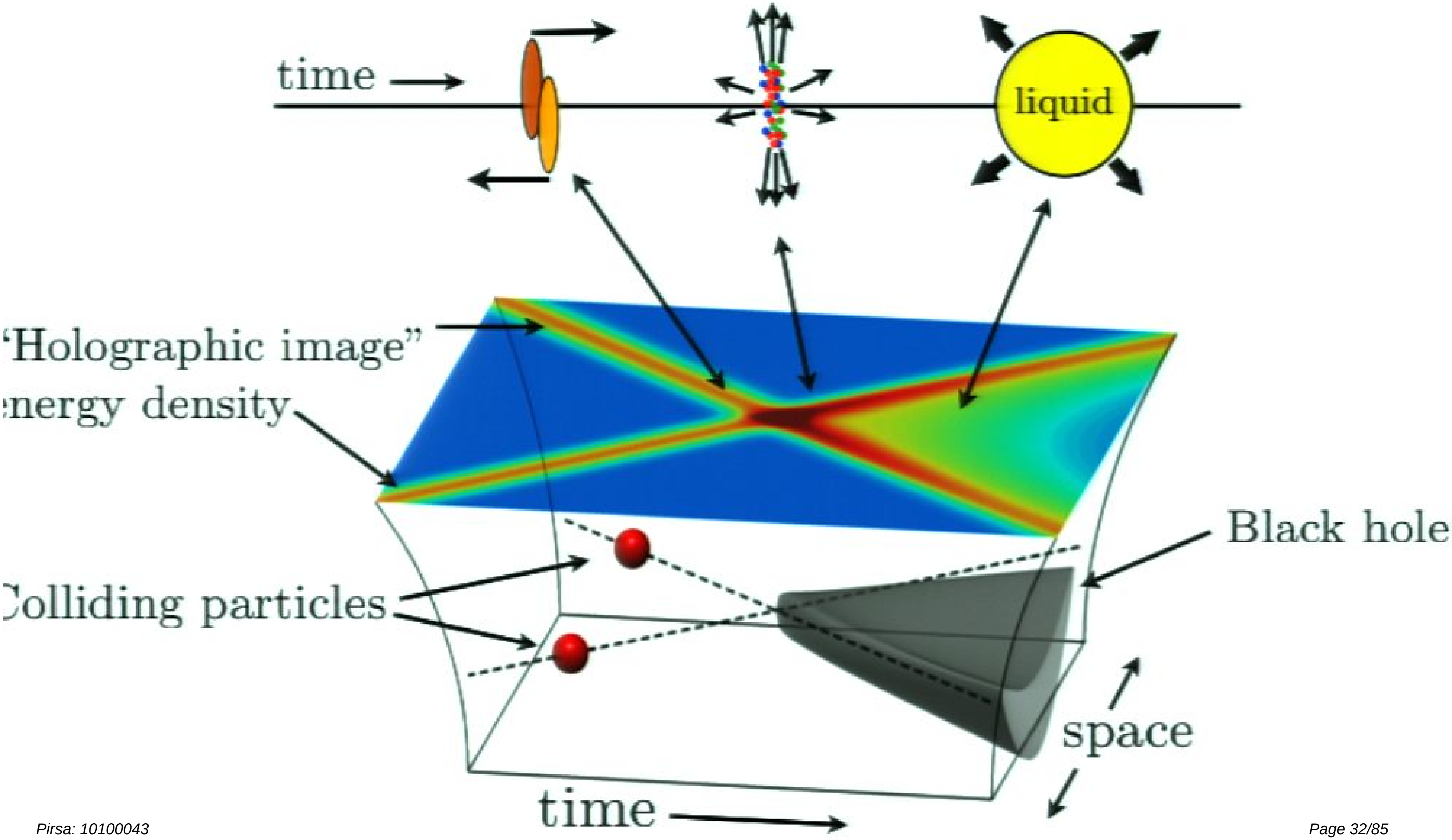


Collisions map into a hard numerical relativity problem ☹

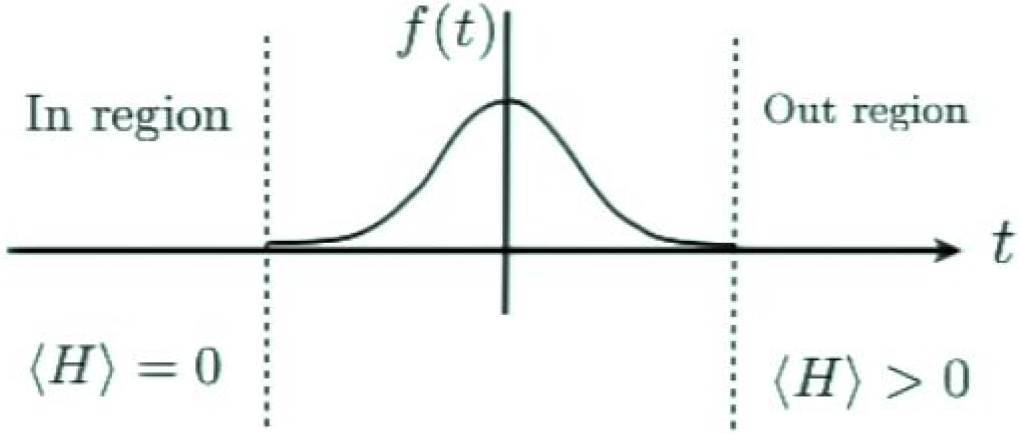
The journey: a simple method for preparing excited states

Recipe:

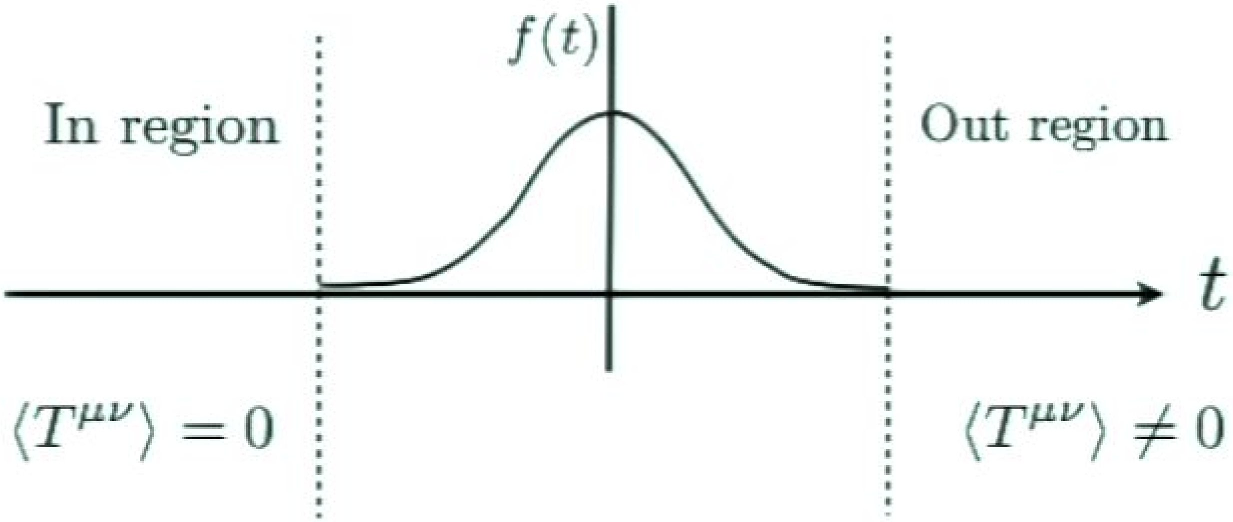
. Start in ground state $|0\rangle$.

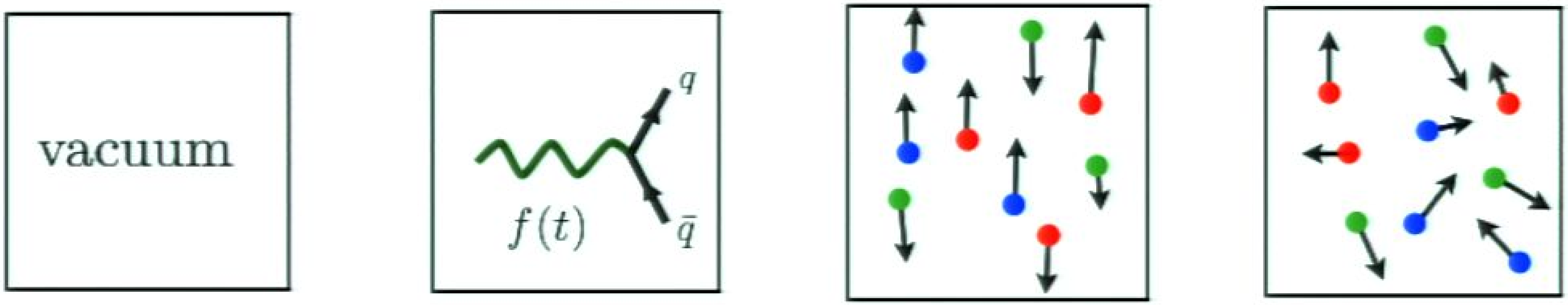
i. Turn on time-dependent background fields to produce excited state.

Example: harmonic oscillator $H = \frac{1}{2}p^2 + \frac{1}{2}\omega^2 x^2 - xf$



Creating excited states with gravity





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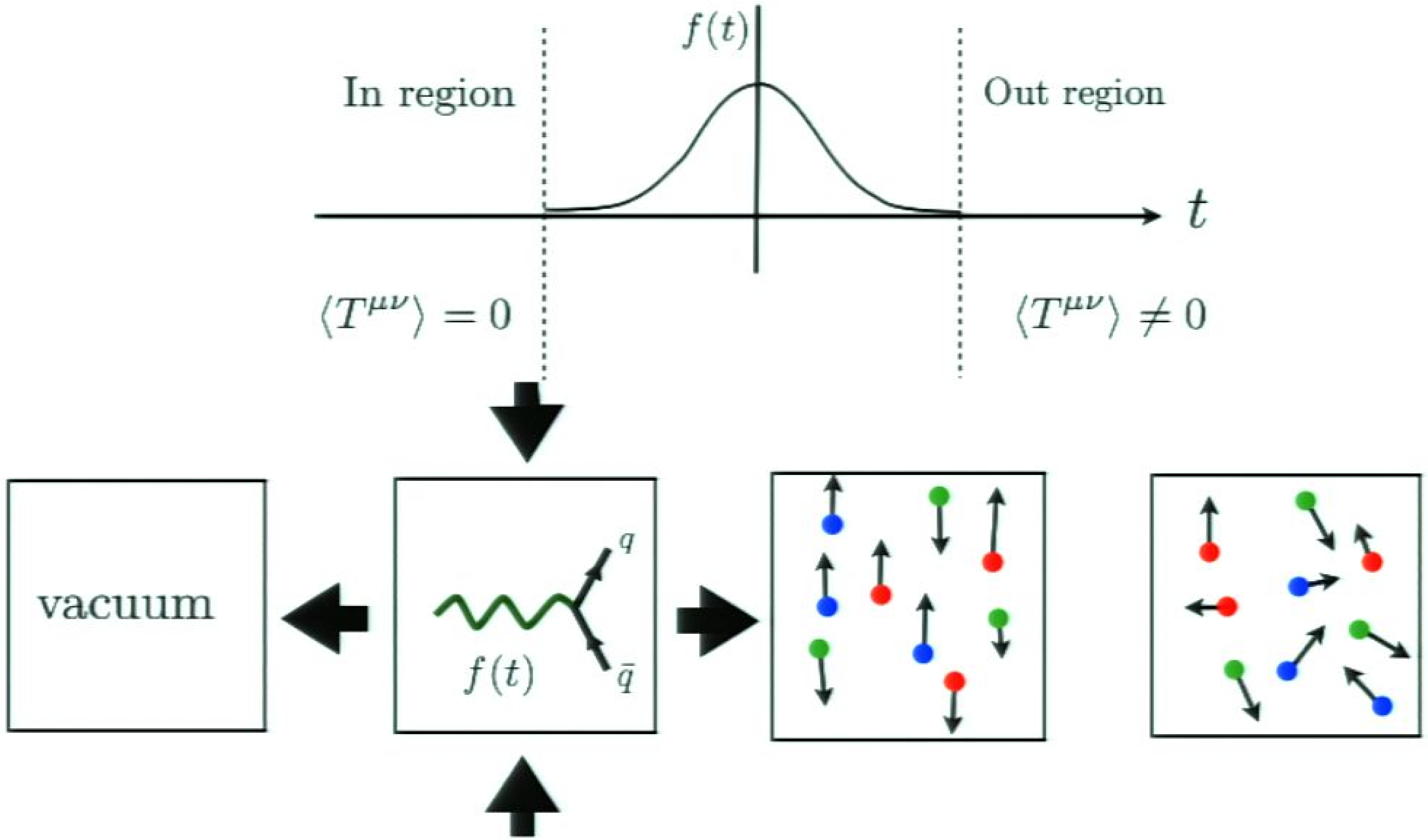
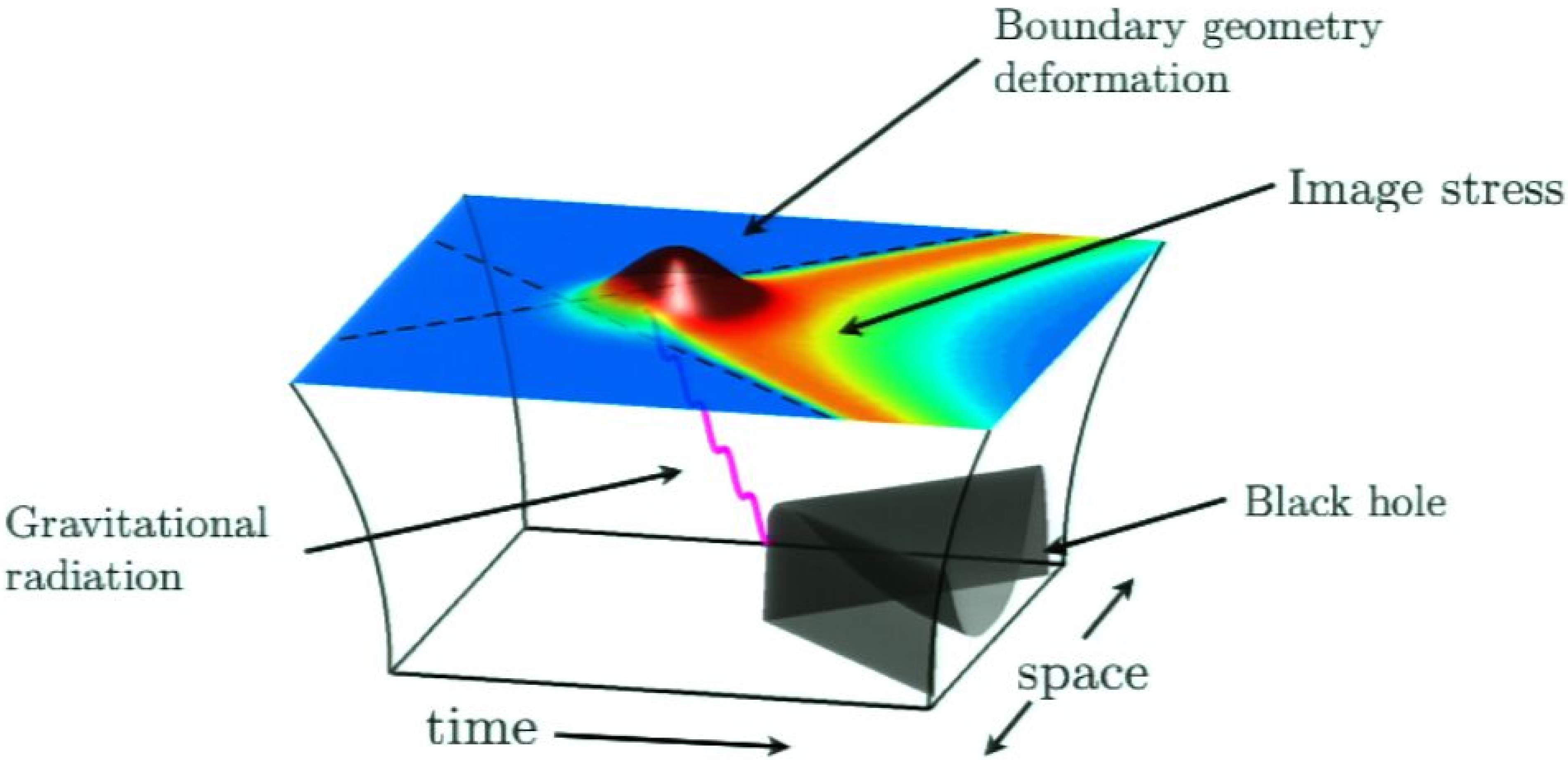


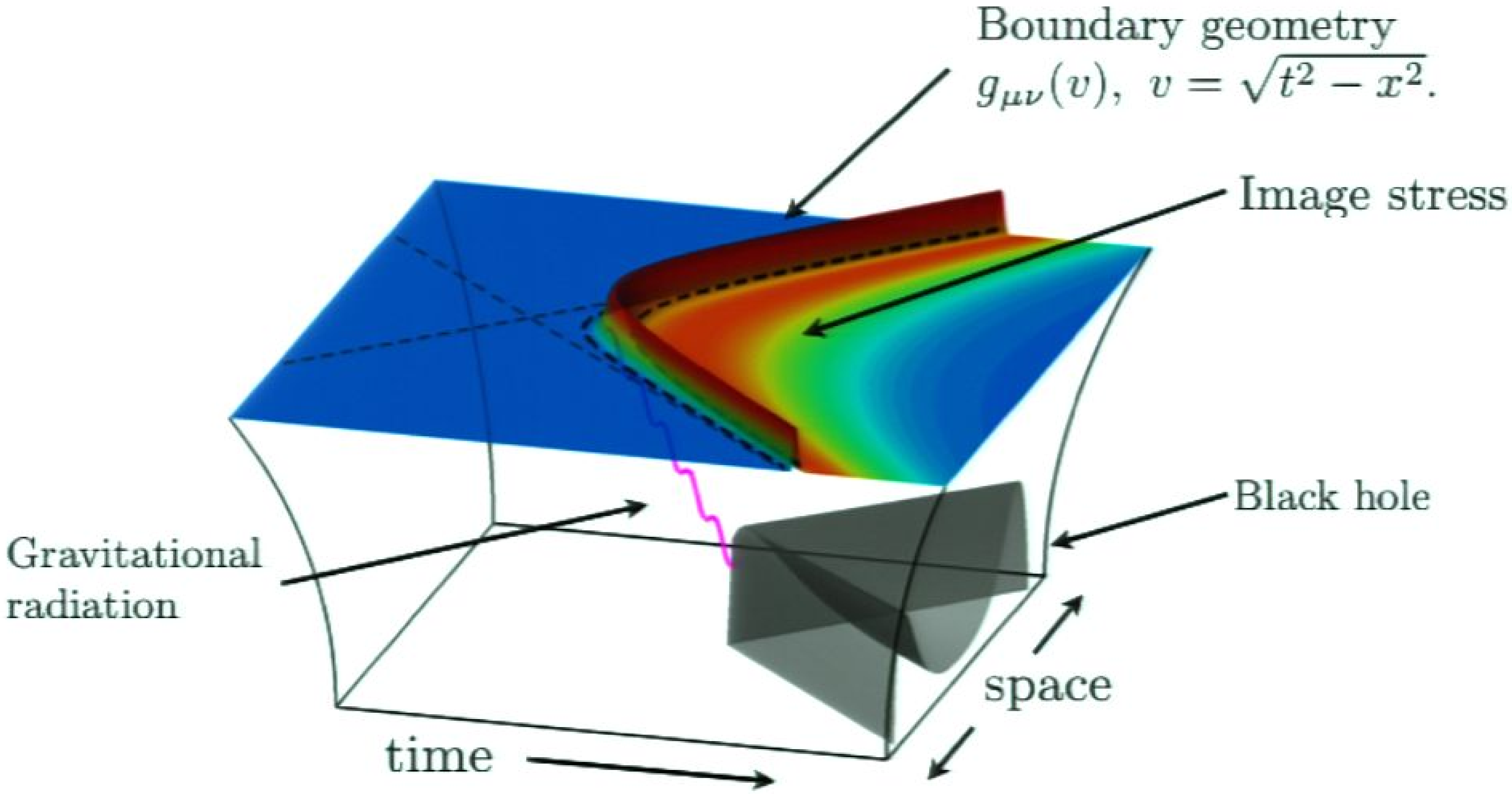
Simplest case: take $f(t)$ to be external gravitational field.

The journey: a simple way to create a black hole



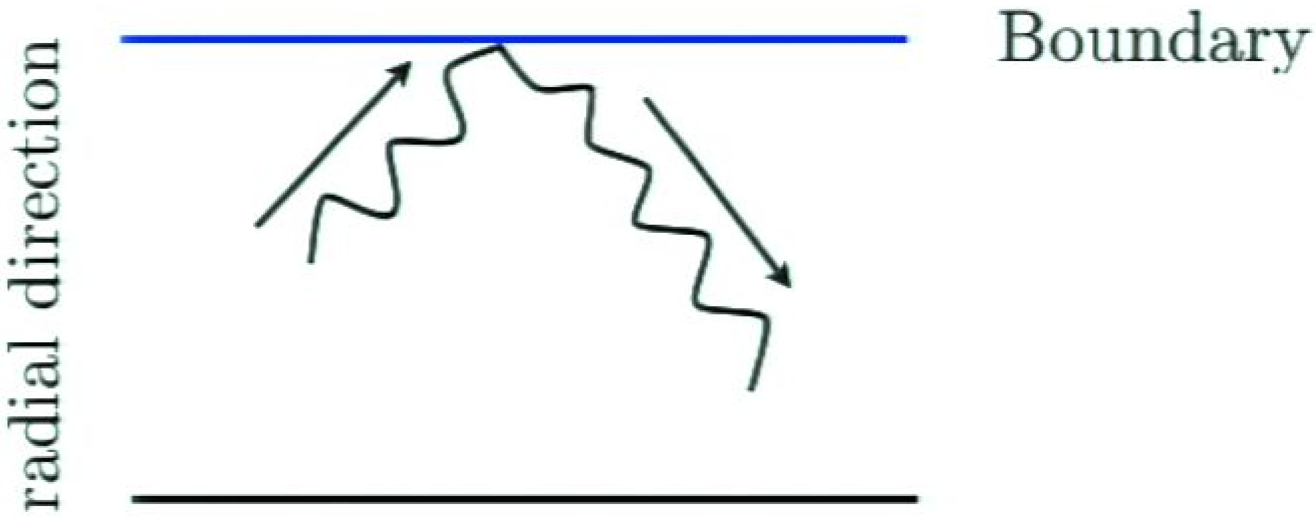
Utility: deformation can have any desired symmetries.

Creating boost invariant black holes



Resulting relativity problem only depends on v and u!

Choosing a coordinate system



Desirable features:

- Natural implementation of BC at boundary.
- Idealized for infalling radiation.
- Regularity at horizon.

Metric ansatz: $ds^2 = -Adv^2 + \Sigma^2 \left[e^B dx_\perp^2 + e^{-2B} dz^2 \right] + 2drdv.$

Einstein's equations

With the metric

$$ds^2 = -A dv^2 + \Sigma^2 \left[e^B dx_\perp^2 + e^{-2B} dz^2 \right] + 2 dr dv,$$

∂_v **always appear in the combination** $\dot{h} = \partial_v h + \frac{1}{2} A h'$.

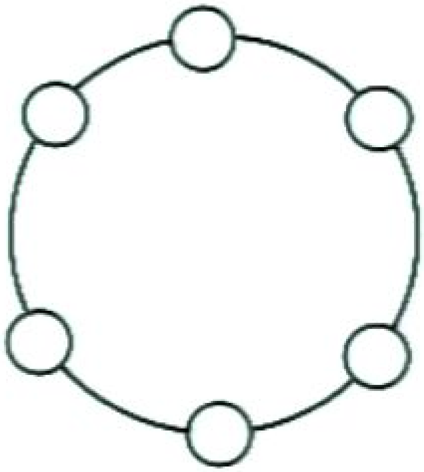
$$\begin{aligned}
0 &= \Sigma\,(\dot{\Sigma})' + 2\Sigma'\,\dot{\Sigma} - 2\Sigma^2\,, \\
0 &= \Sigma\,(\dot{B})' + \tfrac{3}{2}\big(\Sigma'\dot{B} + B'\,\dot{\Sigma}\big)\,, \\
0 &= A'' + 3B'\dot{B} - 12\Sigma'\,\dot{\Sigma}/\Sigma^2 + 4\,, \\
0 &= \ddot{\Sigma} + \tfrac{1}{2}\big(\dot{B}^2\,\Sigma - A'\,\dot{\Sigma}\big)\,, \\
0 &= \Sigma'' + \tfrac{1}{2} B'^2\,\Sigma\,,
\end{aligned}$$

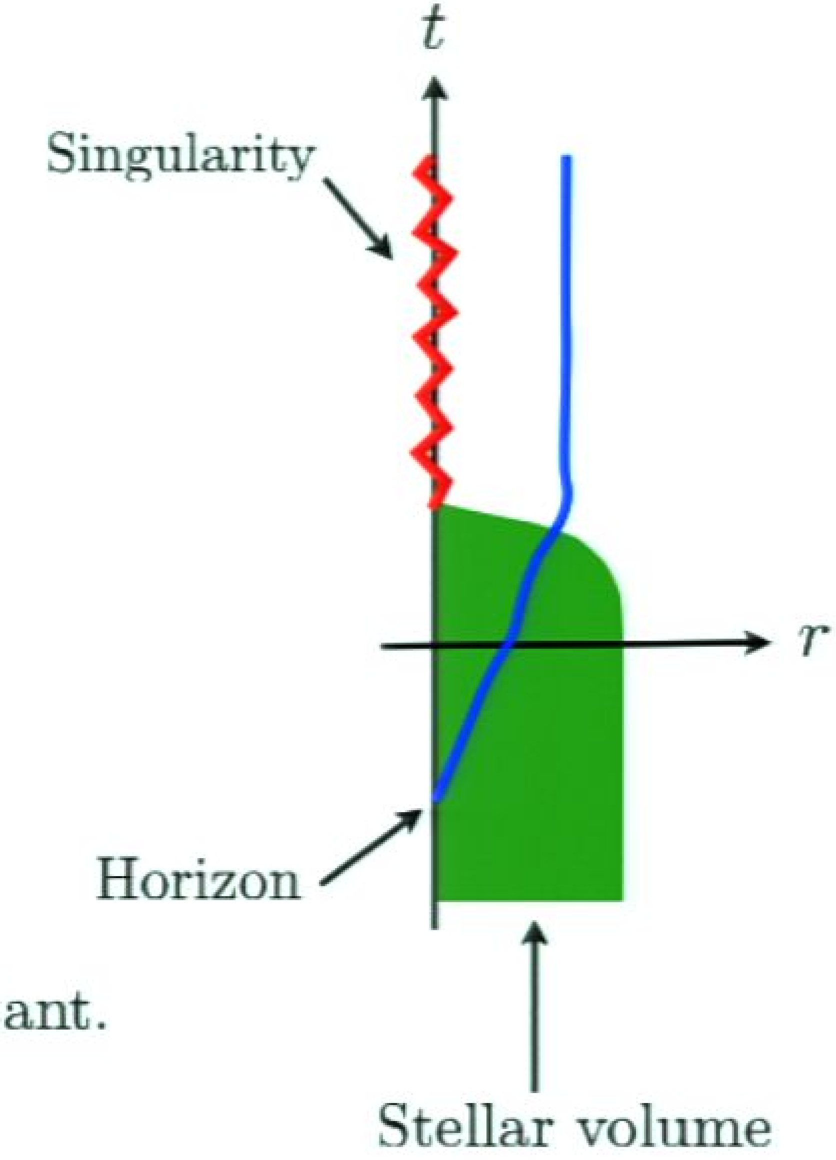
Boundary constraint

Initial value constraint

Singularities and horizon excision

Apparent horizons





Defining condition: area element = constant.

$$\Rightarrow \dot{\Sigma}(v, r = r_h) = 0.$$

Einstein's equations

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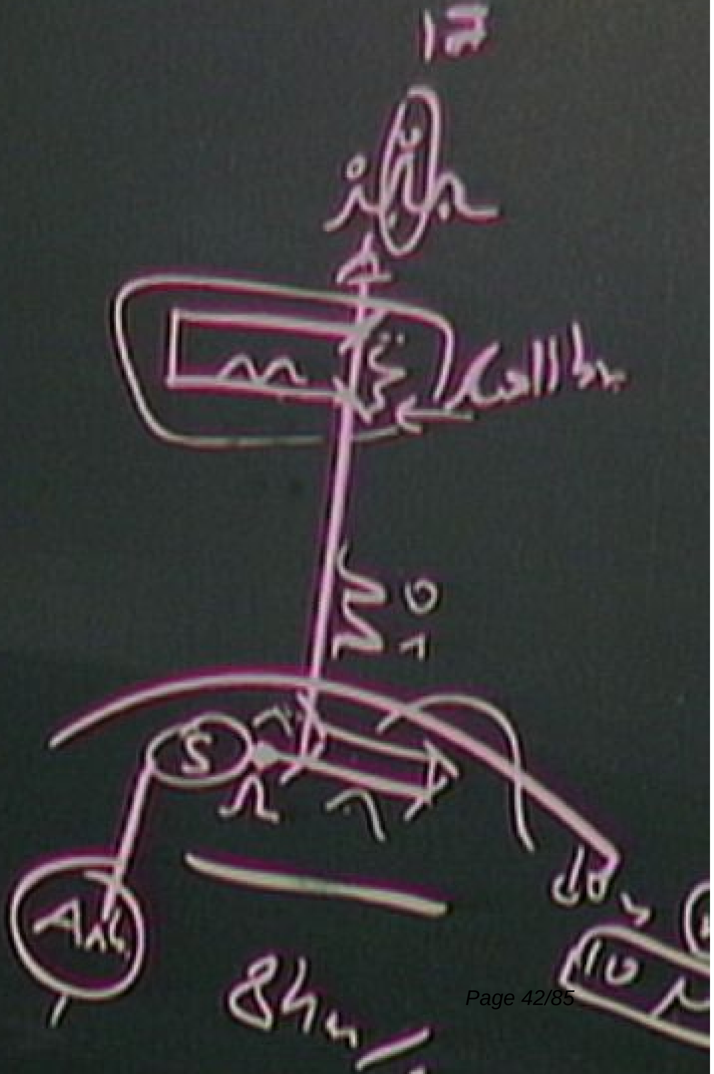
Boundary constraint

Initial value constraint

$$\partial_\mu T^{\mu\nu} = 0$$

$$D_\mu T^{\mu\nu} = 0$$



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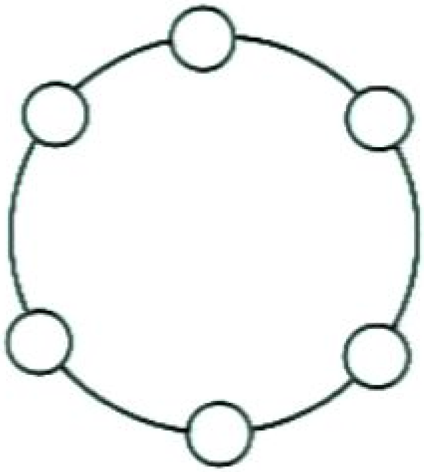

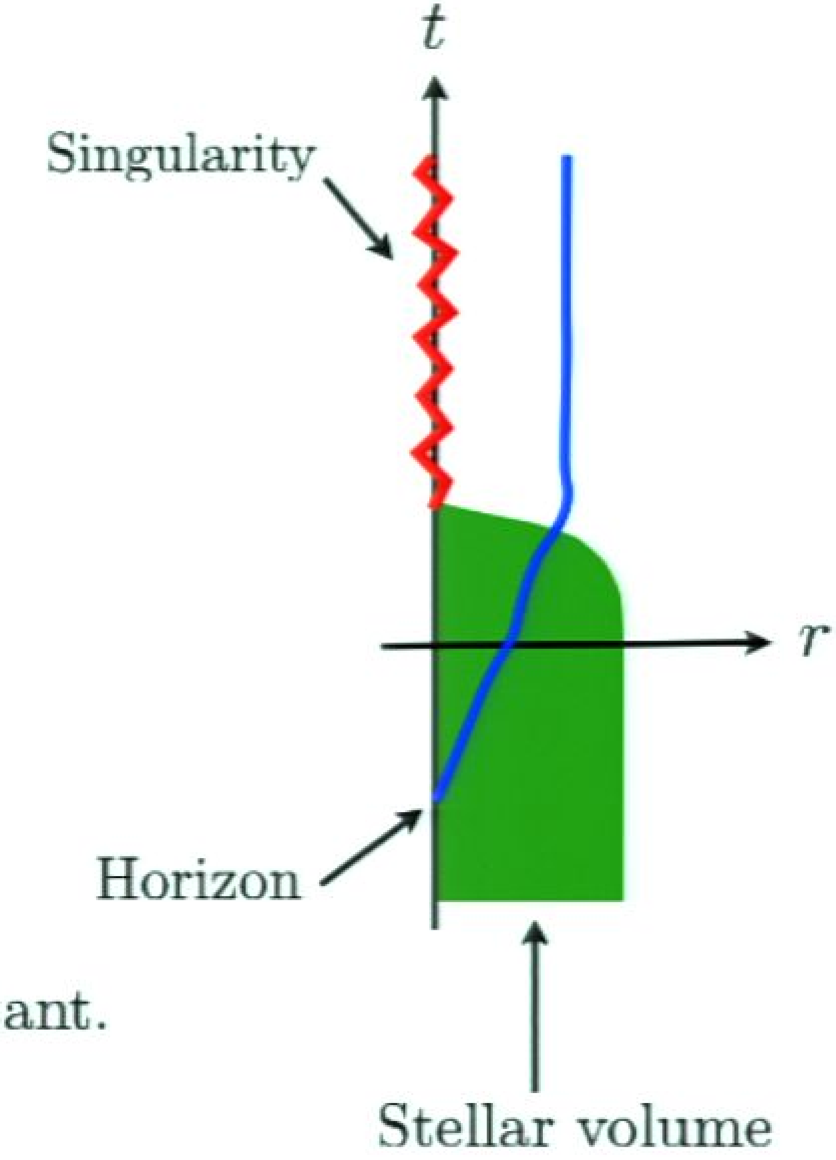

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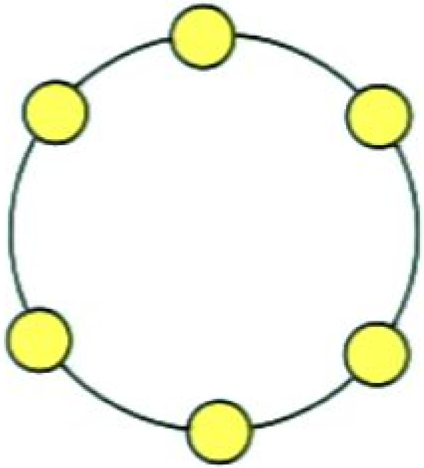

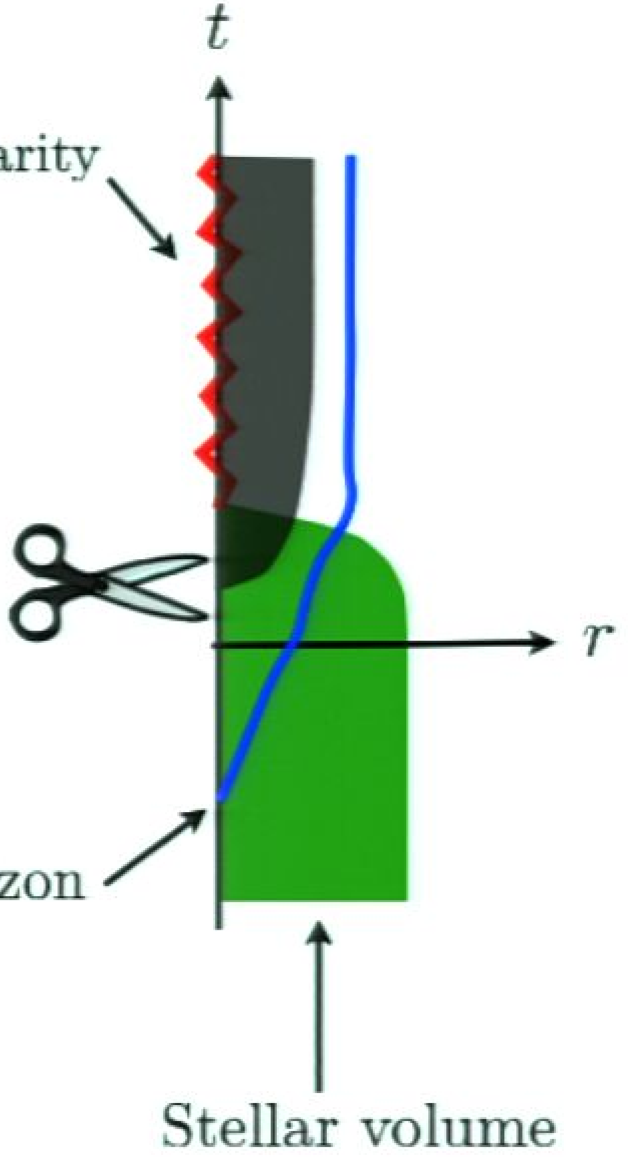

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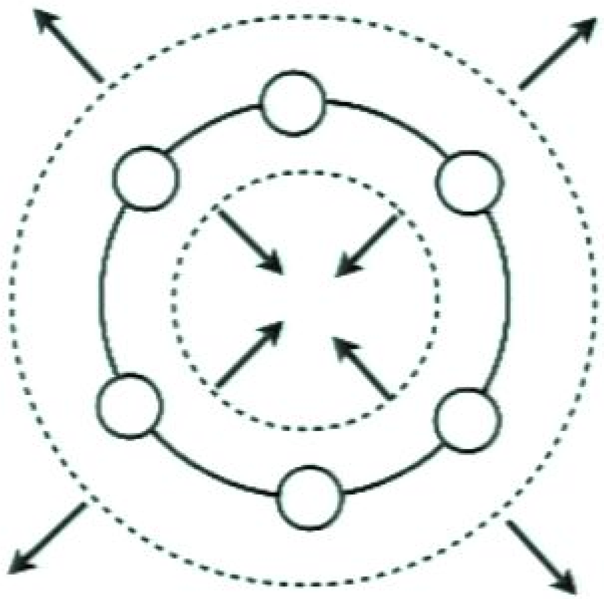

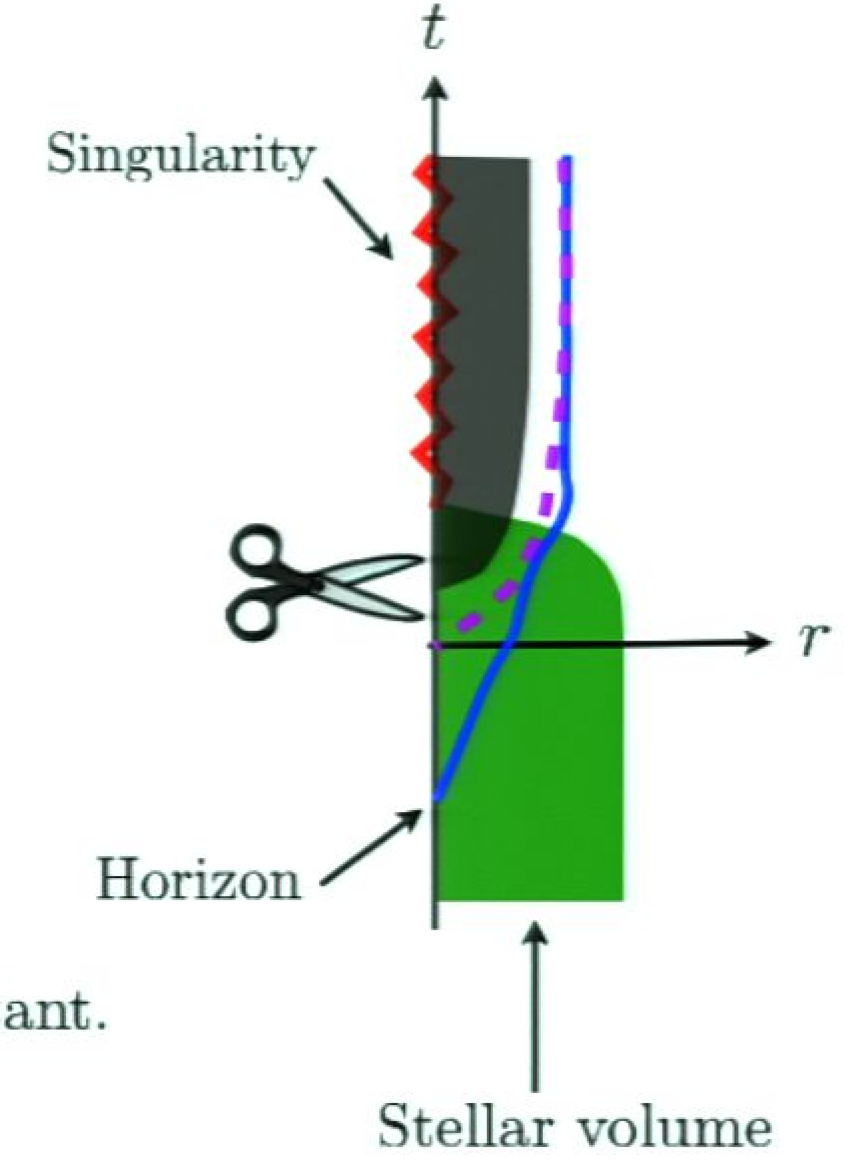

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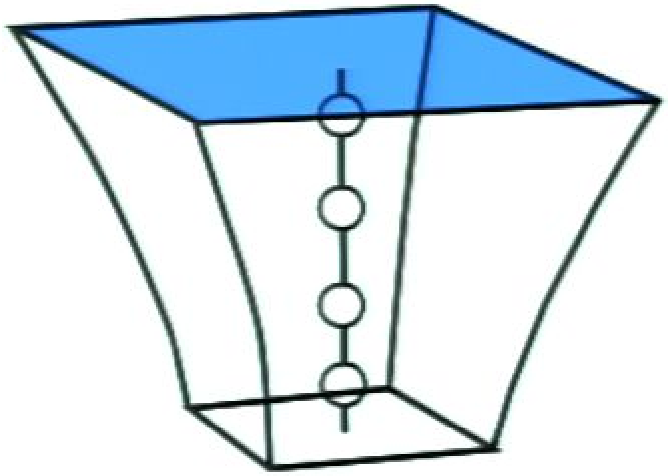
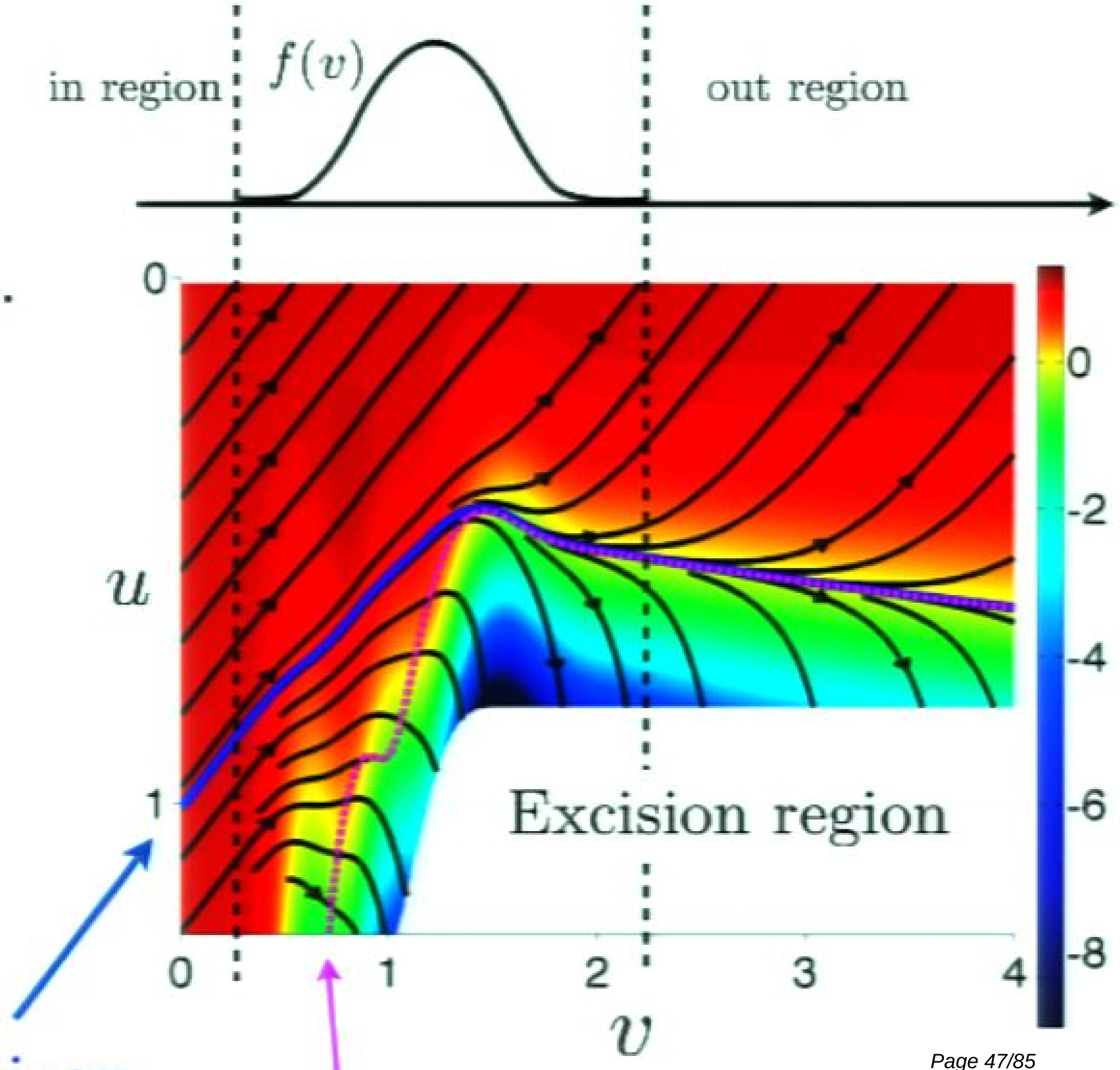
Results illustrated — $5d$ gravitational physics

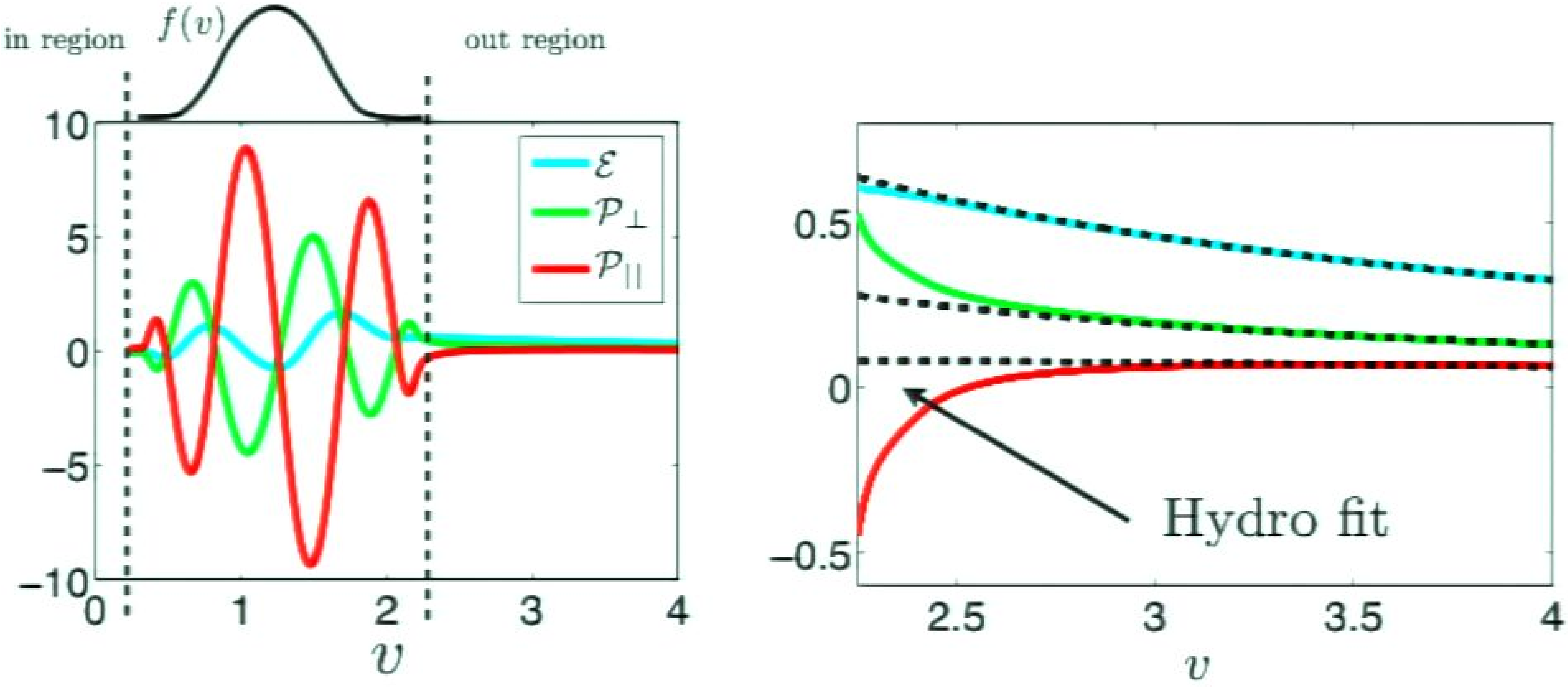
- Coloring $\propto g_{00}$.
- Flow lines = light rays.





Horizon

Results — $4d$ QFT physics



Late time hydro stress: (Janik & Peschanski: hep-th/0512162)
(Kinoshita, Mukohyama, Nakamura & Oda: 0807.3797)

$$\mathcal{E} = \frac{3\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{2C_1}{(\Lambda v)^{2/3}} + \frac{C_2}{(\Lambda v)^{4/3}} + \mathcal{O}(v^{-2})\right],$$

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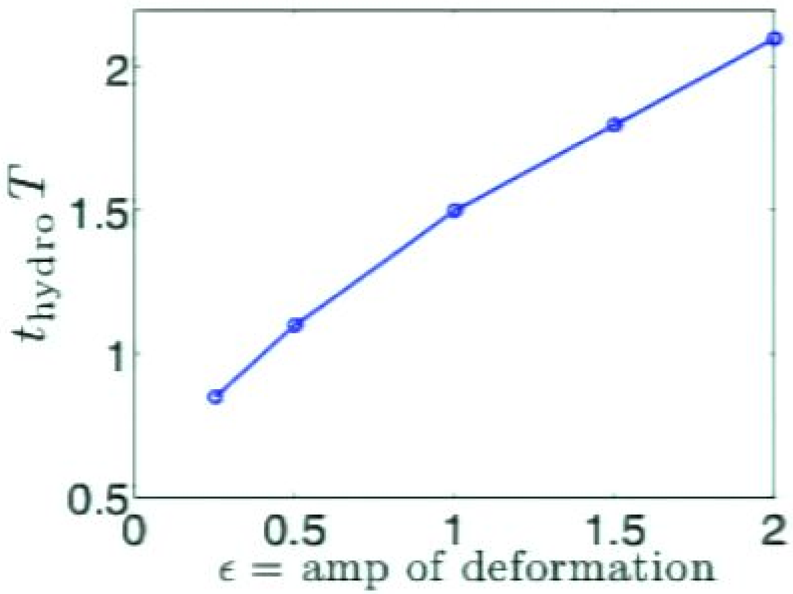
Relaxation time and its gravitational description

- $t_{\text{hydro}} T \gtrsim \frac{1}{2}$.
- For $T = 350$ MeV, $t_{\text{hydro}} \gtrsim 0.3$ fm/c.



Limiting cases

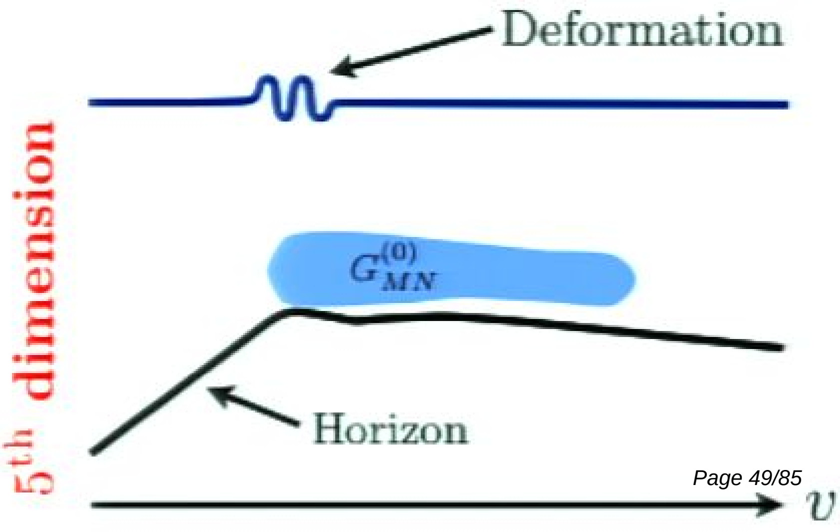
$\epsilon \to \infty$ limit:

- $T \to \infty$.
- $t_{\text{hydro}} \to$ const.

$\epsilon \to 0$ limit:

- $G_{MM} = G^{(0)}_{MN} + \epsilon\, G^{(1)}_{MN} + \epsilon^2 G^{(2)}_{MN} + \ldots$.
- $G^{(0)}_{MN}$ = black hole geometry!

  [Bhattacharyya & Minwalla: 0904.0464]
- $G^{(n)}_{MN}$, $n > 0$ can scatter off $G^{(0)}_{MN}$.



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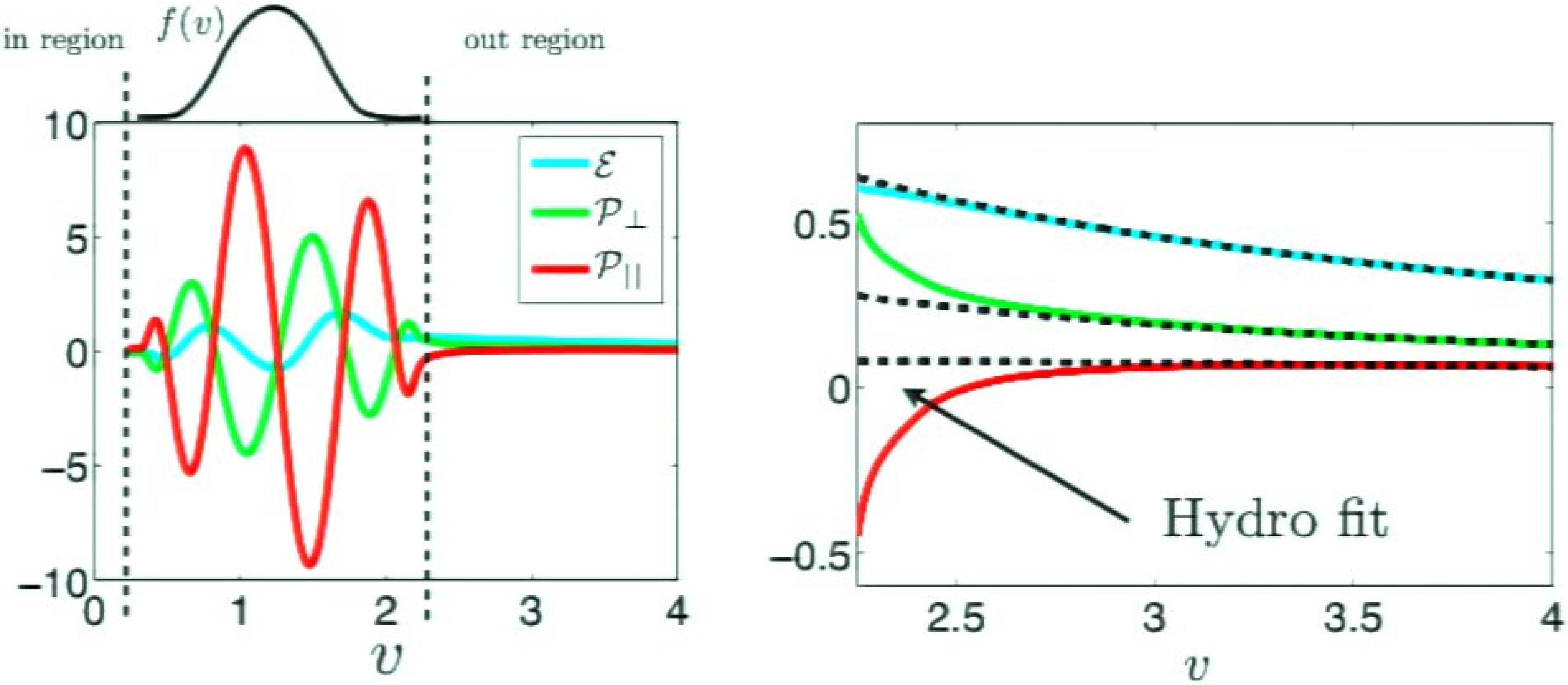

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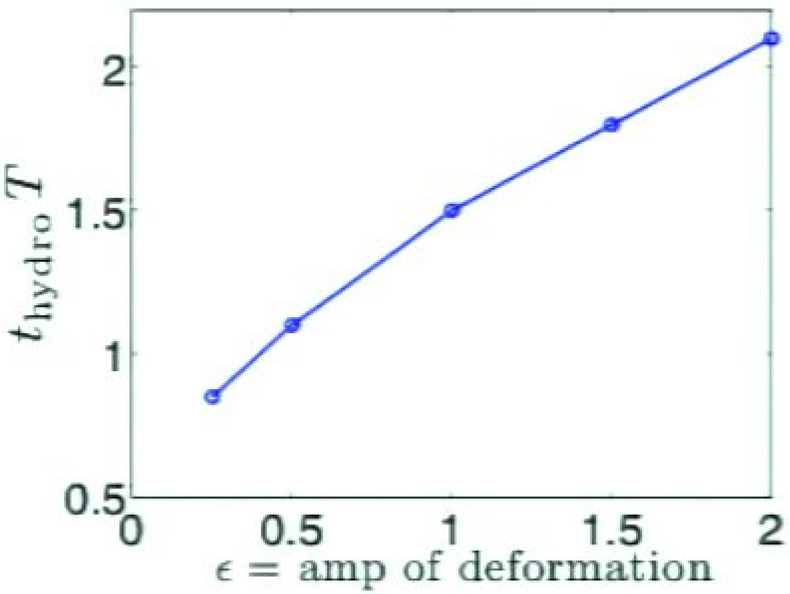

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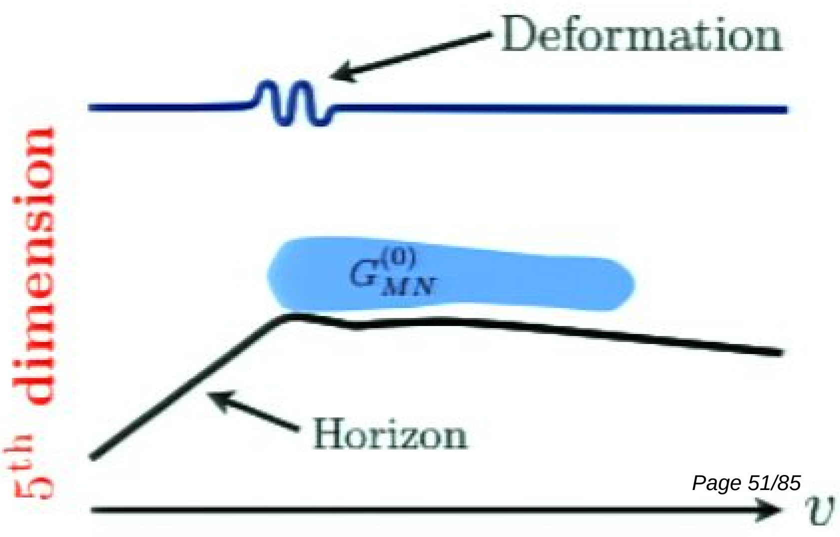

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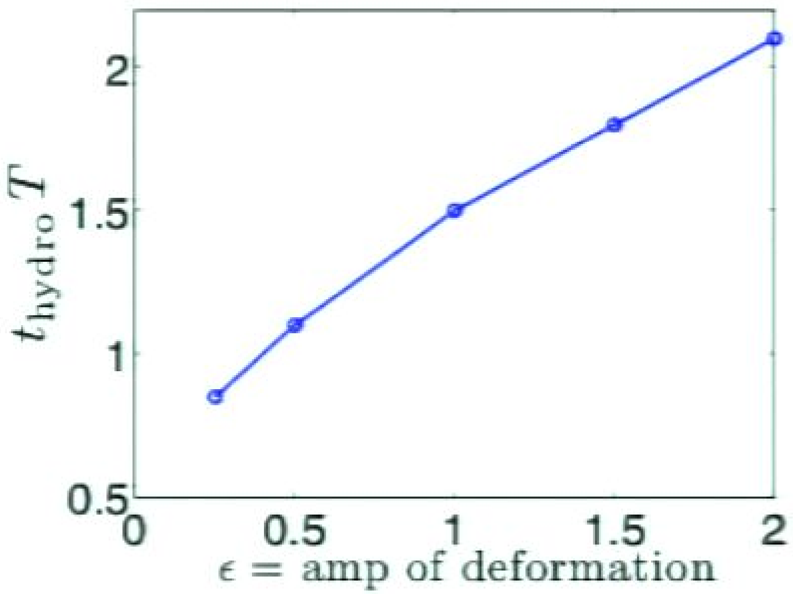

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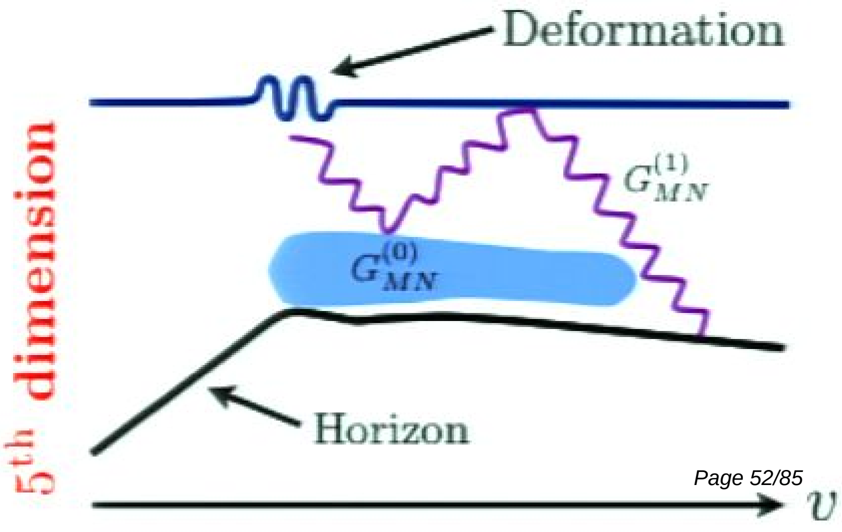
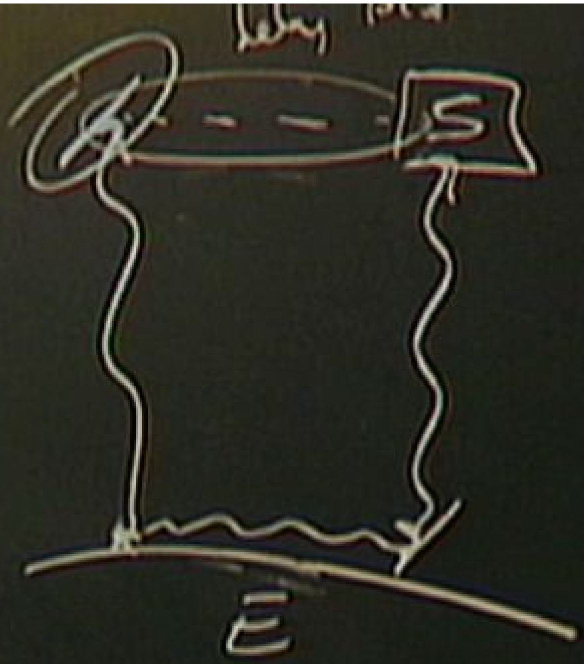
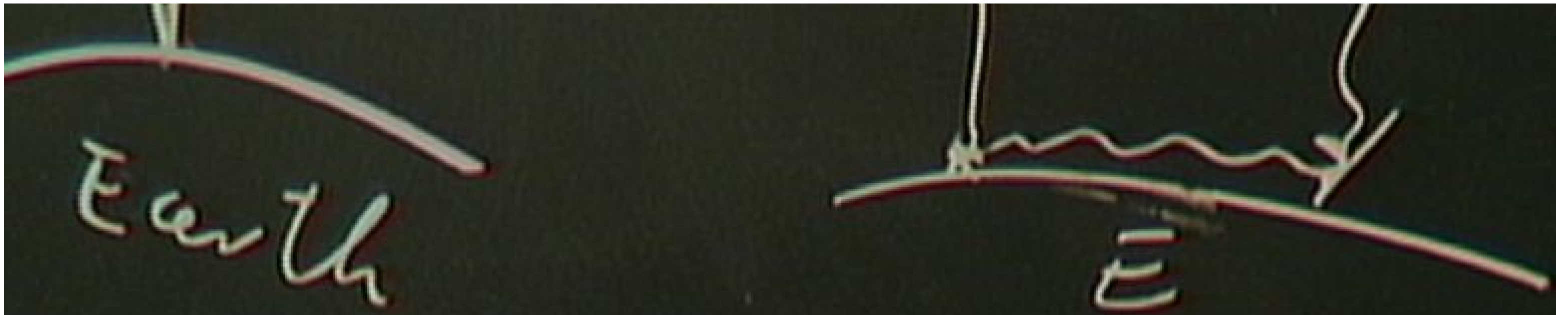




(n~3 in(4)

$$T^{\mu\nu} = \mathrm{diag}(\epsilon, P, P, P), \quad \epsilon = 3P$$



$$T^{\mu\nu} = \text{diag}(\mathcal{E}, P, P, P), \quad \epsilon = 3P$$

$$\mathcal{E} \sim \epsilon^2, \quad T \sim \sqrt{\epsilon}$$

in(t)

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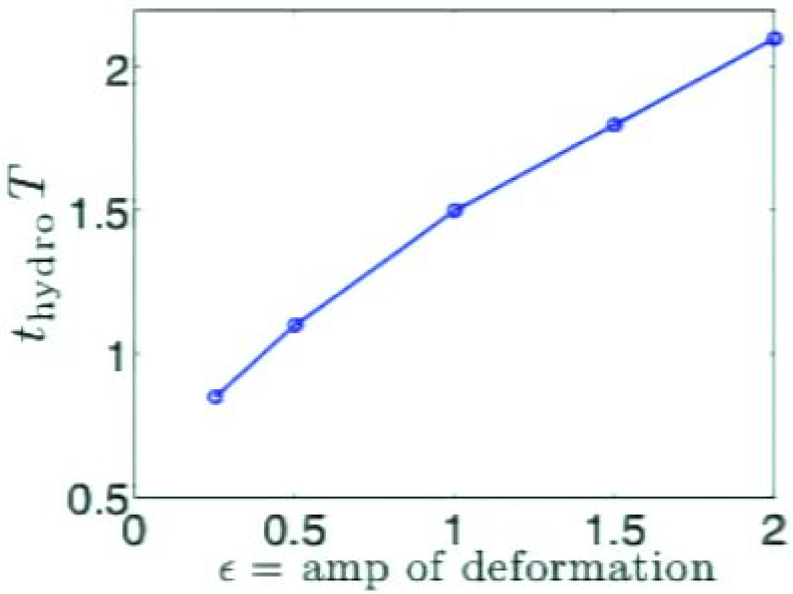

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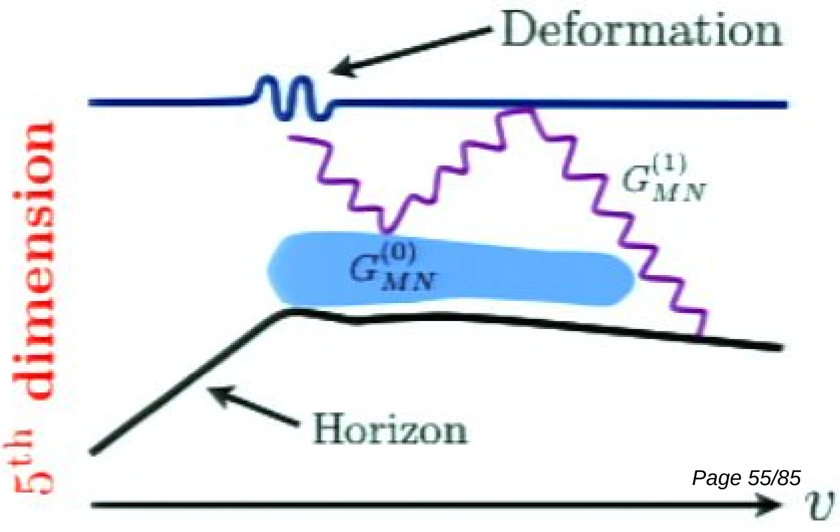
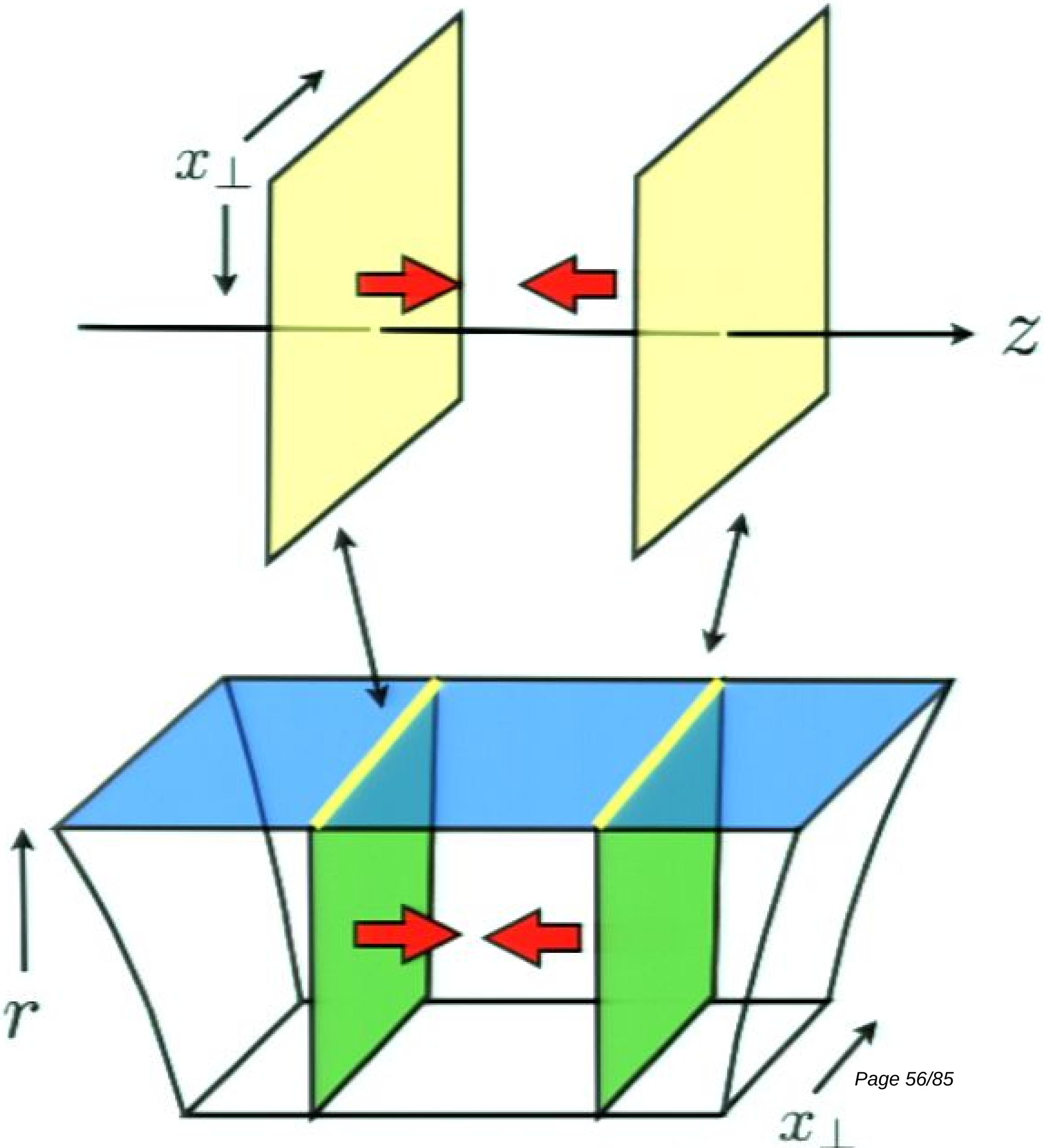


$2 + 1d$ problems — Colliding sheets of matter



CFT description:

- Colliding sheets.
- $\Rightarrow 1 + 1d$ problem.

Gravity description:

- Colliding grav. waves.
- $\Rightarrow 2 + 1d$ problem.

Problems and solutions

Good coordinates for numerical Einstein:

$$ds^2 = -Adv^2 + \Sigma^2\left[e^B dx_\perp^2 + e^{-2B}dz^2\right] + 2drdv + 2Fdzdv.$$

Challenges:

i. Boundary asymtotics & coord trans to EF coordinates:

$$ds_\pm^2 = r^2\left[-dx_+dx_- + \frac{1}{r^4}\varphi(x_\pm)dx_\pm^2 + dx_\perp^2\right] + \frac{dr^2}{r^2},\ x_\pm = t \pm z.$$

ii. $1/r^4$ sickness ⇒ rapid variations in r and z in EF coordinates.

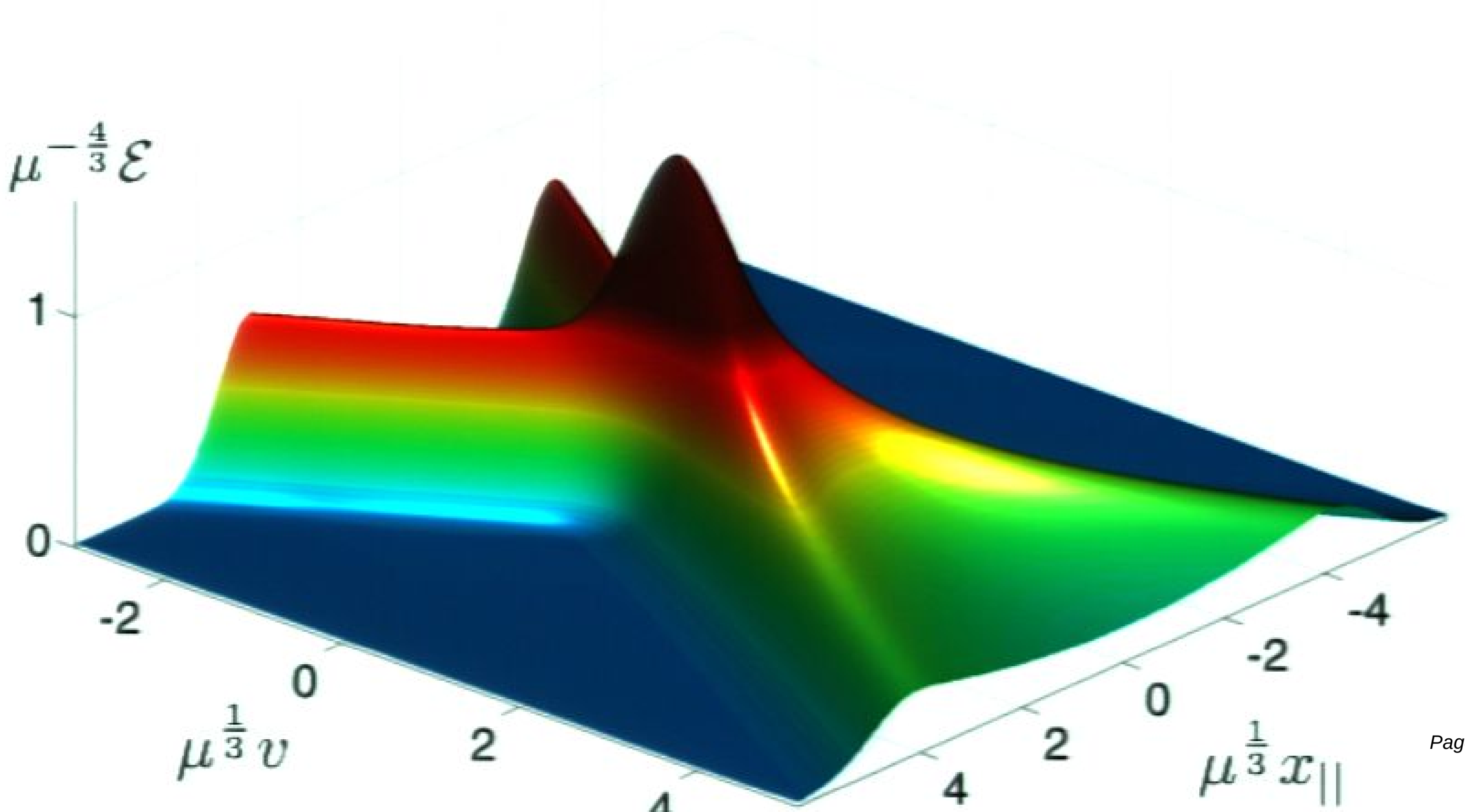
Solutions:

i. Use spectral methods.

Results illustrated

Initial state = gaussian wave packets

- $E/A \equiv \mu$, $\sigma = 0.6\mu^{-\frac{1}{3}}$, $T_{\text{bkgd}} = 0.1\mu^{\frac{1}{3}}$.



Comparing to hydrodynamics

Hydro @ $x_{||} = 0$:

- $\mathcal{P}_{||}^{\text{hydro}} = \frac{1}{3}\mathcal{E} - \frac{1}{3\pi T}\frac{\partial S}{\partial z}.$
- $\mathcal{P}_{\perp}^{\text{hydro}} = \frac{1}{3}\mathcal{E} + \frac{1}{6\pi T}\frac{\partial S}{\partial z}.$
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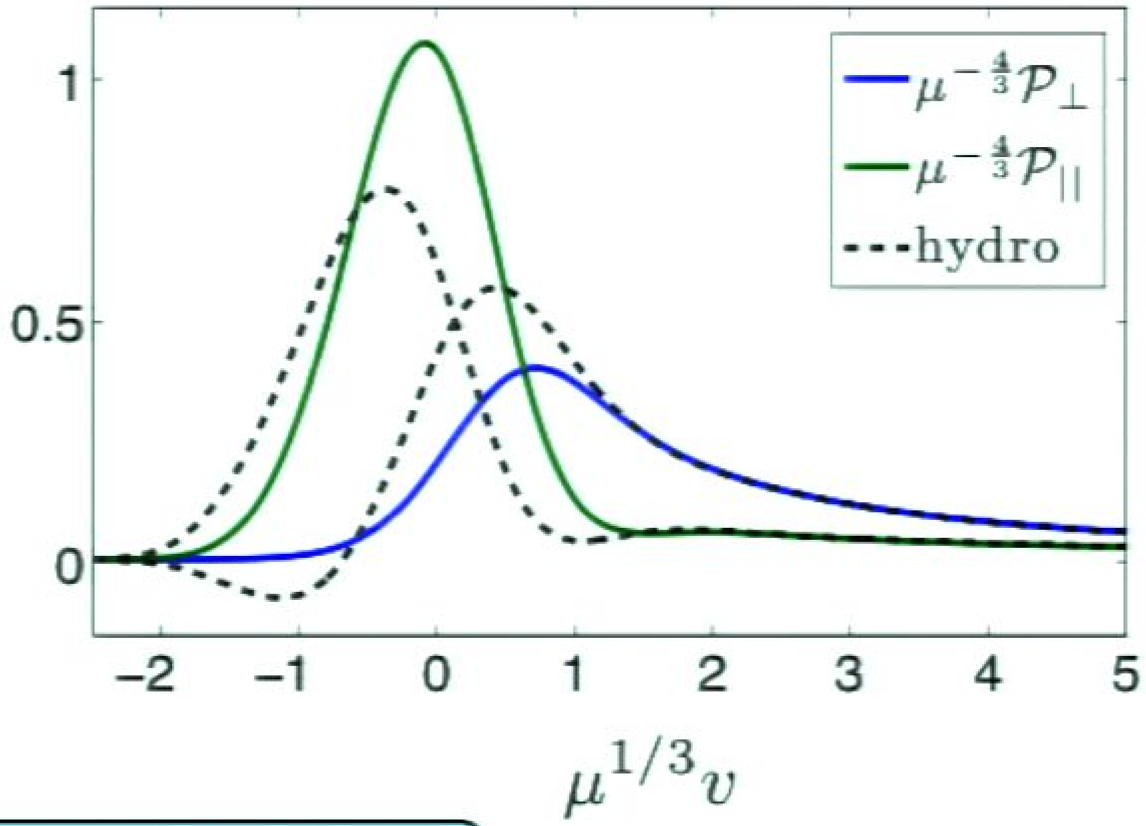

$$v_{\text{therm}} = \frac{3\mu^{-\frac{1}{3}}}{2} = \frac{1}{2T}.$$

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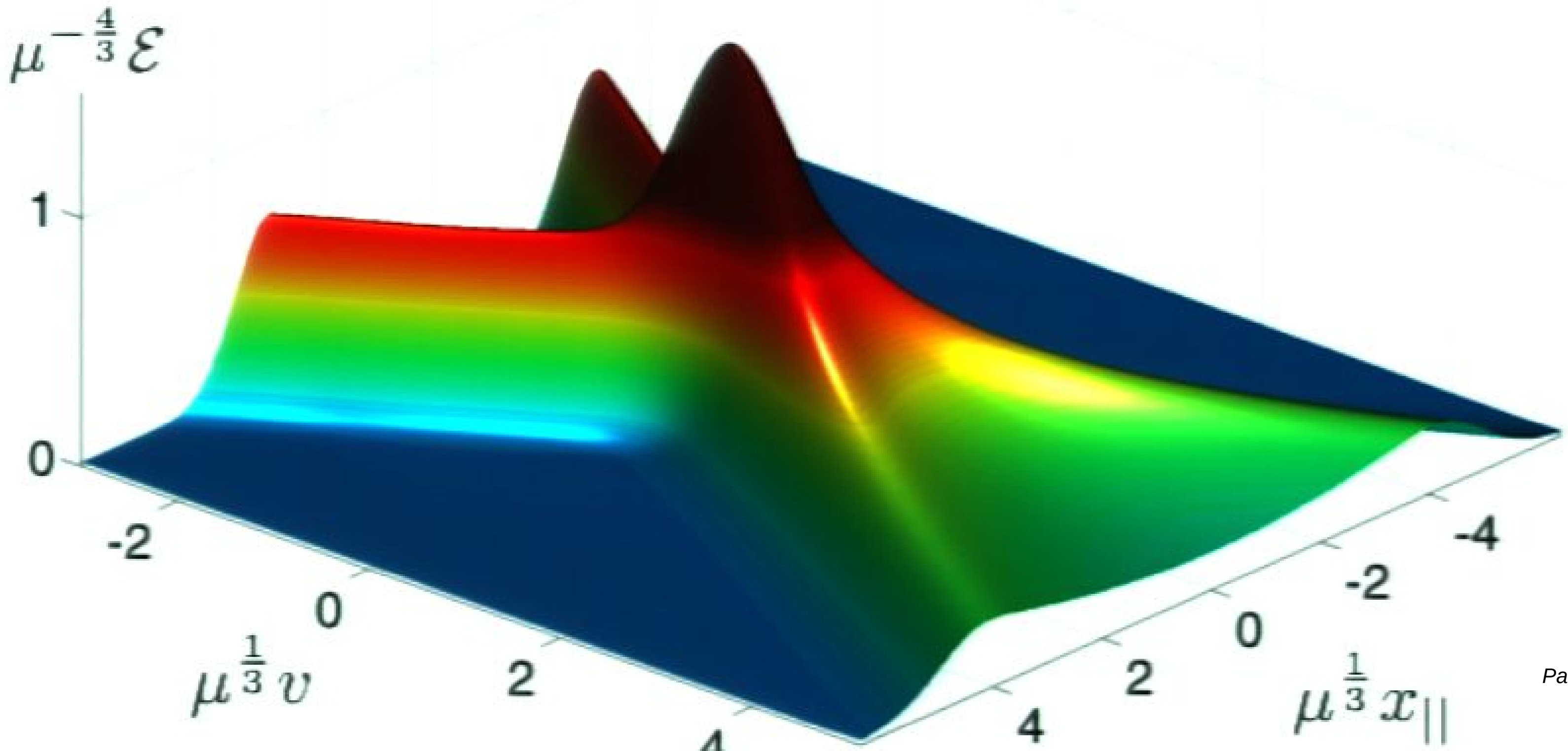

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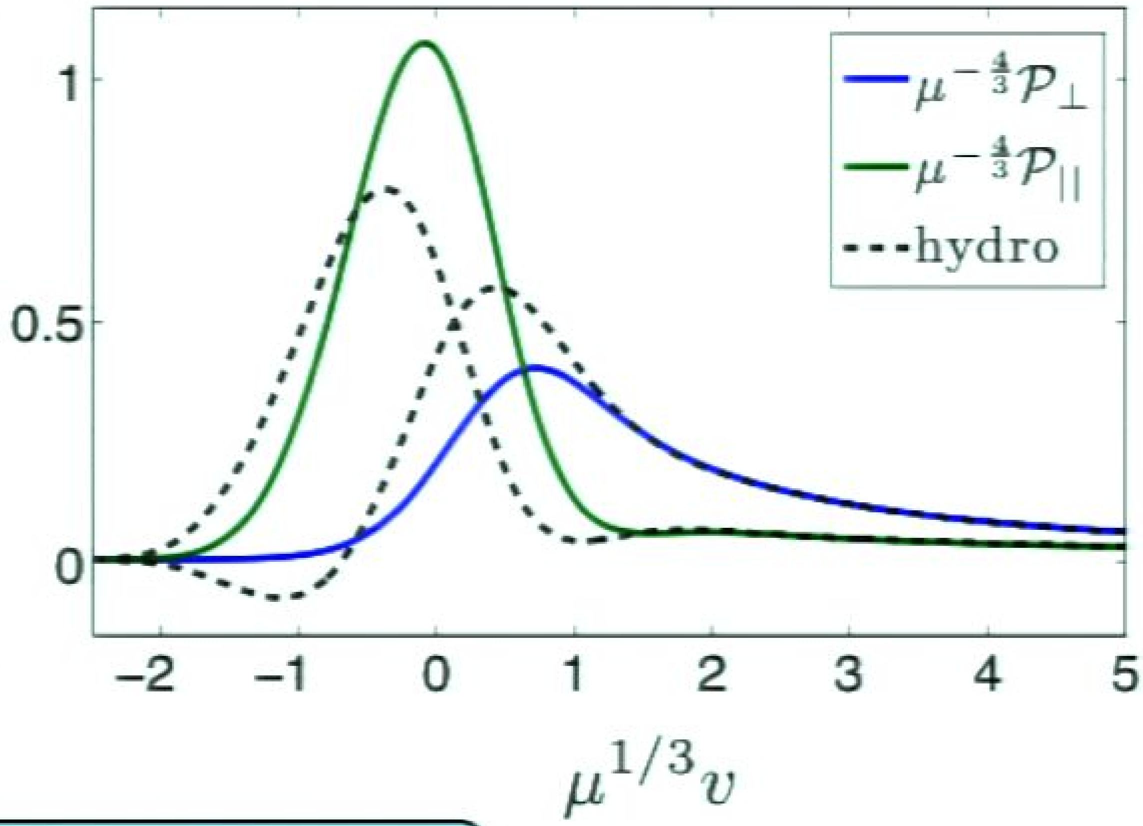

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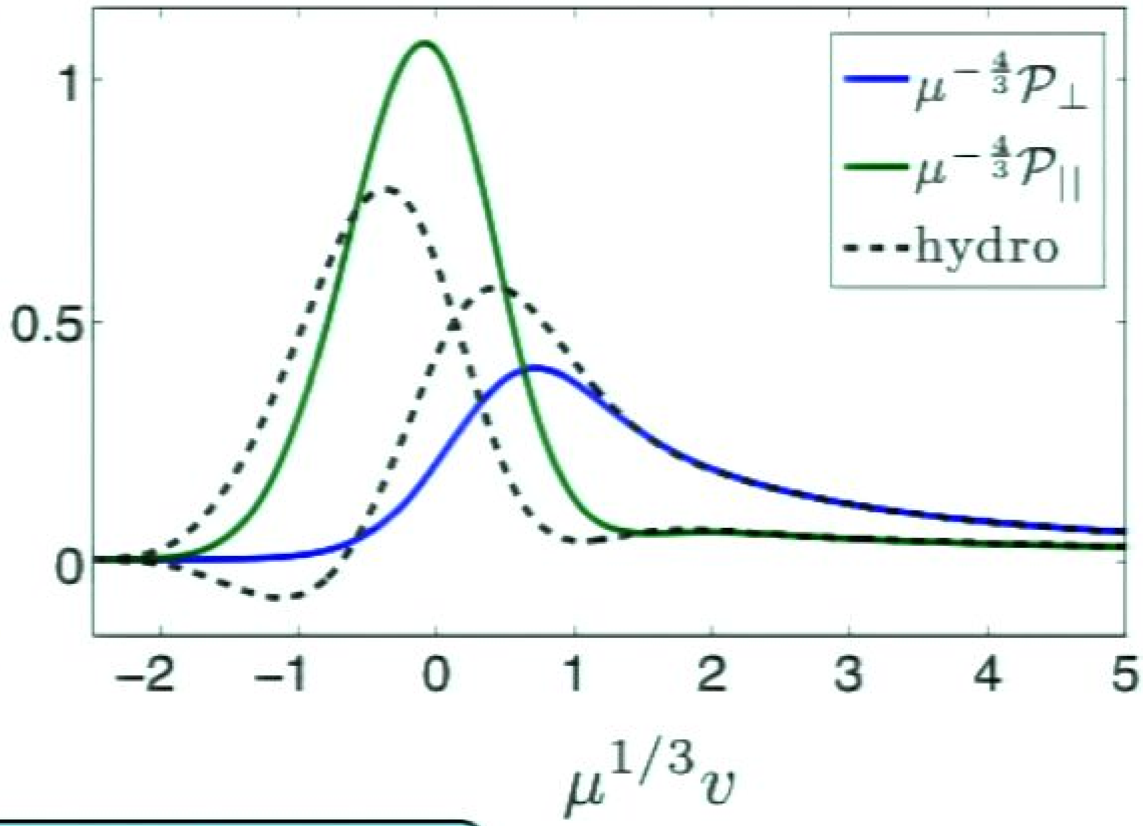

$$v_{\text{therm}} = \frac{3\mu^{-\frac{1}{3}}}{2} = \frac{1}{2T}.$$

Same result as $1+1d$ problem!

$\epsilon, \vec{u}$

LFRF

$T^{00} = \epsilon$

$T^{0i} = 0$

$$T^{ij} = p\delta_{ij} - \eta\left(\nabla_i u_j + \nabla_j u_i - \frac{2}{3}\delta_{ij}\nabla\cdot u\right)$$

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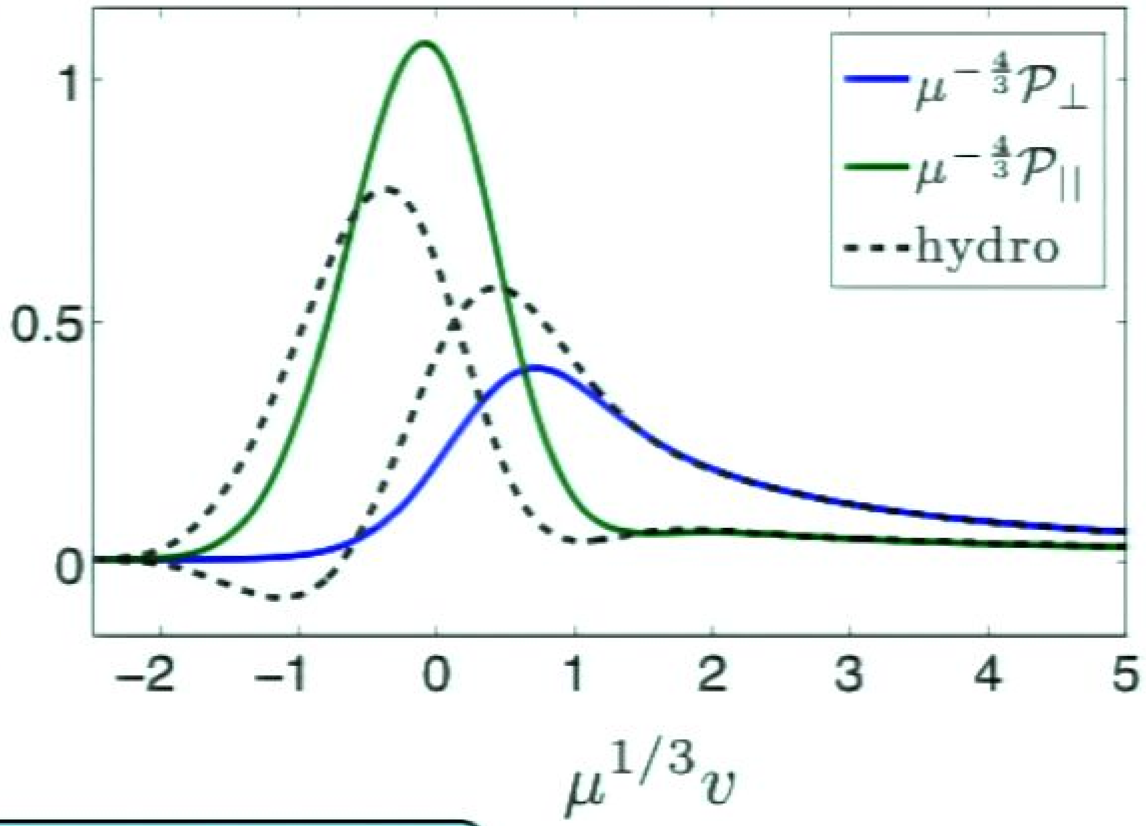

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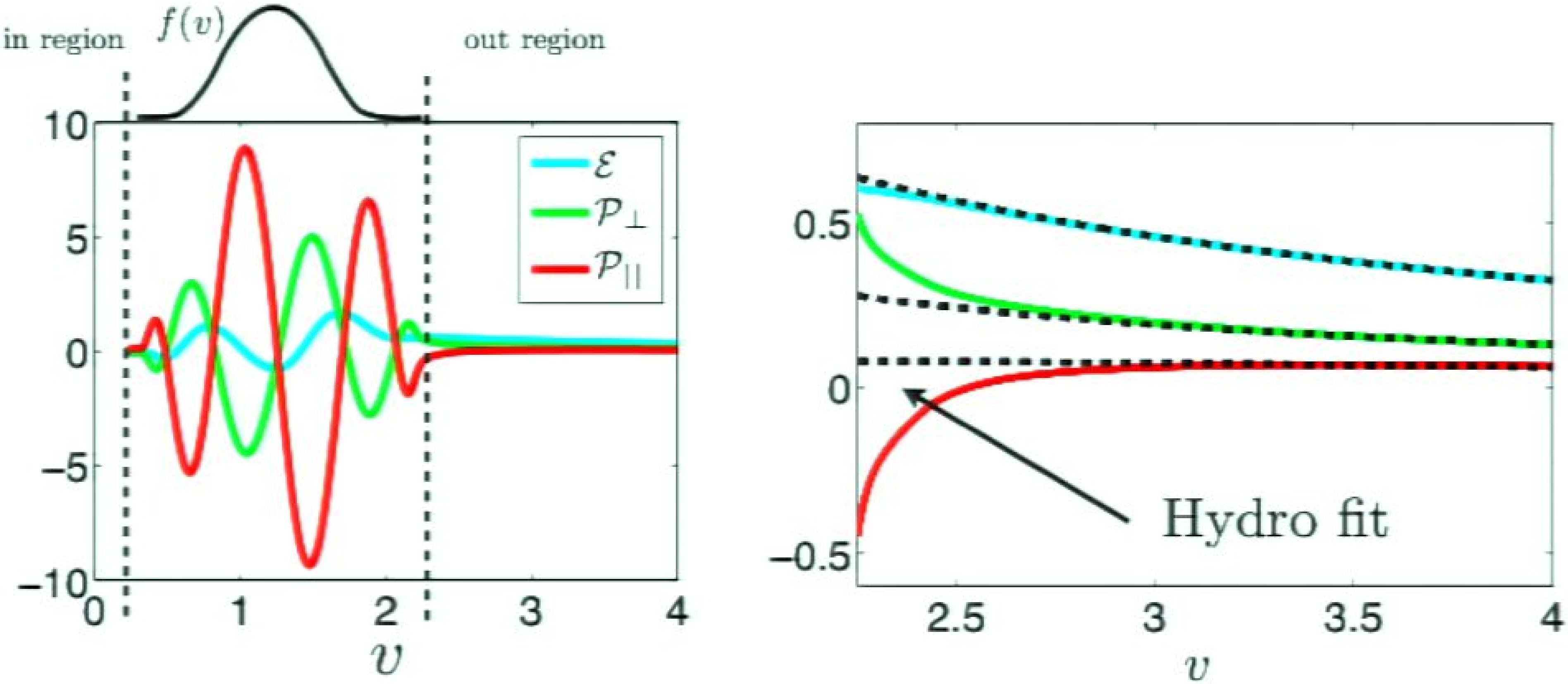
Results — $4d$ QFT physics



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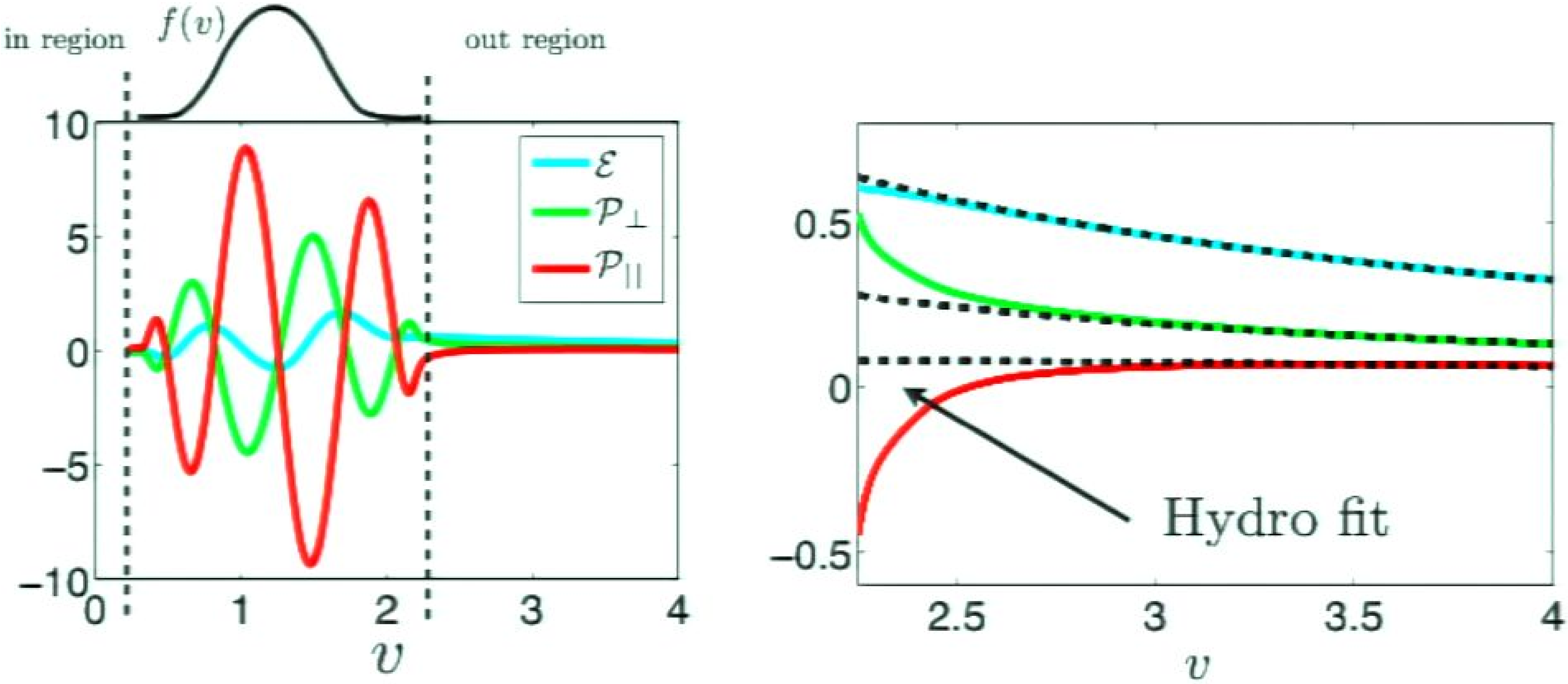
$$\mathcal{P}_{||} = \frac{\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{2C_1}{(\Lambda v)^{2/3}} + \frac{5C_2}{3(\Lambda v)^{4/3}} + \mathcal{O}(v^{-2})\right].$$

Results — $4d$ QFT physics



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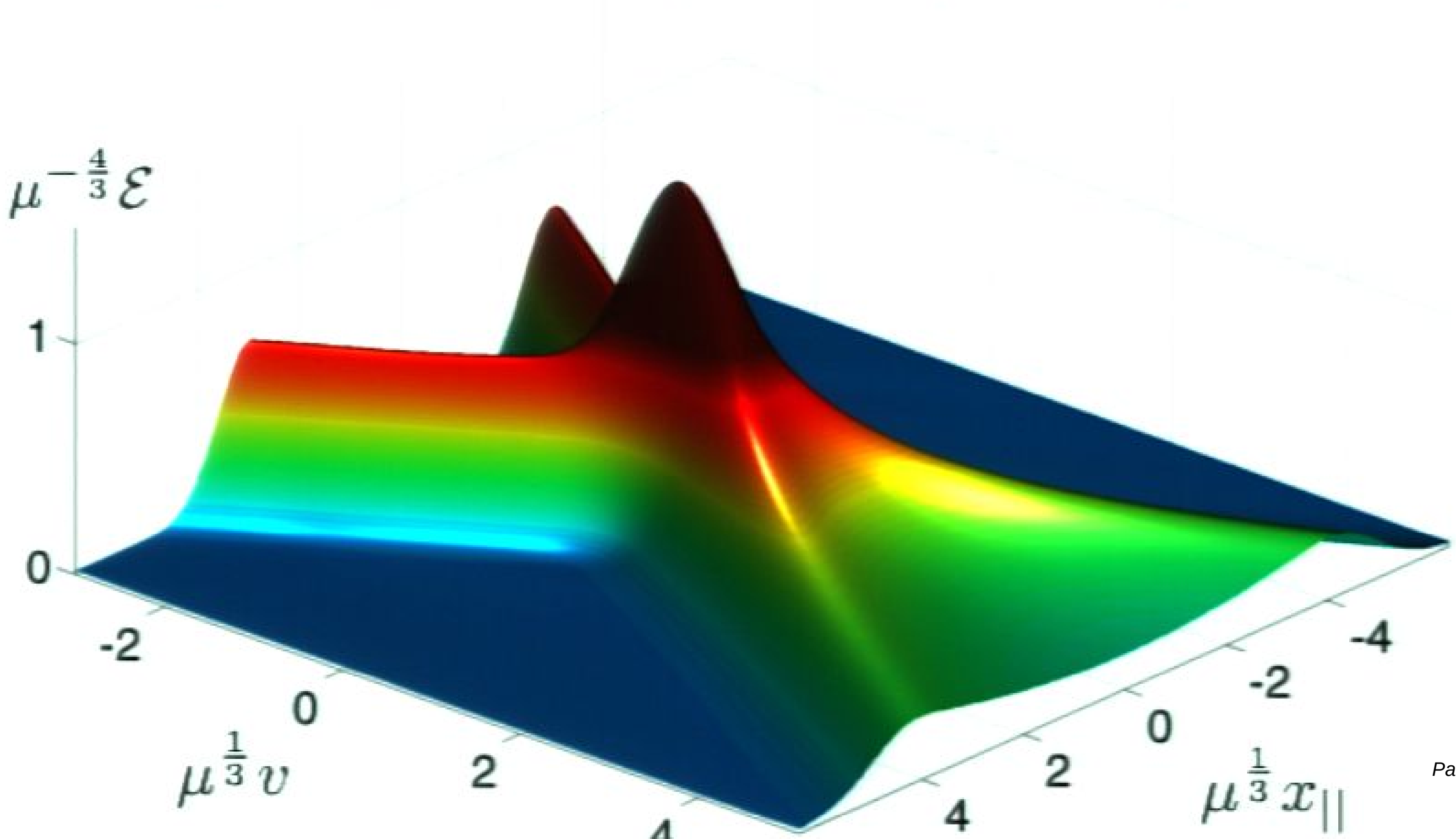
Results illustrated

Initial state = gaussian wave packets

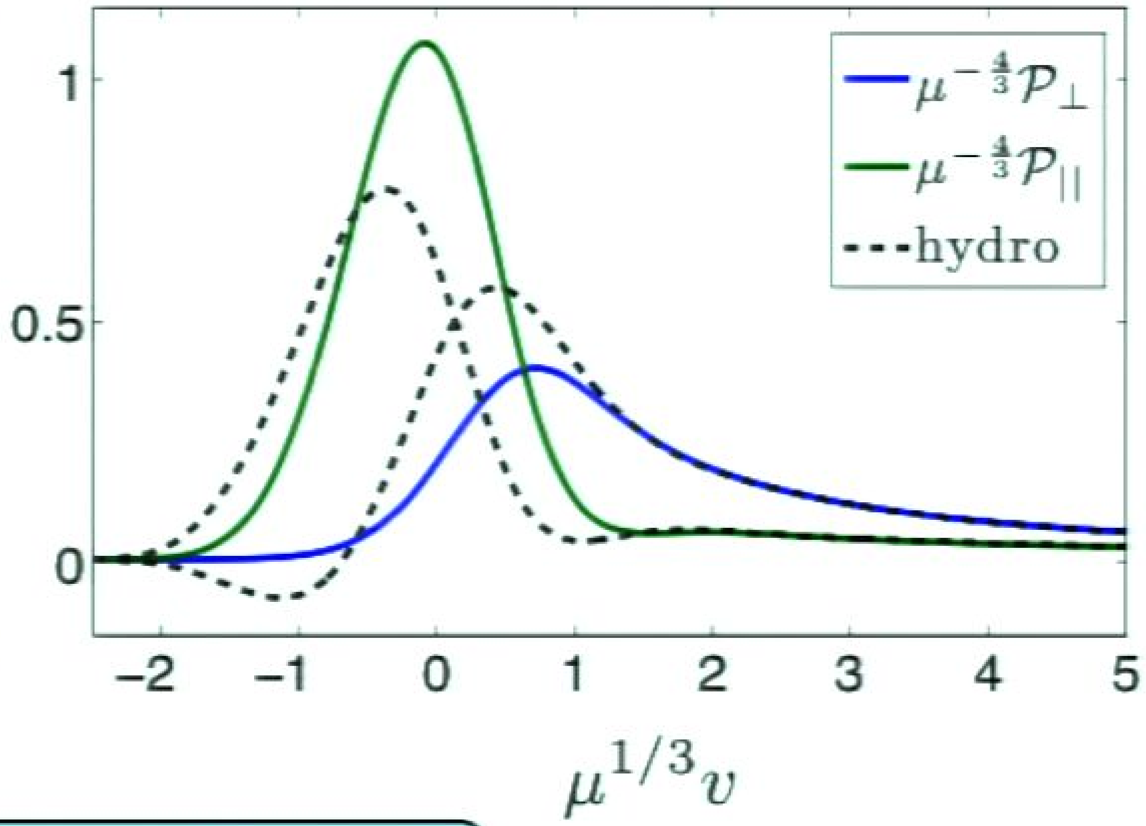
- $E/A \equiv \mu$, $\sigma = 0.6\mu^{-\frac{1}{3}}$, $T_{\rm bkgd} = 0.1\mu^{\frac{1}{3}}$.



Comparing to hydrodynamics

Hydro @ $x_{||} = 0$:

- $\mathcal{P}_{||}^{\text{hydro}} = \frac{1}{3}\mathcal{E} - \frac{1}{3\pi T}\frac{\partial S}{\partial z}$.
- $\mathcal{P}_{\perp}^{\text{hydro}} = \frac{1}{3}\mathcal{E} + \frac{1}{6\pi T}\frac{\partial S}{\partial z}$.
- $T = \left(\frac{4\mathcal{E}}{3\pi^4}\right)^{1/4}$.



$$v_{\text{therm}} = \frac{3\mu^{-\frac{1}{3}}}{2} = \frac{1}{2T}.$$

Summary and open questions

- Difficult QFT problems tractable with holography & numerical relativity.
- Not perfect — no known dual to QCD.
  - ⋆ $t_{CFT} \gtrsim 0.3$ fm/c,
  - ⋆ $t_{\text{experiment}} \lesssim 1$ fm/c,
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Open questions:

- Relevance for heavy ion collisions.
- Other measures of thermalization: correlation functions.
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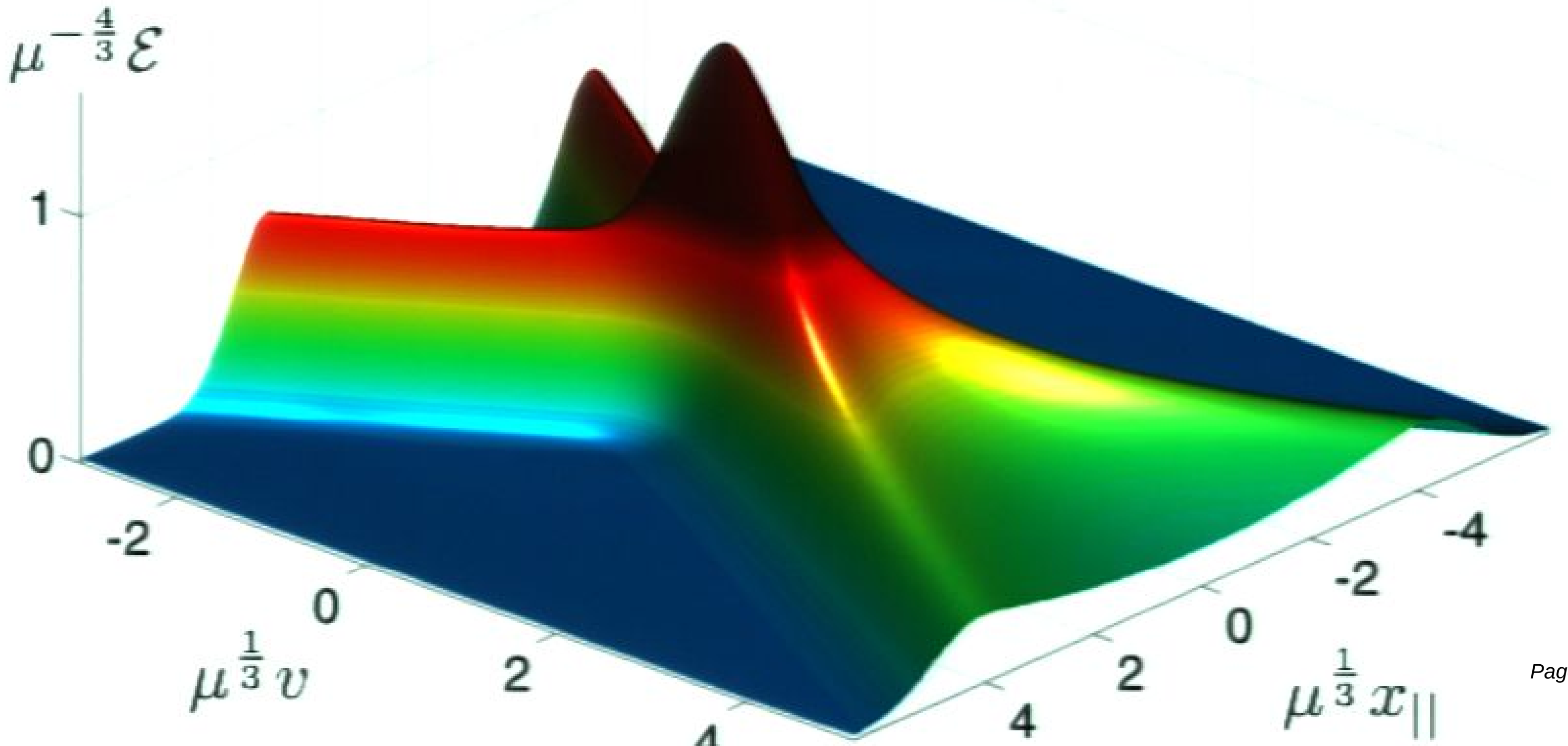

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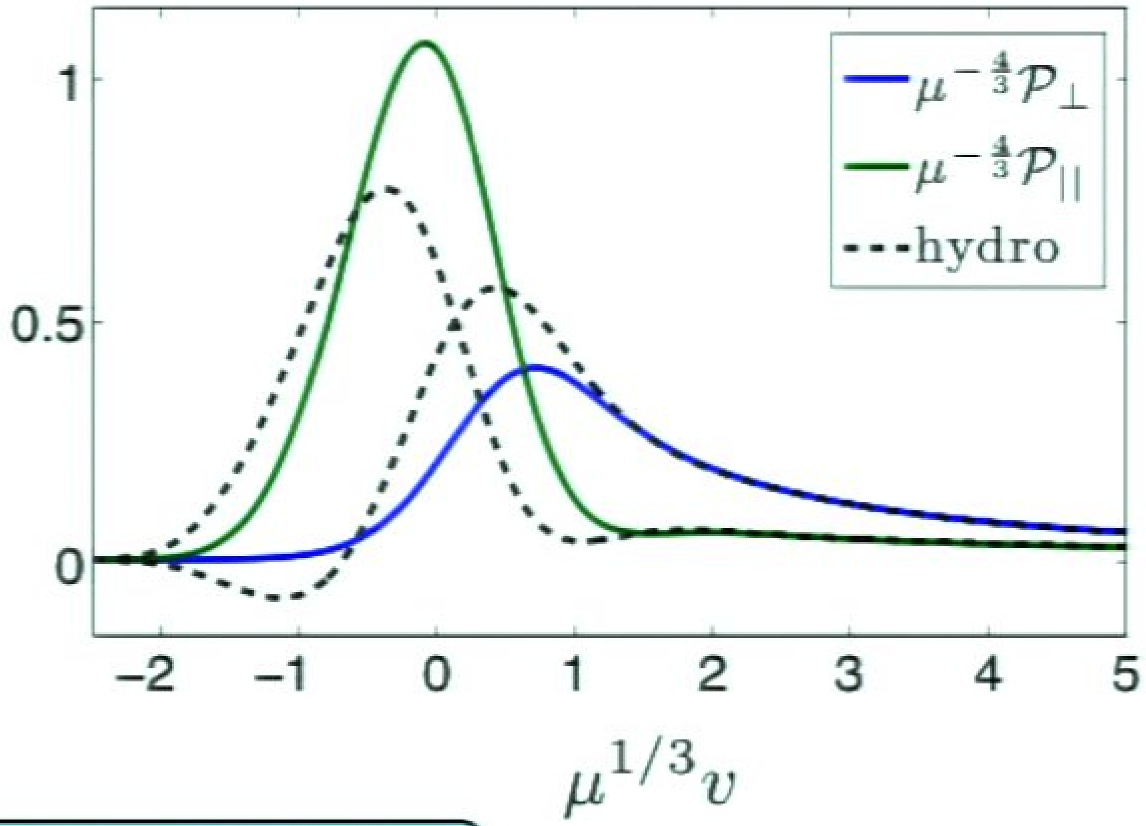

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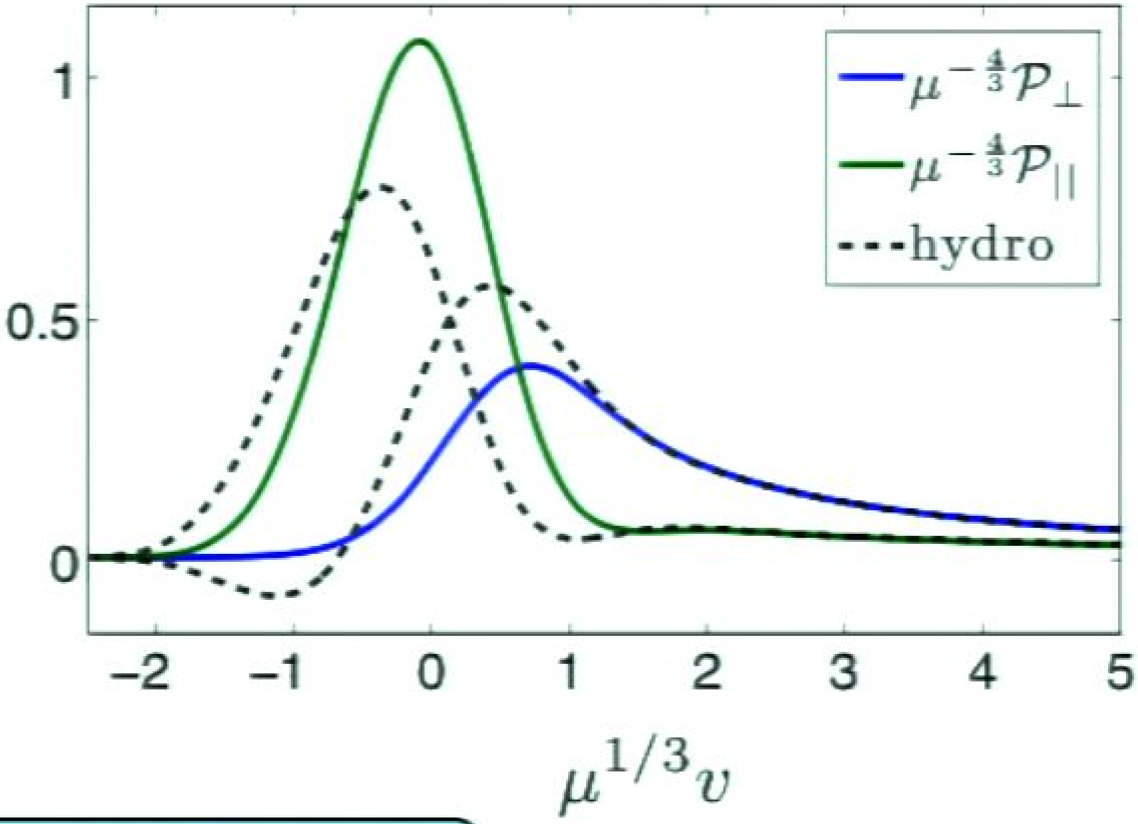

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Same result as $1 + 1d$ problem!

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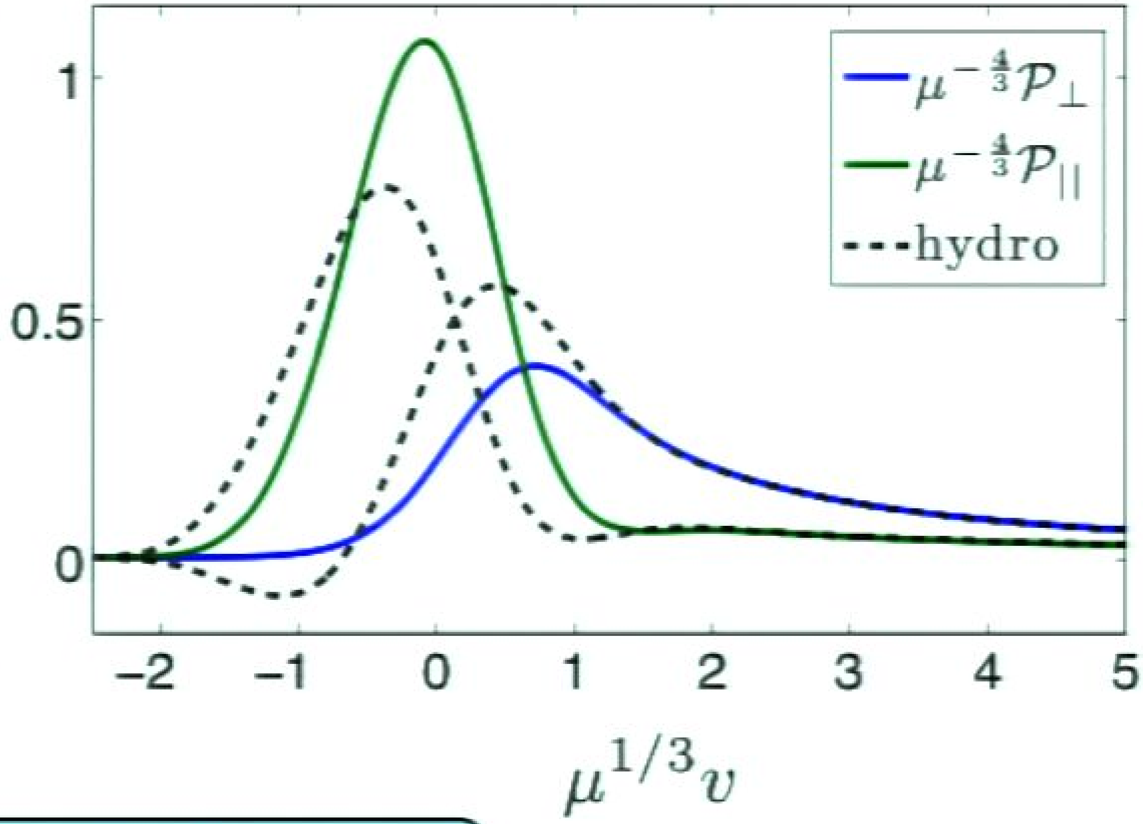

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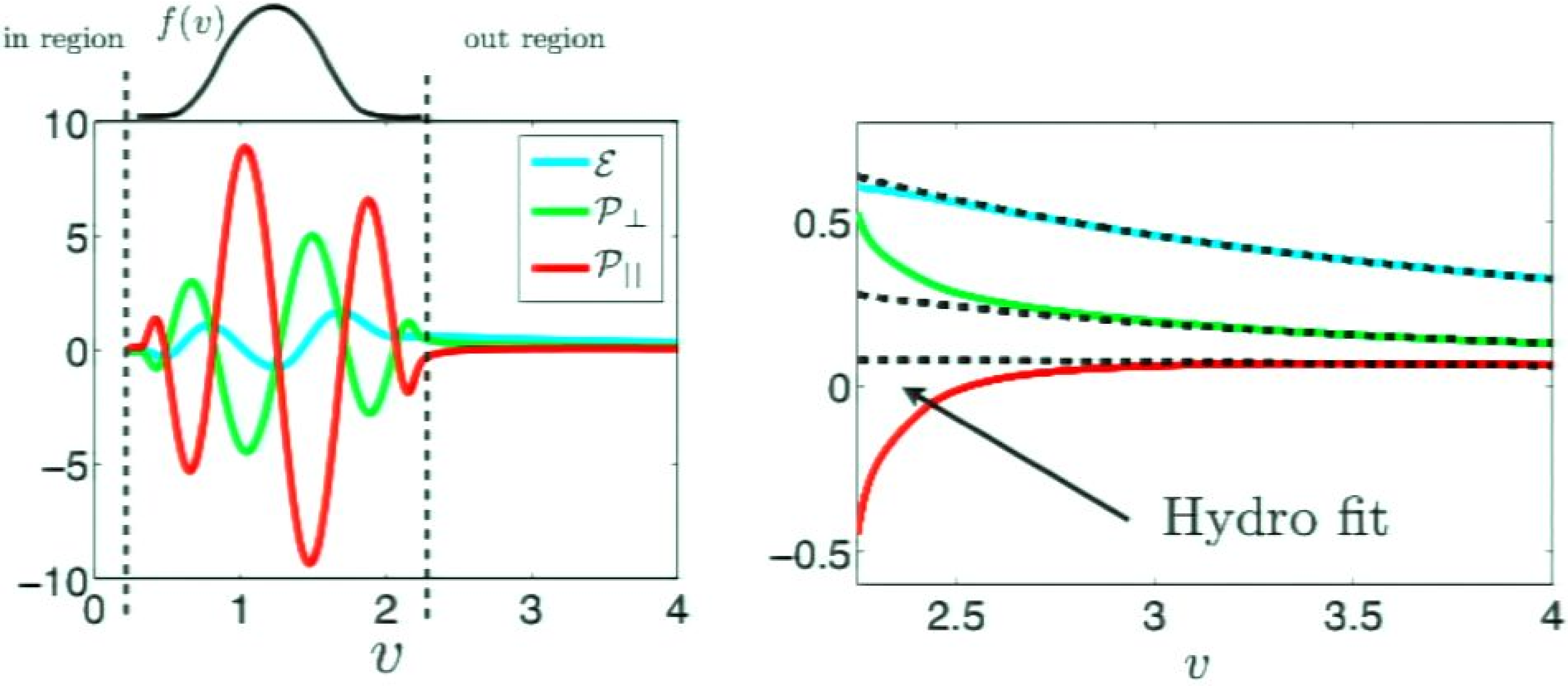
Results — $4d$ QFT physics



Late time hydro stress: (Janik & Peschanski: hep-th/0512162)
(Kinoshita, Mukohyama, Nakamura & Oda: 0807.3797)

$$\mathcal{E} = \frac{3\pi^4\Lambda^4}{4(\Lambda v)^{4/3}}\left[1 - \frac{2C_1}{(\Lambda v)^{2/3}} + \frac{C_2}{(\Lambda v)^{4/3}} + \mathcal{O}(v^{-2})\right],$$

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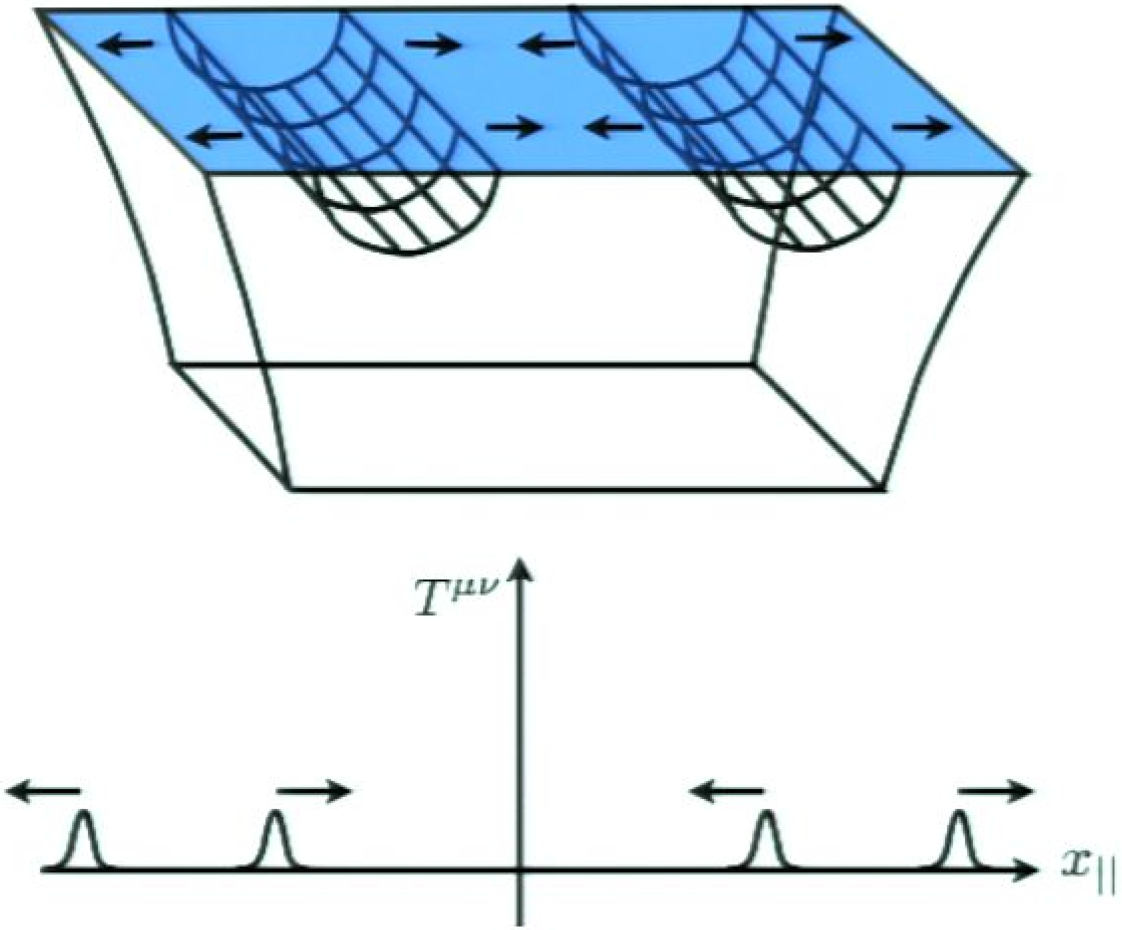
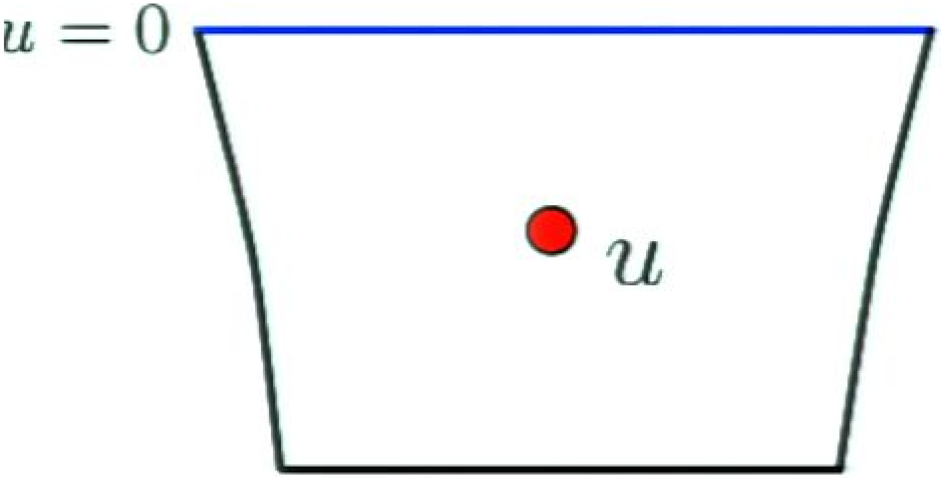
Whats next — colliding gravitational waves



Boundary conditions and expectation values



- $ds^2 = \frac{L^2}{u^2}\left[-dt^2 + d\boldsymbol{x}^2 + du^2\right].$
- distance $= \int_0^u du' \frac{L}{u} = \infty.$
- light travel time $= \int_0^u du' = u.$

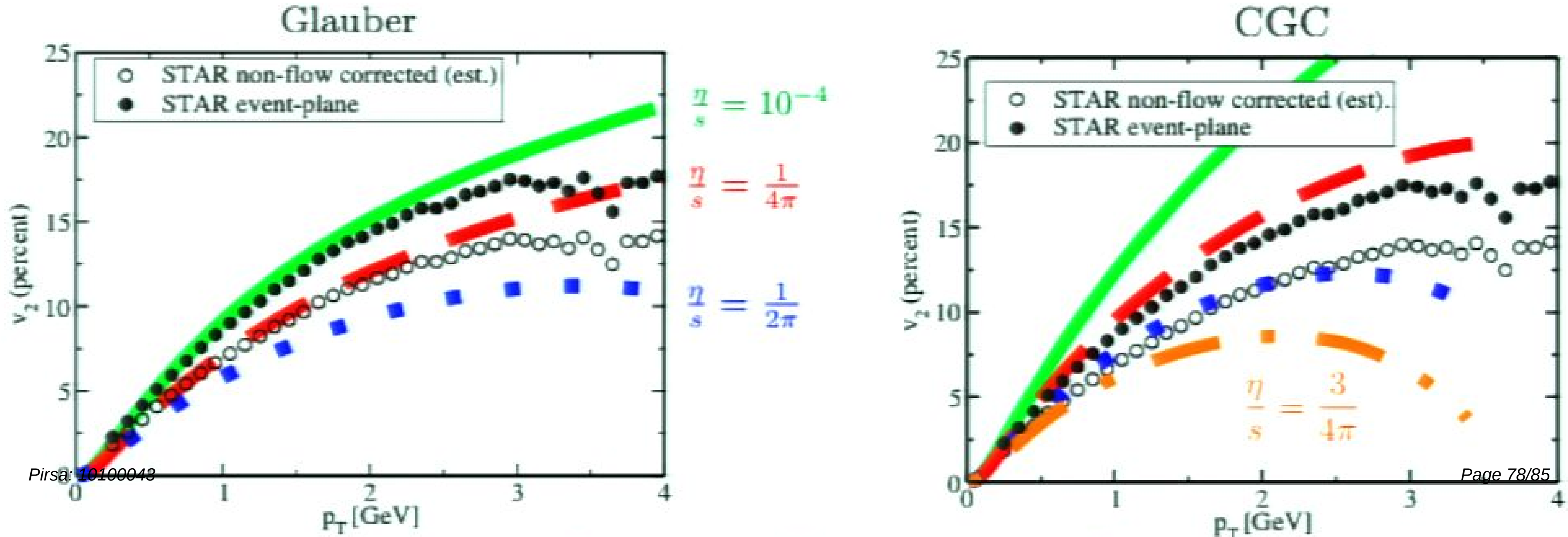
- Boundary conditions: $\lim_{u\to 0} \Phi(x,u) \to \Phi^B(x).$
- QFT observables: $\langle \mathcal{O}(x) \rangle \equiv \frac{\delta S_{\text{SUGRA}}}{\delta \Phi^B(x)}.$
- Ex: EM fields — boundary behaves as conductor.
  - $J^\mu_{\text{induced}} \propto \langle J^\mu \rangle \equiv \frac{\delta S_{\text{SUGRA}}}{\delta A^B_\mu}.$

What have we learned about hydrodynamics?

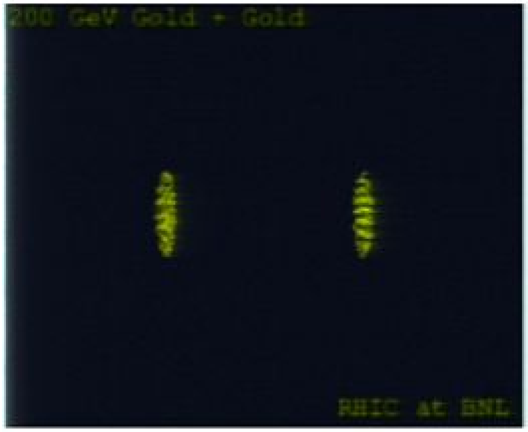
- Transport coefficients
  - Universal result: $\frac{\eta}{s} = \frac{1}{4\pi}$ [Kovtun, Son & Starinets: hep-th/0405231]
- Systematic construction of 2nd order hydrodynamics

$$T_{\mu\nu} = T^{(0)}_{\mu\nu} + T^{(1)}_{\mu\nu} + T^{(2)}_{\mu\nu} + \ldots .$$

ideal viscous causal



A remarkable result from RHIC



- **Many scatterings ⇒ fluid behavior.**
- **Hydrodynamic simulations:**
  - **Applicable 1 fm/c after collision!** [Heinz: nucl-th/0407067]
  - **Viscosity** $\frac{\eta}{s} \lesssim 0.5.$ [Romatschke & Luzum: 0804.4015]

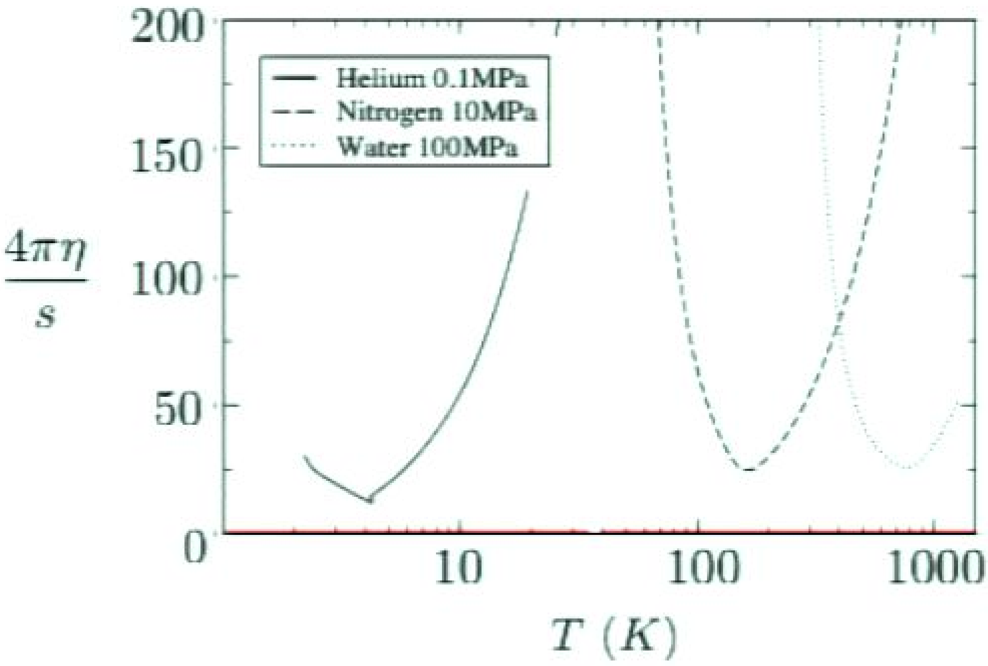
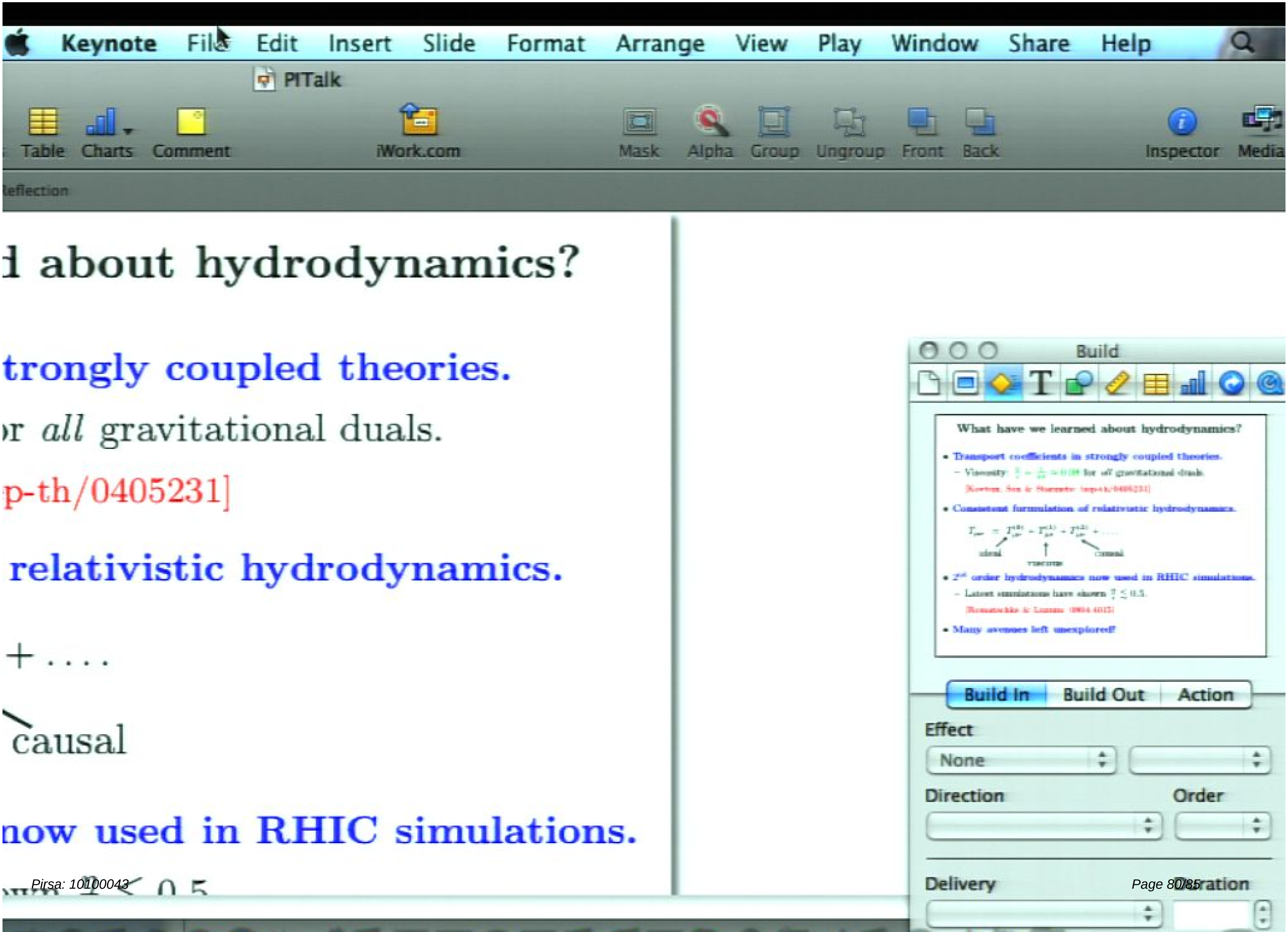


Figure adapted from [Kovtun, Son & Starinets: hep-th/0405231]



about hydrodynamics?

trongly coupled theories.

r *all* gravitational duals.

p-th/0405231]

relativistic hydrodynamics.

$+ \ldots .$

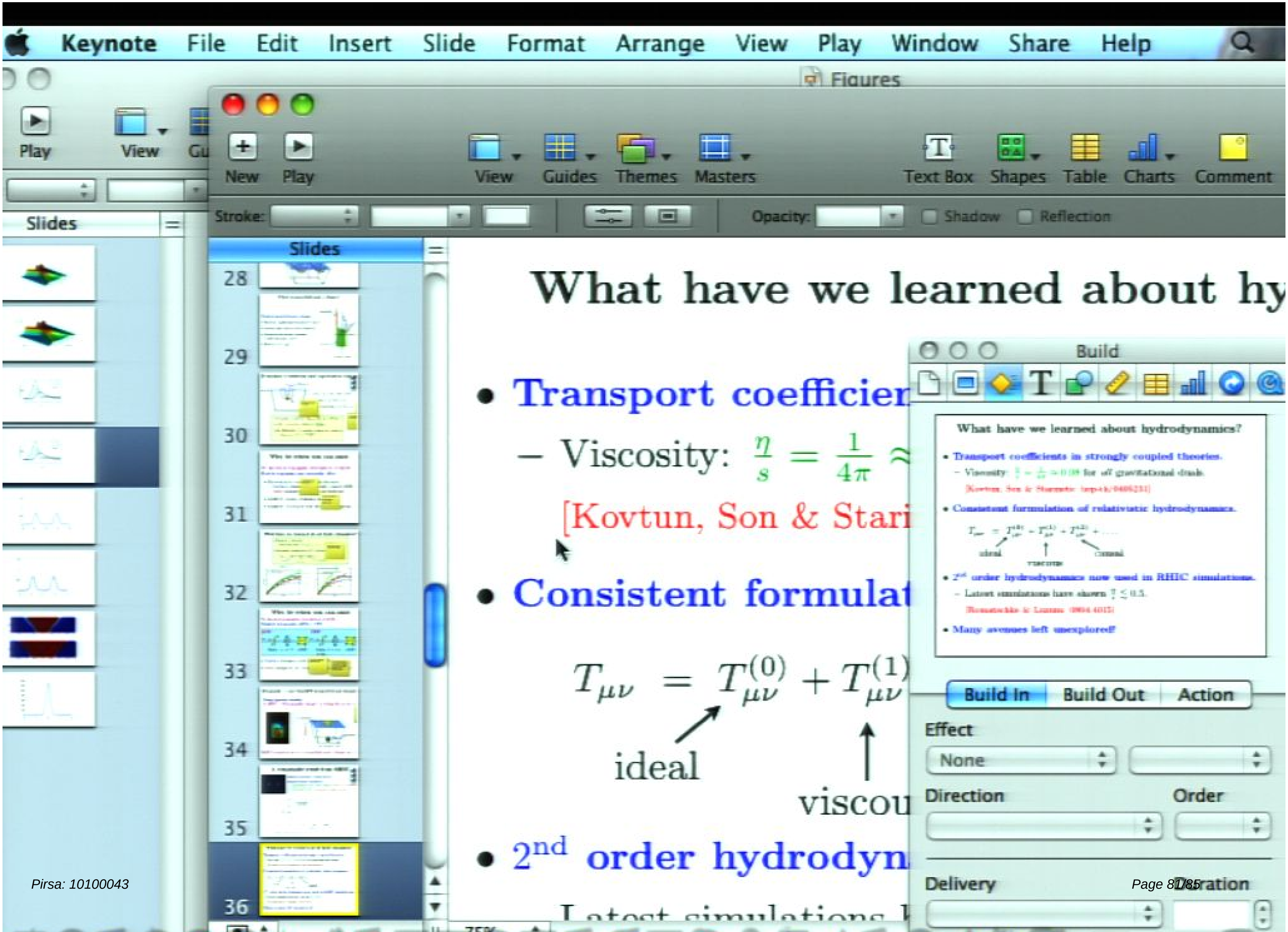
causal

ow used in RHIC simulations.

$\lesssim 0.5$

What have we learned about hydrodynamics?

- Transport coefficients in strongly coupled theories.
  - Viscosity: … for *all* gravitational duals.
- Consistent formulation of relativistic hydrodynamics.
- 2nd order hydrodynamics now used in RHIC simulations.
  - Latest simulations have shown $\lesssim 0.5$.
- Many avenues left unexplored!



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    [Kovtun, Son & Stari
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ideal

viscou

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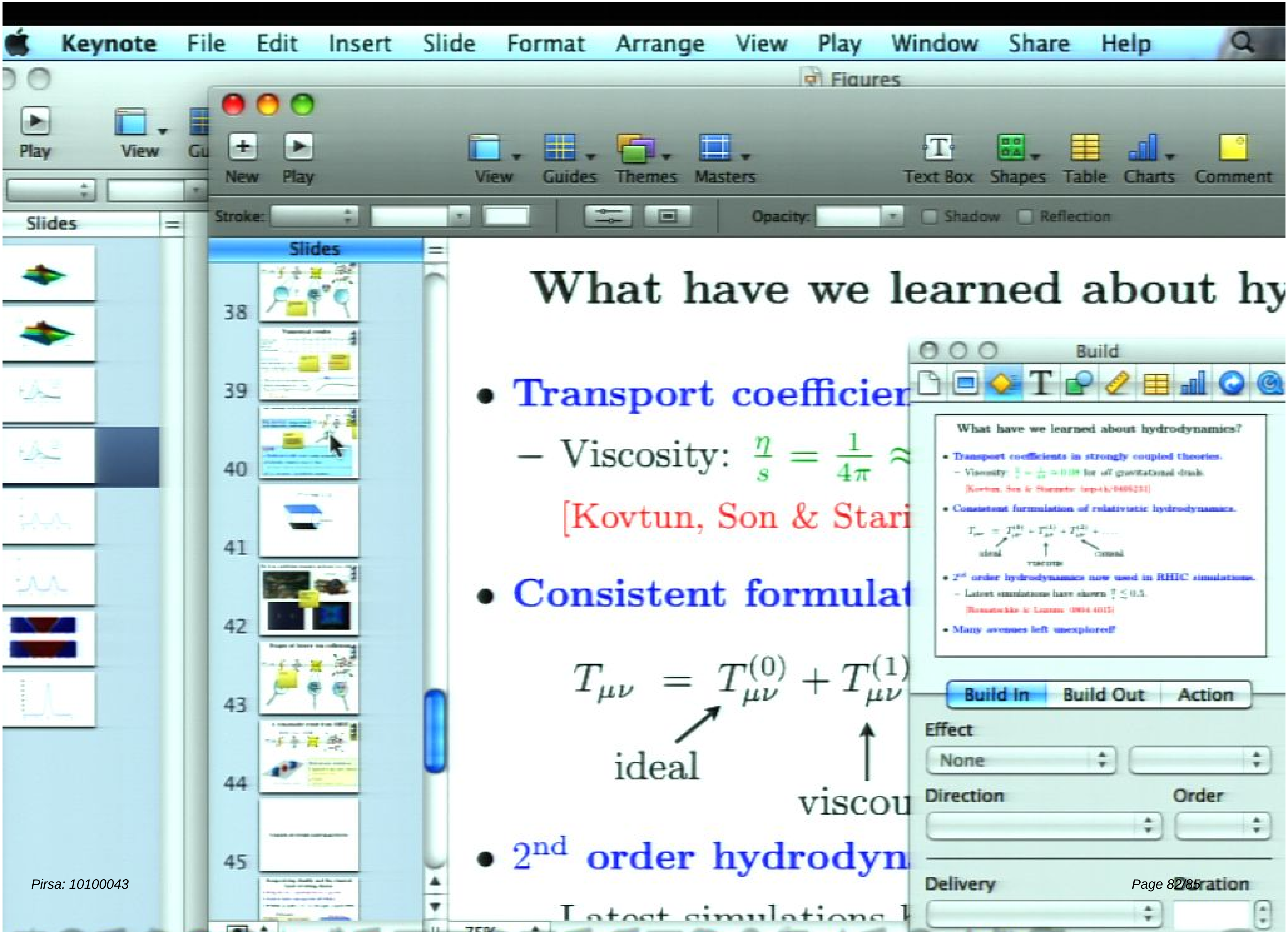

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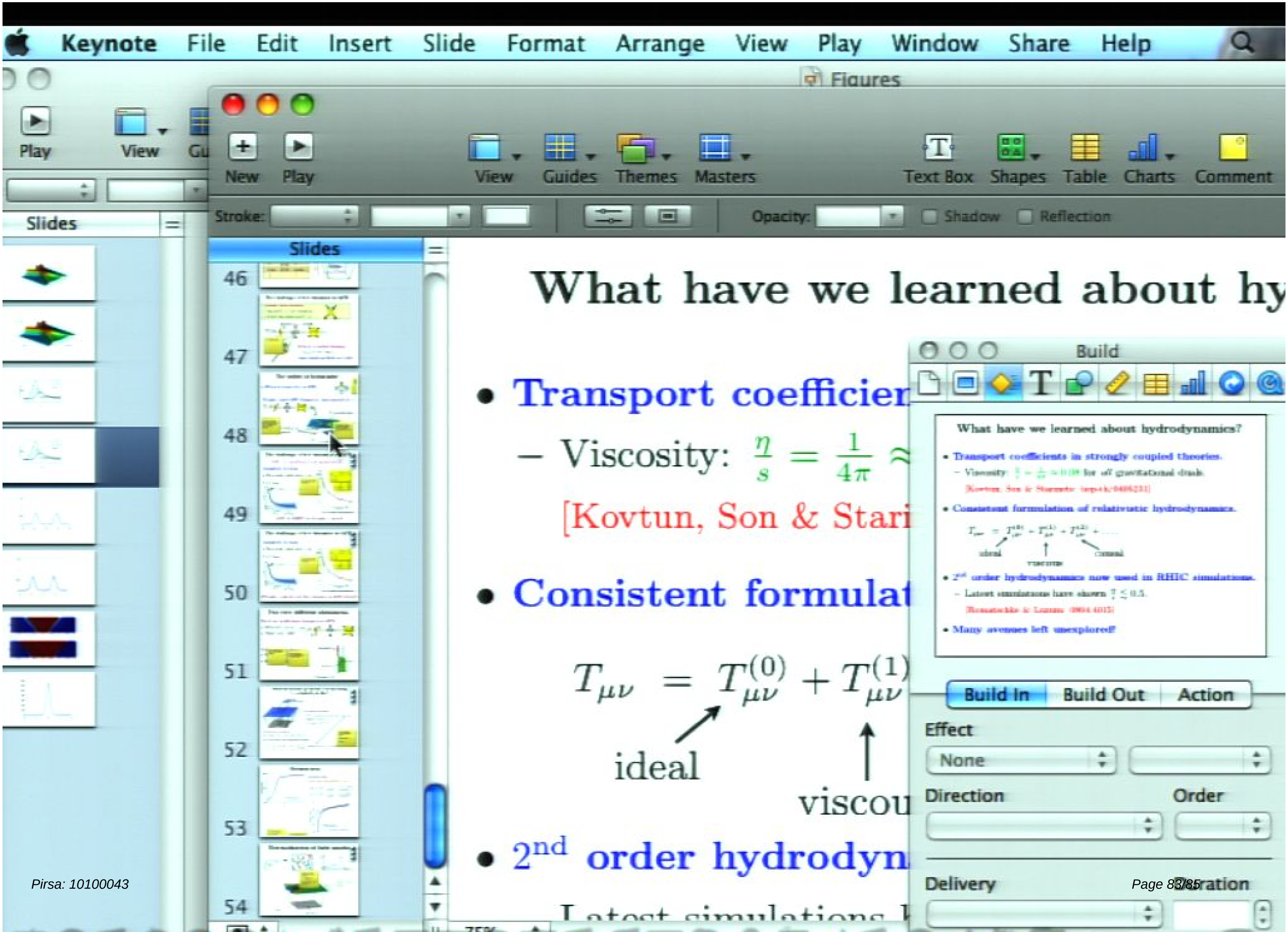

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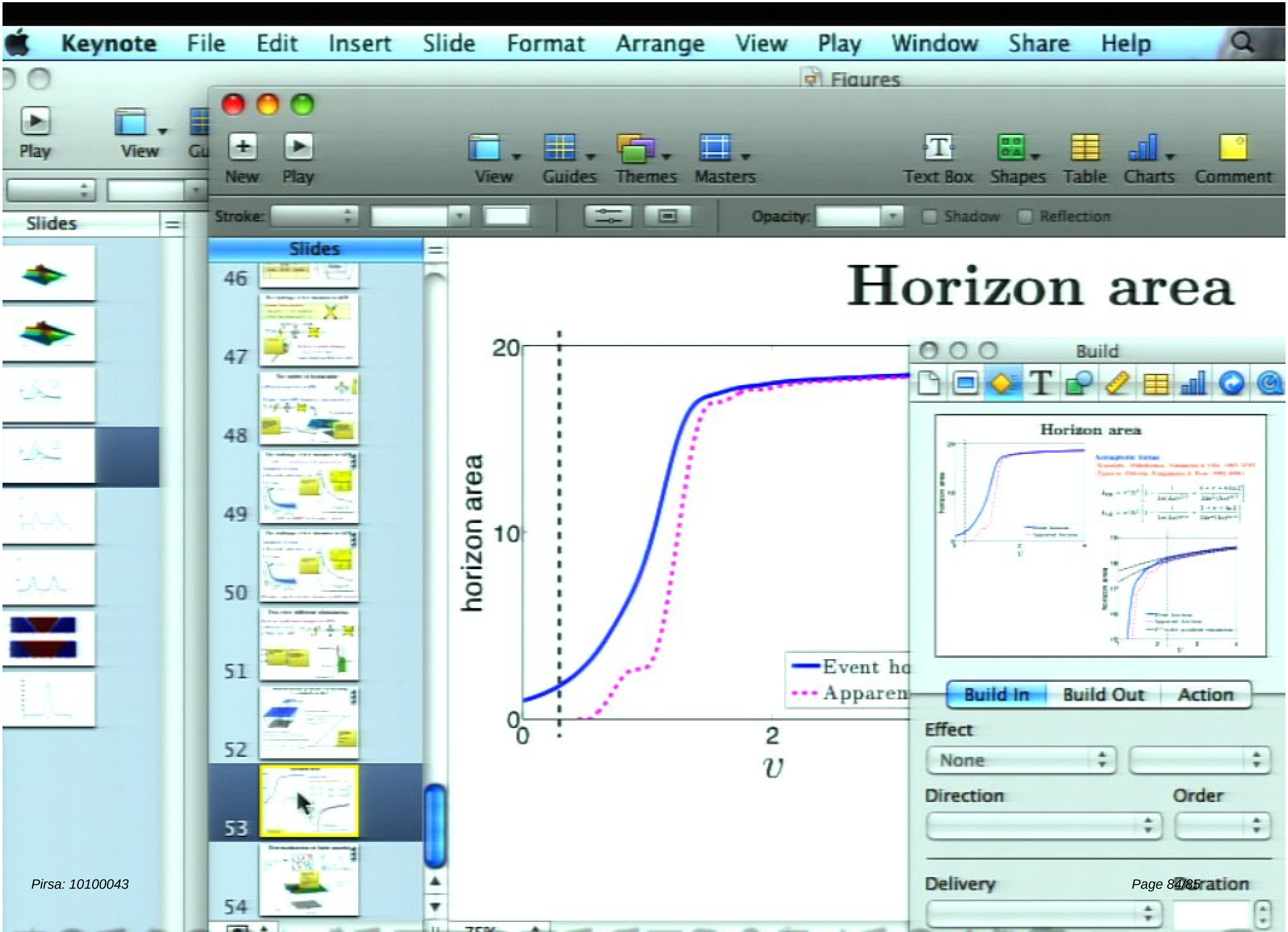
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Masters
Text Box
Shapes
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Charts
Comment
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Opacity:
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Action
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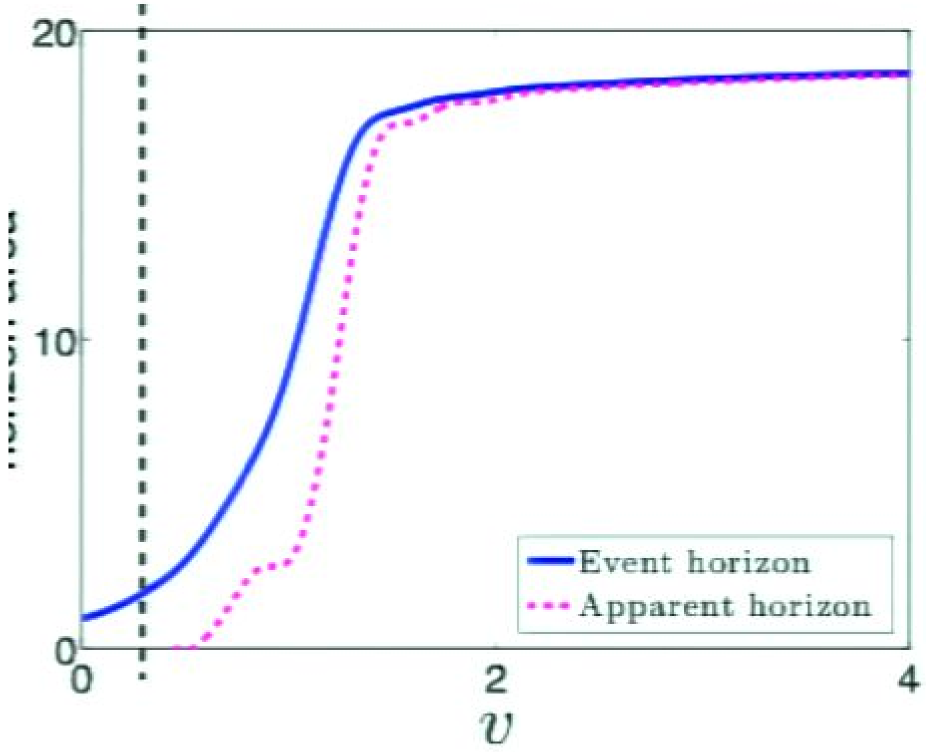
Order
Delivery
Duration

Effect: None

Horizon area



Asymptotic forms:

(Kinoshita, Mukohyama, Nakamura & Oda: 0807.3797)
(Figueras, Hubeny, Rangamani & Ross: 0902.4696)

$$A_{\mathrm{EH}} = \pi^3\Lambda^2\left[1 - \frac{1}{2\pi(\Lambda v)^{2/3}} + \frac{6+\pi+6\ln 2}{24\pi^2(\Lambda v)^{4/3}}\right],$$

$$A_{\mathrm{AH}} = \pi^3\Lambda^2\left[1 - \frac{1}{2\pi(\Lambda v)^{2/3}} + \frac{2+\pi+\ln 2}{24\pi^2(\Lambda v)^{4/3}}\right].$$