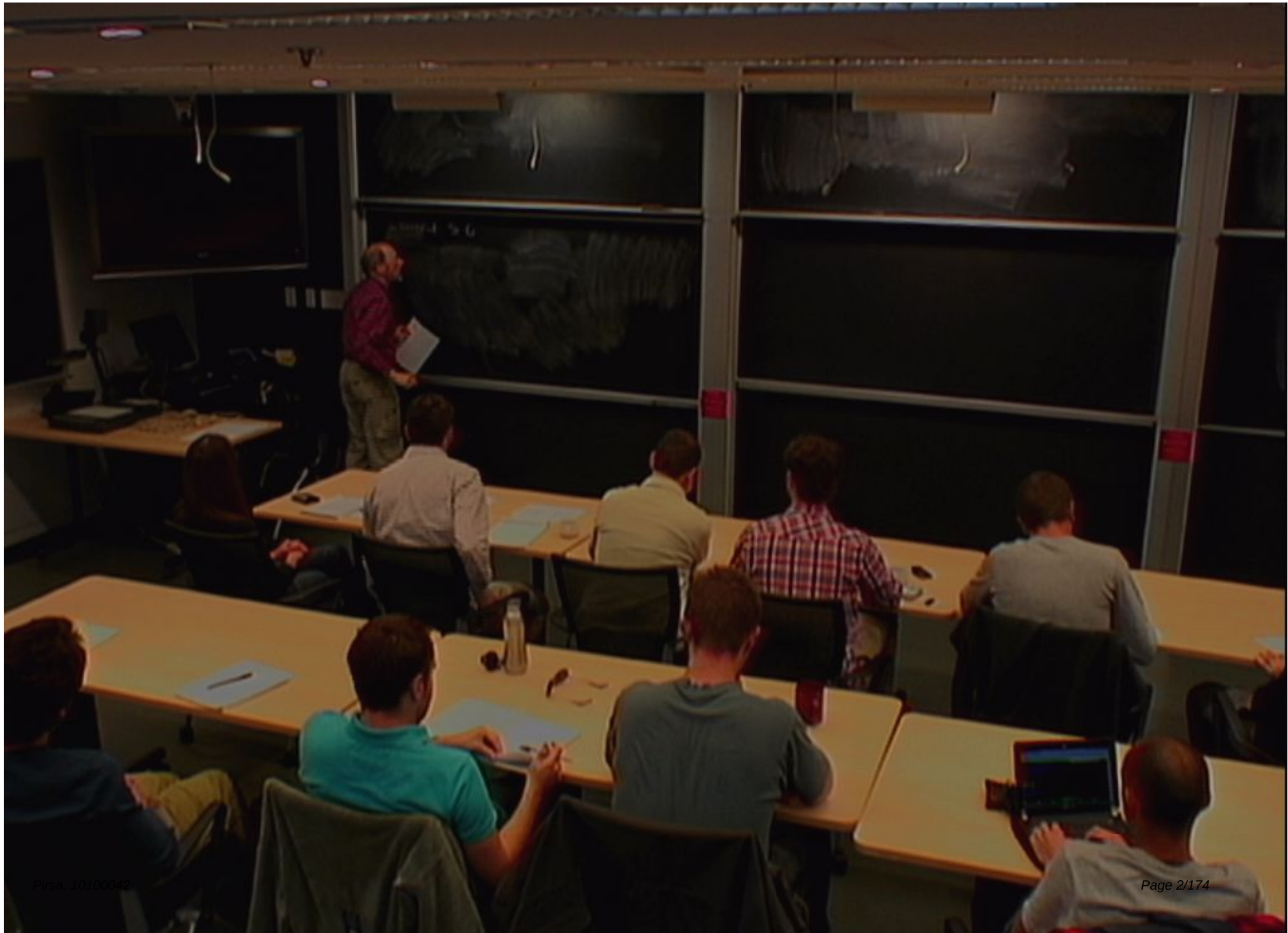


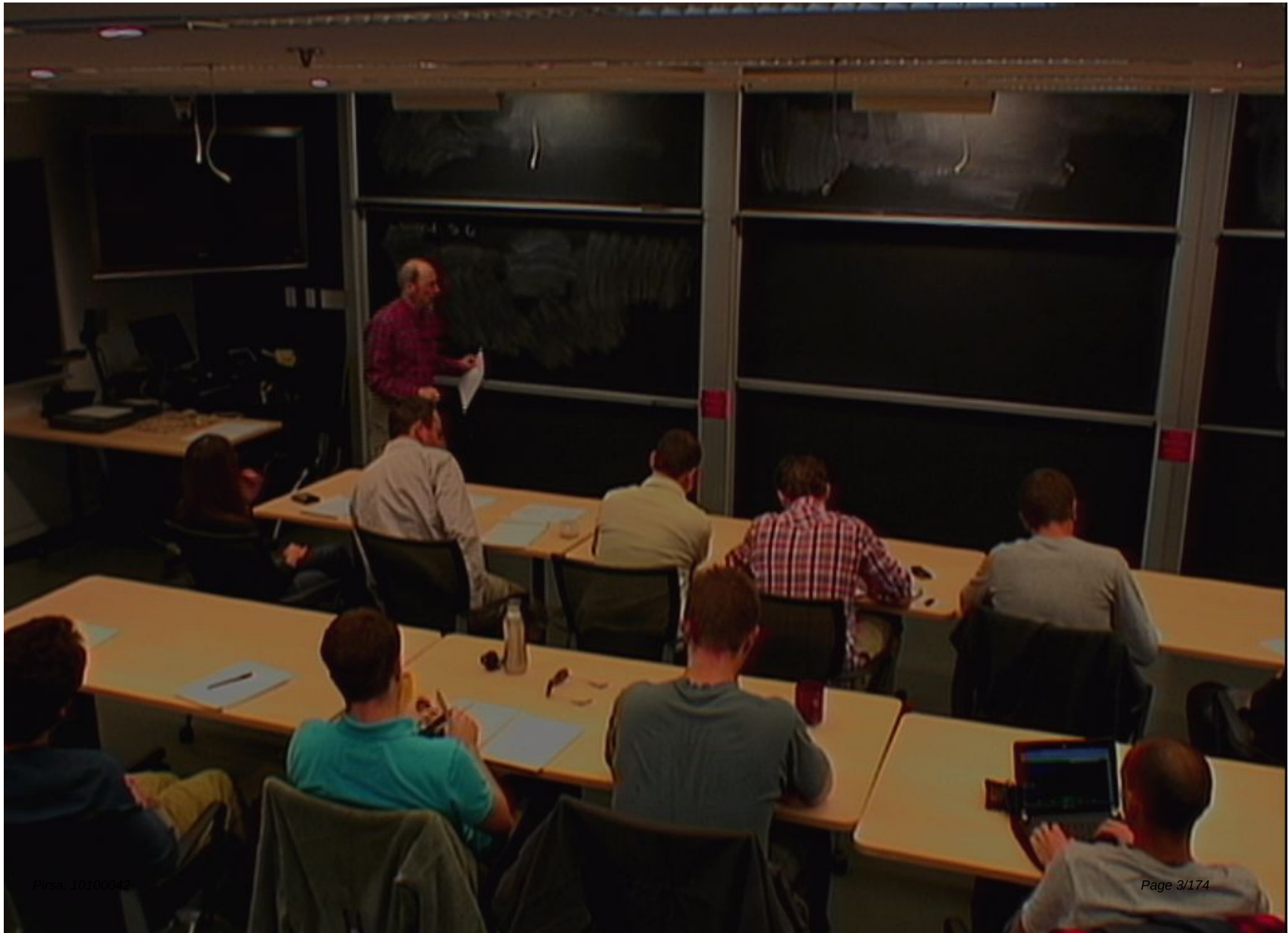
Title: An Invitation to Causal Sets - Lecture 5

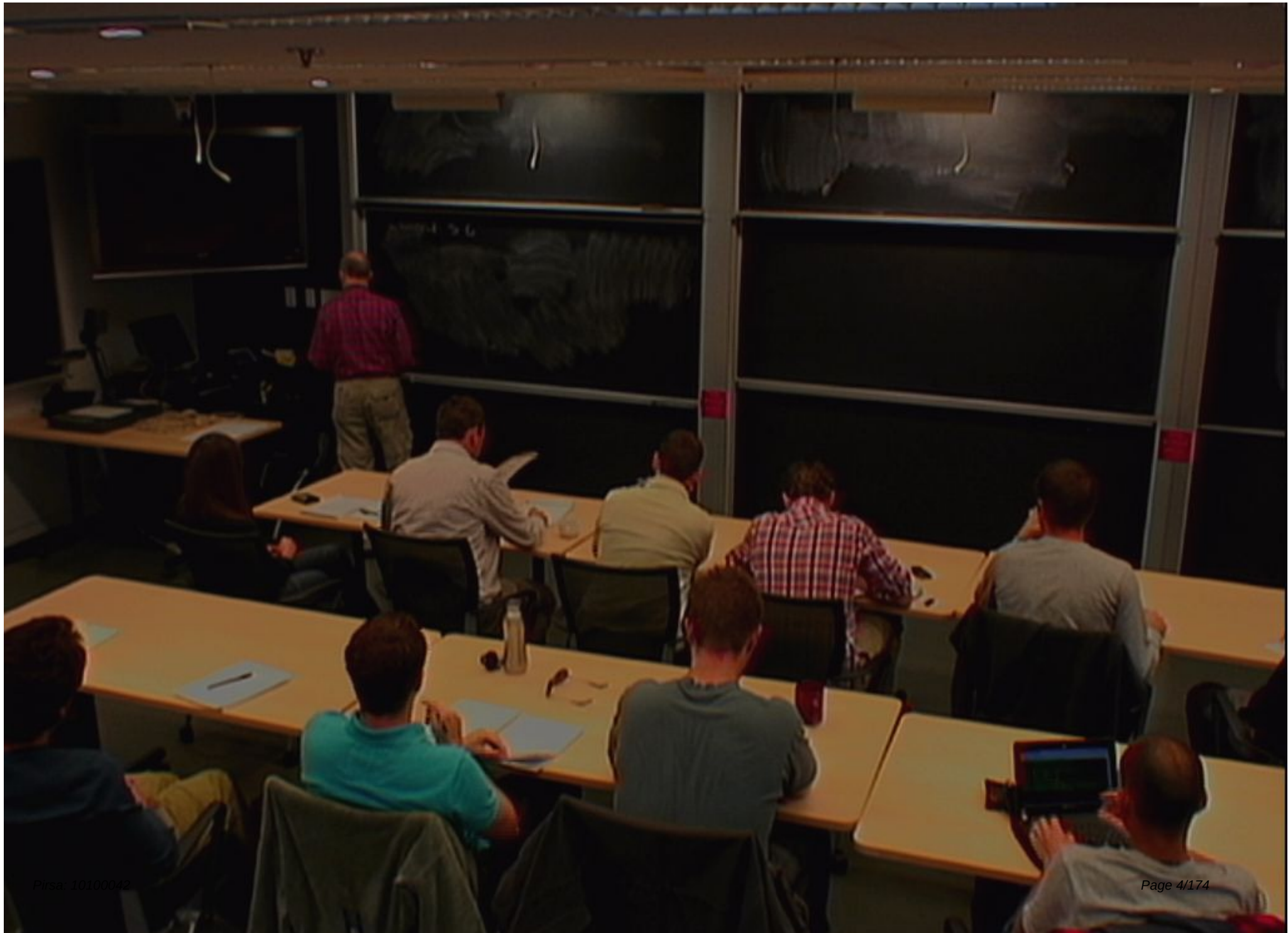
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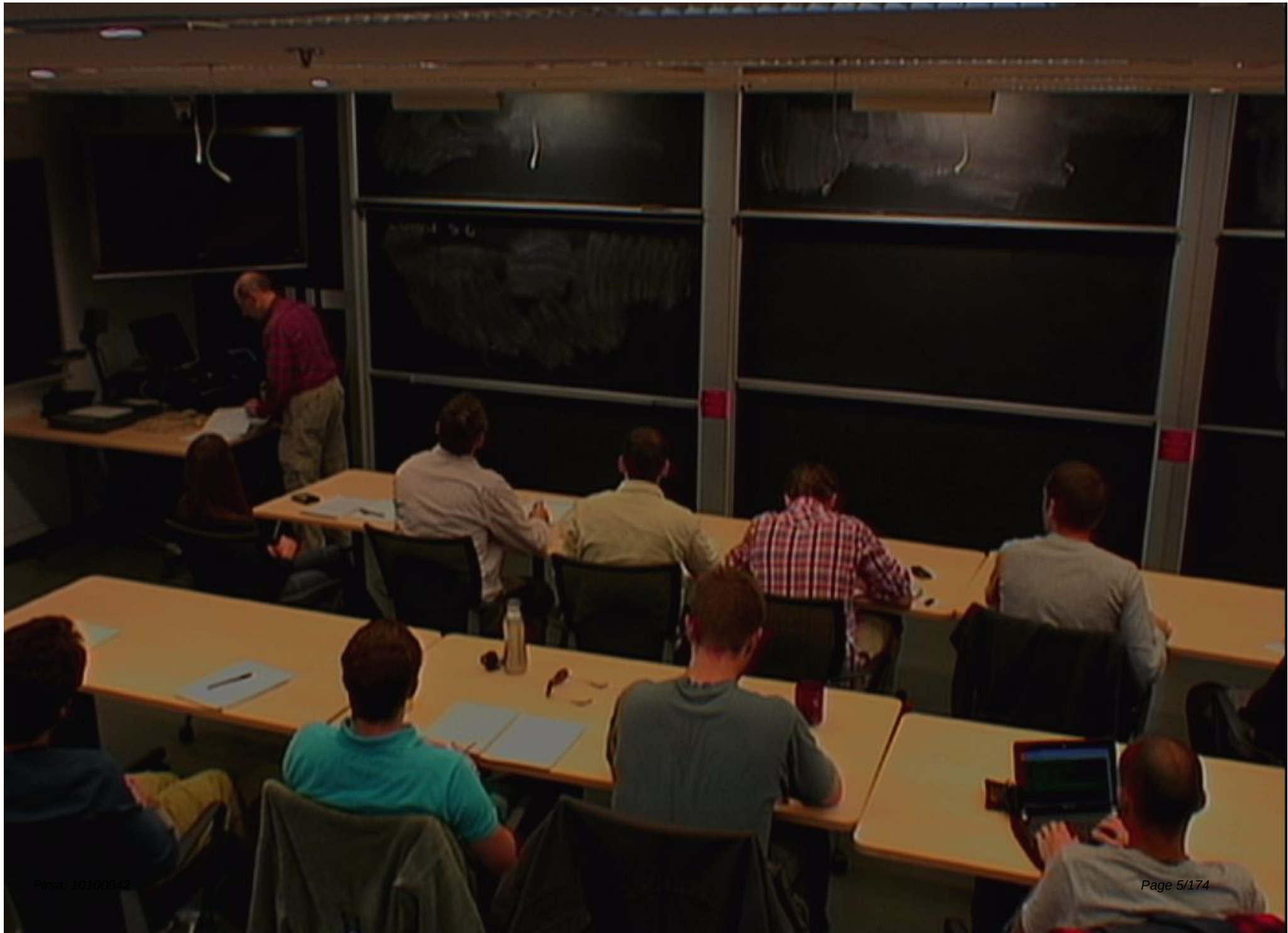
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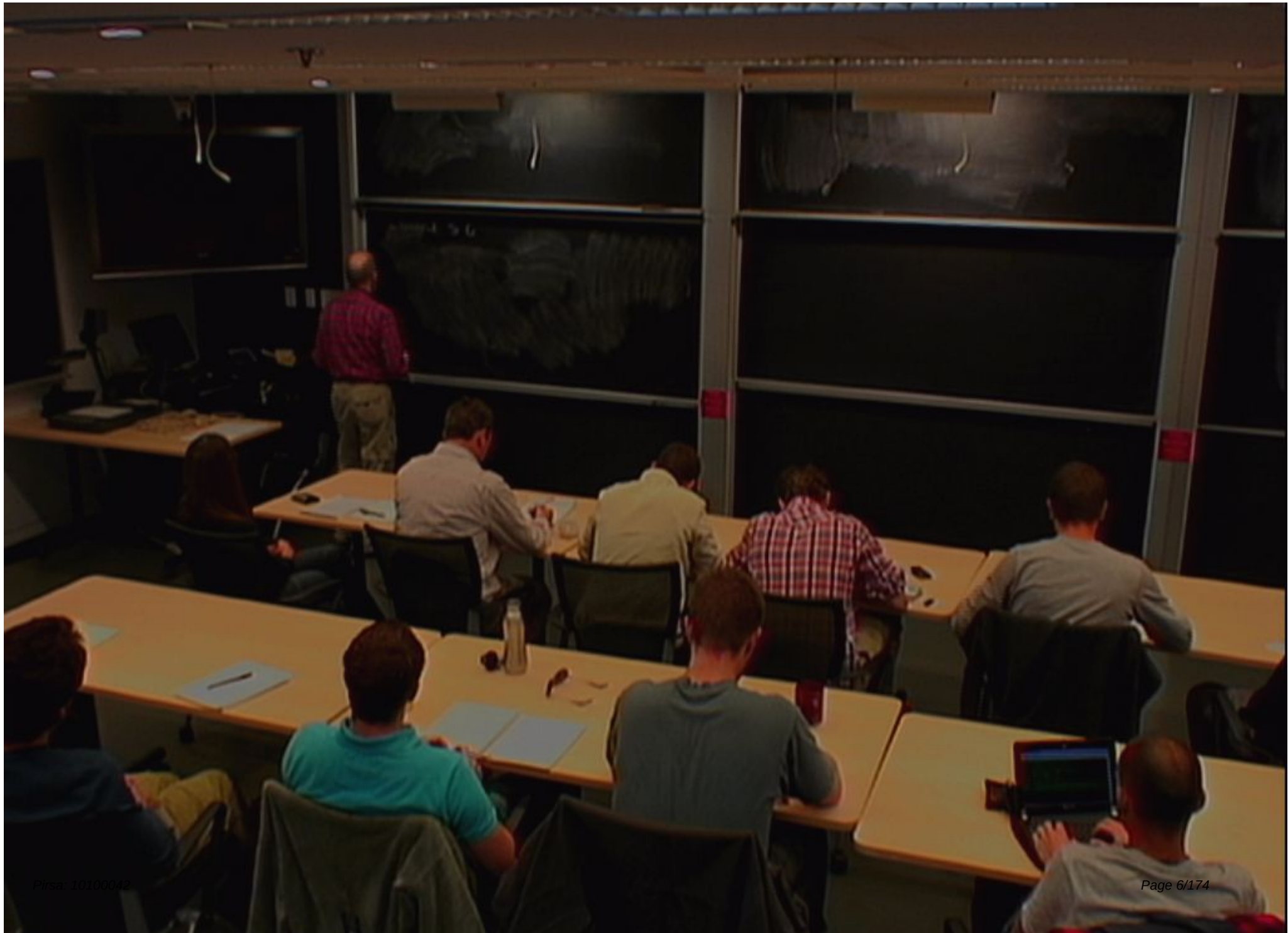
Abstract: Introduction to the causal set approach to quantum gravity and overview of current research in causal set theory

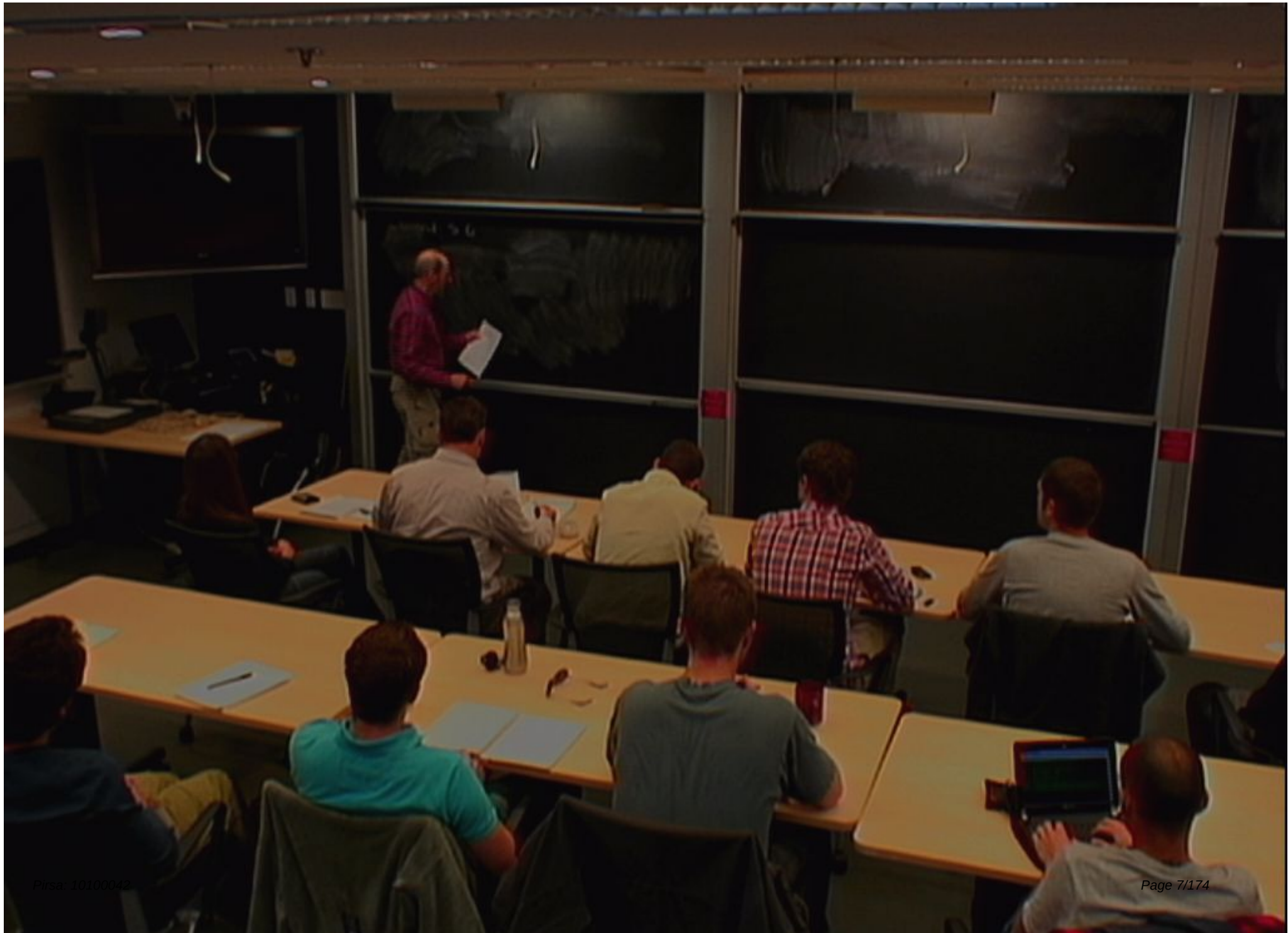


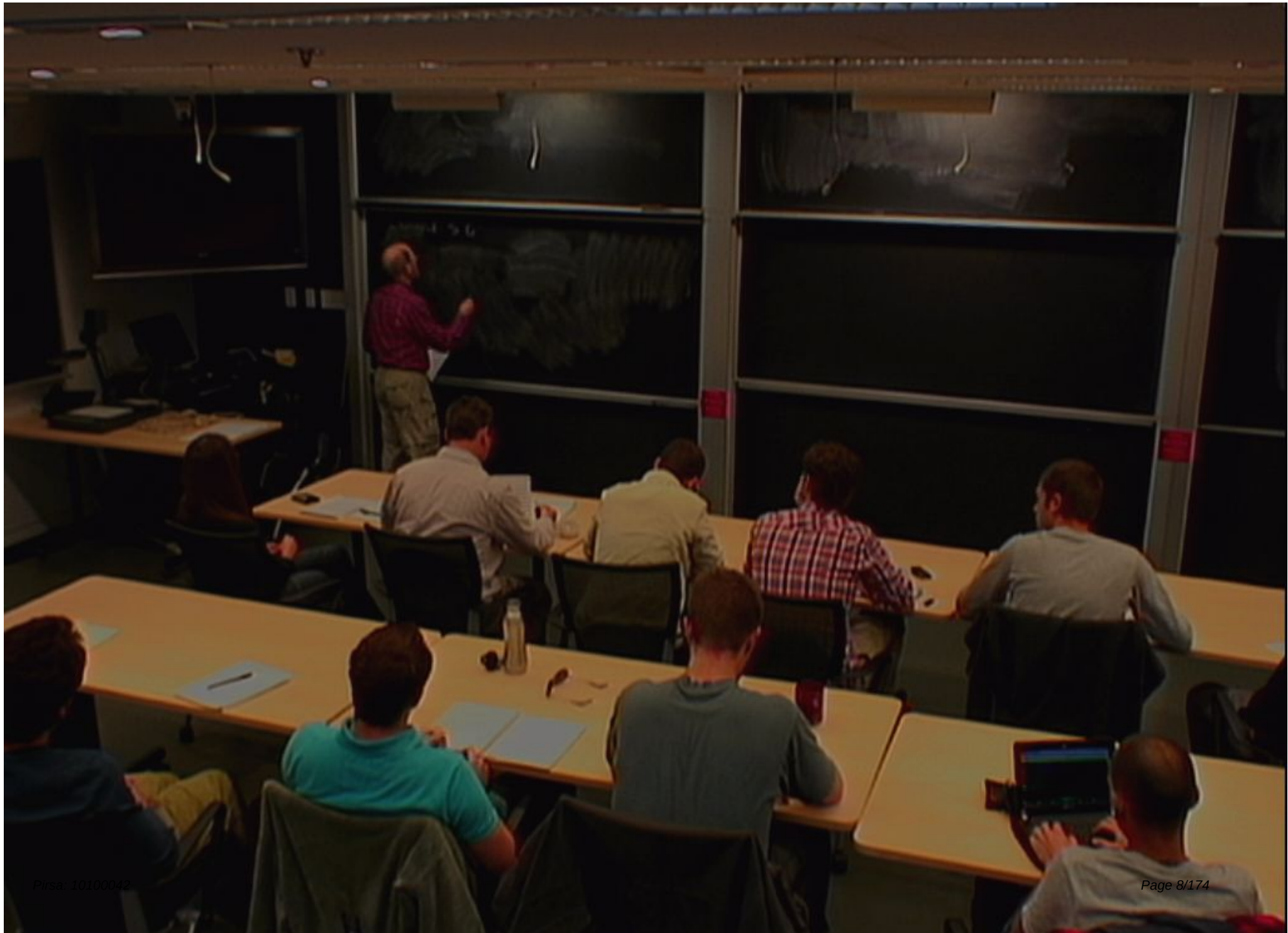


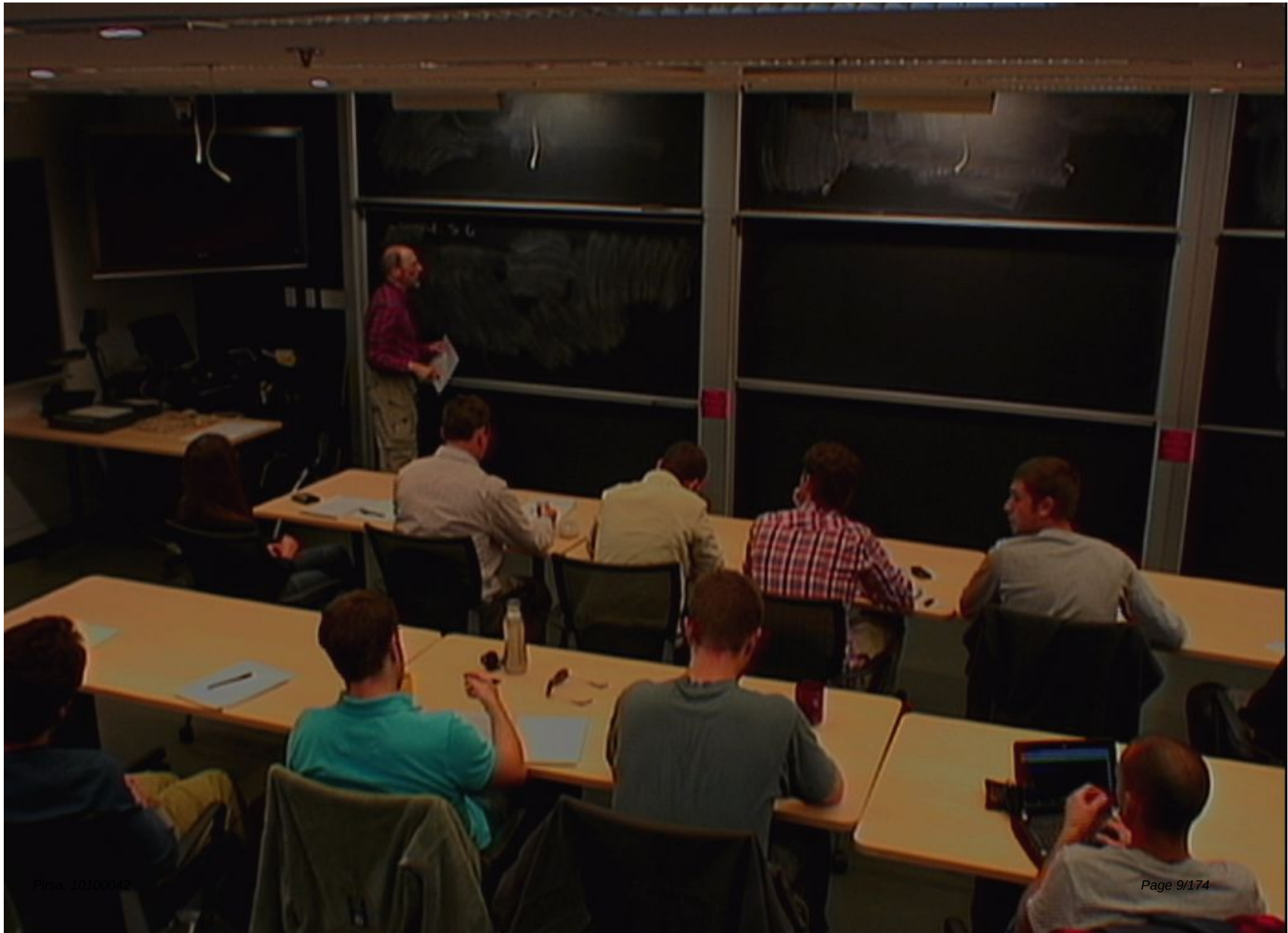


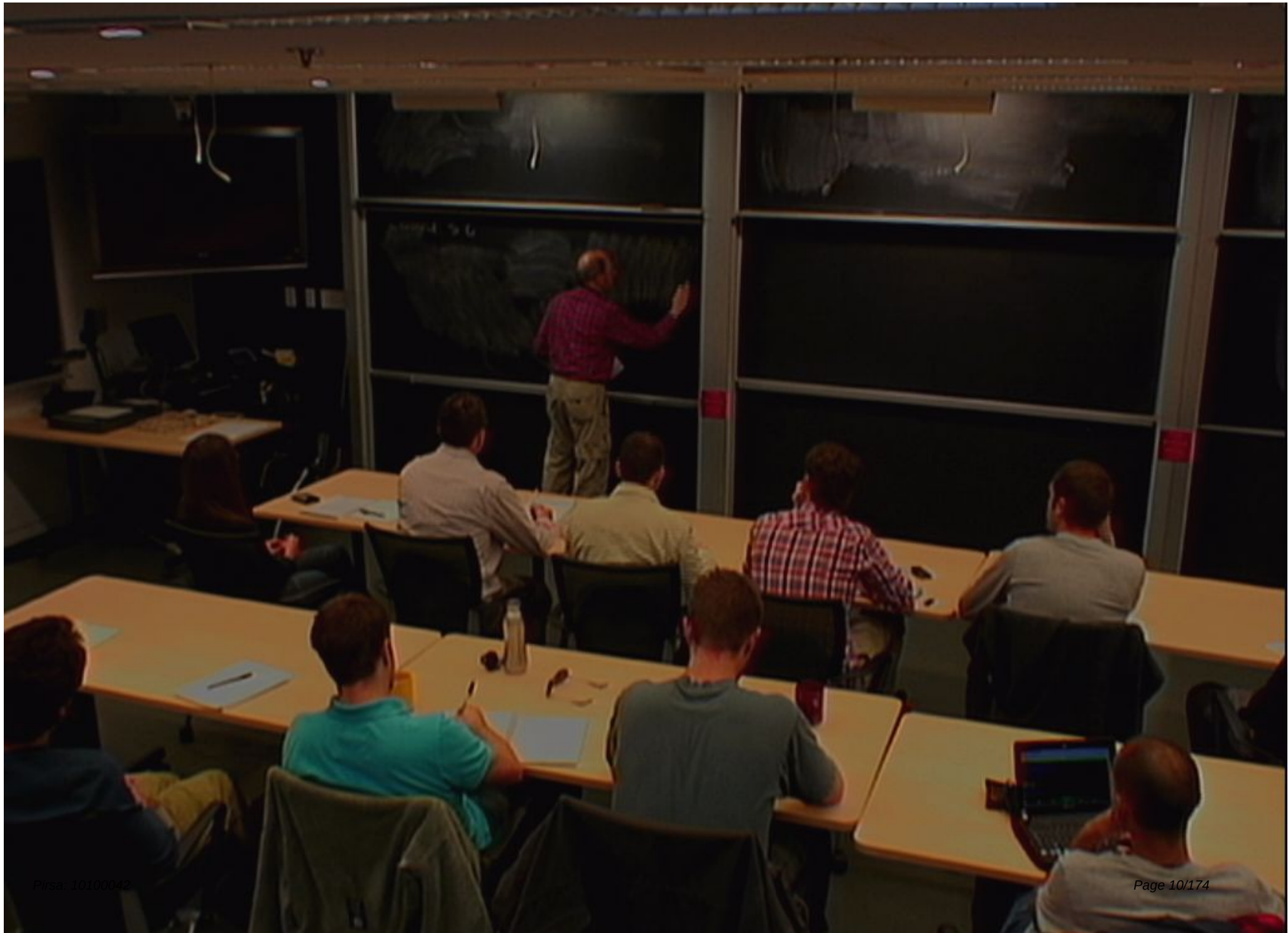


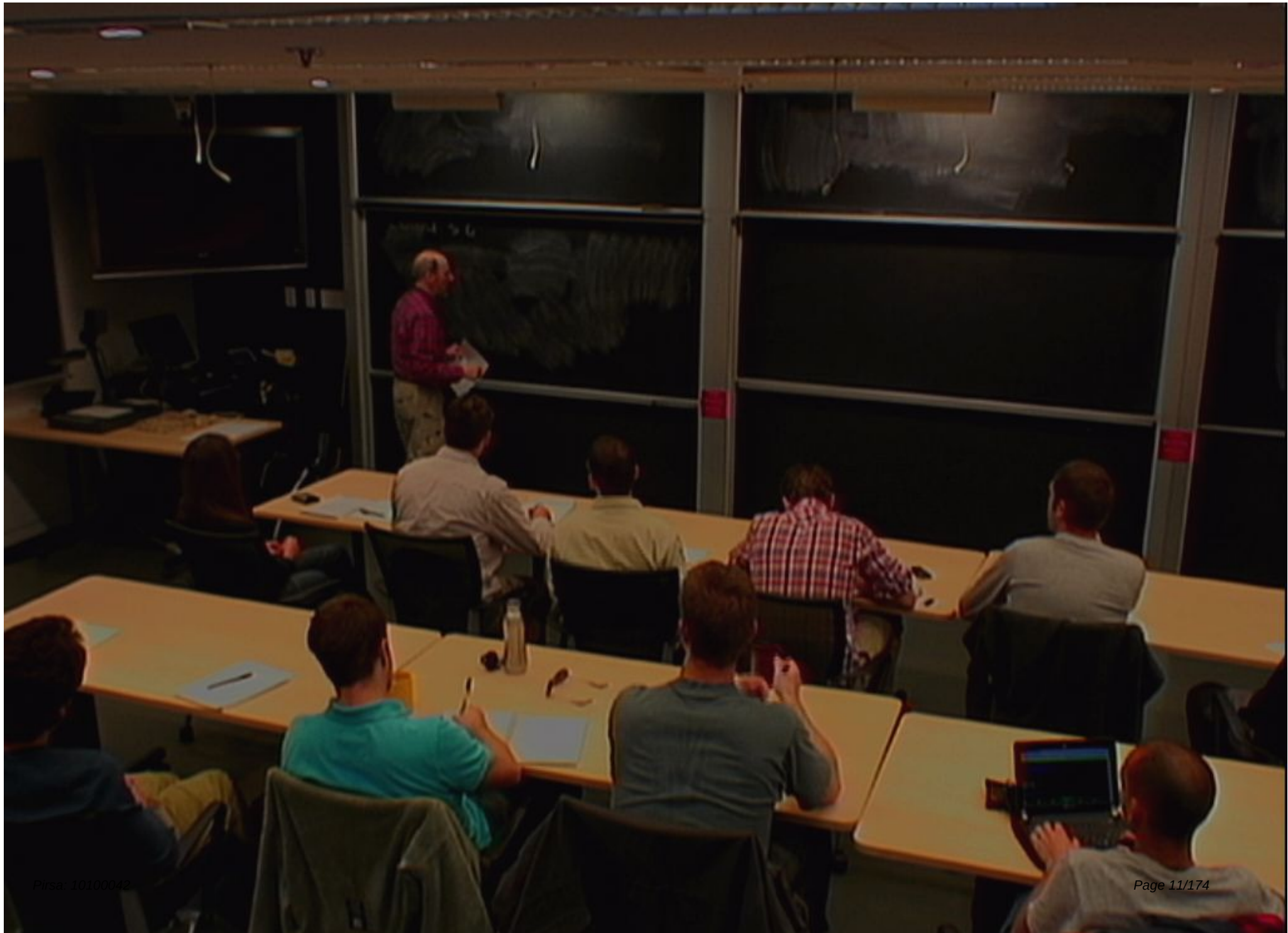








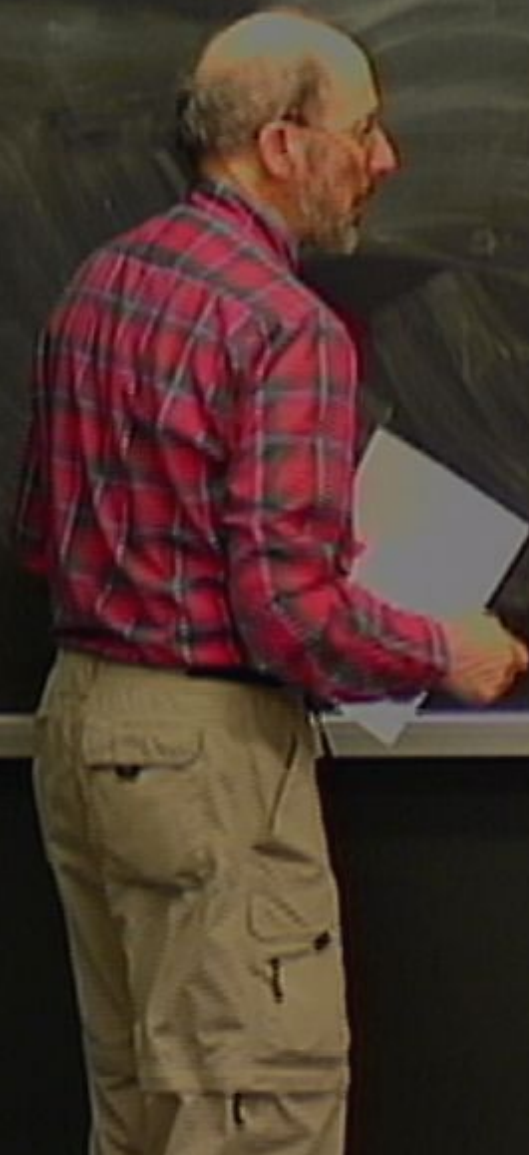


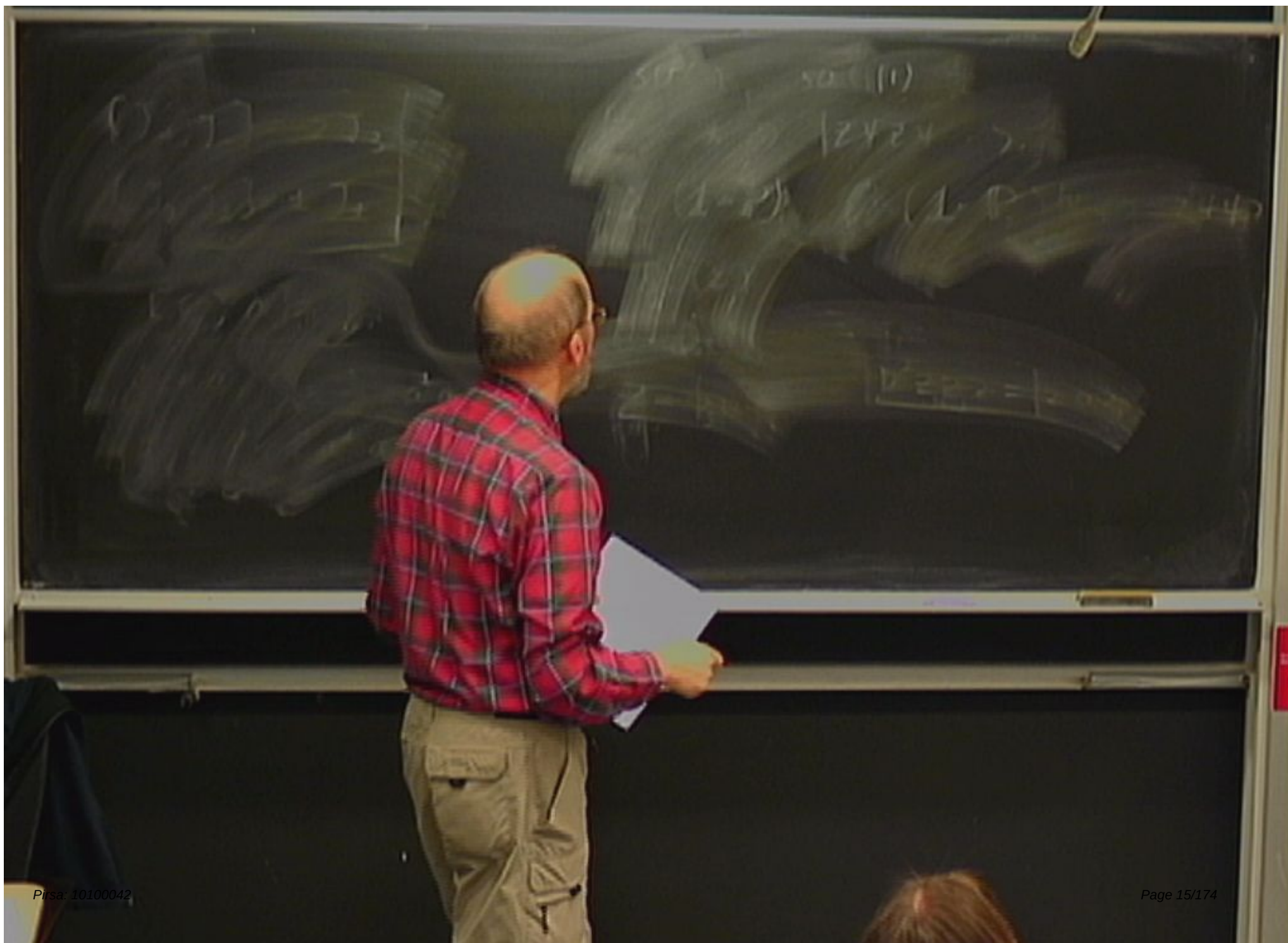


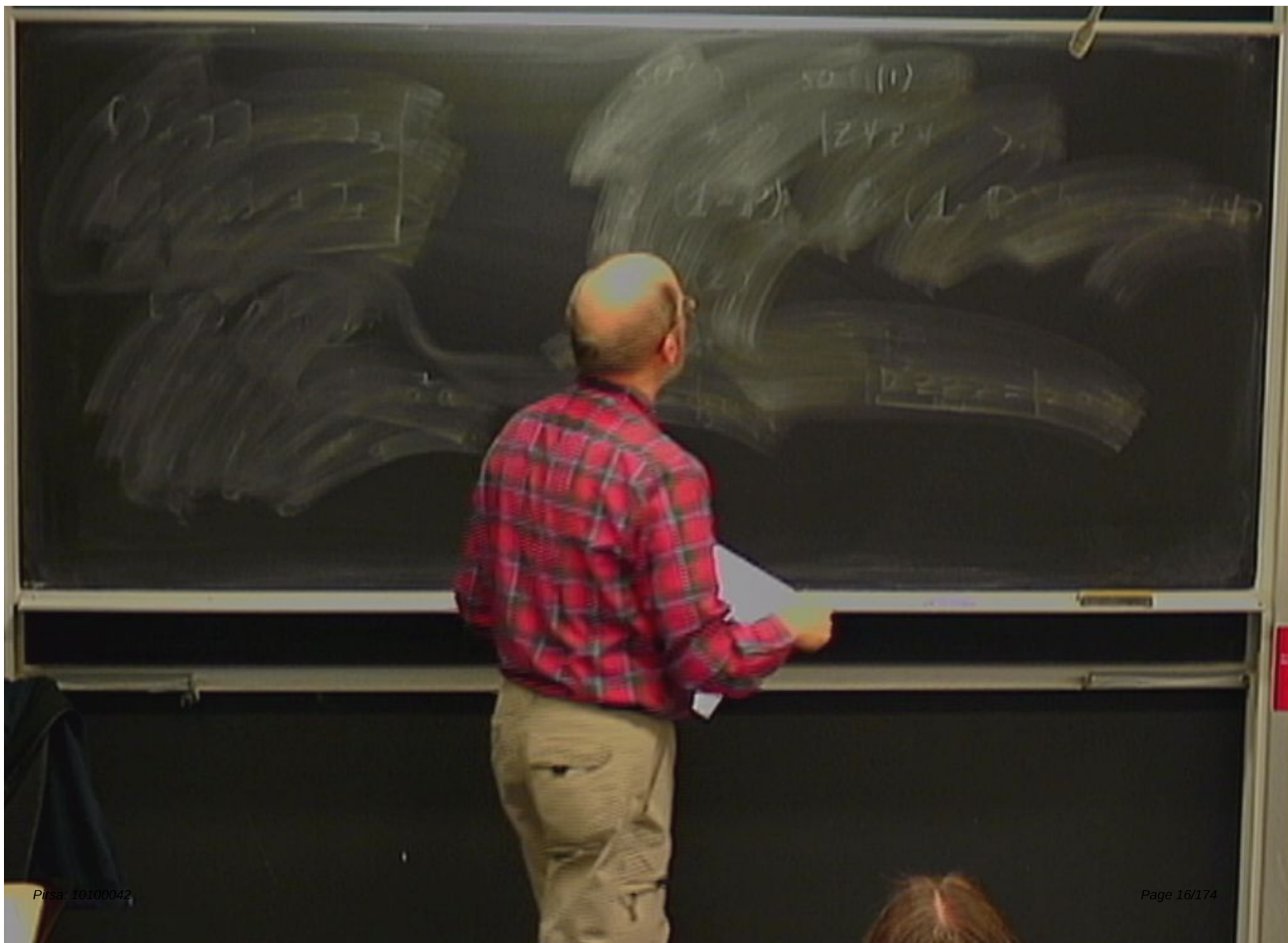
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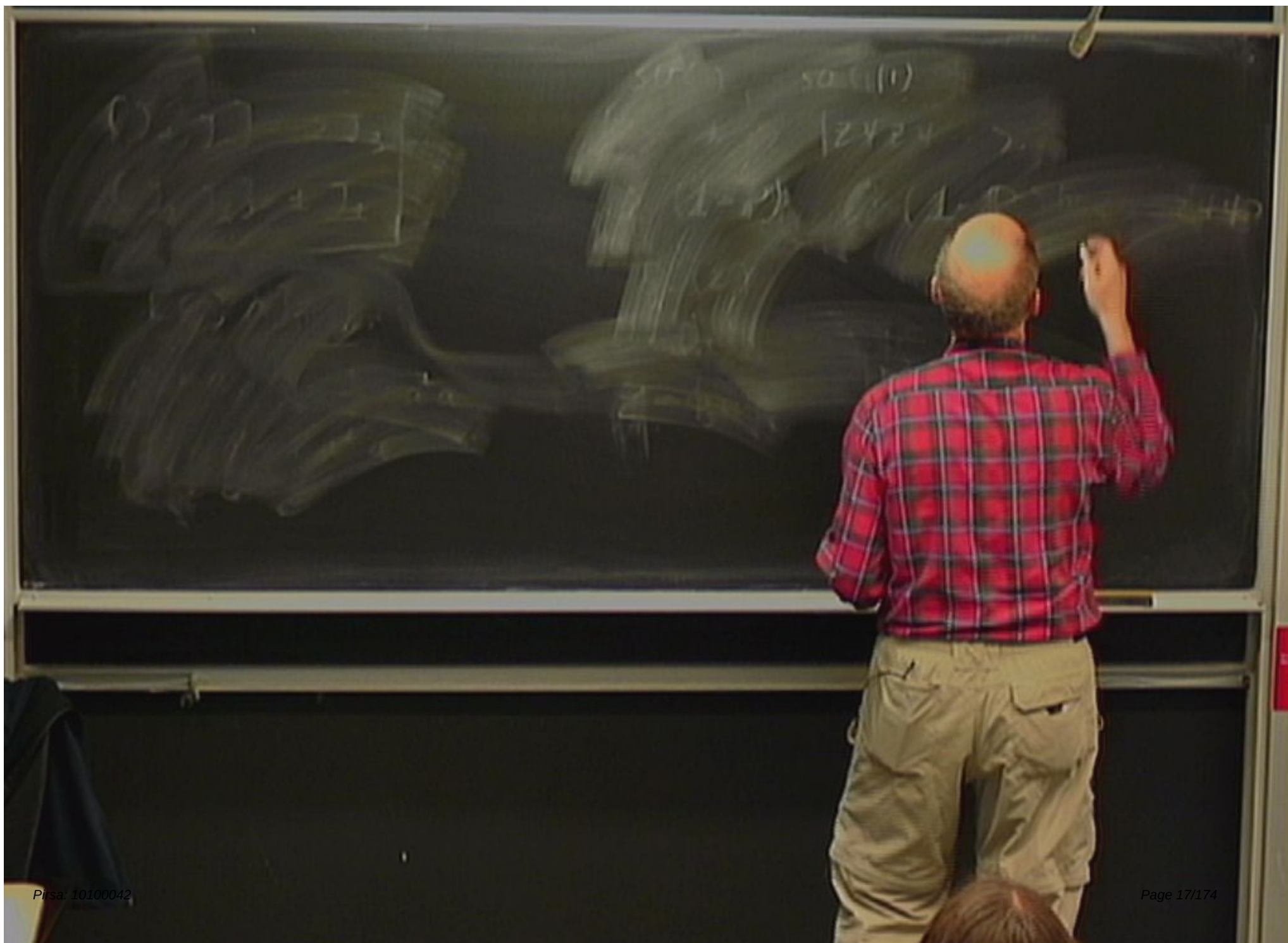
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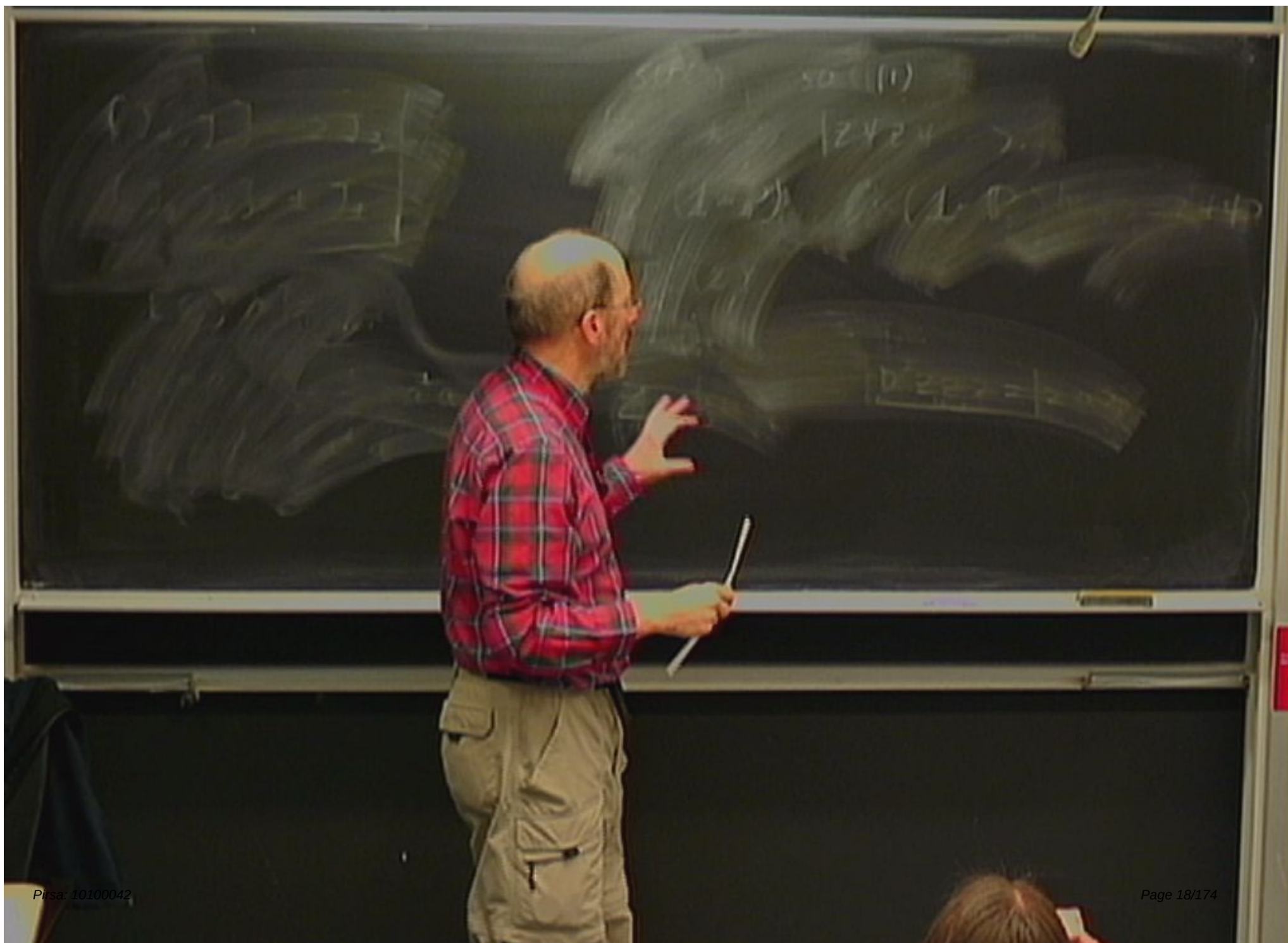
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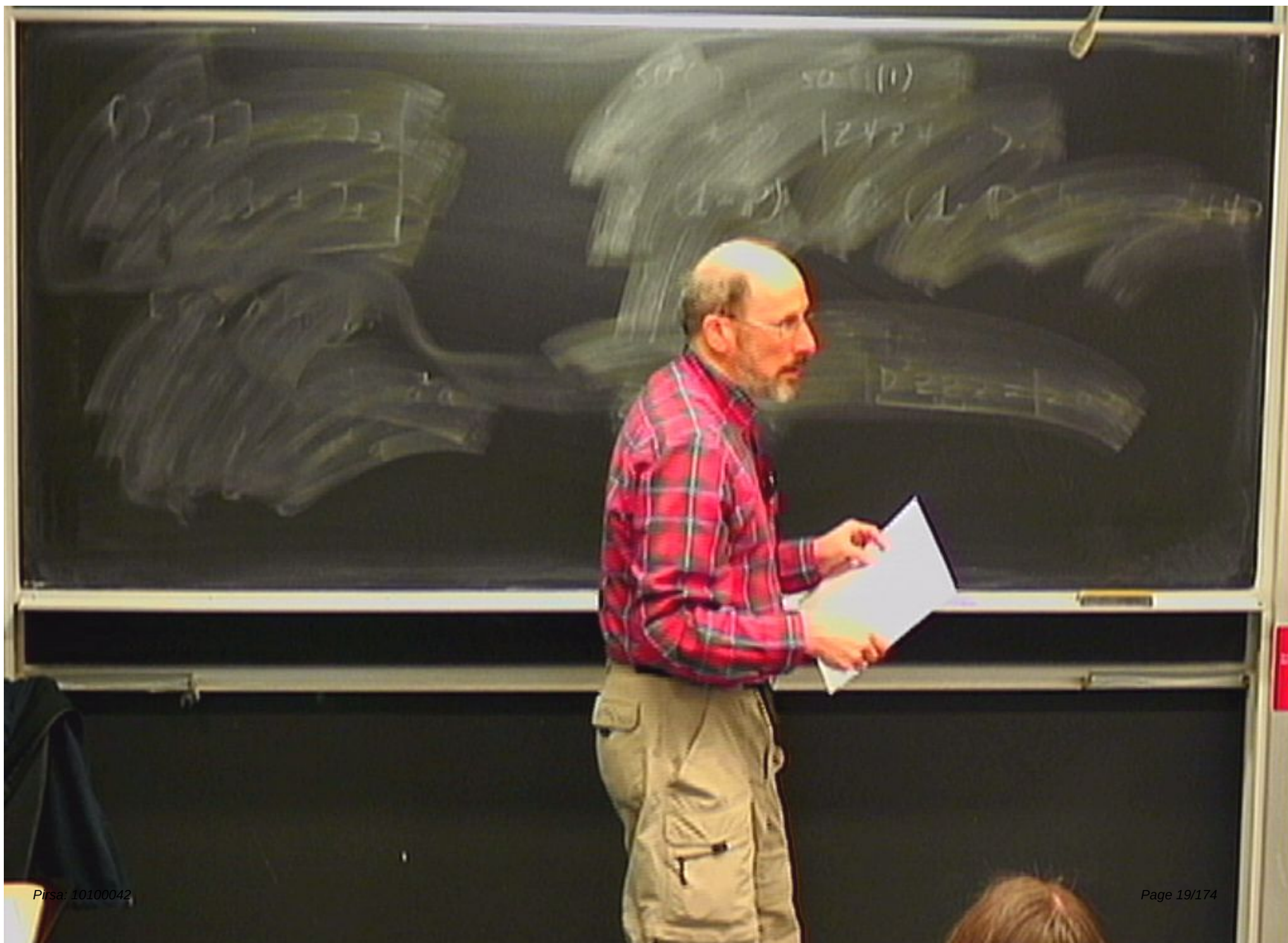


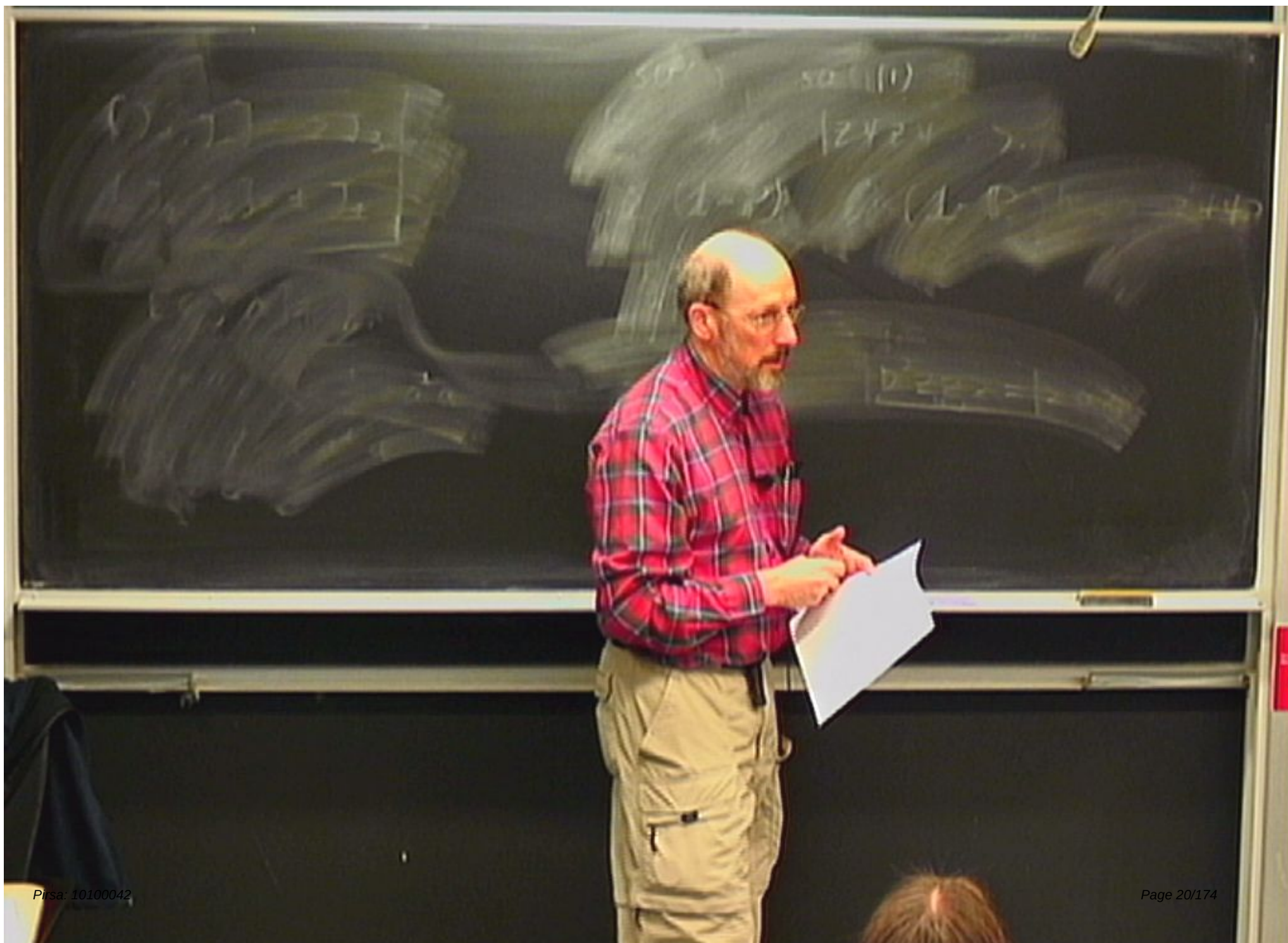


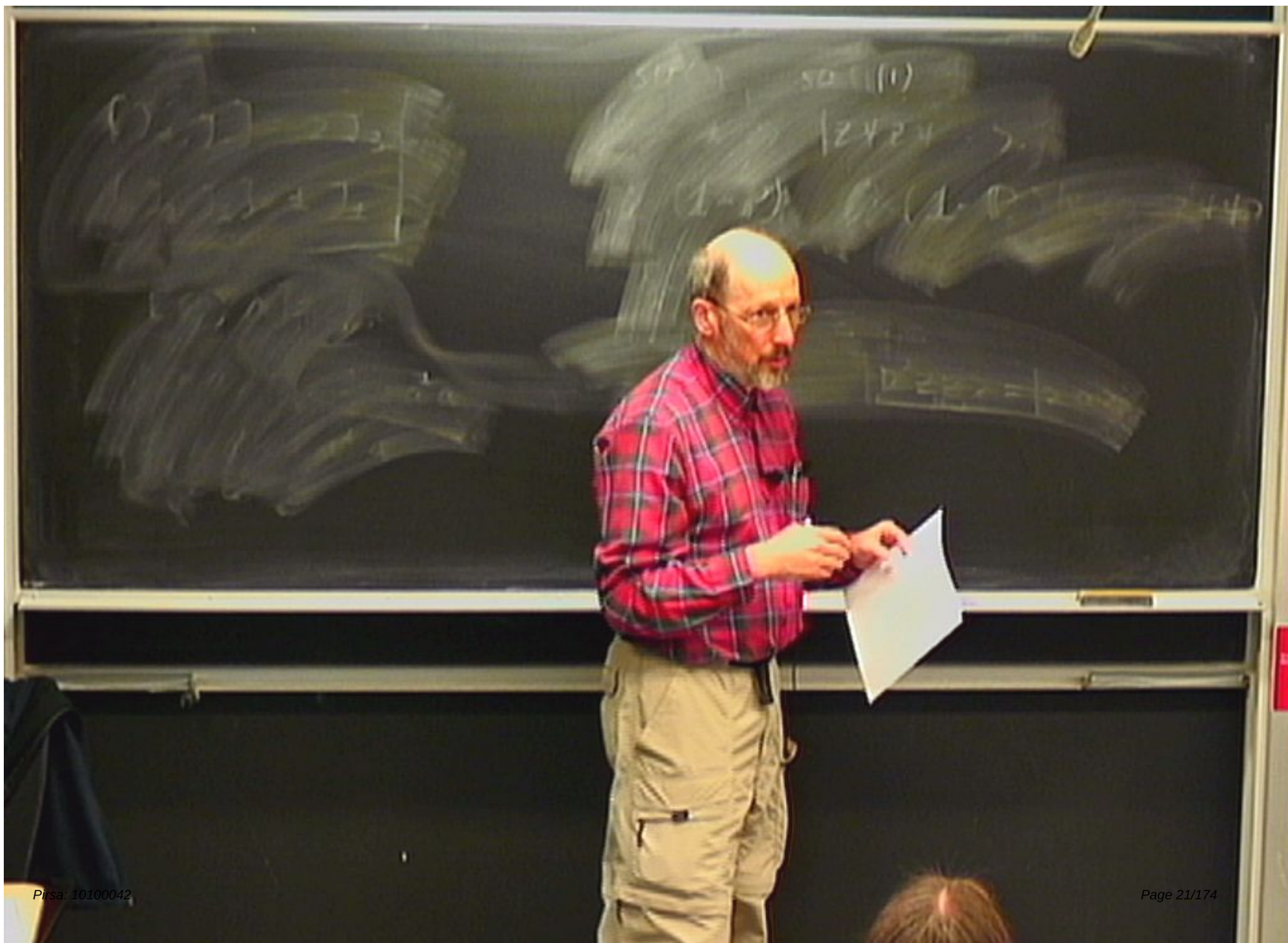


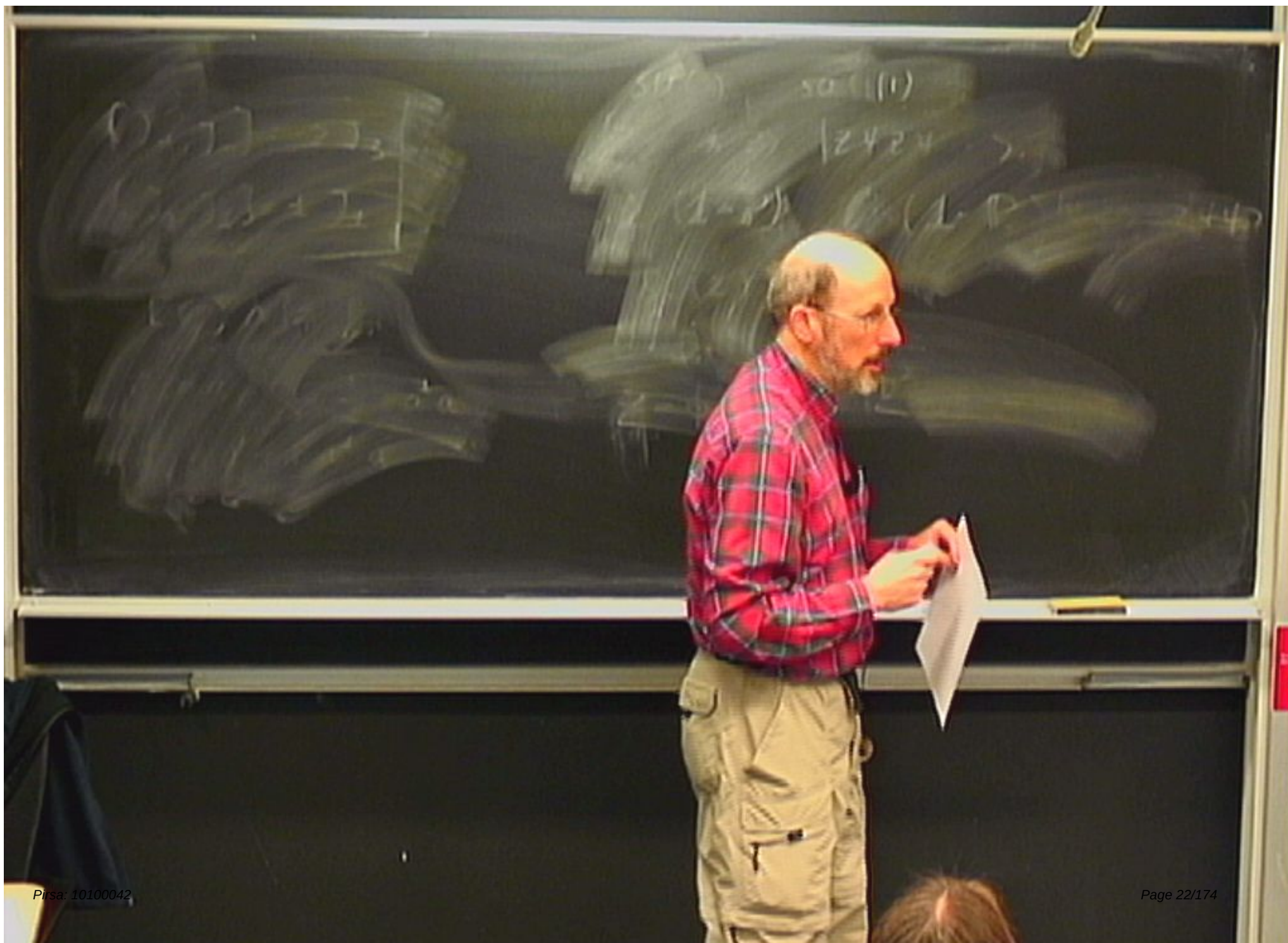


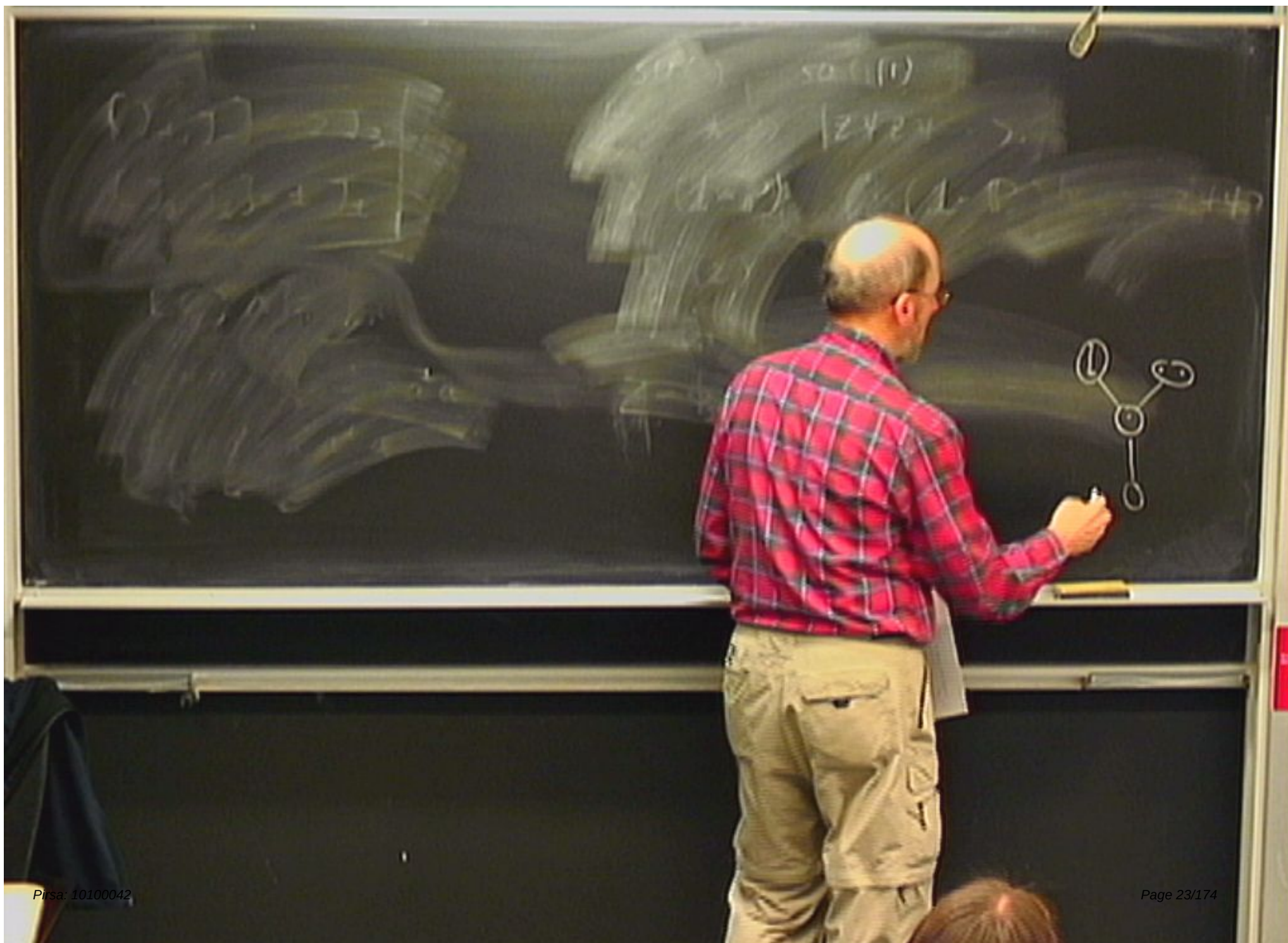


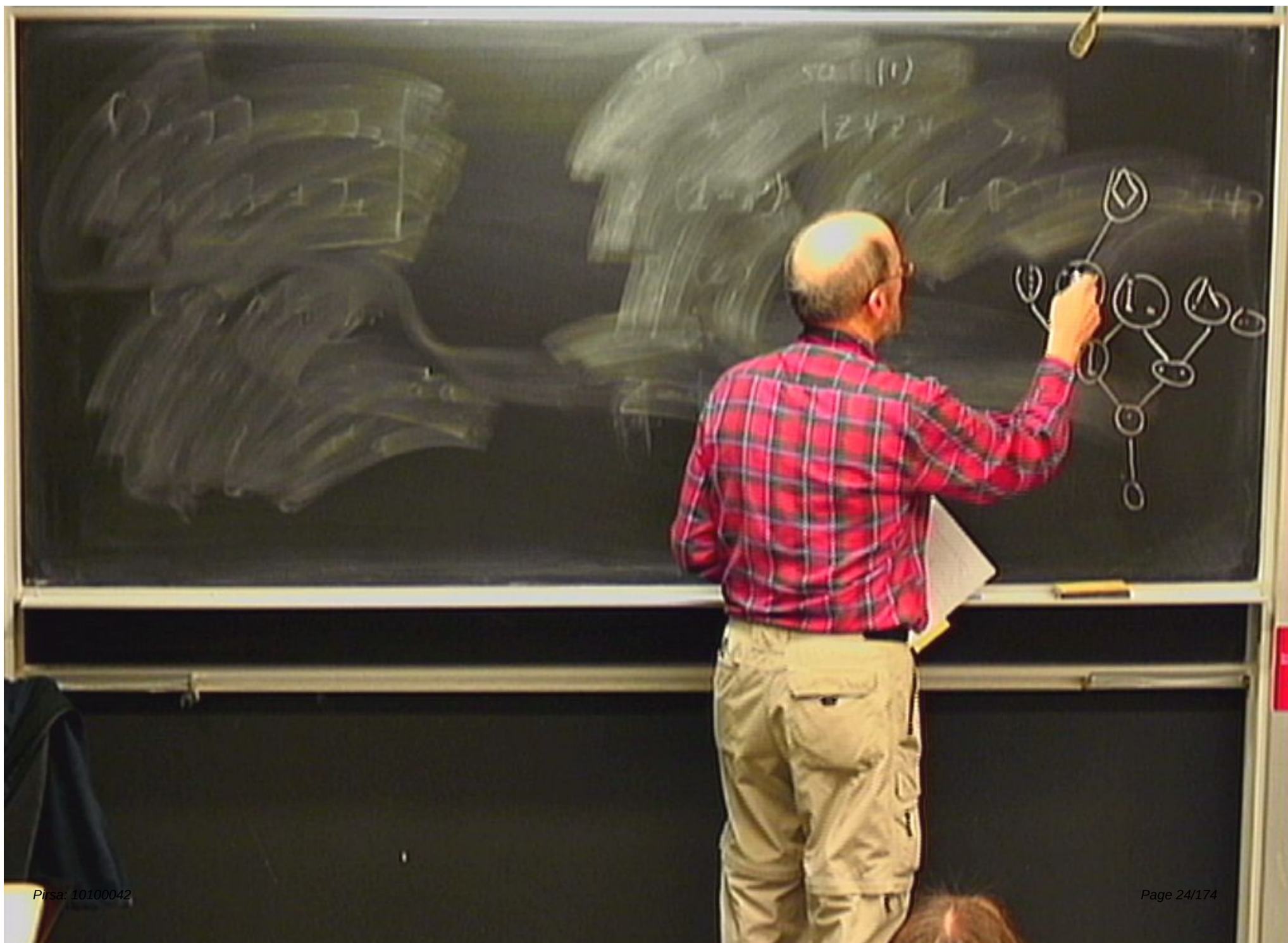


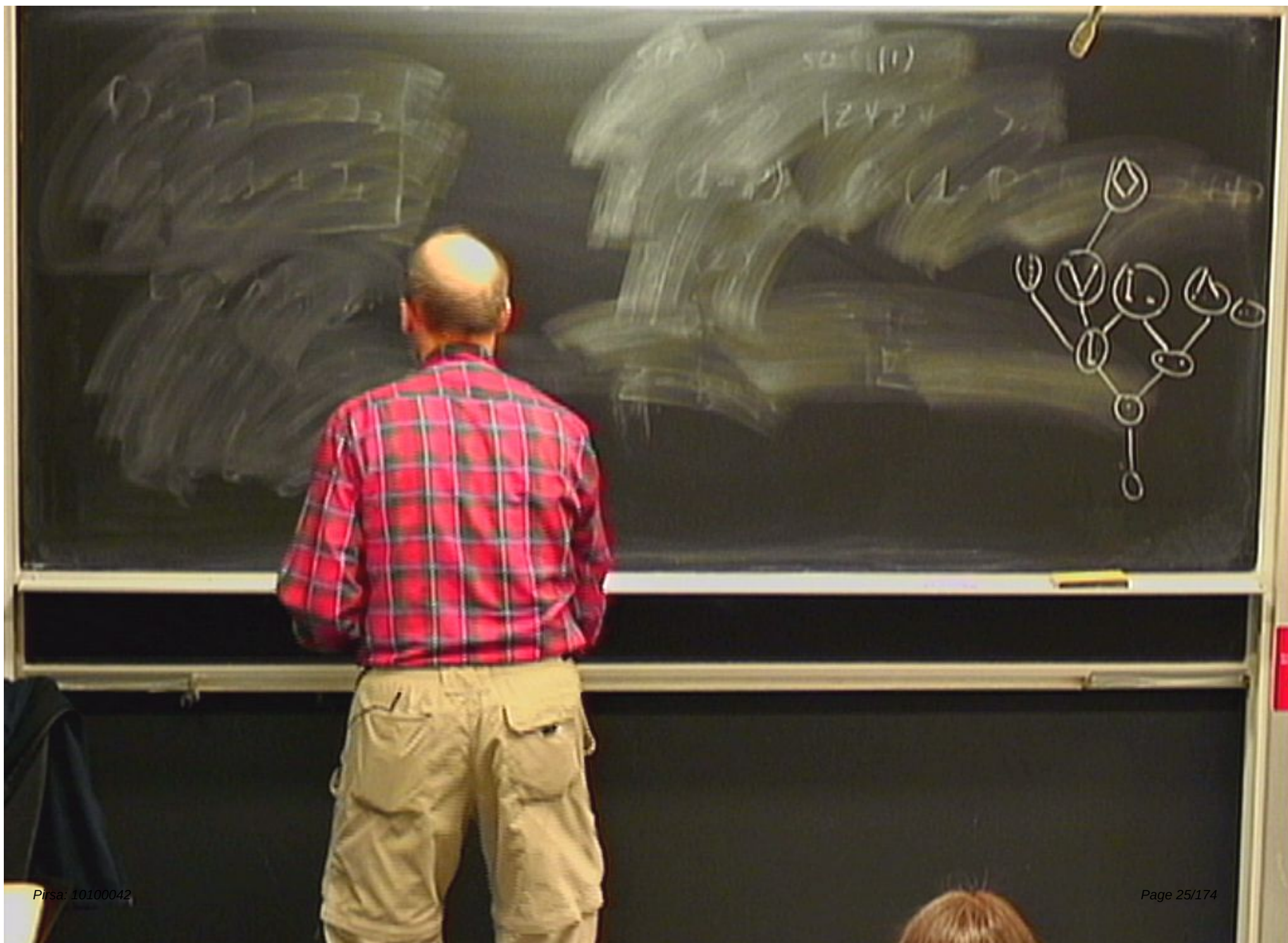


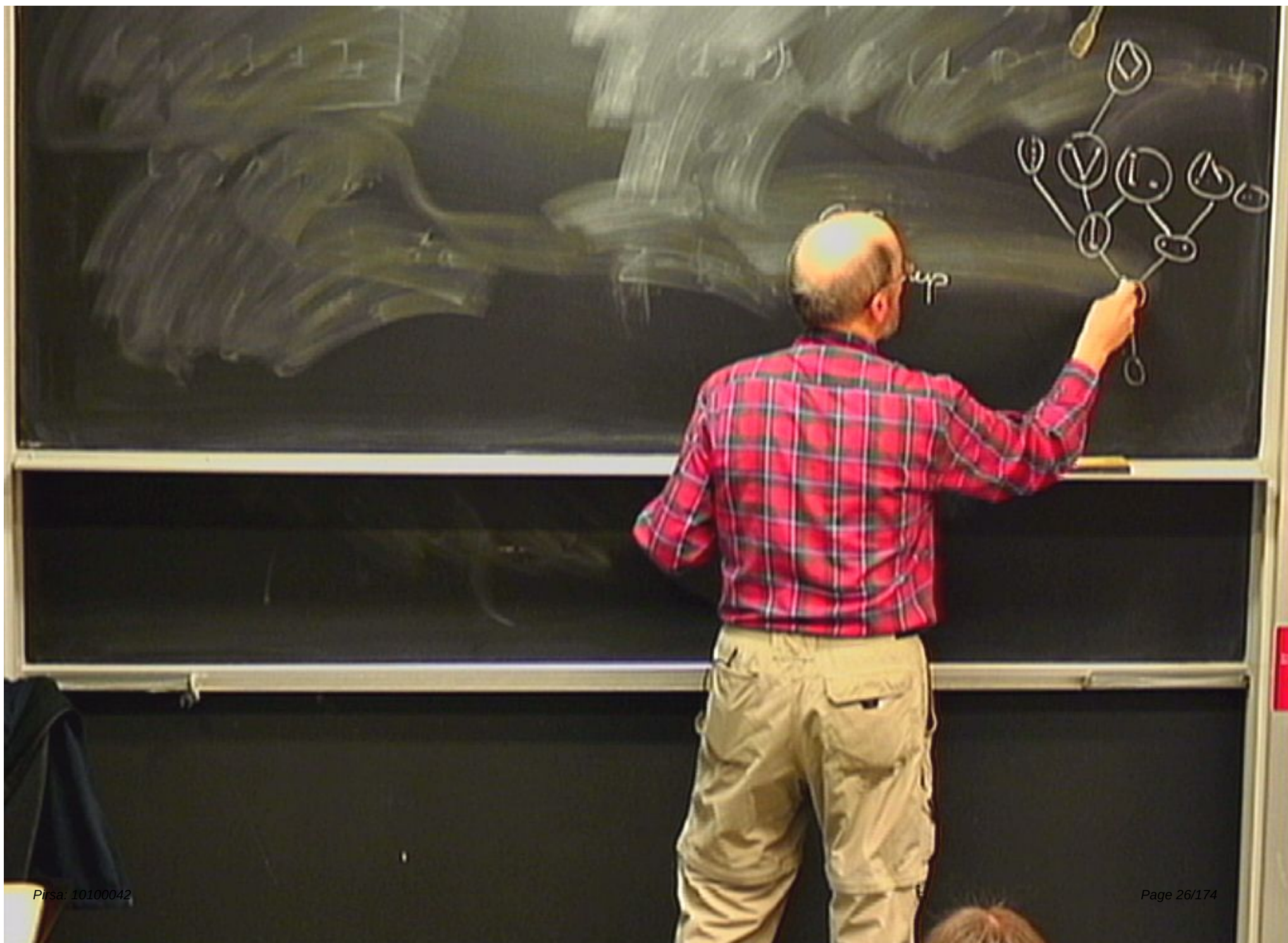


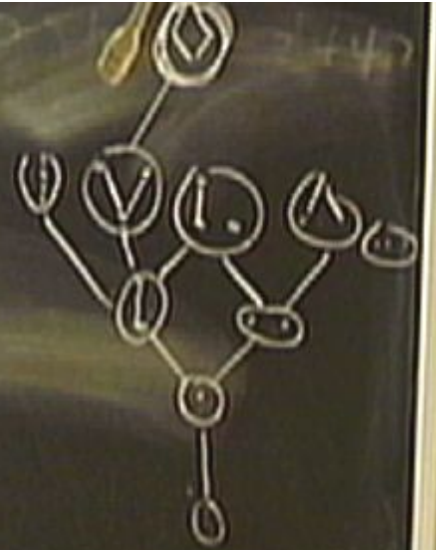


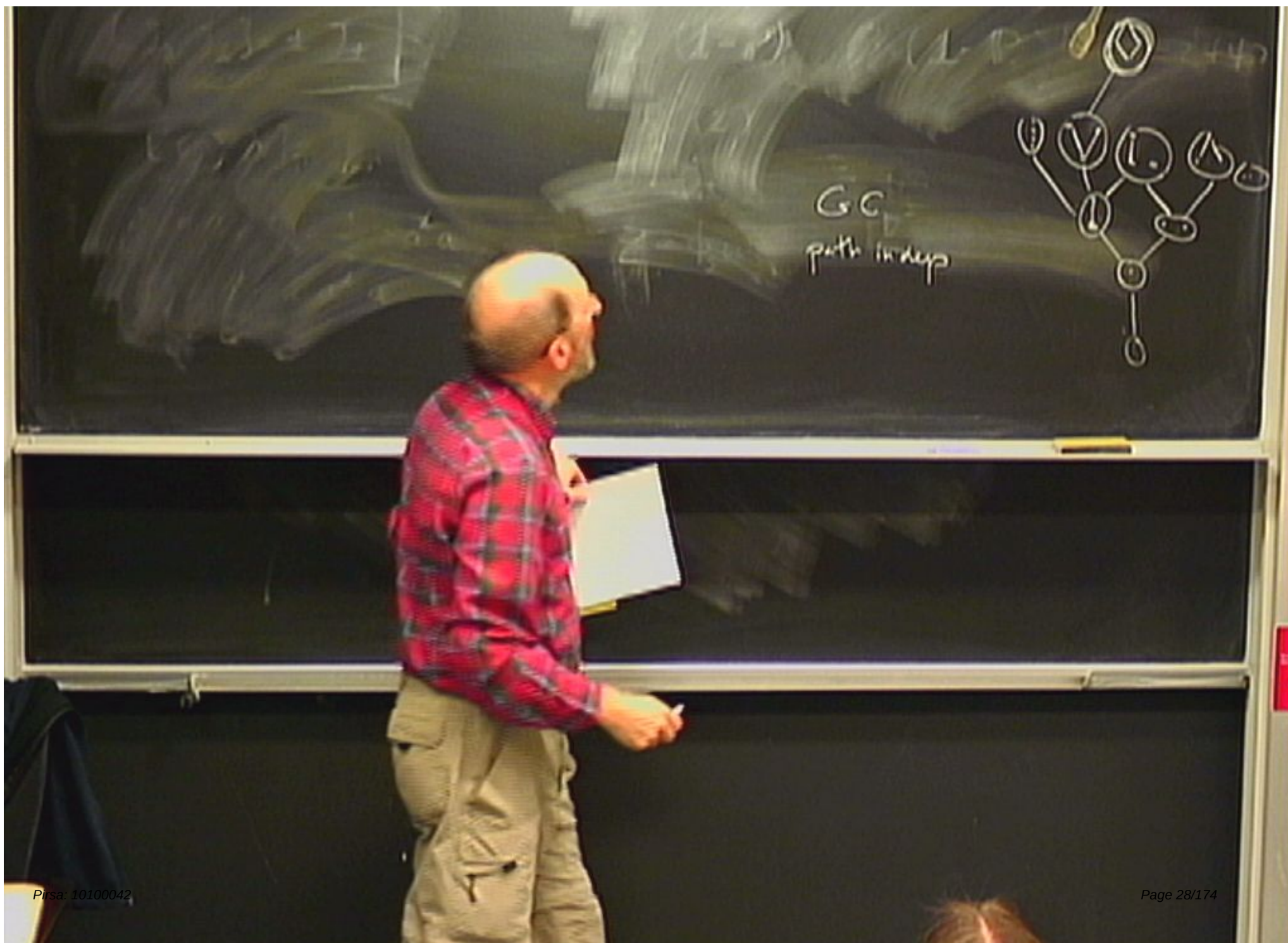


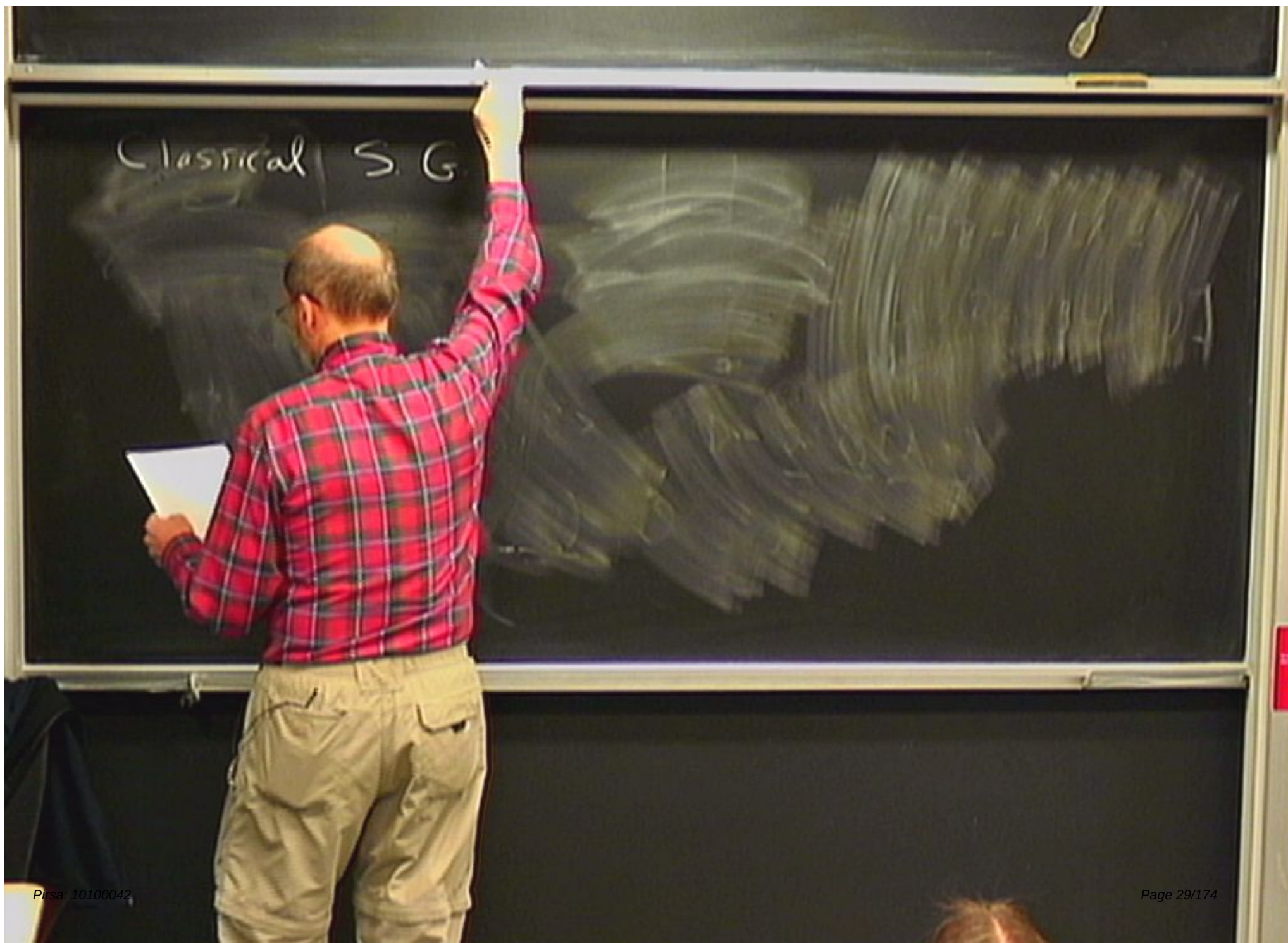


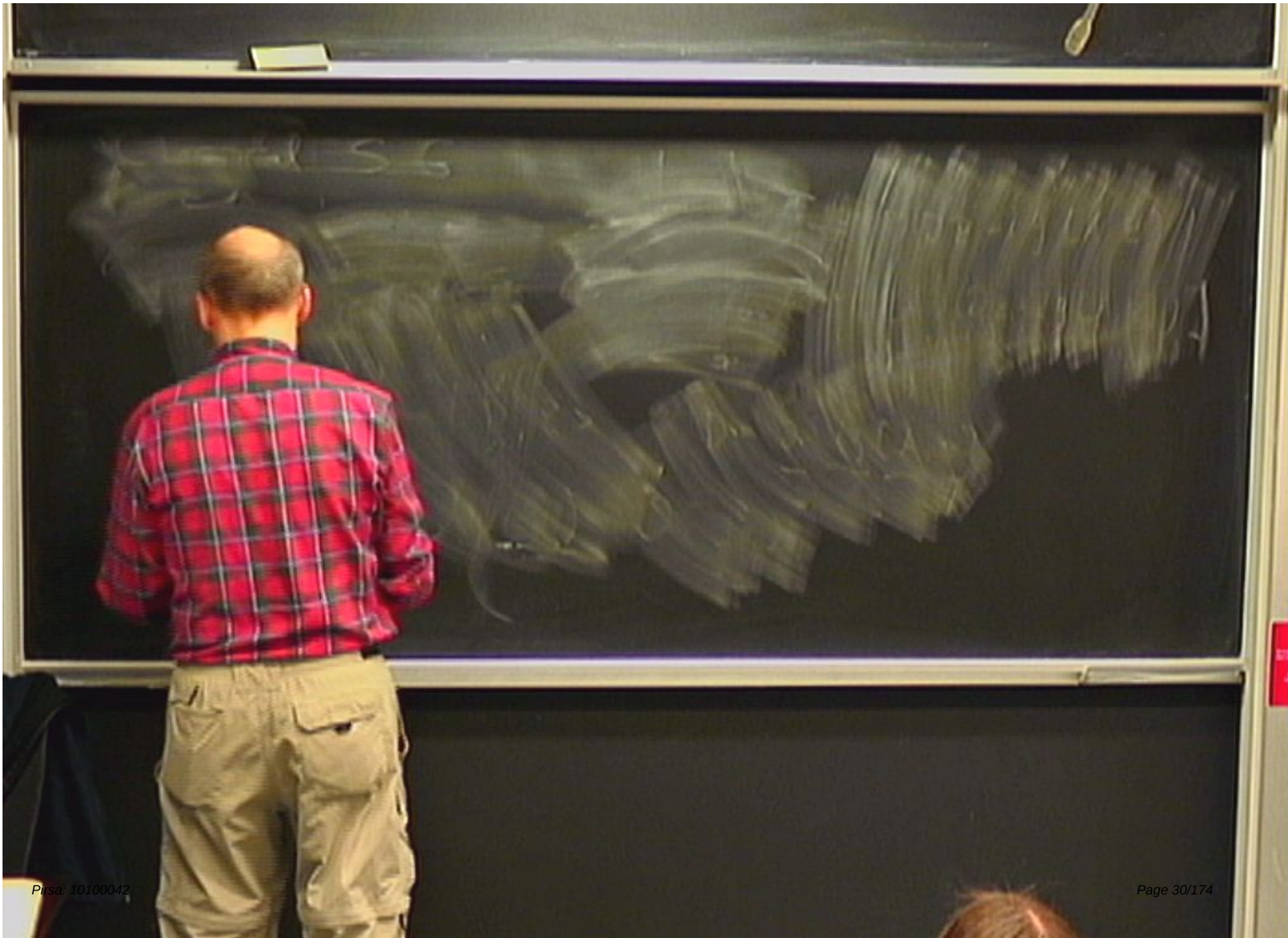


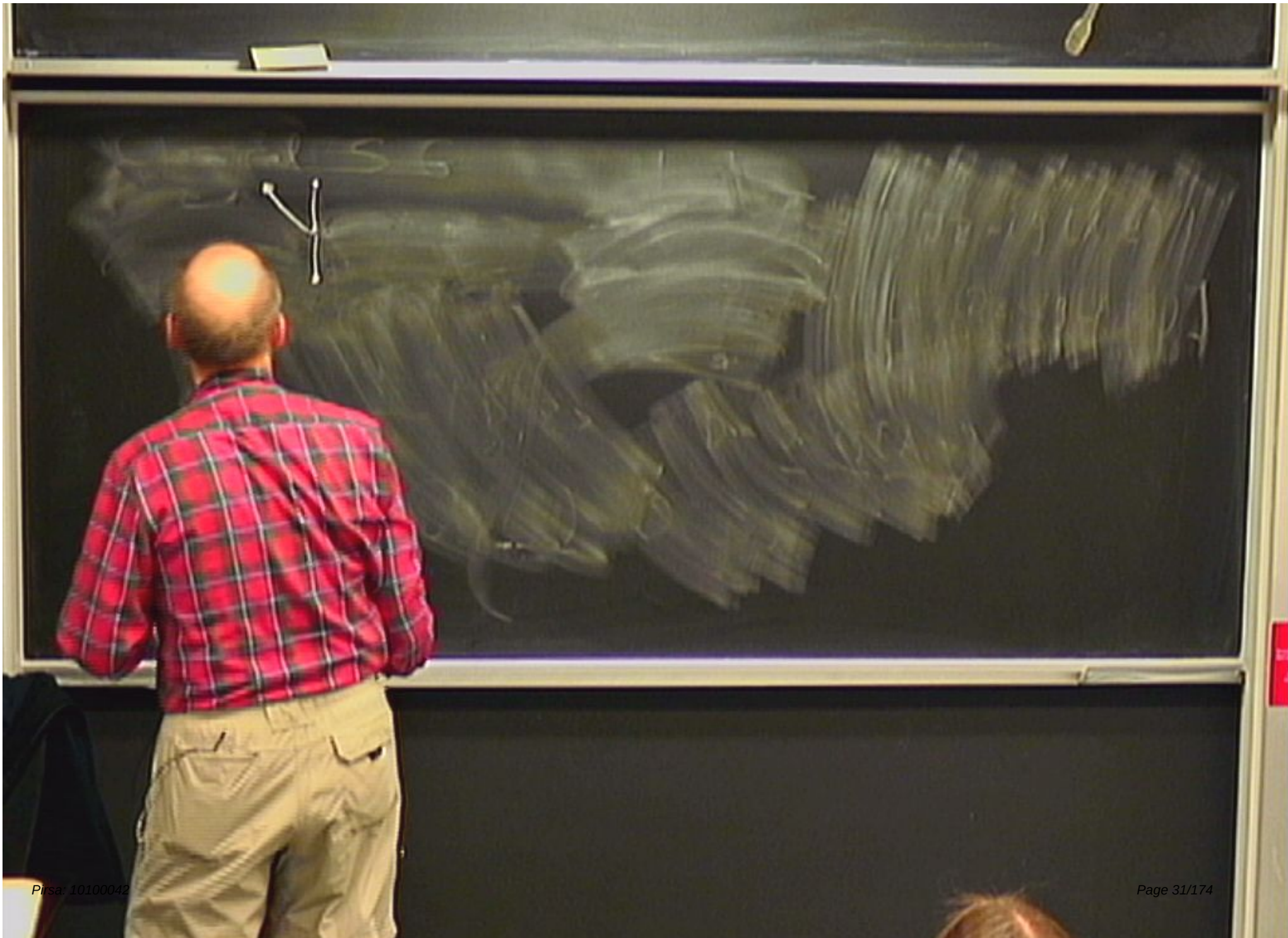




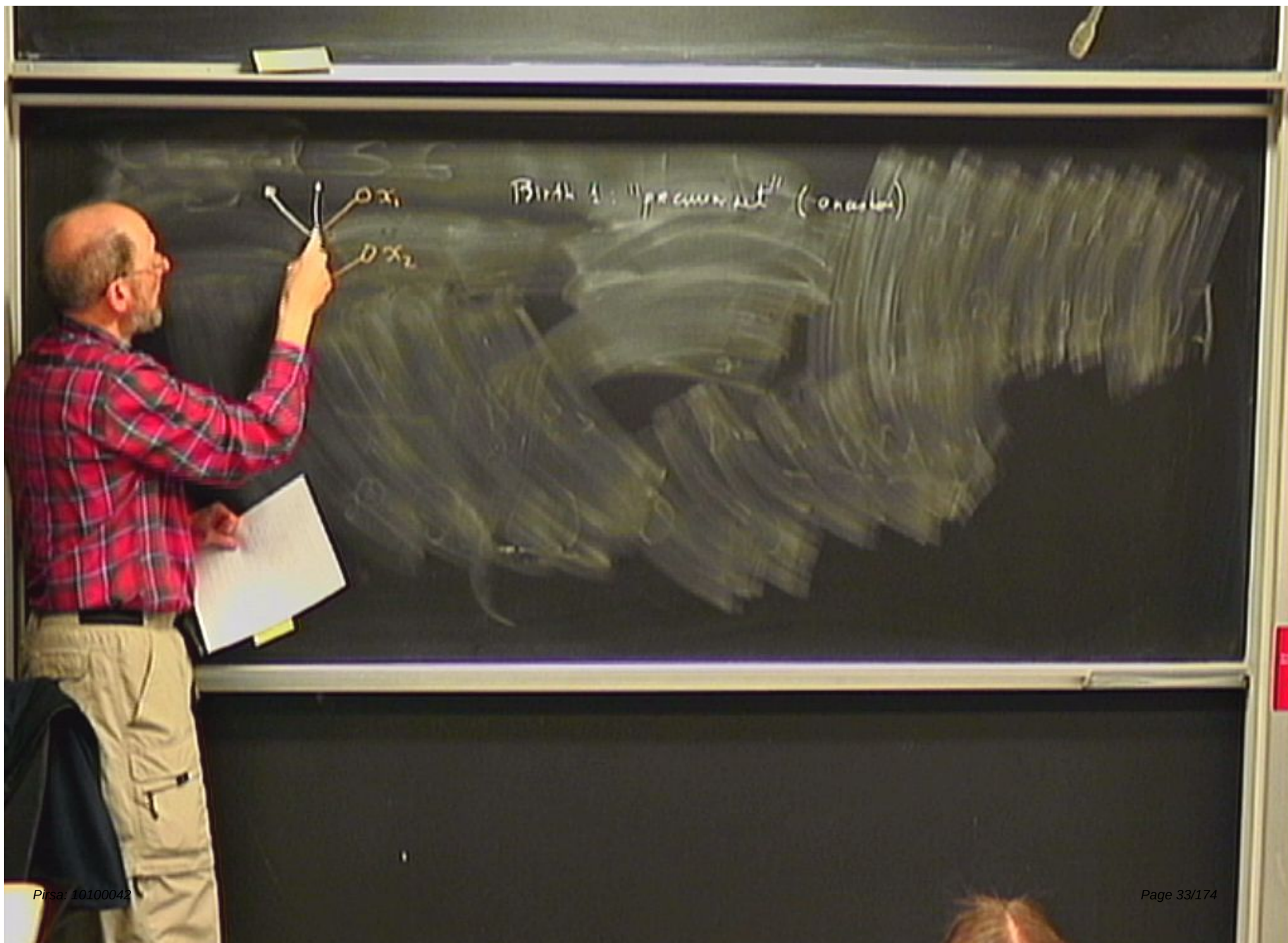


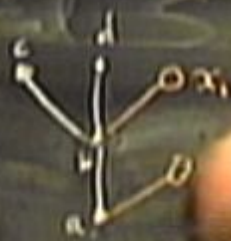








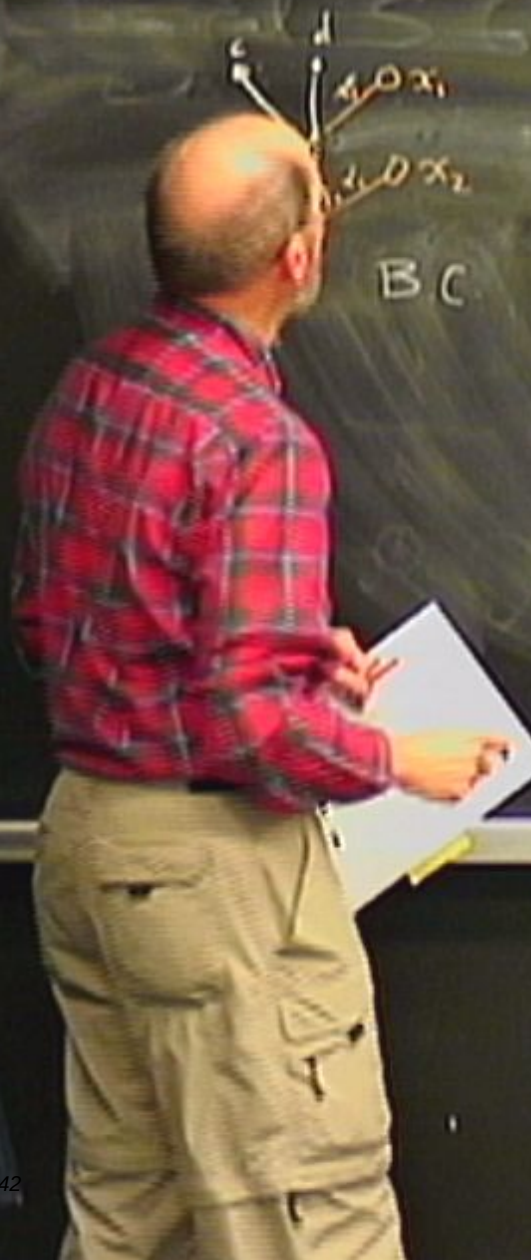




Birth 1: "predecessor" (ancestor) = a, b
 " 2 :

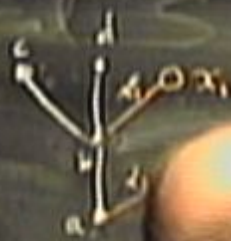


Birth 1: "predecessor" (operator) = a, b
 "2":
 c, d Spectator



Birth 1: "predecessor" (operator) = a, b
" 2 :
 c, d Spectral

BC

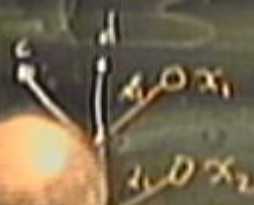


Birth 1: "predecessor" (operator) = a, b

"2"

and spectrum

$\frac{d_1}{d_2}$ independent of spectrum



BC

Birth 1: "predecessor" (operator) = a, b

"2:"

c, d spectators

$\frac{\alpha_1}{\alpha_2}$ independent of spectators



Birth 1: "precurrent" (onset) = a, b

"2"

c, d spectrum

B.C. $\frac{\alpha_1}{\alpha_2}$ independent of spectrum

- Interval Temporal
- Gen. Cov
- B.C
- Markov Sem rule

GC
 = path indep
 $\Rightarrow$ Markov



Solution

$$\lambda(\omega, m) = \sum_{k=m}^n \binom{m-m}{k-m} t_k$$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$\text{trans. prob.} = \frac{\lambda(n, m)}{\lambda(n, 0)}$$

$n \rightarrow n+1$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$\text{trans. prob.} = \frac{\lambda(n, m)}{\lambda(n, 0)}$$

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Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$\text{trans. prob.} = \frac{\lambda(n, m)}{\lambda(n, 0)}$$

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Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

trans. prob.
 $n \rightarrow n+1$

(n, m)
 $(n, 0)$



$n=4$ $\omega=3$
 $m=2$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^n \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$\text{trans. prob.} = \frac{\lambda(n, m)}{\lambda(n, 0)}$$

nb =



$$n=4 \quad \omega=3 \\ m=2$$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

trans. pr
 $n \rightarrow n+1$

$$\frac{\lambda(n, m)}{\lambda(n, 0)}$$

Prob =



$$n=4 \quad \omega=3$$
$$m=2$$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^n \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

trans. pnb. =
 $n \rightarrow n+1$

$$P_{nh}(c) = W(c)$$



$$n=4 \quad \omega=3 \\ m=2$$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$\text{trans. prob.} = \frac{\lambda(n, m)}{\lambda(n, 0)}$$

$$P_{WH}(c) = W(c) \cdot \frac{\prod_{x \in c}}{\quad}$$



$$n=4 \quad \omega=3 \\ m=2$$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^n \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$+ \text{emb.} = \frac{\lambda(\omega, m)}{\lambda(n, 0)}$$

$$F = W(c) \frac{\prod_{x \in c}}{\quad}$$



$$n=4 \quad \omega=3 \\ m=2$$

Solution

$$\lambda(\omega, m) = \sum_{k=m}^{\infty} \binom{\omega-m}{k-m} t_k \quad (t_k \geq 0) \quad t_0, t_1, t_2, \dots$$

$$pnb_i = \frac{\lambda(\omega, m)}{\lambda(n, 0)}$$

$$W(c) = \frac{\prod_{x \in c} \lambda(\omega(x), m(x))}{\prod_{y \in c} \lambda(n, 0)}$$



$$n=4 \quad \omega=3 \\ m=2$$

$$\text{trans. prob.} = \frac{\lambda(D_i, m)}{\lambda(n, 0)}$$

$n \rightarrow n+1$



$n=4 \quad m=3$
 $m=2$

$$P_n W(c) = \frac{\prod_{x \in c} \lambda(\omega(x), m(x))}{\prod_{n \in c} \lambda(n, 0)}$$

$\uparrow$
 path
 length

"Transitive predication"

"Transitive percolation" ("random graph of d degree")

"Transitive percolation" ("random graphs of degree d ")

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q$$

"Transitive percolation" ("random graphs of this")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{1 + p}{q} = \frac{1}{q}\right)$$

"Transitive percolation" ("random graphs of degree")

$$t_n = \left(\begin{matrix} n \\ k \end{matrix} \right) \text{ where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{1+p}{q} = \frac{1}{q} \right)$$

$$\lambda(B) = \sum_k \binom{B-m}{k-m} t_m$$

"Transitive percolation" ("random graph of size")

t_v where $p = 1 - q \in [0, 1]$ $(1 + t = 1 + \frac{p}{q} = \frac{1+p}{q} = \frac{1}{q})$

$$\lambda \left(\sum_k \binom{D-m}{k-m} t_m \right)$$

"Transitive percolation" ("random graphs of this")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1+q \in [0, 1] \quad \left(1+t = 1+\frac{p}{q} = \frac{q+p}{q} = \frac{1}{q}\right)$$

$$\lambda(D, m) = \sum_k \binom{D-m}{k-m} t_p$$

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q + p}{q} = \frac{1}{q}\right)$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m}$$

"Transitive percolation" ("random graph of size")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q + p}{q} = \frac{1}{q}\right)$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m q^n$$

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{q}\right)^n \quad p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q + p}{q} = \frac{1}{q}\right)$$

$$\lambda(n, m) = \binom{m}{n} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

"Transitive percolation" ("random graph of size")

where $p = 1 - q \in [0, 1]$ $(1 + t = 1 + \frac{p}{q} = \frac{1+p}{q} = \frac{1}{q})$

$$\sum_k \binom{D-m}{k-m} \left(\frac{1}{2}\right)^{k-m} \left(\frac{p}{2}\right)^m = \left(1 + \frac{p}{2}\right)^{D-m} \left(\frac{1}{2}\right)^{D-m} = \left(\frac{1}{1}\right)^{D-m} \frac{1}{1^m} = 1^m / 1^D$$

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{1}\right)^n \quad \text{where } p = 1 - \frac{1}{2} \quad \left(1+t = 1+\frac{p}{1} = \frac{1+p}{1} = \frac{1}{1}\right)$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{1}\right)^{k-m} = \left(1 + \frac{p}{1}\right)^{n-m} \left(\frac{p}{1}\right)^{m-m} = \left(\frac{1}{1}\right)^{n-m} \frac{p^m}{1^m} = \frac{p^m}{1^m}$$

$$\lambda(n, 0) = \frac{p^n}{1^n} = \frac{1}{1}$$

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{with } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q + p}{q} = \frac{1}{q}\right)$$

$$\lambda(n, m) = \sum_k \binom{k-m}{2} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

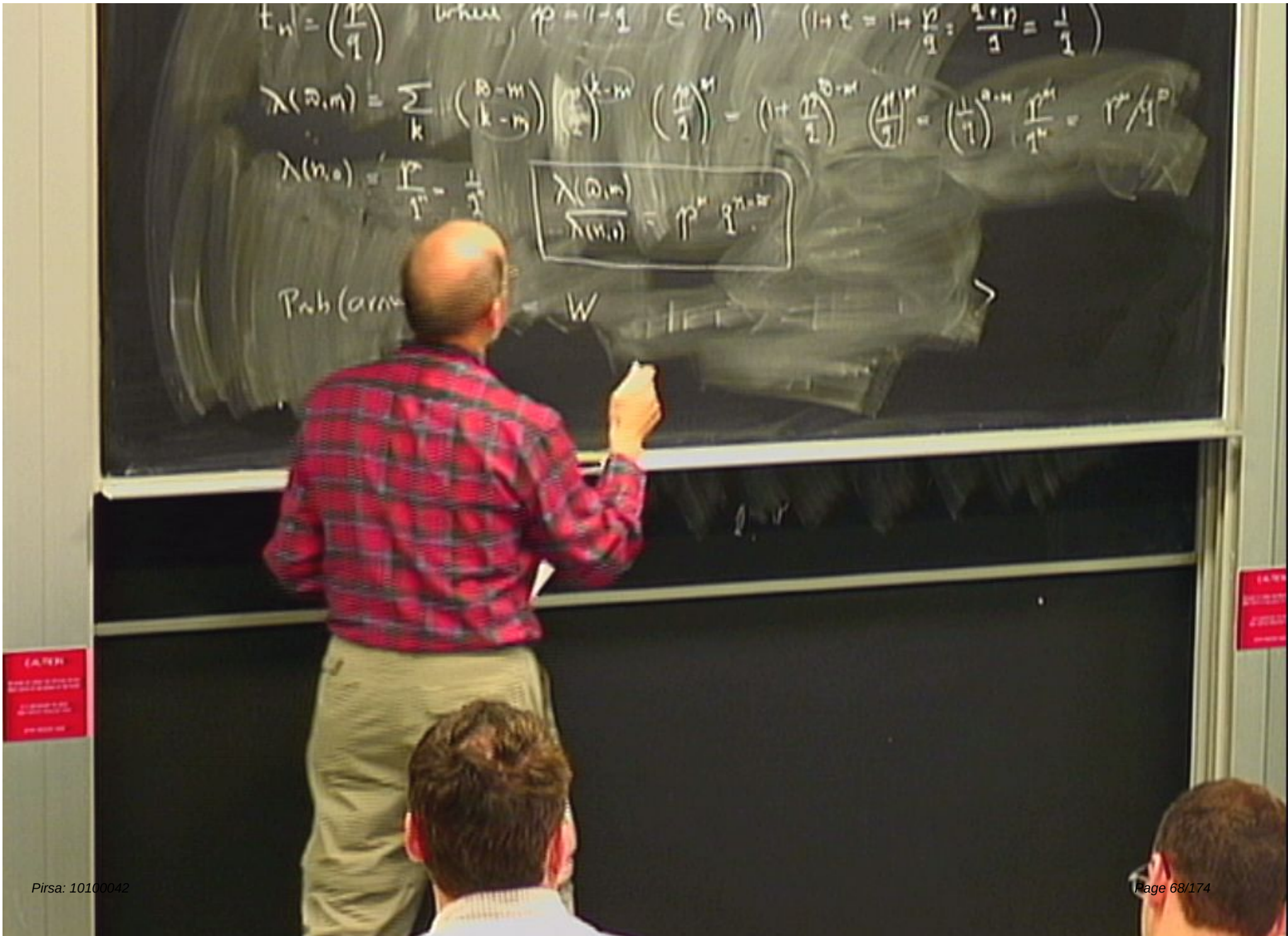
$$\lambda(n, 0) = p^0 / q^n = 1 / q^n$$

$$t_n = \left(\frac{p}{q} \right) \text{ wobei } p = 1+q \in \mathbb{P}_{\geq 1} \quad \left(1+t = 1+\frac{p}{q} = \frac{q+p}{q} = \frac{1}{q} \right)$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q} \right)^{k-m} \left(\frac{p}{q} \right)^m = \left(1 + \frac{p}{q} \right)^{n-m} \left(\frac{p}{q} \right)^m = \left(\frac{1}{q} \right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

$$\lambda(n, 0) = \frac{p^n}{q^n} = \frac{1}{q^n}$$

$$\boxed{\frac{\lambda(n, m)}{\lambda(n, 0)} = p^m q^{n-m}}$$



$$t_n = \binom{n}{q} \text{ where } p = 1 - q \in [0, 1] \quad (1+t = 1 + \frac{p}{q} = \frac{1+p}{q} = \frac{1}{q})$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

$$\lambda(n, 0) = \frac{p^n}{q^n} = \frac{1}{q^n}$$

$$\boxed{\frac{\lambda(n, m)}{\lambda(n, 0)} = p^m q^{n-m}}$$

Prob (arm

W

"Transitive percolation" ("random graph of $\text{dim } \mathcal{H}$ ")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q+p}{q} = \frac{1}{q}\right)$$

$$\lambda(\infty, m) = \sum_k \binom{\infty - m}{k - m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{\infty - m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{\infty - m} \frac{p^m}{q^m} = p^m / q$$

$$\lambda(n, 0) = \frac{p^n}{1^n} = \frac{1}{2}$$

$$\boxed{\frac{\lambda(\infty, m)}{\lambda(n, 0)} = \left(\frac{p}{q}\right)^m q^{n-m}}$$

$$\text{Prob}(\text{arrive at } C) = W$$

"Transitive percolation" ("random graph of d lines")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1+q \in \mathbb{N} \quad \left(1+t = 1+\frac{p}{q} = \frac{q+p}{q} = \frac{1}{q}\right)$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1+\frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} p^m = p^m / q^n$$

$$\lambda(n, 0) = \frac{p^n}{q^n} = \frac{1}{q^n}$$

$$\frac{\lambda(n, m)}{\lambda(n, 0)} = p^m q^{n-m}$$

$$P_{\text{nh}}(\text{argument } C) = W$$

"Transitive percolation" ("random graph of lines")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q+p}{q} = \frac{1}{q}\right)$$

$$\lambda(n, m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

$$\lambda(n, 0) = \frac{p^n}{q^n} = \frac{1}{q}$$

$$\boxed{\frac{\lambda(n, m)}{\lambda(n, 0)} = p^m}$$

$$\text{Prob}(\text{arrive at } C) = W$$

"Transitive percolation" ("random graph of links")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1 - q \in [0, 1] \quad \left(1 + t = 1 + \frac{p}{q} = \frac{q+p}{q} = \frac{1}{q}\right)$$

$$\lambda(w, m) = \sum_k \binom{w-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{w-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{w-m} p^m = p^m / q^w$$

$$\lambda(n, 0) = \frac{p^n}{1^n} = \frac{1}{2^n}$$

$$\boxed{\frac{\lambda(w, m)}{\lambda(n, 0)} = p^m q^{n-w}}$$

$$N = |C|$$

$$\text{Prob}(\text{arrive at } C) = W \underbrace{p^L}_{\text{links}} q^{\binom{N}{2}}$$

"Transitive percolation" ("random graph of links")

$$t_n = \left(\frac{p}{q}\right)^n \text{ where } p = 1+q \in [0,1] \quad \left(1+t = 1+\frac{p}{q} = \frac{q+p}{q} = \frac{1}{q}\right)$$

$$\lambda(n,m) = \sum_k \binom{D-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1+\frac{p}{q}\right)^{D-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{D-m} \frac{p^m}{q^m} = p^m / q^D$$

$$\lambda(n,0) = \frac{p^n}{q^D} = \frac{1}{q^D}$$

$$\boxed{\frac{\lambda(n,m)}{\lambda(n,0)} = p^m q^{n-m}}$$

$$N = |C|$$

$R = \text{relations in } C$

$$\text{Prob(arrive at } C) = W \underbrace{p^L}_{\text{links}} \frac{1}{q^D} = R$$

$$t_n = \left(\frac{p}{q}\right)^n \text{ where } p = 1+q \in [0,1] \quad (1+t = 1+\frac{p}{q} = \frac{q+p}{q} = \frac{1}{q})$$

$$\lambda(n,m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1+\frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

$$\lambda(n,0) = \frac{p^n}{q^n} = \frac{1}{q^n}$$

$$\frac{\lambda(n,m)}{\lambda(n,0)} = \left(\frac{p}{q}\right)^m q^{n-m}$$

$$N = |C|$$

$R = \text{relations in } C$

$$\text{Prob(arrive at } C) = W \underbrace{\frac{p^L}{q^L}}_{\text{links}} \frac{1}{q^N} - R = W \frac{p^L}{q^L} \frac{1}{q^N} - R$$

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$$N = |C|$$

$R = \text{relations in } C$

$$\text{Prob}(\text{arrive at } C) = W \prod_{\text{links}} p_i^L q_i^{N-L} = W q^{\sum L} \exp \left\{ -L \ln \frac{1}{p} + R \ln \frac{1}{q} \right\}$$

of paths $\prod_{n=0}^{\infty} \lambda(n,0)$
 = # of inequivalent natural labellings of C

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{q}\right)^n \text{ where } p = 1 - q \in [0,1] \quad (1+t = 1 + \frac{p}{q} = \frac{q+p}{q} = \frac{1}{q})$$

$$\lambda(n,m) = \sum_k \binom{n-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1 + \frac{p}{q}\right)^{n-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{n-m} \frac{p^m}{q^m} = p^m / q^n$$

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$$N = |C|$$

R = relations in C

$$\text{Prob(arrive at } C) = W \prod_{\text{links}} \frac{p}{q} \exp\left\{-L \ln \frac{1}{p} + R\right\} = W q^L \exp\left\{-L \ln \frac{1}{p} + R\right\}$$

$$\text{trans. prob.} = \frac{\lambda(D, m)}{\lambda(n, 0)}$$

$$n \rightarrow n+1$$

$$\text{Prob}(C) = W(C) \prod_{v \in C} \lambda(\deg(v), m(v))$$



$$n=4, \quad \deg=3$$

"Transitive percolation" ("random graphs of law")

$$t_n = \left(\frac{p}{q}\right)^n \quad \text{where } p = 1+q \in [0, 1] \quad (1+t = 1+\frac{p}{q} = \frac{1+p}{q} = \frac{1}{q})$$

$$\lambda(D, m) = \sum_k \binom{D-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1+\frac{p}{q}\right)^{D-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{D-m} \frac{p^m}{1^m} = p^m / q^D$$

$$\lambda(n, 0) = \frac{p}{1^n} = \frac{1}{1}$$

$$\frac{\lambda(D, m)}{\lambda(n, 0)} = p^m q^{n-D}$$

$$N = |C|$$

$R = \text{relations in } C$

$$\text{Prob(arrive at } C) = W \prod_{\text{links}} p^L q^{(N)-R} = W q^{(N)-R}$$

$$\text{trans. prob:} = \frac{\lambda(\omega, m)}{\lambda(n, 0)}$$

$$n \rightarrow n+1$$



$$n=4 \quad \omega=3$$

$$m=2$$

$$\text{Prob}(C) = W(C) \prod_{v \in C} \lambda(\omega(v), m(v))$$

"Transitive percolation" ("random graphs of size")

$$t_n = \left(\frac{p}{q}\right)^n \text{ where } p = 1+q \in [0, 1] \quad (1+t = 1+\frac{p}{q} = \frac{q+p}{q})$$

$$\lambda(\omega, m) = \sum_k \binom{\omega-m}{k-m} \left(\frac{p}{q}\right)^{k-m} \left(\frac{p}{q}\right)^m = \left(1+\frac{p}{q}\right)^{\omega-m} \left(\frac{p}{q}\right)^m = \left(\frac{1}{q}\right)^{\omega-m} \frac{p^m}{1^m}$$

$$\lambda(n, 0) = \frac{p^n}{1^n} = \frac{1}{1}$$

$$\frac{\lambda(\omega, m)}{\lambda(n, 0)} = p^m q^{n-\omega}$$

$N = |C|$
 $R = \text{relations in } C$

$$\text{Prob(arrive at } C) = W \prod_{\text{links}} \frac{p^L}{q^L} q^{|C| - R} = W$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Prob(C) = W(C) \cdot \frac{\prod_{x \in C} \lambda(\omega(x), m(x))}{\prod_{n=0}^{|\mathcal{C}|-1} \lambda(n, 0)}$$

of paths

= # of inequivalent natural labelings

$$n=4 \quad \omega=3$$

$$m=2$$

"

"

As if each new x picked each pre-existing y at random with p

escape

$$2^{\frac{N^2}{4} + \dots}$$

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$



As if each new x picked each pre-existing y at random with p

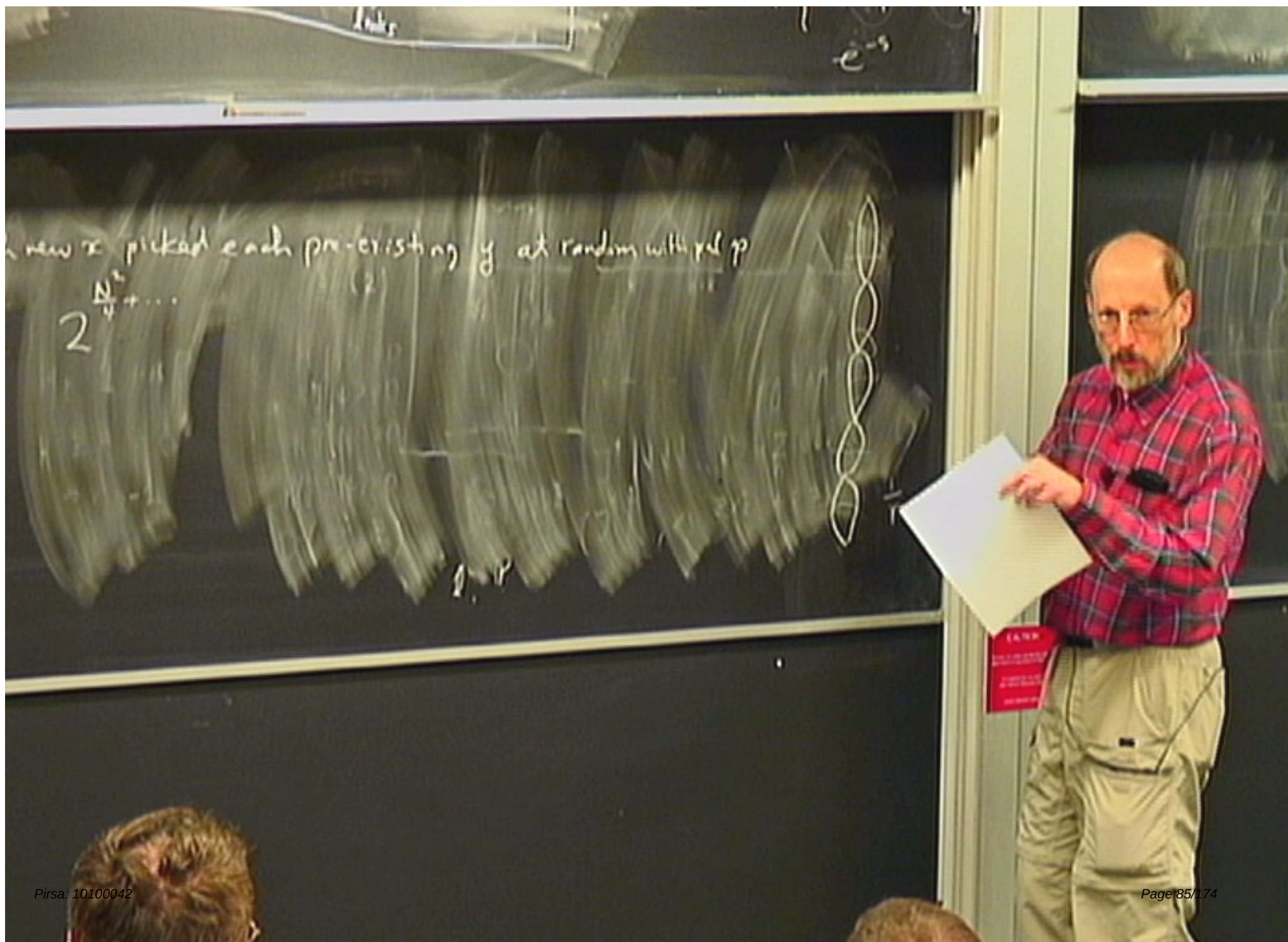
How escape $2^{\frac{N^2}{4} + \dots}$



As if each new x picked each pre-existing y at random with prob p

How escape $2^{\frac{N^2}{4} + \dots}$





As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$



Prob (arrive at C) = $W \frac{1}{1+q} \frac{1}{1+q} - R = W \frac{1}{1+q} \exp \left\{ -L \ln \frac{1}{p} + R \ln \frac{1}{q} \right\}$

How escape $2^{\frac{N}{q} + \dots}$



$$\text{Prob}(\text{arrive at } C) = W \underbrace{\frac{10^L}{\text{links}}}_{\text{links}} \underbrace{\lg(5)}_{\text{links}} - R = W \underbrace{q^{\frac{1}{2}}}_{\text{links}} \exp \left\{ -L \ln \frac{1}{p} + R \ln \frac{1}{q} \right\} e^{-s}$$

How escape

$$\frac{N}{n} + \dots$$

$$\text{Prob}(\text{arrive at } c) = W \frac{10^L}{10^L} \frac{1}{10^L} \frac{1}{10^L} = W \frac{1}{10^L} \exp \left\{ -L \ln \frac{1}{p} + R \ln \frac{1}{q} \right\}$$

R = relations in C

How escape

2

$$\frac{\lambda(\dim)}{\lambda(\dim)} = \left(\frac{1}{p} \right)^{\dim} \frac{1}{q^{\dim}}$$

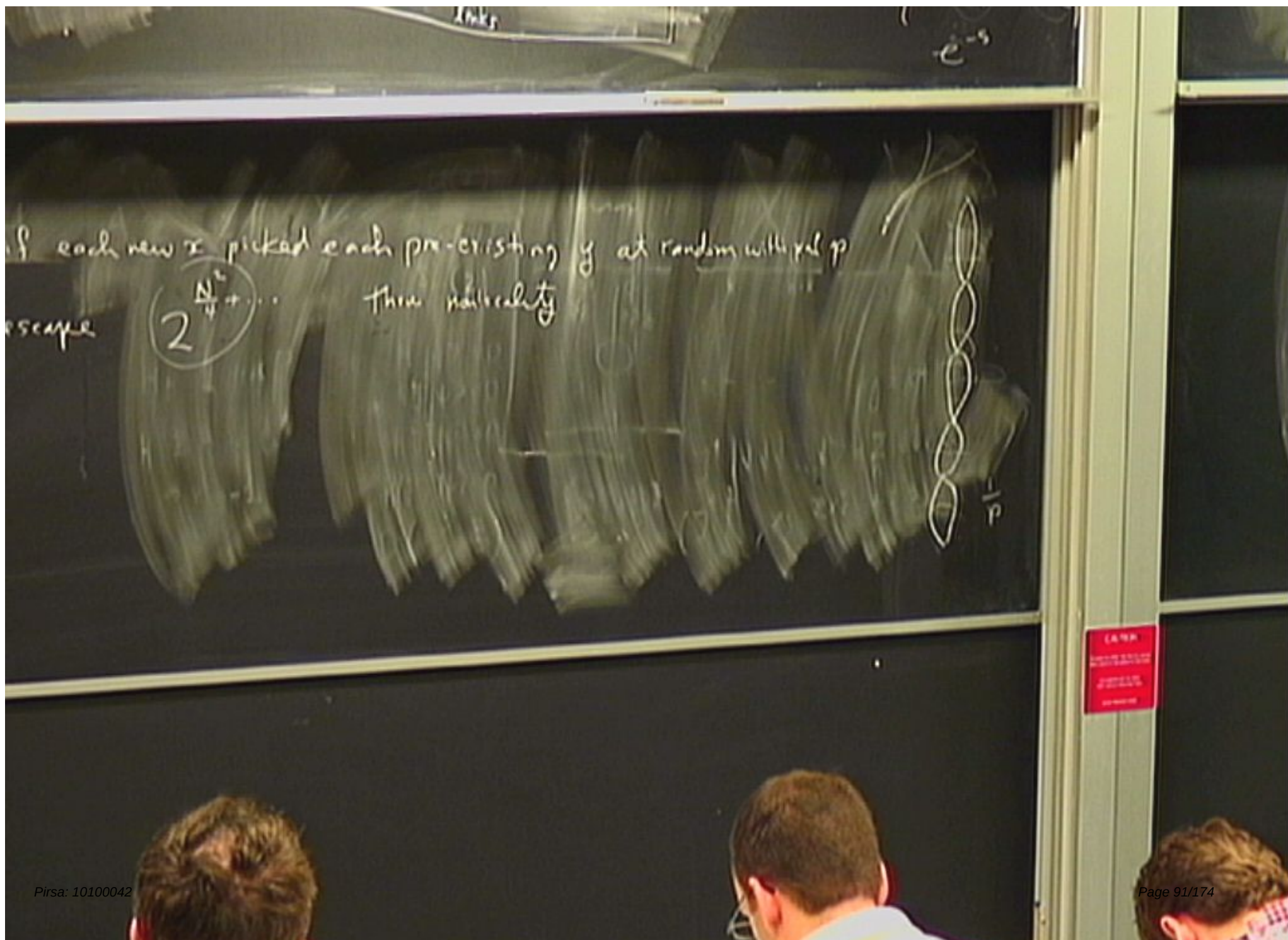
$$N = |C|$$

$$R = \text{relations in } C$$

$$\text{Prob}(\text{argument } C) = W \prod_{\text{links}} \frac{1}{p} \frac{1}{q} = W \prod_{\text{links}} \frac{1}{p} \frac{1}{q} \exp \left\{ -\frac{1}{p} \ln \frac{1}{p} + \frac{1}{q} \ln \frac{1}{q} \right\}$$

Pirsa: 10100042

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if each new x picked each pre-existing y at random with prob p
then reality
escape $2 \frac{N^2}{4} + \dots$



As if each new x picked each pre-existing y at random with p

How escape

$$2^{\frac{N^2}{4} + \dots}$$

then neutrality

Maintain one last copy



As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N}{p} + \dots}$ then neutrality

Maintain one last copy Λ



$\frac{1}{p}$

As if each new x picked each pre-existing y at random with $1/p$

How escape

$$2^{\frac{N^2}{p} + \dots}$$

then naturality

Martin one last case

$\wedge$

$\wedge$



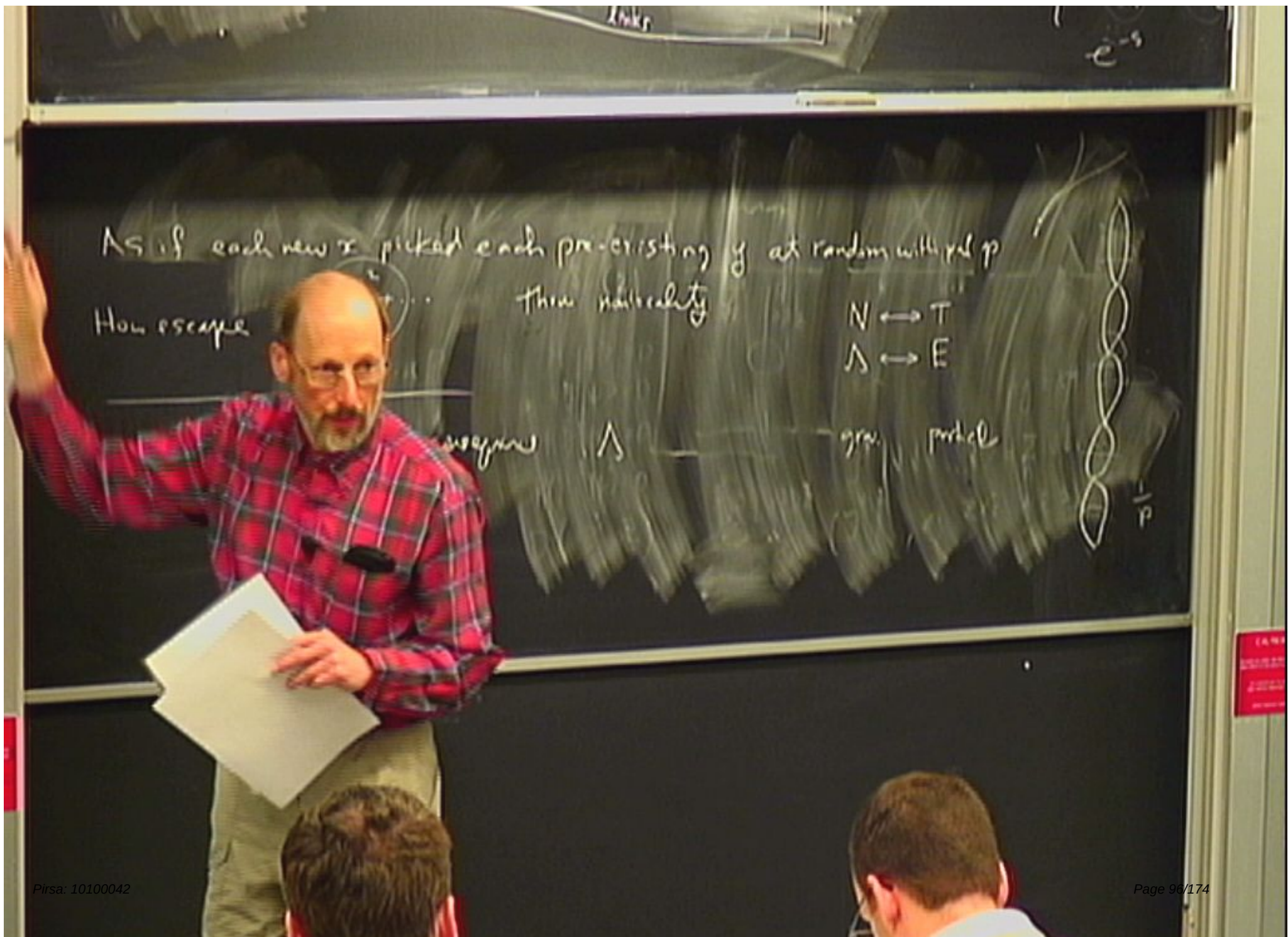
$1/p$

As if each new x picked each pre-existing y at random with prob p

How escape $2^{\frac{N^2}{4} + \dots}$ then nonlocality

Martin one last case Λ

$\Lambda \leftrightarrow E$



As if each new x picked each pre-existing y at random with prob p

How escape

three reality

$N \leftrightarrow T$

$\Lambda \leftrightarrow E$

sequence

Λ

grow

particle

p

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$ then nullability

$V \leftrightarrow N \leftrightarrow T$
 $\Lambda \leftrightarrow E$

Maintain one last case

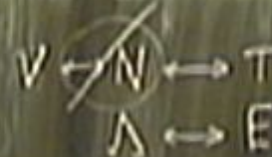
Λ

grow. patch

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$

locality



Martin one last

Λ

10^{60}

grow

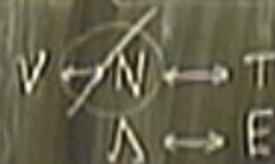
particle

$$\delta \Lambda \sim \frac{\hbar}{\delta V} \sim \frac{\hbar}{\sqrt{N}} \sim \frac{\hbar}{\sqrt{V}}$$

$\frac{1}{p}$

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$ then nullability



Martin one last case

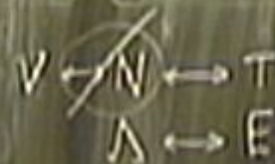
$$\Lambda \sim 10^{60}$$

$$\delta \Lambda \sim \frac{k}{\delta V} \sim \frac{k}{\sqrt{N}} \sim \frac{k}{\sqrt{V}}$$

grav. particle

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$ then nonlocality



Martin one last case

$$\Lambda \sim 10^{60}$$

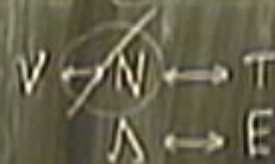
grav. particle

$$\delta \Lambda \sim \frac{\hbar}{\delta V} \sim \frac{\hbar}{\sqrt{V}} \sim \frac{\hbar}{\sqrt{V}} \sim \frac{1}{\sqrt{V}} \sim \frac{1}{10^{40}}$$



As if each new x picked each pre-existing y at random with rel p

How is $2^{\frac{N^2}{4} + \dots}$ then nonlocality



one last case

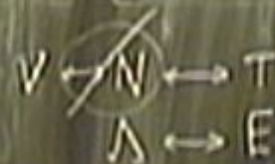
$$\Lambda \sim 10^{60}$$

grav. particle

$$\epsilon \Lambda \sim \frac{\hbar}{\delta V} \sim \frac{\hbar}{\sqrt{N}} \sim \frac{\hbar}{\sqrt{V}} \sim \frac{1}{\sqrt{V^{1/3}}} \sim \frac{1}{P}$$

e^{-s}

As if each new x picked each pre-existing y at random with prob p
 then escape $2^{\frac{N^2}{4} + \dots}$ then nullability



Maintain one last copy



$\Lambda \sim 10^{60}$ grow particle

$$\epsilon \Lambda \sim \frac{\hbar}{\delta V} \sim \frac{\hbar}{\sqrt{N}} \sim \frac{\hbar}{\sqrt{V}} \sim \frac{1}{\sqrt{N^{1/3}}} \sim \frac{1}{10^{19}}$$

$\frac{1}{p}$

Dynamics of Causal Sets

Dynamics of Causal Sets
Dynamics on Causal Sets



Dynamics of Causal Sets

Dynamics on Causal Sets



(opportuniste)
Dynamics of Causal Sets
Dynamics on Causal Sets



Dynamics on Causal Sets

Dynamics on Causal Sets



Dynamics on Causal Sets

④ Have causet analogues, $\Box$, of $\square = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}$

Dynamics on Causal Sets

- ④ Have causet analogues, B , of $\square = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}$ for scalar fields on sprinklings into M^2 (Sorkin & Henson)

Dynamics on Causal Sets

- ④ Have causet analogues, B , of $\square = -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2}$ for scalar fields on sprinklings into M^2 (Sorkin & Hanson)
 M^4 (Benincasa, FD) & $\{3, 5, 6, 7, 8\}$.

Dynamics on Causal Sets

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 M^4 (Benincasa, FD) & $3, 5, 6, 7, 8, \dots$ (Glaser)

- ⊗ Have caused analogues, B , of $\square = -\partial_t^2 + \nabla^2$ scalar fields on sprinklings into M^2 (Sohn & Hanson)
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- ⊗ $\langle B \rangle$

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 M^4 (Benincasa, FD) & $\{3, 5, 6, 7, 8, \dots\}$ (Glaser)
- ⊗ $\langle B\phi \rangle$

(x) Have caused analogues, B , of $\square = \partial_t^2 - \partial_x^2$
 scalar fields on spacetimes into M^2 (Sohn & Hanson)
 M^4 (Benincasa, FD) & $3, 5, 6, 7, 8 \dots$ (Ghiser)

(x) $\langle B \phi \rangle$

- ⊗ Have caused analogies, D, of $\mathbb{R}^2$ $\mathbb{R}^2$
 scalar fields on spacetimes into M^2 (Sorkin & Hanson)
 M^4 (Berencs, FD) & $3, 5, 6, 7, 8, \dots$ (Ghiser)
- ⊗ $\langle B\phi \rangle$ gets approx to $\mathbb{R}^2$, but fluctuations are big.

- ⑧ Have causal analogues, ∂_t & ∂_x scalar fields on spacetimes into M^2 (Sorkin & Hanson) M^4 (Berincasa, FD) & $\{3, 5, 6, 7, 8, \dots\}$ (Ghiser)
- ⑧ $\langle B\phi \rangle$ goes approx to $\mathbb{R}^d$, but fluctuations are big.

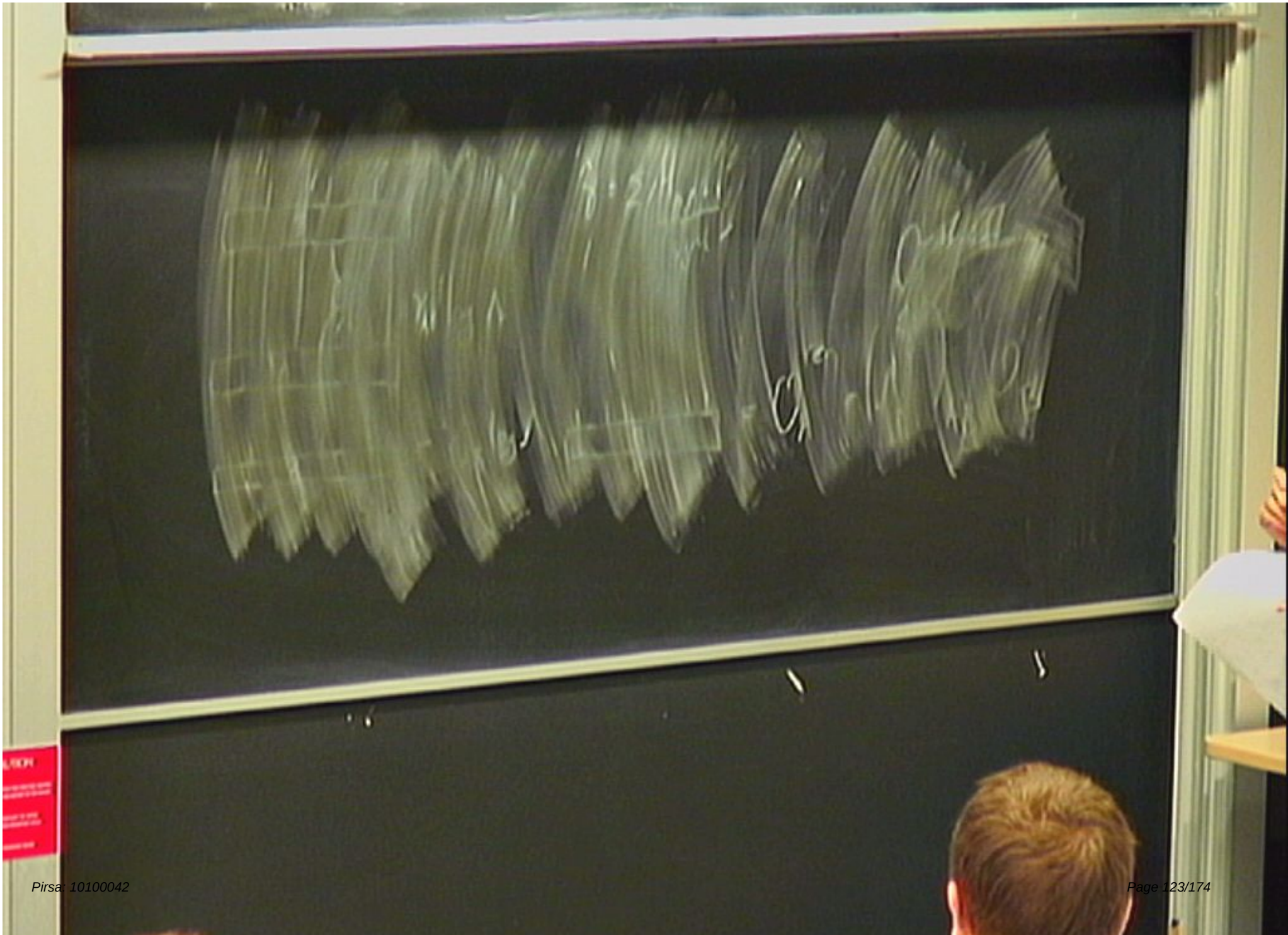
(x) Have caused analogies, D, ϕ ...
 scalar fields on sprinklings into M^2 (Sorkin & Hanson)
 M^4 (Benincasa, FD) & $\{3, 5, 6, 7, 8, \dots\}$ (Ghiser)

(x) $\langle B\phi \rangle$ get approx to $\mathbb{Z}\phi$, but fluctuations are big. To suppress
 these fluctuations, have to introduce a "mesoscale" ℓ

⑧ $\langle B\phi \rangle$ goes approx to $\langle \phi \rangle$, but fluctuations are big. To suppress these fluctuations, have to introduce a "mesoscale" l_k

$\otimes$ $\langle B\phi \rangle$ gets approx to $\langle \beta \rangle$, but fluctuations are big. To suppress
 these fluctuations, have to introduce a "mesoscale" l_k
 $l \ll l_k \ll L \leftarrow$

(x) $\langle B\phi \rangle$ goes approx to $\Box\phi$, but fluctuations are big. To suppress
 these fluctuations, have to introduce a "mesoscale" l_k
 $l \ll l_k \ll L$ ← atomic scale



& a "smeared" causal operator B_k .



& a "smeared" causal operator B_K .

$B_K \phi$ will be close to $B\phi$ for a single sprinkled causal

& a "smeared" causal operator B_k .

$B_k \phi$ will be close to $B\phi$ for a single sprinkled causal

(*)

& a "smeared" causal operator B_k .

$B_k \phi$ will be close to $B\phi$ for a single sprinkled causal

(*) Puncts still nevertheless grow with L &
 $l_k \gtrsim 10^{-12}$ cm (in 4d) if $L \sim \text{Hubble}$

& a "smeared" causal operator B_k .

$B_k \phi$ will be close to $B\phi$ for a single sprinkled caust

(*) Puncts still nevertheless grow in with L &
 $l_k \gtrsim 10^{-12}$ cm (in 4d) if $L \sim$ Hubble radius.

↑
nuclear scale - BIG! Is this already ruled out?

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↑
nuclear scale - BIG! Is this already ruled out?

⊗ Mesocole appears in other quantum gravity approaches : string theory $l_{pl} < l_{string} \ll L$
non-commutative geometry $l_{discrete} < l_{noncommutative} \ll L$

⊗ Mesoscale appears in other quantum gravity approaches : string theory $l_{pl} < l_{string} \ll L$

⊗ We can noncommutative region $l_{discrete} < l_{noncommutative} \ll L$

approaches : string theory $l_{pl} < l_{ny} < L$

⊗ We can propagate fields on causets using $B_k \phi$
→ phenomenology

⊗ "non locality of causal sets emerged"

As if each new x picked each pre-existing y at random with p

How escape $2^{\frac{N^2}{4} + \dots}$ then reality

$$\begin{array}{c} V \leftrightarrow N \leftrightarrow T \\ \Lambda \leftrightarrow E \end{array}$$

Martin one last case

$$\Lambda \sim 10^{60}$$

grav. particle

$$\delta \Lambda \sim \frac{k}{\delta V} \sim \frac{k}{\sqrt{N}} \sim \frac{k}{\sqrt{V}} \sim \frac{1}{\sqrt{V^{1/3}}} \sim \frac{1}{10^{10}}$$



Birth 1: "prevalent" (analog) = a, b

"2":

c, d spectrum

B.C. $\frac{d_1}{d_2}$ independent of spectrum

⊛ Can apply B to fields on causal sets sprinkled into curved



Birth 1: "precurved" (analog) = a, b

"2:

c, d: spectators

B.C.

$\frac{d_1}{d_2}$ independent of spectators

④ Can apply IB to fields on Cauchy sets & provide info on curves

space.

$$\langle B \phi(\omega) \rangle \xrightarrow[\omega \rho \rightarrow \infty]{} \square \phi(\omega) - \frac{1}{2}$$

2

④ Can apply B to fields on Cauchy sets & provide info on curves

space.

$$\langle B \phi(\omega) \rangle \xrightarrow[\omega \rho \rightarrow \infty]{} \square \phi(\omega) = \frac{1}{2} \tau$$

284 d 30
Benincasa
6d
G

Can apply B to fields on Cauchy sets specified by curves

space.

$$\langle B \phi(x) \rangle \xrightarrow{\omega \rho \rightarrow \infty} \begin{cases} \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{cases}$$

224d
Benincasa
6d
Glasow

3, 5, 7
(Glasow)

$\langle d$

Can apply B to fields on causal sets - spinorially curved

space

$$\langle B \phi(x) \rangle \xrightarrow{\omega \rho \rightarrow \infty} \left\{ \begin{array}{l} \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{array} \right.$$

284d
Benincasa
6d
Glassey

$$\langle B \phi(x) \rangle = \square \phi(x) - \frac{1}{2} R \phi + \# \ell^2 R^2 \phi$$

3, 5, 7
(Glassey)

Can apply to fields on causal sets & spinors in curved

space

$$\langle R \phi \rangle$$

$$\xrightarrow{\omega \rho \rightarrow \infty} \begin{cases} \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{cases}$$

2d, 4d
Benincasa
6d
Glasner

3, 5, 7
(Glasner)

$$\rangle = \square \phi(x) - \frac{1}{2} R \phi - \frac{1}{6} \ell^2 R^2 \phi + \dots$$

Can apply B to fields on causal sets > sprinkled in curved

space

$$\langle B \phi(x) \rangle \rightarrow \left\{ \begin{array}{l} \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{array} \right.$$

$\omega \rho \rightarrow \infty$

2&4d
Benincasa
6d
Glaser

$$\square \phi(x) + \frac{1}{2} R(x) \phi(x)$$

3, 5, 7

$$\langle B \phi(x) \rangle = \square \phi(x) - \frac{1}{2} R \phi - \left(\# \ell^2 R^2 \phi \right) + \dots$$

(Glaser)

Can apply to fields on causal sets & spinors in curved

space.

$$\langle R \rangle \xrightarrow{\omega \rho \rightarrow \infty} \begin{cases} \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{cases}$$

2d, 4d
Benincasa
6d
Glasner

3, 5, 7

$$\langle R \rangle = \square \phi(x) - \frac{1}{2} R \phi - \frac{1}{6} \square^2 R^2 \phi + \dots$$

(Glasner)

Can apply B to fields on causal sets & spinors in curved space.

space.

$$\langle B \phi \rangle \xrightarrow{\rho \rightarrow \infty} \begin{cases} (\square \phi(x) - \frac{1}{2} R(x) \phi(x)) & 2, 4, d \\ (\square \phi(x) + \frac{1}{2} R(x) \phi(x)) & 3, 5, 7 \end{cases}$$

Benincasa
6d
Glauber

$$\langle F \rangle \xrightarrow{\rho \rightarrow \infty} \square \phi(x) - \frac{1}{2} R \phi - \sqrt{\#} \ell^2 R^2 \phi + \dots$$

(Glauber)

ρ_{∞}

Can apply B to fields on causal sets sprinkled in curved

space.

$$\langle B \phi(x) \rangle$$



$$\langle B \phi(x) \rangle$$

$$\xrightarrow{\omega \rho \rightarrow \infty} \begin{cases} \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{cases}$$

2d, 4d
Benincasa
6d
Glasner

3, 5, 7
(Glasner)

$$= \square \phi(x) - \frac{1}{2} R \phi - \underbrace{\frac{1}{6} \ell^2 R^2 \phi}_{R_{00}} + \dots$$

Can apply B to fields on causal sets provided they are in the

space.

$$\langle B \phi(x) \rangle \xrightarrow{\omega \rho \rightarrow \infty} \left(\square \phi(x) - \frac{1}{2} R(x) \phi(x) \right)$$



$$\left(\square \phi(x) + \frac{1}{2} R(x) \phi(x) \right)$$

$$\langle B \phi(x) \rangle = \square \phi(x) - \frac{1}{2} R \phi - \sqrt{\# \ell^2 R^2} \phi$$

R_{00}

224 d
B

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set,}$$

$$\otimes 2d \quad S(c)$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set.}$$

$$\otimes 2d \quad S(c)$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set.}$$

$$\otimes 2d \quad (C) =$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set.}$$

$$\otimes 2. S(c) =$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set.}$$

$$\otimes 2d \quad S(C) = -\frac{1}{2} N$$

$$|C| = N$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set.}$$

$$\otimes 2d \quad S(C) = -\frac{1}{2} N + N_1$$

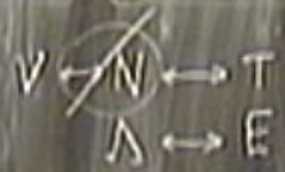
$$|C| = N$$

$$N_1 = \# \text{ links}$$

$N \gg 1$ 7 elements intervals

new x picked each pre-existing y at random with prob p
 then multiplicity

$$2^{\frac{N^2}{4} + \dots}$$



Monte Carlo last case



$$\Lambda \sim 10^{60} \quad \text{grav.} \quad \text{particle}$$

$$\delta \Lambda \sim \frac{k}{\delta V} \sim \frac{k}{\sqrt{N}} \sim \frac{k}{\sqrt{V}} \sim \frac{1}{\sqrt{N^{1/3}}} \sim \frac{1}{10^{10}}$$



$N \approx n$ 4 element intervals

$$S^{(+)}(c) = -N$$

$$l_{pe} = \sqrt{8\pi G h}$$

l

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set.}$$

$$\textcircled{*} \quad S(C) = \frac{1}{2} N + N_1 - 2N_2 + N_3$$

= # links

= # 3-element intervals

= # 4-element intervals.

$N \geq n$ 4 element intervals

$\nwarrow$ # order 1

$$S^{(4)}(c) = -N + N_1 - 9N_2 + 16N_3 - 8N_4$$

Now we can try (opportunistic)

$$l_{pe} = \sqrt{8\pi G h}$$

$$l \sim l_{pe}$$

$N \geq n$ 4 element intervals.

$$S^{(+)}(c) = \overset{\text{# order 1}}{\downarrow} -N + N_1 - 9N_2 + 16N_3 - 8N_4$$

Now we can try (opportunisticly)

$$Z(N) = \sum_{\substack{c \\ |c|=N}} e^{iS}$$

$$l_{pe} = \sqrt{8nGh}$$

$$l \sim l_{pe}$$

$N \geq n$ 4 element intervals.

$$S^{(4)}(c) = \overset{\text{# order 1}}{\downarrow} -N + N_1 - 9N_2 + 16N_3 - 8N_4$$

Now we can try (opportunisticly)

$$Z(N) = \sum_{\substack{C \\ |C|=N}} e^{i S^{(4)}(c)}$$

(Sunya, Rikvent &

$$l_{pe} = \sqrt{8nGh}$$

$$l \sim l_{pe}$$

$N \geq n$ 4 element intervals.

$$S^{(*)}(c) = \overset{\substack{\uparrow \\ \# \text{ order 1}}}{-N} + N_1 - 9N_2 + 16N_3 - 8N_4$$

Now we can try (opportunisticly)

$$Z(N) = \sum_{\substack{C \\ |C|=N}} e^{i S^{(*)}(c)}$$

(Sunya, Rindani & Henson).

$$l_{pe} = \sqrt{8nGh}$$

$$l \sim l_{pe}$$

$N \geq 4$ 4 element intervals

$$S^{(4)}(c) = \overset{\substack{\uparrow \\ \text{4th order}}}{-N} + N_1 - 9N_2 + 16N_3 - 8N_4$$

Now we can try (opportunistic)

$$Z(N) = \sum_{\substack{C \\ |C|=N}} e^{S^{(4)}(C)}$$

(Surya, Rindani & Henson).

$$l_{pe} = \sqrt{8n \ln 5}$$

$$l \sim l_{pe}$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \Rightarrow \text{action for causal set,}$$

$$\textcircled{*} \quad 2d \quad \boxed{S(C) = -\frac{1}{2} N + N_1 - 2N_2 + N_3}$$

$$|C| = N$$

$$N_1 = \# \text{ links}$$

$$N_2 = \# \text{ 3-element intervals}$$

$$N_3 = \# \text{ 4-element intervals}$$

Can apply B to fields on causal set = spinorially curved

space.

$$\langle B \phi(x) \rangle$$



$$\langle B \phi(x) \rangle$$

$$\xrightarrow{\omega \rho \rightarrow \infty} \left\{ \begin{aligned} & \square \phi(x) - \frac{1}{2} R(x) \phi(x) \\ & \square \phi(x) + \frac{1}{2} R(x) \phi(x) \end{aligned} \right.$$

224d
Benincasa
6d
Glasfer

3, 5, 7

(Glasfer)

$$= \square \phi(x) - \frac{1}{2} R \phi - \underbrace{\left(\# \ell^2 R^2 \phi \right)}_{R_{00}} + \dots$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \overset{\text{with constant}}{+} \Rightarrow \text{action for causal set.}$$

$$\textcircled{*} \ 2d \quad \boxed{S(C) = -\frac{1}{2} N + N_1 - 2N_2 + N_3}$$

$$|C| = N$$

$$N_1 = \# \text{ links}$$

$$N_2 = \# \text{ 3-element intervals}$$

$$N_3 = \# \text{ 4-element intervals}$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R^{+k} \Rightarrow \text{action for causal set.}$$

$$\textcircled{*} 2d \quad \boxed{S(C) = -\frac{1}{2} N + N_1 - 2N_2 + N_3}$$

$$|C| = N$$

$$N_1 = \# \text{ links}$$

$$N_2 = \# \text{ 3-element intervals}$$

$$N_3 = \# \text{ 4-element intervals}$$

$$\langle B(\phi=1) \rangle \approx \frac{2}{3} N$$

$$\textcircled{*} 2d \quad S(c) = -\frac{1}{2} N + N_1 - 2N_2 + N_3$$

$$|C| = N$$

$N_1 = \# \text{ links}$

$N_2 = \# \text{ 3-element intervals}$

$N_3 = \# \text{ 4-element intervals}$

$$\sum e^{iS(c)}$$

$$\sum w(c) = e^{-S(c)}$$

$N \geq n$ 4 element intervals.

$$w(c) = e$$

$$S(g, \beta) = -\frac{\beta}{2} N + \frac{1}{\beta} N_1 - 2\beta N_2 + \frac{1}{\beta} N_3$$

$$g \mapsto i\beta''$$

$N \geq n$ 4 element intervals.

$$w(c) = e$$

$$S(c, \beta) = -\frac{\beta}{2} N + \frac{1}{3} N_1 - 2\beta N_2 + \frac{1}{3} N_3$$

$$g \rightarrow g''$$

$$S(c, i) = \left(-\frac{N}{2} - N_1 - 2N_2 - N_3 \right)$$

is
e

$N \geq n$ 4 element intervals.

$$w(c) = e$$

$$S(c, s) = -\frac{3}{2}N + \frac{1}{3}N_1 - 2\frac{3}{5}N_2 + \frac{1}{5}N_3$$

$$g \rightarrow i g$$

$$S(c, i s) = i s \left(-\frac{N}{2} - N_1 - N_3 \right)$$

$$e^{i s(c, s)} = e^{-\frac{1}{2} N - N_1 - N_3}$$

$N \geq n$ 4 element intervals.

$$w(c) = e$$

$$S(c, s) = -\frac{s}{2} N + \frac{1}{s} N_1 - 2s N_2 + \frac{1}{s} N_3$$

$$s \mapsto i s$$

$$S(c, i s) = i s \left(-\frac{N}{2} - N_1 - 2N_2 - N_3 \right) - s \left(-\frac{N}{2} - N_1 - 2N_2 - N_3 \right)$$

$$e^{i S(c, i s)} = e$$

$N \geq n$ 4 element intervals.

$$W(C) = e$$

$$S(C, s) = -\frac{s}{2} N + \frac{1}{s} N_1 - 2s N_2 + \frac{1}{s} N_3$$

$$s \mapsto i s$$

$$S(C, i s) = \frac{i s}{2} (-N) - i s N_1 - 2 i s N_2 + i s N_3$$

$$e^{i S(C, i s)} = e$$

⊗ 2d

$$S(c) = -\frac{1}{2}N + N_1 - 2N_2 + N_3$$

$$|C| = N$$

$N_1 = \# \text{ links}$

$N_2 = \# \text{ 3-element intervals}$

$N_3 = \# \text{ 4-element intervals}$

$$\sum_i e^{iS(c)} = e^{-S_L(c)}$$

$$e^{iS(c)} = e^{-S(-\frac{1}{2}N - N_1 + 2N_2 - N_3)}$$

⊗ 2d

$$S(c) = -\frac{1}{2}N + N_1 - 2N_2 + N_3$$

$$|C| = N$$

$N_1 = \# \text{ links}$

$N_2 = \# \text{ 3-element intervals}$

$N_3 = \# \text{ 4-element intervals}$

$$Z(N, 3)$$

$$\sum$$

$$e^{iS(c)}$$

$$-S(c)$$

$$\sum$$

$$w(c) = e$$

$$S(c, 2) = -\frac{1}{2}N + N_1 - 2N_2 + N_3$$

$$e^{iS(c)} = e$$

⊗ 2d

$$S(c) = -\frac{1}{2}N + N_1 - 2N_2 + N_3$$

$$|C| = N$$

$N_1 = \# \text{ links}$

$N_2 = \# \text{ 3-element intervals}$

$N_3 = \# \text{ 4-element intervals}$

$$Z(N, 3)$$

$$\sum e^{iS(c)}$$

$$\sum w(c) = e$$

$$e^{iS(c)} = e^{-i(-\frac{1}{2}N + N_1 - 2N_2 + N_3)}$$

$$\langle B(\phi=1) \rangle \approx -\frac{1}{2} R \quad \begin{matrix} \text{Higher} \\ \text{order} \end{matrix} \Rightarrow \text{action for causal set.}$$

$$2 \quad \boxed{S(c) = -\frac{1}{2} N + N_1 - 2N_2 + N_3}$$

$$|C| = N$$

$$N_1 = \# \text{ links}$$

$$N_2 = \# \text{ 3-element intervals}$$

$$N_3 = \# \text{ 4-element intervals}$$

$$Z(N, S)$$

$$\sum e^{iS(c)}$$

$$\sum w(c) = e^{-S(c)}$$