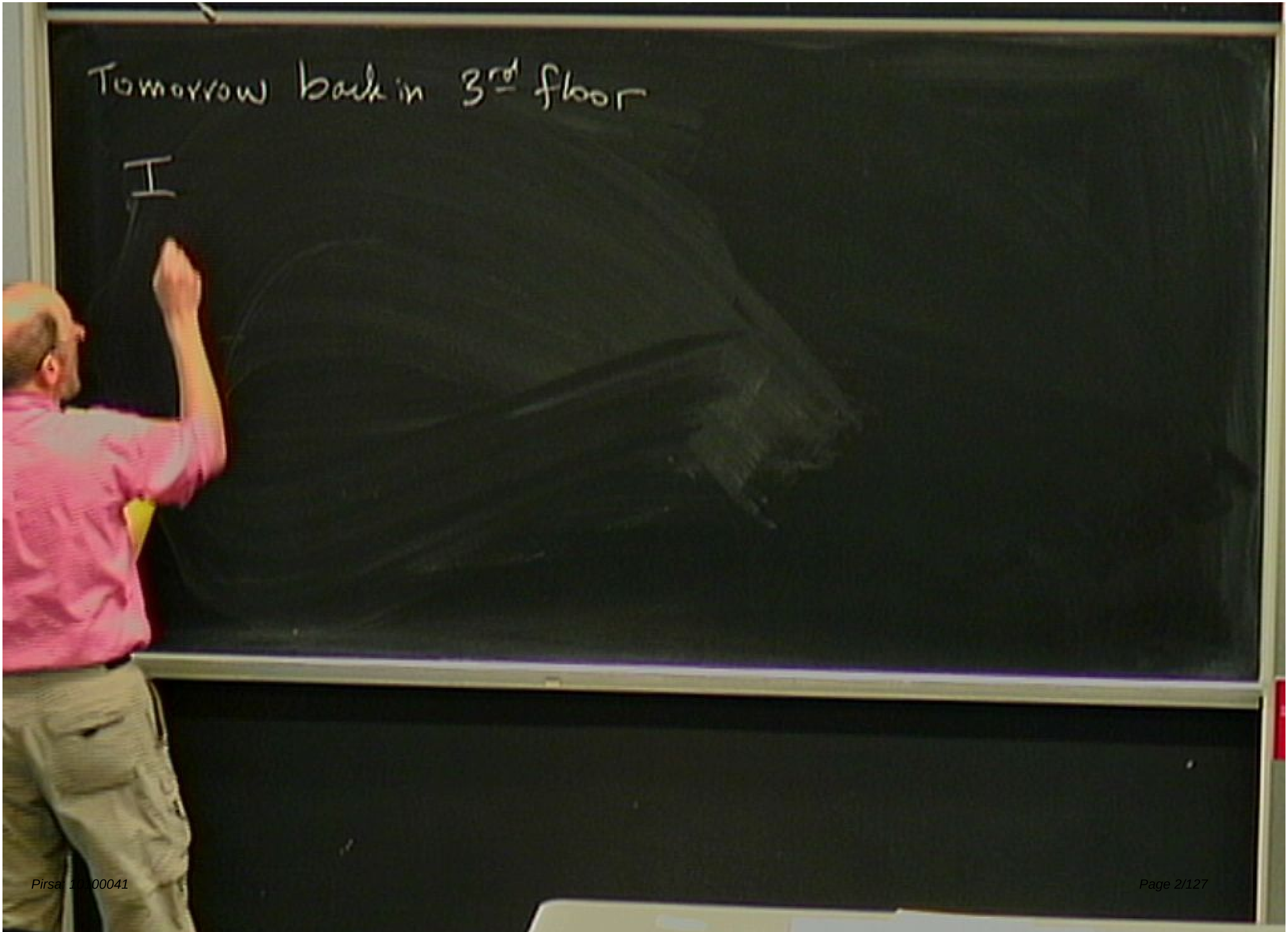
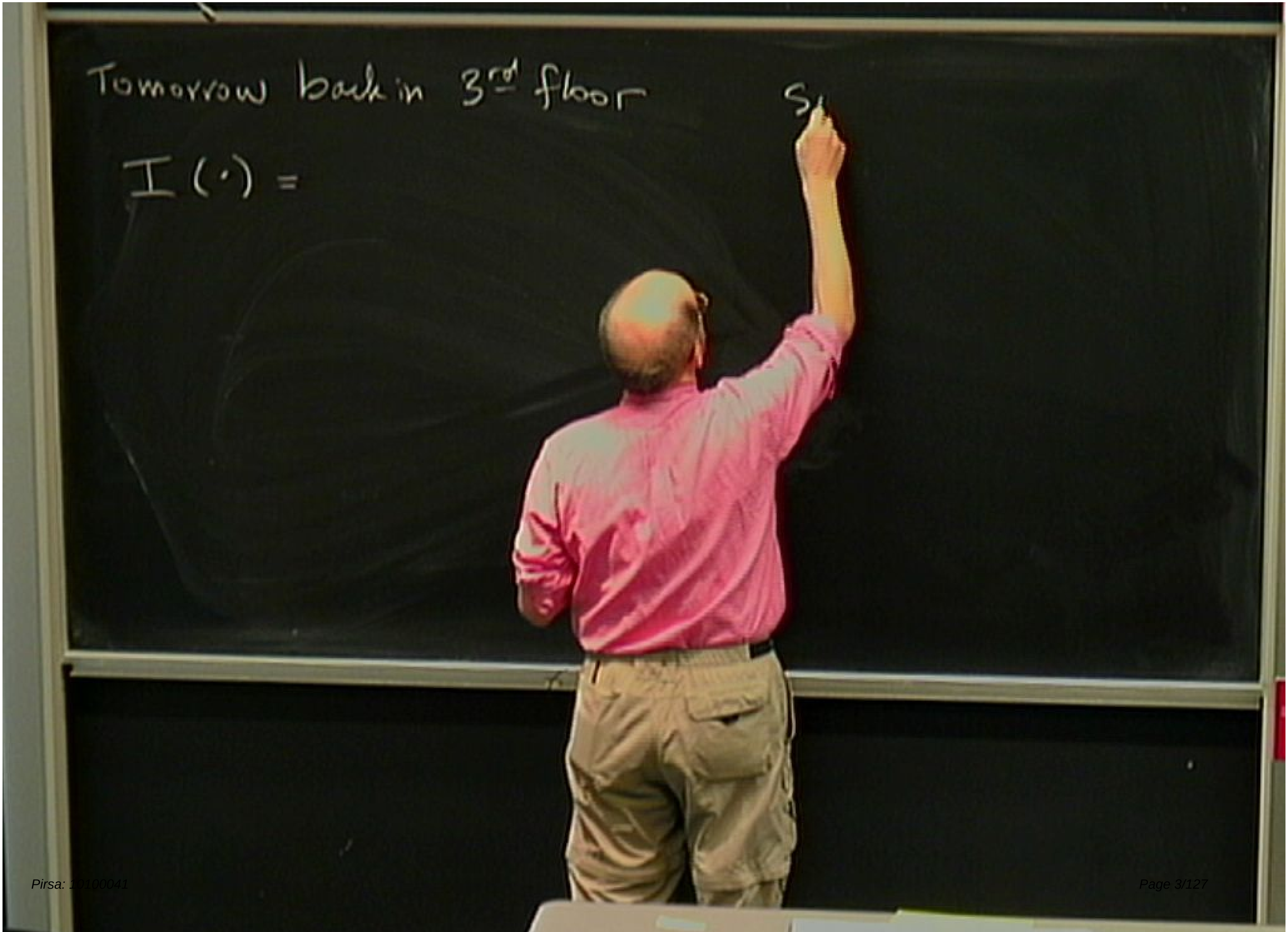
Title: An Invitation to Causal Sets - Lecture 4

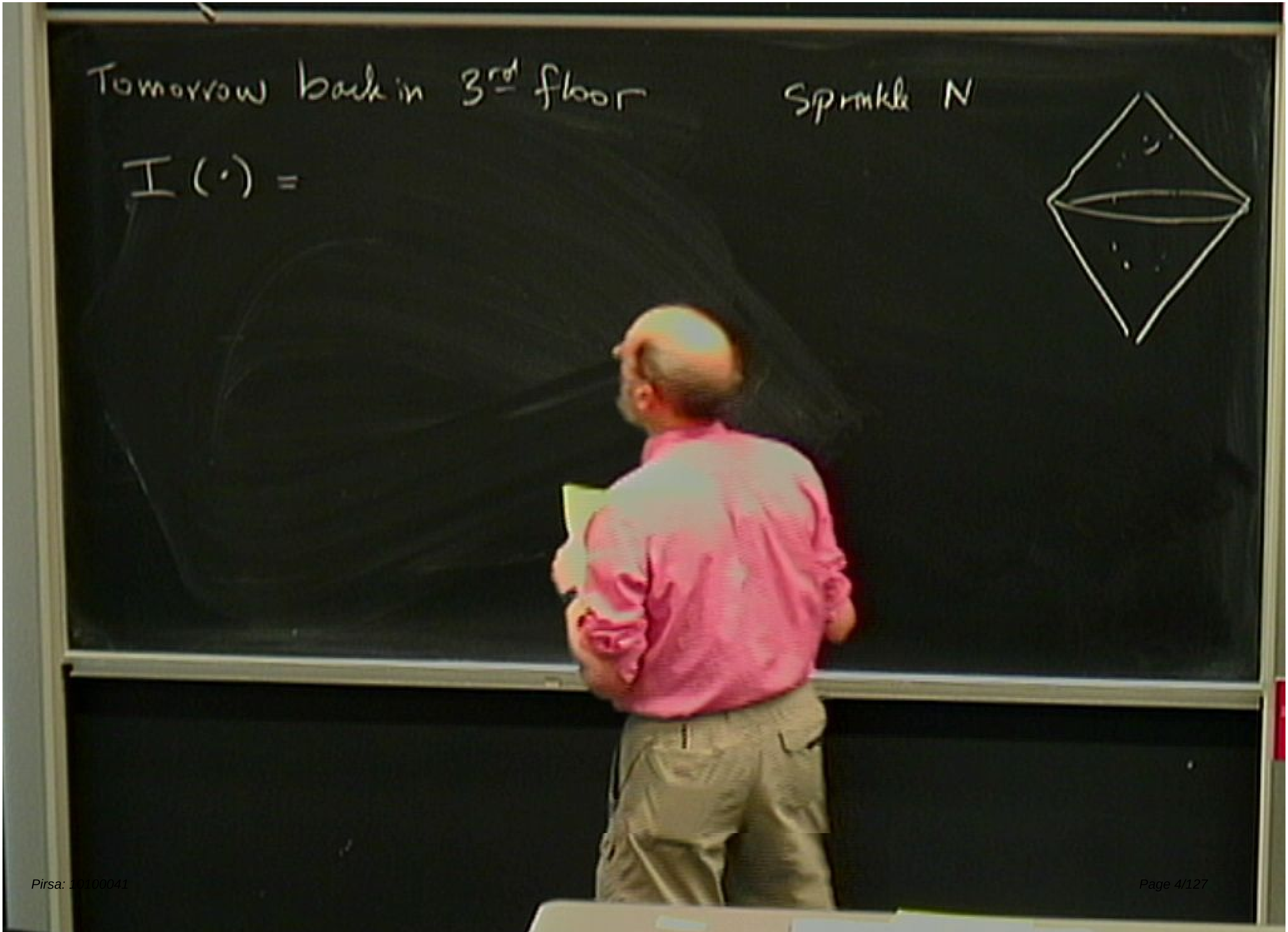
Date: Oct 21, 2010 11:00 AM

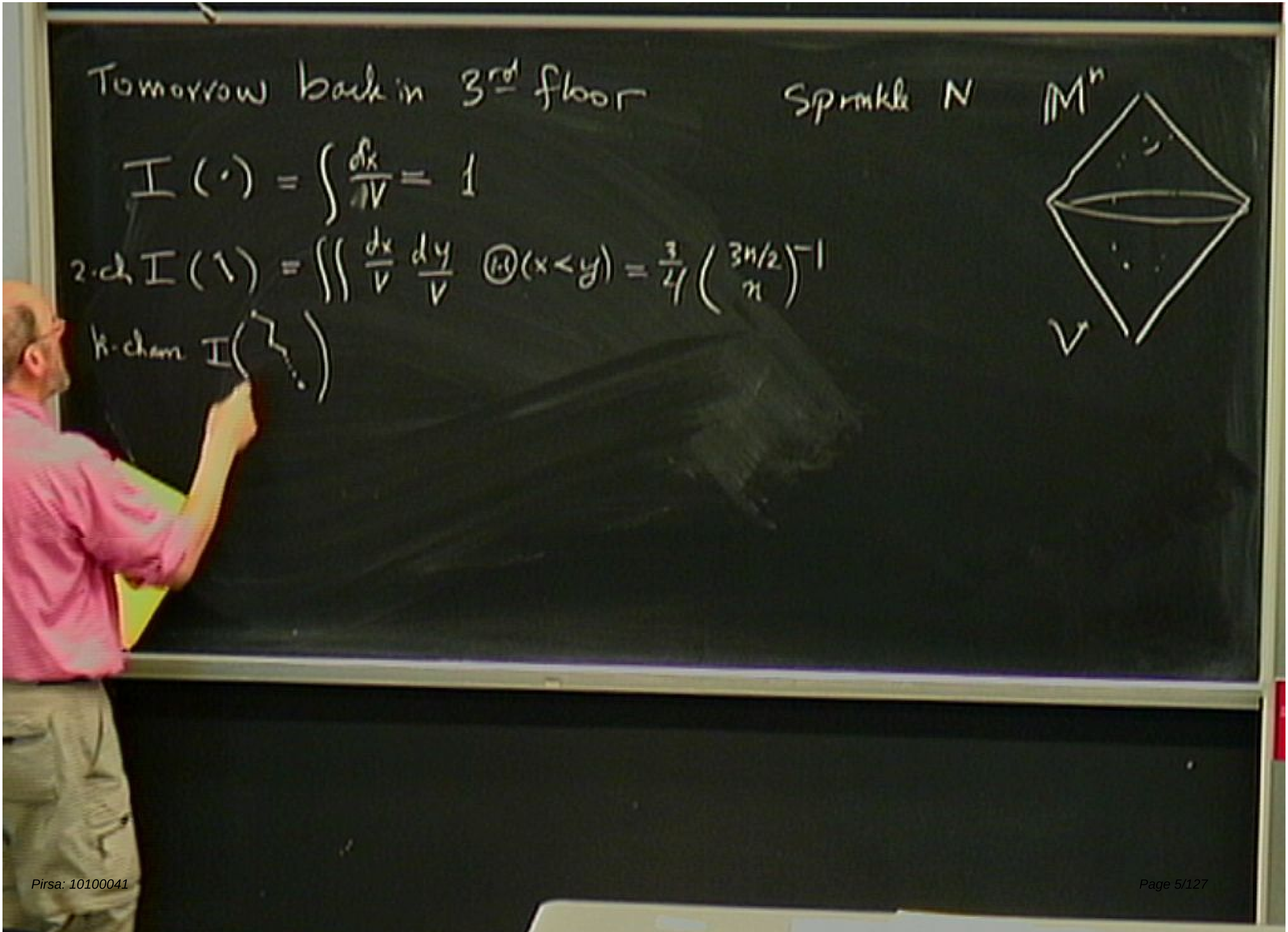
URL: http://pirsa.org/10100041

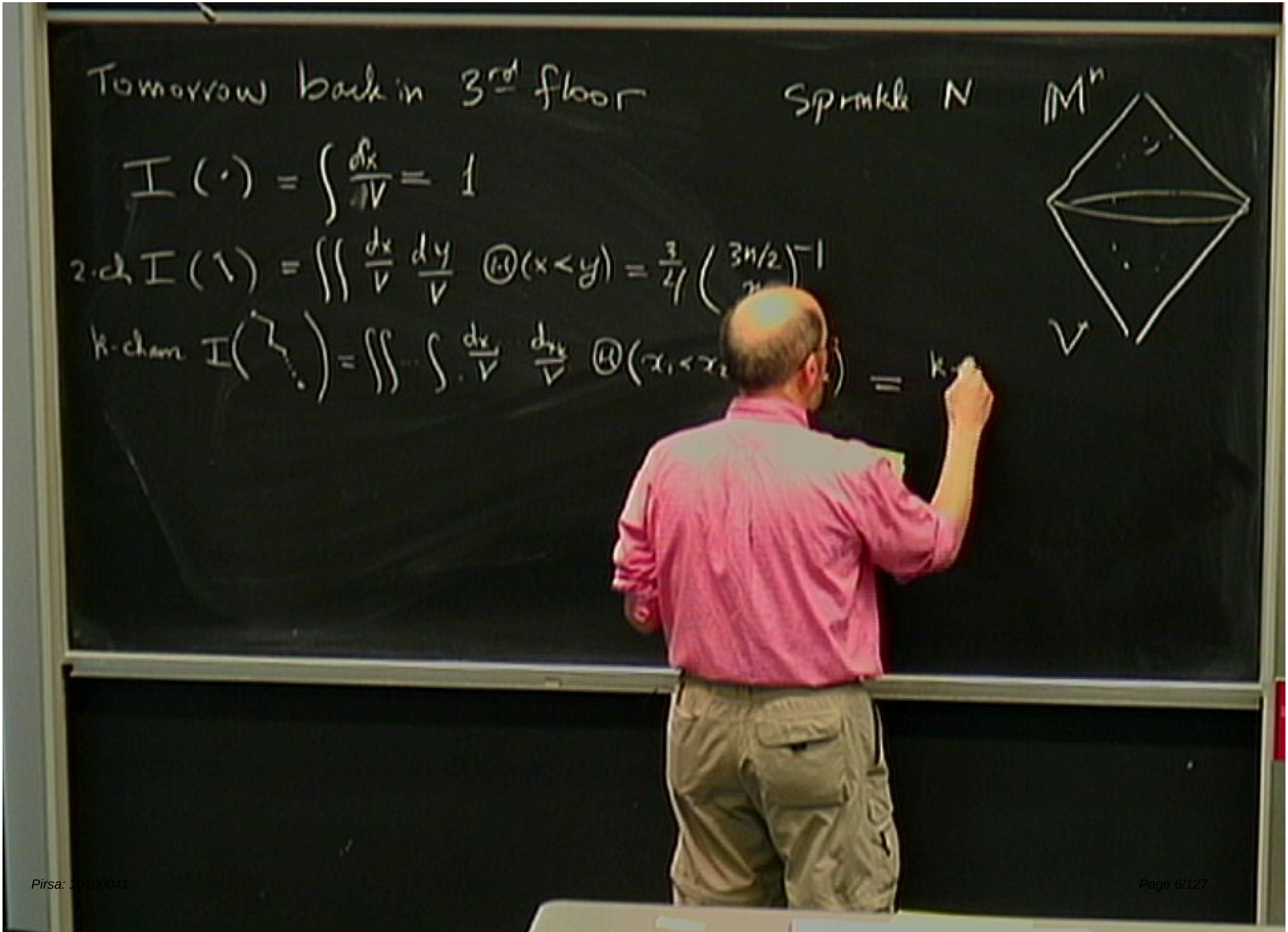
Abstract: Introduction to the causal set approach to quantum gravity and overview of current research in causal set theory


Tomorrow back in 3rd floor
I

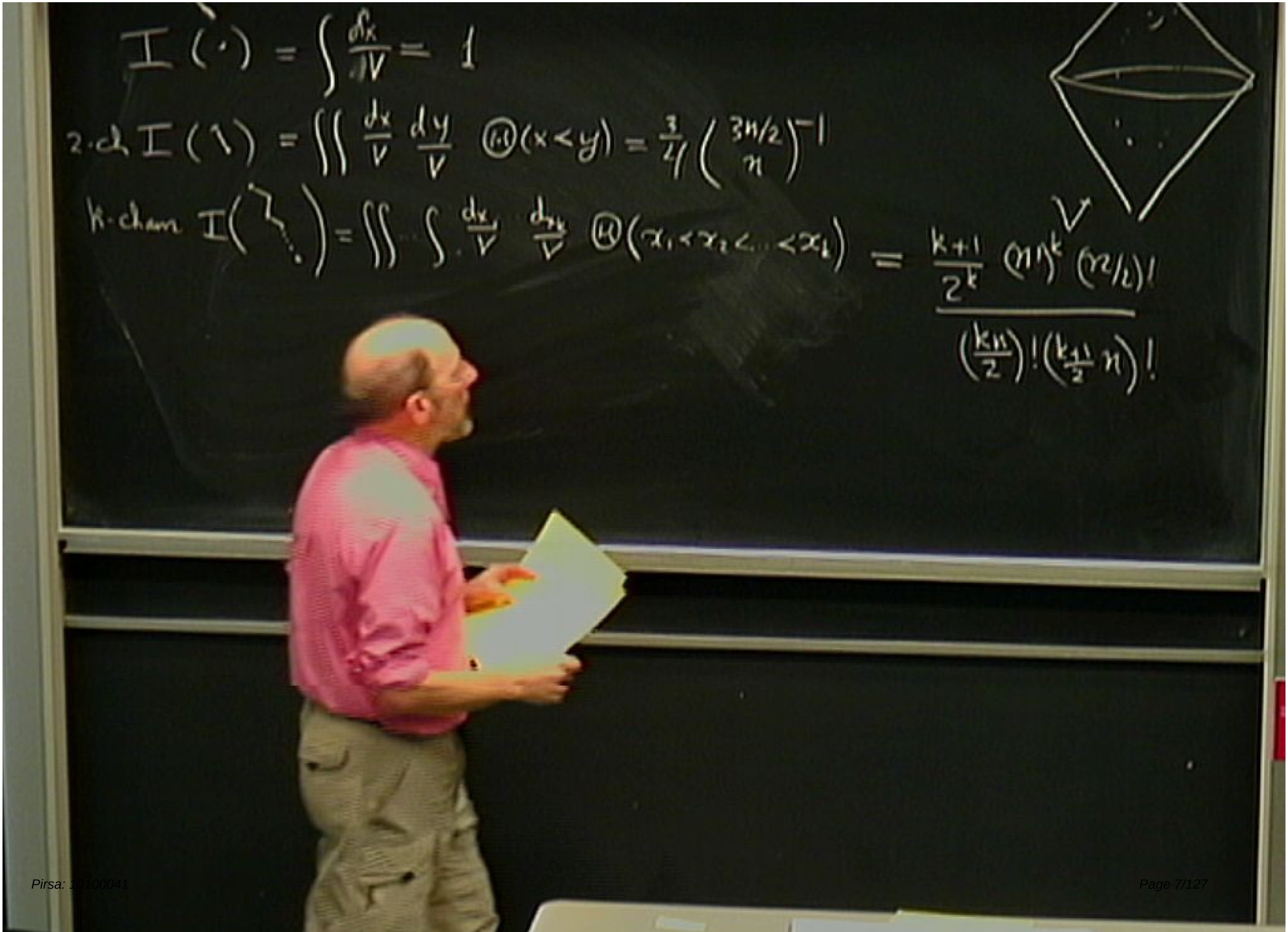

Tomorrow back in 3rd floor
I(·) =


Tomorrow back in 3rd floor
Sprinkle N
I(·) =

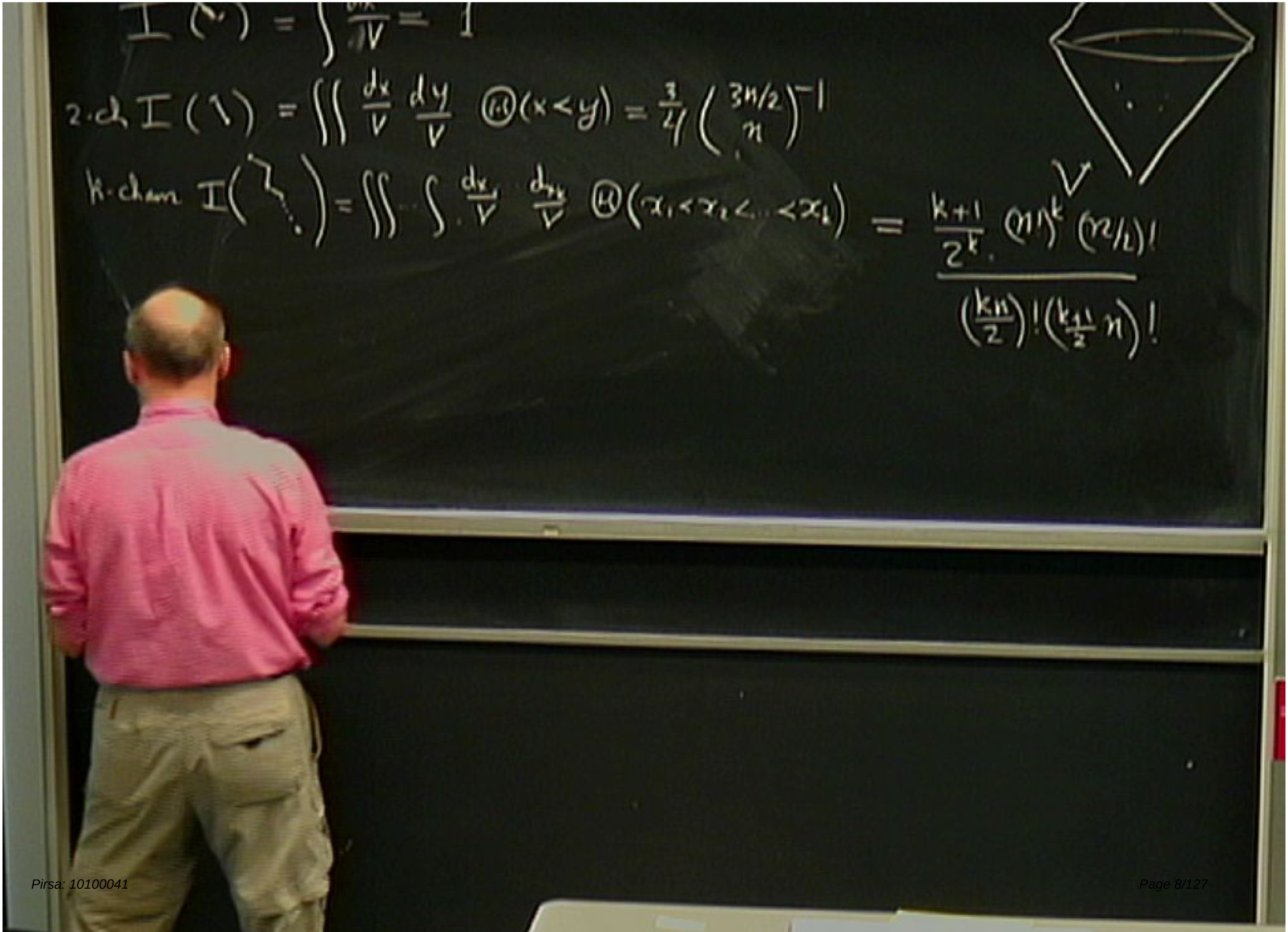

Tomorrow back in 3rd floor
Sprinkle N
M^n
I(·) = ∫ dx/V = 1
2-ch I( ) = ∫∫ dx/V dy/V (x<y) = 3/4 (3n/2 n)^-1
k-chain I( )
V



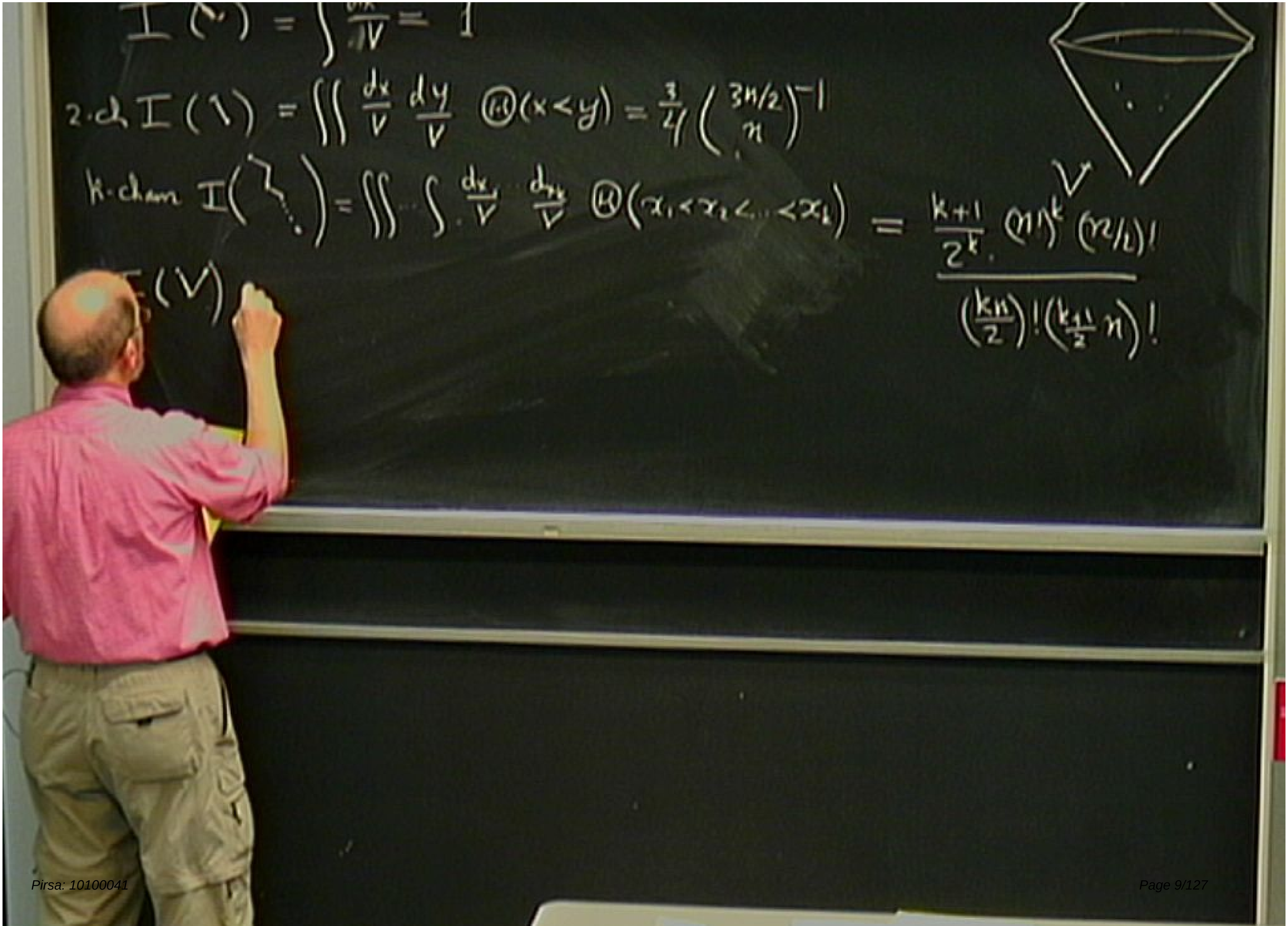
Tomorrow back in 3rd floor
Sprinkle N
M^n
$I(\cdot) = \int \frac{dx}{V} = 1$
2-ch $I(\ ) = \iint \frac{dx}{V}\frac{dy}{V}\,\Theta(x<y) = \frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$
k-chain $I(\ ) = \iint\cdots\int \frac{dx}{V}\cdots\frac{dx_k}{V}\,\Theta(x_1<x_2 \ldots) = k$
V



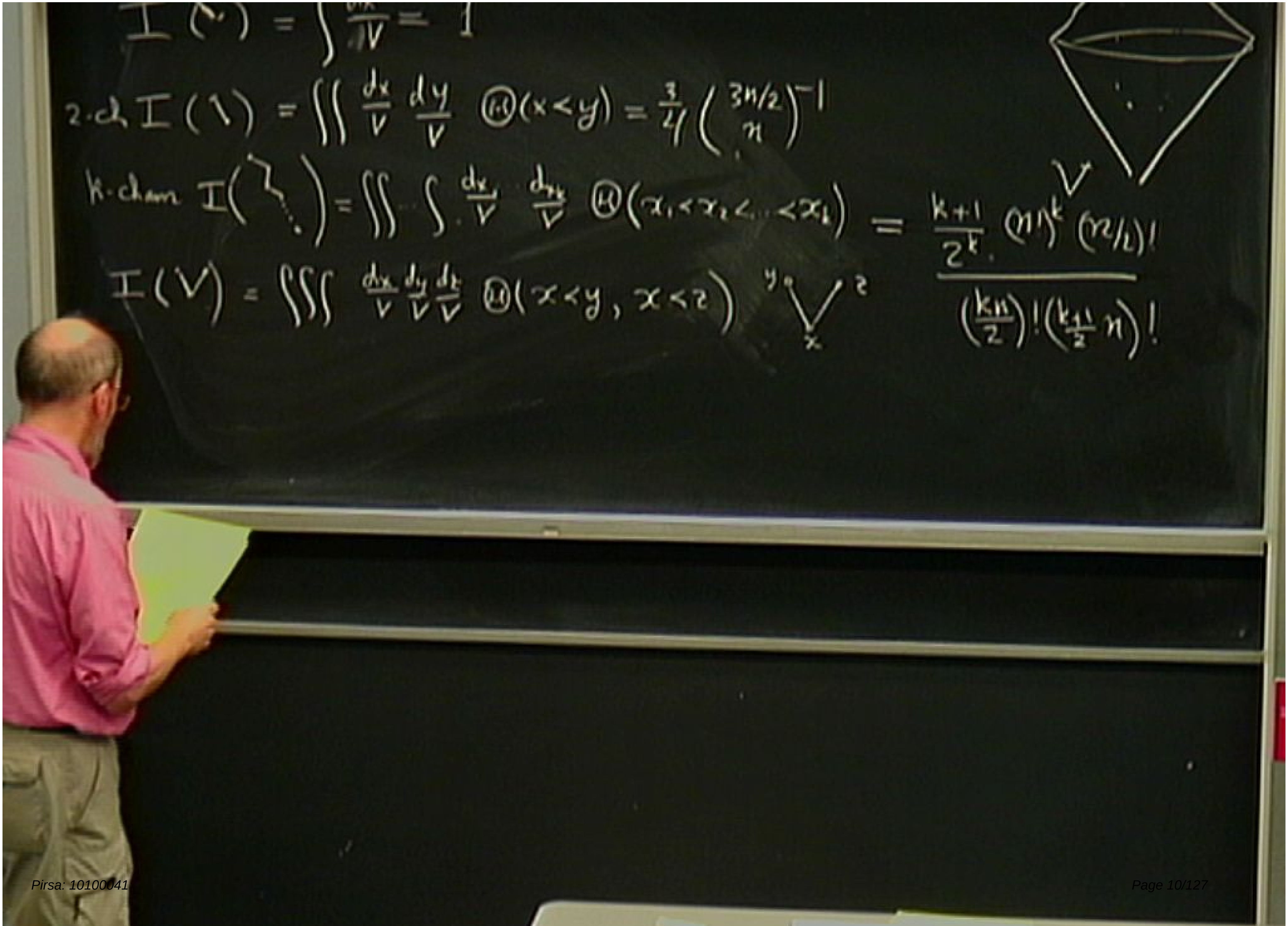
$I(\cdot) = \int \frac{dx}{V} = 1$
$2\text{-ch}\ I(\,\backslash\,) = \iint \frac{dx}{V}\frac{dy}{V}\,\Theta(x<y) = \frac{3}{4}\binom{3n/2}{n}^{-1}$
$k\text{-chain}\ I(\ldots) = \iiint\cdots\int \frac{dx_1}{V}\cdots\frac{dx_k}{V}\,\Theta(x_1<x_2<\ldots<x_k) = \frac{\frac{k+1}{2^k}(n!)^k (n/2)!}{\left(\frac{kn}{2}\right)!\left(\frac{k+1}{2}n\right)!}$
V



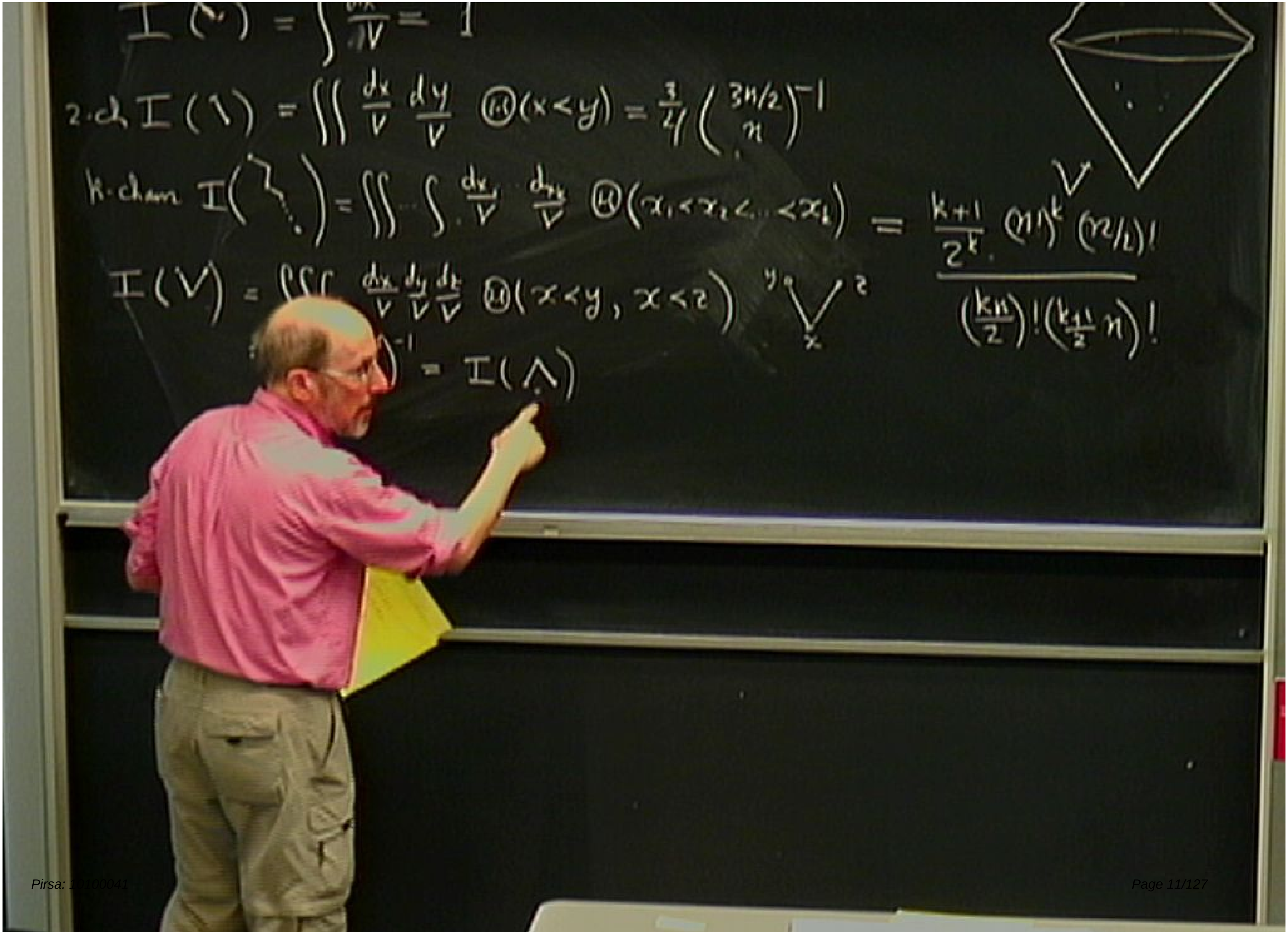
$I(\cdot) = \int \frac{dx}{V} = 1$
$2\text{-ch}\ I(\ ) = \iint \frac{dx}{V}\frac{dy}{V}\,\Theta(x<y) = \frac{3}{4}\binom{3n/2}{n}^{-1}$
$k\text{-chain}\ I(\ ) = \iiint \cdots \int \frac{dx_1}{V} \cdots \frac{dx_k}{V}\,\Theta(x_1<x_2<\cdots<x_k) = \frac{\frac{k+1}{2^k}\cdot (n!)^k\,(n/2)!}{\left(\frac{kn}{2}\right)!\left(\frac{k+1}{2}n\right)!}$
V

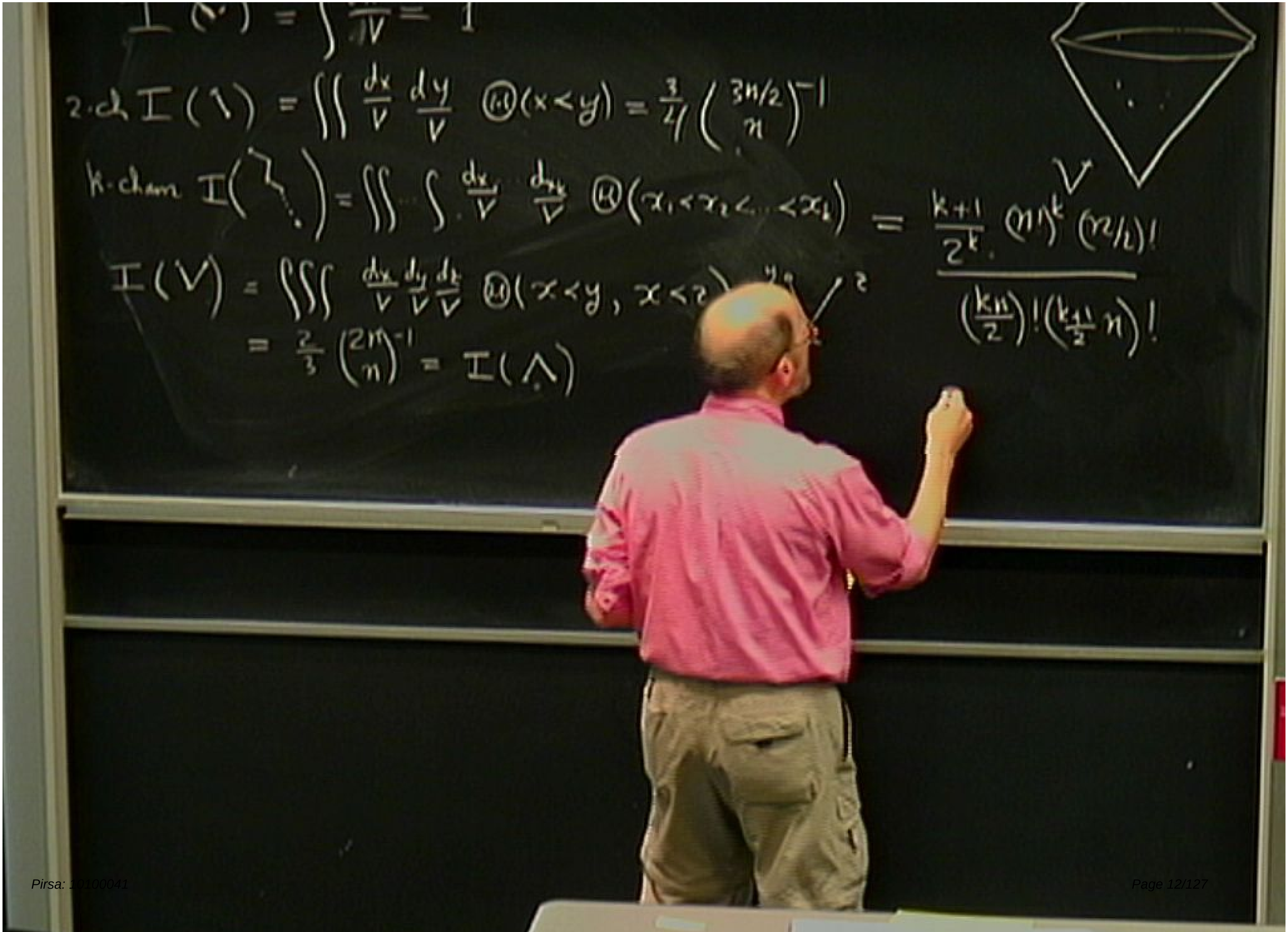


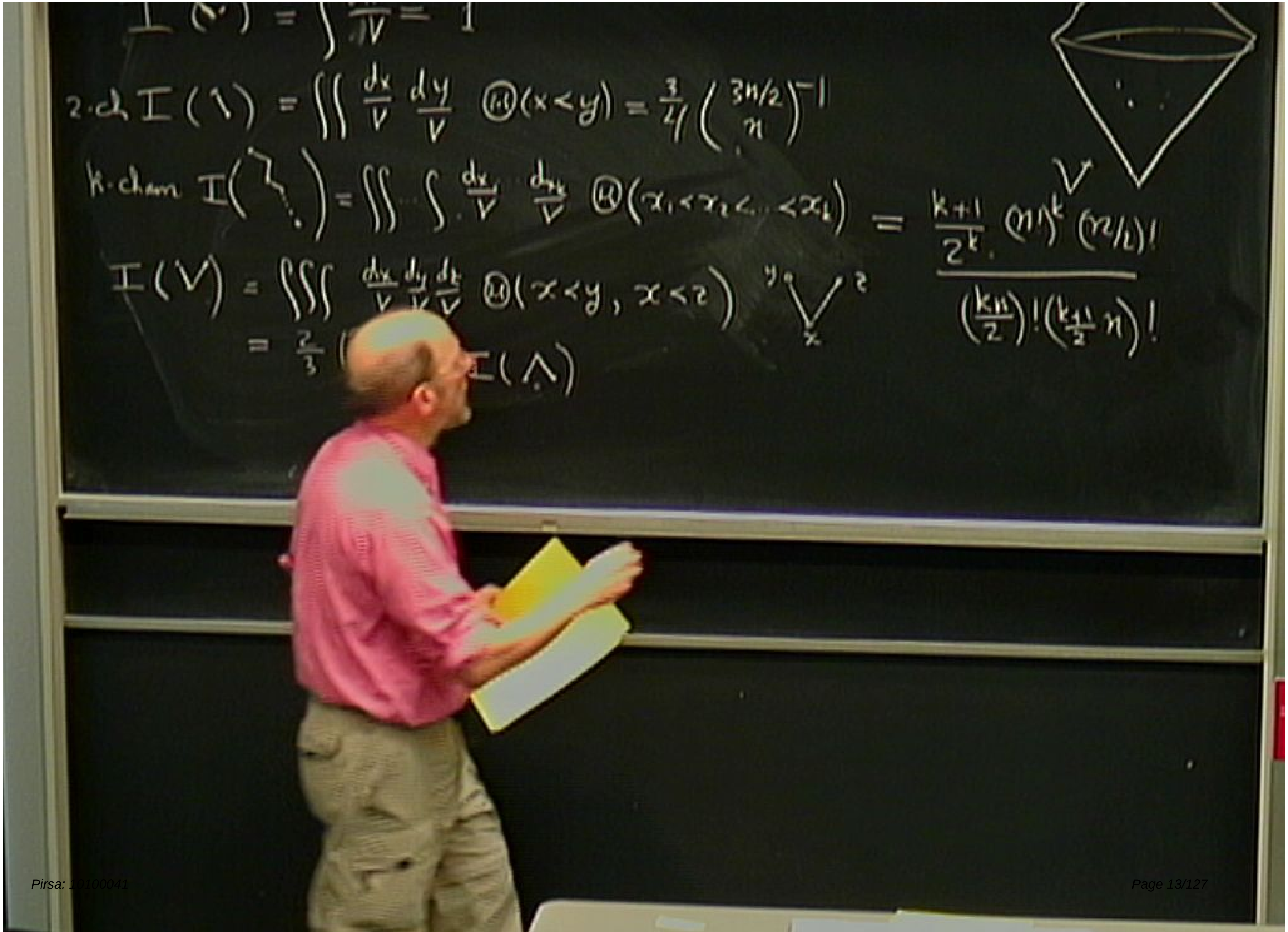
I(·) = ∫ dx/V = 1
2-ch I(\) = ∫∫ dx/V dy/V Θ(x<y) = 3/4 (3n/2 n)^-1
k-chain I( ) = ∫∫…∫ dx/V … dx_k/V Θ(x_1 < x_2 < … < x_k) = (k+1)/2^k (n!)^k (n/2)! / ((kn/2)! ((k+1)/2 n)!)
I(V)
V

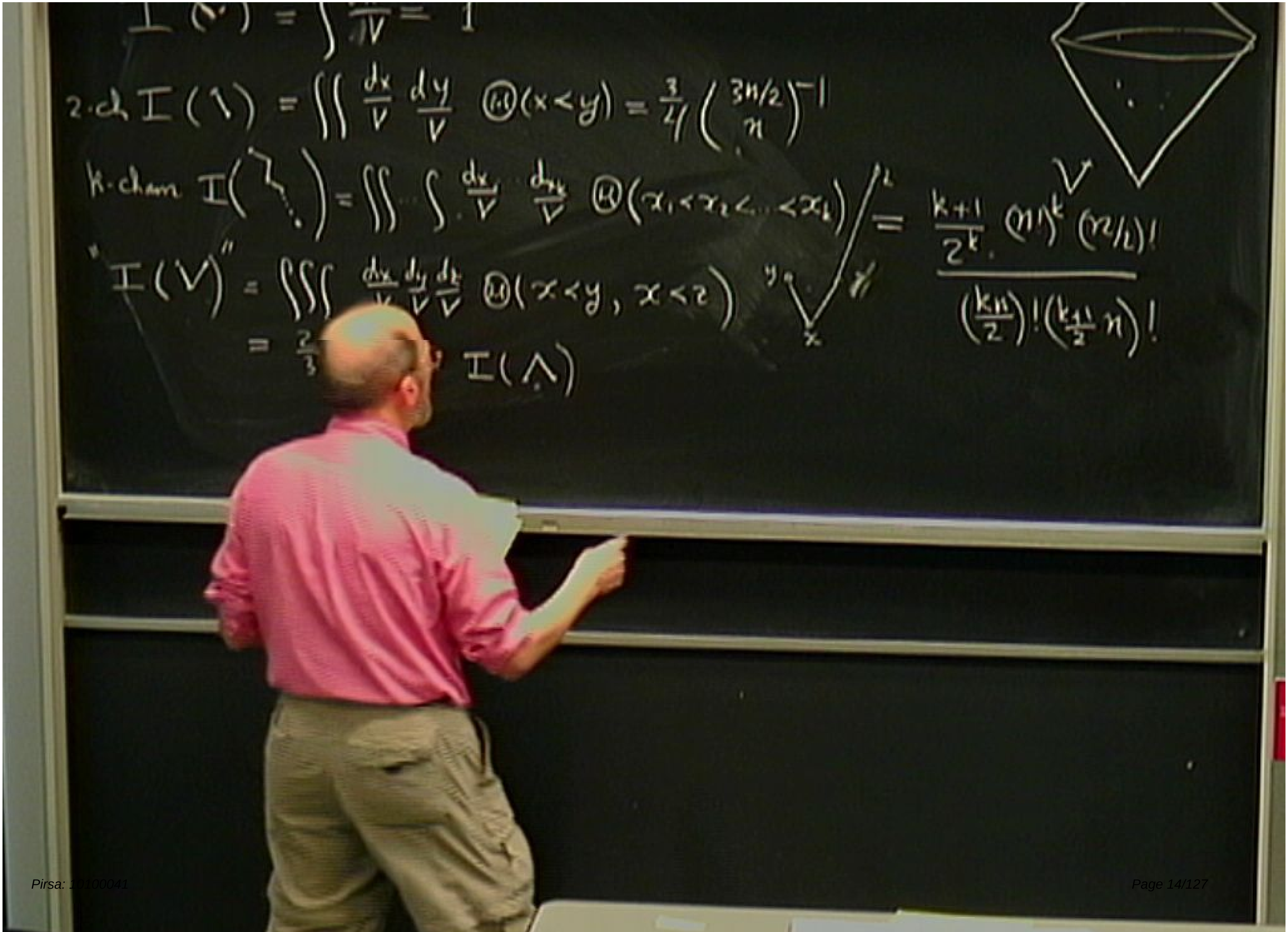


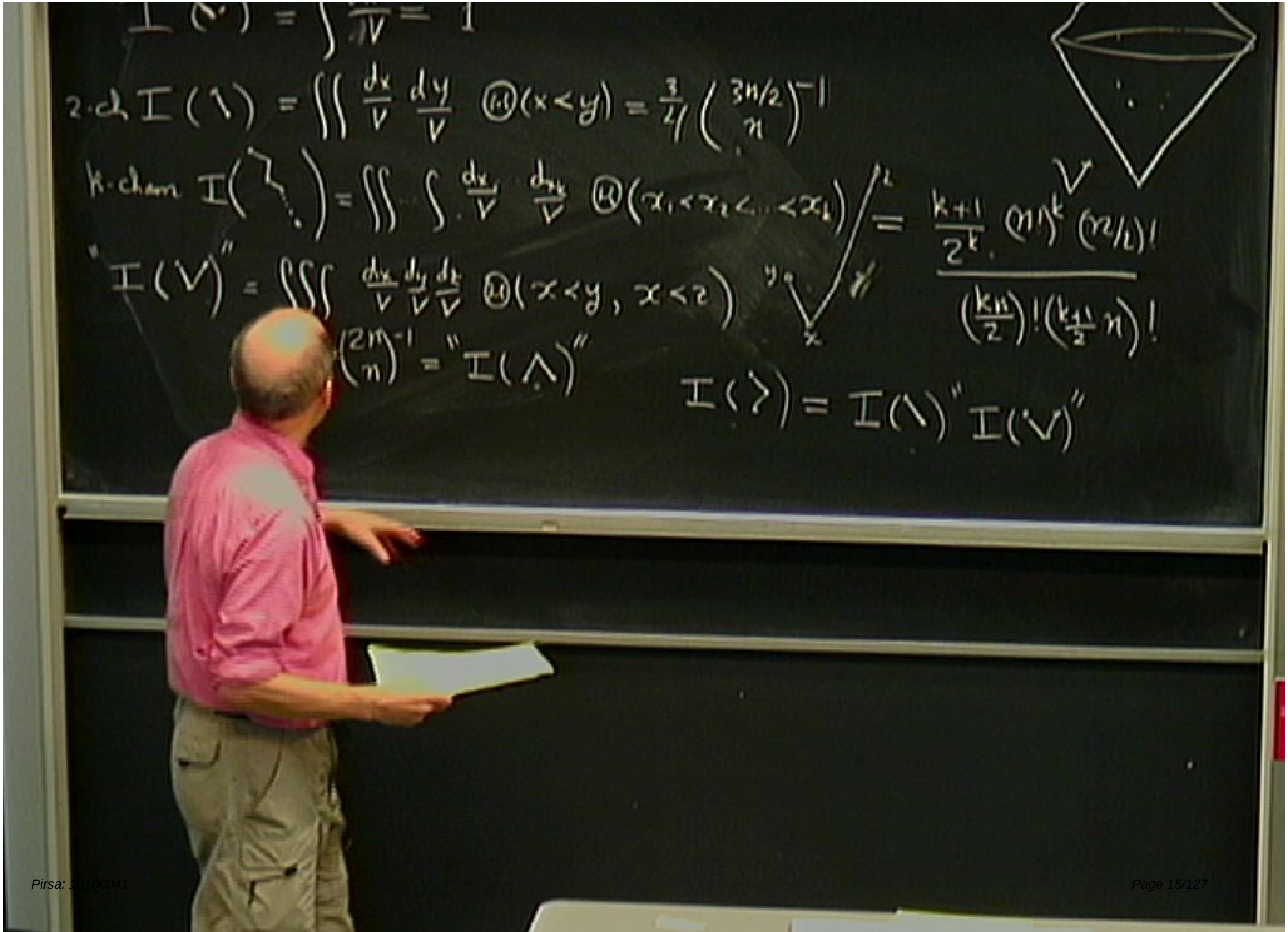
$2\text{-ch}\ I(\backslash) = \iint \frac{dx}{V}\frac{dy}{V}\,\Theta(x<y) = \frac{3}{4}\binom{3n/2}{n}^{-1}$
$k\text{-chain}\ I(\ldots) = \iiint\cdots\int \frac{dx}{V}\cdots\frac{dx_k}{V}\,\Theta(x_1<x_2<\ldots<x_k) = \frac{\frac{k+1}{2^k}(n!)^k(n/2)!}{\left(\frac{kn}{2}\right)!\left(\frac{k+1}{2}n\right)!}$
$I(V) = \iiint \frac{dx}{V}\frac{dy}{V}\frac{dz}{V}\,\Theta(x<y,\ x<z)$
y
z
x
V


2-ch I( ) = ∫∫ dx/V dy/V Θ(x<y) = 3/4 (3n/2 n)^-1
k-chain I( ) = ∫∫∫…∫ dx/V … Θ(x1<x2<…<xk)
I(V) = ∫∫∫ dx/V dy/V dz/V Θ(x<y, x<z)
= I(Λ)

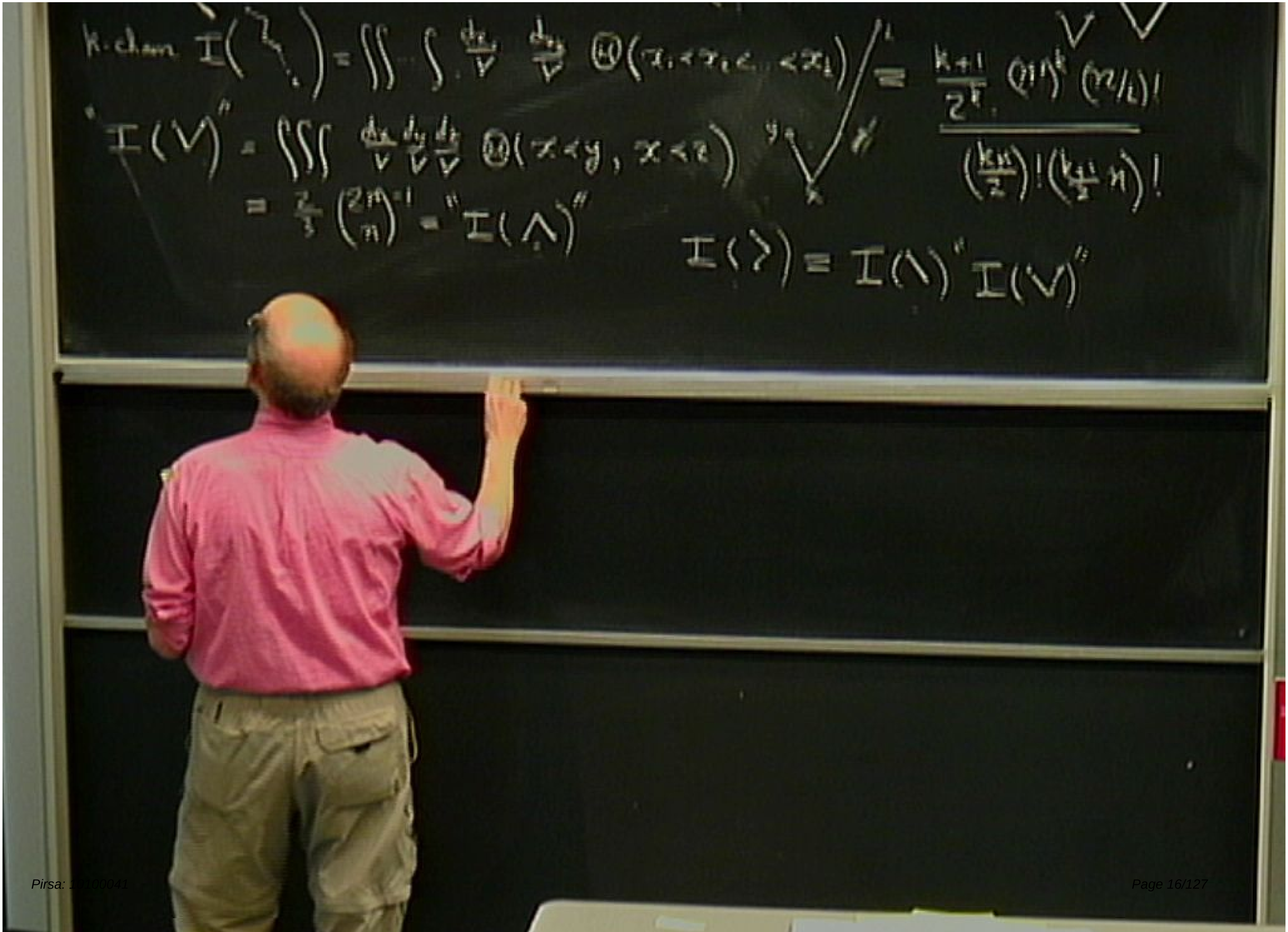

2-ch I( ) = ∫∫ dx/V dy/V Θ(x<y) = 3/4 (3n/2 n)^-1
k-chain I( ) = ∫∫∫ ... ∫ dx₁/V ... dxₖ/V Θ(x₁<x₂<...<xₖ) = (k+1)/2^k (n!)^k (n/2)! / (kn/2)! ((k+1)/2 n)!
I(V) = ∫∫∫ dx/V dy/V dz/V Θ(x<y, x<z)
= 2/3 (2n n)^-1 = I(Λ)

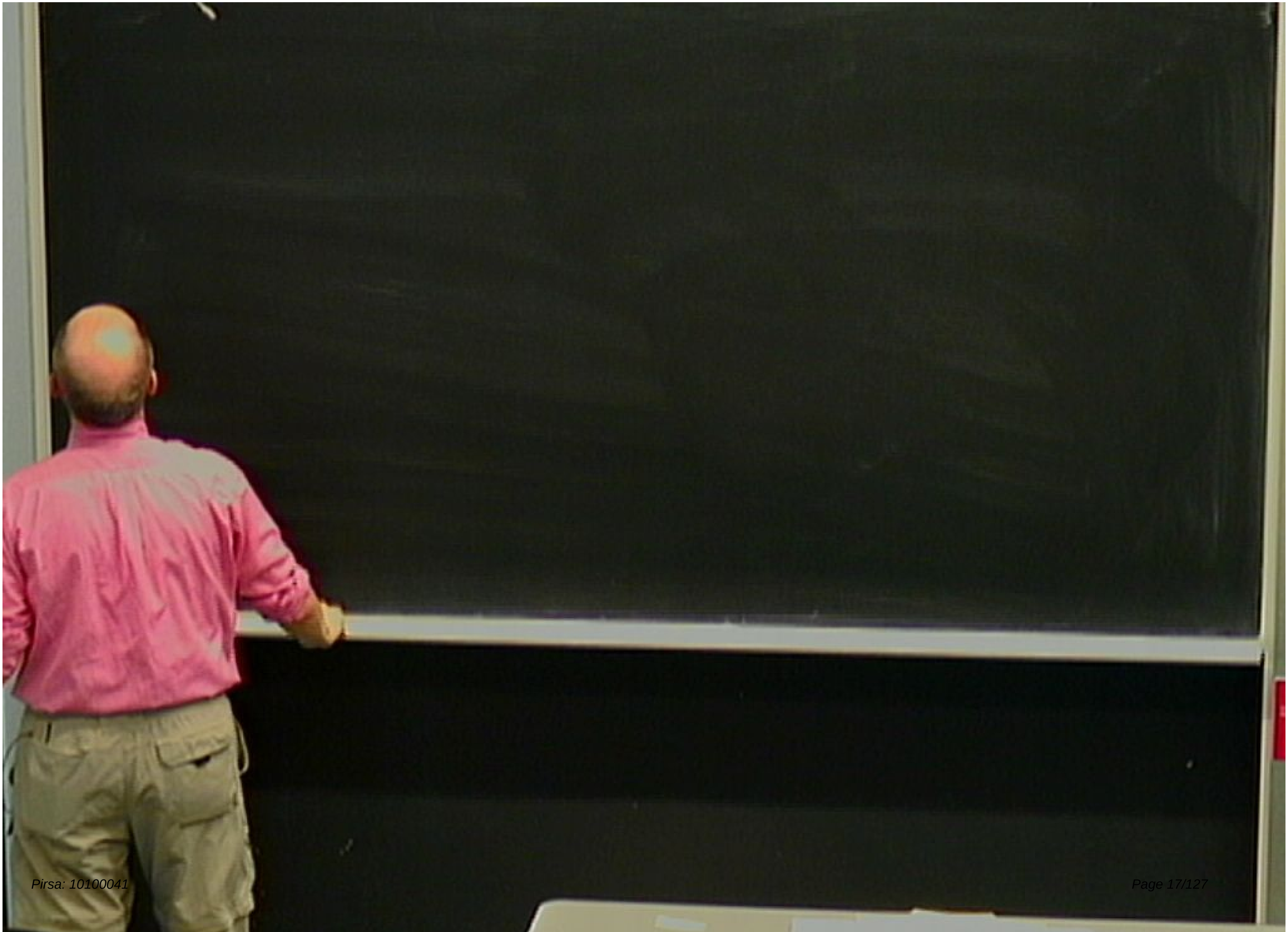

2-ch I( ) = ∬ dx/V dy/V Θ(x < y) = 3/4 (3n/2 n)^-1
k-chain I( ) = ∭ … ∫ dx1/V … dxk/V Θ(x1 < x2 < … < xk) = (k+1)/2^k · (n!)^k (n/2)! / ((kn/2)! ((k+1)/2 n)!)
I(V) = ∭ dx/V dy/V dz/V Θ(x < y, x < z)
= 2/3
I(Λ)
V
y z x



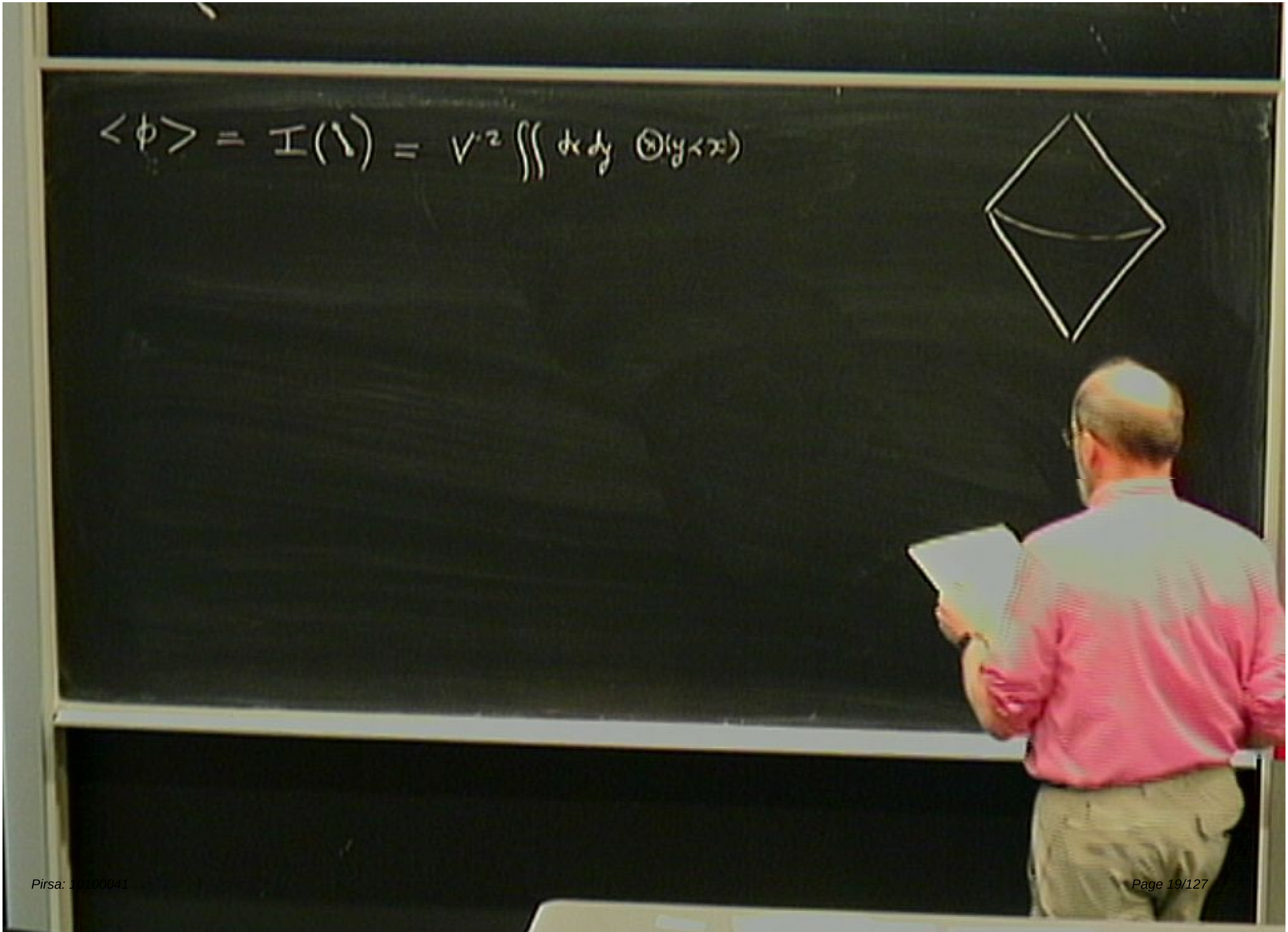


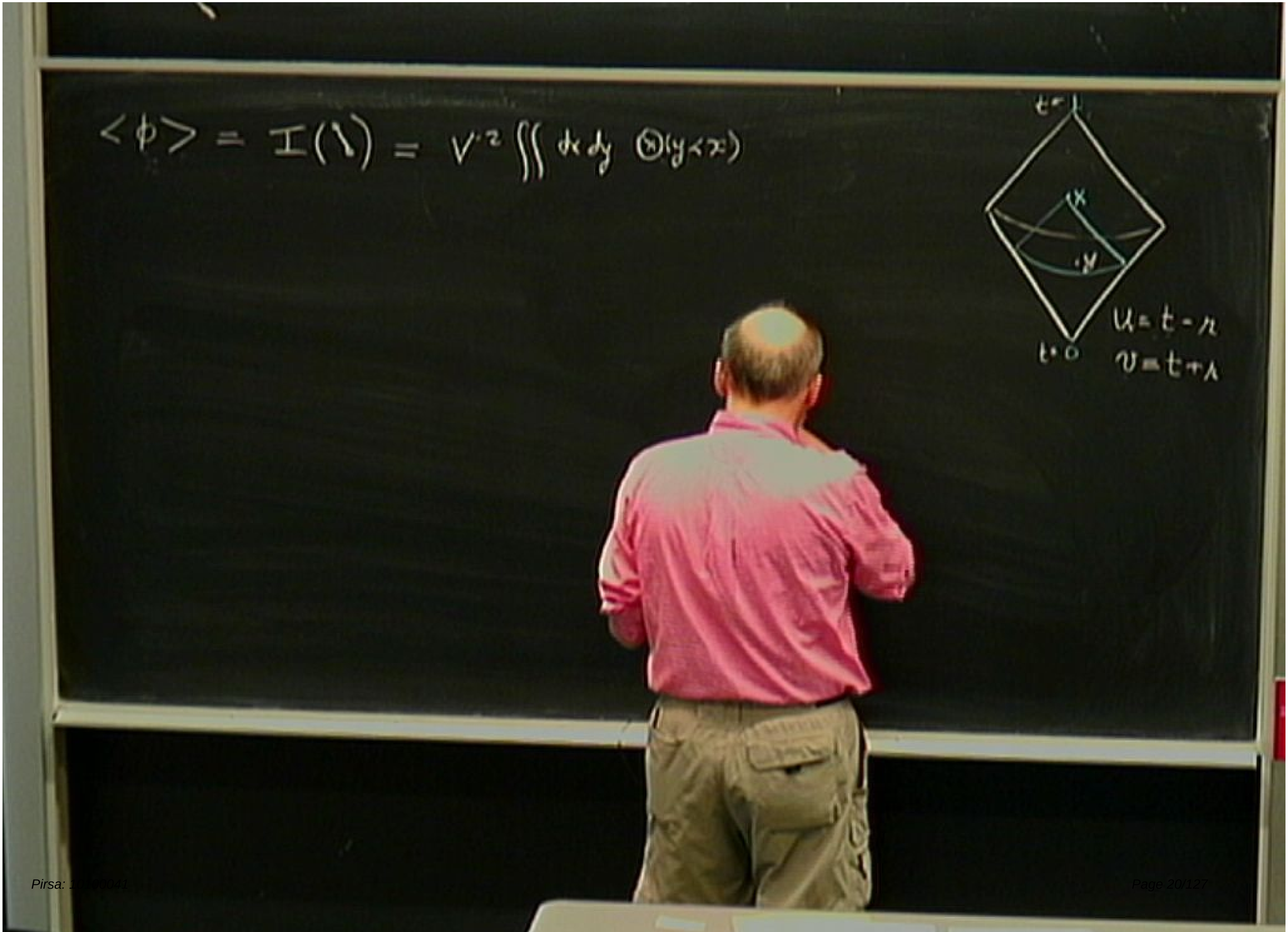
2-ch I( ) = ∬ dx/V dy/V θ(x<y) = 3/4 (3n/2 n)^-1
k-chain I( ) = ∭…∫ dx/V … dx_k/V θ(x_1<x_2<…<x_k) = (k+1)/2^k (n!)^k (n/2)! / ((kn/2)!((k+1)/2 n)!)
"I(V)" = ∭ dx/V dy/V dz/V θ(x<y, x<z)
(2n n)^-1 = "I(Λ)"
I(>) = "I(Λ)" "I(V)"

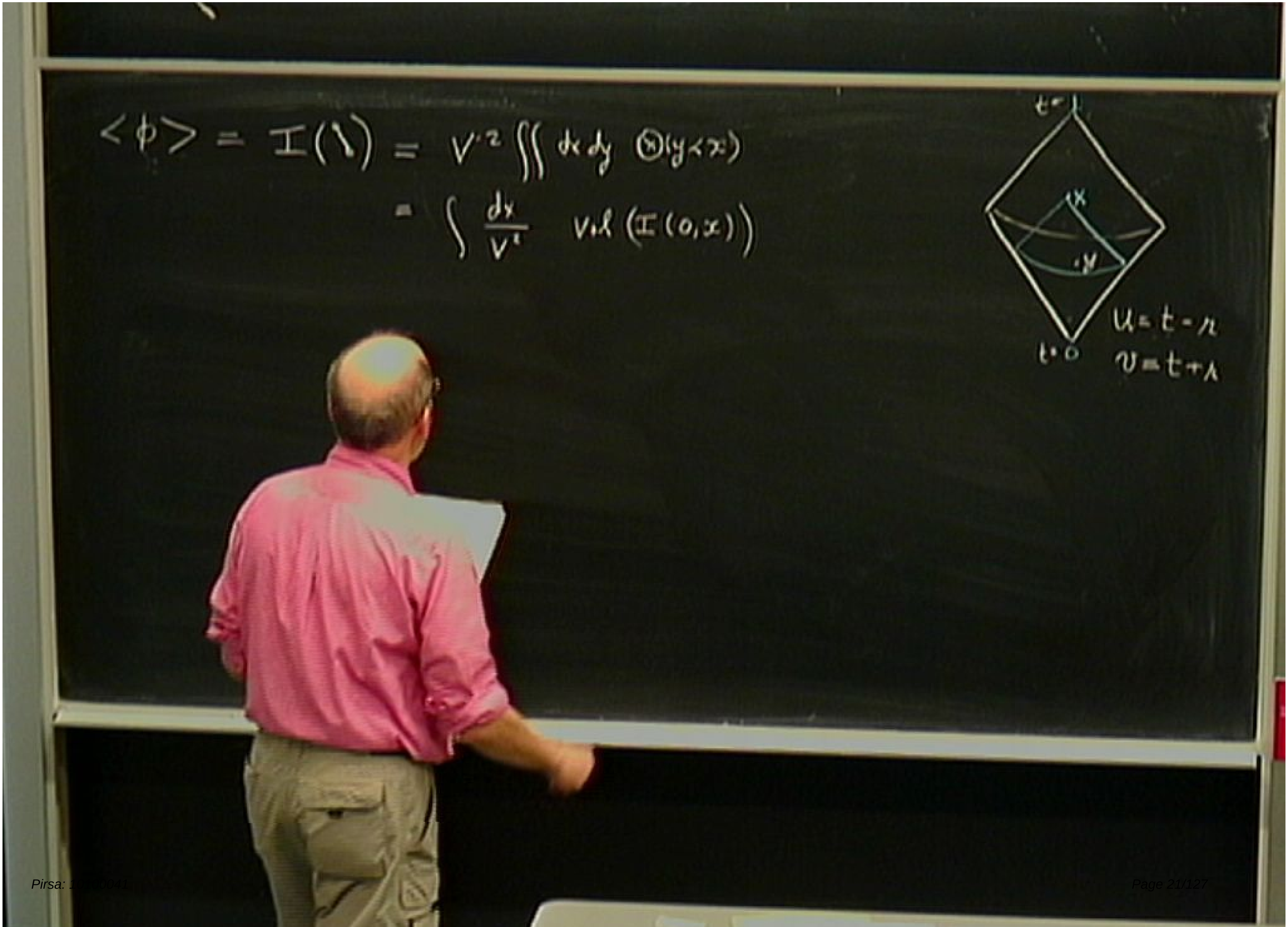

k-chain
I(V)" = ∫∫∫ dx/V dy/V dz/V Θ(x<y, x<z)
= 2/3 (2n n)^-1 = "I(Λ)"
I(>) = I(Λ)" I(V)"



$\langle \phi \rangle$


$\langle \phi \rangle = I(\ ) = V^{-2} \iint dx\, dy\ \Theta(y < x)$


<φ> = I(λ) = V^-2 ∬ dx dy Θ(y<x)
t=1
x
y
t=0
u = t - r
v = t + r


$\langle \phi \rangle = I(\setminus) = V^{-2} \iint dx\, dy\, \Theta(y<x)$
$= \int \frac{dx}{V^2}\, \mathrm{vol}\,(I(0,x))$
t=1
x
y
t=0
$u = t - r$
$v = t + r$

$$\langle \phi \rangle = I(\lambda) = V^{-2} \iint dx\, dy\ \Theta(y < x)$$

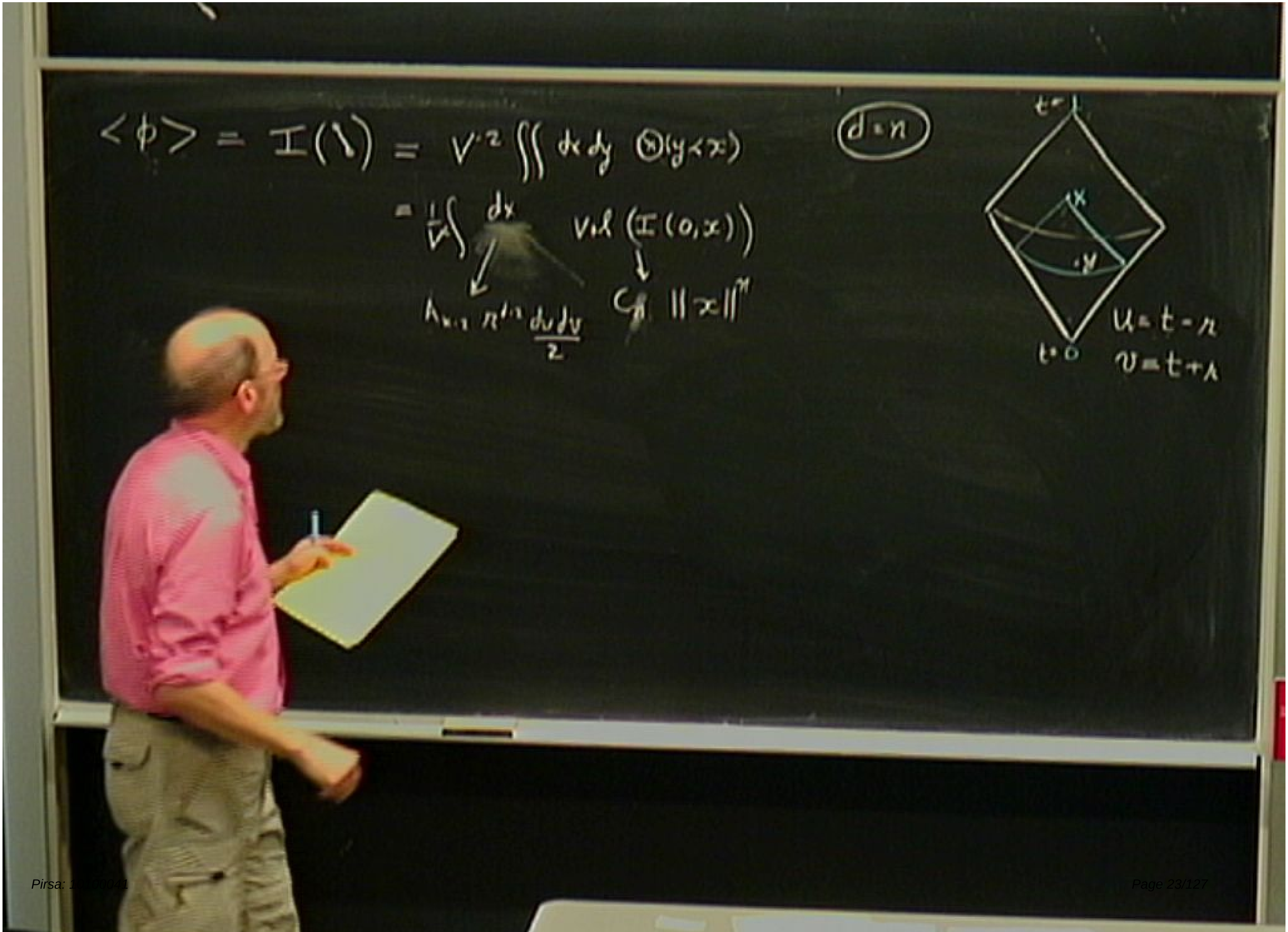
$$= \frac{1}{V} \int \frac{dx}{V}\ \mathrm{vol}\big(I(0,x)\big)$$

$$\mathrm{vol}\big(I(0,x)\big) \to c_n \, \|x\|^n$$

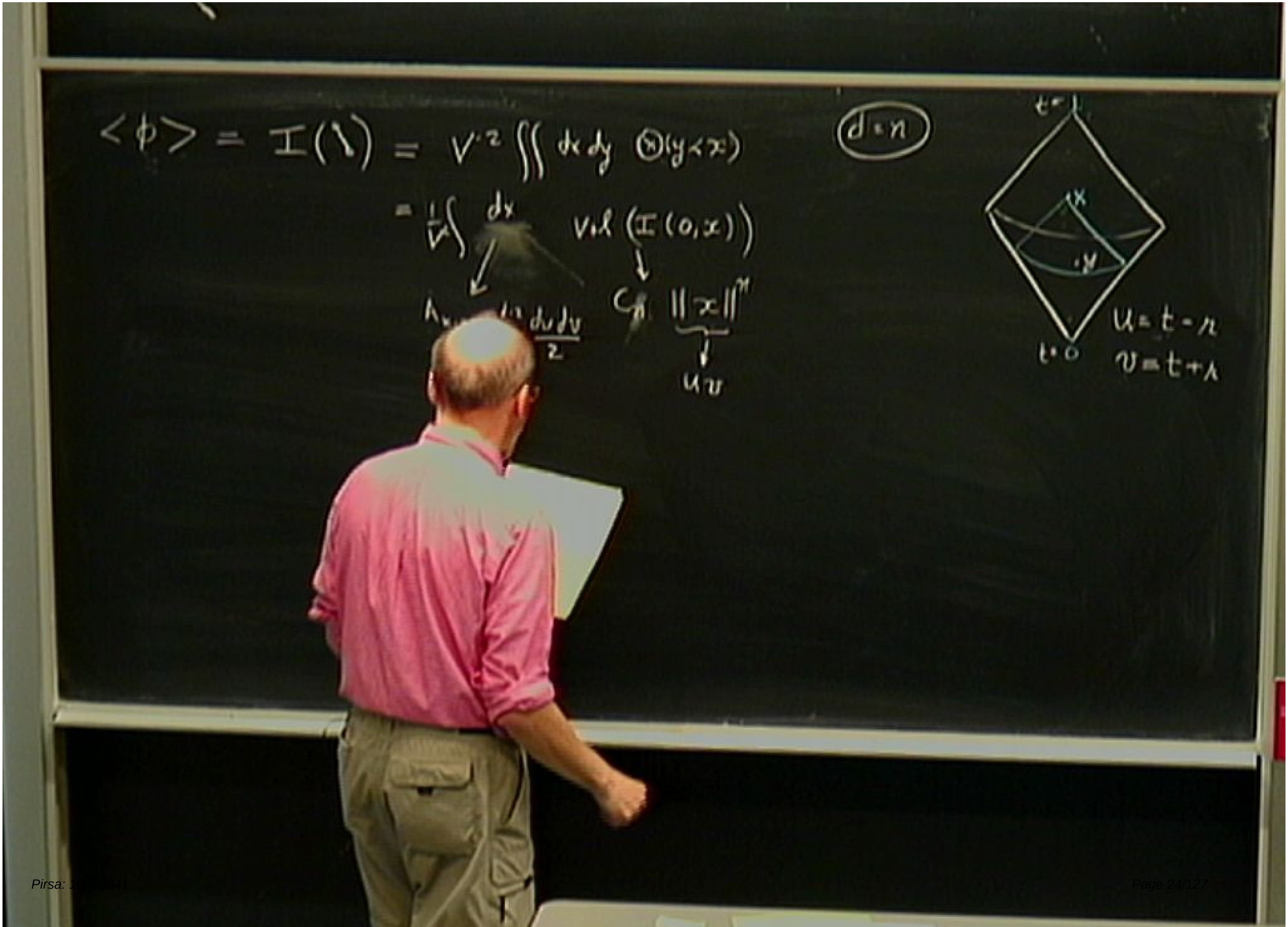
$d = n$

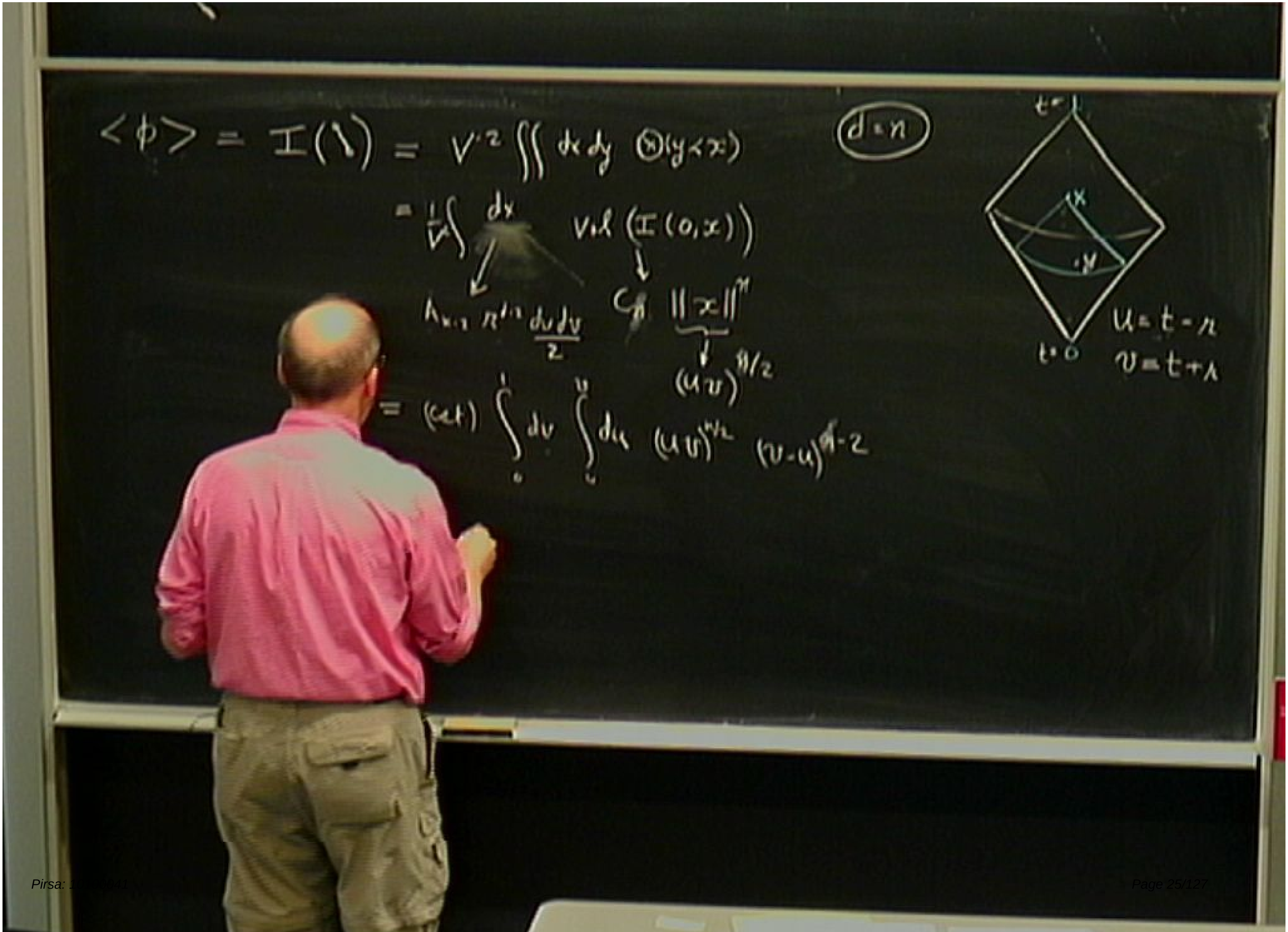
$u = t - r$

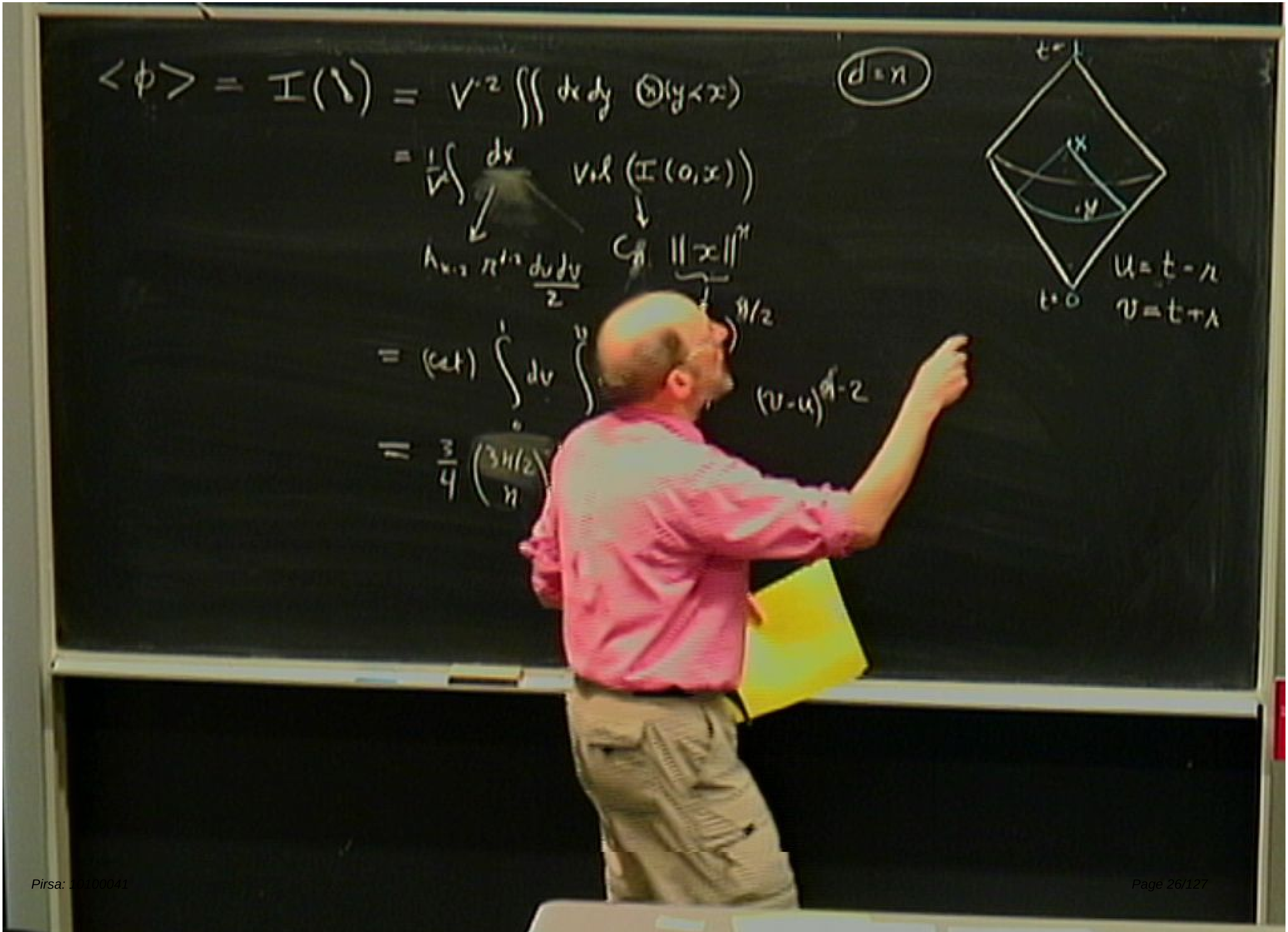
$v = t + r$



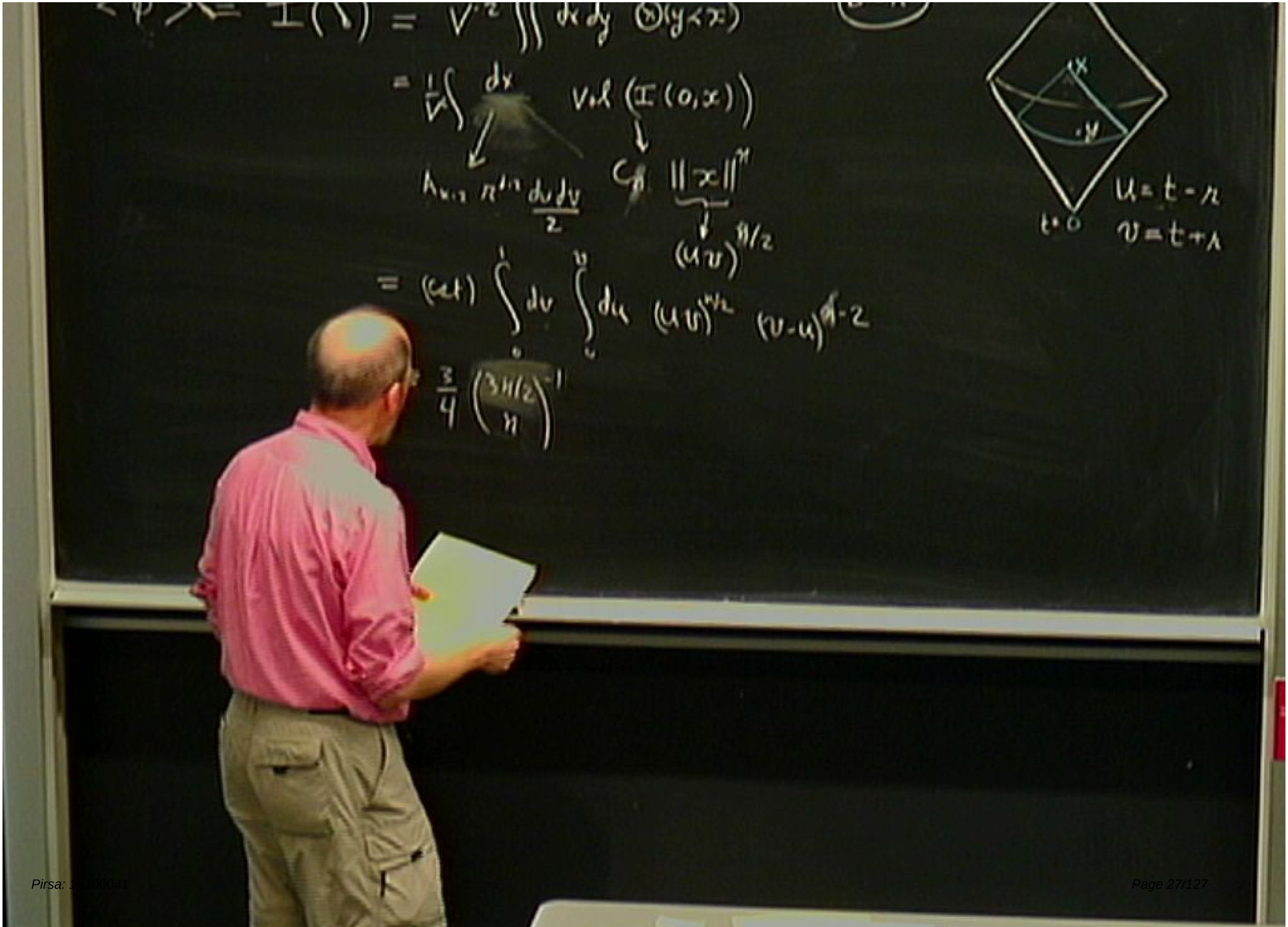
<φ> = I(λ) = V^-2 ∬ dx dy Θ(y<x)
d = 2
= 1/V^2 ∫ dx Vol(I(0,x))
C_d ||x||^d
t=1
t=0
x
y
u = t - r
v = t + r


<φ> = I(λ) = V⁻² ∬ dx dy θ(y<x)
vol (I(0,x))
u = t - r
v = t + r
t = 0


<φ> = I(λ) = V^-2 ∬ dx dy Θ(y<x)
d = n
u = t - r
v = t + r
t = 0



<φ> = I(\) = V^-2 ∬ dx dy Θ(y<x)
d = n
= 1/V ∫ dx Vol (I(0,x))
h_{n-2} r^{n-2} du dv/2
C_n ||x||^n
n/2
= (c·t) ∫ dv ∫
(v-u)^{n-2}
= 3/4 (3n/2 n)
t = 1
x
y
t = 0
u = t - r
v = t + r

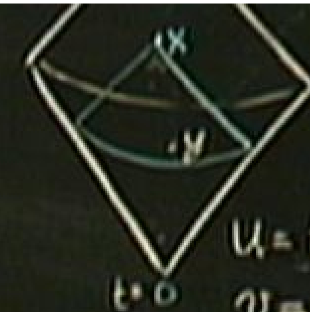

u = t - r
v = t + r
t = 0

$$= \frac{1}{V}\int dx \;\; \mathrm{Vol}\left(I(0,x)\right)$$

$$h_{n-2}\, r^{n-2}\, \frac{du\,dv}{2} \qquad C_n\, \underbrace{\|x\|}_{(uv)^{1/2}}{}^{n}$$

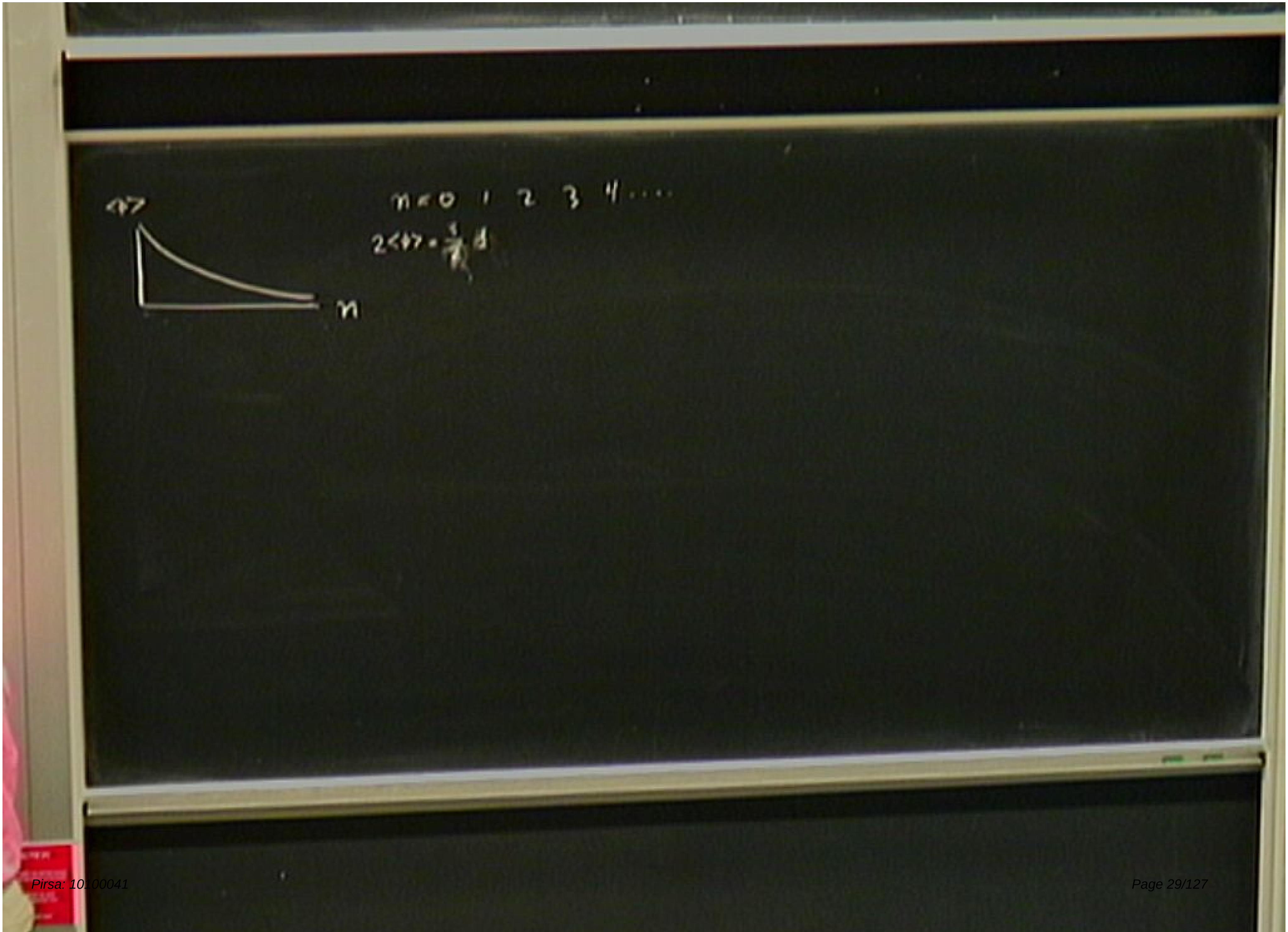
$$= (\mathrm{const}) \int_0^1 dv \int_0^v du \; (uv)^{n/2}\, (v-u)^{n-2}$$

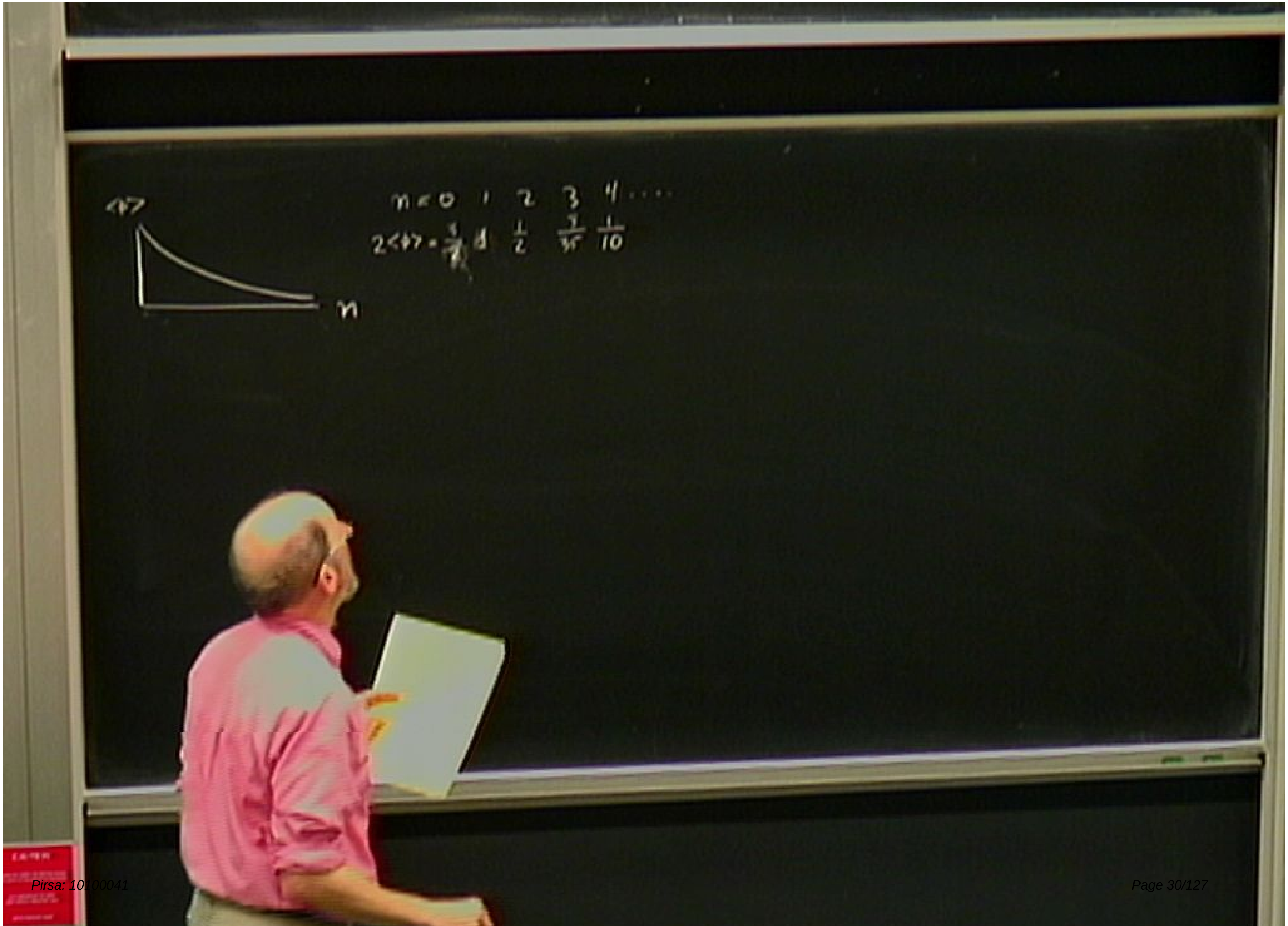
$$= \frac{3}{4}\binom{3n/2}{n}^{-1}$$

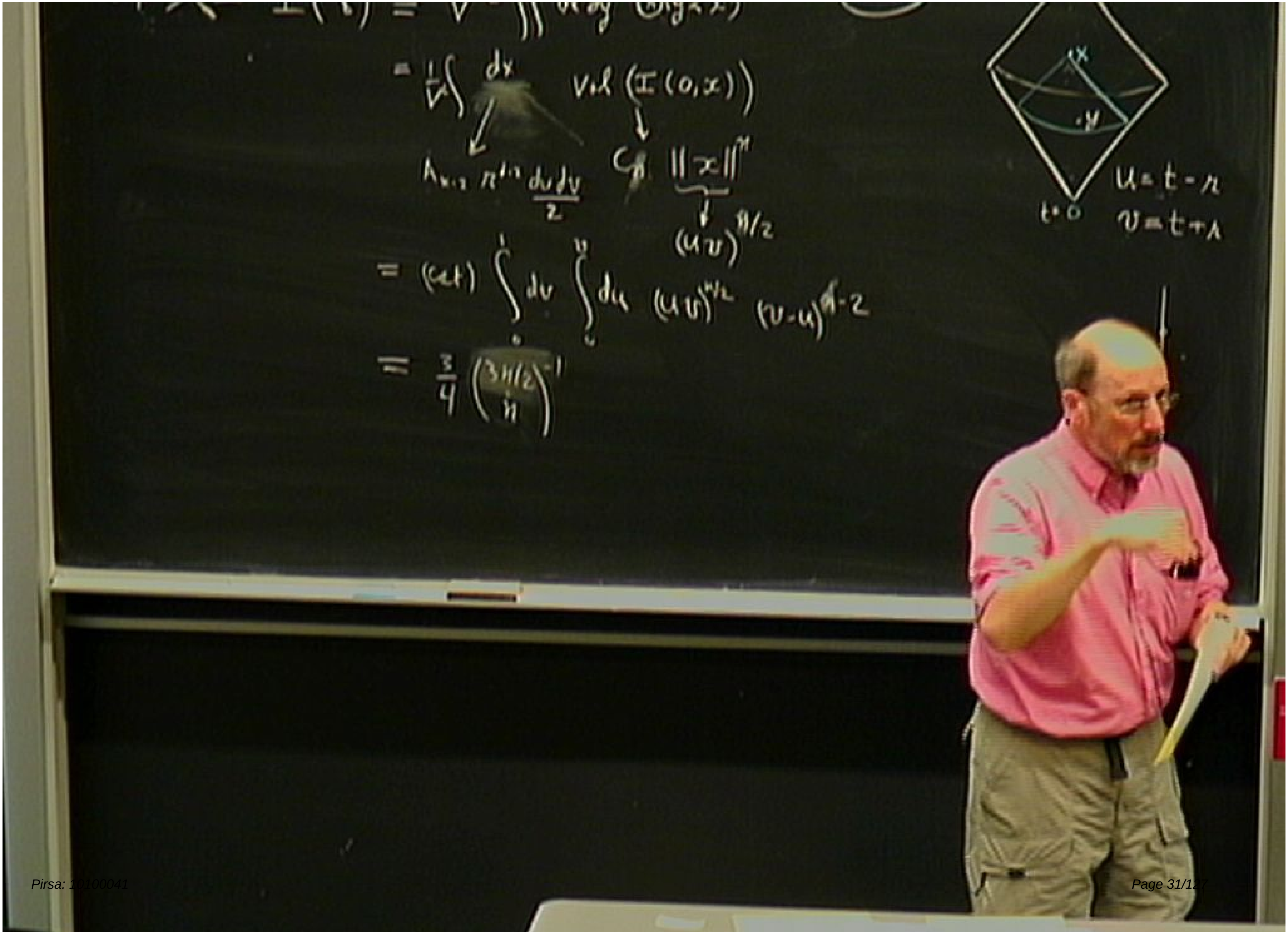


$u = t - r$

$v = t + r$

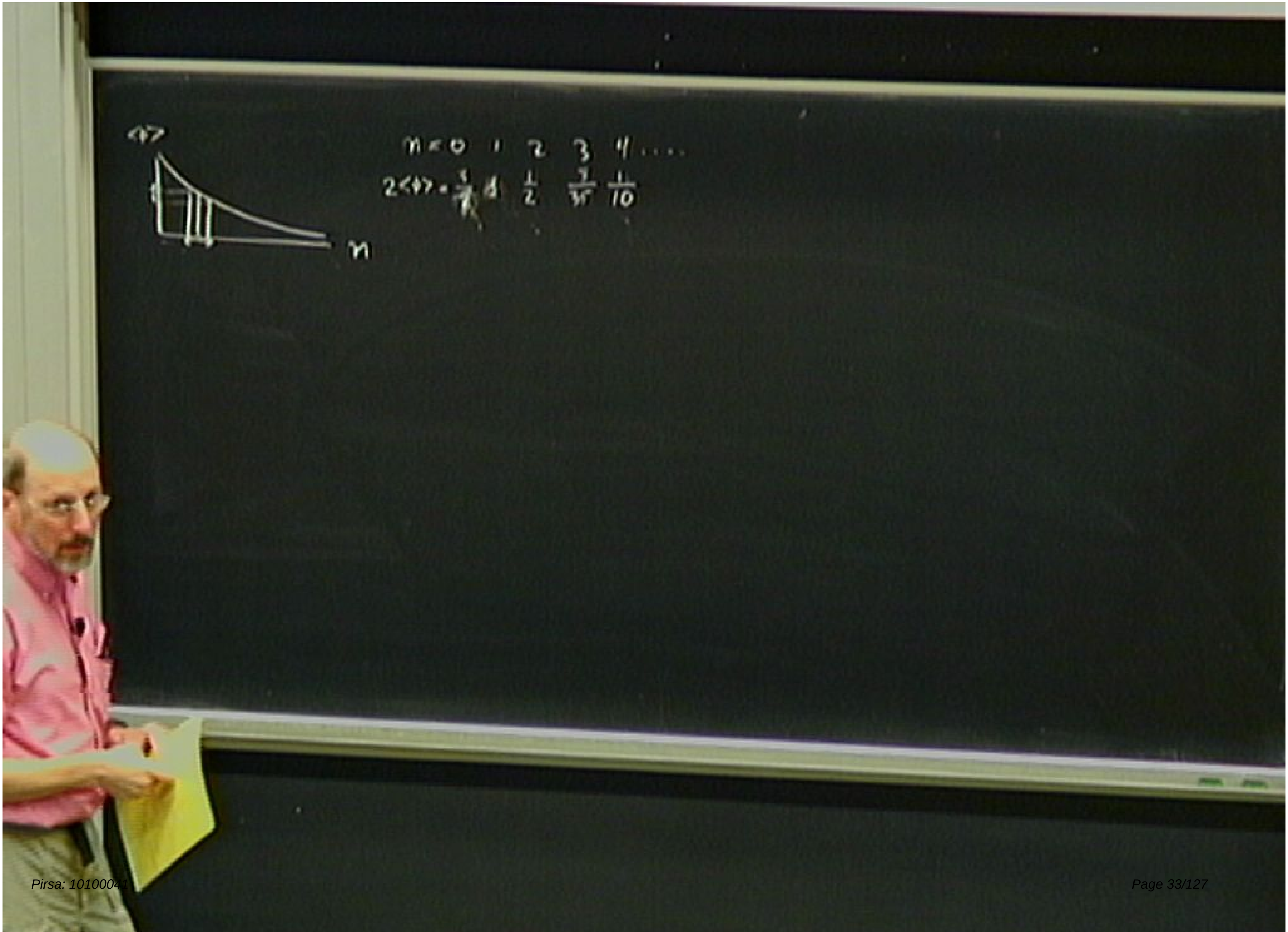

n = 0 1 2 3 4 ....
n

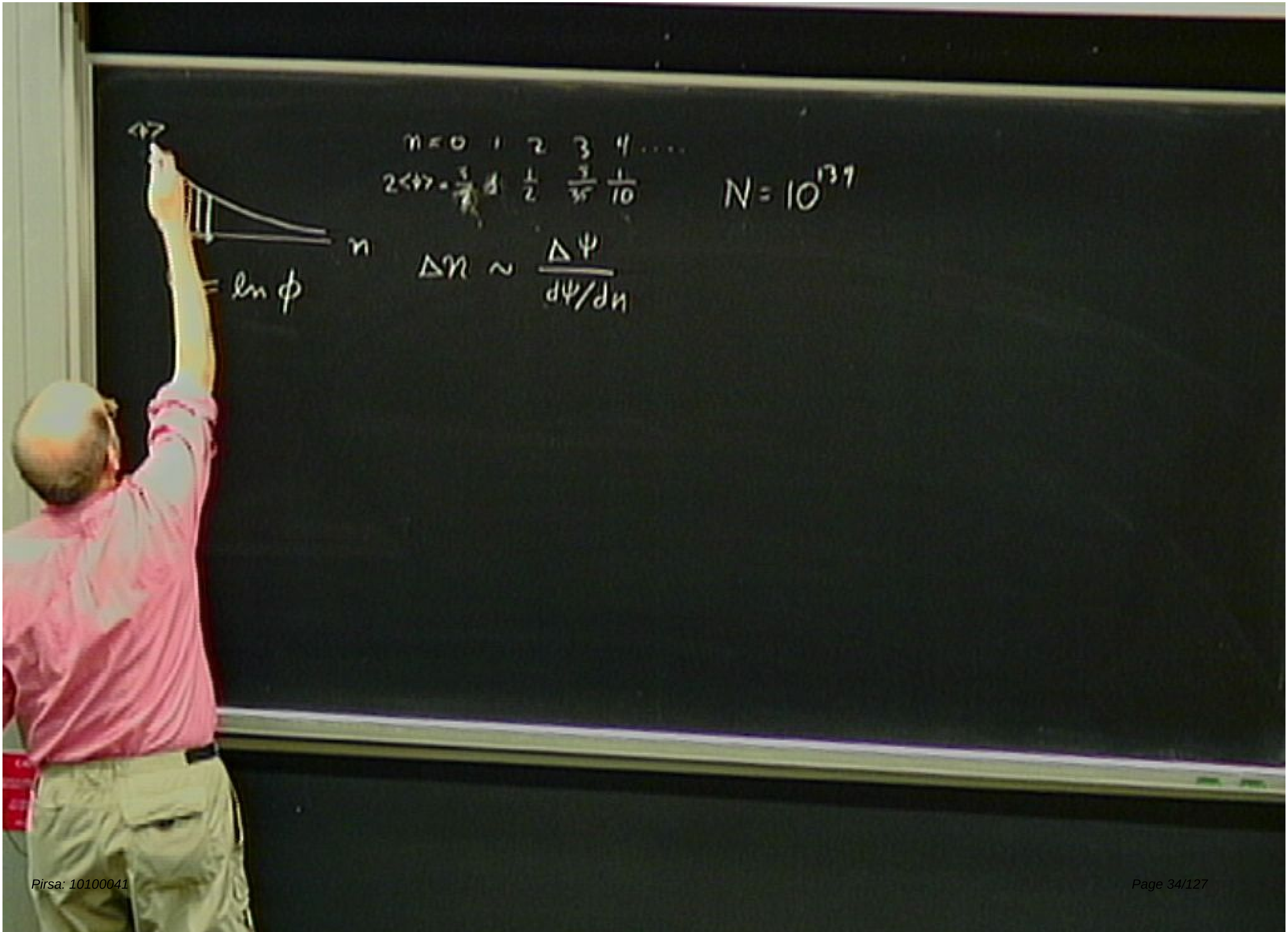

<φ>
n
n = 0 1 2 3 4 . . .
1/2 9/35 1/10

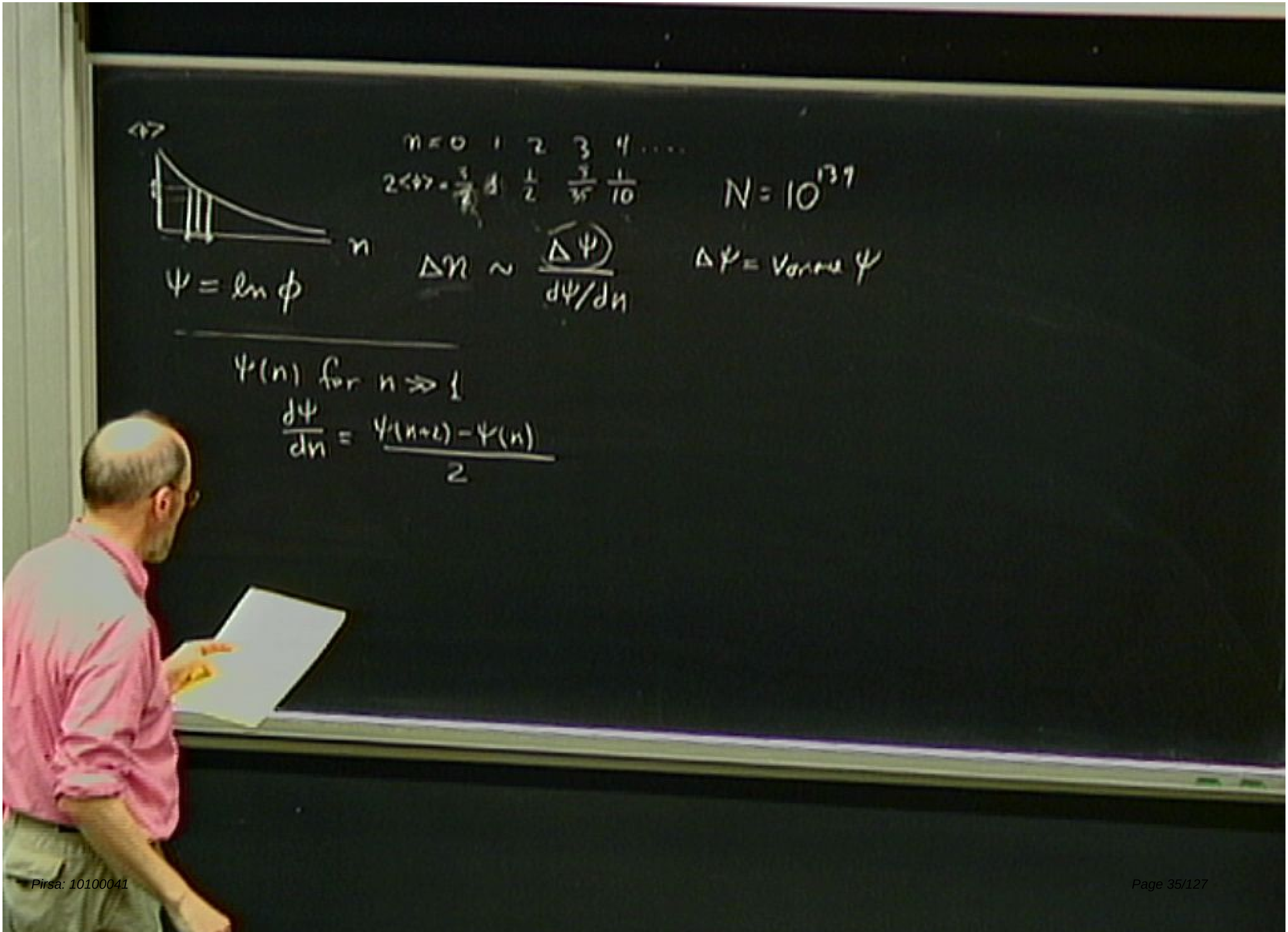

u = t - r
v = t + r
t = 0

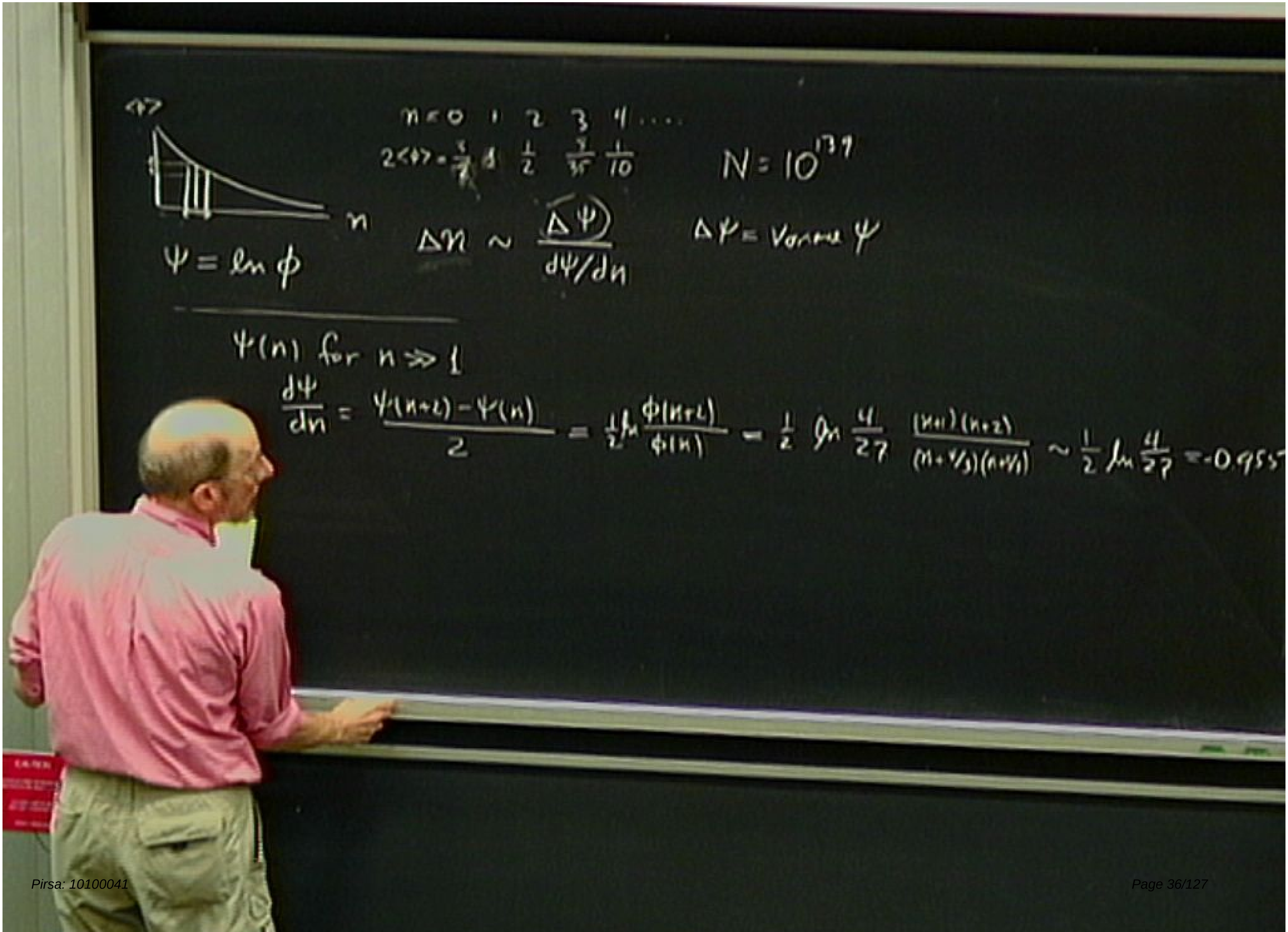
$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \ldots$

$2\langle\phi\rangle = \frac{1}{[illegible]} \quad [illegible] \quad \frac{1}{2} \quad \frac{3}{5} \quad \frac{1}{10}$


n = 0 1 2 3 4 ....
n


n = 0 1 2 3 4 ....
Δn ~ Δψ / (dψ/dn)
n
ln φ


⟨φ⟩
n
ψ = ln φ
n = 0 1 2 3 4 ....
N = 10^137
Δn ~ Δψ / (dψ/dn)
ψ(n) for n ≫ 1
dψ/dn = (ψ(n+2) − ψ(n)) / 2



$\langle\phi\rangle$
n
$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \ldots$
$N = 10^{137}$
$\psi = \ln\phi$
$\Delta n \sim \dfrac{\Delta\psi}{d\psi/dn}$
$\Delta\psi = \text{Variance } \psi$
$\psi(n)$ for $n \gg 1$
$\dfrac{d\psi}{dn} = \dfrac{\psi(n+2)-\psi(n)}{2} = \tfrac{1}{2}\ln\dfrac{\phi(n+2)}{\phi(n)} = \tfrac{1}{2}\ln\dfrac{4}{27}\dfrac{(n+1)(n+2)}{(n+4/3)(n+5/3)} \sim \tfrac{1}{2}\ln\dfrac{4}{27} = -0.955$

$n = 0 \quad 1 \quad 2 \quad 3 \quad 4 \ \ldots$

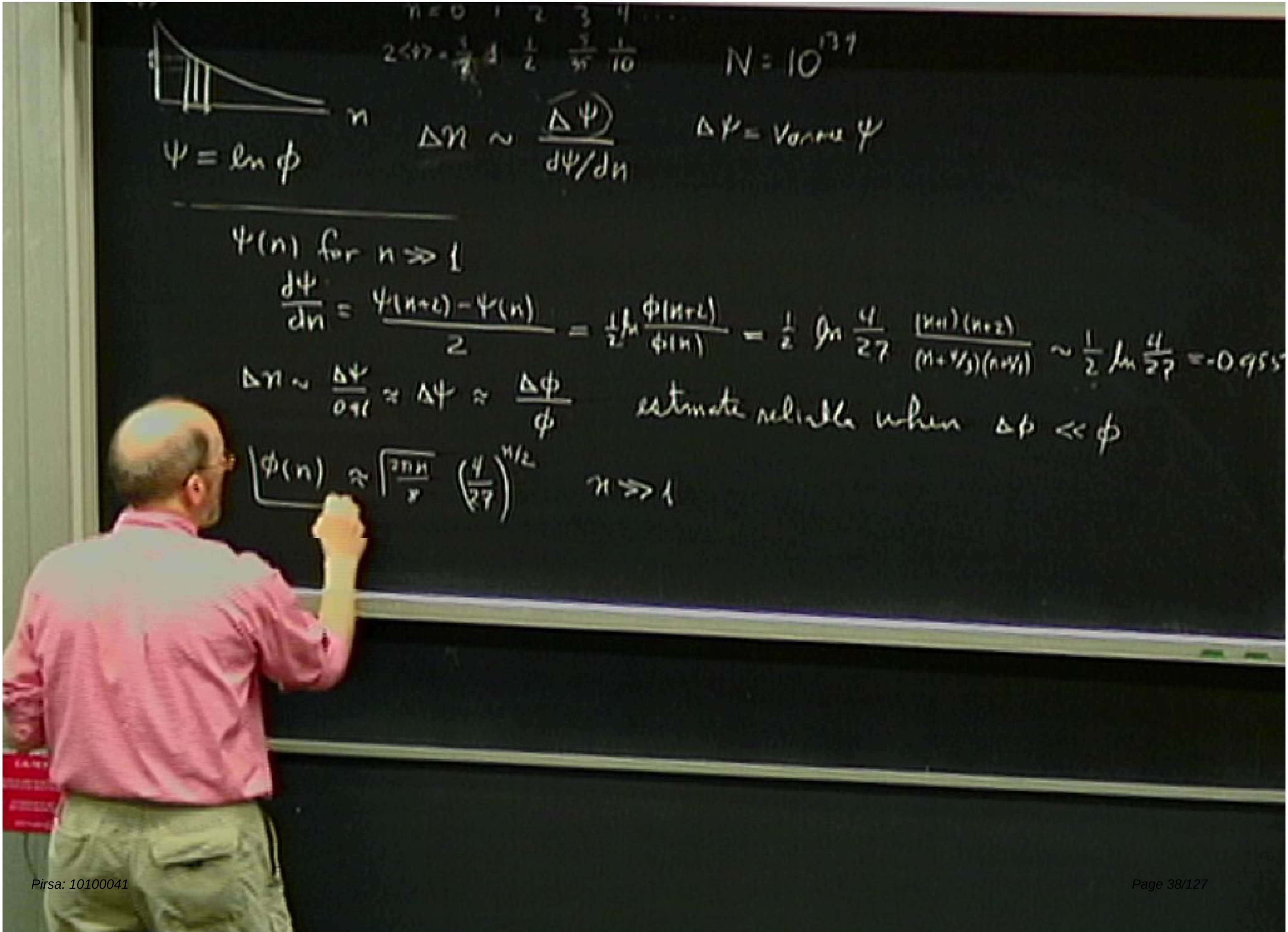
$2\langle\psi\rangle = \frac{3}{4} \quad 1 \quad \frac{1}{2} \quad \frac{3}{35} \quad \frac{1}{10}$ $\qquad N = 10^{137}$

$\psi = \ln\phi \qquad \Delta n \sim \frac{\Delta\psi}{d\psi/dn} \qquad \Delta\psi = \text{Variance } \psi$

$\psi(n)$ for $n \gg 1$

$$\frac{d\psi}{dn} = \frac{\psi(n+2)-\psi(n)}{2} = \frac{1}{2}\ln\frac{\phi(n+2)}{\phi(n)} = \frac{1}{2}\ln\frac{4}{27}\,\frac{(n+1)(n+2)}{(n+4/3)(n+5/3)} \sim \frac{1}{2}\ln\frac{4}{27} = -0.955$$

$\Delta n \sim \frac{\Delta\psi}{d\psi/dn} \approx \Delta\psi \approx \frac{\Delta\phi}{\phi}$ estimate reliable when $\Delta\phi \ll \phi$



N = 10^137
n
Δn ~ Δψ / (dψ/dn)
Δψ = Variance ψ
ψ = ln φ
ψ(n) for n ≫ 1
dψ/dn = (ψ(n+2) − ψ(n)) / 2 = ½ ln φ(n+2)/φ(n) = ½ ln 4/27 (n+1)(n+2)/((n+4/3)(n+5/3)) ~ ½ ln 4/27 = −0.955
Δn ~ Δψ/0.96 ≈ Δψ ≈ Δφ/φ
estimate reliable when Δφ ≪ φ
φ(n) ≈ √(2πn/3) (4/27)^(n/2)
n ≫ 1

$\Delta n \sim \frac{\Delta\psi}{d\psi/dn}$ $\qquad \Delta\psi =$ Variance ψ

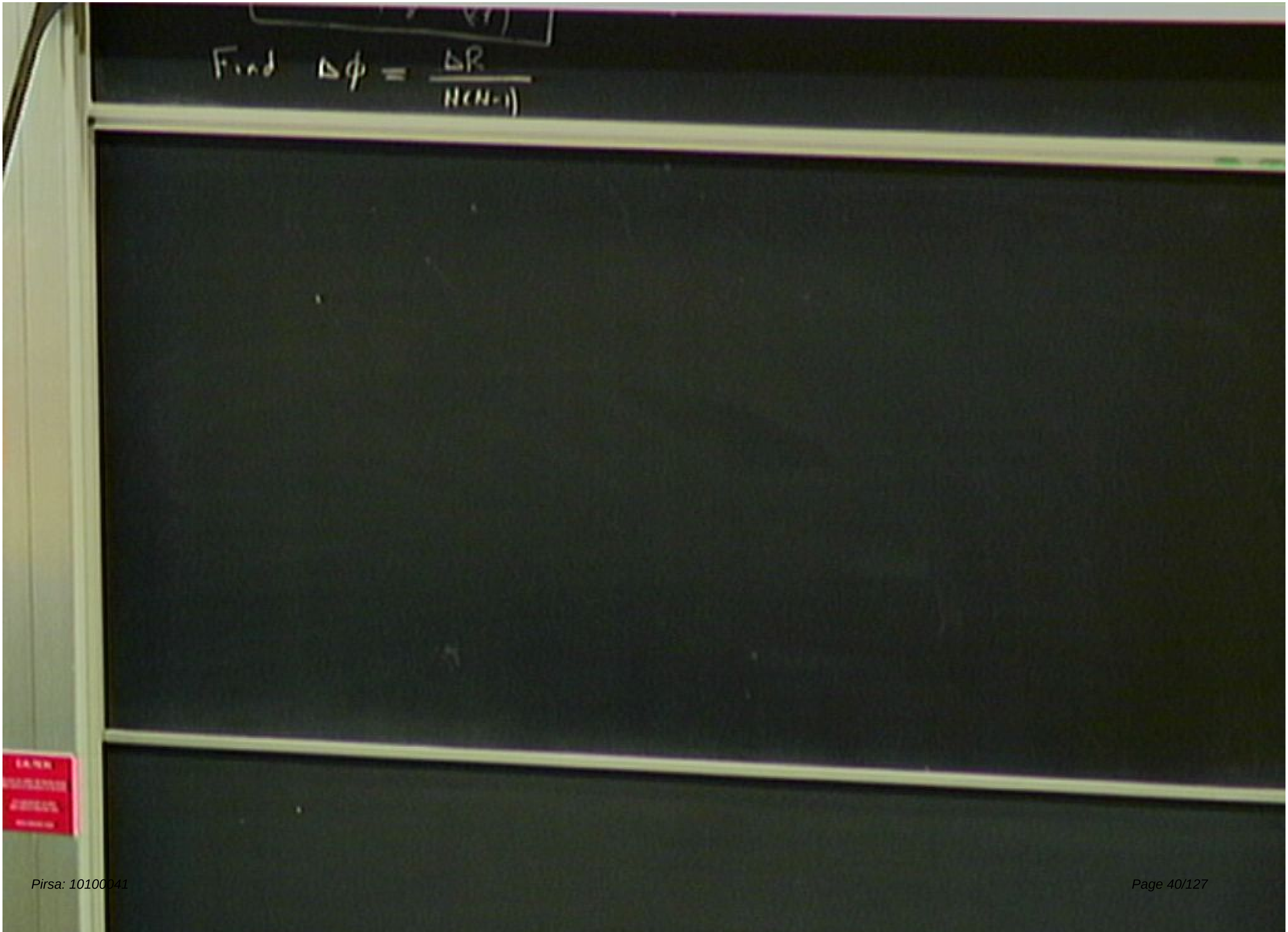
$\psi = \ln\phi$

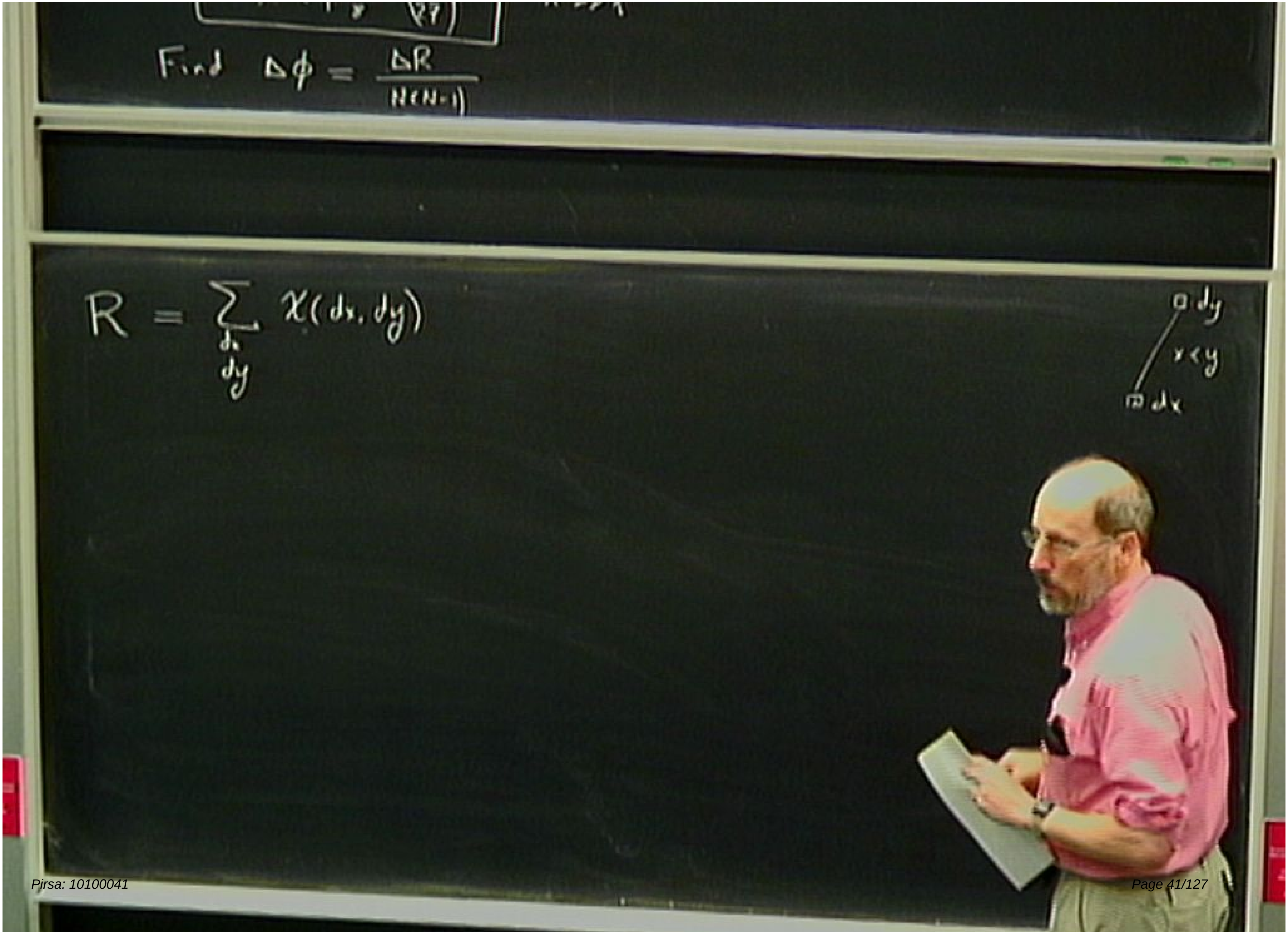
$\psi(n)$ for $n \gg 1$

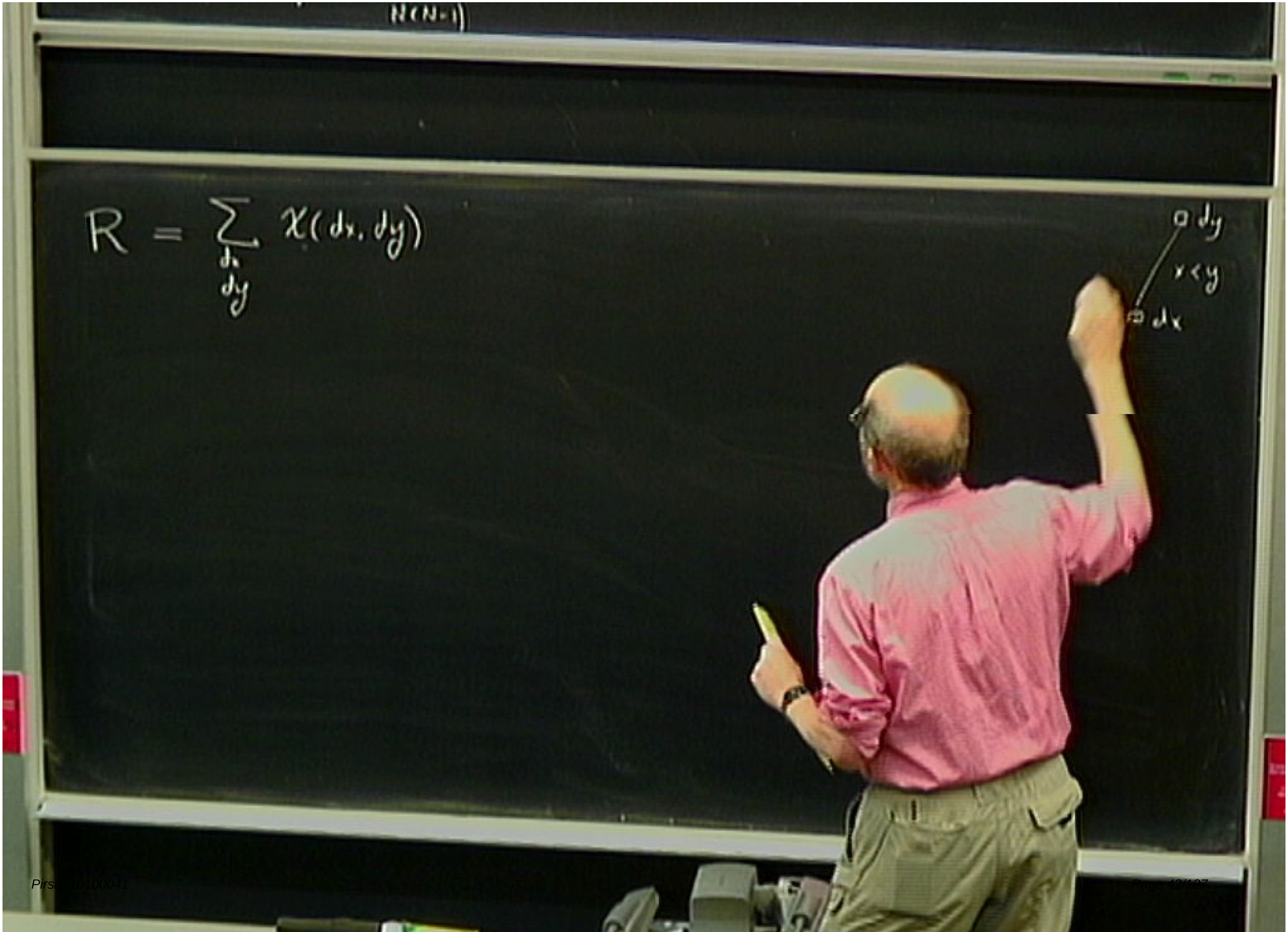
$$\frac{d\psi}{dn} = \frac{\psi(n+2)-\psi(n)}{2} = \frac{1}{2}\ln\frac{\phi(n+2)}{\phi(n)} = \frac{1}{2}\ln\frac{4}{27}\,\frac{(n+1)(n+2)}{(n+4/3)(n+5/3)} \sim \frac{1}{2}\ln\frac{4}{27} = -0.955$$

$$\Delta n \sim \frac{\Delta\psi}{0.96} \approx \Delta\psi \approx \frac{\Delta\phi}{\phi}$$ estimate reliable when $\Delta\phi \ll \phi$

$$\boxed{\phi(n) \approx \sqrt{\frac{2\pi n}{3}}\left(\frac{4}{27}\right)^{n/2}} \qquad n \gg 1$$


Find Δφ = ΔR / N(N-1)


Find Δφ = ΔR / N(N-1)
R = Σ_{dx, dy} χ(dx, dy)
dy
x < y
dx


N(N-1)
$R = \sum_{dx, dy} \chi(dx, dy)$
dy
x < y
dx

$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy) \equiv a + b + c + \dots$$

a, b, c $\Big\{$ $\Box\, dy$, $x < y$, $\Box\, dx$

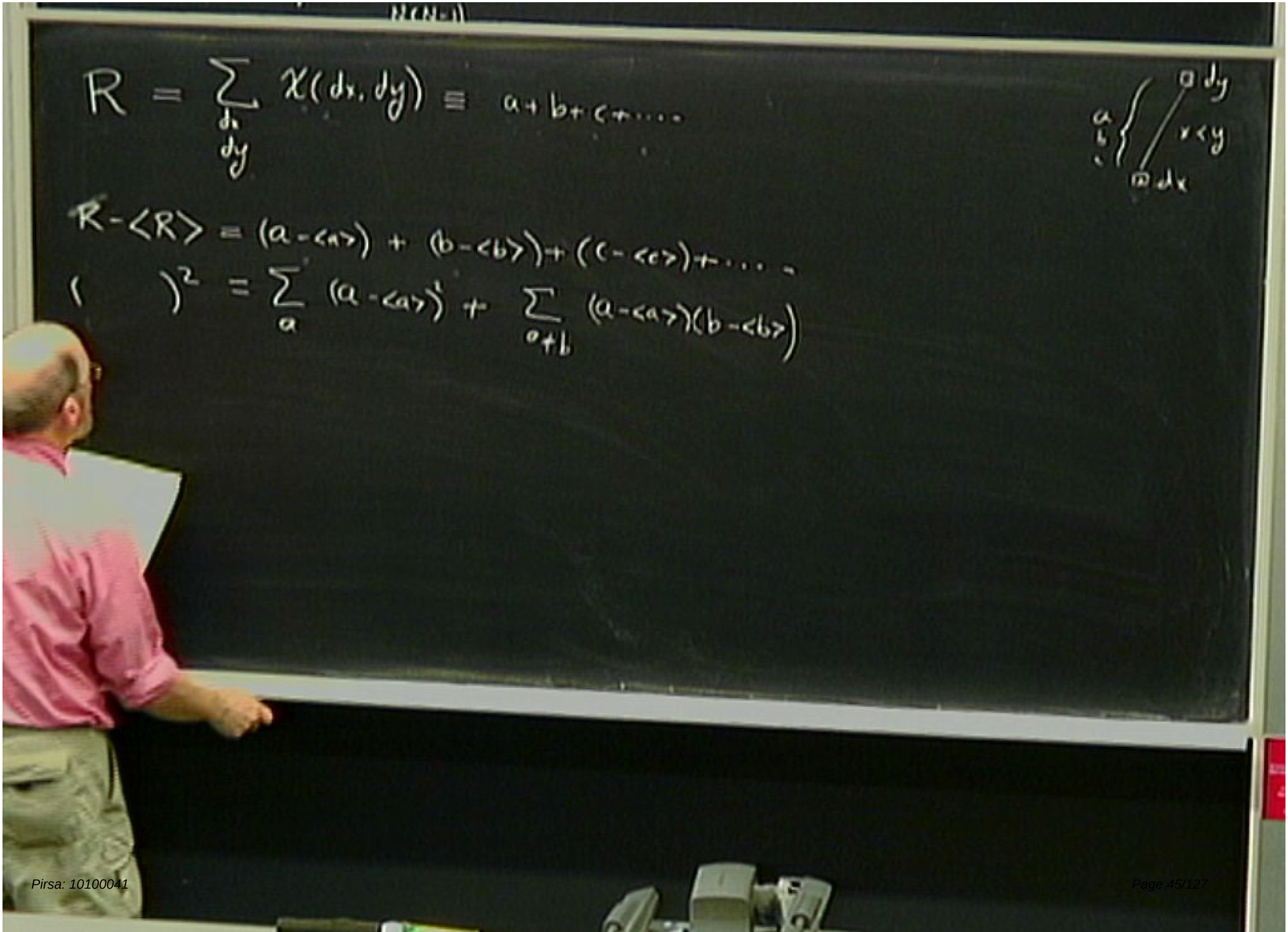
$$R - \langle R \rangle$$

$N(N-1)$

$$R = \sum_{dx,\, dy} \chi(dx, dy) = a + b + c + \ldots$$

$$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \ldots$$

$$(\ \ )^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$$


$R = \sum_{dx, dy} \chi(dx, dy) \equiv a + b + c + \cdots$
a, b, c
@ dy
$x < y$
@ dx
$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \cdots$
$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$

$N(N-1)$

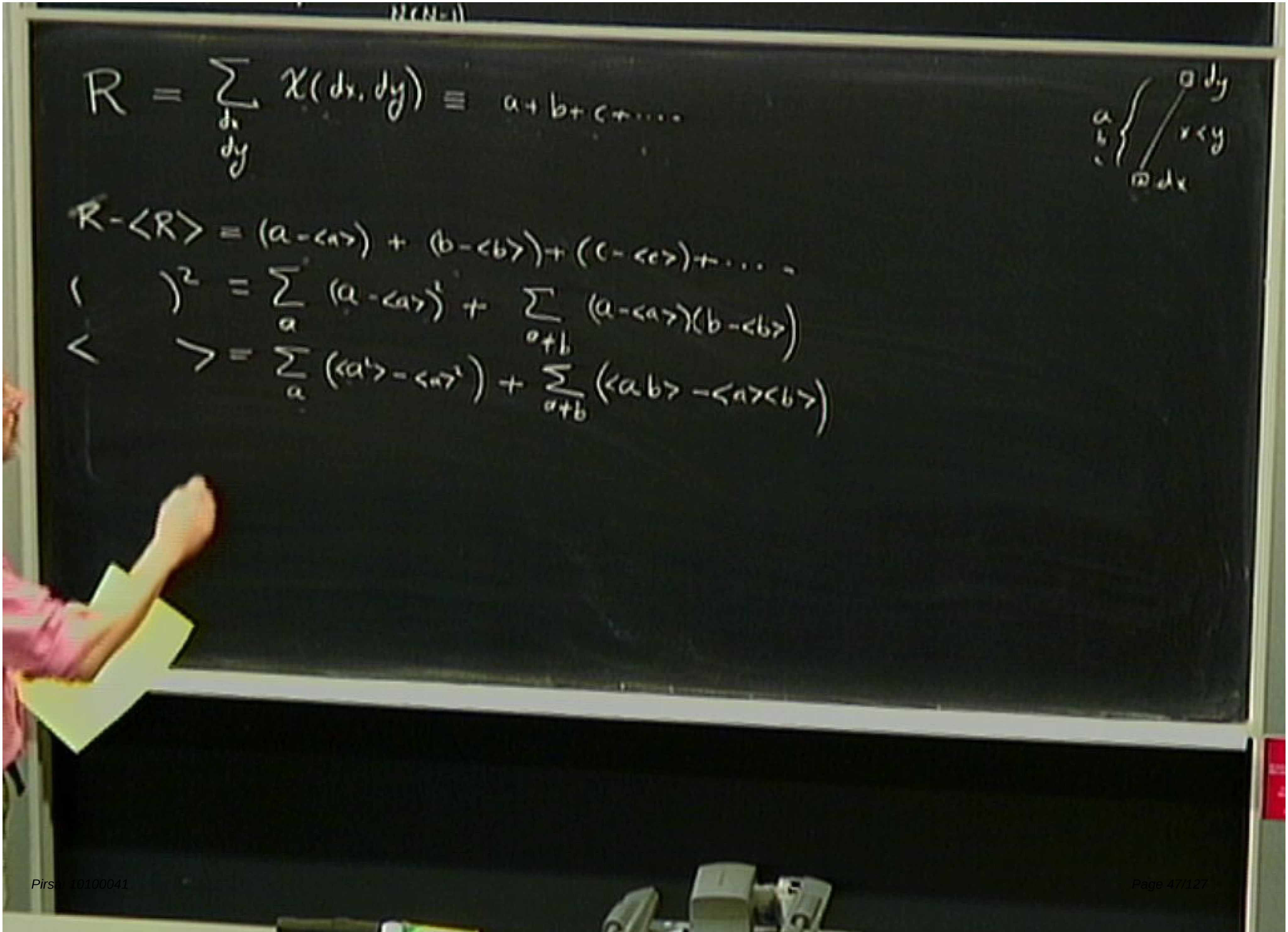
$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy) \equiv a + b + c + \cdots$$

a, b, c — @ dy, $x<y$, @ dx

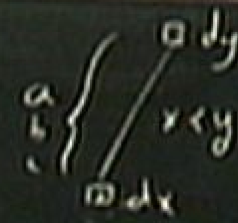
$$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \cdots$$

$$(\qquad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$$

$$\langle \qquad = \sum_a$$


$R = \sum_{dx, dy} \chi(dx, dy) \equiv a + b + c + \dots$
a b c
@ dy
x < y
@ dx
$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \dots$
$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$
$\langle \quad \rangle = \sum_a (\langle a^2 \rangle - \langle a \rangle^2) + \sum_{a \neq b} (\langle a b \rangle - \langle a \rangle \langle b \rangle)$

$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy) \equiv a + b + c + \dots$$

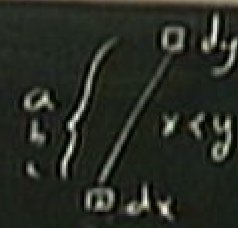


$$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \dots$$

$$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$$

$$\langle \quad \rangle = \sum_a \left(\langle a^2 \rangle - \langle a \rangle^2 \right) + \sum_{a \neq b} \left(\langle a b \rangle - \langle a \rangle \langle b \rangle \right)$$

$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy) \equiv a + b + c + \dots$$

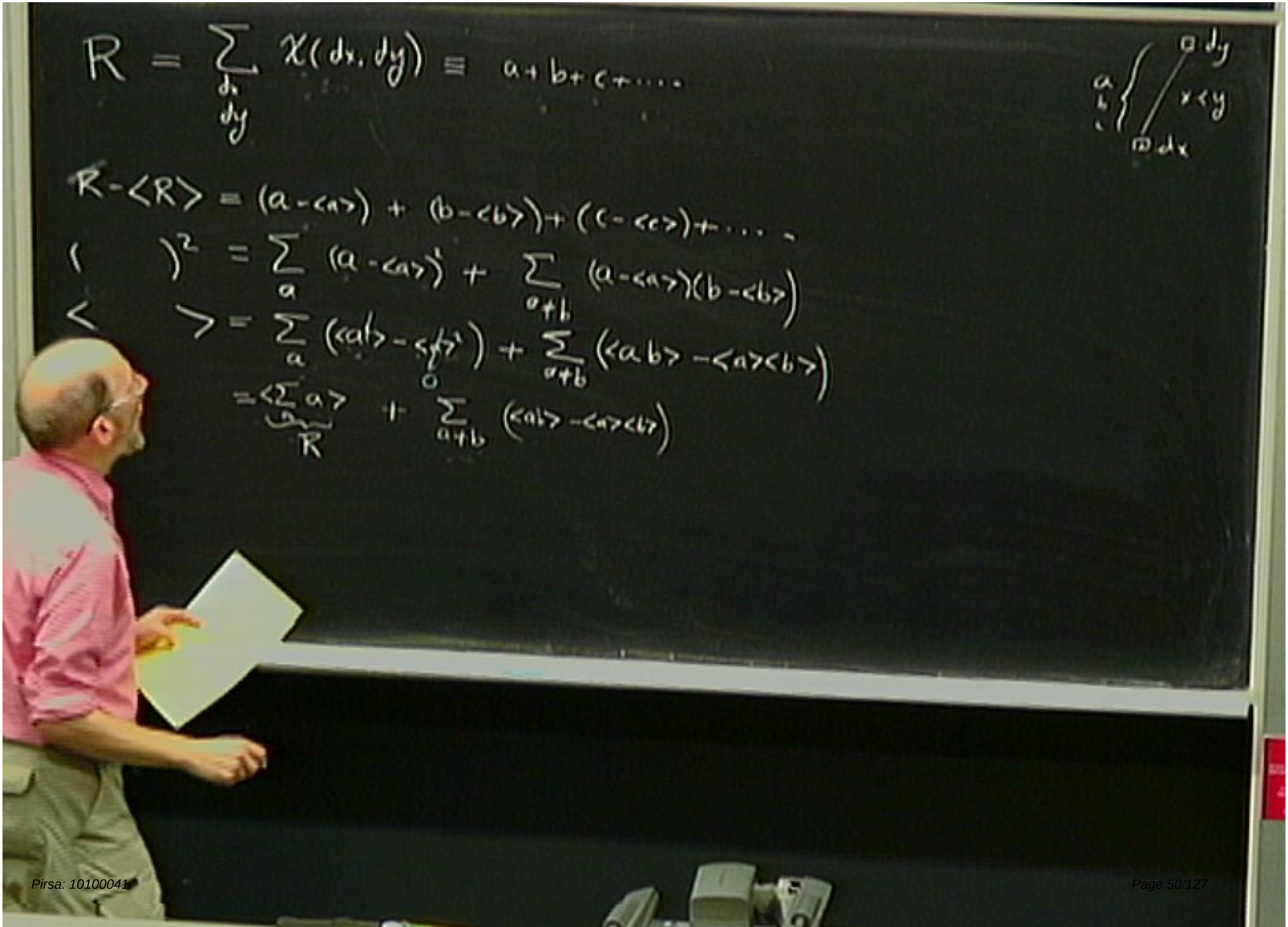


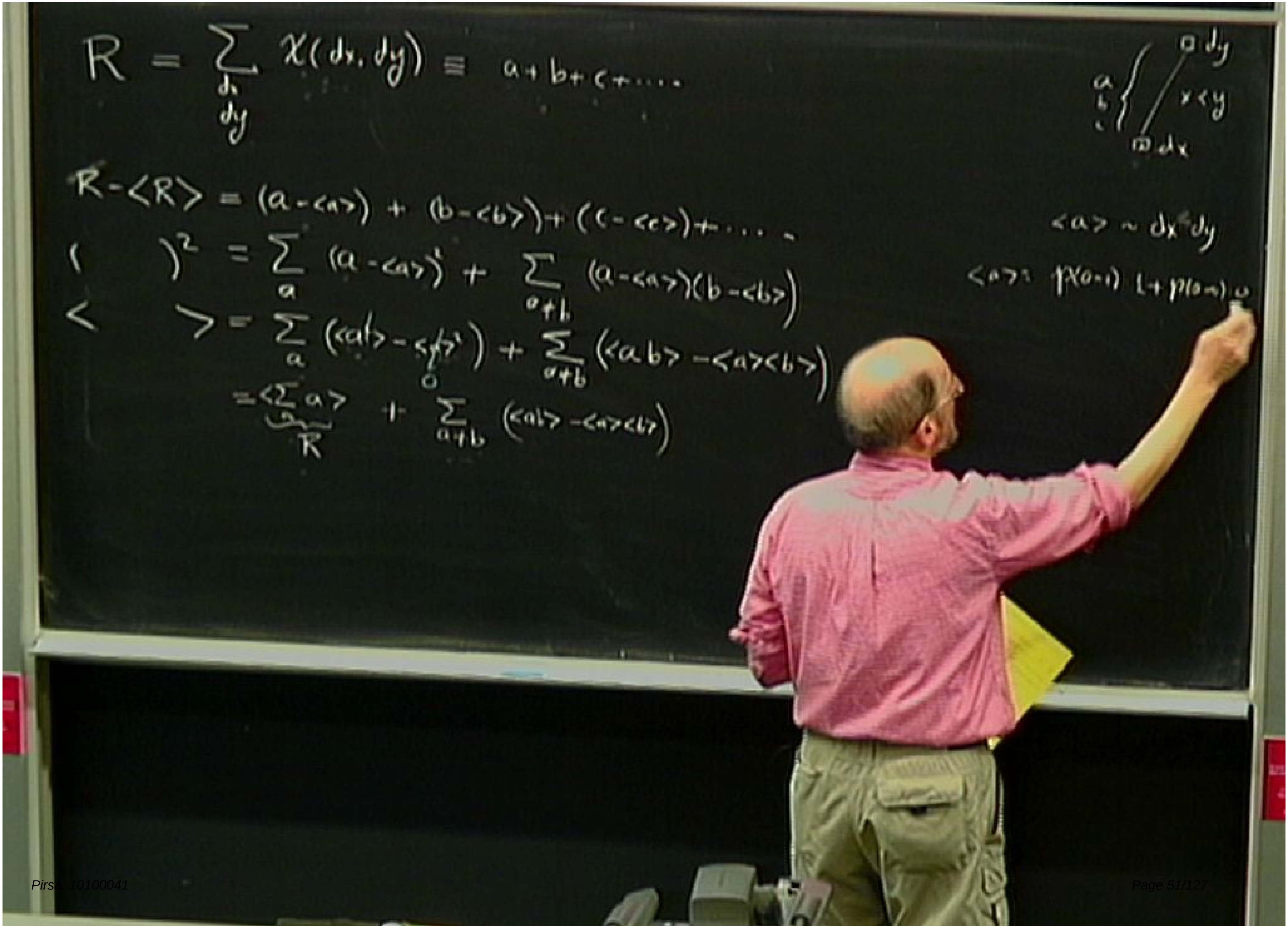
$$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \dots$$

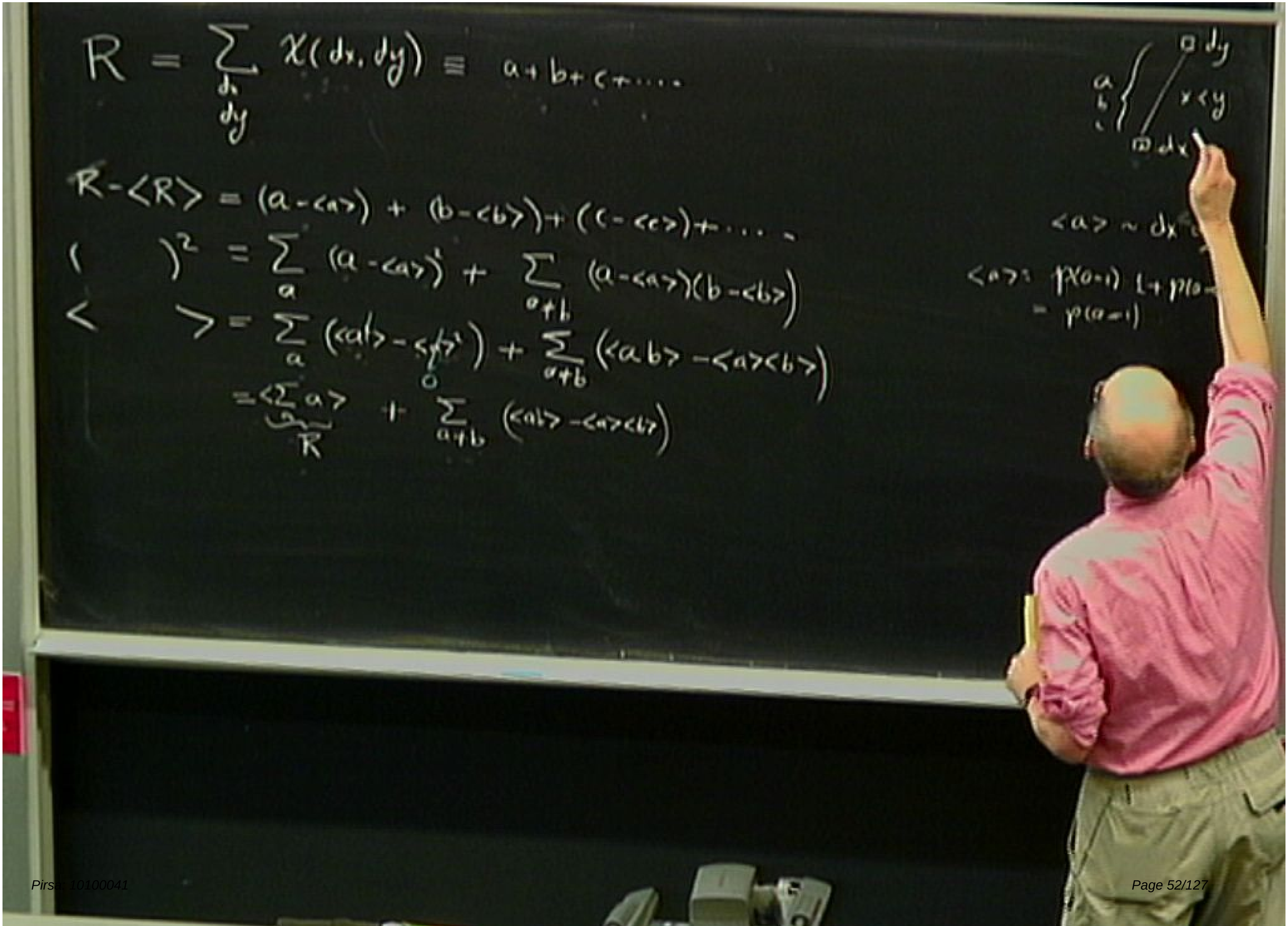
$$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$$

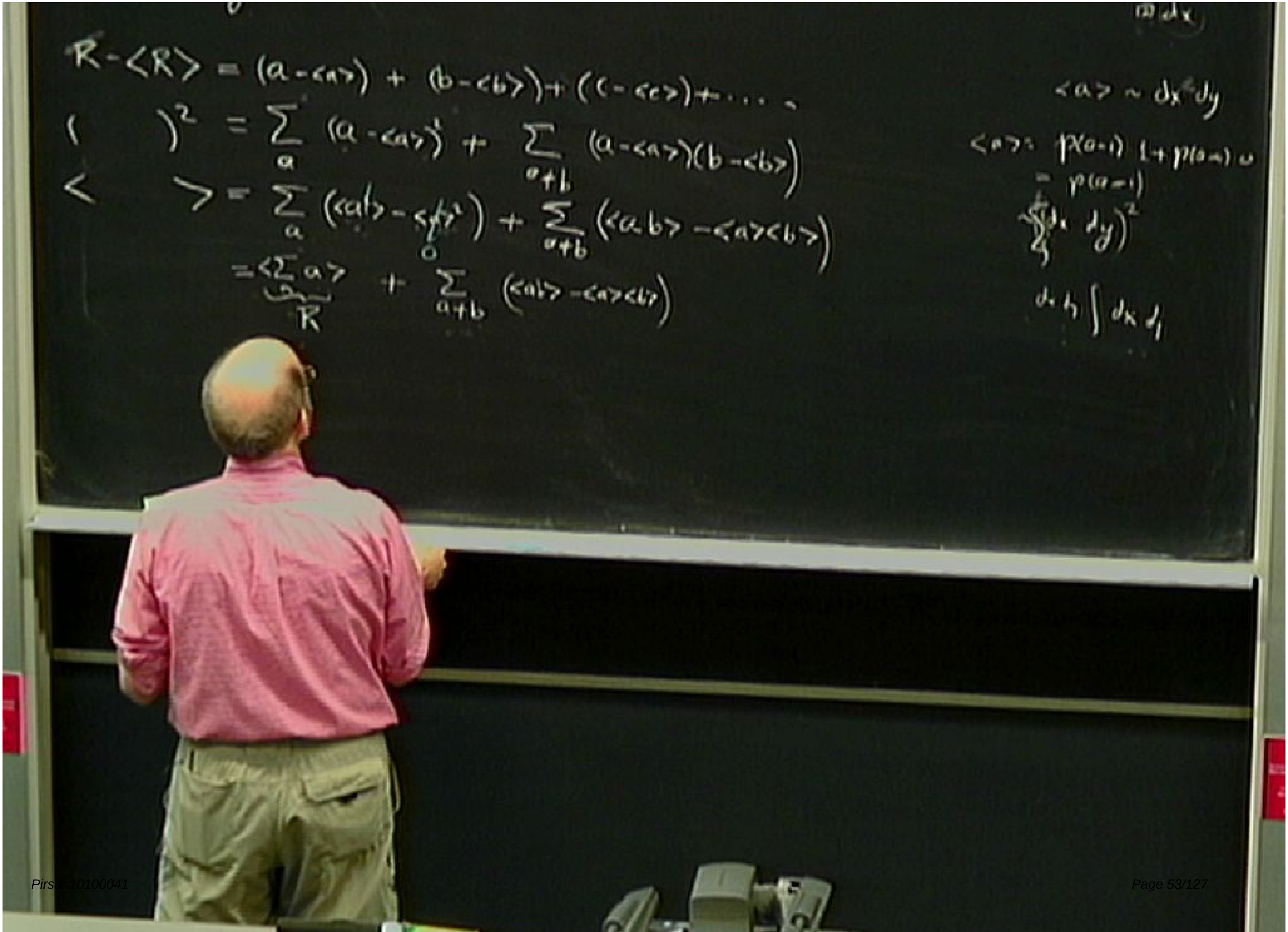
$$\langle \quad \rangle = \sum_a \left(\langle a^2 \rangle - \underset{0}{\cancel{\langle a \rangle}}^2\right) + \sum_{a \neq b} \left(\langle a b \rangle - \langle a \rangle \langle b \rangle\right)$$

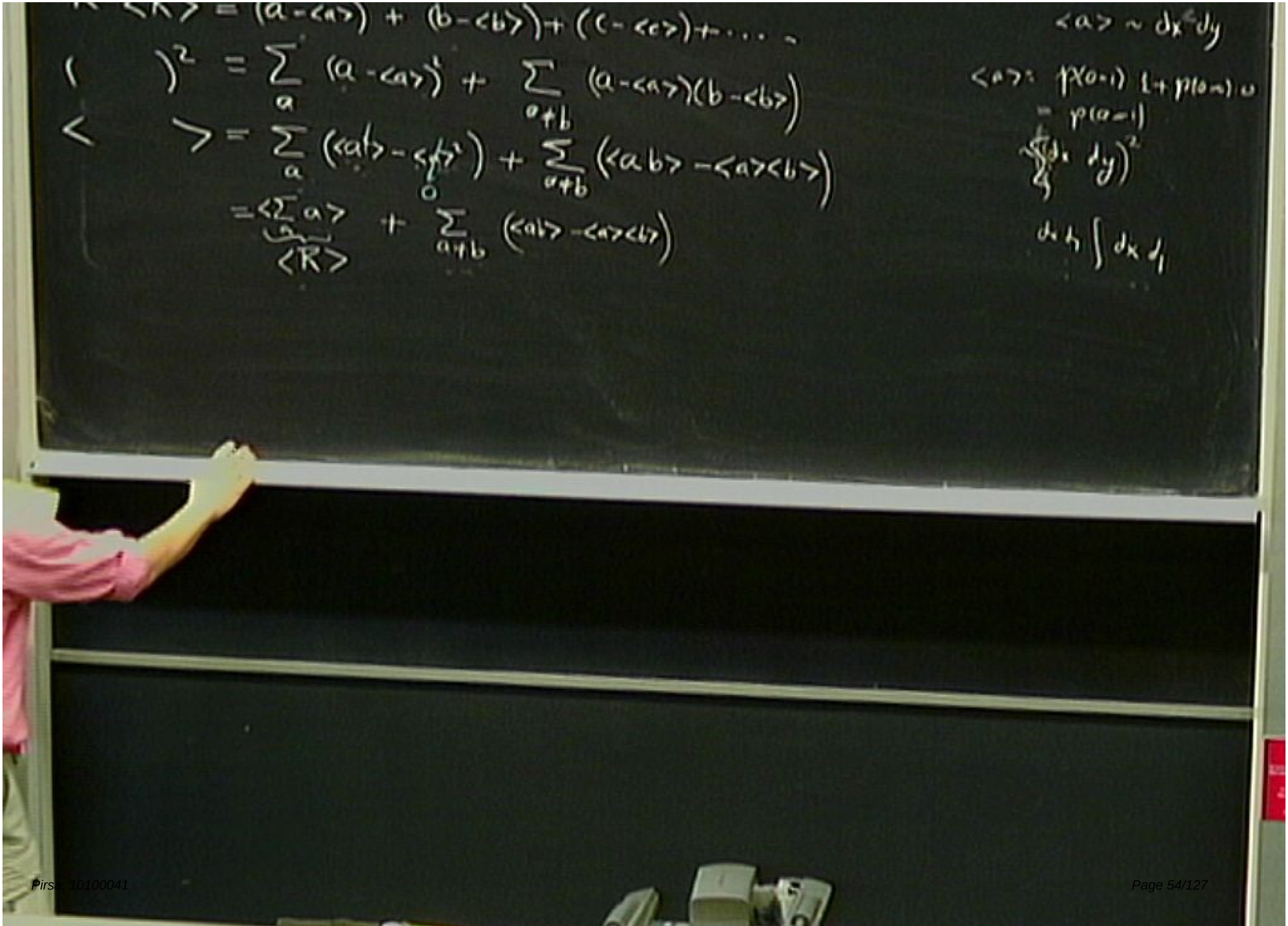
$$=$$


$R = \sum_{dx,\,dy} \chi(dx, dy) \equiv a + b + c + \dots$
a
b
c
dy
x<y
dx
$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \dots$
$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$
$\langle \quad \rangle = \sum_a (\langle a^2 \rangle - \cancel{\langle a \rangle^2}) + \sum_{a \neq b} (\langle ab \rangle - \langle a \rangle \langle b \rangle)$
0
$= \underbrace{\langle \sum a \rangle}_{R} + \sum_{a \neq b} (\langle ab \rangle - \langle a \rangle \langle b \rangle)$

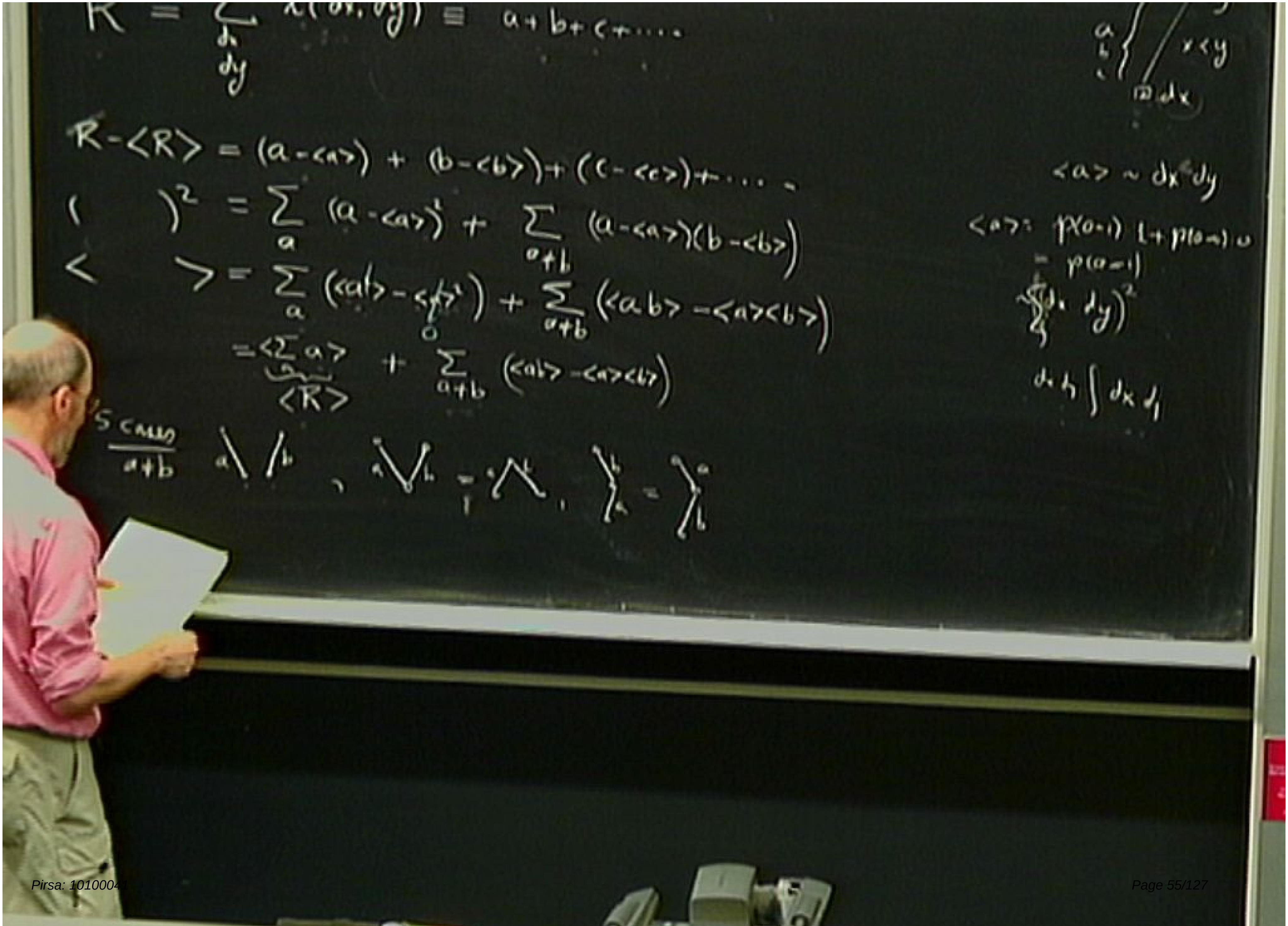

$R = \sum_{dx, dy} \chi(dx, dy) \equiv a + b + c + \dots$
$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \dots$
$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$
$\langle \quad \rangle = \sum_a (\langle a^2 \rangle - \langle a \rangle^2) + \sum_{a \neq b} (\langle a b \rangle - \langle a \rangle \langle b \rangle)$
$= \langle \underbrace{\sum a}_{R} \rangle + \sum_{a \neq b} (\langle ab \rangle - \langle a \rangle \langle b \rangle)$
a b c
dy
x < y
dx
$\langle a \rangle \sim dx\, dy$
$\langle a \rangle = p(a=1)\, 1 + p(a=0)\, 0$


R = Σ_{dx, dy} χ(dx, dy) ≡ a + b + c + ...
R − ⟨R⟩ = (a − ⟨a⟩) + (b − ⟨b⟩) + (c − ⟨c⟩) + ...
( )² = Σ_a (a − ⟨a⟩)² + Σ_{a≠b} (a − ⟨a⟩)(b − ⟨b⟩)
⟨ ⟩ = Σ_a (⟨a²⟩ − ⟨a⟩²) + Σ_{a≠b} (⟨a b⟩ − ⟨a⟩⟨b⟩)
= ⟨Σ a⟩ + Σ_{a≠b} (⟨ab⟩ − ⟨a⟩⟨b⟩)
R
dy
x < y
dx
⟨a⟩ ~ dx
⟨a⟩ = p(a=1)
= p(a=1)

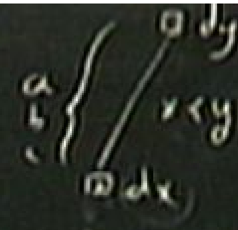

R - <R> = (a - <a>) + (b - <b>) + (c - <c>) + . . .
( )^2 = Σ_a (a - <a>)^2 + Σ_{a≠b} (a - <a>)(b - <b>)
< > = Σ_a (<a^2> - <a>^2) + Σ_{a≠b} (<ab> - <a><b>)
= <Σ a> + Σ_{a≠b} (<ab> - <a><b>)
R
<a> ~ dx dy
<a> = p(a=1) 1 + p(a=0) 0
= p(a=1)
(dx dy)^2
dx dy ∫ dx dy



$\langle R\rangle = (a-\langle a\rangle) + (b-\langle b\rangle) + (c-\langle c\rangle) + \dots$
$(\quad)^2 = \sum_a (a-\langle a\rangle)^2 + \sum_{a\neq b} (a-\langle a\rangle)(b-\langle b\rangle)$
$\langle \quad \rangle = \sum_a (\langle a^2\rangle - \langle a\rangle^2) + \sum_{a\neq b} (\langle ab\rangle - \langle a\rangle\langle b\rangle)$
$= \underbrace{\langle \sum_a a\rangle}_{\langle R\rangle} + \sum_{a\neq b} (\langle ab\rangle - \langle a\rangle\langle b\rangle)$
$\langle a\rangle \sim dx^2 dy$
$\langle a\rangle = p(a=1)\,1 + p(a=0)\,0$
$= p(a=1)$
$(dx\, dy)^2$
$dx\, dy \int dx\, dy$


R = Σ x(dx, dy) ≡ a + b + c + ...
R − ⟨R⟩ = (a − ⟨a⟩) + (b − ⟨b⟩) + (c − ⟨c⟩) + ...
( )² = Σ (a − ⟨a⟩)² + Σ (a − ⟨a⟩)(b − ⟨b⟩)
⟨ ⟩ = Σ (⟨a²⟩ − ⟨a⟩²) + Σ (⟨ab⟩ − ⟨a⟩⟨b⟩)
= ⟨Σ a⟩ + Σ (⟨ab⟩ − ⟨a⟩⟨b⟩)
⟨R⟩
5 cases
a≠b
x < y
⟨a⟩ ~ dx dy
⟨a⟩ = p(a=1) 1 + p(a=0) 0
= p(a=1)

$$R = \sum_{\substack{d_x \\ d_y}} \chi(d_x, d_y) \equiv a + b + c + \cdots$$



$$R - \langle R\rangle = (a - \langle a\rangle) + (b - \langle b\rangle) + (c - \langle c\rangle) + \cdots$$

$$\langle a\rangle \sim d_x^{\epsilon} d_y$$

$$(\quad)^2 = \sum_a (a - \langle a\rangle)^2 + \sum_{a \neq b} (a - \langle a\rangle)(b - \langle b\rangle)$$

$$\langle a\rangle = p(a=1)\,1 + p(a=0)\,0 = p(a=1)$$

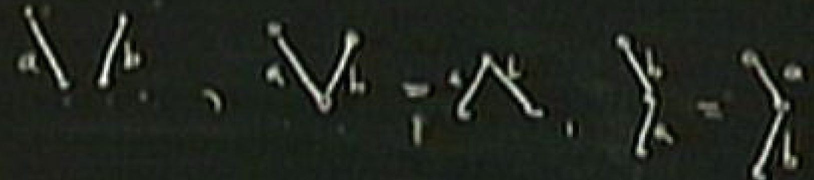
$$\langle \quad \rangle = \sum_a \left(\langle a^2\rangle - \underbrace{\langle a\rangle^2}_{0}\right) + \sum_{a \neq b} \left(\langle a\, b\rangle - \langle a\rangle\langle b\rangle\right)$$

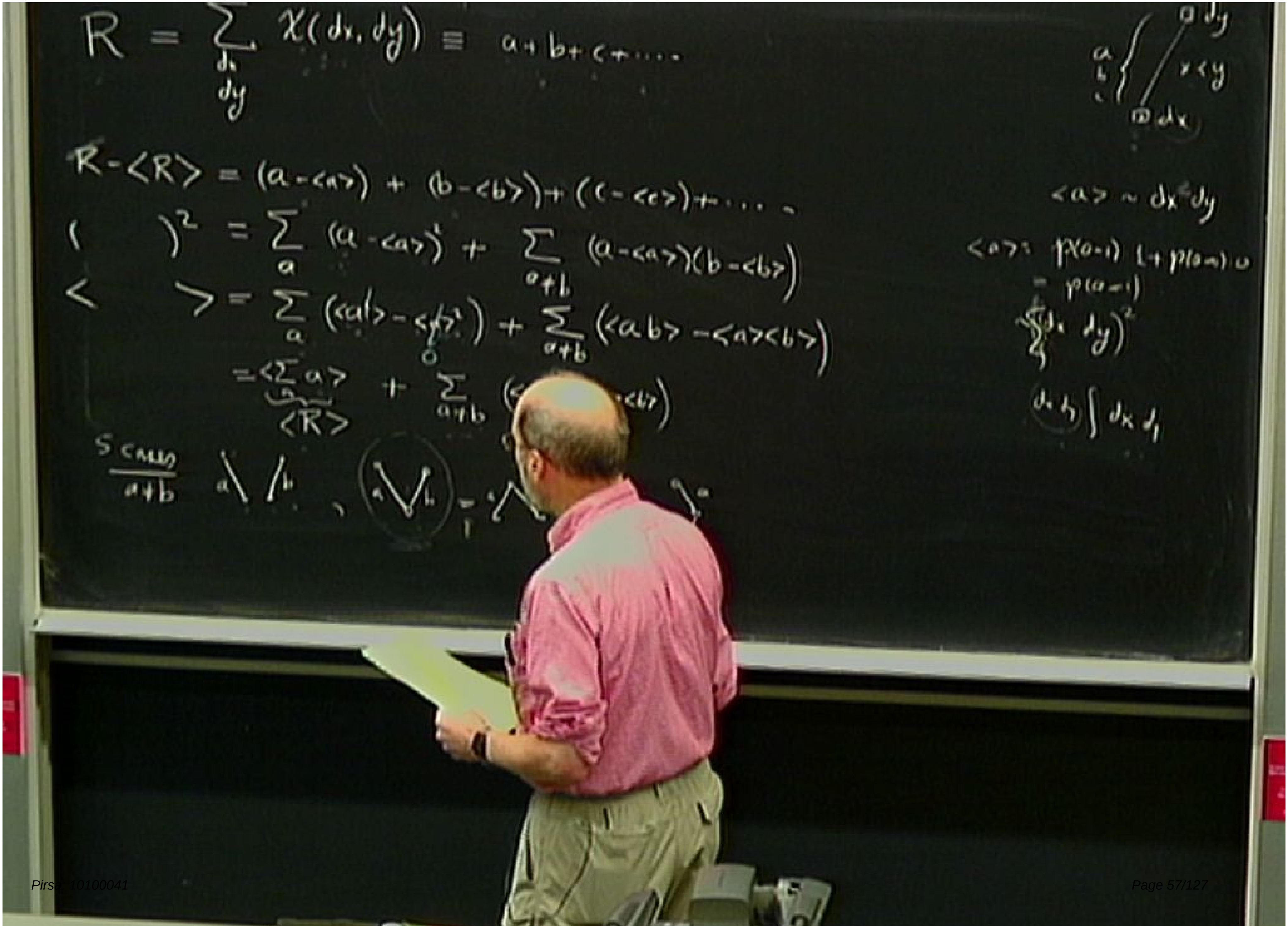
$$\sum (d_x\, d_y)^2$$

$$= \underbrace{\langle \sum_a a\rangle}_{\langle R\rangle} + \sum_{a \neq b} \left(\langle ab\rangle - \langle a\rangle\langle b\rangle\right)$$

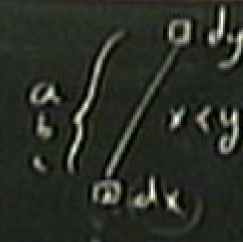
$$d_x\, d_y \int d_x\, d_y$$

$\frac{5 \text{ cases}}{a \neq b}$




R = Σ χ(dx, dy) ≡ a + b + c + . . .
R − <R> = (a − <a>) + (b − <b>) + (c − <c>) + . . .
<a> ~ dx dy
x < y
= p(a=1)

$$R = \sum_{\substack{d_x \\ d_y}} \chi(d_x, d_y) \equiv a + b + c + \cdots$$



$$R - \langle R \rangle = (a - \langle a \rangle) + (b - \langle b \rangle) + (c - \langle c \rangle) + \cdots$$

$$(\quad)^2 = \sum_a (a - \langle a \rangle)^2 + \sum_{a \neq b} (a - \langle a \rangle)(b - \langle b \rangle)$$

$$\langle \quad \rangle = \sum_a \left(\langle a^2 \rangle - \underset{0}{\langle a \rangle^2} \right) + \sum_{a \neq b} \left(\langle a\, b \rangle - \langle a \rangle \langle b \rangle \right)$$

$$= \underbrace{\langle \textstyle\sum a \rangle}_{\langle R \rangle} + \sum_{a \neq b} \left(\langle a b \rangle - \langle a \rangle \langle b \rangle \right)$$

$\langle a \rangle \sim d_x^2 d_y$

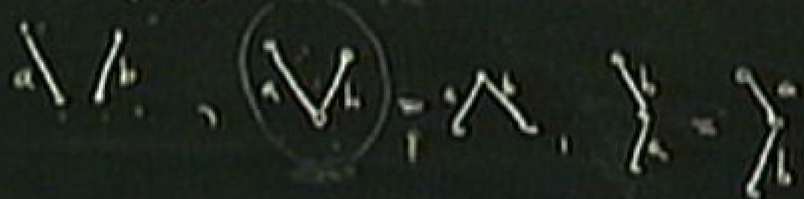
$\langle a \rangle = p(a=1)\, 1 + p(a=0)\, 0$

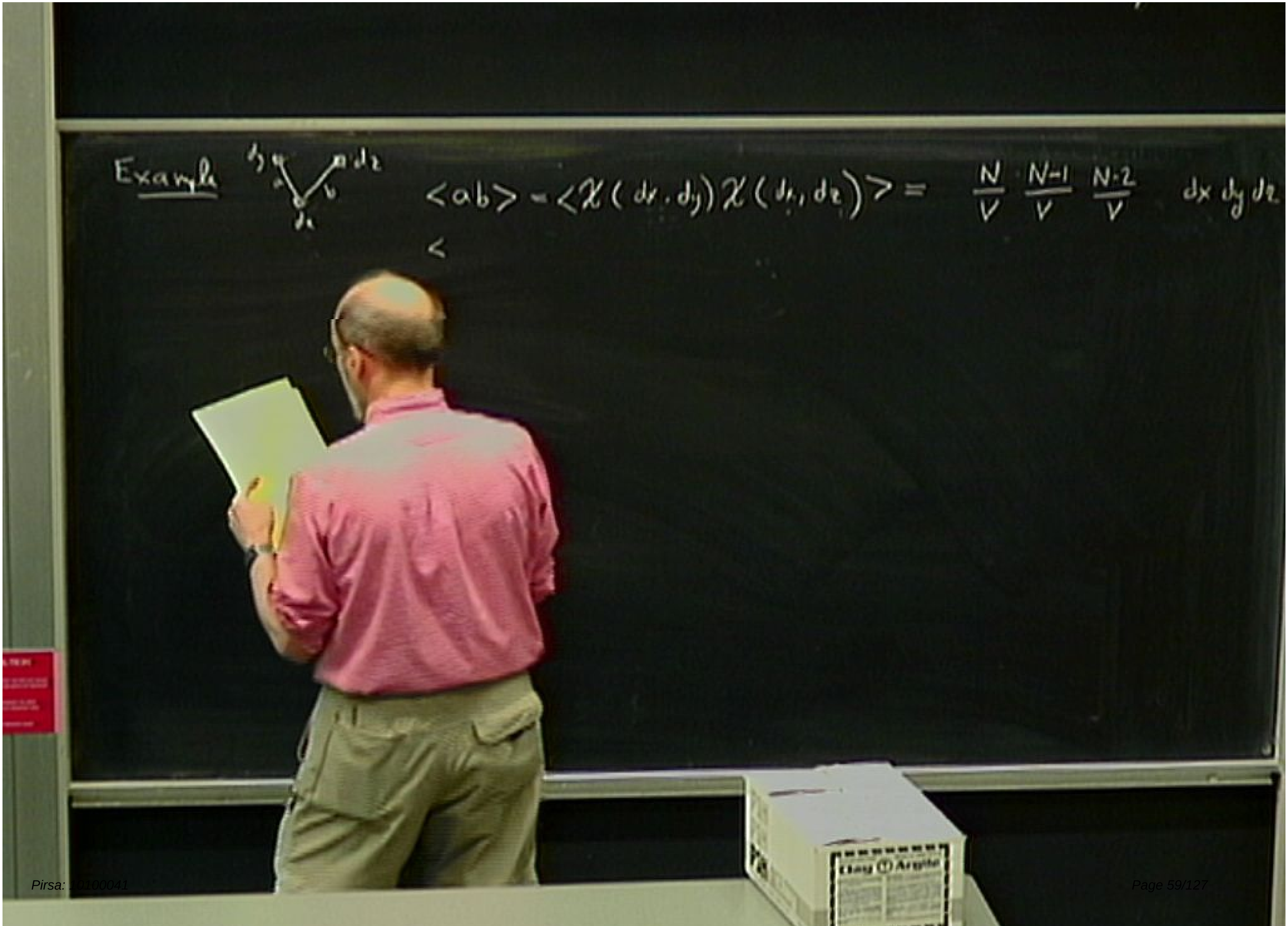
$= p(a=1)$

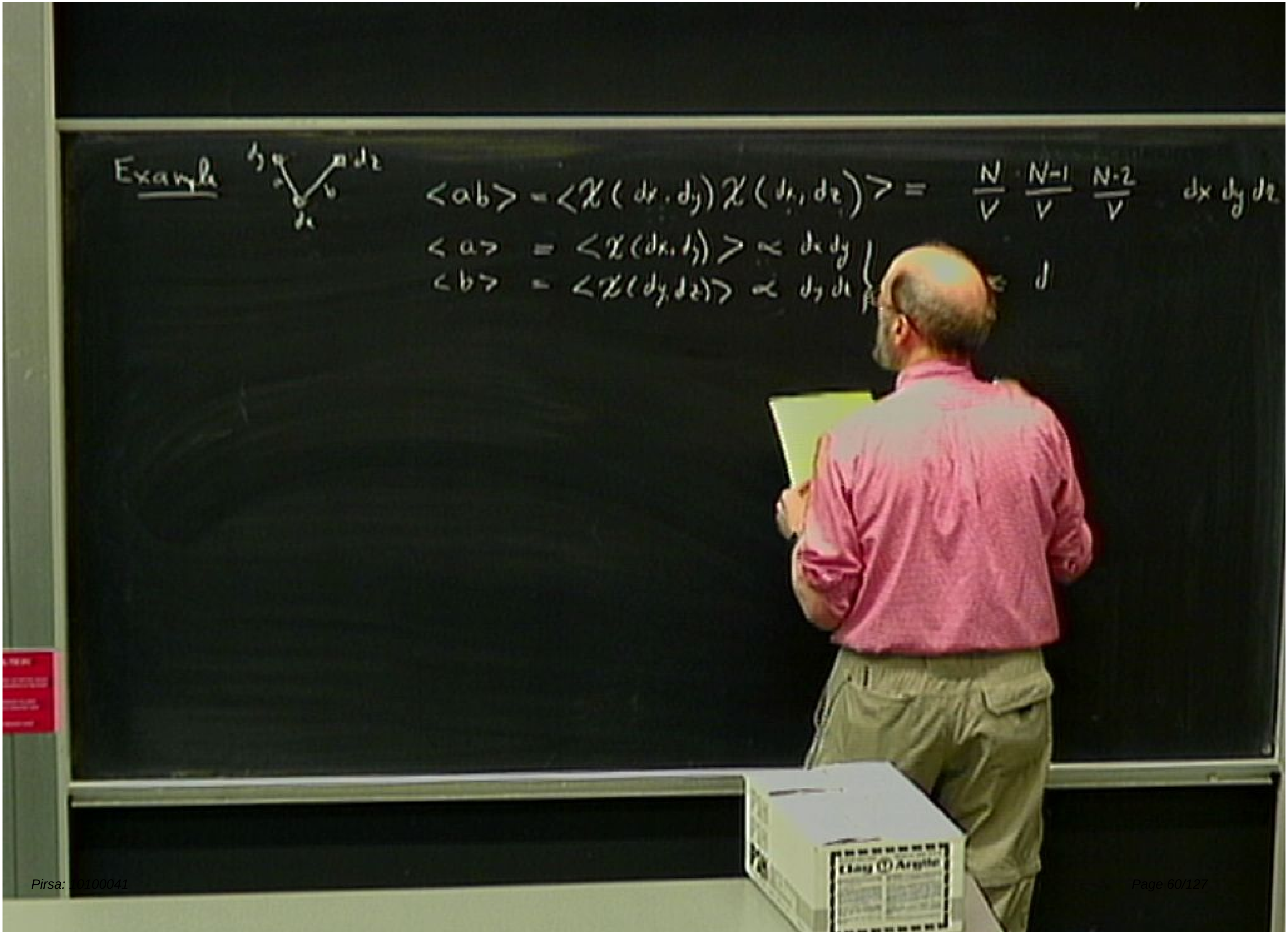
$(d_x \; d_y)^2$

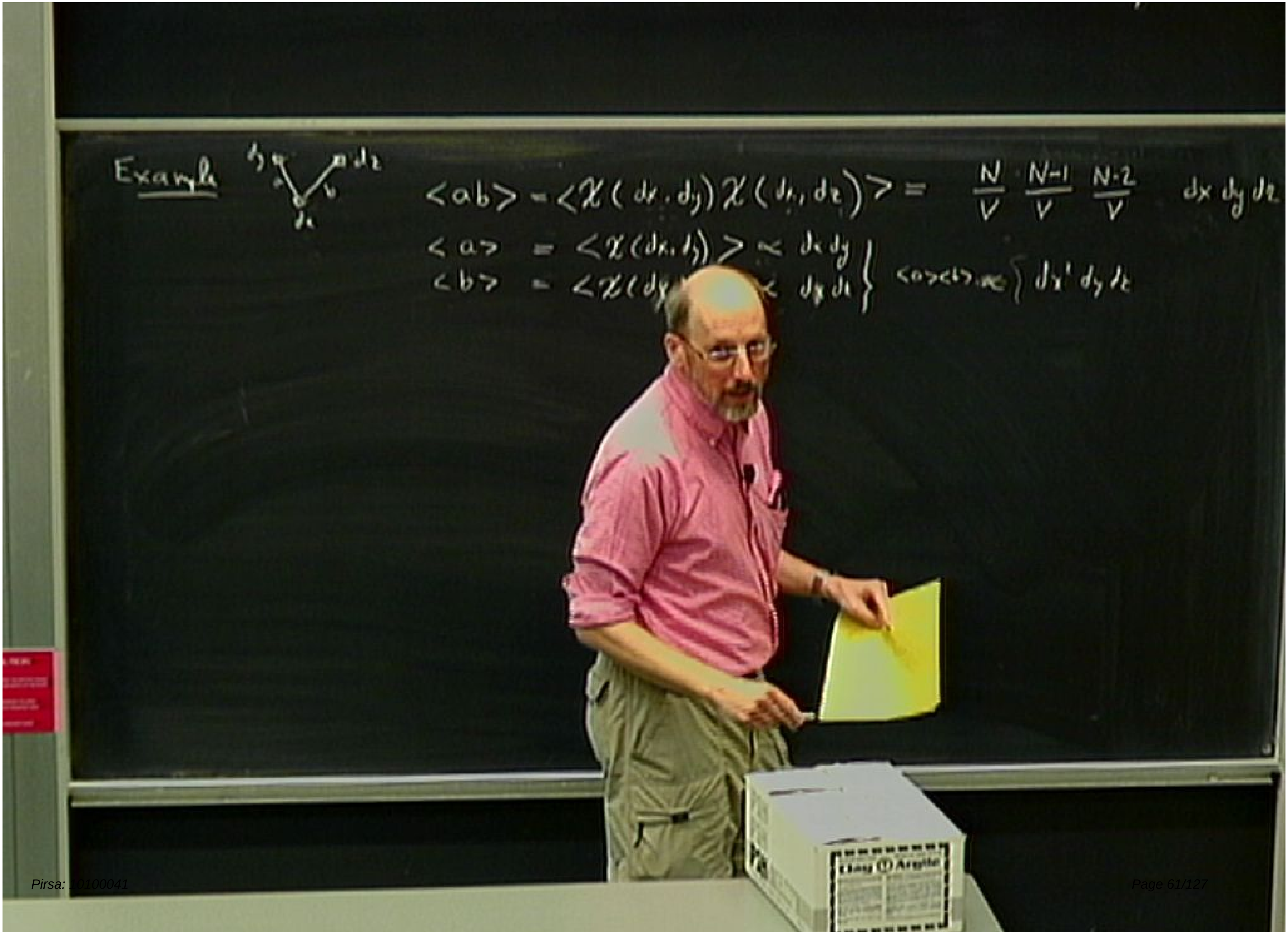
$(d_x\, d_y) \int d_x\, d_y$

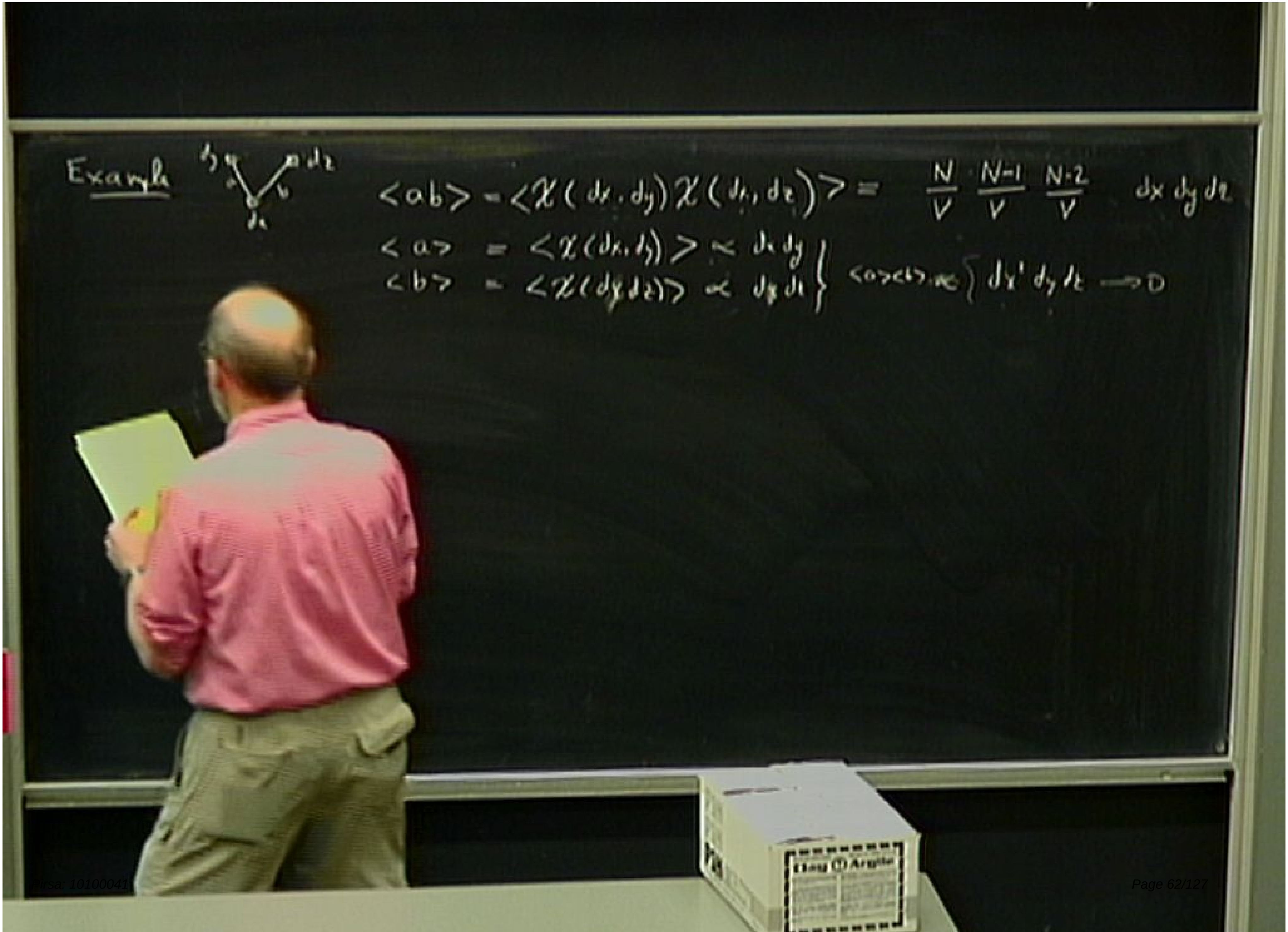
$\dfrac{\text{S cases}}{a \neq b}$



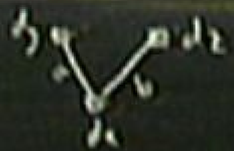

Example
<ab> = <χ(dx, dy) χ(dx, dz)> = N/V N-1/V N-2/V dx dy dz
<


Example
$\langle ab \rangle = \langle \chi(d_x, d_y)\,\chi(d_x, d_z) \rangle = \frac{N}{V}\,\frac{N-1}{V}\,\frac{N-2}{V}\; dx\, dy\, dz$
$\langle a \rangle = \langle \chi(d_x, d_y) \rangle \propto d_x\, d_y$
$\langle b \rangle = \langle \chi(d_y, d_z) \rangle \propto d_y\, d_z$


Example
<ab> = <χ(dx, dy) χ(dx, dz)> = N/V (N-1)/V (N-2)/V dx dy dz
<a> = <χ(dx, dy)> ∝ dx dy
<b> = <χ(dy, dz)> ∝ dy dz
<a><b> ∝ ∫ dx' dy dz


Example
<ab> = <χ(dx, dy) χ(dx, dz)> = N/V · (N-1)/V · (N-2)/V dx dy dz
<a> = <χ(dx, dy)> ∝ dx dy
<b> = <χ(dx, dz)> ∝ dx dz
<a><b> ∝ ∫ dx² dy dz → 0

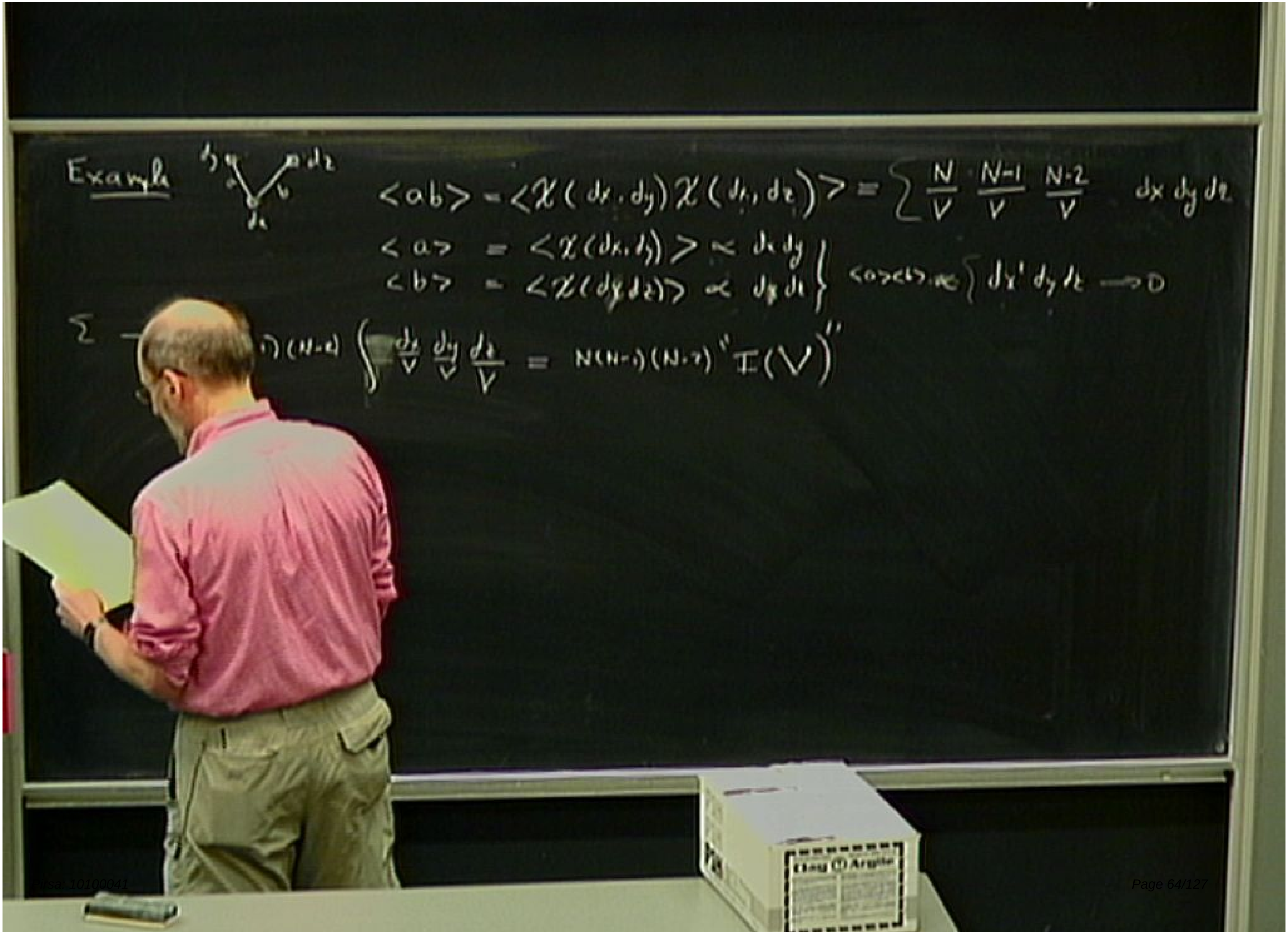
Example

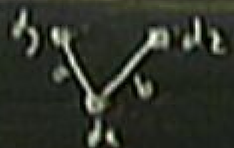


$$\langle ab \rangle = \langle \chi(dx, dy)\, \chi(dx, dz) \rangle = \int \frac{N}{V} \cdot \frac{N-1}{V} \, \frac{N-2}{V} \, dx\, dy\, dz$$

$$\left.\begin{aligned} \langle a \rangle &= \langle \chi(dx, dy) \rangle \propto dx\, dy \\ \langle b \rangle &= \langle \chi(dx, dz) \rangle \propto dx\, dz \end{aligned}\right\} \quad \langle a \rangle \langle b \rangle \propto \int dx^2\, dy\, dz \longrightarrow 0$$

$N(N-1)(N-2) \quad I$

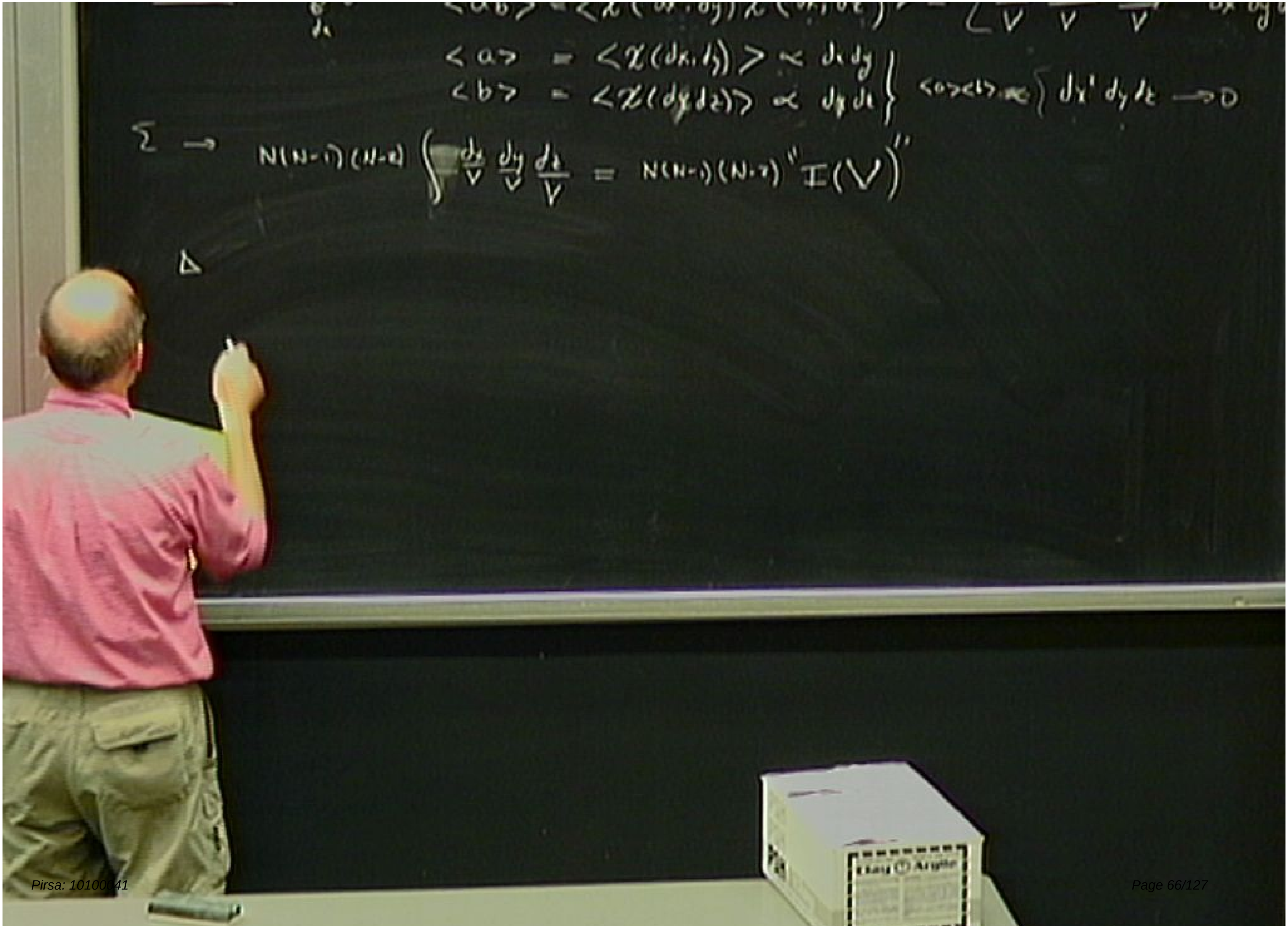

Example
<ab> = <χ(dx, dy) χ(dx, dz)> = ∫ N/V · (N-1)/V · (N-2)/V dx dy dz
<a> = <χ(dx, dy)> ∝ dx dy
<b> = <χ(dx, dz)> ∝ dx dz
<a><b> ∝ ∫ dx² dy dz → 0
Σ (N-2) ∫ dx/V dy/V dz/V = N(N-1)(N-2) "I(V)"

Example 

$$\langle ab \rangle = \langle \chi(dx, dy)\, \chi(dx, dz) \rangle = \int \frac{N}{V} \cdot \frac{N-1}{V} \cdot \frac{N-2}{V} \; dx\, dy\, dz$$

$$\left.\begin{aligned} \langle a \rangle &= \langle \chi(dx, dy) \rangle \propto dx\, dy \\ \langle b \rangle &= \langle \chi(dx, dz) \rangle \propto dx\, dz \end{aligned}\right\} \langle a \rangle \langle b \rangle \propto \int dx^2\, dy\, dz \longrightarrow 0$$

$$\Sigma \longrightarrow N(N-1)(N-2) \int \frac{dx}{V} \frac{dy}{V} \frac{dz}{V} = N(N-1)(N-2)\, \text{"} I(V) \text{"}$$


<a> = <χ(dx, dy)> ∝ dx dy
<b> = <χ(dy dz)> ∝ dy dz
<a><b> ∝ ∫ dx' dy dz → 0
Σ → N(N-1)(N-2) ∫ dx/V dy/V dz/V = N(N-1)(N-2) "I(V)"
Δ

$\langle ab\rangle = \langle \chi(dx,dy)\,\chi(dy,dz)\rangle$

$\langle a\rangle = \langle \chi(dx,dy)\rangle \propto dx\,dy$

$\langle b\rangle = \langle \chi(dy\,dz)\rangle \propto dy\,dz$ $\Big\}$ $\langle a\rangle\langle b\rangle \propto \int dx^{\prime}\,dy\,dz \to 0$

$\Sigma \to N(N-1)(N-2)\int \frac{dx}{V}\frac{dy}{V}\frac{dz}{V} = N(N-1)(N-2)\ \text{"}I(V)\text{"}$

$\langle(\Delta R)^2\rangle = \langle(R-\langle R\rangle)^2\rangle \sim 2\left[I(V)+I(?)-2\,I(V)^2\right]N^3$ $\quad N\to\infty$

$\Delta R \sim N^{3/2}$

$\langle ab \rangle = \langle \chi(dx, dy)\, \chi(dy, dz) \rangle$

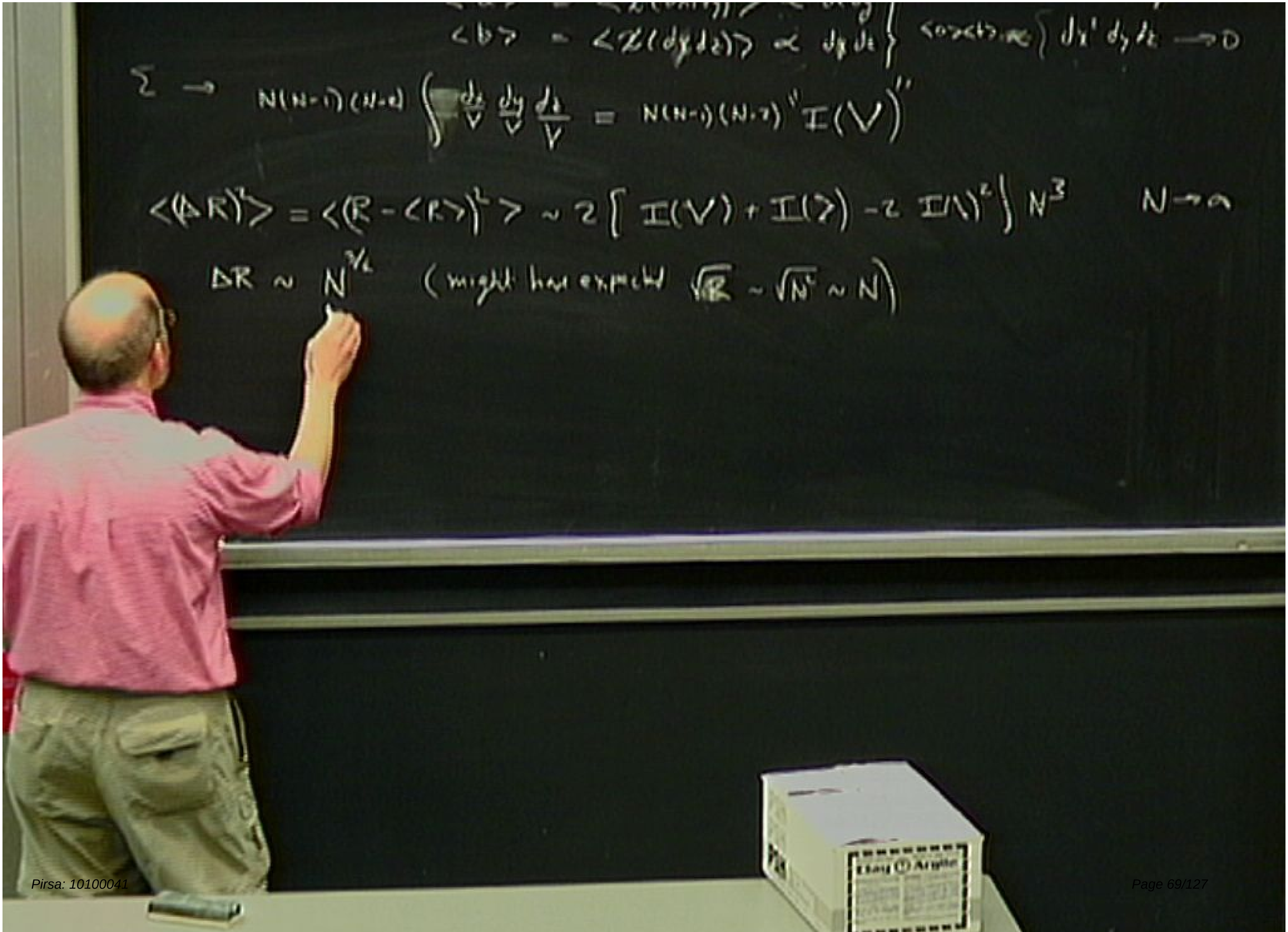
$\langle a \rangle = \langle \chi(dx, dy) \rangle \propto dx\, dy$

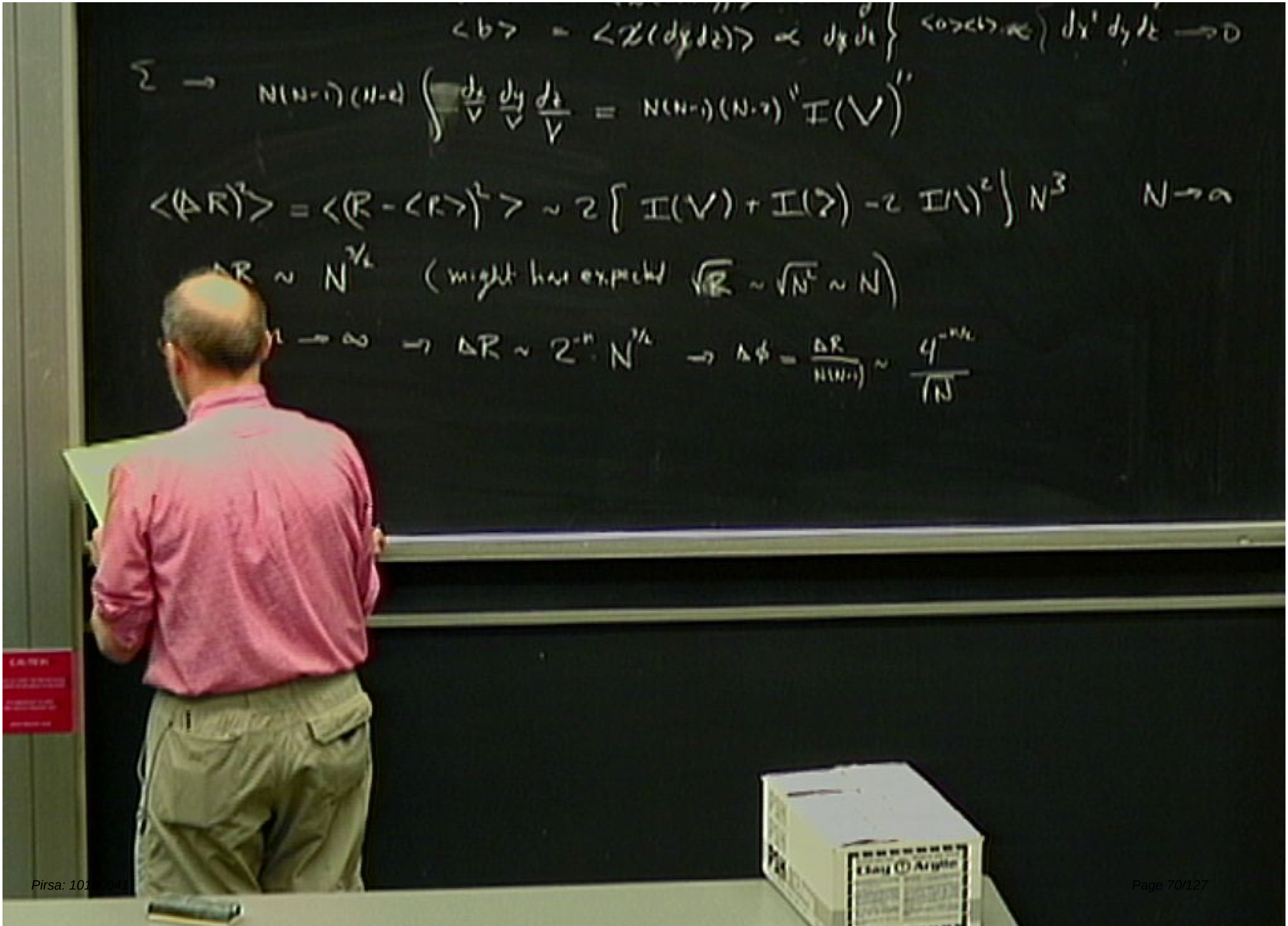
$\langle b \rangle = \langle \chi(dy\, dz) \rangle \propto dy\, dz$ $\Big\}$ $\langle a \rangle \langle b \rangle \propto \int dx'\, dy\, dz \to 0$

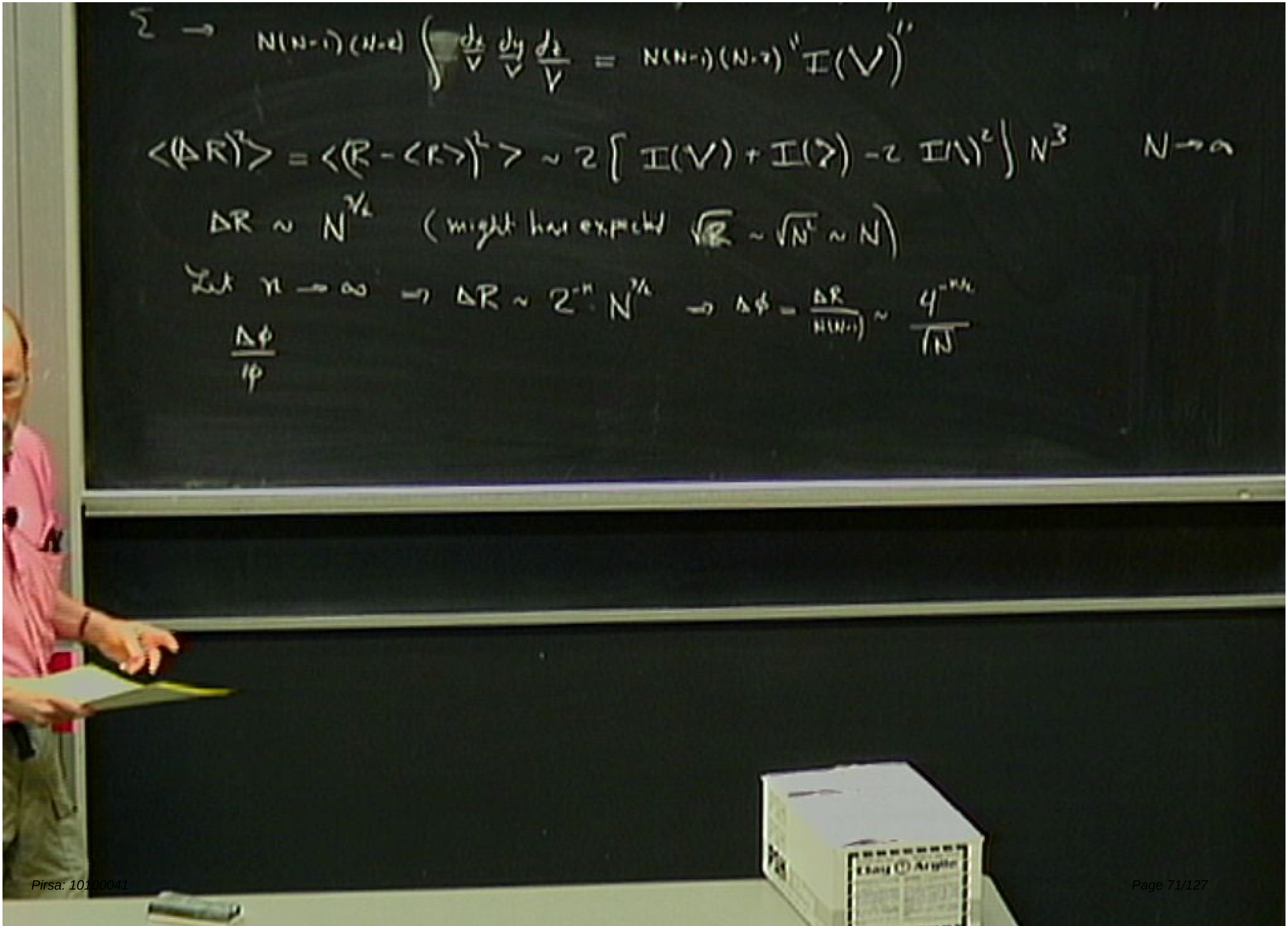
$$\Sigma \to N(N-1)(N-2) \int \frac{dx}{V}\frac{dy}{V}\frac{dz}{V} = N(N-1)(N-2) \text{ "}I(V)\text{"}$$

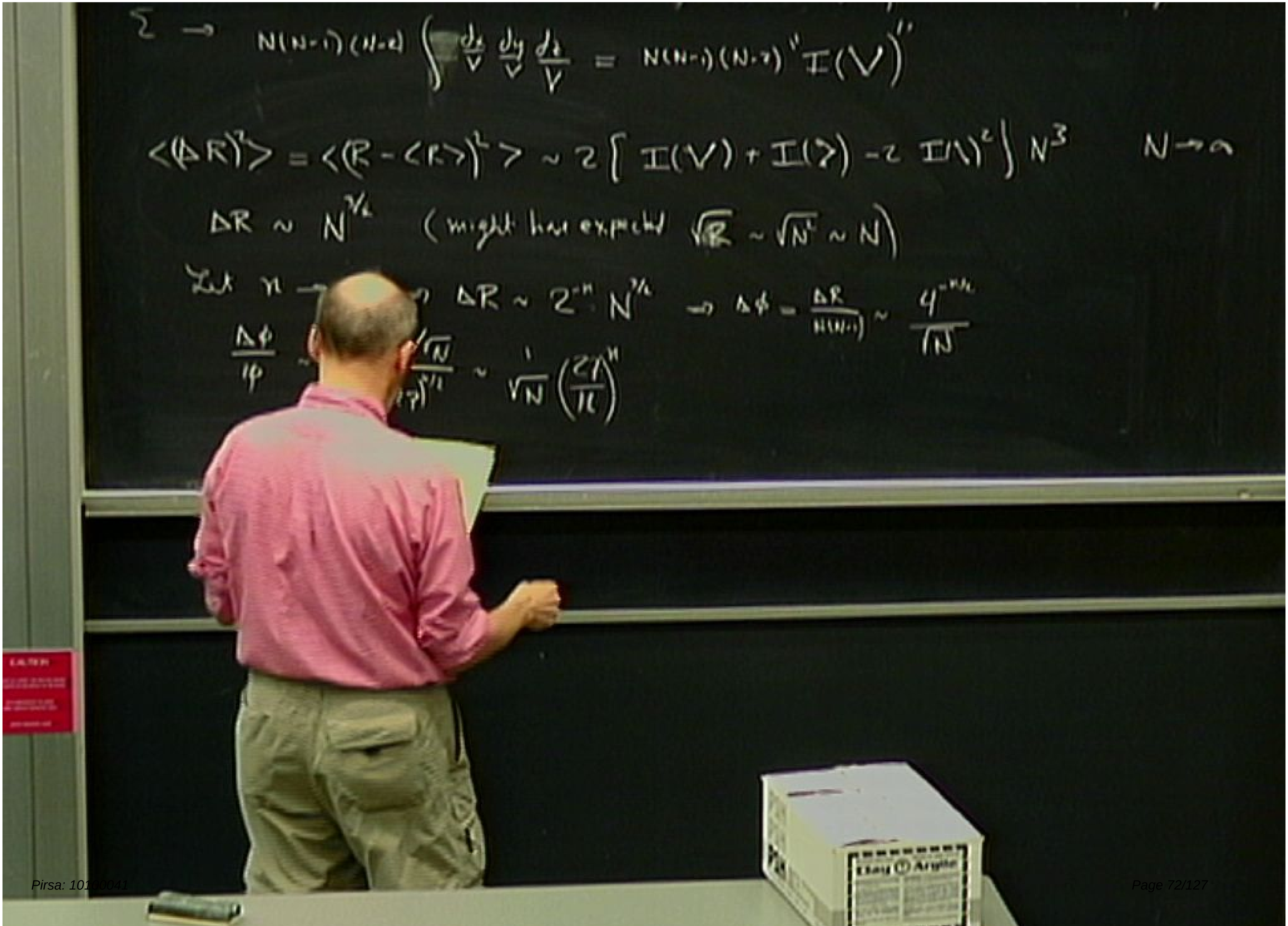
$$\langle (\Delta R)^2 \rangle = \langle (R - \langle R \rangle)^2 \rangle \sim 2\left[I(V) + I(>) - 2\, I(V)^2 \right] N^3 \qquad N \to \infty$$

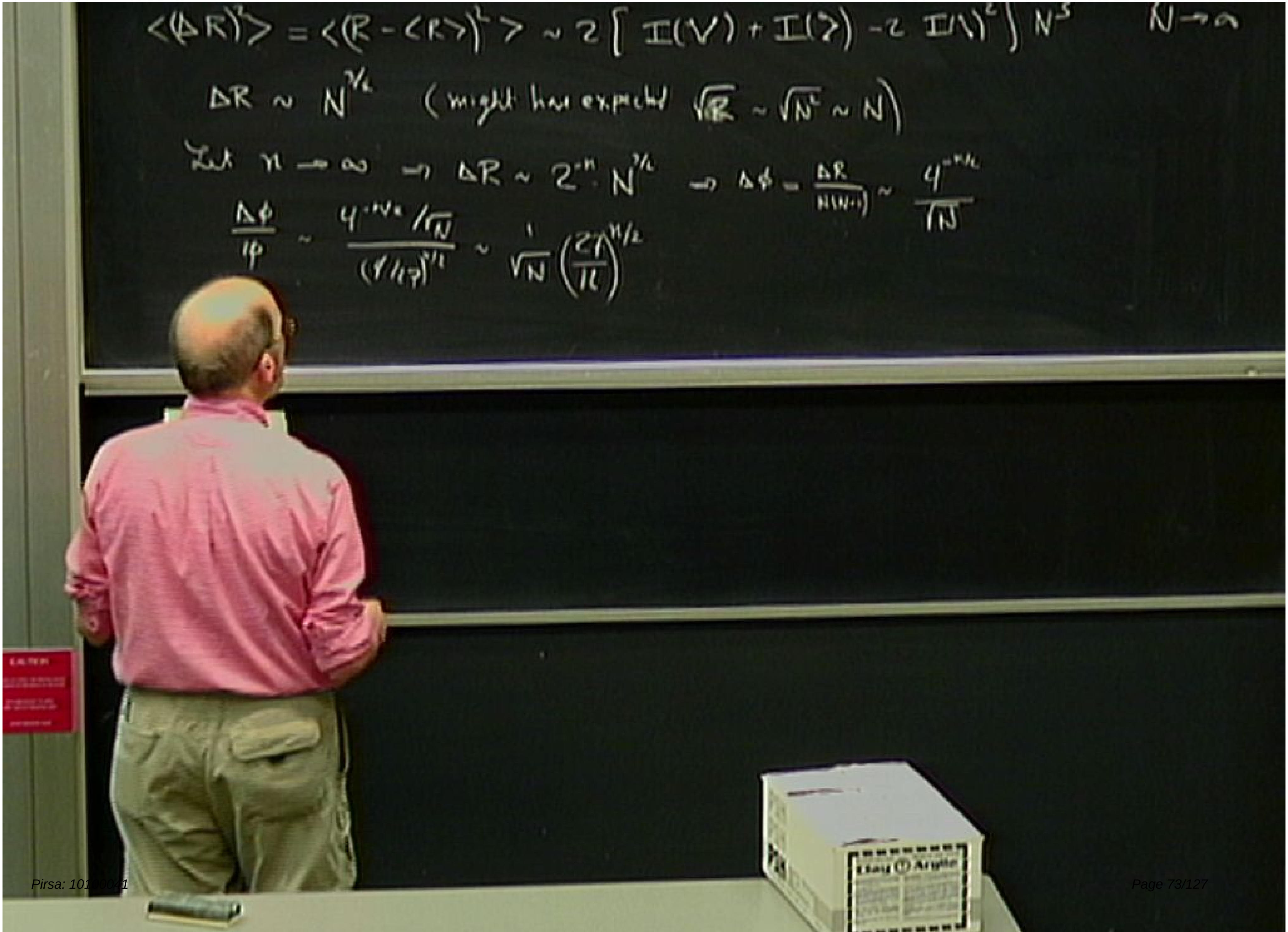
$\Delta R \sim N^{3/2}$ (might have expected $\sqrt{R}$


$\langle b \rangle = \langle Z(dy\,dz) \rangle \propto dy\,dz \}$ $\langle a \rangle \langle b \rangle \propto \int dx'\,dy\,dz \to 0$
$\Sigma \to N(N-1)(N-2) \int \frac{dx}{V} \frac{dy}{V} \frac{dz}{V} = N(N-1)(N-2)$ "$I(V)$"
$\langle (\Delta R)^2 \rangle = \langle (R - \langle R \rangle)^2 \rangle \sim 2 [I(V) + I(?) - 2 I(V)^2] N^3$ $N \to \infty$
$\Delta R \sim N^{3/2}$ (might have expected $\sqrt{R} \sim \sqrt{N^2} \sim N$)

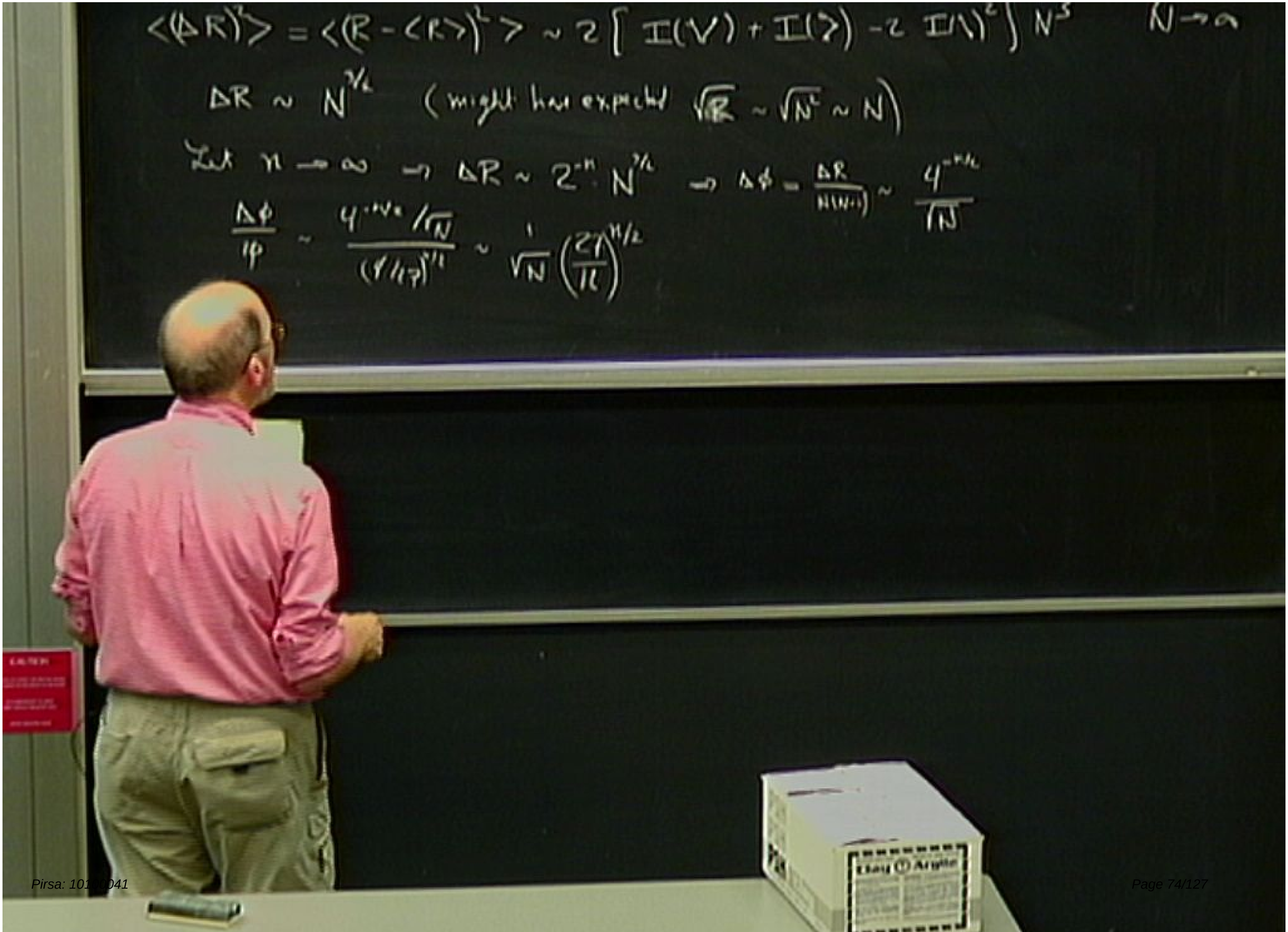

<b> = <χ(dy dz)> ∝ dy dz } <a><b> ∝ ∫ dx' dy dz → 0
Σ → N(N-1)(N-2) ∫ dx/V dy/V dz/V = N(N-1)(N-2) "I(V)"
<(ΔR)^2> = <(R - <R>)^2> ~ 2 { I(V) + I(?) - 2 I(V)^2 } N^3
N → ∞
ΔR ~ N^{3/2} (might have expected √R ~ √N^2 ~ N)
n → ∞ → ΔR ~ 2^{-n} N^{3/2} → Δφ = ΔR / N(N-1) ~ 4^{-n/2} / √N

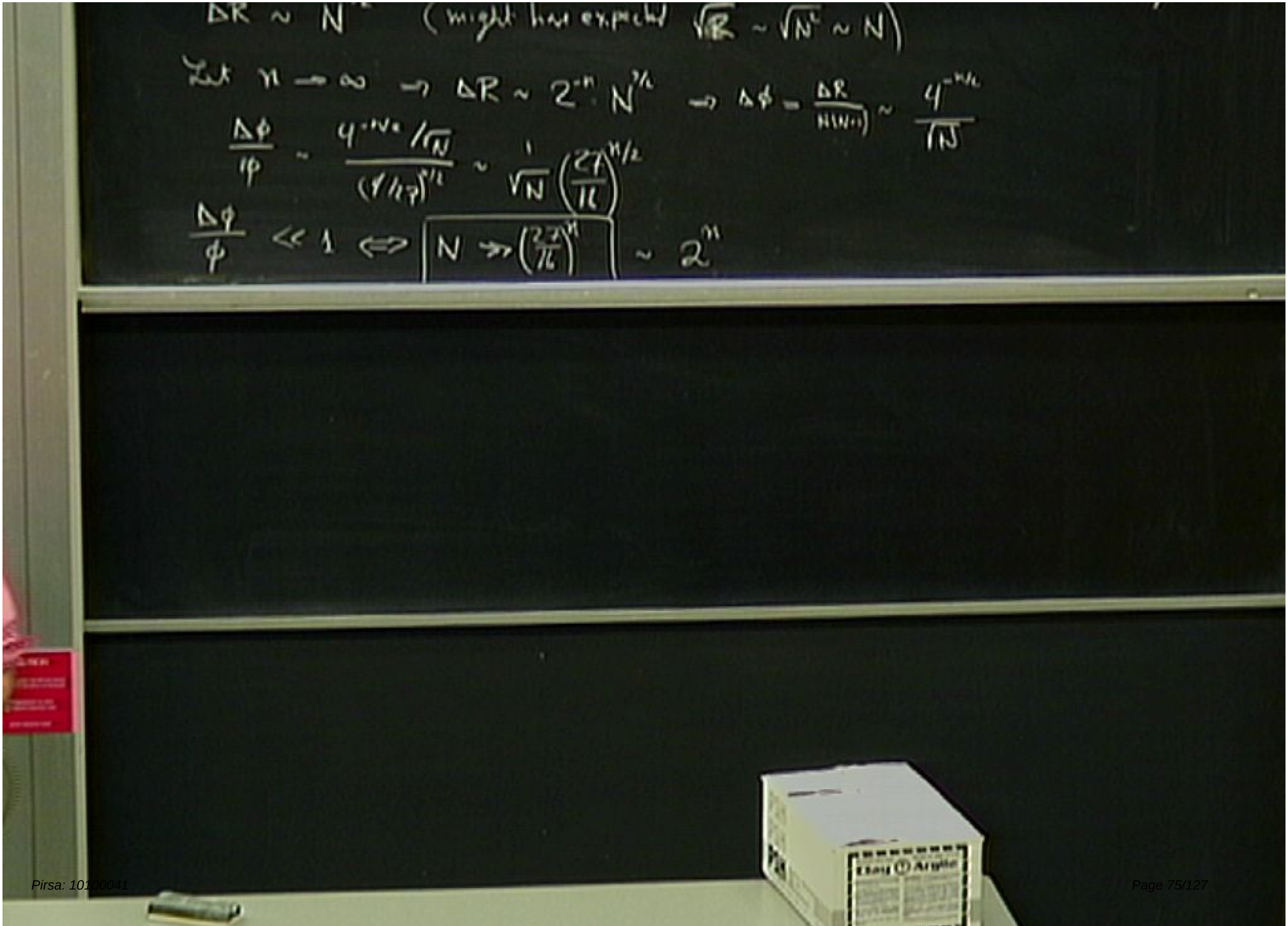

Σ → N(N-1)(N-2) ∫ dx/V dy/V dz/V = N(N-1)(N-2) "I(V)"
<(ΔR)²> = <(R - <R>)²> ~ 2[I(V) + I(?) - 2 I(V)²] N³   N → ∞
ΔR ~ N^{3/2}   (might have expected √R ~ √N² ~ N)
Let n → ∞ ⇒ ΔR ~ 2^{-n} N^{3/2} ⇒ Δφ = ΔR/N(N-1) ~ 4^{-n/2}/√N
Δφ/φ

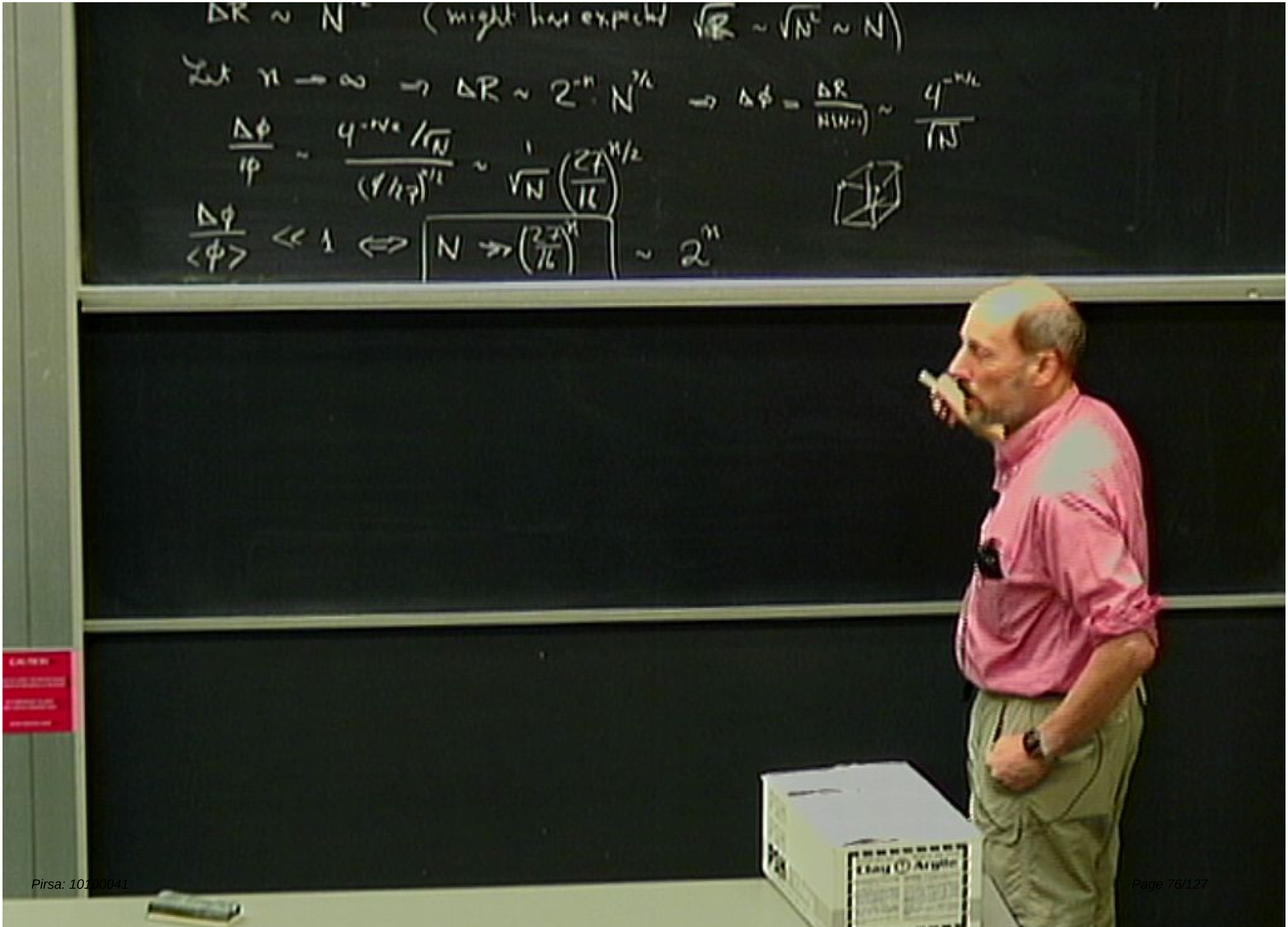

Σ → N(N-1)(N-2) ∫ dx/V dy/V dz/V = N(N-1)(N-2) "I(V)"
<(ΔR)²> = <(R - <R>)²> ~ 2{ I(V) + I(?) - 2 I(V)² } N³
N → ∞
ΔR ~ N^3/2 (might have expected √R ~ √N² ~ N)
Let
ΔR ~ 2^-n N^3/2
Δφ = ΔR / N(N-1)

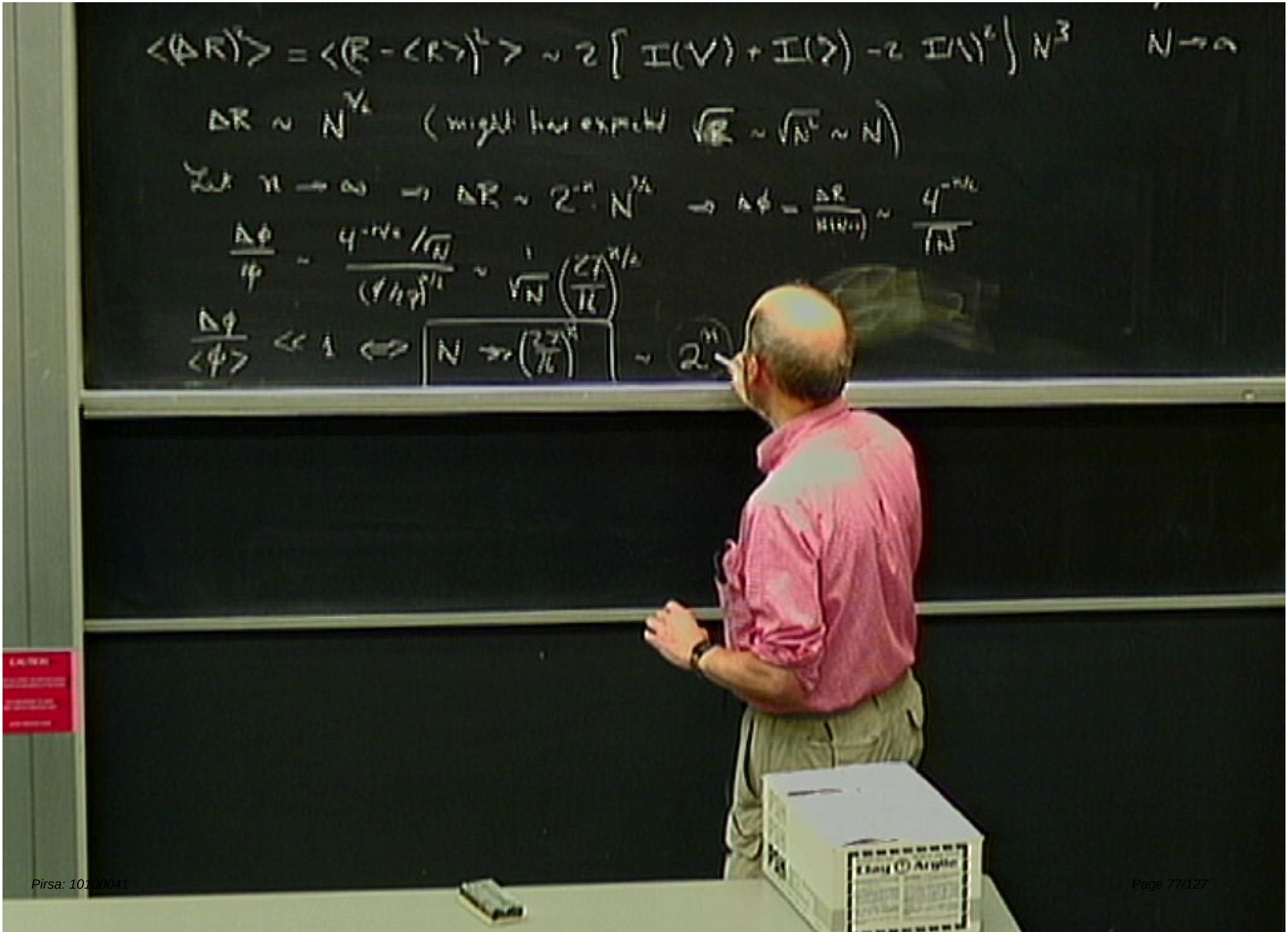


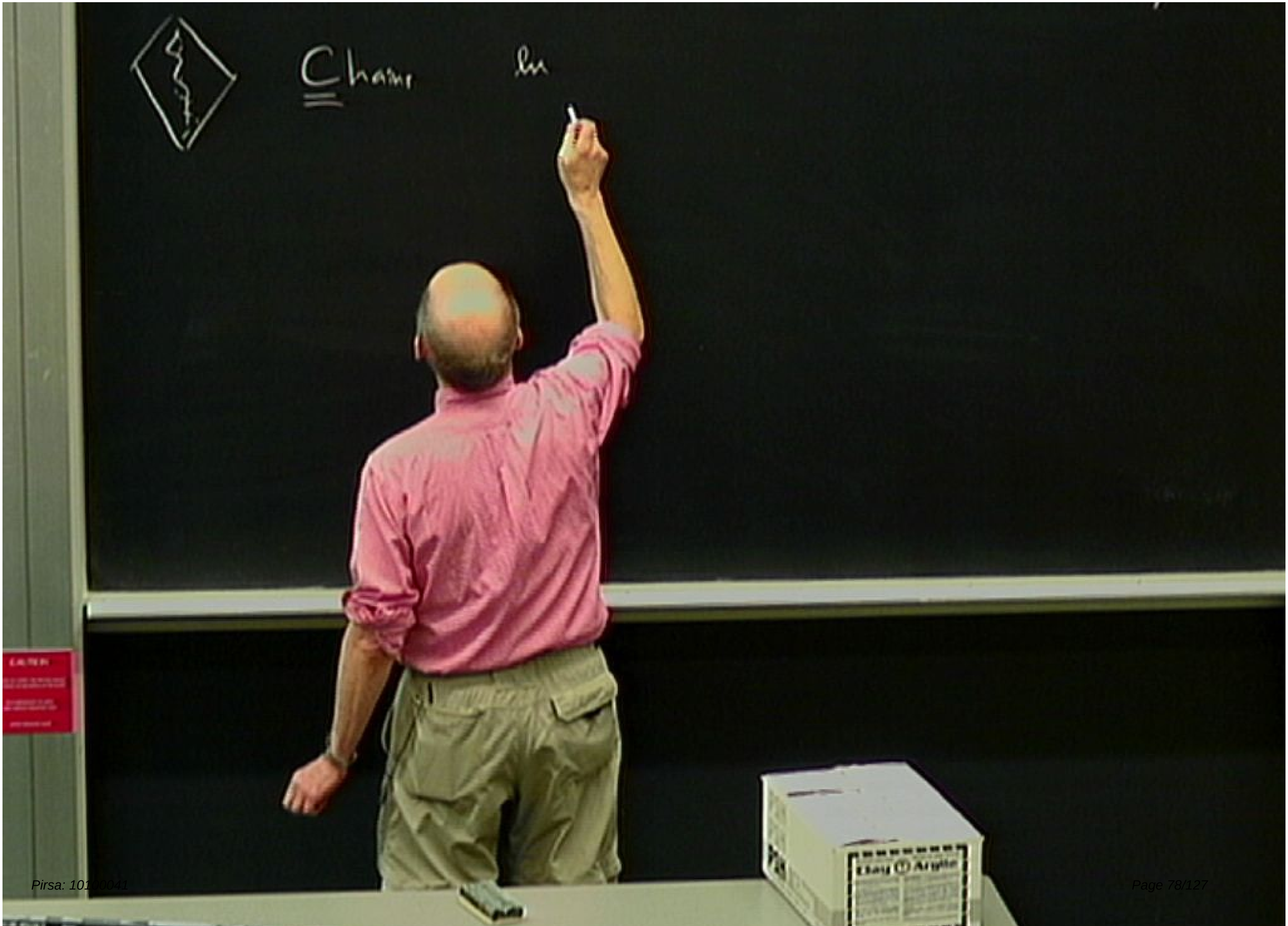
$\langle (\Delta R)^2 \rangle = \langle (R - \langle R \rangle)^2 \rangle \sim 2 \left[I(V) + I(\Lambda) - 2 I(\backslash)^2 \right] N^3$
$N \to \infty$
$\Delta R \sim N^{3/2}$ (might have expected $\sqrt{R} \sim \sqrt{N^2} \sim N$)
Let $n \to \infty \Rightarrow \Delta R \sim 2^{-n} N^{3/2} \Rightarrow \Delta\phi = \frac{\Delta R}{N(N-1)} \sim \frac{4^{-n/2}}{\sqrt{N}}$
$\frac{\Delta\phi}{\phi} \sim \frac{4^{-n/2}/\sqrt{N}}{(4/27)^{n/2}} \sim \frac{1}{\sqrt{N}} \left(\frac{27}{16}\right)^{n/2}$

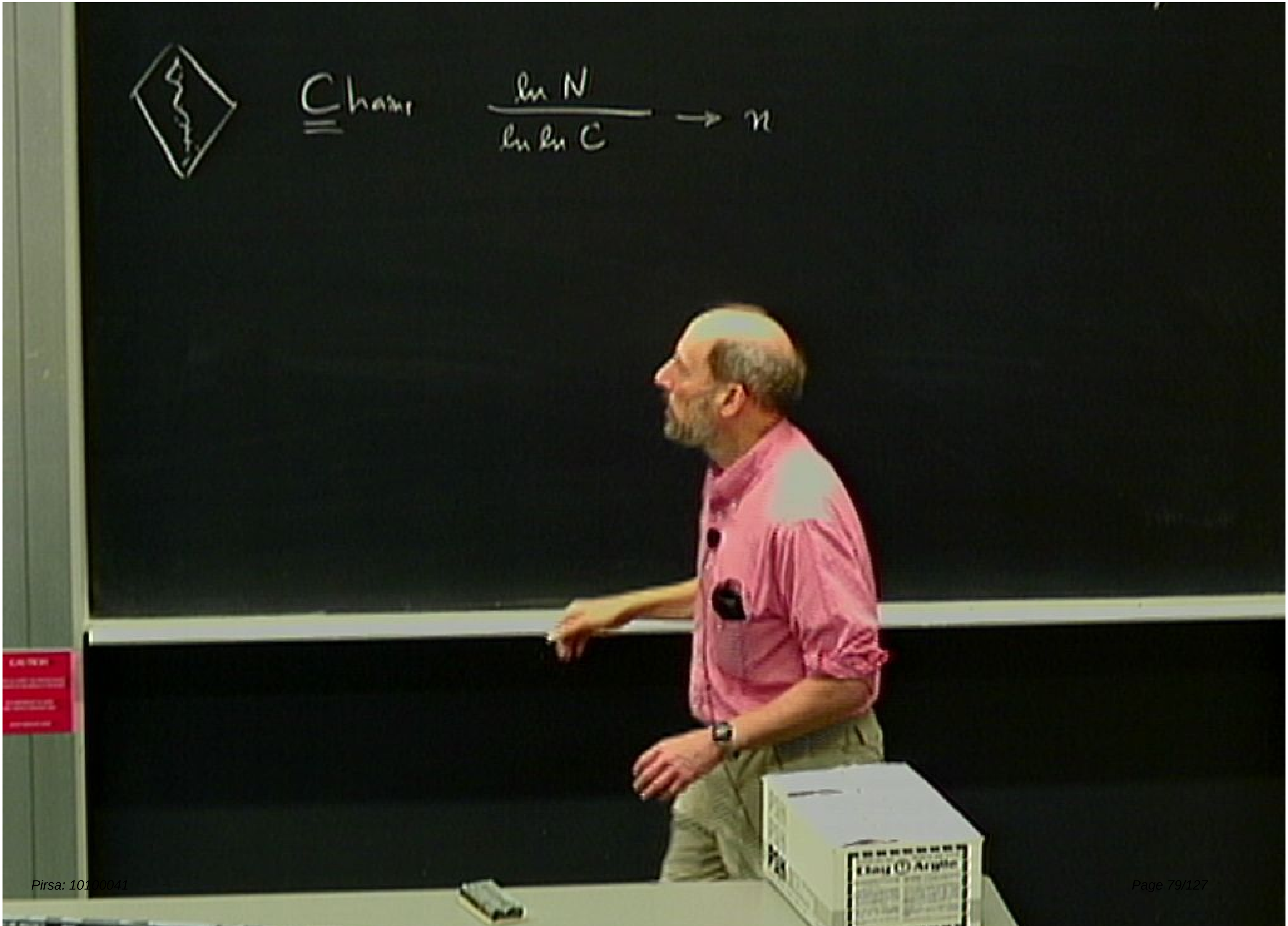

$\langle (\Delta R)^2 \rangle = \langle (R - \langle R \rangle)^2 \rangle \sim 2 [I(V) + I(\Lambda) - 2 I(\backslash)^2] N^3$ $N \to \infty$
$\Delta R \sim N^{3/2}$ (might have expected $\sqrt{R} \sim \sqrt{N^2} \sim N$)
Let $n \to \infty$ $\Rightarrow \Delta R \sim 2^{-n} N^{3/2}$ $\Rightarrow \Delta\phi = \frac{\Delta R}{N(N-1)} \sim \frac{4^{-n/2}}{\sqrt{N}}$
$\frac{\Delta\phi}{\phi} \sim \frac{4^{-n/2}/\sqrt{N}}{(\phi/4\pi)^{n/2}} \sim \frac{1}{\sqrt{N}} \left(\frac{2\pi}{\phi}\right)^{n/2}$

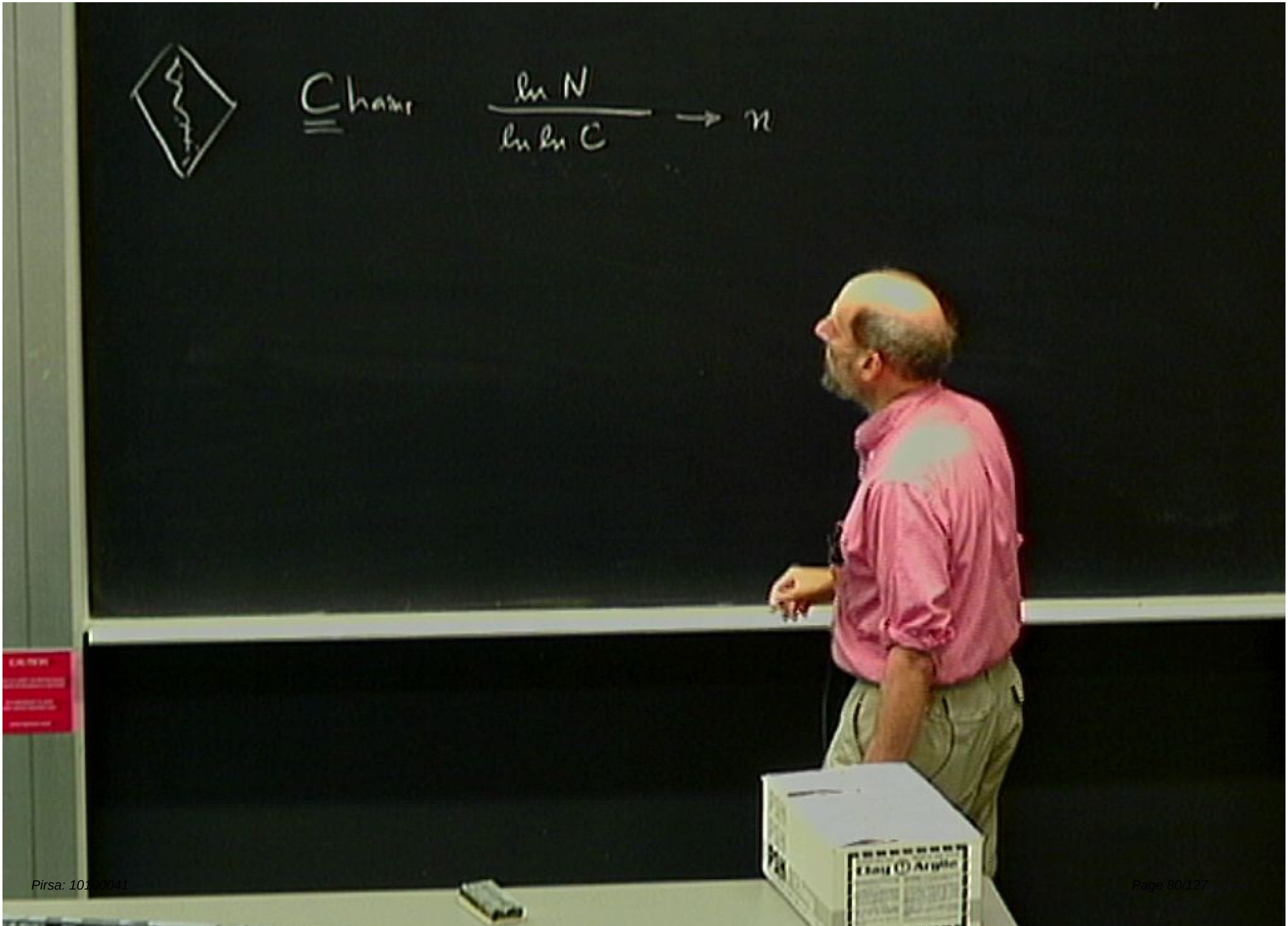

ΔR ~ N (might have expected √R ~ √N² ~ N)
Let n → ∞ ⇒ ΔR ~ 2⁻ⁿ N^{3/2} ⇒ Δφ = ΔR / N(N+1) ~ 4^{-n/2} / √N
Δφ/φ ~ (4^{-n/2} / √N) / (φ/27)^{n/2} ~ 1/√N (27/16)^{n/2}
Δφ/φ ≪ 1 ⇔ N ≫ (27/16)^n ~ 2^n


(might have expected $\sqrt{R} \sim \sqrt{N^2} \sim N$)
Let $n \to \infty \Rightarrow \Delta R \sim 2^{-n} N^{3/2} \Rightarrow \Delta\phi = \frac{\Delta R}{N(N+1)} \sim \frac{4^{-n/2}}{\sqrt{N}}$
$\frac{\Delta\phi}{\phi} \sim \frac{4^{-n/2}/\sqrt{N}}{(\phi/2\pi)^{n/2}} \sim \frac{1}{\sqrt{N}}\left(\frac{27}{\pi}\right)^{n/2}$
$\frac{\Delta\phi}{\langle\phi\rangle} \ll 1 \Leftrightarrow \boxed{N \gg \left(\frac{27}{\pi}\right)^{n}} \sim 2^{n}$


ΔR ~ N^3/2 (might have expected √R ~ √N² ~ N)
N → ∞
Let n → ∞

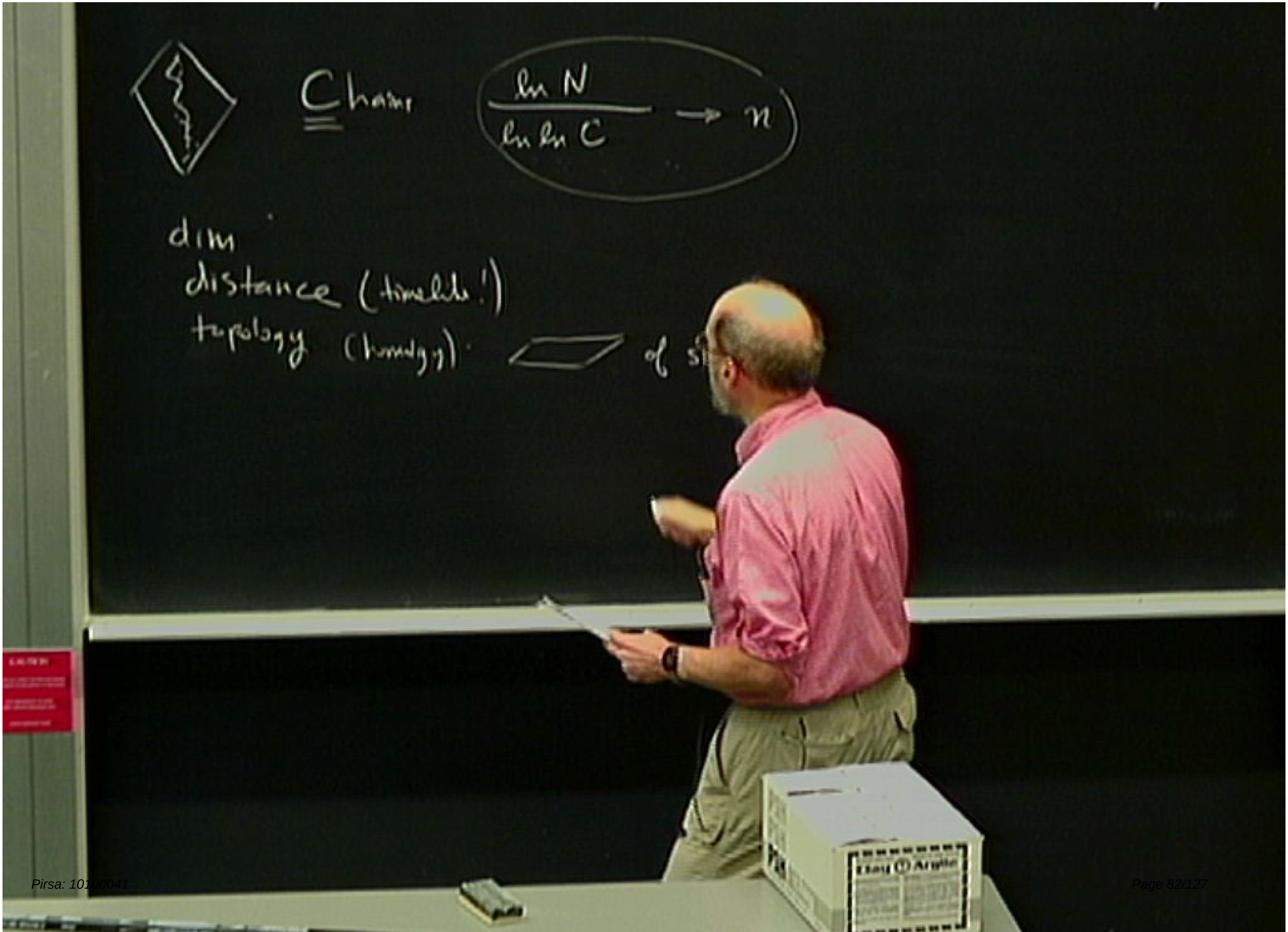


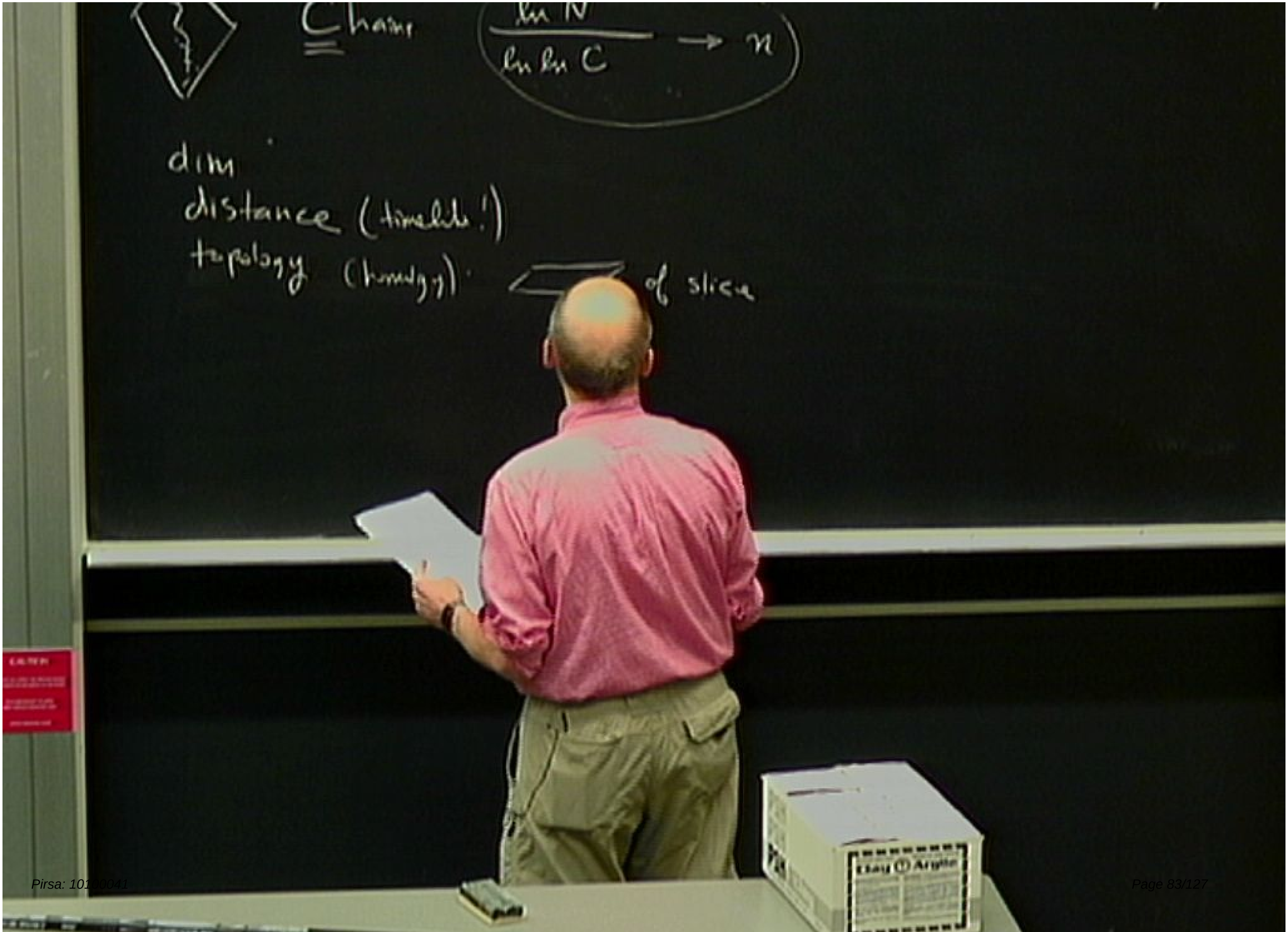

Chains
$\frac{\ln N}{\ln \ln C} \to n$

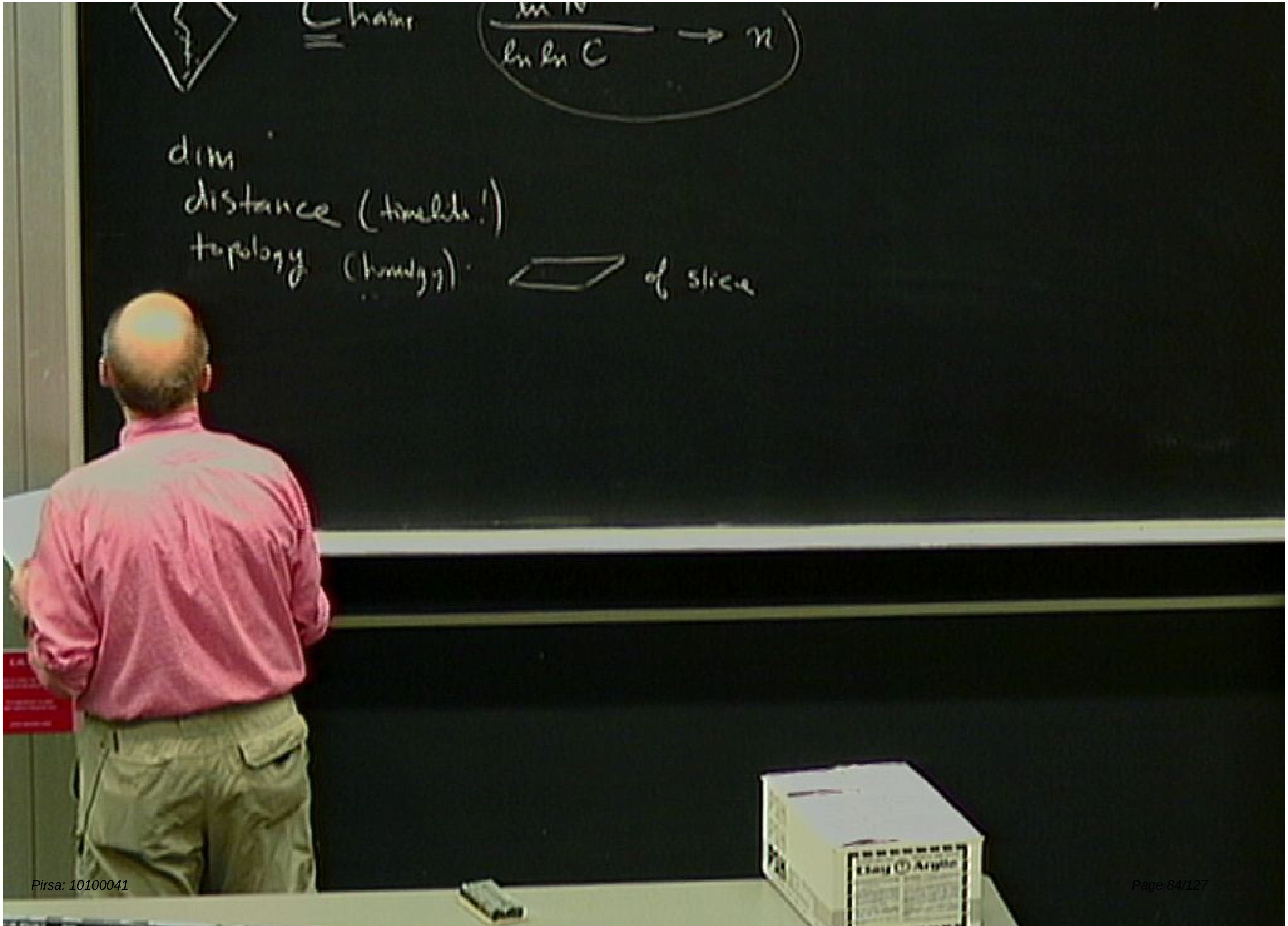

Chains
$\frac{\ln N}{\ln \ln C} \to n$

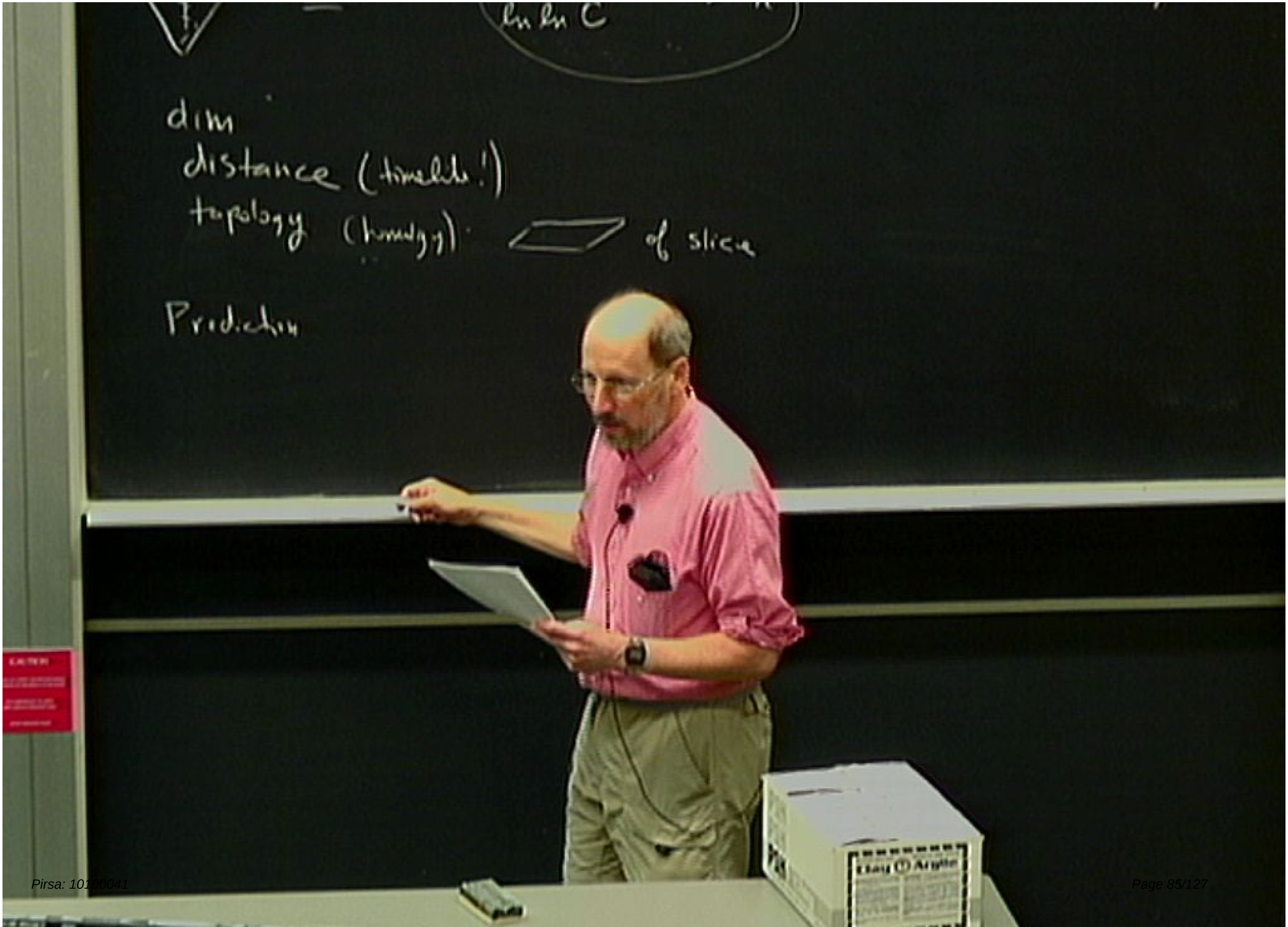
Chains

$$\frac{\ln N}{\ln \ln C} \rightarrow n$$


Chain
ln N / ln ln C → n
dim
distance (timelike!)
topology (homology)
of


Chain
ln N / ln ln C → n
dim
distance (timelike!)
topology (homology)
of slice


Chain
ln ln C
→ n
dim
distance (timelike!)
topology (homology)
of slice


ln ln C
dim
distance (timelike!)
topology (homology)
of slice
Prediction

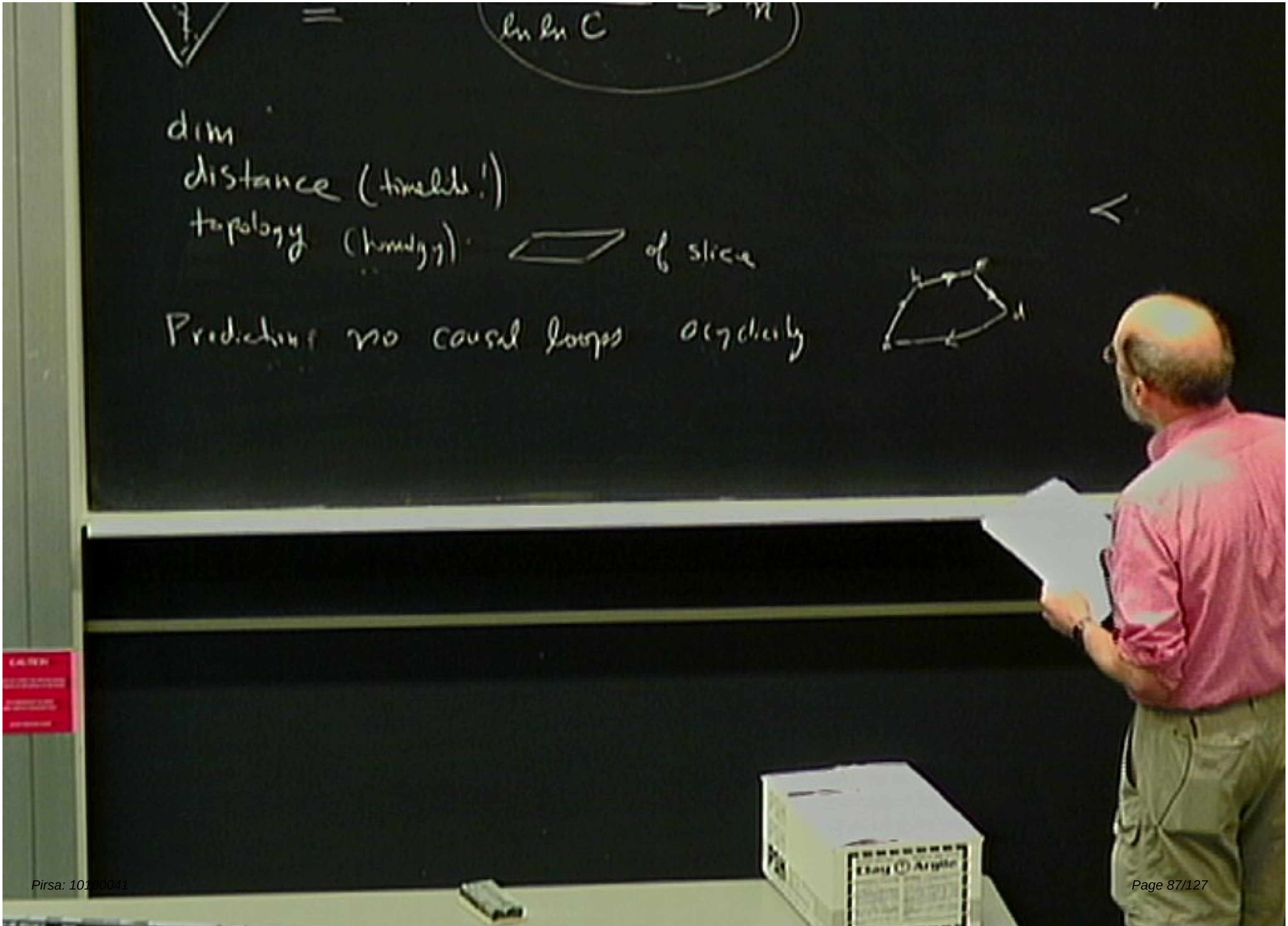
$=$ $\ln \ln C$ $\Rightarrow n$

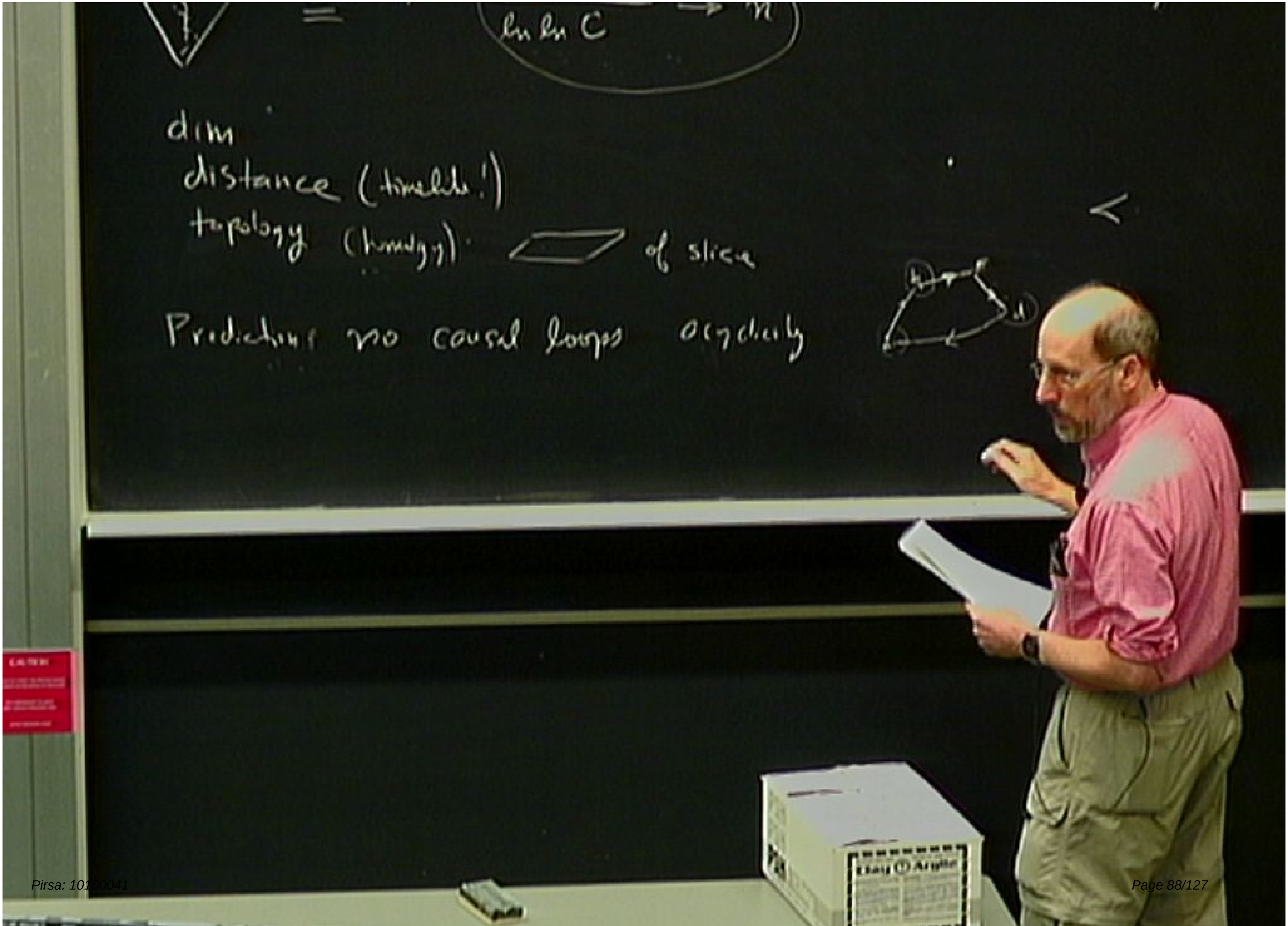
dim

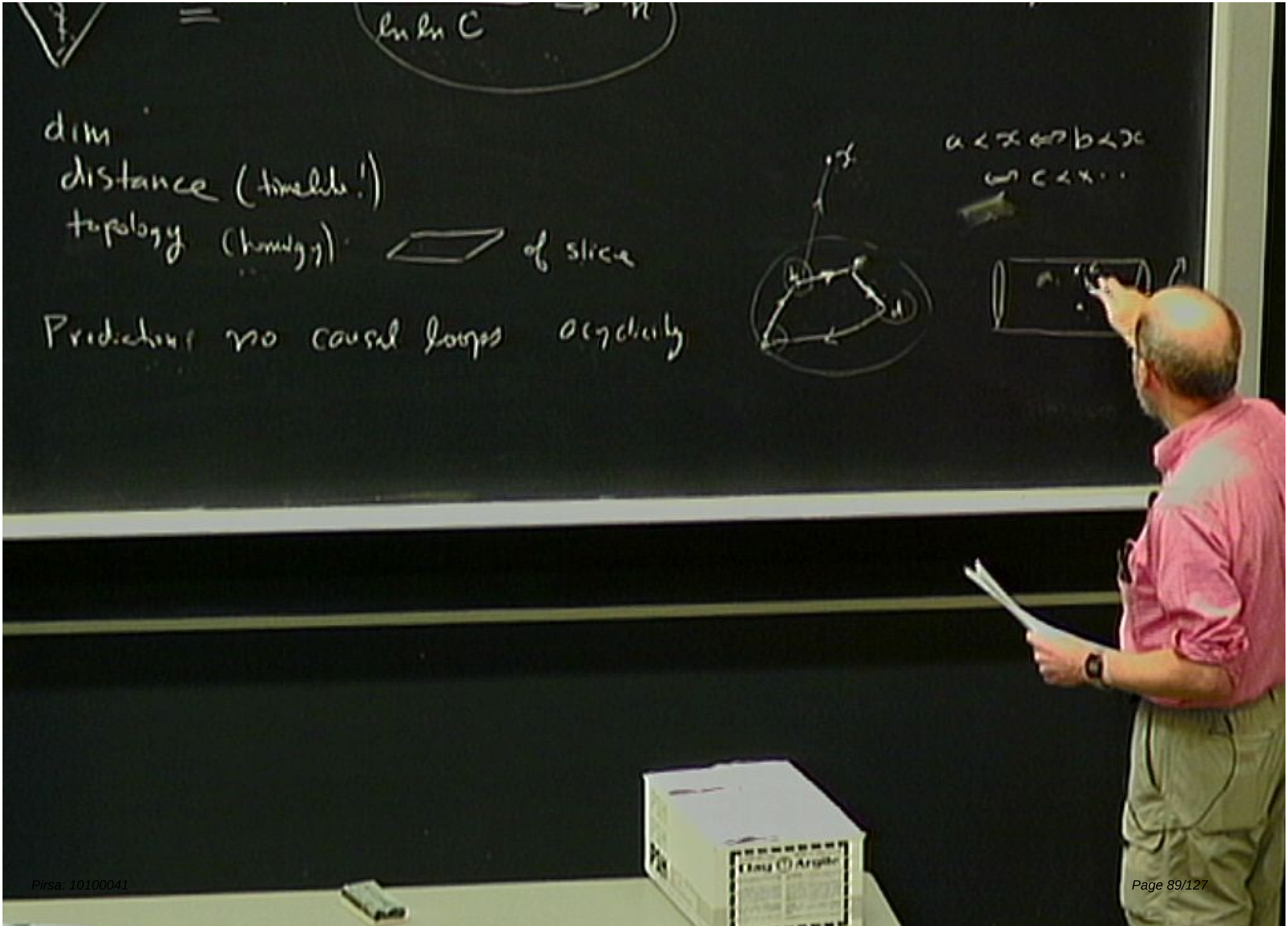
distance (timelike!)

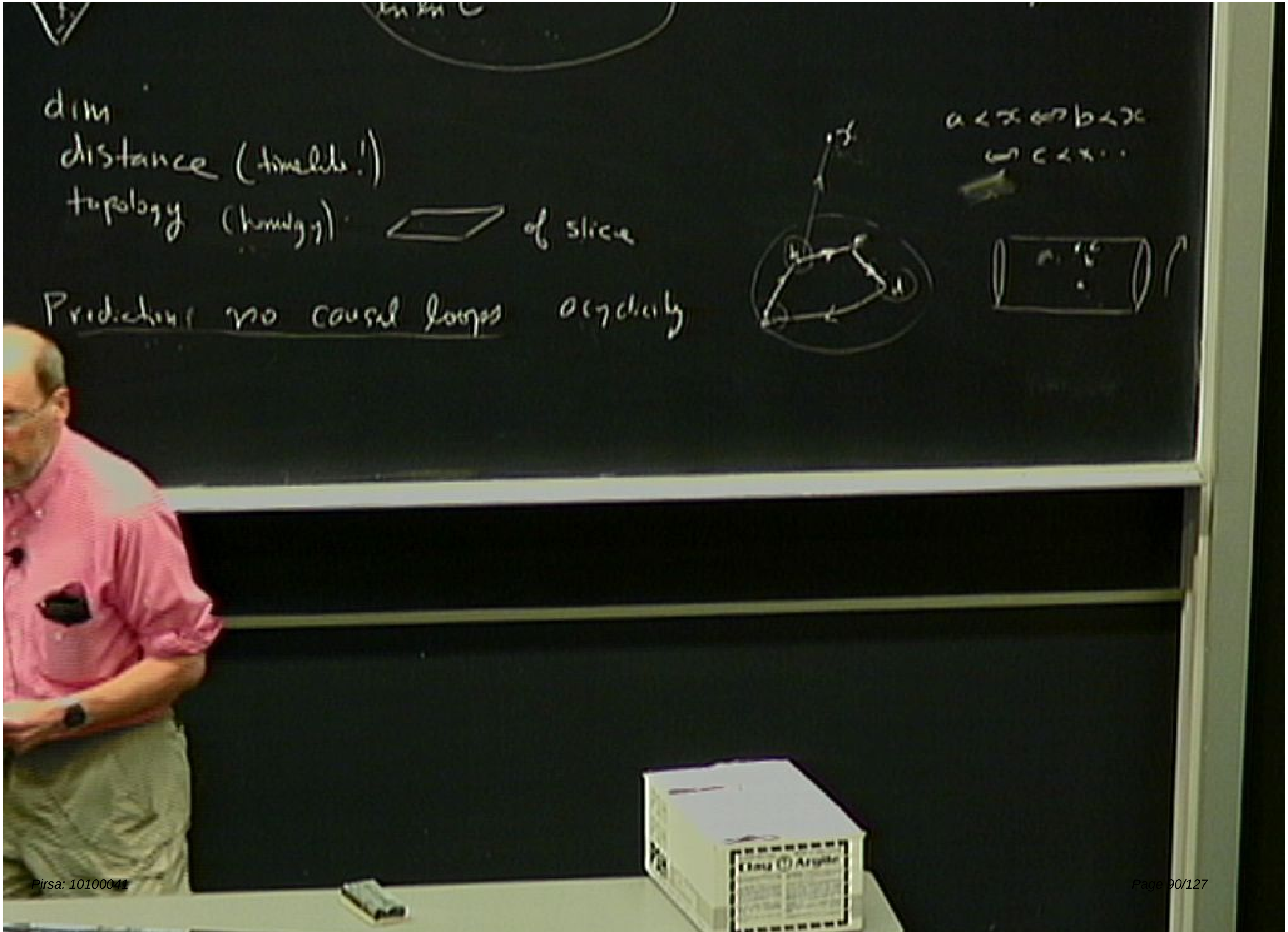
topology (homology) of slice

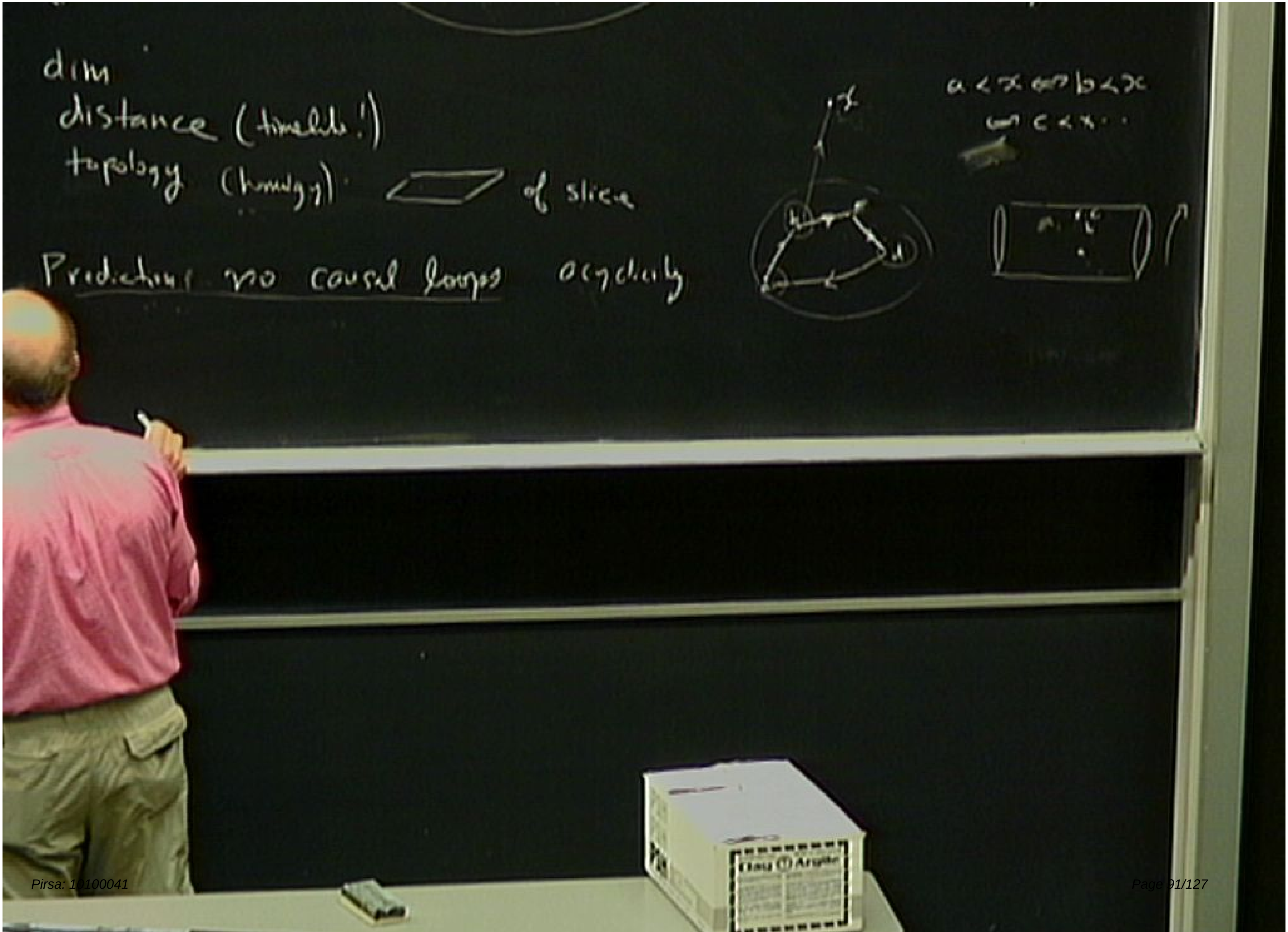
Prediction: no causal loops

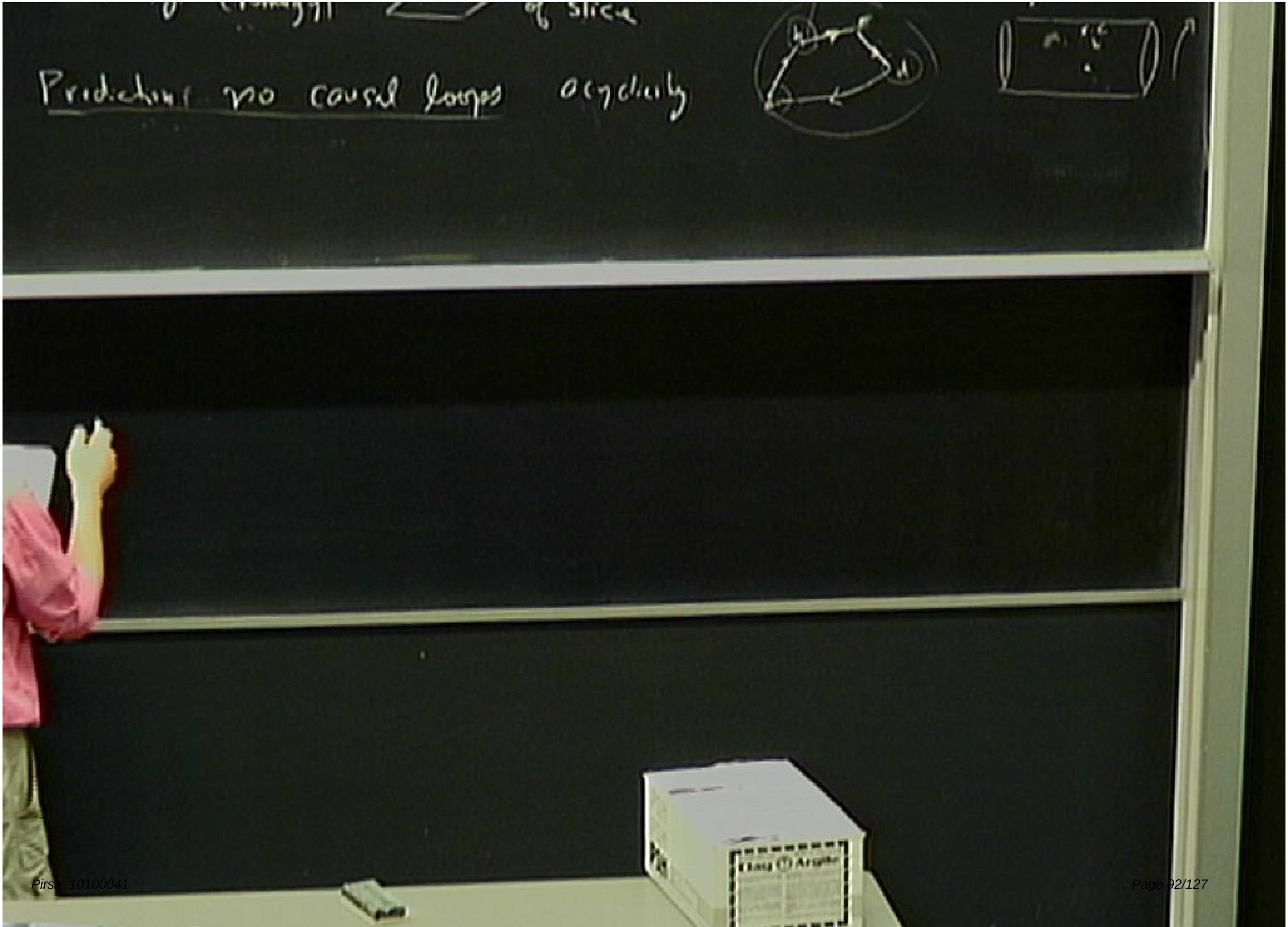

ln ln C
dim
distance (timelike!)
topology (homology)
of slice
Prediction no causal loops acyclicity

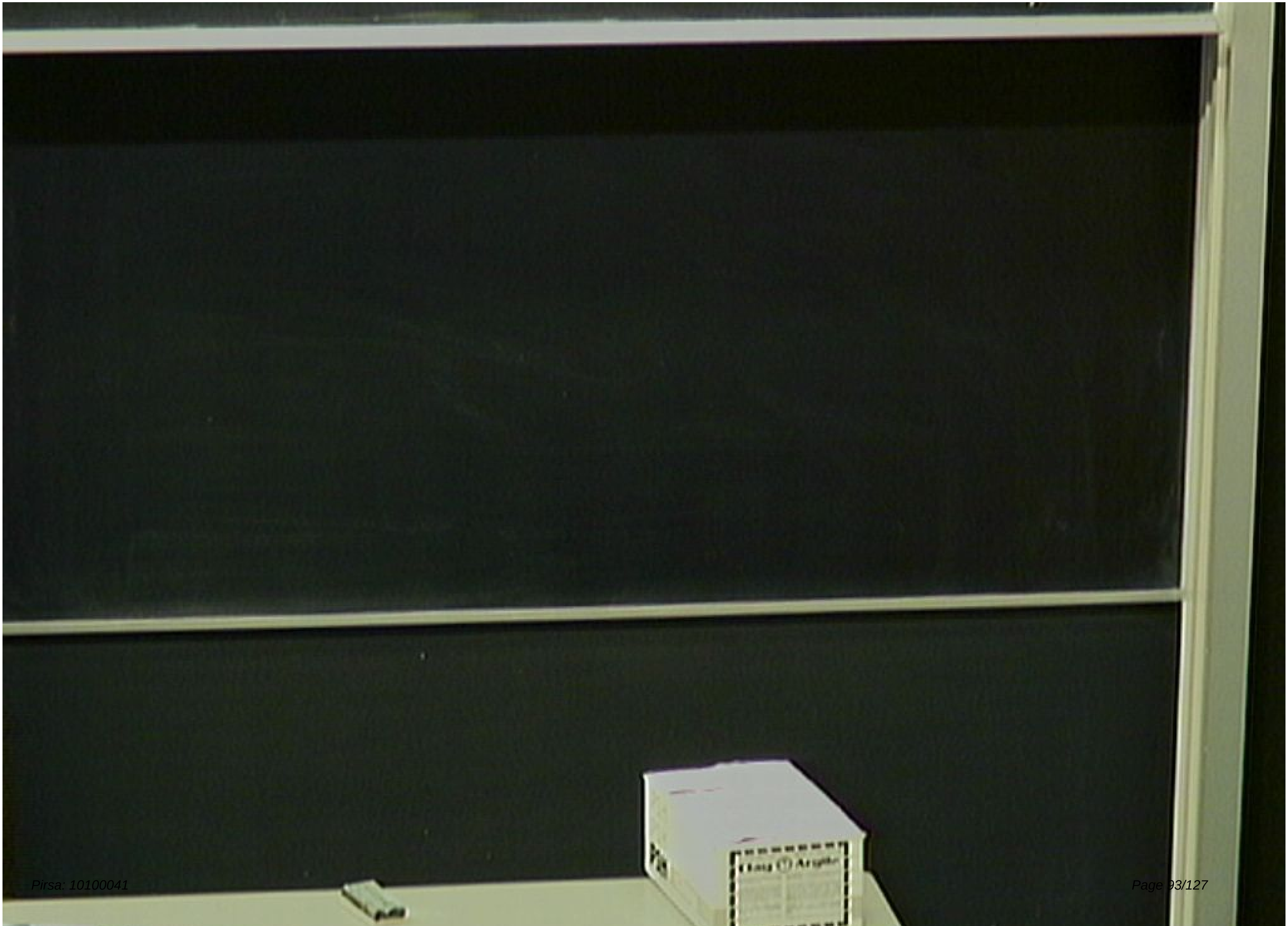

ln ln C
dim
distance (timelike!)
topology (homology)
of slice
Prediction no causal loops acyclicity

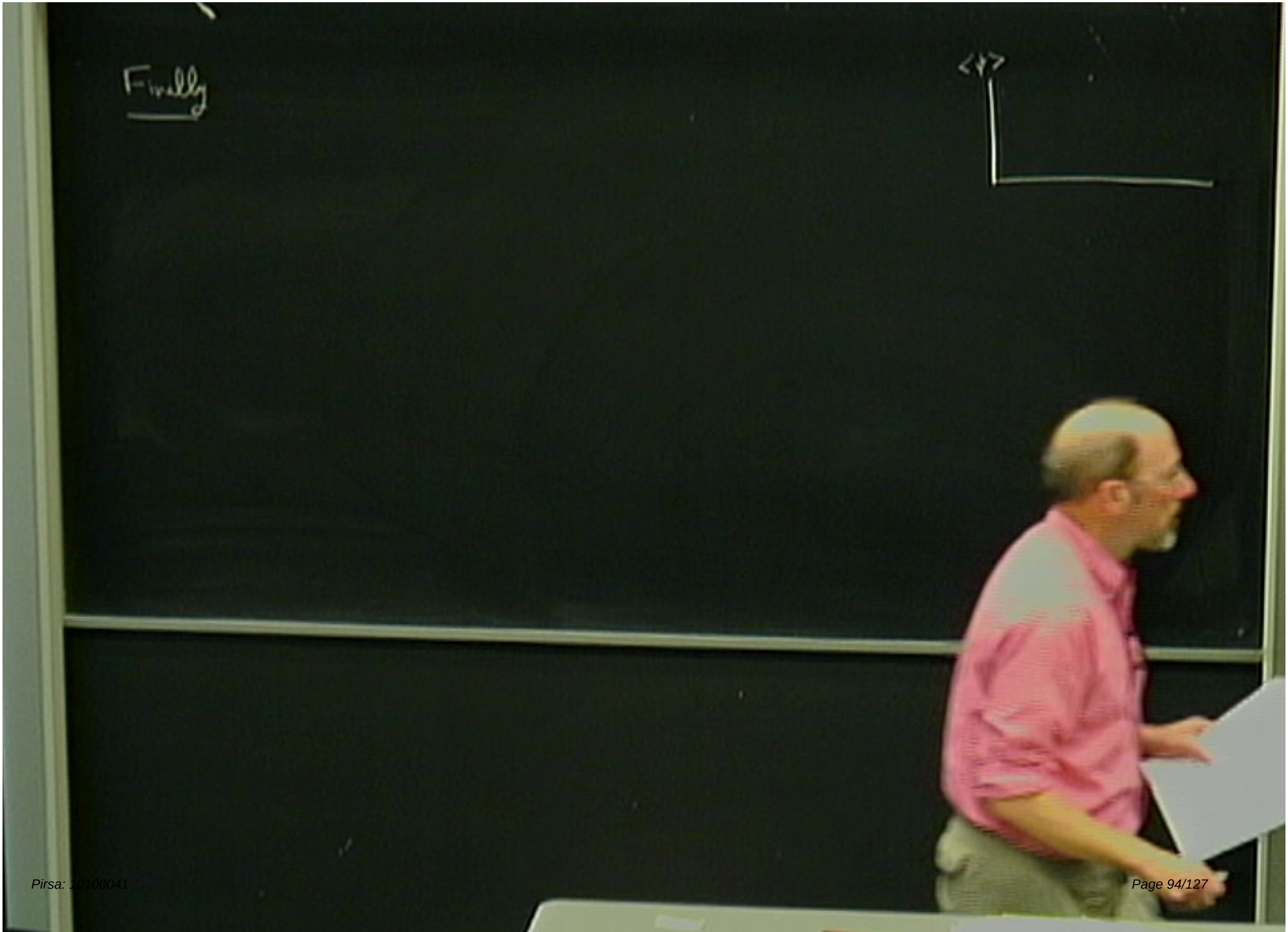

ln ln C
dim
distance (timelike!)
topology (homology)
of slice
Prediction: no causal loops
acyclicity


dim
distance (timelike!)
topology (homology)
of slice
Prediction: no causal loops
acyclicity

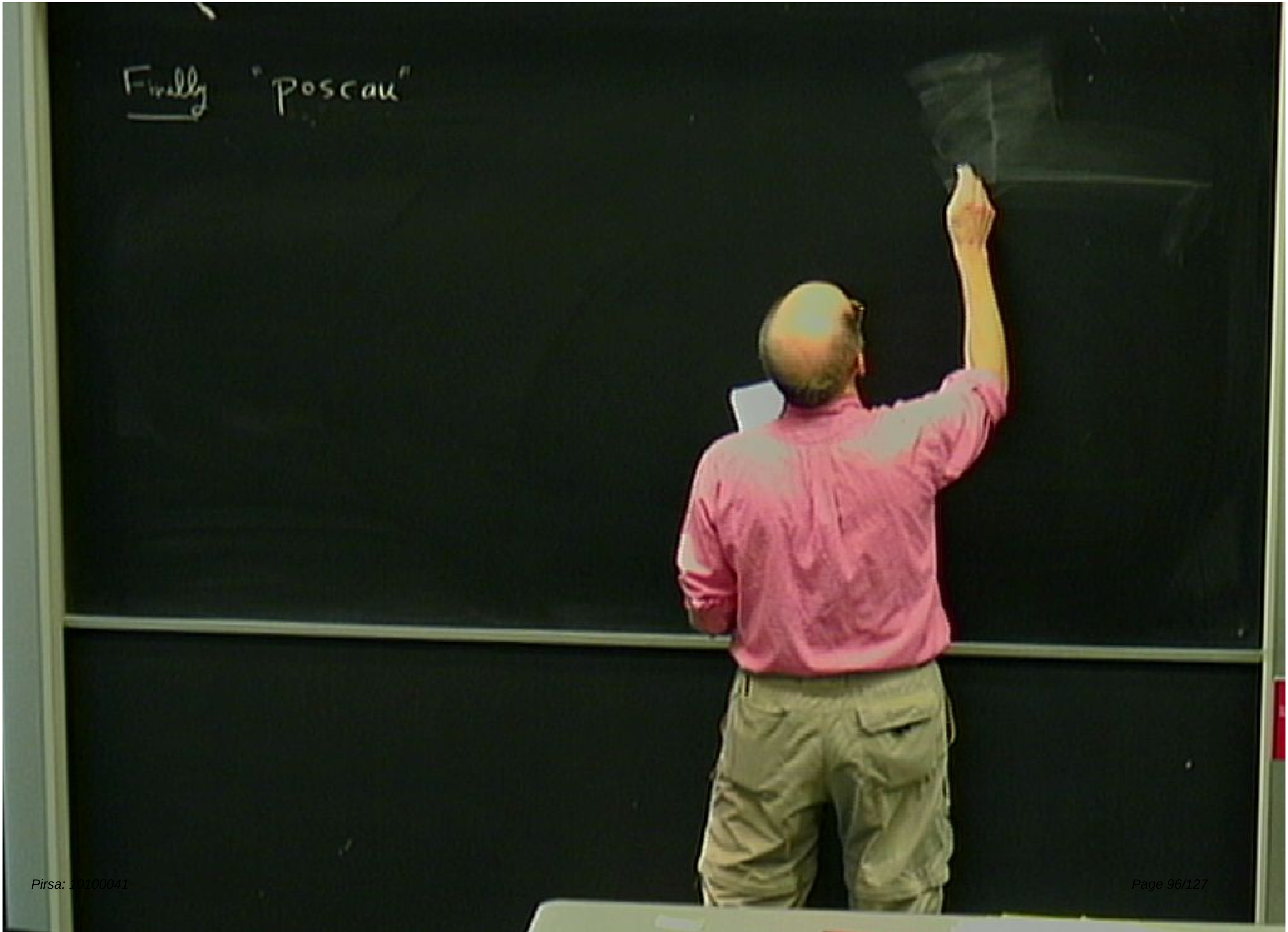

dim
distance (timelike!)
topology (homology)
of slice
Prediction: no causal loops
acyclicity
a < x ⟺ b < x
⟺ c < x ...

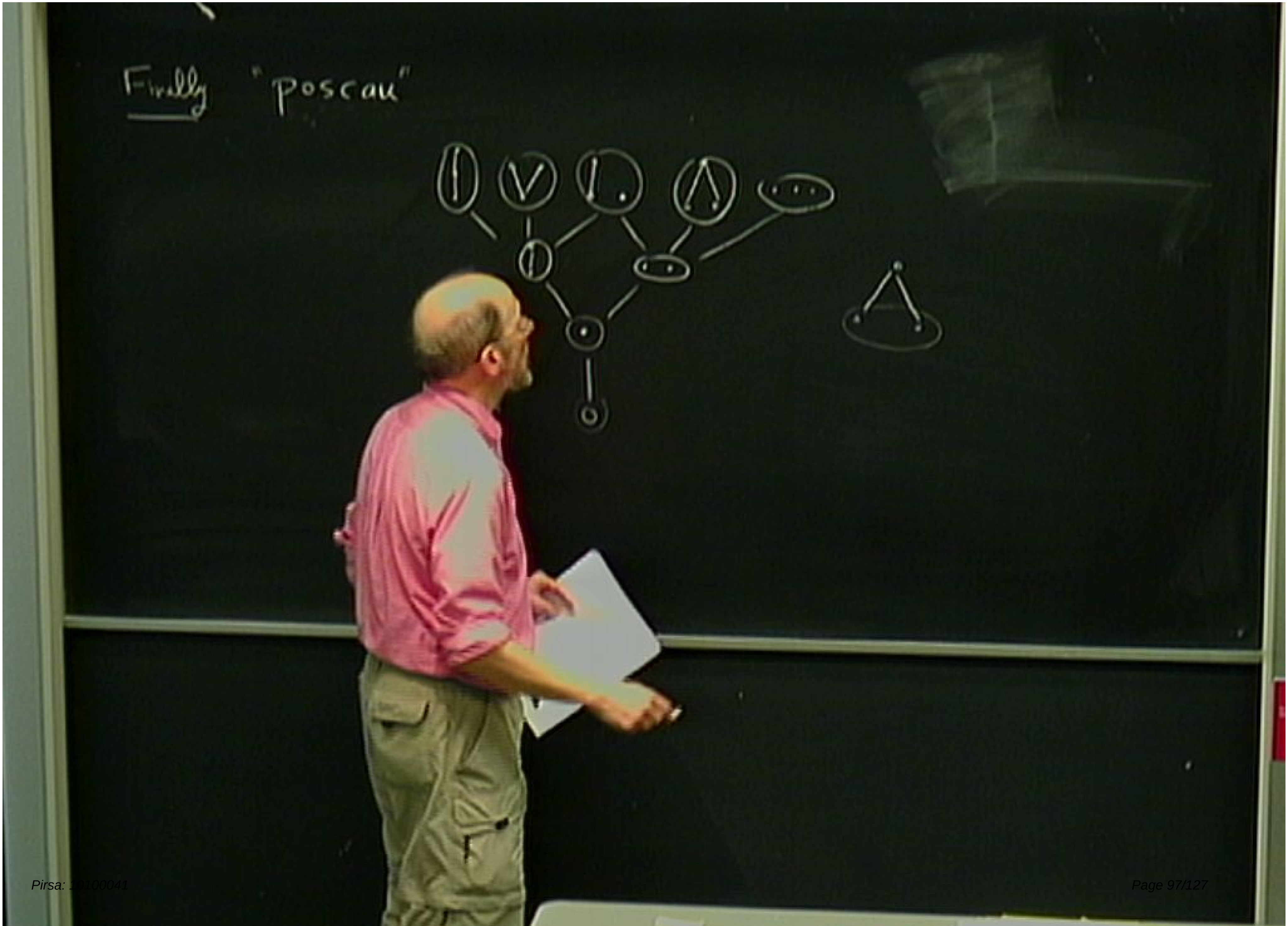

of slice
Prediction: no causal loops
acyclicity

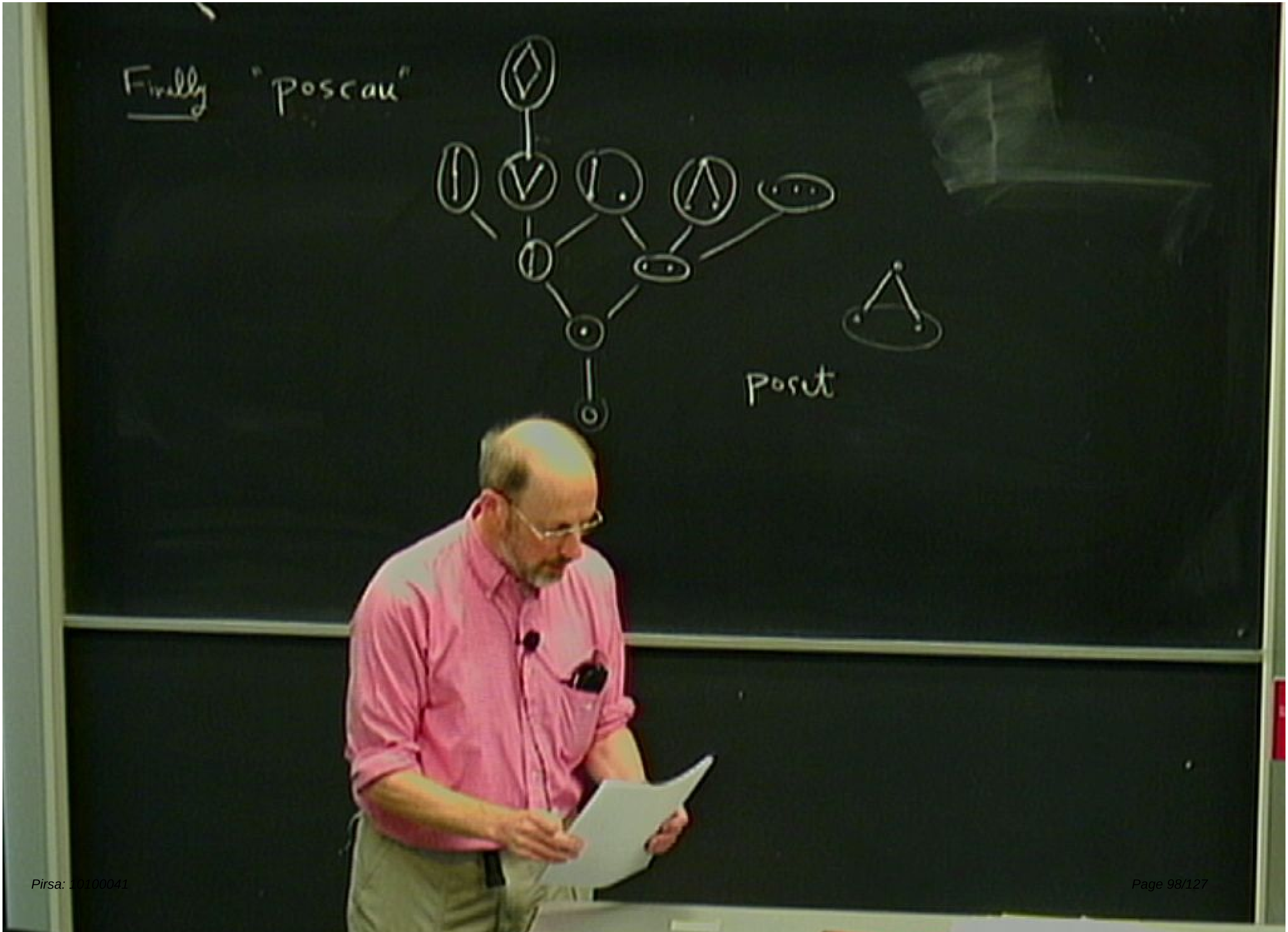


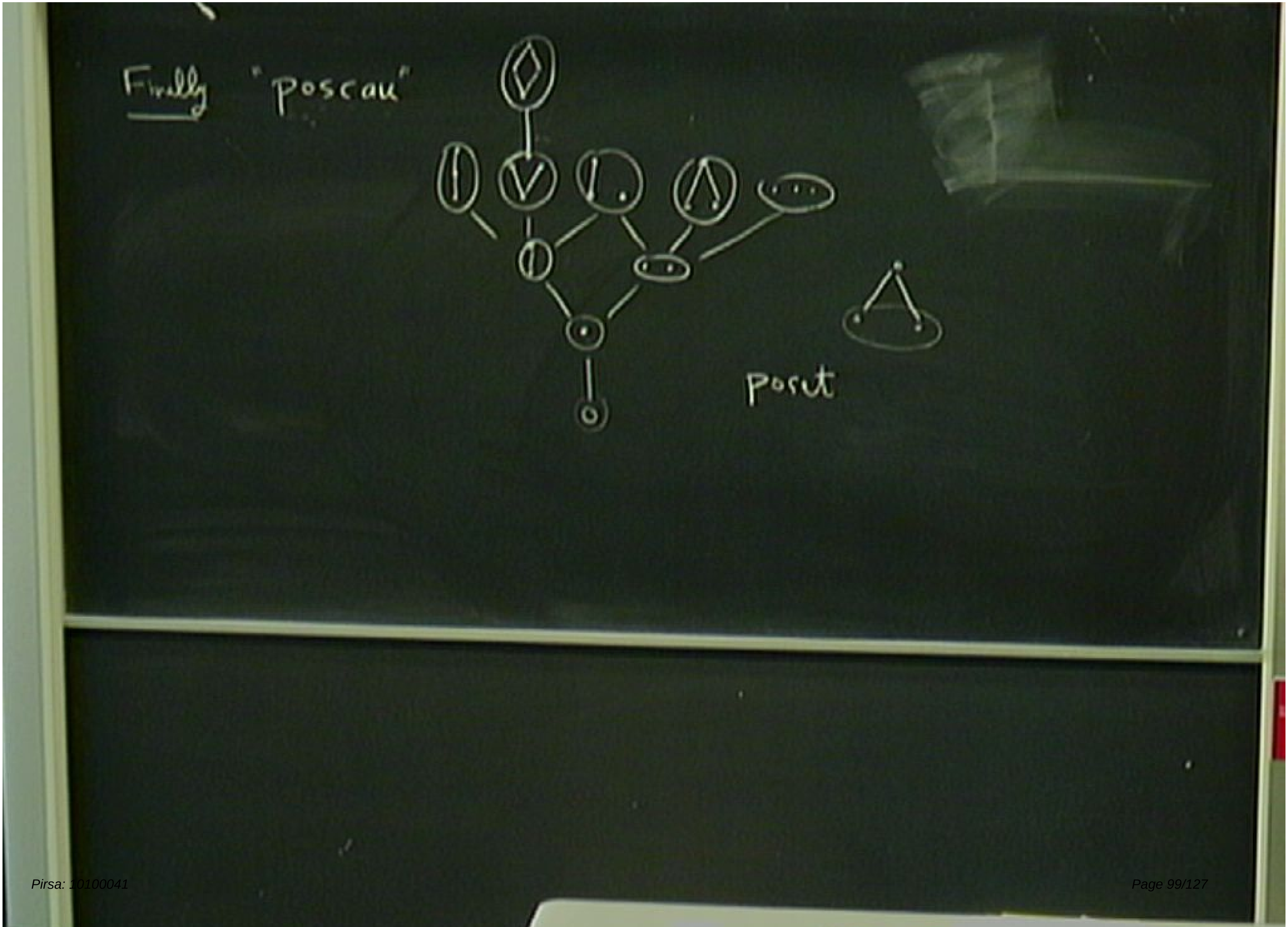

Finally

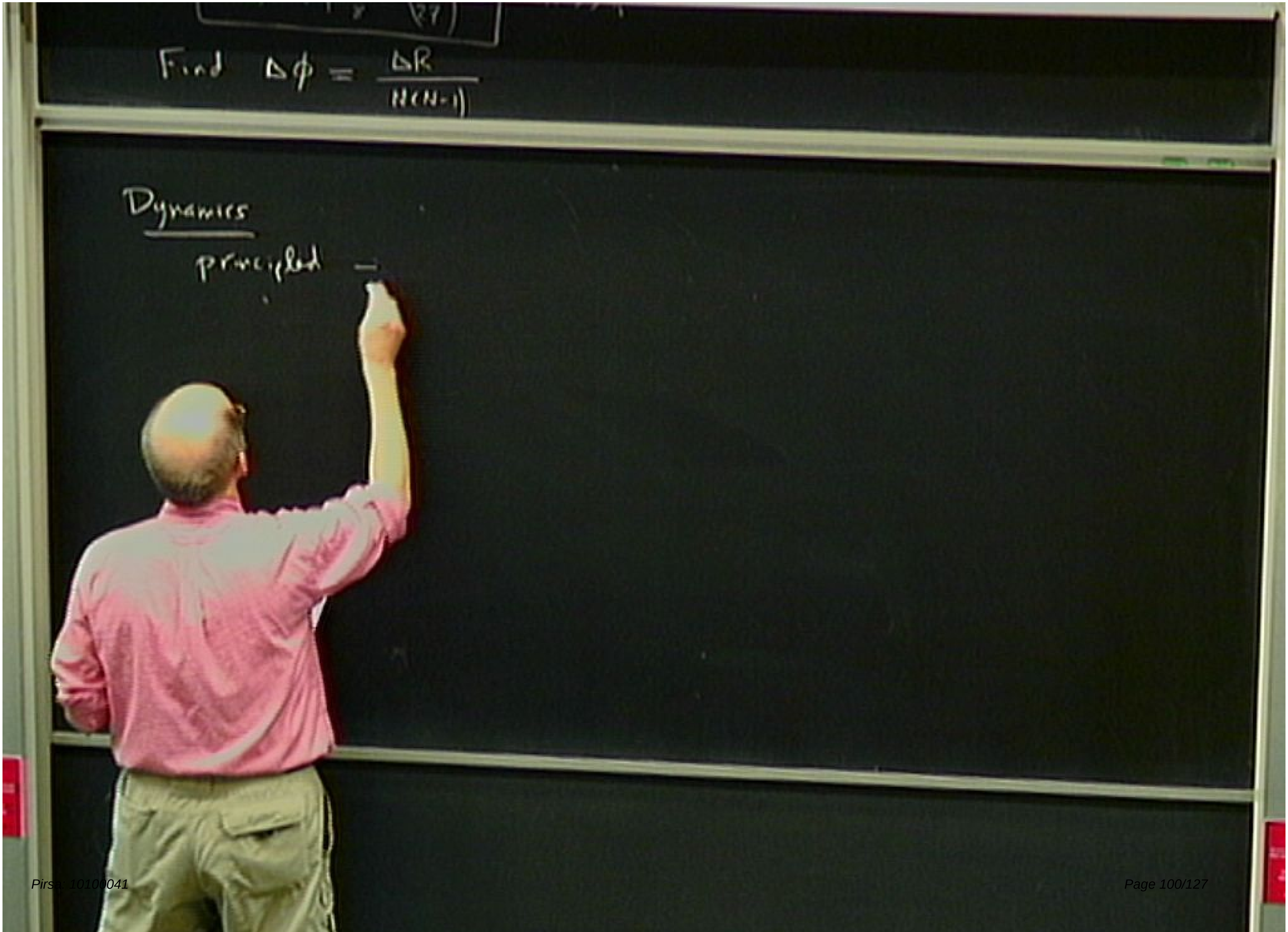
Finally poscau


Finally "poscau"


Finally "poscau"


Finally "poscau"
poset


Finally "poscau"
poset


Find Δφ = ΔR / N(N-1)
Dynamics
principled

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

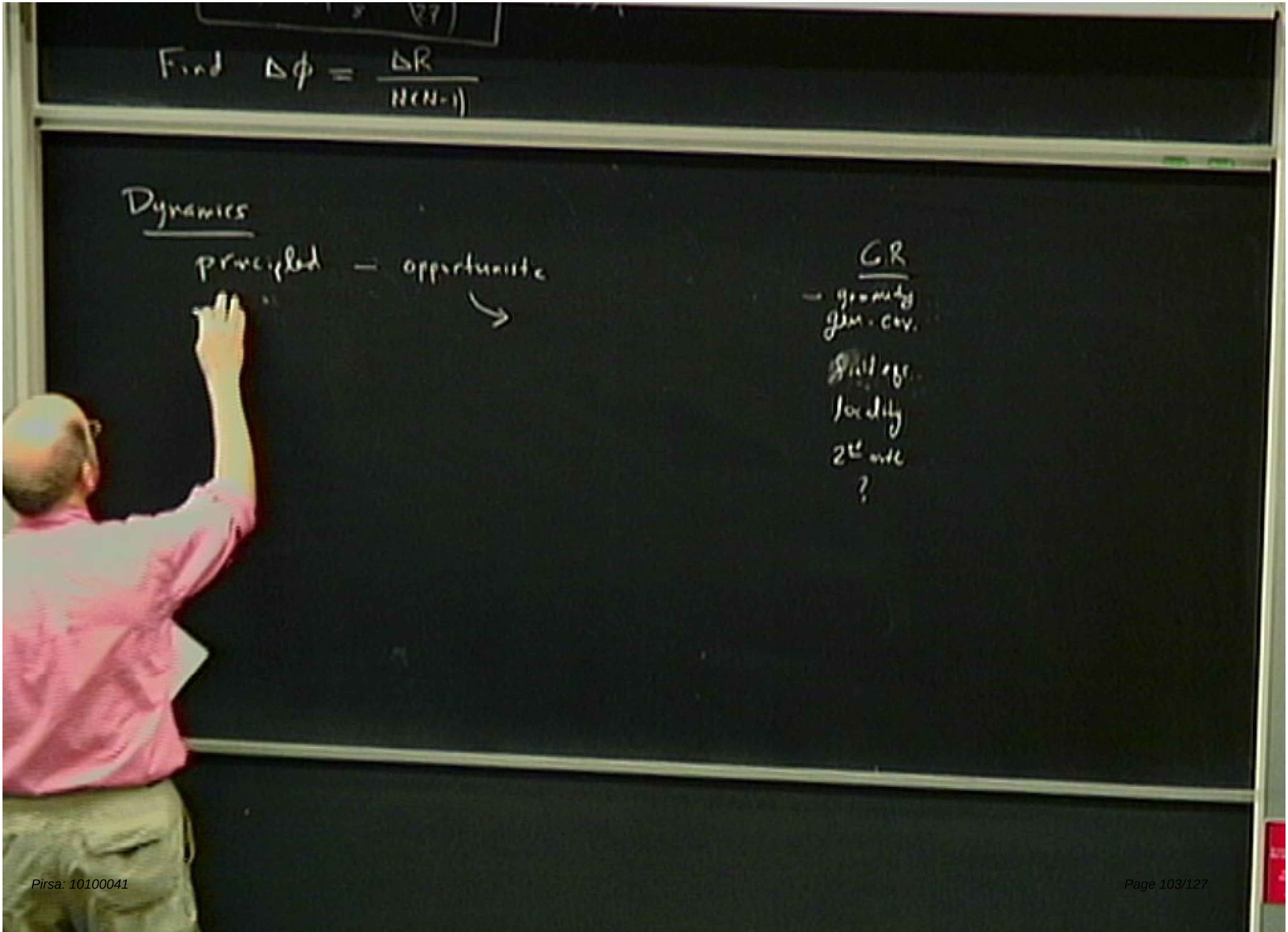
principled — opportunistic

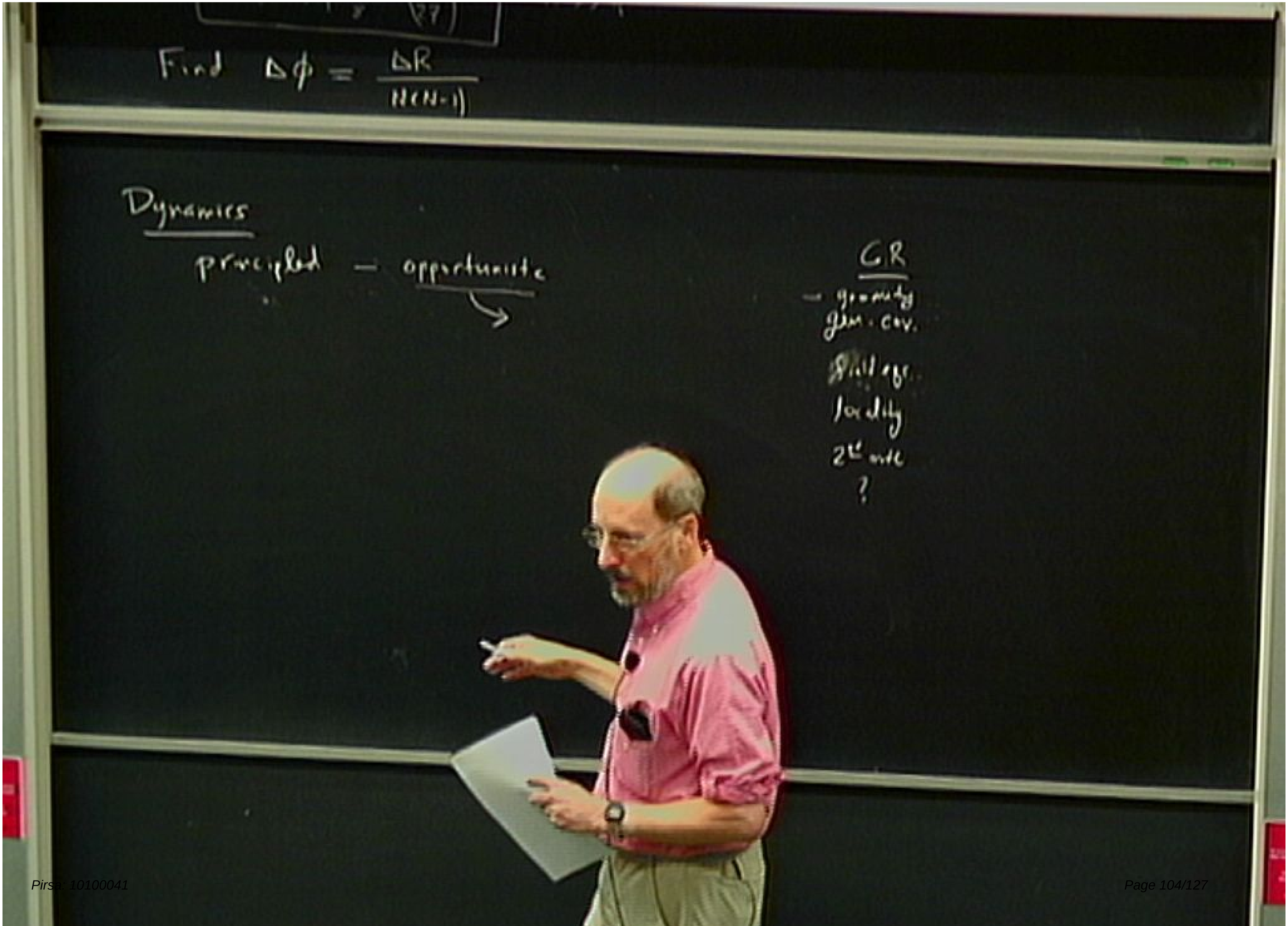
GR

— geometry

gen. cov.

[illegible]


Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$
Dynamics
principled — opportunistic
GR
— geometry
gen. cov.
Diff eqs.
locality
?


Find Δφ = ΔR / N(N-1)
Dynamics
principled — opportunistic
GR
— geometry
gen. cov.
Field eqs.
locality
2nd order
?

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

$\downarrow$

CSG

Fay's

GR

- geometry

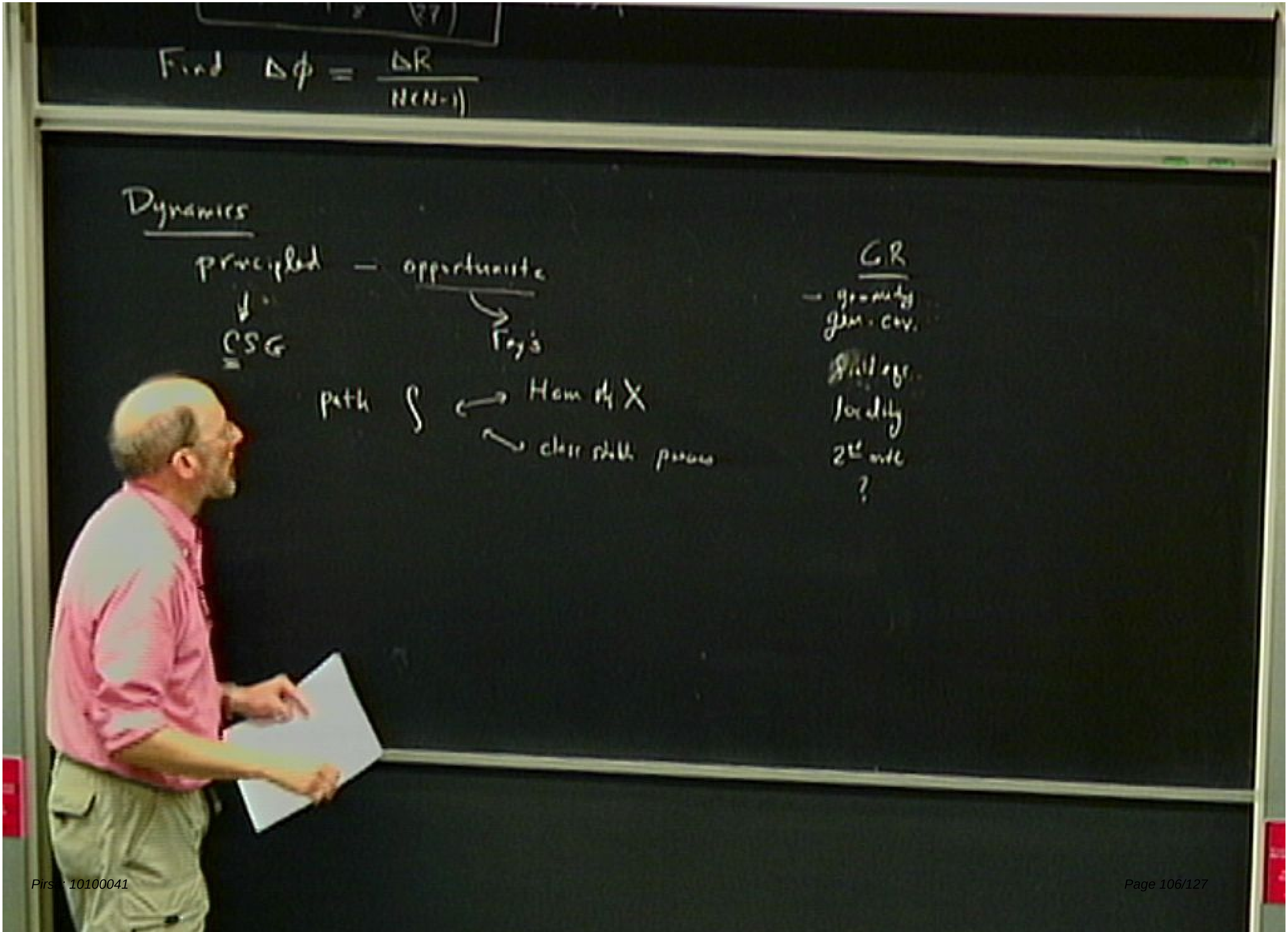
gen. cov.

2nd ... eqs.

locality

2nd order

?


Find Δφ = ΔR / N(N-1)
Dynamics
principled — opportunistic
CSG
Ray's
path ∫
Hom of X
GR
— geometry
gen. cov.
locality
2nd order
?

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

$\downarrow$ CSG

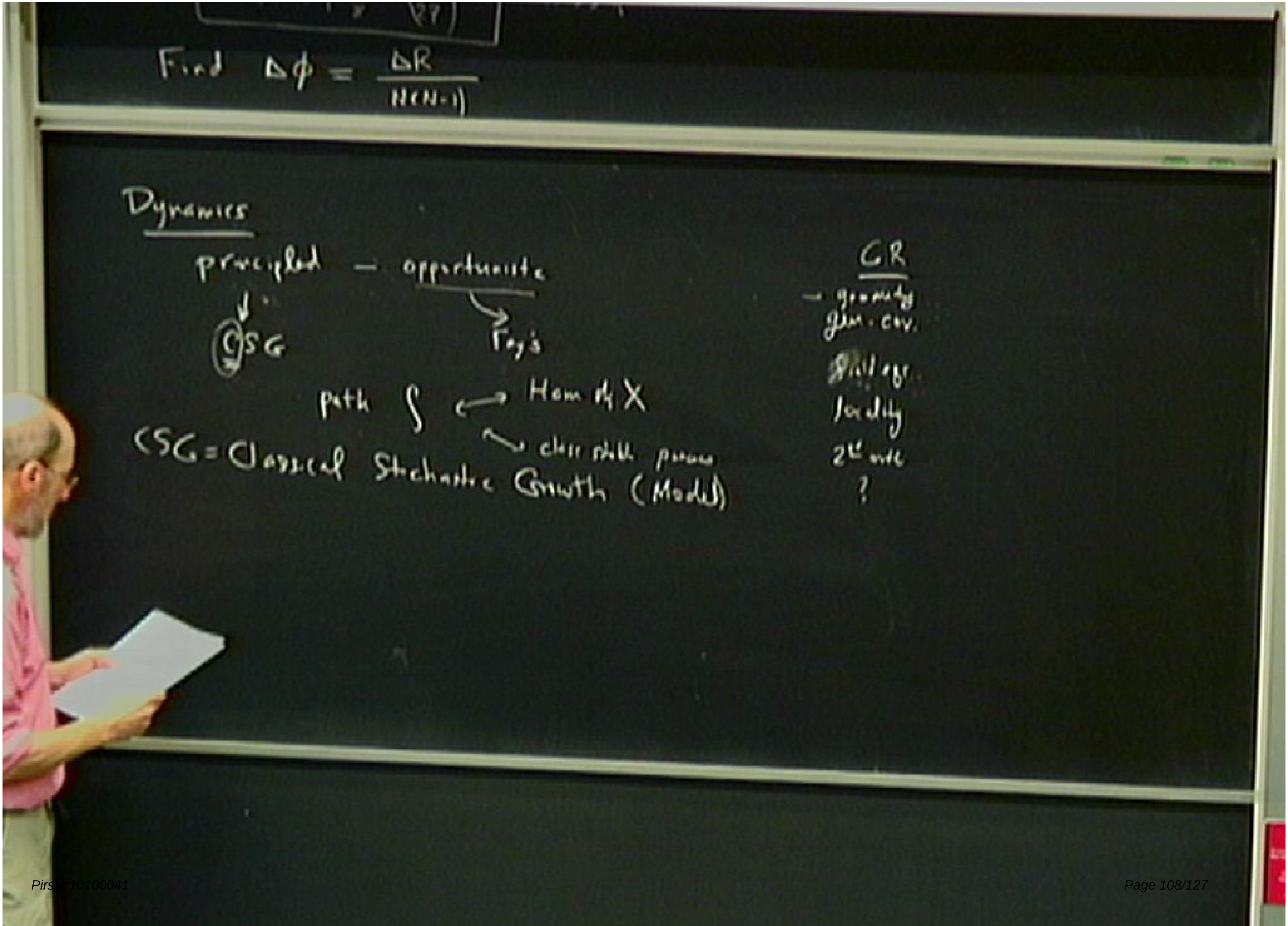
$\to$ Fay's

path $\int$ $\longleftrightarrow$ Hom of X

$\longleftrightarrow$ class stat. [illegible]

GR

- geometry
- gen. cov.
- field eqs.
- locality
- 2nd order
- ?


Find Δφ = ΔR / N(N-1)
Dynamics
principled — opportunistic
CSG
Ray's
path ∫ ⟷ Hom of X
CSG = Classical Stochastic Growth (Model)
GR
geometry
gen. cov.
locality
?

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

CSG

Fay's

path $\int$ $\longleftrightarrow$ Hom of X

$\longleftrightarrow$ class stoch process

CSG = Classical Stochastic Growth (Model)

illuminates

GR

— geometry

gen. cov.

[illegible] eqs.

locality

2nd order

?

Causet

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

↓ CSG   Ray's

path $\int$ ⟷ Hom of X

⟷ class. stoch. process

CSG = Classical Stochastic Growth (Model)

illuminates

• background independent dynamics

GR

— geometry

gen. cov.

field eqs.

locality

2nd order

?

Causets

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

CSG

Ray's

path $\int \longleftrightarrow$ Hom of X

$\longleftrightarrow$ class. stoch. process

CSG = Classical Stochastic Growth (Model)

illuminates

• background independent dynamics (intrinsic)

• " " causality criterion

GR

– geometry

gen. cov.

field eqs.

locality

2nd order

?

Caust

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

CSG

Fay's

path $\int$ ⟷ Ham'n X

⟷ class. stoch. process

CSG = Classical Stochastic Growth (Model)

illuminates

- background independent dynamics (intrinsic)
- " " causality criterion BC

GR

- geometry
- gen. cov.
- field eqs.
- locality
- 2nd order
- ?

Causets

Bell Causality

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

$\downarrow$

CSG

Ray's

path $\int \longleftrightarrow$ Hom of X

$\longleftrightarrow$ class. stoch. process

CSG = Classical Stochastic Growth (Model)

illuminate

- background independent dynamics (intrinsic)
- " " causality criterion BC
- overcomes "entropic catastrophe"

GR

— geometry

gen. cov.

field eqs.

locality

2nd order

?

Causets

Bell Causality

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

CSG — Fay's

path $\int$ ⟷ Ham of X

⟷ class stoch process

CSG = Classical Stochastic Growth (Model)

illustrates

background independent dynamics (intrinsic)

" " causality criterion BS

overcomes "entropic catastrophe" thanks to nonlocality

GR

- geometry

gen. cov.

[illegible]

locality

2nd order

?

Causets

Bell Causality

$2^{\frac{N^2}{4}}$ vs e^N

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled – opportunistic

CSG

Ray's

path $\int$ ⟷ Hom of X

⟷ class. stat. phys.

CSG = Classical Stochastic Growth (Model)

illuminates

- background independent dynamics (intrinsic)
- " " causality criterion BC
- overcomes "entropic catastrophe" – thanks to nonlocality

related Λ

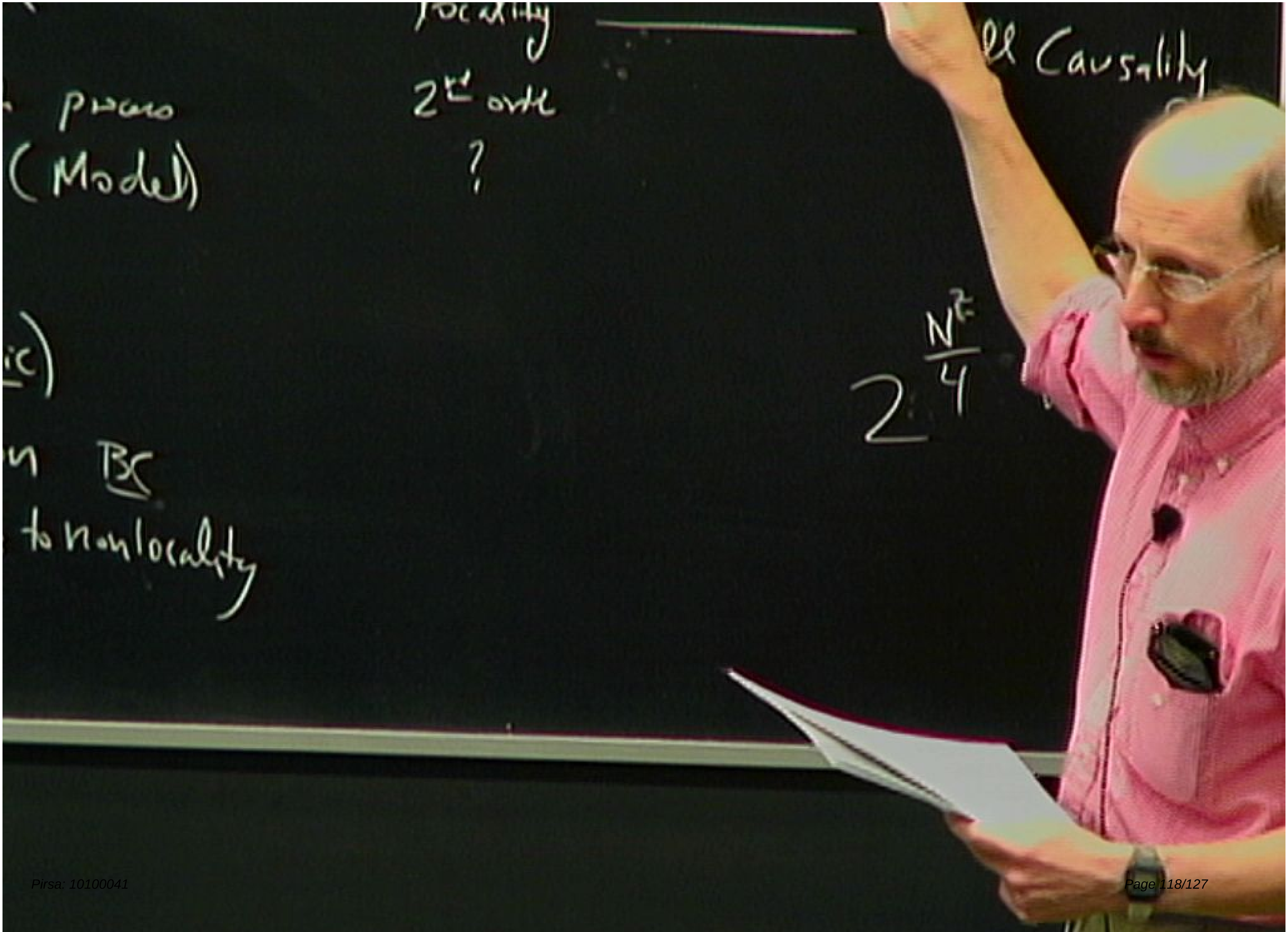
GR — Causets

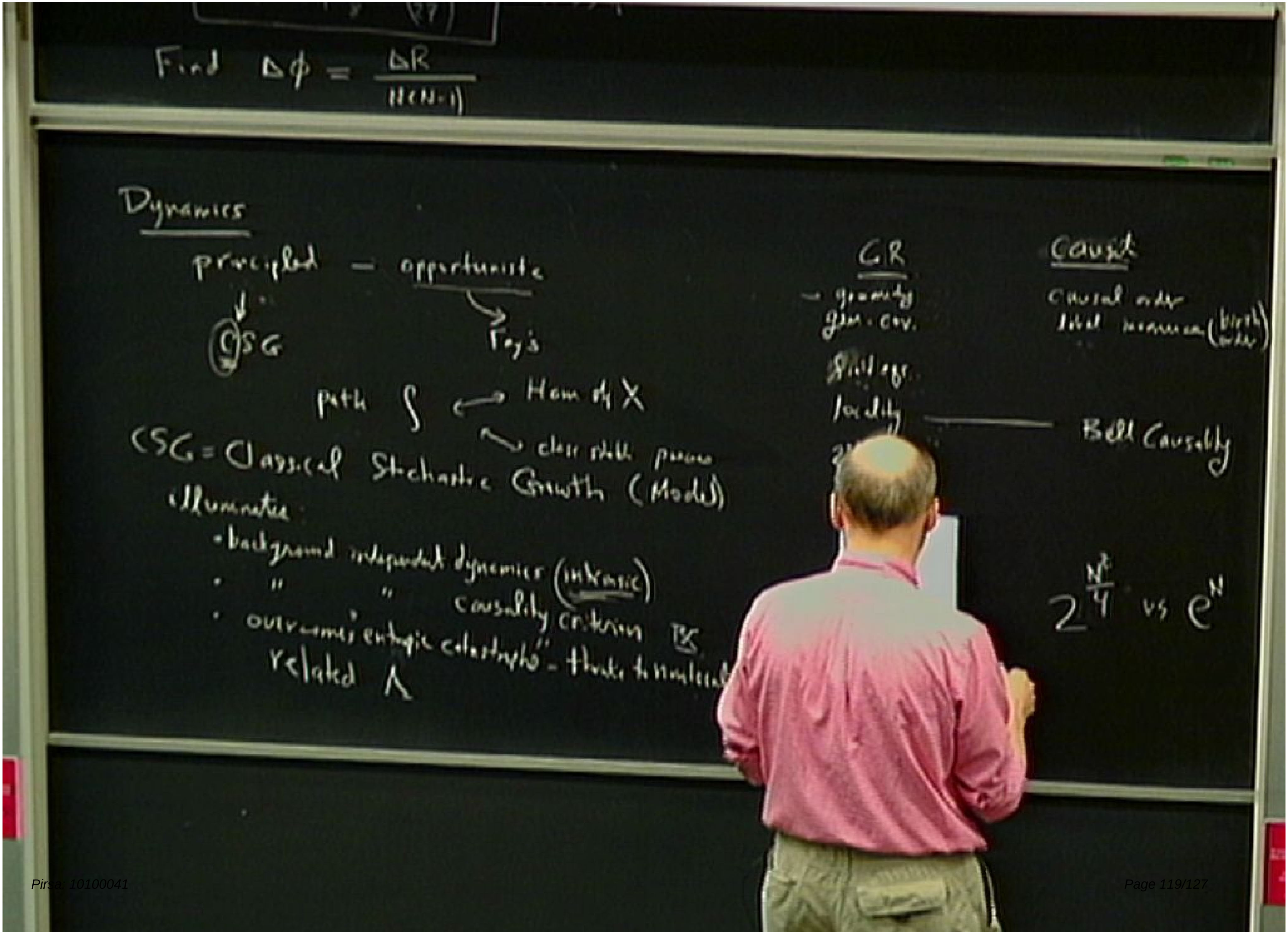
– geometry
gen. cov.
diff eqs.
locality — Bell Causality
2nd order
?

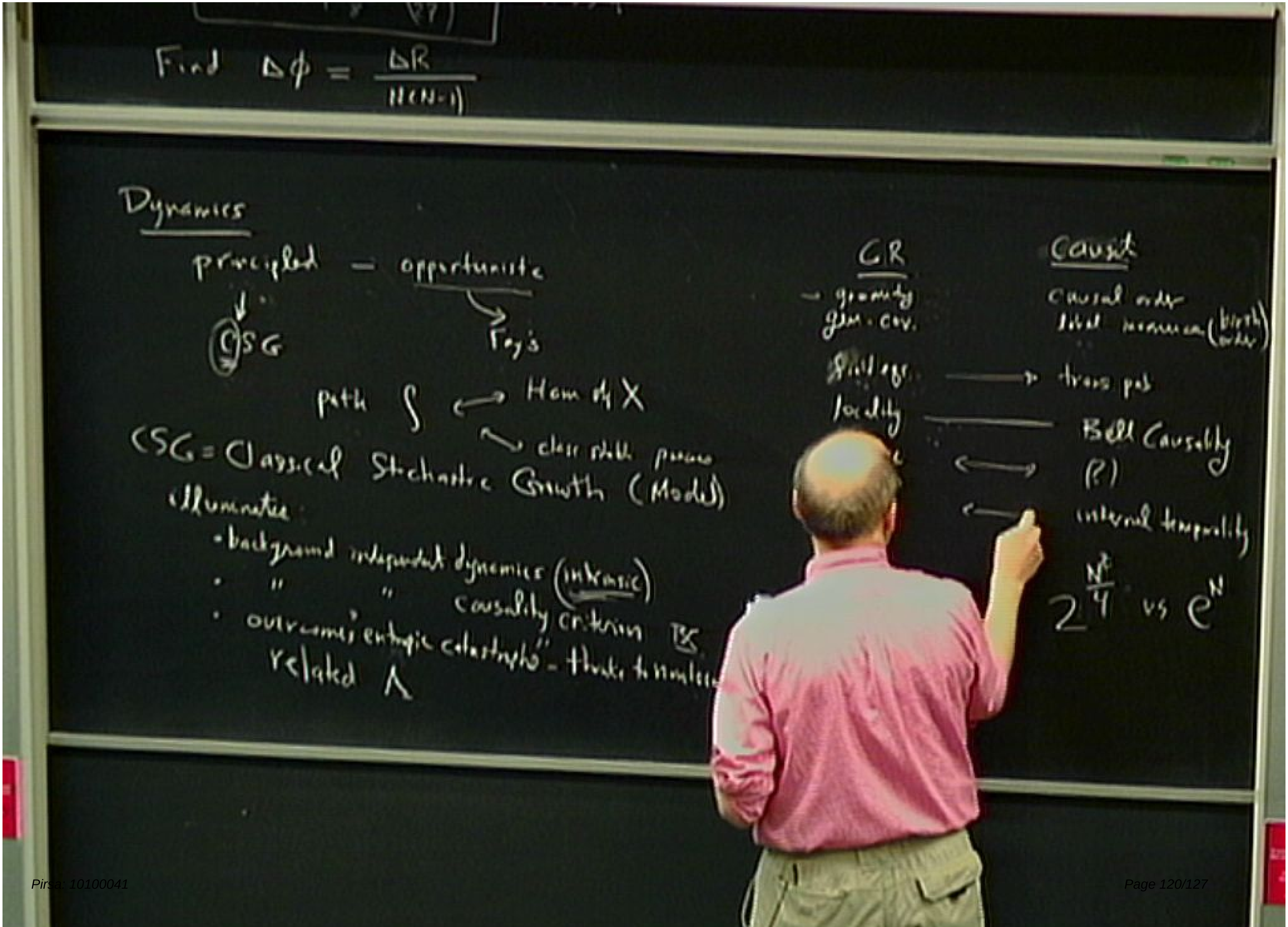
$2^{\frac{N^2}{4}}$ vs e^N

- enticing cosmological scenario
    - cosmic RG (line of fixed points)
    - cyclicing cosmos
- dynamics as <u>growth</u>
- why

- emerging cosmological scenario
  - cosmic RG (line of fixed points)
  - recycling cosmos
- dynamics as <u>growth</u>
- ~~why~~ solve problem of time
- "asynchronous becoming" (things happen)


Causality
2nd order
?
(Model)
BC
to nonlocality
$2^{\frac{N^2}{4}}$


Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$
Dynamics
principled — opportunistic
CSG
path ∫ ⟷ Ham dy X
CSG = Classical Stochastic Growth (Model)
illuminates
• background independent dynamics (intrinsic)
• " " causality criterion
• outcomes' entropic catastrophe
related Λ
GR
geometry
gen. cov.
locality
Causets
causal order
Bell Causality
$2^{\frac{N^2}{4}}$ vs e^N


Find Δφ = ΔR / N(N-1)
Dynamics
principled — opportunistic
CSG
path ∫ ⟷ Hom of X
CSG = Classical Stochastic Growth (Model)
illuminates
• background independent dynamics (intrinsic)
• " " causality criterion
related Λ
GR
gravity
gen. cov.
locality
Causets
causal order
trans pos
Bell Causality
(?)
internal temporality
2^(N²/4) vs e^N

Find $\Delta\phi = \frac{\Delta R}{N(N-1)}$

Dynamics

principled — opportunistic

CSG

Frey's

path $\int \longleftrightarrow$ Hom of X

clas stats phase

CSG = Classical Stochastic Growth (Model)

illuminates

- background independent dynamics (intrinsic)
- " " causality criterion BC
- outcomes' entropic catastrophe" – thanks to nonlocality

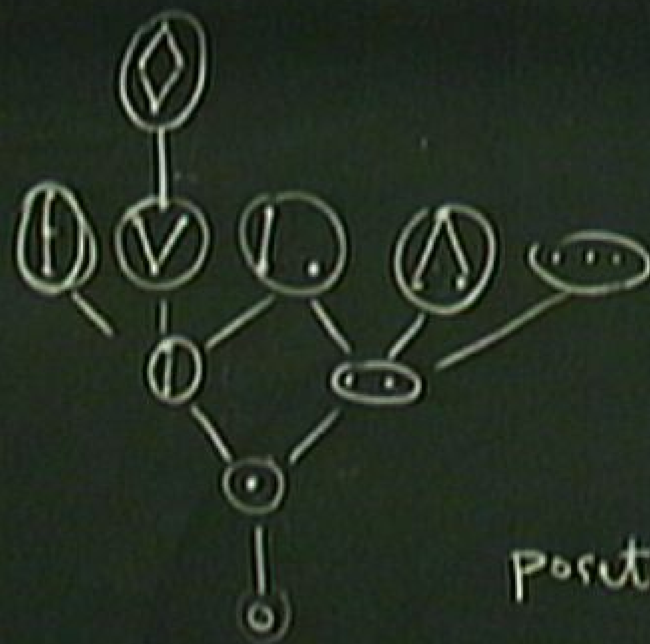
related Λ

GR — Causets

- → geometry — causal order
- gen. cov. — label invariance (birth order)
- [illegible] ⟶ trans pos
- locality ⟶ Bell Causality
- 2nd order ⟷ (?)
- ? ⟷ internal temporality

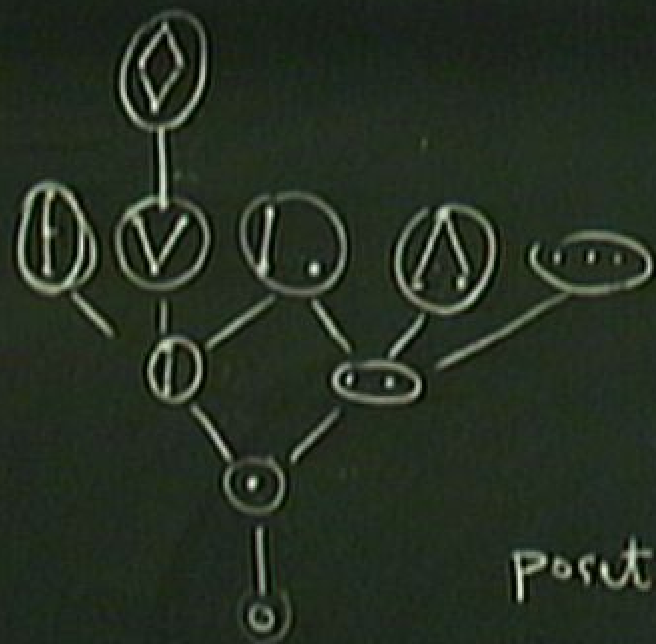
$2^{\frac{N^2}{4}}$ vs e^N

Finally "poscau"

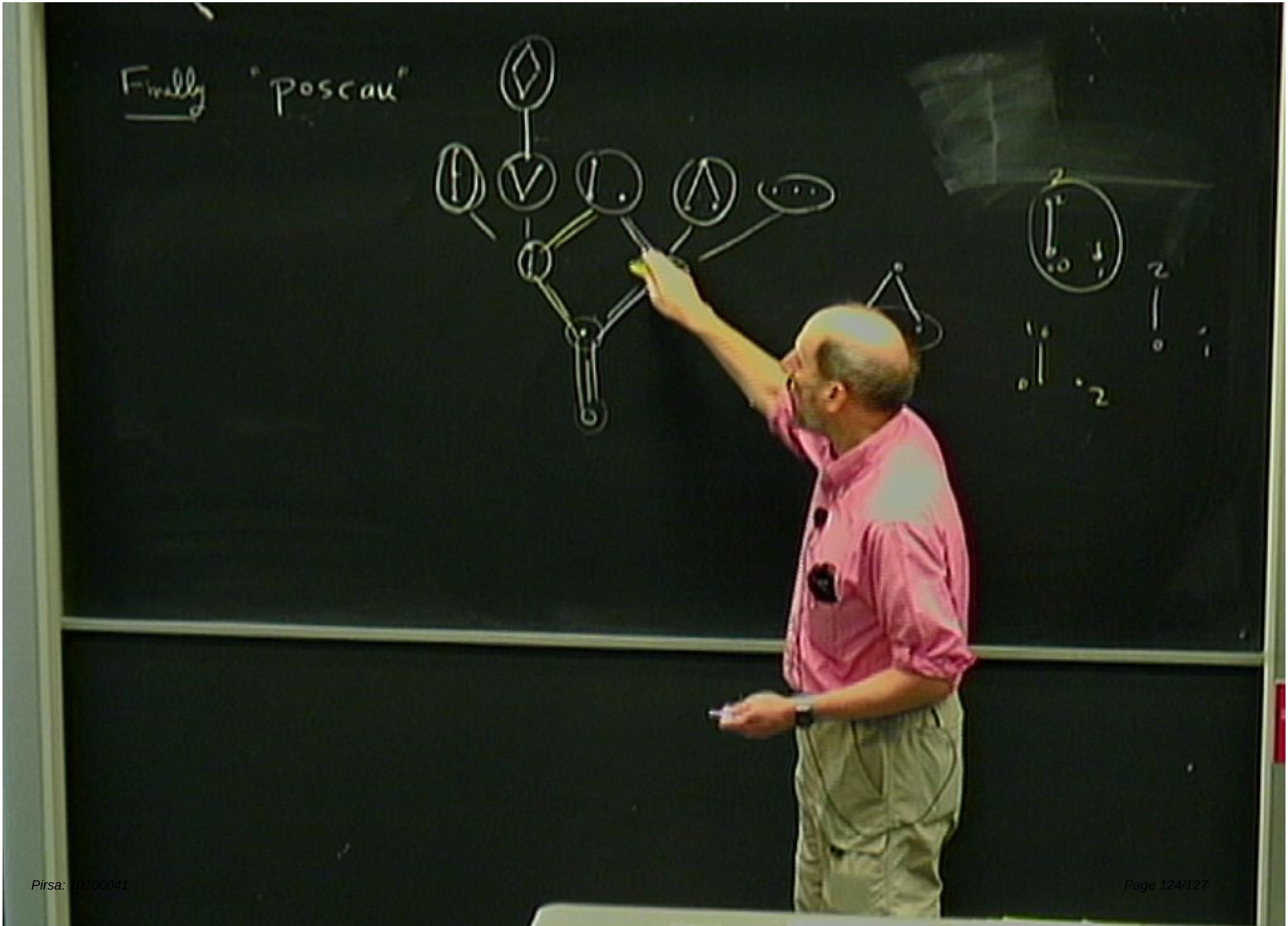


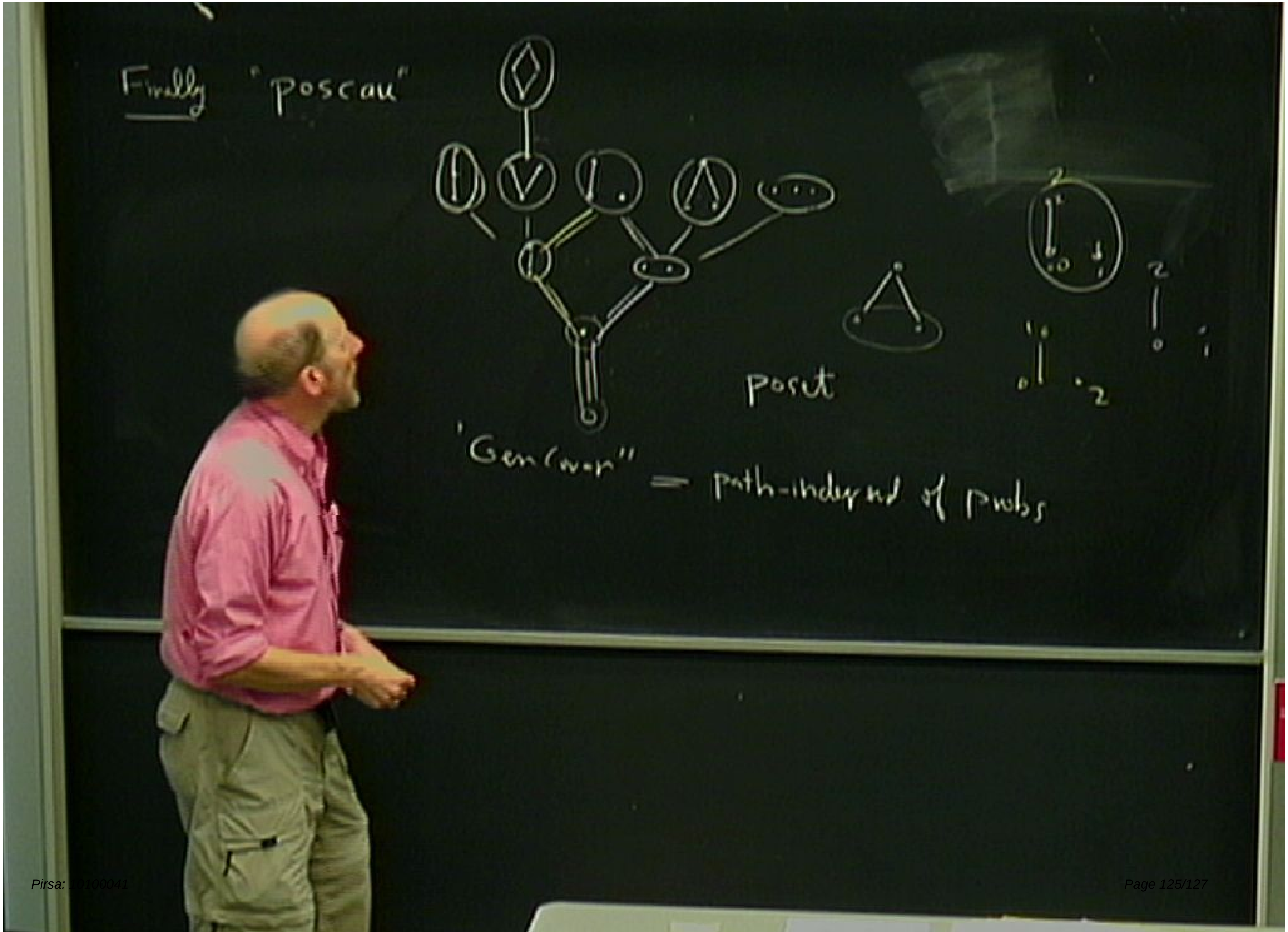
poset

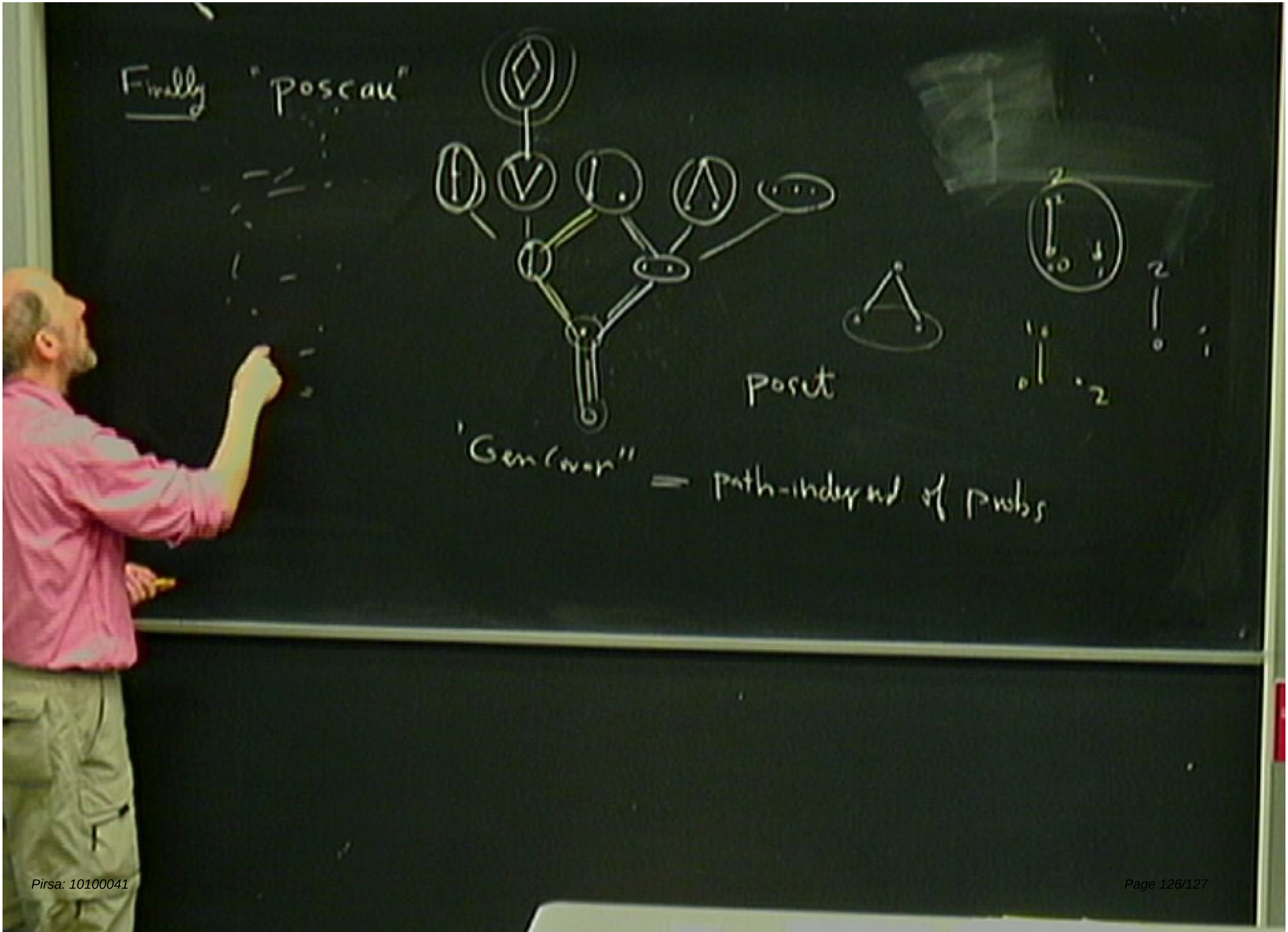
Finally "poscau"

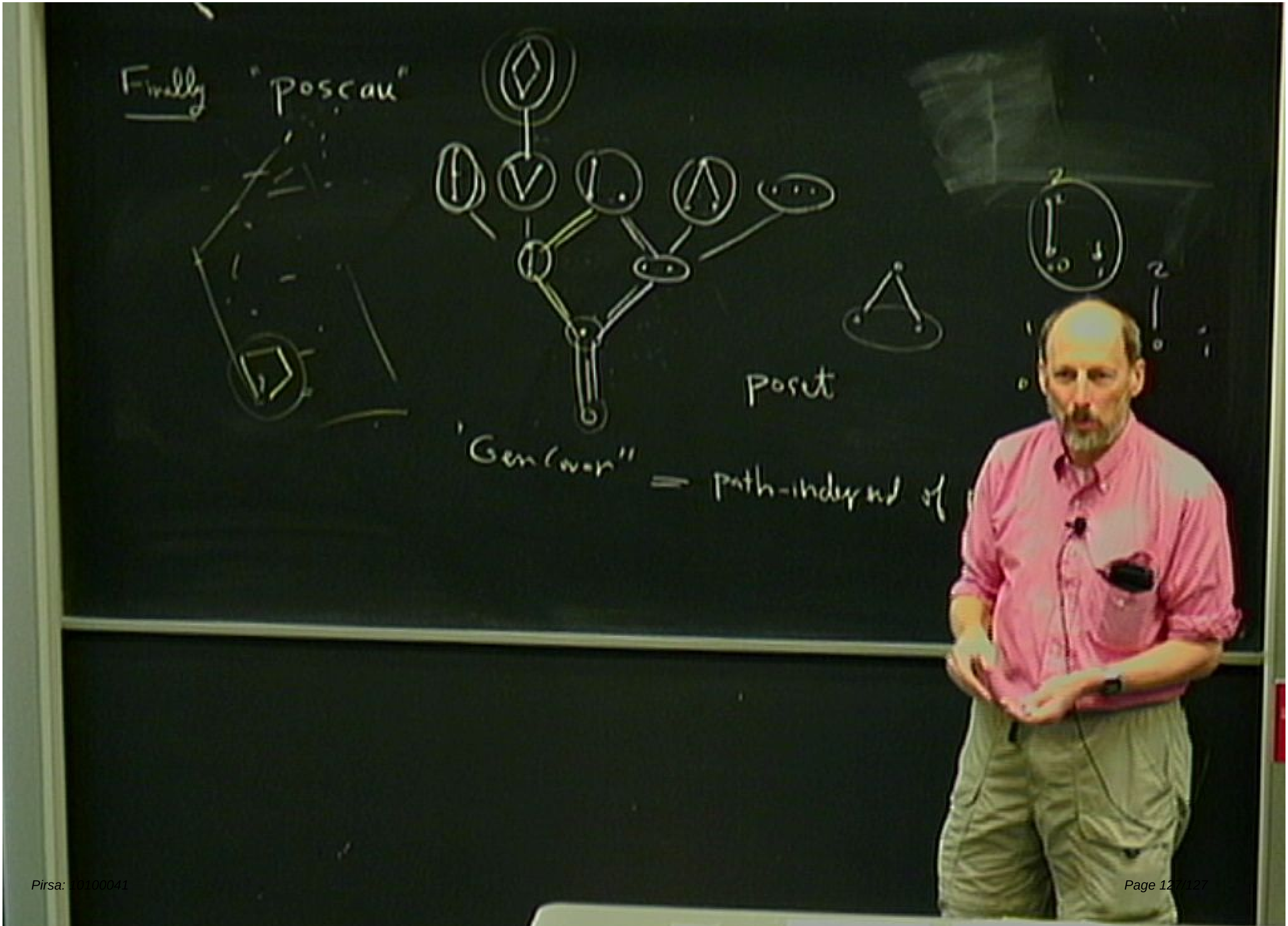


poset


Finally "poscau"


Finally "poscau"
poset
"Gen Cov" = path-indep of probs


Finally "poscau"
poset


Finally "poscau"
poset