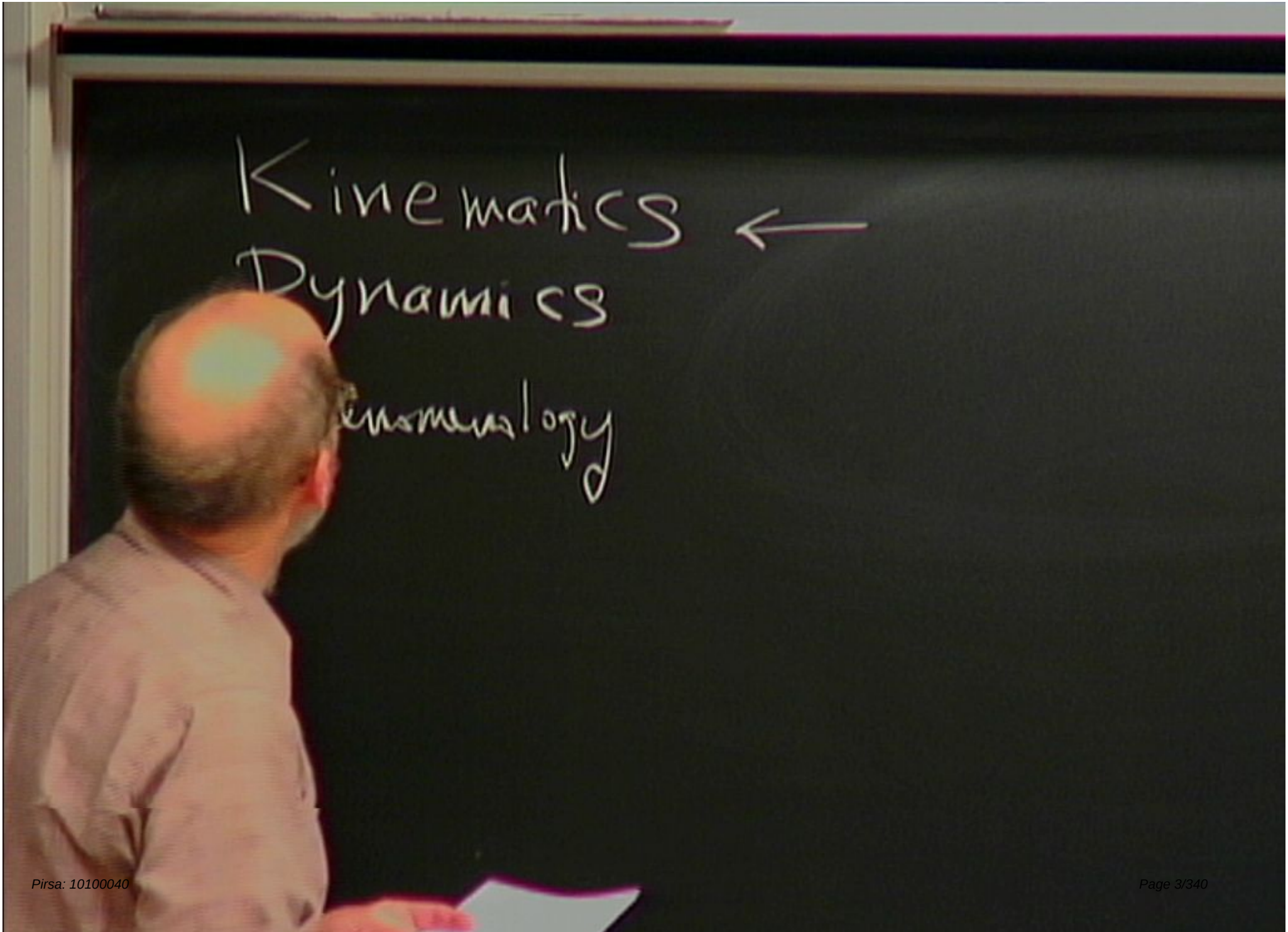
Title: An Invitation to Causal Sets - Lecture 3

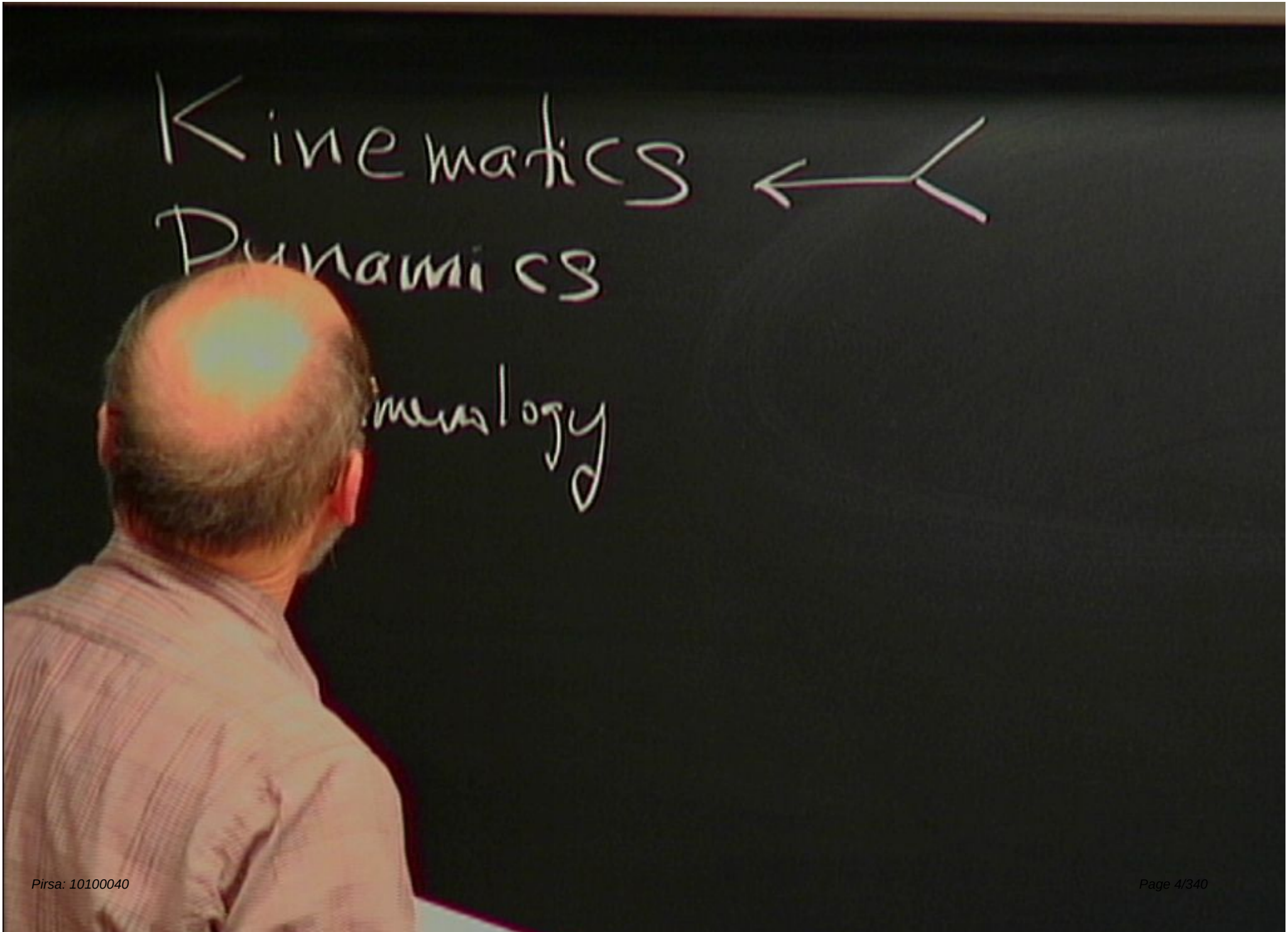
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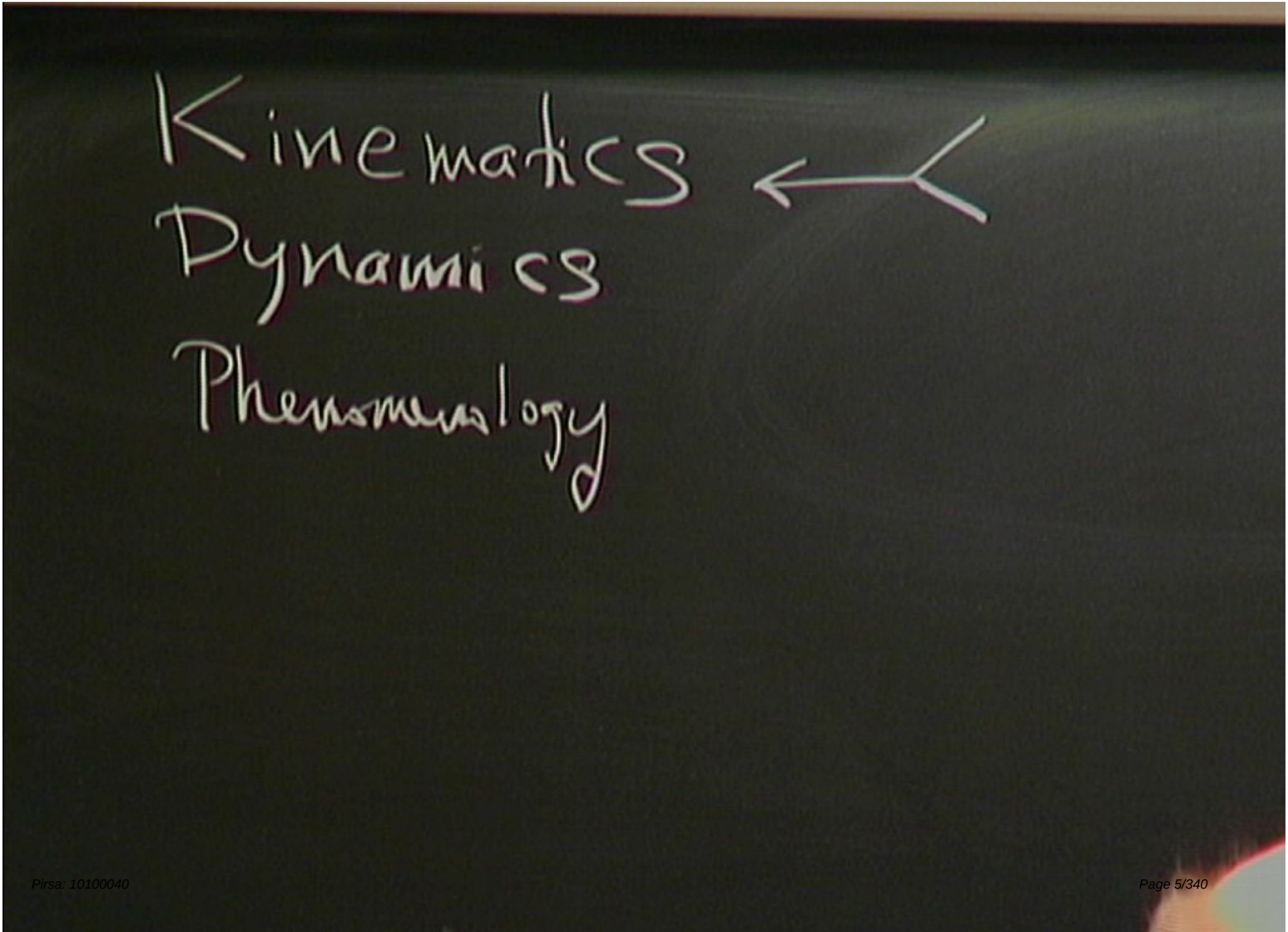
URL: http://pirsa.org/10100040

Abstract: Introduction to the causal set approach to quantum gravity and overview of current research in causal set theory

Joe Henson PIRSA 10090092 (14/3/2010)


Kinematics ←
Dynamics
Phenomenology


Kinematics
Dynamics
Cosmology

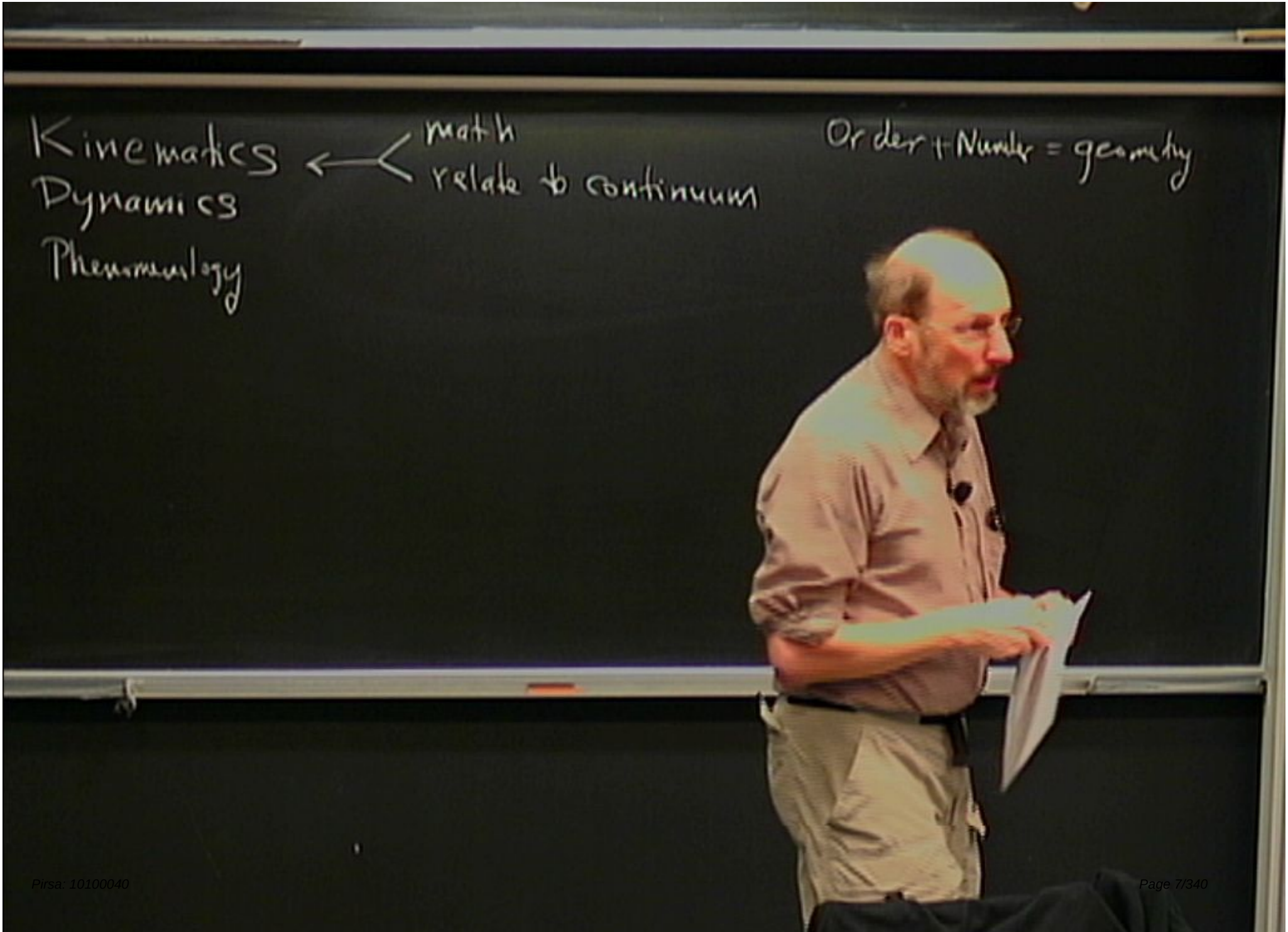

Kinematics
Dynamics
Phenomenology

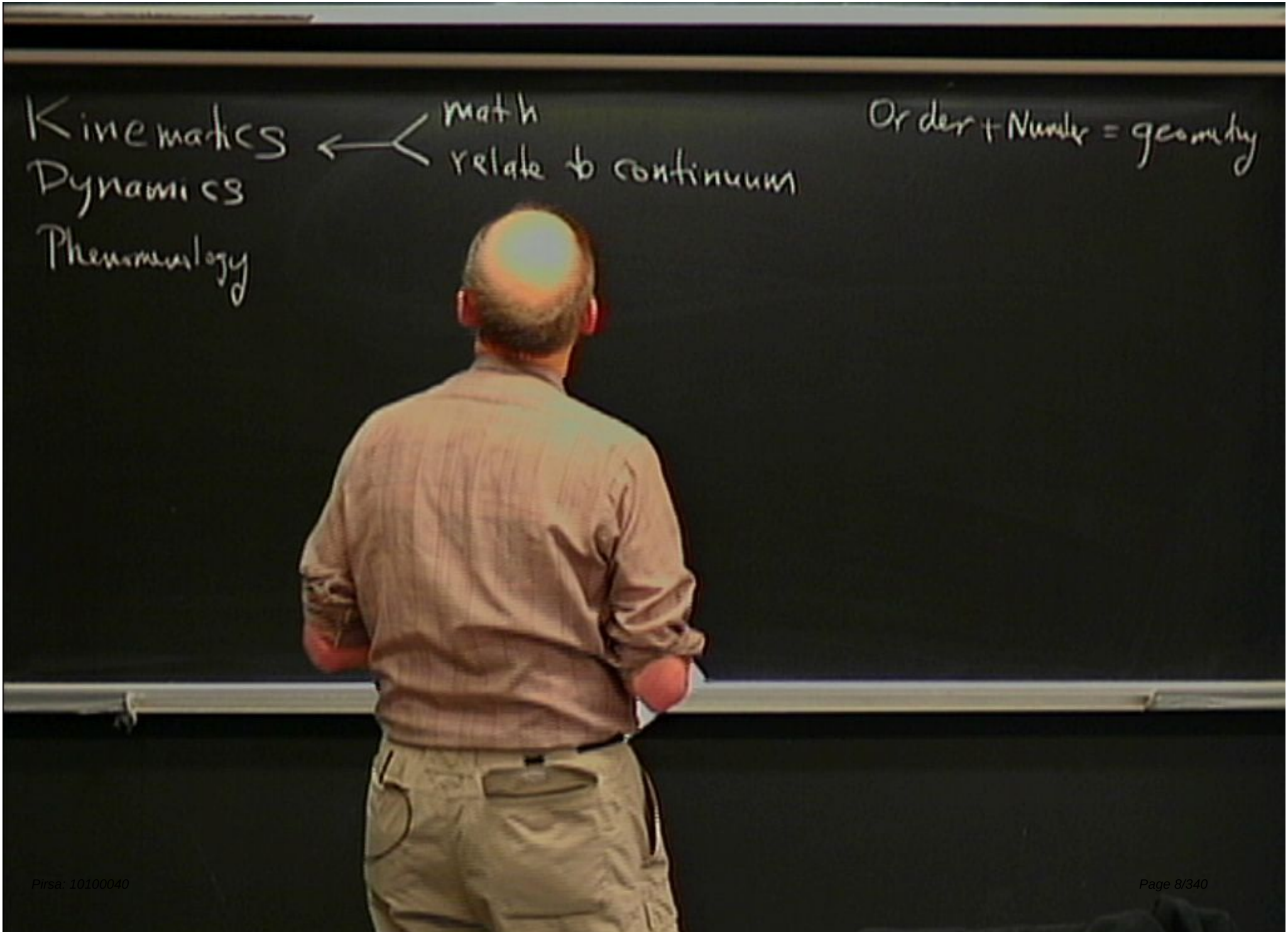
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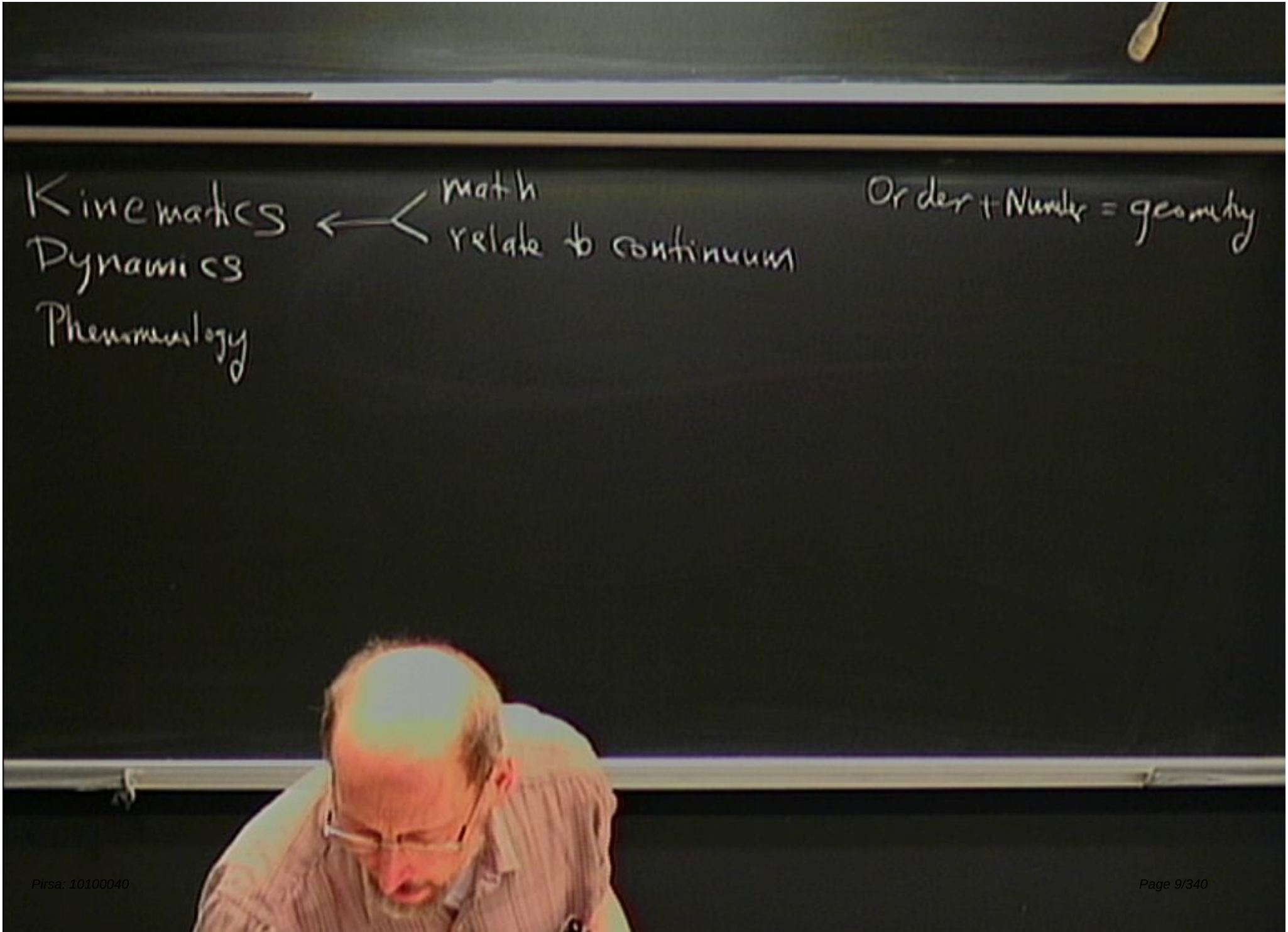
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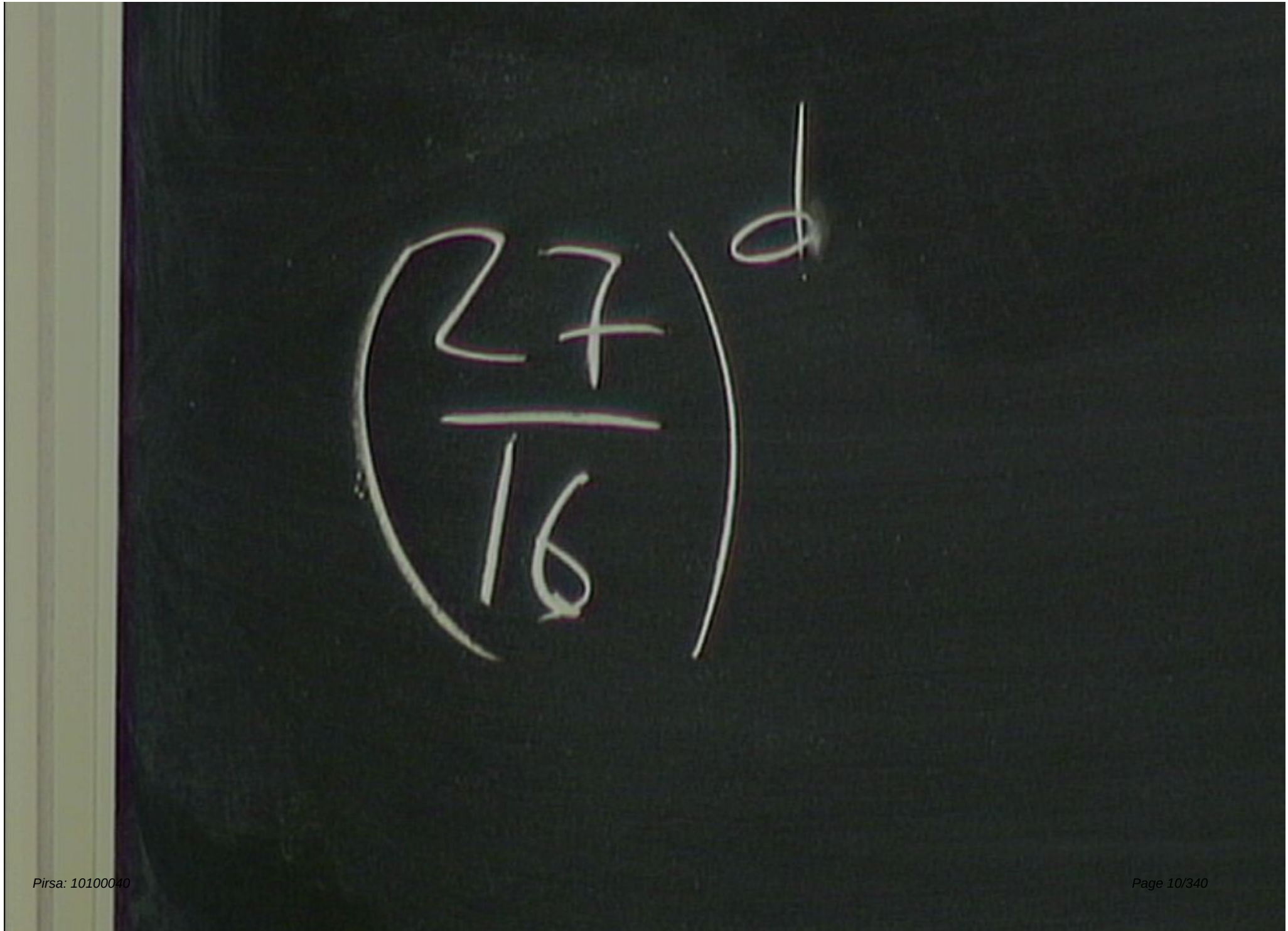
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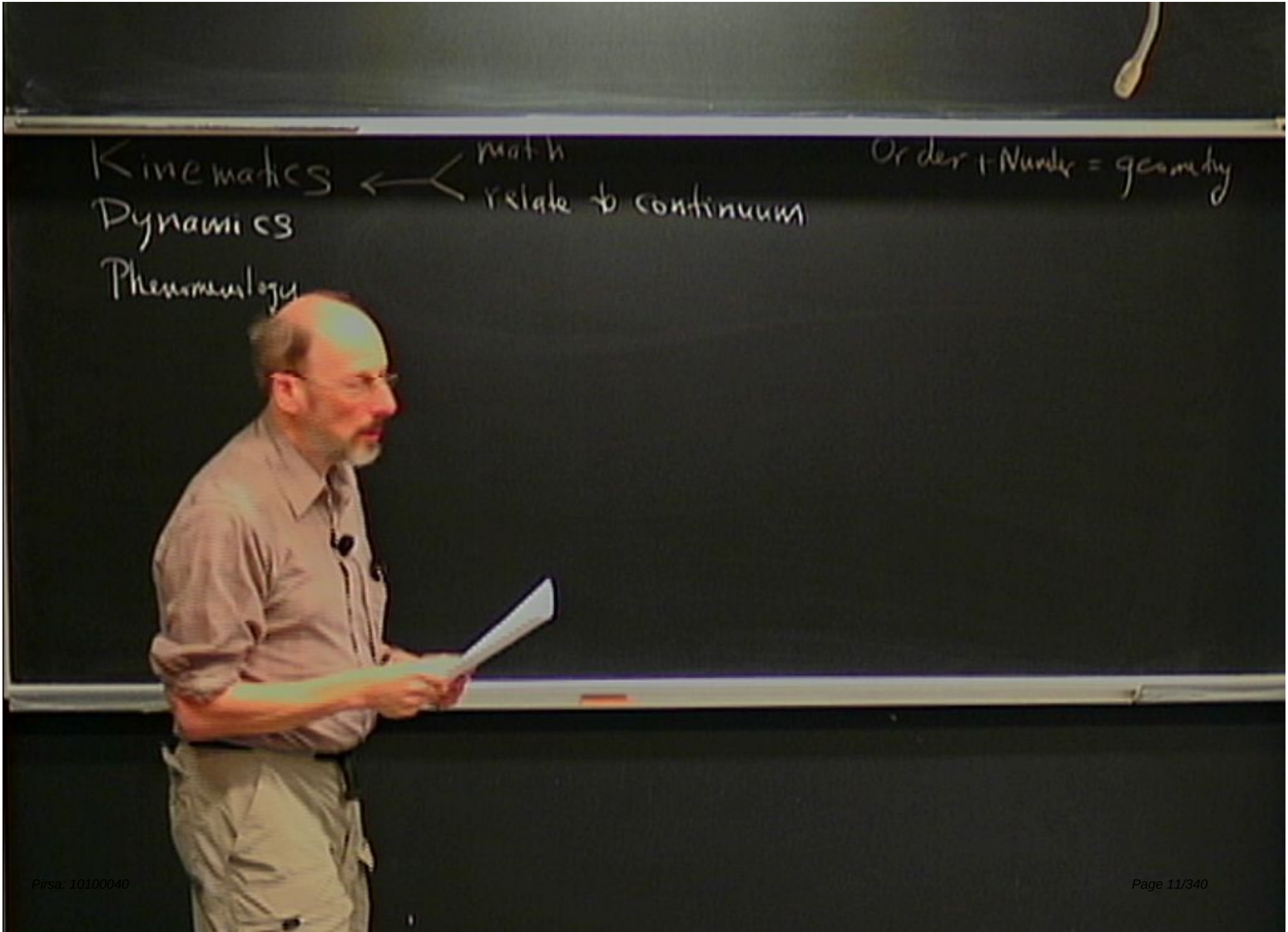


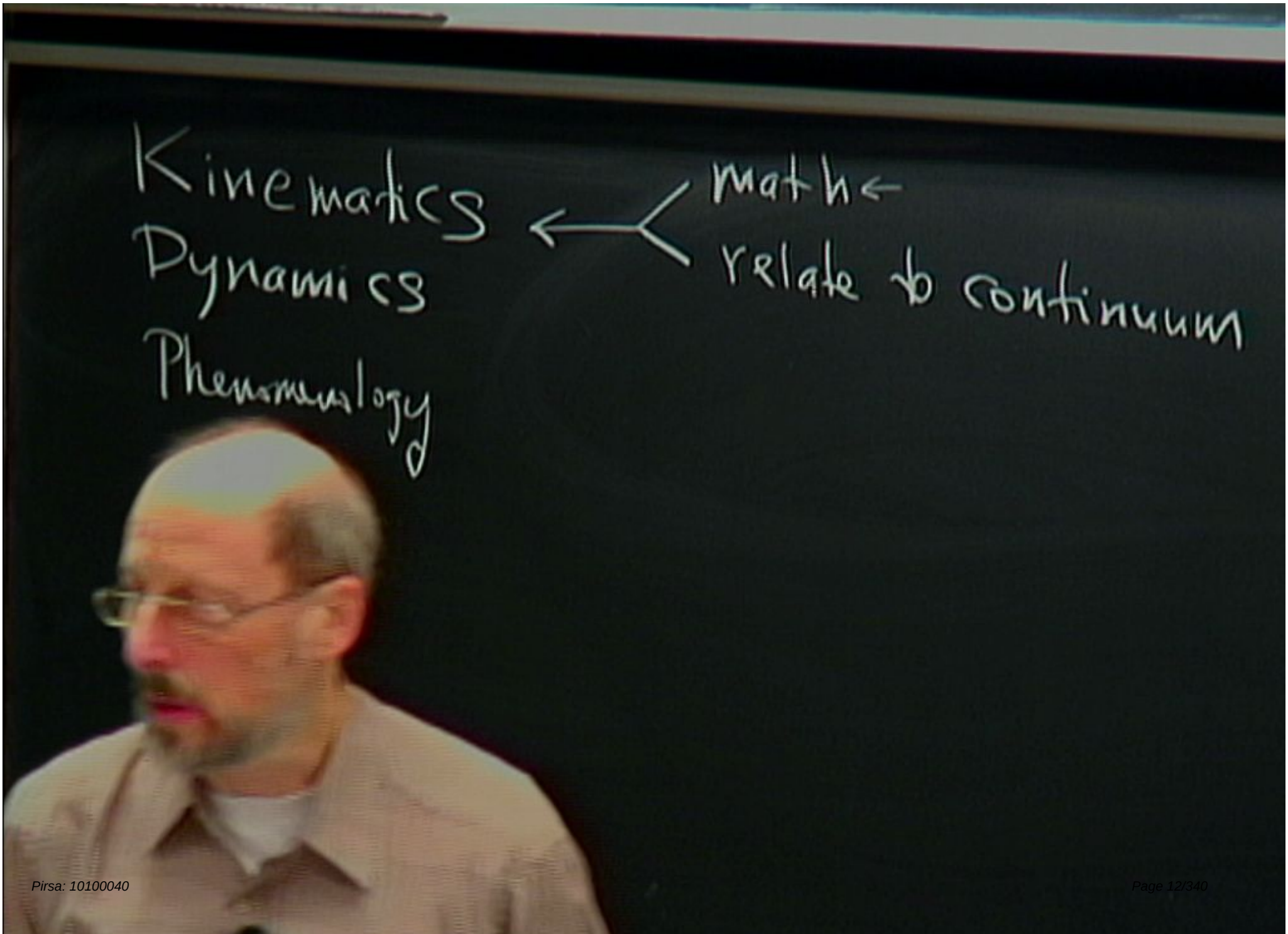

Kinematics
Dynamics
Phenomenology
math
relate to continuum
Order + Number = geometry

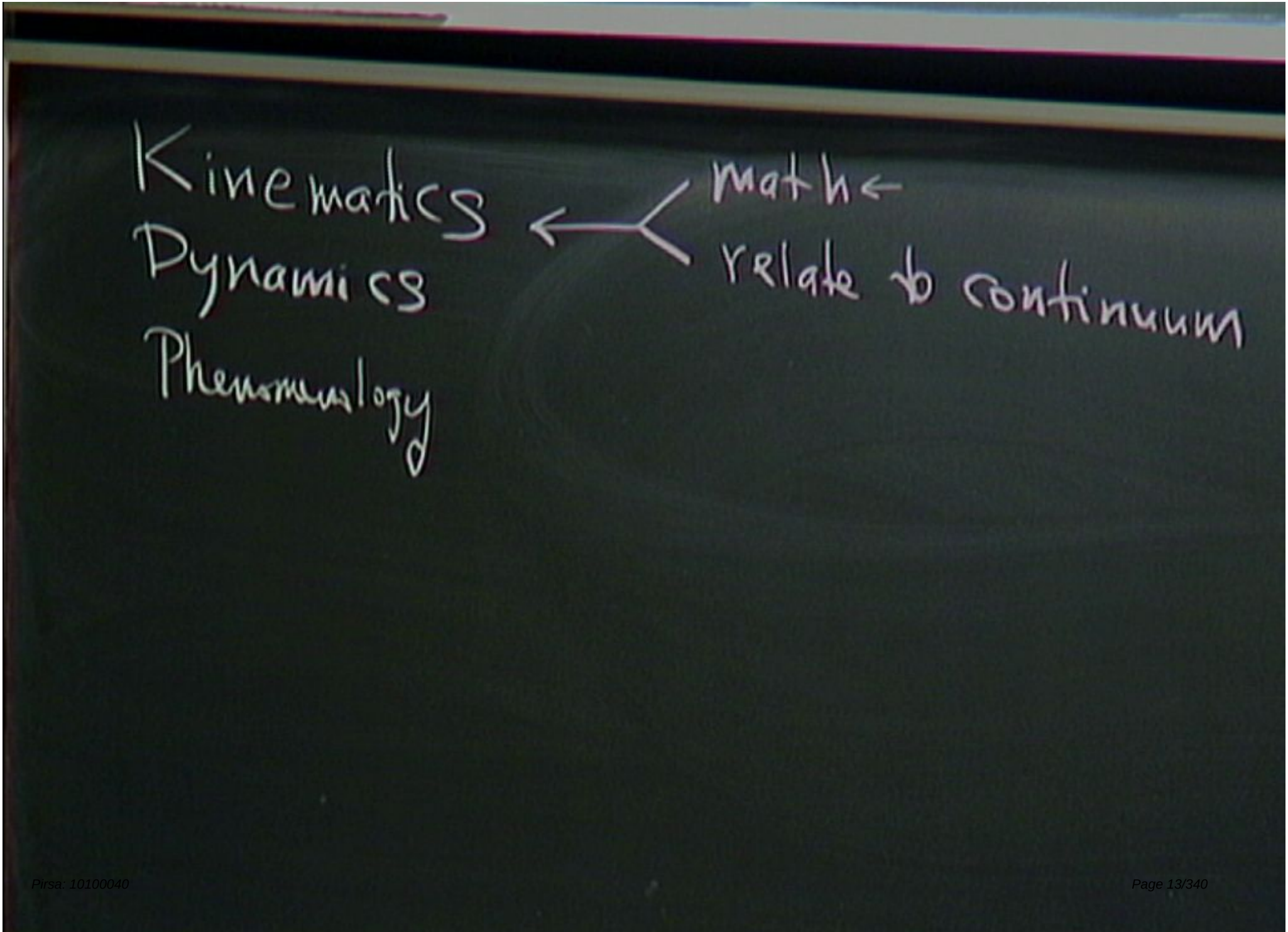

Kinematics
Dynamics
Phenomenology
math
relate to continuum
Order + Number = geometry

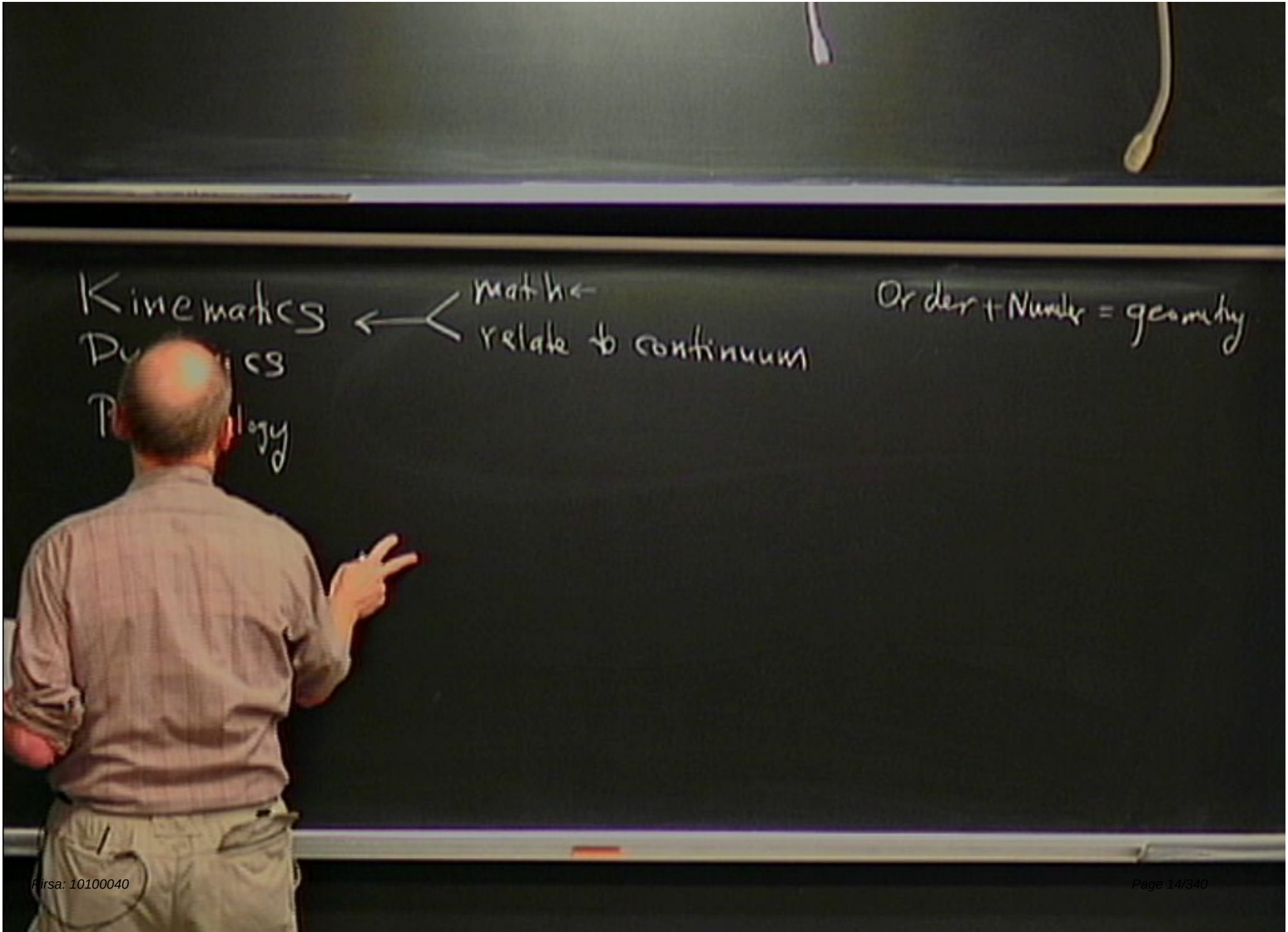

Kinematics
Dynamics
Phenomenology
math
relate to continuum
Order + Number = geometry

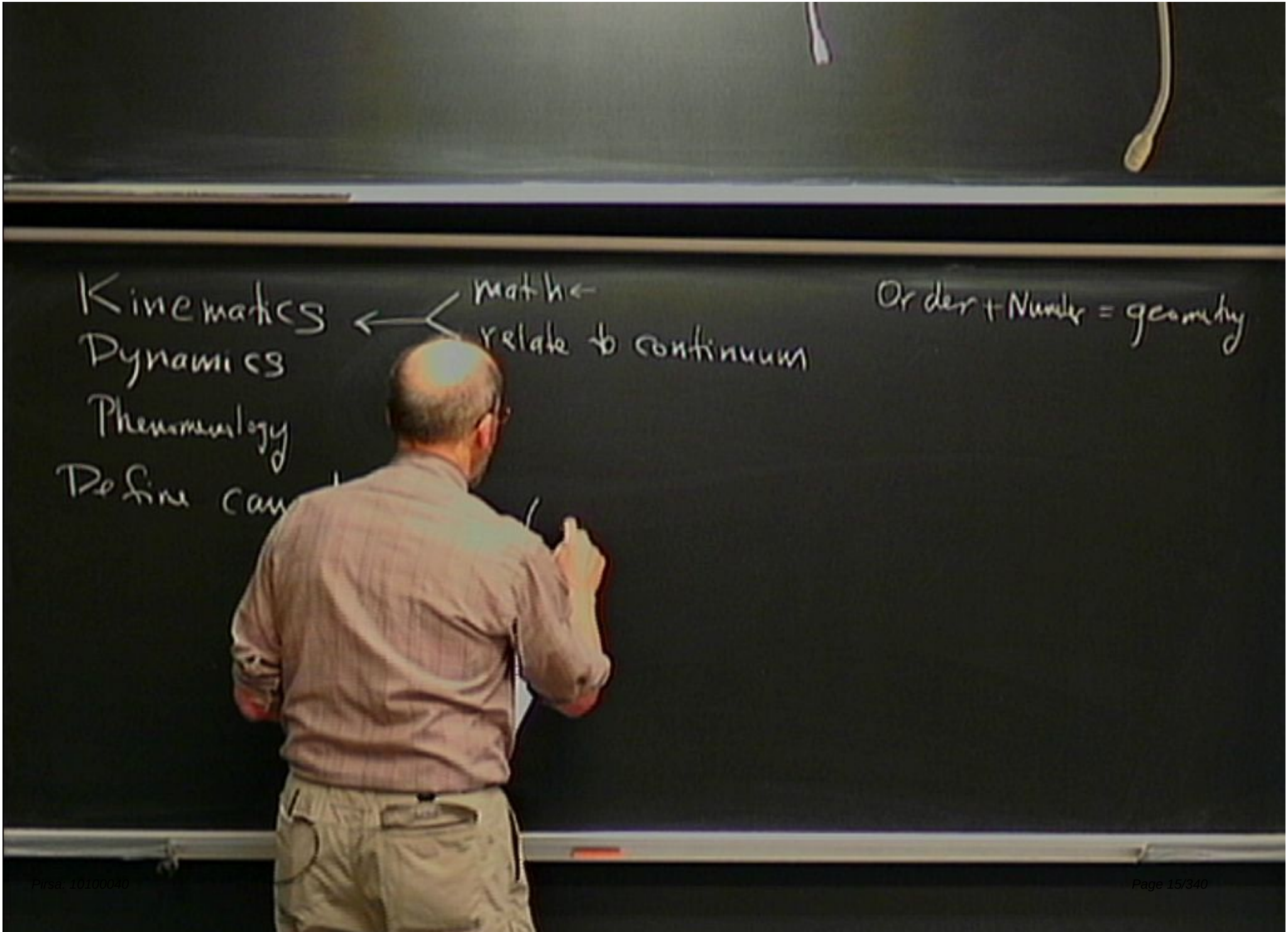

$\left(\frac{27}{16}\right)^d$

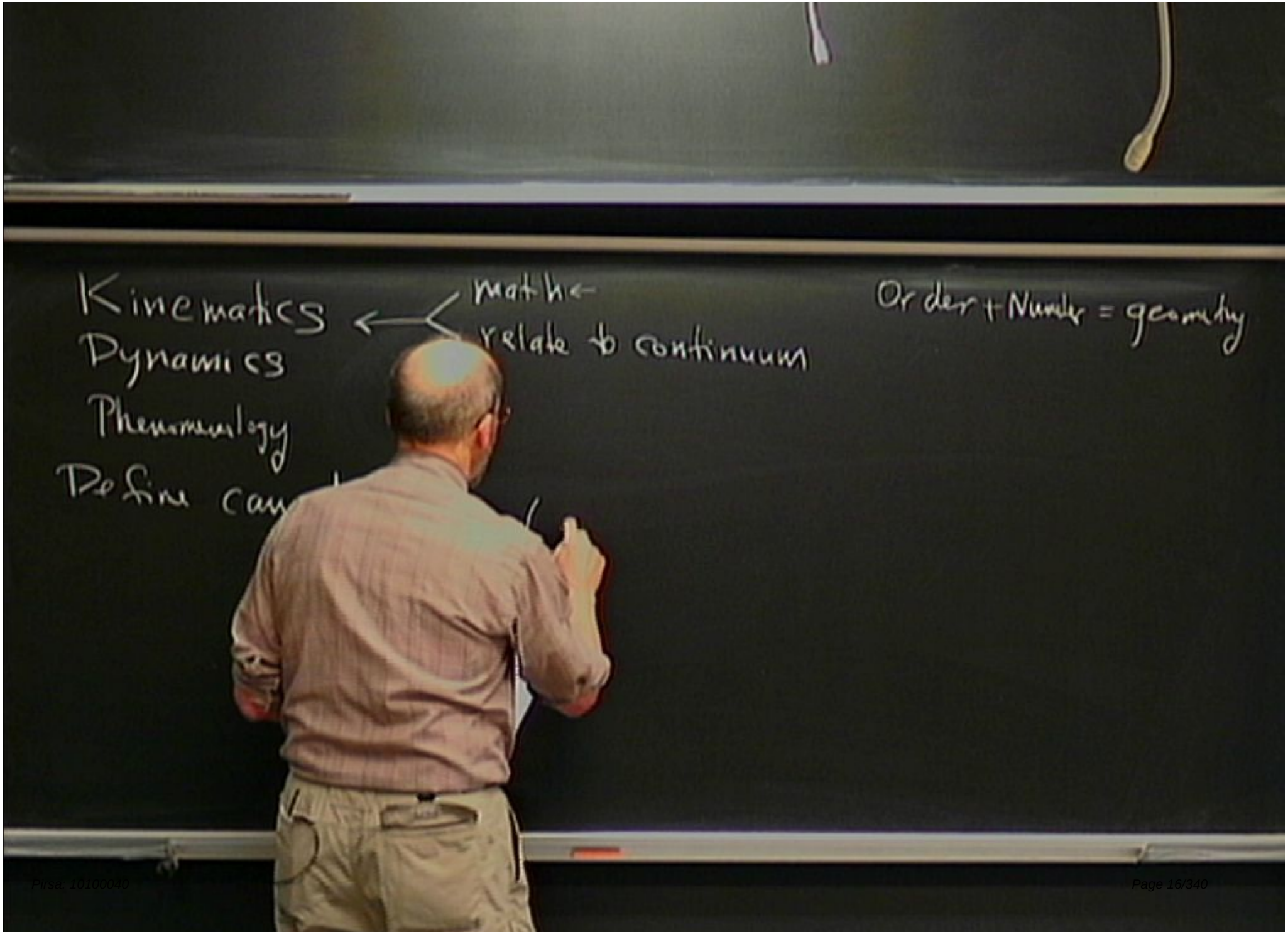

Kinematics
Dynamics
Phenomenology
math
relate to continuum
Order + Number = geometry

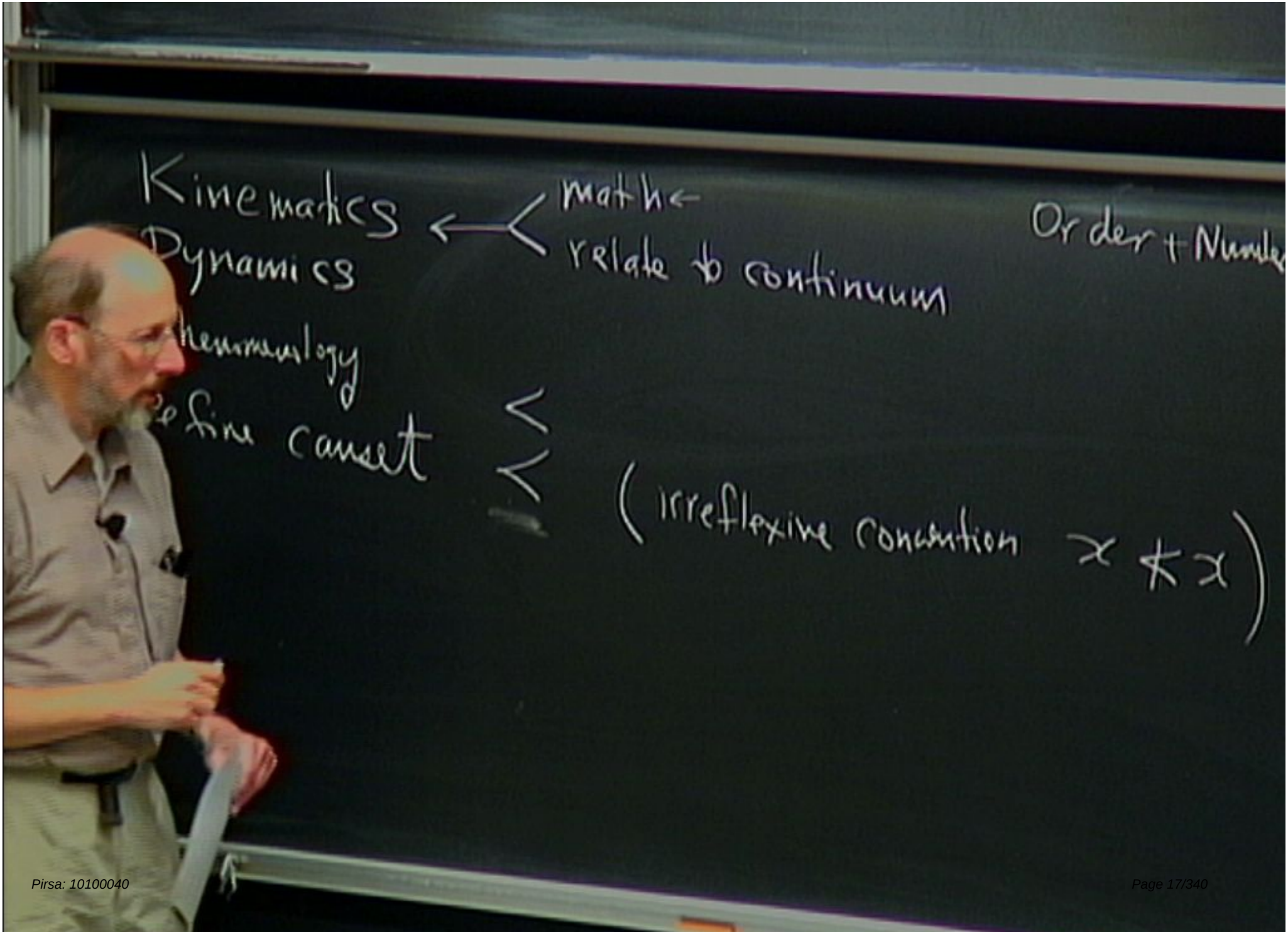

Kinematics
Pynamics
Phenomenology
math ←
relate to continuum

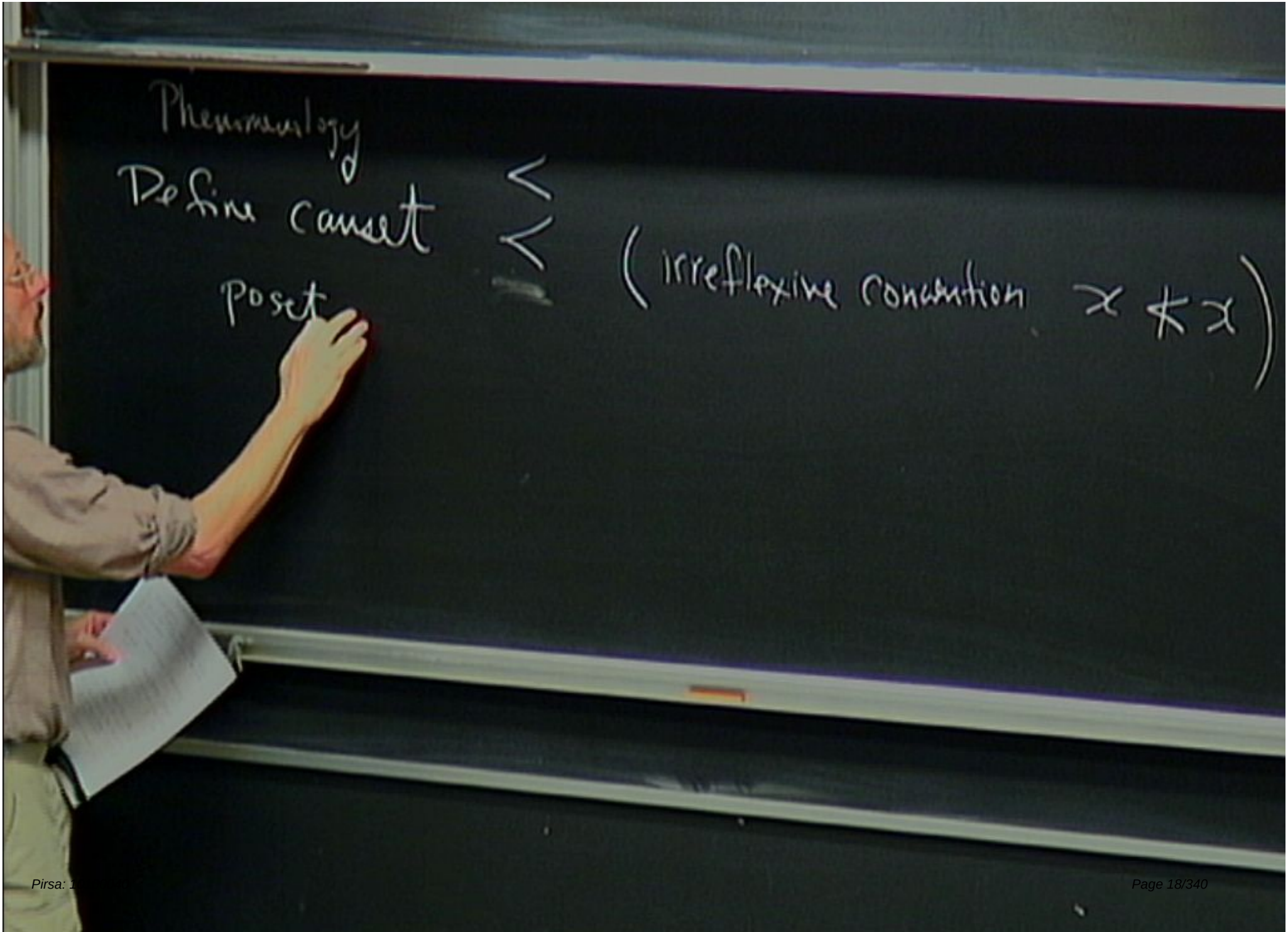

Kinematics
Dynamics
Phenomenology
math ←
relate to continuum

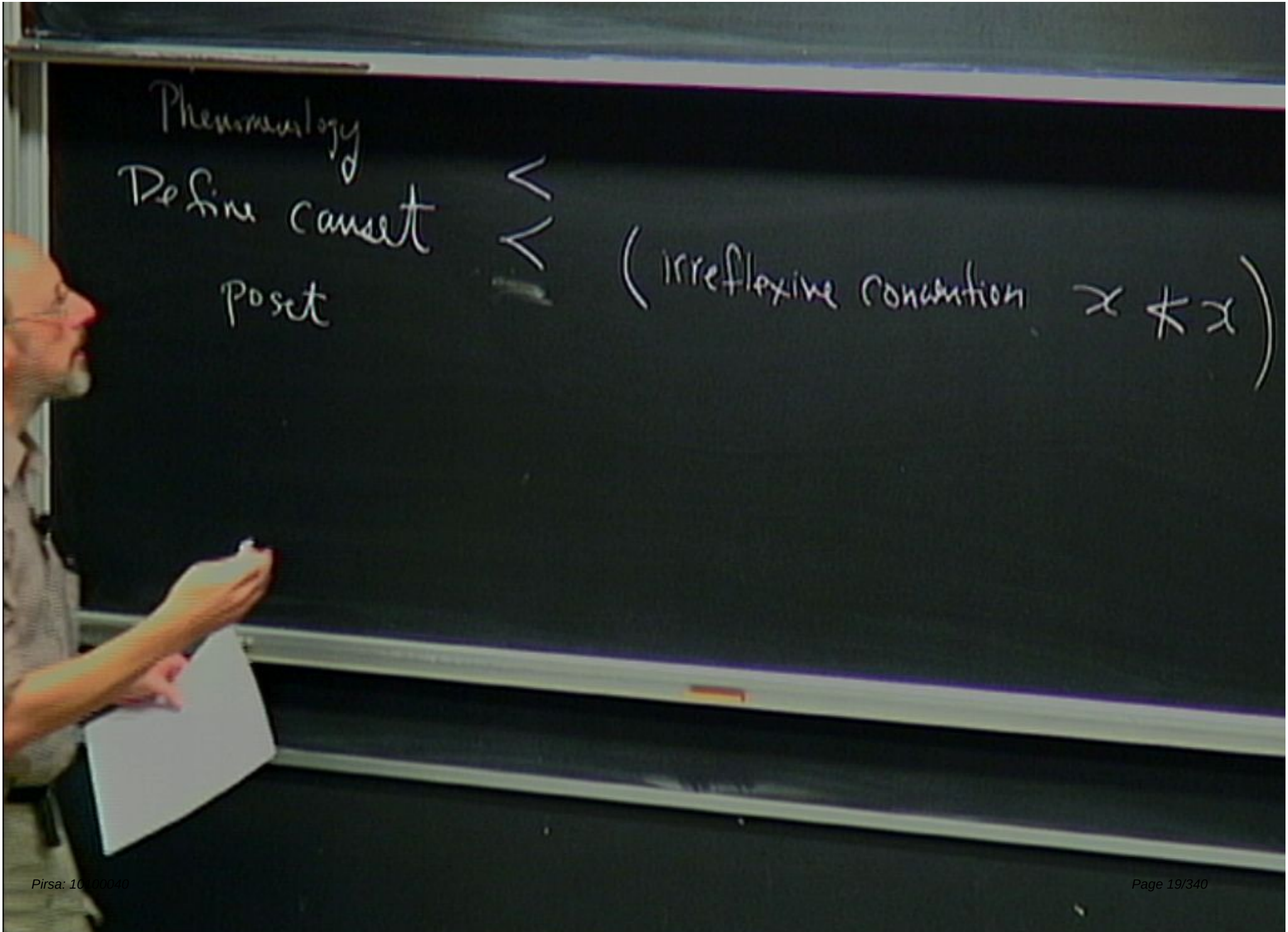

Kinematics
math ←
relate to continuum
Order + Number = geometry

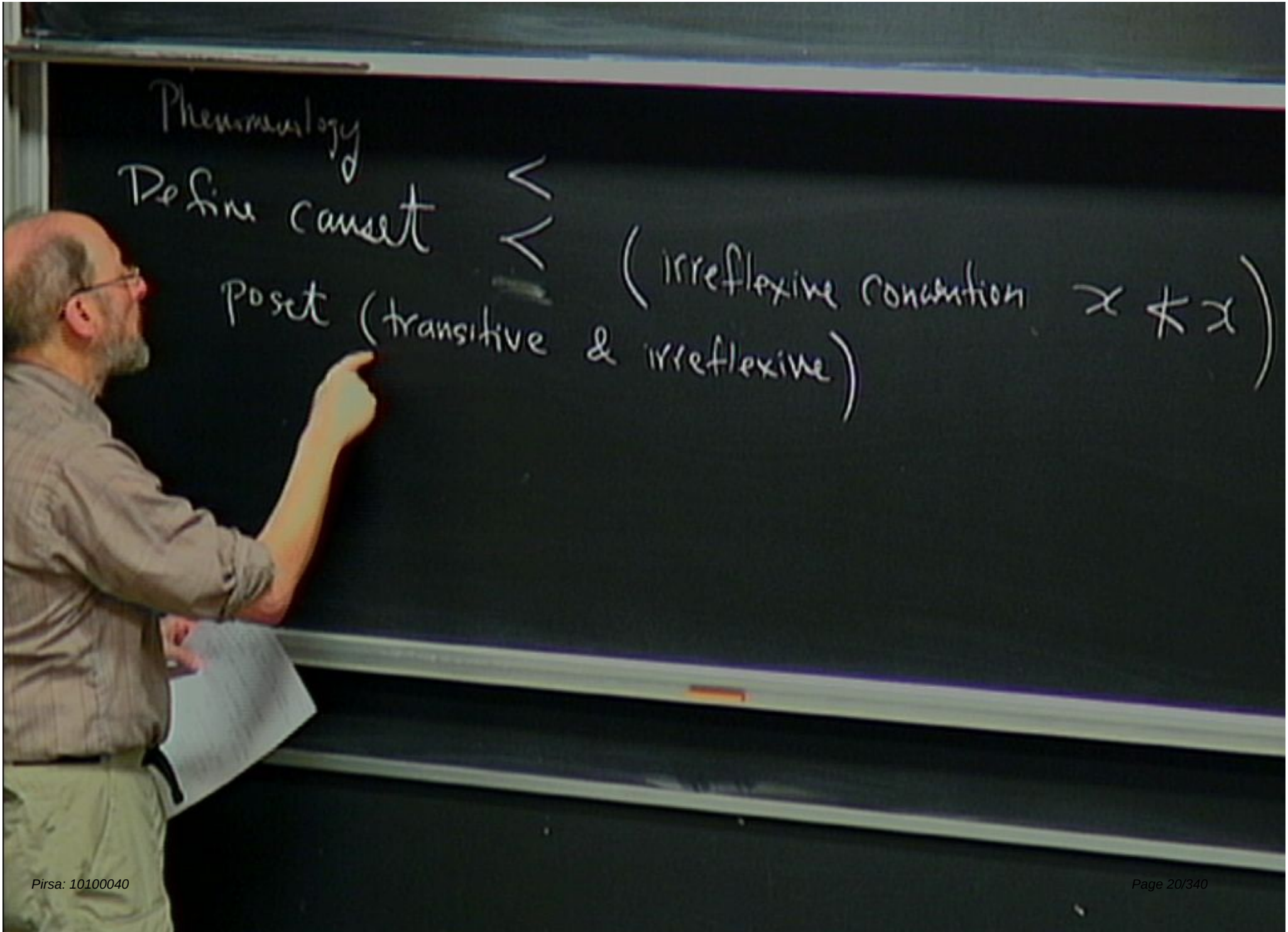

Kinematics
Dynamics
Phenomenology
Order + Number = geometry
relate to continuum

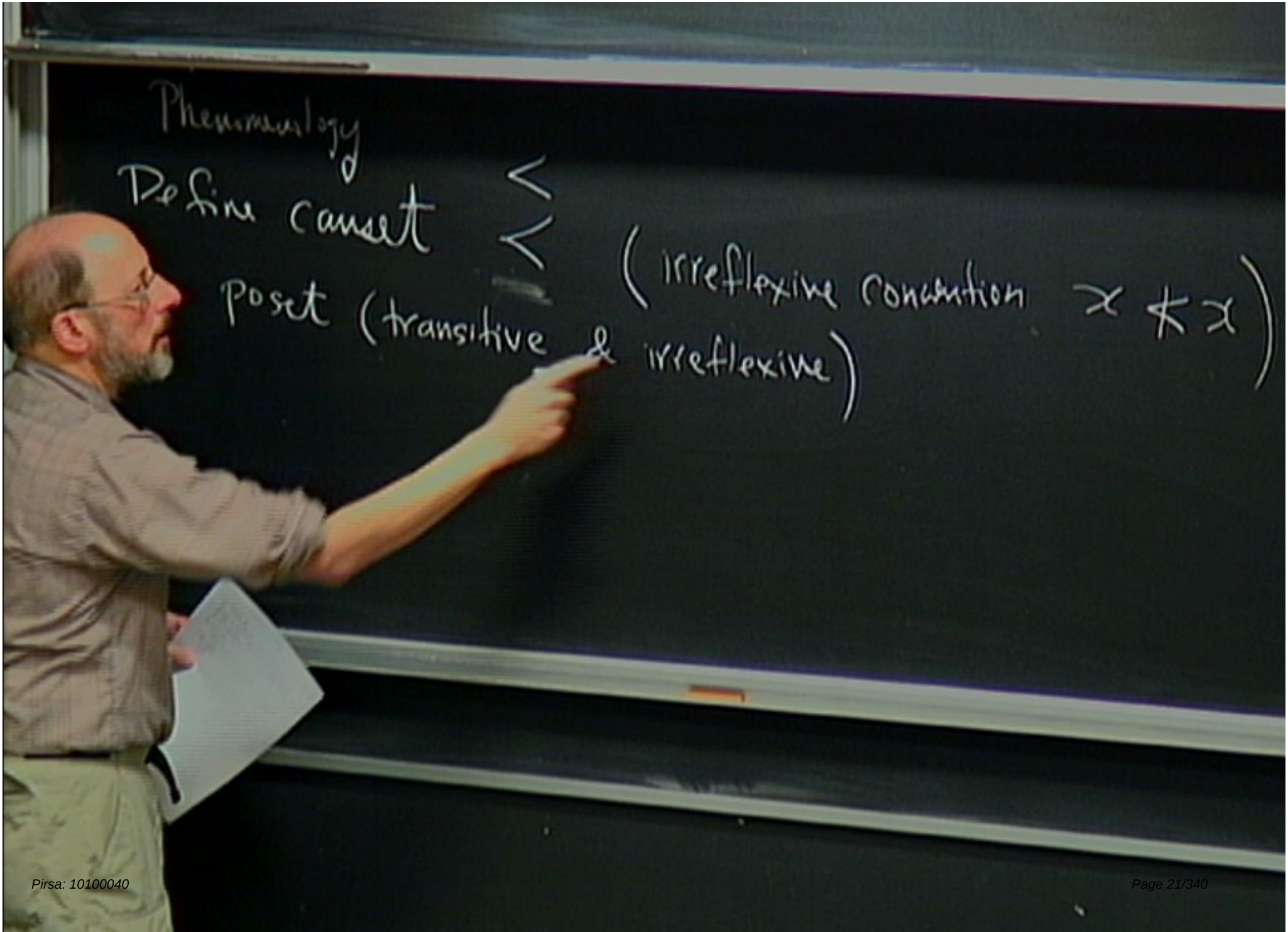

Kinematics
Dynamics
Phenomenology
math ←
relate to continuum
Order + Number = geometry

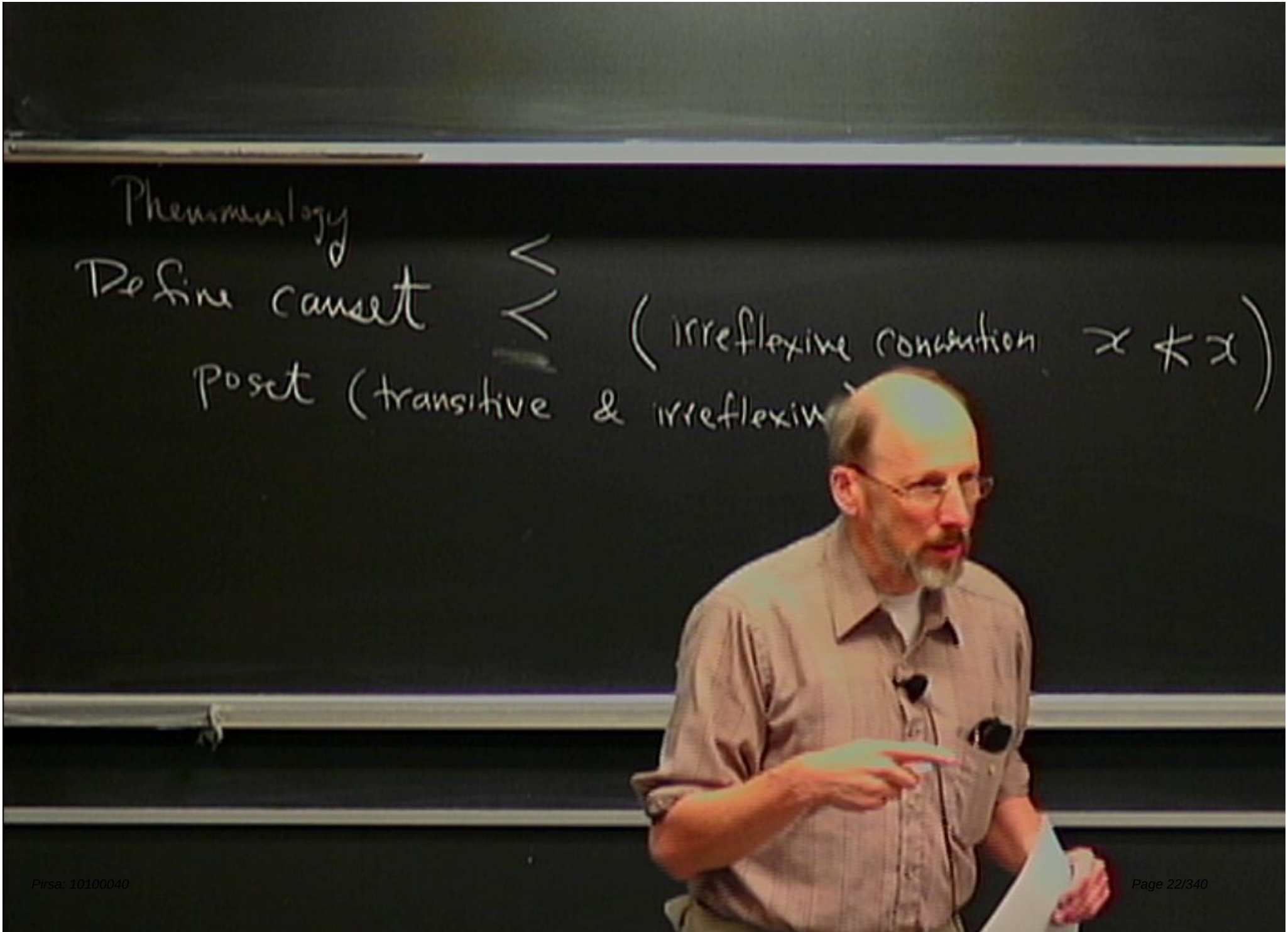

Kinematics
Dynamics
Phenomenology
Define causet
math ←
relate to continuum
Order + Numbe
(irreflexive relation x ≺ x)

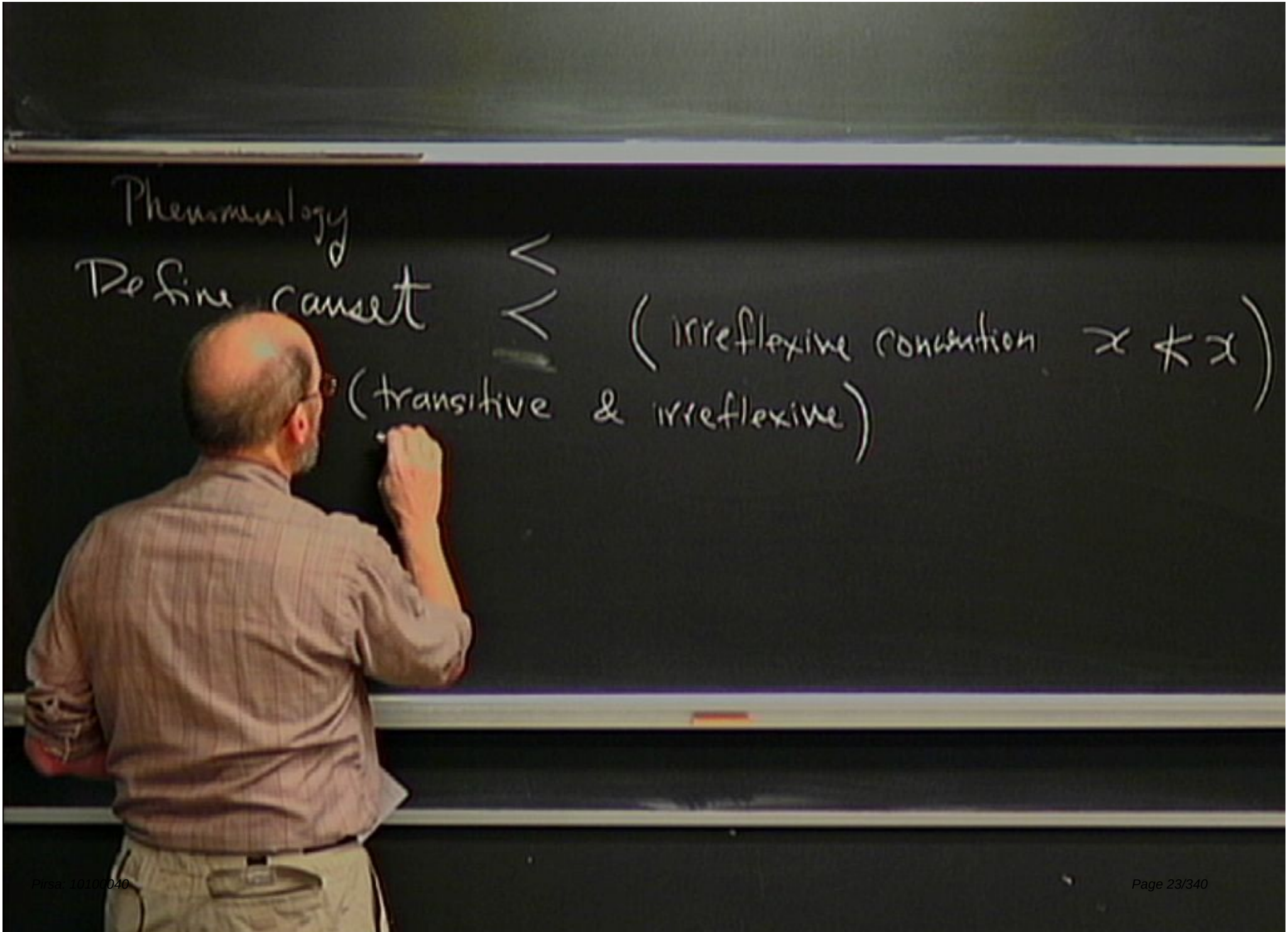

Phenomenology
Define caset
Poset
(irreflexive connection x ≠ x)

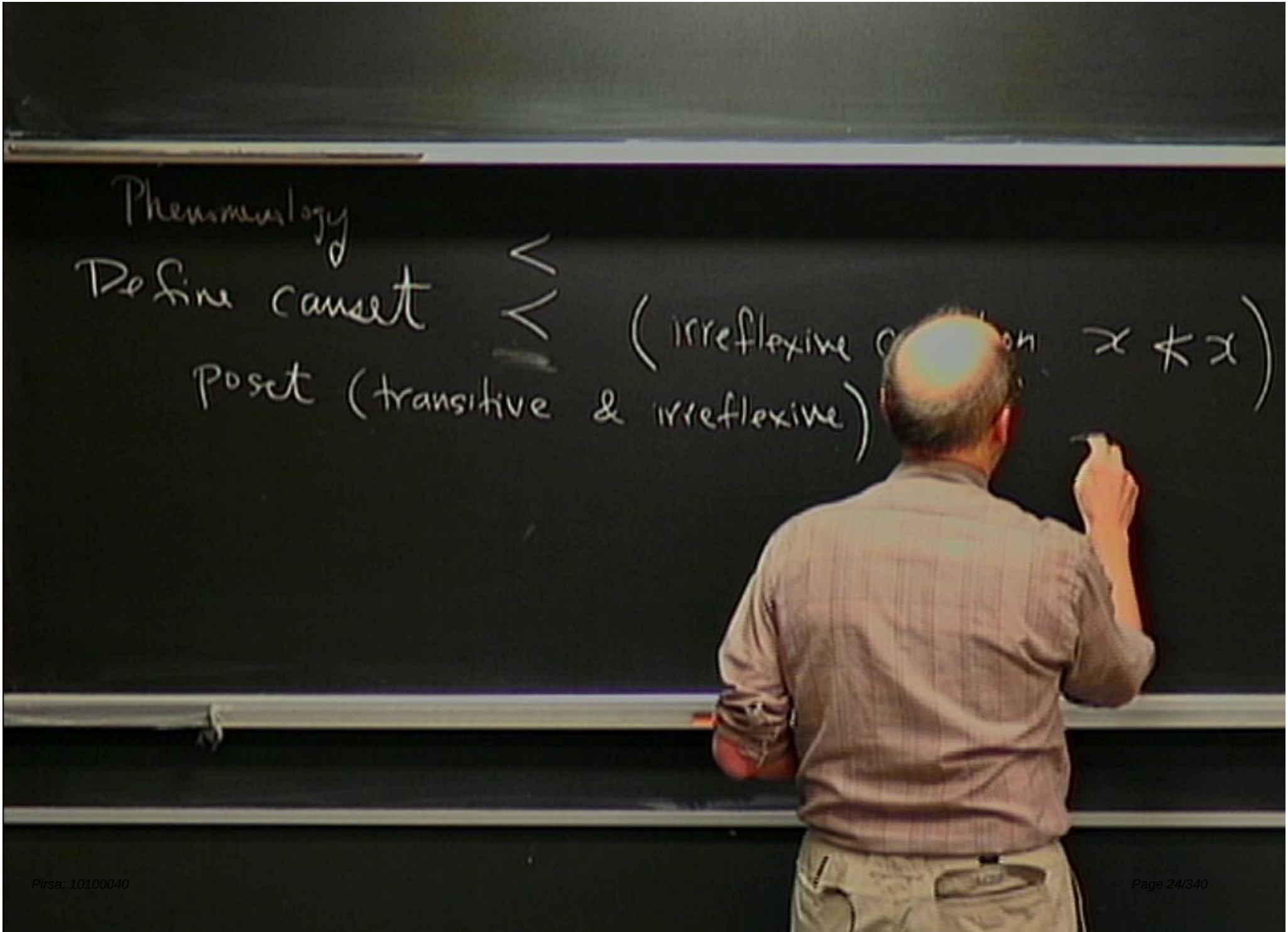

Phenomenology
Define causet
Poset
≤
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(irreflexive relation x ≠ x)

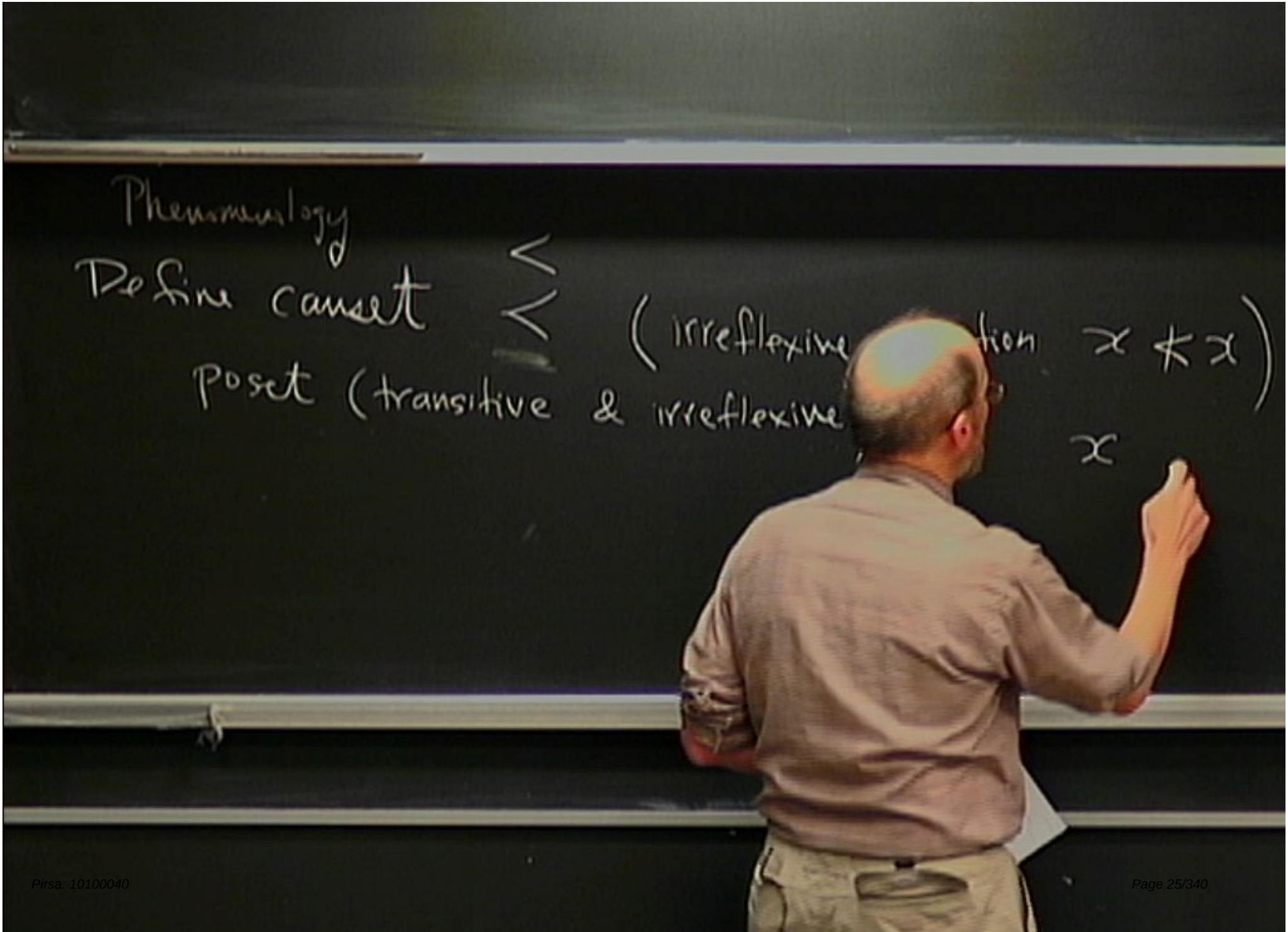

Phenomenology
Define cause
Poset (transitive & irreflexive)
(irreflexive connection x ≠ x)

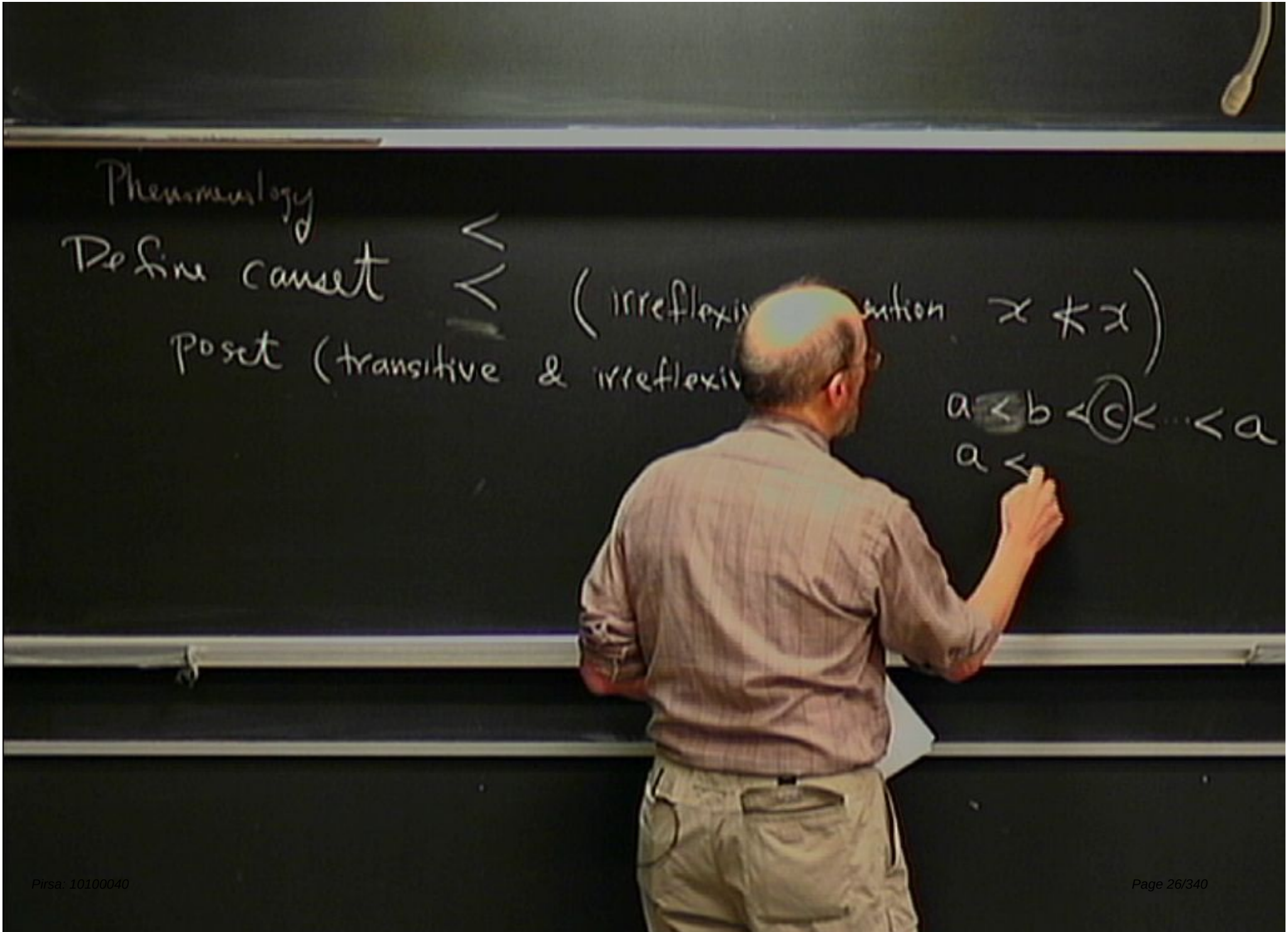

Phenomenology
Define causet
Poset (transitive & irreflexive)
(irreflexive connection x ≠ x)

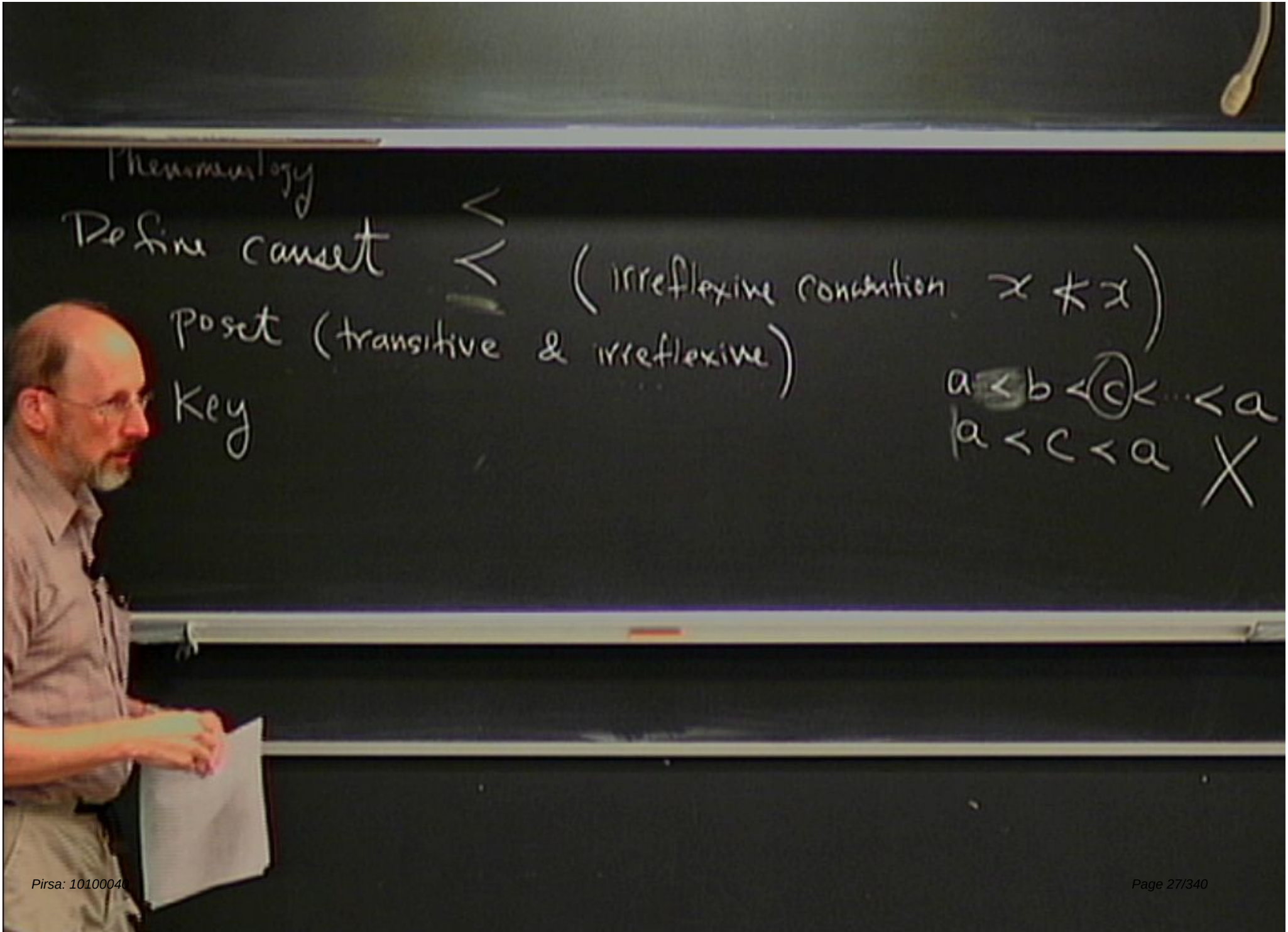

Phenomenology
Define causet
Poset (transitive & irreflexiv
(irreflexive connection x ≠ x)

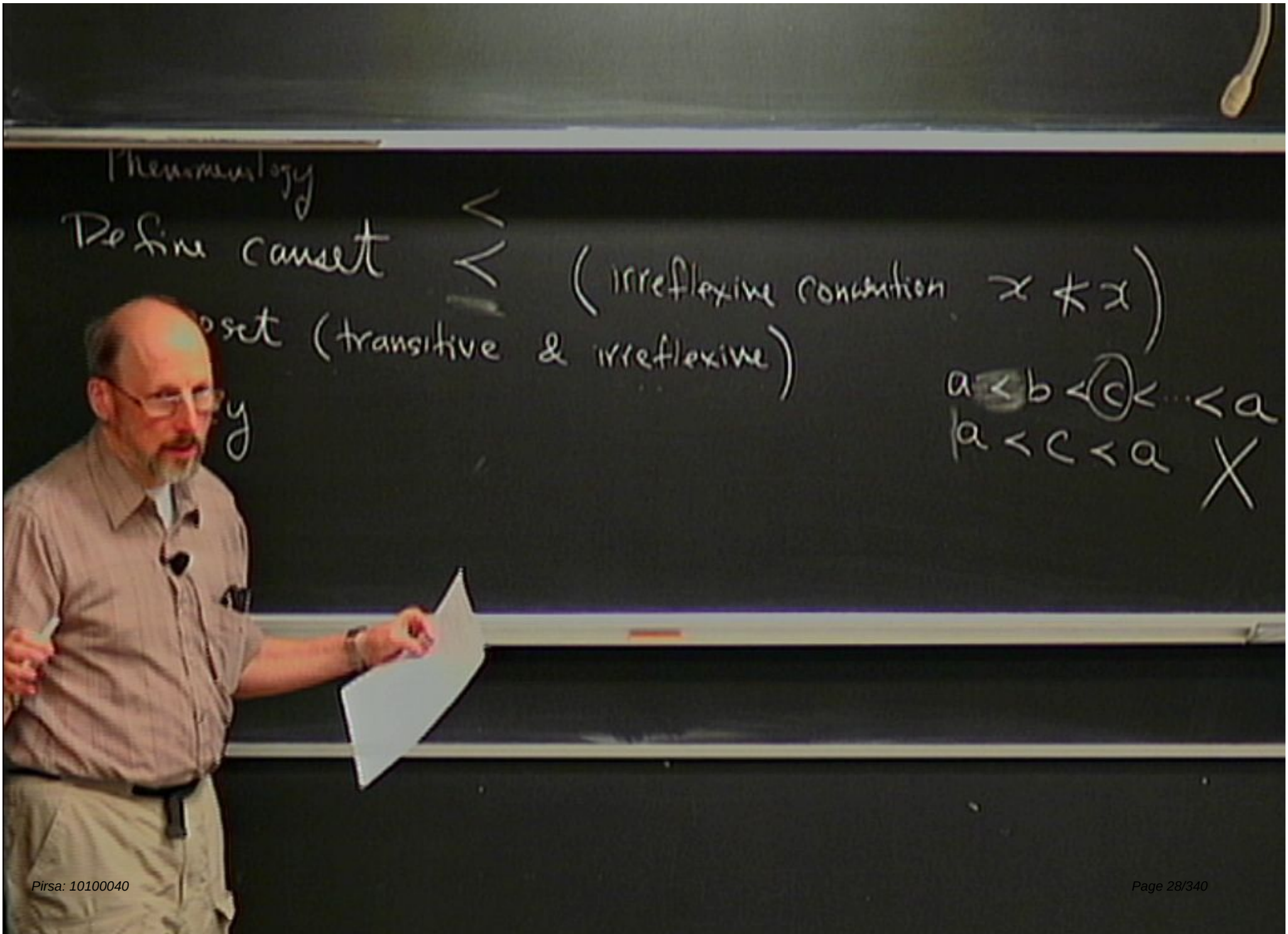

Phenomenology
Define causet
(irreflexive connection $x \neq x$)
(transitive & irreflexive)

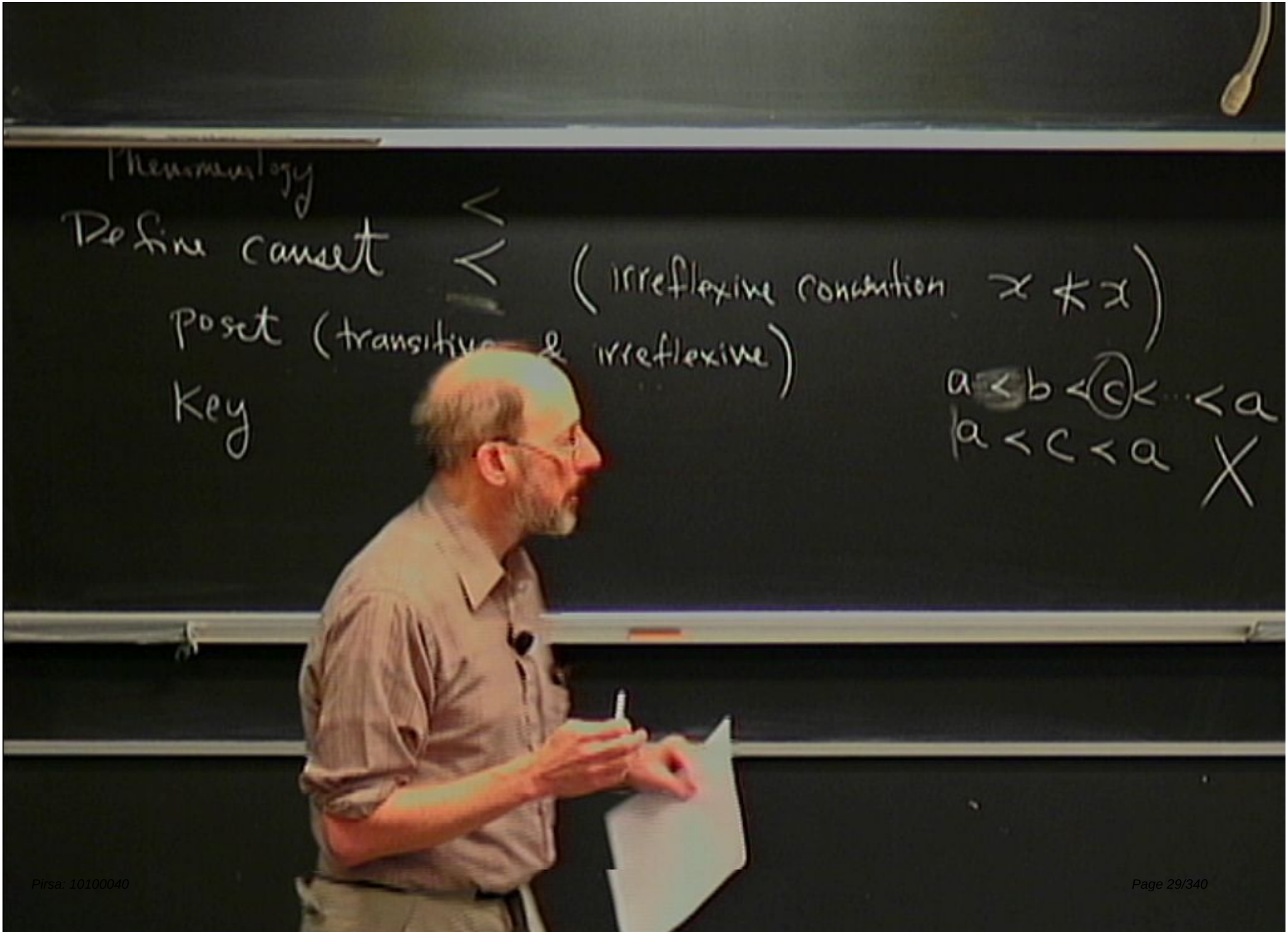

Phenomenology
Define caset
Poset (transitive & irreflexive)

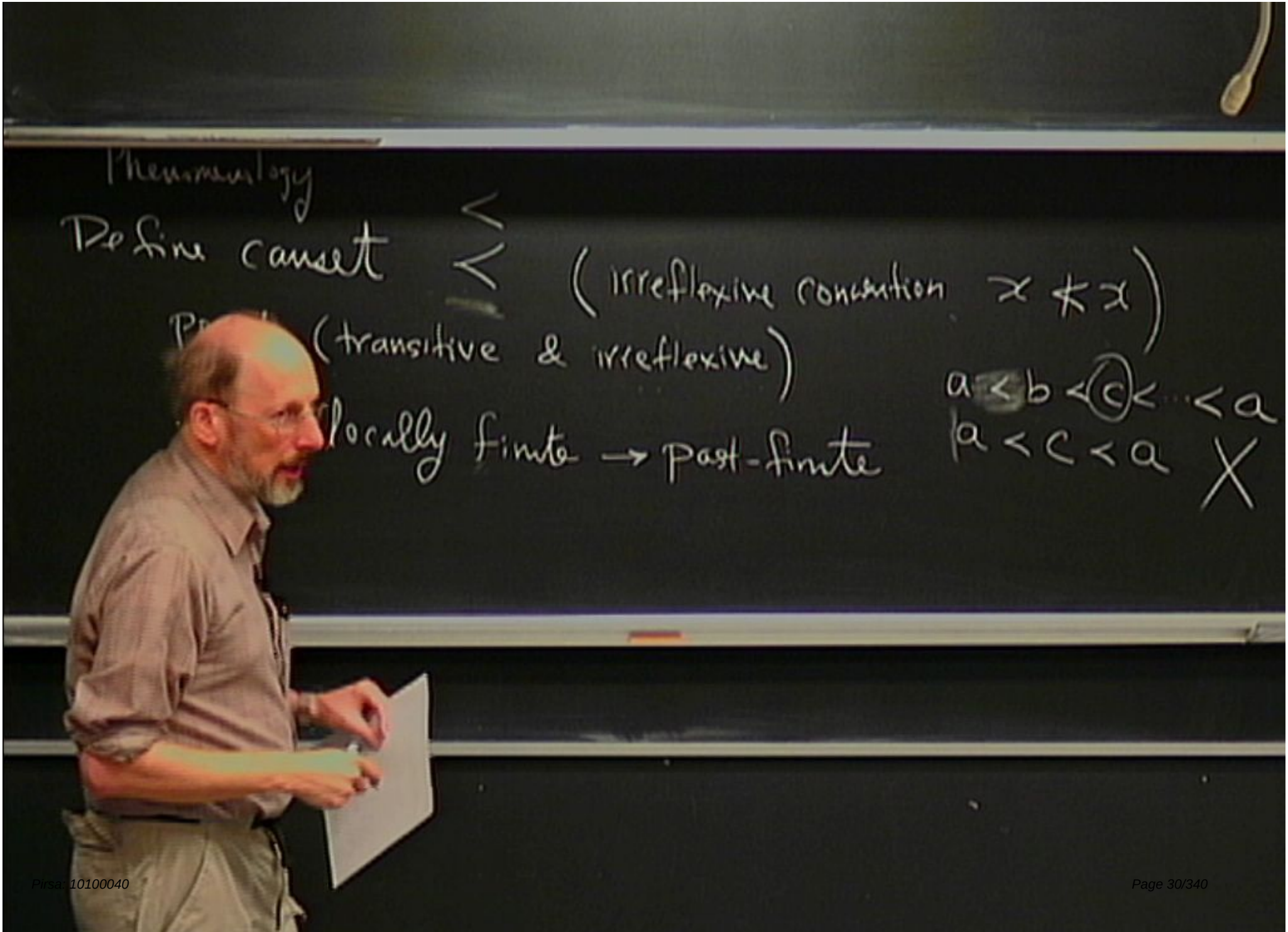

Phenomenology
Define caset
Poset (transitive &
x

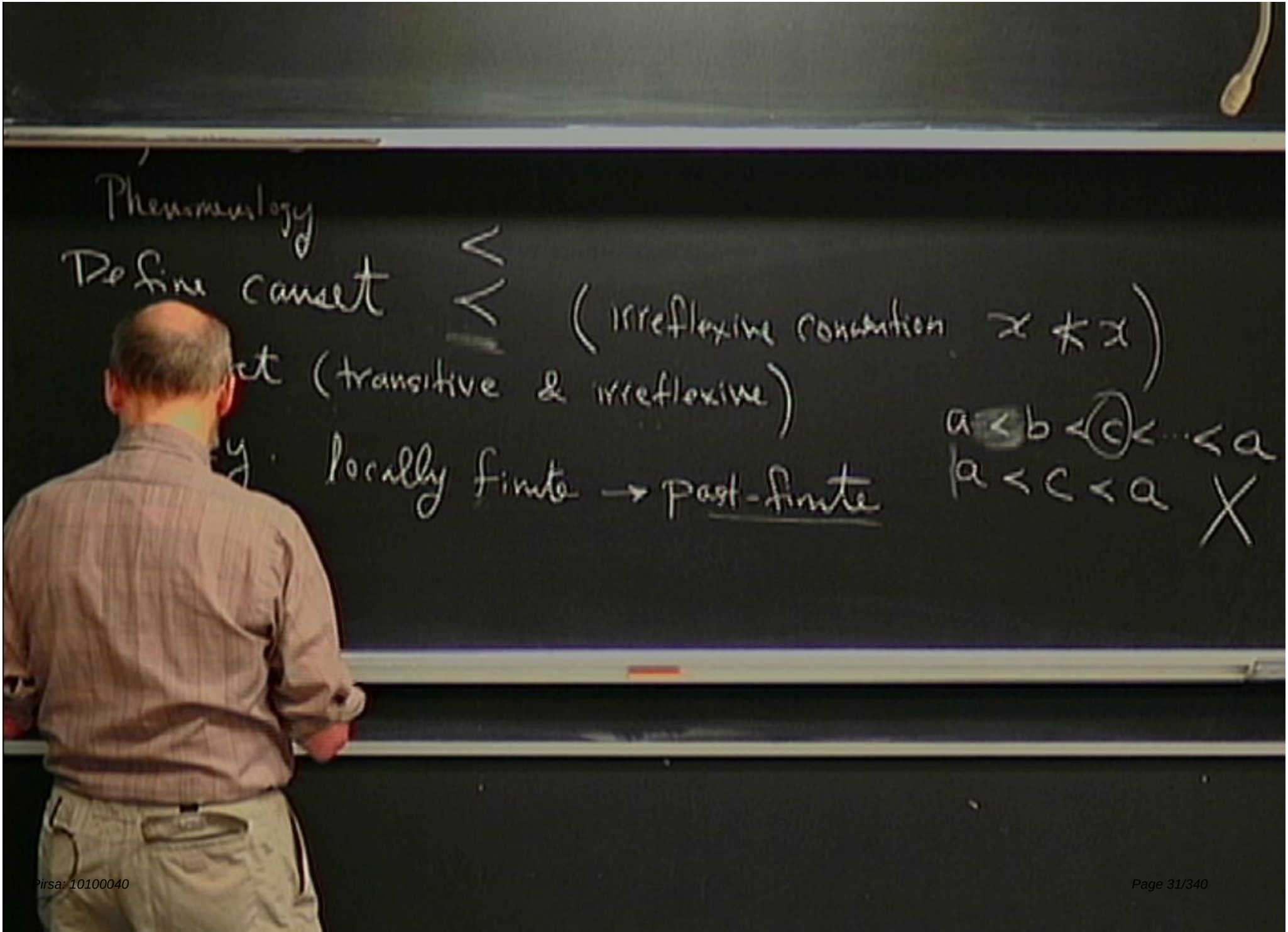

Phenomenology
Define caveat
Poset (transitive &
$a \prec b \prec c \prec \cdots \prec a$

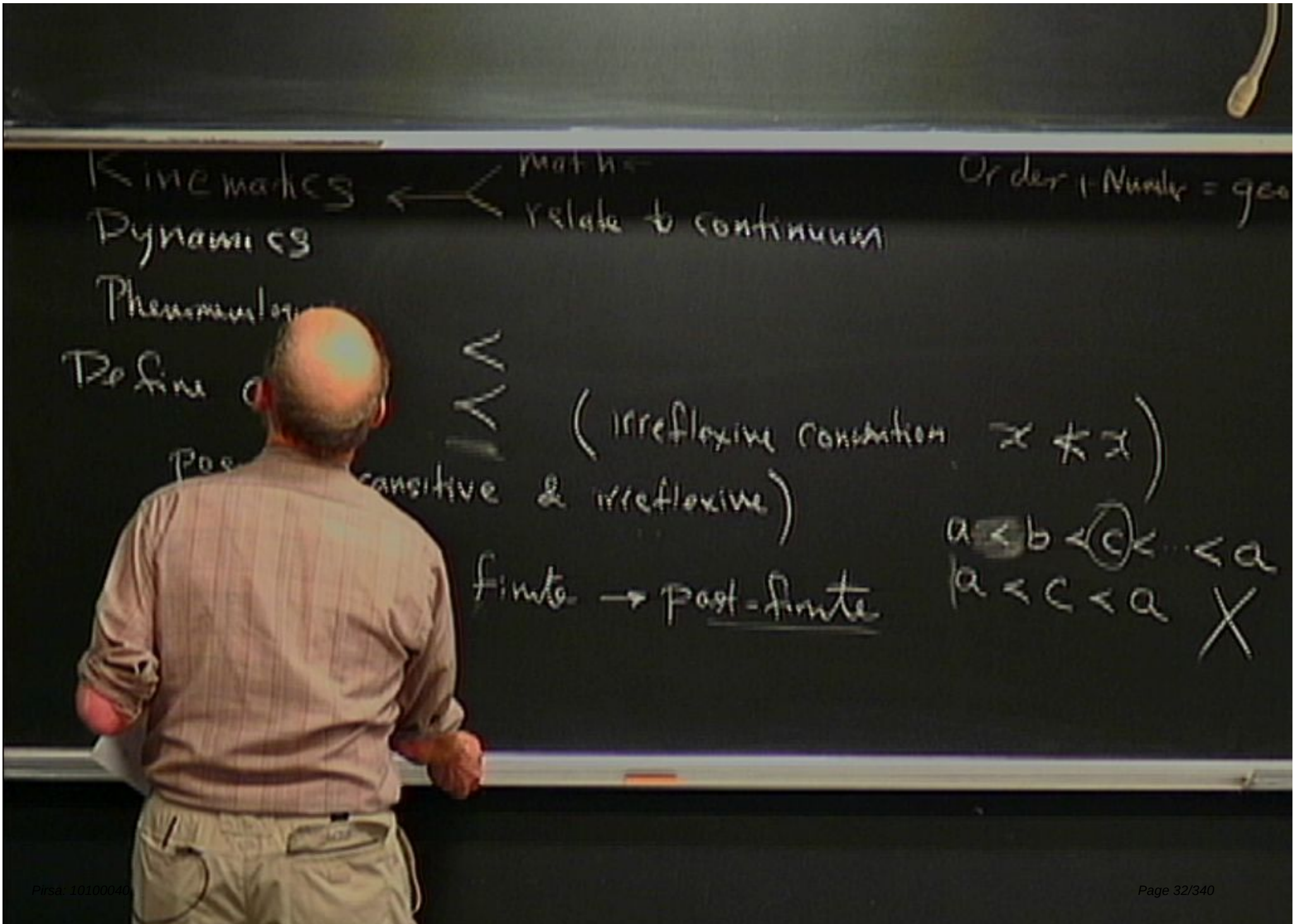

Phenomenology
Define causet $\prec$ (irreflexive connection $x \not\prec x$)
Poset (transitive & irreflexive)
Key
$a \prec b \prec c \prec \cdots \prec a$
$a \prec c \prec a$ X

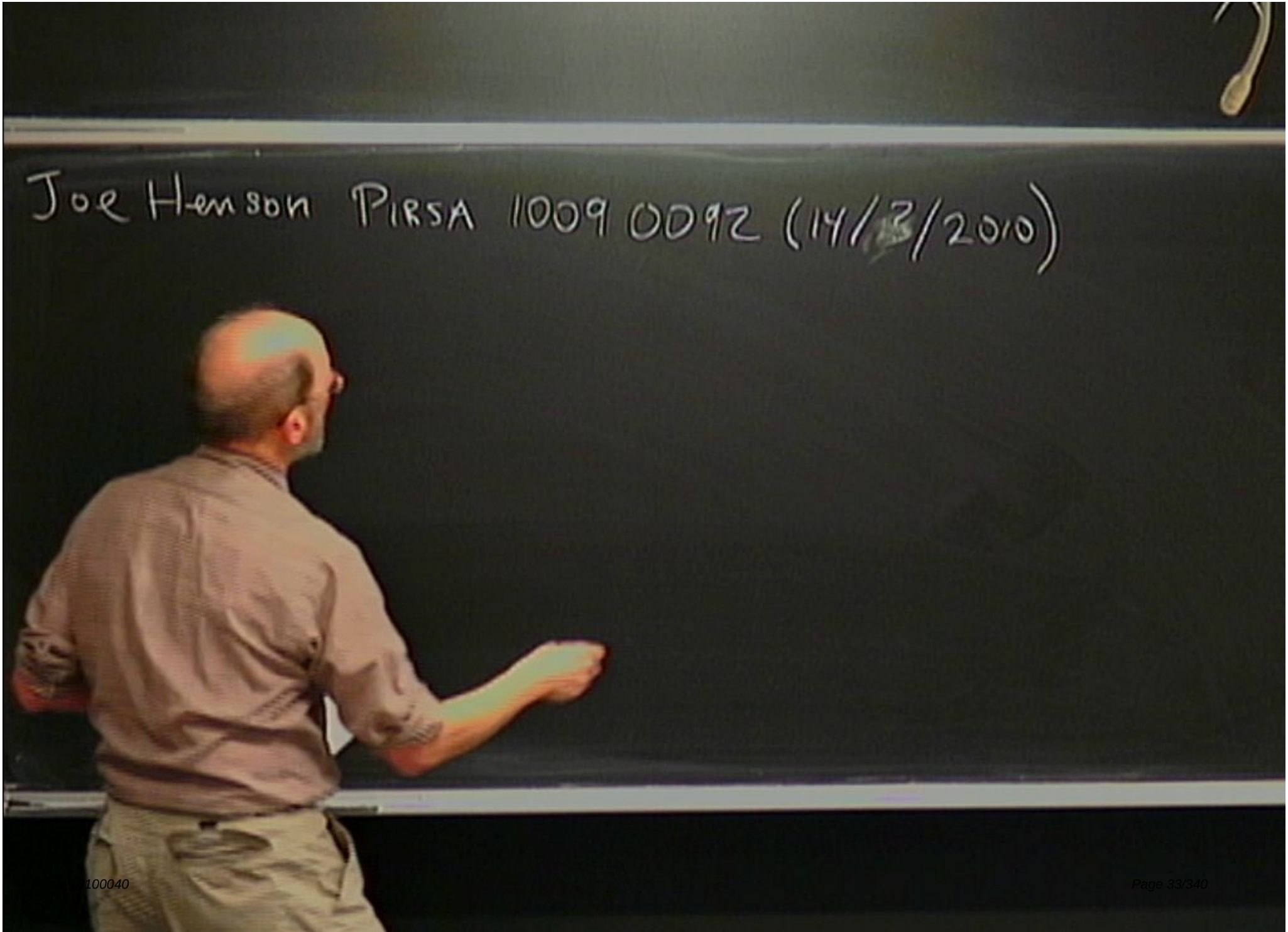

Phenomenology
Define causet
(irreflexive connection $x \not< x$)
set (transitive & irreflexive)
$a < b < c < \cdots < a$
$a < c < a$ X

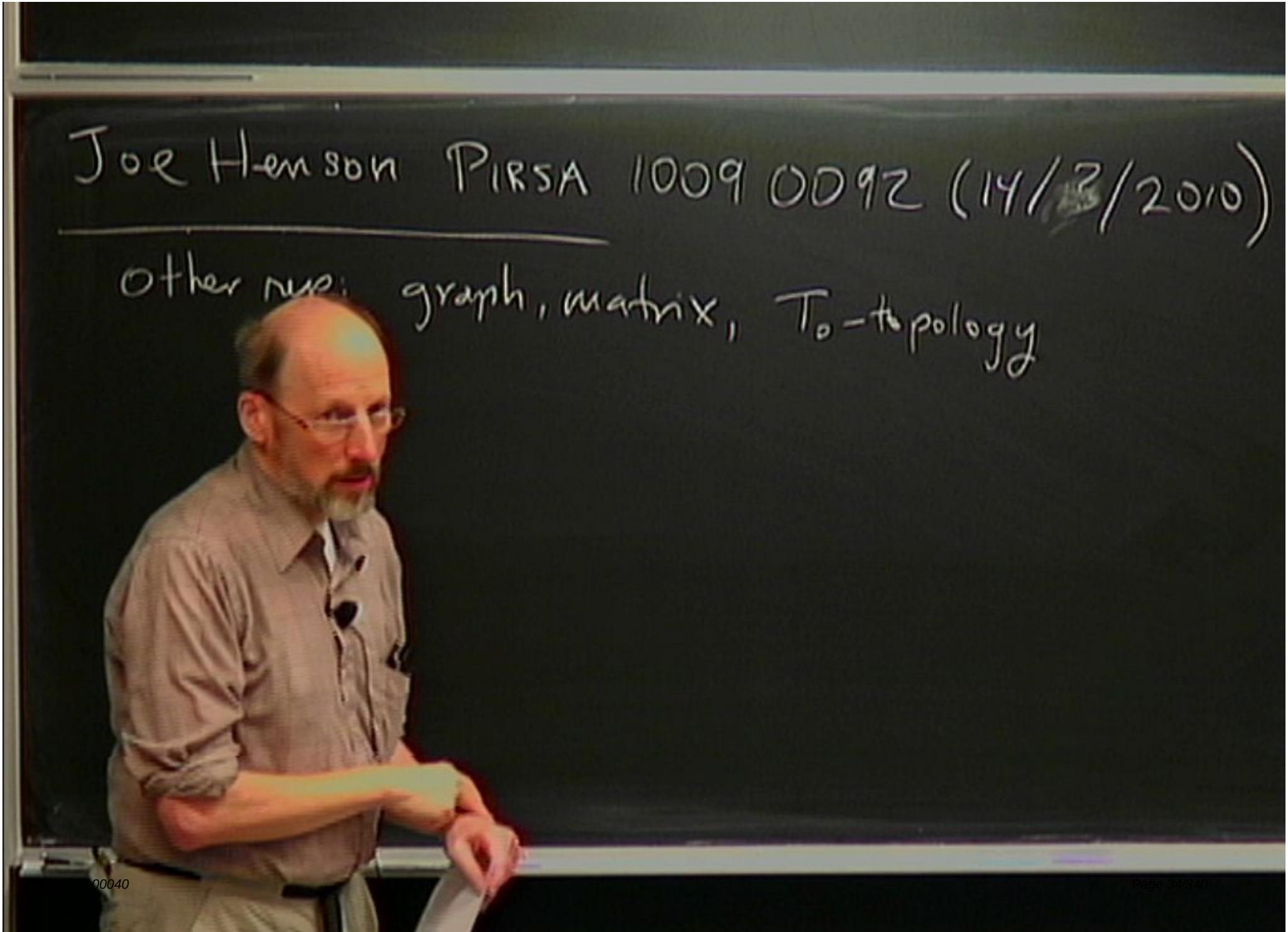

Phenomenology
Define causet
Poset (transitive & irreflexive)
Key
(irreflexive connection x ≠ x)
a < b < c < ... < a
a < c < a
X

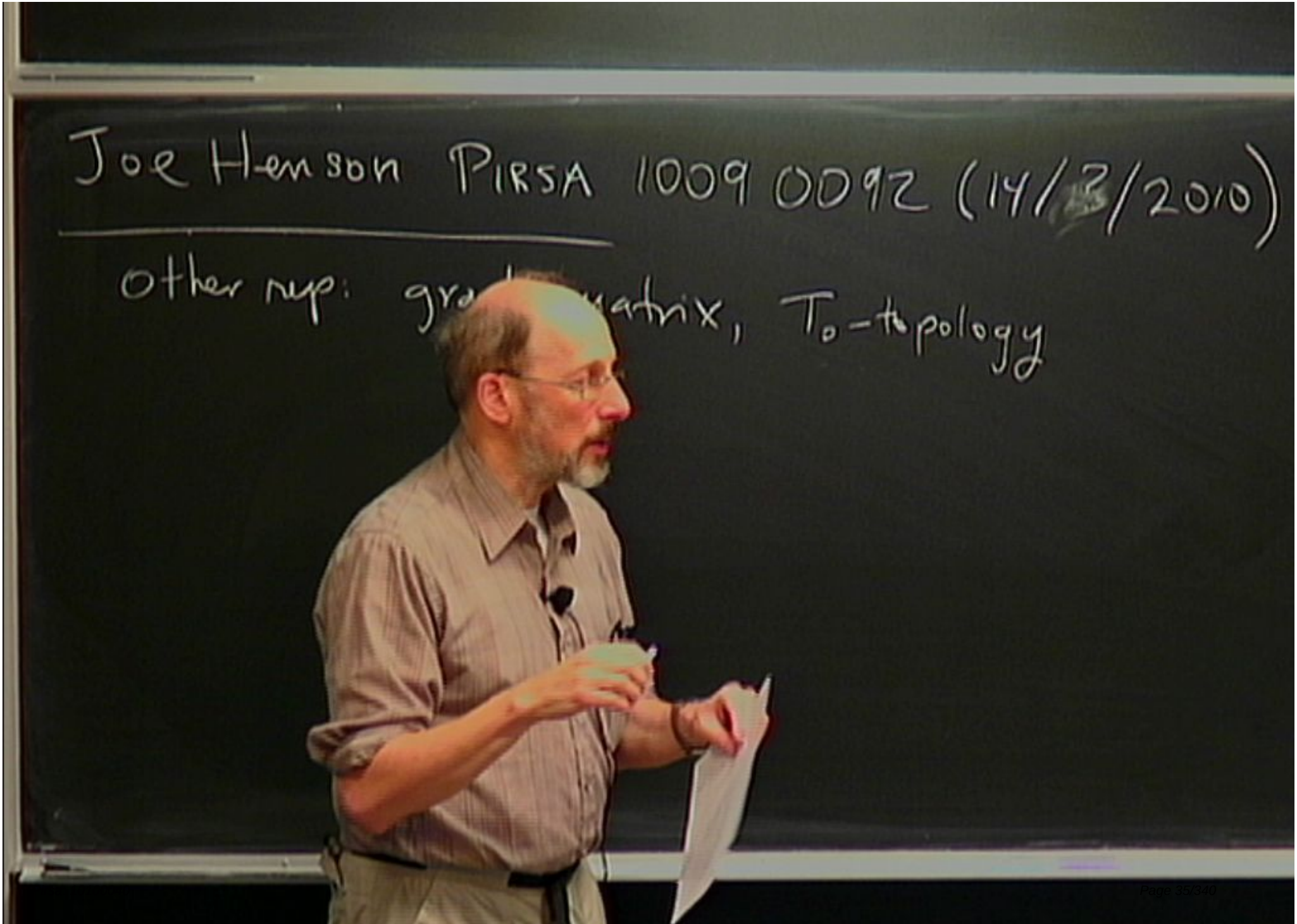

Define causet
(irreflexive connection $x \not< x$)
(transitive & irreflexive)
locally finite → past-finite
$a < b < c < \cdots < a$
$a < c < a$
X

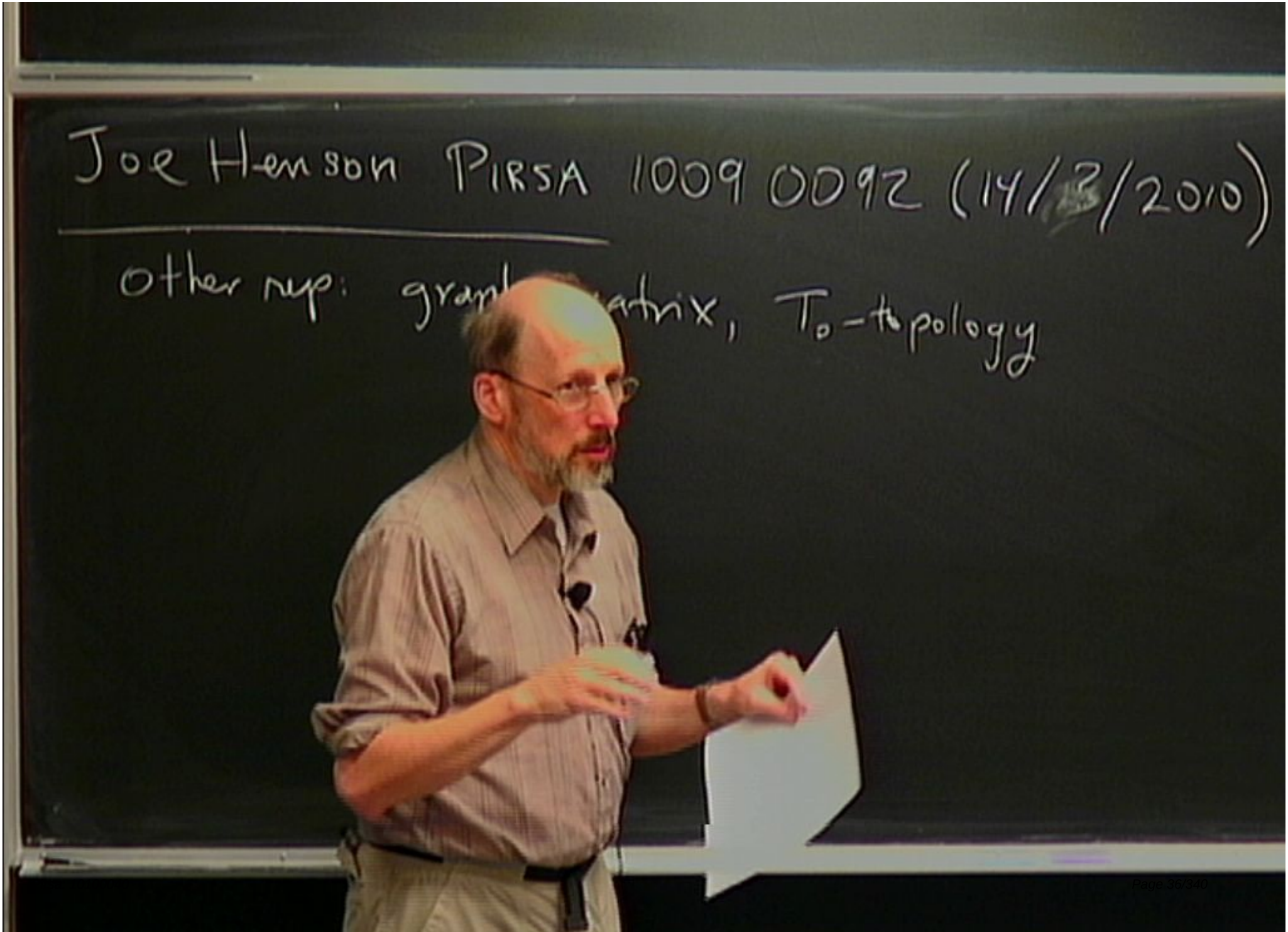

Phenomenology
Define causet ≤ (irreflexive connection x ≠ x)
et (transitive & irreflexive)
y: locally finite → past-finite
a < b < c < ··· < a
a < c < a
X


Kinematics
Dynamics
relate to continuum
Order | Numb
(irreflexive connection x ≠ x)
transitive & irreflexive)
finite → past-finite
a < b < c < ... < a
a < c < a
X


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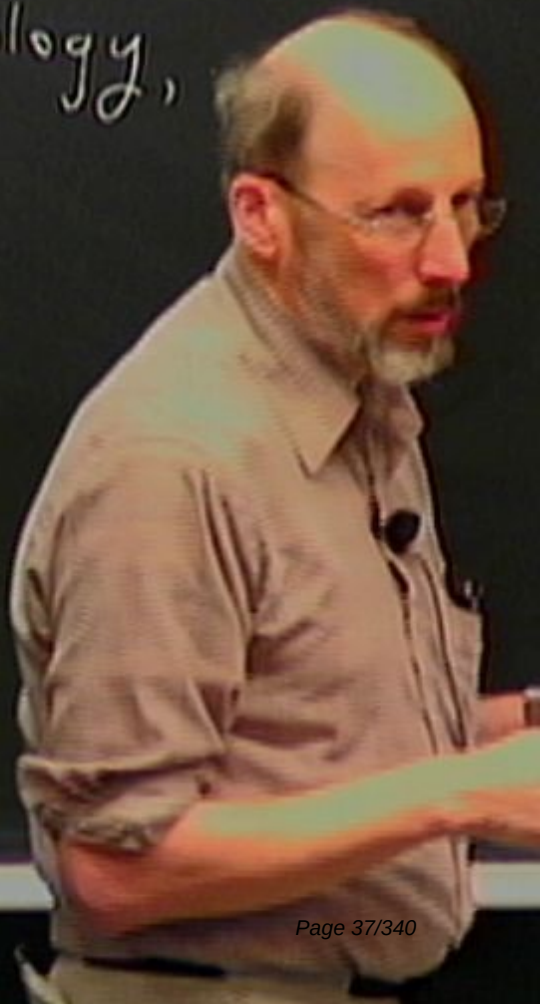

Joe Henson PIRSA 1009 0092 (14/9/2010)
other use: graph, matrix, T_0-topology


Joe Henson PIRSA 1009 0092 (14/9/2010)
other rep: gra matrix, T0-topology


Joe Henson PIRSA 1009 0092 (14/9/2010)
other rep: gram matrix, T_0-topology

Joe Henson PIRSA 1009 0092 (14/9/2010)

other rep: graph, matrix, T_0-topology,



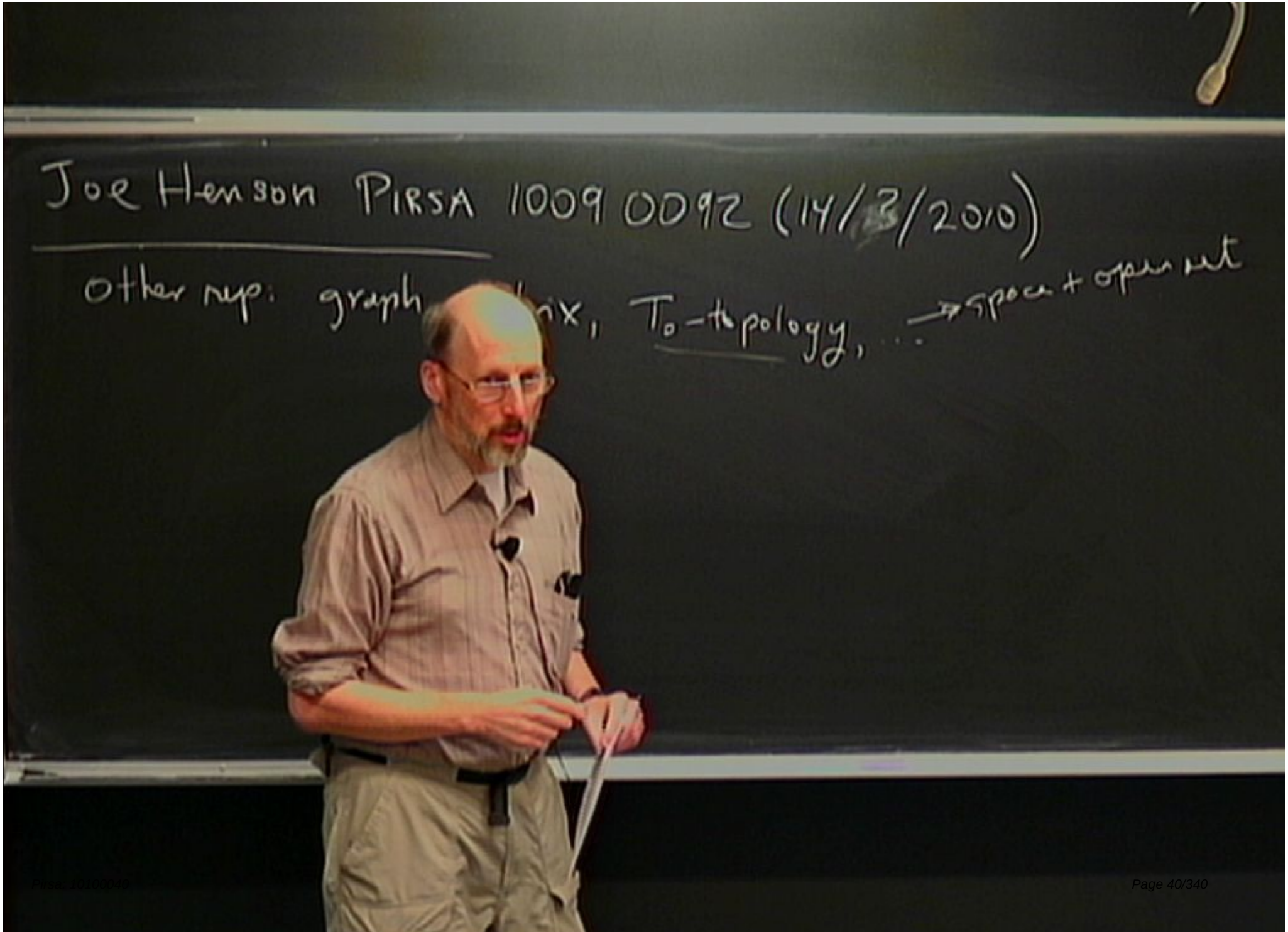
Joe Henson PIRSA 1009 0092 (14/3/2010)

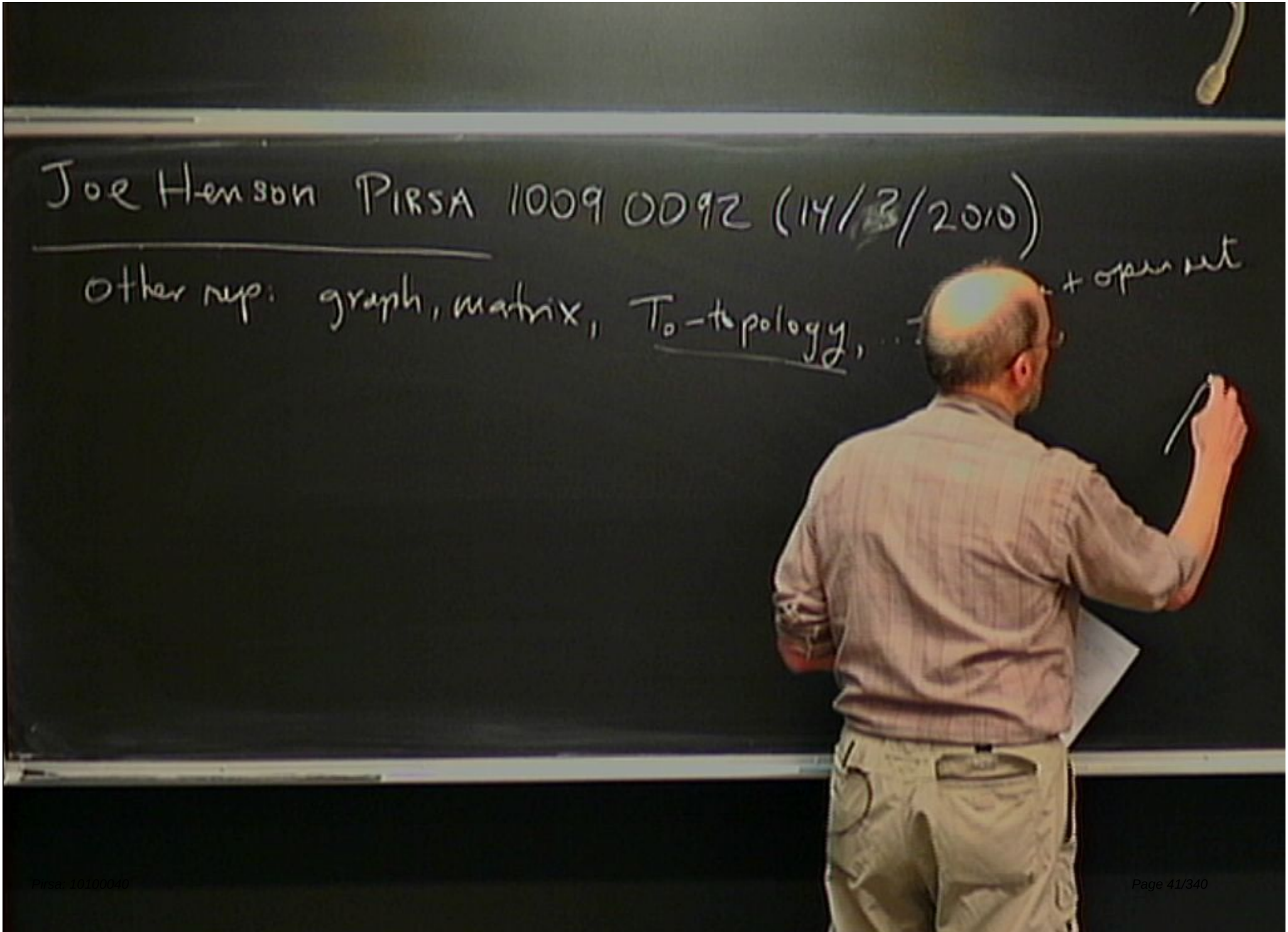
other rep: graph, matrix, T_0-topology, ...



Joe Henson PIRSA 10090092 (14/3/2010)

other rep: graph, matrix, T_0-topology, ... → space + open set

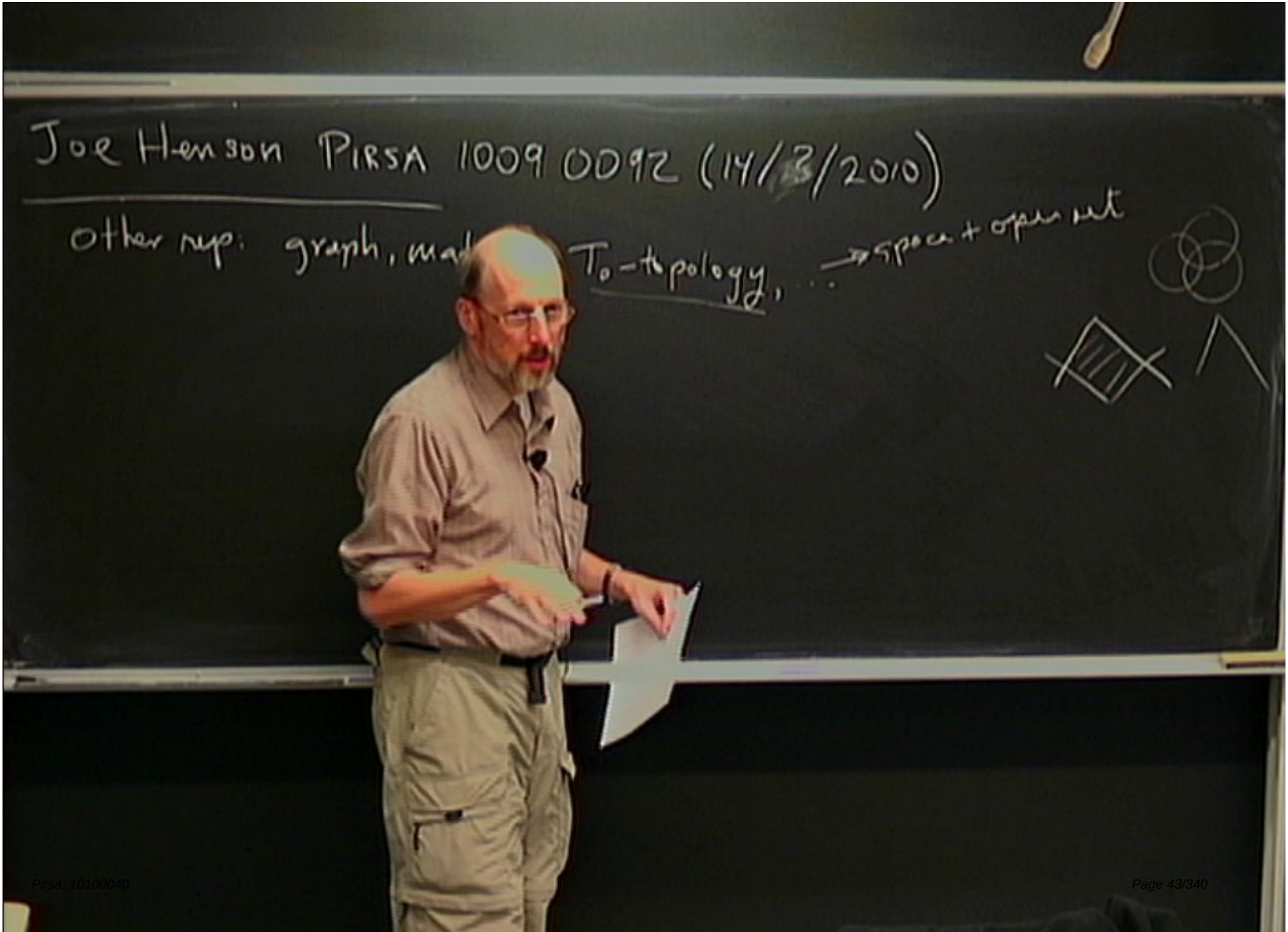

Joe Henson PIRSA 1009 0092 (14/3/2010)
other rep: graph
ix, T0-topology, ... → spacetime + operator

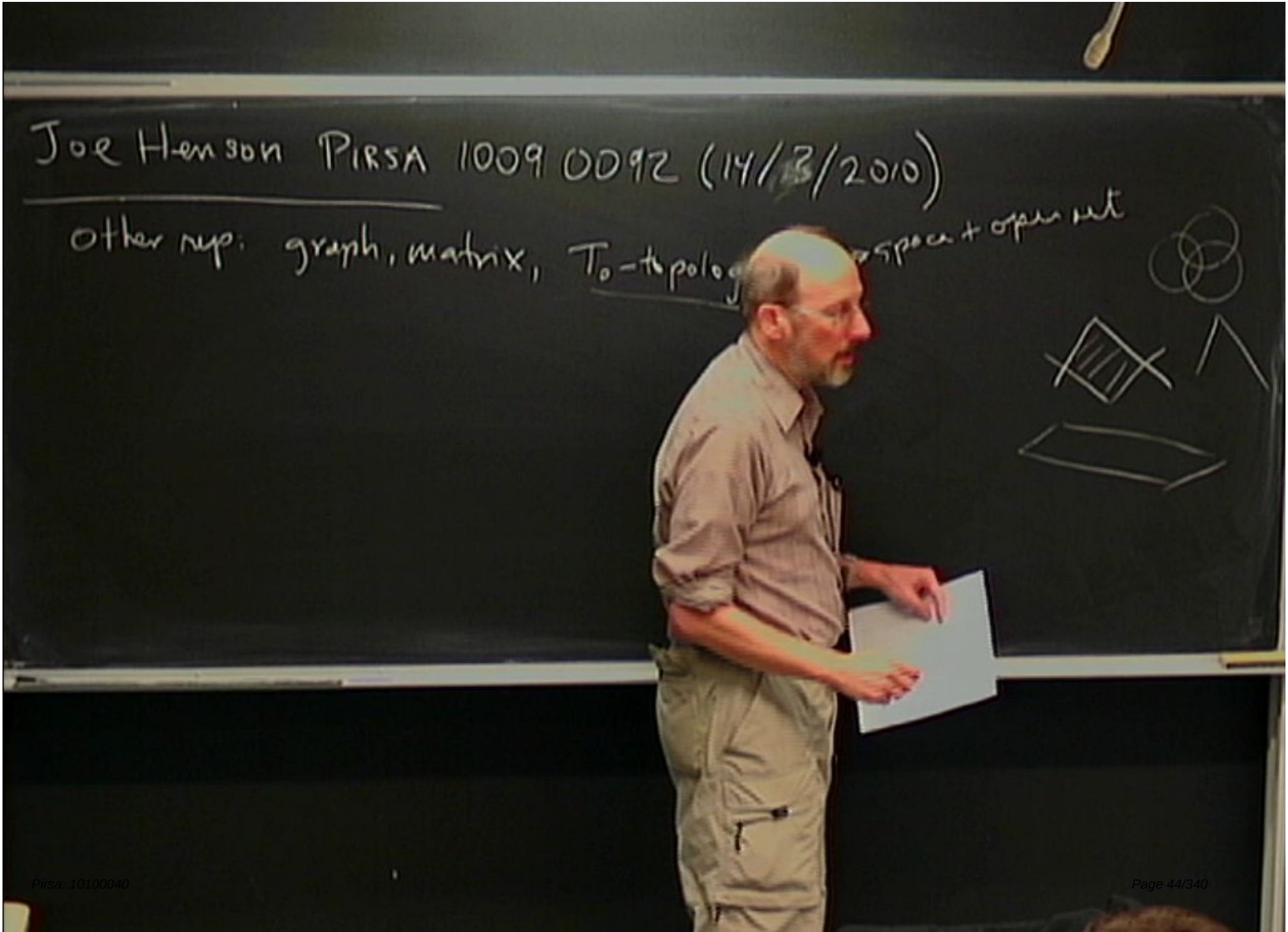

Joe Henson PIRSA 10090092 (14/3/2010)
other rep: graph, matrix, T_0-topology,
+ open set

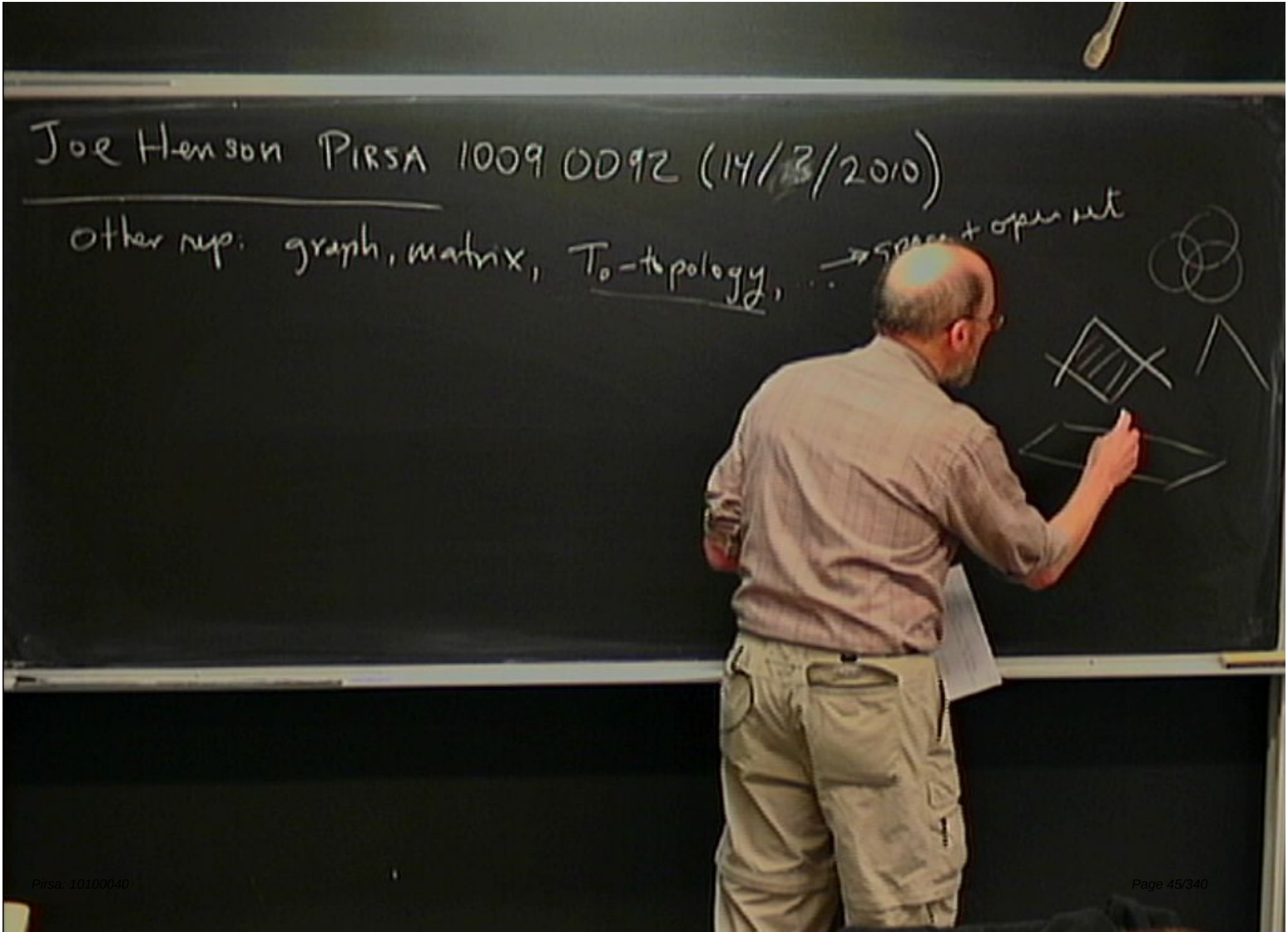
Joe Henson PIRSA 1009 0092 (14/3/2010)

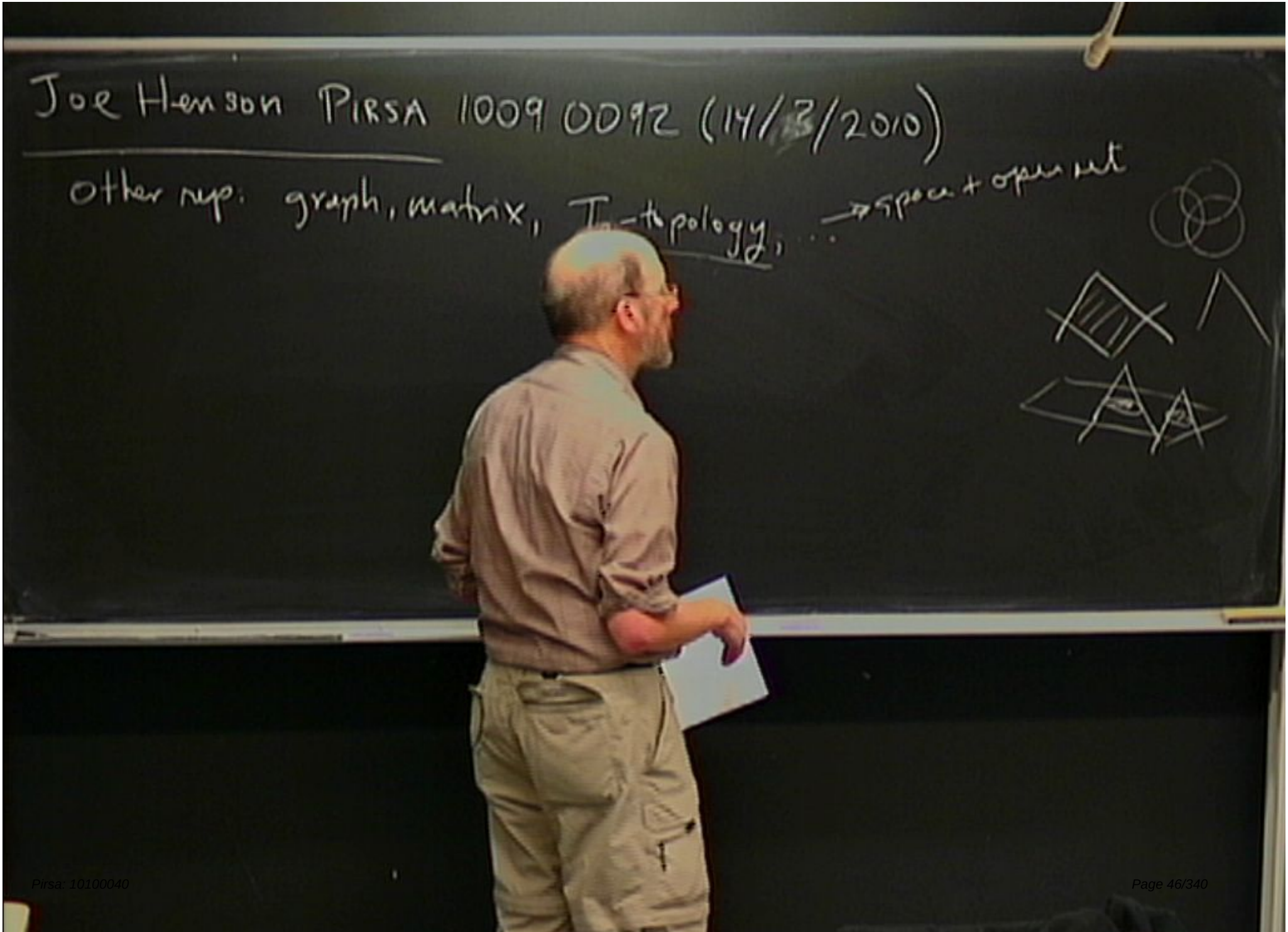
other rep: graph, matrix, T_0-topology, ... + open set

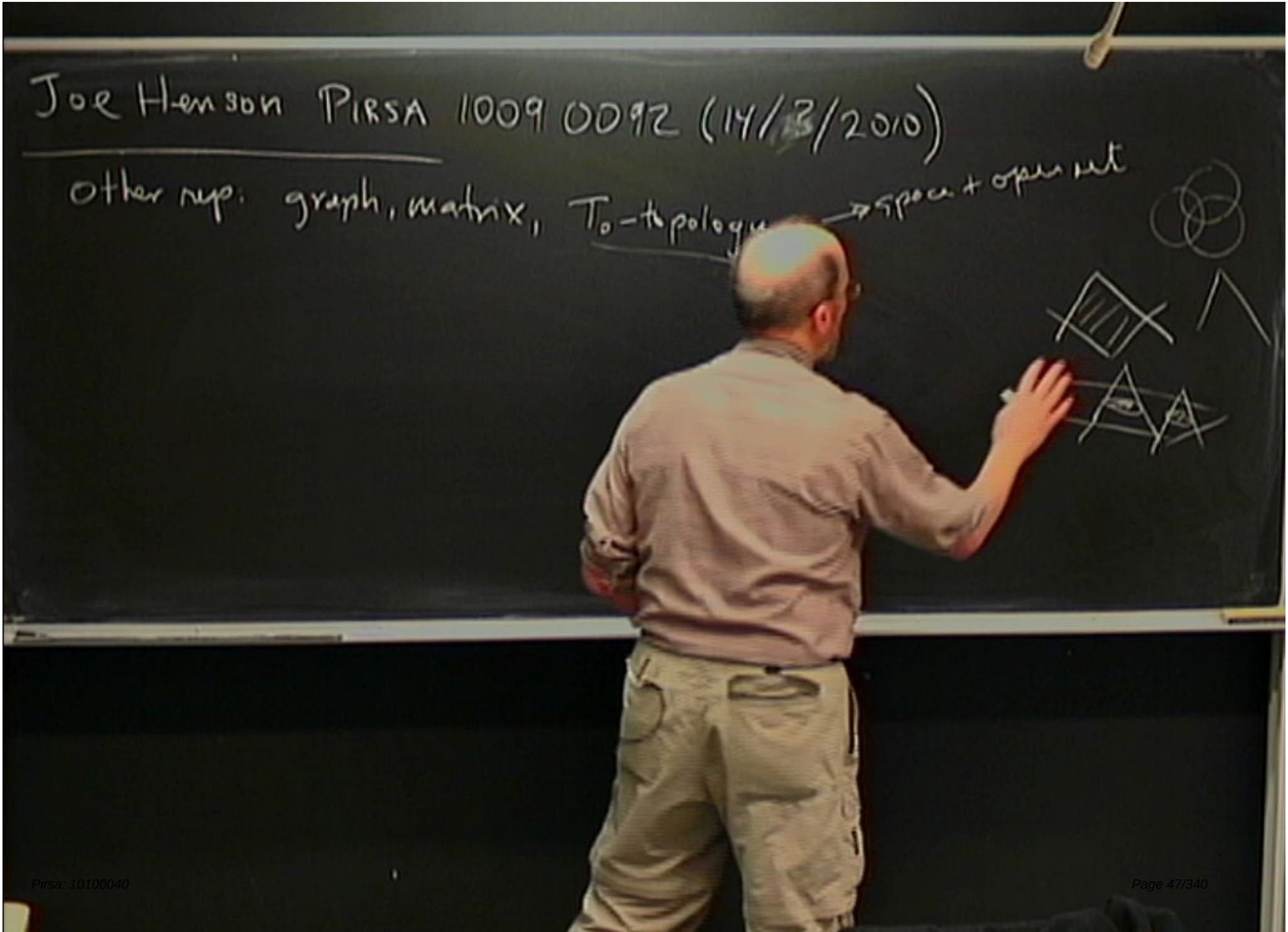


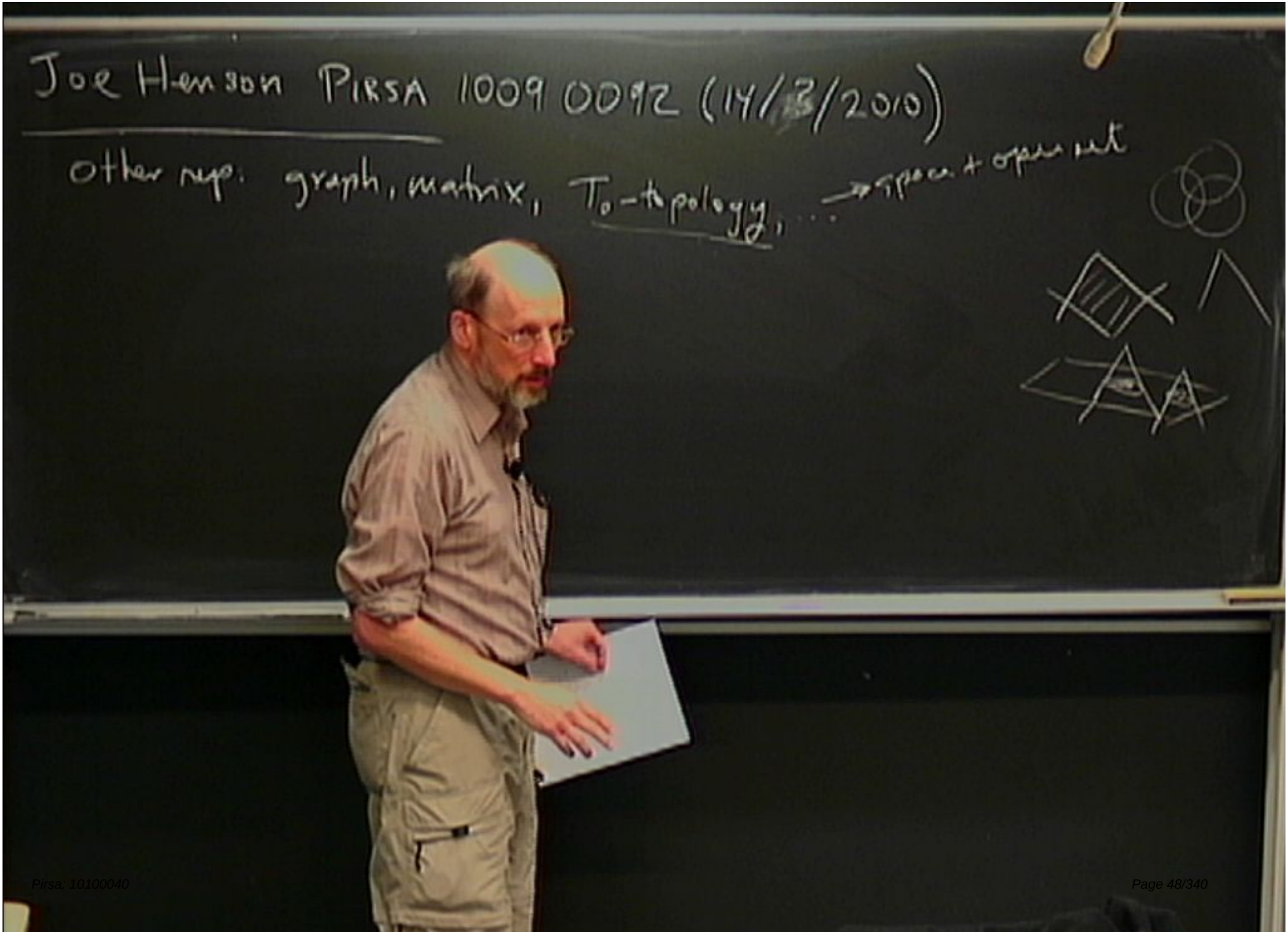

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other rep: graph, mat
T0-topology, ... → space + open set


Joe Henson PIRSA 1009 0092 (14/9/2010)
other rep: graph, matrix, T0-topolog
+ opens set


Joe Henson PIRSA 10090092 (14/8/2010)
other rep: graph, matrix, T_0-topology, ...
+ open set


Joe Henson PIRSA 10090092 (14/3/2010)
other rep: graph, matrix, T_0-topology, ... → spacetime + open set


Joe Henson PIRSA 10090092 (14/8/2010)
other rep: graph, matrix, T0-topolog
spacet + opens set


Joe Henson PIRSA 10090092 (14/12/2010)
other rep: graph, matrix, T_0-topology, ... → Poset + open set

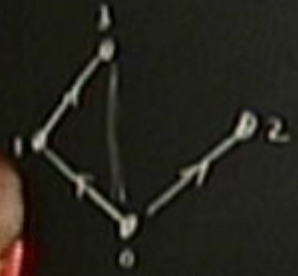
Key. locally finite → Past-finite

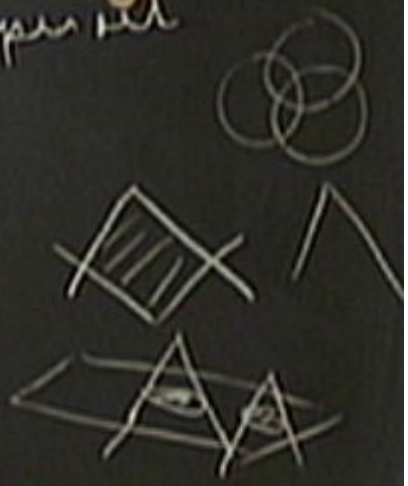
$a \prec b \prec c \prec \cdots \prec a$

$a \prec c \prec a$

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Other rep: (graph), (matrix) T_0-topology, ... → space + open set





Key. locally finite → past-finite

$a < b < c < \cdots < a$

$a < c < a$

Joe Henson PIRSA 10090092 (14/3/2010)

other rep: graph, matrix T_0-topology, ... → space + open set



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_1 < e_2,\ e_0 < e_3$



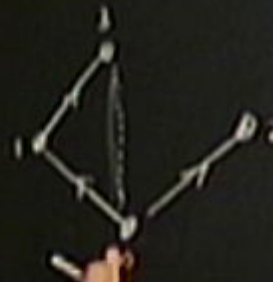
Key. locally finite → past-finite

$a < b < c < \dots < a$

$a < c < a$

Joe Henson PIRSA 1009 0092 (14/3/2010)

Other rep: graph, matrix, T_0-topology, ... → space + open set



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$



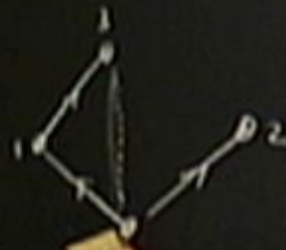
Key. locally finite → past-finite

$a < b < c < \cdots < a$

$a < c < a$ ✗

Joe Henson PIRSA 1009 0092 (14/9/2010)

other rep: (graph), (matrix) T_0-topology, ... → poset + open set



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$

Hasse diagram



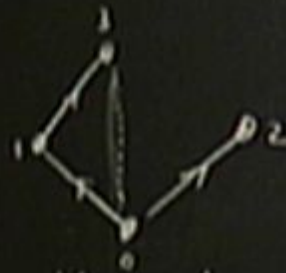
Key. locally finite → past-finite

$a < b < c < \cdots < a$

$a < c < a$ ✗

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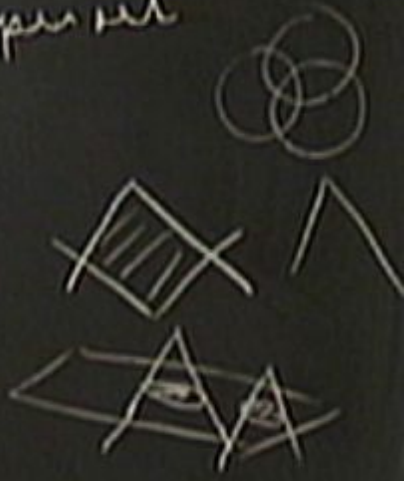
other rep: (graph), (matrix) T_0-topology, ... → space + open set



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

Hasse diagram



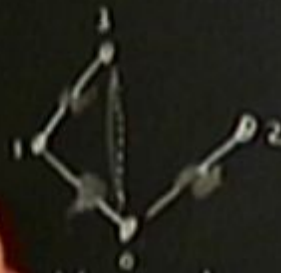
Key. locally finite → past-finite

$a < b < c < \dots < a$

$a < c < a$ ✗

Joe Henson PIRSA 10090092 (14/3/2010)

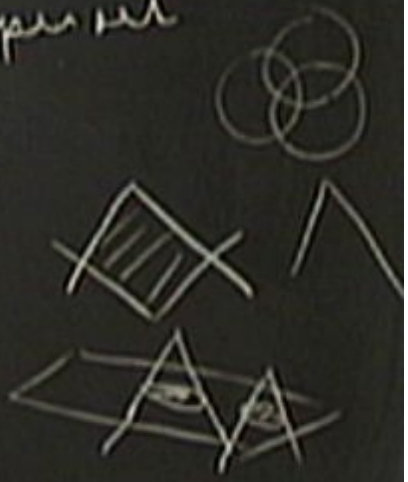
other rep: graph, matrix, T_0-topology, ... → poset + open set



$e_0\ e_1\ e_2\ e_3$

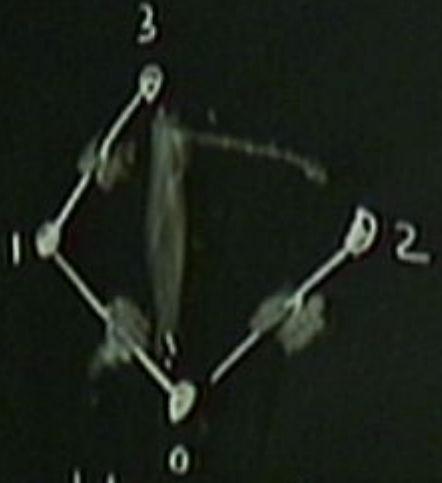
$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

Hasse diagram



Joe Henson PIRSA 1009 0092

other rep: (graph), (matrix) T_0-topo



Hasse diagram

$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

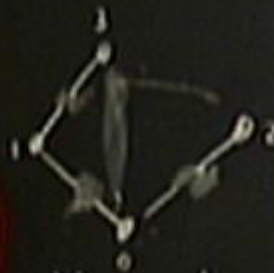
Key. locally finite → past-finite

$a < b < c < \dots < a$

$a < c < a$ ✗

Joe Henson PIRSA 10090092 (14/3/2010)

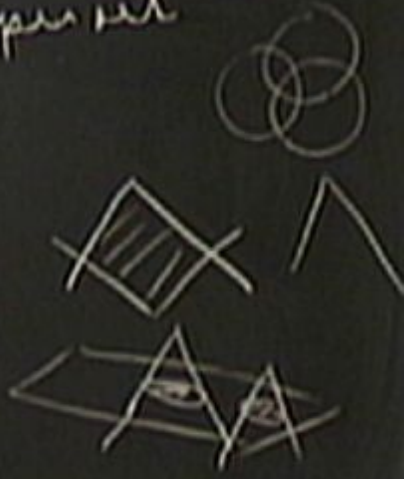
other rep: (graph), (matrix) T_0-topology, ... → poset + open set



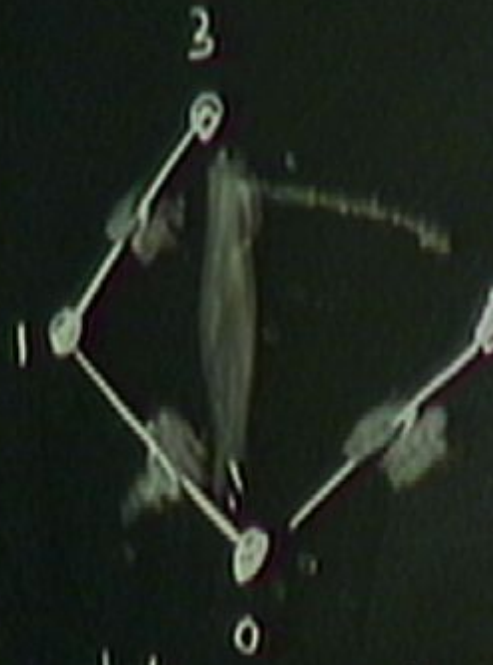
$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$

Hasse diagram
only links drawn



Other rep: graph, matrix

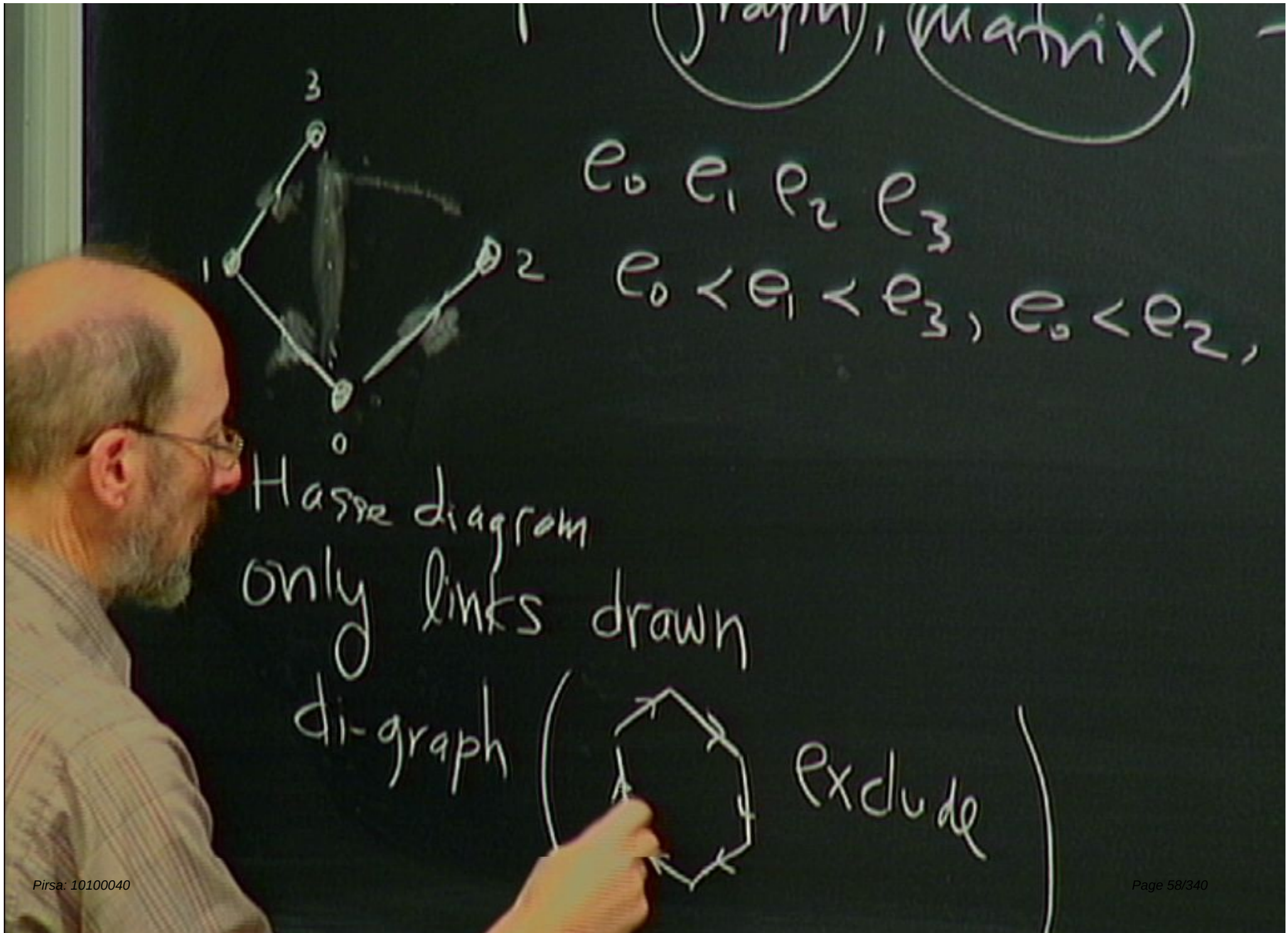


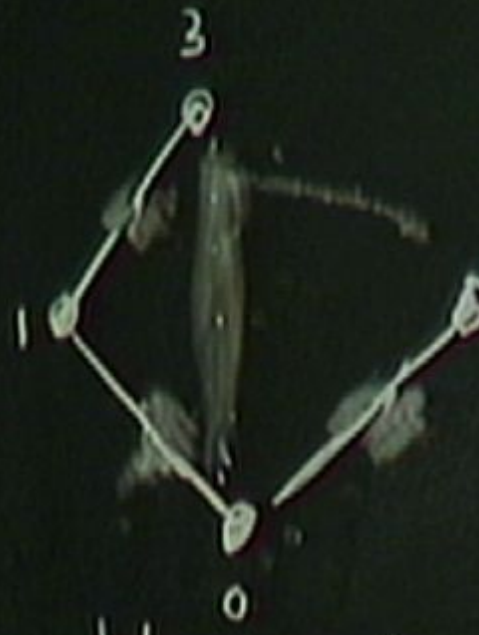
e_0 e_1 e_2 e_3

$e_0 < e_1 < e_3$, $e_0 < e_2$, e

Hasse diagram

only links drawn

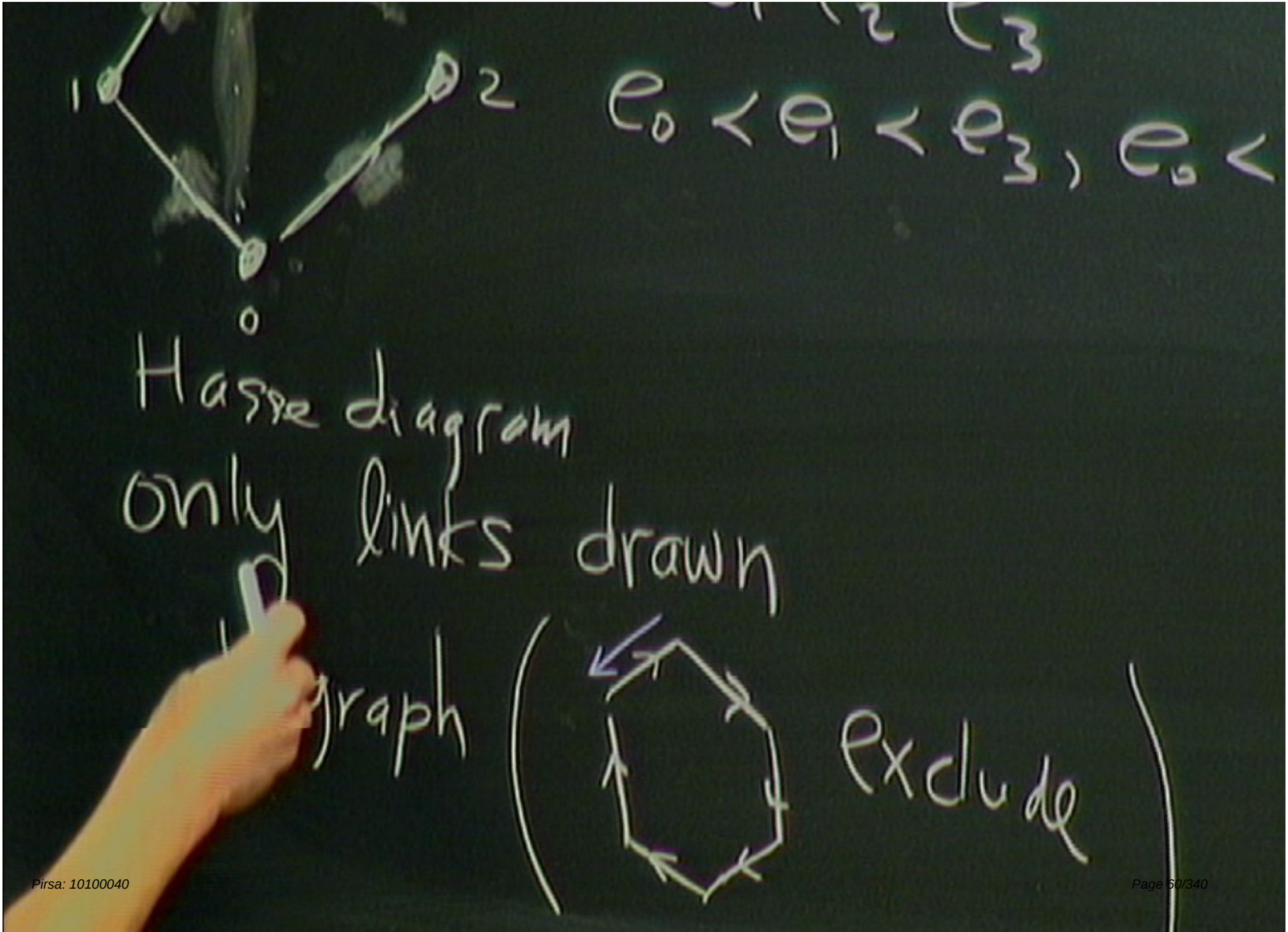

3
1
2
0
e_0 e_1 e_2 e_3
$e_0 < e_1 < e_3$, $e_0 < e_2$,
Hasse diagram
only links drawn
di-graph
exclude

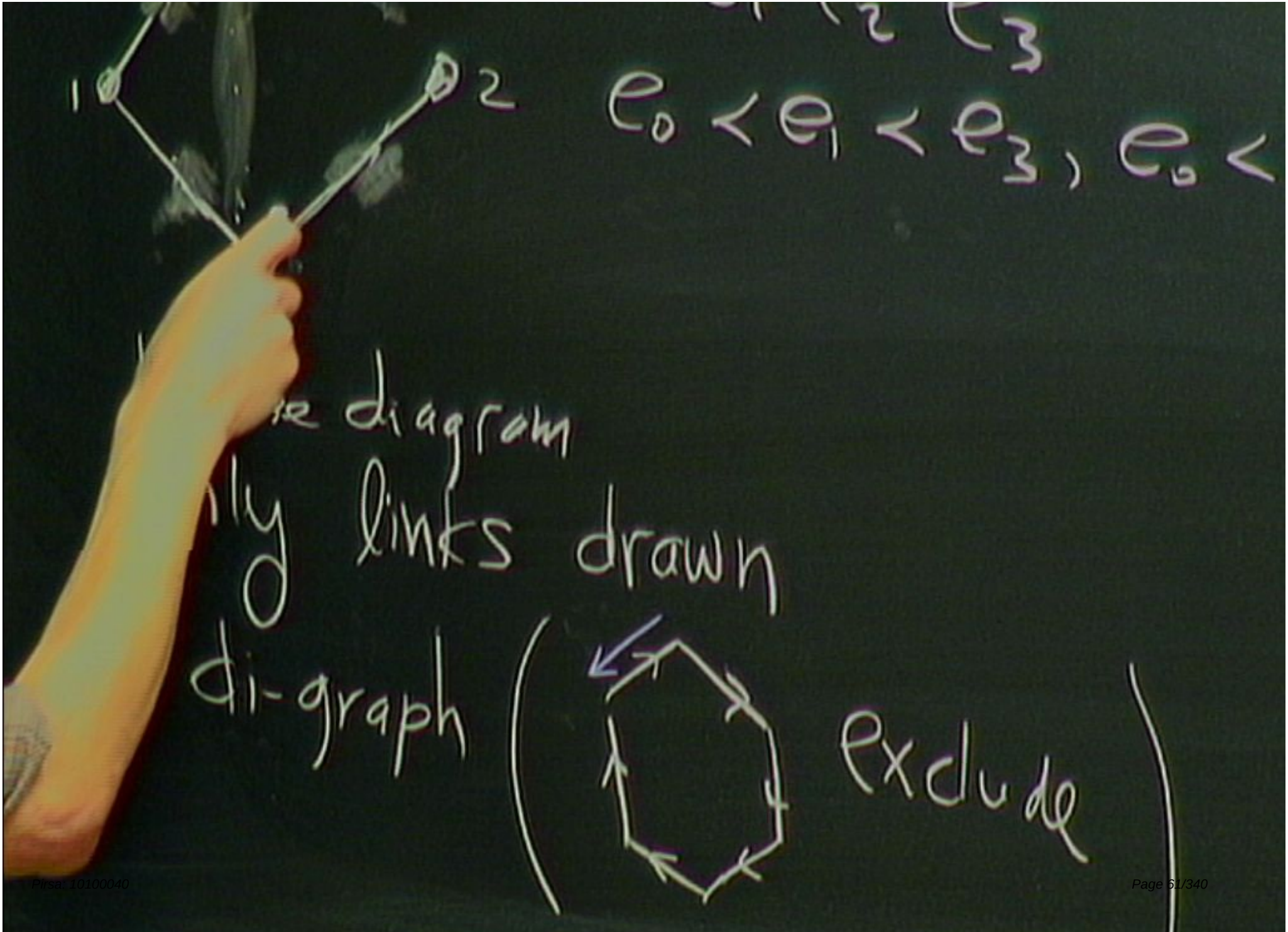


$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,$

Hasse diagram
only links drawn
di-graph (exclude)


1
2
0
$e_0 < e_1 < e_3$, $e_0 <$
Hasse diagram
only links drawn
graph
exclude


1
2
$e_0 < e_1 < e_3$, $e_0 <$
diagram
only links drawn
di-graph
exclude

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix, T_0-topology;



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

Hasse diagram

only links

di-g

exclude

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \left(\begin{array}{cccc} & & & \\ & & & \\ & & & \\ & & & \end{array} \right) \end{array}$$

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



e_0 e_1 e_2 e_3

$e_0 < e_1 < e$ $< e_2$, $e_0 < e_3$

Hasse diagram
only links drawn

di-graph

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0 & 1 & & 1 \\ & & & \\ & & & \end{pmatrix} \end{array}$$

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



e_0 e_1 e_2 e_3

$e_0 < e$, $e_0 < e_2$, $e_0 < e_3$

Hasse diagram
only links dr
di-graph

	0	1	2	3
0	0	1	1	1
1				
2				
3				

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



e_0 e_1 e_2 e_3

$e_0 < e_1$, $e_0 < e_2$, $e_0 < e_3$

	0	1	2	3
0	0	1	1	1
1	0	0		
2				

Hasse diagram only links drawn

di-graph

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



e_0 e_1 e_2 e_3

$e_0 < e_1 < e_2$, $e_0 < e_3$

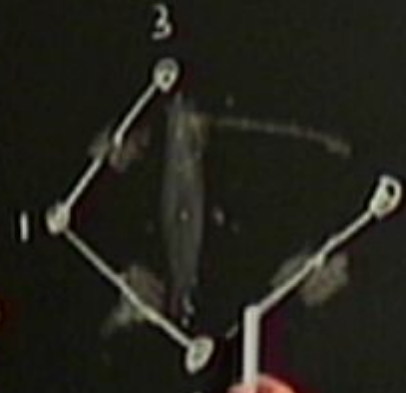
	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0		

Hasse diagram
only links drawn

di-graph

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



e_0 e_1 e_2 e_3

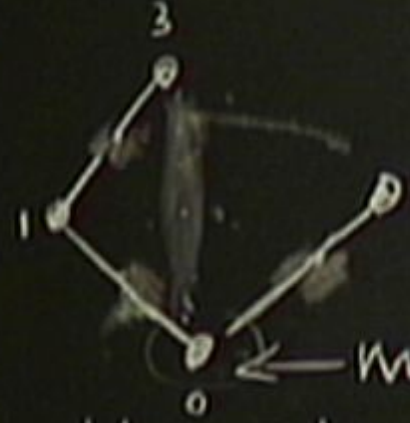
$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$

Ha[illegible]iagram
onl[illegible]links drawn
d[illegible]raph ( exclude)

	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0	0	0

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

← minimal

Hasse diagram

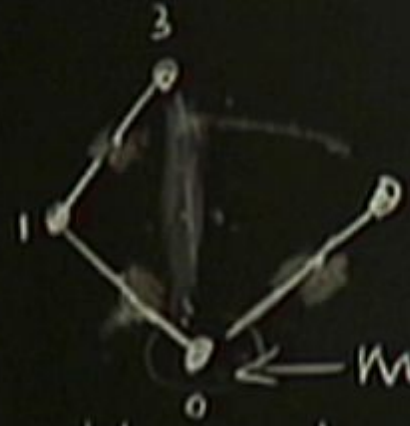
only links drawn

di-graph (exclude)

	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0	0	0

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



e_0 e_1 e_2 e_3

$e_0 < e_1 < e_3$, $e_0 <$ e $e_0 < e_3$

← minimal

Hasse diagram

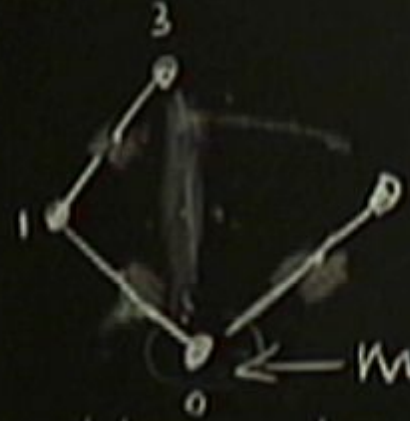
only links drawn

di-graph ( exclu

	1	2	3
	1	1	1
	0	0	1
	0	0	0
	0	0	0

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

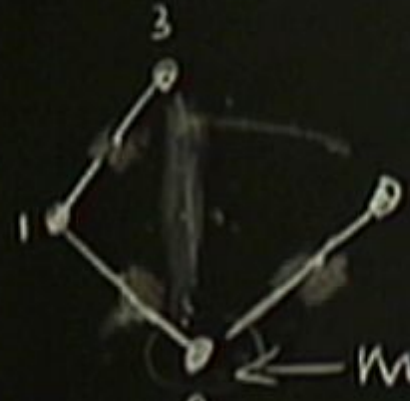
← minima

Hasse diagram
only links
di-graph
exclude

	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0	0	0

Joe Henson PIRSA 1009 0092 (14/

other rep: graph, matrix T_0-topology,



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

← minim

Hasse diagram

only links

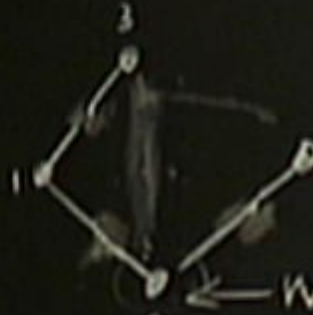
di-graph

exclude

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

Joe Henson PIRSA 1009 0092 (14/3/2010)

other rep: (graph), (matrix) T_0-topology, ... → spectr



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

← minimal

Hasse diagram

only links drawn

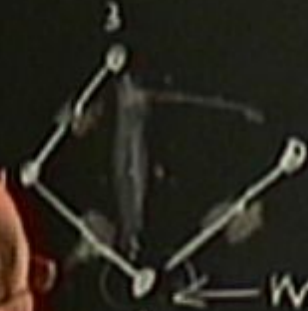
di-graph (exclude)

	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0			
3				

causal matrix

Joe Henson PIRSA 1009 0092 (14/9/2010)

other rep: graph, matrix, T_0-topology, ... → spacet



e_0 e_1 e_2 e_3

$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$

← minimal

Hasse diagram

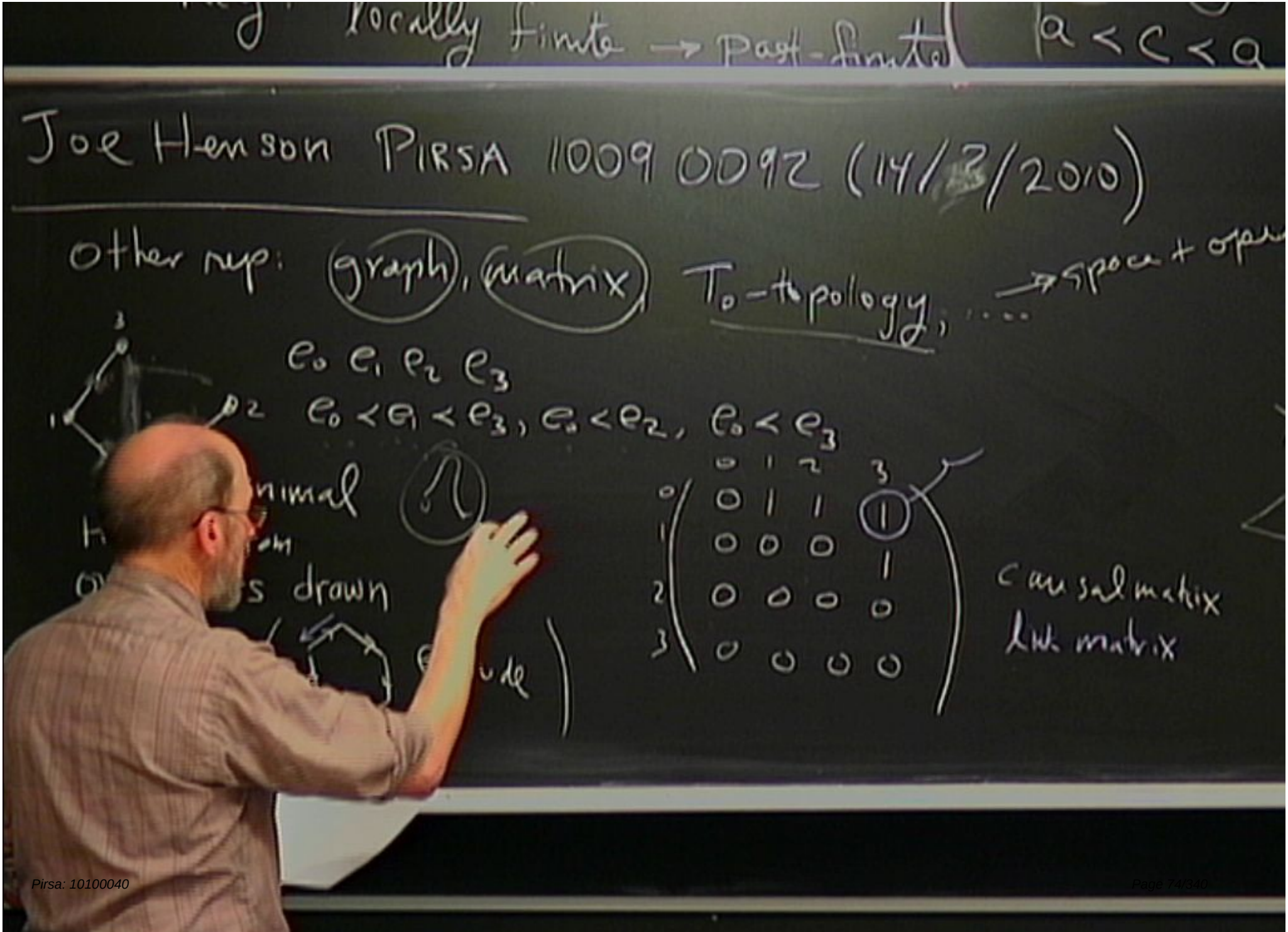
only links drawn

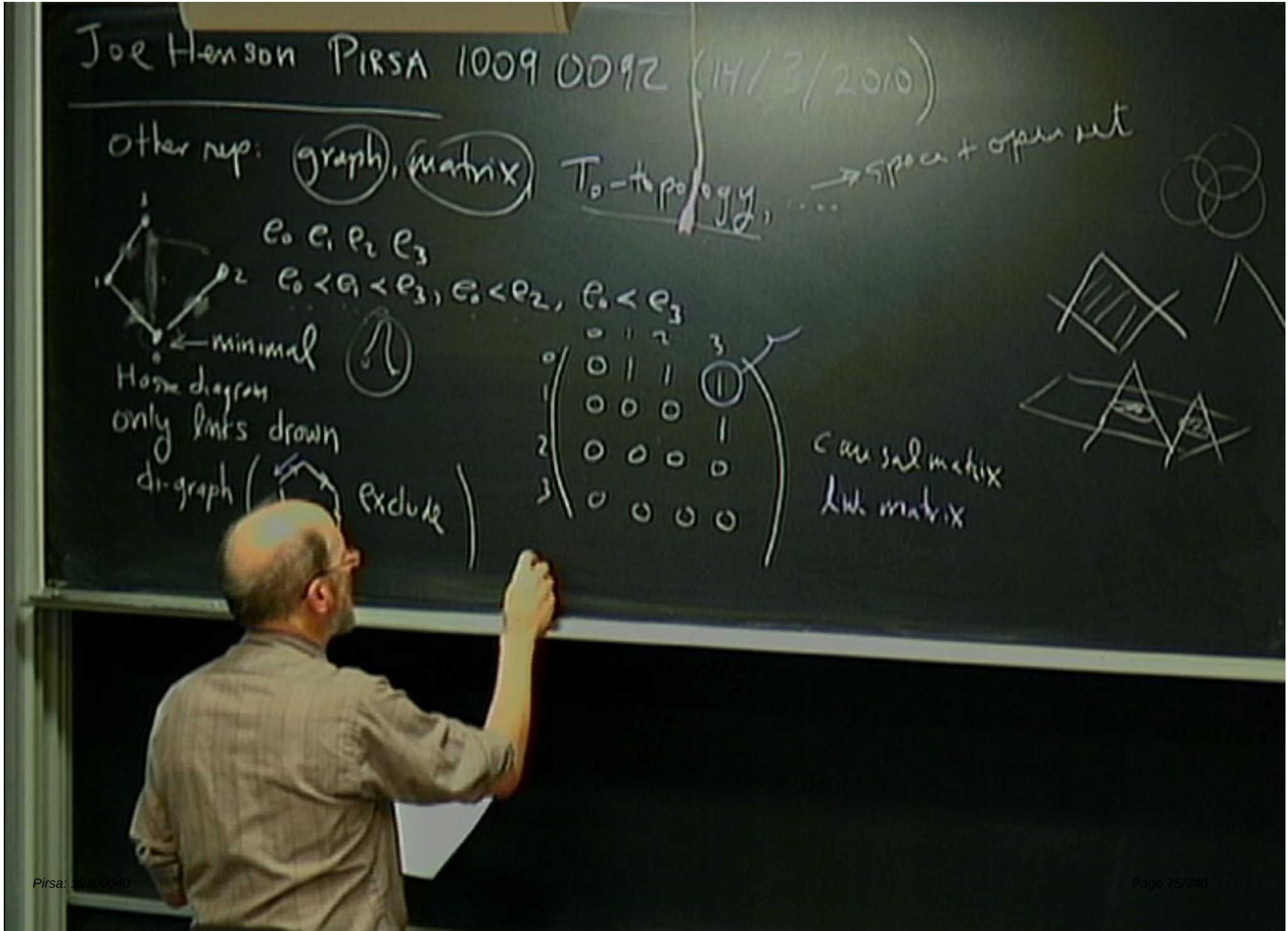
di-graph ( exclude)

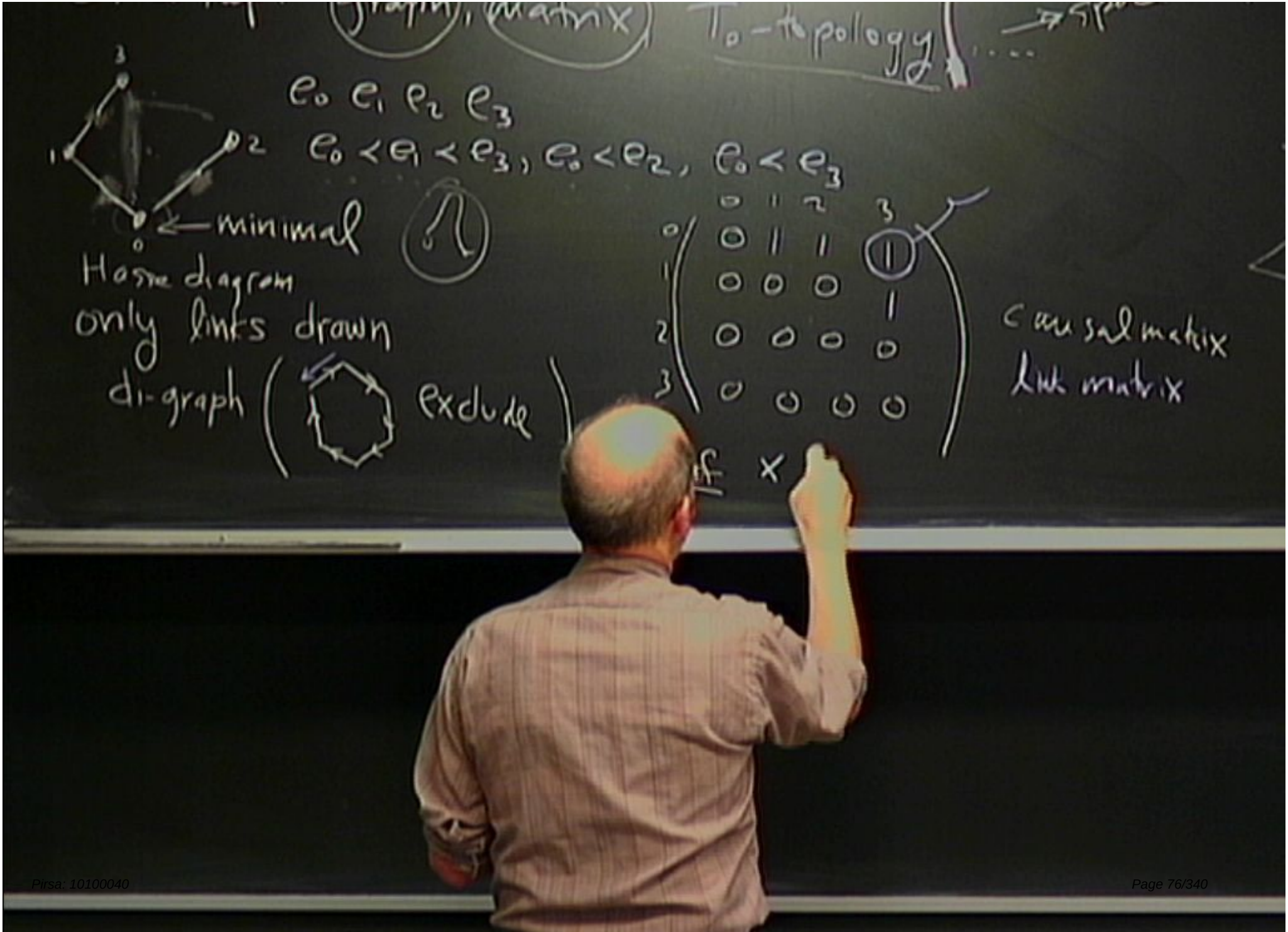
	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0	0	0

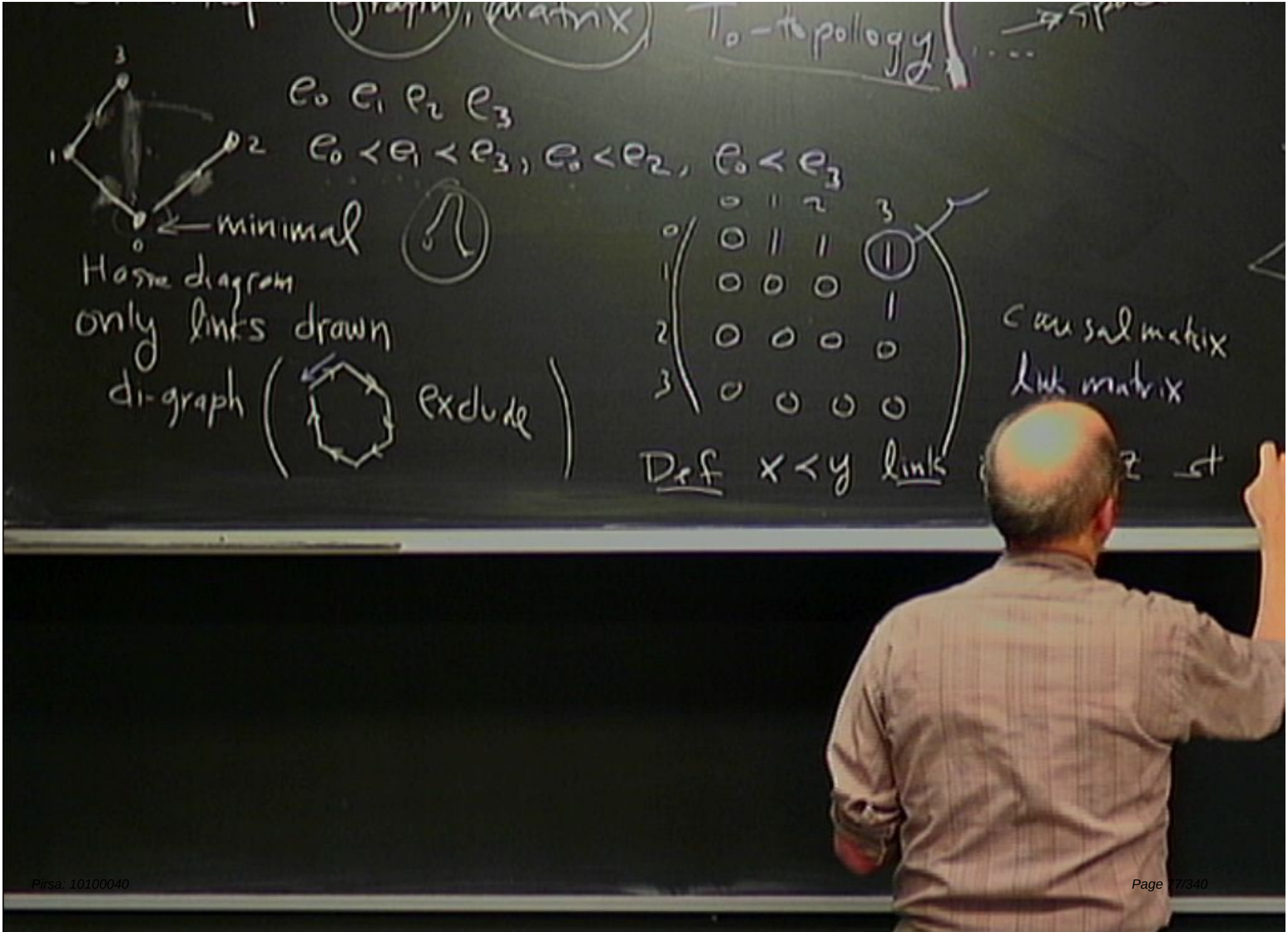
causal matrix

Inc. matrix


locally finite → past-finite
a < c < a
Joe Henson PIRSA 1009 0092 (14/3/2010)
other rep: graph, matrix, T_0-topology, ... → space + opens
e_0 e_1 e_2 e_3
$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$
minimal
drawn
0 1 2 3
0 0 1 1 1
1 0 0 0 1
2 0 0 0 0
3 0 0 0 0
causal matrix
link matrix

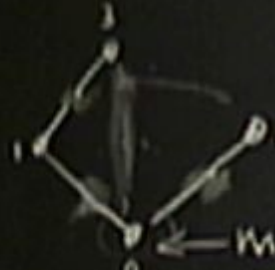

Joe Henson PIRSA 1009 0092 (14/9/2010)
other rep: graph, matrix
T_0-topology, ... → space + open set
e_0 e_1 e_2 e_3
$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$
minimal
Hasse diagram
only links drawn
di-graph
exclude
causal matrix
Inc. matrix


T_0-topology
$e_0\ e_1\ e_2\ e_3$
$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$
minimal
Hasse diagram
only links drawn
di-graph
exclude
causal matrix
link matrix


T_0-topology
$e_0\ e_1\ e_2\ e_3$
$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$
minimal
Hasse diagram
only links drawn
di-graph
exclude
causal matrix
link matrix
Def $x < y$ link

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other rep: graph, matrix, T_0-topology, ... → space + open set



$e_0\ e_1\ e_2\ e_3$

$e_0 \prec e_1 \prec e_3,\ e_0 \prec e_2,\ e_0 \prec e_3$

← minimal

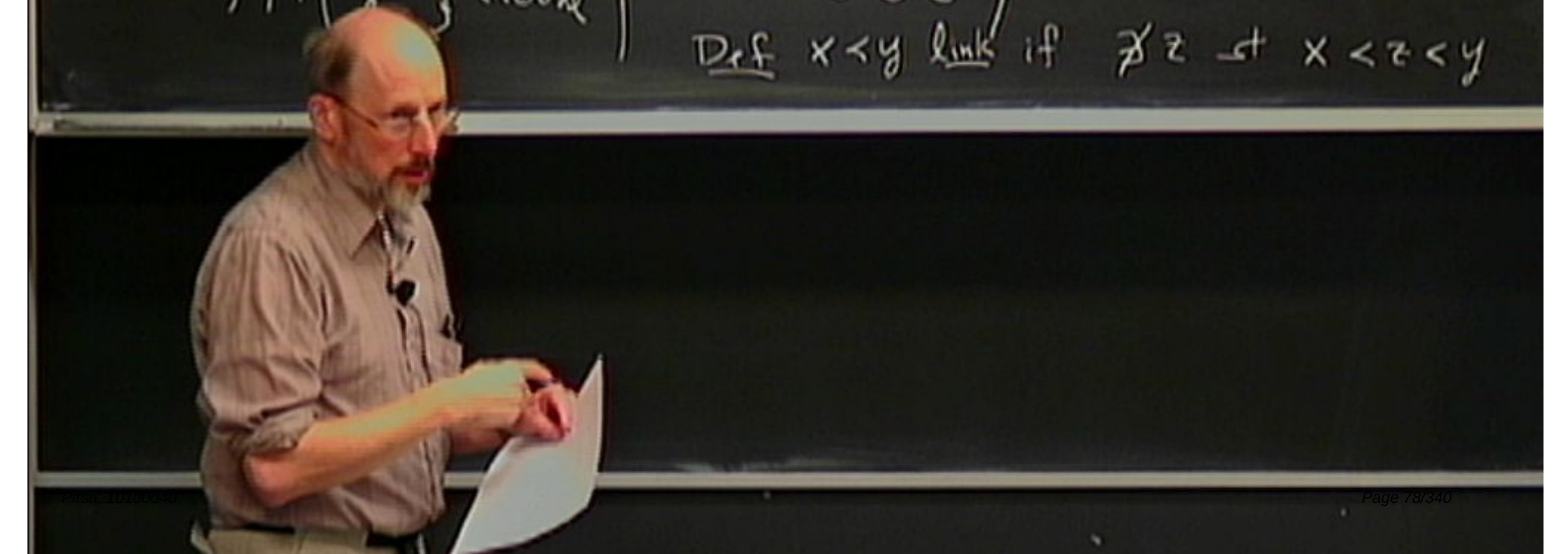
Hasse diagram
only links drawn

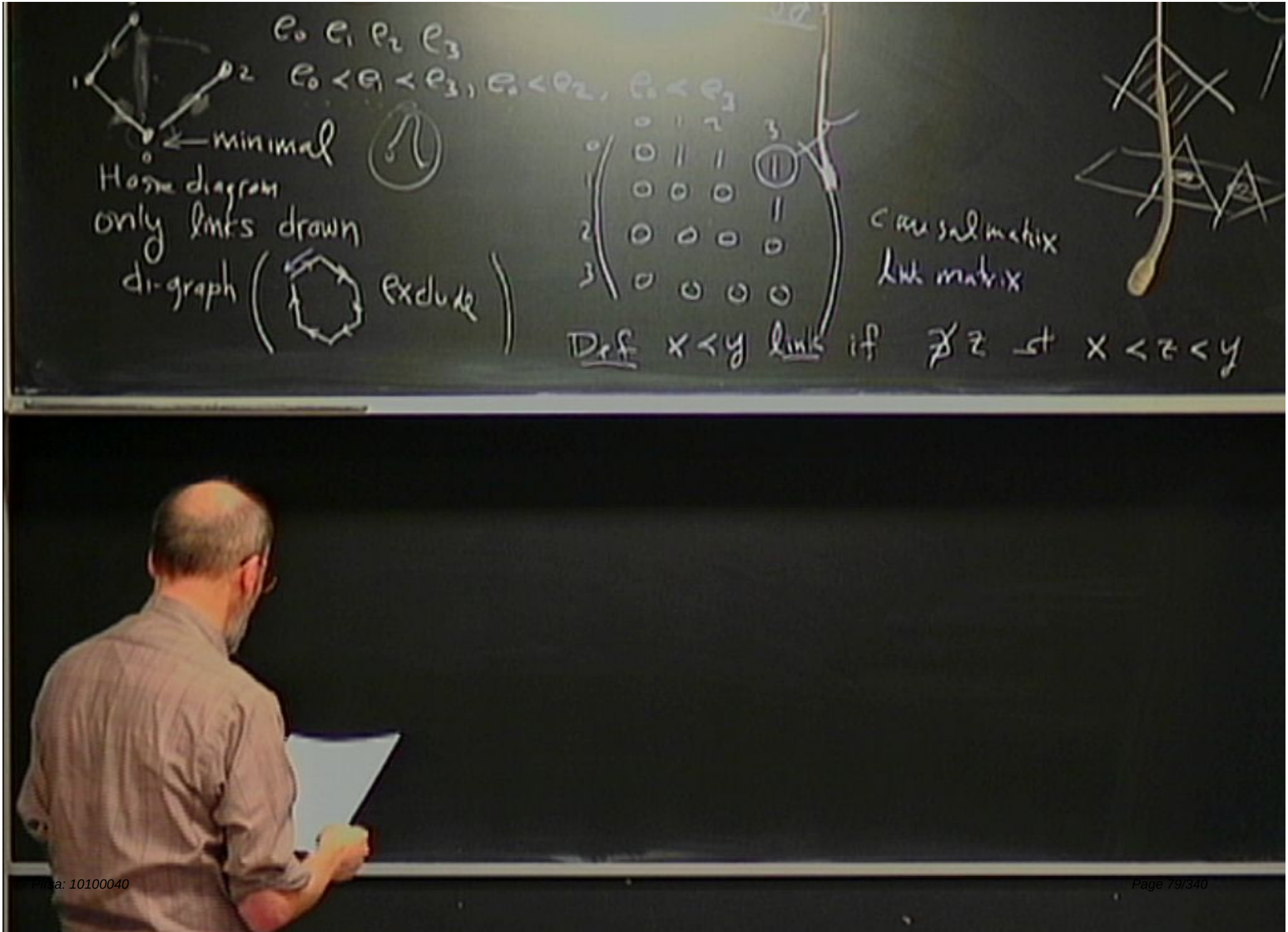
di-graph ( exclude)

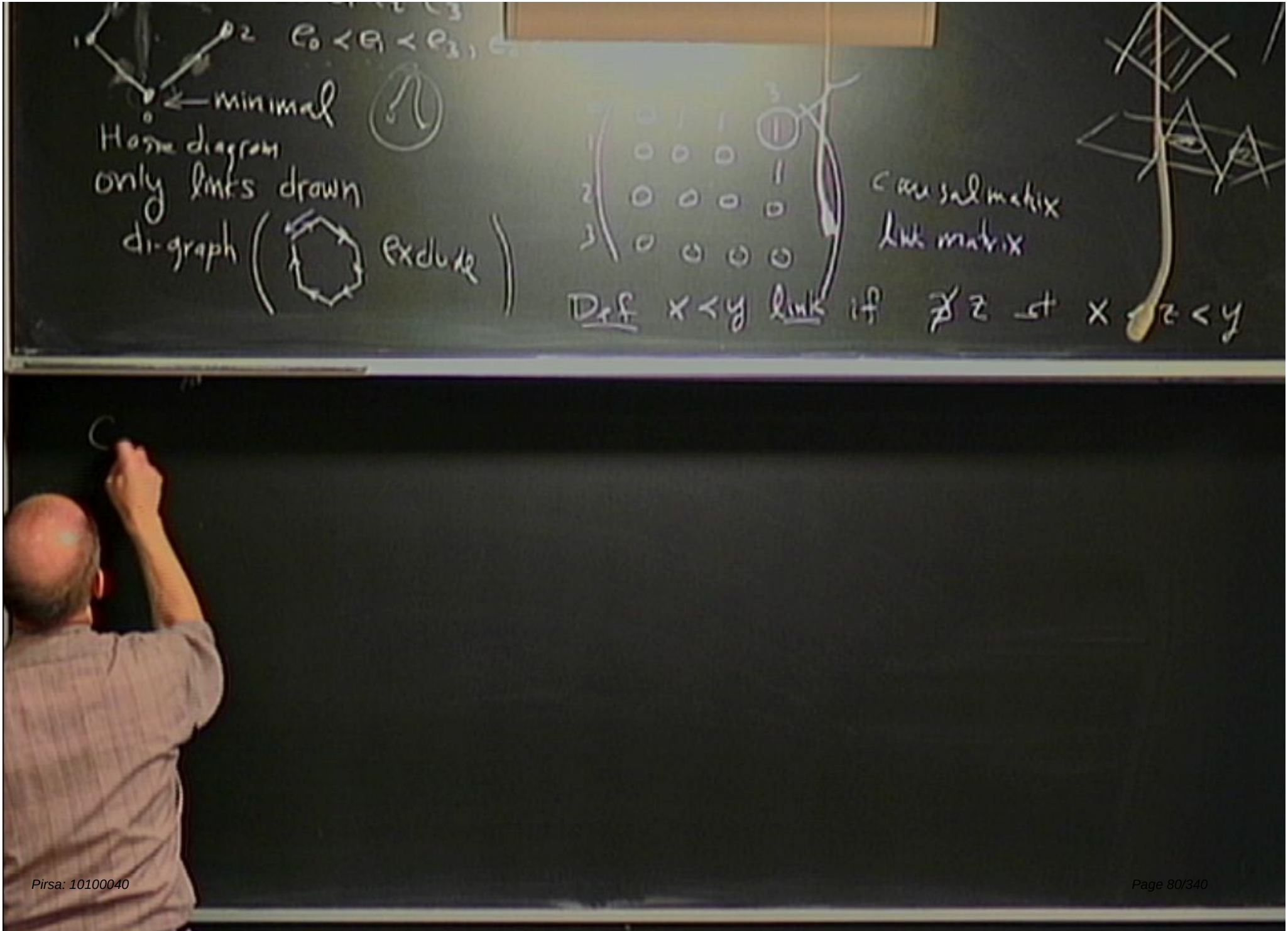
	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0	0	0

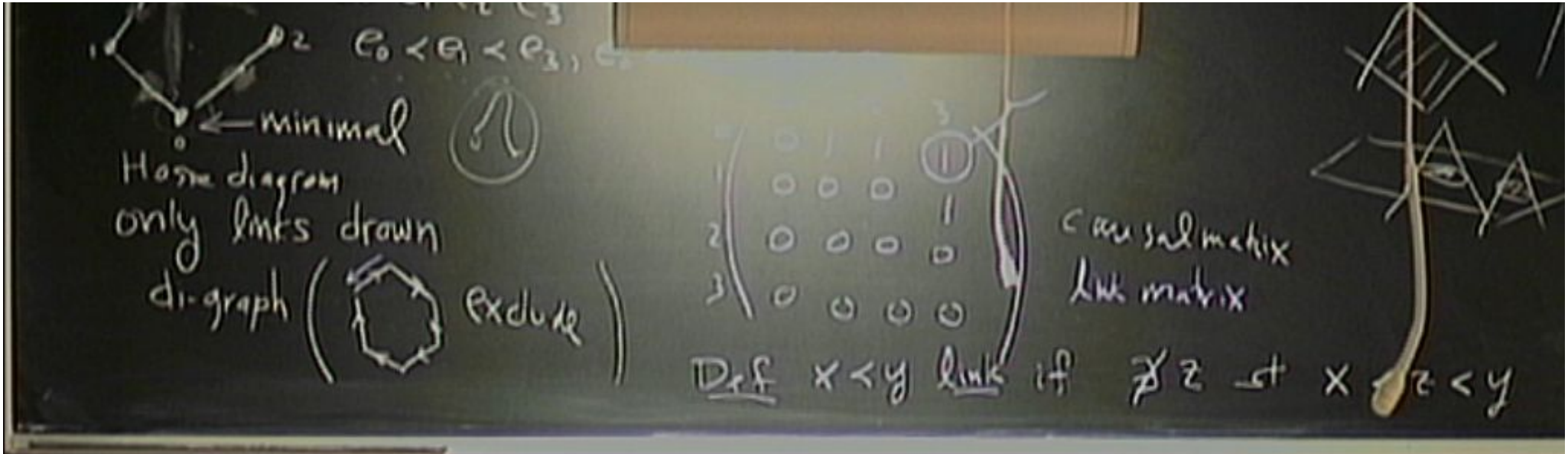
causal matrix
link matrix

Def $x \prec y$ link if $\nexists z$ s.t. $x \prec z \prec y$




e_0 e_1 e_2 e_3
$e_0 < e_1 < e_3$, $e_0 < e_2$, $e_0 < e_3$
minimal
Hasse diagram
only links drawn
di-graph
exclude
0 1 2 3
0 1 1 1
0 0 0 1
0 0 0 1
0 0 0 0
causal matrix
link matrix
Def $x < y$ link if $\nexists z$ s.t. $x < z < y$

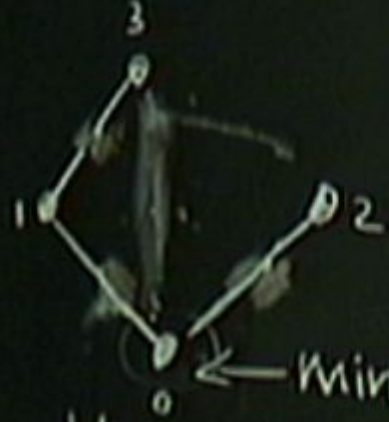

$e_0 < e_1 < e_3$,
minimal
Hasse diagram
only links drawn
di-graph
exclude
Causal matrix
link matrix
Def $x < y$ link if $\nexists z$ s.t. $x < z < y$



Can recover all $<$ from links.

PIRSA 1009 0092 (14/3/2

other rep: graph, matrix T_0-topology, ...



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

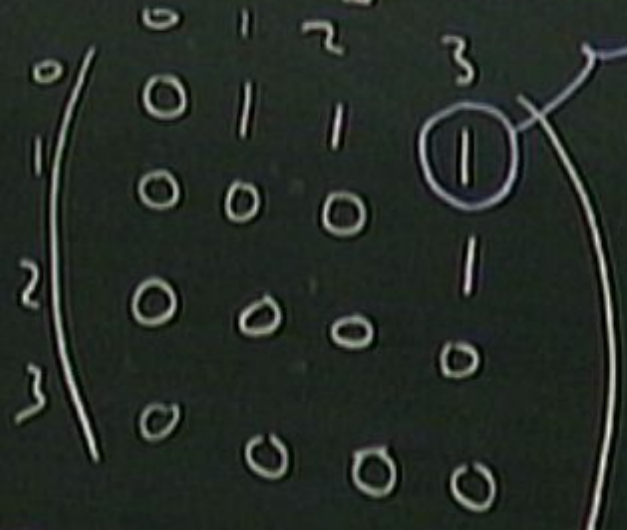
minimal



Hasse diagram

only links drawn

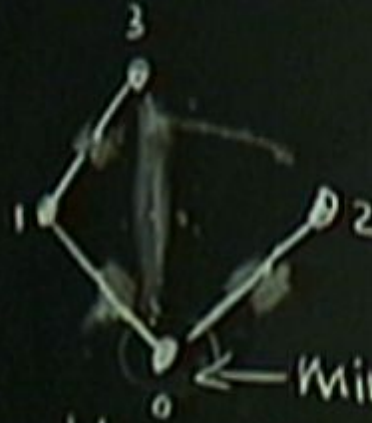
di-graph (exclude)



Def $x < y$ links if

PIRSA 1009 0092 (14/3/2

other rep: graph, matrix T_0-topology, ...



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

← minimal

Hasse diagram
only links drawn



di-graph ( exclude)



Def $x < y$ links if

PIRSA 1009 0092 (14/3/2

other rep: graph, matrix T_0-topology, ...

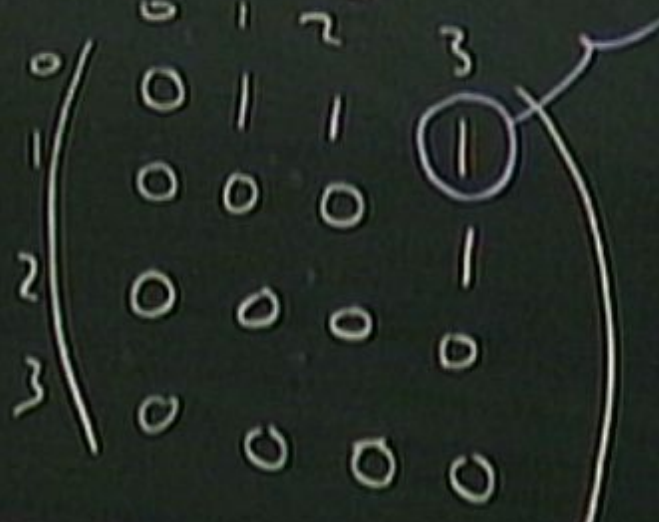


$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

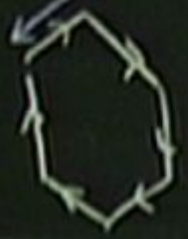
minimal





re diagram

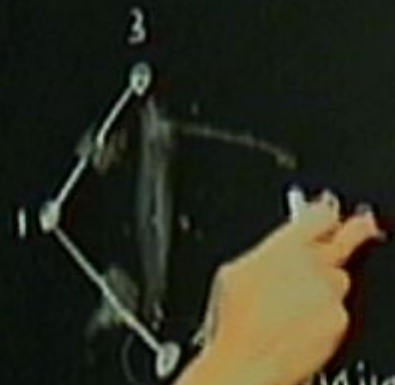
y links drawn

di-graph ( exclude)

Def $x < y$ link if

PIRSA 1009 0092 (14/3/2

other rep: graph, matrix T_0-topology, ...



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

minimal

diagram

links drawn

di-graph



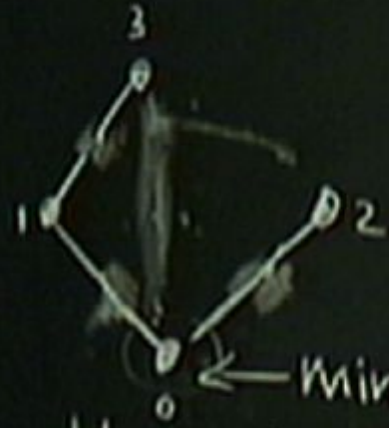
exclude

	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	1
3	0	0	0	0

Def $x < y$ link if

PIRSA 1009 0092 (14/3/2

other rep: graph, matrix T_0-topology, ...



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

← minimal



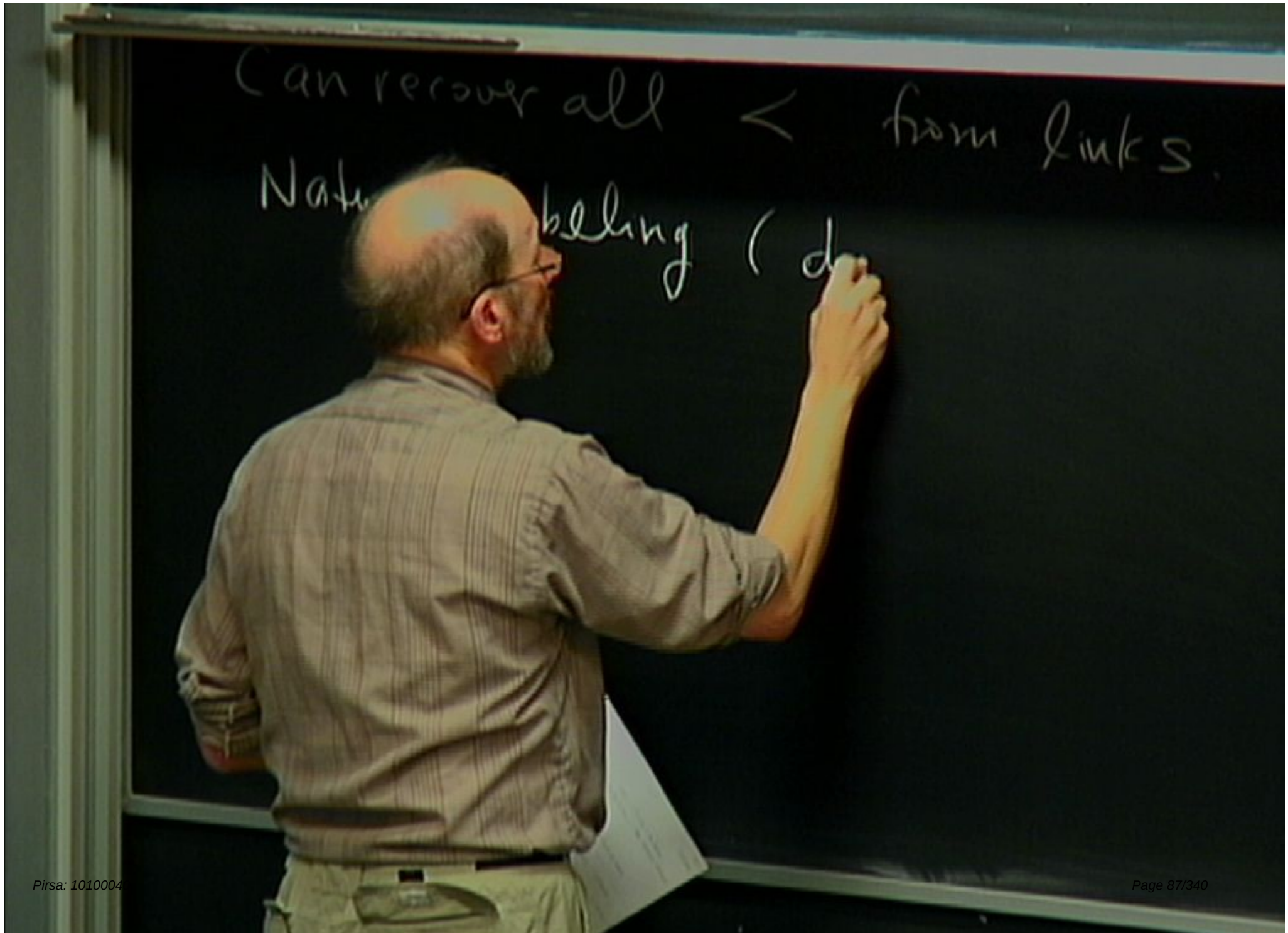
Hasse diagram

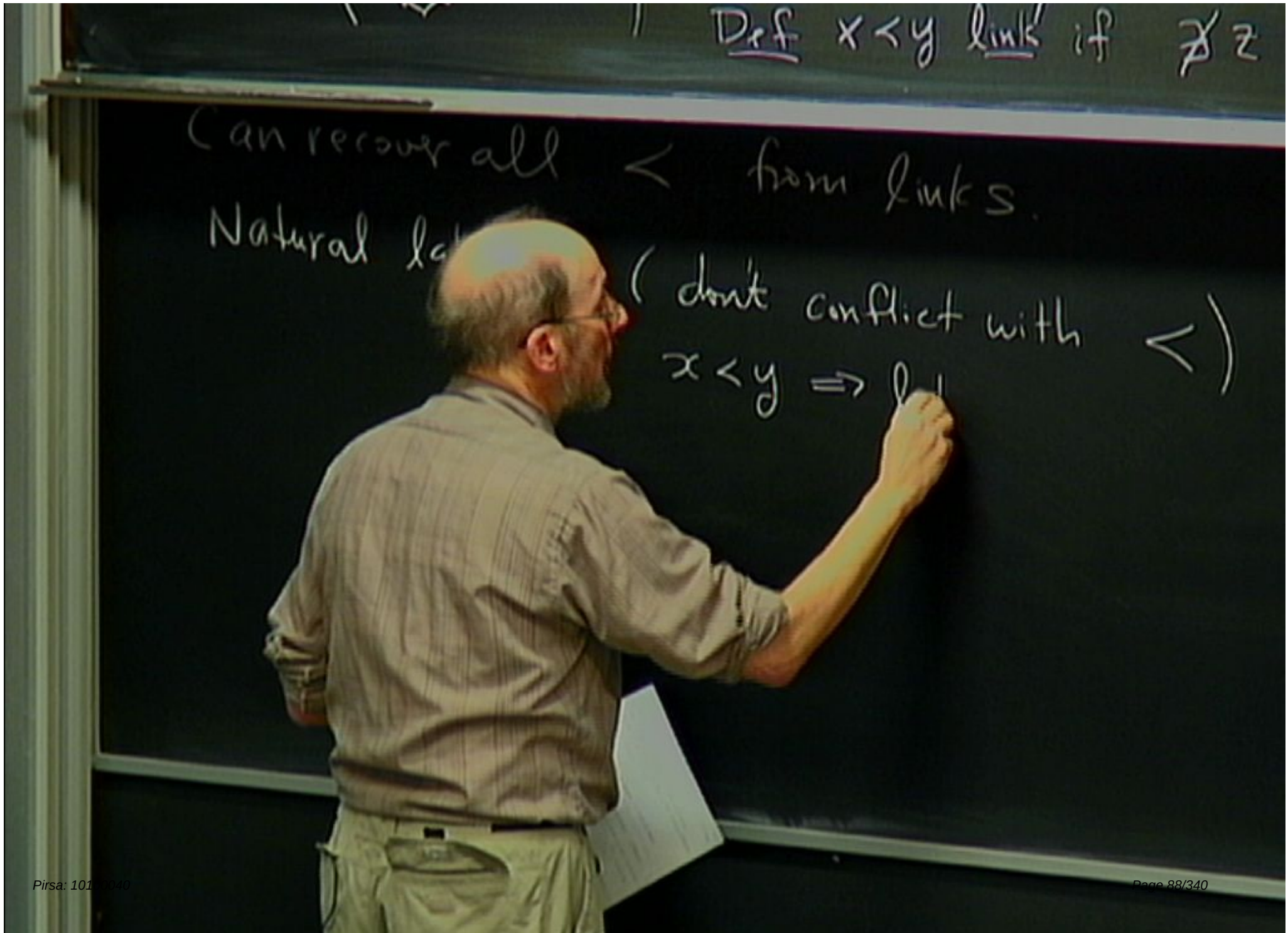
only links drawn

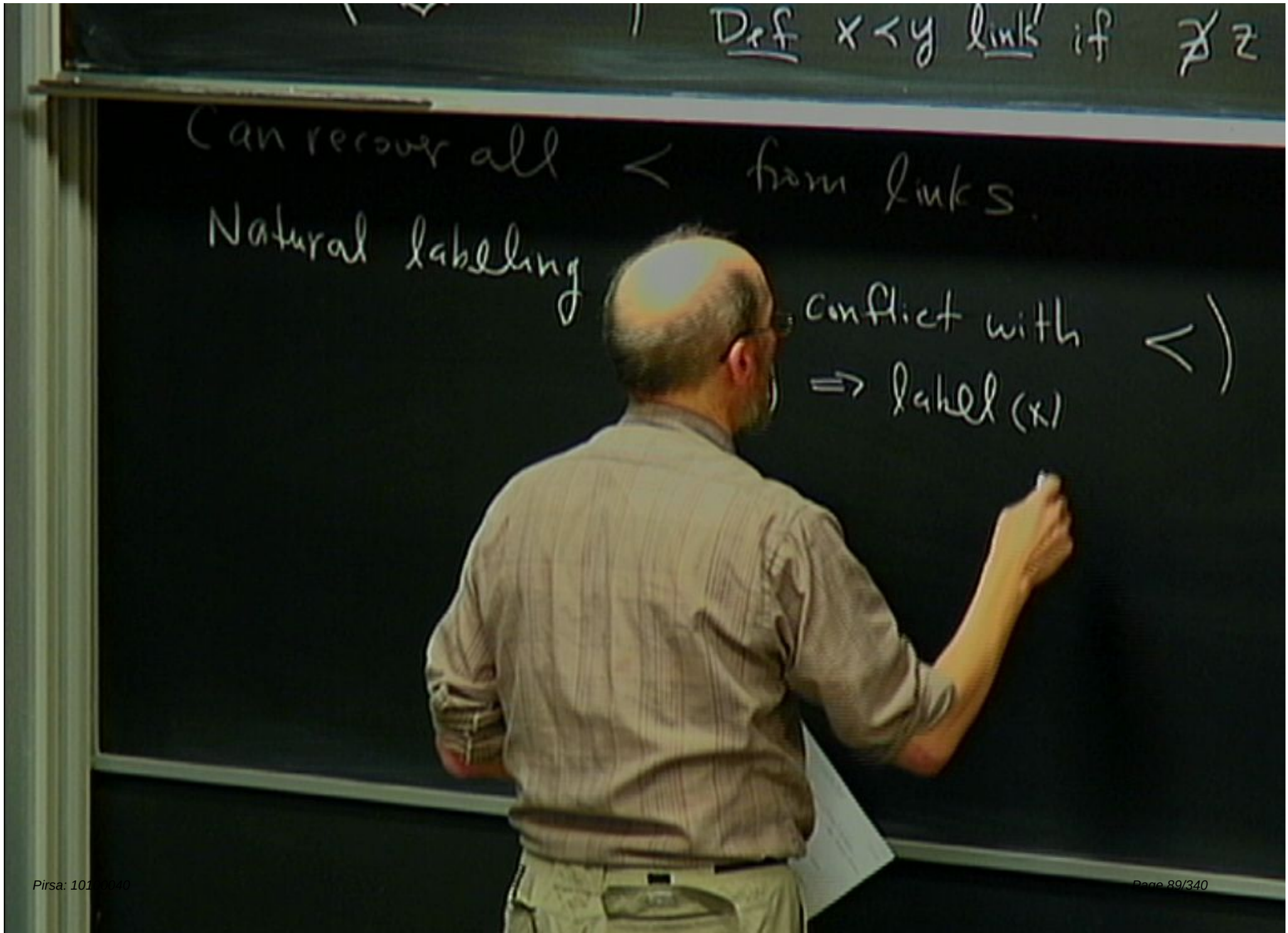
di-graph ( exclude)

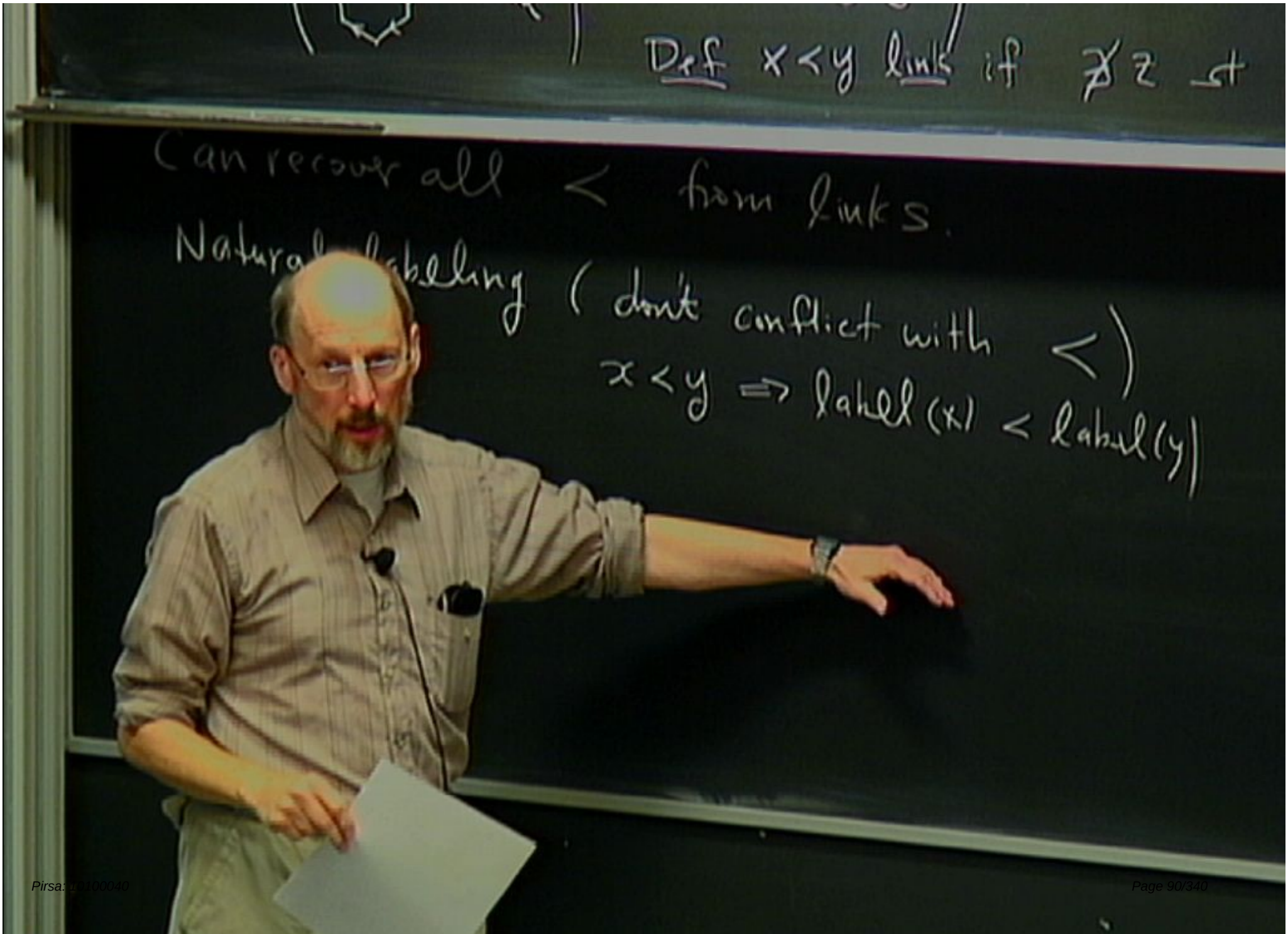
	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	1
3	0	0	0	0

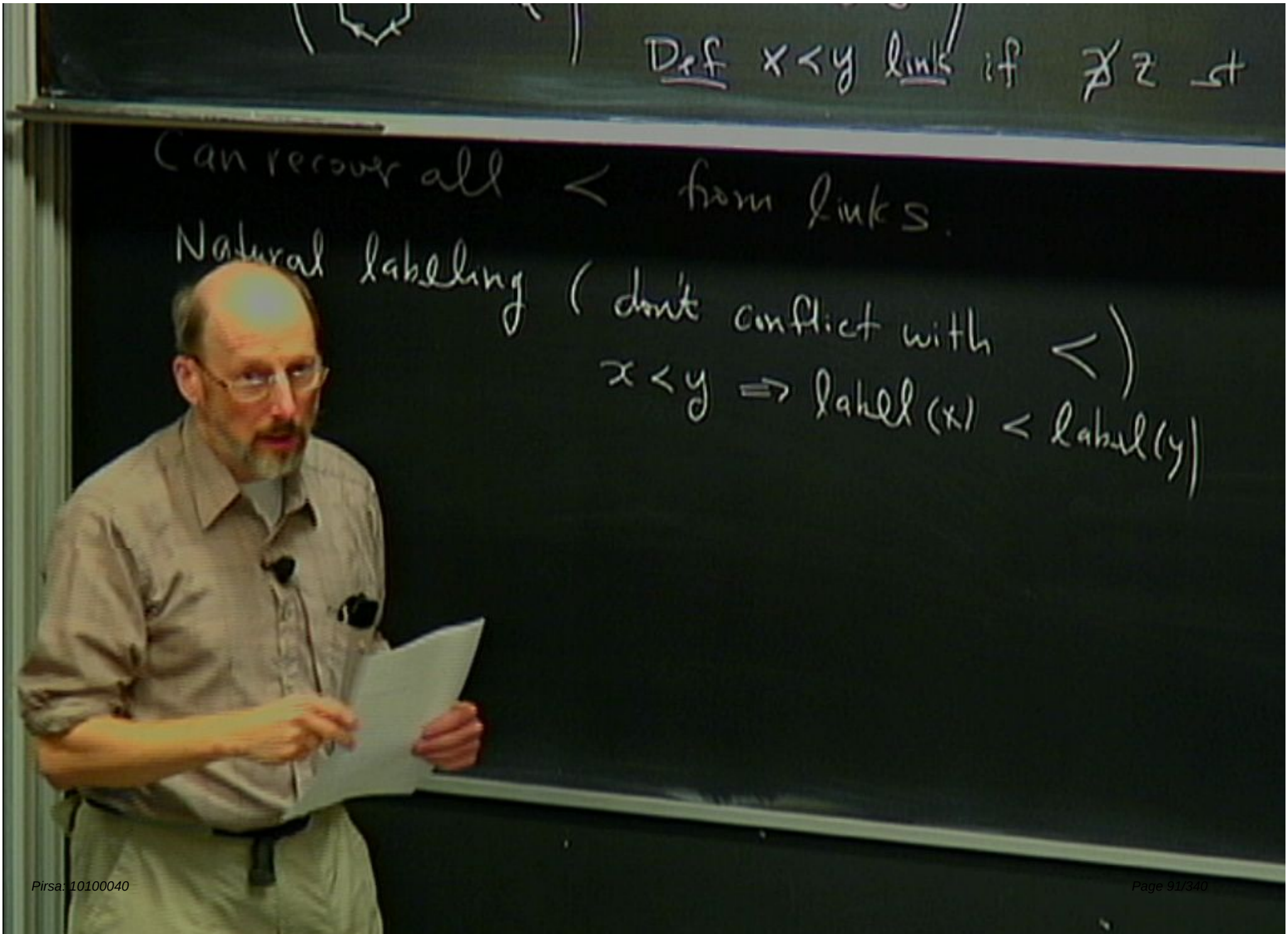
Def $x < y$ link if

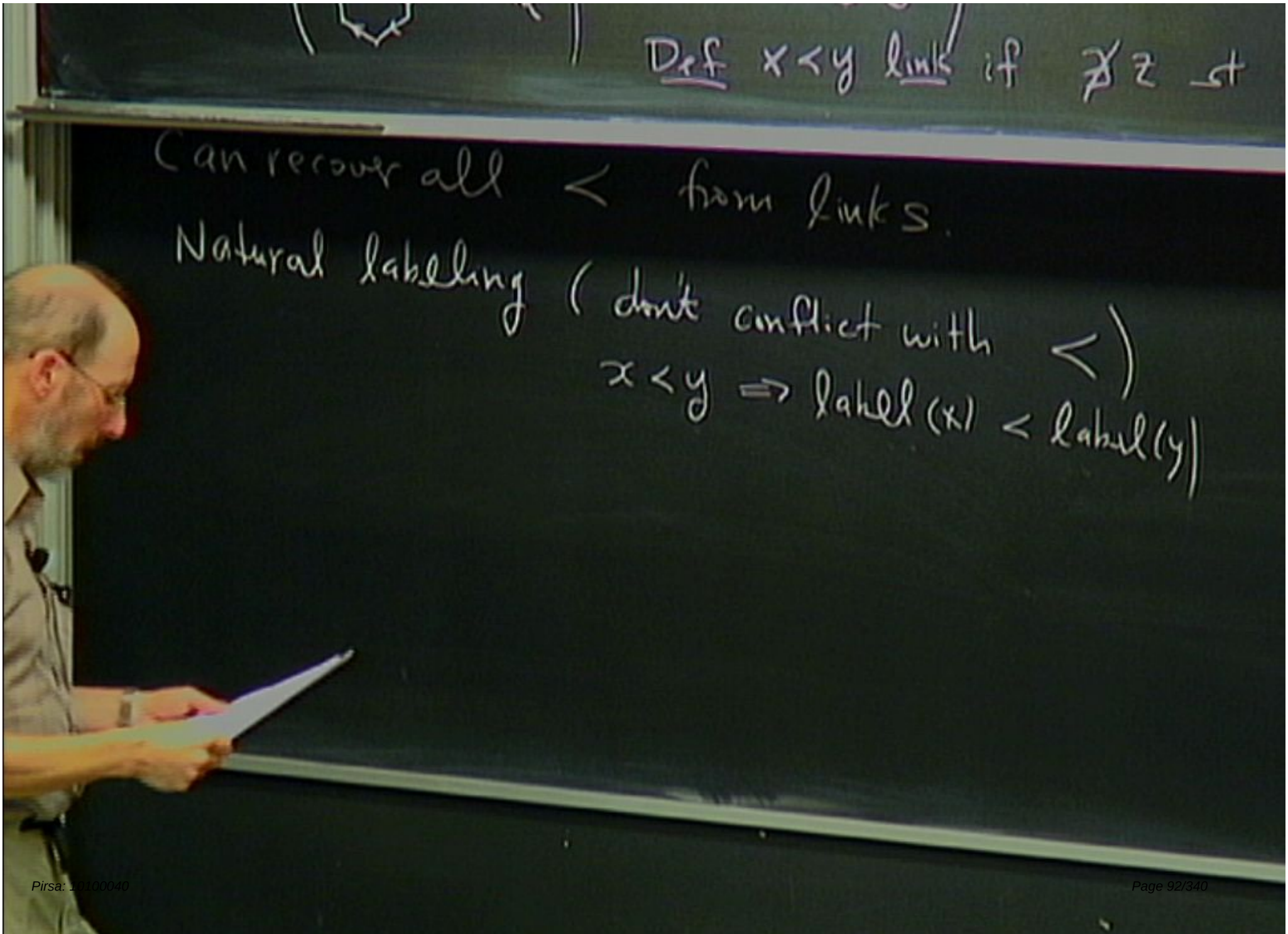

Can recover all < from links.
Nat
belong (d

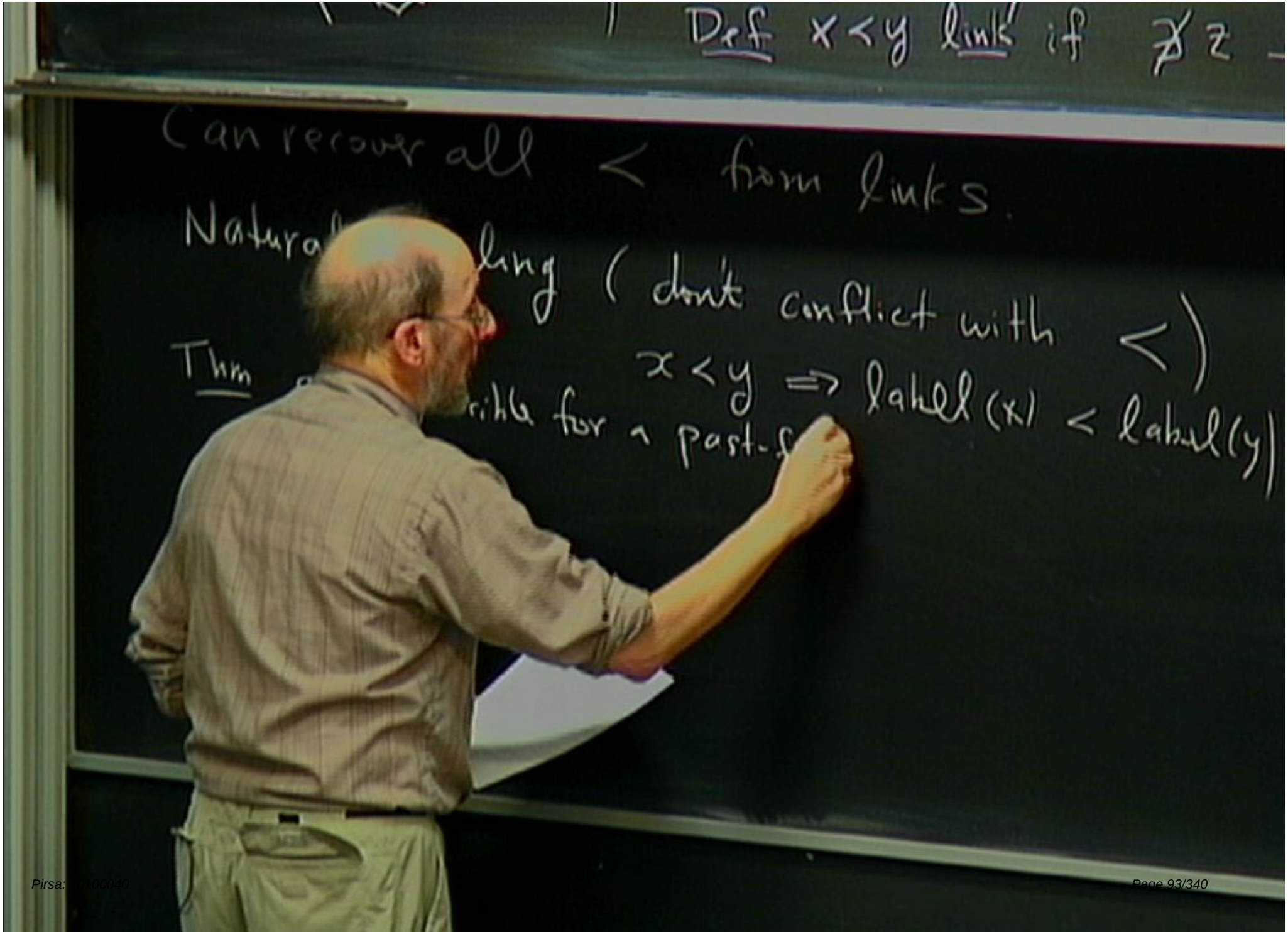

Def $x \prec y$ link if $\not\exists z$
Can recover all $<$ from links.
Natural la
(don't conflict with $<$)
$x \prec y \Rightarrow$

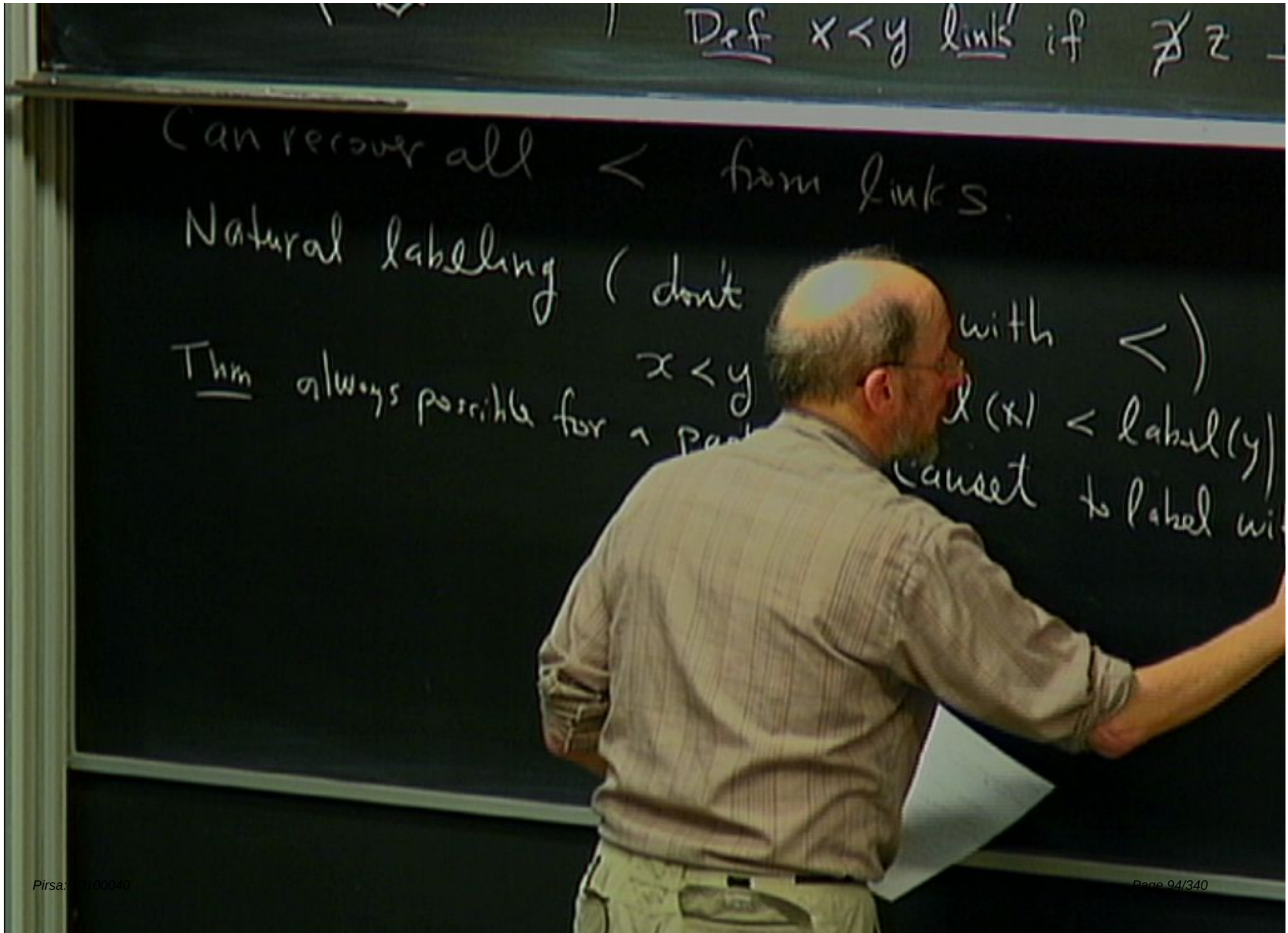

Def x<y link if ≯z
Can recover all < from links.
Natural labeling
conflict with <)
⇒ label (x)

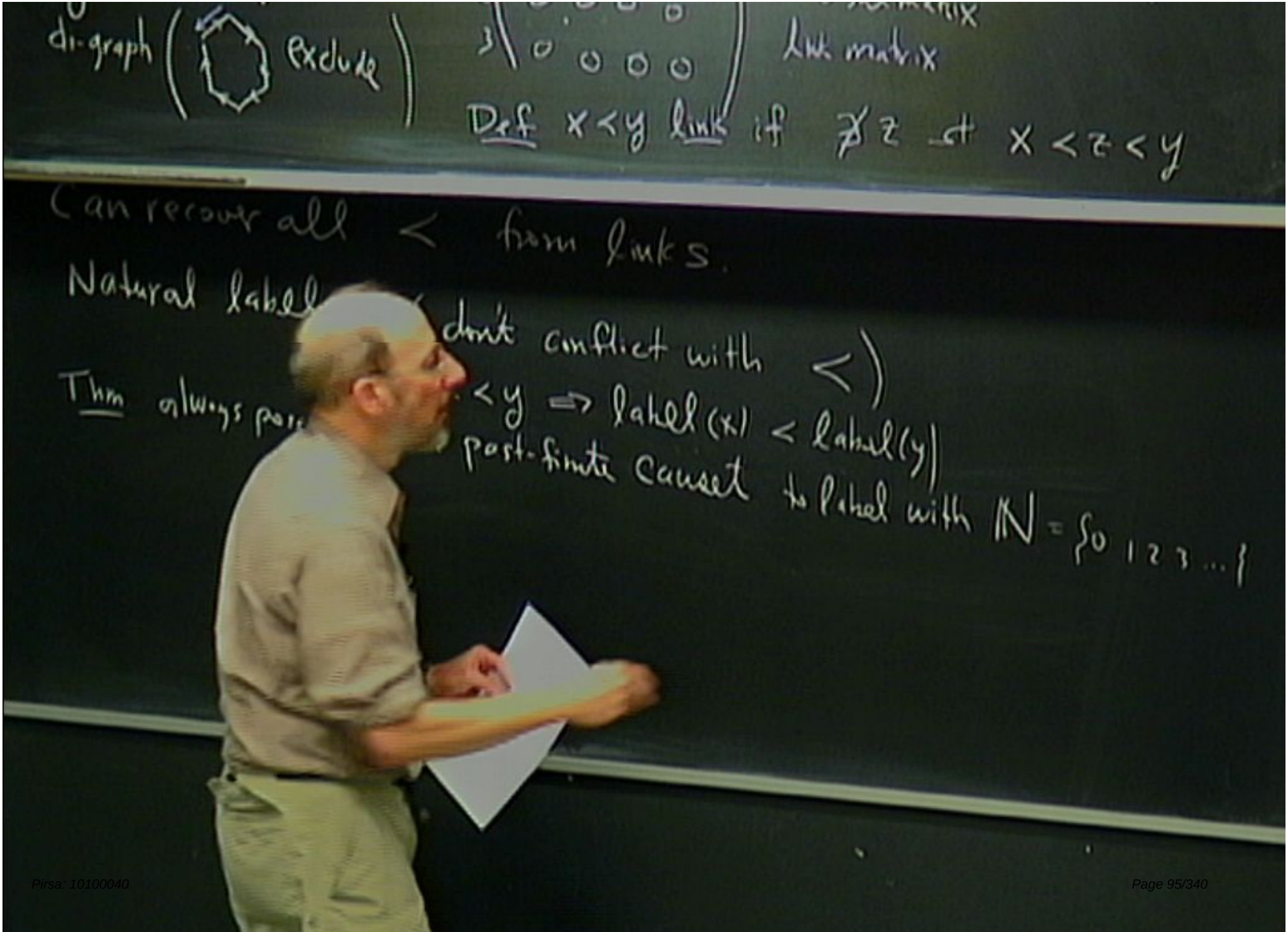

Def $x \prec y$ link if $\nexists z$ s.t.
Can recover all $\prec$ from links.
Natural labeling (don't conflict with $\prec$)
$x \prec y \Rightarrow$ label(x) < label(y)

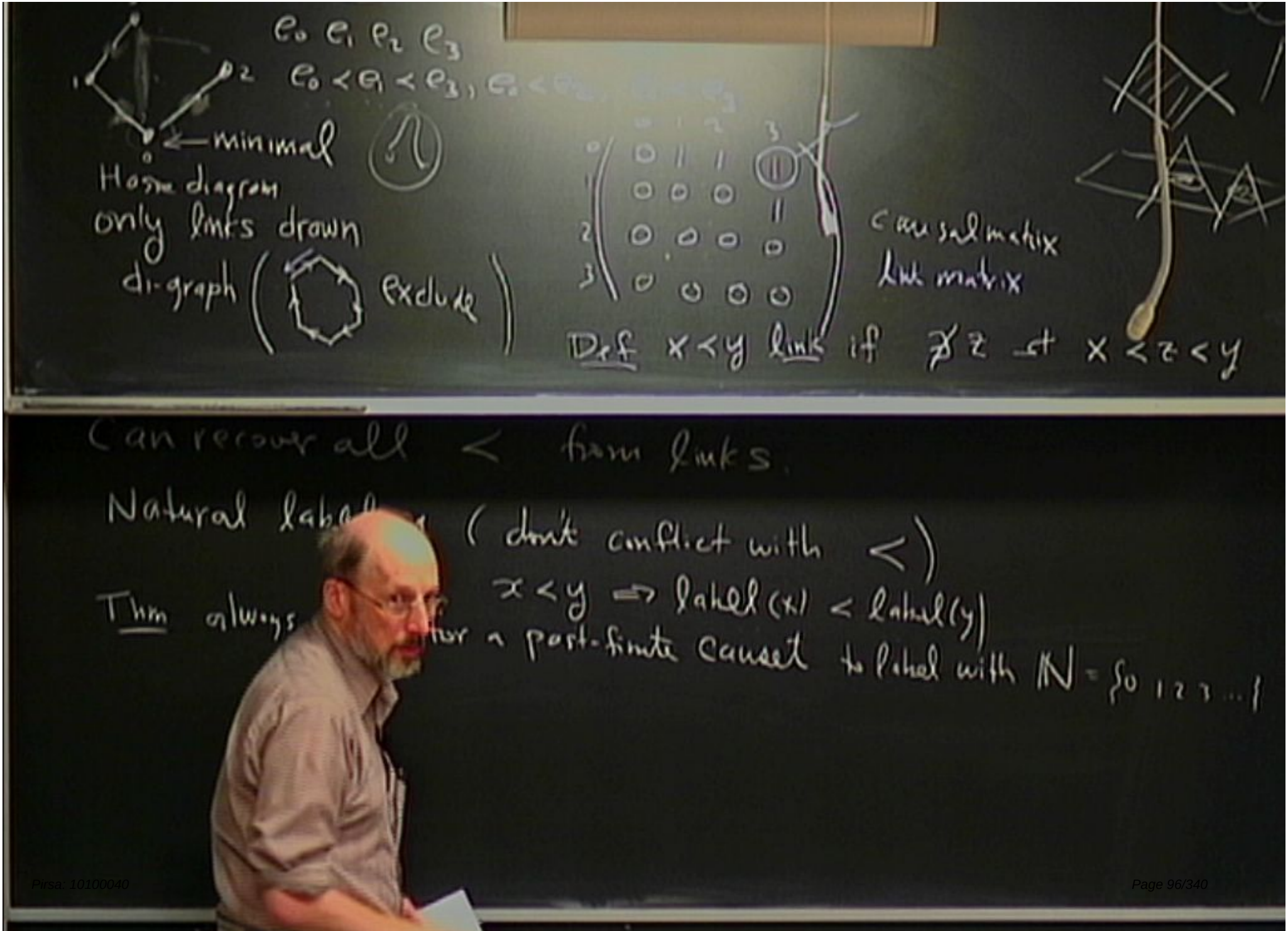

Def $x<y$ link if $\nexists z$ st
Can recover all $<$ from links.
Natural labeling (don't conflict with $<$)
$x<y \Rightarrow$ label(x) < label(y)

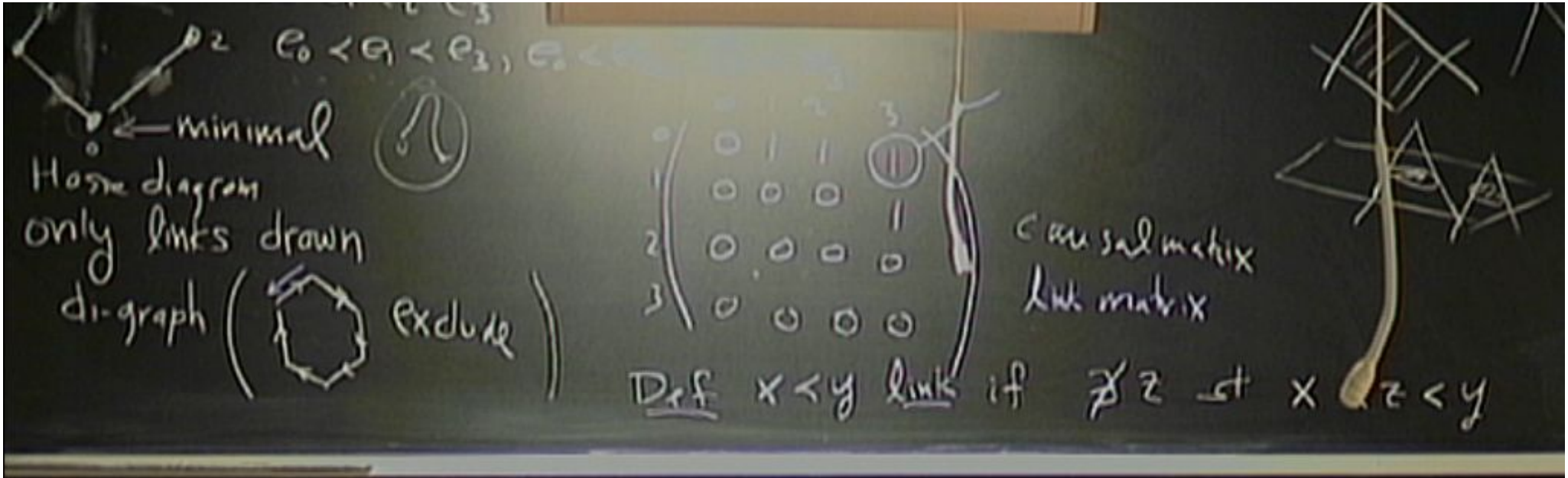

Def $x \prec y$ link if $\nexists z$ s.t.
Can recover all $<$ from links.
Natural labeling (don't conflict with $<$)
$x \prec y \Rightarrow \text{label}(x) < \text{label}(y)$


Def x<y links if ∄z
Can recover all < from links.
Thm
x<y ⇒ label(x) < label(y)


Def x<y link if ≯z
Can recover all < from links.
Natural labeling (don't
with <)
Thm always possible for a
x<y
l(x) < label(y)
cannot to label wi


di-graph
exclude
Def x<y link if ∄z s.t. x<z<y
Can recover all < from links.
Natural label
don't conflict with <)
Thm always
x<y ⇒ label(x) < label(y)
Post-finite can't be labeled with ℕ = {0 1 2 3 ...}


e_0 e_1 e_2 e_3
$e_0 < e_1 < e_3$, $e_0 <$
minimal
Hasse diagram
only links drawn
di-graph
exclude
causal matrix
link matrix
Def $x < y$ link if $\nexists z$ s.t. $x < z < y$
Can recover all $<$ from links.
Natural label
(don't conflict with $<$)
$x < y \Rightarrow$ label(x) < label(y)
Thm always
for a port-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

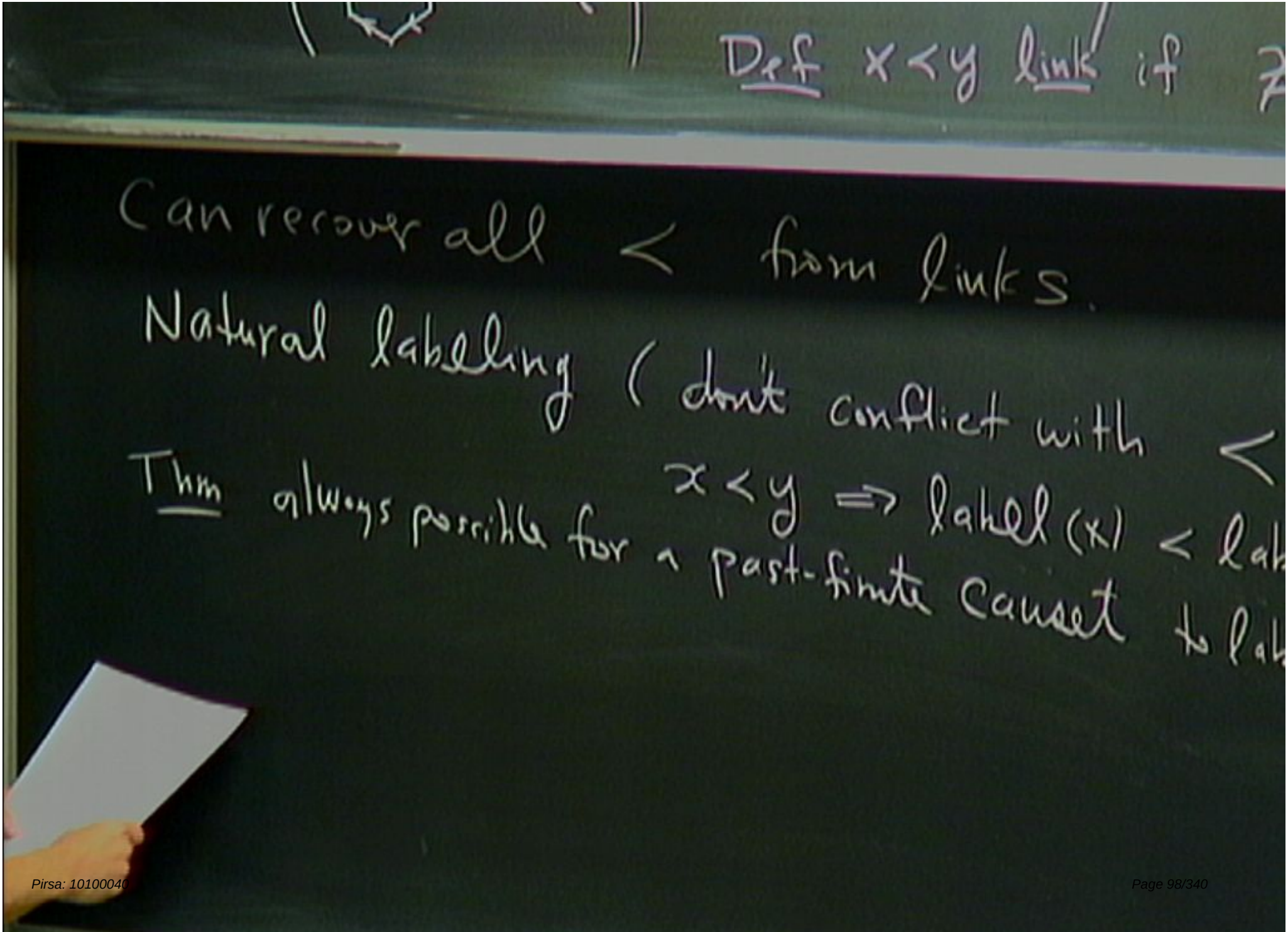


Can recover all $<$ from links.

Natural labeling (don't conflict with $<$)

$x < y \Rightarrow \text{label}(x) < \text{label}(y)$

Thm possible for a part-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$


Def $x \prec y$ link if
Can recover all $<$ from links.
Natural labeling (don't conflict with $<$
Thm always possible for a past-finite causet
$x \prec y \Rightarrow$ label $(x) <$ lab

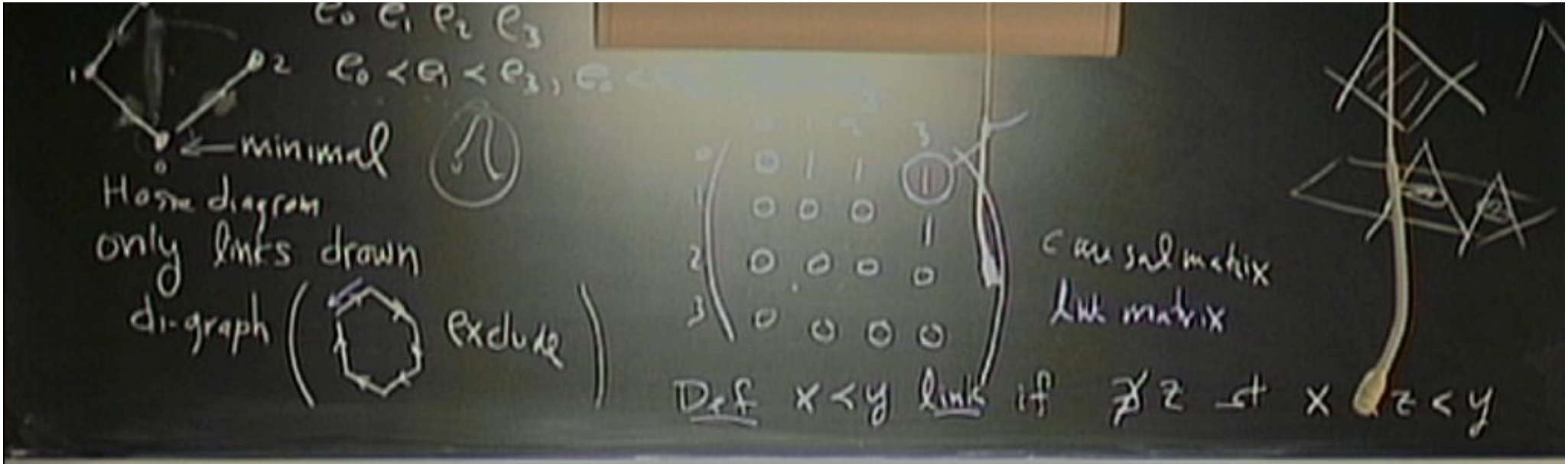
Def $x < y$ link if $\nexists z$ s.t. $x < z < y$

$<$ from links.

$\exists$ (don't conflict with $<$)

$x < y \Rightarrow$ label(x) $<$ label(y)

for a past-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots$

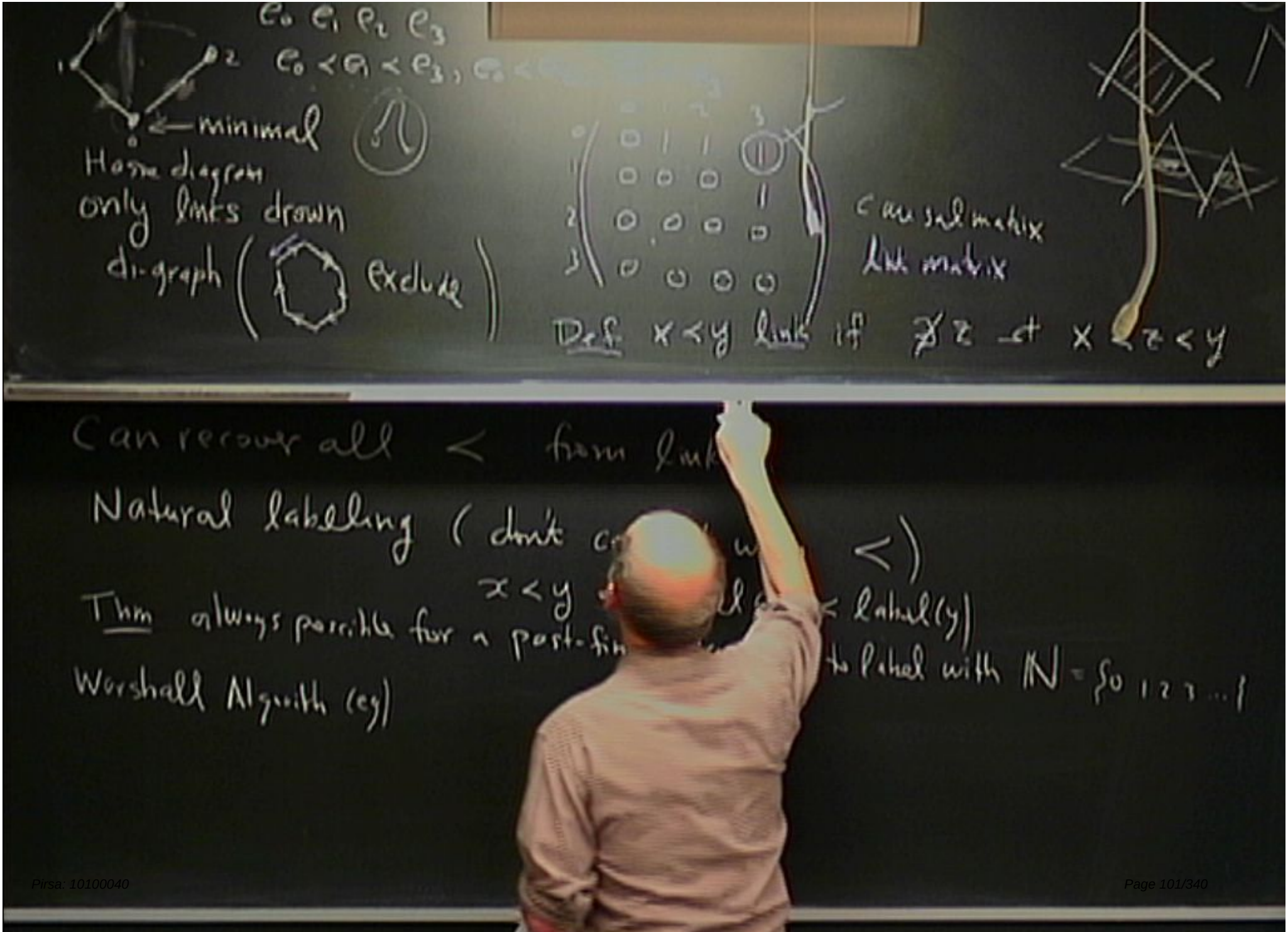


Can recover all < from links.

Natural labeling (don't conflict with <)

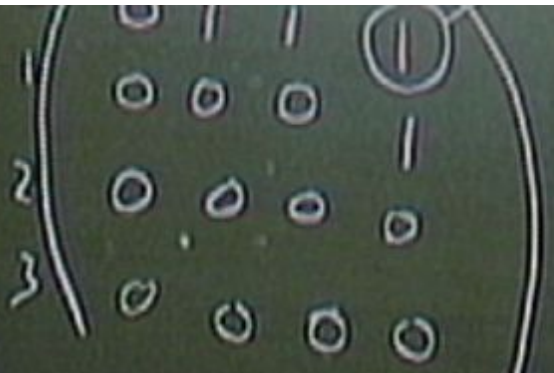
x < y ⇒ label(x) < label(y)

a part-finite causet to label with ℕ = {0 1 2 3 ...}


$e_0\ e_1\ e_2\ e_3$
$e_0 < e_1 < e_3$, $e_0 <$
minimal
Hasse diagram
only links drawn
di-graph
exclude
Def $x < y$ link if $\nexists z$ s.t. $x < z < y$
Can recover all $<$ from link
Natural labelling (don't
$x < y$
$<$ label(y)
Thm always possible for a part-fin
to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
Warshall Algorithm (eg)

Hasse diagram
only links drawn

di-graph ( exclude)



Causal matrix
link matrix

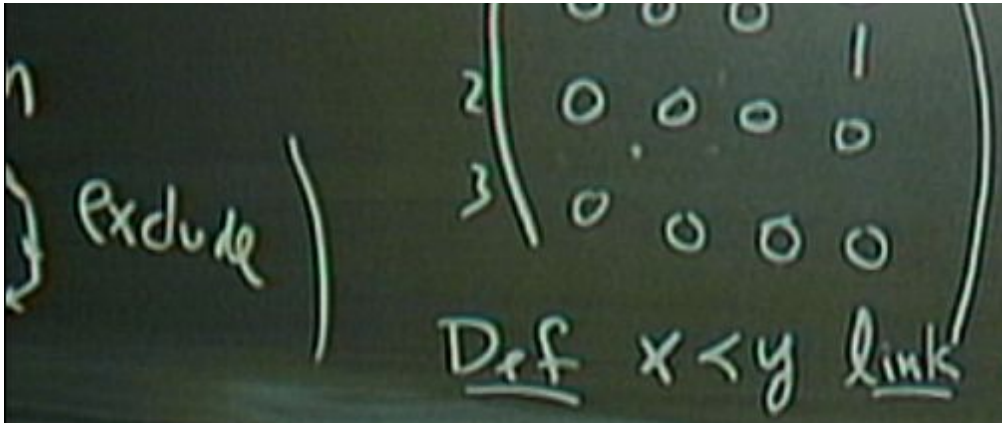
Def $x \prec y$ link if $\nexists z$ st

(don't conflict with $<$)

Thm always possible for a past-finite causet to label with $\mathbb{N}$

$x \prec y \Rightarrow \text{label}(x) < \text{label}(y)$

Warshall Algorithm (eg) transitive closure



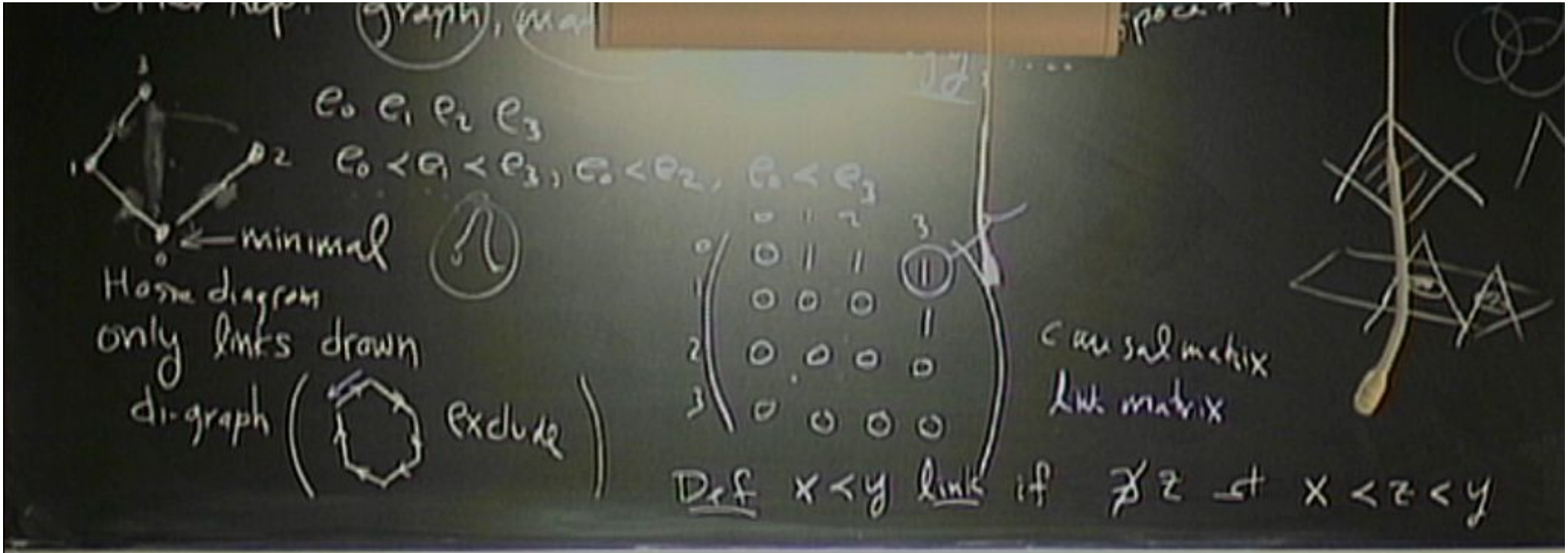
Def $x < y$ link if $\nexists z$ st $x < z < y$

(don't conflict with $<$)

$x < y \Rightarrow \text{label}(x) < \text{label}(y)$

for a past-finite causet to label with $\mathbb{N} = \{0\ 1\ 2\ 3\ \ldots\}$

transitive closure

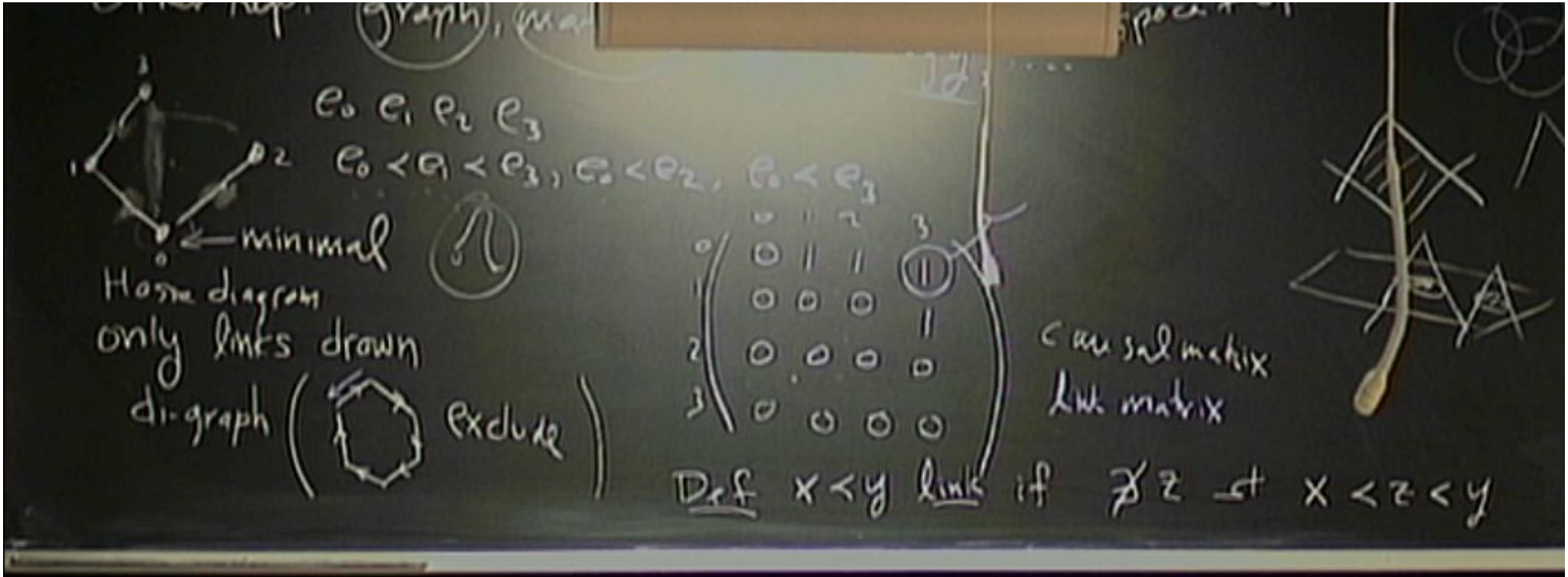


Natural labeling (don't conflict with $<$)

$x < y \Rightarrow$ label(x) $<$ label(y)

always possible for a port-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

[illegible] Algorithm (eg) transitive closure



Natural labeling (don't conflict with $<$)

$x < y \Rightarrow$ label (x) < label (y)

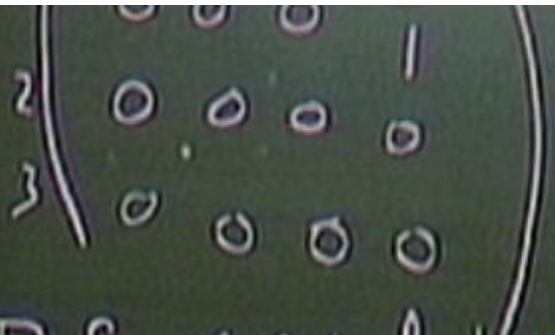
always possible for a post-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Warshall Algorithm (eg) transitive closure

$(\forall y)(\forall x)(\forall z)(x < y < z \rightarrow x < z)$

only links drawn

di-graph ( exclude)



Def $x \prec y$ link if

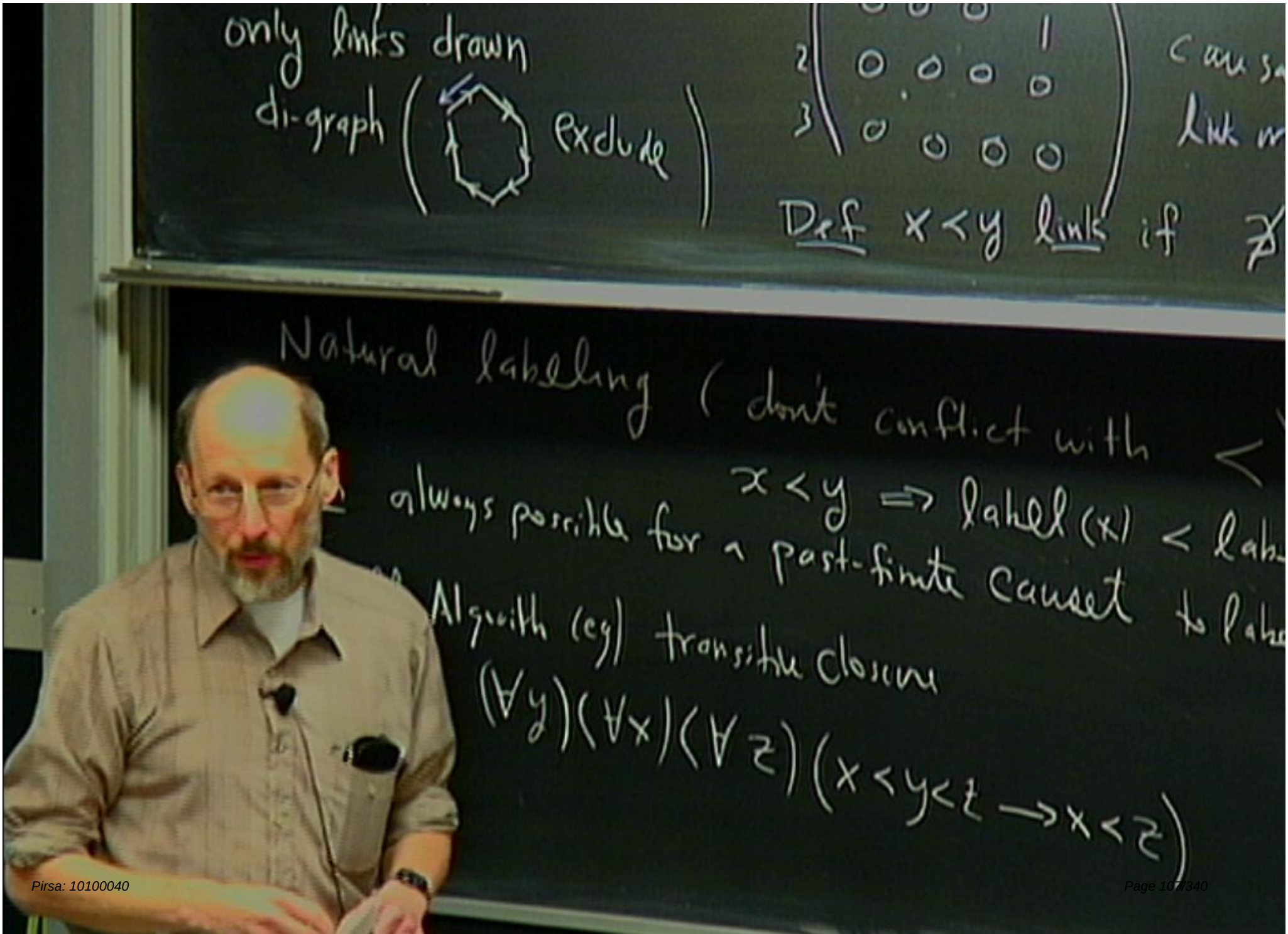
Natural labeling (don't conflict with $\prec$

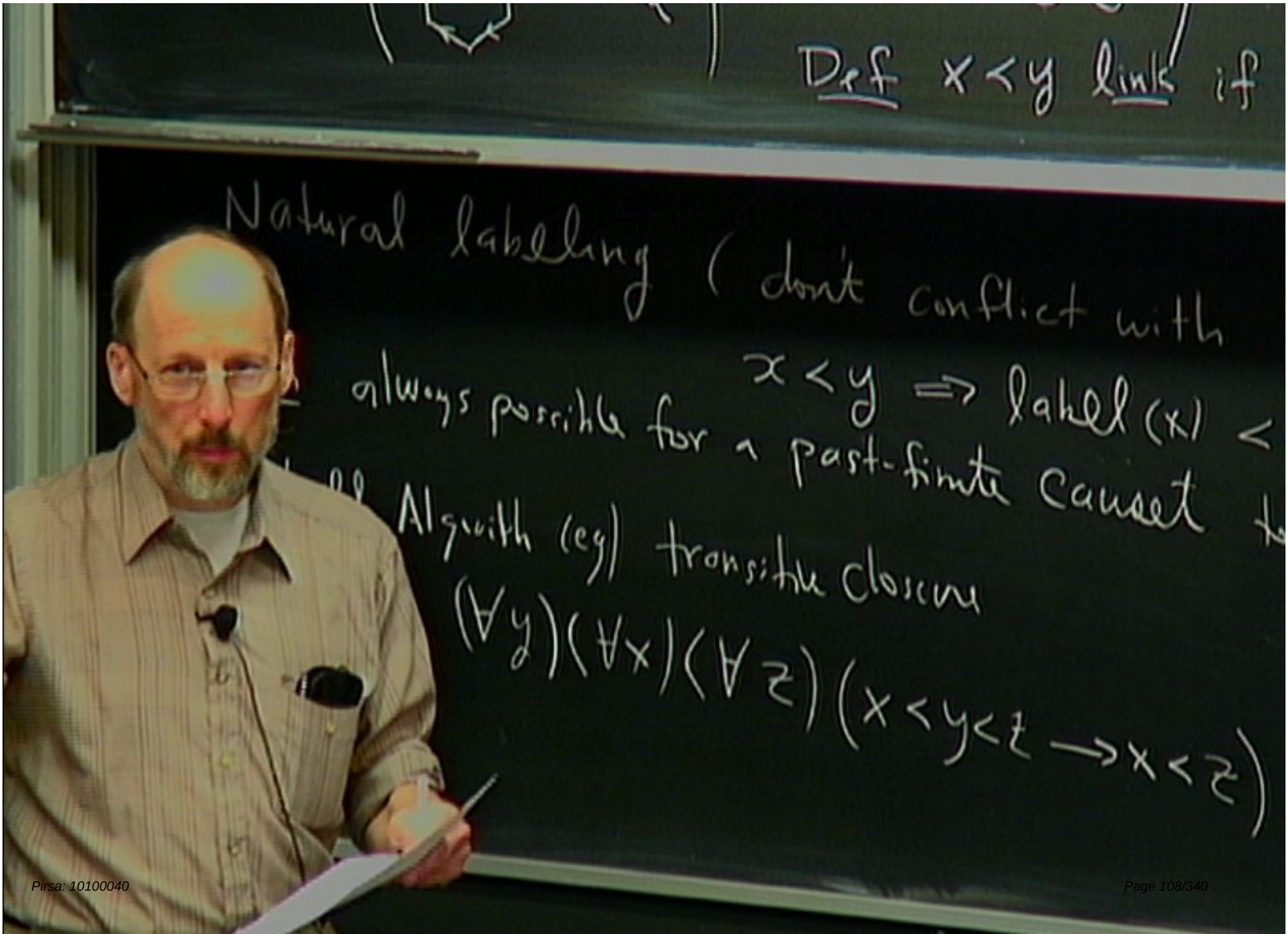
$x \prec y \Rightarrow$ label $(x) <$ lab

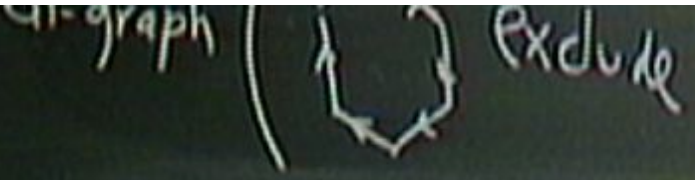
ys possible for a past-finite causet to lab

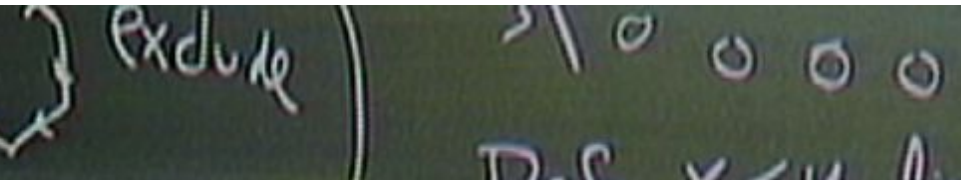
l Algorithm (eg) transitive closure

$$(\forall y)(\forall x)(\forall z)\,(x \prec y \prec z \rightarrow x \prec z)$$


only links drawn
di-graph
exclude
Def x<y link if
Natural labeling (don't conflict with
always possible for a past-finite
Algorithm (eg) transitive closure
(∀y)(∀x)(∀z)(x<y<z → x<z)


Def $x \prec y$ link if
Natural labeling (don't conflict with
$x \prec y \Rightarrow$ label $(x) <$
always possible for a past-finite causet
Algorithm (eg) transitive closure
$(\forall y)(\forall x)(\forall z)(x \prec y \prec z \rightarrow x \prec z)$

di-graph  exclude



link matrix

Def $x \prec y$ link if $\nexists z$ s.t

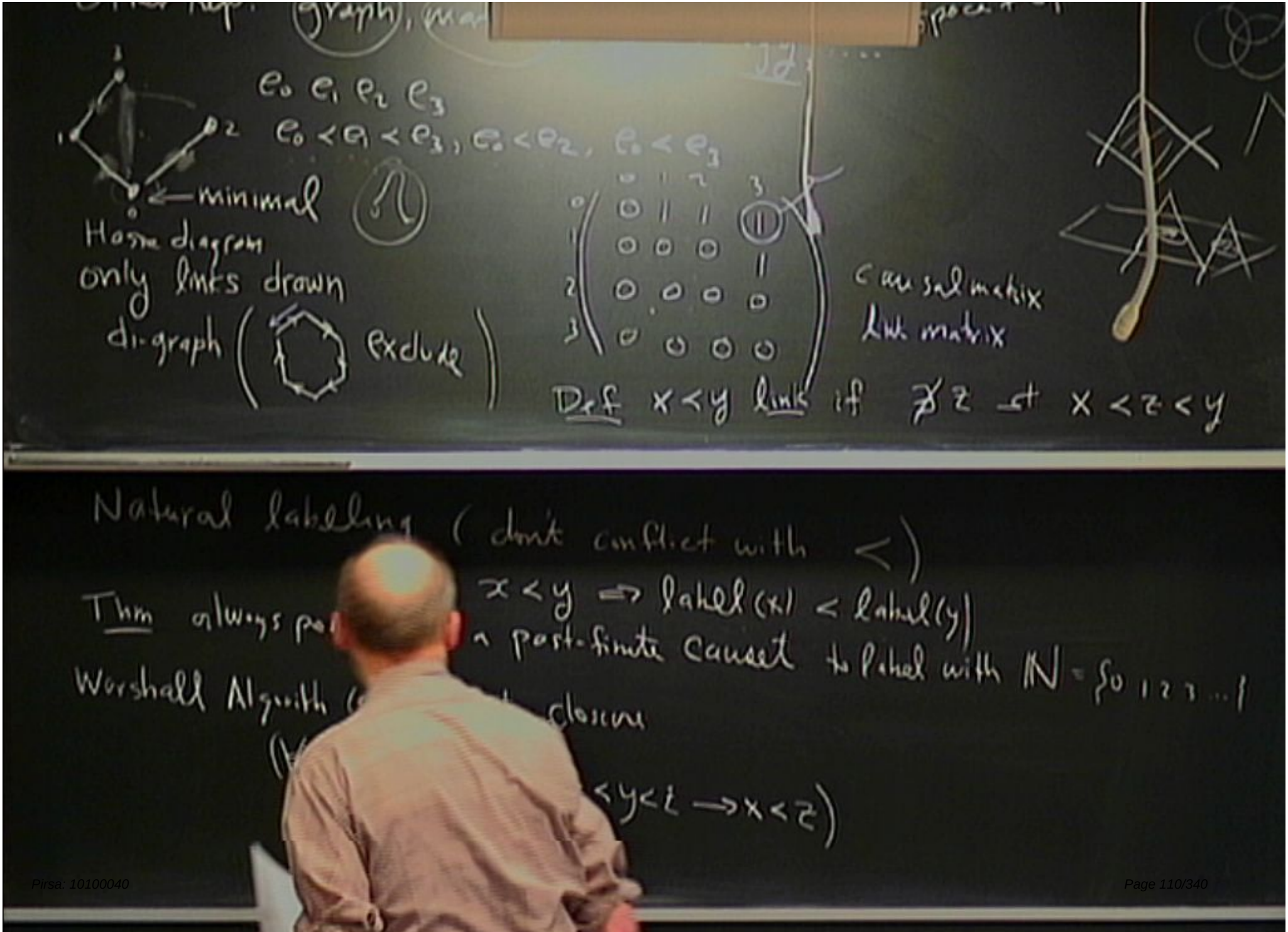
Natural labeling (don't conflict with $<$)

$x < y \Rightarrow \text{label}(x) < \text{label}(y)$

Thm always possible for a past-finite causet to label with

Warshall Algorithm (eg) transitive closure

$$(\forall y)(\forall x)(\forall z)\,(x \prec y \prec z \rightarrow x \prec z)$$


$e_0\ e_1\ e_2\ e_3$
$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$
minimal
Hasse diagram
only links drawn
di-graph
exclude
causal matrix
link matrix
Def $x < y$ link if $\nexists z$ s.t. $x < z < y$
Natural labelling (don't conflict with $<$)
$x < y \Rightarrow$ label(x) < label(y)
Thm always po
a part-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
Warshall Algorith
closure
$< y < i \rightarrow x < z$)

poset (transitive & irreflexive)

Key: locally finite → past-finite

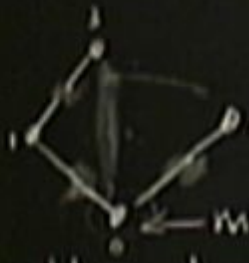
$a < b < c < \dots < a$

$a < c < a$ X

Joe Henson PIRSA 10090092 (14/3/2010)

Stembridge - Maple

other rep: graph, matrix, T_0-topology, ... → spectral ...



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

Hasse diagram

only links drawn

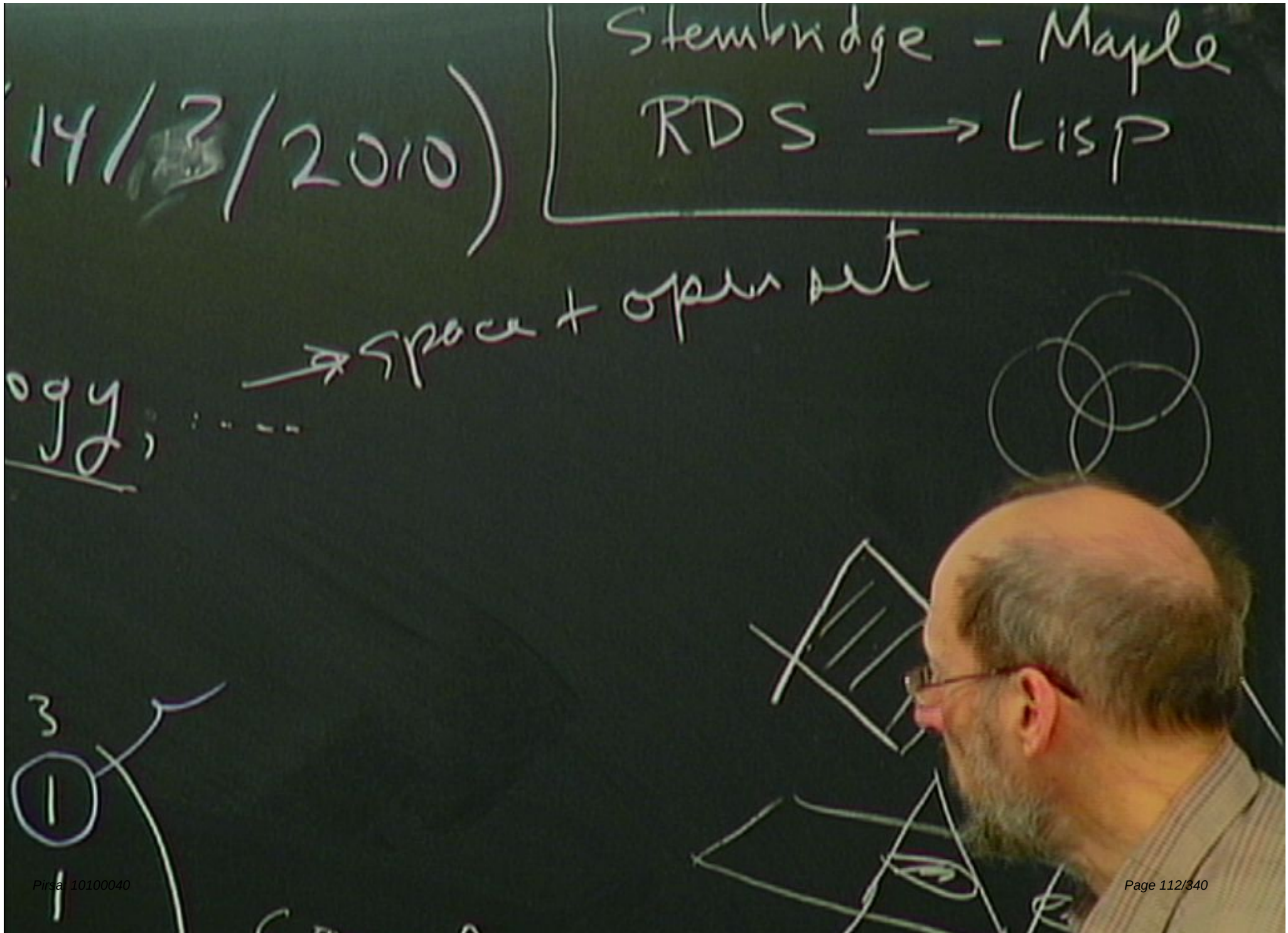
di-graph (exclude)

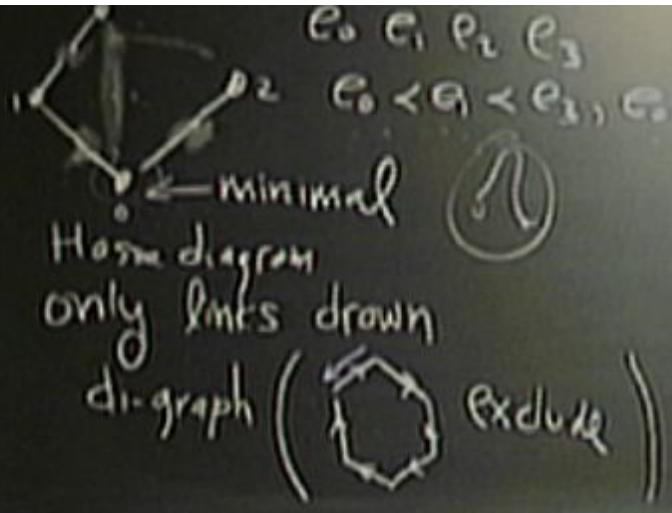
	0	1	2	3
0	0	1	1	1
1	0	0	0	1
2	0	0	0	0
3	0	0	0	0

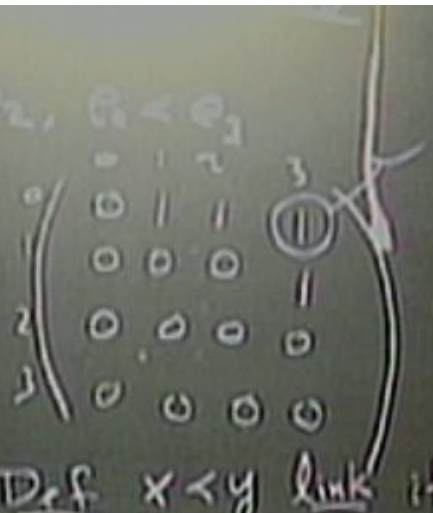
Def $x < y$ link if

$z < y$




14/3/2010
Stembridge - Maple
RDS → Lisp
→ space + open set
3
1
1





causal matrix
link matrix



Def $x < y$ link if $\nexists z$ s.t. $x < z < y$

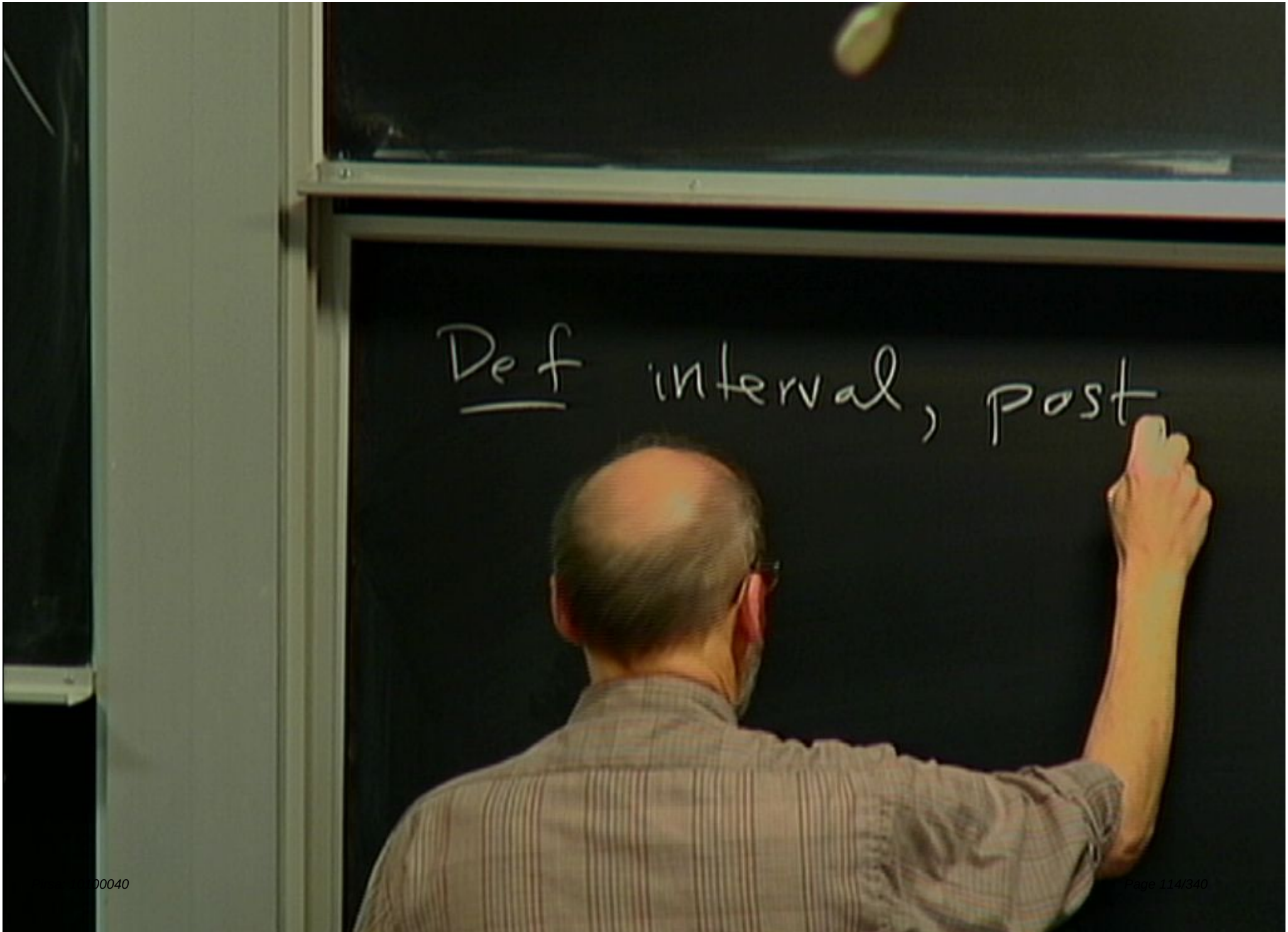
Natural labeling (don't conflict with $<$)

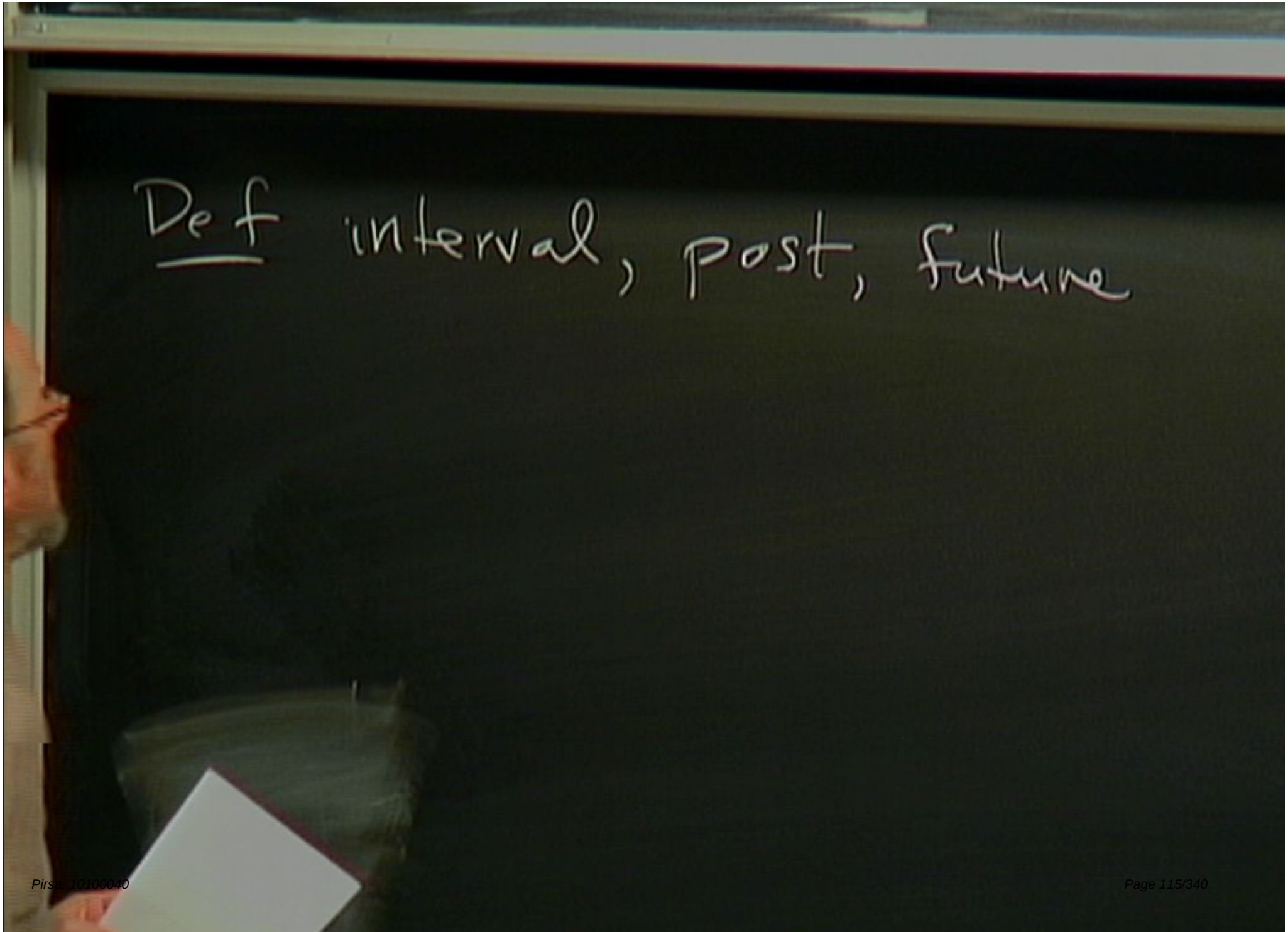
$x < y \Rightarrow \text{label}(x) < \text{label}(y)$

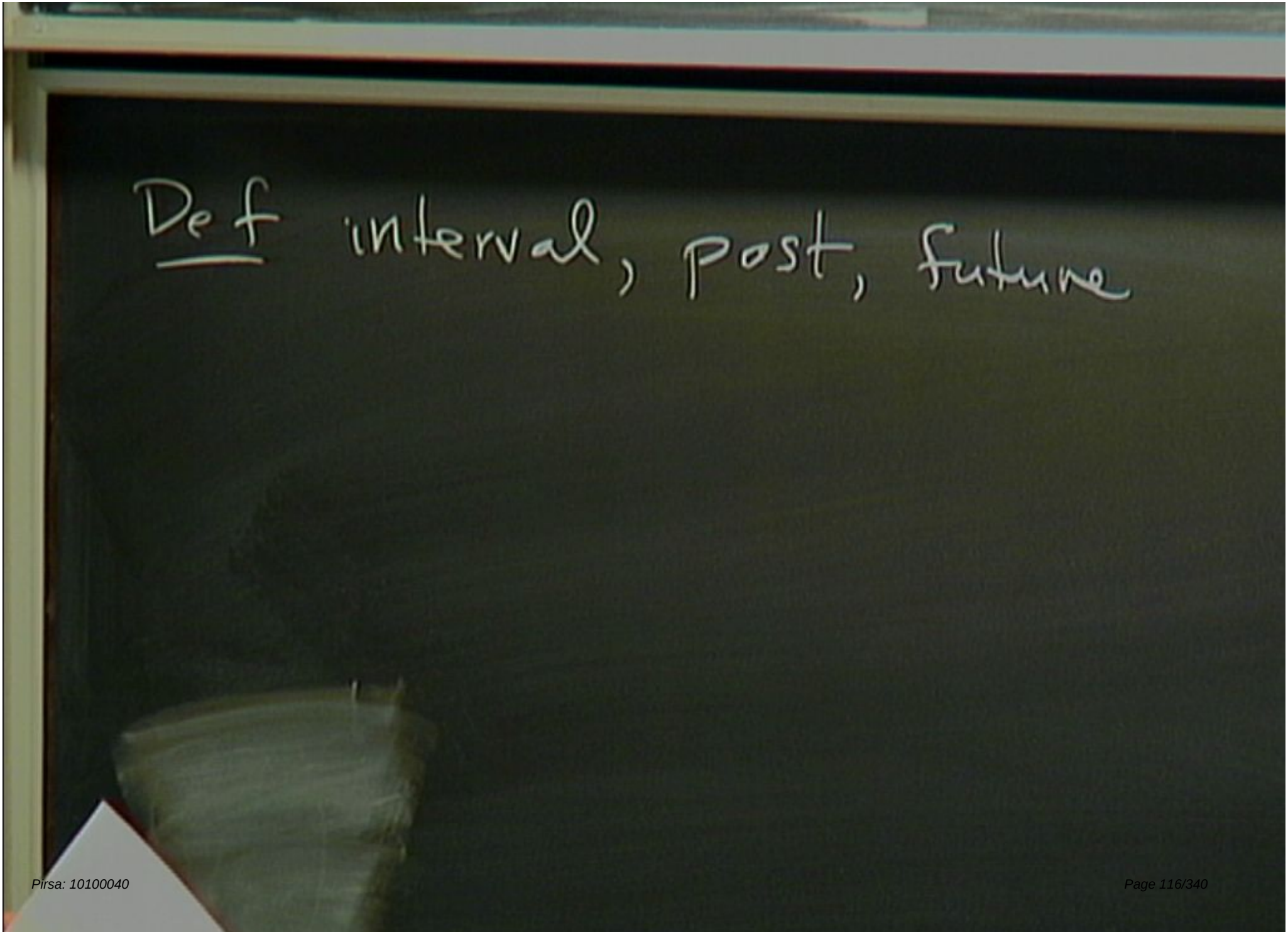
Thm always possible for a past-finite causet to label with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

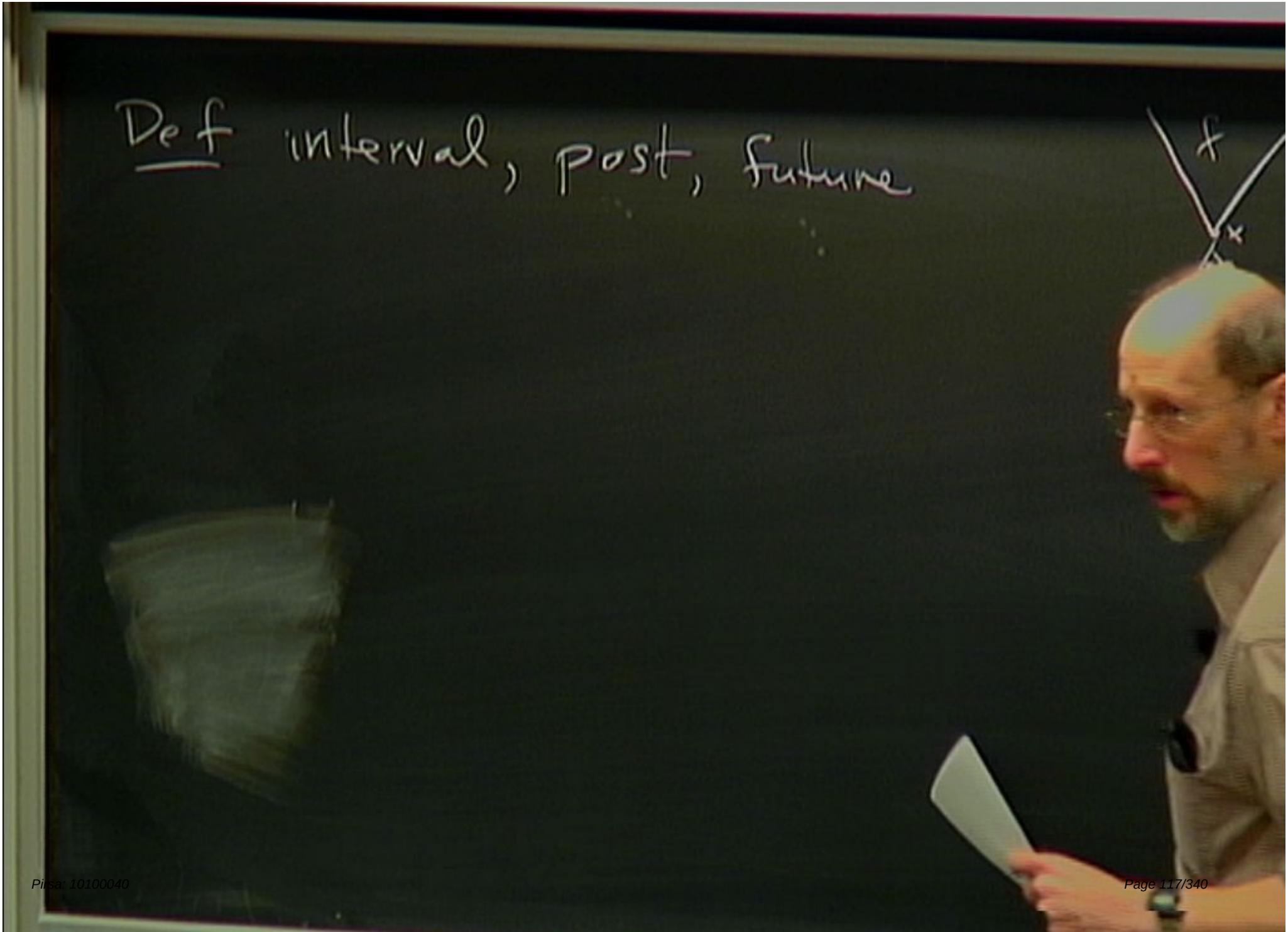
Warshall Algorithm (eg) transitive closure

$$(\forall y)(\forall x)(\forall z)\left(x < y < z \rightarrow x < z\right)$$


Def interval, post


Def interval, past, future


Def interval, past, future


Def interval, past, future
t

Def interval, past, future



Def interval, past, future



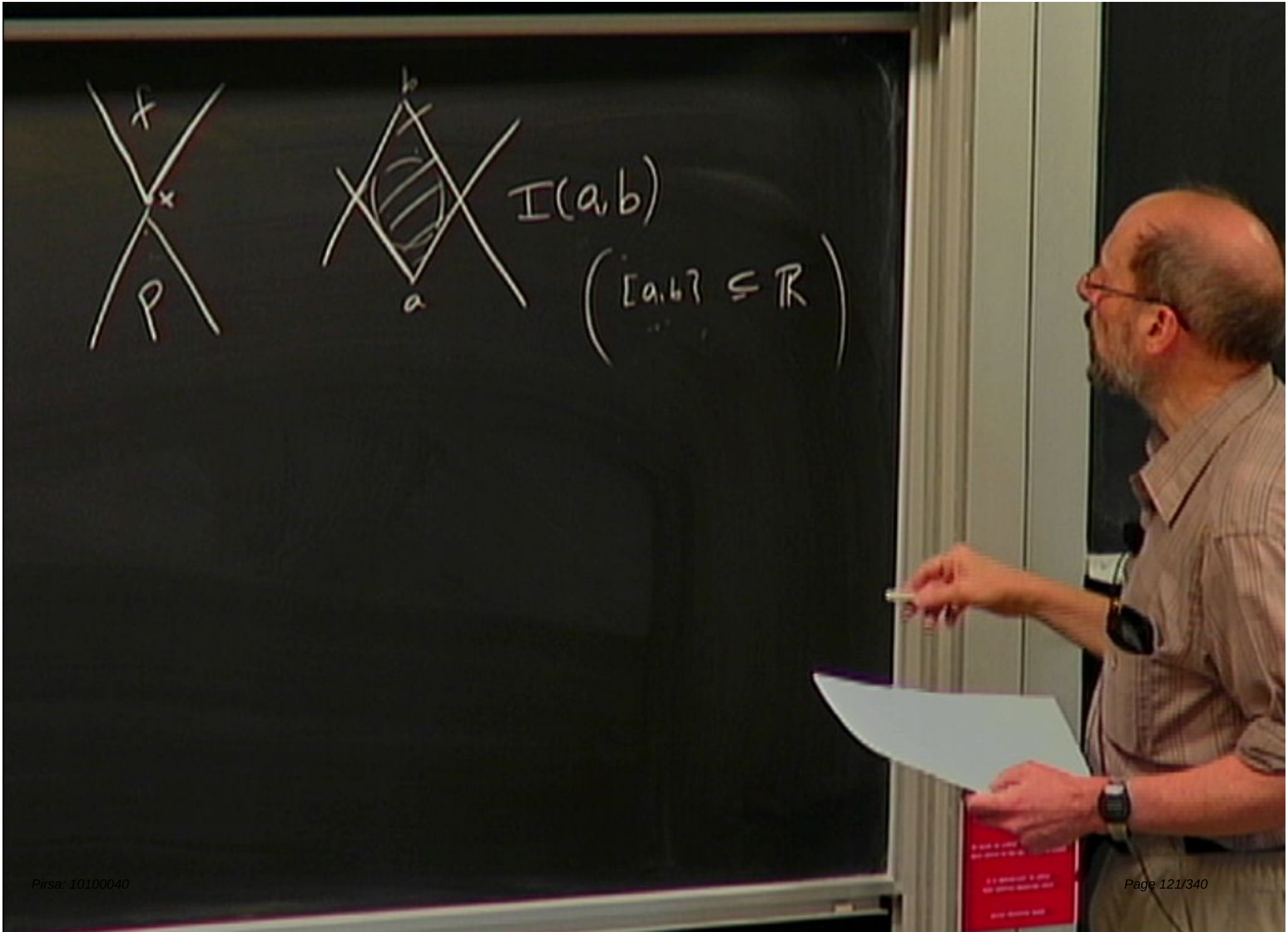


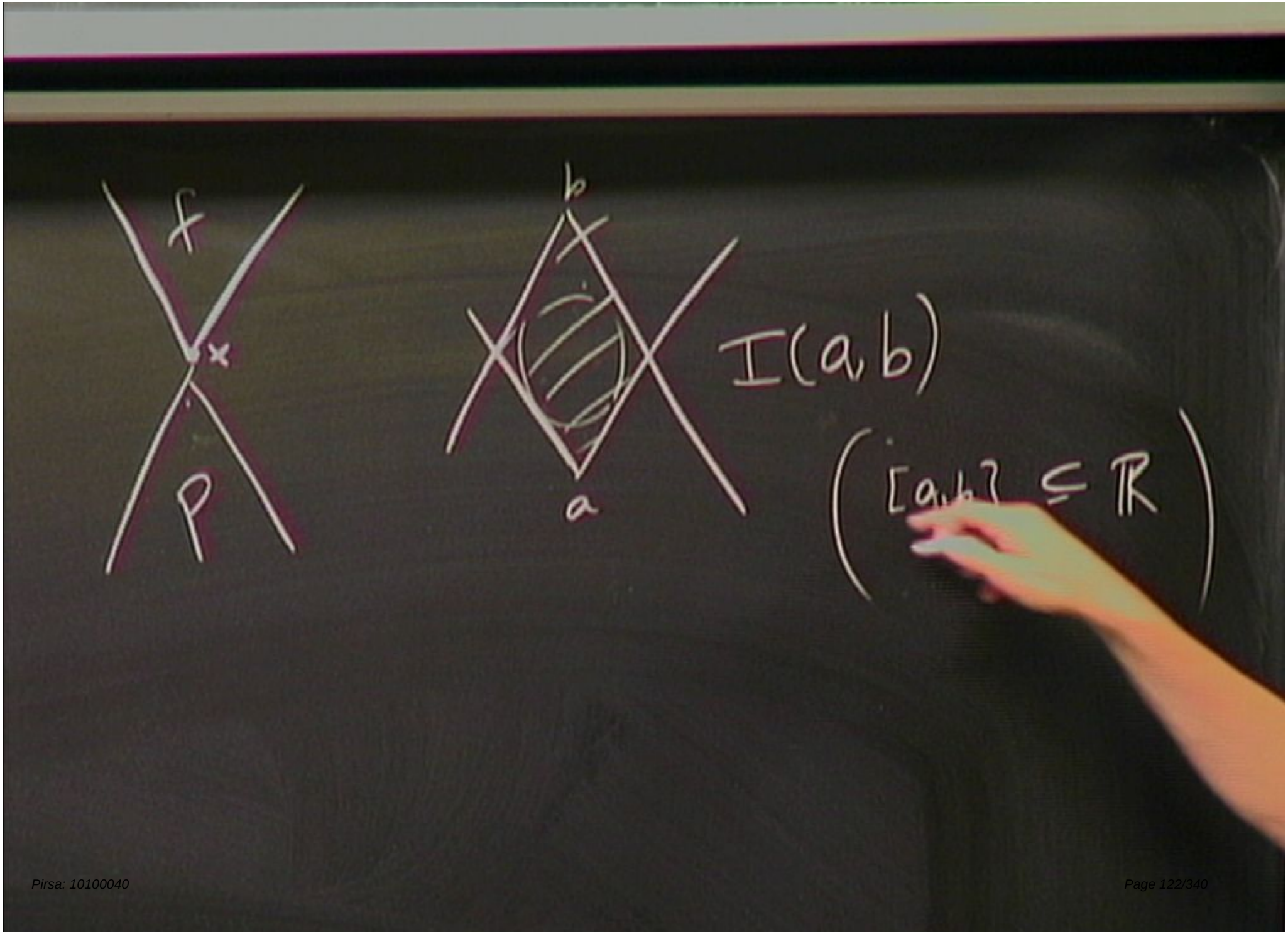
I(a,b)

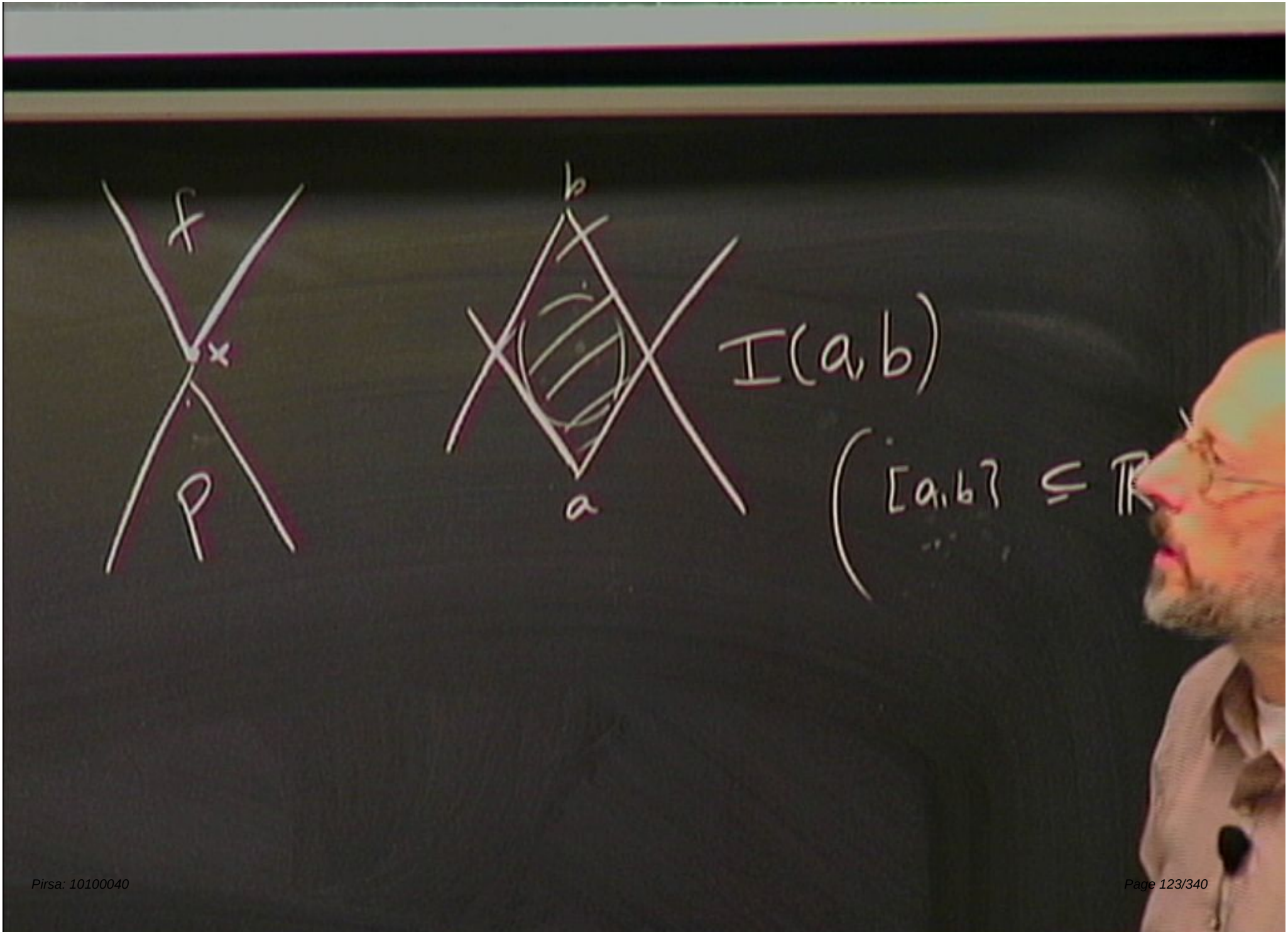
Def interval, past, future






f
x
p
b
a
I(a,b)
([a,b] ⊆ ℝ


f
x
P
b
a
I(a,b)
([a,b] ⊆ ℝ)


f
x
P
b
a
I(a,b)
([a,b] ⊆ ℝ



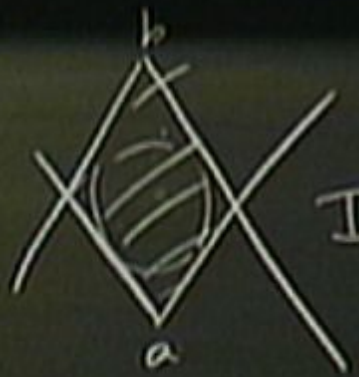


$I(a,b)$

$([a,b] \subseteq \mathbb{R})$

t, future





$I(a,b)$

$([a,b] \subseteq \mathbb{R})$

exclusive int.

$\{x \mid a < x < b\}$





$I(a,b)$

$([a,b] \subseteq \mathbb{R})$

exclusive int.

$\{x \mid a < x < b\}$

Def interval, past, future

Def antichain, chain, height, width

Def interval, past, future

Def antichain, chain, height, width

Def interval, past, future

Def antichain, chain, height, width



Def interval, past, future

Def antichain, chain, height, width

. . . .

invariants



Def interval, past, future

Def antichain, chain, height, width

. . . .

invariants





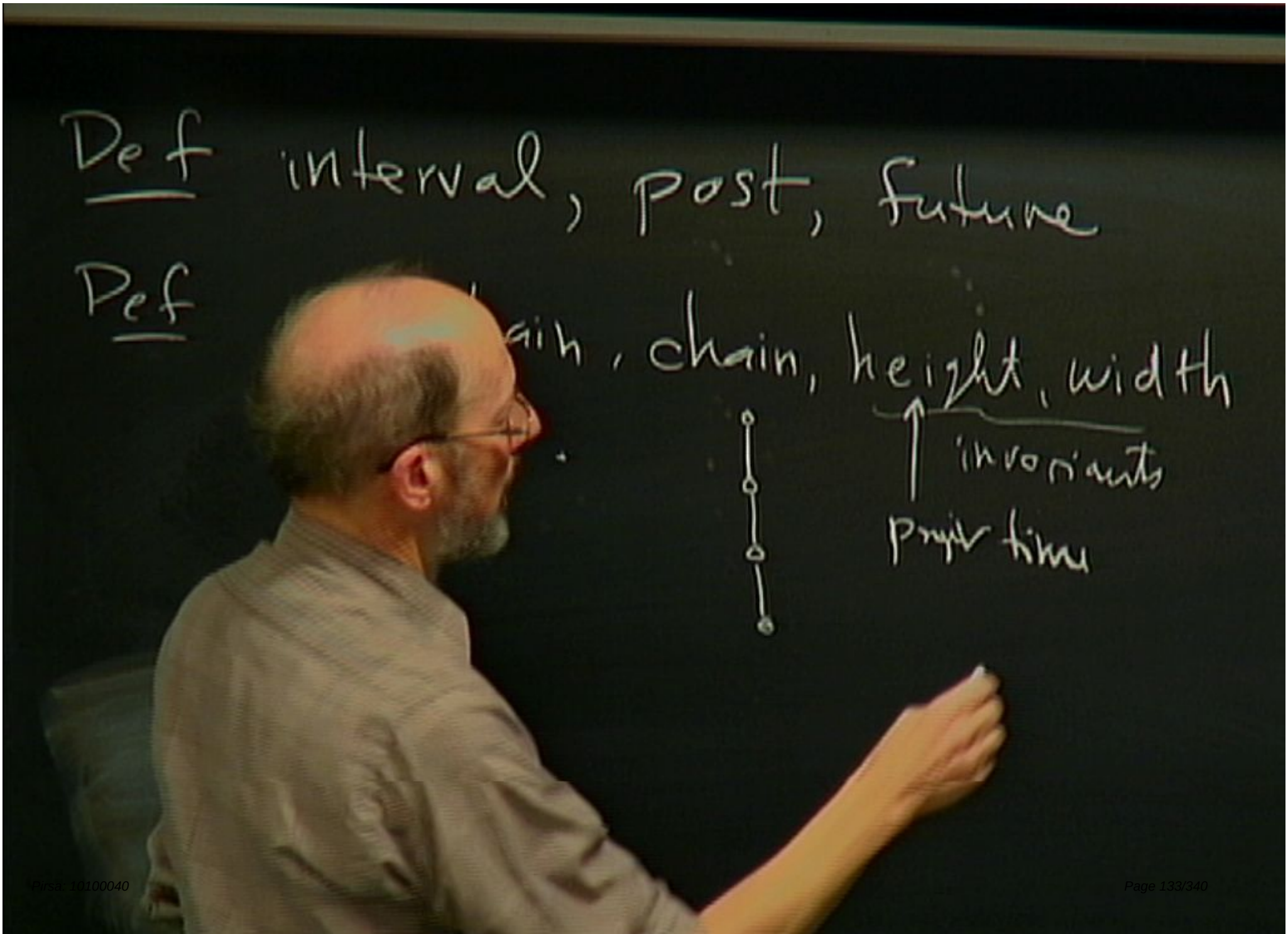


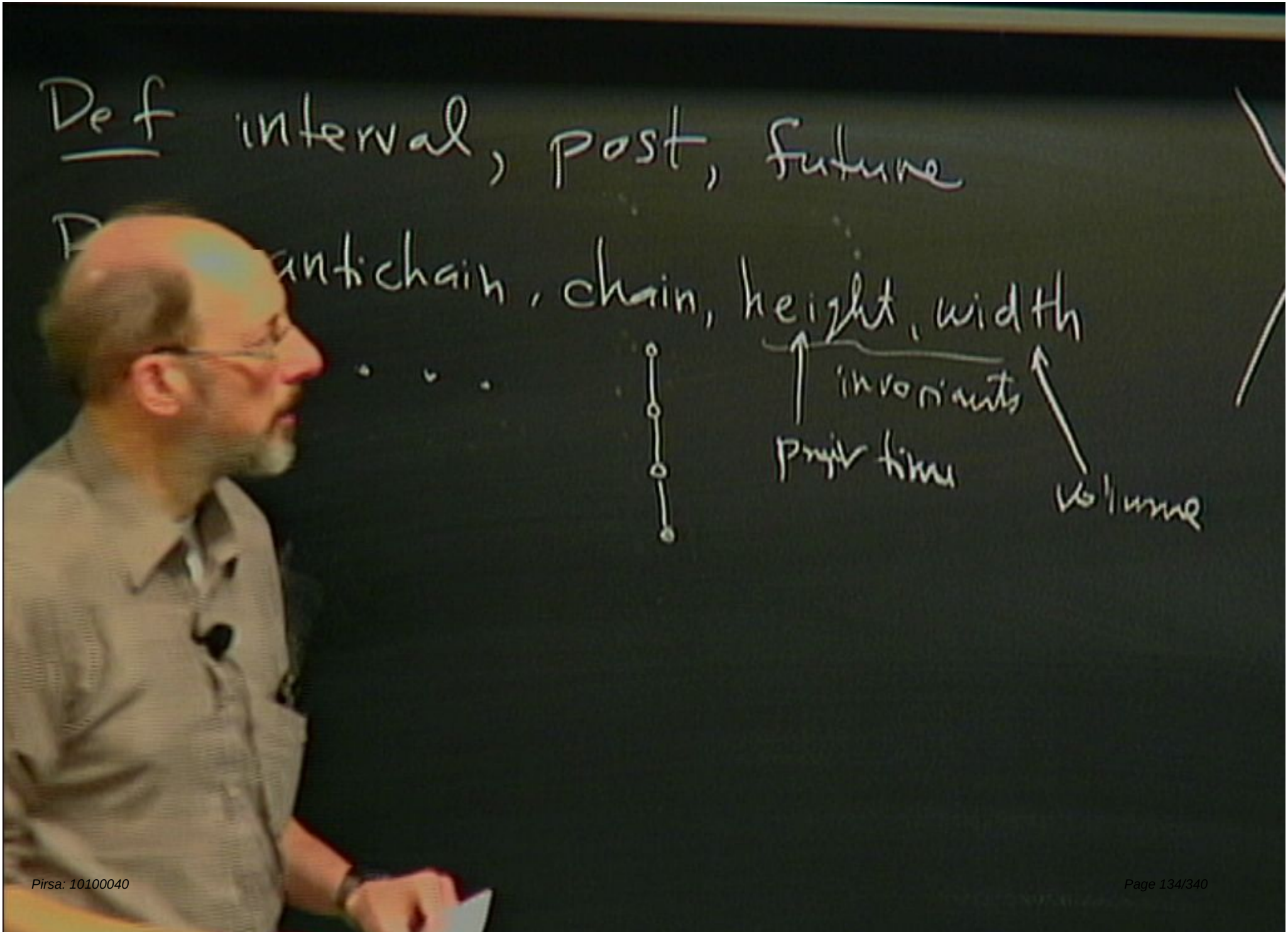
$I(a,b)$

$([a,b] \subseteq \mathbb{R}$

exclusive int.

$\{x \mid a < x < b\}$


Def interval, past, future
Def
ain, chain, height, width
invariants
past time


Def interval, past, future
antichain, chain, height, width
invariants
past time
volume

Def interval, past, future

Def antichain, chain, height, width

. . . .



invariants

past time

3

volume

Def interval, past, future

Def antichain, chain, height, width

. . . .

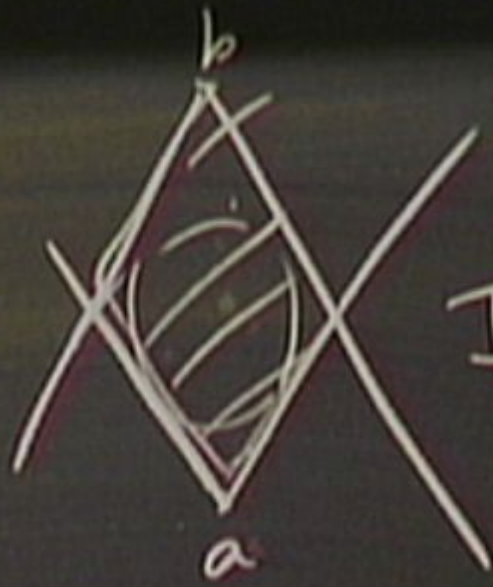


invariants

proper time

3 dimensions





$I(a,b)$

$([a,b] \subseteq \mathbb{R})$

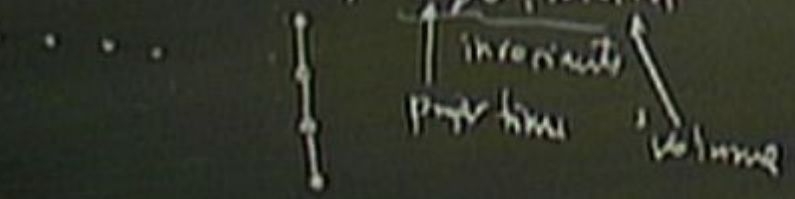
exclusive int.

$\{x \mid a < x < b\}$

Def interval, past, future

Def antichain, chain, height, width

. . . .







$I(a,b)$

$([a,b] \subsetneq \mathbb{R})$

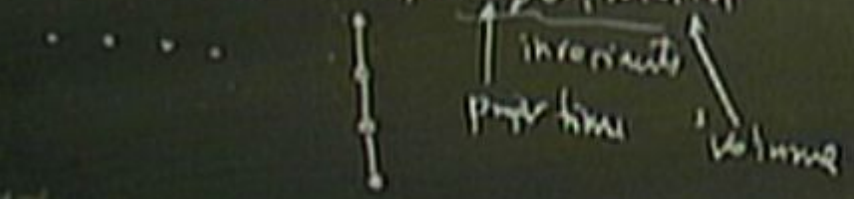
exclusive int.

$\{x \mid a < x < b\}$

Def interval, past, future

Def antichain, chain, height, width

. . . .





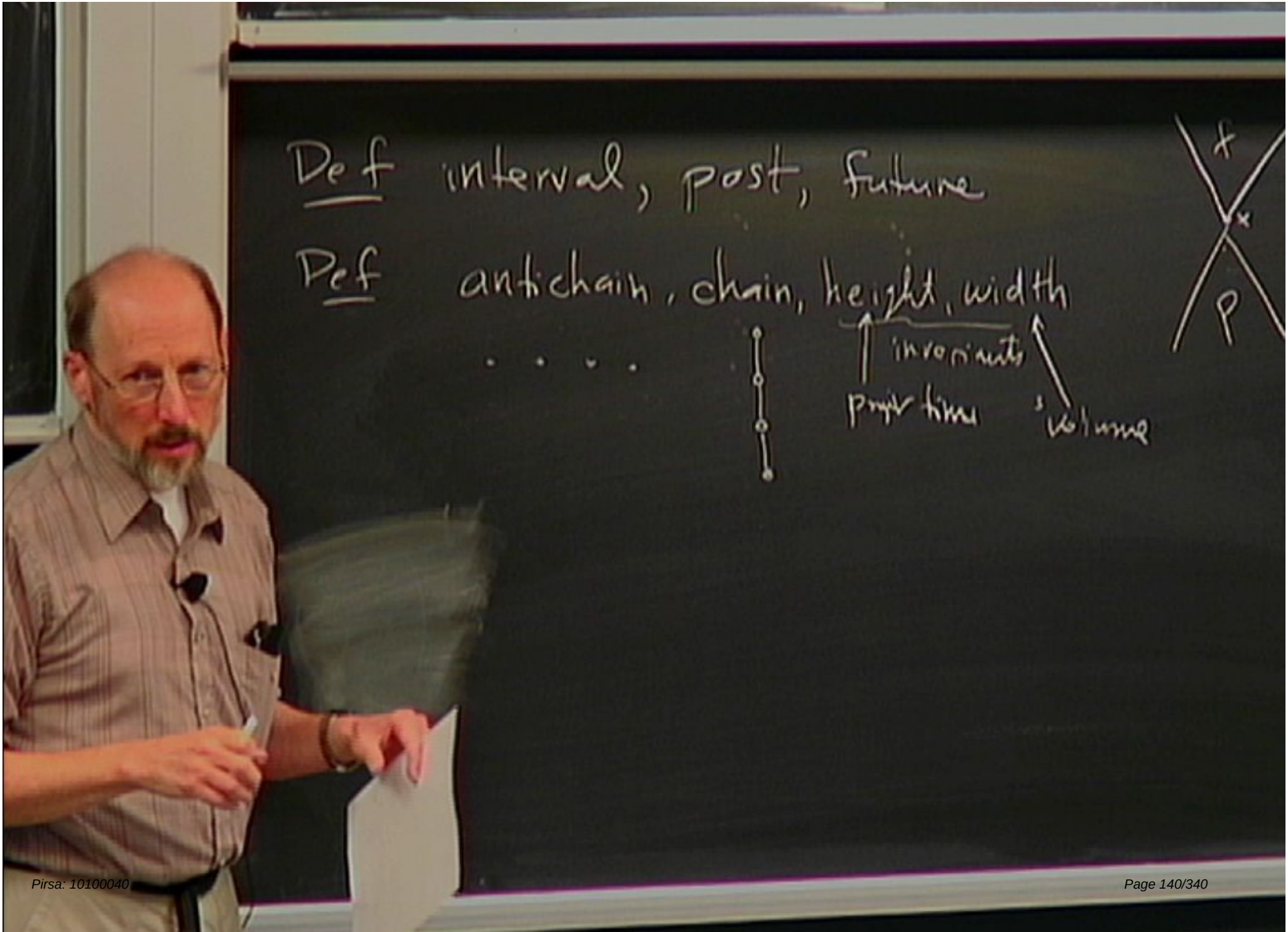


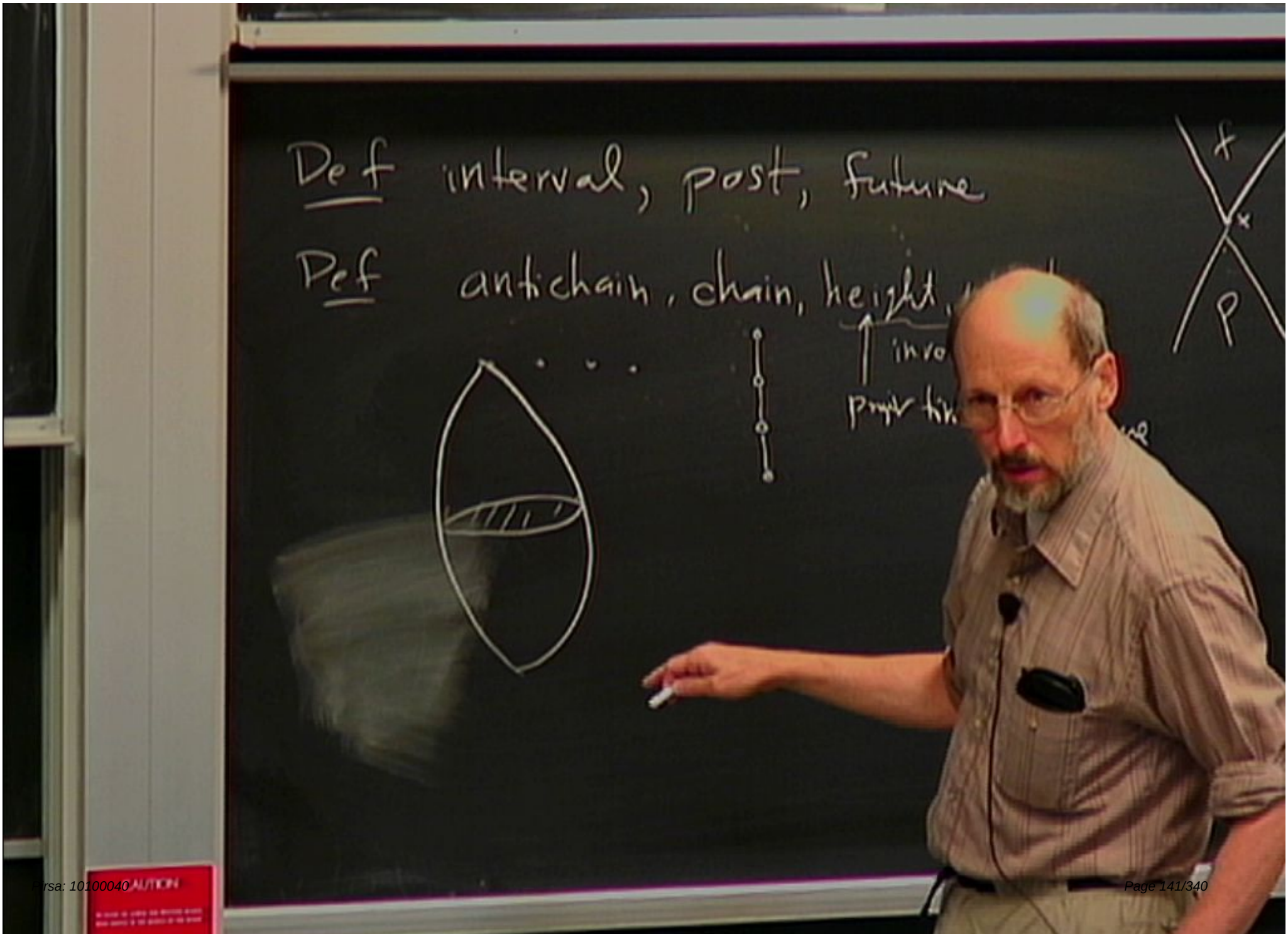
$I(a,b)$

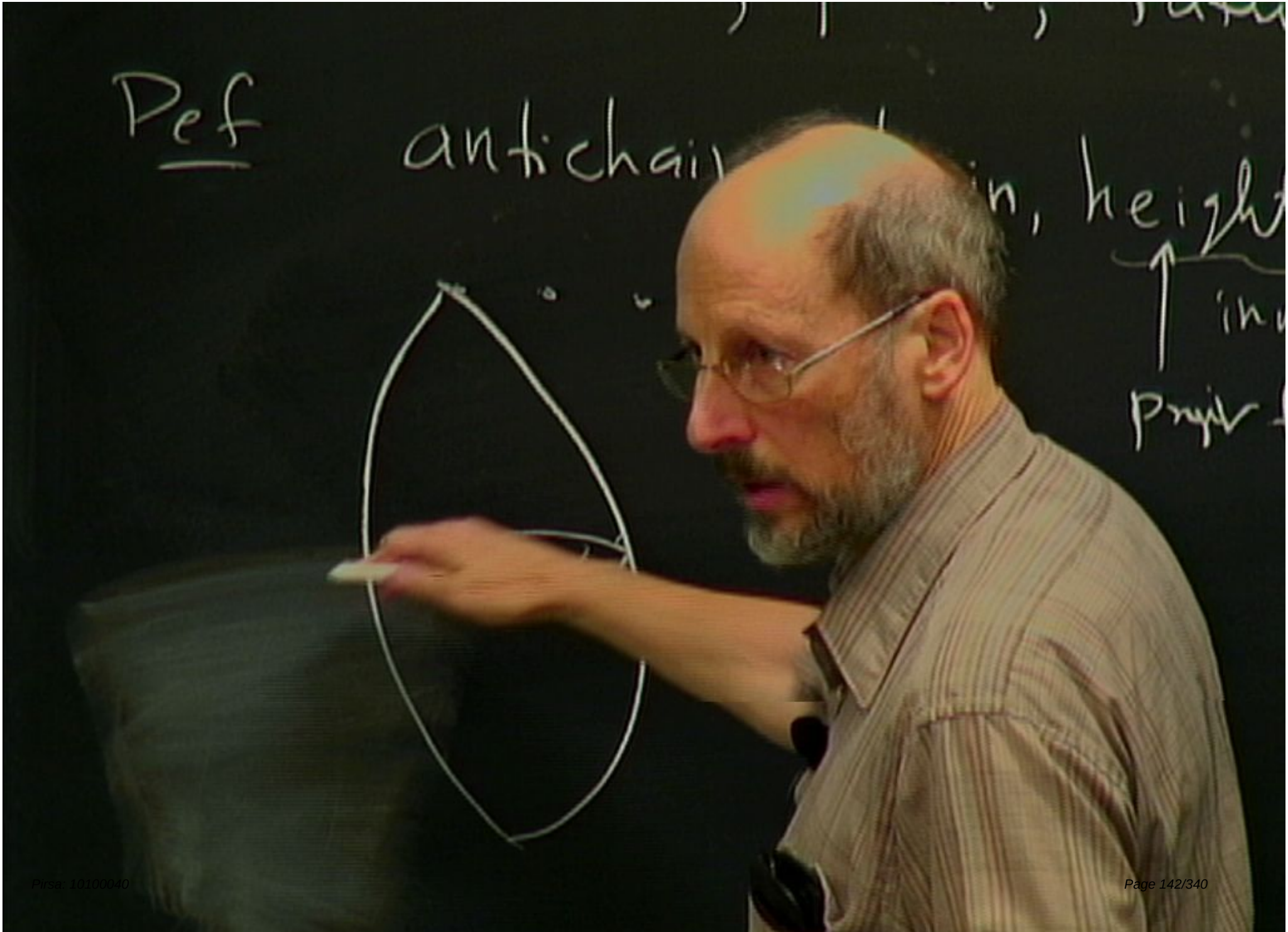
$([a,b] \subseteq \mathbb{R}$

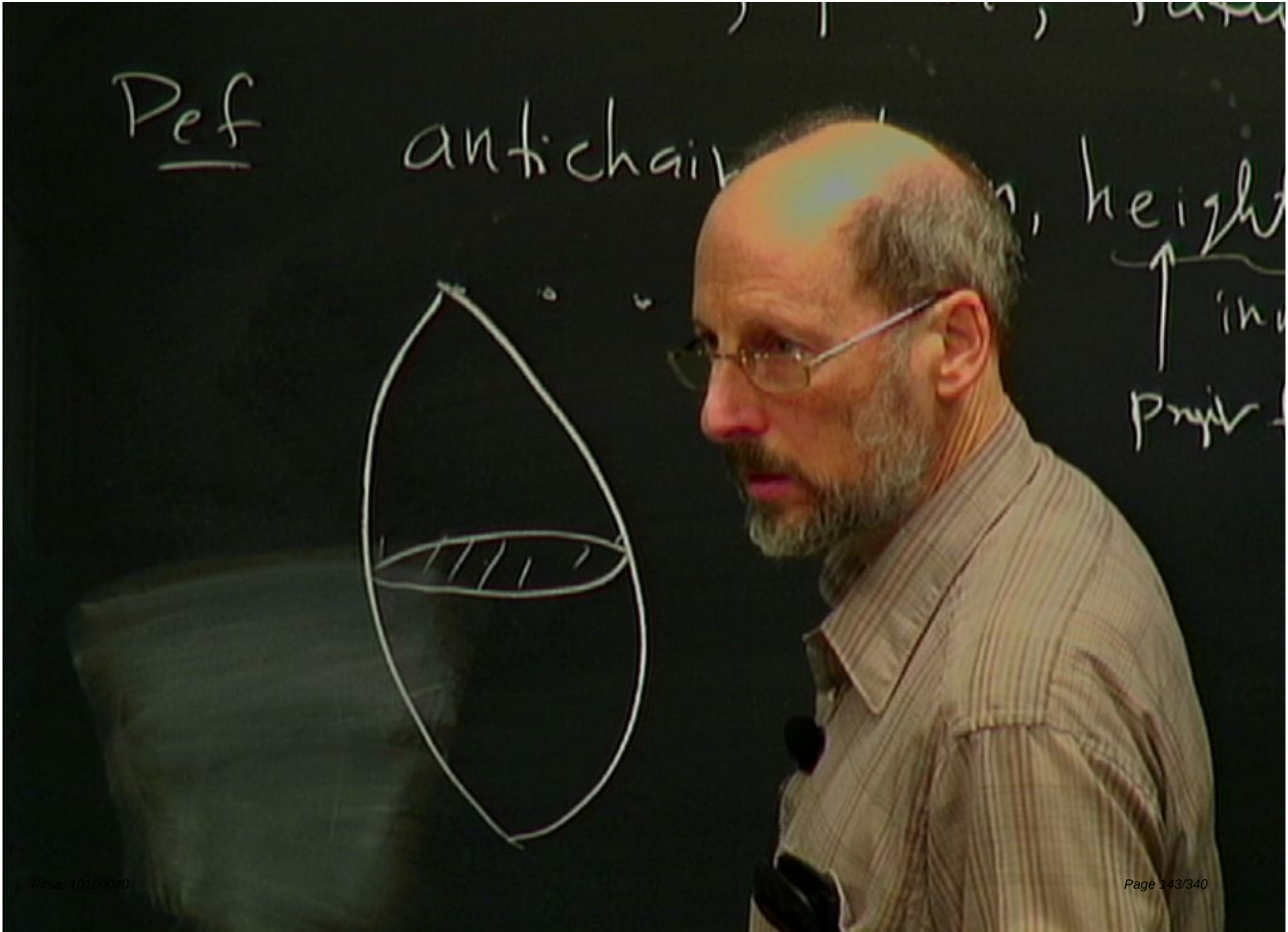
exclusive int.

$\{x \mid a < x < b\}$


Def interval, past, future
Pef antichain, chain, height, width
. . . .
invariants
past time
f
x
P

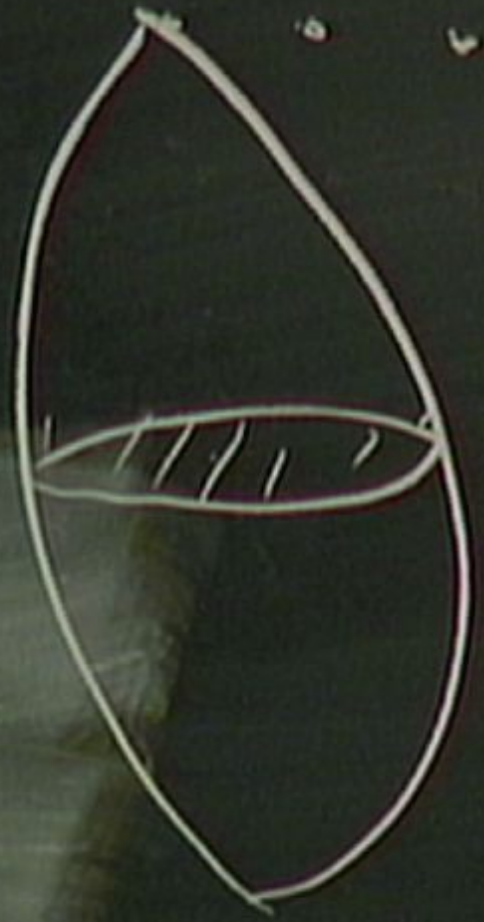

Def interval, past, future
Pef antichain, chain, height,
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x
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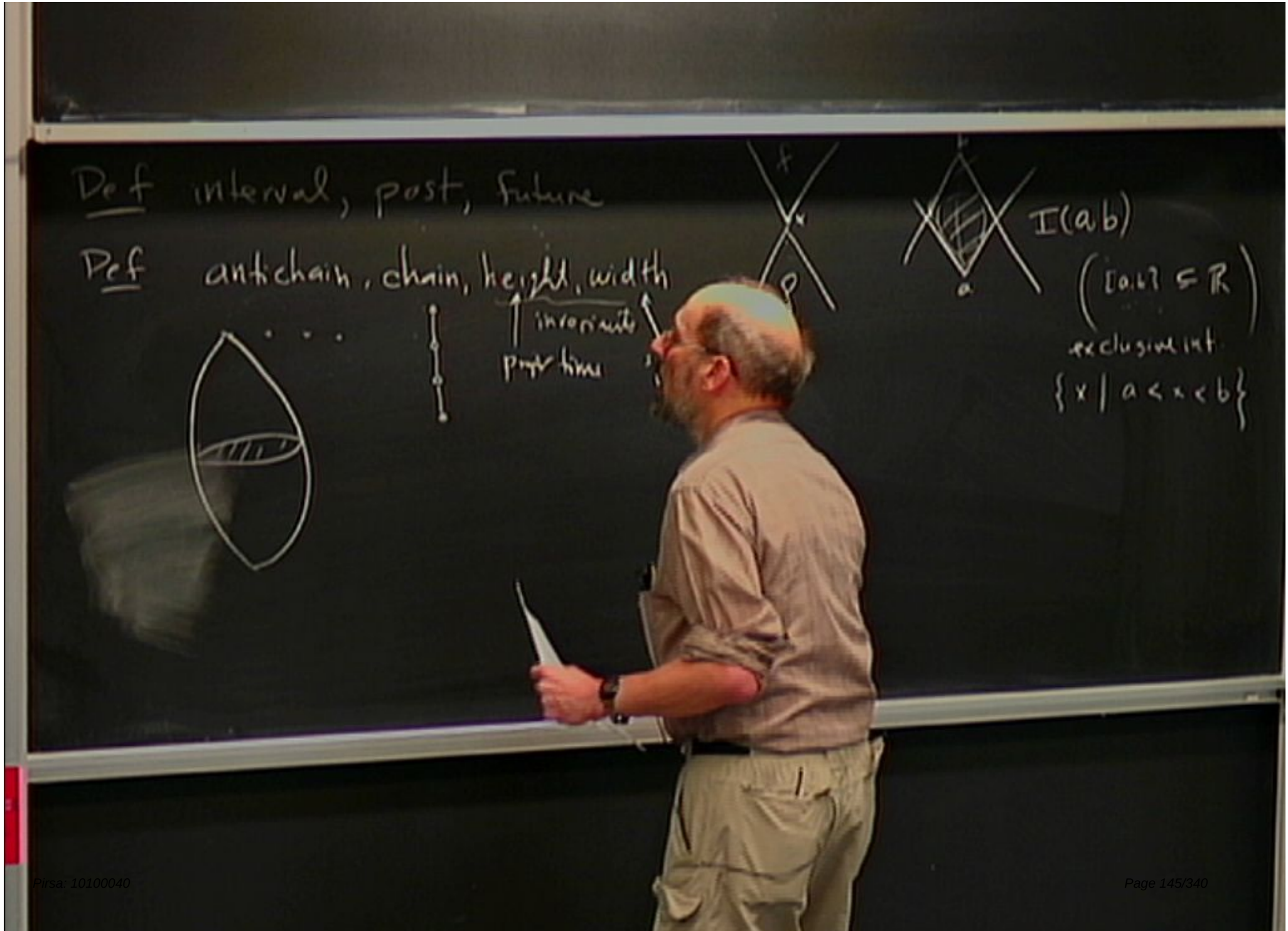

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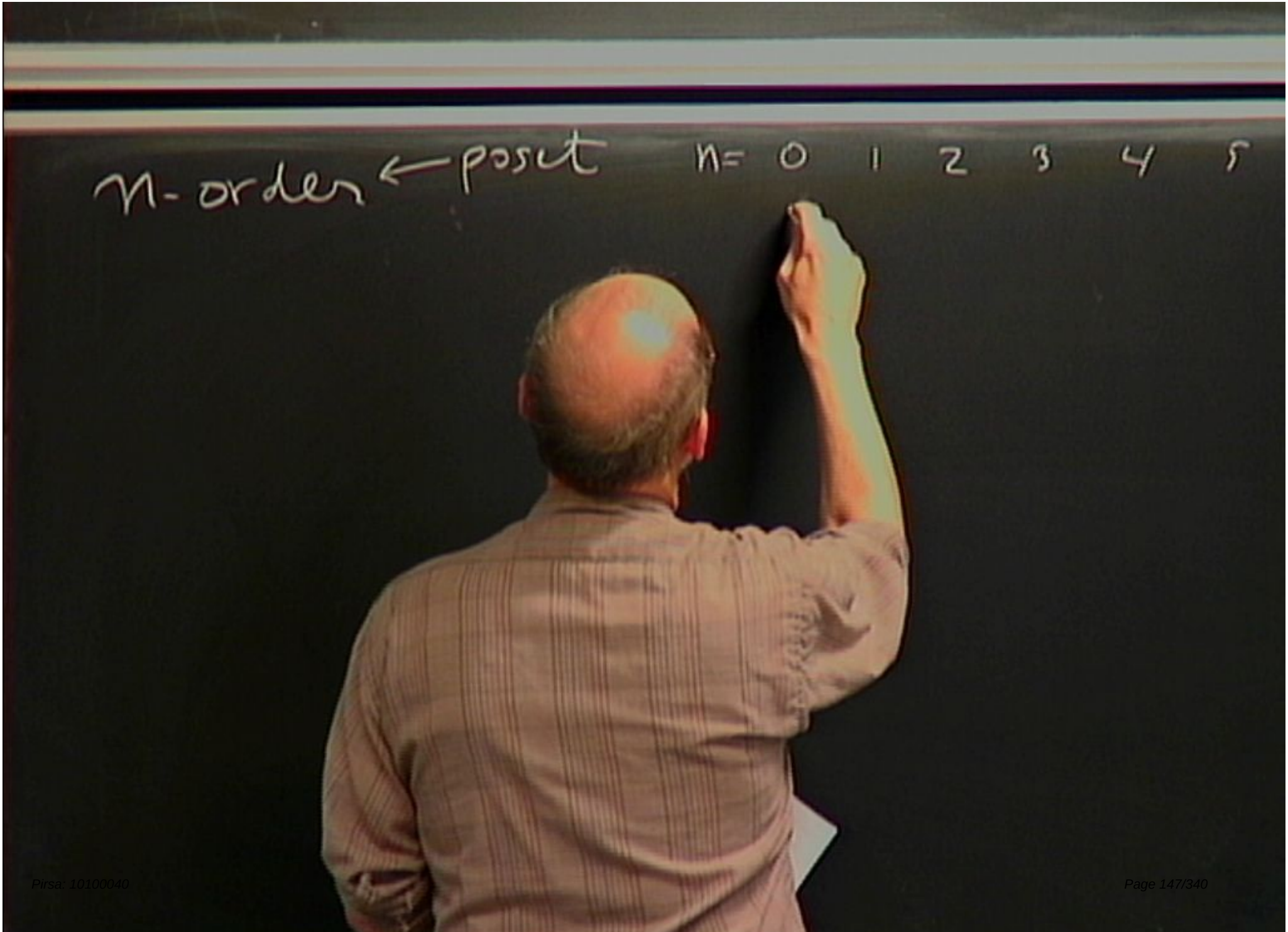
antichain, chain, height

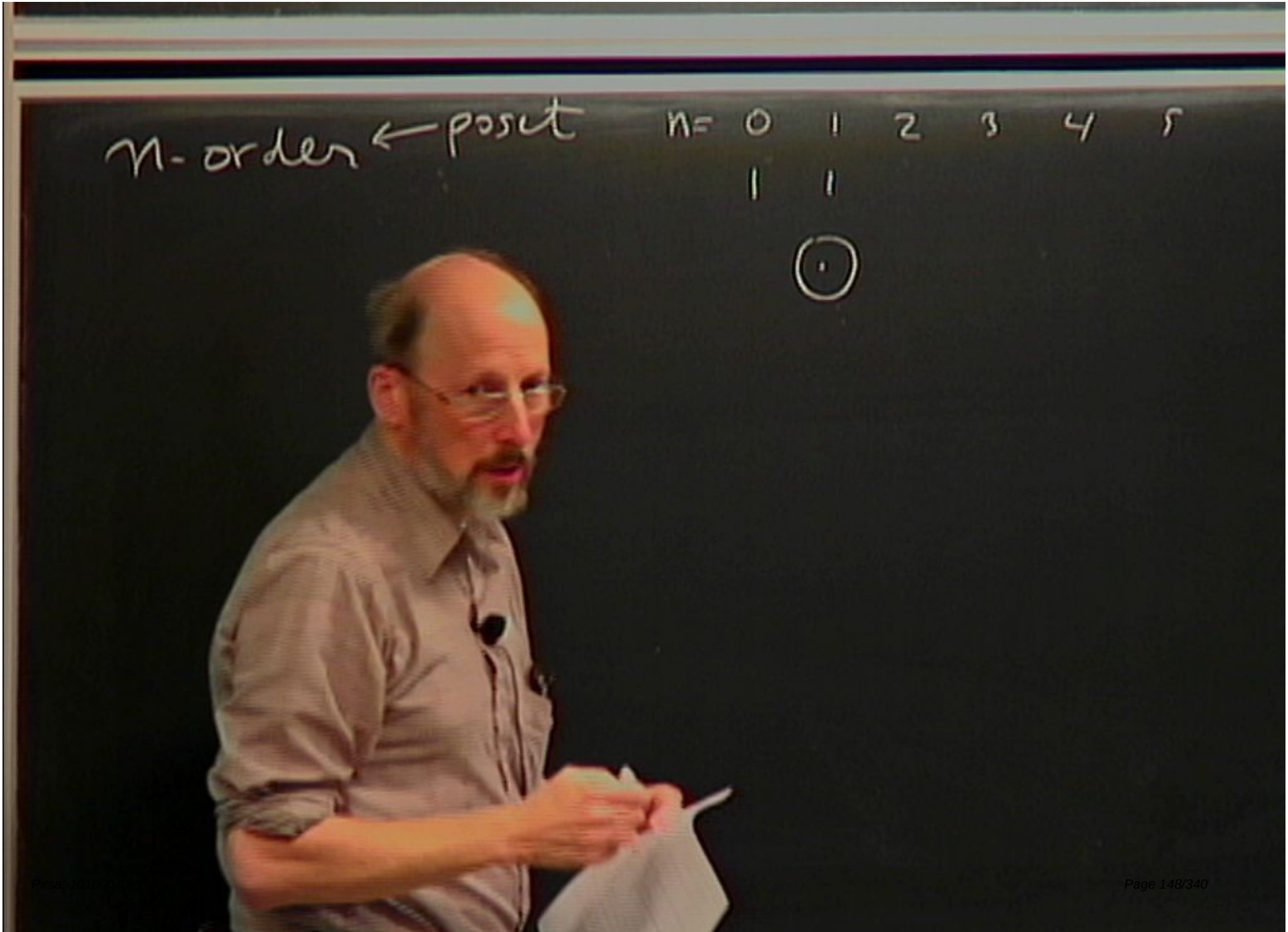


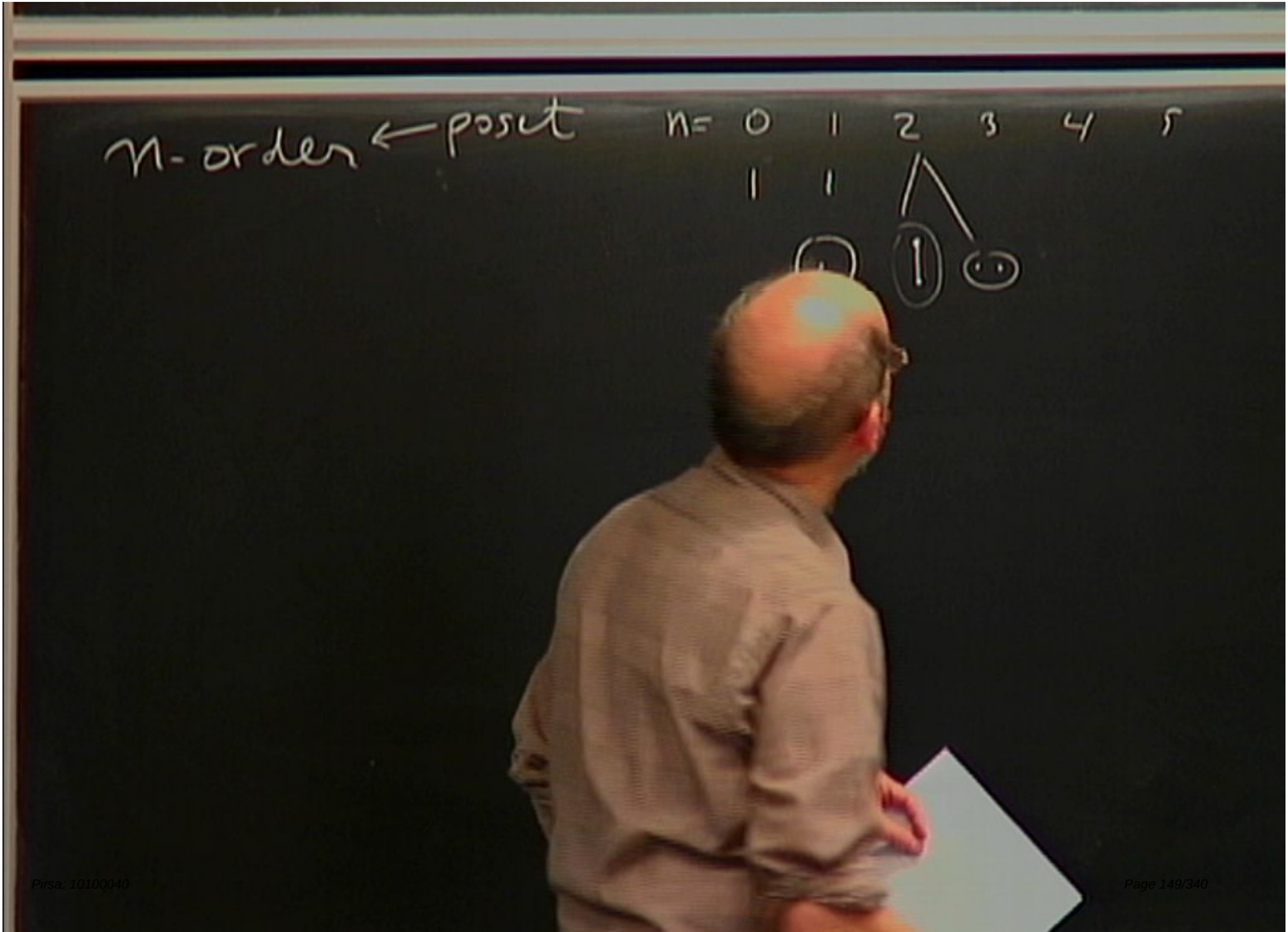


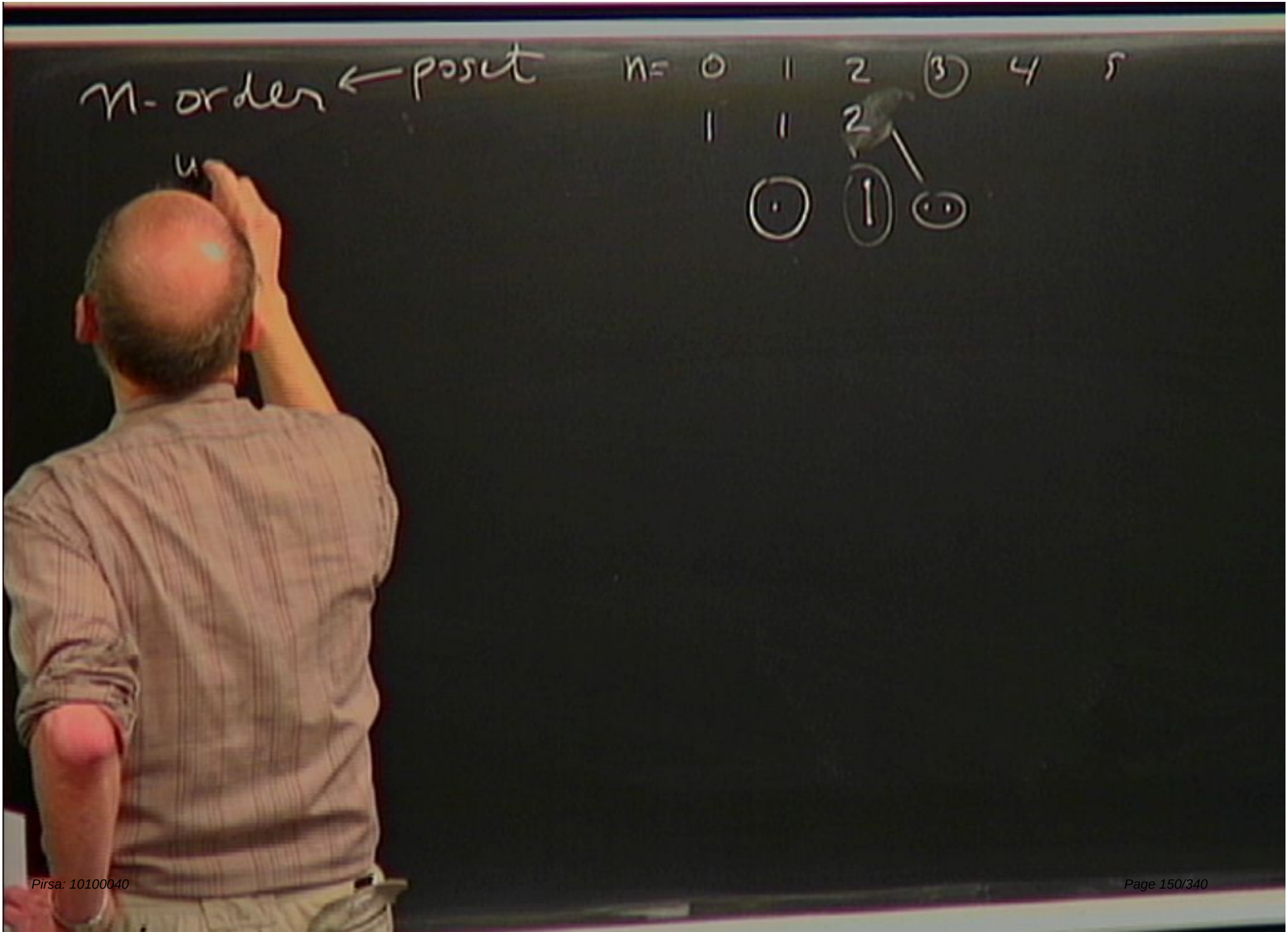

Def interval, past, future
Def antichain, chain, height, width
. . .
invariant
proper time
I(a b)
exclusive int.
{x | a < x < b}

n-order ← poset


n-order ← poset
n= 0 1 2 3 4 5


n-order ← poset
n= 0 1 2 3 4 5
1 1


n-order ← poset
n= 0 1 2 3 4 5
1 1
1


n-order ← poset
n= 0 1 2 3 4 5
1 1 2

n-order ← poset

unlabeled

n=	0	1	2	3	4	5
	1	1	2			

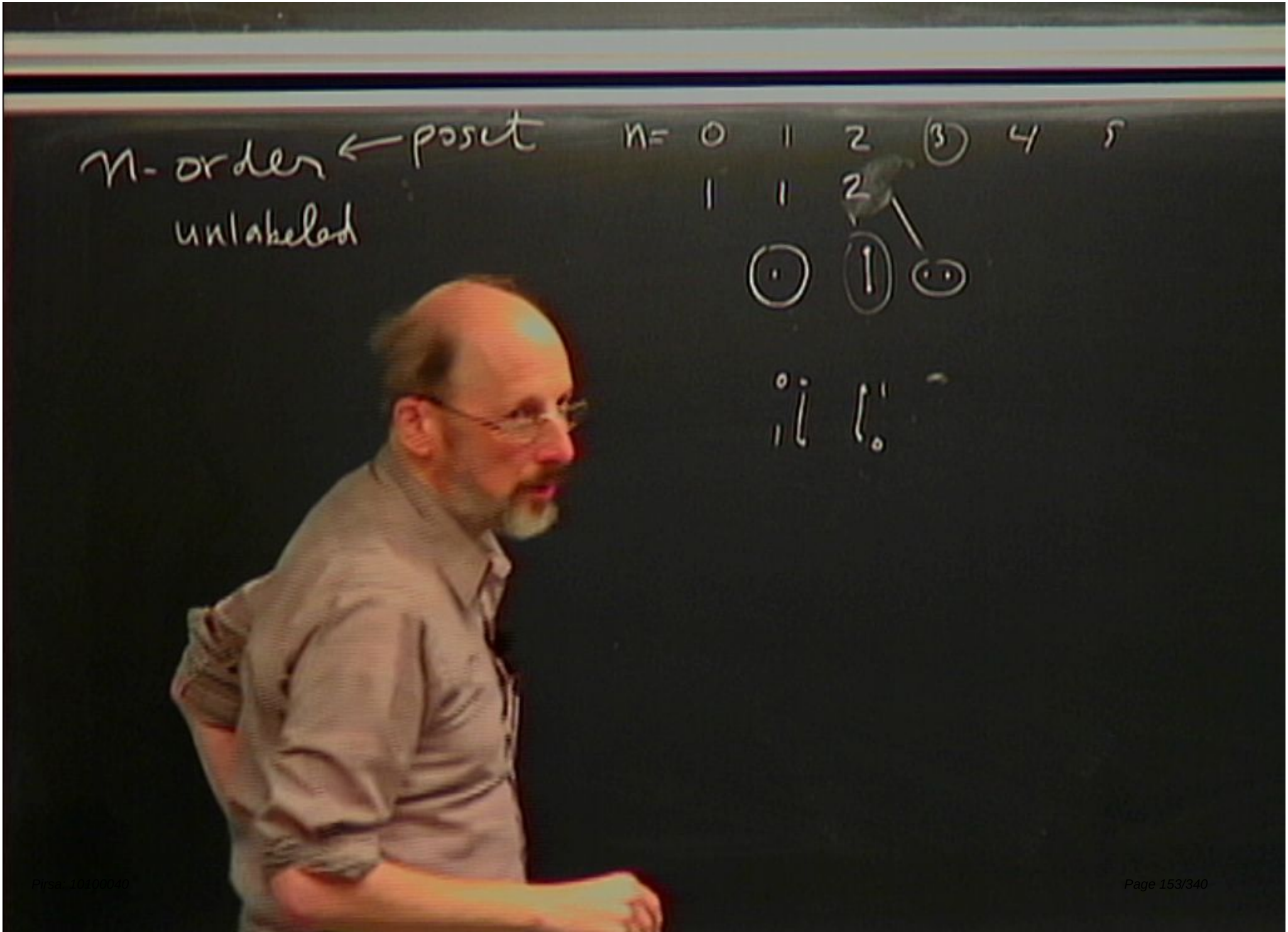


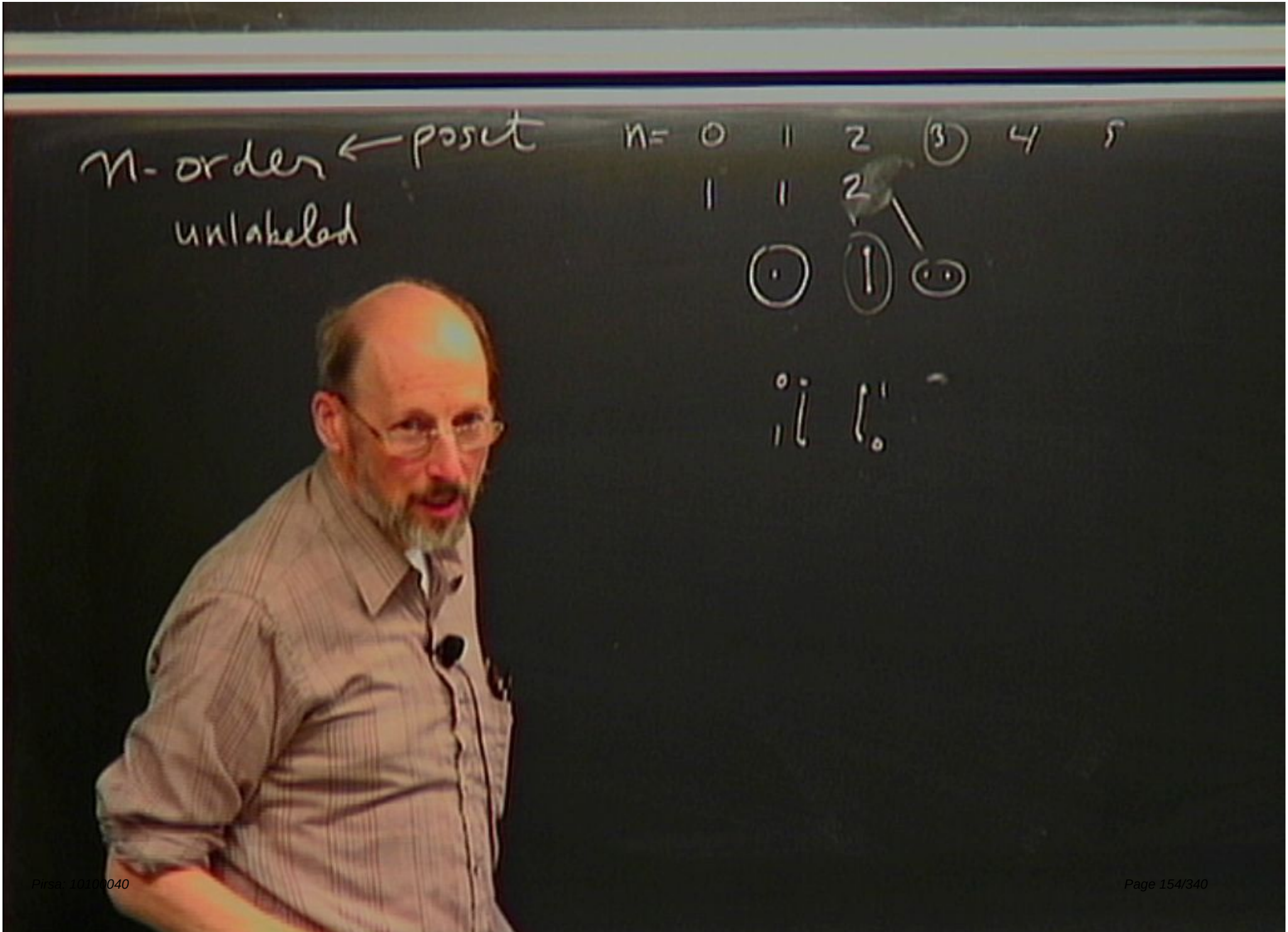
n-order ← poset

unlabeled

n=	0	1	2	3	4	5
	1	1	2			




n-order ← poset
unlabeled
n= 0 1 2 3 4 5
1 1 2
1 1

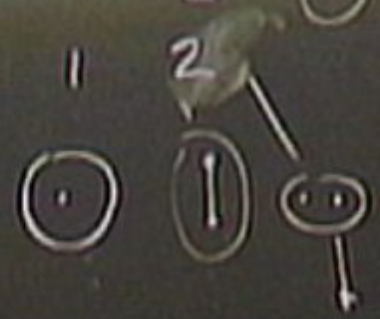

n-order ← poset
unlabeled
n= 0 1 2 3 4 5
1 1 2
1 1

n-order ← poset

unlabeled

n= 0 1 2 3 4 5

1 1 2





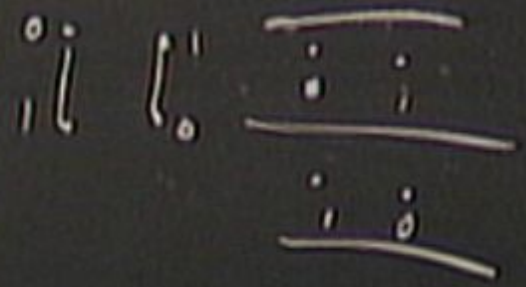
n-order ← poset

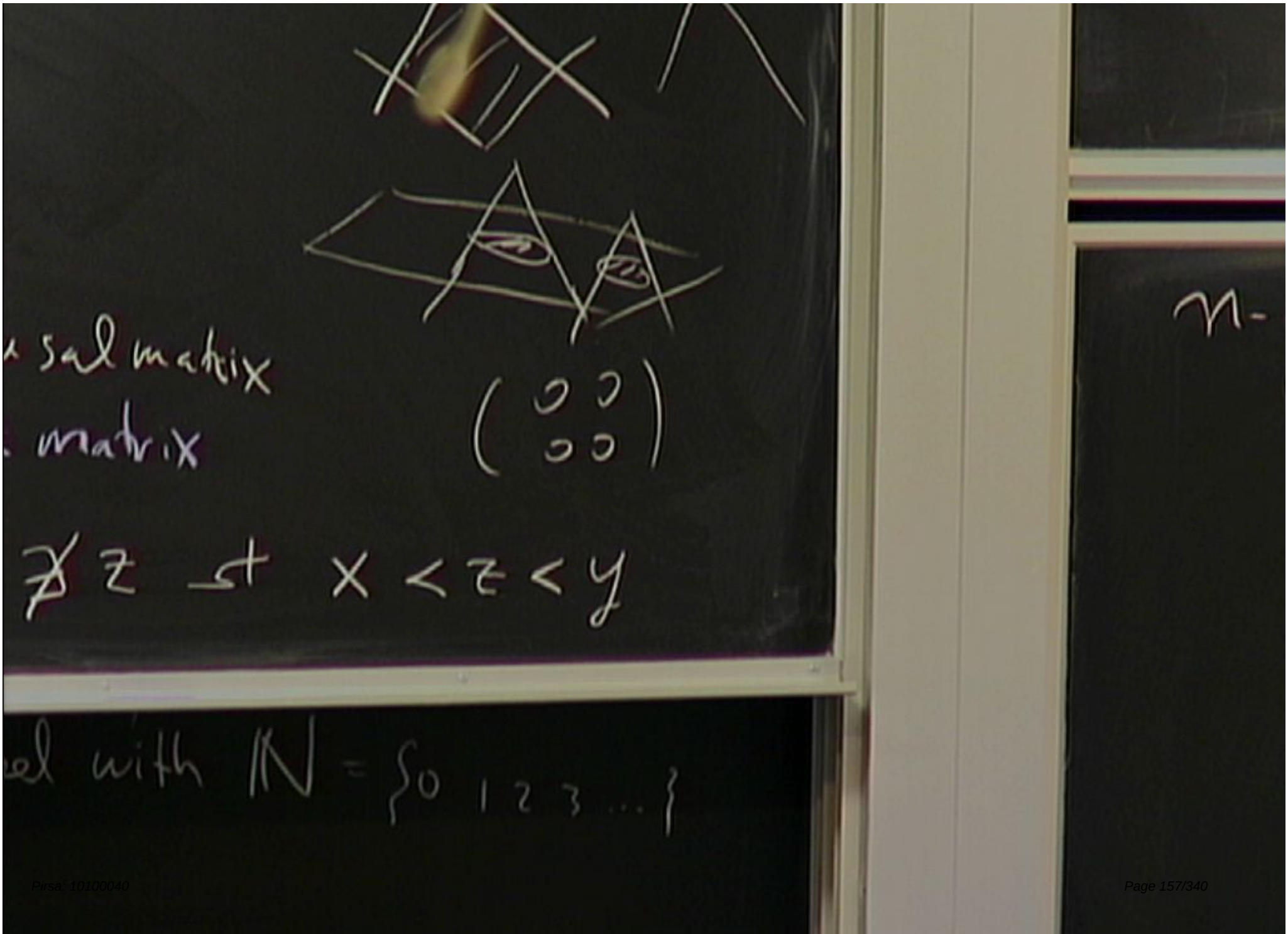
unlabeled

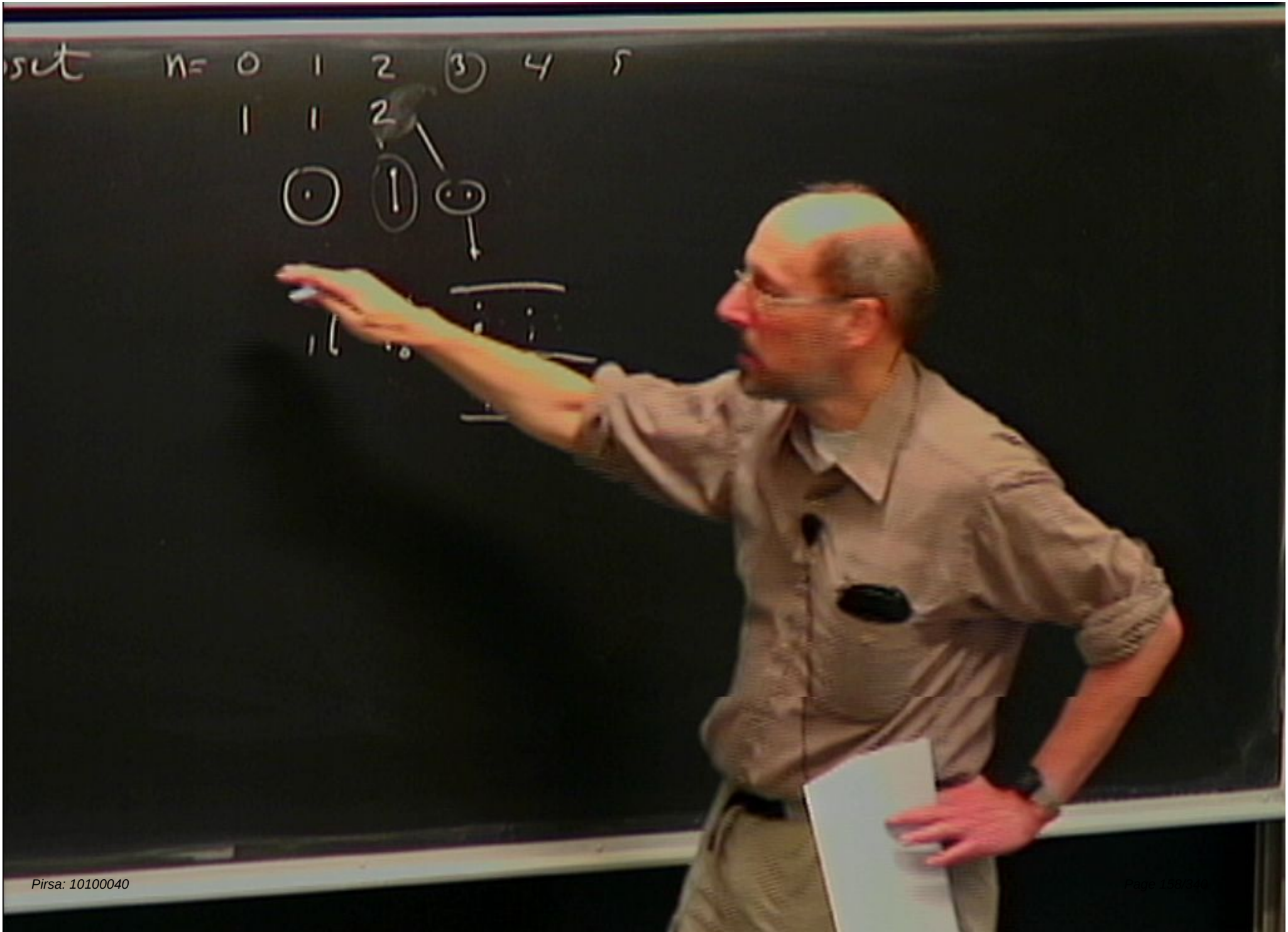
n= 0 1 2 3 4 5

1 1 2






sal matrix
matrix
$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
$\nexists z$ st $x < z < y$
el with $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$
n-

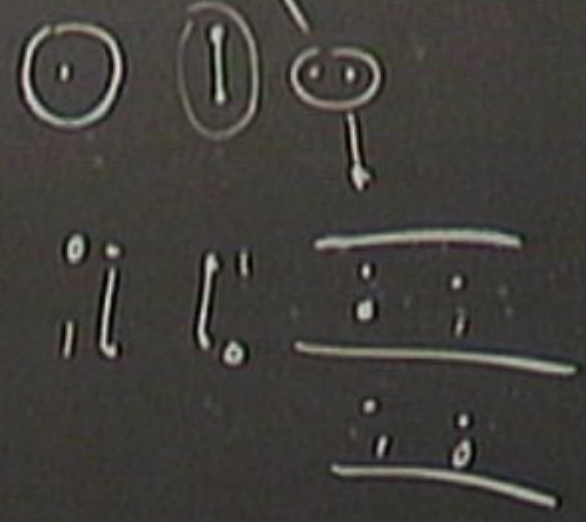

n= 0 1 2 3 4 5
1 1 2

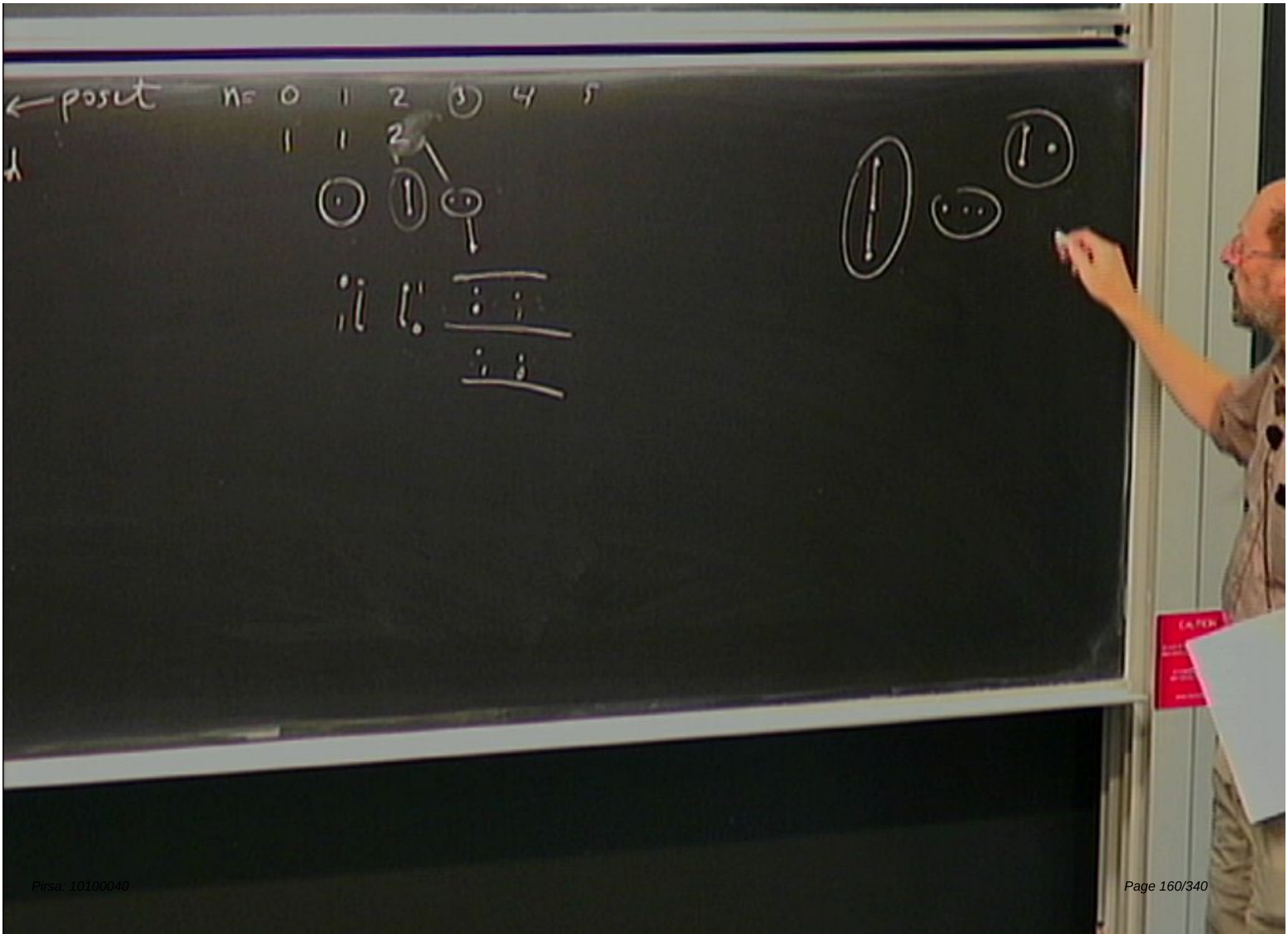
n-order ← poset

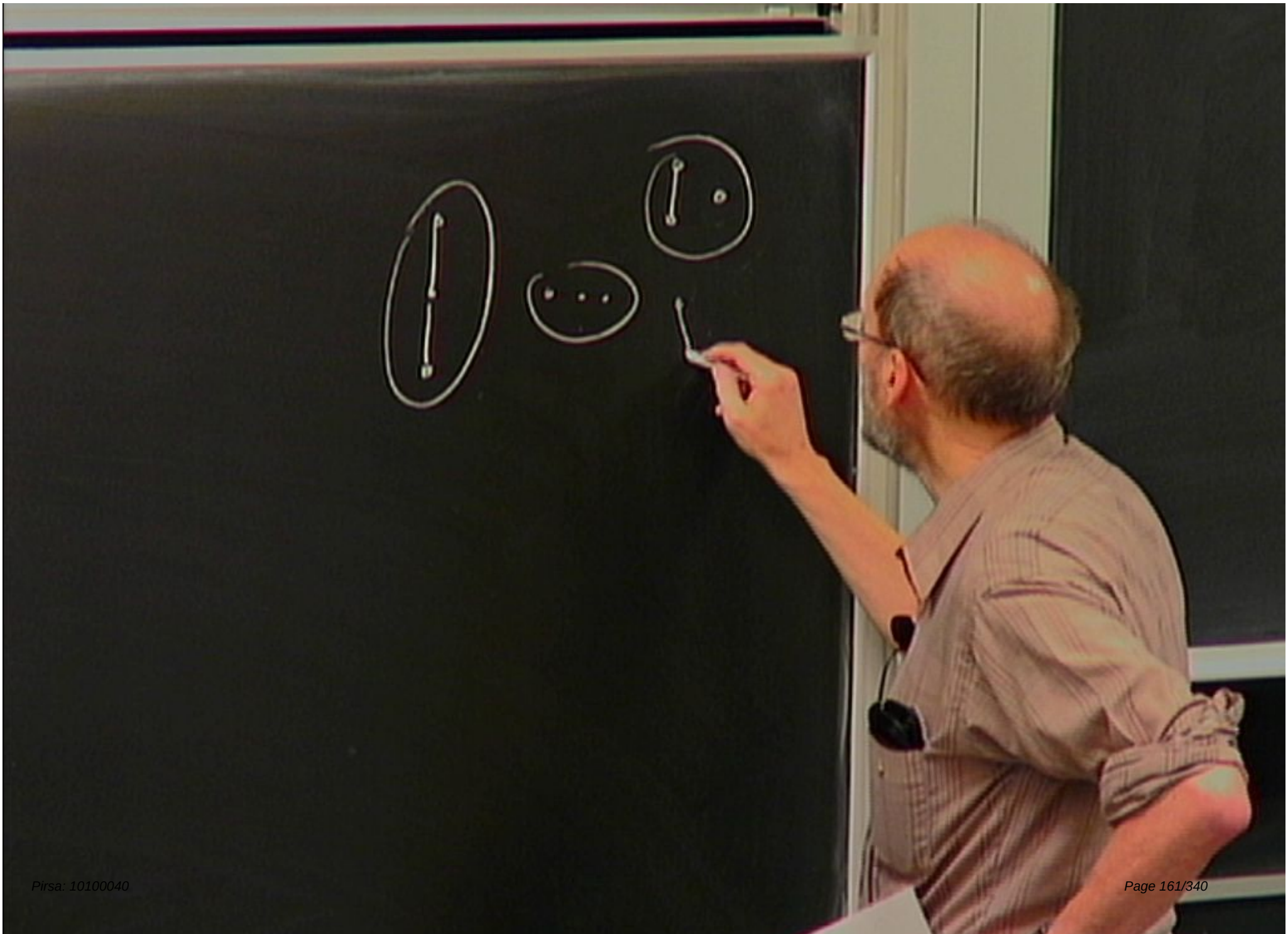
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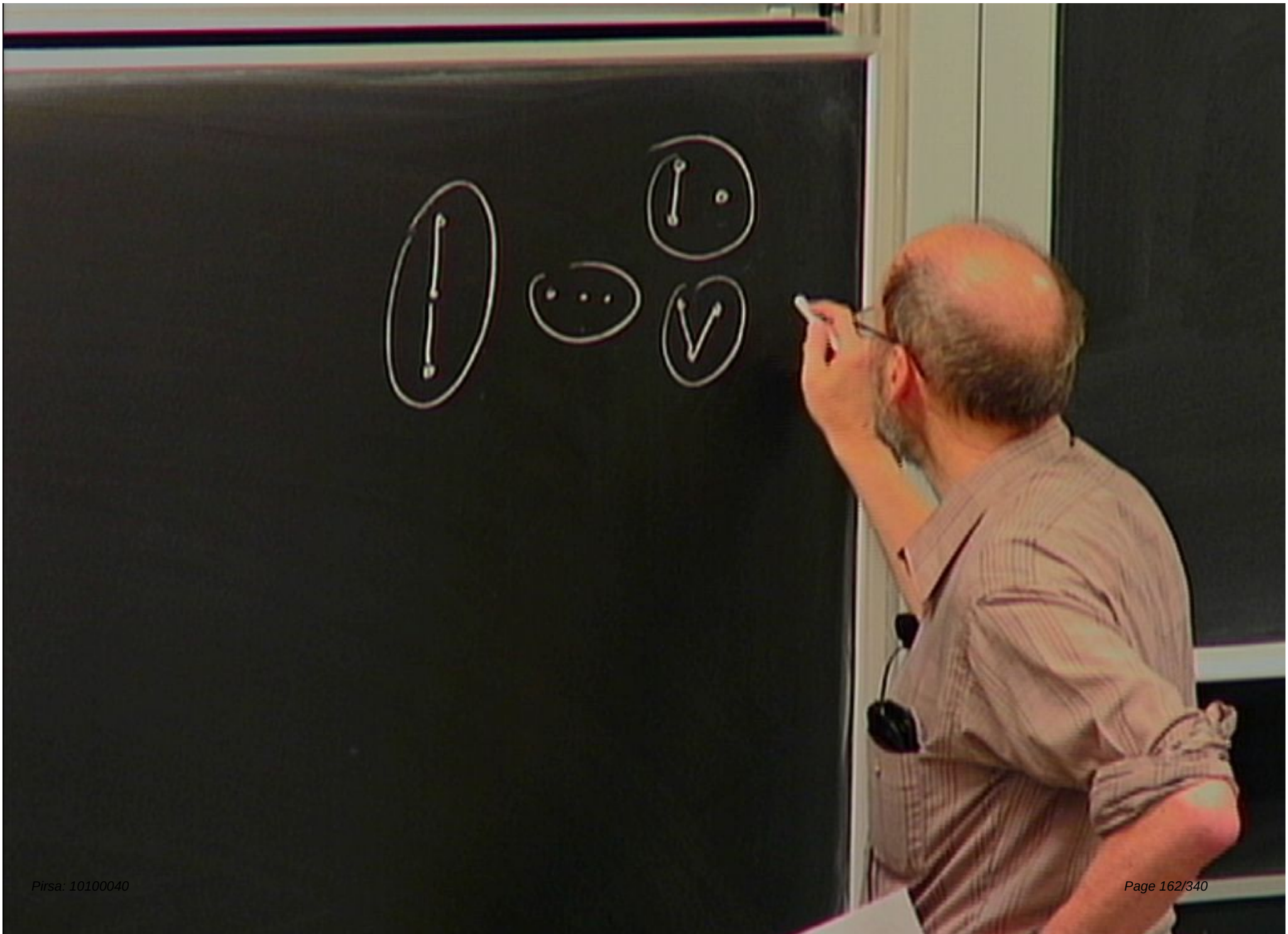
n= 0 1 2 3 4 5

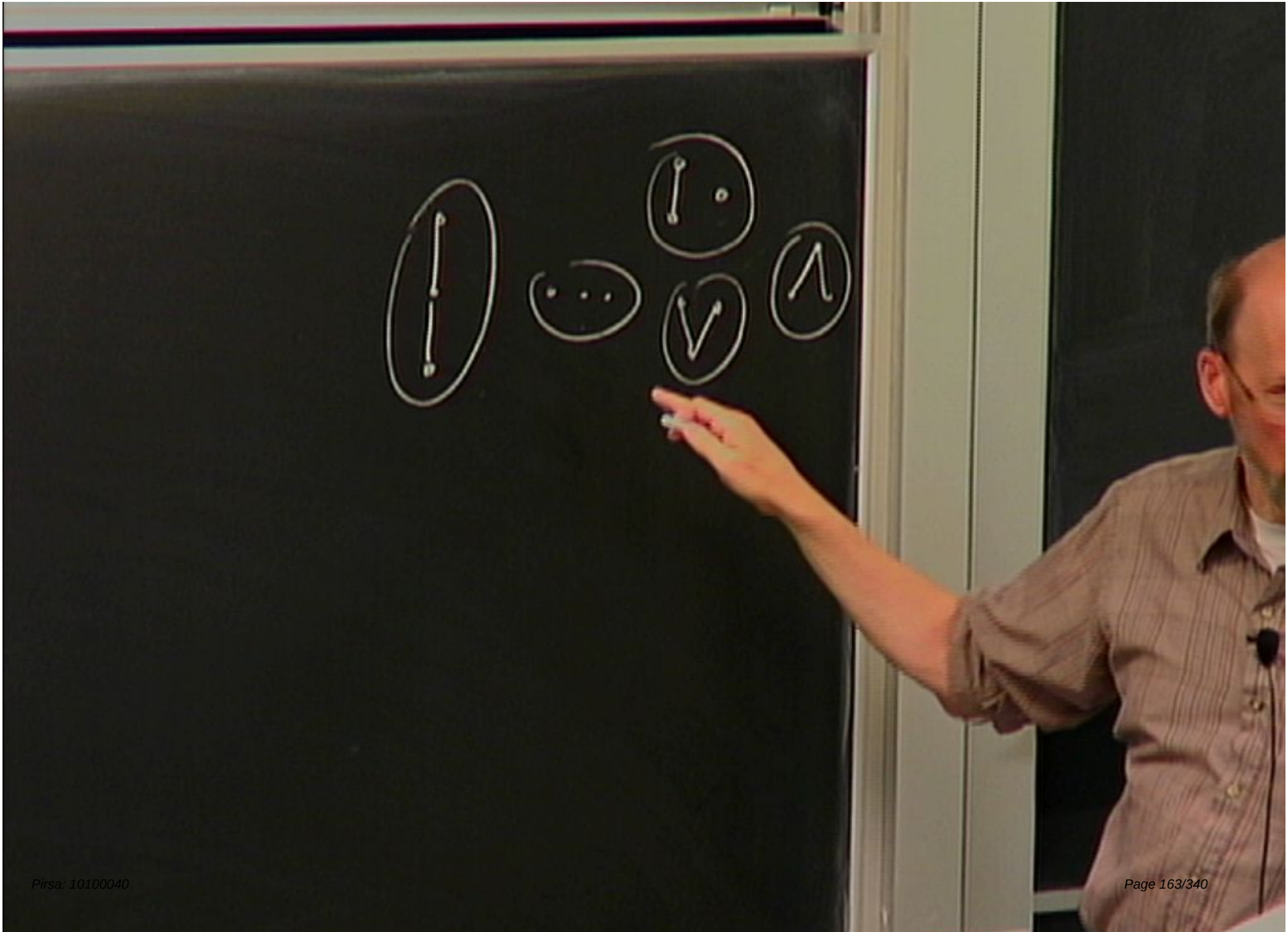
1 1 2

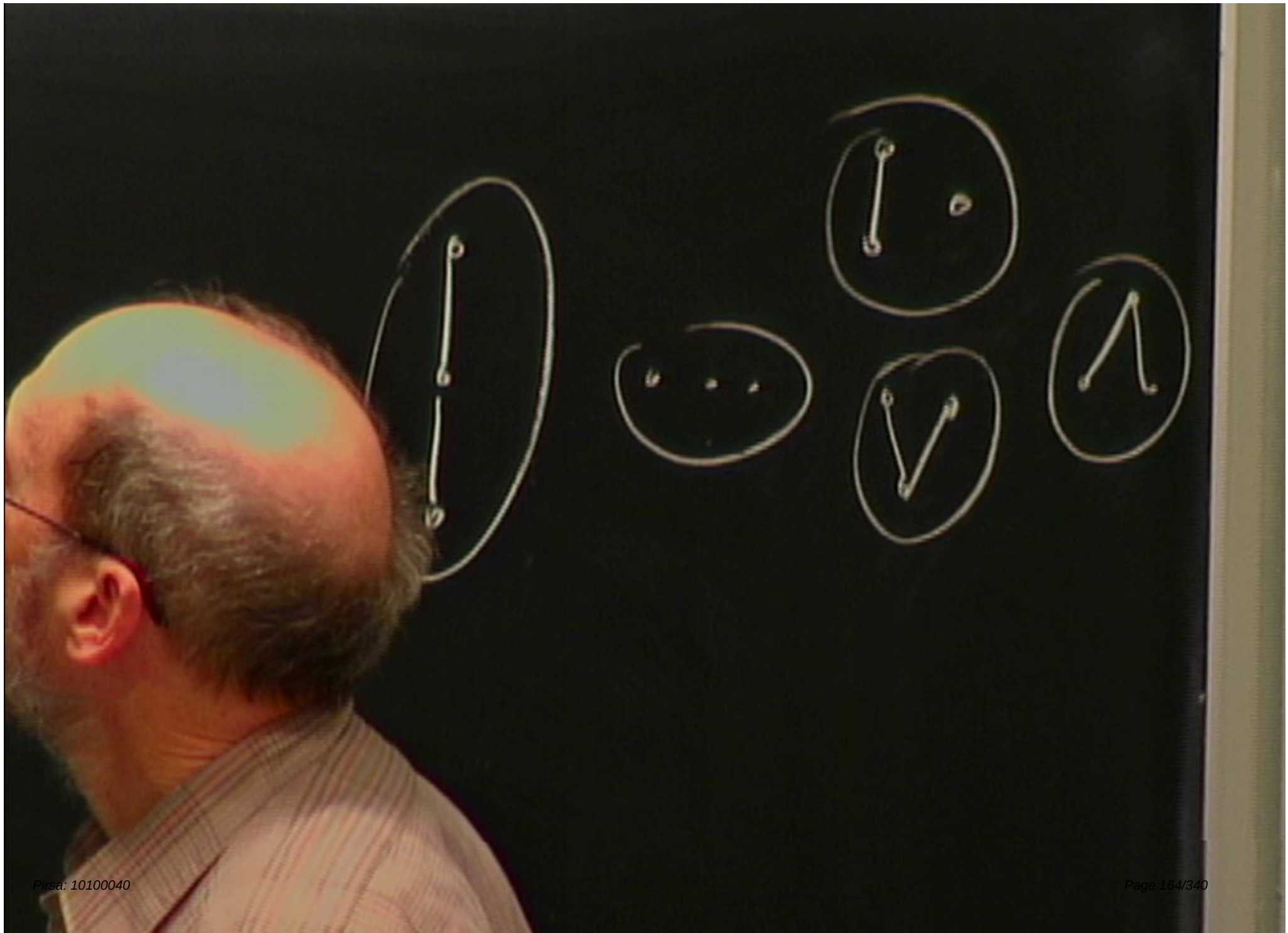


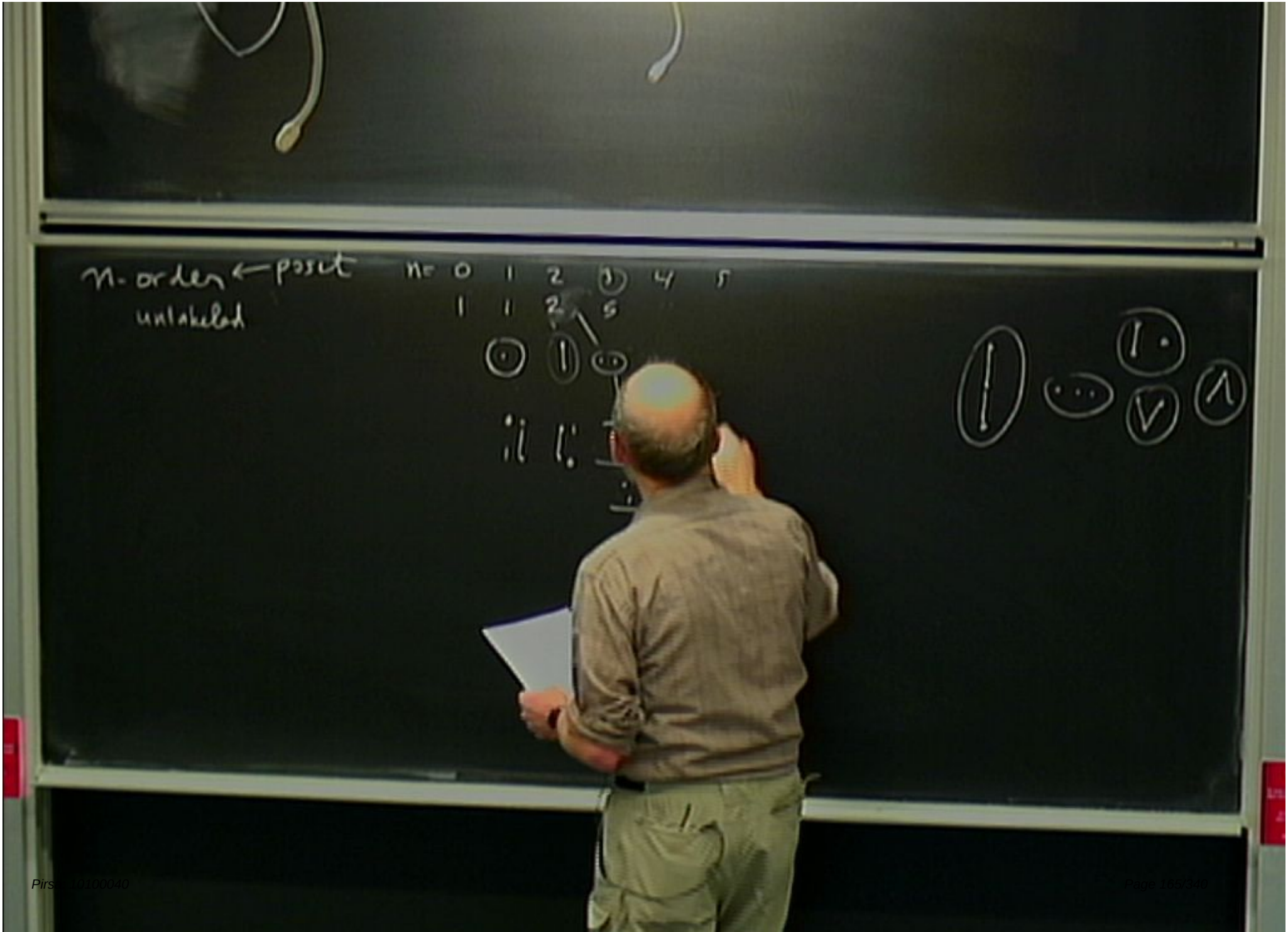

←posets
n= 0 1 2 3 4 5
1 1 2

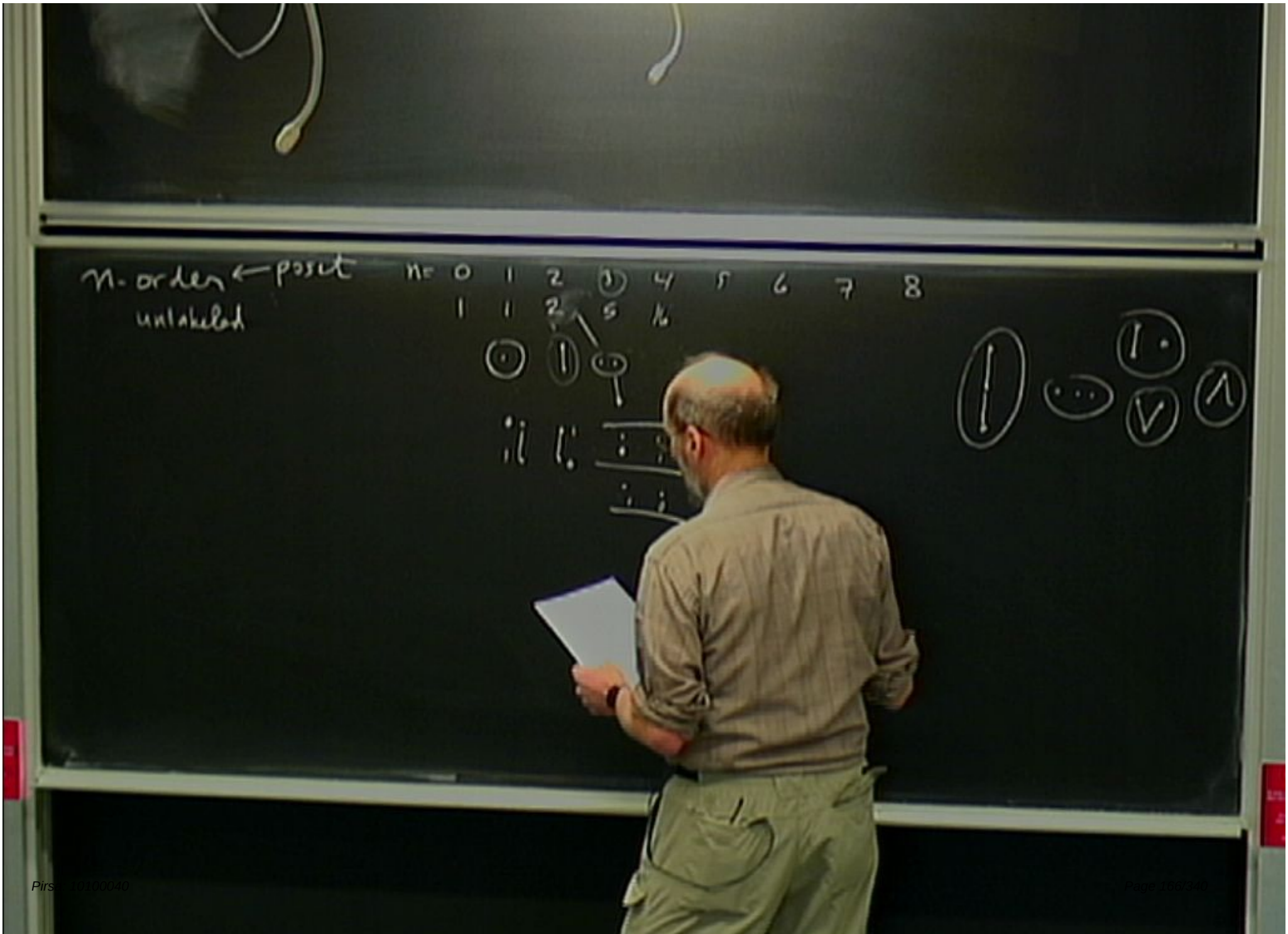


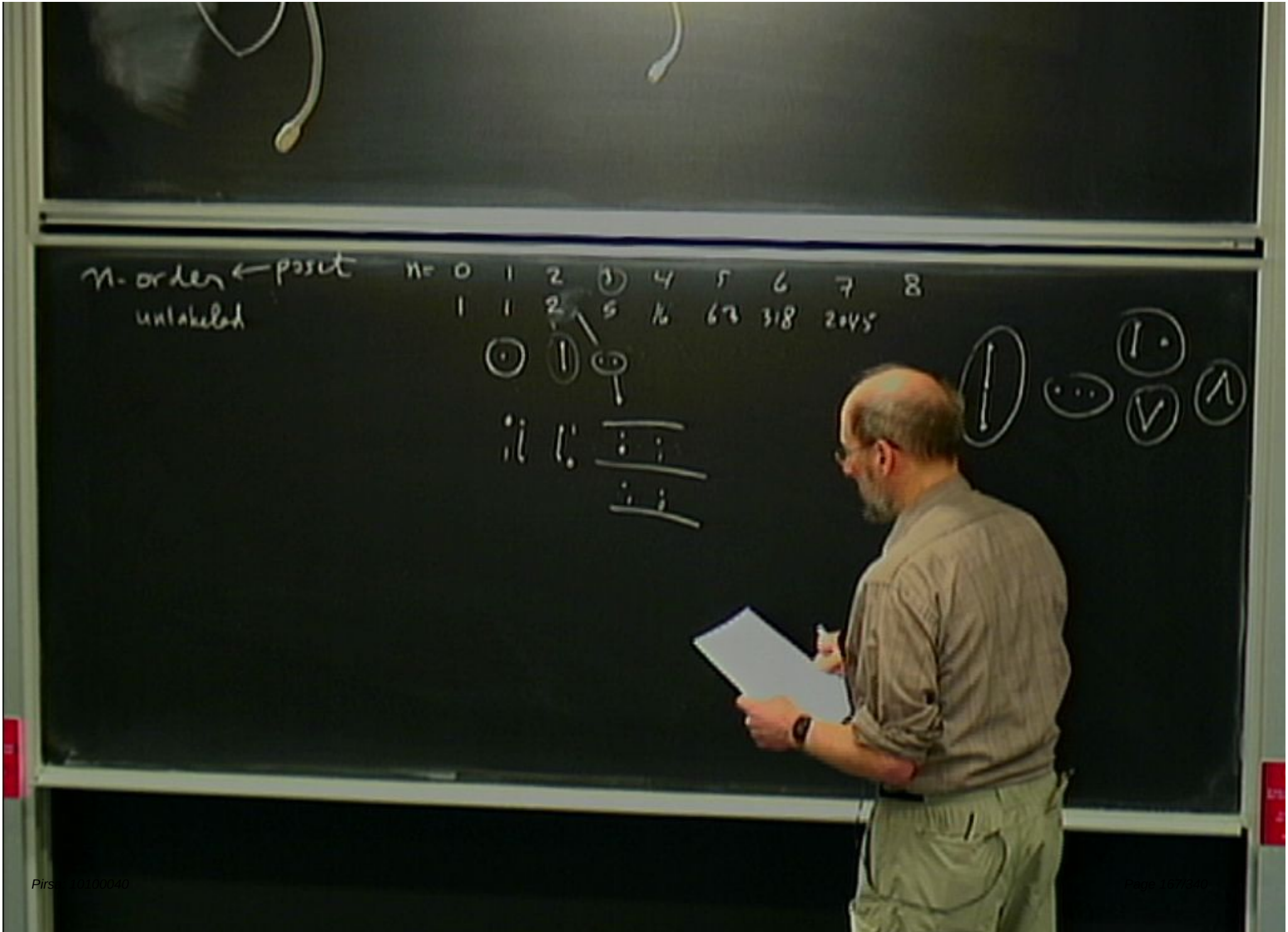


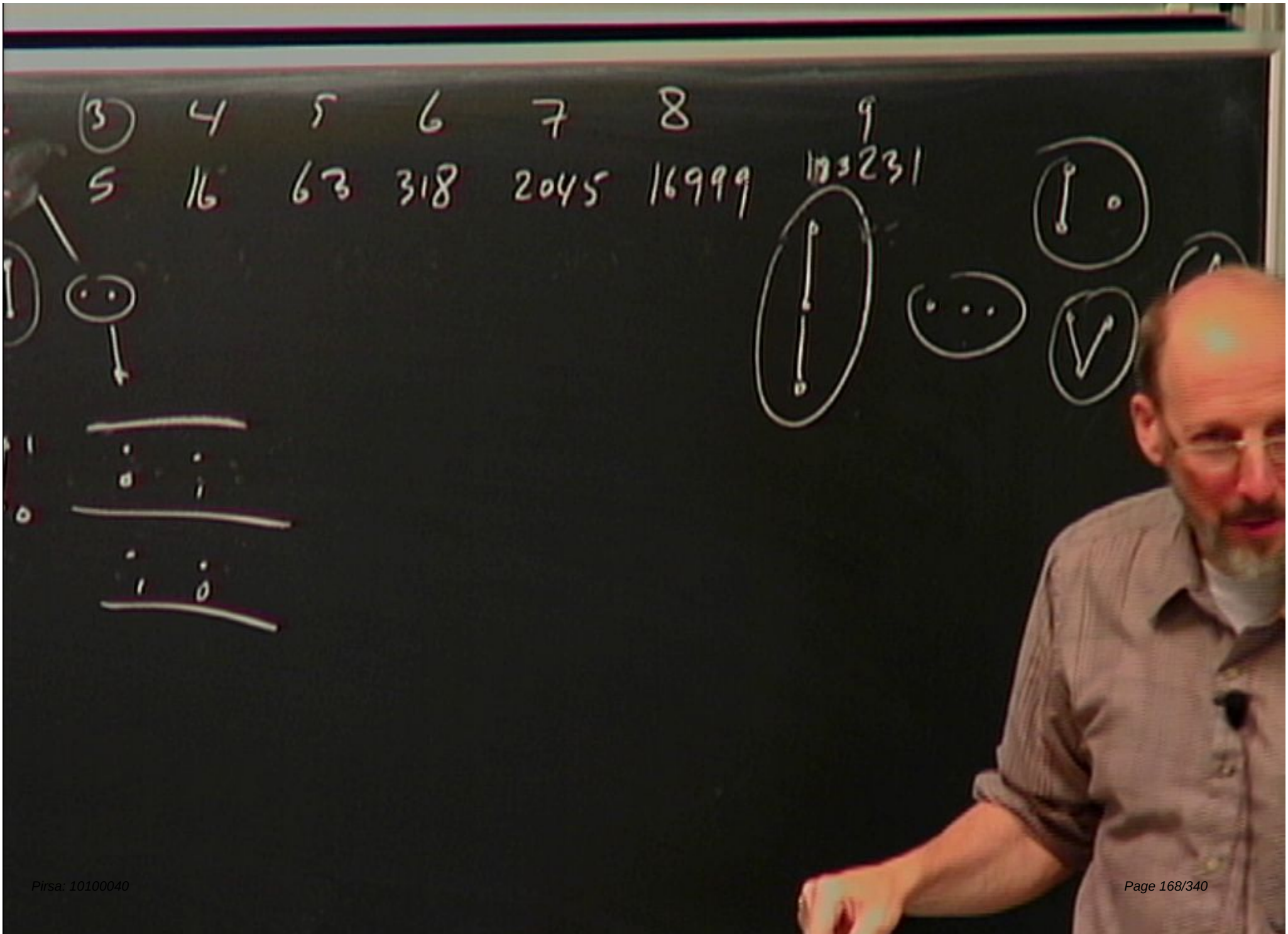


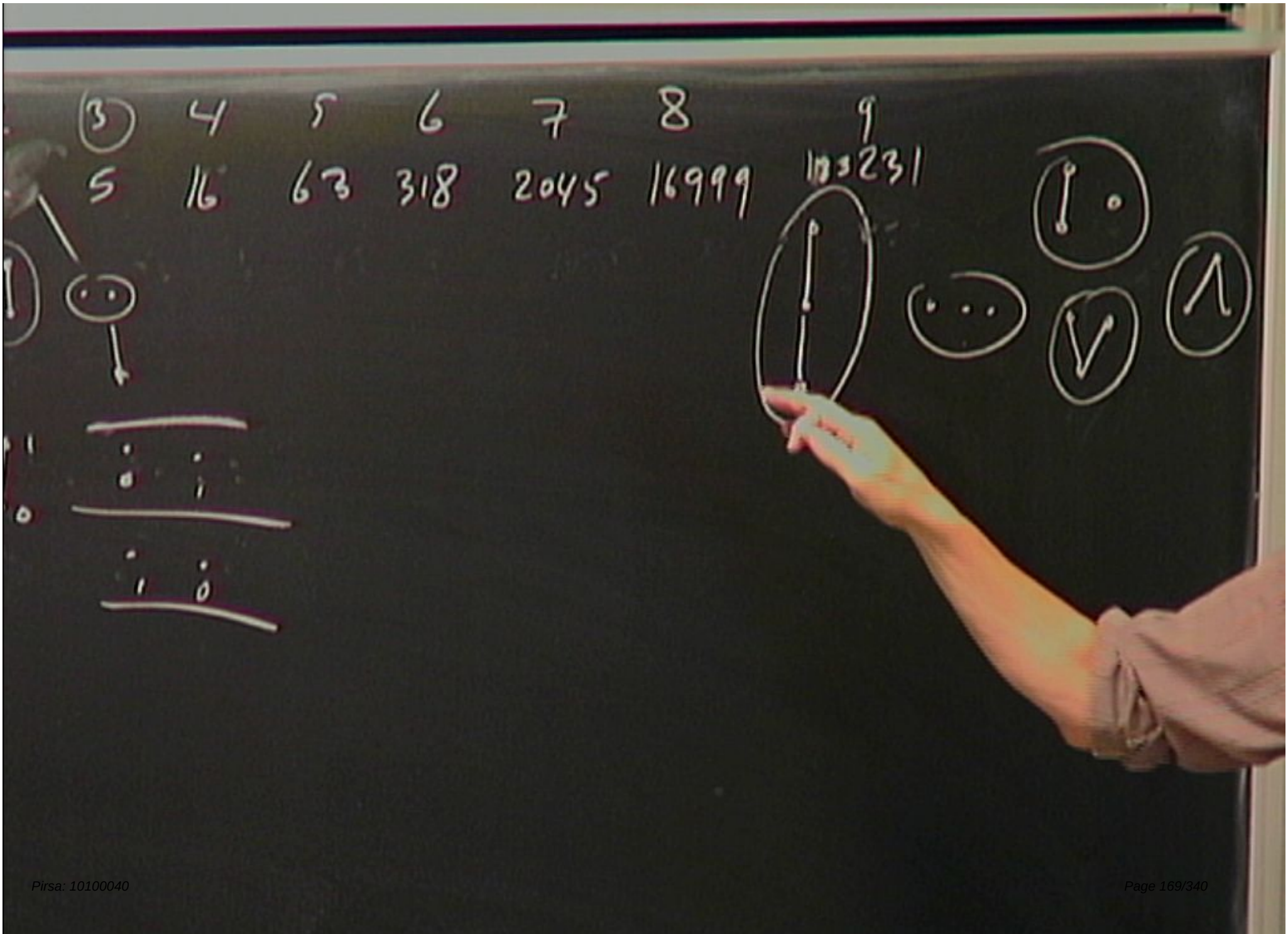


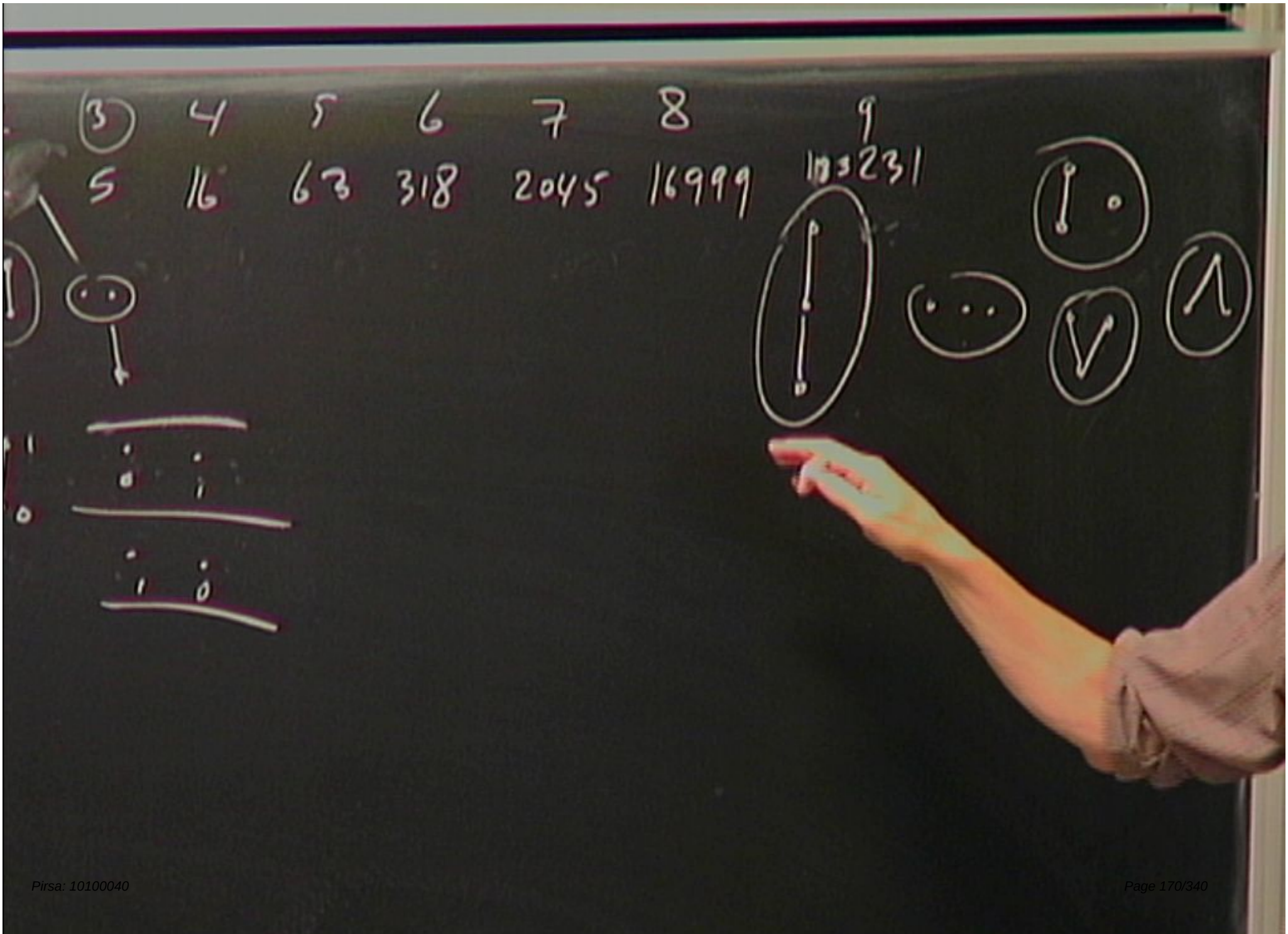

n-orden ← poset
unlabeled
n= 0 1 2 3 4 5
1 1 2 5

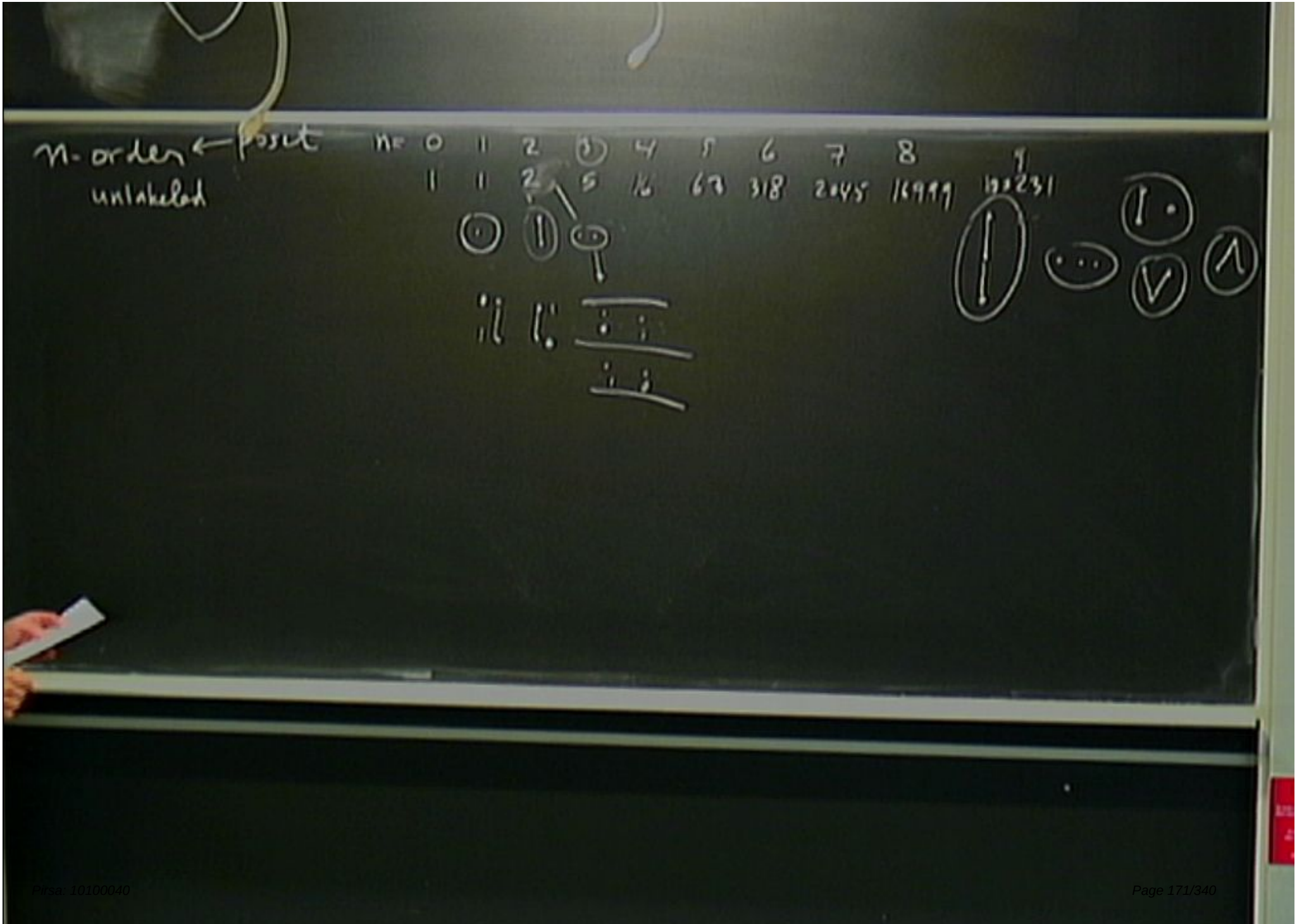

n-orden ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8
1 1 2 5 16

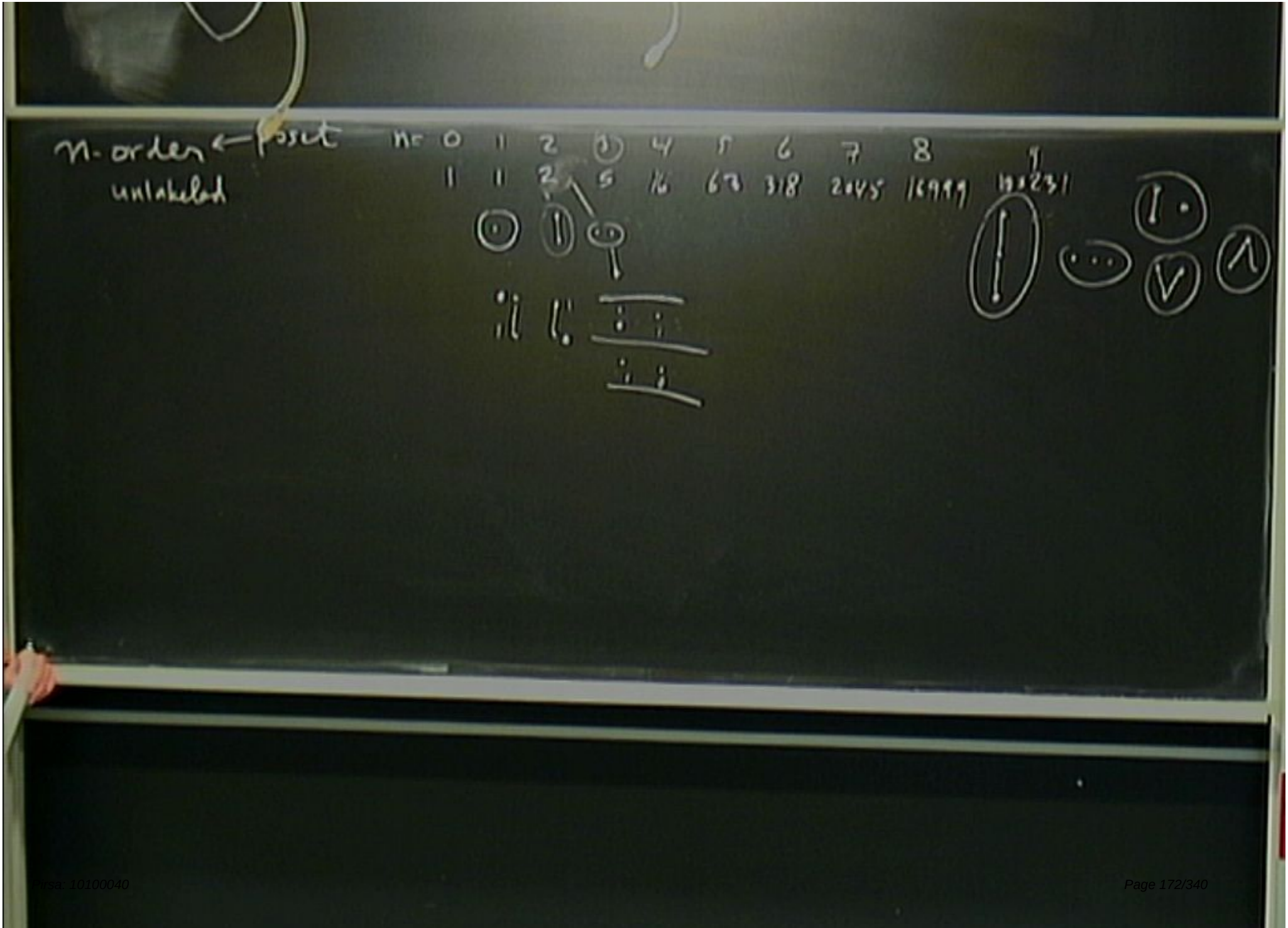

n-orders ← posets
unlabeled
n= 0 1 2 3 4 5 6 7 8
1 1 2 5 16 63 318 2045

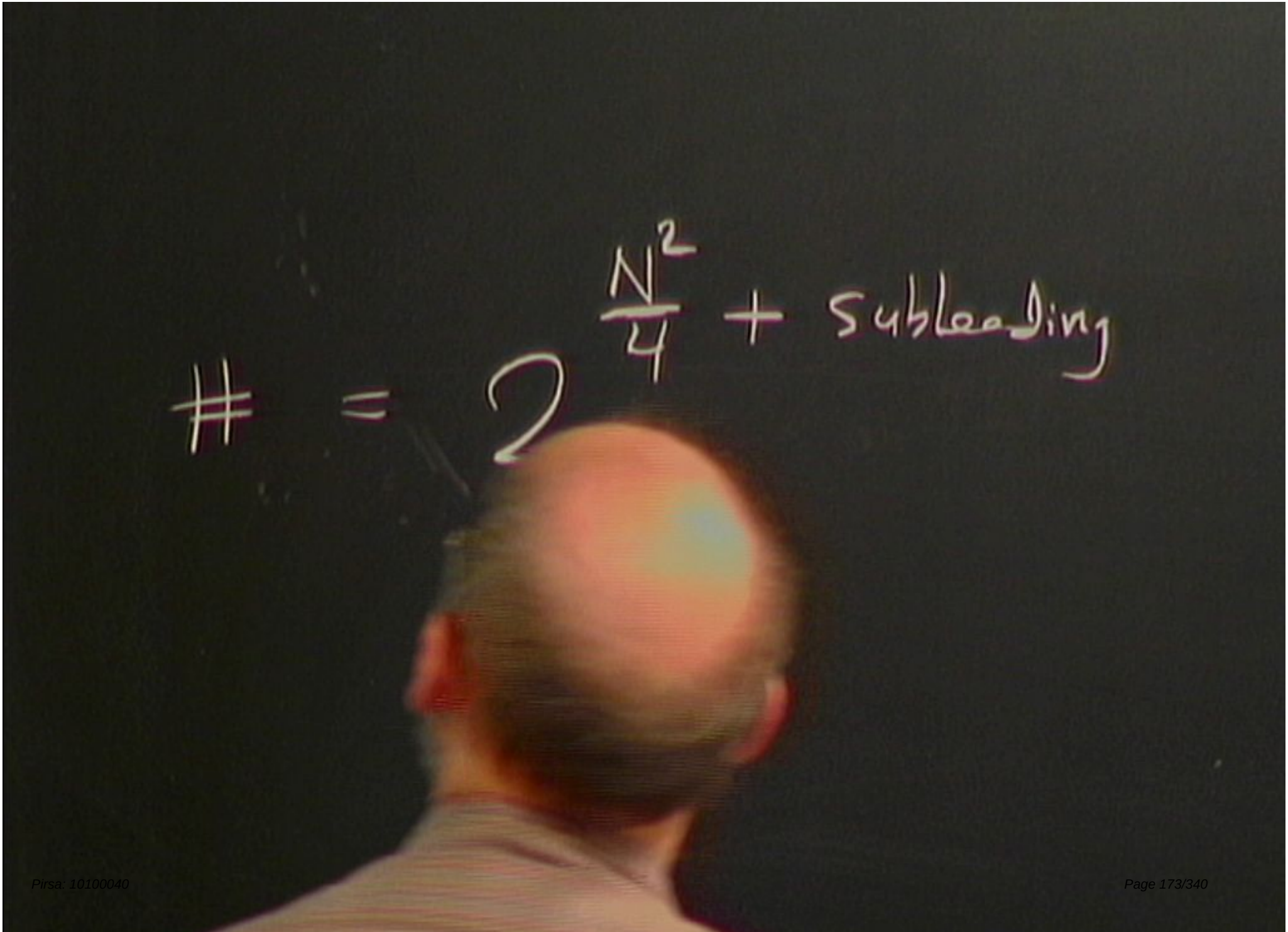

3 4 5 6 7 8 9
5 16 63 318 2045 16999 1m=231


3 4 5 6 7 8 9
5 16 63 318 2045 16999 1048231

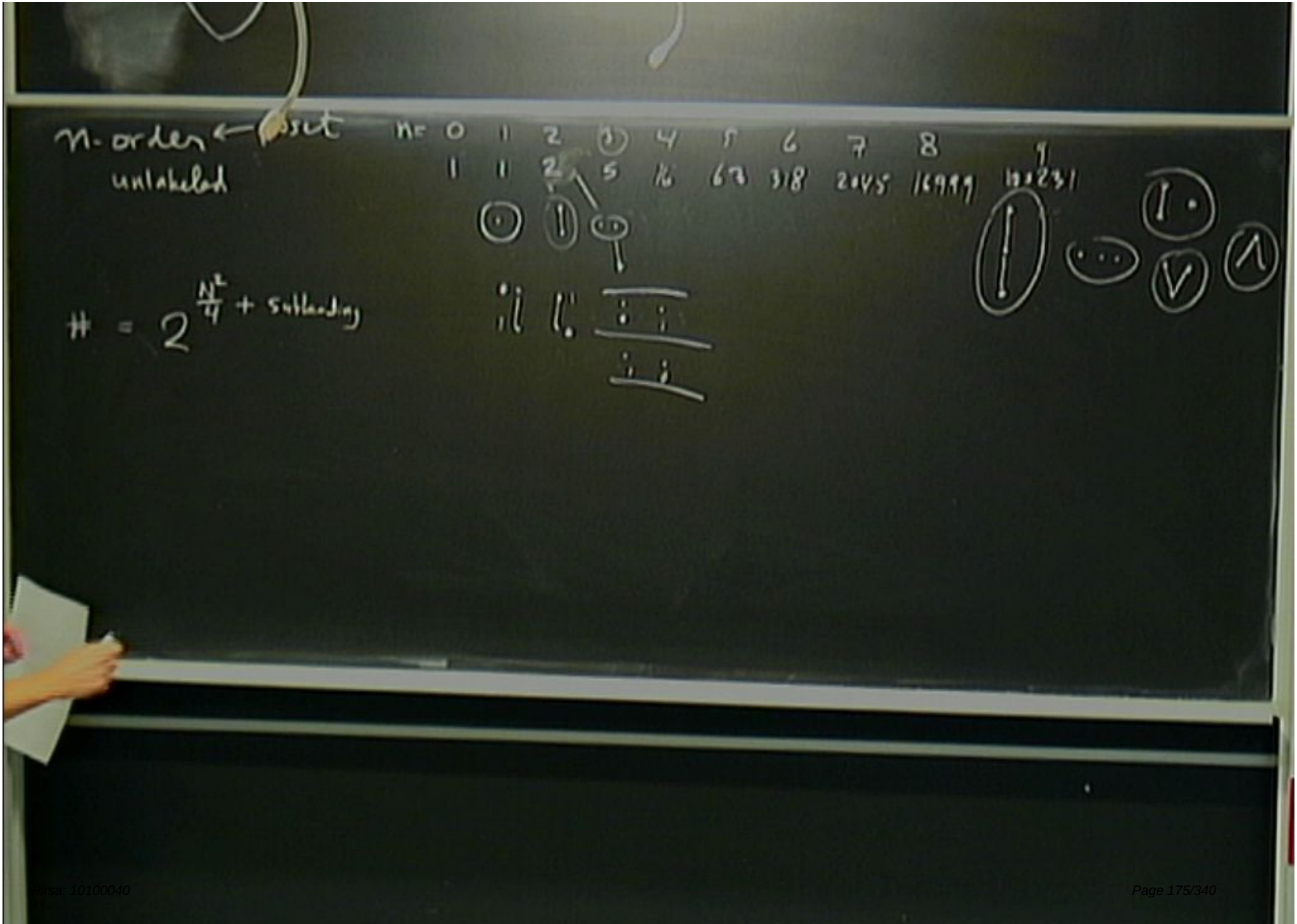

3 4 5 6 7 8 9
5 16 63 318 2045 16999 1048231

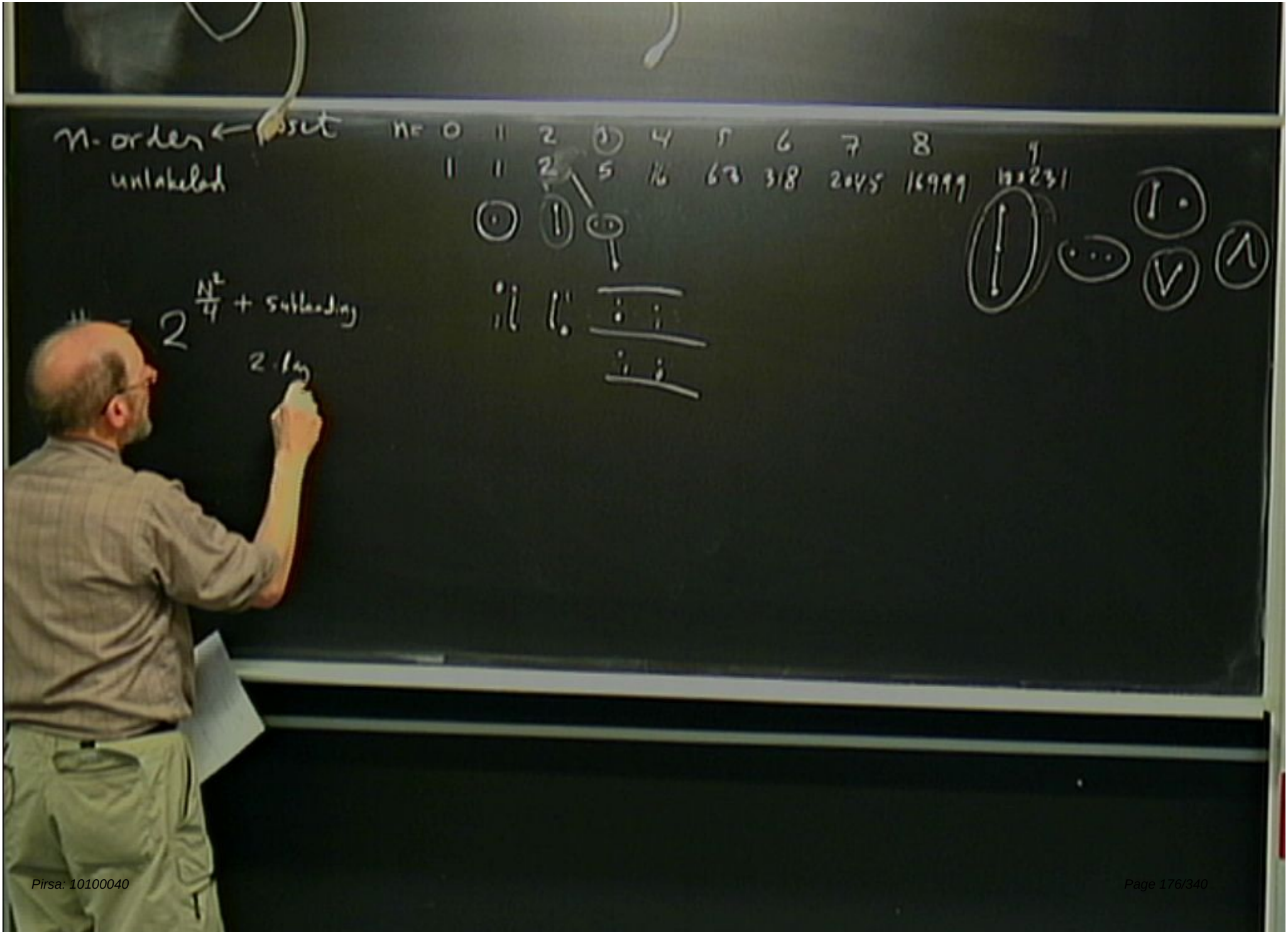

n-orders ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231


n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8
1 1 2 5 16 67 318 2045 16999
1 1 2 5 16 67 318 2045 16999 183231


$\# = 2^{\frac{N^2}{4} + \text{subleading}}$

$$\# = 2^{\frac{N^2}{4} + \text{subleading}}$$


n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 67 318 2045 16999 183231
= $2^{\frac{N^2}{4} + \text{subleading}}$


n-orders ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
2^{N^2/4} + subleading
2 log

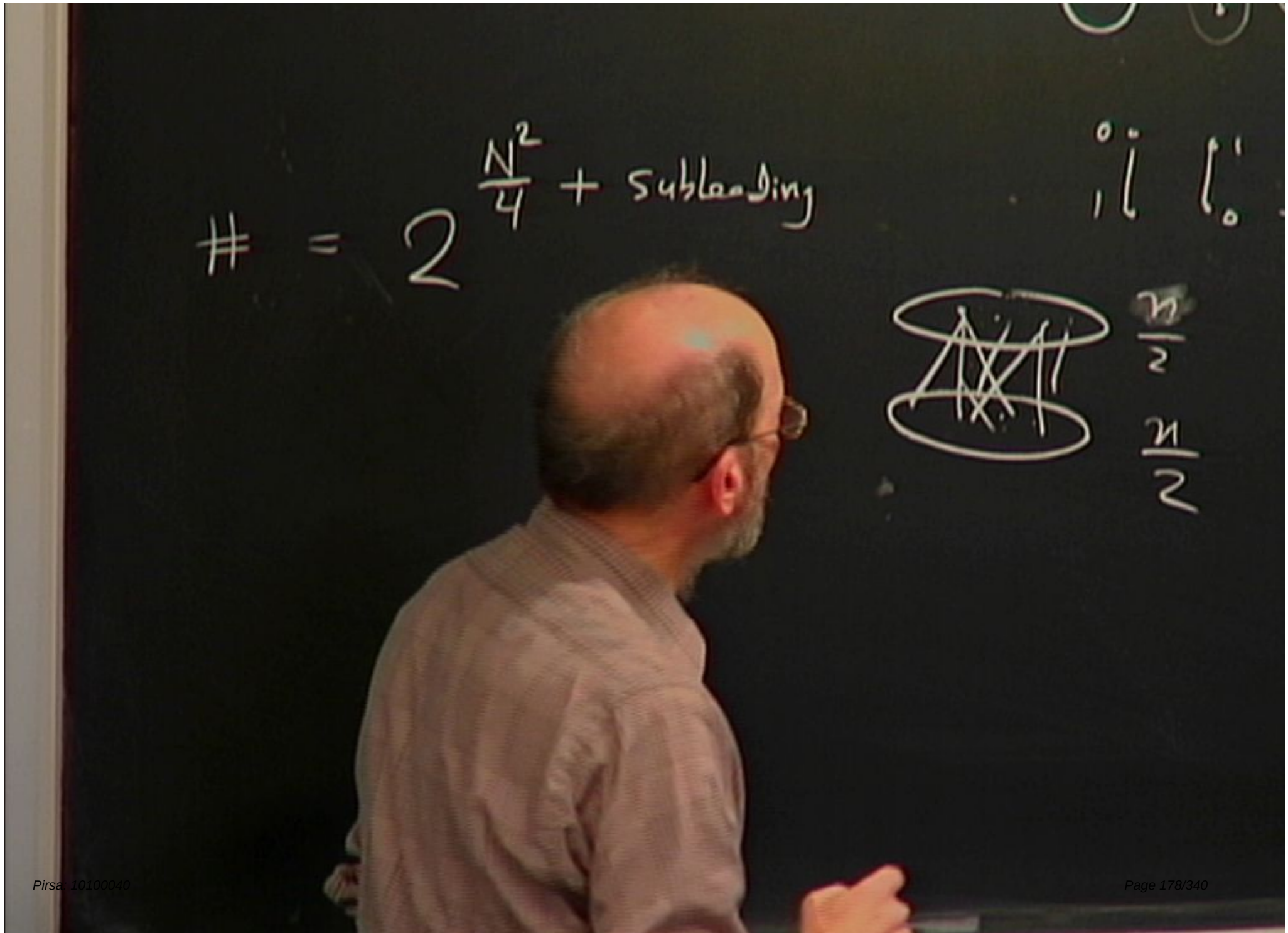
$\# = 2^{\frac{N^2}{4} + \text{subleading}}$

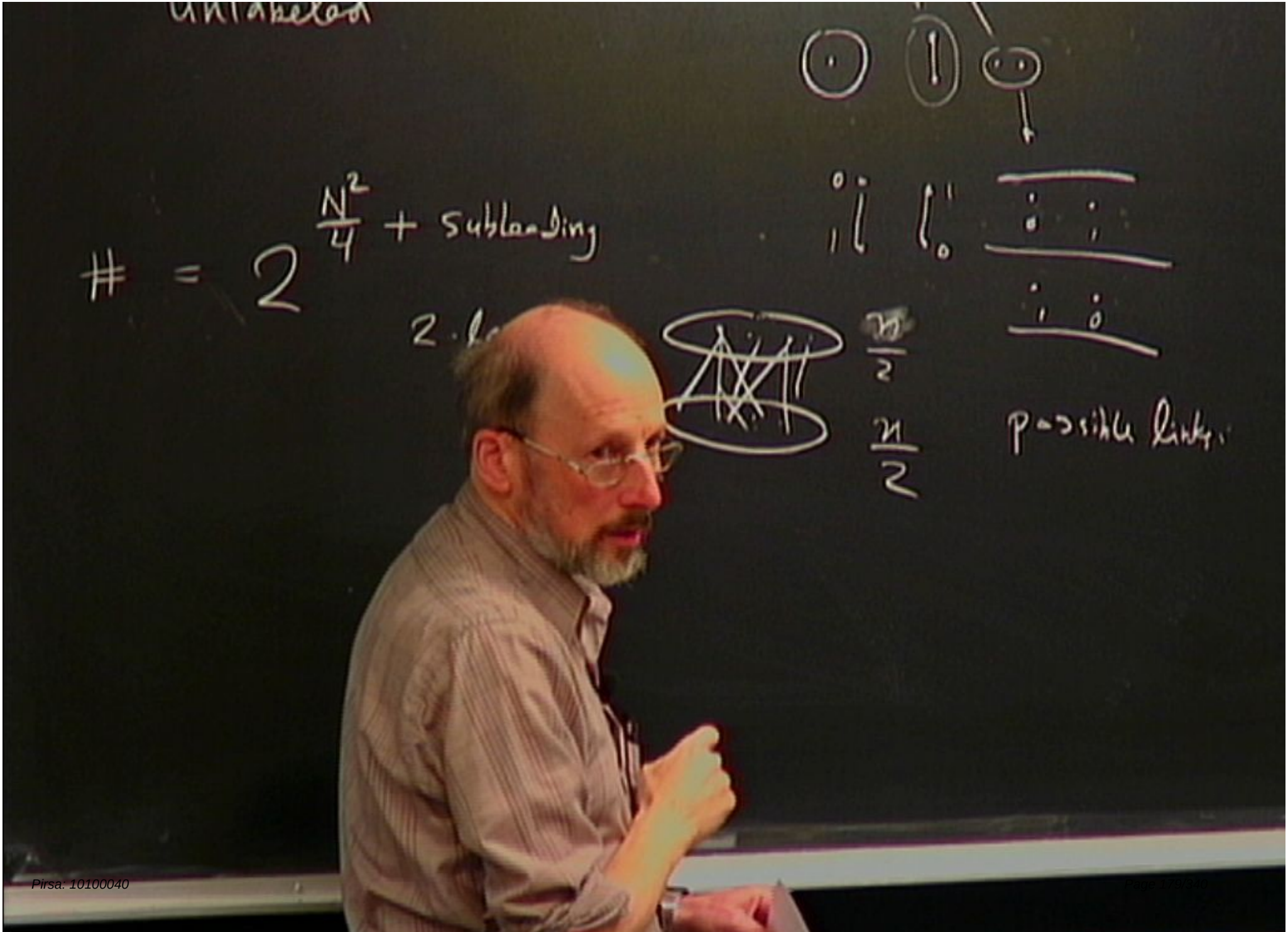
2 layer

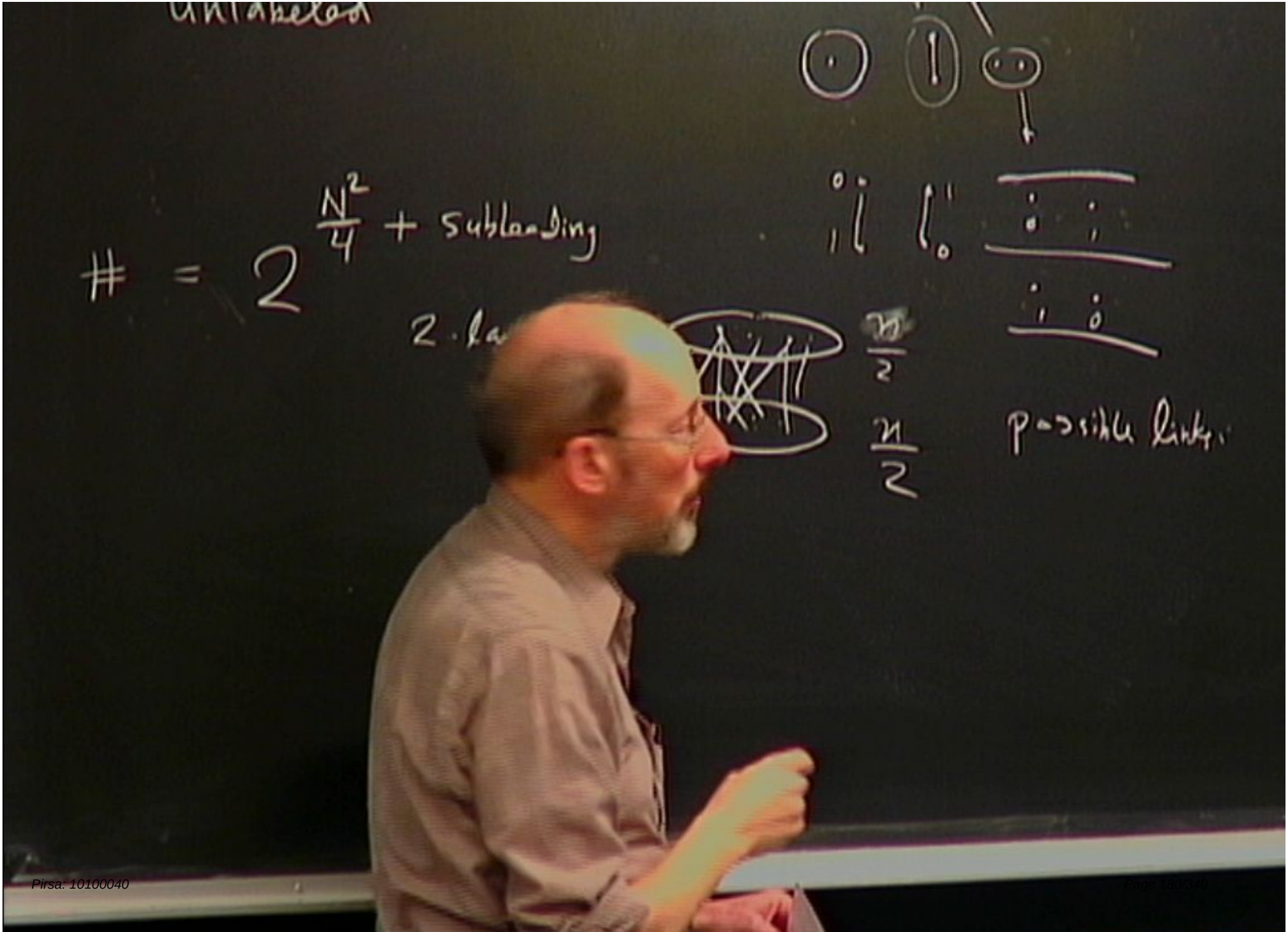


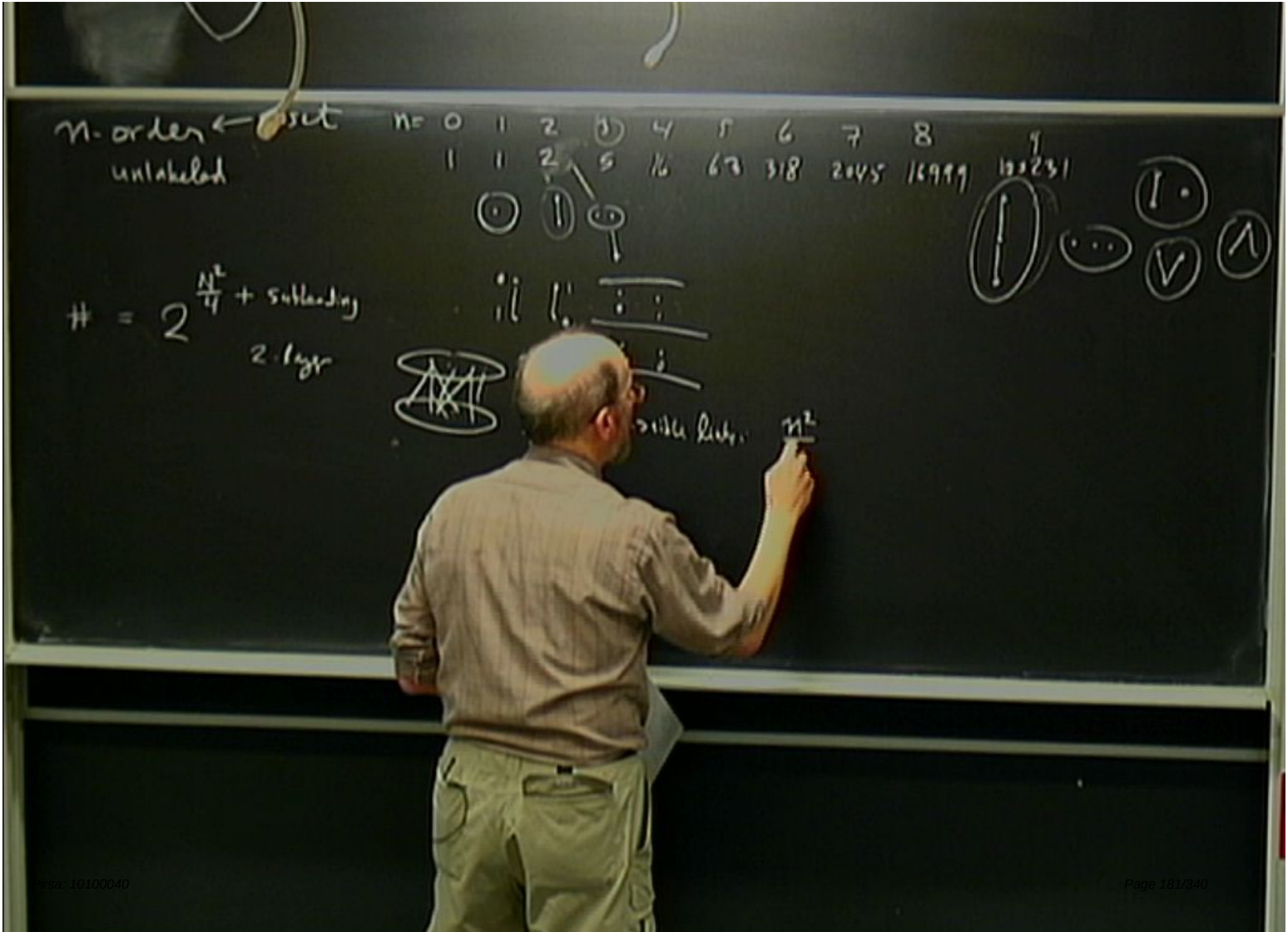
$\frac{n}{2}$

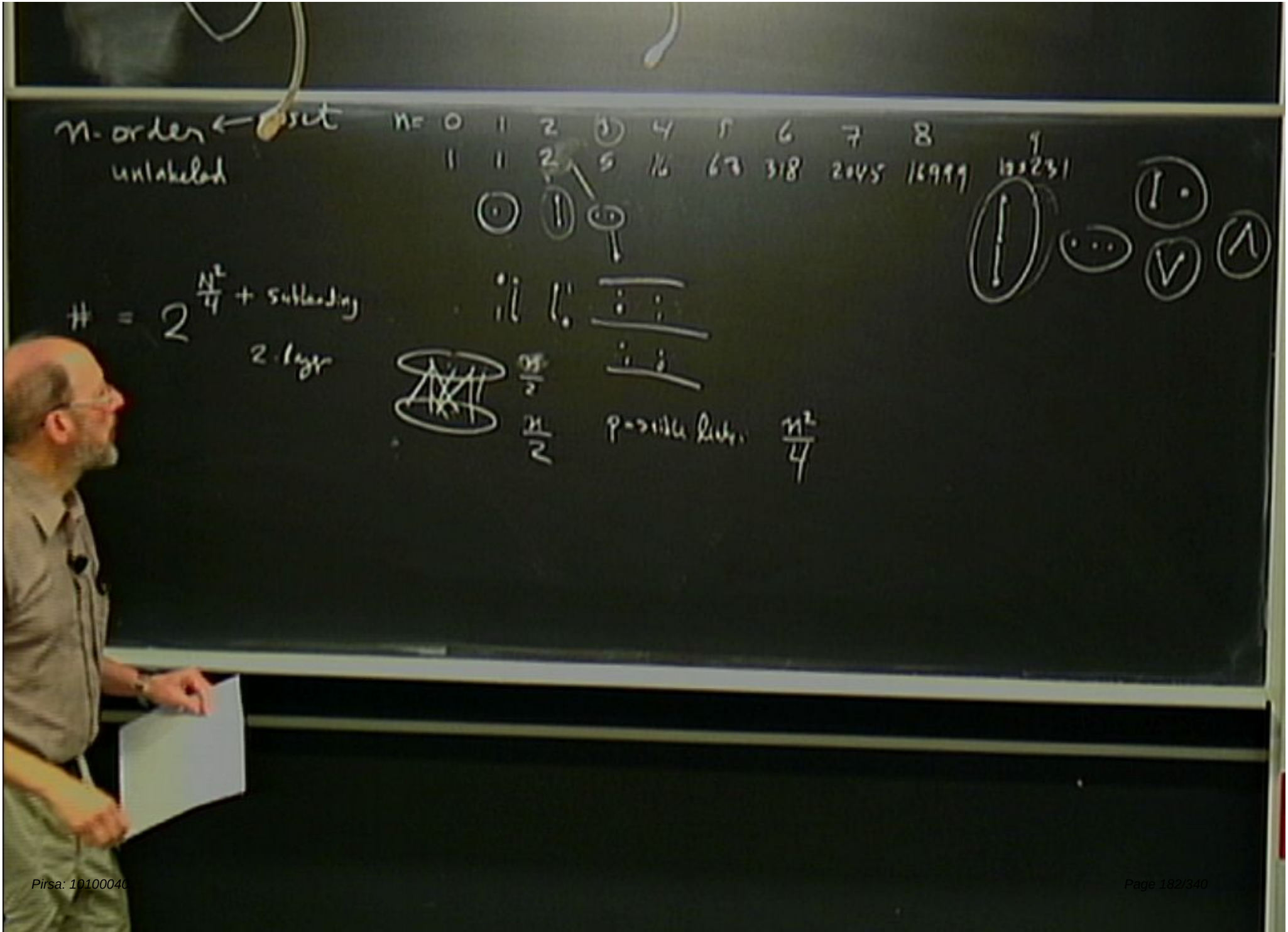
$\frac{n}{2}$

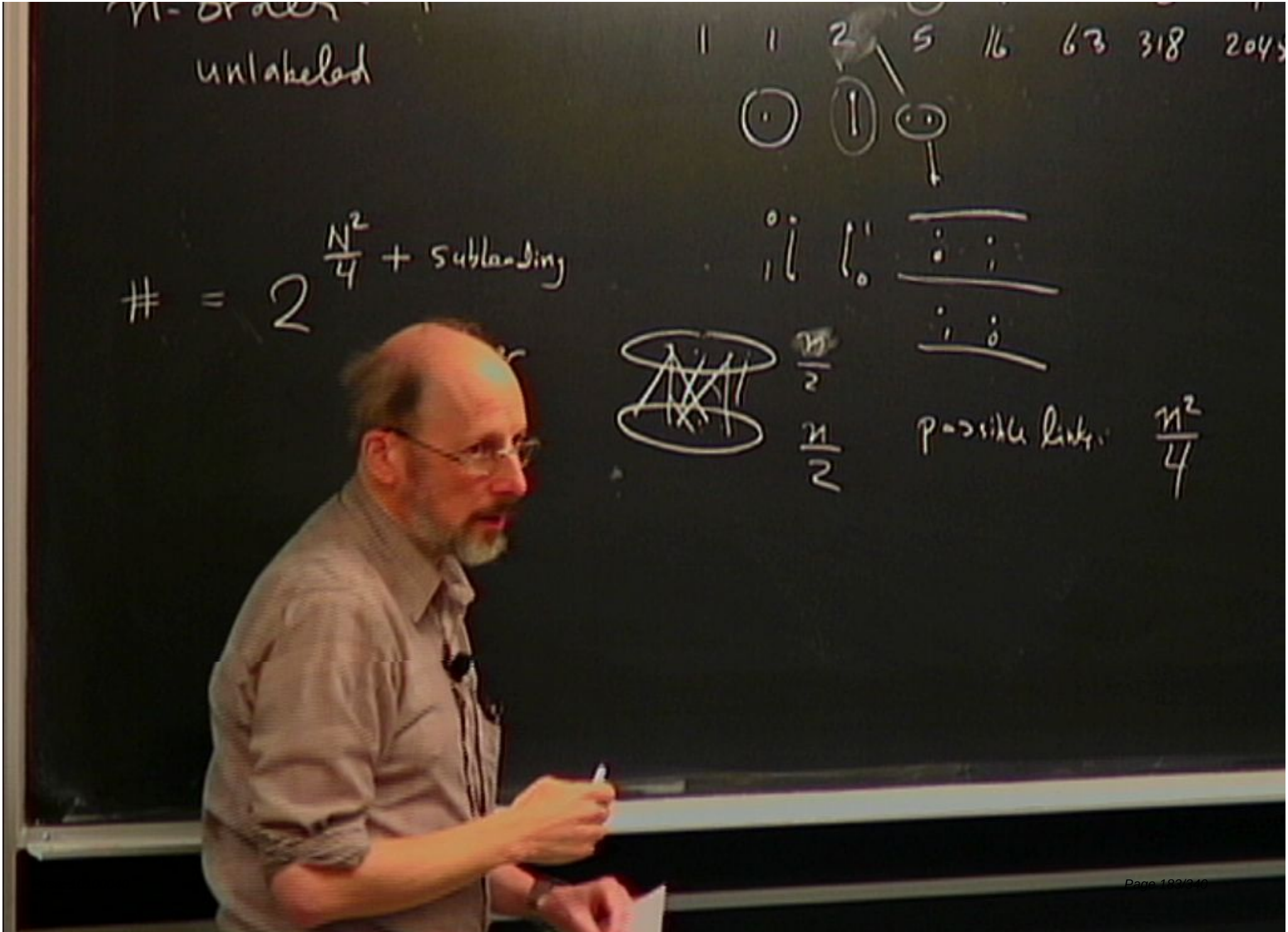

$\# = 2^{\frac{N^2}{4} + \text{subleading}}$
$\frac{n}{2}$
$\frac{n}{2}$

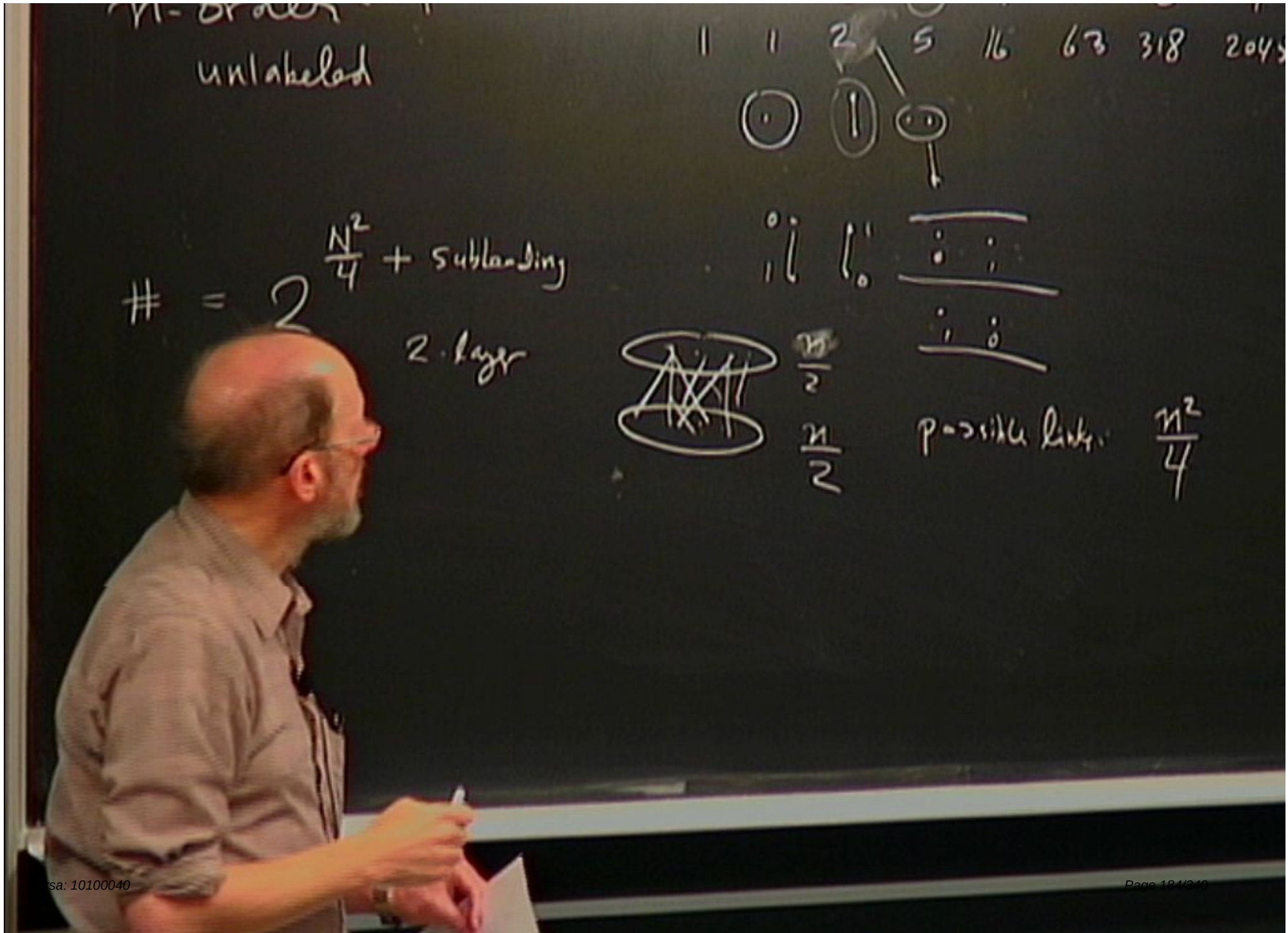

$\# = 2^{\frac{N^2}{4} + \text{subleading}}$
$\frac{2n}{2}$
$\frac{2n}{2}$

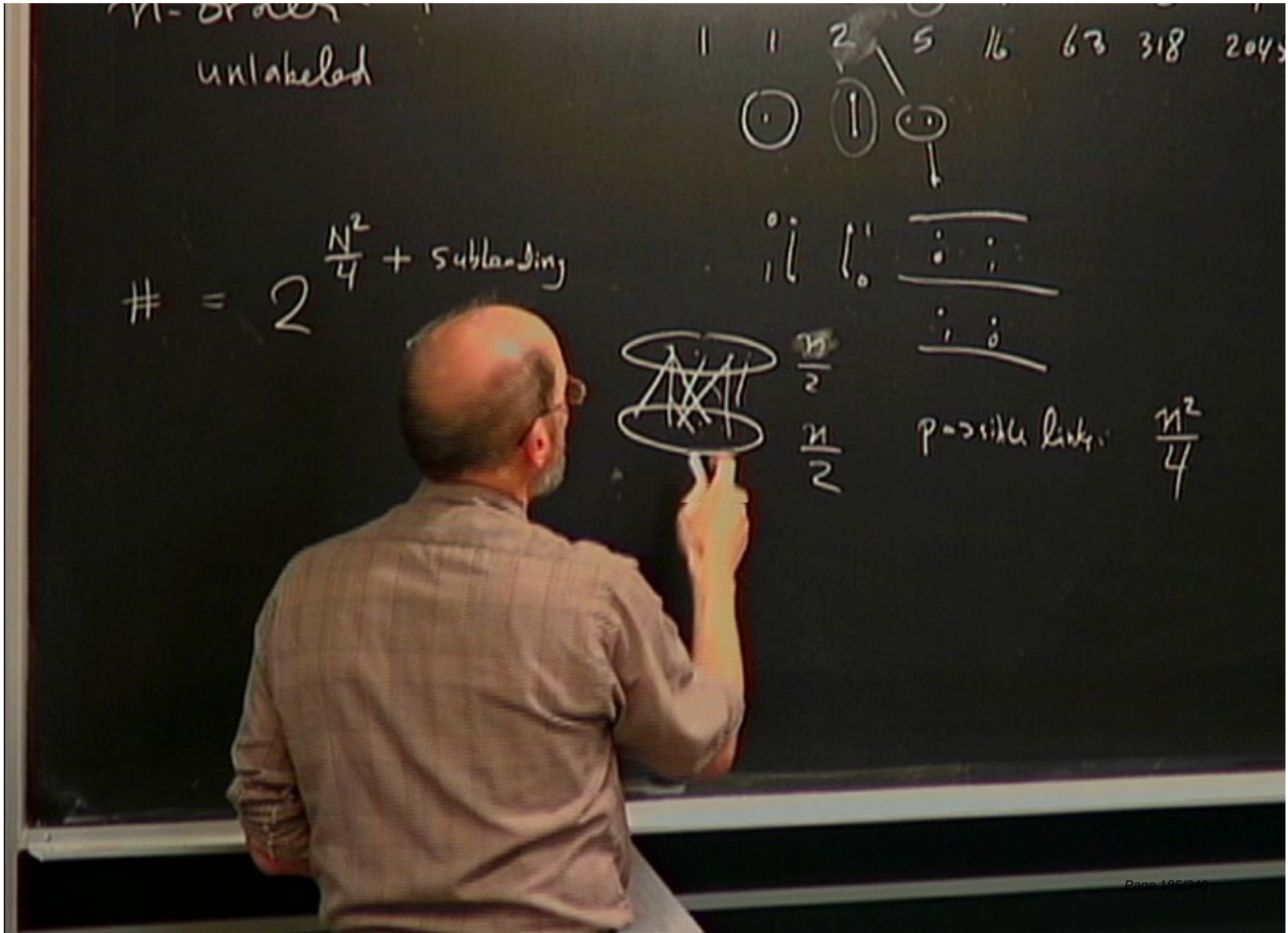

$\# = 2^{\frac{N^2}{4} + \text{subleading}}$
2. la
$\frac{N^2}{2}$
$\frac{N}{2}$
possible links

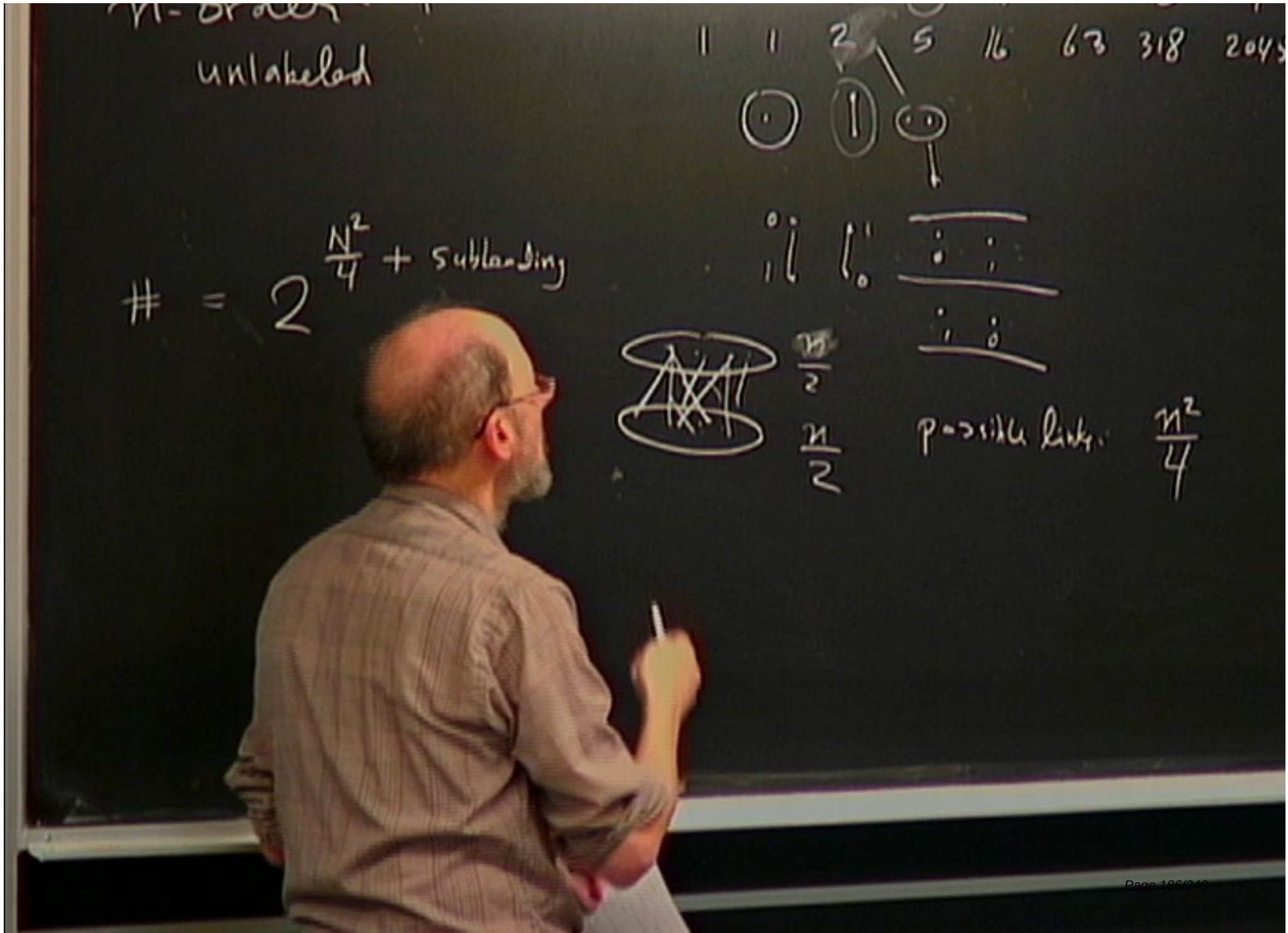

n-order ← set
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
= 2^{n²/4 + subleading}
2 layer

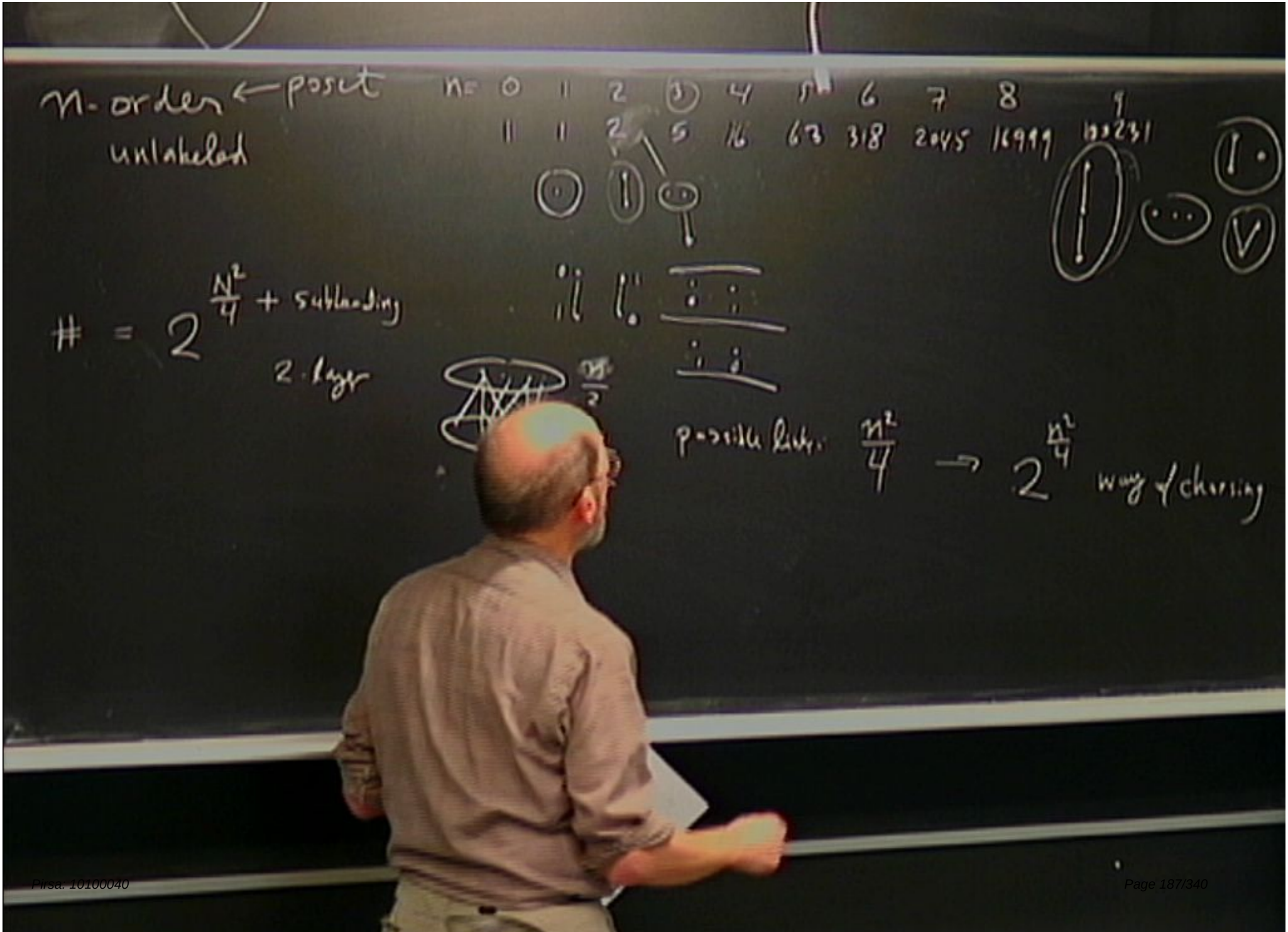

n-order ← set
unlabeled
n= 0 1 2 3 4 5 6 7 8
1 1 2 5 16 63 318 2045 16999
= $2^{\frac{n^2}{4}}$ + subleading
2 layer
$\frac{n}{2}$
$\frac{n}{2}$
possible links
$\frac{n^2}{4}$

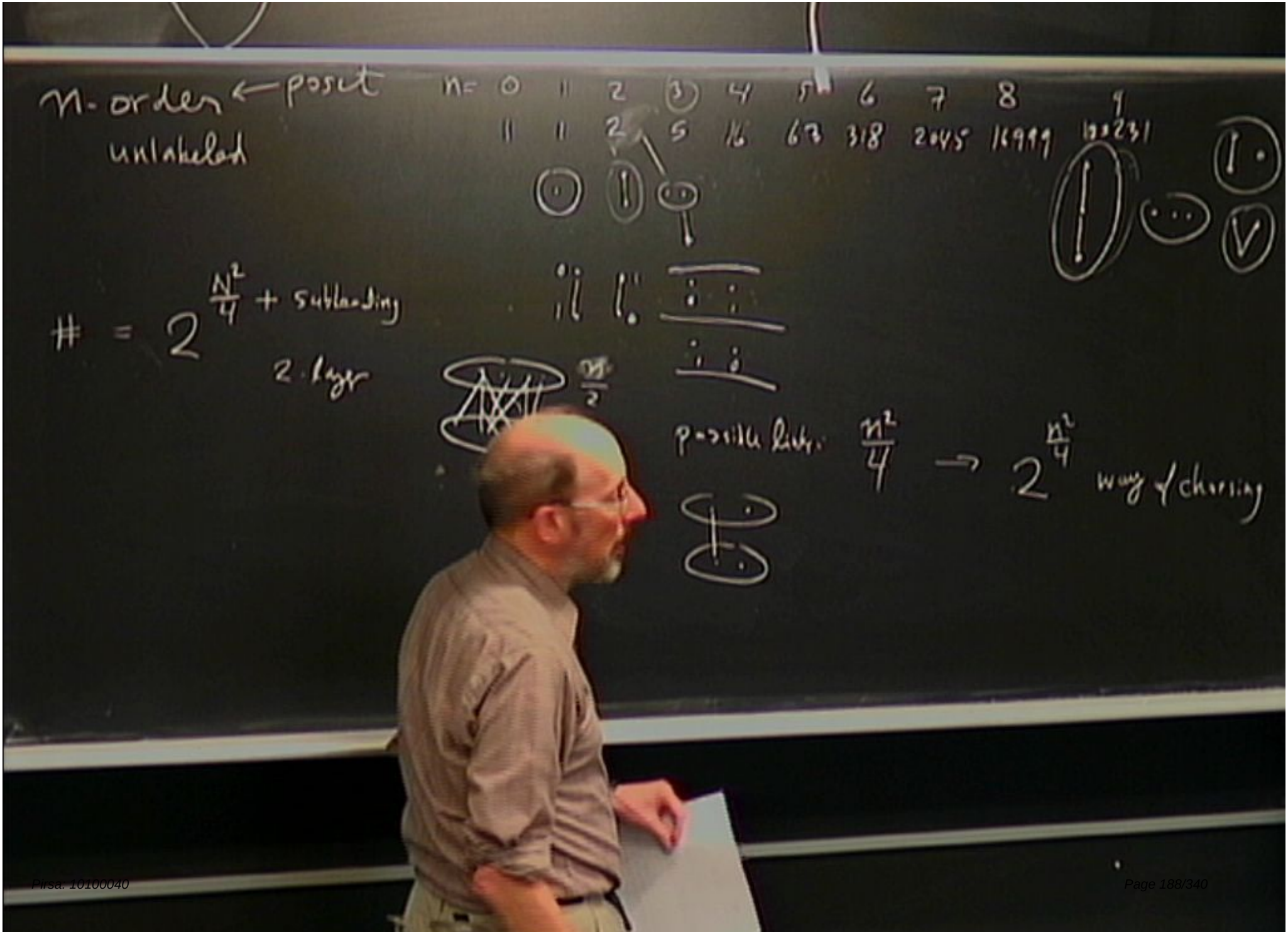

unlabeled
1 1 2 5 16 63 318
$\# = 2^{\frac{N^2}{4}}$ + subleading
$\frac{n}{2}$
$\frac{n}{2}$
possible links
$\frac{n^2}{4}$

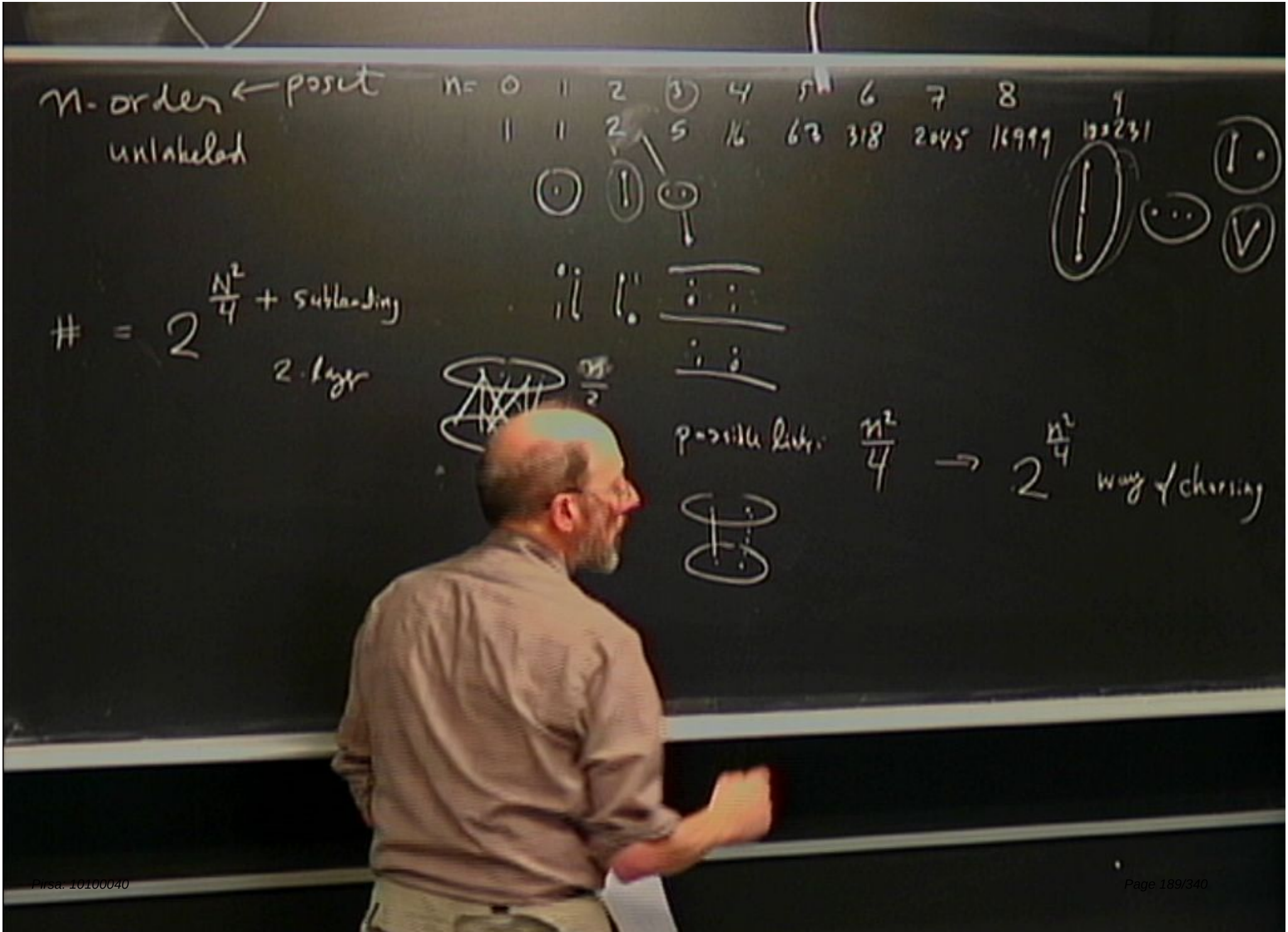

unlabeled
1 1 2 5 16 63 318 2045
= 2^{N²/4} + subleading
2 layer
n/2
n/2
possible links
n²/4


unlabeled
1 1 2 5 16 63 318
$\# = 2^{\frac{N^2}{4} + \text{subleading}}$
$\frac{n}{2}$
$\frac{n}{2}$
possible links.
$\frac{n^2}{4}$


unlabeled
1 1 2 5 16 63 318
$\# = 2^{\frac{N^2}{4}} + \text{subleading}$
$\frac{n}{2}$
$\frac{n}{2}$
possible links.
$\frac{n^2}{4}$


n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
= $2^{\frac{n^2}{4}}$ + subleading
2 layer
$\frac{n^2}{4}$
possible links $\frac{n^2}{4} \to 2^{\frac{n^2}{4}}$ ways of choosing


n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
= $2^{\frac{N^2}{4}}$ + subleading
2 layer
$\frac{N}{2}$
possible links $\frac{n^2}{4}$ → $2^{\frac{n^2}{4}}$ way of choosing


n-order ← posets
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 133231
= $2^{\frac{N^2}{4}}$ + subleading
2 layer
$\frac{N^2}{2}$
possible links
$\frac{n^2}{4}$ → $2^{\frac{N^2}{4}}$
way of choosing

n-order ← posit

unlabeled

n= 0 1 2 3 4 5 6 7 8 9

1 1 2 5 16 63 318 2045 16999 183231

$\# = 2^{\frac{N^2}{4}}$ + subleading

2 layer

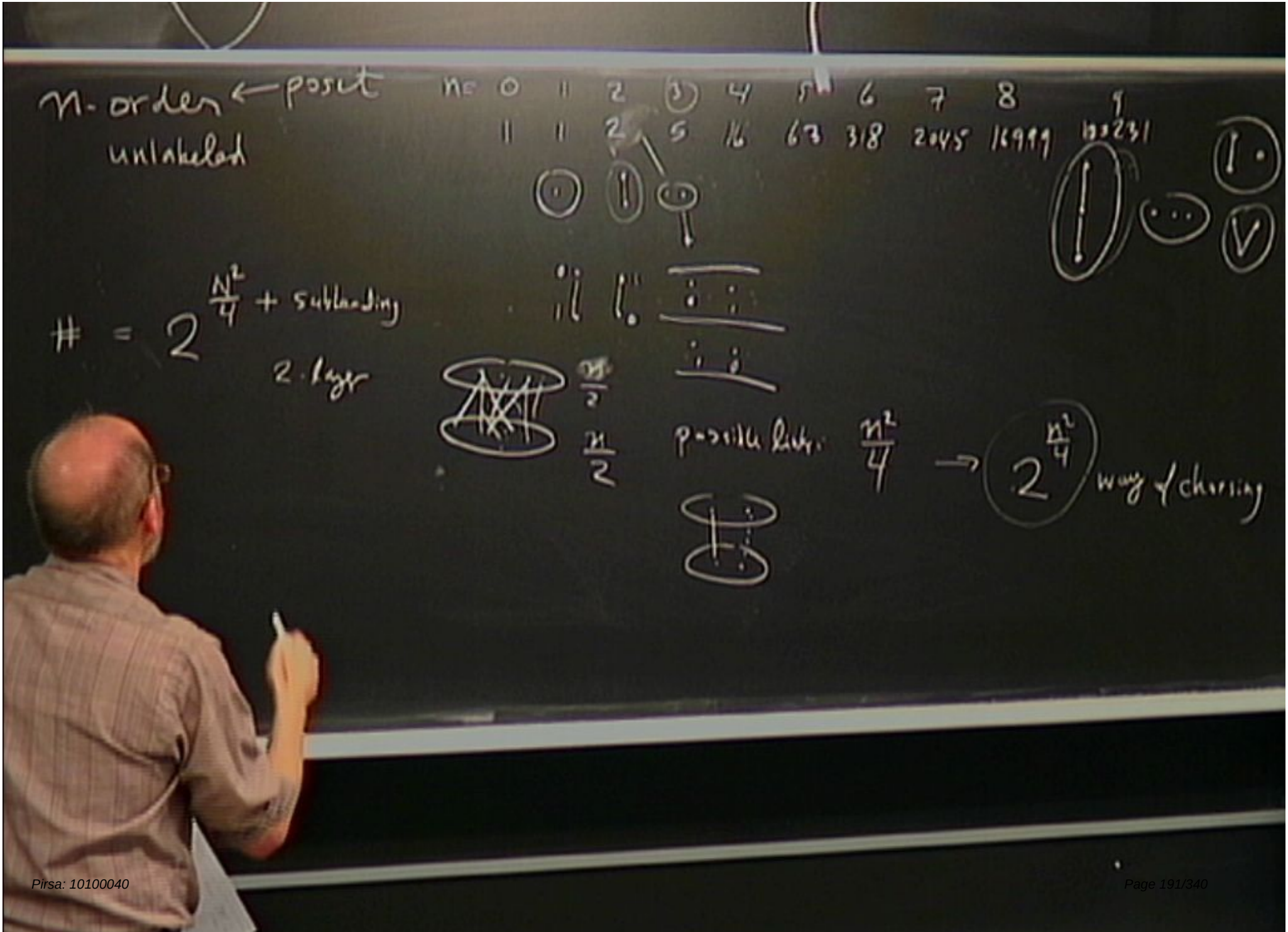


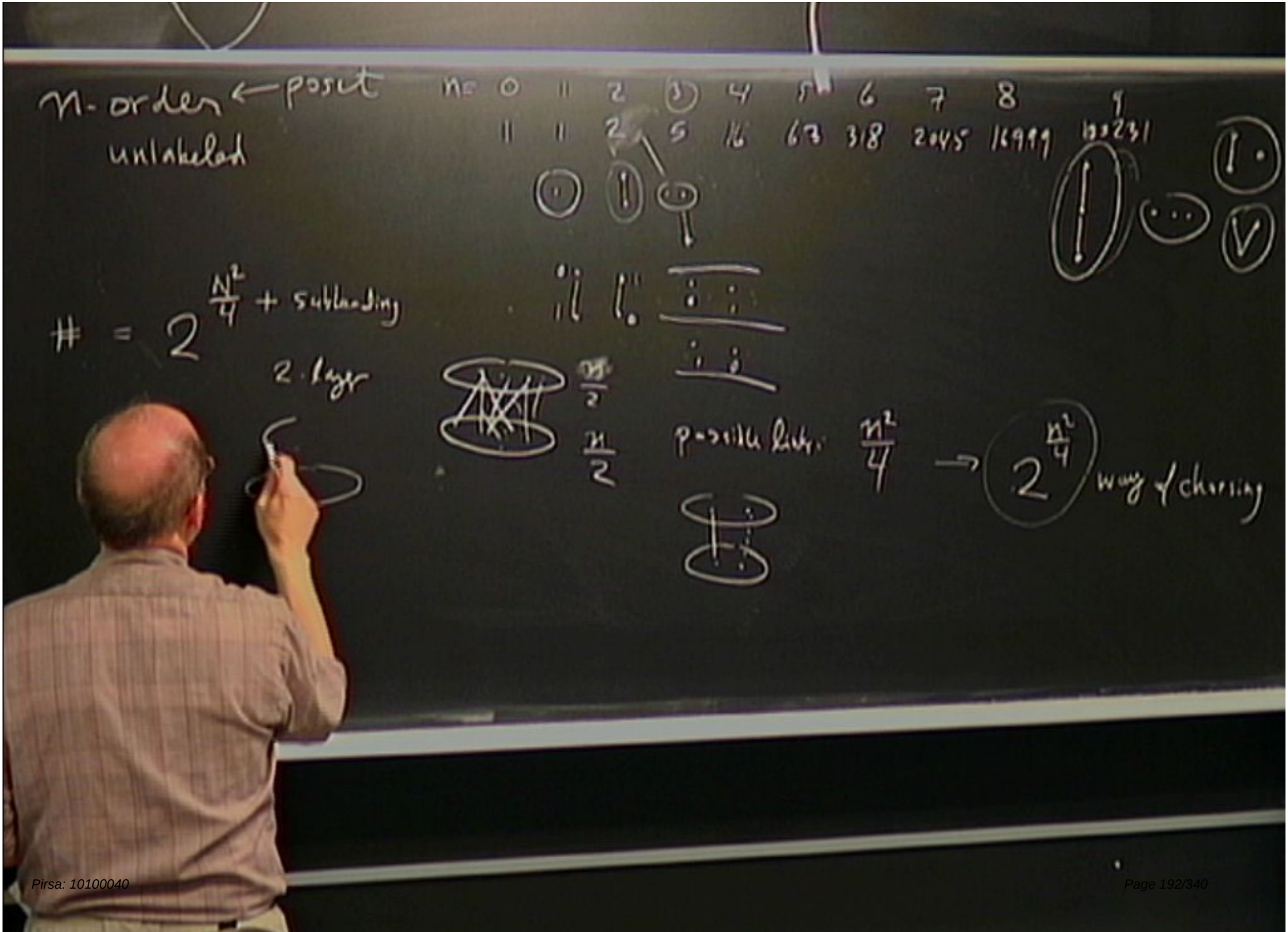
$\frac{n}{2}$

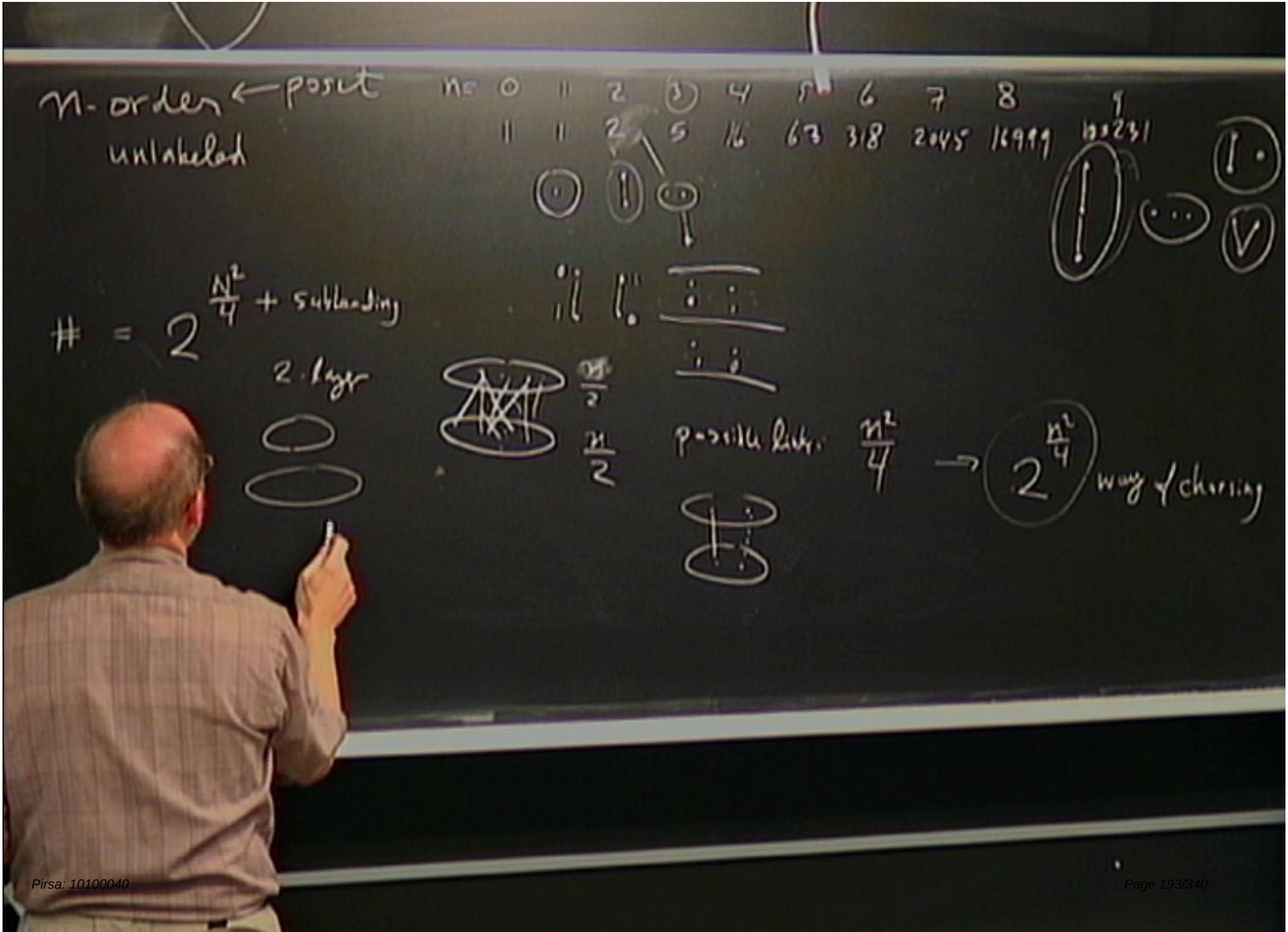
$\frac{n}{2}$

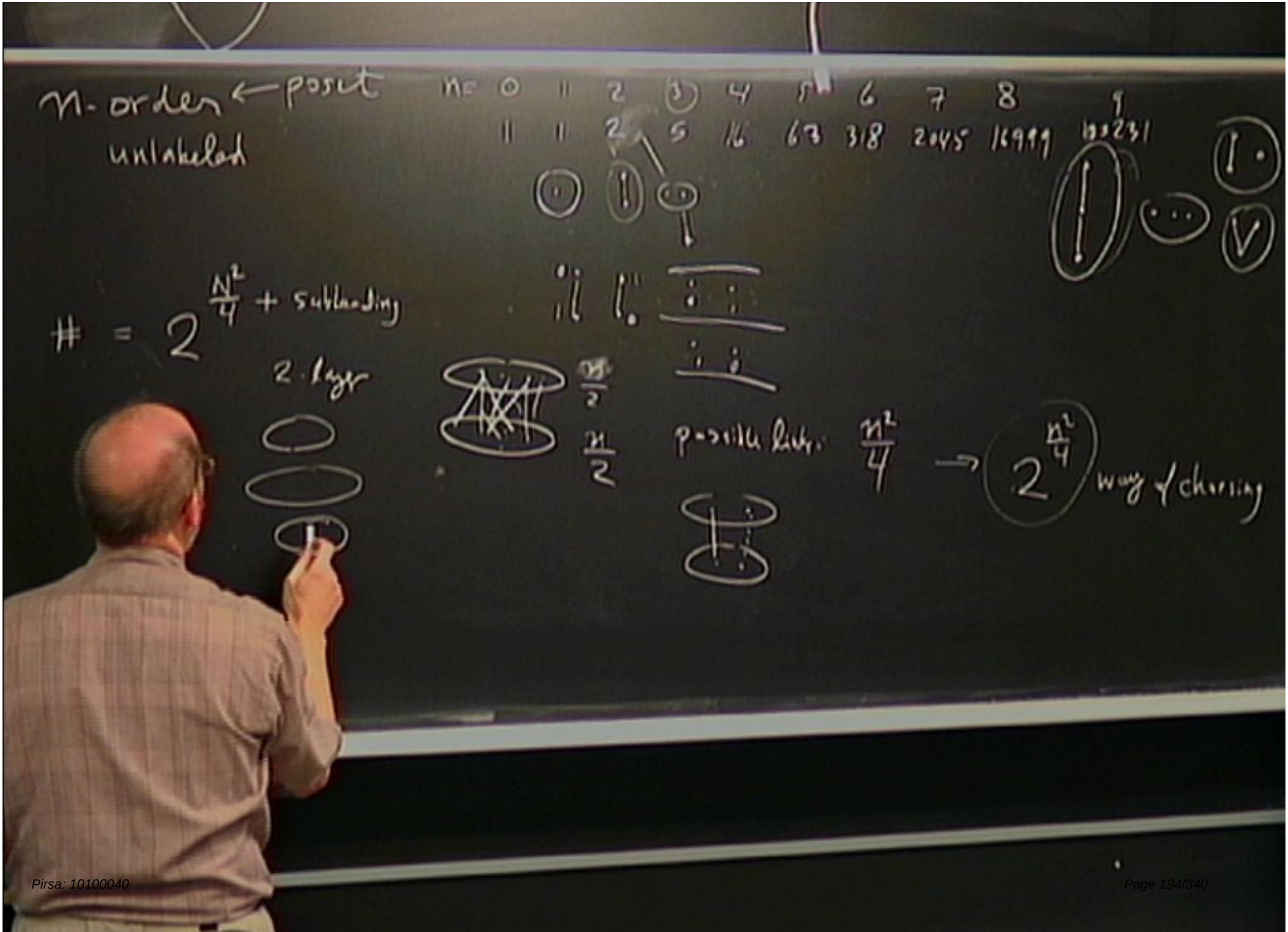
possible links. $\frac{n^2}{4} \rightarrow 2^{\frac{n^2}{4}}$ ways of choosing

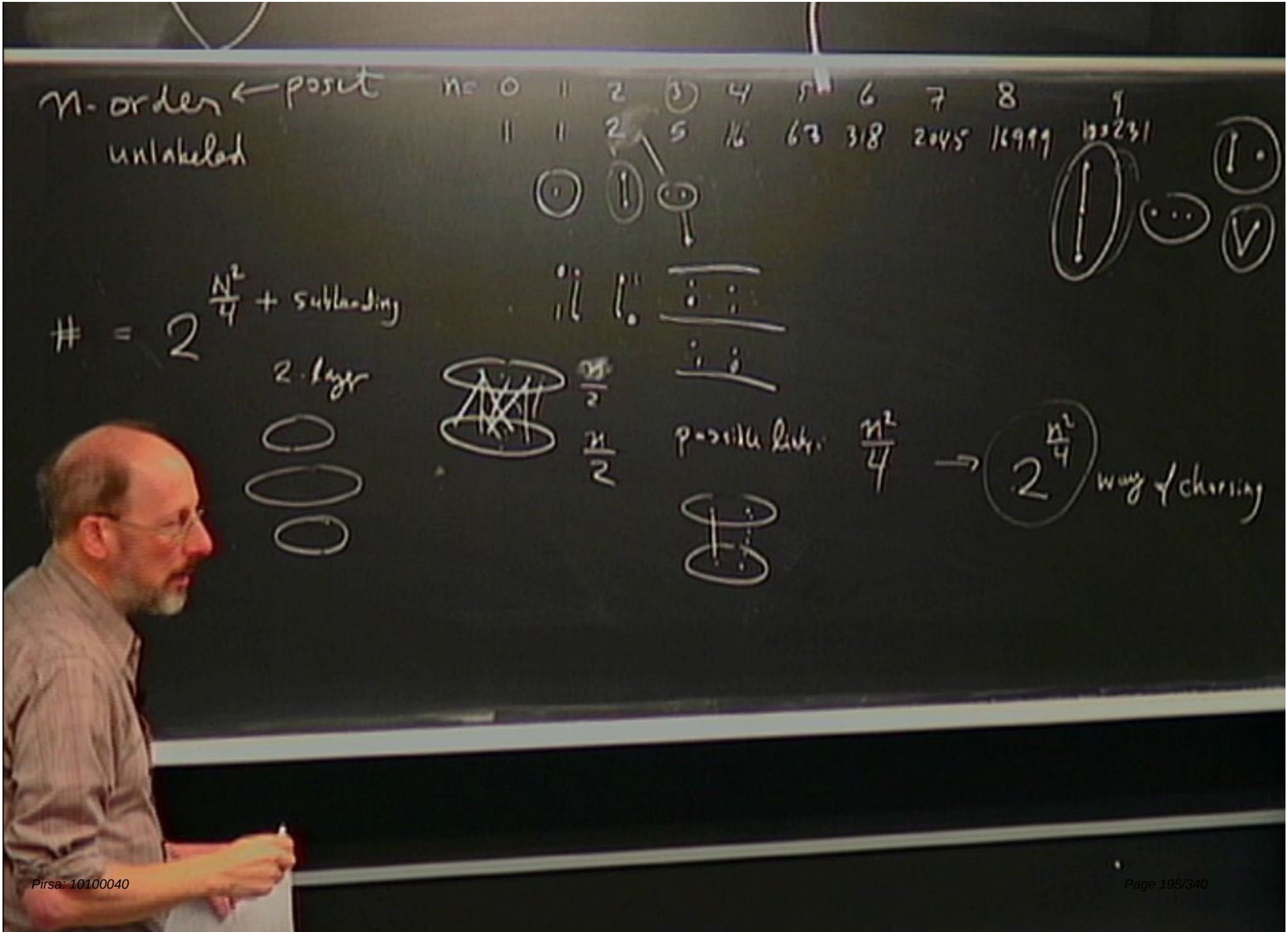



n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
= $2^{\frac{N^2}{4}}$ + subleading
2 layer
$\frac{n}{2}$
$\frac{n}{2}$
possible links $\frac{n^2}{4}$ → $2^{\frac{n^2}{4}}$ way of choosing


n-order ← poset
unlabeled
n = 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 133231
= $2^{\frac{N^2}{4}}$ + subleading
2 layer
$\frac{n}{2}$
$\frac{n}{2}$
possible links
$\frac{n^2}{4}$
$2^{\frac{n^2}{4}}$ way of choosing


n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
= 2^(n²/4) + subleading
2 layer
n/2
n/2
possible links
n²/4
→ 2^(n²/4) ways of choosing


n-order ← poset
unlabeled
n = 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 183231
= $2^{\frac{n^2}{4}}$ + subleading
2 layer
$\frac{n}{2}$
$\frac{n}{2}$
possible links
$\frac{n^2}{4}$
$2^{\frac{n^2}{4}}$ way of choosing


n-order ← posit
unlabeled
n = 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 63 318 2045 16999 133231
= $2^{\frac{N^2}{4}}$ + subleading
2 layer
$\frac{n}{2}$
$\frac{n}{2}$
possible links
$\frac{n^2}{4}$
$2^{\frac{n^2}{4}}$ way of choosing

n-order ← poset

unlabeled

n=	0	1	2	3	4	5	6	7	8	9
	1	1	2	5	16	63	318	2045	16999	153231

$\# = 2^{\frac{N^2}{4}}$ + subleading

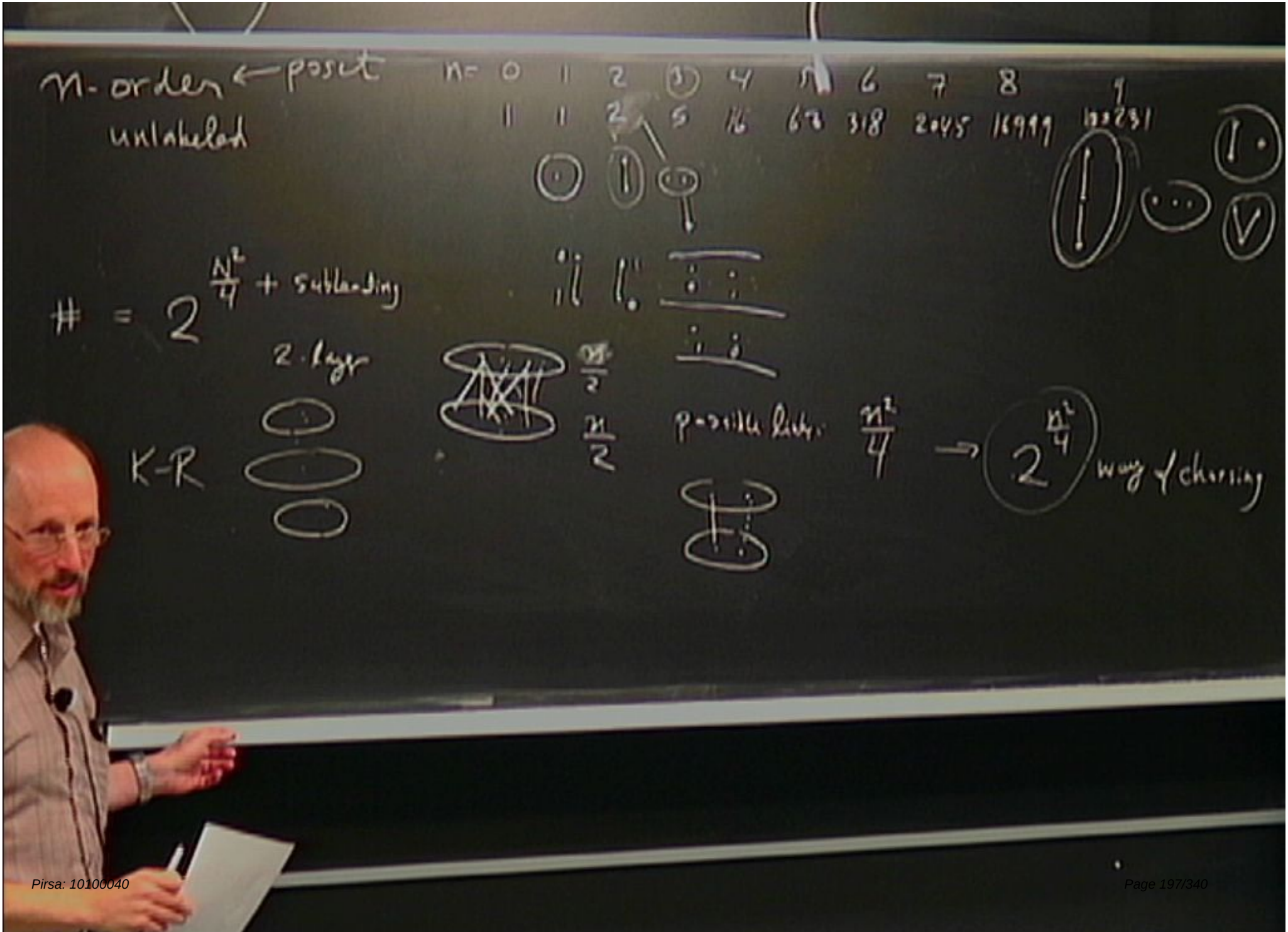
2 layer

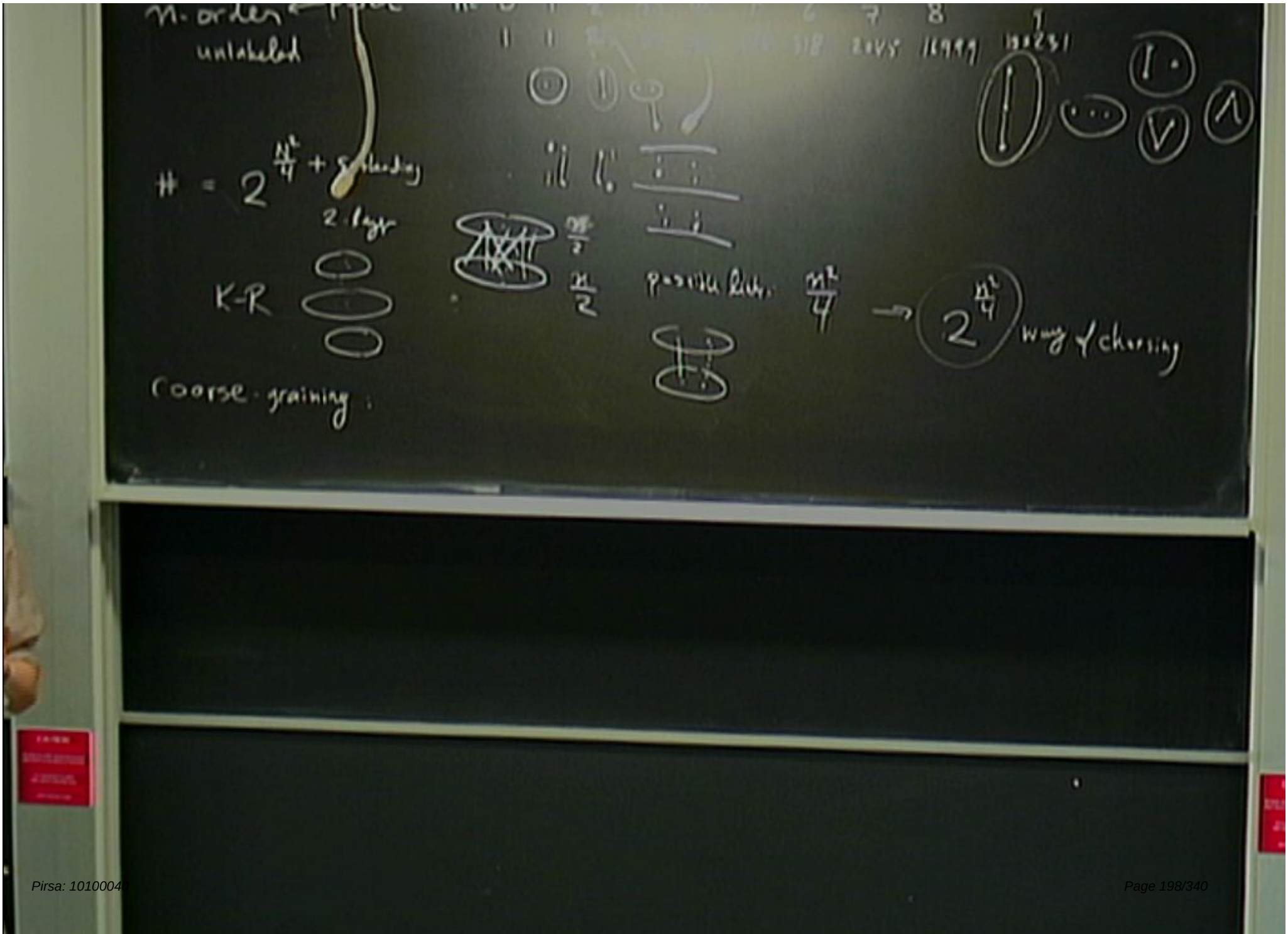
K-R

$\frac{n}{2}$

$\frac{n}{2}$

Possible links: $\frac{n^2}{4}$ → $2^{\frac{n^2}{4}}$ ways of choosing


n-order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8 9
1 1 2 5 16 67 318 2045 16999 183231
= 2^{n²/4} + subleading
2 layer
K-R
n/2
n/2
n²/4
→ 2^{n²/4} way of choosing


unlabeled
K-R
Coarse-graining

n-order ← past

unlabeled

n: 0 1 2 3 4 5 6 7 8

1 1 2 3 4 5 18 2005 16999 1232231

$\# = 2^{\frac{n^2}{4}} + \text{something}$

2 layer

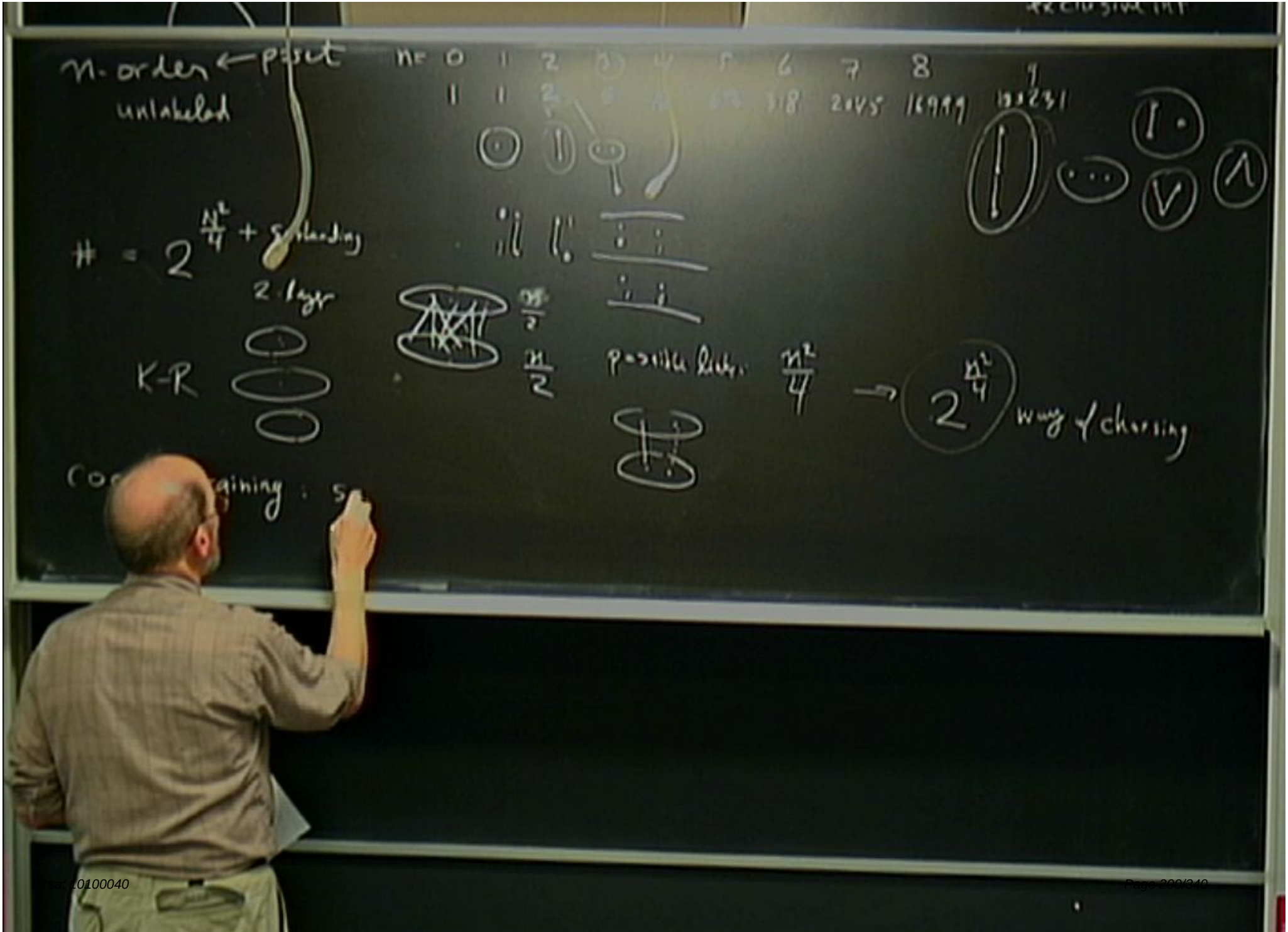
K-R

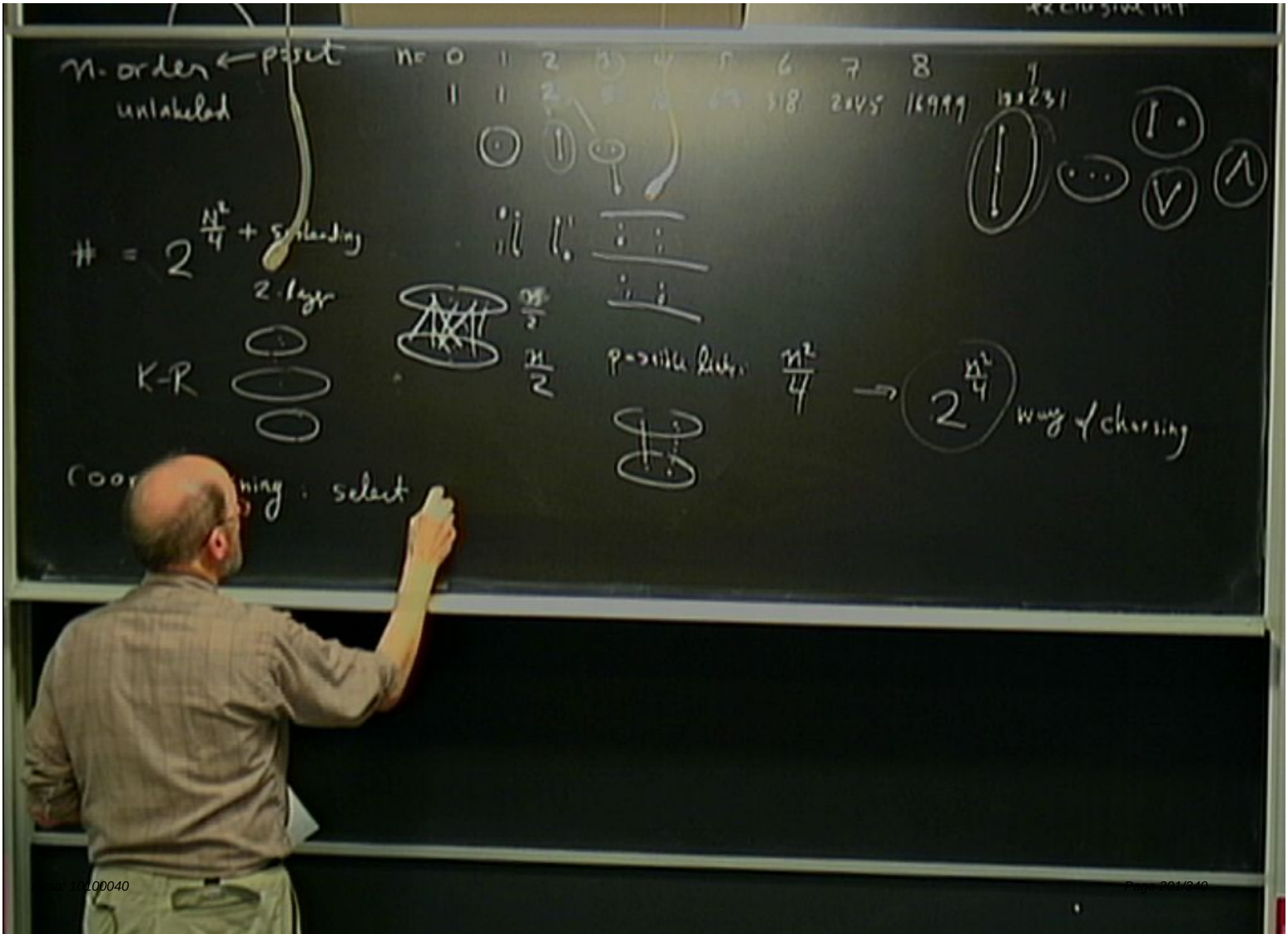
$\frac{n}{2}$

$\frac{n}{2}$

possible links: $\frac{n^2}{4} \rightarrow 2^{\frac{n^2}{4}}$ ways of choosing

Coarse-graining.


n. orders ← past
unlabeled
n: 0 1 2 3 4 5 6 7 8
1 1 2 5 16 318 2V5 16989
2 layer
K-R
possible links
$\frac{n^2}{4}$
$2^{\frac{n^2}{4}}$
ways of choosing


n. order ← poset
unlabeled
n = 0 1 2 3 4 5 6 7 8
1 1 2 5 16 318 2045 ...
2 layer
K-R
Coor... ning : select
... way of choosing

n. order ← poset

unlabeled

n = 0 1 2 3 4 5 6 7 8

1 1 2 5 16 64 318 2045 16999

$\#_n = 2^{\frac{n^2}{4}}$ + something

2 layer

K-R

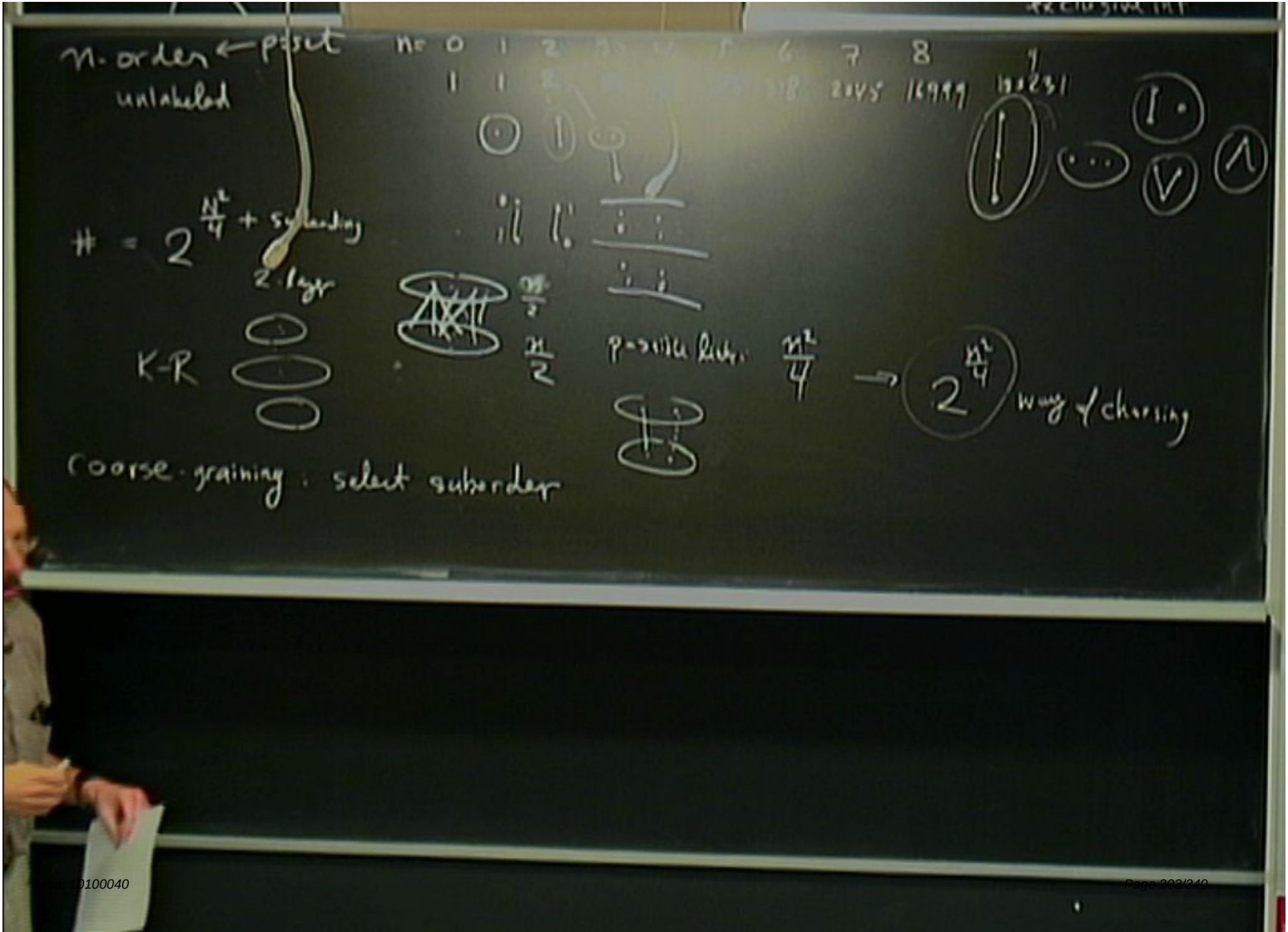
$\frac{n}{2}$

$\frac{n}{2}$

$\frac{n^2}{4}$ → $2^{\frac{n^2}{4}}$ way of choosing

Coarse-graining: select suborder

Visa: 10100040


n. order ← poset
unlabeled
n= 0 1 2 3 4 5 6 7 8
1 1 2 5 16 63 318 2045 16999
= $2^{\frac{n^2}{4}}$ + something
2 layer
K-R
$\frac{n}{2}$
$\frac{n}{2}$
Possible flips
$\frac{n^2}{4}$
$2^{\frac{n^2}{4}}$
ways of choosing
Coarse-graining: select suborder

n. order ← poset

unlabeled

n = 0 1 2 3 4 5 6 7 8

1 1 2 5 16 63 318 2045 16999

$\#_n = 2^{\frac{n^2}{4}}$ + something

2 layer

K-R

$\frac{n}{2}$

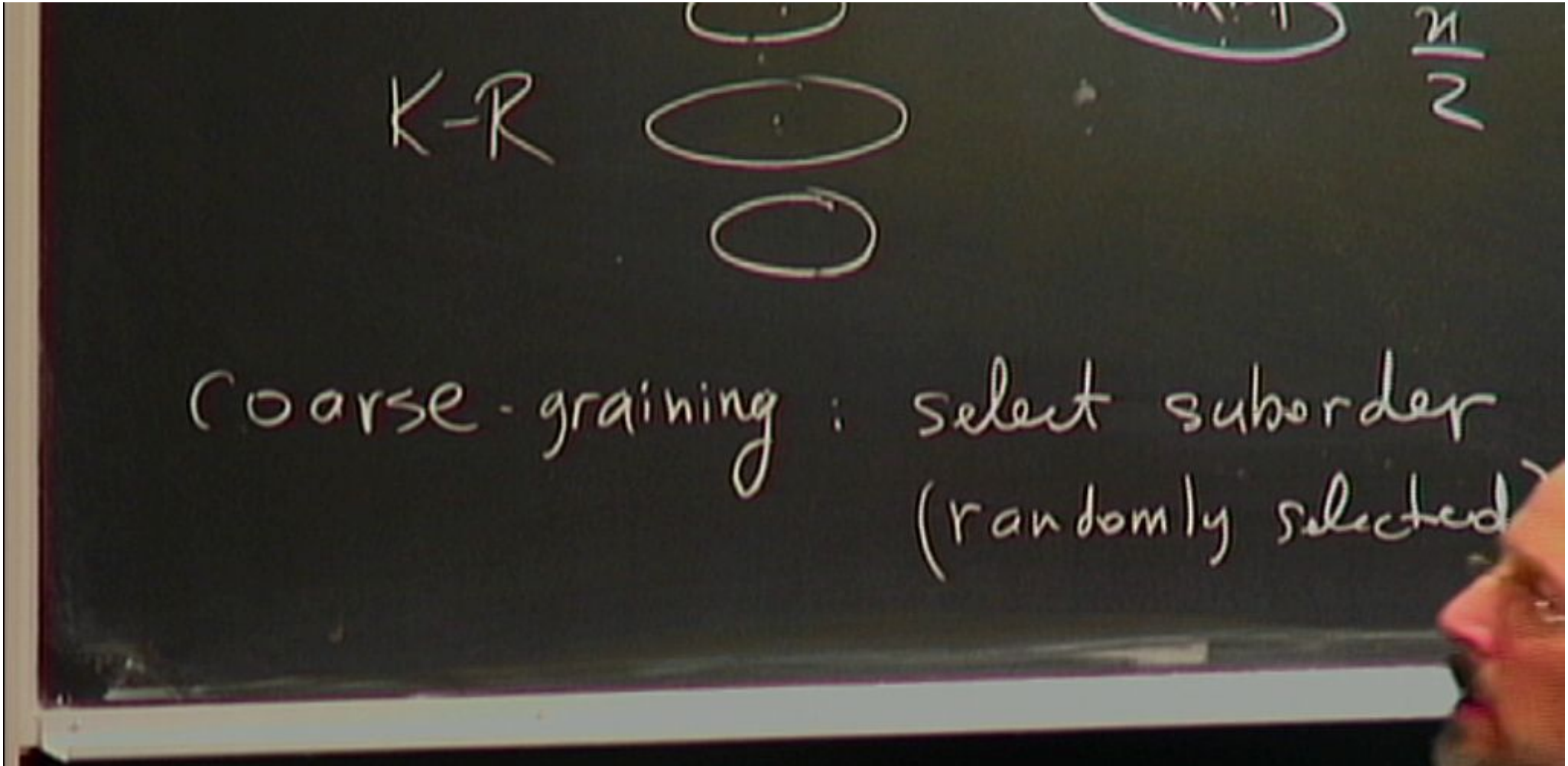
$\frac{n}{2}$

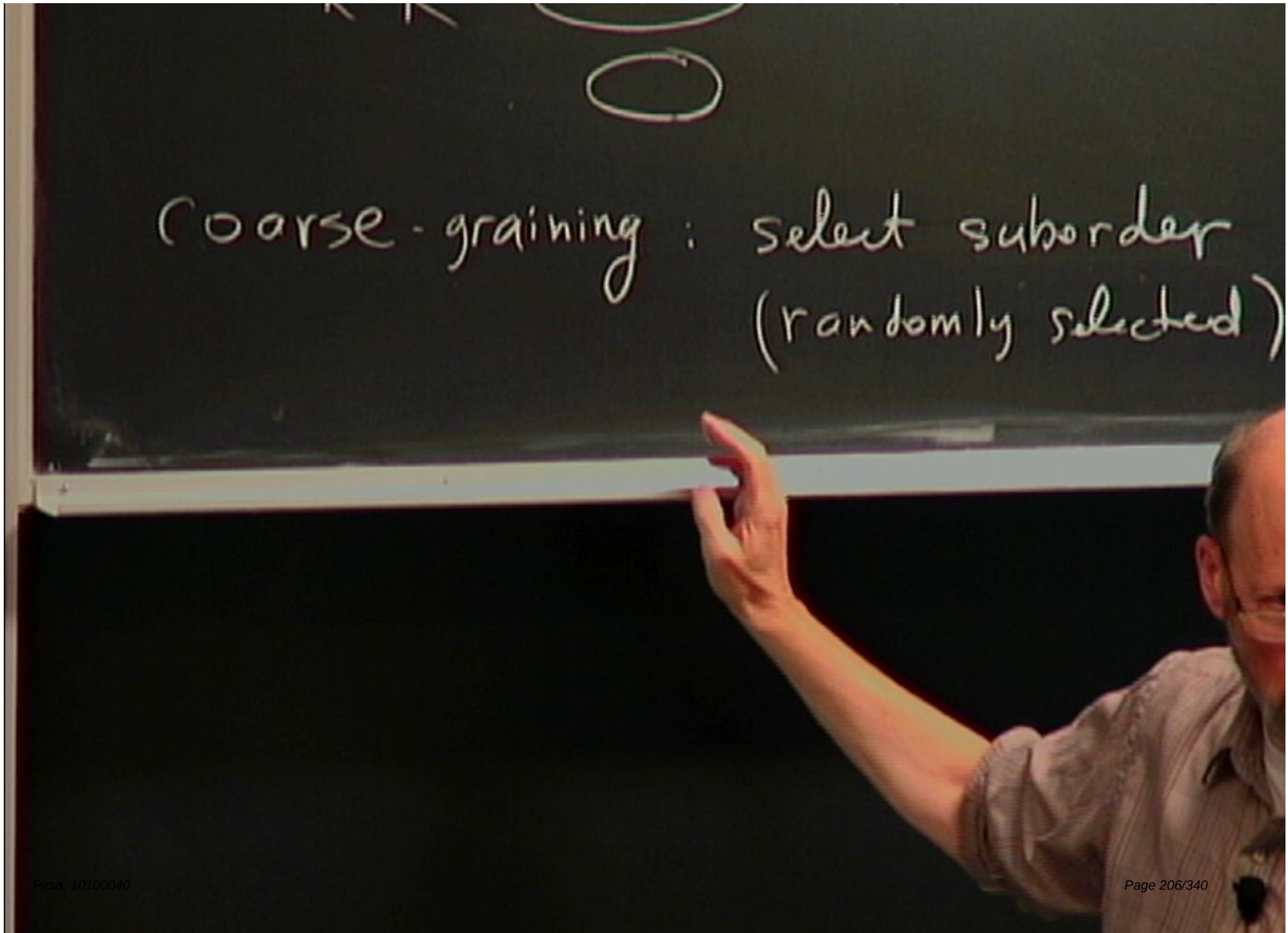
Possible links

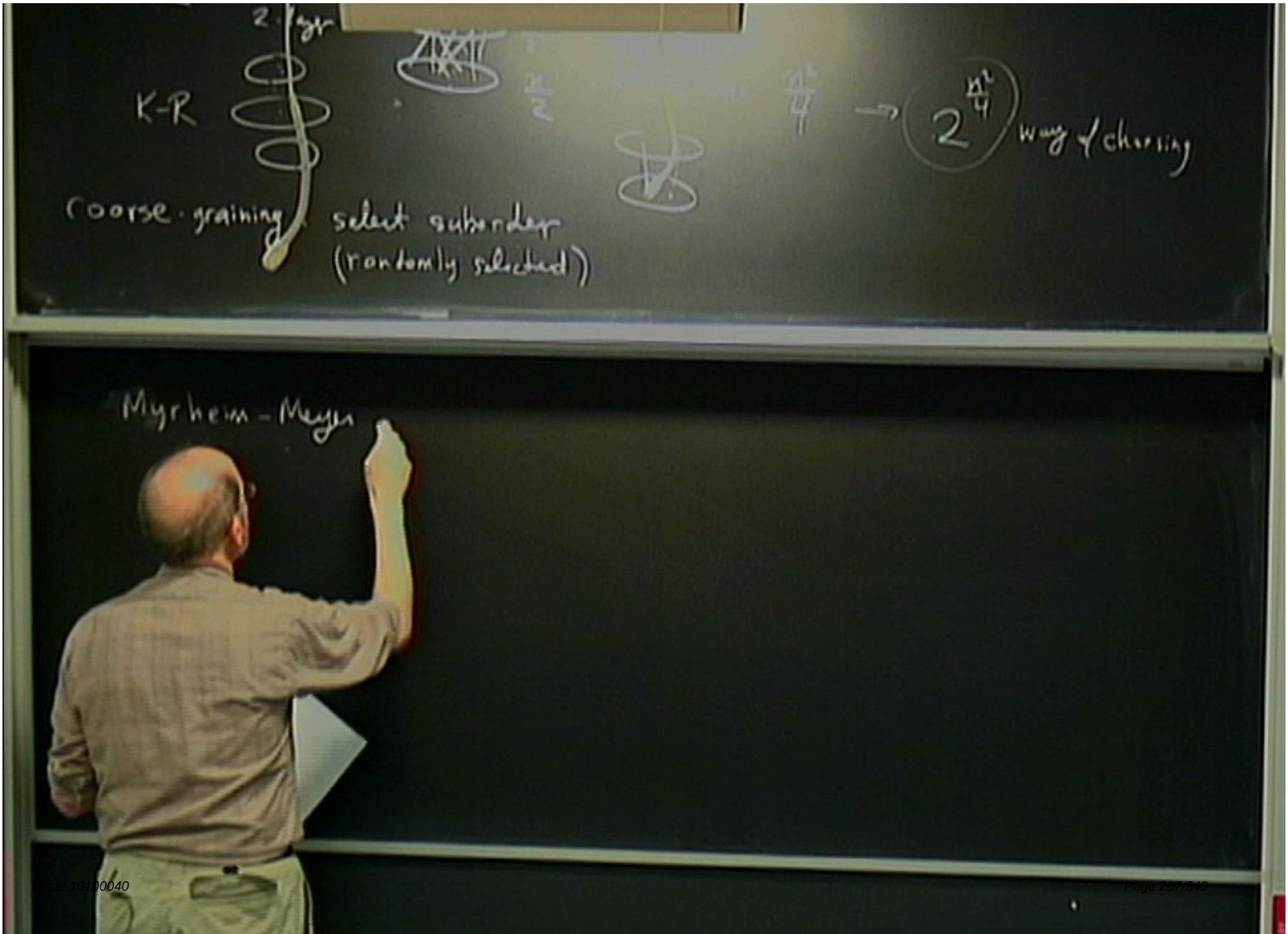
$\frac{n^2}{4}$ → $2^{\frac{n^2}{4}}$ ways of choosing

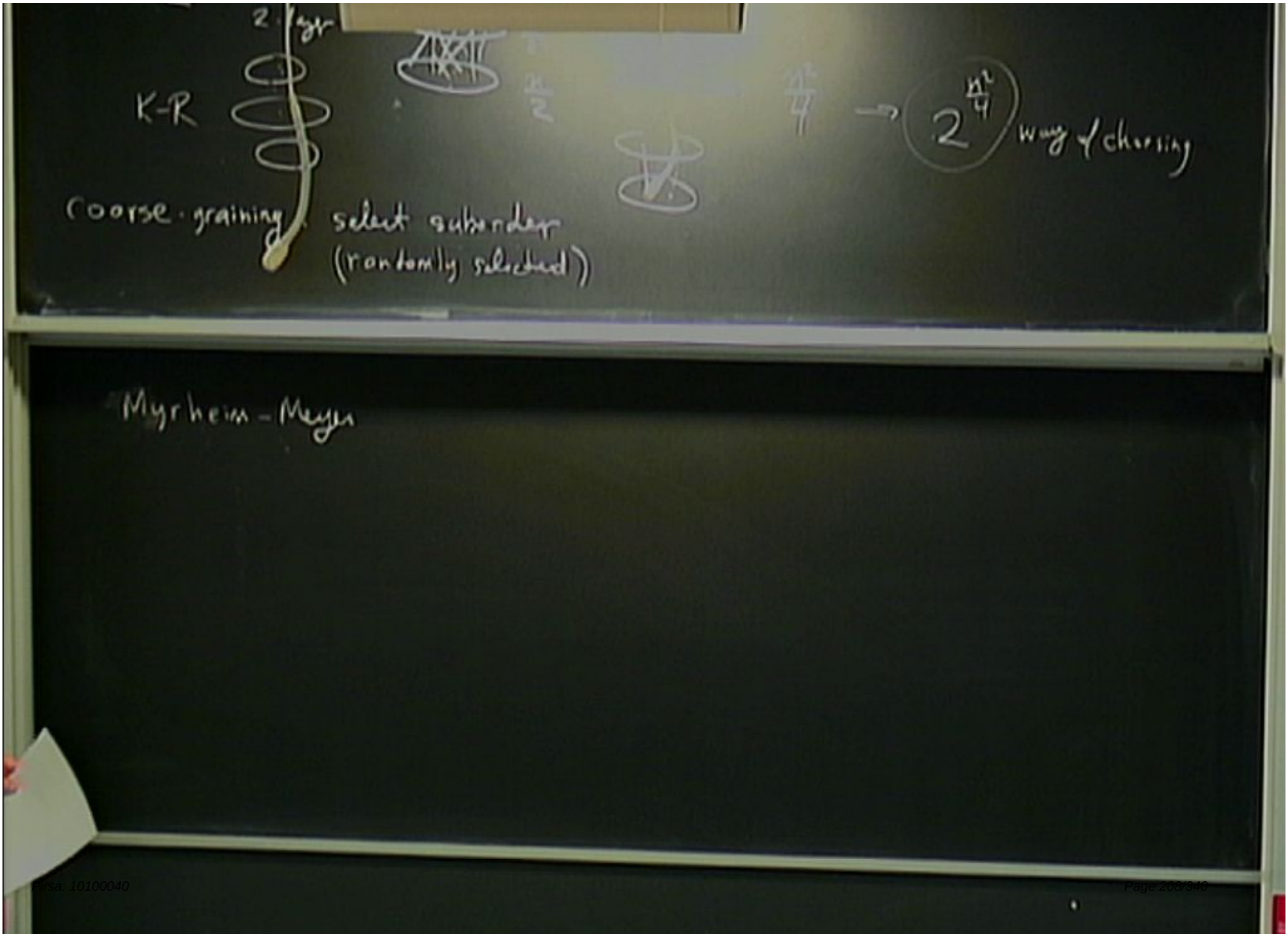
Coarse-graining: select suborder

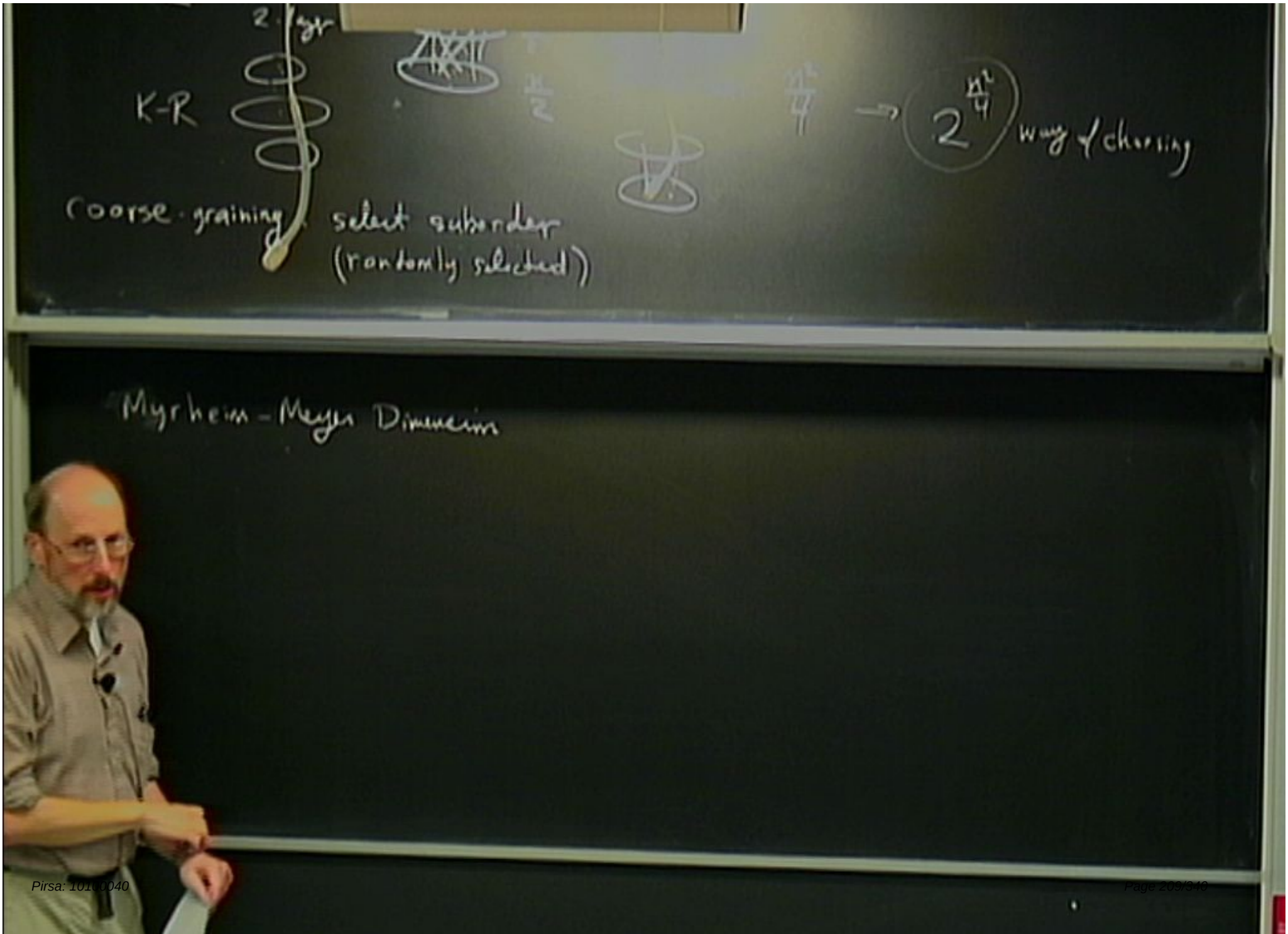
Pirsa: 10100040

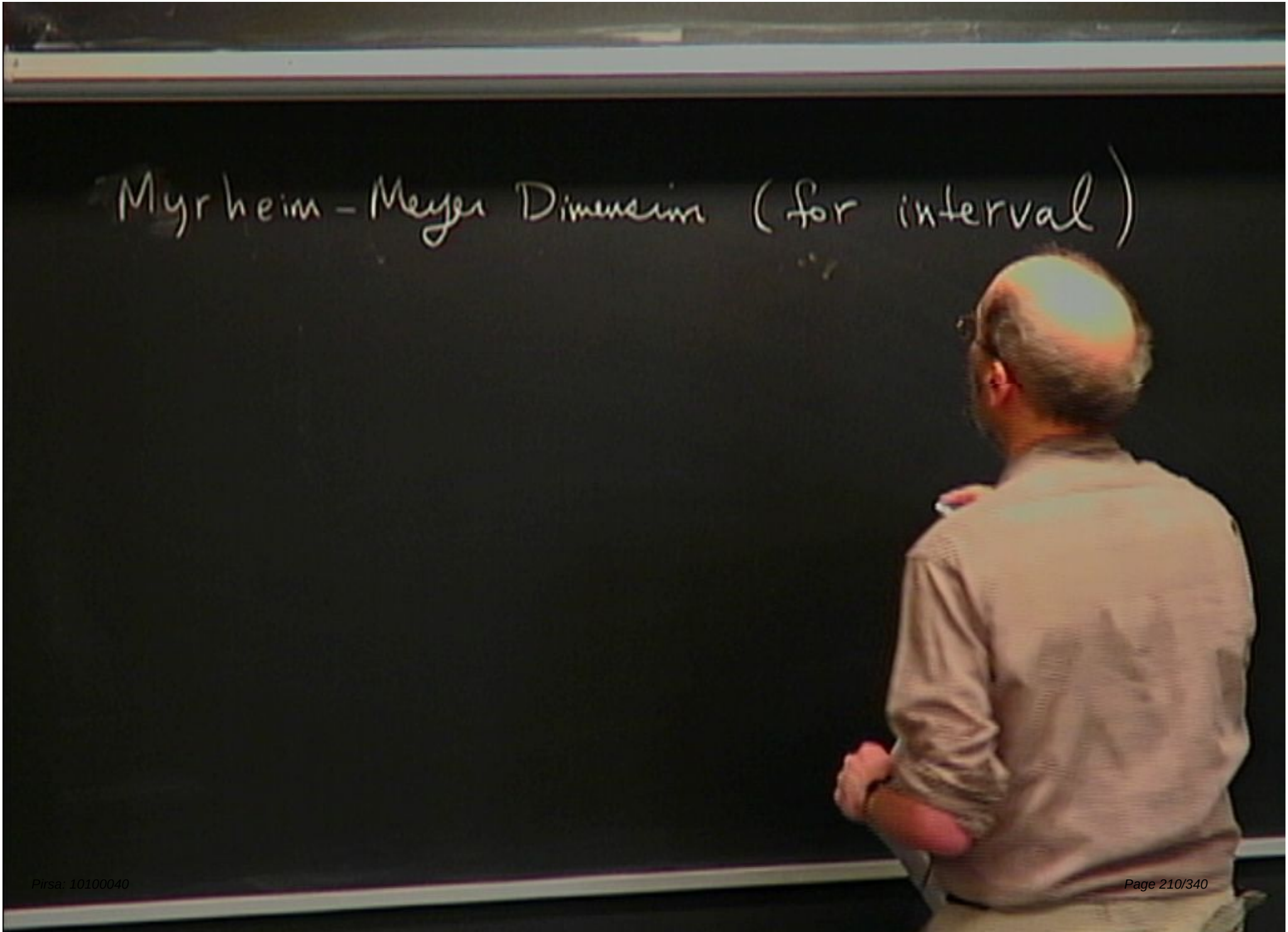

K-R
n/2
Coarse-graining : select suborder
(randomly selected)

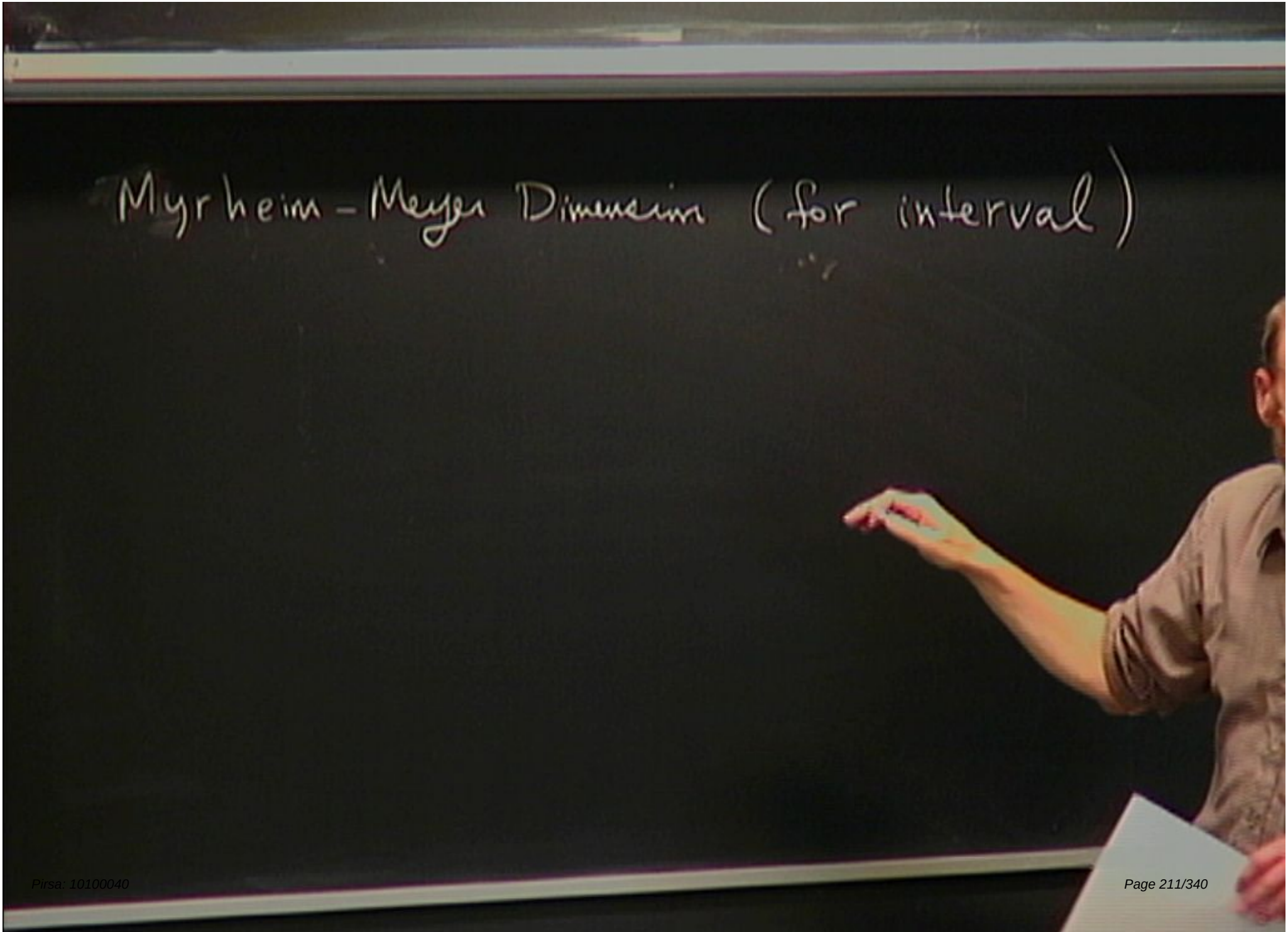

Coarse-graining : select suborder
(randomly selected)

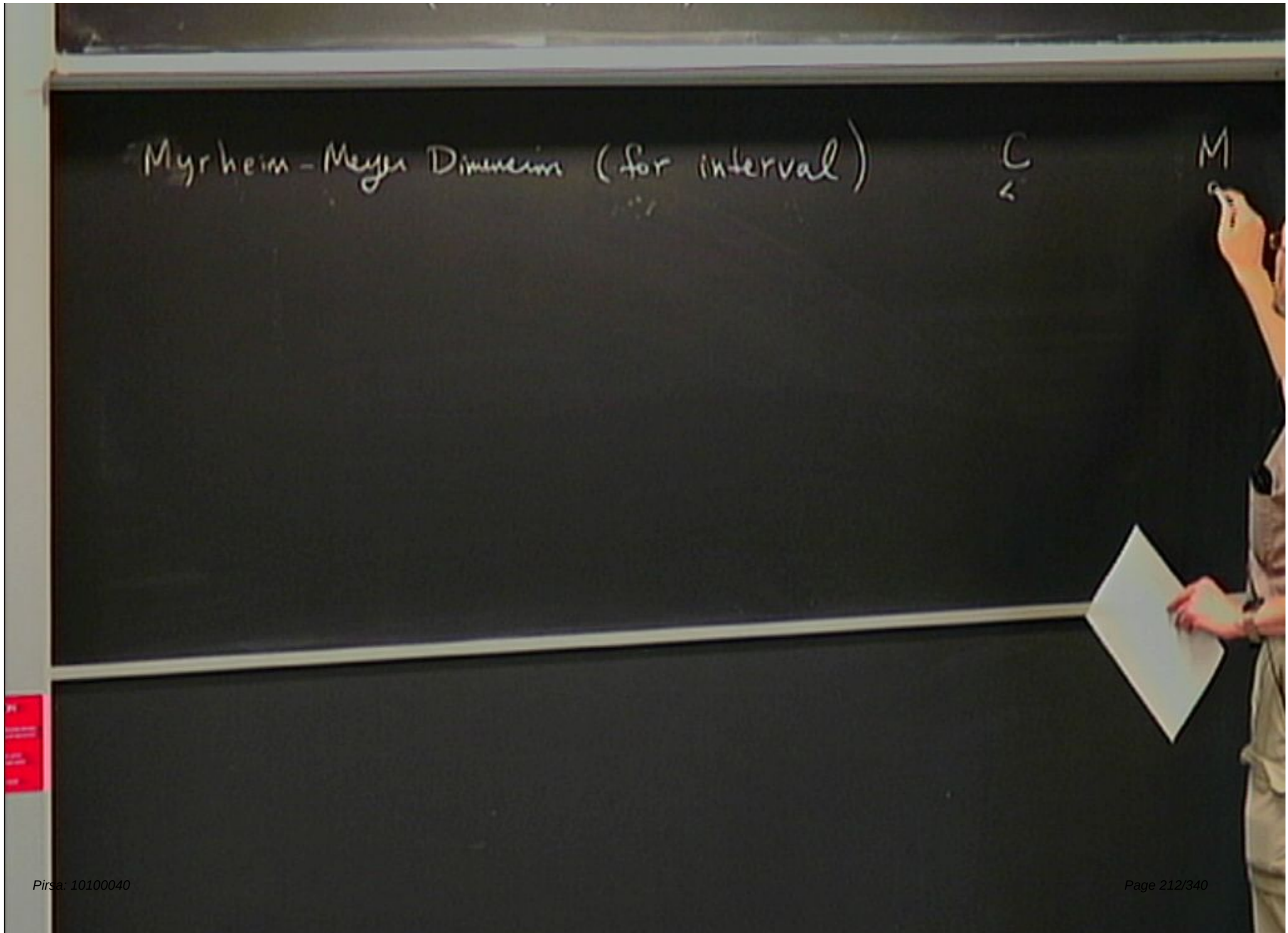

K-R
coarse-graining
select subordering
(randomly selected)
Myrheim-Meyer

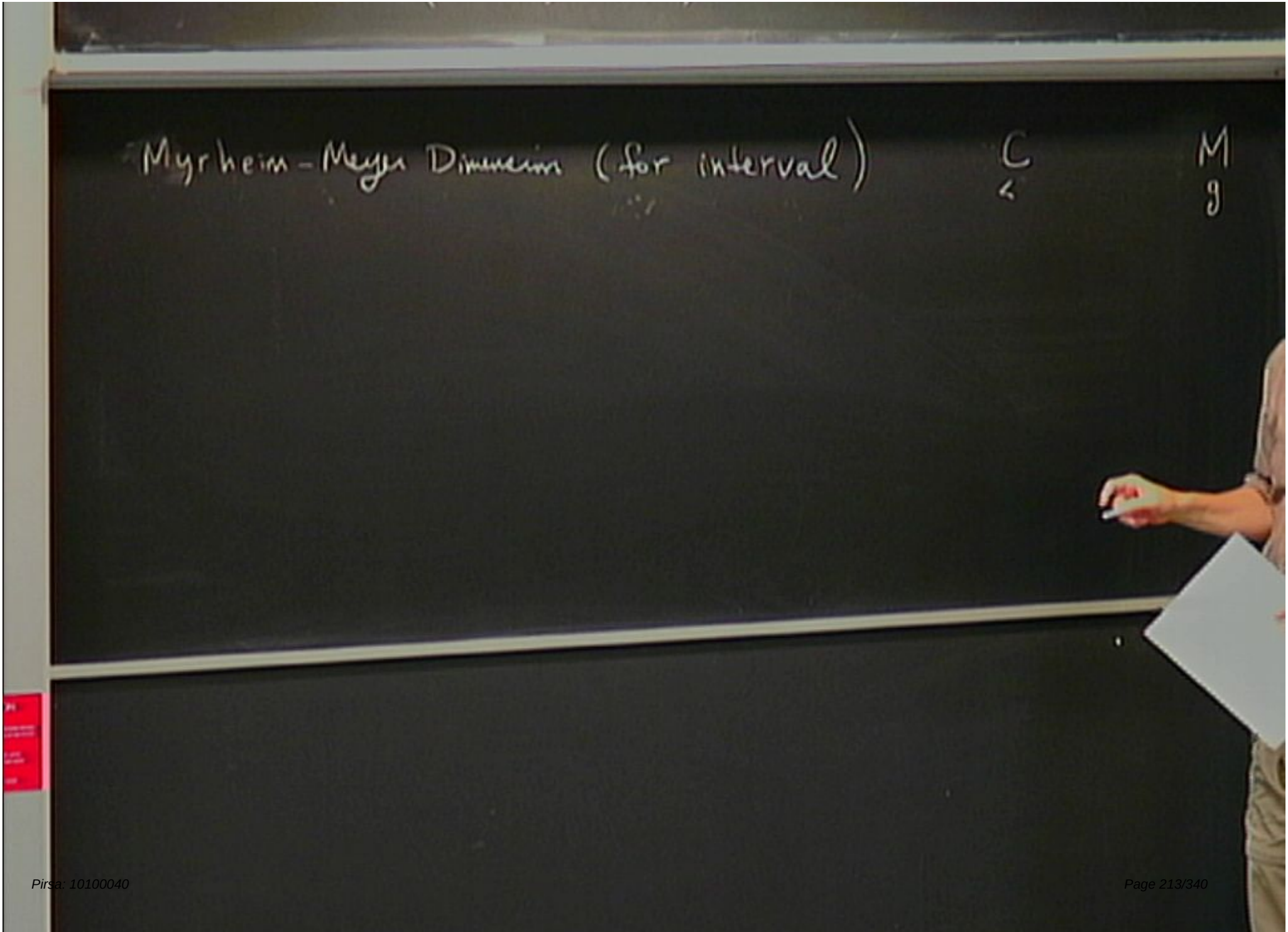

K-R
Coarse-graining
select subordeg
(randomly selected))
Myrheim-Meyer

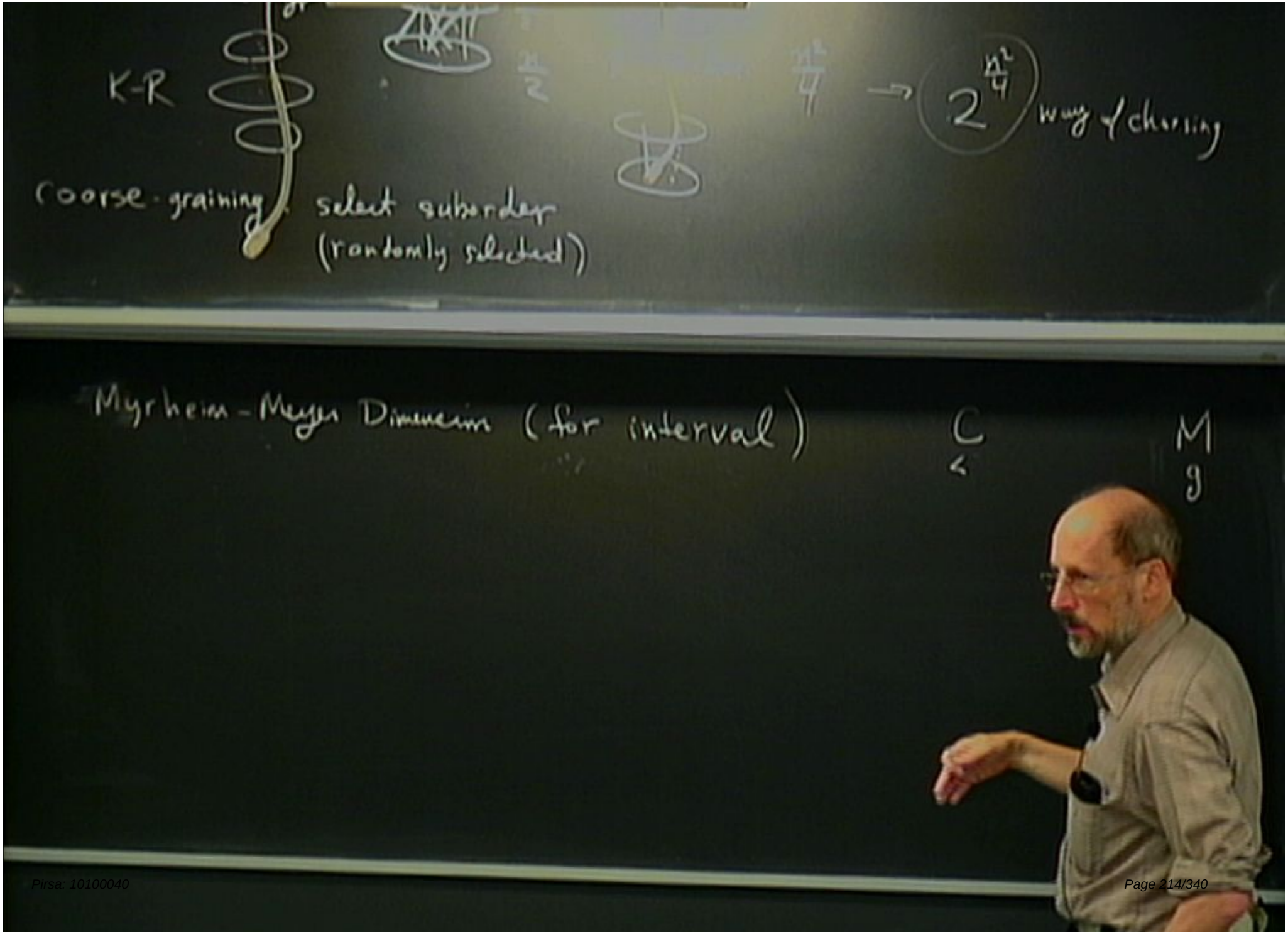

K-R
coarse-graining
select subordinator
(randomly selected))
ways of choosing
Myrheim-Meyer Dimension

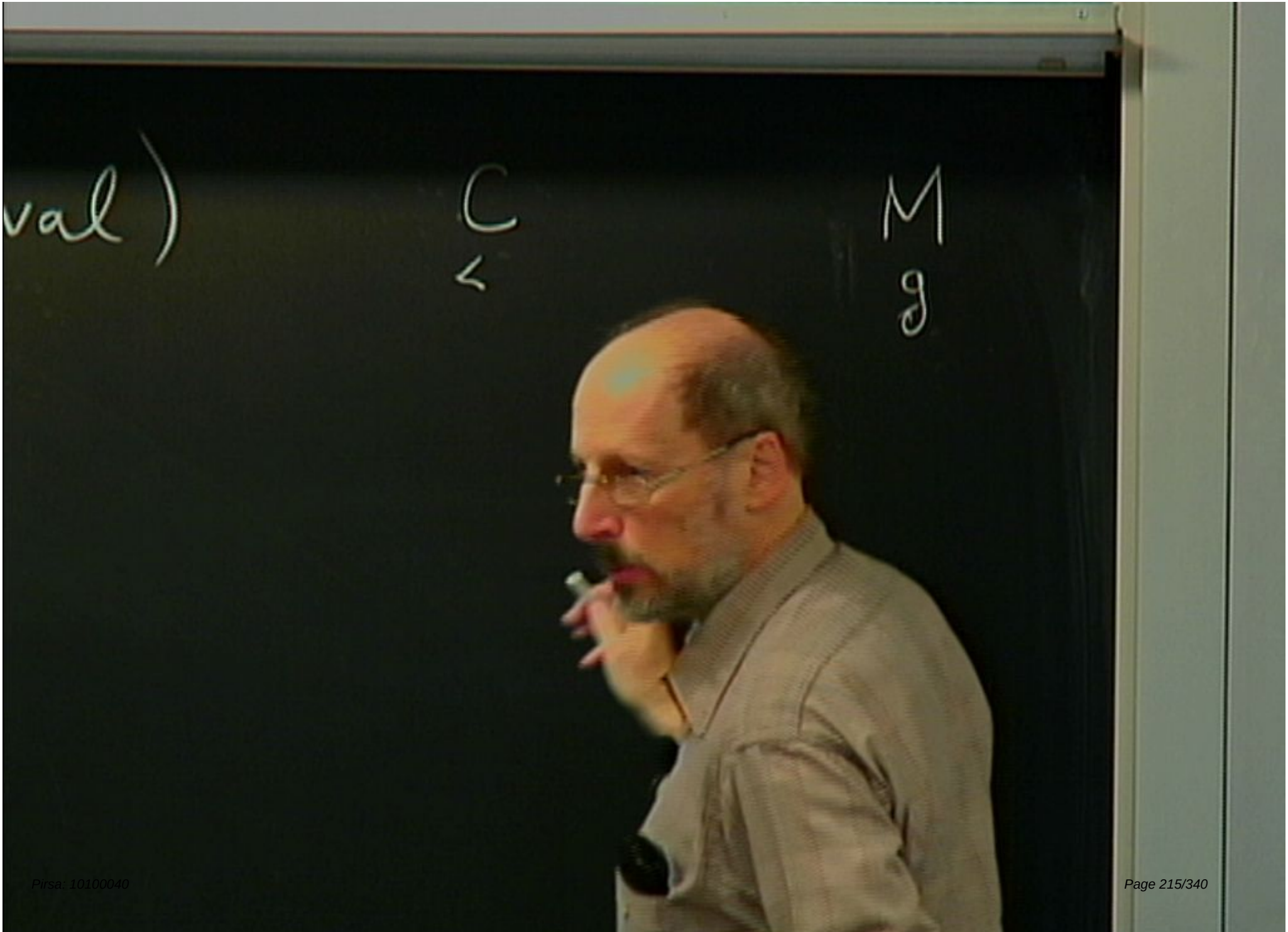

Myrheim-Meyer Dimension (for interval)

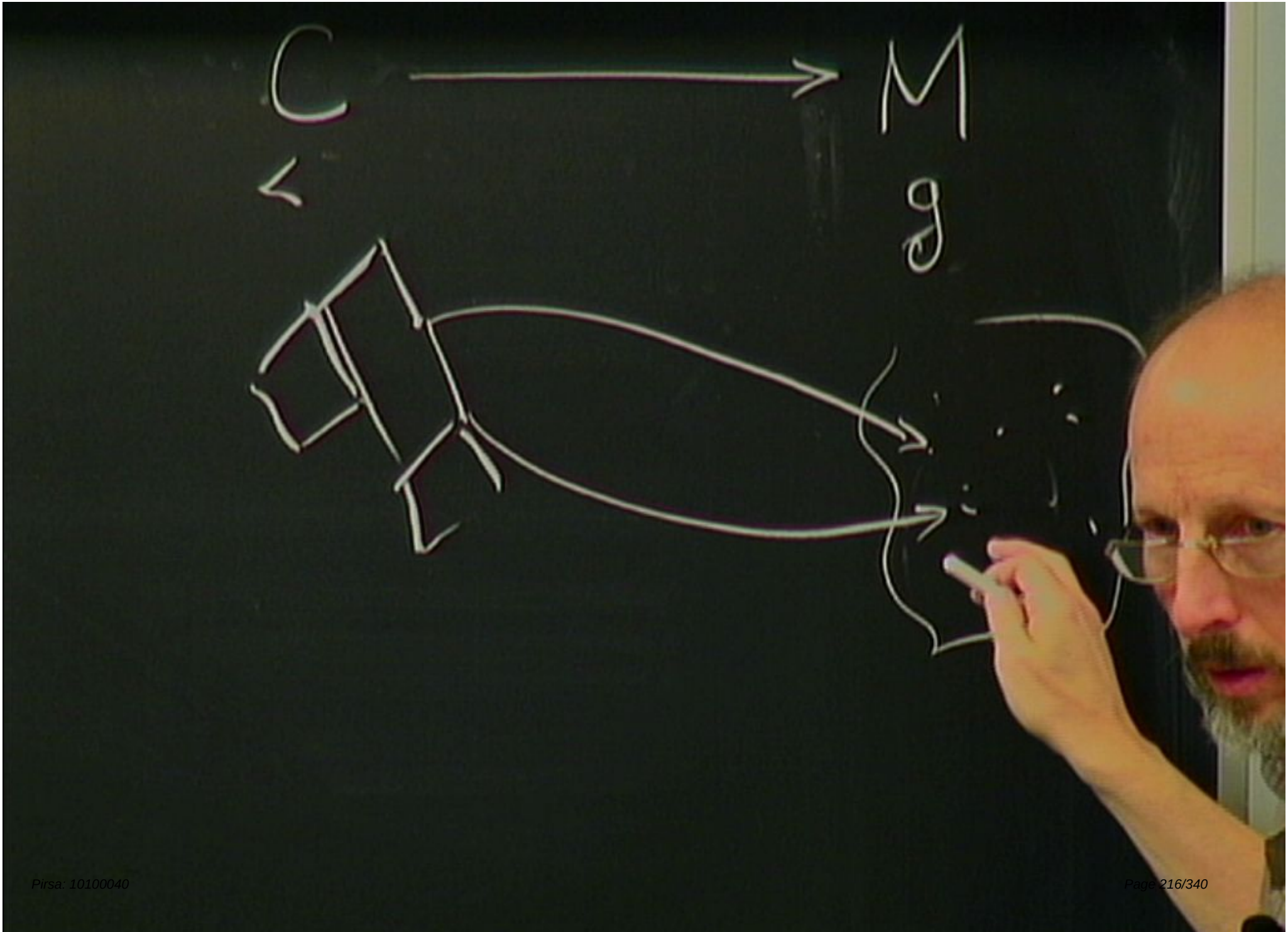

Myrheim - Meyer Dimension (for interval)


Myrheim - Meyer Dimension (for interval)
C
M


Myrheim-Meyer Dimension (for interval)
C
M

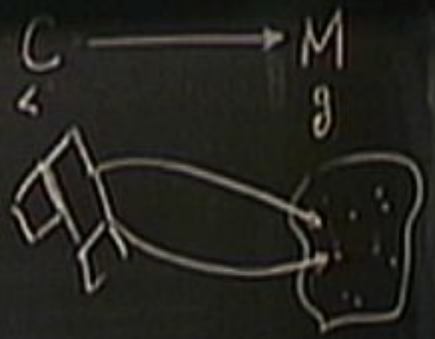

K-R
$\frac{N}{2}$
$\frac{N}{4}$
$2^{\frac{N^2}{4}}$ ways of choosing
Coorse-graining
select subordering
(randomly selected)
Myrheim-Meyer Dimension (for interval)
C
M

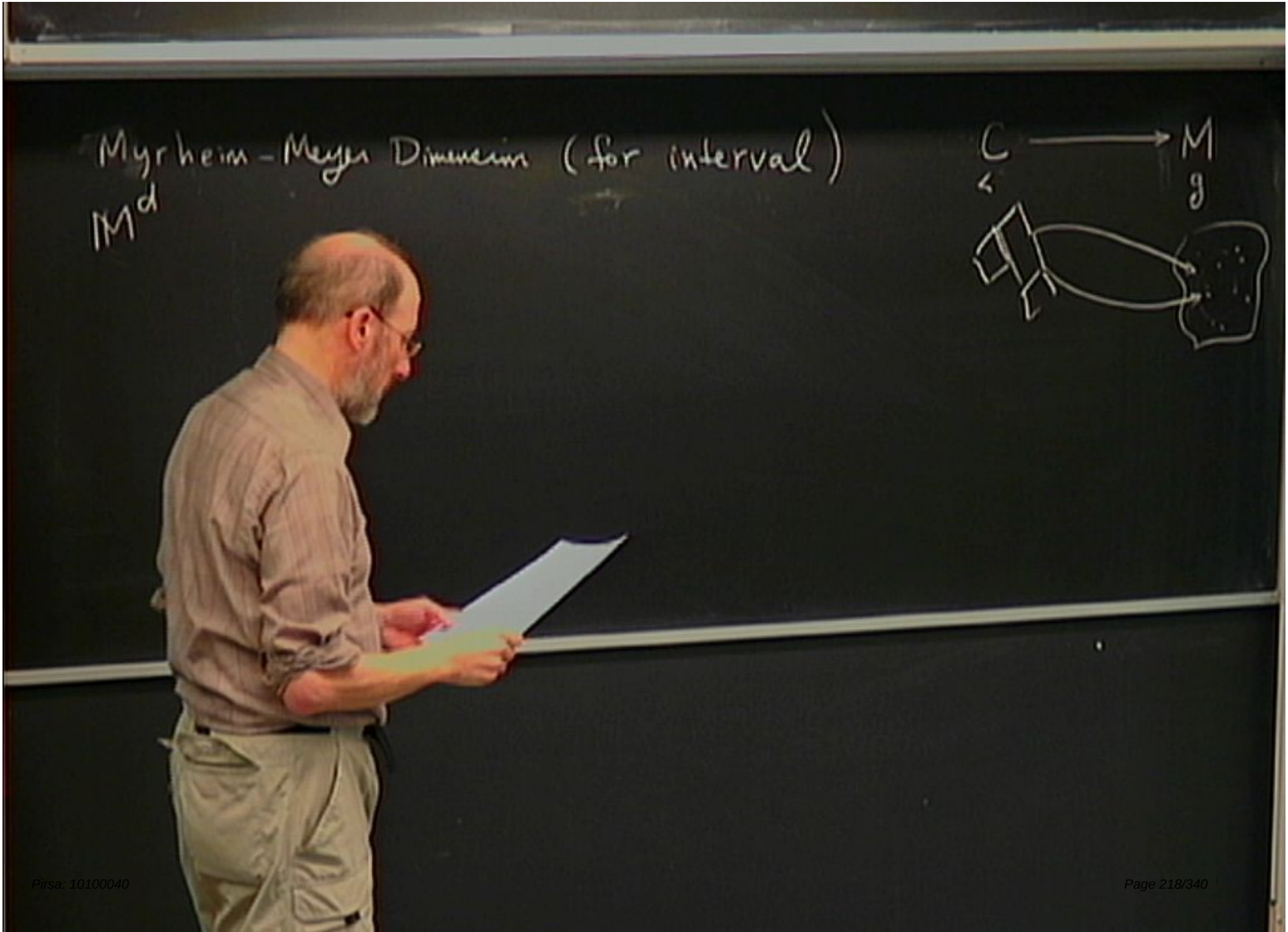

val)
C
M
g

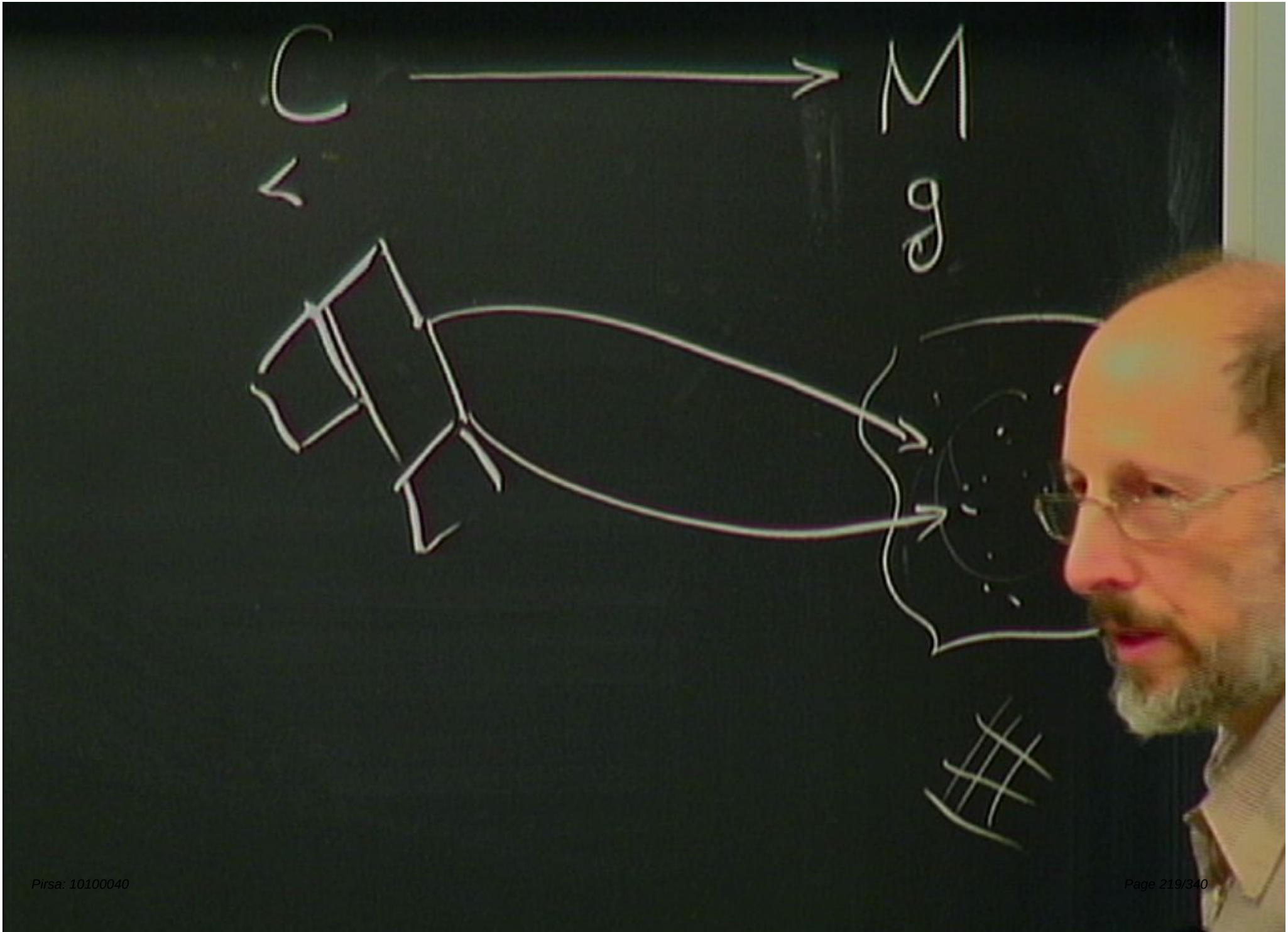

C
M
g

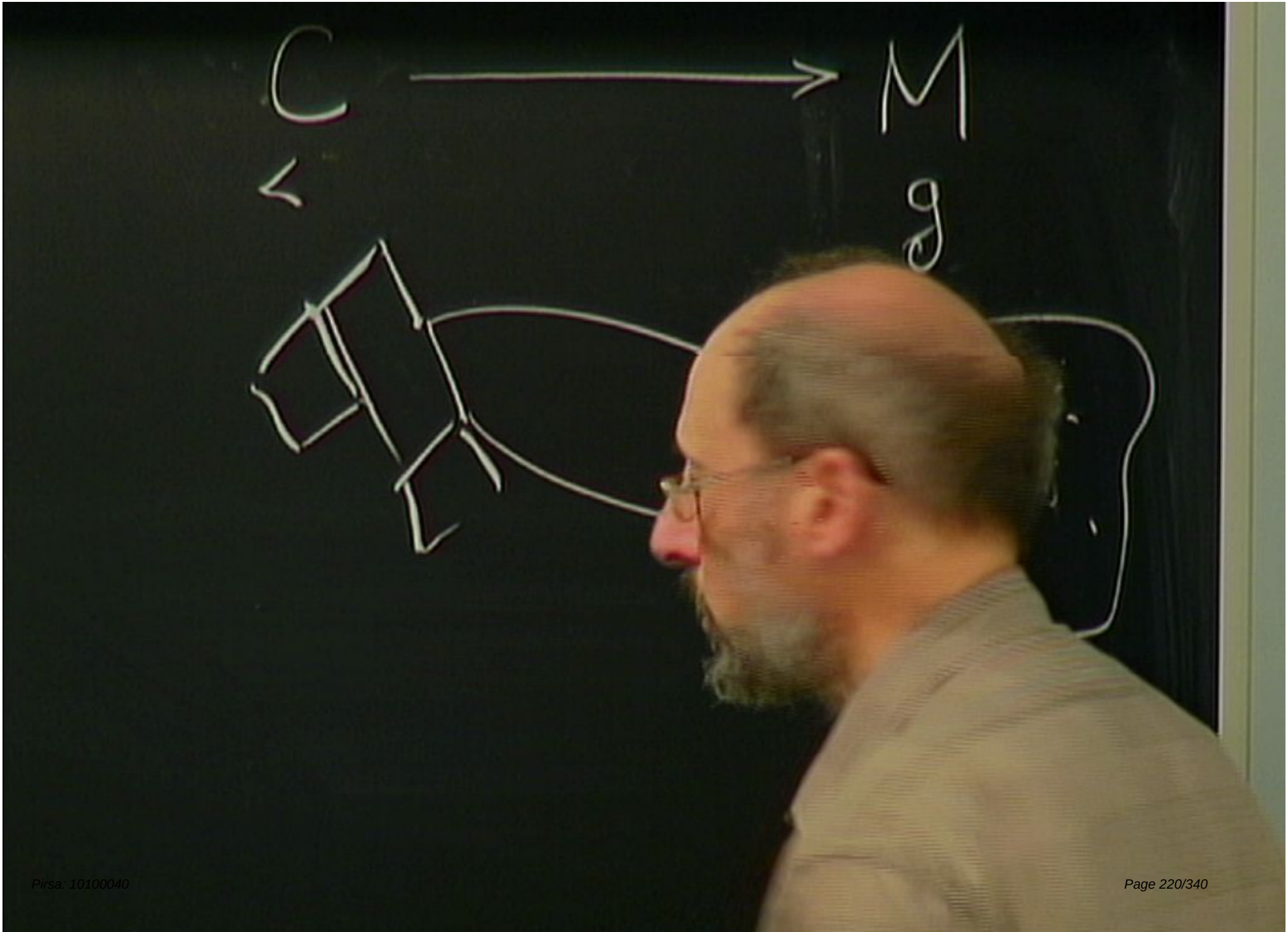
(randomly selected)

Myrheim-Meyer Dimension (for interval)




Myrheim-Meyer Dimension (for interval)
M^d
C
M
g

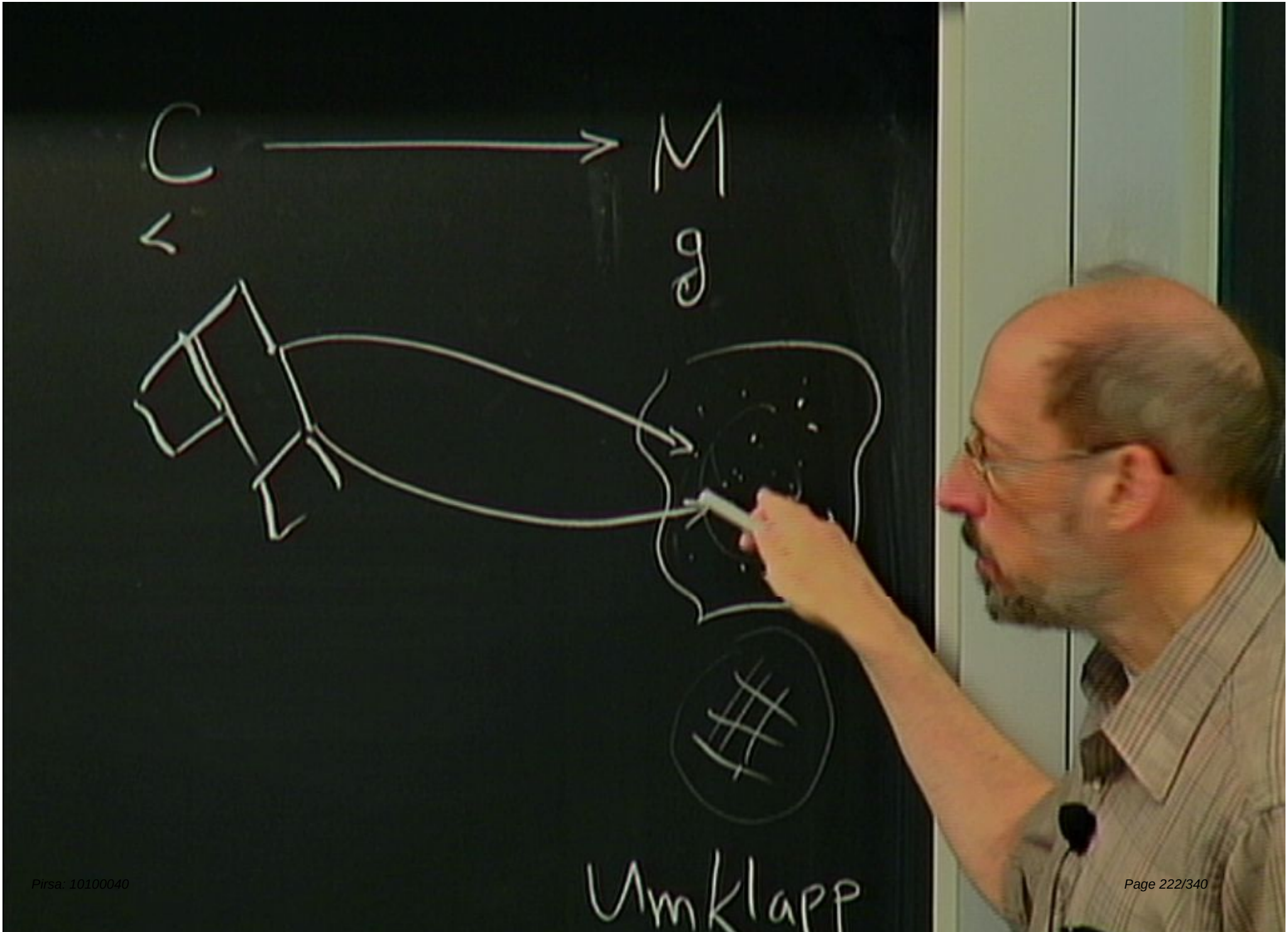

C
M
g

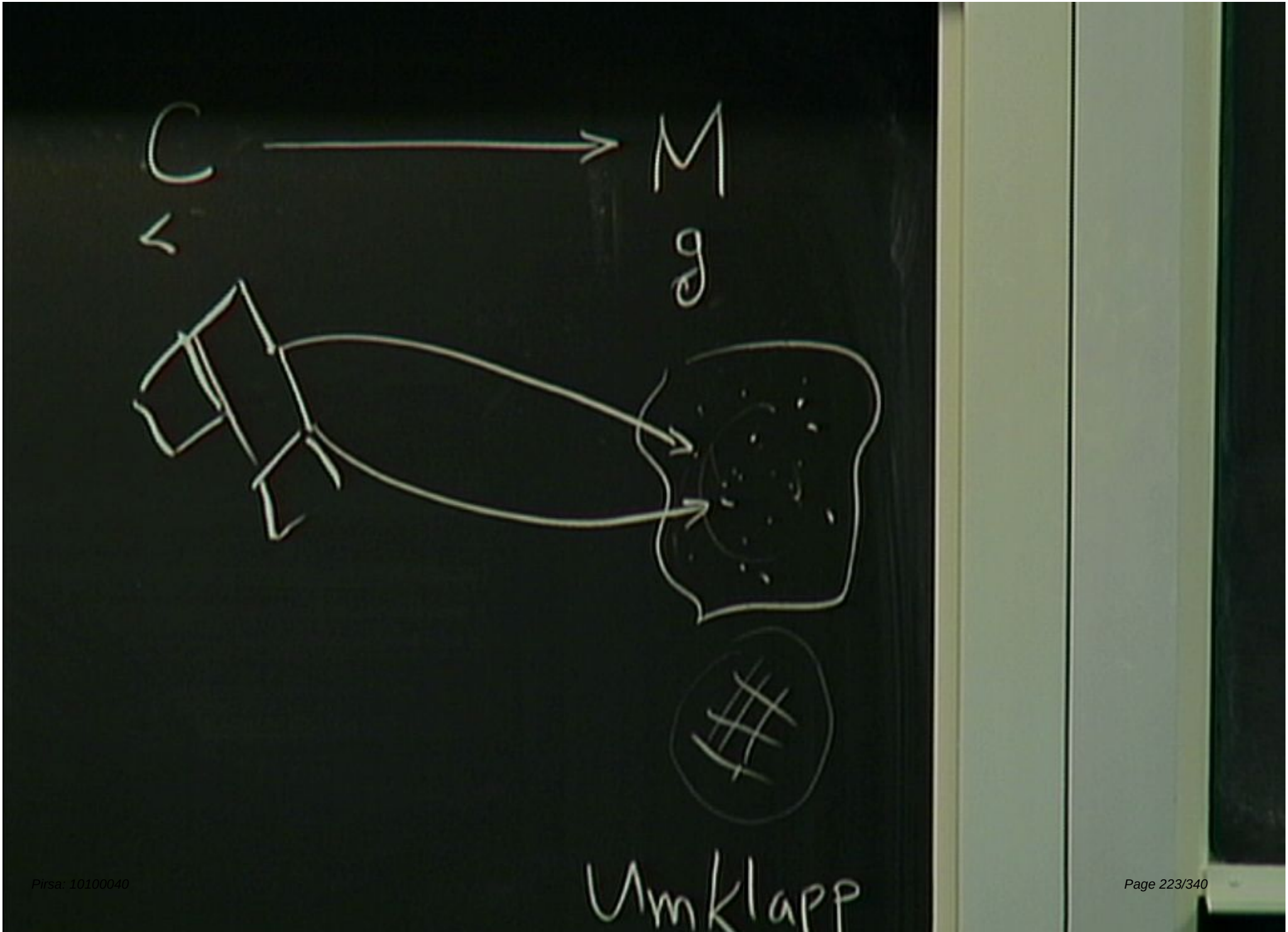

C
M
g

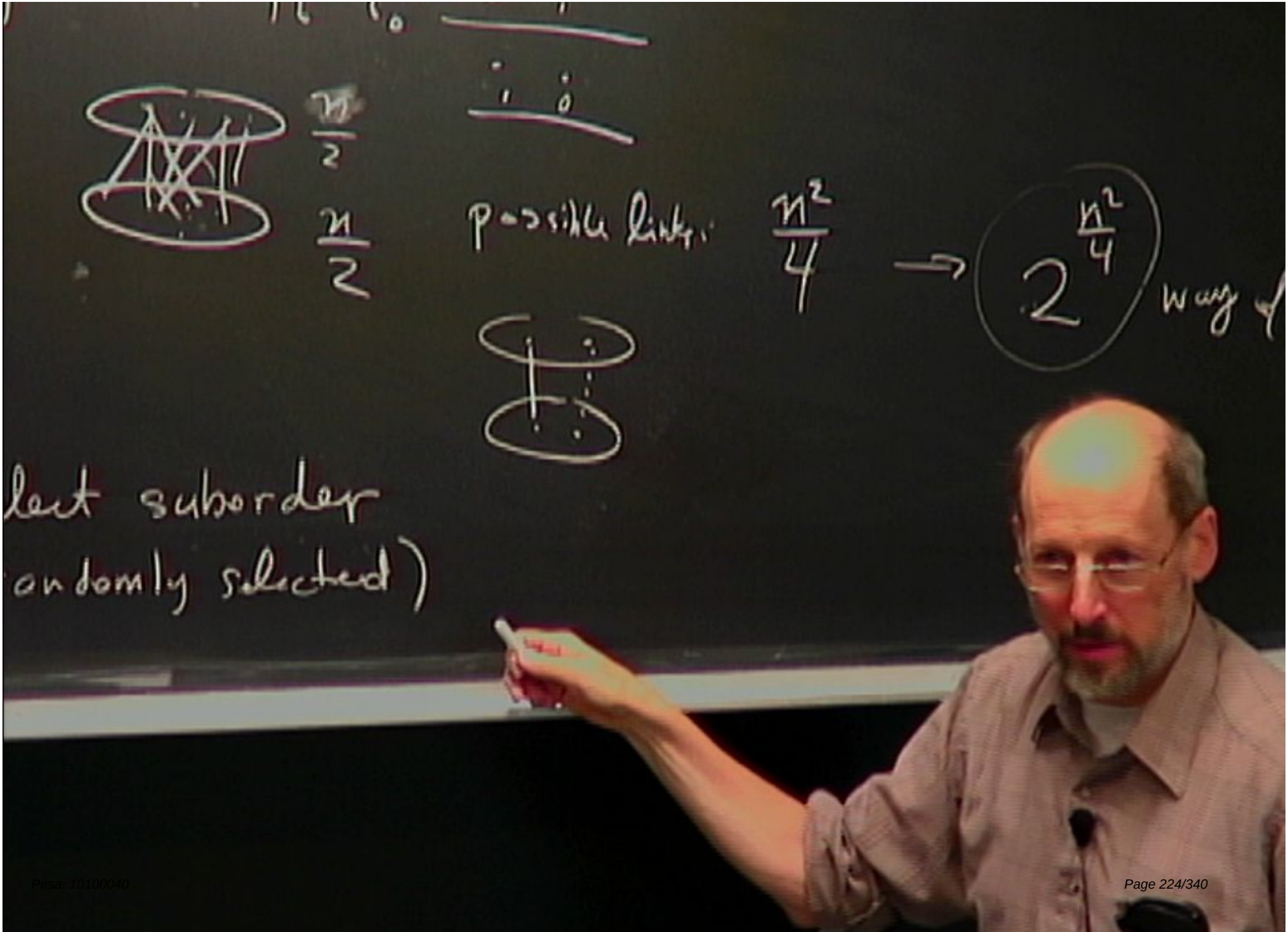
Myrheim-Meyer Dimension (for interval)

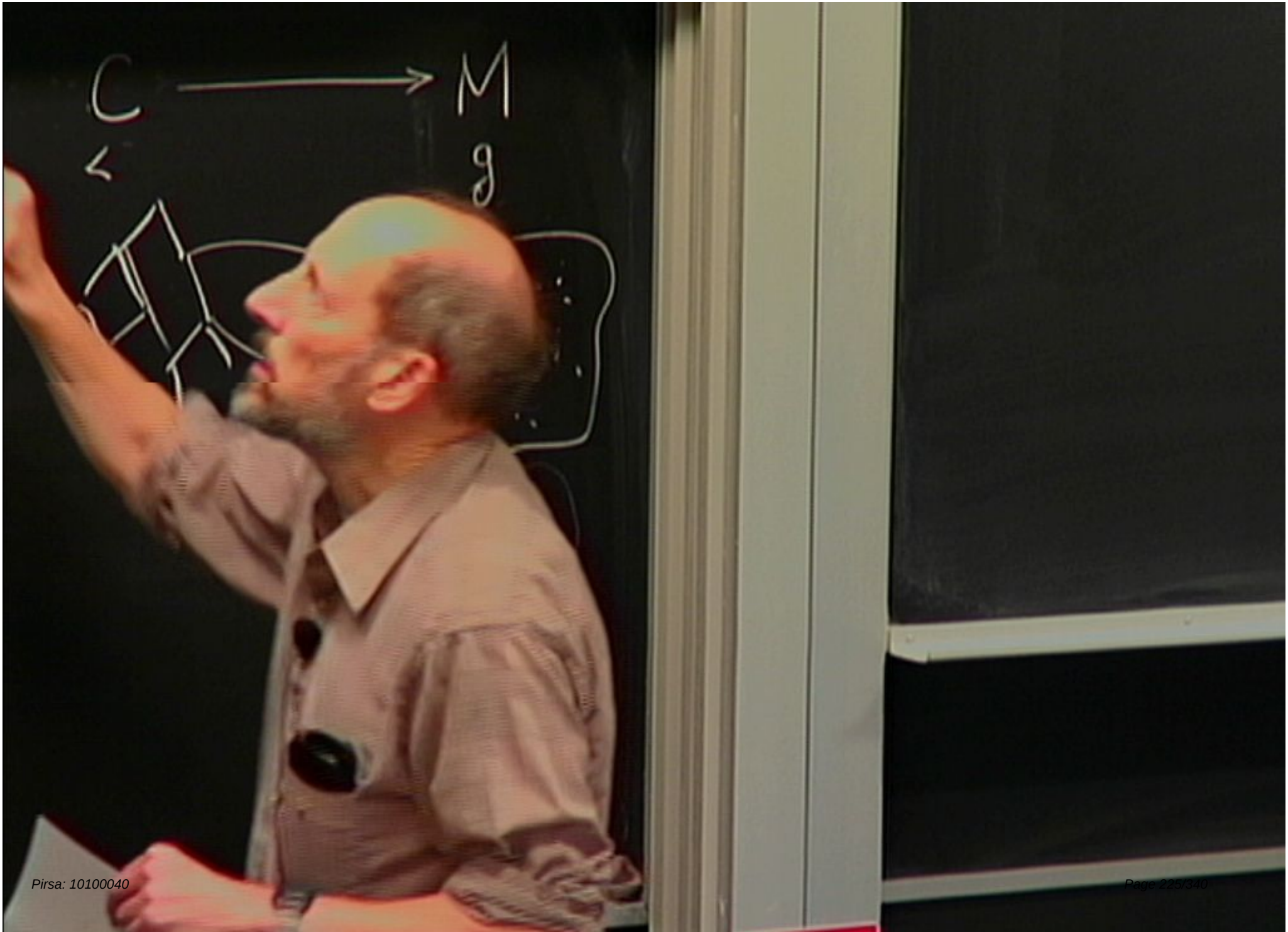
M^d

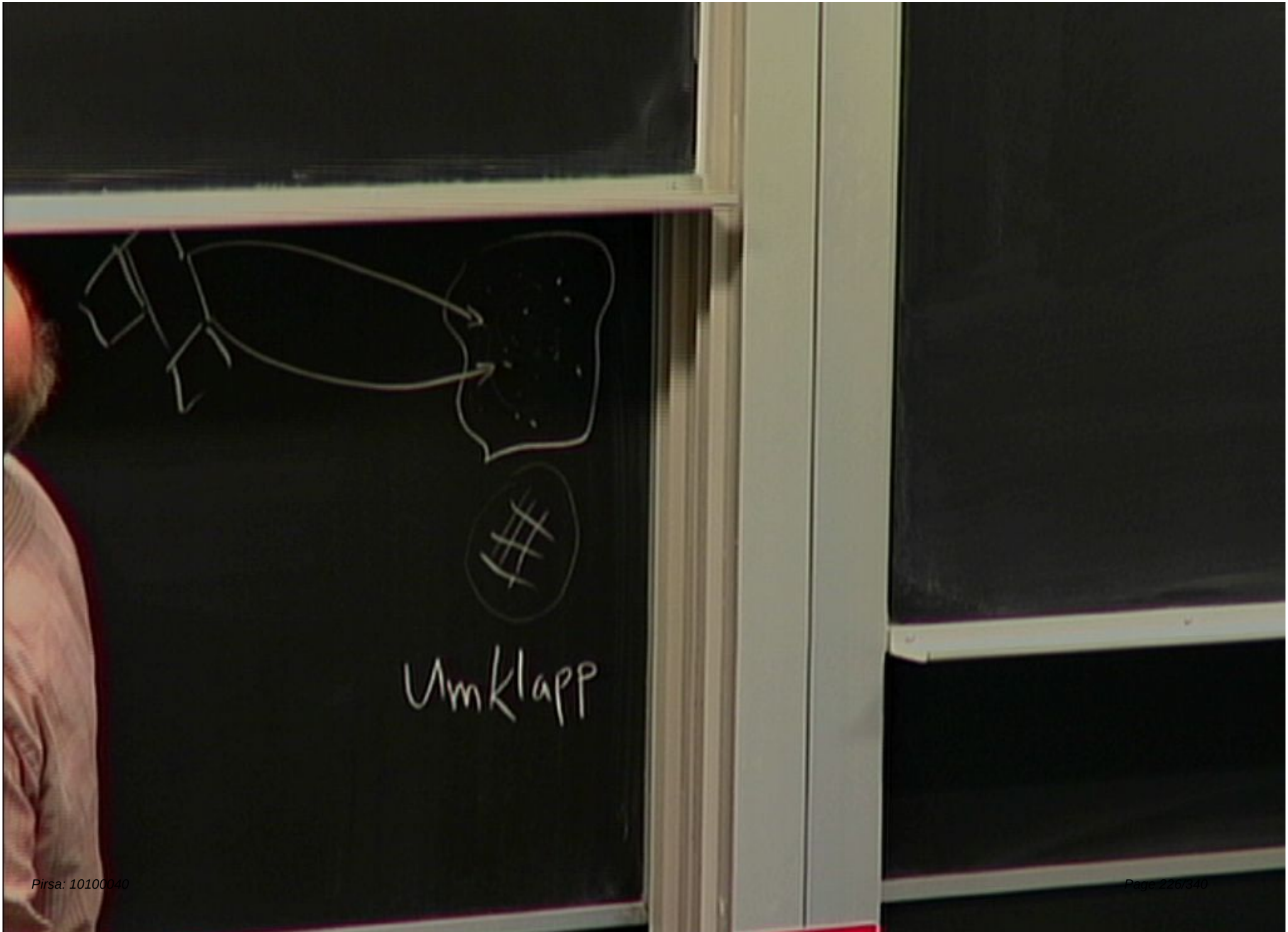


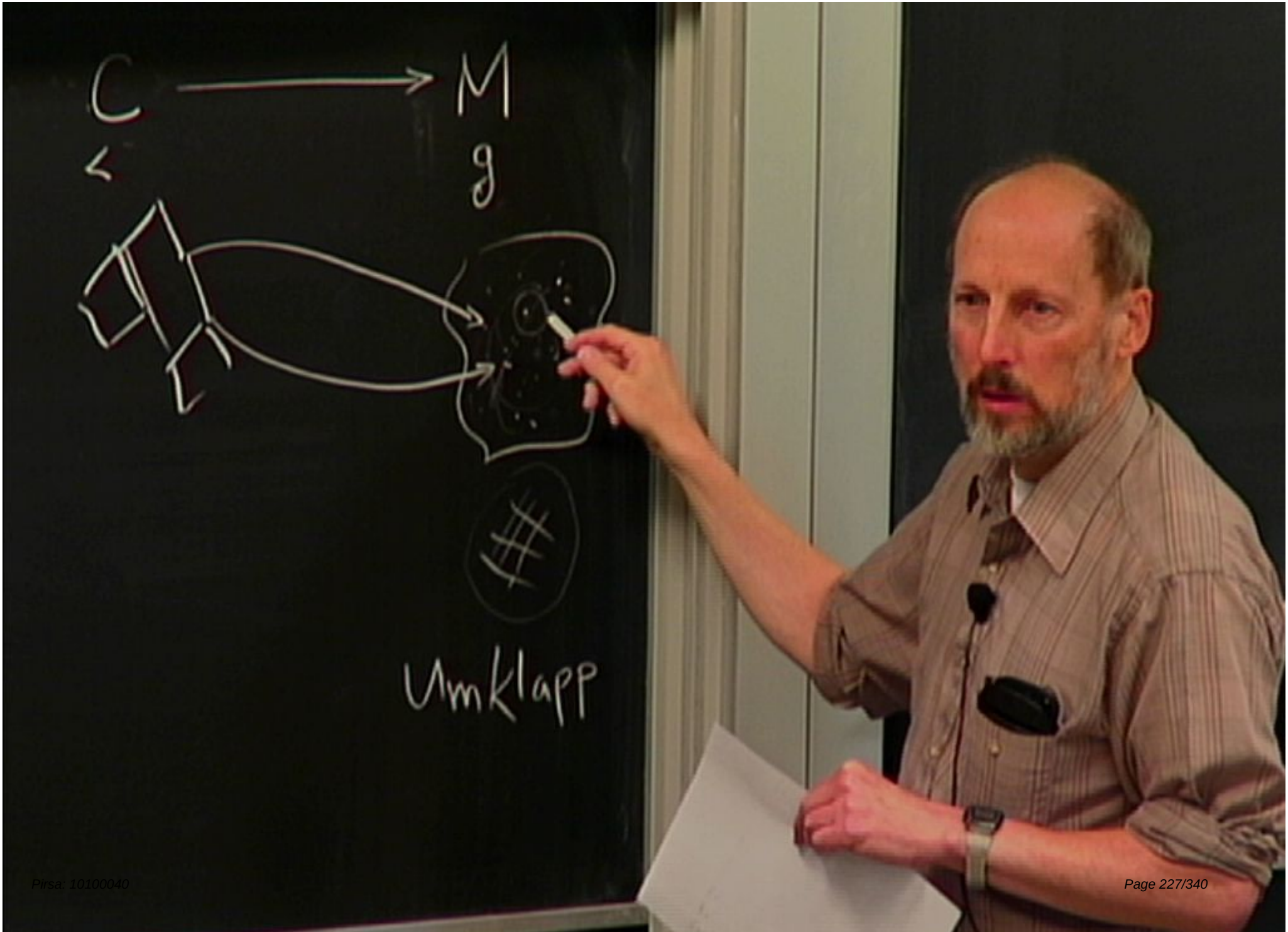

C
M
g
Umklapp

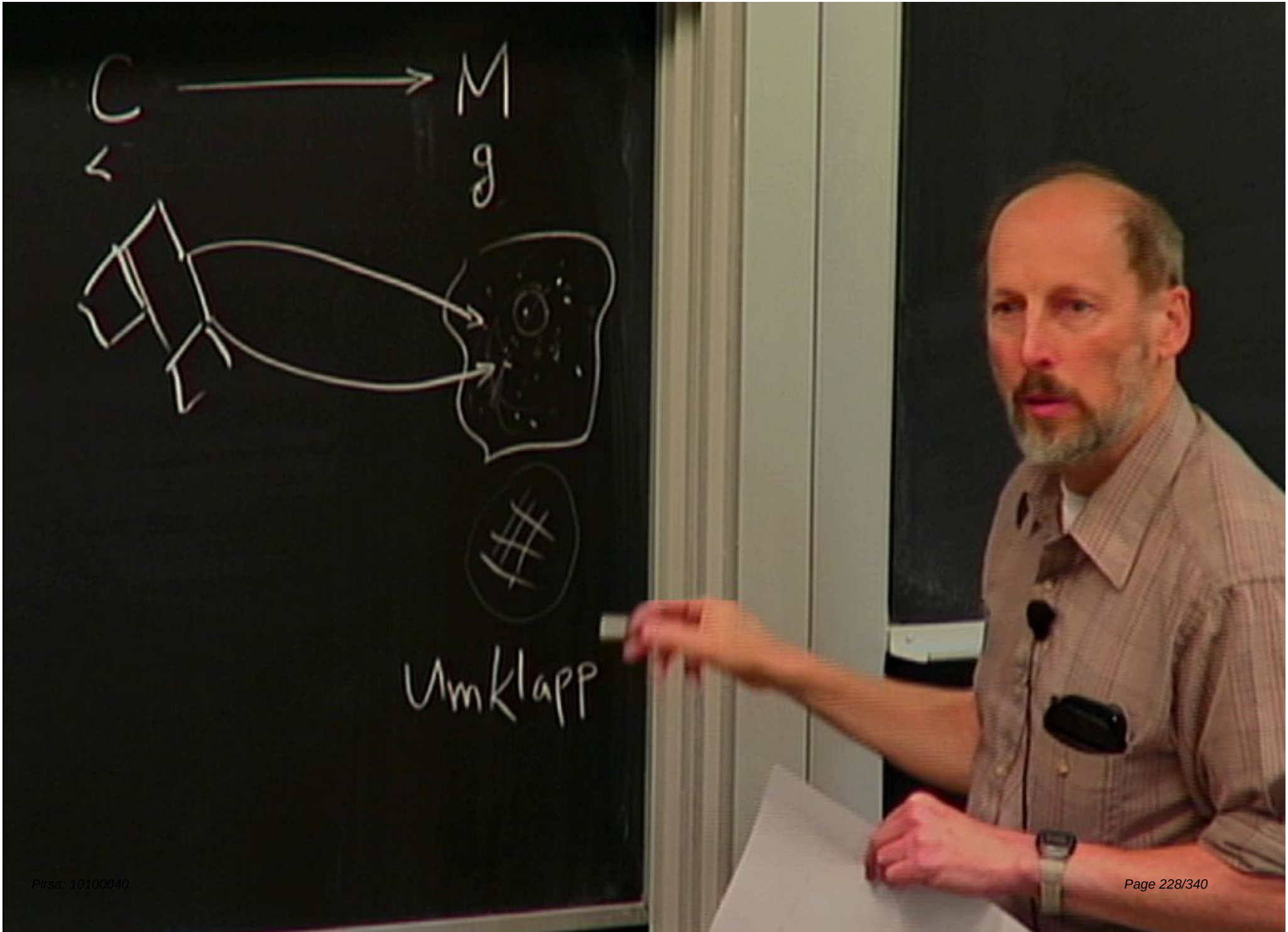

C
M
g
Umklapp

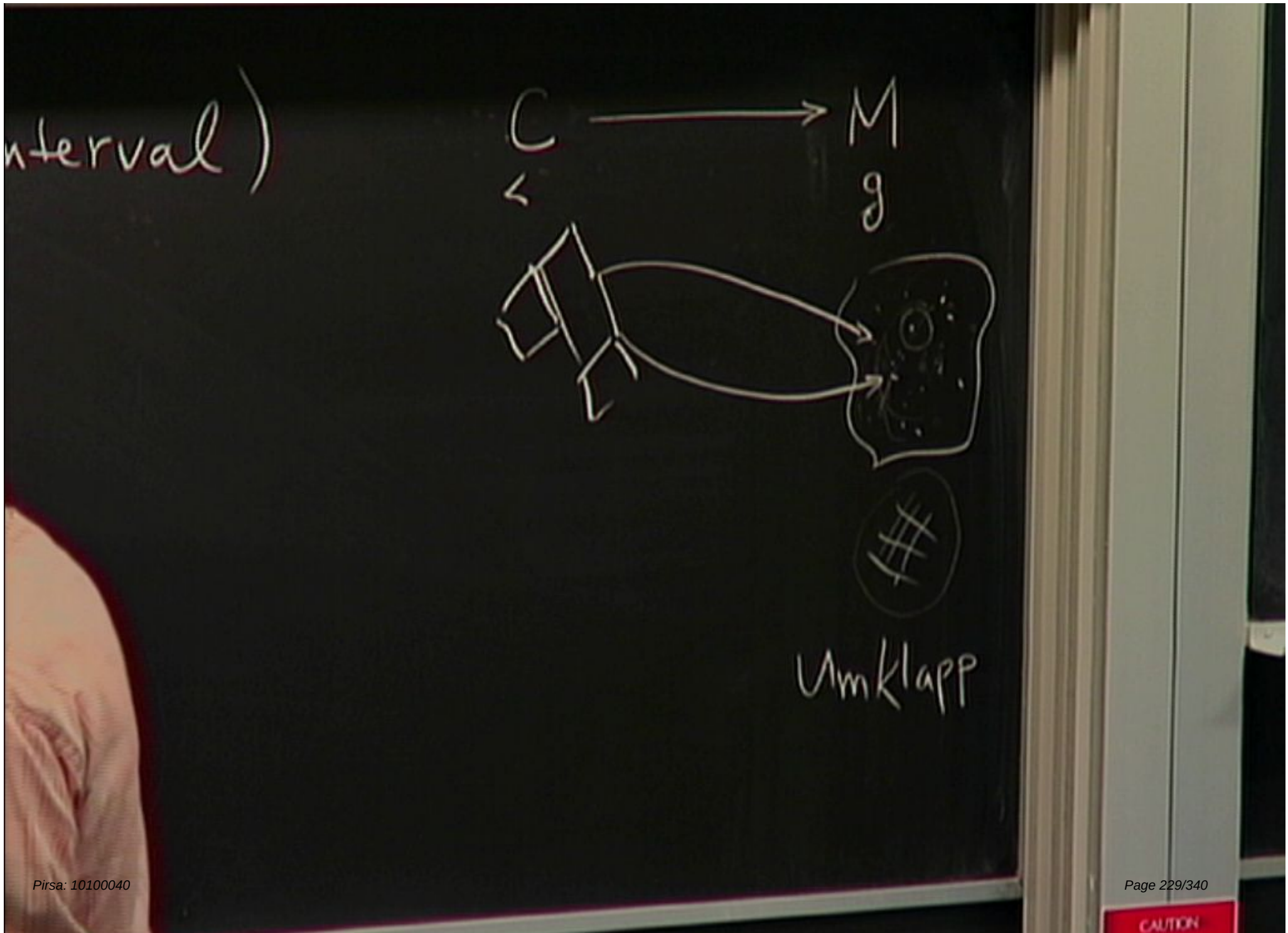

$\frac{n}{2}$
$\frac{n}{2}$
possible links: $\frac{n^2}{4}$
$2^{\frac{n^2}{4}}$
ways
lect suborder
andomly selected)

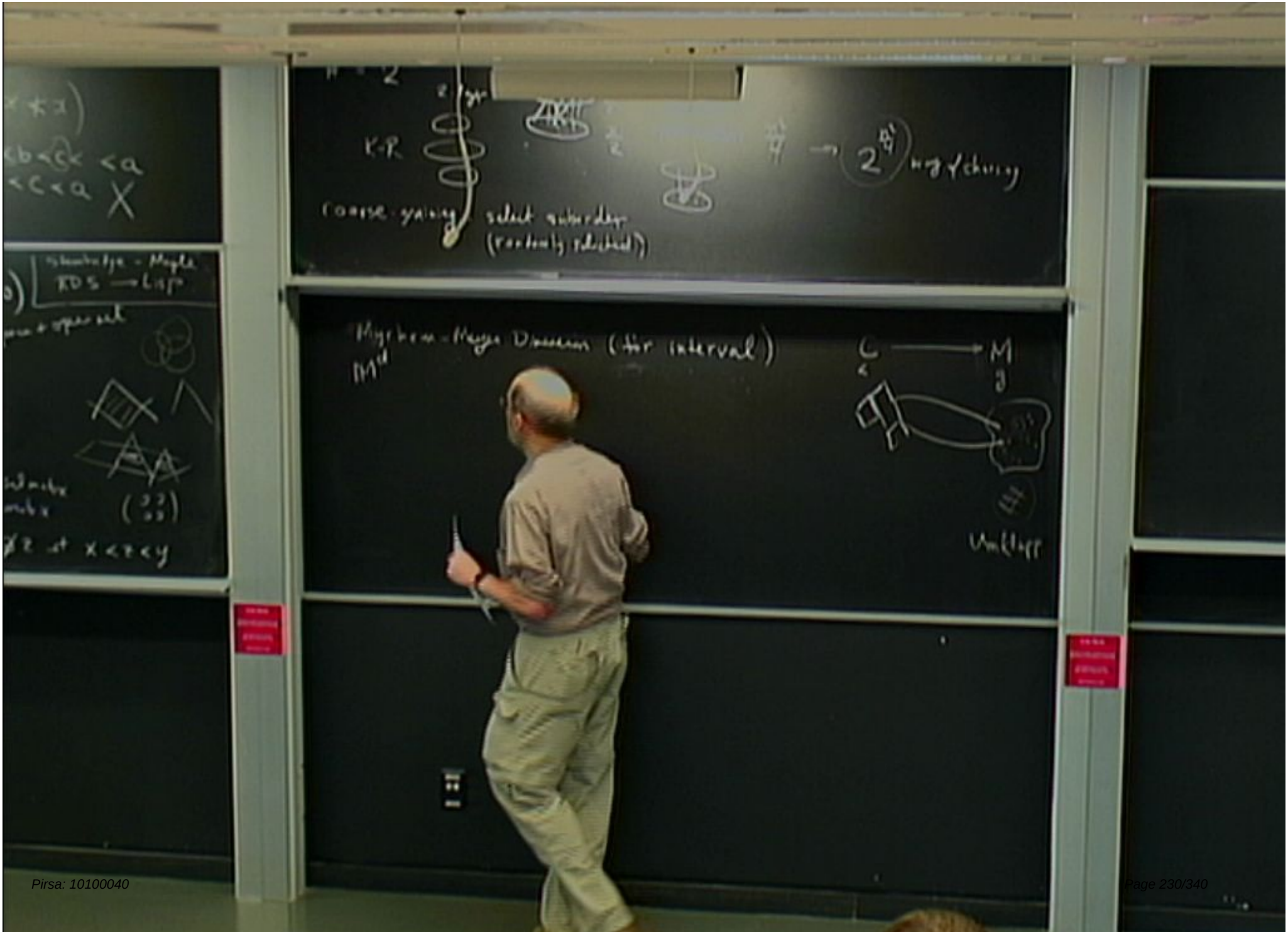

C
M
g


Umklapp


C
M
g
Umklapp


C
M
g
Umklapp


nterval)
C
M
g
Umklapp


K-P.
coarse-graining
select suborders
(randomly relabeled)
Myrheim-Meyer Dimension (thin interval)
M^d
C
M
J
Wolfgr

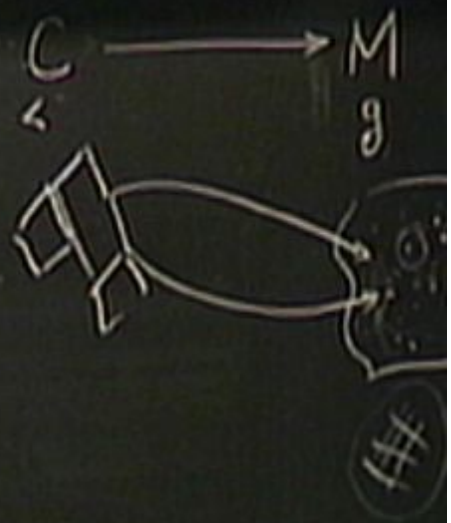
(Pawlowsky Stuckel)

Myrheim-Meyer Dimension (for interval)

M^d



"fractal"



Umklapp

Myrheim-Meyer Dimension (for interval)

M^d



"fractal"



Myrheim-Meyer Dimension (for interval)

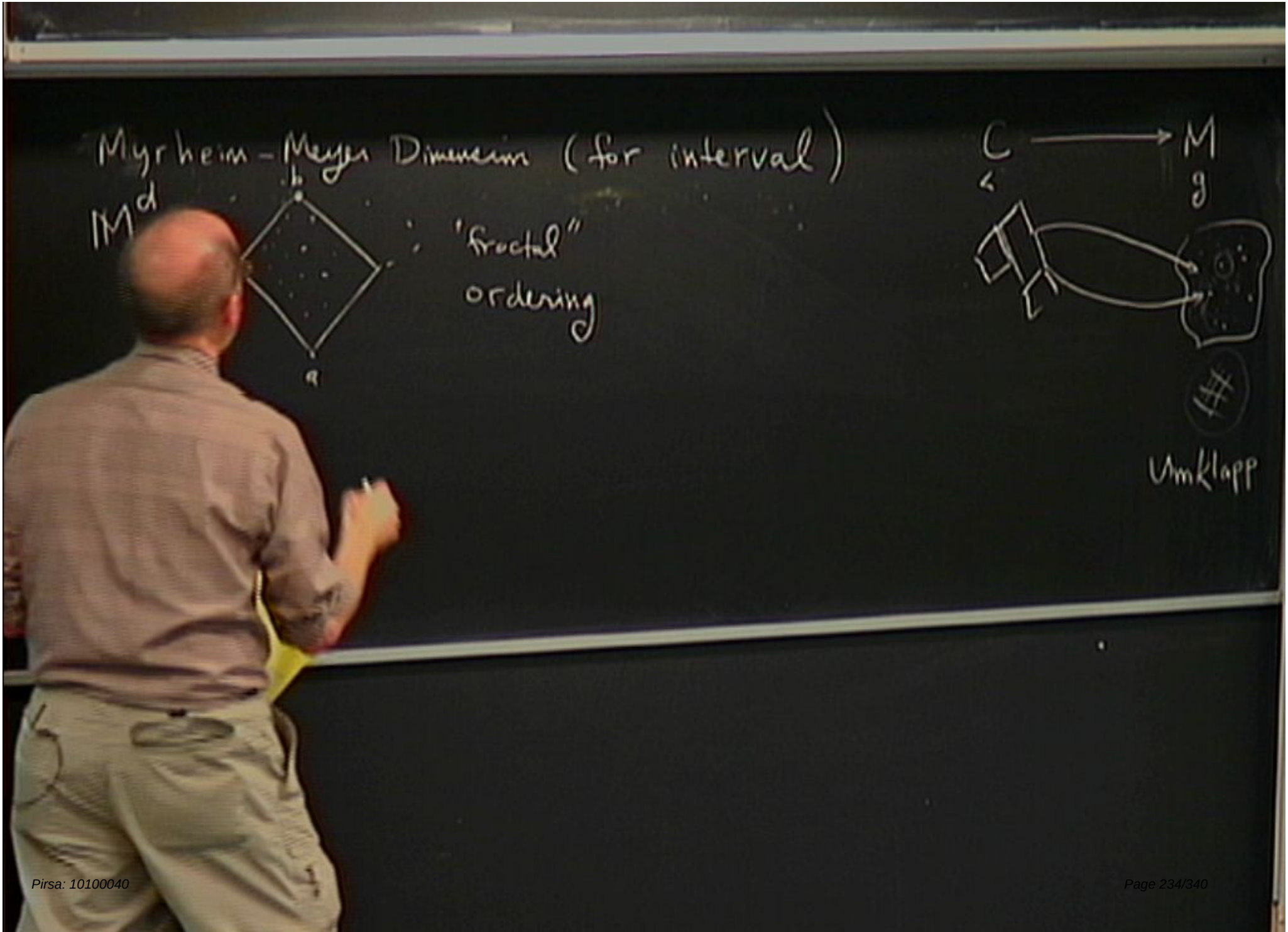
M^d

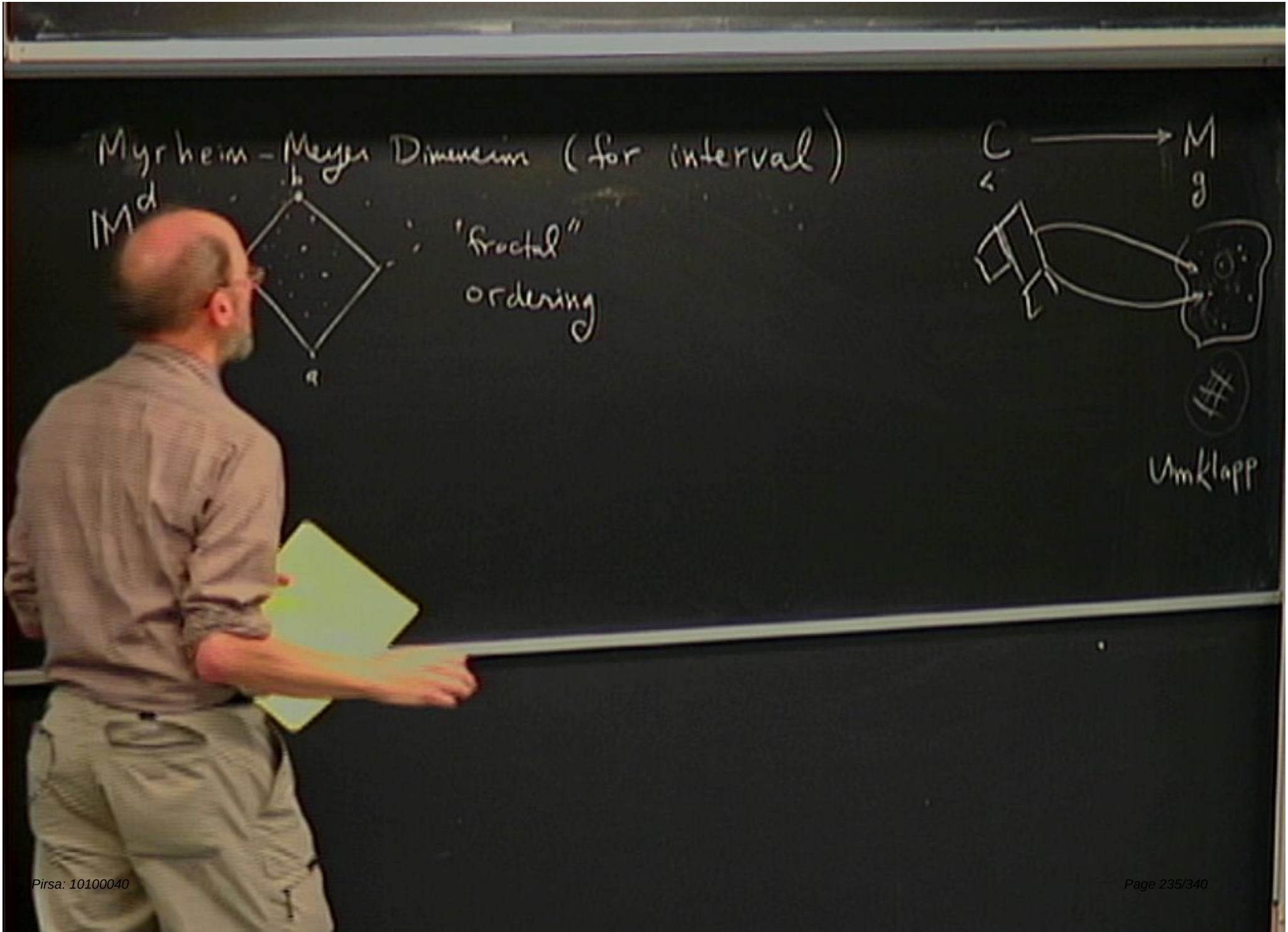


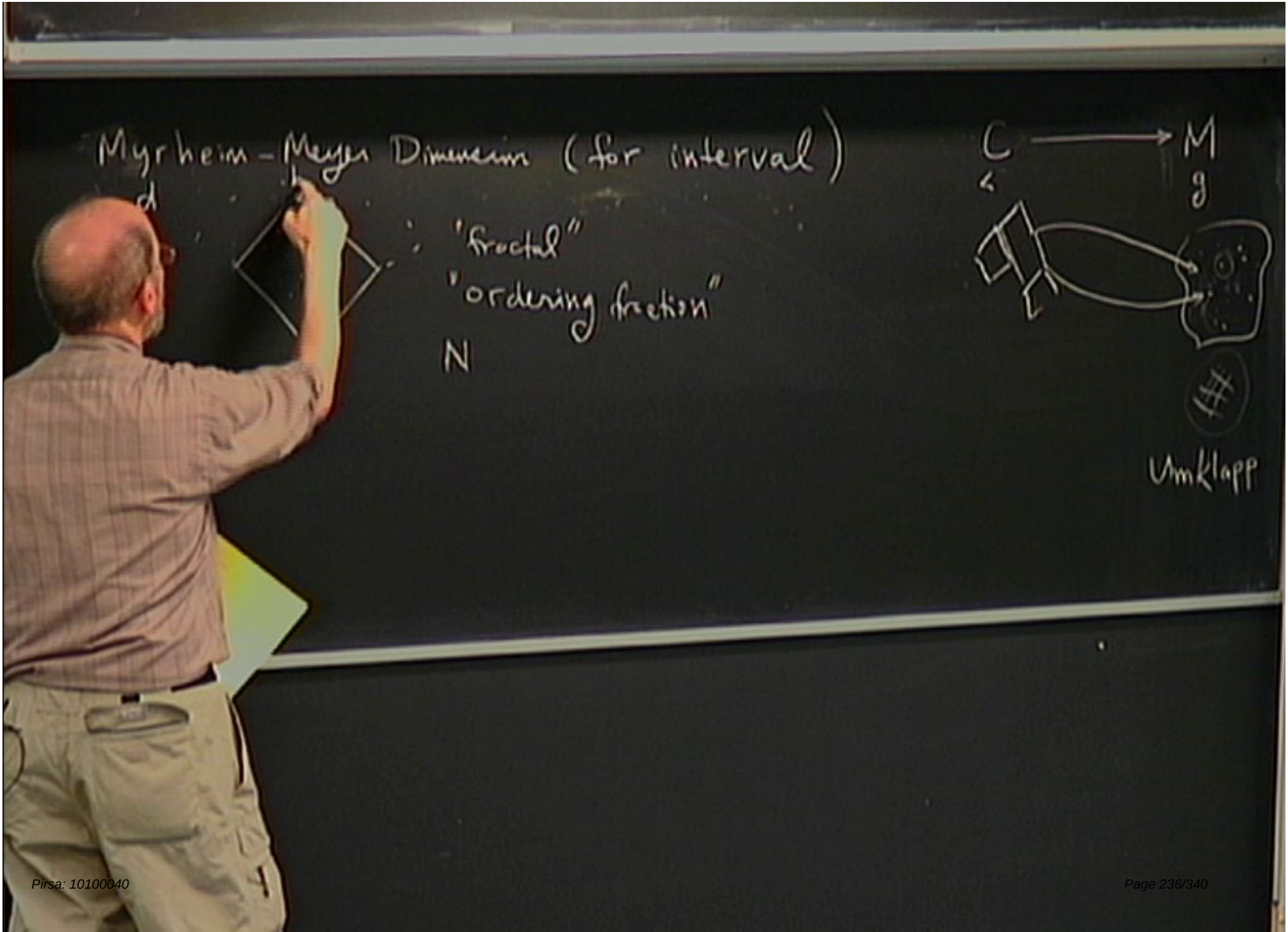
: "fractal"



Umklapp


Myrheim-Meyer Dimension (for interval)
M^d
"fractal" ordering
C
M
g
Umklapp


Myrheim-Meyer Dimension (for interval)
"fractal" ordering
Umklapp


Myrheim - Meyer Dimension (for interval)
d
: "fractal"
"ordering fraction"
N
C
M
g
Umklapp

Myrheim - Meyer Dimension (for interval)

d



"fractal"

"ordering fraction"

N

C → M

g

Umklapp

Myrheim-Meyer Dimension (for interval)

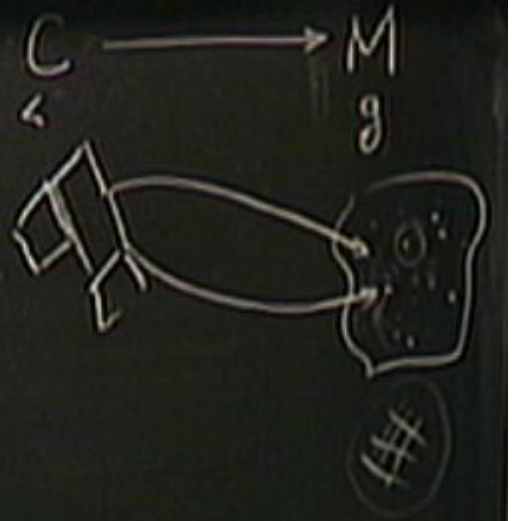
M^d



: "fractal"

"ordering fraction"

N



Umklapp

Myrheim-Meyer Dimension (for interval)

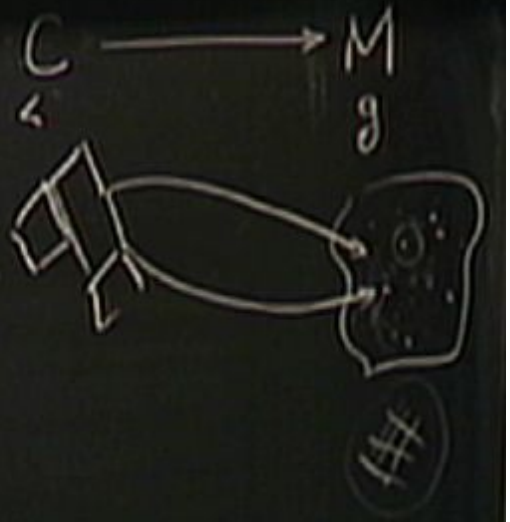


"fractal"

"ordering fraction"

$$\binom{N}{2} = \frac{N(N-1)}{2}$$

R relations



Umklapp

Myrheim-Meyer Dimension (for interval)

M^d



: "fractal"

"ordering fraction"

$\binom{N}{2} = \frac{N(N-1)}{2}$

R relations





Umklapp

Myrheim-Meyer Dimension (for interval)

M^d

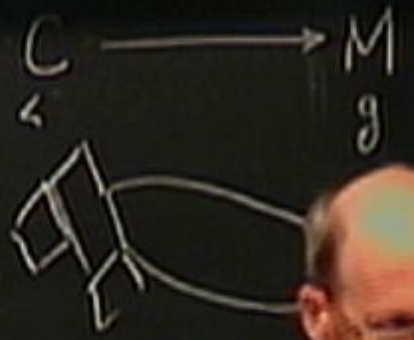


"fractal"

"ordering fraction"

$$\binom{N}{2} = \frac{N(N-1)}{2} \qquad \frac{R}{\frac{N(N-1)}{2}}$$

R relations



(randomly selected)

Myrheim-Meyer Dimension (for [illegible]erval)

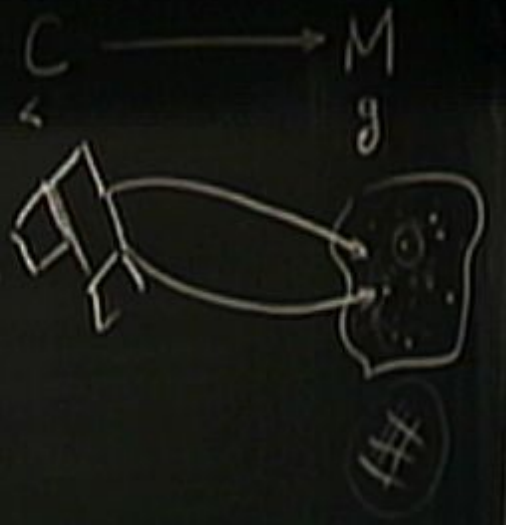
M^d



"fractal"
"ordering fraction"

$$\binom{N}{2} = \frac{N(N-1)}{2}$$

R relations



Umklapp



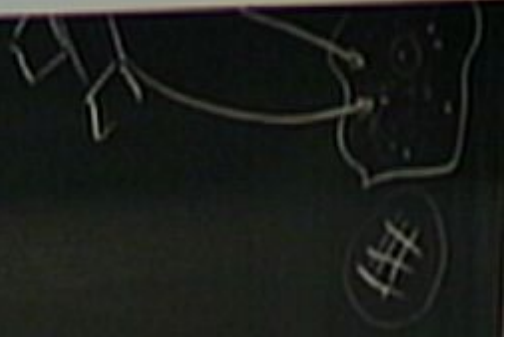
Coorse-graining : select subordering
(randomly selected)



"ordering fraction"

$$\binom{N}{2} = \frac{N(N-1)}{2} \qquad \frac{R}{\frac{N(N-1)}{2}} = \text{order}$$

R relations



Umklapp

Myrheim-Meyer Dimension (for interval)

M^d

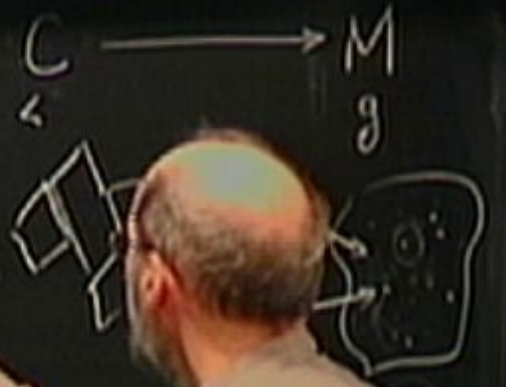


: "fractal"

"ordering fraction"

$\binom{N}{2} = \frac{N(N-1)}{2}$ $\quad \approx \frac{R}{\frac{N(N-1)}{2}}$

R relations



Myrheim-Meyer Dimension (for interval)

M^d



"fractal"

"ordering fraction"

$\binom{N}{2} = \frac{N(N-1)}{2} \approx \frac{R}{\frac{N(N-1)}{2}}$

R relations



Myrheim-Meyer Dimension (for interval)

M^d

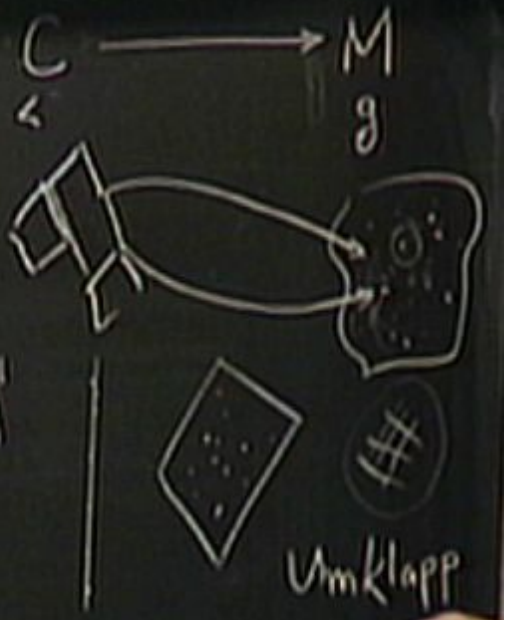


"fractal"

"ordering fraction"

$\binom{N}{2} = \frac{N(N-1)}{2} \approx \frac{R}{\frac{N(N-1)}{2}}$

R relations



Myrheim-Meyer Dimension (for interval)

M^d

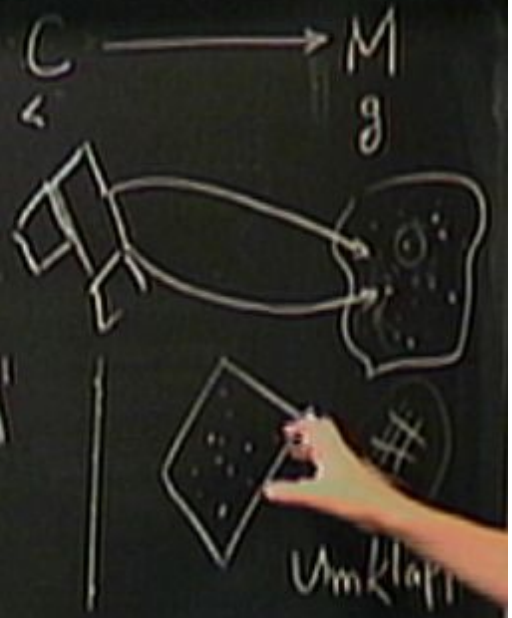


: "fractal"
"ordering fraction"

$\binom{N}{2} = \frac{N(N-1)}{2} \approx \frac{R}{\frac{N(N-1)}{2}}$

R relations

M'



Umklapp

Myrheim - Meyer Dimension (for interval)

M^d



"fractal"

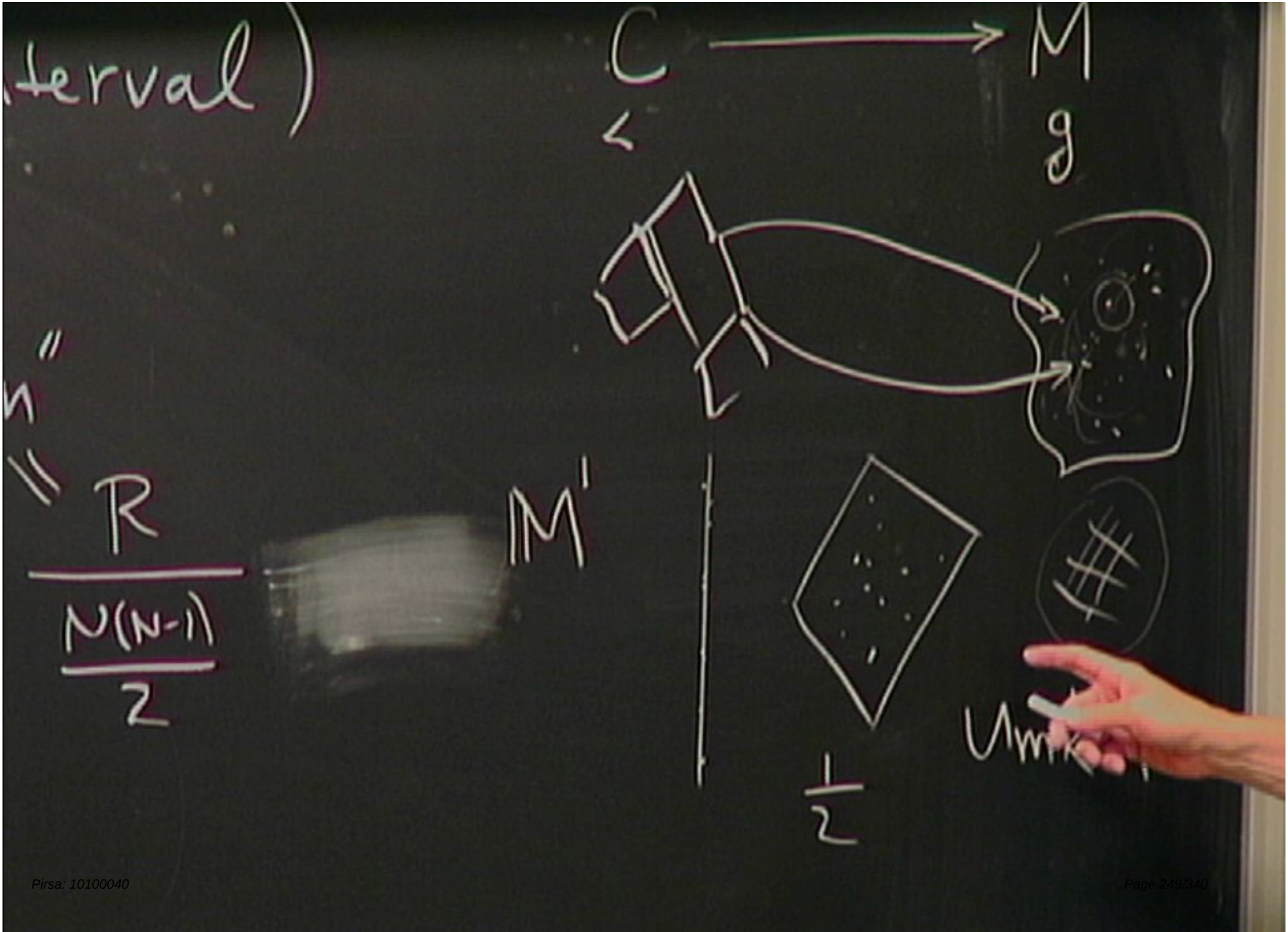
"ordering fraction"

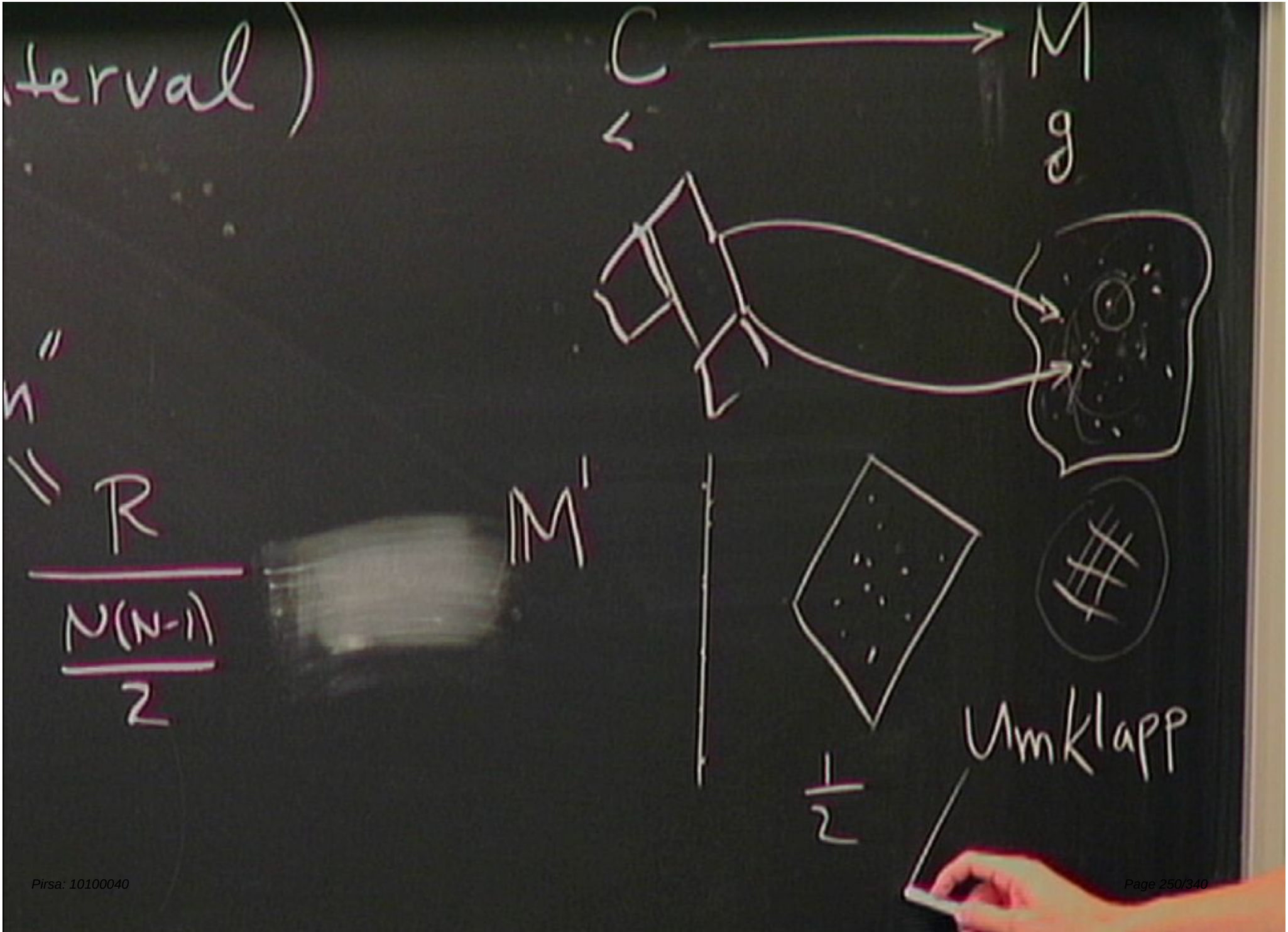
$\binom{N}{2} = \frac{N(N-1)}{2} \approx \frac{R}{\frac{N(N-1)}{2}}$

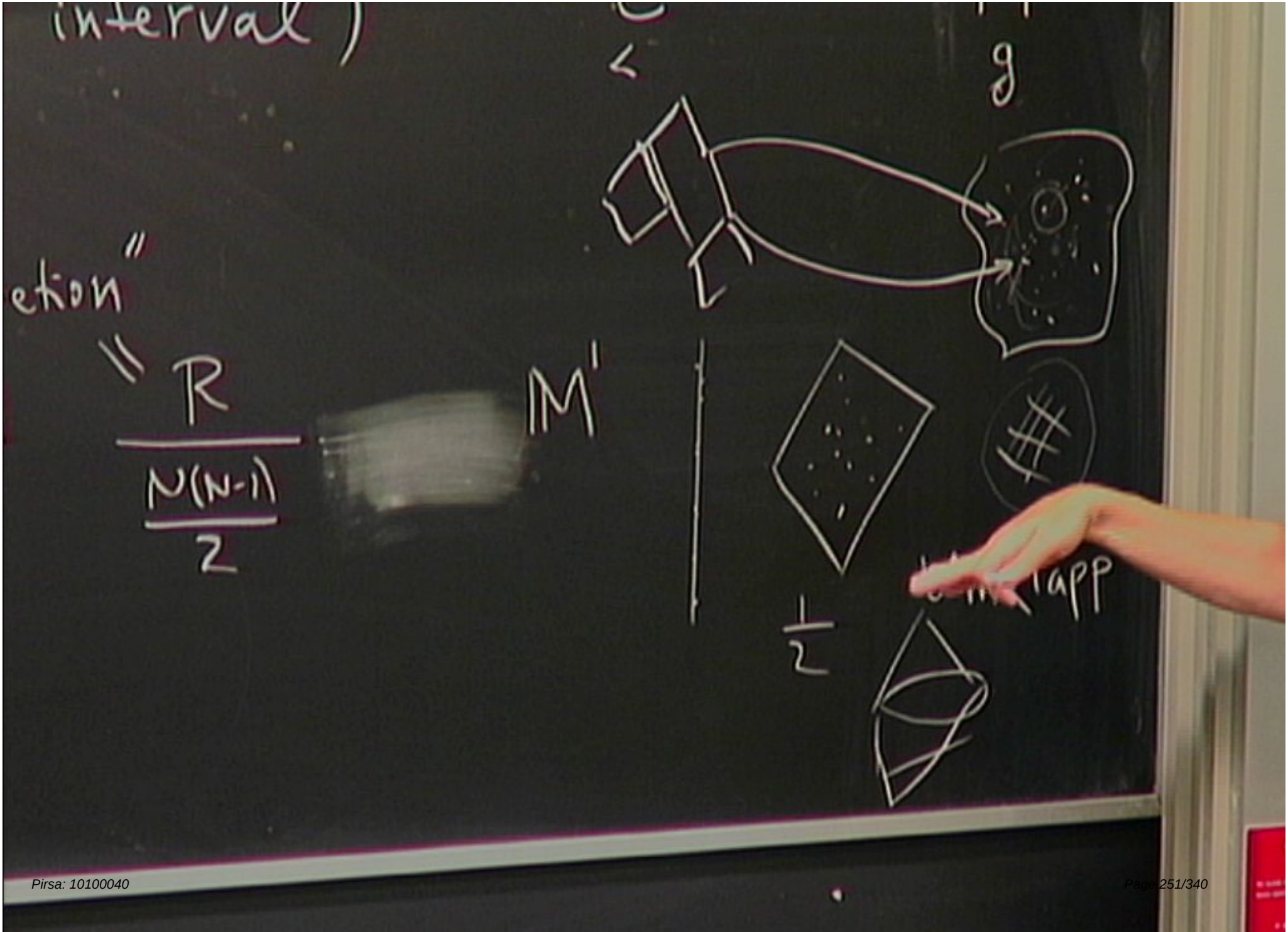
R relations

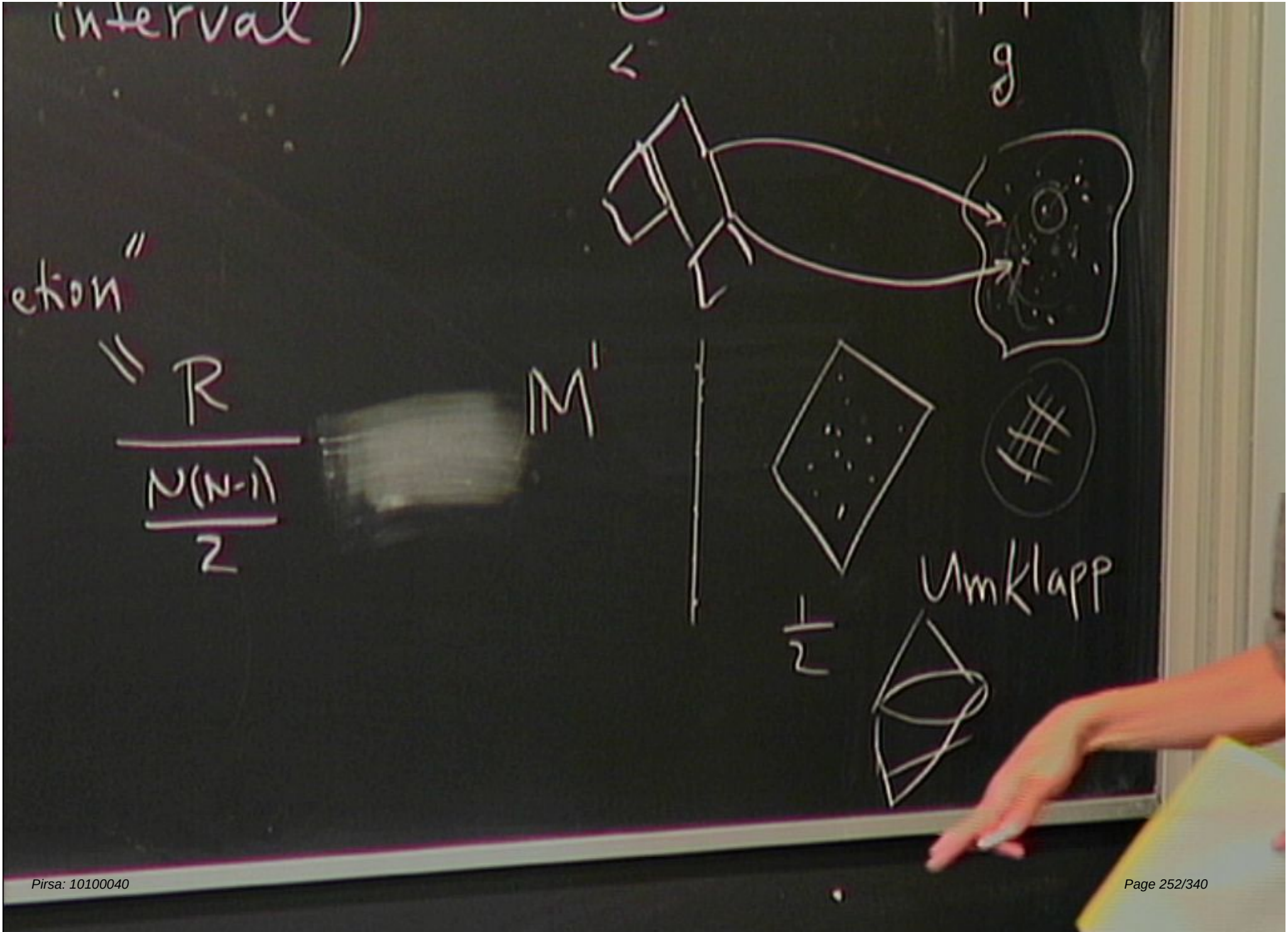
M'

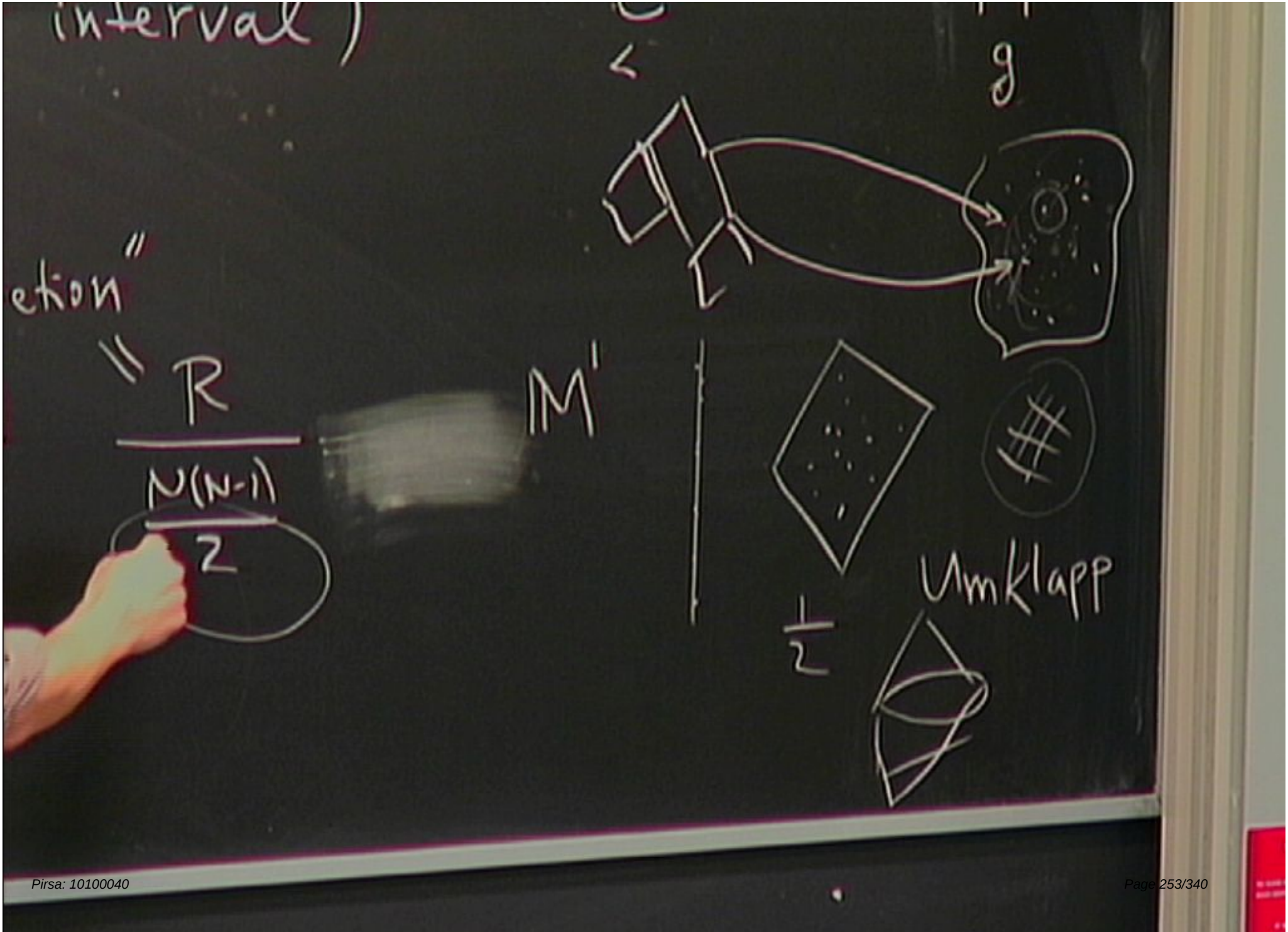



terval)
C
M
g
M'
R
N(N-1)
2
1
2


terval)
C
M
g
R
N(N-1)
2
M'
1
2
Umklapp


interval)
ekon"
R
N(N-1)
2
M'
g
1
2


interval)
ction"
R
N(N-1)
2
M'
g
Umklapp
1
2


interval)
ekon"
R
N(N-1)
2
M'
g
Umklapp
1
2

rheim – Meyer Dimension (for interval)



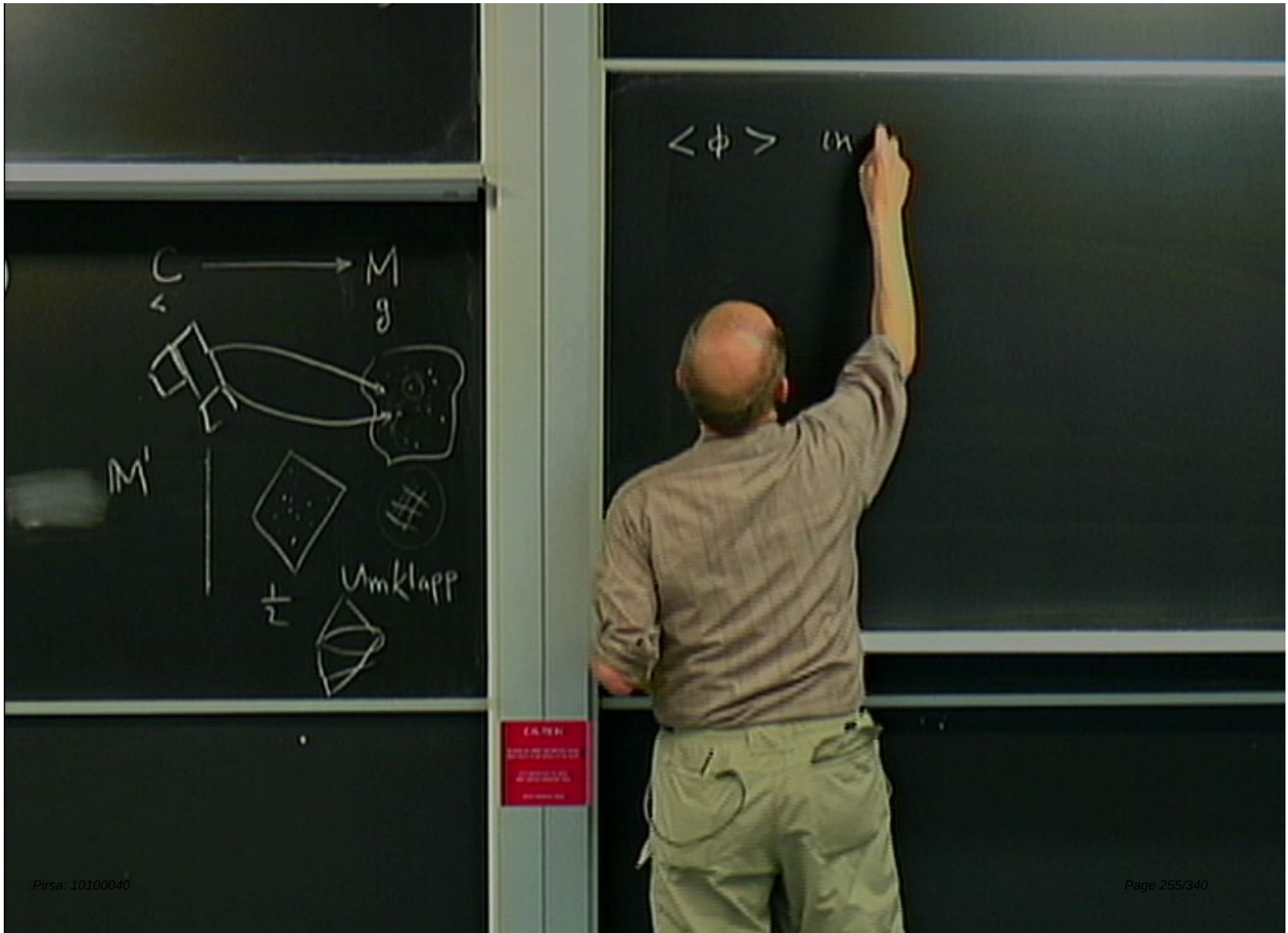
"fractal"

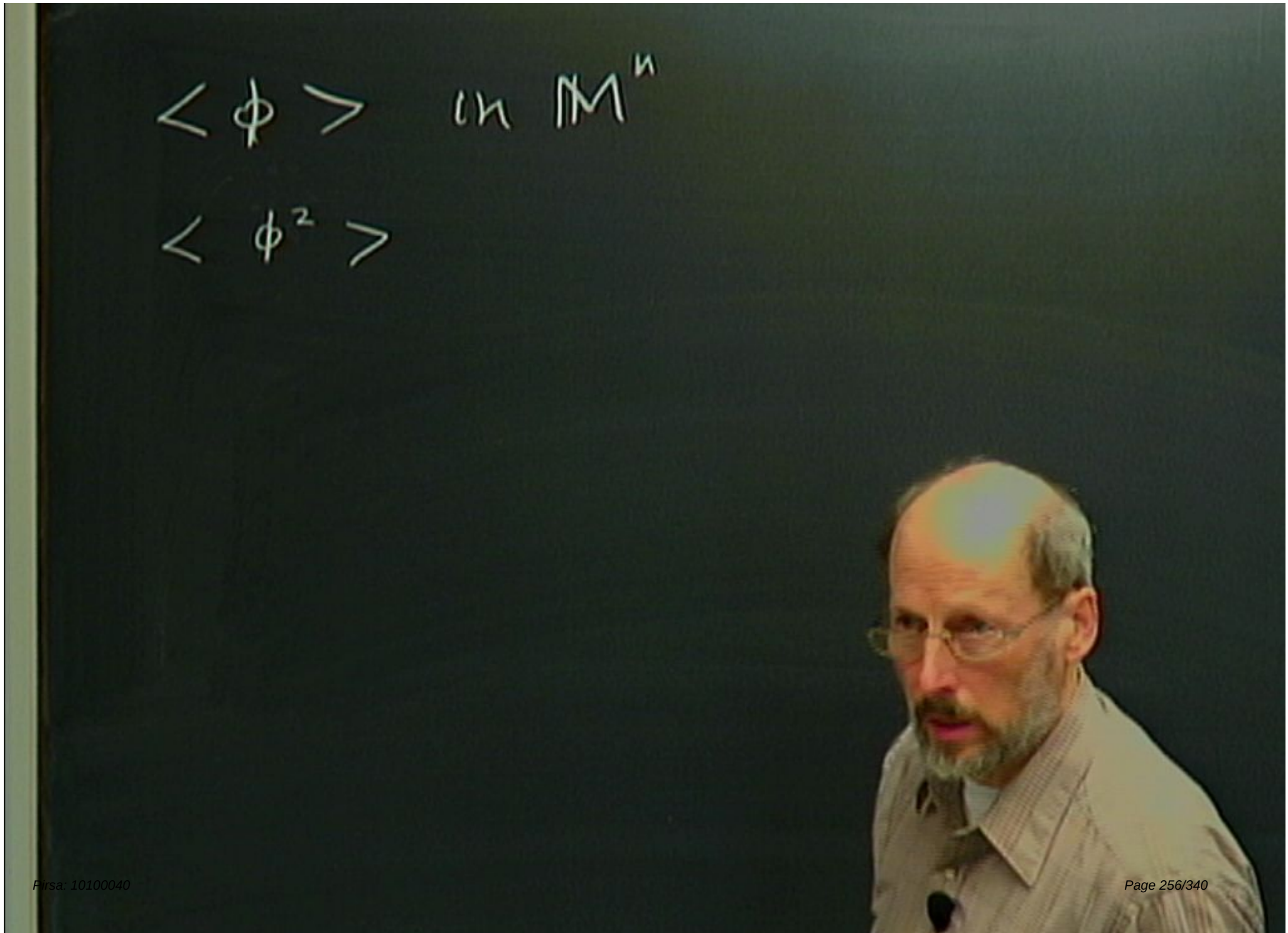
"ordering fraction"

$$\binom{N}{2} = \frac{N(N-1)}{2} \qquad \frac{R}{\frac{N(N-1)}{2}}$$

R relations

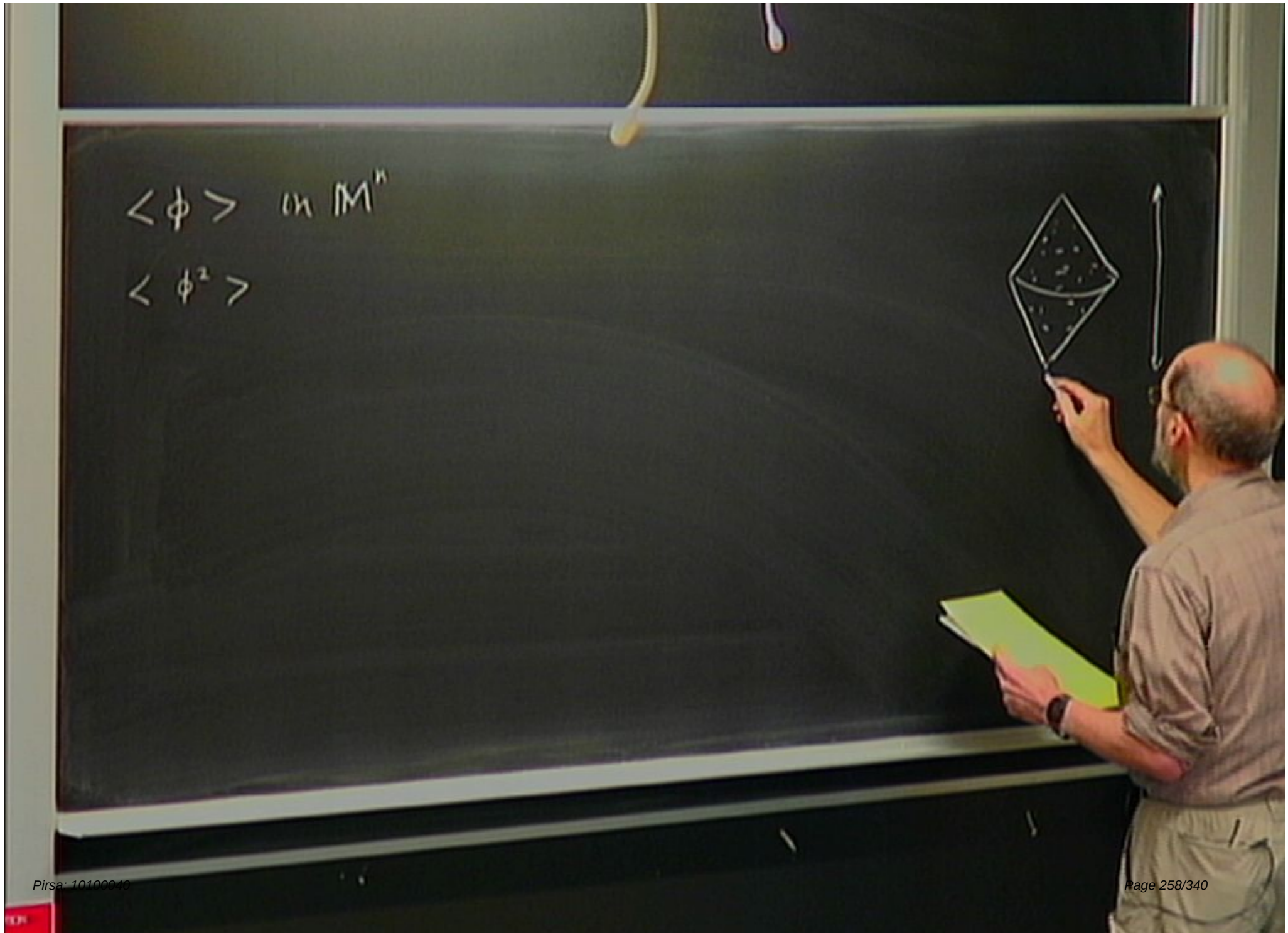
$$\phi = \frac{R}{N(N-1)}$$


C
M
g
M'
Umklapp


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$

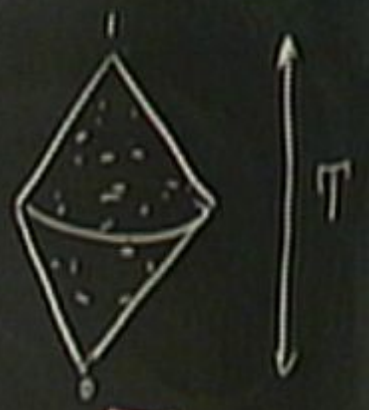
$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$


$\langle \phi \rangle$ on $\mathbb{M}^n$
$\langle \phi^2 \rangle$

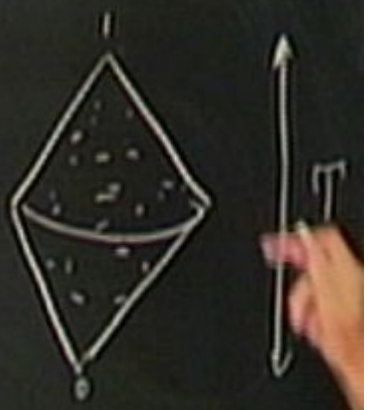
$\langle \phi \rangle$ in $\mathbb{M}^n$

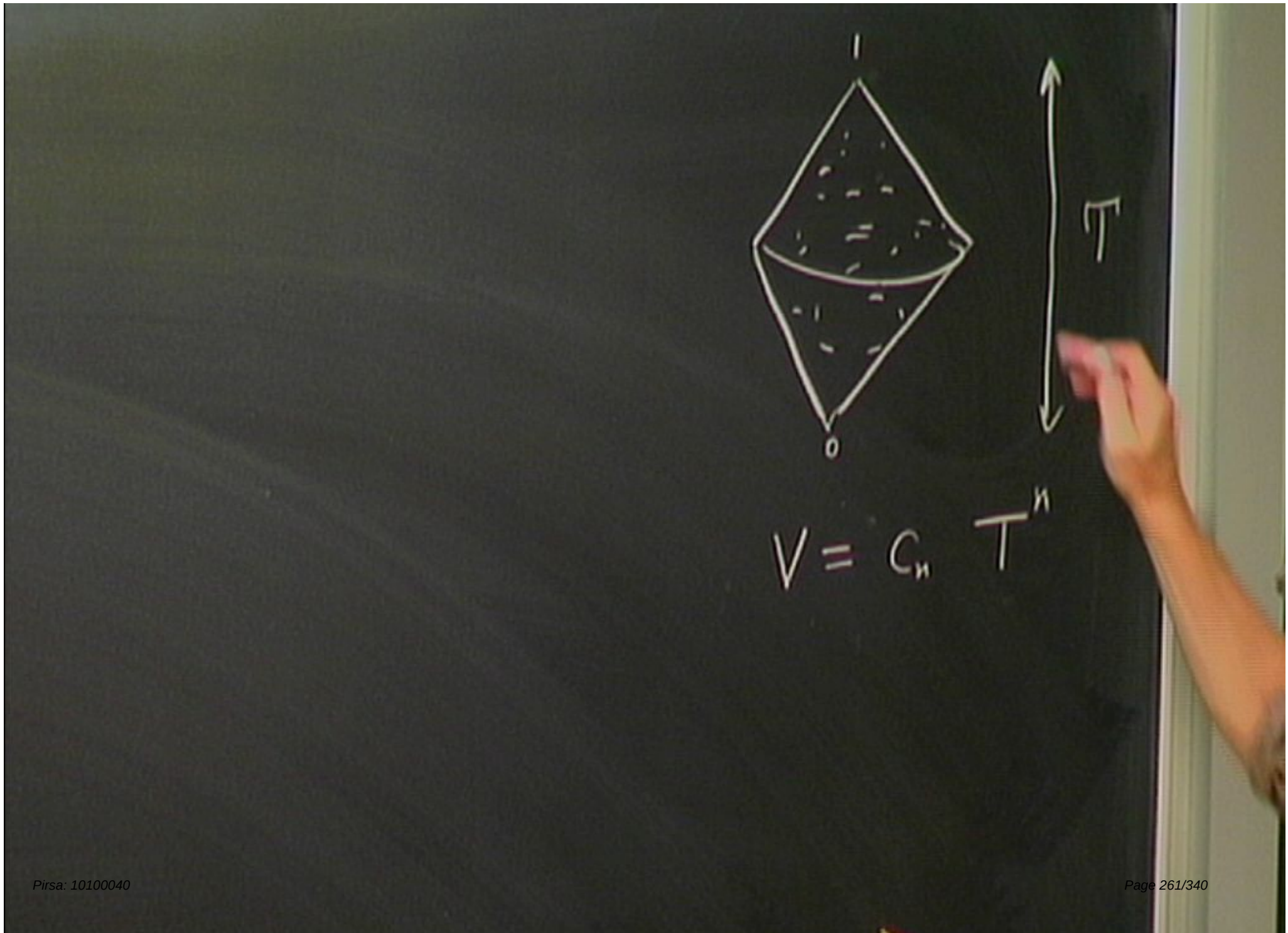
$\langle \phi^2 \rangle$

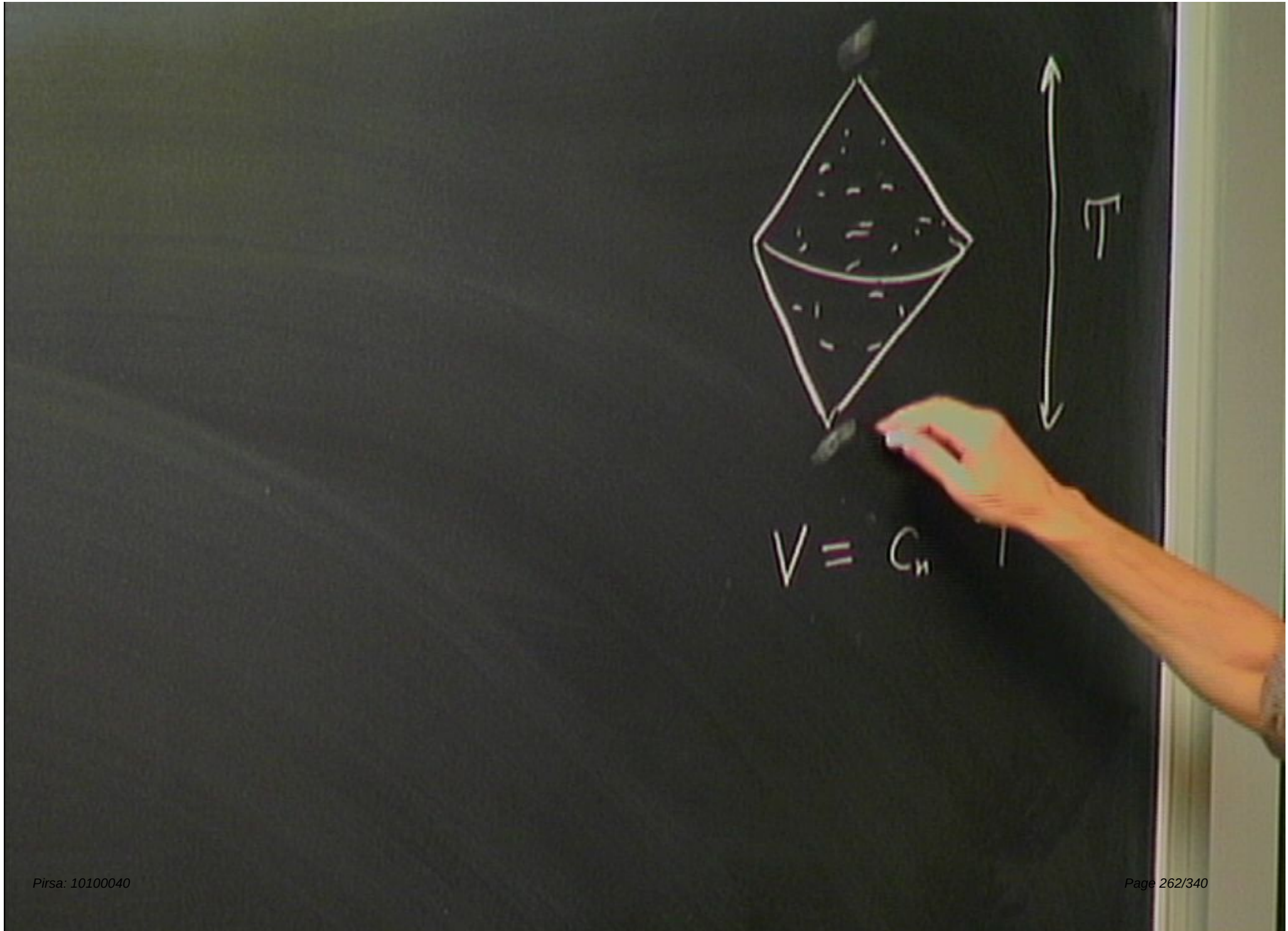


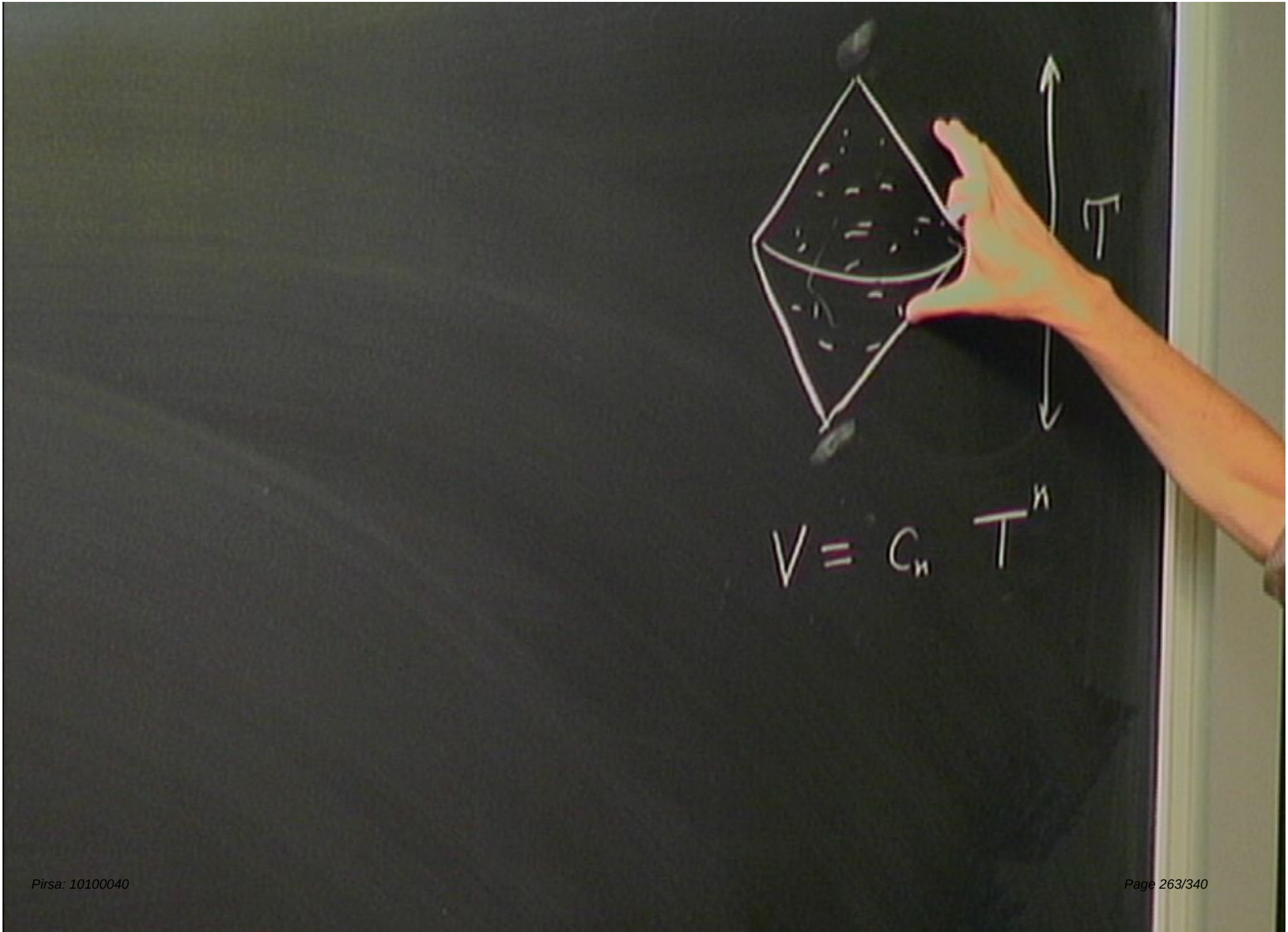
$\langle \phi \rangle$ in $\mathbb{M}^n$

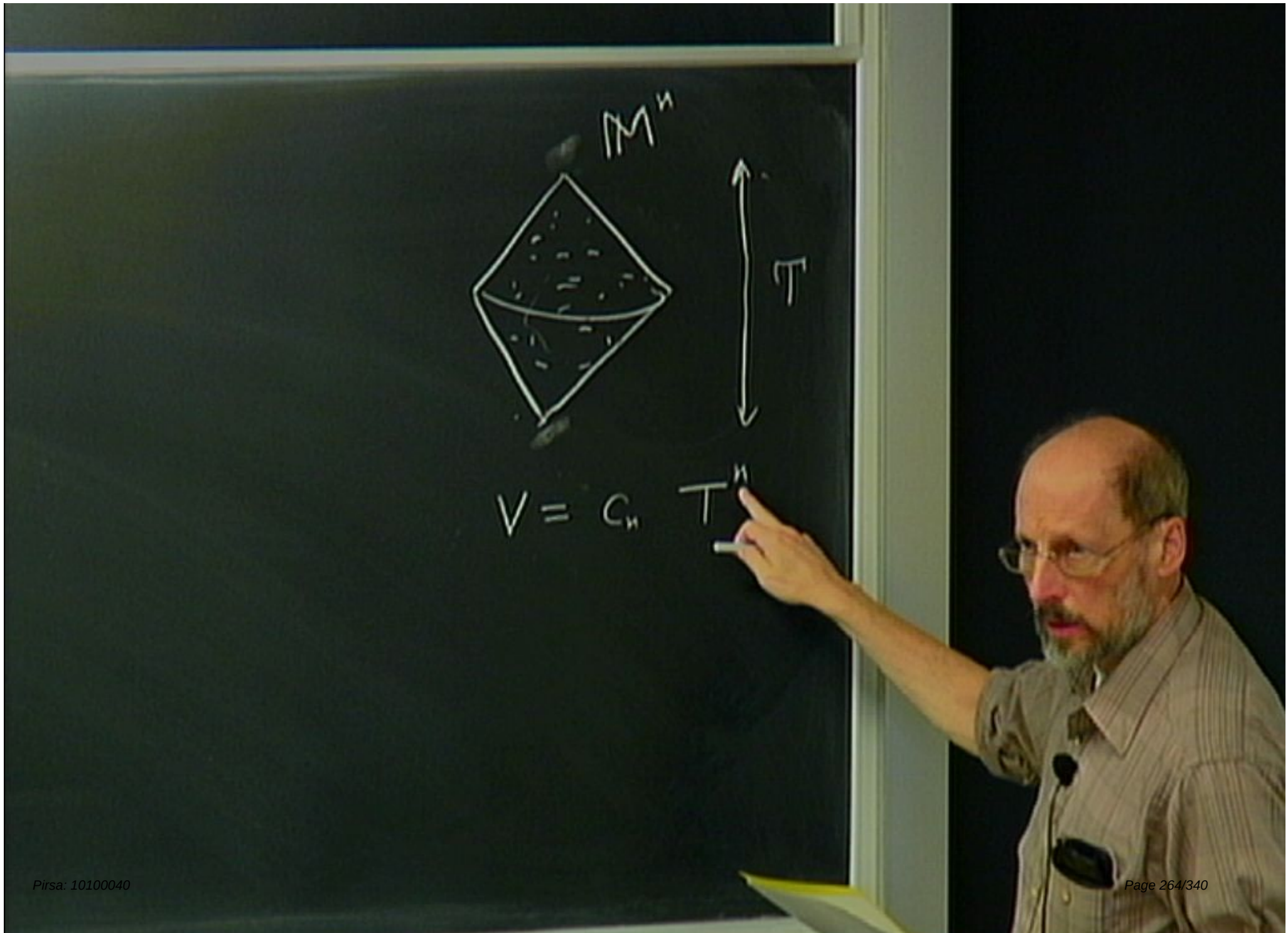
$\langle \phi^2 \rangle$




1
0
T
$V = C_n T^n$


T
V = Cn


T
$V = C_n T^n$


M^n
T
$V = C_n \; T^n$

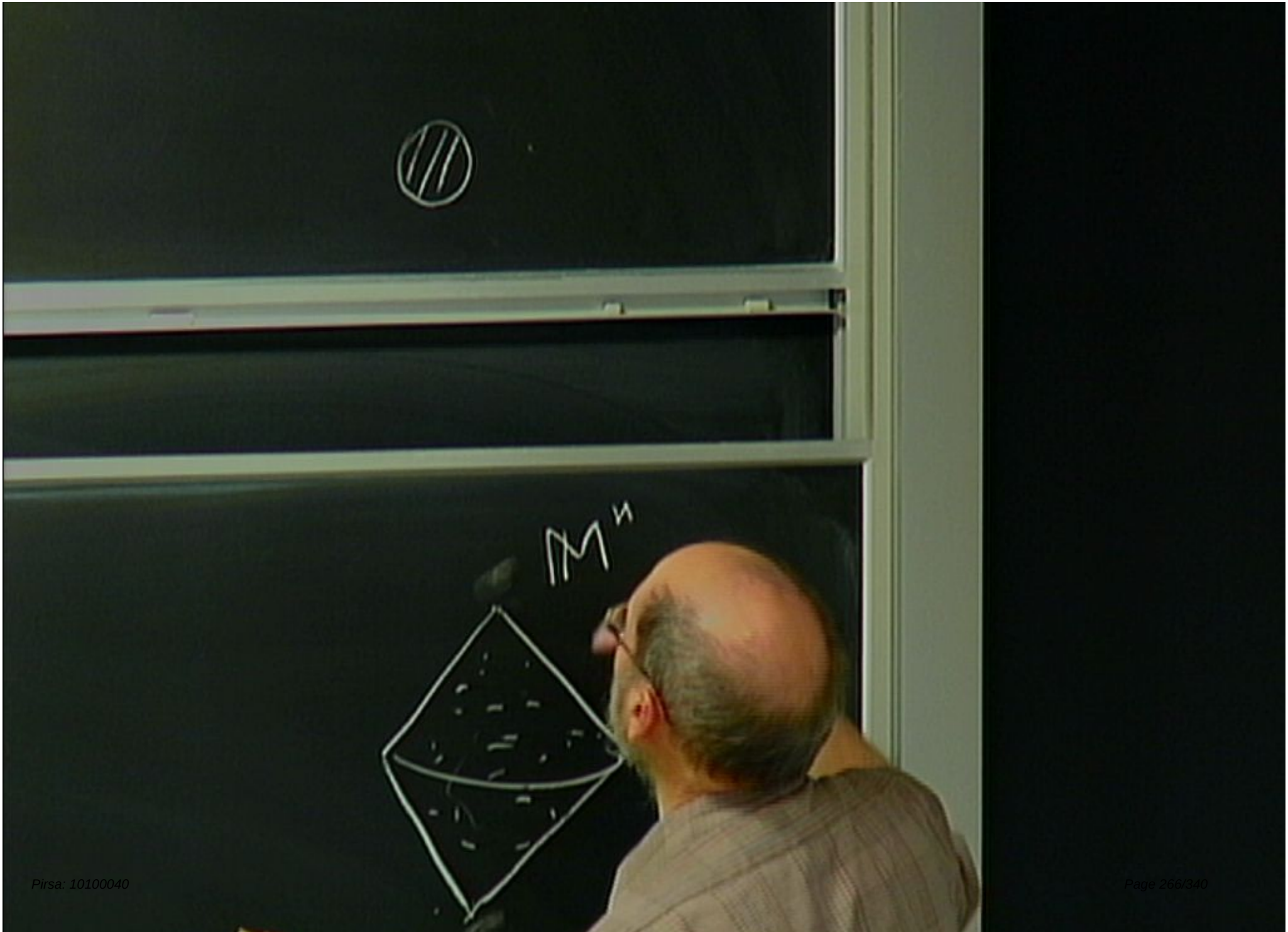
$\langle \phi \rangle$ in M^n

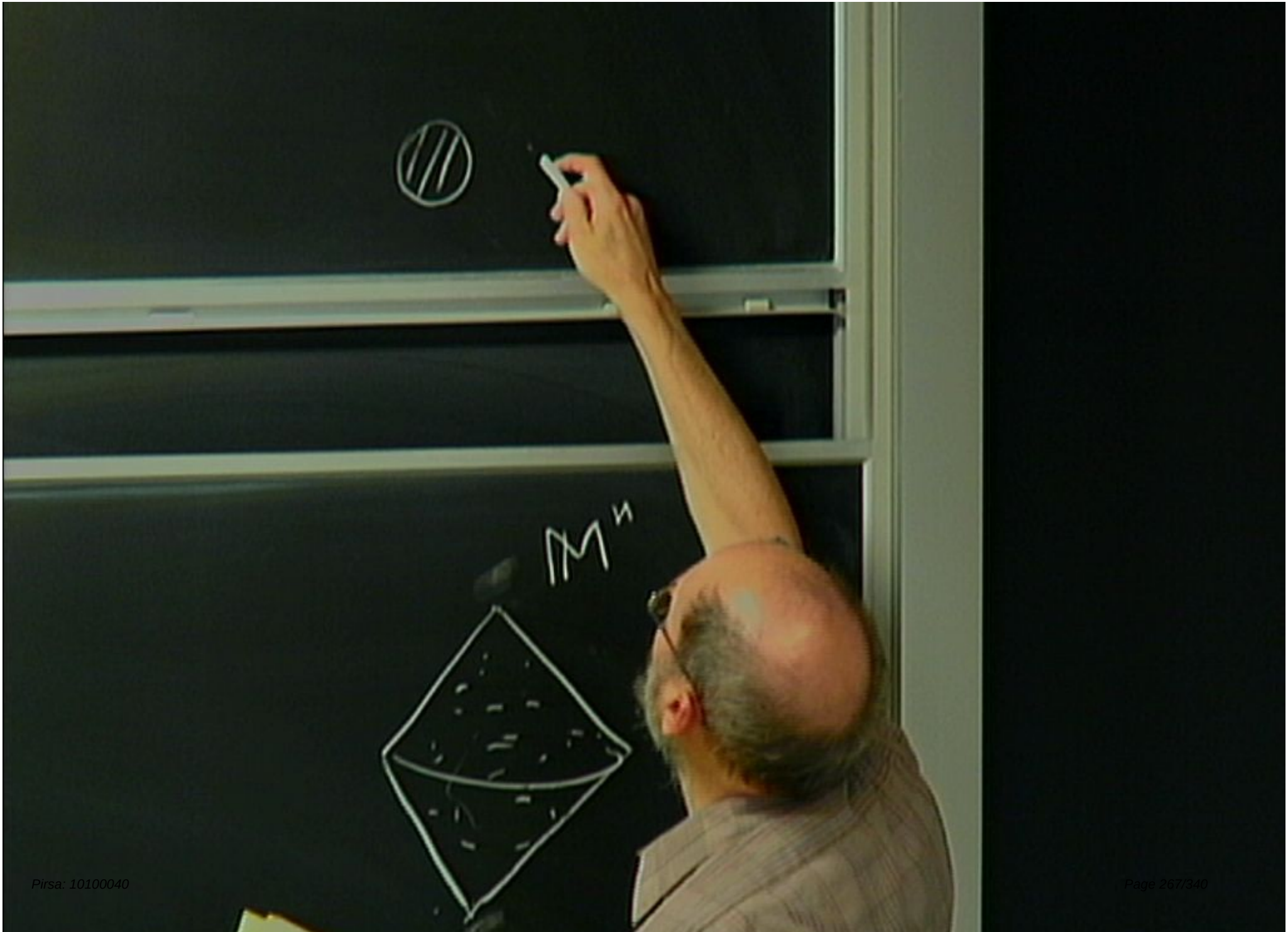
$\langle \phi^2 \rangle$

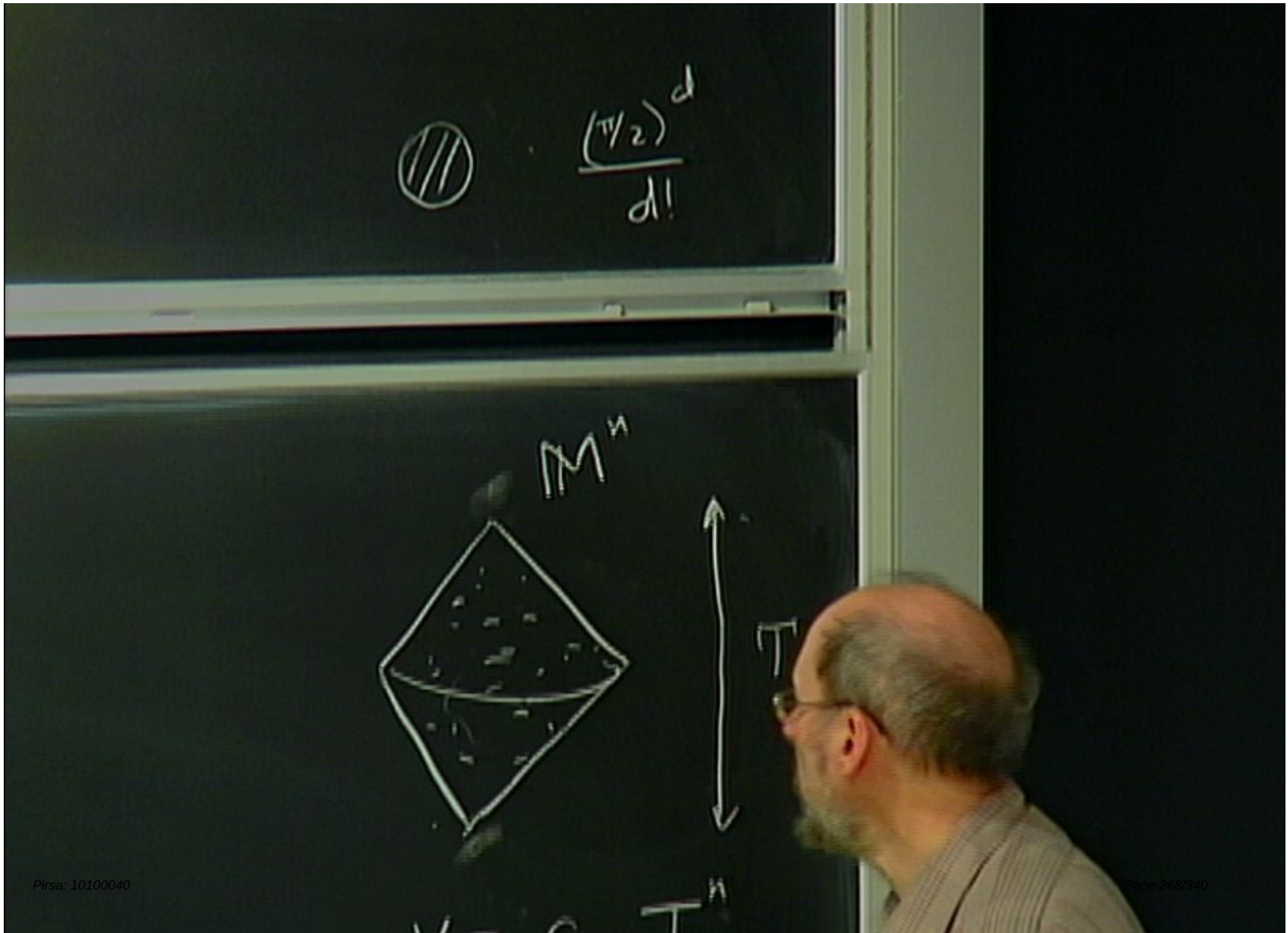


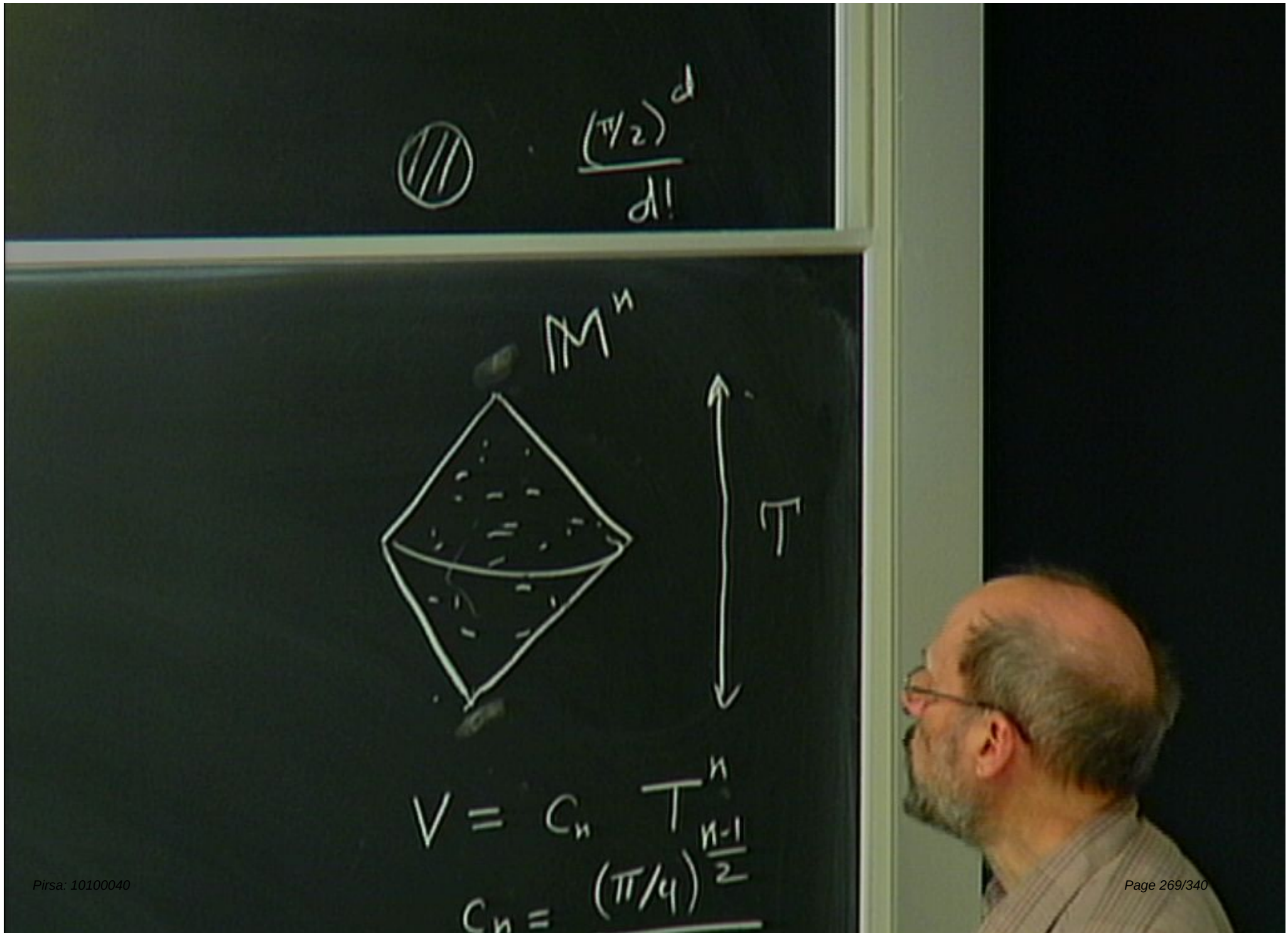
$V = C_n \, T^n$

$C_n = \dfrac{(\pi A)^{\frac{n}{2}}}{n\left(\frac{n}{2}\right)!}$



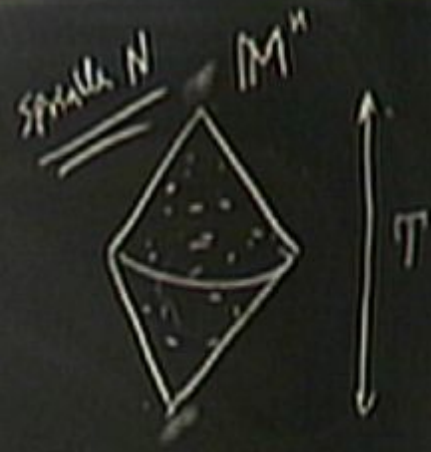



$\frac{(\pi/2)^d}{d!}$
M^n
T
T^n


$\frac{(\pi/2)^d}{d!}$
M^n
T
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{}$

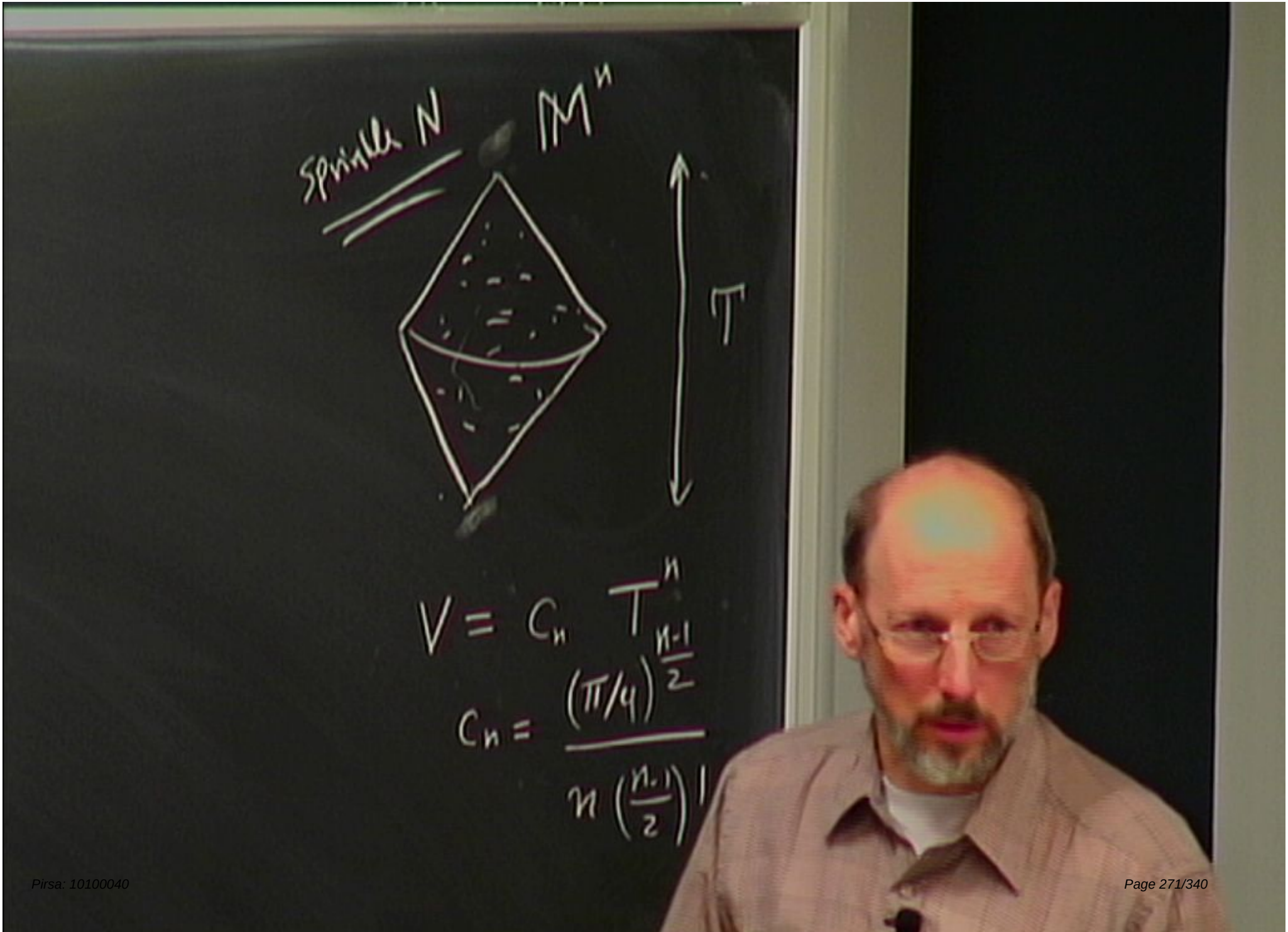
$\langle \phi \rangle$ on $\mathbb{M}^n$

$\langle \phi^2 \rangle$

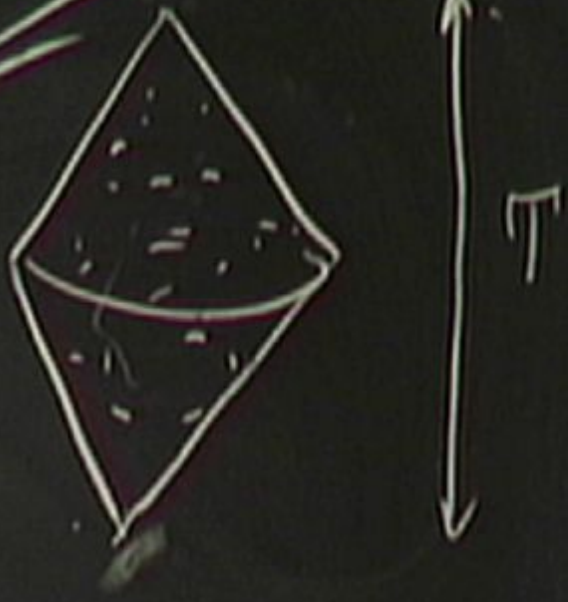


$$V = C_n T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$


Spindle N
M^n
T
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$

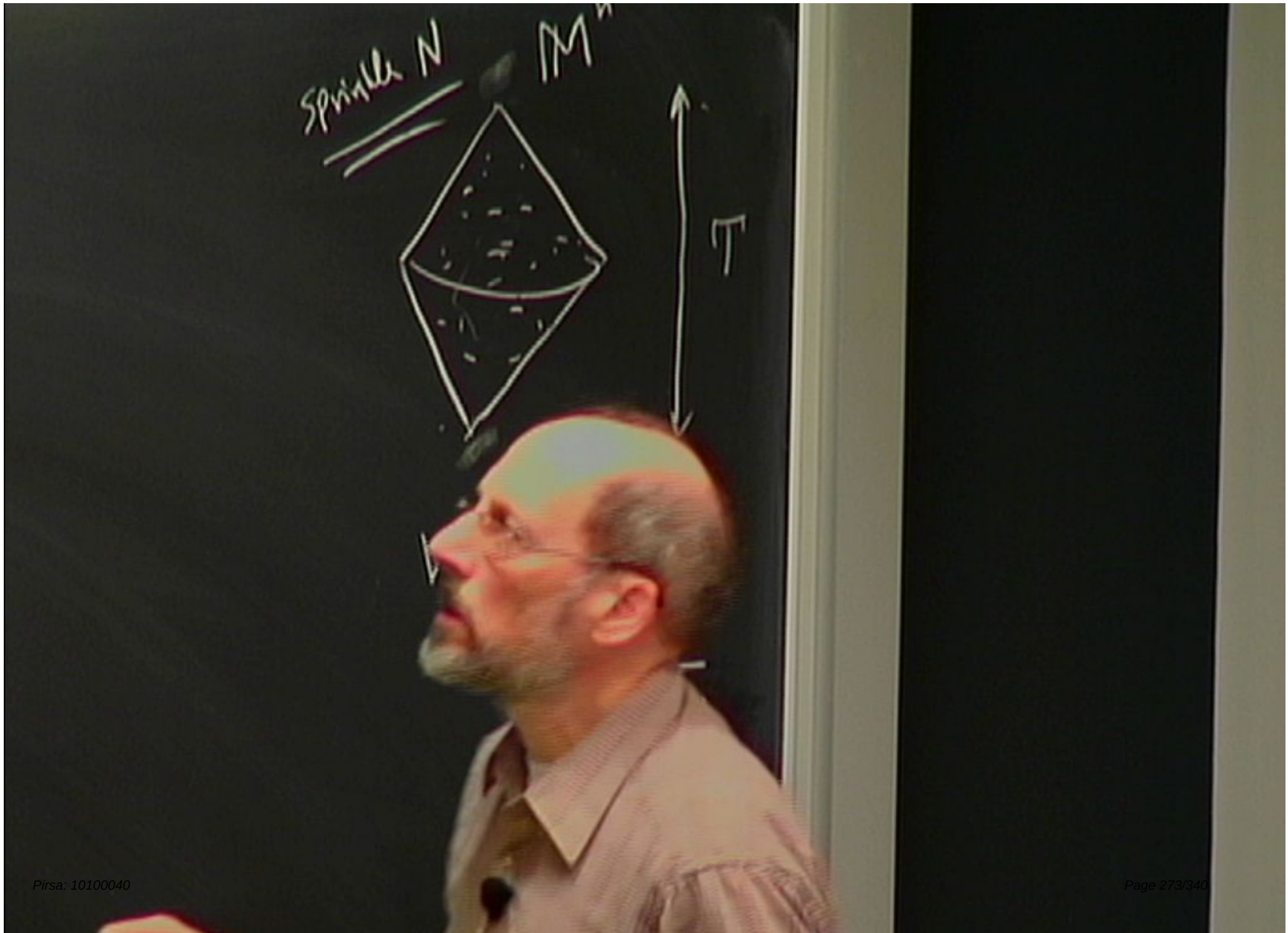
Spinlle N M^n



T

$$V = C_n \, T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$


Spindle N
M
T

Spindle N

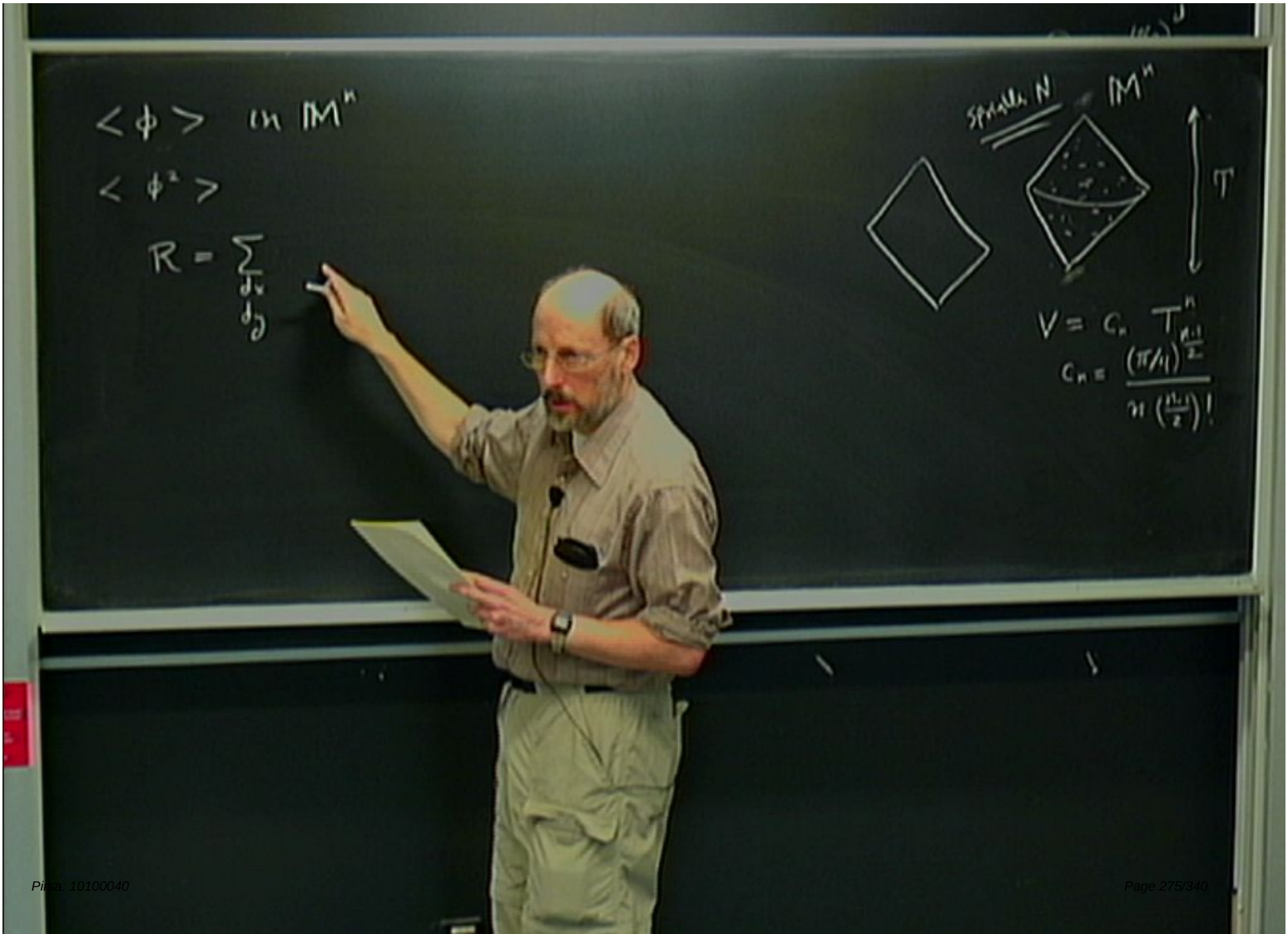
M^n



T

$$V = C_n \, T^n$$

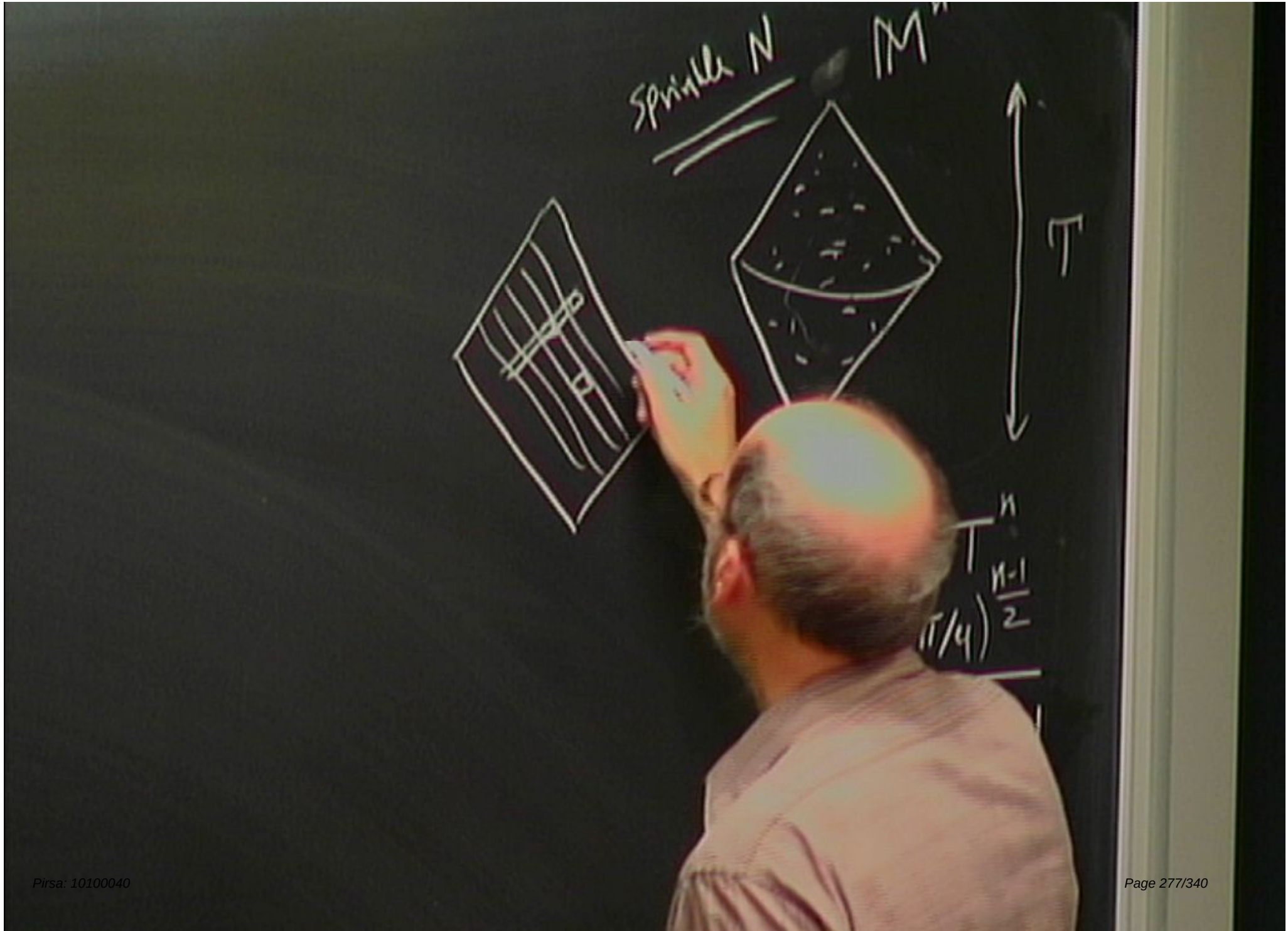
$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$

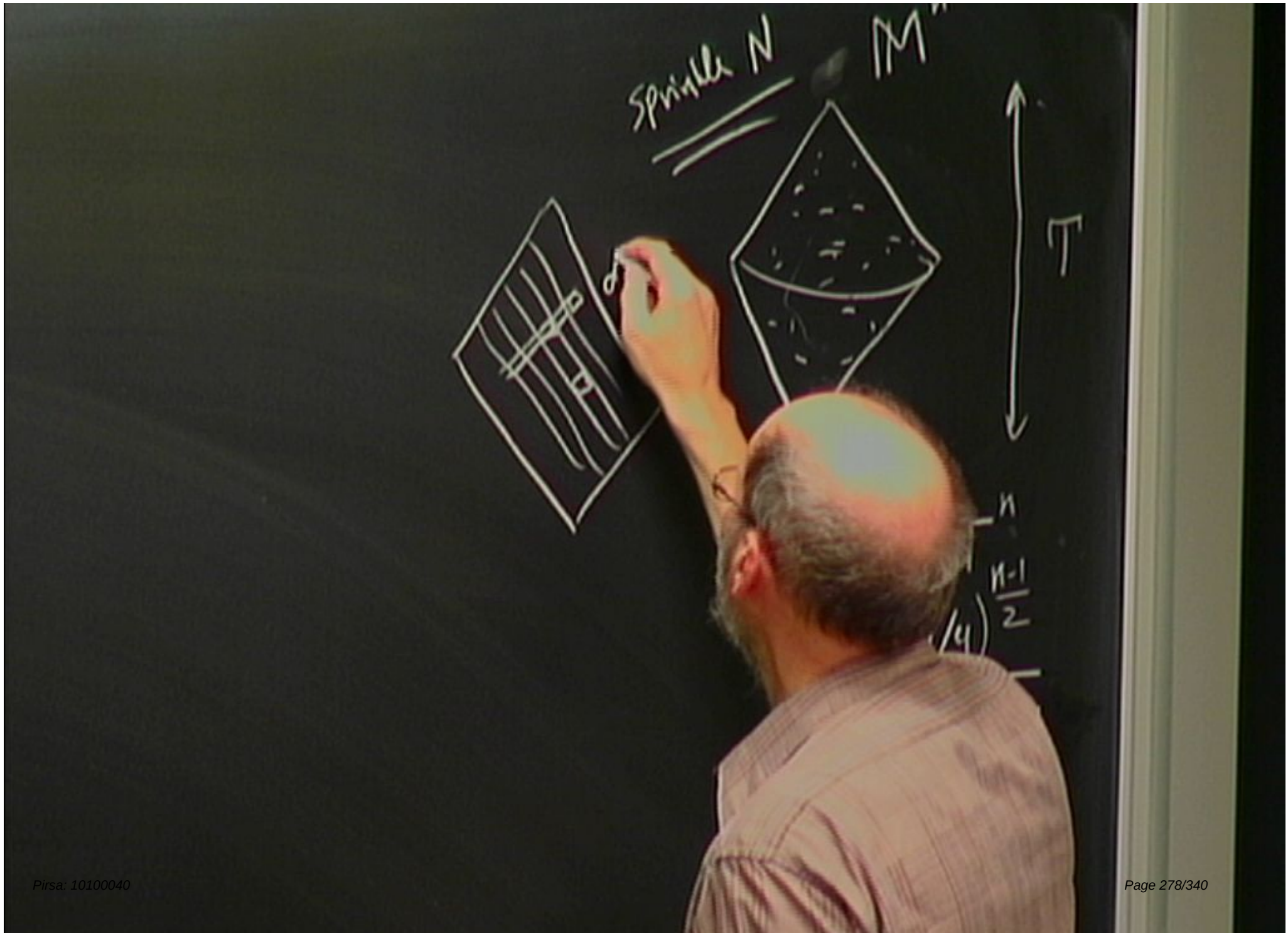

$\langle \phi \rangle$ in M^n
$\langle \phi^2 \rangle$
$R = \sum$
small N
M^n
T
$V = C_n T^n$
$C_n \equiv \frac{(\pi/4)^{\frac{n}{2}}}{n(\frac{n}{2})!}$

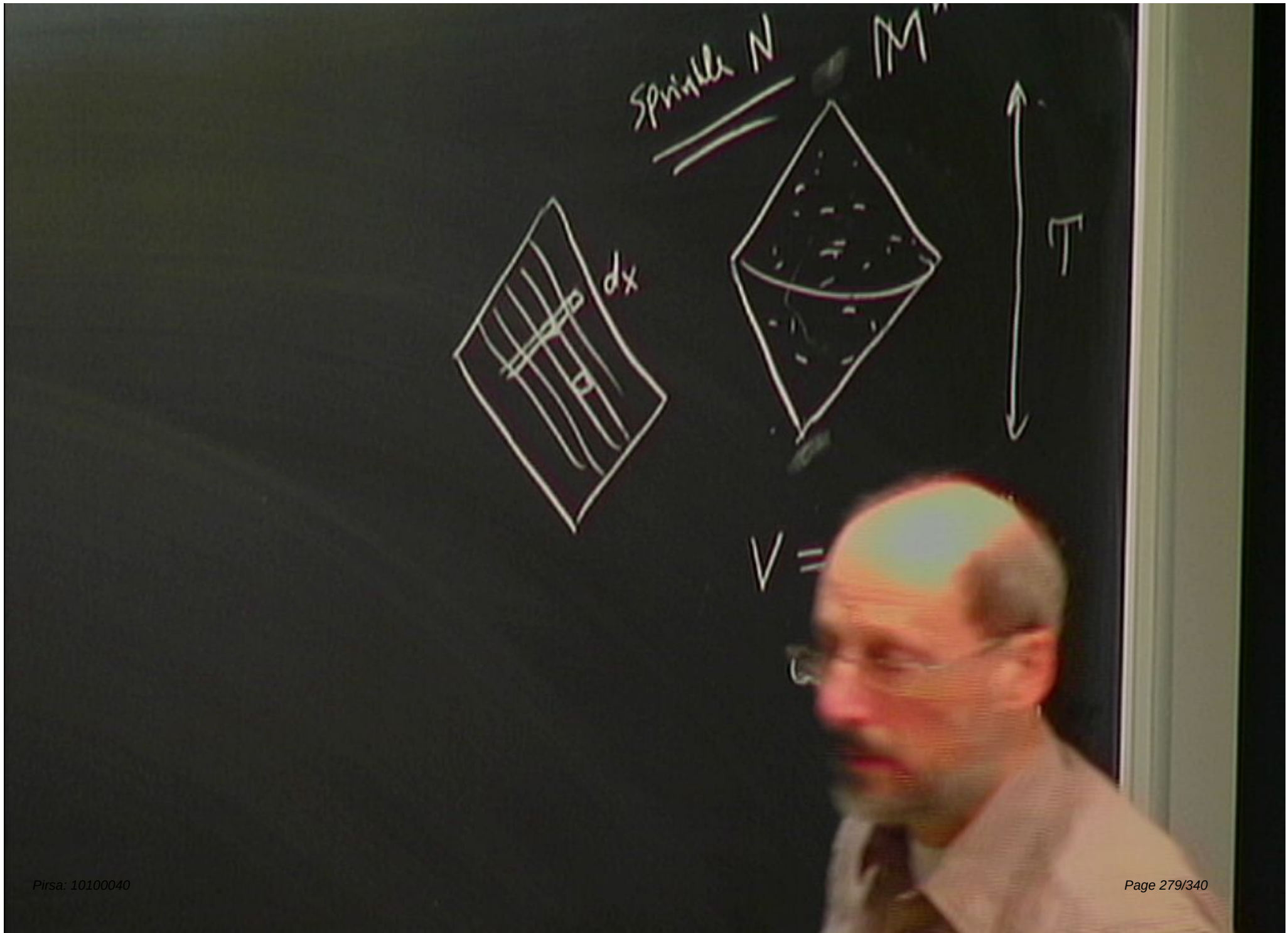
$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$

$$R = \sum_{\substack{d_x \\ d_y}}$$


Spirulli N
T
T^n
(T/4)^(n-1)/2

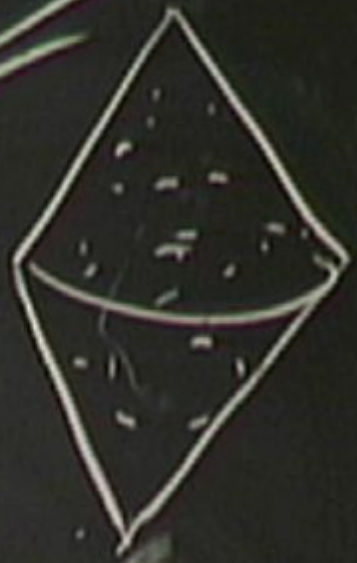

Spindle N
M
T
n
n-1
2


Spirals N
dx
T
V =

Spinlle N

M^n





T

$$V = C_n \, T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$

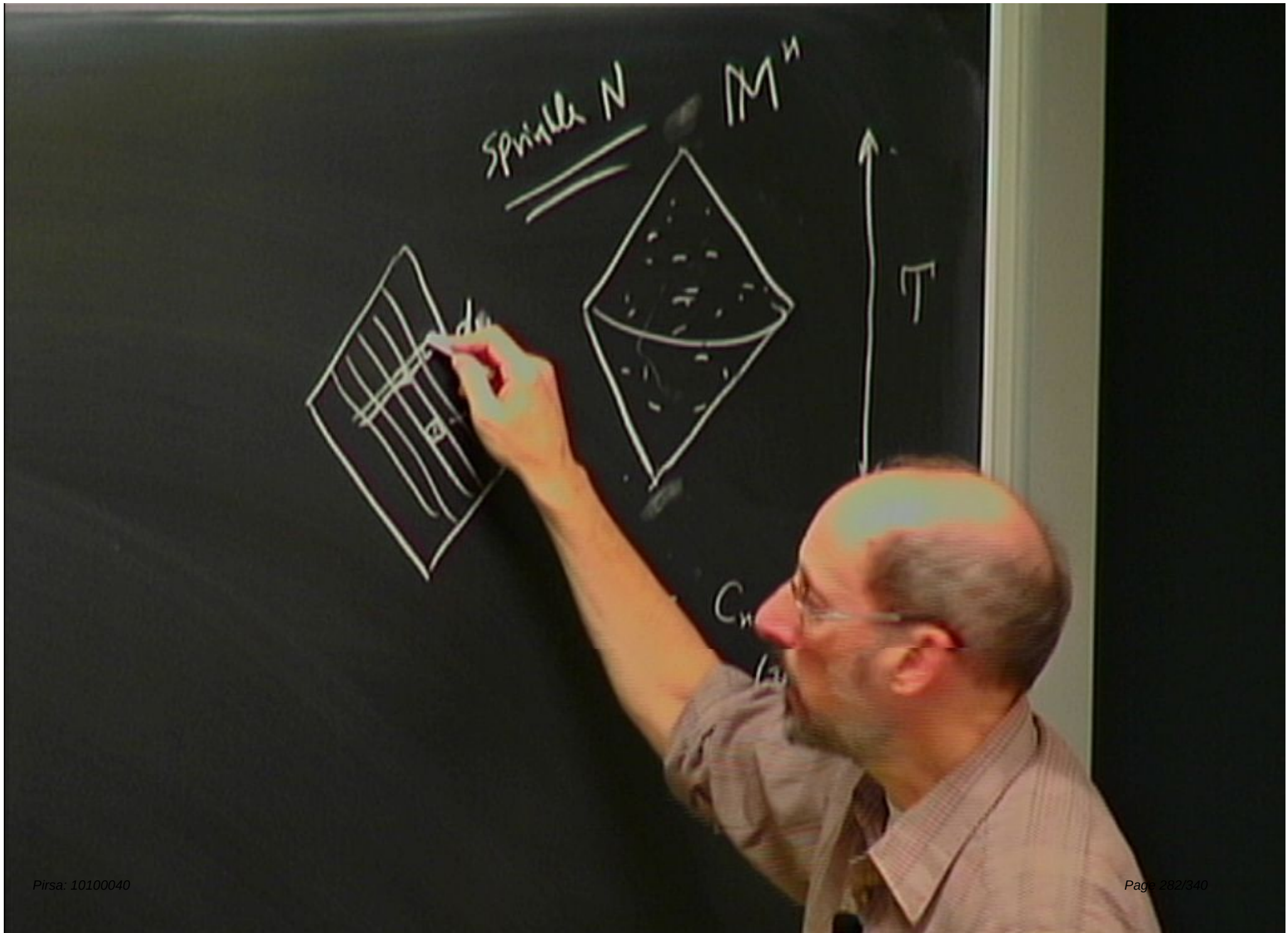
Spinfills N



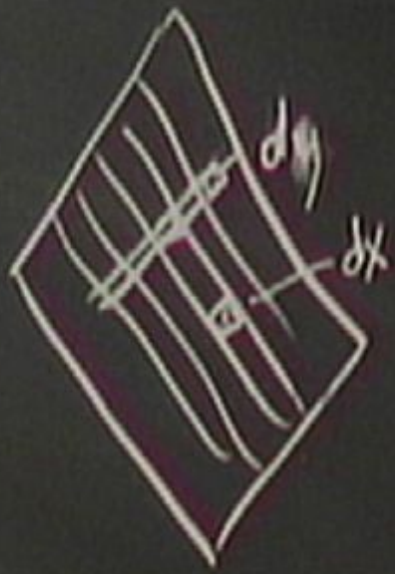


$$V = C_n \, T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$


Spirals N
M^n
T

Sprinkle N M^n

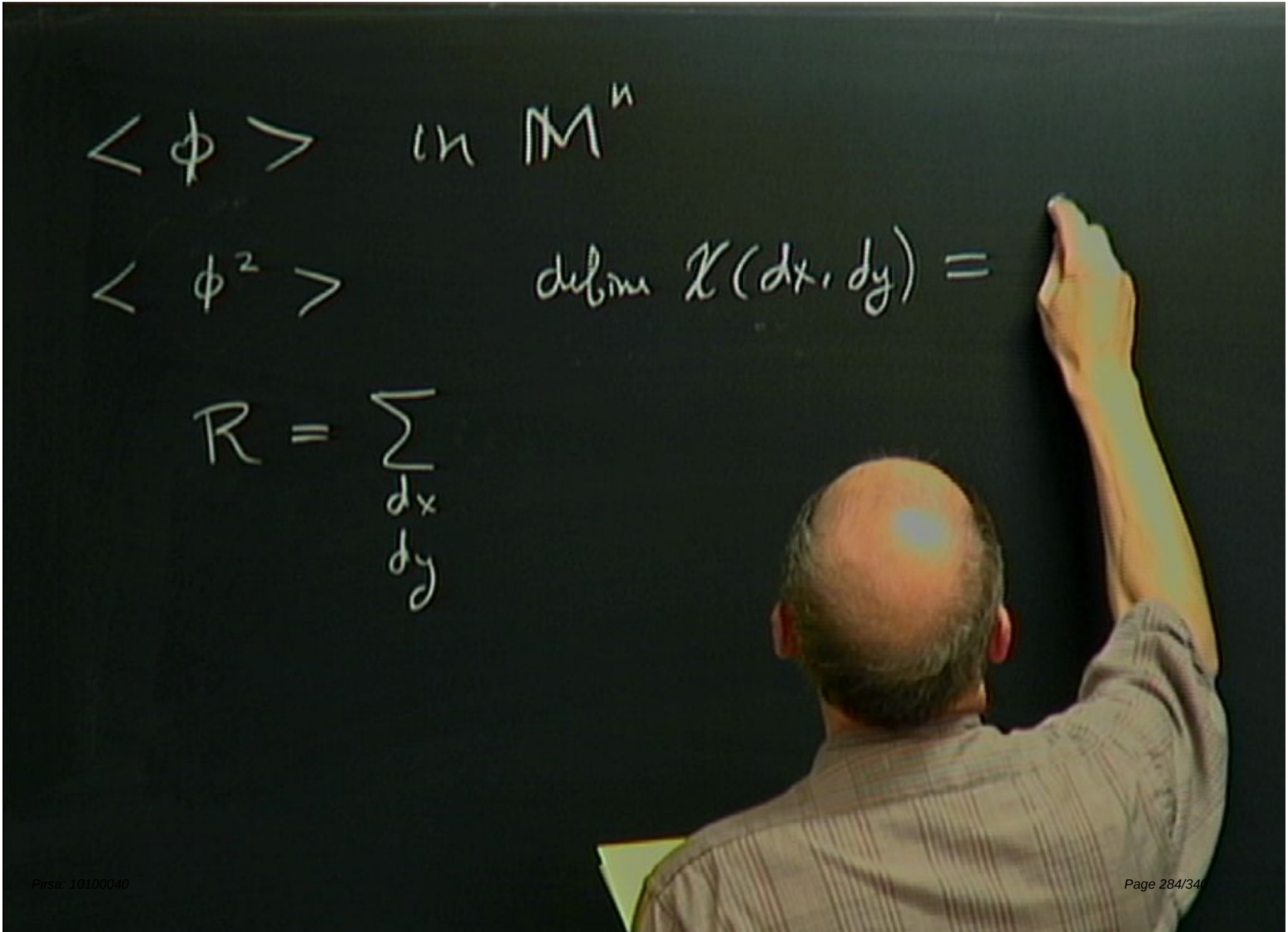






$$V = C_n \, T^n$$

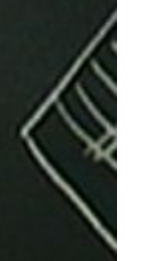
$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$

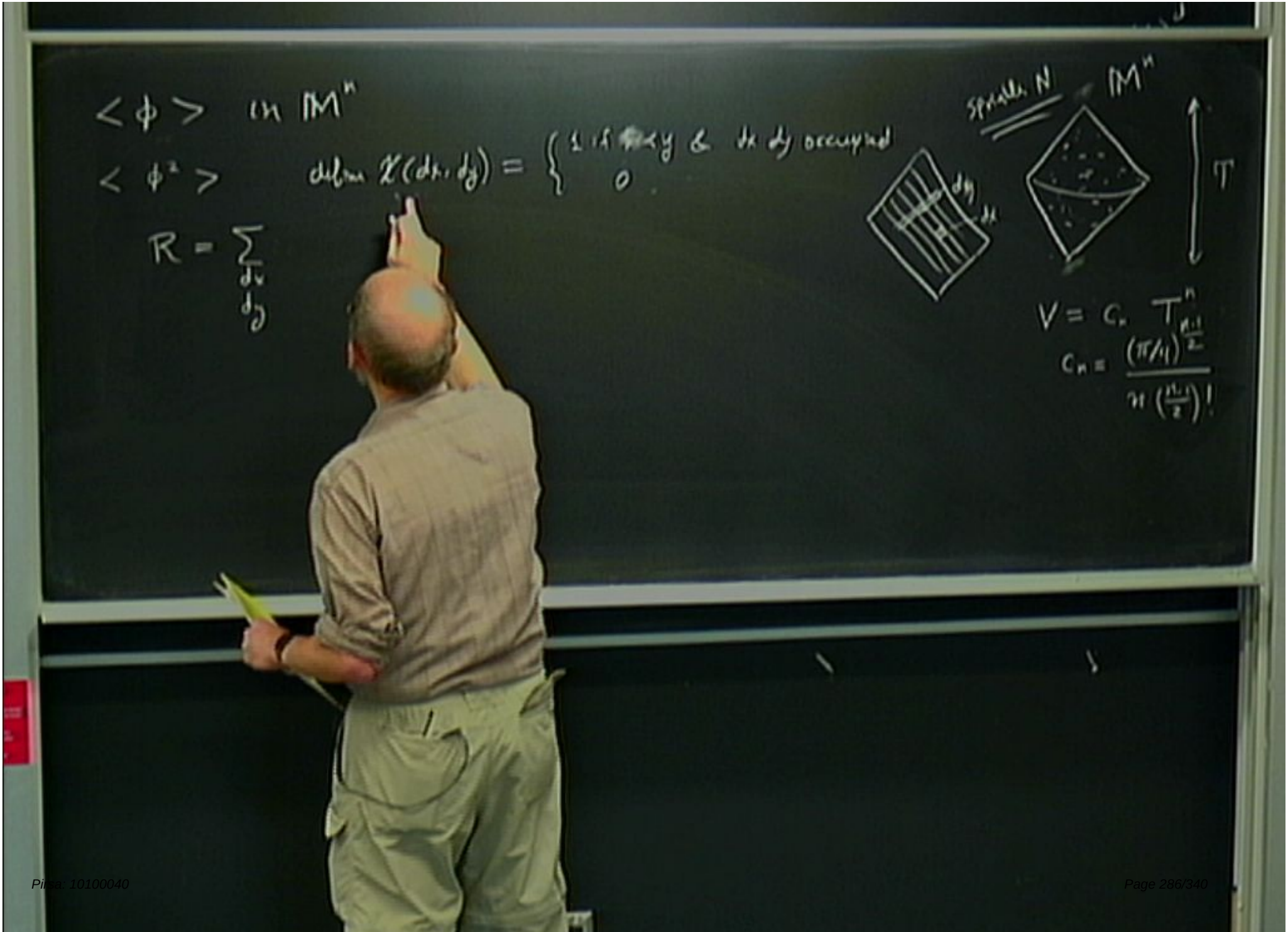

$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
$\lim \mathcal{K}(dx, dy) =$
$R = \sum_{dx\,dy}$

$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x \prec y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$$R = \sum_{\substack{dx \\ dy}}$$

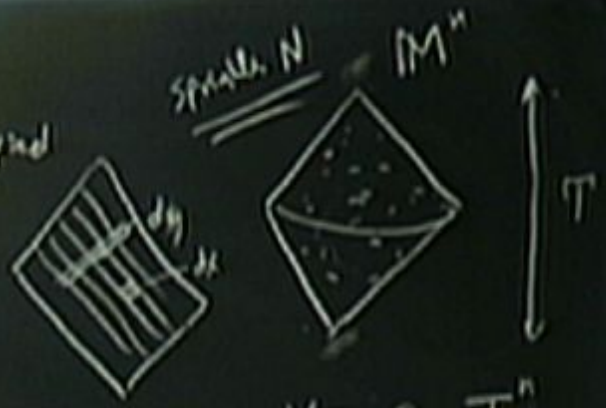



$\langle \phi \rangle$ in M^n
$\langle \phi^2 \rangle$
$R = \sum_{dx\,dy}$
$\chi(dx, dy) = \begin{cases} 1 \text{ if } \\ 0 \end{cases}$ & $dx\,dy$ occupied
sprinkle N
M^n
T
$V = C_n \, T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\,(\frac{n}{2})!}$

$\langle \phi \rangle$ on M^n

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\,dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}}$



$V = C_n \, T^n$

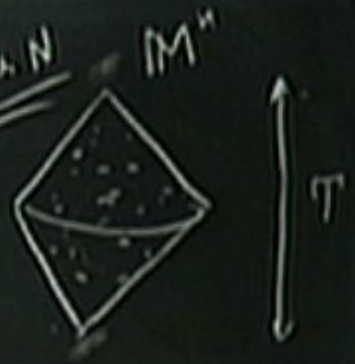
$C_n = \dfrac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$

$\langle \phi \rangle$ in M^n

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\,dy \text{ occupied} \\ 0 \end{cases}$

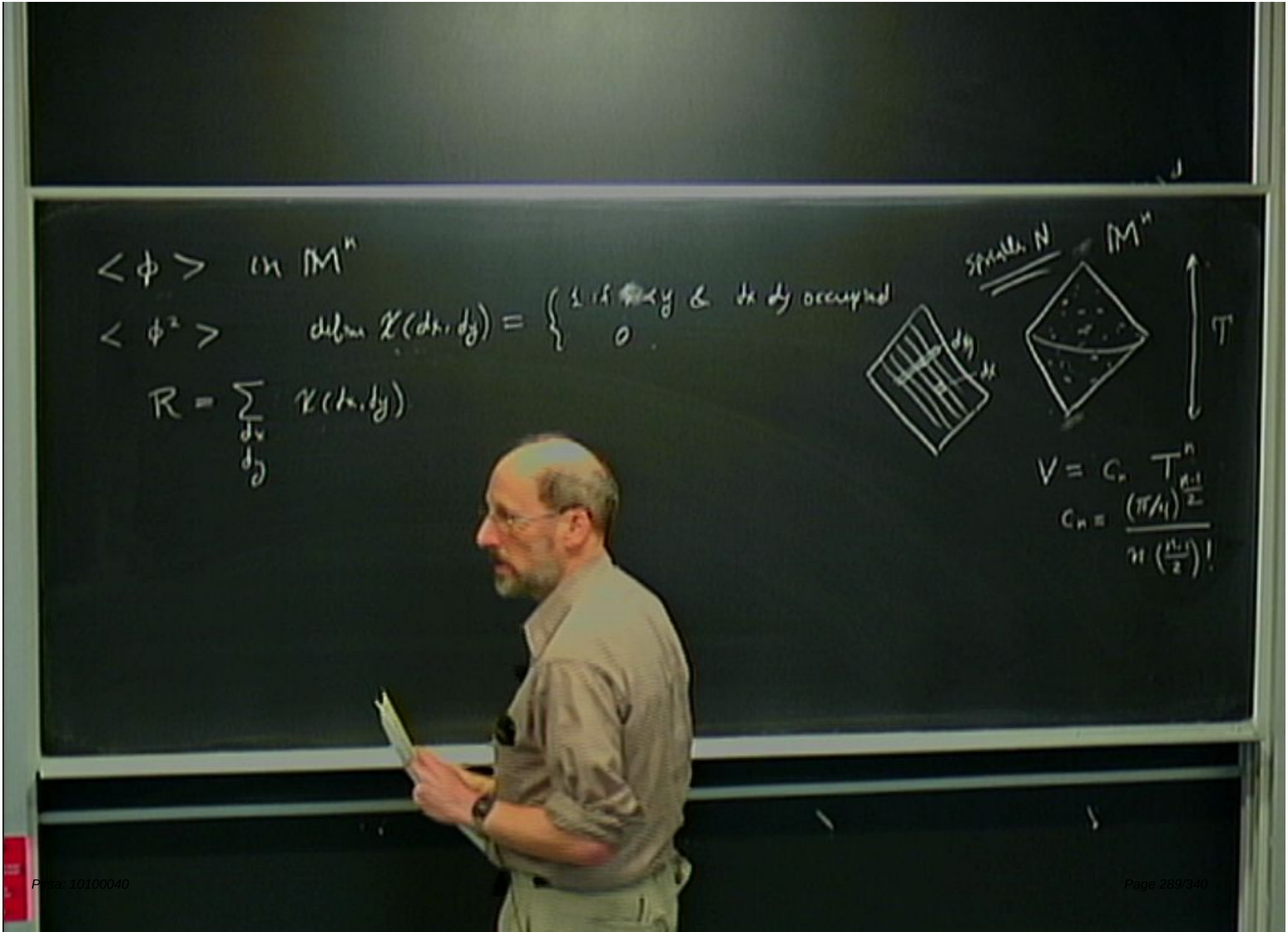
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$





$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\,(\frac{n-1}{2})!}$


$\langle \phi \rangle$ in M^n
$\langle \phi^2 \rangle$
define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x \text{ \& } y \text{ \& } dx\,dy \text{ occupied} \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
smaller N
M^n
T
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\,(\frac{n-1}{2})!}$

$|\phi\rangle$ in $\mathbb{M}^n$

$|\phi^2\rangle$ $\quad$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$$

small N $\quad$ $\mathbb{M}^n$





$V = C_n$

$C_n =$

$|\phi\rangle$ in $\mathbb{M}^n$

$|\phi^2\rangle$ $\quad$ $\dim \mathcal{K}(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$





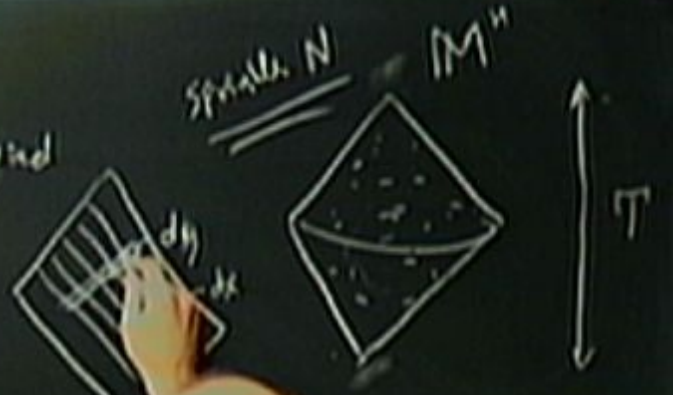
$V = C_n$

$C_n =$

$\langle \phi \rangle$ in $\mathbb{M}^n$

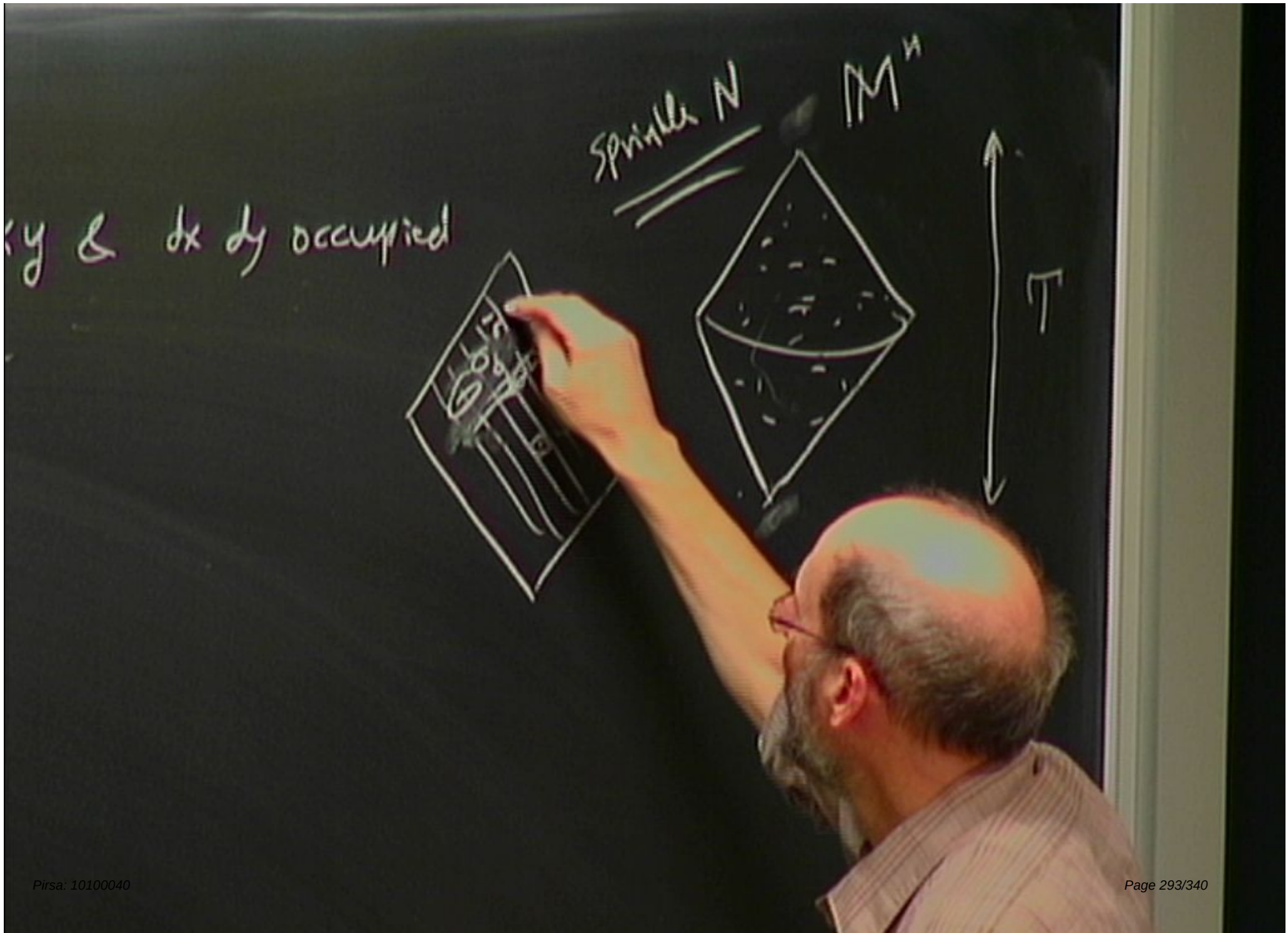
$\langle \phi^2 \rangle$ define $\chi(dx_i, dy_j) = \begin{cases} 1 \text{ if } y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

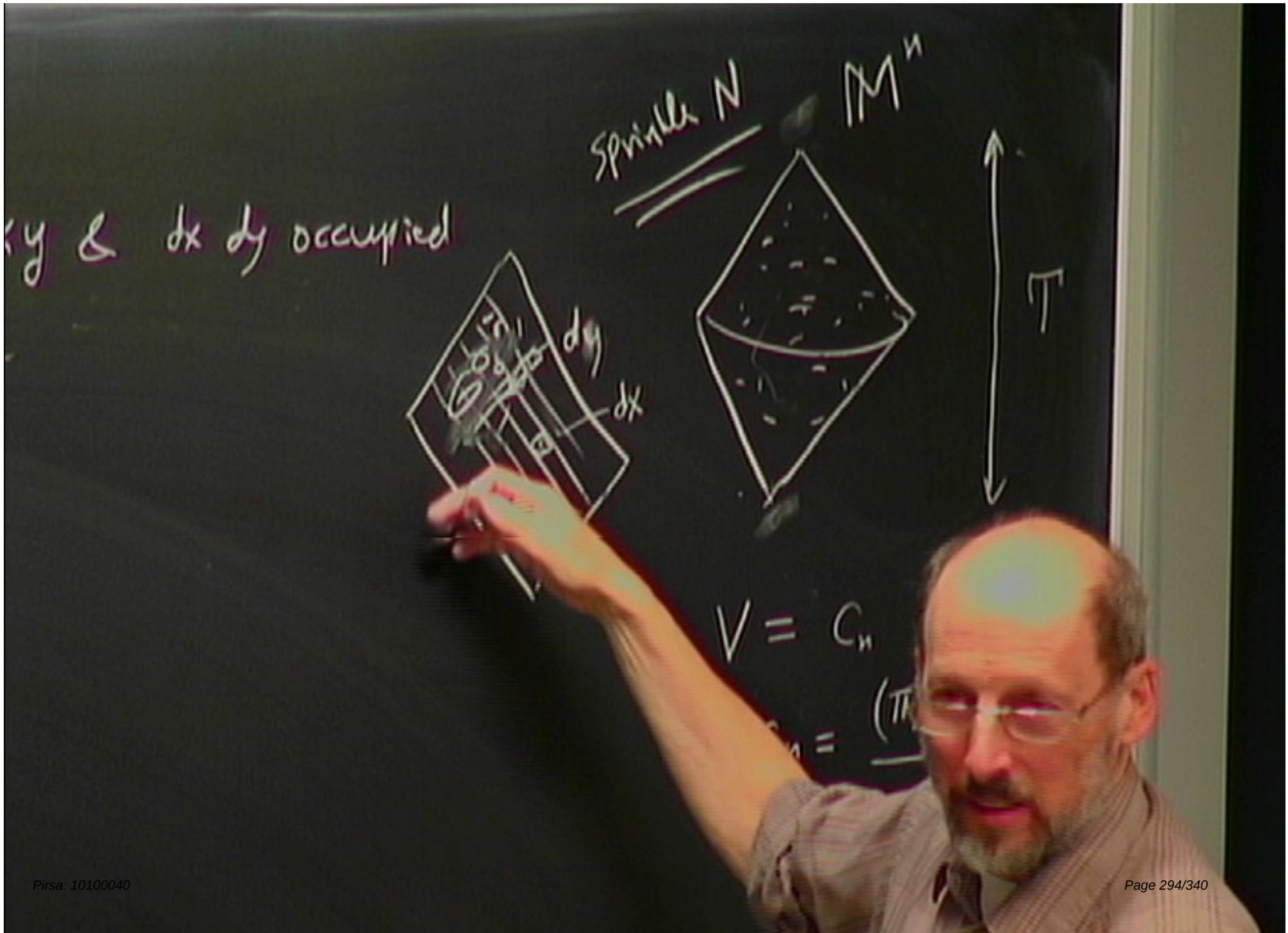
$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$$

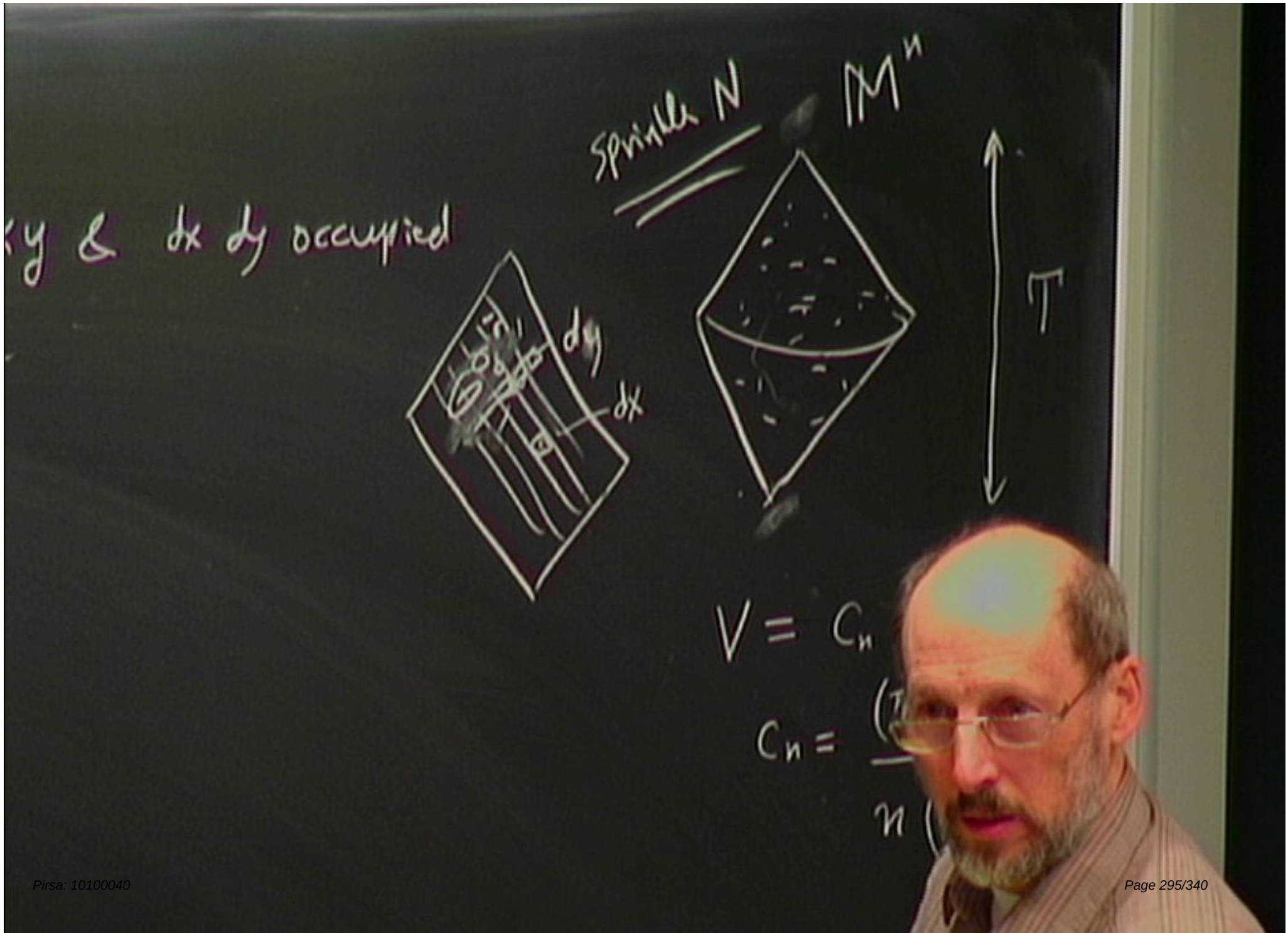


$$= C_n \, T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$$


Spinlike N
M^n
T
occupied


Sprinkle N
M^n
dy
dx
T
$V = C_n$


Spindle N
M^n
dy
dx
T
$V = C_n$
$C_n =$

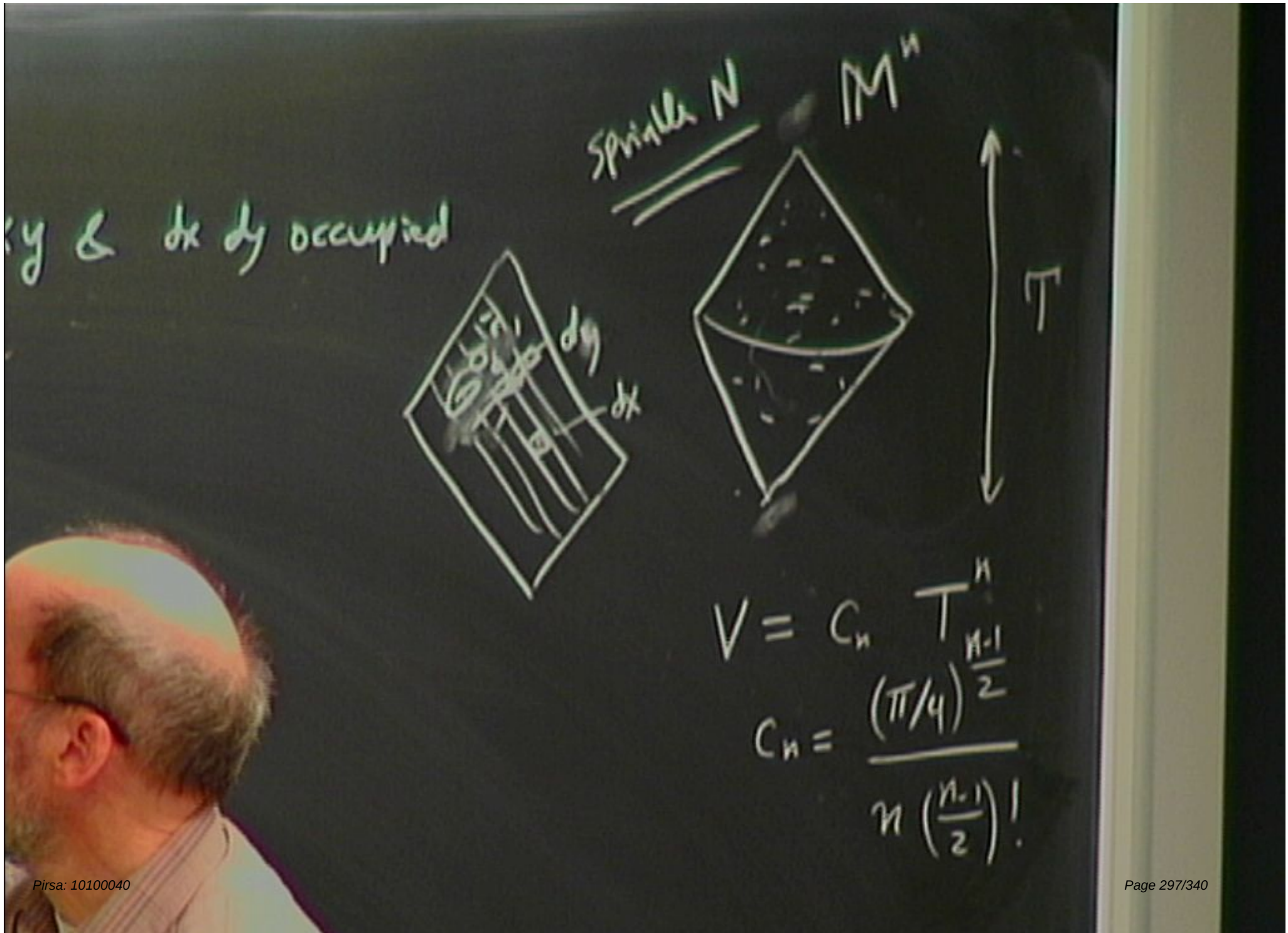
$\langle \phi \rangle$ in $\mathbb{M}^n$

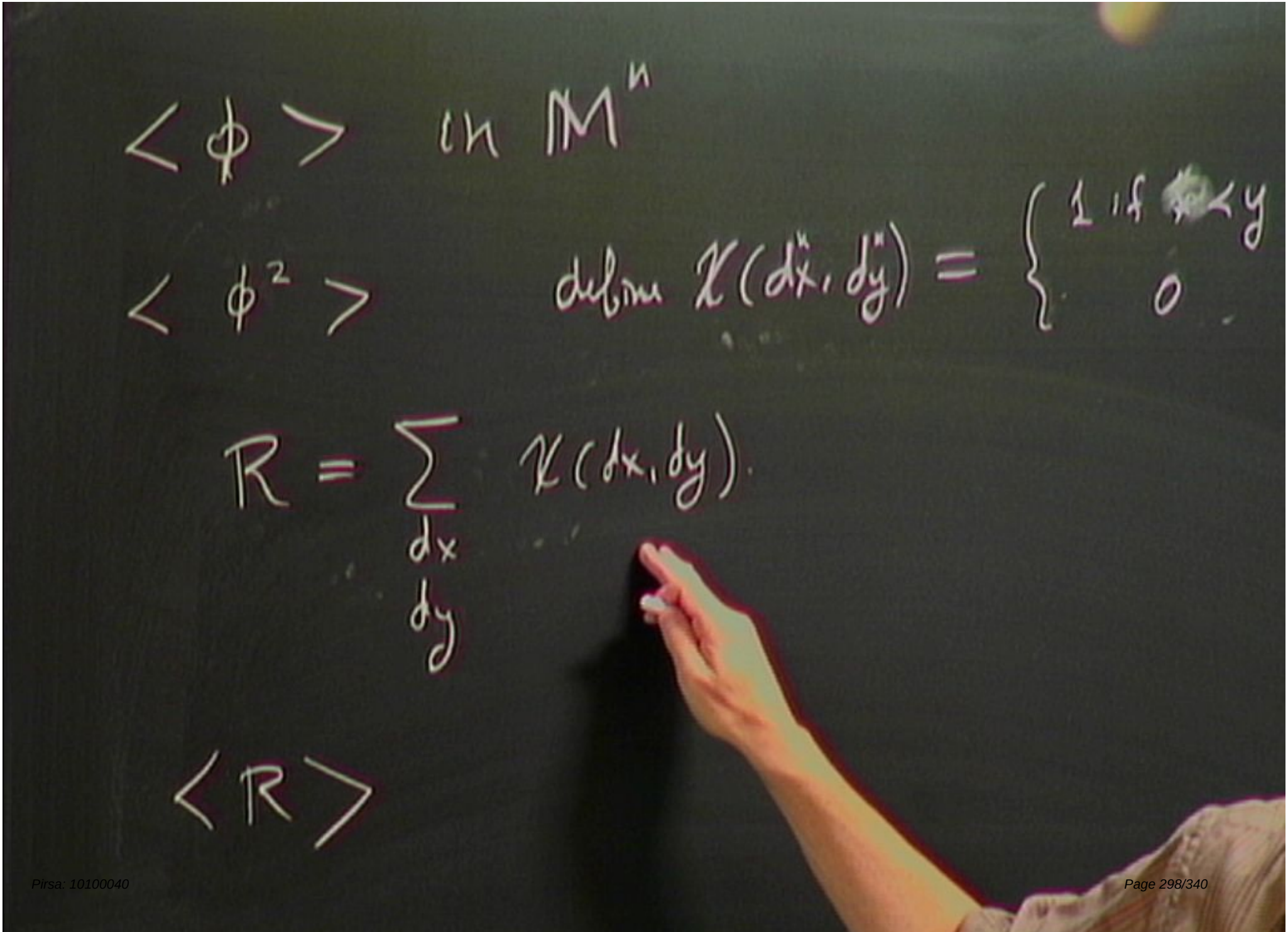
$\langle \phi^2 \rangle$ define $\chi(d_x, d_y) = \begin{cases} 1 \text{ if } x \prec y \\ 0 \end{cases}$

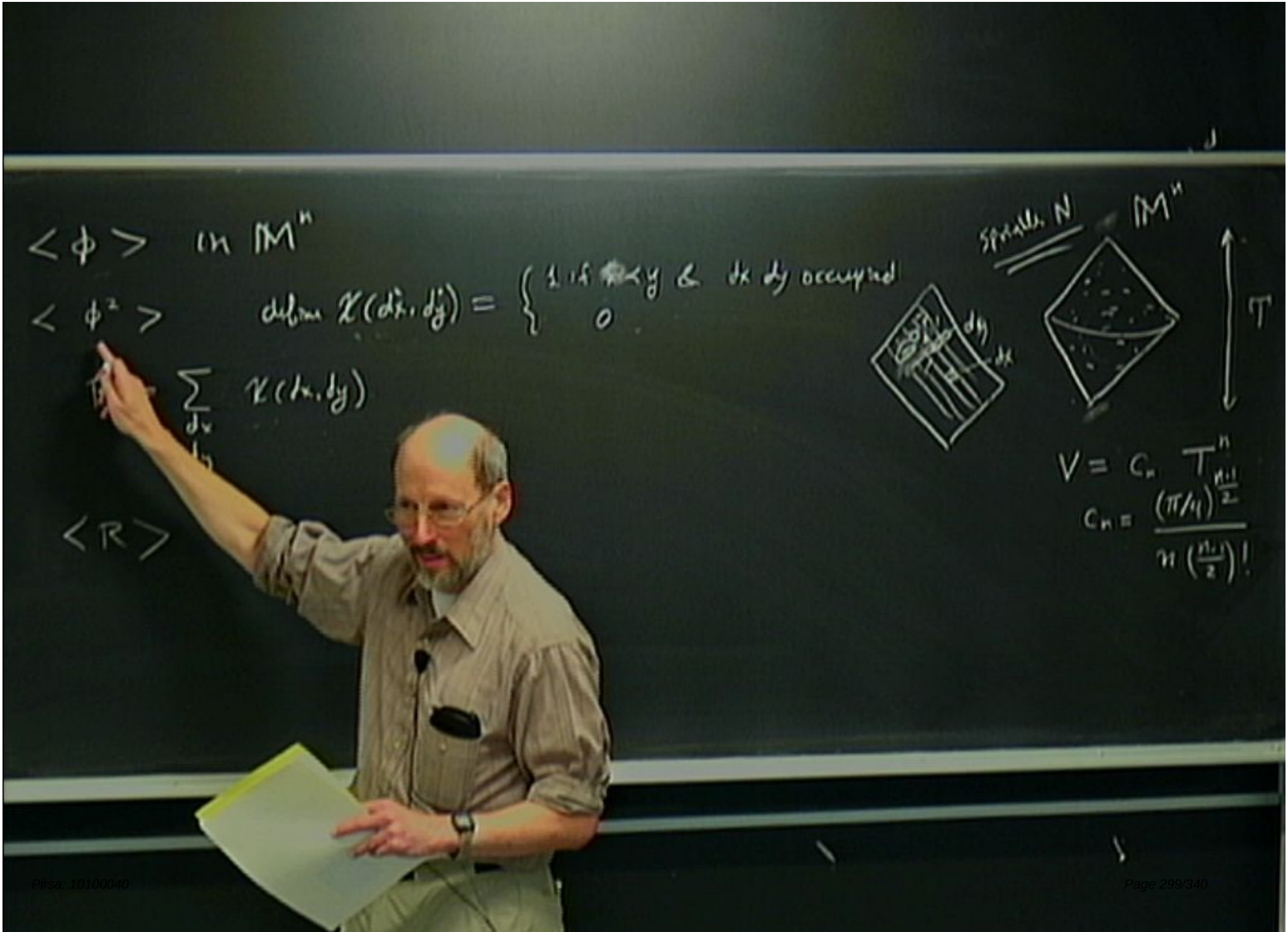
$$R = \sum_{\substack{d_x \\ d_y}} \chi(d_x, d_y).$$

$\langle R \rangle$




Spirale N
M^n
y & dx dy occupied
dy
dx
T
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x \prec y \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
$\langle R \rangle$


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
$\langle R \rangle$
$\sum_{dx}$
$\chi(dx, dy)$
$\mathbb{M}^n$
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n}{2}}}{n\left(\frac{n}{2}\right)!}$

$\langle \phi \rangle$ in M^n

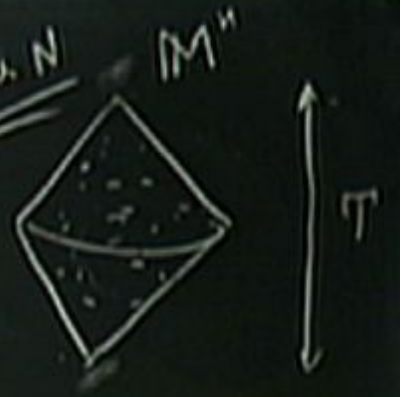
$\langle \phi^2 \rangle$

define $\chi(dx, dy) = \begin{cases} 1 \text{ if } \exists y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$

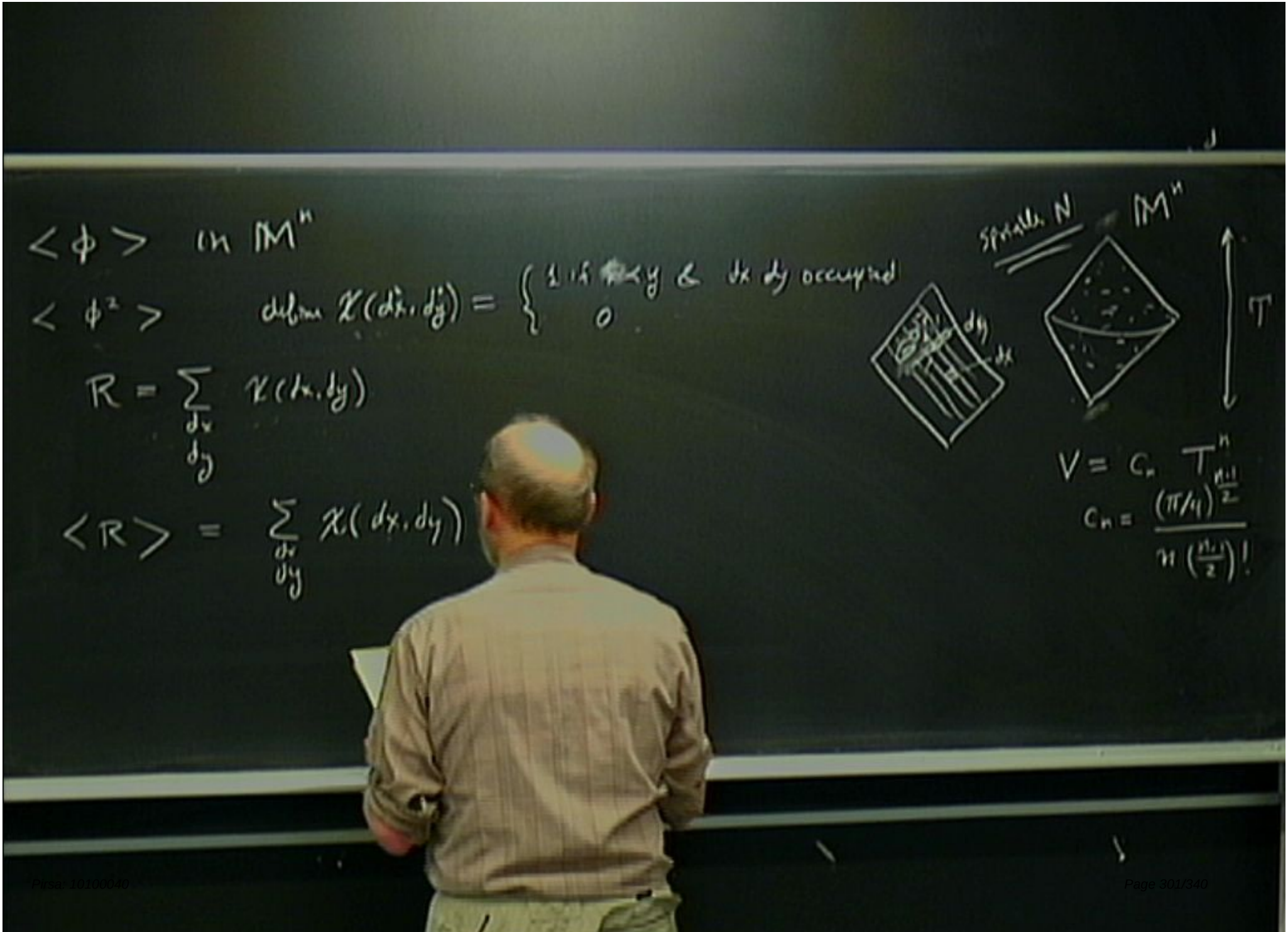
$\langle R \rangle$

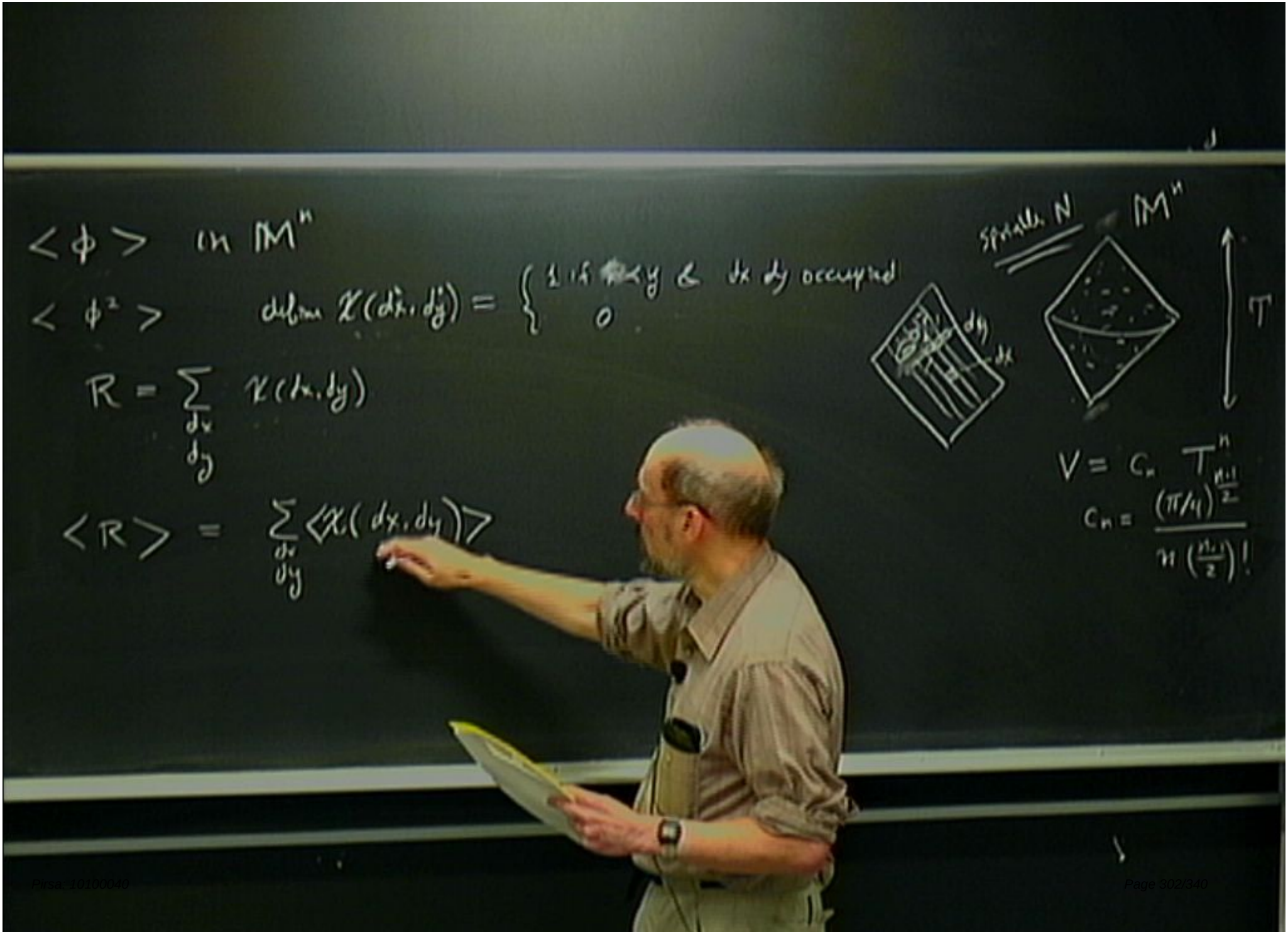




$V = C_n\ T^n$

$C_n = \dfrac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
define $\mathcal{K}(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ & } dx\ dy \text{ occupied} \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \mathcal{K}(dx, dy)$
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \mathcal{K}(dx, dy)$
Sprinkle N
$\mathbb{M}^n$
T
dy
dx
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ & } dx\ dy \text{ occupied} \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$
Spatially N
$\mathbb{M}^n$
T
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$

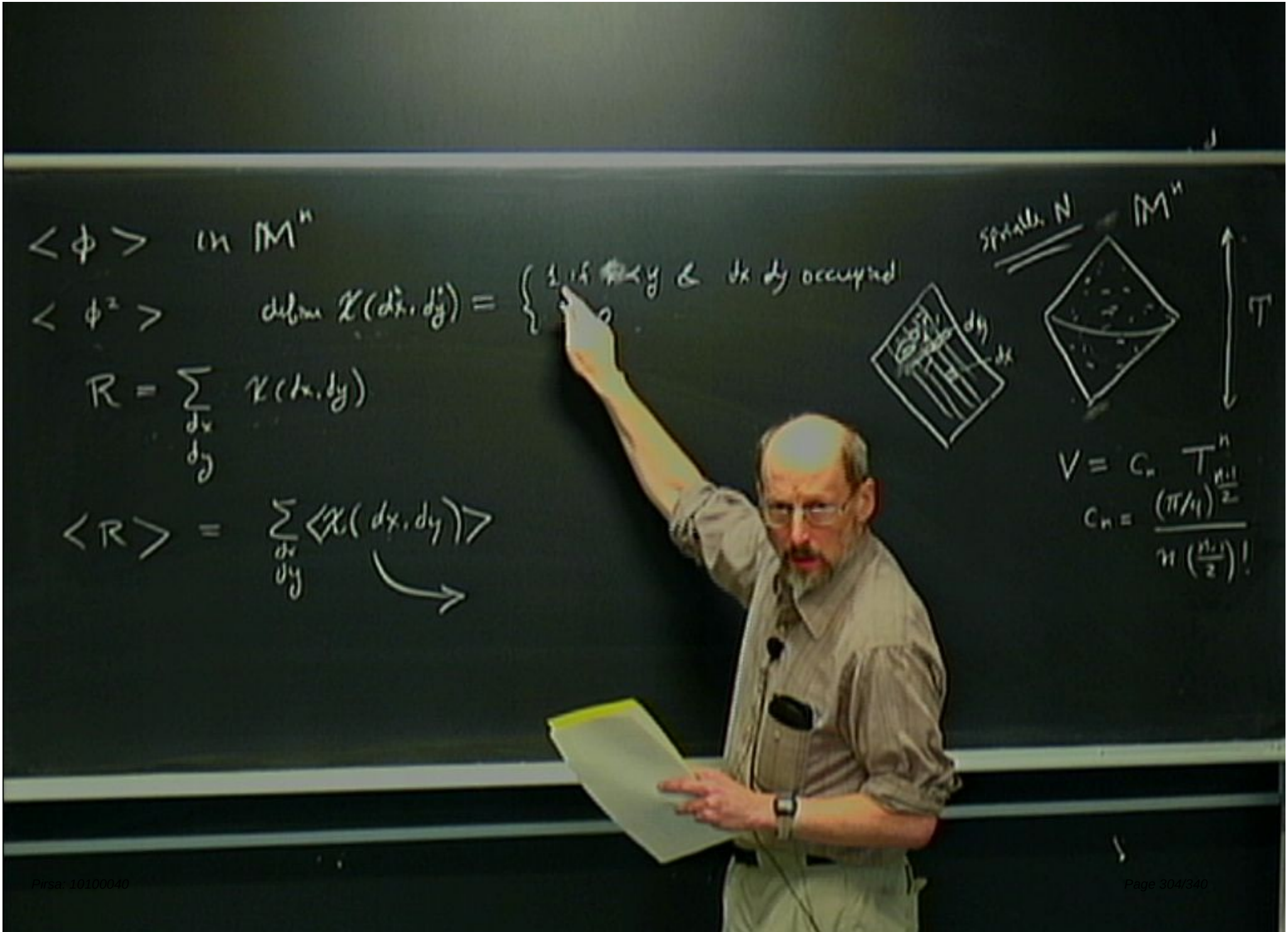
$\langle \phi \rangle$ in $\mathbb{M}^n$

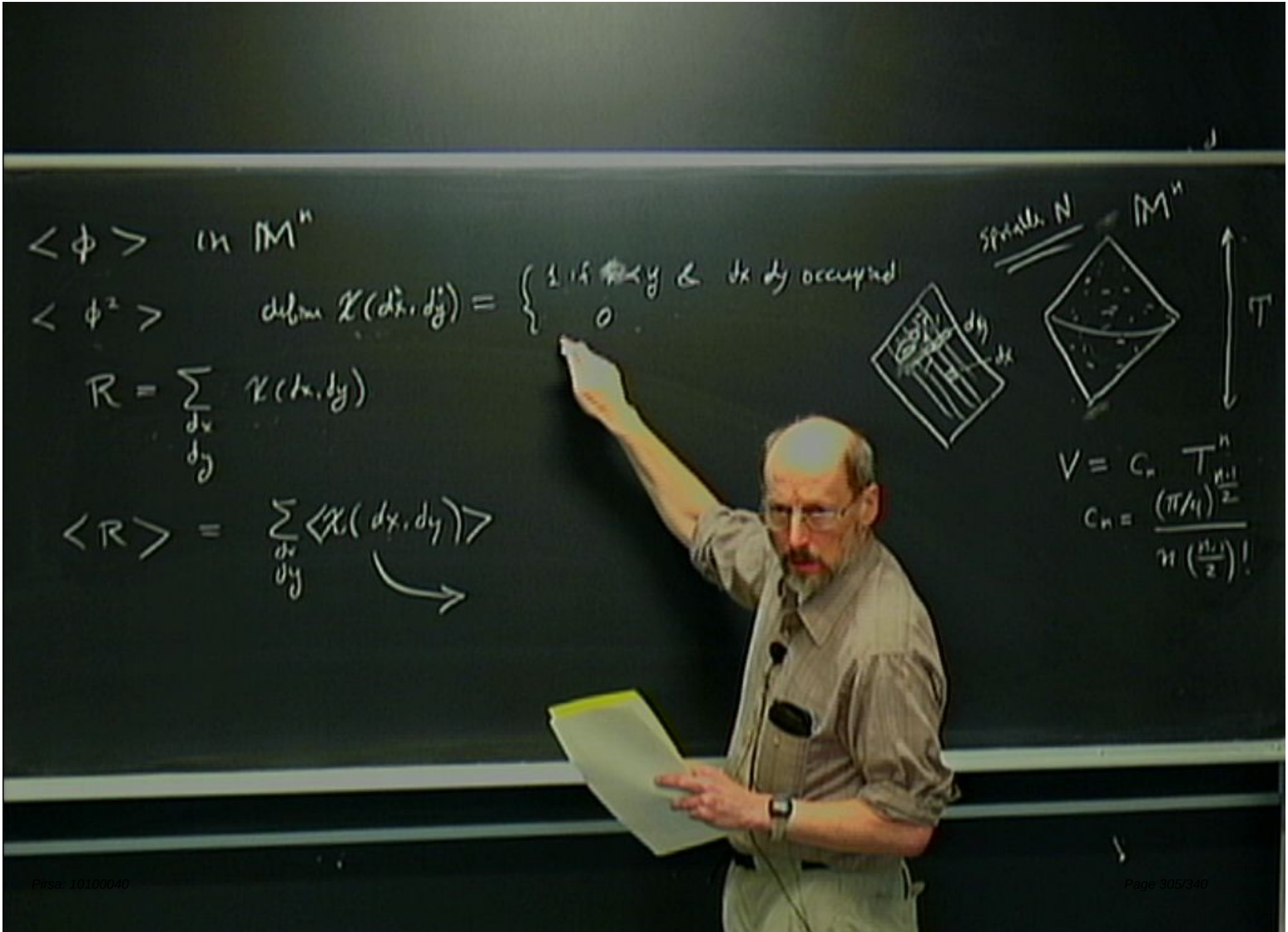
$\langle \phi^2 \rangle$

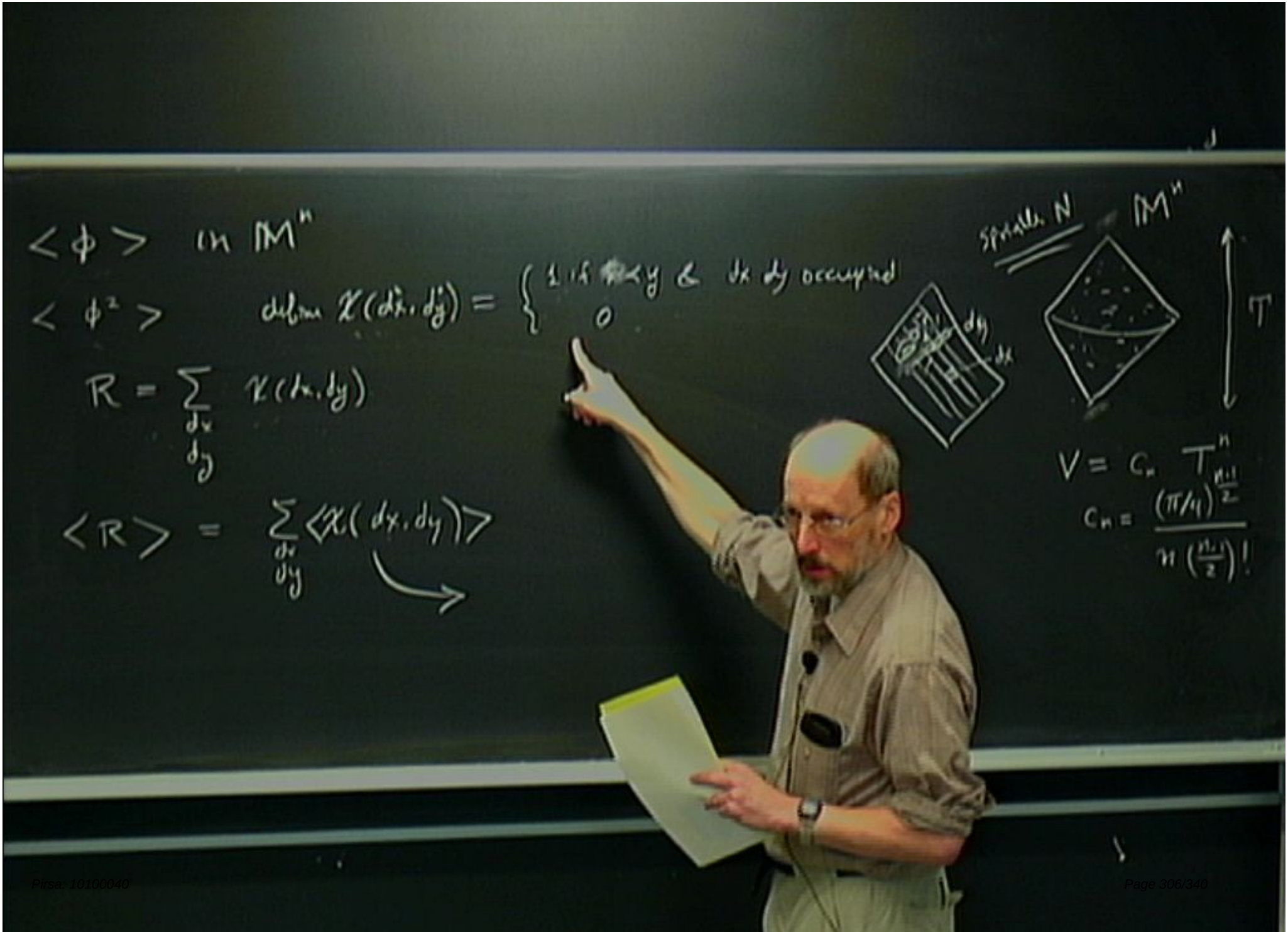
$$\text{define } \chi(d_x^n, d_y^n) = \begin{cases} 1 \text{ if } x \prec y \\ 0 \end{cases}$$

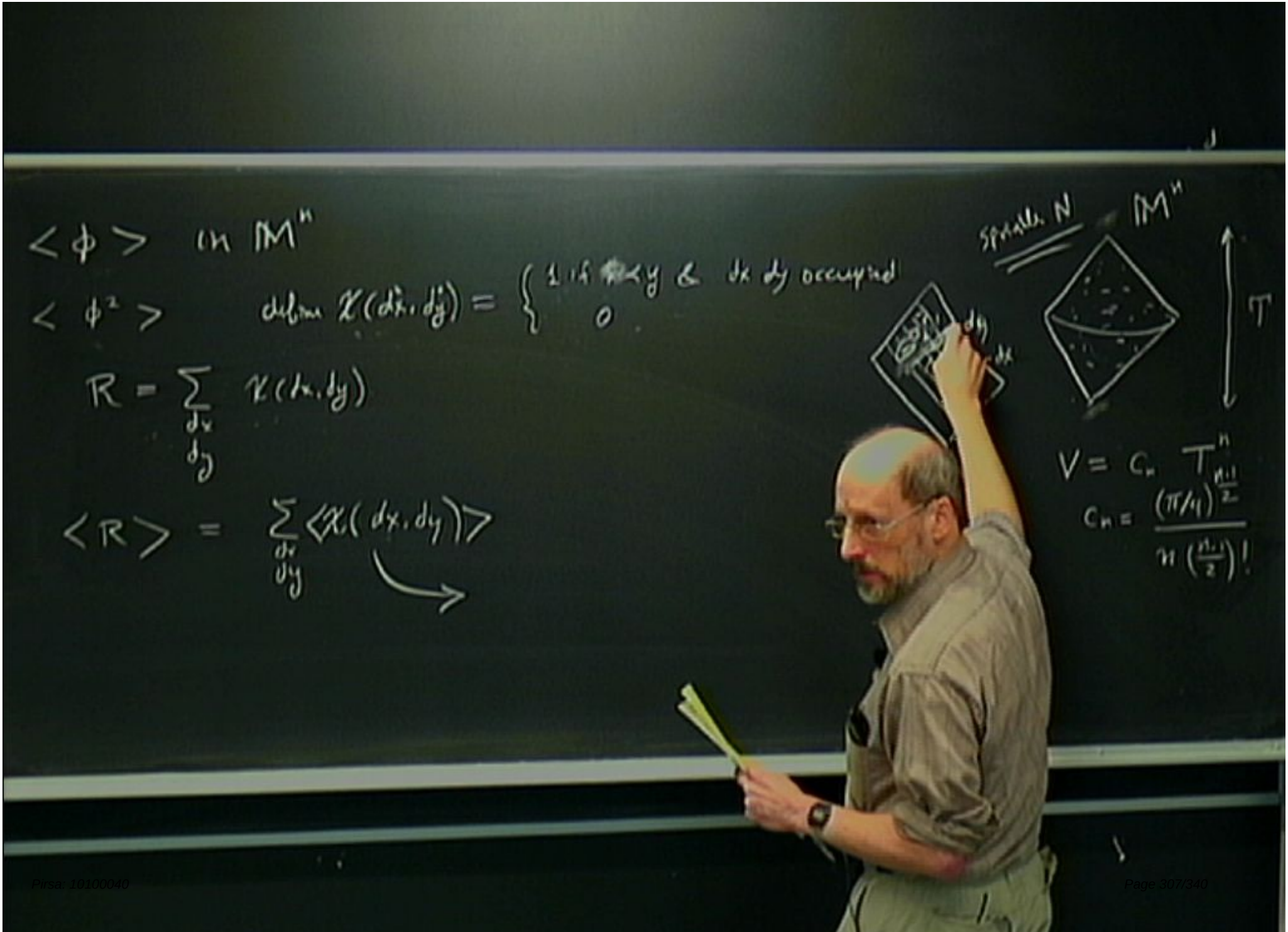
$$R = \sum_{\substack{d_x \\ d_y}} \chi(d_x, d_y)$$

$$\langle R \rangle = \sum_{\substack{d_x \\ d_y}} \langle \chi(d_x, d_y) \rangle$$


$\langle \phi \rangle$ in M^n
$\langle \phi^2 \rangle$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$
Small N
M^n
T
dy
dx
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$


$\langle \phi \rangle$ in M^n
$\langle \phi^2 \rangle$
define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x,y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$
Spatially N
M^n
T
dy
dx
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ & } dx\ dy \text{ occupied} \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$
Sprinkle N
$\mathbb{M}^n$
T
dy
dx
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$


$\langle \phi \rangle$ in $\mathbb{M}^n$
$\langle \phi^2 \rangle$
define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$
$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$
Sprinkle N
$\mathbb{M}^n$
T
$V = C_n T^n$
$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n\left(\frac{n}{2}\right)!}$

$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$

$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$

$\rightarrow \frac{N}{V} dx$

Sprinkle N

$\mathbb{M}^n$

T

$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n \left(\frac{n}{2}\right)!}$

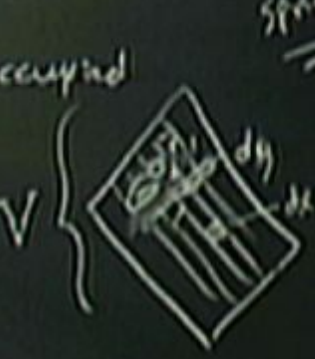
$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$

$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$

$\to \frac{N}{V} dx$





T^n

$\frac{(\pi/\mu)^{\frac{n+1}{2}}}{(\frac{n+1}{2})!}$

$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$

$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle \rightarrow \frac{N}{V} dx$

Sprinkle N

$\mathbb{M}^n$

V

T

$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n \left(\frac{n}{2}\right)!}$

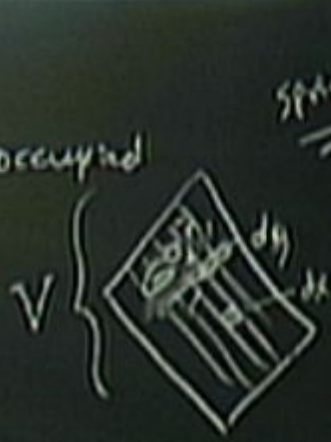
$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$

define $\mathcal{K}(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$$R = \sum_{\substack{dx \\ dy}} \mathcal{K}(dx, dy)$$

$$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \mathcal{K}(dx, dy) \rangle \longrightarrow \frac{N}{V} dx \frac{N-1}{V}$$





$$V = C_n T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$

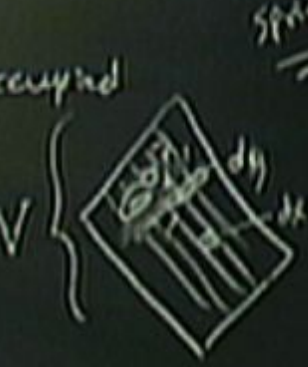
$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$

$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$

$\frac{N}{V} dx \quad \frac{N-1}{V} \quad \frac{dx}{V} \frac{dy}{V}$





$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n+1}{2}}}{n \left(\frac{n}{2}\right)!}$

$\langle \phi \rangle$ on $\mathbb{M}^n$

$\langle \phi^2 \rangle$

define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x, y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

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$V = C_n\ T^n$

$C_n = \dfrac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$

$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$

$\rightarrow \dfrac{N}{V} dx \dfrac{N-1}{V} dy = \dfrac{N(N-1)}{V} \dfrac{dx}{V} \dfrac{dy}{V}$ ⊛(

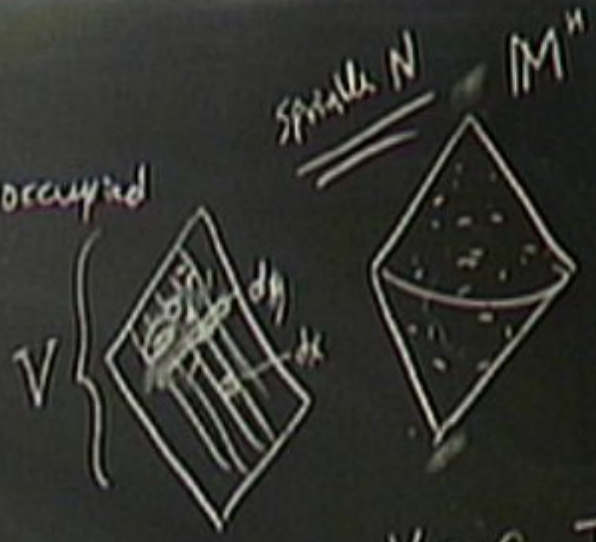
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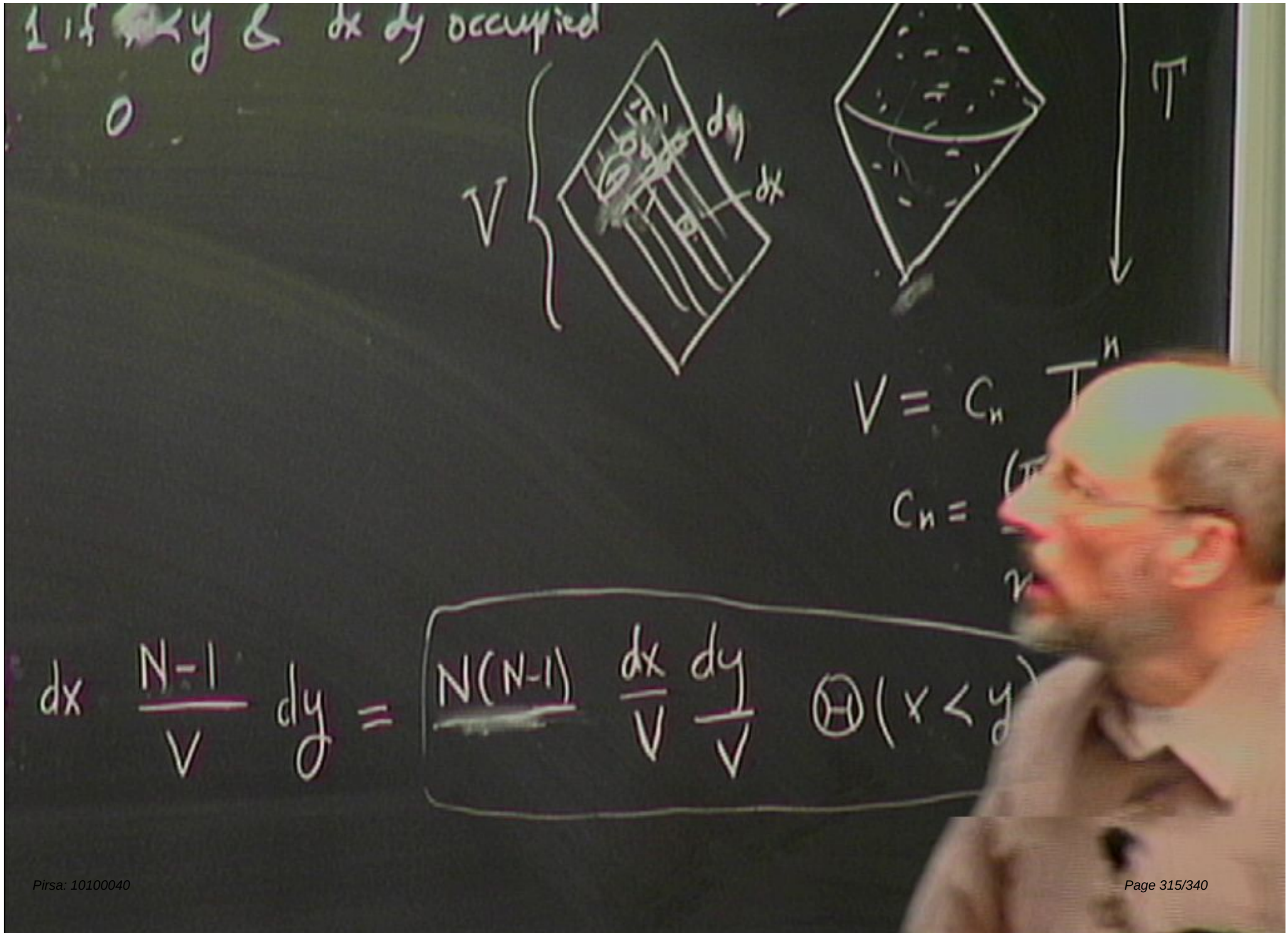
$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle$

$\frac{N}{V} dx \ \frac{N-1}{V} dy = \frac{N(N-1)}{V} \frac{dx}{V} \frac{dy}{V} \ \Theta(x < y)$



$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$


1 if x y & dx dy occupied
0
V
dy
dx
T
$V = C_n T^n$
$C_n =$
$dx \frac{N-1}{V} dy = \frac{N(N-1)}{V} \frac{dx}{V} \frac{dy}{V} \Theta(x<y$

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$\frac{N}{V} dx \ \frac{N-1}{V} dy = \frac{N(N-1)}{V^2} dx\, dy$

small N

$\mathbb{M}^n$

V

dy

dx

T

$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$

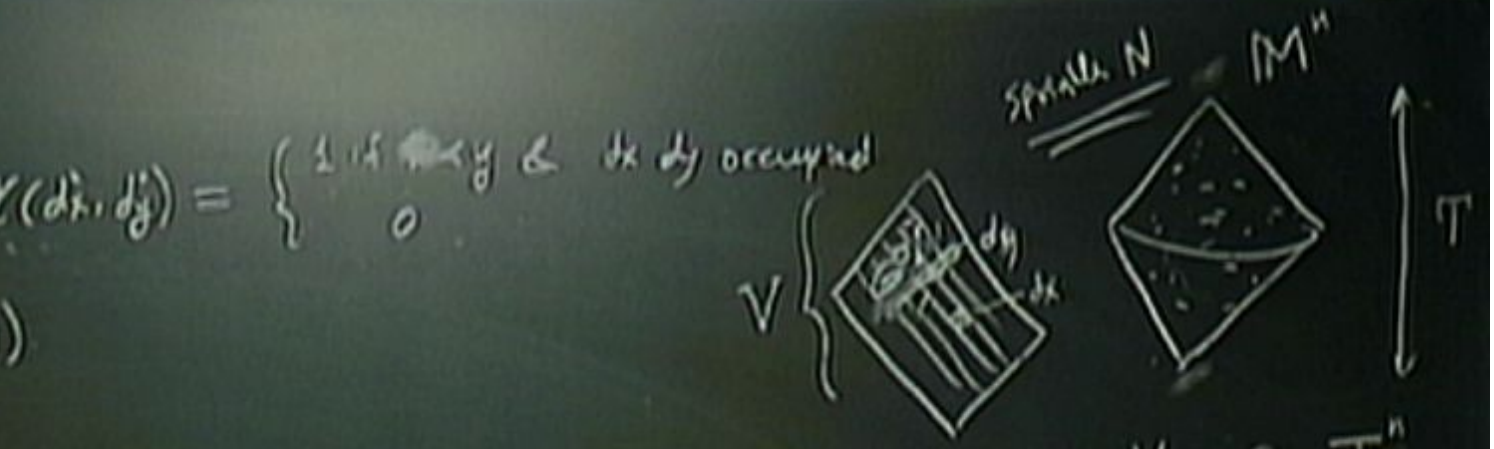
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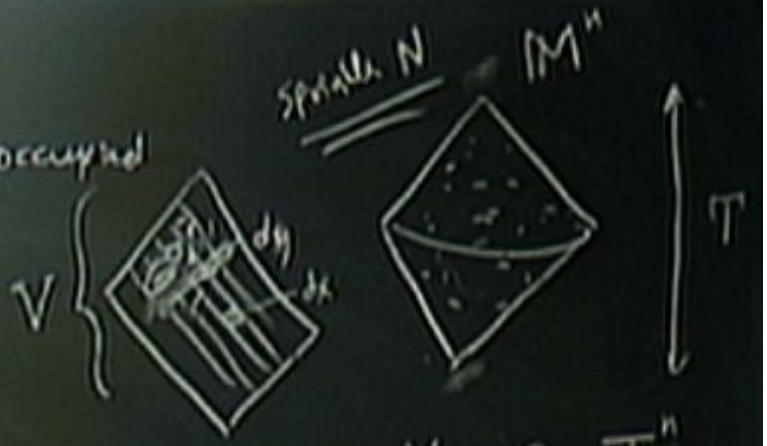
$V = C_n T^n$

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$\langle \phi \rangle$ in $\mathbb{M}^n$

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$\frac{N}{V} dx \ \frac{N-1}{V} dy = \boxed{N(N-1) \frac{dx}{V} \frac{dy}{V} \Theta(x < y)}$

$= \lim N(N-1) \int \int \frac{dx}{V}$

$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$

def. $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

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$\to \frac{N}{V} dx \frac{N-1}{V} dy = \frac{N(N-1)}{V} dx\, dy\, \Theta(x < y)$

$= \lim N(N-1) \iint_{x<y} dx\, dy \frac{dx}{V} \frac{dy}{V}$

Sprinkle N

$\mathbb{M}^n$

T

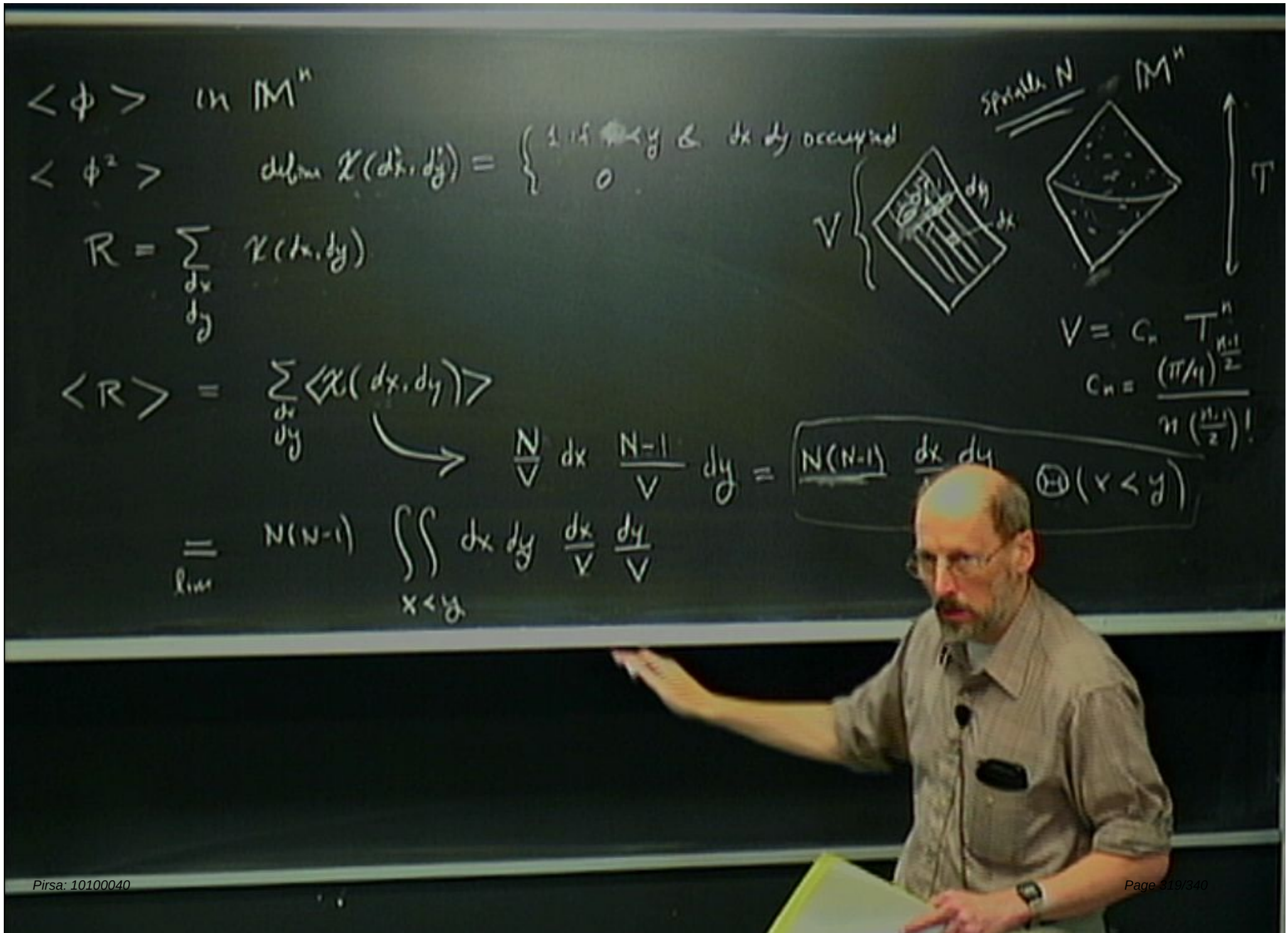
V

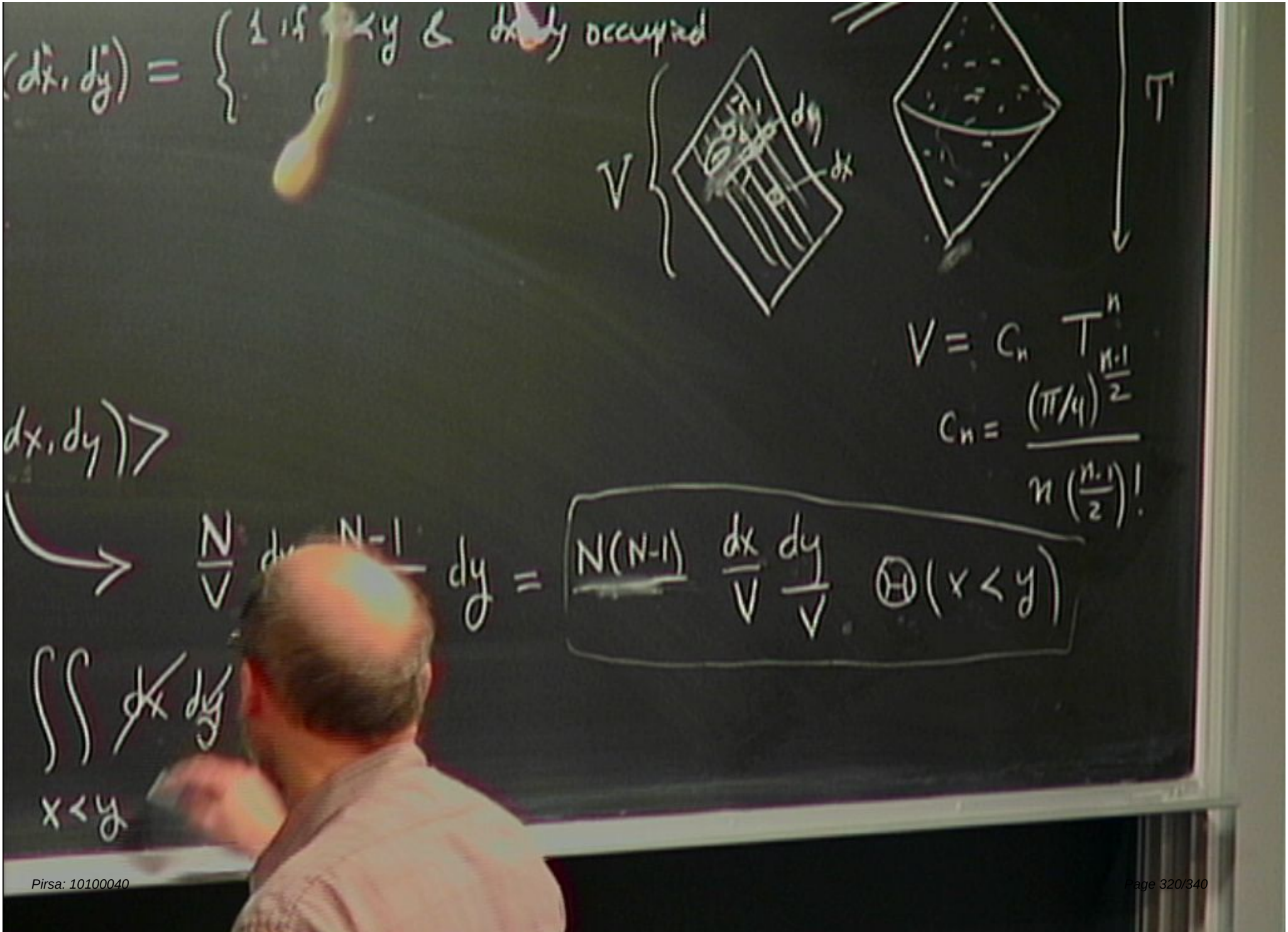
dy

dx

$V = C_n T^n$

$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\,(\frac{n-1}{2})!}$

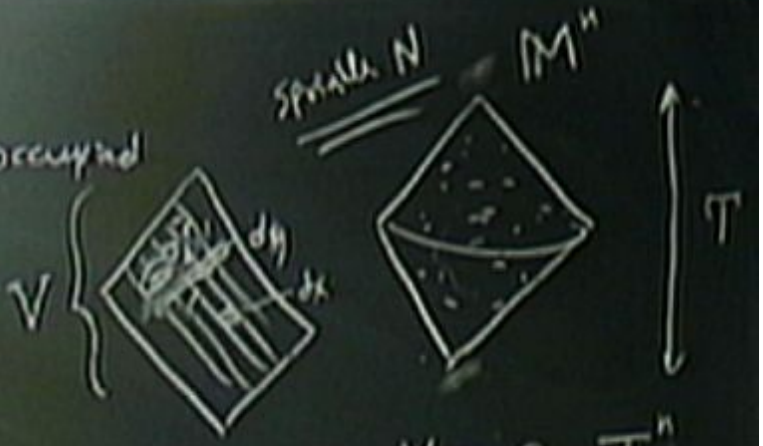



$(dx, dy) = \begin{cases} 1 \text{ if } x<y \ \& \ dx, dy \text{ occupied} \\ \end{cases}$
V
dy
dx
T
$V = C_n \, T^n$
$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$
$dx, dy\rangle$
$\frac{N}{V} \, d\ldots \, \frac{N-1}{\ldots} \, dy = \frac{N(N-1)}{} \, \frac{dx}{V} \frac{dy}{V} \, \Theta(x<y)$
$\iint_{x<y} dx \, dy$

$\langle \phi \rangle$ in $\mathbb{M}^n$

$\langle \phi^2 \rangle$ define $\chi(dx, dy) = \begin{cases} 1 \text{ if } x < y \text{ \& } dx\ dy \text{ occupied} \\ 0 \end{cases}$

$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$$



$$V = C_n\, T^n$$

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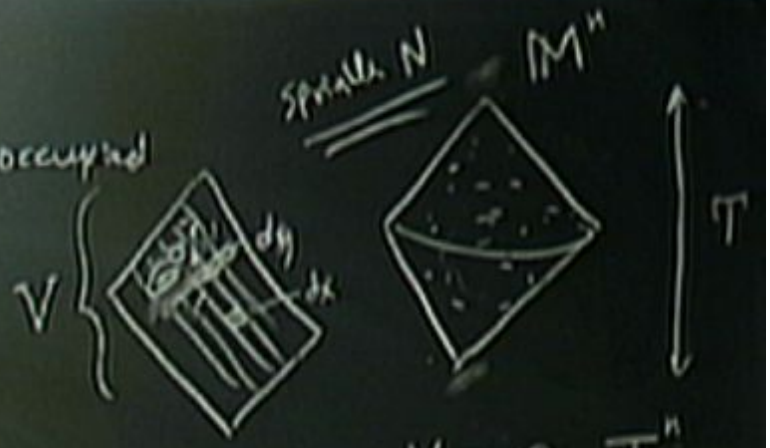
$$\langle R \rangle = \sum_{\substack{dx \\ dy}} \langle \chi(dx, dy) \rangle \longrightarrow \frac{N}{V} dx \, \frac{N-1}{V} dy = \boxed{\frac{N(N-1)}{V} \frac{dx}{V} \frac{dy}{V} \Theta(x < y)}$$

$$\underset{\text{lim}}{=} N(N-1) \iint_{x<y} dx\, dy \, \frac{dx}{V} \frac{dy}{V}$$

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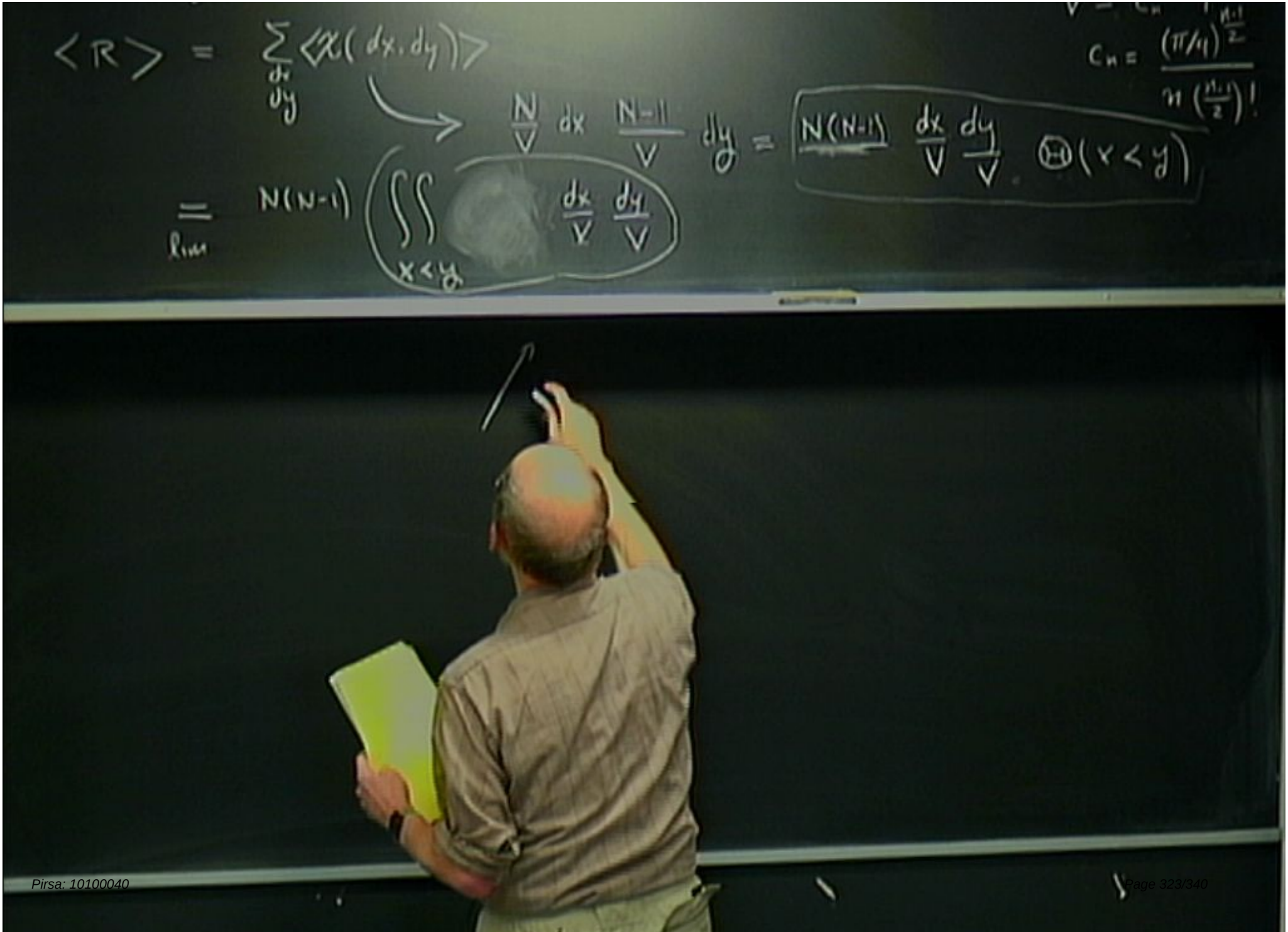


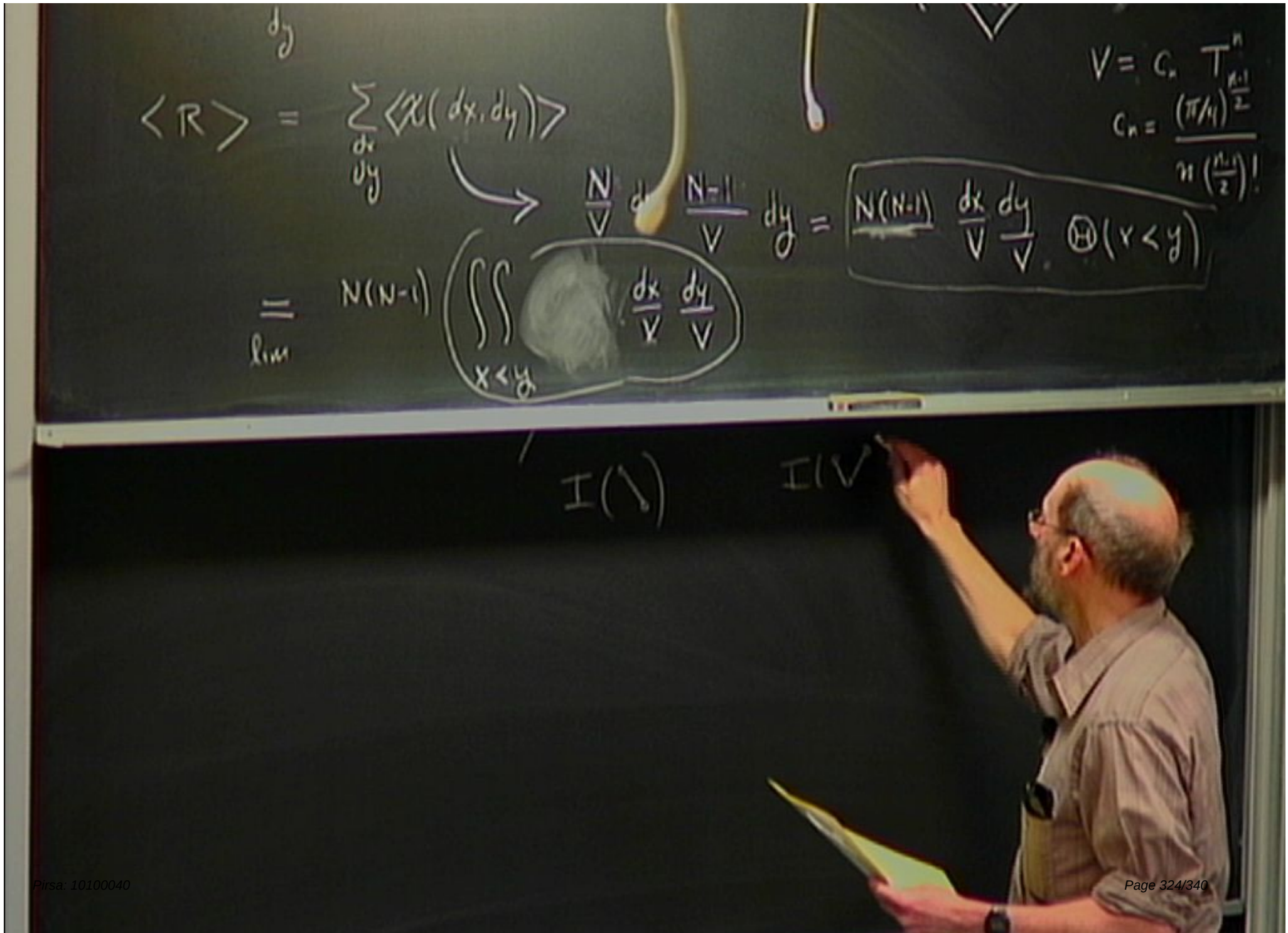
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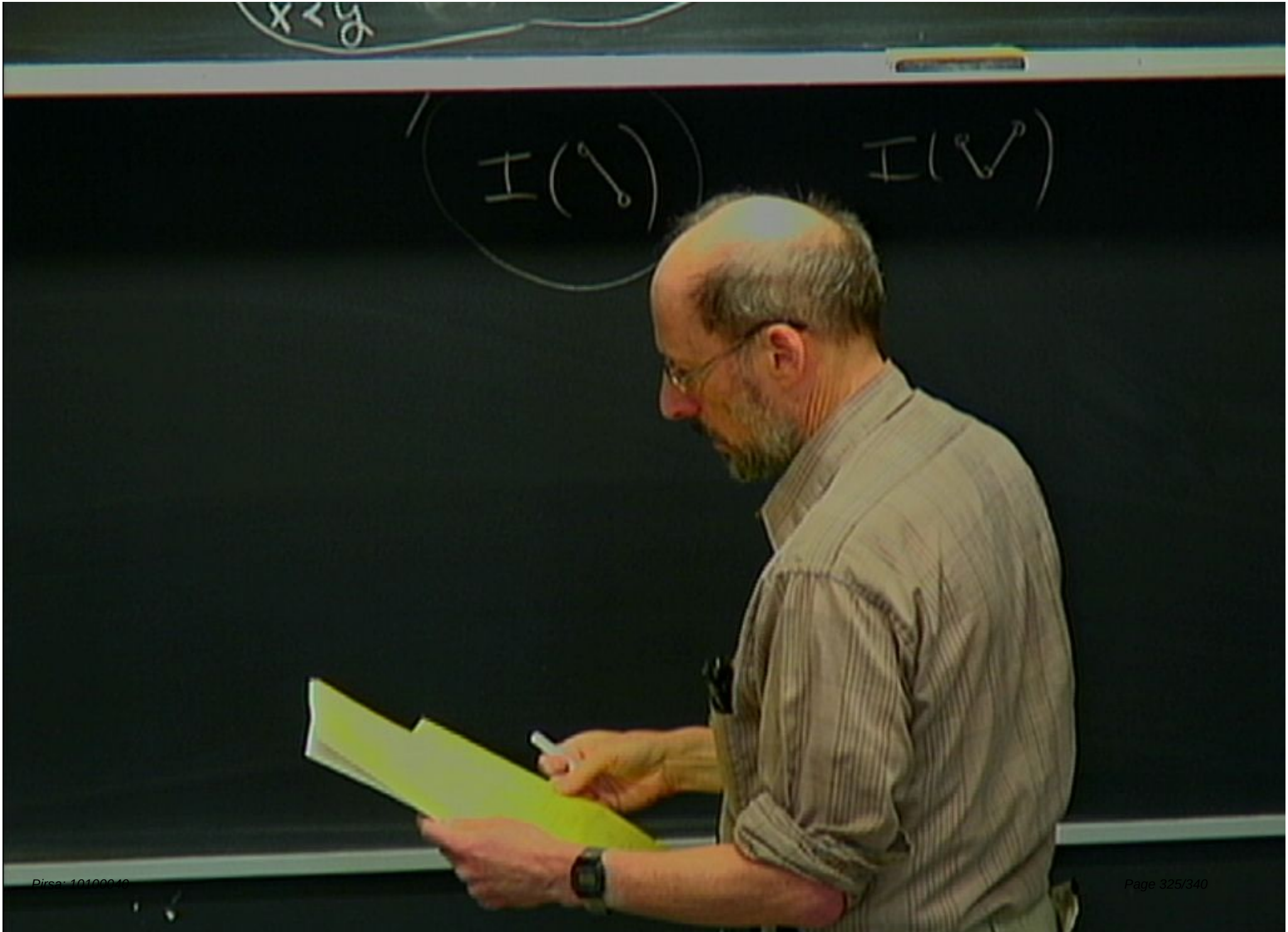
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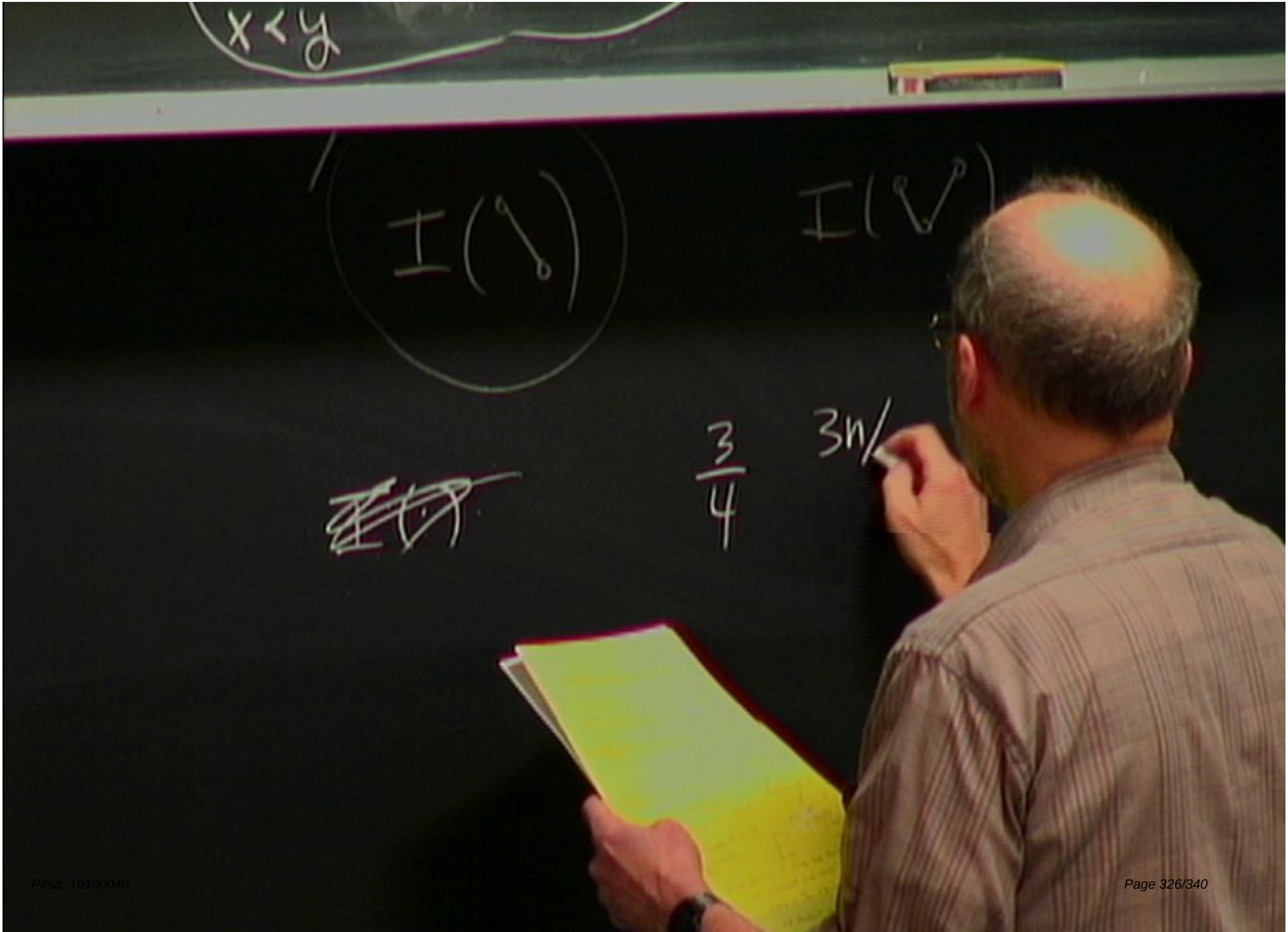
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$= \lim N(N-1) \iint_{x<y} \frac{dx}{V} \frac{dy}{V}$


$\langle R \rangle = \sum_{dx\,dy} \langle \mathcal{R}(dx, dy) \rangle$
$\frac{N}{V} dx \frac{N-1}{V} dy = \frac{N(N-1)}{V} \frac{dx}{V} \frac{dy}{V} \Theta(x<y)$
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$\langle R \rangle = \sum_{dx\,dy} \langle \mathcal{R}(dx, dy) \rangle$
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$\frac{N}{V} \frac{N-1}{V} dy = \frac{N(N-1)}{V} \frac{dx}{V} \frac{dy}{V} \Theta(x<y)$
$= \lim N(N-1) \iint_{x<y} \frac{dx}{V} \frac{dy}{V}$
$I(1)$
$I(V$


x<y
I(λ)
I(ν)


x<y
I(1)
I(V
3/4
3n/

$I(\lambda)$

$I(\nu)$

$\frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$

$I(\eta)$

$I(\rho V)$

$$\frac{3}{4}\binom{3n/2}{n}^{-1}$$

$$\langle \phi \rangle = \frac{\langle RZ}{N(N-1)} = I(\eta)$$

$I(\lambda)$

$I(V)$

$$\frac{3}{4}\binom{3n/2}{n}^{-1}$$



$$\langle \phi \rangle = \frac{\langle RZ}{N(N-1)} = I(\lambda)$$

$I(\nu)$

$I(\nu)$

$$\frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$$



$$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} \equiv I(\nu)$$

$I(\lambda)$

$I(V)$

$$\frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$$



$$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} = I(\theta)$$

$I(1)$

$I(V)$

$$\frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$$



$$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} = I(1)$$

$I(\eta)$ $I(V)$



$$\frac{3}{4}\binom{3n/2}{n}^{-1}$$

$$\langle\phi\rangle = \frac{\langle R^2\rangle}{N(N-1)} = I(\eta)$$

$I(\lambda)$

$I(V)$

$$\frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$$



$$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} = I(\vartheta)$$

$\frac{1}{V}$ $\frac{1}{V}$

$\langle y$

$I(\nu)$ $I(V)$

ϕ



h

$$\frac{3}{4}\binom{3n/2}{n}^{-1}$$

$$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} = I(\vartheta)$$

$$R = \sum_{\substack{dx \\ dy}} \chi(dx, dy)$$

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$$\frac{N}{V} dx \; \frac{N-1}{V} dy = \frac{N(N-1)}{} \; \frac{dx}{V} \frac{dy}{V} \; \Theta(x < y)$$

$$\underset{\text{Lim}}{=} N(N-1) \iint_{x<y} \frac{dx}{V} \frac{dy}{V}$$

$$V = C_n T^n$$

$$C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n \left(\frac{n-1}{2}\right)!}$$

$I(\lambda)$ $I(\lambda')$

$$\frac{3}{4} \binom{3n/2}{n}^{-1}$$

$$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} = I(\lambda)$$

$$R = \sum_{dx\,dy} \chi(dx, dy)$$

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$$\frac{N}{V}\,dx\;\frac{N-1}{V}\,dy = \frac{N(N-1)}{V}\,\frac{dx}{V}\,\frac{dy}{V}\,\Theta(x<y)$$

$$= \lim N(N-1) \iint_{x<y} \frac{dx}{V}\,\frac{dy}{V}$$

$$V = C_n\,T^n \qquad C_n = \frac{(\pi/4)^{\frac{n-1}{2}}}{n\left(\frac{n-1}{2}\right)!}$$

$I(\lambda)$ $\qquad$ $I(V)$

$$\frac{3}{4}\binom{3n/2}{n}^{-1}$$

$$\langle \phi \rangle = \frac{\langle R Z \rangle}{N(N-1)} = I(\lambda)$$

Pirsa: 10100040

$\frac{1}{V}$ $\frac{1}{V}$

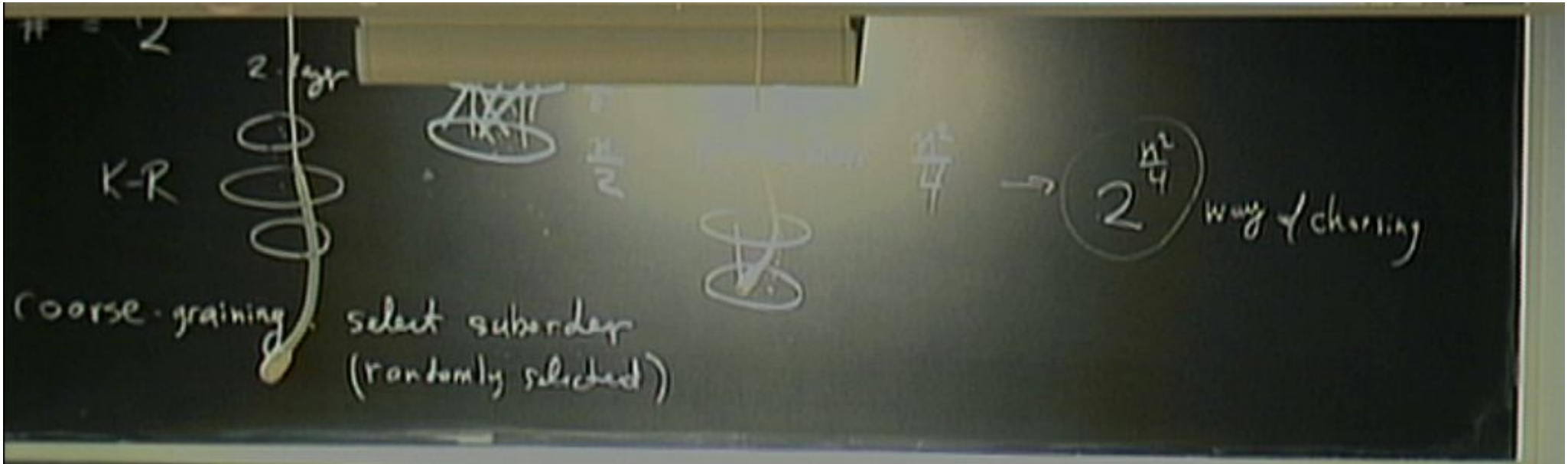
$\langle y$

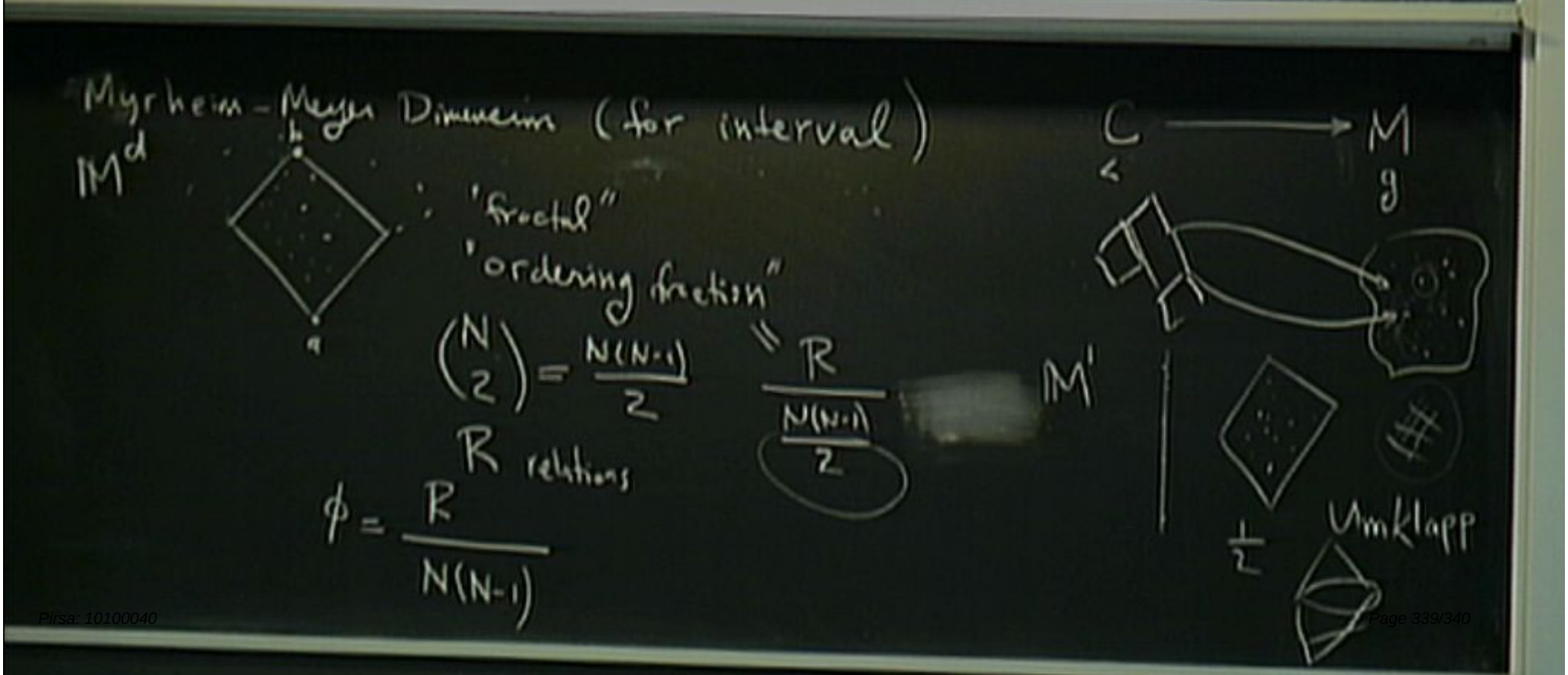
$I(\nu)$ $I(V)$

$\frac{3}{4}\left(\frac{3n/2}{n}\right)^{-1}$



$\langle \phi \rangle = \frac{\langle R \rangle}{N(N-1)} = I(\vartheta)$





Pirsa: 10100040

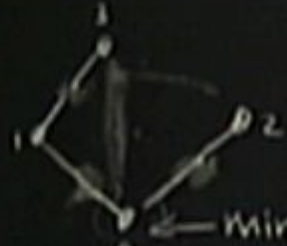
Key: locally finite → past-finite

$a < c < a$

Joe Henson PIRSA 1009 0092 (14/9/2010)

Stembridge - Maple
RDS → Lisp

other rep: graph, matrix, T_0-topology, ... → space + open set



$e_0\ e_1\ e_2\ e_3$

$e_0 < e_1 < e_3,\ e_0 < e_2,\ e_0 < e_3$

minimal (1)

Hasse diagram
only links drawn

di-graph (exclude)



causal matrix
Inf. matrix



Def ... if $\nexists z$ s.t. $x < z < y$