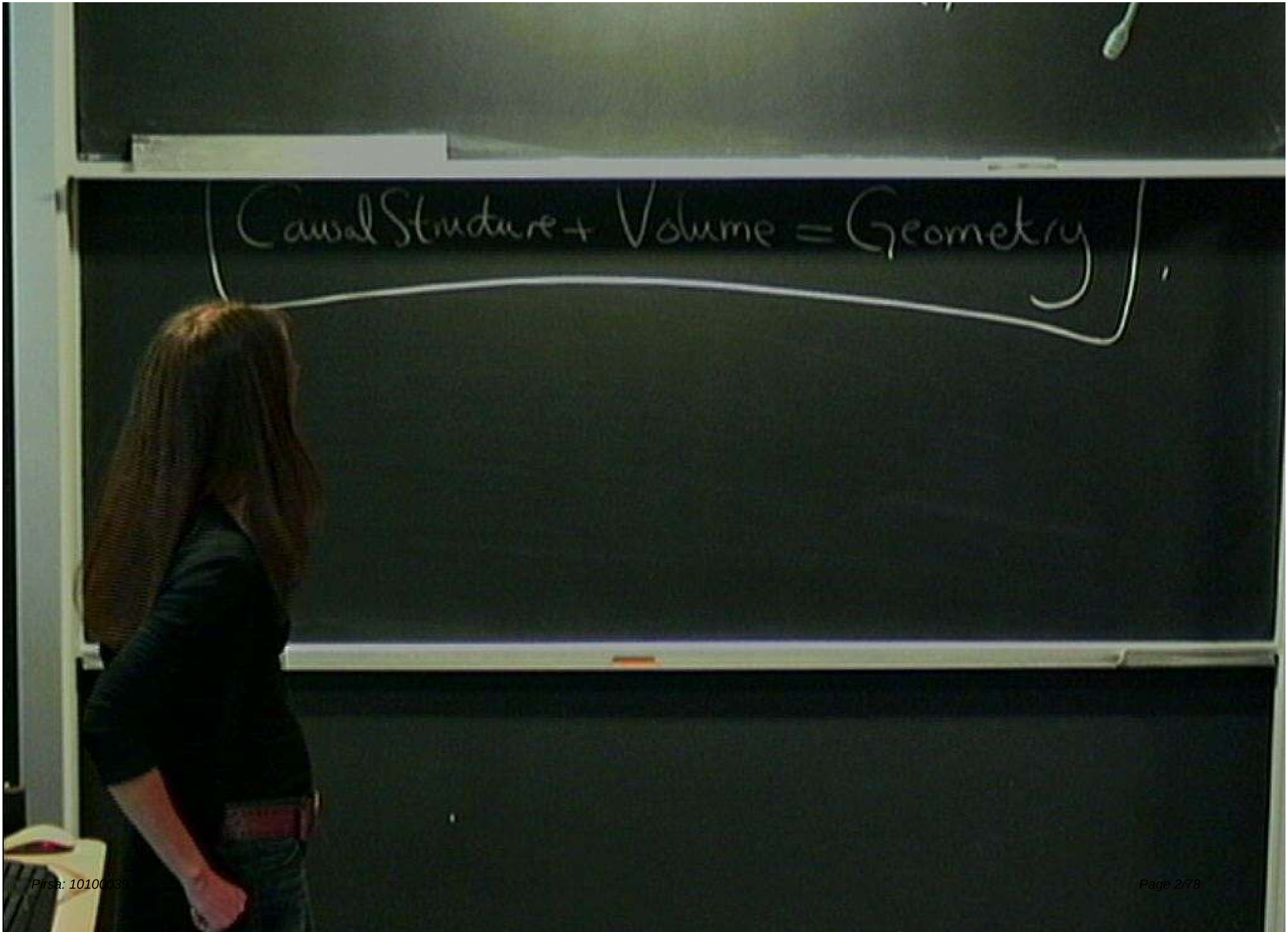
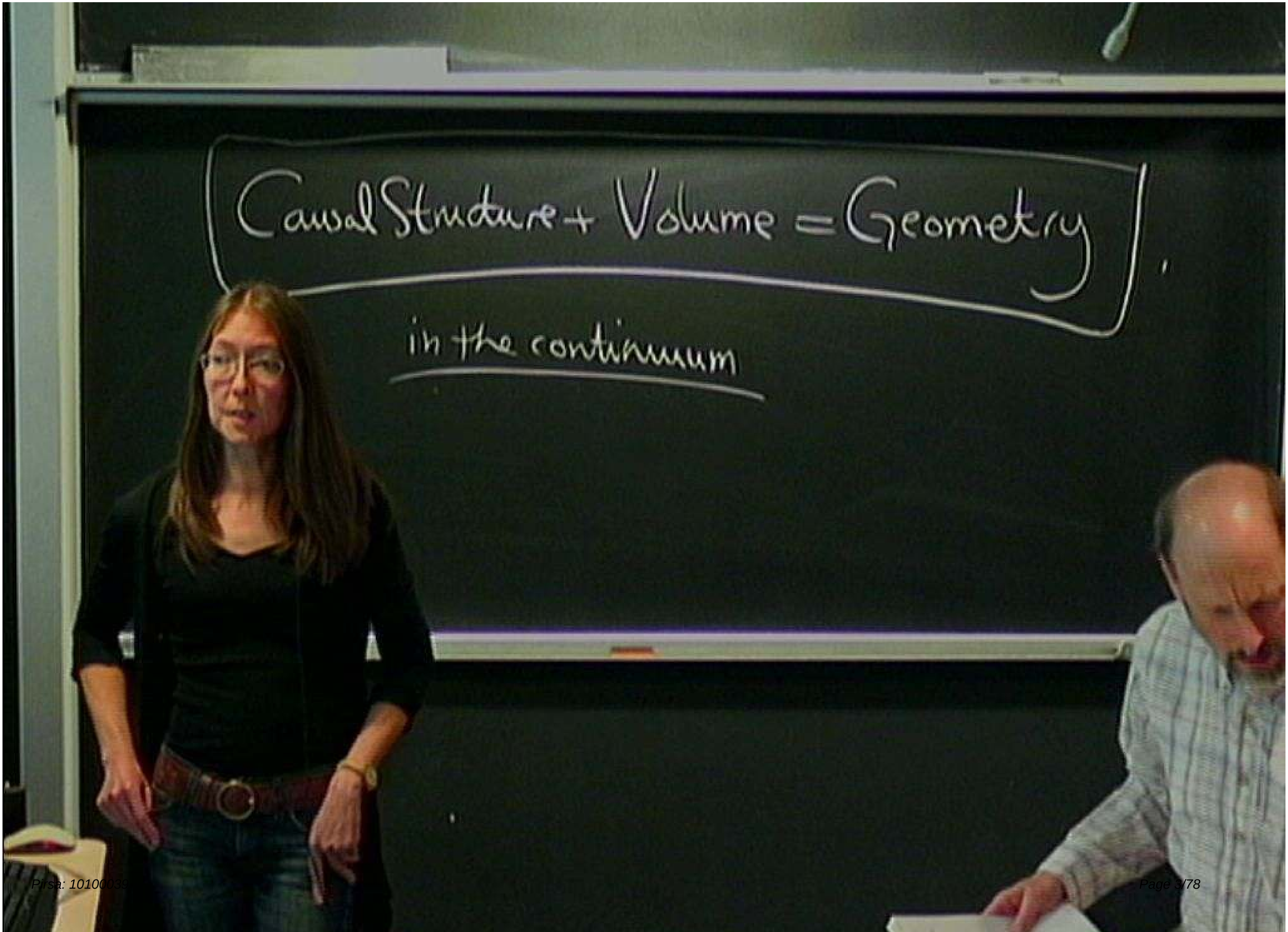
Title: An Invitation to Causal Sets - Lecture 2

Date: Oct 19, 2010 11:30 AM

URL: http://pirsa.org/10100039

Abstract: Introduction to the causal set approach to quantum gravity and overview of current research in causal set theory


Causal Structure + Volume = Geometry


Causal Structure + Volume = Geometry
in the continuum

To discretise spacetime, let us do as we did for the substance-in-a-box: distribute discrete, identical "atoms of spacetime" throughout spacetime in such a way as respects the amount of spacetime in any region, in other words the spacetime volume.

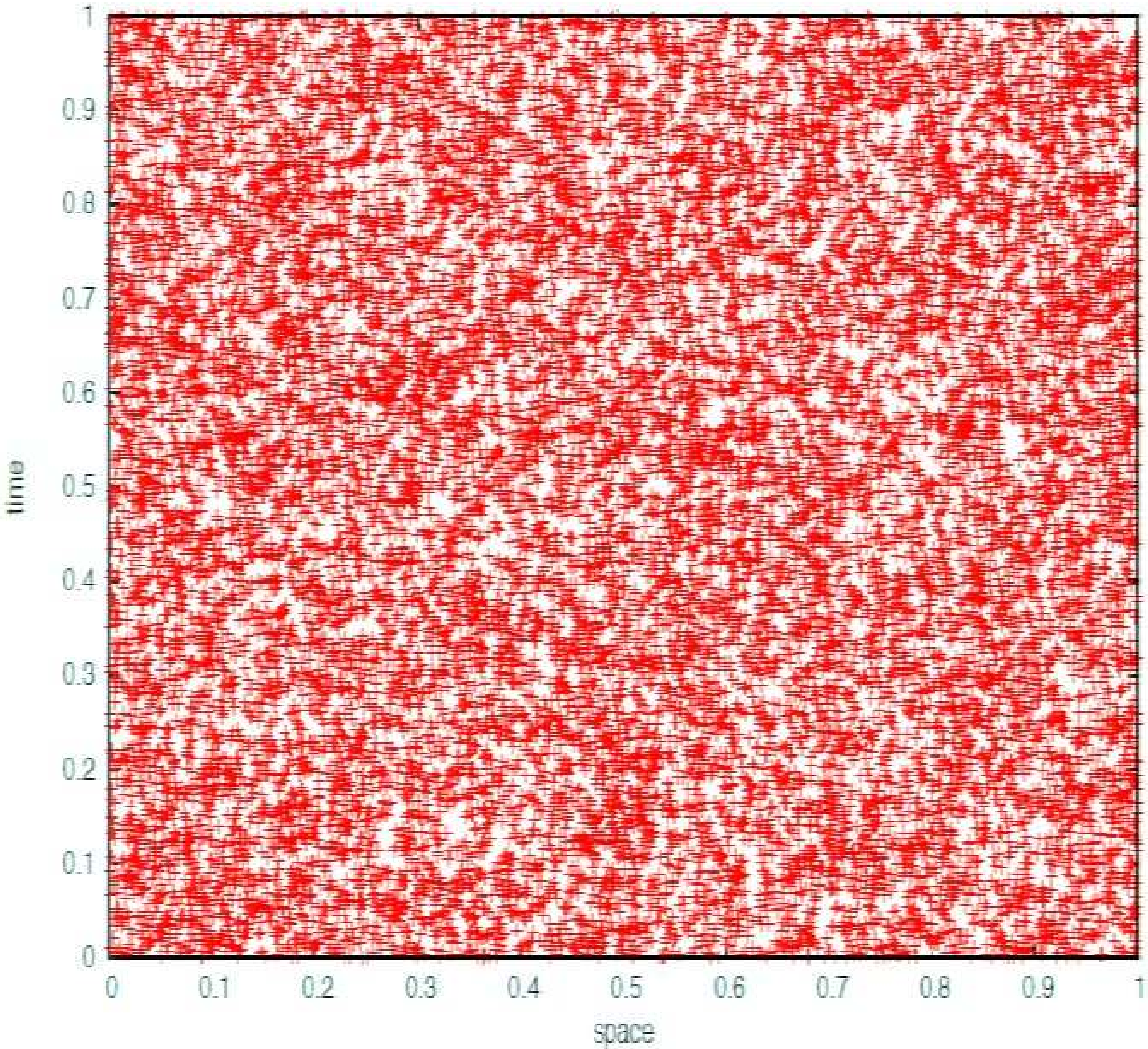
What plays the role of the mass density is $\sqrt{-g(x)}d^4x$ the spacetime volume element and integrating it over any region gives the spacetime volume of that region.

So we place the "atoms" (henceforth elements) in spacetime so that, roughly,

$$N(\text{region}) = \int_{\text{region}} \sqrt{-g(x)}\, d^4x$$

where the volume is measured in fundamental (near Planck) units.
We can satisfy this requirement by distributing the elements randomly by a Poisson process we refer to as "sprinkling".

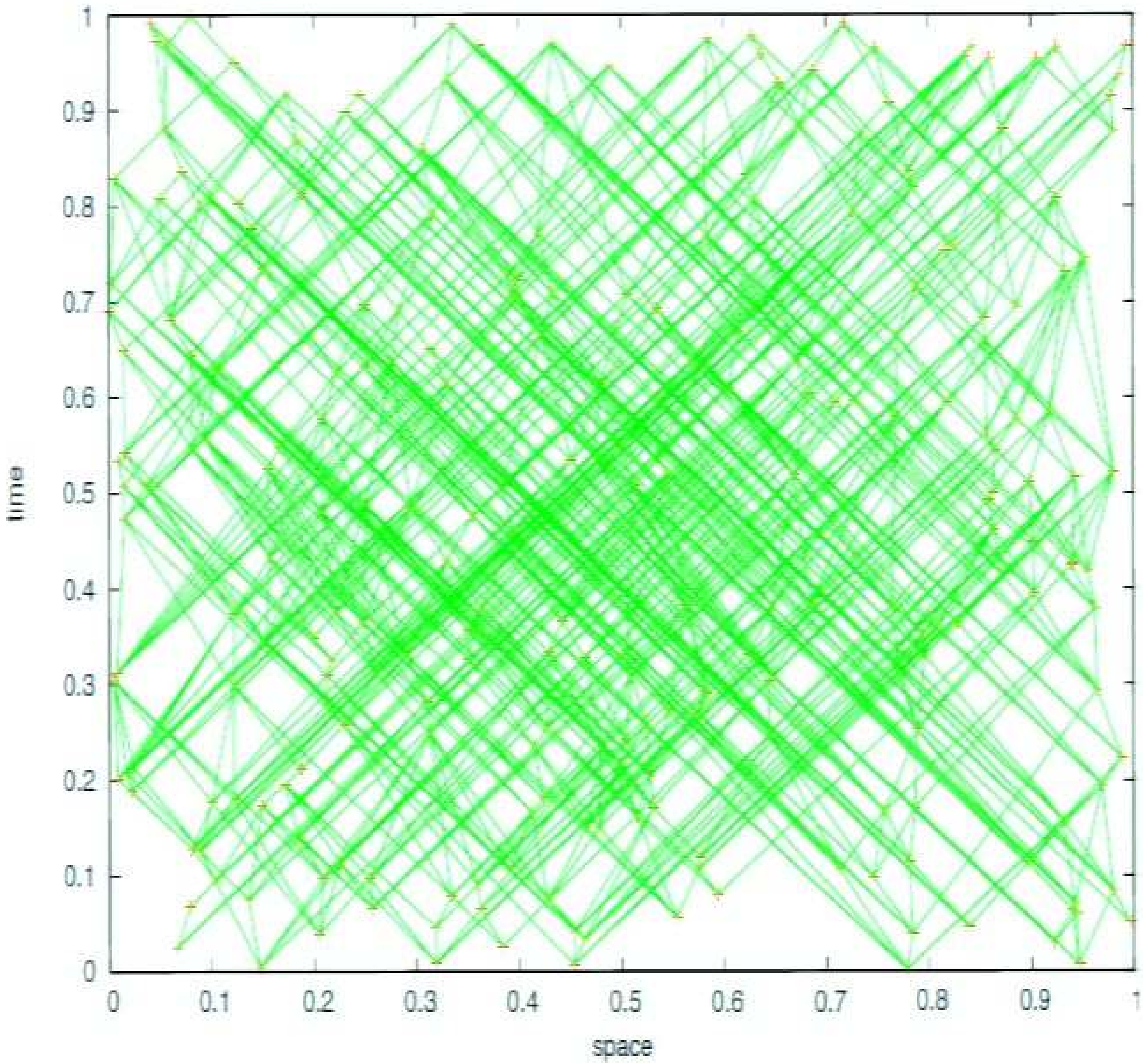
A sprinkling into 1+1 Minkowski spacetime



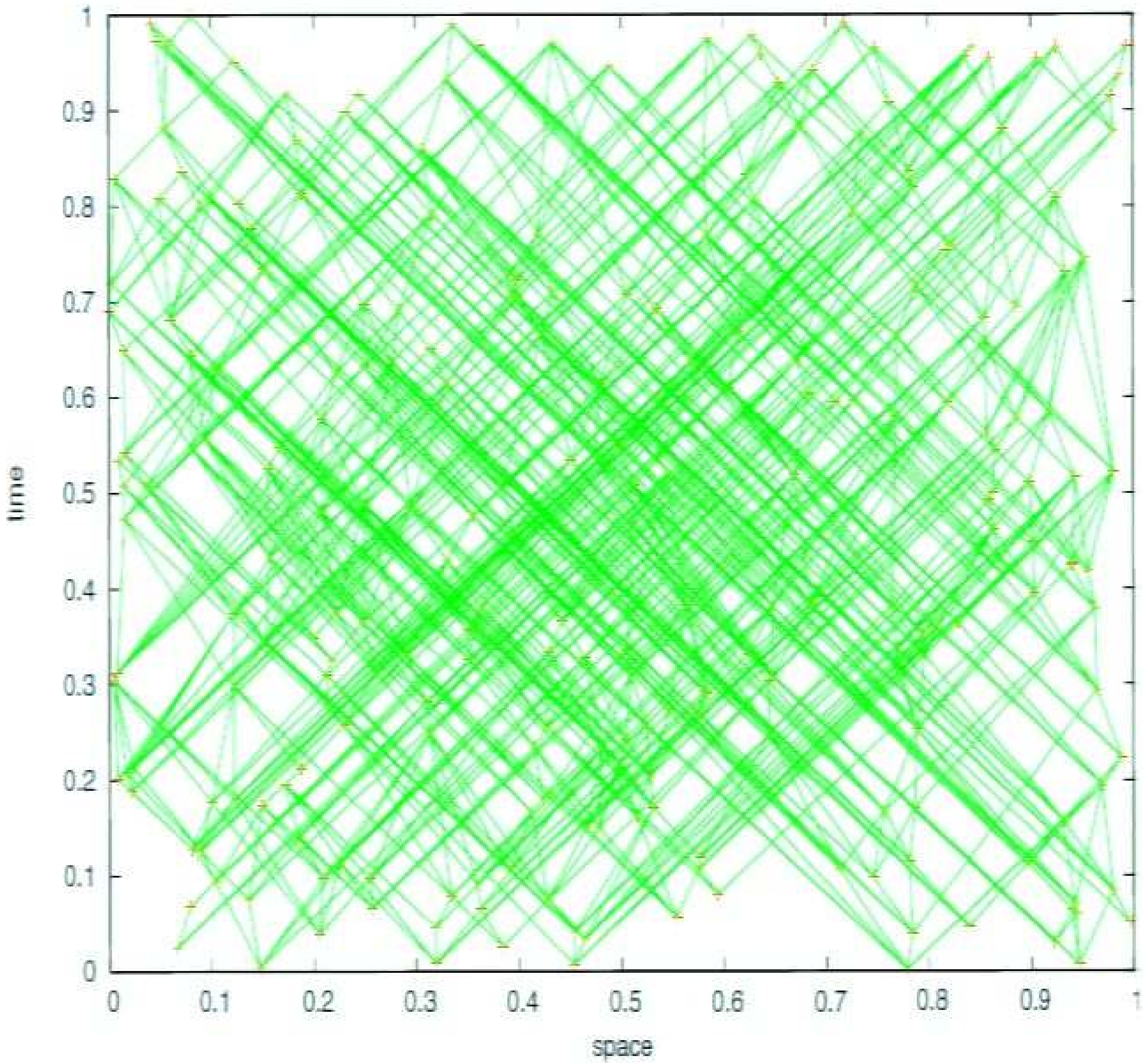
This distribution is Lorentz invariant: it does not pick out a frame

Removing the continuum (spacetime itself) now will result in the elements collapsing in a heap: the set has, as yet, no structure beyond cardinality. What information could we add to the set of atoms to make it "hold its geometric form"? What completes Number/Volume to give Geometry? Causal structure! (I set you up for this!)

So, after sprinkling, we endow the elements with the order relation inherited from the spacetime causal relation:
set $x \preceq y$ iff $x \leq y$ in the continuum



Causal structure is Lorentz invariant so, again, no frame is distinguished



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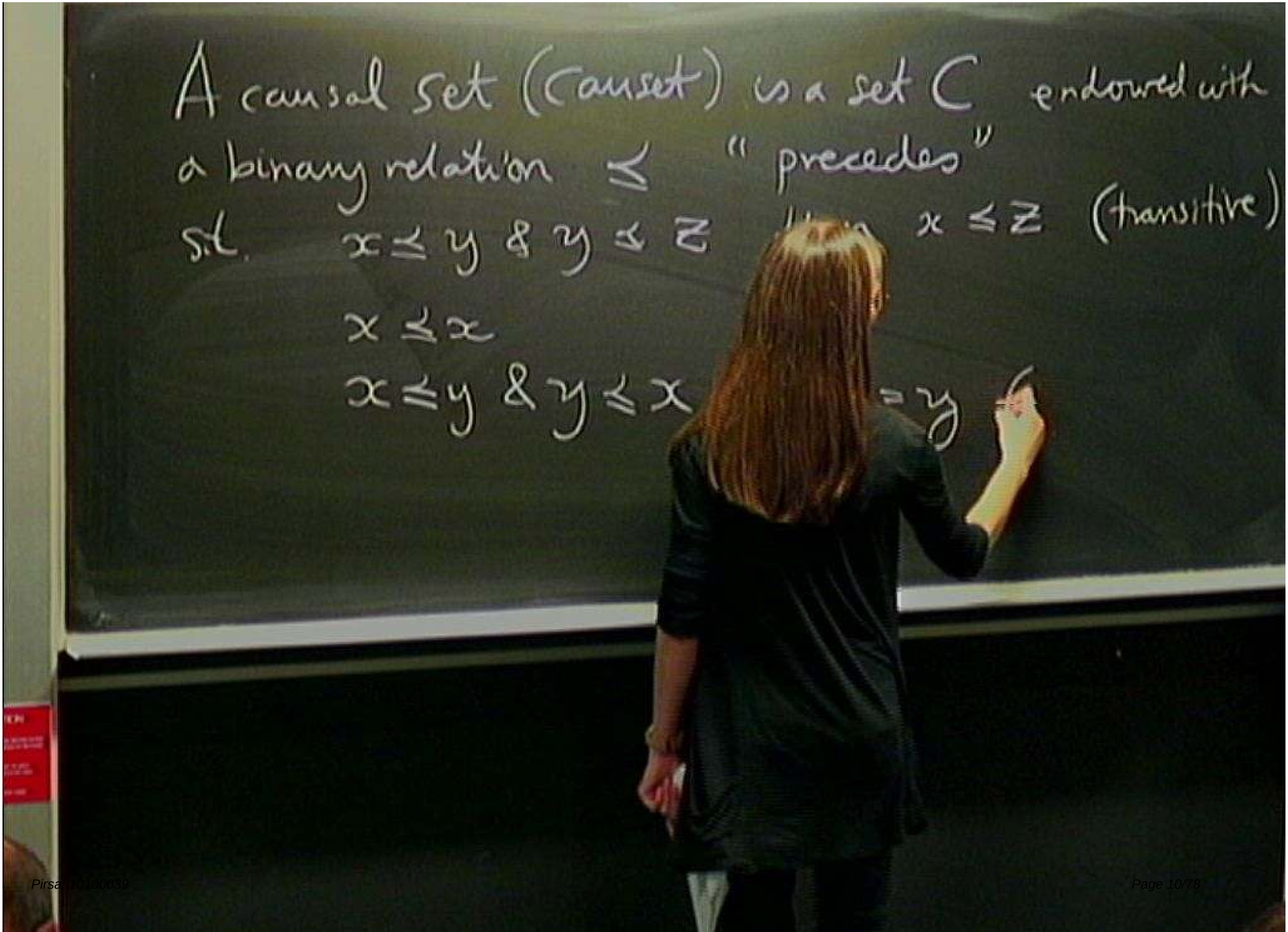
Now throw away the continuum

- What is the structure we have created? (A causal set)
- Does it adequately capture the continuum geometry? (Very likely)
- We propose them as the discrete manifolds in the SOH for quantum gravity (The Causal Set Hypothesis)


A causal set (causet) is a set C endowed with
a binary relation ≤ "precedes"
s.t. x ≤ y & y ≤ z
x ≤ z (transitive)
x ≤ x
x ≤ y & y ≤ x
= y

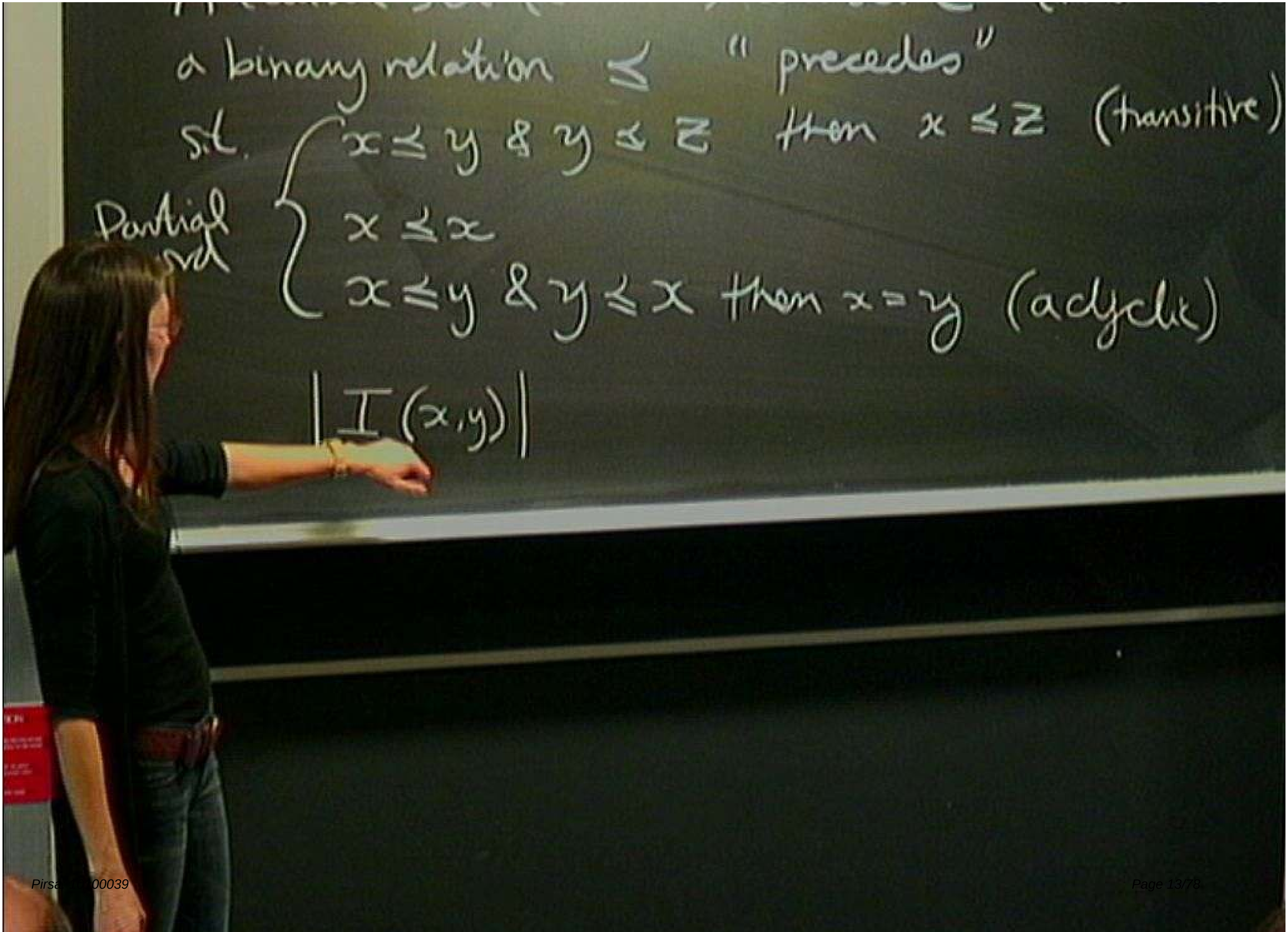
A causal set (causet) is a set C endowed with a binary relation $\preceq$ "precedes"

s.t. $x \preceq y$ & $y \preceq z$ then $x \preceq z$ (transitive)

$x \preceq x$

$x \preceq y$ & $y \preceq x$ then $x = y$ (acyclic) a binary relation $\preceq$ "precedes"

s.t. Partial ord $\begin{cases} x \preceq y \ \& \ y \preceq z \text{ then } x \preceq z \text{ (transitive)} \\ x \preceq x \\ x \preceq y \ \& \ y \preceq x \text{ then } x = y \text{ (acyclic)} \end{cases}$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Partial
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$|I(x,y)|$

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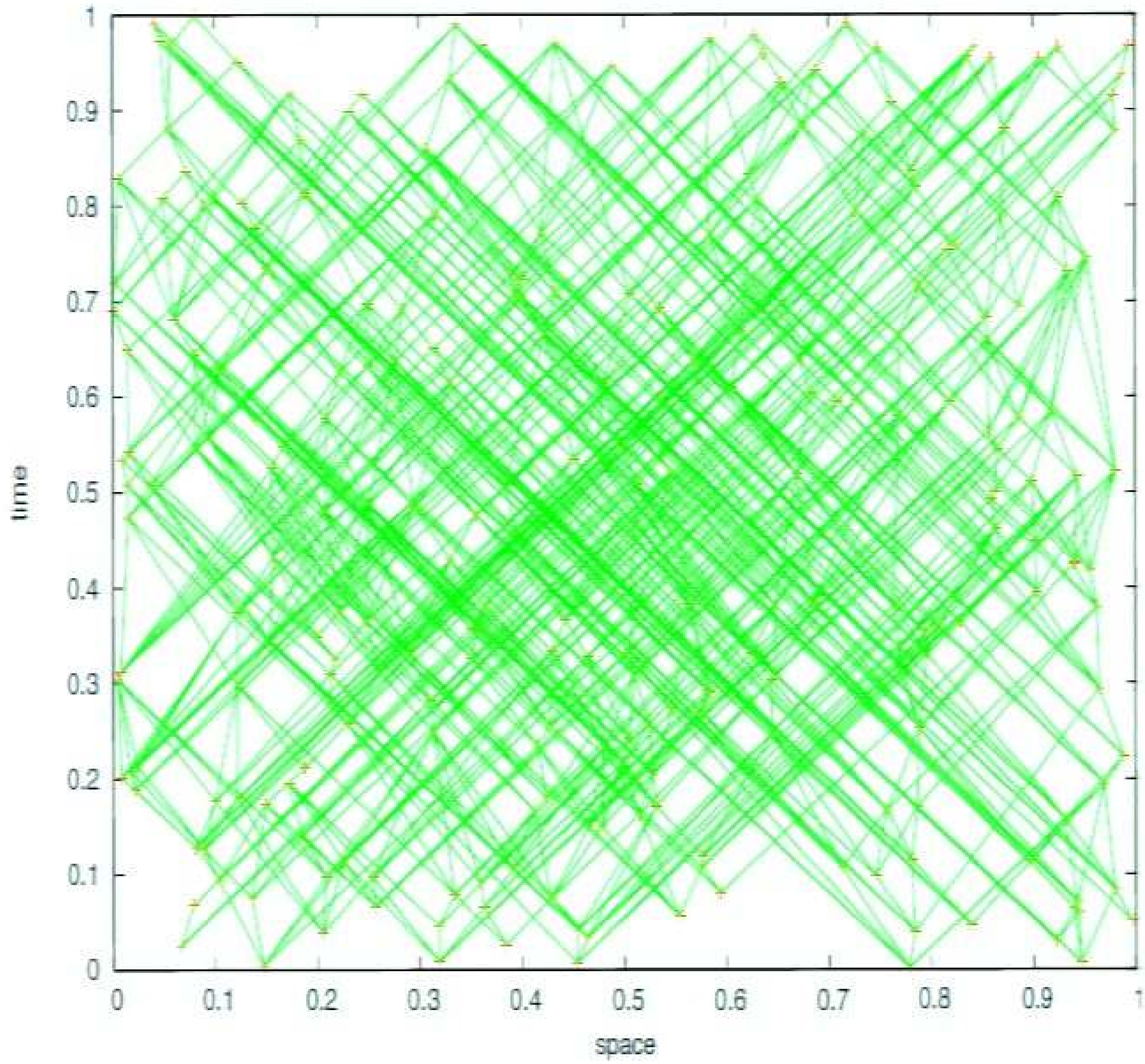
s.t.

Partial ord

$$\begin{cases} x \preceq y \ \& \ y \preceq z \ \text{then} \ x \preceq z & \text{(transitive)} \\ x \preceq x & \\ x \preceq y \ \& \ y \preceq x \ \text{then} \ x = y & \text{(acyclic)} \end{cases}$$

$|I(x,y)| < \infty$ where $I(x,y) = \{z \in C \mid y \preceq z \preceq x\}$

(local finiteness).



Causal structure is Lorentz invariant so, again, no frame is distinguished

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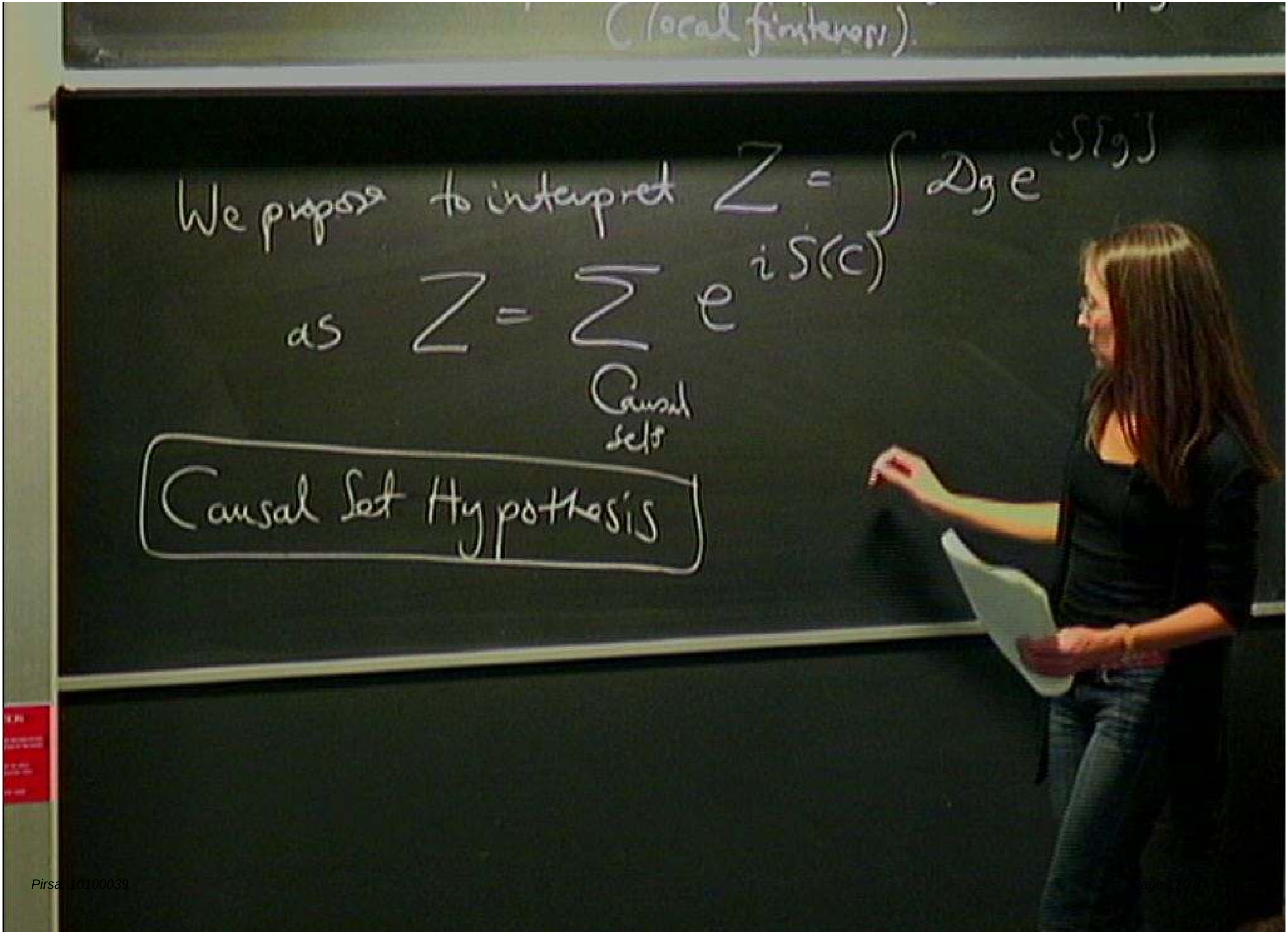
We propose to interpret $Z = \int \mathcal{D}g \, e^{iS[g]}$

as $Z = \sum_{\text{causal sets}} e^{iS(C)}$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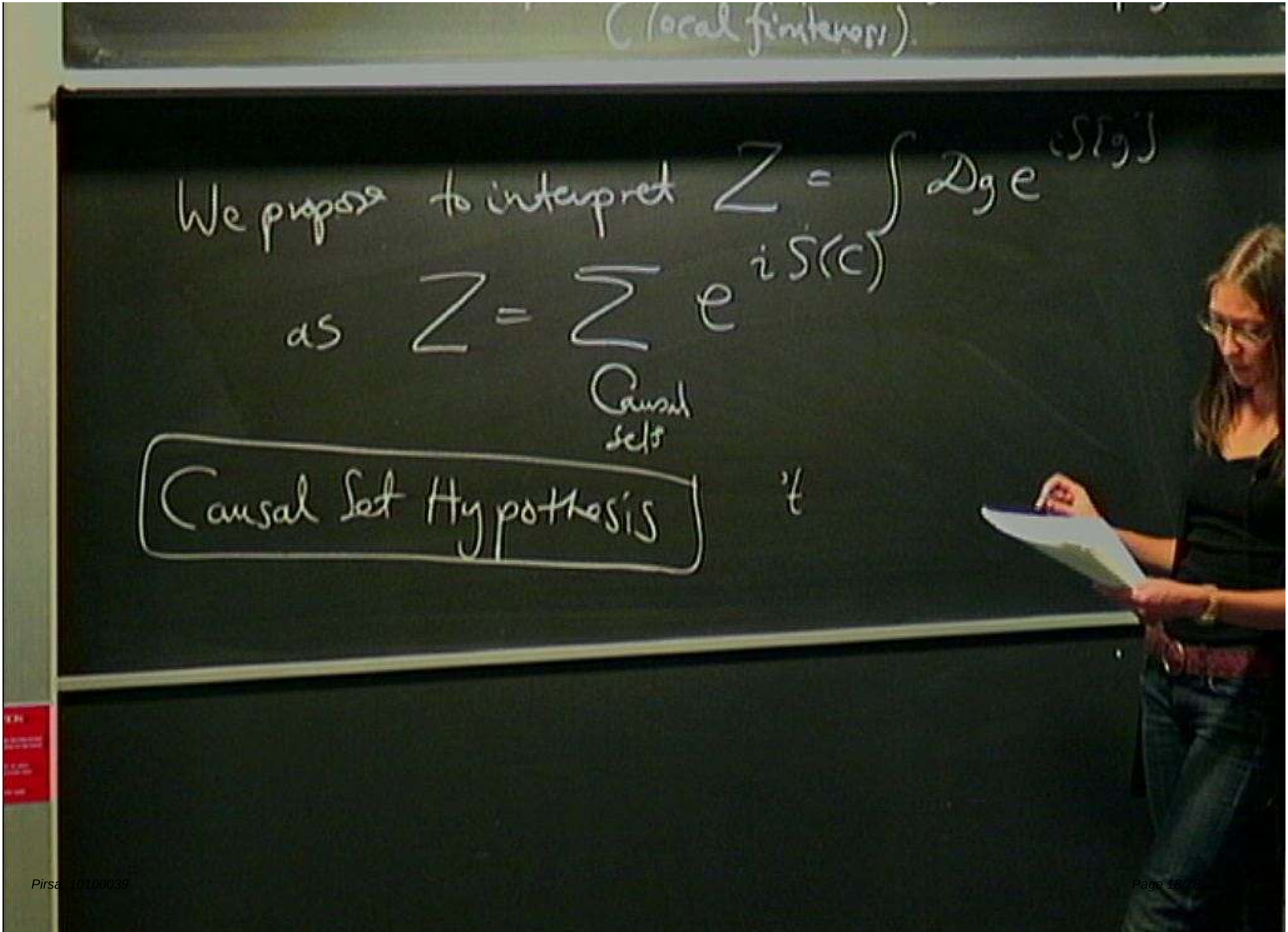
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G. 't Hooft, J. Myrhei

L. Bombelli, J.

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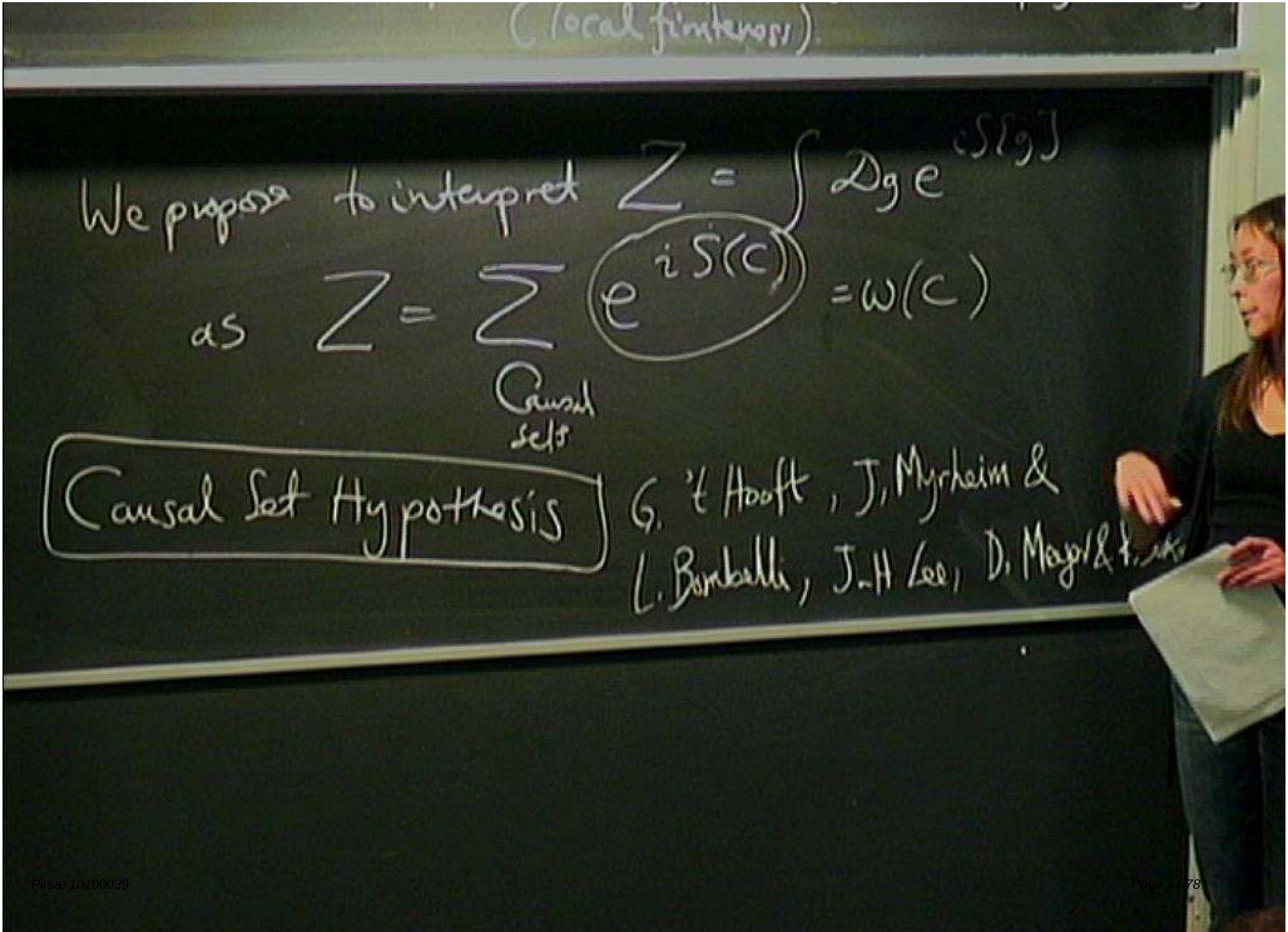
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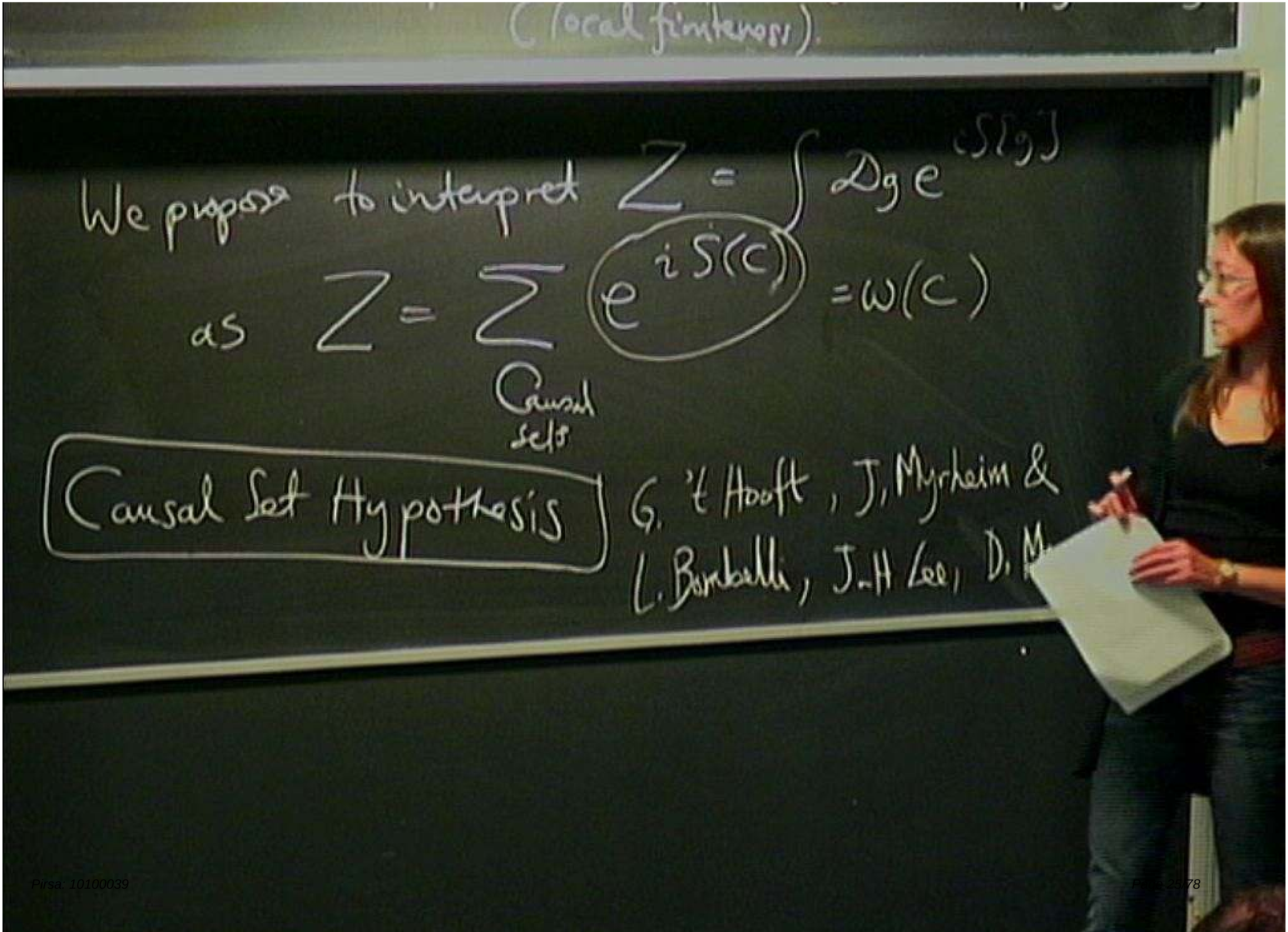
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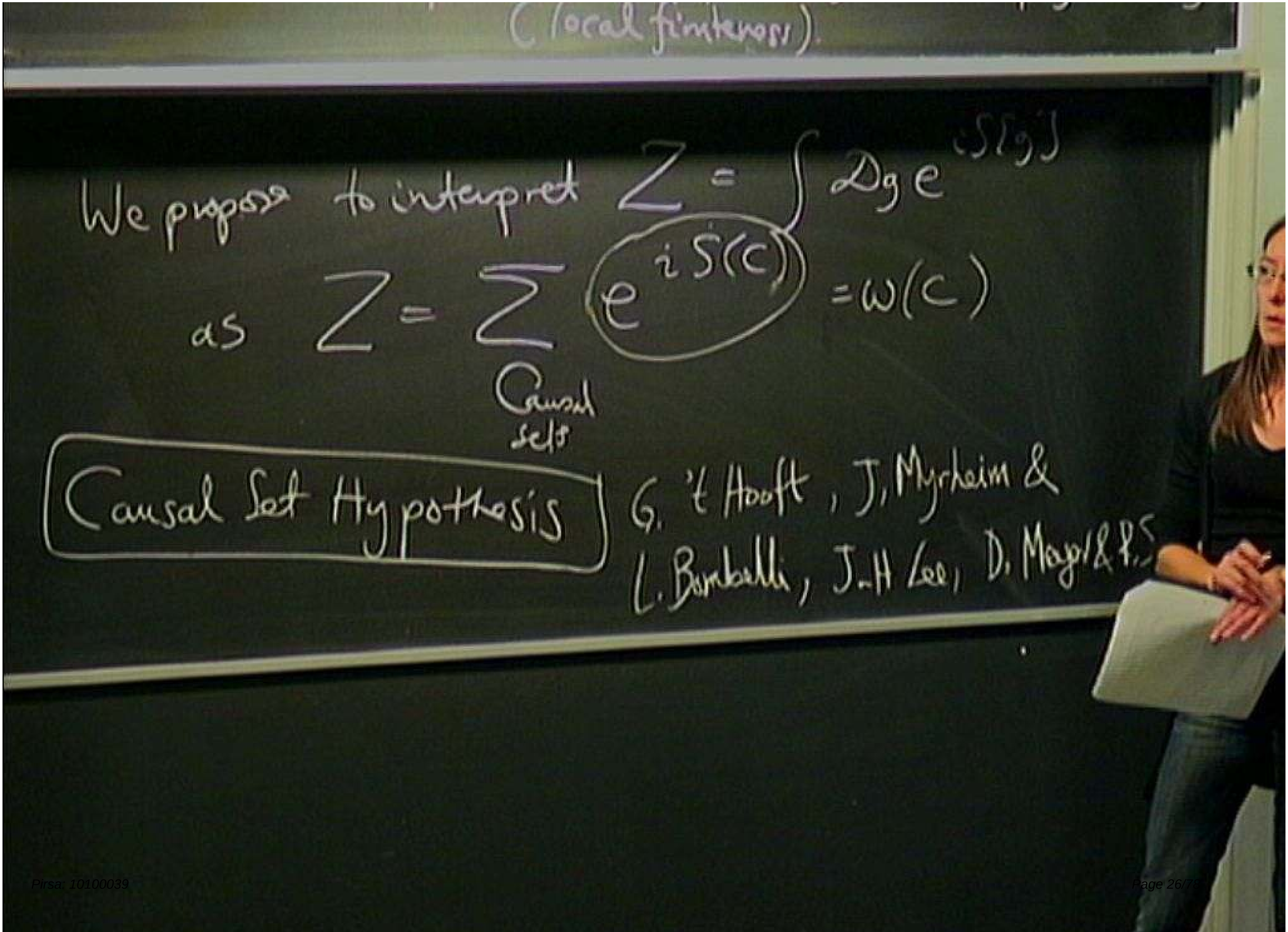
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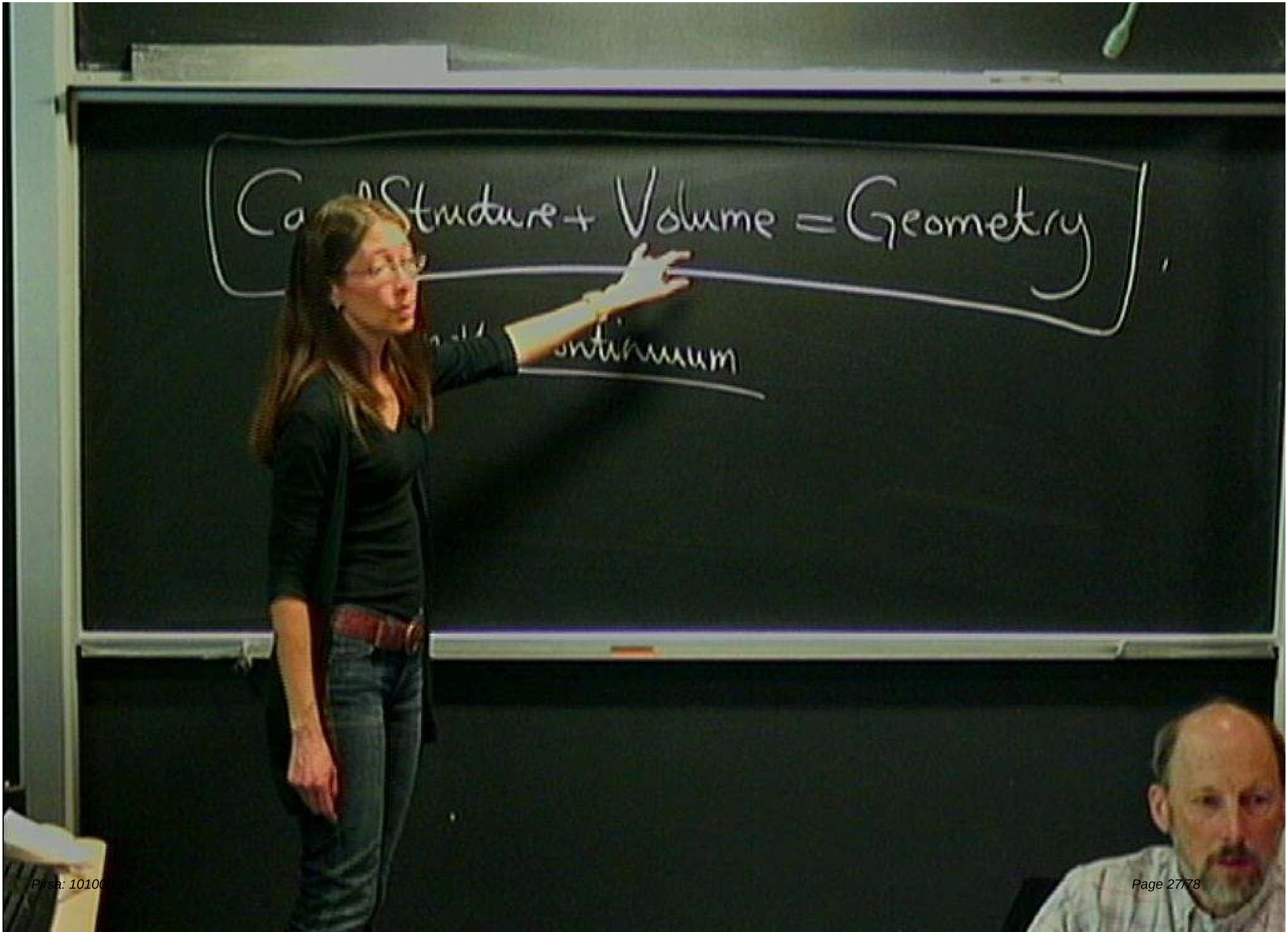
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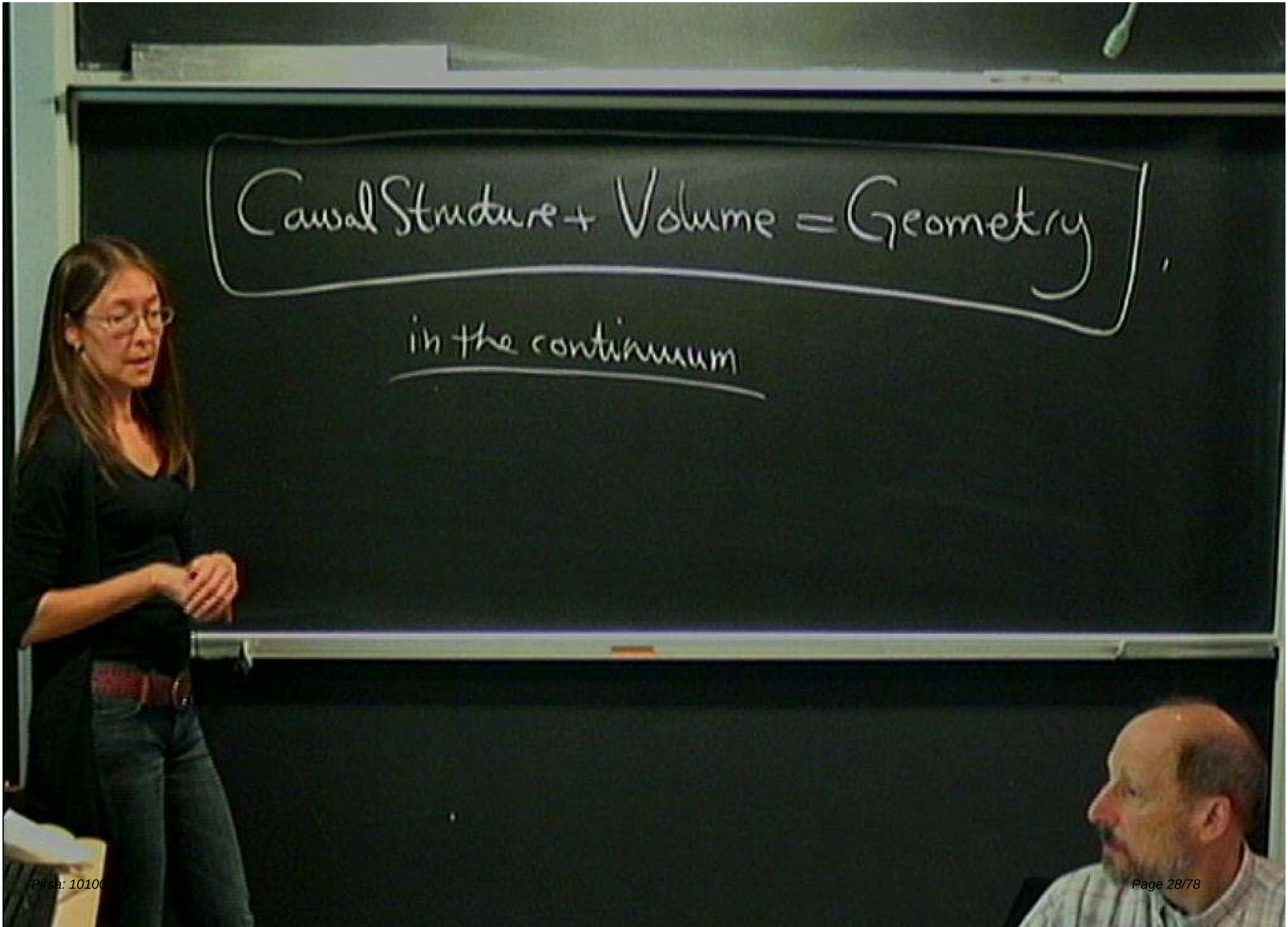
ontinuum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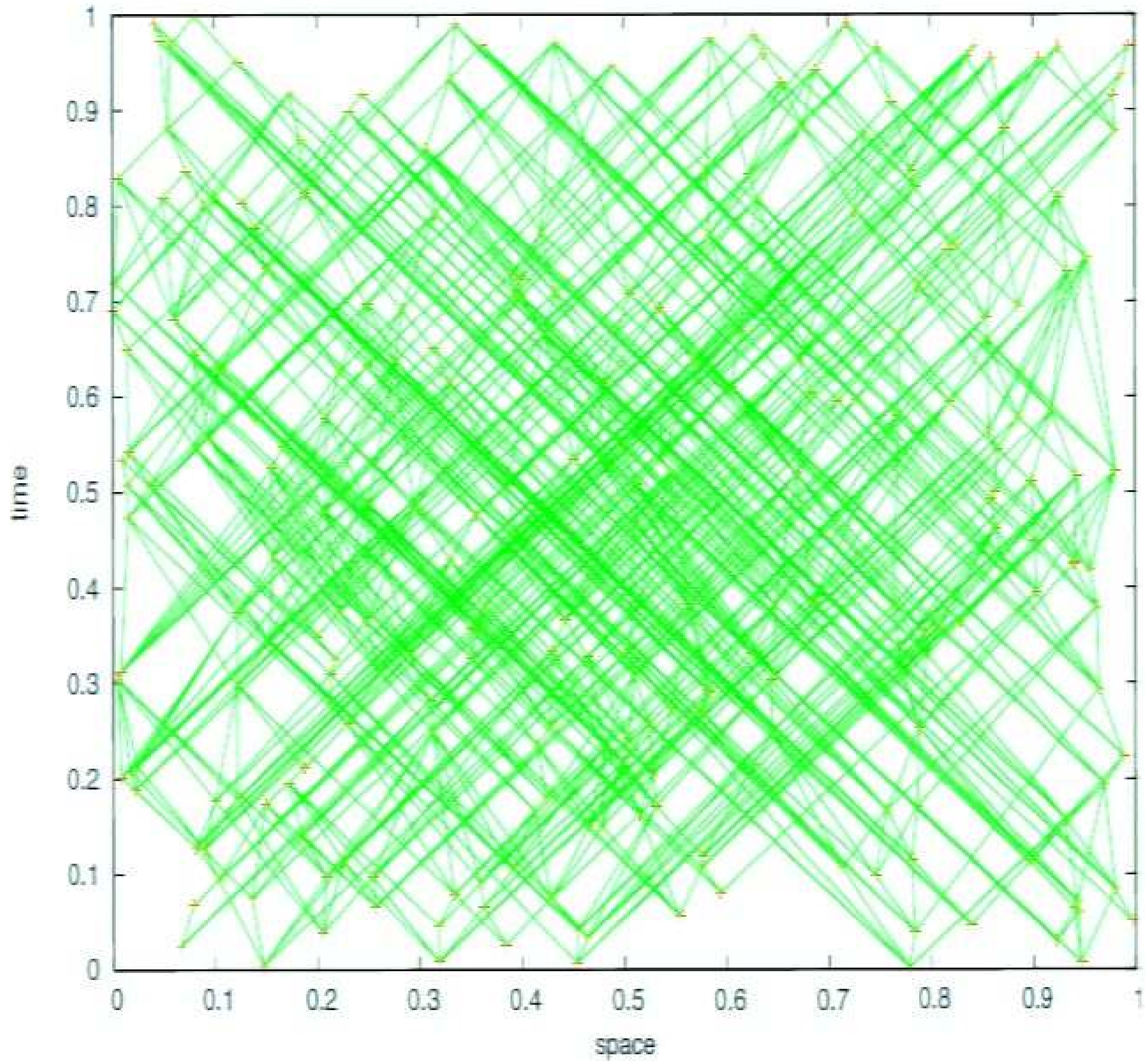

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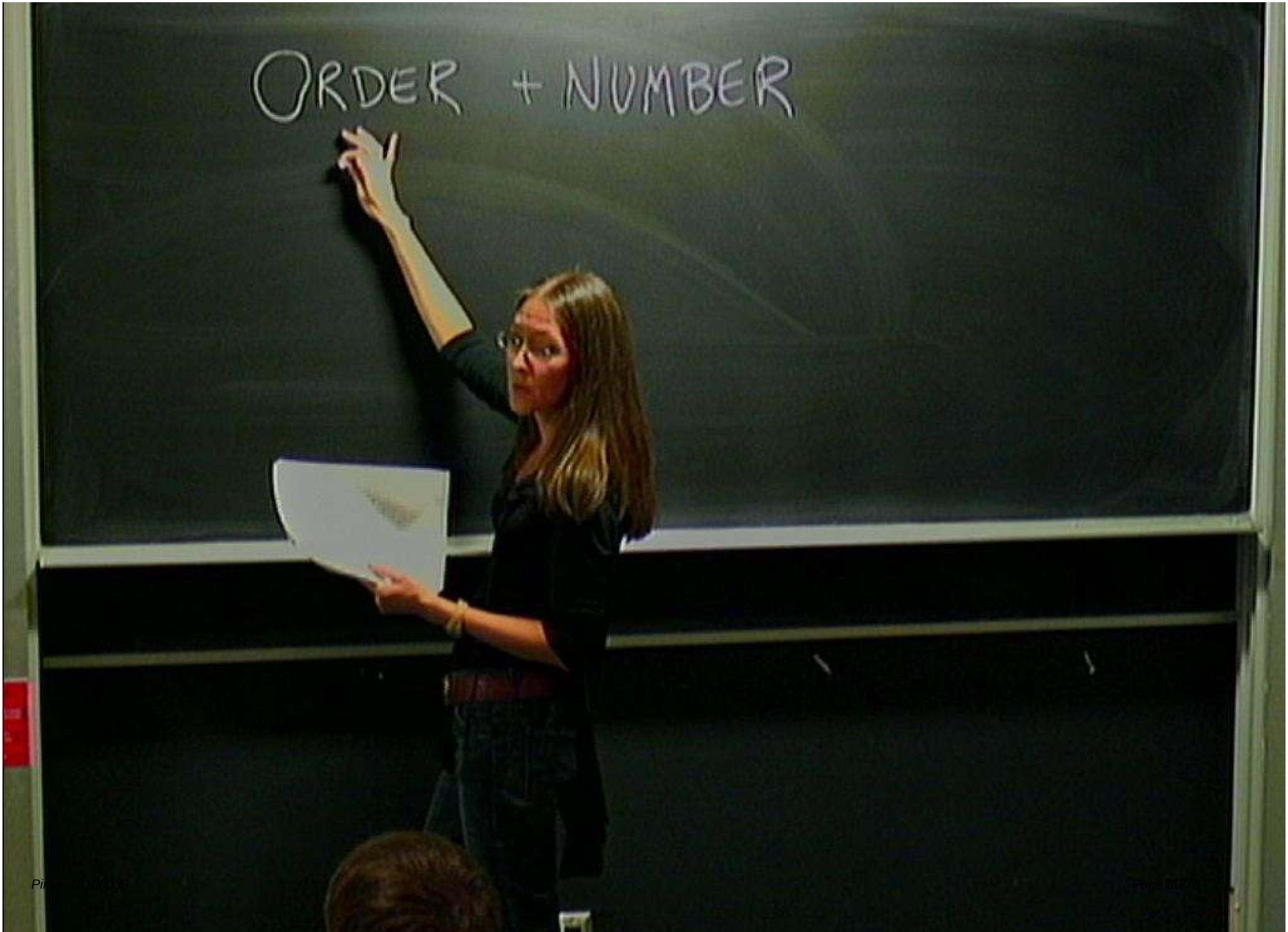
ORDER + NUMBER

ORDER + NUMBER = GEOMETRY

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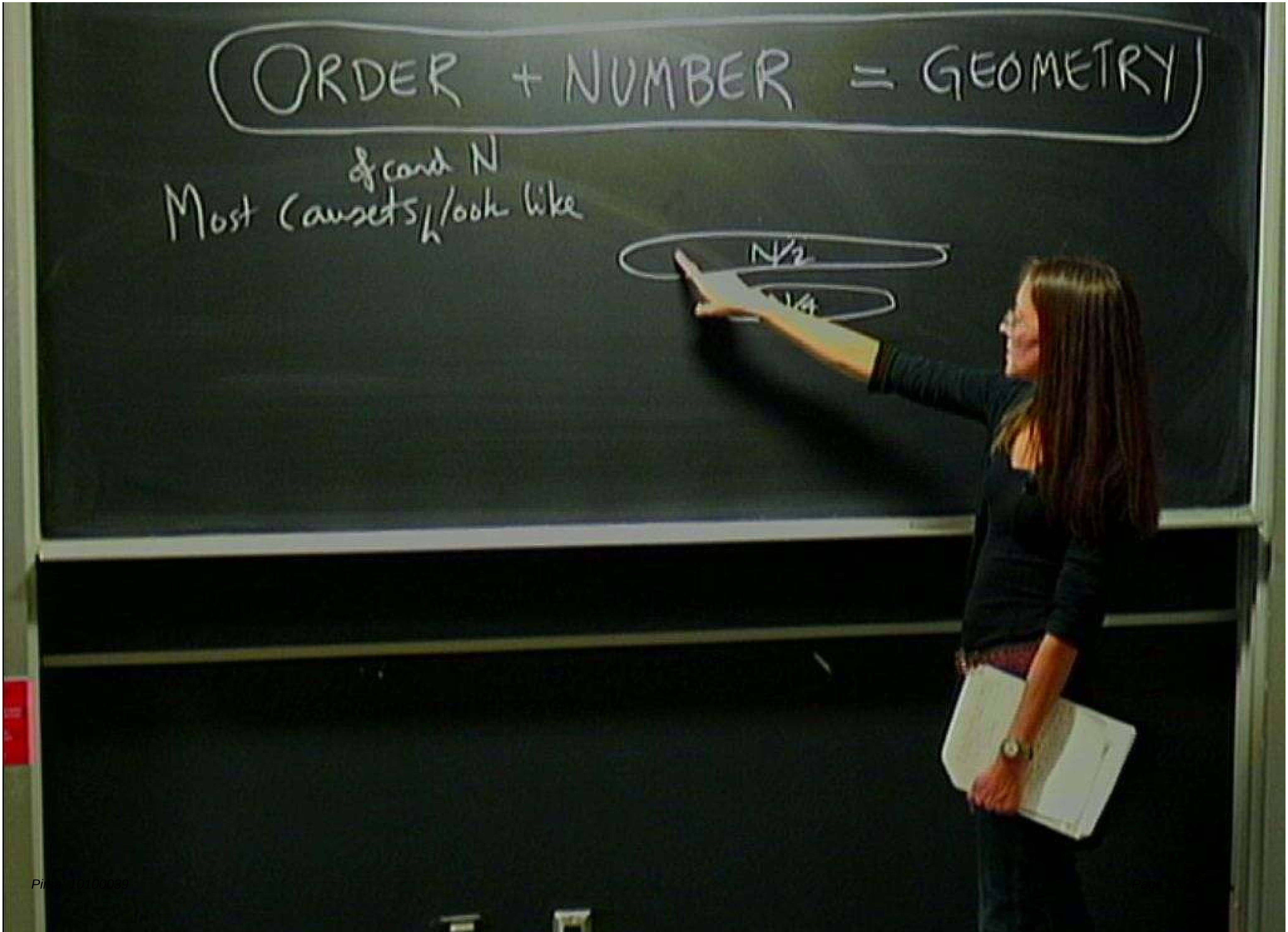
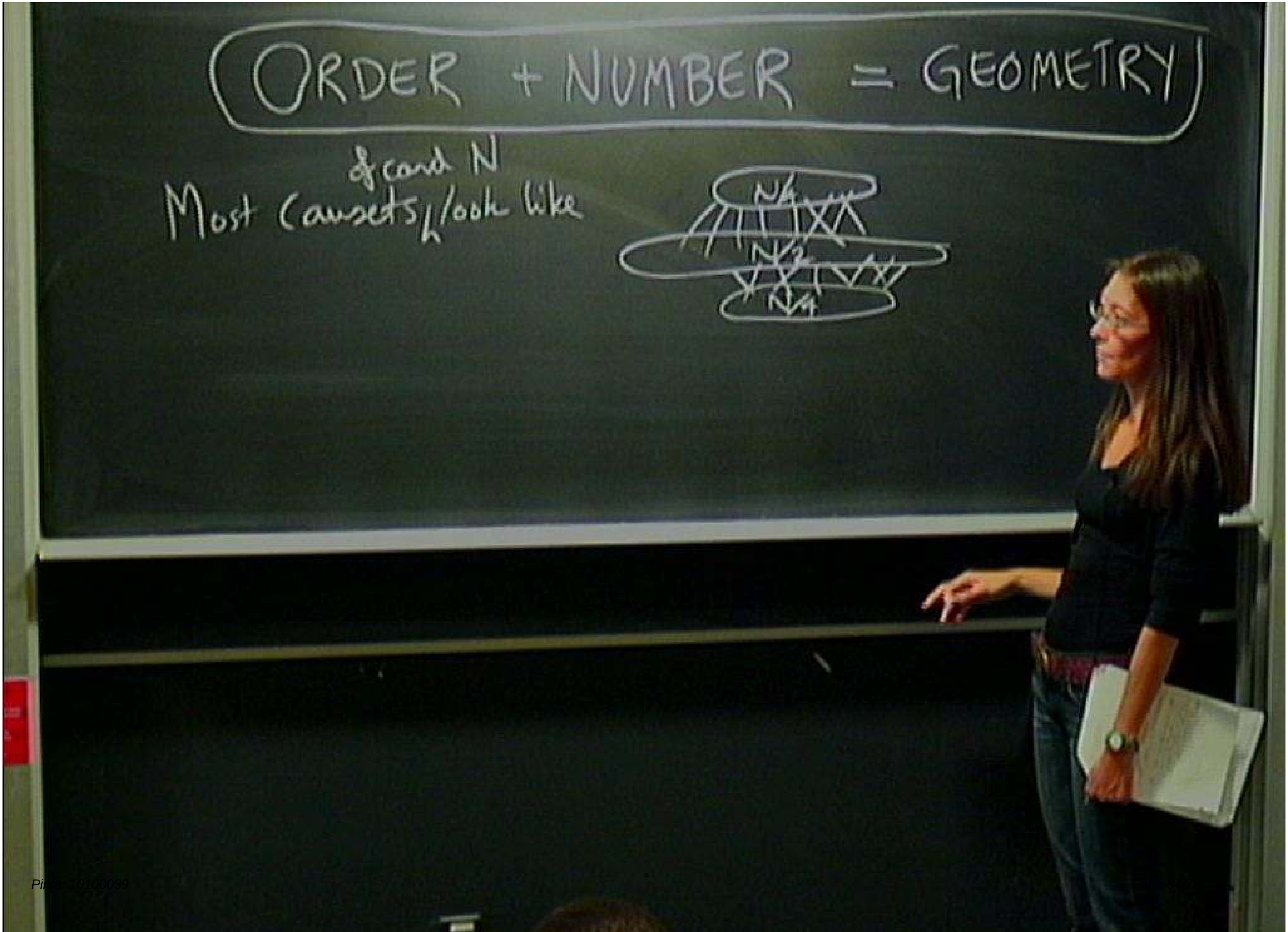

ORDER + NUMBER = GEOMETRY
of card N
Most causets look like
N/2
N/4



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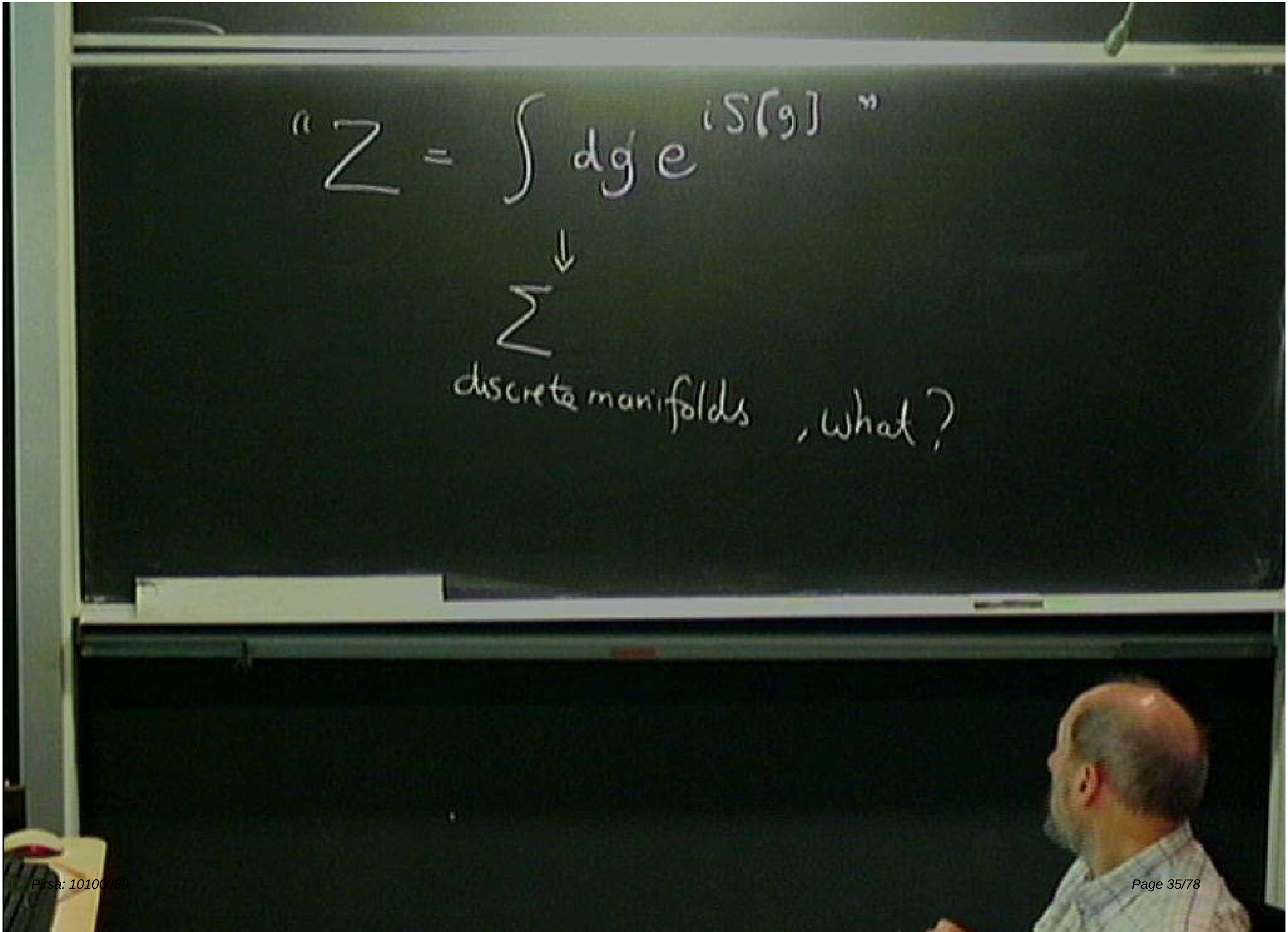
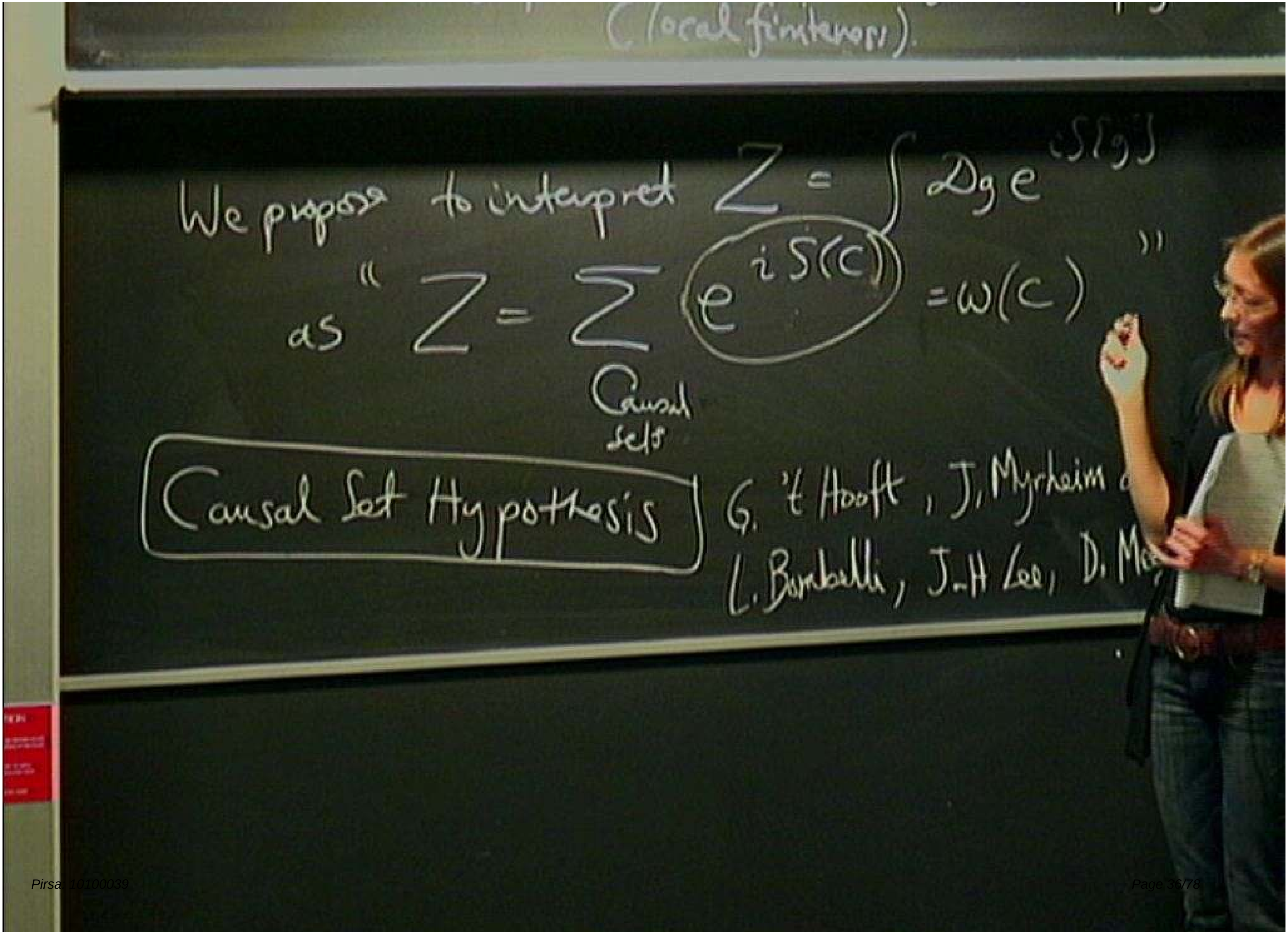

"$Z = \int dg\, e^{iS[g]}$"
$\Sigma_{\text{discrete manifolds}}$, what?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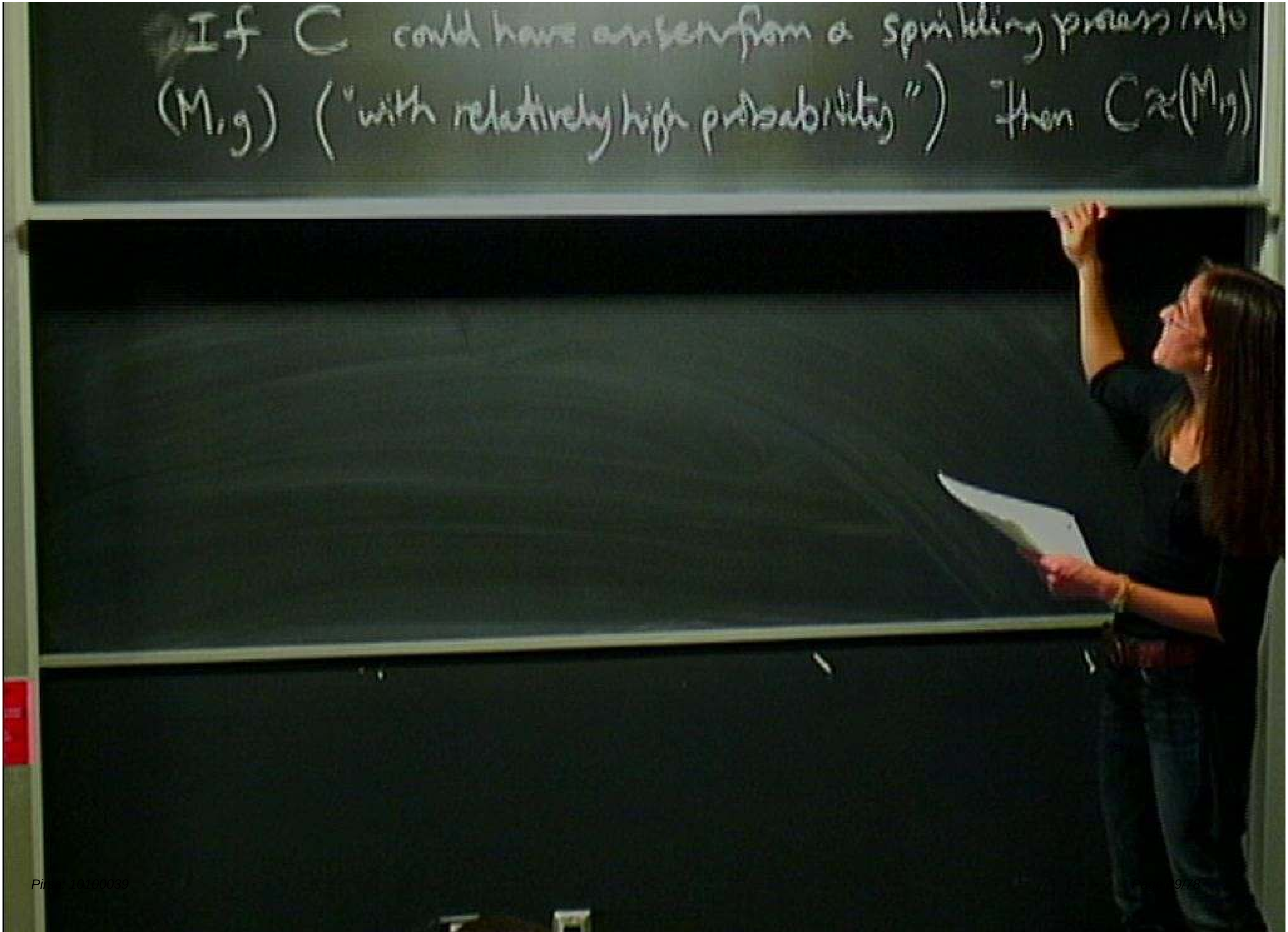
ORDER + NUMBER = GEOMETRY

Most Causets (of card N) look like

If C could have arisen from a sprinkling process in

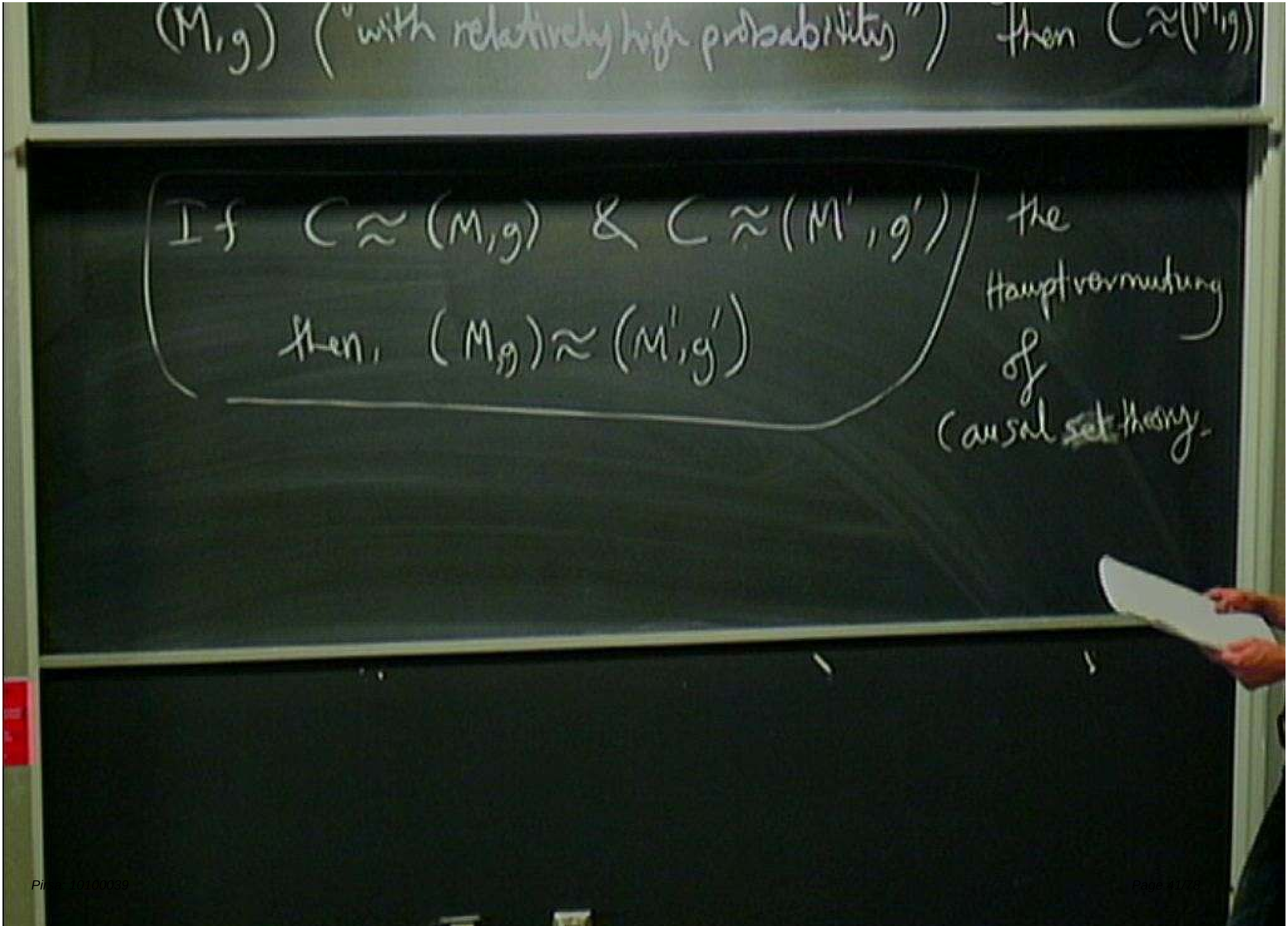
(M,g) ("with relatively high probability")


If C could have arisen from a sprinkling process into
(M,g) ("with relatively high probability") then C ≈ (M,g)

(M,g) ("with relatively high probability") then $C \approx (M,g)$

If $C \approx (M,g)$ & $C \approx (M',g')$

$$(M,g) \approx (M',g')$$


(M,g) ("with relatively high probability") then C ≈ (M,g)
If C ≈ (M,g) & C ≈ (M′,g′)
then, (M,g) ≈ (M′,g′)
the Hauptvermutung of Causal set theory.

(M,g) ("with relatively high probability") then $C \approx (M,g)$

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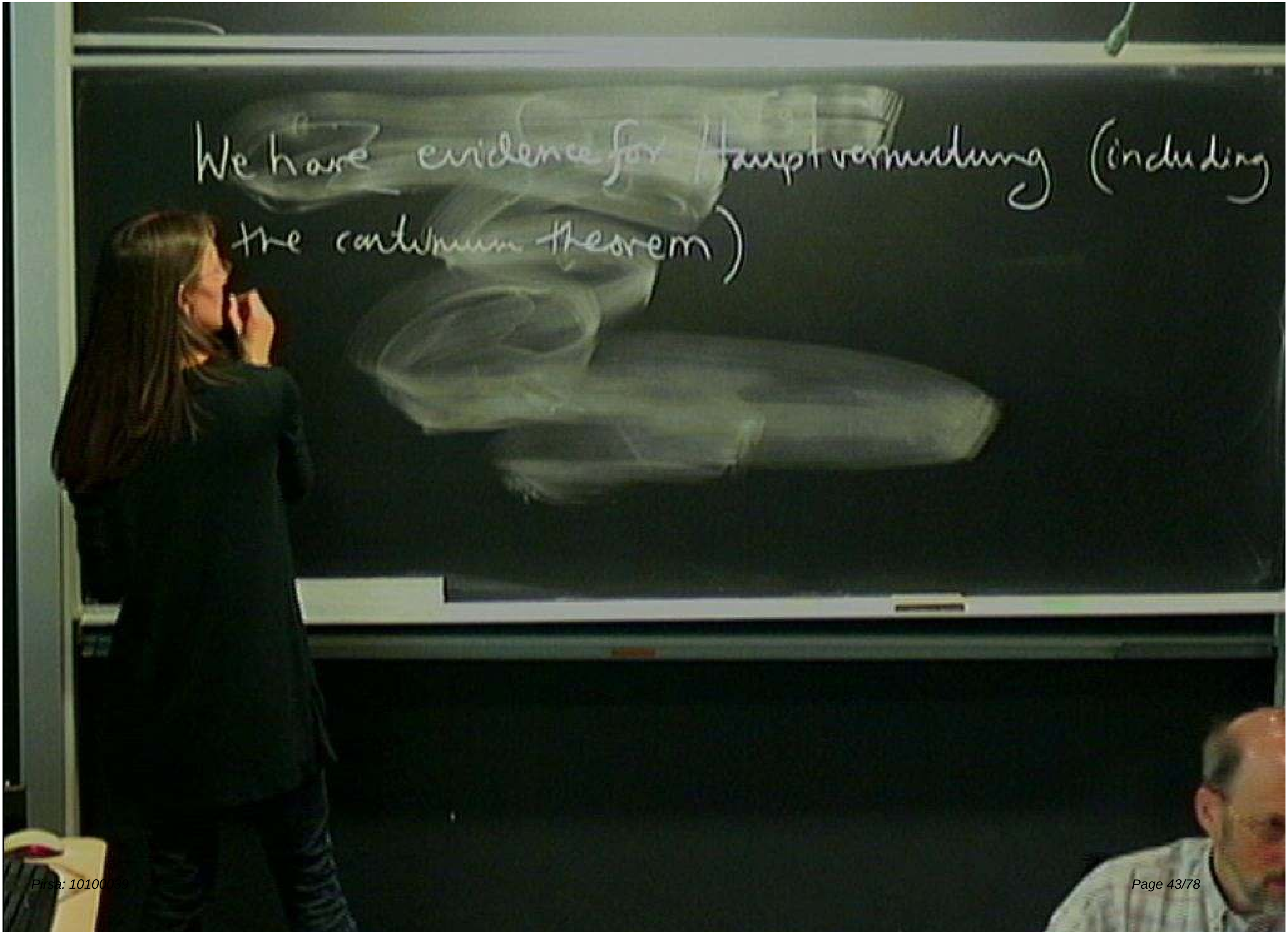
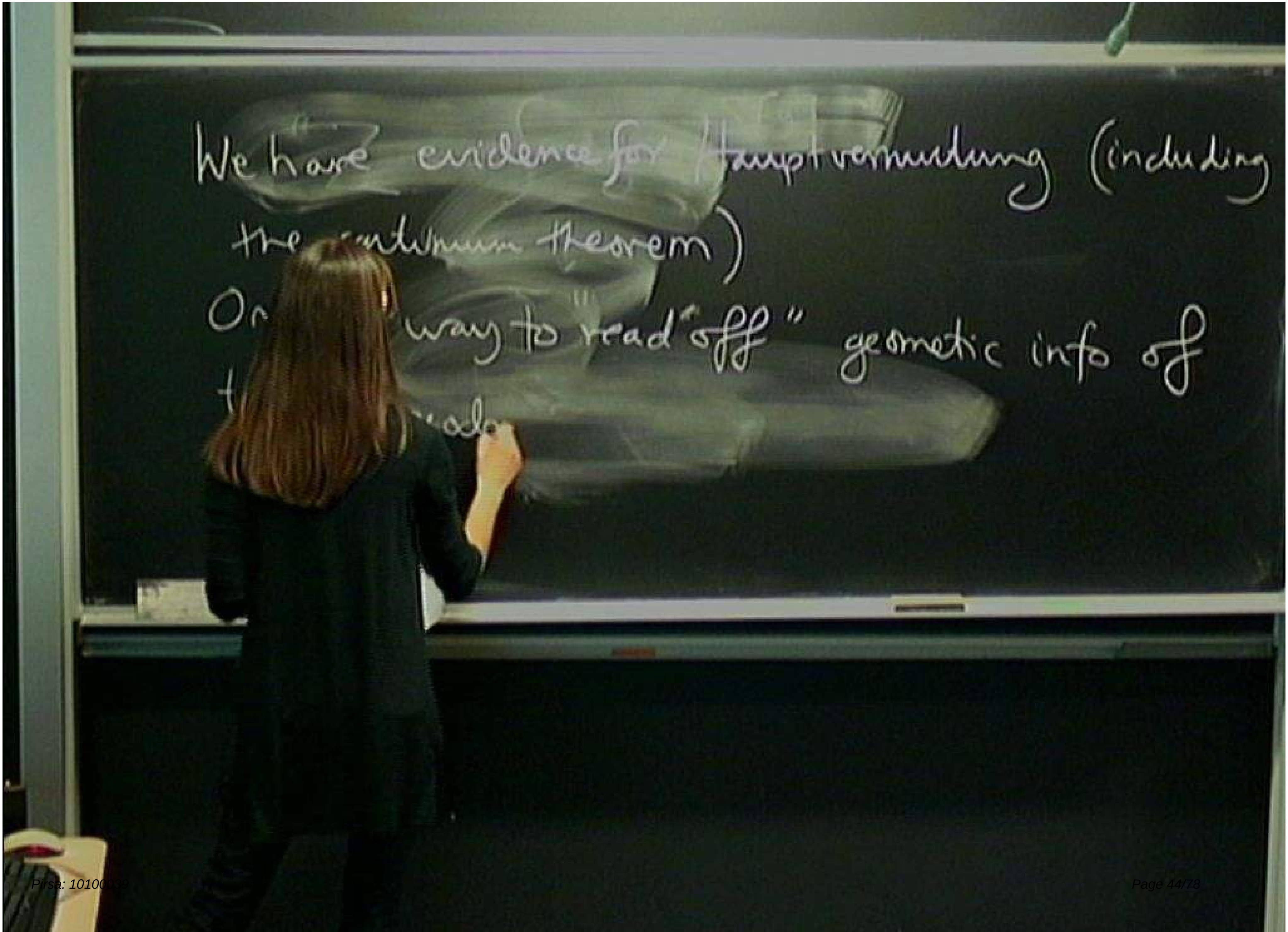

We have evidence for Hauptvermutung (including
the continuum theorem)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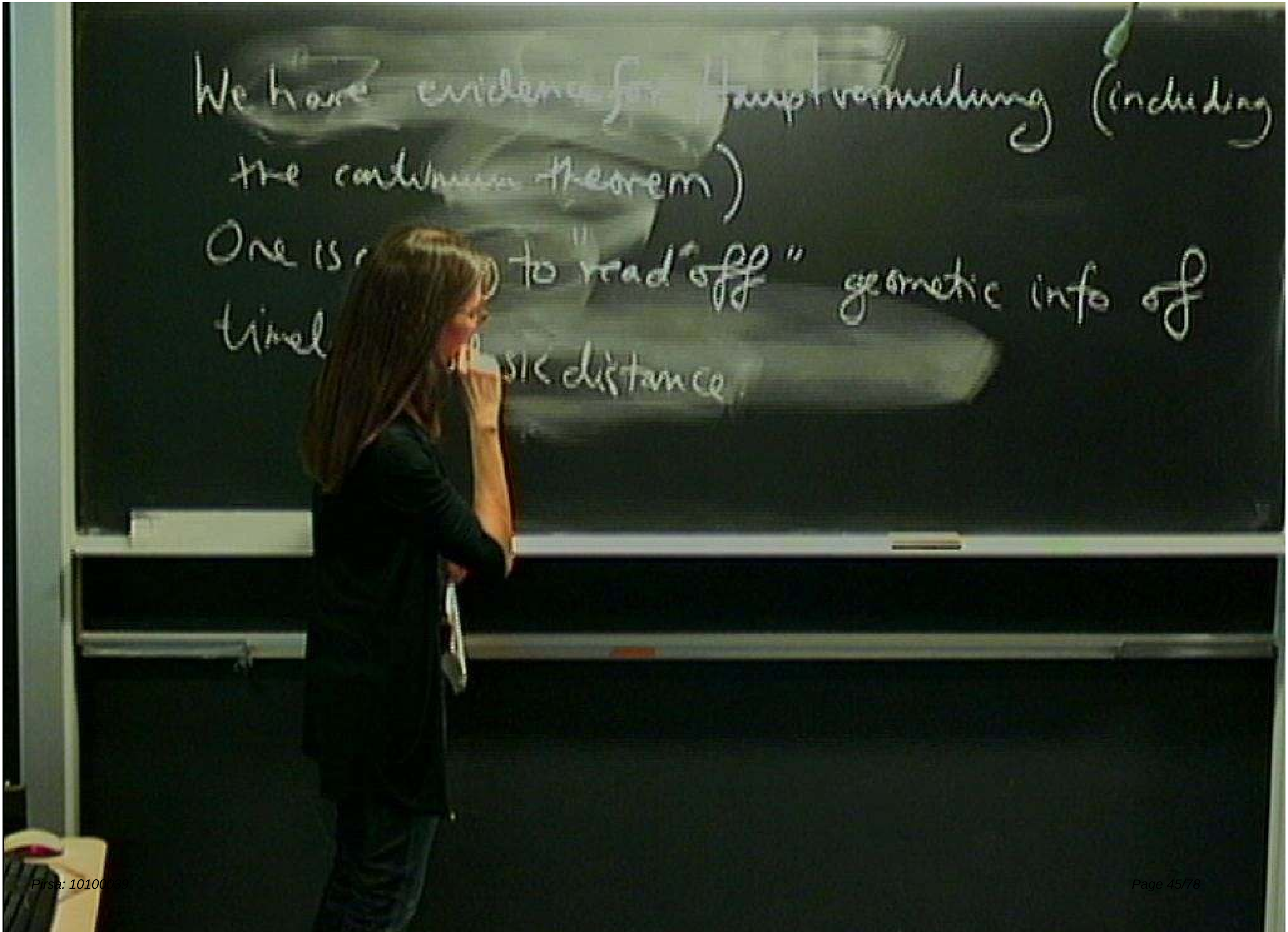
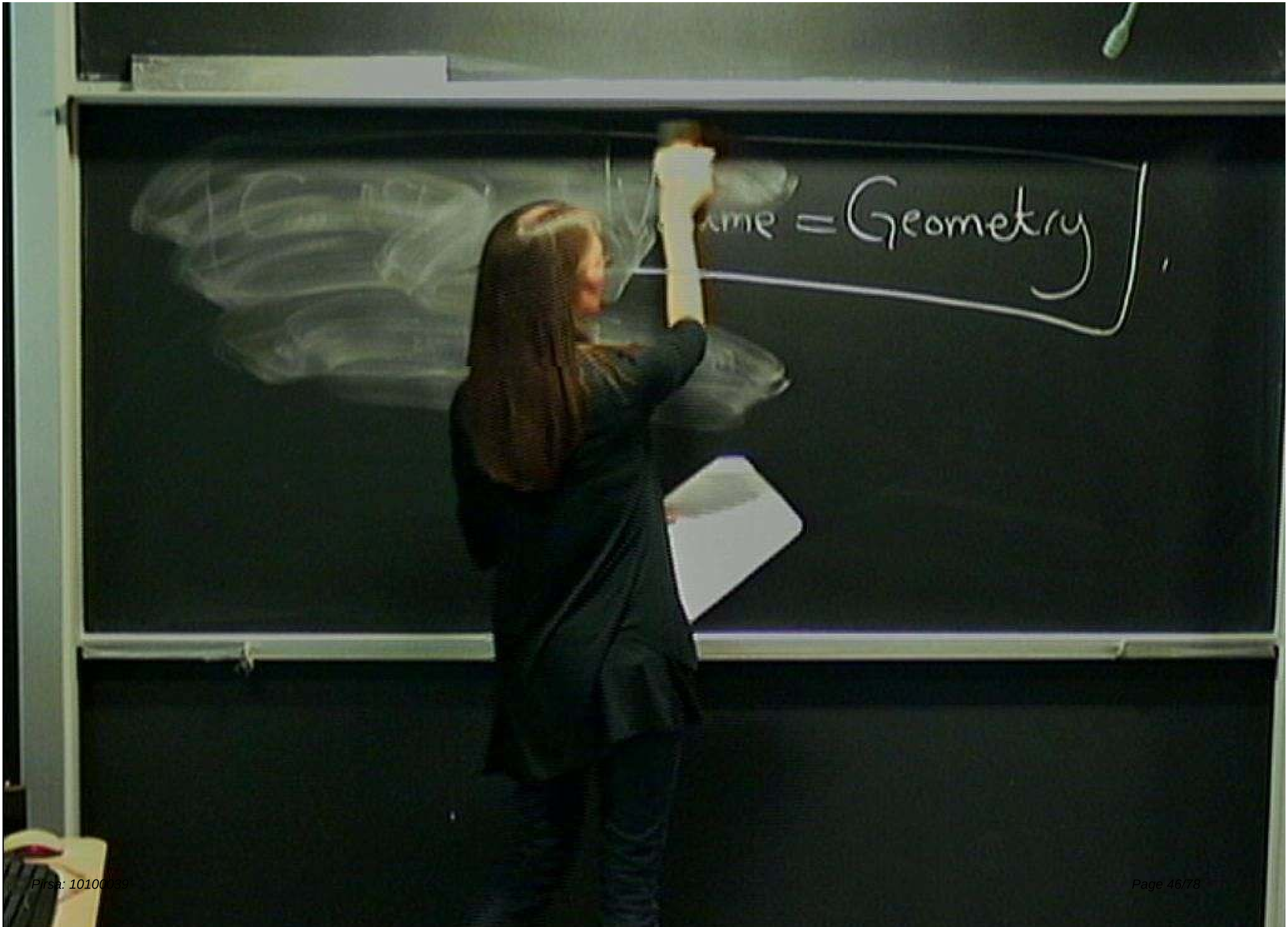
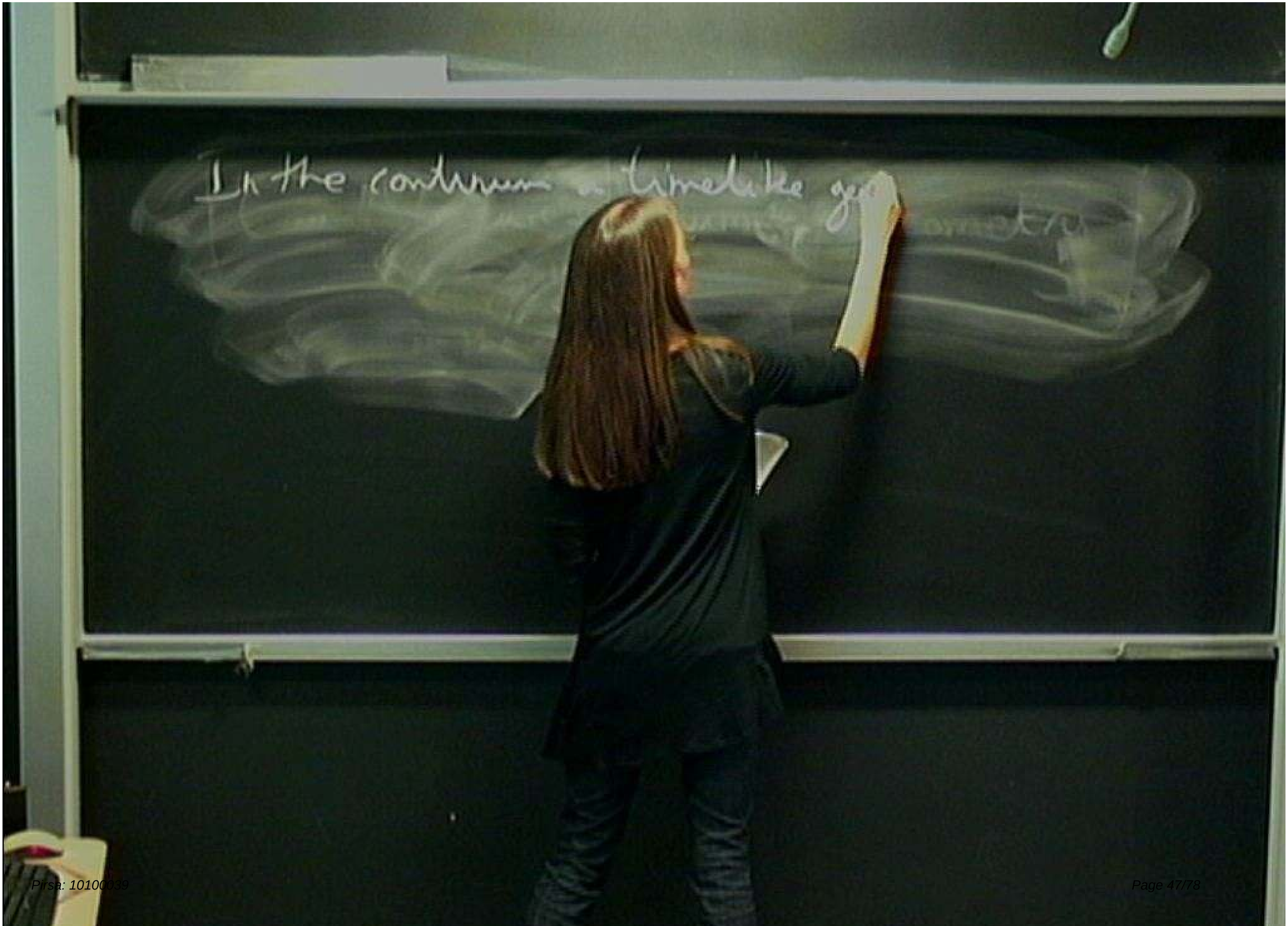
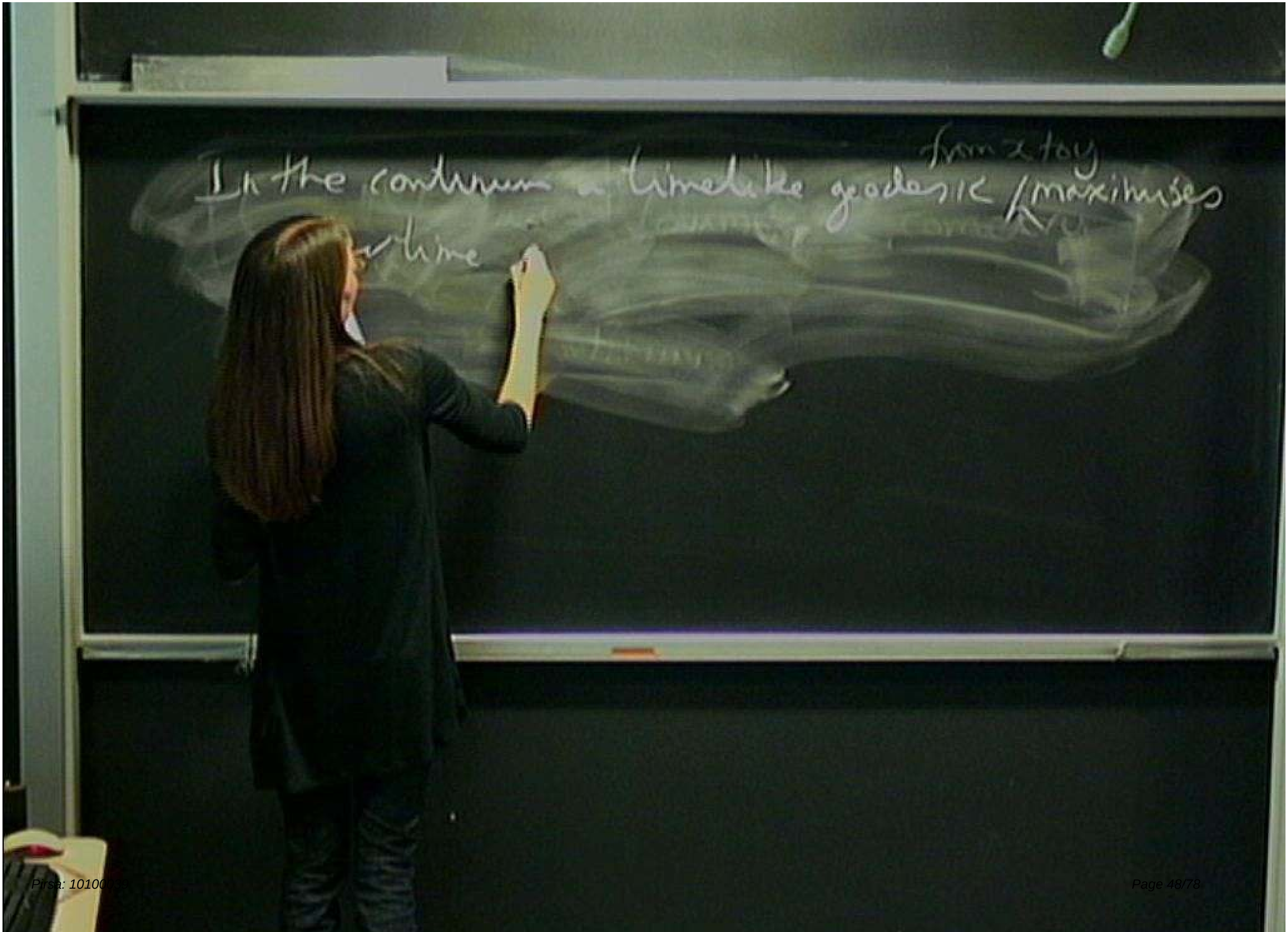
way to read "off" geometric info of

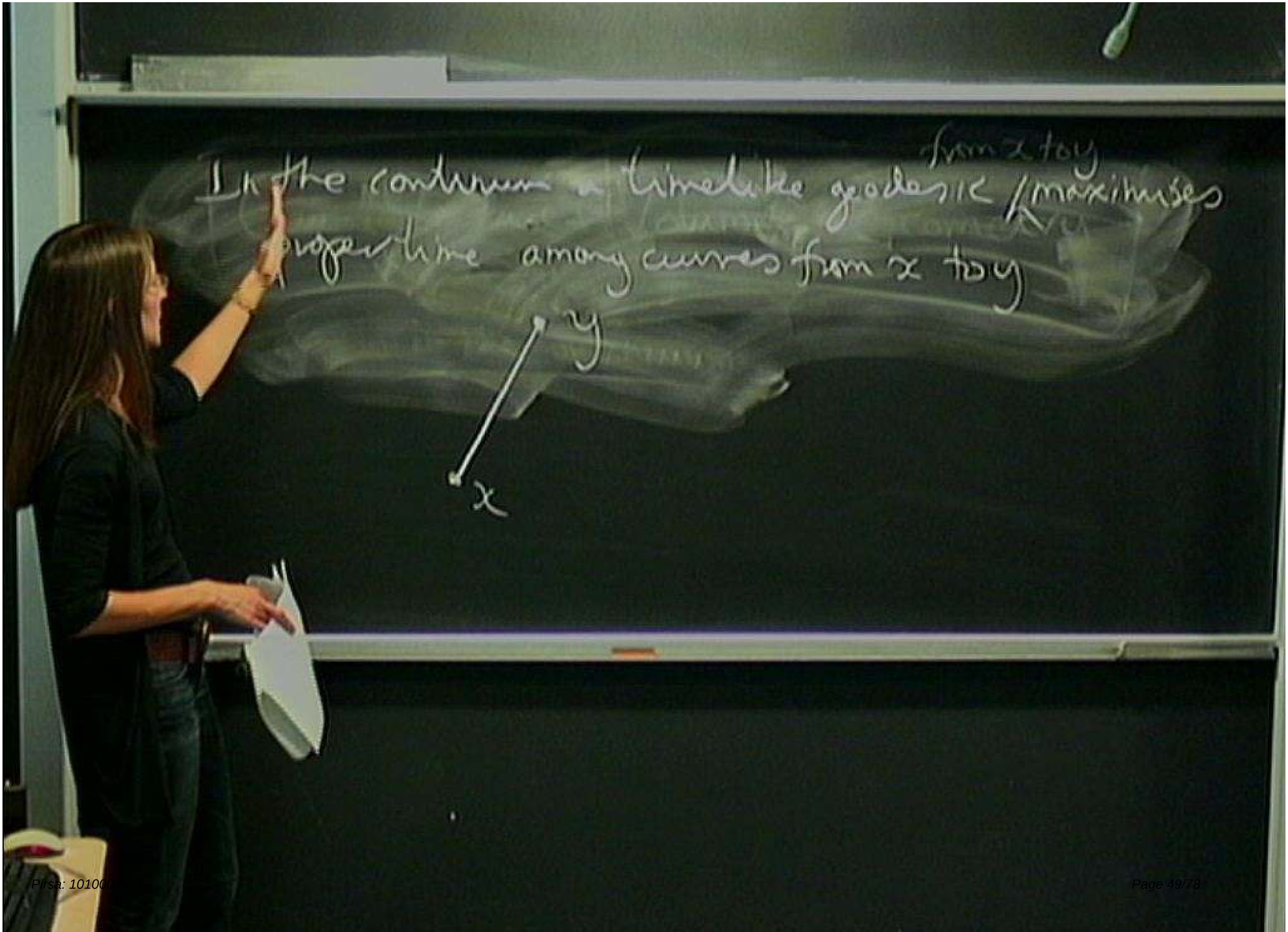


We have evidence for
(including
the continuum theorem)
One is
to "read off" geometric info of
distance


= Geometry


In the continuum
timelike


In the continuum a timelike geodesic from x to y maximises
time


from x to y
In the continuum a timelike geodesic maximises
proper time among curves from x to y
y
x

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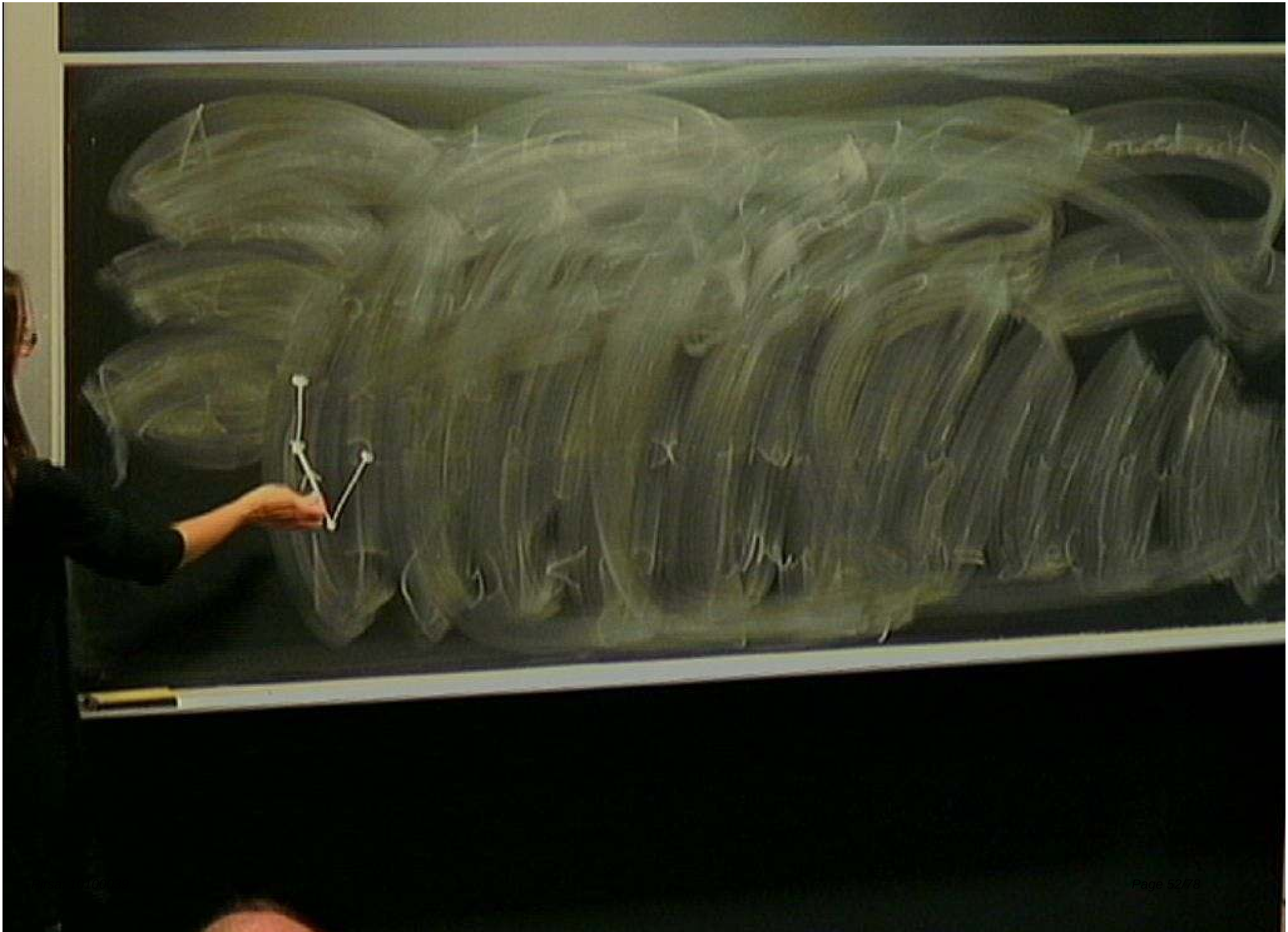
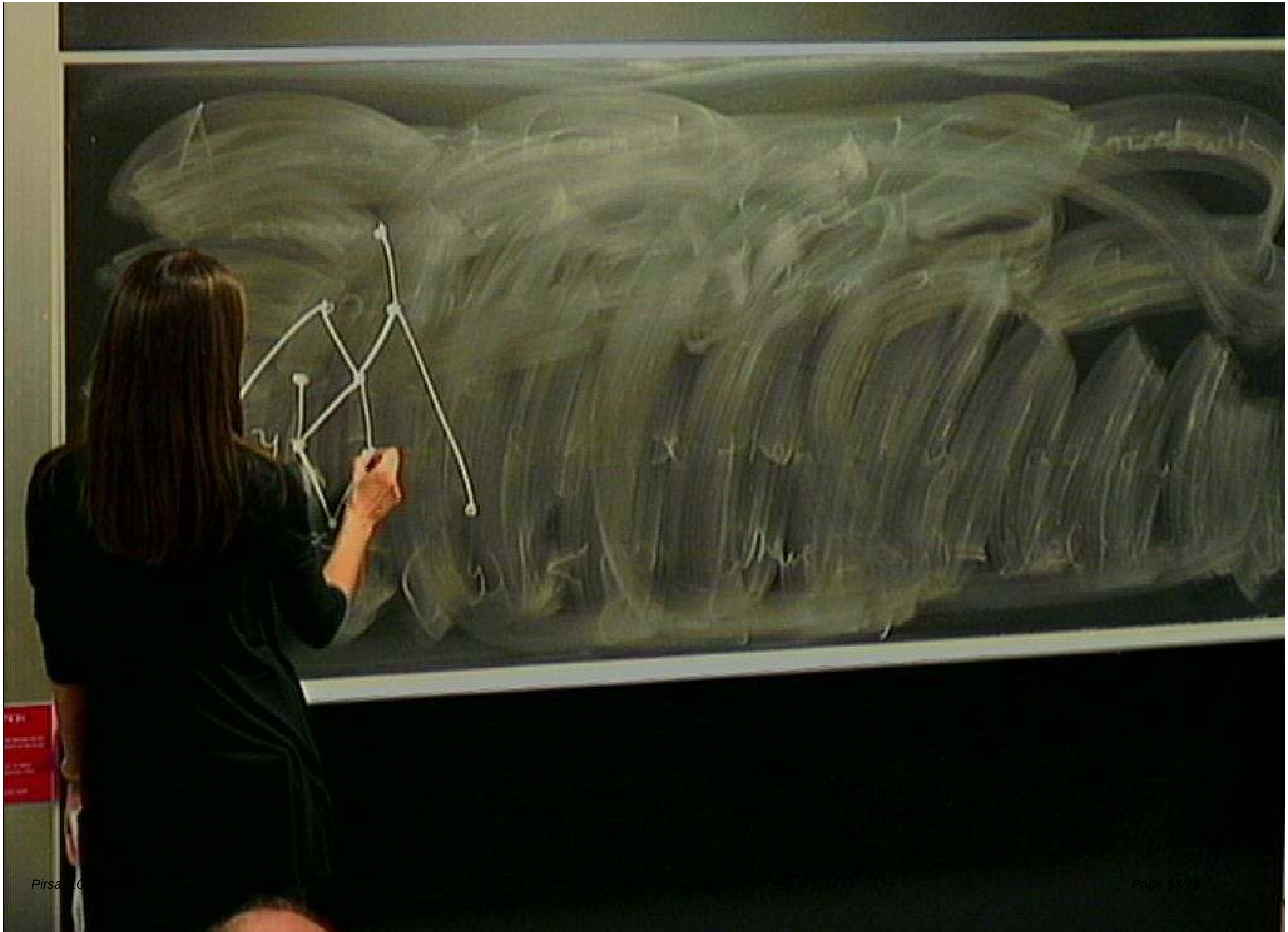
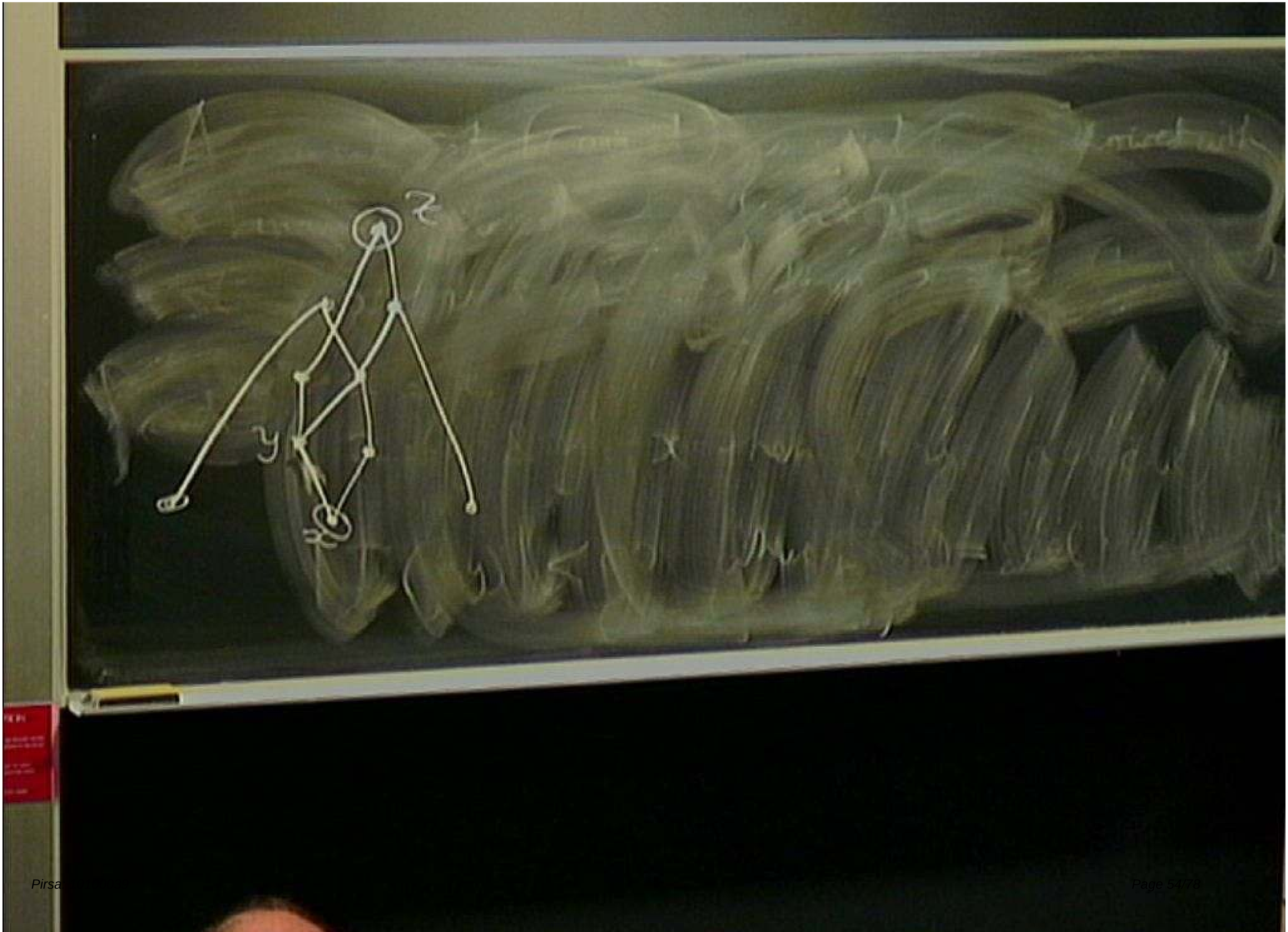
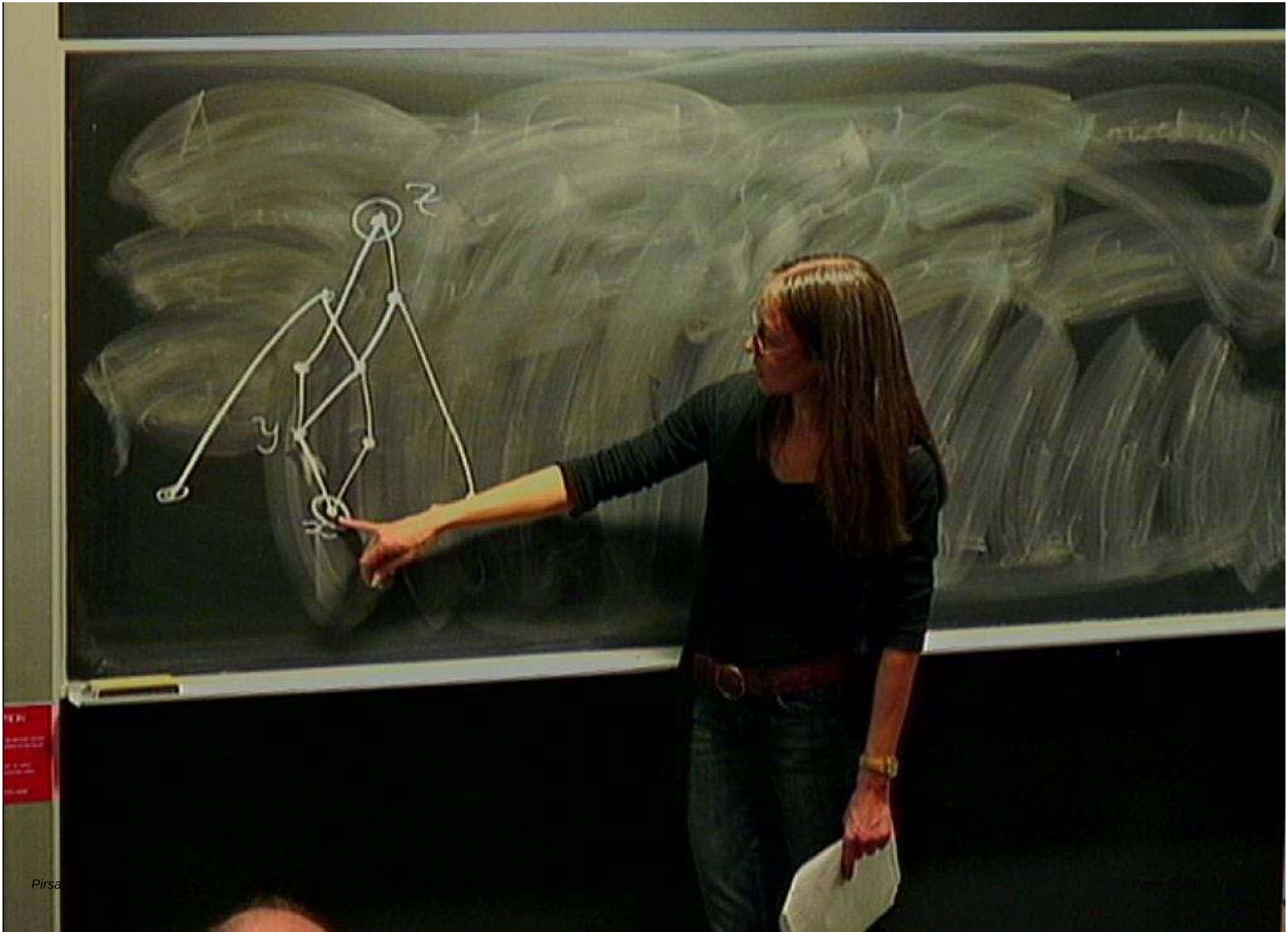
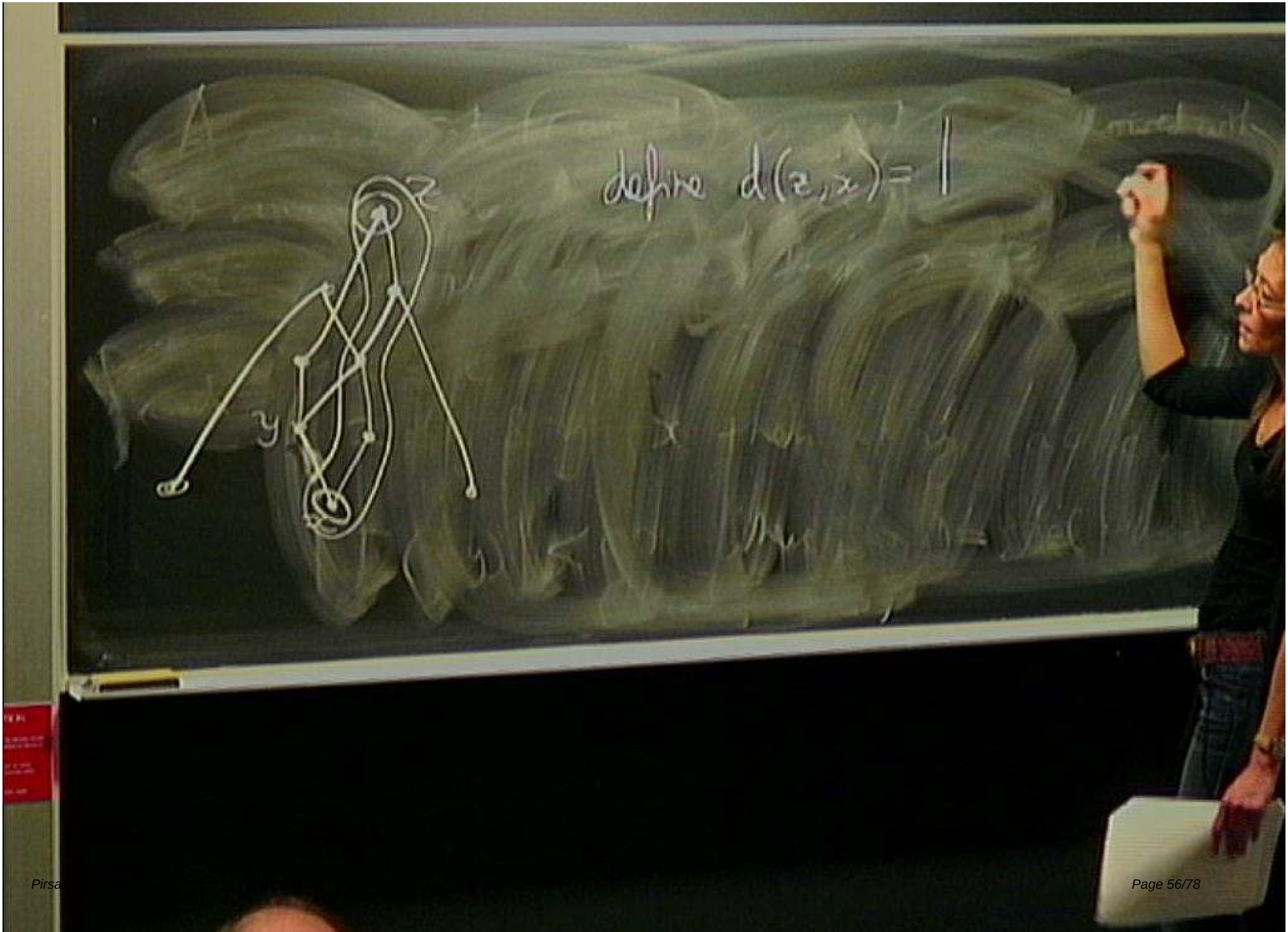
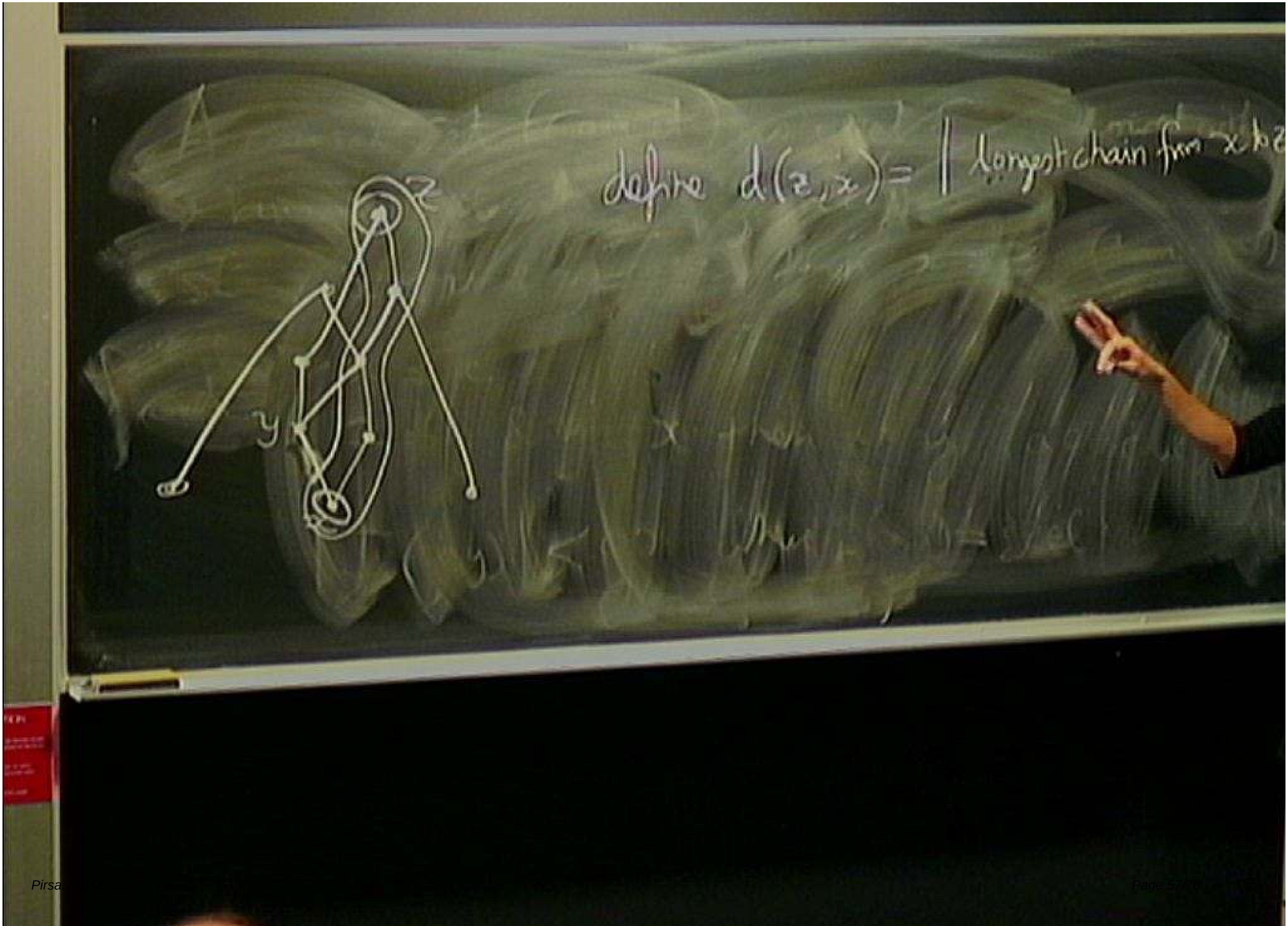


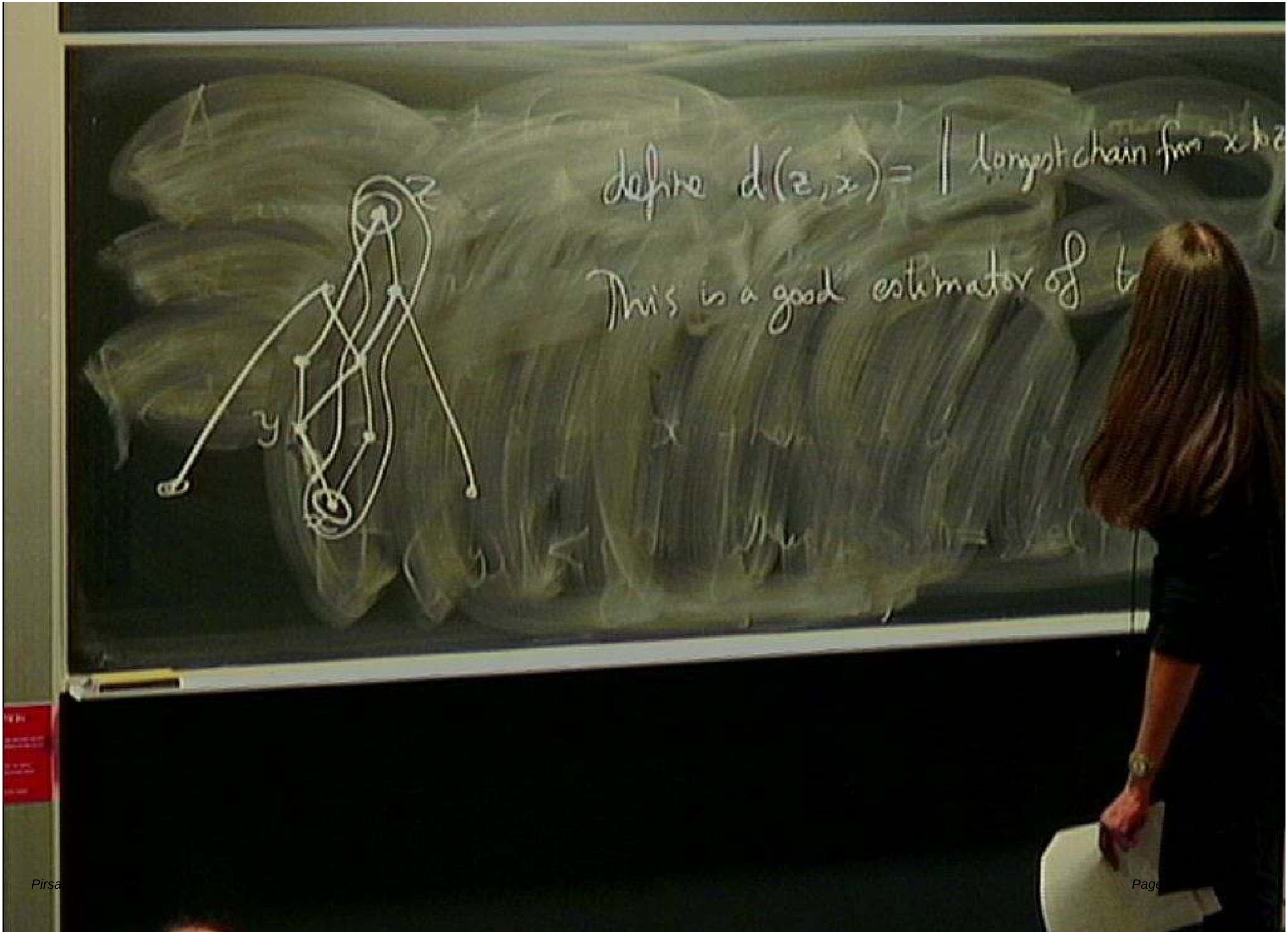


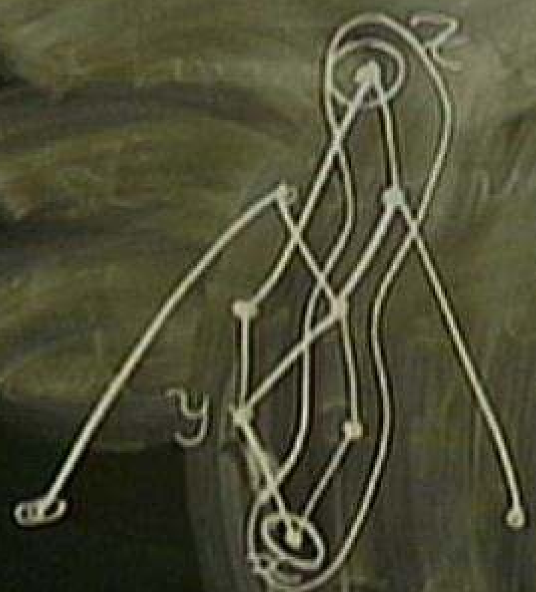

z
y
x


z
y
x


define d(z,x) = 1
z
y
x

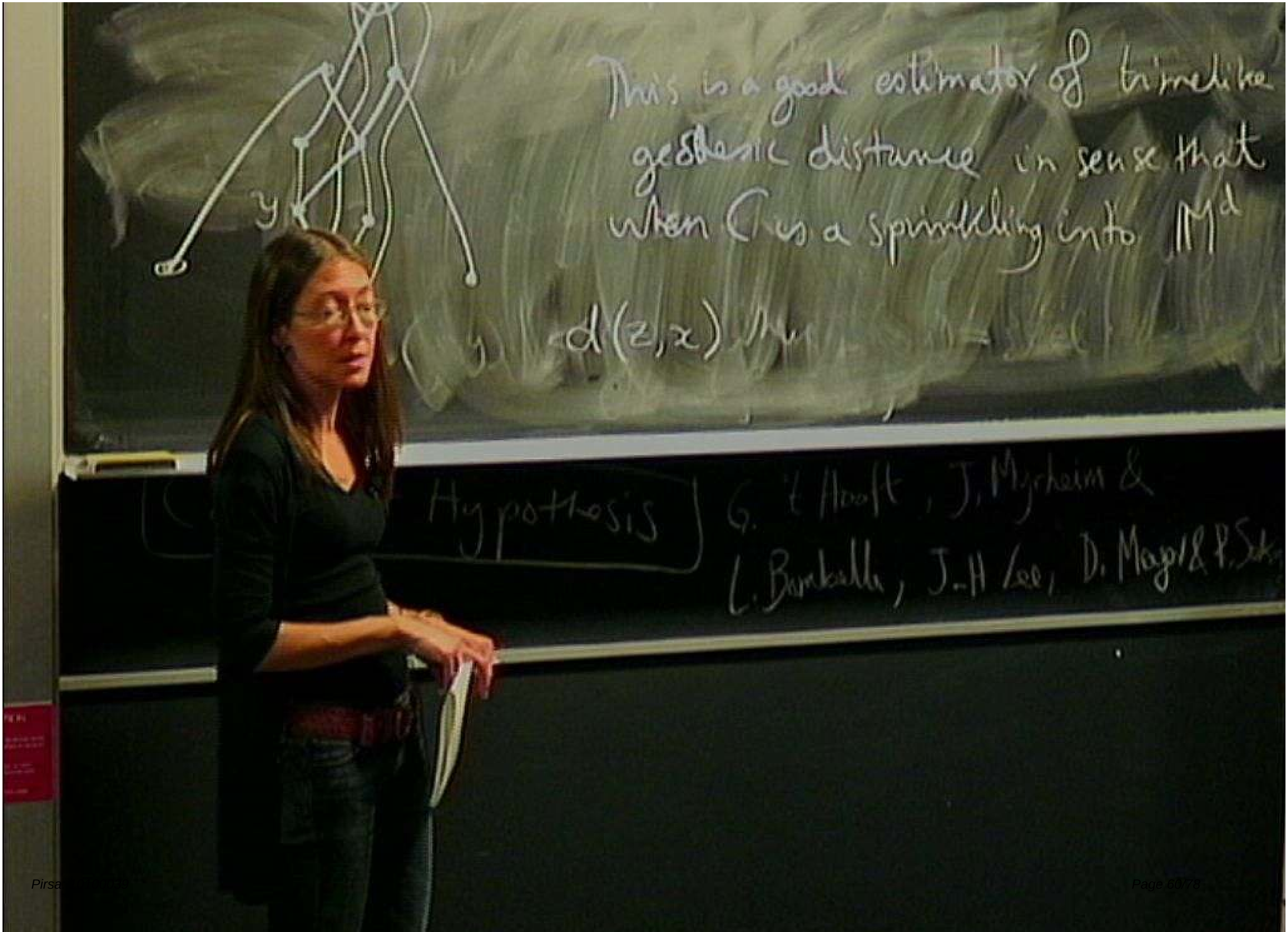

define d(z,x) = | longest chain from x to z
z
y


define d(z, x) = | longest chain from x to z
This is a good estimator of t
z
y



define $d(z,x) = |$ longest chain from x to z

This is a good estimator of timelike geodesic distance in sense that when C is a sprinkling into $\mathbb{M}^d$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Hypothesis
G. 't Hooft, J. Myrheim &
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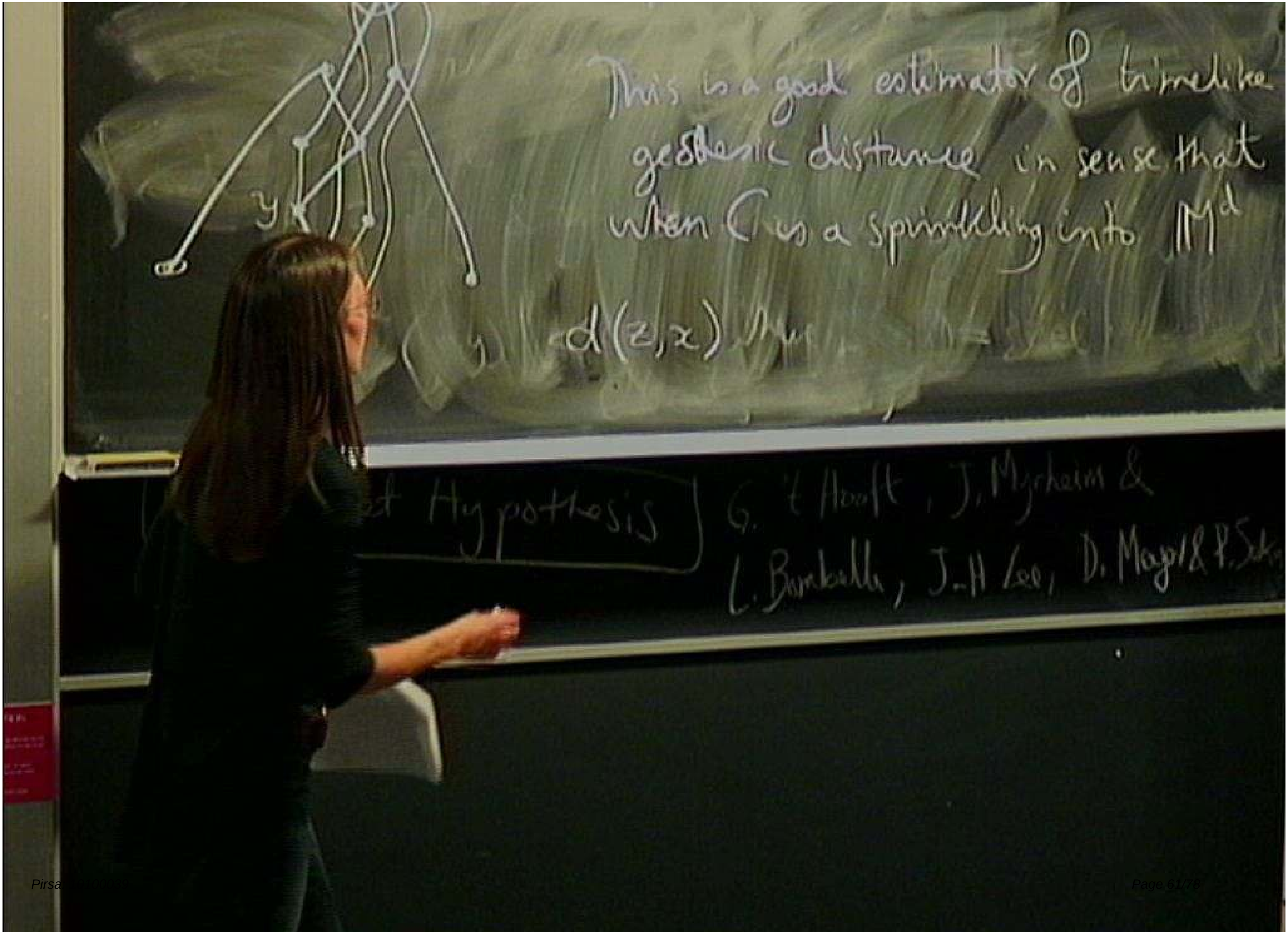
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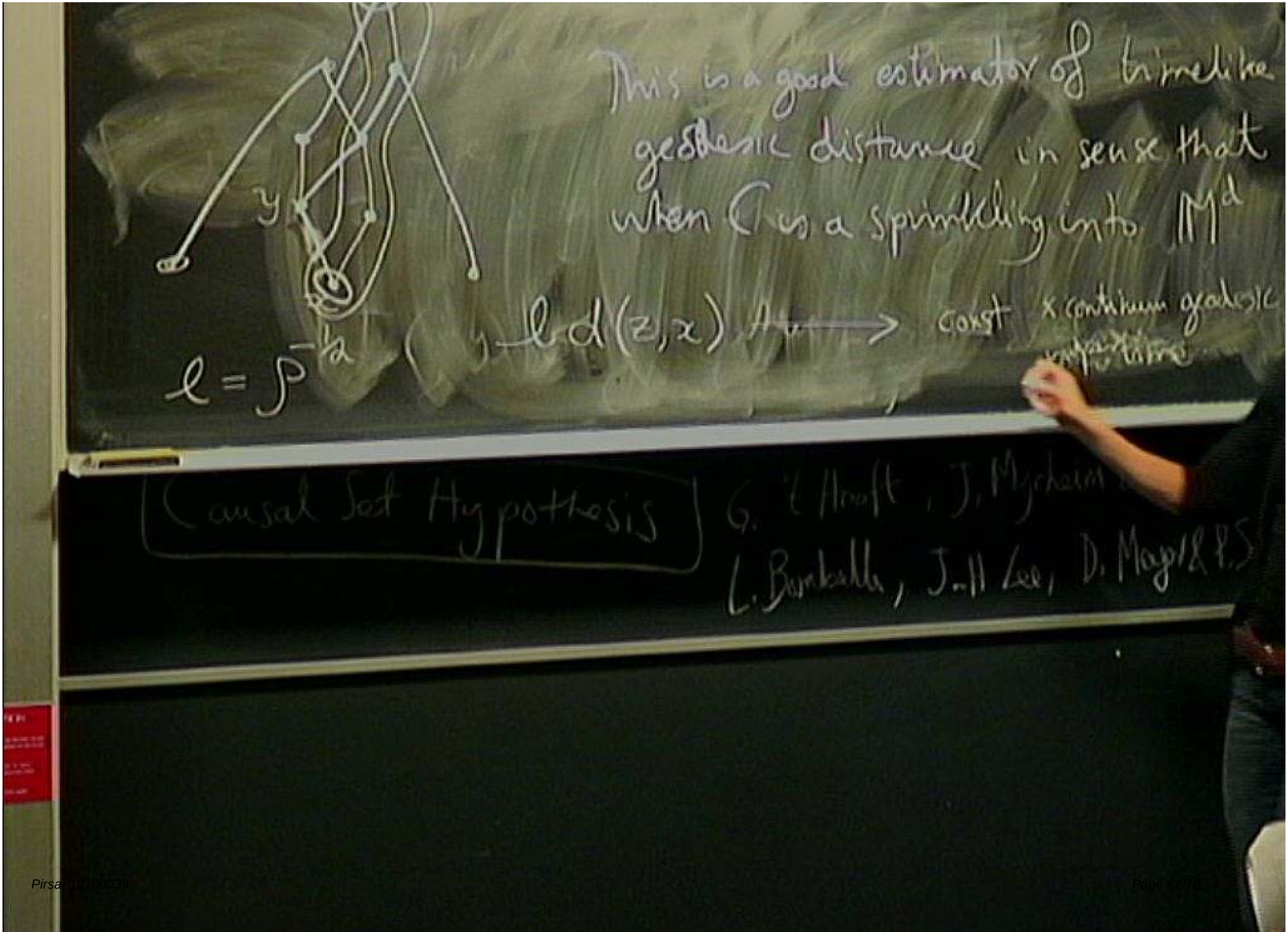
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y
l = ρ^{-1/2}
l d(z,x) → const × continuum geodesic proper time
Causal Set Hypothesis
G. 't Hooft, J. Myrheim
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$$\ell = \rho^{-1/d} \qquad \ell\, d(e,x) \xrightarrow[\rho\to\infty]{} \text{const} \times \text{continuum geodesic proper time}$$

$$\text{as } \text{“}\, Z = \sum_{\text{causal sets}} e^{iS(C)} \,\text{”} \qquad e^{iS(C)} = \omega(C)$$

$\boxed{\text{Causal Set Hypothesis}}$

G. 't Hooft, J. Myrheim &

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Gregory & Brightwell

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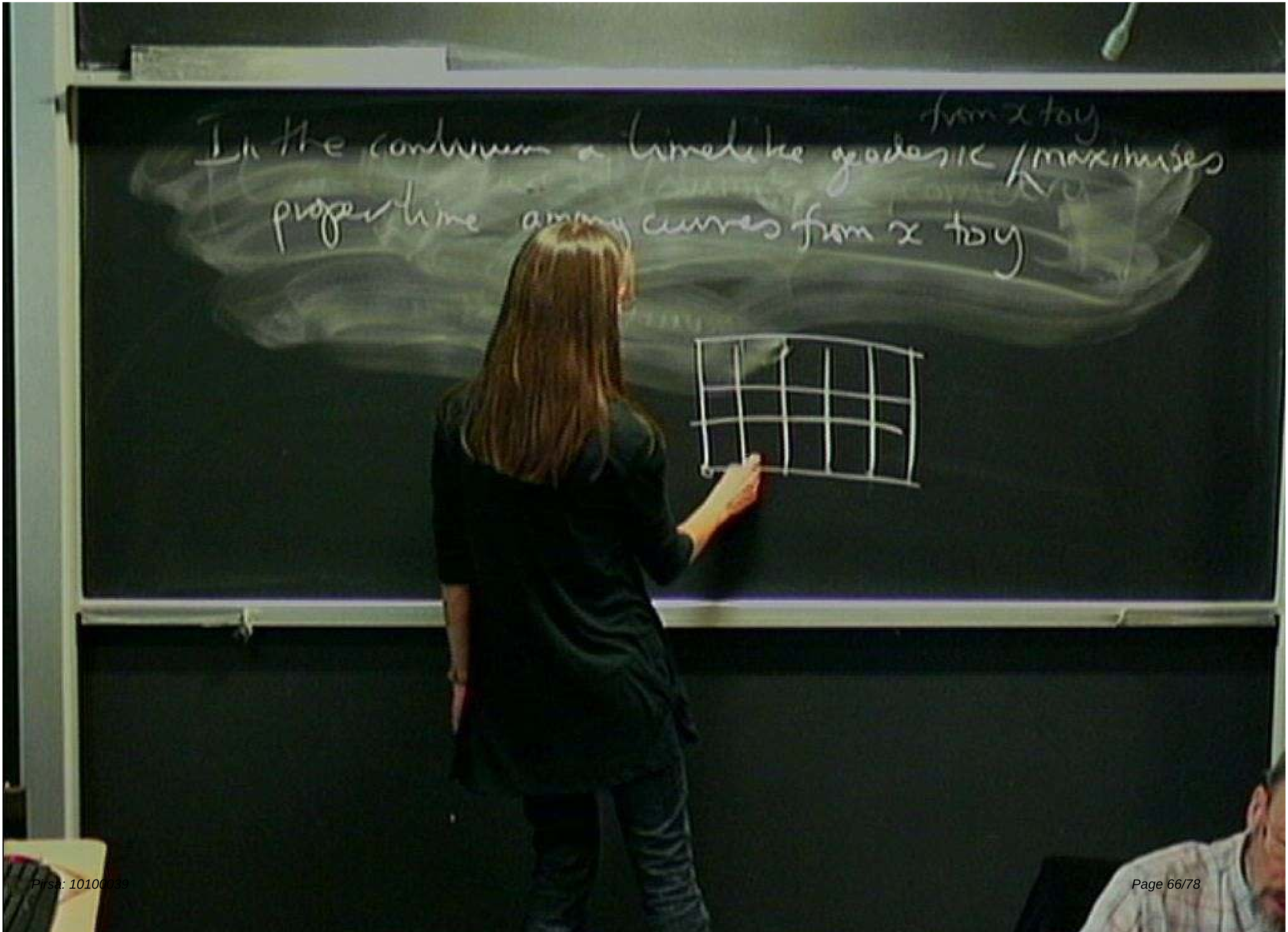
Order + Number = Geometry

- Causal sets are Lorentz invariant
- Causal sets are maximally discrete - combinatorial data only - just need to be able to count
- Carry their own metric information (c.f. Riemann's "discrete manifolds").
- The order relation $\preceq$ unifies within itself topology, differentiable structure, metric and causal structure
- Discreteness and Lorentzian-ness go together like a horse and carriage.
- Randomness of sprinkling is kinematical only: from Number ~ Volume
- Causal sets are fluid-like rather than crystal-like
- Scale dependent topology and geometry (and covariant coarse graining)
- The approach is both conservative and radical at the same time.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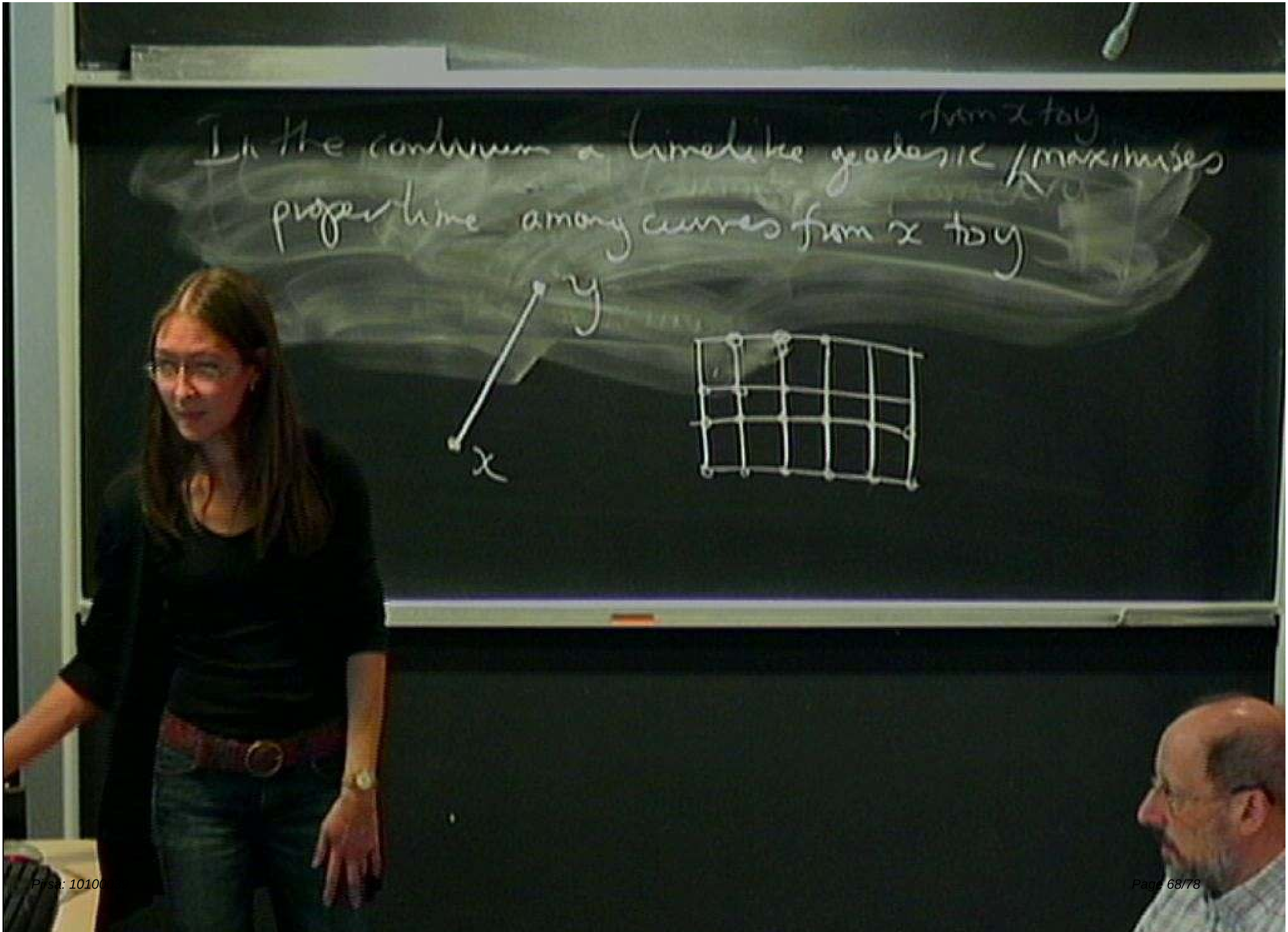
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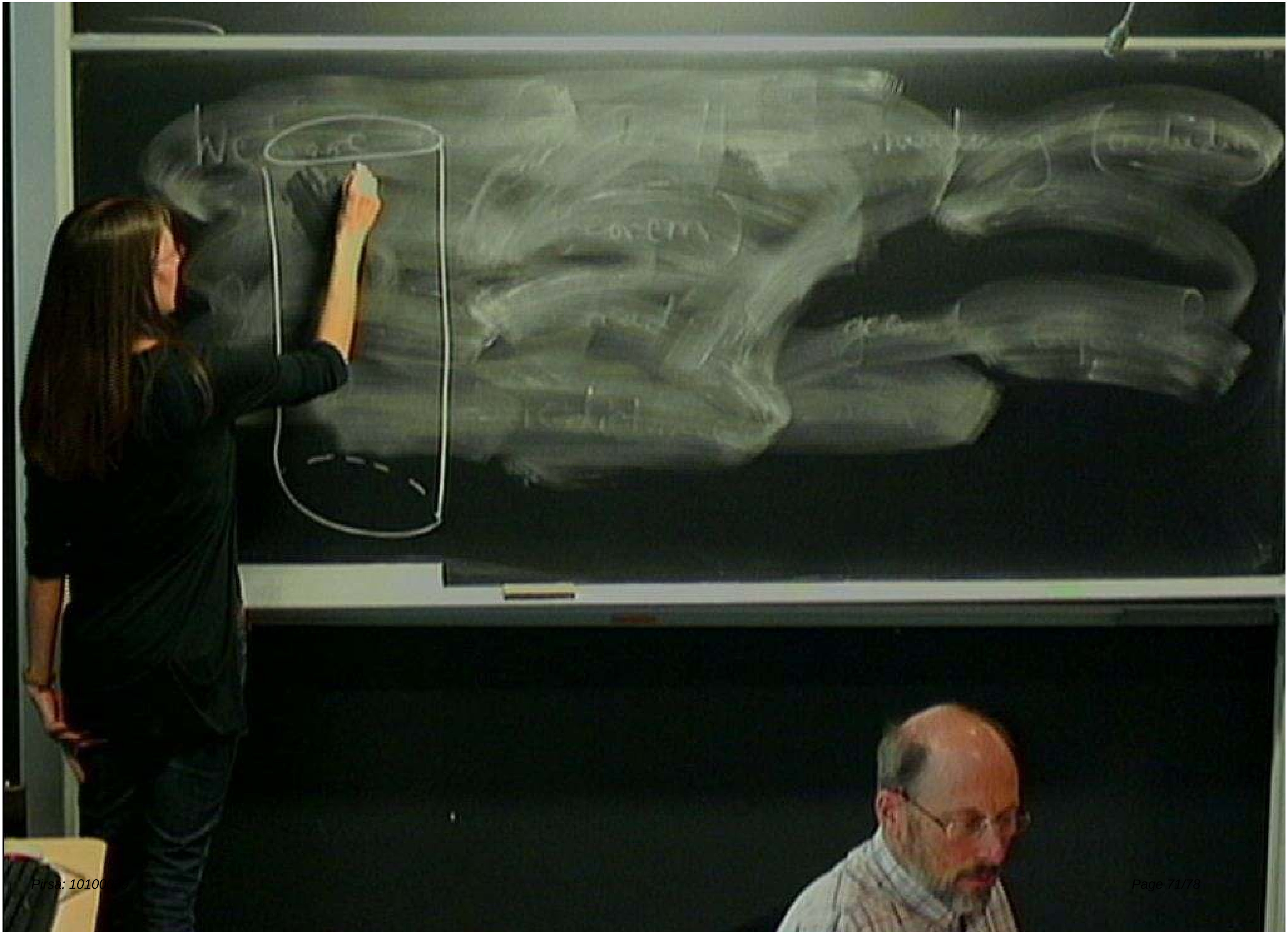

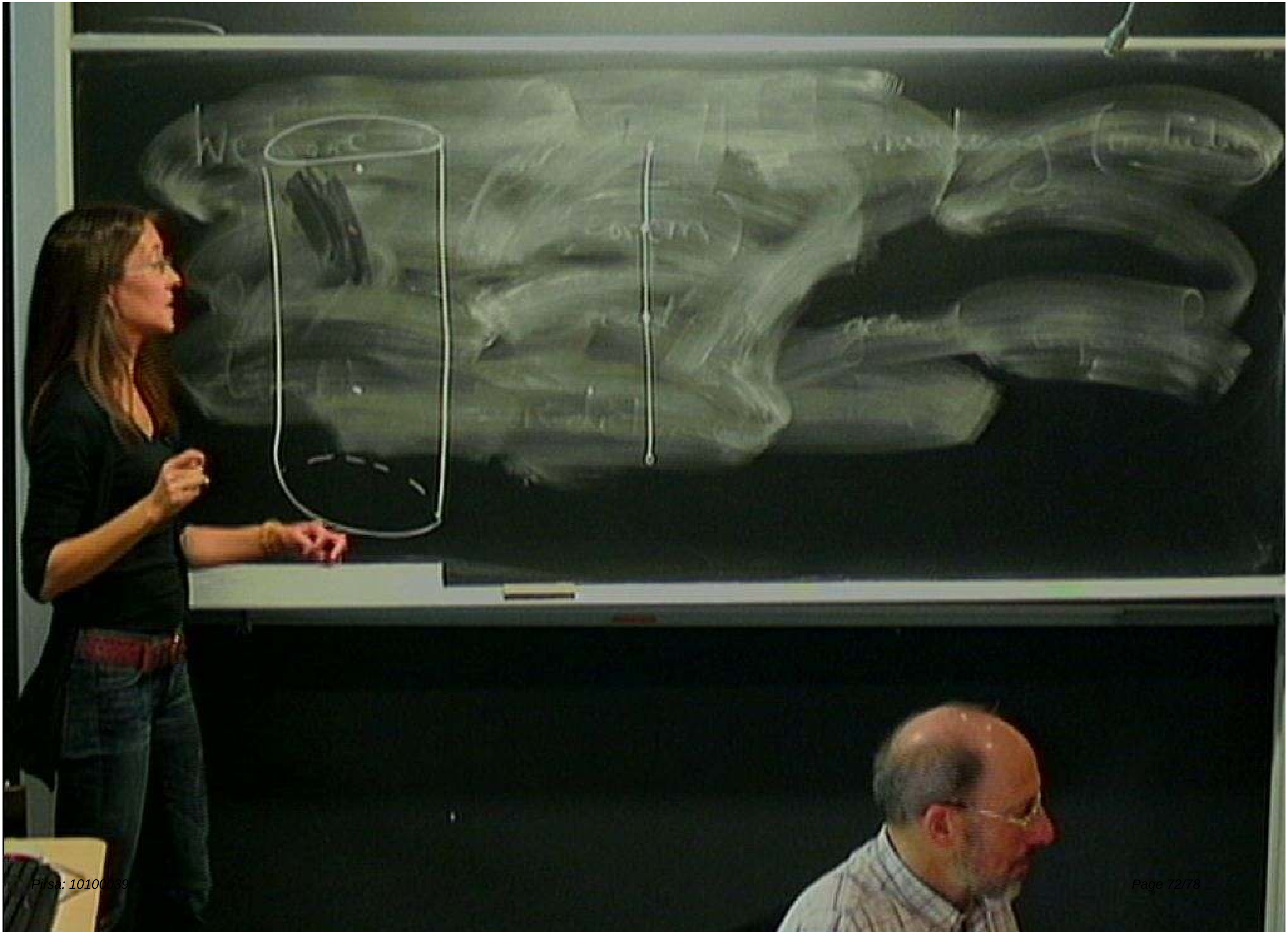

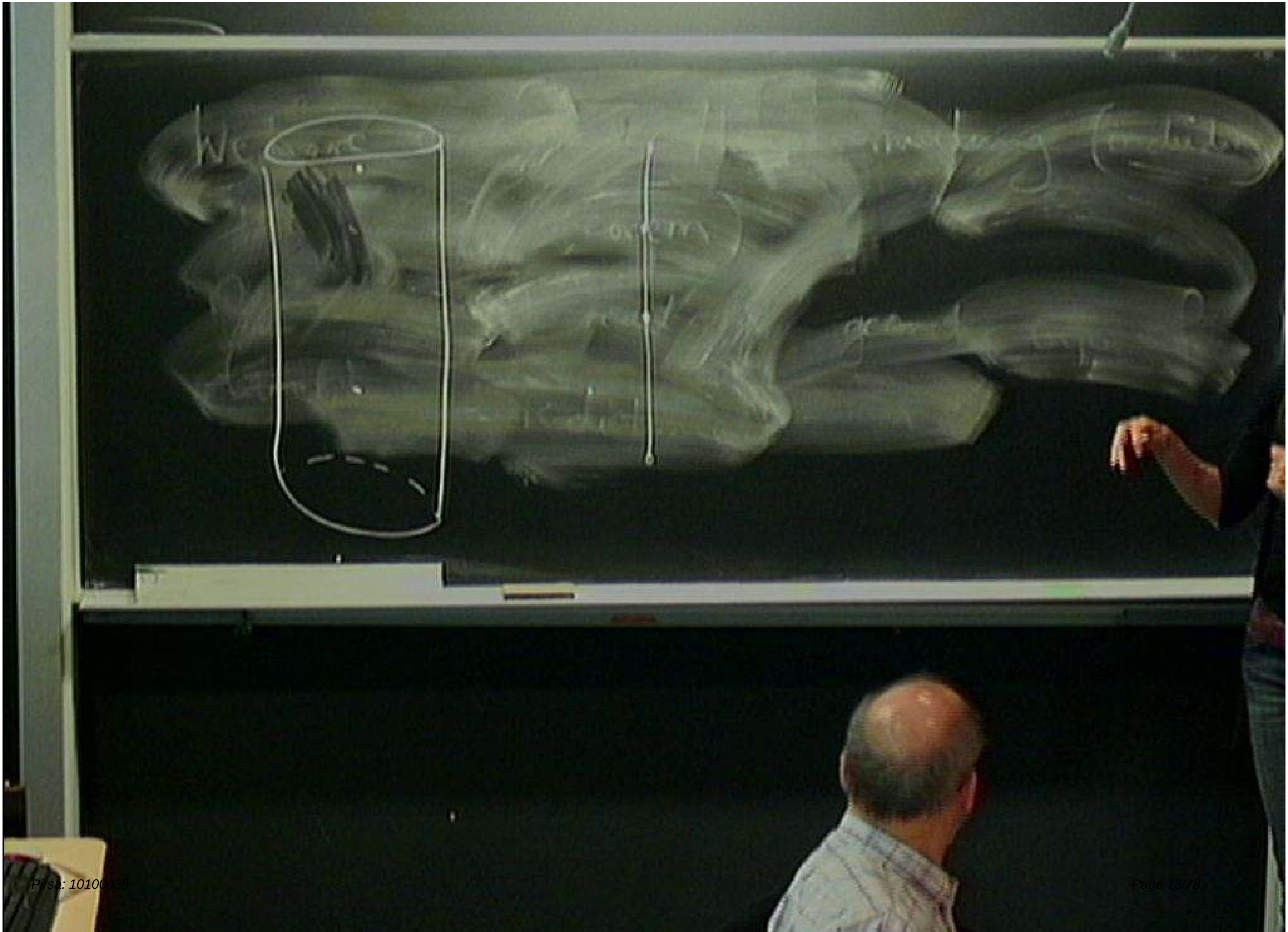

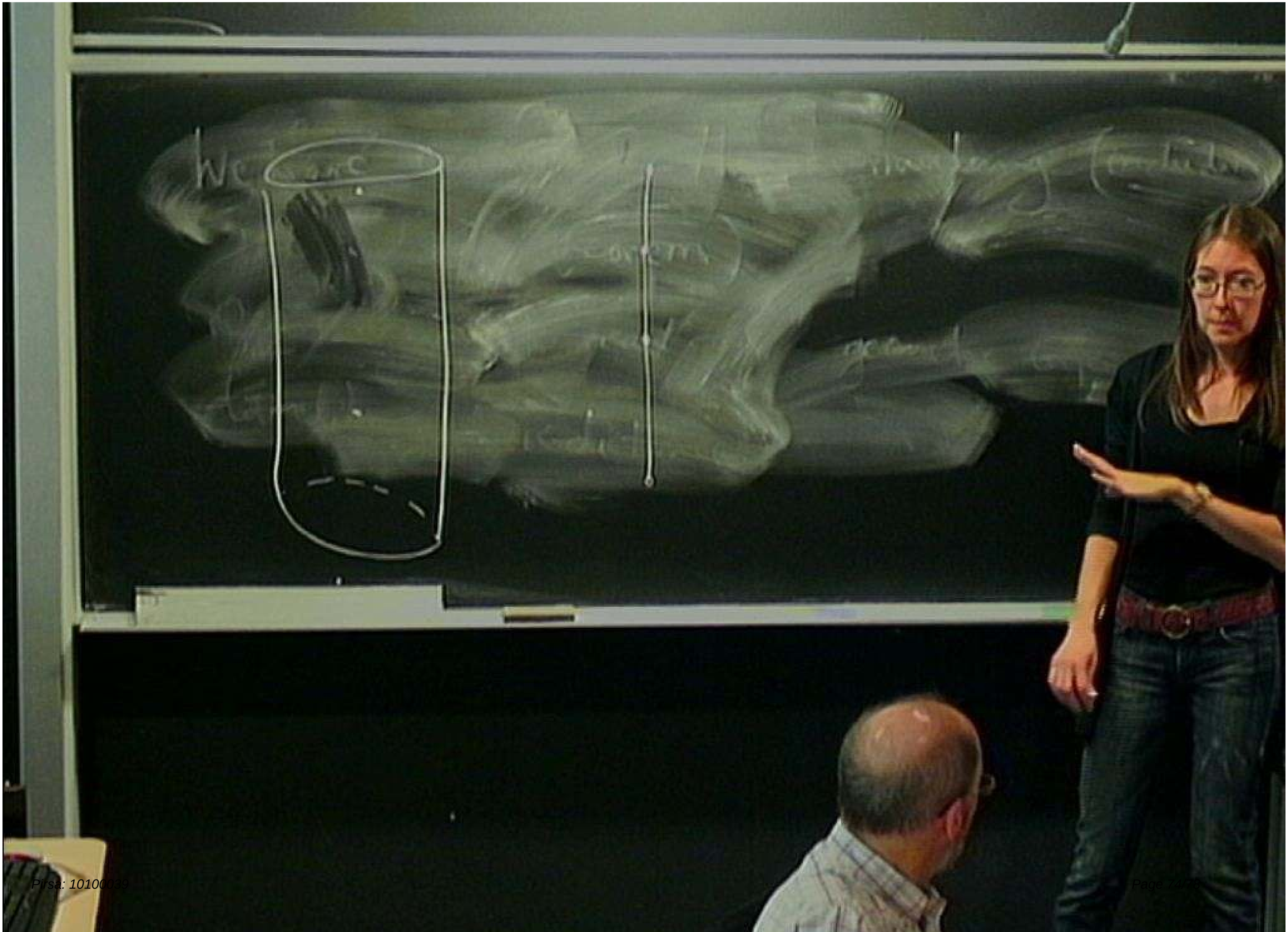

Order + Number = Geometry

- Causal sets are Lorentz invariant
- Causal sets are maximally discrete - combinatorial data only - just need to be able to count
- Carry their own metric information (c.f. Riemann's "discrete manifolds").
- The order relation $\preceq$ unifies within itself topology, differentiable structure, metric and causal structure
- Discreteness and Lorentzian-ness go together like a horse and carriage.
- Randomness of sprinkling is kinematical only: from Number ~ Volume
- Causal sets are fluid-like rather than crystal-like
- Scale dependent topology and geometry (and covariant coarse graining)
- The approach is both conservative and radical at the same time.

A couple of parallel stories

- Can the topology of space change? Spin-half and fermionic statistics from gravity, spin-statistics correlation, topology change, hopes of getting matter from spacetime
- What is the nature of quantum reality? How do we interpret the sum-over-histories? What is the meaning of relativistic causality in a quantum theory? Can we build a relativistically causal quantum dynamics for causal sets?

A partial list of further questions/opportunities

- A different light on the Cosmological Constant Problem
- What about the non-locality?
- What is the quantum dynamics of causal sets e.g. what is the action?
- Is there a Problem of Time for causal sets?
- Is the passage of time physical?
- Why is spacetime 4 dimensional?
- Why are there no holes or boundaries in spacetime?
- What does the black hole entropy count? What about "holography?
- Can we detect the discreteness already/soon? Lorentz Invariant Quantum Gravity phenomenology from causal sets
- What happened before the Big Bang? New ideas with no added scalars!

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