Title: An Invitation to Causal Sets - Lecture 1

Date: Oct 18, 2010 11:00 AM

URL: http://pirsa.org/10100038

Abstract: Introduction to the causal set approach to quantum gravity and overview of current research in causal set theory

Lecture I

A quick jog down one road (not historical) to causal sets

- Very brief comments on (the problem of) "Quantum Gravity"
- In the continuum, Causal Structure = 9/10 metric and Volume = 1/10 (in 4d)
- Go Discrete (for many reasons)! Lorentzian-ness "revealed" by discreteness
- Lorentz invariant (contrast with lattice)
- A (partial) list of things you can think about with a Causal Set Hat on

Lecture 1

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What is "Quantum Gravity"?

- The phrase is a shorthand that names the biggest obstacle we currently face in our search for a unified framework for the whole of physics.
- What Gravity? What Quantum Theory?
- Gravity = GR, Quantum Theory = Sum-Over-Histories (rather than, say, canonical quantization of a classical Hamiltonian theory)

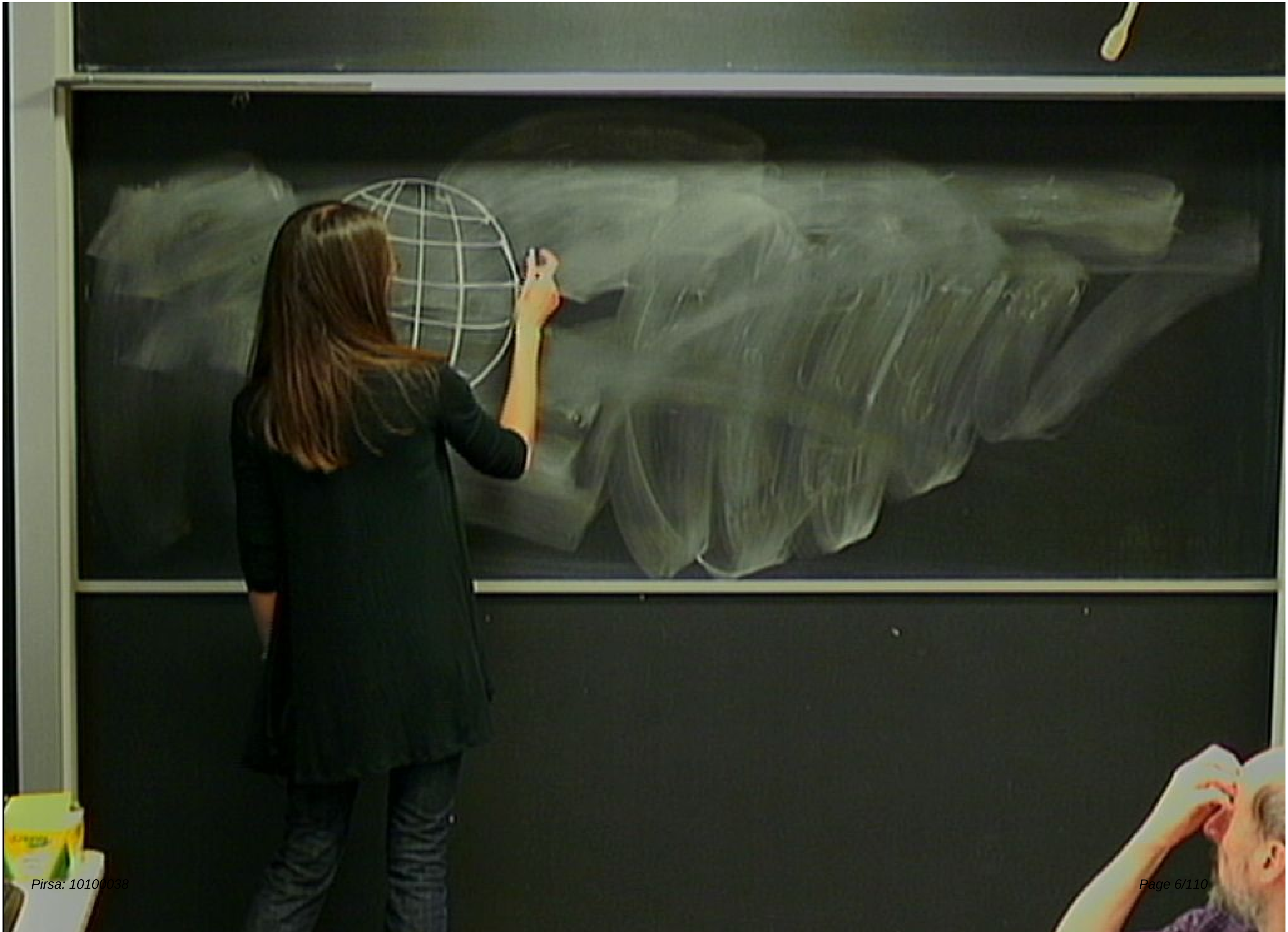
$$Z = \int \mathcal{D}g \; e^{iS[g]}$$

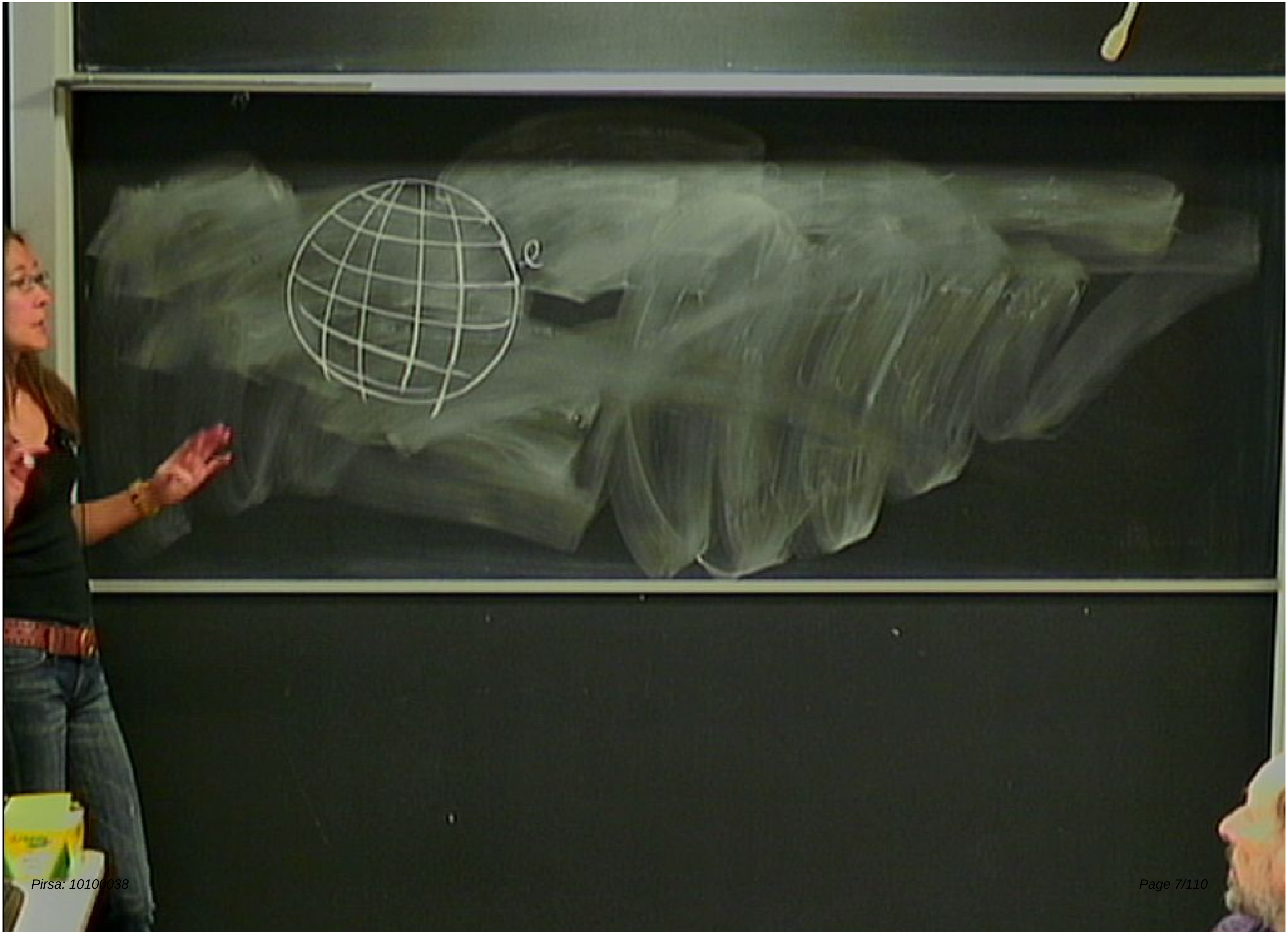
Renate Loll calls this, "A statement of intent"

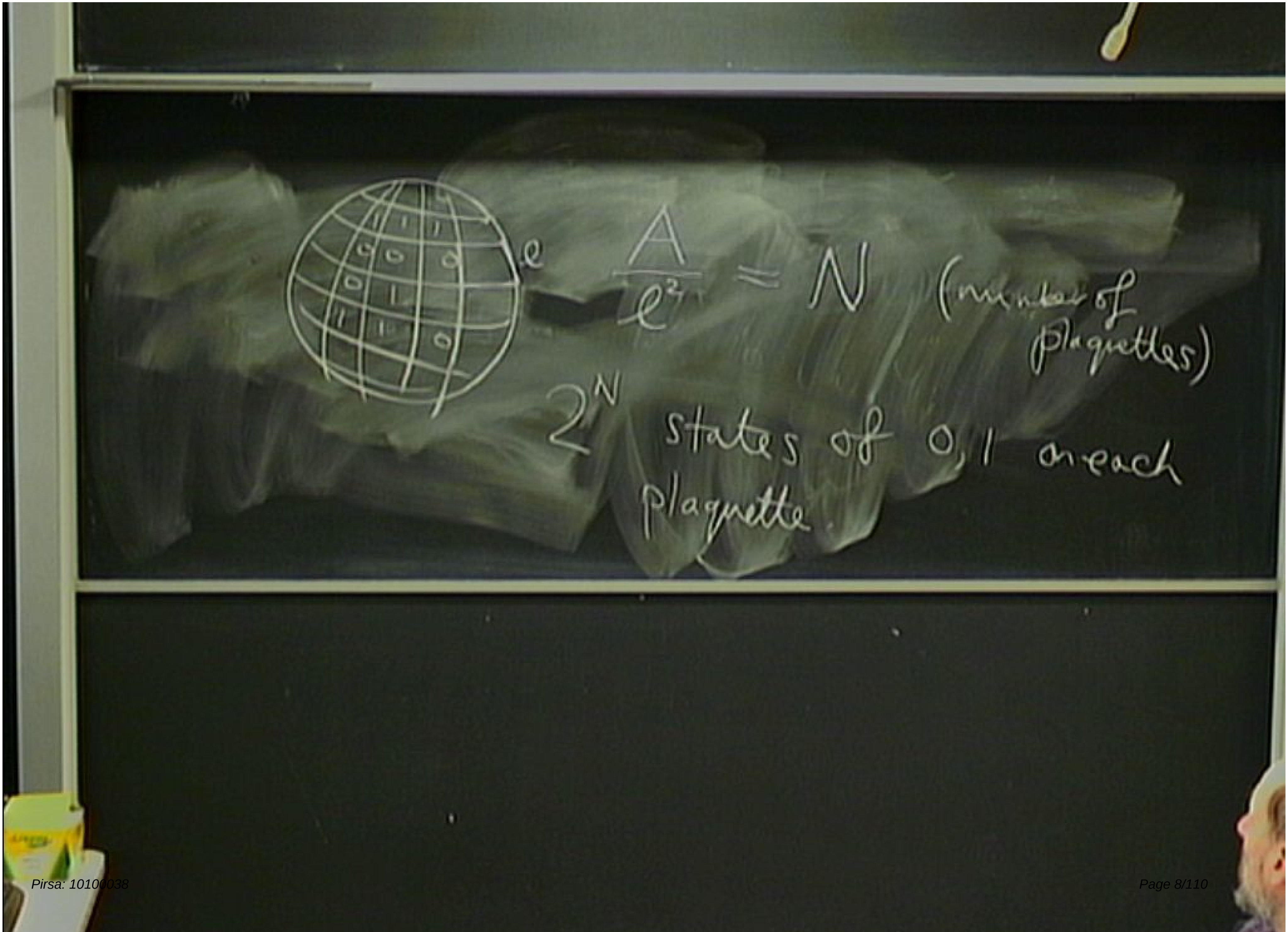
- Technical problems intractable: Go Discrete

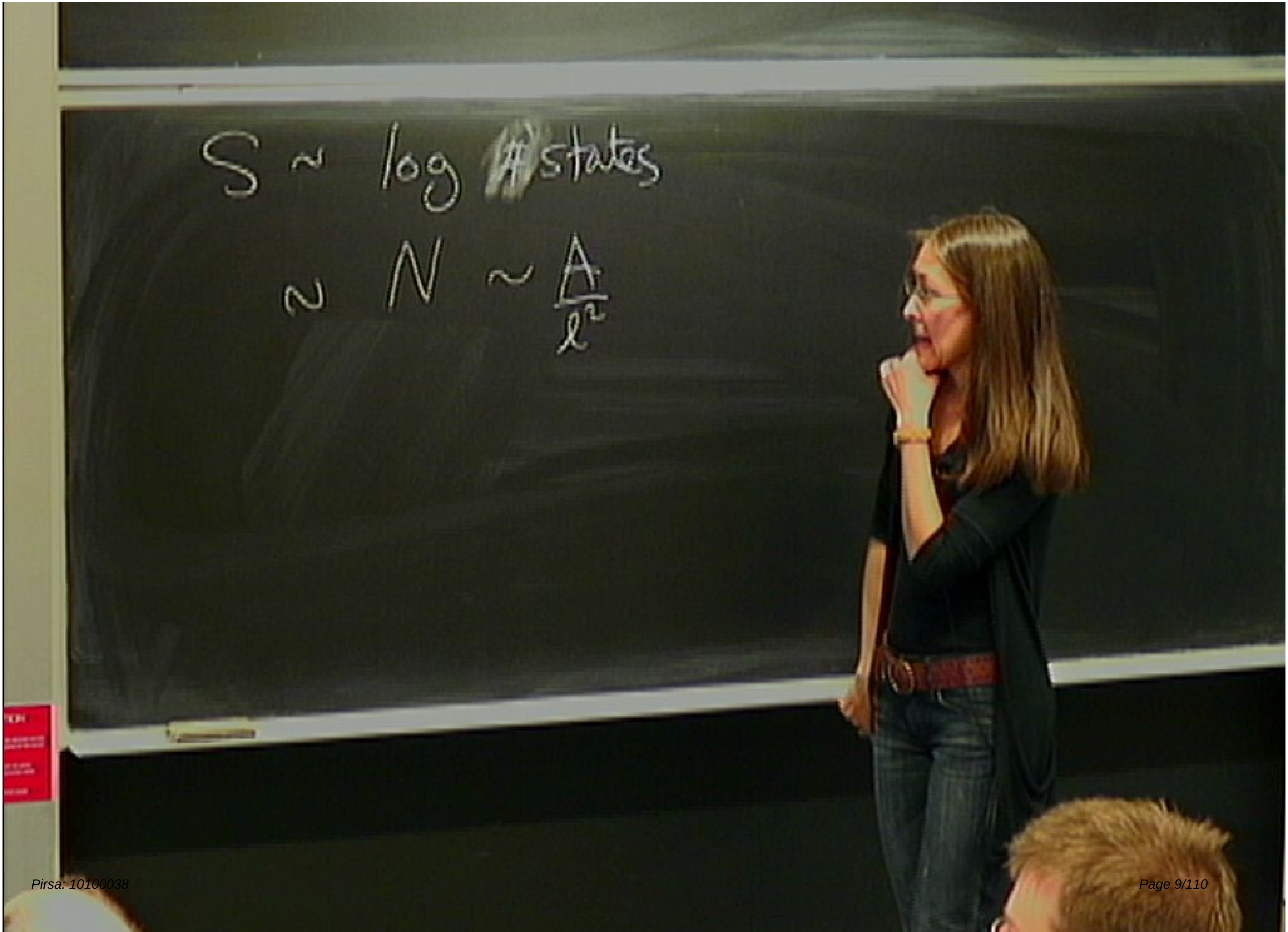
Discreteness: the Zeitgeist

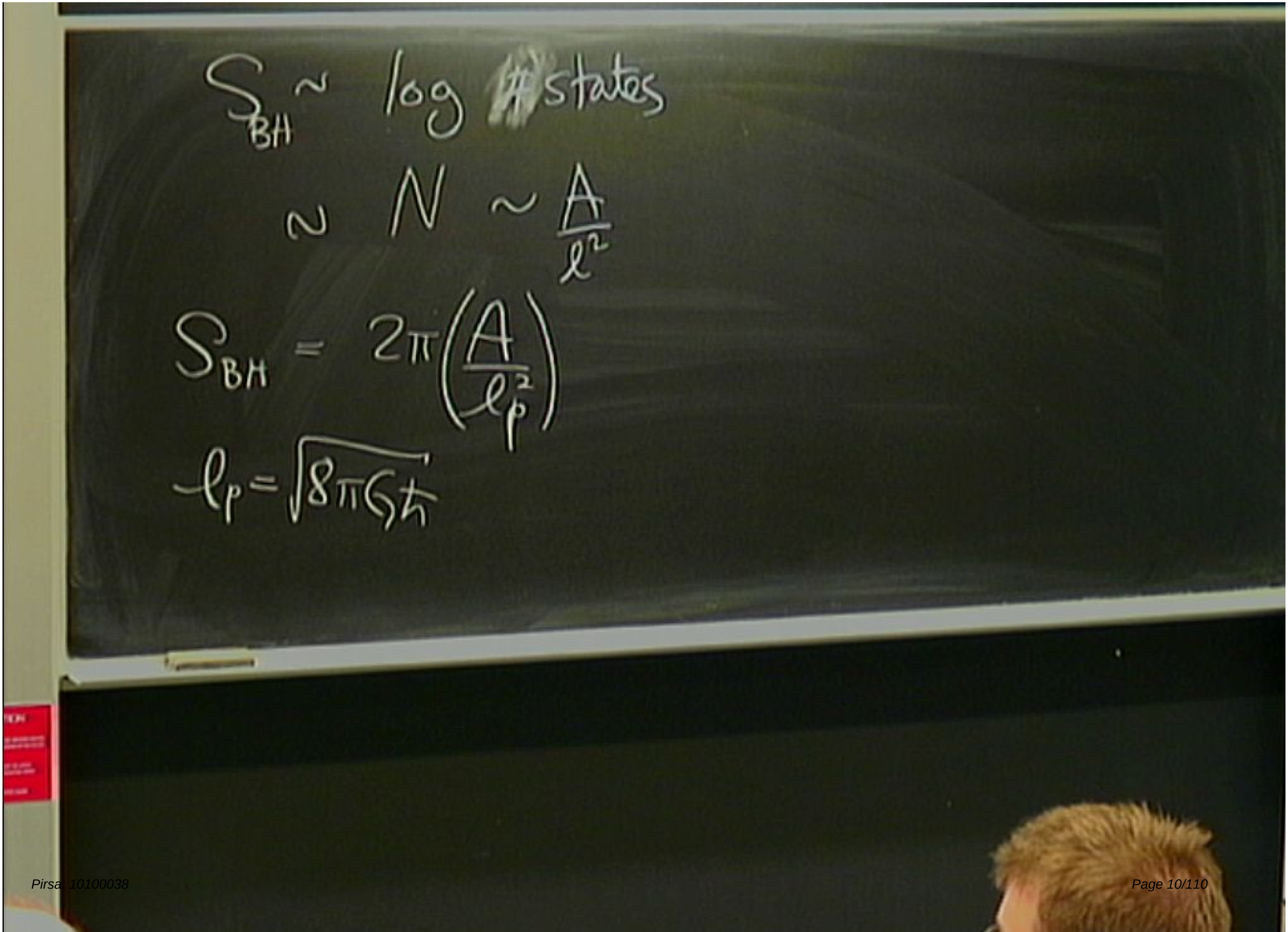
- Physical evidence for discreteness is strong but circumstantial: divergences in continuum QFT and GR. The value of the entropy of a black hole is our best clue: it strongly suggests discreteness of spacetime and that its scale is Planckian.
- Ideas of holography, finite amounts of information in finite regions of spacetime, entropy bounds, entropy density associated with "light sheets" or local Rindler horizons...point to some underlying discrete atomic structure
- Black hole entropy suggests that the discreteness is fundamental and not merely a technical device to calculate Z.
- We will take this seriously: no continuum limit in the fundamental theory
- The question then becomes: What ARE the "discrete manifolds" (Riemann's term) that contribute to the sum-over-histories for quantum gravity?

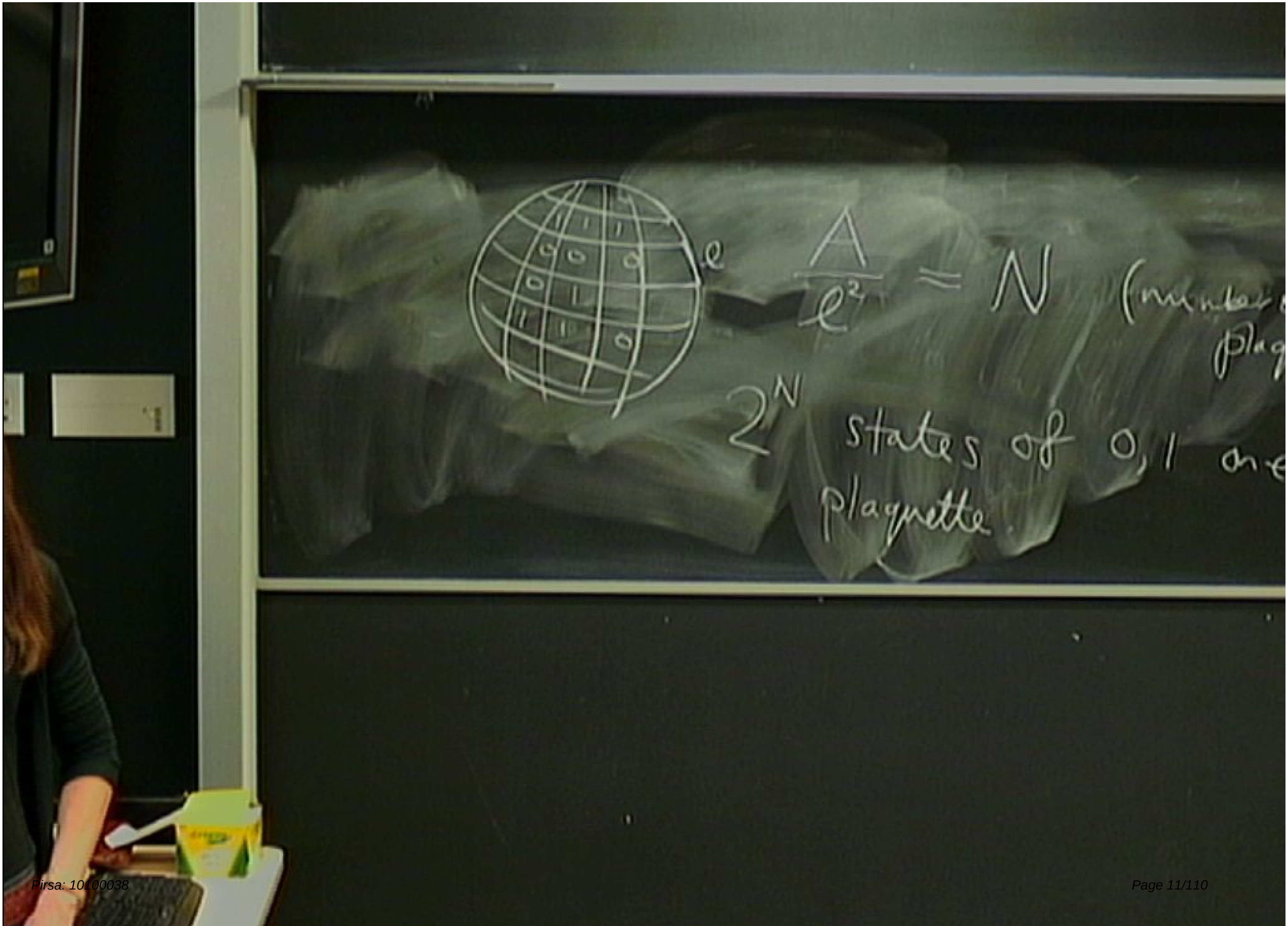





e
A/e² ≈ N (number of plaquettes)
2^N states of 0,1 on each plaquette


S ~ log # states
~ N ~ A/l²


$S_{BH} \sim \log$ # states
$\sim N \sim \frac{A}{\ell^2}$
$S_{BH} = 2\pi\left(\frac{A}{\ell_p^2}\right)$
$\ell_p = \sqrt{8\pi G\hbar}$


A/ℓ² ≃ N
2^N states of 0,1
plaquette.

Discreteness: the Zeitgeist

- Physical evidence for discreteness is strong but circumstantial: divergences in continuum QFT and GR. The value of the entropy of a black hole is our best clue: it strongly suggests discreteness of spacetime and that its scale is Planckian.
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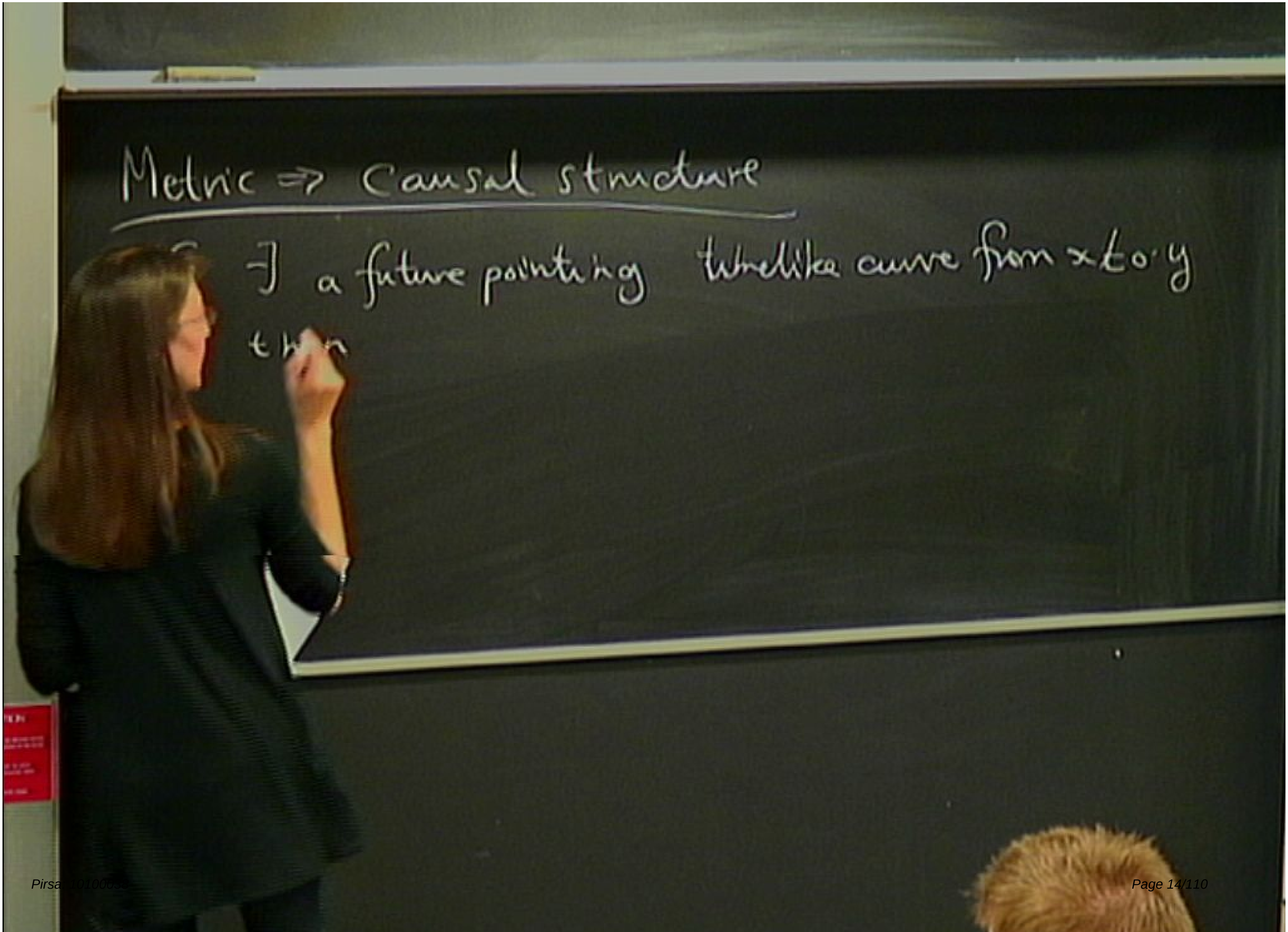
First, another look at the continuum

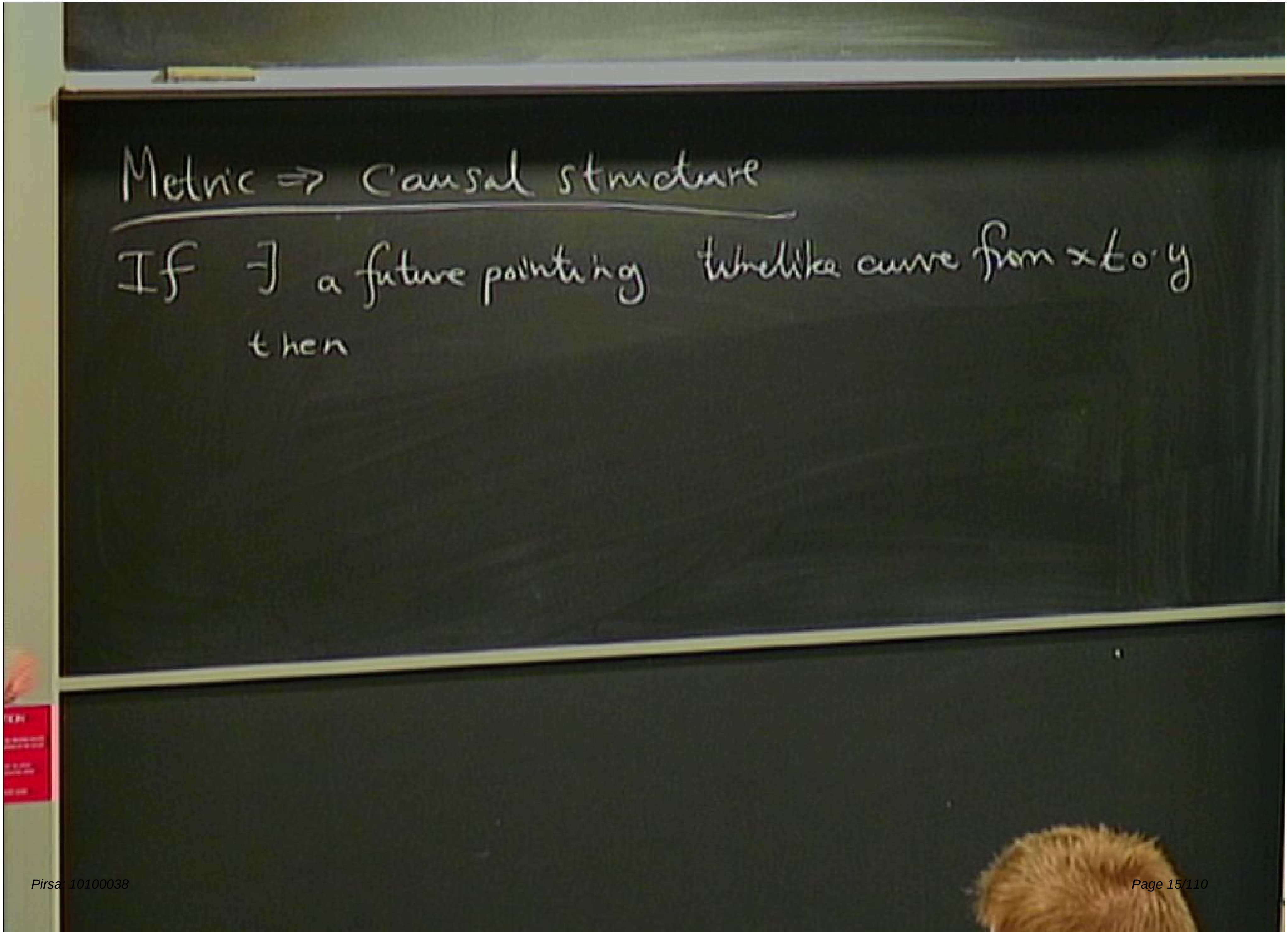
General Relativity (GR) tells us that spacetime is a 4-dimensional differentiable manifold, M, with a metric field, $g_{\mu\nu}$, with signature (- + + +)

- Metric gives the causal structure (we assume spacetimes are causal)
- Causal structure gives 9/10 of the metric (in 4-d)
- The remaining 1/10 is fixed by volume information

We return to the question of the discrete manifolds in the SOH. Some of them, at least, must be approximated by Lorentzian manifolds and they must be "non-ad hoc". Let's get a handle on things by *discretising*.



Metric ⇒ Causal structure
∃ a future pointing timelike curve from x to y


Metric ⇒ Causal structure
If ∃ a future pointing timelike curve from x to y
then

Metric $\Rightarrow$ Causal structure

If $\exists$ a future pointing timelike curve from x to y then $x \ll y$ & define $I^+(x) = \{y \in M \mid x \ll y\}$

Metric ⇒ Causal structure

If $\exists$ a future pointing timelike curve from x to y then $x \ll y$ & define $I^+(x) = \{y \in M \mid x \ll y\}$

If $\exists$ a f. p. spacelike curve from x to y then $x \leq y$ & define $J^+(x) = \{y \in M \mid x \leq y\}$

Metric $\Rightarrow$ Causal structure

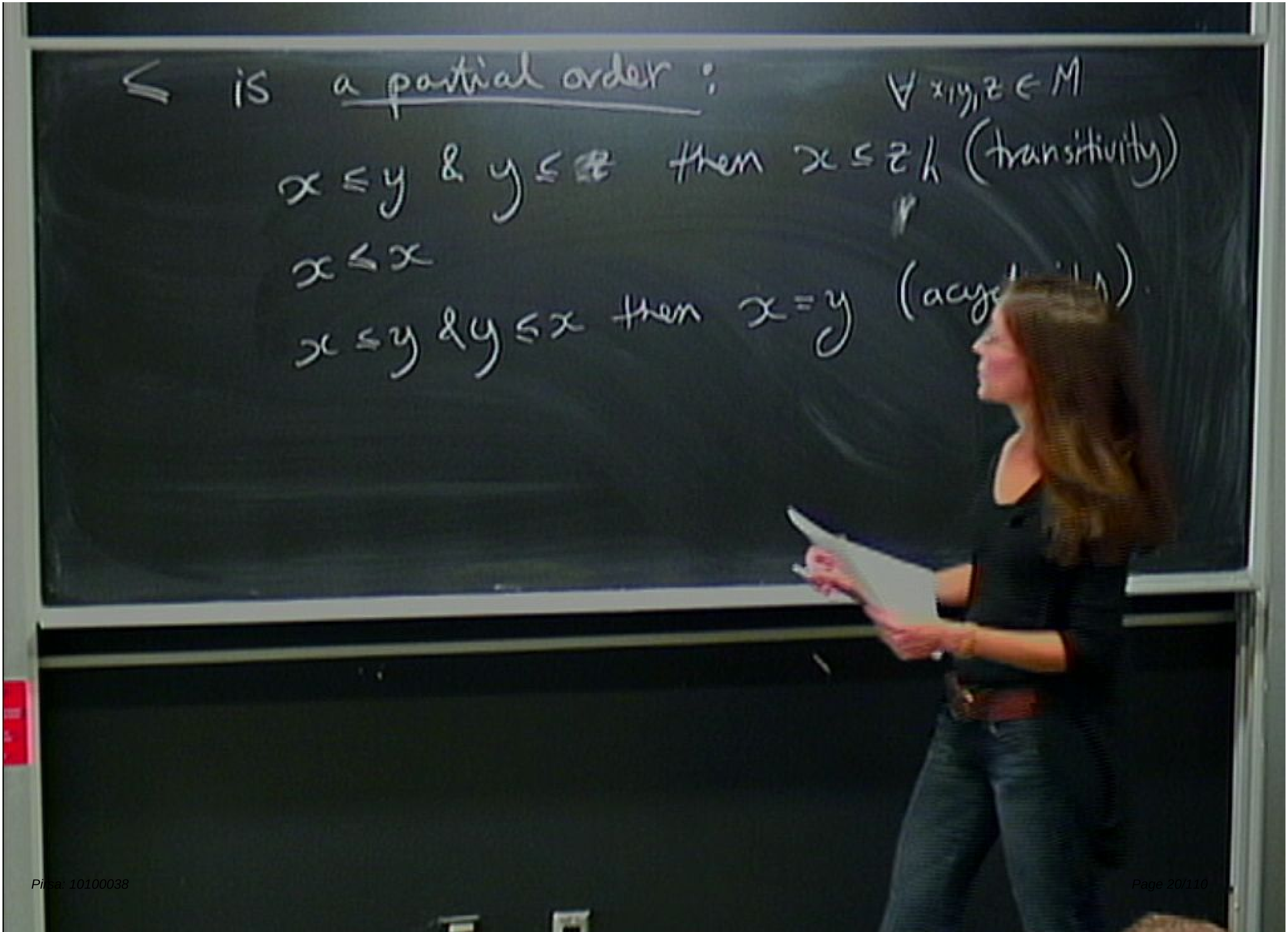
If $\exists$ a future pointing timelike curve from x to y then $x \ll y$ & define $I^+(x) = \{y \in M \mid x \ll y\}$

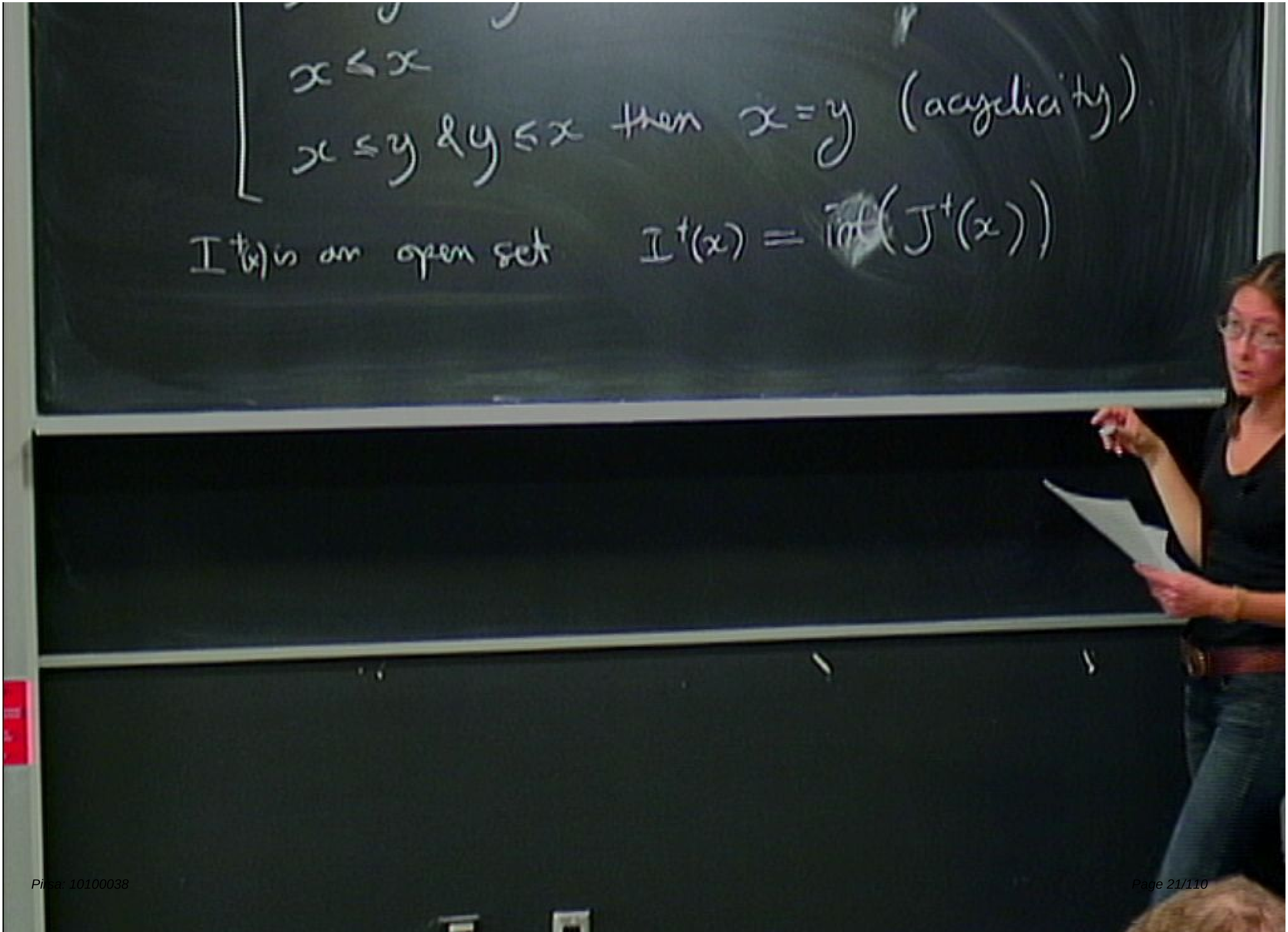
If $\exists$ a f. p. non-spacelike curve from x to y then $x \leq y$ & define $J^+(x) = \{y \in M \mid x \leq y\}$

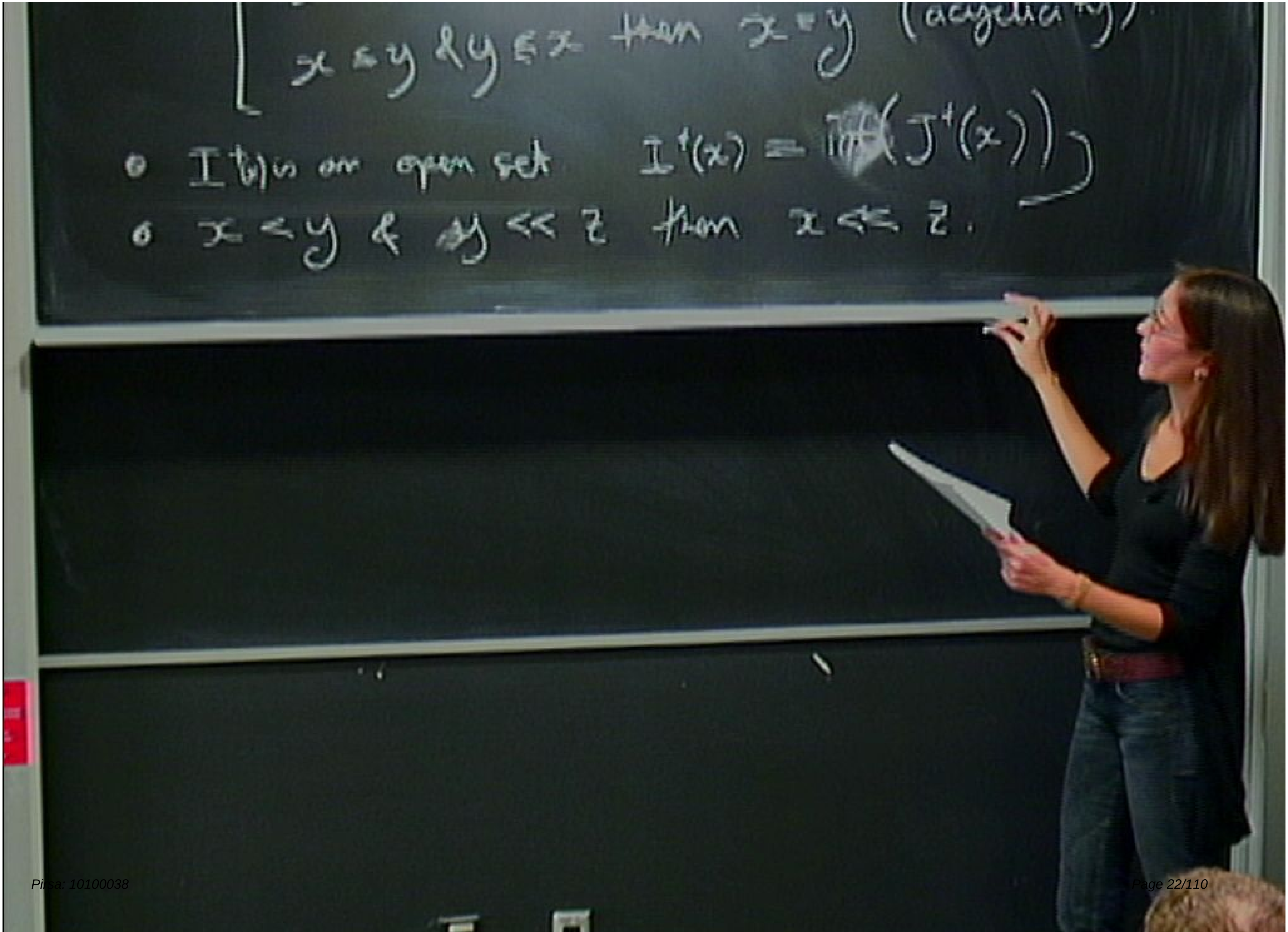
$\leq$ is the causal structure of (M, g)

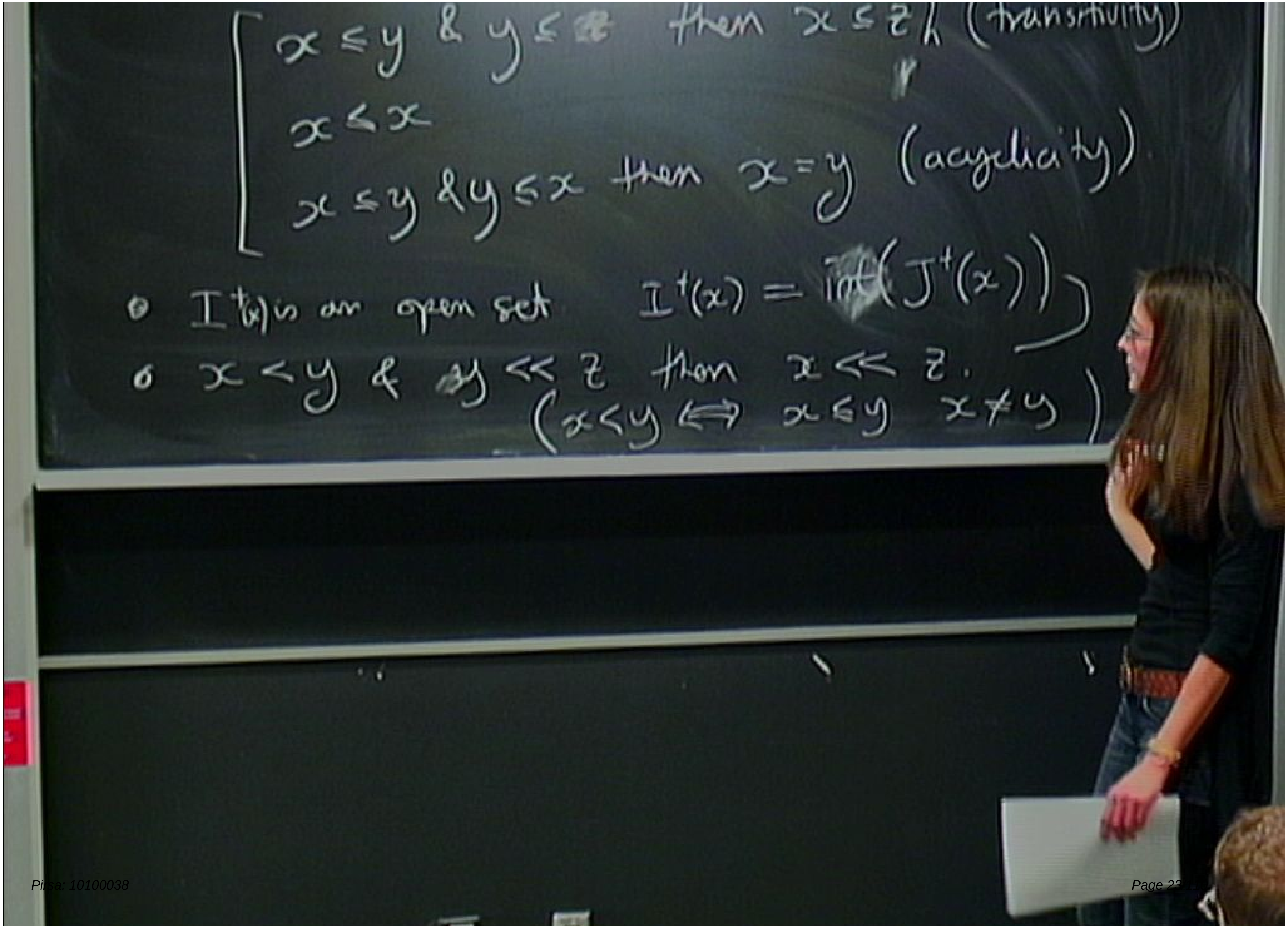
$\leq$ is <u>a partial order</u>:

$x \leq y$ & $y \leq z$ then $x \leq z$ (transitivity)


≤ is a partial order : ∀ x,y,z ∈ M
x ≤ y & y ≤ then x ≤ z (transitivity)
x ≤ x
x ≤ y & y ≤ x then x = y (acyc


x ≤ x
x ≤ y & y ≤ x then x = y (acyclicity)
I⁺(x) is an open set
I⁺(x) = int(J⁺(x))


x ≤ y & y ≤ x then x = y (acyclicity)
I+(x) is an open set
I+(x) = Int(J+(x))
x < y & y << z then x << z.


$x \leq y$ & $y \leq z$ then $x \leq z$ (transitivity)
$x \leq x$
$x \leq y$ & $y \leq x$ then $x = y$ (acyclicity)
• $I^+(x)$ is an open set $I^+(x) = \mathrm{int}(J^+(x))$
• $x < y$ & $y \ll z$ then $x \ll z$.
($x < y \Longleftrightarrow x \leq y$ $x \neq y$)

• $x < y$ & $y \ll z$ then $x \ll z$.
($x < y \Leftrightarrow x \leq y$ $x \neq y$)

Causal structure is 9/10 metric (4d).

<u>Theorem</u> (Hawking, Malament, Levichev)

Let (M,g) & (M',g') be distin... 4d Lorentzian manifolds,
& let $f: M \to M'$ be a causal ...tion i.e.

$$f(x) \leq f(y) \Leftrightarrow$$

- $x < y$ & $y \ll z$ then $x \ll z$.

$(x < y \iff x \le y \;\; x \ne y)$

Causal structure is 9/10 metric (4d).

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Let (M,g) & (M',g') be distinguishing 4d Lorentzian ma

& let $f: M \to M'$ be a causal bijection i.e.

$$f(x) \le f(y) \iff x \le y \quad (f^{-1}(p) \le f^{-1}(q) \iff p$$

- If $x < y$ & $y \ll z$ then $x \ll z$.

  ($x < y \iff x \le y$, $x \ne y$)

Causal structure is 9/10 metric (4d).

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Then f is a smooth conformal isometry i.e. $f_*(g') = \Omega g$

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<u>Theorem</u> (Hawking, Malament, Levichev)
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Causal Structure is 9/10 metric (4d).

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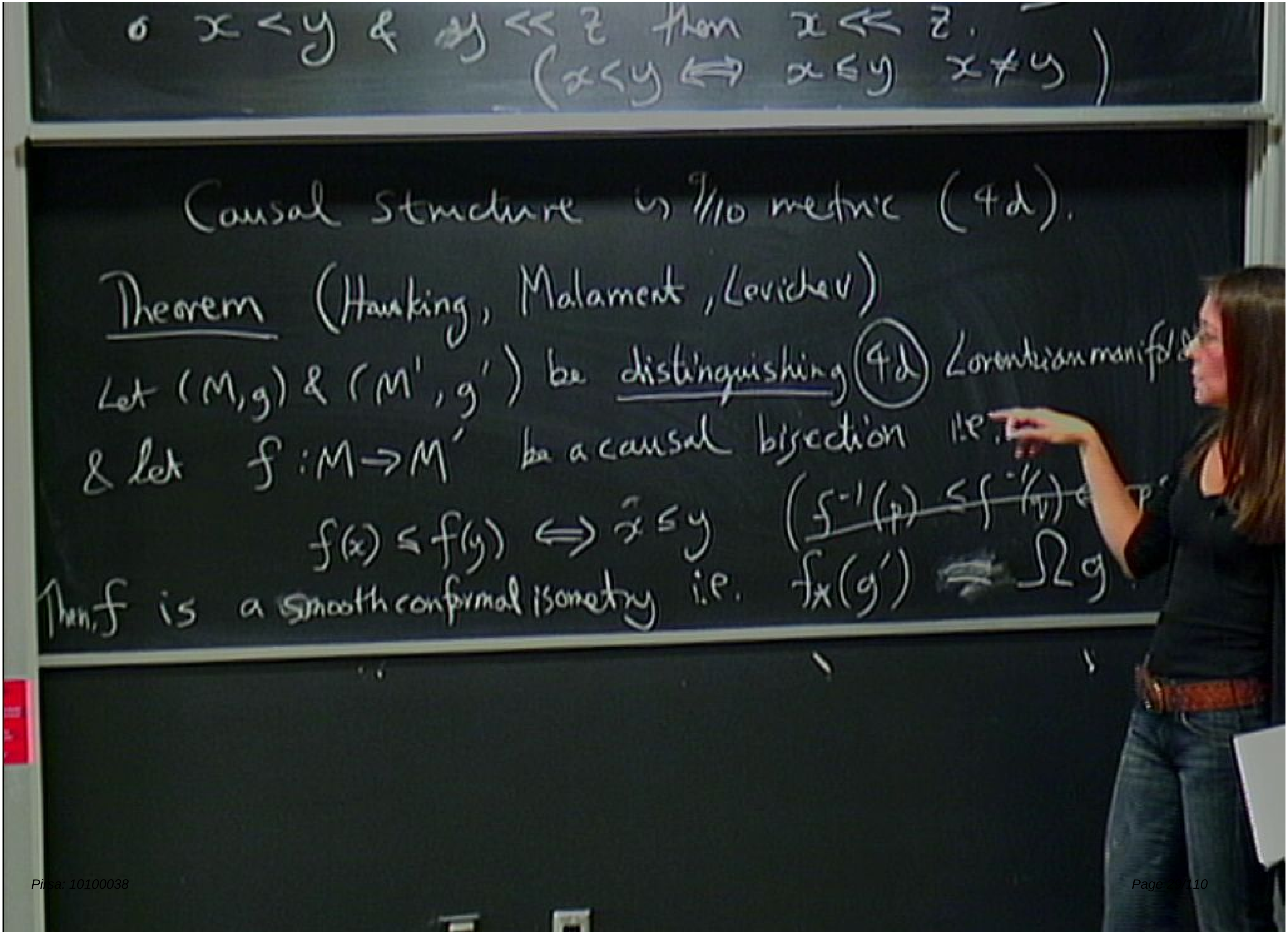
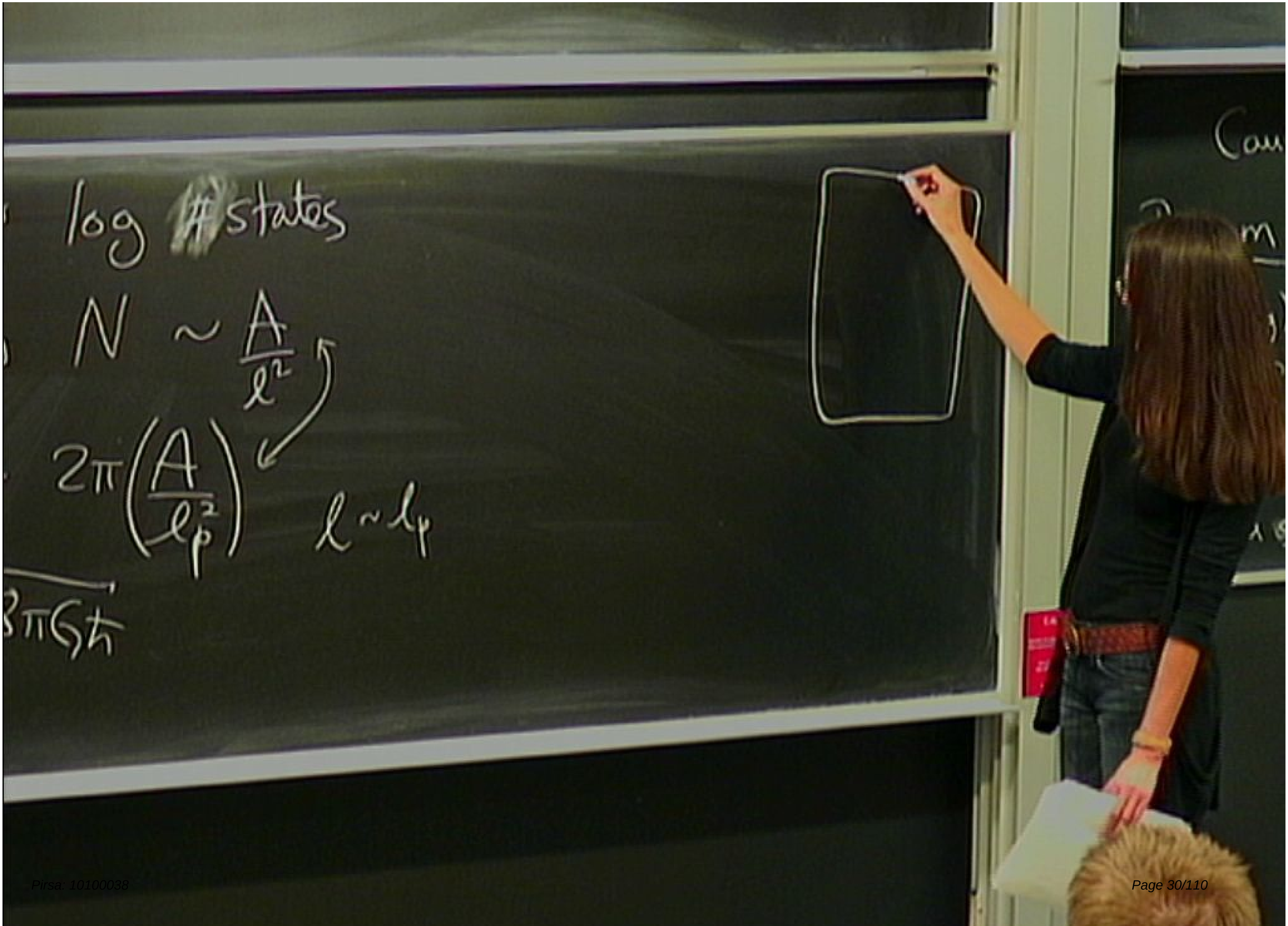
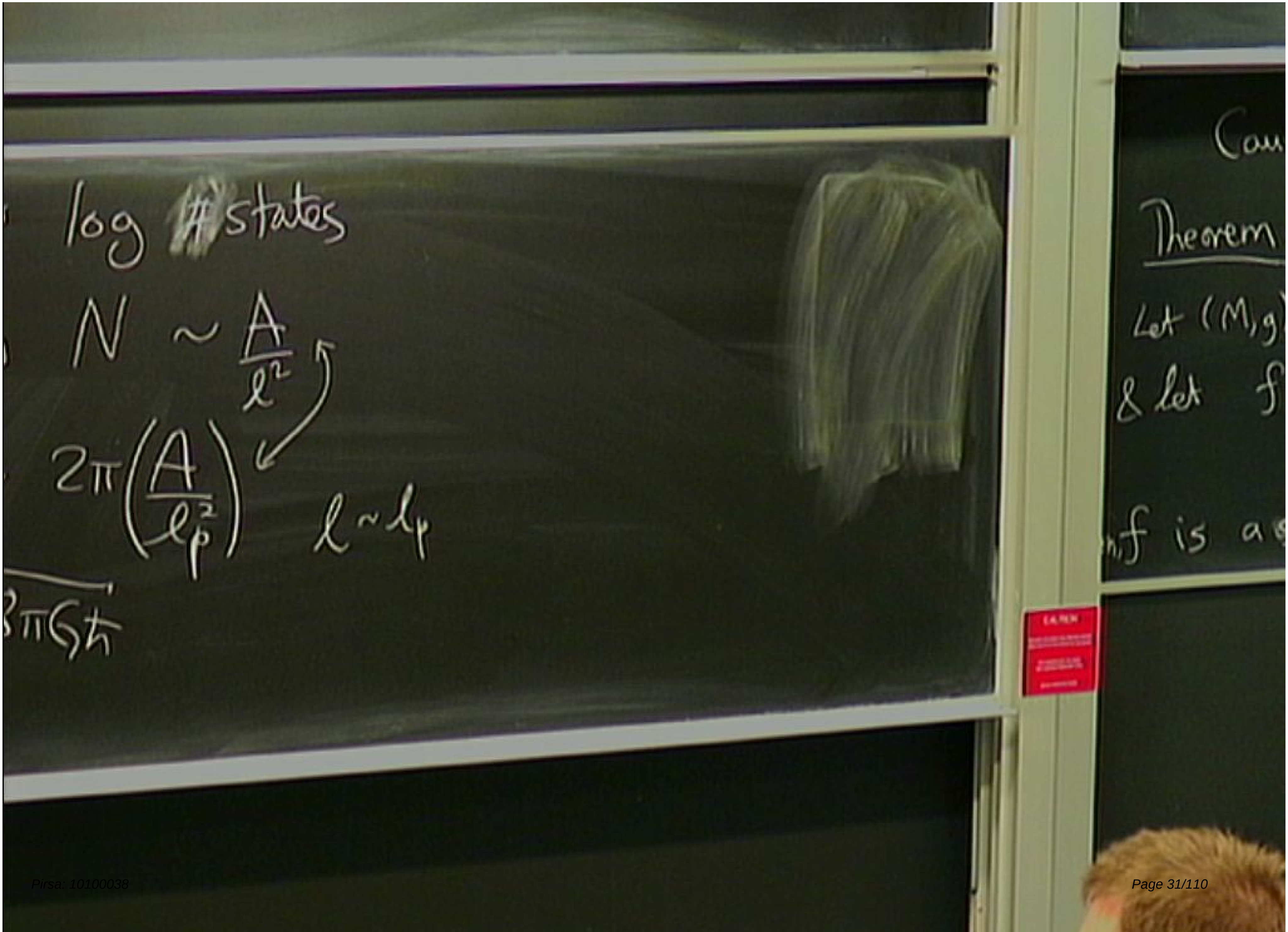
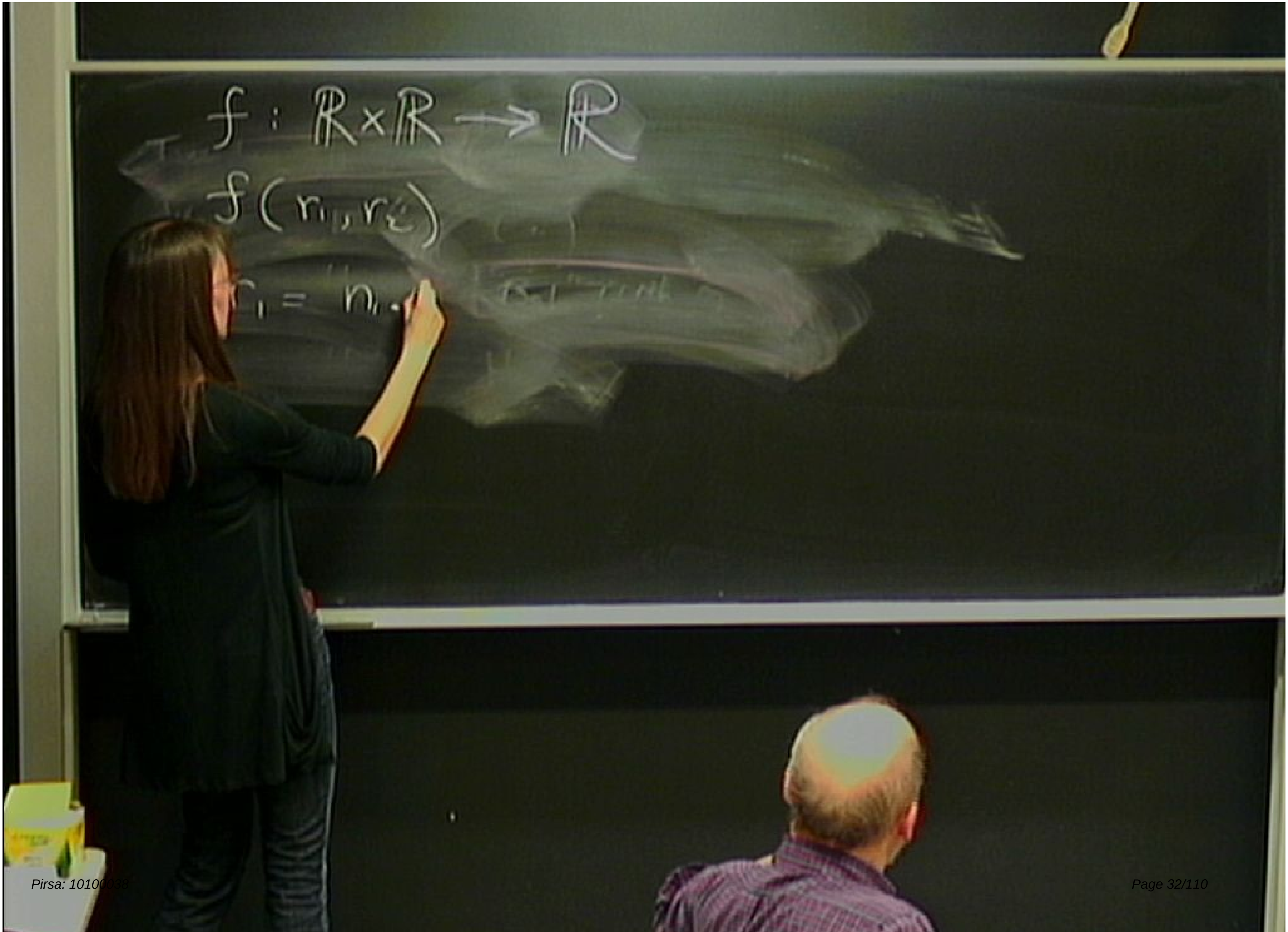
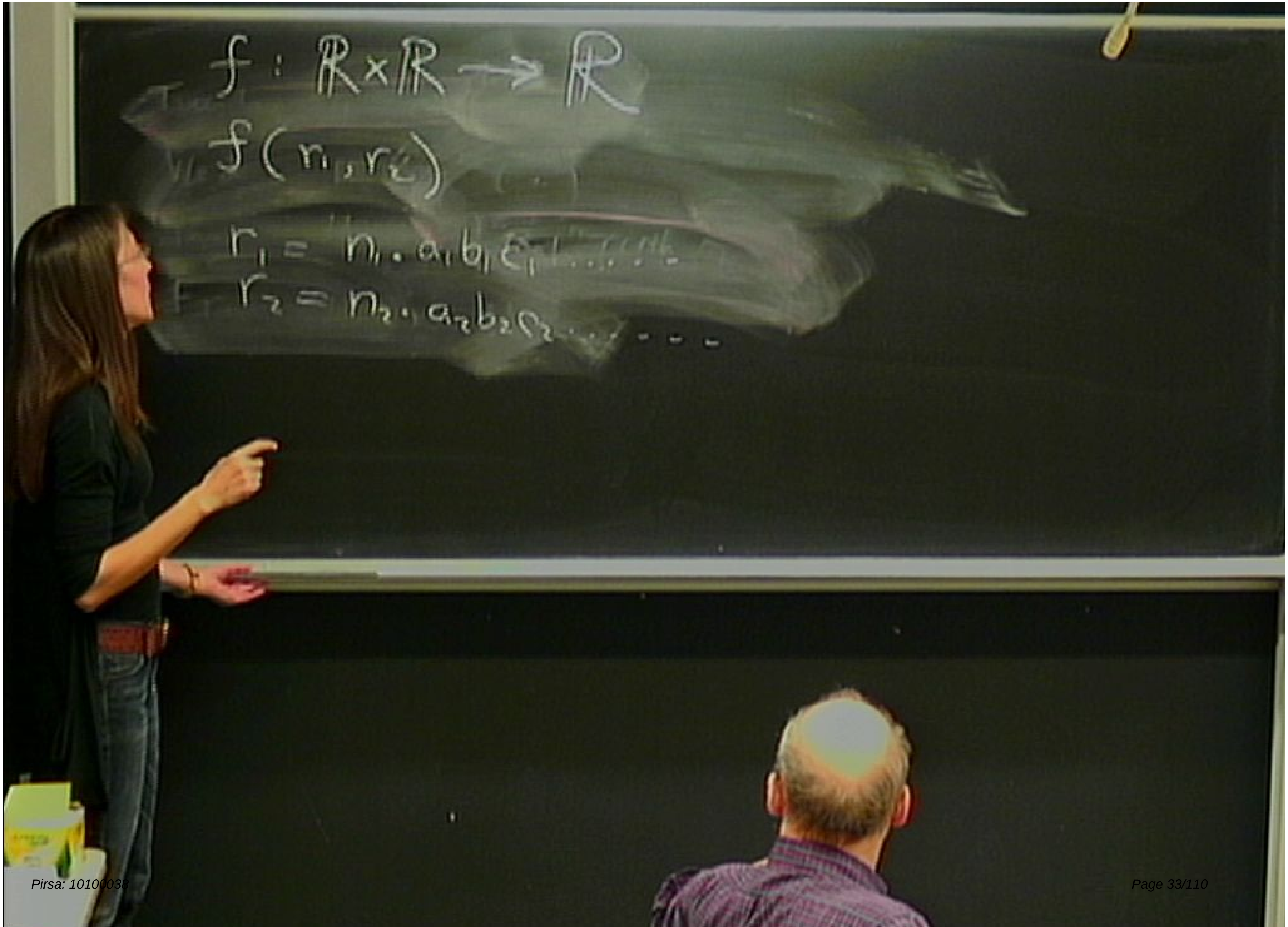


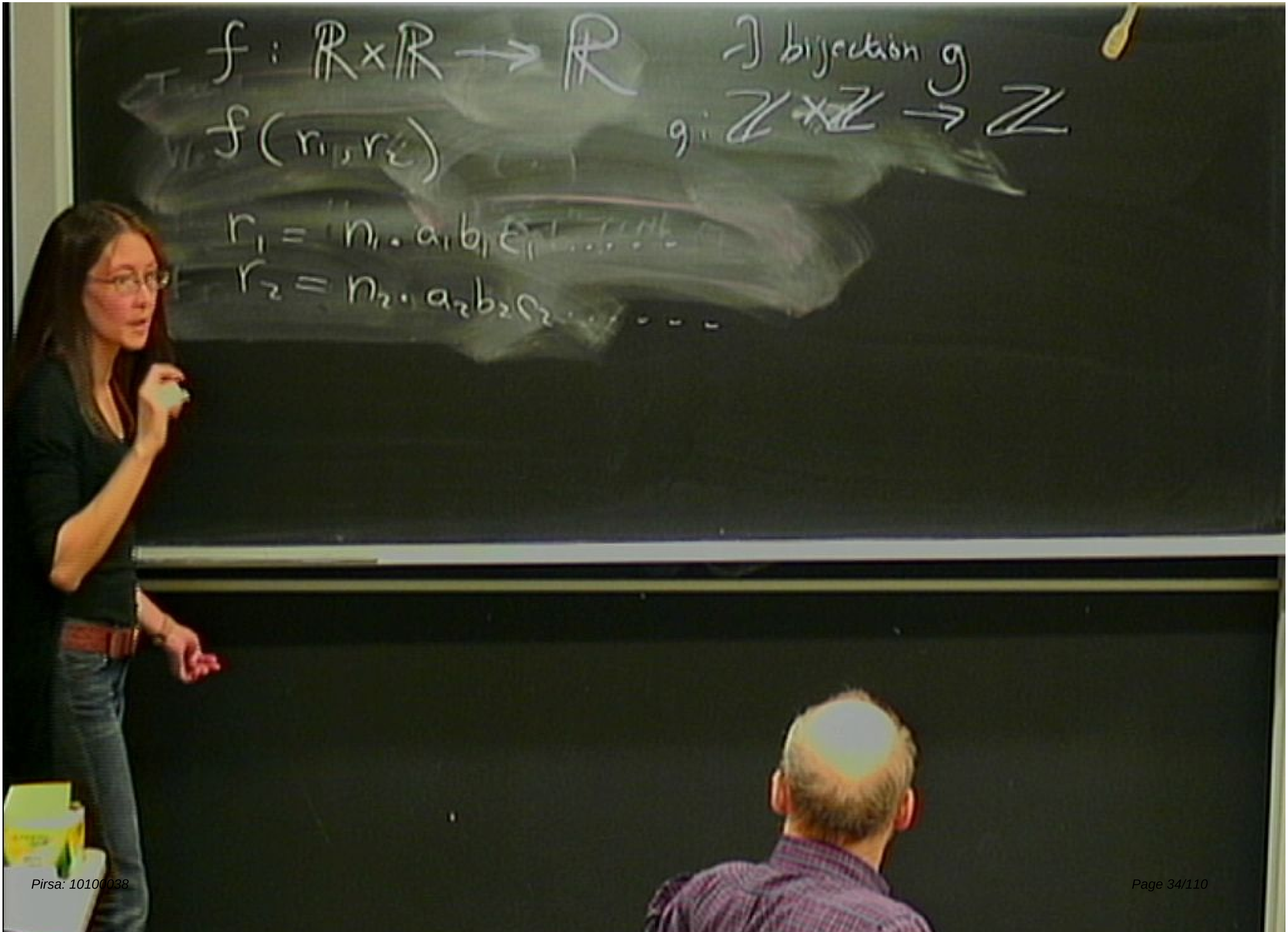
$\circ$ $x < y$ & $y \ll z$ then $x \ll z$.
($x < y \Leftrightarrow x \le y$ $x \ne y$)
Causal structure is 7/10 metric (4d).
Theorem (Hawking, Malament, Levichev)
Let (M,g) & (M',g') be distinguishing (4d) Lorentzian manifolds
& let $f : M \to M'$ be a causal bijection i.e.
$f(x) \le f(y) \Leftrightarrow \tilde{x} \le y$
Then f is a smooth conformal isometry i.e. $f_*(g') = \Omega g$


log #states
N ~ A/l²
2π(A/l_p²)
l ~ l_p
πGħ

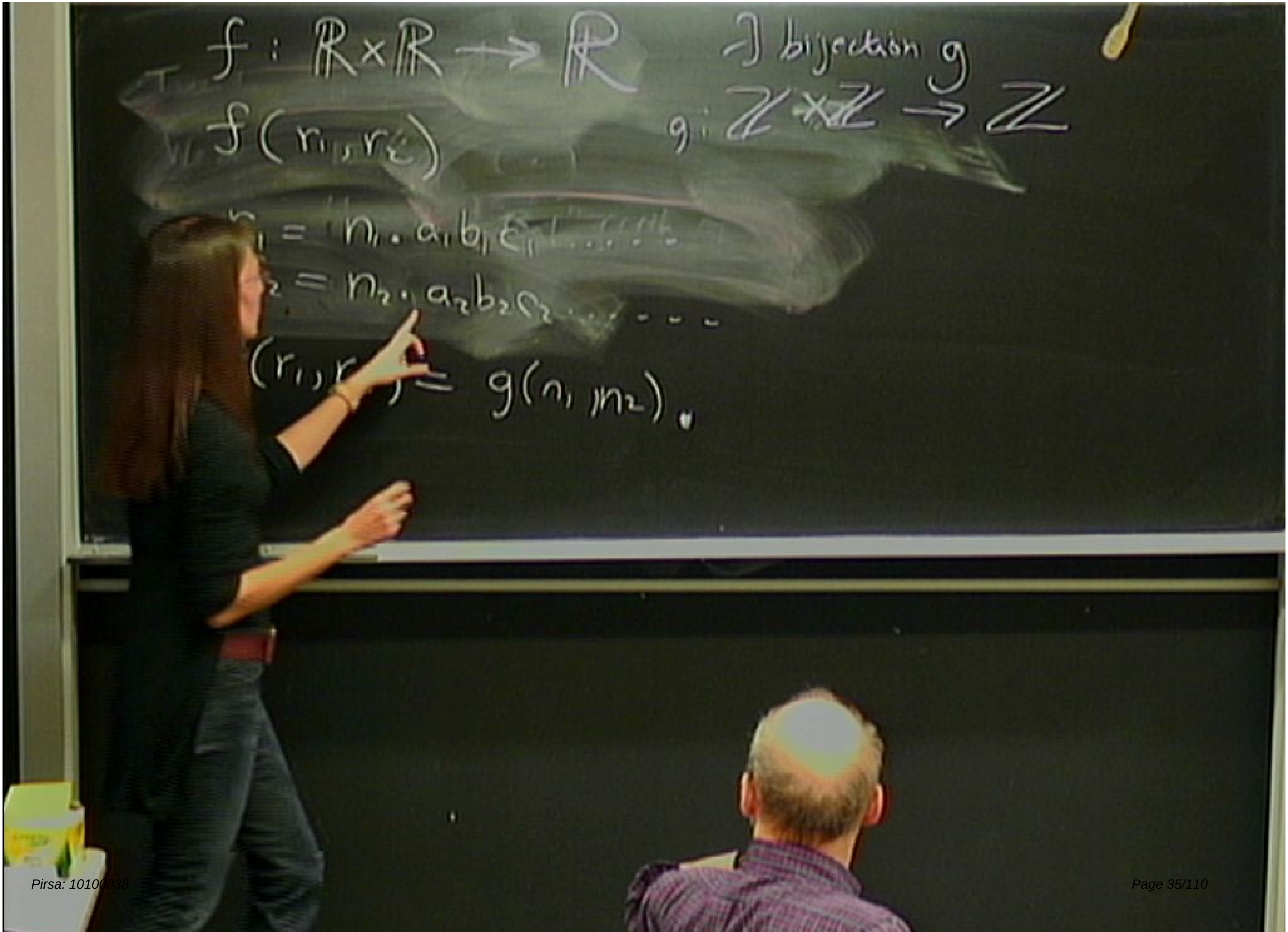

log #states
N ~ A / ℓ²
2π(A / ℓ_p²)
ℓ ~ ℓ_p
πGħ
Theorem
Let (M,g)
& let f
f is a

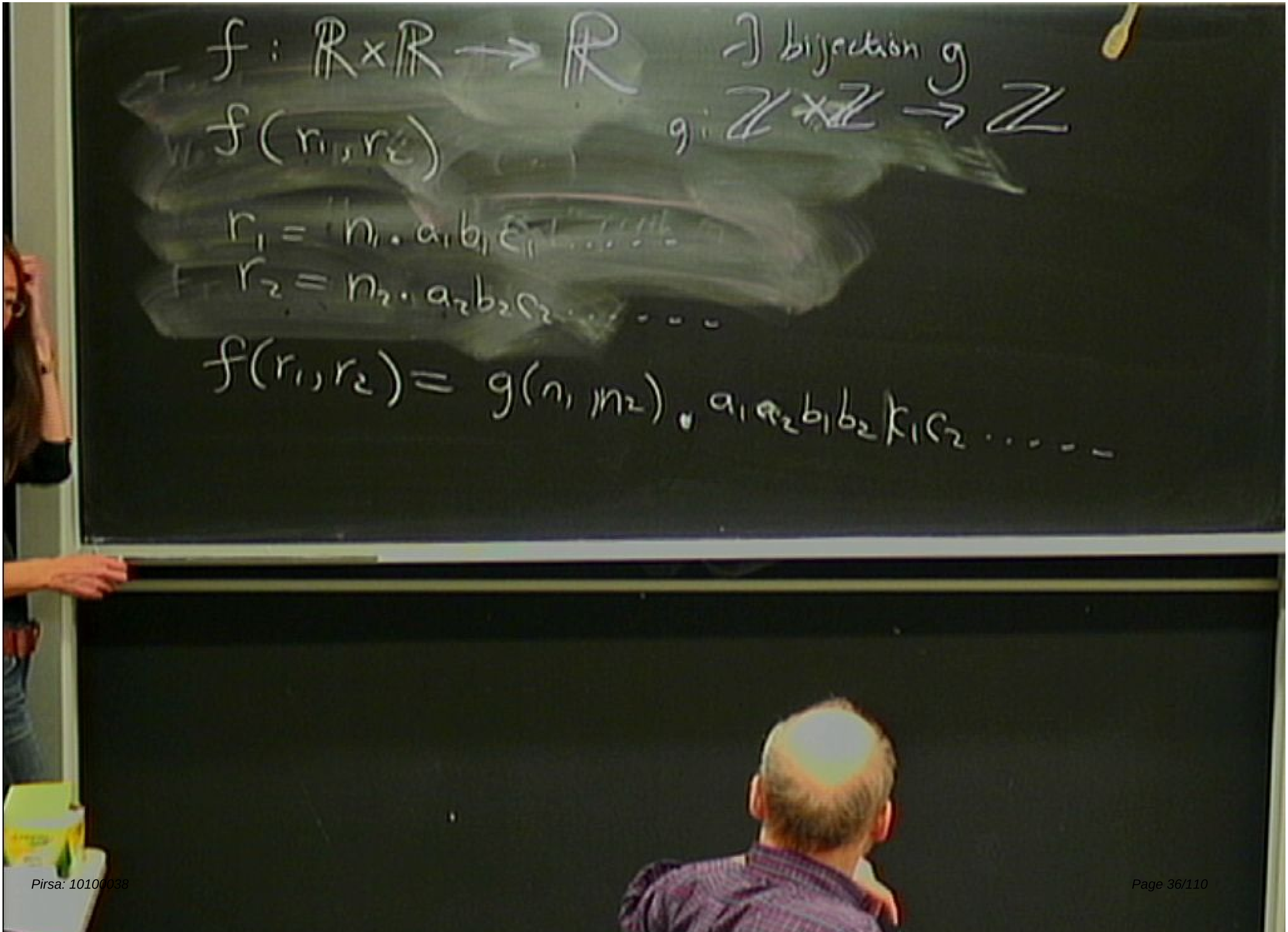

$f : \mathbb{R} \times \mathbb{R} \to \mathbb{R}$
$f(r_1, r_2)$
$r_1 = n_1 \cdot$

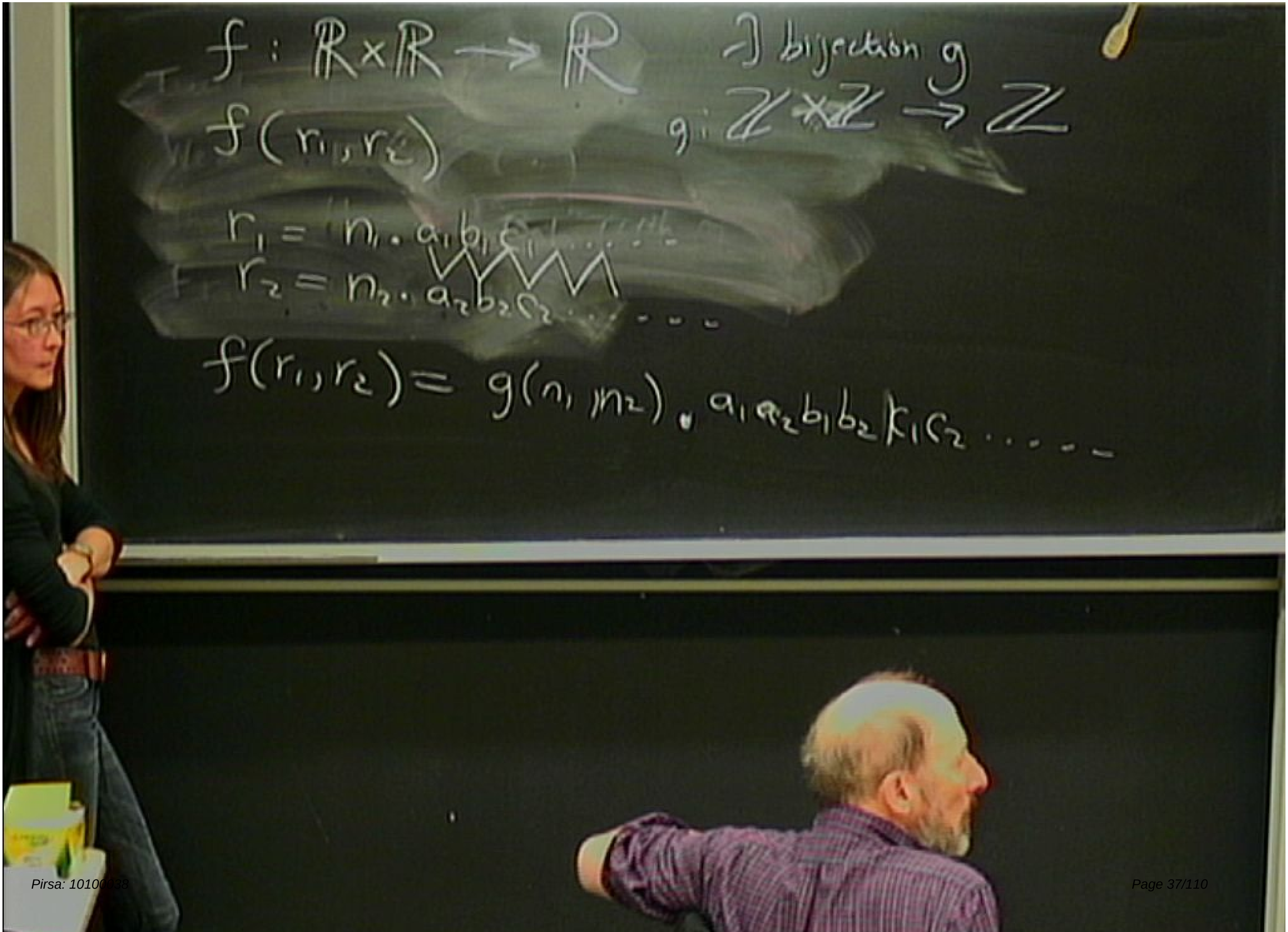

f : ℝ×ℝ → ℝ
f(r1, r2)
r1 = n1 . a1b1c1 ...
r2 = n2 . a2b2c2 ....



f : ℝ×ℝ → ℝ
∃ bijection g
g: ℤ×ℤ → ℤ
f(r1, r2)
r1 = n1 . a1 b1 c1 . . .
r2 = n2 . a2 b2 c2 . . . . . .


f : ℝ×ℝ → ℝ
∃ bijection g
g: ℤ×ℤ → ℤ
f(r₁, r₂)
= n₁ . a₁b₁c₁
= n₂ . a₂b₂c₂
= g(n₁, n₂)


f : ℝ×ℝ → ℝ
∃ bijection g
g: ℤ×ℤ → ℤ
f(r1, r2)
r1 = n1 . a1b1c1 ...
r2 = n2 . a2b2c2 ......
f(r1, r2) = g(n1, n2) . a1a2b1b2c1c2 .....


f : R×R → R
∃ bijection g
g: Z×Z → Z
f(r1, r2)
r1 = n1 . a1b1c1 . . .
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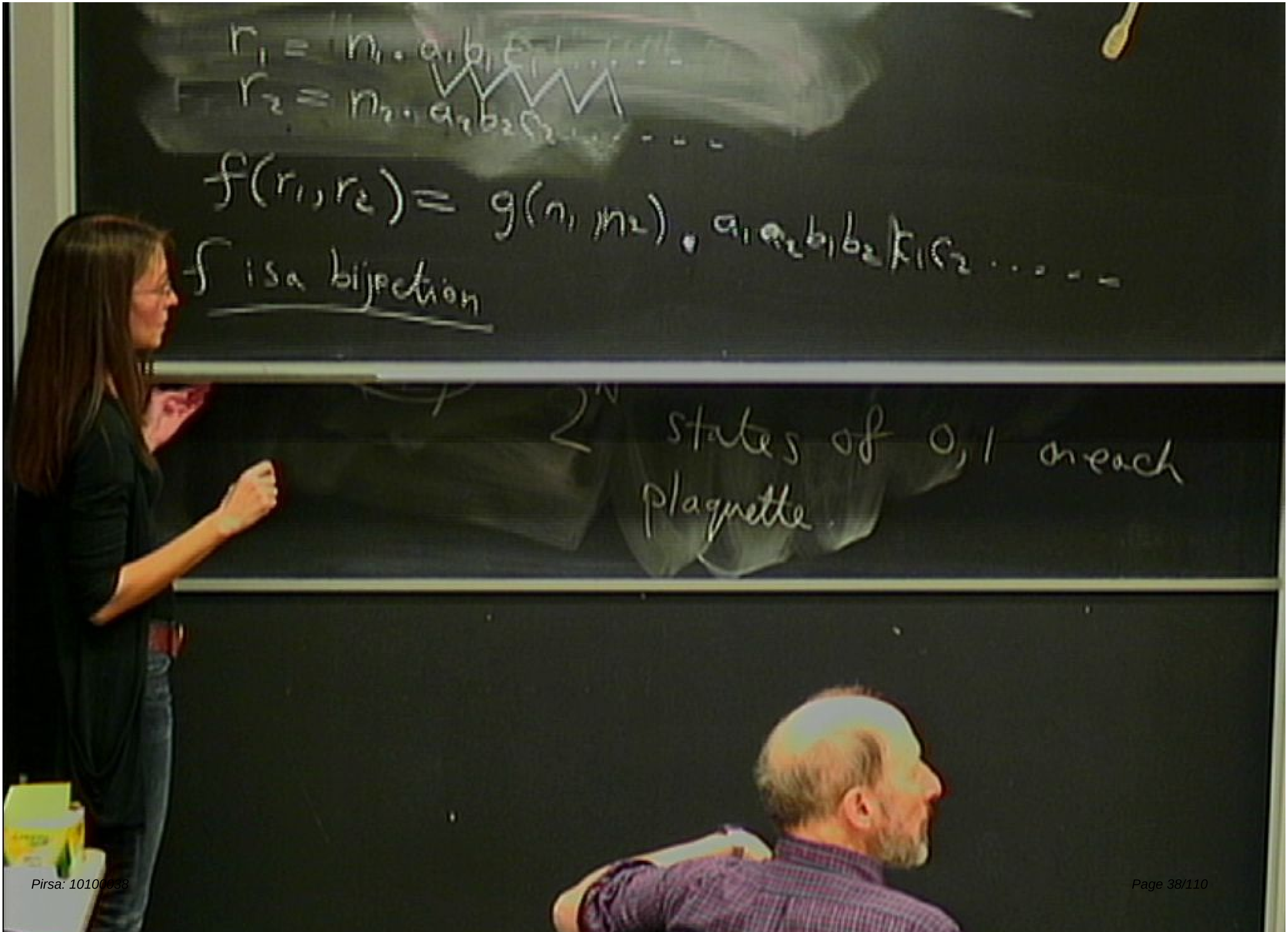
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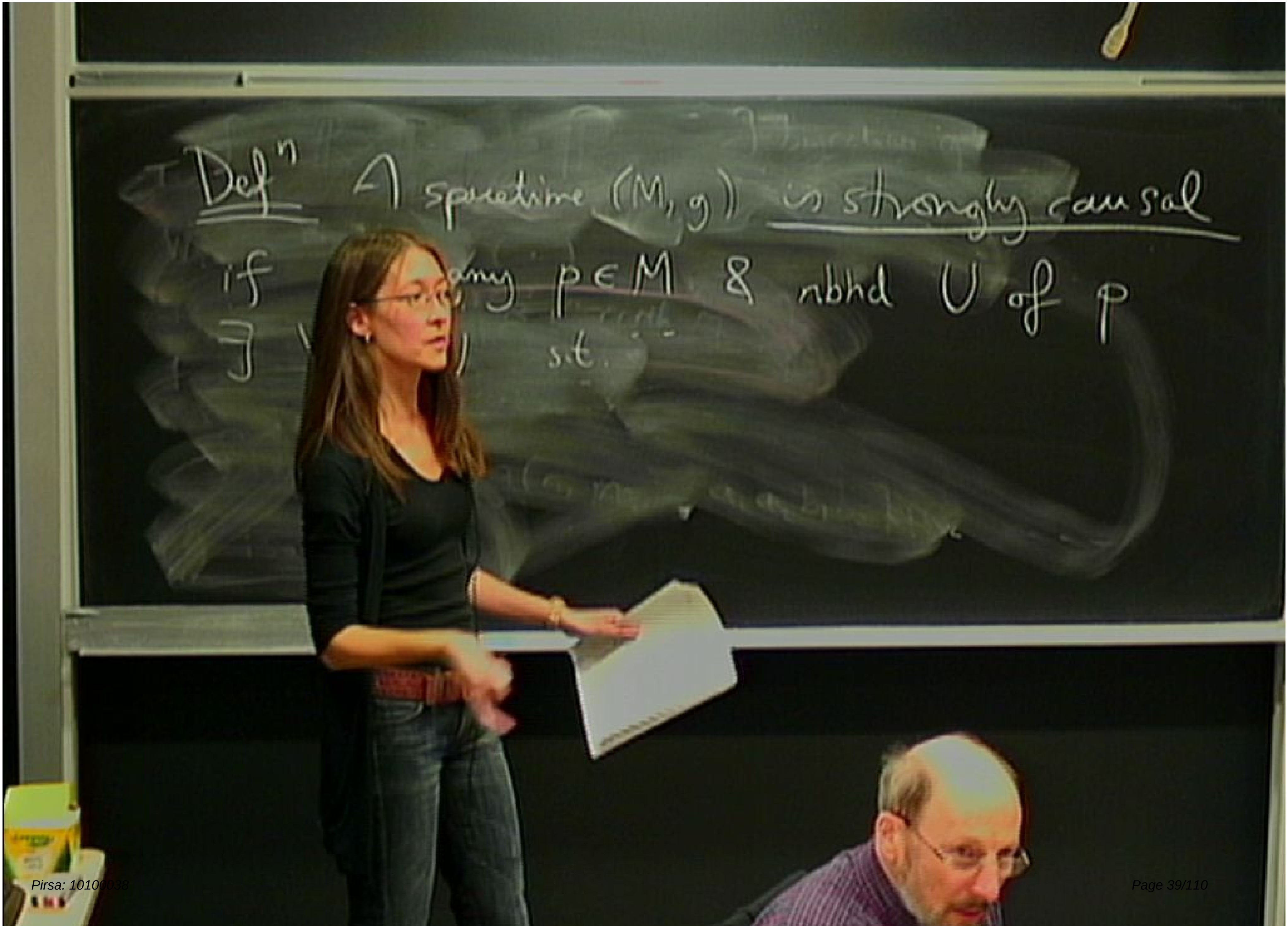
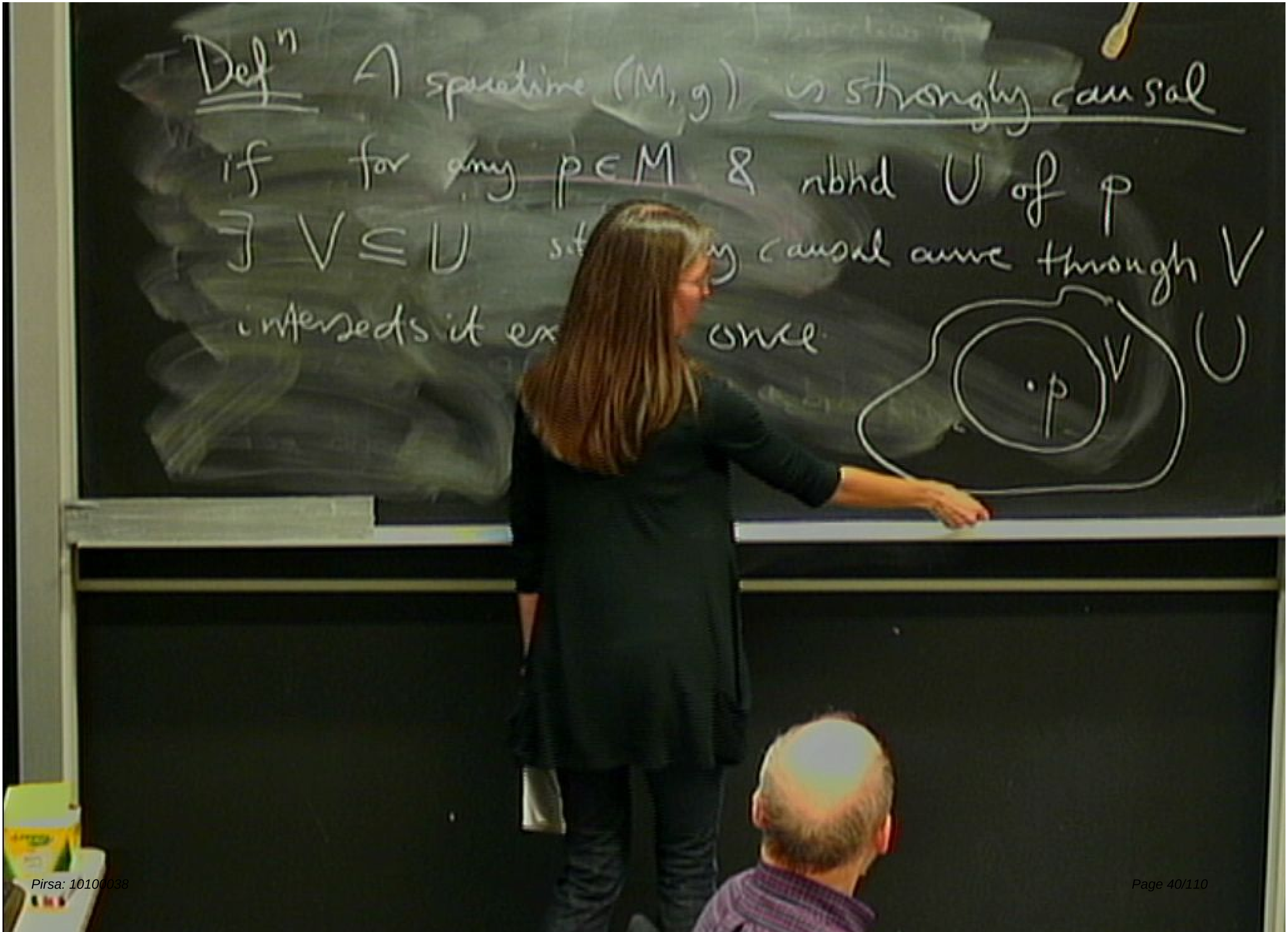
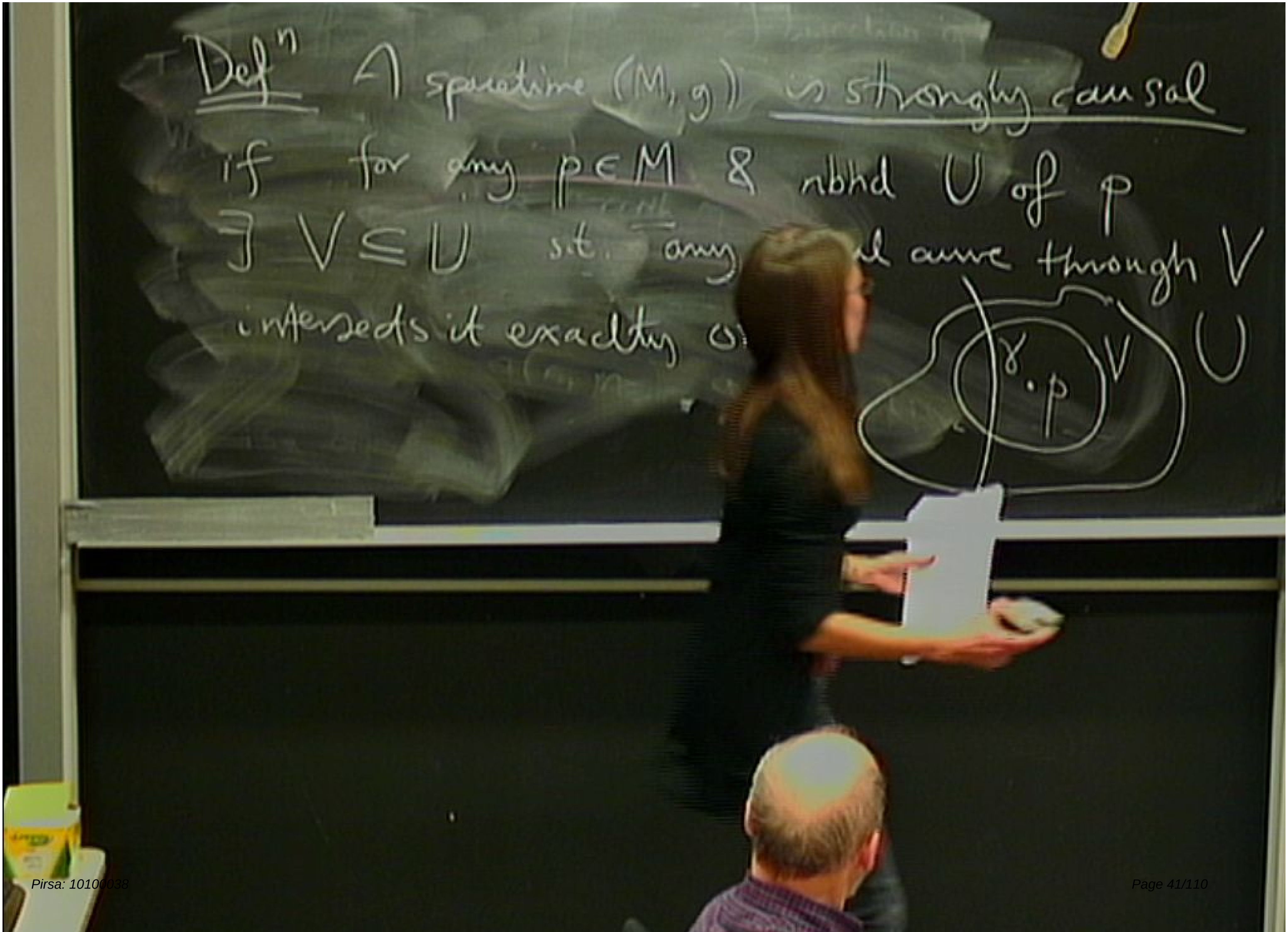
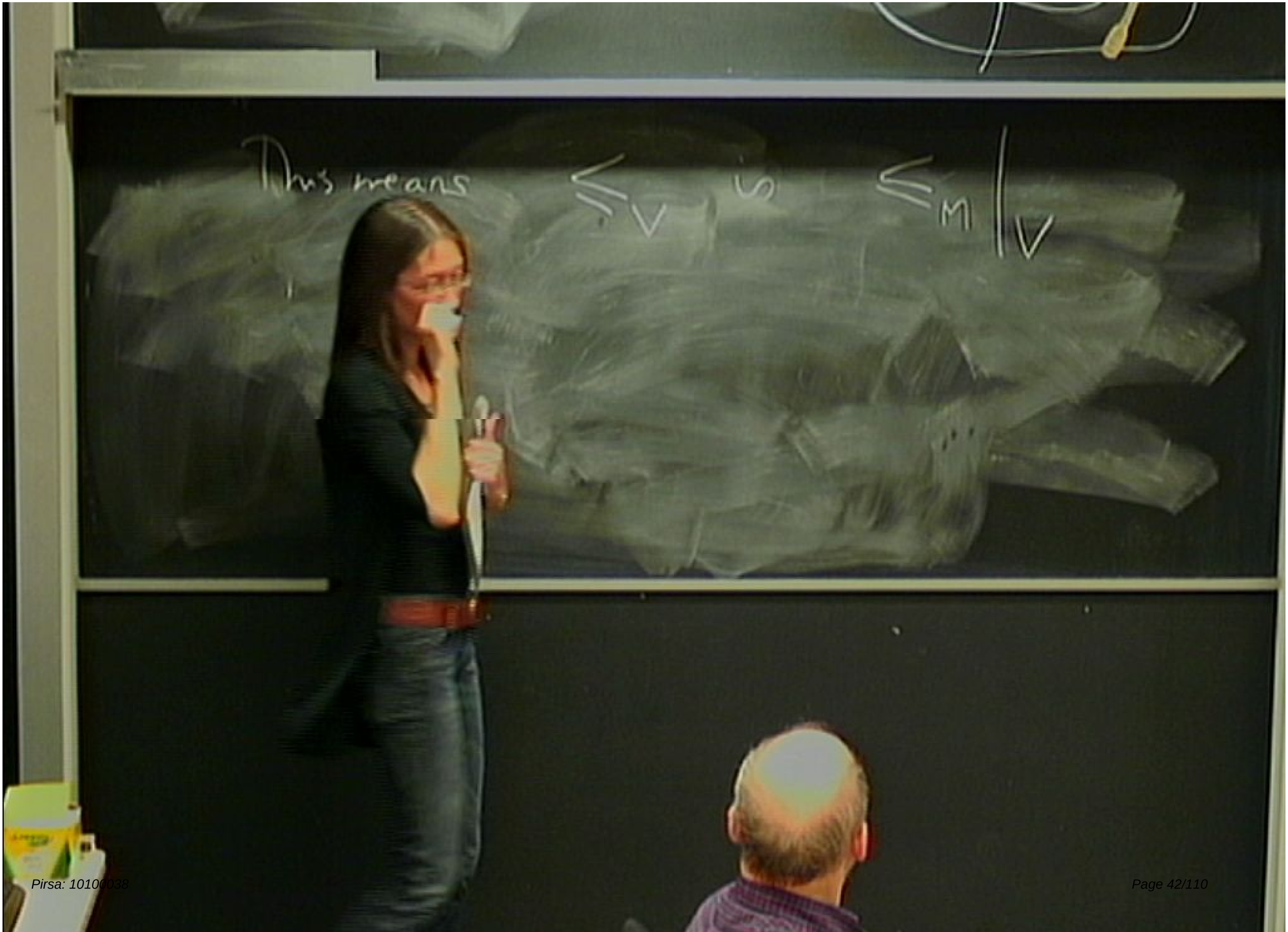
f is a bijection
2^N states of 0, 1 on each plaquette.

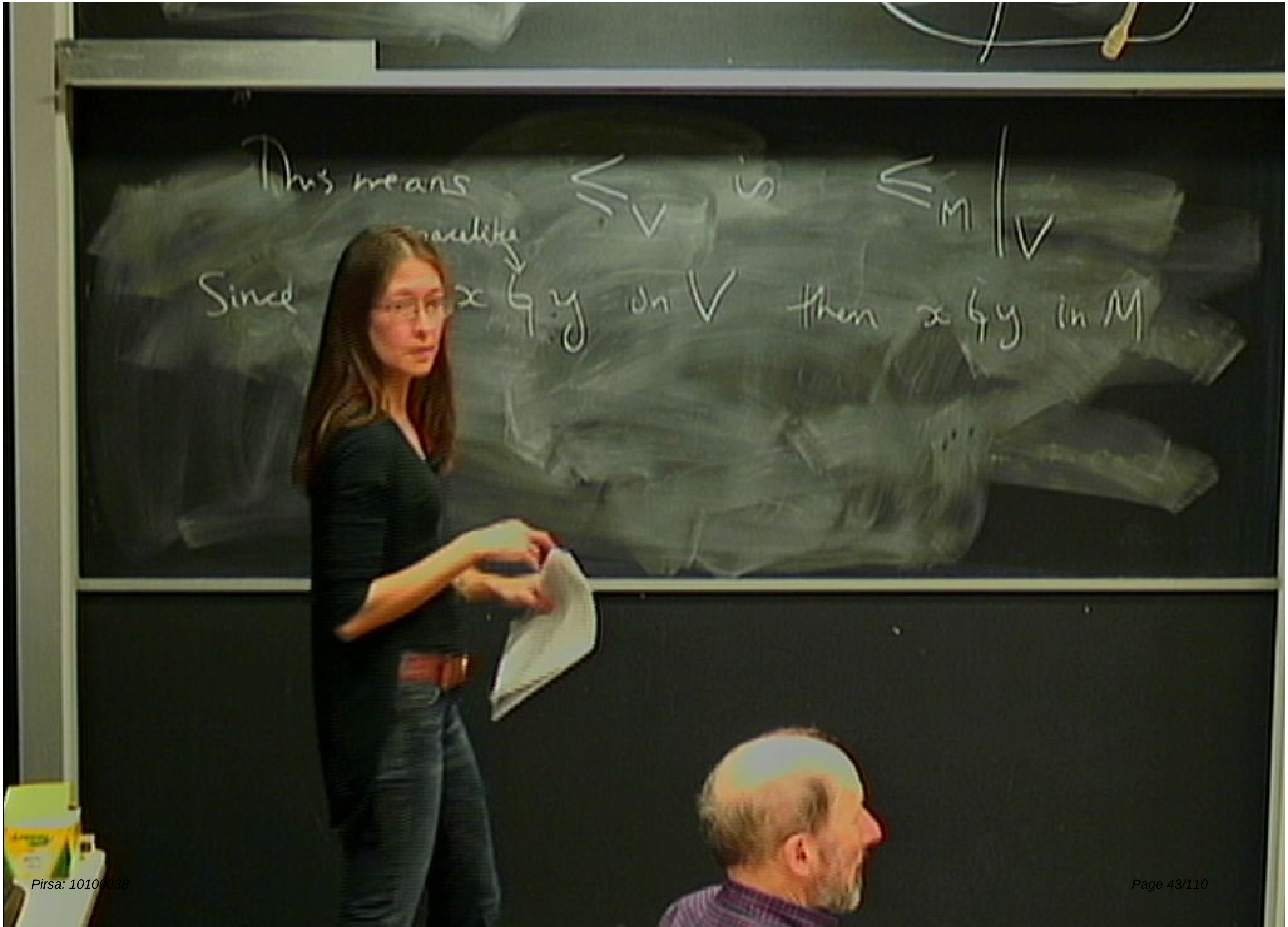

Def^n A spacetime (M, g) is strongly causal
if any p ∈ M & nbhd U of p
∃ V s.t.

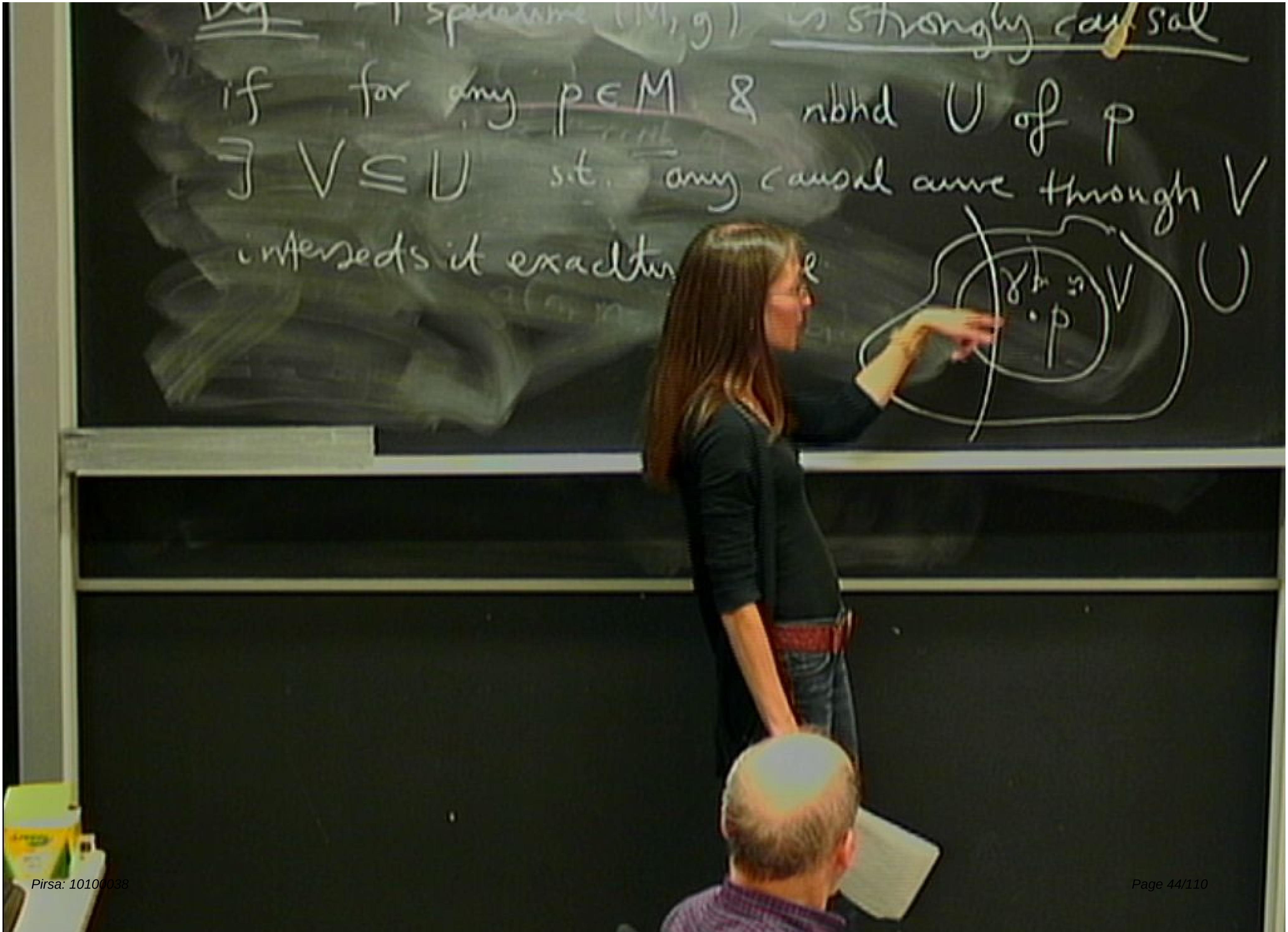


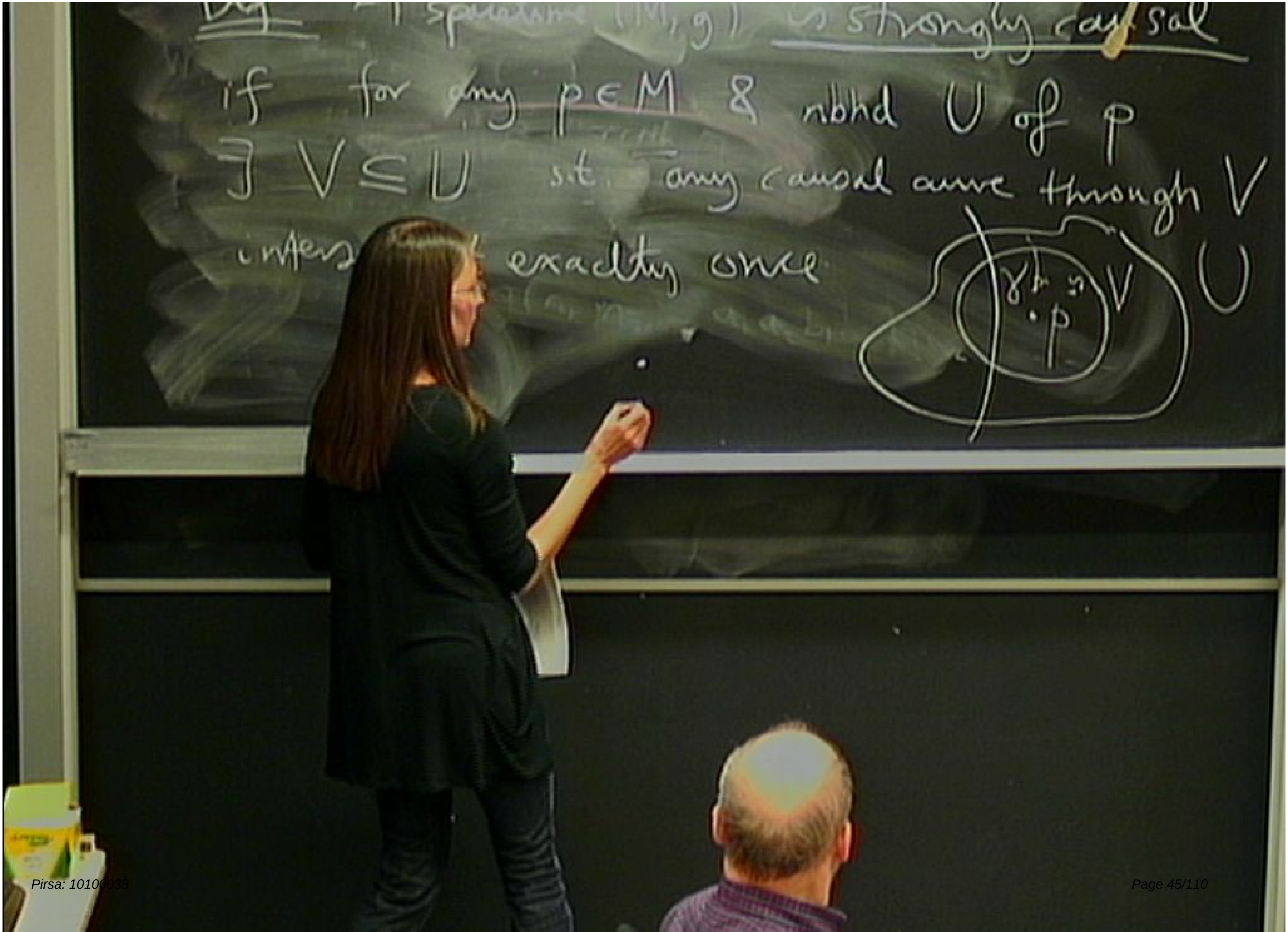
Defn A spacetime (M, g) is strongly causal
if for any p ∈ M & nbhd U of p
∃ V ⊆ U s.t. causal curve through V
intersects it ex once
V
U
p


Defn A spacetime (M, g) is strongly causal
if for any p∈M & nbhd U of p
∃ V⊆U s.t. any
curve through V
intersects it exactly
γ
p
V
U


This means $\leq_V$ is $\leq_M|_V$


This means $\leq_V$ is $\leq_M|_V$
Since $x \lneq y$ in V then $x \lneq y$ in M


spacetime (M,g) is strongly causal
if for any p∈M & nbhd U of p
∃ V⊆U s.t. any causal curve through V
intersects it
V
U
p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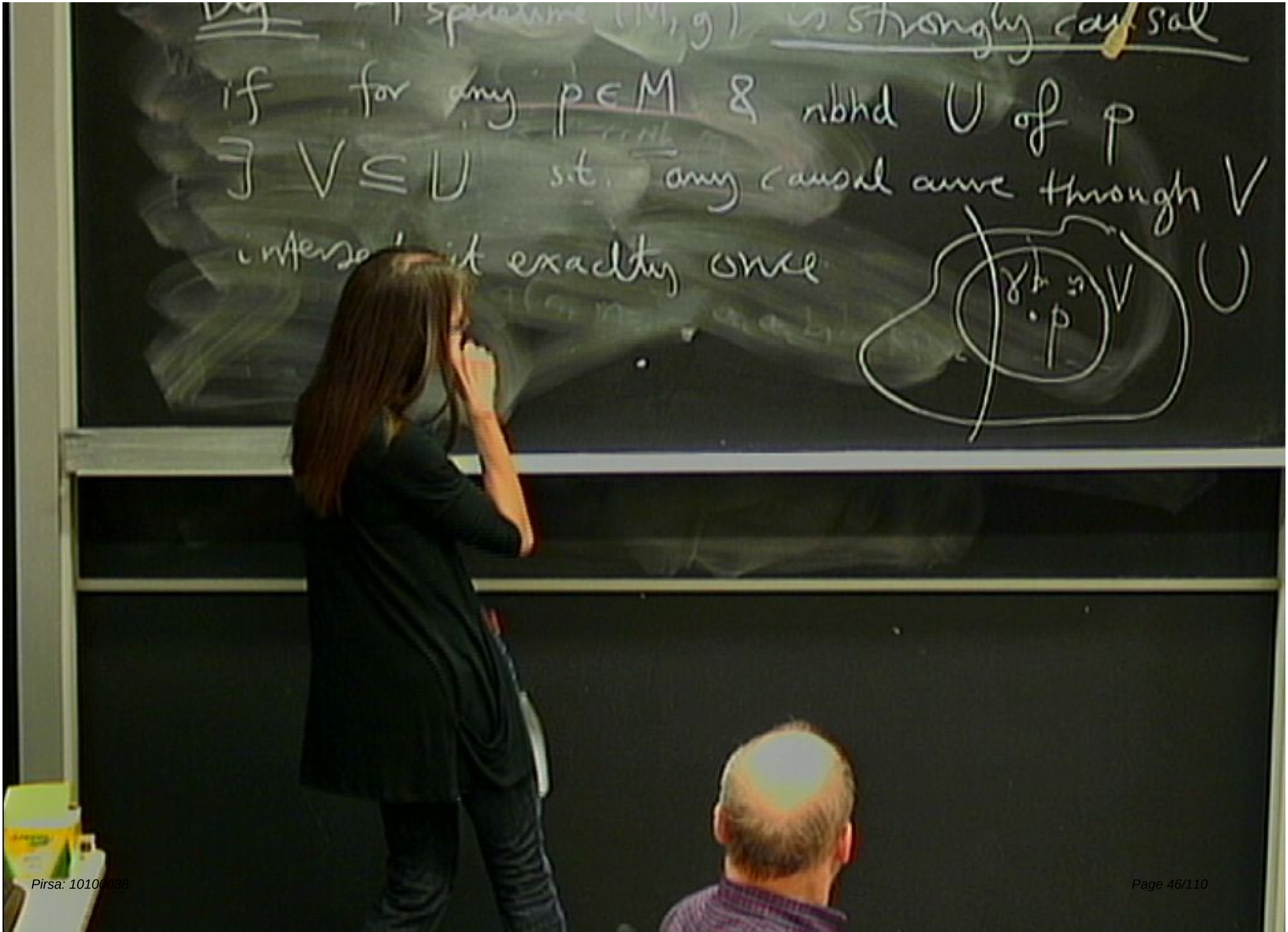
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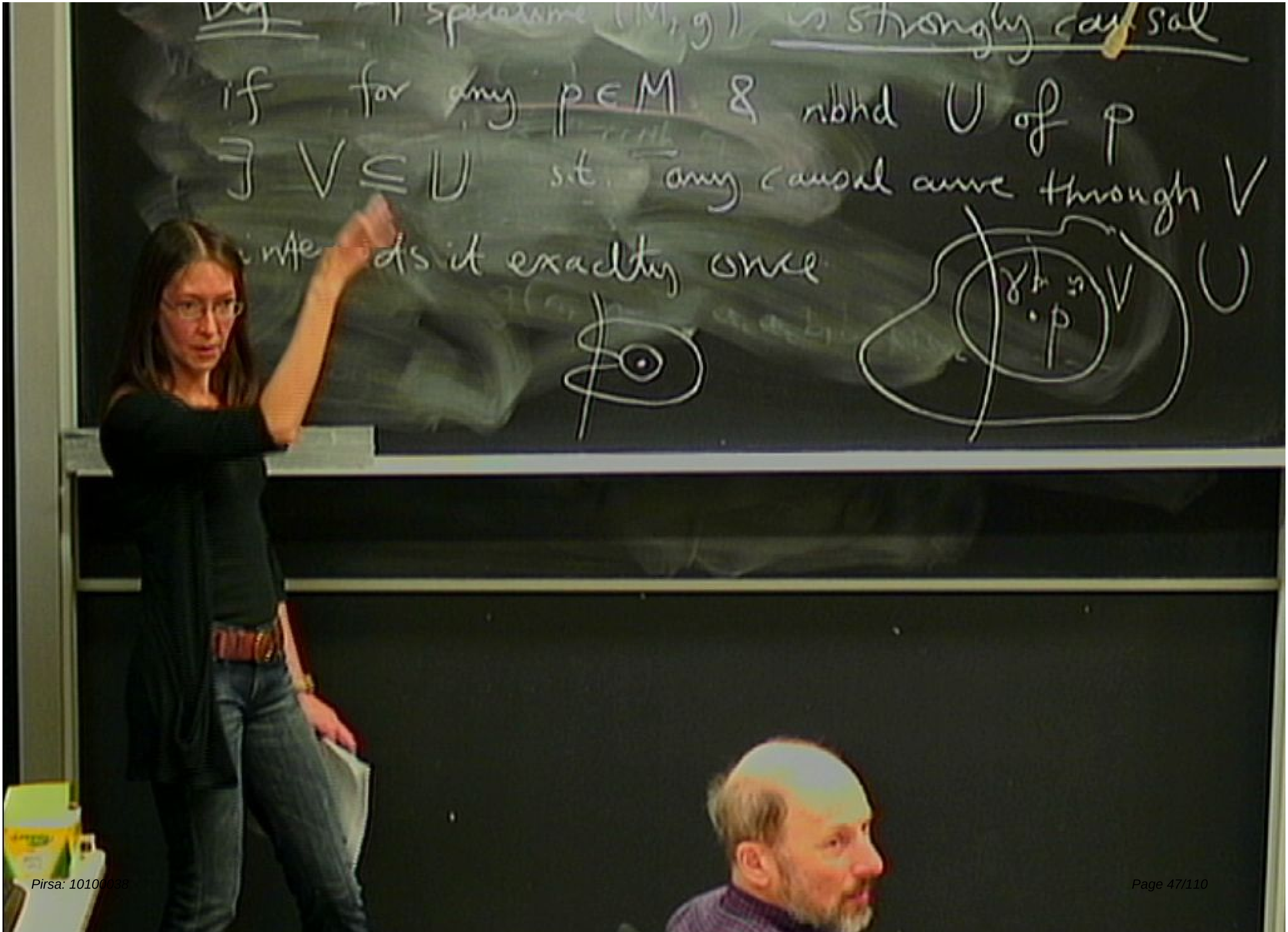
Def A spacetime (M,g) is strongly causal
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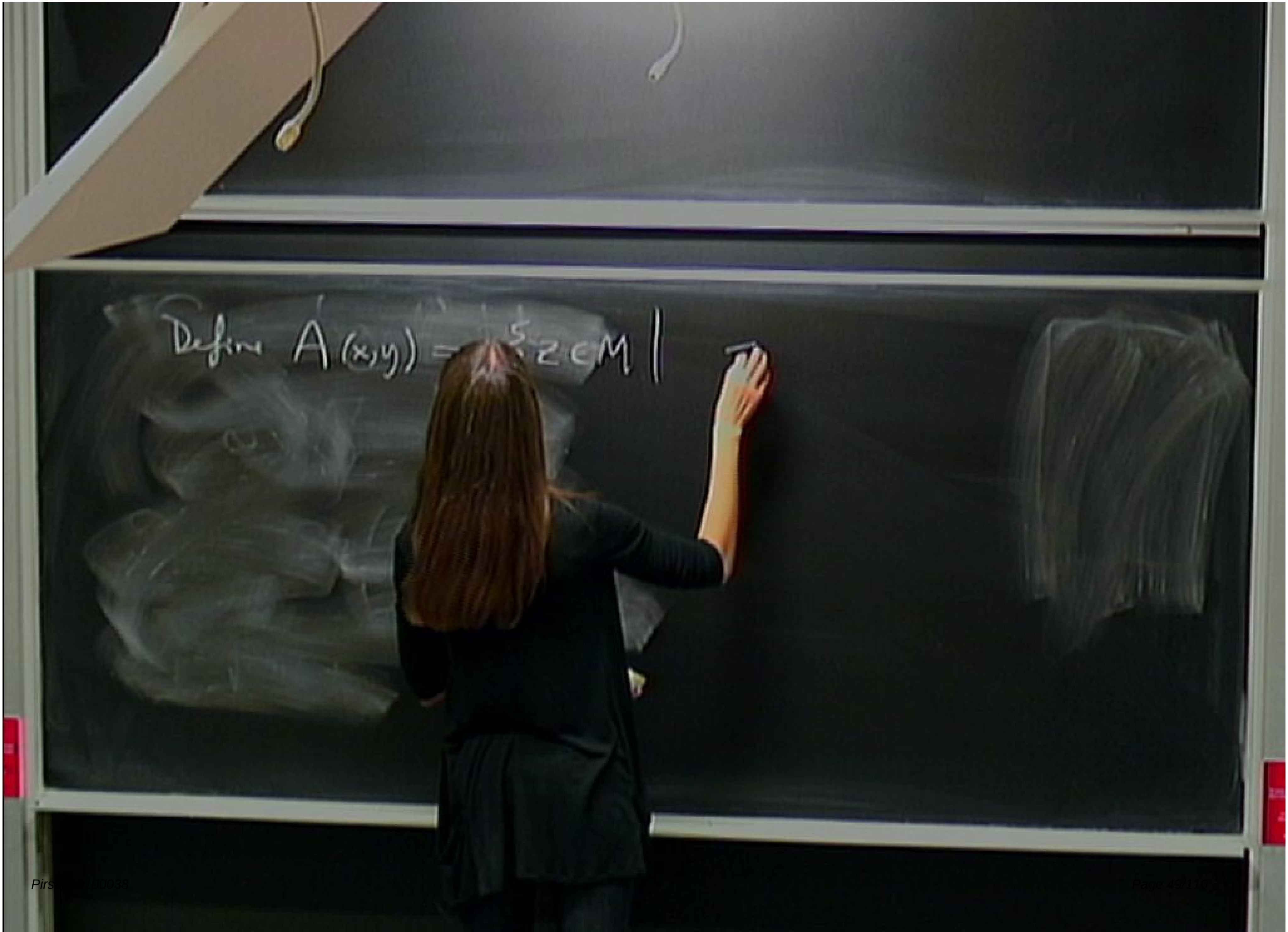
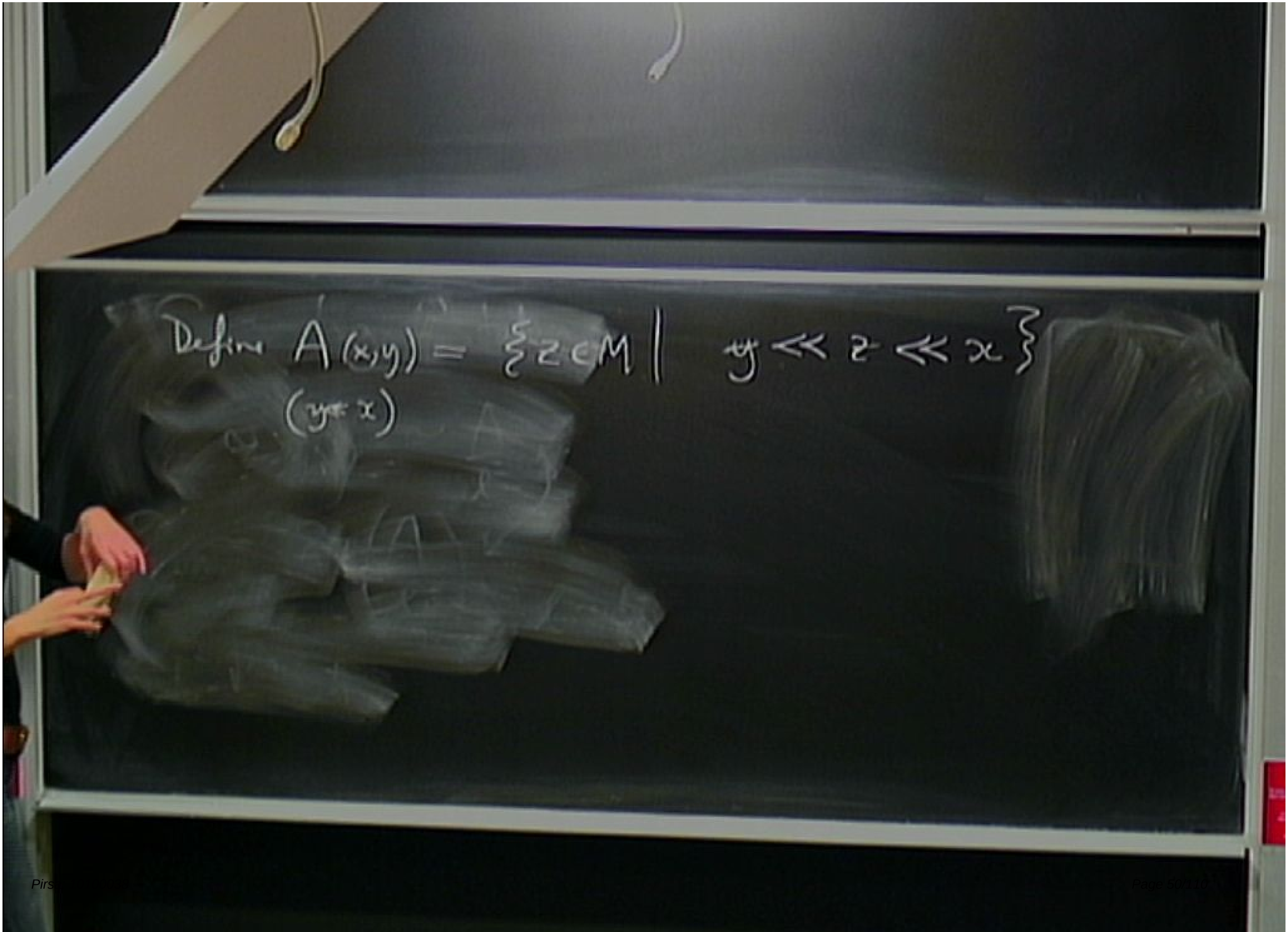

Define A(x,y) = {z ∈ M |



Define A(x,y) = { z ∈ M | y ≪ z ≪ x }
(y ≪ x)

This means $\leq_V$ is $\leq_M \big|_V$

(spacelike)

Since if $x \lneq y$ in V then $x \lneq y$ in M

Def^n A subset S of M is totally ordered if $x, y \in A$ then either $x \leq y$ or $y \leq x$.

$A(x,y) = \{z \in M \mid y \ll z \ll x\}$

$(y \neq x)$

Def^n $I(x,y) := \{z \in M \mid y \leq z \leq x\}$

$\leq$ is the causal structure of (M,g)

$I(x,y) = \{z \in M \mid y \ll z \ll x\}$

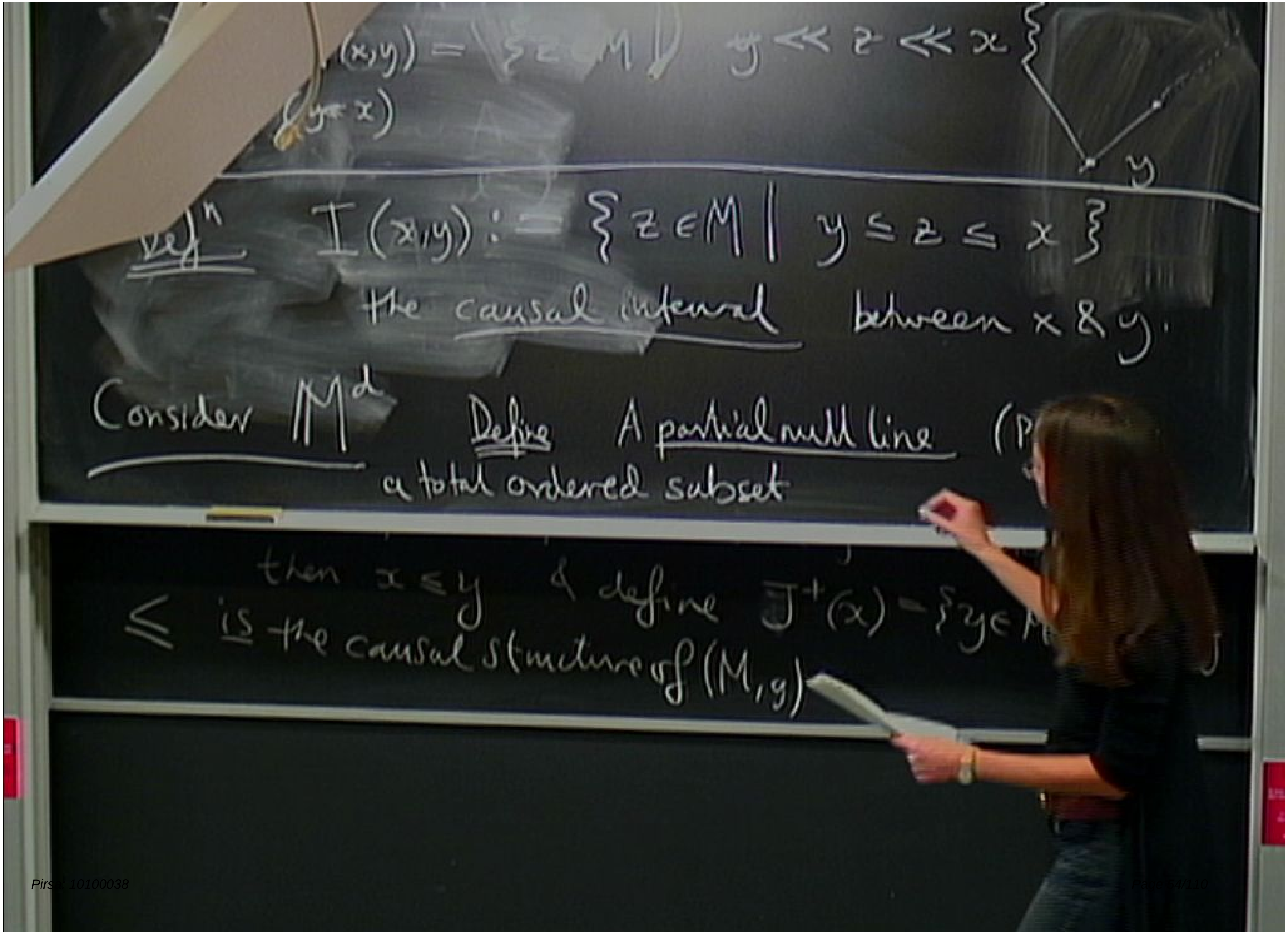
$(y \ll x)$

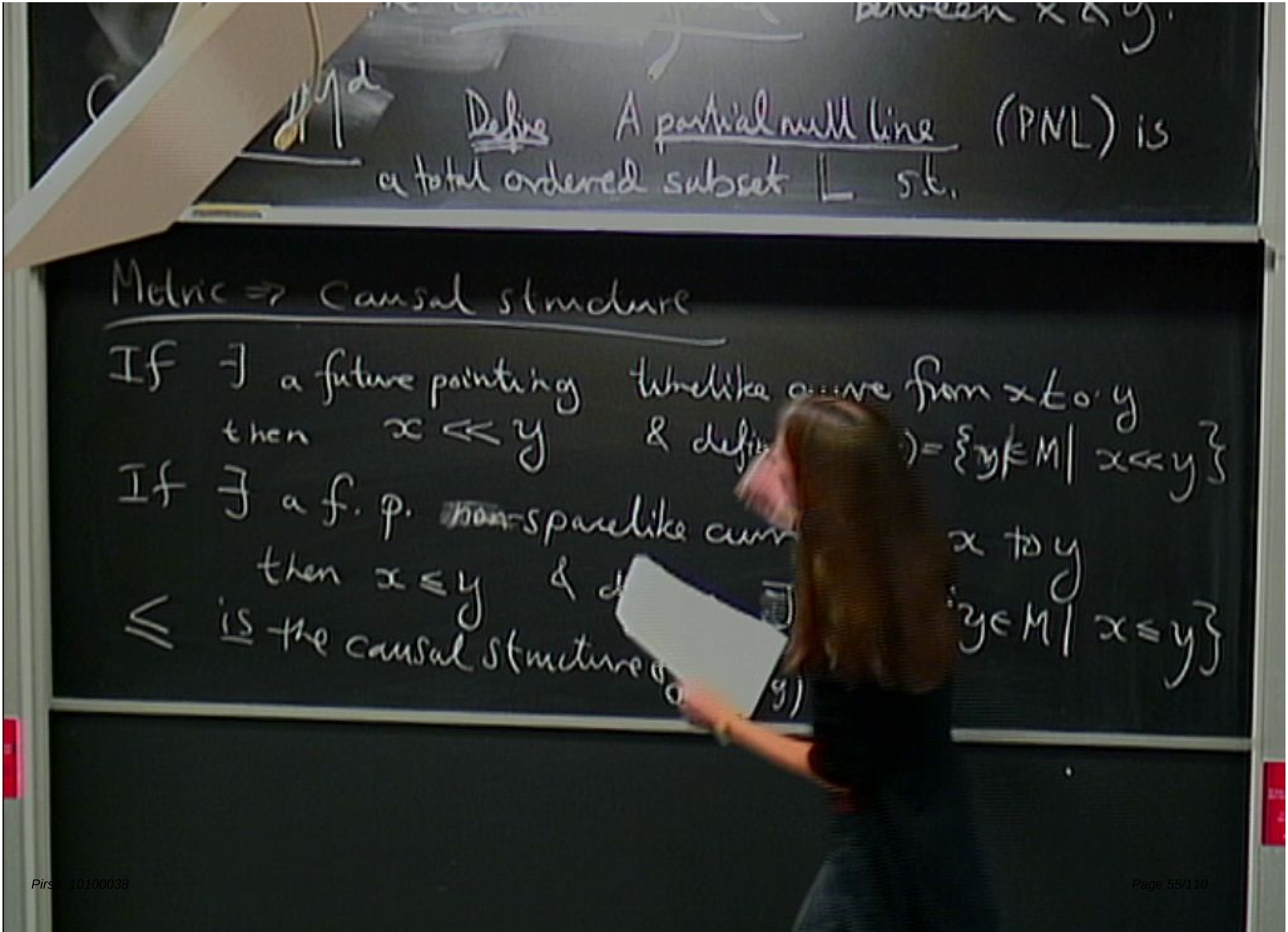
<u>Def^n</u> $I(x,y) := \{z \in M \mid y \le z \le x\}$

the <u>causal interval</u> between x & y.

then $x \le y$ & define $J^+(x) = \{y \in M \mid \ldots\}$

$\le$ <u>is</u> the causal structure of (M, g)


(x,y) = {z ∈ M | y ≪ z ≪ x}
Def^n I(x,y) := {z ∈ M | y ≤ z ≤ x}
the causal interval between x & y.
Consider M^d
Define A partial null line
a total ordered subset
then x ≤ y & define J^+(x) = {y ∈ M
≤ is the causal structure of (M,g)


Define A partial null line (PNL) is
a total ordered subset L s.t.
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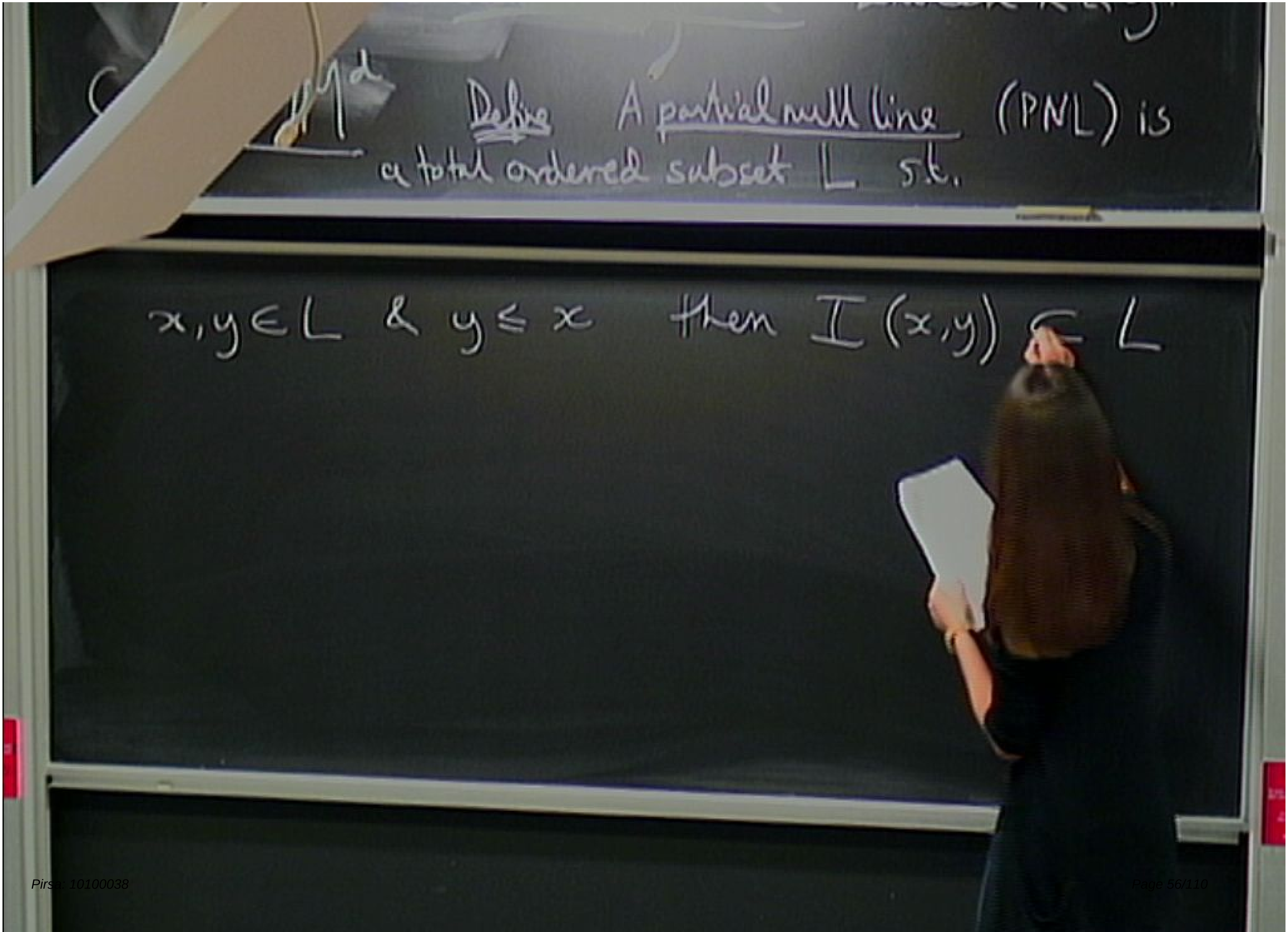
If ∃ a future pointing timelike curve from x to y
then x ≪ y
If ∃ a f. p. non-spacelike curve
x to y
then x ≤ y
≤ is the causal structure


Define A partial null line (PNL) is
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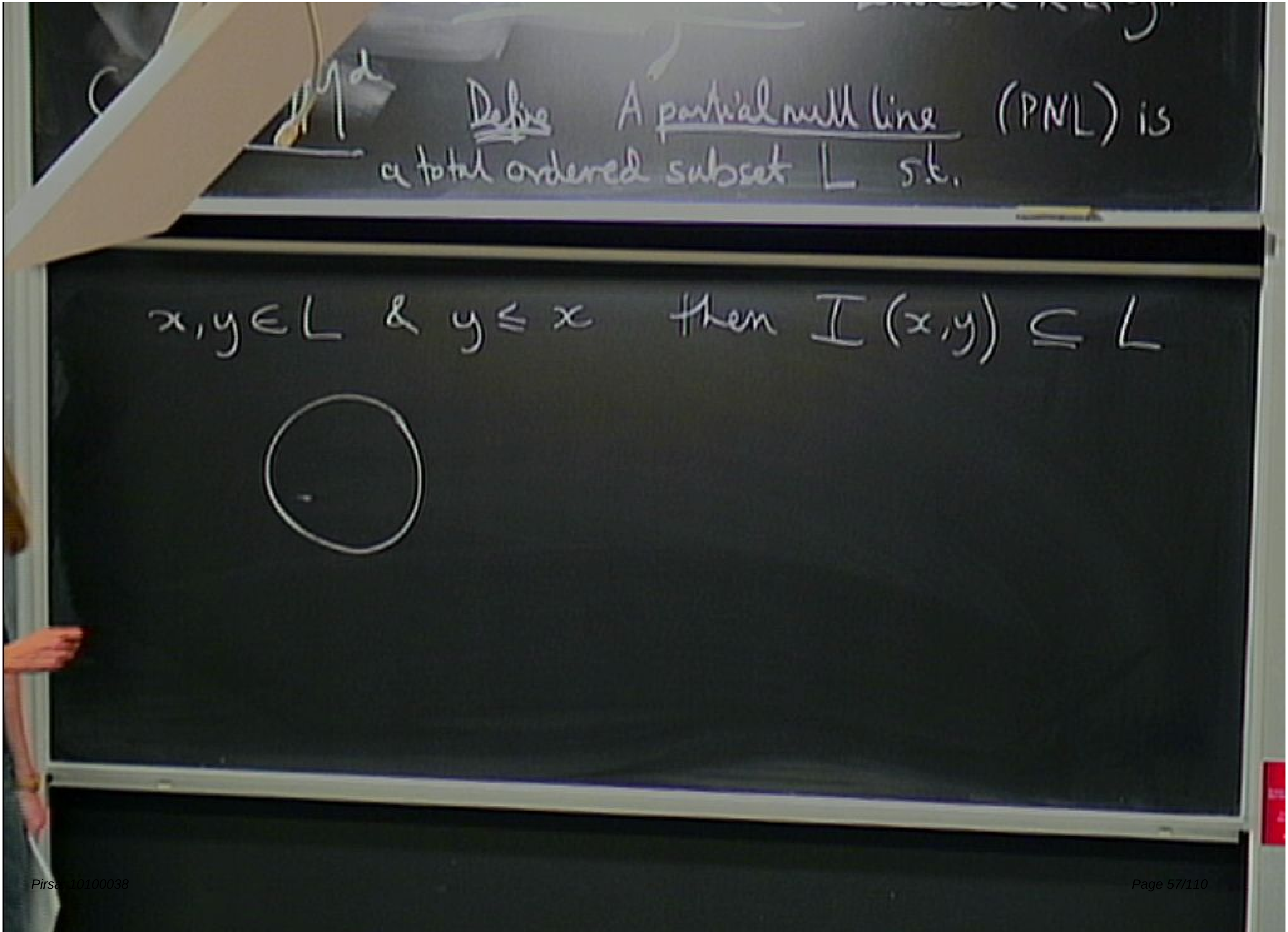
x, y ∈ L & y ≤ x then I (x, y) ⊆ L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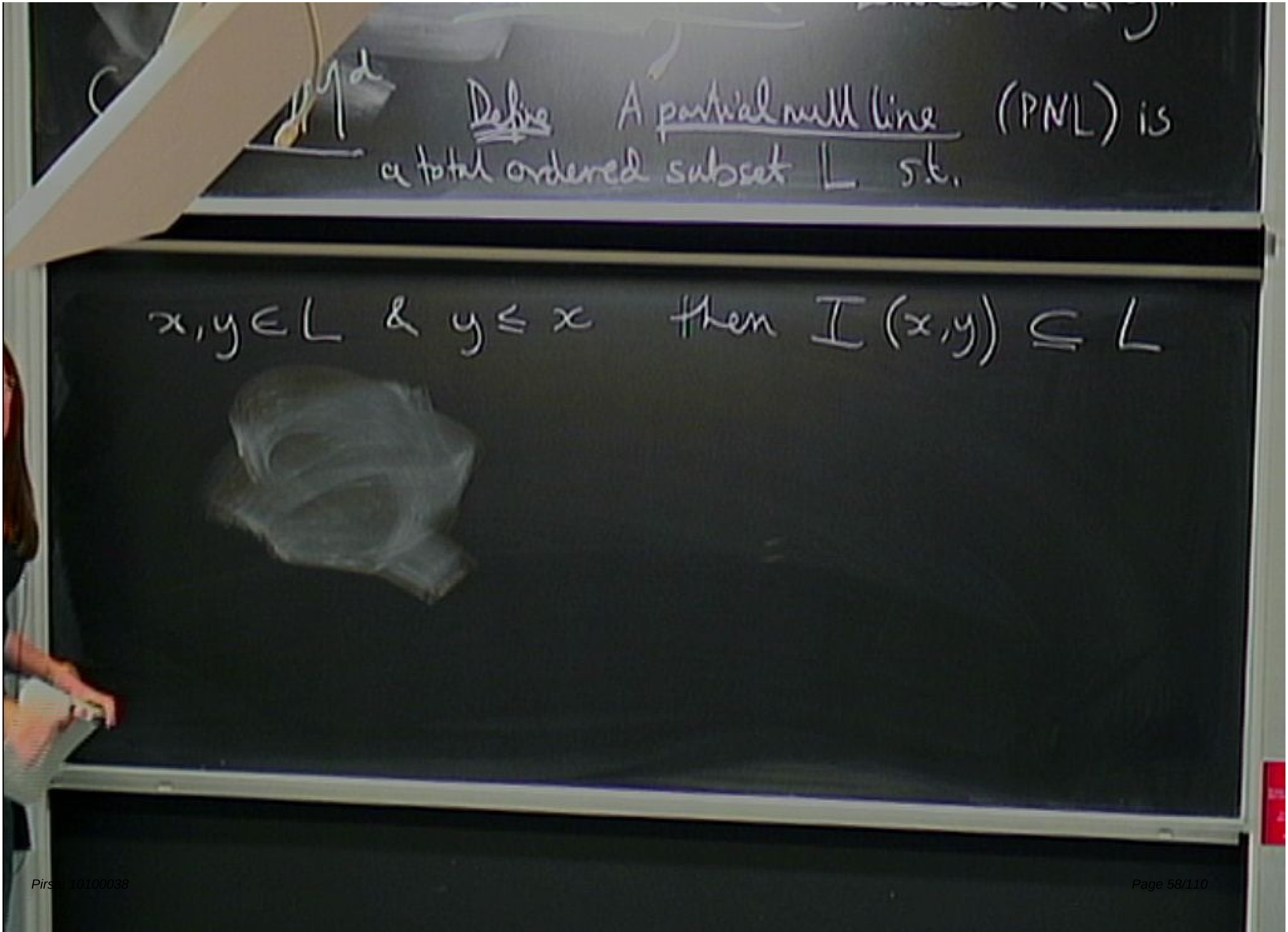
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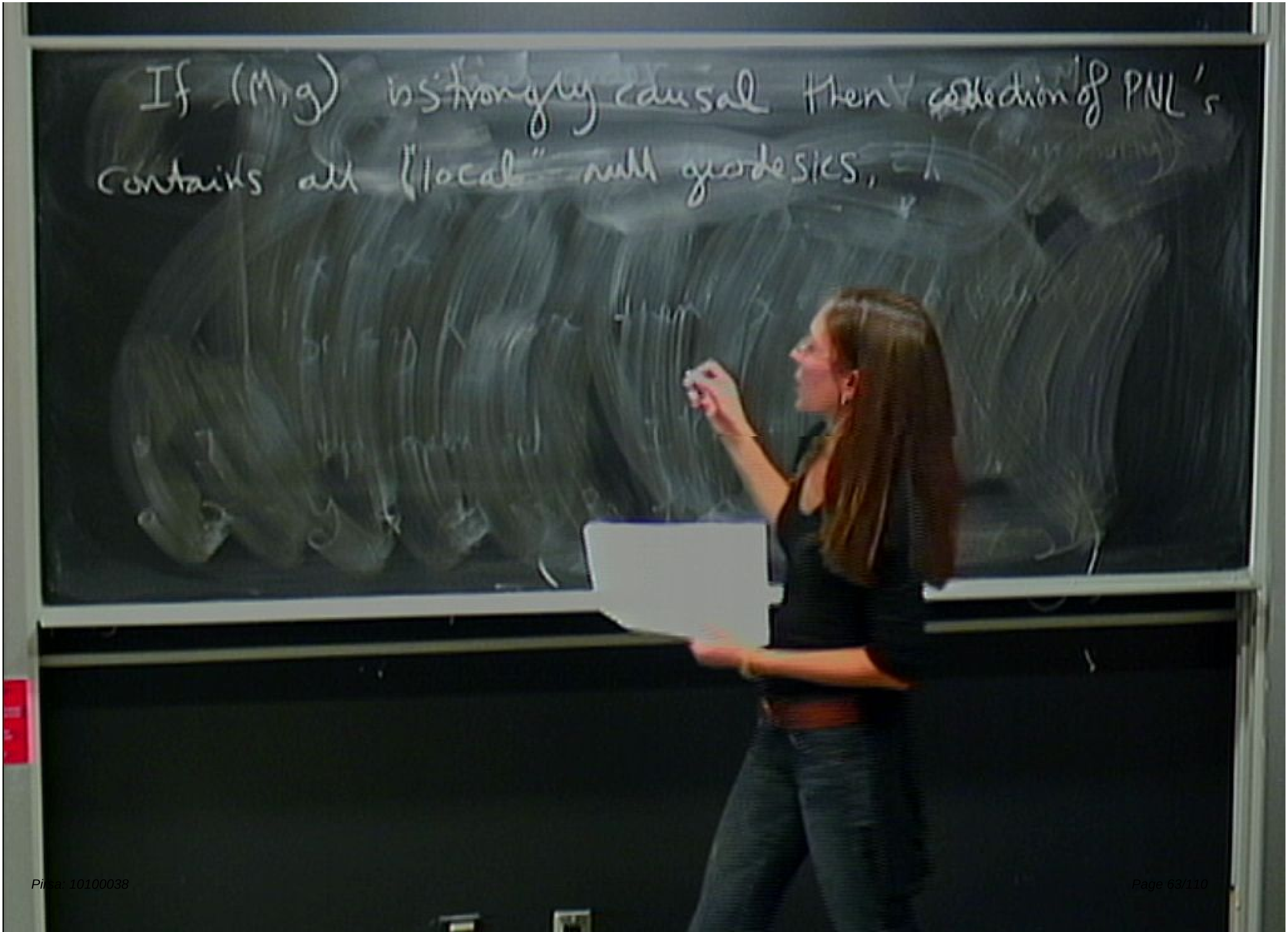
If (M,g) is strongly causal then ∀ collection of PNL's
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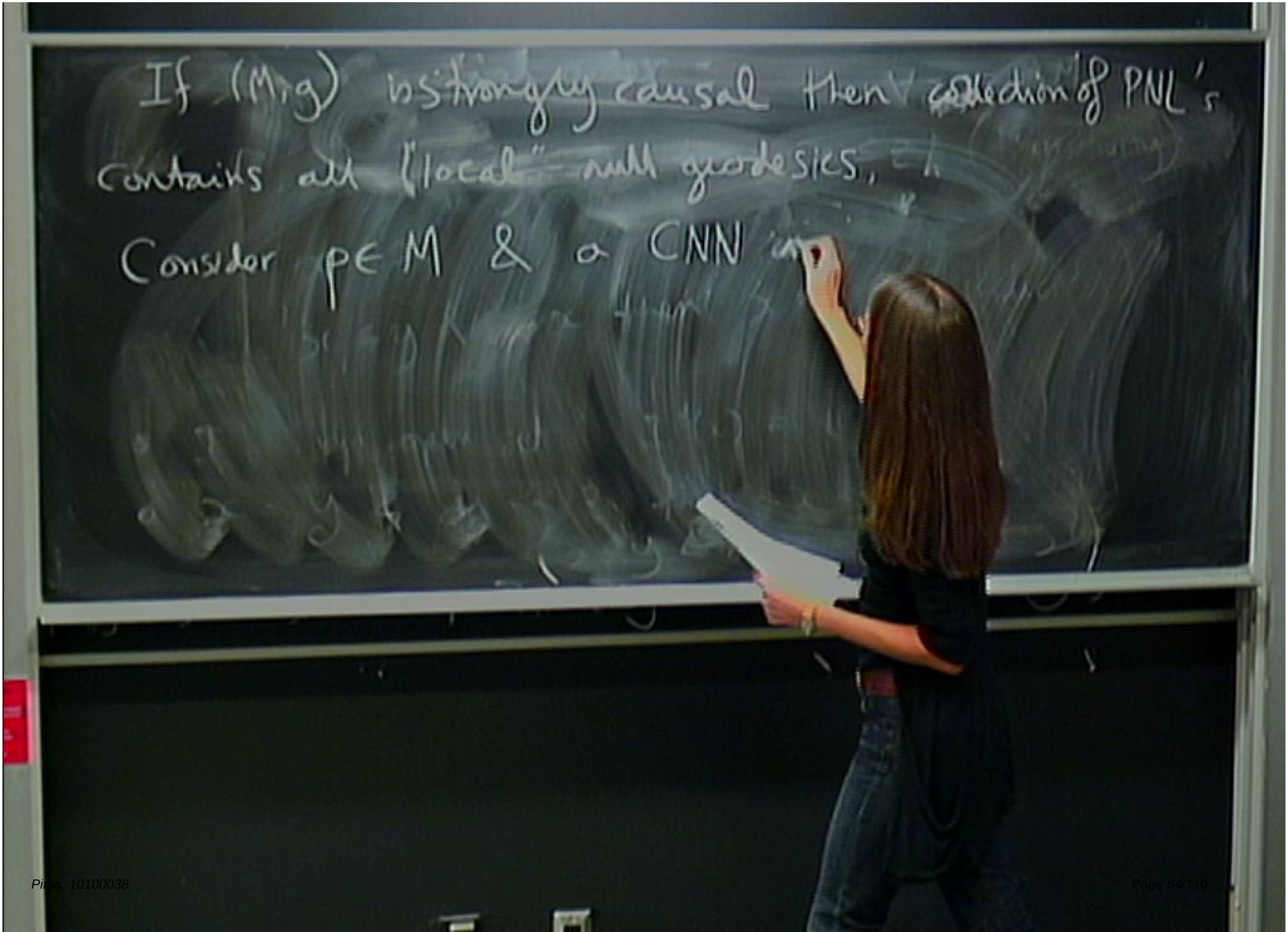
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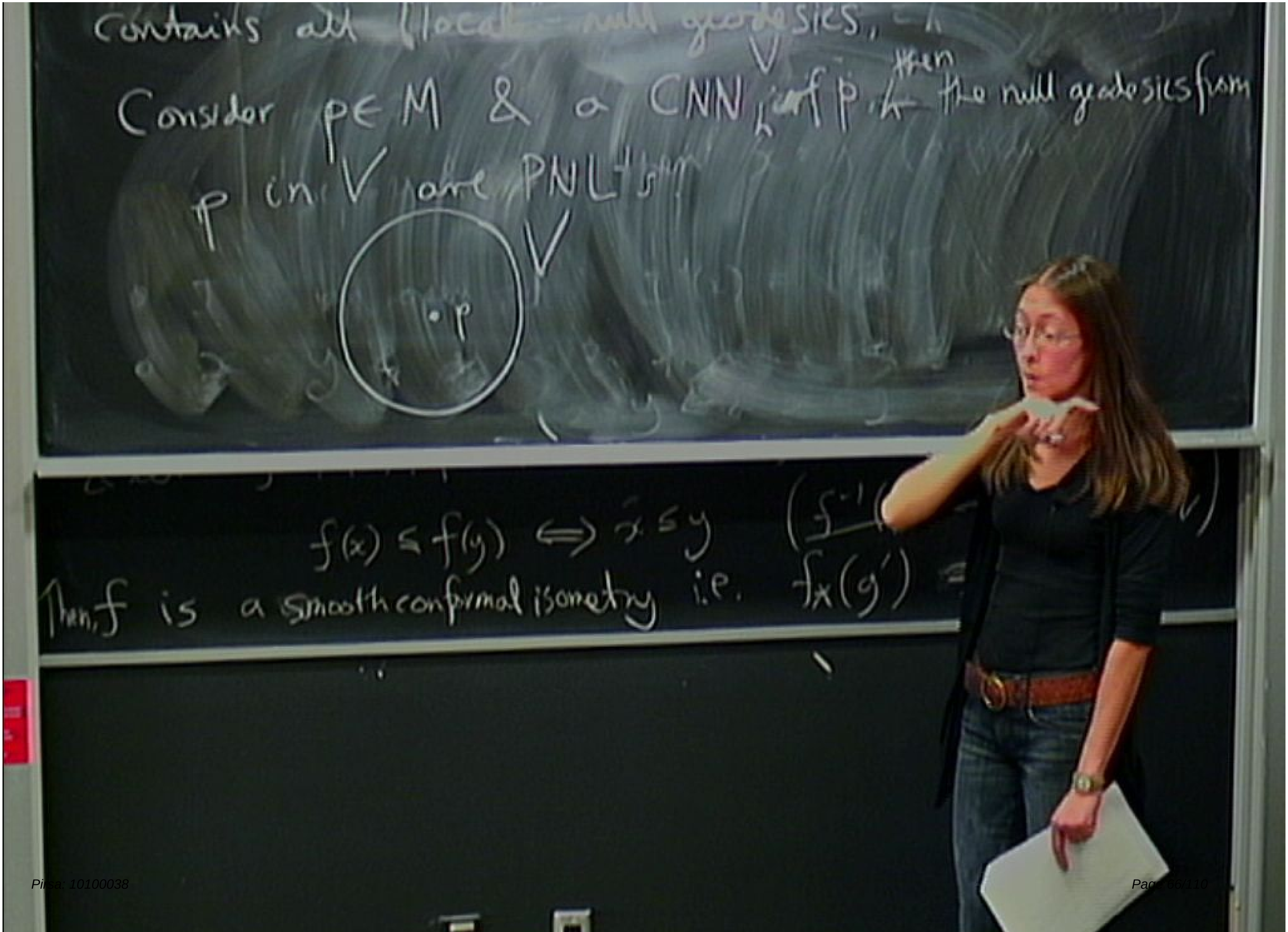
contains all (local) null geodesics,
Consider p ∈ M & a CNN V of p, then the null geodesics from
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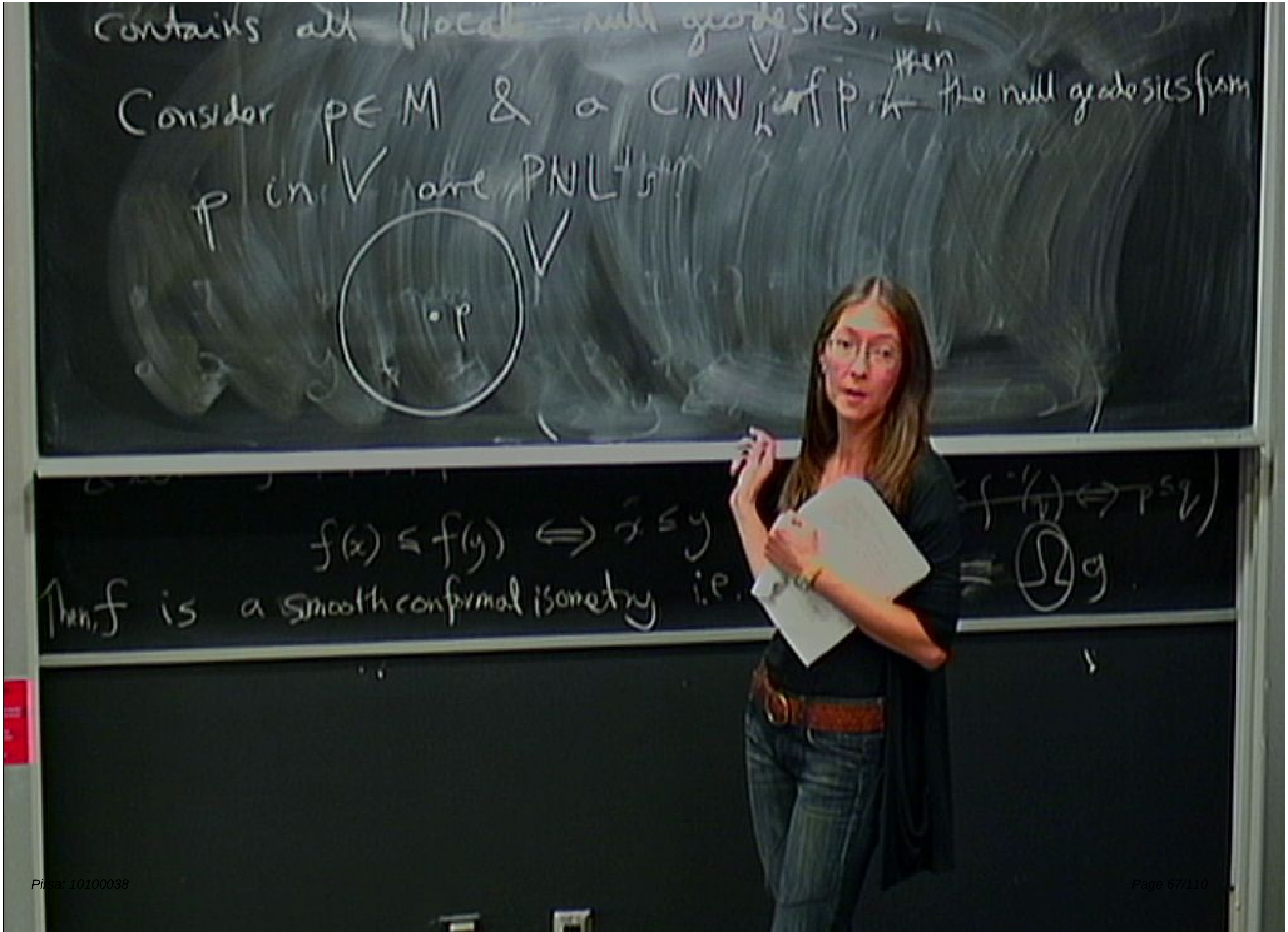
f(x) ≤ f(y) ⟺ x ≤ y
Then, f is a smooth conformal isometry i.e. f*(g')



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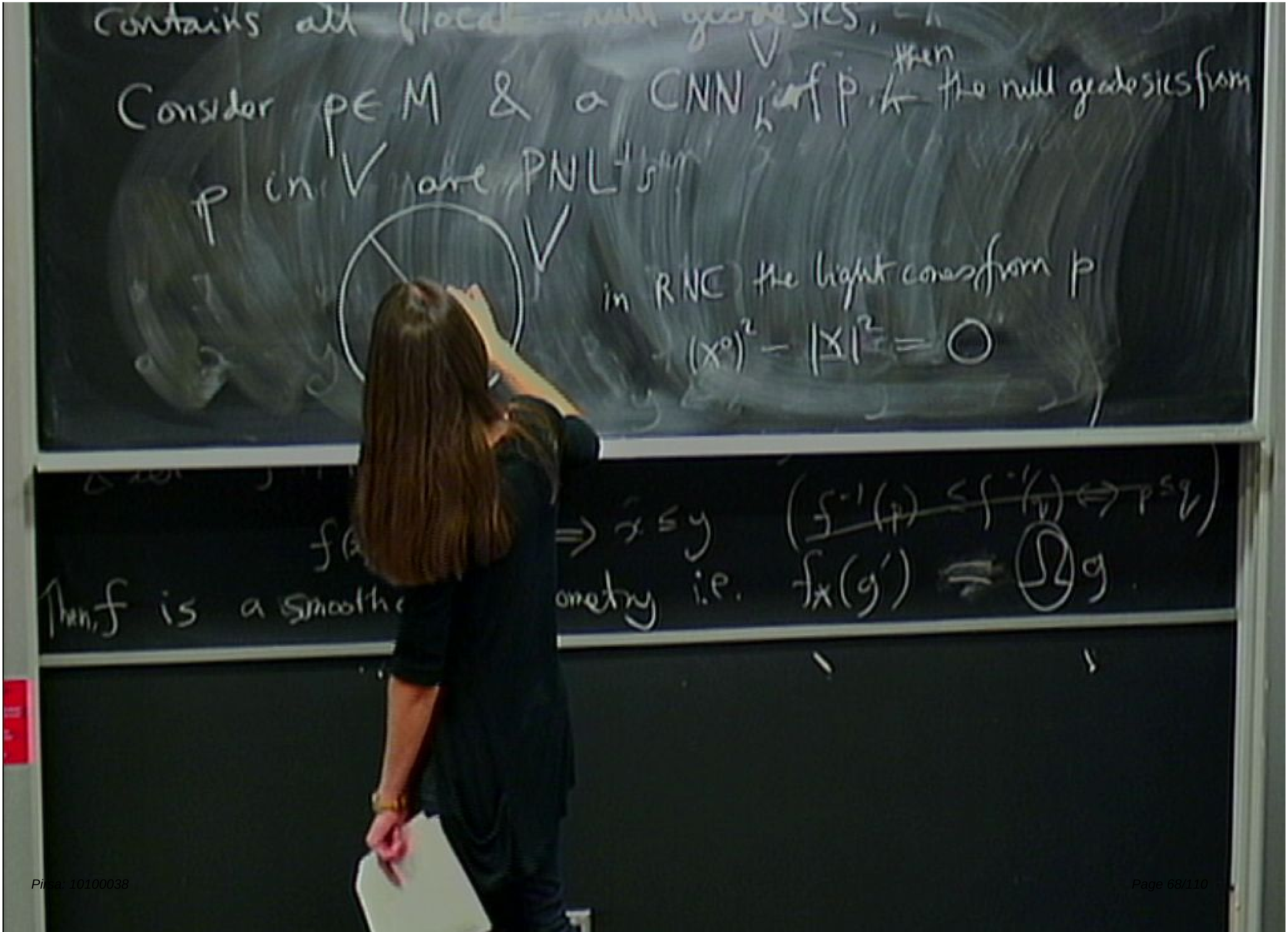
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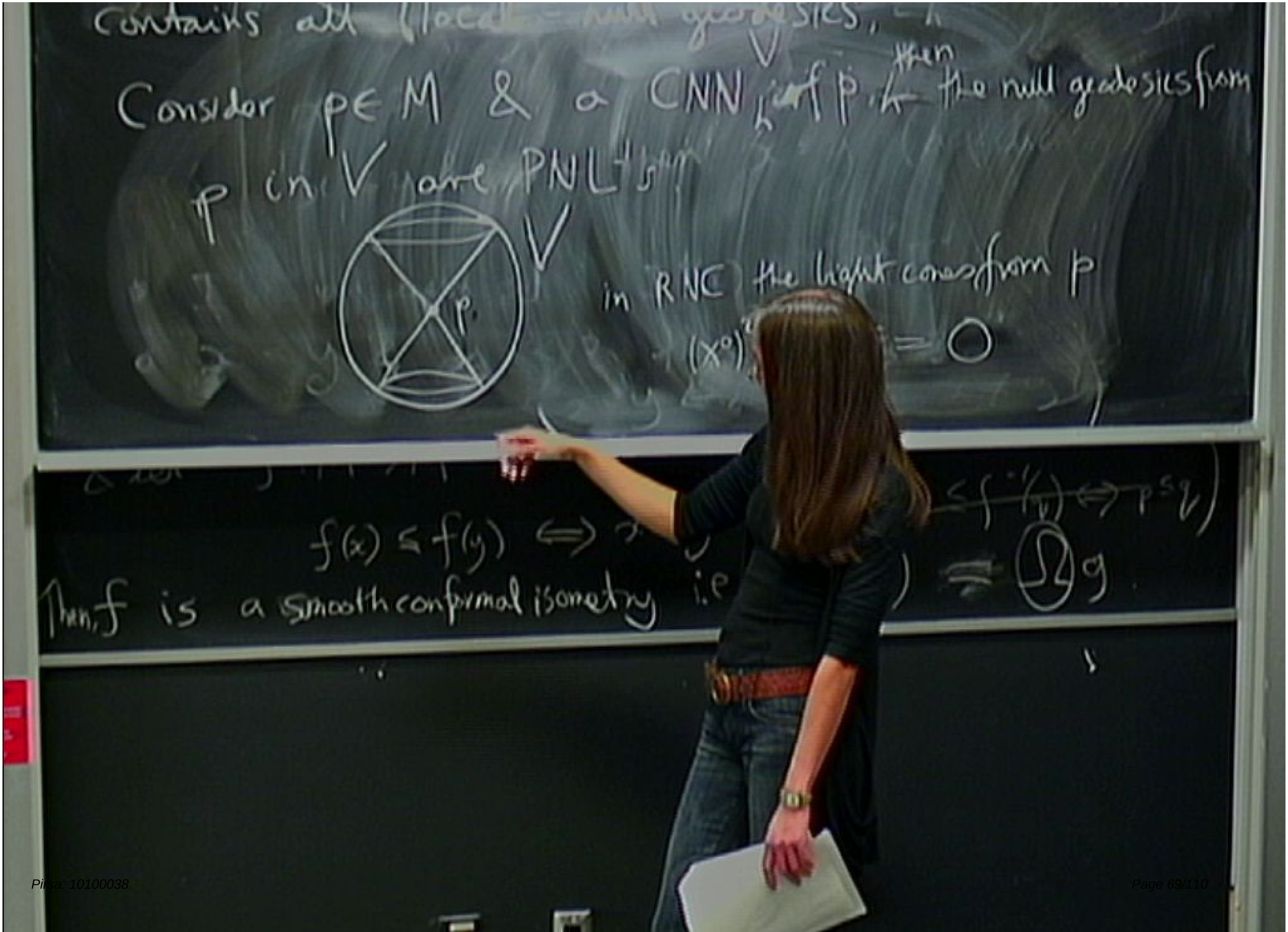
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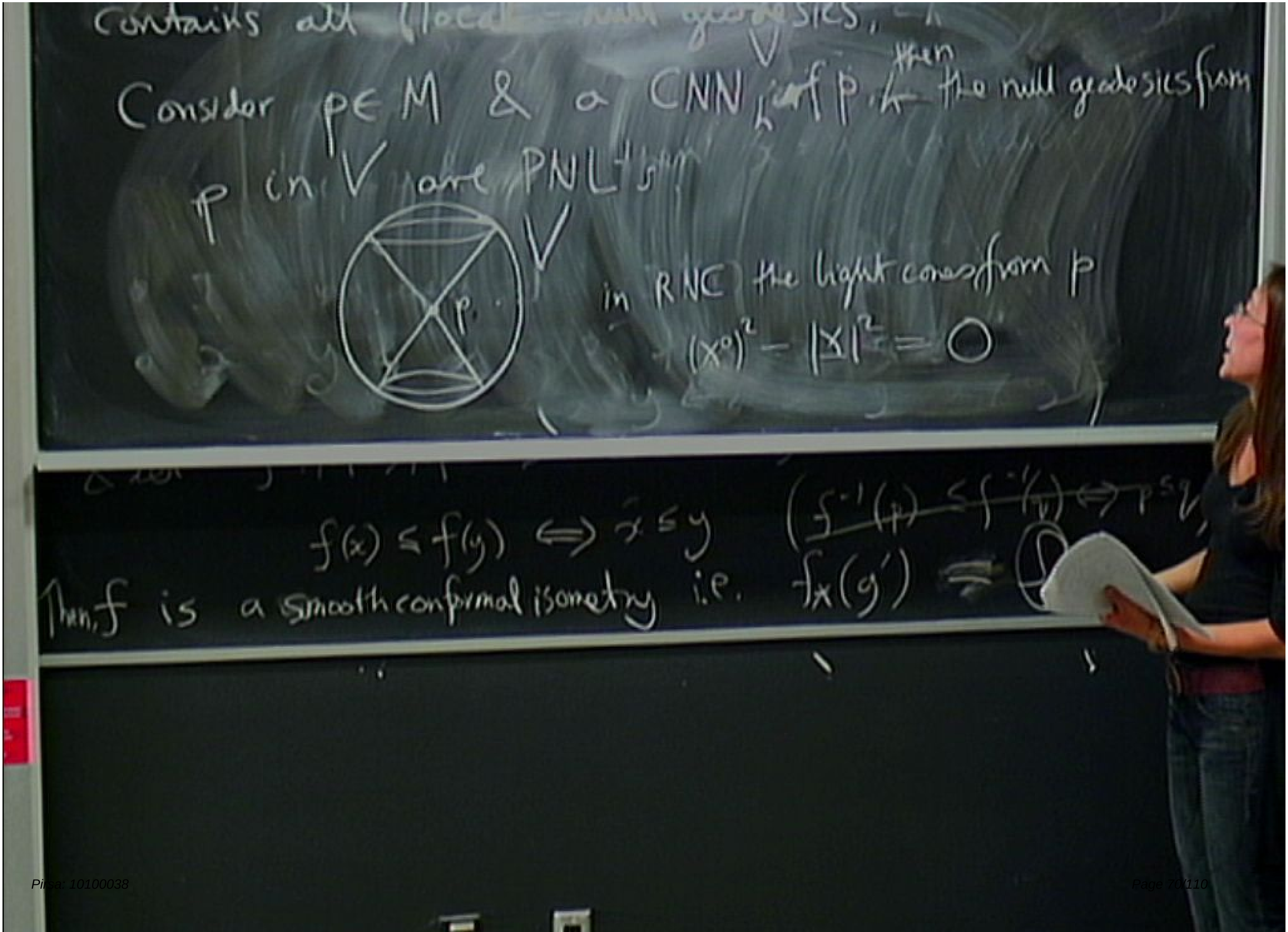
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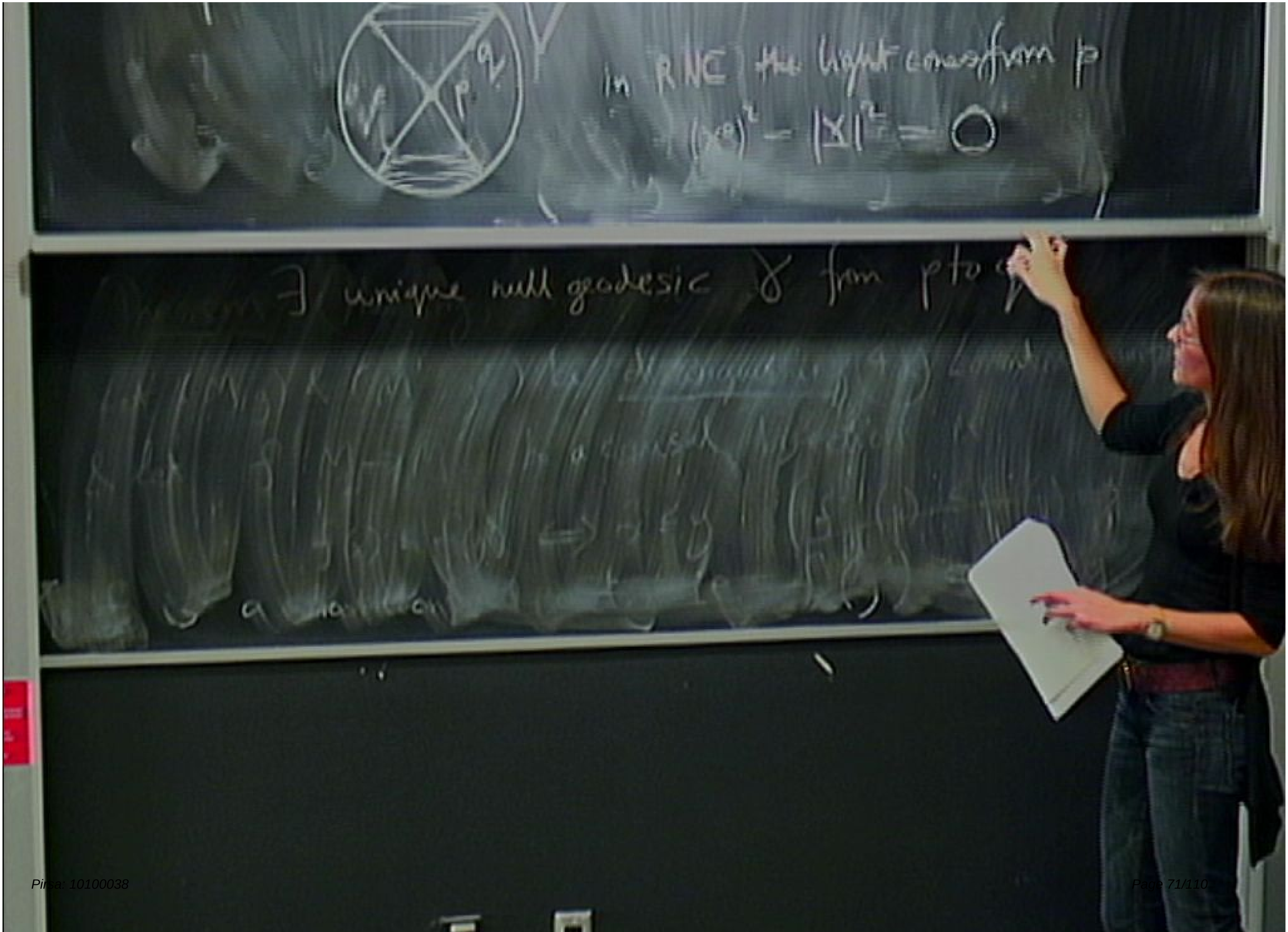
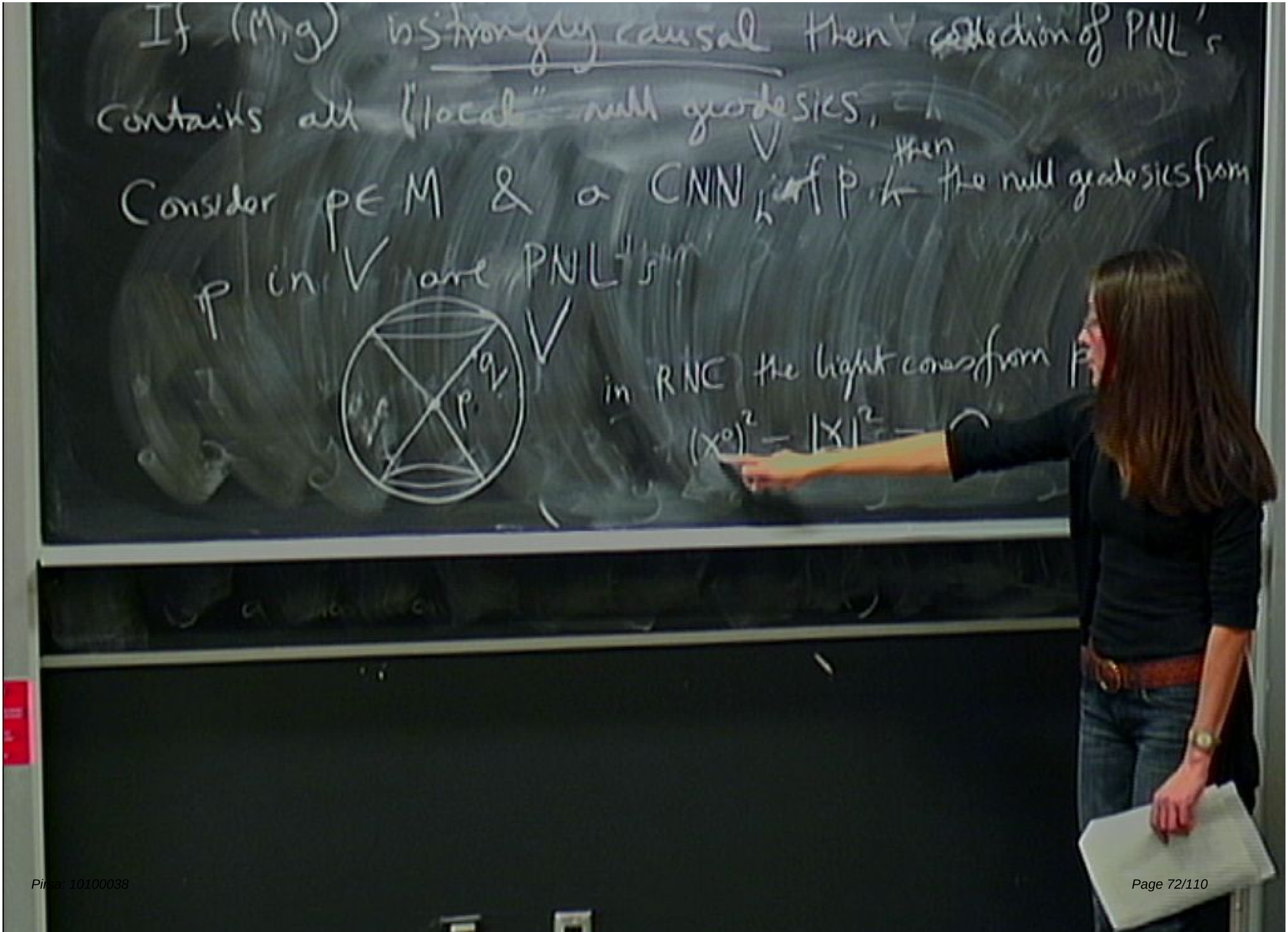

in RNC the light cones from p
$(x^0)^2 - |\underline{x}|^2 = 0$
∃ unique null geodesic γ from p to q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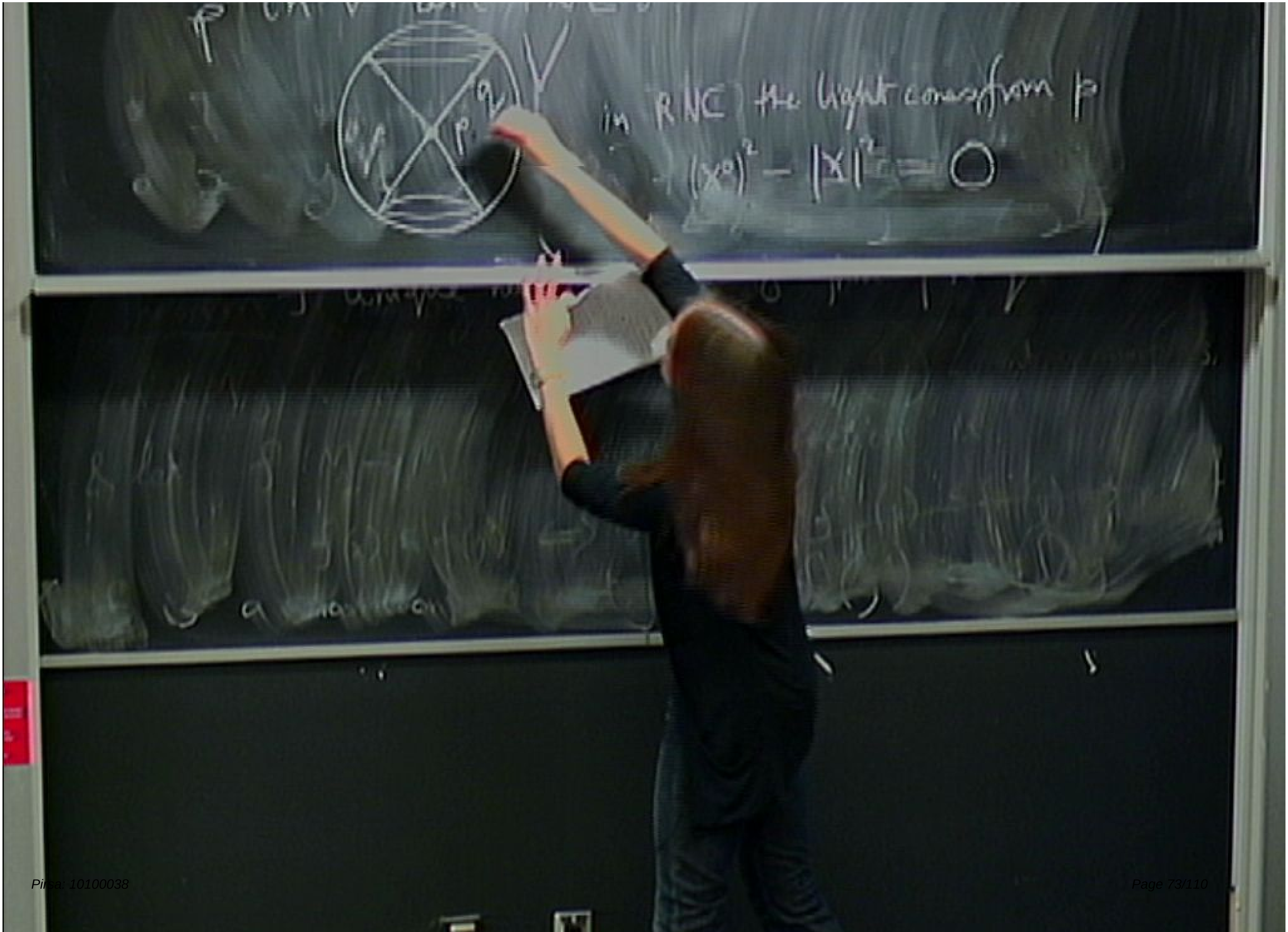
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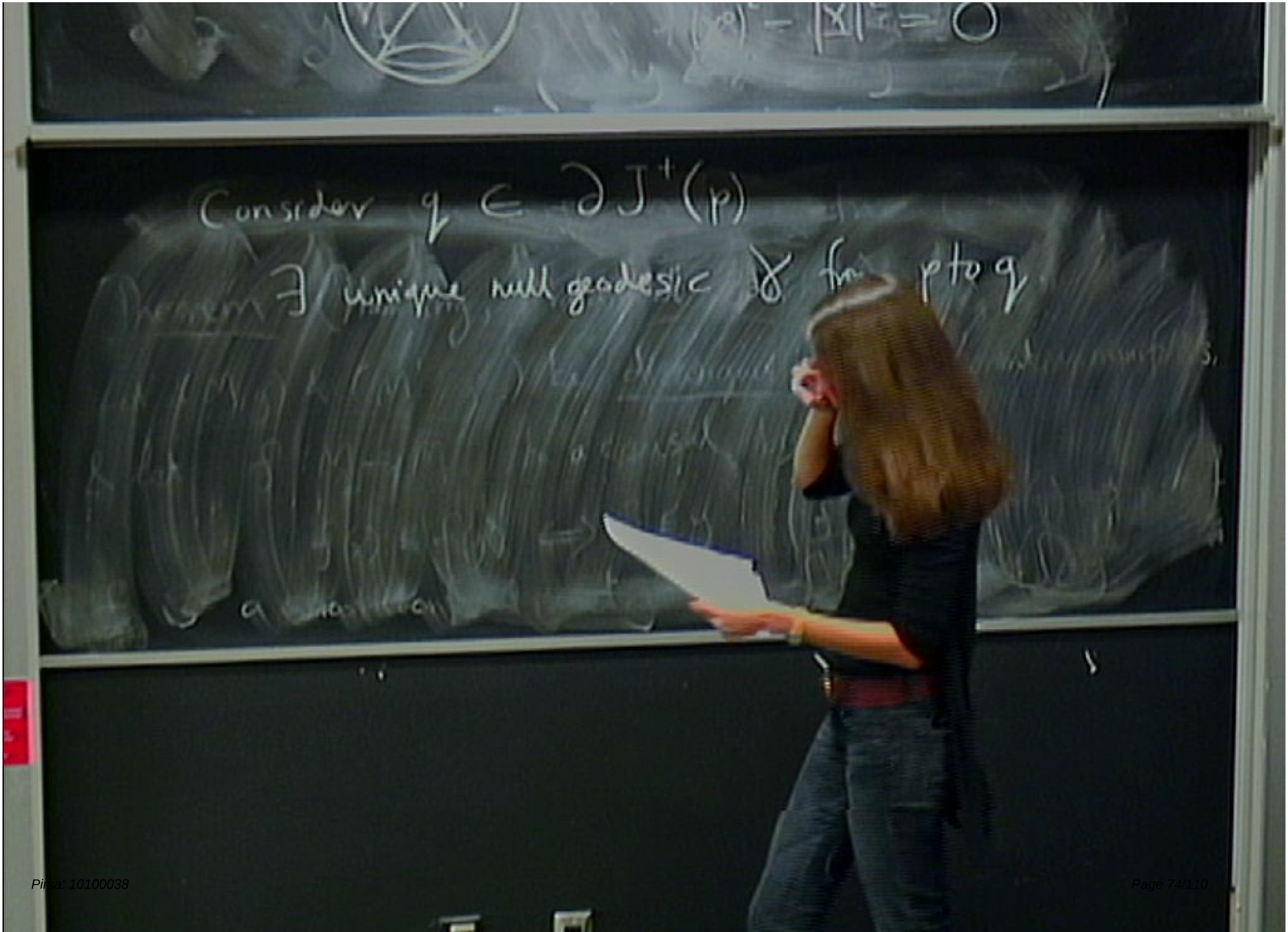
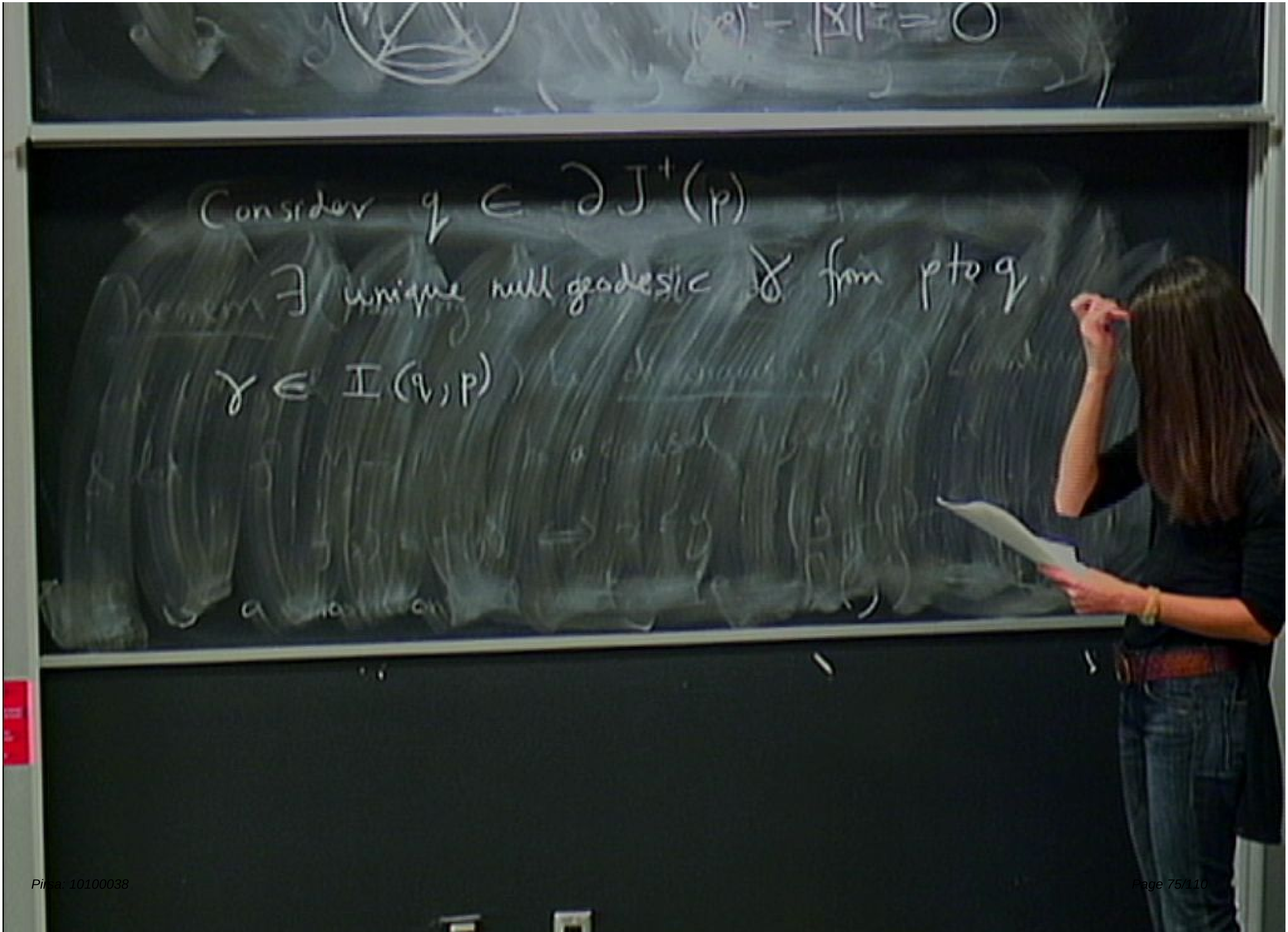
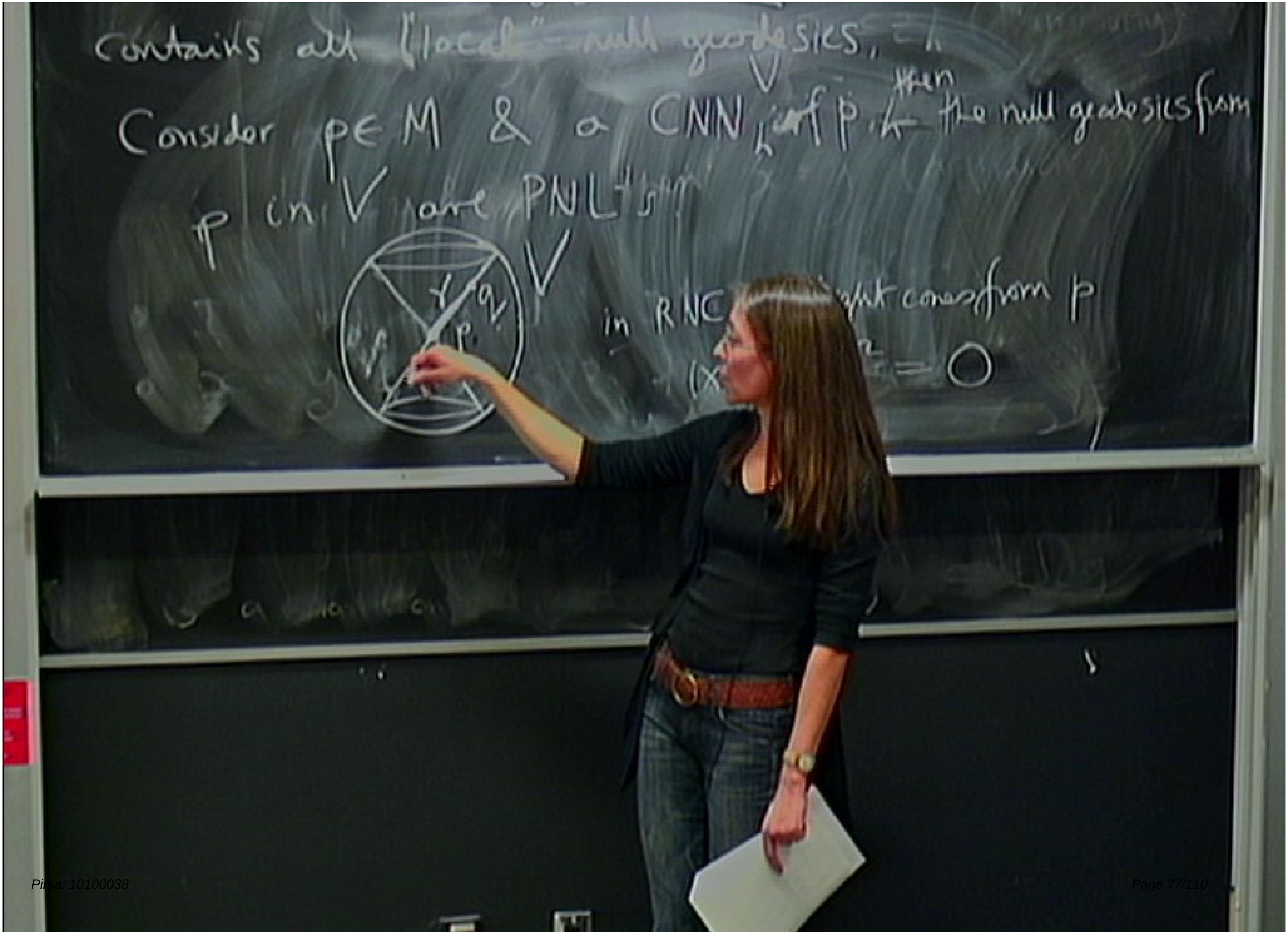

Consider q ∈ ∂J⁺(p)
∃ unique null geodesic γ fm p to q


Consider q ∈ ∂J+(p)
∃ unique null geodesic γ from p to q
γ ∈ I(q, p)

Consider $q \in \partial J^+(p)$

$\exists$ unique null geodesic γ from p to q

$\gamma \in I(q, p)$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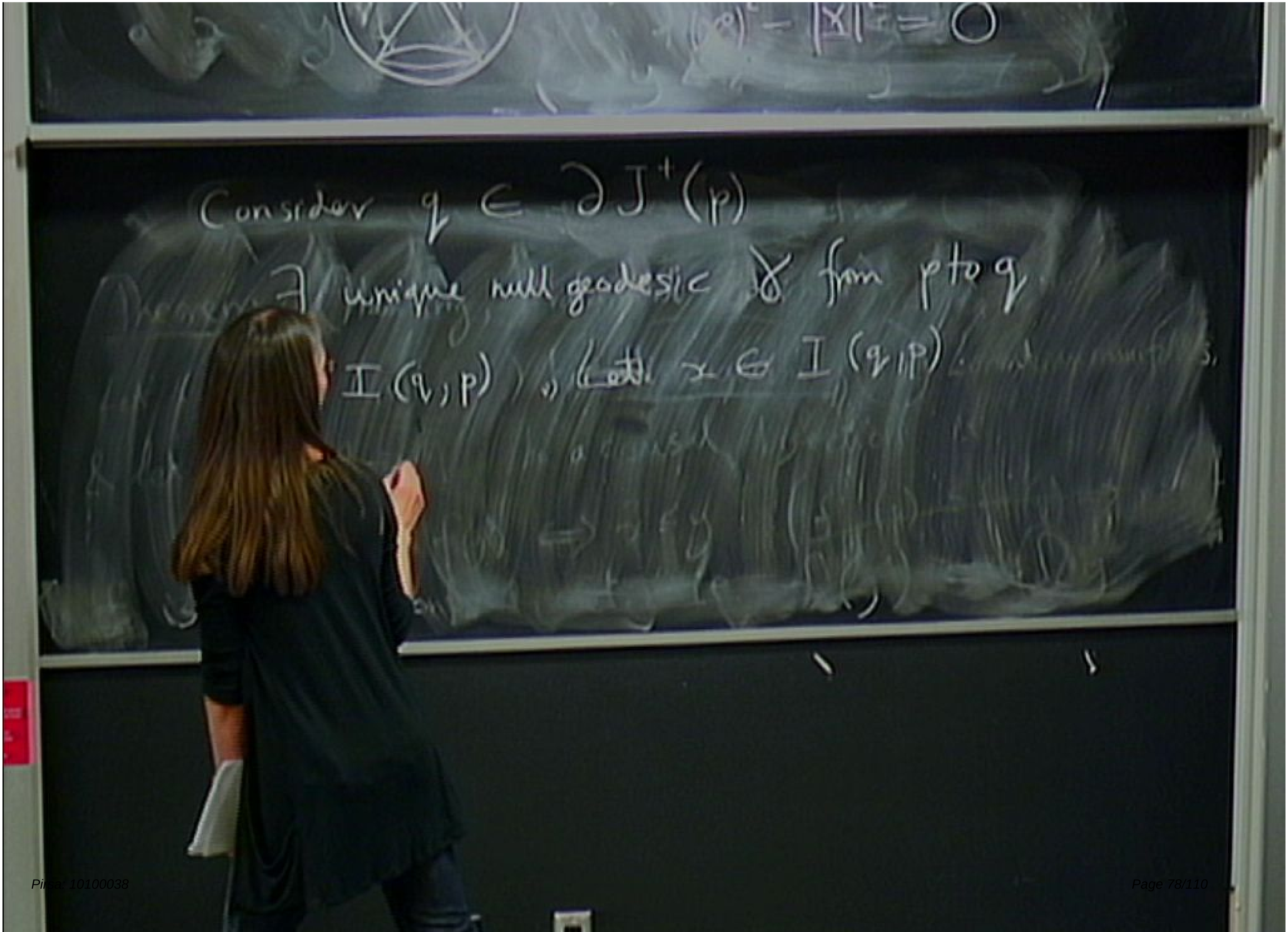
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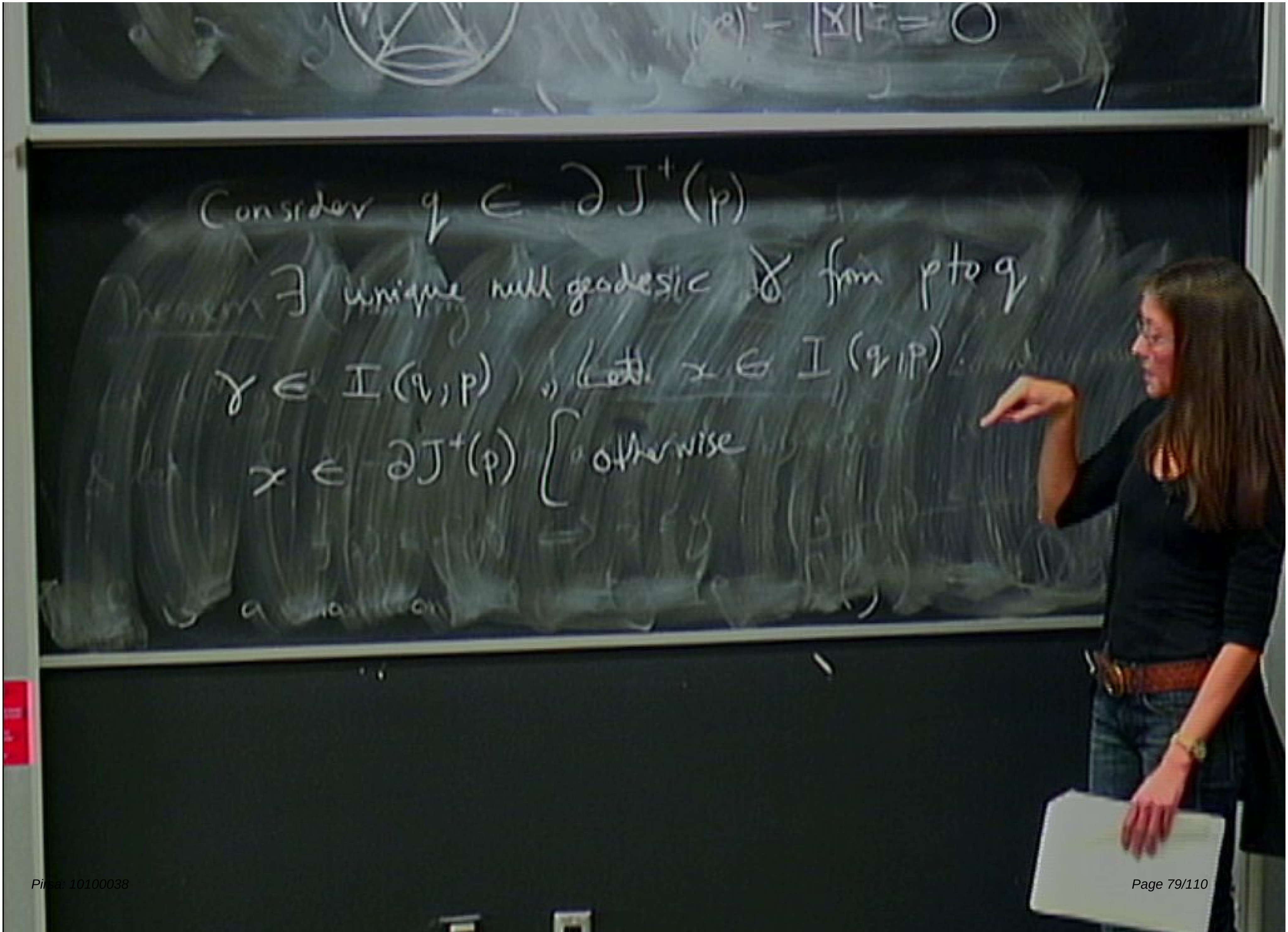
$\exists$ unique null geodesic γ from p to q
$I(q,p)$ Let $x \in I(q,p)$.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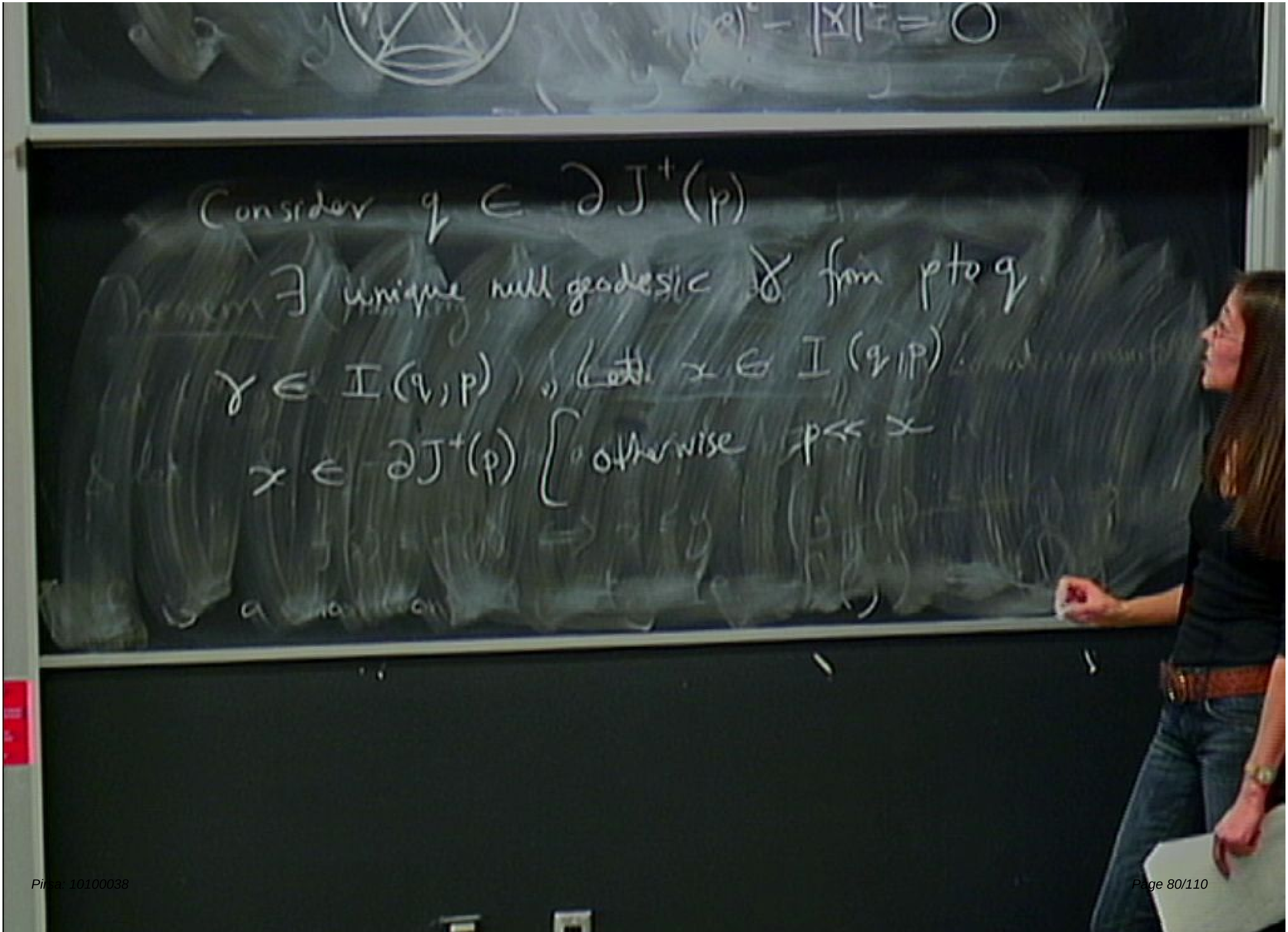
x ∈ ∂J⁺(p) { otherwise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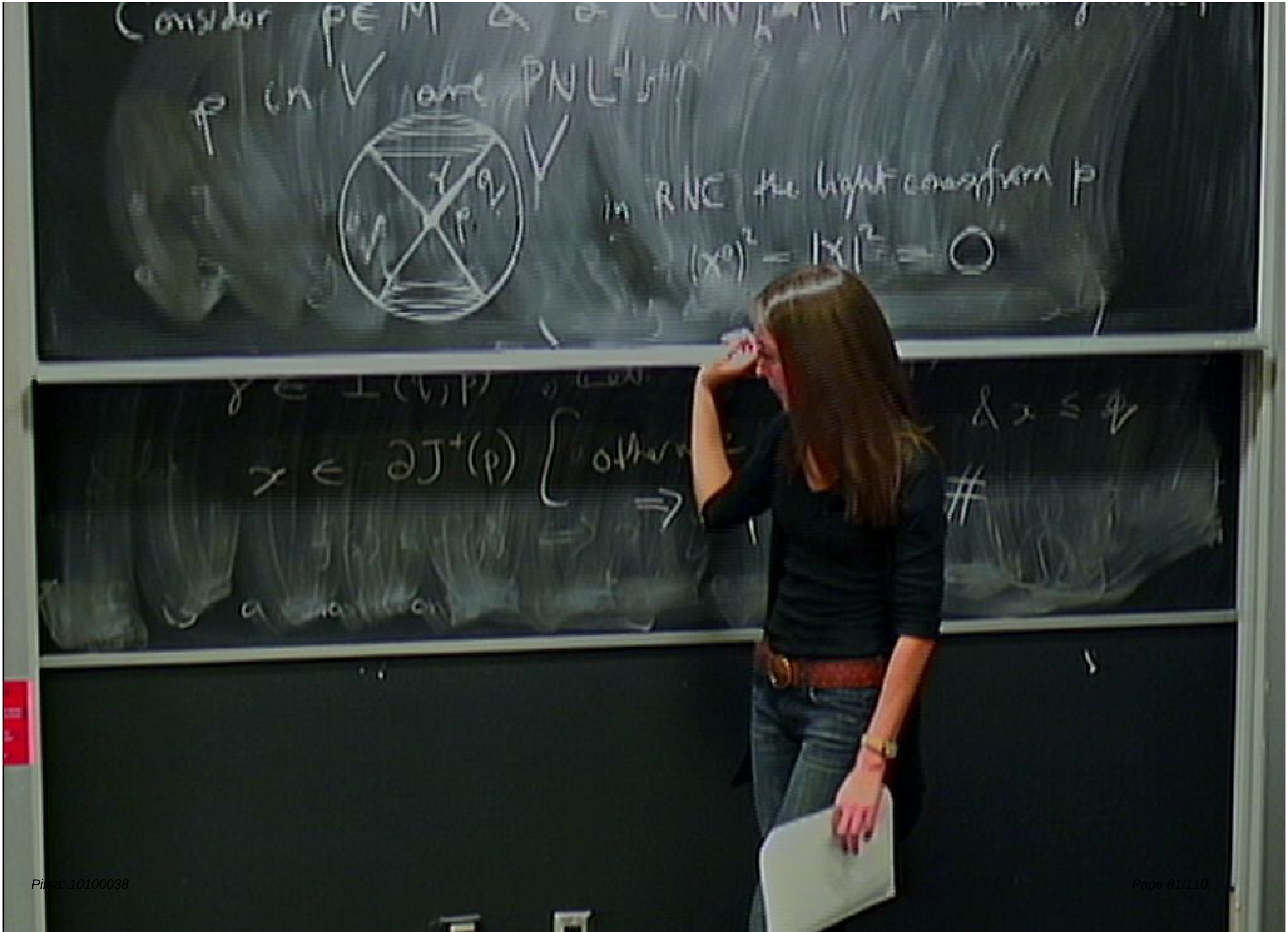
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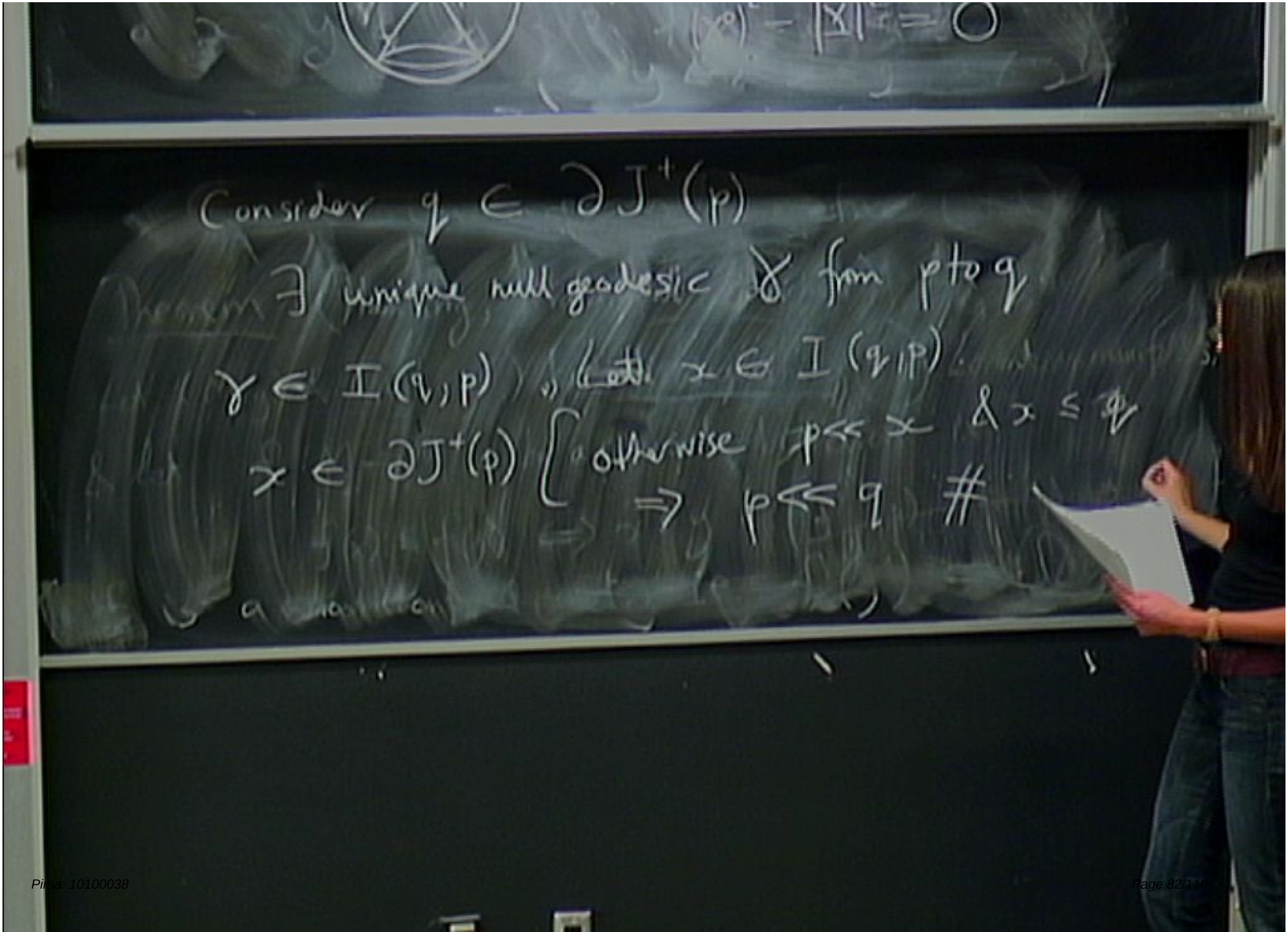
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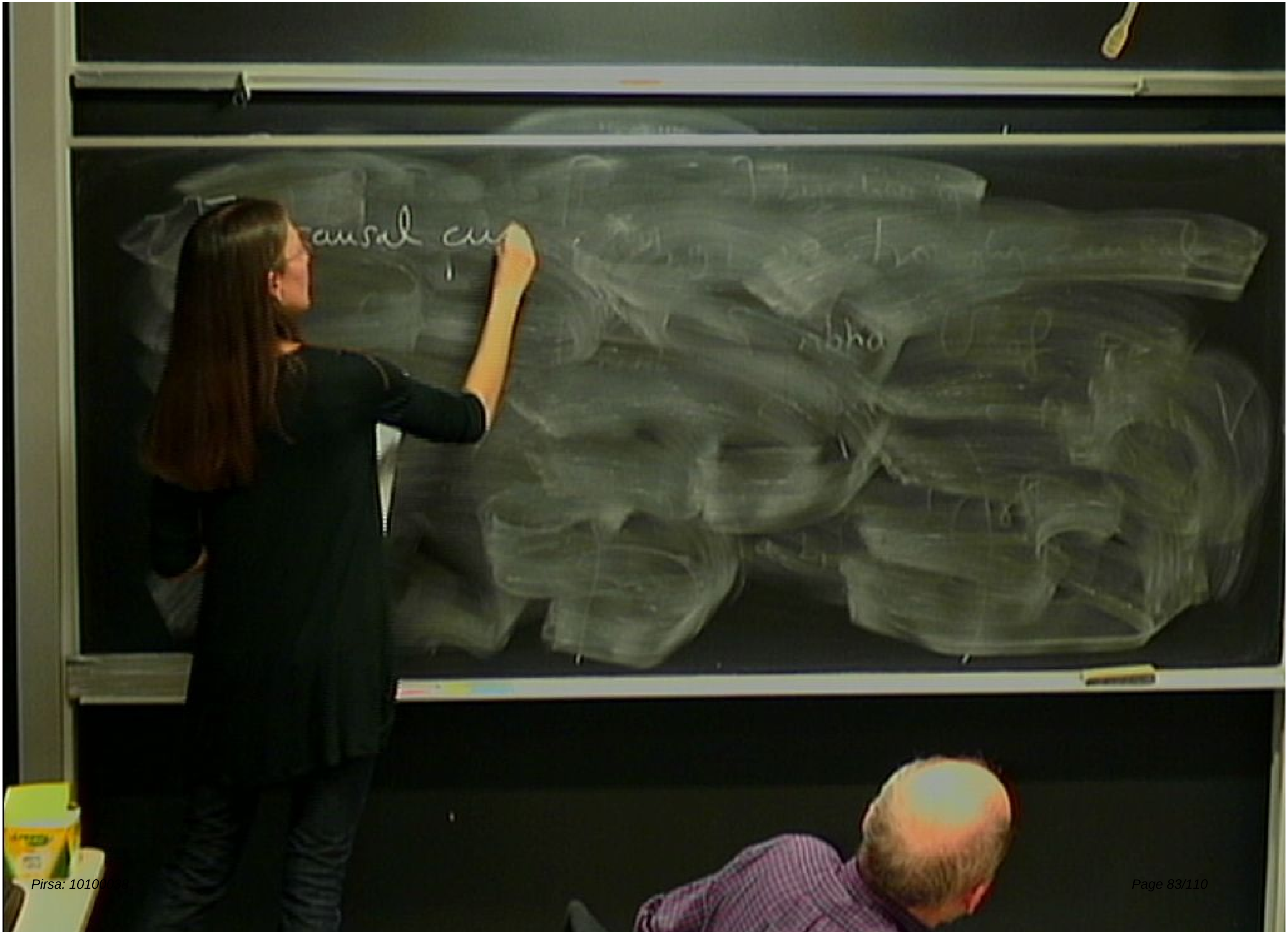
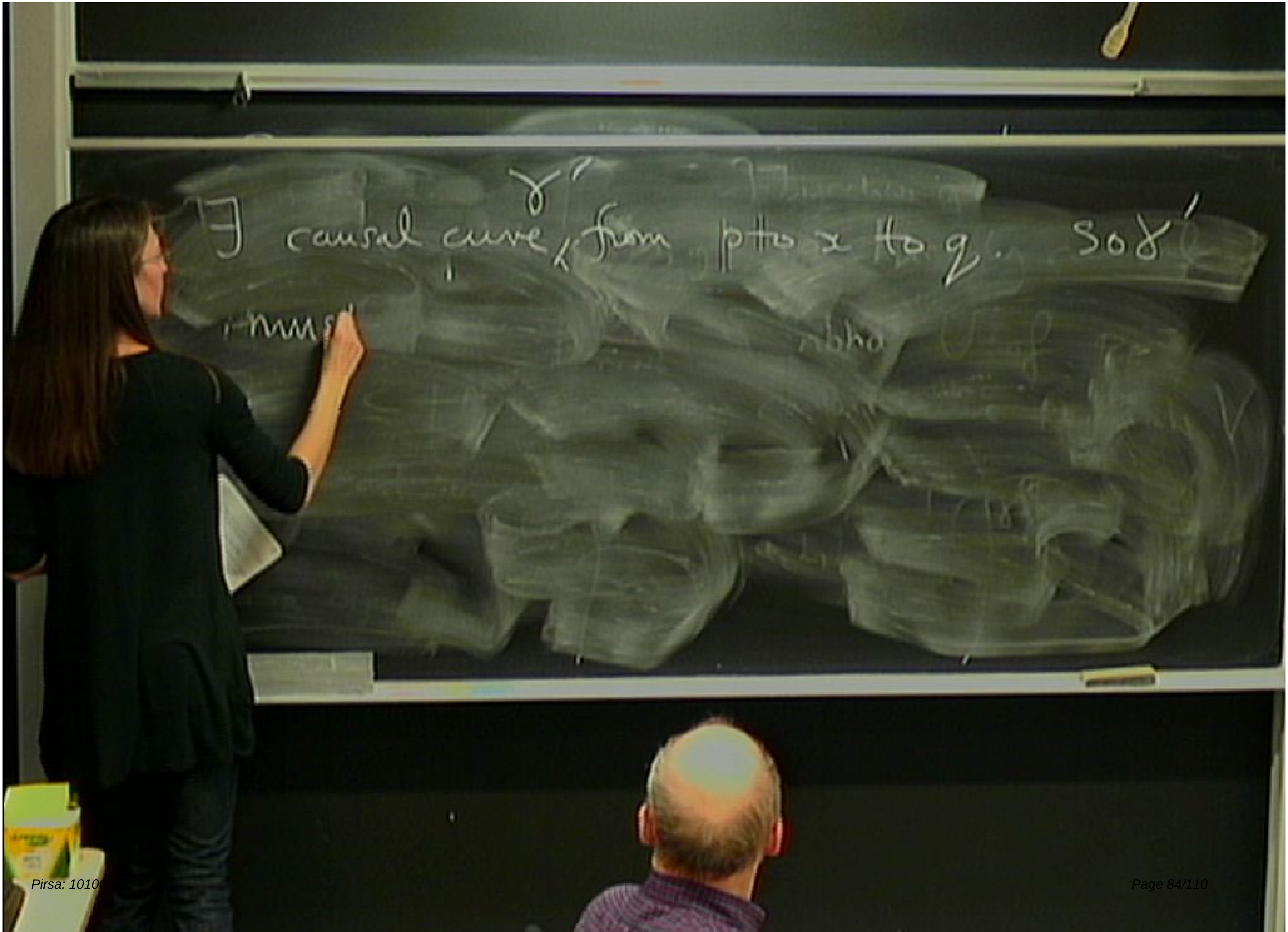
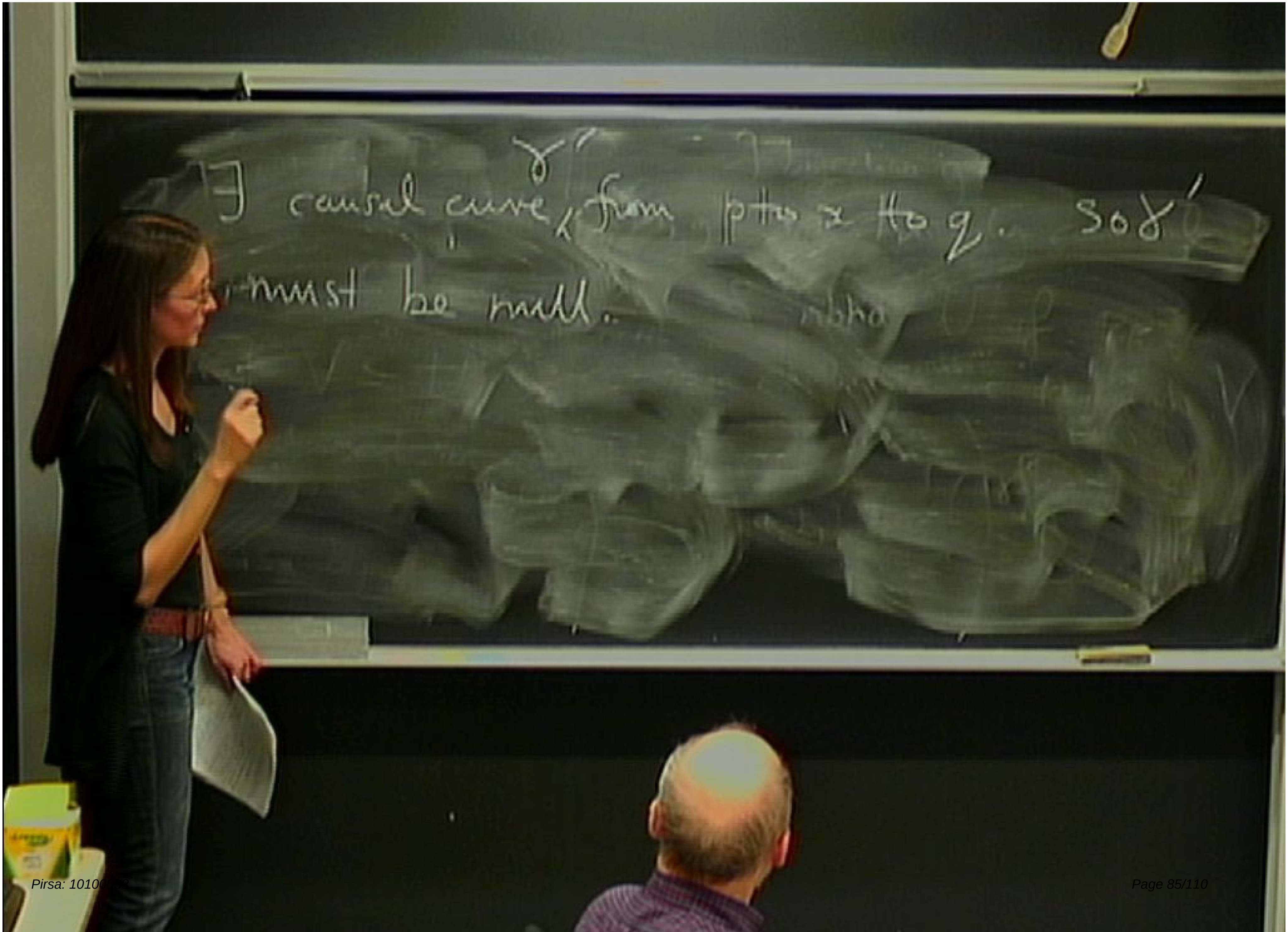
⇒ p ≪ q #


causal cu


γ'
∃ causal curve from p to x to q. so γ'


∃ causal curve γ' from p to x to q. So γ'
must be null.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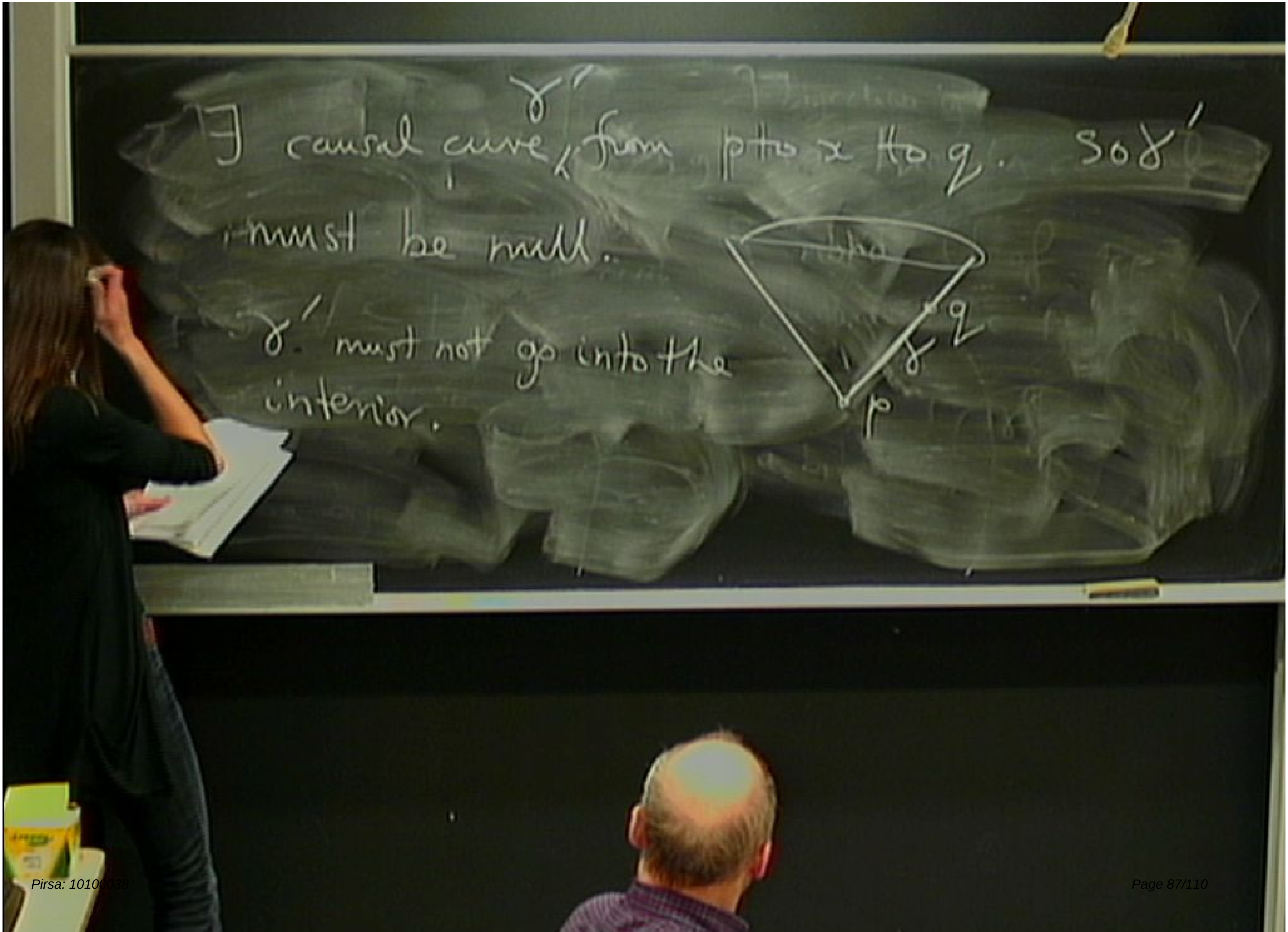
q
γ
p


∃ causal curve γ′ from p to x to q. So γ′
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γ′ must not go into the
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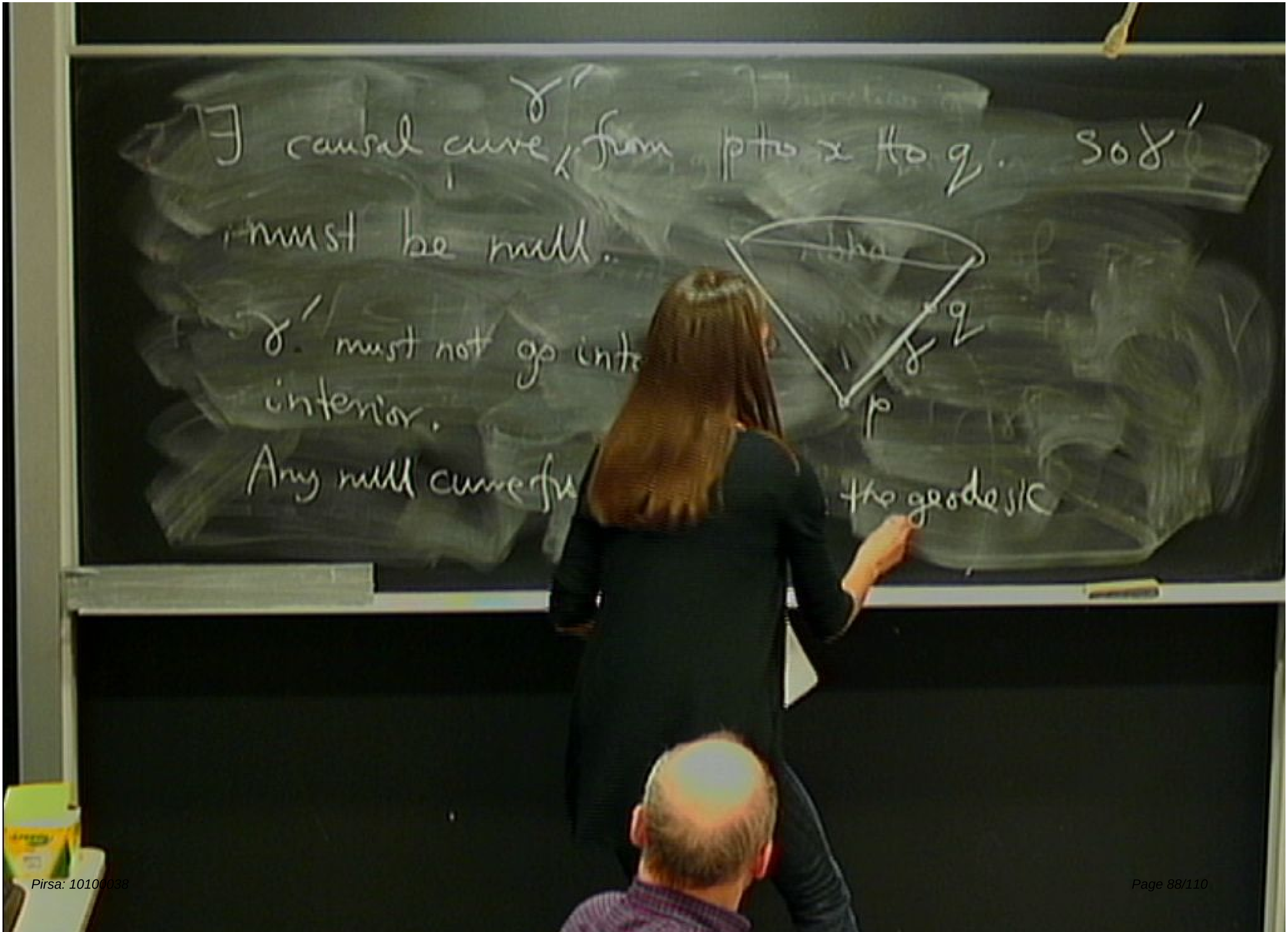
q
γ
p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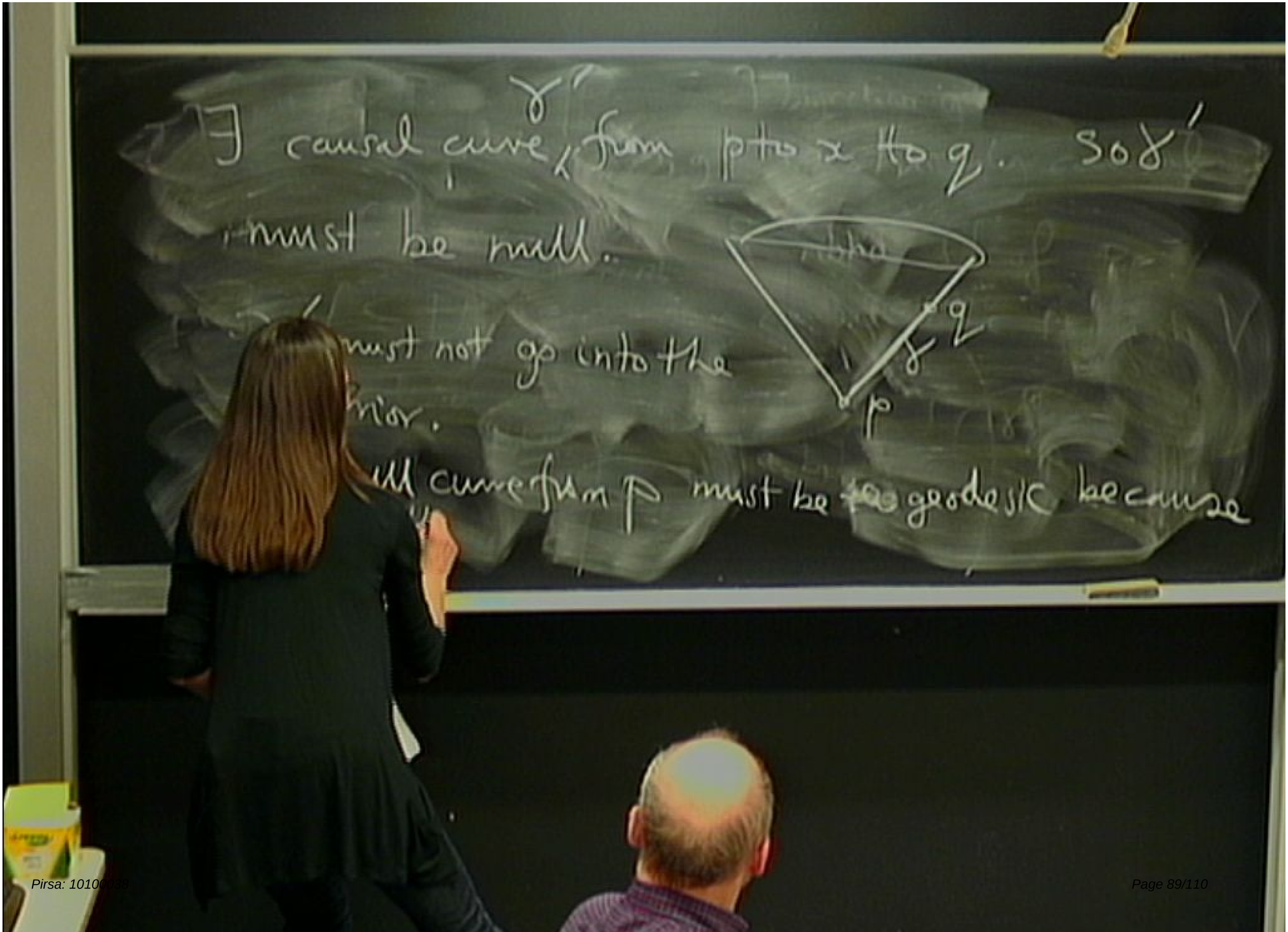
q
γ
p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q
γ
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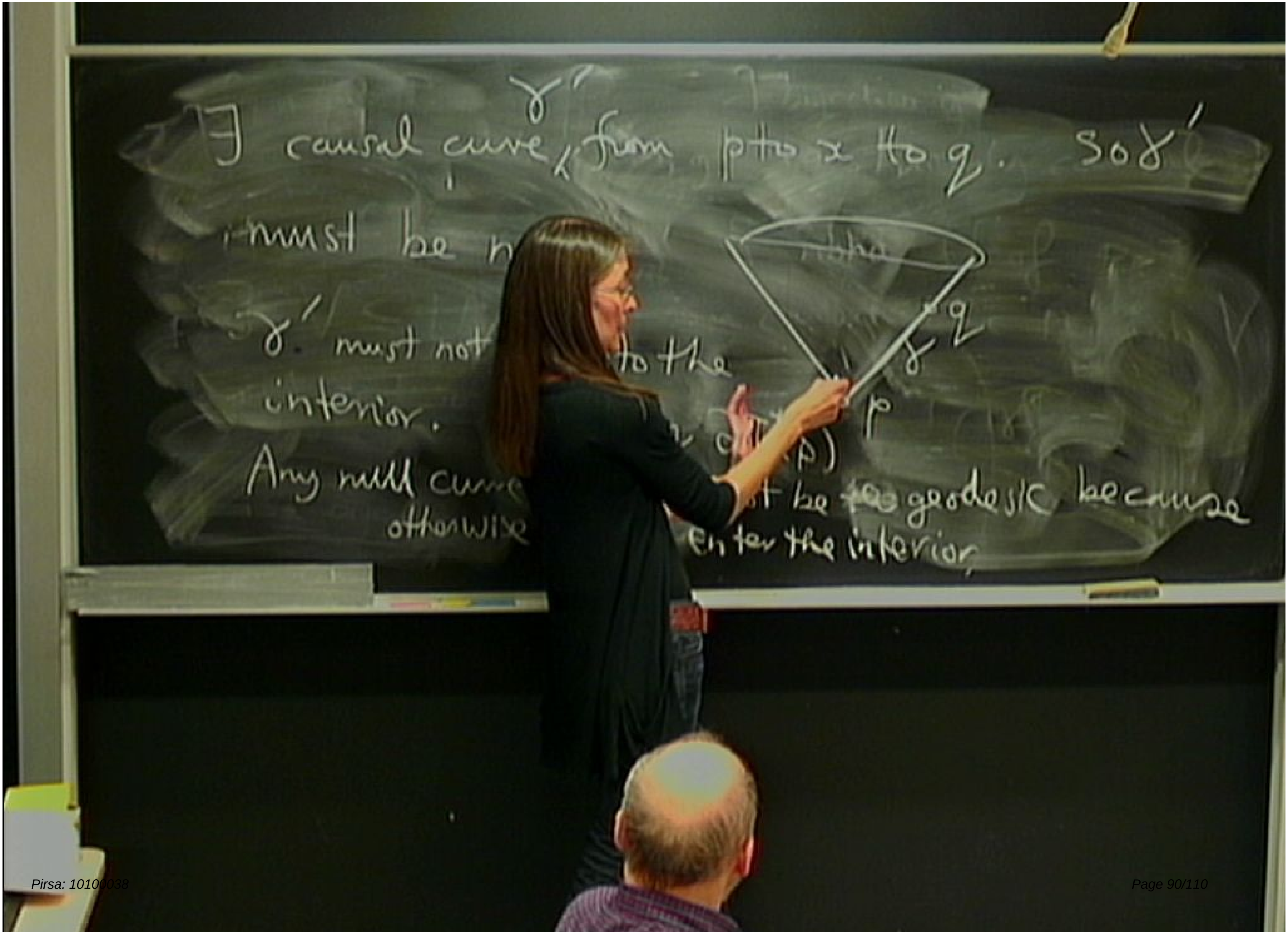

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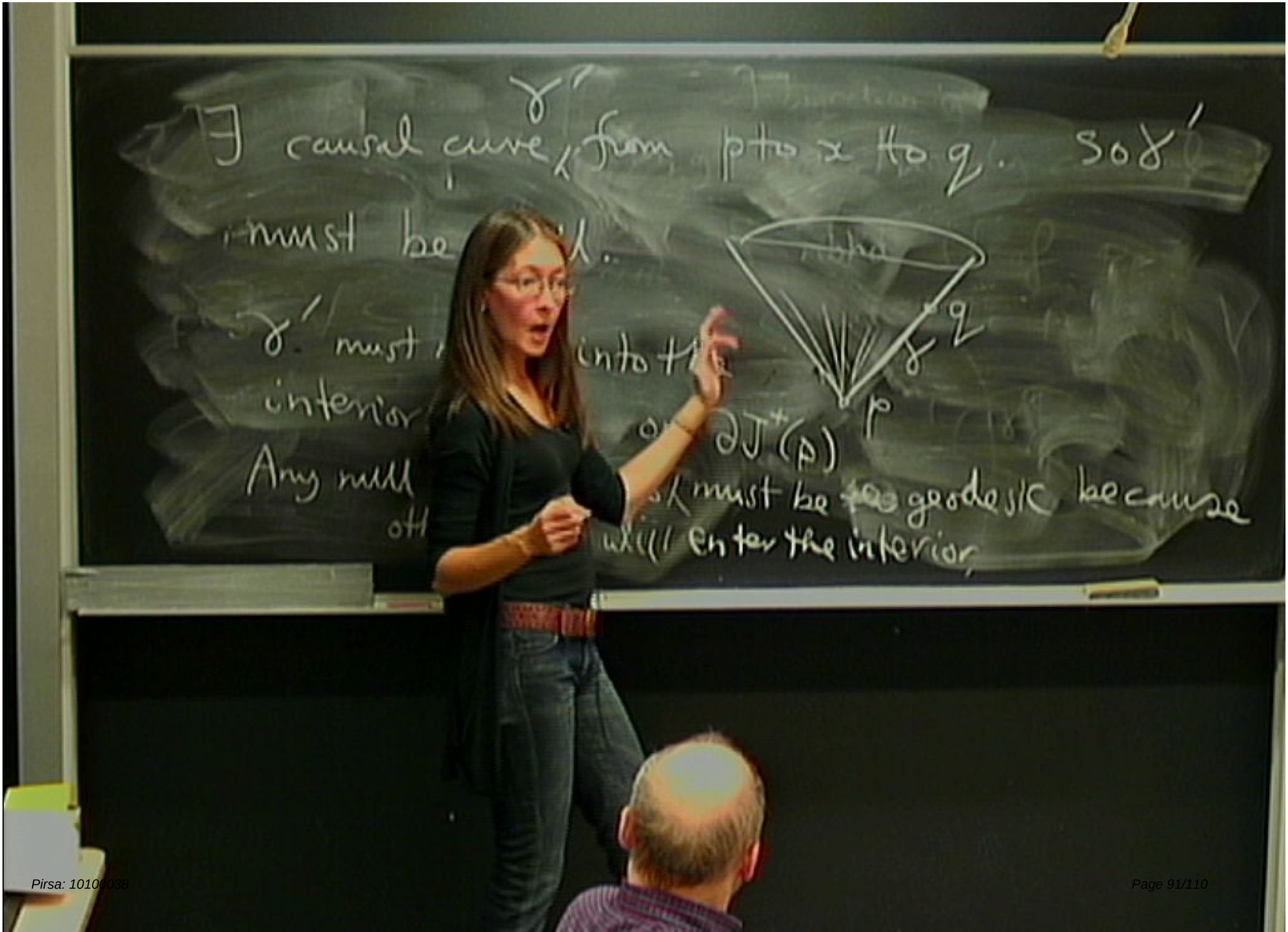
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∂J+(p)
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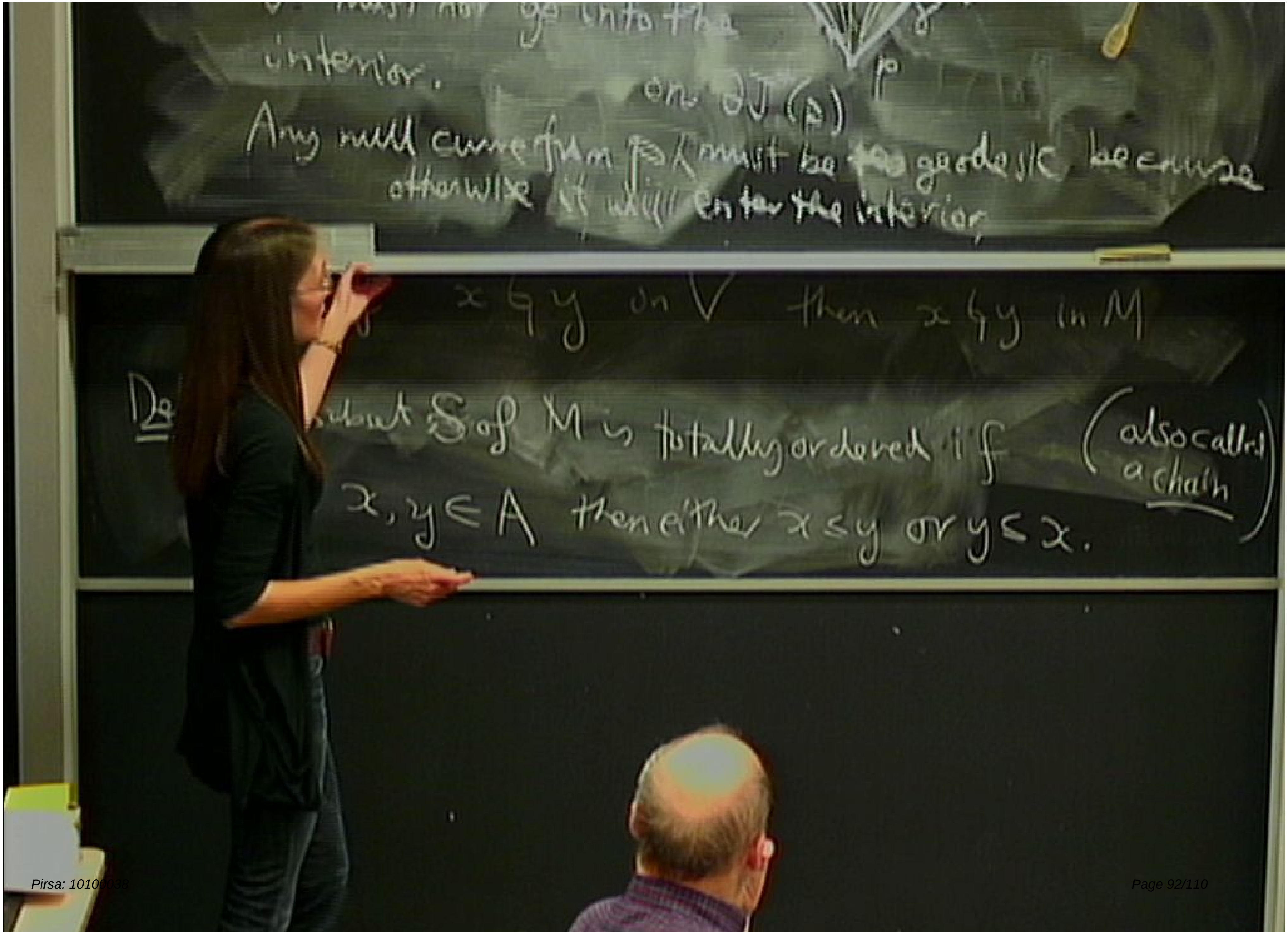
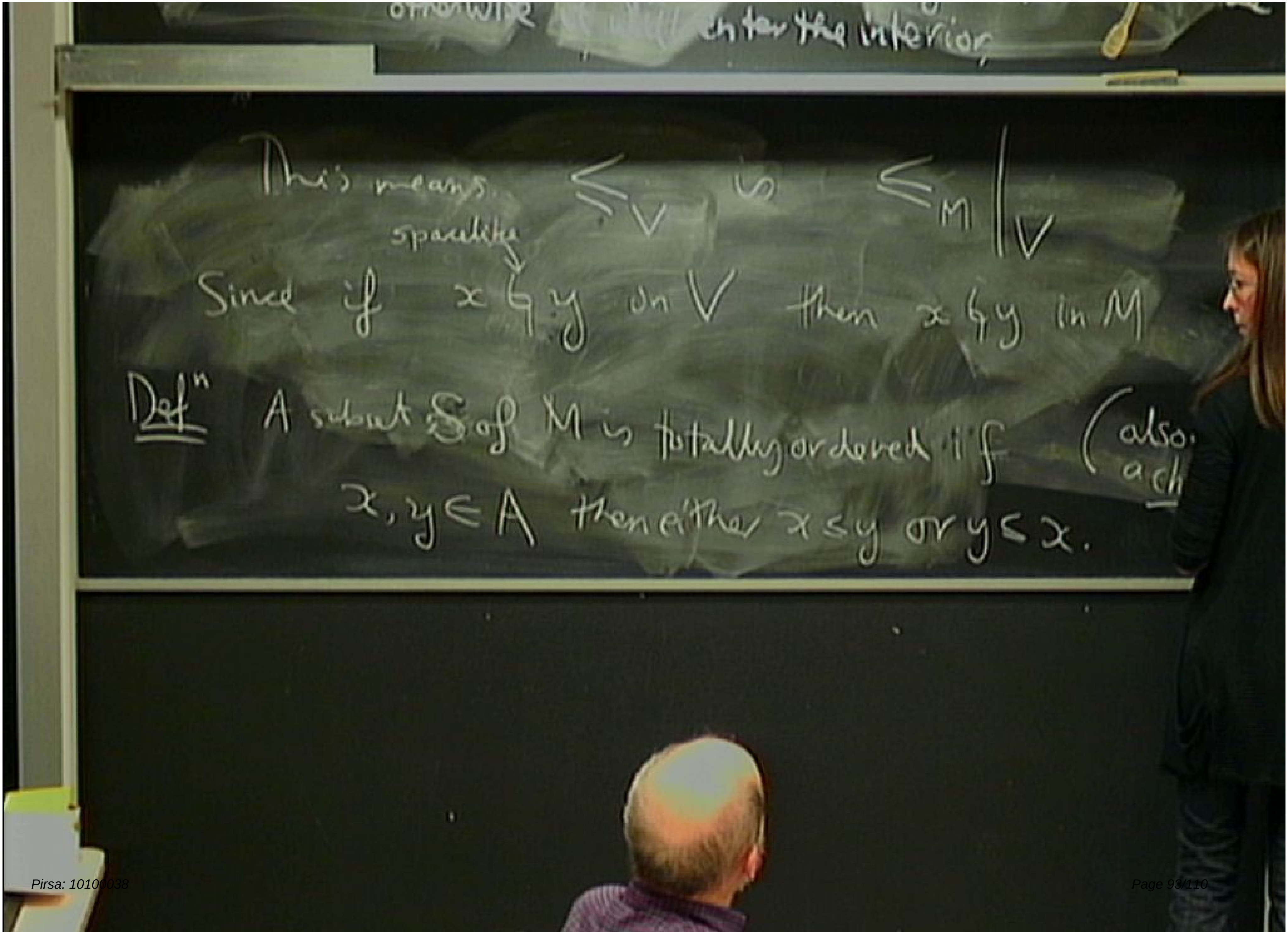


interior.
on ∂J(p)
Any null curve from p must be geodesic because otherwise it will enter the interior.
x ≤ y on V then x ≤ y in M
subset S of M is totally ordered if
(also called a chain)
x, y ∈ A then either x ≤ y or y ≤ x.


enter the interior
This means ≤_V is ≤_M |_V
spacelike
Since if x ≨ y in V then x ≨ y in M
Def^n A subset S of M is totally ordered if
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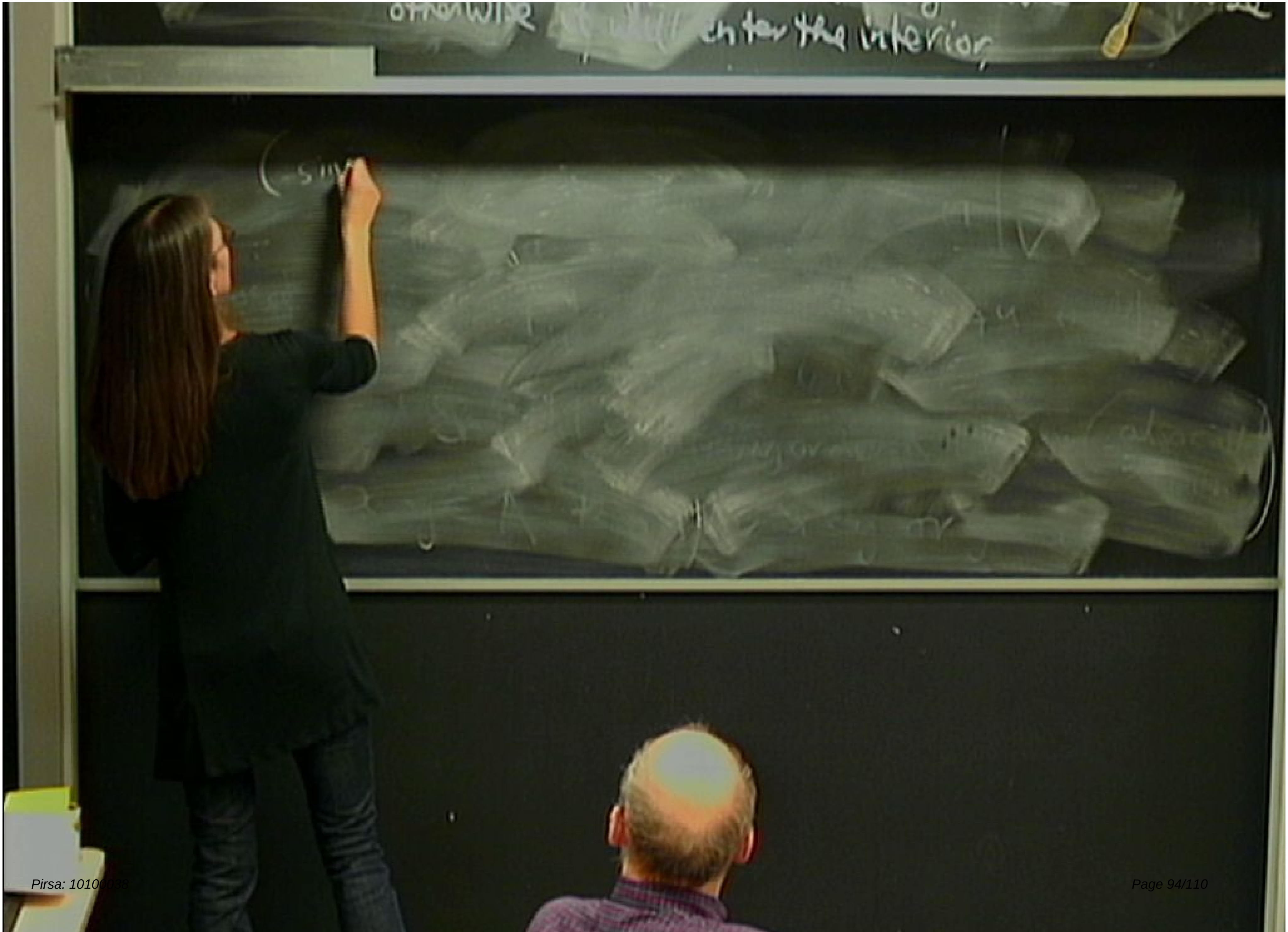
otherwise
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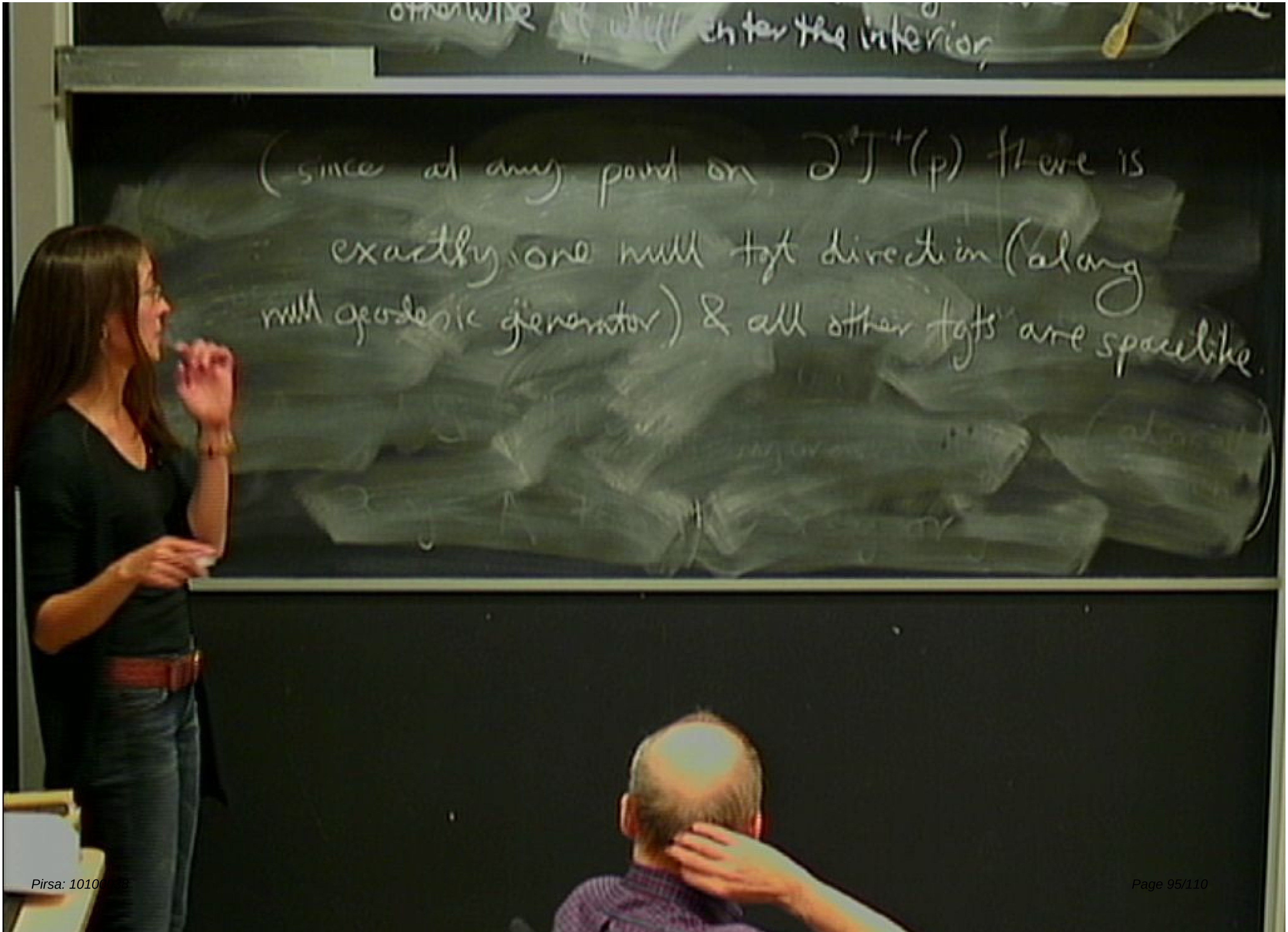
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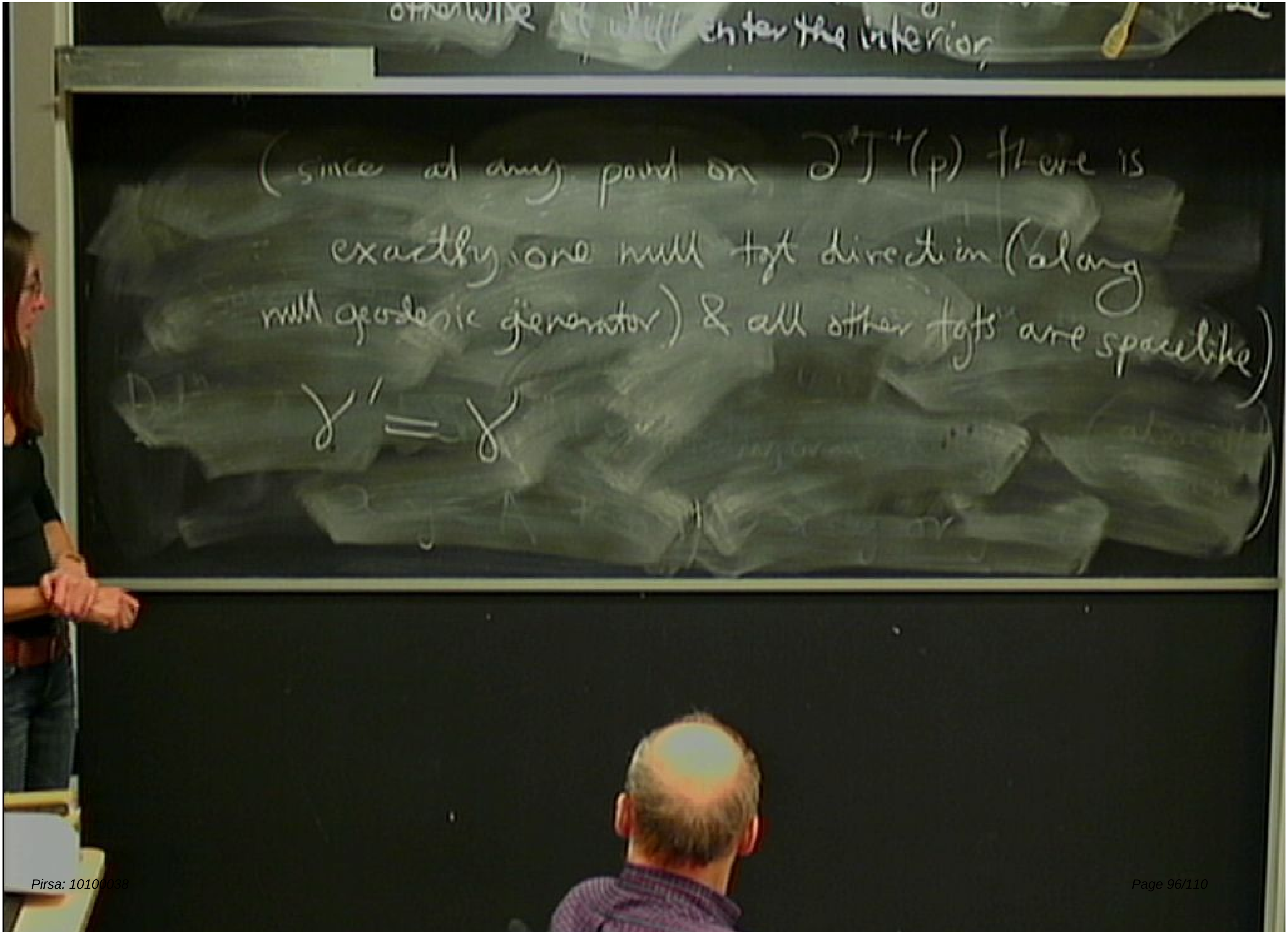
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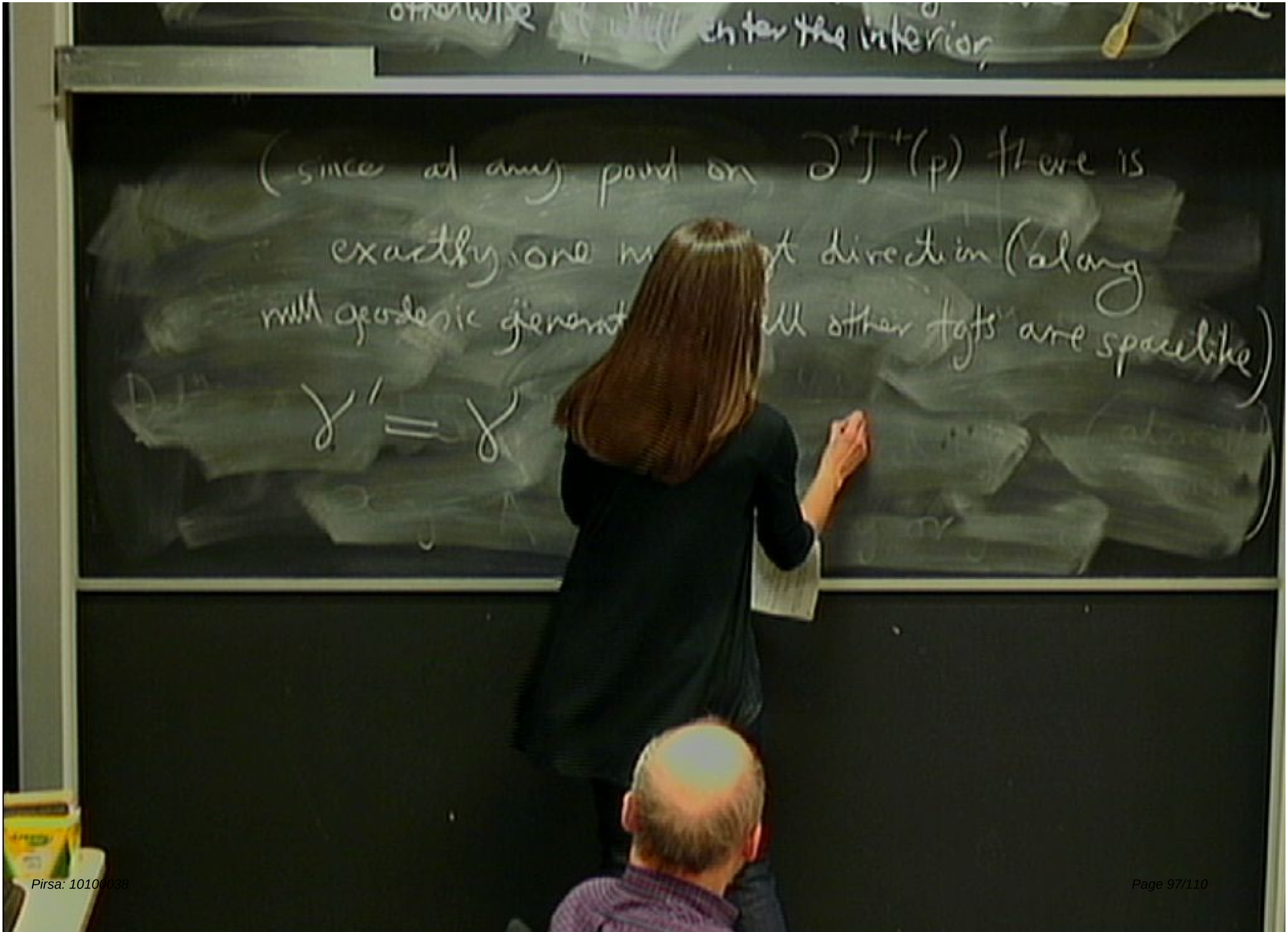
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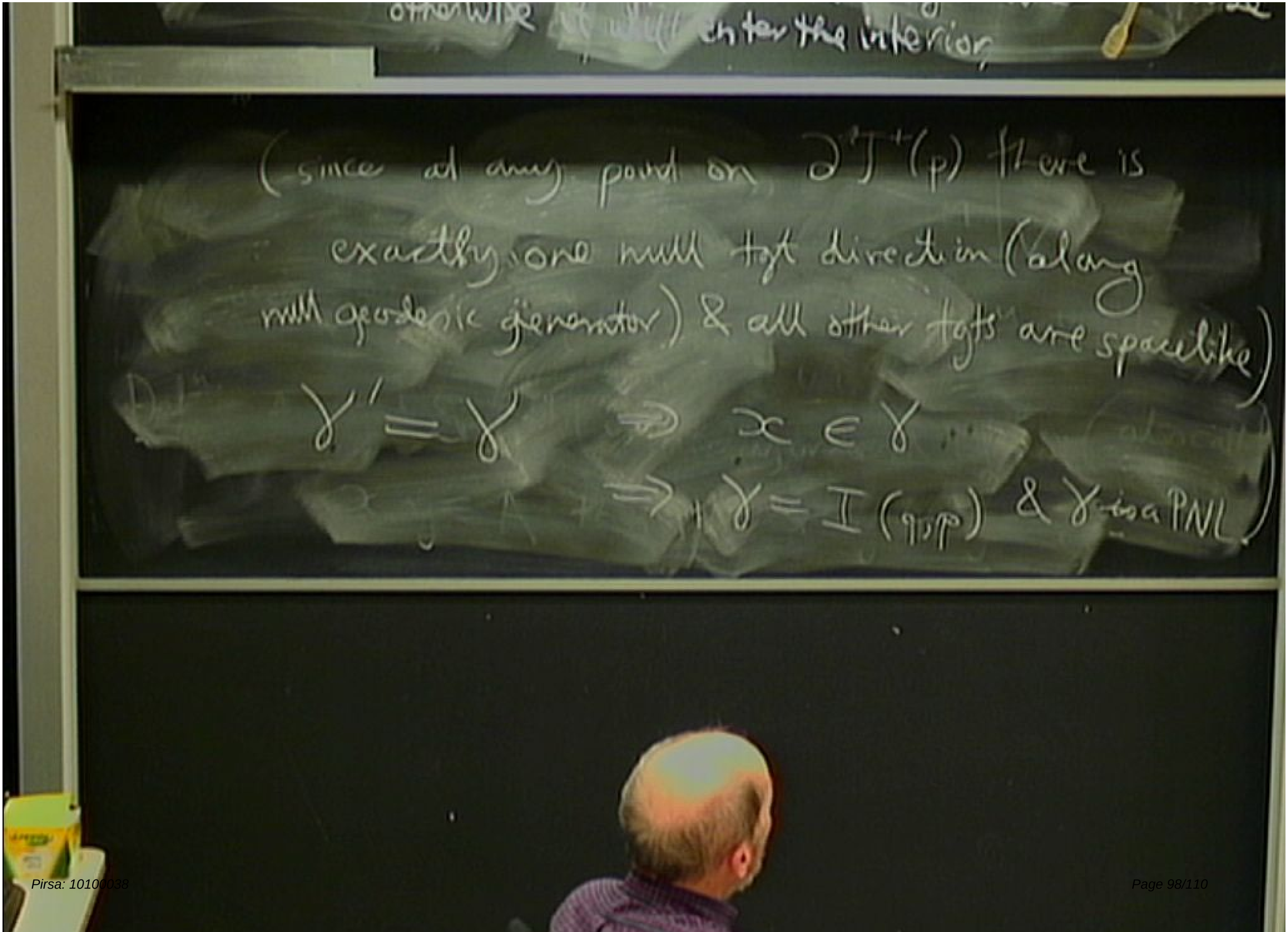
otherwise ... enter the interior,
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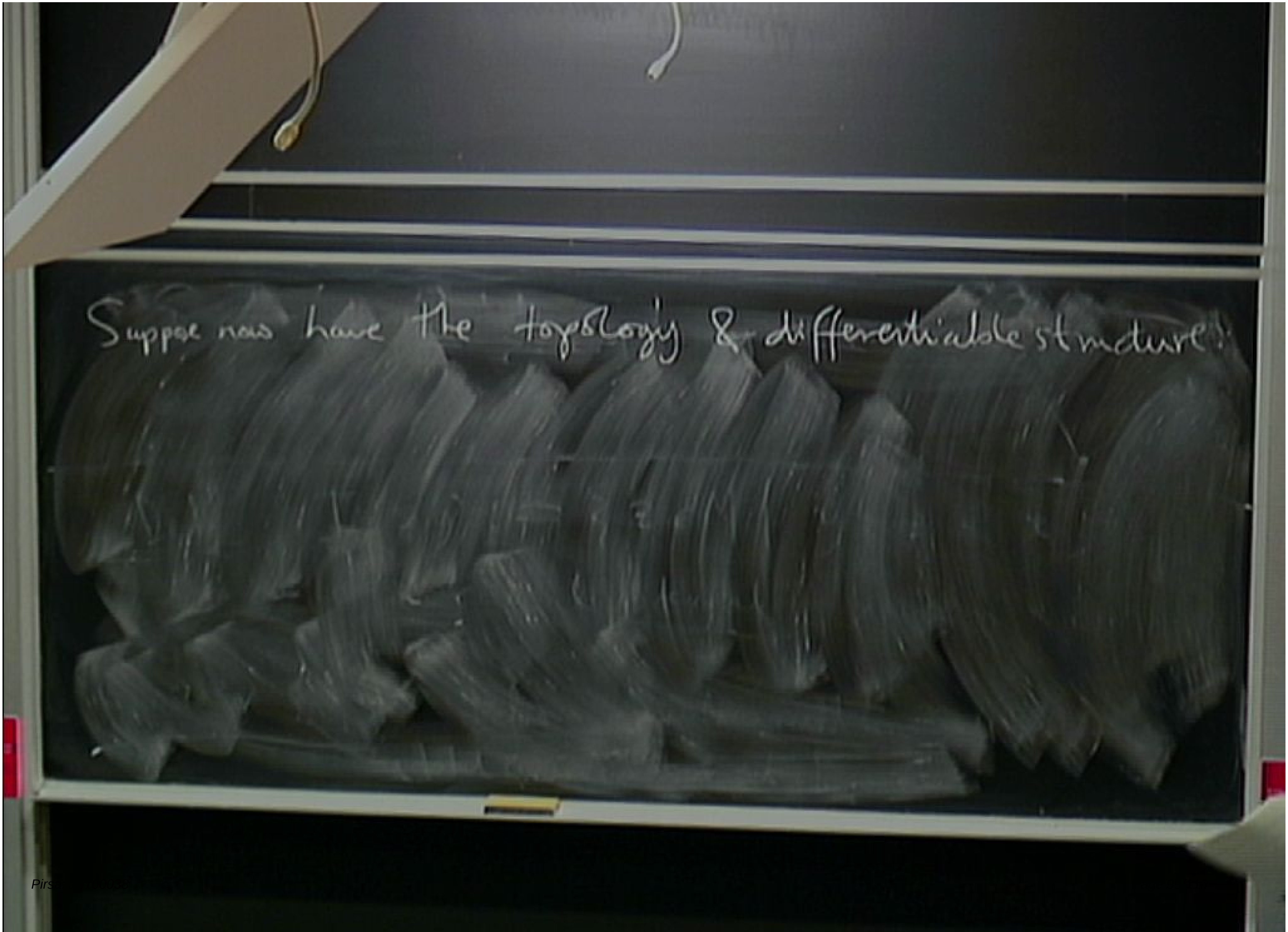
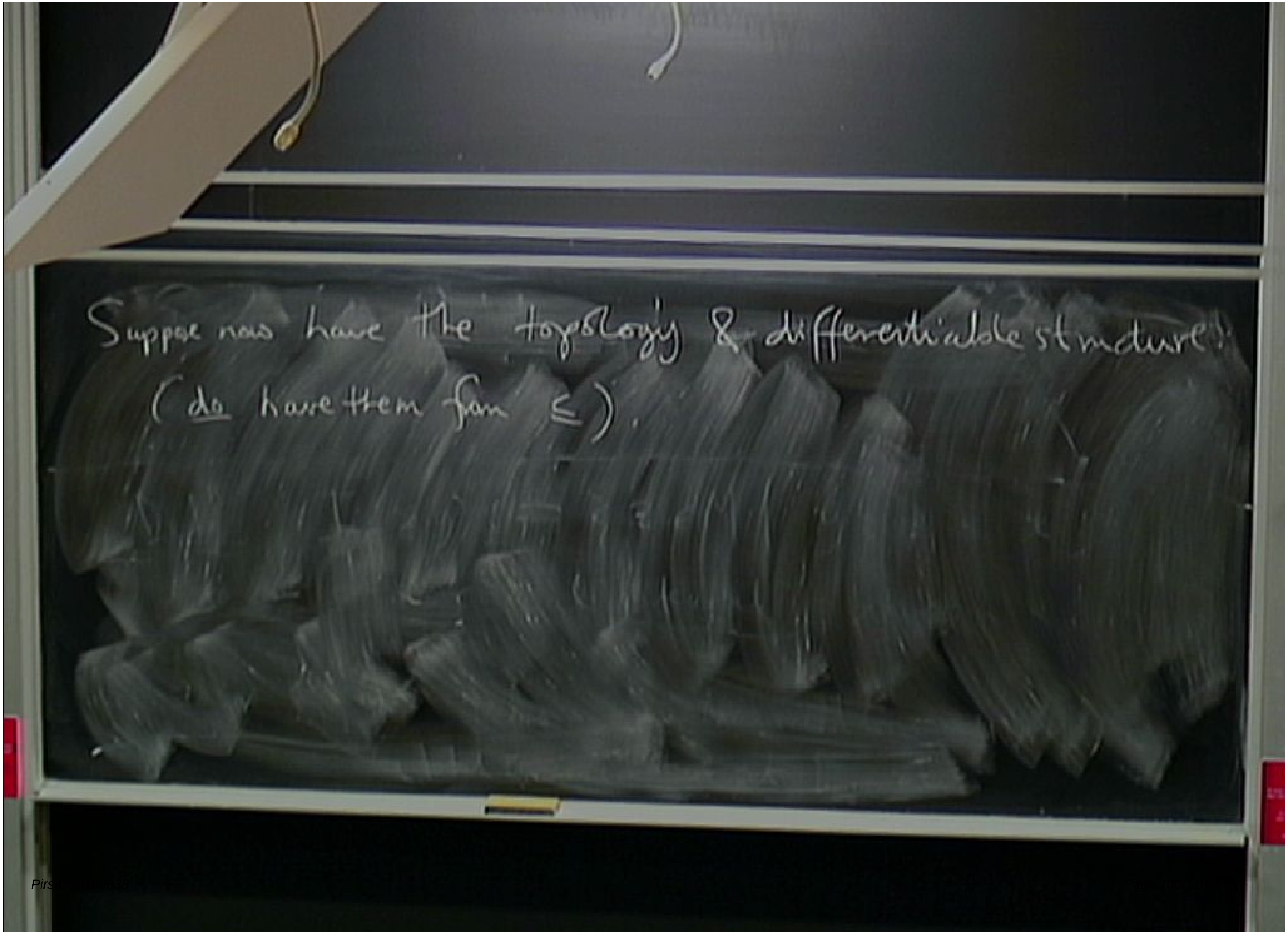
null geodesic generator) & all other tgts are spacelike)
γ' = γ ⇒ x ∈ γ
⇒ γ = I(q,p) & γ is a PNL


Suppose now have the topology & differentiable structure: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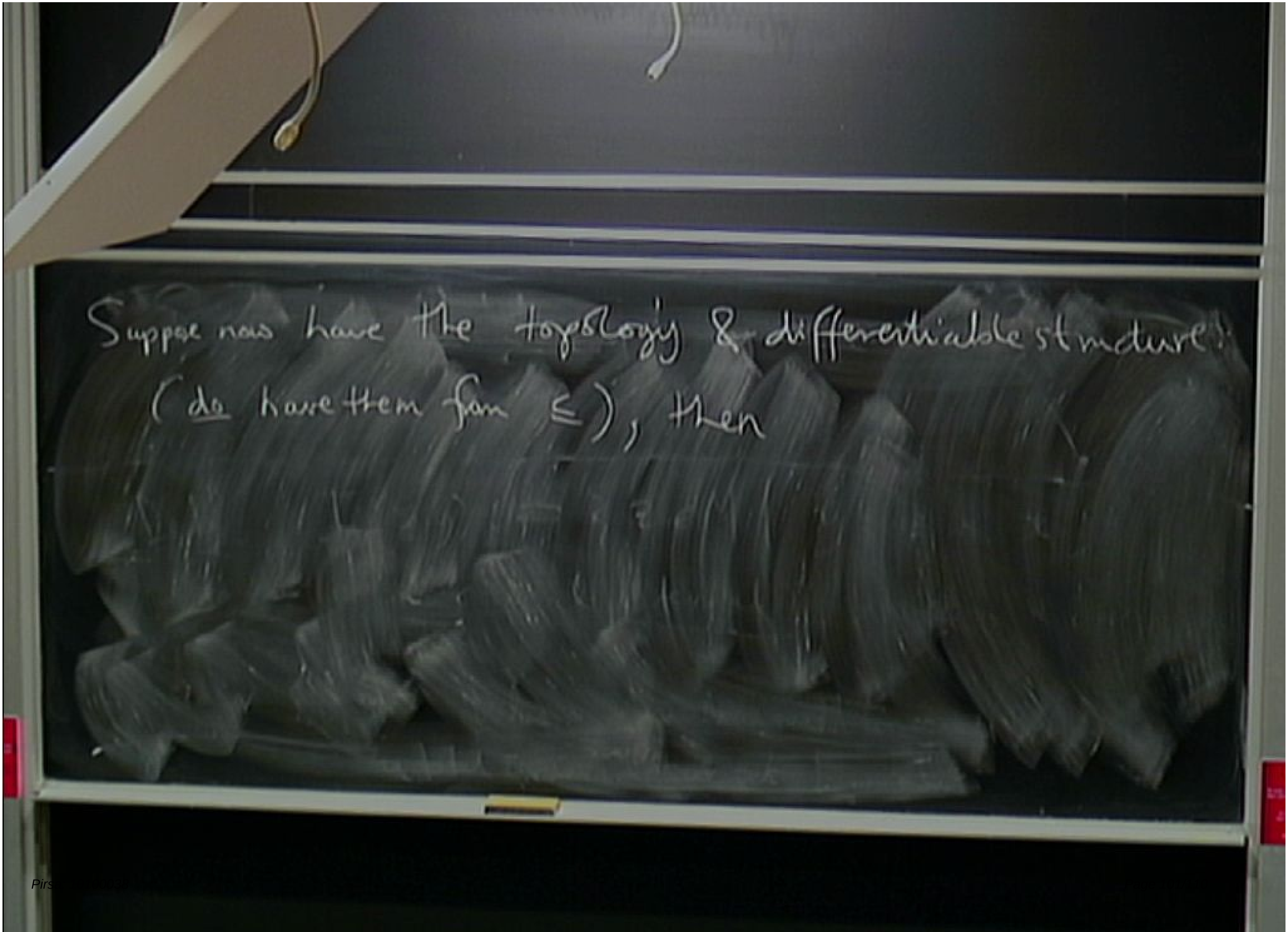
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Su[illegible] have the topology & differentiable structure
[illegible] have them from Σ), then we know $N \subset T_p$
the null cone — set of null vectors in T_p.

(... normal at $P \in L$)

Su... have the topology & differentiable structure (
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First, another look at the continuum

General Relativity (GR) tells us that spacetime is a 4-dimensional differentiable manifold, M, with a metric field, $g_{\mu\nu}$, with signature (- + + +)

- Metric gives the causal structure (we assume spacetimes are causal)
- Causal structure gives 9/10 of the metric (in 4-d)
- The remaining 1/10 is fixed by volume information

We return to the question of the discrete manifolds in the SOH. Some of them, at least, must be approximated by Lorentzian manifolds and they must be "non-ad hoc". Let's get a handle on things by *discretising*.

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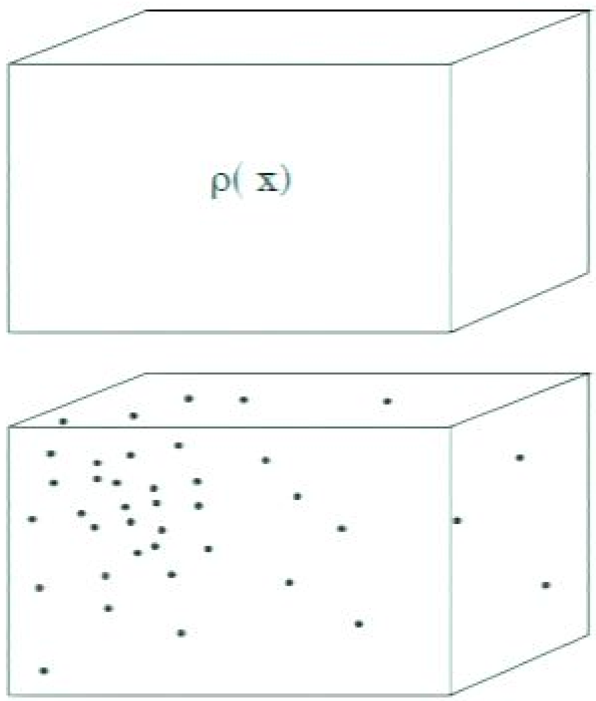
Discretising Spacetime

Let's warm up by considering a simple example: matter in a box in ordinary 3d space described by a continuum mass density, ρ



The atoms are distributed so that, roughly $N(\text{region}) = \int_{\text{region}} \rho(x)\, d^3x$

Now throw away ρ and see if it makes sense to propose the atomic state as fundamental: is it approximated by the (right) continuum density and if so, can it be given a dynamical life of its own ....

To discretise spacetime, let us do as we did for the substance-in-a-box: distribute discrete, identical "atoms of spacetime" throughout spacetime in such a way as respects the amount of spacetime in any region, in other words the spacetime volume.

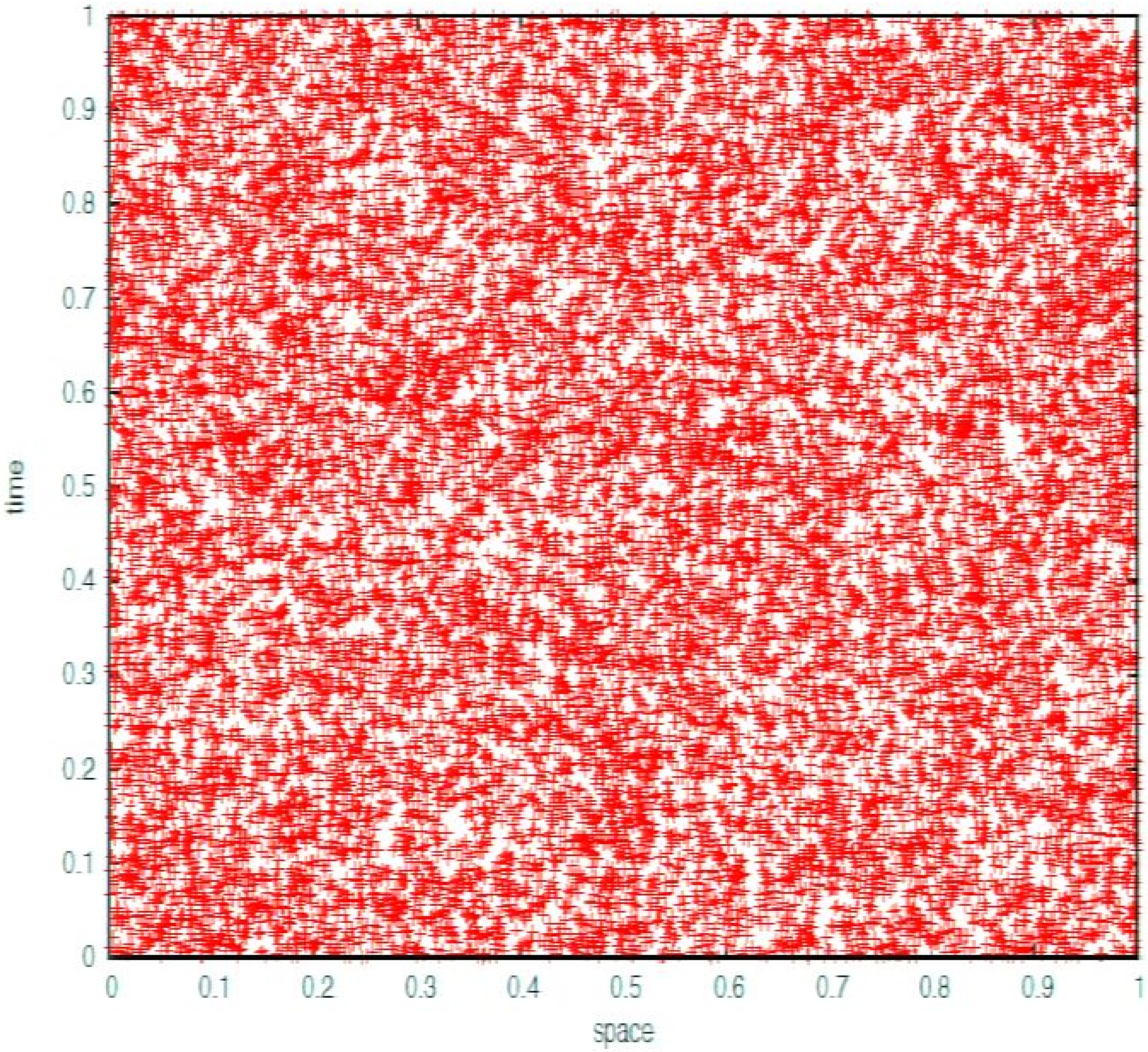
What plays the role of the mass density is $\sqrt{-g(x)}d^4x$ the spacetime volume element and integrating it over any region gives the spacetime volume of that region.

So we place the "atoms" (henceforth elements) in spacetime so that, roughly,

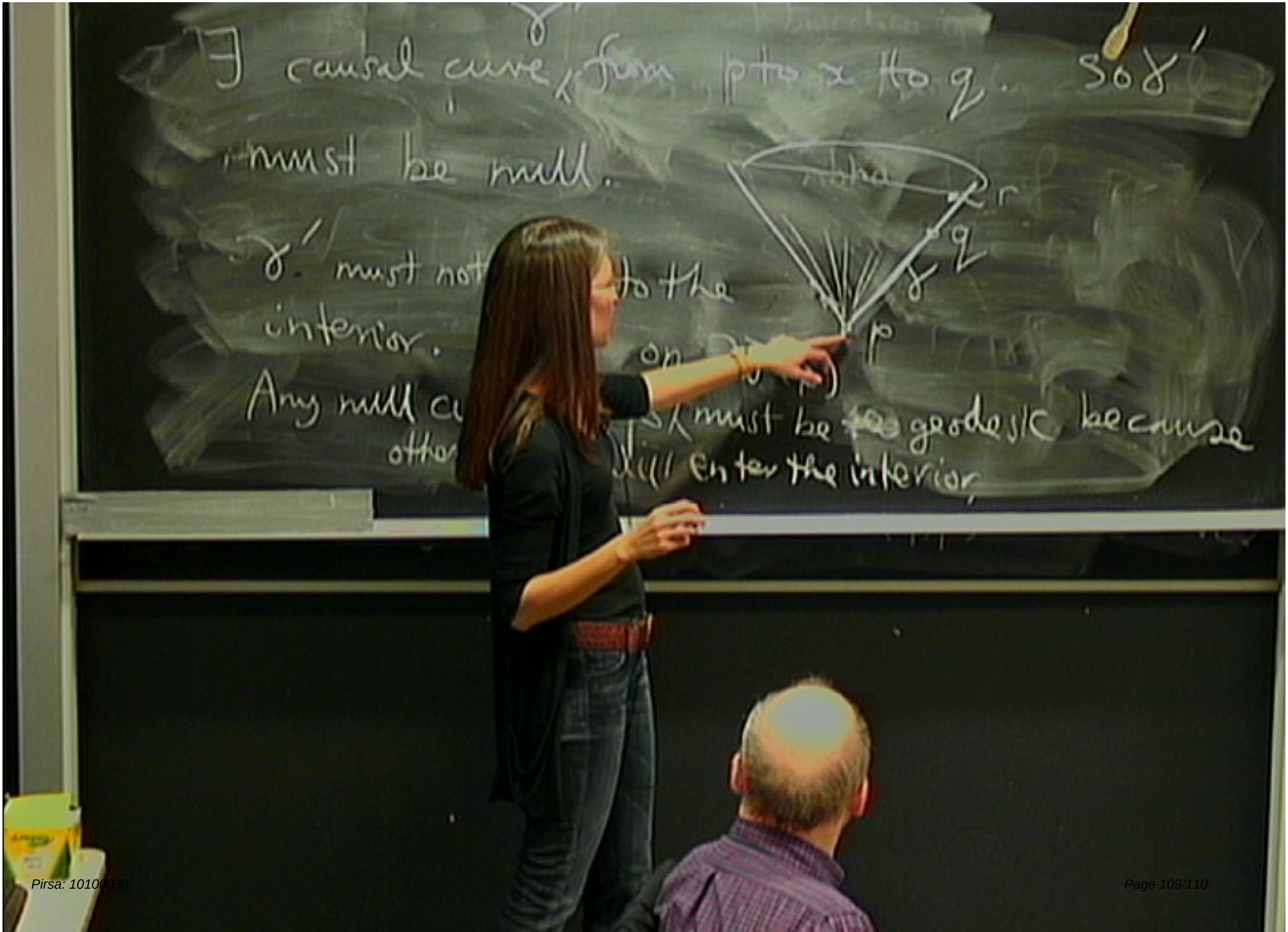
$$N(\text{region}) = \int_{\text{region}} \sqrt{-g(x)}\, d^4x$$

where the volume is measured in fundamental (near Planck) units.
We can satisfy this requirement by distributing the elements randomly by a Poisson process we refer to as "sprinkling".

A sprinkling into 1+1 Minkowski spacetime



This distribution is Lorentz invariant: it does not pick out a frame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r
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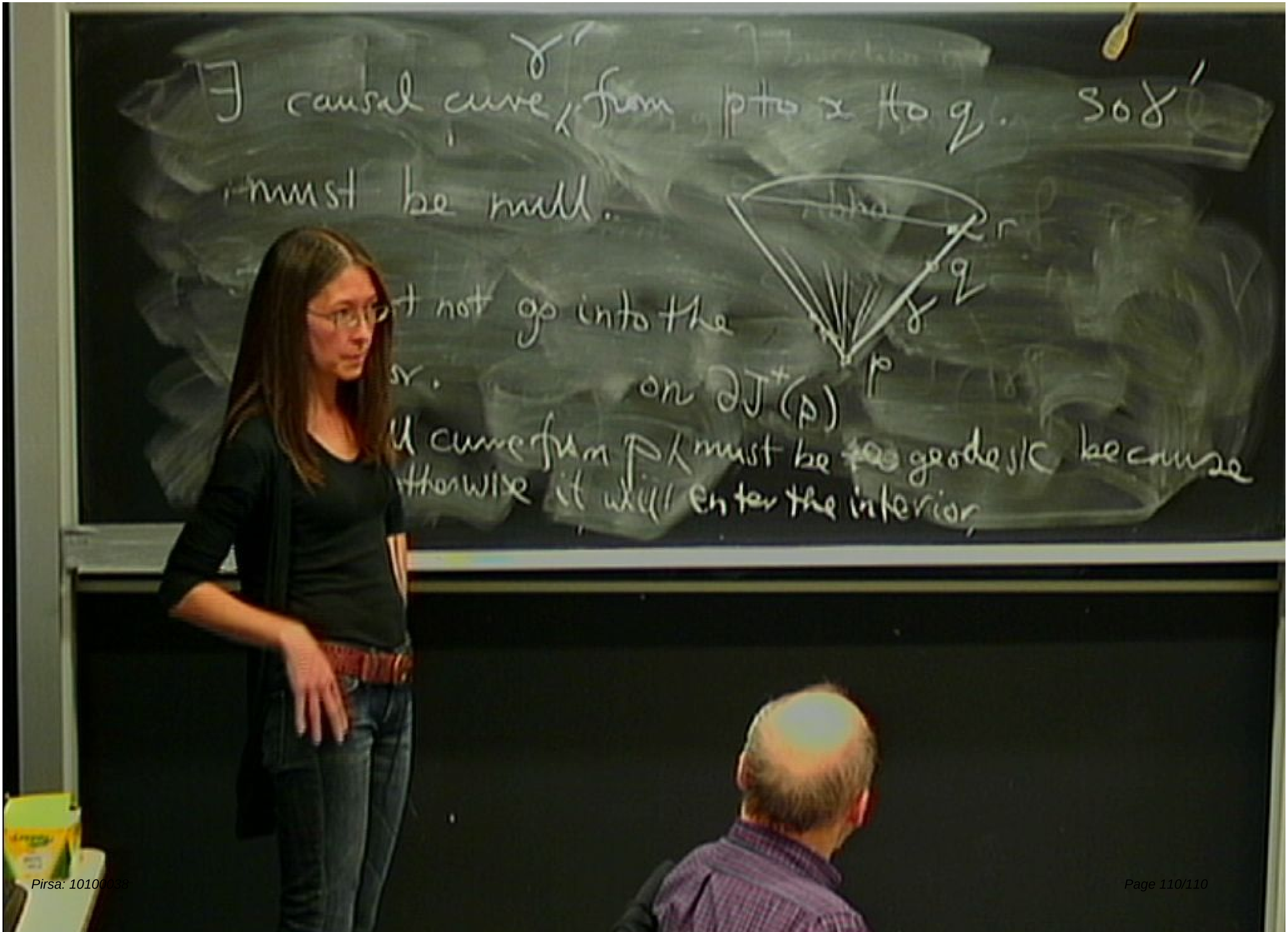
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