Title: A New Perspective on Quantum Field Theory

Date: Oct 20, 2010 04:00 PM

URL: http://pirsa.org/10100037

Abstract: The Exact Renormalization Group (ERG) is a technique which can be fruitfully applied to systems with local interactions that exhibit a large number of degrees of freedom per correlation length. In the first part of the talk I will give a very general overview of the ERG, focussing on its applications in quantum field theory (QFT) and critical phenomena. In the second part I will discuss how a particular extension of the formalism suggests a new understanding of correlation functions in QFTs, in general, and gauge theories in particular.

A New Perspective on Quantum Field Theory

arXiv:1003.1366 [hep-th]

Oliver J. Rosten

Sussex U.

October 2010

Outline of this Lecture

1. Qualitative Aspects of the ERG
2. Renormalizability
3. Correlation Functions in the ERG

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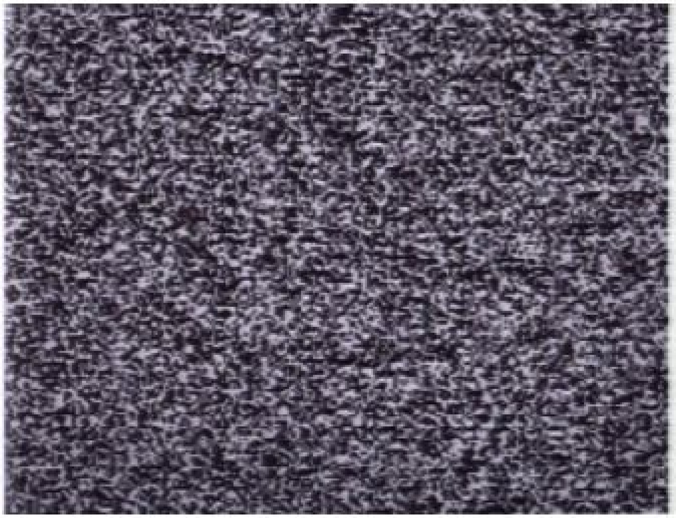

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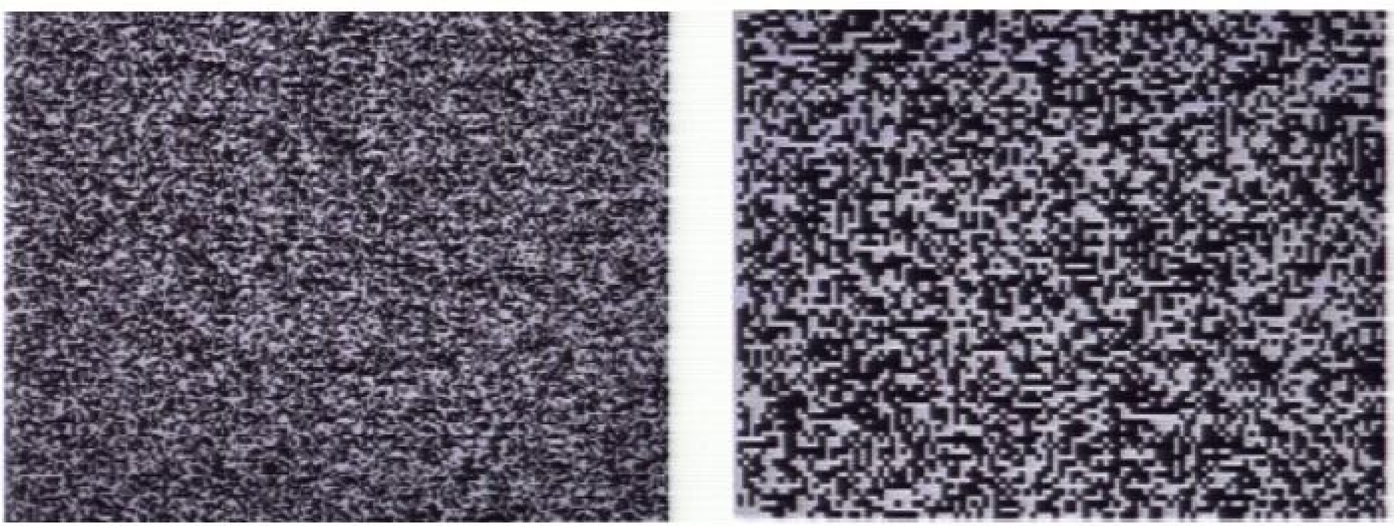

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The ERG is of use for systems with

- A large number of degrees of freedom per correlation length
- Local interactions

Easy Problems (no need for ERG)

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  - The subsystem is easy to understand
  - The subsystem captures the behaviour of the whole system
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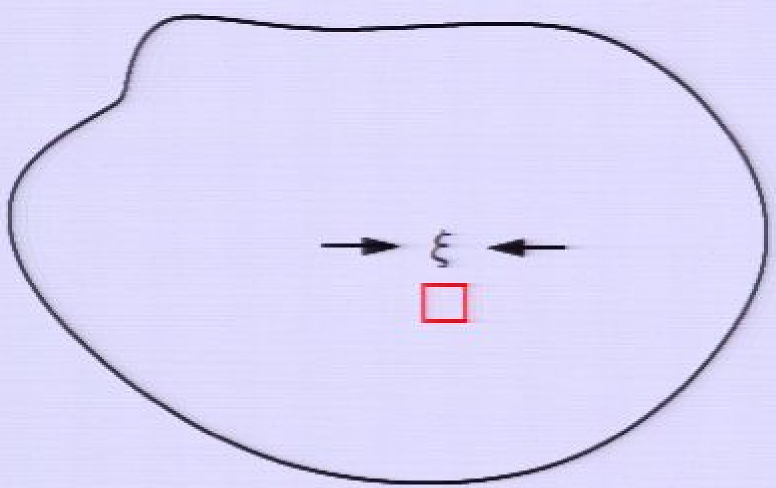

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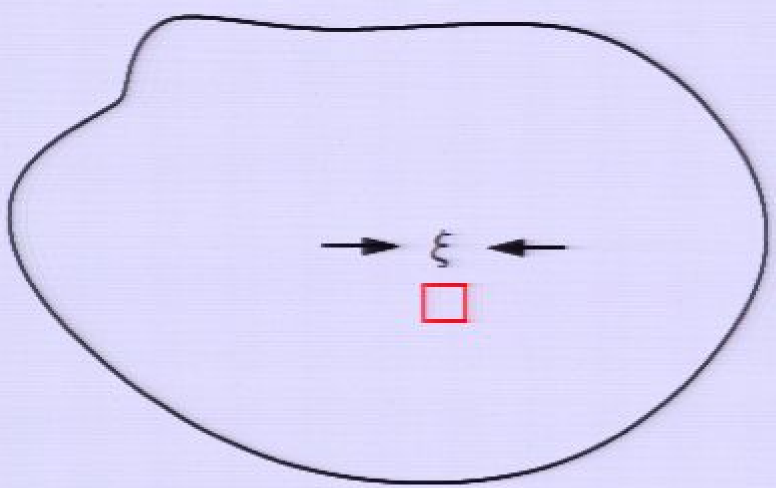

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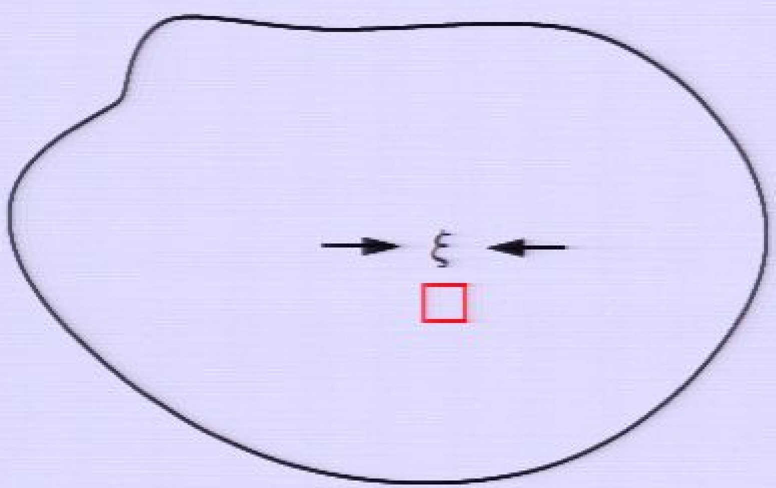

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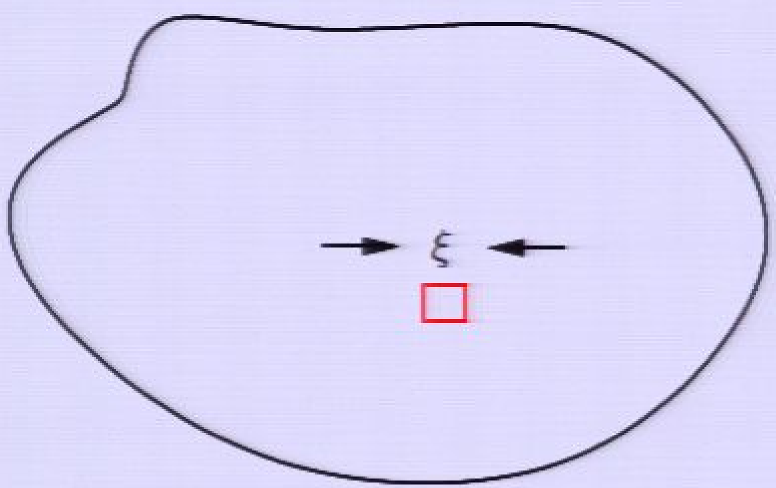

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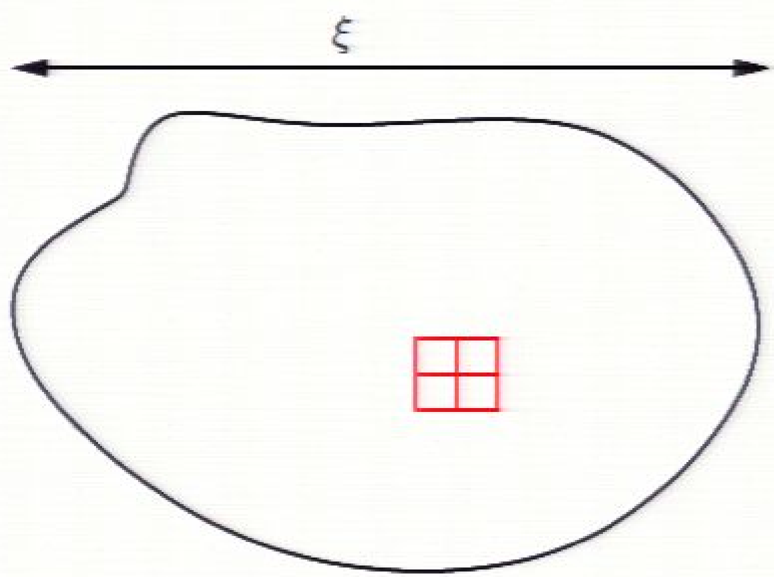

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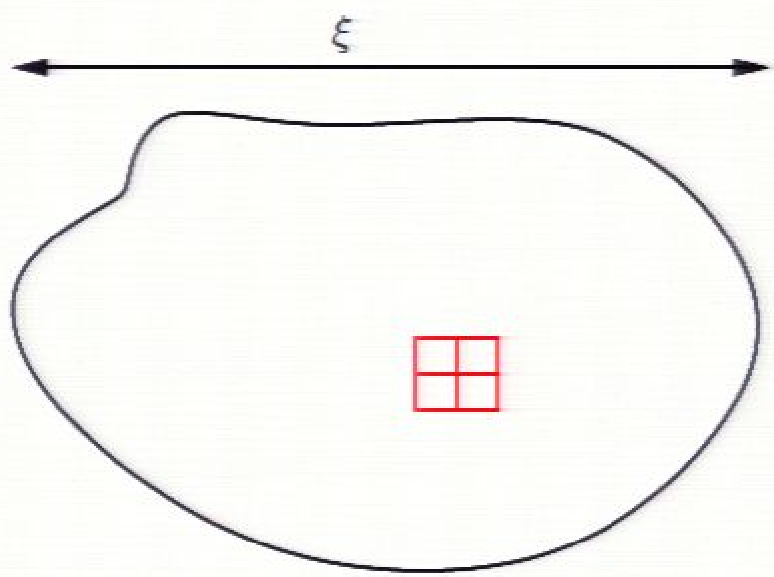

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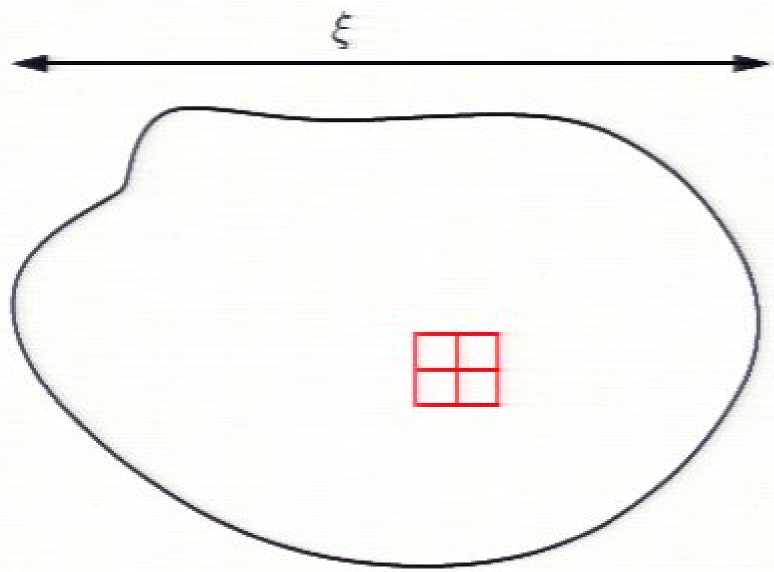

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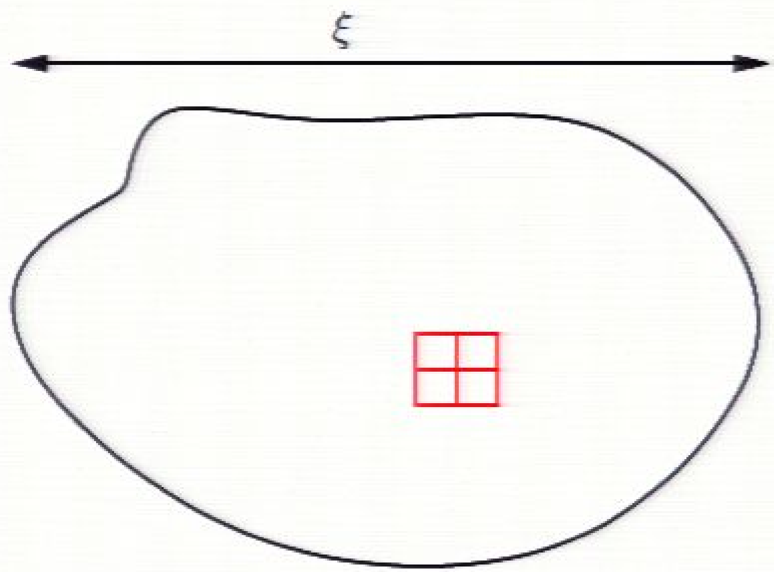

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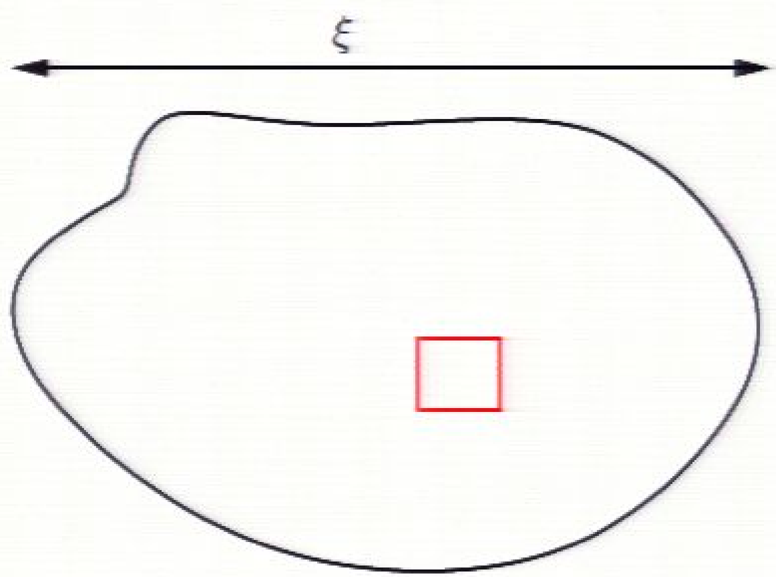

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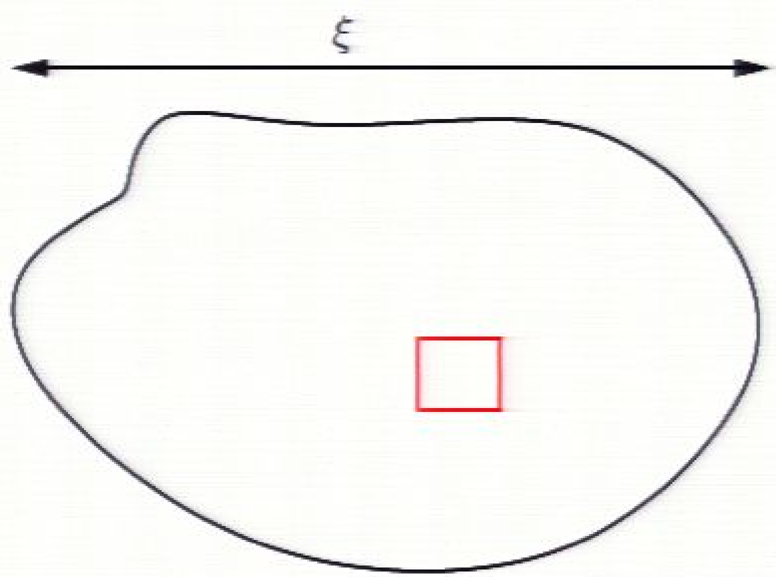

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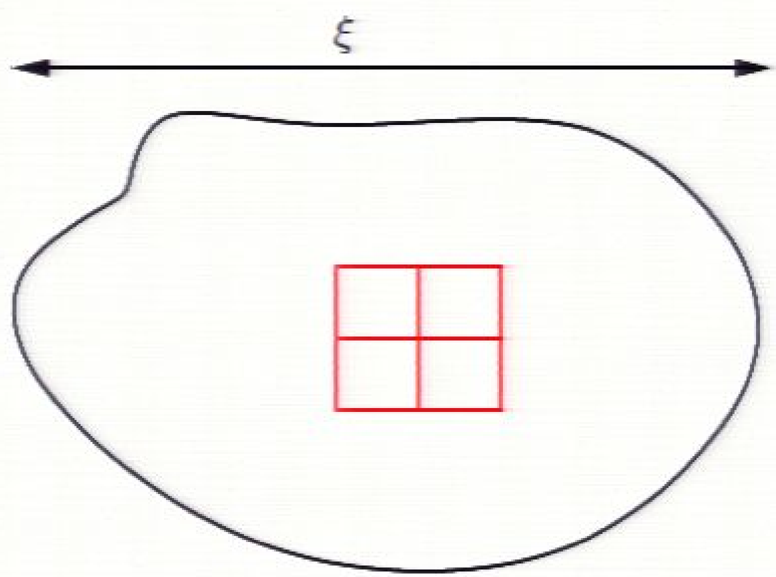

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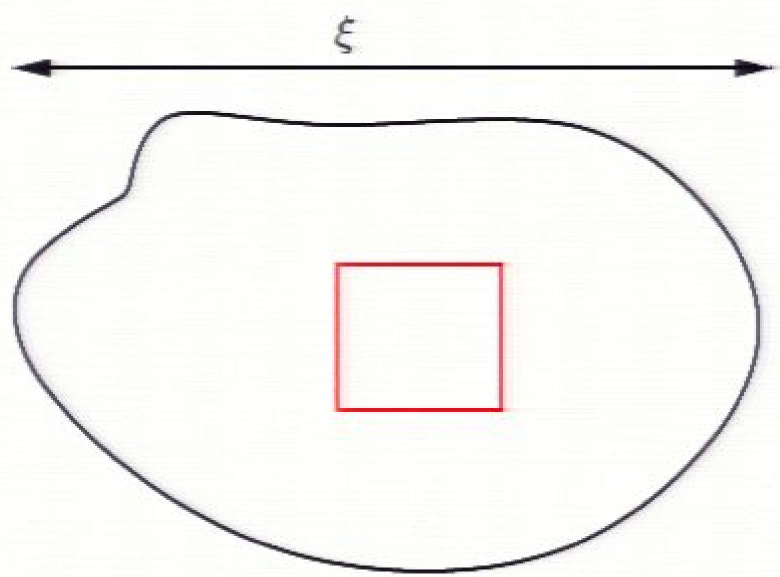

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Applications

- Quantum field theory
- Critical phenomena
- Kondo effect, ultra-cold gases, nuclear physics,. . .

What has the ERG given us?

- A deep understanding of renormalization and universality
- A tool for performing real calculations. . .

What's the catch?

- The coarse graining procedure cannot be done exactly

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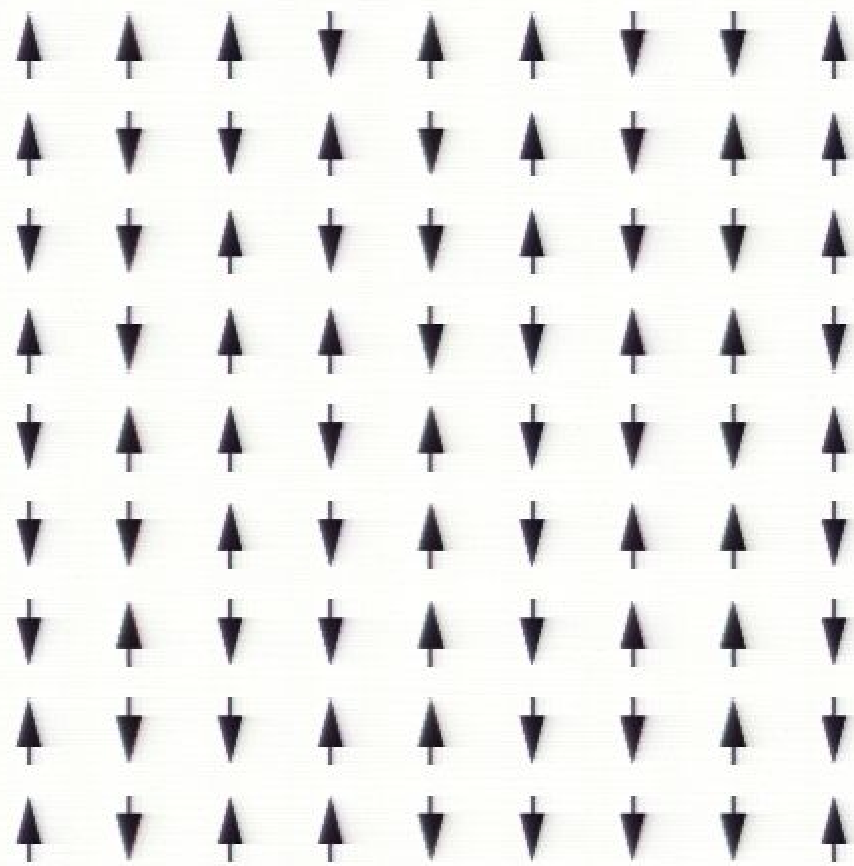

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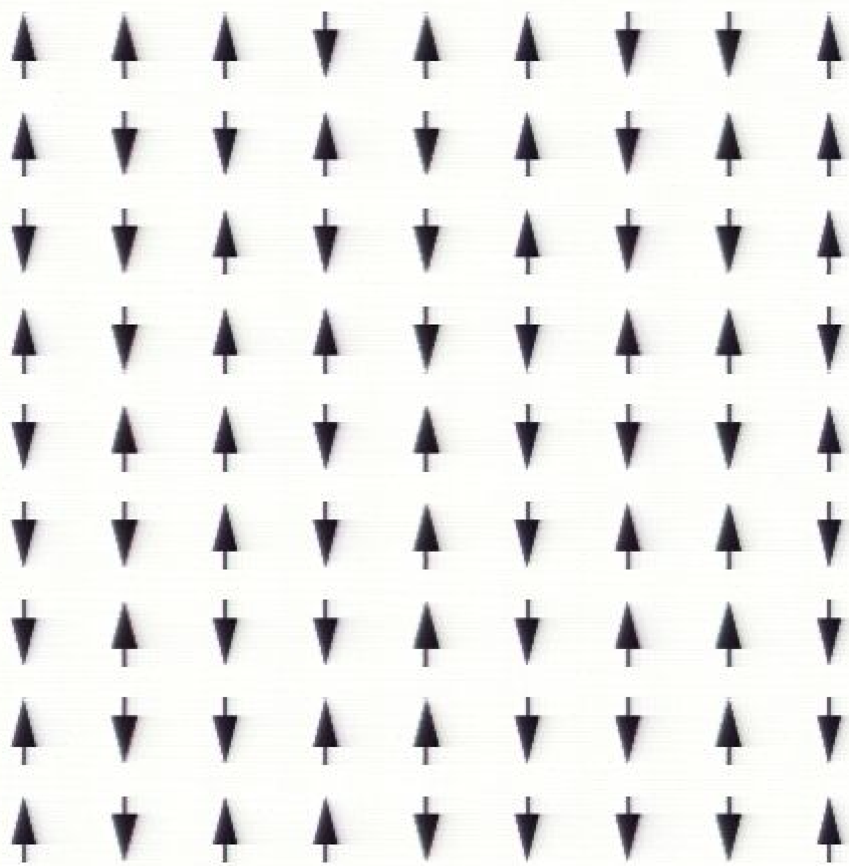

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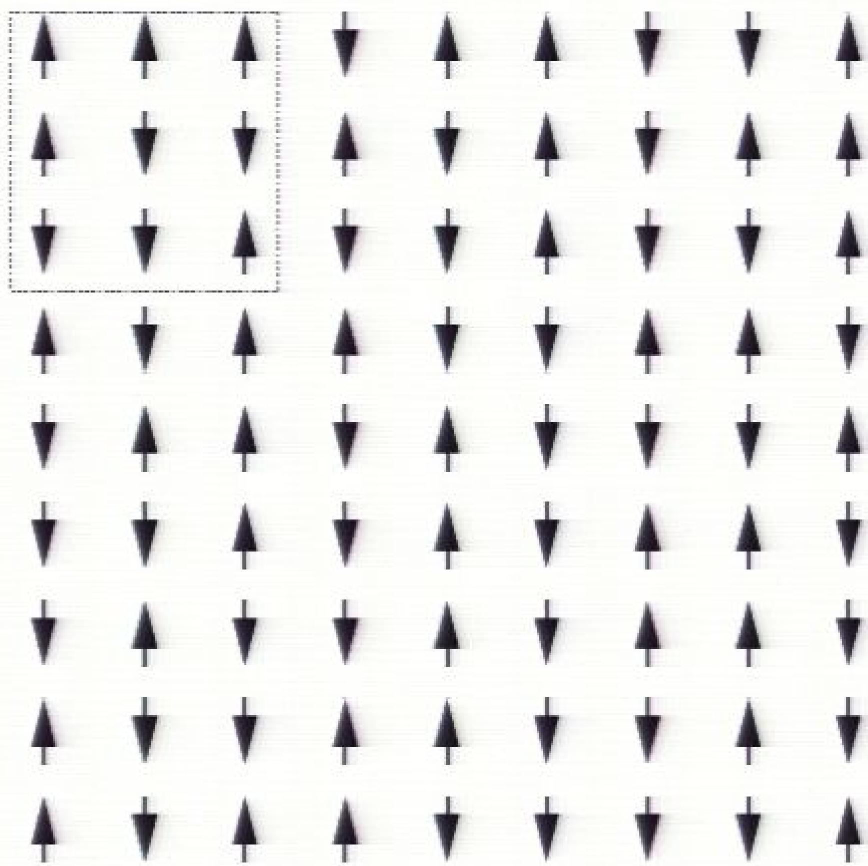

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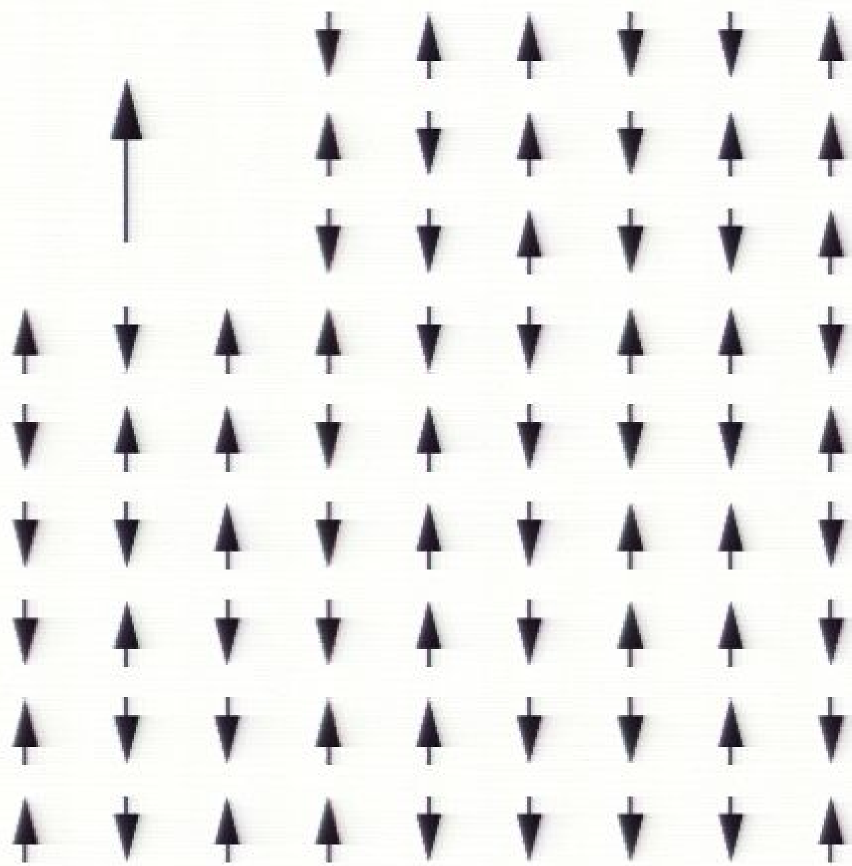

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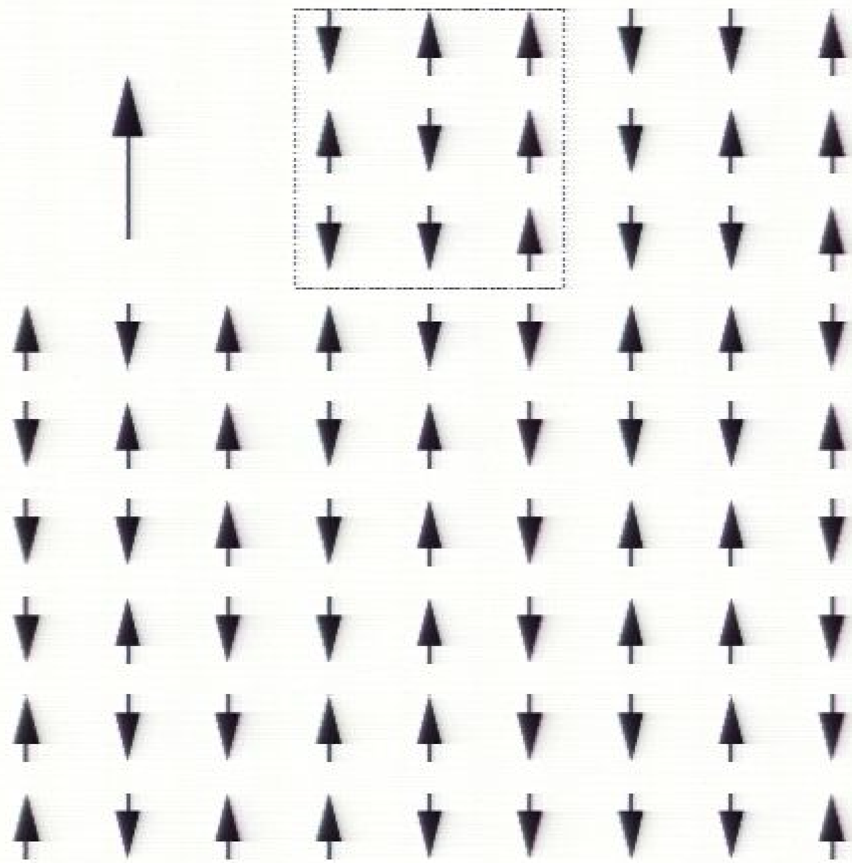

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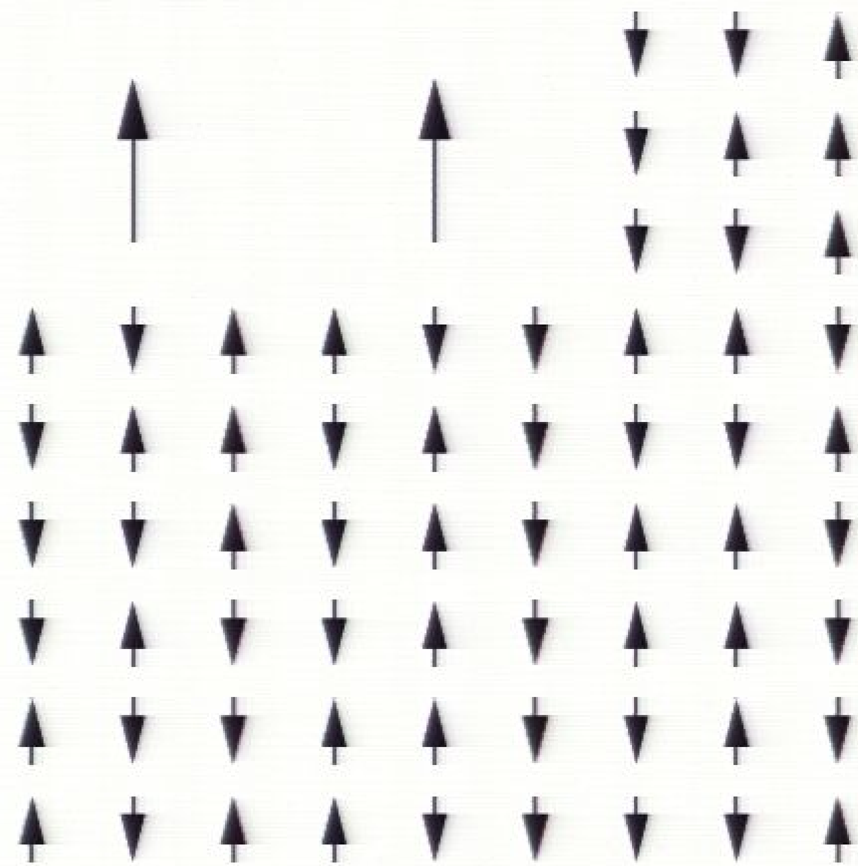

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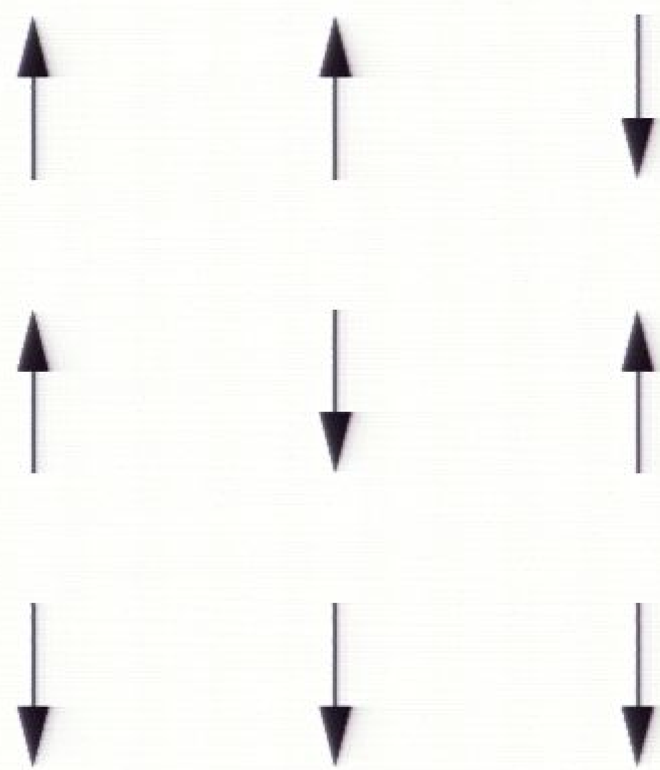

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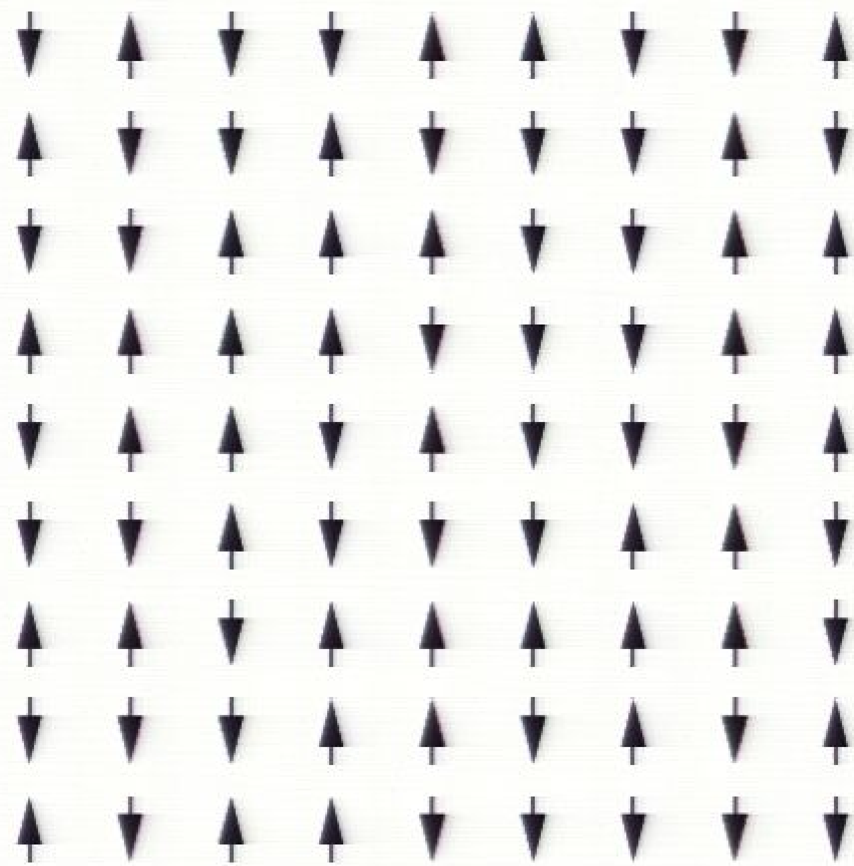

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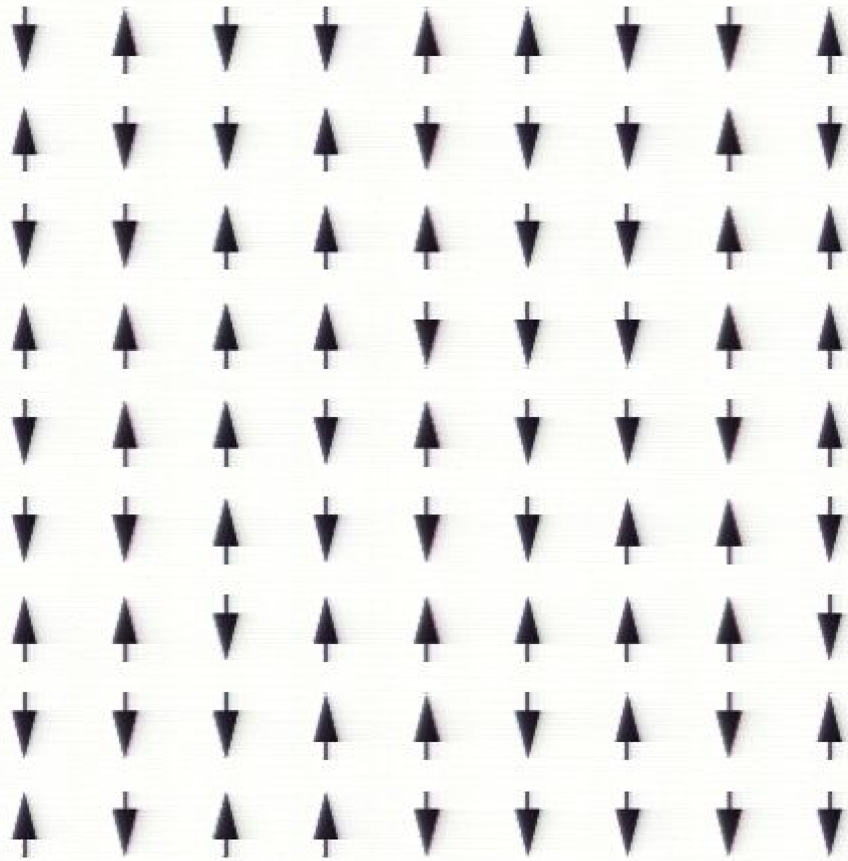

The ERG implements the continuous version of blocking

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- The transformation can have fixed-points

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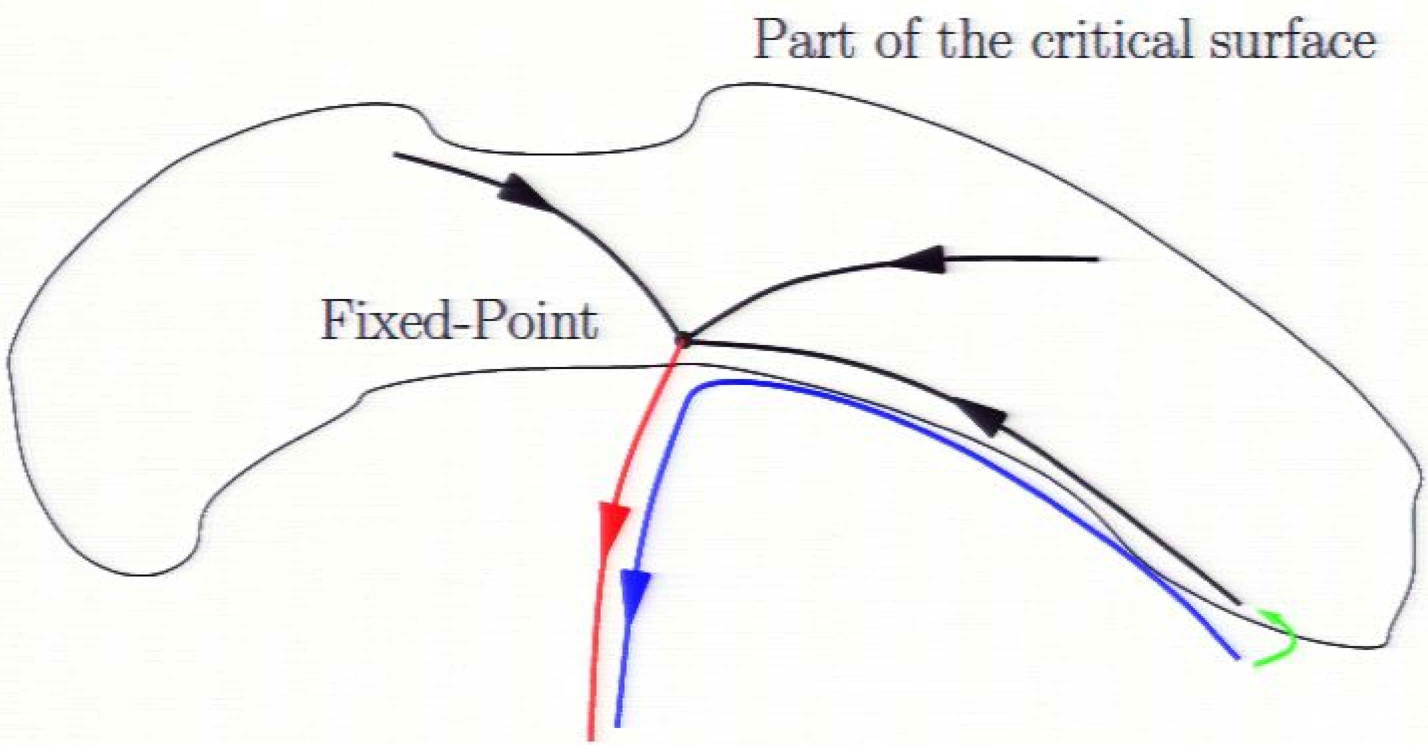

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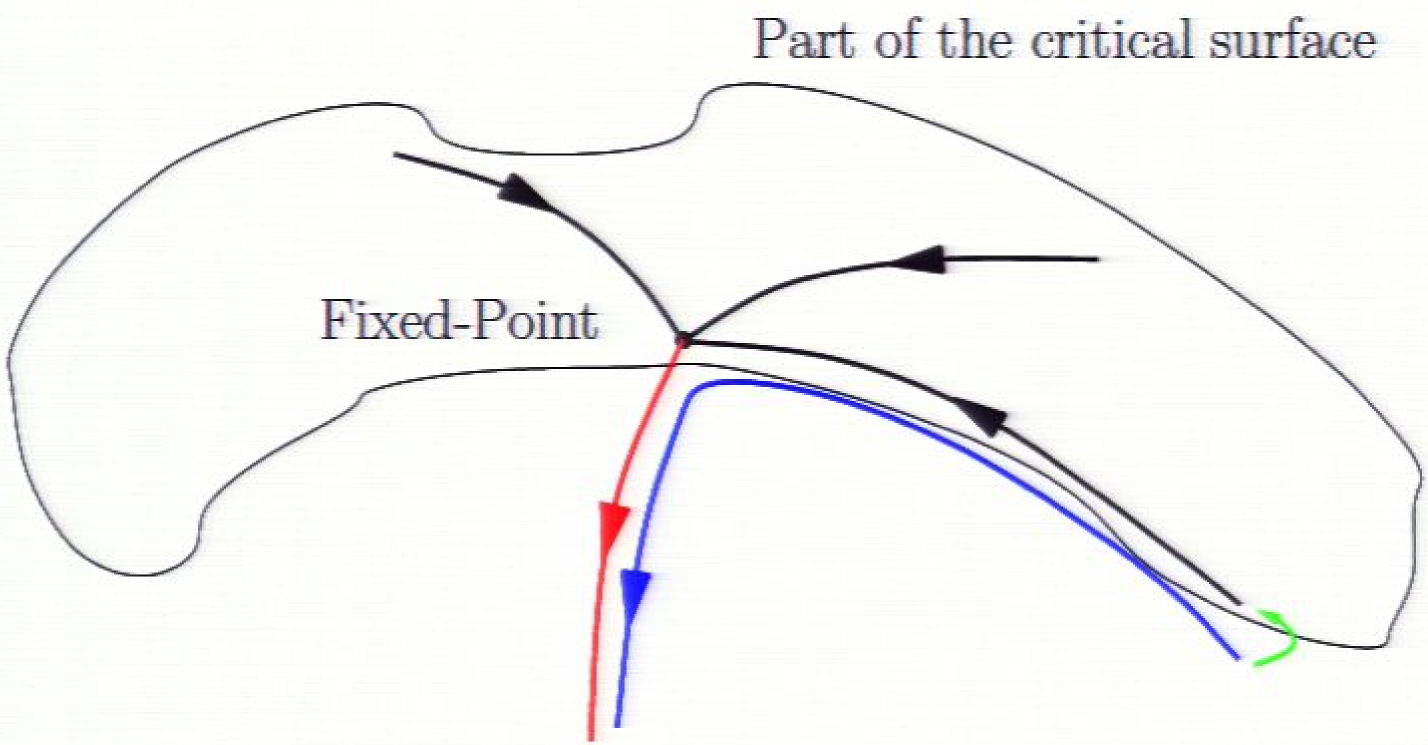

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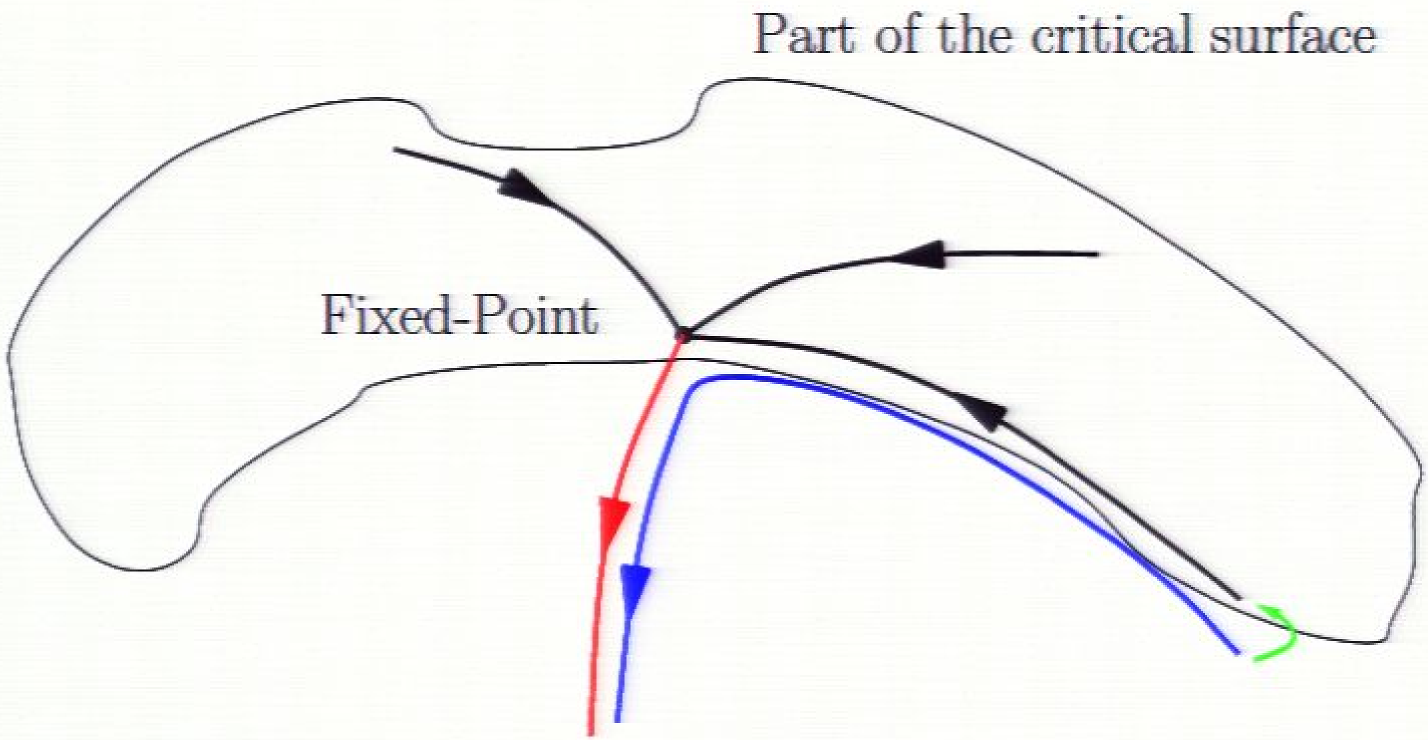

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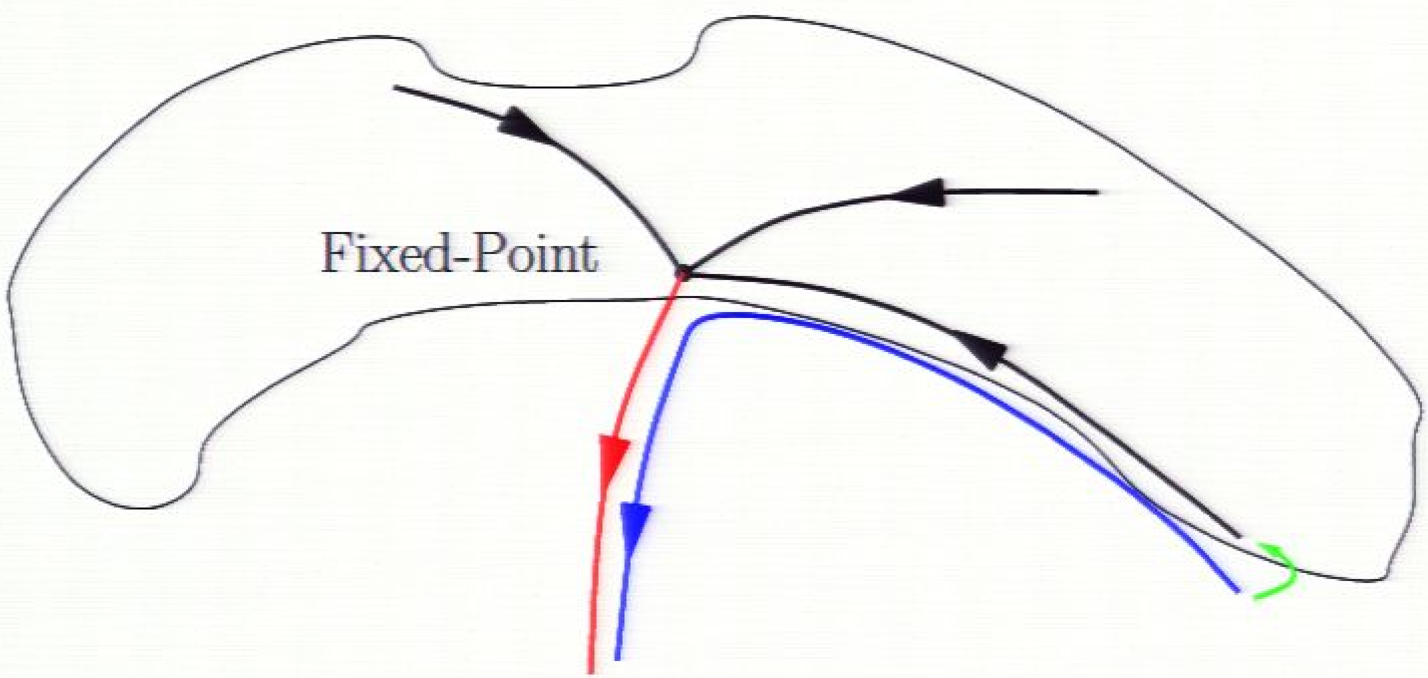

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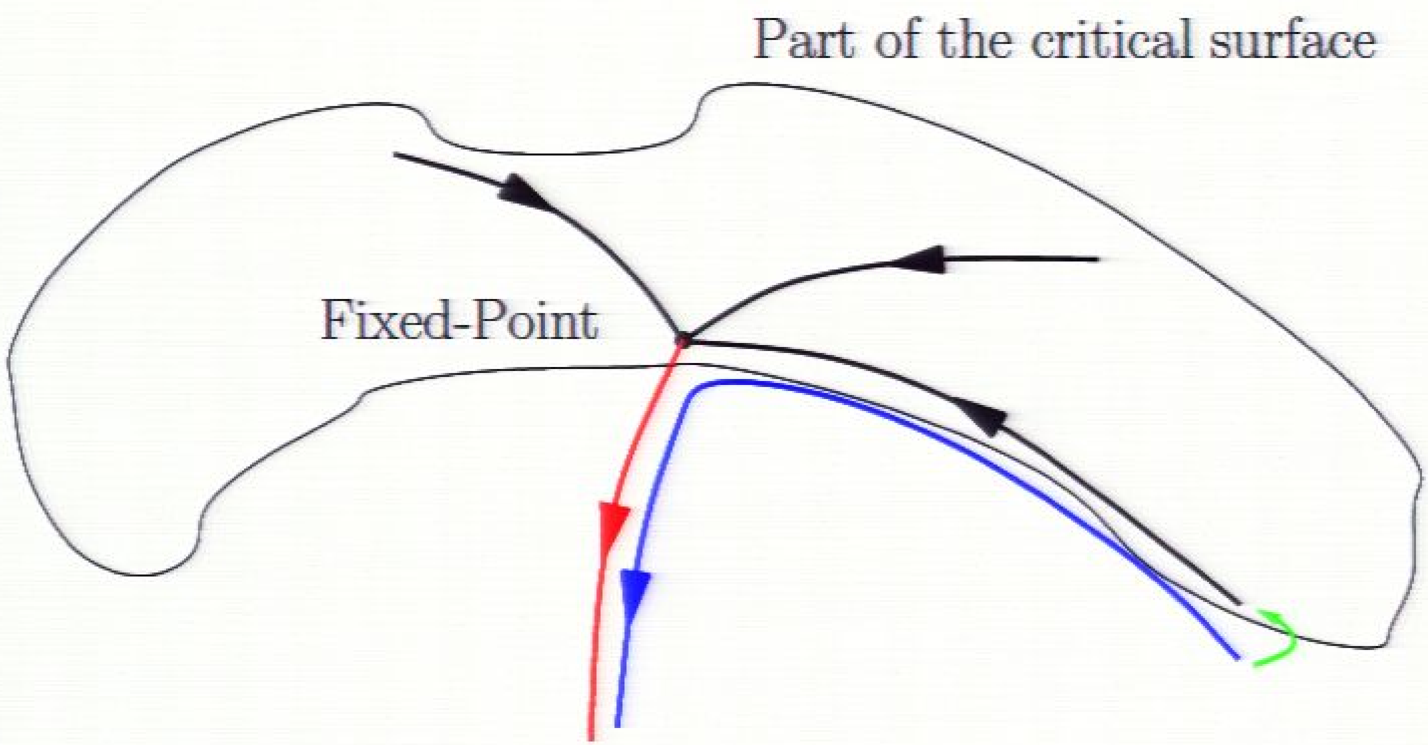

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- If there are *n* relevant directions, then we must tune *n* quantities to get on to the critical surface

The Wilsonian Effective Action

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$$Z = \int_{\Lambda_0} \mathcal{D}\phi e^{-S_{\Lambda_0}[\phi]} = \int_{\Lambda} \mathcal{D}\phi e^{-S_{\Lambda}[\phi]}$$

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- The bare (classical) action
- Integrate out modes between the bare scale and an intermediate scale, Λ

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  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
  - The action evolves $\Rightarrow$ Wilsonian effective action

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- Integrate out modes between the bare scale and an intermediate scale, Λ
  - The partition function stays the same
  - The effects of the high energy modes must be taken into account
  - The action evolves $\Rightarrow$ Wilsonian effective action

The Wilsonian Effective Action

Start with the partition function

$$Z = \int_{\Lambda_0} \mathcal{D}\phi e^{-S_{\Lambda_0}[\phi]} = \int_{\Lambda} \mathcal{D}\phi e^{-S_{\Lambda}[\phi]}$$

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Very General ERGs

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Formulation

$$-\Lambda\partial_\Lambda e^{-S[\phi]} = \int_x \frac{\delta}{\delta\phi(x)}\left(\Psi_x[\phi]e^{-S[\phi]}\right)$$

- effective scale
- set of fields
- Wilsonian effective action
- partition function, $\int \mathcal{D}\phi e^{-S[\phi]}$, invariant under the flow
- Parametrizes blocking procedure
  - huge freedom in precise form—adapt to suit our needs
  - corresponds to a field redefinition

Flow Equation

$$-\Lambda\partial_\Lambda S = \int_x \frac{\delta S}{\delta\phi(x)}\Psi_x - \int_x \frac{\delta\Psi_x}{\delta\phi(x)}$$

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Ingredients of ERG Transformation

- Blocking (coarse-graining)
- Rescaling

Implementing Rescaling

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- Remember to take account of anomalous dimensions!

$$X \to X\Lambda^{\text{full scaling dimension}}$$

- Notation: ϕ dimensionful, φ dimensionless
- $-\Lambda\partial_\Lambda \to \partial_t$, with $t = \ln \mu/\Lambda$

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Relevance/Irrelevance

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- At a fixed-point we have $\partial_t S_\star = 0$
- Consider an infinitesimal perturbation

First order classification

- Operators that grow with t are relevant
- Operators that shrink with t are irrelevant
- Operators that stay the same are marginal

Marginal Operators

- $S_\star + \epsilon \mathcal{O}_{\mathrm{marginal}}$ is a fixed-point up to $\mathcal{O}(\epsilon^2)$
- This might not be true beyond leading order
- Eg the four point coupling in $D = 4$ scalar field theory is marginally irrelevant
- An exactly marginal operator generates a line of fixed-points

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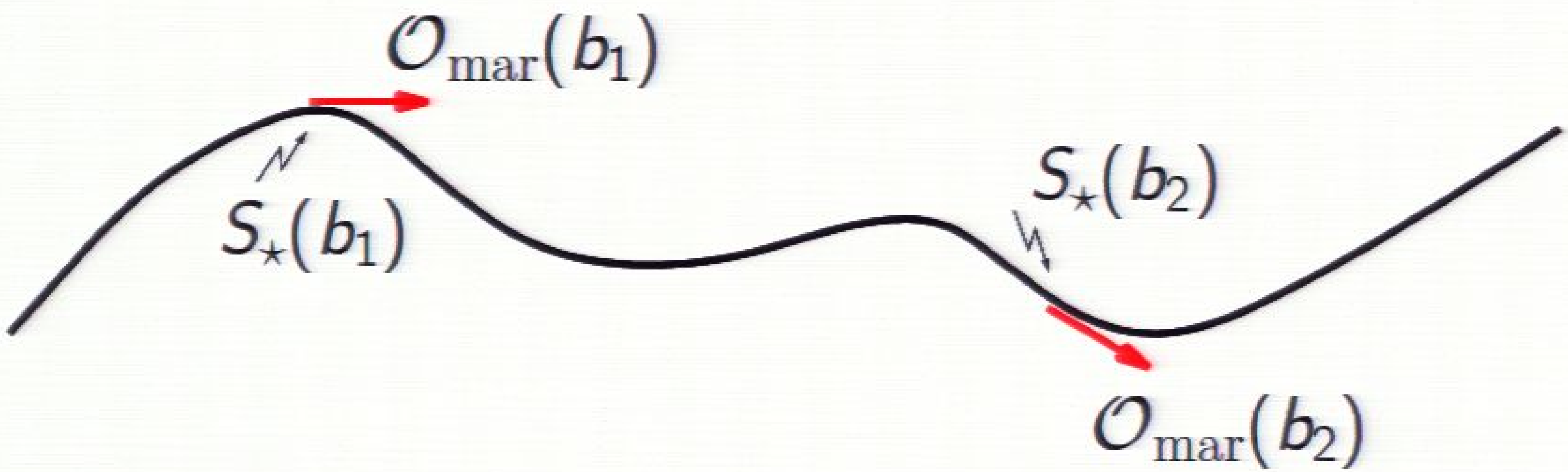

1. Qualitative Aspects of the ERG

2. Renormalizability

3. Correlation Functions in the ERG

Textbook renormalization

Textbook renormalization

- Choose an action *e.g.*

$$S[\phi] = \int d^D x \left[\frac{1}{2}\partial_\mu\phi\partial_\mu\phi + \frac{1}{2}m^2\phi^2 + \frac{\lambda}{4!}\phi^4\right]$$

- Choose a UV regulator
- Start computing the correlation functions

$$\langle\phi(x_1)\cdots\phi(x_n)\rangle = \frac{1}{\mathcal{Z}}\int \mathcal{D}\phi\,\phi(x_1)\cdots\phi(x_n)e^{-S[\phi]}$$

- Adjust the action to absorb UV divergences:

$$S[\phi] \to S[\phi] + \delta S[\phi]$$

- If δS has the same form as S, the theory is renormalizable

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Wilsonian I

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The Question

Are there effective actions $S_{\Lambda,\Lambda_0}[\phi]$ for which we can safely send $\Lambda_0 \to \infty$?

The Simplest Answer

- Rescale all quantities, using Λ
- Only dimensionless variables appear
- Fixed-points of the ERG correspond to continuum limits!
- S_* is independent of all scales, including Λ_0
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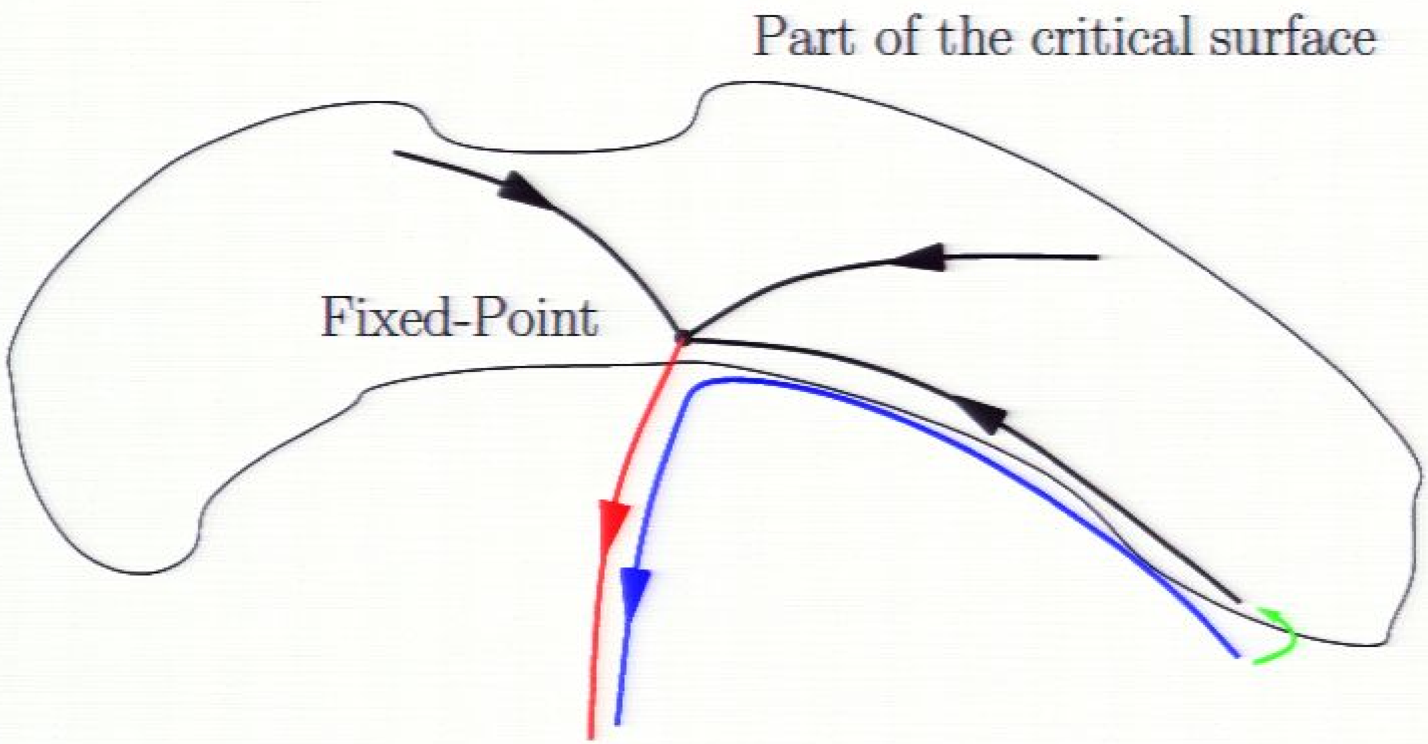

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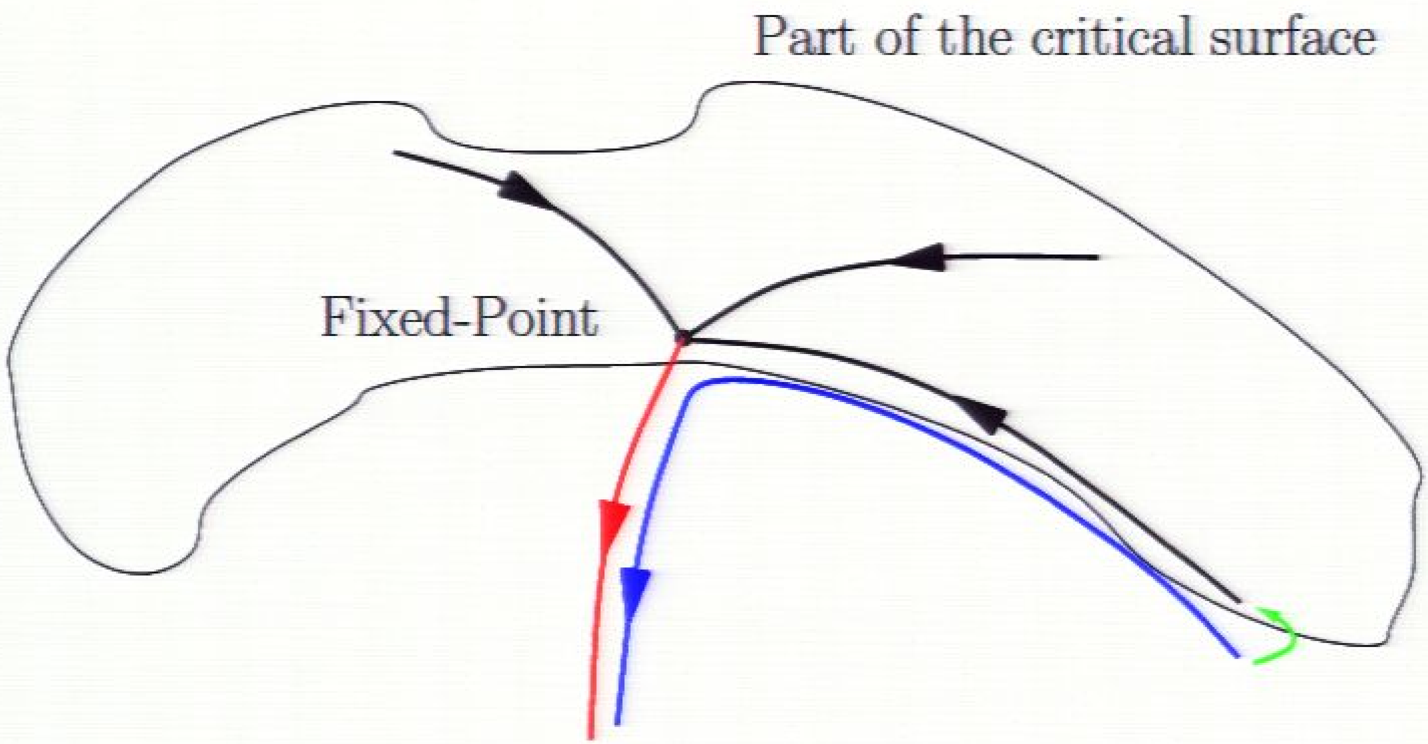

- Tune the trajectory towards the critical surface, as $\Lambda_0 \to \infty$

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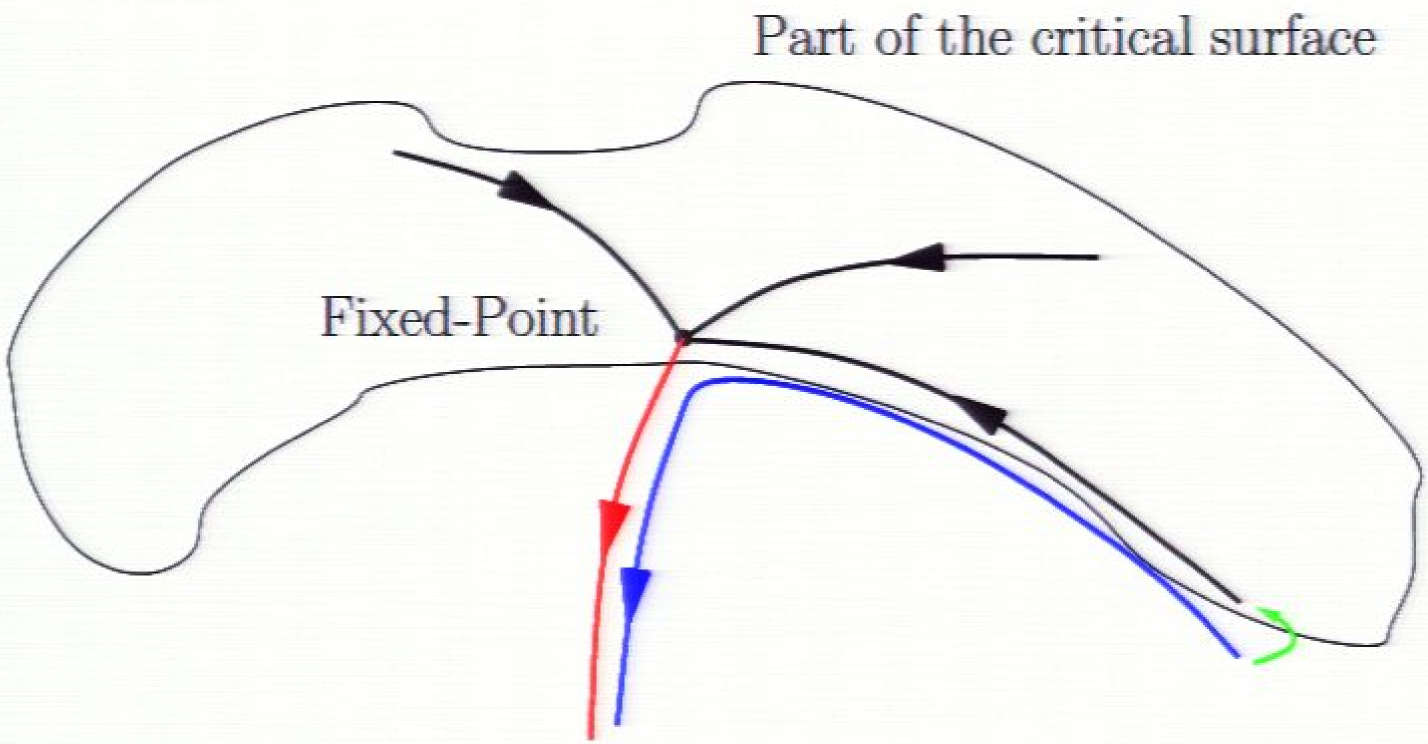

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- The trajectory splits in two:

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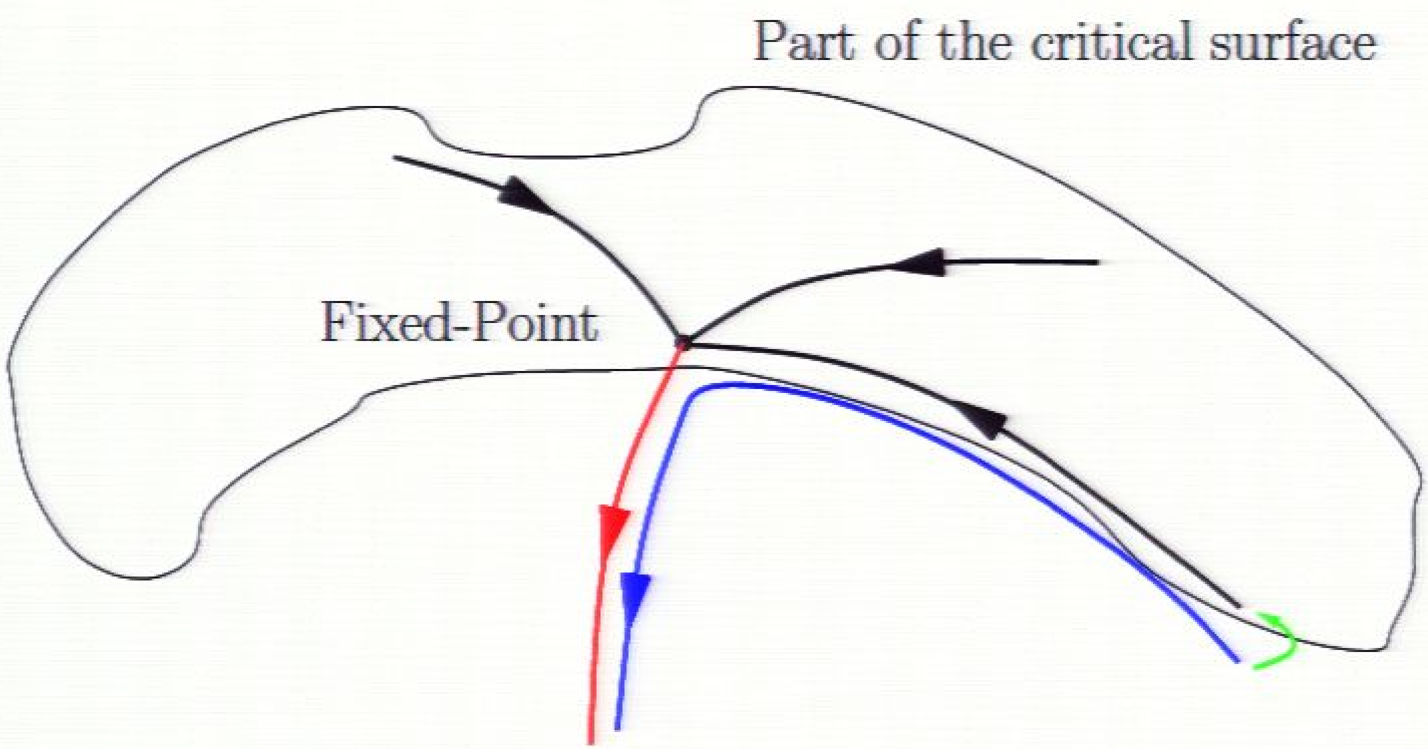

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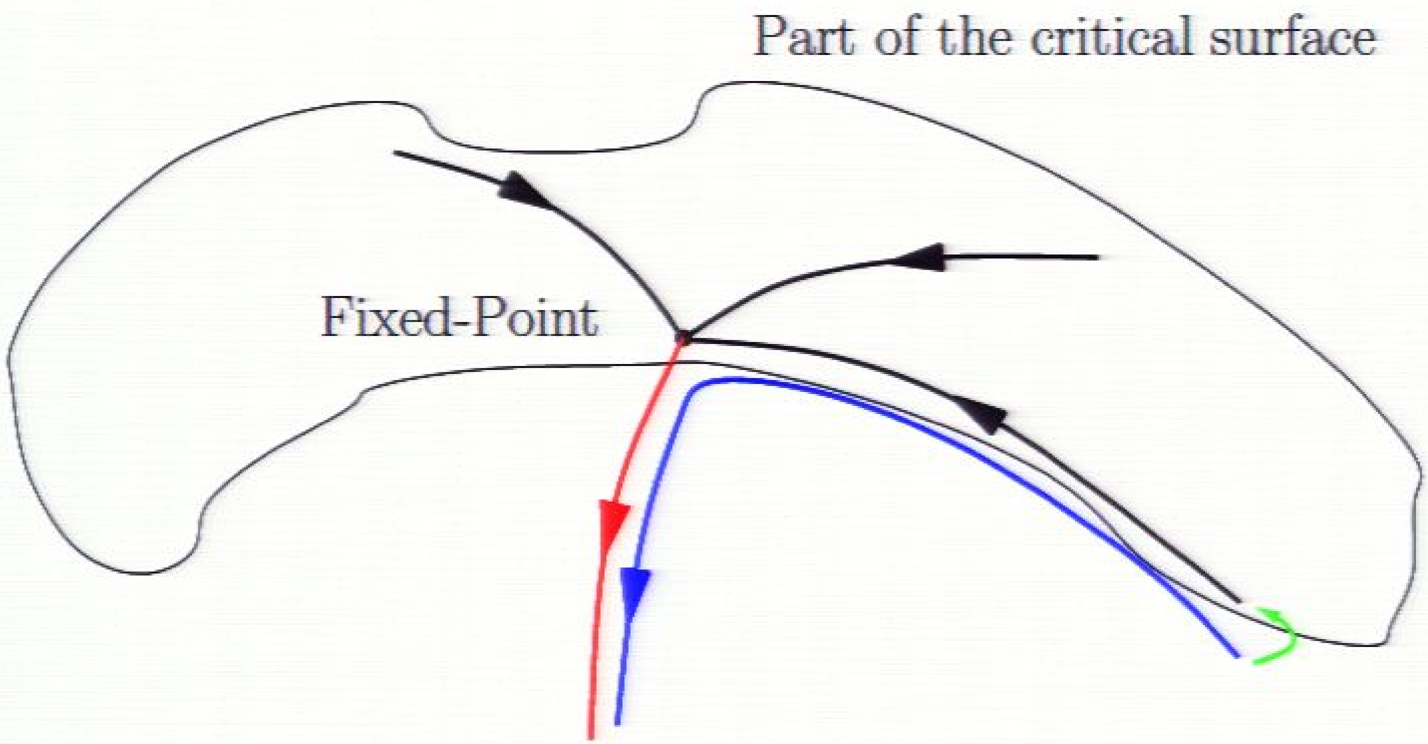

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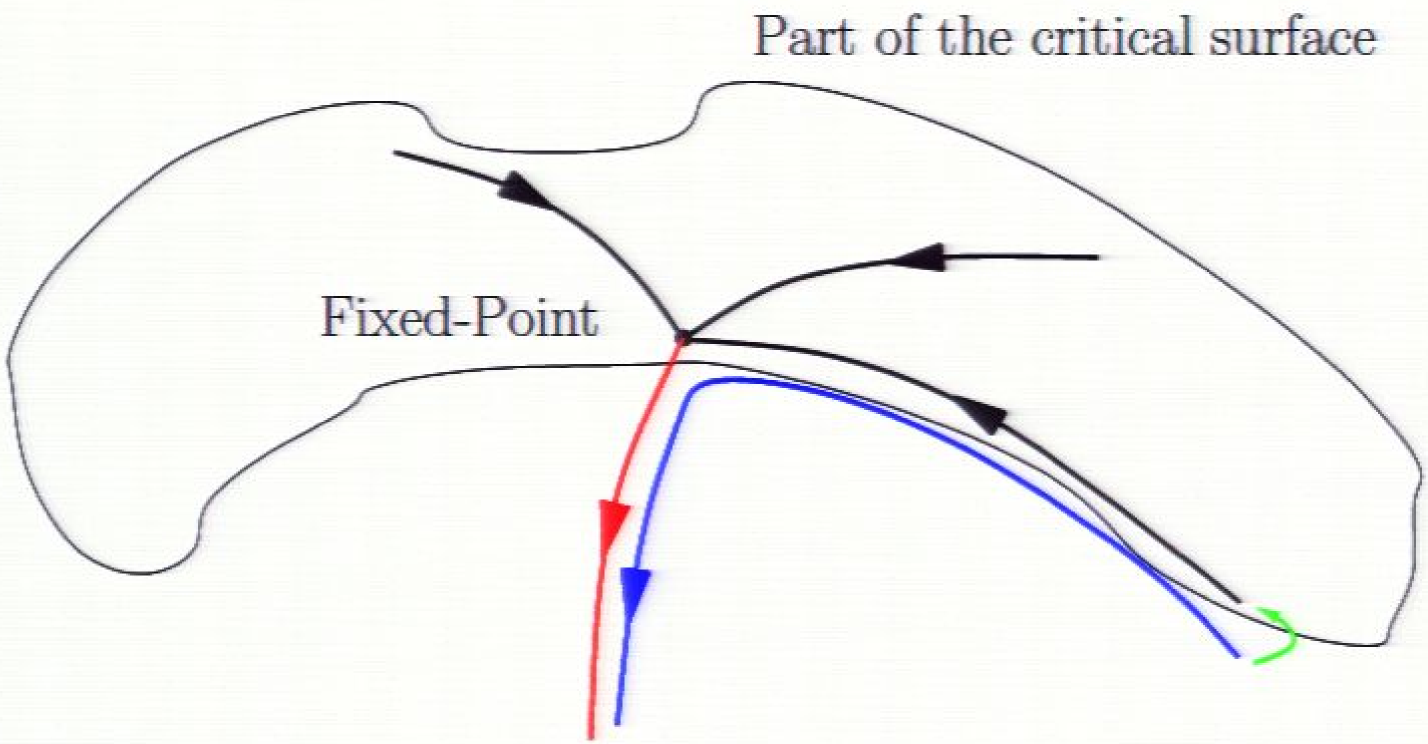

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- Actions on the RT are renormalizable

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Asymptotic Freedom etc.

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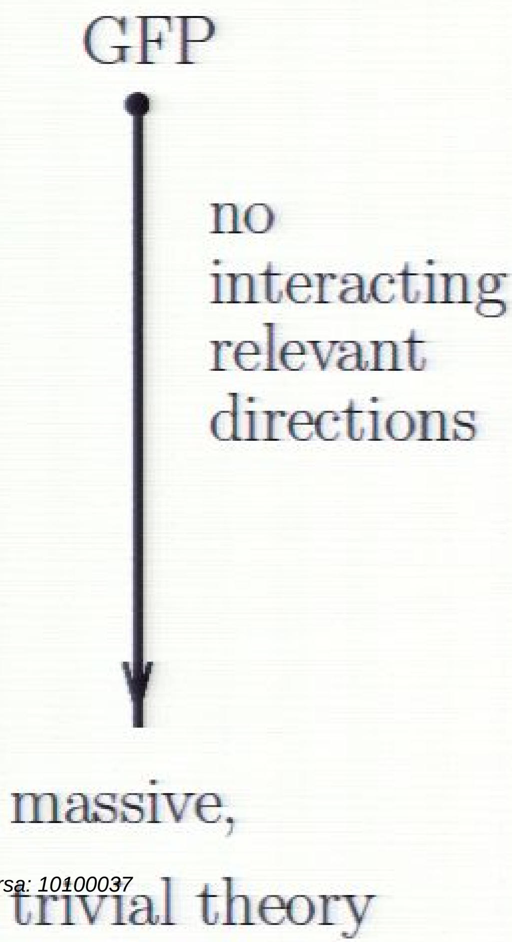
Triviality   Asymptotic Freedom   Asymptotic Safety

Asymptotic Freedom etc.

Triviality | Asymptotic Freedom | Asymptotic Safety



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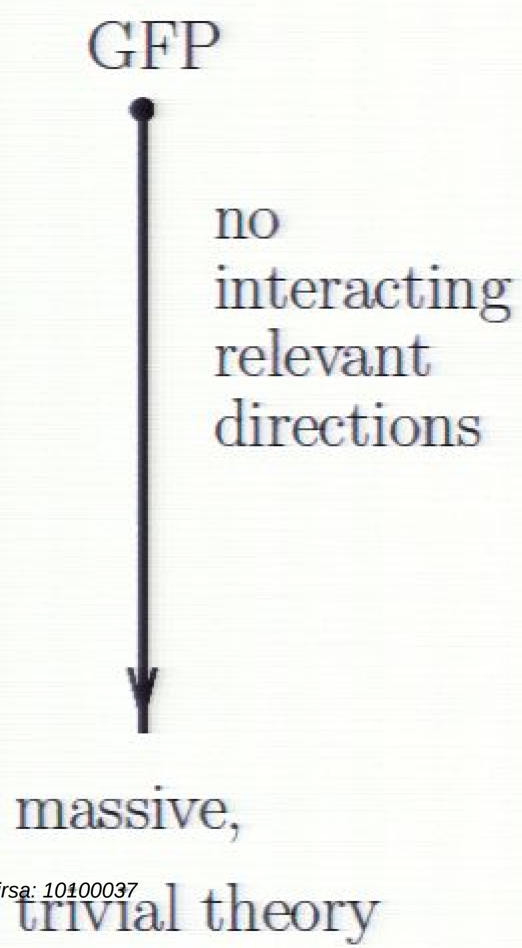

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Triviality

GFP

no interacting relevant directions

massive, trivial theory

Asymptotic Freedom

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interacting relevant directions

interacting, renormalizable theory

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Triviality	Asymptotic Freedom	Asymptotic Safety
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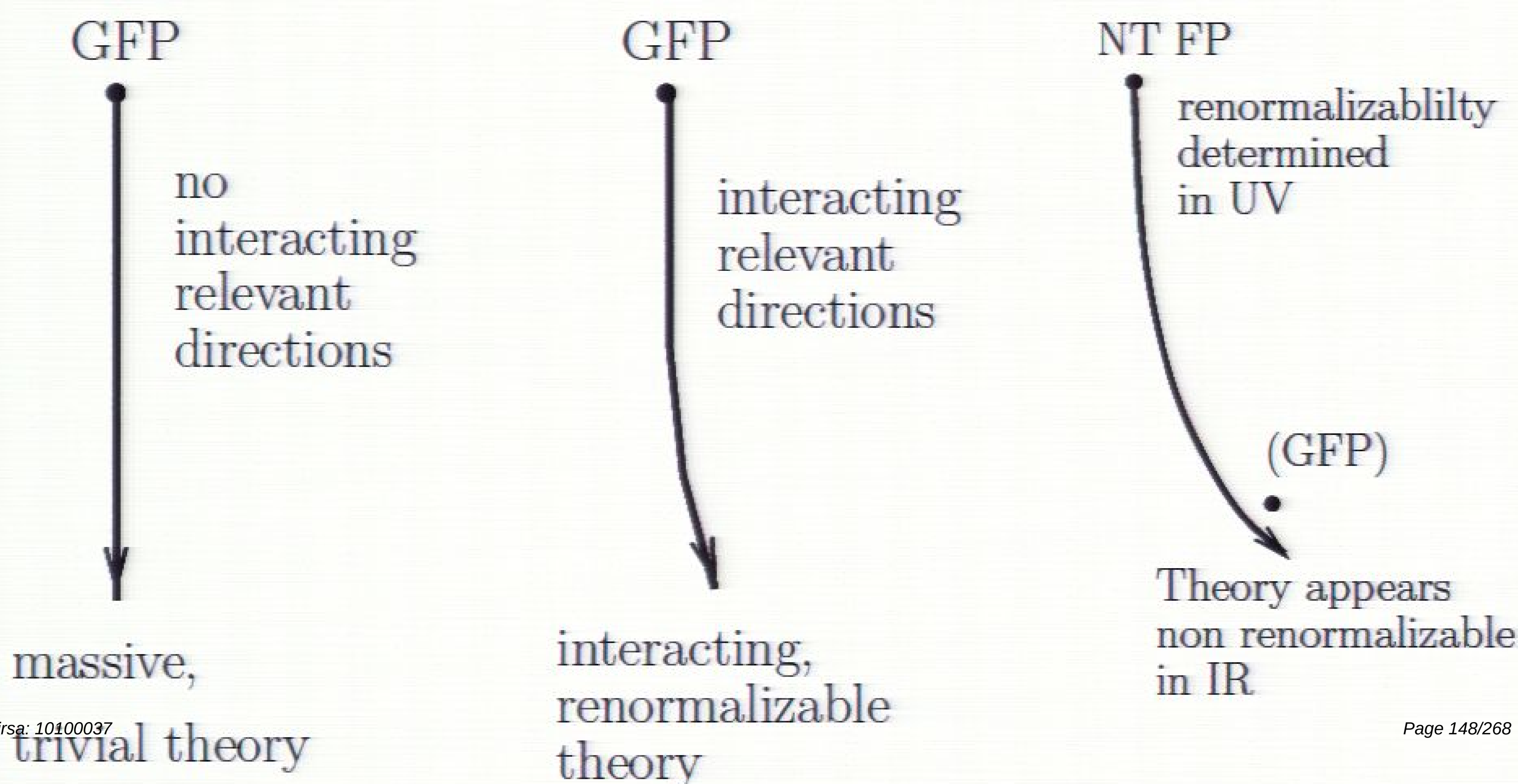

Scalar Field Theory: Four Dimensions

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- Only the Gaussian FP exists
- The mass is relevant
- The four point coupling is marginally irrelevant
- All other couplings are irrelevant
- The only nonperturbatively renormalizable scalar field theories in four dimensions are trivial!

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Gaussian Fixed-point

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- The four-point coupling is relevant
- Non-trivial renormalizable theories exist along the $\lambda\varphi^4$ direction!

Wilson-Fisher Fixed-point

- In addition to the Gaussian FP, there is a non-trivial FP
- The W-F FP possesses a single relevant direction
- This can also be used to construct an RT

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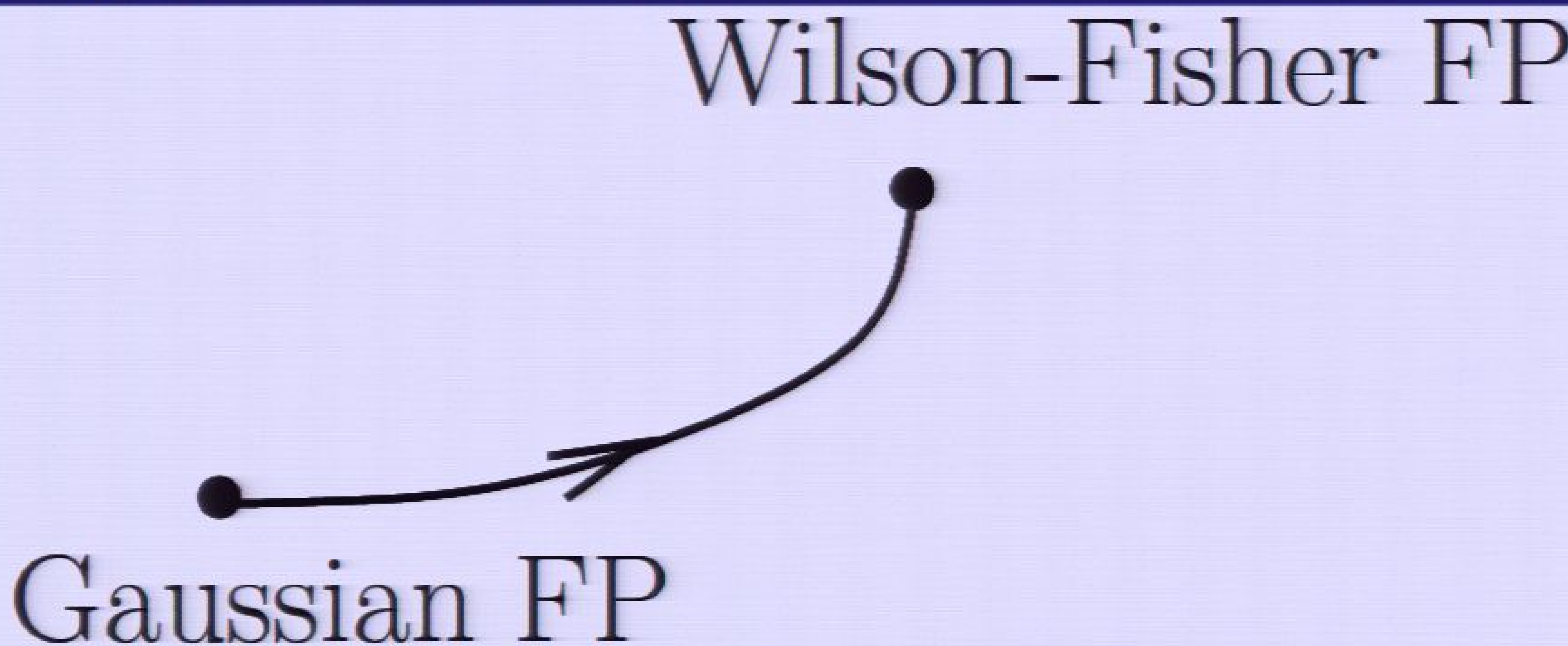
Example of a Continuum Limit in D=3



Textbook versus Wilsonian

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Question: What is the link?

- Textbook formulation is in terms of correlation functions
- Wilsonian formulation is in terms of S_Λ

My aims in the rest of this talk

- To convince you that the question is profound

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1. Qualitative Aspects of the ERG
2. Renormalizability
3. Correlation Functions in the ERG

Polchinski's Equation

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- Polchinski made a particular choice

$$\Psi = \Psi_{\mathrm{Pol}}$$

Pros

- The flow equation is simple
- The correlation functions can be extracted from S
- Renormalizability of $S \Rightarrow$ renormalizability of

Cons

- It is inconvenient for finding fixed-points

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$$\Psi = \Psi_{\mathrm{Pol}} + \psi$$

- Choose

$$\psi = -\frac{1}{2}\eta\phi, \qquad \eta \equiv \Lambda \frac{d \ln Z}{d\Lambda}$$

- Since ψ is a field redefinition, this choice ensures canonical normalization of the kinetic term
- The redundant coupling, Z, is removed from the action

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- Easy to find fixed-points with $\eta_\star \neq 0$

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- The link between S and [illegible] changes

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The Standard Correlation Functions

- Introduce a source term in the bare action

$$\mathcal{Z}[J] = \int \mathcal{D}\phi \, e^{-S_{\Lambda_0}[\phi] + J\cdot\phi}$$

- Extract the connected correlation functions from $W[J] \equiv \ln \mathcal{Z}$

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New ERG Approach

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- Introduce an external field, J, with undetermined scaling dimension, d_J
- Allow for J-dependence of the action

$$S_\Lambda[\phi] \to T_\Lambda[\phi, J]$$

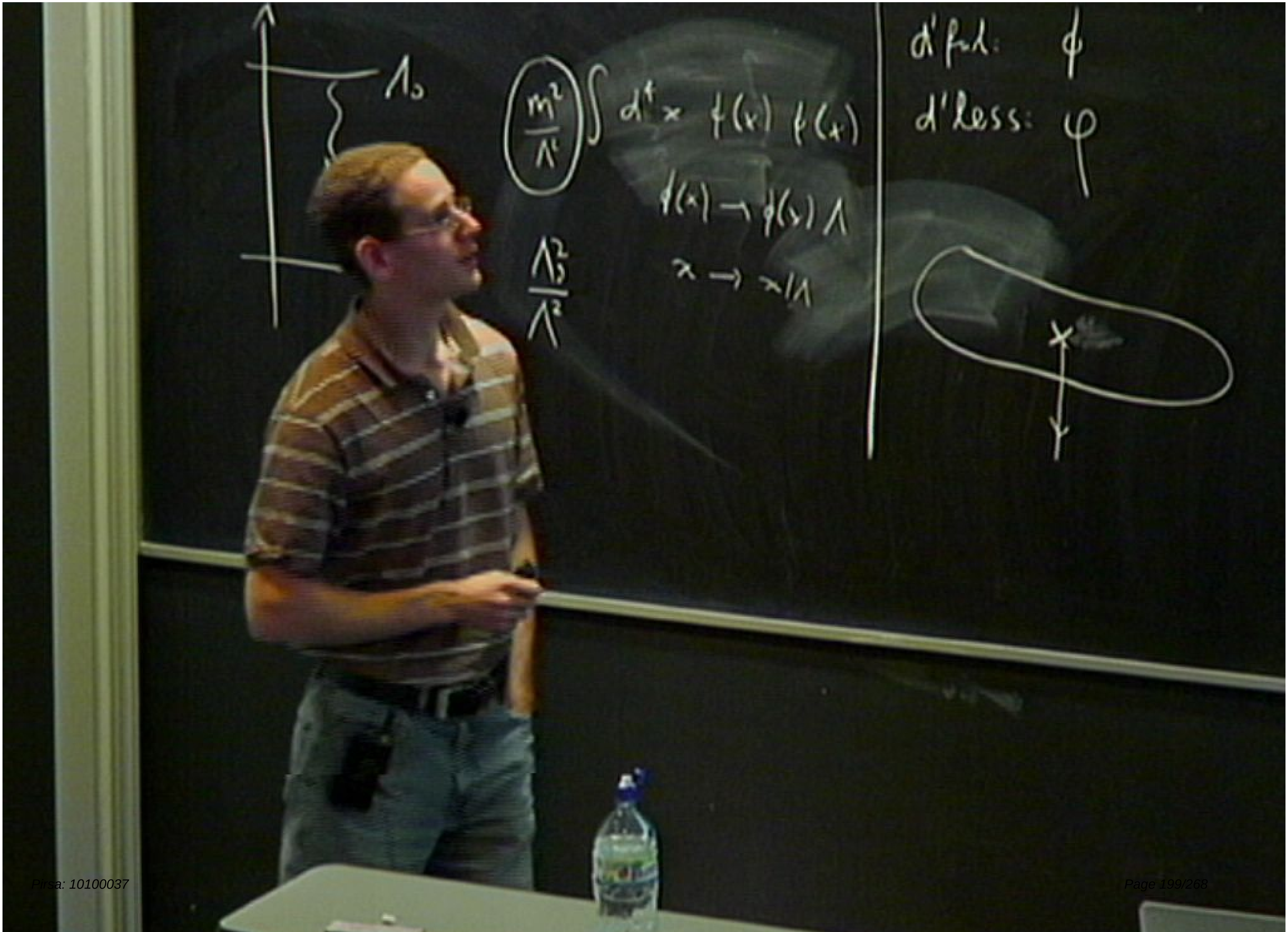
- The flow equation follows as before

$$-\Lambda \partial_\Lambda e^{-T_\Lambda[\phi,J]} = \int d^D x \frac{\delta}{\delta\phi(x)} \left\{ \Psi(x) e^{-T_\Lambda[\phi,J]} \right\}$$

- A sensible boundary condition would be

$$\lim_{\Lambda \to \Lambda_0} T_\Lambda[\phi, J] - S_\Lambda[\phi] = -J \cdot \phi$$

- But we will not implement the bc in this way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Source-Dependent Renormalization

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The game

- Search for renormalizable, source-dependent solutions

The strategy

Nonperturbatively renormalizable solutions follow from fixed points

- Either directly
- Or from relevant (source-dependent) perturbations

Notation

- dimensionful, dimensionless

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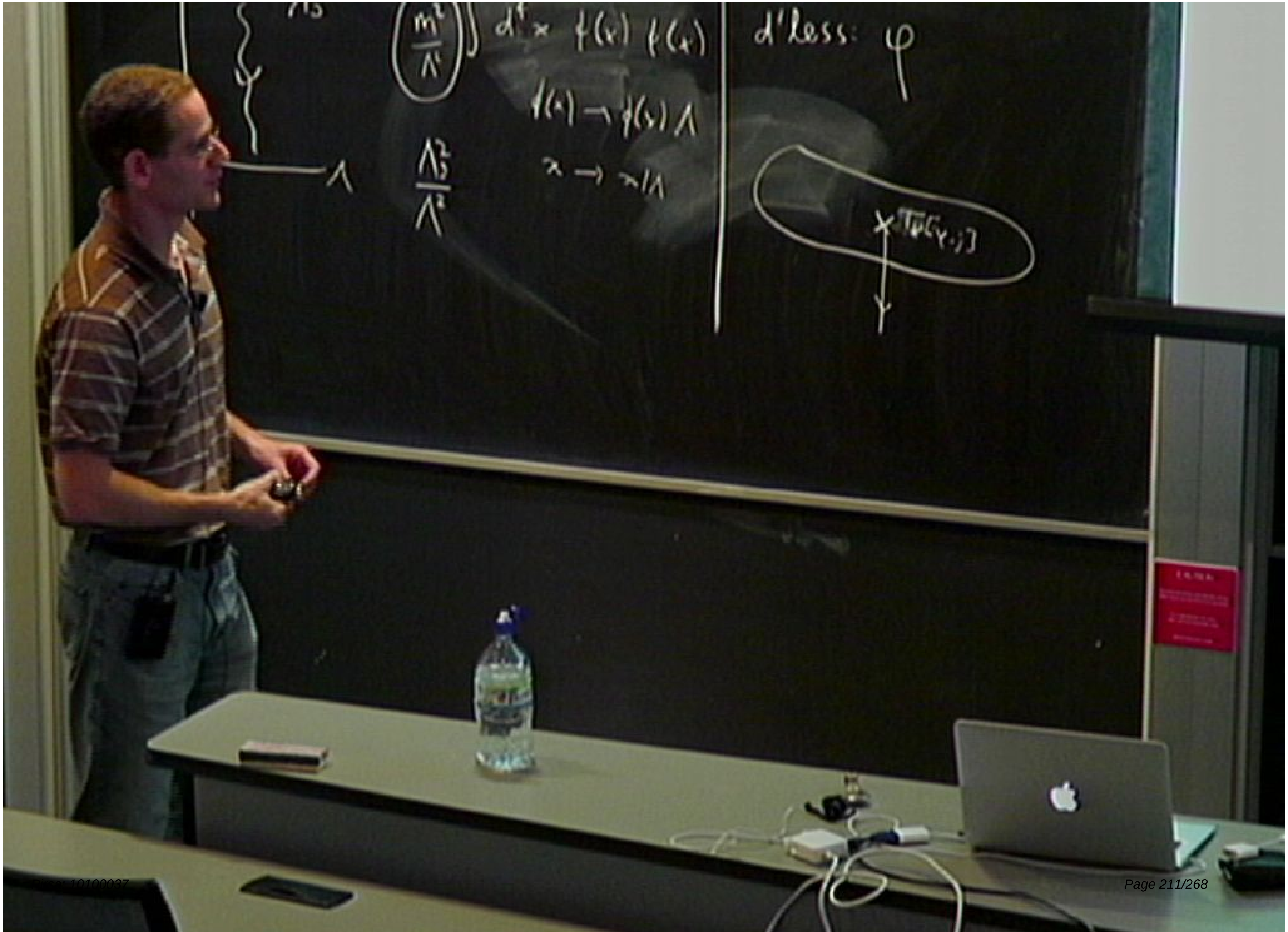
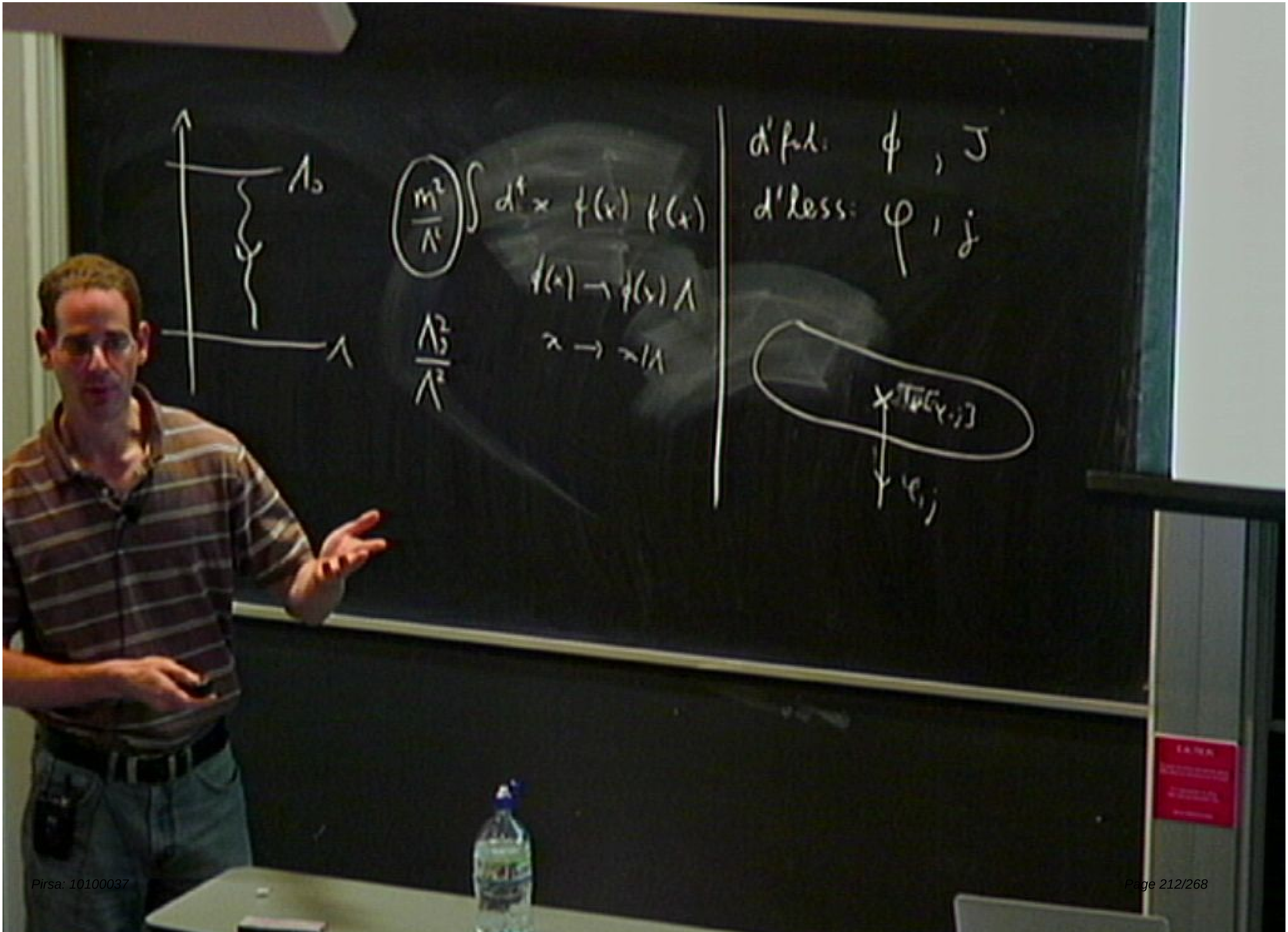

d'less: φ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A Source-Dependent Fixed-Point

- Suppose that we have found a critical fixed-point

$$\partial_t S_\star[\varphi] = 0$$

- Then there is always a source-dependent f-p

$$T_\star[\varphi, j] = S_\star[\varphi] + \left[e^{-\bar{j}\cdot\varrho\cdot\delta/\delta\varphi} - 1\right]\left[S_\star[\varphi] + \frac{1}{2}\varphi \cdot f \cdot \varphi\right]$$

Two crucial points

- The solution only works if $d_J = (D + 2 - \eta_\star)/2$
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And more. . .

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- For each critical f-p, we can find the eigenperturbations

$$S_t[\varphi] = S_\star[\varphi] + \sum_i \alpha_i e^{\lambda_i t}\mathcal{O}_i[\varphi]$$

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A new perspective on QFT?

- Decide which correlation functions to compute
- Introduce appropriate source term *e.g.* $J \cdot \phi$
- Analyse renormalizability of correlation functions
- Allow arbitrary source dependence
- Search for fixed-point solutions
- Deduce the correlation functions to which the solution(s) correspond

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- The Wilsonian effective action is fundamental
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- What happens for other flow equations?
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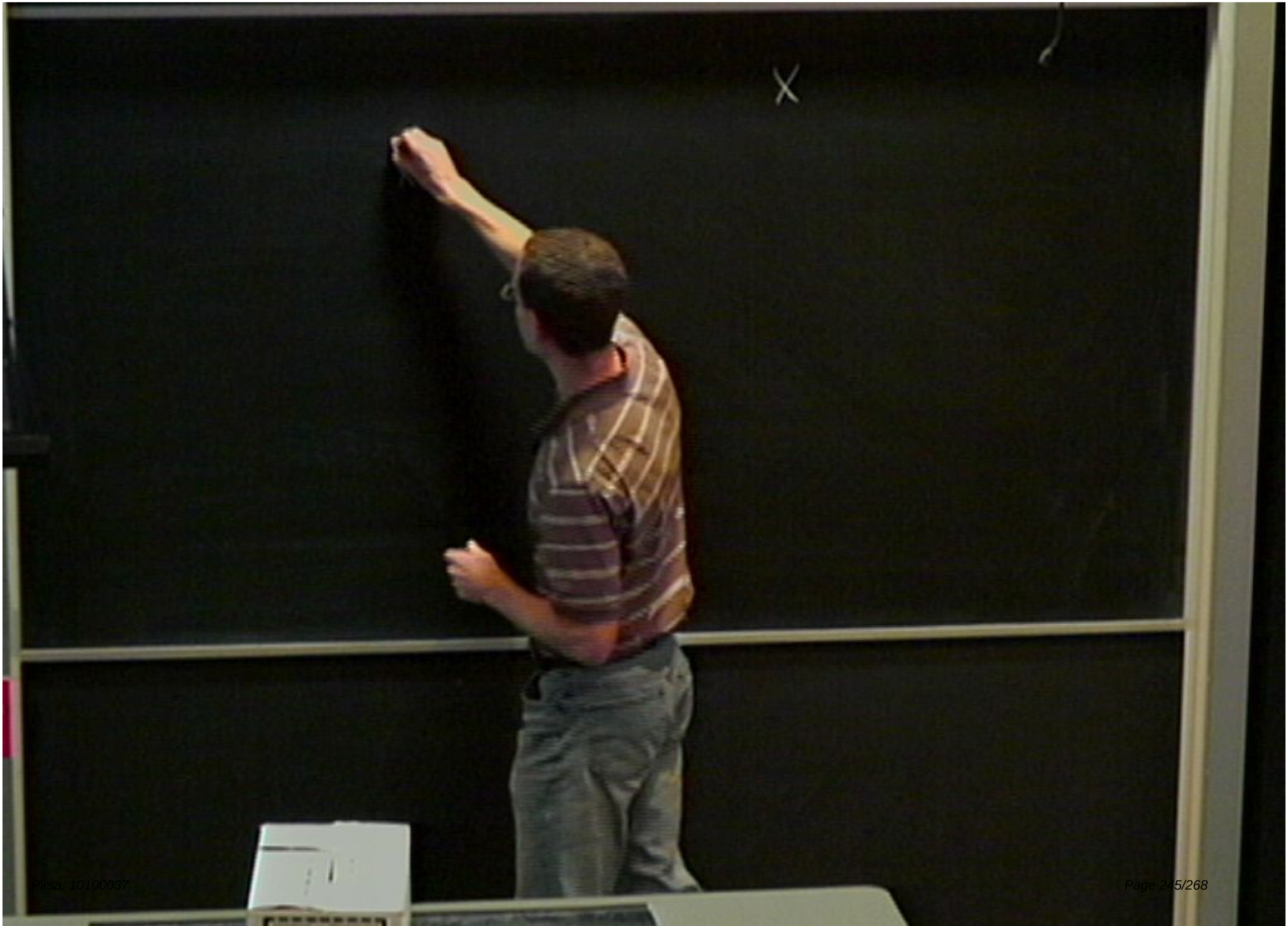

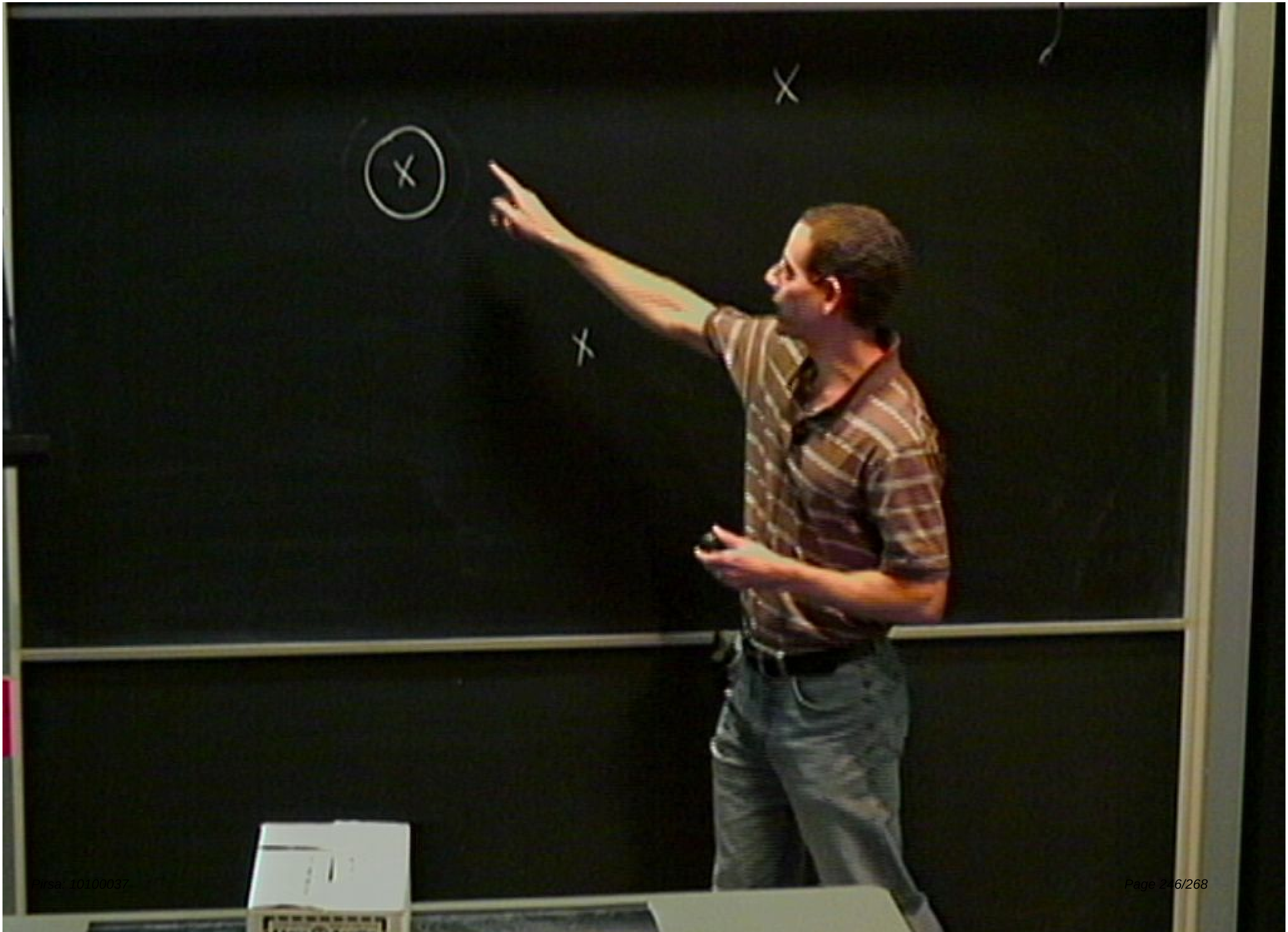

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  - Covariant higher derivatives
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- No gauge fixing is required at any stage!
- The formalism is very complicated

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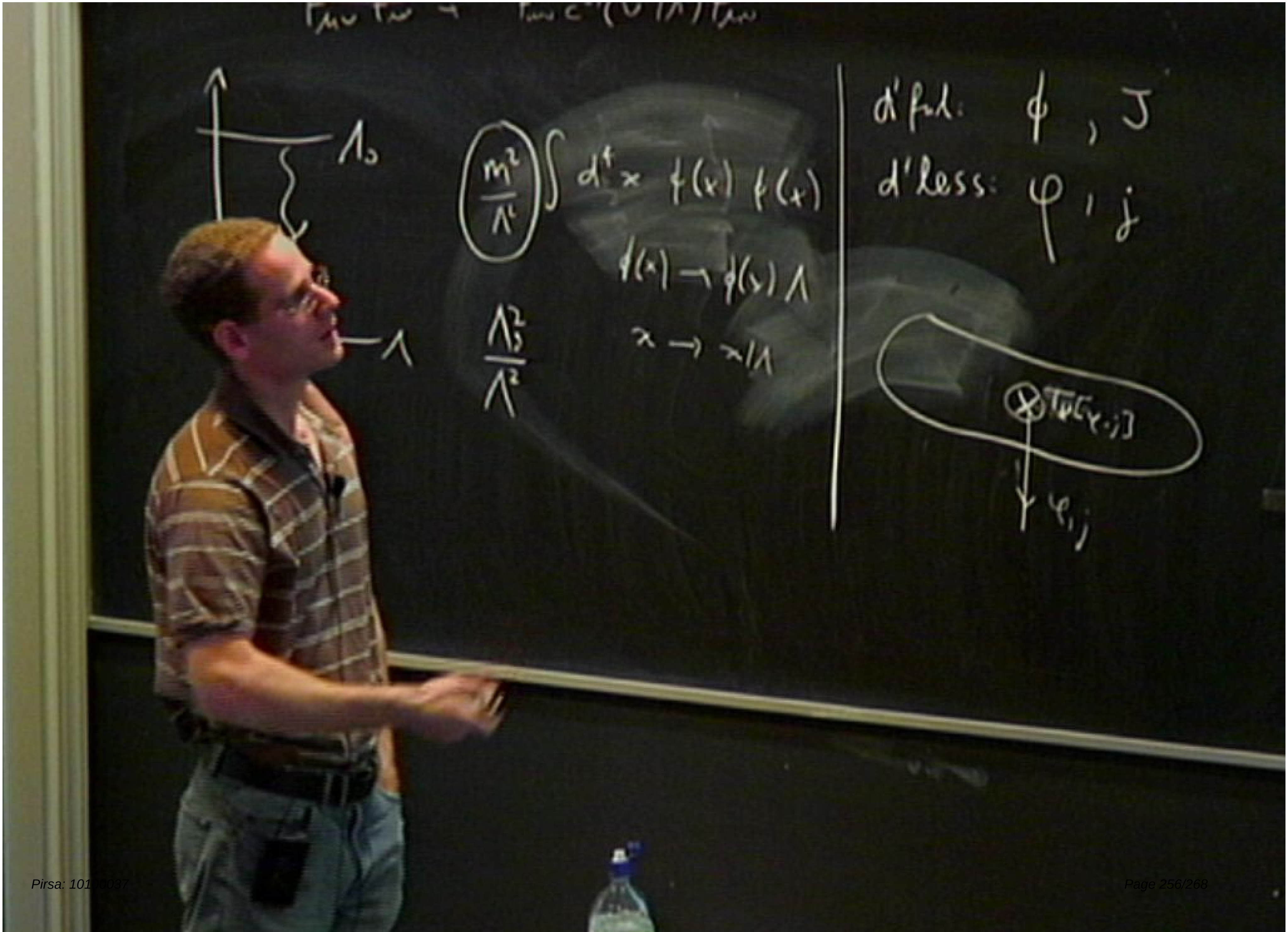
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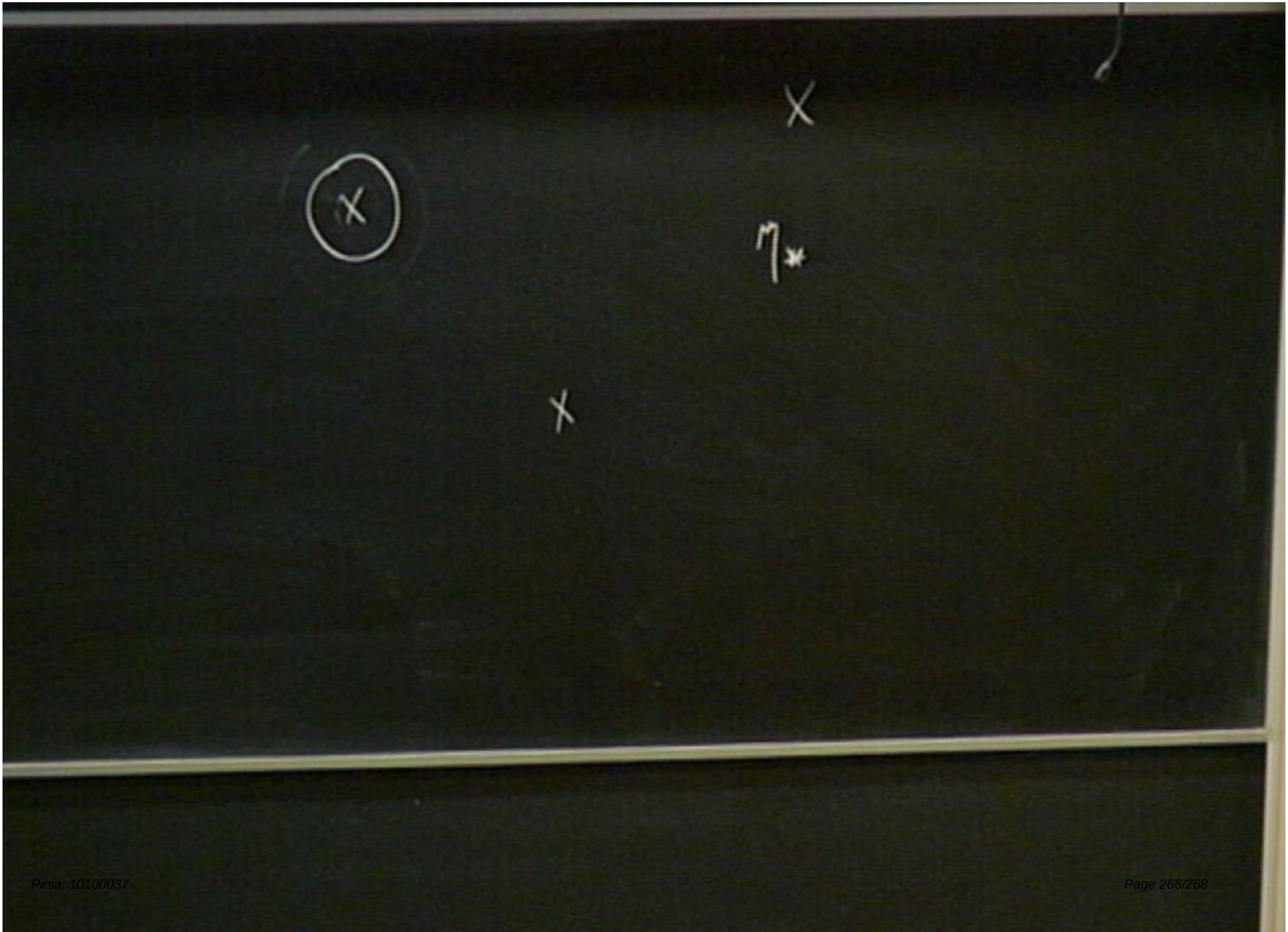
x
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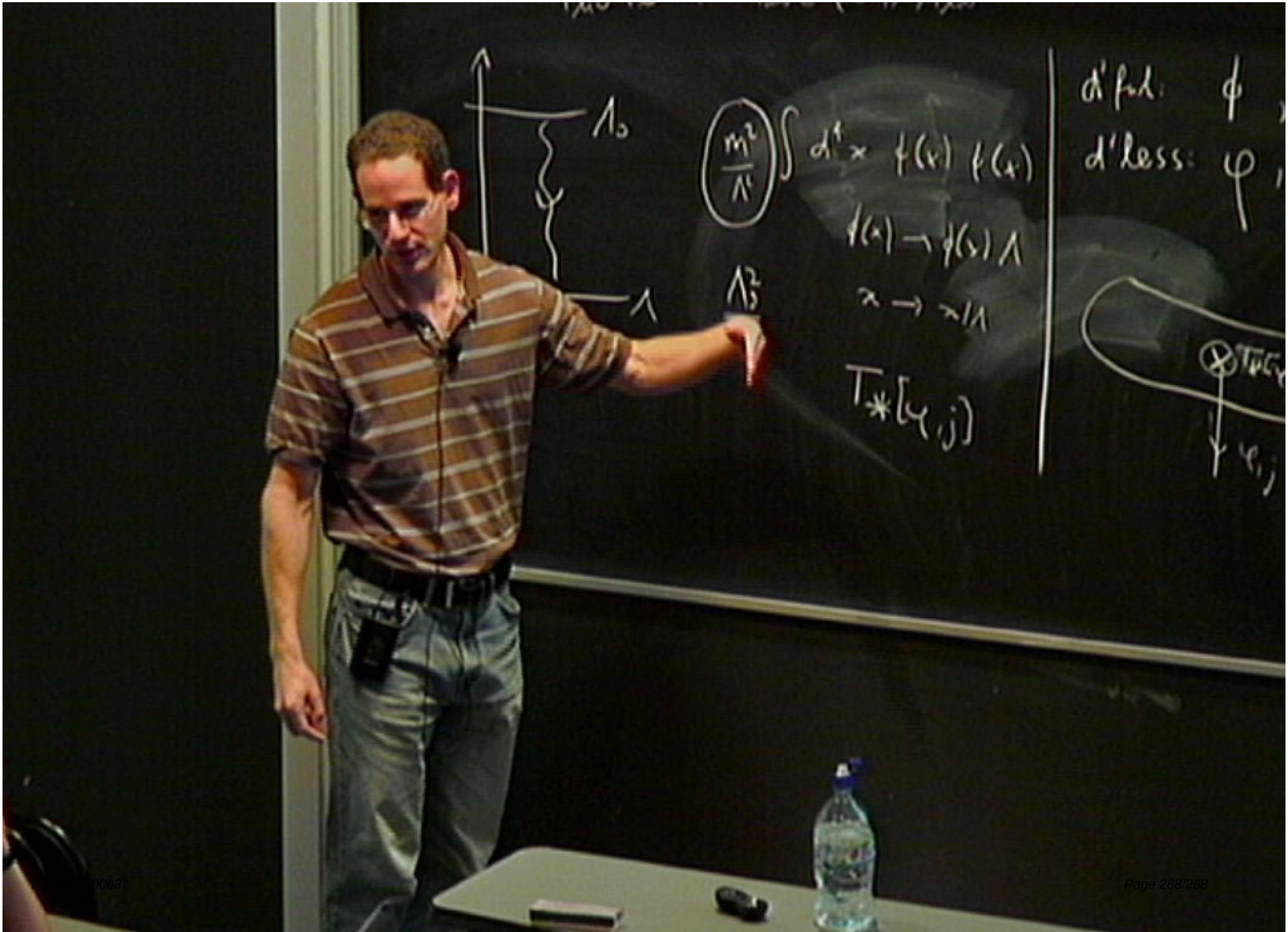
d'ful:
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Λ
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$\phi(x) \to \phi(x)\ \Lambda$
$x \to x/\Lambda$
$T_*[\varphi, j]$