Title: Hawking radiation in AdS_5

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Abstract: We formulate a numerical procedure to calculate Hawking radiation during non-equilibrium black hole formation. The procedure is applied to a static string in thermal AdS and it is shown that for an arbitrary initial state, the final state is an equilibrated heavy quark string. The fluctuations in the quark string are transmitted from the horizon to the boundary leading to Brownian motion in the boundary theory.

Hawking Radiation in AdS_5

Derek Teaney

SUNY Stonybrook and RBRC Fellow

STATE UNIVERSITY OF NEW YORK

- Dam T. Son, DT; arXiv:0901.2338
- Simon Caron-Huot, DT, Paul Chesler – slow and steady

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Three important RHIC Data

(Or how I came to talk about AdS_5)

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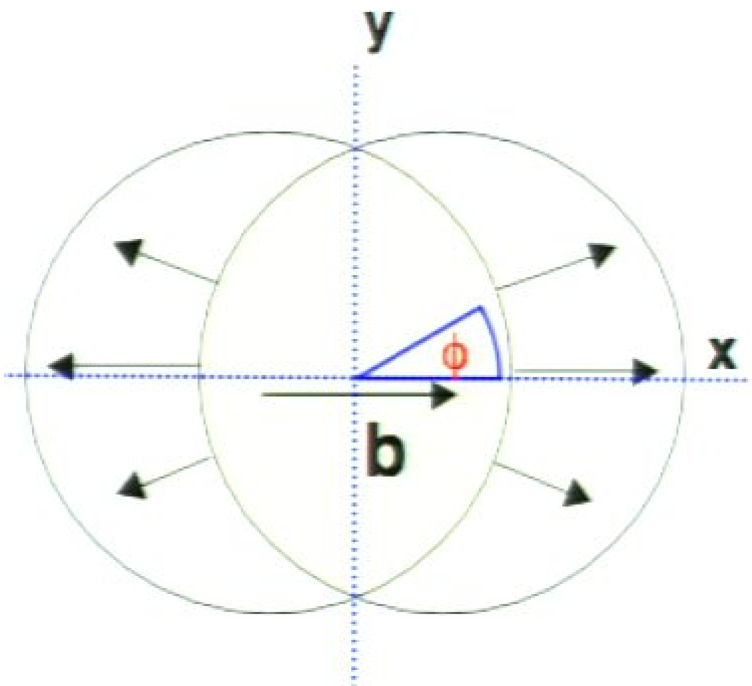
)bservation:



There is a large momentum anisotropy:

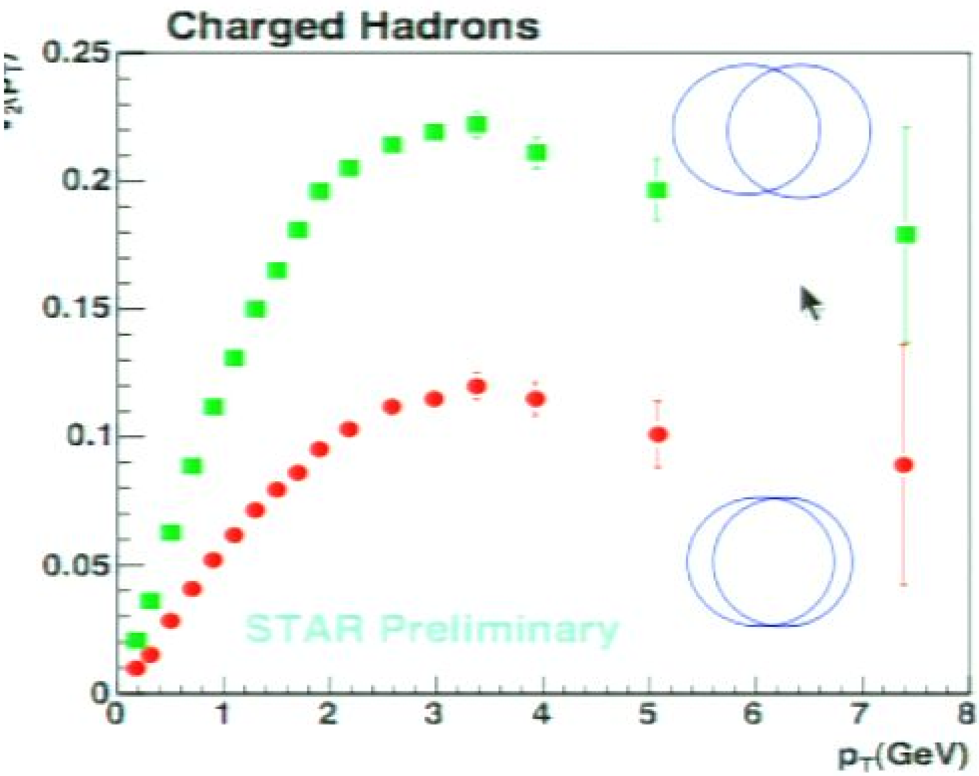
$$v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \approx 20\%$$

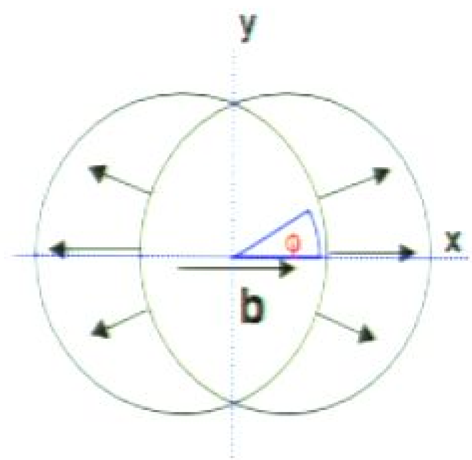
nterpretation

- The medium responds as a fluid to differences in X and Y pressure gradients

Data on Elliptic Flow:

$$\frac{1}{p_T}\frac{dN}{dp_T\,d\phi} = \frac{1}{p_T}\frac{dN}{dp_T}\left(1 + 2\,v_2(p_T)\cos(2\,\phi) + ...\right)$$

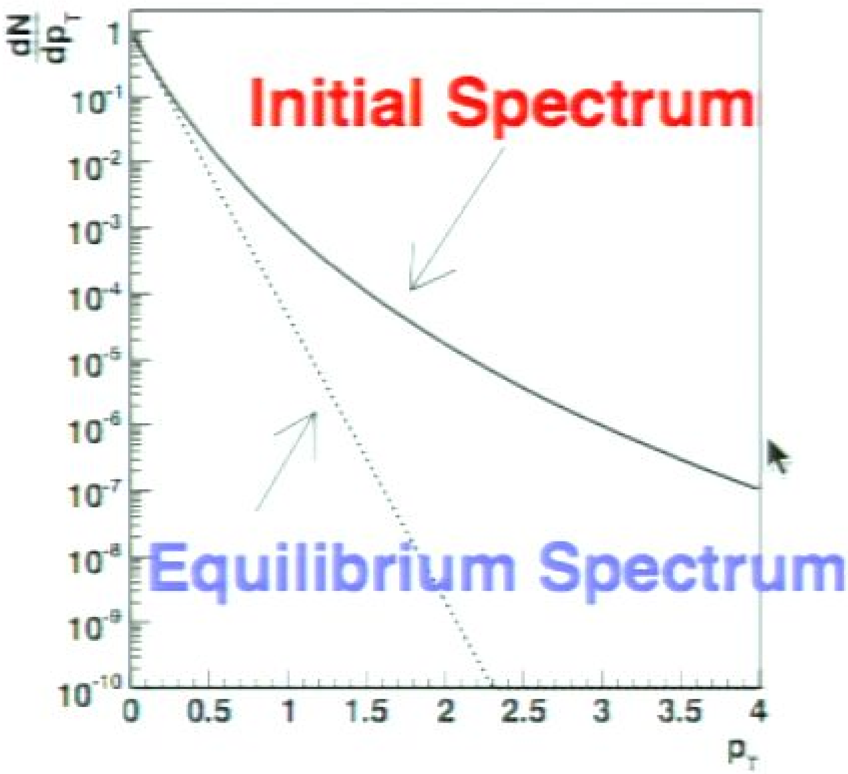




$$X:Y = (1 + \underbrace{2v_2}_{\sim 0.4} : 1 - \underbrace{2v_2}_{\sim 0.4})$$

Elliptic flow is large X:Y $\sim 2.0 : 1$

Energy Loss of Fast Partons – Cartoon



- Power law initial spectrum:

$$\frac{dN}{dp_T} \propto \left(\frac{1}{p_T}\right)^{10}$$

- Exponential equilib. spectrum:

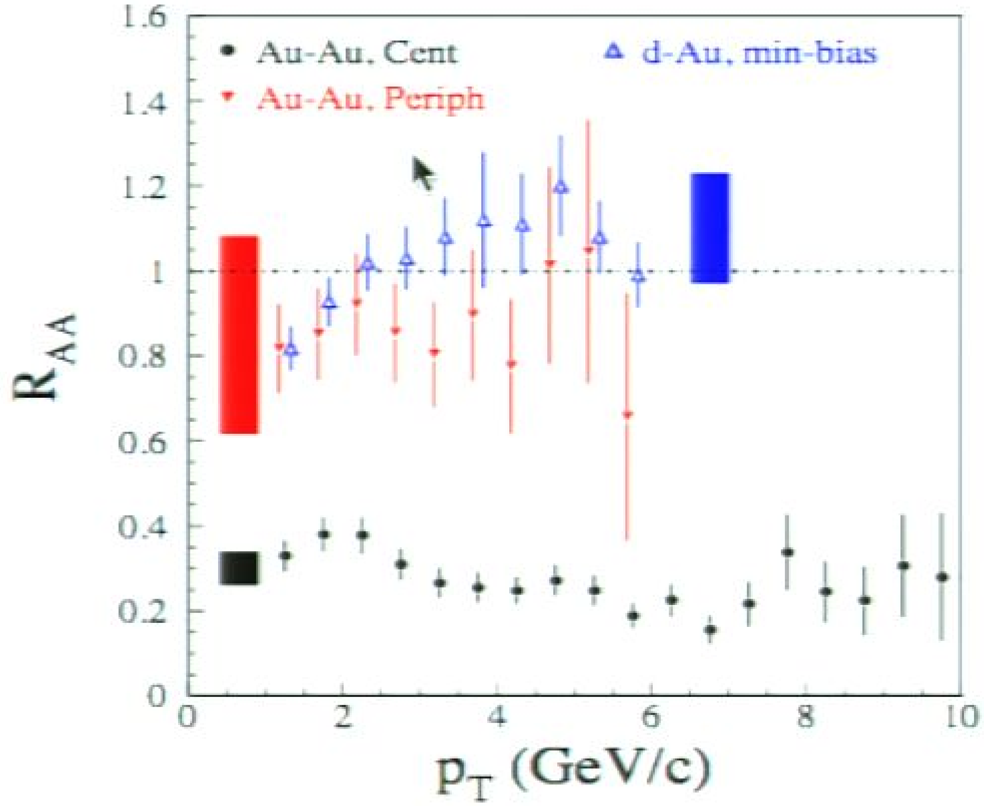
$$\frac{dN}{dp_T} \propto e^{-\frac{p_T}{T}}$$

The initial spectrum will lose energy and approach the equilibrium spectrum

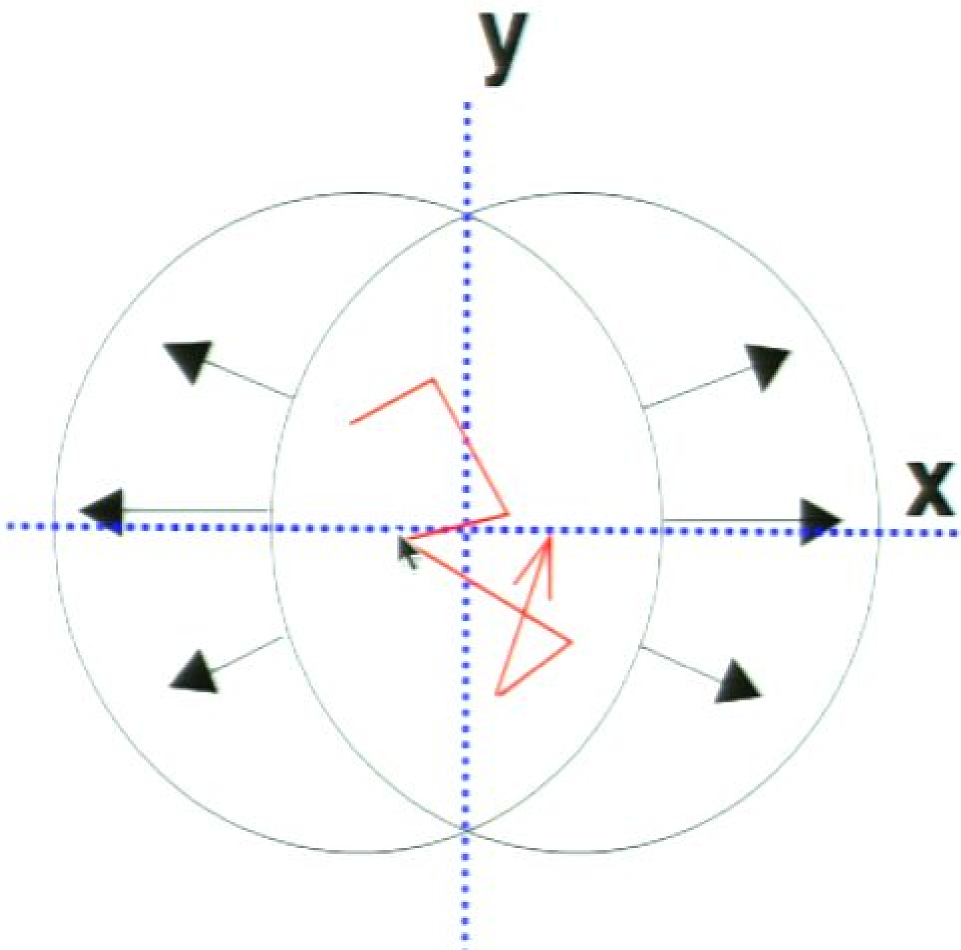
Tells something about density and interaction rates

Data on π^0 p_T spectrum

$$R_{AA} \equiv \frac{\left(\frac{dN}{p_T dp_T}\right)_{\text{In AuAu}}}{N_{\text{coll}}\left(\frac{dN}{p_T dp_T}\right)_{\text{In pp}}}$$



Heavy Quarks



The heavy quarks will either relax to the thermal spectrum and show the same v_2 as all thermal particles or not depending on the Drag/Diffusion coefficients and p_T.

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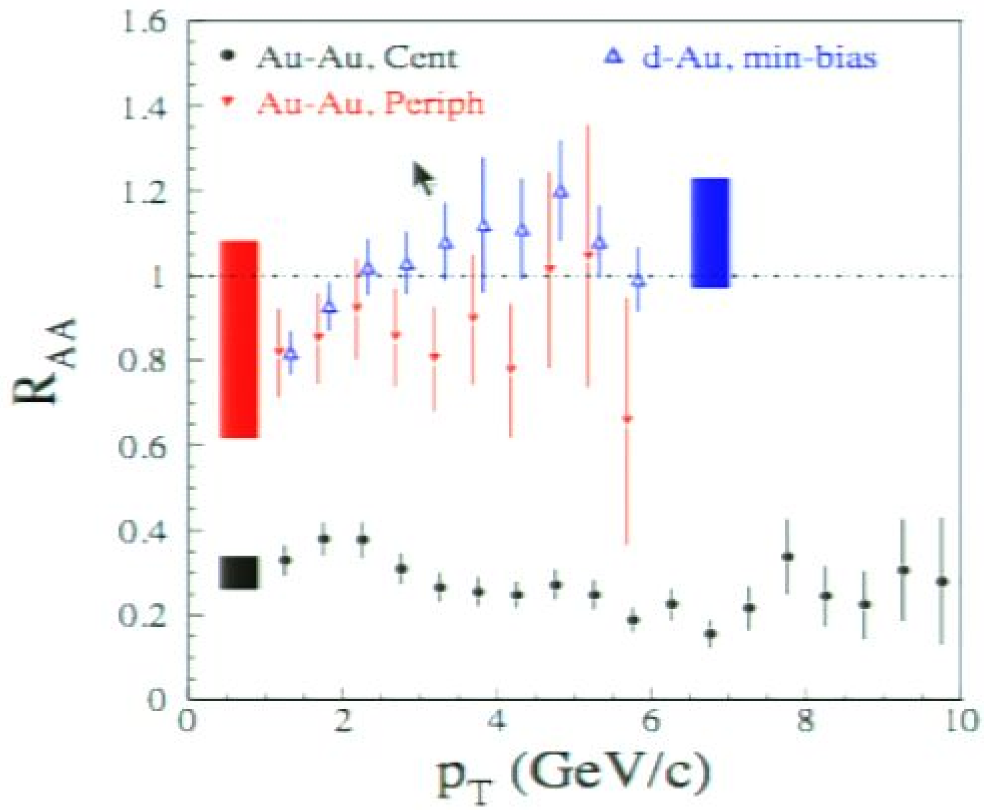

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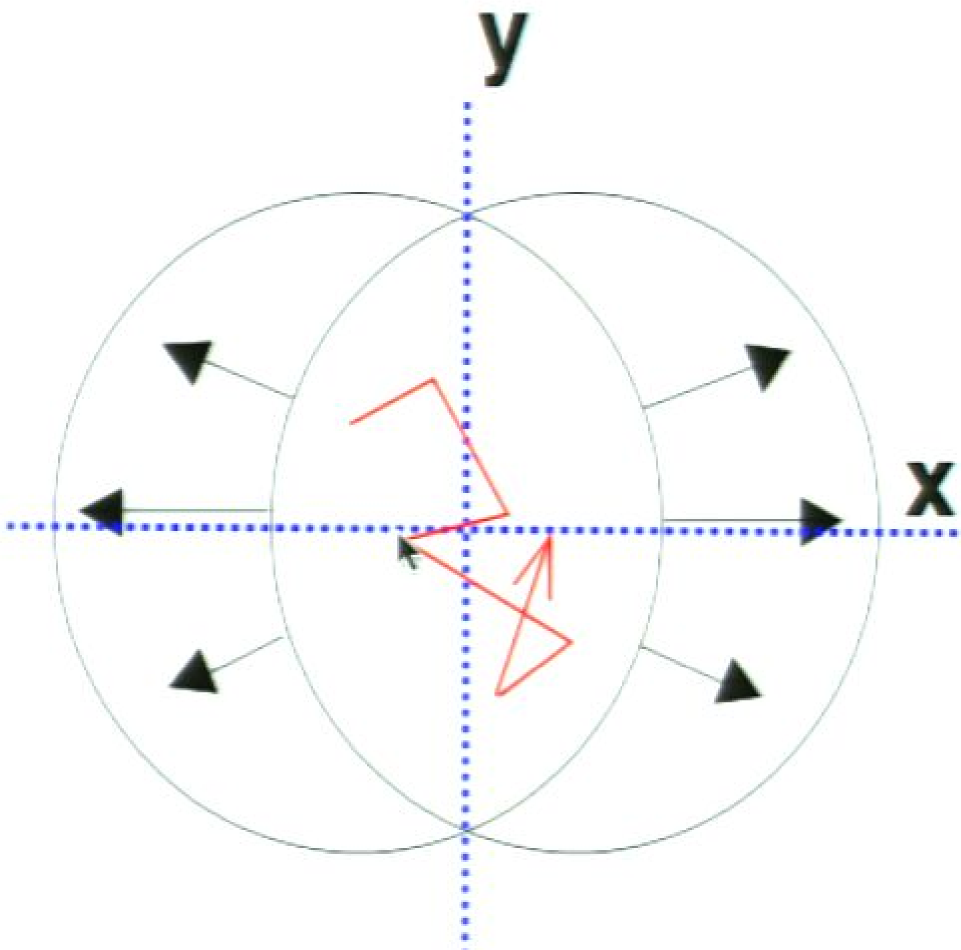

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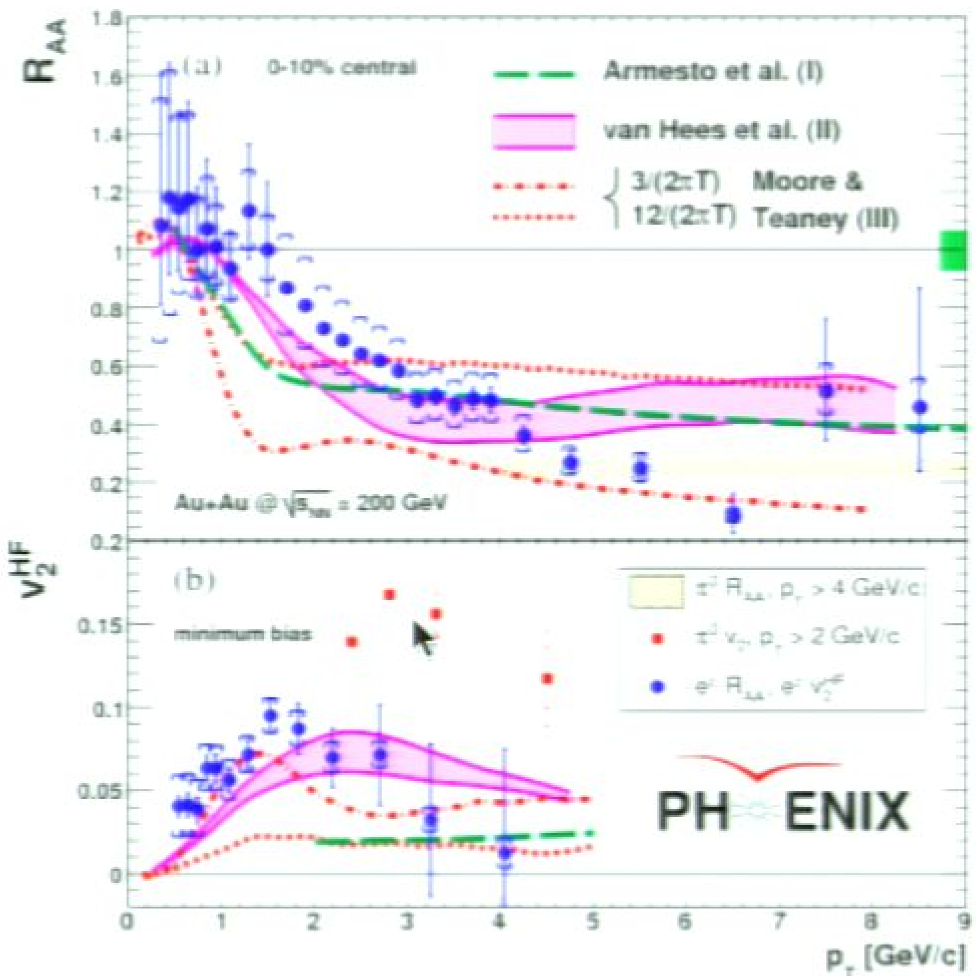


The diffusion coefficient is of order

$$D \sim \frac{1}{T}$$

Data Recapitulation

1. Elliptic Flow – Soft event is strongly modified

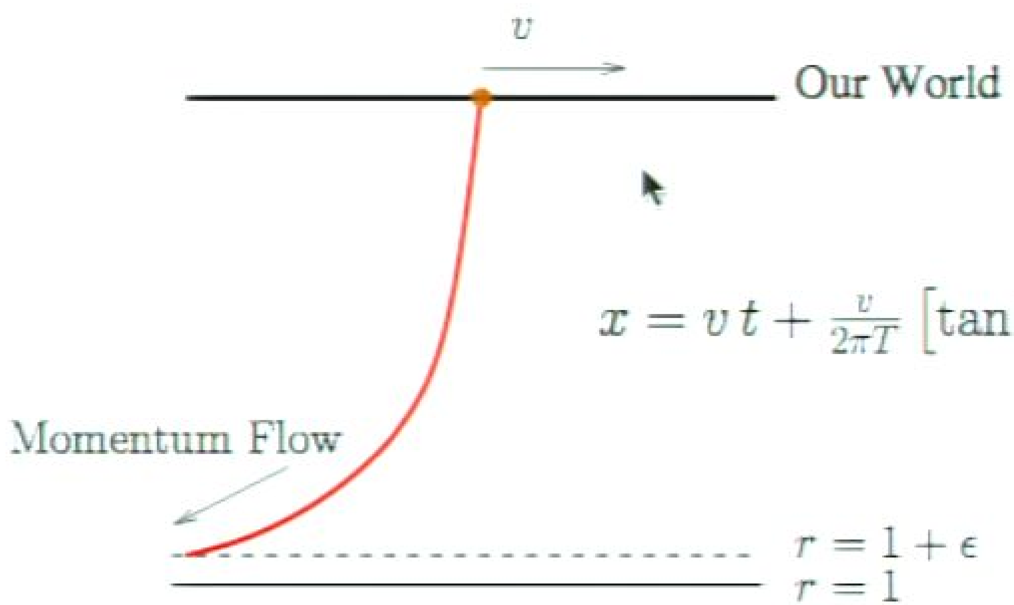
3. Energy loss significant

2. Heavy Quarks – Suppressed and flowing

Heavy Quarks in AdS/CFT

$$\frac{dP}{dt} = -\underbrace{\frac{\pi}{2}\sqrt{\lambda}T^2}_{\equiv \eta}\, v$$

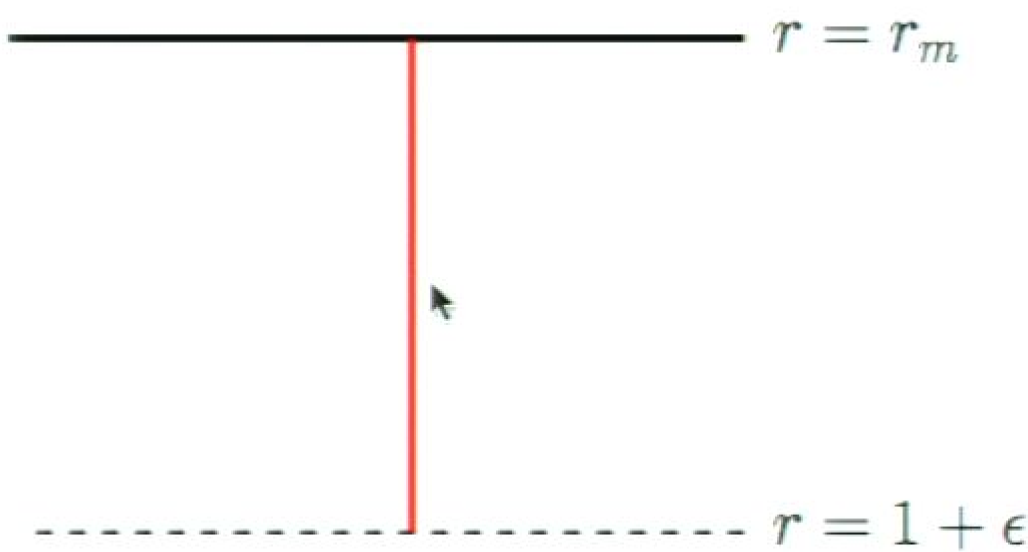
(HKKKY; D.T., J. Casalderrey ; S. Gubser)



$$x = v\,t + \frac{v}{2\pi T}\left[\tan^{-1}(z) - \tanh^{-1}(z)\right]$$

Large drag of quarks in AdS/CFT

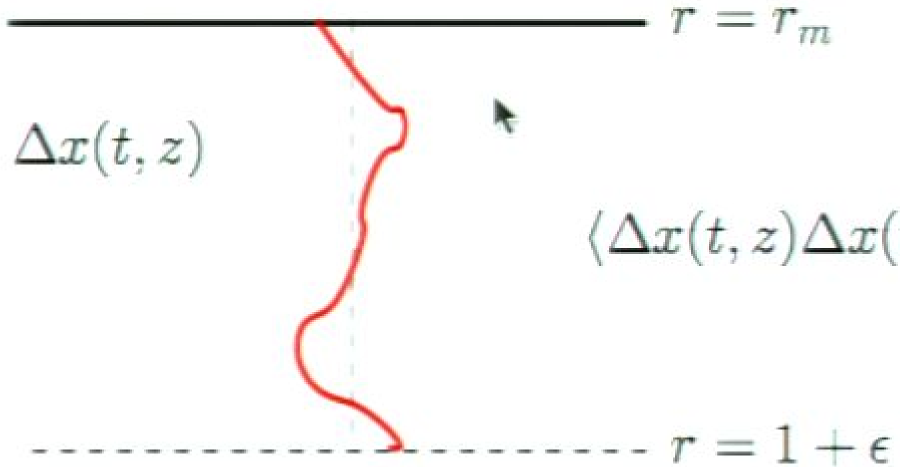
A heavy quark in AdS/CFT



Not the dual of an equilibrated quark!

A heavy quark in AdS/CFT with noise

$$M\frac{d^2\mathbf{x}}{dt^2} = \underbrace{-\eta}_{\text{Drag}}\,\dot{x} + \underbrace{\xi}_{\text{Noise}} \qquad \langle \xi(t)\xi(t')\rangle = 2T\eta\,\delta(t-t')$$



$$\langle \Delta x(t,z)\Delta x(t',z')\rangle = ?$$

Where is the noise? Hawking Radiation?

Brownian Motion

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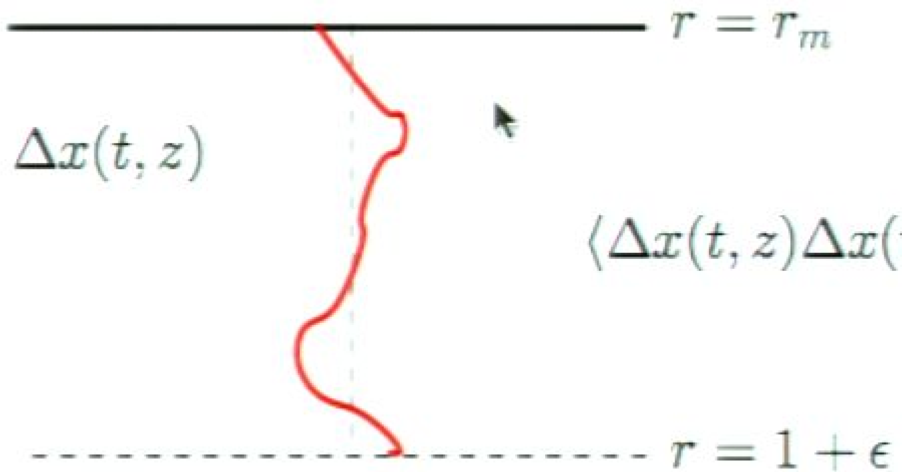

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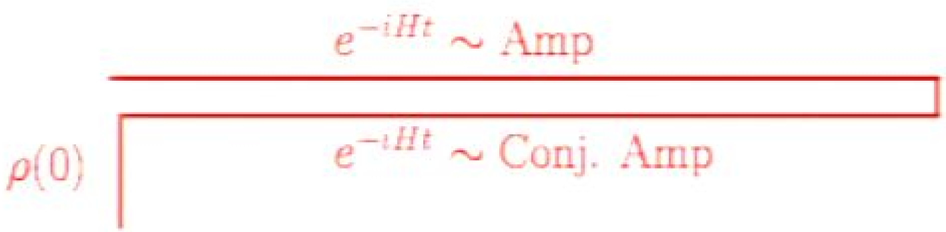
Brownian Motion

$$\rho(t) = e^{iHt}\rho(0)e^{-iHt}$$



- Consider a heavy particle coupled to bath a force on the contour

$$Z_Q = \left\langle \int Dx_1 Dx_2\, e^{i\int \frac{1}{2}Mv_1^2 - i\int \frac{1}{2}Mv_2^2}\, e^{i\int dt_1 F_1 x_1}\, e^{-i\int dt_2 F_2 x_2} \right\rangle_{\text{Bath}}$$

- Different correlators measure different time orderings

$$G_{11}(t,t') = \langle F_1 F_1 \rangle \;=\; \left\langle T[\hat{F}(t)\hat{F}(t')] \right\rangle$$

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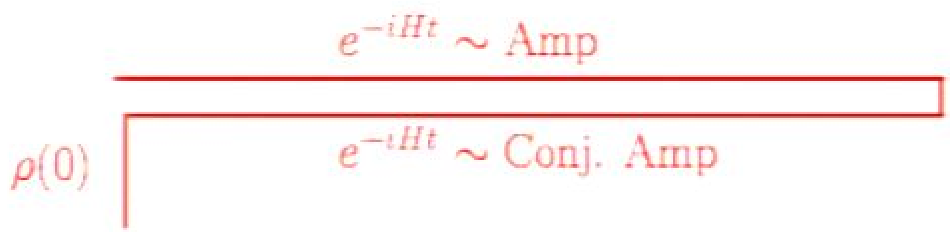

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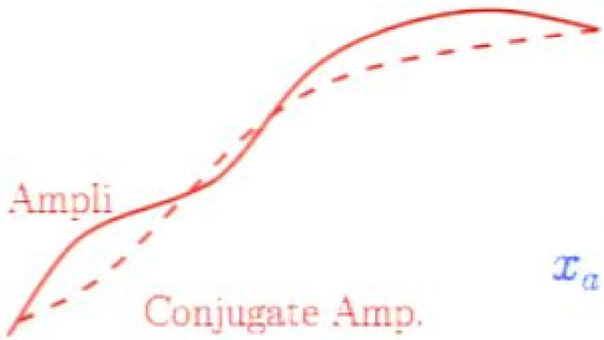
Quasi-Classical Motion – “ra” basis



$x_r \equiv \frac{x_1+x_2}{2}$

$x_a \equiv x_1 - x_2$

x_a is small in classical limit

- Motion classical

$$Z_Q = \left\langle \int Dx_r Dx_a \, e^{i\int M\dot{x}_r\dot{x}_a} \, e^{i\int dt_1 F_r x_a} \, e^{-i\int dt_2 F_a x_r} \right\rangle_{\text{Bath}}$$

- Only two point functions are:

$$\begin{aligned} \langle F_r F_a \rangle = G_R\,(t,t') &= \theta(t)\,\langle [F(t), F(t')] \rangle \\ \langle F_r F_r \rangle = G_{\text{sym}}(t,t') &= \langle \{F(t), F(t')\} \rangle \end{aligned}$$

$$G_{\text{sym}}(\omega) = -(1+2n)\,\text{Im}G_R(\omega)$$

nian Motion

he Partition Function in a heavy quark approximation

$$Z_Q = \int Dx_r Dx_a \, e^{-i\int x_a[-M\omega^2]x_r} \underbrace{e^{-i\int d\omega \, x_a G_R(\omega) x_r} e^{-\frac{1}{2}\int d\omega \, x_a \, G_{\rm sym}(\omega) \, x_a}}_{e^{iS_{\rm eff}}}$$

leplace Gaussian with fourier transform

$$e^{-\frac{1}{2}\int d\omega \, x_a G_{\rm sym}(\omega) x_a} = \int D\xi e^{i\int \xi x_a} e^{-\frac{1}{2}\int \xi G^{-1}_{\rm sym}(\omega)\xi}$$

inally do the integrals over x_a

$$Z_Q = \int Dx_r D\xi \, \delta\left(-M\omega^2 x_r + \underbrace{G_R(\omega)}_{\rm Drag} x_r(\omega) + \underbrace{\xi(\omega)}_{\rm Noise} \right) e^{-\frac{1}{2}\xi G^{-1}_{\rm sym}(\omega)\xi}$$

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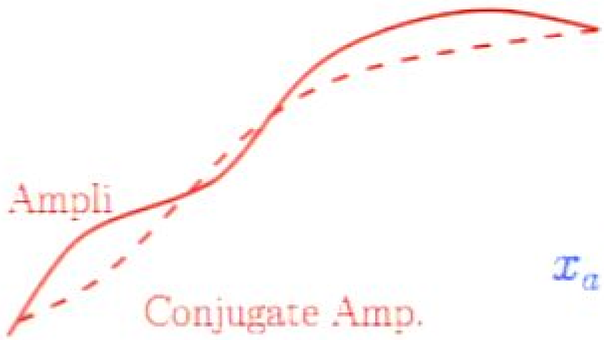

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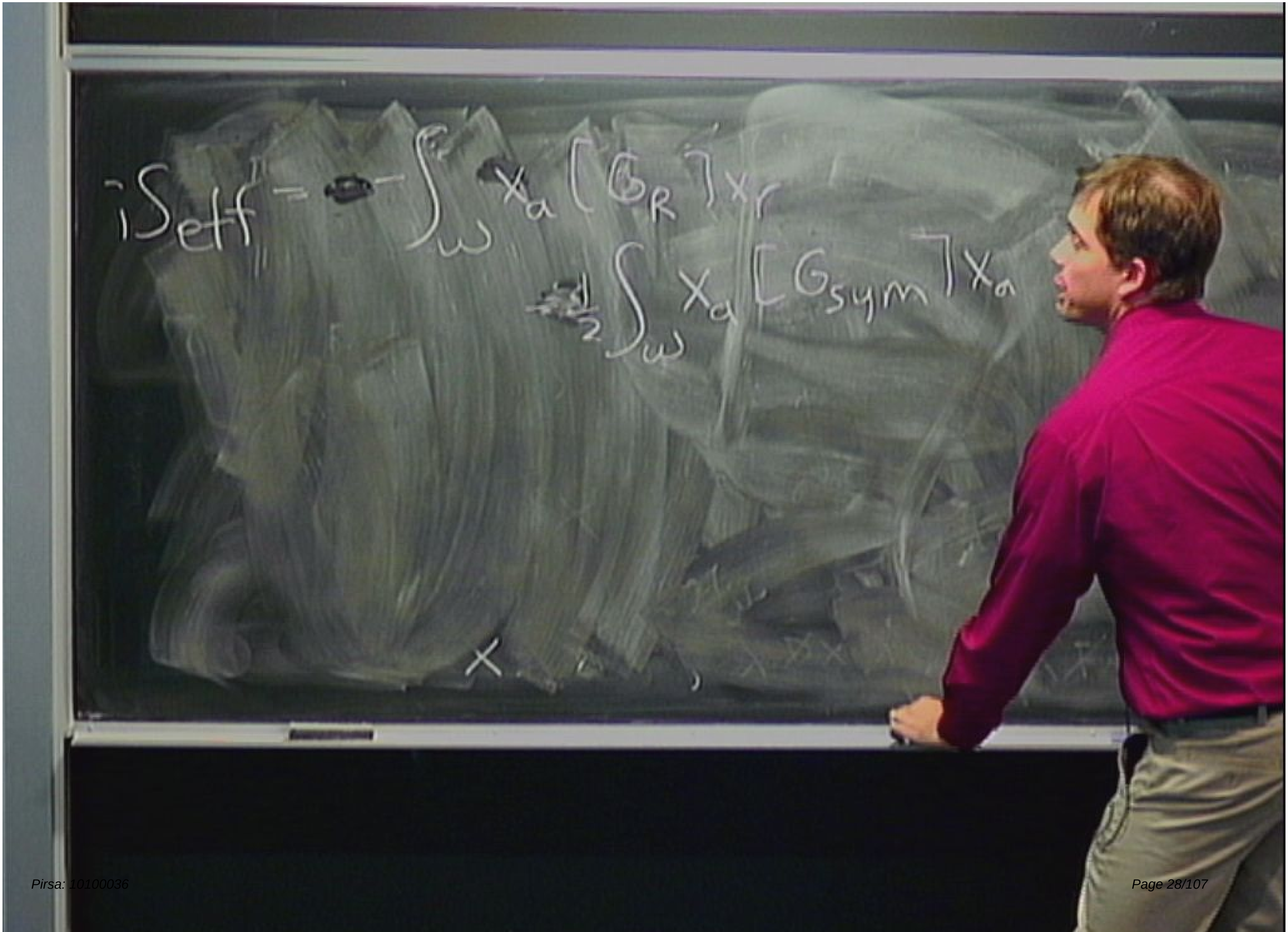
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Result: Generalized Langevin with memory

$$M_Q \frac{d^2 \bar{X}}{dt^2} + \int^t \underbrace{G_R(t-t')}_{\text{Drag}} \bar{X}(t') = \underbrace{\xi(t)}_{\text{noise}}$$

1. Drag = retarded force-force correlator

$$G_R(t) = \theta(t) \langle [F(t), F(0)] \rangle$$

2. Noise= symmetrized force-force correlator

$$\langle \xi(t) \xi(0) \rangle = \langle \{F(t), F(0)\} \rangle$$

3. Fluctuation - Dissipation

$$\langle \xi(t) \xi(0) \rangle = -(1+2n) \mathrm{Im} G_R(\omega)$$

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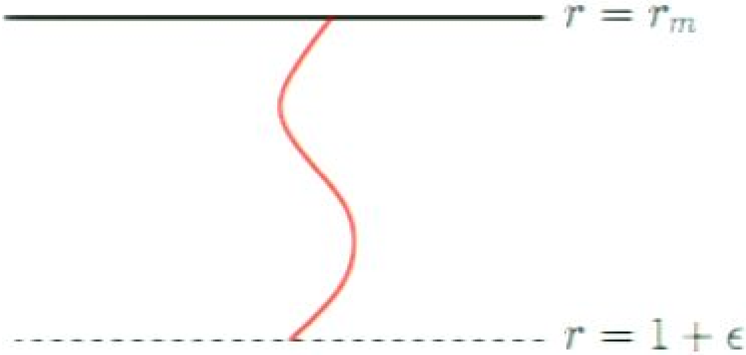
AdS/CFT

Small Fluctuations of the Straight String



- Action

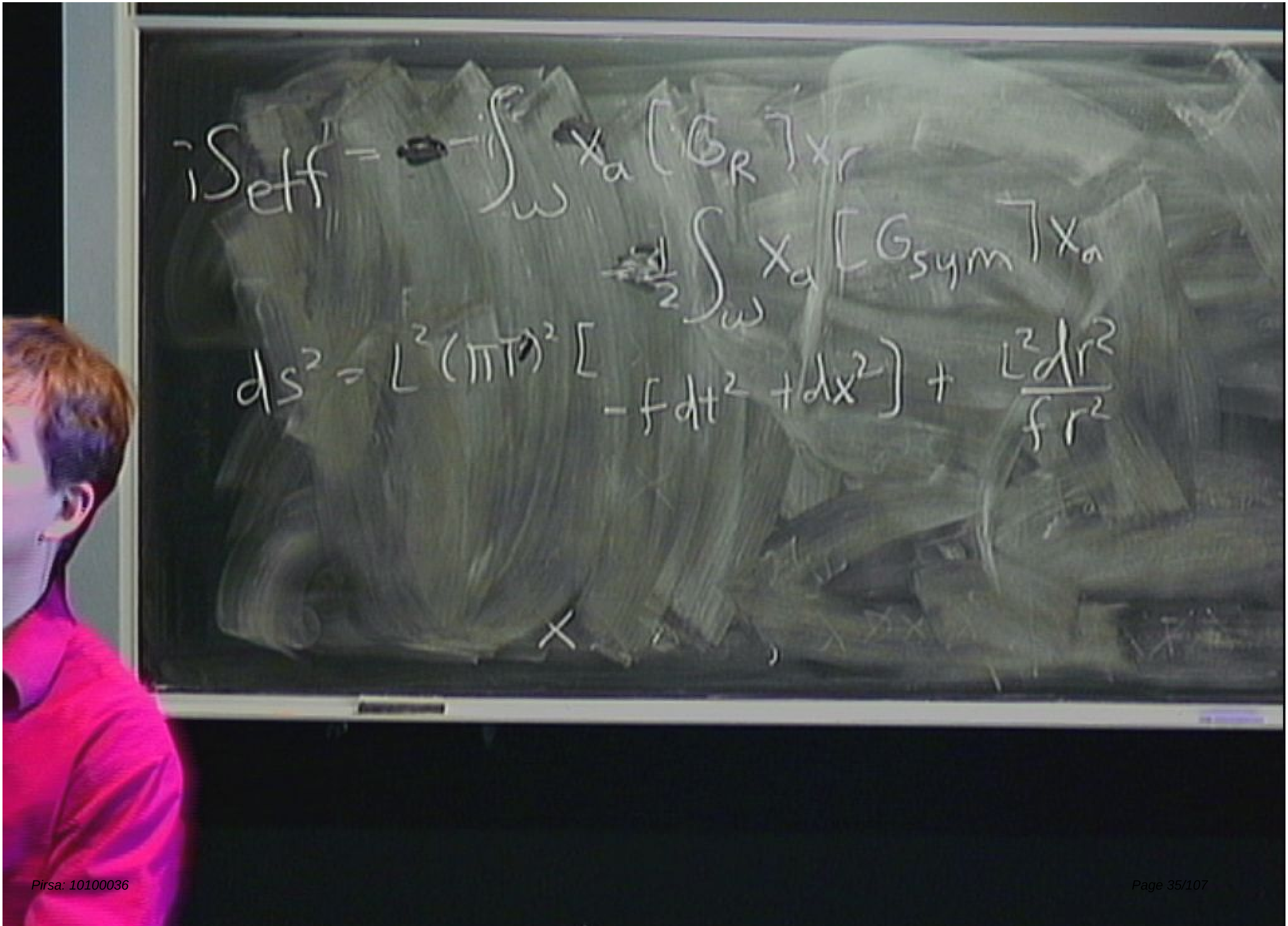
$$S = -\int \mathrm{d}t\mathrm{d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x)^2 - \frac{\overbrace{m}^{\text{mass}}}{2f} (\partial_t x)^2 \right]$$

- Tension depends on radius

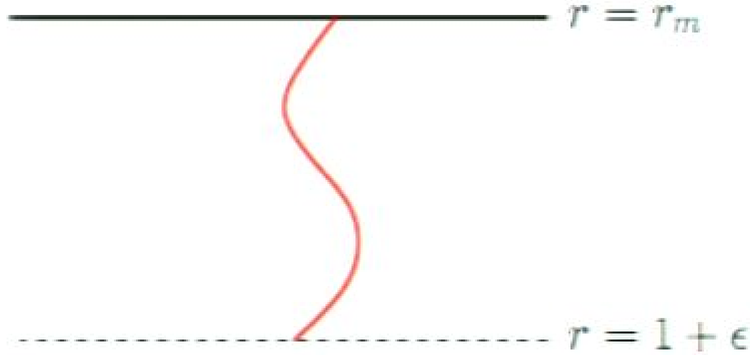
$$\text{Tension} \equiv T_o(r) \propto \sqrt{\lambda}\, f(r) r^4 \to \sqrt{\lambda}\, 4(r-1)$$

- Find two solutions: infalling $(-)$ and one outgoing $(+)$

$$x(\omega, r) \sim (r-1)^{\mp i\omega/4\pi T}$$


$iS_{eff} = -i\int_\omega x_a [G_R] x_r$
$-\frac{1}{2}\int_\omega x_a [G_{sym}] x_a$
$ds^2 = L^2(\pi T)^2 [-f dt^2 + dx^2] + \frac{L^2 dr^2}{f r^2}$

Small Fluctuations of the Straight String



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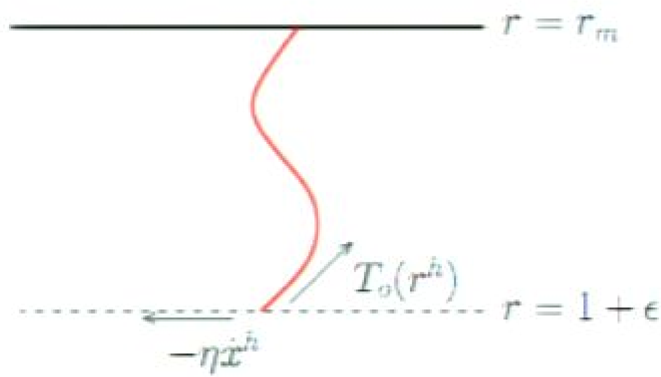
Infalling solution has a physical interpretation:

- Near the horizon we have

$$x(\omega, r) \approx (r-1)^{-i\omega/4\pi T} \quad \rightarrow \quad \overbrace{(r-1)}^{\propto T_o(r)}\, \partial_r x(\omega, r) = \frac{-i\omega}{4\pi T} x(\omega, r)$$

- Multiply by constant $2\pi\sqrt{\lambda}T^3$

$$\underbrace{T_o(r_h)\, \partial_r x(r_h, t)}_{\text{Tension}} = \underbrace{\eta \dot{x}(r_h, t)}_{\text{Drag}}$$



Horizon motion is overdamped – no acceleration on stretched horizon

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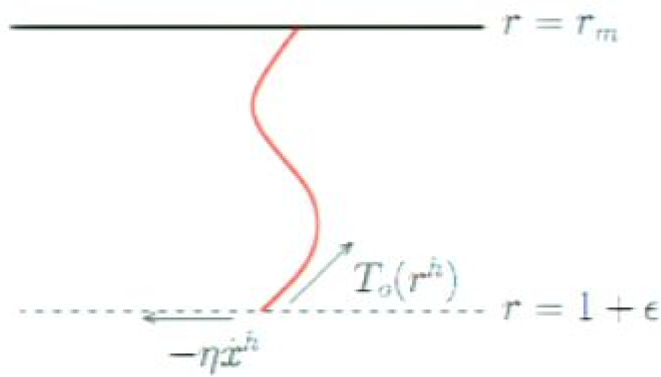
$$\underbrace{T_o(r_h)\,\partial_r x(r_h,t)}_{\text{Tension}} = \underbrace{\eta\dot{x}(r_h,t)}_{\text{Drag}} + \underbrace{???}_{\text{noise}}$$



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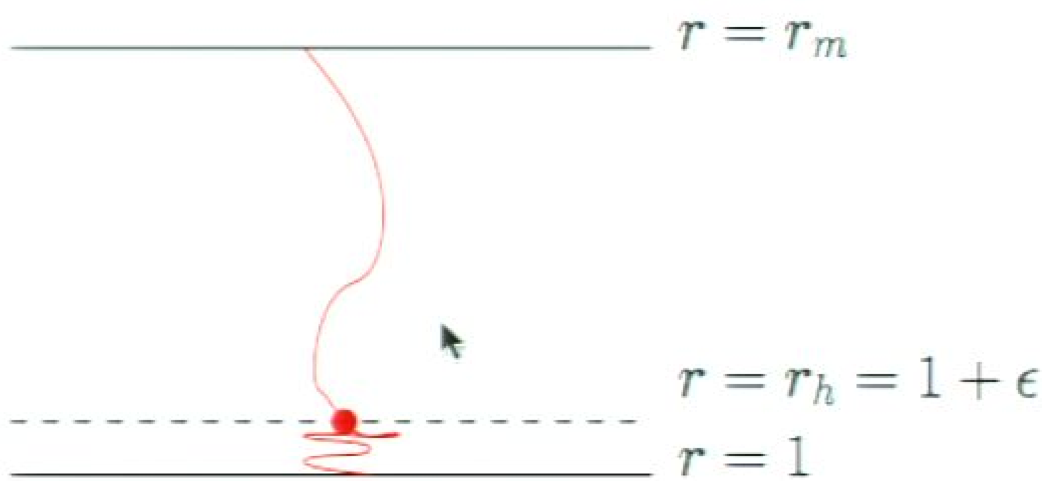
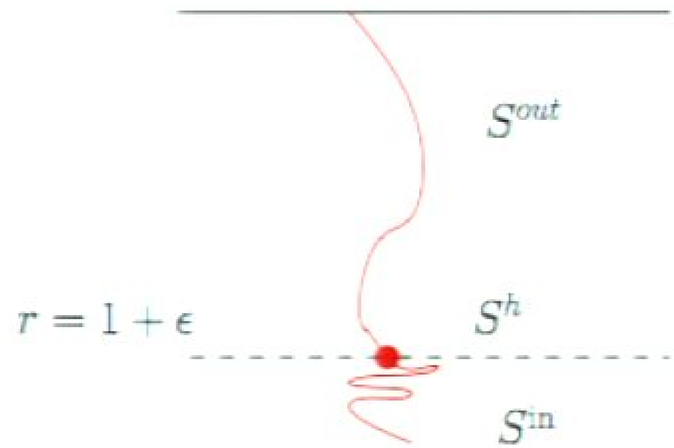
Strategy: Integrate out the fluctuations below the stretched horizon



Find an effective action and an EOM for the horizon endpoint x^h

$$S^h_{\text{eff}}$$

Classical Membrane Paradigm



- Re write the action

$$S = S^{\rm out} + S^{\rm in}$$
$$= (S^{\rm out} + S^h) + (S^{\rm in} - S^h)$$

- Vary the pieces

$$\delta S = (\delta S^{\rm out} + \delta S^h) + (\delta S^{\rm in} - \delta S^h)$$

Choose S^h so that the out and in actions are separately minimized

Actually doing the membrane paradigm

$$S^{\rm in} - S^h = -\int {\rm d}t{\rm d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x_r)(\partial_r x_a) - \overbrace{\frac{m}{2f}}^{\text{mass}} \dot{x}_r \dot{x}_a \right] - S^h$$

- Varying with respect to x_a gives

$$\left[\partial_r \left(T_o(r) \partial_r x_r(\omega, r) \right) + \frac{m\omega^2}{f} x_r(\omega, r) \right] = 0$$

$$T_o(r_h) \partial_r x_r(\omega, r) = \frac{\delta S^h}{\delta x_a} \qquad \Leftarrow \text{Horizon boundary condition}$$

- Recall: the classical solution obeys infalling boundary condtion: $x = C(r-1)^{-i\omega/4\pi T}$

$$T_o(r) \partial_r x_r = -i\omega\eta \, x_r^h \qquad \Leftarrow \text{Infalling bc}$$

So at a classical level the horizon action

$$S^h_{\rm eff} = \int_\omega x_a^h \, [\underbrace{-i\omega\eta}_{G_R(\omega)}] \, x_r^h$$

Actually doing the membrane paradigm

$$S^{\text{in}} - S^h = -\int \mathrm{d}t\mathrm{d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x_r)(\partial_r x_a) - \overbrace{\frac{m}{2f}}^{\text{mass}} \dot{x}_r \dot{x}_a \right] - S^h$$

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Brownian Motion of the horizon

$$iS^h_{\rm eff} = -i\int_\omega x^h_a\Big[\underbrace{-i\omega\eta}_{G_R(\omega)}\Big]x^h_r - \frac{1}{2}\int_\omega x^h_a\Big[\underbrace{(1+2n)\omega\eta}_{G_{\rm sym}(\omega)}\Big]x^h_a$$

1. Dynamics of String in AdS_5

$$Z = \int \left[\mathbb{D}x_1\,\mathrm{D}x^h_1\right]\left[\mathbb{D}x_2\,\mathrm{D}x^h_2\right] e^{iS_1 - iS_2}\, e^{iS^h_{\rm eff}}$$

2. Particle Dynamics

$$Z_Q = \int Dx_r Dx_a\, e^{-i\int x_a[-M\omega^2]x_r}\, \underbrace{e^{-i\int d\omega\, x_a G_R(\omega) x_r}\, e^{-\frac{1}{2}\int d\omega\, x_a\, G_{\rm sym}(\omega)\, x_a}}_{e^{iS_{\rm eff}}}$$

3. Go through the same steps as with the particle

$$\left[\partial_r\left(T_o(r)\partial_r x_r(\omega,r)\right) + \frac{m\omega^2}{f}x_r(\omega,r)\right] = 0 \quad \Leftarrow \text{Bulk equation of motion}$$

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Summary

Brownian Motion of the horizon

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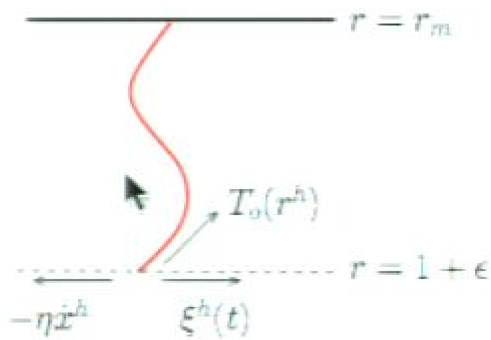
Summary

Hawking radiation and the random force

- Still overdamped motion with a random force

$$\overbrace{T_o(r_h)\partial_r x(t,r_h)}^{\text{Tension}} + \overbrace{\xi^h(t)}^{\text{Random force}} = \overbrace{\eta\,\dot{x}^h(t)}^{\text{Drag}}$$

- Picture



- Random force satisfies a horizon fluctuation dissipation theorem

$$\left\langle \xi^h(t)\xi^h(t') \right\rangle = 2T\eta\delta(t-t')$$

- More Generally when times of order $1/\pi T$

$$\left\langle \xi^h(\omega)\xi^h(-\omega) \right\rangle = (1+2n)\omega\eta$$

Gives rise to random motion on the string

Hawking radiation and the random force

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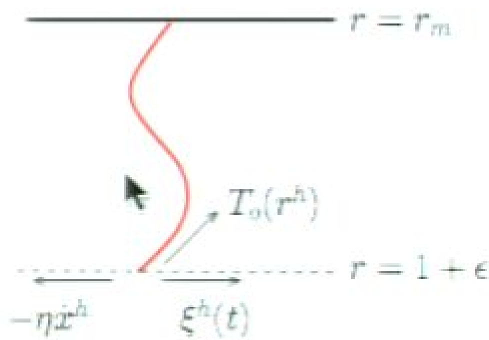

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Gives rise to random motion on the string

Actually doing the membrane paradigm

$$S^{\text{in}} - S^h = -\int \mathrm{d}t\mathrm{d}r \left[\frac{1}{2} \overbrace{T_o(r)}^{\text{Tension}} (\partial_r x_r)(\partial_r x_a) - \frac{\overbrace{m}^{\text{mass}}}{2f} \dot{x}_r \dot{x}_a \right] - S^h$$

- Varying with respect to x_a gives

$$\left[\partial_r \big(T_o(r) \partial_r x_r(\omega, r) \big) + \frac{m\omega^2}{f} x_r(\omega, r) \right] = 0$$

$$T_o(r_h)\partial_r x_r(\omega, r) = \frac{\delta S^h}{\delta x_a} \qquad \Leftarrow \text{Horizon boundary condition}$$

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Brownian Motion of the horizon

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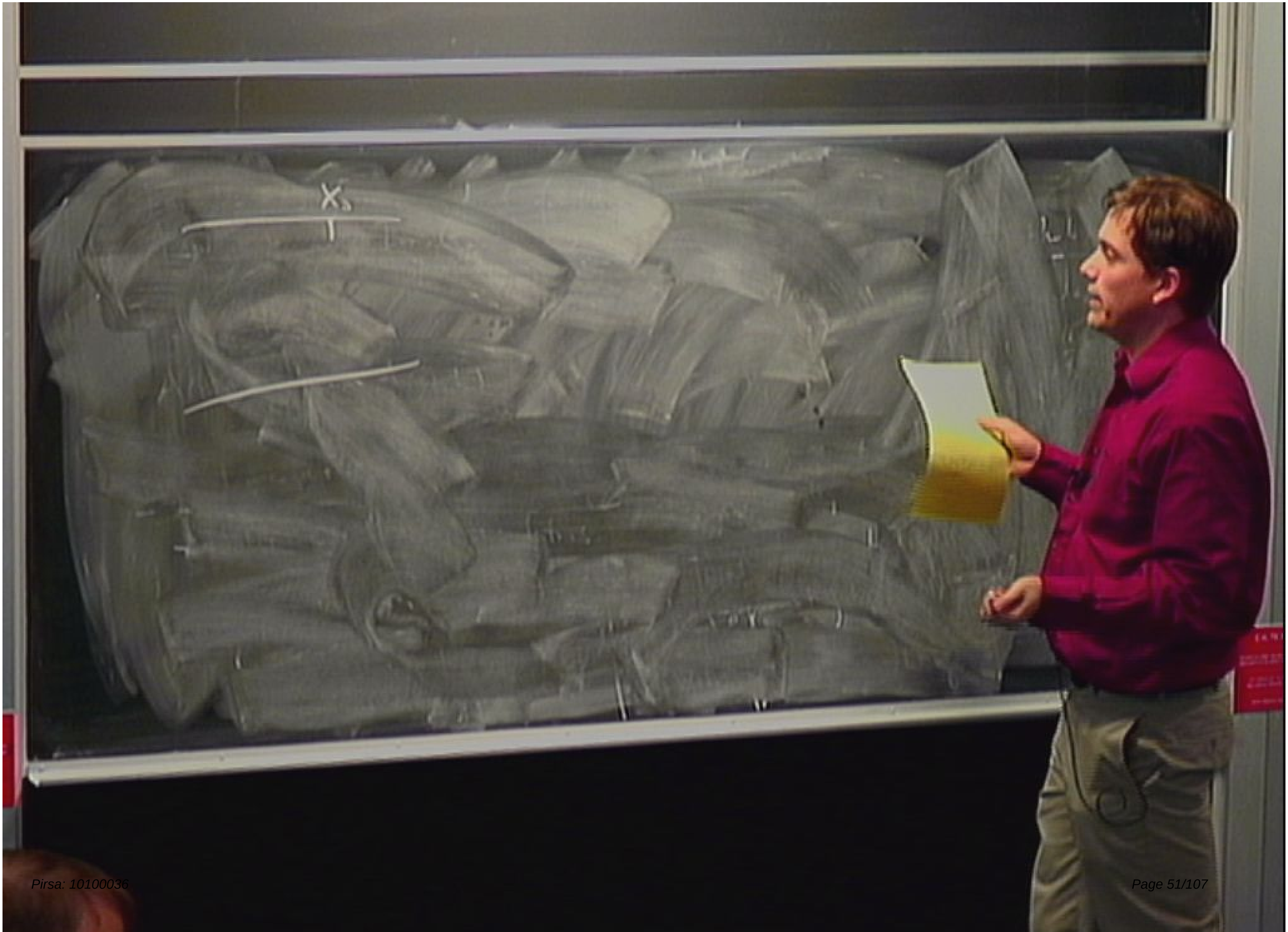
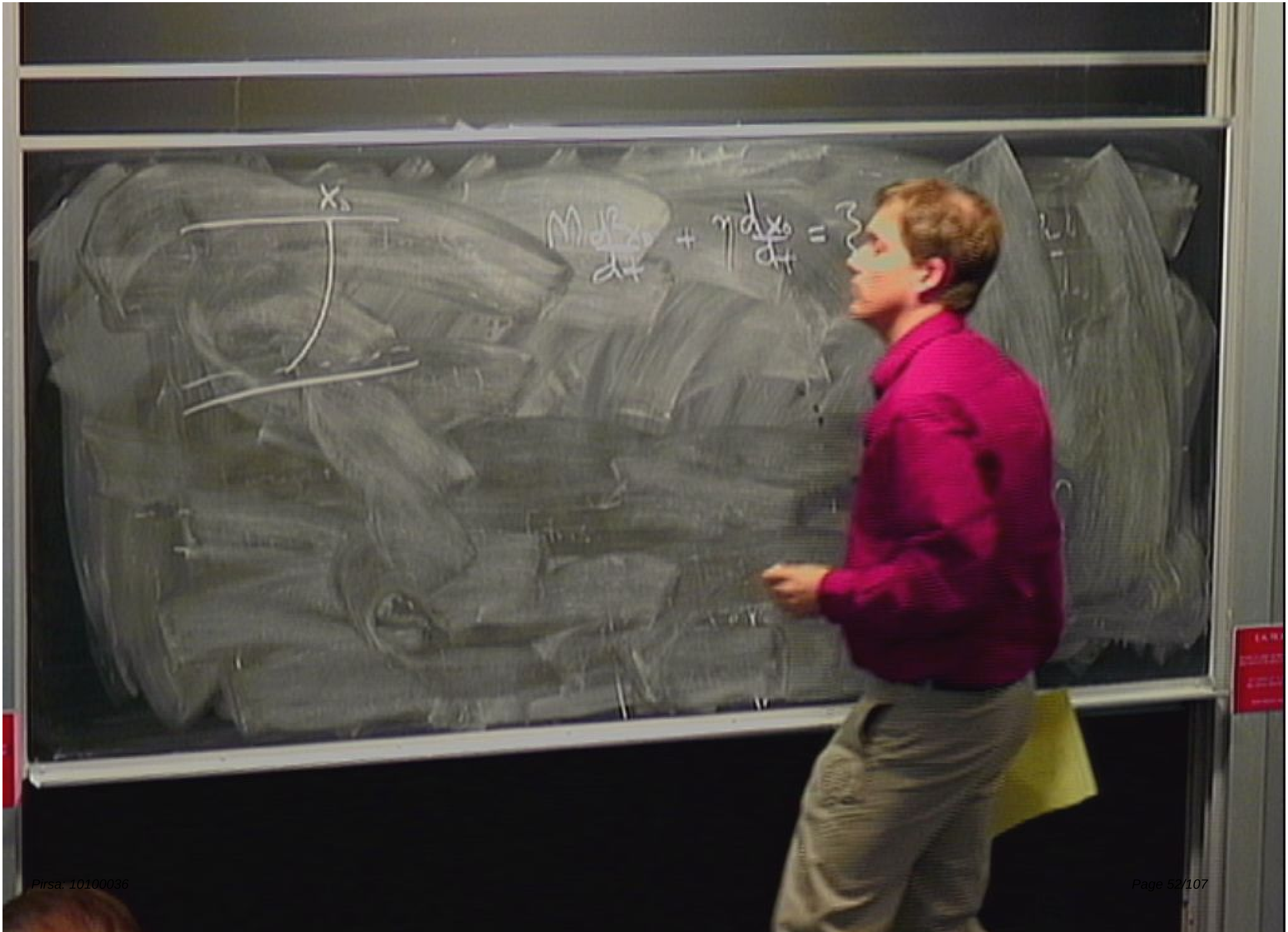
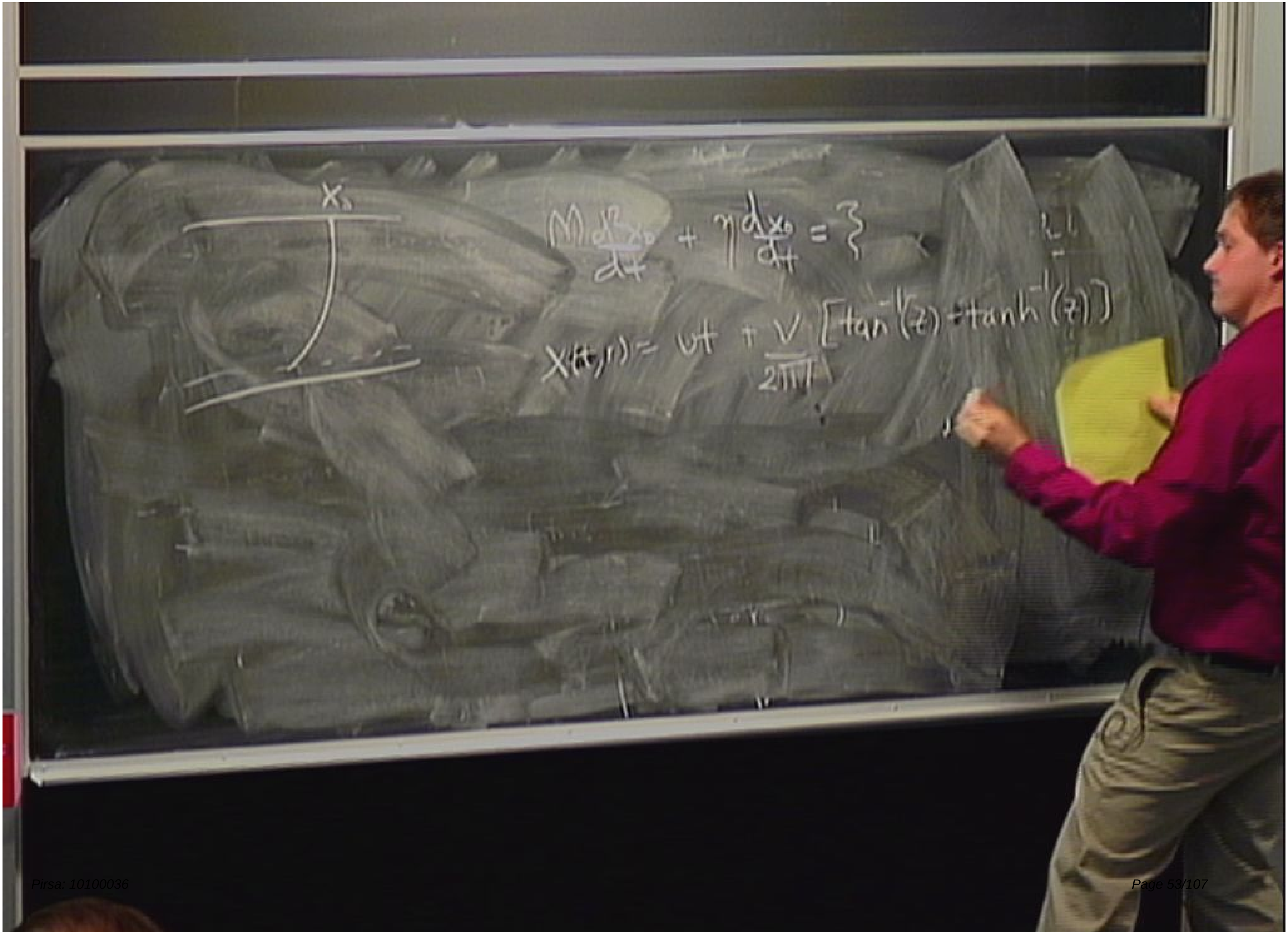
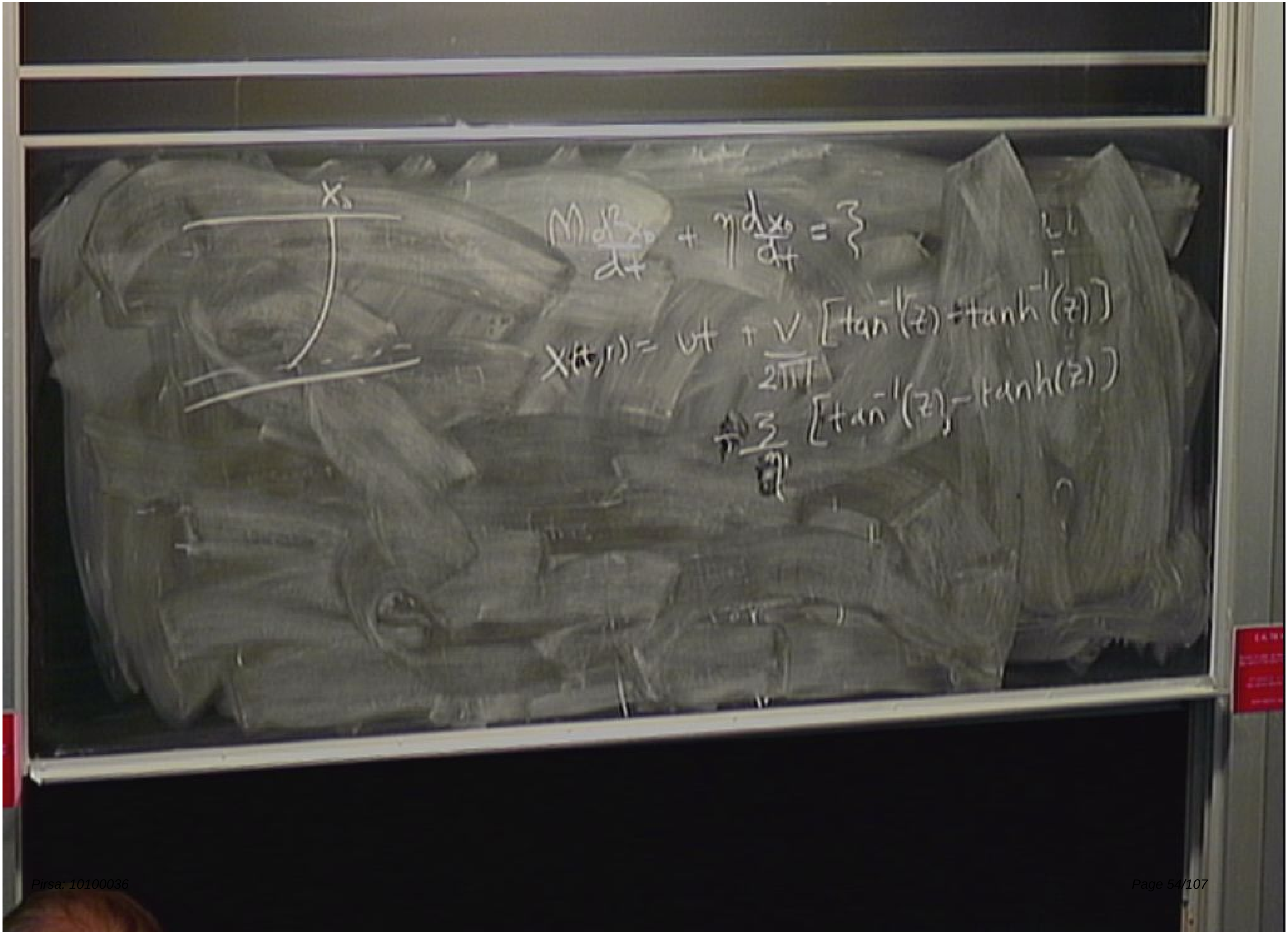



M d²x₀/dt² + η dx₀/dt = ξ


x₀
M d²x₀/dt² + η dx₀/dt = ξ
X(t,r) = vt + V/2πη [tan⁻¹(z) + tanh⁻¹(z)]



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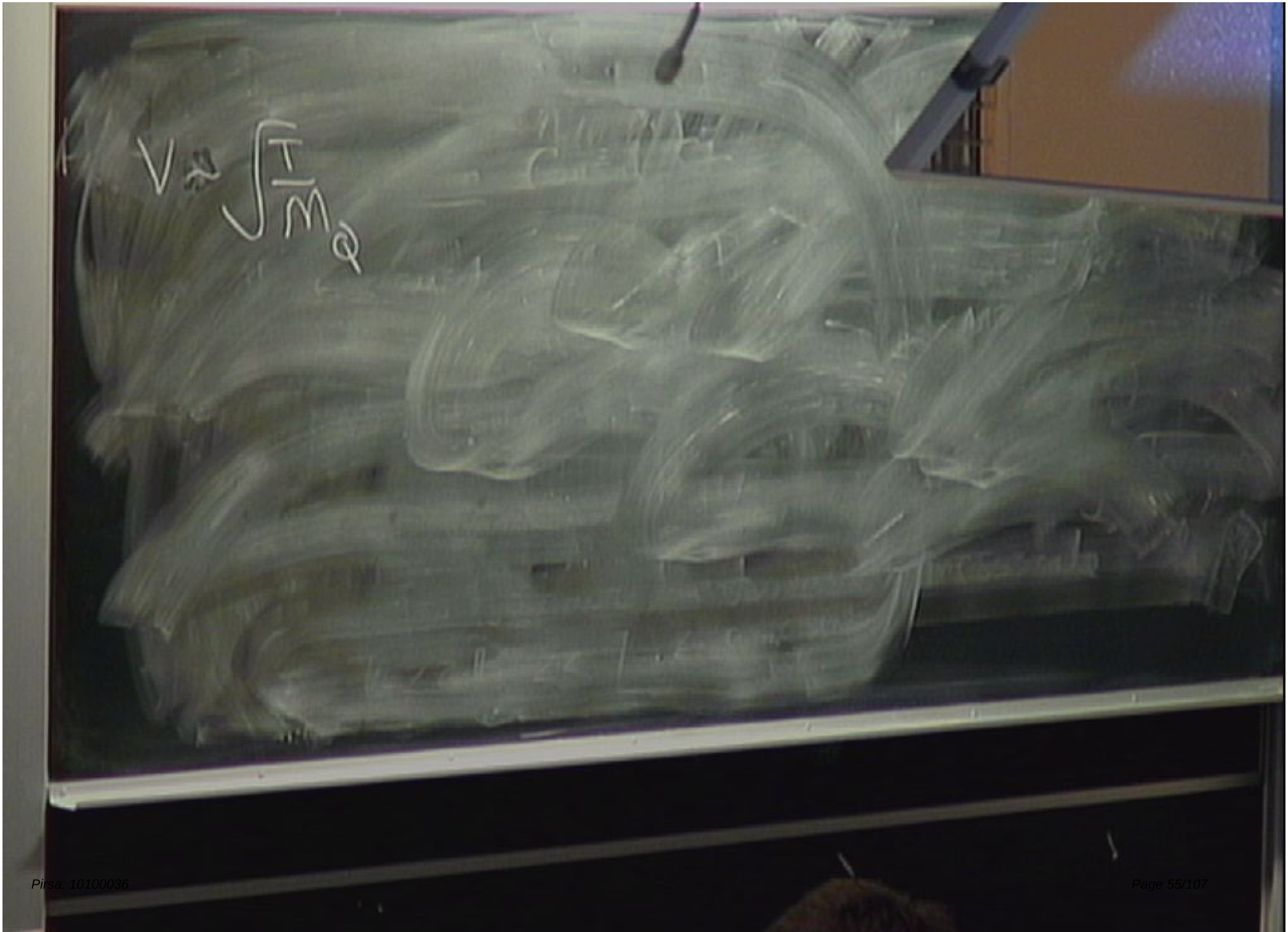
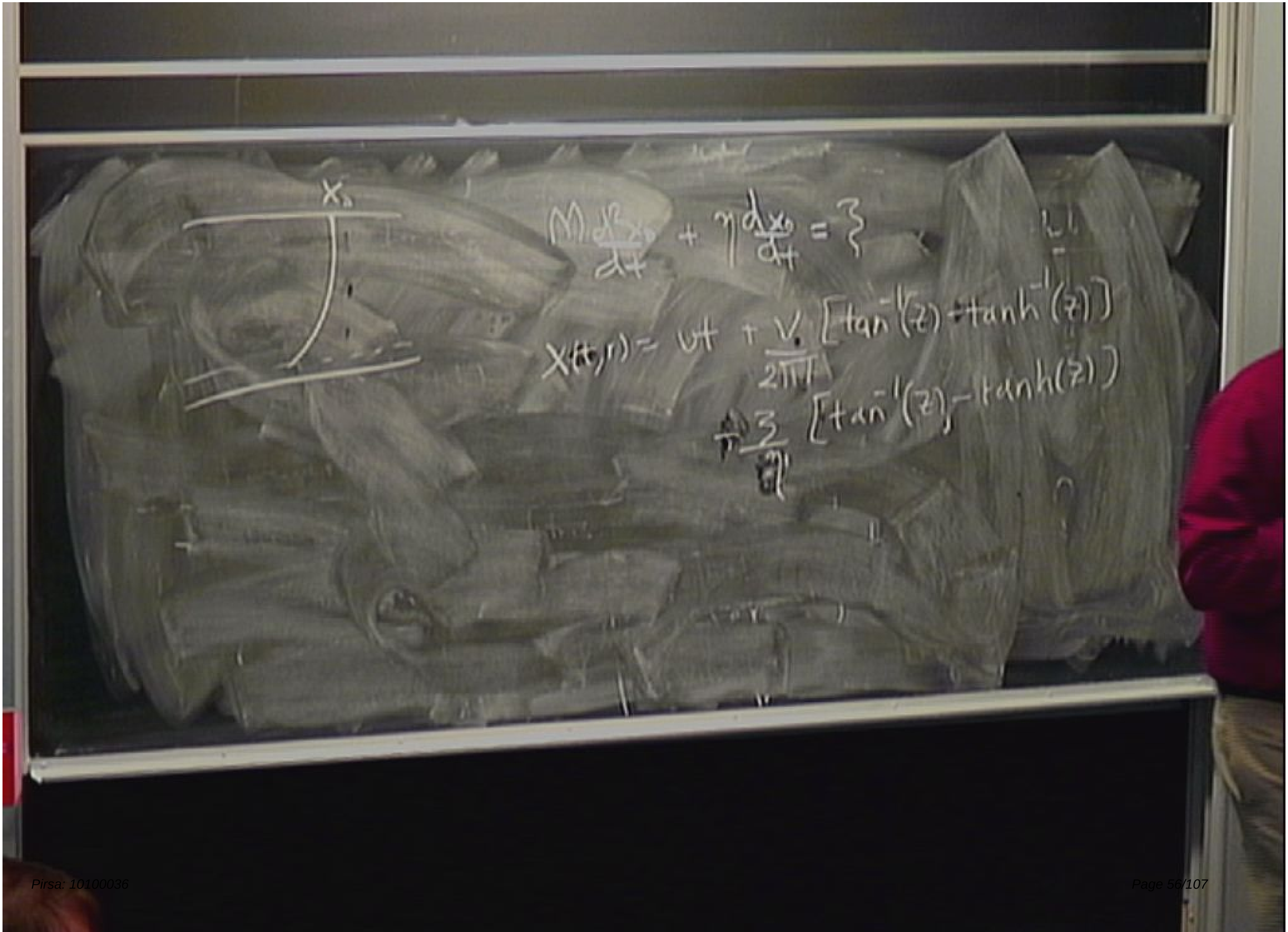

$V \approx \sqrt{\frac{T}{M_Q}}$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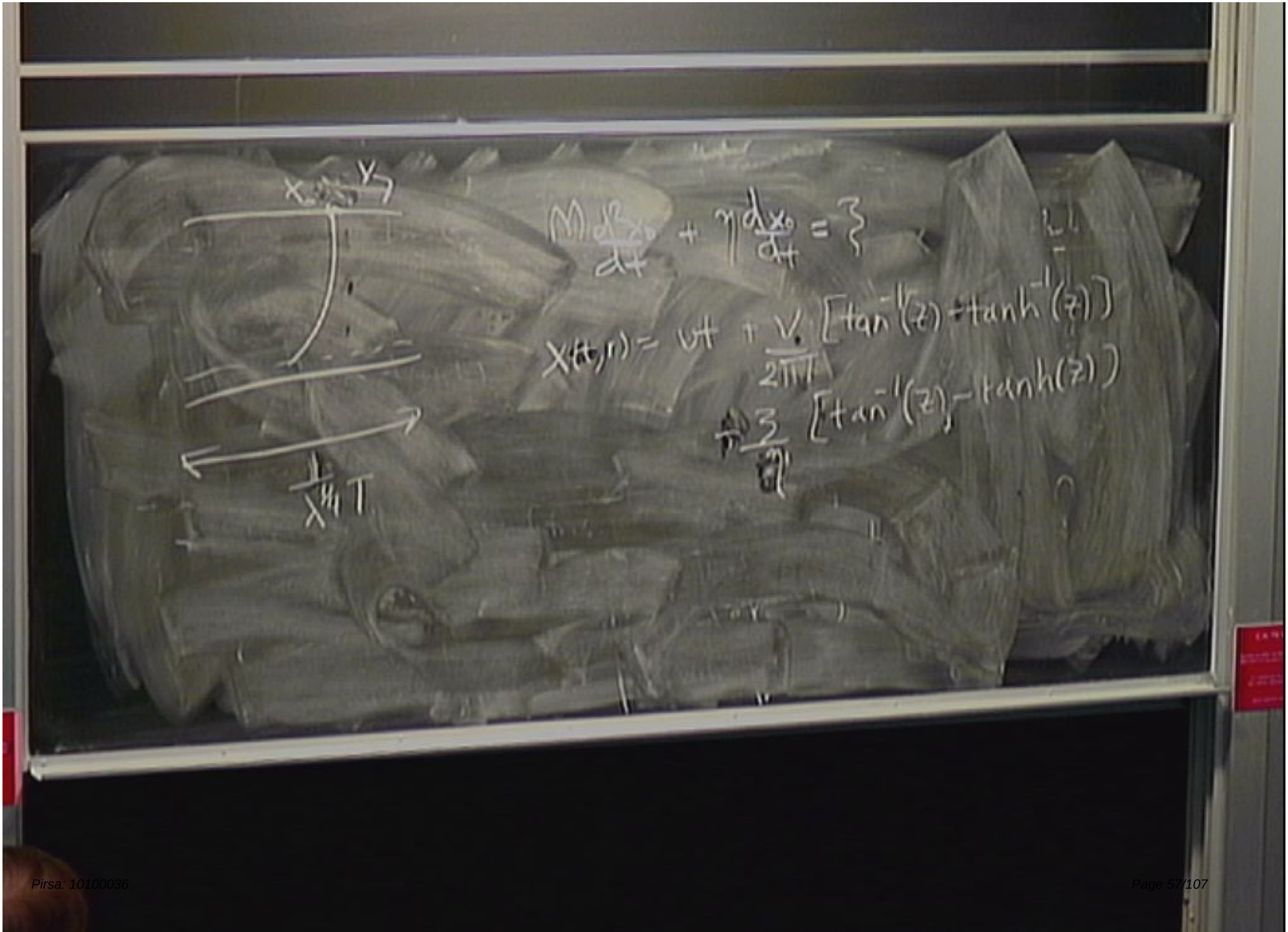
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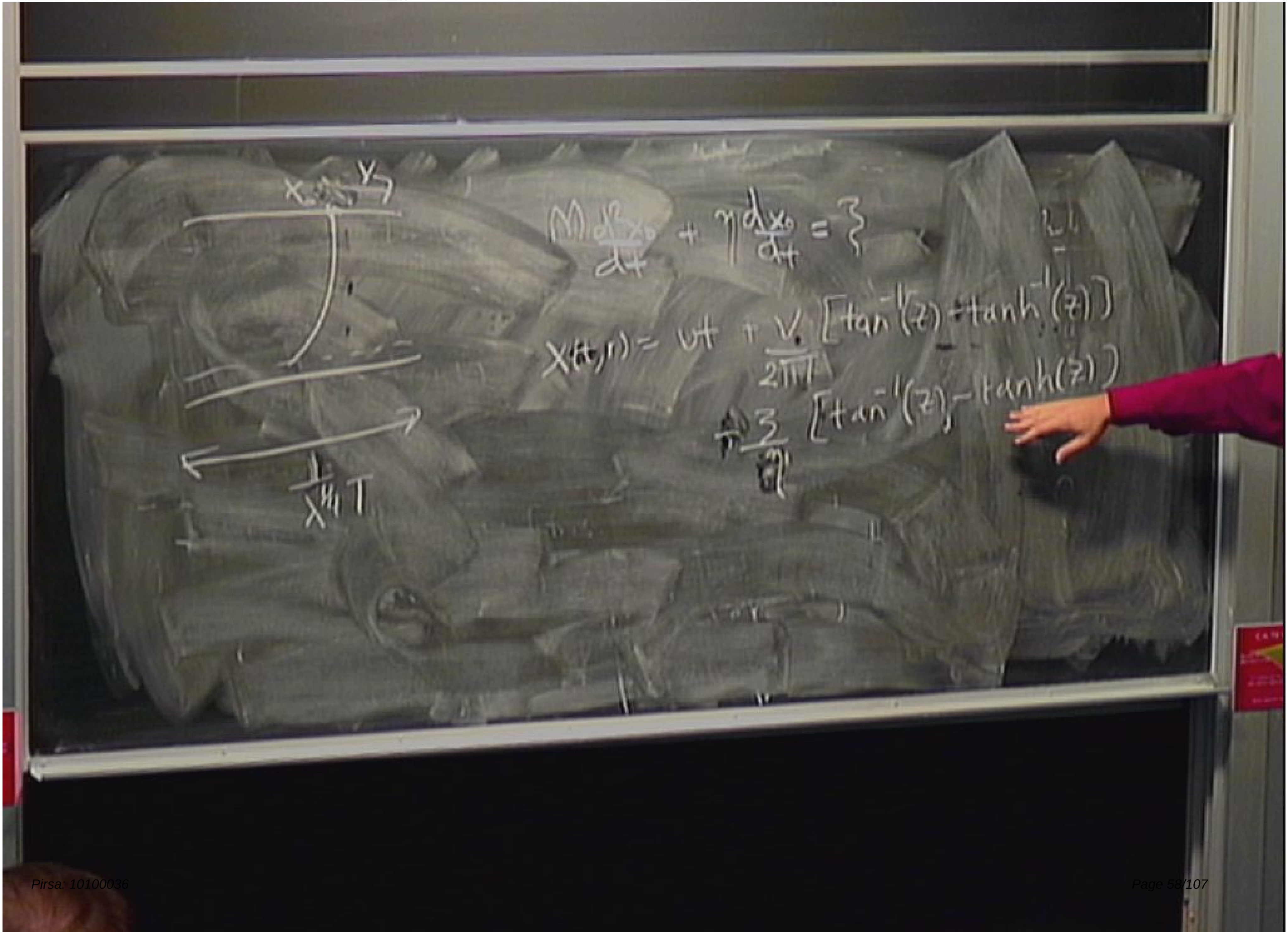
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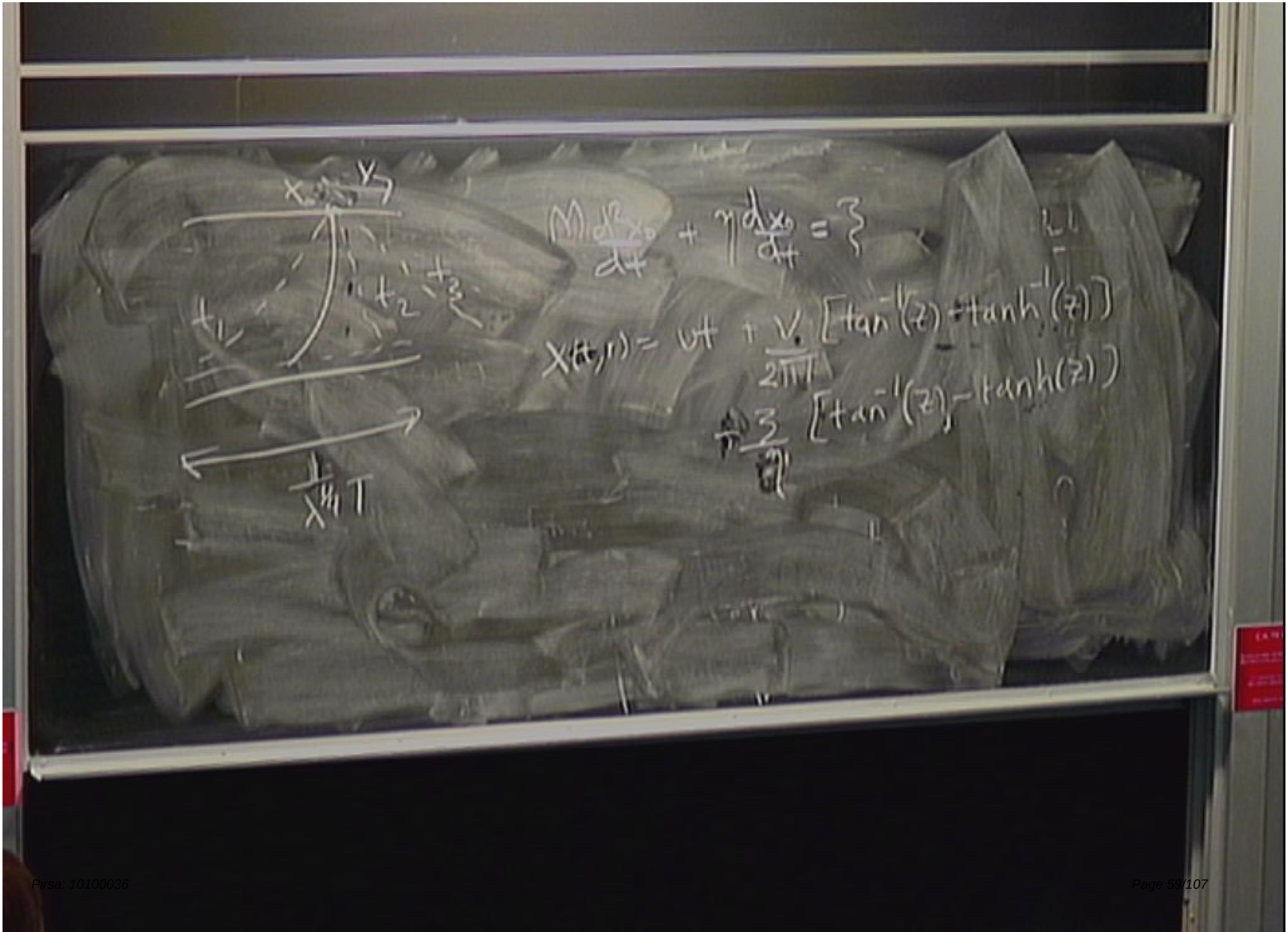

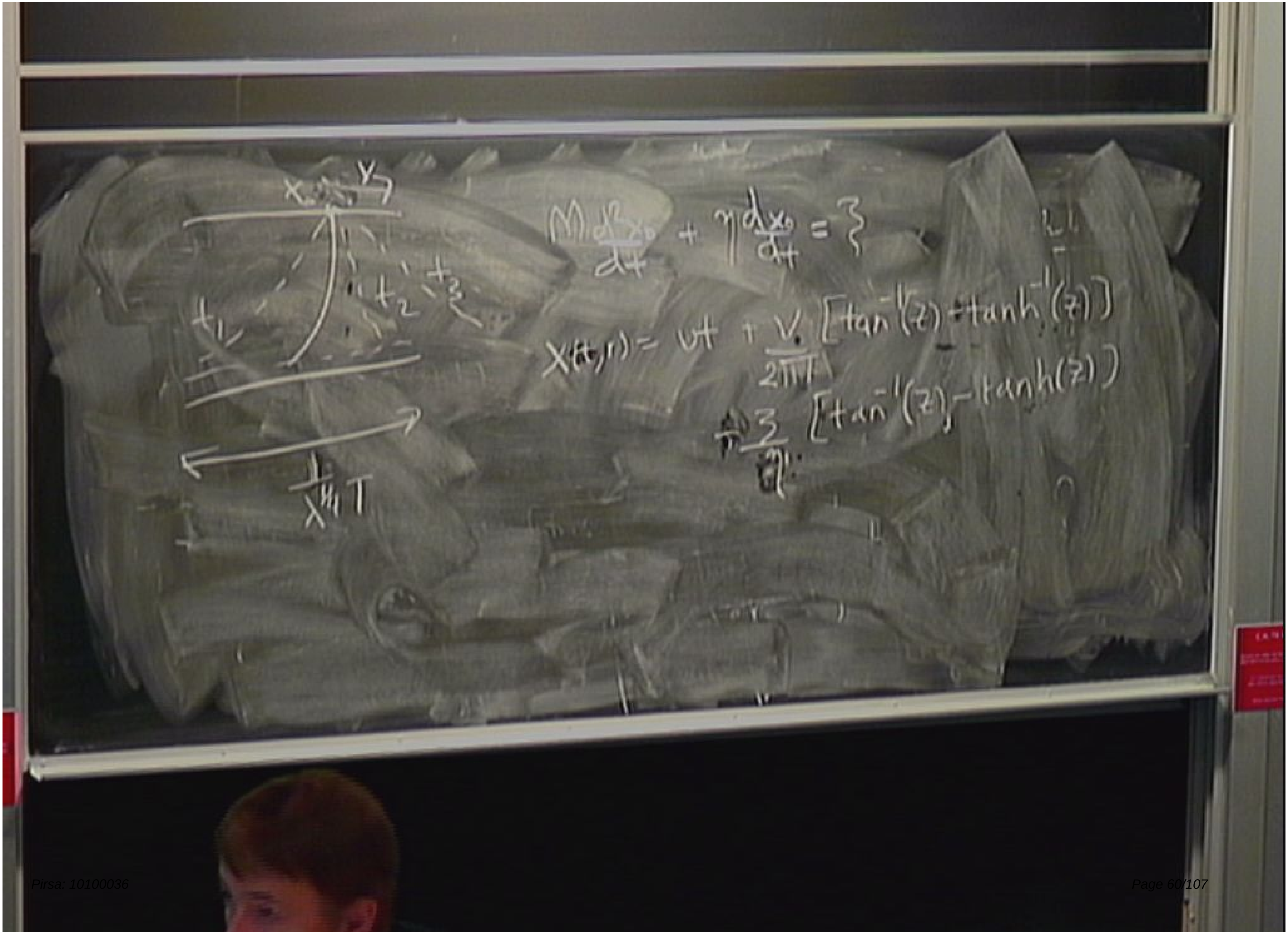

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Brownian Motion of the horizon

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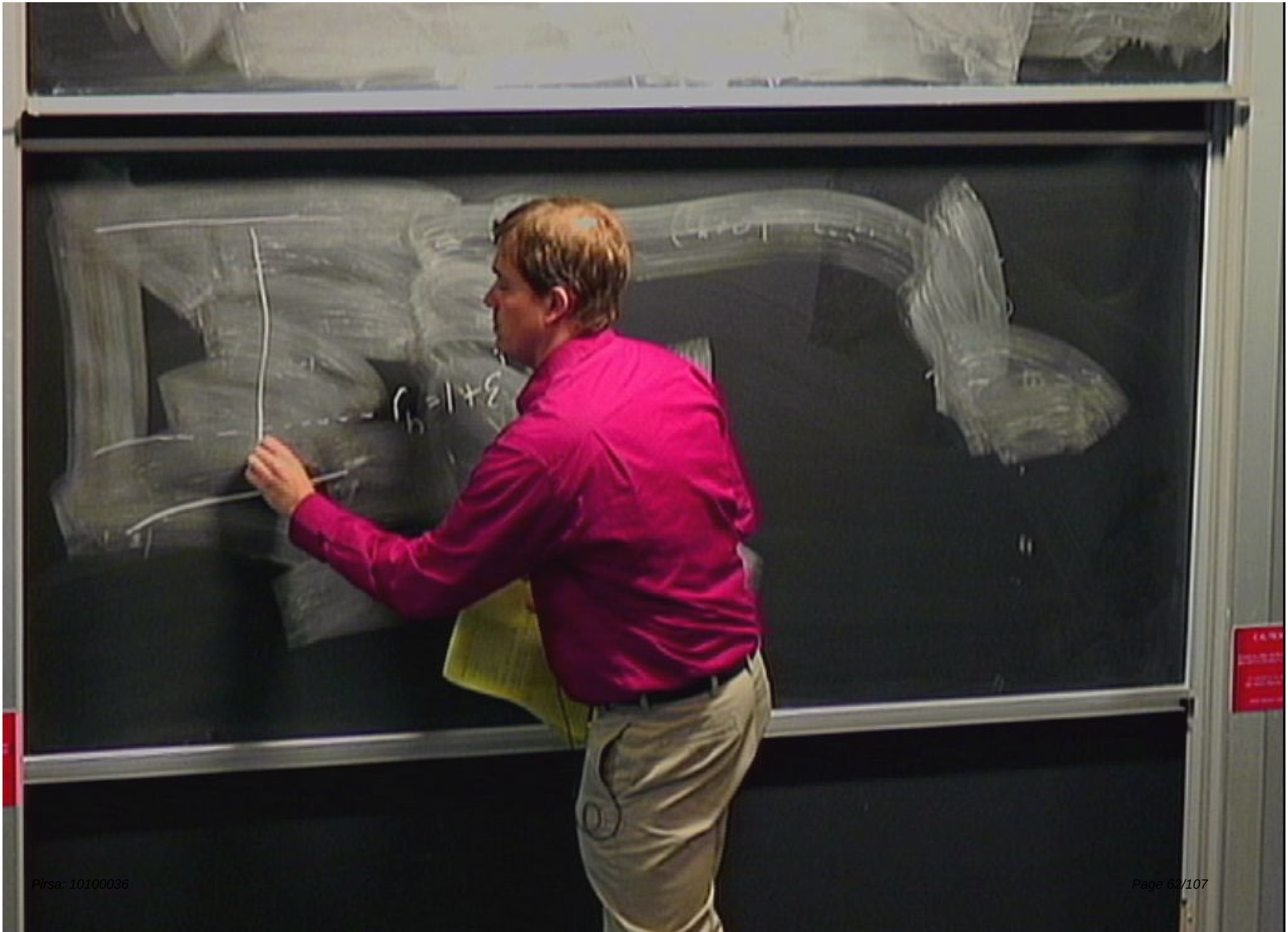
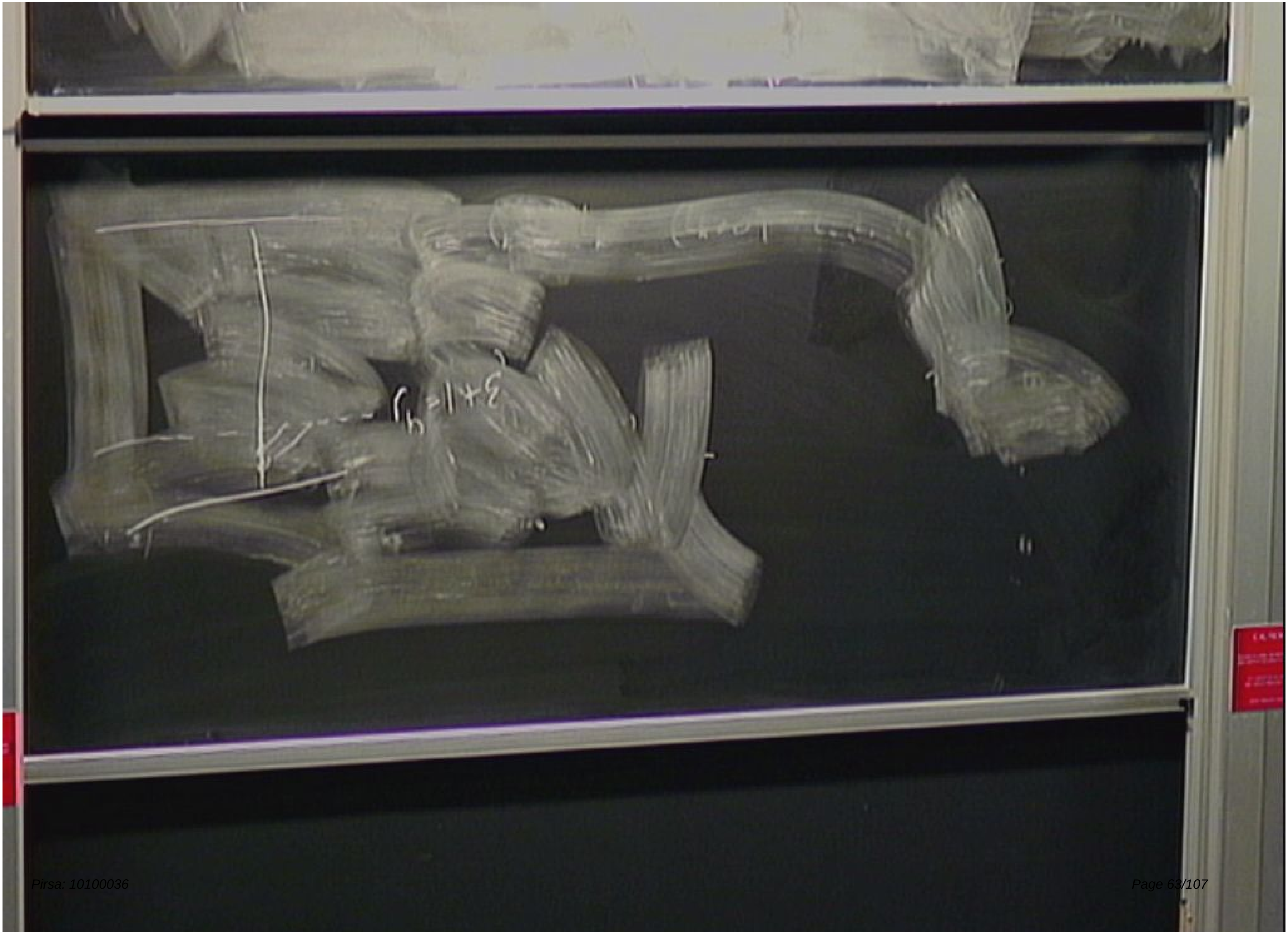
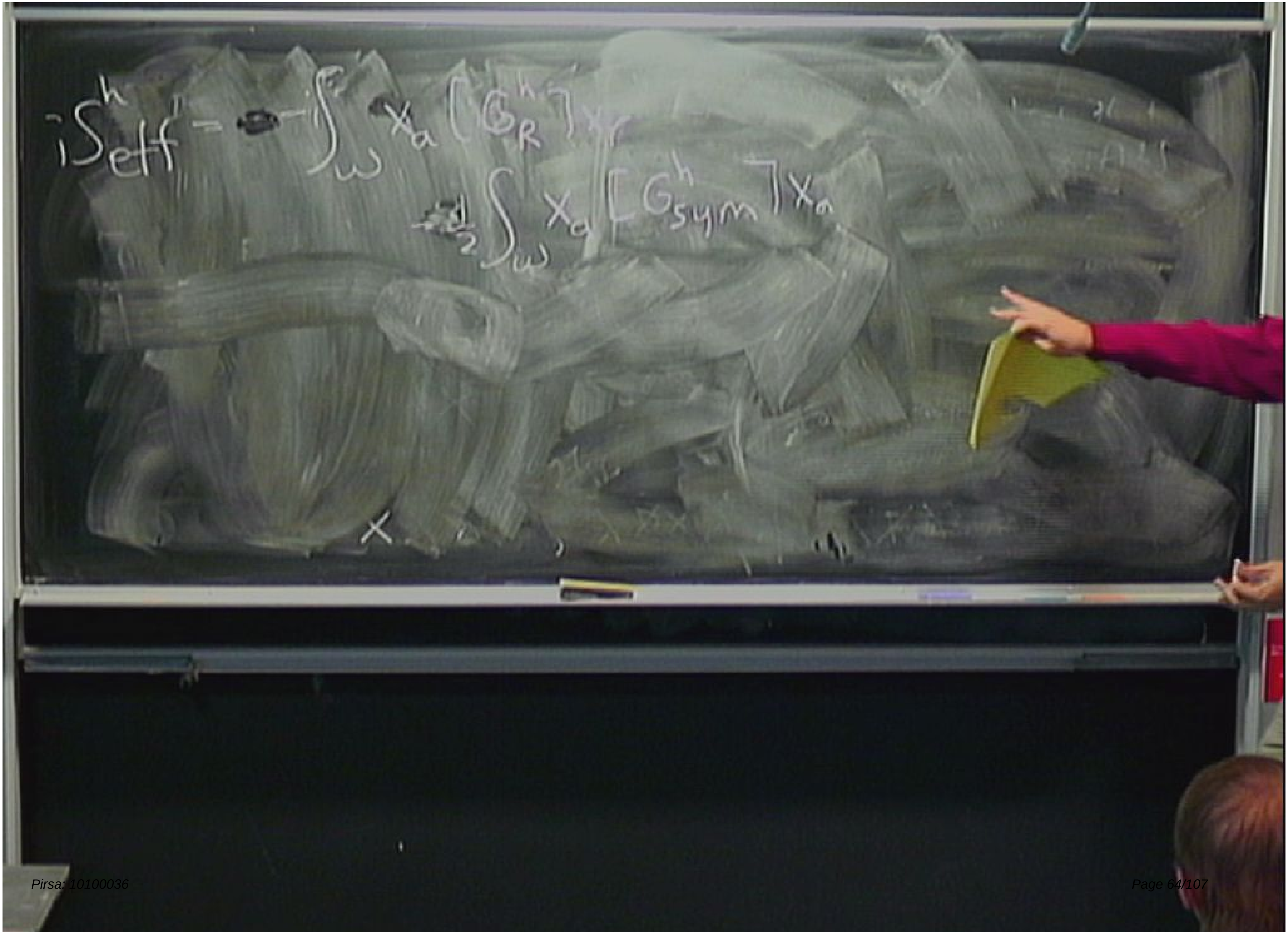





$iS^h_{eff} = -i\int_\omega x_a [G^h_R] x_r$
$\frac{1}{2}\int_\omega x_a [G^h_{sym}] x_a$

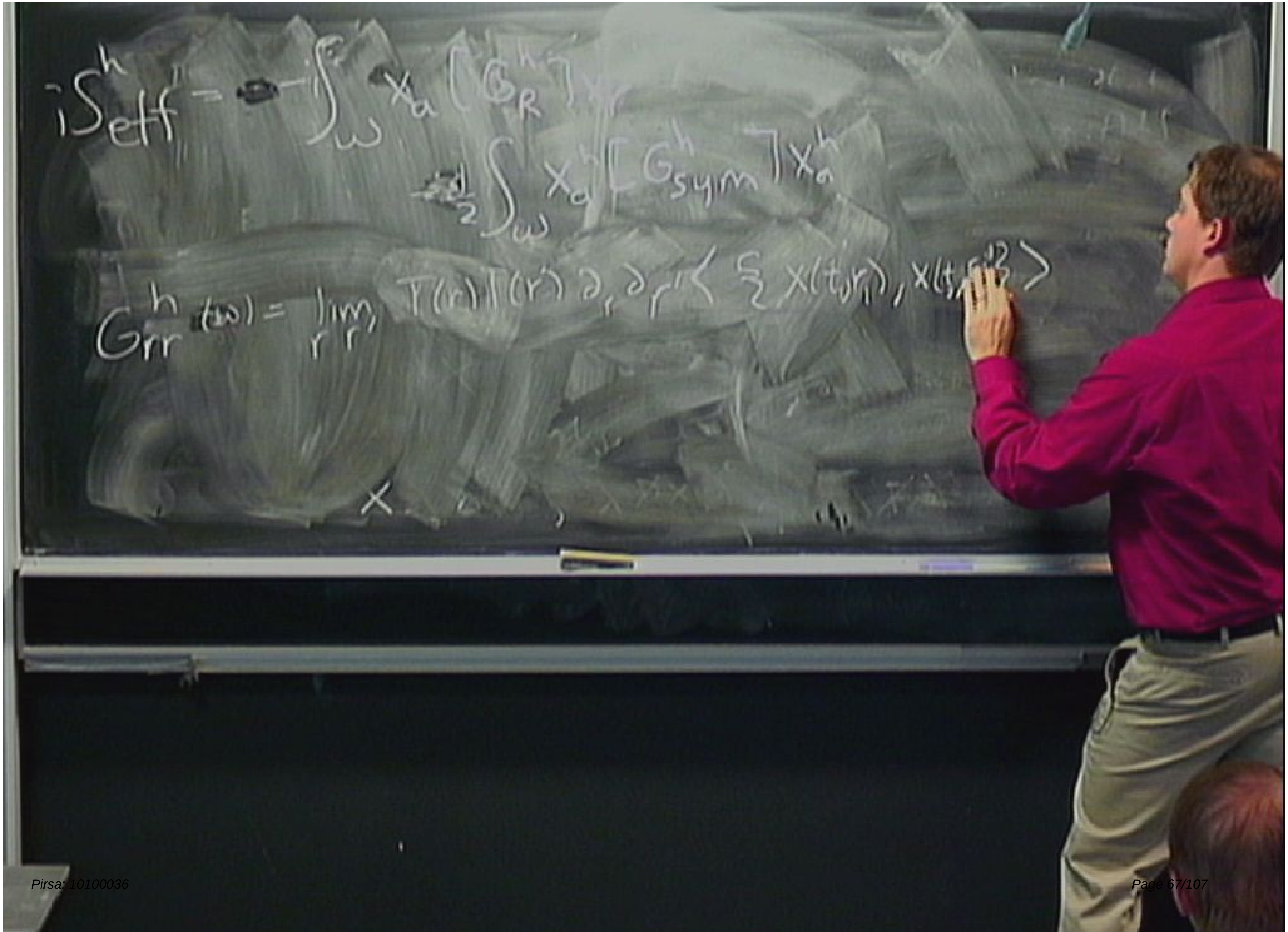
$$iS^h_{eff} = -i\int_\omega x_a [G_R^h] x_r$$

$$\frac{1}{2}\int_\omega x_a^h [G^h_{sym}] x_a^h$$

$$iS^h_{eff} = -i\int_\omega X_a [G^h_R] X_r$$

$$+ \frac{1}{2}\int_\omega X_a [G^h_{sym}] X^h_a$$

$$G^h_{rr}(\omega) = \lim_{r r'} T(r)T(r')\partial_r \partial_{r'} \langle \{ X(t,r), X(t,r') \} \rangle$$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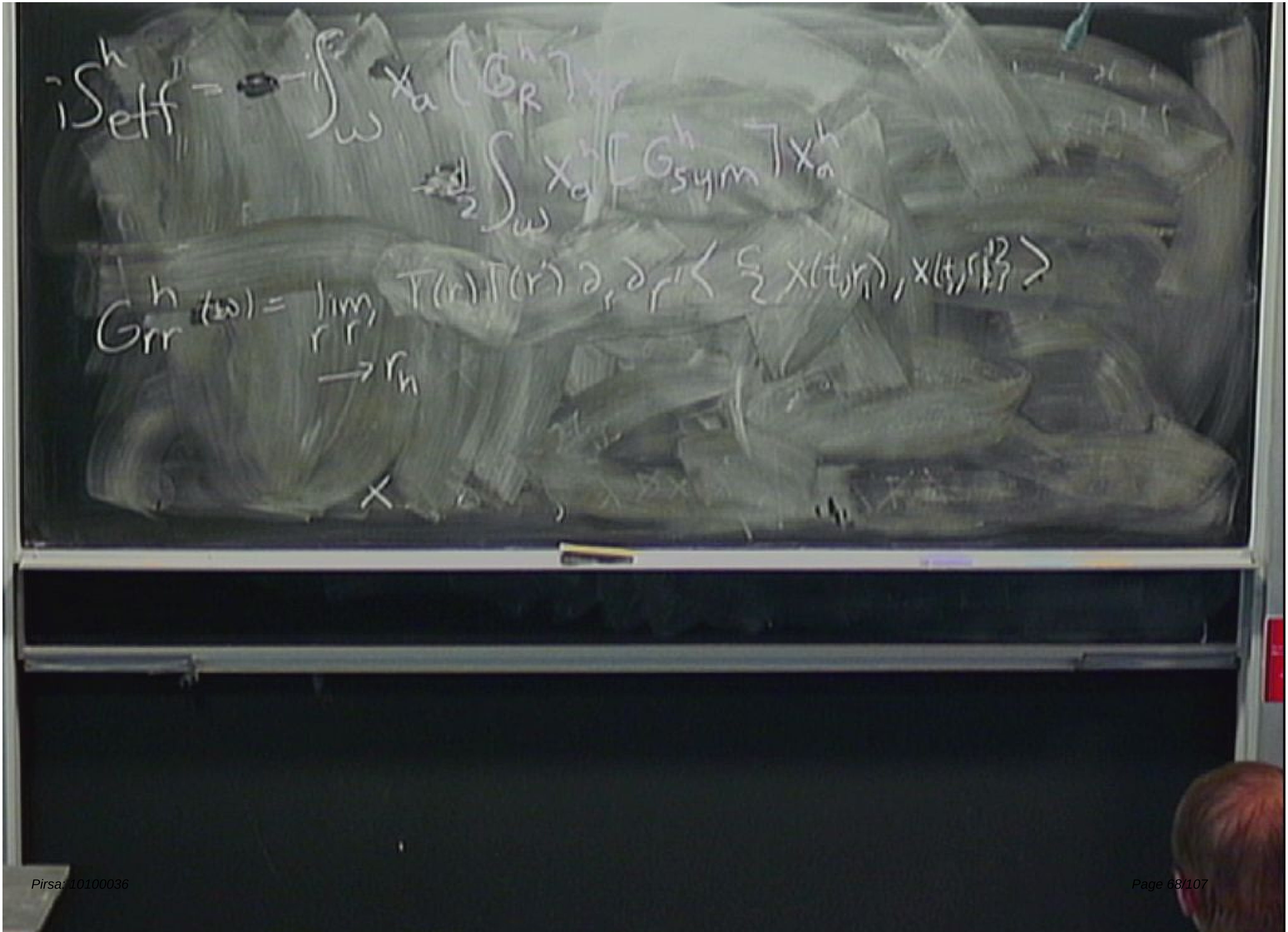
iS^h_eff = -i ∫_ω X_a [G^h_R] X_r
1/2 ∫_ω X_a^h [G^h_sym] X_a^h
G^h_rr(ω) = lim_{r r' → r_h} T(r) T(r') ∂_r ∂_r' < {X(t,r), X(t',r')} >

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$$+ \frac{1}{2}\int_\omega X^h_a [G^h_{sym}] X^h_a$$

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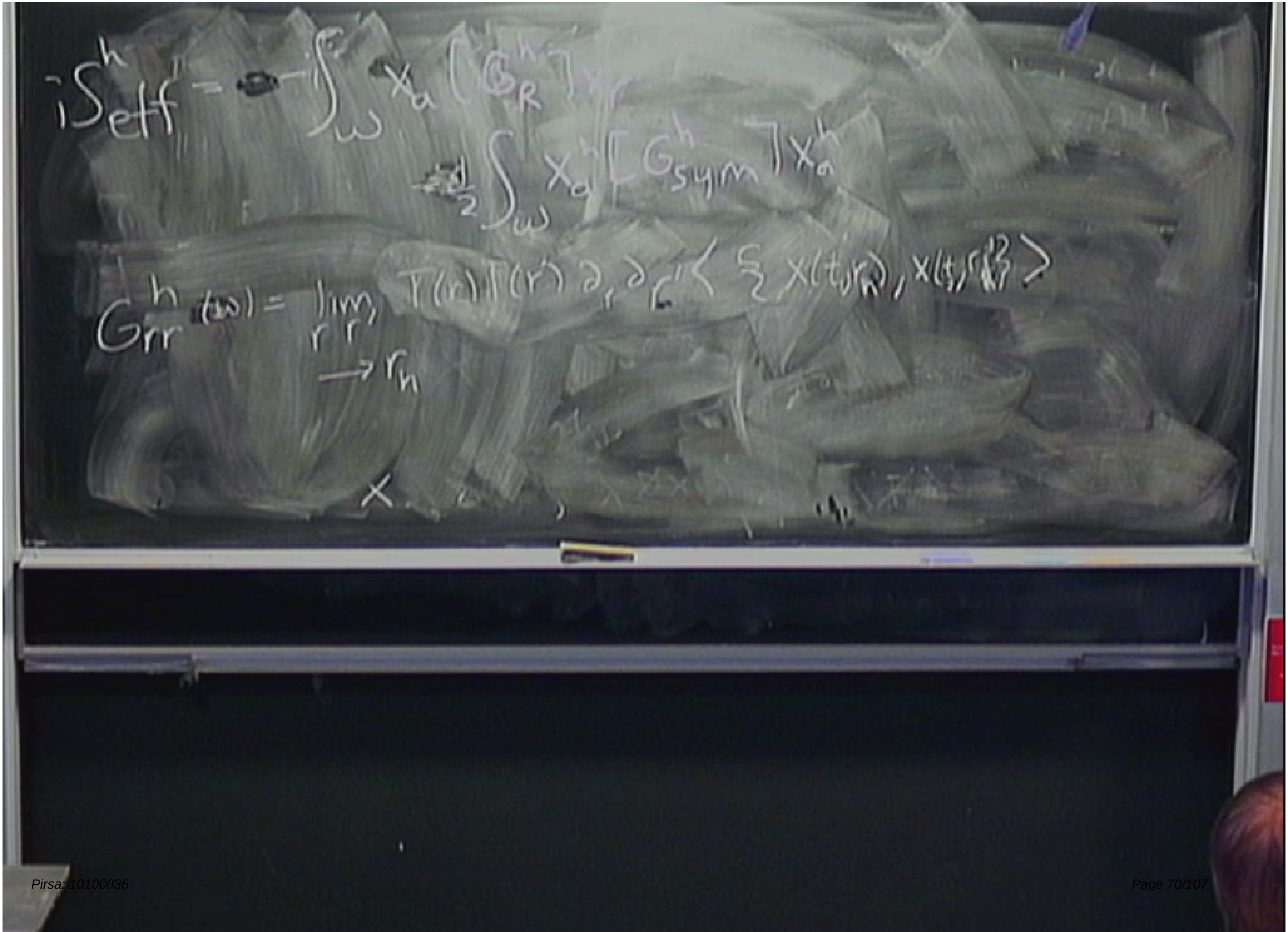
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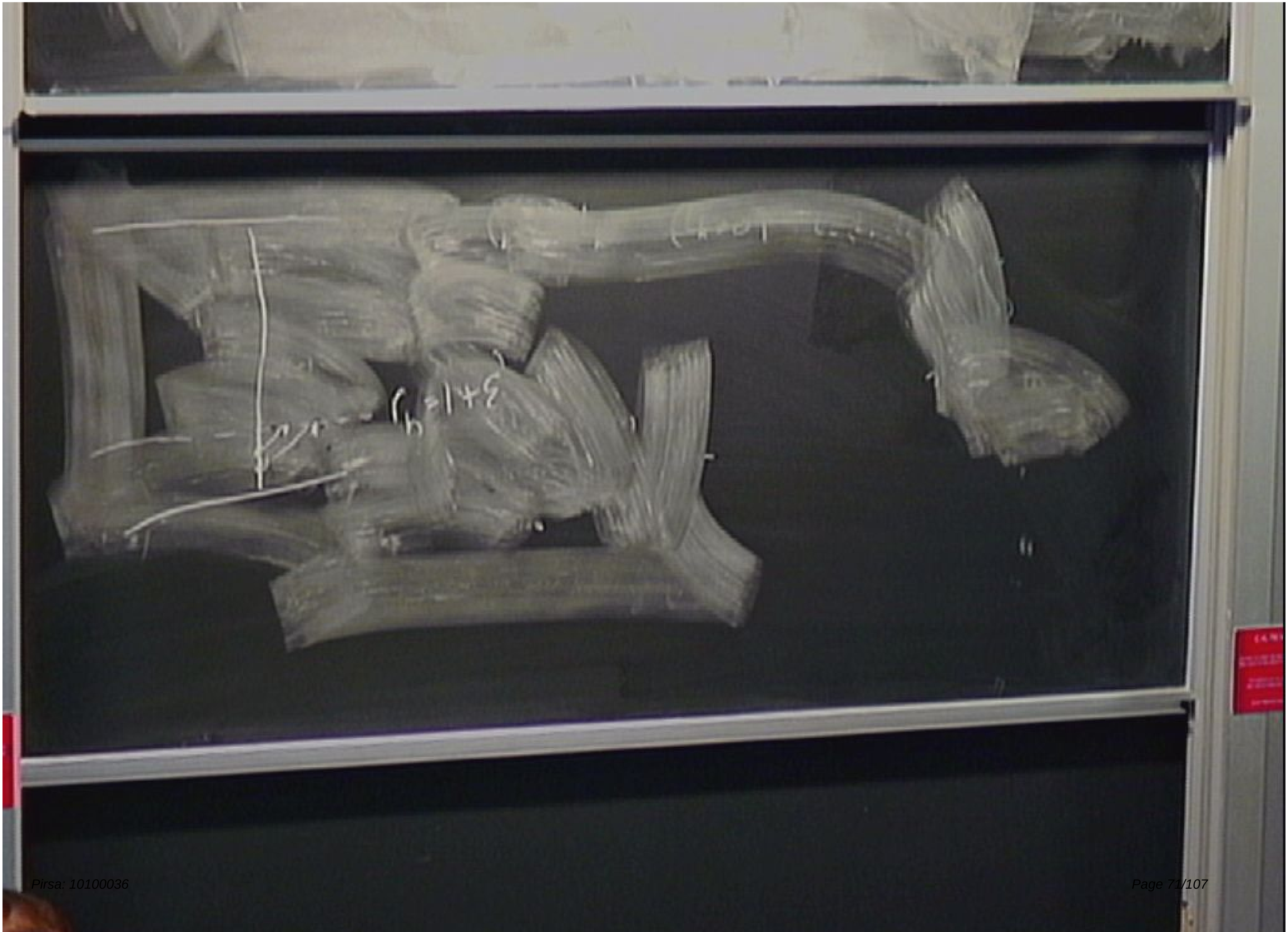

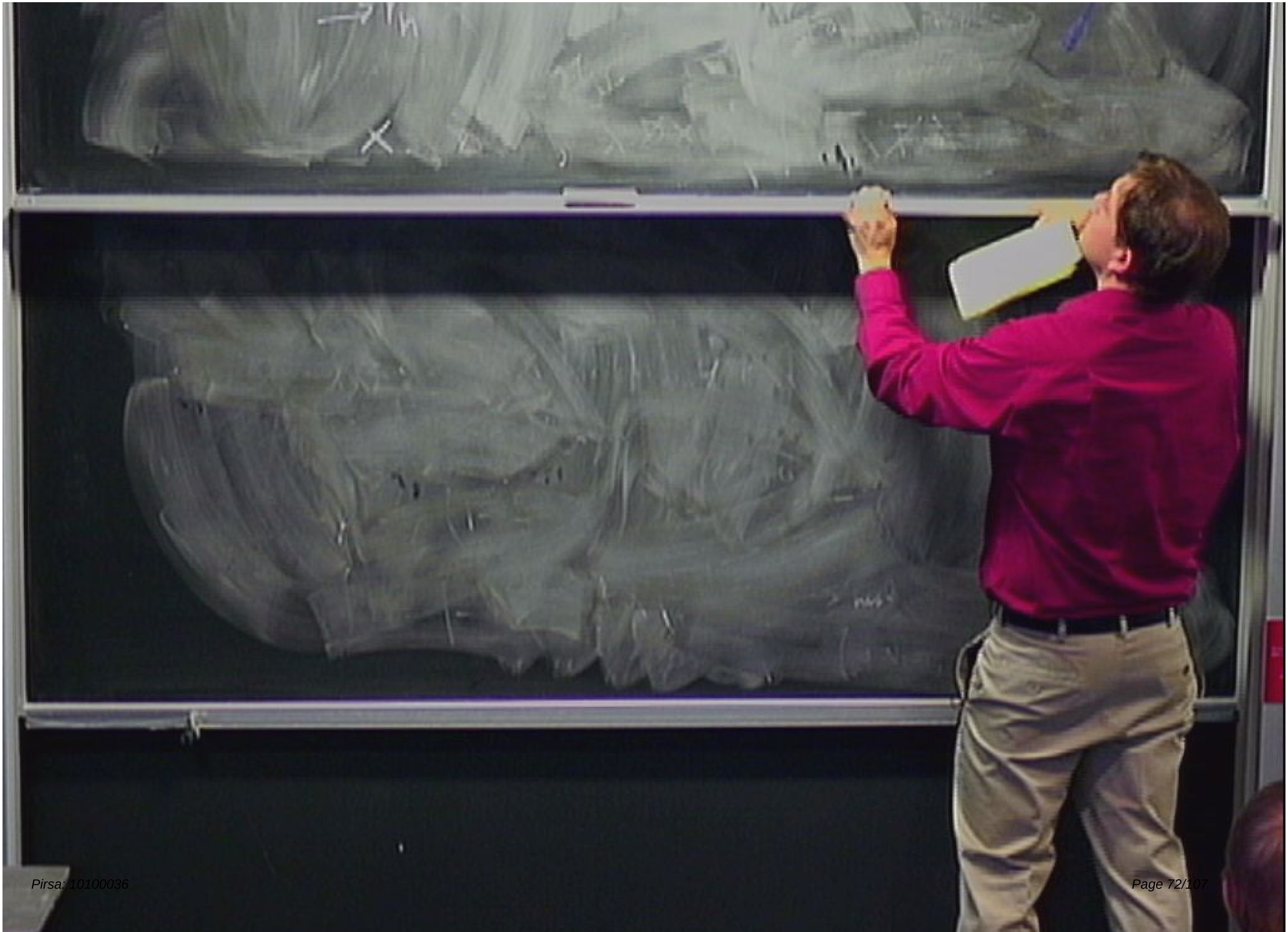

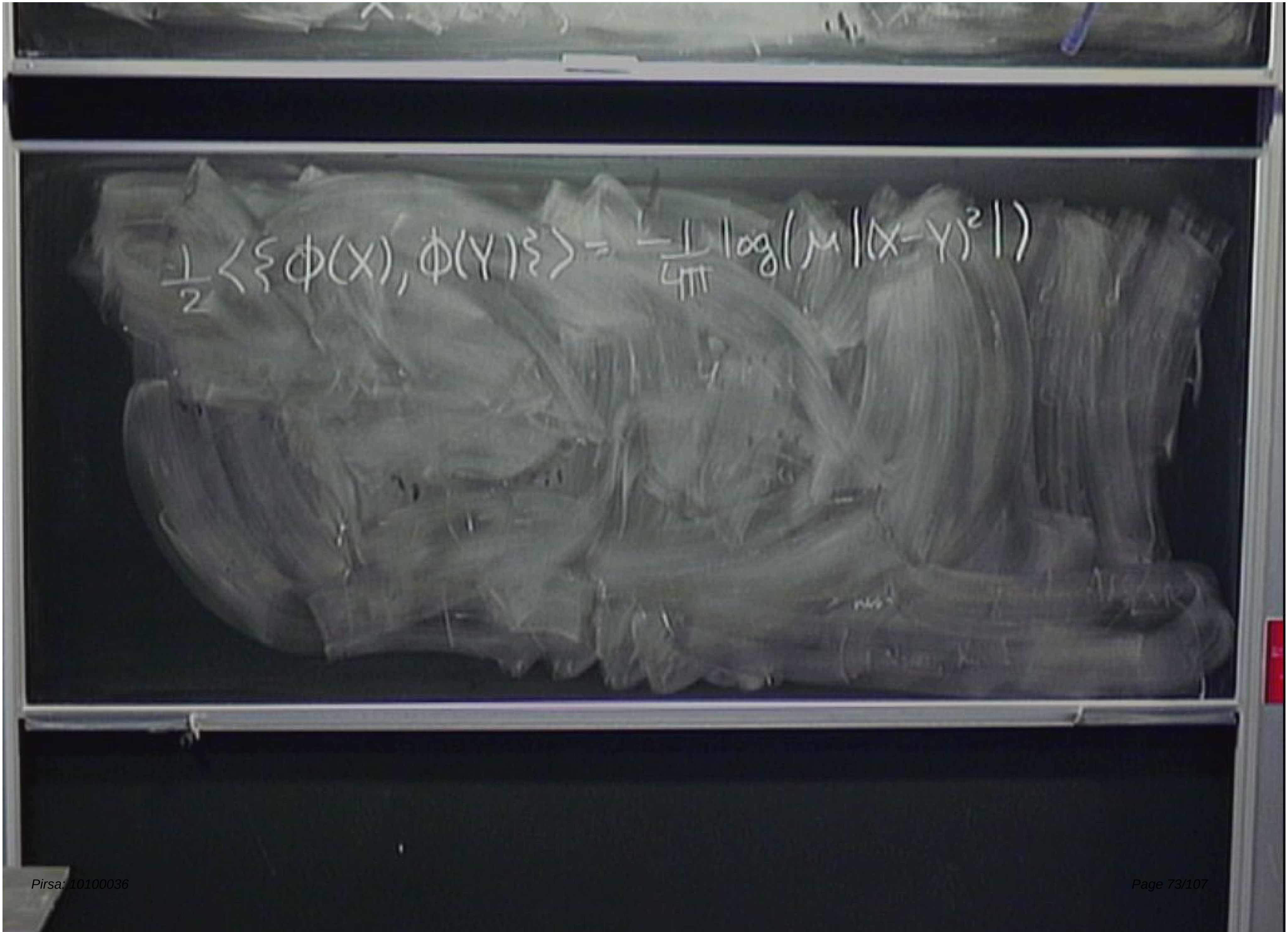
$\frac{1}{2}\langle\{\phi(x),\phi(y)\}\rangle = -\frac{1}{4\pi}\log(\mu|(x-y)^2|)$

massless

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massless

$$\frac{1}{2}\langle\{\phi(X),\phi(Y)\}\rangle = -\frac{1}{4\pi}\log(\mu|(X-Y)^2|)$$

So

$$ds^2 = (\pi T)^2\left[-A\,dv^2 + \frac{2}{\pi T}\,dr\,dv\right]$$

$$V = t + \frac{1}{4\pi T}\log(r-1)$$

massless

$$\frac{1}{2}\langle\{\phi(X),\phi(Y)\}\rangle = -\frac{1}{4\pi}\log(\mu|(X-Y)^2|)$$

So

$$ds^2 \to (\pi T)^2\left[-A\,dv^2 + \frac{2}{\pi T}\,dr\,dv\right]$$

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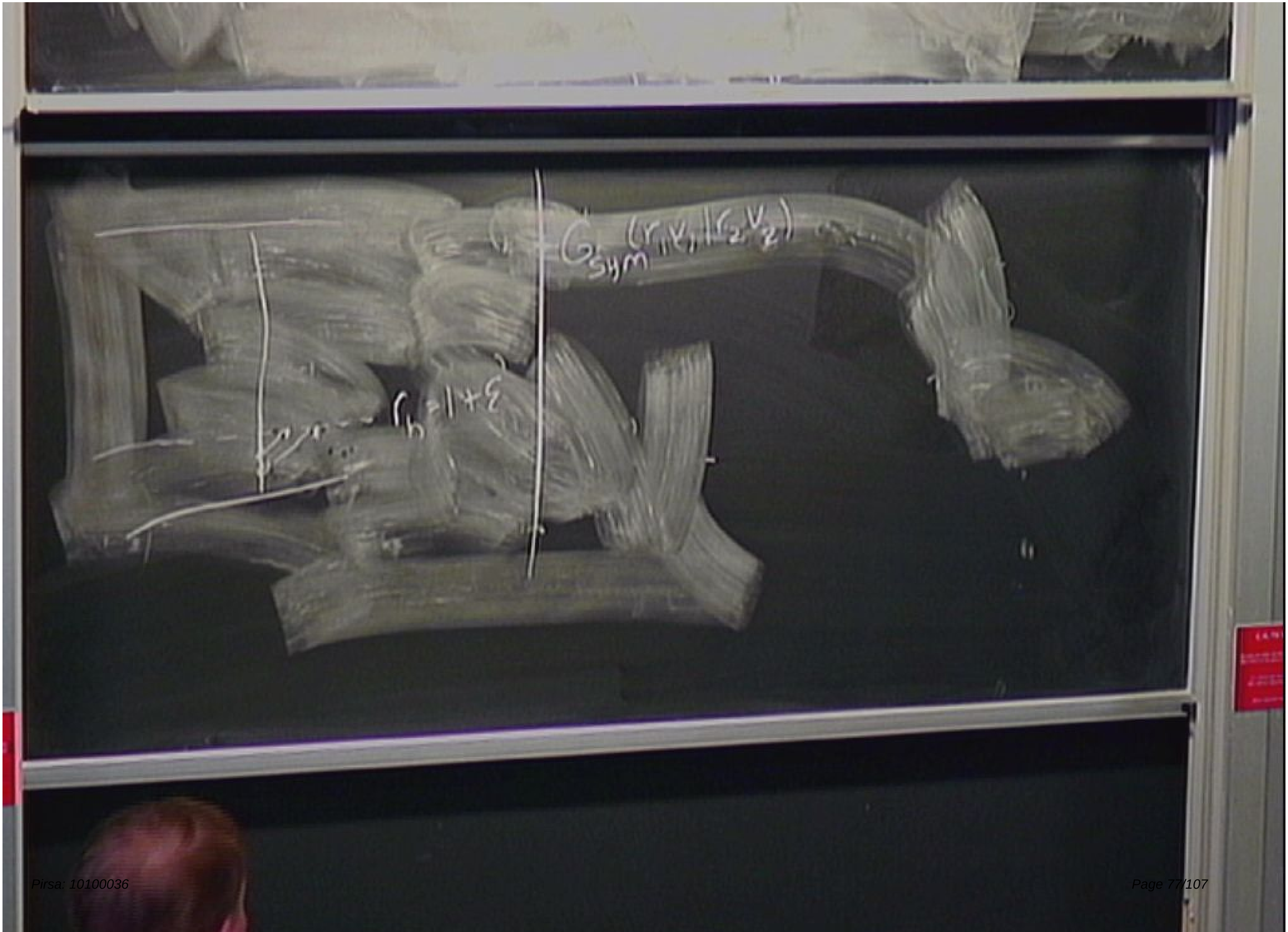
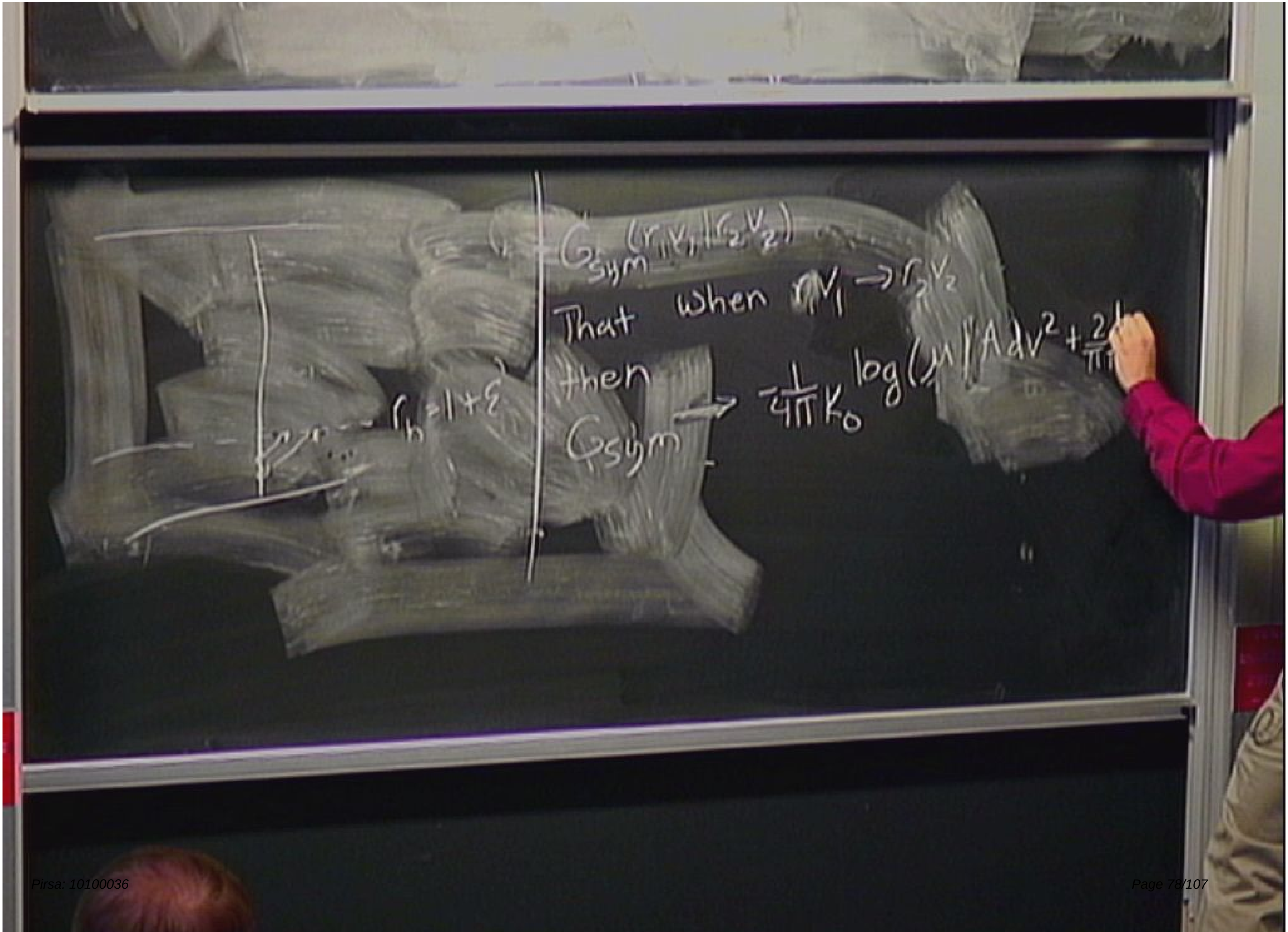
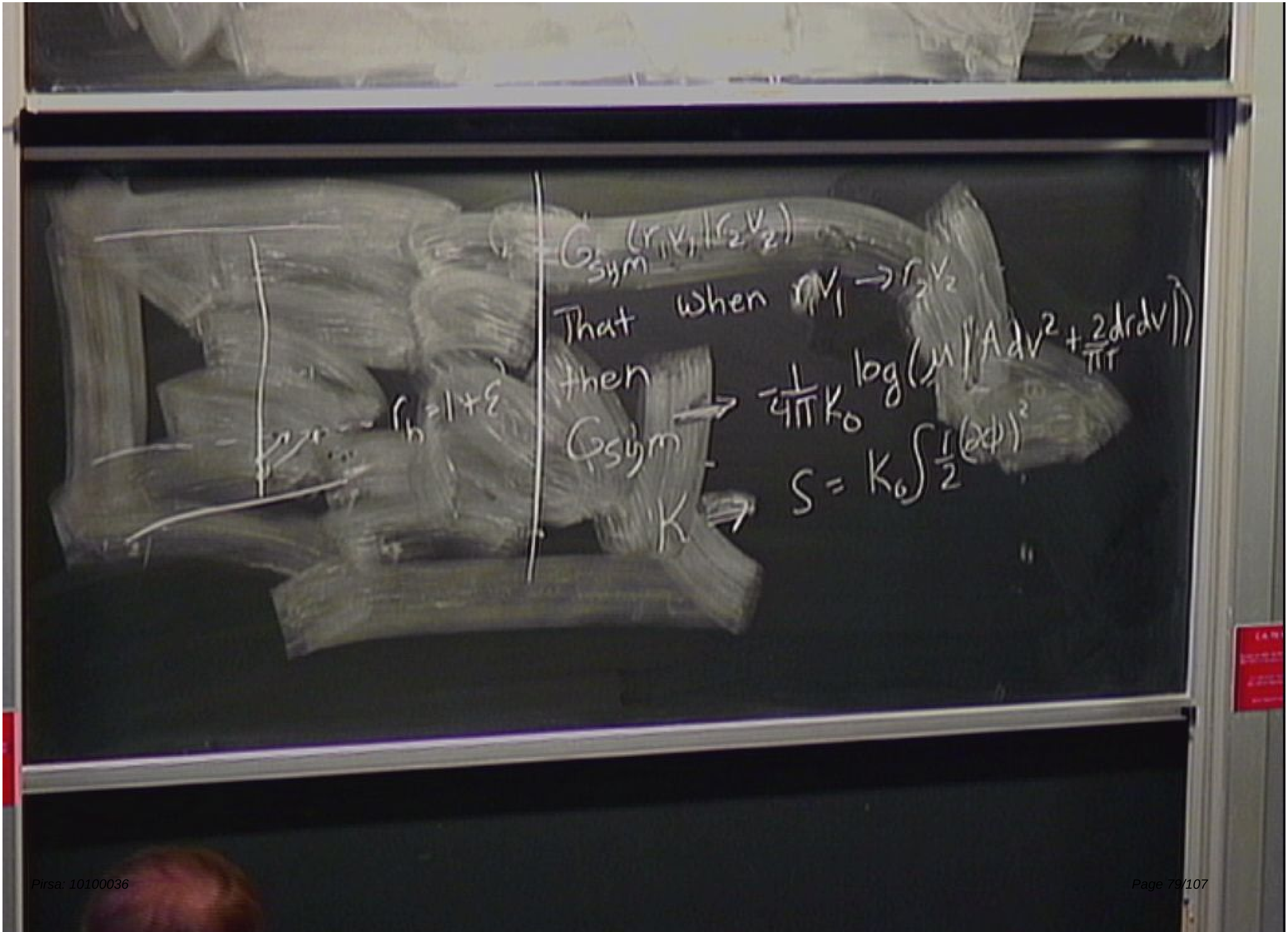
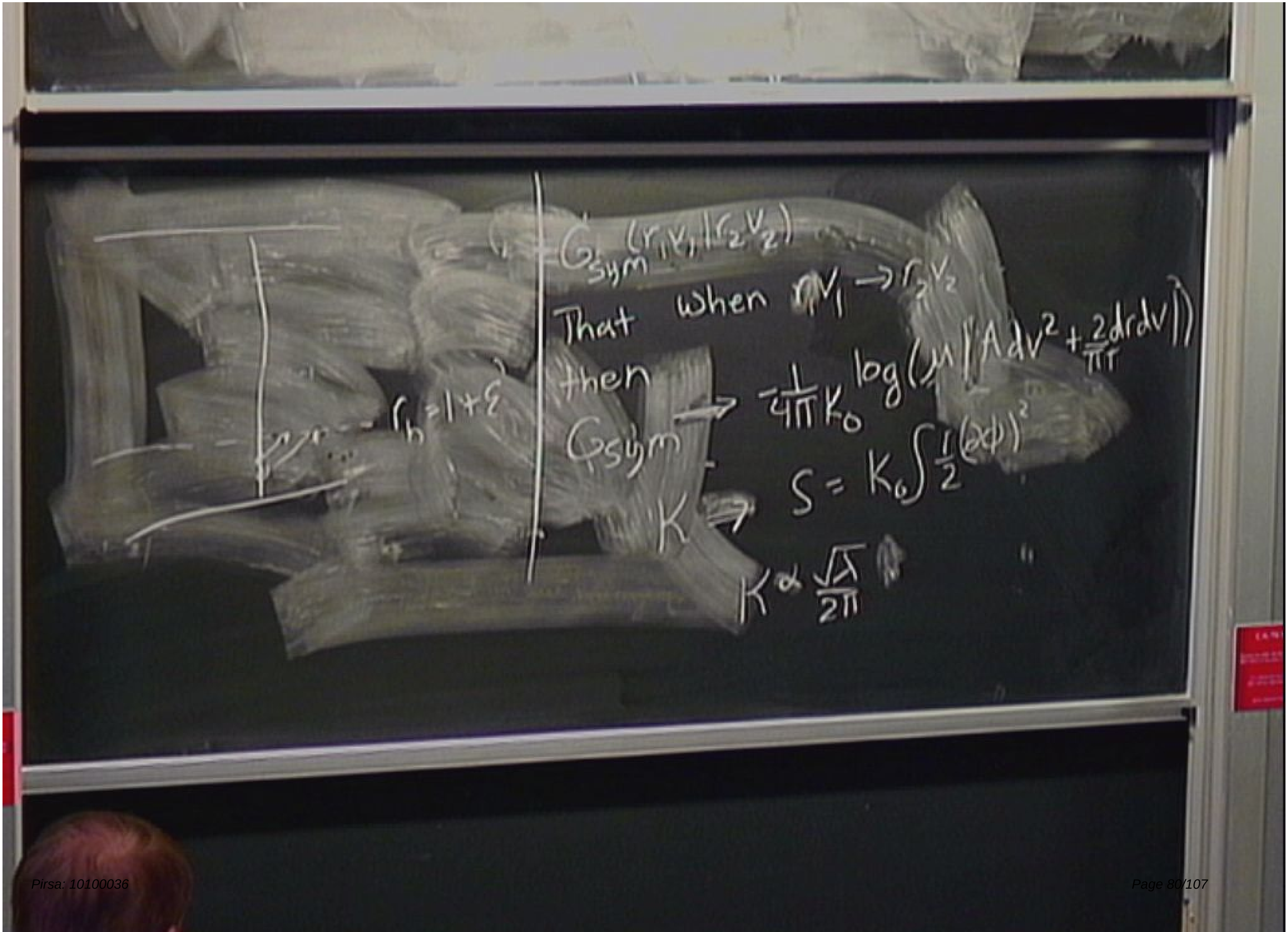
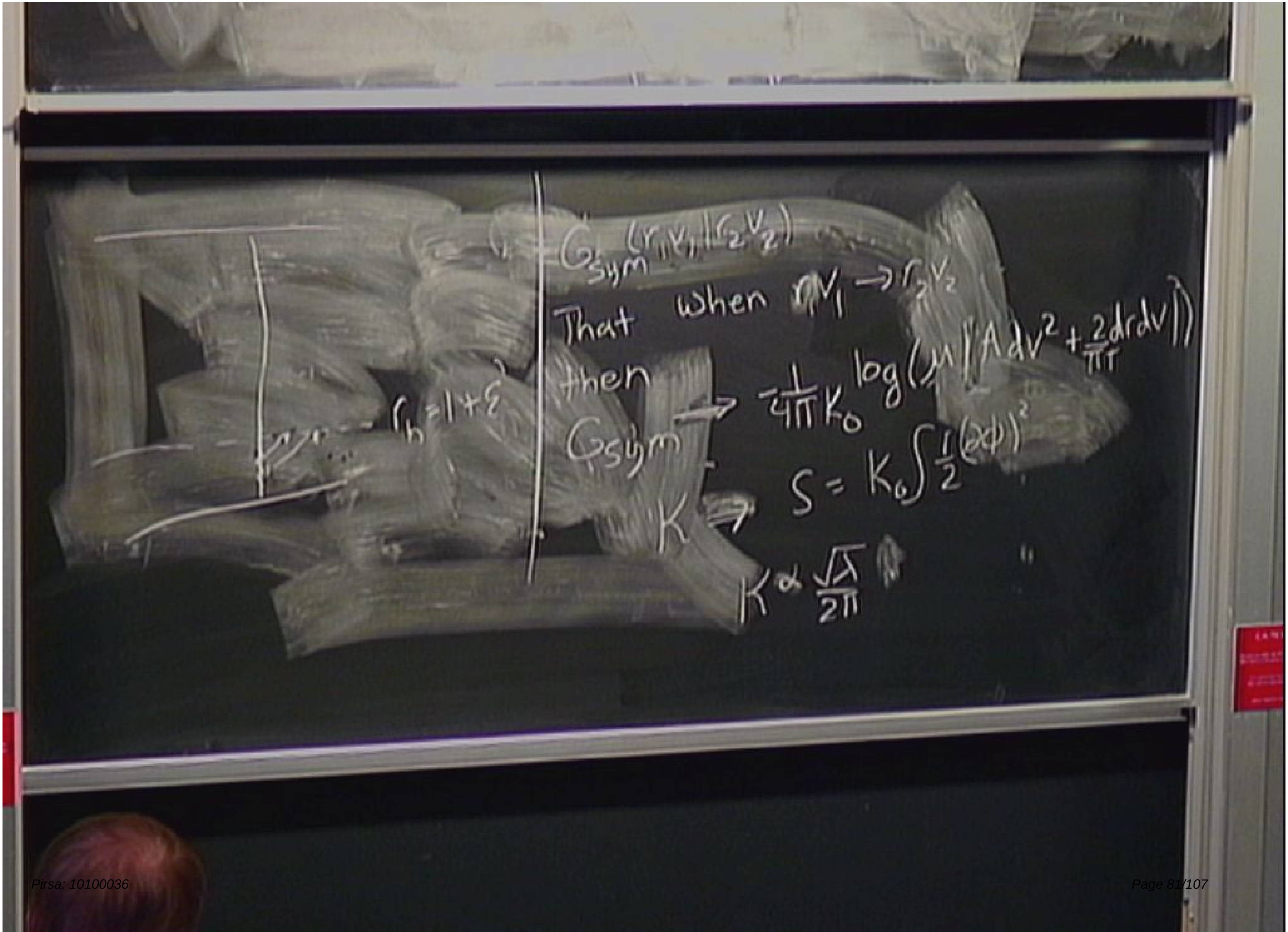

G_sym(r_1 v_1 | r_2 v_2)
r_h = 1 + ε


G_sym (r1v1|r2v2)
That when r1v1 → r2v2
then
G_sym → −1/4π K0 log(μ|Adv² + 2/π


r_h = 1+ε
G_sym (r1 v1 | r2 v2)
That when r1 v1 → r2 v2
then
G_sym → −1/(4π K0) log(μ | A dv² + 2/(πT) dr dv |)
K →
S = K0 ∫ 1/2 (∂φ)²



G_sym (r1 v1 | r2 v2)
That when r1 v1 → r2 v2
then
G_sym → −1/4π K0 log(μ|A dv² + 2/πT dr dv|)
S = K0 ∫ ½ (∂φ)²
K →
K ∝ √λ/2π
r_h = 1+ε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K →
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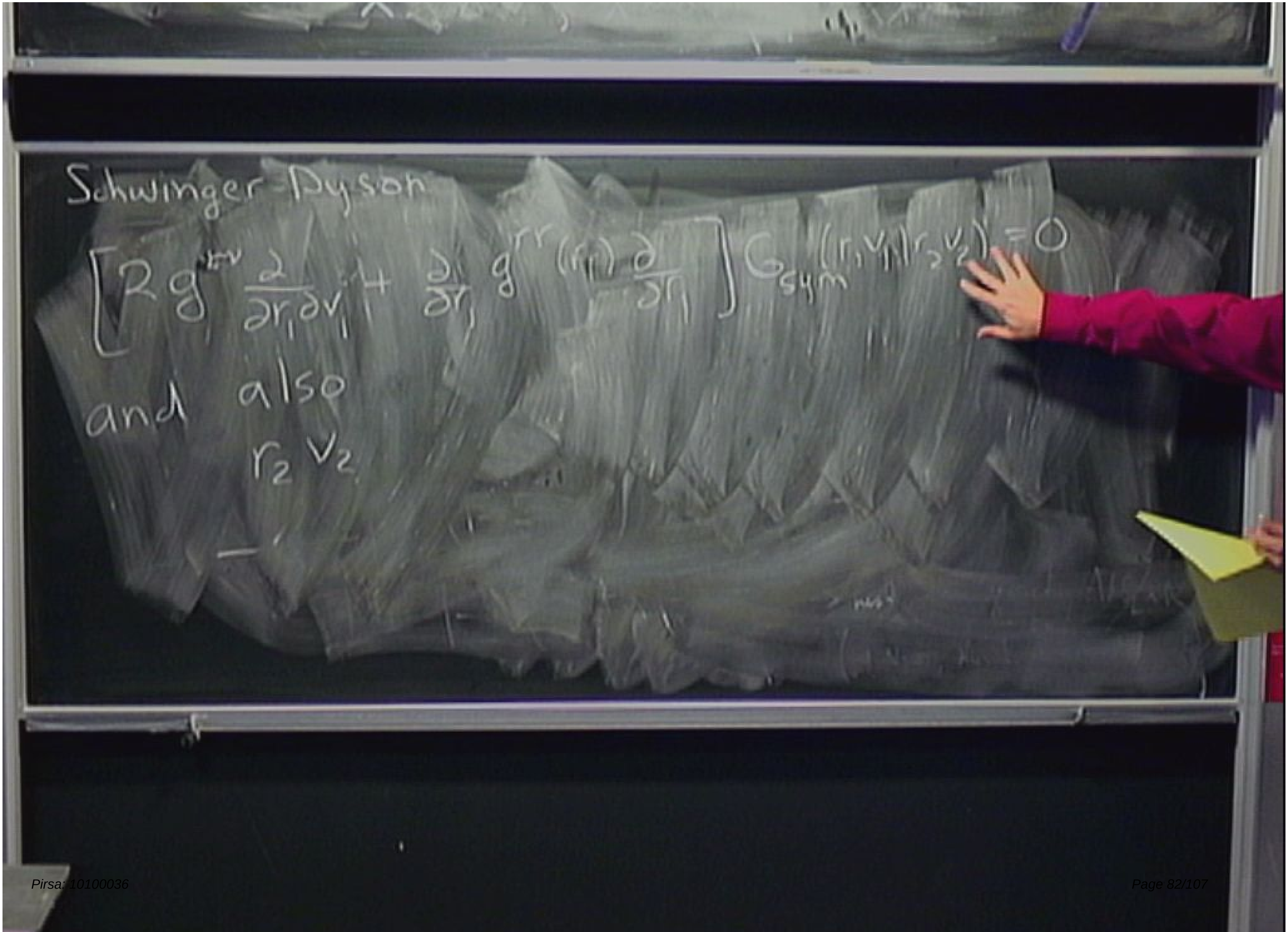

Schwinger-Dyson
[R g^{rv} ∂/∂r1∂v1 + ∂/∂r1 g^{rr}(r1) ∂/∂r1] Gsym (r1v1|r2v2) = 0
and also
r2 v2

Schwinger Dyson

$$\left[R g^{rv} \frac{\partial}{\partial r_1 \partial v_1} + \frac{\partial}{\partial r_1} g^{rr}(r_1) \frac{\partial}{\partial r_1}\right] G_{sym}(r_1 v_1 | r_2 v_2) = 0$$

and also

$r_2 \; v_2$

Schwinger-Dyson

$$\left[R\, g^{rv} \frac{\partial}{\partial r_1 \partial v_1} + \frac{\partial}{\partial r_1} g^{rr}(r_1) \frac{\partial}{\partial r_1} \right] G_{Sym}(r_1 v_1 | r_2 v_2) = 0$$

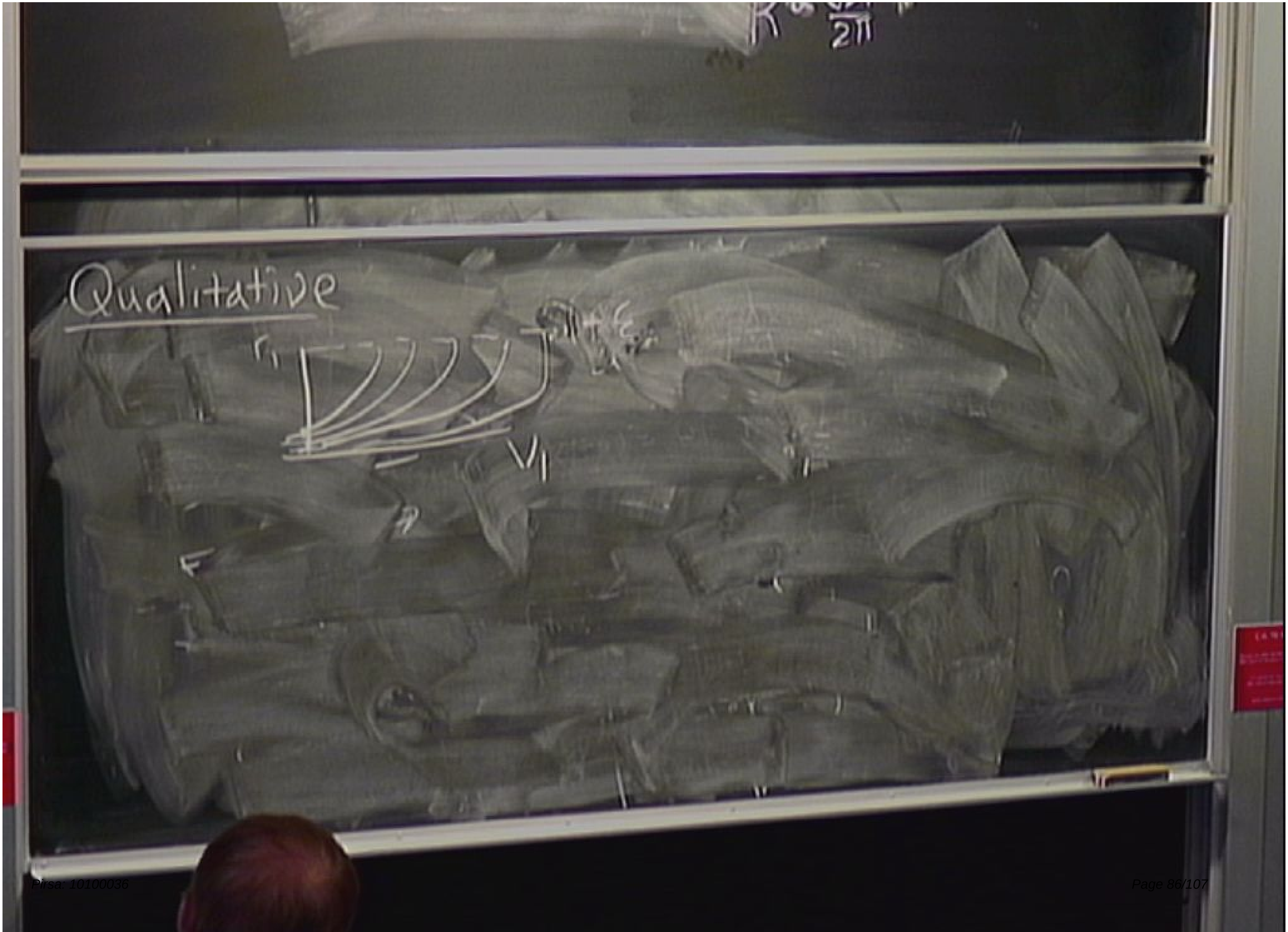
and also

$r_2\, v_2$

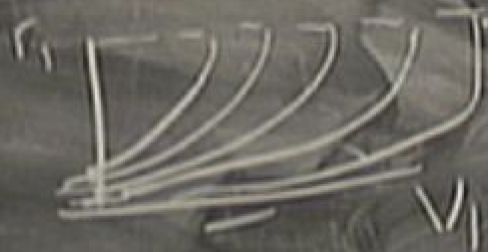
Schwinger-Dyson

$$\left[R g^{rv} \frac{\partial}{\partial r_1 \partial v_1} + \frac{\partial}{\partial r_1} g^{rr}(r_1) \frac{\partial}{\partial r_1}\right] G_{sym}(r_1 v_1 | r_2 v_2) = 0$$

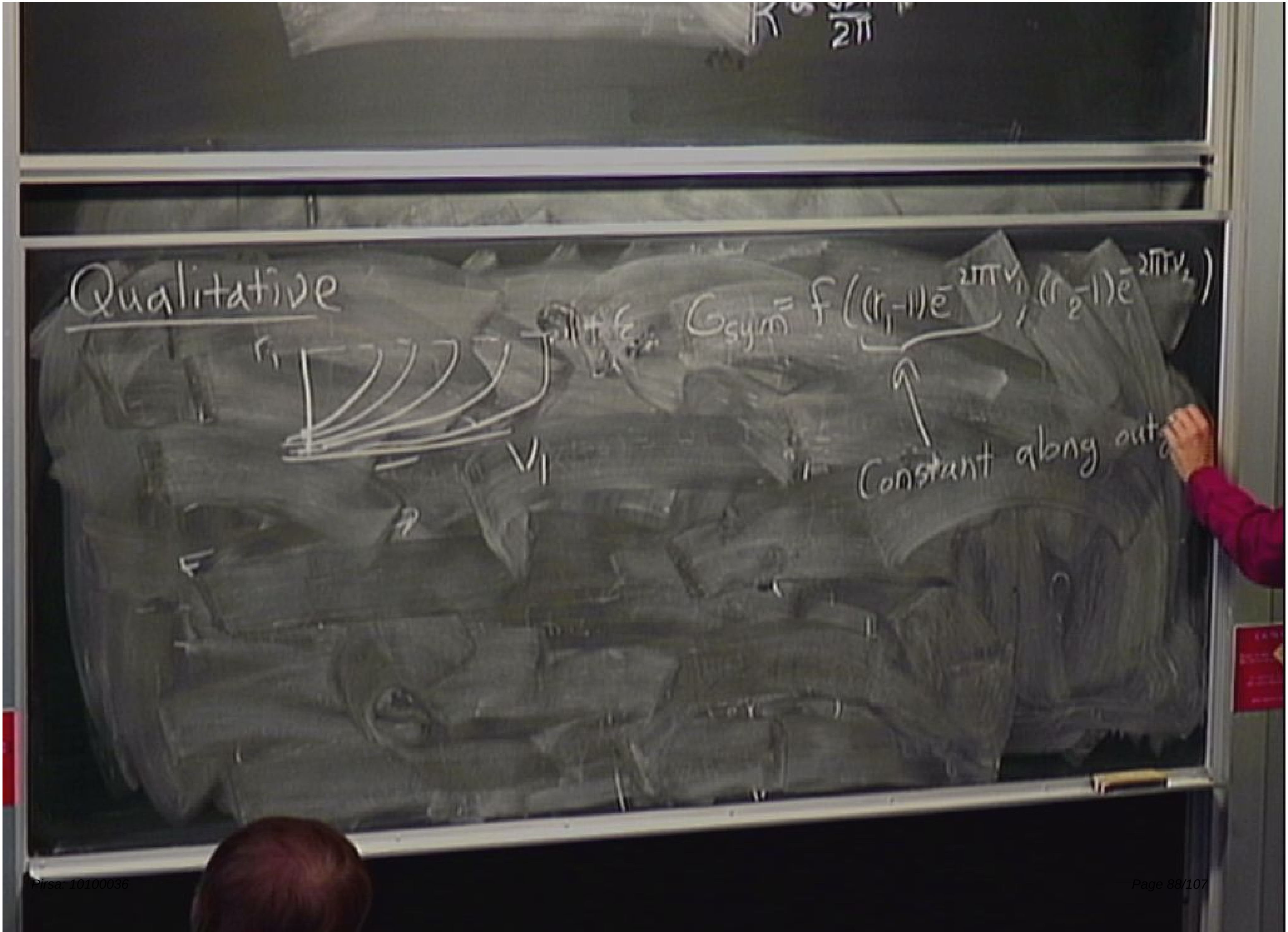
and also $r_2 v_2$

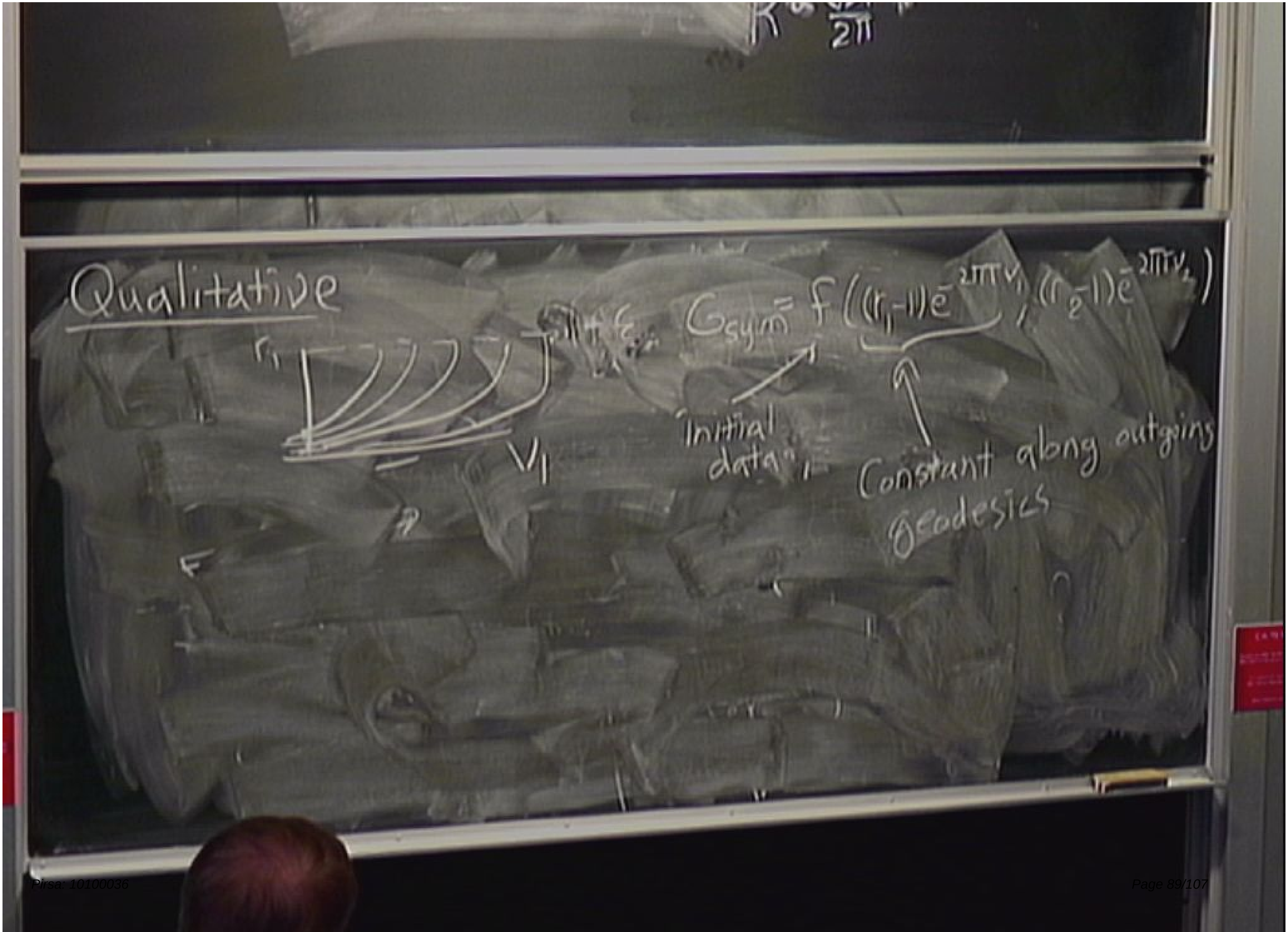

Qualitative

Qualitative

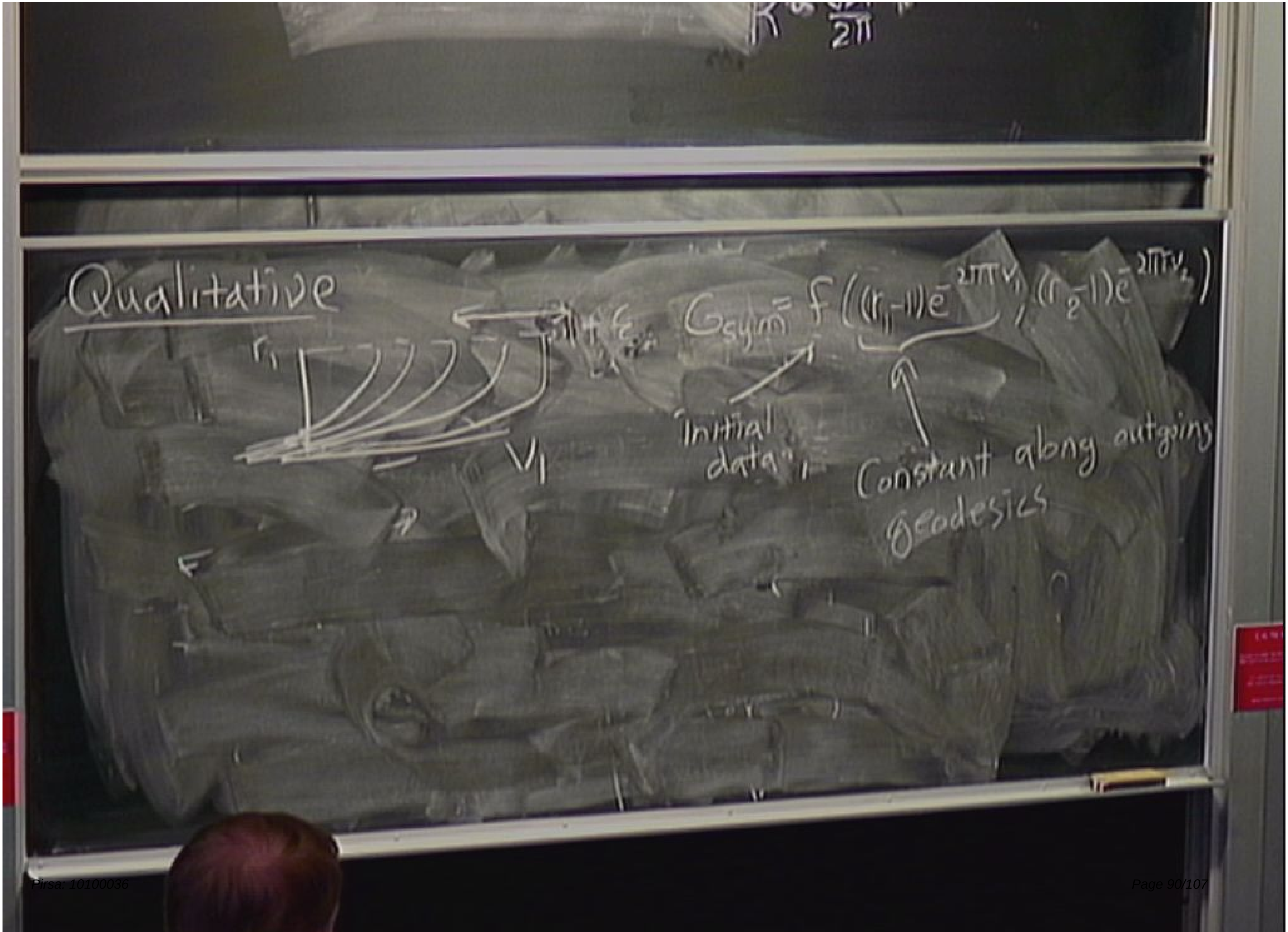


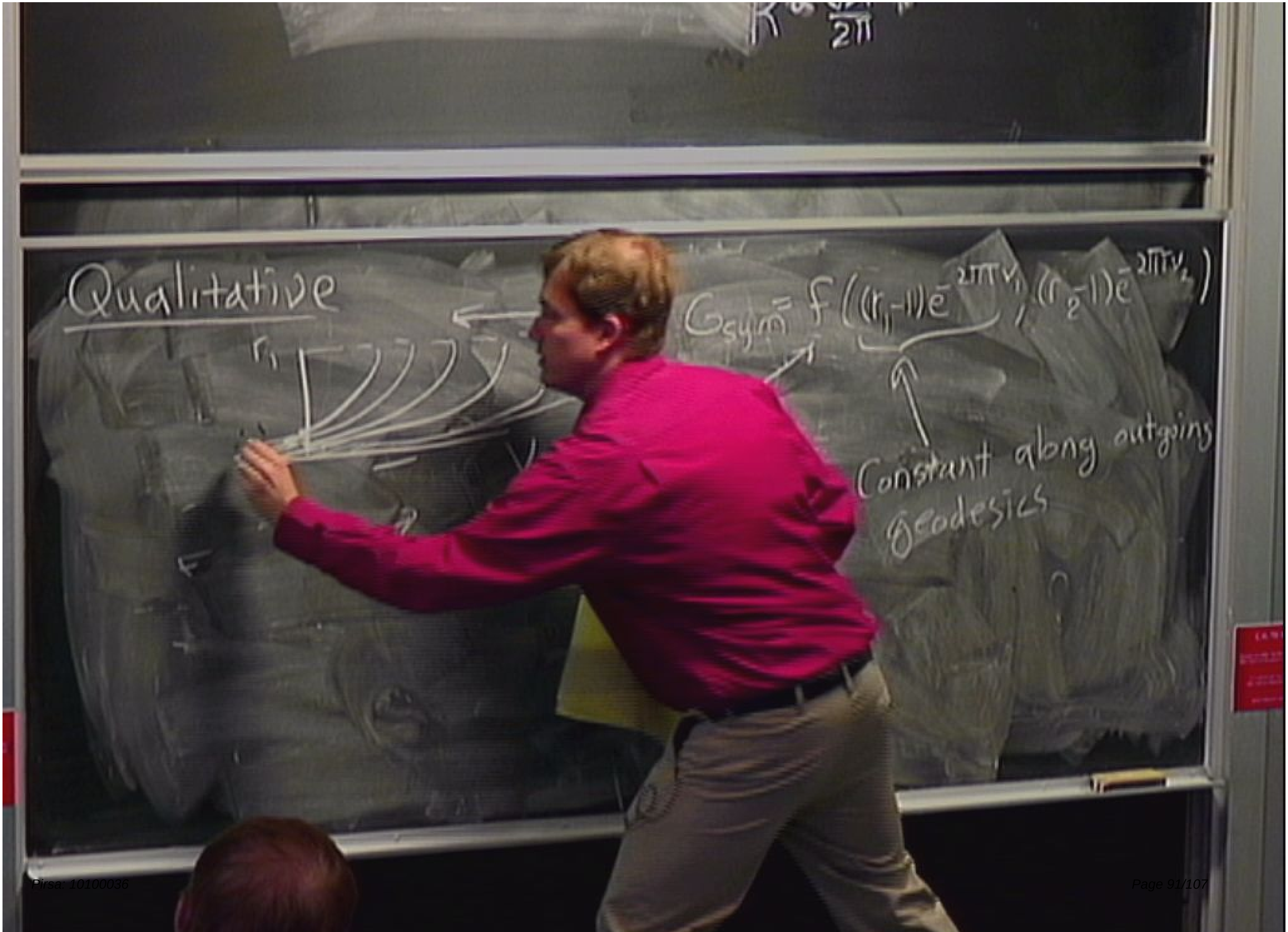
$G_{sym} = f\left((r_1 - 1)e^{-2\pi V_1}, (r_2 - 1)e^{-2\pi \ldots}\right)$

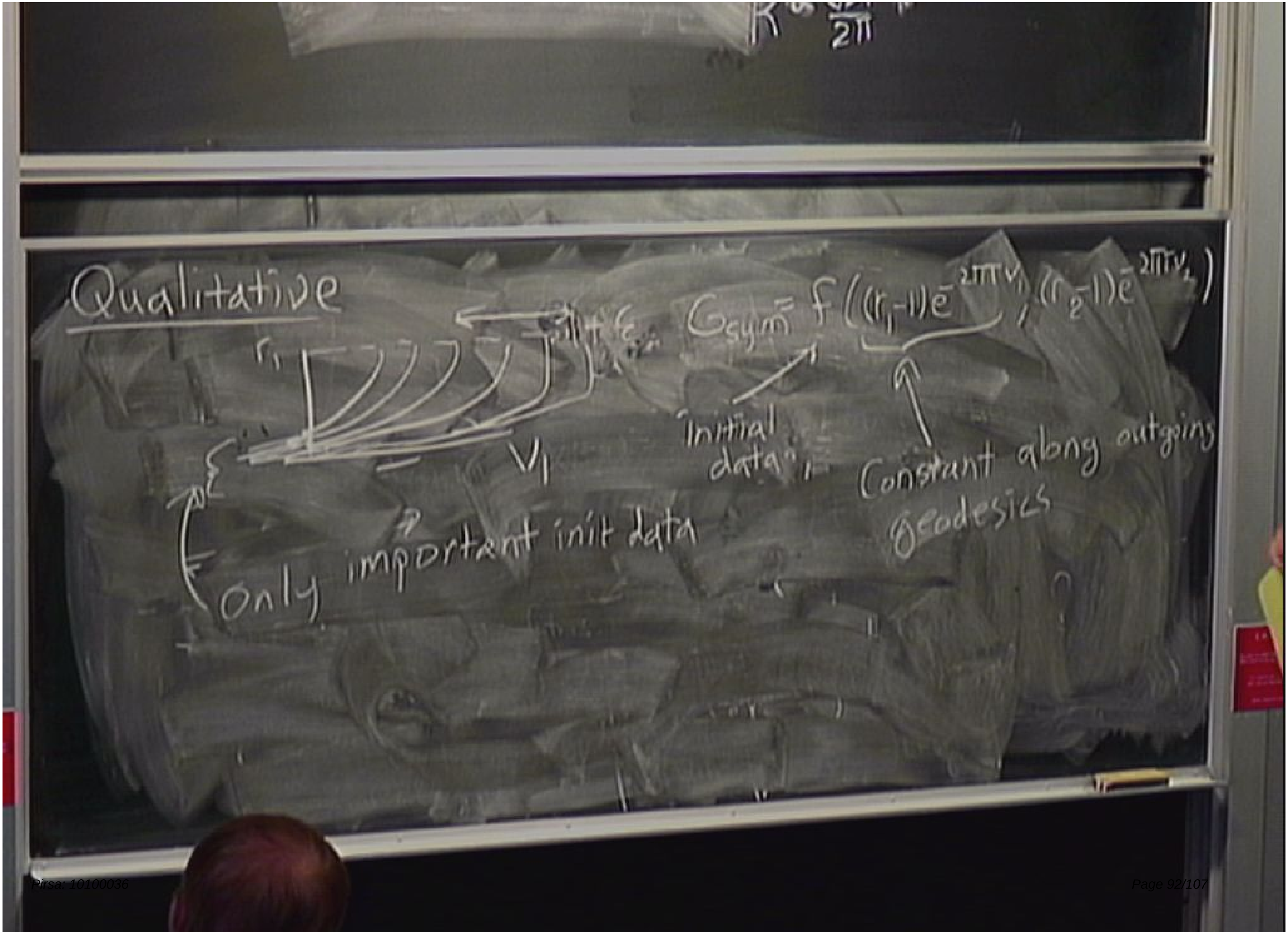

Qualitative
G_sym = f((r_1-1)e^-2πv_1, (r_2-1)e^-2πv_2)
r_1
v_1
Constant along out

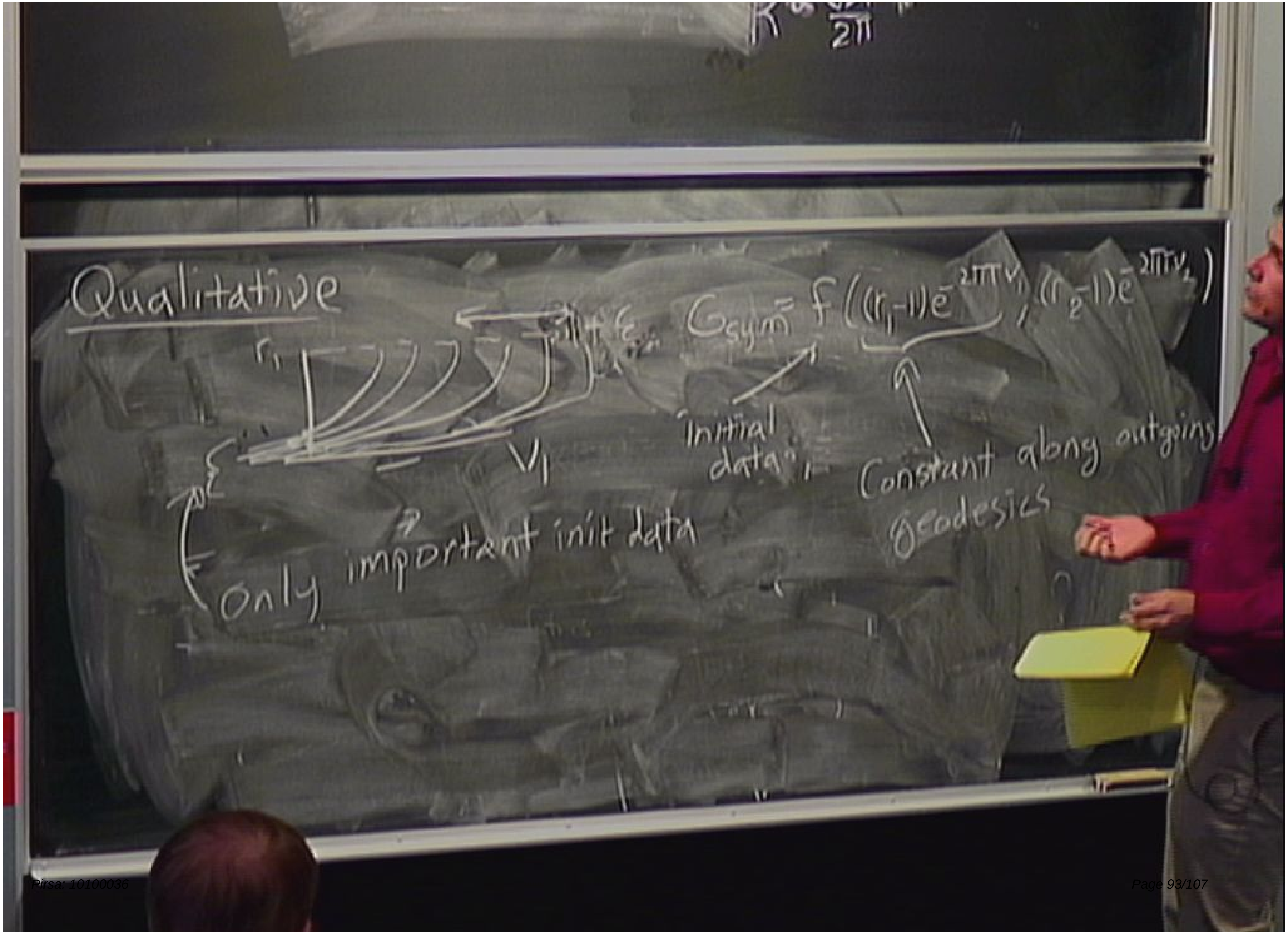


Qualitative
$G_{sym} = f((r_1-1)e^{-2\pi v_1}, (r_2-1)e^{-2\pi v_2})$
r_1
v_1
initial data
Constant along outgoing geodesics


Qualitative
r1
v1
Gsym = f((r1-1)e^-2πv1, (r2-1)e^-2πv2)
"initial data"
Constant along outgoing geodesics


Qualitative
$G_{sym} = f\left((r_1-1)e^{-2\pi v_1}, (r_2-1)e^{-2\pi v_2}\right)$
Constant along outgoing geodesics


Qualitative
$G_{sym} = f\left((r_1-1)e^{-2\pi v_1}, (r_2-1)e^{-2\pi v_2}\right)$
r_1
v_1
initial data
Constant along outgoing geodesics
Only important init data


Qualitative
G_sym = f((r_1-1)e^-2πv_1, (r_2-1)e^-2πv_2)
r_1
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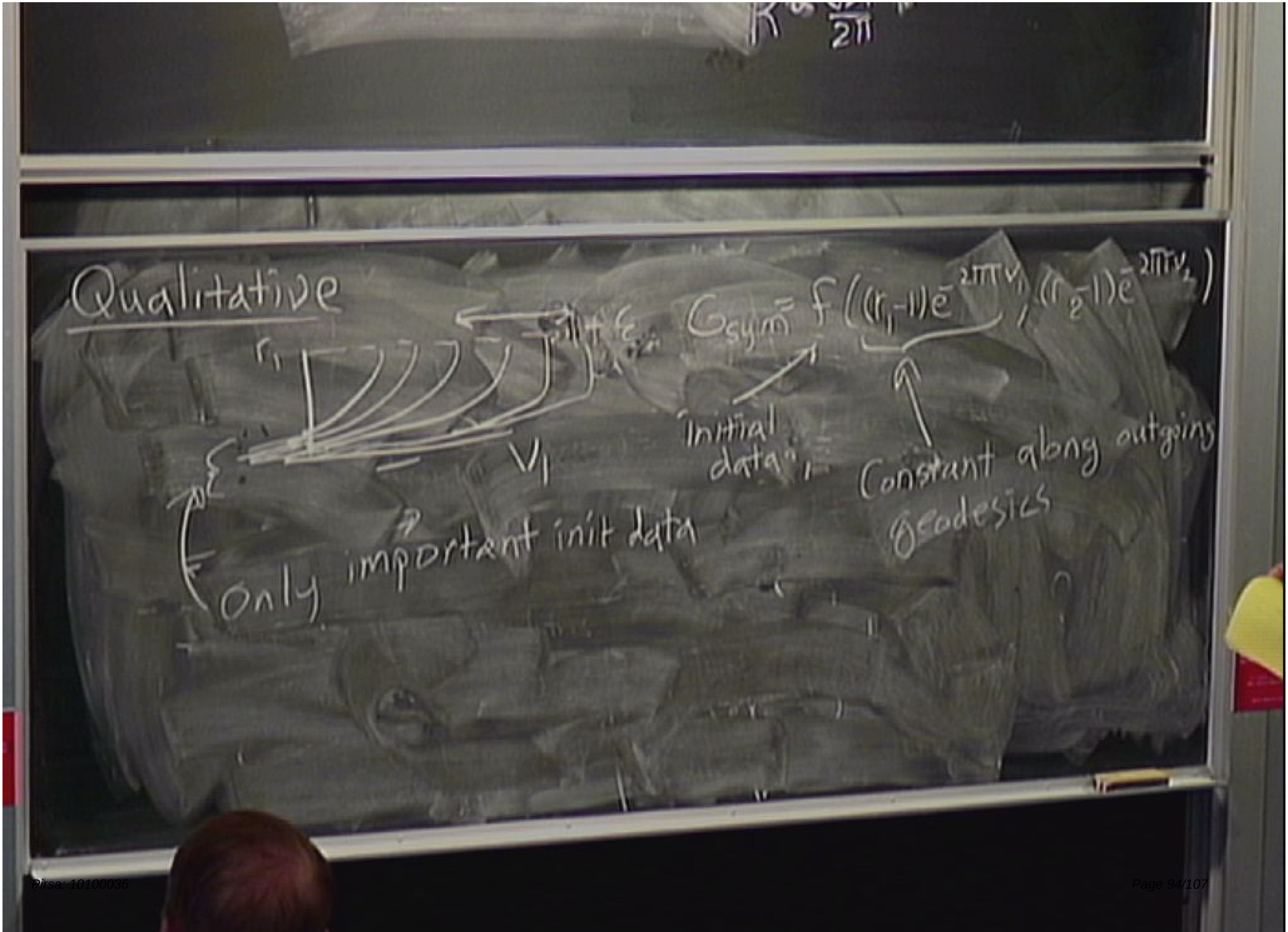
Qualitative
r1
ε
v1
Gsym = f((r1-1)e^-2πv1, (r2-1)e^-2πv2)
initial data
Constant along outgoing geodesics
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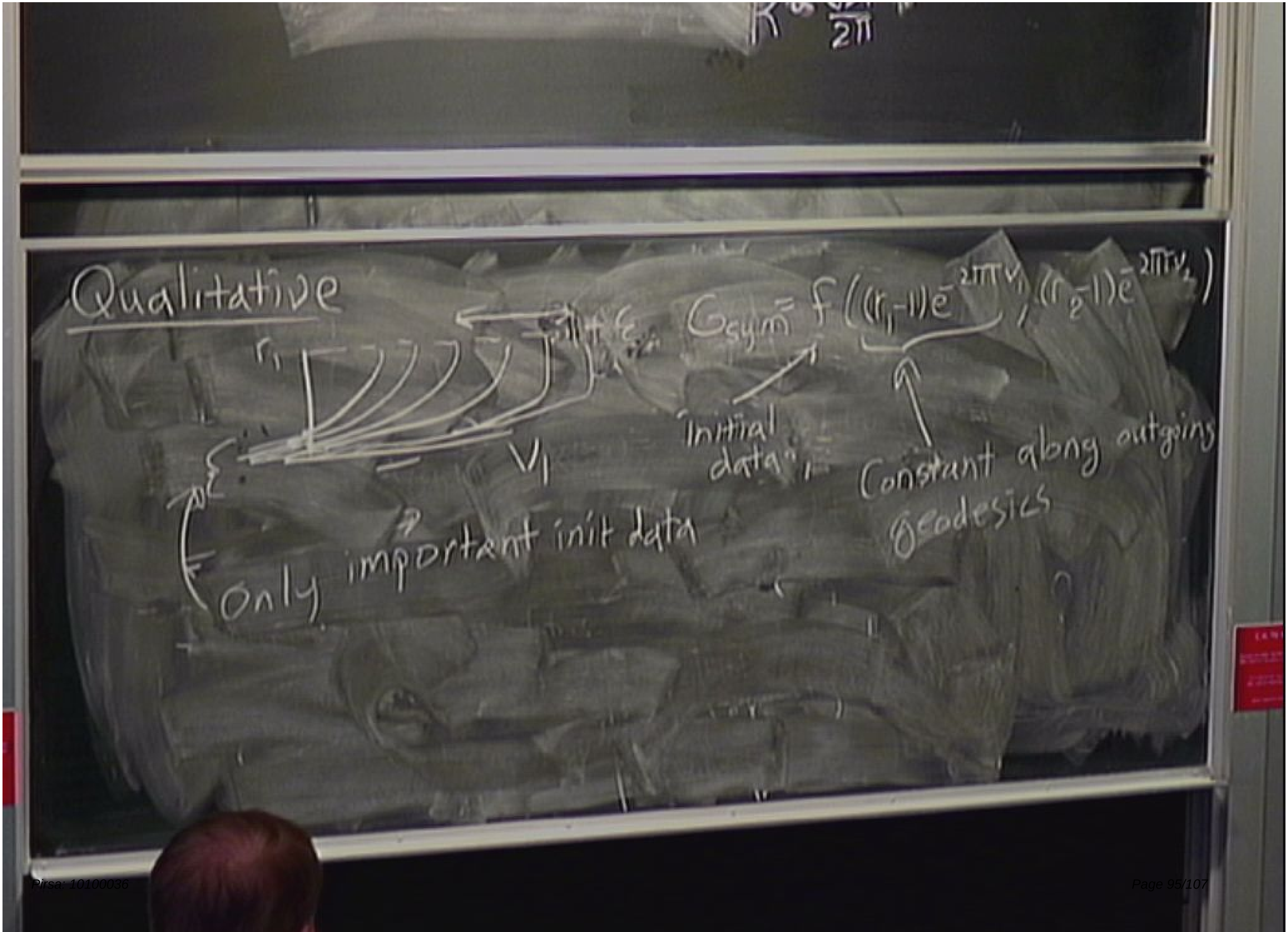
Qualitative
r_1
v_1
$G_{sym} = f((r_1-1)e^{-2\pi v_1}, (r_2-1)e^{-2\pi v_2})$
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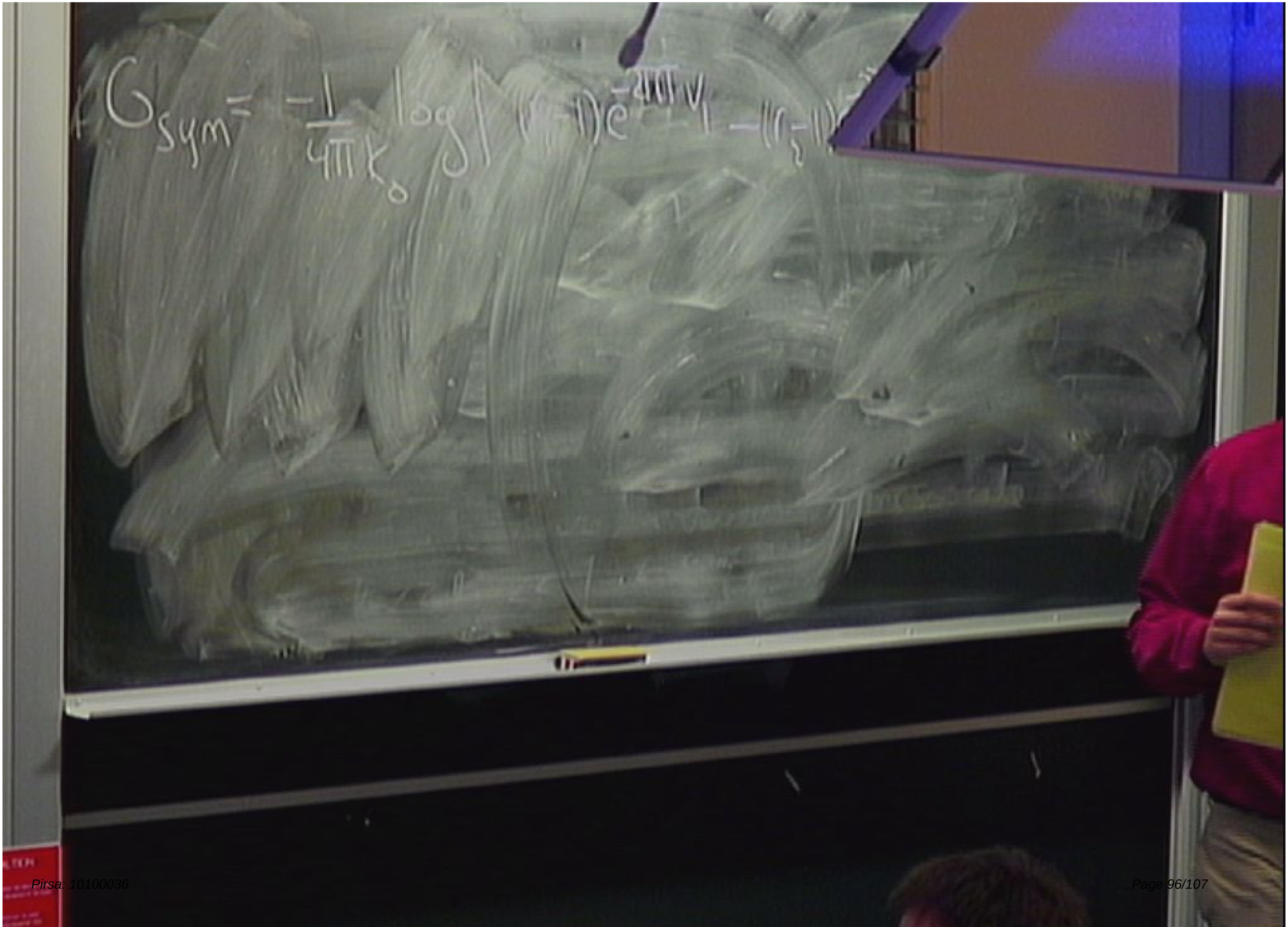
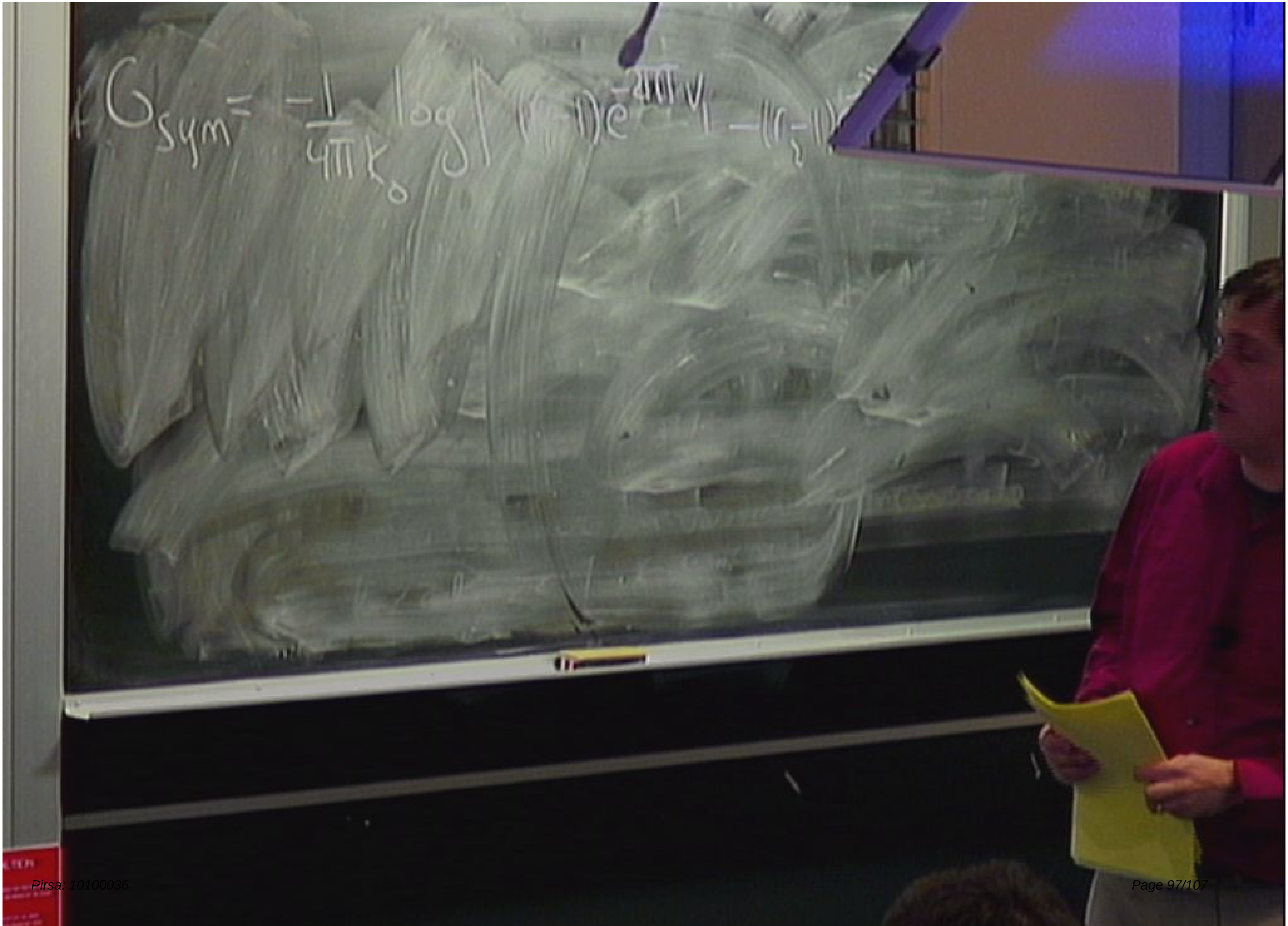
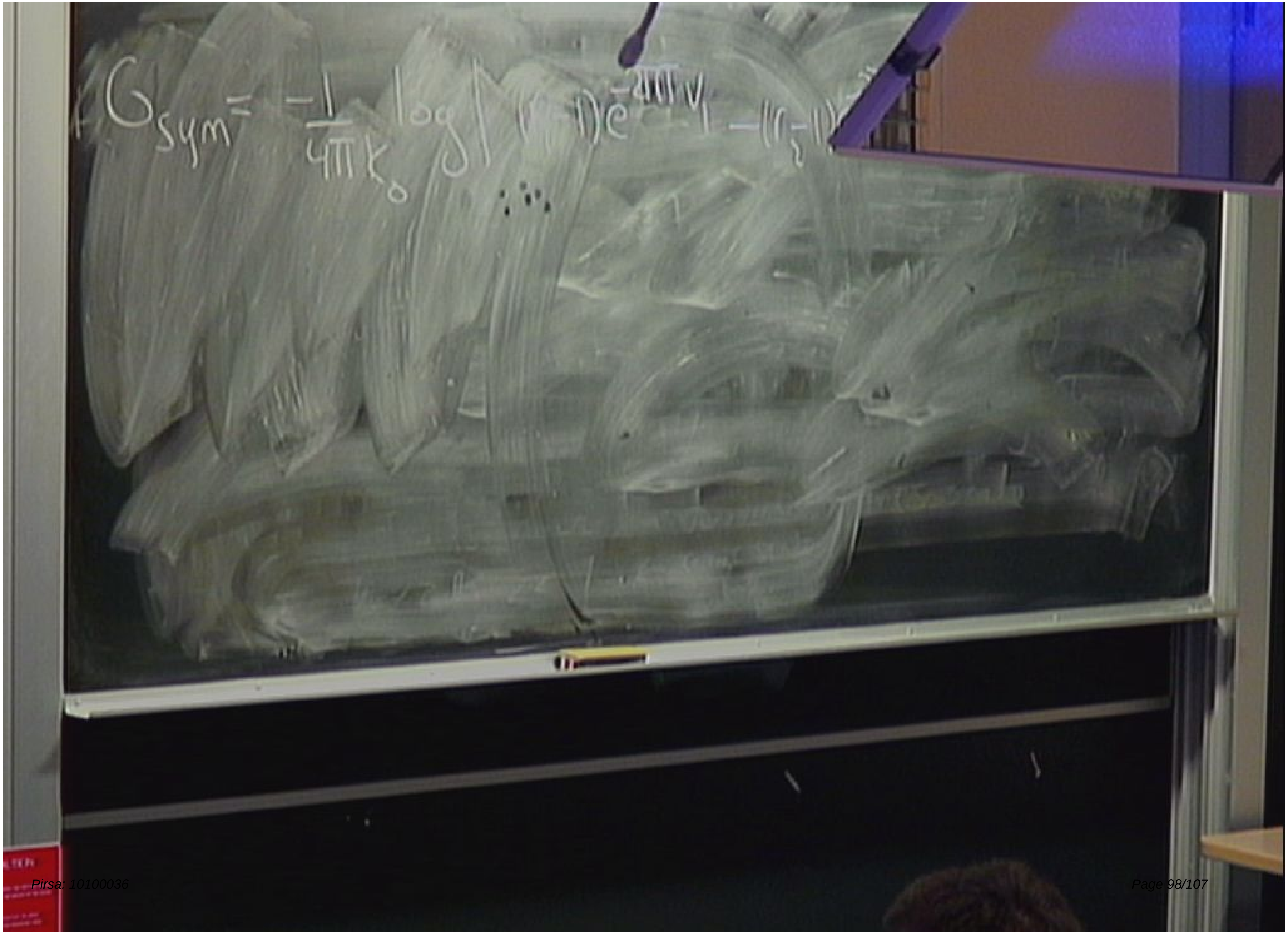
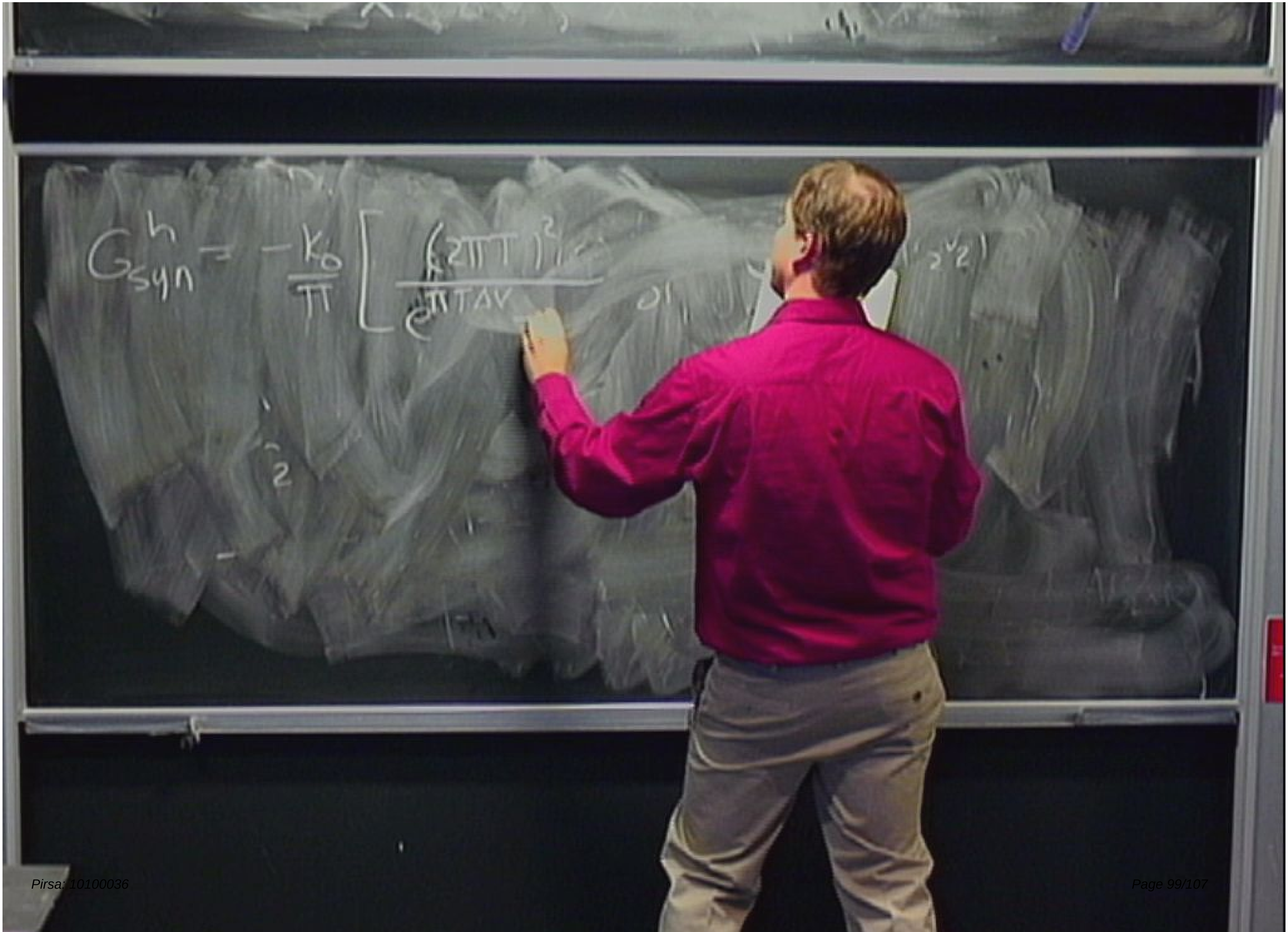
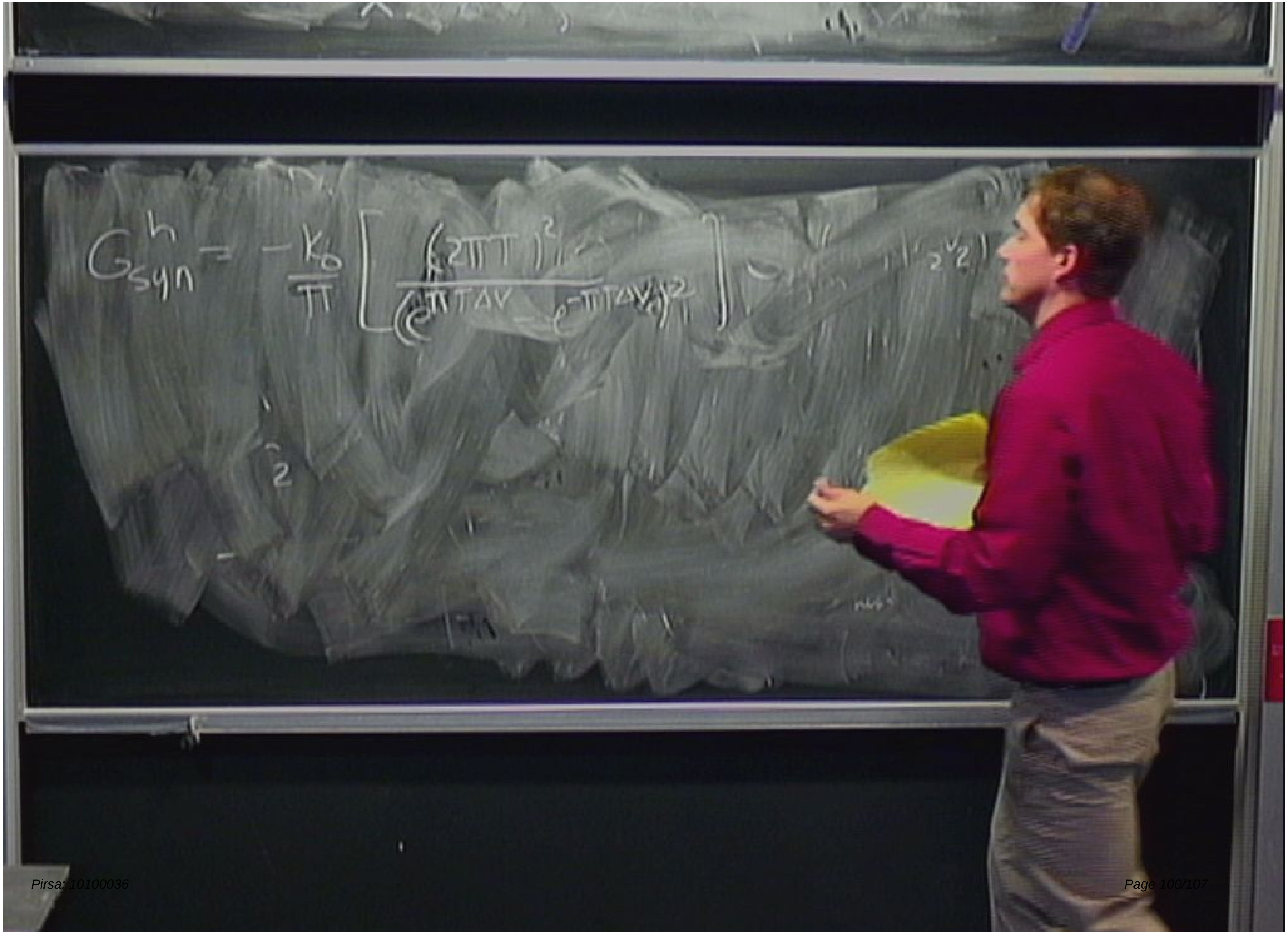
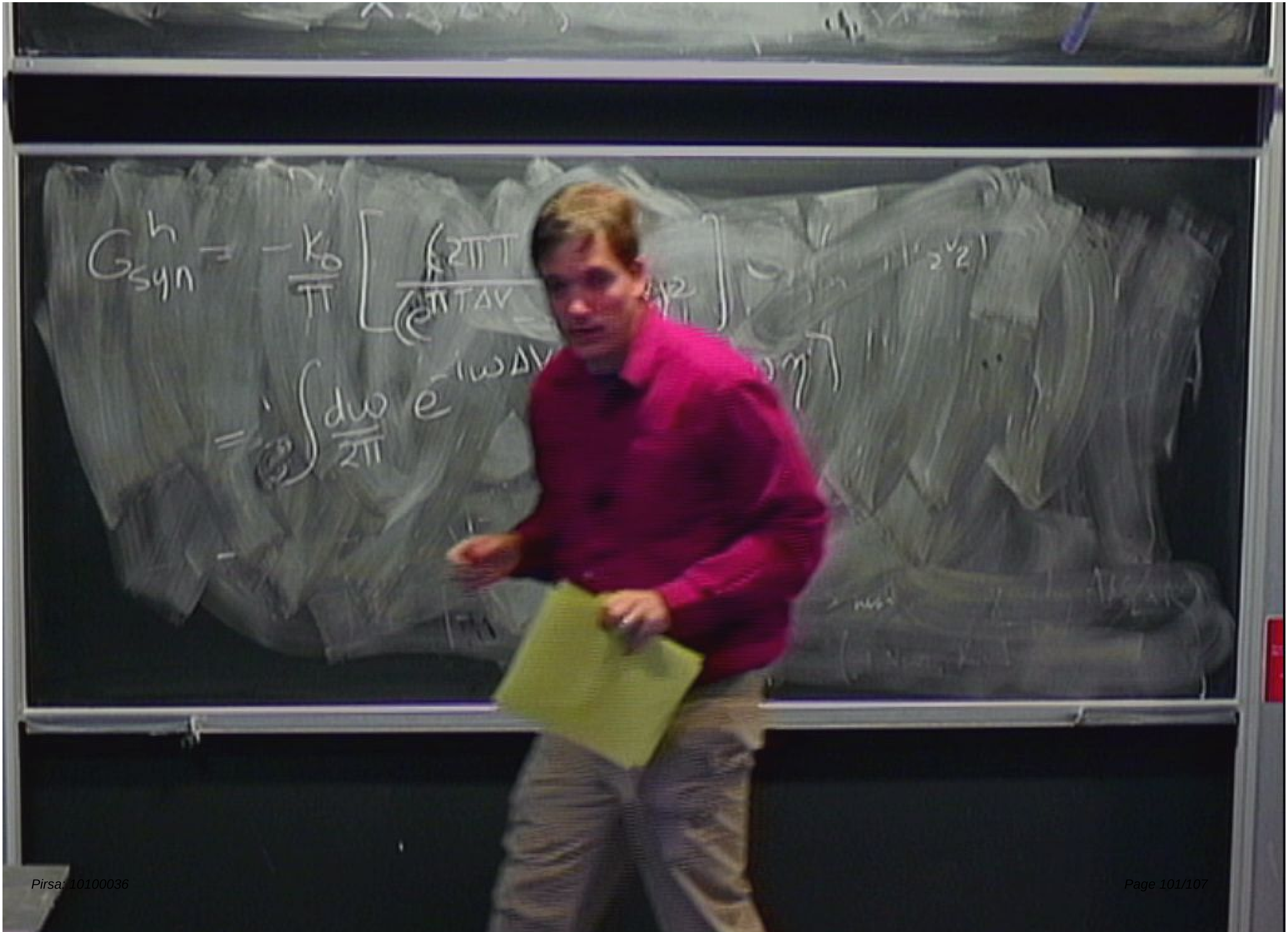


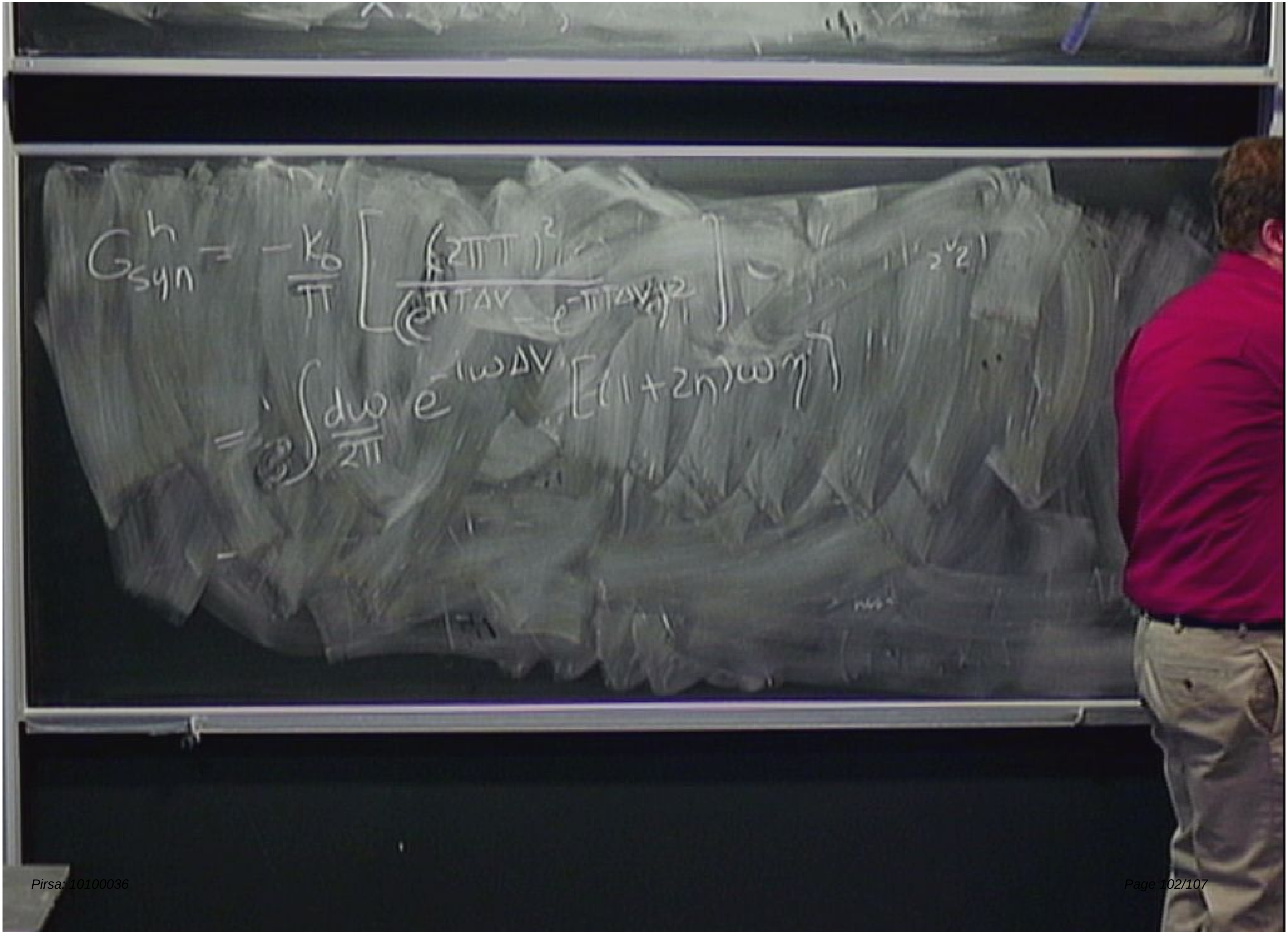


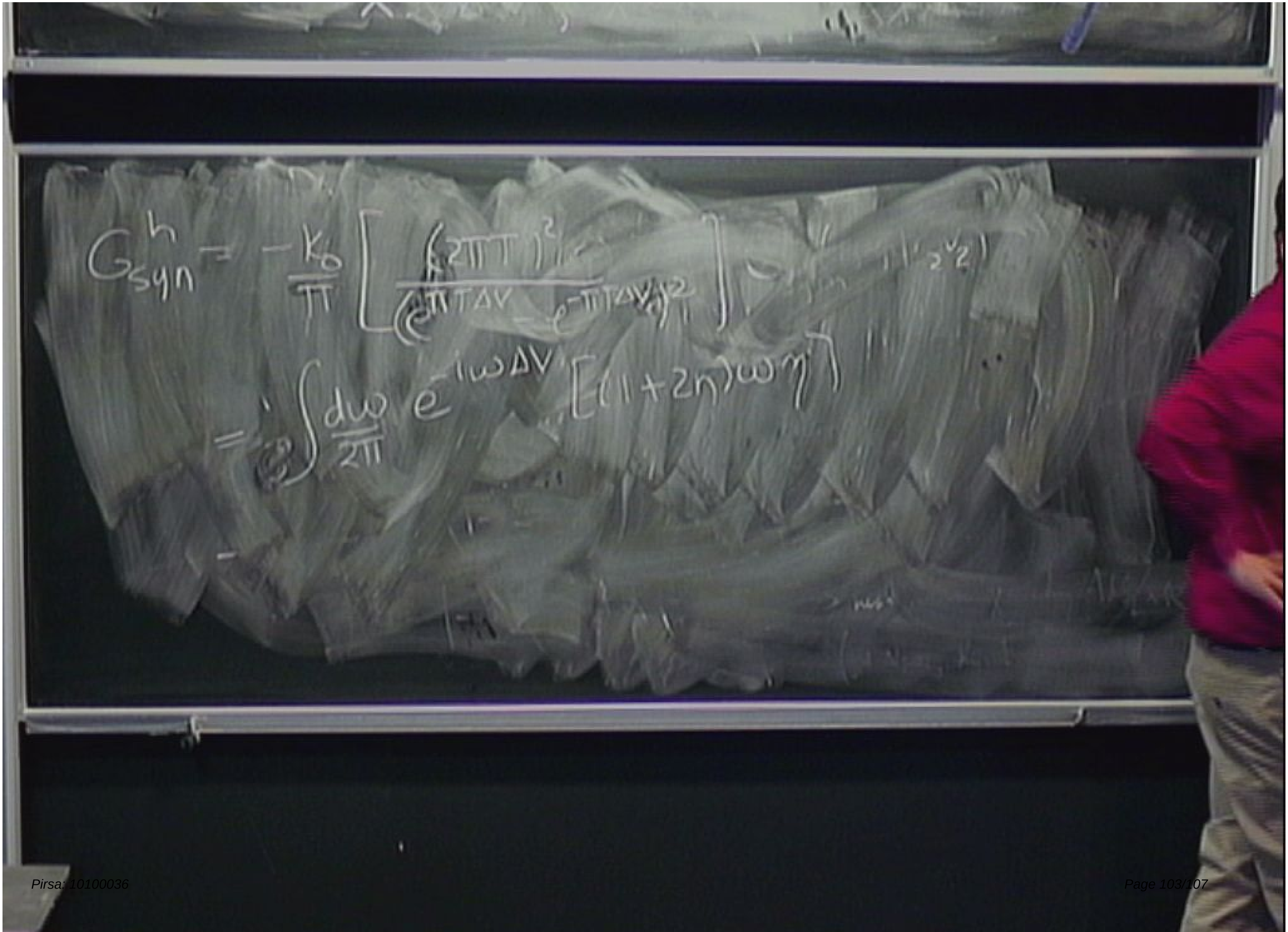


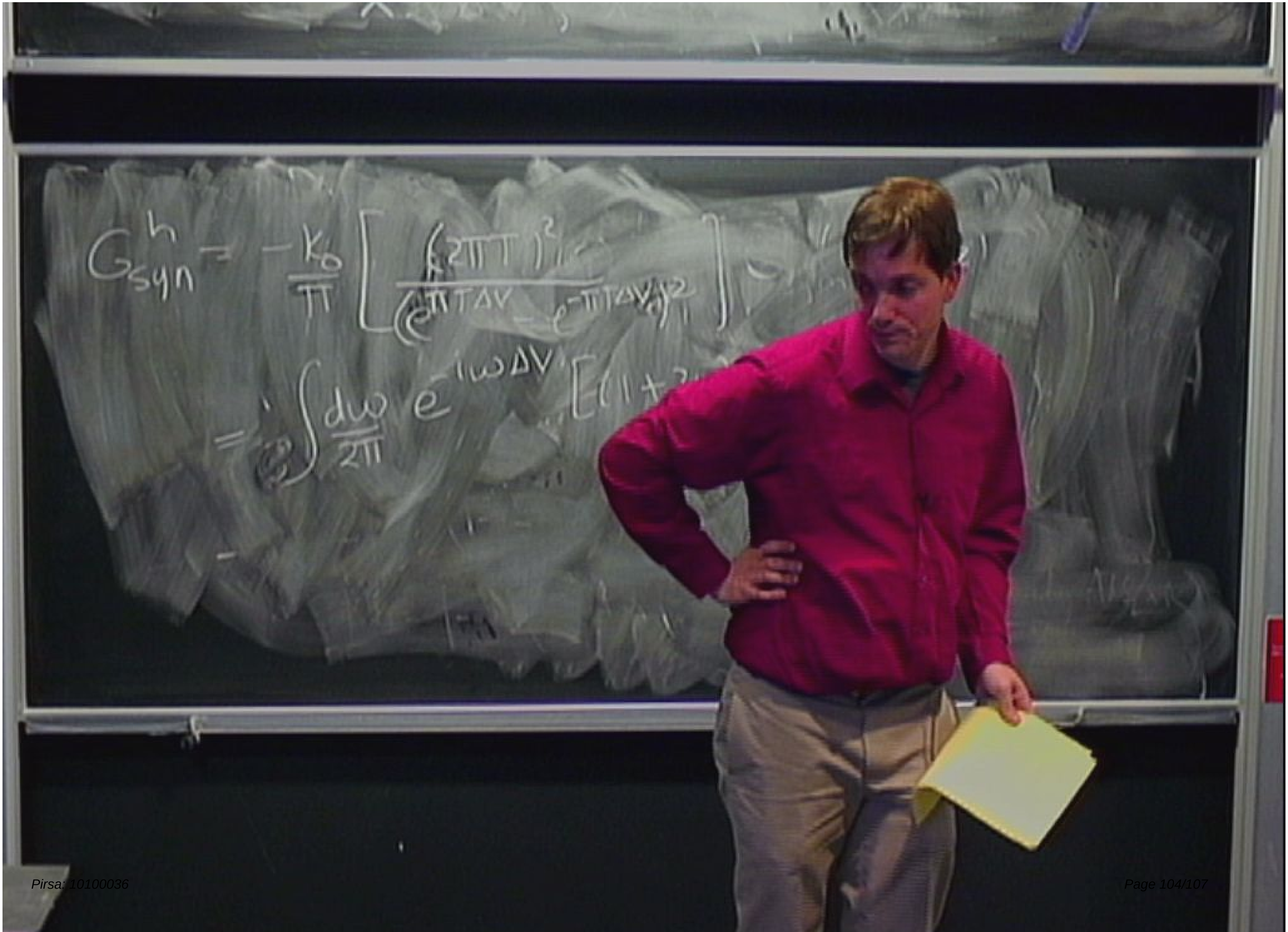

G h syn = − k0 / π [(2πT)² / e^πTΔV






$G^h_{syn} = -\frac{k_0}{\pi}\left[\frac{(2\pi T)^2}{(e^{\pi T \Delta V} - e^{-\pi T \Delta V})^2}\right]$
$= i \int \frac{d\omega}{2\pi} e^{-i\omega \Delta V} [(1+2n)\omega\eta]$


G^h_syn = -k_0/π [(2πT)^2 / (e^{πTΔV} - e^{-πTΔV})^2]
= i ∫ dω/2π e^{-iωΔV} [(1+2n)ωη]



Conclusions

1. Quantum Mechanics of AdS_5 leads to thermal noise
   - Prototypical Example - Brownian Motion
2. Other fields also fluctuate: the dilation ϕ, the graviton $h^{\mu\nu}$, etc, fluctuate
   - Applications?
3. Gave a different derivation of the Hawking flux that extends to non-equilibrium
   1. Stay Tuned!
   2. Fluctuation Dissipation: gravity pulls down, and fields fluctuate

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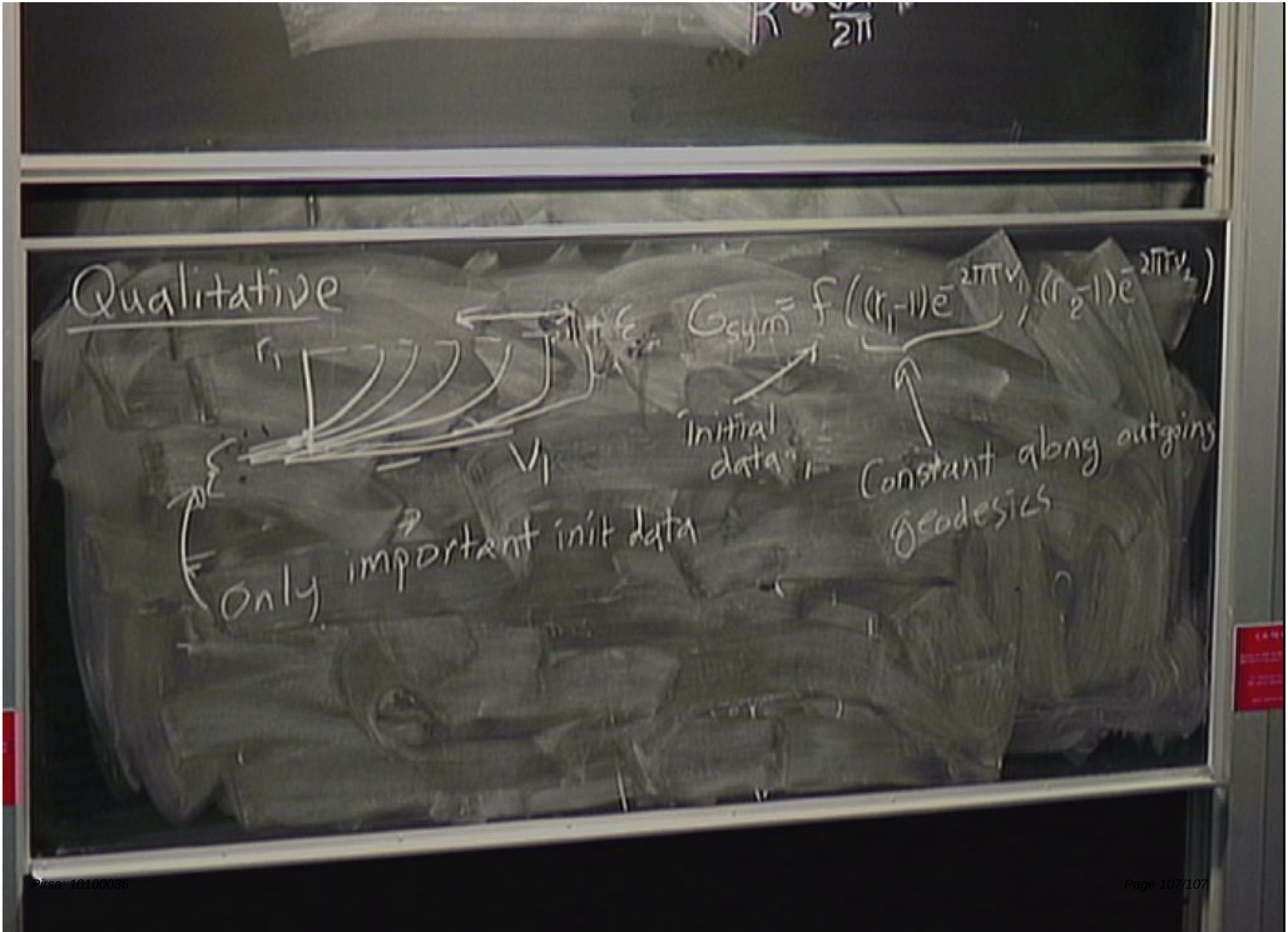
Qualitative
r1
1+ε
G_sym = f((r1-1)e^-2πv1, (r2-1)e^-2πv2)
initial data
Constant along outgoing geodesics
V1
Only important init data
2π