Title: Second-Order Relativistic Hydrodynamics

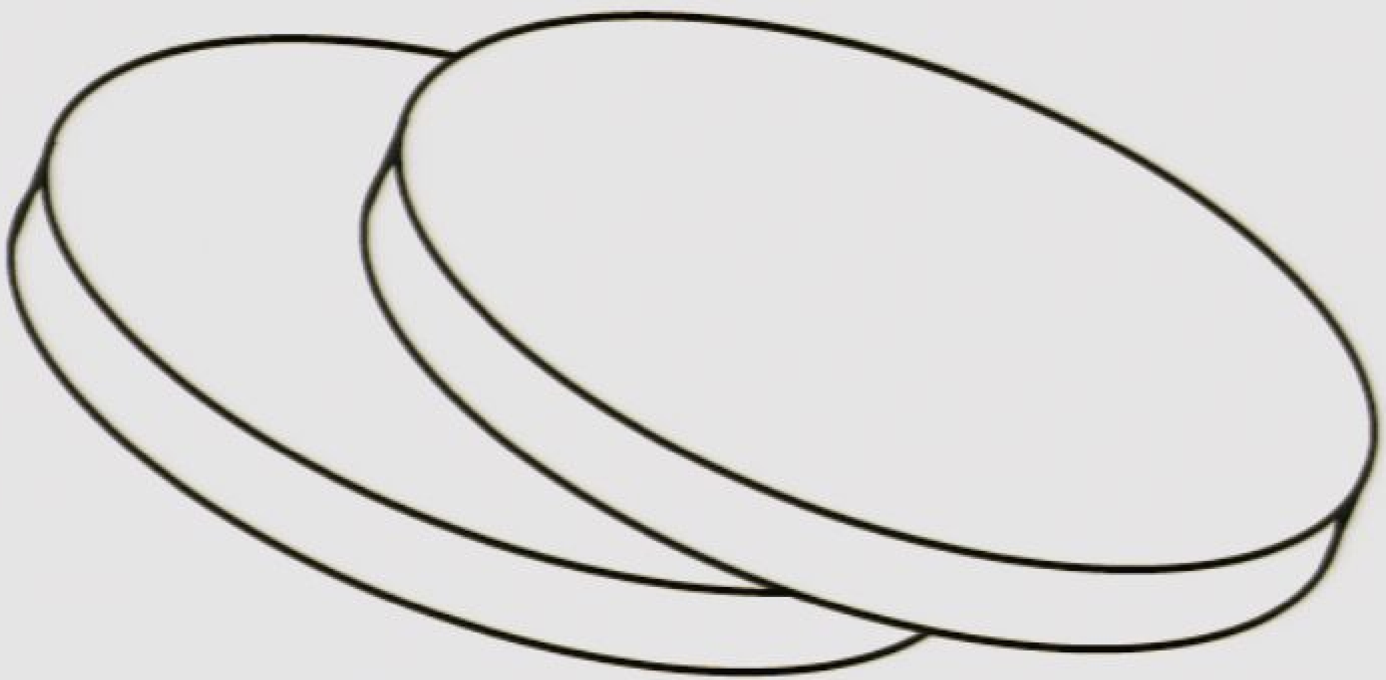
Date: Oct 29, 2010 02:30 PM

URL: http://pirsa.org/10100034

Abstract: Hydrodynamics is the universal theory describing the behavior of fluids when their spacetime variation is on scales longer than any microphysical scale in the fluid. Relativistic hydro has applications in heavy ion collisions and early Universe cosmology, and has seen a surge of interest due to heavy ion experiments and theoretical developments in AdS/CFT. I will explain what second order hydrodynamics is and why it is the minimum theory to study in the relativistic case. Then I discuss some limitations of the theory, including a new bound on how small the viscosity can be and a complication in the rigorous definition of the viscous relaxation time

Heavy ion collisions

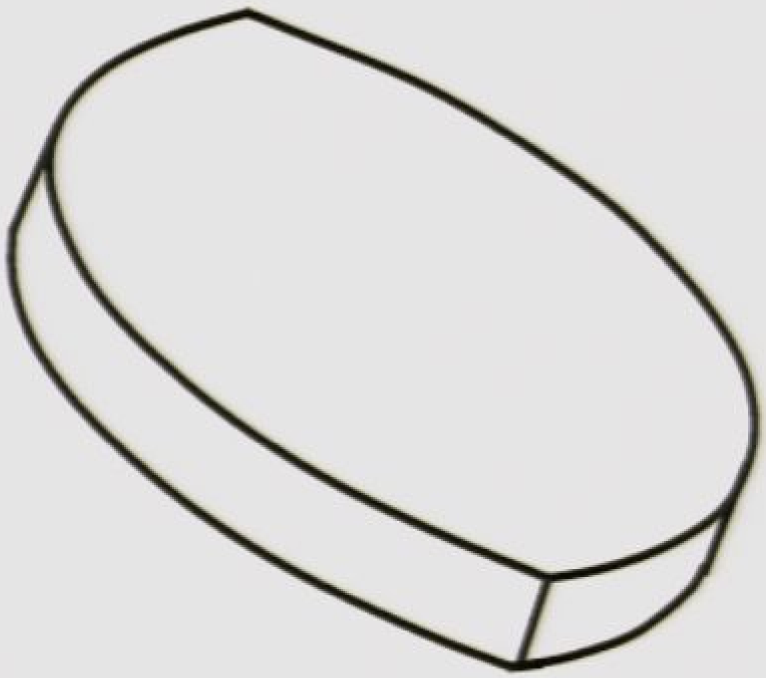
Accelerate two heavy nuclei to high energy, slam together.



Just before: Lorentz contracted nuclei

After the scattering: region where nuclei overlapped:

"Flat almond" shaped region of $q, \bar{q}, g$ which scattered.



$\sim$2 thousand random v quarks+gluons: isotropic in xy plane

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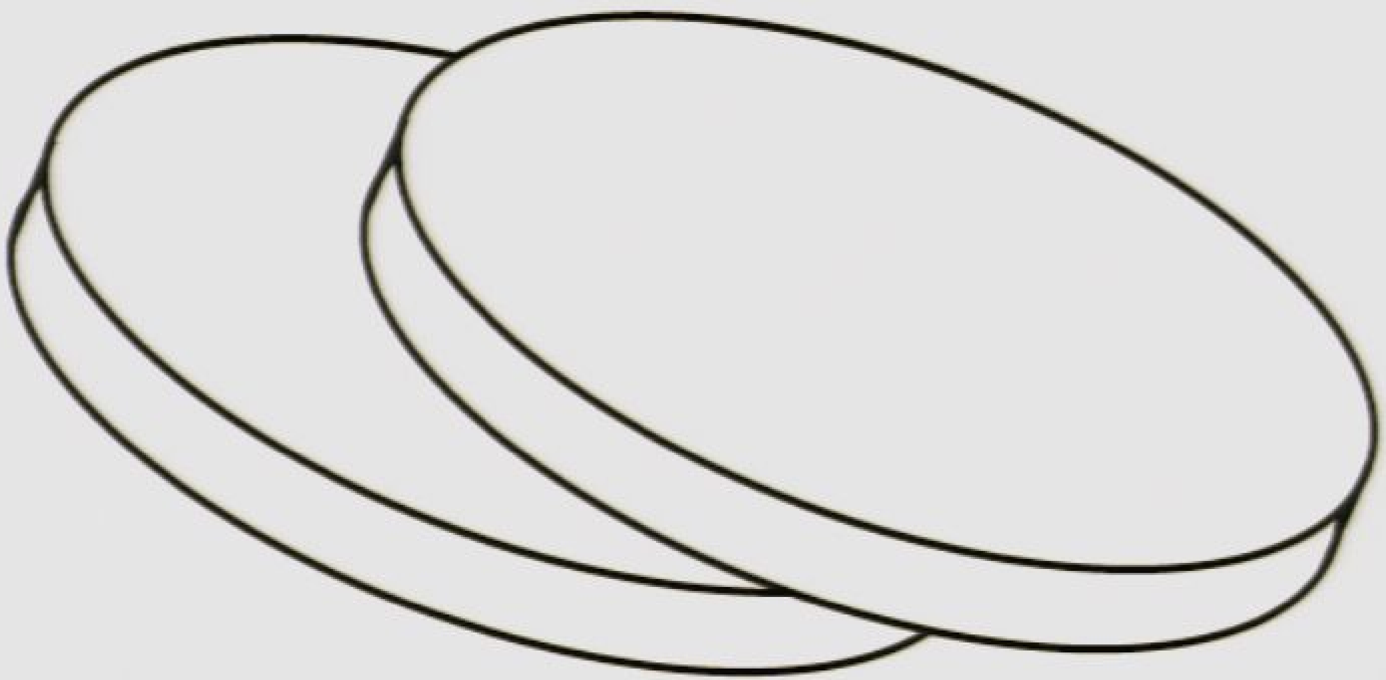

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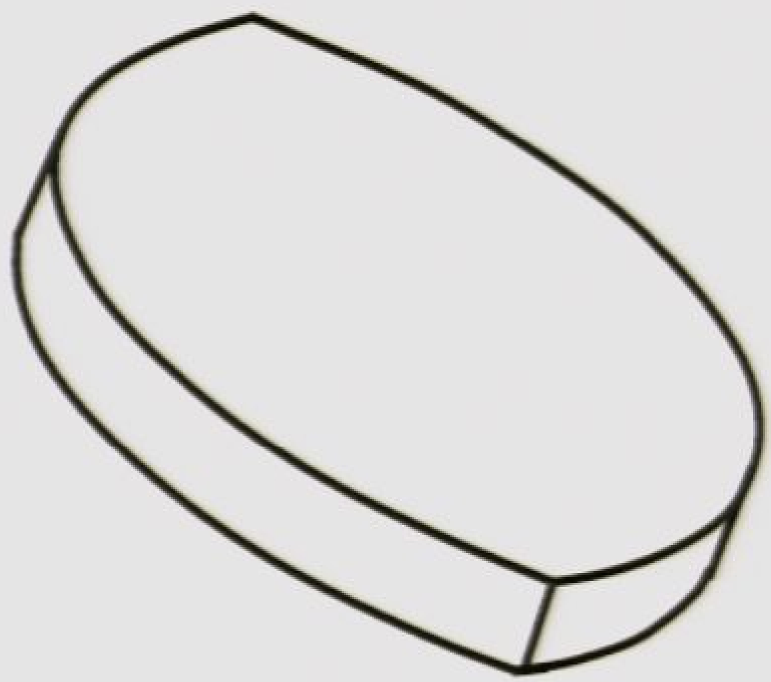

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Free particle propagation:

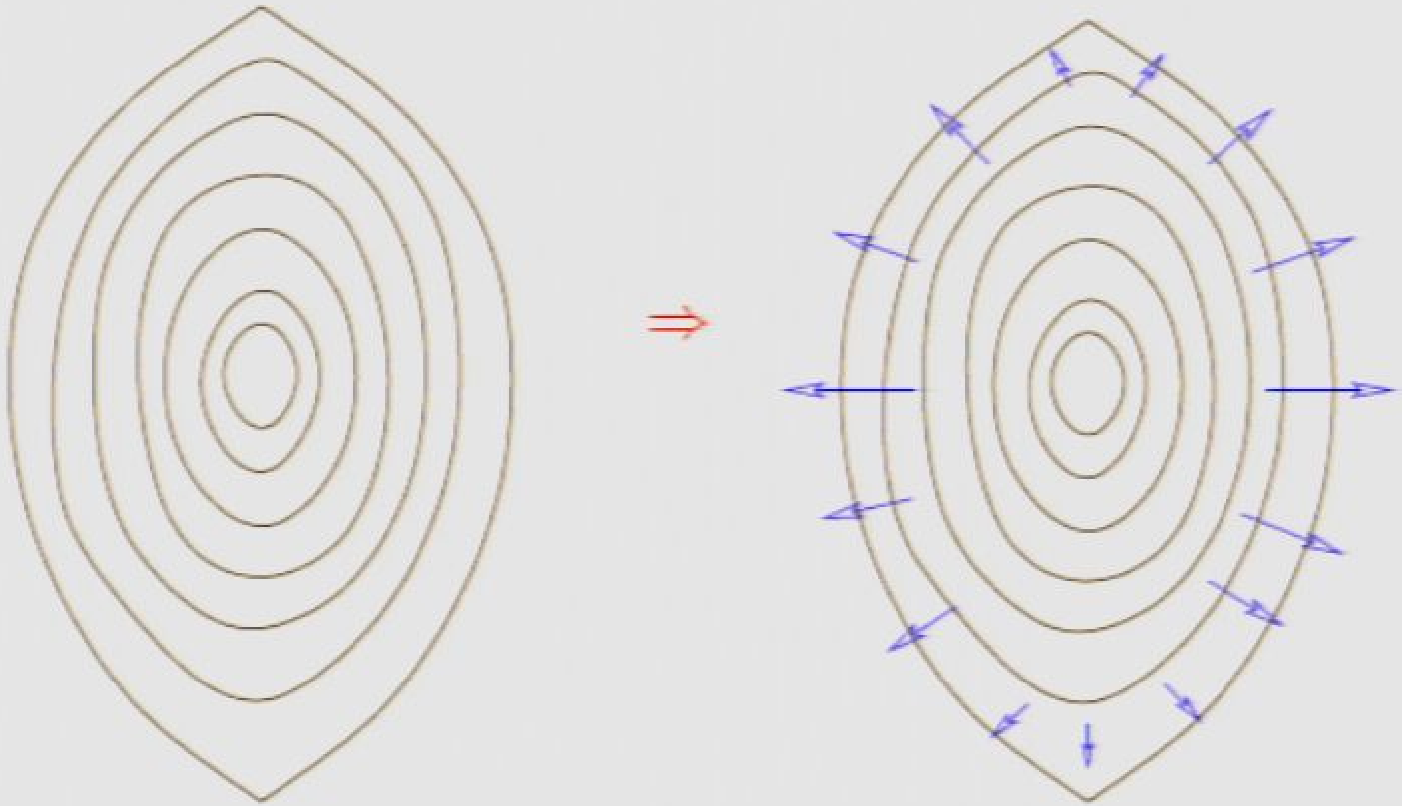
- System-average CM flow velocities $\langle v_{x,\mathrm{CM}}^2 \rangle > \langle v_{y,\mathrm{CM}}^2 \rangle$
- Particle distributions locally triaxial, $\langle v_x^2 \rangle < \langle v_y^2 \rangle$
- Total particle distribution $\langle v_x^2 \rangle = \langle v_y^2 \rangle$

Efficient Equilibration:

- System-average CM flow still has $\langle v_{x,\mathrm{CM}}^2 \rangle > \langle v_{y,\mathrm{CM}}^2 \rangle$
- system changes *locally* towards $\langle v_{x,\mathrm{relative}}^2 \rangle = \langle v_{y,\mathrm{relative}}^2 \rangle$
- Adding these together, $\langle v_{x,\mathrm{tot}}^2 \rangle > \langle v_{y,\mathrm{tot}}^2 \rangle$

Net "Elliptic Flow" $v_2 \equiv \frac{v_x^2 - v_y^2}{v_x^2 + v_y^2}$ measures re-interaction

local CM motions



Pressure contours Expansion pattern

Anisotropy leads to anisotropic (local CM motion) flow.

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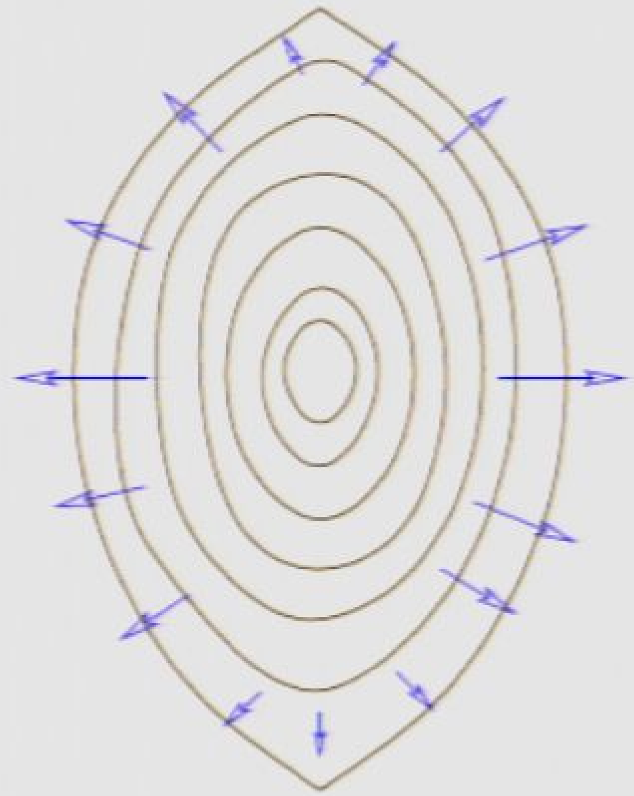

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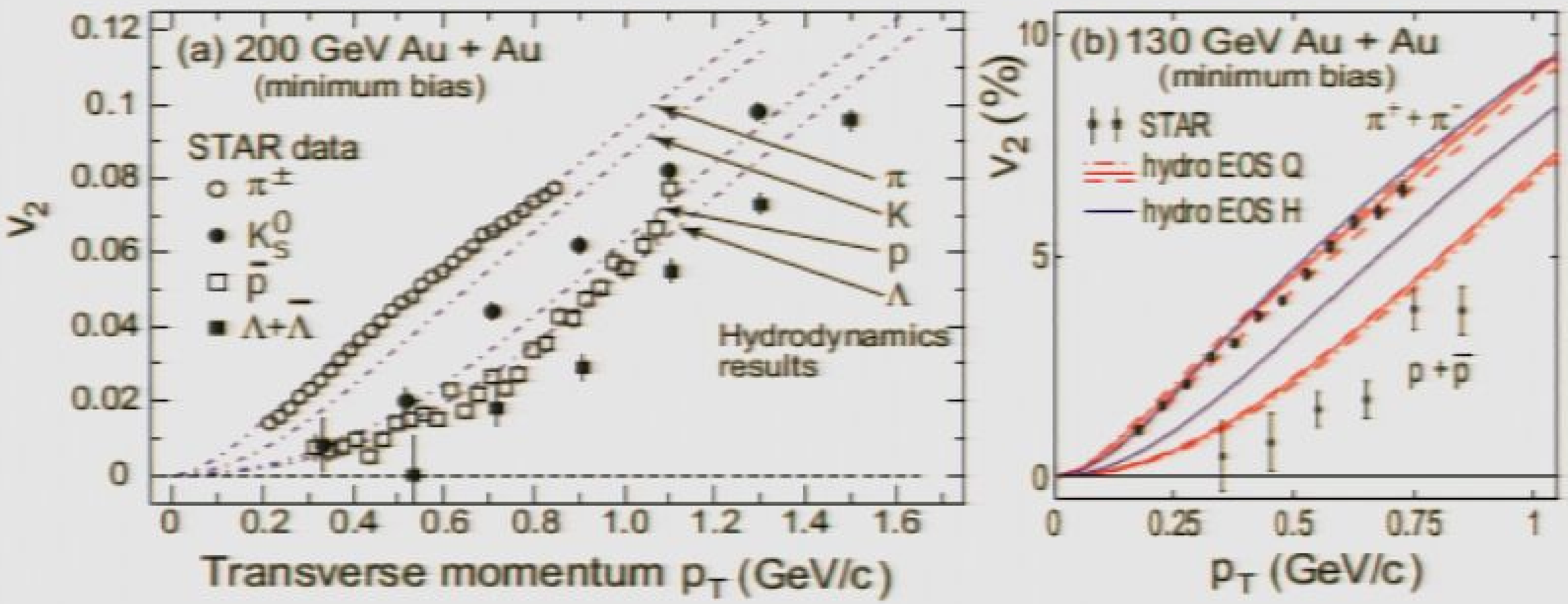
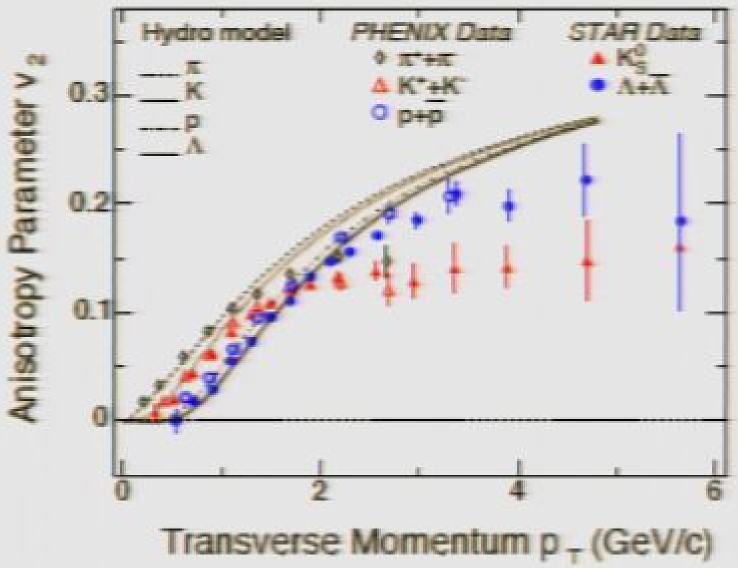
Elliptic flow is measured

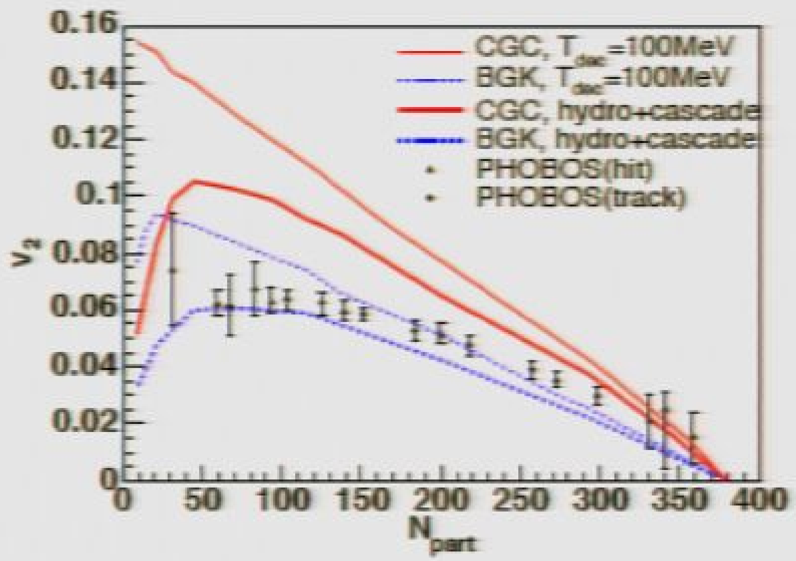


STAR experiment, minimum bias...

We should try to understand it theoretically.

First attempt: ideal hydro





Works, DEPENDING on initial conditions.

Corrections to ideality exist, but are "small" (?)

Can we quantify that?

Ideal Hydrodynamics

Ideal hydro: stress-energy conservation

$$\partial_\mu T^{\mu\nu} = 0 \quad \text{(4 equations, 10 unknowns)}$$

plus local equilibrium *assumption*:

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\text{eq}} = \epsilon u^\mu u^\nu + P(\epsilon)\Delta^{\mu\nu}, \\ u^\mu u_\mu &= -1, \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu \end{aligned}$$

depends on 4 parameters (ϵ, 3 comp of u^μ): closed.

works pretty well for heavy ions. But **quantify** corrections!

Nonideal Hydro

Assume that ideal hydro is "good starting point," look for small systematic corrections.

Near equilibrium iff $t_{\rm therm} \ll t_{\rm vary}, l_{\rm vary}/v$ (so ∂ small)

Allows expansion of corrections in gradients:

$$
\begin{aligned}
T^{\mu\nu} &= T^{\mu\nu}_{\rm eq} + \Pi^{\mu\nu}[\partial, \epsilon, u] \\
\Pi^{\mu\nu} &= \mathcal{O}(\partial\mu, \partial\epsilon) + \mathcal{O}(\partial^2\mu, (\partial\mu)^2, \ldots) + \mathcal{O}(\partial^3 \ldots)
\end{aligned}
$$

For Conformal theory $T^\mu_\mu = 0 = \Pi^\mu_\mu$, 1-order term unique:

$$
\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} , \quad \sigma^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\partial_\alpha u_\beta + \partial_\beta u_\alpha - \frac{2}{3} g_{\alpha\beta}\partial\cdot u\right)
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Coefficient η is shear viscosity.

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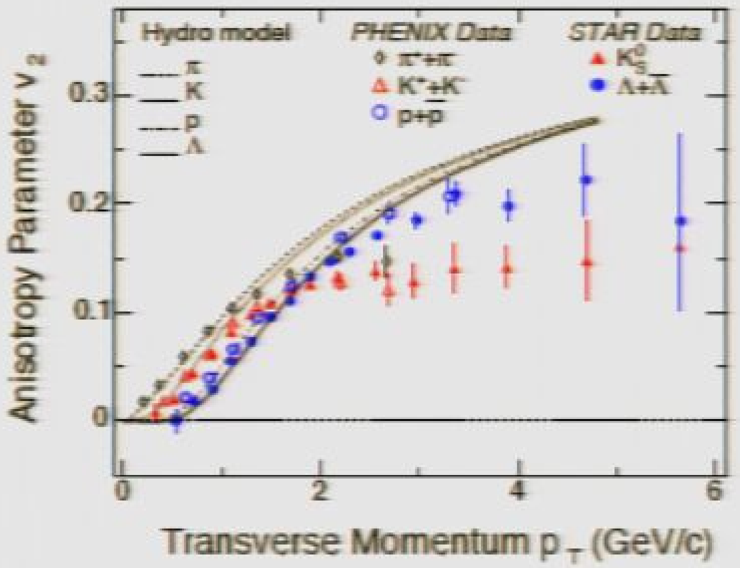

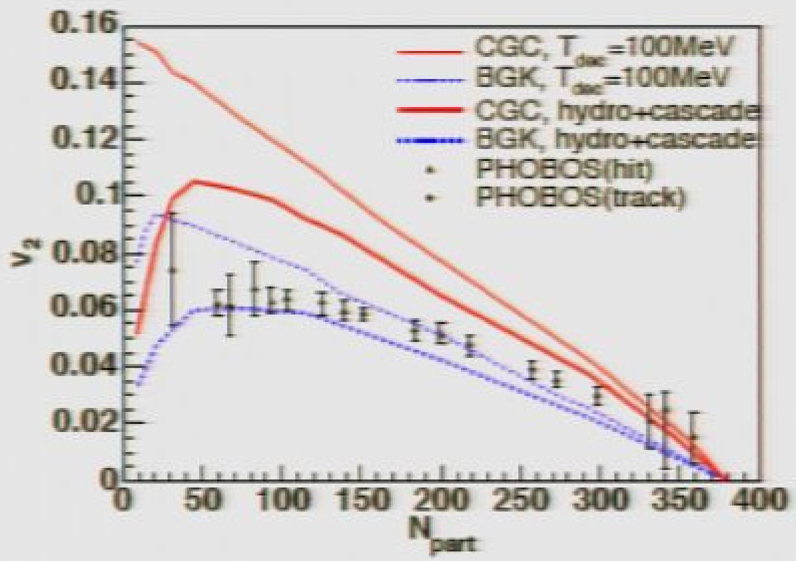

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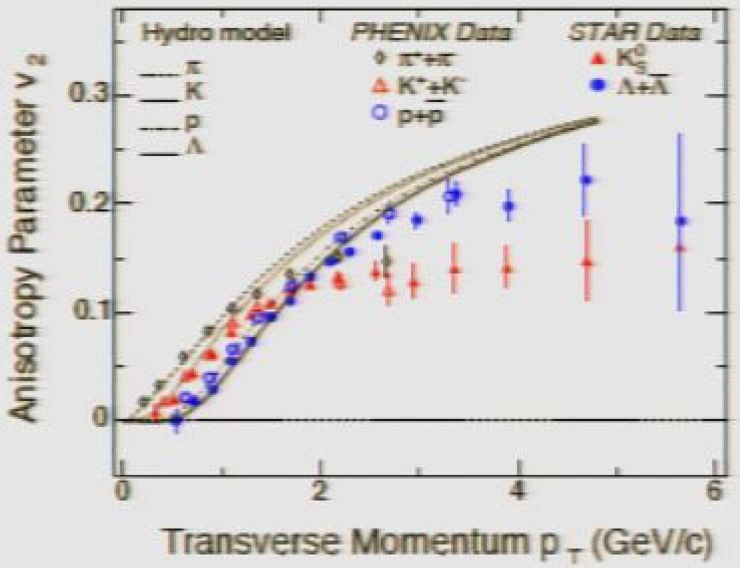

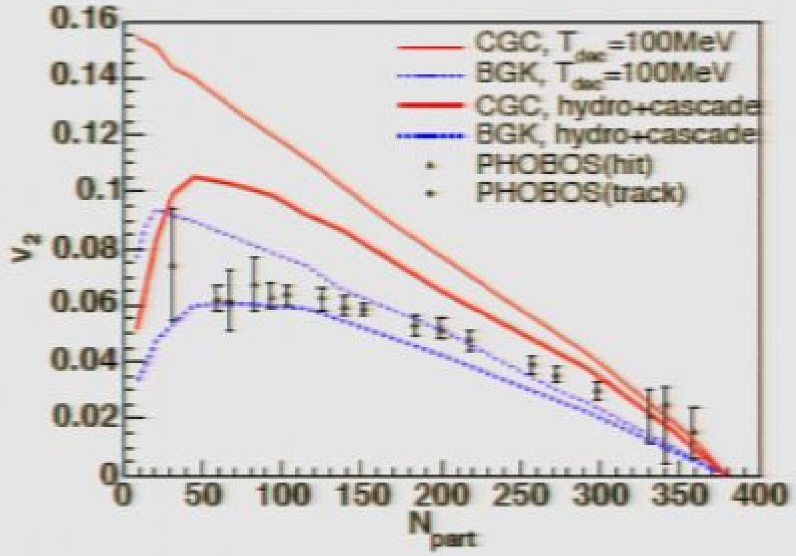

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Coefficient η is shear viscosity.

So why not consider (Navier-Stokes)

$$T^{\mu\nu} = \epsilon u^\mu u^\nu + P\Delta^{\mu\nu} - \eta\sigma^{\mu\nu} \quad ?$$

Because in relativisitc setting, it is

- Acausal: shear viscosity is transverse momentum diffusion. Diffusion $\partial_t P_\perp \sim \nabla^2 P_\perp$ has instantaneous prop. speed. Müller 1967, Israel+Stewart 1976
- Unstable: $v > c$ prop + non-uniform flow velocity → propagate from future into past, exponentially growing solutions. Hiscock 1983

Problem: short length scales, $\eta|\sigma| \sim P$. Numerics must treat these scales (or there's "numerical viscosity")

Israel-Stewart approach

Add one second order term:

$$\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \eta\tau_\pi\, u^\alpha \partial_\alpha \sigma^{\mu\nu}$$

Make (1'st order accurate) $\eta\sigma \to -\Pi$ in order-2 term:

$$\tau_\pi\, u^\alpha \partial_\alpha \Pi^{\mu\nu} \equiv \tau_\pi\, \dot{\Pi}^{\mu\nu} = -\eta\sigma^{\mu\nu} - \Pi^{\mu\nu}$$

Relaxation eq driving $\Pi^{\mu\nu}$ towards $-\eta\sigma^{\mu\nu}$.

Momentum diff. no longer instantaneous.

Causality, stability are restored (depending on τ_π)

But why only one 2'nd order term???

Second order hydrodynamics

It is more consistent to include all possible 2'nd order terms.

Assume *conformality* and *vanishing chem. potentials*:

5 possible terms Baier et al, [arXiv:0712.2451]

$$\Pi^{\mu\nu}_{2\,\mathrm{ord.}} = \eta\tau_\pi\left[u^\alpha\partial_\alpha\sigma^{\mu\nu} + \frac{1}{3}\sigma^{\mu\nu}\partial_\alpha u^\alpha\right] + \lambda_1\left[\sigma^\mu_\alpha\sigma^{\nu\alpha} - (\mathrm{trace})\right]$$
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Let's learn what we can about this theory, its 6 coeff's

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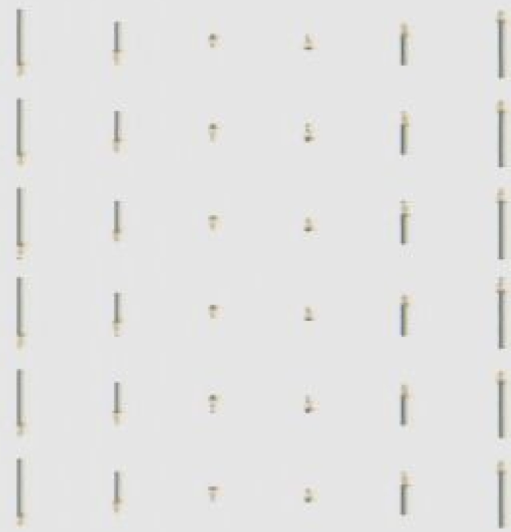
Step 1: What do these mean?

Consider $\partial_y v_x \neq 0$ and $\partial_x v_y \neq 0$:



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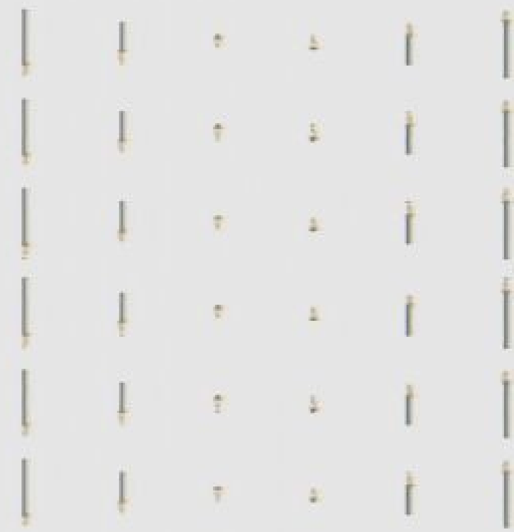

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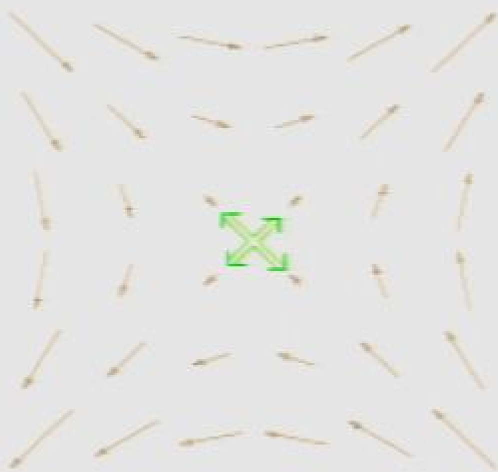
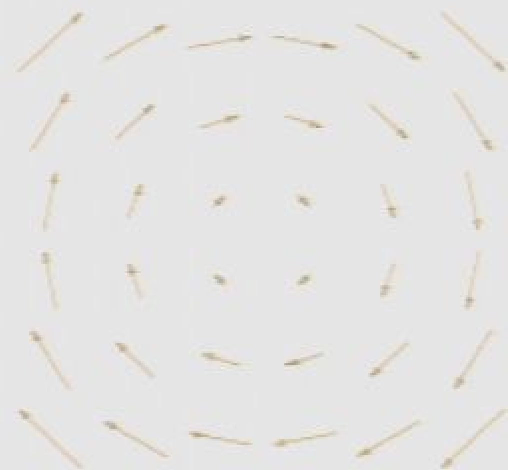


Shear flow



Vorticity

Two basic local measures of flow nonuniformity.

First order: $\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$ as it's symmetric!

Fluid "pushing back" against shear flow

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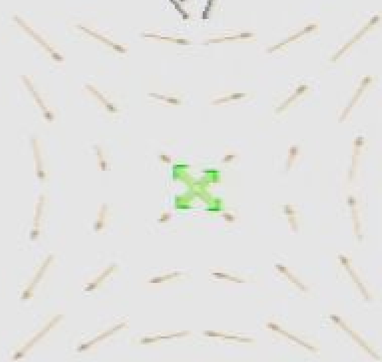

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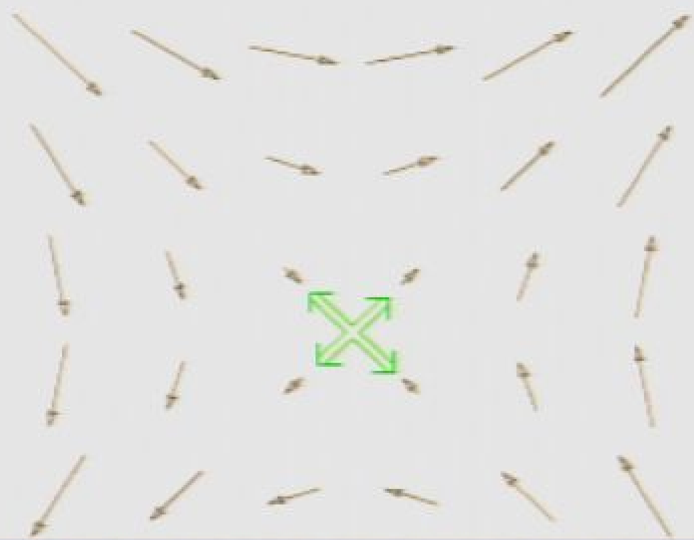

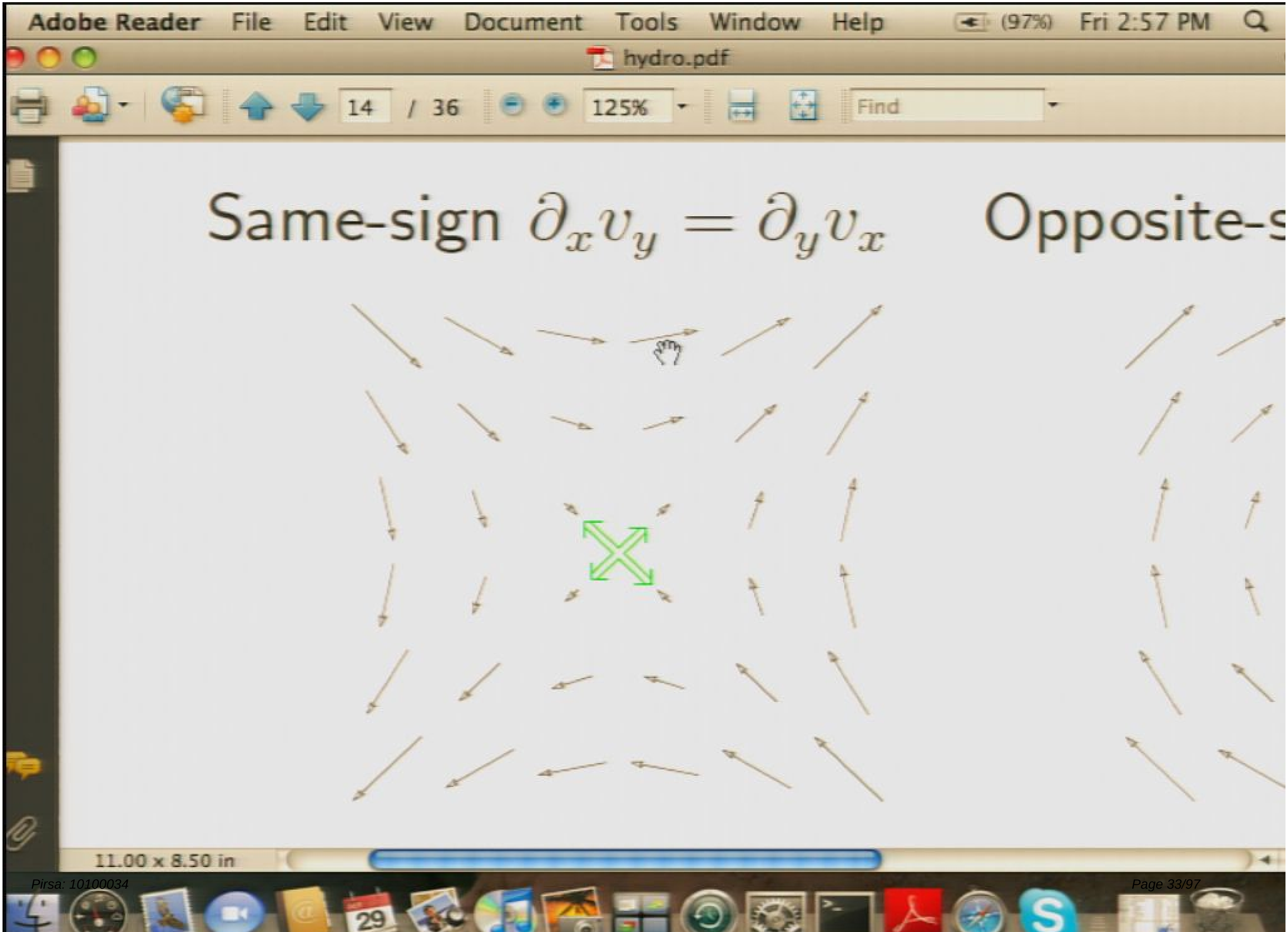
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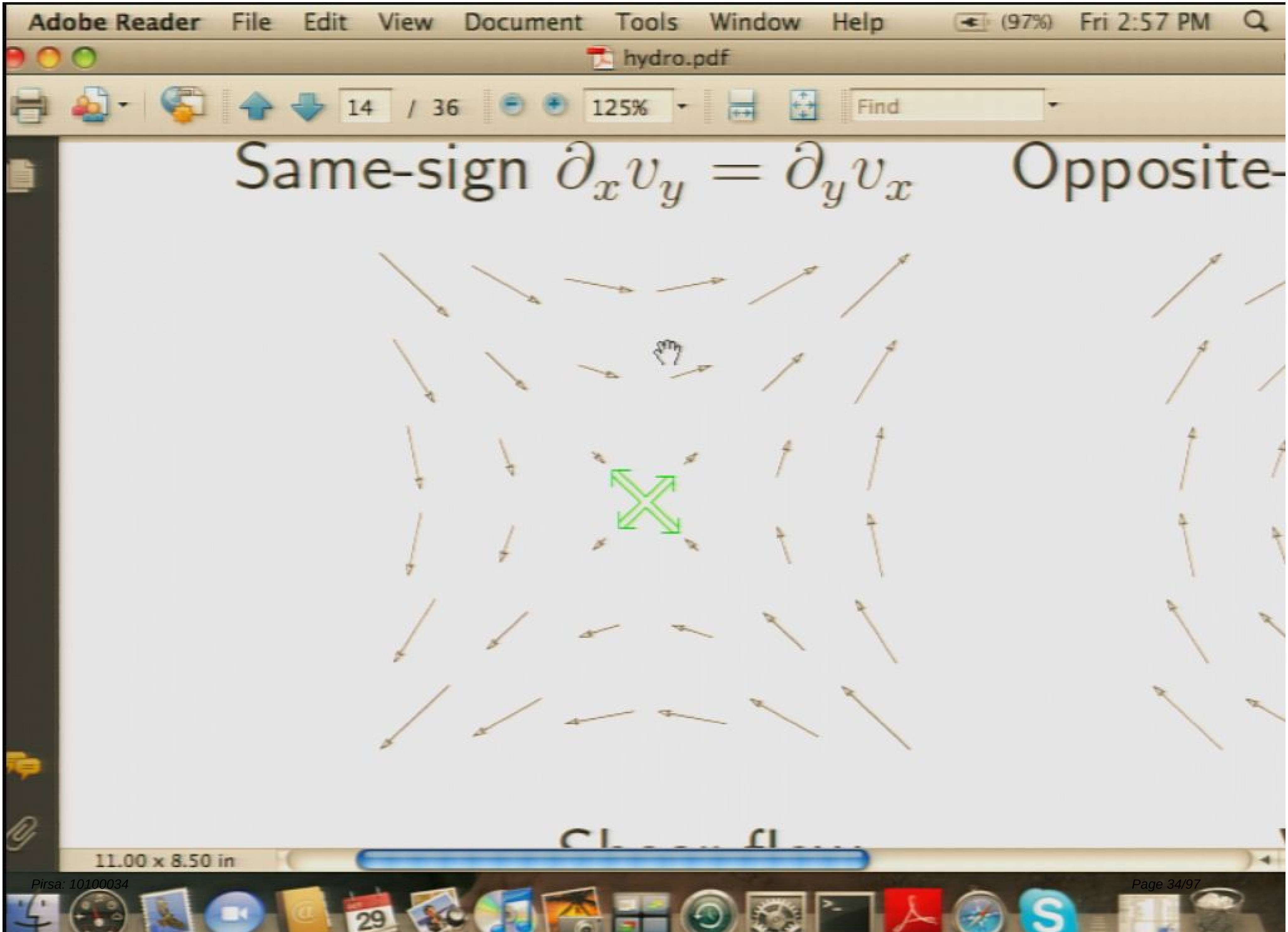
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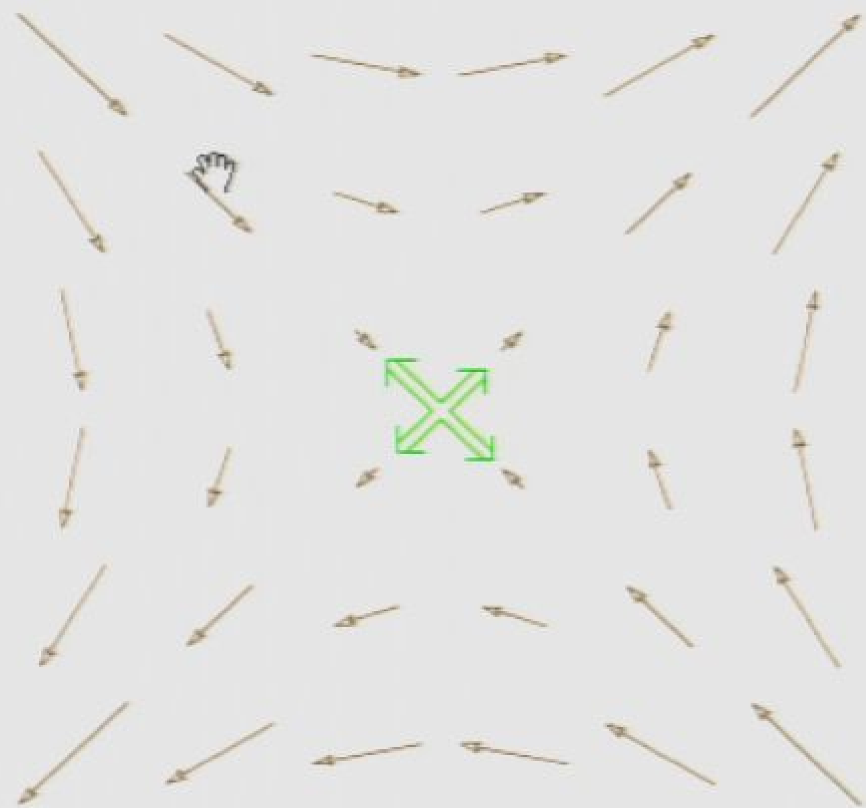

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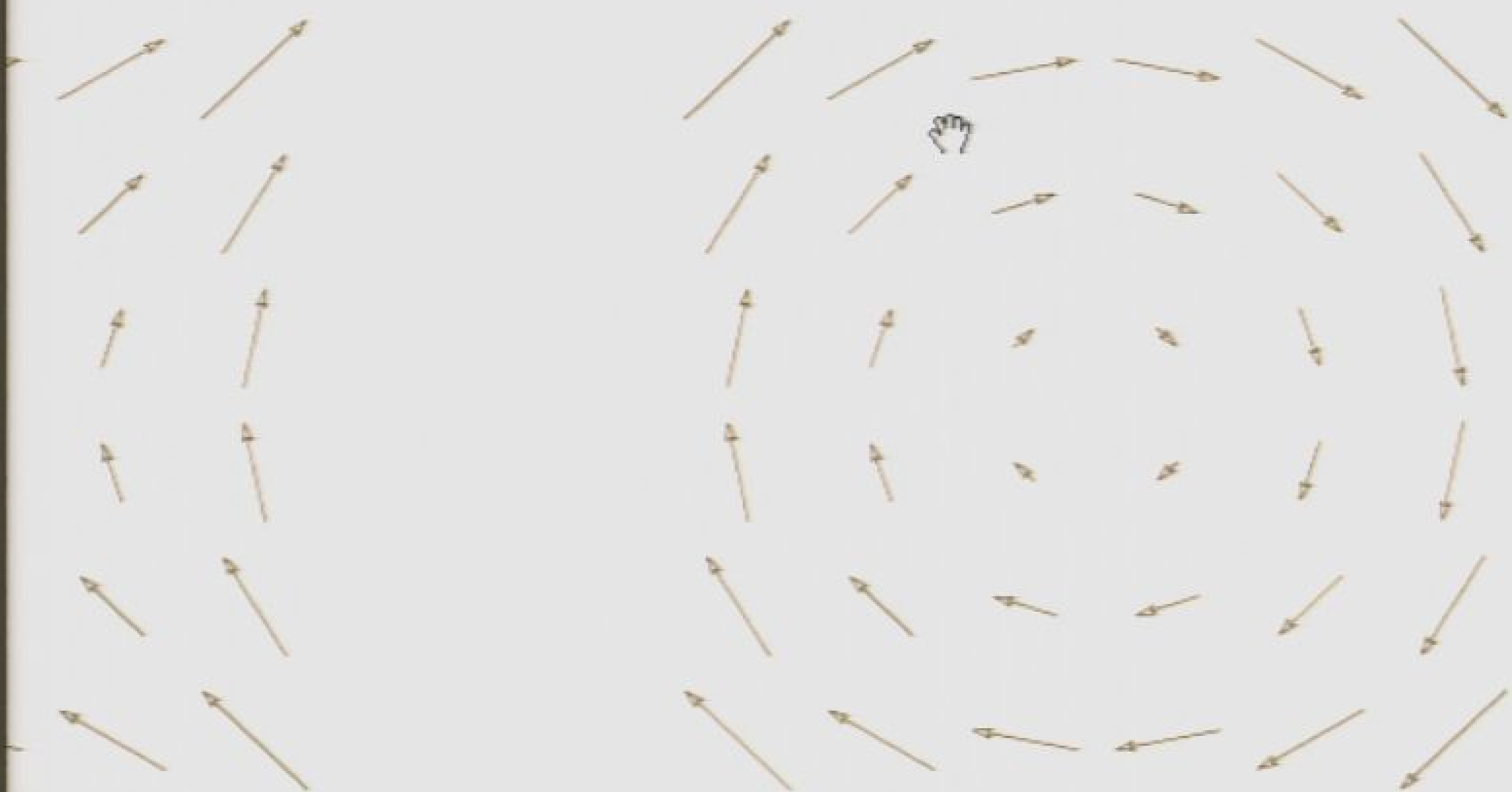

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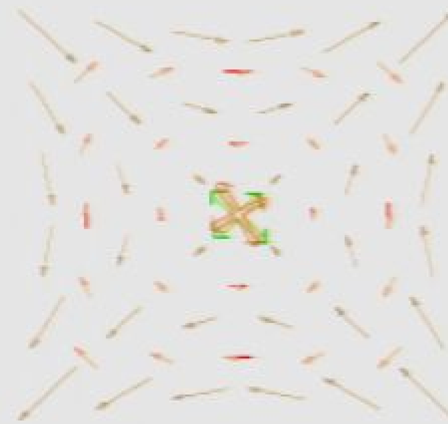
Fluid "pushing back" against shear flow

τ_π: if shear flow $\sigma^{\mu\nu}$ "turns on", delay in $\Pi^{\mu\nu}$ "turning on"

λ_2: if shear makes $\Pi^{\mu\nu} \neq 0$, vorticity rotates $\Pi^{\mu\nu}$ axis from shear axis. Sensible sign if $\lambda_2 < 0$ (sorry)



λ_1: $T^{ij} > -\eta\sigma^{ij}$ (nonlin.) for oblate, or prolate flow?

λ_3: $T^{\mu\nu}$ larger in-plane of vorticity (rotation) than on-axis.

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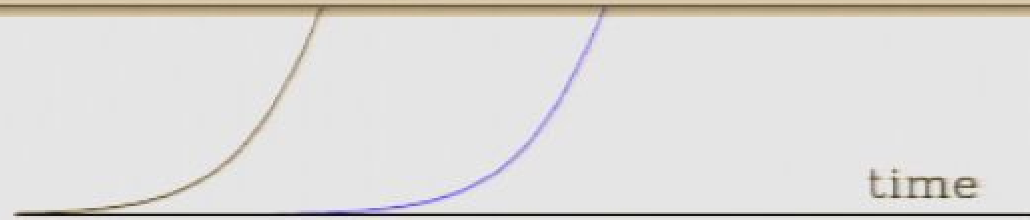

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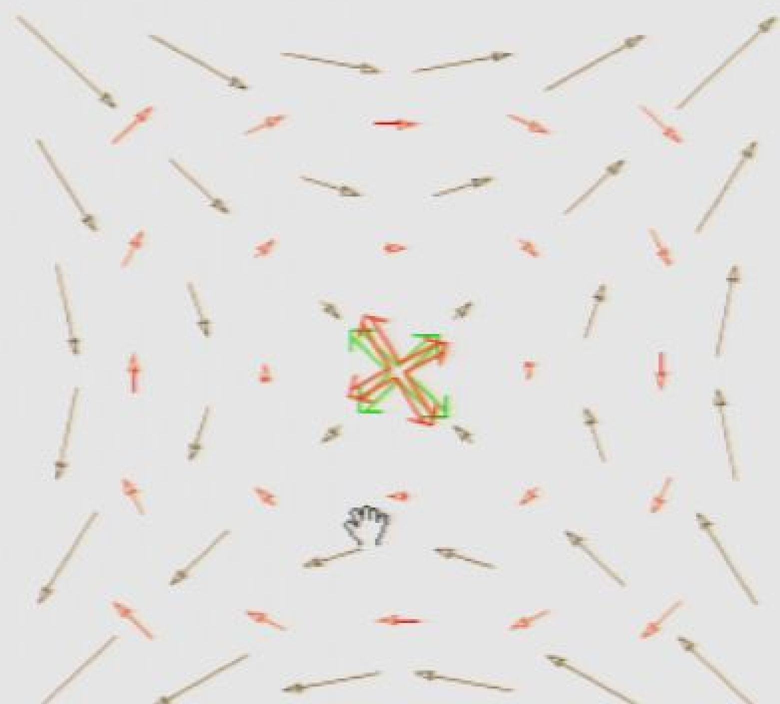

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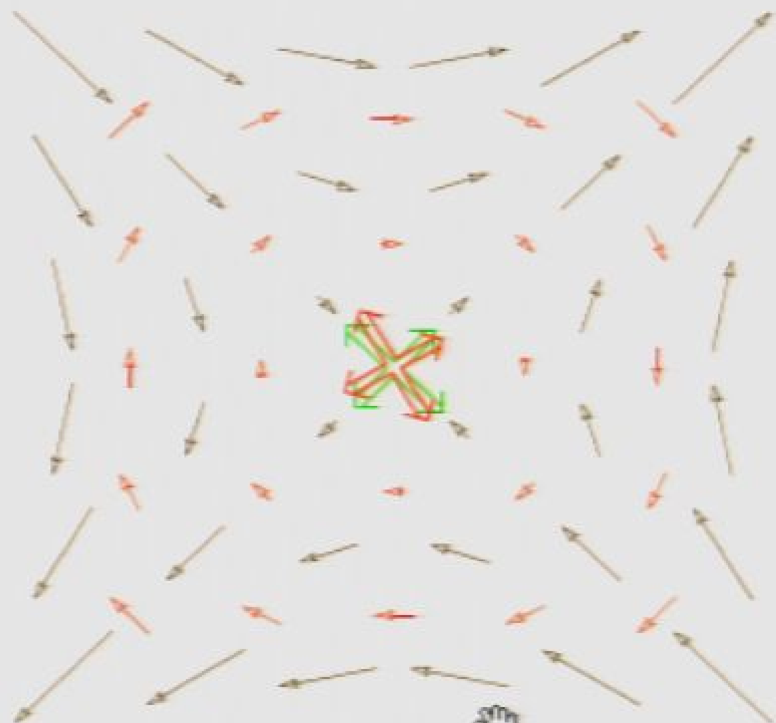

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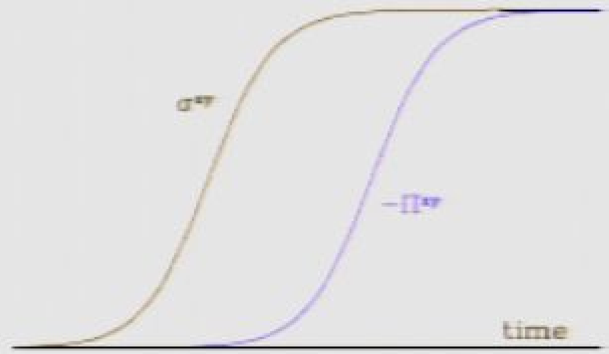
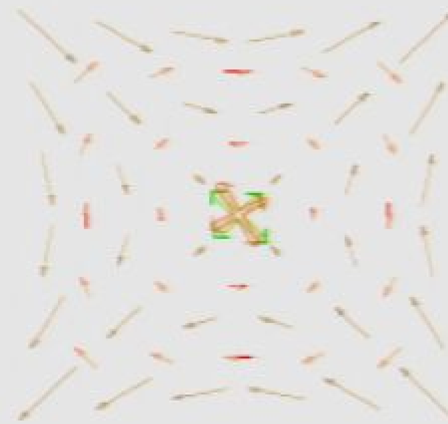


λ_2: if shear makes $\Pi^{\mu\nu} \neq 0$, vorticity rotates $\Pi^{\mu\nu}$ axis from shear axis. Sensible sign if $\lambda_2 < 0$ (sorry)



λ_1: $T^{ij} > -\eta\sigma^{ij}$ (nonlin.) for oblate, or prolate flow?

λ_3: $T^{\mu\nu}$ larger in-plane of vorticity (rotation) than on-axis.

I would like to calculate these coefficients.

Two cases: weak coupling, strong-coupled N=4SYM

That failing, I want a rule for relating them to equilibrium field theory correlators (Kubo relation)

In any case I want to understand consistency or limitations of 2'nd order hydro theory.

Goals of remainder of the talk

Two toy models of QCD

To date, coeff's computed in *toy model* for QCD:
$\mathcal{N}=4$ SYM theory at $N_c, g^2 N_c \to \infty$
(conformal, vast number DOF, many scalars, infinite coupling,...) Baier et al [arXiv:0712.2451], Tata group, [arXiv:0712.2456]

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Leading order calculation: theory conformal, same coeff's
Toolkit for calculation: kinetic theory (valid at leading order)

Kinetic theory

State, all measurables described by particle distrib. $f_a(x,p)$:

$$T^{\mu\nu}(x) = \sum_a \int_p 2P^\mu P^\nu f_a(x,p)\,, \qquad \int_p \equiv \int \frac{d^3p}{(2\pi)^3 2p^0}$$

Dynamics: Boltzmann equation (Schwinger-Dyson eq):

$$2P^\mu \partial_{\mu_x} f(x,p) \;=\; -\mathcal{C}[p, f(x,q)]$$

LHS: particle propagation. $p^0 \equiv \sqrt{\mathrm{p}^2} \equiv p$

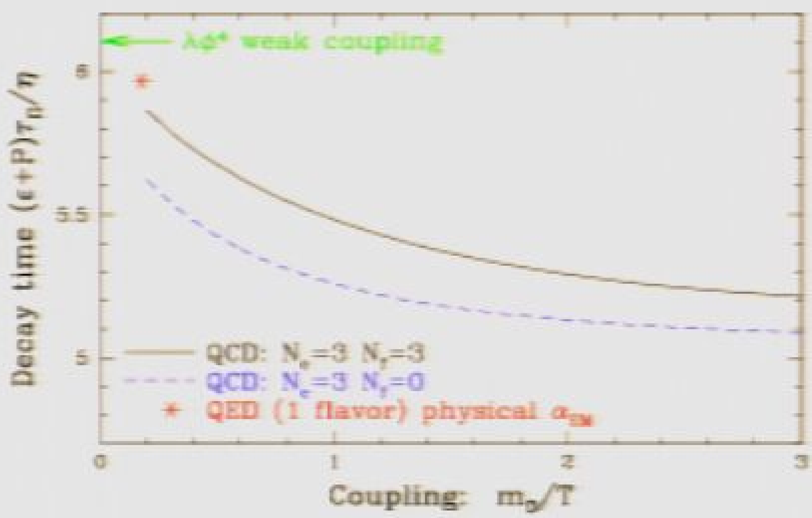
RHS: scattering (Im self-energy). Local in x but not p.

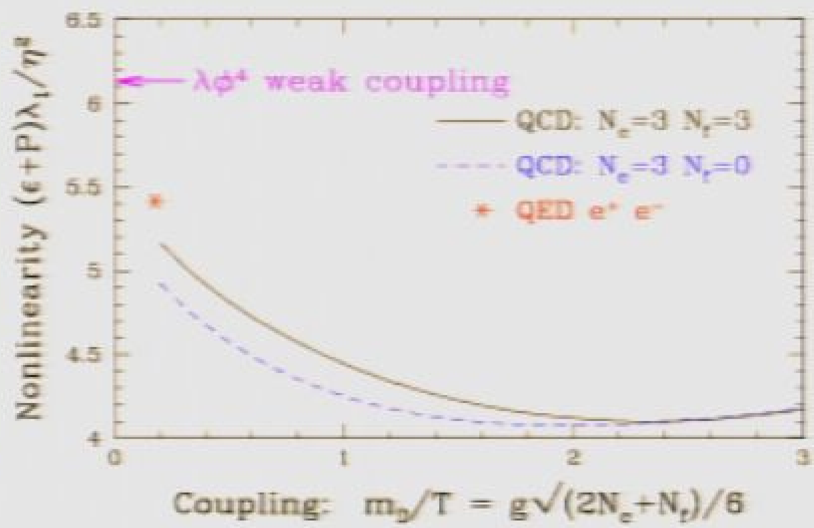
Theory dependence all contained in detailed form of $\mathcal{C}[f]$.

In our case, described in detail in AMY5: `hep-ph:0209353`

Results

$\lambda_3 = \kappa = 0$. $\lambda_2 = -2\eta\tau_\pi$. τ_π, λ_1 nontrivial:





Size of uncertainty is thinner than lines in plots!

Ratios are very stable with value of coupling.

QCD vs SYM comparison

Dimensionless ratios: sensible if one relaxation timescale.

Ratio	QCD value	SYM value
$\frac{\tau_\pi(\epsilon+P)}{\eta}$	5 to 5.9	2.6137
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Good news: Not qualitatively different.

Kinetic theory relation $\lambda_2 = -2\eta\tau_\pi$ not actually general.

Kubo formulae

Find framework where I can compute $T^{\mu\nu}$ using hydro *and* using field theory, both should be valid.

Time-varying geometry does the job:

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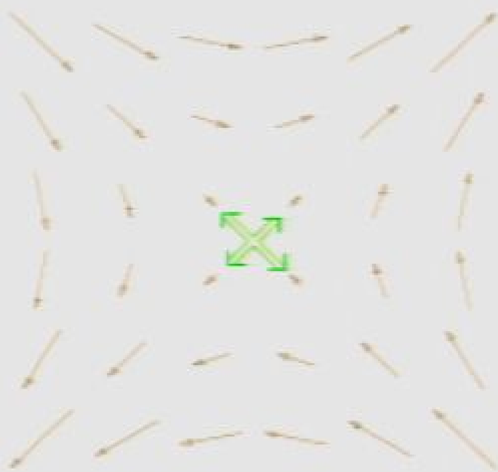
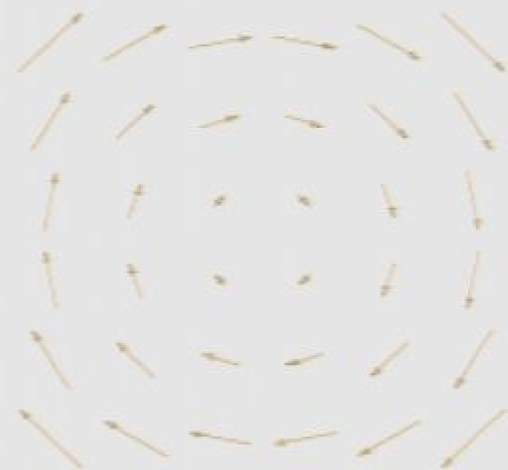
Step 1: What do these mean?

Same-sign $\partial_x v_y = \partial_y v_x$ Opposite-sign $\partial_x v_y = -\partial_y v_x$



Shear flow



Vorticity

Two basic local measures of flow nonuniformity.

First order: $\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu}$ as it's symmetric!

Fluid "pushing back" against shear flow

Second order hydrodynamics

It is more consistent to include all possible 2'nd order terms.

Assume *conformality* and *vanishing chem. potentials*:

5 possible terms Baier et al, [arXiv:0712.2451]

$$\Pi^{\mu\nu}_{2\,\mathrm{ord.}} = \eta\tau_\pi \left[u^\alpha \partial_\alpha \sigma^{\mu\nu} + \frac{1}{3}\sigma^{\mu\nu}\partial_\alpha u^\alpha \right] + \lambda_1 \left[\sigma^\mu_\alpha \sigma^{\nu\alpha} - (\mathrm{trace}) \right]$$
$$+\lambda_2 \left[\frac{1}{2}(\sigma^\mu_\alpha \Omega^{\nu\alpha} + \sigma^\nu_\alpha \Omega^{\mu\alpha}) - (\mathrm{trace}) \right]$$
$$+\lambda_3 \left[\Omega^\mu{}_\alpha \Omega^{\nu\alpha} - (\mathrm{trace}) \right] + \kappa \left(R^{\mu\nu} - \ldots \right),$$
$$\Omega_{\mu\nu} \equiv \frac{1}{2}\Delta_{\mu\alpha}\Delta_{\nu\beta}(\partial^\alpha u^\beta - \partial^\beta u^\alpha) \ \ [\mathrm{vorticity}].$$

Let's learn what we can about this theory, its 6 coeff's

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Two approaches:

1) Choose $h_{\mu\nu}(x^\alpha)$ arbitrary. Find all allowed tensor structures to order $\partial^n h^m$ which are traceless rank-2 tensors invariant to gauge and conformal transformations, and so can contribute to $T^{\mu\nu}$. Expand $\partial^n h^m$ in this tensor basis.

But BRSSS did this work for us – list of allowed terms is list of allowed tensor structures to ∂^2 order

2) Choose $h_{\mu\nu}(x^\alpha)$ just general enough that all of the known allowed terms can appear with independent amplitudes.

We will follow this second, easier approach.
Claim: $h_{xy}(z,t)$ and $h_{0x}(y)$ is general enough.

Give a hydro theorist $h_{xy}(z,t)$, $h_{0x}(y)$ nonzero.

Ask them what $T^{\mu\nu}(0)$ will be.

Answer: $$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} + \Pi^{\mu\nu}$$

First, find ϵ, u: Hydro says

$$\nabla_\mu T^{\mu\nu} = 0 \;\rightarrow\; u^\mu = (1,0,0,0) + \mathcal{O}(\partial^2)\,.$$

Then $u_\mu = (1, h_{0x}, 0, 0)$, $\Gamma^x{}_{yt}$ *etc* nonzero.

They give rise to nonzero σ^{xy}, Ω^{xy}, etc:

$$\sigma^{xy} = \partial_t h_{xy}\,, \qquad \Omega^{xy} = -\partial_y h_{0x}/2$$

Other terms $R^{\langle xy\rangle}$, $u \cdot \nabla \sigma^{xy}$ found similarly.

T^{xy} at $\mathcal{O}(h)$ and $\mathcal{O}(\partial^2)$:

$$T^{xy} = -\eta\partial_t h_{xy} + \eta\tau_\pi\partial_t^2 h_{xy} - \frac{\kappa}{2}\left(\partial_t^2 h_{xy} + \partial_z^2 h_{xy}\right)$$

and T^{xx} at $\mathcal{O}(\partial^2, h^2)$:

$$\begin{aligned}\Pi^{xx} \;=\; & \frac{4}{3}\eta h_{xy}\partial_t h_{xy} + \eta\tau_\pi\left(-\frac{4}{3}h_{xy}\partial_t{}^2 h_{xy} + \partial_t h_{xy}\partial_y h_{0x}\right.\\ & \left.-\frac{1}{3}\left(\partial_t h_{xy}\right)^2\right) + \frac{\kappa}{3}\left(h_{xy}\partial_z{}^2 h_{xy} + 2h_{xy}\partial_t{}^2 h_{xy}\right.\\ & \left.-\partial_y h_{0x}\partial_t h_{xy} - h_{0x}\partial_y{}^2 h_{0x}\right) + \frac{1}{3}\lambda_1\left(\partial_t h_{xy}\right)^2\\ & -\frac{1}{2}\lambda_2\partial_t h_{xy}\partial_y h_{0x} + \frac{1}{12}\lambda_3(\partial_y h_{0x})^2 .\end{aligned}$$

So T^{xy} depends on η, τ_π, κ; T^{xx} is $\mathcal{O}(h^2)$, depends on all 6!

Give field theorist $h_{xy}(z,t)$, $h_{0x}(y)$ nonzero.

Ask them what T^{xy}, T^{xx} will be.

$$\langle T^{\mu\nu}(t)\rangle = \mathrm{Tr}\, e^{-HT} e^{iHt} \hat{T}^{\mu\nu} e^{-iHt}, \quad T^{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\partial \sqrt{-g}\mathcal{L}}{\partial h_{\mu\nu}}$$

with $H = H[h(t')]$! Schwinger-Keldysh in $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$:

$$W \equiv \ln \int_{C=} \mathcal{D}(\Phi_1, \Phi_2, \Phi_3)\, e^{iS_1[h_1,\Phi_1] - iS_2[h_2,\Phi_2] - S_3[\Phi_3]}$$

$S_1[h_1]$, $S_2[h_2]$ depend on independent fields *and metrics!*

$$T_1 = \frac{-2i\delta W}{\delta h_1}, \qquad T_2 = \frac{+2i\delta W}{\delta h_2}$$

Introduce average and difference variables:

$$h_r = \frac{h_1+h_2}{2}\,,\; h_a = h_1 - h_2\,,\quad T_r = \frac{T_1+T_2}{2}\,,\; T_a = T_1 - T_2$$

Note, due to signs $e^{iS_1 - iS_2}$, $T_r = \frac{-2i\delta W}{\delta h_a}$, $\quad T_a = \frac{-2i\delta W}{\delta h_r}$. Take $\delta/\delta h_a \to \langle T\rangle$. Then set $h_a = 0$, $h_r = h$, expand in h:

$$\begin{aligned} \langle T^{\mu\nu}\rangle_h &= G_r^{\mu\nu}(0) - \frac{1}{2}\int d^4x G_{ra}^{\mu\nu,\alpha\beta}(0,x)h_{\alpha\beta}(x) \\ &\quad + \frac{1}{8}\int d^4x d^4y G_{raa}^{\mu\nu,\alpha\beta,\gamma\delta}(0,x,y)h_{\alpha\beta}(x)h_{\gamma\delta}(y) \end{aligned}$$

$$\begin{aligned} G_{ra\ldots}^{\mu\nu,\alpha\beta\ldots}(0,x\ldots) &\equiv \left.\frac{(-i)^{n-1}(-2i)^n\delta^n W}{\delta g_{a,\mu\nu}(0)\delta g_{r,\alpha\beta}(x)\ldots}\right|_{g_{\mu\nu}=\eta_{\mu\nu}} \\ &= (-i)^{n-1}\left\langle T_r^{\mu\nu}(0)T_a^{\alpha\beta}(x)\ldots\right\rangle + \text{c.t.} \end{aligned}$$

Now equate the hydro, field theorist answers:

$$T^{xy} = -\eta\partial_t h_{xy} + \eta\tau_\pi \partial_t^2 h_{xy} - \frac{\kappa}{2}\left(\partial_t^2 h_{xy} + \partial_z^2 h_{xy}\right)$$
$$= -\int d^4x\, G_{ra}^{xy,xy}(0,x) h_{xy}(x)$$

Introduce Fourier transform

$$G_{ra}^{xy,xy}(\omega,k) = \int d^4x e^{i(\omega t - kz)} G_{ra}^{xy,xy}(0,x)$$

and use that h slowly varying, find BRSSS 0712.2451

$$\eta = -i\partial_\omega G_{ra}^{xy,xy}(\omega,k)|_{\omega=0=k}\,,$$
$$\kappa = -\partial_{k_z}^2 G_{ra}^{xy,xy}(\omega,k)|_{\omega=0=k}\,,$$
$$\eta\tau_\pi = \frac{1}{2}\left(\partial_\omega^2 G_{ra}^{xy,xy}(\omega,k) - \partial_{k_z}^2 G_{ra}^{xy,xy}(\omega,k)\right)\Big|_{\omega=0=k}\,.$$

Repeat for T^{xx} and $\mathcal{O}(h^2)$ terms:

$$\begin{aligned}
\Pi^{xx} &= \frac{4}{3}\eta h_{xy}\partial_t h_{xy} + \eta\tau_\pi\left(-\frac{4}{3}h_{xy}{\partial_t}^2 h_{xy} + \partial_t h_{xy}\partial_y h_{0x}\right.\\
&\left.-\frac{1}{3}\left(\partial_t h_{xy}\right)^2\right) + \frac{\kappa}{3}\left(h_{xy}{\partial_z}^2 h_{xy} + 2h_{xy}{\partial_t}^2 h_{xy}\right.\\
&\left.-\partial_y h_{0x}\partial_t h_{xy} - h_{0x}{\partial_y}^2 h_{0x}\right) + \frac{1}{3}\lambda_1\left(\partial_t h_{xy}\right)^2\\
&-\frac{1}{2}\lambda_2\partial_t h_{xy}\partial_y h_{0x} + \frac{1}{12}\lambda_3(\partial_y h_{0x})^2\\
&= \int d^4x d^4y\left(\frac{1}{2}G_{raa}^{xx,xy,xy}(0,x,y)h_{xy}(x)h_{xy}(y)\right.\\
&\quad + G_{raa}^{xx,xy,0x}h_{xy}(x)h_{0x}(y)\\
&\quad \left.+\frac{1}{2}G_{raa}^{xx,0x,0x}(0,x,y)h_{0x}(x)h_{0x}(y)\right)
\end{aligned}$$

Introduce Fourier transforms again:

$$G_{raa}^{xx,xy,0x}(p,q) \equiv \int d^4x d^4y e^{-i(p\cdot x+q\cdot y)} G_{raa}^{xx,xy,0x}(0,x,y) \quad etc$$

Read off 2'nd order Kubo relations:

$$\begin{aligned}
\lambda_1 &= \eta\tau_\pi - \frac{3}{2}\lim_{p,q\to 0}\frac{\partial^2}{\partial p^t \partial q^t} G_{raa}^{xx,xy,xy}(p,q) \\
\lambda_2 &= 2\eta\tau_\pi - \frac{2\kappa}{3} + 2\lim_{p,q\to 0}\frac{\partial^2}{\partial p^t \partial q^y} G_{raa}^{xx,xy,0x}(p,q) \\
\lambda_3 &= -6\lim_{p,q\to 0}\frac{\partial^2}{\partial p^y \partial q^y} G_{raa}^{xx,0x,0x}(p,q) .
\end{aligned}$$

Nature of κ and λ_3

κ and λ_3 have Kubo relations **NOT** involving ∂_t's.

May set frequency $\omega = 0$ from outset:

$$\kappa = -\lim_{\vec{q}\to 0} \frac{\partial^2}{\partial q_z^2} G_{ra}^{xy,xy}(\vec{q}, \omega = 0)$$

$$\lambda_3 = -6 \lim_{\vec{p},\vec{q}\to 0} \frac{\partial^2}{\partial p_y \partial q_y} G_{raa}^{xx,0x,0x}(\vec{p}, \omega_p = 0, \vec{q}, \omega_q = 0)$$

But $G_{ra\ldots}(\omega = 0) = (-)^{n-1} G_{\rm E}(\omega_{\rm E} = 0)$ Euclidean func.

Weak-coupling expansions: $\kappa, \lambda_3 = T^2(\mathcal{O}(1) + \mathcal{O}(g, g^2, \ldots))$

Leading weak-coupling values calculable and *nonzero*

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We said $\Pi^{\mu\nu} = \mathcal{O}(\partial u) + \mathcal{O}(\partial^2 u, (\partial u)^2) + \ldots$

based on assumption thermalization is local, microscopic.

Hydro itself predicts long-lived shear,sound modes:

$$0 = \partial_\mu \Big(T^{\mu\nu} = (\epsilon + P) u^\mu u^\nu + P g^{\mu\nu} - \eta \sigma^{\mu\nu} \Big)$$

fluctuations in u^μ, ϵ: dispersion relations

$$\omega_{\text{shear}} = i \frac{\eta}{\epsilon + P} k^2 \, , \qquad \omega_{\text{sound}} = \pm \frac{k}{\sqrt{3}} + i \frac{2\eta}{3(\epsilon + P)} k^2$$

Small k: long lived, dissipation *not* local,microscopic

Hydro modes' own contrib. to Kubo relations

Above we found

$$G_{ra}^{xy,xy}(\omega) = P - i\eta\omega + \eta\tau_\pi\omega^2 + \dots$$

One contrib. at finite ω: shear, sound modes in loop.
Minimal assumption: thermally occupied (equipartition).
Hydro is IR effective theory, $\eta, \tau_\pi, \lambda_1, \dots$ are Wilson coeff. which can vary with scale.

$$\text{Feynman rules:}\quad T^{ij} = (\epsilon + P)u^i u^j\,,$$

$$\langle u^i u^j(k,\omega)\rangle = \frac{T}{\epsilon+P}\frac{(\delta^{ij} - \hat{k}^i\hat{k}^j)2\gamma_\eta k^2}{(\gamma_\eta k^2 - i\omega)(\gamma_\eta k^2 + i\omega)}$$

$$\left[\gamma_\eta = \tfrac{\eta}{\epsilon+P}, \gamma'_\eta = \tfrac{4}{3}\gamma_\eta\right] \qquad + \frac{T}{\epsilon+P}\frac{(\hat{k}^i\hat{k}^j)2\gamma'_\eta k^2\omega^2}{(\omega^2 - k^2/3)^2 + (\gamma'_\eta k^2\omega)^2}$$

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$\left[\gamma_\eta = \frac{\eta}{\epsilon+P}, \gamma'_\eta = \frac{4}{3}\gamma_\eta\right]$

Computing $G_{ra}^{xy,xy}(\omega, k=0)$

Straightforward application of Feynman rules:

$$G_{ra}^{xy,xy}(\omega)[\text{hydro}] = -i\omega\left(\frac{17Tk_{\max}}{120\pi^2\gamma_\eta}\right) + (i+1)\omega^{\frac{3}{2}}\frac{7+\left(\frac{3}{2}\right)^{\frac{3}{2}}T}{240\pi\gamma_\eta^{3/2}}$$

$k_{\max}$: k-scale above which hydro incorrect/inconsistent.

- $-i\omega$ term: extra contrib. to η
- $i\omega^{3/2}$: effective ω dependence of η.
- $\omega^{3/2}$: like τ_π but *wrong* ω dependence.

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$$\left[\gamma_\eta = \frac{\eta}{\epsilon+P}, \gamma'_\eta = \frac{4}{3}\gamma_\eta\right]$$

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Straightforward application of Feynman rules:

$$G_{ra}^{xy,xy}(\omega)[\text{hydro}] = -i\omega\left(\frac{17Tk_{\text{max}}}{120\pi^2\gamma_\eta}\right) + (i+1)\omega^{\frac{3}{2}}\frac{7+\left(\frac{3}{2}\right)^{\frac{3}{2}}T}{240\pi\gamma_\eta^{3/2}}$$

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Small η: freer propagation of sound, shear modes.
More efficient momentum transport, raising η.

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Expect: scale where it's no longer self-consistent.

Say, $k_{\rm max} \sim \sqrt{3}\omega_{\rm max}$, $\omega_{\rm max} = 1/\tau_\pi$.

For $\frac{\eta}{s} \sim \frac{1}{4\pi}$, this is $\sim 6T$.

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