Title: Statistical Mechanics (PHYS 602) - Lecture 12

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Abstract:

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Extended Singularity

Each universality class shows a connection between a microscopic internal symmetry (e.g. Ising model's up & down) or (rotation in a plane) and the topological properties of a large hunk of space, much larger than the range of the forces. It shows thermodynamic singularities, correlation functions which fall off algebraically, internal parameters, e.g. coherence length =inverse particle mass that have singular behavior.

This connection between macroscopic and microscopic is interesting and quite beautiful.

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The Crucial Ideas-for the revolution

Ideas:

- Criticality: recognized as a subject in itself
- Scaling: Behavior has invariance as length scale is changed
- Universality: Expect that critical phenomena problems can be divided into different "universality classes"
- Running Couplings: Depend on scale. Cf. standard model based on effective couplings of Landau & others.
- effective fields of all sorts: Running couplings are but one example of this.
- Fixed Point: Singularities when couplings stop running. K. Wilson
- Renormalization Group: K. Wilson (1971), calculational method based on ideas above.

Each Item, except RG, is a "consensus" product of many minds

The Outcome of Revolution

Excellent quantitative and qualitative understanding of phase transitions in all dimensions. Information about

- **Universality Classes**

All problems divided into "Universality Classes" based upon dimension, symmetry of order parameter, ...

Different Universality Classes have different critical behavior

e.g. Ising model, ferromagnet, liquid-gas are in same class

XYZ model, with a 3-component spin, is in different class

To get properties of a particular universality class you need only solve one, perhaps very simplified, problem in that class.

moral: theorists should study simplified models. They are close to the problems we wish to understand

Conceptual Advances

First order phase transition represent a choice among several available states or phases. This choice is made by the entire thermodynamic system.

Critical phenomena are the vacillations in decision making as the system chooses its phase.

Information is transferred from place to place via local values of the order parameter.

There are natural thermodynamic variables to describe the process. The system is best described using these variable.

Each variable obeys a simple scaling.

Next: After the Revolution:

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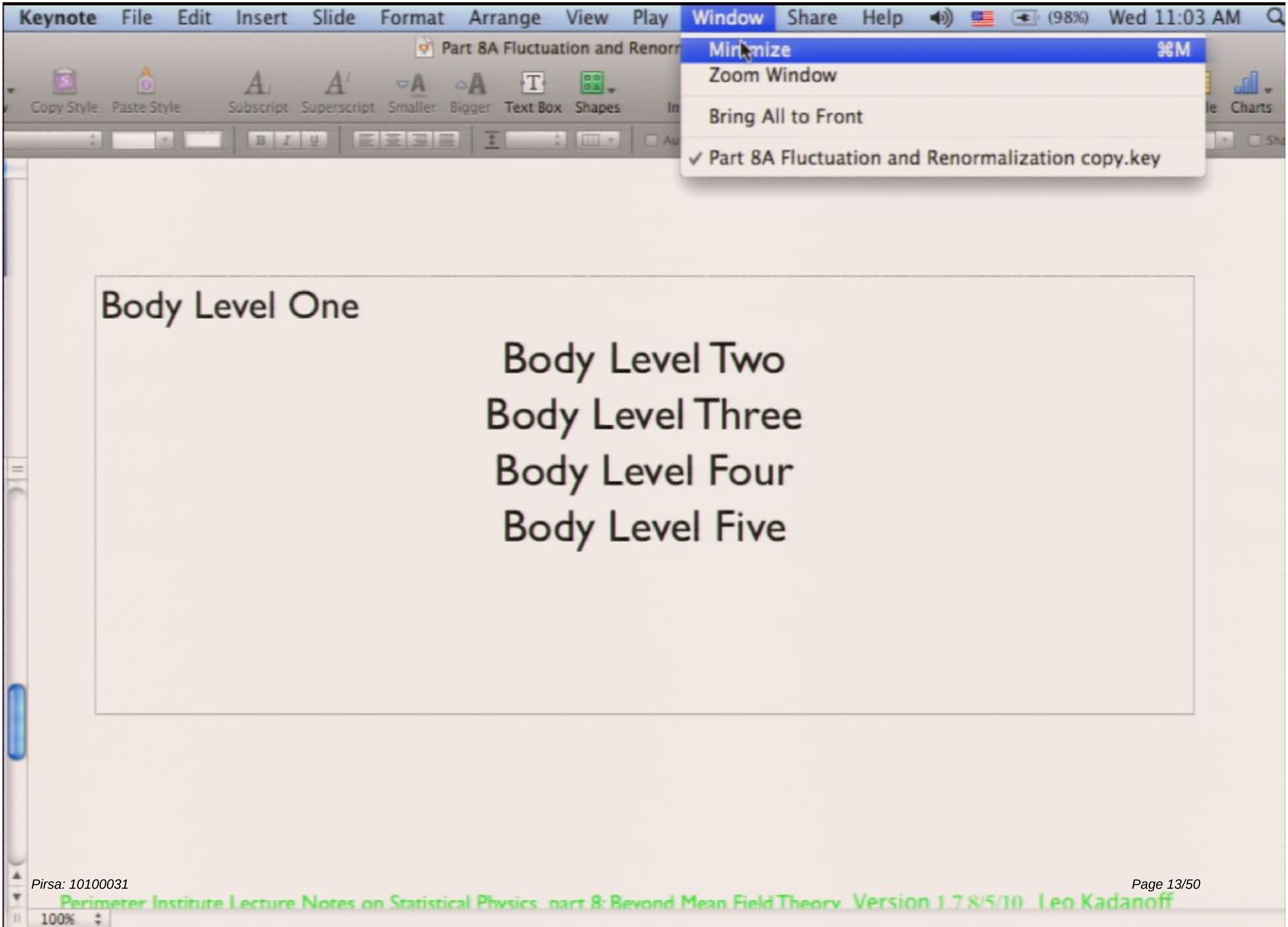
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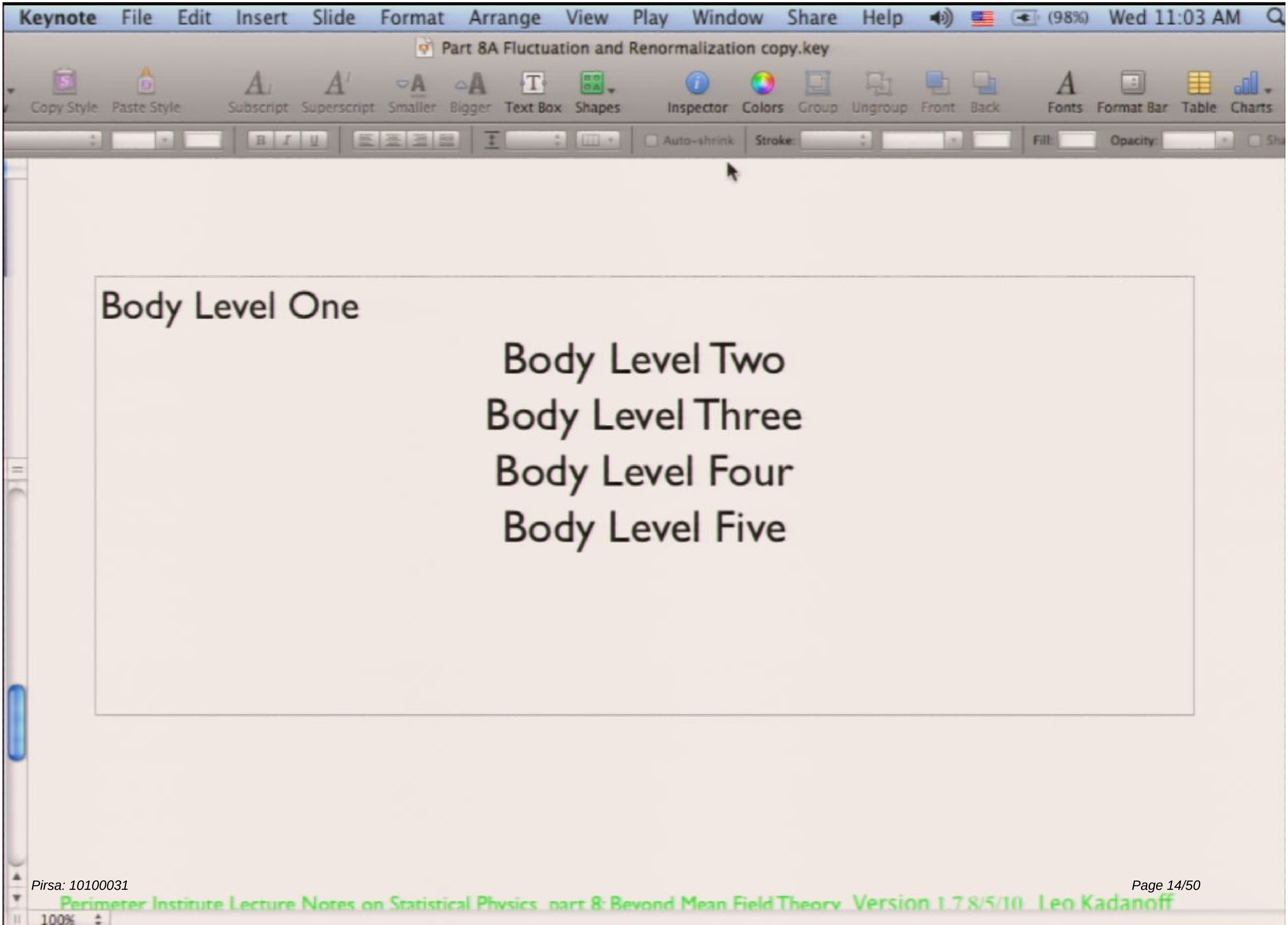
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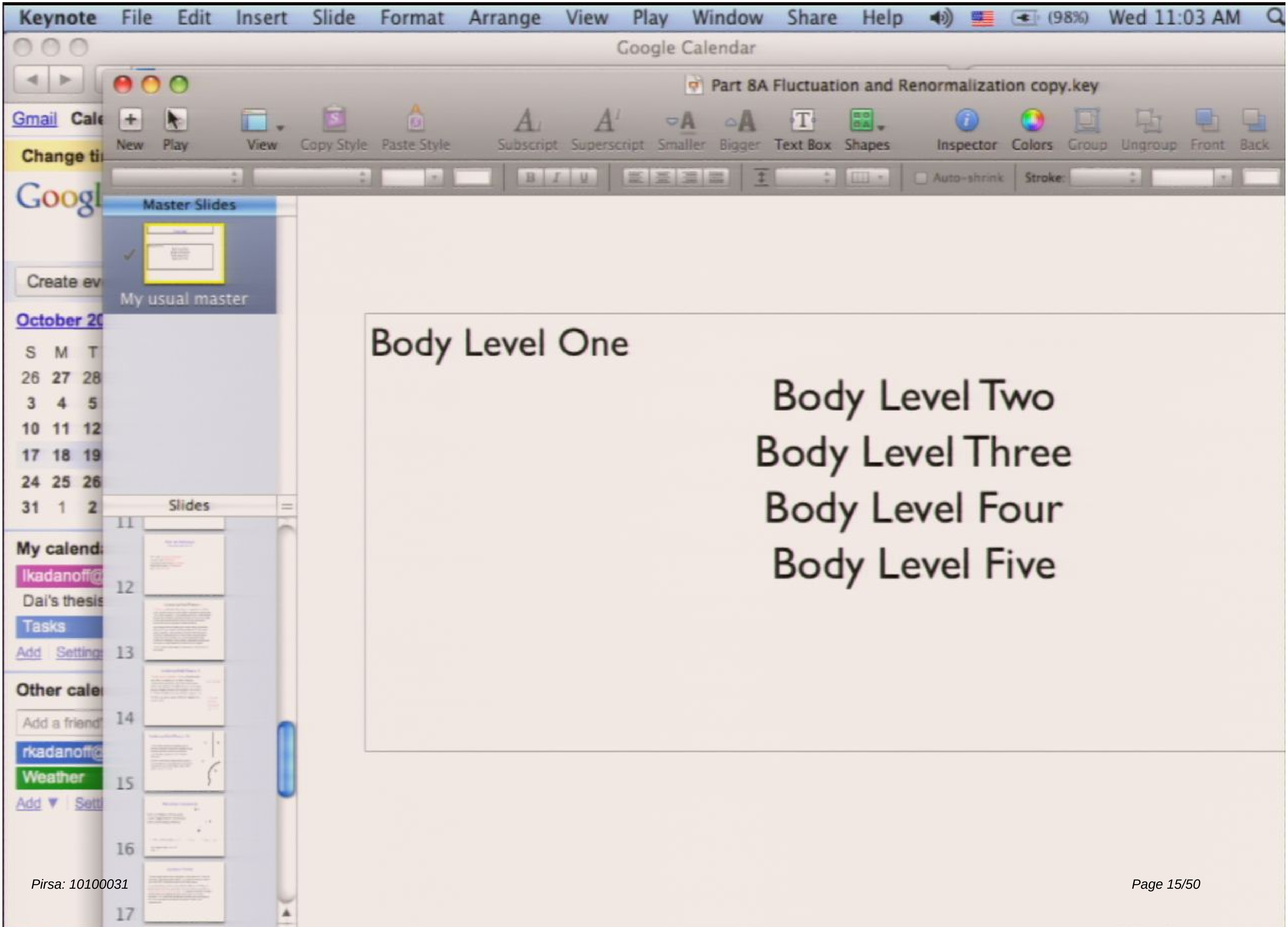
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SLE=Schramm-Loewner-Evolution

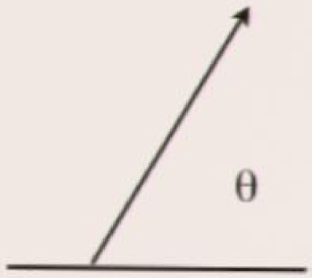
Conformal field theory took us to a

The next step beyond scale invariance is no scale at all. One can formulate "quantum gravity theory" as a classical theory in which one sums over all possible metrics on a given space.

In two dimensions, there are no physical degrees of freedom in gravity theory, and the summation can be carried out exactly.(Gross & Migdal, Douglas & Shenker, Brezin & Kazakov.) In addition to being a solvable gravity model, this approach offers a good start for critical problems. For example B. Duplantier carried out a calculation in which he calculated the spectrum of electric fields in the neighborhood

2D XY model-
Kosterlitz & Thouless (1973)

$\mathbf{S}$= (cos θ, sin θ),



The XY model is a set of two-dimensional spins, described by an angle θ and a nearest neighbor coupling

$K \cos(\theta - \theta')$.

There is an elegant description in terms of free charges and monopoles with electromagnetic interactions between them.

This work is also important because it is a successful calculation involving topological excitations. Milestone.

Start Here

The Renormalization Revolution:

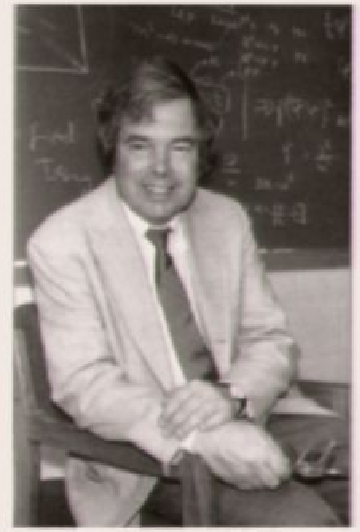
Wilson converts a phenomenology into a calculational method.

- Instead of using a few fields (T-Tc, h), he conceptualizes the use of a whole host of fields $\boldsymbol{K'} = \boldsymbol{R}(\boldsymbol{K})$
- Adds concept of fixed point $\boldsymbol{K_c} = \boldsymbol{R}(\boldsymbol{K_c})$
- He considers repeated transformations

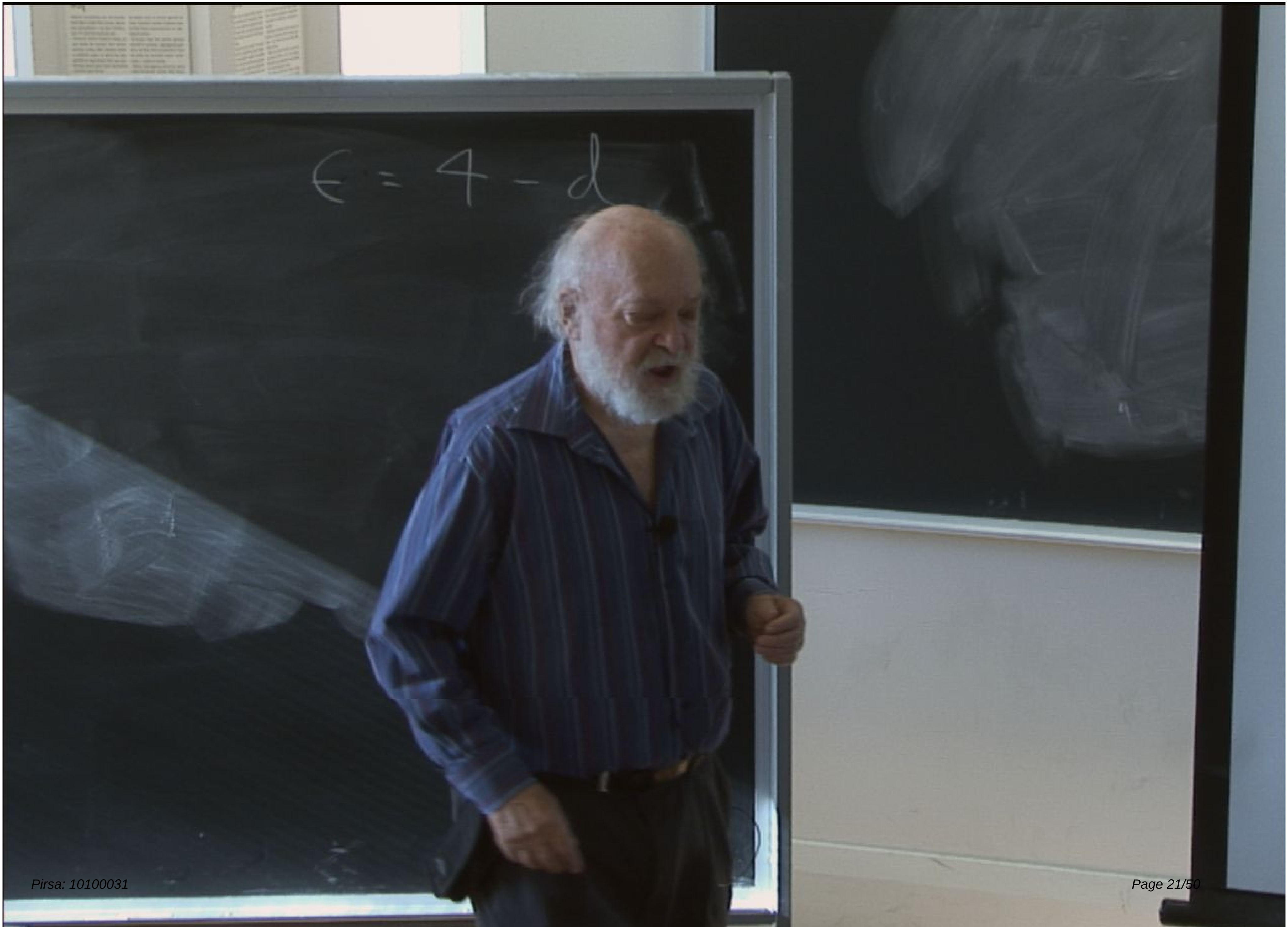
This then provides a method which can be used by many people for many real calculations. The most notable is the epsilon (4-d) expansion of Wilson and Michael Fisher. A similar calculation is used to show renormalizability of QCD ('t Hooft & Veltman)

The ε-expansion theory permits one to calculate everything people wanted to know about 3D critical phenomena and fully matches experiments.

Everything in critical phenomena in both condensed matter and particle physics seems to be explained.



Kenneth G. Wilson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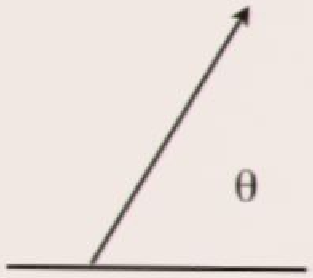

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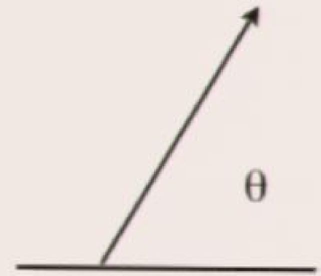

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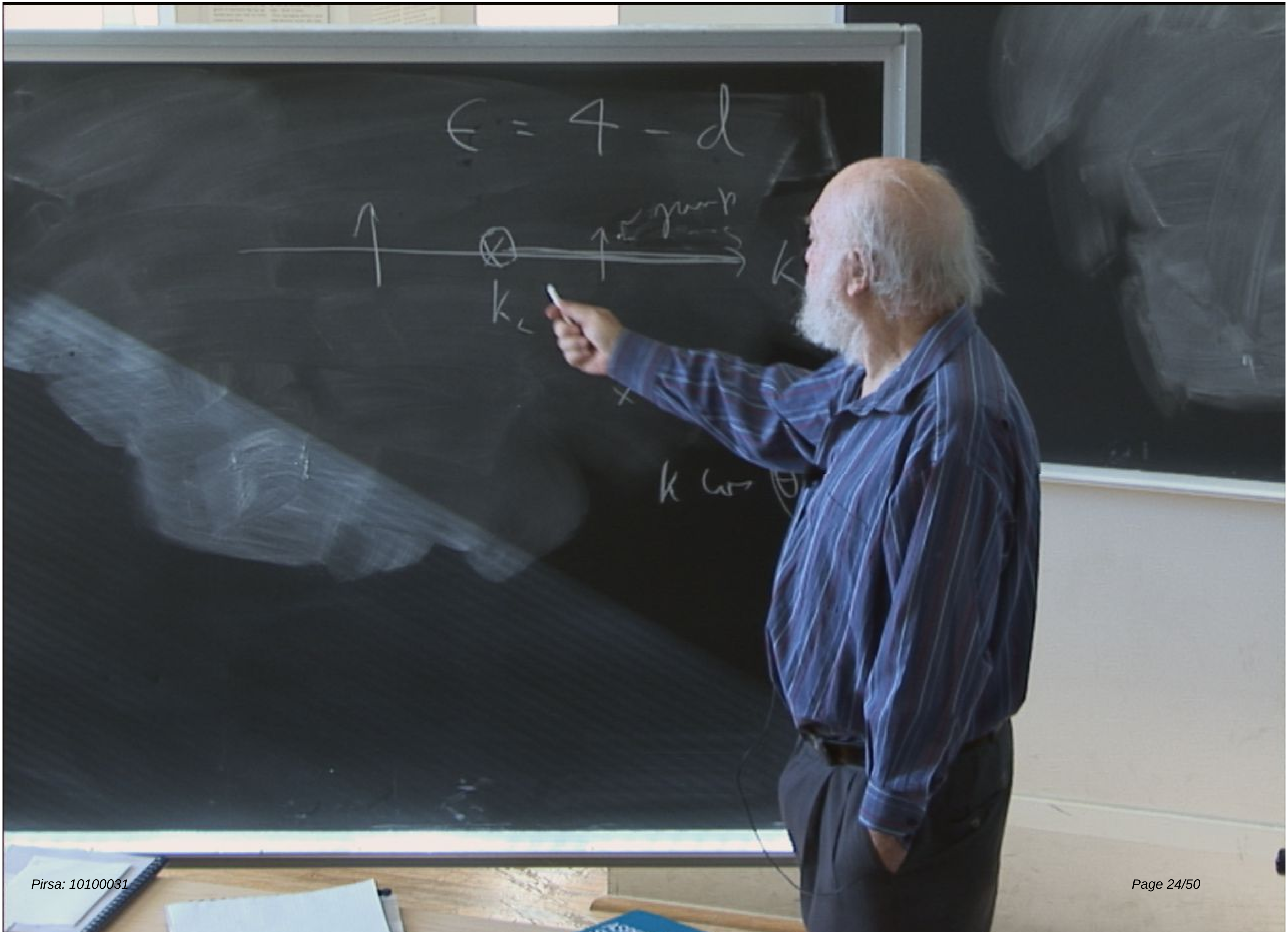
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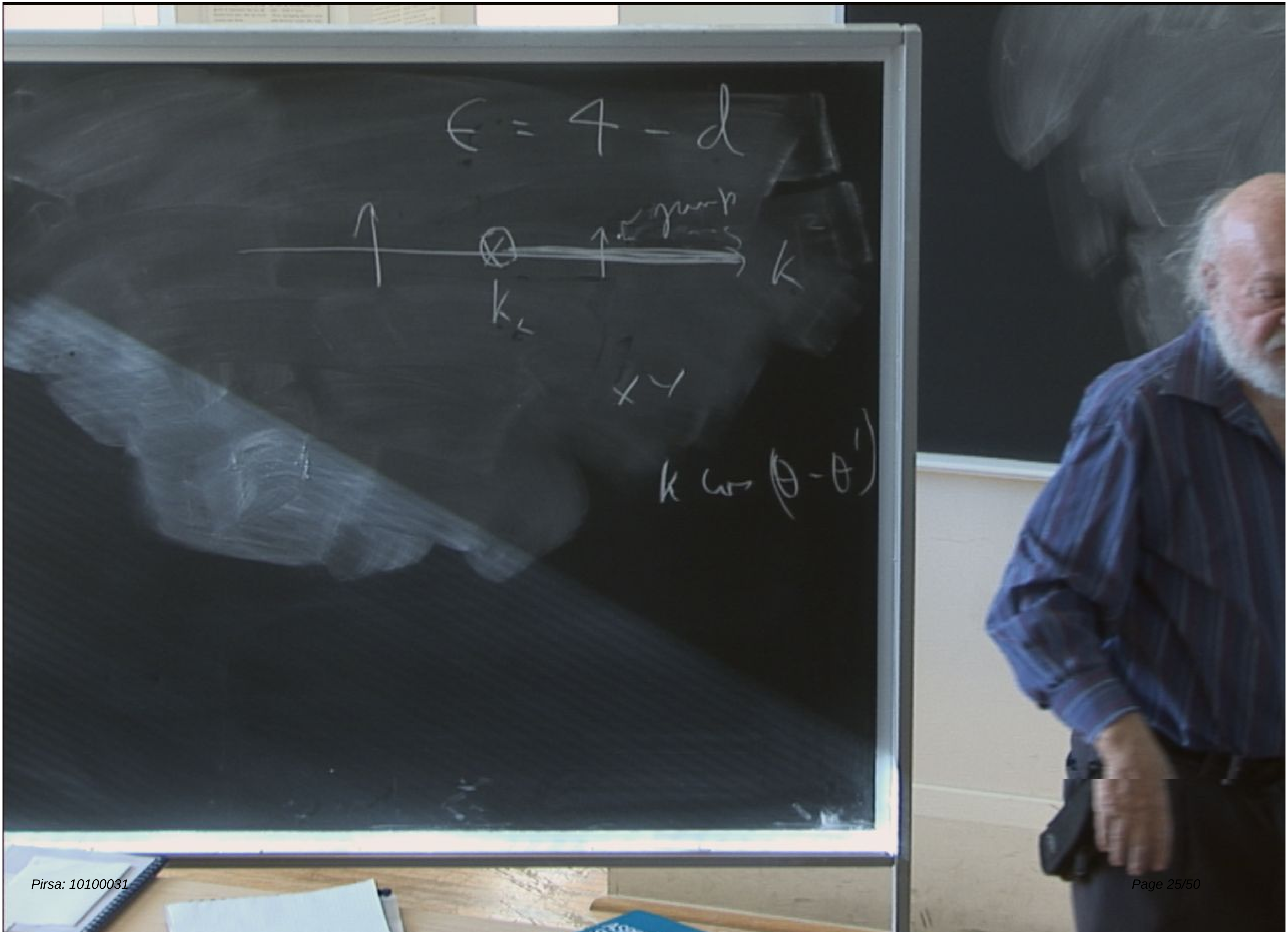
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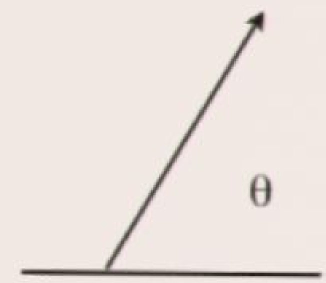

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The RG point of view is Fully Absorbed into Particle Physics

Running coupling constants help define the standard model.

The model is extrapolated back to when weak, electromagnetic and strong interactions are all equal.

Asymptotic freedom--weakening of strong interactions at small distances--permits high energy calculations. Gross, Wilczek, Politzer (1973).

Coulomb gas: B. Nienhuis (~1985)

Nienhuis extended this approach to give a description of correlation functions in terms of a screened coulomb gas (and magnetic monopoles) for many kinds of critical phenomena problems in two dimensions. The list includes:

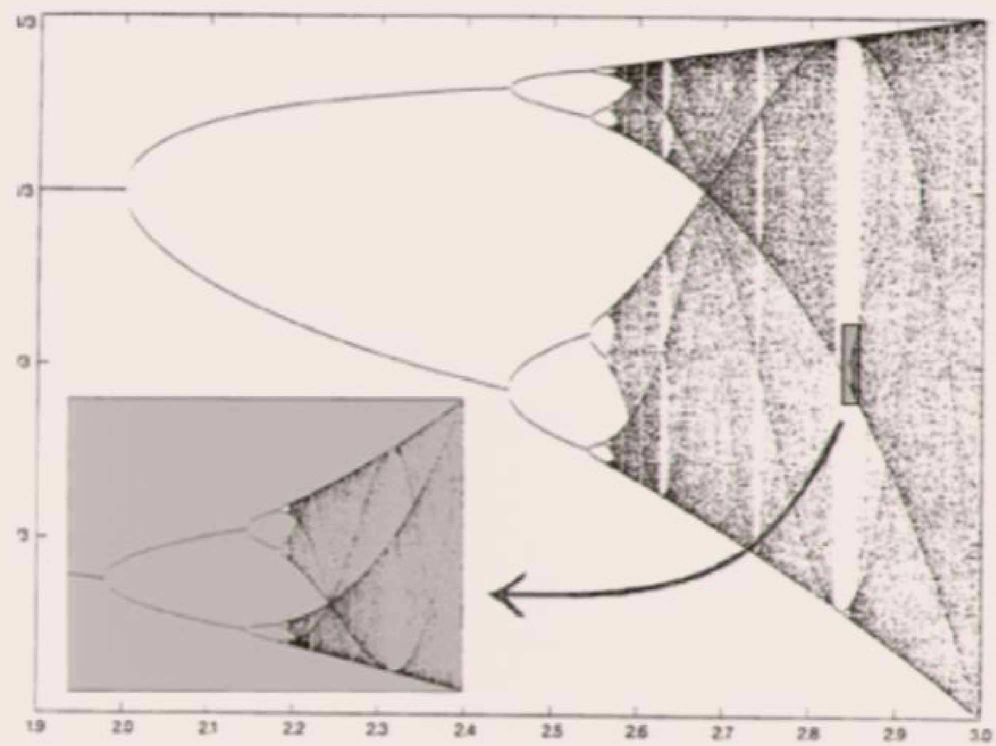
- q-state Potts models (spin takes on q values)
- O_n models (n-component vectors)

These models permit generalizations in which behavior varies continuously with q or n.

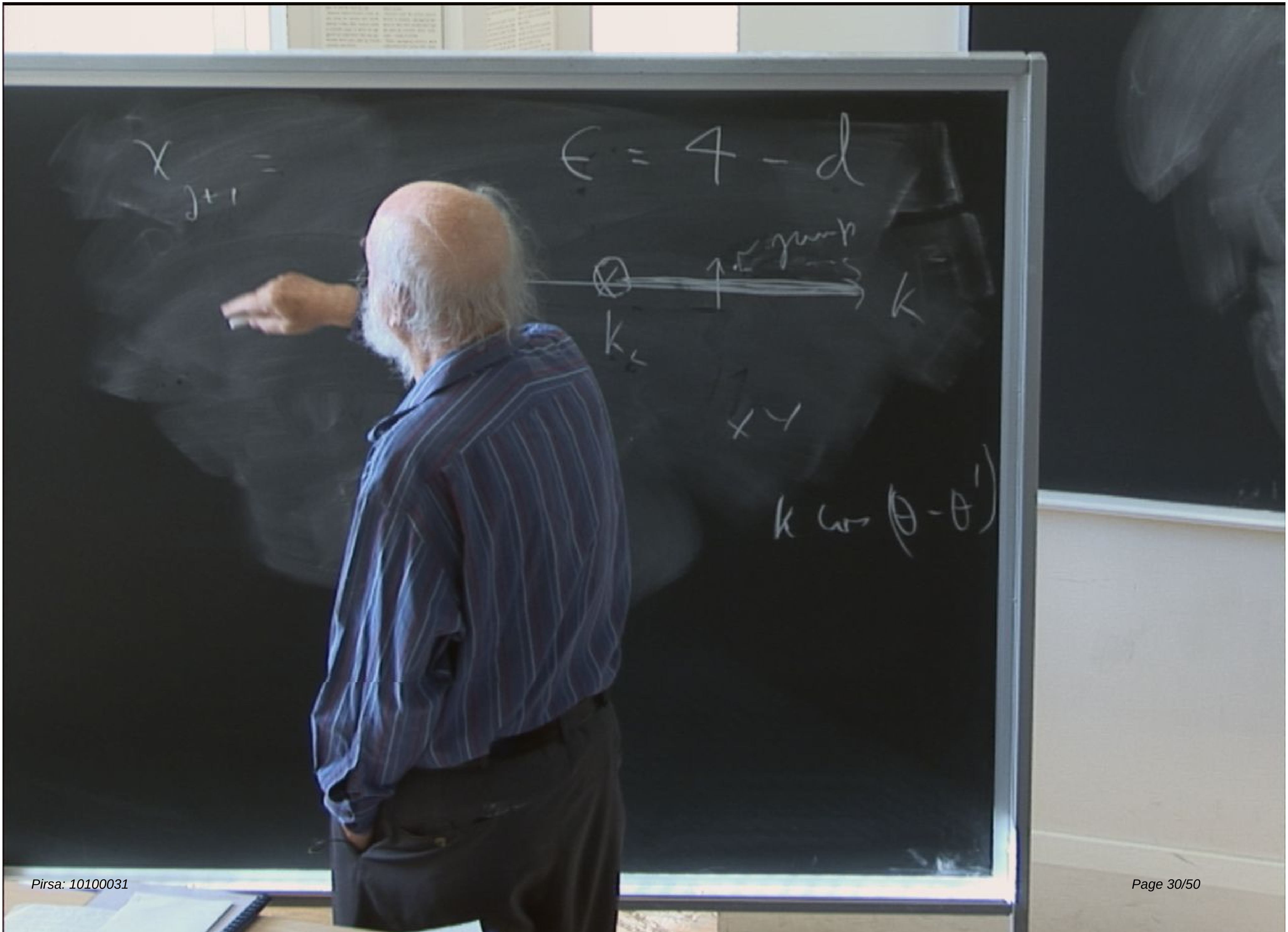
Feigenbaum: Analyzes Route to Chaos (1978)

Feigenbaum used RG, Universality, and Scaling concepts to investigate the period doubling route to chaos via the discrete equation

x(t+1) =r x(t) (1-x(t))



long-term x-values versus r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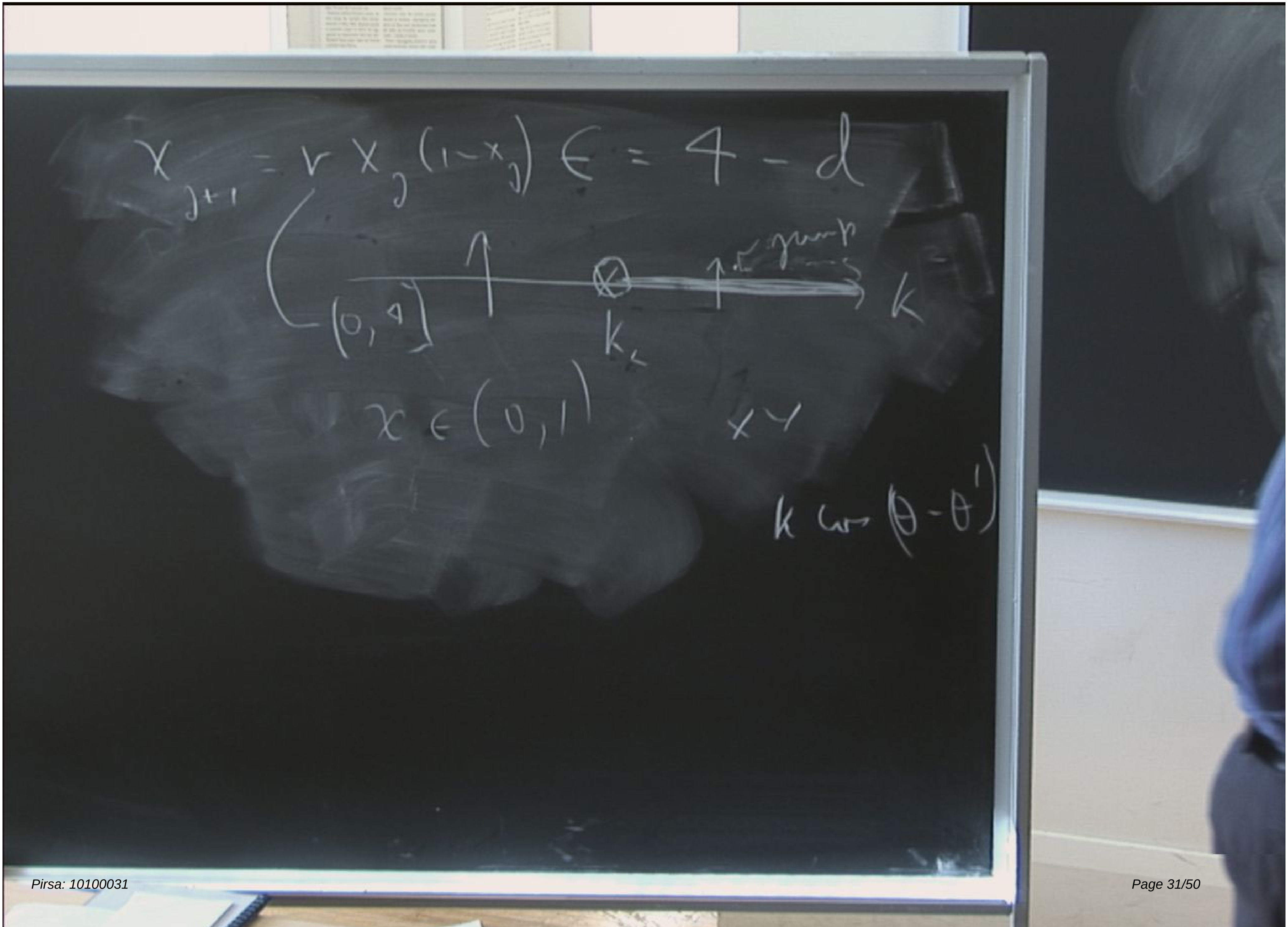
k
k cos (θ − θ')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[0, 4]
k_c
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x ∈ (0, 1)
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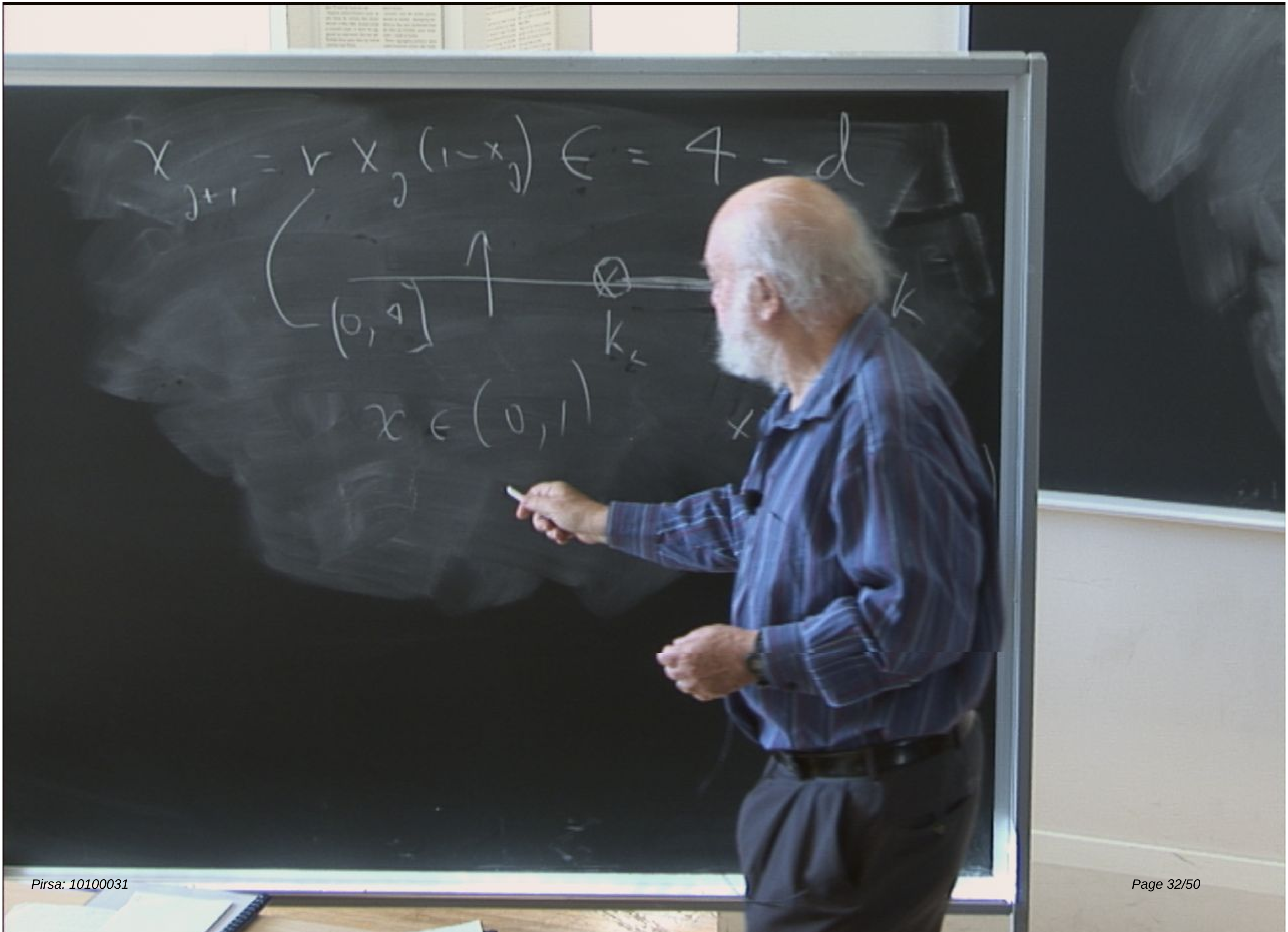
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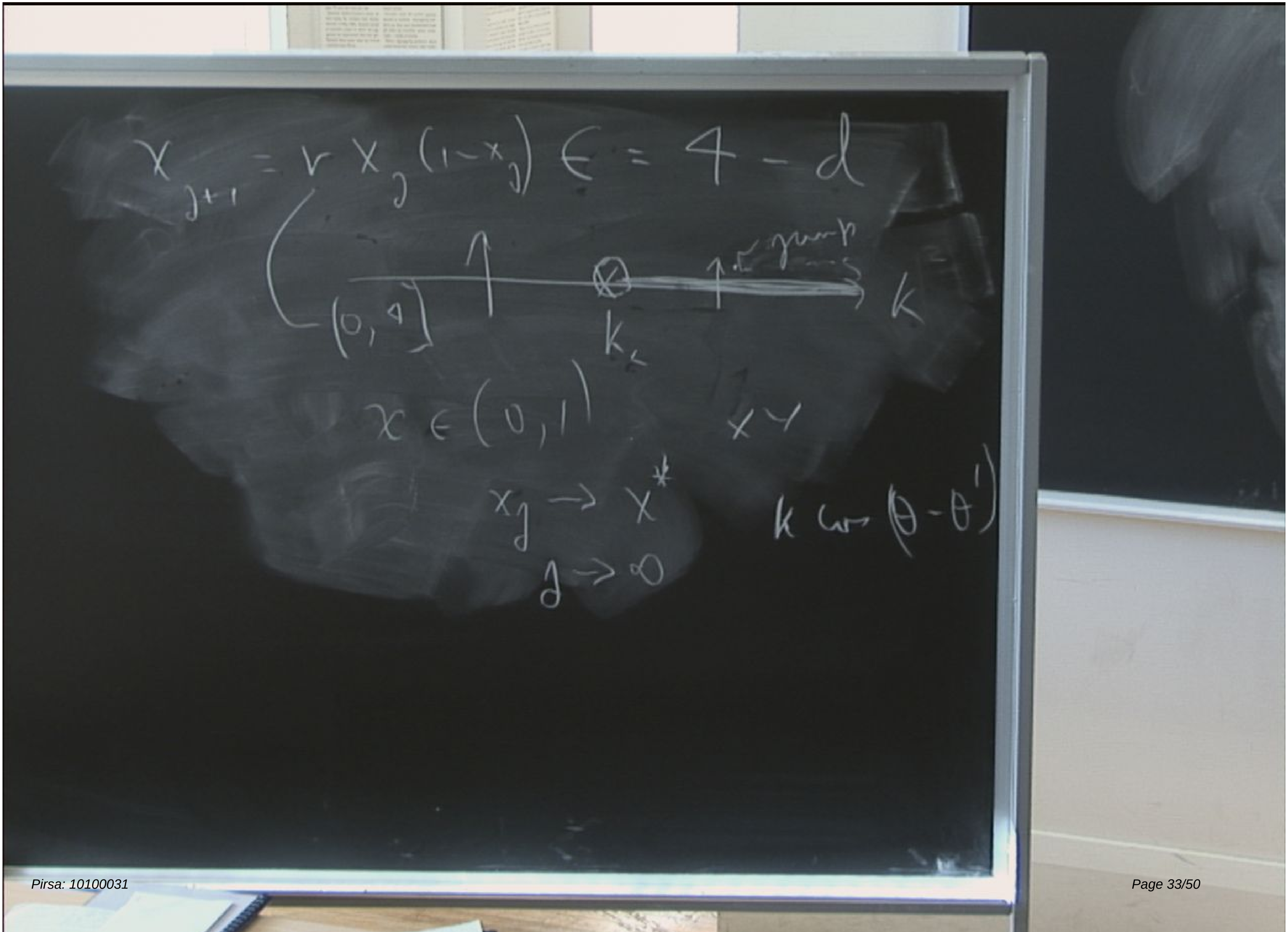
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ε = 4 - d
[0, 4]
k_c
k
x ∈ (0, 1)
x_j → x*
j → ∞
k cos(θ - θ')

Operator Product Expansion: Wilson, Kadanoff

Local operators proved particularly interesting
Products of nearby operators O(R+r/2)O(R-r/2)
can be expanded in terms of the local operators at R.

$$O_\alpha(R+r/2)\, O_\beta(R-r/2)$$

$$= \sum_\gamma A_{\alpha\beta\gamma}(r) O_\gamma(R)$$

This operator product expansion particularly suggests that the operators obey a kind of algebra. Working from the algebra, something deeper might be found.

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Field Theory

Pre-Revolutionary Period:
Onsager solution of 2-D Ising model is free fermion theory

Revolutionary Period:
The connection: Wilson & Fisher (1972) use Ginzburg Landau free energy formulation plus path integral formulation of quantum mechanics to describe critical phenomena theory as a close relative of quantum field theory.

After the Revolution: Polyakov and others reformulate critical point theory as Conformal Field Theory, a field theory for situations invariant under scale transformations but not shear-type distortions

After the Revolution:

New Ideas-mostly for d=2

- XY model- Kosterlitz & Thouless
- Coulomb gas: B. Nienhuis
- Conformal Field Theory: A. Polykov
- Quantum Gravity: B. Duplantier
- SLE: Oded Schramm

Conformal Field Theory: I

A. Polyakov emphasized that there is a special form of field theory which holds at critical points, i.e. places in which there is full scale invariance. In two dimensions this is super-special because the invariance includes all kinds of conformal (angle preserving) transformations which can then be studied through the use of complex variable methods.

Space distortions are based upon stress tensor operators, with the Virasoro algebra being the algebra of local stress tensor densities. Just as spinors, vectors and tensors are derived as representations of the rotation group algebra, equally the local operators of critical phenomena have properties, including critical indices, derivable from the fact that they are representations of the Virasoro algebra.

Continuously varying families of solutions are generated in this fashion

Conformal Field Theory: II

Friedan Qiu & Shenker (1984) showed that unitarity, a quality of all field theories representing possible quantum processes, limits the domain of field theories to include all the known critical 2-D models, e.g. q=2,3, 4 Potts models, but not others (e.g q=1.5).

Further we get a quite different algebra for each model.

Friedan Qiu & Shenker
PRL 52 1575 (1984)

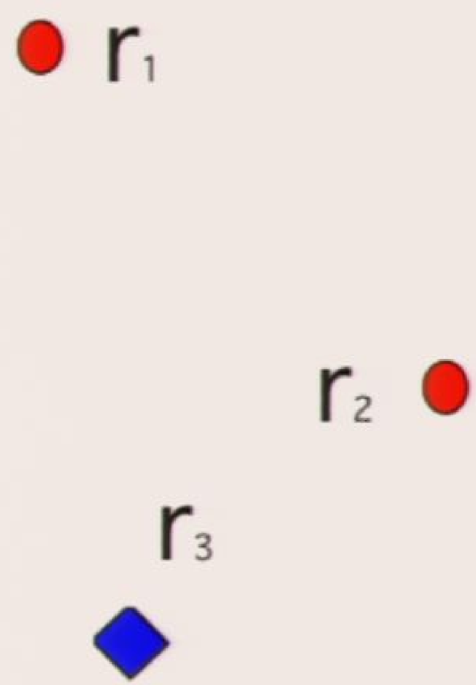
For monograph material see Di Francesco, Mathieu, Senechal Conformal Field Theory, Springer, 1997

Conformal Field Theory: III

Compared to general situations, all the familiar statistical mechanical models are all degenerate and truncated, and permit considerable calculation of correlation functions.

Further conformal transformation permit the calculation of correlations in all kinds of shapes from just a few shapes: plane, half-plane, interior of circle.

Many Exact Calculations



find correlation of two spins
(local magnetization densities)
and a local energy density

$$< \sigma(r_1)\sigma(r_2)\epsilon(r_3) > \quad = C{r_{12}}^{x_\epsilon - 2x}(r_{23}r_{13})^{-x_\epsilon}$$

For Ising model, x=1/8
and x_ϵ = 1

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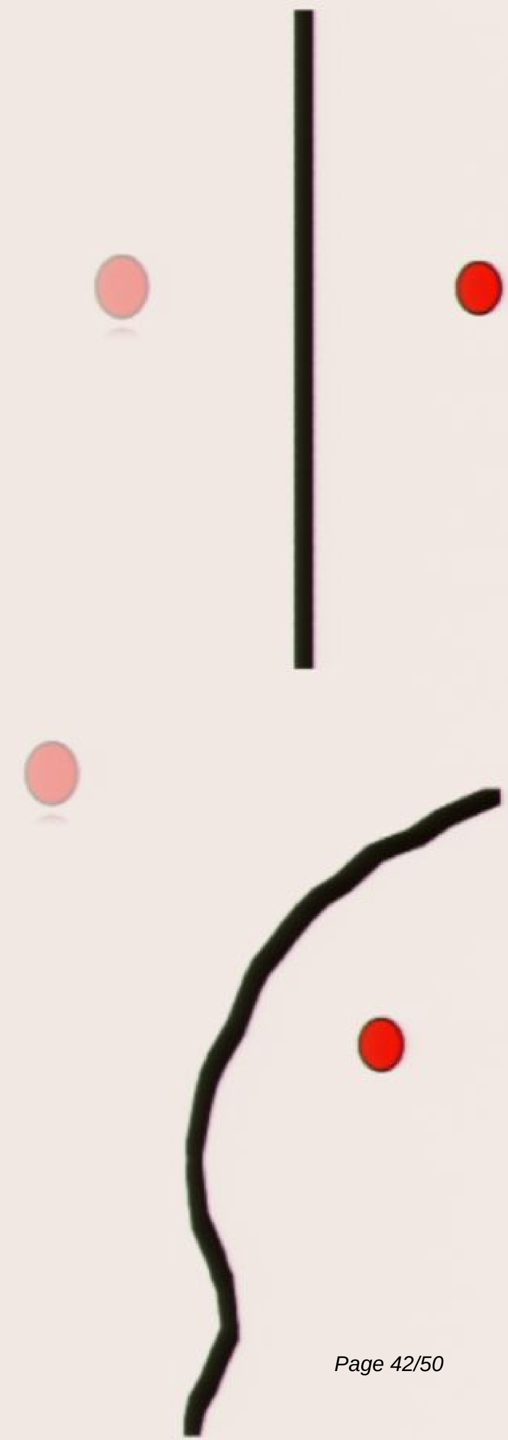

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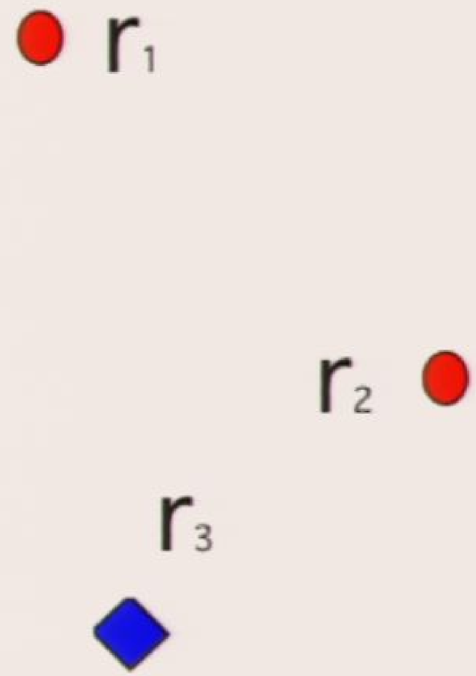

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Quantum Gravity:

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In two dimensions, there are no physical degrees of freedom in gravity theory, and the summation can be carried out exactly.(Gross & Migdal, Douglas & Shenker, Brezin & Kazakov.) In addition to being a solvable gravity model, this approach offers a good start for critical problems. For example B. Duplantier carried out a calculation in which he calculated the spectrum of electric fields in the neighborhood

SLE=Schramm-Loewner-Evolution

Conformal field theory took us to a point at which we could formulate critical phenomena problems on surfaces of various shapes. A recent area of progress arises from work of Oded Schramm, who combined the complex analytic techniques of Loewner with methods of modern mathematical probability theory to gain new insight into the shapes which arise in critical phenomena.

Oded Schramm

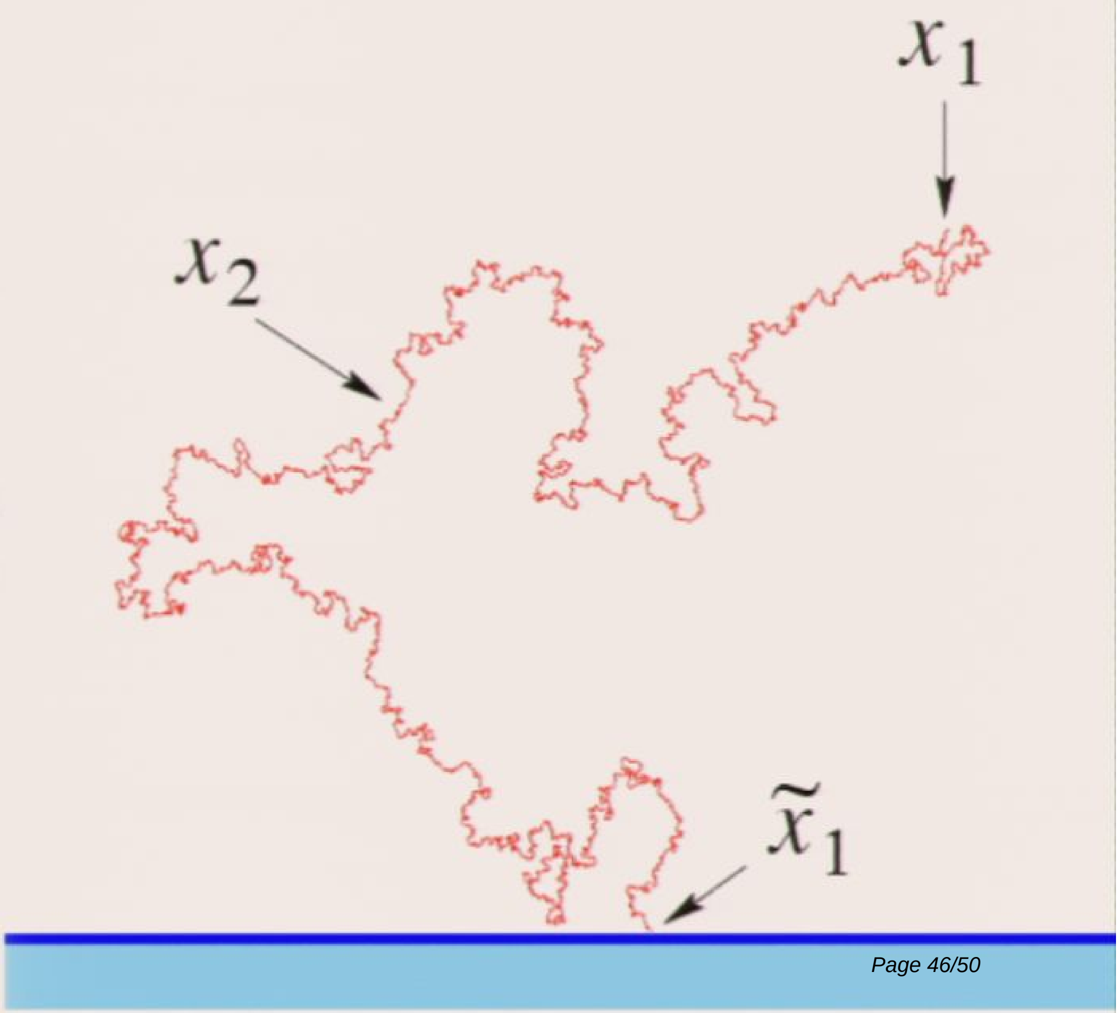
O. Schramm, Israel J. Math. 118 (2000), 221--288.

From Schramm to Critical Shapes

The work of the 20th. Century on critical phenomena was centered on thermodynamics, and correlation functions.

We depicted but did not calculate the shapes of the correlated fractal clusters arising in critical situations.   Schramm provided a constructive technique, involving a differential equation, for making the ensemble of such clusters at critical points.

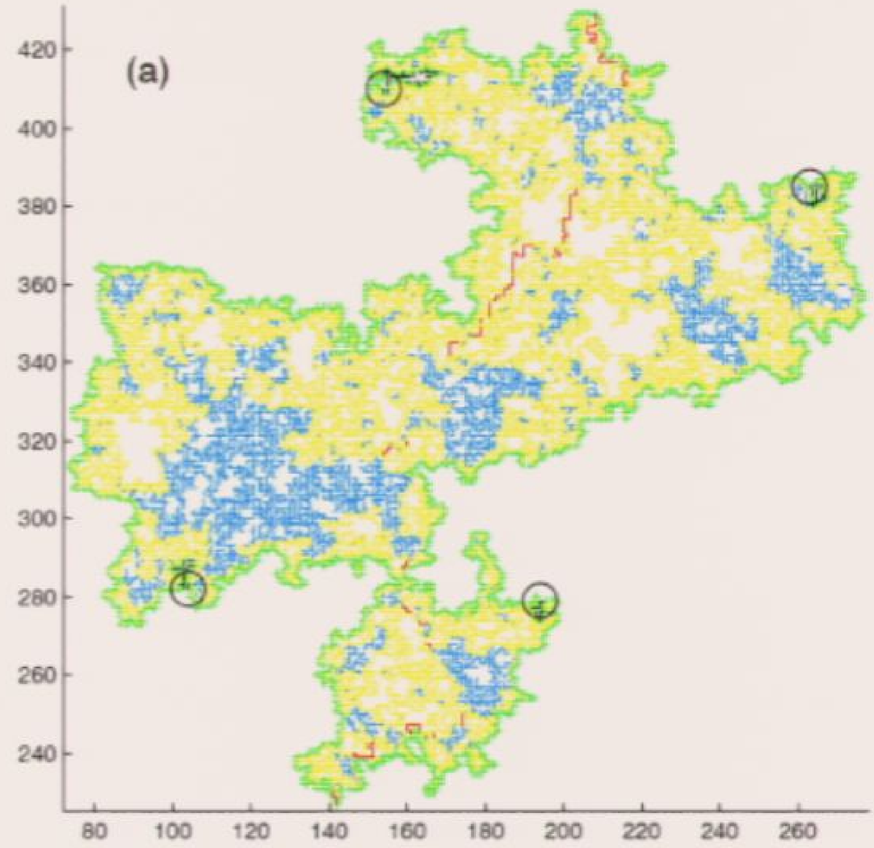
SAW in half plane - 1,000,000 steps



SLE-II

Define a critical cluster as the shape of cluster of spins pointed in the same direction in an Ising model, or of a connected set of occupied sites in a percolation problem.

Problem: Define the ensemble of cluster shapes for any critical situation. Before Schramm's work we ignored this aspect of critical problems.



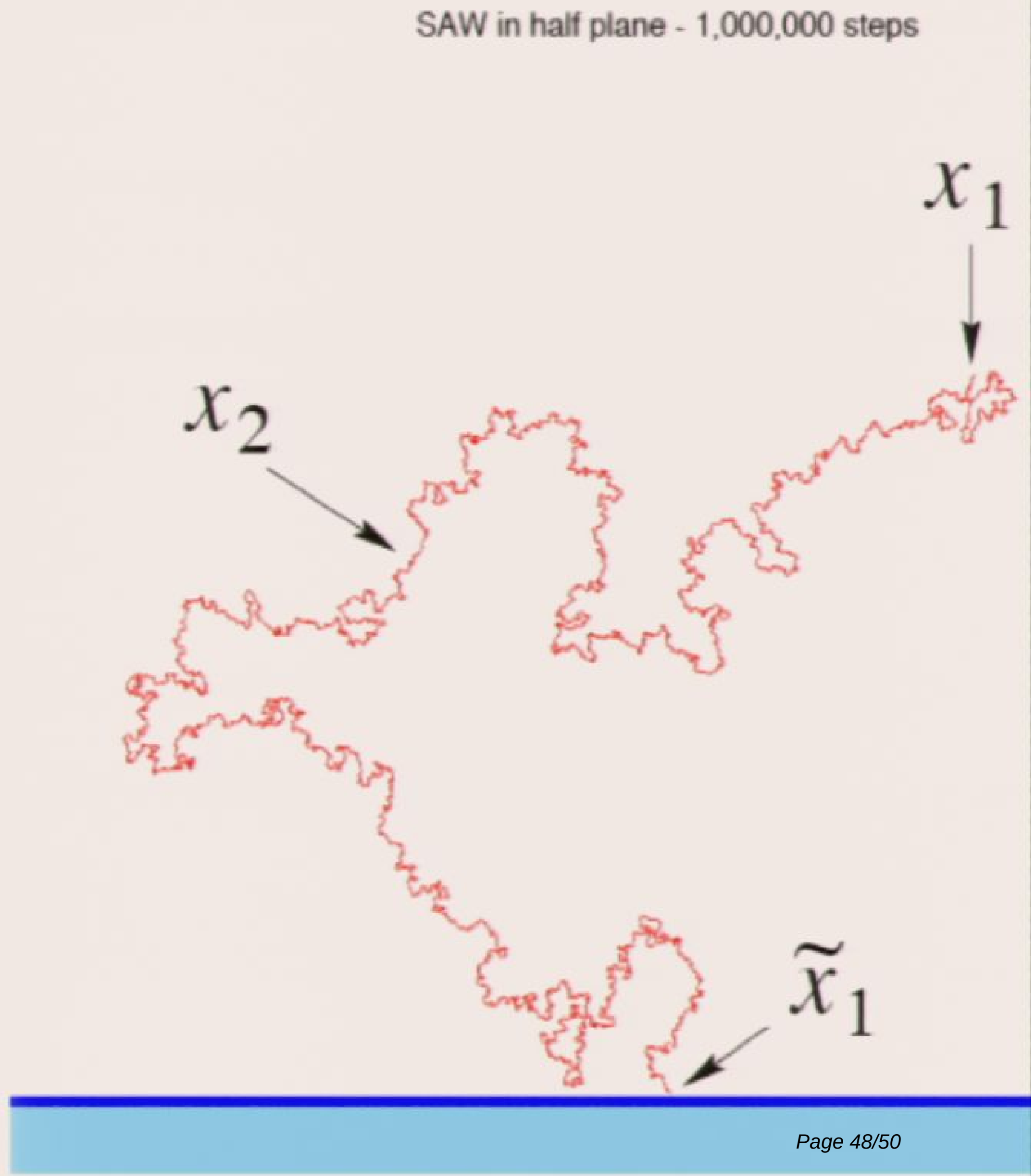
(J. Asikainen et al., 2003)

percolation cluster after Duplantier

Problem: Define the ensemble of cluster shapes for any critical situation.

Answer: Look for the ensemble of shapes formed from the singularities in the solution to the differential equation

where is a random walk on a line with correlations

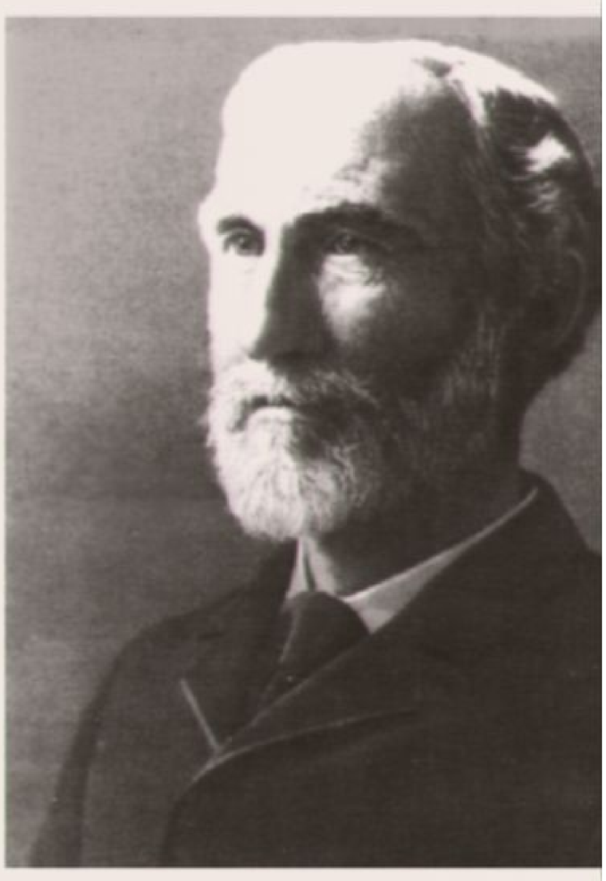


Summary

Critical behavior occurs at but one point of the phase diagram of a typical system. It is anomalous in that it is usually dominated by fluctuations rather than average values. These two facts provide a partial explanation of why it took until the 1960s before it became a major scientific concern. Nonetheless most of the ideas used in the eventual theoretical synthesis were generated in this early period.

Around 1970, these concepts were combined with experimental and numerical results to produce a complete and beautiful theory of critical point behavior.

In the subsequent period the "revolutionary synthesis" radiated outward to (further) inform particle physics, mathematical statistics, various dynamical theories....



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