Title: Relativity (PHYS 604) - Lecture 15A

Date: Oct 01, 2010 10:30 AM

URL: http://pirsa.org/10100002

Abstract:

PERIMETER PI INSTITUTE FOR THEORETICAL PHYSICS

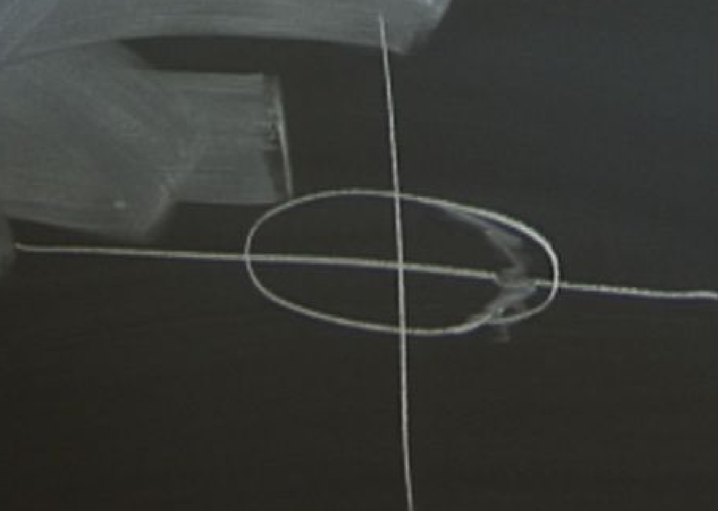
$$-\left(1-\frac{2GMr}{\rho^2c^2}\right)dt^2 - 2\frac{GMar\sin^2\theta}{c^2\rho^2}\left(dt\,d\phi + d\phi\,dt\right)$$

$$+\frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2$$

$$\Delta = r^2 - 2\frac{GMr}{c^2} + a^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$\times$



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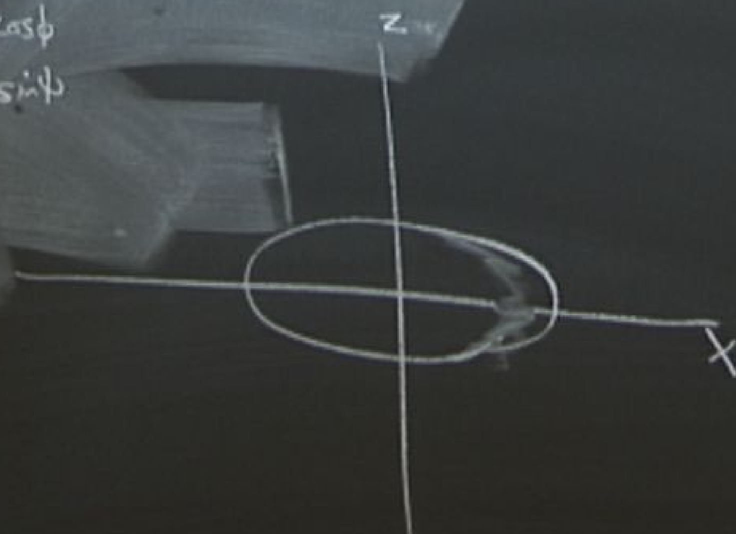

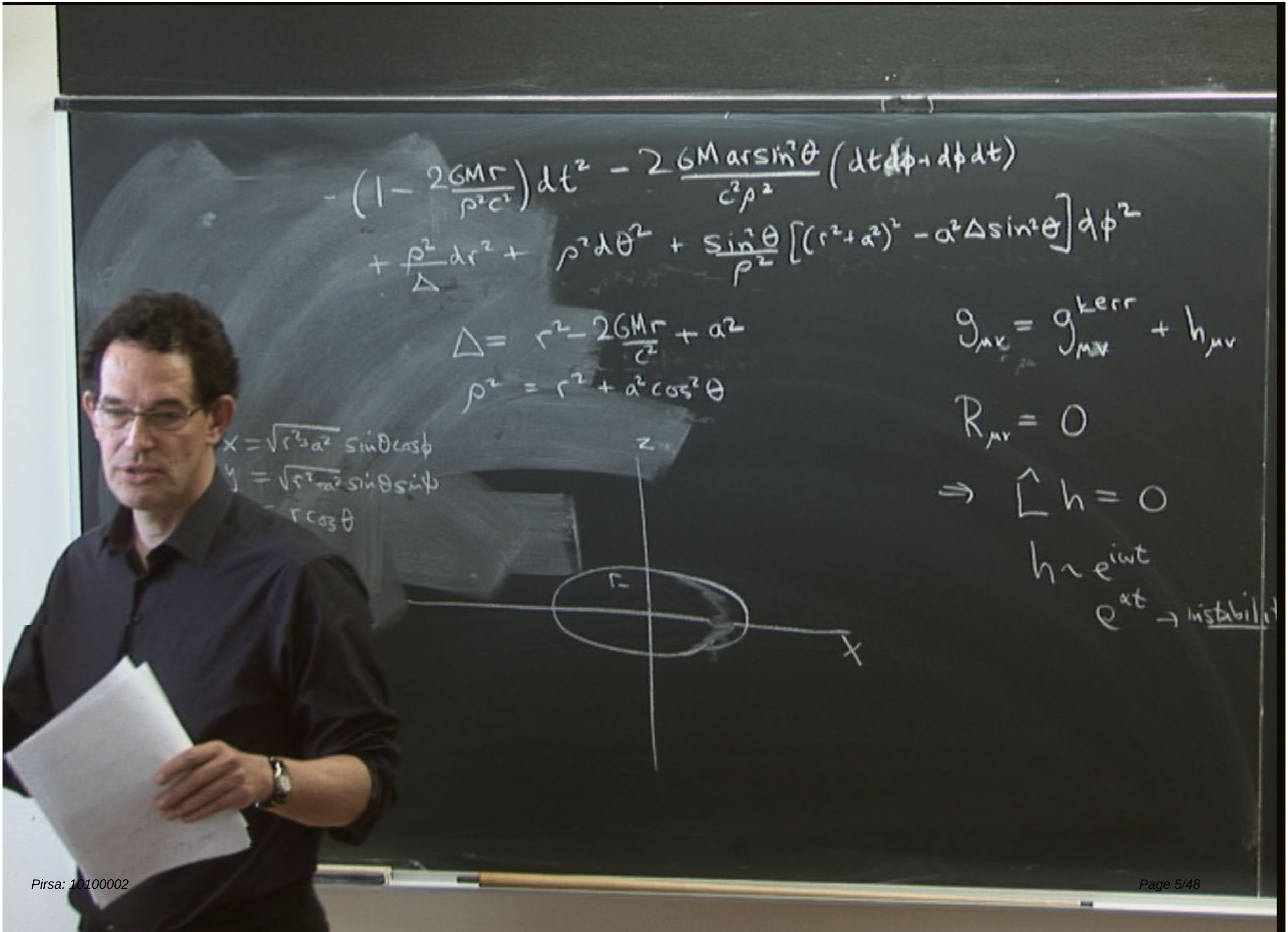
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$r\cos\theta$
$g_{\mu\nu} = g^{kerr}_{\mu\nu} + h_{\mu\nu}$
$R_{\mu\nu} = 0$
$\Rightarrow \Box h = 0$
$h \sim e^{i\omega t}$
$e^{\alpha t} \to$ instability
z
x
r_-

$$-\left(1-2\frac{GMr}{\rho^2c^2}\right)dt^2 - 2\frac{GMar\sin^2\theta}{c^2\rho^2}\left(dt\,d\phi + d\phi\,dt\right)$$

$t \to$

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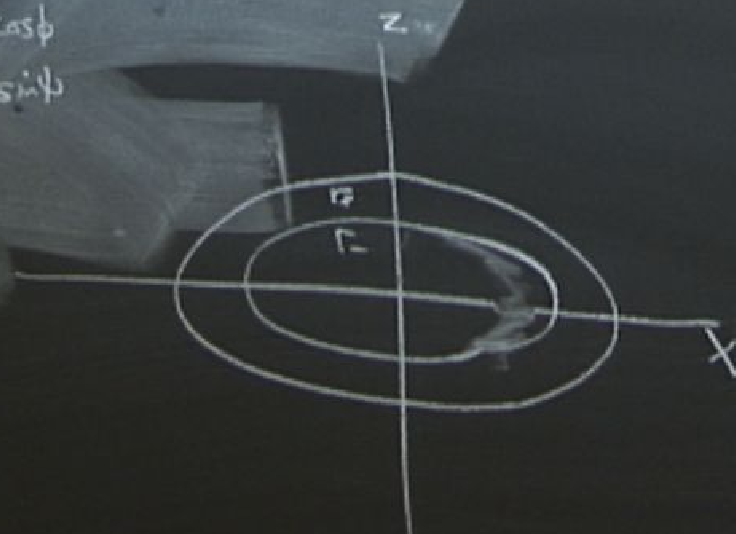

$$g_{\mu\nu} = g^{kerr}_{\mu\nu} + h_{\mu\nu}$$

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$$h \sim e^{i\omega t}$$

$$e^{-\alpha t} \to \text{instability} \quad \alpha > 0$$

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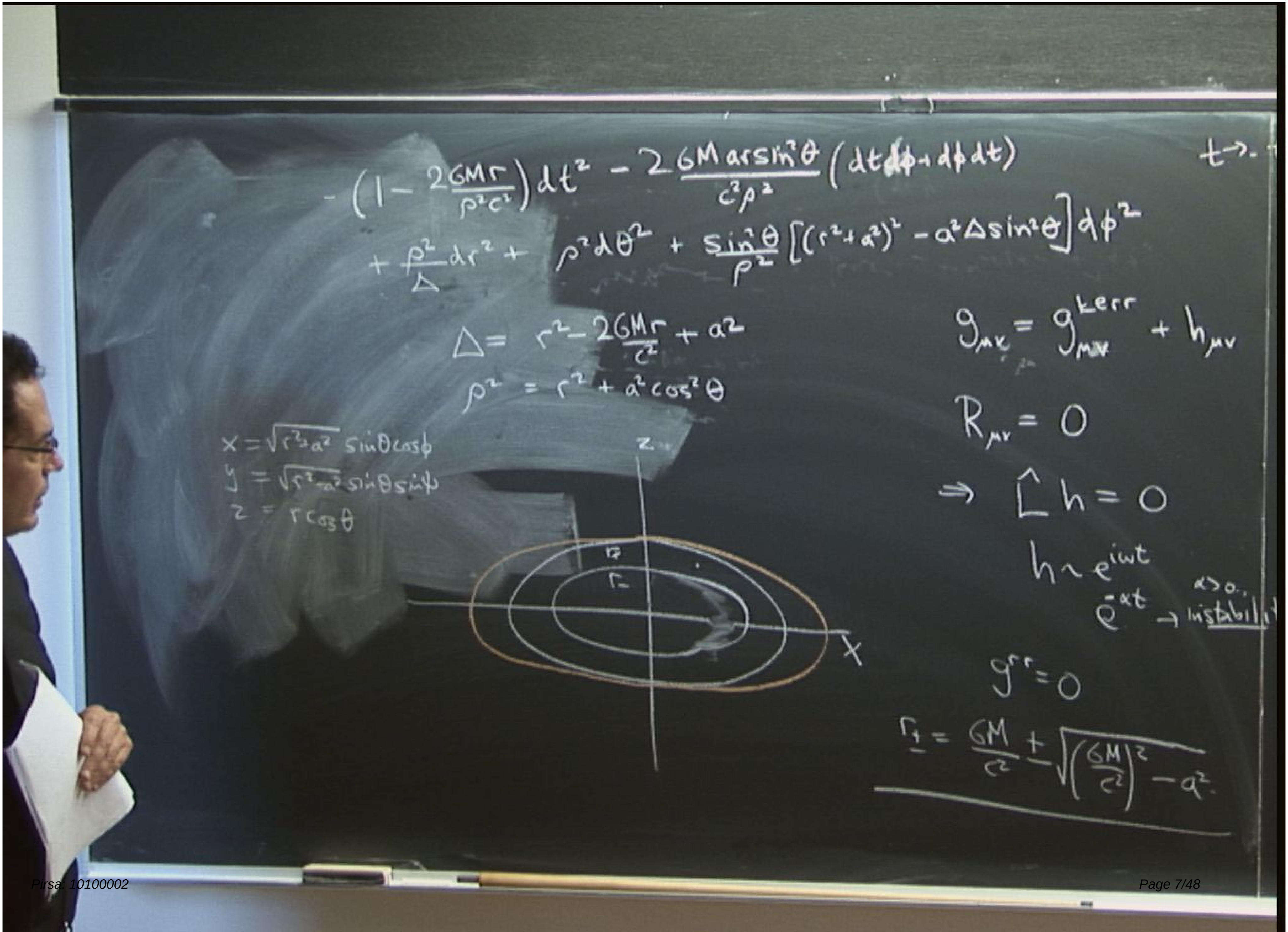
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z
r_+
r_-
x
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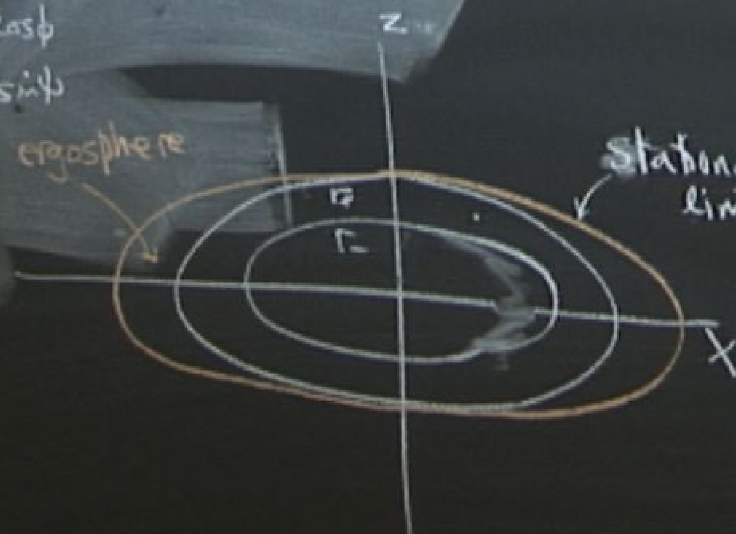

$t \to$

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t→. t

$r_{se}: \quad g_{tt} = 0$

$_{\mu\nu}$

α>0..
instability

a^2.

$t \to t$

$$r_{se}: \quad g_{tt} = 0 \quad : \quad \rho^2 = \frac{2GMr}{c^2}$$

$$\Rightarrow r^2 + a^2\cos^2\theta = \frac{2GMr}{c^2}$$

$$\Rightarrow \left(r_{se} - \frac{GM}{c^2}\right)^2 = \frac{GM}{c^2} - a^2\cos^2\theta$$

$\eta_{\mu\nu}$

$a \to 0$

$t \to t$

r_{se} : $g_{tt} = 0$: $\rho^2 = \frac{2GMr}{c^2}$

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cf. $\left(r_+ - \frac{GM}{c^2}\right)^2 = \left(\frac{GM}{c^2}\right)^2 - a^2$

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$\alpha > 0$.,

instability

$-a^2$.

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cf. $\left(r_+ - \frac{GM}{c^2}\right)^2 = \left(\frac{GM}{c^2}\right)^2 - a^2$

$\Rightarrow r_{se} > r_+$ except at $\theta = 0$.

$\eta_{\mu\nu}$

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$$-\left(1-\frac{2GMr}{\rho^2c^2}\right)dt^2 - 2\frac{GMar\sin^2\theta}{c^2\rho^2}\left(dt\,d\phi + d\phi\,dt\right)$$

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$t \to -t$

$$\Delta = r^2 - \frac{2GMr}{c^2} + a^2$$

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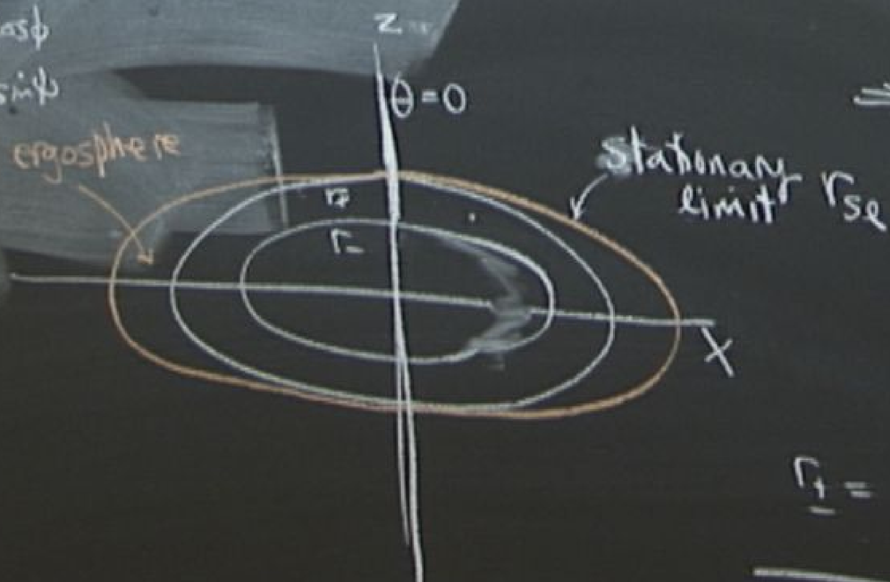

$h \sim e^{i\omega t}$

$e^{-\alpha t} \to$ instability, $\alpha > 0$

$$g^{rr} = 0$$

$$r_\pm = \frac{GM}{c^2} \pm \sqrt{\left(\frac{GM}{c^2}\right)^2 - a^2}$$

$t \to . t$

r_{se} : $g_{tt} = 0$: $\rho^2 = \frac{2GMr}{c^2}$

$\rightarrow r^2 + a^2\cos^2\theta = \frac{2GMr}{c^2}$

$\Rightarrow \left(r_{se} - \frac{GM}{c^2}\right)^2 = \left(\frac{GM}{c^2}\right)^2 - a^2\cos^2\theta$

cf. $\left(r_+ - \frac{GM}{c^2}\right)^2 = \left(\frac{GM}{c^2}\right)^2 - a^2$

$\Rightarrow r_{se} > r_+$ except at $\theta = 0$.

For massive ptlcle,

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$$

$$= g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + \ldots$$

If g_{t}

$t \to t$

$\quad$ $+ h_{\mu\nu}$

$= 0$

$e^{i\omega t}$

$e^{-\alpha t} \xrightarrow{\alpha > 0} \underline{\text{instability}}$

$$\left(\frac{GM}{c^2}\right)^2 - a^2.$$

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If $g_{tt} \geq 0$

then there is no solution

for $\dot{\theta} = \dot{\phi} = \dot{r} = 0$.

$t \to t$

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If $g_{tt} \geq 0$

then there is no solution for $\dot{\theta} = \dot{\phi} = \dot{r} = 0$.

$\Rightarrow$ for $g_{tt} > 0 \Rightarrow$ must have

$+ h_{\mu\nu}$

$= 0$

$e^{i\omega t}$

$e^{-\alpha t} \to$ instability ($\alpha > 0$)

$\left(\frac{GM}{c^2}\right)^2 - a^2.$

$t \to t$

r_{se} : $g_{tt} = 0$: $\rho^2 = \frac{2GMr}{c^2}$

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$\Rightarrow r_{se} > r_+$ except at $\theta = 0$.

$\Rightarrow$ for $g_{tt} > 0 \Rightarrow$ particle must move.

For massive ptikle,

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$e^{-\alpha t} \to$ instability $\alpha > 0$

$\left(\frac{GM}{c^2}\right)^2 - a^2$.

$\frac{a r \sin^2\theta}{c^2\rho^2}(dt\,d\phi + d\phi\,dt)$

$t \to t$

$\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2$

Other properties:
$r = r_{se}$ is also
the infinite redshift surface.

a^2

θ

stationary limit r_{se}

r_{se} : $g_{tt} = 0$: ρ^2

$\to r^2 + a$

For massive particle,

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$$

$$= g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{rr}\dot{r}^2 + \dots$$

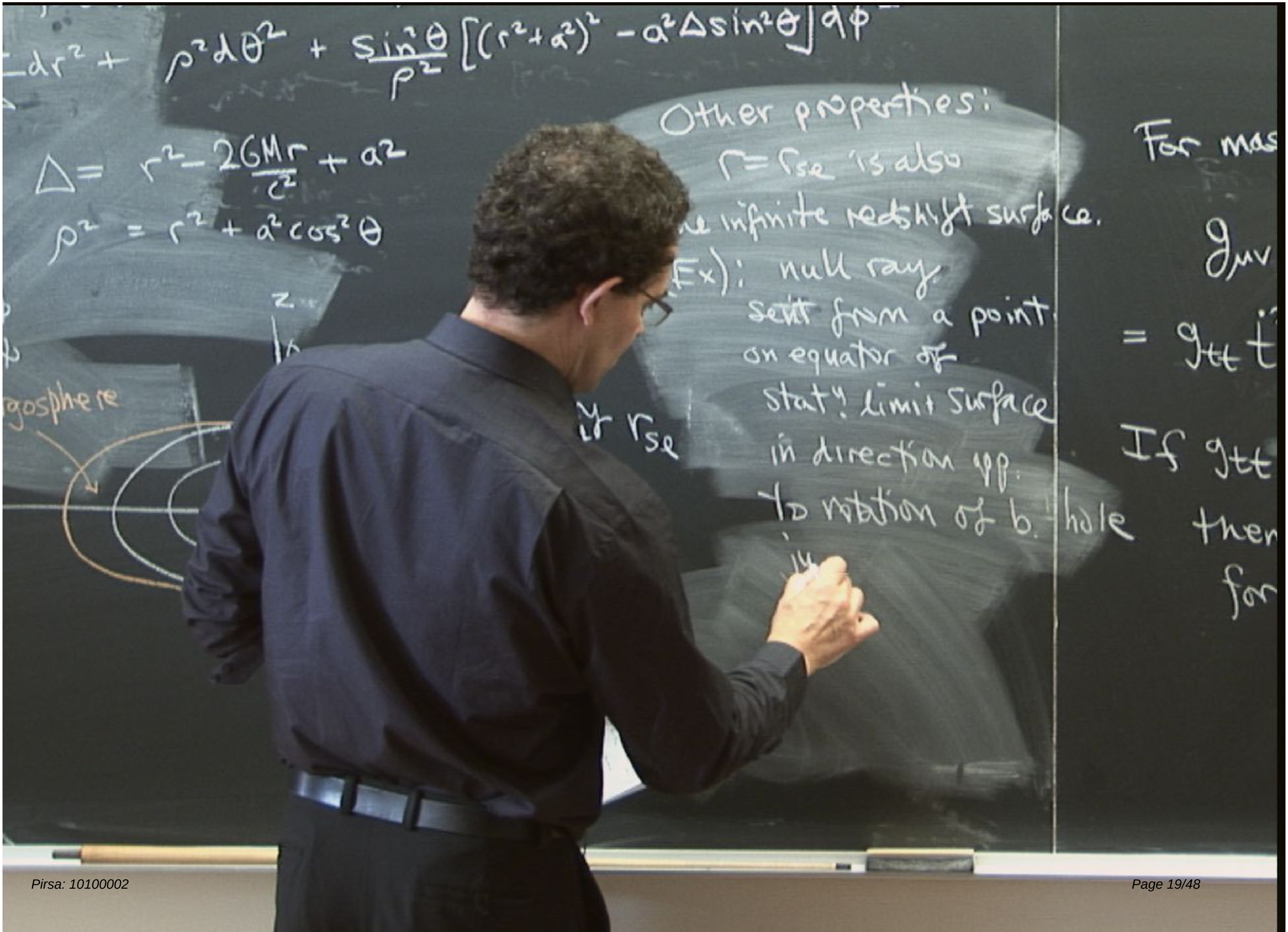
If $g_{tt} \geq 0$
then there is no solution
for $\dot{\theta} = \dot{\phi} = \dot{r} = 0$.

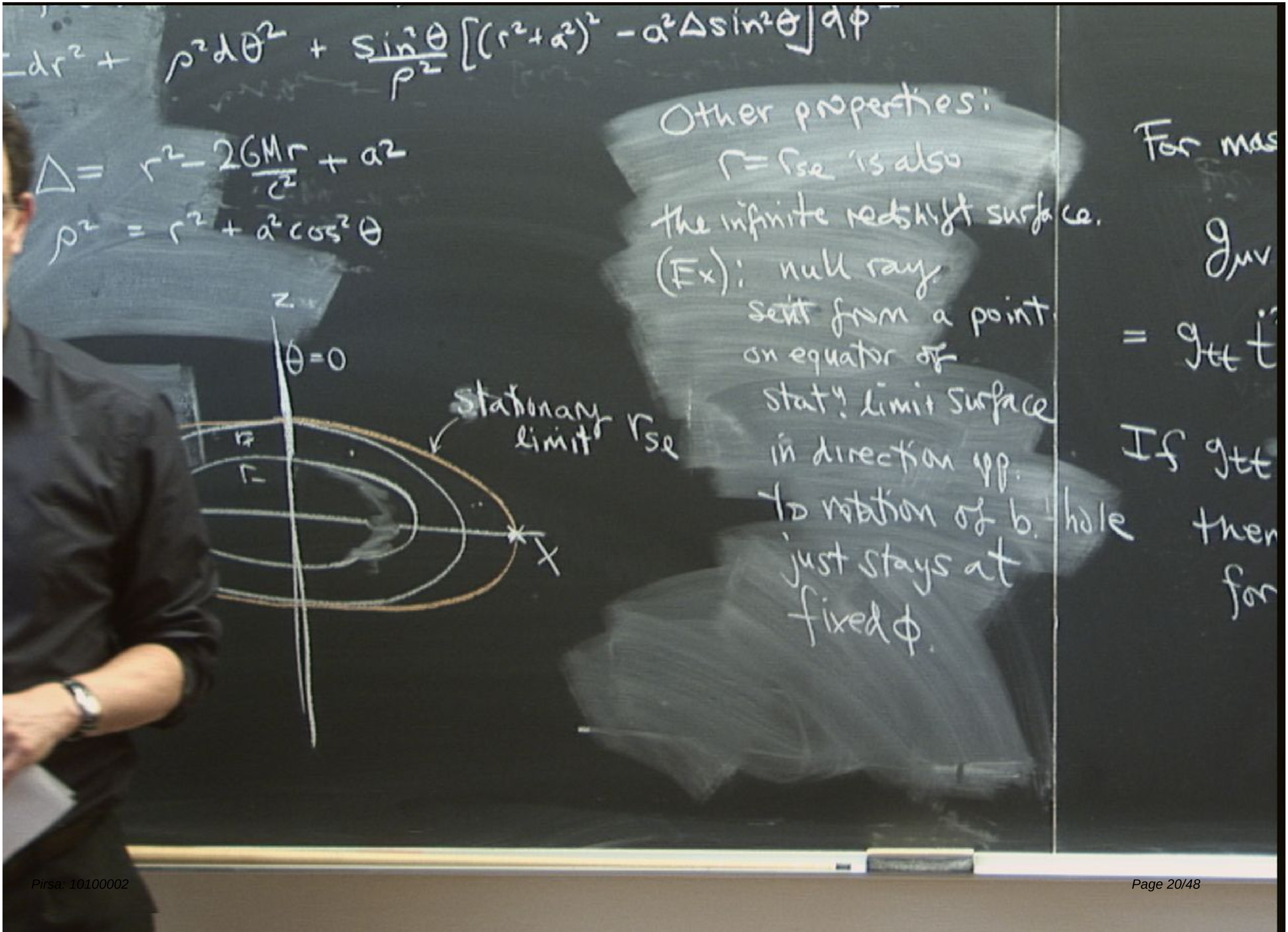
$\Rightarrow (r_{se}$

c.f. $(r_+$

$\Rightarrow r_{se} >$

$\Rightarrow$ for


Other properties:
r = r_se is also
infinite redshift surface.
(Ex): null ray
sent from a point
on equator of
stat'y limit surface
in direction opp.
to rotation of b. hole
For mas
If g_tt
then
for
gosphere
Δ = r² − 2GMr/c² + a²
ρ² = r² + a² cos²θ



$-dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right] d\phi$
$\Delta = r^2 - \frac{2GMr}{c^2} + a^2$
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z
$\theta = 0$
r_+
r_-
stationary limit r_{se}
x
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$g_{\mu\nu}$
$= g_{tt}\dot{t}$
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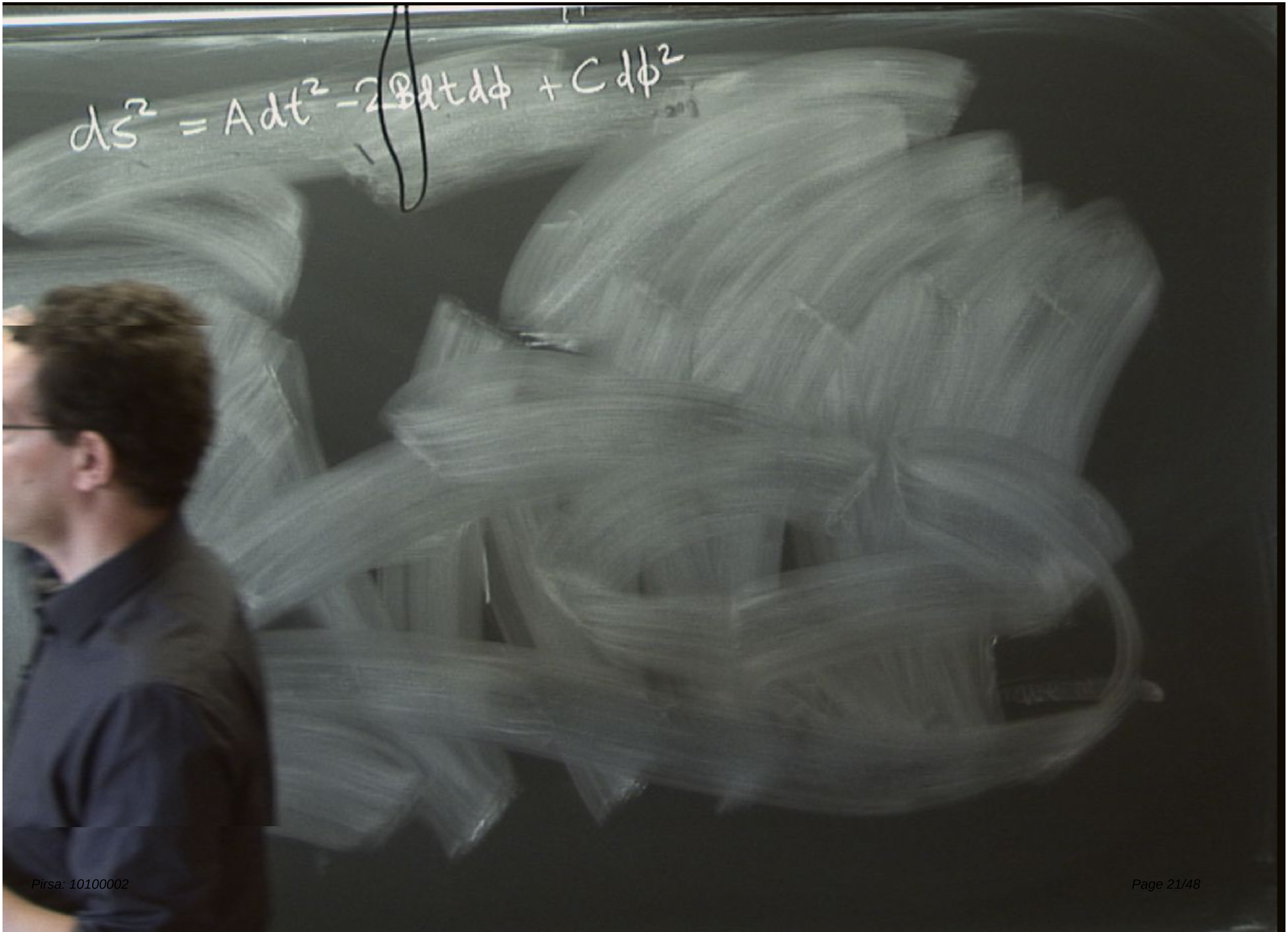
$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$

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Conserved Energy:

$$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$$

Conserved Energy: momentum conjugate to t $\quad p^\mu = \frac{\partial L}{\partial \dot{x}^\mu}$

$$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$$

Conserved Energy: momentum conjugate to t $\quad p_\mu = \frac{\partial L}{\partial \dot{x}^\mu}$

rest mass

$$p_t = \underset{\uparrow}{m}\, g_{tt}\, \dot{t}$$

$$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$$

Conserved Energy: momentum conjugate to t $\quad p_\mu = \frac{\partial L}{\partial \dot{x}^\mu}$

rest mass

$$\frac{m}{2}\int g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \, d\tau$$

$$p_t = m\left(g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\right)$$

$$\dot{p}_t = 0$$

$$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$$

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$$p_t = m\left(g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\right)$$

At $r=\infty$ $\quad g_{tt} = -1 \Rightarrow \underline{p_t = -E}$

$$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$$

Conserved Energy: momentum conjugate to t $\quad p_\mu = \dfrac{\partial L}{\partial \dot{x}^\mu}$

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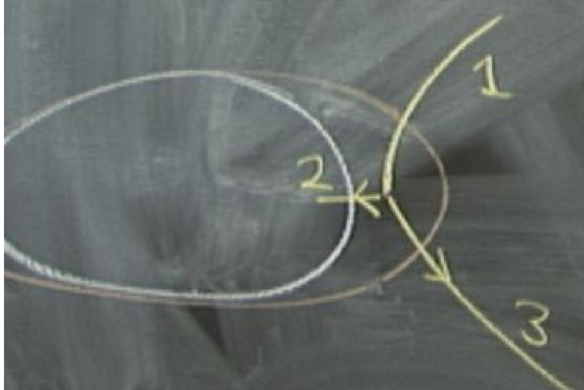

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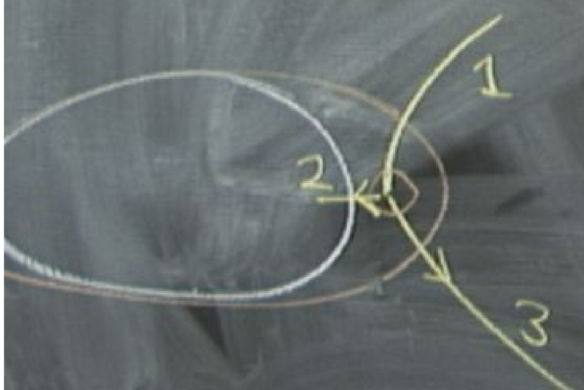

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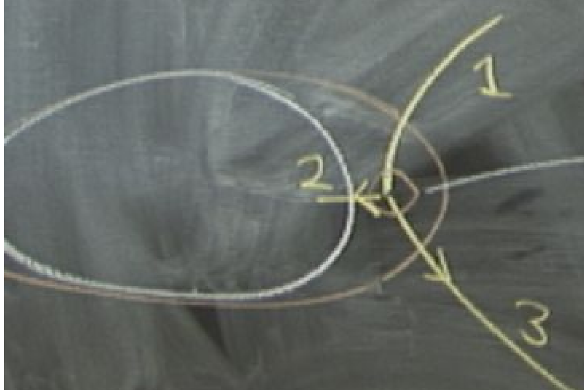
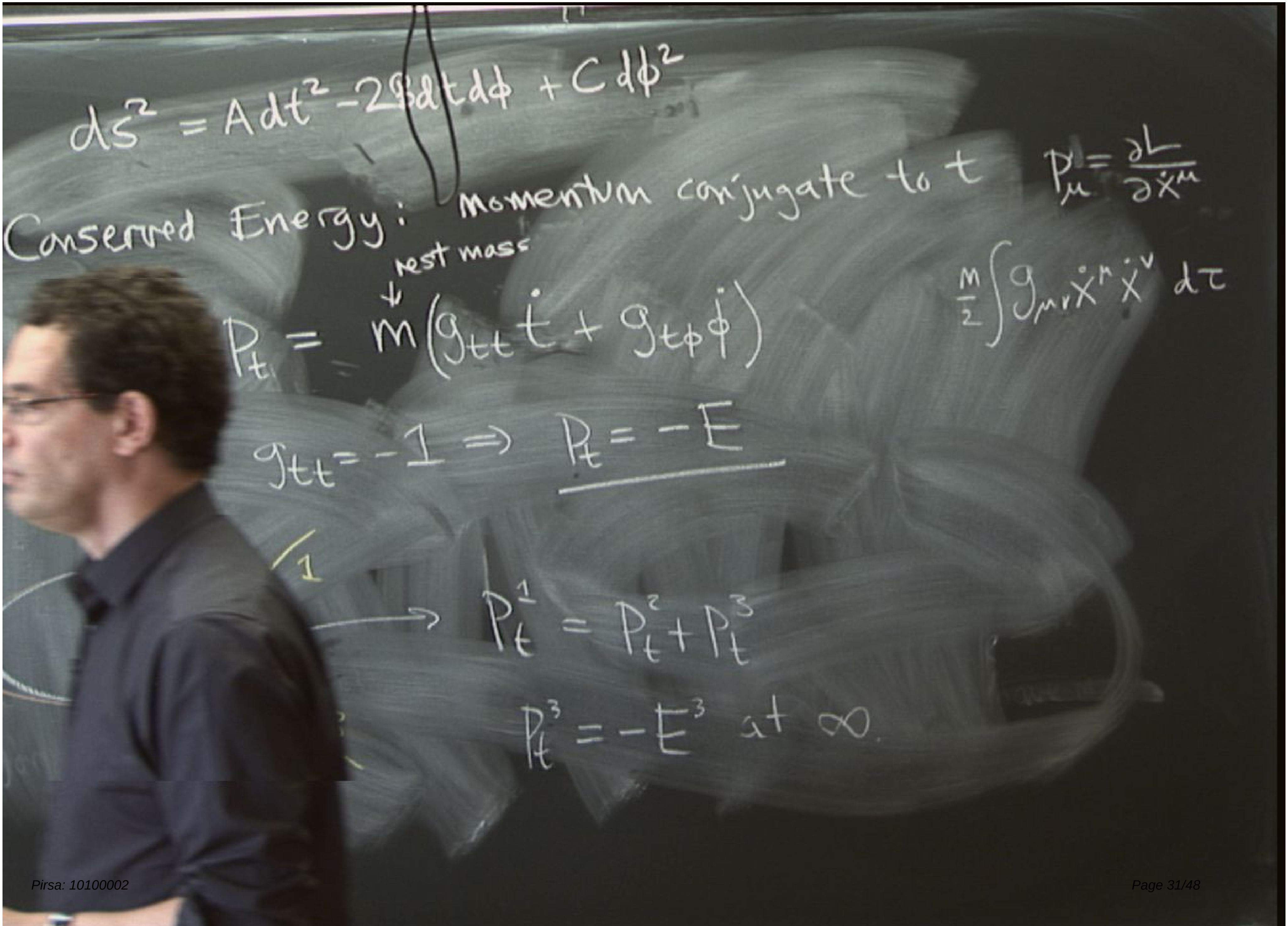


$$\Rightarrow P_t^1 = P_t^2 + P_t^3$$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Conserved Energy: momentum conjugate to t
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rest mass
$\frac{m}{2}\int g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \, d\tau$
$P_t = m(g_{tt}\dot{t} + g_{t\phi}\dot{\phi})$
$g_{tt} = -1 \Rightarrow P_t = -E$
1
$P_t^1 = P_t^2 + P_t^3$
$P_t^3 = -E^3$ at ∞.

$$ds^2 = A\,dt^2 - 2B\,dt\,d\phi + C\,d\phi^2$$

Conserved Energy: momentum conjugate to t $\quad p_\mu = \frac{\partial L}{\partial \dot{x}^\mu}$

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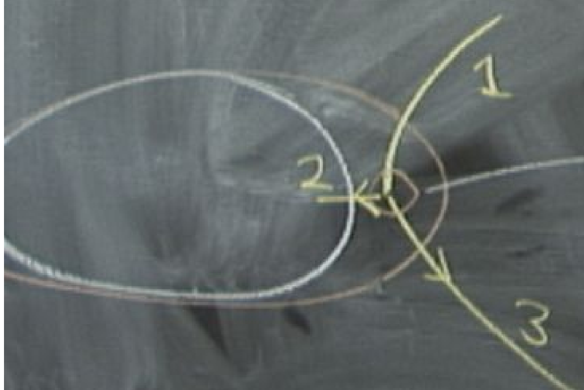

$$p_t^1 = p_t^2 + p_t^3$$

$$p_t^3 = -E^3 \text{ at } \infty$$

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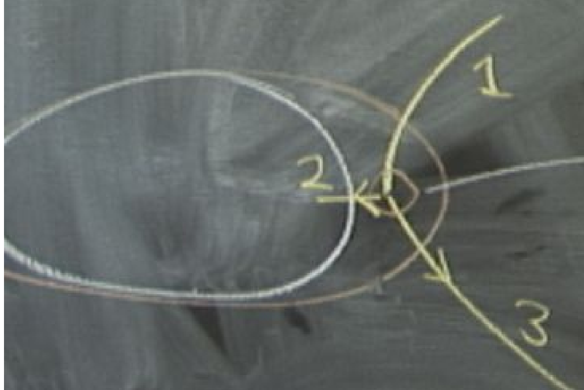
$$\frac{m}{2}\int g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu \, d\tau$$

rest mass

$$p_t = m\left(g_{tt}\dot{t} + g_{t\phi}\dot{\phi}\right)$$

At $r=\infty$ $\quad g_{tt} = -1 \Rightarrow \underline{p_t = -E}$



$$p_t^1 = p_t^2 + p_t^3$$

$$p_t^3 = -E^3 \text{ at } \infty.$$

$$\overbrace{-\left(1-\frac{2GMr}{\rho^2c^2}\right)}^{A}dt^2 - \overbrace{\frac{2GMar\sin^2\theta}{c^2\rho^2}}^{B}\left(dt\,d\phi + d\phi\,dt\right)$$

$$+ \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \underbrace{\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]}_{C}d\phi^2$$

$$\Delta = r^2 - \frac{2GMr}{c^2} + a^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$\overbrace{-\left(1-\frac{2GMr}{\rho^2c^2}\right)}^{A}dt^2 - \overbrace{2\frac{GMar\sin^2\theta}{c^2\rho^2}}^{B}(dt\,d\phi + d\phi\,dt)$$

$$+\frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \underbrace{\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]}_{C} d\phi^2$$

$$\Delta = r^2 - \frac{2GMr}{c^2} + a^2$$

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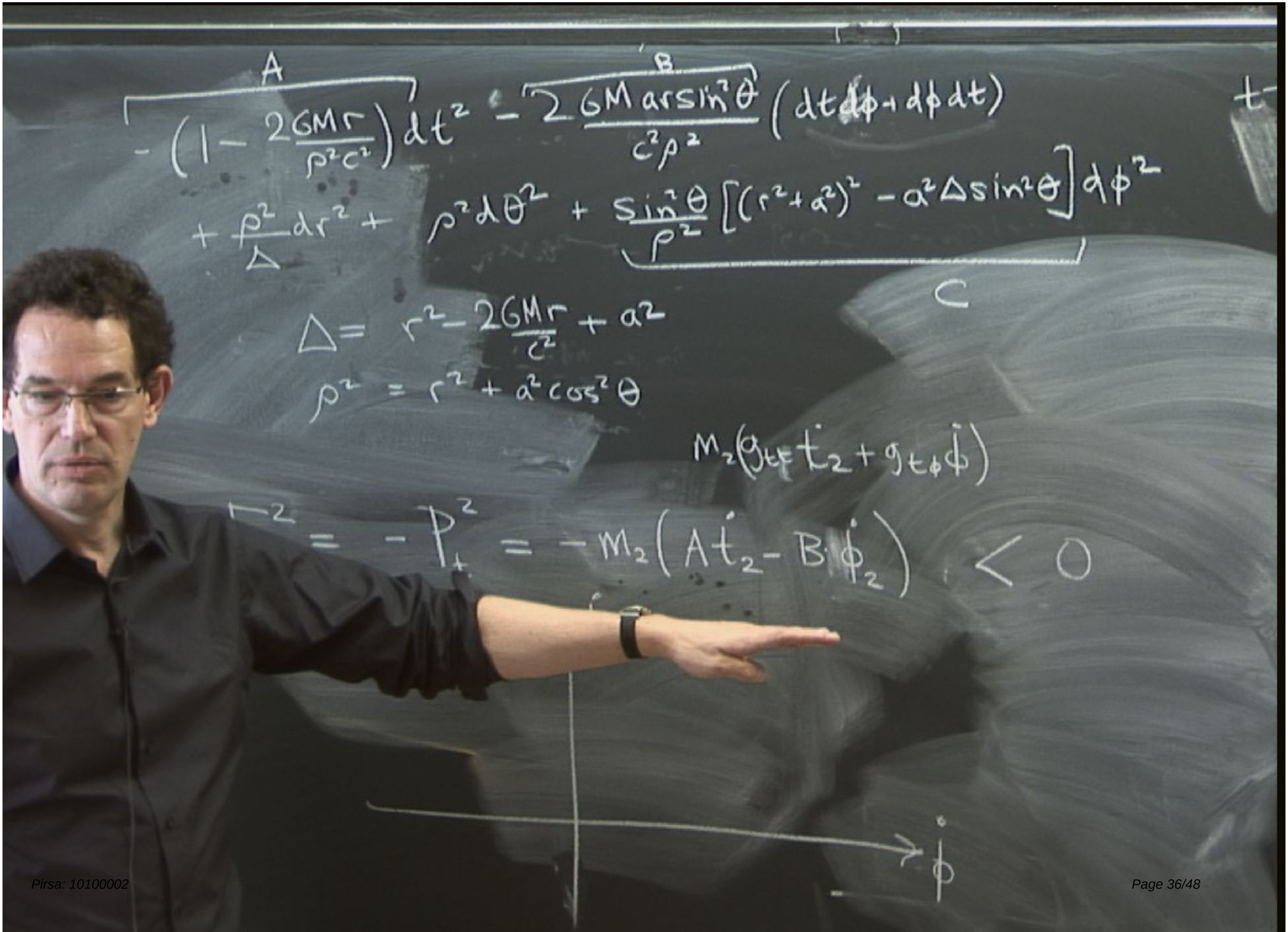
$$m_2(g_{tt}\dot{t}_2 + g_{t\phi}\dot{\phi})$$

$$E^2 = -P_t^2 = -m_2(A\dot{t}_2 - B\dot{\phi}_2)$$


A
B
$-\left(1-\frac{2GMr}{\rho^2c^2}\right)dt^2 - \frac{2GMar\sin^2\theta}{c^2\rho^2}(dt\,d\phi + d\phi\,dt)$
$+\frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]d\phi^2$
C
$\Delta = r^2 - \frac{2GMr}{c^2} + a^2$
$\rho^2 = r^2 + a^2\cos^2\theta$
$M_2(g_{tt}\dot{t}_2 + g_{t\phi}\dot{\phi})$
$= -P_t^2 = -M_2(A\dot{t}_2 - B\dot{\phi}_2) < 0$
$\dot{\phi}$

$$\overbrace{-\left(1-\frac{2GMr}{\rho^2c^2}\right)}^{A}dt^2 - \overbrace{2\frac{GMar\sin^2\theta}{c^2\rho^2}}^{B}(dt\,d\phi+d\phi\,dt)$$

$t-$

$$+\frac{\rho^2}{\Delta}dr^2+\rho^2d\theta^2+\underbrace{\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2-a^2\Delta\sin^2\theta\right]}_{C}d\phi^2$$

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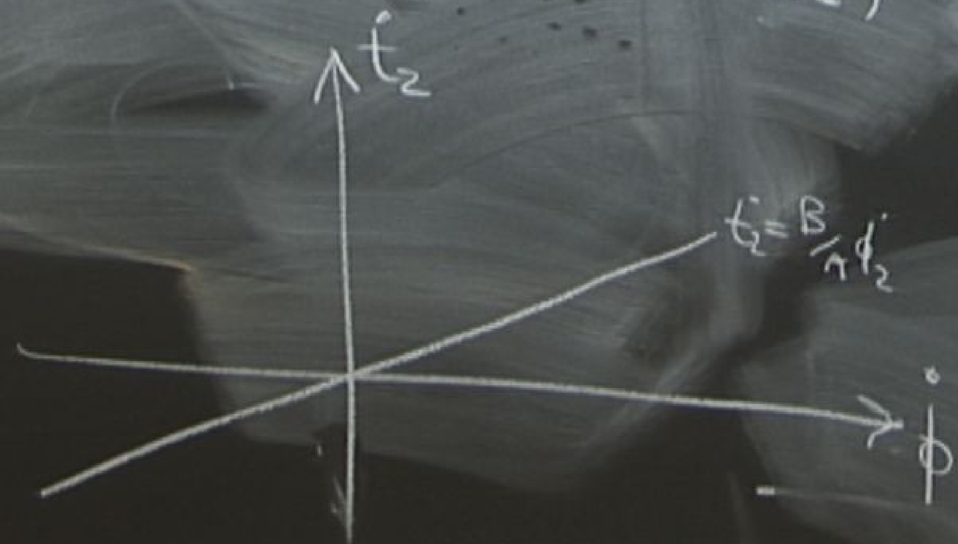
$$E^2 = -P_t^2 = -M_2(A\dot{t}_2 - B\dot{\phi}_2) < 0$$



$$\overbrace{-\left(1-\frac{2GMr}{\rho^2c^2}\right)}^{A}dt^2 - \overbrace{2\frac{GMar\sin^2\theta}{c^2\rho^2}}^{B}\left(dt\,d\phi+d\phi\,dt\right)$$

$$+\frac{\rho^2}{\Delta}dr^2+\rho^2d\theta^2+\underbrace{\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2-a^2\Delta\sin^2\theta\right]}_{C}d\phi^2$$

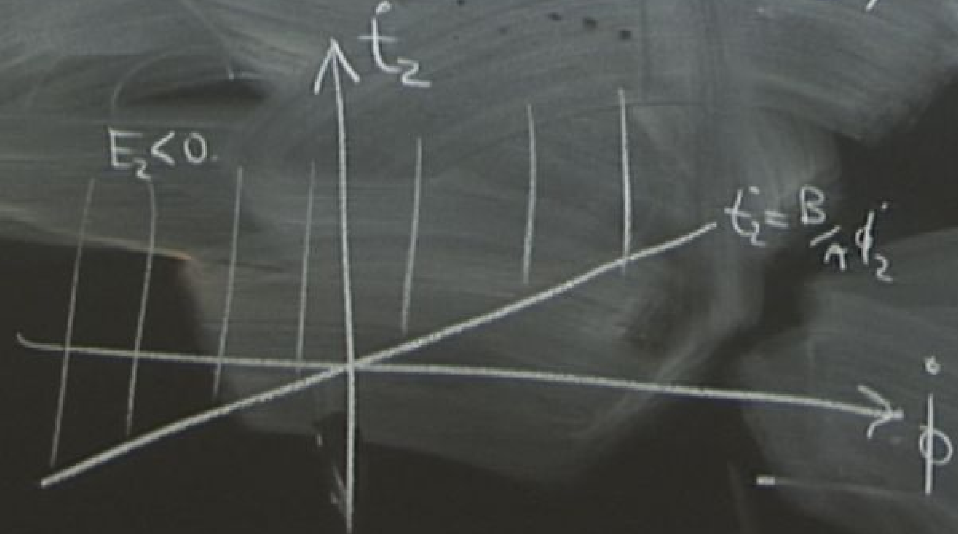
$t-$

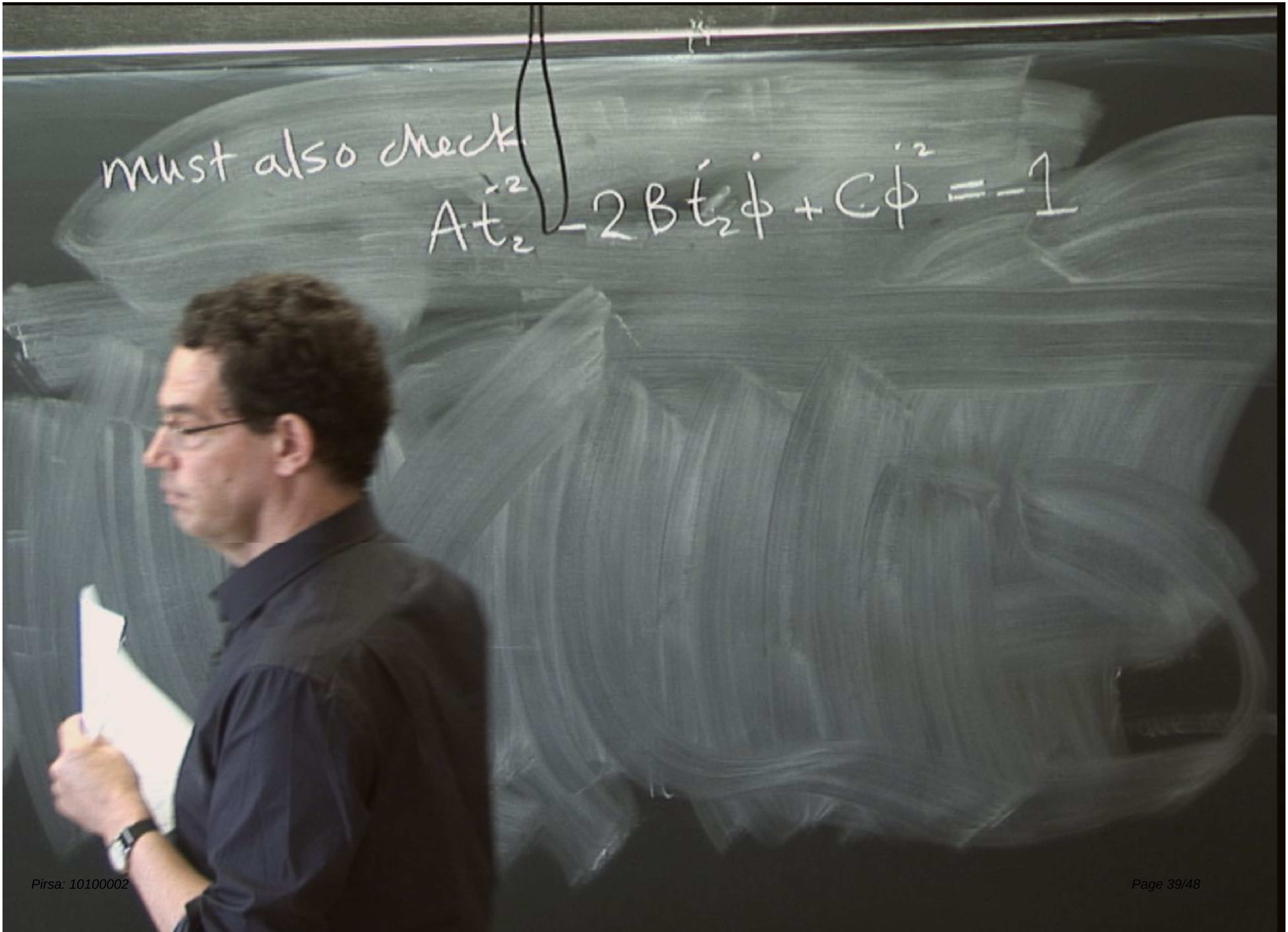
$$\Delta = r^2-2\frac{GMr}{c^2}+a^2$$

$$\rho^2 = r^2+a^2\cos^2\theta$$

$$M_2(g_{tt}\dot t_2+g_{t\phi}\dot\phi)$$

$$E^2 = -P_t = -M_2\left(A\dot t_2 - B\dot\phi_2\right) < 0$$




must also check
$A\dot{t}_2^2 - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 = -1$

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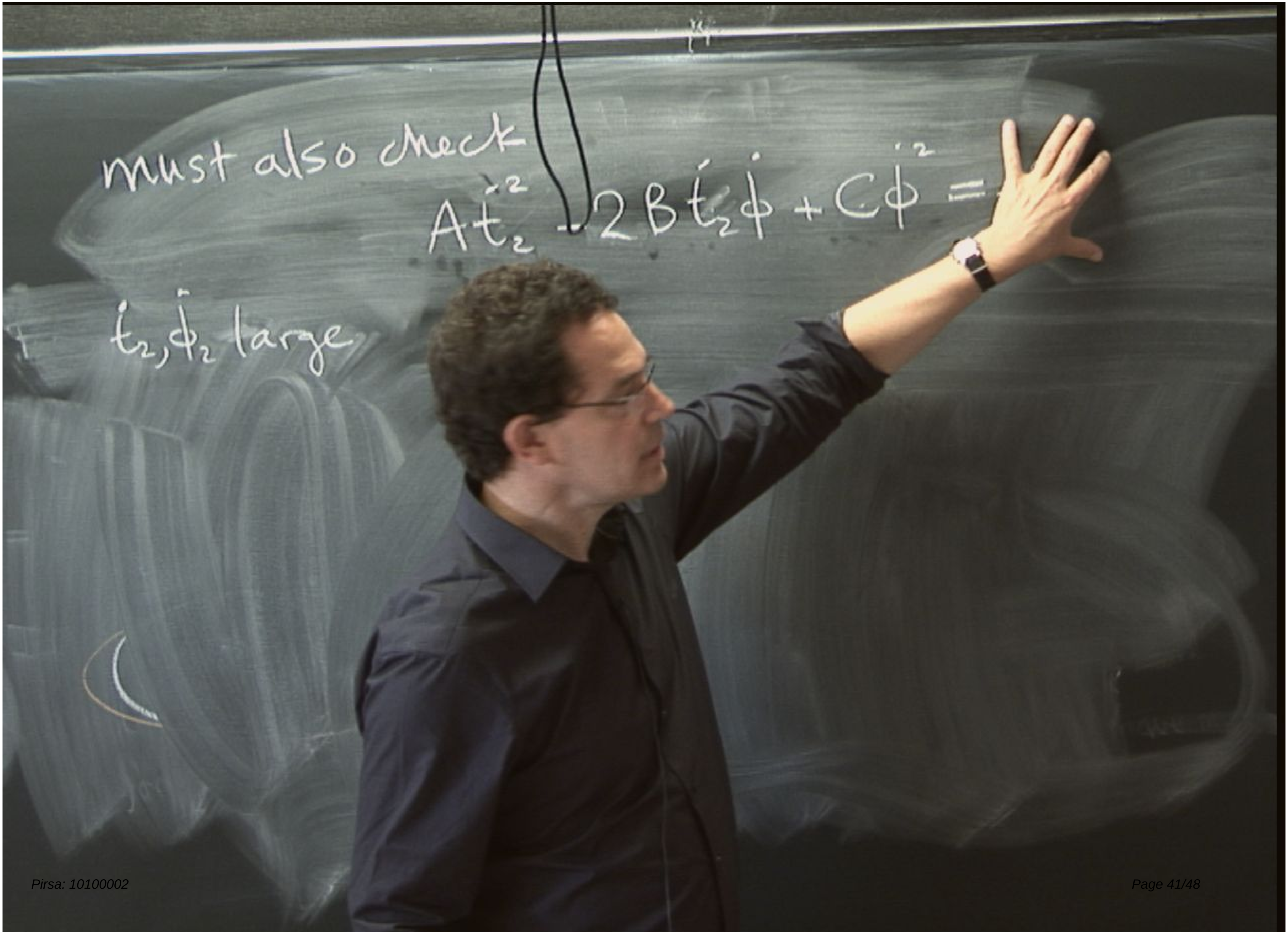

must also check
$A\dot{t}_2^2 - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 =$
$\dot{t}_2, \dot{\phi}_2$ large

must also check

$$A\dot{t}_2^2 - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 = -1$$

$\dot{t}_2, \dot{\phi}_2$ large

hyperbola, asymptotes are

$$\frac{\dot{t}_2}{\dot{\phi}} = \frac{B}{A} \pm \sqrt{\left(\frac{B}{A}\right)^2 - \frac{C}{A}}$$

$$-\overbrace{\left(1-\frac{2GMr}{\rho^2c^2}\right)}^{A}dt^2 - \overbrace{\frac{2GMar\sin^2\theta}{c^2\rho^2}}^{B}(dt\,d\phi + d\phi\,dt)$$

$$+\frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \underbrace{\frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)^2 - a^2\Delta\sin^2\theta\right]}_{C}d\phi^2$$

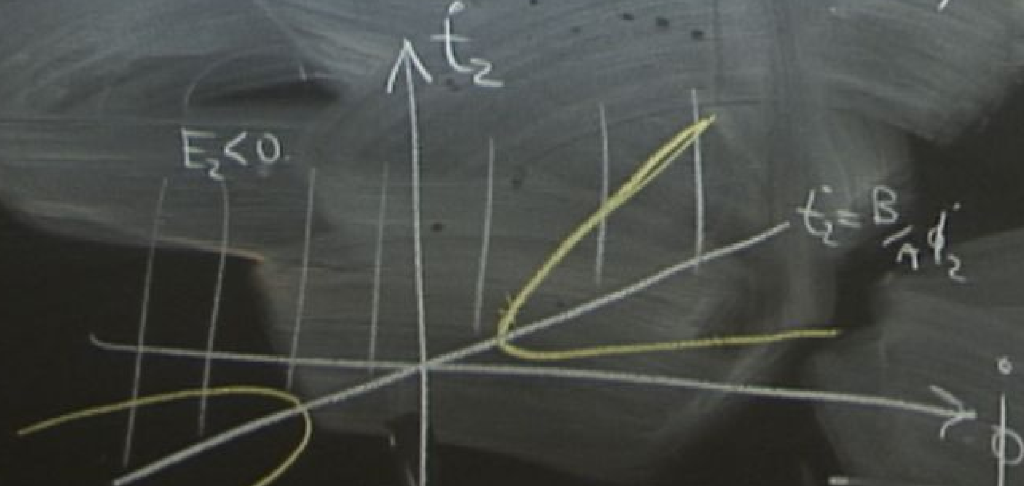
$t \rightarrow$

$$\Delta = r^2 - \frac{2GMr}{c^2} + a^2$$

$$\rho^2 = r^2 + a^2\cos^2\theta$$

$$M_2(g_{tt}\dot{t}_2 + g_{t\phi}\dot{\phi})$$

$$E^2 = -P_t^2 = -M_2(A\dot{t}_2 - B\dot{\phi}_2) < 0$$



must also check

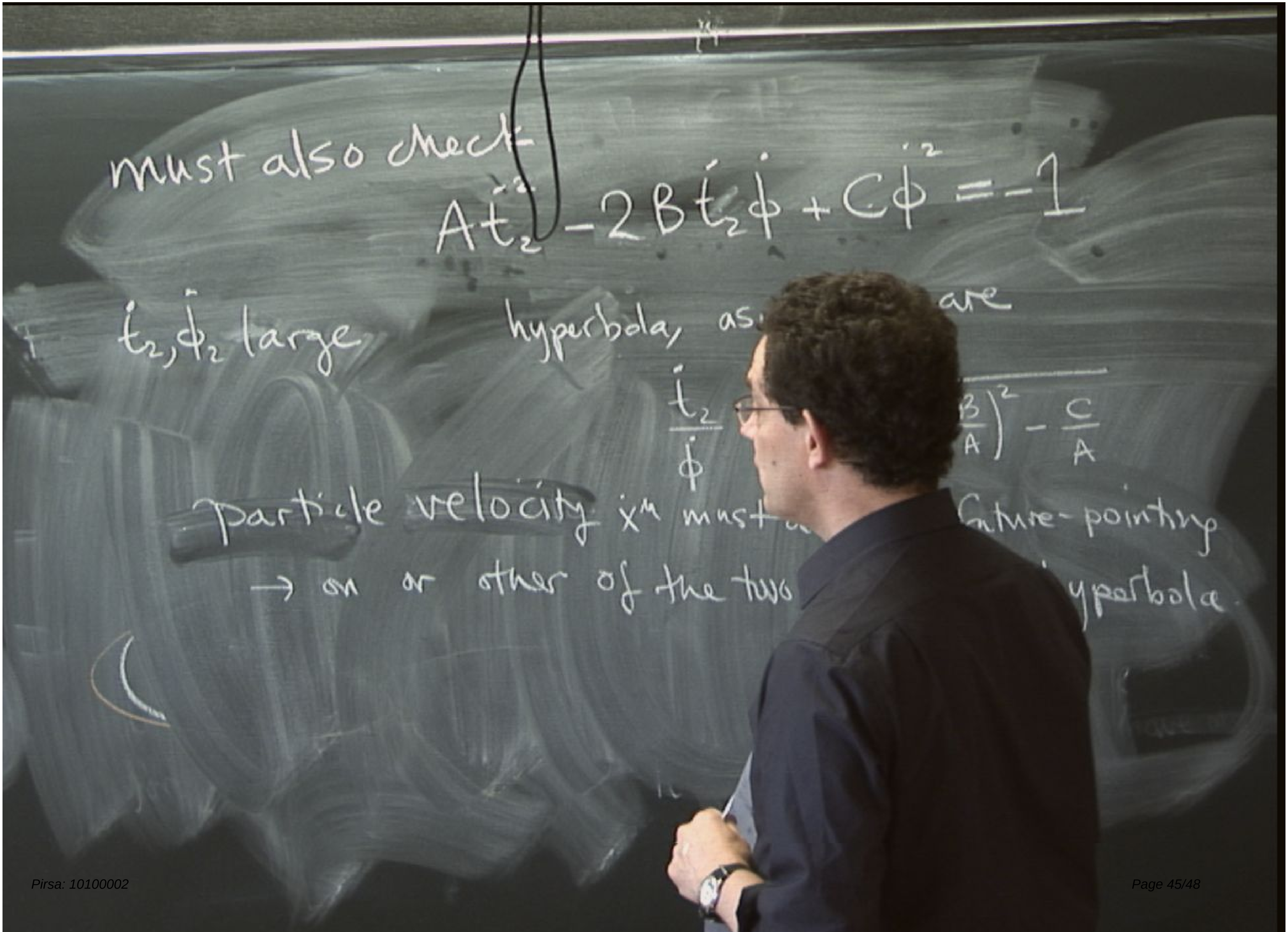
$$A\dot{t}_2^{\,2} - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 = -1$$

$\dot{t}_2, \dot{\phi}_2$ large hyperbola, asymptotes are

$$\frac{\dot{t}_2}{\dot{\phi}} = \frac{B}{A} \pm \sqrt{\left(\frac{B}{A}\right)^2 - \frac{C}{A}}$$

particle velocity $\dot{x}^\mu$ must also be future-pointing

$\rightarrow$ on or other of the two branches of hyperbola


must also check
$A\dot{t}_2^2 - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 = -1$
$\dot{t}_2, \dot{\phi}_2$ large
hyperbola, as
are
$\frac{\dot{t}_2}{\dot{\phi}}$
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$$A\dot{t}_2^2 - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 = -1$$

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particle velocity $\dot{x}^\mu$ must also be future-pointing

$\rightarrow$ on or other of the two branches of hyperbola

also find $L_z^3 > L_z^1 \Rightarrow$ b. hole loses momentum to p

$$\dot{x}_2^\mu = (\dot{t}, 0, 0, \dot{\phi})$$

must also check

$$A\dot{t}_2^2 - 2B\dot{t}_2\dot{\phi} + C\dot{\phi}^2 = -1$$

$\dot{t}_2, \dot{\phi}_2$ large   hyperbola, asymptotes are

$$\frac{\dot{t}_2}{\dot{\phi}} = \frac{B}{A} \pm \sqrt{\left(\frac{B}{A}\right)^2 - \frac{C}{A}}$$

particle velocity $\dot{x}^\mu$ must also be future-pointing

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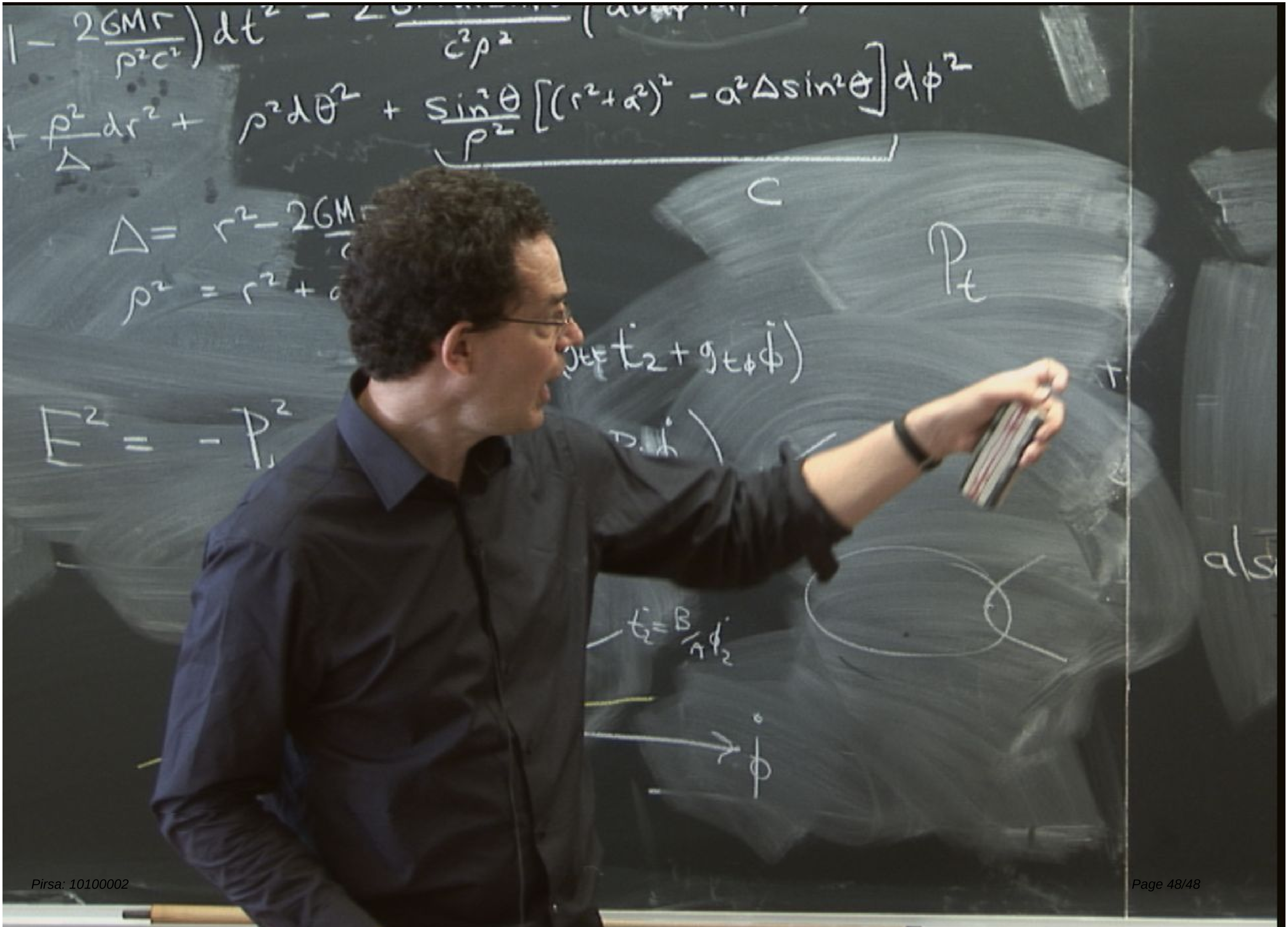
also find $L_z^3 > L_z^1$ ⇒ b. hole loses momentum to p



+ ρ²/Δ dr² + ρ² dθ² + sin²θ/ρ² [(r²+a²)² − a²Δ sin²θ] dφ²
C
Pt
Δ = r² − 2GM
ρ² = r² +
E² = − P
als