Title: Quantum Theory (PHYS 605) - Lecture 15

Date: Oct 01, 2010 09:00 AM

URL: http://pirsa.org/10100001

Abstract:

PERIMETER INSTITUTE FOR THEORETICAL PHYSICS

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NMR for quantum computing

Quantum Theory

PSI -- 1 Oct 2010

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Quantum Theory

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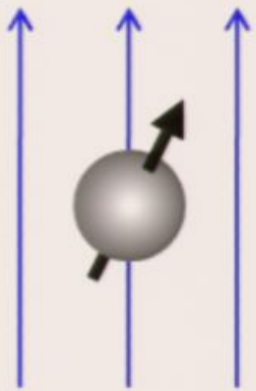
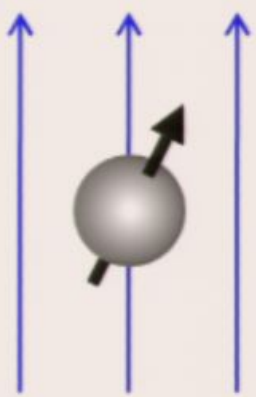
$$\hbar = 1$$

Nuclear spin in a magnetic field



proton in a
10.00 T field

Nuclear spin in a magnetic field



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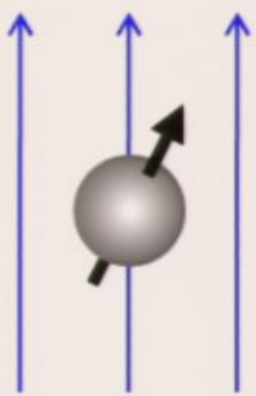
Nuclear magnetic moment couples to external field:

$$\mathsf{H}_0 = -\vec{\mu} \cdot \vec{B}_0 = -\frac{\Omega}{2}\,\mathsf{Z}$$

Nuclear spin in a magnetic field



proton in a
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$\Omega/2\pi = 425.7$ MHz

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Time evolution from external field only:

$$\begin{aligned} \mathsf{U}_0(t) &= \exp(-i\mathsf{H}_0 t) = \exp\left(i\,\frac{\Omega t}{2}\,\mathsf{Z}\right) \\ &= \cos\left(\Omega t/2\right)\,\mathbf{1} + i\sin\left(\Omega t/2\right)\,\mathsf{Z} \end{aligned}$$

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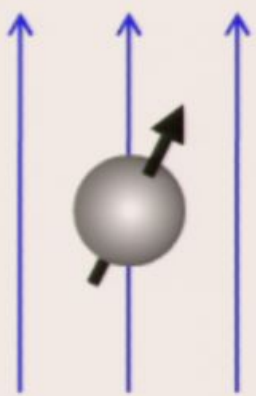

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Actual Hamiltonian includes extra stuff

$$i\frac{d}{dt}\,|\psi(t)\rangle = (\mathsf{H}_0 + \mathsf{H})\,|\psi(t)\rangle$$

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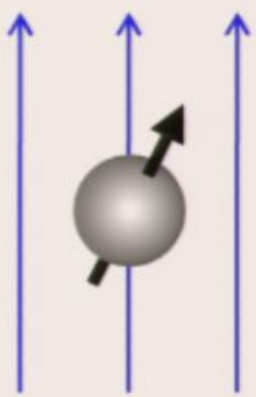

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external field

other effects

Rotating frame picture

Rotating frame picture

Adopt: "Heisenberg" picture for H_0
"Schrödinger" picture for H

$$\left|\hat{\psi}(t)\right\rangle = \mathsf{U}_0(t)^\dagger \left|\psi(t)\right\rangle$$

$$\hat{\mathsf{H}}(t) = \mathsf{U}_0(t)^\dagger \mathsf{H}\, \mathsf{U}_0(t)$$

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This is related to the "interaction picture".

Our "reference frame" precesses at Ω like an unperturbed nuclear spin.

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Possible terms: $\mathsf{H} = \varepsilon_0 \mathbf{1} + \varepsilon_x \mathsf{X} + \varepsilon_y \mathsf{Y} + \varepsilon_z \mathsf{Z}$

What do these look like in the rotating frame picture?

Rotating frame picture

Rotating frame picture

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How does rotating frame U_0 affect Pauli operators?

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Rotating frame Hamiltonian

$$\begin{aligned} \hat{\mathsf{H}}(t) &= \varepsilon_0 \mathbf{1} + (\varepsilon_x \cos \Omega t - \varepsilon_y \sin \Omega t)\mathsf{X} \\ &\quad + (\varepsilon_x \sin \Omega t + \varepsilon_y \cos \Omega t)\mathsf{Y} + \varepsilon_z \mathsf{Z} \end{aligned}$$

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overall E-shift (ignore)

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rapidly oscillating (avg. to zero!)

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Radio pulses

Radio pulses

Radio pulse produces a time-dependent perturbation

$$\mathsf{H} = A(t)\ (\cos(\Omega' t - \phi)\mathsf{X} - \sin(\Omega' t - \phi)\mathsf{Y})$$

$A(t)$ = "envelope" or "shape" of pulse

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This will quickly average to zero unless $\Omega' = \Omega$.
("*Resonance*" is the R in "NMR"!)

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We can control

- How $A(t)$ turns on, off
- Phase shift ϕ

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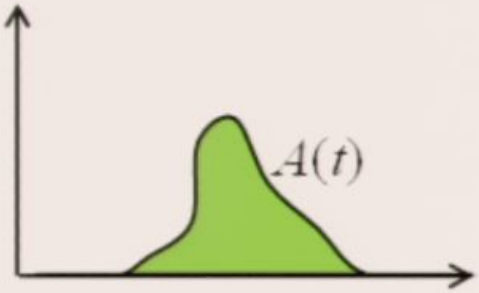

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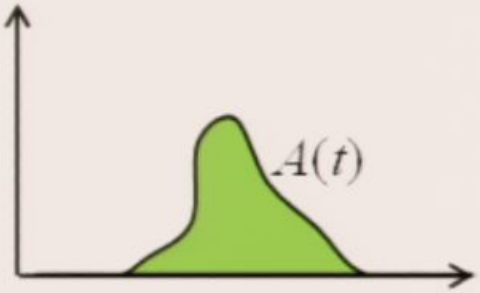

By choosing phases $\phi = 0$ or $\phi = \pi/2$, we can produce

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\hat{\mathsf{R}}_x(\beta) &= e^{-i(\beta/2)\mathsf{X}} = \cos(\beta/2)\mathbf{1} - i\sin(\beta/2)\mathsf{X} \\
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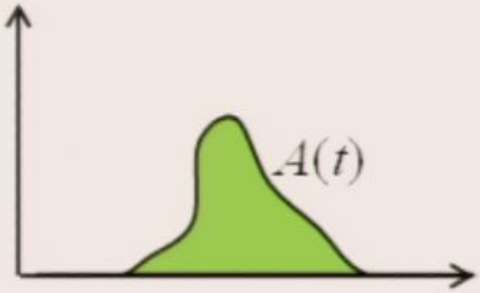

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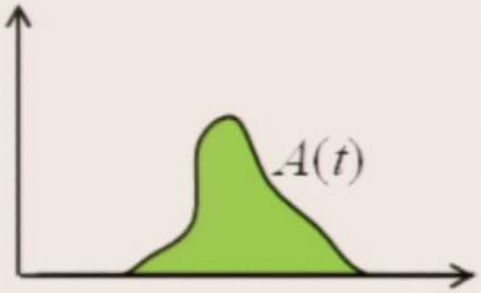

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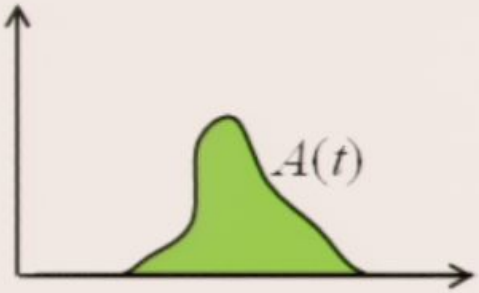

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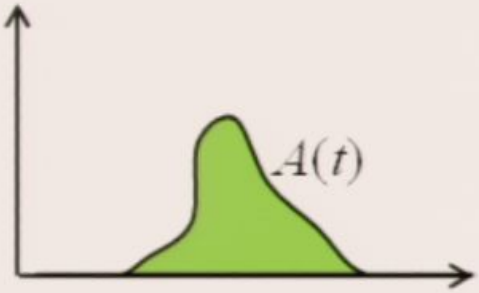

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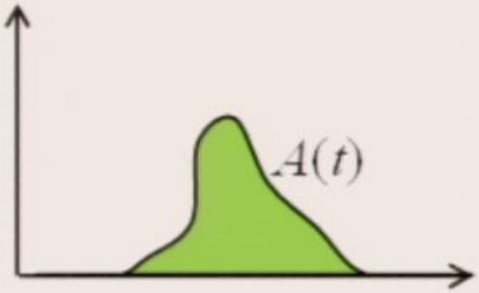

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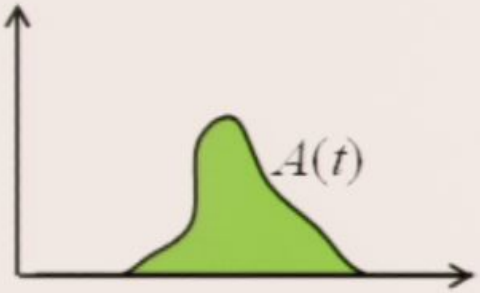

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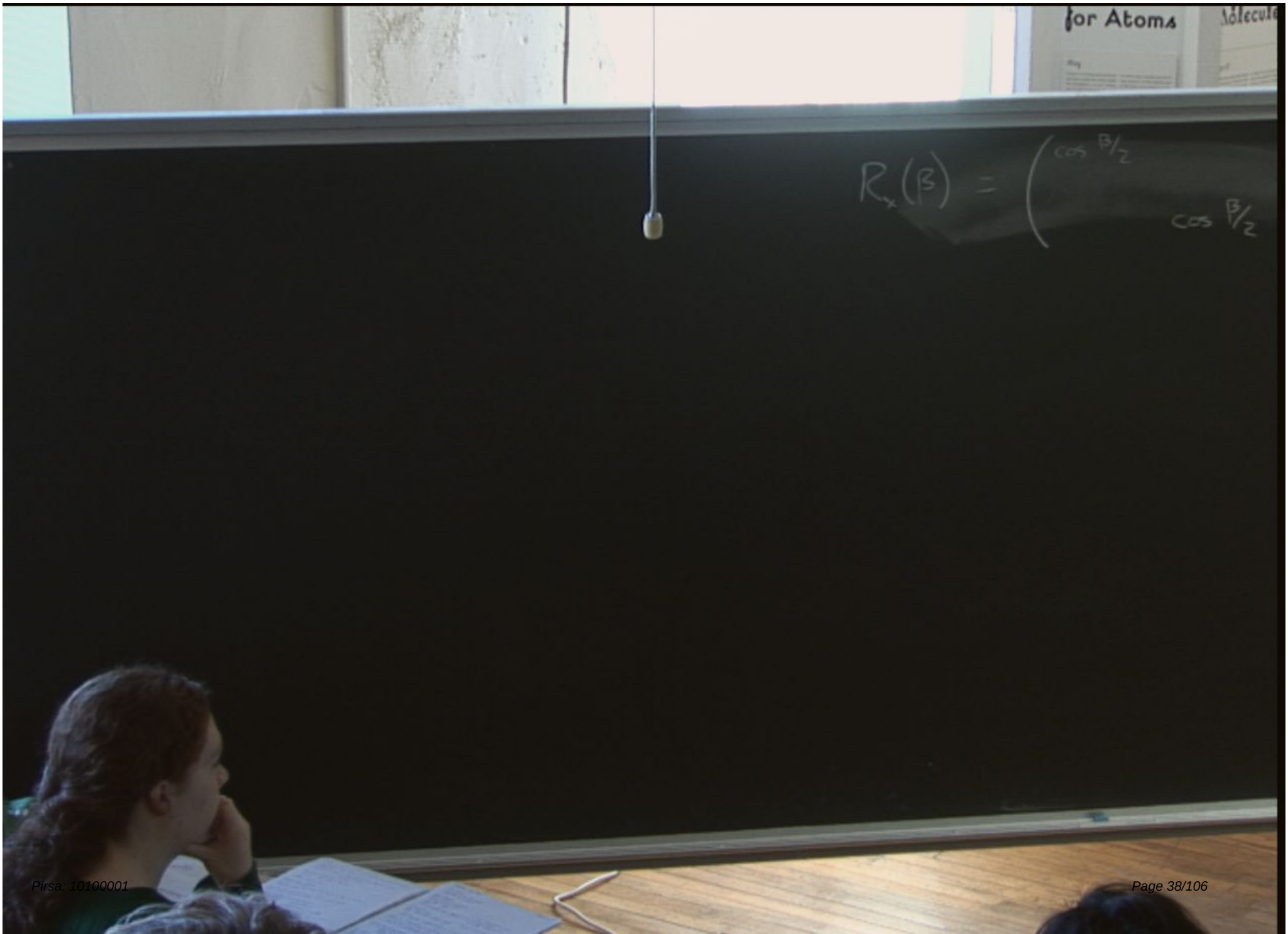
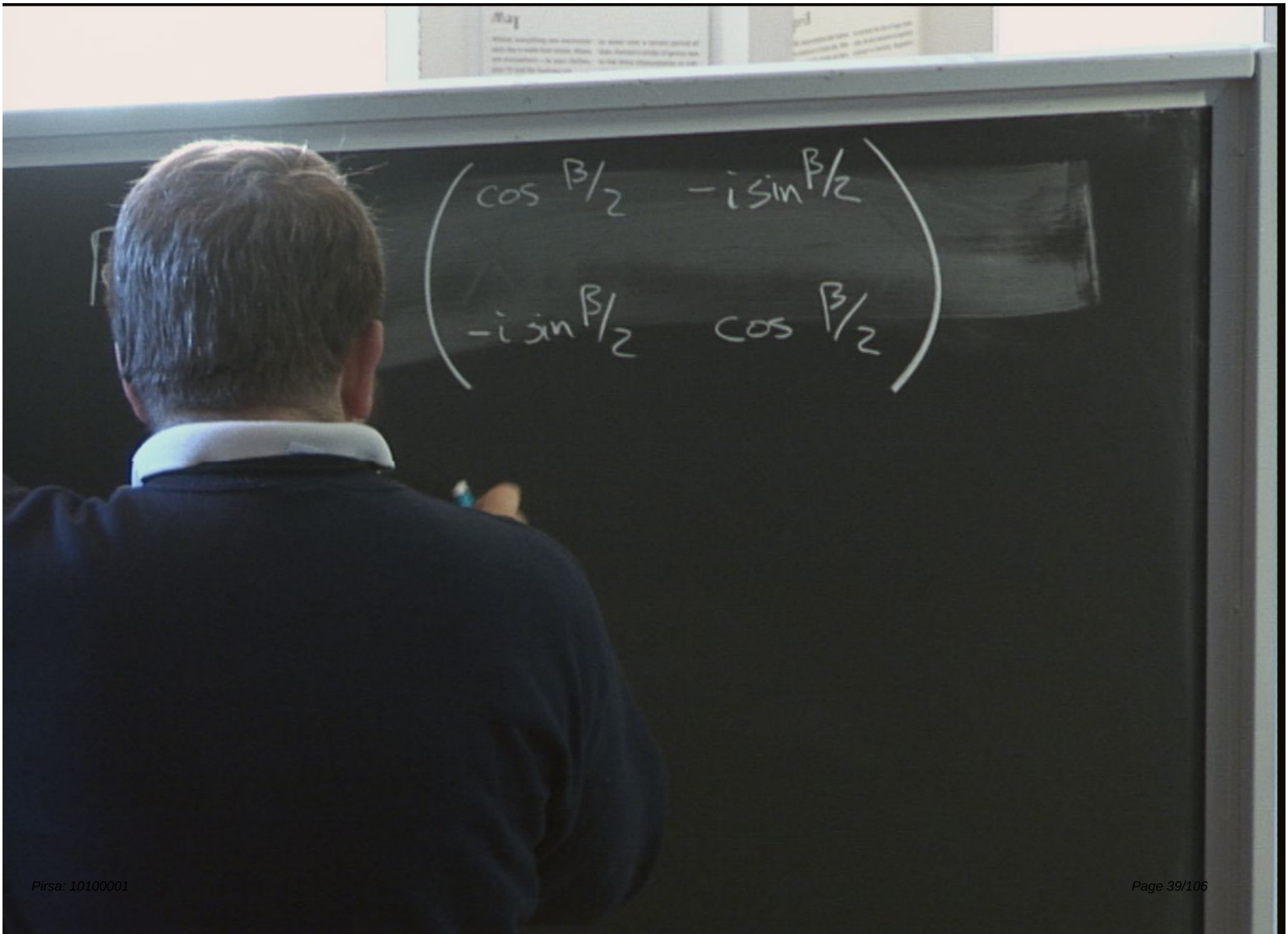
$$\hat{\mathsf{R}}_y(\pi) = -i\mathsf{Y} \qquad \hat{\mathsf{R}}_x(\pi)\hat{\mathsf{R}}_y(\pi/2) = -i\mathsf{H}$$


for Atoms
$R_x(\beta) = \begin{pmatrix} \cos \beta/2 & \\ & \cos \beta/2 \end{pmatrix}$


$\begin{pmatrix} \cos \beta/2 & -i \sin \beta/2 \\ -i \sin \beta/2 & \cos \beta/2 \end{pmatrix}$

$$R_x(\beta) = \begin{pmatrix} \cos \beta/2 & -i \sin \beta/2 \\ -i \sin \beta/2 & \cos \beta/2 \end{pmatrix}$$

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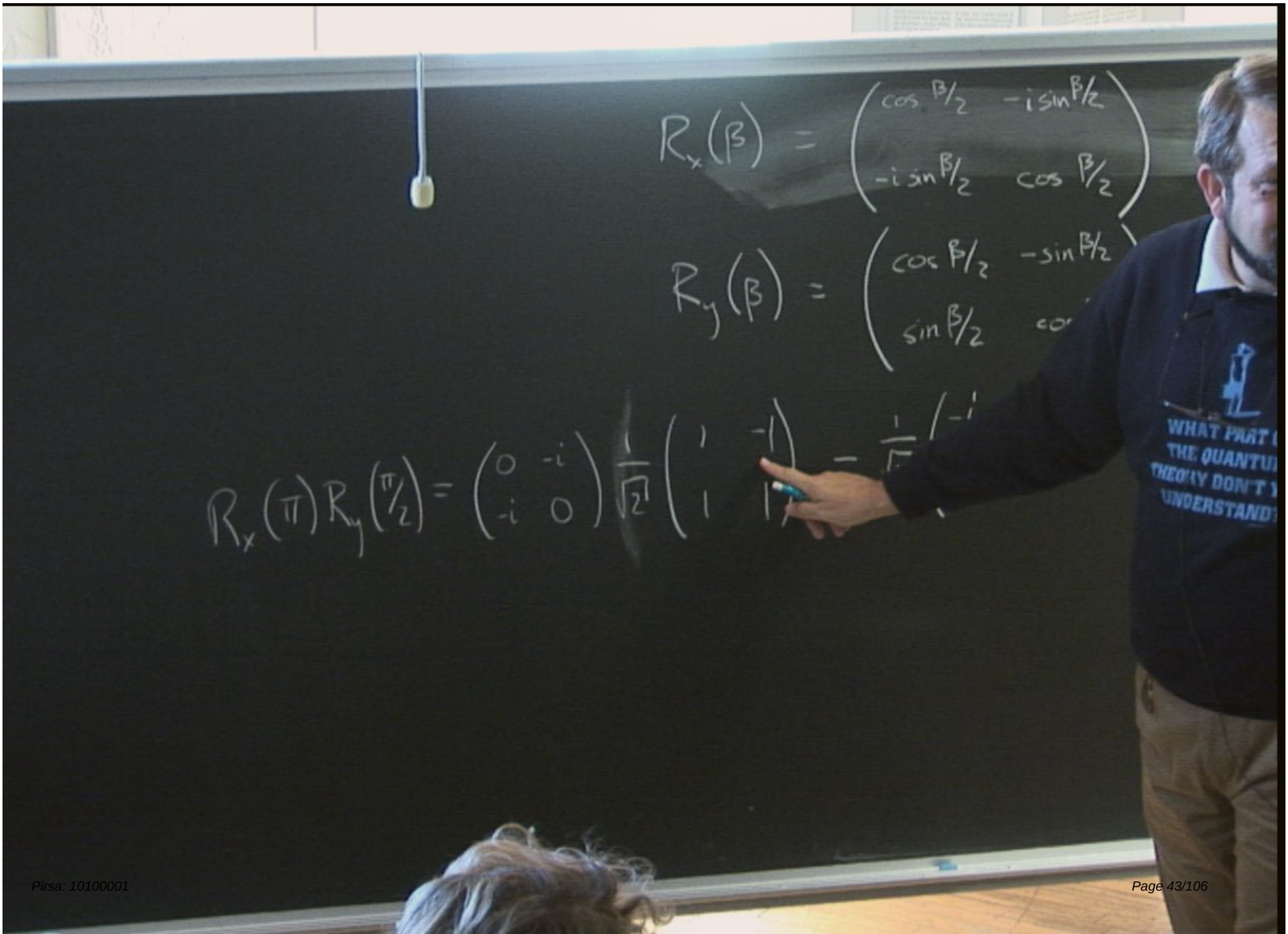
$$R_x(\pi) R_y(\pi/2) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \Big($$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WHAT PART
THE QUANTU
THEORY DON'T
UNDERSTAND

$$R_x(\beta) = \begin{pmatrix} \cos \beta/2 & -i \sin \beta/2 \\ -i \sin \beta/2 & \cos \beta/2 \end{pmatrix}$$

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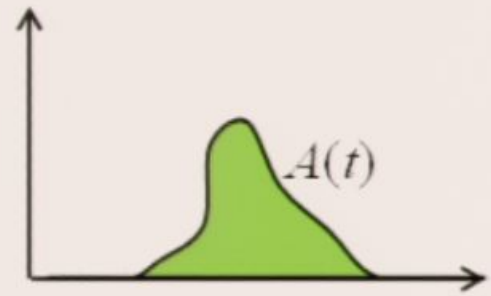
$$= -iH$$

Radio pulses

$$\hat{\mathsf{H}} = A(t)(\cos\phi\,\mathsf{X} + \sin\phi\,\mathsf{Y})$$

Total strength of resonant pulse $\displaystyle\int_{-\infty}^{\infty} A(t)\,dt = \frac{\beta}{2}$



By choosing phases $\phi = 0$ or $\phi = \pi/2$, we can produce

$$\begin{aligned}
\hat{\mathsf{R}}_x(\beta) &= e^{-i(\beta/2)\mathsf{X}} = \cos(\beta/2)\mathbf{1} - i\sin(\beta/2)\mathsf{X} \\
\hat{\mathsf{R}}_y(\beta) &= e^{-i(\beta/2)\mathsf{Y}} = \cos(\beta/2)\mathbf{1} - i\sin(\beta/2)\mathsf{Y}.
\end{aligned}$$

From a sequence of such pulses, we can create any 1-qubit gate!

$$\hat{\mathsf{R}}_x(\pi) = -i\mathsf{X} \qquad \hat{\mathsf{R}}_y(\pi)\hat{\mathsf{R}}_x(\pi) = -i\mathsf{Z}$$

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Note: Overall phases are unimportant

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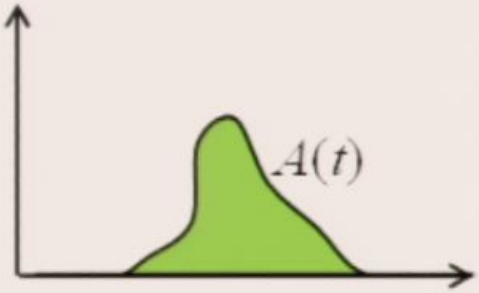

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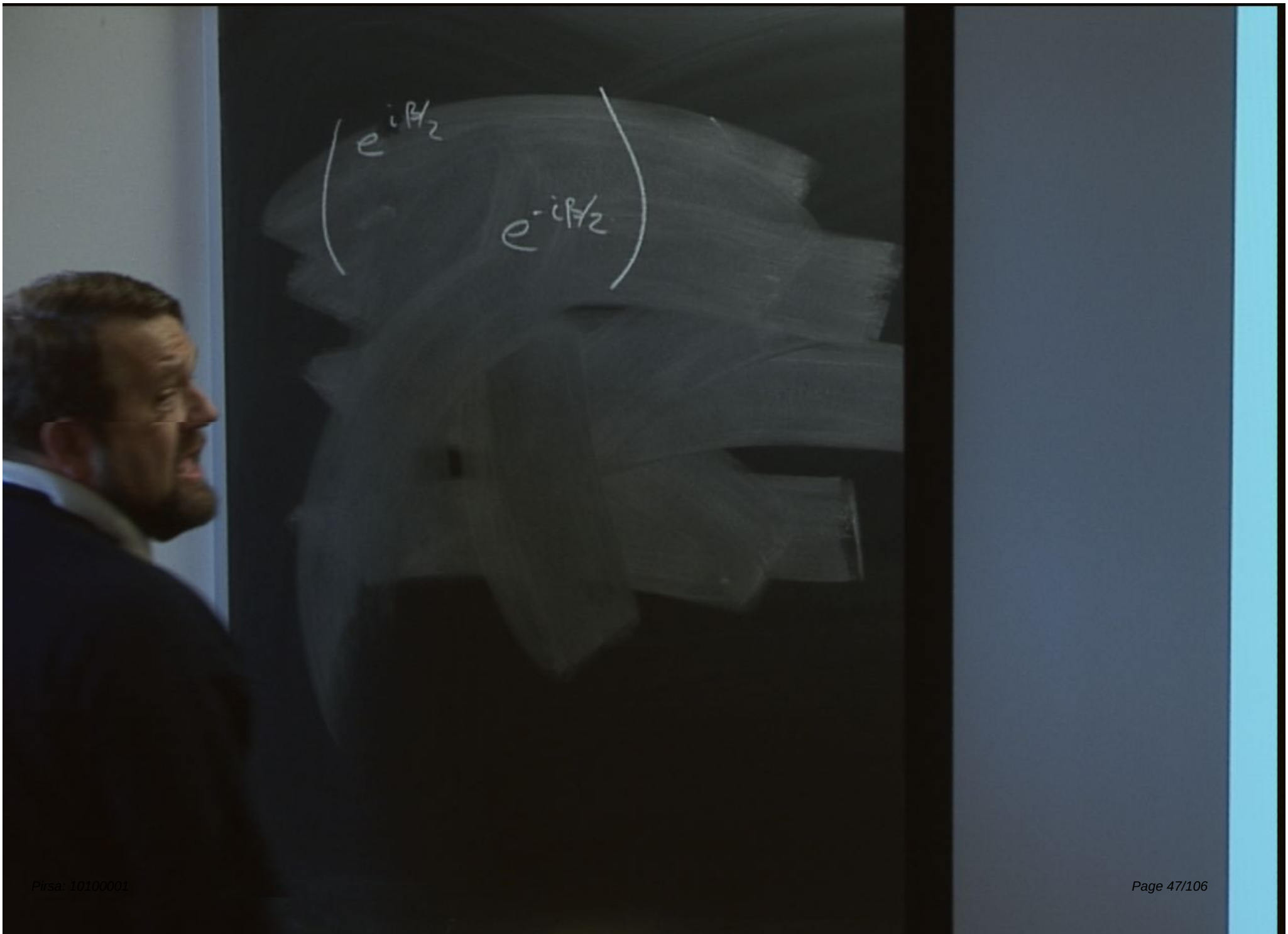
$\begin{pmatrix} e^{i\beta/2} & \\ & e^{-i\beta/2} \end{pmatrix}$

$$e^{-i\beta/2}\begin{pmatrix} e^{i\beta/2} & \\ & e^{-i\beta/2} \end{pmatrix}$$

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Two spins!

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Rotating frame for both spins:

$$\mathsf{U}_0^{(12)}(t) = \exp\left(\frac{i\Omega_1 t}{2}\mathsf{Z}_1\right)\exp\left(\frac{i\Omega_2 t}{2}\mathsf{Z}_2\right)$$

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Terms in $\mathsf{H}^{(12)}$

$$\begin{array}{cccc}
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overall E-shift (ignore)

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$\mathsf{Y}^{(1)} \otimes \mathbf{1}^{(2)}$	$\mathsf{Y}^{(1)} \otimes \mathsf{X}^{(2)}$	$\mathsf{Y}^{(1)} \otimes \mathsf{Y}^{(2)}$	$\mathsf{Y}^{(1)} \otimes \mathsf{Z}^{(2)}$
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J-coupling
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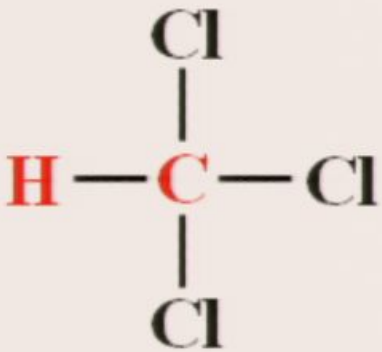

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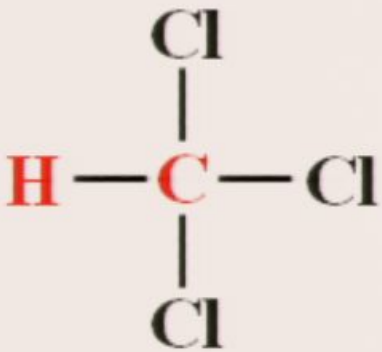

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Chlorine atoms make no average contribution
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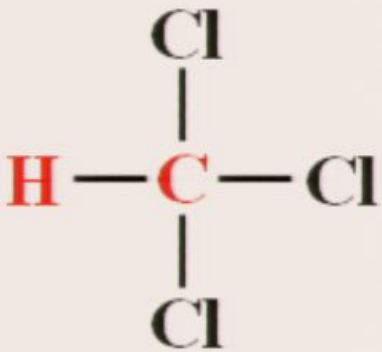

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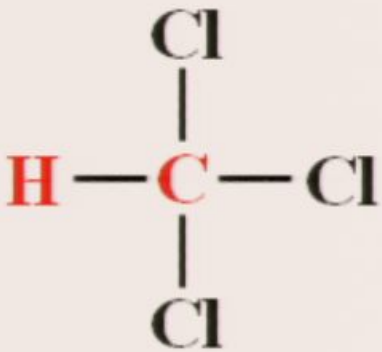

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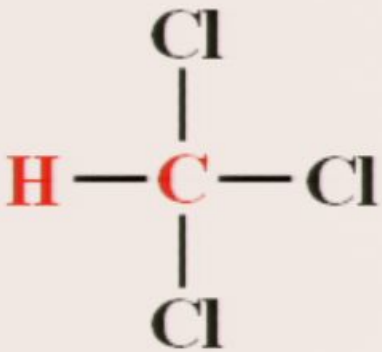

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J-coupling $\varepsilon_{12} = 1350\ s^{-1}$

Refocusing

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Good news: J-coupling is an interaction between spins. We can use it by just introducing a time delay.

Bad news: Everything else in Hamiltonian is also acting!

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Building a 2-qubit gate

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Things we can do:

$$\mathsf{U}_1 = \exp(i\theta \mathsf{Z}^{(1)})$$
$$\mathsf{U}_2 = \exp(i\theta \mathsf{Z}^{(2)})$$

Resonant pulses

$$\mathsf{U}_{12} = \exp\left(-i\theta \mathsf{Z}^{(1)} \otimes \mathsf{Z}^{(2)}\right)$$

J-coupling (refocusing)

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$$\mathsf{U}_{12}: \quad \begin{array}{lcl} |00\rangle & \rightarrow & e^{-i\theta}|00\rangle \\ |01\rangle & \rightarrow & e^{i\theta}|01\rangle \\ |10\rangle & \rightarrow & e^{i\theta}|10\rangle \\ |11\rangle & \rightarrow & e^{-i\theta}|11\rangle \end{array}$$

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Building a 2-qubit gate

Things we can do:

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Resonant pulses

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Controlled Z gate!

NMR quantum computing

1-qubit and 2-qubit gates provide a universal set for quantum computation!

Note: We must be able to address spins *individually* with resonant pulses.

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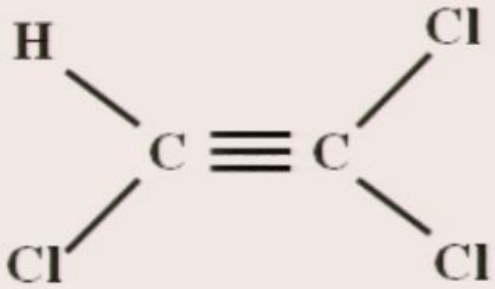

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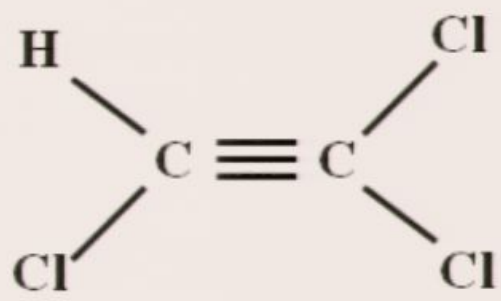

Resonant pulses must be very narrow in frequency -- this limits the speed of 1-qubit gates.

Thermal states

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Use "pseudopure" states, do computation on ~10^{18} molecules.

$$\omega = \left(\frac{1-\eta}{d}\right)\mathbf{1} + \eta\,|\psi\rangle\langle\psi| \qquad \eta = \text{"purity"}$$

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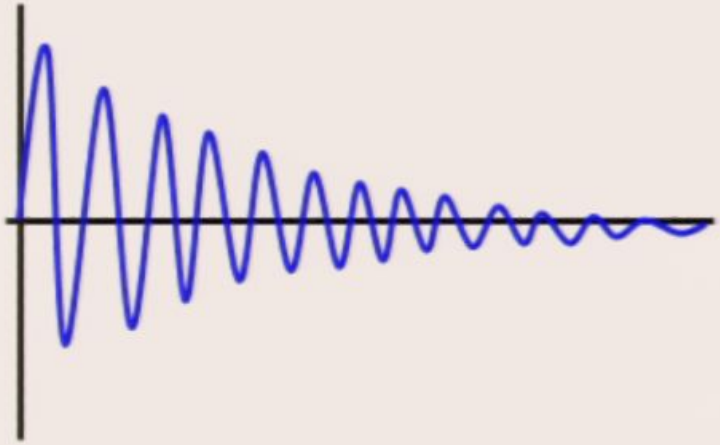
Only transverse (X and Y) components of spins contribute.

From amplitude and phase information in FID, we can determine ensemble averages for $\langle X \rangle$ and $\langle Y \rangle$.



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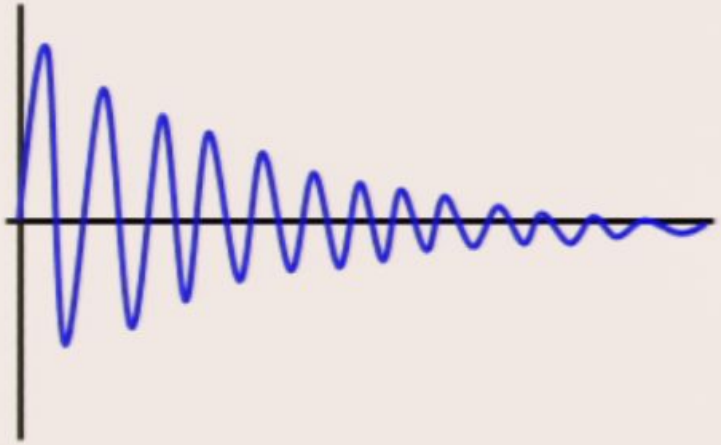

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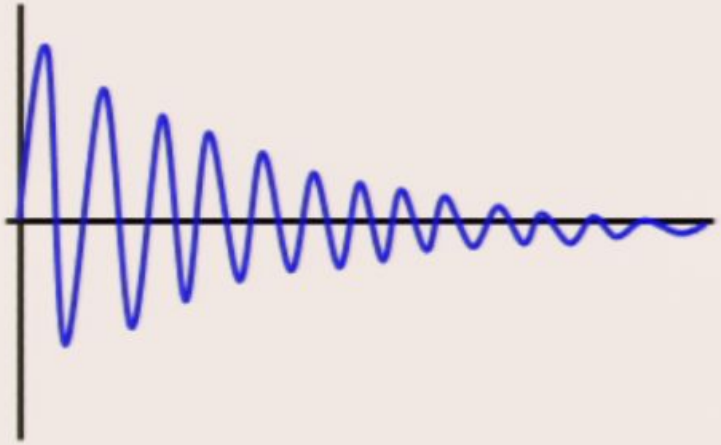

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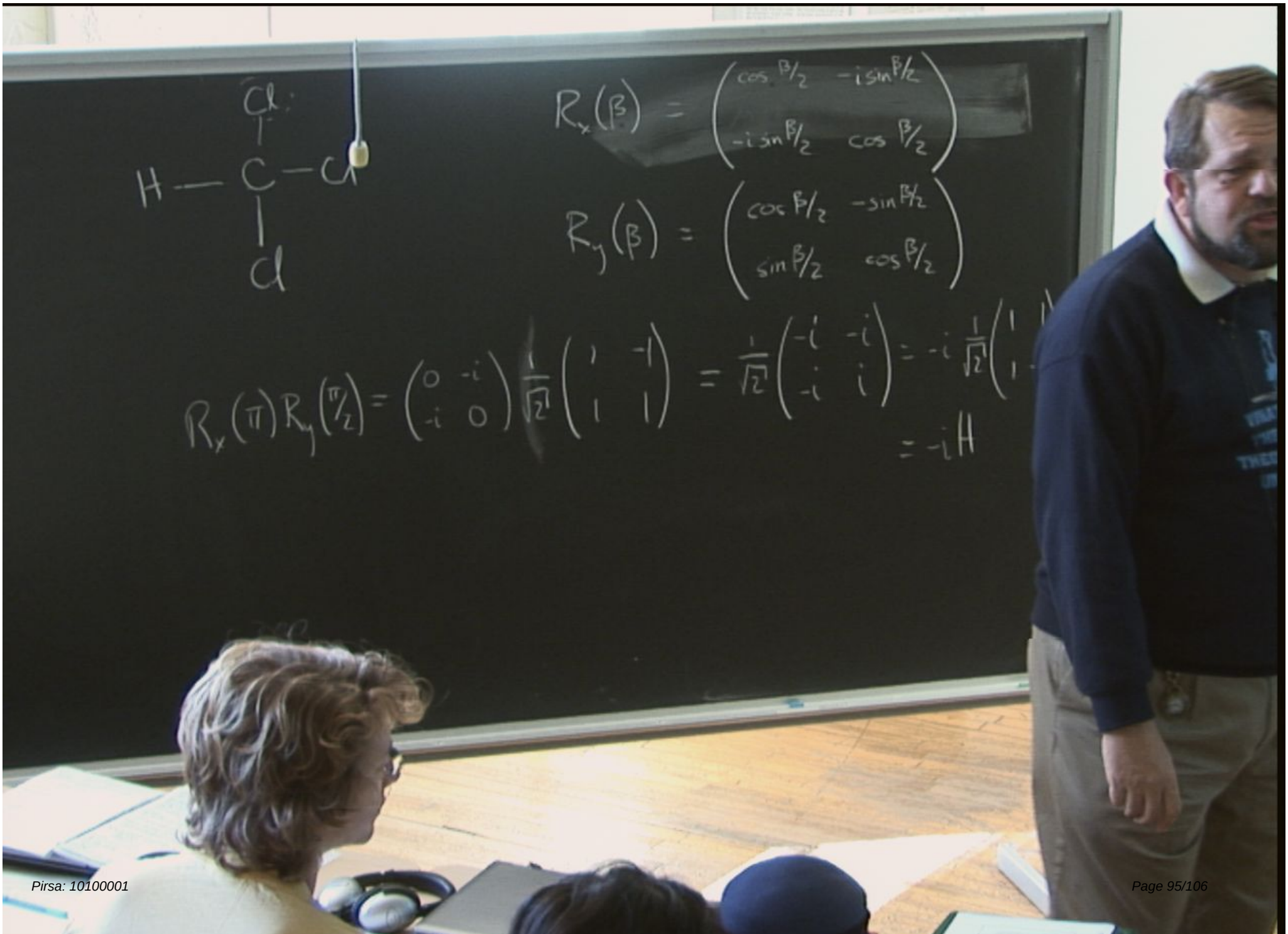
$R_x(\beta) = \begin{pmatrix} \cos\beta/2 & -i\sin\beta/2 \\ -i\sin\beta/2 & \cos\beta/2 \end{pmatrix}$
$R_y(\beta) = \begin{pmatrix} \cos\beta/2 & -\sin\beta/2 \\ \sin\beta/2 & \cos\beta/2 \end{pmatrix}$
$R_x(\pi) R_y(\pi/2) = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} -i & -i \\ -i & i \end{pmatrix} = -i \frac{1}{\sqrt{2}}$
$= -iH$

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- Populations decay toward equilibrium with time constant $\mathbf{T_1}$.
- Coherences decay toward zero with time constant $\mathbf{T_2}$ (faster).
- For ^{13}C in chloroform, $T_2 \approx 0.3$ s -- enough time for many quantum gates!

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